

## Chapter 6 – Student Solutions Manual

1. We do not consider the possibility that the bureau might tip, and treat this as a purely horizontal motion problem (with the person's push  $\vec{F}$  in the  $+x$  direction). Applying Newton's second law to the  $x$  and  $y$  axes, we obtain

$$\begin{aligned}F - f_{s, \max} &= ma \\F_N - mg &= 0\end{aligned}$$

respectively. The second equation yields the normal force  $F_N = mg$ , whereupon the maximum static friction is found to be (from Eq. 6-1)  $f_{s, \max} = \mu_s mg$ . Thus, the first equation becomes

$$F - \mu_s mg = ma = 0$$

where we have set  $a = 0$  to be consistent with the fact that the static friction is still (just barely) able to prevent the bureau from moving.

(a) With  $\mu_s = 0.45$  and  $m = 45$  kg, the equation above leads to  $F = 198$  N. To bring the bureau into a state of motion, the person should push with any force greater than this value. Rounding to two significant figures, we can therefore say the minimum required push is  $F = 2.0 \times 10^2$  N.

(b) Replacing  $m = 45$  kg with  $m = 28$  kg, the reasoning above leads to roughly  $F = 1.2 \times 10^2$  N.

3. We denote  $\vec{F}$  as the horizontal force of the person exerted on the crate (in the  $+x$  direction),  $\vec{f}_k$  is the force of kinetic friction (in the  $-x$  direction),  $F_N$  is the vertical normal force exerted by the floor (in the  $+y$  direction), and  $m\vec{g}$  is the force of gravity. The magnitude of the force of friction is given by  $f_k = \mu_k F_N$  (Eq. 6-2). Applying Newton's second law to the  $x$  and  $y$  axes, we obtain

$$\begin{aligned}F - f_k &= ma \\F_N - mg &= 0\end{aligned}$$

respectively.

(a) The second equation yields the normal force  $F_N = mg$ , so that the friction is

$$f_k = \mu_k mg = (0.35)(55 \text{ kg})(9.8 \text{ m/s}^2) = 1.9 \times 10^2 \text{ N}.$$

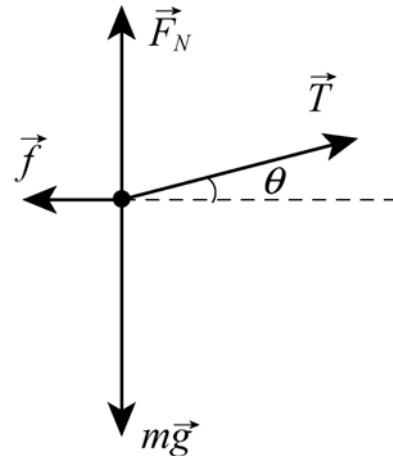
(b) The first equation becomes

$$F - \mu_k mg = ma$$

which (with  $F = 220 \text{ N}$ ) we solve to find

$$a = \frac{F}{m} - \mu_k g = 0.56 \text{ m/s}^2 .$$

13. (a) The free-body diagram for the crate is shown on the right.  $\vec{T}$  is the tension force of the rope on the crate,  $\vec{F}_N$  is the normal force of the floor on the crate,  $m\vec{g}$  is the force of gravity, and  $\vec{f}$  is the force of friction. We take the  $+x$  direction to be horizontal to the right and the  $+y$  direction to be up. We assume the crate is motionless. The equations for the  $x$  and the  $y$  components of the force according to Newton's second law are:



$$\begin{aligned} T \cos \theta - f &= 0 \\ T \sin \theta + F_N - mg &= 0 \end{aligned}$$

where  $\theta = 15^\circ$  is the angle between the rope and the horizontal. The first equation gives  $f = T \cos \theta$  and the second gives  $F_N = mg - T \sin \theta$ . If the crate is to remain at rest,  $f$  must be less than  $\mu_s F_N$ , or  $T \cos \theta < \mu_s (mg - T \sin \theta)$ . When the tension force is sufficient to just start the crate moving, we must have

$$T \cos \theta = \mu_s (mg - T \sin \theta).$$

We solve for the tension:

$$T = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta} = \frac{(0.50)(68 \text{ kg})(9.8 \text{ m/s}^2)}{\cos 15^\circ + 0.50 \sin 15^\circ} = 304 \text{ N} \approx 3.0 \times 10^2 \text{ N}.$$

