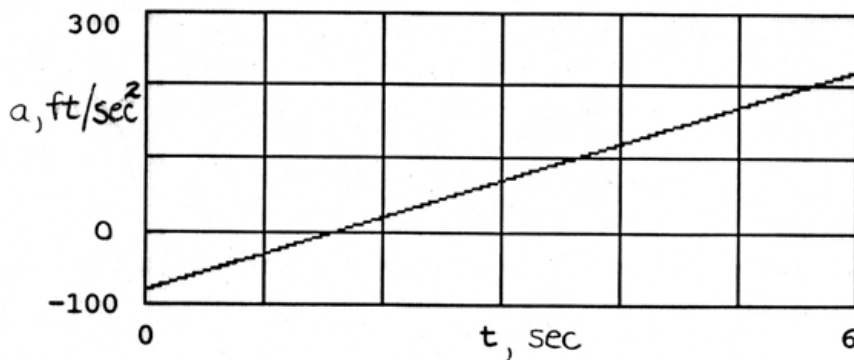
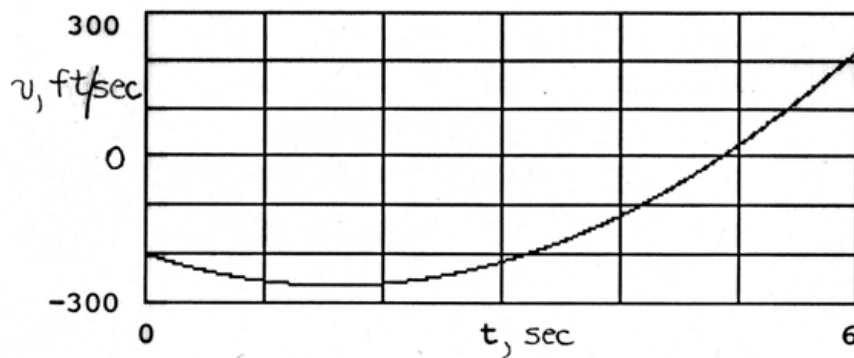


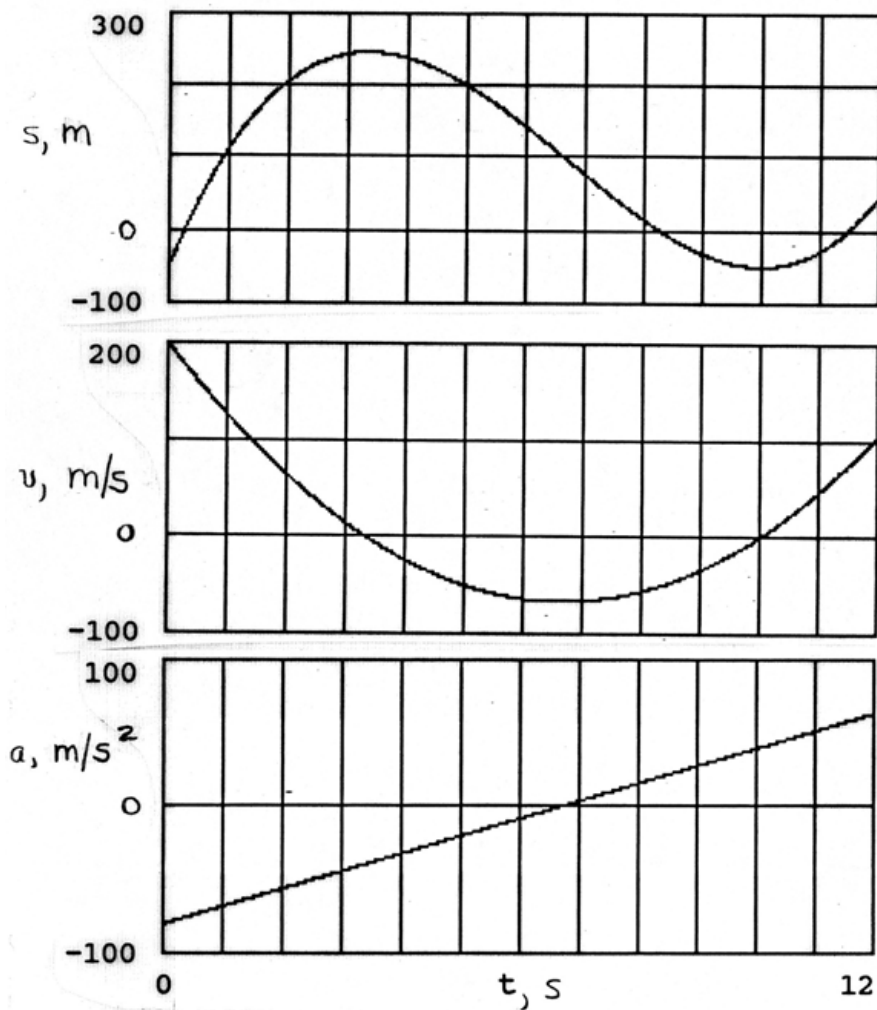
$$\left. \begin{array}{l} 2/1 \\ v = 25t^2 - 80t - 200 \\ a = \frac{dv}{dt} = 50t - 80 \end{array} \right\} \text{See plots}$$

$$a = 0 : 50t - 80 = 0, \quad t = 1.6 \text{ sec}$$

$$\text{At } t = 1.6 \text{ sec, } v = 25(1.6)^2 - 80(1.6) - 200 = \underline{\underline{-264 \frac{\text{ft}}{\text{sec}}}}$$



$$\begin{aligned}
 & \boxed{2/2} \quad \left. \begin{aligned} s &= 2t^3 - 40t^2 + 200t - 50 \\ v &= \frac{ds}{dt} = 6t^2 - 80t + 200 \\ a &= \frac{dv}{dt} = 12t - 80 \end{aligned} \right\} \text{See plots} \\
 & v = 0: 6t^2 - 80t + 200 = 0, \quad t = \frac{80 \pm \sqrt{80^2 - 4(6)(200)}}{-2(6)} \\
 & \quad \quad \quad t = 3.33 \text{ s}, 10 \text{ s}
 \end{aligned}$$

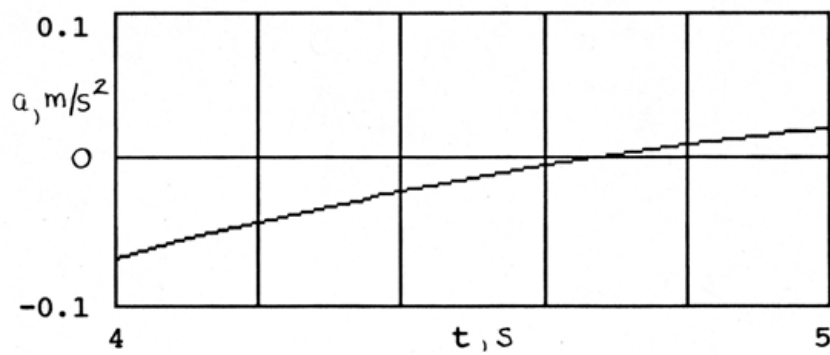
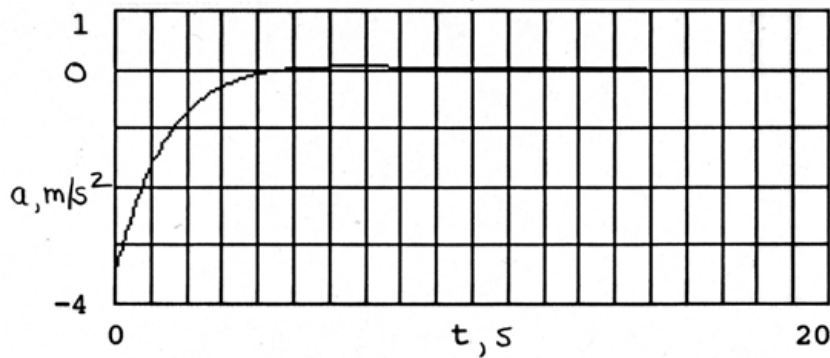
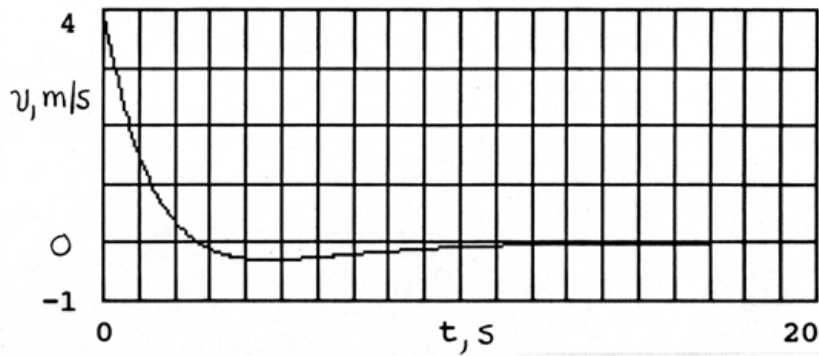
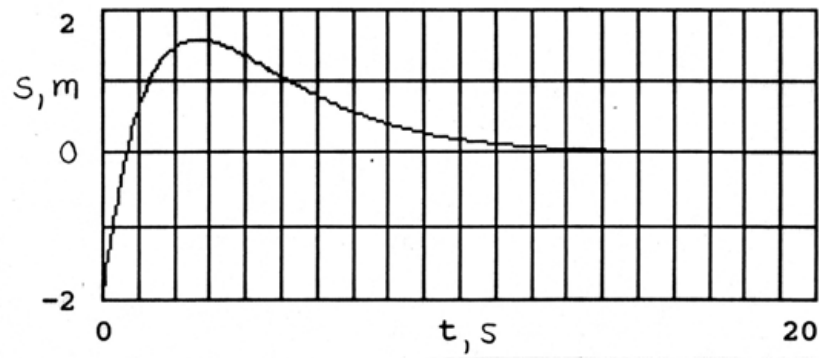


$$\begin{aligned} \underline{2/3} \quad v &= 2 - 4t + 5t^{3/2} \\ a &= \frac{dv}{dt} = -4 + \frac{15}{2} t^{1/2} \\ \frac{ds}{dt} &= 2 - 4t + 5t^{3/2} \\ \int_{s_0=3}^s ds &= \int_0^t (2 - 4t + 5t^{3/2}) dt \end{aligned}$$

$$s = 3 + 2t - 2t^2 + 2t^{5/2}$$

$$\text{At } t = 3s : \begin{cases} s = 22.2 \text{ m} \\ v = 15.98 \text{ m/s} \\ a = 8.99 \text{ m/s}^2 \end{cases}$$

$$\begin{aligned}
 & \boxed{2/4} \quad s = (-2 + 3t)e^{-0.5t} \\
 v &= \frac{ds}{dt} = 3e^{-0.5t} + (-2 + 3t)(-0.5)e^{-0.5t} \\
 &= \underline{(4 - 1.5t)e^{-0.5t}} \\
 a &= \frac{dv}{dt} = -1.5e^{-0.5t} + (4 - 1.5t)(-0.5)e^{-0.5t} \\
 &= \underline{(-3.5 + 0.75t)e^{-0.5t}} \\
 a = 0 &: (-3.5 + 0.75t)e^{-0.5t} = 0, \quad \underline{t = 4.67 \text{ s}}
 \end{aligned}$$



$$\frac{2/5}{a = \frac{dv}{dt} = 2t - 10}$$

$$\int_{v_0=3}^v dv = \int_0^t (2t-10) dt, \quad \underline{v = 3 - 10t + t^2 \text{ (m/s)}}$$

$$\frac{ds}{dt} = 3 - 10t + t^2$$

$$\int_{s_0=-4}^s ds = \int_0^t (3 - 10t + t^2) dt$$

$$\underline{s = -4 + 3t - 5t^2 + \frac{1}{3}t^3 \text{ (m)}}$$

2/6

$$a = v \frac{dv}{ds} = -ks^2$$

$$\int_{v_0}^v v dv = \int_{s_0}^s ks^2 ds \Rightarrow v^2 = v_0^2 - \frac{2}{3}k(s^3 - s_0^3)$$

$$\text{Taking positive sign: } v = \left[v_0^2 - \frac{2}{3}k(s^3 - s_0^3) \right]^{1/2}$$

$$\begin{aligned} \text{Numbers: } v &= \left[10^2 - \frac{2}{3}(0.1)(5^3 - 3^3) \right]^{1/2} \\ &= \underline{9.67 \text{ m/s}} \end{aligned}$$

2/7

$$a = -kv^{1/2} = \frac{dv}{dt}$$
$$-\int_0^t k dt = \int_{v_0}^v \frac{dv}{v^{1/2}} \Rightarrow \underline{v = \left(v_0^{1/2} - \frac{1}{2}kt\right)^2}$$

$$\text{Numbers: } v = \left(7^{1/2} - \frac{1}{2}(0.2)(2)\right)^2 = \underline{5.98 \text{ m/s}}$$

$$\text{Also, } -kv^{1/2} = v \frac{dv}{ds}$$

$$-\int_{s_0}^s k ds = \int v^{1/2} dv \Rightarrow \underline{v = \left[v_0^{3/2} - \frac{3}{2}k(s-s_0)\right]^{2/3}}$$

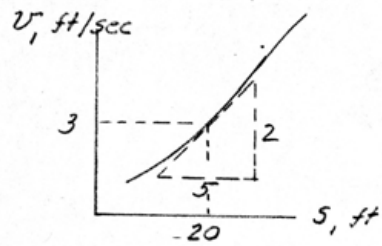
$$\text{Numbers: } v = \left[7^{3/2} - \frac{3}{2}(0.2)(3-1)\right]^{2/3} = \underline{6.85 \text{ m/s}}$$

2/8

$$a = v \frac{dv}{ds} = 10(-3) = \underline{-30 \text{ m/s}^2}$$

2/9

$$a = v \frac{dv}{ds} = 3 \left(\frac{2}{5} \right) = \underline{1.2 \text{ ft/sec}^2}$$



2/10

$$y = v_0 t + \frac{1}{2} a t^2, \quad y = 80t - \frac{1}{2} 32.2 t^2$$

for $y = -200 \text{ ft}$,

$$-200 = 80t - 16.1 t^2$$

$$\text{or } 16.1 t^2 - 80t - 200 = 0$$

$$t = \frac{80 \pm \sqrt{(80)^2 + 4(16.1)(200)}}{2(16.1)} = \underline{6.80 \text{ sec (or } -1.83 \text{ s)}}$$

$$\text{For } v = 0, \quad v^2 = v_0^2 + 2ay, \quad y = h = \frac{0 - 80^2}{-2(32.2)} = \underline{99.4 \text{ ft}}$$

2/11

For constant acceleration,

$$s = \frac{1}{2}at^2, \quad t = \left(\frac{2s}{a}\right)^{1/2} = \left(\frac{2(30\,000)}{1.5(9.81)}\right)$$

$$= \underline{63.9 \text{ s}}$$

$$v = \sqrt{2as} = \sqrt{2(1.5)(9.81)(30\,000)} = \underline{940 \text{ m/s}}$$

2/12

$$v^2 - v_0^2 = 2a(s - s_0)$$

$$0 - \left[50 \frac{5280}{3600} \right]^2 = 2a(100), a = -26.9 \frac{\text{ft}}{\text{sec}^2}$$

$$\text{Then } 0 - \left[70 \frac{5280}{3600} \right]^2 = 2(-26.9)s$$

$$s = \underline{196.0 \text{ ft}}$$

2/13

For $a = \text{constant}$, $v^2 = v_0^2 + 2as$

$$\left[\frac{180(5280)}{3600} \right]^2 = 0^2 + 2a(300)$$

$$a = 116.2 \text{ ft/sec}^2$$

$$\text{or } a = 116.2 / 32.2 = \underline{3.61g}$$

2/14	$B \text{ to } C; \quad t = 10/2 = 5 \text{ s}$ $v = v_i + at; \quad 0 = v_i - 9.8(5), \quad v_i = 49.0 \text{ m/s}$ $v^2 = v_i^2 + 2as; \quad 0 = (49.0)^2 + 2(-9.8)h_2$ $h_2 = 122.6 \text{ m}$
C $\vdots$ h_2 $\vdots$ B $\vdots$ h_1 $\vdots$ A $\uparrow v_i$ $\uparrow a = 40g$	$A \text{ to } B; \quad v^2 = v_0^2 + 2as;$ $(49.0)^2 = 0 + 2(40)(9.8)h_1$ $h_1 = 3.07 \text{ m}$ $h = h_1 + h_2 = \underline{125.7 \text{ m}}$

$$2/15 \quad v^2 = v_0^2 + 2a(s-s_0)$$

$$\left(\frac{200}{3.6}\right)^2 = 0^2 + 2(0.4 \cdot 9.81) s$$

$$s = 393 \text{ m}$$

$$v = v_0 + at : \left(\frac{200}{3.6}\right) = 0 + 0.4(9.81) t$$

$$t = 14.16 \text{ s}$$

2/16

$$\int v dv = \int a ds ; \int_{200/3.6}^{30/3.6} v dv = a \int_0^{600} ds$$

$$200 \text{ km/h}$$

$$= 200/3.6 \text{ m/s}$$

$$30 \text{ km/h}$$

$$= 30/3.6 \text{ m/s}$$

$$\frac{1}{2(3.6)^2} (30^2 - 200^2) = 600 a$$

$$\underline{a = -2.51 \text{ m/s}^2}$$

2/17

$$v = 20e^{-t/10}$$

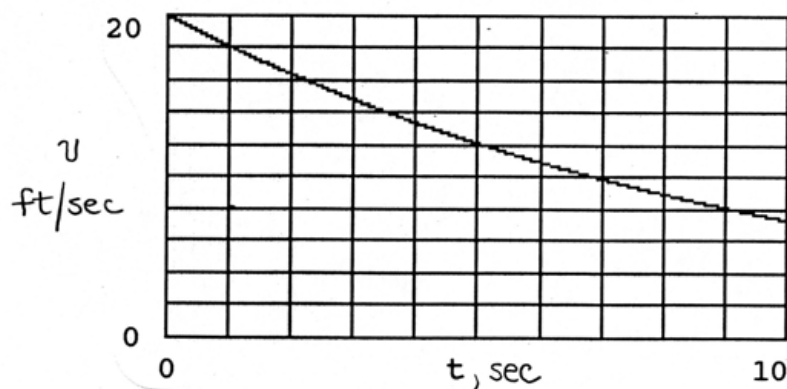
$$a = \dot{v} = -2e^{-t/10}$$

$$\text{When } t = 10 \text{ sec, } a = -2e^{-10/10} = \underline{-0.736 \text{ ft/sec}^2}$$

$$\text{From } v = \frac{ds}{dt} = 20e^{-t/10},$$

$$\int_0^s ds = \int_0^{10} 20e^{-t/10} dt$$

$$s = -200e^{-t/10} \Big|_0^{10} = \underline{126.4 \text{ ft}}$$



$$2/18 \quad v^2 = v_0^2 + 2as, \text{ where } a = g/6$$

$$v^2 = 2^2 + 2\left(\frac{9.81}{6}\right)5, \quad \underline{v = 4.51 \text{ m/s}}$$

2/19

$\Delta s = \text{area under } v-t \text{ curve}$

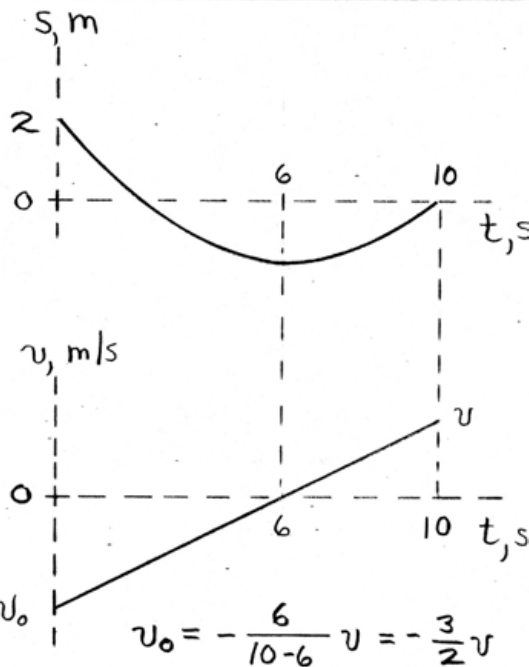
$$0 - 2 = -\frac{3}{2} v\left(\frac{6}{2}\right) + \frac{1}{2} v(4)$$

$$v = 0.8 \text{ m/s}$$

Acceleration

$$a = \frac{dv}{dt} = \frac{0.8}{4}$$

$$= 0.2 \text{ m/s}^2$$



2/20 Acceleration period :

$$v = v_0 + at : \frac{22}{3.6} = 0 + \frac{9.81}{4} t_a, t_a = 2.49 \text{ s}$$

Note that The deceleration time $t_d = t_a$

$$v^2 = v_0^2 + 2a \Delta s : \left(\frac{22}{3.6}\right)^2 = 0^2 + 2 \frac{9.81}{4} \Delta s_a$$

$$\Delta s_a = 7.61 \text{ m} = \Delta s_d$$

$$\text{Cruise period : } \Delta s_c = 350 - \Delta s_a - \Delta s_d = 335 \text{ m}$$

$$\Delta s = v_c t_c : 335 = \frac{22}{3.6} t_c, t_c = 54.8 \text{ s}$$

$$\text{Total run time } t = t_c + t_a + t_d = \underline{59.8 \text{ s}}$$

2/21

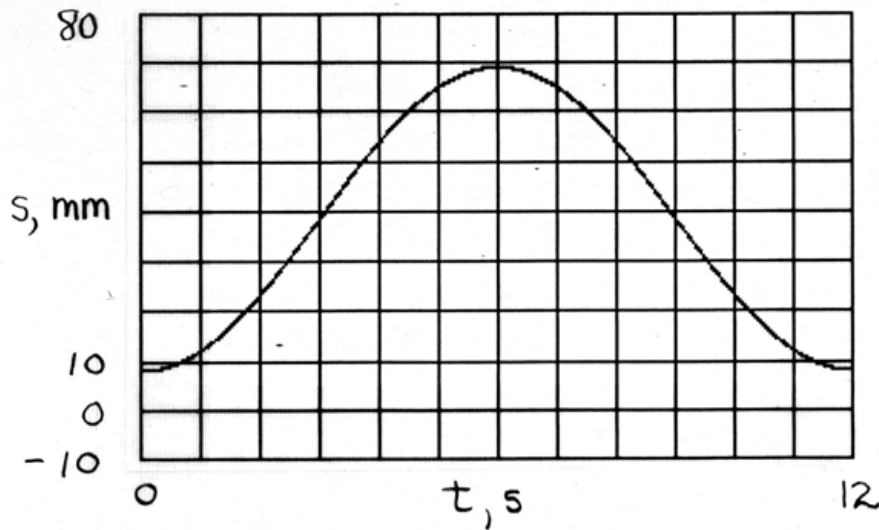
$$v = \frac{ds}{dt} = 16 \sin \frac{\pi t}{6}$$

$$\int_8^s ds = 16 \int_0^t \sin \frac{\pi t}{6} dt$$

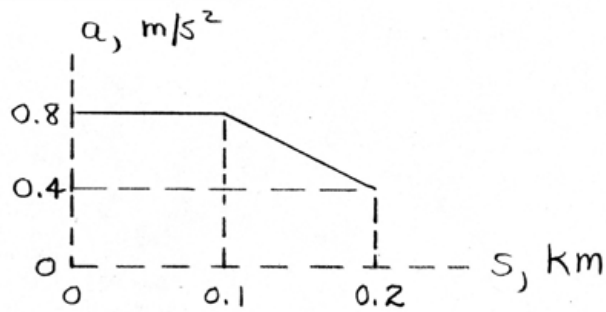
$$s = 8 + 16 \cdot \frac{6}{\pi} \left(-\cos \frac{\pi t}{6} \right) \Big|_0^t = 8 + \frac{96}{\pi} \left[1 - \cos \frac{\pi t}{6} \right]$$

$$s = s_{\max} \text{ when } \cos \frac{\pi t}{6} = -1 \text{ or } t = 6 \text{ s}$$

$$s_{\max} = 8 + \frac{96}{\pi} [1 - (-1)] = \underline{69.1 \text{ mm}}$$



2/22



From $a = v \frac{dv}{ds}$,

$$\int_{v_0}^v v dv = \int_0^{200} a ds = \text{area under } a-s \text{ curve}$$

$$\frac{v^2}{2} - \frac{(40/3.6)^2}{2} = 0.8(100) + 0.6(100)$$

$$\underline{v = 20.1 \text{ m/s}} \quad \text{or} \quad \underline{72.3 \text{ km/h}}$$

2/23

$$s = s_0 + v_0 t + \frac{1}{2} g t^2$$

Sphere ①: $H = \frac{1}{2} g t_2^2$

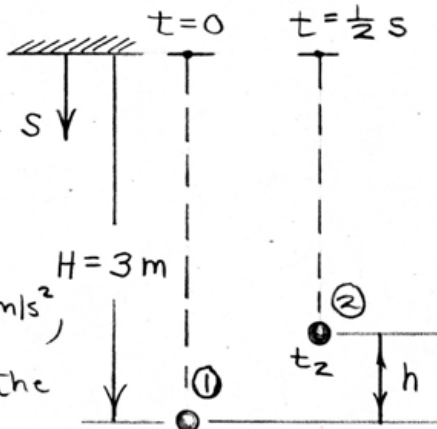
Sphere ②:

$$(H-h) = \frac{1}{2} g \left(t_2 - \frac{1}{2}\right)^2$$

With $H = 3 \text{ m}$ & $g = 9.81 \text{ m/s}^2$,

eliminate t_2 between the

2 equations & obtain $h = 2.61 \text{ m}$. t_2



$$2/24 \quad \Delta v = \int a \, dt = \text{area under } a-t \text{ curve}$$

$$\text{For } t = 4 \text{ s, } v_4 - 100 = -4(9.81) \frac{4-2}{2}, \quad v_4 = \underline{60.8 \text{ m/s}}$$

$$\text{For } t = 8 \text{ s, } v_8 - 60.8 = -4(9.81)(6-4), \quad v_8 = \underline{-17.72 \text{ m/s}}$$

$$\begin{aligned} \boxed{2/25} \quad v^2 &= v_0^2 + 2a(s-s_0) \\ 0 &= 4^2 + 2\left(-\frac{9.81}{4}\right)(s), \quad \underline{s = 3.26 \text{ m}} \\ v &= v_0 + at : 0 = 4 + \left(-\frac{9.81}{4}\right)t_{\text{up}}, \quad t_{\text{up}} = 1.63 \text{ s} \\ t &= 2t_{\text{up}} = 2(1.63) = \underline{3.26 \text{ s}} \end{aligned}$$

$$\boxed{2/26} \quad a = \frac{1}{2} \frac{d(v^2)}{ds} = \frac{1}{2} \frac{900 - 2500}{400 - 100} = -\frac{8}{3} \frac{\text{ft}}{\text{sec}^2}$$

$$\Delta v = \int a dt ; \quad v - 50 = -\frac{8}{3}t \quad (\text{constant})$$

$$\text{At B: } 30 - 50 = -\frac{8}{3}t, \quad t = 7.50 \text{ sec}$$

$$\Delta s = \int v dt = \int_{5.5}^{7.5} (50 - \frac{8}{3}t) dt = \underline{65.3 \text{ ft}}$$

$$2/27 \quad a = 400 - kx, \text{ where } k = \frac{400}{6/12} \text{ sec}^{-2}$$

$$a = 400(1 - 2x) \quad (x \text{ in ft})$$

$$v dv = a dx : \int_0^v v dv = 400 \int_0^x (1 - 2x) dx$$

$$v^2 = 800(x - x^2), \quad v = \frac{dx}{dt} = 20\sqrt{2} \sqrt{x - x^2} \quad (\text{taking + sign})$$

$$\int_0^t dt = \int_0^x \frac{dx}{20\sqrt{2} \sqrt{x - x^2}}$$

$$t = -\frac{1}{20\sqrt{2}} \sin^{-1} \frac{1-2x}{\sqrt{1}} \Big|_0^x = \frac{1}{20\sqrt{2}} \left[\frac{\pi}{2} - \sin^{-1}(1-2x) \right]$$

$$(a) \quad x = \frac{1}{4} \text{ ft} : t = \frac{1}{20\sqrt{2}} \left[\frac{\pi}{2} - \frac{\pi}{6} \right] = \underline{0.0370 \text{ sec}}$$

$$(b) \quad x = \frac{1}{2} \text{ ft} : t = \frac{1}{20\sqrt{2}} \left[\frac{\pi}{2} - 0 \right] = \underline{0.0555 \text{ sec}}$$

2/28

The area under the a-s curve is

$$\int_0^v u \, du = \frac{1}{2} v^2$$

$$\text{Area} \Big|_0^{200 \text{ m}} = \frac{3+6}{2} (100) + \frac{6+4}{2} (100) = 950 \text{ m}^2/\text{s}^2$$

$$\text{So } \frac{1}{2} v^2 = 950, \quad \underline{v = 43.6 \text{ m/s}}$$

$$\frac{dv}{ds} = \frac{a}{v} = \frac{4}{43.6} = \underline{0.0918 \text{ s}^{-1}}$$

$$\underline{2/29} \quad v_0 = 100/3.6 = 27.8 \text{ m/s}$$

$$a = -g \sin \theta = -9.81 \sin \left[\tan^{-1} \frac{6}{100} \right] = -0.588 \text{ m/s}^2$$

$$(a) \quad v = v_0 + at = 27.8 - 0.588(10) = \underline{21.9 \text{ m/s}}$$

$$(b) \quad v^2 = v_0^2 + 2a(s-s_0) = 27.8^2 + 2(-0.588)(100)$$

$$v = \underline{25.6 \text{ m/s}}$$

$$2/30 \quad 0 < t < 4 \text{ s} : a = -3t/4$$

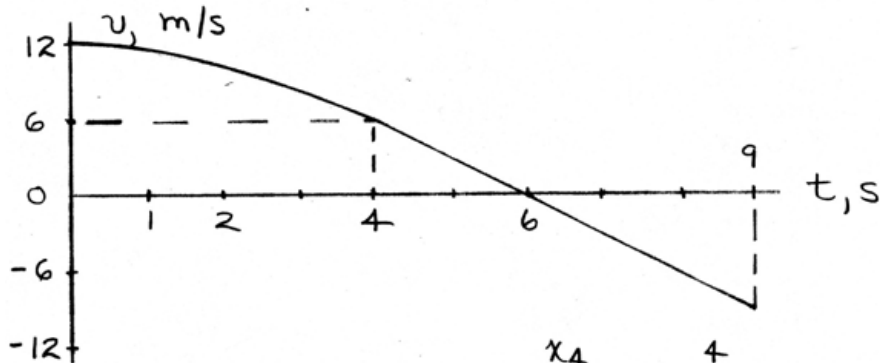
$$a = \frac{dv}{dt} : \int_{12}^v dv = - \int_0^t \frac{3t}{4} dt$$

$$v = 12 - \frac{3}{8}t^2, \quad v_4 = 6 \text{ m/s}$$

$$4 < t < 9 \text{ s} : a = -3 \text{ m/s}^2 = \text{constant}$$

$$v = v_4 + a \Delta t = 6 - 3(t-4) = 18 - 3t$$

$$v_9 = -9 \text{ m/s}$$



$$0 < t < 4 \text{ s} : dx = v dt : \int_0^{x_4} dx = \int_0^4 (12 - \frac{3}{8}t^2) dt$$

$$s_4 = 40 \text{ m}$$

$$4 < t < 9 \text{ s} : x_6 - x_4 = \int_4^6 v dt = \frac{1}{2}(6-4)6 = 6 \text{ m}$$

$$\therefore \Delta x = 40 + 6 = \underline{46 \text{ m}}$$

$$2/31 \quad a = v \frac{dv}{ds} = \frac{1}{2} \frac{d(v^2)}{ds} = \frac{1}{2} \frac{\Delta(v^2)}{\Delta s} = \frac{1}{2} \frac{36-16}{80-30} = \frac{1}{5} \text{ m/s}^2$$

Counting time from A, $v = v_A + at$, $v = 4 + \frac{1}{5}t$

At B, $6 = 4 + \frac{1}{5}t_B$, $t_B = 10 \text{ sec.}$

$$\Delta s = \int v dt = \int_0^{10} (4 + \frac{1}{5}t) dt = 4(2) + \frac{1}{10}(100 - 64)$$

$$\Delta s = \underline{11.6 \text{ m}}$$

$$\boxed{2/32} \quad s_{car} = vt = \frac{120}{3.6} t$$

$$s_{cycle} = v_{av} t_1 + v_{max} t_2 = \frac{1}{2} \frac{150}{3.6} t_1 + \frac{150}{3.6} t_2$$

$$\text{where } t_1 = \frac{v_{max}}{a} = \frac{150}{3.6 \times 6} = 6.94 \text{ s} \quad \& \quad t_2 = t - 6.94 - 2$$

$$s_{car} = s_{cycle}; \quad \frac{120}{3.6} t = \frac{75}{3.6} 6.94 + \frac{150}{3.6} (t - 8.94)$$

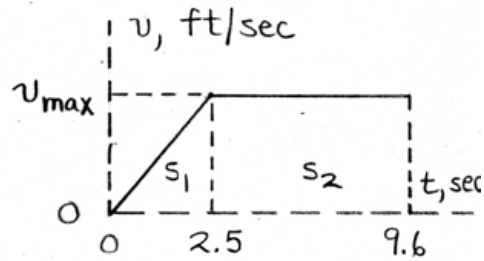
$$30t = 820.8, \quad t = 27.36 \text{ s}$$

$$s = \frac{120}{3.6} (27.36) = \underline{912 \text{ m}}$$

2/33

$$s_1 = \frac{1}{2} (2.5) v_{\max}$$

$$s_2 = (9.6 - 2.5) v_{\max}$$

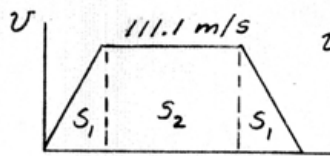


$$s_1 + s_2 = (1.25 + 7.1) v_{\max} = 100(3)$$

$$\underline{v_{\max} = 35.9 \text{ ft/sec}}$$

2/34

$$400 \text{ km/h} \approx \frac{400}{3.6} = 111.1 \text{ m/s}$$



$$v^2 = 2as, \quad S_1 = \frac{(111.1)^2}{2(0.6)(9.81)} = 1049 \text{ m}$$

$$S_2 = 10000 - 2(1049) = 7903 \text{ m}$$

$$\left. \begin{aligned} t_1 &= \frac{v}{a} = \frac{111.1}{0.6(9.81)} = 18.88 \text{ s} \\ t_2 &= \frac{S_2}{v} = \frac{7903}{111.1} = 71.13 \text{ s} \end{aligned} \right\} \begin{aligned} t &= 2t_1 + t_2 \\ &= 2(18.88) + 71.13 \\ &= 108.9 \text{ s} \end{aligned}$$

or $t = 1.81 \text{ min}$

2/35

$$a = g - cy = v \frac{dv}{dy}$$

$$\int_0^{y_m} (g - cy) dy = \int_{v_0}^0 v dv$$

$$\left(gy - c \frac{y^2}{2} \right) \Big|_0^{y_m} = \frac{v^2}{2} \Big|_{v_0}^0$$

$$gy_m - c \frac{y_m^2}{2} = -\frac{v_0^2}{2} \Rightarrow c = \frac{v_0^2 + 2gy_m}{y_m^2}$$

2/36 Particle 1 : $a = -kv$

$$-kv = \frac{dv}{dt}$$
$$-k \int_0^t dt = \int_{v_0}^v \frac{dv}{v} \Rightarrow \underline{v = v_0 e^{-kt}}$$

Then $\frac{ds}{dt} = v_0 e^{-kt}$

$$\int_{s_0=0}^s ds = \int_{t_0=0}^t v_0 e^{-kt} dt \Rightarrow \underline{s = \frac{v_0}{k} (1 - e^{-kt})}$$

Particle 2 : $a = -kt$

$$-kt = \frac{dv}{dt}$$
$$-k \int_0^t dt = \int_{v_0}^v dv \Rightarrow \underline{v = v_0 - \frac{1}{2}kt^2}$$

Then $\frac{ds}{dt} = v_0 - \frac{1}{2}kt^2$

$$\int_0^s ds = \int_0^t (v_0 - \frac{1}{2}kt^2) dt \Rightarrow \underline{s = v_0 t - \frac{1}{6}kt^3}$$

Particle 3 : $a = -ks$

$$-ks = v \frac{dv}{ds}$$

$$-k \int_0^s s \, ds = \int_{v_0}^v v \, dv \Rightarrow v = \pm \sqrt{v_0^2 - ks^2}$$

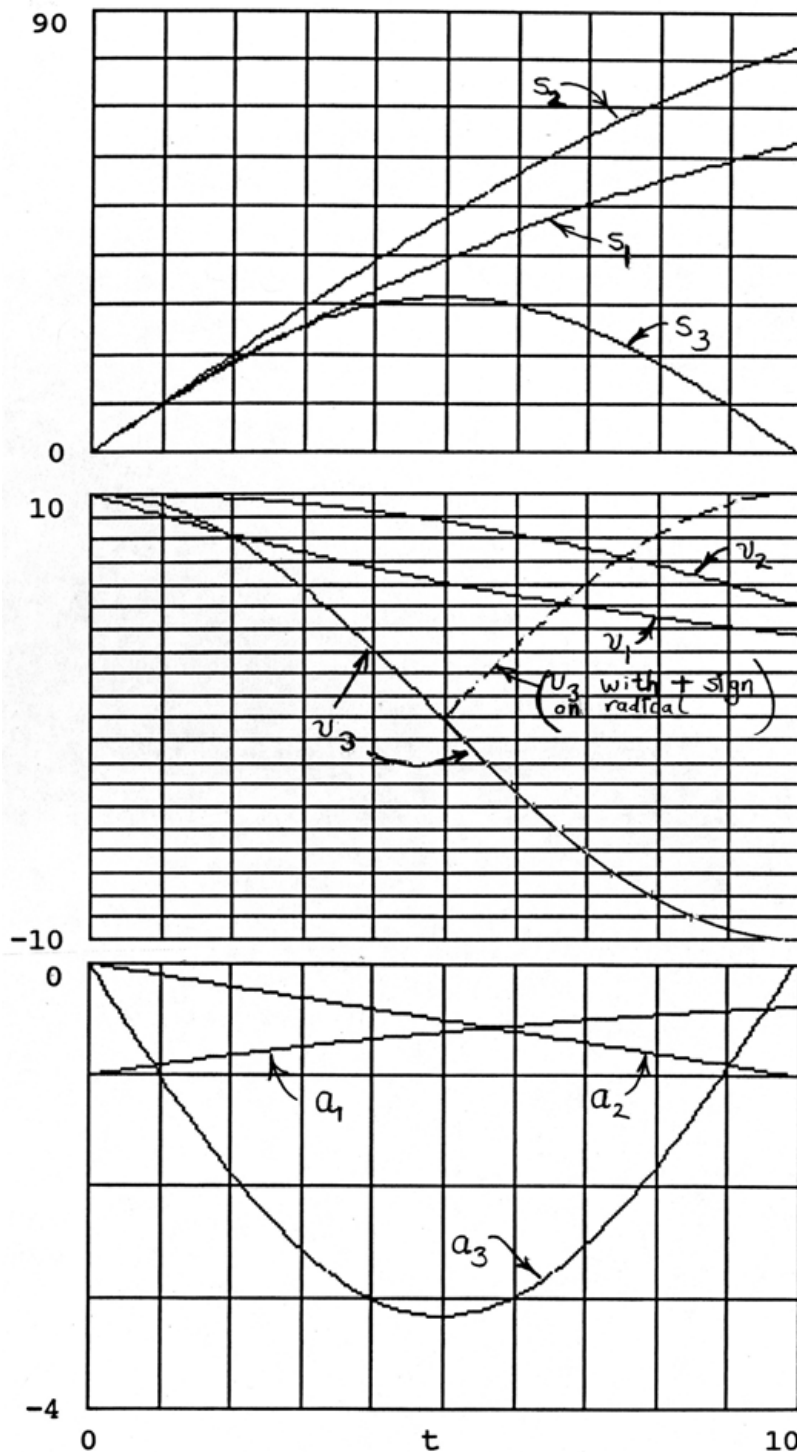
Then

$$\frac{ds}{dt} = \pm \sqrt{v_0^2 - ks^2}$$

$$\int_0^s \frac{ds}{\sqrt{v_0^2 - ks^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{k}} \sin^{-1} \left(\frac{\sqrt{k}}{v_0} s \right) = t \Rightarrow s = \frac{-v_0}{\sqrt{k}} \sin(\sqrt{k} t)$$

Note: Plus sign is chosen until first reversal ($v=0$), thereafter take minus sign, etc.



