

Chapter 8 – Student Solutions Manual

5. The potential energy stored by the spring is given by $U = \frac{1}{2} kx^2$, where k is the spring constant and x is the displacement of the end of the spring from its position when the spring is in equilibrium. Thus

$$k = \frac{2U}{x^2} = \frac{2(25\text{J})}{(0.075\text{m})^2} = 8.9 \times 10^3 \text{ N/m}.$$

9. We neglect any work done by friction. We work with SI units, so the speed is converted: $v = 130(1000/3600) = 36.1 \text{ m/s}$.

(a) We use Eq. 8-17: $K_f + U_f = K_i + U_i$ with $U_i = 0$, $U_f = mgh$ and $K_f = 0$. Since $K_i = \frac{1}{2}mv^2$, where v is the initial speed of the truck, we obtain

$$\frac{1}{2}mv^2 = mgh \Rightarrow h = \frac{v^2}{2g} = \frac{(36.1 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 66.5 \text{ m}.$$

If L is the length of the ramp, then $L \sin 15^\circ = 66.5 \text{ m}$ so that $L = 66.5/\sin 15^\circ = 257 \text{ m}$. Therefore, the ramp must be about $2.6 \times 10^2 \text{ m}$ long if friction is negligible.

(b) The answers do not depend on the mass of the truck. They remain the same if the mass is reduced.

(c) If the speed is decreased, h and L both decrease (note that h is proportional to the square of the speed and that L is proportional to h).

11. (a) If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then $K_f + U_f = K_i + U_i$.

Taking the potential energy to be zero at the bottom of the bowl, then the potential energy at the top is $U_i = mgr$ where $r = 0.220 \text{ m}$ is the radius of the bowl and m is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, we write $\frac{1}{2}mv^2$ for K_f . Energy conservation leads to

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL(1 - \cos \theta).$$

The speed is $v = \sqrt{2gr} = 2.08 \text{ m/s}$.

(b) Since the expression for speed does not contain the mass of the flake, the speed would be the same, 2.08 m/s, regardless of the mass of the flake.

(c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. Since K_i is greater than before, K_f is greater. This means the final speed of the flake is greater.

31. We refer to its starting point as A , the point where it first comes into contact with the spring as B , and the point where the spring is compressed $|x| = 0.055$ m as C . Point C is our reference point for computing gravitational potential energy. Elastic potential energy (of the spring) is zero when the spring is relaxed. Information given in the second sentence allows us to compute the spring constant. From Hooke's law, we find

$$k = \frac{F}{x} = \frac{270 \text{ N}}{0.02 \text{ m}} = 1.35 \times 10^4 \text{ N/m} .$$

(a) The distance between points A and B is $\vec{F}_g$ and we note that the total sliding distance $\ell + |x|$ is related to the initial height h of the block (measured relative to C) by

$$\frac{h}{\ell + |x|} = \sin \theta$$

where the incline angle θ is 30° . Mechanical energy conservation leads to

$$\begin{aligned} K_A + U_A &= K_C + U_C \\ 0 + mgh &= 0 + \frac{1}{2} kx^2 \end{aligned}$$

which yields

$$h = \frac{kx^2}{2mg} = \frac{(1.35 \times 10^4 \text{ N/m})(0.055 \text{ m})^2}{2(12 \text{ kg})(9.8 \text{ m/s}^2)} = 0.174 \text{ m} .$$

Therefore,

$$\ell + |x| = \frac{h}{\sin 30^\circ} = \frac{0.174 \text{ m}}{\sin 30^\circ} = 0.35 \text{ m} .$$

(b) From this result, we find $\ell = 0.35 - 0.055 = 0.29$ m, which means that $\Delta y = -\ell \sin \theta = -0.15$ m in sliding from point A to point B . Thus, Eq. 8-18 gives

$$\begin{aligned} \Delta K + \Delta U &= 0 \\ \frac{1}{2} mv_B^2 + mg\Delta h &= 0 \end{aligned}$$

which yields $v_B = \sqrt{-2g\Delta h} = \sqrt{-(9.8)(-0.15)} = 1.7 \text{ m/s}$.

