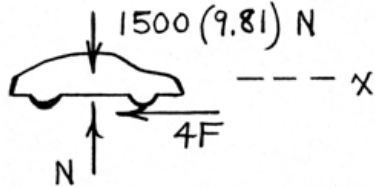


3/1

$$v_2^2 - v_1^2 = 2a(x_2 - x_1)$$

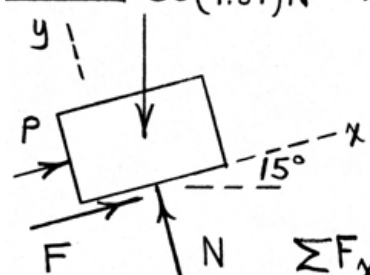
$$0^2 - \left(\frac{100}{3.6}\right)^2 = 2a_x(50), a_x = -7.72 \text{ m/s}^2$$



$$\Sigma F_x = ma_x: -4F = 1500(-7.72)$$

$$F = 2890 \text{ N}$$

3/2



$\Sigma F_y = 0: N - 50(9.81) \cos 15^\circ = 0$
 $N = 474 \text{ N}$ Throughout
 (a) $P = 0$
 Equilibrium check:
 $\Sigma F_x = 0: F - 50(9.81) \sin 15^\circ = 0$
 $F = 127.0 \text{ N}$

$F_{\max} = \mu_s N = 0.2(474) = 94.8 \text{ N} < F: \text{motion} \leftarrow$
 $\Sigma F_x = ma_x: 0.15(474) - 50(9.81) \sin 15^\circ = 50a_x$
 $a_x = -1.118 \text{ m/s}^2$

(b) $P = 150 \text{ N}$; Equilibrium check:

$\Sigma F_x = 0: 150 + F - 50(9.81) \sin 15^\circ = 0$

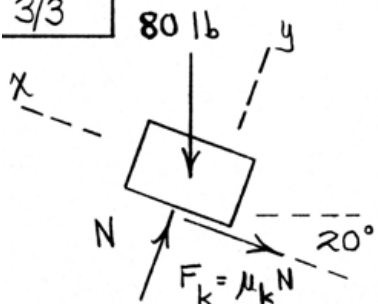
$F = -23.0 \text{ N}, |F| < F_{\max} \text{ so no motion: } \underline{a = 0}$

(c) $P = 300 \text{ N}$; Equilibrium check yields $F = -173.0 \text{ N}$

$|F| > F_{\max}, \text{ so motion } \rightarrow, F = F_k \leftarrow$

$\Sigma F_x = ma_x: 300 - 0.15(474) - 50(9.81) \sin 15^\circ = 50a_x$
 $a_x = 2.04 \text{ m/s}^2$

3/3



$$\sum F_y = 0 : N - 80 \cos 20^\circ = 0$$

$$N = 75.2 \text{ lb}$$

$$\sum F_x = ma_x :$$

$$-0.25(75.2) - 80 \sin 20^\circ = \frac{80}{32.2} a$$

$$a = -18.58 \text{ ft/sec}^2$$

$$v = v_0 + at : 0 = +30 - 18.58t, \quad \underline{t = 1.615 \text{ sec}}$$

$$v^2 = v_0^2 + 2a(s - s_0) : 0^2 = 30^2 + 2(-18.58)d$$

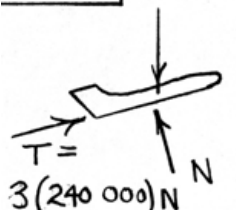
$$\underline{d = 24.2 \text{ ft}}$$

$$v^2 = v_0^2 + 2a(s - s_0) : 15^2 = 30^2 + 2(-18.58)d'$$

$$\underline{d' = 18.17 \text{ ft}}$$

3/4

$$300\,000(9.81)\text{ N}$$

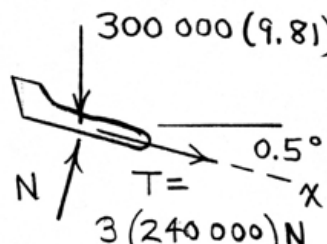


$$\sum F_x = ma_x :$$

$$3(240\,000) - 300\,000(9.81)\sin\frac{1}{2}^\circ = 300\,000 a_x$$

$$a_x = 2.31\text{ m/s}^2$$

$$v^2 = 2a_x s : \left(\frac{220}{3.6}\right)^2 = 2(2.31)s, \quad \underline{s_u = 807\text{ m}}$$



$$\sum F_x = ma_x :$$

$$3(240\,000) + 300\,000(9.81)\sin\frac{1}{2}^\circ = 300\,000 a_x$$

$$a_x = 2.49\text{ m/s}^2$$

$$v^2 = 2a_x s : \left(\frac{220}{3.6}\right)^2 = 2(2.49)s, \quad \underline{s_d = 751\text{ m}}$$

3/5 For entire unit:

$\Sigma F = ma: 20\,000 = 30\,000a$

$a = 0.667 \text{ m/s}^2$

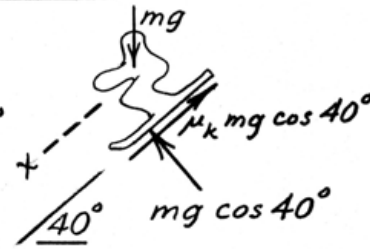
For trailer alone, $T = 20\,000(0.667) = 13\,330 \text{ N}$

or $T = 13.33 \text{ kN}$

3/6

$$\Sigma F_x = ma_x: mg \sin 40^\circ - \mu_k mg \cos 40^\circ = ma$$

$$a = 9.81(\sin 40^\circ - \mu_k \cos 40^\circ) = 6.31 - 7.51\mu_k$$

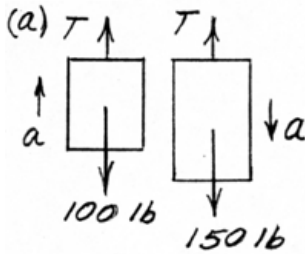


For constant accel. $s = v_0 t + \frac{1}{2} a t^2$:

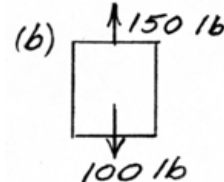
$$20 = 0 + \frac{1}{2} (6.31 - 7.51\mu_k) 2.58^2$$

$$\underline{\mu_k = 0.0395}$$

3/7 $\Sigma F = ma$; $T - 100 = \frac{100}{32.2} a$

(a)  $150 - T = \frac{150}{32.2} a$

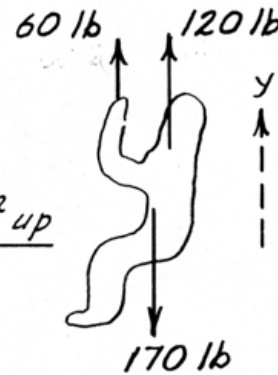
$50 = \frac{250}{32.2} a$, $a = \frac{32.2}{5} = 6.44 \frac{ft}{sec^2}$

(b)  $150 - 100 = \frac{100}{32.2} a$, $a = \frac{32.2}{2} = 16.10 \frac{ft}{sec^2}$

3/8

$$\Sigma F_y = ma_y: 180 - 170 = \frac{170}{32.2} a$$

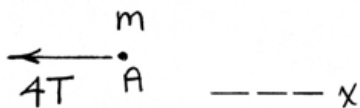
$$\underline{a = 1.894 \text{ ft/sec}^2 \text{ up}}$$



3/9

$\Sigma F_x = m a_x$
 $60 + 120 - 250 \sin 15^\circ = \frac{250}{32.2} a$
 $a = \frac{32.2}{250} (180 - 64.7) = \underline{14.85 \text{ ft/sec}^2}$

$$\frac{3}{10} \quad + \leftarrow \Sigma F = ma : 4(40,000) = \frac{750,000}{32.2} a$$

$$a = 6.87 \text{ ft/sec}^2$$


$$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = -6.87 \underline{i} - 0 = \underline{-6.87 \underline{i} \text{ ft/sec}^2}$$

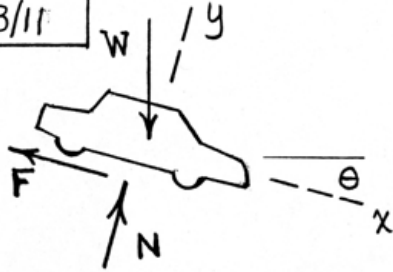
$$\underline{v}_A = (\underline{v}_A)_0 + \underline{a}t = 0 + 6.87(10) = 68.7 \text{ ft/sec}$$

$$\underline{v}_B = 15 \left(\frac{88}{60} \right) = 22 \text{ ft/sec}$$

$$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = -68.7 \underline{i} - 22(\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j})$$

$$= \underline{-87.7 \underline{i} - 11 \underline{j} \text{ ft/sec}}$$

3/11

When $\theta = \theta_1$, $a = 0$:

$$\sum F_x = 0 = -F + W \sin \theta_1$$

$$F = W \sin \theta_1$$

When $\theta = \theta_2$,

$$\sum F_x = ma : W \sin \theta_2 - W \sin \theta_1 = \frac{W}{g} a$$

$$a = g (\sin \theta_2 - \sin \theta_1)$$

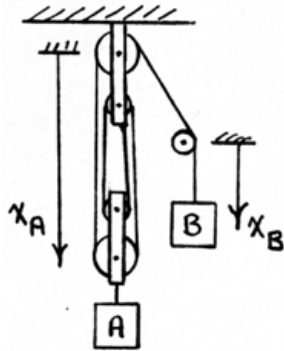
$$\left. \begin{array}{l} \theta_1 = 6^\circ \\ \theta_2 = 2^\circ \end{array} \right\} a = g (\sin 2^\circ - \sin 6^\circ) = \underline{-0.0696 g}$$

$$\underline{(-2.24 \text{ ft/sec}^2 \text{ or } -0.683 \text{ m/s}^2)}$$

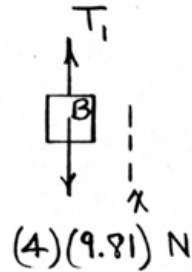
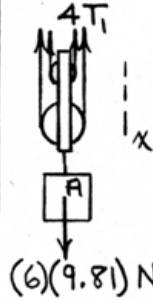
3/12

Kinematics: $4x_A + x_B = L_{\text{rope}} + \text{Constant}$

$$\therefore 4a_A + a_B = 0 \quad (1)$$



Kinetics:



$$A: \Sigma F_x = ma_x: 6(9.81) - 4T_1 = 6a_A \quad (2)$$

$$B: \Sigma F_x = ma_x: 4(9.81) - T_1 = 4a_B \quad (3)$$

$$\text{Solution of Eqs. (1) - (3): } \begin{cases} a_A = -1.401 \text{ m/s}^2 \\ a_B = 5.61 \text{ m/s}^2 \\ T_1 = 16.82 \text{ N} \end{cases}$$

$$\text{Tension in cable above A} \\ T_2 = 4T_1 = 67.3 \text{ N}$$

3/13

$a = 5 \text{ ft/sec}^2$

$\Sigma F_x = ma_x;$

$$P(1 + \cos 30^\circ) - 0.25N - 100 \sin 30^\circ = \frac{100}{32.2}(5)$$

$\Sigma F_y = 0; N + P \sin 30^\circ - 100 \cos 30^\circ = 0$

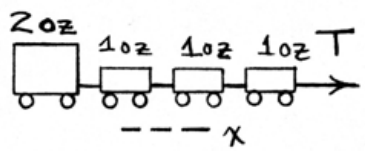
$$\begin{cases} 1.866P - 0.25N = 65.53 \\ 0.5P + N = 86.6 \end{cases} \text{ solve simultaneously}$$

get $N = 64.7 \text{ lb}$

$P = 43.8 \text{ lb}$

3/14 Coupler 1 will fail first, because it must accelerate more mass than any other coupler.

Rear part of train:

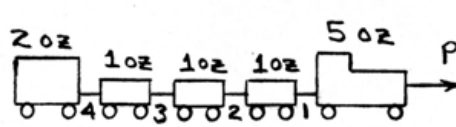


$$\sum F_x = ma_x$$

$$T = 0.2 = \left(\frac{5/16}{32.2} \right) a$$

$$a = 20.6 \text{ ft/sec}^2$$

Whole train:



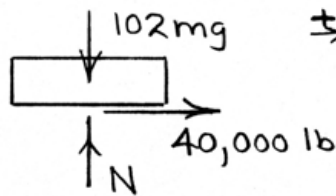
$$\sum F_x = ma_x$$

$$P = \left(\frac{10/16}{32.2} \right) (20.6)$$

$$P = 0.4 \text{ lb}$$

3/15

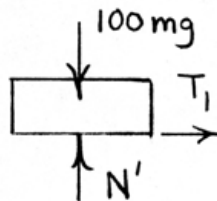
Let m be the mass of each car and $2m$ that of the locomotive.



$$\rightarrow \Sigma F = ma:$$

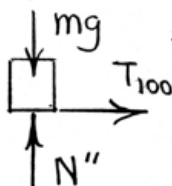
$$40,000 = \frac{102(200,000)}{32.2} a$$

$$a = 0.0631 \text{ ft/sec}^2$$



$$\rightarrow \Sigma F = ma: T_1 = \frac{100(200,000)}{32.2} 0.0631$$

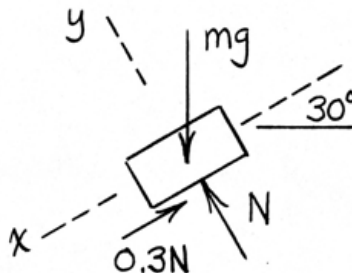
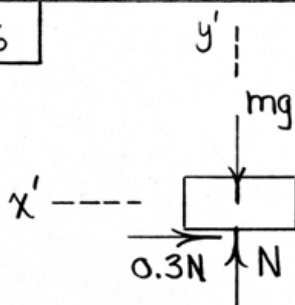
$$\underline{T_1 = 39,200 \text{ lb}}$$



$$\rightarrow \Sigma F = ma: T_{100} = \frac{1(200,000)}{32.2} 0.0631$$

$$\underline{T_{100} = 392 \text{ lb}}$$

3/16



A to B:

$$\Sigma F_y = 0 \Rightarrow N = 0.866 mg$$

$$\Sigma F_x = ma_x : mg \sin 30^\circ - 0.3(0.866 mg) = ma$$

$$a_x = 2.36 \text{ m/s}^2$$

$$v_B^2 = v_A^2 + 2a_x d : v_B^2 = 0.8^2 + 2(2.36)(2)$$

$$v_B = 3.17 \text{ m/s}$$

B to C:

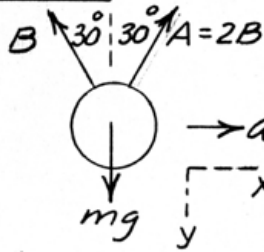
$$\Sigma F_{y'} = 0 \Rightarrow N = mg$$

$$\Sigma F_{x'} = ma_{x'} : -0.3(mg) = ma_{x'} , a_{x'} = -2.94 \text{ m/s}^2$$

$$v_C^2 = v_B^2 + 2a_{x'} s : 0 = 3.17^2 - 2(2.94)s$$

$$s = \underline{1.710 \text{ m}}$$

3/17



$$\Sigma F_x = ma_x; 2B \sin 30^\circ - B \sin 30^\circ = ma$$

$$\Sigma F_y = 0; 2B \cos 30^\circ + B \cos 30^\circ - mg = 0$$

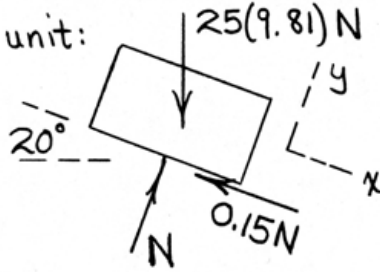
$$\text{Eliminate } B \text{ \& set } \underline{a = g/3\sqrt{3}}$$

3/18

Frame & sphere as a unit:

$$\Sigma F_y = 0: N - 25(9.81) \cos 20^\circ = 0$$

$$N = 230 \text{ N}$$



$$\Sigma F_y = ma_x:$$

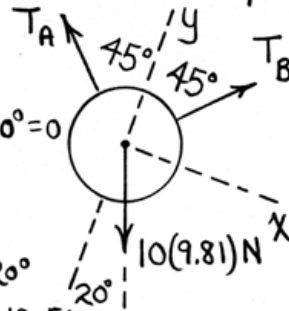
$$25(9.81) \sin 20^\circ - 0.15(230) = 25a, \quad a = 1.973 \text{ m/s}^2$$

Sphere alone:

$$\Sigma F_y = 0: (T_A + T_B) \cos 45^\circ - 10(9.81) \cos 20^\circ = 0$$

$$T_A + T_B = 130.4 \text{ N}$$

$$\Sigma F_x = ma_x: (T_B - T_A) \sin 45^\circ + 98.1 \sin 20^\circ = 10(1.973), \text{ or } T_B - T_A = -19.56 \text{ N}$$



Solution: $\underline{T_A = 75.0 \text{ N}}, \underline{T_B = 55.4 \text{ N}}$

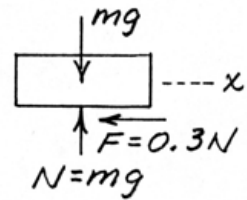
3/19 Let $m = \text{mass of crate}$

$$\Sigma F_x = ma_x; -0.3mg = ma_x$$

$$a_x = -0.3g = -0.3(9.81) = -2.94 \text{ m/s}^2$$

$$\int_v^0 v dv = \int_0^s a_x dx; -\frac{v^2}{2} = a_x s$$

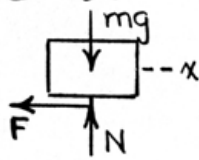
$$s = \frac{-(70/3.6)^2/2}{-2.94} = \underline{64.3 \text{ m}}$$



3/20

$$\text{Truck : } \begin{cases} v^2 - v_0^2 = 2a_T(x - x_0) \\ 0^2 - (19.44)^2 = 2a_T(50 - 0) \\ a_T = -3.78 \text{ m/s}^2 \end{cases}$$

Crate :



$$\Sigma F_x = ma_x : -F = m(-3.78) \\ F = 3.78m$$

$$F_{\max} = \mu_s N = 0.3(m \cdot 9.81) = 2.94m$$

$$F > F_{\max}, \text{ crate slips, } F = \mu_k N$$

$$\therefore \Sigma F_x = ma_x : -0.25mg = ma_c, a_c = -2.45 \text{ m/s}^2$$

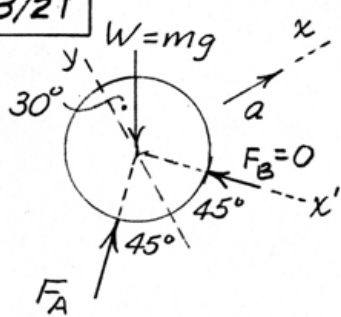
$$a_{c/T} = a_c - a_T = -2.45 - (-3.78) = 1.328 \text{ m/s}^2$$

$$v_{c/T}^2 - v_{c/T_0}^2 = 2a_{c/T}(x_{c/T} - x_{c/T_0})$$

$$v_{c/T}^2 - 0^2 = 2(1.328)(3 - 0), \quad \underline{v_{c/T} = 2.82 \text{ m/s}}$$

(Truck stopping time = 5.14 s, crate impacts at 2.13 s)

3/21



$$\sum F_{x'} = ma_{x'}$$

$$mg \cos(45^\circ + 30^\circ) = ma \cos 45^\circ$$

$$a = g \frac{\cos 75^\circ}{\cos 45^\circ} = 9.81 \frac{0.2588}{0.7071}$$

$$= \underline{0.366g}$$

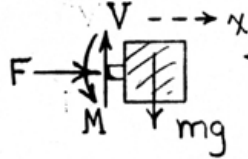
3/22

$$x = X \sin \omega t$$

$$\dot{x} = X \omega \cos \omega t$$

$$\ddot{x} = -X \omega^2 \sin \omega t, \quad \ddot{x}_{\max} = X \omega^2$$

FBD of circuit board:

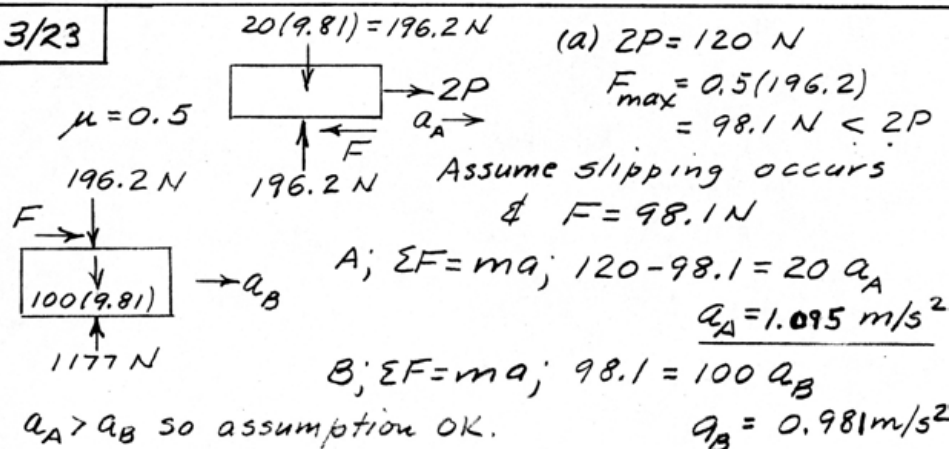


$$\Sigma F_x = m a_x:$$

$$F = m (-X \omega^2 \sin \omega t)$$

$$\underline{F_{\max} = m X \omega^2}$$

3/23



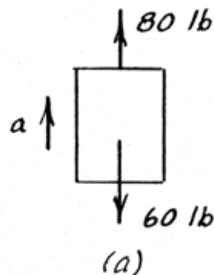
- (b) $2P = 80 \text{ N} < F_{\max}$ so no slipping occurs
 & for block & cart combined,
 $\Sigma F = ma; 80 = 120 a, a_A = a_B = a = 0.667 \text{ m/s}^2$

3/24

(a) $\Sigma F = ma$:

$$80 - 60 = \frac{60}{32.2} a,$$

$$\underline{a = 10.73 \text{ ft/sec}^2}$$



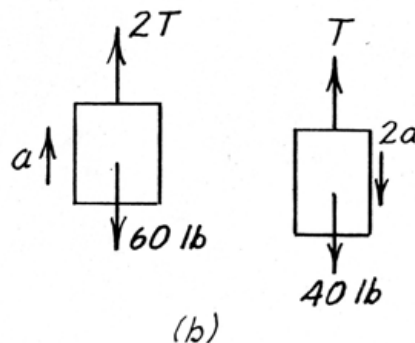
(b) $\Sigma F = ma$:

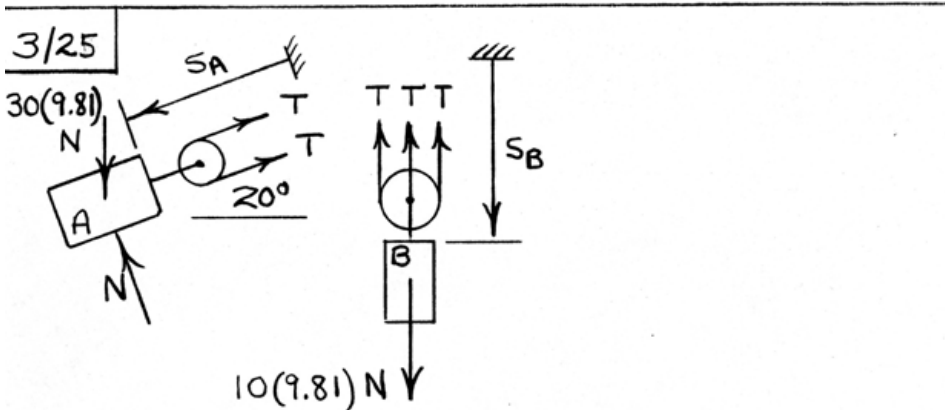
$$[60 - 1b] \quad 2T - 60 = \frac{60}{32.2} a$$

$$[40 - 1b] \quad 40 - T = \frac{40}{32.2} (2a)$$

Solve & get $T = 32.7 \text{ lb}$

$$\underline{a = 2.93 \text{ ft/sec}^2}$$





Kinematic constraint: $L = 2s_A + 3s_B$

$$\Rightarrow 0 = 2a_A + 3a_B \quad (1)$$

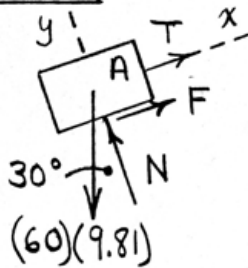
$$+\nearrow \Sigma F = m_A a_A : 30(9.81) \sin 20^\circ - 2T = 30a_A \quad (2)$$

$$+\downarrow \Sigma F = m_B a_B : 10(9.81) - 3T = 10a_B \quad (3)$$

Solution of Eqs. (1)-(3):

$$\begin{cases} a_A = 1.024 \text{ m/s}^2 \\ a_B = -0.682 \text{ m/s}^2 \\ T = 35.0 \text{ N} \end{cases}$$

3/26



Check for motion. Assume static equilibrium. From B,

$T = 196.2 \text{ N}$. Mass A:

$$\Sigma F_x = 0 : 196.2 + F$$

$$-(60)(9.81) \sin 30^\circ = 0, F = 98.1 \text{ N}$$

$$F_{\max} = \mu_s N = (0.25)(60)(9.81)$$

$$x \cos 30^\circ = 127.4 \text{ N (a)}$$

No motion for (a),

$$\text{so } \underline{a = 0}, \underline{T = 196.2 \text{ N}}$$

$$F_{\max} = (0.15)(60)(9.81) \cos 30^\circ$$

$$= 76.5 \text{ N, motion for (b)}$$

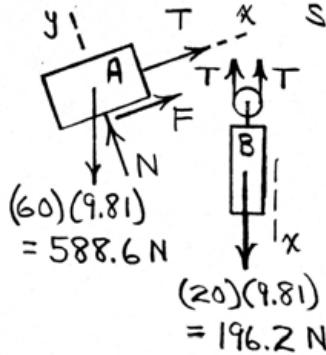
$$\text{A: } \Sigma F_x = ma_x : T - (60)(9.81) \sin 30^\circ + (0.1)(60)(9.81) \cos 30^\circ = 60 a$$

$$\text{B: } \Sigma F_y = ma_y : (20)(9.81) - T = 20 a$$

$$\text{Solution : } \underline{a = -0.589 \text{ m/s}^2}, \underline{T = 208 \text{ N}}$$

3/27

Check for motion by assuming static equilibrium.



$$B: 2T = 196.2, \quad T = 98.1 \text{ N}$$

$$A: \sum F_x = 0: 98.1 - 588.6 \sin 30^\circ + F = 0, \quad F = 196.2 \text{ N}$$

$$F_{\max} = \mu_s N = (0.25)(588.6) \cos 30^\circ = 127.4 \text{ N}$$

$$F > F_{\max} \Rightarrow \text{motion} \quad (\leftarrow)$$

$$\text{From kinematics, } a_A = 2a_B = 2a$$

$$A: \sum F_x = ma_x: T + 0.2(588.6 \cos 30^\circ) - 588.6 \sin 30^\circ = 60(2a)$$

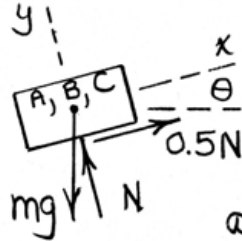
$$B: \sum F_x = ma_x: -2T + 196.2 = 20a$$

$$\text{Solution: } \underline{a = -0.725 \text{ m/s}^2}, \quad \underline{T = 105.4 \text{ N}}$$

3/28

Three-car unit:

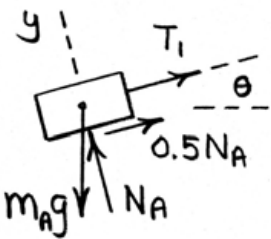
$$(\theta = \tan^{-1}(\frac{5}{100}) = 2.86^\circ)$$



$$\begin{cases} \Sigma F_y = 0 \Rightarrow N = mg \cos \theta \\ \Sigma F_x = ma_x : 0.5mg \cos \theta - mg \sin \theta = ma \end{cases}$$

$$a = g(0.5 \cos \theta - \sin \theta) = \underline{4.41 \text{ m/s}^2}$$

Car A:



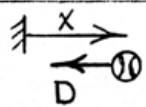
$$\Sigma F_y = 0 : N_A = m_A g \cos \theta$$

$$\Sigma F_x = ma_x : T_1 + 0.5m_A g \cos \theta - m_A g \sin \theta = m_A (4.41), \quad \underline{T_1 = 0}$$

From FBD of Car C, $\underline{T_2 = 0}$

By similar analyses:

	(b)	(c)	(d)
a	2.78 m/s^2	2.78 m/s^2	2.78 m/s^2
T_1	$32\,700 \text{ N (T)}$	$16\,330 \text{ N (C)}$	$16\,330 \text{ N (C)}$
T_2	$16\,330 \text{ N (T)}$	$16\,330 \text{ N (T)}$	$32\,700 \text{ N (C)}$

3/29  (Neglect weight for now)

$$\sum F_x = ma_x: -D = -C_D \frac{1}{2} \rho v^2 S = m v \frac{dv}{dx}$$

$$\int_0^x (-C_D \frac{1}{2} \rho S) dx = m \int_{v_0}^v \frac{dv}{v}$$

$$\Rightarrow v = v_0 e^{(-\frac{1}{2} C_D \rho S x / m)}$$

$$= v_0 e^{(-\frac{1}{2} (0.3) (\frac{0.07530}{32.2}) (\pi) (\frac{9.125/2\pi}{12})^2 x / (16 \cdot 32.2))}$$

$$\underline{v = v_0 e^{-1.623(10^{-3}) x}}$$

For $v_0 = 90$ mi/hr and $x = 60$ ft: $v = 81.7$ mi/hr

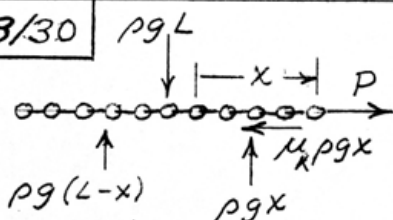
Comment on y-motion. Assume $v = 90$ mi/hr = constant. Time t to plate is

$$t = \frac{60}{90 (\frac{5280}{3600})} = 0.455 \text{ sec}$$

$$v_y = v_{y0} - gt = -32.2(0.455) = -14.64 \text{ ft/sec,}$$

which would not appreciably change $v = \sqrt{v_x^2 + v_y^2}$.

3/30



$$\Sigma F_x = m a_x;$$

$$P - \mu_k \rho g x = \rho L a_x$$

$$\int_0^v v dv = \int_0^L a_x dx$$

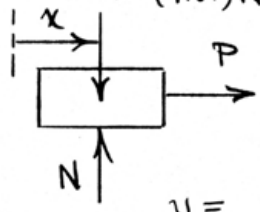
$$\frac{v^2}{2} = \int_0^L \left(\frac{P}{\rho L} - \frac{\mu_k g x}{L} \right) dx = \frac{P}{\rho} - \frac{\mu_k g L}{2}, \quad v = \sqrt{\frac{2P}{\rho} - \mu_k g L}$$

From $\frac{v^2}{2} = \int_0^x \left(\frac{P}{\rho L} - \frac{\mu_k g x}{L} \right) dx$, we obtain

$$v(x) = \sqrt{2 \frac{x}{L} \left(\frac{P}{\rho} - \mu_k g \frac{x}{2} \right)}$$

Note that $v(L) \geq 0$ if $\underline{P \geq \mu_k \rho g \frac{L}{2} = P_{\min}}$

3/31



$$\Sigma F_x = ma_x : P = 10a_x$$

$$\frac{P}{10} = \frac{dv}{dt}, \quad v = \int_0^t \frac{P}{10} dt$$

For $P_1 = 10t$:

$$v = t^2/2, \quad s = t^3/6$$

At $t = 5s$, $\underline{v = 12.5 \text{ m/s}}$, $\underline{s = 20.8 \text{ m}}$

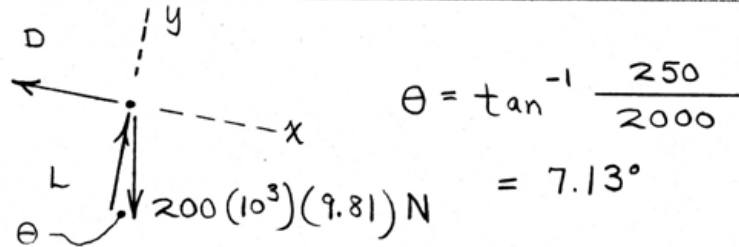
For $P_2 = kt^2$: $50 = k(5)^2$, $k = 2 \text{ N/s}^2$

So $P_2 = 2t^2$

$$v = \int_0^t \frac{2t^2}{10} dt = \frac{t^3}{15}, \quad s = \frac{t^4}{60}$$

At $t = 5s$, $\underline{v = 8.33 \text{ m/s}}$, $\underline{s = 10.42 \text{ m}}$

3/32



$$v_B^2 - v_A^2 = 2a_x (s_B - s_A) :$$

$$\left(\frac{200}{3.6}\right)^2 - \left(\frac{300}{3.6}\right)^2 = 2a_x \left(\frac{2000}{\cos 7.13^\circ}\right), \quad a_x = -0.957 \text{ m/s}^2$$

$$\Sigma F_x = ma_x: -D + 200(10^3)(9.81)\sin 7.13^\circ =$$

$$200(10^3)(-0.957), \quad \underline{D = 435 \text{ kN}}$$

$$\Sigma F_y = 0: L - 200(10^3)(9.81)\cos 7.13^\circ = 0$$

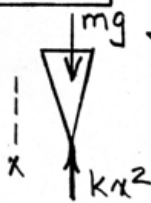
$$\underline{L = 1.947 \text{ MN}}$$

The net aerodynamic force is then

$$R = \sqrt{L^2 + D^2} = \sqrt{1.947^2 + 0.435^2} = \underline{1.995 \text{ MN}}$$

3/33

FBD of cone during penetration:



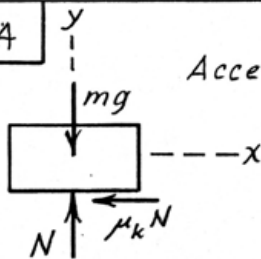
$$\Sigma F_x = ma_x: \quad mg - kx^2 = m v \frac{dv}{dx}$$

$$\int_0^d \left(g - \frac{k}{m} x^2 \right) dx = \int_{v_0}^0 v dv$$

$$gd - \frac{k}{3m} d^3 = -\frac{v_0^2}{2}, \quad \text{where } v_0 = \sqrt{2gh}$$

$$\therefore k = \frac{3mg}{d^3} (h+d)$$

3/34



Accel. down: $\Sigma F_y = ma_y: -mg + N = -ma,$
 $N = m(g - a)$

$\Sigma F_x = ma_x: -\mu_k m(g - a) = ma_x,$
 $a_x = -\mu_k(g - a)$

Accel. up: $\Sigma F_y = ma_y: N - mg = ma,$
 $N = m(g + a)$

$\Sigma F_x = ma_x: -\mu_k m(g + a) = ma_x,$
 $a_x = -\mu_k(g + a)$

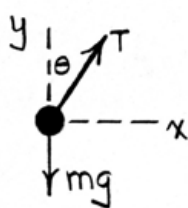
$v^2 = v_0^2 + 2as$: Down: $0 = v^2 - 2\mu_k(g - a)s_1,$

Up: $0 = v^2 - 2\mu_k(g + a)s_2$

Eliminate v^2 & get $(g - a)s_1 = (g + a)s_2,$

$$a = g \frac{s_1 - s_2}{s_1 + s_2}$$

3/35 Mass m :



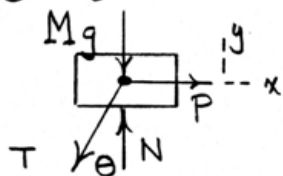
$$\Sigma F_y = 0: T \cos \theta - mg = 0$$

$$T = mg / \cos \theta$$

$$\Sigma F_x = ma_x: T \sin \theta = ma$$

$$\left(\frac{mg}{\cos \theta} \right) \sin \theta = ma, \quad \theta = \tan^{-1} \left(\frac{a}{g} \right)$$

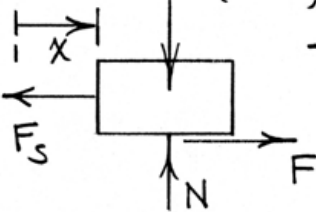
Cart M :



$$\Sigma F_x = ma_x: P - T \sin \theta = Ma$$

$$P = ma + Ma = \underline{(m+M)a}$$

3/36



$$F_s = 150x + 400x^2 \text{ (N)}$$

$$+ \uparrow \Sigma F = 0 \Rightarrow N = 58.9 \text{ N}$$

$$F_{\max} = \mu_s N = 0.30(58.9) = 17.66 \text{ N}$$

(a) $x = 50 \text{ mm}$: $F_s = 150(0.050) + 400(0.050)^2$
 $= 8.5 \text{ N} < F_{\max}$

So $a = 0$

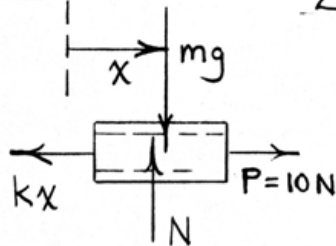
(b) $F_s = 150(0.1) + 400(0.1)^2 = 19 \text{ N} > F_{\max}$

$$\Sigma F_x = ma_x : -19 + 0.25(58.9) = 6a$$

$$\underline{a = -0.714 \text{ m/s}^2}$$

3/37

Eq. pos.



$$\sum F_x = m\ddot{x} : P - kx = m\ddot{x}$$

$$10 - 200x = 2\ddot{x}$$

$$\ddot{x} = 5 - 100x$$

$$5 - 100x = v \frac{dv}{dx}$$

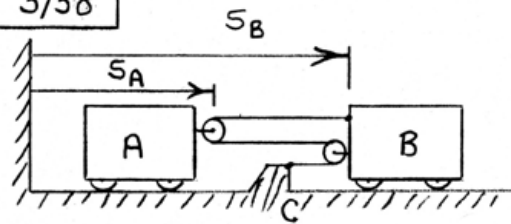
$$\int_0^{0.040} (5 - 100x) dx = \int_0^v v dv, \quad 5x - 50x^2 \Big|_0^{0.040} = \frac{v^2}{2} \Big|_0^v$$

$$v = 0.490 \text{ m/s}$$

$$5x - 50x^2 = 0$$

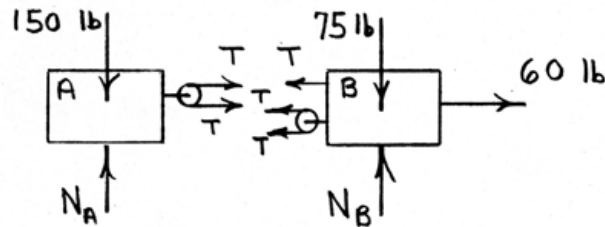
$$x = 0 \text{ (initial condition) or } x = 0.10 \text{ m or } \underline{100 \text{ mm}}$$

3/38



$$L = 2(s_B - s_A) + (s_B - s_C) + \text{constants}$$

$$\Rightarrow 0 = 3a_B - 2a_A \quad (1)$$

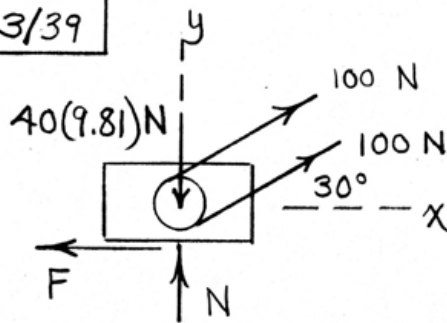


$$\rightarrow \Sigma F = ma: \quad (A) \quad 2T = \frac{150}{32.2} a_A \quad (2)$$

$$(B) \quad 60 - 3T = \frac{75}{32.2} a_B \quad (3)$$

$$\text{Solve Eqs. (1)-(3):} \quad \begin{cases} a_A = 7.03 \text{ ft/sec}^2 \\ a_B = 4.68 \text{ ft/sec}^2 \\ T = 16.36 \text{ lb} \end{cases}$$

3/39



$$\begin{cases} \mu_s = 0.5 \\ \mu_k = 0.4 \end{cases}$$

$$\Sigma F_y = 0: N + 200 \sin 30^\circ - 40(9.81) = 0$$

$$N = 292 \text{ N}$$

Assume static equilibrium:

$$\Sigma F_x = 0: -F + 200 \cos 30^\circ = 0, \quad F = 173.2 \text{ N}$$

$$F_{\max} = \mu_s N = 0.5(292) = 146.2 \text{ N} < F$$

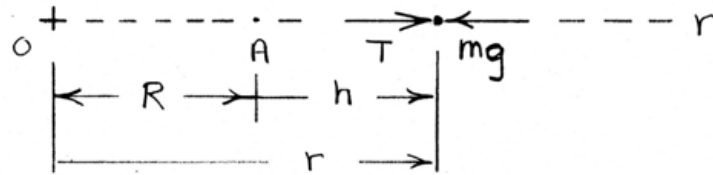
Assumption wrong, motion exists $\rightarrow$.

$$F = \mu_k N = 0.4(292) = 117.0 \text{ N}$$

$$\Sigma F_x = ma_x: -117 + 200 \cos 30^\circ = 40 a_x$$

$$a_x = a = 1.406 \text{ m/s}^2$$

3/40



$$g = g_0 \frac{R^2}{r^2} \quad (\text{all pertaining to moon})$$

$$\Sigma F_r = ma_r: T - mg_0 \frac{R^2}{r^2} = m v \frac{dv}{dr}$$

$$\int_R^{2R} \left(\frac{T}{m} - g_0 \frac{R^2}{r^2} \right) dr = \int_0^v v dv$$

$$\Rightarrow v = \sqrt{\frac{2TR}{m} - g_0 R} = \sqrt{R \left(\frac{2T}{m} - g_0 \right)}$$

Numbers :

$$v = \sqrt{\frac{3476(1000)}{2} \left(\frac{2(2500)}{1200} - 1.62 \right)}$$

$$= \underline{2100 \text{ m/s}}$$

3/41

 mg 

$$\Sigma F_y = ma_y; \quad mg - kv = ma$$

$$a = g - \frac{k}{m}v$$

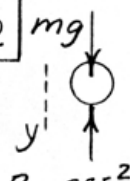
$$R = kv \quad v dv = a dy, \quad \int_0^v \frac{v dv}{g - \frac{k}{m}v} = \int_0^h dy$$

$$\frac{m^2}{k^2} \left[\left(g - \frac{k}{m}v \right) - g \ln \left(g - \frac{k}{m}v \right) \right]_0^v = h$$

$$h = \frac{m^2}{k^2} \left[-\frac{k}{m}v - g \ln \left(1 - \frac{kv}{mg} \right) \right]$$

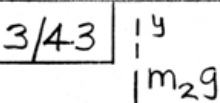
$$h = \frac{m^2}{k^2} g \ln \left(\frac{1}{1 - \frac{kv}{mg}} \right) - \frac{mv}{k}$$

3/42



$\Sigma F_y = ma_y; \quad mg - cv^2 = ma$
 $a = g - \frac{c}{m}v^2$
 $v dv = a dy, \quad \int_0^v \frac{v dv}{g - \frac{c}{m}v^2} = \int_0^h dy$
 $-\frac{m}{2c} \ln \left(g - \frac{c}{m}v^2 \right) \Big|_0^v = h, \quad h = \frac{m}{2c} \ln \left(\frac{mg}{mg - cv^2} \right)$

3/43



$$\begin{cases} \Sigma F_x = m a_x: -F \cos \theta + N \sin \theta = m_2 a \\ \Sigma F_y = 0: F \sin \theta + N \cos \theta - m_2 g = 0 \end{cases}$$

Solve to obtain

$$\begin{cases} F = m_2 (g \sin \theta - a \cos \theta) \\ N = m_2 (a \sin \theta + g \cos \theta) \end{cases}$$

(slipping impends $\rightarrow$)For impending slip, $F = \mu_s N$, or

$$m_2 (g \sin \theta - a \cos \theta) = \mu_s m_2 (a \sin \theta + g \cos \theta)$$

$$\text{Solving for } a: a = g \frac{\sin \theta - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta}$$

$$\text{With numbers, } a = 0.0577g \left(\text{Note: } \tan^{-1} \mu_s = \tan^{-1} 0.3 = 16.70^\circ \right)$$

Let slipping impend up the inclined block (reverse F on above FBD) & obtain

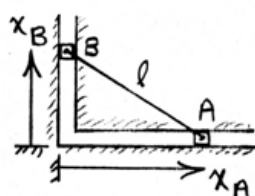
$$a = g \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} = 0.745g$$

$$\begin{aligned} & \text{FBD of System (Sys.):} \\ & \text{Upward force: } (m_1 + m_2)g \\ & \text{Downward force: } P \\ & \text{Normal force: } N_{tot} \\ & \Sigma F_x = m a_x: P = (m_1 + m_2) a \end{aligned}$$

So

$$0.0577(m_1 + m_2)g \leq P \leq 0.745(m_1 + m_2)g$$

3/44



$$x_A^2 + x_B^2 = l^2$$

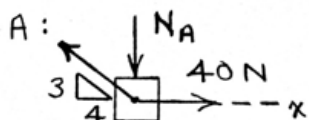
$$2x_A \dot{x}_A + 2x_B \dot{x}_B = 0$$

$$x_A \ddot{x}_A + \dot{x}_A^2 + x_B \ddot{x}_B + \dot{x}_B^2 = 0$$

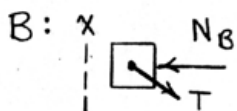
$$\text{So } \dot{x}_B = -\frac{x_A \dot{x}_A}{x_B} = -\frac{(0.4)(0.9)}{0.3} = -1.2 \text{ m/s}$$

$$\ddot{x}_B = \frac{-\dot{x}_B^2 - \dot{x}_A^2 - x_A \ddot{x}_A}{x_B} = \frac{-1.2^2 - 0.9^2 - 0.4 \ddot{x}_A}{0.3}$$

$$= -7.5 - \frac{4}{3} \ddot{x}_A \quad \text{or} \quad a_B = -7.5 - \frac{4}{3} a_A \quad (1)$$



$$\sum F_x = ma_x: 40 - \frac{4}{5}T = 2a_A \quad (2)$$

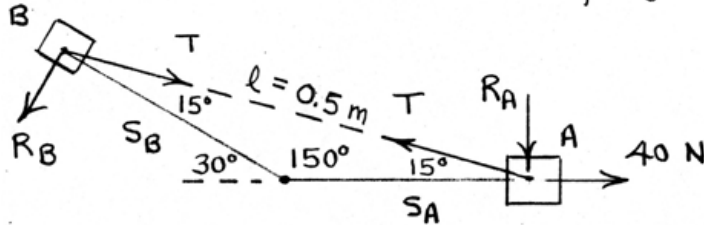


$$\sum F_x = ma_x: -\frac{3}{5}T = 3a_B \quad (3)$$

$$\text{Solution of Eqs. (1)-(3): } \begin{cases} a_A = 1.364 \text{ m/s}^2 \\ a_B = -9.32 \text{ m/s}^2 \\ T = 46.6 \text{ N} \end{cases}$$

3/45

$$\sin 150^\circ / l = \sin 15^\circ / s_B, \quad s_B = s_A = 0.259 \text{ m}$$



Law of cosines: $l^2 = s_A^2 + s_B^2 - 2s_A s_B \cos 150^\circ$

$$2l\dot{l} = 0 = 2s_A v_A + 2s_B v_B - 2\left(-\frac{\sqrt{3}}{2}\right)(s_A v_B + s_B v_A)$$

$$s_A v_A + s_B v_B + \frac{\sqrt{3}}{2}(s_A v_B + v_A s_B) = 0^*$$

With $s_A = s_B = 0.259 \text{ m}$, $v_A = 0.4 \text{ m/s}$: $v_B = -0.4 \text{ m/s}$

Differentiate *: $v_A^2 + s_A a_A + v_B^2 + s_B a_B + \frac{\sqrt{3}}{2}(s_A a_B + v_A v_B + a_A s_B + v_A v_B) = 0$

Numbers: $0.483 a_A + 0.483 a_B + 0.0429 = 0 \quad (1)$

Kinetics:

$$\nearrow \Sigma F = m a_B: -T \cos 15^\circ = 3 a_B \quad (2)$$

$$\rightarrow \Sigma F = m a_A: 40 - T \cos 15^\circ = 2 a_A \quad (3)$$

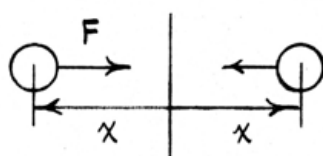
Solution of Eqs. (1)-(3): $T = 25.0 \text{ N}$

$$a_A = 7.95 \text{ m/s}^2$$

$$a_B = -8.04 \text{ m/s}^2$$

3/46

m



$$F = \frac{Gm^2}{x^2}$$

$$m = \rho V = 7210 \left(\frac{4}{3} \pi 0.05^3 \right) = 3.775 \text{ kg}$$

$$\Sigma F_x = ma_x : - \frac{Gm^2}{(2x)^2} = m v \frac{dv}{dx}$$

$$- \frac{Gm}{4} \int_{x_0=0.5}^x \frac{dx}{x^2} = \int_{v_0=0}^v v dv$$

$$v = \sqrt{Gm} \sqrt{\frac{1}{2x} - 1} = \sqrt{(6.673 \times 10^{-11})(3.775)} \sqrt{\frac{1}{2(0.05)} - 1}$$

$$= 4.76 \times 10^{-5} \text{ m/s}$$

$$\text{Now, } \frac{dx}{dt} = - \sqrt{Gm} \sqrt{\frac{1}{2x} - 1}$$

$$x=0.05$$

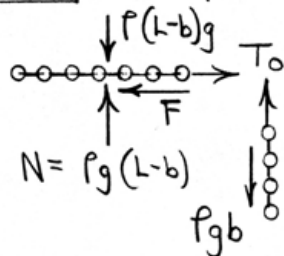
$$\int_{x_0=0.5}^x \frac{\sqrt{x} dx}{\sqrt{\frac{1}{2} - x}} = - \sqrt{Gm} \int_0^t dt$$

$$x_0=0.5$$

$$\left[-\sqrt{x} \sqrt{\frac{1}{2} - x} + \frac{1}{2} \sin^{-1} \sqrt{2x} \right]_{x_0=0.5}^{x=0.05} = -\sqrt{Gm} t$$

$$\text{Solving, } t = 48,800 \text{ s or } t = 13 \text{ hr } 33 \text{ min}$$

► 3/47 | Let $\rho = \text{mass/length}$. Length b to get started:

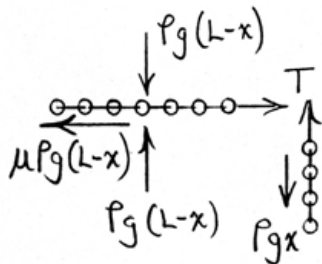


$$F = \mu N = \mu g \rho (L-b)$$

$$\Sigma F = 0: T_0 - \mu g \rho (L-b) = 0$$

$$\text{and } T_0 = \rho g b$$

$$\text{Solve to obtain } b = \frac{\mu L}{1+\mu}$$



$$\Sigma F = ma: T - \mu \rho g(L-x) = \rho(L-x)a$$

$$\text{and } \rho g x - T = \rho x a$$

Eliminate T to obtain

$$a = \ddot{x} = \frac{g}{L} [x(1+\mu) - \mu L]$$

$$v dv = \ddot{x} dx: \int_0^v v dv = \int_b^L \frac{g}{L} [x(1+\mu) - \mu L] dx$$

$$\frac{1}{2} v^2 = \frac{g}{L} \left[\frac{x^2}{2} (1+\mu) - \mu L x \right]_b^L$$

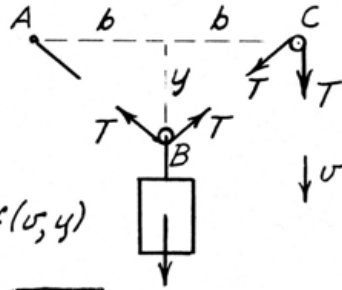
Substitute $b = \frac{\mu L}{1+\mu}$, simplify, and obtain

$$v = \sqrt{\frac{gL}{1+\mu}}$$

3/48

$$+\uparrow \Sigma F = ma: 2T \frac{y}{\sqrt{b^2+y^2}} - mg = ma, \quad a = -\ddot{y}$$

$$T = \frac{m(a+g)\sqrt{b^2+y^2}}{2y} \quad \text{where } a = f(v, y)$$



$$\text{Let } L = \text{length of cable } ABC = 2\sqrt{b^2+y^2} \quad mg$$

$$-v = \dot{L} = 2 \frac{y\dot{y}}{\sqrt{b^2+y^2}}, \quad \ddot{L} = 2 \frac{\sqrt{b^2+y^2}(\dot{y}^2 + y\ddot{y})}{b^2+y^2} - 2 \frac{y\dot{y}(y\dot{y})}{(b^2+y^2)\sqrt{b^2+y^2}} = 0$$

$$\text{so } \sqrt{b^2+y^2} \left(\frac{v^2(b^2+y^2)}{4y^2} + y\ddot{y} \right) = \frac{y^2}{\sqrt{b^2+y^2}} \frac{v^2(b^2+y^2)}{4y^2}$$

$$\text{Simplify and get } \ddot{y} = -\frac{b^2 v^2}{4y^3} = -a$$

$$\text{Thus } T = \frac{m(g + \frac{b^2 v^2}{4y^3})\sqrt{b^2+y^2}}{2y}, \quad T = \frac{m}{2y} \sqrt{b^2+y^2} \left(g + \frac{b^2 v^2}{4y^3} \right)$$

3/49

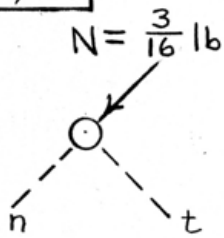
$$\sum F_n = ma_n = m \frac{v^2}{r} :$$

$$N - 2(9.81) = 2 \frac{4^2}{1.5}$$

$$\underline{N = 41.0 \text{ N}} \quad \text{up}$$

Any friction present would not enter the normal equation.