2/37 $a = P/(mv)$, P & m are constant

$$v dv = a ds; \quad v dv = \frac{P}{mv} ds$$

$$\int_{v_1}^{v_2} m v^2 dv = P \int_0^s ds, \quad s = \frac{m}{3P} (v_2^3 - v_1^3)$$

$$dv = a dt; \quad dv = \frac{P}{mv} dt$$

$$m \int_{v_1}^{v_2} v dv = \int_0^t P dt, \quad t = \frac{m}{2P} (v_2^2 - v_1^2)$$

$$\begin{aligned}
 & \boxed{2/38} \quad v dv = a ds ; \quad \frac{v dv}{-Kv^2} = ds, \quad \int_{v_1}^{v_2} \frac{dv}{v} = -K \int_0^s ds \\
 & \ln \frac{v_2}{v_1} = -Ks, \quad K = \frac{1}{s} \ln \frac{v_1}{v_2} = \frac{1}{1500} \ln \frac{100}{20} = \underline{1.073 (10^{-3}) \text{ ft}^{-1}} \\
 & a = \frac{dv}{dt} ; \quad -Kv^2 = \frac{dv}{dt}, \quad \int_{v_1}^{v_2} \frac{dv}{v^2} = -Kt, \quad t = \frac{1}{K} \left(\frac{1}{v_2} - \frac{1}{v_1} \right) \\
 & t = \frac{10^3}{1.073} \left(\frac{1}{20} - \frac{1}{100} \right) \frac{30}{44} = \underline{25.4 \text{ sec}}
 \end{aligned}$$

2/39

$$a = -kv = \frac{dv}{dt}$$

$$-k \int_0^t dt = \int_{v_0}^v \frac{dv}{v} \Rightarrow v = v_0 e^{-kt}$$

$$\text{Given conditions: } 1 = 4e^{-k(2)}, \quad k = 0.693 \text{ s}^{-1}$$

$$\text{So } v = v_0 e^{-0.693t}$$

$$\text{When } v = \frac{v_0}{10}: \quad \frac{v_0}{10} = v_0 e^{-0.693T}, \quad \underline{T = 3.32 \text{ s}}$$

$$\text{Also, } a = -kv = v \frac{dv}{ds}$$

$$-k \int_{s=0}^s ds = \int_{v_0}^v \frac{dv}{v}, \quad v = v_0 - ks$$

$$\text{Given conditions: } \frac{v_0}{10} = v_0 - kD$$

$$\text{With } k = 0.693 \text{ s}^{-1} \text{ and } v_0 = 4 \text{ m/s:}$$

$$\frac{4}{10} = 4 - 0.693D, \quad \underline{D = 5.19 \text{ m}}$$

(We note that T is independent of v_0 ; D is not.)

2/40

$$a = g - cy^2 = v \frac{dv}{dy}$$

$$\int_0^{y_m} (g - cy^2) dy = \int_{v_0}^0 v dv$$

$$\left(gy - c \frac{y^3}{3} \right) \Big|_0^{y_m} = \frac{v^2}{2} \Big|_{v_0}^0$$

$$gy_m - c \frac{y_m^3}{3} = -\frac{v_0^2}{2} \Rightarrow c = \frac{3v_0^2 + 6gy_m}{2y_m^3}$$

2/41

$$v dv = a dx, \quad \int_0^x dx = \int_{v_0}^v \frac{v dv}{-C_1 - C_2 v^2}$$

$$x = \frac{-1}{2C_2} \ln (C_1 + C_2 v^2) \Big|_{v_0}^v = \frac{1}{2C_2} \ln \frac{C_1 + C_2 v_0^2}{C_1 + C_2 v^2}$$

$$\text{When } v=0, \quad x=D = \frac{1}{2C_2} \ln \left(1 + \frac{C_2 v_0^2}{C_1} \right)$$

$$\boxed{2/42} \quad (a) \quad g_0 = 32.2 \text{ ft/sec}^2 = \text{constant}$$

$$v^2 = v_0^2 + 2a(s - s_0) : v^2 = 0^2 + 2(32.2)(500 \cdot 5280)$$

$$v = \underline{13,040 \text{ ft/sec}}$$

$$(b) \quad a = -g_0 \frac{R^2}{r^2} = v \frac{dv}{dr}$$

$$-g_0 R^2 \int_{R+h}^R \frac{dr}{r^2} = \int_{v_0=0}^v v dv$$

$$-g_0 R^2 \left(-\frac{1}{r} \right) \Big|_{R+h}^R = \frac{1}{2} v^2 \Big|_0^v$$

$$\Rightarrow v = \sqrt{\frac{2g_0 R h}{R+h}} = \sqrt{\frac{2(32.2)(3959)(500)(5280)^2}{(3959+500)(5280)}}$$

$$= \underline{12,290 \text{ ft/sec}}$$

$$\boxed{2/43} \quad (a) \quad (g_m)_0 = 5.32 \text{ ft/sec}^2 = \text{constant}$$

$$v^2 = v_0^2 + 2(g_m)_0 (s - s_0) : v^2 = 0^2 + 2(5.32)(750 \cdot 5280)$$

$$v = \underline{6490 \text{ ft/sec}}$$

$$(b) \quad a = -(g_m)_0 \frac{R_m^2}{r^2} = v \frac{dv}{dr} \quad (R_m = \text{moon radius})$$

$$-(g_m)_0 R_m^2 \int_{R_m+h}^{R_m} \frac{dr}{r^2} = \int_{v_0=0}^v v dv$$

$$-(g_m)_0 R_m^2 \left(-\frac{1}{r} \right) \Big|_{R_m+h}^{R_m} = \frac{1}{2} v^2 \Big|_0^v$$

$$v = \sqrt{\frac{2(g_m)_0 R_m h}{R_m+h}} = \sqrt{\frac{2(5.32) \left(\frac{2160}{2} \right) (750) (5280)^2}{\left(\frac{2160}{2} + 750 \right) (5280)}}$$

$$= \underline{4990 \text{ ft/sec}}$$

$$\boxed{2/44} \quad a = \frac{dv}{dt} = -kv, \quad \int \frac{dv}{v} = -k \int_0^t dt$$

$$\ln \frac{v}{v_0} = -kt, \quad \underline{v = v_0 e^{-kt}}$$

$$v = \frac{dx}{dt} = v_0 e^{-kt}, \quad \int_0^x dx = \int_0^t v_0 e^{-kt} dt$$

$$\underline{x = \frac{v_0}{k} [1 - e^{-kt}]}$$

$$v dv = a dx, \quad \frac{v dv}{v} = -k dx$$

$$\int_{v_0}^v dv = -k \int_0^x dx, \quad \underline{v = v_0 - kx}$$

$$\boxed{2/45} \quad a = \frac{dv}{dt} ; \int_0^v \frac{v dv}{g - kv} = \int_0^t dt, \quad \left[-\frac{1}{k} \ln(g - kv) \right]_0^v = t$$

$$kt = \ln \frac{g}{g - kv}, \quad \frac{g}{g - kv} = e^{kt}, \quad \underline{v = \frac{g}{k} (1 - e^{-kt})}$$

$$v = \frac{dy}{dt} ; \int_0^y dy = \frac{g}{k} \int_0^t (1 - e^{-kt}) dt$$

$$y = \frac{g}{k} \left(t + \frac{1}{k} e^{-kt} \right)_0^t, \quad \underline{y = \frac{g}{k} \left[t - \frac{1}{k} (1 - e^{-kt}) \right]}$$

2/46 (a) $a = 2 \text{ m/s}^2 = \text{constant}$

With $v = 250/3.6 = 69.4 \text{ m/s}$, we have

$$v^2 - v_0^2 = 2a(s - s_0) : 69.4^2 - 0^2 = 2(2)s$$

$$s = \underline{1206 \text{ m}}$$

(b) $a = a_0 - kv^2 = v \frac{dv}{ds}$

$$\int_0^s ds = \int_0^v \frac{v dv}{a_0 - kv^2}$$

$$s = -\frac{1}{2k} \ln(a_0 - kv^2) \Big|_0^v$$

$$= -\frac{1}{2k} \ln \left[\frac{a_0 - kv^2}{a_0} \right]$$

$$s = -\frac{1}{2(4)(10^{-5})} \ln \left[\frac{2 - 4(10^{-5})(69.4)^2}{2} \right]$$

$$= \underline{1268 \text{ m}}$$

$$\begin{aligned}
 & \boxed{2/47} \quad a = -kv^2 \quad v \\
 & v dv = a dx, \quad \int_{v_0}^v \frac{v dv}{-kv^2} = \int_0^x dx, \quad x = \left[\frac{-1}{k} \ln v \right]_{v_0}^v \\
 & \quad \quad \quad x = \frac{1}{k} \ln v_0/v \\
 & \text{when } v = v_0/2, \quad x = D = \frac{1}{k} \ln 2 = \underline{0.693/k} \\
 & v = \frac{dx}{dt} \quad \text{where } kx = \ln v_0/v, \quad v = v_0 e^{-kx} \\
 & \quad \text{so } \frac{dx}{v_0 e^{-kx}} = dt \quad \text{or } \int_0^t dt = \frac{1}{v_0} \int_0^x e^{kx} dx \\
 & \quad \quad \quad \& t = \left[\frac{1}{v_0} \frac{1}{k} e^{kx} \right]_0^x = \frac{1}{kv_0} [e^{kx} - 1] \\
 & \text{For } x = D, \quad e^{kx} = 2 \quad \text{so } t = \frac{1}{kv_0} [2 - 1], \quad \underline{t = \frac{1}{kv_0}}
 \end{aligned}$$

2/48

$0 - 60 \text{ mi/hr: } v^2 = v_0^2 + 2as$
 $0 < t < t_1 \quad (88)^2 = 0 + 2a(200), \quad a = 19.36 \text{ ft/sec}^2$
 $v = v_0 + at, \quad v = 0 + 19.36 t$
 $60 - 0; v dv = a ds; \quad a = -kv \text{ so } \frac{v dv}{-kv} = ds$
 $t_1 < t < t_2 \quad \text{or } dv = -k ds \quad \int_{88}^{44} dv = -k \int_0^{400} ds$
 $44 - 88 = -400k, \quad k = 0.11 \text{ 1/sec}$
 $a = dv/dt, \quad \int_{88}^v \frac{dv}{-kv} = \int_{t_1}^t dt, \quad \frac{1}{0.11} \ln \frac{88}{v} = t - t_1$
 $t_1 = \frac{88}{19.36} = 4.55 \text{ sec}, \quad t = \frac{1}{0.11} \ln \frac{88}{44} + 4.55 = \underline{10.85 \text{ sec}}$

$$2/49 \quad a = k/x, \quad v dv = \frac{k}{x} dx$$

$$\int_0^v v dv = k \int_{x_1}^x \frac{dx}{x}; \quad \frac{v^2}{2} = k \ln \frac{x}{x_1}$$

$$\text{Thus } \frac{(600)^2}{2} = k \ln \frac{750}{7.5}, \quad k = \frac{0.36}{2(4.605)} = 0.0391 \text{ (km/s)}^2$$

$$\text{at } x = 375 \text{ mm, } a = \frac{0.0391}{375(10^{-6})} = \underline{104.2 \text{ km/s}^2}$$

2/50

$$a = v \frac{dv}{ds} = 3.22 - 0.004v^2$$

$$\int_0^{v_B} \frac{v dv}{3.22 - 0.004v^2} = \int_0^{600} ds$$

$$\frac{1}{2(-0.004)} \ln(3.22 - 0.004v^2) \Big|_0^{v_B} = 600$$

$$\ln \left[\frac{3.22 - 0.004v_B^2}{3.22} \right] = 600(2)(-0.004)$$

$$\frac{3.22 - 0.004v_B^2}{3.22} = 0.00823$$

$$\underline{v_B = 28.3 \text{ ft/sec}}$$

$$\boxed{2/51} \quad U_p: \quad a_u = -g - kv^2 = v \frac{dv}{dy} \quad \uparrow y$$

$$\int_0^h dy = - \int_{v_0}^0 \frac{v dv}{g + kv^2}$$

$$h = - \frac{1}{2k} \ln [g + kv^2] \Big|_{v_0}^0 = \frac{1}{2k} \ln \left[\frac{g + kv_0^2}{g} \right]$$

$$h = \frac{1}{2(0.002)} \ln \left[\frac{32.2 + 0.002(100)^2}{32.2} \right] = \underline{120.8 \text{ ft}}$$

$$\text{Down:} \quad a_d = -g + kv^2 = v \frac{dv}{dy}$$

$$\int_h^0 dy = \int_0^{v_f} \frac{v dv}{-g + kv^2}$$

$$-h = \frac{1}{2k} \ln [-g + kv^2] \Big|_0^{v_f} = \frac{1}{2k} \ln \left[\frac{g - kv_f^2}{g} \right]$$

$$\Rightarrow v_f = \sqrt{\frac{g}{k} (1 - e^{-2kh})}$$

$$= \sqrt{\frac{32.2}{0.002} (1 - e^{-2(0.002)(120.8)})} = \underline{78.5 \frac{\text{ft}}{\text{sec}}}$$

$$\underline{2/52} \quad U_p : \quad a_u = -g - kv^2 = \frac{dv}{dt}$$

$$\int_0^{t_u} dt = - \int_{v_0}^0 \frac{dv}{g + kv^2}$$

$$t_u = \frac{1}{\sqrt{gk}} \tan^{-1} \left(\frac{v\sqrt{gk}}{g} \right) \Big|_0^{v_0} = \frac{1}{\sqrt{gk}} \tan^{-1} \left(v_0 \sqrt{\frac{k}{g}} \right)$$

$$t_u = \frac{1}{\sqrt{32.2(0.002)}} \tan^{-1} \left(100 \sqrt{\frac{0.002}{32.2}} \right) = \underline{2.63 \text{ sec}}$$

$$(\text{Down}) : \quad a_d = -g + kv^2 = \frac{dv}{dt}$$

$$\int_0^{t_d} dt = \int_0^{v_f} \frac{dv}{-g + kv^2}$$

$$t_d = \frac{1}{\sqrt{gk}} \tanh^{-1} \left(\frac{v\sqrt{gk}}{g} \right) \Big|_0^{v_f} = \frac{1}{\sqrt{gk}} \tanh^{-1} \left(v_f \sqrt{\frac{k}{g}} \right)$$

$$= \frac{1}{\sqrt{32.2(0.002)}} \tanh^{-1} \left(78.5 \sqrt{\frac{0.002}{32.2}} \right)$$

$$= \underline{2.85 \text{ sec}} \quad \left(\begin{array}{l} \text{Refer to solution} \\ \text{of Prob. 2/51} \end{array} \right)$$

$$\boxed{2/53} \quad a = c_1 - c_2 v^2 = v \frac{dv}{ds}$$

$$\int_0^s ds = \int_0^v \frac{v dv}{c_1 - c_2 v^2} = -\frac{1}{2c_2} \int_0^v \frac{-2c_2 v dv}{c_1 - c_2 v^2}$$

$$s = -\frac{1}{2c_2} \ln(c_1 - c_2 v^2) \Big|_0^v = \frac{1}{2c_2} \ln\left(\frac{c_1}{c_1 - c_2 v^2}\right)$$

When $s = 1320 \text{ ft}$, $v = 190 \left(\frac{5280}{3600} \right) = 279 \text{ ft/sec}$:

$$1320 = \frac{1}{2(5)(10^{-5})} \ln\left(\frac{c_1}{c_1 - 5(10^{-5})(279)^2}\right)$$

Solve to obtain $c_1 = 31.4 \text{ ft/sec}^2$

$$\boxed{2/54} \quad a = 31.4 - 5(10^{-5})v^2 = c_1 - c_2v^2 = \frac{dv}{dt}$$

$$\int_0^t dt = \int_0^v \frac{dv}{c_1 - c_2v^2} = \frac{1}{\sqrt{c_1c_2}} \tanh^{-1} \sqrt{\frac{c_2}{c_1}} v \Big|_0^v$$

$$t = \frac{1}{\sqrt{c_1c_2}} \tanh^{-1} \sqrt{\frac{c_2}{c_1}} v$$

$$\text{For } v = 190 \left(\frac{5280}{3600} \right) = 279 \text{ ft/sec,}$$

$$t = \frac{1}{\sqrt{31.4(5)(10^{-5})}} \tanh^{-1} \sqrt{\frac{5(10^{-5})}{31.4}} (279)$$

$$= \underline{9.27 \text{ sec}}$$

2/55 For an acceleration of form $a = -g - kv^2$, we cite the results from Probs. 2/51 & 2/52

$$\begin{cases} t_u = \frac{1}{\sqrt{gk}} \tan^{-1} \left(v_0 \sqrt{\frac{k}{g}} \right) \\ h = \frac{1}{2k} \ln \left[\frac{g + kv_0^2}{g} \right] \end{cases}$$

For the numbers at hand:

$$t_u = \frac{1}{\sqrt{9.81(0.0005)}} \tan^{-1} \left(120 \sqrt{\frac{0.0005}{9.81}} \right) = 10.11 \text{ s}$$

$$h = \frac{1}{2(0.0005)} \ln \left[\frac{9.81 + 0.0005(120)^2}{9.81} \right] = 550 \text{ m}$$

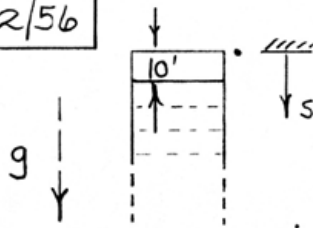
Down ($v = \text{constant}$): $y = y_0 + v_{y0}t$

$$0 = 550 - 4t_d$$

$$t_d = 137.6 \text{ s}$$

$$\text{Flight time } t = t_u + t_d = 10.11 + 137.6 = \underline{147.7 \text{ s}}$$

2/56



$$s = s_0' + v_0' t + \frac{1}{2} g t^2$$

When $s = 10$ ft,

$$10 = \frac{1}{2} (32.2) t_{10'}^2, \quad t_{10'} = 0.788 \text{ sec}$$

Time to pass first story from

the top is $t_1 = t_{10'} - t_0' = 0.788 - 0 = \underline{0.788 \text{ sec}}$

$$10^{\text{th}} \text{ story: } 90 = \frac{1}{2} (32.2) t_{90'}^2, \quad t_{90'} = 2.36 \text{ sec}$$

$$100 = \frac{1}{2} (32.2) t_{100'}^2, \quad t_{100'} = 2.49 \text{ sec}$$

$$t_{10} = t_{100'} - t_{90'} = 2.49 - 2.36 = \underline{0.1279 \text{ sec}}$$

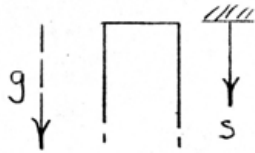
$$100^{\text{th}} \text{ story: } 990 = \frac{1}{2} (32.2) t_{990'}^2$$

$$1000 = \frac{1}{2} (32.2) t_{1000'}^2$$

$$t_{100} = t_{1000'} - t_{990'} = 7.88 - 7.84 = \underline{0.0395 \text{ sec}}$$

2/57

$$a = g - kv^2 = -\frac{dv}{dt}$$



$$\int_0^t dt = \int_0^v \frac{dv}{g - kv^2}$$

$$\begin{aligned} \text{(see Art. C/10): } t &= \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}} v \Big|_0^v \\ &= \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}} v \end{aligned}$$

$$\Rightarrow v = \frac{ds}{dt} = \sqrt{\frac{g}{k}} \tanh(\sqrt{gk} t)$$

$$\int_0^s ds = \sqrt{\frac{g}{k}} \int_0^t \tanh(\sqrt{gk} t) dt$$

$$s = \frac{1}{k} \ln \cosh \sqrt{gk} t$$

$$\text{or } t = \frac{\cosh^{-1}(e^{sk})}{\sqrt{gk}} = \frac{\cosh^{-1}(e^{0.005s})}{0.401}$$

s, ft	t, sec
0	0
10	0.795
90	2.54
100	2.70
990	14.06
1000	14.19

The time t_1 to pass first story

$$\text{is } t_1 = t_{10} - t_0 = 0.795 - 0 = \underline{0.795 \text{ sec}}$$

Similarly,

$$t_{10} = 0.1592 \text{ sec}$$

$$t_{100} = 0.1246 \text{ sec}$$

2/58

$$a = \frac{d^2x}{dt^2} = Kt - k^2x$$

or $\frac{d^2x}{dt^2} + k^2x = Kt$, a second-order,
linear differential equation whose solution is

$$x = x_h + x_p = A \sin kt + B \cos kt + \frac{K}{k^2}t$$

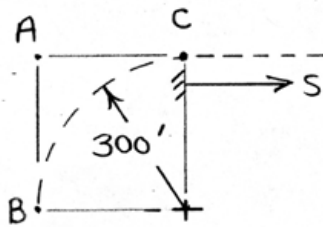
Initial conditions:

$$x(0) = B = 0$$

$$\dot{x}(0) = kA + \frac{K}{k^2} = 0, \quad A = -\frac{K}{k^3}$$

$$\text{So } x = \frac{K}{k^3}(kt - \sin kt)$$

2/59



First, determine B's acceleration time:

$$v = v_0 + at$$

$$65 \left(\frac{44}{30} \right) = 25 \left(\frac{44}{30} \right) + 3.22t$$

$$t = 18.22 \text{ sec}$$

Distance traveled by A in that time:

$$d_A = 65 \left(\frac{44}{30} \right) (18.22) = 1737 \text{ ft}$$

$$\text{Displacement beyond C: } S_A = 1737 - 300 = 1437 \text{ ft}$$

Distance traveled by B in 18.22 sec:

$$d_B = v_0 t + \frac{1}{2} a t^2 = 25 \left(\frac{44}{30} \right) (18.22) + \frac{1}{2} (3.22) (18.22)^2 = 1202 \text{ ft}$$

$$\text{Displacement beyond C: } S_B = 1202 - \frac{\pi(300)}{2} = 731 \text{ ft}$$

So A is ahead of B by $S_A - S_B = 1437 - 731$

$$= \underline{706 \text{ ft}} \quad \left(\begin{array}{c} \text{in the} \\ \text{steady-state} \end{array} \right)$$

2/60 For B, $v_f - v_0 =$ area under $a-t$ curve

$$(65-25) \frac{44}{30} = 3.22t_1 + \frac{1}{2}(3.22)(5)$$

$$t_1 = 15.72 \text{ sec}$$

B reaches 65 mi/hr @ $15.72 + 5 = 20.7 \text{ sec} = t_2$

$$\begin{aligned} \text{Speed of B @ } t_1: v_1 &= 25\left(\frac{44}{30}\right) + 3.22(15.72) \\ &= 87.3 \text{ ft/sec} \end{aligned}$$

The acceleration history ($t_1 < t < t_2$) is $a = 13.34 - 0.644t$

$$\int_{v_1=87.3}^v dv = \int_{t_1=15.72}^t (13.34 - 0.644t) dt \text{ yields}$$

$$v = -42.9 + 13.34t - 0.322t^2$$

$$\text{Then } \int_{s_1}^{s_2} ds = \int_{t_1=15.72}^{t_2=20.7} (-42.9 + 13.34t - 0.322t^2) dt$$

$$\text{yields } (s_2 - s_1) = 463 \text{ ft}$$

Distance traveled by B in 20.7 sec

$$\begin{aligned} d_B &= 25\left(\frac{44}{30}\right)(15.72) + \frac{1}{2}(3.22)(15.72)^2 + 463 \\ &= 1437 \text{ ft} \end{aligned}$$

Displacement beyond C: $s_B = 1437 - \frac{\pi(300)}{2} = 966 \text{ ft}$

Distance traveled by A in 20.7 sec:

$$d_A = 65\left(\frac{44}{30}\right)(20.7) = 1975 \text{ ft}$$

Displacement beyond C: $s_A = 1975 - 300 = 1675 \text{ ft}$

So A is ahead of B by $s_A - s_B = 1675 - 966$
 $= \underline{709 \text{ ft}}$ (in the steady-state)

(Not much more than the 706 ft of Prob. 2/59)

$$\underline{2/61} \quad \underline{a_{av}} = \frac{\Delta \underline{v}}{\Delta t} = \frac{(-0.1\hat{i} + 1.8\hat{j}) - (0.1\hat{i} + 2\hat{j})}{0.1}$$

$$= -2\hat{i} - 2\hat{j} \text{ m/s}$$

$$a_{av} = \sqrt{2^2 + 2^2} = \underline{2.83 \text{ m/s}^2}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-2}{-2}\right) = \underline{225^\circ}$$

2/62 $x = 3t^2 - 4t$, $\dot{x} = 6t - 4$, $\ddot{x} = 6 \text{ mm/s}^2$
 $y = 4t^2 - \frac{1}{3}t^3$, $\dot{y} = 8t - t^2$, $\ddot{y} = 8 - 2t \text{ mm/s}^2$

When $t = 2 \text{ s}$, $\dot{x} = 12 - 4 = 8 \text{ mm/s}$
 $\dot{y} = 16 - 4 = 12 \text{ mm/s}$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{8^2 + 12^2} = 14.42 \frac{\text{mm}}{\text{s}}$$

$$\theta_x = \tan^{-1} \frac{\dot{y}}{\dot{x}} = \tan^{-1} \frac{12}{8} = 56.3^\circ$$

$\ddot{x} = 6 \text{ mm/s}^2$, $\ddot{y} = 8 - 4 = 4 \text{ mm/s}^2$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{6^2 + 4^2} = 7.21 \text{ mm/s}^2$$

$$\theta_x = \tan^{-1} \frac{\ddot{y}}{\ddot{x}} = \tan^{-1} \frac{4}{6} = 33.7^\circ$$

2/63

$$x = t^2 - 4t + 20$$

$$y = 3 \sin 2t$$

$$\dot{x} = 2t - 4$$

$$\dot{y} = 6 \cos 2t$$

$$\ddot{x} = 2$$

$$\ddot{y} = -12 \sin 2t$$

At time $t = 3$ sec :

$$\dot{x} = 2 \text{ in./sec}$$

$$\dot{y} = 5.76 \text{ in./sec}$$

$$\ddot{x} = 2 \text{ in./sec}^2$$

$$\ddot{y} = 3.35 \text{ in./sec}^2$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{2^2 + 5.76^2} = 6.10 \text{ in./sec}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{2^2 + 3.35^2} = 3.90 \text{ in./sec}^2$$

$$\underline{v} = 2\hat{i} + 5.76\hat{j} \text{ in./sec}, \quad \underline{a} = 2\hat{i} + 3.35\hat{j} \text{ in./sec}^2$$

$$\theta = \cos^{-1} \frac{\underline{v} \cdot \underline{a}}{va} = \cos^{-1} \left(\frac{2(2) + 5.76(3.35)}{(6.10)(3.90)} \right)$$

$$= 11.67^\circ$$

$$2/64 \quad x = y^2/6 \quad \& \quad \dot{y} = 3 \text{ in./sec}$$

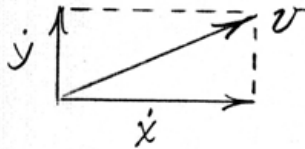
$$\dot{x} = \frac{y}{3} \dot{y} \quad , \quad \ddot{x} = \frac{\dot{y}^2}{3} + \frac{y}{3} \ddot{y} \quad \text{but } \ddot{y} = 0 \quad \& \quad \dot{y} = 3 \text{ in./sec}$$

$$\text{Also when } x = 6 \text{ in.}, \quad y = \sqrt{36} = 6 \text{ in.}$$

$$\text{So } \dot{x} = \frac{6}{3} (3) = 6 \text{ in./sec}$$

$$\text{Hence } v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{6^2 + 3^2} = \underline{3\sqrt{5} \text{ in./sec}}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(3^2/3)^2 + 0} = \underline{3 \text{ in./sec}^2}$$



$$a = \ddot{x}$$

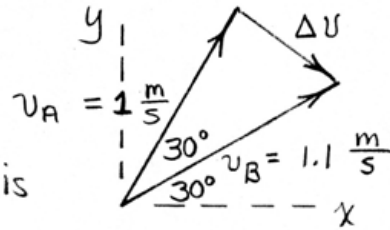
$$2/65 \quad v = \dot{s} = \frac{t}{2}, \quad v_A = \frac{2}{2} = 1 \text{ m/s}, \quad v_B = \frac{2.2}{2} = 1.1 \frac{\text{m}}{\text{s}}$$

$$\Delta v_x = v_{Bx} - v_{Ax} = 1.1 \cos 30^\circ - 1.0 \cos 60^\circ = 0.453 \frac{\text{m}}{\text{s}}$$

$$\Delta v_y = v_{By} - v_{Ay} = 1.1 \sin 30^\circ - 1.0 \sin 60^\circ = -0.316 \frac{\text{m}}{\text{s}}$$

$$\Delta v = \sqrt{0.453^2 + 0.316^2}$$

$$= 0.552 \text{ m/s}$$



The average acceleration is

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{0.552}{0.20} = 2.76 \text{ m/s}^2$$

$$\begin{aligned} \underline{a}_{av} &= \frac{\Delta \underline{v}}{\Delta t} = \frac{0.453\hat{i} - 0.316\hat{j}}{0.20} \\ &= \underline{2.26\hat{i} - 1.580\hat{j} \text{ m/s}^2} \end{aligned}$$

2/66

$$x = 20 + \frac{1}{4}t^2, \quad \dot{x} = \frac{1}{2}t, \quad \ddot{x} = \frac{1}{2} \text{ mm/s}^2$$

$$y = 15 - \frac{1}{6}t^3, \quad \dot{y} = -\frac{1}{2}t^2, \quad \ddot{y} = -t \text{ mm/s}^2$$

For $t = 2 \text{ s}$, $\dot{x} = 1 \text{ mm/s}$

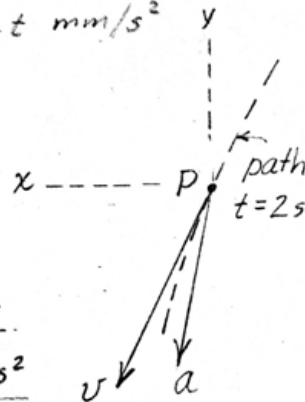
$$\dot{y} = -2 \text{ mm/s}$$

$$\ddot{x} = \frac{1}{2} \text{ mm/s}^2$$

$$\ddot{y} = -2 \text{ mm/s}^2$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{1^2 + (-2)^2} = 2.24 \text{ mm/s}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(\frac{1}{2})^2 + (-2)^2} = 2.06 \text{ mm/s}^2$$



$$\underline{2/67} \quad \underline{r} = \left(\frac{2}{3} t^3 - \frac{3}{2} t^2 \right) \underline{i} + \left(\frac{t^4}{12} \right) \underline{j}$$

$$\underline{v} = \underline{\dot{r}} = (2t^2 - 3t) \underline{i} + \left(\frac{1}{3} t^3 \right) \underline{j}$$

$$\underline{a} = \underline{\dot{v}} = (4t - 3) \underline{i} + (t^2) \underline{j}$$

$$\text{At } t = 2s \quad \begin{cases} \underline{v} = (2 \cdot 2^2 - 3 \cdot 2) \underline{i} + \frac{1}{3} 2^3 \underline{j} = 2 \underline{i} + \frac{8}{3} \underline{j} \\ \underline{a} = (4 \cdot 2 - 3) \underline{i} + 2^2 \underline{j} = 5 \underline{i} + 4 \underline{j} \end{cases} \quad \begin{matrix} \text{mm/s} \\ \text{mm/s}^2 \end{matrix}$$

$$\cos \theta = \frac{\underline{v} \cdot \underline{a}}{v a} = \frac{(2 \underline{i} + \frac{8}{3} \underline{j}) \cdot (5 \underline{i} + 4 \underline{j})}{\sqrt{2^2 + (\frac{8}{3})^2} \sqrt{5^2 + 4^2}}$$

$$\underline{\theta = 14.47^\circ}$$

$$\text{At } t = 3s \quad \begin{cases} \underline{v} = (2 \cdot 3^2 - 3 \cdot 3) \underline{i} + \left(\frac{1}{3} 3^3 \right) \underline{j} = 9 \underline{i} + 9 \underline{j} \quad \frac{\text{mm}}{\text{s}} \\ \underline{a} = (4 \cdot 3 - 3) \underline{i} + (3^2) \underline{j} = 9 \underline{i} + 9 \underline{j} \quad \frac{\text{mm}}{\text{s}^2} \end{cases}$$

$$\underline{v} \parallel \underline{a} \Rightarrow \underline{\theta = 0}$$

$$\frac{2}{68} \begin{cases} x = 3 \cos 4t; & \dot{x} = -12 \sin 4t; & \ddot{x} = -48 \cos 4t \\ y = 2 \sin 4t; & \dot{y} = 8 \cos 4t; & \ddot{y} = -32 \sin 4t \end{cases}$$

At time $t = 1.4$ sec:

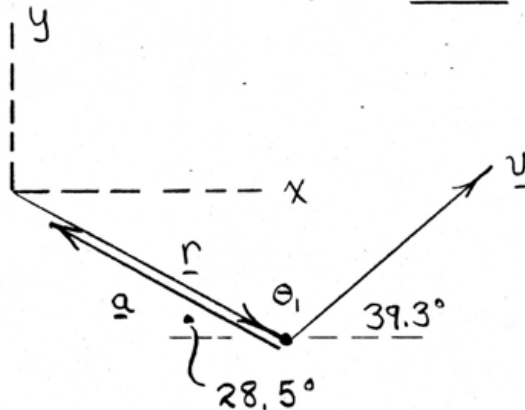
$$\begin{cases} x = 2.33 \text{ ft}; & \dot{x} = 7.58 \text{ ft/sec}; & \ddot{x} = -37.2 \frac{\text{ft}}{\text{sec}^2} \\ y = -1.263 \text{ ft}; & \dot{y} = 6.20 \text{ ft/sec}; & \ddot{y} = 20.2 \frac{\text{ft}}{\text{sec}^2} \end{cases}$$

$$r = 2.65 \text{ ft}; \quad v = 9.79 \text{ ft/sec}; \quad a = 42.4 \frac{\text{ft}}{\text{sec}^2}$$

$$\theta_1 = \cos^{-1} \left[\frac{\underline{a} \cdot \underline{v}}{a v} \right] = \cos^{-1} \left[\frac{-37.2(7.58) + 20.2(6.20)}{42.4(9.79)} \right]$$

$$\theta_1 = 112.2^\circ \quad \text{Similarly, } \theta_2 = \cos^{-1} \left[\frac{\underline{a} \cdot \underline{r}}{a r} \right]$$

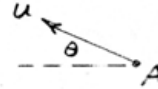
$$\text{Sketch (not to scale):} \quad = 180^\circ$$



2/69

From Sample Prob. 2/6

$$2s = \frac{u^2 \sin 2\theta}{g} = \frac{2(u \cos \theta)(u \sin \theta)}{g}$$



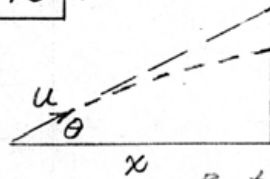
But $2s = 22 \text{ ft}$, $u \cos \theta = 30 \text{ ft/sec}$, $u \sin \theta = v_y$

$$\text{So } v_y = \frac{2sg}{2u \cos \theta} = \frac{22(32.2)}{2(30)} = 11.81 \text{ ft/sec}$$

$$\text{Also, } h = \frac{u^2 \sin^2 \theta}{2g} = \frac{v_y^2}{2g} = \frac{(11.81)^2}{2(32.2)} = 2.16 \text{ ft}$$

2/70

$$x = 1000 \text{ m} \quad u = 600 \text{ m/s} \quad \theta = 20^\circ$$



From Sample Problem 2/6

$$y = x \tan \theta - \frac{gx^2}{2u^2} \sec^2 \theta$$

But $y + \delta = x \tan \theta$, so $\delta = \frac{9.81(1000)^2}{2(600)^2} \frac{1}{(0.9397)^2}$

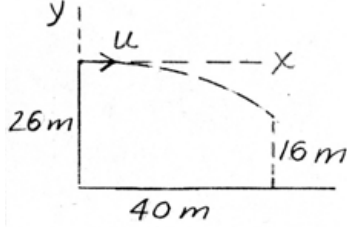
$$\delta = 15.43 \text{ m}$$

2/71 Set up x-y axes at the initial location of G.

$$\left. \begin{aligned} x &= x_0 + v_{x_0} t : 3 = (v_0 \cos \theta) t \\ y &= y_0 + v_{y_0} t - \frac{1}{2} g t^2 : 3.5 = (v_0 \sin \theta) t - 16.1 t^2 \\ v_y &= v_{y_0} - g t : 0 = v_0 \sin \theta - 32.2 t \end{aligned} \right\}$$

$$\text{Solve simultaneously : } \left\{ \begin{aligned} t &= 0.466 \text{ sec} \\ v_0 &= 16.33 \text{ ft/sec} \\ \theta &= 66.8^\circ \end{aligned} \right.$$

$$\frac{2}{72} \quad a_y = -g \text{ so } y = 0 - \frac{1}{2}gt^2, \quad t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(26-16)}{9.81}}$$

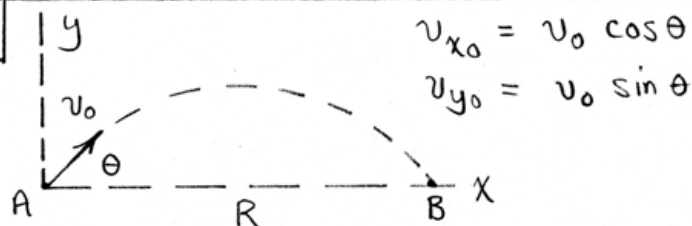


$$= 1.428 \text{ s}$$

$$x = ut; \quad u = 40 / 1.428$$

$$= \underline{28.0 \text{ m/s}}$$

2/73



$$x = x_0 + v_{x0} t \quad @ \quad B: \quad R = 0 + (v_0 \cos \theta) t_f \quad (1)$$

$$y = y_0 + v_{y0} t - \frac{1}{2} g t^2 \quad @ \quad B: \quad 0 = 0 + (v_0 \sin \theta) t_f - \frac{g}{2} t_f^2$$

$$(2): \quad t_f = 0, \quad \frac{2 v_0 \sin \theta}{g} \quad (t=0 \text{ is launch time})^{(2)}$$

$$(1): \quad R = (v_0 \cos \theta) \left(\frac{2 v_0 \sin \theta}{g} \right) = \frac{v_0^2 \sin 2\theta}{g}$$

$$\frac{dR}{d\theta} = 0: \quad \frac{v_0^2}{g} 2 \cos 2\theta = 0 \Rightarrow \underline{\theta = 45^\circ}$$

$$R_{\max} = \frac{v_0^2 \sin (2 \cdot 45^\circ)}{g} = \underline{\underline{\frac{v_0^2}{g}}}$$

2/74 Use x-y coordinates of the figure.