(b) The second law equations for the moving crate are

$$\begin{aligned} T \cos \theta - f &= ma \\ F_N + T \sin \theta - mg &= 0. \end{aligned}$$

Now  $f = \mu_k F_N$ , and the second equation gives  $F_N = mg - T \sin \theta$ , which yields  $f = \mu_k (mg - T \sin \theta)$ . This expression is substituted for  $f$  in the first equation to obtain

$$T \cos \theta - \mu_k (mg - T \sin \theta) = ma,$$

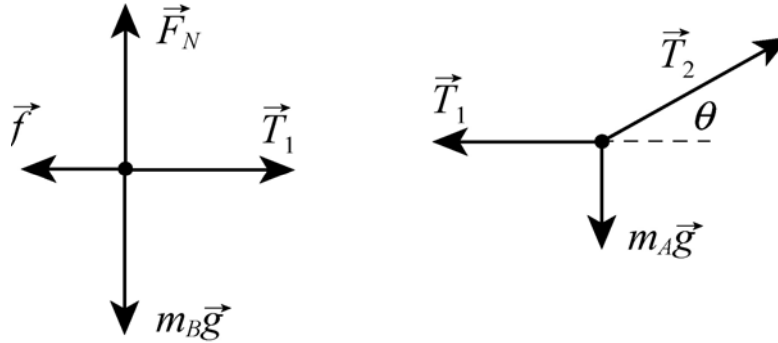
so the acceleration is

$$a = \frac{T(\cos \theta + \mu_k \sin \theta)}{m} - \mu_k g .$$

Numerically, it is given by

$$a = \frac{(304 \text{ N})(\cos 15^\circ + 0.35 \sin 15^\circ)}{68 \text{ kg}} - (0.35)(9.8 \text{ m/s}^2) = 1.3 \text{ m/s}^2 .$$

23. The free-body diagrams for block  $B$  and for the knot just above block  $A$  are shown next.  $\vec{T}_1$  is the tension force of the rope pulling on block  $B$  or pulling on the knot (as the case may be),  $\vec{T}_2$  is the tension force exerted by the second rope (at angle  $\theta = 30^\circ$ ) on the knot,  $\vec{f}$  is the force of static friction exerted by the horizontal surface on block  $B$ ,  $\vec{F}_N$  is normal force exerted by the surface on block  $B$ ,  $W_A$  is the weight of block  $A$  ( $W_A$  is the magnitude of  $m_A \vec{g}$ ), and  $W_B$  is the weight of block  $B$  ( $W_B = 711 \text{ N}$  is the magnitude of  $m_B \vec{g}$ ).



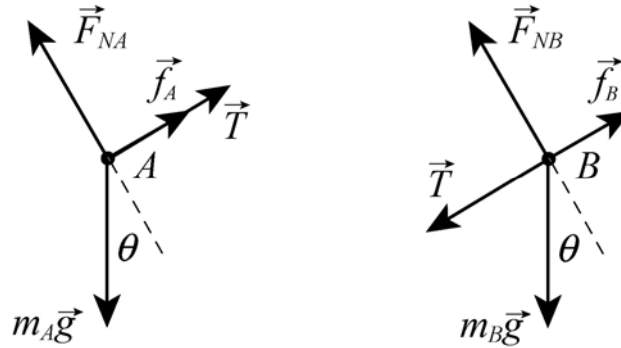
For each object we take  $+x$  horizontally rightward and  $+y$  upward. Applying Newton's second law in the  $x$  and  $y$  directions for block  $B$  and then doing the same for the knot results in four equations:

$$\begin{aligned} T_1 - f_{s,\max} &= 0 \\ F_N - W_B &= 0 \\ T_2 \cos \theta - T_1 &= 0 \\ T_2 \sin \theta - W_A &= 0 \end{aligned}$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). Solving these equations with  $\mu_s = 0.25$ , we obtain  $W_A = 103 \text{ N} \approx 1.0 \times 10^2 \text{ N}$ .