3/50

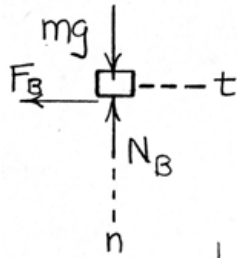


$$\Sigma F_n = m a_n = m \frac{v^2}{\rho} :$$

$$\frac{3}{16} = \frac{2/16}{32.2} \left(\frac{5^2}{\rho} \right)$$

$$\underline{\rho = 0.518 \text{ ft}}$$

3/51

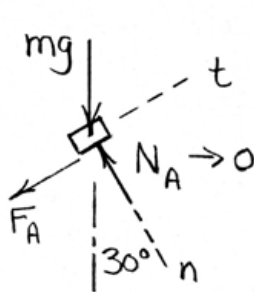


$$\Sigma F_n = ma_n = m \frac{v^2}{r} :$$

$$2(9.81) - N = 2 \frac{3.5^2}{2.4}$$

$$\underline{N_B = 9.41 \text{ N}}$$

loss of contact at A: $N_A \rightarrow 0$

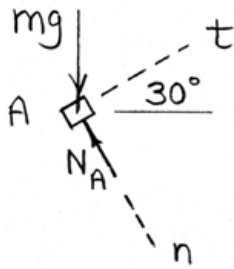


$$\Sigma F_n = ma_n = m \frac{v^2}{r} :$$

$$mg \cos 30^\circ = m \frac{v^2}{2.4}$$

$$\underline{v = 4.52 \text{ m/s}}$$

3/52



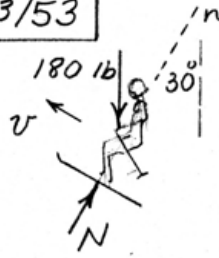
$$\Sigma F_n = ma_n : -N_A + mg \cos 30^\circ = m \frac{v_A^2}{r}$$

$$N_A = m \left(g \cos 30^\circ - \frac{v_A^2}{r} \right)$$
$$= 2 \left(9.81 \cos 30^\circ - \frac{4.5^2}{2.4} \right)$$
$$= \underline{0.1164 \text{ N}}$$

$$\Sigma F_t = ma_t : -mg \sin 30^\circ = ma_t$$

$$a_t = -\frac{g}{2} = \underline{-4.90 \text{ m/s}^2}$$

3/53



$$\Sigma F_n = ma_n; N - 180 \cos 30^\circ = \frac{180}{32.2} \frac{(80)^2}{150}$$

$$N = 180(0.866 + 1.33) = \underline{394 \text{ lb}}$$

3/54

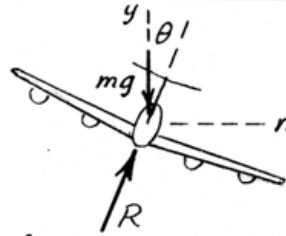
$$\sum F_y = 0: R \cos \theta - mg = 0, R \cos \theta = mg$$

$$\sum F_n = ma_n: R \sin \theta = m v^2 / \rho$$

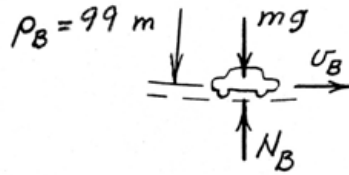
$$\text{Combine \& get } \tan \theta = \frac{v^2}{\rho g}, \theta = \tan^{-1} \frac{v^2}{\rho g}$$

$$\text{where } v = \frac{400 \times 5280}{3600} = 587 \text{ ft/sec}, \rho = 2 \times 5280 = 10,560 \text{ ft}$$

$$\text{so } \theta = \tan^{-1} \frac{587^2}{10,560 \times 32.2} = \tan^{-1} 1.012, \theta = 45.3^\circ$$



3/55



$$\Sigma F_n = m a_n:$$

$$A: mg - N_A = m \frac{v_A^2}{\rho_A}$$

$$B: N_B - mg = m \frac{v_B^2}{\rho_B}$$

$$\text{For } N_B = 2N_A, \quad m \left(\frac{v_B^2}{\rho_B} + g \right) = 2m \left(g - \frac{v_A^2}{\rho_A} \right)$$

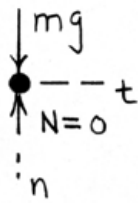
$$v_B^2 = \rho_B g - 2 v_A^2 \frac{\rho_B}{\rho_A} = 99(9.81) - 2 \left(\frac{60 \times 1000}{3600} \right)^2 \frac{99}{101}$$

$$= 427 \text{ m}^2/\text{s}^2$$

$$v_B = 20.7 \text{ m/s} \quad \text{or} \quad \underline{v_B = 74.4 \text{ km/h}}$$

3/56

FBD of object inside airplane:



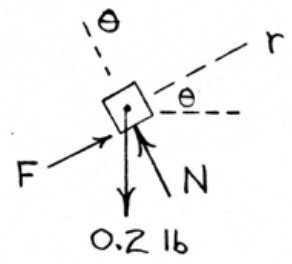
$$\Sigma F_n = ma_n: \quad m/g = m \frac{v^2}{r}$$

$$r = \frac{v^2}{g} = \frac{[(600)(\frac{5280}{3600})]^2}{32.2}$$

$$r = \underline{24,050 \text{ ft}}$$

3/57 $\Sigma F_{\theta} = m a_{\theta} = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$: $N - 0.2 \cos 30^{\circ}$

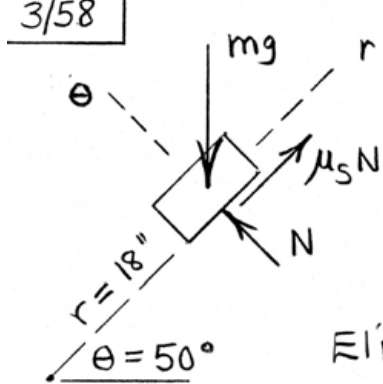
Slider:



$$= \frac{0.2}{32.2} (r\ddot{\theta} + 2(-4)(3))$$

$$\underline{N = 0.024 \text{ lb}}$$

3/58



$$\Sigma F_\theta = ma_\theta: N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

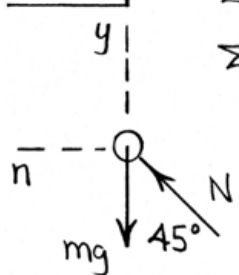
$$\Sigma F_r = mar: \mu_s N - mg \sin \theta = m(0 - r\omega^2)$$

Eliminate N :

$$\mu_s = \tan \theta - \frac{r\omega^2}{g \cos \theta}$$

$$\text{Numbers: } \mu_s = \tan 50^\circ - \frac{(18/12)(3)^2}{32.2 \cos 50^\circ} = \underline{0.540}$$

3/59



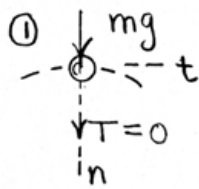
$$\Sigma F_y = 0: N \frac{\sqrt{2}}{2} - mg = 0, \quad N = \frac{2}{\sqrt{2}} mg$$

$$\Sigma F_n = ma_n: N \frac{\sqrt{2}}{2} = m \left(3R + R \frac{\sqrt{2}}{2} \right) \Omega^2$$

$$\frac{2}{\sqrt{2}} mg \left(\frac{\sqrt{2}}{2} \right) = m R \left(3 + \frac{\sqrt{2}}{2} \right) \Omega^2$$

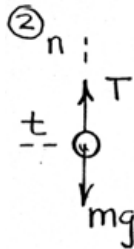
With $R = 0.200 \text{ m}$, $\Omega = 3.64 \frac{\text{rad}}{\text{s}}$

3/60



$$\sum F_n = m \frac{v^2}{r} : mg = m \frac{v^2}{1}$$

$$v = \sqrt{g} = 3.13 \text{ m/s}$$

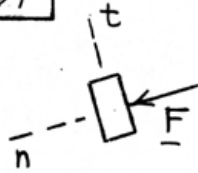


$$\sum F_n = m \frac{v^2}{r} : T - mg = m \frac{g}{1}$$

$$T = 2mg = 2(0.050)(9.81)$$

$$= 0.981 \text{ N}$$

3/6/1



$$a_n = \frac{v^2}{r} = \frac{\left[(35) \left(\frac{5280}{3600} \right) \right]^2}{100}$$

$$= 26.4 \frac{\text{ft}}{\text{sec}^2} \left(\frac{1g}{32.2 \text{ ft/sec}^2} \right)$$

$$= \underline{0.818g}$$

$$\Sigma F_n = m a_n : F = \frac{3000}{32.2} (26.4)$$

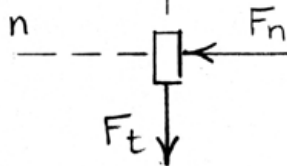
$$= \underline{2460 \text{ lb}}$$

(An average of 614 lb per tire!)

3/62

$$\Sigma F_n = ma_n: F_n = \frac{3000}{32.2} \frac{(25 \cdot \frac{5280}{3600})^2}{100}$$

$$F_n = 1253 \text{ lb}$$



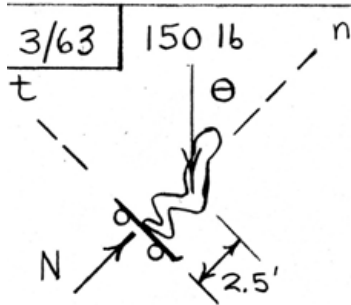
$$\sqrt{F_n^2 + F_t^2} = F_{\text{tot}}$$

$$1253^2 + F_t^2 = 2400^2$$

$$F_t = 2047 \text{ lb}$$

$$\Sigma F_t = ma_t: -2047 = \frac{3000}{32.2} a_t$$

$$a_t = -22.0 \text{ ft/sec}^2$$



$$r = 15 - 2.5 = 12.5'$$

$$\Sigma F_n = ma_n: N - 150 \cos \theta = \frac{150}{32.2} \frac{v^2}{12.5}$$

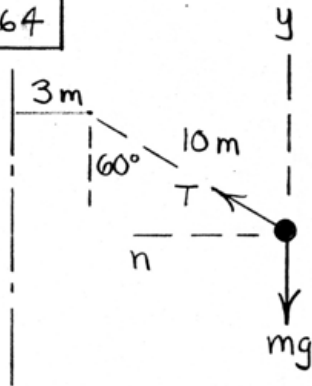
$$N = 150 \left(\cos \theta + \frac{v^2}{402.5} \right)$$

$$\theta = 0(+): N_0 = 150 \left(1 + \frac{28^2}{402.5} \right) = \underline{442 \text{ lb}}$$

$$\theta = 45^\circ: N_{45^\circ} = 150 \left(\frac{\sqrt{2}}{2} + \frac{20^2}{402.5} \right) = \underline{255 \text{ lb}}$$

$$\theta = 90^\circ: \underline{N_{90^\circ} = 0}$$

3/64



$$\Sigma F_n = ma_n:$$

$$T \sin 60^\circ = m[3 + 10 \sin 60^\circ] \omega^2$$

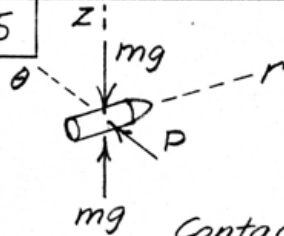
$$\Sigma F_y = 0: T \cos 60^\circ - mg = 0$$

$$\Rightarrow \tan 60^\circ = \frac{3 + 10 \sin 60^\circ}{9.81} \omega^2$$

$$\omega = 1.207 \text{ rad/s}$$

$$N = 1.207 \left(\frac{60}{2\pi} \right) = \underline{11.53 \text{ rev/min}}$$

3/65



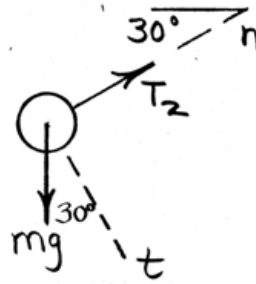
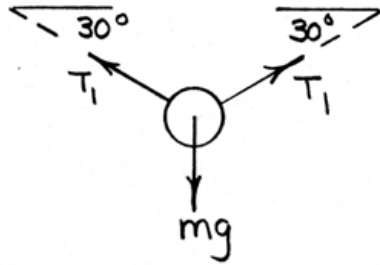
$$\Sigma F_{\theta} = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$P = 0.06(0 + 2[600][0.5])$$

$$= 36 \text{ N}$$

Contact is against right-hand
side of barrel.

3/66

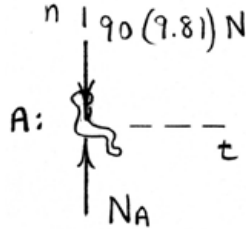


Equilibrium : $\Sigma \underline{F} = \underline{0} \Rightarrow T_1 = mg$

Motion : $\Sigma F_n = ma_n = 0 : T_2 - mg \sin 30^\circ = 0$

$$k = \frac{T_2}{T_1} = \frac{mg \sin 30^\circ}{mg} = \underline{0.5}$$

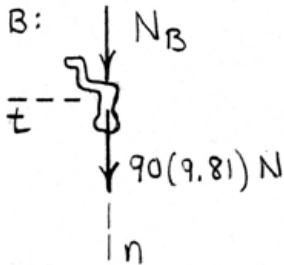
3/67



$$\Sigma F_n = ma_n:$$

$$N_A - 90(9.81) = 90 \frac{(600/3.6)^2}{1000}$$

$$\underline{N_A = 3380 \text{ N}}$$



$$\Sigma F_n = ma_n:$$

$$N_B + 90(9.81) = 90 \frac{(600/3.6)^2}{1000}$$

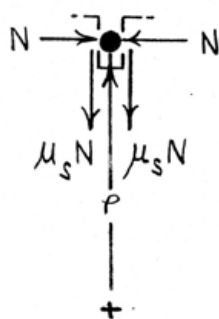
$$\underline{N_B = 1617 \text{ N}}$$

(Note static normal $mg = 90(9.81) = 883 \text{ N}$)

$$\frac{3}{68} \quad \omega = \left(4000 \frac{\text{rev}}{\text{min}}\right) \left(\frac{1 \text{ min}}{60 \text{ sec}}\right) \left(\frac{2\pi \text{ rad}}{\text{sec}}\right)$$

$$= 418.9 \text{ rad/sec}$$

FBD of pebble :



Tire Center

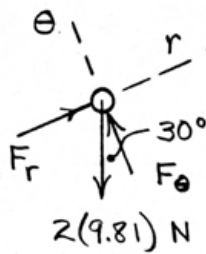
$$\sum F_n = ma_n: 2\mu_s N = mr\omega^2$$

$$N = \frac{mr\omega^2}{2\mu_s} = \frac{(0.010)(0.350)(418.9)^2}{2(0.95)}$$

$$\underline{N = 323 \text{ N}}$$

3/69

F_r and F_θ are the r - and θ -components of the total friction force F .



$$\Sigma F_r = m a_r = m(\ddot{r} - r\dot{\theta}^2):$$

$$F_r - 19.62 \sin 30^\circ = 2[0 - 1(-0.873)^2]$$

$$F_r = 8.29 \text{ N}$$

$$\Sigma F_\theta = m a_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$F_\theta - 19.62 \cos 30^\circ = 2[(1)(3.49) + 2(-0.5)(-0.873)]$$

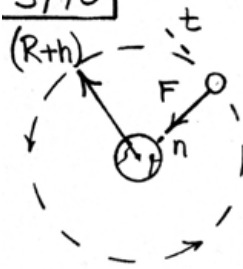
$$F_\theta = 25.7 \text{ N}$$

$$F = \sqrt{F_r^2 + F_\theta^2} = 27.0 \text{ N}$$

$$P = \frac{F/2}{\mu_s} = \frac{27.0/2}{0.5} = 27.0 \text{ N}$$

$$(\text{Static gripping force} = \underline{19.62 \text{ N}})$$

3/70



$$\Sigma F_n = ma_n : F = \frac{Gm_em}{(R+h)^2} = m \frac{v^2}{(R+h)}$$

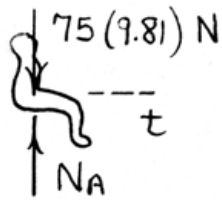
$$\text{But } v = \frac{s}{t} = \frac{2\pi(R+h)}{(23.944)(3600)}$$

Combining the two equations :

$$v = \frac{2\pi(R+h)}{(23.944)(3600)} = \sqrt{\frac{Gm_e}{(R+h)}}$$

Solve for h to obtain $\underline{h = 3.580 \times 10^7 \text{ m}}$
(35,800 km)

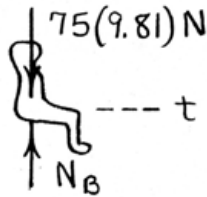
3/7/1

Point A : $\Sigma F_n = ma_n$:

$$N_A - 75(9.81) = 75 \frac{22^2}{40}$$

$$\underline{N_A = 1643 \text{ N}}$$

Point B:

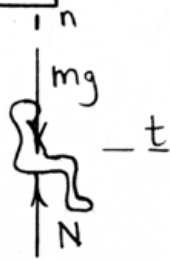
 $\Sigma F_n = ma_n$:

$$75(9.81) - N_B = 75 \frac{12^2}{20}$$

$$\underline{N_B = 195.8 \text{ N}}$$

n (Note static normal of magnitude)
 $(N = mg = 75(9.81) = 736 \text{ N})$

3/72



$$\theta = \frac{\pi}{3} \sin 0.950 t$$

$$\dot{\theta} = \frac{\pi}{3} (0.950) \cos 0.950 t$$

$$\dot{\theta}_{\max} = \frac{\pi}{3} (0.950) = 0.995 \text{ rad/s}$$

when $\theta = 0$.

$$\Sigma F_n = ma_n: N - mg = m r \dot{\theta}^2$$

$$N = mg + m(11)(0.995)^2 = \underline{20.7m} \begin{cases} N \text{ in newtons} \\ \text{when } m \text{ in kg} \end{cases}$$

Riders near center experience the greatest normal force; Those at ends of unit experience the smallest normal force when θ is at a maximum (or minimum).

3/73

$$\sum F_y = 0: N \cos \theta - mg = 0$$

$$N = mg / \cos \theta$$



$$\sum F_n = ma_n: N \sin \theta = m(r \sin \theta) \omega^2$$

$$\left(\frac{mg}{\cos \theta} \right) \sin \theta = m r \sin \theta \omega^2$$

$$\omega = \sqrt{\frac{g}{r \cos \theta}}$$

Note that $\cos \theta = \frac{g}{r \omega^2} \leq 1$

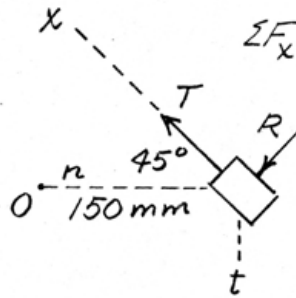
$\therefore \omega^2 \geq \frac{g}{r}$ is a restriction.

3/74

$$a_n = r\dot{\theta}^2 = 0.15 \left(300 \frac{2\pi}{60} \right)^2 = 148.0 \text{ m/s}^2$$

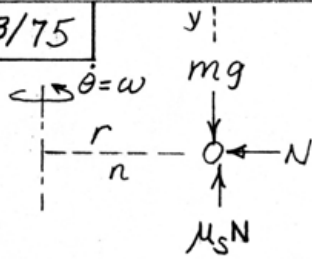
$$\sum F_x = ma_x; T = 3(148.0 \cos 45^\circ)$$

$$= \underline{314 \text{ N}}$$



Direction of rotation does not change accel., hence has no influence on T or R .

3/75

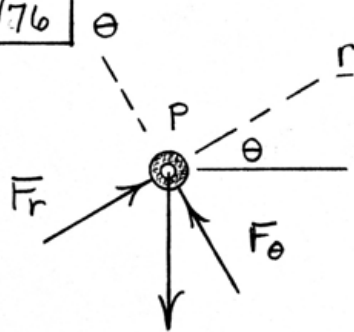


$$\Sigma F_n = m a_n; N = m r \omega^2$$

$$\Sigma F_y = 0; \mu_s (m r \omega^2) = m g$$

$$\omega^2 = \frac{g}{\mu_s r}, \quad \omega = \sqrt{\frac{g}{\mu_s r}}$$

3/76



$$\theta = 30^\circ$$

$$\dot{\theta} = 40 \left(\frac{\pi}{180} \right) = 0.698 \text{ rad/s}$$

$$\ddot{\theta} = 120 \left(\frac{\pi}{180} \right) = 2.09 \text{ rad/s}^2$$

$$r = 1.25 \text{ m}$$

$$\dot{r} = 0.4 \text{ m/s}$$

$$\ddot{r} = -0.3 \text{ m/s}^2$$

$$1.2(9.81) = 11.77 \text{ N}$$

$$\Sigma F_r = m a_r : F_r - 11.77 \sin 30^\circ = 1.2 [-0.3 - 1.25(0.698)^2]$$

$$F_r = 4.79 \text{ N}$$

$$\Sigma F_\theta = m a_\theta : F_\theta - 11.77 \cos 30^\circ = 1.2 [1.25(2.09) + 2(0.4)(0.698)]$$

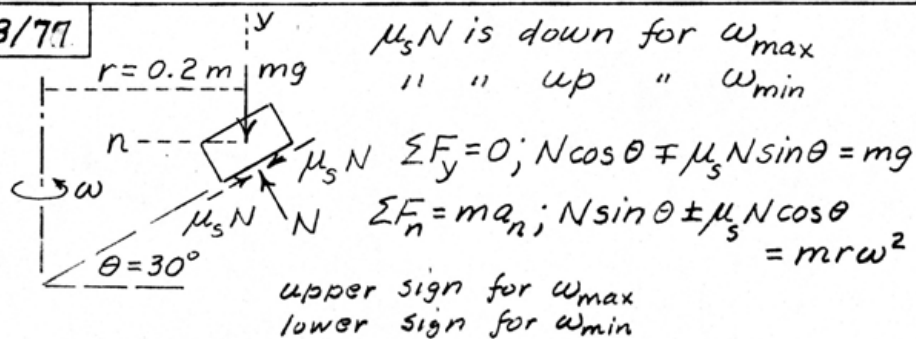
$$F_\theta = 14.00 \text{ N}$$

For static case, set $a_\theta = a_r = 0$ & obtain

$$(F_r)_{st} = 5.89 \text{ N},$$

$$(F_\theta)_{st} = 10.19 \text{ N}$$

3/77



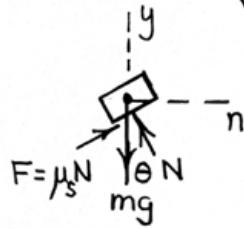
Combine & get $\frac{\sin \theta \pm \mu_s \cos \theta}{\cos \theta \mp \mu_s \sin \theta} = \frac{r \omega^2}{g}$

$$\omega = \sqrt{\frac{g}{r}} \sqrt{\frac{\sin \theta \pm \mu_s \cos \theta}{\cos \theta \mp \mu_s \sin \theta}} = \sqrt{\frac{9.81}{0.2}} \sqrt{\frac{0.5 \pm 0.3(0.866)}{0.866 \mp 0.3(0.5)}}$$

Upper sign $\omega_{\max} = \underline{7.21 \text{ rad/s}}$

Lower sign $\omega_{\min} = \underline{3.41 \text{ rad/s}}$

3/78



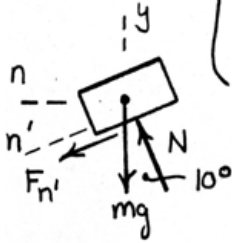
$$\begin{cases} \sum F_y = 0: N \cos \theta - mg + \mu_s N \sin \theta = 0 \\ \sum F_n = ma_n: -N \sin \theta + \mu_s N \cos \theta = mr\omega^2 \end{cases}$$

Solving for ω :

$$\omega = \sqrt{\frac{g}{r} \frac{(\mu_s \cos \theta - \sin \theta)}{(\cos \theta + \mu_s \sin \theta)}} = \underline{2.73 \text{ rad/s}}$$

3/79

Crate:



$$\begin{cases} \sum F_y = 0: N \cos 10^\circ - F_{n'} \sin 10^\circ - mg = 0 \\ \sum F_n = m a_n: F_{n'} \cos 10^\circ + N \sin 10^\circ = m \frac{(2t)^2}{30} \\ \sum F_t = m a_t: F_t = m(2) \end{cases}$$

Solve first two equations for N and $F_{n'}$ to obtain

(t, F_t into paper) $F_{n'} = m \left[\frac{4t^2 \cos 10^\circ}{30} - g \sin 10^\circ \right]$

$$N = m \left[\frac{4t^2 \sin 10^\circ}{30} + g \cos 10^\circ \right]$$

Condition for slipping: $\sqrt{F_t^2 + F_{n'}^2} = \mu_s N$

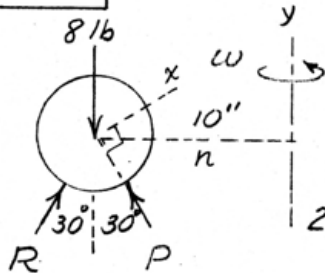
$$\sqrt{2^2 + \left(\frac{4t^2 \cos 10^\circ}{30} - g \sin 10^\circ \right)^2} = 0.3 \left[\frac{4t^2 \sin 10^\circ}{30} + g \cos 10^\circ \right]$$

Square both sides and solve for t :

$$\underline{t = 5.58 \text{ s}}$$

3/80

$$\omega = 30 \times 2\pi / 60 = \pi \text{ rad/sec}$$

SOL. I

$$\sum F_y = 0; (R+P) \cos 30^\circ = 8$$

$$\sum F_n = m a_n; (R-P) \sin 30^\circ = \frac{8}{32.2} \frac{10}{12} (\pi)^2$$

$$2R = 8 \left[\frac{1}{0.866} + \frac{10\pi^2}{32.2(12) 0.5} \right]$$

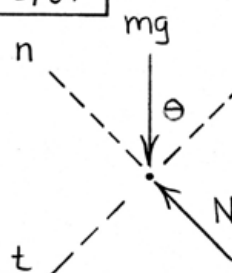
$$R = 4(1.155 + 0.511) = \underline{6.66 \text{ lb}}$$

SOL II (one force equation)

$$\sum F_x = m a_x; R \cos 30^\circ - 8 \sin 30^\circ = \frac{8}{32.2} \frac{10}{12} (\pi)^2 \cos 30^\circ$$

$$R = 8 \left(\tan 30^\circ + \frac{10\pi^2}{32.2 \times 12} \right) = \underline{6.66 \text{ lb}}$$

3/81



Treat the child as a particle.

$$\left\{ \begin{array}{l} \Sigma F_t = ma_t : mg \cos \theta = ma_t \quad (1) \\ \Sigma F_n = ma_n : N - mg \sin \theta = m \frac{v^2}{R} \quad (2) \end{array} \right.$$

$$\text{From (1) : } g \cos \theta = v \frac{dv}{ds} = v \frac{dv}{R d\theta}$$

$$\int_{\theta_0}^{\theta} Rg \cos \theta d\theta = \int_{v_0}^v v dv$$

$$\theta_0 = 20^\circ$$

$$v_0 = 0$$

$$v = [2Rg (\sin \theta - \sin 20^\circ)]^{1/2}$$

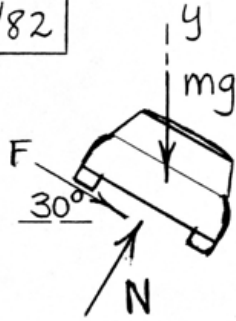
$$(2) : N = m \left(g \sin \theta + \frac{v^2}{R} \right)$$

Numbers ($R = 2.5 \text{ m}$, $g = 9.81 \text{ m/s}^2$)

$$\theta = 30^\circ : \begin{cases} a_t = 8.50 \text{ m/s}^2 \\ v = 2.78 \text{ m/s} \\ N = 280 \text{ N} \end{cases}$$

$$\theta = 90^\circ : \begin{cases} a_t = 0 \\ v = 5.68 \text{ m/s} \\ N = 795 \text{ N} \end{cases}$$

3/82



For no slipping tendency, set F to zero on FBD.

$$\begin{cases} \sum F_y = 0: N \cos 30^\circ - mg = 0 \\ \sum F_n = m \frac{v^2}{r}: N \sin 30^\circ = m \frac{v^2}{1200} \end{cases}$$

Solve: $N = 1.155mg$, $v = 149.4 \text{ ft/sec}$
or $v = 101.8 \text{ mi/hr}$

$v_{\min} = 0$, as $\theta_{\max} = \tan^{-1} \mu_s = \tan^{-1} (0.9)$
 $= 42.0^\circ > 30^\circ$

For $v_{\max}$, set $F = F_{\max} = \mu_s N$:

$$\begin{cases} \sum F_y = 0: N \cos 30^\circ - mg - \mu_s N \sin 30^\circ = 0 \\ \sum F_n = m \frac{v^2}{r}: \mu_s N \cos 30^\circ + N \sin 30^\circ = m \frac{v_{\max}^2}{r} \end{cases}$$

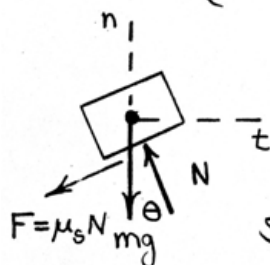
With $\mu_s = 0.9$: $N = 2.40mg$

$v_{\max} = 345 \text{ ft/sec}$ (235 mi/hr)

3/83

Package:

$$\begin{cases} \sum F_t = ma_t : -\mu_s N \cos \theta - N \sin \theta = -m \frac{g}{2} \\ \sum F_n = ma_n : N \cos \theta - \mu_s N \sin \theta - mg = m \left(\frac{19.44^2}{80} \right) \end{cases}$$



First eq. :

$$N = \frac{mg/2}{\sin \theta + \mu_s \cos \theta}$$

Second eq. :

$$\left(\frac{mg/2}{\sin \theta + \mu_s \cos \theta} \right) (\cos \theta - \mu_s \sin \theta) - mg = m(4.726)$$

$$\tan \theta = \left(\frac{1 - 2.9635 \mu_s}{\mu_s + 2.9635} \right)$$

For $\mu_s = 0.2$, $\theta = 7.34^\circ$

For $\mu_s = 0.4$, $\theta = -3.16^\circ$!!

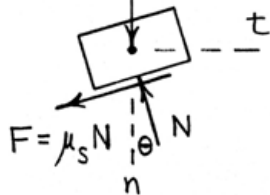
(Note: $N > 0$ for $\theta = -3.16^\circ$)

3/84

$$\Sigma F_t = ma_t: -\mu_s N \cos \theta - N \sin \theta = -mg/2$$

Package:

$$N = \frac{mg/2}{\sin \theta + \mu_s \cos \theta}$$



$$\Sigma F_n = ma_n: -N \cos \theta + \mu_s N \sin \theta + mg = m \left(\frac{19.44^2}{80} \right)$$

$$\left(\frac{mg/2}{\sin \theta + \mu_s \cos \theta} \right) (\mu_s \sin \theta - \cos \theta) + mg = m (4.726)$$

$$\tan \theta = \left[\frac{1 - 1.036 \mu_s}{\mu_s + 1.036} \right]$$

For $\mu_s = 0.2$, $\theta = 32.7^\circ$ and $N > 0$.

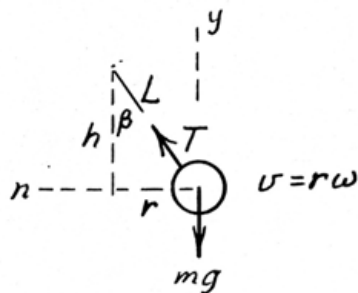
For $\mu_s = 0.4$, $\theta = 22.2^\circ$ and $N > 0$.

3/85

$$\Sigma F_y = 0: T \cos \beta - mg = 0, T \cos \beta = mg$$

$$\Sigma F_n = ma_n: T \sin \beta = m v^2 / r$$

$$\text{Divide \& get } \tan \beta = \frac{v^2}{gr} = \frac{r\omega^2}{g}$$



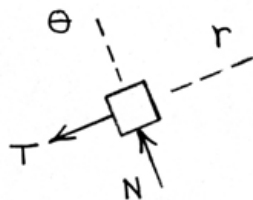
$$\text{But } r = L \sin \beta \text{ so } \tan \beta = L \omega^2 \sin \beta / g$$

$$\text{or } L \cos \beta = g / \omega^2$$

$$\text{And } h = L \cos \beta \text{ so } \underline{h = g / \omega^2} \text{ (depends only on } \omega \text{ \& } g \text{)}$$

$$\text{Then } \underline{T = \frac{mg}{\cos \beta} = \frac{mg}{h/L} = \frac{mgL}{g/\omega^2} = mL\omega^2}$$

3/86



$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2):$$

$$-T = \frac{3}{32.2} \left(0 - \frac{9}{12} 6^2 \right)$$

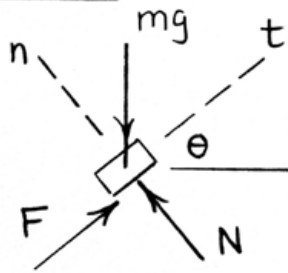
$$\underline{T = 2.52 \text{ lb}}$$

$$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}):$$

$$N = \frac{3}{32.2} \left[\frac{9}{12}(-2) + 2\left(-\frac{2}{12}\right)(6) \right]$$

$$\underline{N = -0.326 \text{ lb}} \quad (\text{Contact on side B})$$

3/87



$$a_n = r\Omega^2 = \frac{13}{12}(7.5)^2 = 60.9 \text{ ft/sec}^2$$

$$\Sigma F_n = ma_n :$$

$$N - mg \cos \theta = 60.9 m \quad (1)$$

$$\Sigma F_t = ma_t :$$

$$F - mg \sin \theta = 0 \quad (2)$$

Slip impends when $F = F_{\max} = \mu_s N$. From

$$(1) \text{ \& } (2): \quad \mu_s = \frac{32.2 \sin \theta}{60.9 + 32.2 \cos \theta}$$

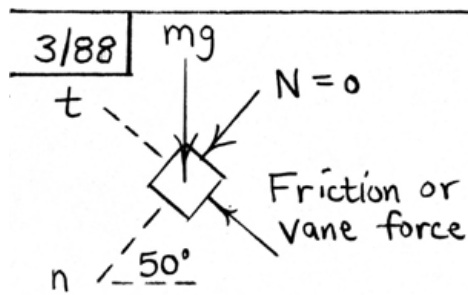
$$(a) \quad \theta = 50^\circ: \quad \mu_s = 0.302$$

$$(b) \quad \theta = 100^\circ: \quad \mu_s = 0.573$$

From (1) $N = m(60.9 + g \cos \theta) > 0$ for all θ .

So contact is maintained.

(Look ahead to solution of Prob.*3/365.)

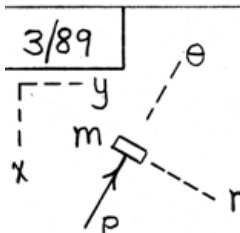


$$\Sigma F_n = ma_n : mg \sin 50^\circ = mr \Omega^2$$

$$\Omega = \sqrt{\frac{g \sin 50^\circ}{r}} = \sqrt{\frac{9.81 \sin 50^\circ}{0.330}} = \underline{4.77 \text{ rad/s}}$$

(45.6 rev/min)

3/89



(Note: mg is static normal $\perp$ to paper)

$$\Sigma F_r = m a_r: 0 = m(\ddot{r} - r\Omega^2) \quad (1)$$

$$\Sigma F_\theta = m a_\theta: P = m(r\ddot{\theta} + 2\dot{r}\Omega) \quad (2)$$

$$(1): \ddot{r} = \dot{r} \frac{d\dot{r}}{dr} = \frac{r}{r} \Omega^2$$

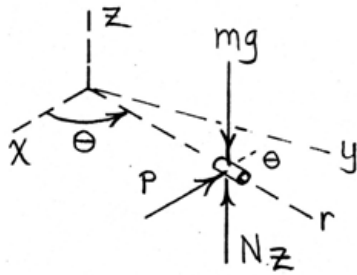
$$\int_{r_0}^{\dot{r}} \dot{r} d\dot{r} = \int_{r_0}^r \Omega^2 r dr \Rightarrow \dot{r}^2 = \dot{r}_0^2 + \Omega^2(r^2 - r_0^2)$$

Numbers: $\dot{r} = \left[60^2 + 7^2 \left(3^2 - \left(\frac{6}{12} \right)^2 \right) \right]^{1/2} = 63.5 \frac{\text{ft}}{\text{sec}}$
(at end of tube)

$$(2): P = m(2\dot{r}\Omega) = \frac{5/16}{32.2} (2)(63.5)(7)$$

$$= \underline{8.62 \text{ lb}}$$

3/90



$$\sum F_z = 0 \Rightarrow N_z = mg$$

From the solution to
Prob. 3/89,

$$\dot{r} = \left\{ \dot{r}_0^2 + \Omega^2 (r^2 - r_0^2) \right\}^{1/2}$$

$$\text{So } \int_{r_0}^r \frac{dr}{\sqrt{\dot{r}_0^2 + \Omega^2 (r^2 - r_0^2)}} = \int_0^t dt$$

$$\frac{1}{\Omega} \ln \left[r + \sqrt{r^2 + \frac{\dot{r}_0^2}{\Omega^2} - r_0^2} \right] \Big|_{r_0}^r = t$$

$$\frac{1}{\Omega} \ln \left[\frac{r + \sqrt{r^2 + \frac{\dot{r}_0^2}{\Omega^2} - r_0^2}}{r_0 + \frac{\dot{r}_0}{\Omega}} \right] = t$$

With numbers ($r = 3 \text{ ft}$, $r_0 = 0.5 \text{ ft}$, $\Omega = 7 \frac{\text{rad}}{\text{sec}}$,
 $\dot{r}_0 = 60 \text{ ft/sec}$),

$$t = 0.0408 \text{ sec}; \text{ Then } \theta = \Omega t = 0.285 \text{ rad}$$

From solution to Prob. 3/89, $P = 8.62 \text{ lb}$

$$P_x = -P \sin \theta = -8.62 \sin 0.285^{\text{R}} = \underline{-2.43 \text{ lb}}$$

$$P_y = P \cos \theta = 8.62 \cos 0.285^{\text{R}} = \underline{8.28 \text{ lb}}$$

3/91 The distance traveled from A to C is

$$(s_c - s_A) = 100 + 250 \left(30 \frac{\pi}{180} \right) = 231 \text{ ft}$$

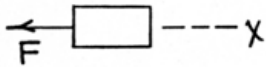
Uniform tangential acceleration: $v_c^2 = v_A^2 + 2a_t(s_c - s_A)$

$$0^2 = \left[60 \frac{5280}{3600} \right]^2 + 2a_t(231), \quad a_t = -16.77 \text{ ft/sec}^2$$

Speed at B: $v_B^2 = v_A^2 + 2a_t(s_B - s_A)$

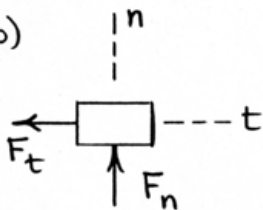
$$v_B^2 = \left[60 \frac{5280}{3600} \right]^2 + 2(-16.77)(100), \quad v_B = 66.3 \text{ ft/sec}$$

(a) $\Sigma F_x = ma_x: -F = \frac{3000}{32.2} (-16.77)$



$$F = 1562 \text{ lb}$$

(b) $\Sigma F_t = ma_t: -F_t = \frac{3000}{32.2} (-16.77)$



$$F_t = 1562 \text{ lb}$$

$$\Sigma F_n = m \frac{v^2}{r}: F_n = \frac{3000}{32.2} \frac{66.3^2}{250}$$

$$F_n = 1636 \text{ lb}$$

$$F = \sqrt{F_t^2 + F_n^2} = 2260 \text{ lb}$$

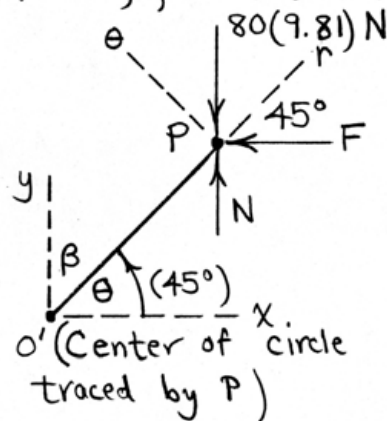
(c) v and therefore F_n go to zero;

$$F = F_t = 1562 \text{ lb}$$

(In all FBDs, there is a weight into the paper and a static normal force out of the paper.)

