

4/1

$$\underline{\bar{r}} = \frac{\sum m_i \underline{r}_i}{\sum m_i} = \frac{m(d\underline{i}) + 2m(2d\underline{j}) + 4m(1.5d\underline{k})}{m + 2m + 4m}$$

$$= \frac{d}{7} (\underline{i} + 4\underline{j} + 6\underline{k})$$

$$\underline{\dot{r}} = \frac{\sum m_i \underline{\dot{r}}_i}{\sum m_i} = \frac{m(2v\underline{j}) + 2m(3v\underline{k}) + 4m(v\underline{i})}{7m}$$

$$= \frac{v}{7} (4\underline{i} + 2\underline{j} + 6\underline{k})$$

$$\underline{\ddot{r}} = \frac{\sum \underline{F}}{\sum m_i} = \frac{\underline{F_k}}{7m}$$

$$T = \sum \frac{1}{2} m_i v_i^2 = \frac{1}{2} [m(2v)^2 + 2m(3v)^2 + 4m(v)^2]$$

$$= \underline{13mv^2}$$

$$\underline{H_0} = \sum \underline{r}_i \times m_i \underline{v}_i = 2mvd\underline{k} + 12mvd\underline{i} + 6mvd\underline{j}$$

$$= \underline{mvd(12\underline{i} + 6\underline{j} + 2\underline{k})}$$

$$\underline{\dot{H}_0} = \sum \underline{\dot{M}_0} = \underline{-F d \underline{j}}$$

4/2 From Eq. 4/10 with $\underline{P}$ replaced by

$$0: \quad \underline{H}_0 = \underline{H}_G + \underline{\bar{r}} \times \sum m_i \underline{\bar{v}}$$

$$\text{or} \quad \underline{H}_G = \underline{H}_0 - \underline{\bar{r}} \times \sum m_i \underline{\bar{v}}$$

$$\begin{aligned} \underline{H}_G &= mvd(12\underline{i} + 6\underline{j} + 2\underline{k}) - \frac{d}{7}(\underline{i} + 4\underline{j} + 6\underline{k}) \\ &\quad \times 7m \cdot \frac{v}{7}(4\underline{i} + 2\underline{j} + 6\underline{k}) \\ &= \underline{\frac{mvd}{7}(72\underline{i} + 24\underline{j} + 28\underline{k})} \end{aligned}$$

($\underline{H}_0$, $\underline{\bar{r}}$, and $\underline{\bar{v}} = \dot{\underline{\bar{r}}}$ from Prob. 4/1)

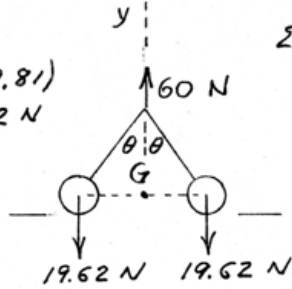
From Eq. 4/11:

$$\underline{\dot{M}}_0 = \underline{H}_G + \underline{\bar{r}} \times \sum m_i \underline{\bar{a}}$$

$$\begin{aligned} \underline{\dot{H}}_G &= \underline{\dot{M}}_0 - \underline{\bar{r}} \times \sum m_i \underline{\bar{a}} \\ &= -Fd\underline{j} - \frac{d}{7}(\underline{i} + 4\underline{j} + 6\underline{k}) \times 7m \left(\frac{F\underline{k}}{7m} \right) \\ &= \underline{-\frac{2Fd}{7}(2\underline{i} + 3\underline{j})} \end{aligned}$$

4/3

$$mg = 2(9.81) \\ = 19.62 \text{ N}$$



$$\Sigma F_y = m\bar{a}_y; 60 - 2(19.62) = 4\bar{a}_y$$

$$\bar{a}_y = \bar{a}_y = 5.19 \text{ m/s}^2$$

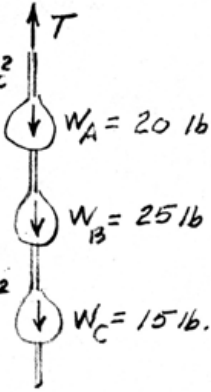
(independent of θ)

4/4

$$a_A = 5 \text{ ft/sec}^2$$

$$a_B = 0$$

$$a_C = 3 \text{ ft/sec}^2$$



$$\Sigma F = \Sigma m_i a_i$$

$$T - (20 + 25 + 15) = \frac{1}{32.2} (20[-5] + 25[0] + 15[3])$$

$$T - 60 = \frac{1}{32.2} (-55)$$

$$T = 60 - 1.708 = \underline{58.3 \text{ lb}}$$

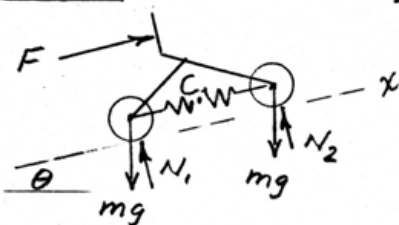
4/5

$$\Sigma F = m\bar{a} : 6.4 = (0.8 + 0.5 + 0.3)\bar{a}$$

$$\bar{a} = 4 \text{ m/s}^2$$

4/6

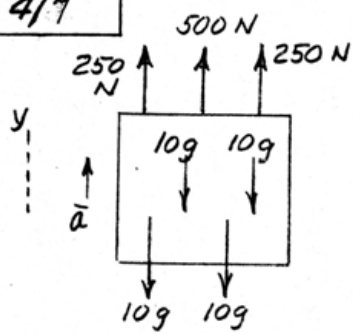
$$\sum F_x = m \bar{a}_x \text{ where } a_c = \bar{a}_x$$



$$F - 2mg \sin \theta = 2ma_c$$

$$a_c = \frac{F}{2m} - g \sin \theta$$

4/7



$$\sum F_y = m\bar{a}_y$$

$$500 + 250 + 250 - 40(9.81) = 40\bar{a}$$

$$40\bar{a} = 1000 - 392$$

$$\bar{a} = \underline{15.19 \text{ m/s}^2}$$

4/8

The principle of the motion of the mass center gives $F = m\bar{a}$ for each case, so the mass-center accelerations are identical. In the two cases of hinged members, however, the mass center is not a point attached to a member, and for these two cases the accelerations of the members will differ.

$$\begin{aligned}
 \boxed{4/9} \quad \underline{F}_{av} &= \frac{\Delta \underline{G}}{\Delta t} = \left[(3.7 - 3.4)\underline{i} + (-2.2 + 2.6)\underline{j} + (4.9 - 4.6)\underline{k} \right] / 0.2 \\
 &= 1.5\underline{i} + 2.0\underline{j} + 1.5\underline{k} \text{ N} \\
 F = |\underline{F}_{av}| &= \sqrt{1.5^2 + 2.0^2 + 1.5^2} = \underline{2.92 \text{ N}}
 \end{aligned}$$

4/10 For system, $\Delta T + \Delta V_g = 0$

$$\Delta T = 3\left(\frac{1}{2}mv^2\right) - 0 = \frac{3}{2}mv^2$$

$$\Delta V_g = 0 - mg\frac{b}{\sqrt{2}} - mg\frac{2b}{\sqrt{2}} = -\frac{3b}{\sqrt{2}}mg$$

$$\text{Thus } \frac{3}{2}mv^2 - \frac{3b}{\sqrt{2}}mg = 0, \quad v^2 = bg\sqrt{2}$$

$$v = \sqrt{bg\sqrt{2}}$$

4/11

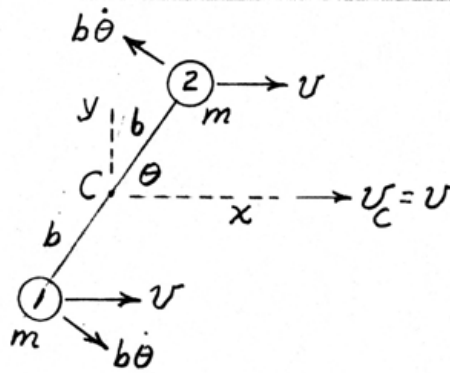
For sphere 1,

$$\underline{G}_1 = m[(v + b\dot{\theta} \sin \theta)\underline{i} - (b\dot{\theta} \cos \theta)\underline{j}]$$

For sphere 2

$$\underline{G}_2 = m[(v - b\dot{\theta} \sin \theta)\underline{i} + (b\dot{\theta} \cos \theta)\underline{j}]$$

$$\underline{G} = \underline{G}_1 + \underline{G}_2 = m[v + v]\underline{i} = \underline{2mv\underline{i}}$$



4/12

$$\begin{aligned}\underline{H}_0 &= \underline{H}_G + \underline{\bar{r}} \times \underline{G}, \quad \underline{G} = 3(3\underline{i} + 4\underline{j}) \text{ kg} \cdot \text{m/s} \\ &= 1.20\underline{k} + (0.4\underline{i} + 0.3\underline{j}) \times 3(3\underline{i} + 4\underline{j}) \\ &= 1.20\underline{k} + 3(1.6\underline{k} - 0.9\underline{k}) \\ &= 1.20\underline{k} + 3(0.7\underline{k}) = \underline{3.3\underline{k} \text{ kg} \cdot \text{m}^2/\text{s}}\end{aligned}$$

4/13

Mass center is center of middle bar, so Eq. 4/1 for the entire system gives

$$\Sigma F = m\bar{a}: 10 = 3 \frac{8}{32.2} a, \quad \underline{a = 13.42 \text{ ft/sec}^2}$$

4/14

$\Sigma M_O = \dot{H}_O$ where O-O is the axis of rotation

$$M = \frac{dH_O}{dt}, \quad \int_0^t M dt = \int_0^{H_O} dH_O = H_O$$

$$Mt = 4m(r\omega)r, \quad t = \frac{4mr^2\omega}{M}$$

$$\boxed{5/15} \quad \Sigma M_O = \dot{H}_O = \frac{dH_O}{dt}, \quad \int \Sigma M_O dt = \Delta H_O$$

$$M_O t = \Delta \left| \Sigma m_i r_i (\dot{r}_i \dot{\theta}) \right| = \Sigma m_i r^2 \Delta \dot{\theta}$$

$$30 \times 5 = [3(0.5)^2 + 4(0.4)^2 + 3(0.6)^2](\dot{\theta}' - 20)$$

$$150 = 2.47(\dot{\theta}' - 20), \quad \dot{\theta}' = 60.7 + 20 = \underline{\underline{80.7 \frac{\text{rad}}{\text{s}}}}$$

4/16

$$\int_0^t M_z dt = H_{z_2} - H_{z_1}, \quad H_z = \sum m_i r_i^2 (\dot{\theta})$$

$$H_z = 2(3)(0.3)^2 \dot{\theta} + 2(3)(0.5)^2 \dot{\theta} = 2.04 \dot{\theta}$$

$$\text{so } 30t = 2.04(20 - [-20]) = 81.6$$

$$\underline{t = 2.72 \text{ s}}$$

4/17

x -----

$v_{rel} = 800 \text{ ft/sec}$

$v_1 = 4 \text{ ft/sec}$

v_2

2000 lb

40 lb

$\Delta G_x = 0, G_1 = G_2$

$$\frac{1}{9}(2000 + 40)4 = \frac{1}{9}(2000 v_2 - 40[800 - v_2])$$

$$8160 = 2040 v_2 - 32000$$

$$\underline{v_2 = 19.69 \text{ ft/sec}}$$

4/18

For entire system $\Delta G_x = 0$, x horiz.

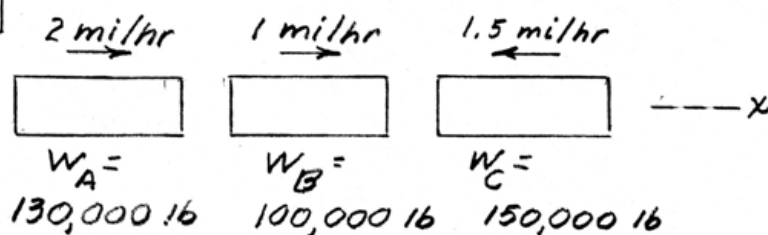
$$(300 + 400 + 100) v$$

$$- (300 \times 0.6 - 400 \times 0.3 + 100 \times 1.2 \cos 30^\circ) = 0$$

$$800 v = 163.9, \quad \underline{v = 0.205 \text{ m/s}}$$

Momentum is conserved regardless of sequence of events, so final velocity would be the same.

4/19



$\sum F_x = 0$ for system so $\Delta G_x = 0$

$$(130 \times 2 + 100 \times 1 - 150 \times 1.5) \frac{44}{30} \frac{10^3}{32.2} - (130 + 100 + 150) v \frac{44}{30} \frac{10^3}{32.2} = 0$$

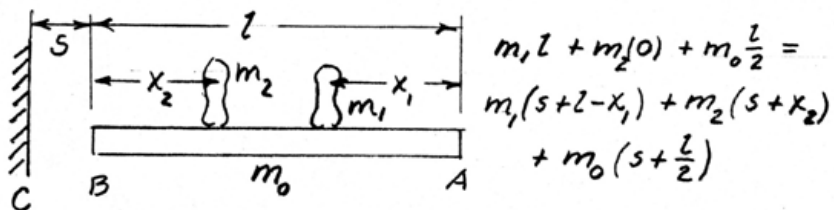
$$v = \frac{260 + 100 - 225}{130 + 100 + 150} = 0.355 \text{ mi/hr}$$

$$\% \text{ loss of energy} = \frac{T_i - T_f}{T_i} 100 = 100 \left(1 - \frac{T_f}{T_i} \right) = n$$

$$n = 100 \left\{ 1 - \frac{\frac{1}{29} (130 + 100 + 150) (0.355)^2}{\frac{1}{29} (130 \times 2^2 + 100 \times 1^2 + 150 \times 1.5^2)} \right\} = 100 \left(1 - \frac{47.96}{957.5} \right)$$

$$\underline{n = 95.0 \%}$$

4/20

With respect to C, $\sum m_i x_i = \text{constant}$ 

$$m_1 l + m_2(0) + m_0 \frac{l}{2} =$$

$$m_1(s+l-x_1) + m_2(s+x_2)$$

$$+ m_0(s+\frac{l}{2})$$

Simplify & get
$$s = \frac{m_1 x_1 - m_2 x_2}{m_0 + m_1 + m_2}$$

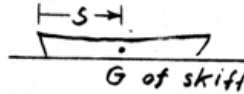
But they meet when $x_2 + x_1 = l$ so

$$s = \frac{(m_1 + m_2) x_1 - m_2 l}{m_0 + m_1 + m_2}$$

4/21

With neglect of hydraulic forces linear momentum is conserved & velocity $U_2 = U_1 = 1 \text{ knot}$. Center of mass does not change position with respect to reference axes moving with constant speed of 1 knot.

$$\text{Thus } (\sum m_i x_i)_1 = (\sum m_i x_i)_2$$



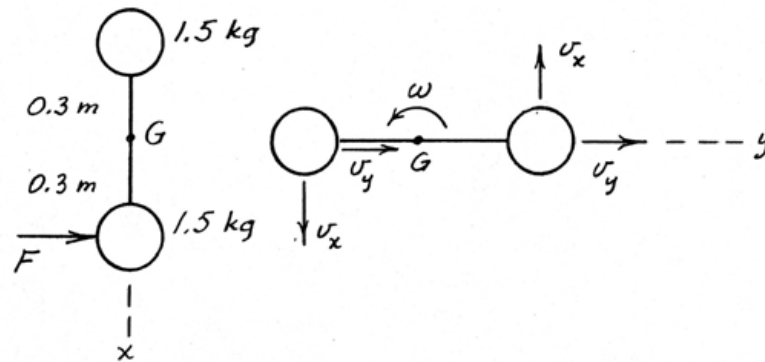
$$\frac{1}{32.2} [120(2) + 180(8) + 160(16) + 300(s)]$$

$$= \frac{1}{32.2} [120(14+x) + 180(4+x) + 160(10+x) + 300(s+x)]$$

$$4240 = 4000 + 760x, \quad x = \frac{240}{760} = 0.316 \text{ ft}$$

Timing & sequence of changed positions does not affect final result because all forces are internal.