(a) $v_0 = 45 \text{ ft/sec}$

$$x = x_0 + v_{x0}t \text{ @ left wall: } 30 = 0 + 45 \cos 60^\circ t$$
$$t = 1.333 \text{ sec}$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2: y = 5 + 45 \sin 60^\circ (1.333) - 16.1 (1.333)^2$$
$$= 28.3 \text{ ft (hits wall)}$$

Ans. : $(x, y) = (30', 28.3')$

(b) $v_0 = 60 \text{ ft/sec}$

Repeat above procedure to find $y = 40.9'$
when $x = 30'$, so water clears left wall.

$$x = x_0 + v_{x0}t \text{ @ right wall: } 50 = 0 + 60 \cos 60^\circ t$$
$$t = 1.667 \text{ sec}$$

y eq. yields $y = 46.9 \text{ ft}$ @ $t = 1.667 \text{ sec}$, so
water clears building! For horizontal range:

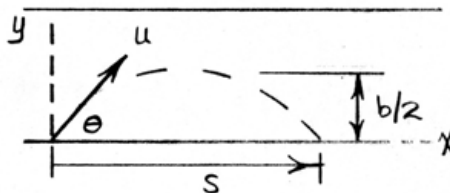
From $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ @ $y = 0$, $y_0 = 5 \text{ ft}$, we
find $t = -0.0935 \text{ s}$ & $t = 3.32 \text{ s}$. From

$$x = x_0 + v_{x0}t: x = 0 + 60 \cos 60^\circ (3.32) = \underline{99.6 \text{ ft}}$$

2/75

$$a_y = -\frac{eE}{m}, \text{ constant}$$

$$a_x = 0$$



$$v_y^2 - v_{y_0}^2 = 2a_y y : \text{ At top, } 0 - (u \sin \theta)^2 = 2 \left(-\frac{eE}{m} \right) \frac{b}{2}$$

$$E = \frac{mu^2 \sin^2 \theta}{eb}$$

$$v_y = v_{y_0} + a_y t : \text{ At top, } 0 = u \sin \theta - \frac{eE}{m} t$$

$$t = \frac{mu \sin \theta}{eE}$$

$$x = v_{x_0} t : s = (u \cos \theta)(2t) = u \cos \theta \left(\frac{2mu \sin \theta}{eE} \right)$$

$$= u \cos \theta \left(\frac{2mu \sin \theta}{e \frac{mu^2 \sin^2 \theta}{eb}} \right) = \underline{2b \cot \theta}$$

2/76

$t = 0$: package dropped at A

t_B, t_C : times package at point B, C

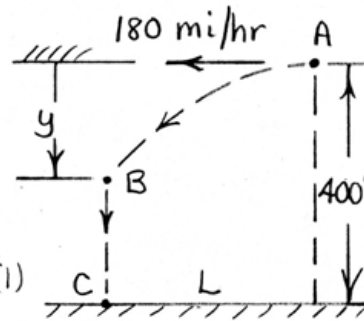
From A to B : $y = \frac{1}{2} g t_B^2$ (1)

From B to C : $(400 - y) = 6(t_C - t_B)$ (2)

Also, $t_C = 37$ sec. Solve (1) & (2) to

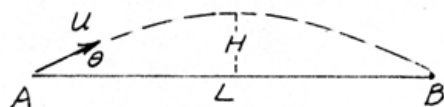
obtain $t_B = 3.52$ sec, $y = 199.1$ ft

$$L = 180 \left(\frac{5280}{3600} \right) (3.52) = \underline{928 \text{ ft}}$$



2/77 From Sample Prob. 2/6, $H = \frac{u^2 \sin^2 \theta}{2g}$ ---- (a)

$L = 2s = \frac{u^2 \sin 2\theta}{g}$ ---- (b)



so $\frac{H}{L} = \frac{\sin^2 \theta}{2(2 \sin \theta \cos \theta)}$
 $= \frac{1}{4} \tan \theta$

Thus $\theta = \tan^{-1}(4H/L)$

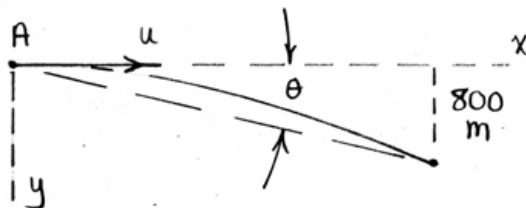
From (a) $\sin \theta = \sqrt{2gH}/u$

" (b) $Lg/u^2 = 2 \sin \theta \cos \theta = 2 \sin \theta \sqrt{1 - \sin^2 \theta}$
 $= 2 \frac{\sqrt{2gH}}{u} \sqrt{1 - \frac{2gH}{u^2}}$

Simplify, solve for u & get $u = \sqrt{2gH} \sqrt{1 + \left(\frac{L}{4H}\right)^2}$

2/78

$$u = \frac{1000}{3.6} = 278 \frac{\text{m}}{\text{s}}$$



$$y\text{-dir.} : y = v_{y0}t + \frac{1}{2}gt^2$$

$$800 = 0 + \frac{1}{2}(9.81)t^2, \quad t = 12.77 \text{ s}$$

$$x\text{-dir.} : x = v_{x0}t + \frac{1}{2}a_x t^2$$

$$= 278(12.77) + \frac{1}{2}\left(\frac{9.81}{2}\right)(12.77)^2$$

$$= 3950 \text{ m}$$

$$\theta = \tan^{-1} \frac{800}{3950} = \underline{11.46^\circ}$$

2/79 Set up x-y coordinates with origin at A.

$$x = x_0 + v_{x_0} t \text{ @ B: } 800 + s \cos 20^\circ = (120 \cos 40^\circ) t \quad (1)$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \text{ @ B:}$$

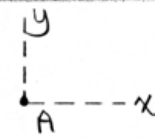
$$-s \sin 20^\circ = (120 \sin 40^\circ) t - \frac{9.81}{2} t^2 \quad (2)$$

Solve (1) & (2) simultaneously to obtain

$$\underline{s = 1057 \text{ m}}, \quad \underline{t = 19.50 \text{ s}}$$

2/80 (a) $v_0 = 140 \text{ ft/sec}$ and $\theta = 8^\circ$:

$$x = x_0 + v_{x_0} t \text{ @ B: } 200 = 0 + (140 \cos 8^\circ) t$$

$$t = \underline{1.443 \text{ sec}}$$


$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \text{ @ B:}$$

$$-(7.5 - h) = 0 + 140 \sin 8^\circ (1.443) - \frac{1}{2} (32.2) (1.443)^2$$

$$h = \underline{2.10 \text{ ft}}$$

(b) $v_0 = 120 \text{ ft/sec}$ and $\theta = 12^\circ$:

$$x = x_0 + v_{x_0} t \text{ @ B: } 200 = 0 + (120 \cos 12^\circ) t$$

$$t = \underline{1.704 \text{ sec}}$$

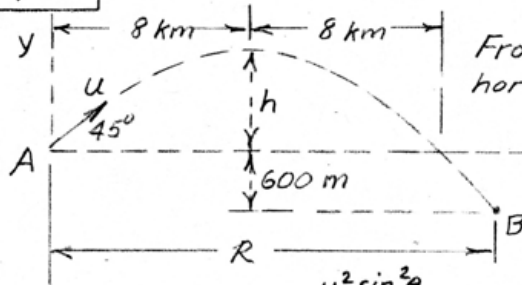
$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \text{ @ B:}$$

$$-(7.5 - h) = 0 + (120 \sin 12^\circ) (1.704) - \frac{1}{2} (32.2) (1.704)^2$$

$$h = \underline{3.27 \text{ ft}}$$

(In baseball, the time of flight is critical;
low trajectories, even with one hop, are better.)

2/81



From Sample Prob. 2/6
horiz. range is $2s = \frac{u^2 \sin 2\theta}{g}$

$$\text{so } 16(10^3) = \frac{u^2 \sin 90^\circ}{9.81}$$

$$u = \sqrt{157000} = 396 \text{ m/s}$$

$$\text{Max. altitude } h = \frac{u^2 \sin^2 \theta}{2g}, \quad H = h + 600 = \frac{(396)^2 \sin^2 45^\circ}{2(9.81)} + 600$$

$$= 4000 + 600 = 4600 \text{ m}$$

$$y = ut \sin \theta - \frac{1}{2} g t^2, \quad -600 = 396(0.7071)t - \frac{1}{2}(9.81)t^2$$

$$t^2 - 57.11t - 122.3 = 0, \quad t = \frac{57.11}{2} \pm \frac{1}{2} \sqrt{3262 + 489}$$

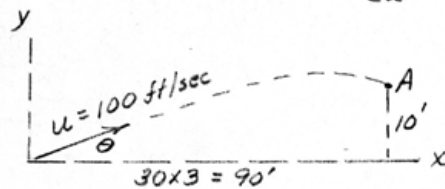
$$= 59.18 \text{ s (or } -2.07 \text{ s)}$$

$$x = ut \cos \theta, \quad R = 396(59.18) \cos 45^\circ = 16579 \text{ m.}$$

$$\text{or } R = 16.58 \text{ km}$$

2/82

$$y = x \tan \theta - \frac{gx^2}{2u^2} \sec^2 \theta$$



$$\text{Let } m = \tan \theta$$

$$\sec^2 \theta = 1 + \tan^2 \theta = 1 + m^2$$

$$y = xm = \frac{gx^2}{2u^2} (1 + m^2), \quad m^2 - \frac{2u^2}{gx} m + \left(\frac{2u^2 y}{gx^2} + 1 \right) = 0$$

$$\text{At A, } m^2 - \frac{2(10^2)^2}{32.2(90)} m + \left(\frac{2(10^2)^2 10}{32.2(90)^2} + 1 \right) = 0$$

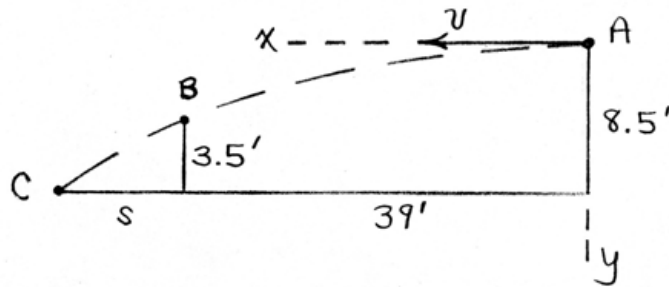
$$m^2 - 6.901m + 1.7668 = 0$$

$$m = \frac{6.901}{2} \pm \frac{1}{2} \sqrt{(6.901)^2 - 4(1.7668)}$$

$$= \frac{6.901 \pm \sqrt{40.56}}{2} = 0.266 \text{ or } 6.635$$

$$\theta = \tan^{-1} m = \underline{14.91^\circ} \quad (\text{or } 81.4^\circ)$$

2/83



$$a_x = 0 : x = v_{x_0} t, \quad 39 = v t_B$$

$$a_y = g : y = v_{y_0} t + \frac{1}{2} g t^2$$

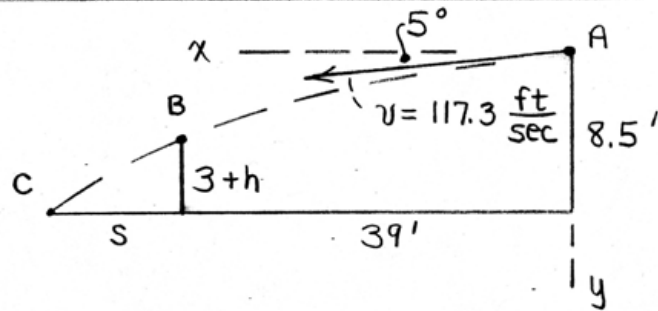
$$\text{At B : } 8.5 - 3.5 = 0 + \frac{1}{2} 32.2 t_B^2, \quad t_B = 0.557 \text{ sec}$$

$$\text{Then } v = \frac{39}{t_B} = \frac{39}{0.557} = 70.0 \text{ ft/sec} \\ \quad \quad \quad \underline{(47.7 \text{ mi/hr})}$$

$$\text{At C : } 8.5 = \frac{1}{2} (32.2) t_c^2, \quad t_c = 0.727 \text{ sec}$$

$$s + 39 = 70.0 (0.727), \quad \underline{s = 11.85 \text{ ft}}$$

2/84



$$a_x = 0, \quad x = v_{x_0} t : 39 = 117.3 \cos 5^\circ t_B, \quad t_B = 0.334 \text{ sec}$$

$$a_y = g, \quad y = v_{y_0} t + \frac{1}{2} g t^2 : \text{ At B,}$$

$$5.5 - h = 117.3 \sin 5^\circ (0.334) + \frac{1}{2} (32.2) (0.334)^2$$

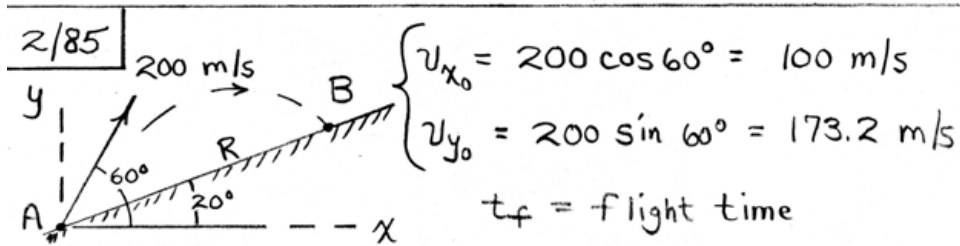
$$h = \underline{0.296 \text{ ft}} \quad \text{or} \quad h = \underline{3.55 \text{ in.}}$$

$$\text{At C: } 8.5 = 117.3 \sin 5^\circ t_c + 16.1 t_c^2$$

$$t_c = 0.475 \text{ sec}$$

$$x\text{-equation at C: } 39 + s = 117.3 \cos 5^\circ (0.475)$$

$$\underline{s = 16.57 \text{ ft}}$$



$$x = x_0 + v_{x_0} t \text{ @ B: } R \cos 20^\circ = 100 t_f \quad (1)$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \text{ @ B: } R \sin 20^\circ = 173.2 t_f - \frac{9.81}{2} t_f^2 \quad (2)$$

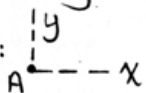
$$(1): t_f = 0.00940 R$$

$$(2): R \sin 20^\circ = 173.2 (0.00940 R) - \frac{9.81}{2} (0.00940 R)^2$$

$$\underline{R = 2970 \text{ m}}$$

2/86

x-y coordinates with origin at release

point : 

$$v_{x_0} = v_0 \sin \theta = 12 \sin \theta$$

$$v_{y_0} = v_0 \cos \theta = 12 \cos \theta$$

$v_y = v_{y_0} - gt$ applied at end of flight:

$$-12 \cos \theta = 12 \cos \theta - 9.81 t_f, \quad t_f = 2.45 \cos \theta$$

$v_x = v_{x_0} - 0.4t$ applied at end of flight:

$$-12 \sin \theta = 12 \sin \theta - 0.4(2.45 \cos \theta)$$

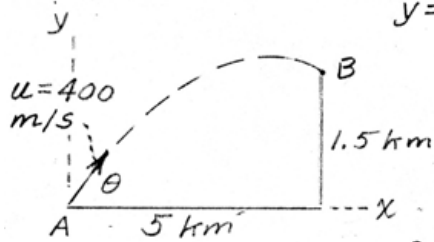
$$24 \sin \theta = 0.979 \cos \theta, \quad \tan \theta = 0.041$$

$$\underline{\theta = 2.33^\circ}$$

2/87

Eq. of trajectory (Sample Problem 2/6)

$$y = x \tan \theta - \frac{gx^2}{2u^2} \sec^2 \theta$$



$$= x \tan \theta - \frac{gx^2}{2u^2} (1 + \tan^2 \theta)$$

Substitute values & get

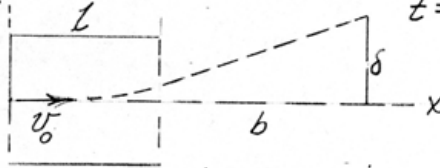
$$1500 = 5000 \tan \theta - \frac{9.81(5000)^2}{2(400)^2} (1 + \tan^2 \theta)$$

$$\text{or } \tan^2 \theta - 6.524 \tan \theta + 2.957 = 0$$

solution gives roots $\theta_1 = 26.1^\circ$ & $\theta_2 = 80.6^\circ$

2/88

Time to travel through plates is

 $t = l/v_0$ since $v_x = v_0 = \text{constant}$ $v_y = a_y t = \frac{eE}{m} \frac{l}{v_0}$ upon emergence from plates

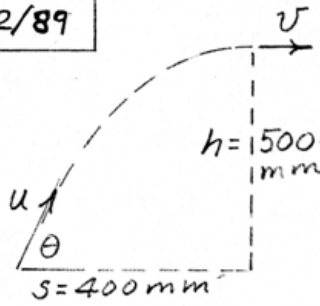
& deflection at this point is

$$y = \frac{1}{2} a_y t^2 = \frac{eE}{2m} \frac{l^2}{v_0^2}$$

Also $\delta - y/b = v_y/v_x$ so $\delta = \frac{eEl^2}{2mv_0^2} + b \frac{eEl}{mv_0} / v_0$

$$\delta = \frac{eEl}{mv_0^2} \left(\frac{l}{2} + b \right)$$

2/89



From results of Sample Problem 2/6

$$h = \frac{u^2 \sin^2 \theta}{2g}, \quad s = \frac{u^2 \sin 2\theta}{2g}$$

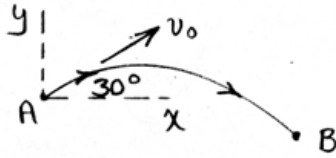
$$\text{so } \frac{h}{s} = \frac{\sin^2 \theta}{\sin 2\theta} = \frac{1}{2} \tan \theta$$

$$\theta = \tan^{-1} \frac{2h}{s} = \tan^{-1} \frac{2(500)}{400} = 68.2^\circ$$

$$u^2 = \frac{2gh}{\sin^2 \theta} = \frac{2(9.81)(0.5)}{(0.9285)^2} = 11.38 \text{ m}^2/\text{s}^2, \quad u = 3.37 \text{ m/s}$$

$$v = u \cos \theta = \text{const.}, \quad v = 3.37(0.3714) = \underline{1.253 \text{ m/s}}$$

2/90 Set up x-y axes at A, target at B:



$$\left. \begin{array}{l} \text{x-eq. : } x_B = (v_0 \cos 30^\circ) t \\ \text{y-eq. : } y_B = (v_0 \sin 30^\circ) t - \frac{1}{2} g t^2 \end{array} \right\}$$

$$\text{For } x_B = 12', y_B = -0.333' : \begin{cases} v_0 = 20.6 \text{ ft/sec} \\ t = 0.672 \text{ sec} \end{cases}$$

$$\text{For } x_B = 14', y_B = -0.333' : \begin{cases} v_0 = 22.4 \frac{\text{ft}}{\text{sec}} \\ t = 0.723 \text{ sec} \end{cases}$$

So the range is $20.6 \leq v_0 \leq 22.4 \text{ ft/sec}$

2/91 Set up x-y coordinates at A

$$x\text{-eq. : } x_B = (36 \cos \theta)t$$

$$y\text{-eq. : } y_B = (36 \sin \theta)t - 16.1t^2$$

Solutions :

$$\text{For } x_B = 40', y_B = -\frac{22}{12}' \text{ (top of stake) :}$$

$$\theta = 34.3^\circ \text{ or } \theta = 53.1^\circ$$

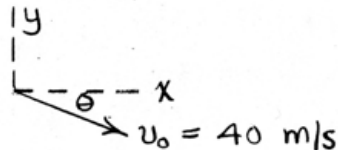
$$\text{For } x_B = 40', y_B = -3' \text{ (bottom of stake):}$$

$$\theta = 31.0^\circ \text{ or } \theta = 54.7^\circ$$

$$\text{Ranges : } 31.0^\circ \leq \theta \leq 34.3^\circ$$

$$\text{or } \underline{53.1^\circ \leq \theta \leq 54.7^\circ}$$

2/92 Set up x-y coordinates with origin at release point:



$$x = x_0 + v_{x_0} t \text{ at mitt: } 20 = (40 \cos \theta) t_f \quad (1)$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \text{ at mitt:}$$

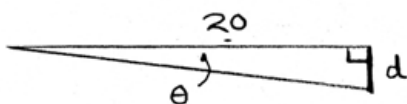
$$-1.8 = 0 + (-40 \sin \theta) t_f - \frac{9.81}{2} t_f^2 \quad (2)$$

$$(1): t_f = \frac{1}{2 \cos \theta}$$

$$(2): -1.8 = -40 \sin \theta \left(\frac{1}{2 \cos \theta} \right) - \frac{9.81}{2} \left(\frac{1}{2 \cos \theta} \right)^2$$

$$\text{Use } \frac{1}{\cos^2 \theta} = \tan^2 \theta + 1: 1.226 \tan^2 \theta + 20 \tan \theta - 0.574 = 0$$

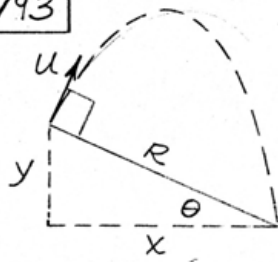
$$\Rightarrow \theta = 1.640^\circ$$



$$d = 20 \tan 1.640^\circ = 0.573'$$

$$h = (2.2 + 0.6) - (0.573 + 1) = \underline{1.227 \text{ m}}$$

2/93



$$y = R \sin \theta = -ut \cos \theta + \frac{1}{2}gt^2$$

$$x = R \cos \theta = ut \sin \theta$$

Eliminate t and get

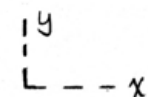
$$R \sin \theta = -R \frac{\cos \theta}{\sin \theta} \cos \theta + \frac{1}{2}g \left(\frac{R \cos \theta}{u \sin \theta} \right)^2$$

$$\frac{1}{\sin \theta} = \frac{g}{2} \frac{R}{u^2 \tan^2 \theta}$$

$$R = \frac{2u^2 \tan^2 \theta}{g \sin \theta} = \underline{\underline{\frac{2u^2}{g} \tan \theta \sec \theta}}$$

2/94

Use x-y coordinates with origin at the

release point : 

$$x = x_0 + v_{x_0} t \quad @ \quad \text{hoop} : \quad 13.75 = 0 + (v_0 \cos 50^\circ) t_f$$

$$t_f = 21.4/v_0$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \quad @ \quad \text{hoop} :$$

$$3 = 0 + v_0 \sin 50^\circ \left(\frac{21.4}{v_0} \right) - 16.1 \left(\frac{21.4}{v_0} \right)^2$$

$$\underline{v_0 = 23.5 \text{ ft/sec}}$$

2/95 Set up x-y coordinates at A: 

From $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ evaluated at D,
we have $0 = 0 + v_0 \sin \alpha t_0 - \frac{1}{2}gt_0^2$ or $t_0 = \frac{2v_0 \sin \alpha}{g}$

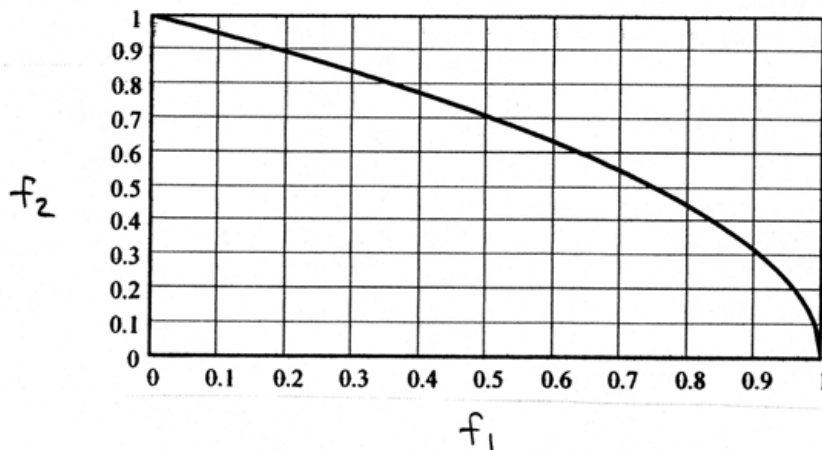
From $v_y^2 = v_{y0}^2 - 2g(y - y_0)$ evaluated at apex, we
have $0 = v_0^2 \sin^2 \alpha - 2gh$ or $h = \frac{v_0^2 \sin^2 \alpha}{2g}$

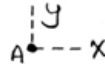
Time to $f_1 h$: $f_1 h = f_1 \frac{v_0^2 \sin^2 \alpha}{2g} = (v_0 \sin \alpha)t - \frac{1}{2}gt^2$

Solve quadratic to obtain $\begin{cases} t_B = \frac{v_0 \sin \alpha (1 - \sqrt{1 - f_1})}{g} \\ t_C = \frac{v_0 \sin \alpha (1 + \sqrt{1 - f_1})}{g} \end{cases}$

So $t_{BC} = t_C - t_B = \frac{2\sqrt{1 - f_1} v_0 \sin \alpha}{g}$

$f_2 = \frac{t_{BC}}{t_D} = \sqrt{1 - f_1}$; $f_2 = \frac{1}{2}$ for $f_1 = \frac{3}{4}$.



2/96 With x-y coordinates, origin at A: 

$$x = x_0 + v_{x_0} t \text{ @ B: } 360 = 0 + (100 \cos \alpha) t_f \quad (1)$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \text{ @ B: } -80 = 0 + (100 \sin \alpha) t_f - \frac{1}{2} (32.2) t_f^2$$

Simultaneous solutions of (1) & (2): (2)

$$\begin{cases} t_f = 4.03 \text{ sec, } \alpha = 26.8^\circ & (a) \end{cases}$$

$$\begin{cases} t_f = 5.68 \text{ sec, } \alpha = 50.7^\circ & (b) \end{cases}$$

Check at corner $[(x, y) = (280', 0)]:$

(a)
$$t_c = \frac{280}{100 \cos 26.8^\circ} = 3.14 \text{ sec}$$

$$y_c = 100 \sin 26.8^\circ (3.14) - \frac{32.2}{2} (3.14)^2 = -16.94 \text{ ft}$$

So conditions (a) are not possible.

(b)
$$t_c = \frac{280}{100 \cos 50.7^\circ} = 4.42 \text{ sec}$$

$$y_c = 100 \sin 50.7^\circ (4.42) - \frac{32.2}{2} (4.42)^2 = 27.5 \text{ ft}$$

Conditions (b) result in clearance at corner

Ans. $\alpha = 50.7^\circ$

$$\boxed{2/97} \quad \left. \begin{aligned} \underline{a} &= -k\underline{v} - g\underline{j} \\ a_x \underline{i} + a_y \underline{j} &= -k(v_x \underline{i} + v_y \underline{j}) - g\underline{j} \end{aligned} \right\} \therefore \begin{aligned} a_x &= -kv_x \\ a_y &= -kv_y - g \end{aligned}$$

$$\underline{x}: \quad a_x = \frac{dv_x}{dt} = -kv_x$$

$$\int_{v_{x0}}^{v_x} \frac{dv_x}{v_x} = -\int_0^t k dt \Rightarrow v_x = v_{x0} e^{-kt}$$

$$\text{or } \underline{v_x = (v_0 \cos \theta) e^{-kt}}$$

$$v_x = \frac{dx}{dt} = v_{x0} e^{-kt}$$

$$\int_0^x dx = \int_0^t v_{x0} e^{-kt} dt$$

$$x = \frac{v_{x0}}{k} [1 - e^{-kt}] = \underline{\frac{v_0 \cos \theta}{k} [1 - e^{-kt}]}$$

$$\underline{y}: \quad a_y = \frac{dv_y}{dt} = -kv_y - g$$

$$\int_{v_{y0}}^{v_y} \frac{dv_y}{kv_y + g} = -\int_0^t dt$$

$$\Rightarrow v_y = \left[v_{y0} + \frac{g}{k} \right] e^{-kt} - \frac{g}{k} = \underline{\left[v_0 \sin \theta + \frac{g}{k} \right] e^{-kt} - \frac{g}{k}}$$

$$v_y = \frac{dy}{dt} = \left[v_{y_0} + \frac{g}{k} \right] e^{-kt} - \frac{g}{k}$$

$$\int_0^y dy = \int_0^t \left\{ \left[v_{y_0} + \frac{g}{k} \right] e^{-kt} - \frac{g}{k} \right\} dt$$

$$y = \frac{1}{k} \left[v_0 \sin \theta + \frac{g}{k} \right] [1 - e^{-kt}] - \frac{g}{k} t$$

Terminal velocity ($t \rightarrow \infty$): $v_x \rightarrow 0$

$$\underline{v_y \rightarrow -\frac{g}{k}}$$

2/98

Use the x - y coordinates of the figure. In the absence of the circular surface B-C, the range over a horizontal surface would be $\frac{v_0^2}{g} \sin 2\theta = \frac{225^2}{32.2} \sin(2 \cdot 30^\circ) = 1362 \text{ ft} > 1000 \text{ ft}$, so the impact point is beyond B.

$$x = x_0 + v_{x_0} t : x = (225 \cos 30^\circ) t \quad (1)$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 : y = (225 \sin 30^\circ) t - 16.1 t^2 \quad (2)$$

Surface constraint:

$$(x - 1000)^2 + (y - 500)^2 = 500^2 \quad (3)$$

Computer solution of Eqs. (1), (2), & (3):

$$\begin{cases} \underline{x = 1242 \text{ ft}} \\ \underline{y = 62.7 \text{ ft}} \end{cases} \quad t = 6.38 \text{ sec}$$

(The nature of the solution indicates impact on the circular surface and not above C.)

2/99 $a_y = g, v_y = gt, y = \frac{1}{2}gt^2$

$$a_x = 2v_y \omega \cos \delta = 2gt \omega \cos \delta$$

$$v_x = gt^2 \omega \cos \delta$$

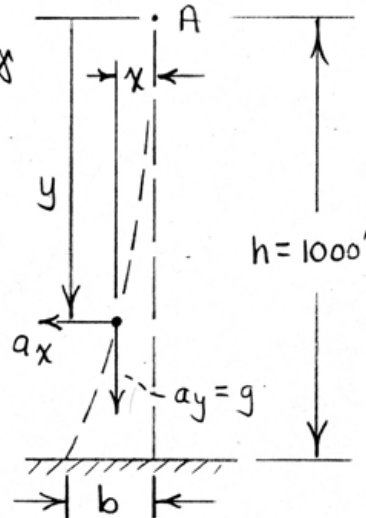
$$x = \frac{1}{3}gt^3 \omega \cos \delta$$

When $y = h, t = \sqrt{\frac{2h}{g}}$

Then

$$x = b = \frac{1}{3}g \left(\frac{2h}{g}\right)^{3/2} \omega \cos \delta$$

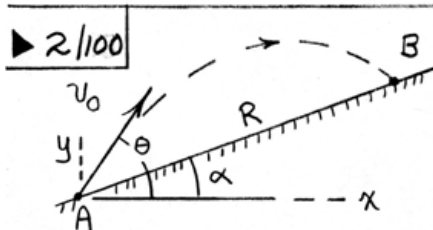
$$b = \frac{2\sqrt{2}}{3} h \sqrt{\frac{h}{g}} \omega \cos \delta$$



For $h = 1000 \text{ ft}$, $\omega = 0.7292(10^{-4}) \text{ rad/sec}$,
and $g = 32.13 \text{ ft/sec}^2$,

$$b = \frac{2\sqrt{2}}{3} (1000) \sqrt{\frac{1000}{32.13}} (0.7292 \cdot 10^{-4}) \cos 30^\circ$$

$$= 0.332 \text{ ft or } \underline{3.99 \text{ in.}}$$



$$x = x_0 + v_{x0}t \text{ @ B: } R \cos \alpha = (v_0 \cos \theta) t_f$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 \text{ @ B: } R \sin \alpha = (v_0 \sin \theta) t_f - \frac{1}{2}g t_f^2$$

$$x\text{-eq: } t_f = \frac{R \cos \alpha}{v_0 \cos \theta}$$

$$y\text{-eq: } R \sin \alpha = (v_0 \sin \theta) \left(\frac{R \cos \alpha}{v_0 \cos \theta} \right) - \frac{1}{2}g \left(\frac{R \cos \alpha}{v_0 \cos \theta} \right)^2$$

$$\Rightarrow R = \frac{2v_0^2 \cos^2 \theta}{g \cos \alpha} (\tan \theta - \tan \alpha)$$

$$\frac{dR}{d\theta} = 0: \frac{4v_0^2 \cos \theta (-\sin \theta)}{g \cos \alpha} (\tan \theta - \tan \alpha) + \frac{2v_0^2 \cos^2 \theta}{g \cos \alpha} \frac{1}{\cos^2 \theta} = 0$$

$$\frac{2v_0^2}{g \cos \alpha} [2 \cos \theta \sin \theta (\tan \alpha - \tan \theta) + 1] = 0$$

$$\Rightarrow 2 \cos \theta \sin \theta \left(\tan \alpha - \frac{\sin \theta}{\cos \theta} \right) + 1 = 0$$

$$(2 \cos \theta \sin \theta) \tan \alpha - 2 \sin^2 \theta + 1 = 0$$

$$\sin 2\theta \tan \alpha - 2 \left(\frac{1}{2} - \frac{1}{2} \cos 2\theta \right) + 1 = 0$$

$$\sin 2\theta \tan \alpha + \cos 2\theta = 0$$

$$\tan 2\theta = -\frac{1}{\tan \alpha}$$

$$\begin{aligned}
 2\theta &= \tan^{-1}\left(-\frac{1}{\tan\alpha}\right) = 180^\circ - \tan^{-1}\left(\frac{1}{\tan\alpha}\right) \\
 &= 180^\circ - (90^\circ - \alpha) = 90^\circ + \alpha \\
 \therefore \theta &= \frac{90^\circ + \alpha}{2}
 \end{aligned}$$

Specific results :

$$\begin{cases} \alpha = 0, & \theta = 45^\circ \\ \alpha = 30^\circ, & \theta = 60^\circ \\ \alpha = 45^\circ, & \theta = 67.5^\circ \end{cases}$$

2/10/ For $a_t = \text{constant}$, $v = v_0 + a_t t$

So $a_t = \frac{v}{t} = \frac{100 (1000)}{3600} / 10 = 2.78 \text{ m/s}^2$

$v_g = 2.78(8) = 22.2 \text{ m/s}$

$a_n = \frac{v^2}{r} = \frac{22.2^2}{80} = 6.17 \text{ m/s}^2$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2.78^2 + 6.17^2} = \underline{6.77 \text{ m/s}^2}$

2/102

a_1 : speed v is increasing, no path curvature.

a_2 : speed v increasing, car turning right.

a_3 : speed v stationary, car turning right.

a_4 : speed v decreasing, car turning right.

a_5 : speed v decreasing, no path curvature.

a_6 : speed v decreasing, car turning left.

a_7 : speed v stationary, car turning left.

a_8 : speed v increasing, car turning left.

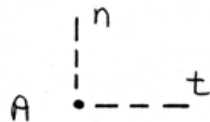
$$2/103 \quad a_n = v^2/r = (0.6)^2/0.3 = 1.2 \text{ m/s}^2$$

$$(a) \quad a_t = \dot{v} = 0 \quad \text{so} \quad \underline{a = a_n = 1.2 \text{ m/s}^2}$$

$$(b) \quad a_t = \dot{v} = 0.9 \text{ m/s}^2 \quad \text{so} \quad a = \sqrt{a_n^2 + a_t^2} = \sqrt{(1.2)^2 + (0.9)^2}$$
$$\underline{a = 1.5 \text{ m/s}^2}$$

$$\begin{aligned}
 2/104 \quad a = a_n = v^2/\rho, \quad v &= \sqrt{\rho a_n} = \sqrt{(100 - 0.6)0.5(9.81)} \\
 &= 22.08 \text{ m/s} \\
 \text{or } v &= 22.08(3.6) = \underline{79.5 \text{ km/h}}
 \end{aligned}$$

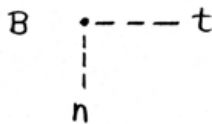
2/105



At A : $a_n = \frac{v_A^2}{r_A}$

$$0.4(9.81) = \frac{v_A^2}{120-0.6}$$

$$v_A = v = 21.6 \text{ m/s}$$



At B : $a_n = \frac{v_B^2}{r_B}$

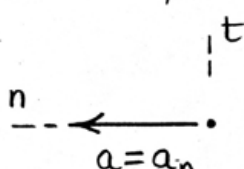
$$0.25(9.81) = \frac{21.7^2}{r_B + 0.6}$$

$$r_B = 190.4 \text{ m}$$

2/106 (a) $a_n = \frac{v^2}{r} = \frac{1.2^2}{0.6} = 2.4 \text{ m/s}^2$

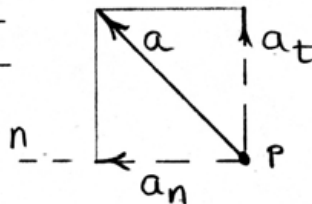
$a_t = 0$

$a = \sqrt{a_n^2 + a_t^2} = 2.4 \text{ m/s}^2$



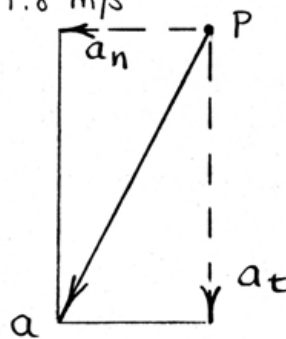
(b) $a_n = 2.4 \text{ m/s}^2$, $a_t = 2.4 \text{ m/s}^2$

$a = \sqrt{2.4^2 + 2.4^2} = 3.39 \text{ m/s}^2$



(c) $a_n = 2.4 \text{ m/s}^2$, $a_t = -4.8 \text{ m/s}^2$

$a = \sqrt{2.4^2 + 4.8^2} = 5.37 \text{ m/s}^2$



2/107

$$\Delta\theta = (25-15) \frac{\pi}{180} = 0.1745 \text{ rad}$$

$$v_B = 14 \text{ ft/sec} \quad v_A = 12 \text{ ft/sec} \quad \Delta v_t \quad v_{av} = \frac{12+14}{2} = 13 \text{ ft/sec}$$

$$a_n = \frac{\Delta v_n}{\Delta t} = \frac{v_{av}(\Delta\theta)}{\Delta t} = \frac{13(0.1745)}{2.62 - 2.4} = \underline{10.31 \frac{\text{ft}}{\text{sec}^2}}$$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{14 - 12}{0.22} = \underline{9.09 \frac{\text{ft}}{\text{sec}^2}}$$

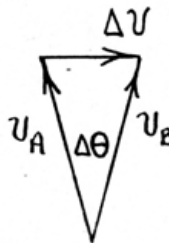
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$$v_A = v_B = v = 2 \text{ m/s}$$

$$\Delta v = 2v \sin \frac{\Delta \theta}{2} = 4 \sin \frac{\Delta \theta}{2} \text{ m/s}$$

$$\Delta t = \frac{r \Delta \theta}{v} = \frac{0.8 \Delta \theta}{2} = 0.4 \Delta \theta \text{ s}$$

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{4 \sin \frac{\Delta \theta}{2}}{0.4 \Delta \theta} = 5 \frac{\sin \frac{\Delta \theta}{2}}{\Delta \theta / 2}$$



$\Delta \theta^\circ$	$\frac{\Delta \theta}{2}^\circ$	$\frac{\Delta \theta}{2} \text{ rad}$	$\sin \frac{\Delta \theta}{2}$	$a_{av}, \text{ m/s}^2$	% diff.
(a) 30°	15°	0.262	0.259	<u>4.94</u>	1.1
(b) 15°	7.5°	0.1309	0.1305	<u>4.99</u>	0.3
(c) 5°	2.5°	0.0436	0.0436	<u>4.998</u>	0.03

$$a_n = \frac{v^2}{r} = \frac{2^2}{0.8} = \underline{5 \text{ m/s}^2}$$

$$\underline{2/109} \quad \text{From } a_n = \frac{v^2}{r}, \quad v = \sqrt{a_n r} = \sqrt{0.8gr}$$

$$v_A = \sqrt{0.8gr_A} = \sqrt{0.8(9.81)(85)} = \underline{25.8 \text{ m/s}}$$

$$v_B = \sqrt{0.8gr_B} = \sqrt{0.8(9.81)(200)} = \underline{39.6 \text{ m/s}}$$

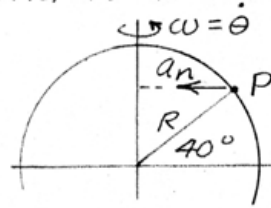
Path BB offers a considerable advantage.

