

5.4 Second Law of Thermodynamics—Irreversible Flow⁵

The second law of thermodynamics formalizes the notion of loss.

The second law of thermodynamics affords us with a means to formalize the inequality

$$\check{u}_2 - \check{u}_1 - q_{\text{net, in}} \geq 0 \quad (5.90)$$

for steady, incompressible, one-dimensional flow with friction (see Eq. 5.73). In this section we continue to develop the notion of loss of useful or available energy for flow with friction. Minimization of loss of available energy in any flow situation is of obvious engineering importance.

5.4.1 Semi-infinitesimal Control Volume Statement of the Energy Equation

If we apply the one-dimensional, steady flow energy equation, Eq. 5.70, to the contents of a control volume that is infinitesimally thin as illustrated in Fig 5.8, the result is

$$\dot{m} \left[d\check{u} + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g(dz) \right] = \delta \dot{Q}_{\text{net, in}} \quad (5.91)$$

For all pure substances including common engineering working fluids, such as air, water, oil, and gasoline, the following relationship is valid (see, for example, Ref. 3).

$$T ds = d\check{u} + pd\left(\frac{1}{\rho}\right) \quad (5.92)$$

where T is the absolute temperature and s is the *entropy* per unit mass.

Combining Eqs. 5.91 and 5.92 we get

$$\dot{m} \left[T ds - pd\left(\frac{1}{\rho}\right) + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g dz \right] = \delta \dot{Q}_{\text{net, in}}$$

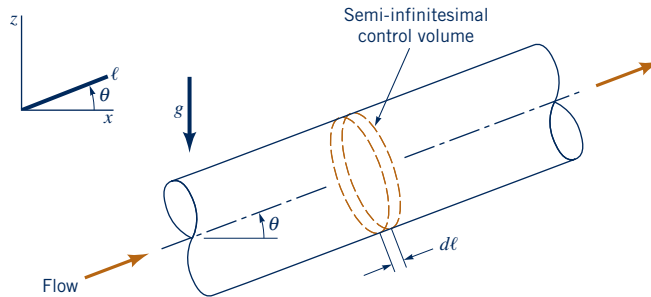
or, dividing through by $\dot{m}$ and letting $\delta q_{\text{net, in}} = \delta \dot{Q}_{\text{net, in}} / \dot{m}$, we obtain

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g dz = -(T ds - \delta q_{\text{net, in}}) \quad (5.93)$$

5.4.2 Semi-infinitesimal Control Volume Statement of the Second Law of Thermodynamics

A general statement of the second law of thermodynamics is

$$\frac{D}{Dt} \int_{\text{sys}} s \rho dV \geq \sum \left(\frac{\delta \dot{Q}_{\text{net, in}}}{T} \right)_{\text{sys}} \quad (5.94)$$



■ **Figure 5.9** Semi-infinitesimal control volume.

⁵This entire section may be omitted without loss of continuity in the text material.

or in words,

The second law of thermodynamics involves entropy, heat transfer, and temperature.

the time rate of increase of the entropy of a system $\geq$ sum of the ratio of net heat transfer rate into system to absolute temperature for each particle of mass in the system receiving heat from surroundings

The right-hand side of Eq. 5.94 is identical for the system and control volume at the instant when system and control volume are coincident; thus,

$$\sum \left(\frac{\delta \dot{Q}_{\text{net}}}{T} \right)_{\text{sys}} = \sum \left(\frac{\delta \dot{Q}_{\text{net}}}{T} \right)_{\text{cv}} \quad (5.95)$$

With the help of the Reynolds transport theorem (Eq. 4.19) the system time derivative can be expressed for the contents of the coincident control volume that is fixed and nondeforming. Using Eq. 4.19, we obtain

$$\frac{D}{Dt} \int_{\text{sys}} s \rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} s \rho dV + \int_{\text{cs}} s \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA \quad (5.96)$$

For a fixed, nondeforming control volume, Eqs. 5.94, 5.95, and 5.96 combine to give

$$\frac{\partial}{\partial t} \int_{\text{cv}} s \rho dV + \int_{\text{cs}} s \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA \geq \sum \left(\frac{\delta \dot{Q}_{\text{net}}}{T} \right)_{\text{cv}} \quad (5.97)$$

At any instant for steady flow

$$\frac{\partial}{\partial t} \int_{\text{cv}} s \rho dV = 0 \quad (5.98)$$

If the flow consists of only one stream through the control volume and if the properties are uniformly distributed (one-dimensional flow), Eqs. 5.97 and 5.98 lead to

$$\dot{m}(s_{\text{out}} - s_{\text{in}}) \geq \sum \frac{\delta \dot{Q}_{\text{net}}}{T} \quad (5.99)$$

For the infinitesimally thin control volume of Fig. 5.8, Eq. 5.99 yields

$$\dot{m} ds \geq \sum \frac{\delta \dot{Q}_{\text{net}}}{T} \quad (5.100)$$

If all of the fluid in the infinitesimally thin control volume is considered as being at a uniform temperature, T , then from Eq. 5.100 we get

$$T ds \geq \delta q_{\text{net, in}}$$

or

$$T ds - \delta q_{\text{net, in}} \geq 0 \quad (5.101)$$

The relationship between entropy and heat transfer rate depends on the process involved.