45. (a) The work done on the block by the force in the rope is, using Eq. 7-7,

$$W = Fd \cos \theta = (7.68 \text{ N})(4.06 \text{ m}) \cos 15.0^\circ = 30.1 \text{ J}.$$

(b) Using f for the magnitude of the kinetic friction force, Eq. 8-29 reveals that the increase in thermal energy is

$$\Delta E_{\text{th}} = fd = (7.42 \text{ N})(4.06 \text{ m}) = 30.1 \text{ J}.$$

(c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f/F_N$ to obtain the coefficient of friction. Place the x axis along the path of the block and the y axis normal to the floor. The x and the y component of Newton's second law are

$$\begin{aligned} x: \quad F \cos \theta - f &= 0 \\ y: \quad F_N + F \sin \theta - mg &= 0, \end{aligned}$$

where m is the mass of the block, F is the force exerted by the rope, and θ is the angle between that force and the horizontal. The first equation gives

$$f = F \cos \theta = (7.68 \text{ N}) \cos 15.0^\circ = 7.42 \text{ N}$$

and the second gives

$$F_N = mg - F \sin \theta = (3.57 \text{ kg})(9.8 \text{ m/s}^2) - (7.68 \text{ N}) \sin 15.0^\circ = 33.0 \text{ N}.$$

Thus,

$$\mu_k = \frac{f}{F_N} = \frac{7.42 \text{ N}}{33.0 \text{ N}} = 0.225.$$

47. (a) We take the initial gravitational potential energy to be $U_i = 0$. Then the final gravitational potential energy is $U_f = -mgL$, where L is the length of the tree. The change is

$$U_f - U_i = -mgL = -(25 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m}) = -2.9 \times 10^3 \text{ J}.$$

(b) The kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}(25 \text{ kg})(5.6 \text{ m/s})^2 = 3.9 \times 10^2 \text{ J}$.

(c) The changes in the mechanical and thermal energies must sum to zero. The change in thermal energy is $\Delta E_{\text{th}} = fL$, where f is the magnitude of the average frictional force; therefore,

$$f = -\frac{\Delta K + \Delta U}{L} = -\frac{3.9 \times 10^2 \text{ J} - 2.9 \times 10^3 \text{ J}}{12 \text{ m}} = 2.1 \times 10^2 \text{ N}$$

69. There is the same potential energy change in both circumstances, so we can equate the kinetic energy changes as well:

$$\Delta K_2 = \Delta K_1 \Rightarrow \frac{1}{2} m v_B^2 - \frac{1}{2} m (4.00)^2 = \frac{1}{2} m (2.60)^2 - \frac{1}{2} m (2.00)^2$$

which leads to $v_B = 4.33 \text{ m/s}$.

75. We note that if the larger mass (block B, $m_B = 2 \text{ kg}$) falls $d = 0.25 \text{ m}$, then the smaller mass (blocks A, $m_A = 1 \text{ kg}$) must increase its height by $h = d \sin 30^\circ$. Thus, by mechanical energy conservation, the kinetic energy of the system is

$$K_{\text{total}} = m_B g d - m_A g h = 3.7 \text{ J}.$$

83. The initial height shown in the figure is the $y = 0$ level in our computations of U_g , and in parts (a) and (b) the heights are $y_a = 0.80 \sin 40^\circ = 0.51 \text{ m}$ and $y_b = 1.00 \sin 40^\circ = 0.64 \text{ m}$, respectively.

(a) The conservation of energy, Eq. 8-17, leads to

$$K_i + U_i = K_a + U_a \Rightarrow 16 + 0 = K_a + mgy_a + \frac{1}{2}k(0.20)^2$$

from which we obtain $K_a = 16 - 5.0 - 4.0 = 7.0 \text{ J}$.

(b) Again we use the conservation of energy

$$K_i + U_i = K_b + U_b \Rightarrow K_i + 0 = 0 + mgy_b + \frac{1}{2}k(0.40)^2$$

from which we obtain $K_i = 6.0 + 16 = 22 \text{ J}$.

87. Since the speed is constant $\Delta K = 0$ and Eq. 8-33 (an application of the energy conservation concept) implies

$$W_{\text{applied}} = \Delta E_{\text{th}} = \Delta E_{\text{th(cube)}} + \Delta E_{\text{th(floor)}}.$$

Thus, if $W_{\text{applied}} = (15)(3.0) = 45 \text{ J}$, and we are told that $\Delta E_{\text{th(cube)}} = 20 \text{ J}$, then we conclude that $\Delta E_{\text{th(floor)}} = 25 \text{ J}$.