27. The free-body diagrams for the two blocks are shown next.  $T$  is the magnitude of the tension force of the string,  $\vec{F}_{NA}$  is the normal force on block  $A$  (the leading block),  $\vec{F}_{NB}$  is the normal force on block  $B$ ,  $\vec{f}_A$  is kinetic friction force on block  $A$ ,  $\vec{f}_B$  is kinetic friction force on block  $B$ . Also,  $m_A$  is the mass of block  $A$  (where  $m_A = W_A/g$  and  $W_A = 3.6 \text{ N}$ ), and

$m_B$  is the mass of block  $B$  (where  $m_B = W_B/g$  and  $W_B = 7.2$  N). The angle of the incline is  $\theta = 30^\circ$ .



For each block we take  $+x$  downhill (which is toward the lower-left in these diagrams) and  $+y$  in the direction of the normal force. Applying Newton's second law to the  $x$  and  $y$  directions of both blocks  $A$  and  $B$ , we arrive at four equations:

$$\begin{aligned} W_A \sin \theta - f_A - T &= m_A a \\ F_{NA} - W_A \cos \theta &= 0 \\ W_B \sin \theta - f_B + T &= m_B a \\ F_{NB} - W_B \cos \theta &= 0 \end{aligned}$$

which, when combined with Eq. 6-2 ( $f_A = \mu_{kA} F_{NA}$  where  $\mu_{kA} = 0.10$  and  $f_B = \mu_{kB} F_{NB}$  where  $\mu_{kB} = 0.20$ ), fully describe the dynamics of the system so long as the blocks have the same acceleration and  $T > 0$ .

(a) From these equations, we find the acceleration to be

$$a = g \left( \sin \theta - \left( \frac{\mu_{kA} W_A + \mu_{kB} W_B}{W_A + W_B} \right) \cos \theta \right) = 3.5 \text{ m/s}^2.$$

(b) We solve the above equations for the tension and obtain

$$T = \left( \frac{W_A W_B}{W_A + W_B} \right) (\mu_{kB} - \mu_{kA}) \cos \theta = 0.21 \text{ N}.$$

Simply returning the value for  $a$  found in part (a) into one of the above equations is certainly fine, and probably easier than solving for  $T$  algebraically as we have done, but the algebraic form does illustrate the  $\mu_{kB} - \mu_{kA}$  factor which aids in the understanding of the next part.

35. We denote the magnitude of the frictional force  $\alpha v$ , where  $\alpha = 70 \text{ N} \cdot \text{s/m}$ . We take the direction of the boat's motion to be positive. Newton's second law gives

$$-\alpha v = m \frac{dv}{dt}.$$

Thus,

$$\int_{v_0}^v \frac{dv}{v} = -\frac{\alpha}{m} \int_0^t dt$$

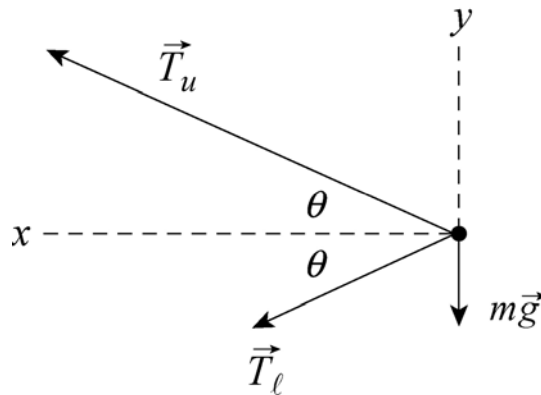
where  $v_0$  is the velocity at time zero and  $v$  is the velocity at time  $t$ . The integrals are evaluated with the result

$$\ln\left(\frac{v}{v_0}\right) = -\frac{\alpha t}{m}$$

We take  $v = v_0/2$  and solve for time:

$$t = \frac{m}{\alpha} \ln 2 = \frac{1000 \text{ kg}}{70 \text{ N} \cdot \text{s/m}} \ln 2 = 9.9 \text{ s}.$$

59. The free-body diagram for the ball is shown below.  $\vec{T}_u$  is the tension exerted by the upper string on the ball,  $\vec{T}_\ell$  is the tension force of the lower string, and  $m$  is the mass of the ball. Note that the tension in the upper string is greater than the tension in the lower string. It must balance the downward pull of gravity and the force of the lower string.