3/92

FBD of rider at P (could be any rider!), treated as a particle:



$$O'P = 6 \text{ m}$$

$$\theta = 45^\circ$$

$$\dot{\theta} = -0.8 \text{ rad/s}$$

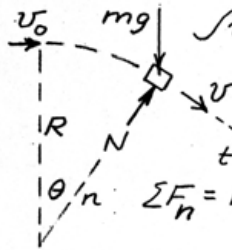
$$\ddot{\theta} = -0.4 \text{ rad/s}^2$$

$$\begin{aligned} \Sigma F_r &= m(\ddot{r} - r\dot{\theta}^2) : -80(9.81)\frac{\sqrt{2}}{2} + N\frac{\sqrt{2}}{2} - F\frac{\sqrt{2}}{2} \\ &= 80[0 - 6(-0.8)^2] \quad (1) \end{aligned}$$

$$\begin{aligned} \Sigma F_\theta &= m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) : -80(9.81)\frac{\sqrt{2}}{2} + N\frac{\sqrt{2}}{2} + F\frac{\sqrt{2}}{2} \\ &= 80[6(-0.4) + 0] \quad (2) \end{aligned}$$

$$\text{Solve (1) \& (2) : } \begin{cases} N = 432 \text{ N} \\ F = 81.5 \text{ N} \end{cases} \parallel \text{Static: } \begin{cases} N_s = 785 \text{ N} \\ F_s = 0 \end{cases}$$

3/93 $\Sigma F_t = ma_t$; $mg \sin \theta = ma_t$, $a_t = g \sin \theta$
 $\int v dv = \int a_t ds$; $\int_{v_0}^v v dv = \int_0^\theta g \sin \theta (R d\theta)$



$v^2 = v_0^2 + 2gR(1 - \cos \theta)$
 $\Sigma F_n = ma_n$; $mg \cos \theta - N = m \frac{v^2}{R}$

$$N = mg \cos \theta - \frac{m}{R} v_0^2 - 2mg(1 - \cos \theta)$$

$$= mg \left(3 \cos \theta - 2 - \frac{v_0^2}{gR} \right)$$

When $N=0$, $\theta = \beta$ so $3 \cos \beta = 2 + \frac{v_0^2}{gR}$

$$\beta = \cos^{-1} \left(\frac{2}{3} + \frac{v_0^2}{3gR} \right)$$

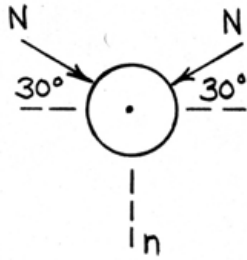
For $v_0=0$, $\beta = \cos^{-1} \left(\frac{2}{3} \right) = \underline{48.2^\circ}$

$$3/94 \quad \Sigma F_n = ma_n = mr\omega^2 :$$

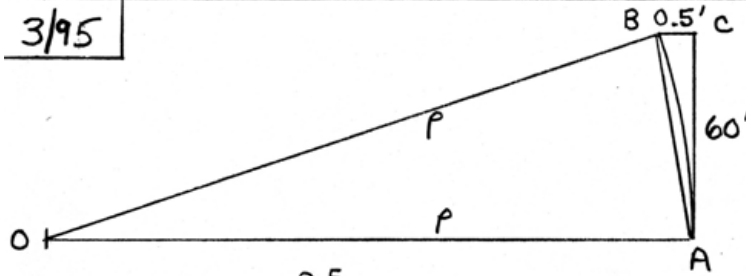
$$2N \sin 30^\circ = 2.5 (0.150) \left[\frac{600(2\pi)}{60} \right]^2$$

$$N = 1480 \text{ N}$$

$$F = 4N \cos 30^\circ = \underline{5130 \text{ N}}$$



3/95



$$\angle BAC = \tan^{-1} \frac{0.5}{60} = 0.477^\circ$$

$$\angle OBA = \angle OAB = (90 - 0.477) = 89.5^\circ$$

$$\angle BOA = 180 - 2(89.5) = 0.955^\circ = 2 \angle BAC$$

$$\overline{AB} = \sqrt{60^2 + 0.5^2} = 60.002'$$

$$\frac{\sin 0.955^\circ}{60.002'} = \frac{\sin 89.5^\circ}{P}, \quad \underline{P = 3600 \text{ ft}}$$

FBD: (horizontal forces)

$$\sum F_n = m a_n : R = \frac{5.125/16}{32.2} \frac{120^2}{3600}$$



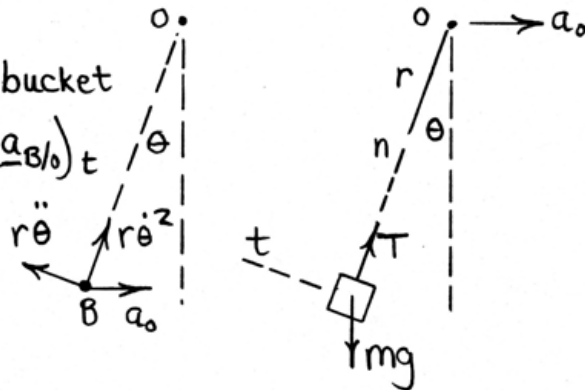
$$\underline{R = 0.0398 \text{ lb}}$$

(Note: $R = 0.637 \text{ oz}$ represents 12.4% of the weight of the baseball)

3/96

Acceleration of bucket

$$\underline{a}_B = \underline{a}_o + (\underline{a}_{B/o})_n + (\underline{a}_{B/o})_t$$



$$\Sigma F_t = ma_t : -mg \sin \theta = m(r\ddot{\theta} - a_o \cos \theta)$$

$$\ddot{\theta} = +\frac{1}{r}(a_o \cos \theta - g \sin \theta)$$

$\dot{\theta}$ is a maximum when $\ddot{\theta} = 0 : a_o \cos \theta = g \sin \theta$

$$\theta = \tan^{-1}\left(\frac{a_o}{g}\right)$$

With $\ddot{\theta} = \dot{\theta} \frac{d\dot{\theta}}{d\theta}$:

$$\int_0^{\dot{\theta}} \dot{\theta} d\dot{\theta} = \int_0^{\theta} \frac{1}{r}(a_o \cos \theta - g \sin \theta) d\theta$$

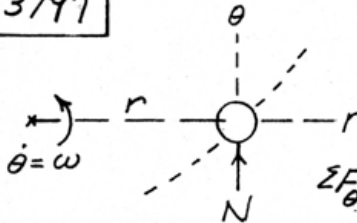
$$\dot{\theta}^2 = \frac{2}{r}(a_o \sin \theta + g \cos \theta - g)$$

$$\Sigma F_n = ma_n : T - mg \cos \theta = m(r\dot{\theta}^2 + a_o \sin \theta)$$

Substitute expression for $\dot{\theta}^2$:

$$T = m(3a_o \sin \theta + 3g \cos \theta - 2g)$$

3/97



$$\sum F_r = ma_r; 0 = m(\ddot{r} - r\omega^2)$$

$$\text{sol. is } r = r_0 \cosh \omega t$$

$$\dot{r} = r_0 \omega \sinh \omega t$$

$$\sum F_\theta = ma_\theta; N = m(0 + 2\dot{r}\omega)$$

$$= 2mr_0\omega^2 \sinh \omega t$$

But $\cosh^2 \omega t - \sinh^2 \omega t = 1$, $\sinh^2 \omega t = \cosh^2 \omega t - 1$

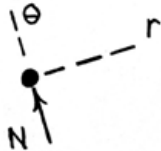
$$\sinh \omega t = \sqrt{\left(\frac{r}{r_0}\right)^2 - 1}$$

So $N = 2mr_0\omega^2 \sqrt{\left(\frac{r}{r_0}\right)^2 - 1} = \underline{2m\omega^2 \sqrt{r^2 - r_0^2}}$

3/98

$$\Sigma F_r = m a_r : 0 = m(\ddot{r} - r\dot{\theta}^2)$$

Particle:



$$\ddot{r} = r\dot{\theta}^2 = r\omega_0^2$$

$$\dot{r} \frac{d\dot{r}}{dr} = r\omega_0^2$$

$$\int_0^{\dot{r}} \dot{r} d\dot{r} = \omega_0^2 \int_{r_0}^r r dr$$

$$\Rightarrow \underline{\dot{r} = \omega_0 \sqrt{r^2 - r_0^2} = v_r}$$

$$\frac{dr}{dt} = \omega_0 \sqrt{r^2 - r_0^2}$$

$$\int_{r_0}^r \frac{dr}{\sqrt{r^2 - r_0^2}} = \omega_0 \int_0^t dt$$

$$\ln[r + \sqrt{r^2 - r_0^2}] \Big|_{r_0}^r = \omega_0 t \Rightarrow \underline{r = \frac{r_0}{2} [e^{-\omega_0 t} + e^{\omega_0 t}]}$$

$$v_\theta = r\dot{\theta} = r\omega_0 = \underline{\frac{r_0 \omega_0}{2} [e^{-\omega_0 t} + e^{\omega_0 t}]}$$

$$\text{As a function of } t, \underline{v_r = \frac{r_0 \omega_0}{2} (e^{\omega_0 t} - e^{-\omega_0 t})}$$

In terms of hyperbolic functions,

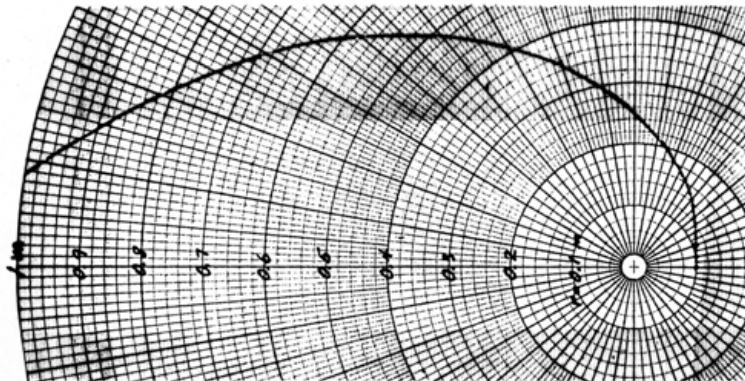
$$v_r = r_0 \omega_0 \sinh \omega_0 t$$

$$r = r_0 \cosh \omega_0 t$$

$$v_\theta = r_0 \omega_0 \cosh \omega_0 t$$

With numbers,

$$\begin{cases} v_r = 0.1 \sinh t \\ r = 0.1 \cosh t \\ v_\theta = 0.1 \cosh t \end{cases}$$

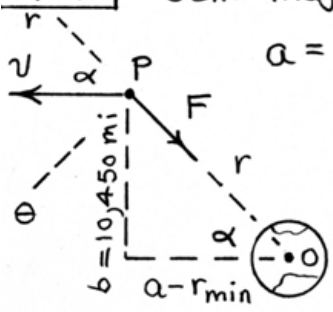


$$(r = 1.0 \text{ m } @ \theta = 171.5^\circ)$$

$\uparrow +\theta$
 $\leftarrow \theta = 0$

3/99

Semi-major axis of ellipse is



$$a = \frac{r_{\max} + r_{\min}}{2} = \frac{26,259 + 4159}{2}$$

$$= 15,209 \text{ mi}$$

$$\alpha = \tan^{-1} \frac{b}{a - r_{\min}}$$

$$= \tan^{-1} \frac{10,450}{15,209 - 4159} = 43.4^\circ$$

$$\dot{r} = v_r = v \cos \alpha = 13,244 \cos 43.4^\circ = \underline{9620 \text{ ft/sec}}$$

$$r\dot{\theta} = v_\theta = v \sin \alpha, \quad \dot{\theta} = \frac{v \sin \alpha}{r}, \quad \text{where}$$

$$r = \sqrt{b^2 + (a - r_{\min})^2} = \sqrt{10,450^2 + (15,209 - 4159)^2}$$

$$= 15,208 \text{ mi}; \quad \text{so } \dot{\theta} = \frac{13,244 \sin 43.4^\circ}{15,208(5280)} = \underline{1.133(10^{-4}) \frac{\text{rad}}{\text{sec}}}$$

$$\Sigma F_r = m a_r: -m \frac{g R^2}{r^2} = m(\ddot{r} - r\dot{\theta}^2)$$

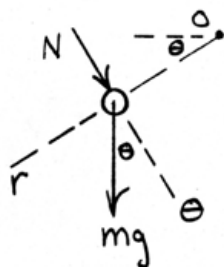
$$\ddot{r} = r\dot{\theta}^2 - \frac{g R^2}{r^2} = 15,208(5280) \left[(1.133)(10^{-4}) \right]^2 - \frac{32.23(3959)^2}{15,208^2}$$

$$= \underline{-1.153 \text{ ft/sec}^2}$$

$$\Sigma F_\theta = m a_\theta: 0 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} = -\frac{2(9620)(1.133)(10^{-4})}{15,208(5280)} = \underline{-2.72(10^{-8}) \frac{\text{rad}}{\text{sec}^2}}$$

3/100



$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2):$$

$$mg \sin \theta = m(\ddot{r} - r\omega_0^2)$$

$$\ddot{r} - \omega_0^2 r = g \sin \omega_0 t$$

Assume $r_h = C e^{st}$ and substitute into equation to obtain $s_1 = -\omega_0$, $s_2 = \omega_0$. Also, assume

a particular solution of form $r_p = D \sin \omega_0 t$, substitute, and obtain $D = -g/2\omega_0^2$.

$$\text{So } r = r_h + r_p = C_1 e^{-\omega_0 t} + C_2 e^{\omega_0 t} - \frac{g}{2\omega_0^2} \sin \omega_0 t$$

Initial conditions :

$$\begin{cases} r(0) = C_1 + C_2 = 0 \\ \dot{r}(0) = -\omega_0 C_1 + \omega_0 C_2 - \frac{g}{2\omega_0} = 0 \end{cases}$$

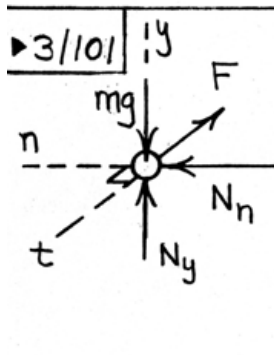
$$\dot{r}(0) = -\omega_0 C_1 + \omega_0 C_2 - \frac{g}{2\omega_0} = 0$$

Solve for C_1 and C_2 to obtain

$$r = -\frac{g}{4\omega_0^2} e^{-\theta} + \frac{g}{4\omega_0^2} e^{\theta} - \frac{g}{2\omega_0^2} \sin \theta$$

$$\text{or } r = \frac{g}{2\omega_0^2} [\sinh \theta - \sin \theta]$$

3/10/14



$$\sum F_y = 0 : N_y = mg$$

$$\sum F_n = ma_n : N_n = m \frac{v^2}{r}$$

$$F = \mu_k N_{tot} = \mu_k \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$$

$$= \frac{\mu_k m}{r} \sqrt{r^2 g^2 + v^4}$$

$$\sum F_t = ma_t : -\frac{\mu_k m}{r} \sqrt{r^2 g^2 + v^4} = m v \frac{dv}{ds}$$

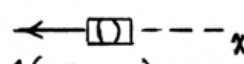
$$-\frac{\mu_k}{r} \int_0^s ds = \int_{v_0}^0 \frac{v dv}{\sqrt{v^4 + r^2 g^2}} = \int_{v_0^2}^0 \frac{\frac{1}{2} dx}{\sqrt{x^2 + r^2 g^2}}$$

where $x = v^2$, $dx = 2v dv$

Integrating,

$$-\frac{\mu_k}{r} s = \frac{1}{2} \ln [x + \sqrt{x^2 + r^2 g^2}] \Big|_{v_0^2}^0$$

$$\text{or } s = \frac{r}{2\mu_k} \ln \left[\frac{v_0^2 + \sqrt{v_0^4 + r^2 g^2}}{r g} \right]$$

3/102 Motion from A to B : 

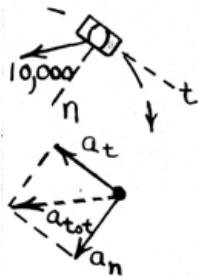
$$\Sigma F_x = ma_x : -4(2500) = 1350a \quad 4(2500\text{N})$$

$$a = -7.407 \text{ m/s}^2, \quad v_B^2 - v_A^2 = 2a(x_B - x_A)$$

$$v_B^2 - 25^2 = 2(-7.407)(10)$$

$$v_B = 21.84 \text{ m/s}$$

Beyond B : $F = ma_{\text{tot}}, \quad a_{\text{tot}} = \frac{10,000}{1350} = 7.407 \text{ m/s}^2$



$$a_{\text{tot}} = \sqrt{a_n^2 + a_t^2} = \sqrt{\frac{v^4}{r^2} + a_t^2}$$

$$a_t = -\sqrt{a_{\text{tot}}^2 - \frac{v^4}{r^2}} = v \frac{dv}{ds}$$

$$\int_{10}^s ds = -r \int_{v_B}^0 \frac{v dv}{\sqrt{r^2 a_{\text{tot}}^2 - v^4}}$$

Let $x = v^2 : s - 10 = -r \int_{v_B^2}^0 \frac{dx/2}{\sqrt{r^2 a_{\text{tot}}^2 - x^2}}$

$$s = 10 + \frac{r}{2} \sin^{-1} \left(\frac{v_B^2}{r a_{\text{tot}}} \right) = \underline{47.4 \text{ m}}$$

3/103

State ① : launch; State ② : apex

$$T_1 + U_{1 \rightarrow 2} = T_2 : \frac{1}{2} m v_0^2 - mgh = 0$$

$$\Rightarrow h = \frac{v_0^2}{2g}$$



$$\text{For } v_0 = 50 \text{ m/s: } h = \frac{50^2}{2(9.81)} = \underline{127.4 \text{ m}}$$

3/104

$$\begin{aligned} (a) \quad U_{1-2} &= \frac{1}{2} k (x_1^2 - x_2^2) \\ &= \frac{1}{2} (3)(12) \left[\left(\frac{6}{12} \right)^2 - \left(\frac{3}{12} \right)^2 \right] = \underline{3.38 \text{ ft-lb}} \end{aligned}$$

$$\begin{aligned} (b) \quad U_{1-2} &= -mgh = -14 \left(\frac{9}{12} \right) \sin 15^\circ \\ &= \underline{-2.72 \text{ ft-lb}} \end{aligned}$$

3/105

$$T_A + U_{A-B} = T_B$$

$$\frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2$$

$$v_B^2 = v_A^2 + 2gh = 4^2 + 2(9.81)(1.8)$$

$$v_B = 7.16 \text{ m/s}$$

Knowledge of the shape of the track is unnecessary, as long as it is known that the cart passes the highest point.

3/106

$$T_A + U_{A-B} = T_B$$

$$\frac{1}{2} m v_A^2 + U_f + mgh = \frac{1}{2} m v_B^2$$

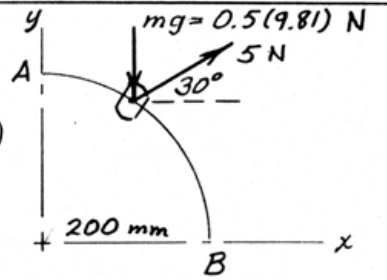
$$U_f = m \left(\frac{v_B^2}{2} - \frac{v_A^2}{2} - gh \right)$$

$$= 3 \left(\frac{6^2}{2} - \frac{4^2}{2} - 9.81(1.8) \right) = \underline{-23.0 \text{ J}}$$

3/107

$$\begin{aligned}
 U = \Delta T: & 5 \cos 30^\circ (0.2) - 5 \sin 30^\circ (0.2) \\
 & + 0.5(9.81)(0.2) \\
 & = \frac{1}{2} 0.5 (v^2 - 0)
 \end{aligned}$$

$$v^2 = 5.39 \text{ (m/s)}^2, \quad \underline{v = 2.32 \text{ m/s}}$$

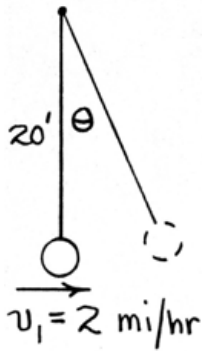


3/108

$$T_1 + U_{1-2} = T_2 : \frac{1}{2} m v_1^2 - mgh = 0$$

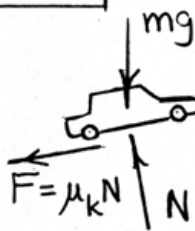
$$\frac{1}{2} \left[2 \frac{5280}{3600} \right]^2 - 32.2 [20(1 - \cos \theta)] = 0$$

$$\theta = 6.63^\circ$$



3/109

$$\theta = \tan^{-1} \frac{6}{100} = 3.43^\circ$$



$$T_A + U_{A-B} = T_B$$

$$\frac{1}{2} m v_0^2 - \mu_k m g \cos \theta s - m g s \sin \theta = 0$$

$$\frac{1}{2} \left(65 \frac{5280}{3600} \right)^2 - 32.2 s [0.6 \cos 3.43^\circ + \sin 3.43^\circ] = 0$$

$$s = 214 \text{ ft}$$

Going downhill ($B \rightarrow A$): $T_B + U_{B-A} = T_A$

$$\frac{1}{2} m v_0^2 - \mu_k m g \cos \theta s + m g s \sin \theta = 0$$

$$\frac{1}{2} \left(65 \frac{5280}{3600} \right)^2 + 32.2 s [-0.6 \cos 3.43^\circ + \sin 3.43^\circ] = 0$$

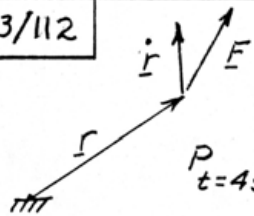
$$s = 262 \text{ ft}$$

$$\begin{aligned} \underline{3/110} \quad & \text{For collar, } U_{1-2} = \Delta T = 0 \\ U_{1-2} = & 50\left(\frac{50-30}{12}\right) - 30 \frac{40}{12} \sin 30^\circ - \frac{1}{2} k \left(\frac{6}{12}\right)^2 = 0 \\ & \underline{k = 267 \text{ lb/ft}} \end{aligned}$$

$$\begin{array}{|l}
 3/111 \\
 \hline
 \end{array}
 \quad U_{1-2} = \Delta T; \quad 2\left(\frac{1}{2} k x^2\right) = \frac{1}{2} m v^2 - 0$$

$$k = \frac{1}{2} \frac{m v^2}{x^2} = \frac{1}{2} \frac{3500 \left(\frac{5}{30} 44\right)^2}{(6/12)^2} \frac{1}{12} = \underline{974 \text{ lb/in.}}$$

3/112



Power $P = \underline{F} \cdot \underline{\dot{r}}$

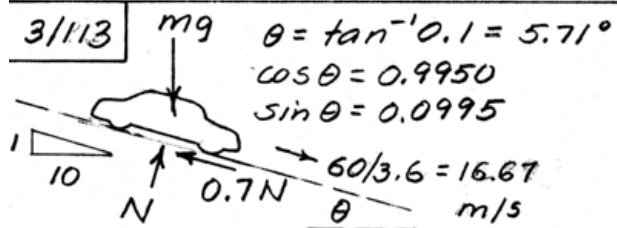
$$P = (40\underline{i} - 20\underline{j} - 36\underline{k}) \cdot (8\underline{i} + 2.4t\underline{j} - 1.5t^2\underline{k})$$

$$P_{t=4s} = (40\underline{i} - 20\underline{j} - 36\underline{k}) \cdot (8\underline{i} + 9.6\underline{j} - 24\underline{k})$$

$$= 320 - 192 + 864 = 992 \text{ W}$$

$$\text{or } \underline{P = 0.992 \text{ kW}}$$

3/113



$$N = mg \cos \theta$$

$$= 0.9950 (9.81) \text{ m}$$

$$= 9.76 \text{ m}$$

$$U = \Delta T; -0.7(9.76 \text{ m})S + 9.81 \text{ m}(0.0995)S = -\frac{m}{2}(16.67)^2$$

$$5.86 S = 138.9, \quad \underline{S = 23.7 \text{ m}}$$

$$3/114 \quad P = W\dot{y} \text{ where } \dot{y} = v \sin \theta$$

$$\theta = \tan^{-1} 0.05 = 2.86^\circ, \sin \theta = 0.0499$$

$$P = 200 \left(\frac{15}{30} 44 \right) 0.0499 = 219.7 \text{ ft-lb/sec}$$

$$\text{or } P = \frac{219.7}{550} = \underline{0.400 \text{ hp}}$$

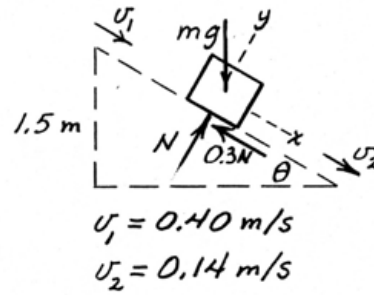
3/115

$$\Sigma F_y = 0: N - mg \cos \theta = 0$$

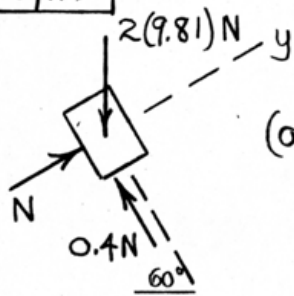
$$U = \Delta T: (mg \sin \theta - 0.3 mg \cos \theta) \frac{1.5}{\sin \theta} = \frac{1}{2} m (0.14^2 - 0.40^2)$$

$$1.5 (9.81) \left(1 - \frac{0.3}{\tan \theta}\right) = -0.0702$$

$$\tan \theta = 0.299, \quad \theta = 16.62^\circ$$



3/116



$$\Sigma F_y = 0: N - 2(9.81) \cos 60^\circ = 0$$

$$N = 9.81 \text{ N}$$

$$(a) \quad U_{1-2} = \Delta T: 2(9.81)(0.5 \sin 60^\circ) - 0.4(9.81)(0.5) = \frac{1}{2} 2 v^2$$

$$v = \underline{2.56 \text{ m/s}}$$

$$(b) \quad U_{1-3} = \Delta T: 2(9.81)(0.5 + x) \sin 60^\circ - 0.4(9.81)(0.5 + x) - \frac{1}{2} (1600)x^2 = 0$$

$$800x^2 - 13.07x - 6.53 = 0$$

$$x = 0.0989 \text{ m} \quad \text{or} \quad \underline{x = 98.9 \text{ mm}}$$

$$3/117 \quad P = Wh / \Delta t$$

$$\text{or } P = \frac{120(9)}{5} / 550 = \frac{0.393 \text{ hp}}{1}$$

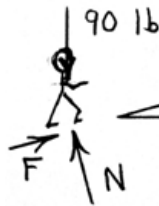
$$\text{Conversions : } h = 9 \text{ ft} \left(\frac{0.3048 \text{ m}}{\text{ft}} \right) = 2.74 \text{ m}$$

$$W = 120 \text{ lb} \left(\frac{4.4482 \text{ N}}{\text{lb}} \right) = 534 \text{ N}$$

$$P = \frac{Wh}{\Delta t} = \frac{534(2.74)}{5} = \underline{293 \text{ watts}}$$

$$\text{Check : } 0.393 \text{ hp} \left(\frac{745.7 \text{ watts}}{\text{hp}} \right) = 293 \text{ watts} \checkmark$$

3/118



$$v_B = 5 \frac{5280}{3600} = 7.33 \text{ ft/sec}$$

$$v_B^2 = 2as, \quad a = \frac{7.33^2}{2(50)} = 0.538 \frac{\text{ft}}{\text{sec}^2}$$

$$\theta = \tan^{-1}(0.1) = 5.71^\circ$$

$$\rightarrow \Sigma F = ma : F - 90 \sin 5.71^\circ = \frac{90}{32.2} (0.538)$$

$$F = 10.46 \text{ lb}$$

$$P = Fv = 10.46 (7.33) = 76.7 \frac{\text{ft-lb}}{\text{sec}}$$

$$\text{or } P = 76.7/550 = \underline{0.1394 \text{ hp}}$$

$$\boxed{3/119} \quad \text{Net power required} = 30(140)(24)/33,000 \\ = 3.05 \text{ hp}$$

$$\text{Mechanical efficiency} = \frac{\text{Power required}}{\text{Power supplied}} = \frac{3.05}{4.00} = \underline{0.764}$$

3/120

$$U_{1-2} = \Delta T; \quad 15(18+2) - \frac{1}{2} 80(2^2) = \frac{1}{2} \frac{15 v^2}{32.2} (12)$$

15 lb

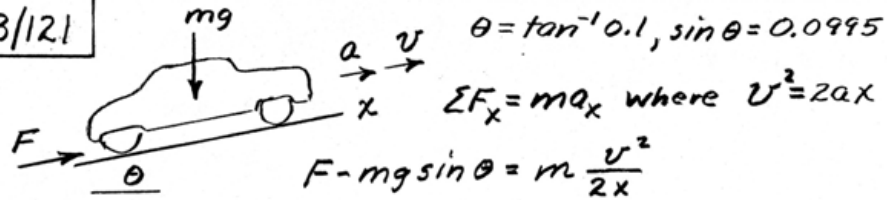


80X

$$300 - 160 = 2.795 v^2, \quad v \text{ in ft/sec}$$

$$v^2 = 50.09, \quad \underline{v = 7.08 \text{ ft/sec}}$$

3/121



$$\theta = \tan^{-1} 0.1, \sin \theta = 0.0995$$

$$\Sigma F_x = ma_x \text{ where } v^2 = 2ax$$

$$F - mg \sin \theta = m \frac{v^2}{2x}$$

$$P = Fv = mgv \sin \theta + \frac{mv^3}{2x}$$

$$= 1500(9.81) \frac{50000}{3600} 0.0995 + \frac{1500 (50000/3600)^3}{2(100)}$$

$$= 20336 + 20094 = 40430 \text{ W}$$

$$\text{or } P = 40.4 \text{ kW}$$

3/122

For $x = 75 \text{ mm}$, $U = \Delta T \neq$

$$\frac{1}{2} (0.075) R_{\max} = \frac{1}{2} (0.25) (600)^2, R_{\max} = 1.2 \text{ MN}$$

For $x = 25 \text{ mm}$, $R = \frac{25}{75} (1.2) = 0.4 \text{ MN}$ or $0.4 (10^6) \text{ N}$

$$U = \Delta T; \frac{1}{2} (0.025) (0.4) 10^6 = \frac{1}{2} (0.25) (600^2 - v^2)$$

$$v^2 = 320 (10^3) \text{ (m/s)}^2, \underline{v = 566 \text{ m/s}}$$

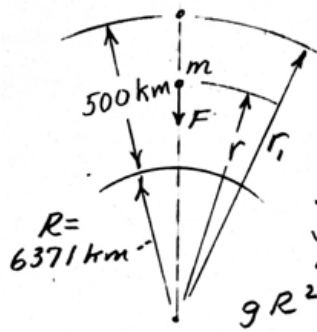
3/123

$$\begin{aligned}\text{Power output} &= \text{rate of doing work} \\ &= 300(9.81)(2) - 100(9.81)(4) \\ &= 1962 \text{ J/s (W)} \\ &= 1.962 \text{ kW}\end{aligned}$$

$$\text{Efficiency } e = \frac{\text{Power output}}{\text{Power input}} = \frac{1.962}{2.20} = \underline{0.892}$$

3/124

$$F = \text{gravitational force} = Gmm_e / r^2 = gR^2m / r^2$$



$$U = \Delta T$$

$$\int_{r_i}^r F(-dr) = \frac{1}{2}mv^2 - 0$$

$$-\int_{r_i}^r \frac{gR^2m}{r^2} dr = \frac{1}{2}mv^2, \quad -gR^2\left(-\frac{1}{r}\right)_{r_i}^r = \frac{v^2}{2}$$

$$gR^2\left(\frac{1}{r} - \frac{1}{r_i}\right) = \frac{v^2}{2}, \quad v = R\sqrt{2g\left(\frac{1}{r} - \frac{1}{r_i}\right)}$$

$$v = 6371 \sqrt{\frac{2(9.825)}{1000} \left(\frac{1}{6371+500-100} - \frac{1}{6371+500} \right)} \quad \frac{\text{km}}{\text{s}}$$

$$= 6371 \sqrt{4.2237(10^{-8})} = \underline{1.309 \text{ km/s}}$$

3/125

$v_A = 0.5 \frac{m}{s}$ $\theta = \sin^{-1} \frac{3}{150} = 1.146^\circ$

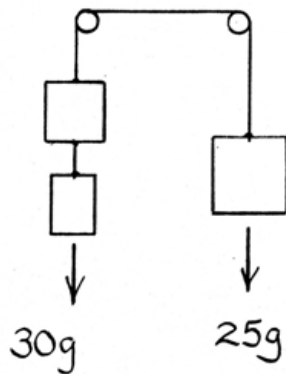
$\overline{AC} = 150 \text{ m}$
 $m = 68 \text{ Mg}$
 $N = mg \cos \theta$
 $F = 32 \text{ kN}$
 $v_C = 3 \frac{m}{s}$

$U = \Delta T; \quad 68(10^3)(9.81)(3) - 32(10^3)x = \frac{1}{2} 68(10^3)(3^2 - 0.5^2)$
 $2001 - 32x = 297.5, \quad x = 53.2 \text{ m}$

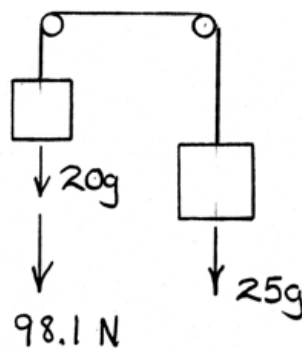
3/126

Active-force diagrams for system:

(a)



(b)

 $U = \Delta T$ for system :

$$(a) (30 - 25)(9.81)2 = \frac{1}{2}(30 + 25)v^2$$

$$v = 1.889 \text{ m/s}$$

$$(b) [(20 - 25)(9.81) + 98.1]2 = \frac{1}{2}(20 + 25)v^2$$

$$v = 2.09 \text{ m/s}$$

3/12.7

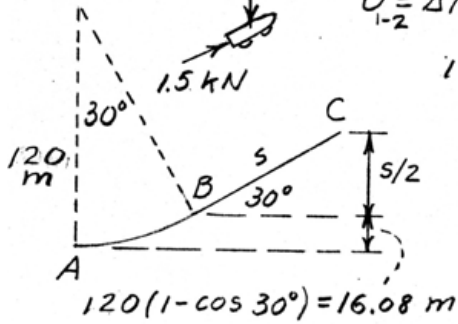
$$mg = 981 \text{ N}$$

$$\bar{AB} = r\theta = 120 \frac{\pi}{6} = 62.8 \text{ m}$$

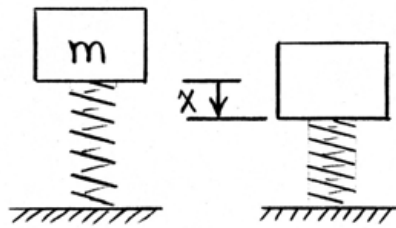
$$U_{1-2} = \Delta T = 0 \text{ since } T_C = T_A = 0$$

$$1500(62.8) - 981(16.08 + \frac{s}{2}) = 0$$

$$s = \frac{2(94248 - 15771)}{981} = 160.0 \text{ m}$$



3/128



Initial

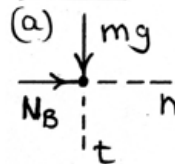
Max. deformation

The maximum force $F = kx$ occurs when x is a maximum with $\dot{x} = 0$.

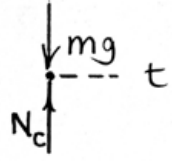
$$U_{1-2} = \Delta T: mgx - \frac{1}{2} kx^2 = 0, x = \frac{2mg}{k}$$

$$\text{So } F = kx = \underline{2mg}$$

3/129 $T_A + U_{A-B} = T_B : 0 + 2mgR = \frac{1}{2}mv_B^2, v_B^2 = 4gR$

(a)  $\Sigma F_n = ma_n : N_B = m \frac{4gR}{R} = \underline{4mg}$

(b) $T_A + U_{A-c} = T_c : 0 + 3mgR = \frac{1}{2}mv_c^2, v_c^2 = 6gR$

 $\Sigma F_n = ma_n : N_c - mg = m \frac{6gR}{R}$
 $N_c = \underline{7mg}$

(c) Call stopping point E :

$$T_A + U_{A-E} = T_E$$

$$0 + 2mgR - mg\left(\frac{1}{2}s\right) - \mu_k \frac{\sqrt{3}}{2} mgs = 0$$

$$s = \frac{4R}{1 + \mu_k \sqrt{3}}$$

(Note: Normal force on incline is)
 $N = mg \cos 30^\circ = \frac{\sqrt{3}}{2} mg$

3/130

Let s = distance down incline before reversal of direction.

$$U_{1-2} = 110(2)(10 + s - s) - 300(10 + s - s)\frac{5}{13} = 1046 \text{ ft-lb}$$

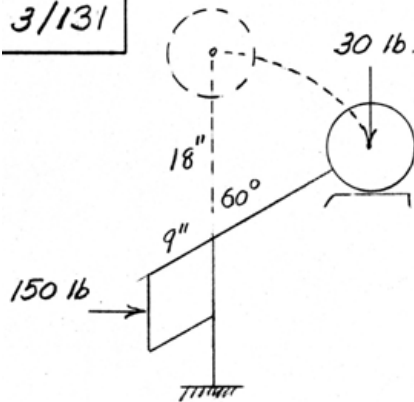
$$\Delta T = \frac{1}{2} \frac{300}{32.2} [v^2 - (\pm 9)^2] = 4.66v^2 - 377 \text{ ft-lb}$$

$$U_{1-2} = \Delta T: 1046 = 4.66v^2 - 377$$

$$v = 17.48 \text{ ft/sec}$$

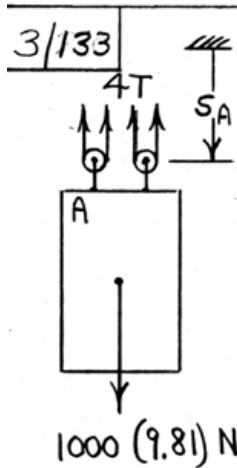
The initial kinetic energy is positive regardless of the velocity direction.

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$$\begin{aligned}
 U_{1-2} &= \Delta T \\
 150 \left(\frac{9}{12} \sin 60^\circ \right) - 30 \frac{18}{12} (1 - \cos 60^\circ) \\
 &= \frac{1}{2} \frac{30}{32.2} (v^2 - 0^2) \\
 v^2 &= 160.8, \quad \underline{v = 12.68 \text{ ft/sec}}
 \end{aligned}$$

$$\begin{aligned}
 \boxed{3/132} \quad U_{1-2} &= \Delta T; \quad mg(0.8 - 1.2 \cos 60^\circ) \\
 &= \frac{1}{2} m (v_C^2 - 3^2) \\
 9.81(0.20) &= \frac{1}{2} (v_C^2 - 9), \quad v_C^2 = 12.92, \quad \underline{v_C = 3.59 \text{ m/s}}
 \end{aligned}$$



$$+\downarrow \Sigma F = 0: 9810 - 4T = 0, T = 2450 \text{ N}$$

Length of cable $L = 4s_A + \text{constants}$

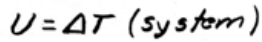
$$\dot{L} = 4v_A = 4(-3) = -12 \text{ m/s}$$

$$P_{\text{out}} = -T\dot{L} = -2450(-12) = 29\,400 \text{ watts}$$

$$\text{or } P_{\text{out}} = 29.4 \text{ kW}$$

$$e = \frac{P_{\text{out}}}{P_{\text{in}}}, \quad P_{\text{in}} = \frac{P_{\text{out}}}{e} = \frac{29.4}{0.8}$$

$$\underline{P_{\text{in}} = 36.8 \text{ kW}}$$


$$30(9.81)1.2 - [50(9.81)0.5 + 127.4] \frac{1.2}{2} = \frac{1}{2} 30 v^2 + \frac{1}{2} 50 (v/2)^2$$

Second interval

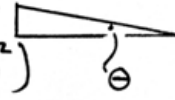
$$15(9.81)(y) - [50(9.81)0.5 + 127.4] \frac{y}{2} = 0 - \frac{1}{2} 15(6.097) - \frac{1}{2} 50 \left(\frac{6.097}{4} \right)$$

$$y = 2.14 \text{ m}, \quad S = \frac{1}{2}(1.2 + 2.14) = \underline{1.67 \text{ m}}$$

$$\begin{aligned}
 3/135 \quad U = \Delta T; \quad - \int_0^4 (3x^2 + 60x) dx &= \frac{1}{2} \frac{48}{32.2} (0 - v^2) 12 \\
 x^3 + 30x^2 \Big|_0^4 &= \frac{288}{32.2} v^2, \quad v \text{ in ft/sec.} \\
 v^2 &= \frac{32.2}{288} (64 + 480) = 60.82 \text{ (ft/sec)}^2, \quad \underline{v = 7.80 \text{ ft/sec}}
 \end{aligned}$$

$$3/136 \quad \theta = \tan^{-1} \frac{6}{100} = 3.43^\circ$$

$$U_{1-2} = \Delta T : U_f + mgh = \frac{1}{2} m (v_2^2 - v_1^2)$$



$$\begin{aligned} U_f &= -1400(9.81)(200 \sin 3.43^\circ) \\ &\quad + \frac{1}{2} 1400 \left[\left(\frac{20}{3.6} \right)^2 - \left(\frac{100}{3.6} \right)^2 \right] \\ &= -683\,000 \text{ J} \quad \text{or} \quad -683 \text{ kJ} \end{aligned}$$

$$\text{Energy lost } Q = \underline{683 \text{ kJ}}$$

3/137 The power output of the drivetrain is

$$P_{\text{out}} = Fv = 560 \left(\frac{90}{3.6} \right) = 14\,000 \text{ W}$$

The power input to the drivetrain:

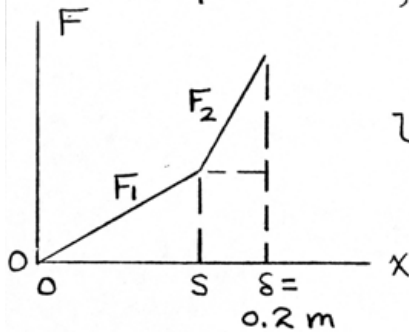
$$P_{\text{in}} = \frac{P_{\text{out}}}{e} = \frac{14\,000}{0.70} = 20\,000 \text{ W}$$

So the motor output $P = 20 \text{ kW}$

3/138

$$F_1 = 300x, \quad F_2 = 300x + 150(x-s)$$

$$= 450x - 150s$$

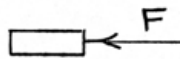


$$U_{1-2} = -\frac{1}{2} 300s^2 - \int_s^{0.2m} (450x - 150s) dx$$

$$= -75s^2 + 30s - 9 \text{ J}$$

$$U = \Delta T :$$

$$-75s^2 + 30s - 9 = \frac{1}{2} (0.5)(0 - 5^2)$$

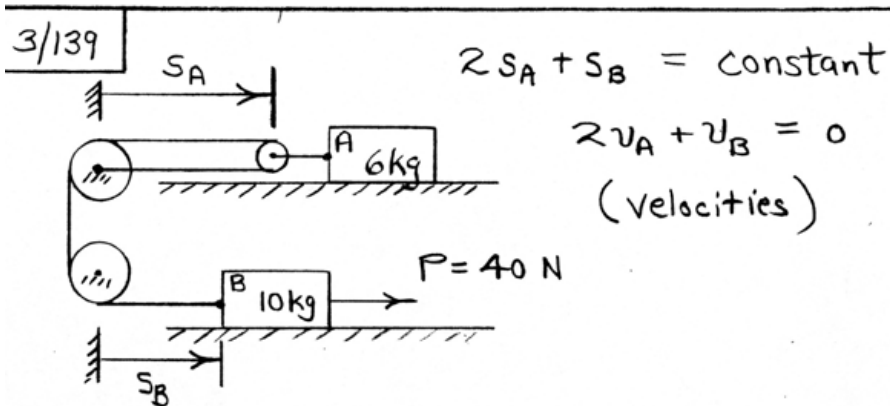


$$75s^2 - 30s + 2.75 = 0$$

$$s = 0.1423 \text{ m or } 0.257 \text{ m}$$

0.257 m > 200 mm, impossible

$$\text{So } \underline{s = 142.3 \text{ mm}}$$



$$T_1 + U_{1-2} = T_2$$

$$0 + 40(0.8) = \frac{1}{2} 6 v_A^2 + \frac{1}{2} 10 (2v_A)^2$$

$$v_A = 1.180 \text{ m/s}$$

$$v_B = 2v_A = 2.36 \text{ m/s}$$

} Speeds

3/140.

6(9.81) N



$$U_{1-2} = \Delta T = 0 :$$

$$6(9.81)(0.1 + \delta) - \int_{0.05}^{0.05 + \delta} 4000x dx = 0$$

$$2000\delta^2 + 141.1\delta - 5.89 = 0$$

$$kx = 4000x \quad \delta = 0.0294\text{m} \text{ or } \underline{\delta = 29.4 \text{ mm}}$$

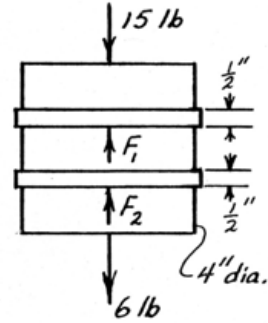
(Positive result taken from quadratic formula)

3/141

$$\begin{aligned}
 F_1 + F_2 &= \mu_k (\text{Area}) p \\
 &= 0.15 \times 2 \times 4\pi \times \frac{1}{2} p \\
 &= 1.885 p \text{ lb}
 \end{aligned}$$

$$U = \Delta T: (15 + 6 - 1.885p) \frac{10}{12} = \frac{1}{2} \frac{6}{32.2} (8^2 - 0^2)$$

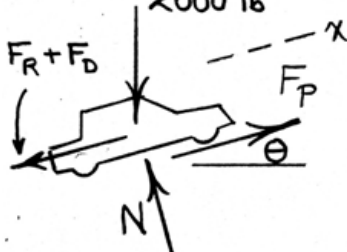
$$\text{Solve \& get } p = \underline{7.34 \text{ lb/in.}^2}$$



3/142

$$mg = 2000 \text{ lb}$$

$$\theta = 0 \text{ or } \theta = \tan^{-1} \frac{6}{100} = 3.43^\circ$$



$$F_D = kv^2; 50 = k(60)^2$$

$$\Rightarrow k = 0.01389 \frac{\text{lb-hr}^2}{\text{mi}^2}$$

$$\dagger F_D = 0.01389 v^2$$

$$\Sigma F_x = 0; F_p - F_R - F_D - mg \sin \theta = 0$$

$$F_p = F_R + F_D + mg \sin \theta$$

$$(a) \theta = 0 \quad v = 30 \text{ mi/hr} : F_D = 0.01389 (30^2) = 12.50 \text{ lb}$$

$$F_p = F_R + F_D = 50 + 12.50 = 62.5 \text{ lb}$$

$$P = Fv = 62.5 \left(30 \frac{5280}{3600} \right) / 550 = 5 \text{ hp}$$

$$v = 60 \text{ mi/hr} : F_D = 50 \text{ lb}, F_p = F_R + F_D = 100 \text{ lb}$$

$$P_{60} = Fv = 100 \left(60 \frac{5280}{3600} \right) / 550 = 16 \text{ hp}$$

$$(b) \theta = 3.43^\circ \quad F_p = 50 + 50 + 2000 \sin 3.43^\circ = 220 \text{ lb}$$

$$P_{up} = 220 \left(60 \frac{5280}{3600} \right) / 550 = 35.2 \text{ hp}$$

$$\text{Down: } F_p = 50 + 50 - 2000 \sin 3.43^\circ = -19.78 \text{ lb}$$

$$P_{down} = -19.78 \left(60 \frac{5280}{3600} \right) / 550 = -3.17 \text{ hp (brakes!)}$$

$$(c) \Sigma F_x = 0: 50 + kv^2 - 2000 \sin 3.43^\circ = 0, \quad v = 70.9 \frac{\text{mi}}{\text{hr}}$$

3/143

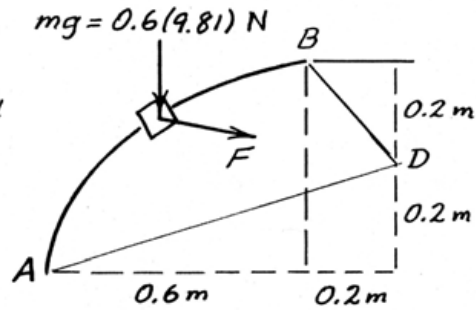
$$U = \Delta T$$

$$\begin{aligned} U &= F(\overline{AD} - \overline{BD}) - 0.6(9.81)0.4 \\ &= F(0.825 - 0.283) - 2.35 \\ &= 0.542F - 2.35 \text{ J} \end{aligned}$$

$$\begin{aligned} \Delta T &= \frac{1}{2}m(v_B^2 - v_A^2) \\ &= \frac{1}{2}0.6(4^2 - 0) = 4.8 \text{ J} \end{aligned}$$

$$\text{Thus } 0.542F - 2.35 = 4.8,$$

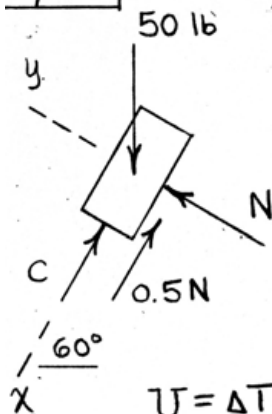
$$\underline{F = 13.21 \text{ N}}$$



$$\overline{AD} = \sqrt{0.8^2 + 0.2^2} = 0.825 \text{ m}$$

$$\overline{BD} = \sqrt{0.2^2 + 0.2^2} = 0.283 \text{ m}$$

3/144



$$\Sigma F_y = 0: N - 50 \cos 60^\circ = 0, N = 25 \text{ lb}$$

Displacement is $3 + \frac{4}{12} = 3.33 \text{ ft}$

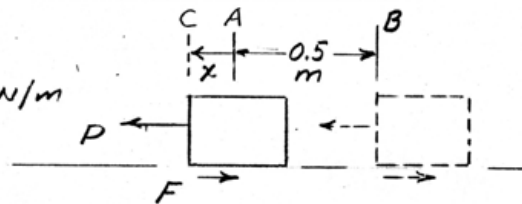
$$\begin{aligned} U_{1-2} &= (50 \sin 60^\circ - 0.5 \cdot 25) 3.33 \\ &\quad - \frac{1}{12} \int_0^{4''} (100x + 9x^2) dx \\ &= 20.0 \text{ ft-lb} \end{aligned}$$

$$U = \Delta T: 20.0 = \frac{1}{2} \frac{50}{32.2} (v^2 - 2^2)$$

$$\underline{v = 5.46 \text{ ft/sec}}$$

3/145

$$k = 300 \text{ N/m}$$



$$\begin{aligned} F &= \mu_k N \\ &= 0.30(10)(9.81) \\ &= 29.43 \text{ N} \end{aligned}$$

(a)

From B to A: $U_{1-2} = \Delta T$

$$\frac{1}{2}(300)(0.5)^2 - 29.43(0.5) = \frac{1}{2}(10)v^2$$

$$v^2 = 4.557 \text{ (m/s)}^2, \quad v = 2.13 \text{ m/s}$$

(b) From A to C: $U_{1-2} = \Delta T$

$$-\frac{1}{2}(300)x^2 - 29.43x = 0 - \frac{1}{2}(10)(4.557)$$

$$x^2 + 0.1962x - 0.1519 = 0$$

$$x = \frac{-0.1962 \pm \sqrt{(0.1962)^2 + 4(0.1519)}}{2}$$

$$= -0.0981 \pm 0.4019, \quad x = 0.304 \text{ m} \quad (x = -0.48)$$

$$\boxed{3/146} \quad P = Fv ; F = ma, \text{ so } P = mav$$

$$\& a = \frac{P}{mv}$$

$$\text{But } v dv = a ds, \text{ so } mv^2 dv = P ds$$

$$\int_{v_1}^{v_2} mv^2 dv = \int_0^s P ds ; \frac{m}{3}(v_2^3 - v_1^3) = Ps$$

$$v_2 = \left(\frac{3Ps}{m} + v_1^3 \right)^{1/3}$$

$$\boxed{3/147} \quad U_{1-2}' = 0 = \Delta T + \Delta V_g + \Delta V_e$$

$$\Delta T = \frac{1}{2} 3 (v^2 - 0) = \frac{3}{2} v^2$$

$$\Delta V_g = -3(9.81)(0.8) = -23.5 \text{ J}$$

$$\Delta V_e = \frac{1}{2} 200 \left[(\sqrt{0.8^2 + 0.6^2} - 0.4)^2 - (0.8 - 0.4)^2 \right]$$

$$= 20 \text{ J}$$

$$\text{So } 0 = \frac{3}{2} v^2 - 23.5 + 20, \quad \underline{v = 1.537 \text{ m/s}}$$

3/148	For the system, $U'_{1-2} = 0$, so $\Delta V_g + \Delta T = 0$
-------	---

$$-mg\left(\frac{18}{12}\right) + \frac{1}{2}(2m)(v^2 - 0) = 0$$
$$v = 6.95 \text{ ft/sec}$$

3/149 Establish datum @ A.

