

Chapter 19 – Student Solutions Manual

7. (a) In solving $pV = nRT$ for n , we first convert the temperature to the Kelvin scale: $T = (40.0 + 273.15) \text{ K} = 313.15 \text{ K}$. And we convert the volume to SI units: $1000 \text{ cm}^3 = 1000 \times 10^{-6} \text{ m}^3$. Now, according to the ideal gas law,

$$n = \frac{pV}{RT} = \frac{(1.01 \times 10^5 \text{ Pa})(1000 \times 10^{-6} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(313.15 \text{ K})} = 3.88 \times 10^{-2} \text{ mol}.$$

(b) The ideal gas law $pV = nRT$ leads to

$$T = \frac{pV}{nR} = \frac{(1.06 \times 10^5 \text{ Pa})(1500 \times 10^{-6} \text{ m}^3)}{(3.88 \times 10^{-2} \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})} = 493 \text{ K}.$$

We note that the final temperature may be expressed in degrees Celsius as 220°C .

13. Suppose the gas expands from volume V_i to volume V_f during the isothermal portion of the process. The work it does is

$$W = \int_{V_i}^{V_f} p dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i},$$

where the ideal gas law $pV = nRT$ was used to replace p with nRT/V . Now $V_i = nRT/p_i$ and $V_f = nRT/p_f$, so $V_f/V_i = p_i/p_f$. Also replace nRT with $p_i V_i$ to obtain

$$W = p_i V_i \ln \frac{p_i}{p_f}.$$

Since the initial gauge pressure is $1.03 \times 10^5 \text{ Pa}$,

$$p_i = 1.03 \times 10^5 \text{ Pa} + 1.013 \times 10^5 \text{ Pa} = 2.04 \times 10^5 \text{ Pa}.$$

The final pressure is atmospheric pressure: $p_f = 1.013 \times 10^5 \text{ Pa}$. Thus

$$W = (2.04 \times 10^5 \text{ Pa})(0.14 \text{ m}^3) \ln \left(\frac{2.04 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}} \right) = 2.00 \times 10^4 \text{ J}.$$

During the constant pressure portion of the process the work done by the gas is $W = p_f(V_i - V_f)$. The gas starts in a state with pressure p_f , so this is the pressure throughout this portion of the process. We also note that the volume decreases from V_f to V_i . Now $V_f = p_i V_i / p_f$, so

$$W = p_f \left(V_i - \frac{p_i V_i}{p_f} \right) = (p_f - p_i) V_i = (1.013 \times 10^5 \text{ Pa} - 2.04 \times 10^5 \text{ Pa}) (0.14 \text{ m}^3) \\ = -1.44 \times 10^4 \text{ J}.$$

The total work done by the gas over the entire process is

$$W = 2.00 \times 10^4 \text{ J} - 1.44 \times 10^4 \text{ J} = 5.60 \times 10^3 \text{ J}.$$

19. According to kinetic theory, the rms speed is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where T is the temperature and M is the molar mass. See Eq. 19-34. According to Table 19-1, the molar mass of molecular hydrogen is $2.02 \text{ g/mol} = 2.02 \times 10^{-3} \text{ kg/mol}$, so

$$v_{\text{rms}} = \sqrt{\frac{3(8.31 \text{ J/mol} \cdot \text{K})(2.7 \text{ K})}{2.02 \times 10^{-3} \text{ kg/mol}}} = 1.8 \times 10^2 \text{ m/s}.$$

21. Table 19-1 gives $M = 28.0 \text{ g/mol}$ for nitrogen. This value can be used in Eq. 19-22 with T in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 19-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using v for v_{rms} :

$$\frac{v_2}{v_1} = \frac{\sqrt{3RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{T_2}{T_1}}.$$

(a) With $T_2 = (20.0 + 273.15) \text{ K} \approx 293 \text{ K}$, we obtain

$$v_2 = (517 \text{ m/s}) \sqrt{\frac{293 \text{ K}}{300 \text{ K}}} = 511 \text{ m/s}.$$

(b) In this case, we set $v_3 = \frac{1}{2} v_2$ and solve $v_3/v_2 = \sqrt{T_3/T_2}$ for T_3 :

$$T_3 = T_2 \left(\frac{v_3}{v_2} \right)^2 = (293 \text{ K}) \left(\frac{1}{2} \right)^2 = 73.0 \text{ K}$$

which we write as $73.0 - 273 = -200^\circ\text{C}$.

(c) Now we have $v_4 = 2v_2$ and obtain

$$T_4 = T_2 \left(\frac{v_4}{v_2} \right)^2 = (293\text{K})(4) = 1.17 \times 10^3 \text{ K}$$

which is equivalent to 899°C.