4/22



$$\int \Sigma F_x dt = 0 \text{ so } \Delta G_x = 0$$

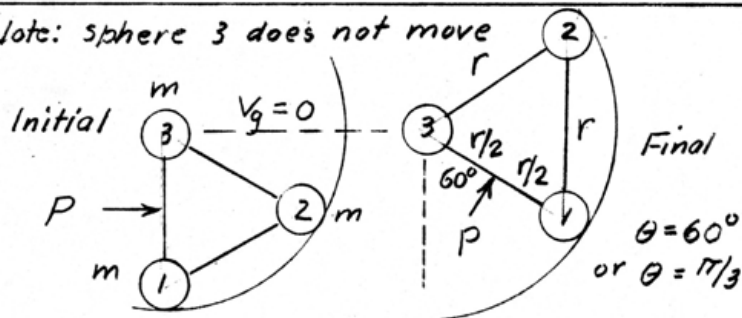
$$\int \Sigma F_y dt = \Delta G_y: 10 = 2(1.5)v_y, v_y = 3.33 \text{ m/s}$$

$$\int \Sigma M_G dt = \Delta H_G: 10(0.3) = 2(1.5)v_x(0.3), v_x = 3.33 \text{ m/s}$$

$$v = 3.33\sqrt{2} = \underline{4.71 \text{ m/s both spheres}}$$

4/23

Note: sphere 3 does not move



$$(a) U' = \Delta V_g : U' = P_{\min} \frac{r}{2} \frac{\pi}{3}$$

$$\Delta V_g = mg \left(r + \frac{r}{2} \right) = \frac{3}{2} mgr$$

$$\text{Thus } \frac{\pi r}{6} P_{\min} = \frac{3}{2} mgr, \quad P_{\min} = \frac{9}{\pi} mg$$

$$(b) U' = \Delta T + \Delta V_g \text{ with } P = 2P_{\min} = \frac{18}{\pi} mg$$

$$U' = \frac{18}{\pi} mg \frac{r}{2} \frac{\pi}{3} = 3mgr, \quad \Delta V_g = \frac{3}{2} mgr$$

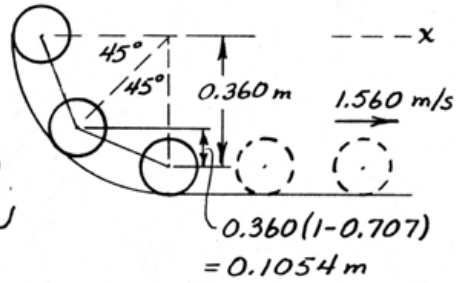
$$\Delta T = 2 \left(\frac{1}{2} m v^2 \right) = m v^2$$

$$\text{Thus } 3mgr = m v^2 + \frac{3}{2} mgr, \quad v = \sqrt{3gr/2}$$

4/24

$$\begin{aligned}
 U'_{1-2} &= \Delta T + \Delta V_g \\
 &= 3\left(\frac{1}{2} \times 2.75 \times 1.560^2\right) - 0 \\
 &\quad - 2.75 \times 9.81(0.360 + 0.1054) \\
 &= 10.04 - 12.56 = -2.52 \text{ J}
 \end{aligned}$$

so loss is $\Delta Q = 2.52 \text{ J}$



$$\begin{aligned}
 I_x &= \int \Sigma F_x dt = \Delta G_x = G_2 - G_1, \quad G_2 = 3mv = 3(2.75)(1.560) \\
 &= 12.87 \text{ N}\cdot\text{s}, \quad G_1 = 0
 \end{aligned}$$

$$\underline{I_x = 12.87 \text{ N}\cdot\text{s}}$$

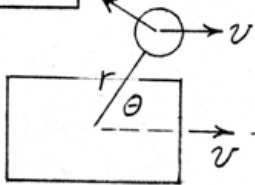
4/25

$$(a) \Sigma F_x = m\bar{a}_x; \quad F = 2m\bar{a}, \quad \bar{a} = F/2m$$

$$(b) H_G = 2m\left(\frac{L}{2}\right)^2\ddot{\theta}, \quad \dot{H}_G = mL^2\ddot{\theta}/2$$

$$\Sigma M_G = \dot{H}_G; \quad Fb = mL^2\ddot{\theta}/2, \quad \ddot{\theta} = \frac{2Fb}{mL^2}$$

4/26



$\Sigma F_x = 0$ for system so $\Delta G_x = 0$

$(G_x)_{\theta=0} = (20 + 5)(0.6) = 15.0 \text{ N}\cdot\text{s}$

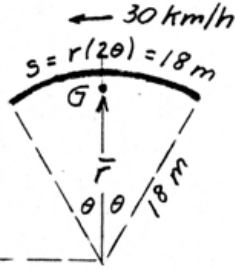
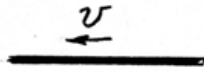
$(G_x)_{\theta=60^\circ} = (20 + 5)v - 5(1.6)\sin 60^\circ$

$= 25v - 6.93 \text{ N}\cdot\text{s}$

$r\dot{\theta} = 0.4(4) = 1.6 \text{ m/s}$

Thus $15.0 = 25v - 6.93$, $v = 21.9/25 = \underline{0.877 \text{ m/s}}$

4/27

 $m = \text{total mass of cars}$ 

$$\begin{aligned} 30 \text{ km/h} \quad \theta &= \frac{s}{2r} = \frac{18}{2(18)} = \frac{1}{2} \text{ rad} \\ &= \frac{1}{2} \frac{180}{\pi} = 28.65^\circ \\ \bar{r} &= \frac{r \sin \theta}{\theta} = \frac{18 \sin 28.65^\circ}{1/2} \\ &= 17.26 \text{ m} \end{aligned}$$

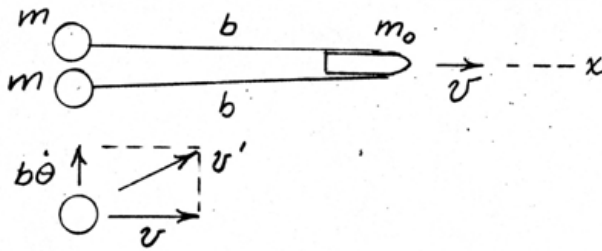
For system $\Delta T + \Delta V_g = 0$

$$\frac{1}{2} m (v^2 - [\frac{30}{3.6}]^2) - mg(17.26) = 0$$

$$\begin{aligned} v^2 &= (30/3.6)^2 + 2(9.81)(17.26) = 69.4 + 338.6 \\ &= 408 \text{ (m/s)}^2 \end{aligned}$$

$$v = 20.2 \text{ m/s} \quad \text{or} \quad v = 20.2(3.6) = \underline{72.7 \text{ km/h}}$$

4/28



For system $\Delta G_x = 0: (m_0 v + 2m v) - m_0 v_0 = 0$

$$v = \frac{m_0}{m_0 + 2m} v_0$$

$U = \Delta T: 0 = \frac{1}{2} m_0 v^2 + 2 \left[\frac{1}{2} m (v^2 + b^2 \dot{\theta}^2) \right] - \frac{1}{2} m_0 v_0^2$

$$(m_0 + 2m) v^2 + 2m b^2 \dot{\theta}^2 = m_0 v_0^2$$

Substitute v & get $\frac{m_0^2 v_0^2}{m_0 + 2m} + 2m b^2 \dot{\theta}^2 = m_0 v_0^2$

Solve for $\dot{\theta}$ & get $\dot{\theta} = \frac{v_0}{b} \sqrt{\frac{m_0}{m_0 + 2m}}$

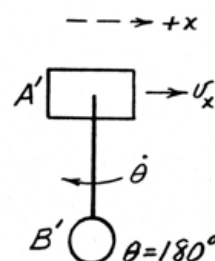
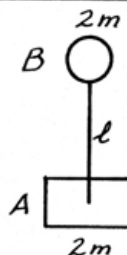
4/29

For entire system,

$$\int \Sigma F_x dt = \Delta G_x = G_{A'} + G_{B'} - 0$$

$$0 = 2m v_x + 2m(v_x - l\dot{\theta})$$

$$2v_x = l\dot{\theta} \quad \text{---- (1)}$$



For entire system,

$$U'_{1-2} = \Delta T + \Delta V_g$$

$$\Delta T = \frac{1}{2}(2m)v_x^2 + \frac{1}{2}(2m)(v_x - l\dot{\theta})^2 - 0$$

$$= m(2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2)$$

$$\Delta V_g = -2mg(2l) = -4mgl$$

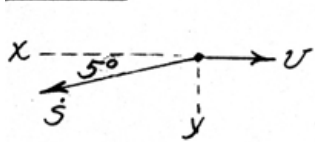
$$U'_{1-2} = 0 \text{ so } 0 = m(2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2) - 4mgl$$

$$\text{or } 2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2 = 4gl \quad \text{--- (2)}$$

$$\text{Combine (1) \& (2): } 2v_x^2 - 4v_x^2 + 4v_x^2 = 4gl, \quad v_x^2 = 2gl, \quad v_x = \sqrt{2gl}$$

$$\dot{\theta} = 2v_x/l = 2\sqrt{2gl}/l = 2\sqrt{\frac{2g}{l}}$$

► 4/30 System is conservative so $\Delta T + \Delta V_g = 0$



$$\begin{aligned} \text{Flatcar; } \Delta T &= \frac{1}{2} m v^2 - 0 \\ &= \frac{1}{2} \frac{50,000}{32.2} v^2 \end{aligned}$$

$$\Delta V_g = 0$$

$$\text{Vehicle; } \Delta T = \frac{1}{2} m v^2 - 0 = \frac{1}{2} \frac{15,000}{32.2} [(\dot{s} \cos 5^\circ - v)^2 + (\dot{s} \sin 5^\circ)^2] - 0$$

$$\Delta V_g = -W \Delta h = -15,000 (40 \sin 5^\circ)$$

$$\begin{aligned} \text{Thus } 776.4 v^2 + 232.9 [(\dot{s} \cos 5^\circ - v)^2 + (\dot{s} \sin 5^\circ)^2] \\ - 52290 = 0 \quad \text{--- (1)} \end{aligned}$$

Also for system, $\sum \vec{F}_x = 0$ so $\Delta G_x = 0$

$$\frac{15,000}{32.2} (\dot{s} \cos 5^\circ - v) - \frac{50,000}{32.2} v = 0$$

$$\begin{aligned} \dot{s} \cos 5^\circ - v &= 3.33 v \quad \& \quad \dot{s} \sin 5^\circ = 4.33 v \tan 5^\circ \\ & & & = 0.379 v \end{aligned}$$

Substitute into (1) & get

$$776.4 v^2 + 232.9 [(3.33 v)^2 + (0.379 v)^2] - 52290 = 0$$

$$v^2 (776.4 + 2588 + 33.5) = 52290$$

$$v^2 = 15.39 \text{ (ft/sec)}^2, \quad \underline{v = 3.92 \text{ ft/sec}}$$

4/31

Initial energy

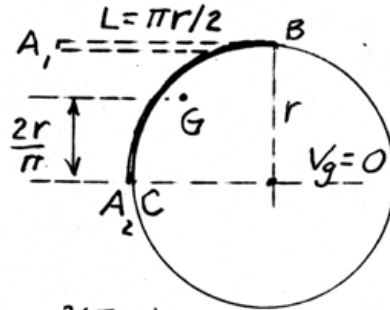
$$V_{g_1} = \rho g \frac{\pi r}{2} r = \rho g \frac{\pi r^2}{2}$$

Final energy

$$V_{g_2} = \rho g \frac{\pi r}{2} \frac{2r}{\pi} = \rho g r^2$$

$$\text{Energy loss } \Delta Q = V_{g_1} - V_{g_2} = \rho g r^2 \left(\frac{\pi}{2} - 1 \right) = \underline{0.571 \rho g r^2}$$

With $\Delta T = 0$ ($T_2 = T_1 = 0$) the loss of potential is dissipated into heat energy due to impact of rope against the drum.



► 4/32

Potential energy of spring

$$\begin{aligned} V_e &= \frac{1}{2} k x^2 \\ &= \frac{1}{2} \frac{200}{20/12} \left(\frac{20}{12} \right)^2 \\ &= 166.7 \text{ ft-lb} \end{aligned}$$

During collapse of spring as $x \rightarrow 0$:

$$v_B = \frac{x'}{20/12} v_A, \quad x' \text{ in feet.}$$

Let ρ = mass per unit length; $dm = \rho dx'$

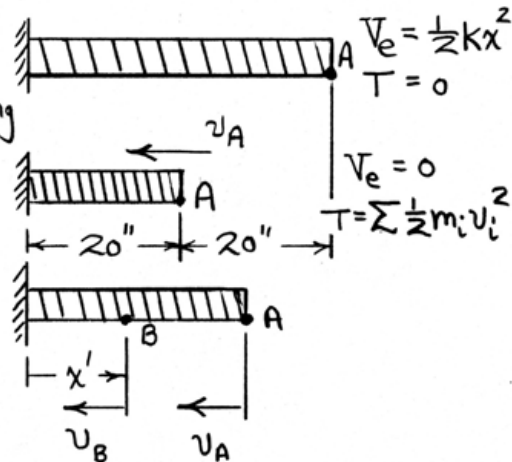
$$T = \int \frac{1}{2} v_B^2 dm = \frac{1}{2} \int_0^{20/12} \frac{x'^2}{(20/12)^2} v_A^2 \rho dx'$$

$$= \frac{1}{6} \rho \frac{20}{12} v_A^2, \quad \text{where } \rho \frac{20}{12} = m$$

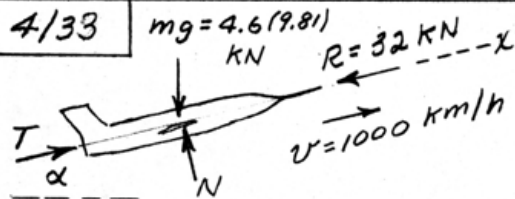
$$\text{So } T = \frac{1}{6} m v_A^2 = \frac{1}{6} \frac{3}{32.2} v_A^2 = \frac{v_A^2}{64.4}$$

$$\text{From } V_e = T: 166.7 = \frac{v_A^2}{64.4}, \quad v_A = 103.6 \text{ ft/sec}$$

(Kinetic energy goes into heat, sound, & deformation.)