(a) $T_A + V_A = T_B + V_B$

$$0 + 0 = \frac{1}{2}mv_B^2 - mgh_B$$

$$v_B = \sqrt{2gh_B} = \sqrt{2(9.81)(4.5)} = \underline{9.40 \text{ m/s}}$$

(b) State F : Spring fully compressed

$$T_A + V_A = T_F + V_F$$

$$0 + 0 = 0 - mgh_f + \frac{1}{2}k\delta^2$$

$$\delta = \sqrt{\frac{2mgh_f}{k}} = \sqrt{\frac{2(1.2)(9.81)(3)}{24000}} = 0.0542 \text{ m}$$

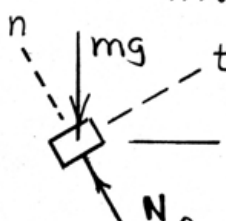
or $\delta = 54.2 \text{ mm}$

3/150 Establish datum @ A.

$$T_A + V_A = T_C + V_C : 0 + 0 = \frac{1}{2}mv_C^2 - mgh_C$$

$$v_C = \sqrt{2gh_C} = \sqrt{2(9.81)(3 + 1.5 \cos 30^\circ)}$$

$$= 9.18 \text{ m/s}$$



$$(a) \Sigma F_n = m \frac{v^2}{r} : N_C - 1.2(9.81) \cos 30^\circ = 1.2 \frac{9.18^2}{1.5}$$

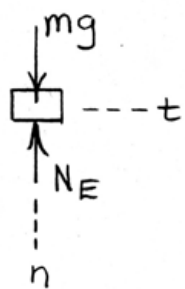
$$\underline{N_C = 77.7 \text{ N}}$$

$$(b) \Sigma F_n = 0 : N_C - 1.2(9.81) \cos 30^\circ = 0$$

$$\underline{N_C = 10.19 \text{ N}}$$

$$T_A + V_A = T_E + V_E : 0 + 0 = \frac{1}{2}mv_E^2 - mgh_E$$

$$v_E = \sqrt{2gh_E} = \sqrt{2(9.81)(3)} = 7.67 \text{ m/s}$$



$$\Sigma F_n = m \frac{v^2}{r} : -N_E + 1.2(9.81) = 1.2 \frac{7.67^2}{1.5}$$

$$\underline{N_E = -35.3 \text{ N (down)}}$$

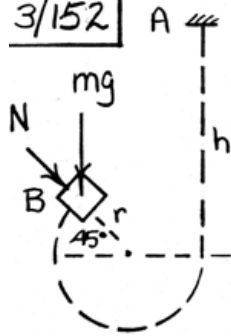
$$\boxed{3/151} \quad \Delta T + \Delta V_e + \Delta V_g = 0, \quad \Delta T = 0$$

$$\Delta V_e = \frac{1}{2} k (x_2^2 - x_1^2) = \frac{1}{2} 500 (0.050^2 - 0.100^2) = -1.875 \text{ J}$$

$$\Delta V_g = mg\Delta h = 2(9.81)h = 19.62h$$

$$\text{Thus } 0 - 1.875 + 19.62h = 0, \quad h = 0.0956 \text{ m or } \underline{h = 95.6 \text{ mm}}$$

3/152



$$U_{1-2} = \Delta T + \Delta V_g = 0$$

$$\frac{1}{2}mv^2 - mg\left(h - \frac{r}{\sqrt{2}}\right) = 0$$

$$v^2 = 2g\left(h - \frac{r}{\sqrt{2}}\right)$$

$$\Sigma F_n = ma_n: N + \frac{mg}{\sqrt{2}} = m \frac{v^2}{r}$$

$$\Rightarrow N = mg\left[\left(\frac{h}{r} - \frac{1}{\sqrt{2}}\right)2 - \frac{1}{\sqrt{2}}\right]$$

$$= mg\left[2\frac{h}{r} - \frac{3}{\sqrt{2}}\right]$$

With $m = 0.25 \text{ kg}$, $r = 0.15 \text{ m}$, & $h = 0.6 \text{ m}$,

$$N = \underline{14.42 \text{ N}}$$

3/153 $T_A + V_A = T_B + V_B$, datum @ B

$$0 + mgR + \frac{1}{2}k[R\sqrt{2} - R]^2 = \frac{1}{2}mv_B^2 + 0$$

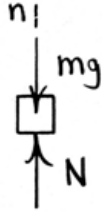
$$v_B = \sqrt{2gR + \frac{kR^2}{m}(3-2\sqrt{2})}$$

$T_A + V_A = T_C + V_C$, datum @ C

$$0 + 2mgR + \frac{1}{2}k[R\sqrt{2} - R]^2 = \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{4gR + \frac{kR^2}{m}(3-2\sqrt{2})}$$

Kinetics at C:



$$\Sigma F_n = ma_n: N - mg = m \frac{v_C^2}{R}$$

$$\Rightarrow N = m \left[5g + \frac{kR}{m}(3-2\sqrt{2}) \right]$$

3/154 For the system, $T_1 + V_1 + U_{1-2}' = T_2 + V_2$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 + 0 = \frac{1}{2}mv^2 + \frac{1}{2}kx_2^2 - mgh,$$

where the datum is the initial position and

h is the drop distance. Note that the

spring deflection runs at twice that of the

cylinder. Numbers:

$$\frac{1}{2} 6(12) \left[\frac{3}{12} \right]^2 = \frac{1}{2} \frac{100}{32.2} v^2 + \frac{1}{2} 6(12) \left[\frac{3+2(\frac{1}{2})}{12} \right]^2 - 100 \left(\frac{1}{2} \right)$$

$$v = 1.248 \text{ ft/sec}$$

$$3/155 \quad (a) \quad \Delta T + \Delta V_g = 0$$

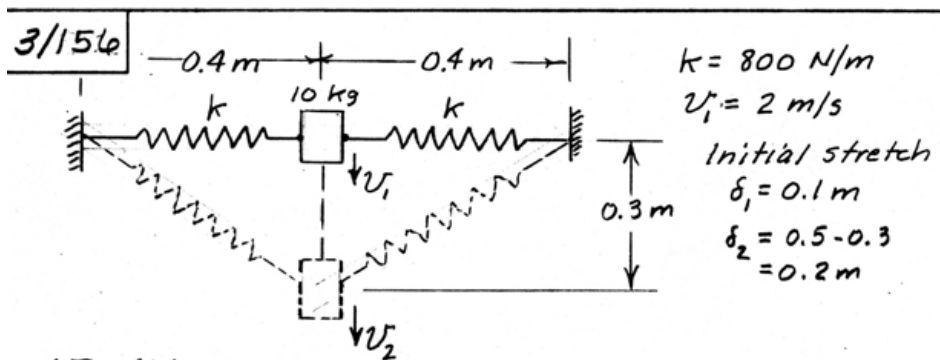
$$\frac{1}{2} \frac{5}{32.2} v^2 + \frac{1}{2} \frac{10}{32.2} \left(\frac{12}{18} v \right)^2 + 5 \frac{18}{12} \sin 60^\circ - 10 \frac{12}{12} \sin 60^\circ = 0$$
$$0.1467 v^2 = 2.165, \quad v^2 = 14.76 \text{ (ft/sec)}^2$$

$$\underline{v = 3.84 \text{ ft/sec}}$$

$$(b) \quad \text{For entire interval } \Delta T = 0, \quad \Delta V_g + \Delta V_e = 0$$

$$-2.165(12) + \frac{1}{2}(200)x^2 = 0, \quad x^2 = 0.2598 \text{ (in)}^2$$

$$\underline{x = 0.510 \text{ in.}}$$



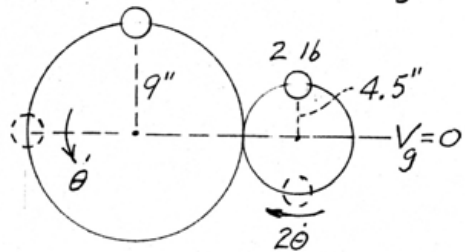
$$\Delta T + \Delta V_g + \Delta V_e = 0$$

$$\frac{1}{2} 10 (v_2^2 - 2^2) - 10 (9.81) (0.3) + \frac{2}{2} 800 (0.2^2 - 0.1^2) = 0$$

$$5v_2^2 = 20 + 29.43 - 24, \quad v_2^2 = 5.086 \text{ (m/s)}^2$$

$$\underline{v_2 = 2.26 \text{ m/s}}$$

3/157



$$\Delta T + \Delta V_g = 0; \quad \Delta T = \frac{1}{2} \frac{3}{32.2} \left(\frac{9}{12} \dot{\theta} \right)^2$$

$$+ \frac{1}{2} \frac{2}{32.2} \left(\frac{4.5}{12} [2\dot{\theta}] \right)^2$$

$$= 0.04367 \dot{\theta}^2 \text{ ft-lb}$$

$$\Delta V_g = -3 \left(\frac{9}{12} \right) - 2 \left(\frac{4.5 + 4.5}{12} \right)$$

$$= -\frac{15}{4} = -3.75 \text{ ft-lb}$$

$$\text{Thus } 0.04367 \dot{\theta}^2 - 3.75 = 0, \quad \dot{\theta}^2 = 85.87 \text{ (rad/sec)}^2$$

$$\dot{\theta} = 9.27 \text{ rad/sec}$$

3/158 Let m be the mass of the car

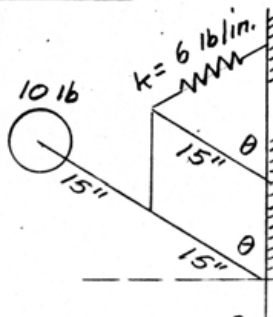
$$U_{1-2}' = \Delta T + \Delta V_g: 0 = \frac{1}{2}m(v^2 - v_0^2) + mgy$$

$$a_n = \frac{v^2}{\rho}: \quad \frac{v_0^2}{\rho_0} = \frac{v_0^2 - 2gy}{\rho}, \quad \rho = \rho_0 \left(1 - \frac{2gy}{v_0^2}\right)$$

For car to remain in contact with the track at the top, $a_n > g$, so for constant

$$a_n, \quad v_0^2/\rho_0 > g \text{ so } \underline{v_{0\min} = \sqrt{\rho_0 g}}$$

3/159 For the interval from $\theta = 60^\circ$ to $\theta = 90^\circ$,



Spring stretch is

$$\delta = 15\sqrt{2} - 15 = 6.21 \text{ in.}$$

$$\Delta V_e = \frac{1}{2} k (\delta)^2 = \frac{1}{2} (6) (6.21)^2 = 115.8 \text{ in.-lb}$$

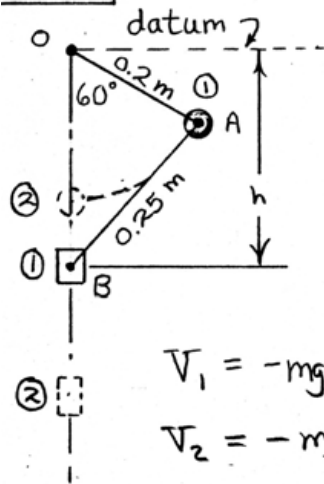
$$\Delta V_g = -mg \Delta h = -10(30) \cos 60^\circ = -150 \text{ in.-lb.}$$

$$\Delta T + \Delta V_g + \Delta V_e = 0$$

$$\frac{1}{2} \frac{10}{32.2(12)} v^2 - 150 + 115.8 = 0, \quad v^2 = 2642, \quad v = 51.4 \frac{\text{in.}}{\text{sec.}}$$

or $v = 4.28 \text{ ft/sec}$

3/160



$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2} m v_A^2$$

Note that

$$h = 0.2 \cos 60^\circ + \sqrt{0.25^2 - (0.2 \sin 60^\circ)^2}$$

$$= 0.280 \text{ m}$$

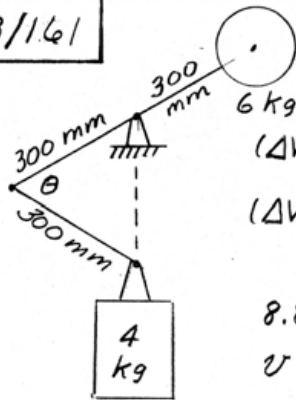
$$V_1 = -mg(0.2 \cos 60^\circ) - mg(0.280) = -0.380 mg$$

$$V_2 = -mg(0.2) - mg(0.45) = -0.650 mg$$

$$50 - 0.380 mg = \frac{1}{2} m v_A^2 - 0.650 mg$$

$$v_A = 2.30 \text{ m/s}$$

3/161



For motion from $\theta = 60^\circ$ to $\theta = 180^\circ$

$$\Delta V_g + \Delta T = 0$$

$$(\Delta V_g)_{6\text{ kg}} = 6(9.81)(0.3)(1 - \sin 30^\circ) = 8.829 \text{ J}$$

$$(\Delta V_g)_{4\text{ kg}} = -4(9.81)(2)(0.3)(1 - \sin 30^\circ) = -11.772 \text{ J}$$

$$8.829 - 11.772 + \frac{1}{2} 6 v^2 + 0 = 0$$

$$v^2 = 0.981 (\text{m/s})^2, \quad \underline{v = 0.990 \text{ m/s}}$$

3/162 Establish datum at release point.

$$T_A + V_A = T_B + V_B$$

$$0 + \frac{1}{2} k_A x_A^2 = 0 + mg(x_A + d + x_B) + \frac{1}{2} k_B x_B^2$$
$$\frac{1}{2} (48)(12) \left(\frac{5}{12} \right)^2 = 14 \left(\frac{5 + 14 + x_B}{12} \right) + \frac{1}{2} (10)(12) \left(\frac{x_B}{12} \right)^2$$

$$\underline{x_B = 6.89 \text{ in.}}$$

The fact that $x_B > x_A$ is due to

The difference in spring stiffnesses (along with the particular value $d = 20 - 6 = 14$).

Note that $d = 14$ is the distance which the collar moves when out of contact with the springs.

3/163 A force analysis reveals that A will move down & B will move up.

Kinematics : $3v_A = 2v_B$ (speeds)

$T_1 + V_1 = T_2 + V_2$, datum @ initial position

$$0 + 0 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B \left(\frac{3}{2} v_A \right)^2 + m_B g h_B - m_A g h_A$$

$$0 = \frac{1}{2} (40) v_A^2 + \frac{1}{2} (8) \frac{9}{4} v_A^2 + 8(9.81)(1) - 40(9.81) \left(\frac{2}{3} (1) \sin 20^\circ \right)$$

$$\underline{v_A = 0.616 \text{ m/s}}, \quad \underline{v_B = \frac{3}{2} v_A = 0.924 \text{ m/s}}$$

3/164

1st interval of motion (0.4 m) $\Delta T + \Delta V_g = 0$ for system

$$\frac{1}{2}(4+6+8)v^2 + 9.81 \times 0.4(8-4-6) = 0, v^2 = 0.872 \text{ (m/s)}^2$$

$$v = 0.934 \text{ m/s}$$

2nd interval for 6- & 8-kg cylinders $\Delta T + \Delta V_g = 0$

$$0 - \frac{1}{2}(6+8)(0.872) + 9.81(h-0.4)(8-6) = 0, h = 0.711 \text{ m}$$

$$\text{or } \underline{h = 711 \text{ mm}}$$

Kinetic energy of collar is dissipated into heat & sound during impact with bracket.

3/165 Constant total energy is $E = T_A + V_A = T_P + V_P$

Thus $\frac{1}{2}mv_A^2 - \frac{mgR^2}{r_A} = \frac{1}{2}mv_P^2 - \frac{mgR^2}{r_P}$

$$v_A^2 = v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right), \quad v_A = \sqrt{v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right)}$$

$$3/166 \quad U'_{1-2} = \Delta T + \Delta V_e + \Delta V_g \text{ for system}$$

$$U'_{1-2} = 50(1.5) \cos 30^\circ = 64.95 \text{ J}$$

$$\Delta T = \frac{1}{2} 2 v^2 = v^2$$

$$\Delta V_e = \frac{1}{2} 30 \left[(\sqrt{2^2 + 1.5^2} - 1.5)^2 - (2 - 1.5)^2 \right] = 11.25 \text{ J}$$

$$\Delta V_g = 2(9.81)(1.5) = 29.43 \text{ J}$$

$$\text{So } 64.95 = v^2 + 11.25 + 29.43, \quad v^2 = 24.27, \quad \underline{v = 4.93 \frac{m}{s}}$$

$$3/167 \quad \Delta T + \Delta V_g = 0, \quad V_g = -\frac{mgR^2}{r}$$

Mean radius of earth is $R = 6371 \text{ km}$

$$g = 9.825 (3600)^2 / 1000 = 127.3 (10^3) \text{ km/h}^2$$

Thus

$$\frac{1}{2}m(v_B^2 - [24000]^2) + 127.3(10^3)(6371)^2m\left(-\frac{1}{6500} + \frac{1}{7000}\right) = 0$$

$$\frac{1}{2}v_B^2 - 288(10^6) + 5167(10^9)(-0.01099)(10^{-3}) = 0$$

$$v_B^2 = 2[288 + 56.8]10^6 = 690(10^6), \quad \underline{v_B = 26300 \text{ km/h}}$$

3/168

$$\Delta T + \Delta V_g + \Delta V_e + U_f = 0$$

Spring elongation at A is $\delta_A = \sqrt{3^2 + 4^2} - 2 = 3 \text{ ft}$

" " " B " $\delta_B = \sqrt{3^2 + 3^2} - 2 = 2.24 \text{ ft}$

$$\Delta V_e = \frac{1}{2} k (\delta_B^2 - \delta_A^2) = \frac{1}{2} 2 (2.24^2 - 3^2) = -3.97 \text{ ft-lb}$$

$$\Delta T = \frac{1}{2} m (v_B^2 - v_A^2) = \frac{1}{2} \frac{5}{32.2} (10^2 - 6^2) = 4.97 \text{ ft-lb}$$

$$\Delta V_g = W \Delta z = 5(0 - 4) = -20 \text{ ft-lb}$$

$$\text{Thus } 4.97 - 20 - 3.97 + U_f = 0, \quad \underline{U_f = 19.00 \text{ ft-lb (loss)}}$$

$$U_f = F_{av} s, \quad F = \frac{19.00}{5} = \underline{3.80 \text{ lb}}$$

3/169

$$\text{Ellipse eccentricity } e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{0.6^2}{0.8^2}} = 0.661$$

$$r_{\min} = a(1-e) = 0.8(1-0.661) = 0.271 \text{ m}$$

$$r_{\max} = a(1+e) = 0.8(1+0.661) = 1.329 \text{ m}$$

$$T_A + V_A = T_C + V_C$$

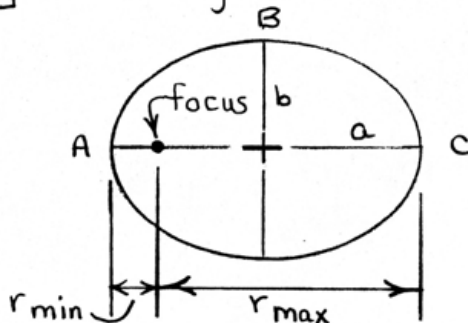
$$\frac{1}{2} m v_A^2 + 0 = 0 + \frac{1}{2} k x_C^2$$

$$\frac{1}{2} (0.4) v_A^2 = \frac{1}{2} (3) [1.329 - 0.271]^2, v_A = 2.90 \text{ m/s}$$

$$\text{Then } T_A + V_A = T_B + V_B$$

$$\frac{1}{2} (0.4) (2.90)^2 + 0 = \frac{1}{2} (0.4) v_B^2 + \frac{3}{2} \left\{ [(0.8 - 0.271)^2 + (0.6)^2]^{1/2} - 0.271 \right\}^2$$

$$\underline{v_B = 2.51 \text{ m/s}}$$



$$\boxed{3/170} \quad U_{1-2}' = \Delta T + \Delta V_g = 0$$

$$\Delta T = \frac{1}{2} m \left[v^2 - \left(2000 \frac{44}{30} \right)^2 \right]$$

$$\Delta V_g = -mgR^2 \left(\frac{1}{R} - \frac{1}{2R} \right) = - \frac{mgR}{2}$$

$$= -\frac{1}{2} m \cdot 5.32 (1080) (5280)$$

$$\text{So } v^2 - \left(2000 \frac{44}{30} \right)^2 = 5.32 (1080) (5280)$$

$$\underline{v = 6240 \text{ ft/sec}} \quad \text{or} \quad \underline{4250 \text{ mi/hr}}$$

$$\underline{3/171} \quad U_{1-2}' = 0 \quad \text{so} \quad T_1 + V_{g1} = T_2 + V_{g2}$$

Take datum $V_g = 0$ at ground level.

$$T_1 = \frac{1}{2} \frac{175 + 10}{32.2} v^2 = 2.87 v^2, \quad T_2 = 0$$

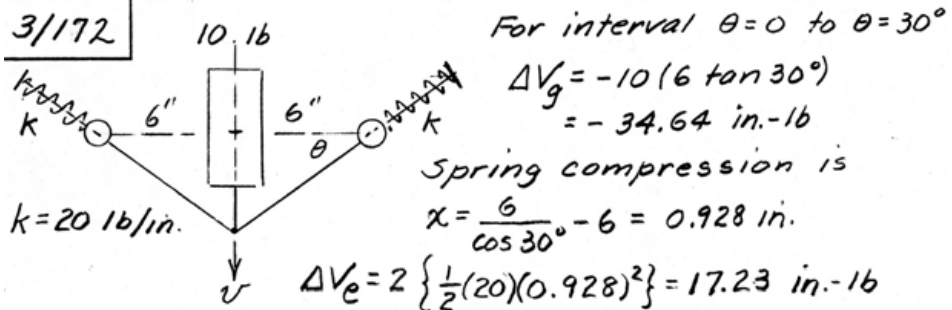
$$V_{g1} = (175 + 10) \frac{42}{12} = 648 \text{ ft-lb}$$

$$V_{g2} = 175(18) + 10(8) = 3230 \text{ ft-lb}$$

$$\text{So } 2.87 v^2 + 648 = 0 + 3230$$

$$\underline{v = 30.0 \text{ ft/sec}} \quad \text{or} \quad \underline{20.4 \text{ mi/hr}}$$

3/172



$$\Delta T = \frac{1}{2} \frac{10}{32.2} \frac{1}{12} v^2, (v \text{ in in./sec})$$

$$= 0.01294 v^2$$

$$\Delta T + \Delta V_g + \Delta V_e = 0; 0.01294 v^2 - 34.64 + 17.24 = 0$$

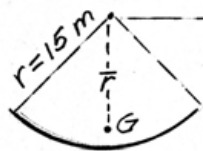
$$v^2 = 1345 (\text{in./sec})^2, v = 36.7 \text{ in./sec}$$

$$\text{or } v = \underline{3.06 \text{ ft/sec}}$$

3/173.

$$\bar{r} = \frac{2\sqrt{2}r}{\pi} = \frac{2\sqrt{2}(15)}{\pi}$$

$$= 13.50 \text{ m}$$



$$v_1 = 90 \text{ km/h}$$

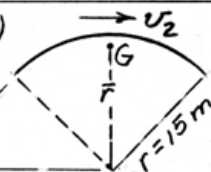
Let m = total mass of train
For system of cars

$$\Delta T + \Delta V_g = 0$$

$$\frac{1}{2}m(v_2^2 - v_1^2) + mg(2\bar{r}) = 0, \quad v_2^2 = v_1^2 - 4g\bar{r}$$

$$v_2^2 = \left[\frac{90(1000)}{3600} \right]^2 - 4(9.81)(13.50) = 625 - 529.9 = 95.07 \text{ (m/s)}^2$$

$$v_2 = 9.75 \text{ m/s or } \underline{v_2 = 35.1 \text{ km/h}}$$



$$\underline{3/174} \quad U' = \Delta T + \Delta V_g = 0 \text{ where } V_g = -\frac{mgR^2}{r}$$

$$\Delta V_g = -9.825 \text{ m} [6371 (10^3)]^2 \left(\frac{1}{(2500+6371)10^3} - \frac{1}{(2200+6371)10^3} \right)$$

$$= 1.573 (10^6) \text{ m}$$

$$\Delta T = \frac{1}{2} m \left(v_B^2 - \left[\frac{25000 \times 10^3}{3600} \right]^2 \right)$$

$$\text{Thus } \frac{1}{2} v_B^2 - \frac{1}{2} \left[\frac{25000}{3.6} \right]^2 + 1.573 (10^6) = 0$$

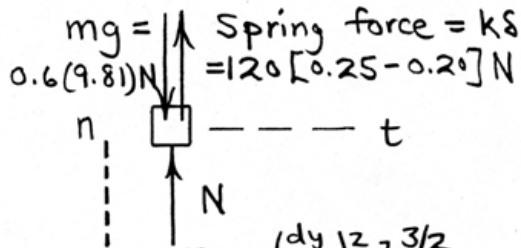
$$v_B^2 = 45.08 (10^6) \left(\frac{\text{m}}{\text{s}} \right)^2, \quad v_B = 6714 \text{ m/s}$$

$$\text{or } \underline{v_B = 24170 \text{ km/h}}$$

$$\begin{aligned} \frac{3}{175} \quad T_A + V_A &= T_B + V_B, \quad \text{datum @ B.} \\ 0 + 0.6(9.81)(0.5) + \frac{1}{2} 120 \left[\sqrt{0.25^2 + 0.5^2} - 0.2 \right]^2 \\ &= \frac{1}{2} (0.6) v_B^2 + \frac{1}{2} 120 [0.25 - 0.2]^2 \end{aligned}$$

$$v_B = 5.92 \text{ m/s}$$

Kinetics at B:



$$\text{Radius of curvature } \rho = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{d^2y/dx^2}$$

$$y = kx^2; \quad 0.5 = k(0.5)^2 \Rightarrow k = 2$$

$$y = 2x^2, \quad \frac{dy}{dx} = 4x, \quad \frac{d^2y}{dx^2} = 4$$

$$\text{When } x=0, \quad \rho = \frac{[1 + 0^2]^{3/2}}{4} = 0.25 \text{ m}$$

$$\begin{aligned} \Sigma F_n = ma_n: \quad N + 120(0.05) - 0.6(9.81) \\ = 0.6 \frac{5.92^2}{0.25} \end{aligned}$$

$$\underline{N = 84.1 \text{ N}}$$

$$\underline{3/176} \quad \Delta T + \Delta V_g + \Delta V_e = 0$$

$$\Delta T = \frac{1}{2} m \dot{y}^2 ; \quad \Delta V_g = -mgy$$

$$\Delta V_e = 2 \left\{ \frac{1}{2} k x^2 \right\} = k (y \sin \theta)^2 = ky^2 (1 - \cos^2 \theta) \\ = ky^2 (1 - c^2/b^2)$$

$$\frac{1}{2} m \dot{y}^2 - mgy + ky^2 (1 - c^2/b^2) = 0$$

$$\dot{y} = \sqrt{2y \left(g - \frac{k}{m} y \frac{b^2 - c^2}{b^2} \right)}$$

$$y_{\max} = y \text{ for } \dot{y} = 0, \text{ so } 2gy - \frac{2k}{m} y^2 (1 - c^2/b^2) = 0$$

$$\text{Hence } (y_{\min} = 0), \quad y_{\max} = \frac{mg}{k} \frac{b^2}{b^2 - c^2}$$

$$\boxed{3/177} \quad x^2 + y^2 = 0.9^2, \quad x\dot{x} + y\dot{y} = 0, \quad v_A = -\dot{y} = \frac{x}{y}\dot{x} = \frac{x}{y}v_B$$

$$\Delta T + \Delta V_g = 0; \quad \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mg(y - \frac{0.9}{\sqrt{2}}) = 0$$

$$\dot{x}^2(1 + \frac{x^2}{y^2}) = 2(9.81)(\frac{0.9}{\sqrt{2}} - y), \quad \dot{x}^2 \frac{x^2 + y^2}{y^2} = 19.62(\frac{0.9}{\sqrt{2}} - y)$$

$$0.9^2 \dot{x}^2 = 19.62(\frac{0.9}{\sqrt{2}} y^2 - y^3)$$

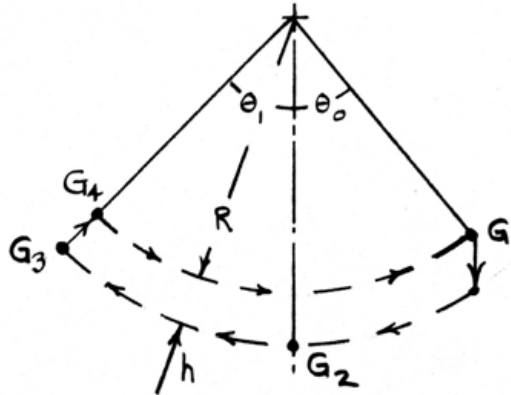
$$\text{For max. } \dot{x}, \quad \frac{d(\dot{x}^2)}{dy} = \frac{19.62}{0.81}(\frac{1.8}{\sqrt{2}}y - 3y^2) = 0$$

$$\text{so } y(\frac{1.8}{\sqrt{2}} - 3y) = 0, \quad y = 0.6/\sqrt{2} \text{ m}$$

$$\dot{x}^2 = \frac{19.62}{0.81}(\frac{0.9}{\sqrt{2}} \frac{0.36}{2} - \frac{0.108}{\sqrt{2}}) = \frac{19.62\sqrt{2}}{30}$$

$$v_{B, \max} = \dot{x} = \sqrt{\frac{19.62\sqrt{2}}{30}} = \underline{0.962 \text{ m/s}}$$

3/178



$$U'_{1-3} = \Delta T + \Delta V_g$$

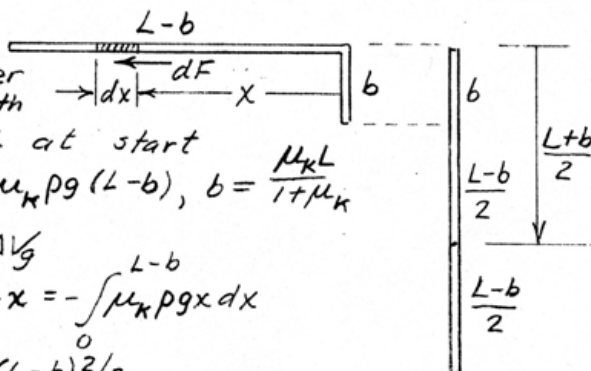
$$U'_{1-2} = U'_{2-3} = 0, \Delta T = 0, \text{ so } V_{g_1} = V_{g_3}$$

$$\text{Thus } R \cos \theta_0 = (R+h) \cos \theta_1$$

$$\theta_1 = \cos^{-1} \left(\frac{R}{R+h} \cos \theta_0 \right)$$

3/179

$\rho = \text{mass per unit length}$



For equil. at start

$$\rho g b = \mu_k \rho g (L-b), \quad b = \frac{\mu_k L}{1+\mu_k}$$

$$U = \Delta T + \Delta V_g$$

$$U = - \int dF \cdot x = - \int_0^{L-b} \mu_k \rho g x dx$$

$$= - \mu_k \rho g (L-b)^2 / 2$$

$$\Delta T = \frac{1}{2} \rho L v^2$$

$$\Delta V_g = - \rho g (L-b) \left(\frac{L+b}{2} \right)$$

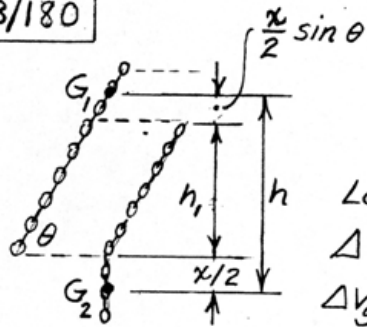
$$\text{Thus } - \mu_k \rho g (L-b)^2 / 2 = \frac{1}{2} \rho L v^2 - \rho g \frac{L^2 - b^2}{2}$$

$$v^2 = g \left(1 - b/L \right) (L+b - \mu_k [L-b]); \text{ Now substitute } b$$

$$\text{so } v^2 = g \left(1 - \frac{\mu_k}{1+\mu_k} \right) \left(L \left[1 + \frac{\mu_k}{1+\mu_k} \right] - \mu_k \left[L - \frac{\mu_k L}{1+\mu_k} \right] \right)$$

$$= \frac{gL}{1+\mu_k}, \quad v = \sqrt{\frac{gL}{1+\mu_k}}$$

3/180



$$h_1 = (L-x) \sin \theta$$

$$h = (L-x) \sin \theta + \frac{x}{2} \sin \theta + \frac{x}{2}$$

$$= L \sin \theta + \frac{x}{2} (1 - \sin \theta)$$

Let ρ = mass per unit length

$$\Delta V_g + \Delta T = 0$$

ΔV_g is that of the length x dropping a distance h

$$\Delta V_g = -\rho g x h = -\rho g \left[Lx \sin \theta + \frac{x^2}{2} (1 - \sin \theta) \right]$$

$$\Delta T = \frac{1}{2} \rho L v^2$$

$$\text{Thus } -\rho g \left[Lx \sin \theta + \frac{x^2}{2} (1 - \sin \theta) \right] + \frac{1}{2} \rho L v^2 = 0$$

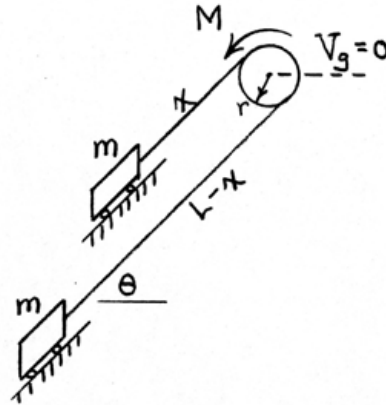
$$v = \sqrt{2gx \left[\sin \theta + \frac{x}{2L} (1 - \sin \theta) \right]}$$

► 3/181

$$U_{1-2}' = \Delta T + \Delta V$$

$$U_{1-2}' = M \frac{x}{r}$$

$$\Delta V_e = 0$$



$$V_{g2} = -g \left[m(L-x) + mx + \rho(L-x) \frac{L-x}{2} + \rho x \frac{x}{2} \right] \sin \theta$$

$$= -g \sin \theta \left\{ mL + \frac{\rho}{2} [(L-x)^2 + x^2] \right\}$$

$$V_{g1} = -g \sin \theta \left\{ mL + \rho L \frac{L}{2} \right\}$$

$$\Delta V_g = -g \sin \theta \left\{ mL + \frac{\rho}{2} [(L-x)^2 + x^2] - mL - \frac{\rho L^2}{2} \right\}$$

$$= -g \sin \theta \left\{ \frac{\rho}{2} [2x^2 - 2Lx] \right\}$$

$$\Delta T = \frac{1}{2} (2m + \rho L) v^2$$

$$\therefore M \frac{x}{r} = \frac{1}{2} (2m + \rho L) v^2 - g \sin \theta \left\{ \frac{\rho}{2} [2x^2 - 2Lx] \right\}$$

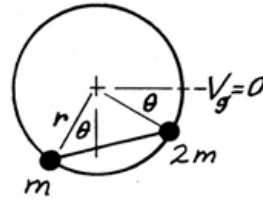
$$\text{Solving, } v = \sqrt{\frac{2}{2m + \rho L}} \sqrt{\frac{Mx}{r} - \rho g x (L-x) \sin \theta}$$

3/182 For the unit $U' = \Delta T + \Delta V_g = 0$

$$\Delta V_g = (-2mgr \sin \theta - mgr \cos \theta) - (-mgr + 0) \\ = mgr(-2 \sin \theta - \cos \theta + 1)$$

$$\text{so } \frac{1}{2} 3m v^2 - 0 + mgr(-2 \sin \theta - \cos \theta + 1) = 0$$

$$\text{or } v^2/gr = \frac{2}{3}(2 \sin \theta + \cos \theta - 1)$$



(a) Rod is horiz. when $\theta = 45^\circ$

$$v^2/gr = \frac{2}{3}(2 \sin 45^\circ + \cos 45^\circ - 1) = 0.748, \quad \underline{v_{45^\circ} = 0.865 \sqrt{gr}}$$

(b) $\frac{d}{d\theta} \left(\frac{v^2}{gr} \right) = \frac{2}{3}(2 \cos \theta - \sin \theta) = 0$ for max v^2 & hence max v
 $\tan \theta = 2, \quad \theta = \tan^{-1} 2 = 63.4^\circ$

$$\text{so } v_{\max}^2/gr = \frac{2}{3}(2 \sin 63.4^\circ + \cos 63.4^\circ - 1) = 0.824 \\ \underline{v_{\max} = 0.908 \sqrt{gr}}$$

(c) $\theta = \theta_{\max}$ when $T = \Delta T = 0$ so $2 \sin \theta + \cos \theta - 1 = 0$

$$2\sqrt{1 - \cos^2 \theta} = 1 - \cos \theta, \quad 5 \cos^2 \theta - 2 \cos \theta - 3 = 0$$

$$\cos \theta = 0.2 \pm 0.8 = 1 \text{ or } -0.6, \quad \theta = 0 \text{ or } \underline{\theta_{\max} = 126.9^\circ}$$

3/183

$$\int \Sigma F dt = \Delta G$$



$$(20\,000)(3 \times 60) = 30\,000 (v - 24\,000) \frac{1000}{3600}$$

$$\underline{v = 24\,400 \text{ km/h}}$$

3/184

$$\int \Sigma F dt = m \Delta v$$



$$[48(10^3) - R]10 = 6450 \left(\frac{250 \times 1000}{3600} - 0 \right)$$

$$R = 3208 \text{ N or } \underline{R = 3.21 \text{ kN}}$$

3/185

$$\int F dt = m \Delta v$$

$$2(26)10^3 t = 90(10^3)[28100 - 28000]/3.6$$

$$t = 48.1 \text{ s}$$

$$\frac{3}{186} \left\{ \begin{aligned} \underline{v} &= 1.5t^3 \underline{i} + (2.4 - 3t^2) \underline{j} + 5 \underline{k} \quad (\text{m/s}) \\ \underline{\dot{v}} &= 4.5t^2 \underline{i} - 6t \underline{j} \quad (\text{m/s}^2) \end{aligned} \right.$$

$$\text{At } t = 2\text{s} : \left\{ \begin{aligned} \underline{v} &= 12 \underline{i} - 9.6 \underline{j} + 5 \underline{k} \quad \text{m/s} \\ \underline{\dot{v}} &= 18 \underline{i} - 12 \underline{j} \quad \text{m/s}^2 \end{aligned} \right.$$

$$\begin{aligned} \text{Then } \underline{G} &= m \underline{v} = 1.2(12 \underline{i} - 9.6 \underline{j} + 5 \underline{k}) \\ &= 14.40 \underline{i} - 11.52 \underline{j} + 6 \underline{k} \quad \text{kg} \cdot \text{m/s} \end{aligned}$$

$$G = \sqrt{14.40^2 + 11.52^2 + 6^2} = \underline{19.39 \text{ kg} \cdot \text{m/s}}$$

$$\begin{aligned} \Sigma \underline{F} &= \underline{\dot{G}} : \underline{R} = m \underline{\dot{v}} = 1.2(18 \underline{i} - 12 \underline{j}) \\ &= \underline{21.6 \underline{i} - 14.4 \underline{j} \text{ N}} \end{aligned}$$

3/187 Conservation of system linear momentum:

$$\rightarrow 0.075(600) = 50.075 v_f, v_f = 0.899 \text{ m/s}$$

$$\text{Initial energy } T_1 = \frac{1}{2}(0.075)(600)^2 = 13\,500 \text{ J}$$

$$\text{Final energy } T_2 = \frac{1}{2}(50.075)(0.899)^2 = 20.2 \text{ J}$$

$$\text{Absolute energy loss } |\Delta E| = T_1 - T_2 = 13\,480 \text{ J}$$

$$\text{Percent lost: } n = \frac{|\Delta E|}{T_1} (100\%) = \underline{99.9\%}$$

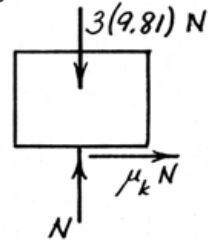
3/188. For system of bullet and block $\Delta G = 0$, $G_1 = G_2$:

$$(0.060)(600) = (0.060)(400) + 3v$$

Initial velocity of block is $v = 4 \text{ m/s}$

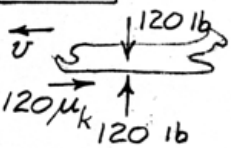
For block $U = \Delta T$: $-\mu_k (3 \times 9.81)(2.70)$
 $= \frac{1}{2} 3(0 - 4^2)$

$$\mu_k = 0.302$$



$$\begin{aligned}
 & \boxed{3/189} \quad \Delta G = 0; \quad 150,000 \times 2 + 120,000 \times 3 \\
 & \quad \quad \quad = (150,000 + 120,000) v, \quad v = 2.44 \text{ mi/hr} \\
 & |\Delta E| = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 - \frac{1}{2} (m_A + m_B) v^2 \\
 & \quad = \frac{1}{2(32.2)} \left(\frac{44}{30} \right)^2 \left[150,000 \times 2^2 + 120,000 \times 3^2 - 270,000 \times 2.44^2 \right] \\
 & \quad = \underline{2230 \text{ ft-lb loss}}
 \end{aligned}$$

3/190 $\Delta G = 0$; $100(15) = 120v$, $v = 12.5 \text{ ft/sec}$



$v^2 = 2as$; $a = F/m = \frac{120\mu_k}{120/g} = \mu_k g$

$32.2\mu_k = \frac{12.5^2}{2(80)}$, $\mu_k = 0.030$

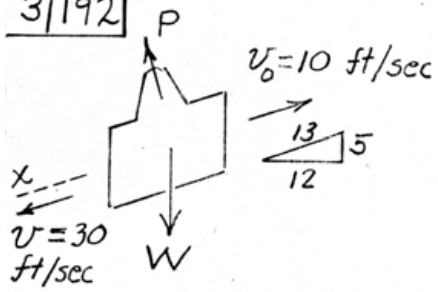
3/19/ No difference between cases (a) & (b).

$$G_1 = G_2: mv = (3m)v', \quad v' = \frac{v}{3}$$

$$T = \frac{1}{2}mv^2, \quad T' = \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \frac{1}{6}mv^2$$

$$n = \frac{T - T'}{T} = \frac{\frac{1}{2}mv^2 - \frac{1}{6}mv^2}{\frac{1}{2}mv^2} = \underline{\underline{\frac{2}{3}}}$$

3/192

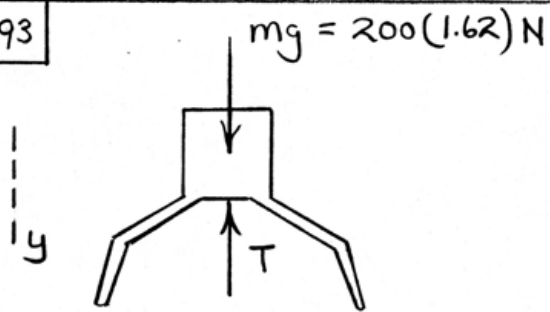


$$\int \Sigma F_x dt = m \Delta v_x$$

$$W \left(\frac{5}{13} \right) t = \frac{W}{32.2} (30 - [-10])$$

$$\underline{t = 3.23 \text{ sec}}$$

3/193



$$\int \Sigma F_y dt = m \Delta v_y :$$

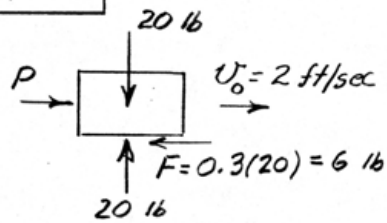
$$200(1.62)(5) - \left[\frac{1}{2} 2(800) + 2(800) \right] = 200(v-6)$$

$$\underline{v = 2.10 \text{ m/s}}$$

$$\boxed{3/194} \quad \Delta G = 0; \quad 600(18000) - \{400v_3 + 200(18060)\} = 0$$

$$\underline{v_3 = 17970 \text{ km/h}}$$

3/195



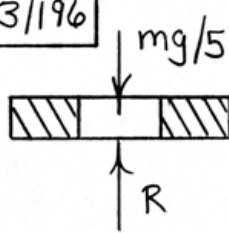
$$\int_0^t \Sigma F dt = m \Delta v$$

$$16(0.2) + 8(0.2) - 6(0.4)$$

$$= \frac{20}{32.2} (v - 2)$$

$$\underline{v = 5.86 \text{ ft/sec}}$$

3/196



Washer :

$$+\downarrow \frac{m}{5} v + \left[\frac{mg}{5} - R \right] \Delta t = 0$$

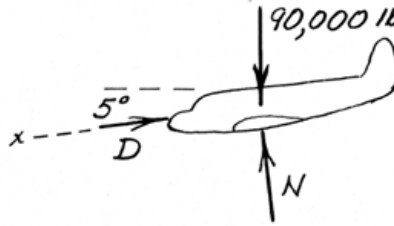
$$R = \frac{m}{5} \left(\frac{v}{\Delta t} + g \right)$$

$$\text{Initial energy : } \frac{1}{2} \frac{6m}{5} v^2 = \frac{3}{5} m v^2$$

$$\text{Final energy : } \frac{1}{2} m v^2$$

$$n = \frac{\frac{3}{5} - \frac{1}{2}}{\frac{3}{5}} (100\%) = \underline{16.67\%}$$

3/197



$$\begin{aligned}\int \Sigma F_x dt &= \Delta G_x : \\ (90,000 \sin 5^\circ - D) 120 \\ &= \frac{90,000}{32.2} (360 - 400) \frac{5280}{3600}\end{aligned}$$

$$\underline{D = 9210 \text{ lb}}$$

$$3/198 \quad \Delta G = 0; \quad (0.140)(600) - [0.140 + 3 \times 0.100]v = 0$$

$$v = 190.9 \text{ m/s}$$

$$|\Delta E| = \frac{1}{2}(0.140)(600)^2 - \frac{1}{2}(0.140 + 0.300)(190.9)^2$$

$$= 25.2(10^3) - 8.018(10^3) = \underline{17.18(10^3)} \text{ J loss}$$

$$\begin{aligned}
 & \boxed{3/199} \int F dt = m \Delta v \\
 & (50,000 \cos 20^\circ) t = \frac{150,000 \times 2240}{32.2} \frac{1 \times 1.151}{1} \frac{44}{30} \\
 & 46,985 t = 17.62 \times 10^6 \\
 & t = 375 \text{ sec or } \underline{t = 6.25 \text{ min}}
 \end{aligned}$$

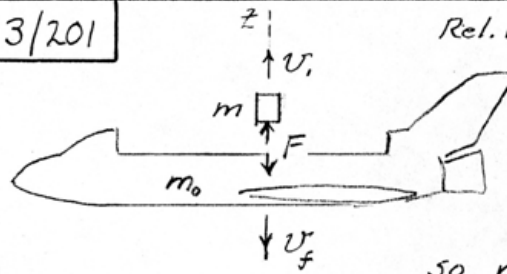
$$3/200 \quad \Delta G = 0; \quad 320(28) - (320 + 20 \times 18) v = 0$$

Initial velocity of chain is $v = 13.18 \text{ m/s}$

$$\int \Sigma F dt = m \Delta v; \quad (20 \times 18) 9.81(0.7) t = (320 + 20 \times 18)(13.18)$$

$$t = \underline{3.62 \text{ s}}$$

3/201



Rel. velocity is

$$v_i + v_f = 0.3 \text{ m/s} \quad \text{---(1)}$$

$$\int F dt = m v_i$$

$$\int -F dt = m_f (-v_f)$$

$$\text{so } m v_i = m_o v_f$$

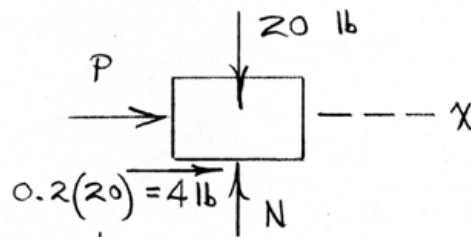
$$800 v_i = 90000 v_f \quad \text{---(2)}$$

$$\text{Solve (1) \& (2) \& get } v_f = 0.3 - \frac{90000}{800} v_f$$

$$v_f = 0.00264 \text{ m/s}$$

$$\text{so } F_{av} \int_0^4 dt = 90000 (0.00264), \quad F_{av} = \frac{90(2.64)}{4} = \underline{59.5 \text{ N}}$$

3/202



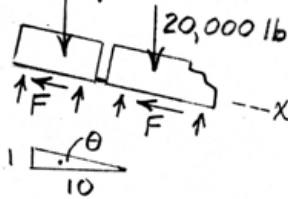
$$\rightarrow m v_1 + \int_0^t \Sigma F dt = 0 :$$

$$-\frac{20}{32.2} (4) + 5(0.2) + 2.5(t-0.2) + 4t = 0$$

$$\underline{t = 0.305 \text{ sec}}$$

3/203

$$\theta = \tan^{-1} 0.1 = 5.71^\circ, \sin \theta = 0.0995$$

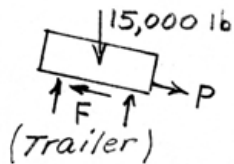


$$\int \Sigma F_x dt = m \Delta V_x$$

$$[35,000 \times 0.0995 - 2F] 5$$

$$= \frac{35,000}{32.2} \left(0 - 20 \frac{44}{30} \right)$$

$$F = 4930 \text{ lb}$$

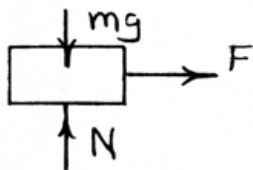


$$[P - 4930 + 15,000 \times 0.0995] 5$$

$$= \frac{15,000}{32.2} \left(0 - 20 \frac{44}{30} \right)$$

$$P = 704 \text{ lb (tension)}$$

3/204



$$\xrightarrow{+} mv_1 + \int \Sigma F dt = mv_2 :$$

$$0 + \int_0^t F_0 e^{-bt} dt = mv$$

$$v = \frac{F_0}{mb} (1 - e^{-bt}) , \quad v \rightarrow \frac{F_0}{mb} \quad \text{as } t \rightarrow \infty$$

$$\frac{ds}{dt} = \frac{F_0}{mb} (1 - e^{-bt})$$

$$\int_{s_0=0}^s ds = \int \frac{F_0}{mb} (1 - e^{-bt}) dt$$

$$s = \frac{F_0}{mb} \left[t + \frac{1}{b} (e^{-bt} - 1) \right]$$

3/205

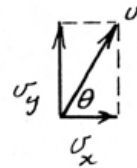
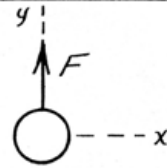
$$\int \Sigma F_y dt = \Delta G_y:$$

$$\int_0^4 \left(2 + \frac{3t^2}{4}\right) dt = 2.4(v_y - [-\frac{3}{5}5])$$

$$2t + \frac{t^3}{4} \Big|_0^4 = 2.4(v_y + 3), \quad v_y = 7 \text{ m/s}$$

$$\int \Sigma F_x dt = \Delta G_x: \quad 0 = 2.4(v_x - \frac{4}{5}5), \quad v_x = 4 \text{ m/s constant}$$

$$v = \sqrt{4^2 + 7^2} = \underline{8.06 \text{ m/s}}, \quad \theta = \tan^{-1} \frac{7}{4} = \underline{60.3^\circ}$$



3/206	Impact velocity $v_0 = \sqrt{2gh} = \sqrt{2(9.81)(1.4)}$ $= 5.24 \text{ m/s}$
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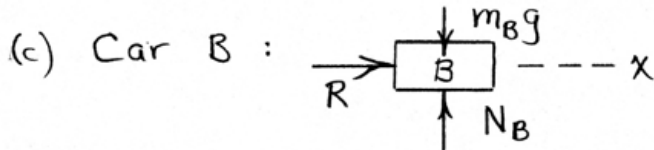
$$\Delta G = 0; 450(5.24) + 0 = (450 + 240)v$$
$$v = 3.42 \text{ m/s}$$

Impulse of weights is negligible compared with impulse of impact forces.

$$3/207 \quad (a) \quad m_A v_A = (m_A + m_B) v'$$

$$v' = \frac{m_A}{m_A + m_B} v_A = \frac{4000/g}{(4000 + 2000)/g} 20$$

$$= 13.33 \text{ mi/hr} \quad (19.56 \text{ ft/sec})$$



$$m_B v_B + R \Delta t = m_B v' : 0 + R(0.1) = \frac{2000}{32.2} (19.56)$$

$$R = 12,150 \text{ lb}$$

(The force which car B exerts on car A is 12,150 lb to the left, by Newton's Third Law.)