35. (a) The average speed is $\bar{v} = \frac{\sum v}{N}$, where the sum is over the speeds of the particles and N is the number of particles. Thus

$$\bar{v} = \frac{(2.0+3.0+4.0+5.0+6.0+7.0+8.0+9.0+10.0+11.0)\text{ km/s}}{10} = 6.5\text{ km/s}.$$

(b) The rms speed is given by $v_{\text{rms}} = \sqrt{\frac{\sum v^2}{N}}$. Now

$$\begin{aligned} \sum v^2 &= [(2.0)^2 + (3.0)^2 + (4.0)^2 + (5.0)^2 + (6.0)^2 \\ &\quad + (7.0)^2 + (8.0)^2 + (9.0)^2 + (10.0)^2 + (11.0)^2] \text{ km}^2/\text{s}^2 = 505 \text{ km}^2/\text{s}^2 \end{aligned}$$

so

$$v_{\text{rms}} = \sqrt{\frac{505 \text{ km}^2/\text{s}^2}{10}} = 7.1 \text{ km/s}.$$

41. (a) The distribution function gives the fraction of particles with speeds between v and $v + dv$, so its integral over all speeds is unity: $\int P(v) dv = 1$. Evaluate the integral by calculating the area under the curve in Fig. 19-24. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or av_0 . Thus,

$$\int P(v)dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0,$$

so $\frac{3}{2}av_0 = 1$ and $av_0 = 2/3 = 0.67$.

(b) The average speed is given by $v_{\text{avg}} = \int vP(v) dv$. For the triangular portion of the distribution $P(v) = av/v_0$, and the contribution of this portion is

$$\frac{a}{v_0} \int_0^{v_0} v^2 dv = \frac{a}{3v_0} v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0,$$

where $2/3v_0$ was substituted for a . $P(v) = a$ in the rectangular portion, and the contribution of this portion is

$$a \int_{v_0}^{2v_0} v dv = \frac{a}{2} (4v_0^2 - v_0^2) = \frac{3a}{2} v_0^2 = v_0.$$

Therefore,

$$v_{\text{avg}} = \frac{2}{9} v_0 + v_0 = 1.22v_0 \Rightarrow \frac{v_{\text{avg}}}{v_0} = 1.22.$$

(c) The mean-square speed is given by $v_{\text{rms}}^2 = \int v^2 P(v) dv$. The contribution of the triangular section is

$$\frac{a}{v_0} \int_0^{v_0} v^3 dv = \frac{a}{4v_0} v_0^4 = \frac{1}{6} v_0^2.$$

The contribution of the rectangular portion is

$$a \int_{v_0}^{2v_0} v^2 dv = \frac{a}{3} (8v_0^3 - v_0^3) = \frac{7a}{3} v_0^3 = \frac{14}{9} v_0^2.$$

Thus,

$$v_{\text{rms}} = \sqrt{\frac{1}{6} v_0^2 + \frac{14}{9} v_0^2} = 1.31v_0 \Rightarrow \frac{v_{\text{rms}}}{v_0} = 1.31.$$

(d) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N \int_{1.5v_0}^{2v_0} P(v) dv$.

The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is $N a(2.0v_0 - 1.5v_0) = 0.5N a v_0 = N/3$, where $2/3v_0$ was substituted for a . In other words, the fraction of particles in this range is $1/3$ or 0.33 .

45. When the temperature changes by ΔT the internal energy of the first gas changes by $n_1 C_1 \Delta T$, the internal energy of the second gas changes by $n_2 C_2 \Delta T$, and the internal energy of the third gas changes by $n_3 C_3 \Delta T$. The change in the internal energy of the composite gas is

$$\Delta E_{\text{int}} = (n_1 C_1 + n_2 C_2 + n_3 C_3) \Delta T.$$

This must be $(n_1 + n_2 + n_3) C_V \Delta T$, where C_V is the molar specific heat of the mixture. Thus

$$C_V = \frac{n_1 C_1 + n_2 C_2 + n_3 C_3}{n_1 + n_2 + n_3}.$$

With $n_1=2.40$ mol, $C_{V1}=12.0$ J/mol·K for gas 1, $n_2=1.50$ mol, $C_{V2}=12.8$ J/mol·K for gas 2, and $n_3=3.20$ mol, $C_{V3}=20.0$ J/mol·K for gas 3, we obtain $C_V=15.8$ J/mol·K for the mixture.

46. Two formulas (other than the first law of thermodynamics) will be of use to us. It is straightforward to show, from Eq. 19-11, that for any process that is depicted as a *straight line* on the pV diagram — the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2} \right) \Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Further, Eq. 19-44 with Eq. 19-51 gives

$$E_{\text{int}} = n \left(\frac{f}{2} \right) RT = \left(\frac{f}{2} \right) pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in Joules, pressure should be in Pascals (N / m²) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is $f = 5$.