2/110

$$a = a_n = r\dot{\theta}^2 = R \cos \gamma \dot{\theta}^2$$

$$= \frac{12.742 (10^6)}{2} \cos 40^\circ (0.729 \times 10^{-4})^2$$

$$= \underline{0.0259 \text{ m/s}^2}$$



$$\begin{aligned}
 \underline{2/111} \quad v &= v_0 + a_t t = 0 + 1.8(5) = 9 \text{ m/s} \\
 a_n &= \frac{v^2}{r} = \frac{9^2}{40} = 2.025 \text{ m/s}^2 \\
 a &= \sqrt{a_t^2 + a_n^2} = \sqrt{(1.8)^2 + (2.025)^2} = \underline{2.71 \frac{\text{m}}{\text{s}^2}}
 \end{aligned}$$

$$\begin{aligned} 2/112 \quad a_n &= v^2/\rho \quad ; \quad \rho = s/\theta = \frac{300}{\pi/4} = 382 \text{ ft} \\ v &= 45 \frac{44}{30} = 66 \text{ ft/sec} \\ a &= a_n = 66^2/382 = \underline{11.40 \text{ ft/sec}^2} \end{aligned}$$

$$\begin{aligned} \underline{2/113} \quad a_n = g = \frac{v^2}{r} &= \frac{[17,369 (5280/3600)]^2}{(3959 + 150)(5280)} \\ &= \underline{29.91 \text{ ft/sec}^2} \end{aligned}$$

$$\begin{aligned} \text{Check: } g &= g_0 \left(\frac{R}{R+h} \right)^2 = 32.22 \left(\frac{3959}{3959+150} \right)^2 \\ &= \underline{29.91 \text{ ft/sec}^2} \quad \checkmark \end{aligned}$$

2/114 The radius of Jupiter is

$$R = \frac{142984}{2} (10^3) = 7.15(10^7) \text{ m}$$

$$\text{So } p = R+h = 7.15(10^7) + 10^6 \text{ m} = 7.25(10^7) \text{ m}$$

From the gravitational law

$$a_n = g = g_0 \frac{R^2}{(R+h)^2} = g_0 \frac{R^2}{p^2} = 24.85 \frac{[7.15(10^7)]^2}{[7.25(10^7)]^2}$$
$$= 24.2 \text{ m/s}^2$$

$$\text{From } a_n = \frac{v^2}{p}, \quad v^2 = a_n p = (24.2)(7.25 \cdot 10^7)$$

$$v = 41900 \text{ m/s} \quad \text{or} \quad v = 150700 \text{ km/h}$$

2/115

$$a_t = 20/3.6 = 5.56 \text{ m/s}^2$$

$$a^2 = a_n^2 + a_t^2, \quad a_n^2 = \overline{3(9.81)}^2 - \overline{5.56}^2 = 835.2$$

$$a_n = 28.90 \text{ m/s}^2$$

$$a_n = v^2/\rho, \quad \rho = \frac{(800/3.6)^2}{28.90} = \underline{1709 \text{ m}}$$

$$2/116 \quad a_t = -0.6 \frac{\text{m}}{\text{s}^2}, \text{ constant}$$

$$v_B^2 = v_A^2 + 2a_t s = 16^2 - 2(0.6)(120)$$

$$v_B = 10.58 \text{ m/s}$$

$$a_n = v_B^2 / \rho = 10.58^2 / 60 = 1.867 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6^2 + 1.867^2} = \underline{1.961 \frac{\text{m}}{\text{s}^2}}$$

$$\begin{array}{|l} 2/117 \\ \hline \end{array} \quad a_n = v\dot{\beta} = g, \quad \dot{\beta} = \frac{9.79}{800(10^3)/3600} = 0.04406 \text{ rad/s}$$

$$\text{or } \dot{\beta} = 0.04406 \frac{180}{\pi} = \underline{2.52 \text{ deg/s}}$$

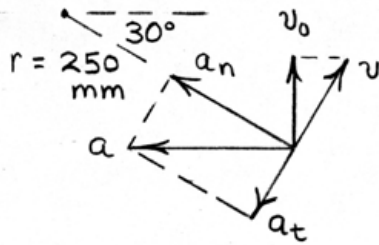
2/118

$$v = v_0 / \cos 30^\circ = 2 / \cos 30^\circ = 2.31 \text{ m/s}$$

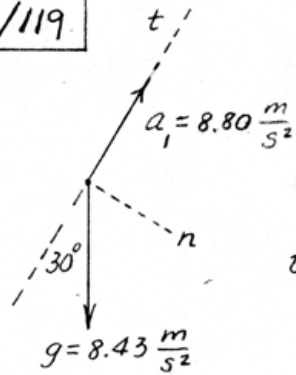
$$a_n = v^2 / r = 2.31^2 / 0.250$$

$$= 21.3 \text{ m/s}^2$$

$$a_t = -a_n \tan 30^\circ = -12.32 \frac{\text{m}}{\text{s}^2}$$



2/119



$$a_n = g \sin 30^\circ = 8.43 (0.5) = 4.22 \text{ m/s}^2$$

$$\text{or } a_n = \frac{4.22}{1000} (3600)^2 = 54630 \text{ km/h}^2$$

$$a_n = \frac{v^2}{\rho}, \quad \rho = \frac{v^2}{a_n} = \frac{(30000)^2}{54630} = 16480 \text{ km}$$

$$\dot{v} = a_t = 8.80 - 8.43 \cos 30^\circ = 1.499 \text{ m/s}^2$$

2/120

$$v = r\dot{\theta}, \quad v_B = \frac{8}{4} 2 = 4 \text{ ft/sec}$$

$$a_n = v^2/r, \quad a_n = 4^2/8/12 = 24 \text{ ft/sec}^2$$

$$a_t = r\ddot{\theta}; \quad a_t = \frac{8}{4} 6 = 12 \text{ ft/sec}^2$$

$$a_B = \sqrt{24^2 + 12^2} = \underline{26.8 \text{ ft/sec}^2}$$

2/121 Relative to space station, $a_n = r\dot{\theta}^2$

where $a_n = 32.17 \text{ ft/sec}^2$.

Thus $32.17 = (240 + 20) \dot{\theta}^2$, $\dot{\theta} = 0.352 \frac{\text{rad}}{\text{sec}}$

$N = 0.352 \left(\frac{60}{2\pi} \right) = \underline{3.36 \text{ rev/min}}$

$$\underline{2/122} \quad a_t = \frac{v_f - v_i}{\Delta t} = \frac{6 - 3}{2} = 1.5 \text{ m/s}^2$$

Halfway through time interval, $v = 4.5 \text{ m/s}$

$$a_{P_1} = \sqrt{a_t^2 + a_n^2} = \sqrt{1.5^2 + \left(\frac{4.5^2}{0.060}\right)^2}$$
$$= \underline{338 \text{ m/s}^2} \quad (34.4g!)$$

$$a_{P_2} = a_t = \underline{1.5 \text{ m/s}^2}$$

$$2/123 \quad v^2 = v_0^2 + 2a_t \Delta s, \quad 18^2 = 2^2 + 2a_t(8)$$

$$a_t = 20 \text{ m/s}^2$$

$$a_n = v^2/r = 3^2/0.150 = 60 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20^2 + 60^2} = \underline{63.2 \text{ m/s}^2}$$

$$\boxed{2/124} \quad v_{x=50'} = \sqrt{2g \left(\frac{50}{20}\right)^2} = \frac{5}{2} \sqrt{2g} \frac{\text{ft}}{\text{sec}}$$

$$a_n = \frac{v^2}{\rho}, \text{ where } \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{\left[1 + \left(\frac{x}{200}\right)^2\right]^{3/2}}{1/200}$$

$$\rho_{x=50'} = 219 \text{ ft}$$

$$a_n = \frac{25}{4} \cdot 2(32.2) / 219 = \underline{1.838 \text{ ft/sec}^2}$$

$$2/125 \quad a_n = v^2/r = 4^2/0.120 = 133.3 \text{ m/s}^2$$

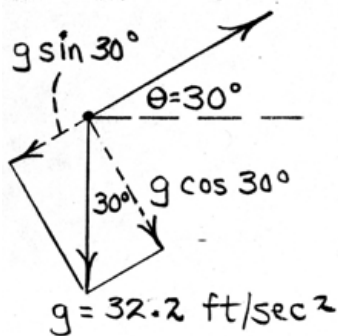
$$a_t = -a_n \tan 4^\circ = -133.3 \tan 4^\circ = -1907 \text{ m/s}^2$$

$$\text{With } a_t \text{ const, } v_f = v_i + a_t t, \quad 0 = 4 - 1907 t$$

$$t = \frac{4}{1907} = \underline{2.10(10^{-3}) \text{ s}}$$

2/126

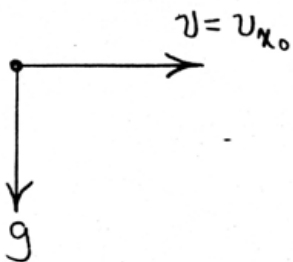
$$v_0 = 100 \text{ ft/sec}$$



$$(a) a_n = g \cos 30^\circ = \frac{v^2}{r}$$

$$r = \frac{100^2}{g \cos 30^\circ} = \underline{359 \text{ ft}}$$

$$\dot{v} = -g \sin 30^\circ = \underline{-16.1 \text{ ft/sec}^2}$$



$$(b) a_n = g = \frac{v^2}{r}$$

$$r = \frac{(100 \cos 30^\circ)^2}{32.2} = \underline{233 \text{ ft}}$$

$$\underline{\dot{v} = 0}$$

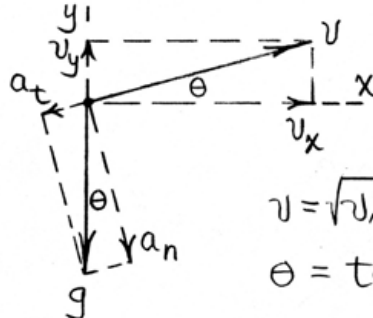
2/127

The time t_{up} to apex is found from

$$v_y = v_{y_0} - gt : 0 = 100 \sin 30^\circ - 32.2 t_{up}, t_{up} = 1.553 \text{ sec}$$

So $t = 1 \text{ sec}$ is before apex and $t = 2.5 \text{ sec}$ is after.

(a) $t = 1 \text{ sec}$



$$v_x = 100 \cos 30^\circ = 86.6 \text{ ft/sec}$$

$$v_y = 100 \sin 30^\circ - 32.2(1) = 17.80 \text{ ft/sec}$$

$$v = \sqrt{v_x^2 + v_y^2} = 88.4 \text{ ft/sec}$$

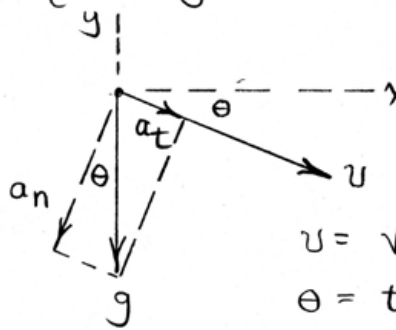
$$\theta = \tan^{-1} \frac{v_y}{v_x} = 11.61^\circ$$

$$a_n = g \cos \theta = 32.2 \cos 11.61^\circ = 31.5 \text{ ft/sec}^2$$

$$r = \frac{v^2}{a_n} = \frac{88.4^2}{31.5} = 248 \text{ ft}$$

$$a_t = -g \sin \theta = -32.2 \sin 11.61^\circ = -6.48 \text{ ft/sec}^2$$

(b) $t = 2.5 \text{ sec}$



$$v_x = 86.6 \text{ ft/sec}$$

$$v_y = 100 \sin 30^\circ - 32.2(2.5) = -30.5 \frac{\text{ft}}{\text{sec}}$$

$$v = \sqrt{v_x^2 + v_y^2} = 91.8 \text{ ft/sec}$$

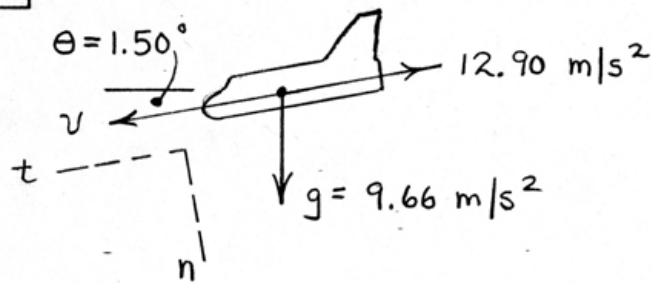
$$\theta = \tan^{-1} \frac{|v_y|}{v_x} = 19.40^\circ$$

$$a_n = g \cos \theta = 32.2 \cos 19.40^\circ = 30.4 \text{ ft/sec}^2$$

$$r = \frac{v^2}{a_n} = \frac{91.8^2}{30.4} = 278 \text{ ft}$$

$$a_t = +g \sin \theta = +32.2 \sin 19.40^\circ = 10.70 \text{ ft/sec}^2$$

2/128



$$\dot{v} = a_t = 9.66 \sin 1.50^\circ - 12.90 = \underline{-12.65 \text{ m/s}^2}$$

$$a_n = g \cos \theta = 9.66 \cos 1.5^\circ = 9.657 \text{ m/s}^2$$

$$a_n = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{a_n} = \frac{(15\,450/3.6)^2}{9.657}$$

$$\underline{r = 1907 \text{ km}}$$

$$\underline{2/129} \quad a_n = 0.8g = \frac{v^2}{r} \Rightarrow v = \sqrt{0.8gr}$$

$$\text{Car A: } v_A = \sqrt{0.8(9.81)(88)} = 26.3 \text{ m/s}$$

$$\text{Car B: } v_B = \sqrt{(0.8)(9.81)72} = 23.8 \text{ m/s}$$

$$t_A = \frac{s_A}{v_A} = \frac{\pi(88)}{26.3} = \underline{10.52 \text{ s}}$$

$$t_B = \frac{s_B}{v_B} = \frac{\pi(72) + 2(16)}{23.8} = \underline{10.86 \text{ s}}$$

Car A will win the race!

2/130 Evaluate $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ at C:

$$0 = 150 + 0 - \frac{1}{2}(9.81)t^2, \quad t = 3.05 \text{ sec}$$

Evaluate $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$ at C:

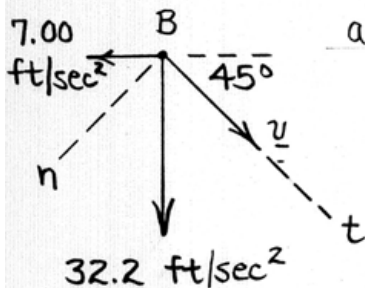
$$120 = 0 + 50(3.08) + \frac{1}{2}a_x (3.08)^2, \quad a_x = -7.00 \text{ ft/sec}^2$$

$$\text{At B: } \begin{cases} v_x = v_{x0} + a_x t : v_x = 50 - 7.00t \\ v_y = v_{y0} - gt : |v_y| = 32.2t \end{cases}$$

Set $v_x = |v_y|$ & obtain $t = 1.275 \text{ sec}$

$$\text{So at B: } v_x = |v_y| = 32.2(1.276) = 41.1 \text{ ft/sec}$$

The speed at B is $v = 41.1\sqrt{2} = 58.1 \text{ ft/sec}$



$$\begin{aligned} a_n &= 32.2 \cos 45^\circ + 7.00 \cos 45^\circ \\ &= 27.7 \text{ ft/sec}^2 \end{aligned}$$

$$\text{From } a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(58.1)^2}{27.7} = \underline{\underline{121.7 \text{ ft}}}$$

2/131 For $q_t = \text{const.}$, $v_c^2 = v_A^2 + 2q_t \Delta s_{A-c}$

$$v_A = \frac{250}{3.6} \text{ m/s}, v_C = \frac{200}{3.6} \text{ m/s}, q_t = \frac{(200)^2 - (250)^2}{(3.6)^2 2(300)} = -2.89 \text{ m/s}^2$$

$$v_B^2 = v_A^2 + 2q_t \Delta s_{A-B} = \left(\frac{250}{3.6}\right)^2 + 2(-2.89)(150) = 3954 \text{ (m/s)}^2$$

$$v_B = 62.9 \text{ m/s}$$

at B, $a_n = \frac{v_B^2}{\rho} = \frac{3954}{500} = 7.91 \text{ m/s}^2$

$$a = \sqrt{a_n^2 + q_t^2} = \sqrt{(7.91)^2 + (-2.89)^2} = \underline{8.42 \text{ m/s}^2}$$

2/132

$$x = 16 - 12t + 4t^2$$

$$y = 2 + 15t - 3t^2$$

$$\dot{x} = 8t - 12$$

$$\dot{y} = 15 - 6t$$

$$\ddot{x} = 8$$

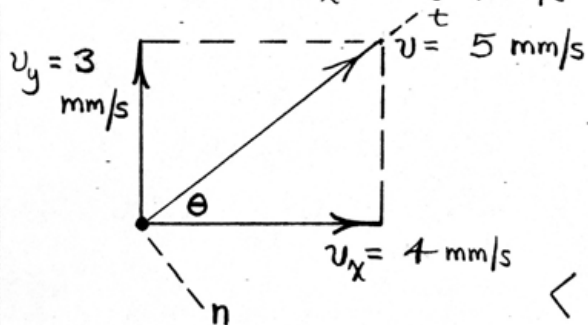
$$\ddot{y} = -6$$

$$\text{At } t = 2 \text{ s : } \dot{x} = 4 \text{ mm/s}$$

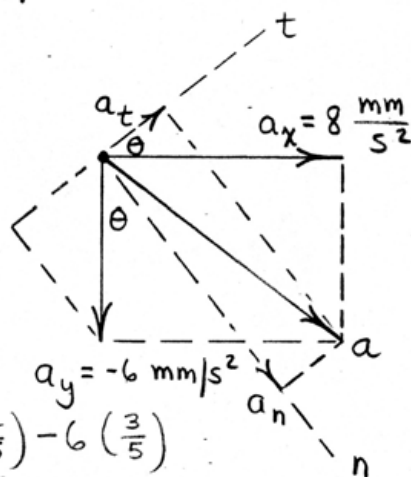
$$\dot{y} = 15 - 12 = 3 \frac{\text{mm}}{\text{s}}$$

$$\ddot{x} = 8 \text{ mm/s}^2$$

$$\ddot{y} = -6 \text{ mm/s}^2$$



$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$



$$a_t = 8 \cos \theta - 6 \sin \theta = 8\left(\frac{4}{5}\right) - 6\left(\frac{3}{5}\right) = 2.8 \text{ mm/s}^2$$

$$a_n = 6 \cos \theta + 8 \sin \theta = 6\left(\frac{4}{5}\right) + 8\left(\frac{3}{5}\right) = 9.6 \text{ mm/s}^2$$

$$a_n = \frac{v^2}{r}, \quad r = \frac{v^2}{a_n} = \frac{5^2}{9.6} = \underline{2.60 \text{ mm}}$$

2/133 $y - 10 = kx^2$ $k = -\frac{1}{10} \text{ in.}^{-1}$

So $y = 10(1 - \frac{x^2}{100}) \text{ in.}$

$\dot{x} = 15 \text{ in./sec}$, $\ddot{x} = 0$

$\dot{y} = -x\dot{x}/5$, $\ddot{y} = -\dot{x}^2/5$

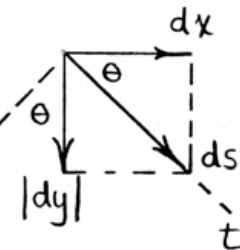
So $a = a_y = \ddot{y} = -\frac{15^2}{5} = -45 \text{ in./sec}^2$

$\frac{dy}{dx} = -\frac{x}{5} = -\frac{6}{5} \text{ @ } x = 6 \text{ in.}$

$\theta = \tan^{-1} \frac{6}{5} = 50.2^\circ$

$a_n = a \cos \theta = 45 \cos 50.2^\circ = 28.8 \text{ in./sec}^2$

$a_t = a \sin \theta = 45 \sin 50.2^\circ = 34.6 \text{ in./sec}^2$



For $x = 6 \text{ in.}$, $\dot{y} = -6(15)/5 = -18 \text{ in./sec}$

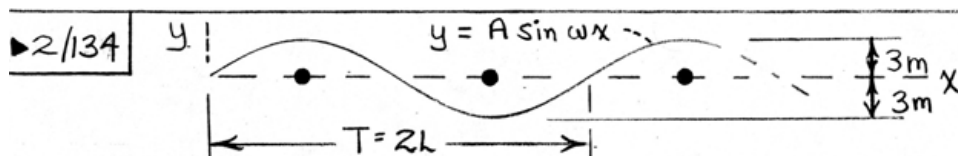
$v = \sqrt{15^2 + 18^2} = 23.4 \text{ in./sec}$

$a_n = \frac{v^2}{\rho}$, $\rho = \frac{23.4^2}{28.8} = 19.06 \text{ in.}$

Check : $\rho_{xy} = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2} = \frac{[1 + (-6/5)^2]^{3/2}}{-1/5}$

$= -19.06 \text{ in.} \checkmark$





$$y = A \sin \omega x, \text{ where } A = 3m \text{ \& } \omega = \frac{2\pi}{T}$$

$$\text{Radius of curvature } \rho = \frac{[1 + (\frac{dy}{dx})^2]^{3/2}}{d^2y/dx^2}$$

$$\frac{dy}{dx} = A\omega \cos \omega x, \quad \frac{d^2y}{dx^2} = -A\omega^2 \sin \omega x$$

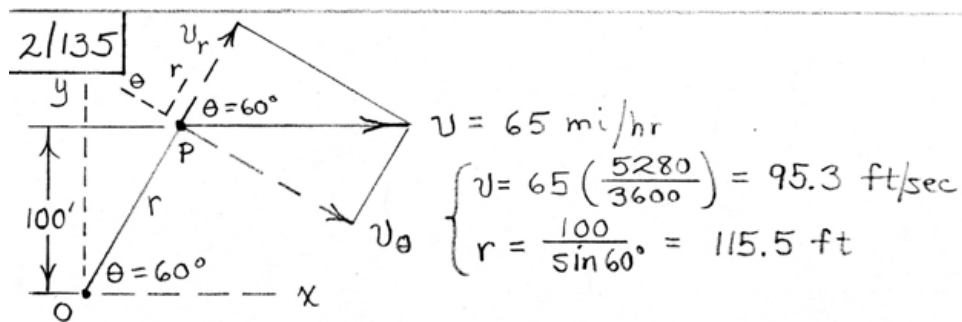
$$\text{Set } \frac{d\rho}{dx} = 0 \text{ to show that } |\rho| \text{ is a min @ } x = \frac{T}{4}$$

$$\& \ x = \frac{3T}{4}.$$

$$\rho_{\min} = \frac{[1 + \{A \frac{2\pi}{T} \cos(\frac{2\pi}{T} \cdot \frac{T}{4})\}^2]}{+ A (\frac{2\pi}{T})^2 \sin(\frac{2\pi}{T} \cdot \frac{T}{4})} = \frac{T^2}{4\pi^2 A}$$

$$a_n = \frac{v^2}{\rho} : -0.7(9.81) = \frac{(80/3.6)^2}{T^2/(4\pi^2 \cdot 3)}$$

$$T = 92.3 \text{ m} = 2L, \quad \underline{L = 46.1 \text{ m}}$$



$$v_r = \dot{r} = v \cos \theta = 95.3 \cos 60^\circ = \underline{47.7 \text{ ft/sec}}$$

$$v_\theta = r\dot{\theta} = -v \sin \theta$$

$$\dot{\theta} = -\frac{v \sin \theta}{r} = -\frac{95.3 \sin 60^\circ}{115.5} = -0.715 \text{ rad/sec}$$

$$\text{or } \dot{\theta} = -0.715 \left(\frac{180}{\pi} \right) = \underline{-41.0 \text{ deg/sec}}$$

$$\underline{2/136} \quad v_r = \dot{r} = 0.5 \text{ ft/sec}$$

$$v_\theta = r\dot{\theta} = 35 \left(2 \frac{\pi}{180} \right) = 1.222 \text{ ft/sec}$$

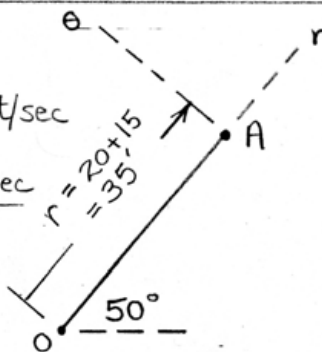
$$v = \sqrt{0.5^2 + 1.222^2} = \underline{1.320 \text{ ft/sec}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 35 \left(2 \frac{\pi}{180} \right)^2$$

$$= -0.0426 \text{ ft/sec}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(0.5) \left(2 \frac{\pi}{180} \right) = 0.0349 \frac{\text{ft}}{\text{sec}^2}$$

$$a = \sqrt{0.0426^2 + 0.0349^2} = \underline{0.0551 \text{ ft/sec}^2}$$



2/137

$$v_r = \dot{r} = 40 \text{ mm/s}$$

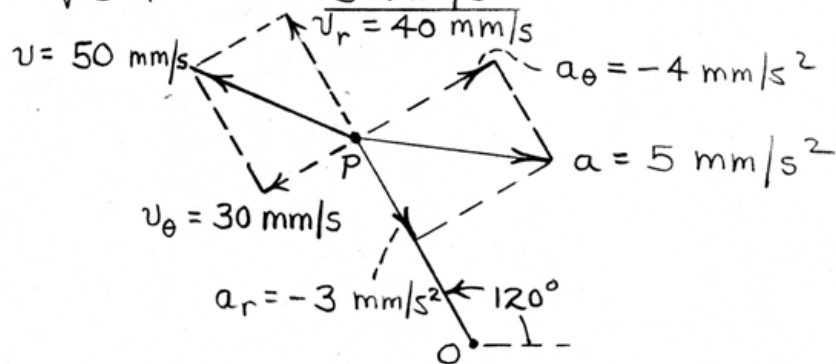
$$v_\theta = r\dot{\theta} = 300(0.1) = 30 \text{ mm/s}$$

$$v = \sqrt{40^2 + 30^2} = 50 \text{ mm/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300(0.1)^2 = -3 \text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(-0.04) + 2(40)(0.1) = -4 \text{ mm/s}^2$$

$$a = \sqrt{3^2 + 4^2} = 5 \text{ mm/s}^2$$



Position	r	$\dot{r}$	$\ddot{r}$	θ	$\dot{\theta}$	$\ddot{\theta}$
A	+	-	+	+	+	+
B	+	0	+	+	+	0
C	+	+	+	+	+	-

- Notes :
- (1) $r \geq 0$, always, by definition
 - (2) $\dot{r}$ determined by inspection
 - (3) $\ddot{r}$ found from $a_r = \ddot{r} - r\dot{\theta}^2 = 0$
 - (4) $\theta \geq 0$, by definition in figure
 - (5) $\dot{\theta} > 0$ here, by inspection
 - (6) $\ddot{\theta}$ found from $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$$\begin{aligned} \underline{v} &= \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta = 1.5 \underline{e}_r + (24+7)(5 \frac{\pi}{180}) \underline{e}_\theta \\ &= 1.5 \underline{e}_r + 2.71 \underline{e}_\theta \quad \text{ft/sec} \end{aligned}$$

$$\begin{aligned} \underline{a} &= (\ddot{r} - r \dot{\theta}^2) \underline{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \underline{e}_\theta \\ &= [-4 - 31(5 \frac{\pi}{180})^2] \underline{e}_r + [31(2 \frac{\pi}{180}) + 2(1.5)(5 \frac{\pi}{180})] \underline{e}_\theta \\ &= -4.24 \underline{e}_r + 1.344 \underline{e}_\theta \quad \text{ft/sec}^2 \end{aligned}$$

2/140 $\underline{\dot{r}} \neq \underline{v}$, $\underline{\dot{r}} \neq \underline{v}$, $\underline{\ddot{r}} \neq \underline{a}$, and $\underline{\ddot{r}} \neq \underline{a}$ because scalars are NEVER equal to vectors.

$\underline{\dot{r}} \neq \underline{v}$ because $\underline{v} \neq (v_r = \dot{r})$.

$\underline{\ddot{r}} \neq \underline{a}$ because $\underline{\ddot{r}}$ is only one part of the magnitude of a_r (and therefore of a).

$\underline{\dot{r}} \neq \dot{r} \underline{e}_r$ because $\underline{v} = \underline{\dot{r}}$ also contains an $\underline{e}_\theta$ -Component.

$\underline{\ddot{r}} \neq \ddot{r} \underline{e}_r$ because $\underline{a} = \underline{\ddot{r}}$ contains another $\underline{e}_r$ -Component and also an $\underline{e}_\theta$ -Component.

$\underline{\dot{r}} \neq r \dot{\theta} \underline{e}_\theta$ because $\underline{\dot{r}} = \underline{v}$ also contains an $\underline{e}_r$ -Component.

2/14/1

$$\textcircled{A} \quad \underline{v}_A = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta = \underline{v}_r + l \Omega \underline{e}_\theta$$

$$\underline{a}_A = (\ddot{r} - r \dot{\theta}^2) \underline{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \underline{e}_\theta$$
$$= -l \Omega^2 \underline{e}_r + 2v \Omega \underline{e}_\theta$$

$$\textcircled{B} \quad \underline{v}_B = 4v \underline{e}_r + 2l \Omega \underline{e}_\theta$$

$$\underline{a}_B = -2l \Omega^2 \underline{e}_r + 8v \Omega \underline{e}_\theta$$

$$\boxed{2/142} \quad r = 375 + 125 = 500 \text{ mm}, \quad \dot{r} = \dot{d} = -150 \frac{\text{mm}}{\text{s}}$$

$$\ddot{r} = 0, \quad \dot{\theta} = 60 \left(\frac{\pi}{180} \right) = \frac{\pi}{3} \text{ rad/s}, \quad \ddot{\theta} = 0$$

$$v_r = \dot{r} = -150 \frac{\text{mm}}{\text{s}}, \quad v_\theta = r\dot{\theta} = 500 \left(\frac{\pi}{3} \right) = 524 \frac{\text{mm}}{\text{s}}$$

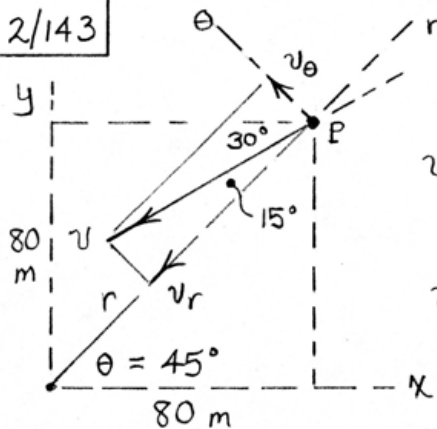
$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-150)^2 + (524)^2} = \underline{545 \frac{\text{mm}}{\text{s}}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 500 \left(\frac{\pi}{3} \right)^2 = -548 \text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-150) \left(\frac{\pi}{3} \right) = -314 \text{ mm/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-548)^2 + (-314)^2} = \underline{632 \text{ mm/s}^2}$$

2/143



$$\begin{cases} v = 100 \text{ m/s} \\ r = 80\sqrt{2} = 113.1 \text{ m} \end{cases}$$

$$v_r = \dot{r} = -v \cos 15^\circ = -100 \cos 15^\circ$$

$$= \underline{-96.6 \text{ m/s}}$$

$$v_\theta = r\dot{\theta} = v \sin 15^\circ$$

$$\dot{\theta} = \frac{100 \sin 15^\circ}{113.1}$$

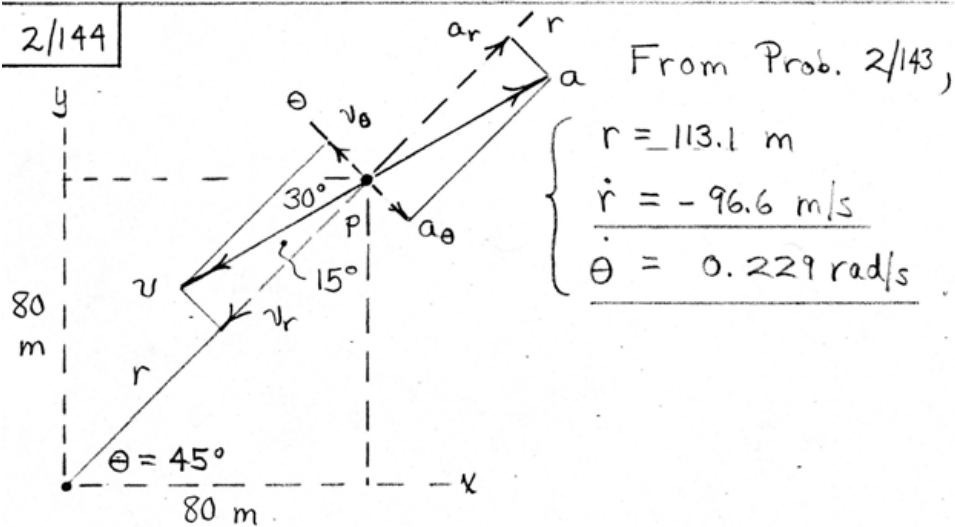
$$= \underline{0.229 \text{ rad/s}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0, \quad \ddot{r} = r\dot{\theta}^2 = 113.1(0.229)^2 = \underline{5.92 \frac{\text{m}}{\text{s}^2}}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0, \quad \ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r}$$

$$= -\frac{2(-96.6)(0.229)}{113.1} = \underline{-0.391 \frac{\text{rad}}{\text{s}^2}}$$

2/144



$$a_r = \ddot{r} - r\dot{\theta}^2 = 20 \cos 15^\circ$$

$$\ddot{r} = 113.1 (0.229)^2 + 20 \cos 15^\circ = \underline{25.2 \text{ m/s}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -20 \sin 15^\circ$$

$$\ddot{\theta} = \frac{-2(-96.6)(0.229) - 20 \sin 15^\circ}{113.1} = \underline{0.345 \text{ rad/s}^2}$$

2/145 From $\underline{a} = [\ddot{r} - r\dot{\theta}^2]\underline{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\underline{e}_\theta$

we have, for $\ddot{r} = \ddot{\theta} = 0$, $\dot{\theta} = \Omega$, and $r = l$:

$$a = \sqrt{(l\Omega^2)^2 + (2\dot{l}\Omega)^2} = \Omega \sqrt{l^2\Omega^2 + 4\dot{l}^2}$$

$$0.011 = 0.05 \sqrt{[4.2(0.05)]^2 + 4\dot{l}^2}$$

$$\text{Solve for } \dot{l} : \quad \underline{\dot{l} = 0.0328 \text{ m/s}}$$

$$\text{or } \underline{\dot{l} = 32.8 \text{ mm/s}}$$

2/146

$$\begin{cases} r = r_0 \cosh Kt \\ \dot{r} = r_0 K \sinh Kt \\ \ddot{r} = r_0 K^2 \cosh Kt \end{cases}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = r_0 K^2 \cosh Kt - (r_0 \cosh Kt) K^2 = 0$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2r_0 K \sinh Kt (K)$$

$$= 2r_0 K^2 \sinh Kt$$

$$\text{But } \cosh^2 Kt - \sinh^2 Kt = 1, \quad \sinh Kt = \sqrt{\cosh^2 Kt - 1}$$

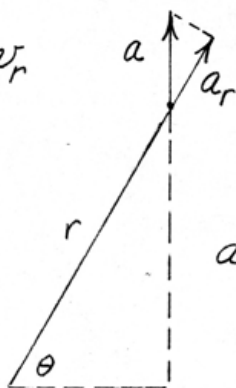
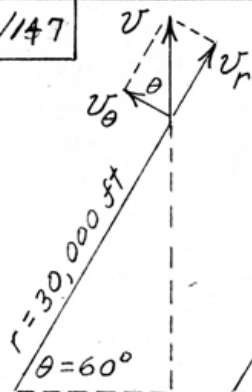
$$= \sqrt{\left(\frac{r}{r_0}\right)^2 - 1}$$

$$\text{When } r = R, \quad \sinh Kt = \sqrt{\left(\frac{R}{r_0}\right)^2 - 1}$$

So at the instant of leaving the vane,

$$a = a_\theta = 2r_0 K^2 \sqrt{\left(\frac{R}{r_0}\right)^2 - 1} = \underline{2K^2 \sqrt{R^2 - r_0^2}}$$

2/147



$$v_{\theta} = r\dot{\theta} = 30(10^3)(0.020)$$

$$= 600 \text{ ft/sec}$$

$$v = v_{\theta} / \cos 60^{\circ}$$

$$= 600 / 0.5 = \underline{1200 \text{ ft/sec}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

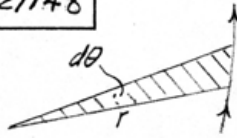
$$= 70 - 30(10^3)(0.020)^2$$

$$= 58 \text{ ft/sec}^2$$

$$a = a_r / \sin 60^{\circ}$$

$$= 58 / 0.866 = \underline{67.0 \text{ ft/sec}^2}$$

2/148



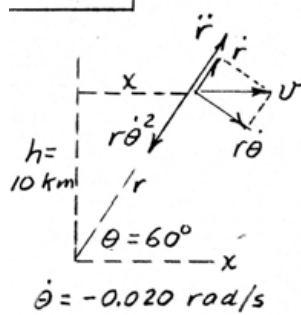
$$dA = \frac{1}{2} r d\theta (r) = \frac{1}{2} r^2 d\theta$$

$$\dot{A} = \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \dot{\theta} = \text{constant}$$

$$\text{Since } q_{\theta} = \frac{1}{r} \frac{d}{dt} (r^2 \dot{\theta}) = 0$$

2/149

Acceleration in all directions is zero, so



$$\dot{\theta} = -0.020 \text{ rad/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0, \quad \ddot{r} = r\dot{\theta}^2$$

$$r = h/\sin \theta, \quad r = \frac{10}{\sqrt{3}/2} = 11.55 \text{ km}$$

$$\ddot{r} = 11.55 (-0.020)^2 = 0.00462 \text{ km/s}^2$$

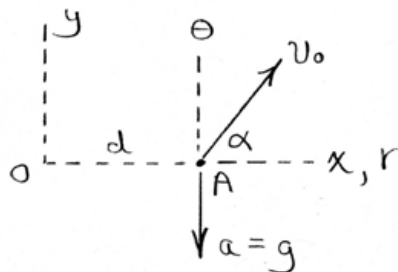
$$= 4.62 \text{ m/s}^2$$

$$v = |r\dot{\theta}|/\sin \theta = h\dot{\theta}/\sin^2 \theta$$

$$= \frac{10(-0.020)}{(\sqrt{3}/2)^2} = 0.267 \text{ km/s}$$

$$\text{or } v = 0.267 (3600) = \underline{960 \text{ km/h}}$$

2/150



$$\underline{r = d, \quad \theta = 0}$$

$$v_r = \dot{r} = v_0 \cos \alpha$$

$$v_\theta = r\dot{\theta} = v_0 \sin \alpha,$$

$$\dot{\theta} = \frac{v_0 \sin \alpha}{d}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0,$$

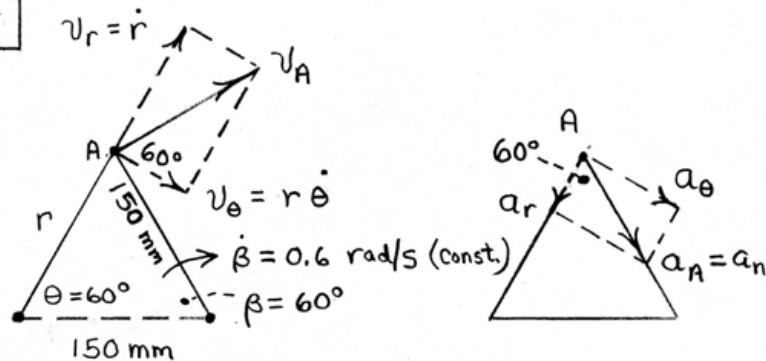
$$\ddot{r} = r\dot{\theta}^2 = \frac{v_0^2 \sin^2 \alpha}{d}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -g, \quad \ddot{\theta} = -\frac{2\dot{r}\dot{\theta} + g}{r}$$

$$\text{or } \ddot{\theta} = -\frac{2(v_0 \cos \alpha)\left(\frac{v_0 \sin \alpha}{d}\right) + g}{d}$$

$$= -\frac{1}{d} \left(\frac{2v_0^2}{d} \cos \alpha \sin \alpha + g \right)$$

2/151



For $\beta = 60^\circ$, $\theta = 60^\circ$, $r = 150 \text{ mm}$

$$v_A = 150(0.6) = 90 \text{ mm/s}$$

$$v_\theta = r\dot{\theta} = -v_A \cos 60^\circ, \quad \dot{\theta} = \frac{-90 \cos 60^\circ}{150} = -0.3 \frac{\text{rad}}{\text{s}}$$

$$v_r = \dot{r} = v_A \sin 60^\circ = 90 \sin 60^\circ = \underline{77.9 \text{ mm/s}}$$

$$a_A = a_n = 150(0.6)^2 = 54 \text{ mm/s}^2$$

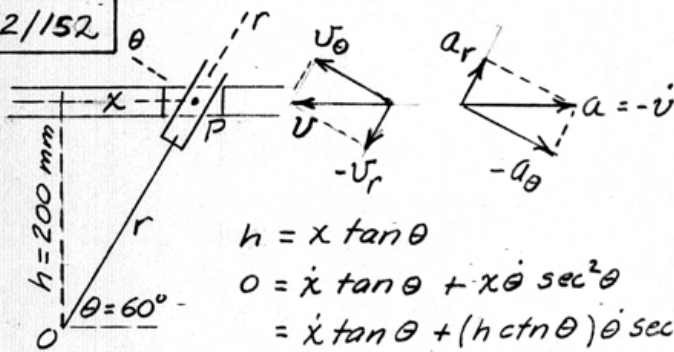
$$a_r = \ddot{r} - r\dot{\theta}^2 : -54 \cos 60^\circ = \ddot{r} - 150(-0.3)^2$$

$$\ddot{r} = -13.5 \text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : -54 \sin 60^\circ = 150\ddot{\theta} + 2(77.9)(-0.3)$$

$$\underline{\ddot{\theta} = 0}$$

2/152



$$h = x \tan \theta$$

$$0 = \dot{x} \tan \theta + x \dot{\theta} \sec^2 \theta$$

$$= \dot{x} \tan \theta + (h \cot \theta) \dot{\theta} \sec^2 \theta$$

$$v = -\dot{x} = h \dot{\theta} \cot \theta \sec^2 \theta / \tan \theta$$

$$= h \dot{\theta} \csc^2 \theta = 200 (2) \left(\frac{2}{\sqrt{3}} \right)^2 = 533 \text{ mm/s}$$

$$v_r = -v \cos \theta = -533 (1/2) = -267 \text{ mm/s}$$

$$-a = \dot{v} = h \dot{\theta}^2 \csc \theta (-\cot \theta \csc \theta) \dot{\theta} = -2h \dot{\theta}^2 \cot \theta \csc^2 \theta$$

$$a = 2(200) 2^2 \frac{1}{\sqrt{3}} \left(\frac{2}{\sqrt{3}} \right)^2 = 1232 \text{ mm/s}^2$$

$$a_r = a \cos \theta = 1232 (1/2) = 616 \text{ mm/s}^2$$

(Alternatively obtain $\dot{r} = v_r$ & $\ddot{r}$ from $r = h \csc \theta$)