109. The connection between angle θ (measured from vertical) and height h (measured from the lowest point, which is our choice of reference position in computing the

gravitational potential energy mgh) is given by $h = L(1 - \cos \theta)$ where L is the length of the pendulum.

(a) Using this formula (or simply using intuition) we see the initial height is $h_1 = 2L$, and of course $h_2 = 0$. We use energy conservation in the form of Eq. 8-17.

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + mg(2L) &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

This leads to $v = 2\sqrt{gL}$. With $L = 0.62$ m, we have

$$v = 2\sqrt{(9.8 \text{ m/s}^2)(0.62 \text{ m})} = 4.9 \text{ m/s}.$$

(b) The ball is in circular motion with the center of the circle above it, so $\vec{a} = v^2 / r$ upward, where $r = L$. Newton's second law leads to

$$T - mg = m \frac{v^2}{r} \Rightarrow T = m \left(g + \frac{4gL}{L} \right) = 5 mg.$$

With $m = 0.092$ kg, the tension is given by $T = 4.5$ N.

(c) The pendulum is now started (with zero speed) at $\theta_i = 90^\circ$ (that is, $h_i = L$), and we look for an angle θ such that $T = mg$. When the ball is moving through a point at angle θ , then Newton's second law applied to the axis along the rod yields

$$T - mg \cos \theta = m \frac{v^2}{r}$$

which (since $r = L$) implies $v^2 = gL(1 - \cos \theta)$ at the position we are looking for. Energy conservation leads to

$$\begin{aligned} K_i + U_i &= K + U \\ 0 + mgL &= \frac{1}{2}mv^2 + mgL(1 - \cos \theta) \\ gL &= \frac{1}{2}(gL(1 - \cos \theta)) + gL(1 - \cos \theta) \end{aligned}$$

where we have divided by mass in the last step. Simplifying, we obtain

$$\theta = \cos^{-1} \left(\frac{1}{3} \right) = 71^\circ.$$

(d) Since the angle found in (c) is independent of the mass, the result remains the same if the mass of the ball is changed.

111. (a) At the top of its flight, the vertical component of the velocity vanishes, and the horizontal component (neglecting air friction) is the same as it was when it was thrown. Thus,

$$K_{\text{top}} = \frac{1}{2}mv_x^2 = \frac{1}{2}(0.050 \text{ kg})((8.0 \text{ m/s})\cos 30^\circ)^2 = 1.2 \text{ J}.$$

(b) We choose the point 3.0 m below the window as the reference level for computing the potential energy. Thus, equating the mechanical energy when it was thrown to when it is at this reference level, we have (with SI units understood)

$$\begin{aligned} mgy_0 + K_0 &= K \\ m(9.8)(3.0) + \frac{1}{2}m(8.0)^2 &= \frac{1}{2}mv^2 \end{aligned}$$

which yields (after canceling m and simplifying) $v = 11 \text{ m/s}$.

(c) As mentioned, m cancels — and is therefore not relevant to that computation.

(d) The v in the kinetic energy formula is the magnitude of the velocity vector; it does not depend on the direction.

119. (a) During the final $d = 12 \text{ m}$ of motion, we use

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 + f_k d \\ \frac{1}{2}mv^2 + 0 &= 0 + 0 + f_k d \end{aligned}$$

where $v = 4.2 \text{ m/s}$. This gives $f_k = 0.31 \text{ N}$. Therefore, the thermal energy change is $f_k d = 3.7 \text{ J}$.

(b) Using $f_k = 0.31 \text{ N}$ we obtain $f_k d_{\text{total}} = 4.3 \text{ J}$ for the thermal energy generated by friction; here, $d_{\text{total}} = 14 \text{ m}$.

(c) During the initial $d' = 2 \text{ m}$ of motion, we have

$$K_0 + U_0 + W_{\text{app}} = K_1 + U_1 + f_k d' \Rightarrow 0 + 0 + W_{\text{app}} = \frac{1}{2}mv^2 + 0 + f_k d'$$

which essentially combines Eq. 8-31 and Eq. 8-33. This leads to the result $W_{\text{app}} = 4.3 \text{ J}$, and — reasonably enough — is the same as our answer in part (b).