4/33



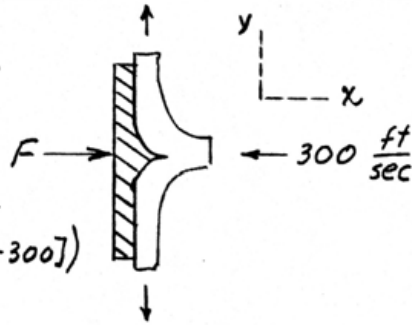
$$\begin{aligned}
 T &= m_a'(u-v) + m_f' u \\
 &= 106(680 - 1000/3.6) \\
 &\quad + 4(680) \\
 &= 45400 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 \Sigma F_x = m a_x = 0; \quad 45.4 - 32 - 4.6(9.81) \sin \alpha &= 0 \\
 \sin \alpha &= 0.2960, \quad \alpha = 17.22^\circ
 \end{aligned}$$

4/34

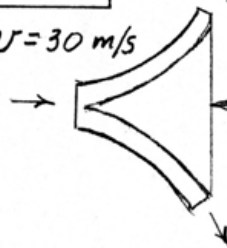
$$\begin{aligned}
 m' &= \frac{\mu}{g} Q \\
 &= \frac{0.0753}{32.2} (6.50) \\
 &= 0.0152 \text{ lb} \cdot \text{sec} / \text{ft}
 \end{aligned}$$

$$\begin{aligned}
 \sum F_x &= m' \Delta v_x: F = 0.0152 (0 - [-300]) \\
 &= \underline{4.56 \text{ lb}}
 \end{aligned}$$



4/35

$U = 30 \text{ m/s}$



60°

$$m' = \rho Q = 1000(0.05) = 50 \text{ kg/s}$$

$$\Sigma F_x = m' \Delta v_x; -F = 50(30 \cos 60^\circ - 30)$$

$$\underline{F = 750 \text{ N}}$$

4/36

Resistance R equals net thrust T

where $T = m'(u - v)$

$$\text{Nozzle velocity } u = Q/A = \frac{0.082}{\pi(0.050)^2} = 41.8 \text{ m/s}$$

Density of salt water, Table D-1, $\rho = 1030 \text{ kg/m}^3$

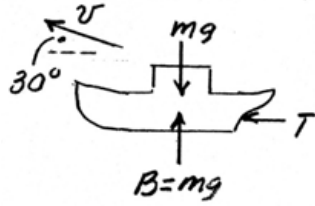
$$m' = \rho Q = 1030(0.082) = 84.5 \text{ kg/s}$$

$$v = 70 \frac{1000}{3600} = 19.44 \text{ m/s}$$

$$R = T = 84.5(41.8 - 19.44) = \underline{1885 \text{ N}}$$

4/37

$$v = 40 \text{ m/s}, m' = \rho Q = 1030 (0.080) \\ = 82.4 \text{ kg/s}$$

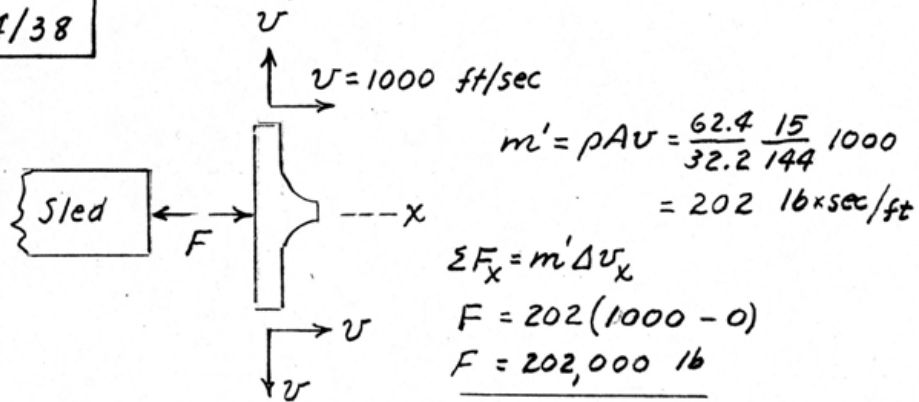


$$\Sigma F = m' \Delta v:$$

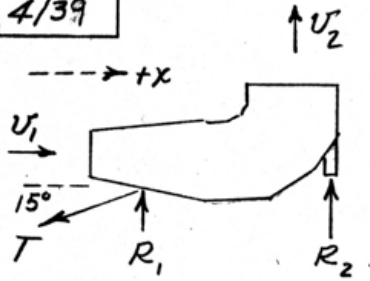
$$T = 82.4 (40 \cos 30^\circ - 0) = 2850 \text{ N}$$

$$\text{or } \underline{T = 2.85 \text{ kN}}$$

4/38



4/39



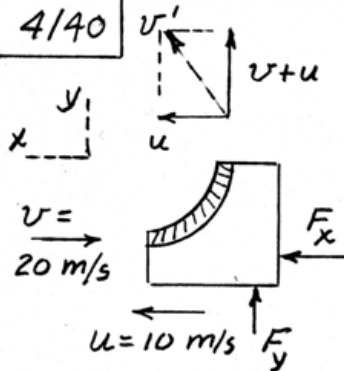
$$\sum F_x = m' \Delta v_x$$

$$-T \cos 15^\circ = (43 + 0.8)(0 - 720)$$

$$T = 32600 \text{ N}$$

$$\text{or } \underline{T = 32.6 \text{ kN}}$$

4/40



Final velocity of diverted water is $\underline{v}' = \underline{u} + \underline{v}_{rel}$

where $v_{rel} = v + u$

$$\dot{m}' = \rho A v_{rel}$$

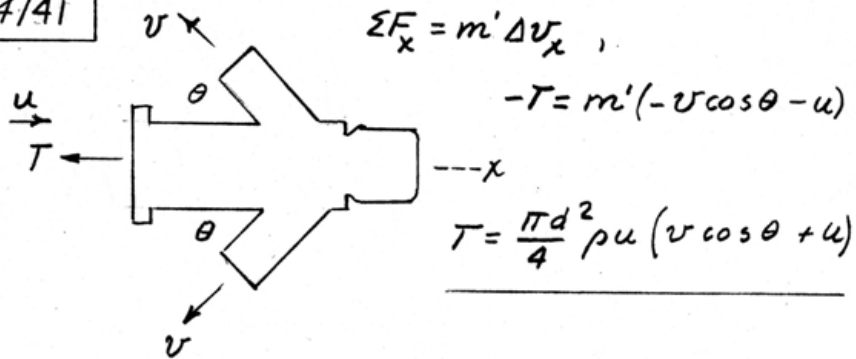
$$= 1000 \frac{\pi}{4} (0.025)^2 (20 + 10)$$

$$= 14.73 \text{ kg/s}$$

$$\sum F_x = \dot{m}' \Delta v_x: F_x = 14.73 (10 - [-20]) = \underline{442 \text{ N}}$$

$$\sum F_y = \dot{m}' \Delta v_y: F_y = 14.73 (20 + 10 - 0) = \underline{442 \text{ N}}$$

4/41



4/42

Ball & stream just under it:

$$\Sigma F_y = m' \Delta v_y:$$

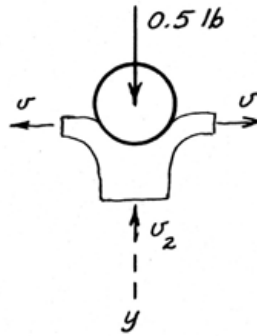
$$m' \text{ at ball} = m' \text{ at nozzle}$$

$$= \rho A v = \frac{62.4}{32.2} \frac{\pi (0.5)^2 \left(\frac{1}{12}\right)^2 \cdot 35}{4}$$

$$= 0.925 \text{ lb-sec/ft}$$

$$\text{so } 0.5 = 0.925 (0 - [-v_2])$$

$$v_2 = 5.41 \text{ ft/sec}$$



For water stream $\Delta V_g + \Delta T = 0$:

$$mgh + \frac{1}{2}m(v_2^2 - v_1^2) = 0,$$

$$h = \frac{1}{2 \times 32.2} (35^2 - 5.41^2) = \underline{18.57 \text{ ft}}$$

4/43

$$\Sigma F = \Sigma m'u$$

With reversers in place,

$$T_R = m'_g u \sin 30^\circ + m'_a v$$

$$T_R = (50 + 0.65)(650) \sin 30^\circ + 50(55.6 - 0)$$

$$= 16460 + 2780$$

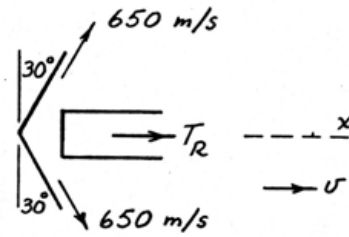
$$= 19240 \text{ N}$$

Without reversers $T = m'_g u - m'_a v$

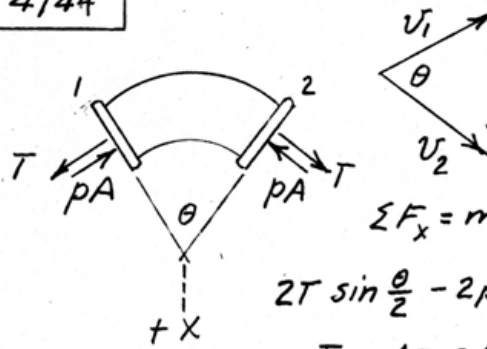
$$T = (50 + 0.65)650 - 50(55.6)$$

$$= 32900 - 2780 = 30100 \text{ N}$$

$$\text{so } n = \frac{19240}{30100} = \underline{0.638}$$



$$v = 200/3.6 = 55.6 \text{ m/s}$$



The diagram shows a fluid element of length Δx in a curved pipe with radius R . The pipe is bent at an angle θ . The fluid element has a cross-sectional area A and is subjected to tension T at both ends and pressure pA on the inner and outer surfaces. A velocity vector triangle is shown to the right, with vectors $\underline{v}_1$ and $\underline{v}_2$ at an angle θ , and the resultant vector $\Delta \underline{v}$ opposite the angle. The magnitude of the resultant is $\Delta v = 2v \sin \frac{\theta}{2}$, where $v = |\underline{v}_1| = |\underline{v}_2|$.

$\Delta v = 2v \sin \frac{\theta}{2}$
 where $v = |\underline{v}_1| = |\underline{v}_2|$

$\Sigma F_x = m' \Delta v_x$ entire system

$2T \sin \frac{\theta}{2} - 2pA \sin \frac{\theta}{2} = \rho A v (2v \sin \frac{\theta}{2})$

so $T - pA = \rho A v^2$, $T = A(p + \rho v^2)$
 independent of θ

4/45 Continuity requires $\rho_A A v_A = \rho_B A v$
 so $v_A = v \rho_B / \rho_A$
 $m' = \rho A v = \rho_B A v$ at B

$$\Sigma F = m' \Delta v; \quad R + p_A A - p_B A = \rho_B A v (v - v_A)$$

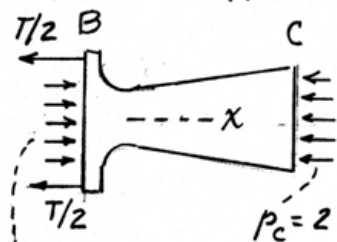
$$= \rho_B A v^2 (1 - \rho_B / \rho_A)$$

Also, $A = \pi d^2 / 4$, so

$$R = \rho_B \frac{\pi d^2}{4} v^2 (1 - \frac{\rho_B}{\rho_A}) + (p_B - p_A) \frac{\pi d^2}{4}$$

$$R = \frac{\pi d^2}{4} \left[\rho_B (1 - \frac{\rho_B}{\rho_A}) v^2 + (p_B - p_A) \right]$$

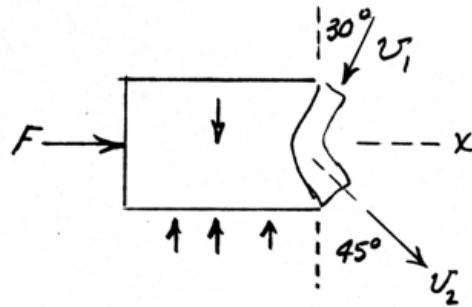
4/46 $A_c = \frac{\pi 4^2}{4(144)} = 0.0873 \text{ ft}^2, A_B = 4A_c = 0.349 \text{ ft}^2$



$m' = \rho A U$
 $m'_B = \frac{0.840}{32.2} (0.349) 50 = 0.455 \frac{\text{lb-sec}}{\text{ft}}$
 $m'_C = \frac{0.0760}{32.2} (0.0873) U_C = 2.06 (10^{-4}) U_C$
 $m'_B = m'_C \text{ so } U_C = \frac{0.455}{2.06 (10^{-4})}$
 $\rightarrow U_B = 50 \text{ ft/sec} \quad = 2210 \text{ ft/sec}$

$\Sigma F_x = m' \Delta U_x: 150(0.349)(144) - 2(0.0873)(144) - T$
 $= 0.455(2210 - 50)$
 $T = 6530 \text{ lb}$

4/47



$$Q = AV:$$

$$\frac{1}{2}(231) = \frac{0.01^2 \pi U_1}{4}$$

$$U_1 = 1.471(10^6) \text{ in./min}$$

$$\text{or } U_1 = 2042 \text{ ft/sec} \quad \& \quad U_2 = 0.60(2042) = 1225 \text{ ft/sec}$$

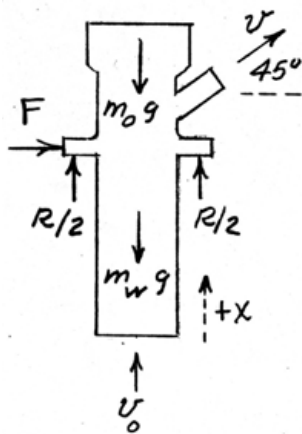
$$\Sigma F_x = m' \Delta U_x: \quad m' = \frac{231}{2} \frac{1}{60} \frac{1}{1728} \frac{68}{32.2} = 2.35(10^{-3}) \text{ lb-sec/ft} \quad (\text{slugs/sec})$$

$$F = 2.35(10^{-3})(1225 \sin 45^\circ - [-2042 \sin 30^\circ])$$

$$F = 4.44 \text{ lb}$$

4/48

$$\Sigma F_x = m' \Delta v_x :$$



$$R - m_o g - m_w g = \rho Q (v \cos 45^\circ - v_o)$$

$$m_o = 310 \text{ kg}$$

$$\begin{aligned} \text{Mass of water } m_w &= \rho V \\ &= 1000 \frac{\pi}{4} (0.2)^2 (6) \\ &= 188.5 \text{ kg} \end{aligned}$$

$$Q = 0.125 \text{ m}^3/\text{s}$$

$$A = \frac{\pi}{4} (0.1)^2 = 0.00785 \text{ m}^2$$

$$A_o = \frac{\pi}{4} (0.25)^2 = 0.0491 \text{ m}^2$$

$$v = Q/A = 0.125/0.00785 = 15.92 \text{ m/s}$$

$$v_o = Q/A_o = 0.125/0.0491 = 2.55 \text{ m/s}$$

$$\begin{aligned} \text{Thus } R - (310 + 188.5) 9.81 &= 1000 (0.125) (15.92 \cos 45^\circ - 2.55) \\ &= 1088 \text{ N} \\ \underline{R} &= \underline{5980 \text{ N}} \end{aligned}$$