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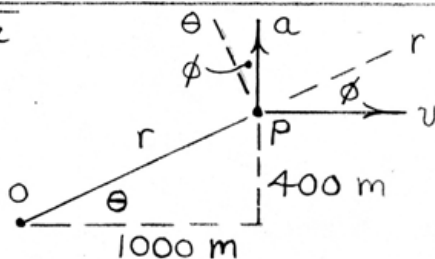
$$r = \sqrt{1000^2 + 400^2}$$

$$= 1077 \text{ m}$$

$$\theta = \tan^{-1} \frac{400}{1200} = 21.8^\circ$$

$$v = \frac{600}{3.6} = 166.7 \text{ m/s}$$

$$a = a_n = \frac{v^2}{r} = \frac{166.7^2}{1200} = 23.1 \text{ m/s}^2$$



$$v_r = \dot{r} = v \cos \theta = 166.7 \cos 21.8^\circ = 154.7 \text{ m/s}$$

$$v_\theta = r\dot{\theta} : -166.7 \sin 21.8^\circ = 1077\dot{\theta}, \dot{\theta} = -0.0575 \frac{\text{rad}}{\text{s}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 : 23.1 \sin 21.8^\circ = \ddot{r} - 1077(-0.0575)^2$$

$$\ddot{r} = 12.15 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 23.1 \cos 21.8^\circ = 1077\ddot{\theta} + 2(154.7)(-0.0575)$$

$$\ddot{\theta} = 0.0365 \text{ rad/s}^2$$

2/154

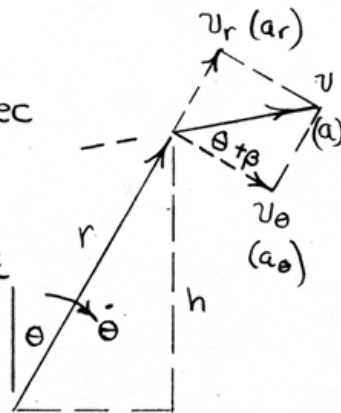
$$\dot{\theta} = 2.20 \left(\frac{\pi}{180} \right) = 0.0384 \text{ rad/sec}$$

$$v_r = \dot{r} = 360 \text{ ft/sec}$$

$$v_\theta = r\dot{\theta} = 12,000(0.0384) = 461 \frac{\text{ft}}{\text{sec}}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = 585 \text{ ft/sec}$$

$$\text{or } v = \underline{399 \text{ mi/hr}}$$



$$30 + \beta = \tan^{-1} \frac{360}{461} = 38.0^\circ, \quad \underline{\beta = 8.00^\circ}$$

$$h = r \cos 30^\circ = 12,000 \cos 30^\circ = \underline{10,390 \text{ ft}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 19.60 - 12,000(0.0384)^2 = 1.908 \frac{\text{ft}}{\text{sec}^2}$$

$$a = \frac{a_r}{\sin(\theta + \beta)} = \frac{1.908}{\sin 38.0^\circ} = \underline{3.10 \text{ ft/sec}^2}$$

$$a_\theta = \frac{a_r}{\tan(\theta + \beta)} = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

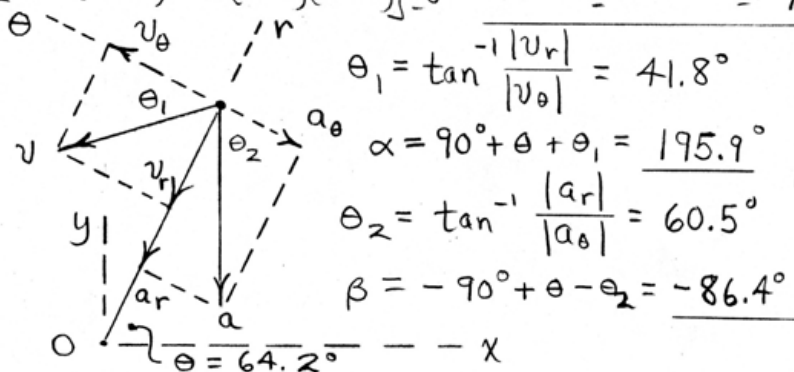
$$\frac{1.908}{\tan 38.0^\circ} = 12,000 \ddot{\theta} + 2(360)(0.0384), \quad \underline{\ddot{\theta} = -0.00210 \frac{\text{rad}}{\text{sec}^2}}$$

$$\begin{array}{l|l} 2/155 \quad \theta = 0.4 + 0.12t + 0.06t^3 & r = 0.8 - 0.1t - 0.05t^2 \\ \dot{\theta} = 0.12 + 0.18t^2 & \dot{r} = -0.1 - 0.1t \\ \ddot{\theta} = 0.36t & \ddot{r} = -0.1 \end{array}$$

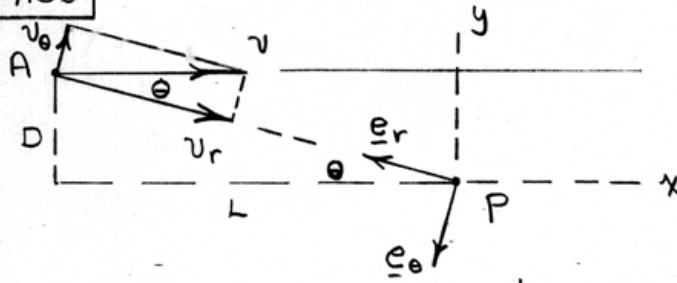
$$\text{At } t = 2 \text{ s : } \begin{cases} \theta = 1.12 \text{ rad} \\ \dot{\theta} = 0.84 \text{ rad/s} \\ \ddot{\theta} = 0.72 \text{ rad/s}^2 \end{cases} \quad \begin{cases} r = 0.4 \text{ m} \\ \dot{r} = -0.3 \text{ m/s} \\ \ddot{r} = -0.1 \text{ m/s}^2 \end{cases}$$

$$\begin{aligned} \underline{v} &= \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = -0.3\underline{e}_r + 0.4(0.84)\underline{e}_\theta \\ &= -0.3\underline{e}_r + 0.336\underline{e}_\theta \text{ m/s} \end{aligned}$$

$$\begin{aligned} \underline{a} &= (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta = [-0.1 - 0.4(0.84)^2]\underline{e}_r \\ &+ [0.4(0.72) + 2(-0.3)(0.84)]\underline{e}_\theta = -0.382\underline{e}_r - 0.216\underline{e}_\theta \text{ m/s}^2 \end{aligned}$$



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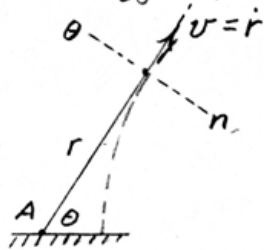
$$v' = |v_r| = v \cos \theta = v \frac{L}{\sqrt{L^2 + D^2}}$$

$$\text{Numbers: } v' = 70 \frac{500}{\sqrt{500^2 + 20^2}} = \underline{69.9 \text{ mi/hr}}$$

The factor of $\cos \theta$ is the basis for the statement that, kinematically, radar can yield an accurate or low, but not high, speed measurement. As can be seen, however, adherence to the speed limit (not reliance upon $\cos \theta$) is the best policy!

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Radial line r must be tangent to trajectory for $\dot{\theta} = 0$. Thus $+\theta$ direction is in the opposite sense to the normal n -direction of the curve.



$$r = 35,000 \text{ ft}, \dot{r} = 1600 \text{ ft/sec}$$

$$\dot{\theta} = 0, \ddot{\theta} = -7.20(10^{-3}) \text{ rad/sec}^2$$

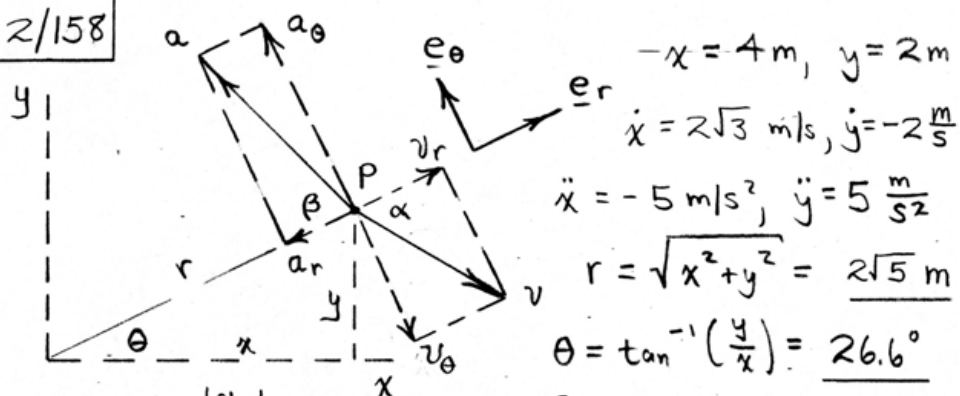
$$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 35,000(-7.20)(10^{-3}) + 0$$

$$= -252 \text{ ft/sec}^2$$

$$-a_{\theta} = a_n = v^2/\rho, \quad \rho = v^2/-a_{\theta}$$

$$= \frac{(1600)^2}{252} = 10.16(10^3) \text{ ft}$$

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$$\alpha = \tan^{-1} \frac{|v_y|}{|v_x|} + \theta = \tan^{-1} \frac{2}{2\sqrt{3}} + 26.6^\circ = 56.6^\circ$$

$$\beta = \tan^{-1} \left| \frac{a_y}{a_x} \right| + \theta = \tan^{-1} \frac{5}{5} + 26.6^\circ = 71.6^\circ$$

$$v = \sqrt{v_y^2 + v_x^2} = \sqrt{2^2 + (2\sqrt{3})^2} = 4 \text{ m/s}$$

$$a = \sqrt{a_y^2 + a_x^2} = \sqrt{5^2 + 5^2} = 7.07 \text{ m/s}^2$$

$$\dot{r} = v_r = v \cos \alpha = 4 \cos 56.6^\circ = 2.20 \text{ m/s}$$

$$v_\theta = -v \sin \alpha = -4 \sin 56.6^\circ = -3.34 \text{ m/s}$$

$$v_\theta = r \dot{\theta} : -3.34 = 2\sqrt{5} \dot{\theta}, \quad \dot{\theta} = -0.746 \text{ rad/s}$$

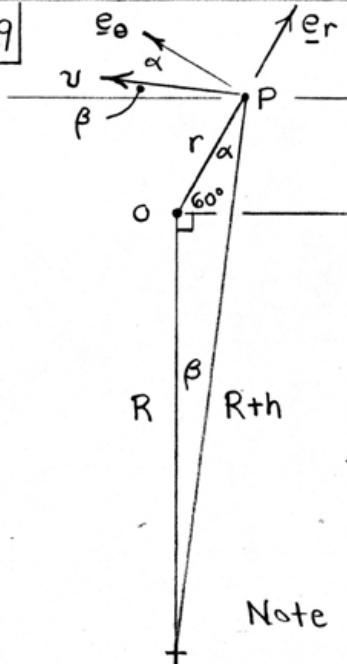
$$a_r = -a \cos \beta = -7.07 \cos 71.6^\circ = -2.24 \text{ m/s}^2$$

$$a_r = \ddot{r} - r \dot{\theta}^2 : -2.24 = \ddot{r} - 2\sqrt{5} (0.746)^2, \quad \ddot{r} = 0.255 \frac{\text{m}}{\text{s}^2}$$

$$a_\theta = a \sin \beta = 7.07 \sin 71.6^\circ = 6.71 \text{ m/s}^2$$

$$a_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} : 6.71 = 2\sqrt{5} \ddot{\theta} + 2(2.20)(-0.746), \quad \ddot{\theta} = 2.24 \frac{\text{rad}}{\text{s}^2}$$

2/159



$$\frac{\sin 150^\circ}{R+h} = \frac{\sin \alpha}{R}$$

$$\frac{\sin 150^\circ}{3959+150} = \frac{\sin \alpha}{3959}$$

$$\alpha = 28.8^\circ$$

$$\alpha + \beta + 150^\circ = 180^\circ \Rightarrow \beta = 1.200^\circ$$

$$v_r = \dot{r} = -12,272 = -v \sin \alpha$$

$$-12,272 = -v \sin 28.8^\circ$$

$$v = \underline{25,474 \text{ ft/sec}}$$

Note: Because v is nearly parallel to the horizontal at O ($\beta = 1.200^\circ$), one obtains a close approximation ($v = 24,544 \frac{\text{ft}}{\text{sec}}$) to the correct answer by neglecting β (assuming a flat earth).

$$2/160 \quad r = r_0 + b_0 \sin 2\pi n t, \quad \dot{r} = 2\pi n b_0 \cos 2\pi n t$$

$$\ddot{r} = -4\pi^2 b_0 n^2 \sin 2\pi n t$$

$$\dot{\theta} = \omega, \quad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \quad a_r = -4\pi^2 b_0 n^2 \sin 2\pi n t - (r_0 + b_0 \sin 2\pi n t)\omega^2$$

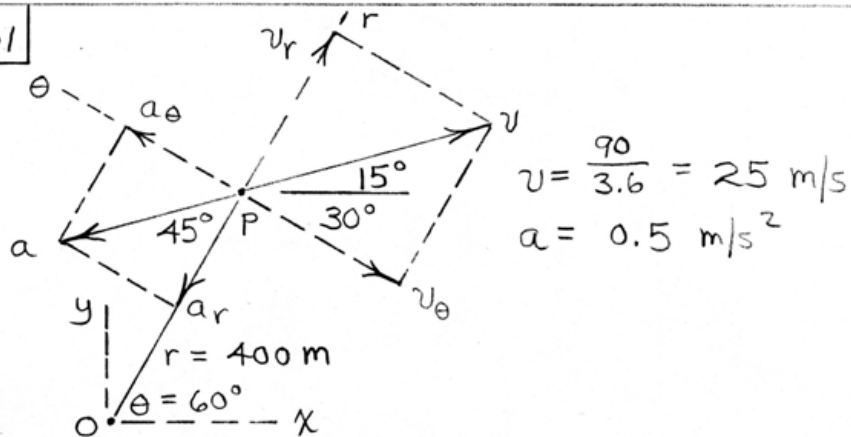
$$= -(4\pi^2 n^2 + \omega^2)b_0 \sin 2\pi n t - r_0 \omega^2$$

$$|a_r|_{\max} = \underline{(4\pi^2 n^2 + \omega^2)b_0 + r_0 \omega^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}, \quad a_\theta = 0 + 4\pi b_0 n \omega \cos 2\pi n t$$

$$|a_\theta|_{\max} = \underline{4\pi b_0 n \omega}$$

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$$v_r = \dot{r} = v \sin 45^\circ = 25 \frac{\sqrt{2}}{2} = 17.68 \text{ m/s}$$

$$v_\theta = r\dot{\theta} : -25 \cos 45^\circ = 400\dot{\theta}, \quad \dot{\theta} = -0.0442 \text{ rad/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 : -0.5 \cos 45^\circ = \ddot{r} - 400(-0.0442)^2$$

$$\ddot{r} = 0.428 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 0.5 \sin 45^\circ = 400\ddot{\theta} + 2(17.68)(-0.0442)$$

$$\ddot{\theta} = 0.00479 \text{ rad/s}^2$$

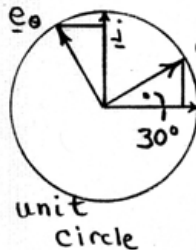
$$\begin{cases} r = 0.75 + 0.5 = 1.25 \text{ m} & \theta = 30^\circ \\ \dot{r} = 0.2 \text{ m/s} & \dot{\theta} = 0.1745 \frac{\text{rad}}{\text{s}} \\ \ddot{r} = -0.3 \text{ m/s}^2 & \ddot{\theta} = 0 \end{cases}$$

$$\begin{aligned} \underline{v} &= v_r \underline{e}_r + v_\theta \underline{e}_\theta = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta \\ &= 0.2 \underline{e}_r + 1.25(0.1745) \underline{e}_\theta = 0.2 \underline{e}_r + 0.218 \underline{e}_\theta \frac{\text{m}}{\text{s}} \end{aligned}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \underline{0.296 \text{ m/s}}$$

$$\begin{aligned} \underline{a} &= a_r \underline{e}_r + a_\theta \underline{e}_\theta = (\ddot{r} - r \dot{\theta}^2) \underline{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \underline{e}_\theta \\ &= [-0.3 - 1.25(0.1745)^2] \underline{e}_r + [1.25(0) + 2(0.2)(0.1745)] \underline{e}_\theta \\ &= -0.338 \underline{e}_r + 0.0698 \underline{e}_\theta \text{ m/s}^2 \end{aligned}$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \underline{0.345 \text{ m/s}^2}$$



$$\underline{e}_r = \underline{i} \cos 30^\circ + \underline{j} \sin 30^\circ$$

$$\underline{e}_\theta = -\underline{i} \sin 30^\circ + \underline{j} \cos 30^\circ$$

$$\begin{aligned} \underline{v} &= 0.2 [\underline{i} \cos 30^\circ + \underline{j} \sin 30^\circ] + 0.218 [-\underline{i} \sin 30^\circ + \underline{j} \cos 30^\circ] \\ &= \underline{0.064 \underline{i} + 0.289 \underline{j} \text{ m/s}} \end{aligned}$$

$$\begin{aligned} \underline{a} &= -0.338 [\underline{i} \cos 30^\circ + \underline{j} \sin 30^\circ] + 0.0698 [-\underline{i} \sin 30^\circ + \underline{j} \cos 30^\circ] \\ &= \underline{-0.328 \underline{i} - 0.1086 \underline{j} \text{ m/s}^2} \end{aligned}$$

$$\boxed{2/163} \quad r = \overline{BD} = 2R \sin \frac{\theta}{2}, \quad \dot{r} = R\dot{\theta} \cos \frac{\theta}{2}$$

$$\ddot{r} = -\frac{R}{2} \dot{\theta}^2 \sin \frac{\theta}{2}$$

For $\theta = 30^\circ$:

$$\begin{cases} r = 2(15) \sin 15^\circ = 7.76 \text{ in.} \\ \dot{r} = 15(4) \cos 15^\circ = 58.0 \text{ in./sec} \\ \ddot{r} = -\frac{15}{2} 4^2 \sin 15^\circ = -31.1 \text{ in./sec}^2 \end{cases}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -31.1 - 7.76(4)^2 = -155.3 \text{ in./sec}^2$$

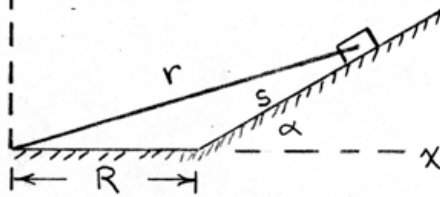
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(58.0)(4) = 464 \text{ in./sec}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \underline{489 \text{ in./sec}^2}$$

2/164

$$x = R + s \cos \alpha = R + (s_0 + v_0 t + \frac{1}{2} a t^2) \cos \alpha$$

y



$$= R + \frac{1}{2} a t^2 \cos \alpha$$

$$y = s \sin \alpha$$

$$= \frac{1}{2} a t^2 \sin \alpha$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(R + \frac{1}{2} a t^2 \cos \alpha)^2 + (\frac{1}{2} a t^2 \sin \alpha)^2}$$

$$= \sqrt{R^2 + R a t^2 \cos \alpha + \frac{1}{4} a^2 t^4}$$

$$\dot{r} = \frac{1}{2} (R^2 + R a t^2 \cos \alpha + \frac{1}{4} a^2 t^4)^{-1/2} [2 R a t \cos \alpha + a^2 t^3]$$

$$= \frac{\frac{1}{2} a t (2 R \cos \alpha + a t^2)}{\sqrt{R^2 + R a t^2 \cos \alpha + \frac{1}{4} a^2 t^4}}$$

2/165

From the solution to Prob. 2/164 :

$$x = R + \frac{1}{2}at^2 \cos \alpha$$

$$y = \frac{1}{2}at^2 \sin \alpha$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right) = \tan^{-1} \left[\frac{\frac{1}{2}at^2 \sin \alpha}{R + \frac{1}{2}at^2 \cos \alpha} \right]$$

$$\dot{\theta} = \frac{\frac{(R + \frac{1}{2}at^2 \cos \alpha)(at \sin \alpha) - (\frac{1}{2}at^2 \sin \alpha)(at \cos \alpha)}{(R + \frac{1}{2}at^2 \cos \alpha)^2}}{1 + \left[\frac{\frac{1}{2}at^2 \sin \alpha}{R + \frac{1}{2}at^2 \cos \alpha} \right]^2}$$

Simplify to

$$\dot{\theta} = \frac{Rat \sin \alpha}{R^2 + Rat^2 \cos \alpha + \frac{1}{4}a^2 t^4}$$

$$\begin{aligned}
 2/166 \quad r &= 1.6 + 0.3 \sin \frac{\pi t}{2} \text{ m} ; & \theta &= \frac{\pi}{4} + \frac{\pi}{8} \sin \frac{\pi t}{2} \\
 \dot{r} &= \frac{0.3\pi}{2} \cos \frac{\pi t}{2} \text{ m/s} ; & \dot{\theta} &= \frac{\pi^2}{16} \cos \frac{\pi t}{2} \text{ rad/s} \\
 \ddot{r} &= -\frac{0.3\pi^2}{4} \sin \frac{\pi t}{2} \text{ m/s}^2 ; & \ddot{\theta} &= -\frac{\pi^3}{32} \sin \frac{\pi t}{2} \text{ rad/s}^2
 \end{aligned}$$

$$v_r = \dot{r} = \frac{0.3\pi}{2} \cos \frac{\pi t}{2}$$

$$v_\theta = r\dot{\theta} = (1.6 + 0.3 \sin \frac{\pi t}{2}) \left(\frac{\pi^2}{16} \cos \frac{\pi t}{2} \right)$$

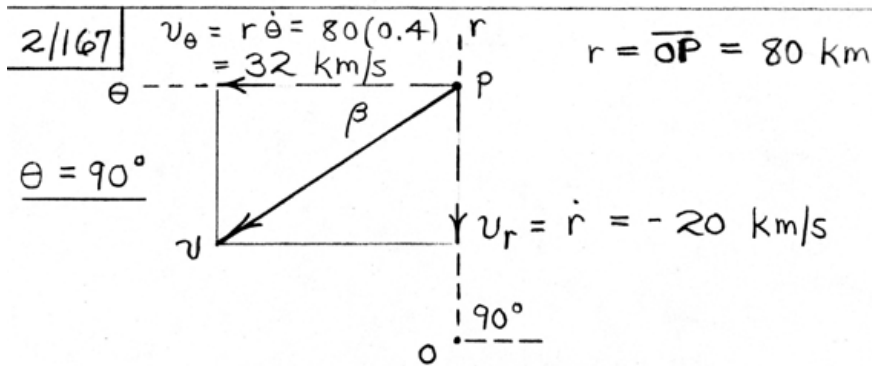
$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{0.3\pi^2}{4} \sin \frac{\pi t}{2} - (1.6 + 0.3 \sin \frac{\pi t}{2}) \left(\frac{\pi^2}{16} \cos \frac{\pi t}{2} \right)^2$$

$$\begin{aligned}
 a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} &= (1.6 + 0.3 \sin \frac{\pi t}{2}) \left(-\frac{\pi^3}{32} \sin \frac{\pi t}{2} \right) \\
 &\quad + 2 \left(\frac{0.3\pi}{2} \cos \frac{\pi t}{2} \right) \left(\frac{\pi^2}{16} \cos \frac{\pi t}{2} \right)
 \end{aligned}$$

$$\text{At } t=1\text{ s: } \left. \begin{aligned} v_r &= 0 \\ v_\theta &= 0 \end{aligned} \right\} v = \sqrt{v_r^2 + v_\theta^2} = \underline{0}$$

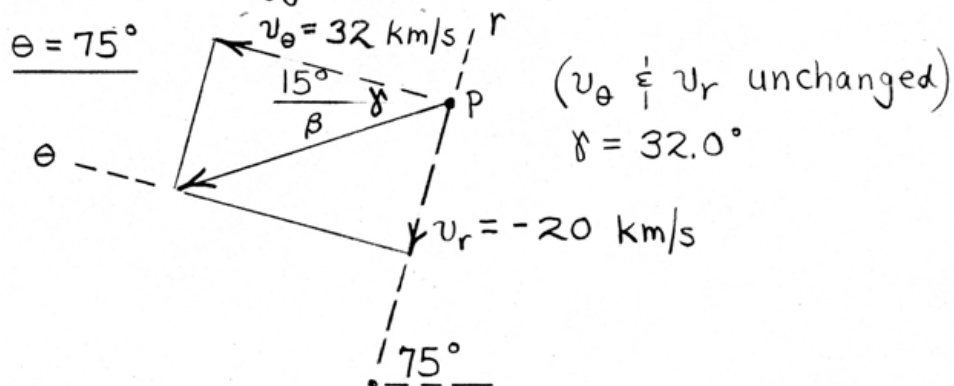
$$\left. \begin{aligned} a_r &= -0.740 \text{ m/s}^2 \\ a_\theta &= -1.841 \text{ m/s}^2 \end{aligned} \right\} a = \sqrt{0.740^2 + 1.841^2} \\
 &= \underline{1.984 \text{ m/s}^2}$$

$$\begin{aligned}
 \text{At } t=2\text{ s: } \left. \begin{aligned} v_r &= -0.471 \text{ m/s} \\ v_\theta &= 0.987 \text{ m/s} \end{aligned} \right\} v &= \underline{1.094 \text{ m/s}} \\
 \left. \begin{aligned} a_r &= -0.609 \text{ m/s}^2 \\ a_\theta &= 0.581 \text{ m/s}^2 \end{aligned} \right\} a &= \underline{0.842 \text{ m/s}^2}
 \end{aligned}$$



$$v = \sqrt{v_\theta^2 + v_r^2} = \sqrt{32^2 + 20^2} = \underline{37.7 \text{ km/s}}$$

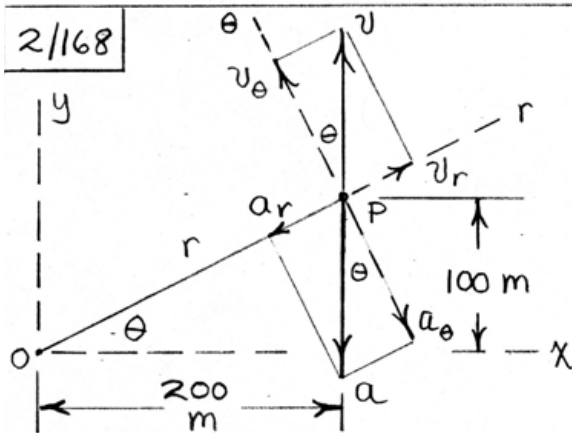
$$\beta = \tan^{-1} \frac{|v_r|}{v_\theta} = \tan^{-1} \frac{20}{32} = \underline{32.0^\circ}$$



$$v = 37.7 \text{ km/s} \text{ (unchanged)}$$

$$\beta = \gamma - 15^\circ = 32.0 - 15^\circ = \underline{17.01^\circ}$$

2/168



$$r = \sqrt{200^2 + 100^2}$$

$$= 224 \text{ m}$$

$$\theta = \tan^{-1} \frac{100}{200}$$

$$= 26.6^\circ$$

$$v_r = \dot{r} = v \sin \theta = 15 \sin 26.6^\circ = \underline{6.71 \text{ m/s}}$$

$$v_\theta = r\dot{\theta}: 15 \cos 26.6^\circ = 224\dot{\theta}, \dot{\theta} = \underline{0.06 \text{ rad/s}}$$

$$a = -g - kv^2 = -9.81 - 0.01(15)^2 = -12.06 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2: -12.06 \sin 26.6^\circ = \ddot{r} - 224(0.06)^2$$

$$\ddot{r} = \underline{-4.59 \text{ m/s}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}: -12.06 \cos 26.6^\circ = 224\ddot{\theta} + 2(6.71)(0.06)$$

$$\ddot{\theta} = \underline{-0.0518 \text{ rad/s}^2}$$

2/169

$$v = 12,149 \left(\frac{5280}{3600} \right) = 17,819 \frac{\text{ft}}{\text{sec}}$$

$$v_{\theta} = r\dot{\theta} : 17,819 \cos 30^{\circ} = 8400(5280)\dot{\theta}$$

$$\dot{\theta} = 3.48(10^{-4}) \text{ rad/sec}$$

$$v_r = \dot{r} : 17,819 \sin 30^{\circ} = \dot{r}$$

$$\dot{r} = 8910 \frac{\text{ft}}{\text{sec}}$$

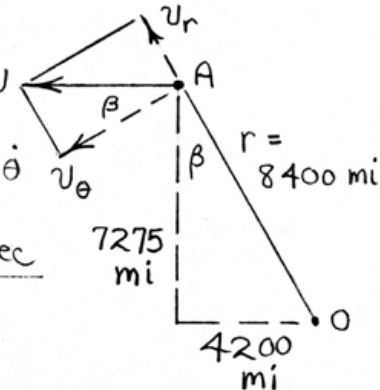
$$\beta = \tan^{-1} \left(\frac{4200}{7275} \right) = 30^{\circ}$$

$$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 0 = 8400(5280)\ddot{\theta} + 2(8910)(3.48)(10^{-4})$$

$$\ddot{\theta} = -1.398(10^{-7}) \text{ rad/sec}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 : -7.159 = \ddot{r} - 8400(5280)(3.48 \times 10^{-4})^2$$

$$\ddot{r} = -1.790 \text{ ft/sec}^2$$



2/170

$x = x_0 + v_{x_0} t$
 $= 0 + 100 \cos 30^\circ (0.5)$
 $= 43.3 \text{ ft}$

$\dot{x} = v_{x_0} = 100 \cos 30^\circ = 86.6 \frac{\text{ft}}{\text{sec}}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 = 6 + 100 \sin 30^\circ (0.5) - 16.1 (0.5)^2 = 27.0 \text{ ft}$

$\dot{y} = v_{y_0} - g t = 100 \sin 30^\circ - 32.2 (0.5) = 33.9 \text{ ft/sec}$

$r = \sqrt{x^2 + y^2} = \sqrt{43.3^2 + 27.0^2} = 51.0 \text{ ft}$

$\theta = \tan^{-1} \left(\frac{y}{x} \right) = \tan^{-1} (27.0/43.3) = 31.9^\circ$

$\alpha = \tan^{-1} (v_y/v_x) = \tan^{-1} (33.9/86.6) = 21.4^\circ$

$\beta = \theta - \alpha = 10.54^\circ, v = \sqrt{\dot{x}^2 + \dot{y}^2} = 93.0 \text{ ft/sec}$

$v_r = v \cos \beta = 93.0 \cos 10.54^\circ = 91.4 \text{ ft/sec} = \dot{r}$

$v_\theta = -v \sin \beta = -93.0 \sin 10.54^\circ = -17.02 \text{ ft/sec}$

$-17.02 = r \dot{\theta} = 51.0 \dot{\theta}, \quad \dot{\theta} = -0.334 \text{ rad/sec}$

$a = \ddot{y} = -32.2 \text{ ft/sec}^2$

$a_r = -g \sin \theta = -32.2 \sin 31.9^\circ = -17.03 \text{ ft/sec}^2$

$-17.03 = \ddot{r} - r \dot{\theta}^2 = \ddot{r} - 51.0 (-0.334)^2, \quad \ddot{r} = -11.35 \text{ ft/sec}^2$

$a_\theta = -g \cos \theta = -32.2 \cos 31.9^\circ = -27.3 \text{ ft/sec}^2$

$-27.3 = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 51.0 \ddot{\theta} + 2(91.4)(-0.334)$

$\ddot{\theta} = 0.660 \text{ rad/sec}^2$

$$\underline{2/171} \left\{ \begin{array}{l} x = .30 \cos 2t; \quad y = 40 \sin 2t; \quad z = 20t + 3t^2 \\ \dot{x} = -60 \sin 2t; \quad \dot{y} = 80 \cos 2t; \quad \dot{z} = 20 + 6t \\ \ddot{x} = -120 \cos 2t; \quad \ddot{y} = -160 \sin 2t; \quad \ddot{z} = 6 \end{array} \right.$$

At $t = 2 \text{ s}$:

$$\left\{ \begin{array}{l} x = -19.61 \text{ mm} \quad ; \quad y = -30.3 \text{ mm} \quad ; \quad z = 52 \text{ mm} \\ \dot{x} = 45.4 \text{ mm/s} \quad ; \quad \dot{y} = -52.3 \text{ mm/s} \quad ; \quad \dot{z} = 32 \text{ mm/s} \\ \ddot{x} = 78.4 \text{ mm/s}^2 \quad ; \quad \ddot{y} = 121.1 \text{ mm/s}^2 \quad ; \quad \ddot{z} = 6 \text{ mm/s}^2 \end{array} \right.$$

$$r = (x^2 + y^2 + z^2)^{1/2} = 63.3 \text{ mm} \quad \parallel \quad a = (\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2)^{1/2}$$

$$v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} = 76.3 \text{ mm/s} \quad \parallel \quad = 144.4 \text{ mm/s}^2$$

$$\theta_1 = \cos^{-1} \left[\frac{\underline{r} \cdot \underline{v}}{rv} \right] = \cos^{-1} \left[\frac{-19.61(45.4) - 30.3(-52.3) + 52(32)}{(63.3)(76.3)} \right]$$

$$= \underline{60.8^\circ}$$

$$\theta_2 = \cos^{-1} \left[\frac{\underline{r} \cdot \underline{a}}{ra} \right] = \cos^{-1} \left[\frac{-19.61(78.4) - 30.3(121.1) - 52(6)}{(63.3)(144.4)} \right]$$

$$= \underline{122.4^\circ}$$

$$\underline{2/172} \quad v_{z_0} = 600 \sin 60^\circ = 520 \text{ ft/sec}$$

$$v_{xy_0} = 600 \cos 60^\circ = 300 \text{ ft/sec}$$

$$v_z = v_{z_0} - gt = 520 - 32.2(20) = \underline{-124.4 \text{ ft/sec}}$$

$$v_{xy} = v_{xy_0} = 300 \text{ ft/sec} = \text{constant}$$

$$v_x = -v_{xy} \sin 20^\circ = -300 \sin 20^\circ = \underline{-102.6 \text{ ft/sec}}$$

$$v_y = v_{xy} \cos 20^\circ = 300 \cos 20^\circ = \underline{282 \text{ ft/sec}}$$

$$d_{xy} = v_{xy} t = 300(20) = 6000 \text{ ft}$$

$$x = -d_{xy} \sin 20^\circ = -6000 \sin 20^\circ = \underline{-2050 \text{ ft}}$$

$$y = d_{xy} \cos 20^\circ = 6000 \cos 20^\circ = \underline{5640 \text{ ft}}$$

$$z = v_{z_0} t - \frac{1}{2} g t^2 = 520(20) - 16.1(20)^2 = \underline{3950 \text{ ft}}$$

$$\underline{a_x = a_y = 0, \quad a_z = -g = -32.2 \text{ ft/sec}^2}$$

$$\underline{2/173} \quad \underline{v} = 4\underline{i} - 2\underline{j} - \underline{k} \text{ m/s}, \quad v = \sqrt{4^2 + 2^2 + 1^2} = 4.58 \frac{\text{m}}{\text{s}}$$

$$a_n = a \sin 20^\circ = 8 \sin 20^\circ = 2.74 \text{ m/s}^2$$

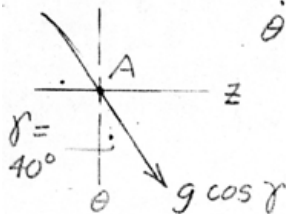
$$\text{From } a_n = \frac{v^2}{r}, \quad r = \frac{v^2}{a_n} = \frac{4.58^2}{2.74} = 7.67 \text{ m}$$

$$\dot{v} = a_t = a \cos 20^\circ = 8 \cos 20^\circ = \underline{7.52 \text{ m/s}^2}$$

2/174

$$v_{\theta} = r\dot{\theta} \quad \& \quad v_{\theta} = v \cos \gamma \quad \text{so} \quad \dot{\theta} = \frac{v}{r} \cos \gamma$$

$$\dot{\theta} = \frac{15}{5} (0.7660) = 2.298 \text{ rad/s}$$



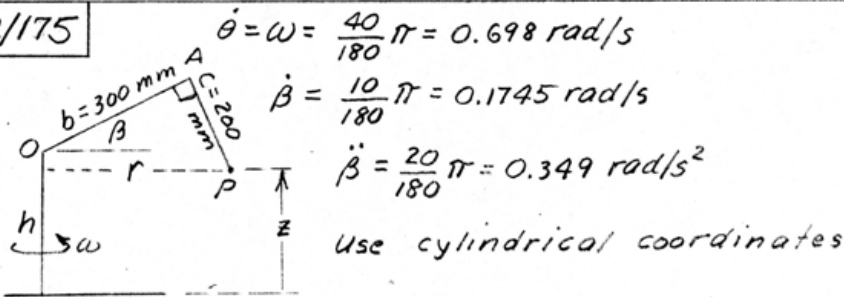
$$a_{\theta} = g \cos^2 \gamma = 9.81 (0.7660)^2 = 5.76 \text{ m/s}^2$$

$$a_z = g \cos \gamma \sin \gamma = 9.81 (0.7660)(0.6428) = 4.83 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(2.298)^2 = -26.41 \text{ m/s}^2$$

$$a = \sqrt{26.41^2 + 5.76^2 + 4.83^2} = \underline{27.5 \text{ m/s}^2}$$

2/175



$$r = b \cos \beta + c \sin \beta, \quad \dot{r} = (-b \sin \beta + c \cos \beta) \dot{\beta}$$

$$\ddot{r} = (-b \cos \beta - c \sin \beta) \dot{\beta}^2 + (-b \sin \beta + c \cos \beta) \ddot{\beta}$$

$$z = h + b \sin \beta - c \cos \beta, \quad \dot{z} = (b \cos \beta + c \sin \beta) \dot{\beta}$$

$$\ddot{z} = (-b \sin \beta + c \cos \beta) \dot{\beta}^2 + (b \cos \beta + c \sin \beta) \ddot{\beta}$$

$$\text{For } \beta = 30^\circ, \quad \dot{r} = (-300 \times 0.5 + 200 \times 0.866)(0.1745) = 4.050 \text{ mm/s}$$

$$\ddot{r} = (-300 \times 0.866 - 200 \times 0.5)(0.1745)^2 + (-300 \times 0.5 + 200 \times 0.866)(0.349) = -2.860 \text{ mm/s}^2$$

$$\ddot{z} = (-300 \times 0.5 + 200 \times 0.866)(0.1745)^2 + (300 \times 0.866 + 200 \times 0.5)(0.349) = 126.30 \text{ mm/s}^2$$

$$\begin{aligned} a_r &= \ddot{r} - r \dot{\theta}^2 = -2.860 - 359.8(0.698)^2 = -178.23 \text{ mm/s}^2 \\ a_\theta &= r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0 + 2(4.049)(0.698) = 5.65 \text{ mm/s}^2 \\ a_z &= \ddot{z} = 126.30 \text{ mm/s}^2 \end{aligned} \quad \left| \quad \begin{aligned} a &= \sqrt{a_r^2 + a_\theta^2 + a_z^2} \\ &= \underline{219 \text{ mm/s}^2} \end{aligned} \right.$$

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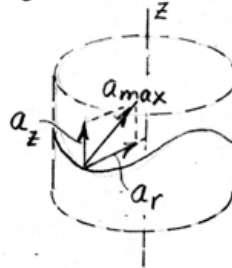
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\omega^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$$

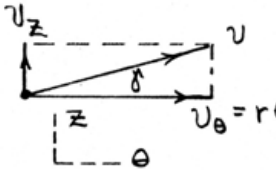
$$a_z = \frac{d^2}{dt^2}(z_0 \sin 2\pi n t) = -4n^2\pi^2 z_0 \sin 2\pi n t$$

$$a = \sqrt{(-r\omega^2)^2 + (-4n^2\pi^2 z_0 \sin 2\pi n t)^2}$$

$$a_{\max} = \sqrt{r^2\omega^4 + 16n^4\pi^4 z_0^2}$$



2/177 Helix angle is $\gamma = \tan^{-1} \frac{10}{24\pi} = 7.55^\circ$



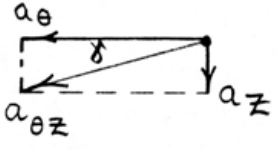
A velocity vector diagram showing a vector v at an angle γ to the horizontal axis z . The vertical component is v_z and the horizontal component is $v_\theta = r\dot{\theta}$. The angle γ is between the vector v and the horizontal axis.

$$v = 15 \left(\frac{5280}{3600} \right) = 22 \text{ ft/sec}$$

$$v_z = v \sin \gamma = 22 \sin 7.55^\circ = 2.89 \text{ ft/sec}$$

$$v_\theta = v \cos \gamma = 22 \cos 7.55^\circ = 21.8 \text{ ft/sec}$$

From $v_\theta = r\dot{\theta}$, $\dot{\theta} = \frac{v_\theta}{r} = \frac{21.8}{24} = 0.909 \text{ rad/sec}$



An acceleration vector diagram showing a vector a at an angle γ to the horizontal axis z . The vertical component is a_z and the horizontal component is a_θ . The angle γ is between the vector a and the horizontal axis.

$$a_\theta = 2 \left(\frac{5280}{3600} \right) = 2.93 \text{ ft/sec}^2$$

$$a_\theta = -2.93 \cos 7.55^\circ = -2.91 \text{ ft/sec}^2$$

$$a_z = -2.93 \sin 7.55^\circ = -0.386 \text{ ft/sec}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 24(0.909)^2 = -19.82 \text{ ft/sec}^2$$

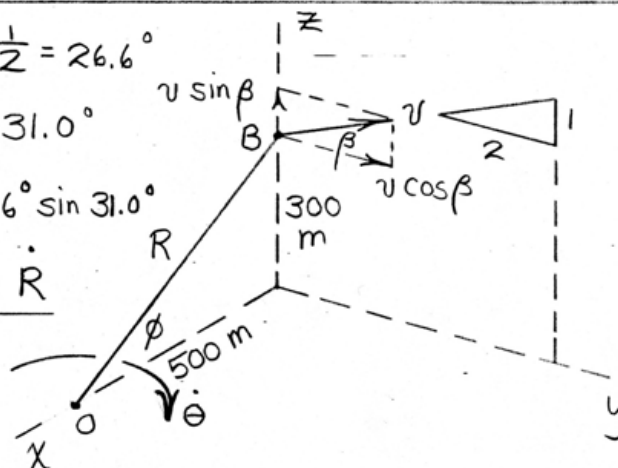
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$$\beta = \tan^{-1} \frac{1}{2} = 26.6^\circ$$

$$\phi = \tan^{-1} \frac{3}{5} = 31.0^\circ$$

$$v_R = 400 \sin 26.6^\circ \sin 31.0^\circ$$

$$= 92.0 \text{ km/h} = \dot{R}$$



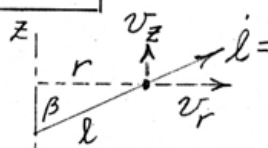
$$v_\theta = R \dot{\theta} \cos \phi : \frac{400}{3.6} \cos 26.6^\circ = 500 \dot{\theta}$$

$$\dot{\theta} = 0.1988 \text{ rad/s}$$

$$v_\phi = R \dot{\phi} : \frac{400}{3.6} \sin 26.6^\circ \cos 31.0^\circ = \frac{500}{\cos 31.0^\circ} \dot{\phi}$$

$$\dot{\phi} = 0.0731 \text{ rad/s}$$

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$$v_r = \dot{l} \sin \beta = c \sin \beta$$

$$v_\theta = r \dot{\theta} = (l \sin \beta) K = K l \sin \beta$$

$$v_z = \dot{l} \cos \beta = c \cos \beta$$

$$\dot{\theta} = K \quad v = \sqrt{(c \sin \beta)^2 + (K l \sin \beta)^2 + (c \cos \beta)^2}$$

$$= \sqrt{c^2 + K^2 l^2 \sin^2 \beta}$$

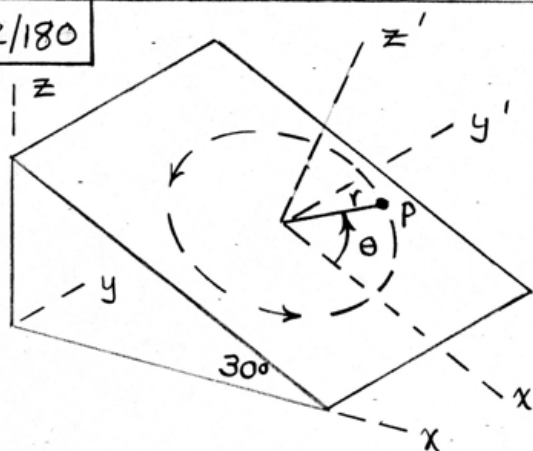
$$a_r = \ddot{r} - r \dot{\theta}^2 = 0 - l \sin \beta (K^2) = -K^2 l \sin \beta$$

$$a_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0 + 2 c \sin \beta (K) = 2 K c \sin \beta$$

$$a_z = \ddot{z} = 0$$

$$a = \sqrt{(-K^2 l \sin \beta)^2 + (2 K c \sin \beta)^2} = K \sin \beta \sqrt{K^2 l^2 + 4 c^2}$$

2/180



$$\theta = \omega t = \frac{v}{r} t$$

$$\dot{\theta} = \frac{v}{r}$$

$$\begin{cases} x' = r \cos \theta \\ y' = r \sin \theta \\ z' = 0 \end{cases}$$

$$\begin{cases} \dot{x}' = -r \dot{\theta} \sin \theta \\ \dot{y}' = r \dot{\theta} \cos \theta \\ \dot{z}' = 0 \end{cases}$$

$$\ddot{x}' = -r \dot{\theta}^2 \cos \theta, \ddot{y}' = -r \dot{\theta}^2 \sin \theta, \ddot{z}' = 0$$

Component relationships:
$$\begin{cases} x = x' \cos 30^\circ + z' \sin 30^\circ \\ y = y' \\ z = -x' \sin 30^\circ + z' \cos 30^\circ \end{cases}$$

(Similar for $\dot{x}, \ddot{x}$, etc.)