(a) We take the  $+x$  direction to be leftward (toward the center of the circular orbit) and  $+y$  upward. Since the magnitude of the acceleration is  $a = v^2/R$ , the  $x$  component of Newton's second law is

$$T_u \cos \theta + T_\ell \cos \theta = \frac{mv^2}{R},$$

where  $v$  is the speed of the ball and  $R$  is the radius of its orbit. The  $y$  component is

$$T_u \sin \theta - T_\ell \sin \theta - mg = 0.$$

The second equation gives the tension in the lower string:  $T_\ell = T_u - mg / \sin \theta$ . Since the triangle is equilateral  $\theta = 30.0^\circ$ . Thus

$$T_\ell = 35.0 \text{ N} - \frac{(1.34 \text{ kg})(9.80 \text{ m/s}^2)}{\sin 30.0^\circ} = 8.74 \text{ N}.$$

(b) The net force has magnitude

$$F_{\text{net,str}} = (T_u + T_\ell) \cos \theta = (35.0 \text{ N} + 8.74 \text{ N}) \cos 30.0^\circ = 37.9 \text{ N}.$$

(c) The radius of the path is

$$R = ((1.70 \text{ m})/2) \tan 30.0^\circ = 1.47 \text{ m}.$$

Using  $F_{\text{net,str}} = mv^2/R$ , we find that the speed of the ball is

$$v = \sqrt{\frac{RF_{\text{net,str}}}{m}} = \sqrt{\frac{(1.47 \text{ m})(37.9 \text{ N})}{1.34 \text{ kg}}} = 6.45 \text{ m/s}.$$

(d) The direction of  $\vec{F}_{\text{net,str}}$  is leftward (“radially inward”).

65. (a) Using  $F = \mu_s mg$ , the coefficient of static friction for the surface between the two blocks is  $\mu_s = (12 \text{ N})/(39.2 \text{ N}) = 0.31$ , where  $m_t g = (4.0 \text{ kg})(9.8 \text{ m/s}^2) = 39.2 \text{ N}$  is the weight of the top block. Let  $M = m_t + m_b = 9.0 \text{ kg}$  be the total system mass, then the maximum horizontal force has a magnitude  $Ma = M\mu_s g = 27 \text{ N}$ .

(b) The acceleration (in the maximal case) is  $a = \mu_s g = 3.0 \text{ m/s}^2$ .

77. The magnitude of the acceleration of the cyclist as it moves along the horizontal circular path is given by  $v^2/R$ , where  $v$  is the speed of the cyclist and  $R$  is the radius of the curve.

(a) The horizontal component of Newton’s second law is  $f = mv^2/R$ , where  $f$  is the static friction exerted horizontally by the ground on the tires. Thus,

$$f = \frac{(85.0 \text{ kg})(9.00 \text{ m/s})^2}{25.0 \text{ m}} = 275 \text{ N}.$$

(b) If  $F_N$  is the vertical force of the ground on the bicycle and  $m$  is the mass of the bicycle and rider, the vertical component of Newton's second law leads to  $F_N = mg = 833 \text{ N}$ . The magnitude of the force exerted by the ground on the bicycle is therefore

$$\sqrt{f^2 + F_N^2} = \sqrt{(275 \text{ N})^2 + (833 \text{ N})^2} = 877 \text{ N}.$$

81. (a) If we choose “downhill” positive, then Newton's law gives

$$m_A g \sin \theta - f_A - T = m_A a$$

for block A (where  $\theta = 30^\circ$ ). For block B we choose leftward as the positive direction and write  $T - f_B = m_B a$ . Now

$$f_A = \mu_{k, \text{incline}} F_{NA} = \mu' m_A g \cos \theta$$

using Eq. 6-12 applies to block A, and

$$f_B = \mu_k F_{NB} = \mu_k m_B g.$$

In this particular problem, we are asked to set  $\mu' = 0$ , and the resulting equations can be straightforwardly solved for the tension:  $T = 13 \text{ N}$ .