4/49

$$Kx = m' \Delta v, \quad m' = \rho A v = 1000 \frac{\pi}{4} (0.030)^2 v$$
$$= 0.7069 v$$

$$15000(0.150) = 0.7069 v(v - 0)$$

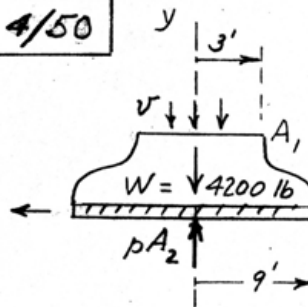
$$v^2 = 3183, \quad \underline{v = 56.4 \text{ m/s}}$$

$$\Sigma M_A = m' v d; \quad M = 15(150)(15 \sin 75^\circ - 4.8 \cos 75^\circ)$$

$$= 2250(13.25) = 29800 \text{ N}\cdot\text{m}$$

$$\text{or } \underline{M = 29.8 \text{ kN}\cdot\text{m}}$$

4/50



$$\Sigma F_y = m' \Delta v_y ; \quad m' = \rho A_1 v$$

$$\Delta v_y = 0 - (-v) = v$$

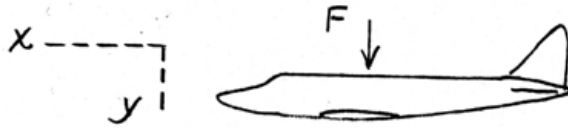
$$\text{so } pA_2 - W = \rho A_1 v^2$$

$$p(\pi \times 9^2)(144) - 4200 = \frac{0.076}{32.2} \pi \times 3^2 (150^2)$$

$$p = 0.1556 \text{ lb/in.}^2$$

4/51

Consider motion as measured from the inertial reference of the aircraft.



$$F = m' \Delta v_y \quad \text{where } \Delta v_y = 0 - 20 = -20 \text{ ft/sec}$$

Volume of water striking horizontal surface per second is $Q = A v_y = (2960 \text{ ft}^2) \left(\frac{1}{12} \frac{\text{ft}}{\text{hr}} \right) \left(\frac{1}{3600} \frac{1}{\text{sec/hr}} \right)$

$$= 0.0685 \text{ ft}^3/\text{sec}$$

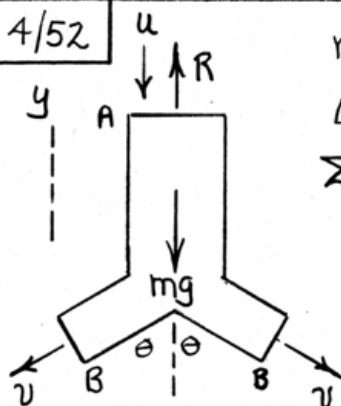
$$m' = \rho Q = \frac{62.4}{32.2} (0.0685) = 0.1328 \text{ lb-sec/ft}$$

(slugs/sec)

$$F = m' |\Delta v_y| \quad F = 0.1328 (20)$$

$$= \underline{2.66 \text{ lb}}$$

4/52



$$m' = \rho A v_A = \rho \frac{\pi d^2}{4} u$$

$$\Delta v_y = -v \cos \theta - (-u) = u - v \cos \theta$$

$$\Sigma F_y = m' \Delta v_y:$$

$$R - mg = \rho \frac{\pi d^2}{4} u (u - v \cos \theta)$$

$$R = mg + \rho \frac{\pi d^2}{4} u (u - v \cos \theta)$$

4/53

$$Q = 1.6 \text{ ft}^3/\text{sec}$$

$$v_1 = \frac{Q}{A_1} = \frac{1.6}{20/144} = 11.52 \text{ ft/sec}$$

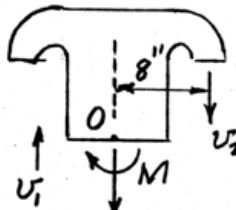
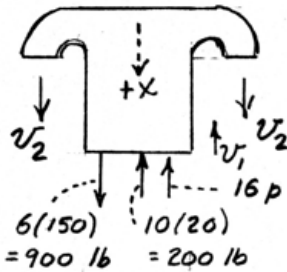
$$v_2 = \frac{Q}{A_2} = \frac{1.6/2}{3.2/144} = 36 \text{ ft/sec}$$

$$m' = \rho Q = \frac{64.4}{32.2} 1.6 = 3.2 \text{ lb-sec/ft}$$

$$\sum F_x = m' \Delta v_x:$$

$$900 - 200 - 16p = 3.2(36 - [-11.52])$$

$$p = 34.2 \text{ lb/in.}^2$$



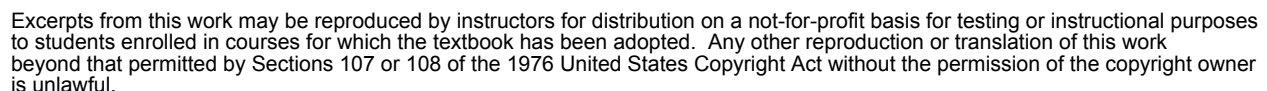
For left side blocked off & with

$$Q = 1.6/2 = 0.8 \text{ ft}^3/\text{sec},$$

$$v_2 = \frac{Q}{A_2} = \frac{0.8}{3.2/144} = 36 \frac{\text{ft}}{\text{sec}}, m' = \frac{3.2}{2} = 1.6 \text{ lb-sec/ft}$$

$$\sum F \quad \sum M_o = m'(v_2 d_2 - v_1 d_1)$$

$$M = 1.6(36 \times 8 - 0) = 461 \text{ lb-in.}$$



4/55 For the truck and plow as a system:

$$\Sigma F_x = m' \Delta v_x: P = \frac{60000}{60} \left[\frac{20}{3.6} - 0 \right] = 5560 \text{ N}$$

$$\text{or } \underline{P = 5.56 \text{ kN}}$$

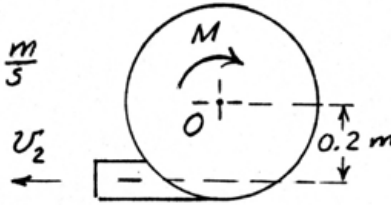
$$\Sigma F_y = m' \Delta v_y: R = \frac{60000}{60} [12 \cos 45^\circ - 0] = 8490 \text{ N}$$

$$\text{or } \underline{R = 8.49 \text{ kN}}$$

$$4/56 \quad M = M_0 = m'(v_2 d_2 - 0)$$

$$v_2 = \frac{Q}{A} = \frac{16}{\pi(0.150)^2/4} \cdot \frac{1}{60} = 15.09 \frac{m}{s}$$

From Table D-1, air density is 1.206 kg/m^3



$$\text{so } m' = \rho Q = 1.206(16)/60 = 0.322 \text{ kg/s}$$

$$M_0 = 0.322(15.09 \times 0.2 - 0) = 0.971 \text{ N}\cdot\text{m}$$

$$P = 0.32 + M_0 \omega / 1000 = 0.32 + \frac{0.971(3450 \times 2\pi/60)}{1000}$$

$$P = 0.32 + 0.351 = \underline{0.671 \text{ kW}}$$

4/57

$$v = 40 \text{ m/s}, u = 420 \text{ m/s}$$

$$pA = -1.8(10^3)(0.1320) \\ = -238 \text{ N}$$

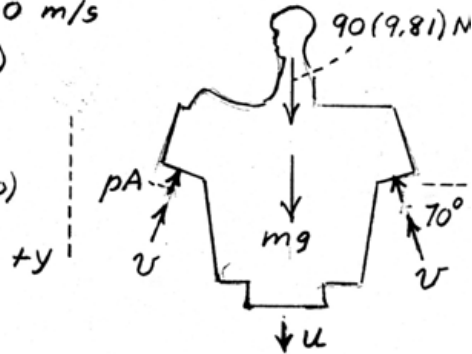
$$m' = \rho A v \\ = 1.206(0.1320)(40) \\ = 6.37 \text{ kg/s}$$

For steady flow

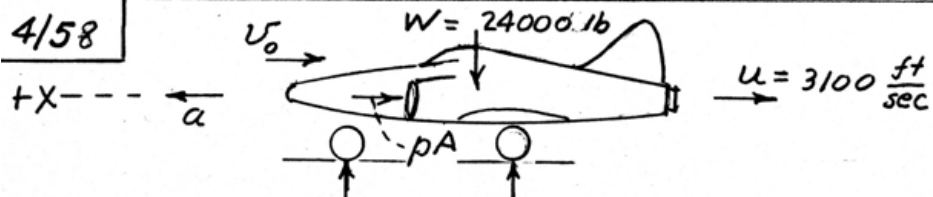
$$\sum F_y = m' \Delta v_y$$

$$(90 + m)9.81 - (-238 \sin 70^\circ) = 6.37(420 - [-40 \sin 70^\circ])$$

$$9.81m + 1106 = 2914, \quad \underline{m = 184.3 \text{ kg}}$$



4/58



$$m'_{air} = \frac{106}{32.2} \frac{\text{lb/sec}}{\text{ft/sec}^2} = 3.29 \text{ lb} \times \text{sec}/\text{ft}$$

$$m'_{fuel} = 3.29/18 = 0.1829 \text{ lb} \times \text{sec}/\text{ft}$$

$$\text{air intake velocity } U_0 = \frac{m'_{air}}{\rho A} = \frac{3.29}{(0.0753/32.2)(1800/144)}$$

$$m'_{exhaust} = 3.29 + 0.1829 = 112.6 \text{ ft/sec}$$

$$= 3.47 \text{ lb} \times \text{sec}/\text{ft}$$

$$pA = -0.30(1800) = -540 \text{ lb}$$

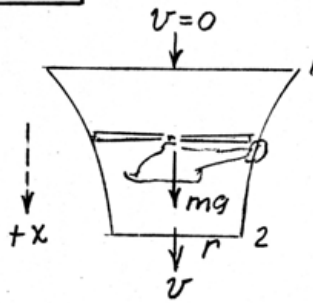
$$\text{Net thrust } T = m'_{ex} u - m'_{air} U_0 - pA$$

$$= 3.47(3100) - 3.29(112.6) - (-540)$$

$$= 10,940 \text{ lb}$$

$$\Sigma F_x = ma_x: 10940 = \frac{24000}{32.2} a, \underline{a = 14.68 \text{ ft/sec}^2}$$

4/59



mg = weight of helicopter
 = force on system of
 air stream & helicopter

For system between sections 1 & 2

$$\Sigma F_x = m' \Delta v_x$$

$$mg = \rho \pi r^2 v (v - 0)$$

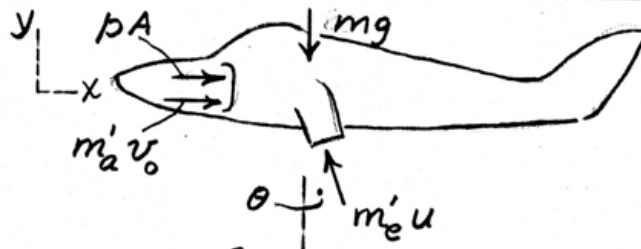
$$v = \frac{1}{r} \sqrt{\frac{mg}{\pi \rho}}$$

Power = rate of increase of kinetic energy

$$P = \frac{1}{2} m' (v_2^2 - v_1^2) = \frac{1}{2} m' v^2 = m' v \frac{v}{2} = mg \frac{v}{2}$$

$$P = \frac{mg}{2r} \sqrt{\frac{mg}{\pi \rho}}$$

4/60

Simulated
FBD

$$mg = 8600(9.81) = 84.4(10^3) \text{ N}$$

$$\text{mass rate of air} = m'_a = 90 \text{ kg/s}$$

$$\text{" " " fuel} = 90/18 = 5 \text{ kg/s}$$

$$\text{" " " exhaust} = m'_e = 95 \text{ kg/s}$$

$$pA = -2(10^3)(1.10) = -2200 \text{ N}$$

$$v_o = m'_a / \rho A = \frac{90}{1.206(1.10)} = 67.8 \text{ m/s}, m'_a v_o = 90(67.8) = 6110 \text{ N}$$

$$m'_e u = 95(1020) = 96900 \text{ N}$$

$$\text{For vertical take off } \Sigma F_x = 0: 6110 - 2200 - 96900 \sin \theta$$

$$\sin \theta = 0.0403, \theta = 2.31^\circ = 0$$

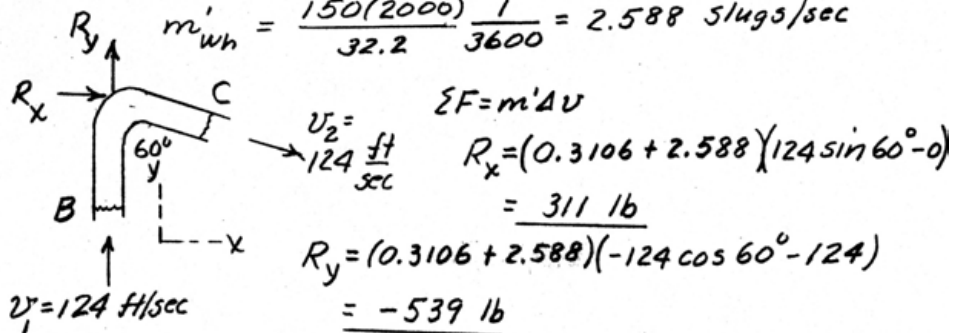
$$\Sigma F_y = ma_y: 96900 \cos 2.31^\circ - 84400 = 8600 a_y$$

$$a_y = 1.448 \text{ m/s}^2$$

4/61

$$\dot{m}'_{air} = \frac{18(2000)}{32.2} \frac{1}{3600} = 0.3106 \text{ slugs/sec}$$

$$\dot{m}'_{wh} = \frac{150(2000)}{32.2} \frac{1}{3600} = 2.588 \text{ slugs/sec}$$



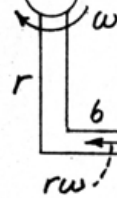
Forces acting on pipe bend & mass within it

- 1) tension $pA = 4.42 \frac{\pi(14)^2}{4} = 680 \text{ lb}$ due to vacuum
- 2) tension in pipe at B
- 3) " " " " C
- 4) weight of bend
- 5) balance of external support forces from crane
- 6) shear force and bending moment at C

4/62

For entire system $\Sigma M = m'(v_2 d_2 - v_1 d_1)$

Let u = velocity of water relative to nozzle = $\frac{Q}{4A}$



$$m' = \rho Q$$

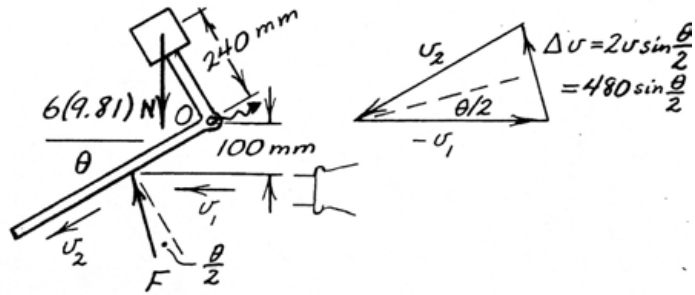
$$-M = \rho Q (r^2 \omega + b^2 \omega - \frac{Q}{4A} r - 0)$$

$$M = \rho Q \left(\frac{Qr}{4A} - [r^2 + b^2] \omega \right)$$

$$\text{For } M = 0, \quad \omega = \omega_0 = \frac{Qr}{4A(r^2 + b^2)}$$

Components of
absolute velocity
of water at exit

4/63



$$F = m' \Delta u: m' = \rho A u = 1.206 \frac{\pi \times 0.040^2}{4} 240 = 0.364 \text{ kg/s}$$

$$F = 0.364 \times 480 \sin \frac{\theta}{2} = 174.6 \sin \frac{\theta}{2} \text{ N}$$

For vane:

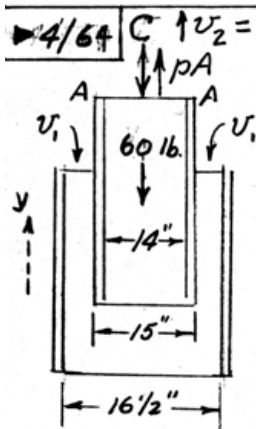
$$\sum M_O = 0: 174.6 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \left(\frac{0.100}{\sin \theta} \right) - 6(9.81)(0.240 \sin \theta) = 0$$

$$87.3 \times 0.100 = 6(9.81)(0.240 \sin \theta)$$

$$\sin \theta = 0.618, \theta = 38.2^\circ$$

Assumption: Entire air stream is diverted downward along the vane, with no flow toward O .