Thus

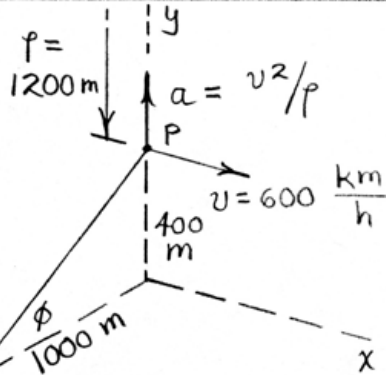
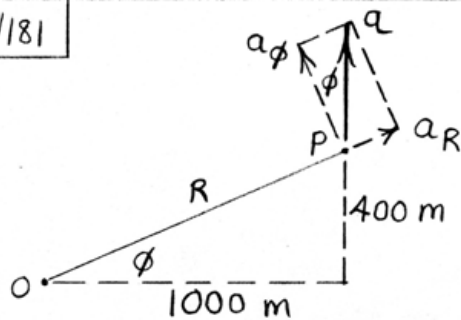
$$x = \frac{\sqrt{3}}{2} r \cos \frac{v}{r} t, y = r \sin \frac{v}{r} t, z = -\frac{1}{2} r \cos \frac{v}{r} t$$

$$\dot{x} = -\frac{\sqrt{3}}{2} v \sin \frac{v}{r} t, \dot{y} = v \cos \frac{v}{r} t, \dot{z} = \frac{1}{2} v \sin \frac{v}{r} t$$

$$\ddot{x} = -\frac{\sqrt{3}}{2} \frac{v^2}{r} \cos \frac{v}{r} t, \ddot{y} = -\frac{v^2}{r} \sin \frac{v}{r} t, \ddot{z} = \frac{1}{2} \frac{v^2}{r} \cos \frac{v}{r} t$$

Note that position relationships do not include the constants associated with the origin positions.

2/181



$$\phi = \tan^{-1} \frac{400}{1000} = 21.8^\circ$$

$$a = \frac{v^2}{r} = \frac{(600/3.6)^2}{1200} = 23.1 \text{ m/s}^2$$

$$\dot{R} = 0, \quad \dot{\phi} = 0, \quad \dot{\theta} = \frac{v_\theta}{R \cos \phi} = \frac{600(1000)/3.6}{1000} = 0.1667 \frac{\text{rad}}{\text{s}}$$

$$\text{Eqs. 2/19: } a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2 \cos^2 \phi$$

$$23.1 \sin 21.8^\circ = \ddot{R} - 0 - \frac{1000}{\cos 21.8^\circ} (0.1667)^2 \cos^2 21.8^\circ$$

$$\ddot{R} = 34.4 \text{ m/s}^2$$

$$a_\phi = \frac{1}{R} \frac{d}{dt} (R^2 \dot{\phi}) + R\dot{\theta}^2 \sin \phi \cos \phi = 2R\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2 \sin \phi \cos \phi$$

$$23.1 \cos 21.8^\circ = 0 + \frac{1000}{\cos 21.8^\circ} \ddot{\phi} + \frac{1000}{\cos 21.8^\circ} (0.1667)^2 \sin 21.8^\circ \cos 21.8^\circ$$

$$\ddot{\phi} = 0.01038 \text{ rad/s}^2$$

$$2/182 \quad R = 0.75 + 0.5 = 1.25 \text{ m}, \quad \dot{R} = 0.2 \text{ m/s}, \quad \ddot{R} = -0.3 \frac{\text{m}}{\text{s}^2}$$

$$\phi = 30^\circ, \quad \dot{\phi} = 10 \left(\frac{\pi}{180} \right) \text{ rad/s}, \quad \ddot{\phi} = 0, \quad \dot{\theta} = 20 \left(\frac{\pi}{180} \right) \text{ rad/s}, \quad \ddot{\theta} = 0$$

$$\begin{cases} v_R = \dot{R} = 0.2 \text{ m/s} \\ v_\theta = R\dot{\theta} \cos \phi = 1.25 \left(20 \frac{\pi}{180} \right) \cos 30^\circ = 0.378 \frac{\text{m}}{\text{s}} \\ v_\phi = R\dot{\phi} = 1.25 \left(10 \frac{\pi}{180} \right) = 0.218 \text{ m/s} \end{cases}$$

$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{0.480 \text{ m/s}}$$

$$\begin{aligned} a_R &= \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2 \cos^2 \phi \\ &= -0.3 - 1.25 \left(10 \frac{\pi}{180} \right)^2 - 1.25 \left(20 \frac{\pi}{180} \right)^2 \cos^2 30^\circ \\ &= -0.4523 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} a_\theta &= \cos \phi [2\dot{R}\dot{\theta} + R\ddot{\theta}] - 2R\dot{\theta}\dot{\phi} \sin \phi \\ &= \cos 30^\circ [2(0.2) \left(20 \frac{\pi}{180} \right) + 1.25(0)] \\ &\quad - 2(1.25) \left(10 \frac{\pi}{180} \right) \left(20 \frac{\pi}{180} \right) \sin 30^\circ = 0.0448 \frac{\text{m}}{\text{s}^2} \end{aligned}$$

$$\begin{aligned} a_\phi &= 2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2 \sin \phi \cos \phi \\ &= 2(0.2) \left(10 \frac{\pi}{180} \right) + 1.25(0) + 1.25 \left(20 \frac{\pi}{180} \right)^2 0.5 \frac{\sqrt{3}}{2} \\ &= 0.1358 \text{ m/s}^2 \end{aligned}$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.474 \text{ m/s}^2}$$

2/183 Spherical coordinates

$$v_R = \dot{R} = 0.5 \text{ m/s}$$

$$v_\theta = R\dot{\theta} \cos \phi = 15 \left(10 \frac{\pi}{180}\right) \cos 30^\circ = 2.27 \text{ m/s}$$

$$v_\phi = R\dot{\phi} = 15 \left(7 \frac{\pi}{180}\right) = 1.833 \text{ m/s}$$

$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{2.96 \text{ m/s}}$$

$$a_R = \ddot{R} - R\dot{\theta}^2 - R\dot{\phi}^2 \cos^2 \phi$$

$$= 0 - 15 \left(7 \frac{\pi}{180}\right)^2 - 15 \left(10 \frac{\pi}{180}\right)^2 \cos^2 30^\circ = -0.567 \text{ m/s}^2$$

$$a_\theta = \frac{\cos \phi}{R} [R^2 \ddot{\theta} + 2R\dot{R}\dot{\theta}] - 2R\dot{\theta}\dot{\phi} \sin \phi$$

$$= \frac{\cos 30^\circ}{15} [0 + 2(15)(0.5) \left(10 \frac{\pi}{180}\right)] - 2(15) \left(10 \frac{\pi}{180}\right) \left(7 \frac{\pi}{180}\right) \sin 30^\circ$$

$$= -0.1687 \text{ m/s}^2$$

$$a_\phi = \frac{1}{R} [R^2 \ddot{\phi} + 2R\dot{R}\dot{\phi}] + R\dot{\theta}^2 \sin \phi \cos \phi$$

$$= \frac{1}{15} [0 + 2(15)(0.5) \left(7 \frac{\pi}{180}\right)] + 15 \left(10 \frac{\pi}{180}\right)^2 \sin 30^\circ \cos 30^\circ$$

$$= 0.320 \text{ m/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.672 \text{ m/s}^2}$$

2/184 Use Eq. 2/19 where $\dot{\phi} = -\dot{\beta}$, $R = L$, $\dot{\theta} = \omega$

$$a_r = 0 - 1.2 \left(-\frac{3}{2}\right)^2 - 1.2(2)^2 \frac{1}{2} = \underline{-5.10 \text{ m/s}^2}$$

$$a_\theta = \frac{\sin \beta}{L} (2L\dot{L}\omega + 0) + 2L\omega\dot{\beta}\cos\beta = 2\omega(L\dot{\beta}\sin\beta + L\dot{\beta}\cos\beta)$$

$$= 2(2) \left(0.9 \frac{1}{\sqrt{2}} + 1.2 \left(-\frac{3}{2}\right) \frac{1}{\sqrt{2}} \right) = \frac{10.8}{\sqrt{2}} = \underline{7.64 \text{ m/s}^2}$$

$$a_\phi = -2\dot{L}\dot{\beta} + L\omega^2 \cos\beta \sin\beta = -2(0.9)\frac{3}{2} + 1.2(2^2)\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$$
$$= -2.7 + 2.4 = \underline{-0.3 \text{ m/s}^2}$$

2/185

$$R = 200 + 50 \sin 4\pi t \quad \dot{\theta} = 120 \left(\frac{2\pi}{60} \right) = 4\pi \text{ rad/s}$$

$$\dot{R} = 200\pi \cos 4\pi t \quad \ddot{\theta} = 0$$

$$\ddot{R} = -800\pi^2 \sin 4\pi t \quad \phi = \frac{\pi}{2} - \beta = 60^\circ, \dot{\phi} = \ddot{\phi} = 0$$

For $\dot{R}$ maximum, $\cos 4\pi t = 1$ and $\sin 4\pi t = 0$

$$\text{Eq. 2/19: } a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2 \cos^2 \phi$$

$$= 0 + (200+0)0 - (200+0)(4\pi)^2 \cos^2 60^\circ = -800\pi^2 \frac{\text{mm}}{\text{s}^2}$$

$$a_\theta = \frac{\cos \phi}{R} \frac{d}{dt} (R^2 \dot{\theta}) - 2R\dot{\theta}\dot{\phi} \sin \phi$$

$$= 2\dot{R}\dot{\theta} \cos \phi - 2R\dot{\theta}\dot{\phi} \sin \phi$$

$$= 2(200\pi \cdot 1)(4\pi) \cos 60^\circ - 2(200+0)(4\pi)(0) \sin 60^\circ$$

$$= 800\pi^2 \text{ mm/s}^2$$

$$a_\phi = \frac{1}{R} \frac{d}{dt} (R^2 \dot{\phi}) + R\dot{\theta}^2 \sin \phi \cos \phi$$

$$= 0 + (200+0)(4\pi)^2 \sin 60^\circ \cos 60^\circ = 800\sqrt{3} \pi^2 \text{ mm/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = 17660 \text{ mm/s}^2 \text{ or } \underline{17.66 \text{ m/s}^2}$$

► 2/186 $R = \text{const}$ $\theta = \omega t$ $\sin \phi = z/R$
 $z = \frac{h}{2}(1 - \cos 2\theta)$, $\dot{z} = \omega h \sin 2\theta$ where $\dot{\theta} = \omega$
 $(\cos \phi) \dot{\phi} = \frac{1}{R} \dot{z}$, $\dot{\phi} = \frac{\omega h \sin 2\theta}{R \cos \phi}$

$$v_R = \dot{R} = 0$$

$$v_\theta = R \dot{\theta} \cos \phi = R \omega \sqrt{1 - \sin^2 \phi} = R \omega \sqrt{1 - \left(\frac{h}{2R}[1 - \cos 2\theta]\right)^2}$$

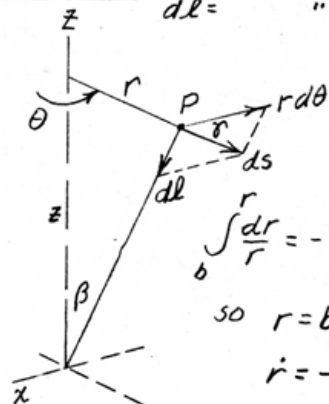
$$v_\phi = R \dot{\phi} = \frac{\omega h \sin 2\theta}{\cos \phi} = h \omega \frac{\sin 2\theta}{\sqrt{1 - \left(\frac{h}{2R}[1 - \cos 2\theta]\right)^2}}$$

When $\theta = \omega t = \pi/4$, $1 - \cos 2\theta = 1$ so that

$$\underline{v_\theta = R \omega \sqrt{1 - (h/2R)^2}}, \quad \underline{v_\phi = \frac{h \omega}{\sqrt{1 - (h/2R)^2}}}, \quad \underline{v_R = 0}$$

► 2/187

ds = differential distance along curve
 $d\ell$ = " " " in direction of cone element



$$r = \frac{b}{h} z, \tan \beta = b/h$$

$$d\ell = r d\theta \tan \gamma$$

$$dr = -d\ell \sin \beta = -r d\theta \tan \gamma \sin \beta$$

$$\int_b^r \frac{dr}{r} = -\tan \gamma \sin \beta \int_0^\theta d\theta, \ln r \Big|_b^r = -\tan \gamma \sin \beta \theta$$

$$\text{so } r = b e^{-\tan \gamma \sin \beta \theta}, r = b e^{-K\theta} \text{ where } K = \tan \gamma \sin \beta$$

$$\dot{r} = -b K \dot{\theta} e^{-K\theta}, \ddot{r} = b K^2 \dot{\theta}^2 e^{-K\theta}, \ddot{\theta} = 0$$

$$\text{Thus } a_r = \ddot{r} - r \dot{\theta}^2 = b \dot{\theta}^2 e^{-K\theta} (K^2 - 1)$$

$$\text{or } a_r = b \dot{\theta}^2 (\tan^2 \gamma \sin^2 \beta - 1) e^{-\theta \tan \gamma \sin \beta}$$

$$\text{where } \beta = \tan^{-1} \frac{b}{h}$$

2/188 The terms appearing in Eq. 2/19 are

$$R = 50 + 200(1/2) = 100 \text{ mm}, \quad \dot{R} = 400t = 400(1/2) = 200 \text{ mm/s} \\ \ddot{R} = 400 \text{ mm/s}^2$$

$$\theta = \omega t = \frac{\pi}{3}(\frac{1}{2}) = \pi/6 \text{ rad}, \quad \dot{\theta} = \pi/3 \text{ rad/s}, \quad \ddot{\theta} = 0$$

$$\phi = \dot{\phi} t = \frac{2\pi}{3}(\frac{1}{2}) = \pi/3 \text{ rad}, \quad \dot{\phi} = 2\pi/3 \text{ rad/s}, \quad \ddot{\phi} = 0$$

$$\sin \theta = 1/2, \quad \cos \theta = \sqrt{3}/2, \quad \sin \phi = \sqrt{3}/2, \quad \cos \phi = 1/2$$

$$\frac{d}{dt}(R^2\ddot{\theta}) = 2R\dot{R}\ddot{\theta} + R^2\ddot{\ddot{\theta}} = 2(0.1)(0.2)\pi/3 + 0 = \frac{0.04\pi}{3} \text{ (m/s)}^2$$

$$\frac{d}{dt}(R^2\ddot{\phi}) = 2R\dot{R}\ddot{\phi} + R^2\ddot{\ddot{\phi}} = 2(0.1)(0.2)2\pi/3 + 0 = \frac{0.08\pi}{3} \text{ (m/s)}^2$$

Thus the components of a from Eq. 2/19 become

$$a_R = 0.40 - 0.10(2\pi/3)^2 - 0.10(\pi/3)^2(1/2)^2 = -0.0661 \text{ m/s}^2$$

$$a_\theta = \frac{1/2}{0.10} \frac{0.04\pi}{3} - 2(0.10)(\pi/3)(2\pi/3)(\sqrt{3}/2) = -0.1704 \text{ m/s}^2$$

$$a_\phi = \frac{1}{0.10} \frac{0.08\pi}{3} + 0.10(\pi/3)^2(\sqrt{3}/2)(1/2) = 0.885 \text{ m/s}^2$$

The magnitude of the acceleration is, then,

$$a = \sqrt{(-0.0661)^2 + (-0.1704)^2 + (0.885)^2} = \underline{0.904 \text{ m/s}^2}$$

$$\underline{2/189} \quad \underline{v_{B/A} = v_B - v_A = 40\hat{i} - (-80\hat{i}) = 120\hat{i} \frac{\text{km}}{\text{h}}}$$

$$\underline{a_{B/A} = a_B - a_A = 0 - 2\hat{i} = -2\hat{i} \text{ m/s}^2}$$

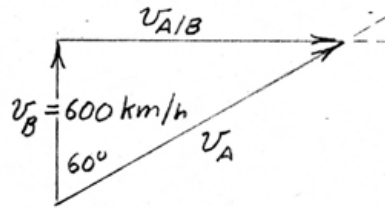
— — — — x

2/190

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$v_A = \frac{600}{\cos 60^\circ} = 1200 \text{ km/h}$$

$$\begin{aligned} v_{A/B} &= 600 \tan 60^\circ \\ &= 1039 \text{ km/h} \end{aligned}$$

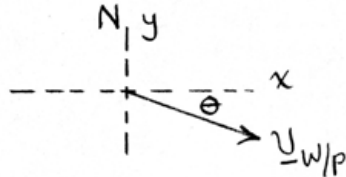


2/19/1

$$(a) \quad \underline{v}_{w/p} = \underline{v}_w - \underline{v}_p \\ = 3\left(\frac{\sqrt{2}}{2}\underline{i} - \frac{\sqrt{2}}{2}\underline{j}\right) - (-4\underline{i}) = 6.12\underline{i} - 2.12\underline{j} \frac{\text{mi}}{\text{hr}}$$

$$\text{or } v_{w/p} = (6.12^2 + 2.12^2)^{1/2} = \underline{6.48 \text{ mi/hr}}$$

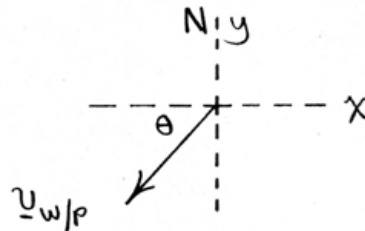
$$\text{at } \theta = \tan^{-1} \frac{2.12}{6.12} = \underline{19.11^\circ \text{ south of east}}$$



$$(b) \quad \underline{v}_{w/p} = \underline{v}_w - \underline{v}_p = 3\left(\frac{\sqrt{2}}{2}\underline{i} - \frac{\sqrt{2}}{2}\underline{j}\right) - 4\underline{i} = -1.879\underline{i} - 2.12\underline{j} \frac{\text{mi}}{\text{hr}}$$

$$\text{or } v_{w/p} = (1.879^2 + 2.12^2)^{1/2} = \underline{2.83 \text{ mi/hr}}$$

$$\text{at } \theta = \tan^{-1} \frac{2.12}{1.879} = \underline{48.5^\circ \text{ south of west}}$$



2/192

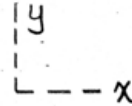
$$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B$$

$$= 120 [\cos 15^\circ \underline{i} + \sin 15^\circ \underline{j}] - 90 [\cos 60^\circ \underline{i} + \sin 60^\circ \underline{j}]$$

$$= 70.9 \underline{i} - 46.9 \underline{j} \quad \text{km/h}$$

$$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \underline{0} - 3(-\cos 60^\circ \underline{i} - \sin 60^\circ \underline{j})$$

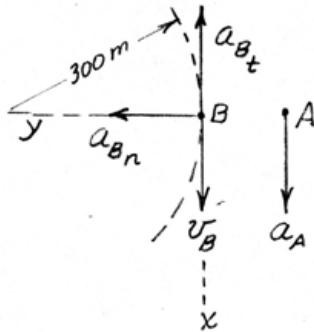
$$= 1.5 \underline{i} + 2.60 \underline{j} \quad \text{m/s}^2$$



2/193

$$a_{B_t} = \frac{8}{3.6} = 2.22 \text{ m/s}^2, \quad a_A = \frac{8}{3.6} = 2.22 \text{ m/s}^2$$

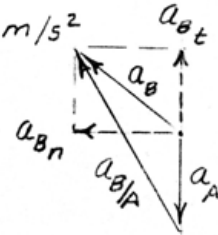
$$a_{B_n} = \frac{v_B^2}{\rho} = \frac{(100/3.6)^2}{300} = 2.57 \text{ m/s}^2$$



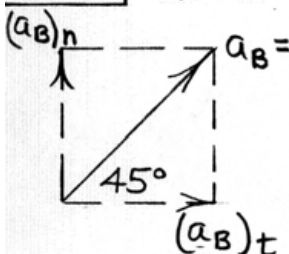
$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

$$-2.22\hat{i} + 2.57\hat{j} = 2.22\hat{i} + \underline{a}_{B/A}$$

$$\underline{a}_{B/A} = -4.44\hat{i} + 2.57\hat{j} \text{ m/s}^2$$



2/194 With $\underline{a}_{B/A} = \underline{0}$, $\underline{a}_A = \underline{a}_B$



$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{\left(45 \frac{ft}{30}\right)^2}{600}$$

$$= 7.26 \text{ ft/sec}^2$$

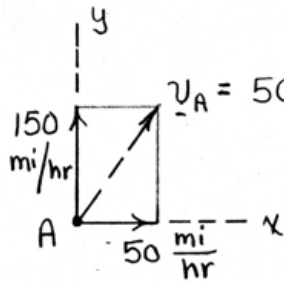
$$\dot{v}_B = (a_B)_t = 7.26 \text{ ft/sec}^2$$

$$a_A = a_B = 7.26\sqrt{2} = 10.27 \frac{ft}{sec^2}$$

$$\begin{aligned}
 \boxed{2/195} \quad \underline{v_{A/B}} &= \underline{v_A} - \underline{v_B}, \quad \Omega = 3(2\pi/60) = 0.314 \frac{\text{rad}}{\text{s}} \\
 &= \frac{18}{3.6} \underline{i} - 9(0.314)(\cos 45^\circ \underline{i} - \sin 45^\circ \underline{j}) \\
 &= \underline{3.00 \underline{i} + 1.999 \underline{j} \text{ m/s}}
 \end{aligned}$$

$$\begin{aligned}
 \underline{a_{A/B}} &= \underline{a_A} - \underline{a_B} = 3 \underline{i} - 9(0.314)^2(-\cos 45^\circ \underline{i} - \sin 45^\circ \underline{j}) \\
 &= \underline{3.63 \underline{i} + 0.628 \underline{j} \text{ m/s}^2}
 \end{aligned}$$

2/196



$$\underline{v}_B = (180 - 50)(-\underline{i}) = -130 \underline{i} \frac{\text{mi}}{\text{hr}}$$

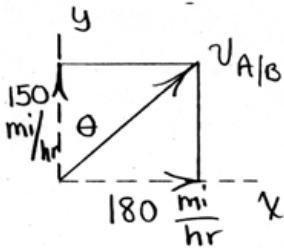
$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} : 50 \underline{i} + 150 \underline{j} = -130 \underline{i} + \underline{v}_{A/B}$$

$$\underline{v}_{A/B} = 180 \underline{i} + 150 \underline{j} \text{ mi/hr}$$

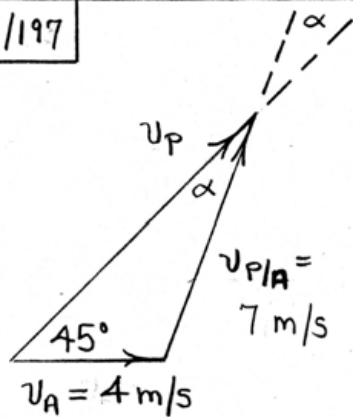
$$v_{A/B} = \sqrt{180^2 + 150^2} = \underline{234 \text{ mi/hr}}$$

$$\theta = \tan^{-1}\left(\frac{180}{150}\right) = \underline{50.2^\circ}$$

(east of north)



2/197



With P being the puck:

$$\underline{v}_P = \underline{v}_A + \underline{v}_{P/A}$$

Law of sines :

$$\frac{\sin 45^\circ}{7} = \frac{\sin \alpha}{4}$$

$$\underline{\alpha = 23.8^\circ}$$

2/198

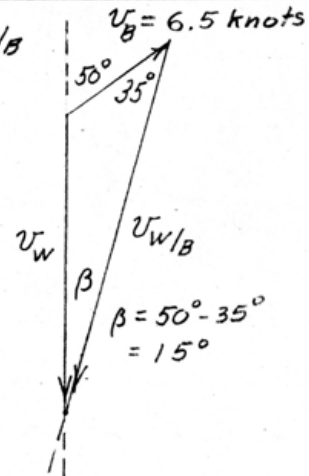
$W = \text{wind}$
 $B = \text{boat}$

$$\vec{v}_W = \vec{v}_B + \vec{v}_{W/B}$$

Law of sines,

$$\frac{v_W}{\sin 35^\circ} = \frac{6.5}{\sin 15^\circ}$$

$$v_W = \frac{6.5(0.5736)}{0.2588} = \underline{14.40 \text{ knots}}$$



2/199 $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$ $\frac{v_{B/A}}{\sin 135^\circ} = \frac{15}{\sin \beta}$

The diagram shows a velocity triangle. Vector $\vec{v}_A$ is horizontal and labeled "15 knots". Vector $\vec{v}_B$ is at an angle of 45° below the horizontal. Vector $\vec{v}_{B/A}$ connects the tip of $\vec{v}_A$ to the tip of $\vec{v}_B$. The angle between $\vec{v}_A$ and $\vec{v}_{B/A}$ is 20° . The angle between $\vec{v}_{B/A}$ and $\vec{v}_B$ is β . The angle between $\vec{v}_A$ and $\vec{v}_B$ is 135° .

where $\beta = 180 - 20 - 135 = 25^\circ$

$t = \frac{\text{Dist.}}{\text{Vel.}} = \frac{10}{25.1} = 0.398 \text{ hr}$
or 23.9 min

$v_{B/A} = \frac{0.7071}{0.4226} 15 = 25.1 \text{ knots}$

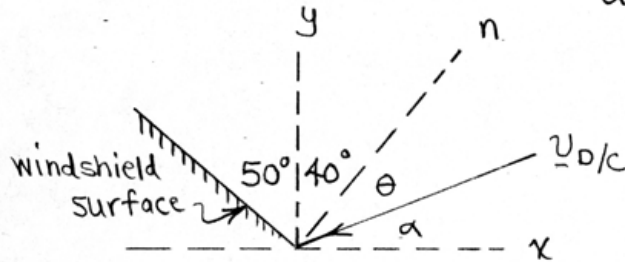
So collision would occur at 2:24 pm

2/200 Drop: $v_D = \sqrt{2gh} = \sqrt{2(9.81)(6)} = 10.85 \text{ m/s}$

Car: $v_C = 100/3.6 = 27.8 \text{ m/s}$

$\underline{v}_{D/C} = \underline{v}_D - \underline{v}_C = -10.85\hat{j} - 27.8\hat{i} \text{ m/s}$

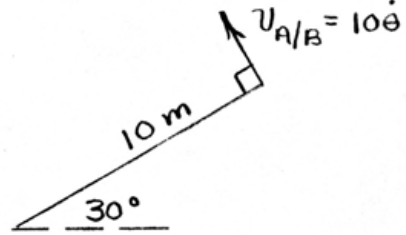
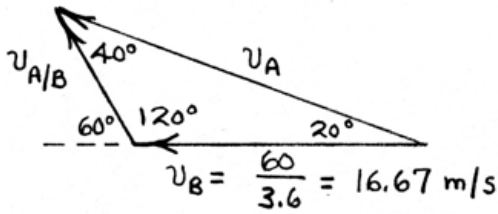
$\alpha = \tan^{-1} \frac{10.85}{27.8}$
 $= 21.3^\circ$



$40^\circ + \theta + \alpha = 90^\circ \Rightarrow \underline{\theta = 28.7^\circ \text{ below normal}}$

2/201

$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$$



Law of sines : $\frac{16.67}{\sin 40^\circ} = \frac{v_A}{\sin 120^\circ}$

$$v_A = 22.5 \text{ m/s} \quad \text{or} \quad \underline{v_A = 80.8 \text{ km/h}}$$

$$v_{A/B} = 16.67 \frac{\sin 20^\circ}{\sin 40^\circ} = 10\dot{\theta}, \quad \underline{\dot{\theta} = 0.887 \frac{\text{rad}}{\text{s}}}$$

2/202

Let s = satellite, A = observer.

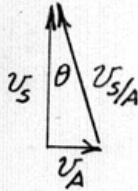
$$\vec{v}_s = \vec{v}_A + \vec{v}_{s/A}, \quad v_A = R\omega$$

$$= 6378(0.729)(10^{-4})(3600)$$

$$= 1674 \text{ km/h (East)}$$

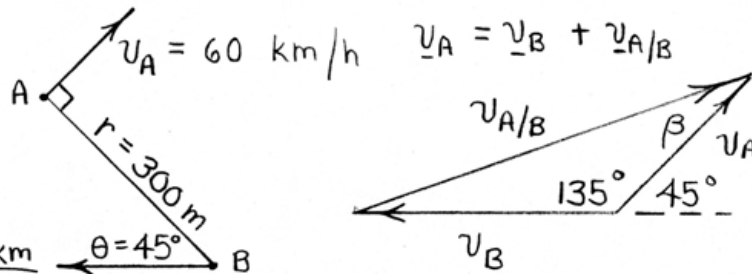
$$v_s = 27940 \text{ km/h (North)}$$

$$\theta = \tan^{-1} \frac{1674}{27940} = 3.43^\circ$$



Satellite appears to travel 3.43°
west of north

2/203



$$v_B = 80 \frac{\text{km}}{\text{h}} \quad \theta = 45^\circ$$

$$v_{A/B}^2 = 60^2 + 80^2 - 2(60)(80)\cos 135^\circ$$

$$v_{A/B} = 129.6 \text{ km/h} \quad \text{or} \quad \underline{36.0 \text{ m/s}}$$

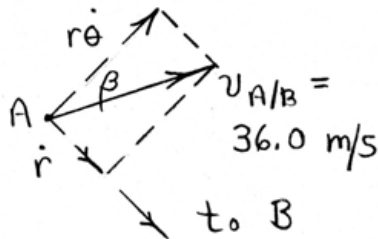
$$\frac{36.0}{\sin 135^\circ} = \frac{80/3.6}{\sin \beta} \quad \beta = 25.9^\circ$$

$$r\dot{\theta} = v_{A/B} \cos \beta : 300 \dot{\theta} = 36.0 \cos 25.9^\circ$$

$$\underline{\dot{\theta} = 0.1079 \text{ rad/s}}$$

$$\dot{r} = -v_{A/B} \sin \beta : \dot{r} = -36.0 \sin 25.9^\circ$$

$$\underline{\dot{r} = -15.71 \text{ m/s}}$$



2/204

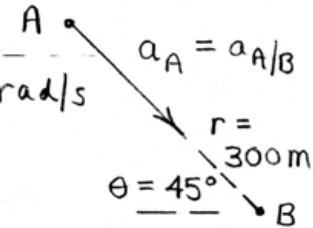
From Prob. 2/203,

$$\dot{r} = -15.71 \text{ m/s}, \quad \dot{\theta} = 0.1079 \text{ rad/s}$$

$$a_A = a_B + a_{A/B}$$

$$a_A = \frac{v_A^2}{r} = \frac{(60/3.6)^2}{300}$$

$$= 0.926 \text{ m/s}^2 = a_{A/B}$$



$$(a_{A/B})_r = \ddot{r} - r\dot{\theta}^2 : -0.926 = \ddot{r} - 300(0.1079)^2$$

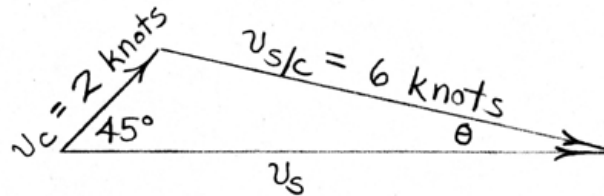
$$\ddot{r} = 2.57 \text{ m/s}^2$$

$$(a_{A/B})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 0 = 300\ddot{\theta} + 2(-15.71)(0.1079)$$

$$\ddot{\theta} = 0.01130 \text{ rad/s}^2$$

$$\boxed{2/205} \begin{cases} \underline{v}_s : \text{true velocity of ship (A to B)} \\ \underline{v}_c : \text{true velocity of current (2 knots NE)} \\ \underline{v}_{s/c} : \text{velocity of ship relative to current} \\ \quad \quad \quad (\text{magnitude 6 knots}) \end{cases}$$

$$\underline{v}_s = \underline{v}_c + \underline{v}_{s/c}$$



$$\text{Law of sines : } \frac{2}{\sin \theta} = \frac{6}{\sin 45^\circ}, \theta = 13.63^\circ$$

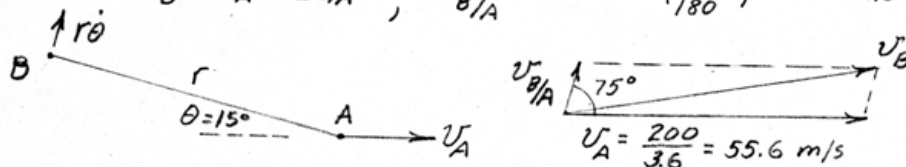
$$\text{So heading } H = 90^\circ + 13.63^\circ \approx \underline{104^\circ}$$

$$v_s = 2 \cos 45^\circ + 6 \cos 13.63^\circ = 7.25 \text{ knots}$$

$$\text{Time } t = \frac{10}{7.25} = 1.380 \text{ hr or } \underline{t = 1 \text{ hr } 23 \text{ min}}$$

2/206

$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}, \quad v_{B/A} = r\dot{\theta} = 60\left(\frac{5}{180}\pi\right) = 5.24 \text{ m/s}$$



$$v_B^2 = (5.24)^2 + (55.6)^2 + 2(5.24)(55.6)\cos 75^\circ = 3264 \text{ (m/s)}^2$$

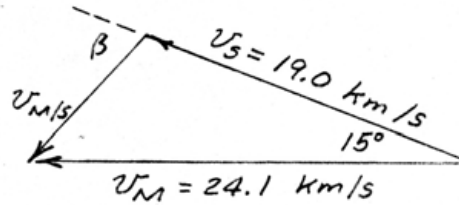
$$v_B = 57.1 \text{ m/s} \text{ or } v_B = 57.1(3.6) = \underline{206 \text{ km/h}}$$

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}, \quad a_A = 0, \quad a_{B/A} = r\dot{\theta}^2 = 60\left(\frac{5\pi}{180}\right)^2 = 0.457 \text{ m/s}^2$$

$$\text{Thus } a_B = a_{B/A} = \underline{0.457 \text{ m/s}^2} \text{ from B to A}$$

2/207 Mars will appear to be approaching the spacecraft head on when $v_{M/s}$ is along the line of sight M-S.

$$\underline{v}_M = \underline{v}_S + \underline{v}_{M/s}$$

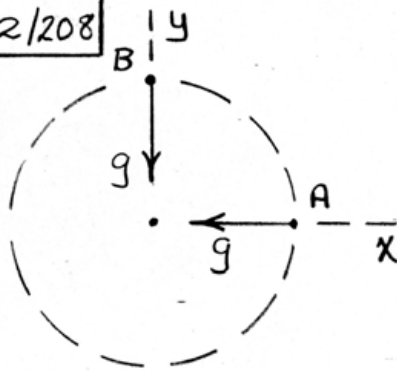


$$(v_{M/s})^2 = (19.0)^2 + (24.1)^2 - 2(19.0)(24.1)\cos 15^\circ = 57.2 \text{ (km/s)}^2$$

$$v_{M/s} = 7.56 \text{ km/s}, \quad \frac{24.1}{\sin(\pi - \beta)} = \frac{7.56}{\sin 15^\circ}$$

$$\sin(\pi - \beta) = \sin \beta = 0.8246, \quad \underline{\beta = 55.6^\circ}$$