121. We use Eq. 8-20.

(a) The force at $x = 2.0$ m is

$$F = -\frac{dU}{dx} \approx -\frac{-(17.5) - (-2.8)}{4.0 - 1.0} = 4.9 \text{ N.}$$

(b) The force points in the $+x$ direction (but there is some uncertainty in reading the graph which makes the last digit not very significant).

(c) The total mechanical energy at $x = 2.0$ m is

$$E = \frac{1}{2}mv^2 + U \approx \frac{1}{2}(2.0)(-1.5)^2 - 7.7 = -5.5$$

in SI units (Joules). Again, there is some uncertainty in reading the graph which makes the last digit not very significant. At that level (-5.5 J) on the graph, we find two points where the potential energy curve has that value — at $x \approx 1.5$ m and $x \approx 13.5$ m. Therefore, the particle remains in the region $1.5 < x < 13.5$ m. The left boundary is at $x = 1.5$ m.

(d) From the above results, the right boundary is at $x = 13.5$ m.

(e) At $x = 7.0$ m, we read $U \approx -17.5$ J. Thus, if its total energy (calculated in the previous part) is $E \approx -5.5$ J, then we find

$$\frac{1}{2}mv^2 = E - U \approx 12 \text{ J} \Rightarrow v = \sqrt{\frac{2}{m}(E - U)} \approx 3.5 \text{ m/s}$$

where there is certainly room for disagreement on that last digit for the reasons cited above.

123. Converting to SI units, $v_0 = 8.3$ m/s and $v = 11.1$ m/s. The incline angle is $\theta = 5.0^\circ$. The height difference between the car's highest and lowest points is $(50 \text{ m}) \sin \theta = 4.4$ m. We take the lowest point (the car's final reported location) to correspond to the $y = 0$ reference level.

(a) Using Eq. 8-31 and Eq. 8-33, we find

$$f_k d = -\Delta K - \Delta U \Rightarrow f_k d = \frac{1}{2}m(v_0^2 - v^2) + mgy_0.$$

Therefore, the mechanical energy reduction (due to friction) is $f_k d = 2.4 \times 10^4$ J.

(b) With $d = 50$ m, we solve for f_k and obtain 4.7×10^2 N.

127. (a) When there is no change in potential energy, Eq. 8-24 leads to

$$W_{\text{app}} = \Delta K = \frac{1}{2} m (v^2 - v_0^2) .$$

Therefore, $\Delta E = 6.0 \times 10^3 \text{ J}$.

(b) From the above manipulation, we see $W_{\text{app}} = 6.0 \times 10^3 \text{ J}$. Also, from Chapter 2, we know that $\Delta t = \Delta v/a = 10 \text{ s}$. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{6.0 \times 10^3}{10} = 600 \text{ W} .$$

(c) and (d) The constant applied force is $ma = 30 \text{ N}$ and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 300 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 900 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

We note that the average of these two values agrees with the result in part (b).

131. The power generation (assumed constant, so average power is the same as instantaneous power) is

$$P = \frac{mgh}{t} = \frac{(3/4)(1200 \text{ m}^3)(10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(100 \text{ m})}{1.0 \text{ s}} = 8.80 \times 10^8 \text{ W} .$$

133. (a) Sample Problem 8-3 illustrates simple energy conservation in a similar situation, and derives the frequently encountered relationship: $v = \sqrt{2gh}$. In our present problem, the height change is equal to the rod length L . Thus, using the suggested notation for the speed, we have $v_o = \sqrt{2gL}$.

(b) At B the speed is (from Eq. 8-17)

$$v = \sqrt{v_o^2 + 2gL} = \sqrt{4gL} .$$

The direction of the centripetal acceleration ($v^2/r = 4gL/L = 4g$) is upward (at that moment), as is the tension force. Thus, Newton's second law gives

$$T - mg = m(4g) \Rightarrow T = 5mg .$$

(c) The difference in height between C and D is L , so the “loss” of mechanical energy (which goes into thermal energy) is $-mgL$.

(d) The difference in height between B and D is $2L$, so the total “loss” of mechanical energy (which all goes into thermal energy) is $-2mgL$.