► 4/64



$$C \uparrow v_2 = 124 \text{ ft/sec}$$

$$\text{Air inlet area} = \frac{\pi}{4} ([16.5]^2 - [15]^2) = 37.1 \text{ in.}^2$$

$$= 37.1 / 144 = 0.258 \text{ ft}^2$$

$$\text{Exit area} = \frac{\pi}{4} (14)^2 = 153.9 \text{ in.}^2$$

$$= 153.9 / 144 = 1.069 \text{ ft}^2$$

$$pA = -(-4.42)(153.9) = 680 \text{ lb}$$

$$m'_{\text{air}} = \frac{18(2000)}{32.2} \frac{1}{3600} = 0.3106 \text{ slugs/sec}$$

$$m'_{\text{wh}} = \frac{150(2000)}{32.2} \frac{1}{3600} = 2.588 \text{ slugs/sec}$$

$$v_1 = m'_{\text{air}} / \rho A = \frac{0.3106}{\left(\frac{0.075}{32.2}\right)(0.258)} = 517 \text{ ft/sec}$$

$$\Sigma F_y = m' \Delta v_y; -C + 680 - 60 = 0.3106(124 - [-517]) + 2.588(124 - 0)$$

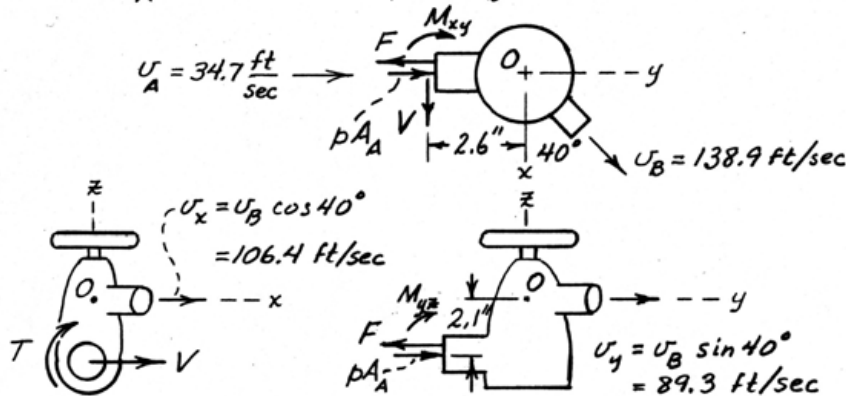
$$= 199.2 + 320.9$$

$$C = 100.3 \text{ lb}$$

► 4/65 Flow rate $Q = \frac{340 \times 231}{1728 \times 60} = 0.758 \frac{\text{ft}^3}{\text{sec}}$, $m' = \rho Q = \frac{62.4}{32.2} 0.758 = 1.468 \frac{\text{lb-sec}}{\text{ft}}$

Flow area $A_A = \frac{\pi 2^2}{4} / 144 = 0.0218 \text{ ft}^2$, $A_B = \frac{\pi 1^2}{4} / 144 = 0.00545 \text{ ft}^2$

Velocity $v_A = \frac{Q}{A_A} = \frac{0.758}{0.0218} = 34.7 \frac{\text{ft}}{\text{sec}}$, $v_B = \frac{Q}{A_B} = \frac{0.758}{0.00545} = 138.9 \frac{\text{ft}}{\text{sec}}$



$(x-y) \sum F_x = m' \Delta v_x: 150 \left(\frac{\pi 2^2}{4} \right) - F = 1.468 (89.3 - 34.7), \quad F = 391 \text{ lb}$

$\sum F_y = m' \Delta v_y: V = 1.468 (106.4 - 0), \quad V = 156.2 \text{ lb}$

$\sum M_{A-A} = m' \Delta (vd): M_{xy} = 1.468 \left(106.4 \times \frac{2.6}{12} \right) = 33.8 \text{ lb-ft}$

$(y-z) \sum M_{A-A} = m' \Delta (vd): M_{yz} = 1.468 \left(89.3 \times \frac{2.1}{12} \right) = 22.9 \text{ lb-ft}$

$M = \sqrt{M_{xy}^2 + M_{yz}^2} = (33.8^2 + 22.9^2)^{1/2} = 40.9 \text{ lb-ft}$

$(x-z) \sum M_O = 0: T - Vd = 0, T = 156.2 \left(\frac{2.1}{12} \right) = 27.3 \text{ lb-ft}$

► 4/66 From Part (b) of Sample Problem $m' = \rho A(v-u)$
 $= (1000) \frac{\pi \times 0.140^2}{4} (150-u)$
 $= 15.39 (150-u) \text{ kg/s}$

$\& F = \rho A(v-u)^2 (1 - \cos 120^\circ), \theta = 90^\circ + 30^\circ = 120^\circ$

$= 15.39 (150-u)^2 (1 - (-0.5))$

$= 23.1 (150-u)^2$

$\Sigma F = m\dot{u}: 23.1 (150-u)^2 - 1373$

$= 1400 \dot{u}$

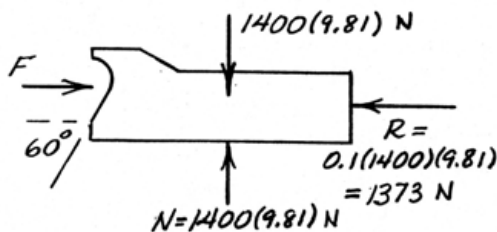
$\int_0^u \frac{du}{0.01649 (150-u)^2 - 0.981} = \int_0^3 dt$

To integrate let $w = 150-u$, $\int_{u=0}^{u=u} \frac{dw}{0.981 - 0.01649 w^2} = 3$

$\frac{1}{2\sqrt{0.01649}\sqrt{0.981}} \ln \left| \frac{0.990 + 0.1284 (150-u)}{0.990 - 0.1284 (150-u)} \right|_0^u = 3$

$3.93 \ln \frac{1 - 0.00634u}{1 - 0.00703u} = 3, \frac{1 - 0.00634u}{1 - 0.00703u} = 2.145$

Solve for u & get $u = 131.0 \text{ m/s}$

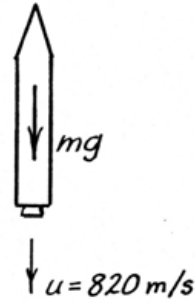


4/67

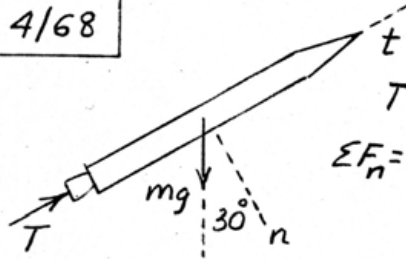
$$\Sigma F_y = ma + \dot{m}u: -9.81m = 6.80m - 220(820) \quad +y \uparrow$$

$$m = 10.86 (10^3) \text{ kg}$$

$$\text{or } \underline{m = 10.86 \text{ Mg}}$$



4/68



$$mg = 3(10^3)(9.60) = 28.8(10^3) \text{ N}$$

$$T = m \Delta v = 130(600) = 78(10^3) \text{ N}$$

$$\Sigma F_n = ma_n: 28.8(10^3) \cos 30^\circ = 3(10^3) a_n$$

$$a_n = 8.31 \text{ m/s}^2$$

$$\Sigma F_t = ma_t: 78(10^3) - 28.8(10^3) \sin 30^\circ = 3(10^3) a_t$$

$$a_t = 21.2 \text{ m/s}^2$$

4/69



$$mg = 2.04(10^6)(9.81) = 20.0(10^6) \text{ N}$$

$$3P_1 = 3(2.00)(10^6) = 6.00(10^6) \text{ N}$$

$$2P_2 = 2(11.80)(10^6) = 23.6(10^6) \text{ N}$$

$$\text{Specific impulse } I = \frac{u}{g} = 455 \text{ s}$$

$$\text{So } u = 455(9.81) = 4460 \text{ m/s}$$

$$\Sigma F_y = ma_y: (6.00)10^6 + (23.6)10^6 - 20.0(10^6) = 2.04(10^6) a$$

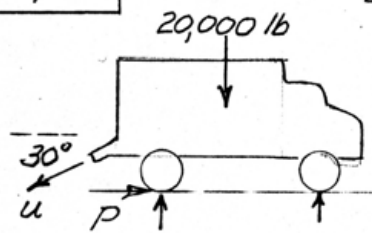
$$a = 4.70 \text{ m/s}^2$$

$$P_1 = m' u, 2.00(10^6) = m'(4460)$$

$$m' = 448 \text{ kg/s}$$

4/70

$$\Sigma F_x = m\dot{v} + \dot{m}u$$



$$u = 60 \text{ ft/sec}$$

$$\dot{m} = -\frac{80}{32.2} = -2.48 \frac{\text{slugs}}{\text{sec}}$$

$$m = \frac{20,000}{32.2} = 621 \text{ slugs}$$

$$a = 2 \text{ ft/sec}^2$$

$$\begin{aligned} \text{(a) Water on; } P &= 621(2) - 2.48(60)\cos 30^\circ \\ &= 1242 - 129 = \underline{1113 \text{ lb}} \end{aligned}$$

$$\text{(b) Water off; } \dot{m} = 0, P = \underline{1242 \text{ lb}}$$

4/71

$$m = 50 + 80 = 130 \text{ kg}$$

$$\dot{m} = \rho A u = \rho A (V + V_0)$$

$$= 1000 (2000) 10^{-6} (2 + 1.5)$$

$$= 7.0 \text{ kg/s}$$

$$u = V + V_0 = 3.5 \text{ m/s}$$

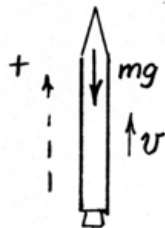
$$\sum F_x = m \dot{u} + \dot{m} u ; \quad P = 130(0.4) + 7.0(3.5)$$

$$= 52 + 24.5 = \underline{76.5 \text{ N}}$$

4/72

$$\Sigma F = m\ddot{u} + \dot{m}u ; -mg = ma + \dot{m}u$$

$$-m(a+g) = \dot{m}u$$

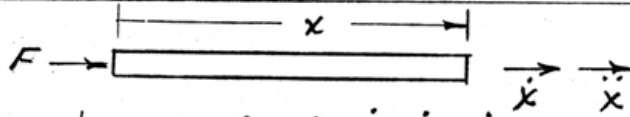


$$u \frac{dm}{dt} = -m(a+g)$$

$$\int_{m_0}^m \frac{dm}{m} = -\frac{a+g}{u} \int_0^t dt, \ln \frac{m}{m_0} = -\frac{a+g}{u} t$$

$$m = m_0 e^{-\frac{a+g}{u} t}$$

4/73



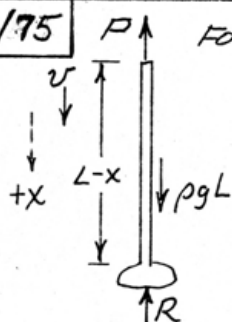
$$\Sigma F = m\dot{v} + \dot{m}u \quad \text{where } m = \rho x, v = \dot{x}, \dot{m} = \rho \dot{x}, u = \dot{x}$$

$$\text{Thus } F = \rho x \ddot{x} + \rho \dot{x} \dot{x}, \quad \underline{F = \rho(x \ddot{x} + \dot{x}^2)}$$

4/74

mg With added moisture particles
 initially at rest, relative velocity
 of attachment of mass is $u = v$
 Thus with $\Sigma F = m\dot{v} + m\dot{u}$
 we have $\Sigma F = m\dot{v} + m\dot{v} = \frac{d}{dt}(mv)$
 $\downarrow v$ where $\Sigma F = mg - R$

4/75



For $\dot{x} = v = \text{const}$, $P = \text{weight of descending links}$
 $= \rho g (L - x)$

$$\sum F_x = \frac{dG_x}{dt}; \quad \rho g L - R - \rho g (L - x) = \frac{d}{dt} (\rho [L - x] v)$$

$$\rho g x - R = -\rho v \dot{x} = -\rho v^2$$

$$\text{so } R = \rho g x + \rho v^2$$

4/76 $\Sigma F_x = m\ddot{u} + \dot{m}u:$

$\Sigma F_x = 380 - 200 = 180 \text{ lb}$

$m = \frac{12000 + 4(220)}{32.2}$

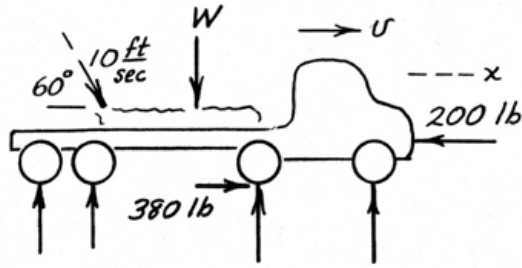
$= 400 \text{ lb-sec}^2/\text{ft}$

at $t = 4 \text{ sec.}$

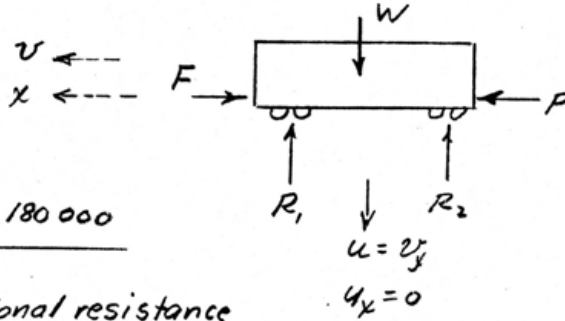
$\dot{m} = 220/32.2 = 6.83 \text{ lb-sec/ft} \quad 1.5 \text{ mi/hr} = 2.20 \text{ ft/sec}$

$u = 2.20 - 10 \cos 60^\circ = -2.80 \text{ ft/sec}$

So $180 = 400\ddot{u} + 6.83(-2.80), a = \ddot{u} = \underline{0.498 \text{ ft/sec}^2}$



4/77



$$\sum F_x = m\ddot{u}_x + m\dot{u}_x$$

$$F = 4 \frac{54,600 + \frac{1}{2} 180,000}{2000}$$

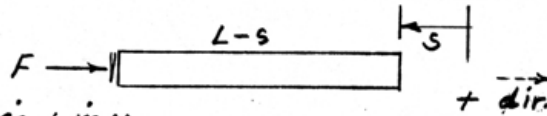
= 289 lb frictional resistance

$u_x =$ rel. velocity with respect to the car in x-dir. = 0

$$\text{Thus } P - 289 = \frac{54,600 + \frac{1}{2} 180,000}{32.2} 0.15 + 0$$

$$\text{So } P = 674 + 289, \underline{P = 963 \text{ lb}}$$

4/78



Eq. 4/20 $\Sigma F = m\dot{v} + \dot{m}u$

where $\Sigma F = F$, $m = \rho(L-s)$, $\dot{m} = -\rho\dot{s}$, $\dot{v} = -\ddot{s}$, $u = -\dot{s}$

Thus $F = \rho(L-s)(-\ddot{s}) + (-\rho\dot{s})(-\dot{s})$

$F = \rho\dot{s}^2 + \rho(L-s)\ddot{s}$

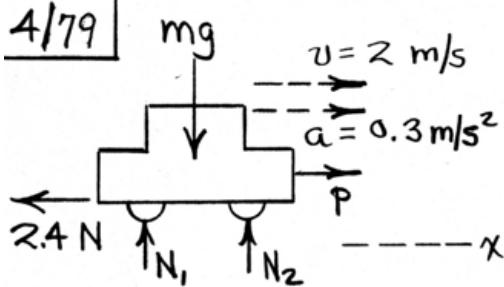
Eq. 4/6 for entire system $\Sigma F = \dot{G}$

$G = \rho(L-s)(-\dot{s}) = -\rho(L-s)\dot{s}$

$\dot{G} = -\rho(L-s)\ddot{s} - \rho(-\dot{s})\dot{s} = \rho\dot{s}^2 - \rho(L-s)\ddot{s}$

Thus $F = \rho\dot{s}^2 - \rho(L-s)\ddot{s}$

4/79



$$m = 40 + 30(1.2)$$

$$= 76 \text{ kg}$$

$$\dot{m} = -\rho v = -1.2(2)$$

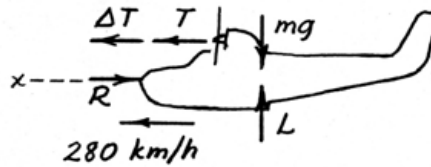
$$= -2.4 \text{ kg/s}$$

$$\sum F_x = m\dot{v} + \dot{m}u : P - 2.4 = 76(0.3) - 2.4(2)$$

$$\underline{P = 20.4 \text{ N}}$$

4/80

For constant initial speed
propeller thrust T
= drag R .