2/208



$$g = g_0 \left(\frac{R}{R+h} \right)^2$$

$$= 9.825 \left(\frac{6371}{6371+1500} \right)^2$$

$$= 6.44 \text{ m/s}^2$$

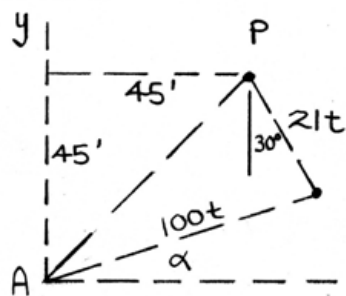
$$\underline{a}_{B/A} = \underline{a}_B - \underline{a}_A$$

$$= -6.44 \underline{j} - (-6.44 \underline{i})$$

$$= 6.44 \underline{i} - 6.44 \underline{j} \text{ m/s}^2$$

$$a_{B/A} = 6.44 \sqrt{2} = \underline{9.10 \text{ m/s}^2}$$

2/209



$$\begin{cases} 100t \cos \alpha = 45 + 21t \sin 30^\circ \\ 100t \sin \alpha = 45 - 21t \cos 30^\circ \end{cases}$$

$$\text{or} \begin{cases} 100 \cos \alpha = \frac{45}{t} + 21 \sin 30^\circ \\ 100 \sin \alpha = \frac{45}{t} - 21 \cos 30^\circ \end{cases}$$

Subtract 2nd from 1st:

$$100 (\cos \alpha - \sin \alpha) = 21 \left(\frac{1}{2} + \frac{\sqrt{3}}{2} \right) \text{ or } \cos \alpha - \sin \alpha = 0.287$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = 0.287 + \sin \alpha$$

$$\text{SQBS \& rearrange: } 2 \sin^2 \alpha + 0.574 \sin \alpha - 0.918 = 0$$

$$\text{Positive solution: } \alpha = 33.3^\circ; \text{ Then } t = 0.616 \text{ sec}$$

$$\begin{aligned} \underline{v_{A/B}} &= \underline{v_A} - \underline{v_B} = 100 [\cos \alpha \underline{i} + \sin \alpha \underline{j}] \\ &\quad - 21 [\sin 30^\circ \underline{i} - \cos 30^\circ \underline{j}] \end{aligned}$$

$$= \underline{73.1 \underline{i} + 73.1 \underline{j} \text{ ft/sec}}$$

► 2/210

$$v_{B/A} = v_B - v_A$$

$$= (1500 - 1000) / 3.6 = 138.9 \frac{\text{m}}{\text{s}}$$

$$(v_{B/A})_r = \dot{r} = 138.9 \cos 30^\circ$$

$$= 120.3 \text{ m/s}$$

$$(v_{B/A})_\theta = r\dot{\theta} = -138.9 \sin 30^\circ = \frac{6000}{\sin 30^\circ} \dot{\theta}$$

$$\dot{\theta} = -0.00579 \text{ rad/s}$$

$$a_{B/A} = a_B - a_A = 0 - 1.2 = -1.2 \text{ m/s}^2$$

$$(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2$$

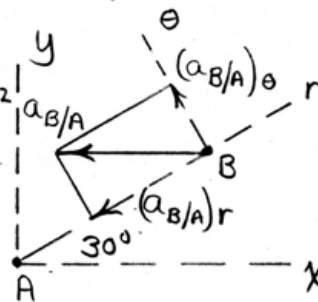
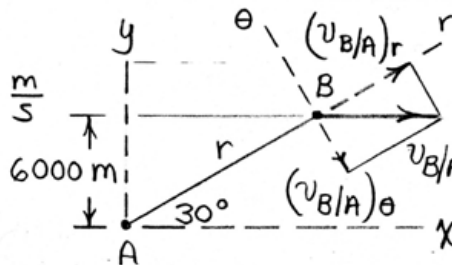
$$-1.2 \cos 30^\circ = \ddot{r} - 12000(-0.00579)^2$$

$$\ddot{r} = -0.637 \text{ m/s}^2$$

$$(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$1.2 \sin 30^\circ = 12000 \ddot{\theta} + 2(120.3)(-0.00579)$$

$$\ddot{\theta} = 0.1660 (10^{-3}) \text{ rad/s}^2$$



► 2/21/ Find flight time t :

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 : 7 = 3 + 100\sin 30^\circ t - 16.1t^2$$

Solve to obtain 0.0822 sec (discard) & $t = 3.02$ sec

$$\text{Range } R = x_0 + v_{x0}t = 0 + 100\cos 30^\circ (3.02) \\ = 262 \text{ ft}$$

Fielder must run $262 - 220 = 41.8$ ft

$$\text{in } (3.02 - 0.25) \text{ sec} \Rightarrow v_B = \frac{41.8}{2.77} = 15.08 \text{ ft/sec}$$

Velocity components of ball when caught:

$$v_x = v_{x0} = 100 \cos 30^\circ = 86.6 \text{ ft/sec}$$

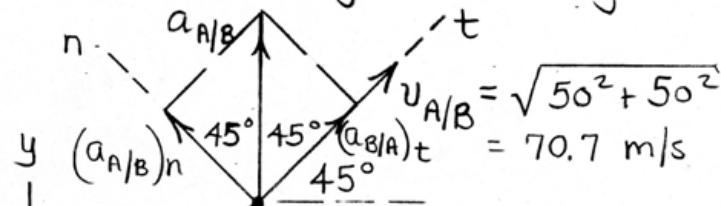
$$v_y = v_{y0} - gt = 100 \sin 30^\circ - 32.2(3.02) = -47.4 \frac{\text{ft}}{\text{sec}}$$

$$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = (86.6\hat{i} - 47.4\hat{j}) - 15.08\hat{i} \\ = \underline{71.5\hat{i} - 47.4\hat{j} \text{ ft/sec}}$$

► 2/2/12 (a) $\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 50\hat{i} - (-50\hat{j}) = 50\hat{i} + 50\hat{j} \text{ m/s}$

$$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \frac{v_A^2}{r_A} \hat{j} - 0 = \frac{50^2}{2000} \hat{j} = 1.250 \hat{j} \text{ m/s}^2$$

(b) Use the results of part (a) for a normal-tangential analysis:



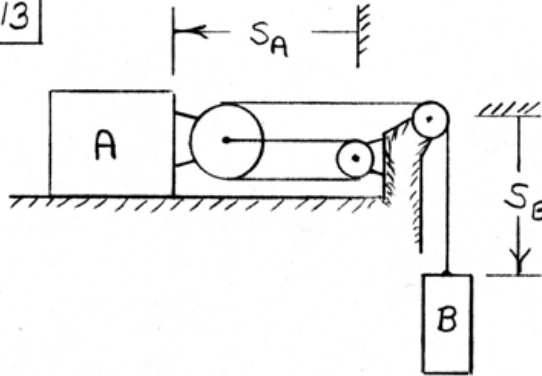
$$\dot{v}_r = (a_{B/A})_t = a_{A/B} \cos 45^\circ = 1.250 \frac{\sqrt{2}}{2} = 0.884 \text{ m/s}^2$$

$$(a_{B/A})_n = a_{B/A} \sin 45^\circ = \frac{v_{A/B}^2}{r}$$

$$1.250 \frac{\sqrt{2}}{2} = \frac{70.7^2}{r}, \quad r = 5660 \text{ m}$$

(in n-direction from A)

2/213



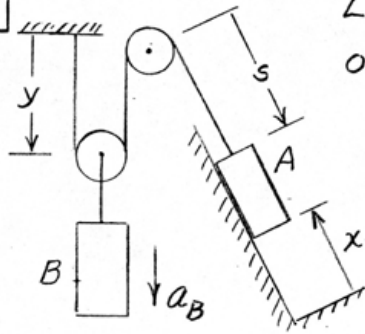
Length of cable $L = 3s_A + s_B + \text{constant}$

Differentiate: $0 = 3v_A + v_B$

$$v_B = -3v_A = -3(-3.6)$$

$$= \underline{10.8 \text{ ft/sec (down)}}$$

2/214



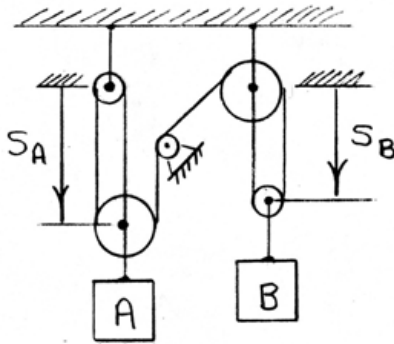
$$L = 2y + s + \text{const.}$$

$$0 = 2\dot{y} + \dot{s}, \quad 0 = 2\ddot{y} + \ddot{s}$$

$$a_B = \ddot{y} = -\frac{\ddot{s}}{2} = +\frac{\ddot{x}}{2}$$

$$a_B = \frac{0.044}{2} = \underline{0.022 \text{ m/s}^2}$$

2/2/5



The length of the main cable is

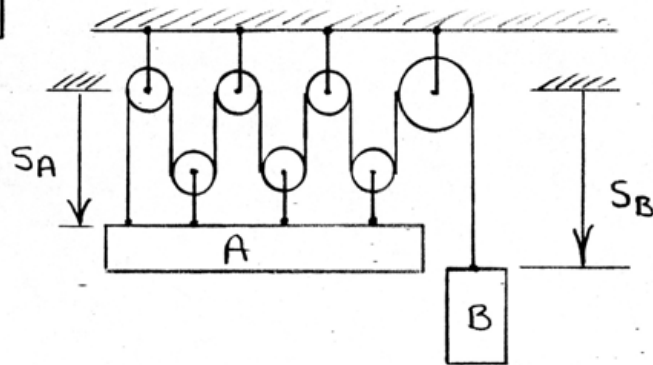
$$L = 3s_A + 2s_B + \text{constants}$$

$$\Rightarrow 0 = 3v_A + 2v_B ; \quad 0 = 3a_A + 2a_B$$

$$\text{So } v_B = -\frac{3}{2}v_A = -\frac{3}{2}(0.8) = \underline{-1.2 \text{ m/s}} \quad (\text{up})$$

$$\text{and } a_B = -\frac{3}{2}a_A = -\frac{3}{2}(-2) = \underline{3 \text{ m/s}^2} \quad (\text{down})$$

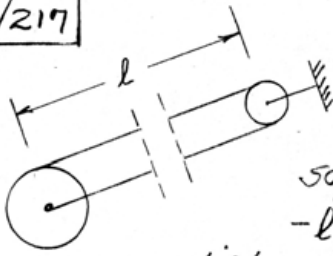
2/216



$$L = 7s_A + s_B + \text{constants}$$

$$0 = 7a_A + a_B \quad (\text{for coordinates shown})$$

2/217



Length of cable is

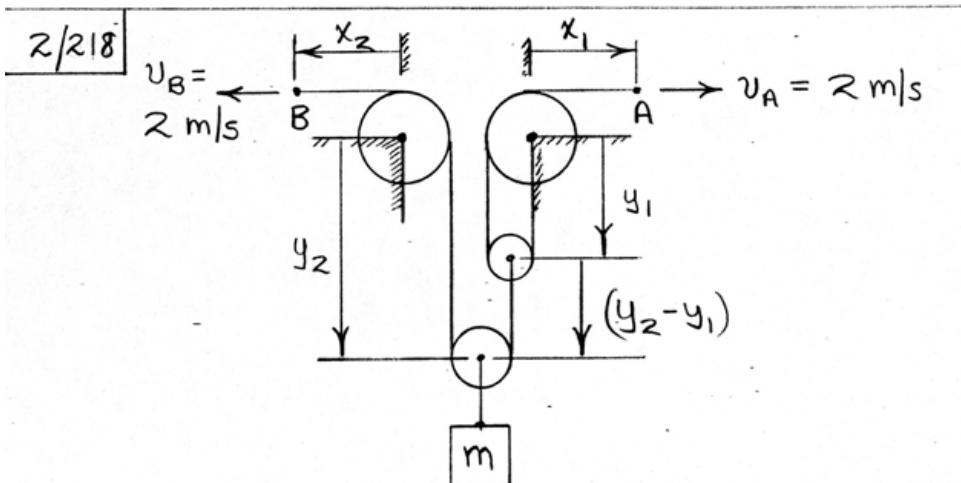
$$L = 2l$$

$$\dot{L} = 2\dot{l}$$

So velocity of truck is

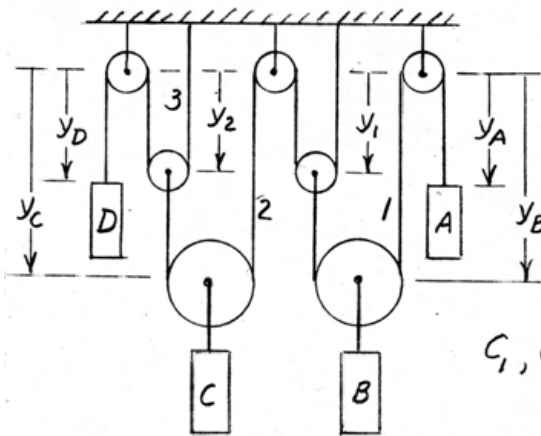
$$-\dot{l} = \frac{1}{2}(-\dot{L}) = \frac{1}{2}(40) = 20 \text{ mm/s}$$

$$\text{time } t = \frac{\text{distance}}{\text{velocity}} = \frac{4(10^3)}{20} = 200 \text{ s or } \underline{3 \text{ min } 20 \text{ s}}$$



$$\begin{aligned}
 x_1 + 2y_1 &= \text{constant}; \quad \dot{x}_1 + 2\dot{y}_1 = 0, \quad \dot{y}_1 = -\frac{\dot{x}_1}{2} \\
 x_2 + y_2 + (y_2 - y_1) &= \text{constant}; \quad \dot{x}_2 + 2\dot{y}_2 - \dot{y}_1 = 0 \\
 \dot{x}_2 + 2\dot{y}_2 - \left(-\frac{\dot{x}_1}{2}\right) &= 0 \\
 \dot{x}_2 + 2\dot{y}_2 + \frac{\dot{x}_1}{2} &= 0, \quad \dot{y}_2 = -\frac{\dot{x}_2}{2} - \frac{\dot{x}_1}{4} = -\frac{2}{2} - \frac{2}{4} = -1.5 \text{ m/s} \\
 &\text{or } 1.5 \text{ m/s up}
 \end{aligned}$$

2/219



3 degrees of freedom

C_1, C_2, C_3 are constant lengths

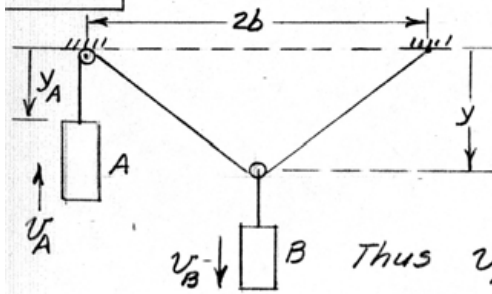
$$L_1 = y_B + y_A + (y_B - y_1) + C_1; \quad 0 = 2\dot{y}_B + \dot{y}_A - \dot{y}_1$$

$$L_2 = y_C + 2y_1 + (y_C - y_2) + C_2 \quad 0 = 2\dot{y}_C + 2\dot{y}_1 - \dot{y}_2$$

$$L_3 = 2y_2 + y_D + C_3, \quad 0 = 2\dot{y}_2 + \dot{y}_D$$

Eliminate $\dot{y}_1$ & $\dot{y}_2$ & get $4\dot{y}_A + 8\dot{y}_B + 4\dot{y}_C + \dot{y}_D = 0$

2/220



Cable length is

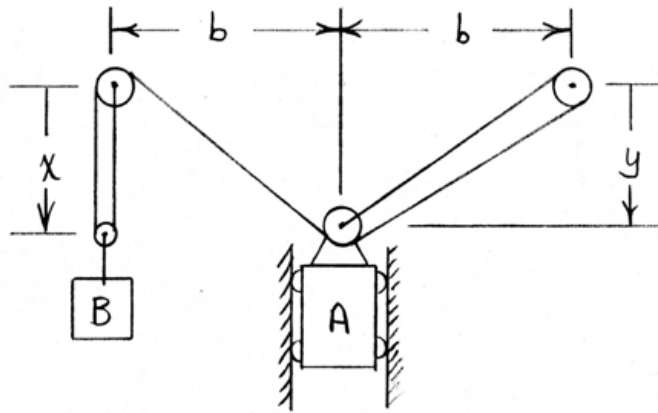
$$L = y_A + 2\sqrt{y_B^2 + b^2}$$

$$0 = \dot{y}_A + \frac{2y\dot{y}}{\sqrt{y^2 + b^2}}$$

But $v_A = -\dot{y}_A$, $v_B = \dot{y}$

Thus $v_A = \frac{2y}{\sqrt{y^2 + b^2}} v_B$

2/221



The total length of the cable is

$$L = 2x + 3\sqrt{y^2 + b^2} + \text{constant}.$$

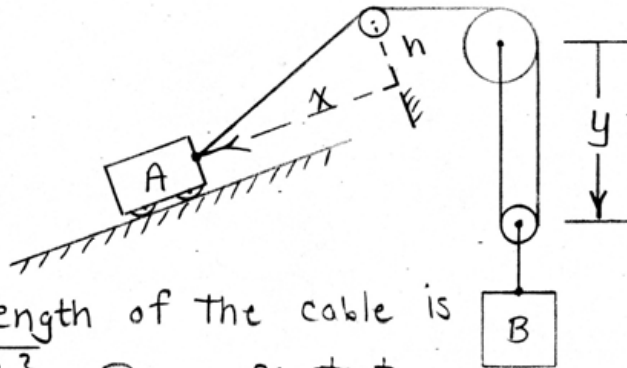
Differentiate to obtain

$$\dot{L} = 0 = 2\dot{x} + 3 \frac{y\dot{y}}{\sqrt{y^2 + b^2}}$$

With $\dot{x} = v_B$ & $\dot{y} = v_A$, we have

$$v_B = -\frac{3y}{2\sqrt{y^2 + b^2}} v_A$$

2/222



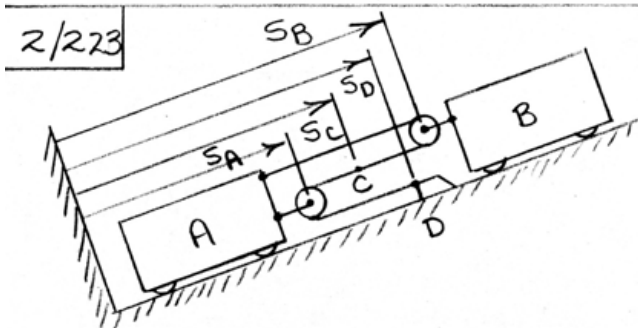
The total length of the cable is

$$L = \sqrt{x^2 + h^2} + 2y + \text{constant}$$

$$\dot{L} = 0 = \frac{1}{2} \frac{2x\dot{x}}{\sqrt{x^2 + h^2}} + 2\dot{y}$$

With $v_A = \dot{x}$ & $v_B = -\dot{y}$:

$$v_A = + \frac{2\sqrt{x^2 + h^2}}{x} v_B$$



The cable length is $L = 2(s_B - s_A) + s_D - s_A$
 + Constants
 Differentiating:

$$0 = 2v_B - 3v_A \quad ; \quad 0 = 2a_B - 3a_A$$

$$\text{So } v_A = \frac{2}{3}v_B = \frac{2}{3}(3) = 2 \text{ ft/sec}$$

$$a_A = \frac{2}{3}a_B = \frac{2}{3}(6) = 4 \text{ ft/sec}^2$$

$$v_{B/A} = v_B - v_A = 3 - 2 = \underline{1 \text{ ft/sec}}$$

$$a_{B/A} = a_B - a_A = 6 - 4 = \underline{2 \text{ ft/sec}^2}$$

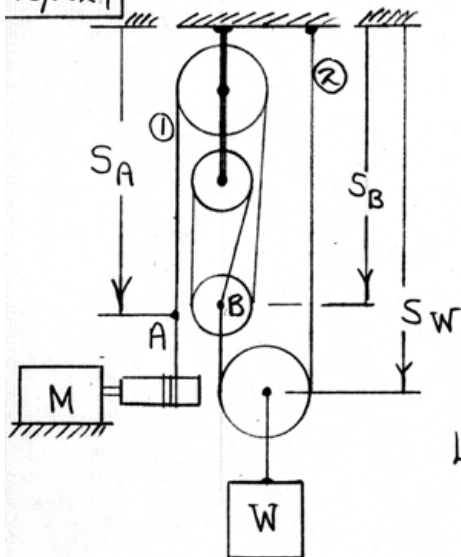
The length of cable between A and C is

$$L' = (s_B - s_A) + (s_B - s_C) = 2s_B - s_A - s_C + \text{constants}$$

$$\Rightarrow 0 = 2v_B - v_A - v_C; \quad v_C = 2v_B - v_A = 2(3) - 2 = \underline{4 \text{ ft/sec}}$$

(All answers are quantities directed up in cline.)

2/224



Let A be a point on cable 1, and let B be the indicated pulley.

$$L_1 = S_A + 3S_B + \text{Constant}$$

$$0 = v_A + 3v_B \quad (1)$$

$$L_2 = S_W - S_B + S_W$$

$$= 2S_W - S_B$$

$$0 = 2v_W - v_B \quad (2)$$

Combine (1) & (2) : $v_A + 6v_W = 0$

So $v_W = -\frac{1}{6}v_A$

Hence W rises $\frac{1}{6}(180)(10) = \underline{300 \text{ mm} = h}$

2/225

$$\text{Length } L_1 = l_1 + 2(l_1 - l_2) + \text{const.}$$

$$\dot{L}_1 = -r\omega = 3\dot{l}_1 - 2\dot{l}_2$$

$$\text{Length } L_2 = l_2 + l_1 + \text{const.}$$

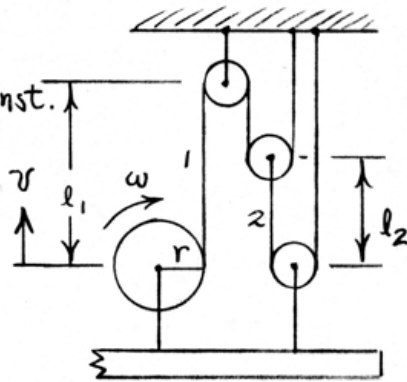
$$\dot{L}_2 = 0 = \dot{l}_2 + \dot{l}_1, \quad -\dot{l}_1 = \dot{l}_2$$

$$\text{But } v = -\dot{l}_1, \text{ so}$$

$$-r\omega = 3(-v) - 2v, \quad r\omega = 5v$$

$$v = \frac{r\omega}{5} = \frac{0.1(40)\left(\frac{2\pi}{60}\right)}{5} = 0.0838 \frac{\text{m}}{\text{s}}$$

$$\text{or } \underline{v = 83.8 \text{ mm/s}}$$



2/226

Length $L_1 = h + 2(l_1 - l_2) + \text{const.}$

$$\dot{L}_1 = -r\omega = 0 + 2\dot{q}_1 - 2\dot{q}_2$$

But $v = -\dot{l}_1$

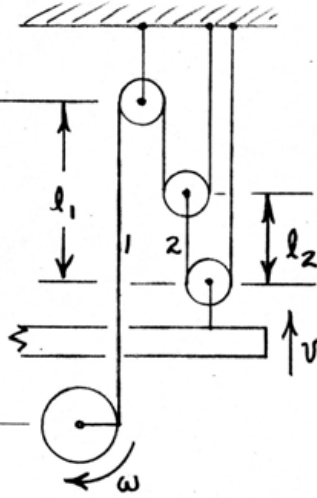
$$\text{So } -rw = -2v - 2\dot{l}_2$$

Length $L_2 = l_1 + l_2 + \text{const.}$

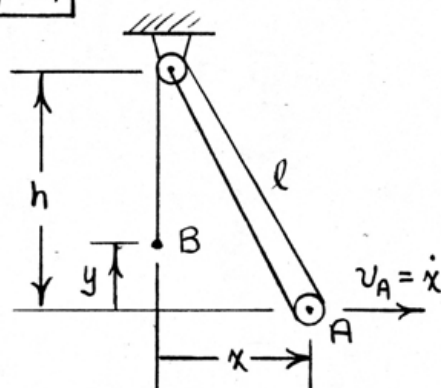
$$L_z = 0 = \dot{l}_1 + \dot{l}_2, \quad v = \dot{l}_2$$

$$\therefore r\omega = 2v + 2v, \quad v = \frac{r\omega}{4} = \frac{0.1(40)(2\pi/60)}{4}$$

$$v = 0.1047 \text{ m/s} \text{ or } v = 104.7 \frac{\text{mm}}{\text{s}}$$



2/227



Length of cable is

$$L = h - y + 2l + \text{constant} = h - y + 2\sqrt{x^2 + h^2} + \text{const.}$$

$$\dot{L} = 0 = -\dot{y} + \frac{2x\dot{x}}{\sqrt{x^2 + h^2}}$$

Substitute v_A for $\dot{x}$ and v_B for $\dot{y}$:

$$v_B = \frac{2x}{\sqrt{x^2 + h^2}} v_A \quad \left(\begin{array}{l} \text{4 times as fast as} \\ \text{with Sample Prob. 2/15} \end{array} \right)$$

2/228

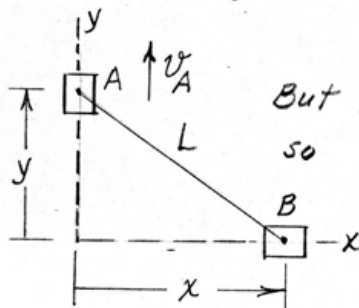
$$x^2 + y^2 = L^2; \quad x\dot{x} + y\dot{y} = 0$$

$$\neq \dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0$$

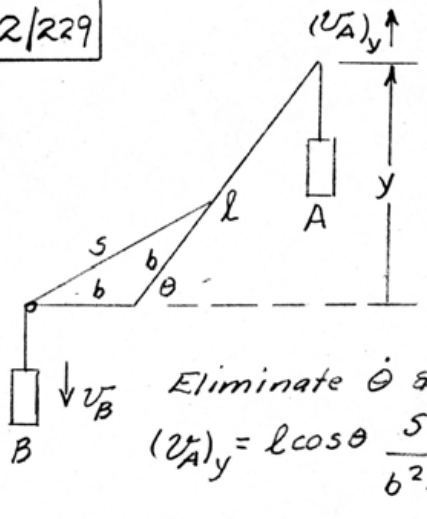
$$\text{But } \dot{y} = v_A, \quad \ddot{y} = 0$$

$$\text{so } a_x = \ddot{x} = -\frac{\dot{x}^2 + \dot{y}^2}{x} = -\frac{\dot{y}^2 \frac{y^2}{x^2} + \dot{y}^2}{x}$$

$$a_x = -\frac{L^2 \dot{y}^2}{x^3} = -\frac{L^2 v_A^2}{(L^2 - y^2)^{3/2}}$$



2/229



$$s^2 = 2b^2 + 2b^2 \cos \theta$$

$$= 2b^2(1 + \cos \theta)$$

$$2s\dot{s} = 2b^2(-\dot{\theta} \sin \theta)$$

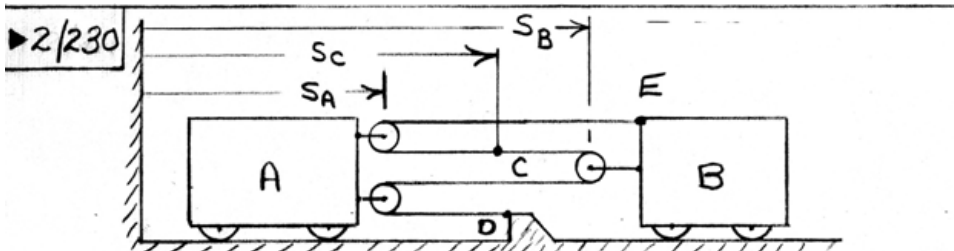
$$v_B = -\dot{s} = \frac{b^2}{s} \dot{\theta} \sin \theta$$

$$y = l \sin \theta$$

$$(v_A)_y = \dot{y} = l \dot{\theta} \cos \theta$$

Eliminate $\dot{\theta}$ & get

$$(v_A)_y = l \cos \theta \frac{s v_B}{b^2 \sin \theta} = \frac{l \sqrt{2(1 + \cos \theta)}}{b \tan \theta} v_B$$



Cable length $L = 3(s_B - s_A) + (s_D - s_A)$

$$0 = 3v_B - 4v_A, \quad 0 = 3a_B - 4a_A$$

$$v_A = \frac{3}{4}v_B = \frac{3}{4}(2) = 1.5 \text{ m/s}$$

$$a_A = \frac{3}{4}a_B = \frac{3}{4}(3) = 2.25 \text{ m/s}^2$$

$$v_{B/A} = v_B - v_A = 2 - (1.5) = \underline{0.5 \text{ m/s}}$$

$$a_{B/A} = a_B - a_A = 3 - (2.25) = \underline{0.75 \text{ m/s}^2}$$

Length of cable between points E and C:

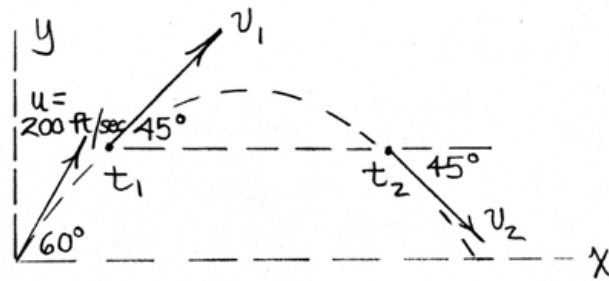
$$L' = (s_B - s_A) + (s_C - s_A) + \text{constants}$$

$$0 = v_B - 2v_A + v_C \Rightarrow v_C = 2v_A - v_B$$

$$\text{or } v_C = 2(1.5) - 2 = \underline{1 \text{ m/s}}$$

(All answers are quantities directed to right)

2/231



$$\dot{x} = u \cos \theta = 200 \cos 60^\circ = 100 \text{ ft/sec}$$

$$\dot{y} = u \sin \theta - gt = 200 \sin 60^\circ - 32.2t = 173.2 - 32.2t$$

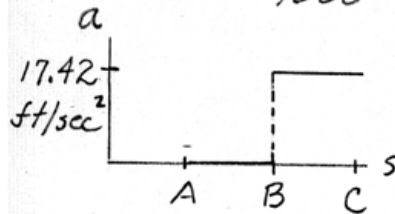
$$\text{At } t_1: \dot{x} = \dot{y}: 100 = 173.2 - 32.2t_1, \quad \underline{t_1 = 2.27 \text{ sec}}$$

$$\text{At } t_2: \dot{x} = -\dot{y}: 100 = -173.2 + 32.2t_2, \quad \underline{t_2 = 8.48 \text{ sec}}$$

2/232

From A to B $a = 0$ From B to C $a = a_n = v^2/\rho$

$$a_n = \frac{(90 \times 44 / 30)^2}{1000} = 17.42 \text{ ft/sec}^2$$



Abrupt acceleration would cause abrupt forces which would be uncomfortable for passengers.

A transition section to change curvature gradually over an interval of track would be required.

2/233

$$r = r_0 + b \sin \frac{2\pi t}{\tau}, \quad \dot{r} = \frac{2\pi}{\tau} b \cos \frac{2\pi t}{\tau}$$

$$\ddot{r} = -\frac{4\pi^2}{\tau^2} b \sin \frac{2\pi t}{\tau}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{4\pi^2}{\tau^2} b \sin \frac{2\pi t}{\tau} - r\dot{\theta}^2 = 0$$

$$\Rightarrow r = r_0 \frac{1}{1 + \left(\frac{\tau\dot{\theta}}{2\pi}\right)^2}$$

$$\boxed{2/234} \quad \dot{x} = 20 \text{ mm/s}, \quad \ddot{x} = 0$$

$$y = x^2/160, \quad \dot{y} = x\dot{x}/80, \quad \ddot{y} = (\dot{x}^2 + x\ddot{x})/80$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{\dot{x}^2 + (x\dot{x}/80)^2} = \dot{x} \sqrt{1 + (x/80)^2}$$

$$\text{For } x = 60 \text{ mm}$$

$$v = 20 \sqrt{1 + (60/80)^2} = \underline{25 \text{ mm/s}}$$

$$a = \ddot{y} = \dot{x}^2/80 \text{ since } \ddot{x} = 0, \quad a = (20)^2/80 = \underline{5 \text{ mm/s}^2}$$

2/235 y-dir, $v^2 = v_i^2 + 2as$; $0 = u_y^2 - 2g(12)$ A to B

$$u_y = \sqrt{2(9.81)(12)} = 15.34 \text{ m/s}$$

$$s = v_i t + \frac{1}{2} a t^2; -30 = 15.34 t - \frac{9.81}{2} t^2$$

$$t^2 - 3.128 t - 6.116 = 0,$$

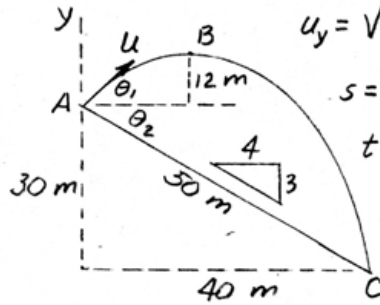
$$t = 4.49 \text{ s} \quad (t = -1.36 \text{ s})$$

$$x\text{-dir, } 40 = u_x (4.49), u_x = 8.91 \text{ m/s}$$

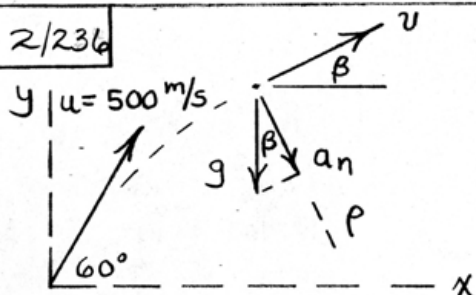
$$u = \sqrt{(8.91)^2 + (15.34)^2} = 17.74 \text{ m/s}$$

$$\theta_1 = \tan^{-1} \frac{15.34}{8.91} = 59.86^\circ; \theta_2 = \tan^{-1} \frac{3}{4} = 36.87^\circ$$

$$\theta = \theta_1 + \theta_2 = 59.86 + 36.87 = 96.7^\circ$$



2/236



$$v_x = 500 \cos 60^\circ = 250 \text{ m/s}$$

$$v_y = v_{y_0} - gt = 500 \sin 60^\circ - 9.81(30) = 138.7 \frac{\text{m}}{\text{s}}$$

$$v^2 = v_x^2 + v_y^2 = 250^2 + 138.7^2 = 81.7(10^3) \text{ m}^2/\text{s}^2$$

$$\beta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{138.7}{250} = 29.0^\circ$$

$$a_n = g \cos \beta = 9.81 \cos 29.0^\circ = 8.58 \text{ m/s}^2$$

$$a_n = v^2/\rho \Rightarrow \rho = \frac{v^2}{a_n} = \frac{81.7(10^3)}{8.58} = 9529 \text{ m}$$

$$\text{or } \underline{\underline{\rho = 9.53 \text{ km}}}$$

$$\boxed{2/237} \quad \theta = 4 [t + 30e^{-0.03t} - 30] \quad (\text{rad})$$

$$\dot{\theta} = 4 [1 - 0.9e^{-0.03t}] \quad (\text{rad/sec})$$

$$\ddot{\theta} = 0.1080e^{-0.03t} \quad (\text{rad/sec}^2)$$

$$r\dot{\theta}^2 = 30 [4(1 - 0.9e^{-0.03t})]^2 = 32.2(10)$$

$$(1 - 0.9e^{-0.03t})^2 = 0.671$$

$$(1 - 0.9e^{-0.03t}) = \pm 0.819$$

Take (+) as (-) will result in $t < 0$:

$$(1 - 0.9e^{-0.03t}) = 0.819 \Rightarrow \underline{t = 53.5 \text{ sec}}$$

$$\ddot{\theta} = 0.1080e^{-0.03(53.5)} = 0.0217 \text{ rad/sec}^2$$

$$r\ddot{\theta} = 30(0.0217) = 0.651 \text{ ft/sec}^2 \quad (0.020g)$$

Thus a_t can be neglected.