(a) The internal energy change is

$$\begin{aligned} E_{\text{int } c} - E_{\text{int } a} &= \frac{5}{2} (p_c V_c - p_a V_a) = \frac{5}{2} ((2.0 \times 10^3 \text{ Pa})(4.0 \text{ m}^3) - (5.0 \times 10^3 \text{ Pa})(2.0 \text{ m}^3)) \\ &= -5.0 \times 10^3 \text{ J.} \end{aligned}$$

(b) The work done during the process represented by the diagonal path is

$$W_{\text{diag}} = \left(\frac{p_a + p_c}{2} \right) (V_c - V_a) = (3.5 \times 10^3 \text{ Pa})(2.0 \text{ m}^3)$$

which yields $W_{\text{diag}} = 7.0 \times 10^3$ J. Consequently, the first law of thermodynamics gives

$$Q_{\text{diag}} = \Delta E_{\text{int}} + W_{\text{diag}} = (-5.0 \times 10^3 + 7.0 \times 10^3) \text{ J} = 2.0 \times 10^3 \text{ J.}$$

(c) The fact that ΔE_{int} only depends on the initial and final states, and not on the details of the “path” between them, means we can write $\Delta E_{\text{int}} = E_{\text{int } c} - E_{\text{int } a} = -5.0 \times 10^3$ J for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{\text{indirect}} = (5.0 \times 10^3 \text{ Pa})(2.0 \text{ m}^3) + 0 = 1.0 \times 10^4 \text{ J.}$$

Now, the first law of thermodynamics leads to

$$Q_{\text{indirect}} = \Delta E_{\text{int}} + W_{\text{indirect}} = (-5.0 \times 10^3 + 1.0 \times 10^4) \text{ J} = 5.0 \times 10^3 \text{ J}.$$

53. (a) Since the process is at constant pressure, energy transferred as heat to the gas is given by $Q = nC_p \Delta T$, where n is the number of moles in the gas, C_p is the molar specific heat at constant pressure, and ΔT is the increase in temperature. For a diatomic ideal gas $C_p = \frac{7}{2}R$. Thus,

$$Q = \frac{7}{2}nR\Delta T = \frac{7}{2}(4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(60.0 \text{ K}) = 6.98 \times 10^3 \text{ J}.$$

(b) The change in the internal energy is given by $\Delta E_{\text{int}} = nC_v \Delta T$, where C_v is the specific heat at constant volume. For a diatomic ideal gas $C_v = \frac{5}{2}R$, so

$$\Delta E_{\text{int}} = \frac{5}{2}nR\Delta T = \frac{5}{2}(4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(60.0 \text{ K}) = 4.99 \times 10^3 \text{ J}.$$

(c) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, so

$$W = Q - \Delta E_{\text{int}} = 6.98 \times 10^3 \text{ J} - 4.99 \times 10^3 \text{ J} = 1.99 \times 10^3 \text{ J}.$$

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2}nR\Delta T = \frac{3}{2}(4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(60.0 \text{ K}) = 2.99 \times 10^3 \text{ J}.$$

67. In this solution we will use non-standard notation: writing ρ for *weight*-density (instead of mass-density), where ρ_c refers to the cool air and ρ_h refers to the hot air. Then the condition required by the problem is

$$F_{\text{net}} = F_{\text{buoyant}} - \text{hot-air-weight} - \text{balloon-weight}$$

$$2.67 \times 10^3 \text{ N} = \rho_c V - \rho_h V - 2.45 \times 10^3 \text{ N}$$

where $V = 2.18 \times 10^3 \text{ m}^3$ and $\rho_c = 11.9 \text{ N/m}^3$. This condition leads to $\rho_h = 9.55 \text{ N/m}^3$. Using the ideal gas law to write ρ_h as PMg/RT where $P = 101000$ Pascals and $M = 0.028 \text{ kg/m}^3$ (as suggested in the problem), we conclude that the temperature of the enclosed air should be 349 K.

69. (a) By Eq. 19-28, $W = -374 \text{ J}$ (since the process is an adiabatic compression).

(b) $Q = 0$ since the process is adiabatic.

(c) By first law of thermodynamics, the change in internal energy is $\Delta E_{\text{int}} = Q - W = +374$ J.

(d) The change in the average kinetic energy per atom is $\Delta K_{\text{avg}} = \Delta E_{\text{int}}/N = +3.11 \times 10^{-22}$ J.