Added power = $\Delta T \cdot v$,

$$\Delta T \times \frac{280 \times 1000}{3600} = 223.8 (10^3) \text{ watts (joules/second)}$$

$$\Delta T = 2880 \text{ N}$$

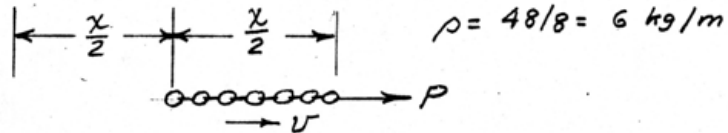
$\Sigma F_x = m\ddot{x} + \dot{m}u$ where $\dot{m} = 4.5 \times 1000 / 12 = 375 \text{ kg/s}$,

$$u = v = \frac{280 \times 1000}{3600} = 77.8 \text{ m/s}$$

$$\text{So } 2880 = 16.4(10^3) \ddot{x} + 375(77.8), \ddot{x} = a = -1.603 \text{ m/s}^2$$

(deceleration)

4/81



$$\dot{m} = \rho \frac{d}{dt} \left(\frac{x}{2} \right) = \frac{1}{2} \rho \dot{x} = \frac{1}{2} \rho v = \frac{1}{2} (6) (1.5) = 4.5 \text{ kg/s}$$

$$u = v = 1.5 \text{ m/s}$$

$$\Sigma F = m \dot{v} + \dot{m} u$$

$$(a) \dot{v} = 0; \quad P = 0 + 4.5 (1.5) = \underline{6.75 \text{ N}}$$

$$(b) m = \rho \frac{x}{2} = 6 \left(\frac{4}{2} \right) = 12 \text{ kg}$$

$$20 = 12 \dot{v} + 4.5 (1.5), \quad a = \dot{v} = \underline{1.104 \text{ m/s}^2}$$

4/82 $\Sigma F_x = ma_x: T - mg \sin \theta = ma_x$

$T = m'u = \frac{2}{32.2} (400) = 24.8 \text{ lb constant}$

$m = m_0 - m't = \frac{1}{32.2} (125 - 2t) \text{ lb-sec}^2/\text{ft}$

Propulsion time $t = \frac{20}{2} = 10 \text{ sec}$

So $m'u - (m_0 - m't)g \sin \theta = (m_0 - m't) \frac{dv}{dt}$

$\int_0^t \left[\frac{m'u}{m_0 - m't} - g \sin \theta \right] dt = \int_0^v dv$

$\Rightarrow v = u \ln \left(\frac{m_0}{m_0 - m't} \right) - gt \sin \theta$

When $t = 10 \text{ sec}$, $v = 400 \ln \left(\frac{125}{125 - 20} \right) - 32.2(10) \sin 10^\circ$
 $= \underline{13.83 \text{ ft/sec}}$

4/83

With $v = \text{const.}$, $\dot{v} = \text{accel.} = 0$ so $\Sigma F_y = 0$ for all bodies:

$$P + \rho g y - T = 0 \quad \text{--- (1)}$$

$$\rho v^2 + \rho g h - T = 0 \quad \text{--- (2)}$$

Eliminate T & get

$$P = \rho v^2 + \rho g(h - y)$$

Left-hand portion (constant mass,
upper end moving up):

$$\Sigma F_x = \dot{G}_x: T + R - \rho g h - \rho g(L - h - y) = \frac{d}{dt}(\rho h v)$$

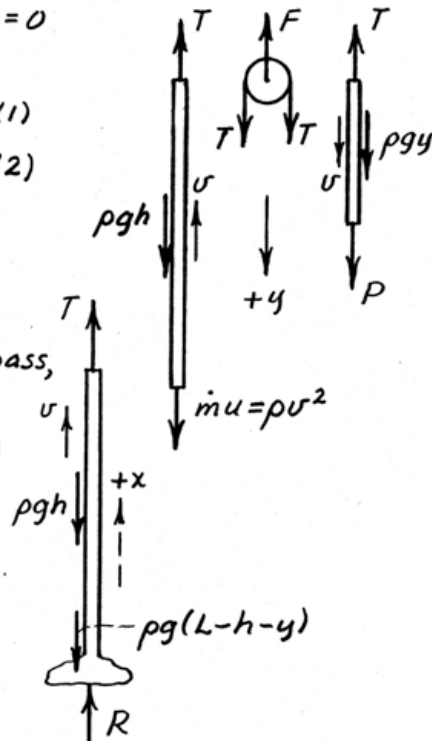
$$\rho v^2 + \rho g h + R - \rho g h - \rho g(L - h - y) = \frac{d}{dt}(\rho h v) = \rho v^2$$

where $\dot{h} = v$

$$\rho v^2 + R - \rho g(L - h - y) = \rho v^2$$

$$R = \rho g(L - h - y) = \text{weight}$$

of pile of chain



4/84 Let $m_0 = \text{initial mass of car} = 25(10^3) \text{ kg}$
 $\dot{m} = 4(10^3) \text{ kg/s}$

The car acquires mass which has zero initial horizontal velocity, so for horizontal x -dir, $\sum F_x = \frac{d}{dt}(mv)$

$$0 = \frac{d}{dt}(m_0 + \dot{m}t)v \quad (m_0 + \dot{m}t)a + \dot{m}v = 0$$

$$a = \frac{dv}{dt} = - \frac{\dot{m}v}{m_0 + \dot{m}t}$$

$$\int_{v_0}^v \frac{dv}{v} = - \int_0^t \frac{\dot{m}}{m_0 + \dot{m}t} dt \Rightarrow v = \frac{dx}{dt} = \frac{m_0 v_0}{m_0 + \dot{m}t}$$

$$\text{Then } \int_0^x dx = \int_0^t \frac{m_0 v_0}{m_0 + \dot{m}t} dt \Rightarrow x = \frac{m_0 v_0}{\dot{m}} \ln \left(\frac{m_0 + \dot{m}t}{m_0} \right)$$

$$\text{With } t = \frac{32}{4} = 8 \text{ s, } x = \frac{25(10^3)(1.2)}{4(10^3)} \ln \left(\frac{25 + 4(8)}{25} \right)$$

$$\underline{x = 6.18 \text{ m}}$$

4/85 $m = m_0 - \rho x$

T is not transmitted to cart so $\Sigma F = ma$

$$P = (m_0 - \rho x)a$$

$$a = \frac{P}{m_0 - \rho x}$$

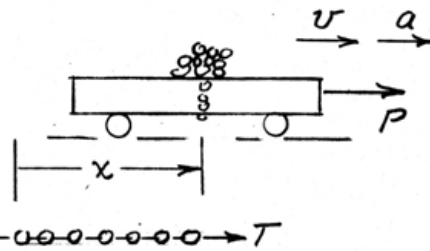
$$v dv = a dx, \quad \int_{v_0}^v v dv = \int_0^x \frac{P dx}{m_0 - \rho x}$$

$$\frac{v^2}{2} \Big|_{v_0}^v = -\frac{P}{\rho} \ln(m_0 - \rho x) \Big|_0^x = \frac{P}{\rho} \ln \frac{m_0}{m_0 - \rho x}$$

$$\text{Thus } v = \sqrt{v_0^2 + \frac{2P}{\rho} \ln \frac{m_0}{m_0 - \rho x}}$$

$$T = m' \Delta v, \quad T = \rho v(v), \quad \underline{T = \rho v^2}$$

There is no reaction (R in Fig. 4/6) between departing links & cart, so $m'u$ is zero & $\Sigma F = ma$



4/86 Let x be the displacement of the chain & T be the tension in the chain at the corner.

Horiz. part $\Sigma F_x = ma_x$:

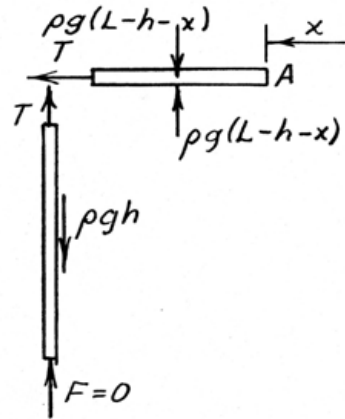
$$T = \rho(L-h-x)\ddot{x}$$

Vert. part $\Sigma F_y = ma_y$:

$$\rho gh - T = \rho h \ddot{x}$$

Eliminate T & get

$$\ddot{x} = \frac{gh}{L-x}$$



$$\dot{x} d\dot{x} = \ddot{x} dx: \int_0^{v_1^2} \frac{1}{2} d(\dot{x}^2) = \int_0^{L-h} \frac{gh}{L-x} dx$$

$$\frac{\dot{x}^2}{2} \Big|_{\dot{x}=0}^{v_1} = -gh \ln(L-x) \Big|_0^{L-h}, \quad \frac{v_1^2}{2} = gh \ln(L-x) \Big|_{L-h}^0 = gh \ln \frac{L}{h}$$

(a) $v_1 = \sqrt{2gh \ln(L/h)}$

(b) Free fall of end A gives $v_2^2 = v_1^2 + 2gh = 2gh \ln \frac{L}{h} + 2gh$

$v_2 = \sqrt{2gh(1 + \ln[L/h])}$

(c) $Q = \text{loss of potential energy since } \Delta T = 0$

$Q = \rho gh \frac{h}{2} + \rho g(L-h)h, \quad Q = \rho gh(L - \frac{h}{2}) \text{ loss}$

4/87 For airplane plus moving portion of chains

$$\Sigma F = 0 = m\dot{v} + \dot{m}u = (m + 2\rho\frac{x}{2})\dot{v} + [2\frac{d}{dt}(\rho\frac{x}{2})]v$$

$$-(m + \rho x)\frac{dv}{dt} = \rho v\frac{dx}{dt} \quad \frac{dv}{v} = -\frac{\rho dx}{m + \rho x}$$

$$\int_{v_0}^v \frac{dv}{v} = -\int_0^x \frac{\rho dx}{m + \rho x}; \quad \ln \frac{v}{v_0} = -\ln \frac{m + \rho x}{m}, \quad \frac{v}{v_0} = \frac{m}{m + \rho x}$$

$$\text{or } v = \frac{v_0}{1 + \rho x/m} \quad \& \text{ for } x = 2L, \quad v = \frac{v_0}{1 + \frac{2\rho L}{m}}$$

$$\text{Also, } v = \frac{dx}{dt} \text{ so } \int_0^x (1 + \frac{\rho x}{m}) dx = \int_0^t v_0 dt$$

$$x + \frac{\rho x^2}{2m} = v_0 t, \quad x^2 + \frac{2m}{\rho} x - \frac{2mv_0 t}{\rho}$$

$$x = -\frac{m}{\rho} \pm \frac{1}{2} \sqrt{\frac{4m^2}{\rho^2} + \frac{8mv_0 t}{\rho}}, \quad x = \frac{m}{\rho} \left[\sqrt{1 + \frac{2v_0 t \rho}{m}} - 1 \right]$$

for + root

$$V_g = 0$$

There is no force on moving part other than weight so acceleration $\ddot{x} = g = \text{constant}$.

$$\text{Thus, } v^2 = 2gx$$

$$T_1 = m' \Delta v : T_1 = \left[\frac{d}{dt} \left(\rho \frac{L-x}{2} \right) \right] [0-v]$$

$$= \frac{1}{2} \rho v^2 = \frac{1}{2} \rho (2gx) = \underline{\rho g x}$$

Equilibrium of links at rest:

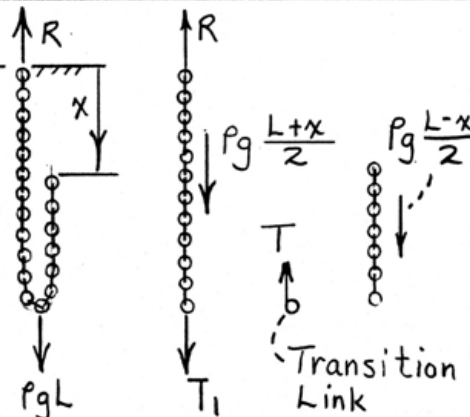
$$\Sigma F_x = 0 : T_1 + \rho g \frac{L+x}{2} - R = 0$$

$$\Rightarrow \underline{R = \frac{1}{2} \rho g (L+3x)}$$

$$\text{Loss } Q = |V_{g_1} - V_{g_2}| = \left| \rho g L \left(-\frac{L}{4} \right) - \rho g L \left(-\frac{L}{2} \right) \right|$$

$$= \underline{\frac{1}{4} \rho g L^2}$$

When $x \rightarrow L$, $v \rightarrow \infty$. The loss of potential energy equals the gain in kinetic energy, so the gain is concentrated in the last element and is lost during impact when the last element is abruptly brought to rest.