2/238

$$\underline{v}_A = \underline{v}_W + \underline{v}_{A/W} = -48\underline{i} + 220\underline{j} = 172\underline{j} \frac{\text{km}}{\text{h}}$$

y

L --- x

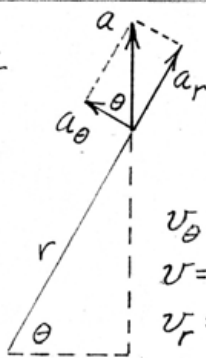
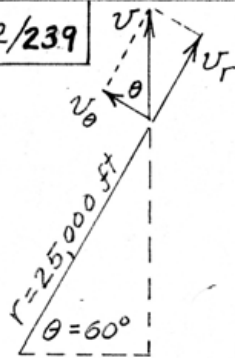
On descent: $\underline{v}_A = 172(\cos 10^\circ \underline{i} - \sin 10^\circ \underline{j})$
km/h

$$\underline{v}_{A/c} = \underline{v}_A - \underline{v}_c = 172(\cos 10^\circ \underline{i} - \sin 10^\circ \underline{j}) - 30\underline{i}$$

$$= 139.4\underline{i} - 29.9\underline{j} \text{ km/h}$$

$$\beta = \tan^{-1} \left(\frac{29.9}{139.4} \right) = \underline{12.09^\circ}$$

2/239

Given $r = 25,000 \text{ ft}$ $\dot{\theta} = 0.03 \text{ rad/sec}$ $\theta = 60^\circ$ $a = 64 \text{ ft/sec}^2$

$$v_\theta = r\dot{\theta} = 25(10^3)(0.03) = 750 \text{ ft/sec}$$

$$v = 750 / \cos 60^\circ = 1500 \text{ ft/sec}$$

$$v_r = \dot{r} = 1500 \cos 30^\circ = 1299 \text{ ft/sec}$$

$$a_\theta = a \cos 60^\circ = 64(0.5) = 32 \text{ ft/sec}^2$$

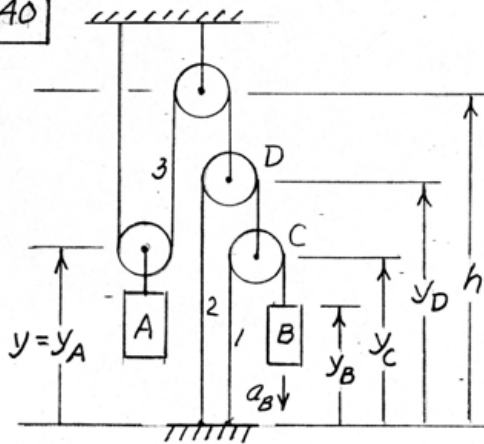
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}, \text{ so } r\ddot{\theta} = 32 - 2(1299)(0.03) = -45.94$$

$$\ddot{\theta} = -\frac{45.94}{25,000} = -1.838(10^{-3}) \text{ rad/sec}^2$$

$$a_r = a \sin 60^\circ = 64(\sqrt{3}/2) = 55.43 \text{ ft/sec}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \text{ so } \ddot{r} = 55.43 + 25(10^3)(0.03)^2 = 77.9 \text{ ft/sec}^2$$

2/240



$$y = y_A = t^2/4 \text{ m}$$

$$\dot{y}_A = t/2 \text{ m/s}$$

$$a_A = \ddot{y}_A = 1/2 \text{ m/s}^2$$

one degree of freedom

Cable lengths

$$L_1 = y_C + (y_C - y_B) + C_1 \quad 0 = 2\ddot{y}_C - \ddot{y}_B \quad \text{where } a_B = -\ddot{y}_B$$

$$L_2 = y_D + (y_D - y_C) + C_2 \quad 0 = 2\ddot{y}_D - \ddot{y}_C$$

$$L_3 = 2(h - y_A) + h - y_D + C_3, \quad 0 = -2\ddot{y}_A - \ddot{y}_D$$

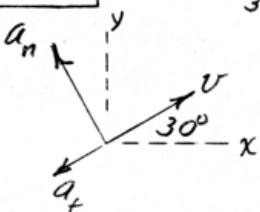
$$\text{Eliminate } \ddot{y}_C \text{ \& } \ddot{y}_D \text{ \& get } \ddot{y}_B = -8\ddot{y}_A \text{ or } a_B = 8\ddot{y}_A = 4 \text{ m/s}^2$$

(By inspection of pulley displacements

$$-dy_B = 8dy_A, \text{ so } a_B = 8a_A = 8(1/2) = 4 \text{ m/s}^2)$$

2/241-

$$v = \frac{1000}{3.6} = 278 \text{ m/s}, \quad a_t = \frac{15}{3.6} = 4.17 \text{ m/s}^2$$

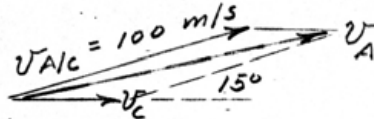
$$a_n = \frac{v^2}{\rho} = \frac{(278)^2}{1500} = 51.4 \text{ m/s}^2$$


$$\ddot{x} = -51.4 \sin 30^\circ - 4.17 \cos 30^\circ = \underline{-29.3 \text{ m/s}^2}$$

$$\ddot{y} = 51.4 \cos 30^\circ - 4.17 \sin 30^\circ = \underline{42.5 \text{ m/s}^2}$$

2/242 Carrier deck has a constant velocity; so may be used as an inertial coordinate base.
velocity of aircraft relative to carrier is

$$v_{A/C}^2 = 2as = 2(50)(100) = 10\,000 \text{ (m/s)}^2, v_{A/C} = 100 \text{ m/s}$$

$$\vec{v}_A = \vec{v}_C + \vec{v}_{A/C}$$


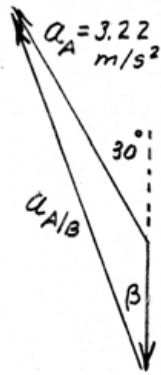
$v_{A/C} = 100 \text{ m/s}$
 15°

$$v_C = 30(1.852)/3.6 = 15.43 \text{ m/s}$$

$$v_A^2 = (100)^2 + (15.43)^2 + 2(100)(15.43)\cos 15^\circ = 13\,220 \text{ (m/s)}^2$$

$$v_A = 115.0 \text{ m/s or } v_A = v = 115.0(3.6) = \underline{414 \text{ km/h}}$$

2/243 $\underline{a_A = a_B + a_{A/B}}$, $a_A = v_A^2/\rho = (50/3.6)^2/60 = 3.22 \text{ m/s}^2$



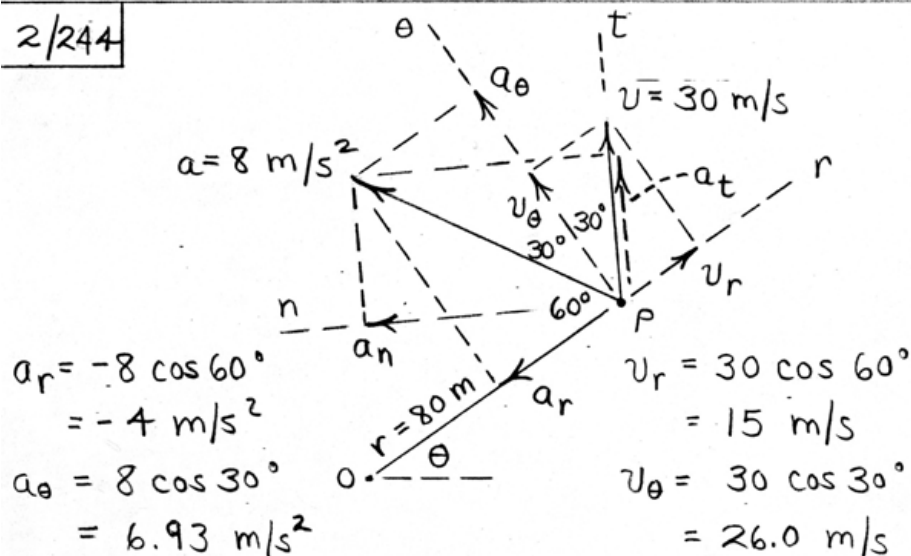
$$a_{A/B} = \sqrt{\left(3.22 \frac{\sqrt{3}}{2} + 1.5\right)^2 + \left(3.22 \left[\frac{1}{2}\right]\right)^2}$$

$$= 4.58 \text{ m/s}^2$$

$$\beta = \tan^{-1} \frac{1.608}{4.28} = \tan^{-1} 0.3752$$

$$= 20.6^\circ \text{ west of north}$$

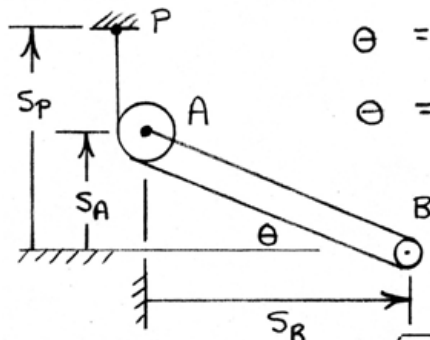
2/244



$$\begin{aligned}
 [r-\theta] \quad v_r &= \dot{r} = 15 \text{ m/s} \\
 v_\theta &= r\dot{\theta} : 26.0 = 80\dot{\theta}, \quad \dot{\theta} = 0.325 \text{ rad/s} \\
 a_r &= \ddot{r} - r\dot{\theta}^2 : -4 = \ddot{r} - 80(0.325)^2, \quad \ddot{r} = 4.44 \text{ m/s}^2 \\
 a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} : 6.93 = 80\ddot{\theta} + 2(15)(0.325), \quad \ddot{\theta} = -0.0352 \text{ rad/s}^2
 \end{aligned}$$

$$\begin{aligned}
 [n-t] : a_n &= 8 \cos 30^\circ = 6.93 \text{ m/s}^2 \\
 a_t &= 8 \sin 30^\circ = 4 \text{ m/s}^2 \\
 a_n &= \frac{v^2}{r}, \quad r = \frac{v^2}{a_n} = \frac{30^2}{6.93} = 129.9 \text{ m}
 \end{aligned}$$

2/245



$$\theta = 45^\circ : s_B/s_A = 1$$

$$\theta = 30^\circ : s_B/s_A = \sqrt{3}$$

$$\theta = 15^\circ : s_B/s_A = 3.73$$

$$L = (s_P - s_A) + 2\sqrt{s_A^2 + s_B^2}$$

$$0 = -v_A + 2(s_A v_A + s_B v_B) / \sqrt{s_A^2 + s_B^2}$$

$$\Rightarrow v_B = \left[\sqrt{1 + (s_B/s_A)^2} - 2 \right] \frac{v_A}{2(s_B/s_A)}$$

$$\theta = 45^\circ : v_B = -0.293 v_A = -0.293(-1) = \underline{0.293 \frac{m}{s}}$$

$$\theta = 30^\circ : v_B = \underline{0}$$

$$\theta = 15^\circ : v_B = 0.250 v_A = 0.250(-1) = \underline{-0.250 \frac{m}{s}}$$

$$\begin{aligned}
 & \boxed{2/246} \left\{ \begin{aligned} x &= 50 \text{ ft} & \dot{x} &= -10 \text{ ft/sec} & \ddot{x} &= -10 \text{ ft/sec}^2 \\ y &= 25 \text{ ft} & \dot{y} &= 10 \text{ ft/sec} & \ddot{y} &= 5 \text{ ft/sec}^2 \end{aligned} \right. \\
 v &= \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(-10)^2 + 10^2} = 10\sqrt{2} \text{ ft/sec} \\
 a &= \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(-10)^2 + 5^2} = 11.18 \text{ ft/sec}^2 \\
 \underline{e}_t &= \underline{v}/v = (-10\underline{i} + 10\underline{j})/10\sqrt{2} = \frac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) \\
 \underline{a}_t &= \underline{a} \cdot \underline{e}_t = (-10\underline{i} + 5\underline{j}) \cdot \frac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) = 10.61 \text{ ft/sec}^2 \\
 \underline{a}_t &= \underline{a}_t \underline{e}_t = 10.61 \frac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) = -7.5\underline{i} + 7.5\underline{j} \text{ ft/sec}^2 \\
 \underline{a}_n &= \underline{a} - \underline{a}_t = (-10\underline{i} + 5\underline{j}) - (-7.5\underline{i} + 7.5\underline{j}) = -2.5(\underline{i} + \underline{j}) \text{ ft/sec}^2 \\
 a_n &= \sqrt{2.5^2 + 2.5^2} = 3.54 \text{ ft/sec}^2 \\
 \rho &= v^2/a_n = (10\sqrt{2})^2/3.54 = 56.6 \text{ ft} \\
 \underline{e}_n &= \underline{a}_n/a_n = -2.5(\underline{i} + \underline{j})/3.54 = -\frac{\sqrt{2}}{2}(\underline{i} + \underline{j}) \\
 \underline{e}_r &= \underline{r}/r = 50\underline{i} + 25\underline{j}/\sqrt{50^2 + 25^2} = 0.894\underline{i} + 0.447\underline{j} \\
 \underline{e}_\theta &= \underline{e}_r \text{ rotated CCW } 90^\circ = -0.447\underline{i} + 0.894\underline{j} \\
 \underline{v}_r &= \underline{v} \cdot \underline{e}_r = (-10\underline{i} + 10\underline{j}) \cdot (0.894\underline{i} + 0.447\underline{j}) = -4.47 \text{ ft/sec} \\
 \underline{v}_r &= v_r \underline{e}_r = -4.47(0.894\underline{i} + 0.447\underline{j}) = -4\underline{i} - 2\underline{j} \text{ ft/sec} \\
 \underline{v}_\theta &= \underline{v} \cdot \underline{e}_\theta = (-10\underline{i} + 10\underline{j}) \cdot (-0.447\underline{i} + 0.894\underline{j}) = 13.42 \text{ ft/sec} \\
 \underline{v}_\theta &= v_\theta \underline{e}_\theta = 13.42(-0.447\underline{i} + 0.894\underline{j}) = -6\underline{i} + 12\underline{j} \text{ ft/sec} \\
 \underline{a}_r &= \underline{a} \cdot \underline{e}_r = (-10\underline{i} + 5\underline{j}) \cdot (0.894\underline{i} + 0.447\underline{j}) = -6.71 \text{ ft/sec}^2 \\
 \underline{a}_r &= a_r \underline{e}_r = -6.71(0.894\underline{i} + 0.447\underline{j}) = -6\underline{i} - 3\underline{j} \text{ ft/sec}^2
 \end{aligned}$$

$$a_\theta = \underline{a} \cdot \underline{e}_\theta = (-10\hat{i} + 5\hat{j}) \cdot (-0.447\hat{i} + 0.894\hat{j}) = \underline{8.94 \text{ ft/sec}^2}$$

$$\underline{a}_\theta = a_\theta \underline{e}_\theta = 8.94(-0.447\hat{i} + 0.894\hat{j}) = \underline{-4\hat{i} + 8\hat{j} \text{ ft/sec}^2}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{50^2 + 25^2} = \underline{55.9 \text{ ft}}$$

$$\dot{r} = v_r = \underline{-4.47 \text{ ft/sec}}$$

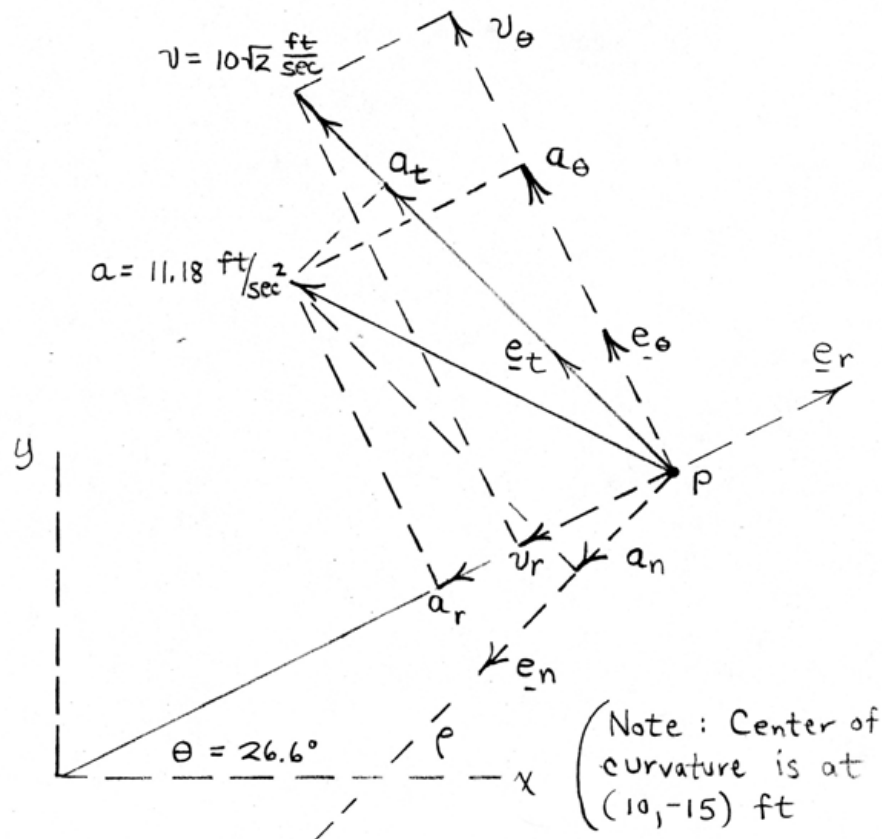
$$v_\theta = r\dot{\theta}, \dot{\theta} = v_\theta/r = 13.42/55.9 = \underline{0.240 \text{ rad/sec}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \ddot{r} = a_r + r\dot{\theta}^2 = -6.71 + 55.9(0.240)^2 = \underline{-3.49 \text{ ft/sec}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}, \ddot{\theta} = \frac{1}{r}(a_\theta - 2\dot{r}\dot{\theta})$$

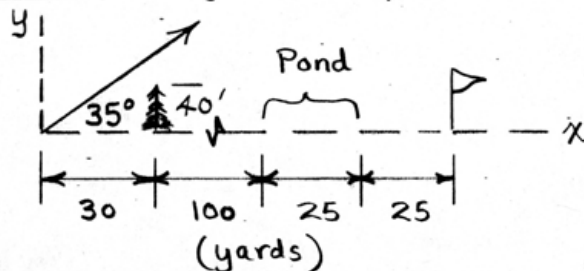
$$= \frac{1}{55.9} [8.94 - 2(-4.47)(0.240)] = \underline{0.1984 \text{ rad/sec}^2}$$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(25/50) = \underline{26.6^\circ}$$



2/247

$$v_0 = 125 \text{ ft/sec}$$



Time to tree: $x = x_0 + v_{x_0} t$: $90 = 0 + 125 \cos 35^\circ t$

$$t = 0.879 \text{ sec}$$

Altitude : $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$$y = 0 + 125 \sin 35^\circ (0.879) - 16.1 (0.879)^2 = 50.6 \text{ ft}$$

So ball clears (slender) tree.

Flight time (y -eq.) : $0 = 0 + 125 \sin 35^\circ t_f - 16.1 t_f^2$

$$t_f = 0 \text{ (launch time) or } t = 4.45 \text{ sec (impact time)}$$

Range (x -eq.) : $R = 0 + 125 \cos 35^\circ (4.45)$

$$= 456 \text{ ft or } \underline{152.0 \text{ yd}}$$

Ball lands in water hazard!

$$\frac{2}{248} \quad v_0 = 27000/3.6 = 7500 \text{ m/s}$$

$$H = 35(10^4) \text{ m}$$

$$g_0 = 9.832 \text{ m/s}^2 \text{ (Fig. 1/1)}$$

$$R = 6371 \text{ km or } 6.371(10^6) \text{ m}$$

$$g = g_0 \left(\frac{R}{R+H+y} \right)^2$$

From $a = v \frac{dv}{dy}$, we have

$$\int_{v_0}^0 v dv = \int_0^h -g_0 \left(\frac{R}{R+H+y} \right)^2 dy$$

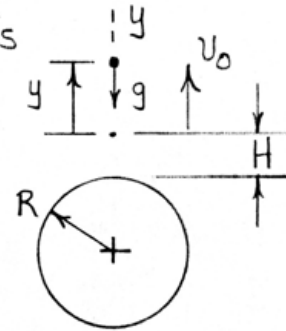
$$-\frac{1}{2} v_0^2 = g_0 R^2 \frac{1}{R+H+y} \Big|_0^h$$

$$v_0^2 = 2g_0 R^2 \frac{h}{(R+H)(R+H+h)}$$

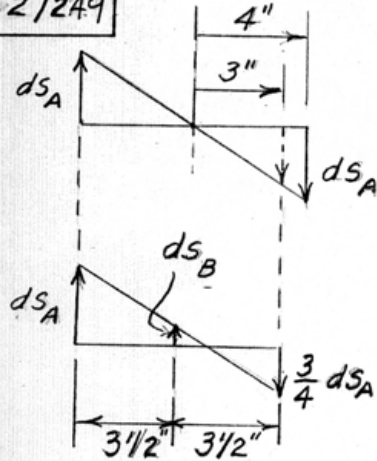
$$\text{Solve for } h: h = \frac{(R+H)^2 v_0^2}{2g_0 R^2 - (R+H)v_0^2}$$

Substitute numerical values and obtain

$$h = 6048(10^3) \text{ m or } \underline{h = 6048 \text{ km}}$$



2/249



$$ds_B = \text{average of } ds_A \text{ and } -\frac{3}{4}ds_A$$

$$= \frac{ds_A + (-\frac{3}{4}ds_A)}{2} = \frac{1}{8}ds_A$$

$$\text{so } a_B = \frac{1}{8}a_A$$

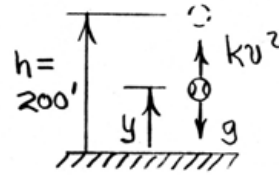
$$= \frac{2}{8} = 0.25 \text{ ft/sec}^2$$

*2/250 $a = v \frac{dv}{dy} = -g + kv^2$

$$\int_0^v \frac{v dv}{-g + kv^2} = \int_h^y dy$$

$$\frac{1}{2k} \ln [-g + kv^2]_0^v = y \Big|_h^y$$

$$\frac{1}{2k} \ln \left[\frac{-g + kv^2}{-g} \right] = y - h \Rightarrow v = \sqrt{\frac{g}{k} [1 - e^{2k(y-h)}]}$$



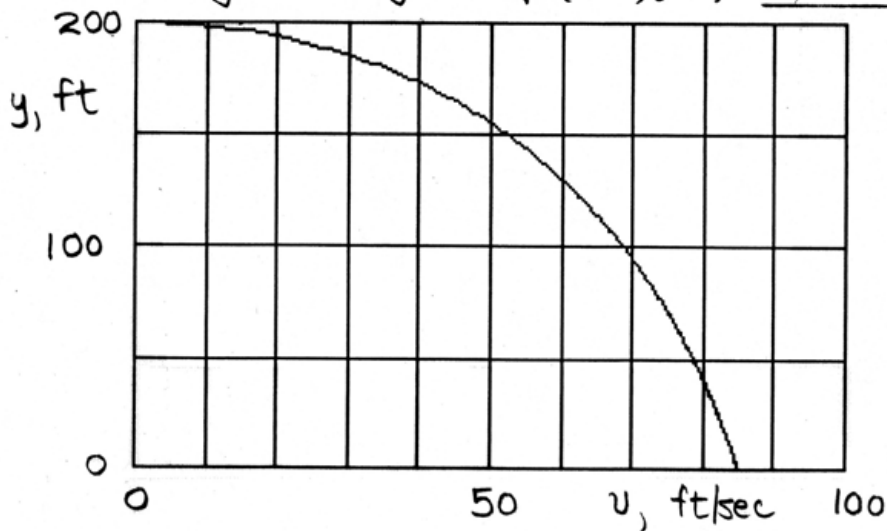
Given numbers : $85 = \sqrt{\frac{32.2}{k} [1 - e^{2k(0-200)}]}$

Numerical solution : $k = 0.00323 \text{ ft}^{-1}$

Terminal speed : $g = kv^2 : 32.2 = 0.00323 v_t^2$

$v_t = 99.8 \text{ ft/sec}$

Without drag : $v' = \sqrt{2gh} = \sqrt{2(32.2)(200)} = 113.5 \text{ ft/sec}$



$$*2/251 \quad a_t = \frac{dv}{dt} = g \cos \theta - \frac{k}{m} v$$

$$\text{With } v = r\dot{\theta}: \quad \frac{d}{dt}(r\dot{\theta}) = g \cos \theta - \frac{k}{m} (r\dot{\theta})$$

$$\text{or } \frac{d^2\theta}{dt^2} + \frac{k}{m} \frac{d\theta}{dt} - \frac{g}{r} \cos \theta = 0$$

This is a nonlinear, second-order differential equation, so a numerical integration is in order.

To switch to first order form, we let

$$x_1 = \theta \quad \& \quad x_2 = \dot{\theta}$$

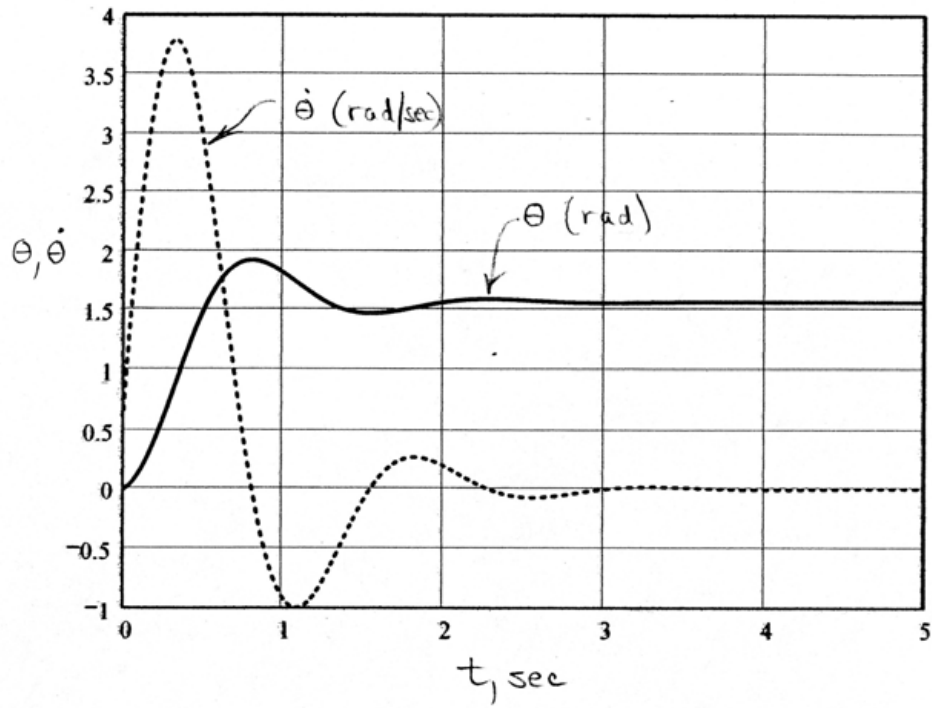
$$\text{So } \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m} x_2 + \frac{g}{r} \cos x_1 \end{cases} \quad \left\| \begin{array}{l} x_{10} = \theta_0 = 0 \\ x_{20} = \dot{\theta}_0 = \frac{v_0}{r} \end{array} \right.$$

The plots below show θ and $\dot{\theta}$ as functions of t .

$$\underline{\theta_{\max} = 110.4^\circ \text{ @ } t = 0.802 \text{ sec}}$$

$$\underline{\dot{\theta}_{\max} = 3.79 \text{ rad/sec @ } t = 0.324 \text{ sec}}$$

$$\underline{\theta = 90^\circ \text{ @ } t = 0.526 \text{ sec}}$$



*2/252 $\ddot{\theta} = \dot{\theta} \frac{d\dot{\theta}}{d\theta} = \frac{g}{l} \cos \theta$

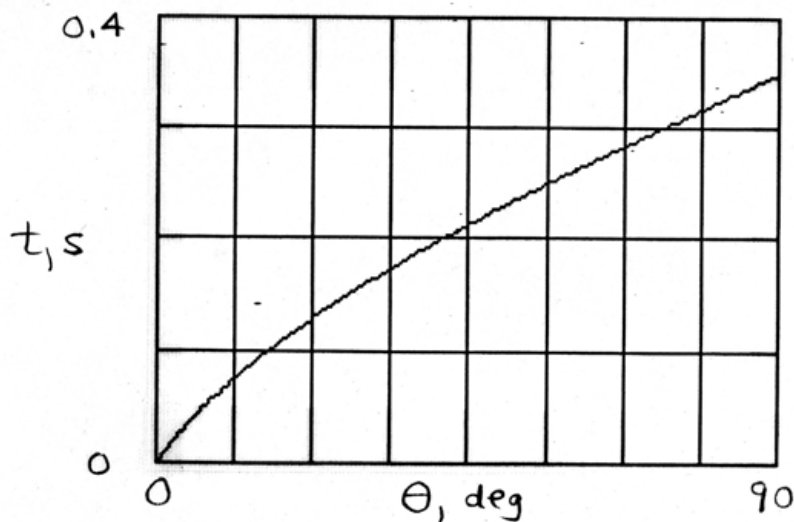
$$\int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta} d\dot{\theta} = \frac{g}{l} \int_0^{\theta} \cos \theta d\theta$$

$$\dot{\theta} = \left[\dot{\theta}_0^2 + \frac{2g}{l} \sin \theta \right]^{1/2}$$

Then $\dot{\theta} = \frac{d\theta}{dt} = \left[\dot{\theta}_0^2 + \frac{2g}{l} \sin \theta \right]^{1/2}$

$$t = \int_0^{\theta} \frac{d\theta}{\sqrt{\dot{\theta}_0^2 + \frac{2g}{l} \sin \theta}}$$

With $\dot{\theta}_0 = 2 \text{ rad/s}$, $l = 0.6 \text{ m}$, $g = 9.81$, $\theta = \frac{\pi}{2}$,
a numerical integration yields $t' = 0.349 \text{ s}$.



*2/253 $v dv = a ds, \quad a = \frac{T - 4.50v^2}{m}$

$$\int_0^v \frac{m v dv}{T - 4.50v^2} = \int_0^s ds; \quad s = \frac{m}{9.00} \ln \frac{T}{T - 4.50v^2}$$

$$e^{9.00s/m} = \frac{T}{T - 4.50v^2}, \quad v = \sqrt{\frac{T}{4.50} (1 - e^{-9.00s/m})}$$

where $T = 250 \text{ kN}$, $s = \text{distance in meters}$, $m = 16000 \text{ tons (metric)}$
 (1 metric ton = 1000 kg)

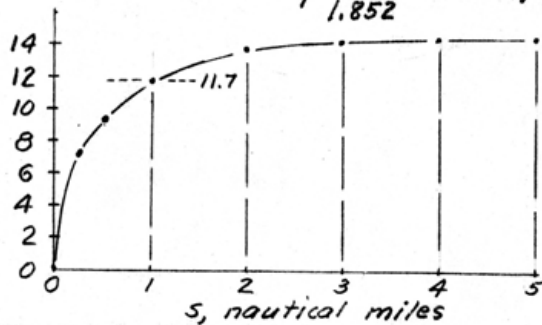
$v = \text{speed in m/s}$

or if $v = \text{speed in knots}$ & $s = \text{distance in nautical miles}$, then

$$v = \sqrt{\frac{250}{4.50} (1 - e^{-9.00 \times 1852s / 16000})} \frac{3.6}{1.852} = 14.49 \sqrt{1 - e^{-1.042s}}$$

$$v_{1 \text{ mi}} = 11.66 \text{ knots}$$

$$v_{\text{max}} = 14.49 \text{ knots}$$



*2/254 Let $\omega_n = \sqrt{g/l}$:
$$\begin{cases} \theta = \theta_0 \sin \omega_n t \\ \dot{\theta} = \theta_0 \omega_n \cos \omega_n t \\ \ddot{\theta} = -\theta_0 \omega_n^2 \sin \omega_n t \end{cases}$$

$$a_t = l\ddot{\theta} = -l\theta_0 \omega_n^2 \sin \omega_n t = -g\theta_0 \sin \omega_n t$$

$$a_n = l\dot{\theta}^2 = l\theta_0^2 \omega_n^2 \cos^2 \omega_n t = g\theta_0^2 \cos^2 \omega_n t$$

$$a = \sqrt{a_t^2 + a_n^2} = g\theta_0 \sqrt{\sin^2 \omega_n t + \theta_0^2 \cos^2 \omega_n t}$$

a^2 (and therefore a) is an extreme when

$$\frac{da^2}{d\theta} = 0 = g^2 \theta_0^2 [2 \sin \omega_n t (\cos \omega_n t) + \theta_0^2 4 \cos^3 \omega_n t (-\sin \omega_n t)]$$

$$\Rightarrow [1 - 2\theta_0^2 \cos^2 \omega_n t] = 0$$

$$[1 - 2\left(\frac{\pi}{3}\right)^2 \cos^2 \omega_n t] = 0, \quad \omega_n t = 0.830 \text{ rad}$$

$$\text{With } \omega_n = \sqrt{g/l} = \sqrt{\frac{9.81}{0.8}} = 3.50 \text{ s}^{-1}, t = 0.237 \text{ s}$$

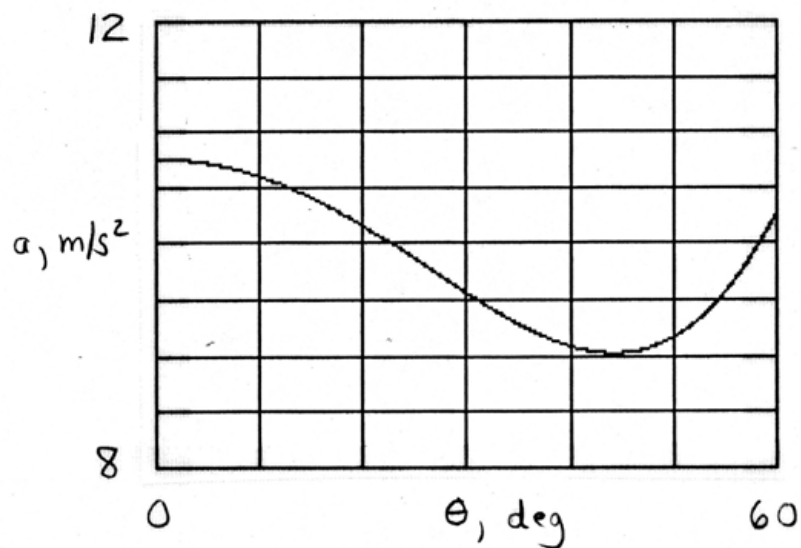
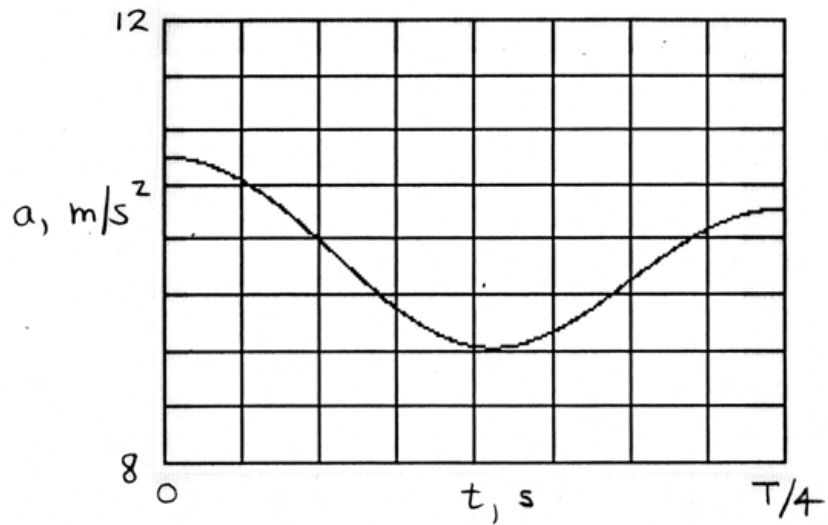
$$\theta = \frac{\pi}{3} \sin(0.830) = 0.772 \text{ rad } (44.3^\circ)$$

As can be seen from the plots below,

the above represents a minimum:

$$a_{\min} = 9.03 \text{ m/s}^2 \text{ @ } \theta = 44.3^\circ \text{ \& } t = 0.237 \text{ s}$$

$$a_{\max} = 10.76 \text{ m/s}^2 \text{ @ } \theta = 0 \text{ \& } t = 0$$



Note : Period $T = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.50} = 1.794 \text{ s}$

$$\frac{T}{4} = 0.449 \text{ s}$$

*2/255

$$a = \frac{dv}{dt} = c_1 - c_2 v^2$$

$$\int_0^t dt = \int_0^v \frac{dv}{c_1 - c_2 v^2} = \frac{1}{\sqrt{c_1 c_2}} \tanh^{-1} \sqrt{\frac{c_2}{c_1}} v \Big|_0^v$$

$$t = \frac{1}{\sqrt{c_1 c_2}} \tanh^{-1} \sqrt{\frac{c_2}{c_1}} v$$

$$\text{Then } v = \frac{ds}{dt} = \sqrt{\frac{c_1}{c_2}} \tanh \sqrt{c_1 c_2} t$$

$$\int_0^s ds = \sqrt{\frac{c_1}{c_2}} \int_0^t \tanh \sqrt{c_1 c_2} t dt$$

$$s = \sqrt{\frac{c_1}{c_2}} \cdot \frac{1}{\sqrt{c_1 c_2}} \ln (\cosh \sqrt{c_1 c_2} t) \Big|_0^t$$

$$= \frac{1}{c_2} \ln (\cosh \sqrt{c_1 c_2} t)$$

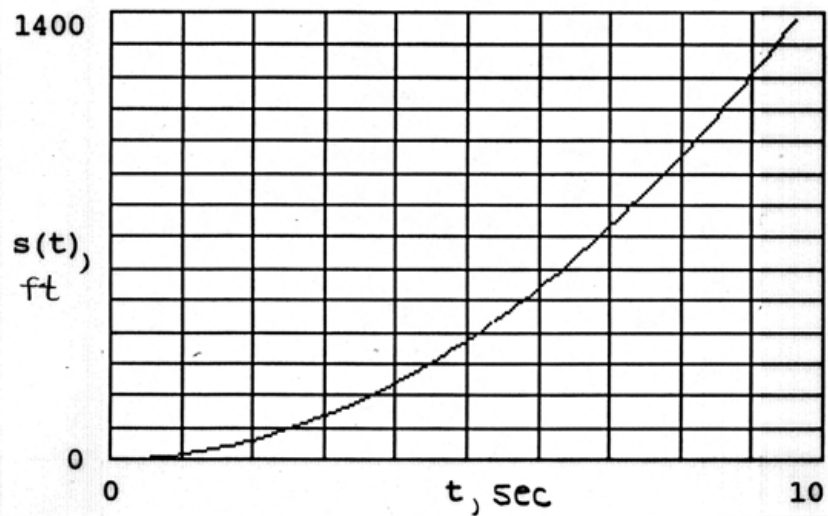
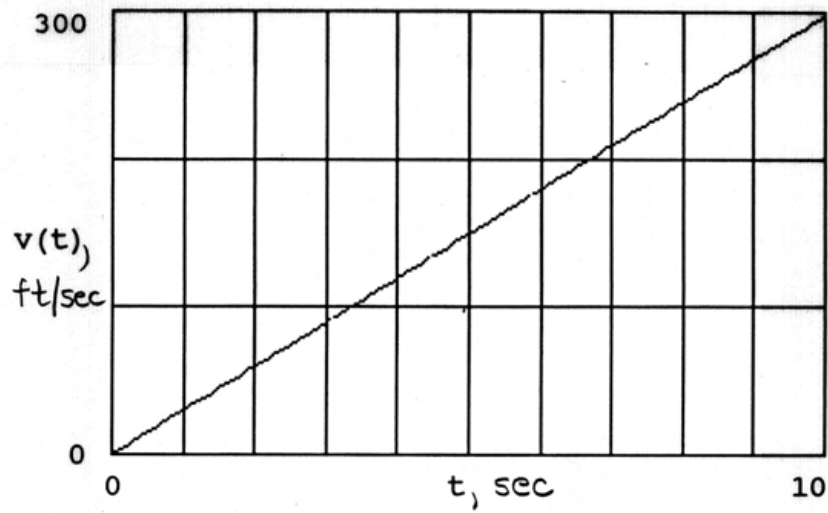
With $s = 1320$ ft, $c_1 = 30$ ft/sec², $\dagger t = 9.4$ sec:

$$1320 = \frac{1}{c_2} \ln (\cosh \sqrt{30 c_2} \cdot 9.4)$$

$$= \frac{1}{c_2} \ln (\cosh 51.5 \sqrt{c_2})$$

Numerical solution: $c_2 = 9.28 (10^{-6}) \text{ ft}^{-1}$

(On plots below, note that v vs. t appears linear, but it is not!)



*2/256 $a = v \frac{dv}{dy} = -g - kv^2$

$$\int_{v_0}^0 \frac{v dv}{-g - kv^2} = \int_0^h dy$$

Let $u = g + kv^2$, $du = 2kv dv$: $\int_{v=v_0}^{v=0} -\frac{1}{2k} \frac{du}{u} = h$

$$h = -\frac{1}{2k} \ln(g + kv^2) \Big|_{v_0}^0 = -\frac{1}{2k} \ln\left(\frac{g}{g + kv_0^2}\right)$$

$$2kh = \ln\left(1 + \frac{kv_0^2}{g}\right): 2(5280)k = \ln\left(1 + \frac{2000^2 k}{32.2}\right)$$

Solve numerically to obtain $k = 3.63 (10^{-4}) \text{ ft}^{-1}$

*2/257 $y = x^2/4$, x & y in inches ; $x = 4 \sin 2t$, t in seconds

$$\dot{y} = \frac{x\dot{x}}{2} = 2 \sin 2t (8 \cos 2t) = 16 \sin 2t \cos 2t \quad \text{in./sec}$$

$$\dot{x} = 8 \cos 2t$$

$$v^2 = \dot{x}^2 + \dot{y}^2 = 64 \cos^2 2t + 256 \sin^2 2t \cos^2 2t$$

$$= 64 \cos^2 2t (1 + 4 \sin^2 2t) \quad (\text{in./sec})^2$$

$$v = 8 \cos 2t \sqrt{1 + 4 \sin^2 2t}$$

$$\frac{dv}{dt} = 0 \quad \text{gives}$$

$$1 - 2 \sin^2 2t = 1/4$$

$$\sin 2t = \sqrt{3/8}$$

$$\cos 2t = \sqrt{5/8}$$

$$v = 8 \sqrt{\frac{5}{8}} \sqrt{1 + 4(3/8)}$$

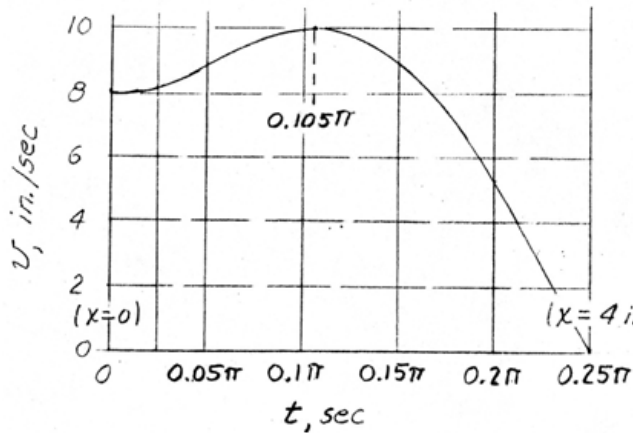
$$= 10 \text{ in./sec}$$

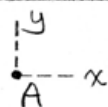
$$2t = 0.659 \text{ rad}$$

$$t = 0.330 \text{ sec}$$

$$(\text{= } 0.105 \pi \text{ sec})$$

$$\text{@ } x = 2.45 \text{ in.}$$



*2/258 Set up x-y coordinates @ A : 
 Let coordinates of B be (R, h) .

$$x = x_0 + v_{x_0} t \text{ @ B : } R = 0 + (v_0 \cos \alpha) t_f \quad (1)$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \text{ @ B : } h = 0 + (v_0 \sin \alpha) t_f - \frac{1}{2} g t_f^2 \quad (2)$$

Solve (1) & (2) for R & t_f :

$$\left. \begin{aligned} R &= \frac{v_0 \cos \alpha}{g} \left[v_0 \sin \alpha + \sqrt{v_0^2 \sin^2 \alpha - 2gh} \right] \\ t_f &= \frac{1}{g} \left[v_0 \sin \alpha + \sqrt{v_0^2 \sin^2 \alpha - 2gh} \right] \end{aligned} \right\}$$

where the + sign has been taken to maximize R .

Set $\frac{dR}{d\alpha} = 0$ (done by computer) to find

$$\underline{\alpha = 48.5^\circ} \quad (\text{for } h = 10 \text{ m, } v_0 = 30 \text{ m/s,} \\ \text{and } g = 9.81 \text{ m/s}^2)$$

The corresponding value of R is

$$\underline{R = 81.1 \text{ m}}$$

So the 10-m plateau is indeed achieved,
 as assumed (because $R > 50 \text{ m}$).