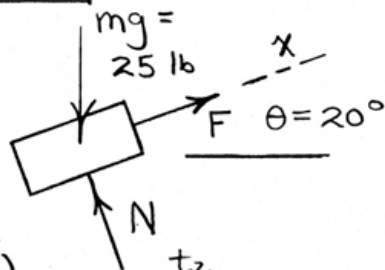
$$(b) \quad a_A = \frac{\Delta v}{\Delta t} = \frac{19.56 - 20 \left(\frac{5280}{3600} \right)}{0.1} = -97.8 \frac{\text{ft}}{\text{sec}^2}$$

$$a_B = \frac{\Delta v}{\Delta t} = \frac{19.56 - 0}{0.1} = 195.6 \frac{\text{ft}}{\text{sec}^2}$$

3/208

$\Delta G_x = 0; \frac{3200}{g} (30) = \frac{(3200+3400)}{g} v_x$
 $v_x = 14.55 \text{ mi/hr}$
 $\Delta G_y = 0; \frac{3400 (20)}{g} = \frac{(3200+3400)}{g} v_y$
 $v_y = 10.30 \text{ mi/hr}$
 $v = \sqrt{(14.55)^2 + (10.30)^2} = 17.82 \text{ mi/hr}$
 $\theta = \tan^{-1} \frac{v_x}{v_y} = \tan^{-1} \frac{14.55}{10.30} = 54.7^\circ$

3/209



$$F = b + 10 \sin 6t$$

(a) $m v_{x1} + \int_{t_1}^{t_2} \sum F_x dt = m v_{x2} :$

$$0 - (mg \sin \theta) \Delta t + \int_0^{\Delta t} F dt = m v$$

$$- 25 \sin 20^\circ (1.5) + \left[5t - \frac{10}{6} \cos 6t \right]_0^{1.5} = \frac{25}{32.2} v$$

$$v = -2.76 \text{ ft/sec}$$

(b) b must equal $mg \sin \theta$

$$\text{or } b = 25 \sin 20^\circ = \underline{8.55 \text{ lb}}$$

$$\begin{aligned}
 \underline{3/210} \quad \underline{G_1 = G_2 : m_s \underline{v}_s + m_m \underline{v}_m = (m_s + m_m) \underline{v}} \\
 1000(2000)\underline{j} + 10(5000) \left[\frac{+5\underline{i} - 4\underline{j} - 2\underline{k}}{\sqrt{5^2 + 4^2 + 2^2}} \right] = (1000 + 10) \underline{v} \\
 \underline{\underline{\underline{v} = 36.9\underline{i} + 1951\underline{j} - 14.76\underline{k} \quad \text{m/s}}}
 \end{aligned}$$

The angle between $\underline{v}_s$ and $\underline{v}$ is

$$\begin{aligned}
 \beta &= \cos^{-1} \frac{\underline{v} \cdot \underline{v}_s}{v v_s} \\
 &= \cos^{-1} \left[\frac{(36.9\underline{i} + 1951\underline{j} - 14.76\underline{k}) \cdot 2000\underline{j}}{\sqrt{36.9^2 + 1951^2 + 14.76^2} \cdot 2000} \right] \\
 &= \underline{\underline{1.167^\circ}}
 \end{aligned}$$

$$\boxed{3/211} \quad \int \underline{F} dt = \underline{F} t = m \Delta \underline{v}$$

$$\underline{F} = \frac{0.20}{0.04} \left([18 \cos 20^\circ] \underline{i} + [18 \sin 20^\circ] \underline{j} - [-12 \underline{i}] \right)$$

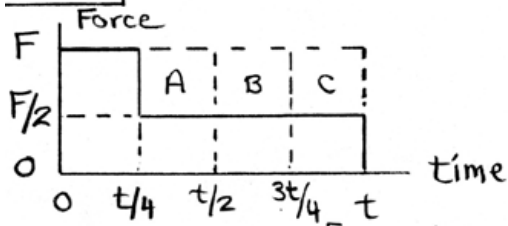
$$= 5 (18 \times 0.9397 \underline{i} + 18 \times 0.3420 \underline{j} + 12 \underline{i})$$

$$= 30 (4.819 \underline{i} + 1.026 \underline{j}) \text{ N}$$

$$F = 30 \sqrt{4.819^2 + 1.026^2} = \underline{147.8 \text{ N}}$$

$$\beta = \tan^{-1} v_y / v_x = \tan^{-1} \frac{1.026}{4.819} = \underline{12.02^\circ}$$

3/21/2



Solid area is $\frac{5}{8}$ of nominal area, so $n = 62.5\%$

In order to compensate, areas A, B, & C must be added after time t , so the extra time $t' = \frac{3}{4}t$.

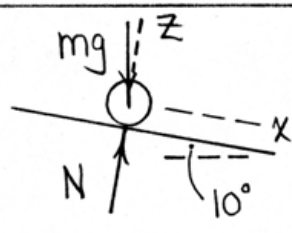
3/213 $\int F_x dt = m \Delta v_x :$

$$(mg \sin 10^\circ)Z = m[v_x - (-3 \sin 15^\circ)]$$

$$v_x = 2.63 \text{ m/s}$$

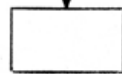
$\int F_y dt = m \Delta v_y: \quad 0 = m[v_y - 3 \cos 15^\circ]$

$$v_y = 2.90 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \underline{3.91 \text{ m/s}}$$


3/214

$$10(9.81) \text{ N}$$



$$N = 98.1 \text{ N}$$

$$F_s = \mu_s N = 0.6(98.1) = 58.9 \text{ N}$$

$$F_k = \mu_k N = 0.4(98.1) = 39.2 \text{ N}$$

Block does not move until

$$P = F_s \text{ or } 25t = 58.9, t = 2.35 \text{ s}$$

Then F becomes 39.2 N

$$\int_0^4 F dt = m \Delta v; \int_{2.35}^4 (25t - 39.2) dt = 10(v - 0)$$

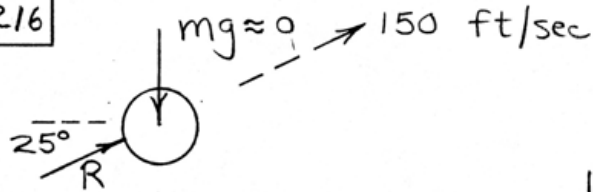
$$\left[\frac{25t^2}{2} - 39.2t \right]_{2.35}^4 = 10v, \quad 10v = 66.1, \quad \underline{v = 6.61 \text{ m/s}}$$

$$\boxed{3/215} \quad \int F_x dt = m \Delta v_x : 0.2t = \frac{1.2}{32.2} [v_x - (-10 \sin 30^\circ)]$$

$$v_x = \frac{dx}{dt} = 5.37t - 5$$

$$\int_0^0 dx = \int_0^t (5.37t - 5) dt, \quad \underline{t = 1.863 \text{ sec}}$$

3/2/16



$$\rightarrow R \Delta t = mv : R(0.001) = \frac{1.62/16}{32.2} (150)$$
$$R = 472 \text{ lb}$$

$$\rightarrow R = ma : 472 = \frac{1.62/16}{32.2} a$$

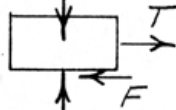
$$a = 150,000 \text{ ft/sec}^2 (4660g)$$

$$v^2 - v_0^2 = 2ad : 150^2 - 0^2 = 2(150,000) d$$

$$d = 0.075 \text{ ft or } 0.900 \text{ in.}$$

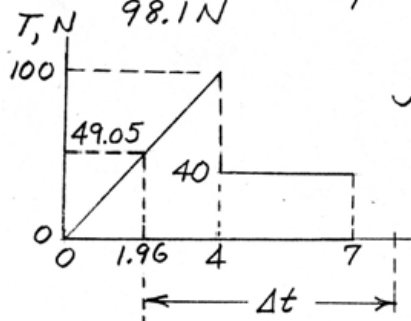
3/217

10(9.81) N Block begins to move
when $T = F = \mu W = 0.5(98.1)$
 $= 49.05 \text{ N}$



which occurs at

$$t_1 = \frac{49.05}{100} 4 = 1.96 \text{ s}$$



$$\int \Sigma F dt = m \Delta v$$

Max. velocity reached by
block occurs at $t = 4 \text{ s}$

$$\frac{(100 - 49.05)}{2} (4 - 1.96) = 10(v - 0)$$

$$\underline{v_{\max} = 5.19 \text{ m/s}}$$

For total motion $\Delta v = 0$, so

$$\frac{100 + 49.05}{2} (4 - 1.96) + 40(7 - 4) - 49.05 \Delta t = 0$$

$$\underline{\Delta t = 5.54 \text{ s}}$$

$$\underline{3/218} \quad \Delta G = 0, \quad G_1 = G_2$$

$$\left(\frac{2/16}{32.2} + 0 \right) 2000 = \frac{2/16 + 50}{32.2} v_2, \quad v_2 = 4.99 \text{ ft/sec}$$

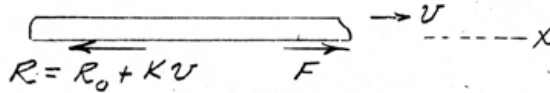
$$U = \Delta T: v_2 = \sqrt{2gh}, \quad 4.99^2 = 2(32.2)(6)(1 - \cos \theta) \text{ where } h = 6(1 - \cos \theta)$$

$$\cos \theta = 0.936, \quad \theta = 20.7^\circ$$

$$\% \text{ energy loss} = \frac{\frac{1}{2} m_1 v_1^2 - (m_1 + m_2)gh}{\frac{1}{2} m_1 v_1^2} \times 100\% = \left(1 - \frac{m_1 + m_2}{m_1} \frac{2gh}{v_1^2} \right) 100\%$$

$$= \left[1 - \frac{2/16 + 50}{2/16} \frac{2(32.2)6(1 - 0.936)}{2000^2} \right] 100\% = \underline{99.8\%}$$

3/219



$$\Sigma F dt = m dv, (F - R_0 - Kv) dt = m dv$$

$$\int_0^t dt = \int_0^v \frac{m dv}{F - R_0 - Kv}; t = -\frac{m}{K} \ln(F - R_0 - Kv) \Big|_0^v$$

$$t = -\frac{m}{K} \ln \frac{F - R_0 - Kv}{F - R_0}$$

$$t = \frac{m}{K} \ln \frac{F - R_0}{F - R_0 - Kv}$$

$$\boxed{3/220} \quad \text{For plug: } \Delta T + \Delta V_g = 0; \quad \frac{1}{2} m_A v^2 - m_A g r = 0$$

$$v = \sqrt{2gr}$$

$$\text{Plug \& block: } \Delta G = 0; \quad m_A \sqrt{2gr} = (m_A + m_C) v'$$

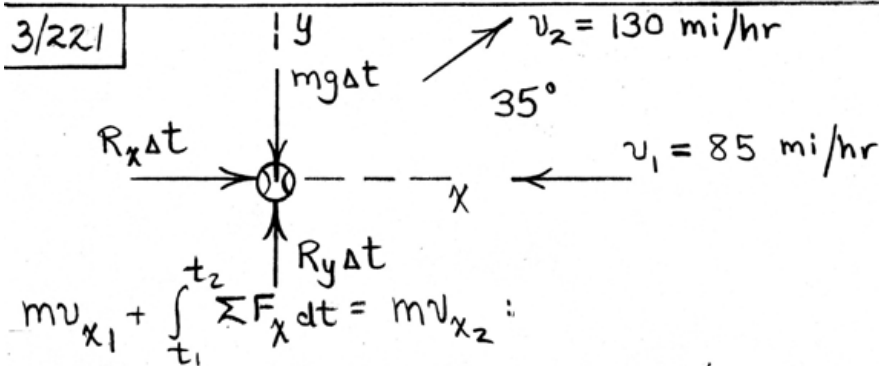
where v' = velocity of block & plug after impact

$$\text{Friction force } F = \mu_k (m_A + m_C) g$$

$$\text{Deceleration } a = F / (m_A + m_C) = \mu_k g$$

$$v'^2 = 2as, \quad s = \left(\frac{m_A}{m_A + m_C} \right)^2 2gr \frac{1}{2\mu_k g} = \frac{r}{\mu_k} \left(\frac{m_A}{m_A + m_C} \right)^2$$

3/22/1



$$mv_{x1} + \int_{t_1}^{t_2} \Sigma F_x dt = mv_{x2} :$$

$$-\frac{5.125/16}{32.2} \left(85 \frac{5280}{3600} \right) + R_x (0.005) = \frac{5.125/16}{32.2} \left(130 \frac{5280}{3600} \cos 35^\circ \right)$$

$$\underline{R_x = 559 \text{ lb}}$$

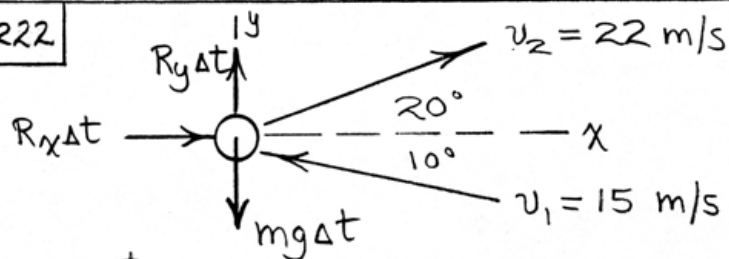
$$mv_{y1} + \int_{t_1}^{t_2} \Sigma F_y dt = mv_{y2} :$$

$$0 + R_y (0.005) - \frac{5.125}{16} (0.005) = \frac{5.125/16}{32.2} \left(130 \frac{5280}{3600} \sin 35^\circ \right)$$

$$\underline{R_y = 218 \text{ lb}}$$

If mg is neglected during impact, $R_y = 218 \text{ lb}$ - a good assumption! The quantity mg may not be neglected thereafter - otherwise we obtain a record home-run distance!

3/222



$$mv_{x1} + \int_{t_1}^{t_2} \Sigma F_x dt = mv_{x2}$$

$$0.060(-15 \cos 10^\circ) + R_x(0.05) = 0.060(22 \cos 20^\circ)$$

$$R_x = 42.5 \text{ N}$$

$$mv_{y1} + \int_{t_1}^{t_2} \Sigma F_y dt = mv_{y2}$$

$$0.060(15 \sin 10^\circ) + R_y(0.05) - 0.060(9.81)(0.05) = 0.060(22 \sin 20^\circ)$$

$$R_y = 6.49 \text{ N}$$

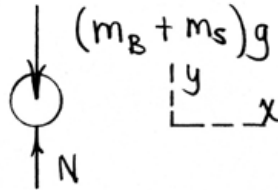
Weight $mg = 0.060(9.81) = 0.589 \text{ N}$ is about 9% of R_y - no need to omit mg from analysis!

$$R = \sqrt{R_x^2 + R_y^2} = \underline{43.0 \text{ N}}$$

$$\beta = \tan^{-1} \frac{R_y}{R_x} = \underline{8.68^\circ}$$

3/223

System :



$$m_B v_{Bx} + m_S v_{Sx}^{\nearrow 0} = (m_B + m_S) v$$

$$v = \frac{m_B v_{Bx}}{(m_B + m_S)} = \frac{80/32.2}{90(32.2)} (16 \cos 30^\circ)$$

$$= \underline{12.32 \text{ ft/sec}}$$

$$m_B v_{By} + m_S v_{Sy}^{\nearrow 0} + \int_0^{\Delta t} [N - (m_B + m_S)g] dt = 0$$
$$- \frac{80}{32.2} (16 \sin 30^\circ) + N(0.05) - 90(0.05) = 0$$

$$\underline{N = 488 \text{ lb}}$$

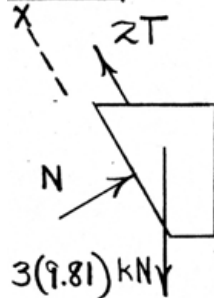
3/224

Skip begins to move when

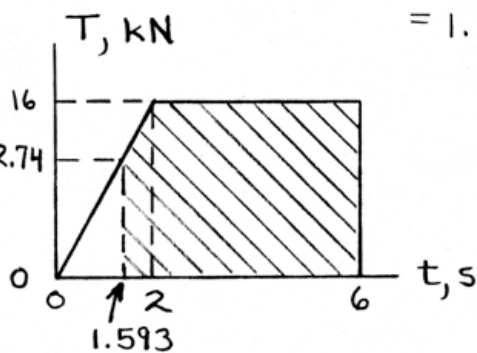
$$2T = 3(9.81) \frac{\sqrt{3}}{2}, \quad T = 12.74 \text{ kN}$$

$$\text{which occurs at } t = \frac{12.74}{16}$$

$$= 1.593 \text{ s}$$



60°



$$\int \Sigma F_x dt = m \Delta v_x :$$

$$2 \left[\frac{16 + 12.74}{2} (2 - 1.593) + 16(6 - 2) \right] - 3(9.81) \frac{\sqrt{3}}{2} (6 - 1.593)$$

$$= 3(v - 0)$$

$$\underline{v = 9.13 \text{ m/s}}$$

$$\boxed{3/225} \quad T_y = 600 \cos \theta ; \quad \dot{\theta} = \pi/10 \text{ rad/s, so } dt = \frac{10}{\pi} d\theta$$

$$\int \Sigma F_y dt = m \Delta v_y ; \quad \int_0^{\pi/2} 600 \cos \theta \left(\frac{10}{\pi} d\theta \right) = 260 (v_y - 0)$$

$$\frac{6000}{\pi} \sin \theta \Big|_0^{\pi/2} = 260 v_y , \quad v_y = \frac{6000}{260\pi} = \underline{7.35 \text{ m/s}}$$

3/226 (a) $\Delta G = 0; m(4) + 0 = m v_A + m v_B$

$$v_A + v_B = 4$$

$$\Delta T = 0.4T; \frac{1}{2}m(4^2) - \left[\frac{1}{2}m v_A^2 + \frac{1}{2}m v_B^2 \right] = 0.4 \left[\frac{1}{2}m(4^2) \right]$$

$$v_A^2 + v_B^2 = 9.6$$

Solve simultaneously & get $(4 - v_B)^2 + v_B^2 = 9.6$

$$\text{or } v_B^2 - 4v_B + 3.2 = 0, v_B = \frac{4}{2} \pm \frac{1}{2}\sqrt{16 - 4(3.2)}$$

$$= 2 \pm 0.894$$

$$(\text{Sol. I}) v_B = 2.894 \text{ ft/sec}, v_A = 4 - 2.894 = 1.106 \text{ ft/sec}$$

$$(\text{Sol. II}) v_B = 1.106 \text{ ft/sec}, v_A = 4 - 1.106 = 2.894 \text{ ft/sec}$$

Sol. II is ruled out since distance between A & B would be decreasing so that $v_B > v_A$

Thus $v_B = 2.89 \text{ ft/sec}$

(b) For initial to final condition

$$\Delta G = 0; m(4) + 0 = 2m v_C, \underline{v_C = 2 \text{ ft/sec}}$$

3/227 (a) $\underline{H}_O = \underline{r} \times m \underline{v}$

$$\underline{H}_O = (-6\underline{i} + 8\underline{j}) \times 2(7)(-\sin 30^\circ \underline{i} - \cos 30^\circ \underline{j})$$

$$= 128.7 \underline{k} \text{ kg} \cdot \text{m}^2/\text{s}$$

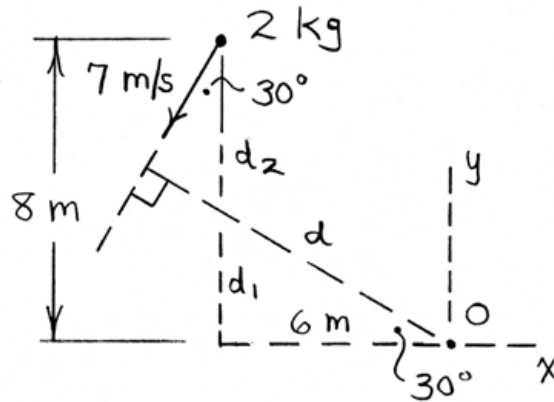
So $H_O = 128.7 \text{ kg} \cdot \text{m}^2/\text{s}$

$$d_1 = 6 \tan 30^\circ$$

$$= 3.46 \text{ m}$$

$$d_2 = 8 - d_1$$

$$= 4.54 \text{ m}$$



$$d = \frac{6}{\cos 30^\circ} + 4.54 \sin 30^\circ = 9.20 \text{ m}$$

∴ $H_O = mvd = 2(7)(9.20) = 128.7 \text{ kg} \cdot \text{m}^2/\text{s}$

$$\begin{aligned} \underline{3/228} \quad (a) \quad \underline{G} &= m\underline{v} = 3 \cdot 5 (-\cos 30^\circ \underline{i} - \sin 30^\circ \underline{j}) \\ &= -12.99 \underline{i} - 7.5 \underline{j} \quad \text{kg} \cdot \text{m/s} \end{aligned}$$

$$\begin{aligned} (b) \quad \underline{H}_o &= \underline{r} \times m\underline{v} = \underline{r} \times \underline{G} \\ &= 2 (\cos 15^\circ \underline{i} - \sin 15^\circ \underline{j}) \times (-12.99 \underline{i} - 7.5 \underline{j}) \\ &= -21.2 \underline{k} \quad \text{kg} \cdot \text{m}^2/\text{s} \end{aligned}$$

$$(c) \quad T = \frac{1}{2} m v^2 = \frac{1}{2} (3) (5)^2 = \underline{37.5 \text{ J}}$$

3/229

$$\begin{aligned}\underline{H}_o &= \underline{r} \times m \underline{v} \\ &= (a\underline{i} + b\underline{j} + c\underline{k}) \times m v \underline{k} \\ &= \underline{mv (b\underline{i} - a\underline{j})}\end{aligned}$$

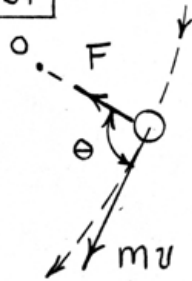
$$\begin{aligned}\dot{\underline{H}}_o &= \underline{M}_o = (a\underline{i} + b\underline{j} + c\underline{k}) \times F \underline{j} \\ &= \underline{F (-c\underline{i} + a\underline{k})}\end{aligned}$$

3/230 Angular momentum about O is conserved:

$$H_{O_1} = H_{O_2} : \quad 3mv(L) + 2mv(L) = 3mL^2\omega$$

$$\omega = \frac{5}{3} \frac{v}{L}$$

3/231



$$\sum M_o = \dot{H}_o = 0, \text{ so } H_o = \text{const.}$$

$$H_{oA} = H_{oB}$$

$$m(4)(0.350 \sin 54^\circ) =$$

$$mv_B (0.230 \sin 65^\circ)$$

$$v_B = 5.43 \text{ m/s}$$

$$\boxed{3/232} \quad (a) \quad v_B = \sqrt{2gr}$$
$$H_o = mrv_B = \underline{mr\sqrt{2gr}} \quad , \quad \underline{\dot{H}_o = mgr}$$

$$(b) \quad v_C = \sqrt{2g(2r)} = 2\sqrt{gr}$$

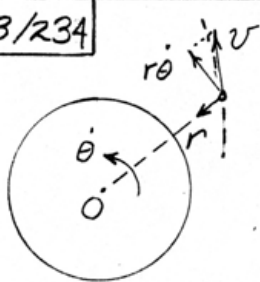
$$H_o = mrv_C = \underline{2mr\sqrt{gr}} \quad , \quad \underline{\dot{H}_o = 0}$$

$$\boxed{3/233} \quad H_1 + \int_{t_1}^t \dot{M}^2 dt = H_2$$

$$0 + 20(0.1) t = 4(3)(0.4)^2 \left[150 \left(\frac{1}{60} \right) (2\pi) \right]$$

$$\underline{t = 15.08 \text{ s}}$$

3/234



$$\Sigma M_O = \dot{H}_O ; 0 = \frac{d}{dt} (m r \dot{\theta} \times r)$$

$$\text{or } \frac{d}{dt} (r^2 \dot{\theta}) = 0$$

$$\text{so } r^2 \dot{\theta} = \text{const.}$$

3/235

$$T_A + U_{A-C} = T_C$$

$$\frac{1}{2} m v_A^2 + m g h_{A-C} = \frac{1}{2} m v_C^2$$

$$v_C^2 = v_A^2 + 2 g h_{A-C}$$

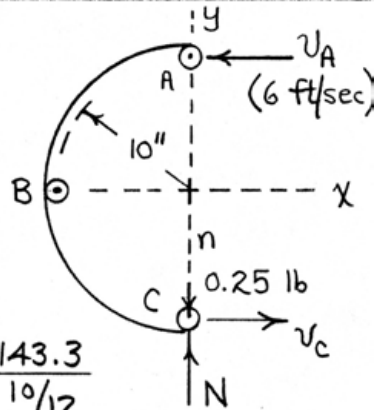
$$= 6^2 + 32.2 \left(\frac{20}{12} \right) (2)$$

$$= 143.3 \text{ ft}^2/\text{sec}^2$$

$$\sum F_y = m a_y : N - 0.25 = \frac{0.25}{32.2} \frac{143.3}{10/12}$$

$$N = 1.585 \text{ lb}$$

$$\underline{H}_B = \underline{M}_B = (1.585 - 0.25) \frac{10}{12} \underline{k} = 1.113 \underline{k} \text{ lb-ft}$$



3/236

Velocity of plug upon impact is

$$v = \sqrt{2gh} = \sqrt{2(9.81)(0.6)} = 3.43 \text{ m/s}$$

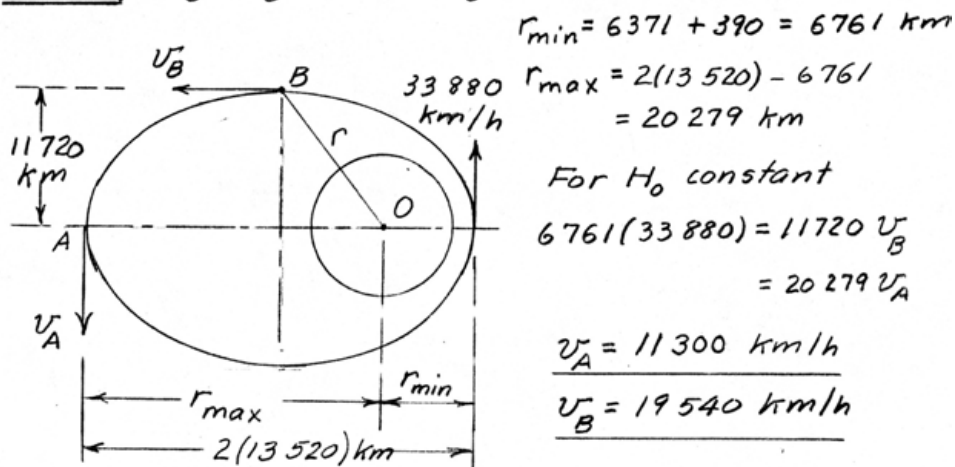
For system, $\Delta H_o = 0$. Take C.W. positive

$$\begin{aligned} \text{Initial } H_o &= -4(0.5)^2(2) - 6(0.3)^2(2) + 2(3.43)(0.5) \\ &= -2 - 1.08 + 3.43 = 0.351 \text{ N}\cdot\text{m}\cdot\text{s} \end{aligned}$$

$$\begin{aligned} \text{Final } H_o &= [(4+2)(0.5)^2 + 6(0.3)^2] \omega \\ &= 2.04 \omega \end{aligned}$$

$$\text{So } 0.351 = 2.04 \omega, \quad \underline{\omega = 0.1721 \text{ rad/s CW}}$$

3/237 $\sum M_O = \dot{H}_O = 0$ so $H_O = \text{constant}$



$$r_{\min} = 6371 + 390 = 6761 \text{ km}$$

$$r_{\max} = 2(13520) - 6761 = 20279 \text{ km}$$

For H_O constant

$$6761(33880) = 11720 v_B = 20279 v_A$$

$$v_A = 11300 \text{ km/h}$$

$$v_B = 19540 \text{ km/h}$$

3/238 For the entire system, $\Sigma M_o = \dot{H}_o = 0$,

So angular momentum is conserved.

$$H_{o1} = H_{o2} : 2mr^2\omega_o + 0 = 2mr^2\omega + 2m(2r)^2\omega$$

$$\underline{\omega = \omega_o/5}$$

Kinetic energy loss $\Delta Q = T_1 - T_2$

$$\begin{aligned}\Delta Q &= 2\left(\frac{1}{2}mr^2\omega_o^2\right) - \left\{2\left(\frac{1}{2}mr^2\omega^2\right) + 2\left(\frac{1}{2}m(2r)^2\omega^2\right)\right\} \\ &= mr^2\omega_o^2 - mr^2\left(5\left(\frac{\omega_o}{5}\right)^2\right) = \frac{4}{5}mr^2\omega_o^2\end{aligned}$$

$$\text{So } n = \frac{\Delta Q}{T_1}(100\%) = \frac{\frac{4}{5}mr^2\omega_o^2}{2\left(\frac{1}{2}mr^2\omega_o^2\right)}(100\%) = \underline{80\%}$$

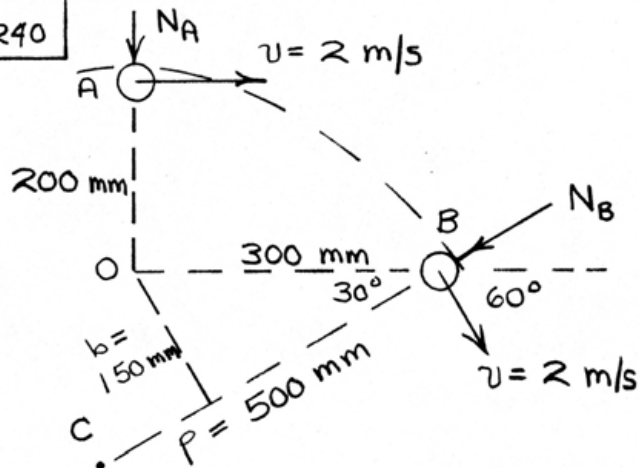
$$\underline{3/239} \quad \Delta H = 0; \quad 2m r \omega_0(r) - 2m(2r) \omega(2r) = 0$$

$$\underline{\omega = \omega_0/4}$$

$$\Delta T = 2 \left(\frac{1}{2} m [r \omega_0]^2 \right) - 2 \left(\frac{1}{2} m \left[2r \frac{\omega_0}{4} \right]^2 \right) = m r^2 \omega_0^2 (3/4)$$

$$n = \Delta T / T = \frac{3}{4} m r^2 \omega_0^2 / m r^2 \omega_0^2 = \underline{3/4}$$

3/240



$$\sum M_{O_z} = \dot{H}_{O_z}$$

$$\text{At A, } \sum M_{O_z} = 0, \text{ so } \dot{H}_{O_z} = 0$$

$$\text{At B, } \sum M_{O_z} = -N_b b, \text{ where } N_b = m \frac{v^2}{r} = 0.1 \frac{2^2}{0.5} = 0.8 \text{ N}$$

$$\text{So } \dot{H}_{O_z} = -N_b b = -0.8(0.150) = -0.120 \text{ N}\cdot\text{m} \\ \text{(or } -0.120 \text{ kg}\cdot\text{m}^2/\text{s}^2 \text{)}$$

3/241 $\Sigma M_O = \dot{H}_O = 0$, so angular momentum is conserved: $H_{O1} = H_{O2}$ (O: any point on axis)

$$0.2 (0.3 \cos 30^\circ)^2 4 = 0.2 (0.2 \cos 30^\circ)^2 \omega$$

$$\omega = 9 \text{ rad/s}$$

$$U'_{1-2} = \Delta T + \Delta V_g$$

$$\Delta T = \frac{1}{2} (0.2) [(0.2 \cos 30^\circ \cdot 9)^2 - (0.3 \cos 30^\circ \cdot 4)^2]$$
$$= 0.1350 \text{ J}$$

$$\Delta V_g = 0.2 (9.81) (0.1 \sin 30^\circ) = 0.0981 \text{ J}$$

$$\text{So } U'_{1-2} = 0.1350 + 0.0981 = \underline{0.233 \text{ J}}$$

$$\boxed{3/242} \quad \int \Sigma M_o dt = \Delta H_o = H_{o_B} - H_{o_A}$$

$$H_{o_A} = 0.02(4)(0.090) \sin 30^\circ = 0.0036 \text{ kg}\cdot\text{m}^2/\text{s}$$

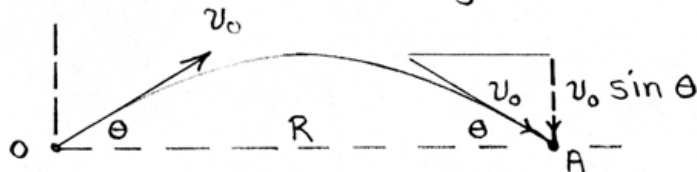
$$H_{o_B} = 0.02(6)(0.180) \sin 60^\circ = 0.01871 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$\Delta H_o = 0.01871 - 0.0036 = 0.01511 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$M_{o_{av}} \times 0.5 = 0.01511, \quad \underline{M_{o_{av}} = 0.0302 \text{ N}\cdot\text{m}}$$

3/243 (a) $H_0 = 0$ when projectile is at 0.

(b) Range $R = \frac{2v_0^2 \cos \theta \sin \theta}{g}$

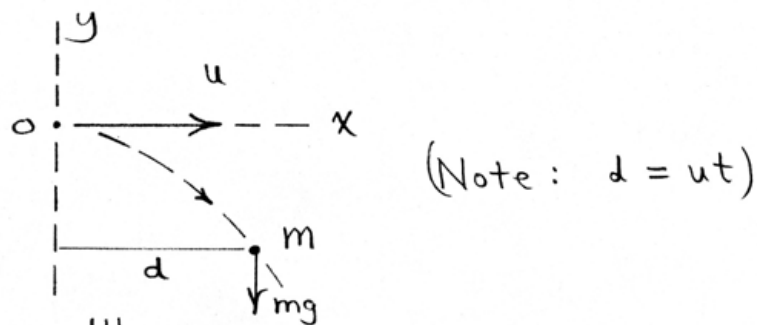


$$H_0 = mv_y R = mv_0 \sin \theta \frac{2v_0^2 \cos \theta \sin \theta}{g}$$

$$= \frac{2mv_0^3 \sin^2 \theta \cos \theta}{g}$$

The moment of the projectile weight about point O is always increasing the angular momentum about O.

3/244



$$\dot{H}_o = \frac{dH_o}{dt} = \underline{M}_o = -mg d \underline{k}$$

$$\int_0^{H_o} d\underline{H}_o = - \int_0^t mg d \underline{k} dt = - \int_0^t mg u t \underline{k} dt$$

$$\Rightarrow \underline{H}_o = - \frac{1}{2} mg u t^2 \underline{k}$$

3/245 Conservation of angular momentum :

$$(mrv_{\theta})_A = (mrv_{\theta})_B$$

$$50(10^6)(188,500) = 75(10^6)v_{\theta}$$

$$\underline{v_{\theta} = 125,700 \text{ ft/sec @ B}}$$

Energy conservation $T_A + V_A = T_B + V_B$

$$\frac{1}{2}mv_A^2 - \frac{Gm_s m}{r_A} = \frac{1}{2}mv_B^2 - \frac{Gm_s m}{r_B}$$

$$\frac{1}{2}(188,500)^2 - \frac{1}{2}v_B^2 =$$

$$3.439(10^{-8})(333,000)(4.095)10^{23} \left[\frac{1}{50(10^6)} - \frac{1}{75(10^6)} \right] \frac{1}{5280}$$

$$v_B = 153,900 \text{ ft/sec}$$

$$v_r = \sqrt{v_B^2 - v_{\theta}^2} = \sqrt{153,900^2 - 125,700^2} = \underline{88,870 \frac{\text{ft}}{\text{sec}}}$$

3/246

$$\Sigma M_o = \dot{H}_o : mgl \cos \theta = \frac{d}{dt} (ml^2 \dot{\theta})$$

$$= ml^2 \ddot{\theta}$$

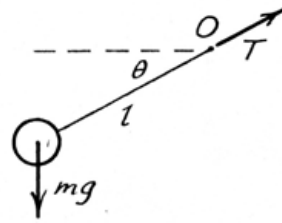
$$\ddot{\theta} = \frac{g}{l} \cos \theta$$

$$\text{From } \int \dot{\theta} d\dot{\theta} = \int \ddot{\theta} d\theta, \quad \frac{\dot{\theta}^2}{2} \Big|_0^{\dot{\theta}} = \int_0^{\theta} \frac{g}{l} \cos \theta d\theta,$$

$$\dot{\theta}^2 = \frac{2g}{l} \sin \theta, \quad \dot{\theta}_{\theta=90^\circ} = \sqrt{\frac{2g}{l}}$$

$$\text{so at } \theta = 90^\circ, \quad v = l\dot{\theta} = \sqrt{2gl}$$

$$\text{By work-energy } U = \Delta T, \quad mgl = \frac{1}{2}mv^2, \quad v = \sqrt{2gl}$$



3/247 Forces on particle exert no moment about the central axis, so angular momentum is conserved about this axis. Thus $\Delta H_z = 0$ &

$$m v_0 \cos \beta (r) = m v \cos \theta (r), v_0 \cos \beta = v \cos \theta$$

Also energy is conserved so that

$$\Delta T + \Delta V_g = 0; \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 - m g h = 0$$

Eliminate v & get $\cos \theta = \frac{v_0 \cos \beta}{\sqrt{v_0^2 + 2 g h}}$

or $\theta = \cos^{-1} \frac{\cos \beta}{\sqrt{1 + \frac{2 g h}{v_0^2}}}$

3/248 System angular momentum conserved

during impact: $\vec{r} \times H_{O1} = H_{O2}$:

$$0.050(300)(0.4 \cos 20^\circ) - 3.2(0.2)^2 6 - 3.2(0.4)^2 6 \\ = (0.050 + 3.2)(0.4)^2 \omega' + 3.2(0.2)^2 \omega'$$

$$\omega' = 2.77 \text{ rad/s (CCW)}$$

Energy considerations after impact:

$T' + V' = T^0 + V^0$, choose datum @ 0:

$$\frac{1}{2}(0.05 + 3.2)[0.4(2.77)]^2 + \frac{1}{2}(3.2)[0.2(2.77)]^2 \\ + [3.2(0.2) - (3.2 + 0.05)(0.4)]9.81 = 0 + \\ [3.2(0.2) - (3.2 + 0.05)(0.4)]9.81 \cos \theta \\ \theta = 52.1^\circ$$

3/249 Path form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a = 5$ ft
 $b = 4$ ft)

Angular momentum about O is conserved:

$$mr_A v_A = mr_B v_B : v_B = \frac{r_A}{r_B} v_A = \frac{a}{b} v_A$$

$$= \frac{5}{4} (8) = 10 \text{ ft/sec}$$

$$y = b \left[1 - \left(\frac{x}{a} \right)^2 \right]^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2} b \left[1 - \left(\frac{x}{a} \right)^2 \right]^{-1/2} \cdot \left(-\frac{2x}{a^2} \right) = -\frac{bx}{a^2} \left[1 - \left(\frac{x}{a} \right)^2 \right]^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{b}{a^2} \left[1 - \left(\frac{x}{a} \right)^2 \right]^{-1/2} - \frac{bx}{a^2} \left(-\frac{1}{2} \right) \left[1 - \left(\frac{x}{a} \right)^2 \right]^{-3/2} \left(-\frac{2x}{a^2} \right)$$

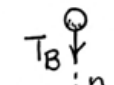
$$= -\frac{b}{a^2} \left[1 - \left(\frac{x}{a} \right)^2 \right]^{-1/2} - \frac{bx^2}{a^4} \left[1 - \left(\frac{x}{a} \right)^2 \right]^{-3/2}$$

Now, $\left. \frac{dy}{dx} \right|_{x=0} = 0$ and $\left. \frac{d^2y}{dx^2} \right|_{x=0} = -\frac{b}{a^2}$

$$= -\frac{4}{5^2} = -\frac{4}{25}$$

$$r_{xy} = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{d^2y/dx^2} = \frac{[1+0]^{3/2}}{-4/25} = -6.25'$$

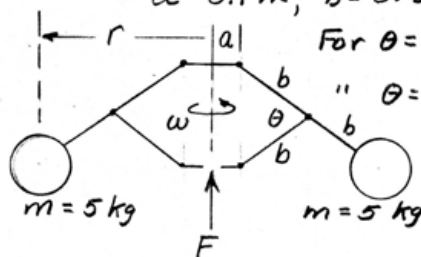
So $P = 6.25'$; $\Sigma F_n = m \frac{v^2}{P} : T_B = \frac{1.5}{32.2} \frac{10^2}{6.25} = 0.745 \text{ lb}$



► 3/250

$$\omega_0 = 40(2\pi)/60 = 4.19 \text{ rad/s}$$

$$a = 0.1 \text{ m}, b = 0.3 \text{ m}$$



$$\text{For } \theta = 90^\circ, r_0 = 0.1 + 2(0.3) \cos 45^\circ = 0.524 \text{ m}$$

$$\text{" } \theta = 60^\circ, r = 0.1 + 2(0.3) \cos 30^\circ = 0.620 \text{ m}$$

$$\Delta H = 0; 2mr_0^2\omega_0 - 2mr^2\omega = 0$$

$$\omega = \frac{r_0^2}{r^2} \omega_0 = \left(\frac{0.524}{0.620} \right)^2 (4.19)$$

$$= 3.00 \text{ rad/s}$$

$$\left(\text{or } \frac{3.00}{2\pi} 60 = 28.6 \text{ rev/min} \right)$$

$$U = \Delta T + \Delta V_g = 2\left(\frac{1}{2}m\right)(r^2\omega^2 - r_0^2\omega_0^2) + 2mg\Delta h$$

$$\text{where } \Delta h = 2b(\sin 45^\circ - \sin 30^\circ)$$

$$= 2(0.3)(0.7071 - 0.5) = 0.1243 \text{ m}$$

$$U = 5\left([0.620 \times 3.00]^2 - [0.524 \times 4.19]^2\right) + 2(5)(9.81)(0.1243)$$

$$= -6.850 + 12.190 = \underline{5.34 \text{ J}}$$

$$\begin{aligned}
 &\boxed{3/251} \quad v = \sqrt{2gh} \quad , \quad v' = \sqrt{2gh'} \\
 &e = \frac{v'}{v} = \sqrt{\frac{h'}{h}} = \sqrt{\frac{1100}{2100}} = \underline{0.724} \\
 &n = \frac{mgh - mgh'}{mgh} (100\%) = \frac{2100 - 1100}{2100} (100\%) \\
 &= \underline{47.6\%}
 \end{aligned}$$

3/252 System linear momentum :

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$
$$\frac{1.5}{32.2} (0.8) + \frac{2}{32.2} (-2.4) = \frac{1.5}{32.2} v_1' + \frac{2}{32.2} v_2'$$

Restitution : $e = \frac{v_2' - v_1'}{v_1 - v_2} : 0.5 = \frac{v_2' - v_1'}{0.8 - (-2.4)}$

Solve the two equations to obtain

$$v_1' = -1.943 \text{ ft/sec}$$

$$v_2' = -0.343 \text{ ft/sec}$$

Original energy : $T_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$$T_1 = \frac{1}{2} \frac{1.5}{32.2} (0.8)^2 + \frac{1}{2} \frac{2}{32.2} (2.4)^2$$
$$= 0.1938 \text{ ft-lb}$$

$$T_2 = \frac{1}{2} \frac{1.5}{32.2} (1.943)^2 + \frac{1}{2} \frac{2}{32.2} (0.343)^2$$
$$= 0.0916 \text{ ft-lb}$$

$$n = \frac{T_1 - T_2}{T_1} (100\%) = \frac{0.1938 - 0.0916}{0.1938} (100\%)$$
$$= \underline{52.7\%}$$

3/253 System momentum:

$$\frac{1.5}{32.2}(0.8) + \frac{2}{32.2} v_2 = \frac{1.5}{32.2} v_1' + 0$$

$$\text{Restitution: } \frac{-v_1'}{0.8 - v_2} = 0.5$$

$$\text{Solve to obtain } \begin{cases} v_1' = -1.120 \text{ ft/sec} \\ v_2 = -1.440 \text{ ft/sec} \end{cases}$$

(Note: v_2 assumed totally unknown above -
no leftward direction assumed.)

3/254 Consider the case $v_2' = v_1$. Conservation

of system linear momentum:

$$m_1 v_1 + m_2 v_2^0 = m_1 v_1' + m_2 v_2' = m_1 v_1' + m_2 v_1$$

$$v_1' = \frac{(m_1 - m_2)}{m_1} v_1$$

$$\text{Restitution: } e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{v_1 - \left(\frac{m_1 - m_2}{m_1}\right)v_1}{v_1}$$

$$\frac{m_1}{m_2} = \frac{1}{e}$$

$$\text{So for } v_2' > v_1, \quad \underline{\frac{m_1}{m_2} > \frac{1}{e}}$$

3/255

System linear momentum :

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \rightarrow$$

$$m v + 0 = m v_A' + p m v_B' \quad (1)$$

$$\text{Restitution : } e = \frac{v_B' - v_A'}{v_A - v_B} : 0.1 = \frac{v_B' - v_A'}{v - 0} \quad (2)$$

Solve (1) & (2) to obtain

$$v_A' = \left(\frac{1 - 0.1p}{1 + p} \right) v, \quad v_B' = \frac{1.1}{1 + p} v$$

$$\text{For } p = \frac{1}{2} : \quad v_A' = 0.633 v, \quad v_B' = 0.733 v$$

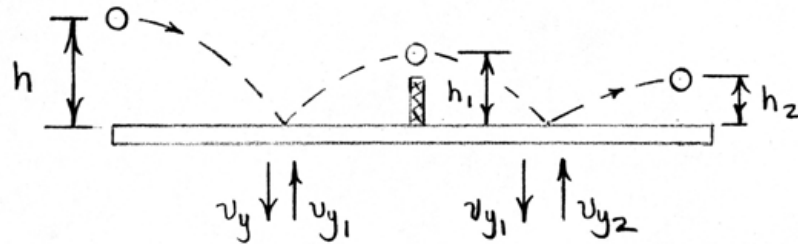
$$\boxed{3/256} \quad \text{Impact velocity } v = \sqrt{2gh} = \sqrt{2(32.2)(4)} \\ = 16.05 \text{ ft/sec}$$

$$\Delta G = 0; 500(16.05) + 0 = 0 + 800 v'$$

$$v' = 10.03 \text{ ft/sec}$$

$$e = \frac{v'}{v} = \frac{10.03}{16.05} = \underline{0.625}$$

3/257



(Note: Drop distances reduced by $r = 0.75''$)

$$v_y = \sqrt{2g(h-r)}, \quad v_{y1} = e v_y = e \sqrt{2g(h-r)} \\ = \sqrt{2g(h_1-r)}$$

$$\therefore 0.90^2 (2g)(h-0.75) = (2g)(9-0.75), \quad h = 10.94 \text{ in.}$$

$$v_{y2} = e v_{y1} : \quad \sqrt{2g(h_2-r)} = 0.9 \sqrt{2g(h_1-r)}$$

$$\text{With } r = 0.75 \text{ in. } \uparrow h_1 = 9 \text{ in.} : \quad \underline{h_2 = 7.43 \text{ in.}}$$

$$\boxed{3/258} \quad \Delta G = 0; \quad m_A v_A + 0 = m_A v'_A + m_B v'_B$$

$$e = 0; \quad v'_A = v'_B$$

$$\text{Thus } m_A v_A = (m_A + m_B) v'_A$$

$$|\Delta T| = -\frac{1}{2} m_A v'^2_A - \frac{1}{2} m_B v'^2_B + \frac{1}{2} m_A v_A^2$$

$$= -\frac{1}{2} m_A \left(\frac{m_A}{m_A + m_B} v_A \right)^2 - \frac{1}{2} m_B \left(\frac{m_A}{m_A + m_B} v_A \right)^2 + \frac{1}{2} m_A v_A^2$$

$$= -\frac{1}{2} \left(\frac{m_A}{m_A + m_B} v_A \right)^2 (m_A + m_B) + \frac{1}{2} m_A v_A^2$$

$$= \frac{1}{2} \frac{m_A m_B}{m_A + m_B} v_A^2 \quad (\text{loss})$$

$$\frac{|\Delta T|}{T} = \frac{1}{2} \frac{m_A m_B}{m_A + m_B} v_A^2 \frac{1}{\frac{1}{2} m_A v_A^2} = \frac{m_B}{m_A + m_B}$$

3/259

$$v = \sqrt{2gH} = \sqrt{2 \times 32.2 \times 3}$$

$$= 13.90 \text{ ft/sec}$$

At impact $\sum F_x = 0$ so $\Delta G_x = 0$ so

$$v' \cos(\beta + 10^\circ) - 13.90 \sin 10^\circ = 0 \quad \text{--- (a)}$$

$$e = 0.7 = \frac{v' \sin(\beta + 10^\circ)}{13.90 \cos 10^\circ} \quad \text{--- (b)}$$

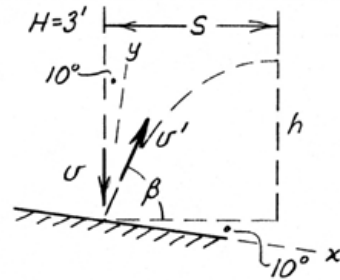
Combine & get $\tan(\beta + 10^\circ) = 3.97$

$$\beta + 10^\circ = 75.9^\circ, \quad \beta = 65.9^\circ$$

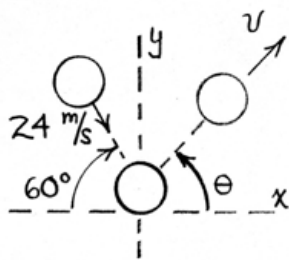
$$\text{From Eq. (a)} \quad v' = \frac{13.90 \sin 10^\circ}{\cos 75.9^\circ} = 9.88 \text{ ft/sec}$$

$$\text{From Sample Prob. 2/6, } h = \frac{v'^2 \sin^2 \beta}{2g} = \frac{9.88^2 \sin^2 65.9^\circ}{2 \times 32.2} = 1.263 \text{ ft}$$

$$s = \frac{v'^2 \sin 2\beta}{2g} = \frac{9.88^2 \sin 131.7^\circ}{2 \times 32.2} = 1.132 \text{ ft}$$



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During impact $\Sigma F_x = 0$ so no change in x velocity component.

$$v \cos \theta = 24(0.5) = 12 \text{ m/s}$$

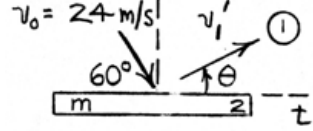
$$\text{In y-dir.}, e = \frac{v \sin \theta}{24 \cos 30^\circ} = 0.8$$

$$\tan \theta = \frac{16.63}{12} = 1.386$$

$$\theta = 54.2^\circ$$

$$v = \frac{12}{\cos 54.2^\circ} = \underline{20.5 \text{ m/s}}$$

3/26/1



t-momentum conserved:

$$\text{Ball: } m_1 v_{1t} = m_1 v'_{1t}$$

$$v'_{1t} = v_{1t} = 24 \cos 60^\circ = 12 \text{ m/s}$$

$$\text{Plate: } m_2 v_{2t} = m_2 v'_{2t}, \quad v'_{2t} = v_{2t} = 0$$

n-momentum:

$$m_1 v_{1n} + m_2 v_{2n} = m_1 v'_{1n} + m_2 v'_{2n}$$

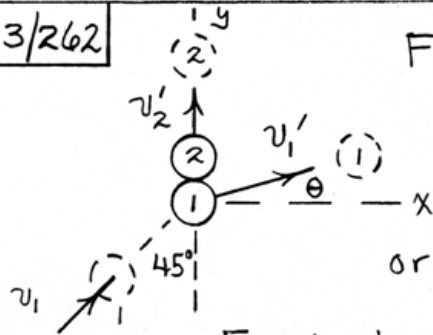
$$-24 \sin 60^\circ = v'_{1n} + v'_{2n}$$

$$\text{Restitution: } e = \frac{v'_{2n} - v'_{1n}}{v_{1n} - v_{2n}}, \quad 0.8 = \frac{v'_{2n} - v'_{1n}}{-24 \sin 60^\circ - 0}$$

$$\text{Solve to find } v'_{1n} = -2.08 \text{ m/s}, \quad v'_{2n} = -18.71 \text{ m/s}$$

$$v'_1 = \sqrt{v'^2_{1t} + v'^2_{1n}} = 12.20 \text{ m/s}, \quad \theta = \tan^{-1}\left(\frac{v'_{1n}}{v'_{1t}}\right) = -9.83^\circ$$

3/262



For 1 & 2 together :

$$mv_1 \cos 45^\circ = mv_1' \sin \theta + mv_2'$$

$$\text{or } v_1' \sin \theta + v_2' = v_1 / \sqrt{2} \quad (1)$$

$$\text{For 1 alone : } mv_1 \sin 45^\circ = mv_1' \cos \theta$$

$$\text{or } v_1' \cos \theta = v_1 / \sqrt{2} \quad (2)$$

$$\text{Restitution : } v_2' - v_1' \sin \theta = e v_1 \cos 45^\circ$$

$$\text{or } v_2' - v_1' \sin \theta = 0.9 v_1 / \sqrt{2} \quad (3)$$

$$(1) \div (2) : v_1' \sin \theta = 0.0354 v_1 ; \text{ Divide by (2) : } \theta = 2.86^\circ$$

$$n = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{\frac{1}{2} m v_2'^2 + \frac{1}{2} m v_1'^2}{\frac{1}{2} m v_1^2}$$

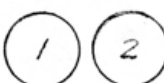
$$= 1 - \frac{v_2'^2 + v_1'^2}{v_1^2}, \text{ where } v_1' = 0.708 v_1, v_2' = 0.672 v_1$$

$$\text{So } n = 1 - \frac{0.672^2 + 0.708^2}{1} = \underline{0.0475}$$

$$\boxed{3/263} \quad v_1 \rightarrow \quad \Delta G = 0; \quad m v_1 = -m v_1' + m v_2'$$

Before 

$$v_2' = v_1 + v_1'$$

After 

$$e = \frac{v_2' + v_1'}{v_1}, \quad v_1' = e v_1 - v_2'$$

$$v_1' \leftarrow \quad \rightarrow v_2' \quad \text{Combine \& get}$$

$$v_2' = v_1 + e v_1 - v_2'$$

$$\text{or } v_2' = \frac{1+e}{2} v_1$$

It follows that $v_3' = \frac{1+e}{2} v_2' = \left(\frac{1+e}{2}\right)^2 v_1$

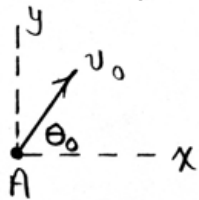
$$v_4' = \frac{1+e}{2} v_3' = \left(\frac{1+e}{2}\right)^3 v_1$$

$$\vdots$$

so $v_n = \left(\frac{1+e}{2}\right)^{n-1} v_1$

3/264

Let the launch conditions at A be speed v_0 , launch angle θ_0 :



The range L_1 is

$$L_1 = \frac{2v_0^2 \sin \theta_0 \cos \theta_0}{g}$$

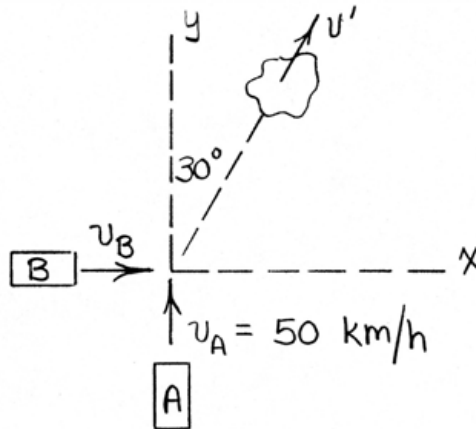
and the velocity components coming into B are $\begin{cases} v_x = v_0 \cos \theta_0 \\ v_y = -v_0 \sin \theta_0 \end{cases}$

The velocity components after impact at B

are $v_x = v_0 \cos \theta_0$, $v_y = e v_0 \sin \theta_0$, which result in the range $L_2 = \frac{2e v_0^2 \sin \theta_0 \cos \theta_0}{g}$

So $L_2 = e L_1$.