► 4/89

$$\Delta V_g = \rho g \frac{L-x}{2} (-x) + \rho g \frac{x}{2} (-\frac{x}{2})$$

$$= -\frac{1}{2} \rho g x (L - \frac{x}{2})$$

$$\Delta T = \frac{1}{2} \rho \frac{L-x}{2} v^2$$

$$\Delta V_g + \Delta T = 0$$

$$\frac{1}{4} \rho (L-x) v^2 = \frac{1}{2} \rho g x (L - \frac{x}{2})$$

$$v^2 = 2gx \frac{L-x/2}{L-x} \quad \text{--- (1)}$$

$$v dv = a dx \quad \text{so} \quad a = \frac{1}{2} \frac{dv^2}{dx} = g \frac{(L-x)(L-x) - x(L-x/2)(-1)}{(L-x)^2}$$

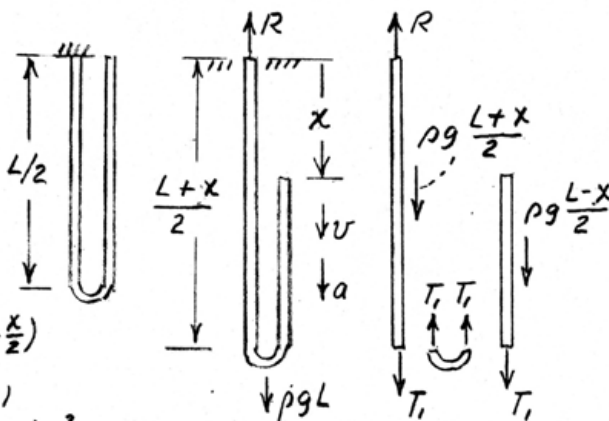
$$a = g \left(1 + \frac{x(L-x/2)}{(L-x)^2} \right) \quad \text{--- (2)}$$

For entire rope

$$\Sigma F_x = \dot{G}_x: \rho g L - R = \frac{d}{dt} \left(\rho \frac{L-x}{2} v \right) = \frac{\rho}{2} [(L-x)a - v^2]$$

$$\text{Sub. (1) \& (2) \& get} \quad R = \frac{1}{2} \rho g \left[(L+x) + \frac{x(L-x/2)}{L-x} \right]$$

$$\text{Equil of fixed part gives} \quad T_1 = \frac{1}{2} \rho g x (L-x/2) / (L-x)$$



► 4/90

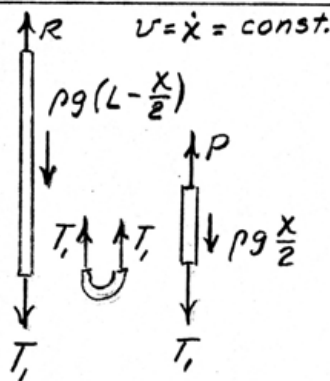
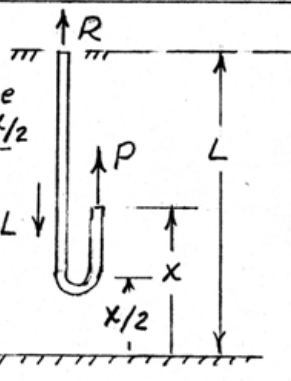
For entire rope

$$V_g = \rho g \left(L - \frac{x}{2}\right) \frac{L + x/2}{2}$$

$$+ \rho g \frac{x}{2} \frac{3x}{4}$$

$$= \frac{\rho g}{2} \left(L^2 + \frac{x^2}{2}\right)$$

$V_g = 0$



$v = \dot{x} = \text{const.}$

Entire rope: $\sum F_x = \dot{G}_x: R + P - \rho g L = \frac{d}{dt} \left(\rho \frac{x}{2} v \right) = \frac{1}{2} \rho v^2 \quad \dots (1)$

Work-energy: $dU' = dT + dV_g: P dx = d \left(\frac{1}{2} \rho \frac{x}{2} v^2 \right) + d \left\{ \frac{\rho g}{2} \left(L^2 + \frac{x^2}{2} \right) \right\}$

$= \frac{1}{4} \rho v^2 dx + \frac{1}{2} \rho g x dx$

$P = \frac{1}{4} \rho v^2 + \frac{1}{2} \rho g x \quad \dots (2)$

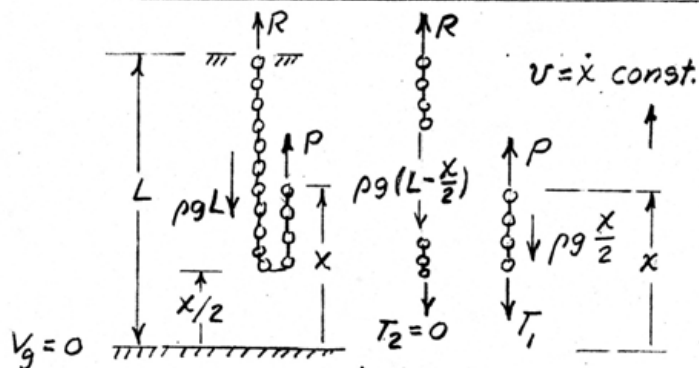
Sub. (2) into (1)

$R = \frac{1}{4} \rho v^2 + \rho g \left(L - \frac{x}{2} \right)$

Equil. of part
not moving

$\sum F_y = 0: R - \rho g \left(L - \frac{x}{2} \right) - T_1 = 0, \quad T_1 = \frac{1}{4} \rho v^2$

► 4/91



Entire chain: $\Sigma F_x = \dot{G}_x$ $R + P - \rho g L = \frac{d}{dt} \left(\rho \frac{x}{2} v \right)$

$R + P = \frac{1}{2} \rho v^2 + \rho g L$ (same as for rope)

$T_1 = m' \Delta v$: $T_1 = \frac{d}{dt} \left(\rho \frac{x}{2} v \right)$, $T_1 = \frac{1}{2} \rho v^2$

Moving part: $\Sigma F_x = 0$: $P - \rho g \frac{x}{2} - \frac{1}{2} \rho v^2 = 0$, $P = \frac{1}{2} \rho (v^2 + gx)$

Equil. of part not moving $\Sigma F_x = 0$: $R - \rho g (L - \frac{x}{2}) = 0$, $R = \rho g (L - \frac{x}{2})$

$U_{1-2} = \Delta T + \Delta V_g + Q$, $\Delta T = \frac{1}{2} \rho \frac{x}{2} v^2$, $\Delta V_g = \rho g \frac{x}{2} \frac{x}{2}$

$U = P x = \frac{1}{2} \rho (v^2 x + gx^2)$ so $Q = \frac{1}{4} \rho x (v^2 + gx)$

► 4/92 For falling part $\Sigma F = m\ddot{u} + \dot{m}u$

Where $\Sigma F = \rho g x$, $m = \rho x$, $\dot{m} = \rho v$, $u = v = \dot{x}$

Thus $\rho g x = \rho x \ddot{v} + \rho v \dot{x}$, $g x dt = x dv + v dx$

or $g x dt = d(xv)$; $g x^2 v dt = x v d(xv)$

$$\text{so } g x^2 dx = \frac{1}{2} d[(xv)^2] \text{ \& } g \int_0^x x^2 dx = \frac{1}{2} \int_0^{(xv)^2} d[(xv)^2]$$

$$\frac{gx^3}{3} = \frac{1}{2} (xv)^2, \quad v = \sqrt{\frac{2gx}{3}}$$

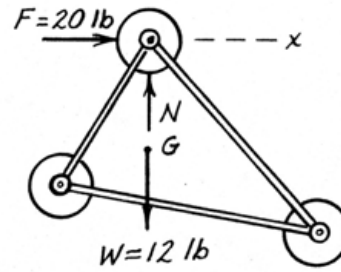
$$a = \ddot{v} = \sqrt{\frac{2g}{3}} \frac{1}{2} x^{-1/2} \dot{x} = \sqrt{\frac{2g}{3}} \frac{1}{2\sqrt{x}} \sqrt{\frac{2gx}{3}}, \quad a = \frac{g}{3} \text{ constant}$$

$$Q = -\Delta V_g - \Delta T = + \frac{\rho g L^2}{2} - \frac{\rho L}{2} v_{x=L}^2 = \frac{\rho g L^2}{2} - \frac{\rho g L^2}{3} = \underline{\underline{\frac{\rho g L^2}{6}}}$$

4/93

$$\Sigma F_x = m\bar{a}_x : 20 = \frac{12}{32.2} \bar{a}_x$$

$$\bar{a}_x = \bar{a} = \underline{53.7 \text{ ft/sec}^2}$$



4/94 For the system, $\Sigma M_o = \dot{H}_o = 0$, so

H_o is conserved:

$$\frac{2}{16} (1000) \frac{10}{12} = \frac{2}{16} \left(\frac{10}{12} \right)^2 \omega + 3 \left(\frac{20}{12} \right)^2 \omega$$

$$\omega = 12.37 \text{ rad/sec}$$

A large horizontal force is exerted on the rod by the bearing so that $\Sigma F \neq 0$ in the horizontal direction. Thus $\dot{G}_x \neq 0$ and the linear momentum of the bullet-pendulum system is not conserved.

4/95

$$F = m' \Delta v_x: \Delta v_x = v \cos 20^\circ$$

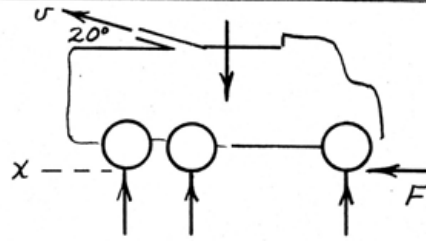
$$Q = Av: \frac{1400 \times 231}{1728} \frac{1}{60} \frac{\text{ft}^3}{\text{sec}}$$

$$= \frac{\pi \times 2^2 / 4}{144} v, v = 143.0 \text{ ft/sec}$$

$$\Delta v_x = 143.0 \cos 20^\circ - 0 = 134.4 \text{ ft/sec}$$

$$m' = \rho Q = \frac{62.4}{32.2} \frac{1400 \times 231}{1728 \times 60} = 6.04 \text{ lb-sec/ft}$$

$$F = 6.04 (134.4) = \underline{812 \text{ lb}}$$



4/96

$$m = m_0 - m't$$

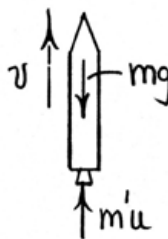
$$\Sigma F = ma: m'u - (m_0 - m't)g = (m_0 - m't)a$$

$$a = \frac{dv}{dt} = \frac{m'u}{m_0 - m't} - g$$

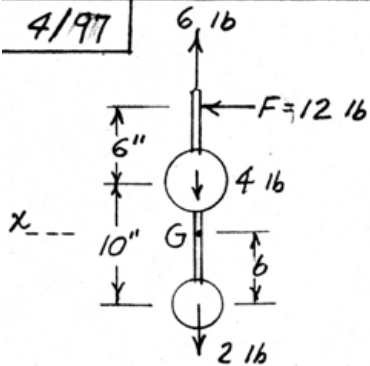
$$\int_0^v dv = \int_0^t \frac{m'u}{m_0 - m't} dt - \int_0^t g dt$$

$$v = -u \ln(m_0 - m't) \Big|_0^t - gt \Big|_0^t$$

$$v = u \ln \left(\frac{m_0}{m_0 - m't} \right) - gt$$



4/97



$$\Sigma F_x = m\bar{a}_x; 12 = \frac{4+2}{32.2} \bar{a}$$

$$\bar{a} = 64.4 \text{ ft/sec}^2$$

$$4(10-b) = 2b, \quad b = 6.67 \text{ in.}$$

$$H_G = \Sigma mr^2 \ddot{\theta} = \frac{4(3.33)^2 + 2(6.67)^2}{32.2(12)^2} \ddot{\theta}$$

$$= 0.0288 \ddot{\theta} \text{ lb-ft-sec}$$

$$\Sigma M_G = \dot{H}_G; 12 \frac{(6+3.33)}{12} = 0.0288 \ddot{\theta}$$

$$\ddot{\theta} = \frac{9.33}{0.0288} = 325 \text{ rad/sec}^2$$

4/98

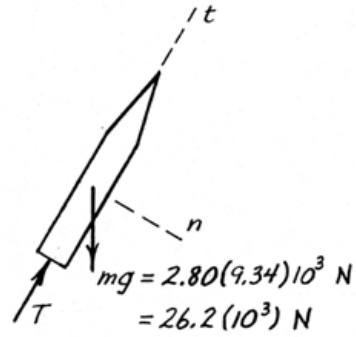
$$T = m'u = 120(640) = 76.8(10^3) \text{ N}$$

$$\begin{aligned}\Sigma F_t = ma_t: 76.8(10^3) - 26.2(10^3) \cos 30^\circ \\ = 2.80(10^3) a_t\end{aligned}$$

$$\underline{a_t = 19.34 \text{ m/s}^2}$$

$$\begin{aligned}\Sigma F_n = ma_n: 26.2(10^3) \sin 30^\circ \\ = 2.80(10^3) a_n\end{aligned}$$

$$\underline{a_n = 4.67 \text{ m/s}^2}$$



4/99

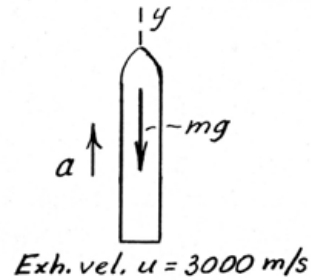
$$\dot{m} = -\dot{m}' = -5.2 \text{ kg/s}$$

$$m = 200 + 1200 - 5.2t = 1400 - 5.2t \text{ kg}$$

$$\Sigma F = m\ddot{u} + \dot{m}u: -mg = ma - 5.2(3000)$$

$$(1400 - 5.2t)(a + 8.70) = 15600$$

$$a = \frac{15600}{1400 - 5.2t} - 8.70 \text{ m/s}^2$$



$$\text{When } t = 60 \text{ s, } a = \frac{15600}{1400 - 5.2(60)} - 8.70 = 14.34 - 8.70 = \underline{5.64 \text{ m/s}^2}$$

Max. accel. occurs when $5.2t = 1200$, $t = 231 \text{ s}$

$$a_{\text{max}} = \frac{15600}{1400 - 5.2(231)} - 8.70 = 78.0 - 8.70 = \underline{69.3 \text{ m/s}^2}$$

4/100

Vertical drop of
spheres is

$$h_1 = 20''$$

$$h_2 = 15''$$

$$h_3 = 10''$$

For system

$$\Delta V_g = -mg(h_1 + h_2 + h_3)$$

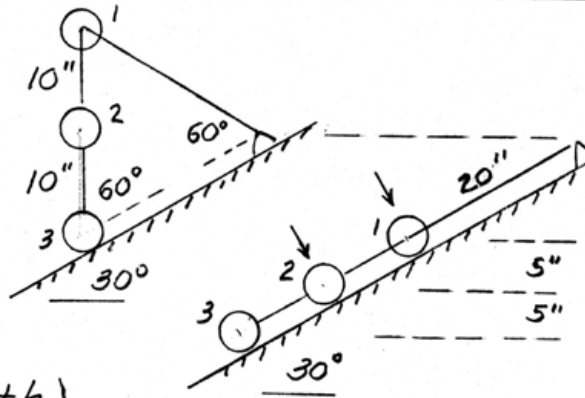
$$\Delta T = \frac{1}{2}m(v_1^2 + v_2^2 + 0) = \frac{1}{2}m(v^2 + [\frac{v}{2}]^2) = \frac{5}{8}mv^2$$

$$U' = \Delta T + \Delta V : 0 = \frac{5}{8}mv^2 - mg(h_1 + h_2 + h_3)$$

$$v^2 = \frac{8}{5} (32.2) \frac{20+15+10}{12} = 193.2 \text{ (ft/sec)}^2$$

$$v = 13.90 \text{ ft/sec}$$

Potential energy loss goes into impact energy loss



4/101

$$\Sigma F_x = m' \Delta v_x$$

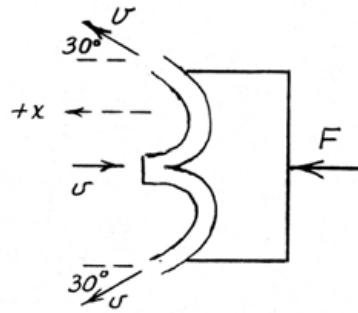
$$m' = \rho A v = \frac{62.4}{32.2} \left(\frac{\pi}{4} \frac{(3/4)^2}{144} \right) 120$$

$$= 0.713 \text{ lb-sec/ft}$$

$$\Delta v_x = v \cos 30^\circ - (-v) = v (1 + \cos 30^\circ)$$

$$= 120(1 + 0.866) = 224 \text{ ft/sec}$$

$$F = 0.713 \times 224 = \underline{159.8 \text{ lb}}$$



4/102 $dU = dT; P dx = d(\frac{1}{2} \rho \frac{x}{2} v^2) = \frac{1}{4} \rho v^2 dx$