The equality is for any reversible (frictionless) process; the inequality is for all irreversible (friction) processes.

5.4.3 Combination of the Equations of the First and Second Laws of Thermodynamics

Combining Eqs. 5.93 and 5.101, we conclude that

$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2} \right) + g dz \right] \geq 0 \quad (5.102)$$

The equality is for any steady, reversible (frictionless) flow, an important example being flow for which the Bernoulli equation (Eq. 3.7) is applicable. The inequality is for all steady, irreversible (friction) flows. The actual amount of the inequality has physical significance. It represents the extent of loss of useful or available energy which occurs because of irreversible flow phenomena including viscous effects. Thus, Eq. 5.102 can be expressed as

$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g dz\right] = \delta(\text{loss}) = (T ds - \delta q_{\text{net in}}) \quad (5.103)$$

The irreversible flow loss is zero for a frictionless flow and greater than zero for a flow with frictional effects. Note that when the flow is frictionless, Eq. 5.103 multiplied by density, ρ , is identical to Eq. 3.5. Thus, for steady frictionless flow, Newton's second law of motion (see Section 3.1) and the first and second laws of thermodynamics lead to the same differential equation,

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g dz = 0 \quad (5.104)$$

If some shaft work is involved, then the flow must be at least locally unsteady in a cyclical way and the appropriate form of the energy equation for the contents of an infinitesimally thin control volume can be developed starting with Eq. 5.67. The resulting equation is

$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g dz\right] = \delta(\text{loss}) - \delta w_{\text{shaft net in}} \quad (5.105)$$

Equations 5.103 and 5.105 are valid for incompressible and compressible flows. If we combine Eqs. 5.92 and 5.103, we obtain

$$d\tilde{u} + p d\left(\frac{1}{\rho}\right) - \delta q_{\text{net in}} = \delta(\text{loss}) \quad (5.106)$$

For incompressible flow, $d(1/\rho) = 0$ and, thus, from Eq. 5.106,

$$d\tilde{u} - \delta q_{\text{net in}} = \delta(\text{loss}) \quad (5.107)$$

Applying Eq. 5.107 to a finite control volume, we obtain

$$\tilde{u}_{\text{out}} - \tilde{u}_{\text{in}} - q_{\text{net in}} = \text{loss}$$

which is the same conclusion we reached earlier (see Eq. 5.78) for incompressible flows.

For compressible flow, $d(1/\rho) \neq 0$, and thus when we apply Eq. 5.106 to a finite control volume we obtain

$$\tilde{u}_{\text{out}} - \tilde{u}_{\text{in}} + \int_{\text{in}}^{\text{out}} p d\left(\frac{1}{\rho}\right) - q_{\text{net in}} = \text{loss} \quad (5.108)$$

indicating that $u_{\text{out}} - u_{\text{in}} - q_{\text{net in}}$ is not equal to loss.

5.4.4 Application of the Loss Form of the Energy Equation

Steady flow along a pathline in an incompressible and frictionless flow field provides a simple application of the loss form of the energy equation (Eq. 5.105). We start with Eq. 5.105 and integrate it term by term from one location on the pathline, section (1), to another one downstream, section (2). Note that because the flow is frictionless, loss = 0. Also, because the flow is steady throughout, $w_{\text{shaft net in}} = 0$. Since the flow is incompressible, the density is constant. The control volume in this case is an infinitesimally small diameter streamtube (Fig. 5.7). The resultant equation is

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 \quad (5.109)$$

which is identical to the Bernoulli equation (Eq. 3.7) already discussed in Chapter 3.

Zero loss is associated with the Bernoulli equation.

If the frictionless and steady pathline flow of the fluid particle considered above was compressible, application of Eq. 5.105 would yield

$$\int_1^2 \frac{dp}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{V_1^2}{2} + gz_1 \quad (5.110)$$

To carry out the integration required, $\int_1^2 (dp/\rho)$, a relationship between fluid density, ρ , and pressure, p , must be known. If the frictionless compressible flow we are considering is adiabatic and involves the flow of an ideal gas, it is shown in Section 11.1 that

$$\frac{p}{\rho^k} = \text{constant} \quad (5.111)$$

where $k = c_p/c_v$ is the ratio of gas specific heats, c_p and c_v , which are properties of the fluid. Using Eq. 5.111 we get

$$\int_1^2 \frac{dp}{\rho} = \frac{k}{k-1} \left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} \right) \quad (5.112)$$

Thus, Eqs. 5.110 and 5.112 lead to

$$\frac{k}{k-1} \frac{p_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 = \frac{k}{k-1} \frac{p_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 \quad (5.113)$$

Note that this equation is identical to Eq. 3.24. An example application of Eqs. 5.109 and 5.113 follows.