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$$G_{1x} = G_{2x}: m_B v_B + 0 = (m_A + m_B) v' \sin 30^\circ$$

$$1600 v_B = 2800 v' \left(\frac{1}{2} \right) \quad (1)$$

$$G_{1y} = G_{2y}: m_A v_A + 0 = (m_A + m_B) v' \cos 30^\circ$$

$$1200 (50) = 2800 v' (0.866) \quad (2)$$

$$\text{From (2): } v' = 24.7 \text{ km/h}$$

$$\text{From (1): } \underline{v_B = 21.7 \text{ km/h}}$$

3/266 System linear momentum is conserved:

$$m_s \underline{v}_s + m_m \underline{v}_m = (m_s + m_m) \underline{v}'$$

$$(1000)(2000 \underline{i}) + 100(-v_m \underline{j}) = (1000 + 100)[v'(\cos 20^\circ \underline{i} - \sin 20^\circ \underline{j})]$$

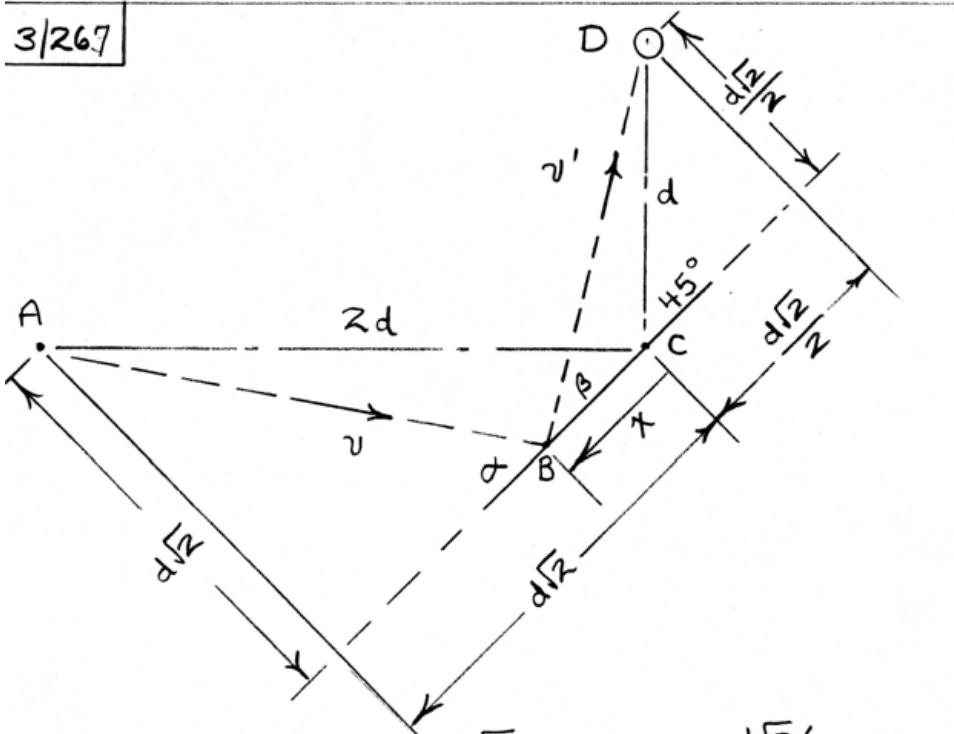
Equating coefficients:

$$\underline{i} : (1000)(2000) = 1100 \cos 20^\circ v'$$
$$v' = 1935 \text{ m/s}$$

$$\underline{j} : -100 v_m = -(1100)(1935) \sin 20^\circ$$

$$\underline{v}_m = \underline{7280 \text{ m/s}}$$

3/267 D 



$$\tan \alpha = \frac{1}{e} \tan \beta : \frac{\frac{d\sqrt{2}}{2}}{d\sqrt{2} - x} = \frac{1}{0.8} \frac{\frac{d\sqrt{2}}{2}}{\left(\frac{d\sqrt{2}}{2} + x\right)}$$

Solve for $x = 0.1088d$

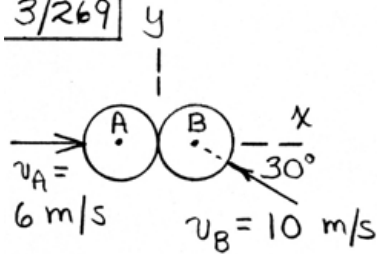
3/268 Let v_s and v_b stand for rebound velocities from steel and brass plates.

$$\text{Impact speed} = \sqrt{2gh} = \sqrt{2(9.81)(0.15)} = 1.716 \text{ m/s}$$

$$\left. \begin{array}{l} 0.6 = \frac{v_s}{1.716}, v_s = 1.029 \text{ m/s} \\ 0.4 = \frac{v_b}{1.716}, v_b = 0.686 \text{ m/s} \end{array} \right\} \omega = \frac{1.029 - 0.686}{0.60} = 0.572 \text{ rad/s}$$

CCW

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$$v_{Ay}' = v_{Ay} = 0$$

$$v_{By}' = v_{By} = 10 \sin 30^\circ = 5 \text{ m/s}$$

$$m_A v_{Ax} + m_B v_{Bx} = m_A v_{Ax}' + m_B v_{Bx}'$$

$$6 - 10 \cos 30^\circ = v_{Ax}' + v_{Bx}' \quad (1)$$

$$e = \frac{v_{Bx}' - v_{Ax}'}{v_{Ax} - v_{Bx}} : 0.75 = \frac{v_{Bx}' - v_{Ax}'}{6 - (-10 \cos 30^\circ)} \quad (2)$$

Solve Eqs. (1) & (2) :

$$\begin{cases} v_{Ax}' = -6.83 \text{ m/s} \\ v_{Bx}' = 4.17 \text{ m/s} \end{cases}$$

Magnitudes and
directions $\nearrow \theta - x$

$$v_A' = 6.83 \frac{\text{m}}{\text{s}} @ \theta_A = 180^\circ$$

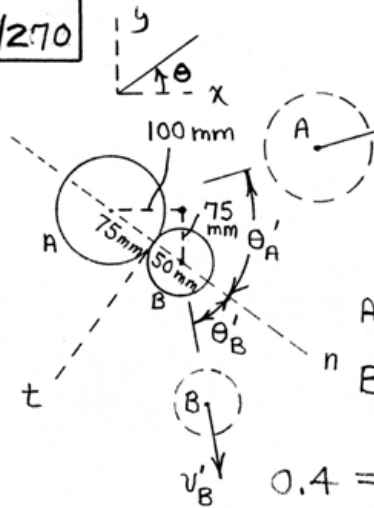
$$v_B' = 6.51 \frac{\text{m}}{\text{s}} @ \theta_B = 50.2^\circ$$

$$\text{Initial: } T_1 = \frac{1}{2} m (6^2 + 10^2) = 68 \text{ m}$$

$$\text{Final: } T_2 = \frac{1}{2} m (6.83^2 + 6.51^2) = 44.5 \text{ m}$$

$$n = \frac{68 - 44.5}{68} (100\%) = \underline{34.6\%}$$

3/270



For system, $\Delta G_n = 0$

$$23v'_A \cos \theta'_A + 4v'_B \cos \theta'_B = 23(4)\left(\frac{4}{5}\right) - 4(12)\left(\frac{4}{5}\right) \quad \text{--- (a)}$$

For each sphere, $\Delta G_t = 0$

$$A: 4\left(\frac{3}{5}\right) = v'_A \sin \theta'_A \quad \text{--- (b)}$$

$$B: 12\left(\frac{3}{5}\right) = v'_B \sin \theta'_B \quad \text{--- (c)}$$

$$0.4 = \frac{v'_B \cos \theta'_B - v'_A \cos \theta'_A}{12\left(\frac{4}{5}\right) + 4\left(\frac{4}{5}\right)} \quad \text{--- (d)}$$

Solve (a), (b), (c), (d): $v'_A = 2.46 \text{ m/s}$, $\theta'_A = 77.2^\circ$

$$v'_B = 9.16 \text{ m/s}, \theta'_B = 51.8^\circ$$

Relative to the +x-axis, the directions of the

$$\text{final velocities are } \begin{cases} \theta_A = 77.2 - 36.9 = 40.3^\circ \\ \theta_B = -51.8 - 36.9 = -88.7^\circ \end{cases}$$

3/27/11

Diagram showing two spheres, A (10 kg) and B (2 kg), colliding. Sphere A is moving at $v_A = 3 \text{ m/s}$ at a 45° angle. Sphere B is moving at $v_B = 12 \text{ m/s}$ at a 30° angle. The collision is analyzed using a coordinate system where the x -axis is at 20° and the y -axis is at 30° . The normal direction is perpendicular to the line of impact, and the tangential direction is parallel to it.

Initial velocities in the normal direction:

$$v_{A_t}' = v_{A_t} = 3 \sin 45^\circ = 2.12 \frac{\text{m}}{\text{s}}$$

$$v_{B_t}' = v_{B_t} = -12 \sin 30^\circ = -6 \frac{\text{m}}{\text{s}}$$

Conservation of momentum in the normal direction:

$$m_A v_{A_n} + m_B v_{B_n} = m_A v_{A_n}' + m_B v_{B_n}'$$

$$10(3 \cos 45^\circ) + 2(-12 \cos 30^\circ) = 10 v_{A_n}' + 2 v_{B_n}' \quad (1)$$

Coefficient of restitution:

$$e = \frac{v_{B_n}' - v_{A_n}'}{v_{A_n} - v_{B_n}} : 0.5 = \frac{v_{B_n}' - v_{A_n}'}{3 \cos 45^\circ - (-12 \cos 30^\circ)} \quad (2)$$

Solve (1) & (2) : $v_{A_n}' = -1.007 \text{ m/s}$, $v_{B_n}' = 5.25 \text{ m/s}$

Then:

$$\begin{cases} v_{A_x}' = -2.12 \sin 20^\circ - 1.007 \cos 20^\circ = -1.672 \text{ m/s} \\ v_{A_y}' = 2.12 \cos 20^\circ - 1.007 \sin 20^\circ = 1.649 \text{ m/s} \\ v_{B_x}' = -(-6 \sin 20^\circ) + 5.25 \cos 20^\circ = 6.99 \text{ m/s} \\ v_{B_y}' = -6 \cos 20^\circ + 5.25 \sin 20^\circ = -3.84 \text{ m/s} \end{cases}$$

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For system, $\Delta G_y = 0$ so

$$v_2' \uparrow \quad v_2 = 4 \frac{\text{ft}}{\text{sec}} \quad [m(6 \cos 30^\circ) - m(4)] - [m(-v_1' \sin \theta_1') + m v_2'] = 0$$

$$e = 0.60 \quad \text{or} \quad v_2' - v_1' \sin \theta_1' = 1.196 \quad (1)$$

$$v_1 = 6 \frac{\text{ft}}{\text{sec}} / 30^\circ \quad v_1' \cos \theta_1' = 6(\frac{1}{2}), \quad v_{2x}' = 0$$

$$\text{Also} \quad e = 0.60 = \frac{v_2' + v_1' \sin \theta_1'}{4 + 6 \cos 30^\circ}, \quad v_2' + v_1' \sin \theta_1' = 5.518 \quad (2)$$

$$\text{Combine (1) \& (2) \& get } 2v_2' = 1.196 + 5.518, \quad v_2' = 3.36 \frac{\text{ft}}{\text{sec}}$$

$$\& \quad v_1' \sin \theta_1' = 2.16; \text{ Divide by } v_1' \cos \theta_1' = 3$$

$$\& \text{ get } \theta_1' = \tan^{-1} 0.7203 = 35.8^\circ \& \quad v_1' = \frac{3}{\cos 35.8^\circ} = 3.70 \frac{\text{ft}}{\text{sec}}$$

$$\text{Initial kinetic energy} = \frac{1}{2} m (6^2 + 4^2) = \frac{1}{2} m (52)$$

$$\text{Final} \quad \quad \quad = \frac{1}{2} m (3.70^2 + 3.36^2) = \frac{1}{2} m (24.9)$$

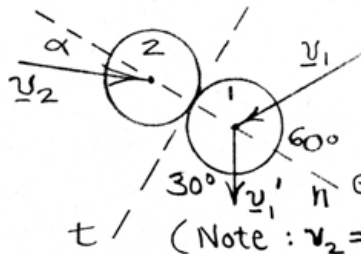
$$\% \text{ loss} = \frac{52 - 24.9}{52} = 0.520 \text{ or } \underline{52.0\%}$$

3/273

Conservation of n-momentum :

$$m(-v_1 \cos 60^\circ) + m(v_2 \cos \alpha) =$$

$$mv'_{1n} + mv'_{2n} \quad (a)$$



Restitution :

$$e = 0.8 = \frac{v'_{2n} - v'_{1n}}{-v_1 \cos 60^\circ - v_2 \cos \alpha} \quad (b)$$

(Note : $v_2 = v_1$)

Simultaneous solution of Eqs. (a) and (b) :

$$v'_{1n} = v_1 [0.9 \cos \alpha - 0.05]$$

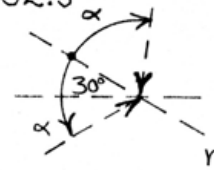
$$v'_{1t} = v_{1t} = v_1 \sin 60^\circ = \frac{\sqrt{3}}{2} v_1$$

$$\tan 30^\circ = \frac{v'_{1n}}{v'_{1t}} = \frac{v_1 [0.9 \cos \alpha - 0.05]}{\frac{\sqrt{3}}{2} v_1}$$

$$\text{Solving, } \cos \alpha = 0.611, \quad \alpha = \pm 52.3^\circ$$

$$\text{So } \theta = 30^\circ + 52.3^\circ = 82.3^\circ$$

$$\text{or } \theta = 30^\circ - 52.3^\circ = -22.3^\circ$$



$$\alpha = \tan^{-1} \frac{10.268}{13.144}$$

$$= 38.0^\circ$$

$$2 \sin 30^\circ = 1''$$

$$2 \cos 30^\circ = 1.732''$$

$$28 - 13.856 - 1 = 13.144$$

$$12 - 1.732 = 10.268''$$

$$v_1 \sin 68.0^\circ = (v_1')_n + (v_2')_n$$

$$0.9 = \frac{(v_2')_n - (v_1')_n}{v_1 \sin 68.0^\circ - 0}$$

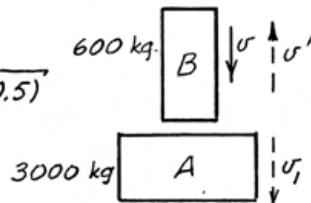
Also, $(v_1')_t = (v_1)_t = v_1 \cos 68.0^\circ = 0.375 v_1$

$$\beta = 30^\circ - \theta_1' = 22.95^\circ, \tan \beta = \frac{h}{13.856 - 1 + 1} = 0.423$$

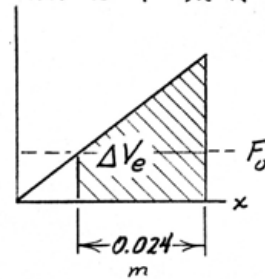
$$h = 5.87". \text{ Then } x = 24 - 1.732 - 5.87 = 16.40 \text{ in.}$$

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Hammer: $v = \sqrt{2gh}$
 $= \sqrt{2(9.81)(0.5)}$
 $= 3.13 \text{ m/s}$



$F = kx = 2.8(10^6)x \text{ N}$



For anvil: $\Delta T + \Delta V_e + \Delta V_g = 0$

$\Delta T = 0 - \frac{1}{2} 3000 v_1'^2 \text{ J}$

$\Delta V_e = \frac{1}{2} (2.8 \times 10^6) (0.024)^2$
 $+ 29.4 (10^3) (0.024) \text{ J}$

$F_0 = mg = 3000(9.81) = 29.4(10^3) \text{ N}$

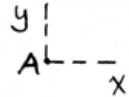
$\Delta V_g = -29.4 (10^3) (0.024) \text{ J}$

Substitute & get $-\frac{1}{2} (3000) v_1'^2 + \frac{1}{2} (2.8 \times 10^6) (0.024)^2 = 0$,
 $v_1' = 0.733 \text{ m/s}$

Hammer & anvil impact: $\Delta G_x = 0: 3000(0.733) - 600 v' - 600(3.13) = 0$
 $v' = 0.534 \text{ m/s}$

Hammer after impact: $v' = \sqrt{2gh}$, $h = \frac{0.534^2}{2 \times 9.81} = 0.01453 \text{ m}$
 or $h = 14.53 \text{ mm}$

$e = \frac{0.534 - (-0.733)}{3.13} = 0.405$

3/276 $v_{xA} = 50 \cos \alpha, v_{yA} = 50 \sin \alpha$ 

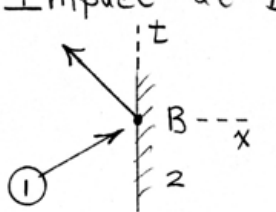
$$t_{AB} = \frac{10}{v_{xA}} = \frac{10}{50 \cos \alpha} = \frac{1}{5 \cos \alpha}$$

$$v_{xB} = v_{xA} = 50 \cos \alpha$$

$$v_{yB} = v_{yA} - gt = 50 \sin \alpha - \frac{g}{5 \cos \alpha}$$

$$y_B = y_A + v_{yA}t - \frac{1}{2}gt^2 = 0 + (50 \sin \alpha)\left(\frac{1}{5 \cos \alpha}\right) - \frac{g}{2}\left(\frac{1}{25 \cos^2 \alpha}\right) = 10 \tan \alpha - \frac{g}{50 \cos^2 \alpha}$$

Impact at B:



$$v'_{yB} = v_{yB} = 50 \sin \alpha - \frac{g}{5 \cos \alpha}$$

$$e = \frac{v'_{2x} - v'_{1x}}{v_{1x} - v_{2x}} = \frac{0 - v'_{1x}}{50 \cos \alpha - 0} = 0.5$$

$$v'_{1x} = -25 \cos \alpha$$

$$t_{BA} = \frac{10}{25 \cos \alpha} = \frac{2}{5 \cos \alpha}$$

$$y_A = y_B + v'_{yB}t - \frac{1}{2}gt^2$$

$$0 = \left(10 \tan \alpha - \frac{g}{50 \cos^2 \alpha}\right) + \left(50 \sin \alpha - \frac{g}{5 \cos \alpha}\right)\left(\frac{2}{5 \cos \alpha}\right) - \frac{g}{2}\left(\frac{2}{5 \cos \alpha}\right)^2$$

Collect terms :

$$30 \tan \alpha - \frac{9g}{50} \frac{1}{\cos^2 \alpha} = 0$$

Use $\frac{1}{\cos^2 \alpha} = (\tan^2 \alpha + 1)$ to obtain

$$5.796 \tan^2 \alpha - 30 \tan \alpha + 5.796 = 0$$

Quadratic solution : $\tan \alpha = 0.201, 4.97$
 $\Rightarrow \underline{\alpha = 11.37^\circ \text{ or } 78.6^\circ}$

3/277

From A to B

$$\Delta T + \Delta V_g = 0: \frac{1}{2} m v_1^2 - mgh = 0,$$

$$v_1 = \sqrt{2gh}$$

During impact

$$\Sigma F_{x'} \geq 0: R - mg \cos \theta + N \cos \theta \geq 0$$

$$R < mg \cos \theta$$

During small time of impact, impulses
of R & mg are negligible

$$\text{so } \int \Sigma F_x dt \approx 0 \text{ \& } \Delta G_x \approx 0:$$

$$m v_1 \cos \theta = m v_2 \cos \beta \quad (1)$$

$$\int \Sigma F_y dt \neq 0 \text{ \& } v_2 \sin \beta = e v_1 \sin \theta \quad (2)$$

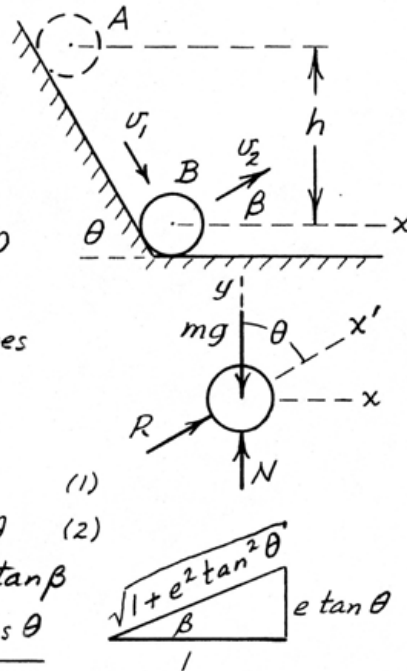
Divide (2) by (1) & get $e \tan \theta = \tan \beta$

$$v_x = v_2 \cos \beta = v_1 \cos \theta, \quad v_x = \sqrt{2gh} \cos \theta$$

$$n = \frac{|\Delta T|}{T} = \frac{\frac{1}{2} m v_1^2 - \frac{1}{2} m v_2^2}{\frac{1}{2} m v_1^2} = 1 - \frac{v_2^2}{v_1^2} = 1 - \frac{(e v_1 \sin \theta / \sin \beta)^2}{v_1^2}$$

$$n = 1 - \frac{e^2 \sin^2 \theta}{\sin^2 \beta} = 1 - (\cos^2 \theta + e^2 \sin^2 \theta) \text{ where } \sin^2 \beta = \frac{e^2 \tan^2 \theta}{1 + e^2 \tan^2 \theta}$$

For a rounded corner of radius greater than that of the sphere, there would
be no discontinuity in the magnitude of the velocity and, hence, no impact.



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$\Delta G_x = 0; 2(10) + 0 = -2v' \cos \theta + 10v_o$ (For system) (1)
 $\Delta G_t = 0$ (for sphere)
 $2(10 \sin 30^\circ) = 2v' \sin(\theta - 30^\circ)$ (2)
 Coefficient of restitution applies to velocity components in n-dir.
 $0.6 = \frac{v_o \sin 60^\circ + v' \cos(\theta - 30^\circ)}{10 \cos 30^\circ + 0}$ (3)

Eq. (3) is $5.196 = 0.866 v_o + 0.866 v' \cos \theta + 0.5 v' \sin \theta$
 Sub. Eq. (1) to eliminate v_o & get
 $5.196 = 0.866(2 + 0.2 v' \cos \theta) + 0.866 v' \cos \theta + 0.5 v' \sin \theta$
 or $1.039 v' \cos \theta + 0.5 v' \sin \theta = 3.464$ (4)
 Eq. (2) becomes $0.866 v' \sin \theta - 0.5 v' \cos \theta = 5$ (5)
 Solve (4) & (5) & get $v' = 6.04 \text{ m/s}, \theta = 85.9^\circ$
 From Eq. (1) $v_o = 2.087 \text{ m/s}$
 For carriage $\Delta T + \Delta V_e = 0; -\frac{1}{2} \cdot 10 (2.087)^2 + \frac{1}{2} \cdot 1600 \delta^2 = 0$
 $\delta^2 = 0.02722, \delta = 0.1650 \text{ m}$ or $\delta = 165.0 \text{ mm}$

3/279

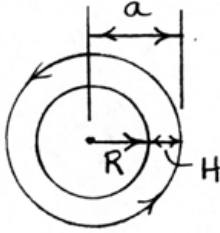
$$v = \sqrt{\frac{G m_s}{r}} = \sqrt{\frac{(3.439 \times 10^{-8})(333,000)(4.095 \times 10^{23})}{(93 \times 10^6)(5280)}}$$

$$= 97,725 \text{ ft/sec} = \underline{18.51 \text{ mi/sec}}$$

3/280 For a circular orbit, $r_{\min} = r_{\max}$
and $a = R+h$, so Eq. 3/48 becomes

$$v = R \sqrt{\frac{g}{R+h}} = 6371(10^3) \sqrt{\frac{9.825}{(6371+590)10^3}}$$
$$= \underline{7569 \text{ m/s}} \text{ or } \underline{27250 \text{ km/h}}$$

3/281

Eq. 3/47 with $r = a = R + H$:

$$v^2 = 2gR^2 \left(\frac{1}{a} - \frac{1}{2a} \right) = \frac{gR^2}{R+H}$$

$$= \frac{1.62 (3476/2)^2}{\frac{3476}{2} + 80} (1000)$$

$$v = \underline{1641 \text{ m/s}} \quad \text{or} \quad \underline{5910 \text{ km/h}}$$

$$\begin{aligned}
 & \frac{3/282}{\text{Moon } m} \quad \frac{F_s}{F_e} = \frac{G m_s m / d_{m-s}^2}{G m_e m / d_{m-e}^2} \\
 & \begin{array}{c} \leftarrow \text{O} \rightarrow \\ F_e \quad F_s \end{array} = \left(\frac{d_{m-e}}{d_{m-s}} \right)^2 \frac{m_s}{m_e} \\
 & = \left(\frac{384\,398}{149.6(10^6) - 384\,398} \right)^2 333\,000 = 2.21
 \end{aligned}$$

Therefore, the acceleration of the moon is toward the sun, and thus the path is concave toward the sun!

3/283 From Eq. 3/47 for a circular orbit of altitude H & with $a=r=R+H$

$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2r}\right) = gR^2/(R+H), \quad v = R\sqrt{g/(R+H)}$$

$$v_{\text{escape}}^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{\infty}\right) = 2gR^2/(R+H), \quad v = R\sqrt{2g/(R+H)}$$

$$\Delta v = v_{\text{escape}} - v = R\sqrt{\frac{g}{R+H}} (\sqrt{2} - 1) = (\sqrt{2} - 1)v = 0.414 v$$

For $R = 3959 \text{ mi}$, $g = 32.23 \text{ ft/sec}^2$, $H = 200 \text{ mi}$,

$$v = 3959 \sqrt{\frac{32.23/5280}{3959 + 200}} = 4.80 \text{ mi/sec}, \quad \Delta v = 0.414 (4.80) = \underline{1.987 \frac{\text{mi}}{\text{sec}}}$$

$$3/284 \quad r_{\min} = 6371 + 240 = 6611 \text{ km}$$

$$r_{\max} = 6371 + 400 = 6771 \text{ km}$$

$$\text{From Eq. 3/43} \quad \frac{r_{\min}}{r_{\max}} = \frac{1-e}{1+e}$$

$$\text{So } (1+e)6611 = (1-e)6771, \quad \underline{e = 0.01196}$$

$$\text{From Eq. 3/44 with } a = \frac{1}{2}(r_{\max} + r_{\min}) = 6691 \text{ km}$$

$$T = 2\pi \frac{(6691 \times 10^3)^{3/2}}{(6371 \times 10^3) \sqrt{9.824}} = \underline{5446 \text{ s or}}$$

$$\underline{T = 1 \text{ h } 30 \text{ min } 46 \text{ s}}$$

$$\underline{3/285} \quad F = G \frac{m_1 m_2}{r^2}$$

$$= 6.673(10^{-11}) \frac{1.490(10^{23})(1.900)(10^{27})}{(1.070 \times 10^9)^2} = \underline{16.50(10^{21}) \text{ N}}$$

$$F = mr\omega^2, \quad \omega = \sqrt{\frac{F}{mr}} = \sqrt{\frac{16.50(10^{21})}{1.490(10^{23})/1.070(10^9)}}$$

$$= 1.017(10^{-5}) \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega} = \frac{2\pi}{1.017(10^{-5})} = \underline{6.18(10^5) \text{ s}} \text{ or } \underline{7.17 \text{ days}}$$

$$a_n = \frac{F}{m_1} = \frac{16.50(10^{21})}{1.490(10^{23})} = \underline{110.7(10^{-3}) \text{ m/s}^2}$$

$$\boxed{3/286} \quad r_{\min} = 2R, \quad r_{\max} = 3R$$

$$a = \frac{r_{\min} + r_{\max}}{2} = 2.5R$$

$$v_p = R \sqrt{\frac{g}{a}} \sqrt{\frac{r_{\max}}{r_{\min}}} = R \sqrt{\frac{g}{2.5R}} \sqrt{\frac{3R}{2R}} = \sqrt{\frac{3gR}{5}}$$

The velocity in the original circular orbit is

$$v_c = R \sqrt{\frac{g}{a}} = R \sqrt{\frac{g}{2.5R}} = \sqrt{\frac{1}{2}gR}$$

$$\Delta v = v_p - v_c = \sqrt{gR} \left(\sqrt{\frac{3}{5}} - \sqrt{\frac{1}{2}} \right) = 0.0675 \sqrt{gR}$$

$$\begin{aligned} \text{Numbers : } \Delta v &= 0.0675 \sqrt{9.825(6371)(1000)} \\ &= \underline{534 \text{ m/s}} \end{aligned}$$

(Δv to occur opposite B)

$$\underline{3/287} \quad r=a = 6371 + 300 = 6671 \text{ km} = 6.671(10^6) \text{ m}$$

$$\begin{aligned} \tau &= \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi (6.671 \times 10^6)^{3/2}}{6.371 \times 10^6 \sqrt{9.825}} \\ &= 5421 \text{ s} \end{aligned}$$

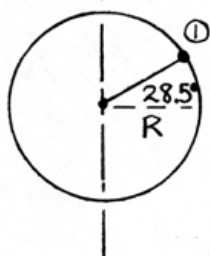
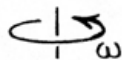
Speed of ground point on equator

$$v_e = R_e \omega_e = (6378)(7.292 \times 10^{-5}) = 0.4651 \text{ km/s}$$

$$\begin{aligned} \text{Required distance } d &= v_e \tau = (0.4651)(5421) \\ &= \underline{2520 \text{ km}} \end{aligned}$$

3/288

① On ground, speed $v_1 = (R \cos 28.5^\circ) \omega$



$$= 6371(1000) \cos 28.5^\circ (0.7292 \cdot 10^{-4})$$

$$= 408 \text{ m/s}$$

$$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} (80000) 408^2$$

$$= 6.67 (10^9) \text{ J}$$

$$V_1 = -\frac{mgR^2}{R} = -80000 (9.825) (6371 \cdot 1000) = -5.01 (10^{12}) \text{ J}$$

② In circular orbit: $v_2 = R \sqrt{\frac{g}{r}}$

$$= 6371 (10^3) \sqrt{\frac{9.825}{(6371+300)(1000)}} = 7.73 (10^3) \text{ m/s}$$

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} 80000 [7.73 (10^3)]^2 = 2.39 (10^{12}) \text{ J}$$

$$V_2 = -\frac{mgR^2}{r} = -\frac{80000 (9.825) (6371 \cdot 1000)^2}{(6371+300) \cdot 1000} = -4.78 (10^{12}) \text{ J}$$

$$\Delta E = T_2 + V_2 - (T_1 + V_1) = \underline{2.61 (10^{12}) \text{ J}}$$

$$\underline{3/289} \quad (a) \quad v = R \sqrt{\frac{g}{r}} = 6371(1000) \sqrt{\frac{9.825}{(6371+637)(1000)}} \\ = \underline{7544 \text{ m/s}}$$

$$(b) \text{ From } r_{\min} = a(1-e), \quad a = \frac{r_{\min}}{1-e} = \frac{1.1(6371)}{1-0.1} \\ = 7787 \text{ km}$$

$$v_p = R \sqrt{\frac{g}{a}} \sqrt{\frac{1+e}{1-e}} = 6371(1000) \sqrt{\frac{9.825}{7787(1000)}} \sqrt{\frac{1+0.1}{1-0.1}} \\ = \underline{7912 \text{ m/s} = v}$$

$$(c) \quad a = \frac{r_{\min}}{1-e} = \frac{1.1(6371)}{1-0.9} = 70081 \text{ km}$$

$$v_p = 6371(1000) \sqrt{\frac{9.825}{70081(1000)}} \sqrt{\frac{1+0.9}{1-0.9}} \\ = \underline{10398 \text{ m/s} = v}$$

$$(d) \text{ Eq. 3/47 with } a \rightarrow \infty: \quad v = R \sqrt{\frac{2g}{r}}$$

This is $\sqrt{2}$ times answer for part (a), so

$$v = \sqrt{2} (7544) = \underline{10668 \text{ m/s}}$$

$$\frac{3}{290} v_A = R \sqrt{\frac{g}{r}} = (3959)(5280) \sqrt{\frac{32.23}{(4759)(5280)}}$$

$$= 23,676 \text{ ft/sec}$$

$$v_B = R \sqrt{\frac{g}{a}} \sqrt{\frac{r_{\max}}{r_{\min}}}$$

$$= (3959)(5280) \sqrt{\frac{32.23}{\frac{(2(3959)+1800)(5280)}{2}}} \sqrt{\frac{4959}{4759}}$$

$$= 23,917 \text{ ft/sec}$$

Momentum conservation during impact:

$$m_A v_A + m_B v_B = (m_A + m_B) v_C. \text{ But } m_A = m_B, \text{ so}$$

$$v_C = \frac{1}{2} (v_A + v_B) = 23,796 \text{ ft/sec}$$

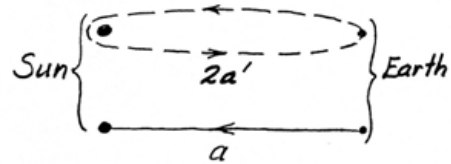
$$\text{From } v_p = R \sqrt{\frac{g}{a}} \sqrt{\frac{r_{\max}}{r_{\min}}}$$

$$r_{\max} = \frac{r_{\min}}{\left(\frac{2gR^2}{v_p^2 r_{\min}} - 1 \right)} = \frac{2.5652 \times 10^7 \text{ ft}}{(4858 \text{ mi})}$$

$$h_{\max} = r_{\max} - R = 4858 - 3959 = \underline{899 \text{ mi}}$$

3/291

Radius of actual orbit around the sun is a , which is the major axis $2a'$ of the degenerate ellipse.



R = radius of sun

g = gravitational accel. on surface of sun

Orbital period Eq. 3/44

For actual orbit $\tau = 2\pi \frac{a^{3/2}}{R\sqrt{g}}$

For degenerate ellipse $\tau' = 2\pi \frac{(a/2)^{3/2}}{R\sqrt{g}}$

so $\frac{\tau'}{\tau} = \frac{(\frac{1}{2})^{3/2}}{1}$

But time t to fall is $t = \frac{1}{2}\tau' = \frac{1}{2}\left(\frac{1}{2}\right)^{3/2}\tau = \frac{1}{4\sqrt{2}}365.26$
 $= \underline{64.6 \text{ days}}$

3/292 The apogee speed at C is

$$v_a = R \sqrt{\frac{g}{a}} \sqrt{\frac{r_{\min}}{r_{\max}}}$$

$$= 6371(10^3) \sqrt{\frac{9.825}{(2 \cdot 6371 + 240 + 320)1000/2}} \sqrt{\frac{6371 + 240}{6371 + 320}}$$
$$= 7697 \text{ m/s}$$

The circular orbit speed at $h = 320 \text{ km}$ is

$$v_{\text{circ}} = R \sqrt{\frac{g}{r_{\max}}} = 7720 \text{ m/s}$$

$$\Delta v = v_{\text{circ}} - v_a = 7720 - 7697 = 23.25 \text{ m/s}$$

$$F \Delta t = m \Delta v: 2(30000)(\Delta t) = 85000(23.25)$$

$$\Delta t = 32.9 \text{ s}$$

The burn to increase speed is at C.

3/293 The linear impulses from drag and from the thruster must be equal in magnitude, or

$$Dt = \sum T t_{\text{burn}}$$

$$t = 10\tau, \text{ where } \tau = 2\pi \frac{a^{3/2}}{R\sqrt{g}}$$

$$\text{or } \tau = 2\pi \frac{(6.571 \times 10^6)^{3/2}}{6.371 \times 10^6 \sqrt{9.825}} = 5300 \text{ s}$$

$$t = 10\tau = 53,000 \text{ s}$$

$$D = \frac{\sum T t_{\text{burn}}}{t} = \frac{2(300)}{53,000} = \underline{0.01132 \text{ N}}$$

3/294 From $r_{\min} = a(1-e)$:

$$7959 = [4000 + 3959 + 16,000](1-e), e = 0.668$$

$$b = a\sqrt{1-e^2} = 23,959\sqrt{1-0.668^2} = 17,833 \text{ mi}$$

$$\text{At B, } r = \sqrt{16,000^2 + 17,833^2} = 23,959 \text{ mi}$$

$$\begin{aligned} v_B^2 &= 2gR^2 \left(\frac{1}{r} - \frac{1}{2a} \right) \\ &= 2(32.23)3959^2(5280) \left[\frac{1}{23,959} - \frac{1}{2(23,959)} \right] \end{aligned}$$

$$\underline{v_B = 10,551 \text{ ft/sec}}$$

$$\underline{3/295} \quad \text{Eq. 3/44: } \tau_f = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi a^{3/2}}{\sqrt{G m_A}}$$

$$= 2\pi \frac{[200(10^9)]^{3/2}}{\sqrt{6.673(10^{-11}) 10^{31}}} = \underline{21,760,000 \text{ s}}$$

$$\text{Eq. 3/49b: } \tau_{nf} = \frac{2\pi a^{3/2}}{\sqrt{G(m_A + m_B)}}$$

$$= 2\pi \frac{[200(10^9)]^{3/2}}{\sqrt{6.673(10^{-11})(10^{31} + 10^{30})}} = \underline{20,740,000 \text{ s}}$$

(-4.7 percent difference)

3/29/6 Speed in circular orbit is

$$v = R \sqrt{\frac{g}{a}} = (3959)(5280) \sqrt{\frac{32.23}{(4459)(5280)}}$$
$$= 24,458 \text{ ft/sec}$$

Time required for B to return to C's

burn position: $t = \frac{2\pi r - 1000(5280)}{v}$

$$= 5832 \text{ s}$$

$$T = \frac{2\pi a^{3/2}}{R\sqrt{g}}, \quad a = \left(\frac{TR\sqrt{g}}{2\pi} \right)^{2/3}$$

$$a = \left(\frac{(5832)(3959)(5280)\sqrt{32.23}}{2\pi} \right)^{2/3} = 2.29799 \cdot (10)^7 \text{ ft}$$

$$\text{At apogee, } v_c = \sqrt{2gR^2 \left[\frac{1}{r} - \frac{1}{2a} \right]} = 24,156 \text{ ft/sec}$$

$$\Delta v = v - v_c = 24,458 - 24,156 = \underline{302 \text{ ft/sec}}$$

(Can check to ensure that C does not strike the earth by finding $r_{\min} = 2.242 \times 10^7 \text{ ft}$
 $> R = 2.090 \times 10^7 \text{ ft}$!)

3/297 From previous solution, the circular orbit speed is $v = 24,458 \text{ ft/sec}$.