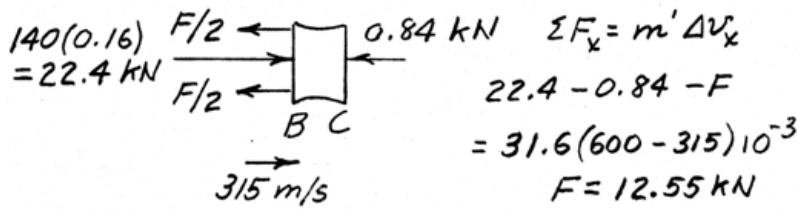
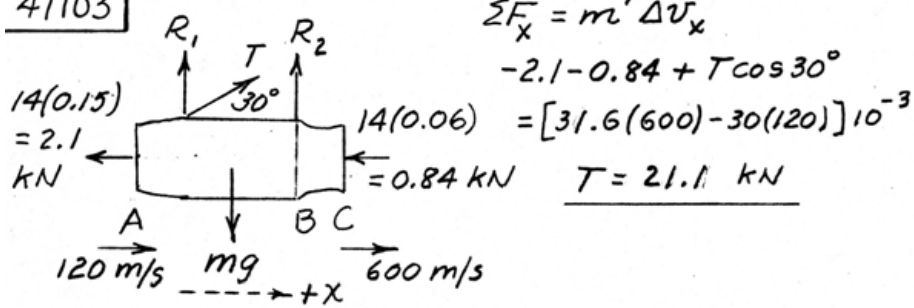
$x \xrightarrow{x/2} P \xrightarrow{v \text{ const.}} R = P$ $P = \frac{1}{4} \rho v^2$

$\begin{array}{c} P \xrightarrow{\quad} P \\ P \xrightarrow{\quad} R = P \end{array}$ or $\Sigma F_x = \dot{G}_x$

$P + R = \frac{d}{dt} (\rho \frac{x}{2} v)$

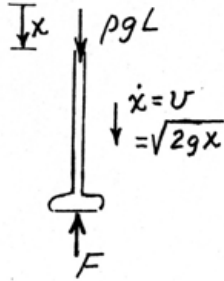
$R = \frac{1}{2} \rho v^2 - \frac{1}{4} \rho v^2 = \frac{1}{4} \rho v^2$

4/103



4/104

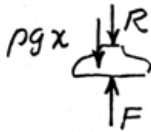
$$G_x = \rho(L-x)\sqrt{2gx} = \rho\sqrt{2g}(Lx^{1/2} - x^{3/2})$$


 $\Sigma F_x = \dot{G}_x$ for entire system

$$\begin{aligned}\rho g L - F &= \rho\sqrt{2g}\left(\frac{1}{2}Lx^{-1/2} - \frac{3}{2}x^{1/2}\right)\dot{x} \\ &= \rho\sqrt{2g}\left(\frac{1}{2}L\sqrt{2g} - \frac{3}{2}\sqrt{2g}x\right) \\ &= \rho g L - 3\rho g x\end{aligned}$$

So that $F = 3\rho g x$

Alternatively

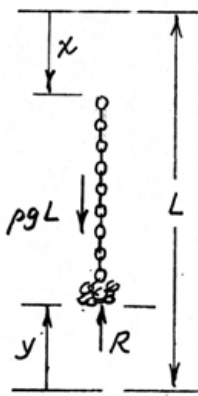


$$R = m'\Delta v = \rho\dot{x}(\dot{x}) = \rho\dot{x}^2 = \rho(2gx) = 2\rho g x$$

$$\Sigma F_x = 0; \quad 2\rho g x + \rho g x - F = 0, \quad \underline{F = 3\rho g x}$$

4/105

Take entire chain as system (constant mass)



$$\ddot{x} = g, \quad \ddot{y} = a, \quad G_x = \rho(L-x-y)\ddot{x} - \rho(x+y)\ddot{y}$$

$$\dot{x} = gt, \quad \dot{y} = at$$

$$x = \frac{1}{2}gt^2, \quad y = \frac{1}{2}at^2$$

$$\Sigma F_x = \dot{G}_x; \quad \rho gL - R = \rho[(-\dot{x}-\dot{y})\dot{x} + (L-x-y)\ddot{x}] - \rho[(\dot{x}+\dot{y})\dot{y} + (x+y)\ddot{y}]$$

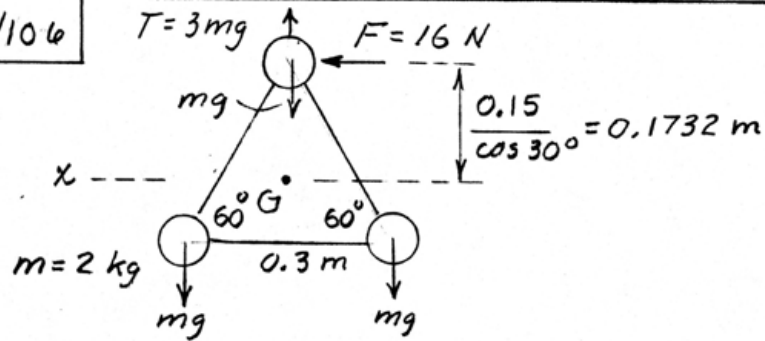
$$\rho gL - R = \rho[-(\dot{x}+\dot{y})^2 - (x+y)(\ddot{x}+\ddot{y}) + L\ddot{x}]$$

$$= \rho[-(a+g)^2 t^2 - \frac{1}{2}(a+g)^2 t^2 + Lg]$$

$$= -\frac{3}{2}\rho(a+g)^2 t^2 + \rho gL$$

$$\text{so } R = \frac{3}{2}\rho(a+g)^2 t^2$$

4/106



For system: $\sum F_x = m\bar{a}_x$: $16 = 3(2)\bar{a}$, $\bar{a} = 2.67 \frac{\text{m}}{\text{s}^2}$

$$\sum M_G = \dot{H}_G: 16(0.1732) = \frac{d}{dt}(3 \times 2 \times 0.1732^2 \dot{\theta})$$

$$\ddot{\theta} = \frac{16(0.1732)}{6(0.1732)^2} = 15.40 \frac{\text{rad}}{\text{s}^2}$$

For top sphere

$$a = \bar{a} + \bar{r}\ddot{\theta} = 2.67 + 0.1732(15.40) = 5.33 \text{ m/s}^2$$

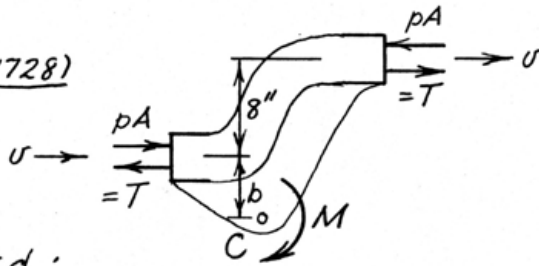
4/107

$$Q = A v$$

$$v = Q/A = \frac{5000 \times 231}{(60 \times 1728)}$$

$$= \frac{\frac{\pi \times 4^2}{4} \frac{1}{144}}{144}$$

$$= 127.7 \text{ ft/sec}$$



$$\Sigma M_o = \dot{H}_o = m' v_2 d_2 - m' v_1 d_1 :$$

$$m' = \frac{\mu}{g} Q = \frac{62.4}{32.2} \frac{5000 \times 231}{60 \times 1728} = 21.6 \text{ lb-sec/ft}$$

$$\text{so } M = 21.6 (127.7) (8/12) = \underline{1837 \text{ lb-ft}}$$

4/108

System is conservative, so $\Delta V_g + \Delta T = 0$.

$$- \rho g x \frac{x}{2} + \frac{1}{2} \rho L \dot{x}^2 = 0, \quad \frac{g}{L} x^2 = \dot{x}^2, \quad \dot{x} = \sqrt{\frac{g}{L}} x$$

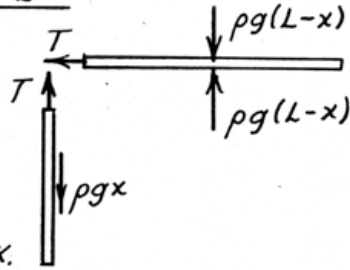
$$(a) \text{ accel } a = \ddot{x} = \sqrt{\frac{g}{L}} \dot{x} = \sqrt{\frac{g}{L}} \sqrt{\frac{g}{L}} x, \text{ so } \underline{a = \frac{g}{L} x}$$

$$(b) \Sigma F = ma: T = \rho(L-x) \frac{g}{L} x$$

$$\underline{T = \rho g x \left(1 - \frac{x}{L}\right)}$$

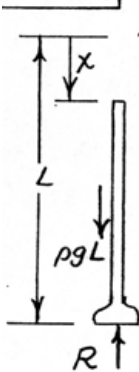
Check from vertical part

$$\rho g x - T = \rho x \frac{g}{L} x, \quad T = \rho g x \left(1 - \frac{x}{L}\right), \text{ OK.}$$



$$(c) v dv = a_x dx: \int_0^v v dv = \frac{g}{L} \int_0^L x dx, \quad \frac{v^2}{2} = \frac{g}{L} \frac{L^2}{2}, \quad \underline{v = \sqrt{gL}}$$

► 4/109 For entire rope of constant mass



$$\Sigma F_x = \dot{G}_x; \rho g L - R = \frac{d}{dt}(\rho[L-x]\dot{x} + 0) \quad \text{--- (1)}$$

$$\Delta T + \Delta V_g = 0; \frac{1}{2}\rho(L-x)\dot{x}^2 = \rho g x(L - \frac{x}{2})$$

$$\dot{x}^2 = \frac{x(2L-x)}{L-x}g \quad \text{or} \quad \dot{x}(L-x) = \frac{x}{\dot{x}}(2L-x)g$$

Substitute into (1) & get

$$\begin{aligned} \rho g L - R &= \frac{d}{dt} \left(\frac{x(2L-x)}{\dot{x}} \rho g \right) = \rho g \frac{\dot{x}[2L-2x]\dot{x} - x[2L-x]\ddot{x}}{\dot{x}^2} \\ &= 2\rho g(L-x) - \rho g x(2L-x) \frac{\ddot{x}}{\dot{x}^2} \end{aligned}$$

Differentiate $\dot{x}^2$ & get $\ddot{x} = \left[1 + \frac{x(L-\frac{x}{2})}{(L-x)^2} \right] g$. Substitute & get $\rho g L - R = 2\rho g(L-x) - \rho g x(2L-x) \left[1 + \frac{x(L-\frac{x}{2})}{(L-x)^2} \right] g / \dot{x}^2$

Substitute $\dot{x}^2$, simplify, & solve for R

& get

$$R = \rho g x \frac{4L-3x}{2(L-x)}, \quad \left(\text{Less than } R_{a=0} \text{ of Prob. 4/105 with } x < \frac{2L}{3} \right)$$

► 4/110

$$T_1 = p_1 A, T_2 = p_2 A$$

$$\text{Power } P = M\omega$$

$$M = \frac{40(10^3)}{900(2\pi/60)} = 424 \text{ N}\cdot\text{m}$$

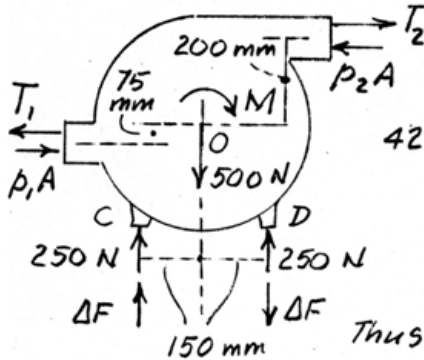
$$\Sigma M_O = m'(v_2 d_2 - v_1 d_1)$$

$$424 + 0.3\Delta F = \frac{20}{60}(1000)[18(0.2) - (18)(-0.075)]$$

$$\Delta F = \frac{1650 - 424}{0.3} = 4090 \text{ N}$$

$$\text{Thus } C = 250 + 4090 = \underline{4340 \text{ N (up)}}$$

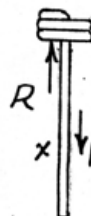
$$D = 4090 - 250 = \underline{3840 \text{ N (down)}}$$



► 4/III

Entire system is conservative so $\Delta V_g + \Delta T = 0$

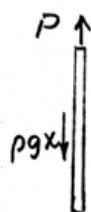
$$- \rho g x \frac{x}{2} + \frac{1}{2} \rho x v^2 = 0, \quad v = \sqrt{g x}, \quad a = \dot{v} = \frac{1}{2} \frac{\sqrt{g}}{\sqrt{x}} \sqrt{g x} = g/2$$



For entire system $\Sigma F_x = \dot{G}_x$

$$\rho g L - R = \frac{d}{dt}(\rho x v) = \frac{d}{dt}(\rho \sqrt{g x} x^{3/2}) = \frac{3}{2} \rho g x$$

$$\text{Thus } R = \rho g L - \frac{3}{2} \rho g x, \quad \underline{R = \rho g (L - \frac{3}{2} x)}$$

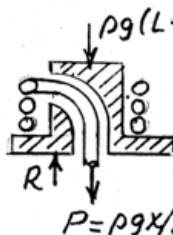


Explanation of $R=0$ for $x = 2L/3$:

For vertical section $\Sigma F_x = m a_x$; $\rho g x - P = \rho x \frac{g}{2}$

$P = \rho g x/2$. For idealized flow, a frictionless guide must be introduced. Guide & coil of length $L-x$ is isolated. $\Sigma F_x = m' \Delta v_x$

(or $\Sigma F_x = m(0) + m'u$) gives



$$\rho g x/2 + \rho g (L-x) - R = \rho v(v) = \rho (g x)$$

$$R = \rho g \left(\frac{x}{2} + L - x - x \right) = \rho g (L - \frac{3x}{2})$$

& $R=0$ when $x = 2L/3$ as before.

When $x > 2L/3$, the rate of momentum change requires more force than $P + \rho g (L-x)$ so R reverses.

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With neglect of mass of pulley & weight of small portion of chain in contact with pulley

$$\Sigma M_O \approx 0 \text{ so } T_1 = T_2 = T$$

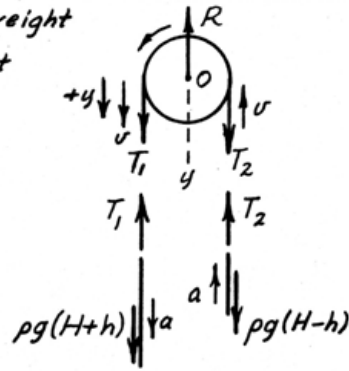
$$\Sigma F = ma \text{ for chains}$$

$$\rho g(H+h) - T = \rho(H+h)a \quad \text{--- (1)}$$

$$T - \rho g(H-h) = \rho(H-h)a \quad \text{--- (2)}$$

Eliminate T from (1) & (2) & get

$$a = \frac{h}{H} g \quad (3)$$



$$\text{So } T = \rho(H-h)\frac{h}{H}g + \rho g(H-h), T = \rho g(H - \frac{h^2}{H}) \quad (4)$$

Pulley & chain on it: steady flow gives

$$\Sigma F_y = m' \Delta v_y: 2T - R = \rho v(v - [-v]), R = 2T - 2\rho v^2 \quad (5)$$

$$\text{But } \int_0^v v dv = \int_0^h \frac{g}{H} h dh, v^2 = \frac{g}{H} h^2, v = \sqrt{\frac{g}{H}} h \quad (6)$$

Substitute (4) & (6) into (3) & get

$$R = 2\rho g(H - \frac{h^2}{H}) - 2\rho \frac{g}{H} h^2, R = 2\rho g(H - \frac{2h^2}{H})$$