(b) Similarly, finding the value of  $a$  is straightforward:

$$a = g(m_A \sin \theta - \mu_k m_B) / (m_A + m_B) = 1.6 \text{ m/s}^2.$$

85. The mass of the car is  $m = (10700/9.80) \text{ kg} = 1.09 \times 10^3 \text{ kg}$ . We choose “inward” (horizontally towards the center of the circular path) as the positive direction.

(a) With  $v = 13.4 \text{ m/s}$  and  $R = 61 \text{ m}$ , Newton's second law (using Eq. 6-18) leads to

$$f_s = \frac{mv^2}{R} = 3.21 \times 10^3 \text{ N}.$$

(b) Noting that  $F_N = mg$  in this situation, the maximum possible static friction is found to be

$$f_{s, \text{max}} = \mu_s mg = (0.35)(10700 \text{ N}) = 3.75 \times 10^3 \text{ N}$$

using Eq. 6-1. We see that the static friction found in part (a) is less than this, so the car rolls (no skidding) and successfully negotiates the curve.

91. We apply Newton's second law (as  $F_{\text{push}} - f = ma$ ). If we find  $F_{\text{push}} < f_{\text{max}}$ , we conclude “no, the cabinet does not move” (which means  $a$  is actually 0 and  $f = F_{\text{push}}$ ), and

if we obtain  $a > 0$  then it moves (so  $f = f_k$ ). For  $f_{\max}$  and  $f_k$  we use Eq. 6-1 and Eq. 6-2 (respectively), and in those formulas we set the magnitude of the normal force equal to 556 N. Thus,  $f_{\max} = 378$  N and  $f_k = 311$  N.

(a) Here we find  $F_{\text{push}} < f_{\max}$  which leads to  $f = F_{\text{push}} = 222$  N.

(b) Again we find  $F_{\text{push}} < f_{\max}$  which leads to  $f = F_{\text{push}} = 334$  N.

(c) Now we have  $F_{\text{push}} > f_{\max}$  which means it moves and  $f = f_k = 311$  N.

(d) Again we have  $F_{\text{push}} > f_{\max}$  which means it moves and  $f = f_k = 311$  N.

(e) The cabinet moves in (c) and (d).

99. Replace  $f_s$  with  $f_k$  in Fig. 6-5(b) to produce the appropriate force diagram for the first part of this problem (when it is sliding downhill with zero acceleration). This amounts to replacing the static coefficient with the kinetic coefficient in Eq. 6-13:  $\mu_k = \tan \theta$ . Now (for the second part of the problem, with the block projected uphill) the friction direction is reversed from what is shown in Fig. 6-5(b). Newton's second law for the uphill motion (and Eq. 6-12) leads to

$$-mg \sin \theta - \mu_k mg \cos \theta = ma.$$

Canceling the mass and substituting what we found earlier for the coefficient, we have

$$-g \sin \theta - \tan \theta g \cos \theta = a.$$

This simplifies to  $-2g \sin \theta = a$ . Eq. 2-16 then gives the distance to stop:  $\Delta x = -v_o^2/2a$ .

(a) Thus, the distance up the incline traveled by the block is  $\Delta x = v_o^2/(4g \sin \theta)$ .

(b) We usually expect  $\mu_s > \mu_k$  (see the discussion in section 6-1). Sample Problem 6-2 treats the "angle of repose" (the minimum angle necessary for a stationary block to start sliding downhill):  $\mu_s = \tan(\theta_{\text{repose}})$ . Therefore, we expect  $\theta_{\text{repose}} > \theta$  found in part (a). Consequently, when the block comes to rest, the incline is not steep enough to cause it to start slipping down the incline again.

105. Probably the most appropriate picture in the textbook to represent the situation in this problem is in the previous chapter: Fig. 5-9. We adopt the familiar axes with +x rightward and +y upward, and refer to the 85 N horizontal push of the worker as  $P$  (and assume it to be rightward). Applying Newton's second law to the x axis and y axis, respectively, produces

$$\begin{aligned} P - f_k &= ma \\ F_N - mg &= 0. \end{aligned}$$

Using  $v^2 = v_0^2 + 2a\Delta x$  we find  $a = 0.36 \text{ m/s}^2$ . Consequently, we obtain  $f_k = 71 \text{ N}$  and  $F_N = 392 \text{ N}$ . Therefore,  $\mu_k = f_k / F_N = 0.18$ .