Time required for B to return to C's burn position over almost two circular orbits:

$$t = \frac{4\pi r - (1000)(5280)}{v} = 11,881 \text{ s}$$

$$a = \left(\frac{TR\sqrt{g}}{2\pi} \right)^{2/3} = \left[\frac{\left(\frac{11,881}{2} \right) (3959)(5280) \sqrt{32.23}}{2\pi} \right]^{2/3}$$
$$= 2.32626 (10^7) \text{ ft}$$

$$\text{At apogee, } v_c = \sqrt{2gR^2 \left(\frac{1}{r} - \frac{1}{2a} \right)} = 24,309 \text{ ft/sec}$$

$$\Delta v = v - v_c = 24,458 - 24,309 = \underline{148 \text{ ft/sec}}$$

3/298 Circular orbit speed

$$v_o = R \sqrt{\frac{g}{a}} = R \sqrt{\frac{g}{3R}} = \sqrt{\frac{1}{3} g R}$$

Speed at A (apogee) in elliptical orbit:

$$v_A = R \sqrt{\frac{g}{a}} \sqrt{\frac{r_{\min}}{r_{\max}}} = R \sqrt{\frac{g}{2R}} \sqrt{\frac{R}{3R}} = \sqrt{\frac{1}{6} g R}$$

$$v_r = v_o - v_A = \sqrt{gR} \left[\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{6}} \right] = 0.1691 \sqrt{gR}$$

$$\begin{aligned} \text{Numbers: } v_r &= 0.1691 \sqrt{1.62 \frac{3476}{2} (1000)} \\ &= \underline{284 \text{ m/s (directed rearward)}} \end{aligned}$$

Call the circular orbit period τ_o and the elliptical orbit period τ_{AB}

$$\tau_o = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi (3R)^{3/2}}{R\sqrt{g}}; \tau_{AB} = \frac{2\pi (2R)^{3/2}}{R\sqrt{g}}$$

$$\theta = \left(\frac{\tau_{AB}/2}{\tau_o/2} \right) \pi = \left(\frac{2}{3} \right)^{3/2} \pi = 1.710 \text{ rad or } \underline{98.0^\circ}$$

3/299 Circular orbit : $v = R \sqrt{\frac{g}{r}}$

$$v = (3959)(5280) \sqrt{\frac{32.23}{(4159)(5280)}} = 25,324 \text{ ft/sec}$$

$$\text{During burn : } a_t = \frac{F}{m} = \frac{2(6000)}{(175,000)/32.2} = 2.208 \frac{\text{ft}}{\text{sec}^2}$$

$$v_a = v - a_t t = 25,324 - 2.208(150) = 24,993 \frac{\text{ft}}{\text{sec}}$$

$$v^2 = 2gR^2 \left[\frac{1}{r} - \frac{1}{2a} \right]$$

Substitute conditions at B to find $a = 2.1403(10^7) \text{ ft}$

$$\text{Use } v_A = R \sqrt{\frac{g}{a}} \sqrt{\frac{1-e}{1+e}} \text{ to obtain } e = 0.02599$$

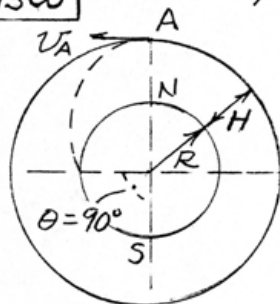
$$r = \frac{a(1-e^2)}{1+e \cos \theta} \text{ utilized at point C:}$$

$$(3959)(5280) = \frac{(2.1403 \times 10^7)(1-0.02599^2)}{1 + 0.02599 \cos \theta}$$

$$\theta = 26.7^\circ$$

$$\beta = 180 - \theta = \underline{153.3^\circ}$$

3/300

From Eq. 3/43 $\frac{1}{r} = \frac{1+e\cos\theta}{a(1-e^2)}$ when $\theta = 90^\circ$, $r = R$ " $\theta = 180^\circ$, $r = R+H$ Thus $\frac{1}{R} = \frac{1}{a(1-e^2)}$

$$\& \frac{1}{R+H} = \frac{1-e}{a(1-e^2)} = \frac{1}{a(1+e)}$$

Solve & get $1-e = \frac{R}{R+H}$ & $a(1+e) = R+H$

From Eq. 3/48

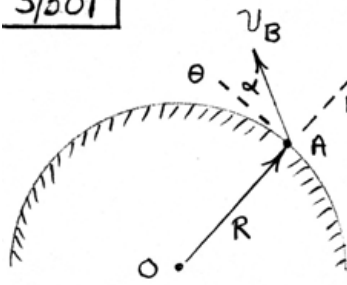
$$v_A = R \sqrt{\frac{g(1-e)}{a(1+e)}} = R \sqrt{g} \sqrt{\frac{R}{R+H} \frac{1}{R+H}} = \frac{R \sqrt{g} R}{R+H}$$

For circular orbit

$$v = R \sqrt{\frac{g}{R+H}} \quad \text{so } \Delta v_A = R \sqrt{\frac{g}{R+H}} - \frac{R \sqrt{g} R}{R+H}$$

$$= R \sqrt{\frac{g}{R+H}} \left(1 - \sqrt{\frac{R}{R+H}}\right)$$

3/301



$$v_{\theta} = v \cos \alpha = 2000 \cos 30^{\circ} = 1732 \text{ m/s}$$

$$v_r = v \sin \alpha = 2000 \sin 30^{\circ} = 1000 \text{ m/s}$$

$$v^2 = 2gR^2 \left(\frac{1}{r} - \frac{1}{2a} \right)$$

Using conditions at B:

$$a = 3.2906 \times 10^6 \text{ m}$$

$$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} m (2000)^2 = 2 \times 10^6 \text{ m}$$

$$V_B = \frac{-mgR^2}{r} = \frac{-m(9.825)(6.371 \times 10^6)^2}{6.371 \times 10^6}$$

$$= -6.2595 \times 10^7 \text{ m}$$

$$E = T_B + V_B = -6.0595 \times 10^7 \text{ m}$$

$$h = r v_{\theta} = 6.371(10^6)(1732) = 1.1035 \times 10^{10}$$

Now use $e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}}$ to get $e = 0.9525$

Finally, $r_{\max} = a(1+e) = 3.2906(10^6)(1+0.9525)$
 $= 6.4249 \times 10^6 \text{ m}$

$$h_{\max} = r_{\max} - R = 53900 \text{ m or } \underline{53.9 \text{ km}}$$

3/302 Point A is the apogee, so we have $r_{\max} = \frac{3R}{2} = a(1+e)$.

$$r = \frac{a(1-e^2)}{1+e\cos\theta}; \text{ At B : } R = \frac{a(1-e^2)}{1+e\cos(135^\circ)}$$

Solving, $e = 0.6306$, $a = 0.9199R$

$$\text{Now, } v_B^2 = 2gR^2 \left(\frac{1}{r} - \frac{1}{2a} \right)$$

$$\text{At A : } v_B^2 = 2(9.825)(6.371 \times 10^6)^2 \times \left(\frac{1}{6.371 \times 10^6} - \frac{1}{2(0.9199)(6.371 \times 10^6)} \right)$$

$$\underline{v_B = 7560 \text{ m/s}}$$

3/303

$$v_A = R \sqrt{\frac{g}{a}} \sqrt{\frac{1-e}{1+e}} = R \sqrt{\frac{9.825}{0.9199 R}} \sqrt{\frac{1-0.6306}{1+0.6306}}$$

$$= 1.555 \sqrt{R}$$

$$h = r_A v_A = \frac{3}{2} R (1.555 \sqrt{R}) = 2.3332 R^{3/2}$$

Conservation of angular momentum requires

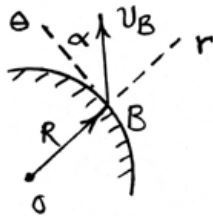
$$h = r_B v_{B\theta} = R v_{B\theta} = 2.3332 R^{3/2}$$

$$v_{B\theta} = 2.3332 R^{1/2}$$

$$= 2.3332 (6.371 \times 10^6)^{1/2} = 5889 \text{ m/s}$$

$$v_{B\theta} = v_B \cos \alpha$$

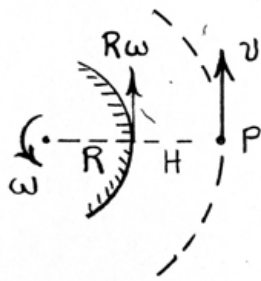
$$\alpha = \cos^{-1} \left(\frac{v_{B\theta}}{v_B} \right) = \cos^{-1} \left(\frac{5889}{7560} \right) = \underline{38.8^\circ}$$



(Value of v_B from previous solution)

3/304

$$a = \frac{1}{2} [2(6371) + 150 + 1500] = 7196 \text{ km}$$



Eq. 3/47 at perigee P:

$$v^2 = 2(9.825) [6371(10^3)]^2 \times \left[\frac{1}{(6371+150)10^3} - \frac{1}{2(7196)10^3} \right]$$

$$v = 8179 \text{ m/s}$$

$$R\omega = 6371(10^3)(0.7292 \cdot 10^{-4}) = 465 \text{ m/s}$$

$$\text{Absolute dish angular velocity } P_a = \frac{v - R\omega}{H}$$

$$\text{Relative dish angular velocity } p = P_a - \omega$$

$$p = \frac{v - R\omega}{H} - \omega = \frac{8179 - 465}{150(10^3)} - 0.7292(10^{-4})$$

$$= \underline{0.0514 \text{ rad/s}}$$

3/305

At perigee,
 $a = a_n = \frac{v_p^2}{f_p}$

so $f_p = \frac{v_p^2}{a_n}$

From Eq. 3/48,

$$v_p^2 = R^2 \frac{g}{a} \frac{r_{\max}}{r_{\min}}$$

But from Eqs. 3/43: $r_{\min} + r_{\max} = 2a$, so

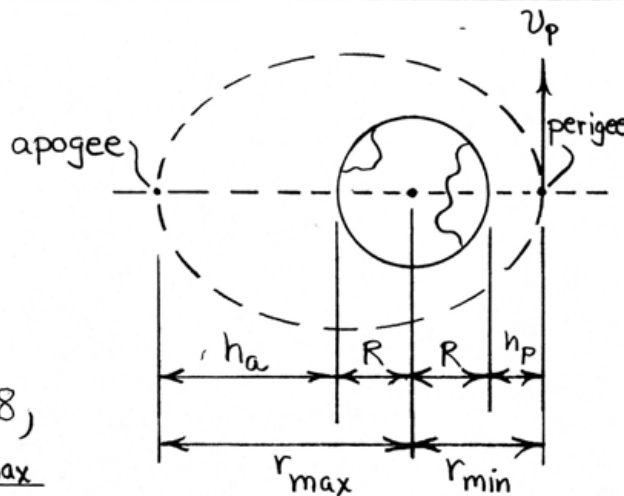
$$v_p^2 = g R^2 \frac{r_{\max}}{r_{\min}} \frac{2}{r_{\min} + r_{\max}}$$

Also, $a_n = g_{\text{perigee}} = g \left(\frac{R}{r_{\min}} \right)^2$ (from Chapter 1)

Thus $f_p = 2gR^2 \frac{r_{\max}}{r_{\min}} \frac{1}{r_{\min} + r_{\max}} \bigg/ g \frac{R^2}{r_{\min}^2}$

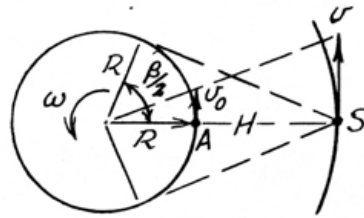
$$= 2 \frac{r_{\max} r_{\min}}{r_{\min} + r_{\max}}$$

or $f_p = 2 \frac{(R+h_a)(R+h_p)}{2R+h_a+h_p}$



3/306

Path is limited to an equatorial orbit in order to remain above a point A on the equator.



$$\frac{v}{R+H} = \omega \text{ \& for circular orbit}$$

Eq. 3/4.7 with $a=r=R+H$
gives $v = R \sqrt{\frac{g}{R+H}}$

$$\omega = 0.7292 \times 10^{-4} \text{ rad/s}$$

$$R = 6371 \text{ km}$$

$$\text{Combine \& get } R+H = \sqrt[3]{\frac{gR^2}{\omega^2}}, H = \sqrt[3]{\frac{9.825(6371 \times 10^3)^2}{(0.7292 \times 10^{-4})^2}} - 6371 \times 10^3$$

$$= (42\,170 - 6371) \times 10^3 = 35.8 \times 10^6 \text{ m}$$

or $H = 35\,800 \text{ km}$

$$\frac{\beta}{2} = \cos^{-1} \frac{R}{R+H} = \cos^{-1} \frac{6371}{42\,170} = 81.3^\circ, \underline{\beta = 162.6^\circ \text{ of longitude}}$$

$$3/307 \quad Ft = m \Delta v$$

For circular orbit, $v_1 = R\sqrt{g/a_1}$

$$= 6371(10^3) \sqrt{\frac{9.825}{12371(10^3)}}$$

$$= 5678 \text{ m/s}$$

For elliptical orbit at apogee A

$$v_A = R \sqrt{\frac{g}{a_2}} \sqrt{\frac{r_{\min}}{r_{\max}}}$$

where $r_{\min} = 6371 + 3000 = 9371 \text{ km}$

$$r_{\max} = 6371 + 6000 = 12371 \text{ km}$$

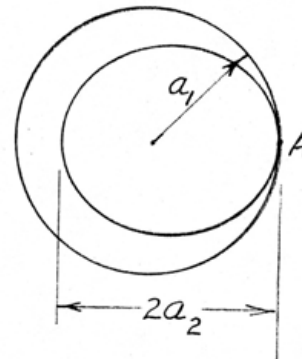
so $v_A = 6371(10^3) \sqrt{\frac{9.825}{10871(10^3)}} \sqrt{\frac{9371}{12371}}$

$$= 5271 \text{ m/s}$$

Thus $\Delta v = 5678 - 5271 = 406 \text{ m/s}$

so $2000 t = 800(406)$

$$t = 162 \text{ s}$$



$$a_1 = R + 6000$$

$$= 6371 + 6000$$

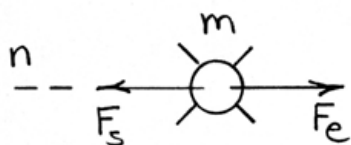
$$= 12371 \text{ km}$$

$$2a_2 = 2a_1 - 3000$$

$$= 21742 \text{ km}$$

$$a_2 = 10871 \text{ km}$$

*3/308



F_s : force exerted on spacecraft by sun

F_e : force exerted on spacecraft by earth

$$\Sigma F_n = ma_n: F_s - F_e = m \frac{v^2}{r} = m r \omega^2 \\ = m(D-h) \left(\frac{2\pi}{T} \right)^2$$

where D is the earth-sun distance and T is the earth orbital period.

$$\frac{G m_s m}{(D-h)^2} - \frac{G m_e m}{h^2} = m(D-h) \left(\frac{2\pi}{T} \right)^2$$

With $G = 3.439(10^{-8}) \frac{\text{ft}^4}{\text{lb-sec}^2}$, $m_s = 333,000 m_e$,
 $m_e = 4.095(10^{23})$ slugs, $D = 92.96(10^6)(5280)$ ft,
and $T = 365.26(24)(3600)$ sec, solve numerically
for h as $h = 4.87(10^9)$ ft or 922,000 mi

►3/309 For 1, $v_1 = R\sqrt{g/r_1}$; for 2, $v_2 = R\sqrt{g/r_2}$

For transfer ellipse at A, $v_1' = R\sqrt{g/a} \sqrt{r_2/r_1}$ } $a = \frac{r_1 + r_2}{2}$

For transfer ellipse at B, $v_2' = R\sqrt{g/a} \sqrt{r_1/r_2}$ } (Eq. 3/48)

$$\text{At A, } \Delta v_A = v_1' - v_1 = R\sqrt{g/a} \sqrt{r_2/r_1} - R\sqrt{g/r_1} = R\sqrt{g/r_1} \left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1 \right)$$

$$\text{At B, } \Delta v_B = v_2 - v_2' = R\sqrt{g/r_2} - R\sqrt{g/a} \sqrt{r_1/r_2} = R\sqrt{g/r_2} \left(1 - \sqrt{\frac{2r_1}{r_1+r_2}} \right)$$

$$\Delta v_A = 6371(10^3) \sqrt{\frac{9.825(10^3)}{6871}} \left(\sqrt{\frac{2(42171)}{6871+42171}} - 1 \right) = \underline{2370 \text{ m/s}}$$

$$\Delta v_B = 6371(10^3) \sqrt{\frac{9.825(10^3)}{42171}} \left(1 - \sqrt{\frac{2(6871)}{6871+42171}} \right) = \underline{1447 \text{ m/s}}$$

► 3/310 Eq. 3/47: $v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

$$7400^2 = 2(9.825)(6371 \cdot 1000)^2 \left[\frac{1}{7371 \cdot 1000} - \frac{1}{2a} \right]$$

$$\underline{a = 7462 \text{ km}}$$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(7400)^2 = 27.38(10^6) \text{ m}$$

$$V = -\frac{mgR^2}{r} = -\frac{m(9.825)(6371 \cdot 1000)^2}{7371(1000)} = -54.1(10^6) \text{ m}$$

$$E = T + V = -26.7(10^6) \text{ m (in Joules)}$$

$$h = r v_{\theta} = 7371(10^3)(7400 \cos 2^\circ) = 5.45(10^{10}) \frac{\text{m}^2}{\text{s}}$$

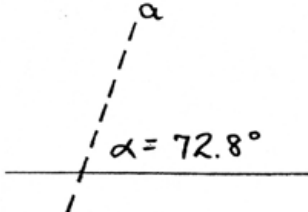
$$e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}} = \sqrt{1 + \frac{2(-26.7)10^6 \text{ m} \cdot 5.45^2 10^{20}}{m(9.825)^2(6371 \cdot 1000)^4}}$$

$$= 0.0369$$

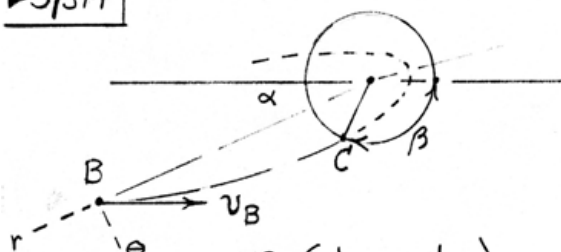
$$\text{From } r = \frac{a(1-e^2)}{1+e \cos \theta} : 7371 = \frac{7462(1-0.0369^2)}{1+0.0369 \cos \theta}$$

$$\theta = \pm 72.8^\circ \quad \text{So } \underline{\alpha' = 72.8^\circ}$$

$$r_{\min} = a(1-e) = 7462(1-0.0369) = 7186 > R = 6371 \text{ m}$$

 $\therefore$ Does not strike earth

►3/3/1



$$\begin{aligned} \text{At B, } r &= \sqrt{29} R \\ \alpha &= \tan^{-1} \left(\frac{2R}{5R} \right) \\ &= 21.8^\circ \end{aligned}$$

$$v^2 = 2gR^2 \left(\frac{1}{r} - \frac{1}{2a} \right)$$

$$\text{At B: } 3200^2 = 2(9.825)(6.371 \times 10^6)^2 \left[\frac{1}{\sqrt{29} 6.371(10^6)} - \frac{1}{2a} \right]$$

$$a = 3.066 \times 10^7 \text{ m}$$

$$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} m (3200)^2 = 5.120 \times 10^6 \text{ m}$$

$$V_B = -\frac{mgR^2}{r_B} = -m \frac{(9.825)(6.371 \times 10^6)^2}{\sqrt{29} (6.371 \times 10^6)} = -1.162 \times 10^7 \text{ m}$$

$$E = T_B + V_B = -6.504 \times 10^6 \text{ m}$$

$$v_\theta = 3200 \sin \alpha = 1188.5 \text{ m/s}$$

$$h = r v_\theta = \sqrt{29} (6.371 \times 10^6) (1188.5) = 4.077 \times 10^{10} \text{ kg-m}^2/\text{s}$$

$$e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}}$$

$$e = \sqrt{1 + \frac{2(-6.504 \text{ m})(4.077 \times 10^{10})^2}{m(9.825)^2 (6.371 \times 10^6)^4}}$$

$$= 0.9295$$

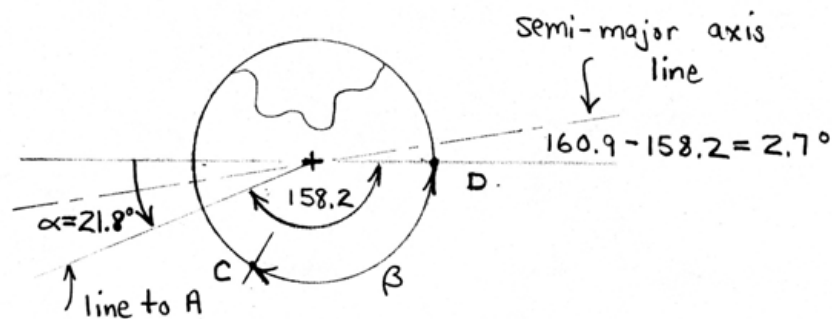
$$r = \frac{a(1-e^2)}{1+e \cos \theta}$$

$$\text{At B: } \sqrt{29}(6.371 \times 10^6) = \frac{(3.066 \times 10^7)(1-0.9295^2)}{1+0.9295 \cos \theta}$$

$$\theta = 160.9^\circ$$

$$\text{At C: } 6371(10^6) = \frac{(3.066 \times 10^7)(1-0.9295^2)}{1+e \cos \theta}$$

$$\theta = 111.8^\circ$$



$$\beta = 111.8 - 2.7 = \underline{109.1^\circ}$$

►3/312 The speed of the orbiter is

$$v_o = \sqrt{\frac{G m_e}{r}} = \sqrt{\frac{6.673(10^{-11})(5.976)(10^{24})}{(6371+200)(1000)}} = 7790 \text{ m/s}$$

The speed of the satellite is

$$v = \sqrt{v_o^2 + v_{s/o}^2} = 7791 \text{ m/s}$$

$$\text{Eq. 3/47: } v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$$

$$(7791)^2 = 2(9.825)(6371 \cdot 1000)^2 \left[\frac{1}{6571(1000)} - \frac{1}{2a} \right]$$

$$a = 6572 \text{ km}$$

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}} = 5301 \text{ s}$$

$$\text{Energy } E = \frac{1}{2}mv^2 - \frac{Gm_em}{r} = -30.3 \text{ m}(10^6) \text{ J}$$

$$h = r v_\theta = 6571(1000) 7790 = 5.12(10^{10}) \text{ m}^2/\text{s}$$

$$\text{Eq. 3/45: } e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}} = 0.01284$$

$$\text{From } \frac{1}{r} = \frac{1 + e \cos \theta}{a(1 - e^2)}, \quad \theta = 90^\circ \text{ exactly}$$

(semimajor axis is parallel to x -axis)

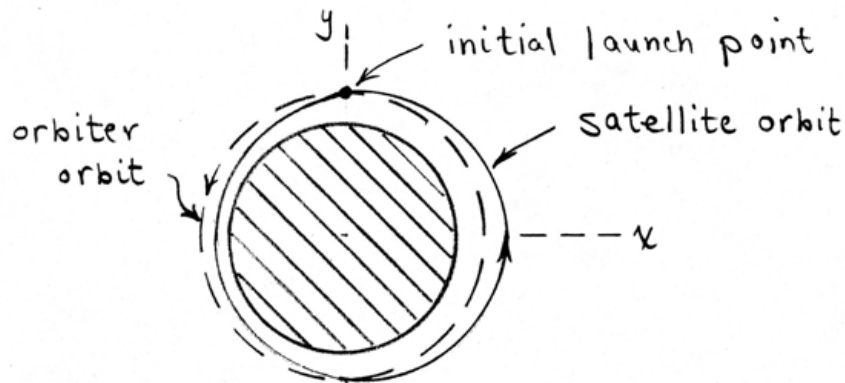
$$r_{\min} = a(1-e) = \underline{6.49(10^6) \text{ m}}$$

$$r_{\max} = a(1+e) = \underline{6.66(10^6) \text{ m}}$$

$$v_p = R\sqrt{\frac{g}{a}} \sqrt{\frac{r_{\max}}{r_{\min}}} = \underline{7890 \text{ m/s}}$$

$$v_a = R\sqrt{\frac{g}{a}} \sqrt{\frac{r_{\min}}{r_{\max}}} = \underline{7690 \text{ m/s}}$$

Sketch (not to scale):



3/313

Truck bed is a constant-velocity frame of reference
so that $U_{\text{rel}} = \Delta T_{\text{rel}}$ holds.

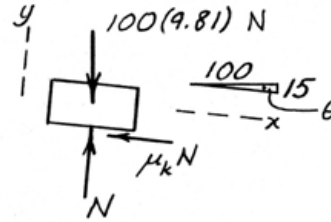
$$\Sigma F_y = 0: N - 981 \cos 8.53^\circ = 0$$

$$N = 970 \text{ N}$$

$$U_{\text{rel}} = \Delta T_{\text{rel}}: (981 \sin 8.53^\circ - 970 \mu_k) 2$$

$$= \frac{1}{2} 100 (0 - 3^2)$$

$$\mu_k = 0.382$$



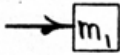
$$\theta = \tan^{-1} 0.15 = 8.53^\circ$$

3/314

$$\Sigma F_x = m a_x : k\delta = m_1 a_{x1}$$

$$-k\delta = m_2 a_{x2}$$

$k\delta$



-----x

$$a_{1/2} = a_{x1} - a_{x2} = \frac{k\delta}{m_1} - \left(-\frac{k\delta}{m_2}\right)$$

$$\text{or } a_{1/2} = a_{rel} = k\delta \left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

$$\boxed{3/315} \quad v_{rel} = l\dot{\theta} = 0.5(2) = 1 \text{ m/s} \rightarrow$$

$$v = v + v_{rel} = 2 + 1 = 3 \text{ m/s} \rightarrow$$

$$\underline{G} = m\underline{v} = 3(3\underline{i}) = \underline{9\underline{i} \text{ kg}\cdot\text{m/s}}$$

$$\underline{G}_{rel} = m\underline{v}_{rel} = 3(1\underline{i}) = \underline{3\underline{i} \text{ kg}\cdot\text{m/s}}$$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}(3)(3)^2 = \underline{13.5 \text{ J}}$$

$$T_{rel} = \frac{1}{2}mv_{rel}^2 = \frac{1}{2}(3)(1)^2 = \underline{1.5 \text{ J}}$$

$$\underline{H}_o = -lmv\underline{k} = -(0.5)(3)(3)\underline{k} = \underline{-4.5\underline{k} \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

$$\underline{H}_{Brel} = -lmv_{rel}\underline{k} = -(0.5)(3)(1)\underline{k} = \underline{-1.5\underline{k} \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

3/316 Rel. to carrier $U_{rel} = \Delta T_{rel}$

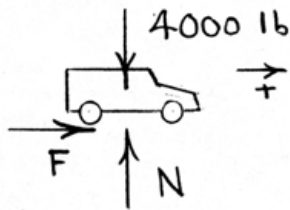
$$(22 + P)(10^3)75 = \frac{1}{2}(3)(10^3)\left[(240/3.6)^2 - 0\right]$$

$$22 + P = 88.9 \text{ kN}, \quad \underline{P = 66.9 \text{ kN}}$$

3/317 Barge-fixed frame is Newtonian.

$$v^2 = v_0^2 + 2a(s-s_0) : \left(15 \frac{5280}{3600}\right)^2 = 2a(80)$$

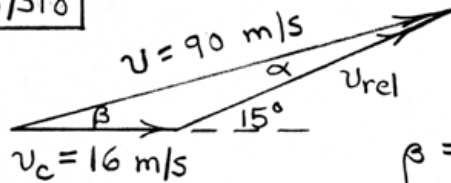
$$a = 3.03 \text{ ft/sec}^2 = a_{\text{rel}}$$



$$\rightarrow \Sigma F = ma : F = \frac{4000}{32.2} (3.03)$$

$$\underline{F = 376 \text{ lb}}$$

3/318



$$\frac{\sin 165^\circ}{90} = \frac{\sin \alpha}{16}$$

$$\alpha = 2.64^\circ$$

$$\beta = 180 - 165 - \alpha = 12.36^\circ$$

$$\frac{\sin \beta}{v_{rel}} = \frac{\sin 165^\circ}{90} \quad , \quad v_{rel} = 74.4 \text{ m/s}$$

$$U_{rel} = \Delta T_{rel} : F_d = \frac{1}{2} m (v_{rel}^2 - 0)$$

$$F(100) = \frac{1}{2} 7000 (74.4^2)$$

$$F = 194\,000 \text{ N or } 194.0 \text{ kN}$$

3/319 | --- x

$$\text{For truck, } a_T = -0.9g, \quad t_{\text{stop}} = \frac{15 \text{ m/s}}{0.9(9.81 \text{ m/s}^2)} = 1.699 \text{ s}$$

$$\text{For crate, } a_c = -0.7g$$

$$a_{c/T} = a_c - a_T = -0.7g - (-0.9g) = 0.2g$$

(As long as truck is moving)

$$\text{At } t = t_{\text{stop}},$$

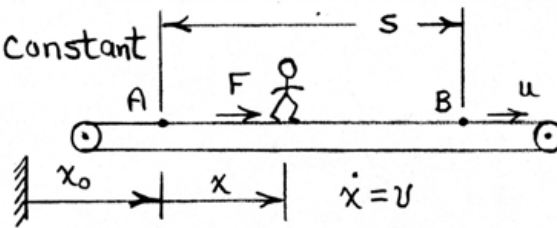
$$\begin{aligned} x_{c/T} &= (x_{c/T})_0 + (v_{c/T})_0 t_{\text{stop}} + \frac{1}{2} a_{c/T} t_{\text{stop}}^2 \\ &= 0 + 0 + \frac{1}{2} (0.2g) (1.699)^2 = 2.83 \text{ m} \end{aligned}$$

$$v_{c/T} = (v_{c/T})_0 + a_{c/T} t_{\text{stop}} = 0 + (0.2g)(1.699) = 3.33 \frac{\text{m}}{\text{s}}$$

$$\begin{aligned} \text{Then: } v_c^2 &= v_{c_0}^2 + 2a_c(x - x_0) \\ &= (3.33)^2 + 2(-0.7g)(3.2 - 2.83) \end{aligned}$$

$$\underline{v_c = 2.46 \text{ m/s}}$$

3/320

 $F = \text{constant}$ 

Absolute: $U = \Delta T : F(s + \Delta x_0) = \frac{1}{2}m(u+v)^2 - \frac{1}{2}mu^2$

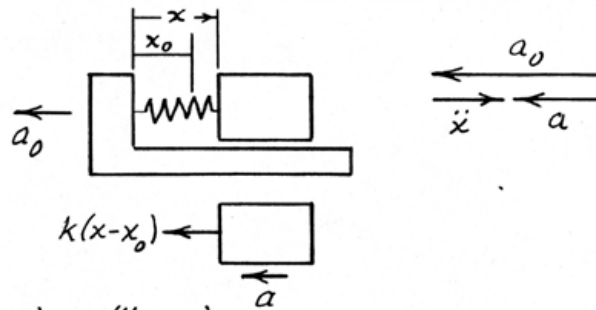
$$Fs + F\Delta x_0 = \frac{1}{2}mv^2 + muv \quad (1)$$

Relative to walkway: $U_{\text{rel}} = \Delta T_{\text{rel}} : Fs = \frac{1}{2}mv^2 - 0 \quad (2)$

Subtract (2) from (1): $F\Delta x_0 = muv$

The term muv represents the work done by force F due only to the movement of the walkway.

3/321



$$\sum F_x = ma_x: -k(x-x_0) = m(\ddot{x} - a_0)$$

$$\dot{x} d\dot{x} = \ddot{x} dx \text{ so } \int_0^{\dot{x}} \dot{x} d\dot{x} = \int_{x_0}^x \left[a_0 - \frac{k}{m}(x-x_0) \right] dx$$

$$\frac{1}{2} \dot{x}^2 = \left(a_0 - \frac{kx_0}{m} \right) (x-x_0) - \frac{k}{2m} (x^2 - x_0^2)$$

$$\frac{d}{dx} \left(\frac{\dot{x}^2}{2} \right) = a_0 + \frac{kx_0}{m} - \frac{kx}{m} = 0 \text{ for max. } \frac{\dot{x}^2}{2} \text{ \& hence max } \dot{x}$$

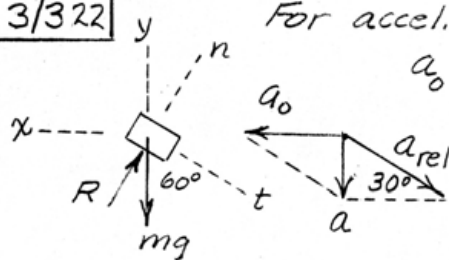
$$\text{so } \frac{kx}{m} = a_0 + \frac{kx_0}{m}, \quad x = x_0 + \frac{ma_0}{k}$$

Thus

$$\begin{aligned} (v_{rel})_{max}^2 &= \dot{x}_{max}^2 = 2 \left(a_0 + \frac{kx_0}{m} \right) \left(x_0 + \frac{ma_0}{k} - x_0 \right) - \frac{k}{m} \left(x_0^2 + \frac{2ma_0}{k} x_0 + \frac{m^2 a_0^2}{k^2} - x_0^2 \right) \\ &= \frac{ma_0^2}{k} \end{aligned}$$

$$\underline{(v_{rel})_{max} = a_0 \sqrt{m/k}}$$

3/322



For accel. a vertically down,

$$a_0 = a_{rel} \cos 30^\circ$$

$$a_x = 0 \text{ so } \Sigma F_x = 0$$

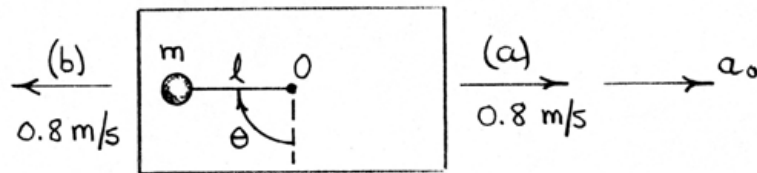
$$-R \sin 30^\circ = 0, \underline{R = 0}$$

$$\Sigma F_y = ma_y, mg = ma$$

$$a = g \text{ \& } a_0 = a\sqrt{3} = g\sqrt{3}$$

$$= 9.8/\sqrt{3} = \underline{16.99 \frac{m}{s^2}}$$

3/323



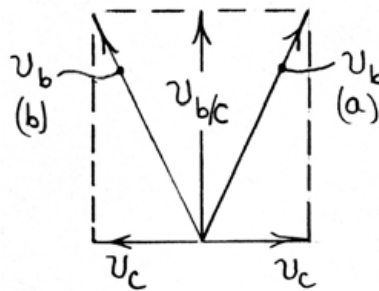
$$m_b = 10 \text{ kg}$$

$$l = 0.8 \text{ m}$$

$$\theta = 90^\circ$$

$$m_c = 250 \text{ kg}$$

$$\dot{\theta} = 3 \text{ rad/s}$$



$$v_{b/c} = l \dot{\theta} = 0.8(3) = 2.4 \text{ m/s}$$

$$T_b = \frac{1}{2} m_b v_b^2 = \frac{1}{2} (10) [0.8^2 + 2.4^2] = 32 \text{ J}$$

(same for cases (a) and (b))

$$T_c = \frac{1}{2} m_c v_c^2 = \frac{1}{2} (250) (0.8)^2 = 80 \text{ J}$$

$$T = T_b + T_c = 32 + 80 = \underline{112 \text{ J}} \text{ for both cases}$$

3/324 $\Sigma \underline{F} = m(\underline{a}_0 + \underline{a}_{rel})$. In t-dir., $\Sigma F_t = 0$,

so $a_t = l\ddot{\theta} - a_0 \cos \theta = 0$

$\ddot{\theta} = \frac{a_0}{l} \cos \theta$ (1)

In n-dir., $\Sigma F_n = ma_n$

$T = m(l\dot{\theta}^2 + a_0 \sin \theta)$ (2)

Integrate (1): $\ddot{\theta} = \dot{\theta} \frac{d\dot{\theta}}{d\theta} = \frac{a_0}{l} \cos \theta$

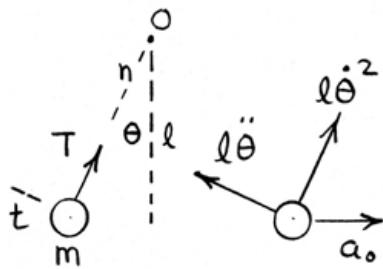
$\int \dot{\theta} d\dot{\theta} = \int \frac{a_0}{l} \cos \theta d\theta$

$\frac{1}{2} \dot{\theta}^2 = \frac{a_0}{l} \sin \theta$

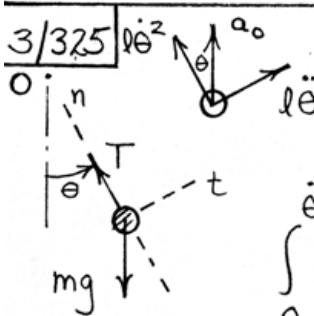
From (2): $T = m[2a_0 \sin \theta + a_0 \sin \theta]$

or $T = 3ma_0 \sin \theta$

For $\theta = \frac{\pi}{2}$, $T_{\pi/2} = 3ma_0 = 3(10)(3) = \underline{90 \text{ N}}$



3/325



$$\Sigma F_t = ma_t:$$

$$-mg \sin \theta = m(l\ddot{\theta} + a_0 \sin \theta)$$

$$\ddot{\theta} = -\left(\frac{a_0 + g}{l}\right) \sin \theta$$

$$\int_0^{\dot{\theta}} \dot{\theta} d\dot{\theta} = -\int_{\theta_0}^{\theta} \left(\frac{a_0 + g}{l}\right) \sin \theta d\theta$$

$$\dot{\theta}^2 = 2\left(\frac{a_0 + g}{l}\right)(\cos \theta - \cos \theta_0)$$

$$\Sigma F_n = ma_n: T - mg \cos \theta = m(l\dot{\theta}^2 + a_0 \cos \theta)$$

$$T = m \left[g(3 \cos \theta - 2 \cos \theta_0) + a_0(3 \cos \theta - 2 \cos \theta_0) \right]$$

$$\text{When } \theta = 0, \quad = m(g + a_0)(3 \cos \theta - 2 \cos \theta_0)$$

$$T_0 = m(g + a_0)(3 - 2 \cos \theta_0)$$

$$\text{If } \theta_0 = \pi/2,$$

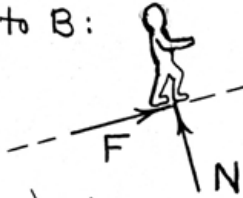
$$T_0 = 3m(g + a_0)$$

3/326 For motion from A to B:

Absolute: $U'_{abs} = \Delta T + \Delta V_g$:

$$F(\Delta x_0 + s) = \frac{1}{2} m (v_r + u)^2 - \frac{1}{2} m u^2 + mg(\Delta x_0 + s) \sin \theta$$

$$= \frac{1}{2} m v_r^2 + m v_r u + mg(\Delta x_0 + s) \sin \theta$$



Relative: $U'_{rel} = \Delta T_{rel} + \Delta V_{g_{rel}}$: $Fs = \frac{1}{2} m v_r^2 + mgs \sin \theta$

Work done by walkway: $U'_{abs} - U'_{rel} = m v_r u + mg \Delta x_0 \sin \theta$
 $m v_r u$ represents the work done by the belt due only to the motion of the walkway.

For $m = \frac{150}{32.2}$ slugs, $v_r = 2.5$ ft/sec, $u = 2$ ft/sec,

$\theta = 10^\circ$, $s = 30$ ft:

$$\Sigma F_x = m a_{x_{rel}}: a_{x_{rel}} = \frac{v_r^2}{2s} = \frac{2.5^2}{2(30)} = 0.1042 \frac{\text{ft}}{\text{sec}^2}$$

$$F - 150 \sin 10^\circ = \frac{150}{32.2} (0.1042), \quad F = 26.5 \text{ lb}$$

$$\text{Power by bag: } P_{rel} = F v_r = 26.5 (2.5) = 66.3 \frac{\text{ft-lb}}{\text{sec}}$$

$$\text{or } P_{rel} = 66.3 / 550 = 0.1206 \text{ hp}$$

3/327

(a) $a = 0$, elevator is Newtonian frame

$$v' = ev = e\sqrt{2gh_1} = \sqrt{2gh_2}$$

$$h_2 = e^2 h_1$$

$$(b) a = \frac{g}{4} \text{ up}$$

Let B = ball, E = elevator, $\downarrow +$

$$\text{At impact, } S_B = S_E: S_{B_0} + v_{B_0}t + \frac{1}{2}gt^2 = S_{E_0} + v_{E_0}t - \frac{1}{2}\frac{g}{4}t^2$$

$$S_{B_0} + v_0 t + \frac{1}{2}gt^2 = (S_{B_0} + h_1) + v_0 t - \frac{1}{8}gt^2, \quad t = 2\sqrt{\frac{2h_1}{5g}}$$

$$v_{B/E} = v_B - v_E = \left(v_0 + g \cdot 2\sqrt{\frac{2h_1}{5g}}\right) - \left(v_0 - \frac{g}{4} \cdot 2\sqrt{\frac{2h_1}{5g}}\right) \\ = \sqrt{\frac{5h_1 g}{2}}$$

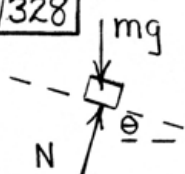
$$\text{After collision, } v'_{B/E_0} = -e\sqrt{\frac{5h_1 g}{2}} \quad (\text{up})$$

$$v'_{B/E} = v'_{B/E_0} + a_{B/E}t = -e\sqrt{\frac{5h_1 g}{2}} + \frac{5}{4}gt$$

$$\text{When } v'_{B/E} = 0, \quad t = 2e\sqrt{\frac{2h_1}{5g}}$$

$$S'_{B/E} = S'_{B/E_0} + v'_{B/E_0}t + \frac{1}{2}\frac{5}{4}gt^2 \\ = 0 - e\sqrt{\frac{5h_1 g}{2}} \cdot 2e\sqrt{\frac{2h_1}{5g}} + \frac{5}{8}g \cdot 4e^2 \frac{2h_1}{5g} \\ = -e^2 h_1 \Rightarrow \underline{h_2 = e^2 h_1}$$

3/328



$$U_{\text{rel}} = \Delta T_{\text{rel}}$$

$$mg l \sin \theta = \frac{1}{2} m v_{\text{rel}}^2 - 0$$

$$v_{\text{rel}}^2 = 2gl \sin \theta$$

$$U = \Delta T: mg l \sin \theta + (N \sin \theta) d = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$$

where d is the horizontal distance traveled by the block.

$$\text{Time to slide from B to C: } l = \frac{1}{2} a t^2 = \frac{1}{2} g \sin \theta t^2$$

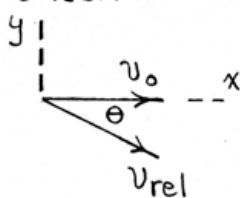
$$t = \left(\frac{2l}{g \sin \theta} \right)^{1/2}. \text{ So } d = v_0 t = v_0 \sqrt{\frac{2l}{g \sin \theta}}$$

$$\text{Also, } N = mg \cos \theta$$

Solving the work-energy equation for v^2 :

$$v_A = \left(v_0^2 + 2gl \sin \theta + 2v_0 \cos \theta \sqrt{2gl \sin \theta} \right)^{1/2}$$

Check:



$$\underline{v}_A = \underline{v}_0 + \underline{v}_{\text{rel}}$$

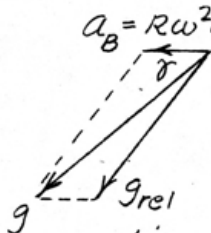
$$= v_0 \underline{i} + \sqrt{2gl \sin \theta} (\cos \theta \underline{i} - \sin \theta \underline{j})$$

$$= (v_0 + \sqrt{2gl \sin \theta} \cos \theta) \underline{i} - \sqrt{2gl \sin^3 \theta} \underline{j}$$

$$v_A^2 = (v_0 + \sqrt{2gl \sin \theta} \cos \theta)^2 + (2gl \sin^3 \theta)$$

$$\checkmark v_A^2 = v_0^2 + 2gl \sin \theta + 2v_0 \cos \theta \sqrt{2gl \sin \theta}$$

► 3/329 From law of cosines



$$a_B = R\omega^2 \cos \gamma \quad g_{rel}^2 = g^2 + a_B^2 - 2g a_B \cos \gamma$$

$$= g^2 \left(1 + \left[\frac{a_B}{g} \right]^2 - 2 \frac{a_B}{g} \cos \gamma \right)$$

$$g_{rel} = g \left[1 + \frac{a_B}{g} \left(\frac{a_B}{g} - 2 \cos \gamma \right) \right]^{1/2}$$

use binomial expansion for 1st two terms
 $(1+x)^n = 1 + nx + \dots$ & get

$$g_{rel} = g \left[1 + \frac{a_B}{g} \left(\frac{a_B}{2g} - \cos \gamma \right) + \dots \right]$$

$$= g + a_B \left(\frac{a_B}{2g} - \cos \gamma \right) + \dots$$

$$g_{rel} = g - R\omega^2 \cos^2 \gamma \left(1 - \frac{R\omega^2}{2g} \right) + \dots$$

$$R\omega^2 = 6.371(10^6)(0.7292 \times 10^{-4})^2 = 0.03388 \text{ m/s}^2$$

$$g_{rel} = 9.825 - 0.03388 \left(1 - \frac{0.03388}{2 \times 9.825} \right) \cos^2 \gamma + \dots$$

$$= \underline{9.825 - 0.03382 \cos^2 \gamma} \text{ m/s}^2$$

Case (a): Orbital speed is constant so that $\ddot{x}$ is both the absolute and relative acceleration in the x -direction.
Hence $F = m\ddot{x}$ holds.

Case (b): Orbital speed is decreasing in the position shown so that a component of acceleration in the negative x -direction exists so that the true (absolute) acceleration in the x -direction is $\ddot{x}$ minus the tangential orbital deceleration. Consequently $F \neq m\ddot{x}$. Only at the perigee and apogee positions where $\dot{v} = 0$ would $F = m\ddot{x}$ be true.

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$$(Up) \sum F_x = ma_x: -0.20(0.940 W)$$

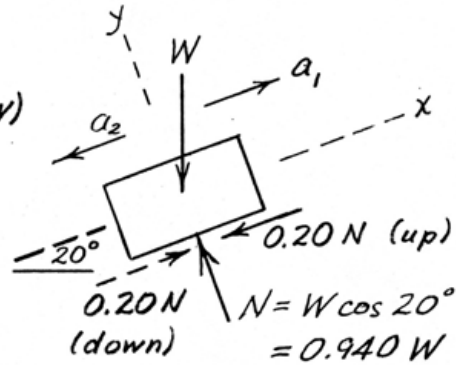
$$-W \sin 20^\circ = \frac{W}{g} a_1$$

$$a_1 = -17.06 \text{ ft/sec}^2$$

$$v^2 = v_1^2 + 2a_1 s:$$

$$0 = 20^2 + 2(-17.06)s$$

$$s = 11.72 \text{ ft}$$



$$(Down) \sum F_x = ma_x: -W \sin 20^\circ + 0.20(0.940 W) = \frac{W}{g} (-a_2)$$

$$a_2 = -4.96 \text{ ft/sec}^2$$

$$v^2 = v_0^2 + 2a_2 s: v_2^2 = 0^2 + 2(4.96)(11.72)$$

$$= 116.3 \text{ (ft/sec)}^2$$

$$v_2 = \underline{10.78 \text{ ft/sec}}$$

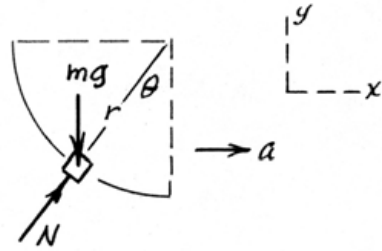
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$$\Sigma F_y = 0: N \cos \theta - mg = 0, N \cos \theta = mg$$

$$\Sigma F_x = ma_x: N \sin \theta = ma$$

Divide & get $\tan \theta = a/g$

$$\theta = \tan^{-1} \frac{a}{g}$$



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Critical condition will occur when



weight is at bottom position.

$$\sum F_n = ma_n: F - mg = mr\omega^2$$

$$80 - 0.030(9.81) = 0.030(0.175)\omega^2$$

$$\omega = 123.2 \frac{\text{rad}}{\text{s}}$$

$$N = \omega \left(\frac{60}{2\pi} \right) = \underline{1177 \text{ rev/min}}$$

$$\frac{3}{334} \quad v^2 = 2gh = 2(9.81)(0.4 + 0.4 \cos 30^\circ)$$

$$= 14.64 \text{ m}^2/\text{s}^2$$

$$\Sigma F_n = ma_n: T - 2(9.81) \cos 30^\circ = 2 \frac{14.64}{0.4}$$

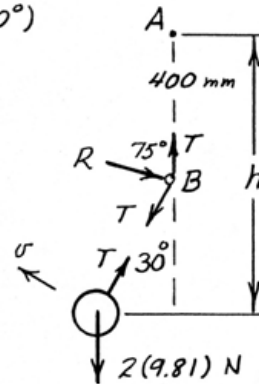
$$T = 90.2 \text{ N}$$

Equil. of forces at B:

$$R = 2T \cos 75^\circ$$

$$= 2(90.2)(0.259)$$

$$= \underline{46.7 \text{ N}}$$



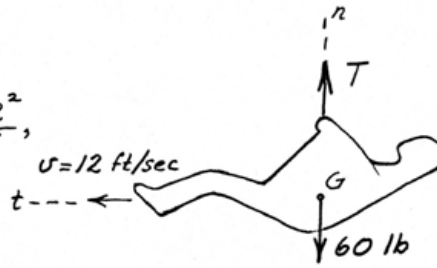
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System:

$$\Sigma F_n = m \frac{v^2}{r}: T - 60 = \frac{60}{32.2} \frac{12^2}{15},$$

$$T = 60(1 + 0.298)$$

$$= \underline{77.9 \text{ lb}}$$

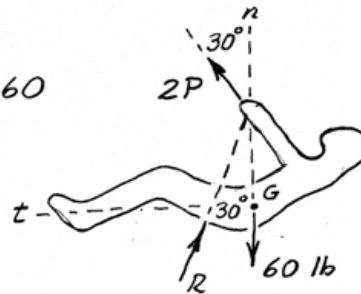


Girl:

$$\Sigma F_n = m \frac{v^2}{r}: 2P \cos 30^\circ + R \cos 30^\circ - 60$$

$$= \frac{60}{32.2} \frac{12^2}{15} = 17.89 \text{ lb}$$

$$\Sigma F_t = 0: 2P \sin 30^\circ - R \sin 30^\circ = 0$$



Solve & get

$$\underline{P = 22.5 \text{ lb}}, \quad \underline{R = 45.0 \text{ lb}}$$

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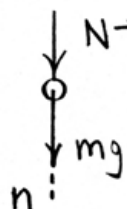
← F ---- x

$$\int \Sigma F_x dt = m \Delta v_x$$

$$-\frac{1}{2}(0.4)(8) - \frac{1}{2}(0.4)(10) = 2(v - 4), \quad \underline{v = 2.2 \text{ m/s}}$$

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Dynamics at B (top of loop)



$$\sum F_n = ma_n: mg = m \frac{v_B^2}{R}$$

$$v_B^2 = gR$$

Work- kinetic energy from A to B:

$$T_A + U_{A-B} = T_B: 0 + \frac{1}{2}kS^2 - mg\mu_k R - mg(2R) = \frac{1}{2}m(gR)$$

$$S = \sqrt{\frac{mgR(5+2\mu_k)}{k}}$$

3/338 Possibilities $\left\{ \begin{array}{l} \text{(a) 2 masses with speed } v_1 \\ \text{considered} \\ \text{(b) 1 mass with speed } 2v_1 \end{array} \right.$

Both (a) and (b) conserve system momentum
since $2(1mv_1) = 1(2m)v_1$.

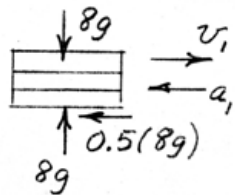
But with $e=1$, kinetic energy must also be conserved.

$$\text{Initial: } T = 2\left(\frac{1}{2}mv_1^2\right) = mv_1^2$$

$$\text{Final: } \begin{cases} T'_a = 2\left(\frac{1}{2}mv_1^2\right) = mv_1^2 \\ T'_b = 1\left(\frac{1}{2}m(2v_1)^2\right) = 2mv_1^2 \end{cases}$$

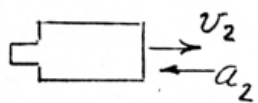
So choice (b) is ruled out.

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$$F = ma; a_1 = 0.5g$$

$$v_1 = \sqrt{2a_1 s_1} = \sqrt{2(0.5 \times 9.81) \times 0.8} = 2.80 \text{ m/s}$$

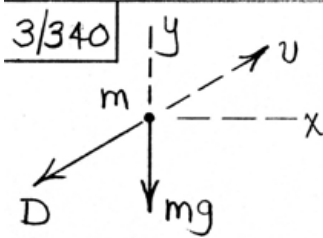


$$a_2 = 0.5g$$

$$v_2 = \sqrt{2a_2 s_2} = \sqrt{2(0.5 \times 9.81) \times 1.2} = 3.43 \text{ m/s}$$

$$\Delta G = 0; 0.060v + 0 = 8(2.80) + (6 + 0.06)3.43$$

$$v = 720 \text{ m/s}$$



$$\Sigma \underline{F} = m \underline{a} : -C_D \frac{1}{2} \rho v^2 S' \frac{\underline{v}}{v} - mg \underline{j} = m(a_x \underline{i} + a_y \underline{j})$$

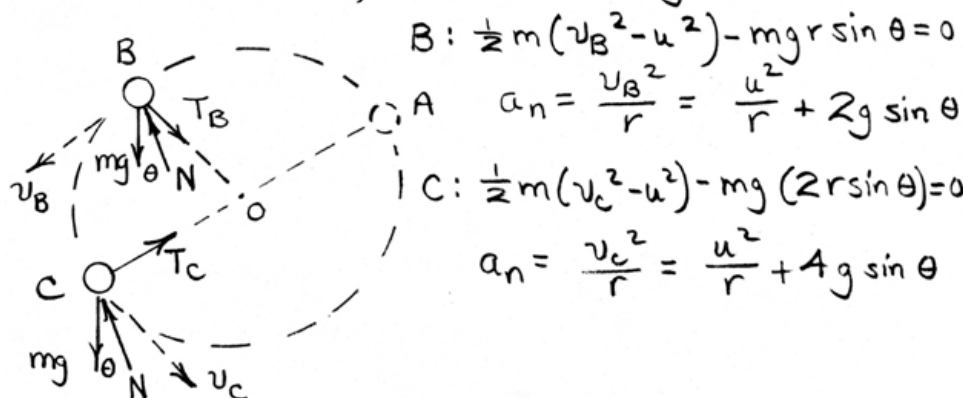
$$-C_D \frac{1}{2} \rho v S' (v_x \underline{i} + v_y \underline{j}) - mg \underline{j} = m(a_x \underline{i} + a_y \underline{j})$$

$$\text{So } \begin{cases} a_x = -C_D \frac{1}{2} \rho S' v v_x / m \\ a_y = -C_D \frac{1}{2} \rho S' v v_y / m - g \end{cases}$$

where $v = \sqrt{v_x^2 + v_y^2}$

The two acceleration expressions are coupled through the speed term. And The expressions are nonlinear.