71. This is very similar to Sample Problem 19-4 (and we use similar notation here) except for the use of Eq. 19-31 for v_{avg} (whereas in that Sample Problem, its value was just assumed). Thus,

$$f = \frac{\text{speed}}{\text{distance}} = \frac{v_{\text{avg}}}{\lambda} = \frac{p}{k} \frac{d^2}{\left(\frac{16\pi R}{MT}\right)}.$$

Therefore, with $p = 2.02 \times 10^3$ Pa, $d = 290 \times 10^{-12}$ m and $M = 0.032$ kg/mol (see Table 19-1), we obtain $f = 7.03 \times 10^9$ s⁻¹.

77. (a) The final pressure is

$$p_f = \frac{p_i V_i}{V_f} = \frac{(32 \text{ atm})(1.0 \text{ L})}{4.0 \text{ L}} = 8.0 \text{ atm},$$

(b) For the isothermal process the final temperature of the gas is $T_f = T_i = 300$ K.

(c) The work done is

$$\begin{aligned} W &= nRT_i \ln\left(\frac{V_f}{V_i}\right) = p_i V_i \ln\left(\frac{V_f}{V_i}\right) = (32 \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})(1.0 \times 10^{-3} \text{ m}^3) \ln\left(\frac{4.0 \text{ L}}{1.0 \text{ L}}\right) \\ &= 4.4 \times 10^3 \text{ J}. \end{aligned}$$

For the adiabatic process $p_i V_i^\gamma = p_f V_f^\gamma$. Thus,

(d) The final pressure is

$$p_f = p_i \left(\frac{V_i}{V_f}\right)^\gamma = (32 \text{ atm}) \left(\frac{1.0 \text{ L}}{4.0 \text{ L}}\right)^{5/3} = 3.2 \text{ atm}.$$

(e) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(3.2 \text{ atm})(4.0 \text{ L})(300 \text{ K})}{(32 \text{ atm})(1.0 \text{ L})} = 120 \text{ K}.$$

(f) The work done is

$$\begin{aligned}
 W &= Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}} = -\frac{3}{2}nR\Delta T = -\frac{3}{2}(p_f V_f - p_i V_i) \\
 &= -\frac{3}{2}[(3.2 \text{ atm})(4.0 \text{ L}) - (32 \text{ atm})(1.0 \text{ L})](1.01 \times 10^5 \text{ Pa/atm})(10^{-3} \text{ m}^3/\text{L}) \\
 &= 2.9 \times 10^3 \text{ J} .
 \end{aligned}$$

If the gas is diatomic, then $\gamma = 1.4$.

(g) The final pressure is

$$p_f = p_i \left(\frac{V_i}{V_f} \right)^\gamma = (32 \text{ atm}) \left(\frac{1.0 \text{ L}}{4.0 \text{ L}} \right)^{1.4} = 4.6 \text{ atm} .$$

(h) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(4.6 \text{ atm})(4.0 \text{ L})(300 \text{ K})}{(32 \text{ atm})(1.0 \text{ L})} = 170 \text{ K} .$$

(i) The work done is

$$\begin{aligned}
 W &= Q - \Delta E_{\text{int}} = -\frac{5}{2}nR\Delta T = -\frac{5}{2}(p_f V_f - p_i V_i) \\
 &= -\frac{5}{2}[(4.6 \text{ atm})(4.0 \text{ L}) - (32 \text{ atm})(1.0 \text{ L})](1.01 \times 10^5 \text{ Pa/atm})(10^{-3} \text{ m}^3/\text{L}) \\
 &= 3.4 \times 10^3 \text{ J} .
 \end{aligned}$$

81. It is recommended to look over §19-7 before doing this problem.

(a) We normalize the distribution function as follows:

$$\int_0^{v_o} P(v) dv = 1 \Rightarrow C = \frac{3}{v_o^3} .$$

(b) The average speed is

$$\int_0^{v_o} v P(v) dv = \int_0^{v_o} v \left(\frac{3v^2}{v_o^3} \right) dv = \frac{3}{4} v_o .$$

(c) The rms speed is the square root of

$$\int_0^{v_0} v^2 P(v) dv = \int_0^{v_0} v^2 \left(\frac{3v^2}{v_0^3} \right) dv = \frac{3}{5} v_0^2.$$

Therefore, $v_{\text{rms}} = \sqrt{3/5} v_0 \approx 0.775 v_0$.

83. (a) The temperature is $10.0^\circ\text{C} \rightarrow T = 283 \text{ K}$. Then, with $n = 3.50 \text{ mol}$ and $V_f/V_0 = 3/4$, we use Eq. 19-14:

$$W = nRT \ln \left(\frac{V_f}{V_0} \right) = -2.37 \text{ kJ}.$$

(b) The internal energy change ΔE_{int} vanishes (for an ideal gas) when $\Delta T = 0$ so that the First Law of Thermodynamics leads to $Q = W = -2.37 \text{ kJ}$. The negative value implies that the heat transfer is from the sample to its environment.