EXAMPLE 5.29 Energy—Comparison of Compressible and Incompressible Flow

GIVEN Air steadily expands adiabatically and without friction from stagnation conditions of 100 psia and 520 °R to 14.7 psia.

FIND Determine the velocity of the expanded air assuming (a) incompressible flow, (b) compressible flow.

SOLUTION

(a) If the flow is considered incompressible, the Bernoulli equation, Eq. 5.109, can be applied to flow through an infinitesimal cross-sectional streamtube, like the one in Fig. 5.7, from the stagnation state (1) to the expanded state (2). From Eq. 5.109 we get

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2} = \frac{p_1}{\rho} + \frac{\cancel{V_1^2}}{2} + \cancel{gz_1} \quad (1)$$

0 (1 is the stagnation state)

0 (changes in gz are negligible for air flow)

or

$$V_2 = \sqrt{2 \left(\frac{p_1 - p_2}{\rho} \right)}$$

We can calculate the density at state (1) by assuming that air behaves like an ideal gas,

$$\rho = \frac{p_1}{RT_1} = \frac{(100 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)}{(1716 \text{ ft} \cdot \text{lb}/\text{slug} \cdot \text{°R})(520 \text{ °R})} = 0.0161 \text{ slug}/\text{ft}^3 \quad (2)$$

Thus,

$$V_2 = \sqrt{\frac{2(100 \text{ psia} - 14.7 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)}{(0.016 \text{ slug}/\text{ft}^3)[1 (\text{lb} \cdot \text{s}^2)/(\text{slug} \cdot \text{ft})]}} = 1240 \text{ ft/s} \quad (\text{Ans})$$

The assumption of incompressible flow is not valid in this case since for air a change from 100 psia to 14.7 psia would undoubtedly result in a significant density change.

(b) If the flow is considered compressible, Eq. 5.113 can be applied to the flow through an infinitesimal cross-sectional control volume, like the one in Fig. 5.7, from the stagnation state (1) to the expanded state (2). We obtain

$$\frac{k}{k-1} \frac{p_2}{\rho_2} + \frac{V_2^2}{2} + \cancel{gz_2} = \frac{k}{k-1} \frac{p_1}{\rho_1} + \frac{\cancel{V_1^2}}{2} + \cancel{gz_1} \quad (3)$$

0 (1 is the stagnation state)

0 (changes in gz are negligible for air flow)

or

$$V_2 = \sqrt{\frac{2k}{k-1} \left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} \right)} \quad (4)$$

Given in the problem statement are values of p_1 and p_2 . A value of ρ_1 was calculated earlier (Eq. 2). To determine ρ_2 we need to make use of a property relationship for reversible (frictionless) and adiabatic flow of an ideal gas that is derived in Chapter 11; namely,

$$\frac{p}{\rho^k} = \text{constant} \quad (5)$$

where $k = 1.4$ for air. Solving Eq. 5 for ρ_2 we get

$$\rho_2 = \rho_1 \left(\frac{p_2}{p_1} \right)^{1/k}$$

or

$$\rho_2 = (0.0161 \text{ slug/ft}^3) \left[\frac{14.7 \text{ psia}}{100 \text{ psia}} \right]^{1/1.4} = 0.00409 \text{ slug/ft}^3$$

Then, from Eq. 4, with $p_1 = 100 \text{ lb/in.}^2 (144 \text{ in.}^2/\text{ft}^2) = 14,400 \text{ lb/ft}^2$ and $p_2 = 14.7 \text{ lb/in.}^2 (144 \text{ in.}^2/\text{ft}^2) = 2117 \text{ lb/ft}^2$,

$$\begin{aligned} V_2 &= \sqrt{\frac{(2)(1.4)}{1.4-1} \left(\frac{14,400 \text{ lb/ft}^2}{0.0161 \text{ slug/ft}^3} - \frac{2117 \text{ lb/ft}^2}{0.00409 \text{ slug/ft}^3} \right)} \\ &= 1620 (\text{lb} \cdot \text{ft/slug})^{1/2} [(1 \text{ slug} \cdot \text{ft/s}^2)/\text{lb}]^{1/2} \end{aligned}$$

or

$$V_2 = 1620 \text{ ft/s} \quad (\text{Ans})$$

COMMENT A considerable difference exists between the air velocities calculated assuming incompressible and compressible flow. In Section 3.8.1, a discussion of when a fluid flow may be appropriately considered incompressible is provided. Basically, when flow speed is less than a third of the speed of sound in the fluid involved, incompressible flow may be assumed with only a small error.