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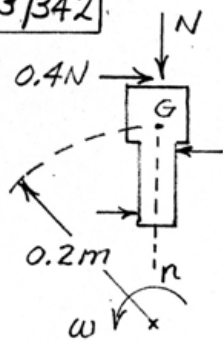
 $U_{1-2}' = 0$, so $\Delta T + \Delta V_g = 0$  $\Sigma F = ma_n$:

$$B: T_B = m \left(\frac{u^2}{r} + 2g \sin \theta \right)$$

$$C: T_C - mg \sin \theta = m \left(\frac{u^2}{r} + 4g \sin \theta \right)$$

$$T_C = m \left(\frac{u^2}{r} + 5g \sin \theta \right)$$

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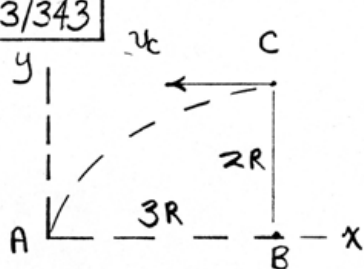


$$\omega = \frac{3000(2\pi)}{60} = 314.2 \text{ rad/s}$$

$$\Sigma F_n = ma_n; \quad N = 2(0.2)(314.2)^2 = 39.5(10^3) \text{ N}$$

$$M = 4\mu_k N r = 4(0.4)(39.5)(10^3)(0.3) = \underline{18.96 \text{ kN}\cdot\text{m}}$$

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$$y_A = y_C + v_{y_C} t - \frac{1}{2} g t^2 ;$$

$$0 = 2R + 0(t) - \frac{1}{2} g t^2$$

$$t = 2\sqrt{R/g}$$

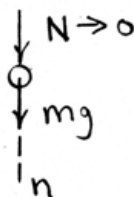
$$x_A = x_C + v_{x_C} t ;$$

$$0 = 3R - v_C 2\sqrt{R/g}$$

$$v_C = \frac{3}{2} \sqrt{gR}$$

$$T_B + U_{B-C} = T_C : \frac{1}{2} m u^2 - mg(2R) = \frac{1}{2} m \left[\frac{3}{2} \sqrt{gR} \right]^2$$

$$u = \frac{5}{2} \sqrt{gR}$$



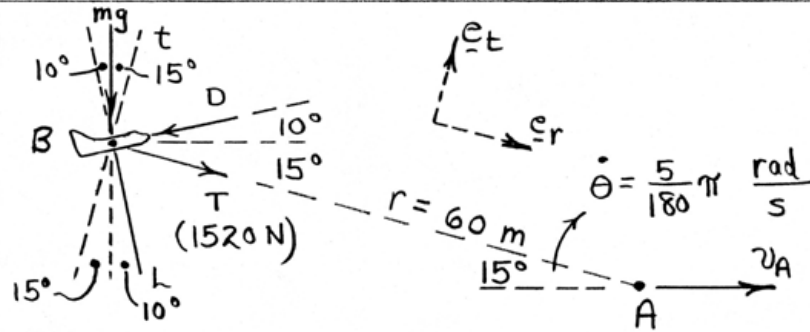
$$\Sigma F_n = m a_n : mg = m \frac{v_C^2}{R}, v_C = \sqrt{gR}$$

$$x_A = x_C + v_{x_C} t ;$$

$$0 = x_{\min} - \sqrt{gR} 2\sqrt{R/g}$$

$$\underline{x_{\min} = 2R}$$

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$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = \underline{0} + r\dot{\theta}^2 \underline{e}_r + 0 \underline{e}_t$$

$$a_B = 60 \left[\frac{5\pi}{180} \right]^2 = 0.457 \text{ m/s}^2$$

$$\Sigma F_t = ma_t: L \cos 25^\circ - D \sin 25^\circ - 200(9.81) \cos 15^\circ = 0$$

$$\Sigma F_r = ma_r: 1520 + 200(9.81) \sin 15^\circ - L \sin 25^\circ - D \cos 25^\circ = 200(0.457)$$

Solve the above two equations to obtain

$$\underline{D = 954 \text{ N}}$$

$$\underline{L = 2540 \text{ N}}$$

3/345 Velocity of plug at bottom is

$$\sqrt{2gh} = \sqrt{2(32.2)6} = 19.66 \text{ ft/sec}$$

$$\Delta G = 0; \frac{2(19.66)}{g} - \frac{(2+4)v}{g} = 0, \underline{v = 6.55 \text{ ft/sec}}$$

$$\Delta T + \Delta V_e = 0; \frac{1}{2} \frac{6}{32.2} (0 - 6.55^2) + \frac{1}{2} 80 (x^2 - 0) = 0$$

$$x^2 = 0.100 \text{ ft}^2, \underline{x = 0.316 \text{ ft}}$$

$$n = \frac{\Delta T}{T}; n = \left[\frac{1}{2} \frac{2}{g} (19.66)^2 - \frac{1}{2} \frac{6}{g} (6.55)^2 \right] / \frac{1}{2} \frac{2}{g} (19.66)^2$$

$$= 1 - \frac{6}{2} \left(\frac{6.55}{19.66} \right)^2 = 1 - 3(0.1111) = \underline{0.667}$$

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The method of work-energy cannot handle forces which are functions of time; the impulse-momentum method cannot accept forces which vary with displacement. Newton's Second Law gives the acceleration as

$$a = -\frac{k}{m}x + \frac{F(t)}{m}$$

which is not easily integrated by standard (non-numerical) methods.

3/347 Final Skidding : $U_{1-2} = \Delta T$

(Prime denotes speed after impact) $-\mu_k mgd = 0 - \frac{1}{2} mv'^2$
 $v' = \sqrt{2\mu_k g d}$

$$A: v_A' = \sqrt{2(0.9)(32.2)(50)} = 53.8 \text{ ft/sec}$$

$$B: v_B' = \sqrt{2(0.9)(32.2)(100)} = 76.1 \text{ ft/sec}$$

$$\begin{aligned} \text{Collision: } m_A v_A + m_B v_B &= m_A v_A' + m_B v_B' \\ \frac{4000}{g} v_A + 0 &= \frac{4000}{g} (53.8) + \frac{2000}{g} (76.1) \\ v_A &= 91.9 \text{ ft/sec} \end{aligned}$$

Initial Skidding : $U_{1-2} = \Delta T$

$$-\mu_k mgd = \frac{1}{2} m (v_A^2 - v_{A0}^2)$$

$$-(0.9)(32.2)(50) = \frac{1}{2} (91.9^2 - v_{A0}^2), \quad v_{A0} = 106.5 \frac{\text{ft}}{\text{sec}}$$

$$(\text{Speed limit was exceeded!}) \quad \underline{= 72.6 \text{ mi/hr}}$$

3/348

For the system of man and cord for full fall

$$(a) U'_{1-2} = 0 = \Delta V_g + \Delta V_e : 0 = 80(9.81)(-44) + \frac{1}{2}k(44-20)^2,$$

$$k = 119.9 \text{ N/m}$$

$$(b) U'_{1-2} = 0 = \Delta T + \Delta V_g + \Delta V_e : 0 = \frac{1}{2}80v^2 - 80(9.81)(20+y) + \frac{1}{2}119.9y^2$$

where y = elongation of bungee cord.

$$40 \frac{d(v^2)}{dy} = 80(9.81) - 119.9y = 0 \text{ for max } v^2, y = 6.55 \text{ m}$$

$$\frac{1}{2}v_{\max}^2 = \frac{1}{40} \left\{ 80(9.81)(20+6.55) - \frac{1}{2}119.9(6.55)^2 \right\} = 457 \text{ m}^2/\text{s}^2$$

$$\underline{v_{\max} = 21.4 \text{ m/s}}$$

(c) Max. acceleration occurs at bottom where tension is greatest



$$T_{\max} = Ky = 119.9(44-20) = 2880 \text{ N}$$

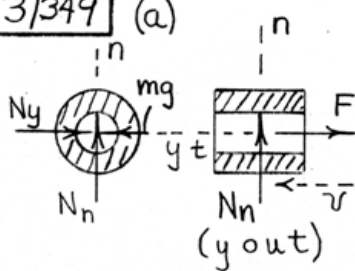
$$\uparrow \Sigma F_y = ma_{\max} : 2880 - 80(9.81) = 80 a_{\max}$$

$$\underline{a_{\max} = 26.2 \text{ m/s}^2 \text{ or } \frac{8}{3}g}$$

$$80(9.81) \text{ N}$$

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(a)



$$\Sigma F_y = 0: N_y = mg$$

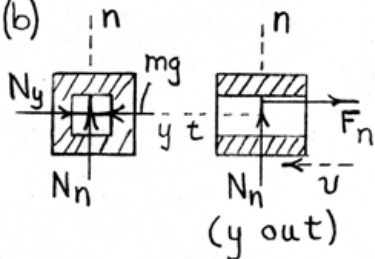
$$\Sigma F_n = m \frac{v^2}{\rho}: N_n = m \frac{v^2}{\rho}$$

$$N_{tot} = \sqrt{N_y^2 + N_n^2} = m \sqrt{g^2 + \frac{v^4}{\rho^2}}$$

$$F = \mu_k N_{tot} = \mu_k m \sqrt{g^2 + \frac{v^4}{\rho^2}}$$

$$\Sigma F_t = ma_t: -\mu_k m \sqrt{g^2 + \frac{v^4}{\rho^2}} = m a_t, \quad a_t = -10.75 \frac{m}{s^2}$$

(b)



As in part (a), $N_y = mg$
and $N_n = m \frac{v^2}{\rho}$.

But $F_y = \mu_k N_y = \mu_k mg$
and $F_n = \mu_k N_n = \mu_k m \frac{v^2}{\rho}$.

$$a_t = -\frac{F_y + F_n}{m} = -\mu_k g - \mu_k \frac{v^2}{\rho} = -14.89 \text{ m/s}^2$$

3/350 (Roman numeral: process; Arabic number: state)

I. Engine moves 1 ft: $U = \Delta T: Fd = \frac{1}{2} m (v_2^2 - v_1^2)$
(State ① $\rightarrow$ State ②) $40,000(1) = \frac{1}{2} \frac{400,000}{32.2} (v_2^2 - 0^2)$

$$v_2 = 2.54 \text{ ft/sec}$$

II. "Collision" with A: $m_L v_2 = (m_L + m_A) v_3$
(② $\rightarrow$ ③) $400,000(2.54) = 600,000 v_3, v_3 = 1.692 \frac{\text{ft}}{\text{sec}}$

III. L & A move 1 ft: $40,000(1) = \frac{1}{2} \frac{600,000}{32.2} (v_4^2 - 1.692^2)$
(③ $\rightarrow$ ④) $v_4 = 2.67 \text{ ft/sec}$

IV. "Collision" with B: $(m_L + m_A) v_4 = (m_L + m_A + m_B) v_5$
(④ $\rightarrow$ ⑤) $600,000(2.67) = 800,000 v_5$

$$v_5 = 2.01 \text{ ft/sec}$$

V. L, A, & B move 1 ft: $40,000(1) = \frac{800,000}{32.2} (v_6^2 - 2.01^2)$
(⑤ $\rightarrow$ ⑥) $v_6 = 2.69 \text{ ft/sec}$

VI. "Collision" with C: (⑥ $\rightarrow$ ⑦)

$$(m_L + m_A + m_B) v_6 = (m_L + m_A + m_B + m_C) v_7$$

$$800,000(2.69) = 1,000,000 v_7$$

(a)

$$\underline{v_7 = 2.15 \text{ ft/sec} = v}$$

With no slack,

$$U = \Delta T: 40,000(3) = \frac{1}{2} \frac{10^6}{32.2} (v'^2 - 0^2)$$

(b)

$$\underline{v' = 2.78 \text{ ft/sec}}$$

$$\begin{aligned} \underline{3/35/} \quad D \text{ to } E : \quad y &= y_0 + v_{y0}t - \frac{1}{2}gt^2 \\ -p &= -\frac{1}{2}gt^2, \quad t = \sqrt{\frac{2p}{g}} \\ x &= x_0 + v_{x0}t : \quad d = v_D \sqrt{\frac{2p}{g}}, \quad v_D = d \sqrt{\frac{g}{2p}} \end{aligned}$$

$$A \text{ to } D : \quad U = \Delta T$$

$$\frac{1}{2}k\delta^2 - \mu_k mgp - mgp = \frac{1}{2}m\left(d^2 \frac{g}{2p}\right) - 0$$

$$\delta = \sqrt{\frac{mg}{k}} \sqrt{\frac{d^2}{2p} + 2p(1 + \mu_k)}$$

But speed at top of hill must be ≥ 0 :

$$U = \Delta T : \quad \frac{1}{2}k\delta^2 - \mu_k mgp - 3mgp = \frac{1}{2}mv^2 - 0 \geq 0$$

$$\text{or } \delta \geq \sqrt{\frac{2mgp}{k}} (3 + \mu_k)$$

$$\therefore \frac{mg}{k} \left(\frac{d^2}{2p} + 2p[1 + \mu_k] \right) \geq \frac{2mgp}{k} (3 + \mu_k)$$

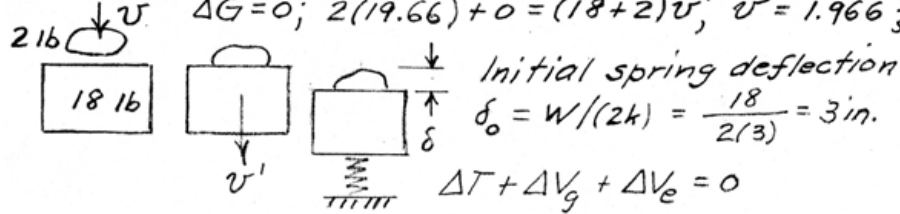
$$\text{or } \underline{d \geq 2\sqrt{2}p}$$

3/352 For a minimum escape orbit (parabolic) $e=1$ & $a \rightarrow \infty$
so from Eq. 3/47

$$v_{esc} = R \sqrt{\frac{2g}{R+H}} = 6371 \sqrt{\frac{2 \times 9.825 \times 10^{-3}}{6371+2000}} \times 3600$$
$$= 35\,140 \text{ km/h}$$

$$\text{Thus } \Delta v = 35140 - 26140 = \underline{9000 \text{ km/h}}$$

3/353 $v = \sqrt{2gh} = \sqrt{2(32.2)(6)} = 19.66 \text{ ft/sec}$
 $\Delta G = 0; 2(19.66) + 0 = (18+2)v'; v' = 1.966 \frac{\text{ft}}{\text{sec}}$



$$\Delta T = 0 - \frac{1}{2} \frac{20}{32.2} (1.966)^2 = -1.200 \text{ ft-lb}$$

$$\Delta V_g = -20 \delta / 12 = -1.667\delta \text{ ft-lb where } \delta \text{ is in inches}$$

$$\Delta V_e = \frac{1}{2} (2)(3) \left[(3+\delta)^2 - 3^2 \right] \frac{1}{12} = \frac{(3+\delta)^2 - 9}{4} \text{ ft-lb}$$

$$\text{Thus } -1.200 - 1.667\delta + \frac{(3+\delta)^2 - 9}{4} = 0$$

$$\text{or } \delta^2 - 0.667\delta - 4.800 = 0$$

$$\delta = \frac{0.667}{2} \pm \frac{1}{2} \sqrt{0.444 + 19.20} = 0.333 \pm 2.216$$

$$\delta = 2.55 \text{ in. (or } \delta = -1.88 \text{ in.)}$$

3/354

$$\rightarrow G_1 = G_2$$

$$m v_A + m v_B = 2 m v'$$

$$3 - 5 = 2 v', \quad v' = -1 \text{ m/s (left)}$$

$$H_{G_1} = H_{G_2} : \quad m r v_A + m r v_B = 2 m r \dot{\theta}'$$

$$\dot{\theta}' = \frac{v_A + v_B}{2r} = \frac{3 + 5}{2(0.4)}$$

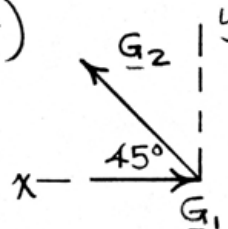
$$= \underline{10 \text{ rad/s (CCW)}}$$

$$3/355 \quad \text{From S.P. 2/6, } 2s = \frac{v^2 \sin 2\theta}{g}$$

$$350 = \frac{v^2 \sin 90^\circ}{32.2}, \quad v = 106.2 \text{ ft/sec}$$

$$\underline{G}_1 = m \underline{v}_1 = \frac{5/16}{32.2} \left(90 \frac{5280}{3600} \right) (-\underline{i}) = -1.281 \underline{i} \text{ lb-sec}$$

$$\begin{aligned} \underline{G}_2 = m \underline{v} &= \frac{5/16}{32.2} 106.2 \left(\frac{\underline{i}}{\sqrt{2}} + \frac{\underline{j}}{\sqrt{2}} \right) \\ &= 0.729 (\underline{i} + \underline{j}) \text{ lb-sec} \end{aligned}$$



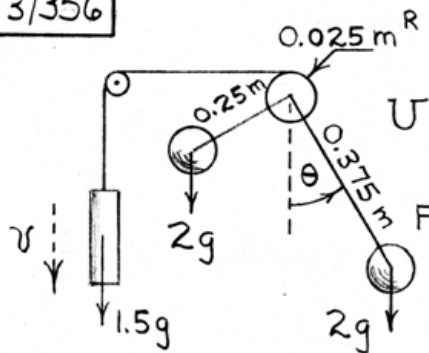
$$\underline{F}_{av} \Delta t = \underline{G}_2 - \underline{G}_1 : \underline{F}_{av} (0.005) = 0.729(\underline{i} + \underline{j}) - (-1.281 \underline{i})$$

$$\underline{F}_{av} = 402 \underline{i} + 145.7 \underline{j} \text{ lb}$$

$$F_{av} = \sqrt{402^2 + 145.7^2} = \underline{428 \text{ lb}}$$

(Note: The weight of the baseball is ignored during its impact with the bat. With the weight included, F_{av} still rounds to 428 lb!)

3/356



Work done during angular displacement θ is

$$U = 1.5g(0.025\theta) + 2g(0.25\sin\theta) - 2g(0.375)(1 - \cos\theta)$$

$$\text{For } \theta = 30^\circ, U = 1.659 \text{ J}$$

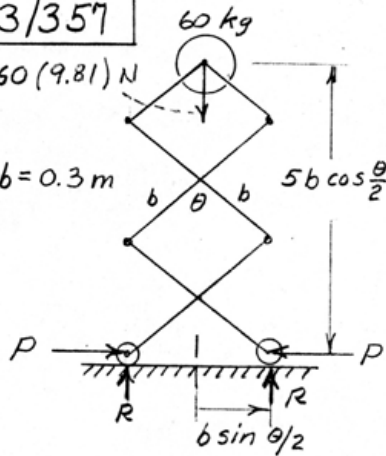
$$\Delta T = \frac{1}{2} \left[1.5 v^2 + 2 \left(\frac{0.25}{0.025} v \right)^2 + 2 \left(\frac{0.375}{0.025} v \right)^2 \right]$$

$$= 325.8 v^2. \quad U_{1-2} = \Delta T \text{ yields } \underline{0.0714 \text{ m/s} \left(71.4 \frac{\text{mm}}{\text{s}} \right)}$$

3/357

$$60(9.81) \text{ N}$$

$$b = 0.3 \text{ m}$$



$$U_{1-2}' = \Delta V_g + \Delta T$$

$$\theta_1 = 120^\circ, v_1 = 0$$

$$\theta_2 = 60^\circ, v_2 = 3 \text{ m/s}$$

$$U_{1-2}' = 2Pb \left(\sin \frac{\theta_1}{2} - \sin \frac{\theta_2}{2} \right)$$

$$= 0.6P(\sin 60^\circ - \sin 30^\circ)$$

$$= 0.2196P \text{ J}$$

$$\Delta V_g = mg \Delta h$$

$$= mg(5b) \left(\cos \frac{\theta_2}{2} - \cos \frac{\theta_1}{2} \right)$$

$$= 60(9.81)(1.5)(\cos 30^\circ - \cos 60^\circ)$$

$$= 323.2 \text{ J}$$

$$\Delta T = \frac{1}{2} 60(3)^2 - 0 = 270 \text{ J}$$

$$\text{Thus } 0.2196P = 323.2 + 270, \quad P = \frac{323.2 + 270}{0.2196} = 2701 \text{ N}$$

$$\text{or } P = 2.70 \text{ kN}$$

For complete system

$$\sum F = ma; \quad 2R - 60(9.81) = 60(20)$$

$$R = 894 \text{ N}$$

3/358 | Drop of A (state ① → state ②) :

$$T_1 + U_{1-2} = T_2 : 0 + m_A g 1.8 (1 - \cos 60^\circ) = \frac{1}{2} m_A v_{A2}^2$$

$$v_{A2} = 4.20 \text{ m/s}$$

Collision (② → ③) :

$$\begin{cases} m_A v_{A2} + m_B v_{B2}^{\nearrow 0} = m_A v_{A3} + m_B v_{B3} & (1) \end{cases}$$

$$\begin{cases} v_{B3} - v_{A3} = 0.7 (v_{A2} - v_{B2}^{\nearrow 0}) & (2) \end{cases}$$

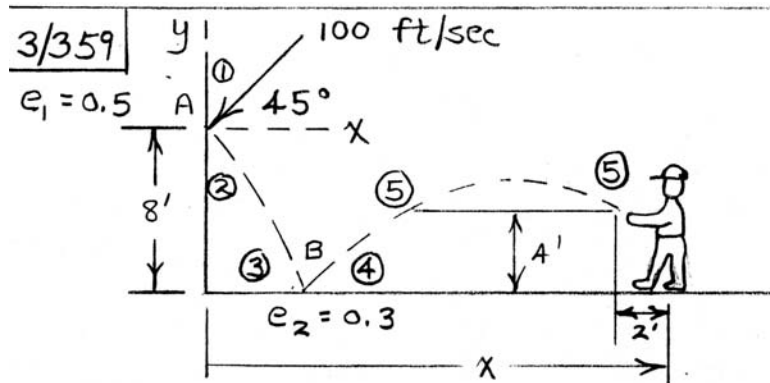
$$\text{Solution : } v_{A3} = 2.42 \text{ m/s}, v_{B3} = 5.36 \text{ m/s}$$

Rise of B (③ → ④) :

$$T_3 + U_{3-4} = T_4 :$$

$$\frac{1}{2} m_B (5.36)^2 - m_B (9.81) [2.4(1 - \cos 30^\circ) + s \sin 30^\circ] = 0$$

$$\underline{s = 2.28 \text{ m}}$$



Use coordinates & states ① - ⑤ shown.

$$v_{1x} = -100 \cos 45^\circ = -70.7 \text{ ft/sec}$$

$$v_{1y} = -100 \sin 45^\circ = -70.7 \text{ ft/sec}$$

$$v_{2x} = -e_1 v_{1x} = -0.5(-70.7) = 35.4 \text{ ft/sec}$$

$$v_{2y} = v_{1y} = -70.7 \text{ ft/sec}$$

$$v_{3x} = v_{2x} = 35.4 \text{ ft/sec}$$

$$v_{3y} = -\sqrt{v_{2y}^2 + 2g(8)} = -\sqrt{70.7^2 + 2(32.2)(8)} = -74.3 \frac{\text{ft}}{\text{sec}}$$

$$v_{3y} = v_{2y} - gt_3 : -74.3 = -70.7 - 32.2 t_3, \quad t_3 = 0.1104 \text{ sec}$$

$$v_{4x} = v_{3x} = 35.4 \text{ ft/sec}$$

$$v_{4y} = -e_2 v_{3y} = -0.3(-74.3) = 22.3 \text{ ft/sec}$$

$$y_5 = y_4 + v_{y4} t_5 - \frac{1}{2} g t_5^2 :$$

$$-4 = -8 + 22.3 t_5 - 16.1 t_5^2 : \quad t_5 = 0.212, 1.172 \text{ sec}$$

$$\text{Then } x = x_3 + v_{4x} t_5 + 2, \text{ where } x_3 = v_{2x} t_3$$

$$= 35.4(0.1104) = 3.73 \text{ ft}$$

$$\text{Thus } x = 3.73 + 35.4(0.212) + 2 = \underline{13.40 \text{ ft}}$$

$$\text{or } x = 3.73 + 35.4(1.172) + 2 = \underline{47.3 \text{ ft}}$$

3/360 Results of Prob. 3/309 :

$$\Delta v_A = R \sqrt{\frac{g}{r_1}} \left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1 \right)$$

Nominally,

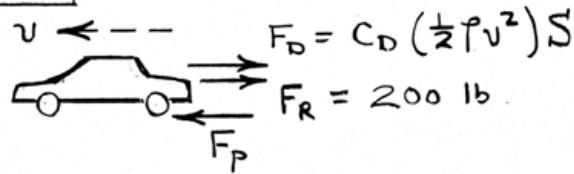
$$(\Delta v_A)_n = (3959)(5280) \sqrt{\frac{32.23}{(3959+170)(5280)}} \times \\ \left(\sqrt{\frac{2(3959+22,300)}{(3959+170)+(3959+22,300)}} - 1 \right) = 7997 \frac{\text{ft}}{\text{sec}}$$

Actually,

$$(\Delta v_A)_a = (3959)(5280) \sqrt{\frac{32.23}{(3959+170)(5280)}} \times \\ \left(\sqrt{\frac{2(3959+700)}{(3959+170)+(3959+700)}} - 1 \right) = 755 \frac{\text{ft}}{\text{sec}}$$

$$\frac{(\Delta v_A)_a}{(\Delta v_A)_n} = \frac{t'}{t}, \quad t' = \frac{(\Delta v_A)_a}{(\Delta v_A)_n} t = \frac{755}{7997} (90) = \underline{8.50 \text{ sec}}$$

► 3/361



$$\leftarrow \sum F = 0 : F_P = F_D + F_R$$

$$\text{Undamaged : } F_P = 0.3 \left[\frac{1}{2} \frac{0.07530}{32.2} \left(200 \cdot \frac{5280}{3600} \right)^2 \right] 30 + 200 = 1105 \text{ lb}$$

$$\begin{aligned} \text{Power required : } P &= F_P \cdot v = 1105 \cdot \left(200 \cdot \frac{5280}{3600} \right) \\ &= 324(10^3) \text{ ft-lb/sec} \end{aligned}$$

Damaged (power available is unchanged)

$$P = F_P' \cdot v' : 324(10^3) = \left[0.4 \left(\frac{1}{2} \frac{0.07530}{32.2} v'^2 \right) 30 + 200 \right] v'$$

$$\text{Solve cubic : } v' = 268 \text{ ft/sec or } \underline{182.9 \text{ mi/hr}}$$

$$\triangleright 3/362 \left\{ \begin{aligned} F_R &= -k_1 v, & k_1 &= 0.833 \frac{\text{lb-hr}}{\text{mi}} = 0.5682 \frac{\text{lb-sec}}{\text{ft}} \\ F_D &= -k_2 v^2, & k_2 &= 0.0139 \frac{\text{lb-hr}^2}{\text{mi}^2} = 0.006457 \frac{\text{lb-sec}^2}{\text{ft}^2} \end{aligned} \right.$$

$$(a) P_{30} = Fv = [0.833(30) + 0.0139(30)^2] \left[30 \left(\frac{5280}{3600} \right) \right]$$

$$= 1650 \frac{\text{ft-lb}}{\text{sec}} = \underline{3 \text{ hp}}$$

$$P_{60} = Fv = [0.833(60) + 0.0139(60)^2] \left[60 \left(\frac{5280}{3600} \right) \right]$$

$$= 8800 \frac{\text{ft-lb}}{\text{sec}} = \underline{16 \text{ hp}}$$

$$(b) -k_1 v - k_2 v^2 = m \frac{dv}{dt}$$

$$\int_0^t dt = -m \int_{v_1}^{v_2} \frac{dv}{v(k_1 + k_2 v)}$$

$$t = -\frac{m}{k_1} \ln \left[\frac{v_2 (k_1 + k_2 v_1)}{v_1 (k_1 + k_2 v_2)} \right]$$

$$t = -\frac{2000/32.2}{0.5682} \ln \left[\frac{7.33(0.5682 + 0.006457(88))}{88(0.5682 + 0.006457(7.33))} \right]$$

$$= \underline{205 \text{ sec}}$$

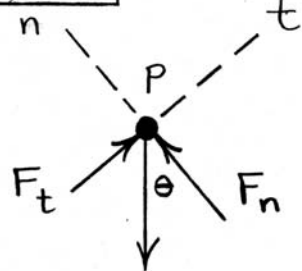
$$-k_1 v - k_2 v^2 = m v \frac{dv}{ds}$$

$$\int_0^s ds = -m \int_{v_1}^{v_2} \frac{dv}{k_1 + k_2 v}$$

$$s = -\frac{m}{k_2} \ln (k_1 + k_2 v) \Big|_{v_1}^{v_2}$$

$$= -\frac{m}{k_2} \ln \left[\frac{k_1 + k_2 v_2}{k_1 + k_2 v_1} \right] = \underline{5898 \text{ ft}}$$

*3/363



$$\theta = t^2, \quad \dot{\theta} = 2t, \quad \ddot{\theta} = 2 \text{ rad/s}^2$$

$$0.4(9.81) = 3.924 \text{ N}$$

$$\sum F_n = m a_n: F_n - 3.924 \cos t^2 = 0.4(1.5)(2t)^2$$

$$F_n = 3.924 \cos t^2 + 2.4 t^2 \quad (\text{N})$$

$$\sum F_t = m a_t: F_t - 3.924 \sin t^2 = 0.4(1.5)(2)$$

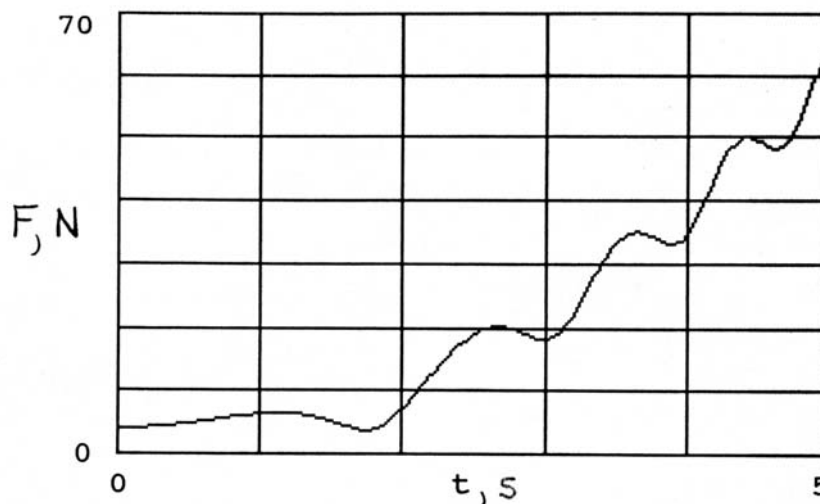
$$F_t = 3.924 \sin t^2 + 1.2 \quad (\text{N})$$

$$F = \sqrt{F_n^2 + F_t^2}; \text{ simplify to}$$

$$F = \sqrt{16.84 + 18.84 t^2 \cos^2 t^2 + 9.42 \sin^2 t^2 + 5.76 t^4}$$

$$\text{When } F = 30 \text{ N}, \quad t = 3.40 \text{ s (numerically)}$$

$$\theta = 3.40^2 = 11.57 \text{ rad or } \theta = 663^\circ$$



*3/364 Power $P = Fv$ where $F = k(b-x)$

$$F = 1.8(10^3)(0.1-x) \text{ N}$$

$$U = \Delta T: U = \int_0^x F dx = \int_0^x 1.8(10^3)(0.1-x) dx$$

$$= 1.8(10^3)(0.1x - \frac{x^2}{2}) \text{ J}$$

$$\Delta T = \frac{1}{2}mv^2 - 0 = \frac{1}{2}3v^2 \text{ J}$$

$$\text{Thus } v^2 = \frac{2}{3} 1.8(10^3)(0.1x - \frac{x^2}{2})$$

$$P^2 = F^2 v^2 = (1.8)^2(10^6)(0.1-x)^2 \times \frac{2}{3} 1.8(10^3)(0.1x - \frac{x^2}{2})$$

$$= 3.89(10^9)(0.1-x)^2(0.1x - \frac{x^2}{2}) \text{ watts}^2 \text{ with } x \text{ in meters}$$

$$\frac{d(P^2)}{dx} = 3.89(10^9) \{ 2(0.1-x)(-1)(0.1x - \frac{x^2}{2}) + (0.1-x)^2(0.1-x) \}$$

$$= 3.89(10^9)(-0.2x + x^2 + 0.01 - 0.2x + x^2)(0.1-x)$$

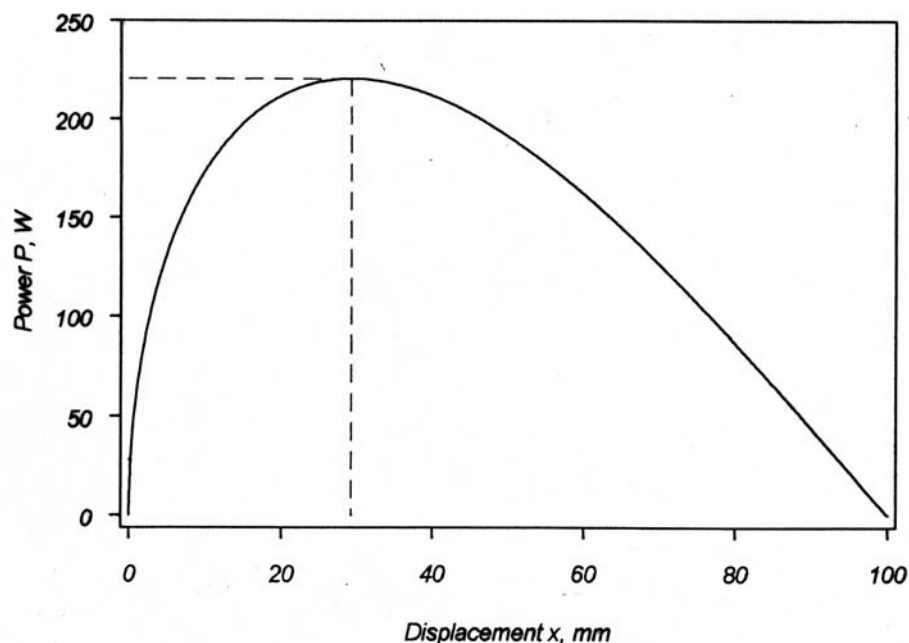
$$= 3.89(10^9)(0.1-x)(x^2 - 0.2x + 0.005) \times 2 = 0 \text{ for max or min}$$

$$\text{so } x = 0.1 \text{ or } x = 0.1 \pm 0.0707 = 0.1707 \text{ m or } \underline{x = 0.0293 \text{ m}}$$

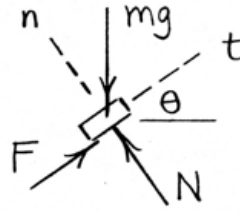
$$P = \sqrt{3.89(10^8)(0.1-x)} \sqrt{0.1x - 0.5x^2} \text{ W}$$

Substitute x from above & get

$$\underline{P_{\max} = 220 \text{ W}}$$



*3/365



$$a_n = r\Omega^2 = \frac{13}{12} (7.5)^2 = 60.9 \frac{\text{ft}}{\text{sec}^2}$$

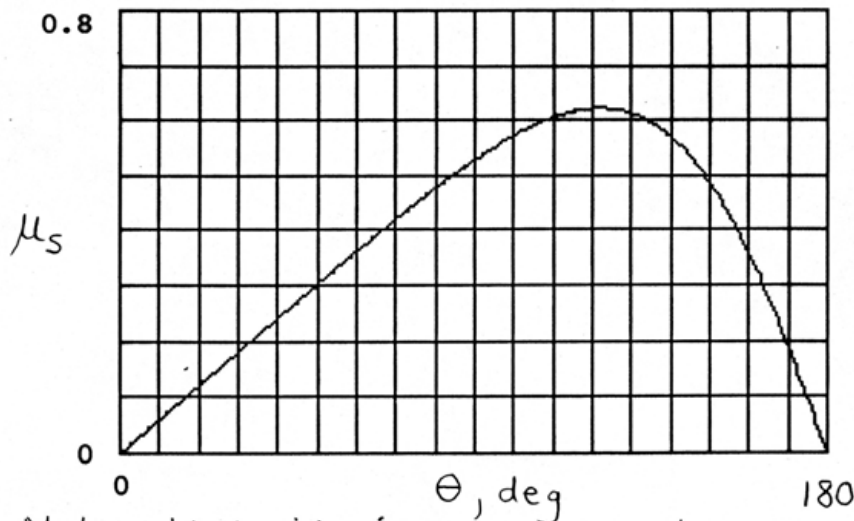
$$\begin{cases} \sum F_n = ma_n : N - mg \cos \theta = 60.9 m \\ \sum F_t = ma_t : F - mg \sin \theta = 0 \end{cases}$$

Slipping impends when $F = F_{\max} = \mu_s N$

Simultaneous solution: $\mu_s = \frac{32.2 \sin \theta}{60.9 + 32.2 \cos \theta}$

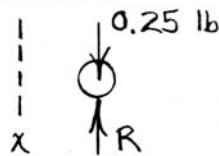
See plot of μ_s vs. θ below. Set $\frac{d\mu_s}{d\theta} = 0$ or numerically determine that

$$\mu_{\min} = 0.622 @ \theta = 121.9^\circ$$



Note that it is impossible to see slippage first occur at any angle greater than $\theta = 121.9^\circ$!

*3/366



$$\Sigma F_x = ma_x: 0.25 - R = \frac{0.25}{32.2} a$$

$$(0.25 - R) dx = \frac{0.25}{32.2} a dx$$

$$\text{But } a dx = v dv, \text{ so}$$

$$(0.25 - R) dx = \frac{0.25}{32.2} v dv = 0.00388 d(v^2)$$

$$\text{For small intervals: } (0.25 - R) \Delta x = 0.00388 \Delta(v^2)$$

$$\text{or } \Delta(v^2) = (64.4 - 258R) \Delta x$$

Set up program to produce the following table:

x ft	Δx ft	R lb	$64.4 - 258R$ ft/sec ²	Δv^2 (ft/sec) ²	v^2 (ft/sec) ²	v ft/sec
0		0	64.4		0	0
	1			64.4		
1		0.04	54.1		64.4	8.02
	1			54.1		
2		0.08	43.7		118.5	10.9
	1			43.7		
3		0.13	30.9		162.2	12.7
	1			30.9		
4		0.16	23.1		193.1	13.9
	1			23.1		
5		0.19	15.4		216.2	14.7
	1			15.4		
6		0.21	10.2		231.6	15.2
	1			10.2		
7		0.22	7.6		241.8	15.6
	1			7.6		
8		0.23	5.1		249.4	15.8
	1			5.1		
9		0.25	0		254.5	16.0
	1			0		
10					254.5	16.0

$$\text{So } v = 16.0 \text{ ft/sec}$$

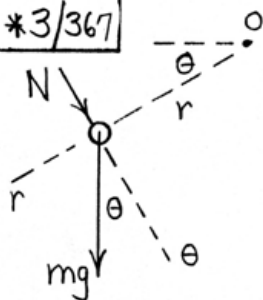
$$\text{For } R = kv^2, W - kv^2 = \frac{W}{g} a$$

$$\int_0^x \frac{g}{W} dx = \int_0^v \frac{v dv}{W - kv^2}$$

$$\Rightarrow v = \sqrt{\frac{W}{k} (1 - e^{-2gkx/W})}$$

$$\text{With numbers, } v = 16.3 \text{ ft/sec}$$

*3/367



$$\Sigma F_r = m a_r = m(\ddot{r} - r\dot{\theta}^2):$$

$$mg \sin \theta = m(\ddot{r} - r\omega_0^2)$$

$$\ddot{r} - \omega_0^2 r = g \sin \omega_0 t$$

Assume $r_h = C e^{st}$ to obtain

$$s_1 = -\omega_0, \quad s_2 = \omega_0. \quad \text{Assume a}$$

particular solution of form $r_p = D \sin \omega_0 t$

and find $D = -\frac{g}{2\omega_0^2}$. So

$$r = r_h + r_p = C_1 e^{-\omega_0 t} + C_2 e^{\omega_0 t} - \frac{g}{2\omega_0^2} \sin \omega_0 t$$

Use the initial conditions $r(0) = \dot{r}(0) = 0$ to find C_1 and C_2 , allowing us to write the solution as

$$r = \frac{g}{4\omega_0^2} (-e^{-\theta} + e^{\theta} - 2 \sin \theta)$$

Now, set $r = 1$ m and $\omega_0 = 0.5$ rad/s

and use Newton's method to solve for

θ as $\theta = 0.535$ rad, or 30.6°. From

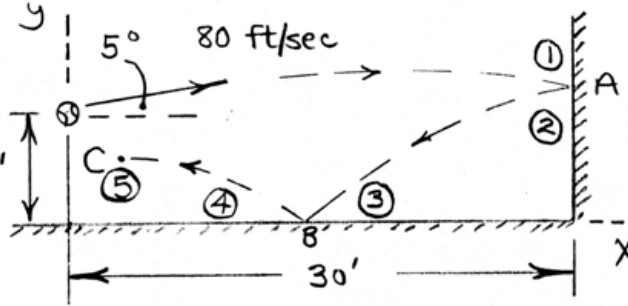
$$\theta = \omega_0 t, \quad t = \frac{0.535}{0.5} = \underline{1.069 \text{ s}}.$$

*3/368

Define states

①-⑤ as shown. 3'

$$\begin{cases} v_0 = 80 \text{ ft/sec} \\ \theta = 5^\circ \end{cases}$$



$$v_{0x} = v_0 \cos \theta = 80 \cos 5^\circ = 79.7 \text{ ft/sec}$$

$$v_{0y} = v_0 \sin \theta = 80 \sin 5^\circ = 6.97 \text{ ft/sec}$$

$$t_{01} = \frac{30}{79.7} = 0.376 \text{ sec}$$

$$y_1 = 3 + 6.97(0.376) - 16.1(0.376)^2 = 3.34 \text{ ft}$$

$$v_{1x} = v_{0x} = 79.7 \text{ ft/sec}$$

$$v_{1y} = v_{0y} - g t_{01} = 6.97 - 32.2(0.376) = -5.15 \text{ ft/sec}$$

Now program the following numbered equations:

$$v_{2x} = -e v_{1x} \quad (1)$$

$$v_{2y} = v_{1y} \quad (2)$$

$$v_{3x} = v_{2x} \quad (3)$$

$$v_{3y} = -\sqrt{v_{2y}^2 + 2gy_1} \quad (4)$$

$$t_{23} = (v_{2y} - v_{3y})/g \quad (5)$$

$$x_3 = 30 + v_{2x} t_{23} \quad (6)$$

$$v_{4x} = v_{3x} \quad (7)$$

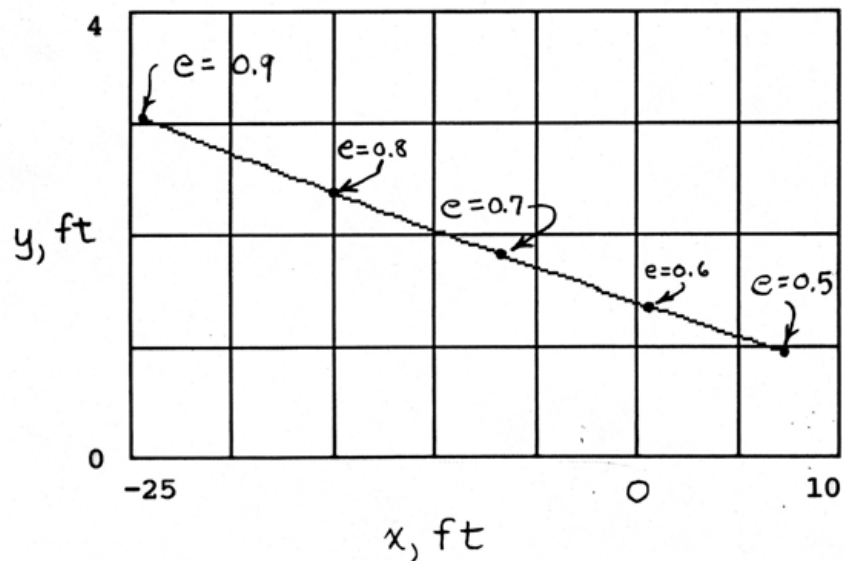
$$v_{4y} = -e v_{3y} \quad (8)$$

$$t_{45} = v_{4y} / g \quad (9)$$

$$x_5 = x_3 + v_{4x} t_{45} = x \quad (10)$$

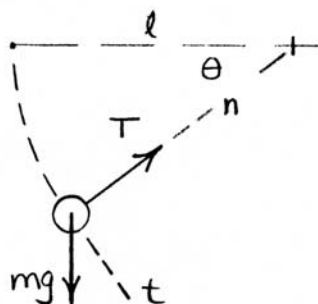
$$y_5 = v_{4y} t_{45} - \frac{1}{2} g t_{45}^2 = y \quad (11)$$

Solve Eqs. (1)-(11) for $0.5 \leq e \leq 0.9$ to obtain the following plot.



For $x=0$, $e=0.610$, $y=1.396$ ft

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$$\begin{aligned} v_0 &= l \dot{\theta}_0 \\ &= 0.5 (0.2) \\ &= 0.1 \text{ m/s} \end{aligned}$$

$$\Sigma F_t = ma_t : mg \cos \theta = m l \ddot{\theta}, \quad \ddot{\theta} = \frac{g}{l} \cos \theta$$

$$v dv = a_t ds : v dv = l \ddot{\theta} (l d\theta) = g l \cos \theta d\theta$$

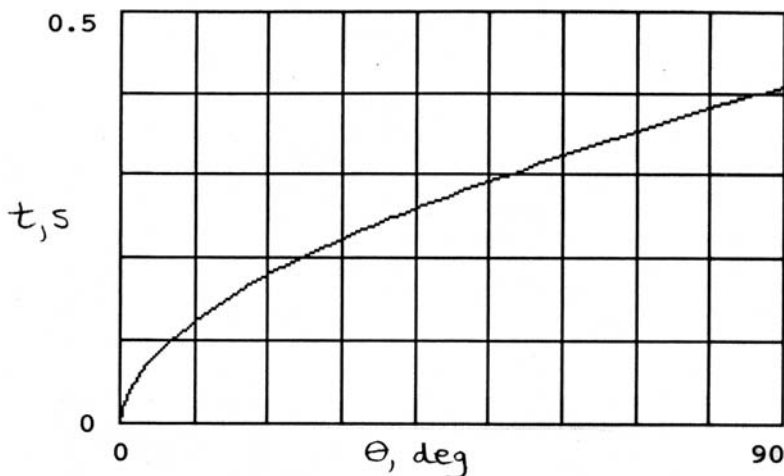
$$\int_{v_0=0.1}^v v dv = \int_{\theta_0=0}^{\theta} g l \cos \theta d\theta, \quad v^2 = 2gl \sin \theta + v_0^2$$

$$v = \frac{ds}{dt} = \frac{l d\theta}{dt} : \sqrt{2gl \sin \theta + v_0^2} = l \frac{d\theta}{dt}$$

$$\text{Rearrange to } \int_{t_0=0}^t dt = \int_{\theta_0=0}^{\theta} \frac{l d\theta}{\sqrt{2gl \sin \theta + v_0^2}}$$

$$\text{So } t = 0.5 \int_0^{\theta} \frac{d\theta}{\sqrt{9.81 \sin \theta + 0.01}}$$

Set up a numerical integration scheme (see Appendix C/12) and integrate the above for various upper limits ($0 \leq \theta \leq \pi/2$)



$$\text{When } \theta = 90^\circ, \quad t = 0.409 \text{ s.}$$

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$$V_g = \rho g \left(\frac{\pi r}{2} - r\theta \right) \bar{r} \sin \alpha - \rho g r \theta \frac{r\theta}{2}$$

$$\text{where } \bar{r} = \frac{r \sin \alpha}{\alpha} = r \frac{\sin(\frac{\pi}{4} - \frac{\theta}{2})}{\frac{\pi}{4} - \frac{\theta}{2}}$$

$$V_g = \rho g r^2 \left\{ \left(\frac{\pi}{2} - \theta \right) \frac{\sin^2(\frac{\pi}{4} - \frac{\theta}{2})}{\frac{\pi}{4} - \frac{\theta}{2}} - \frac{\theta^2}{2} \right\}$$

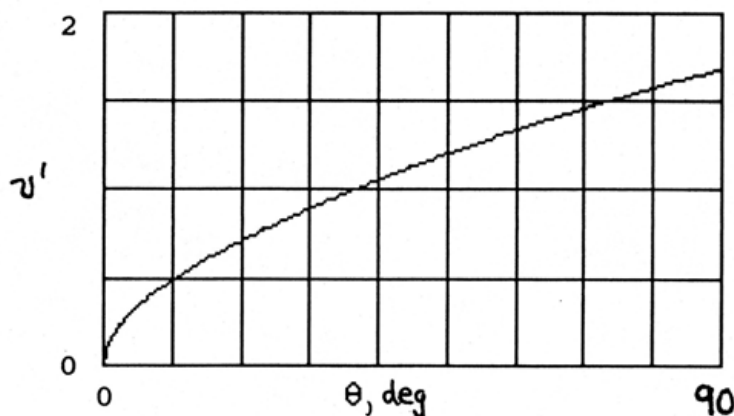
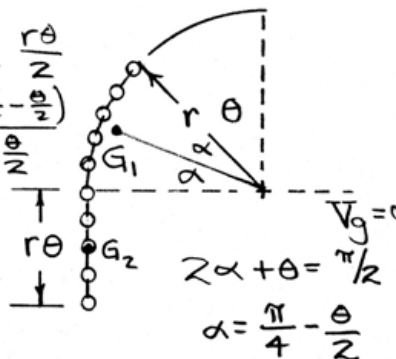
$$\text{For } \theta = 0, V_g = \rho g \frac{\pi r}{2} \frac{2r}{\pi} = \rho g r^2$$

$$\text{Thus } \Delta V_g = (V_g)_\theta - (V_g)_{\theta=0}$$

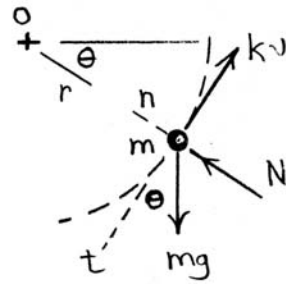
$$= 3 \rho g r^2 \left\{ 1 + \frac{\theta^2}{2} - \left(\frac{\pi}{2} - \theta \right) \frac{\sin^2(\frac{\pi}{4} - \frac{\theta}{2})}{\frac{\pi}{4} - \frac{\theta}{2}} \right\}$$

$$U'_{1-2} = 0 = \Delta T + \Delta V_g; \text{ with } \Delta T = \frac{1}{2} \rho \frac{\pi r}{2} v^2, \text{ we get}$$

$$v' = \frac{v}{\sqrt{g r}} = \sqrt{2} \sqrt{\frac{1}{\pi} (2 + \theta^2) - \left(1 - \frac{2\theta}{\pi} \right) \frac{\sin^2(\frac{\pi}{4} - \frac{\theta}{2})}{\frac{\pi}{4} - \frac{\theta}{2}}}$$



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$$\Sigma F_t = ma_t :$$

$$mg \cos \theta - kv = mv \frac{dv}{ds}$$

But $ds = r d\theta$, so

$$mg \cos \theta - kv = mv \frac{dv}{r d\theta}$$

$$\frac{dv}{d\theta} = \frac{gr \cos \theta}{v} - \frac{kr}{m}$$

It is not possible to separate variables, so we numerically integrate to obtain v as a function of θ .

$$\Sigma F_n = m \frac{v^2}{r} : N - mg \sin \theta = m \frac{v^2}{r}$$

$$N = m \left[g \sin \theta + \frac{v^2}{r} \right]$$

where v is available from the previously mentioned numerical integration. Plots of both v and N as functions of θ are shown below.

The maxima are

$$v_{\max} = 5.69 \text{ ft/sec @ } \theta = 50.8^\circ$$

$$N_{\max} = 2.75 \text{ lb @ } \theta = 66.2^\circ$$

