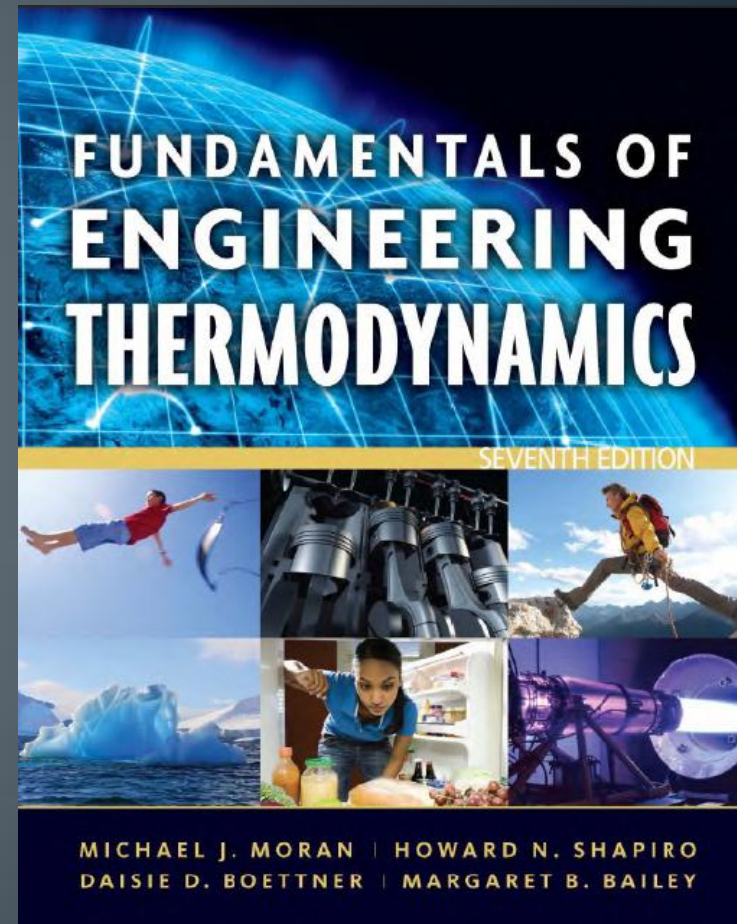
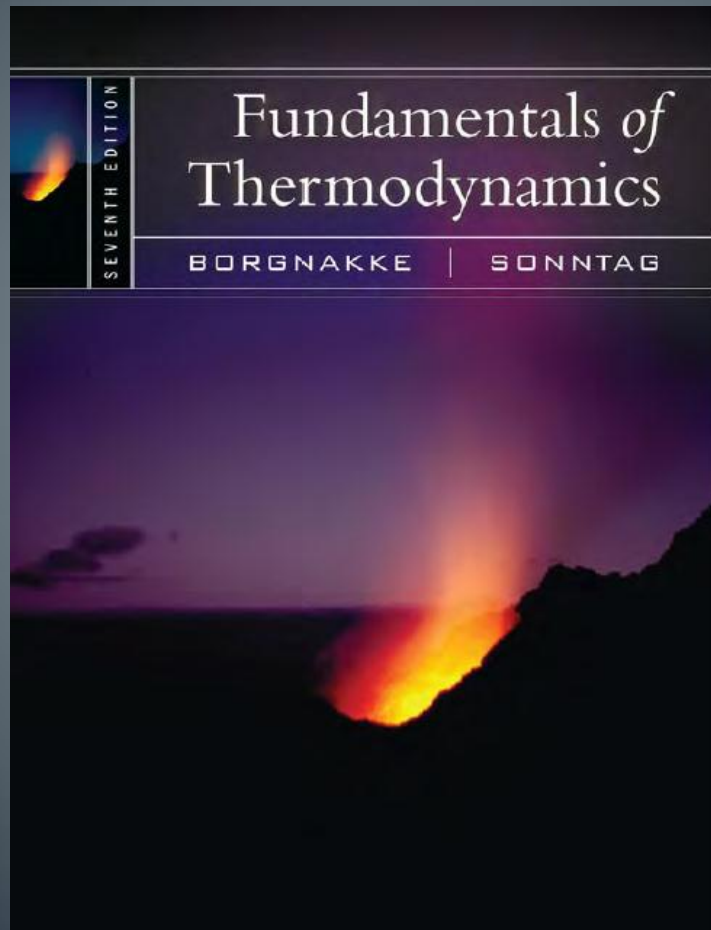


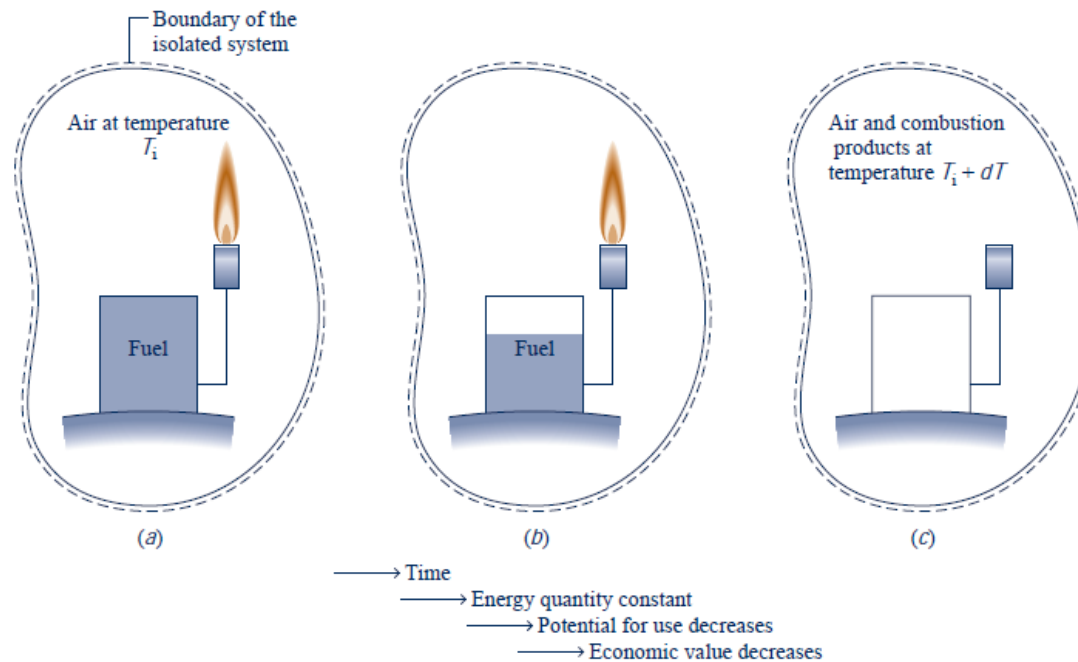
THERMODYNAMICS II

Farbod Fakhrabadi

Islamic Azad University of Qazvin

Textbooks

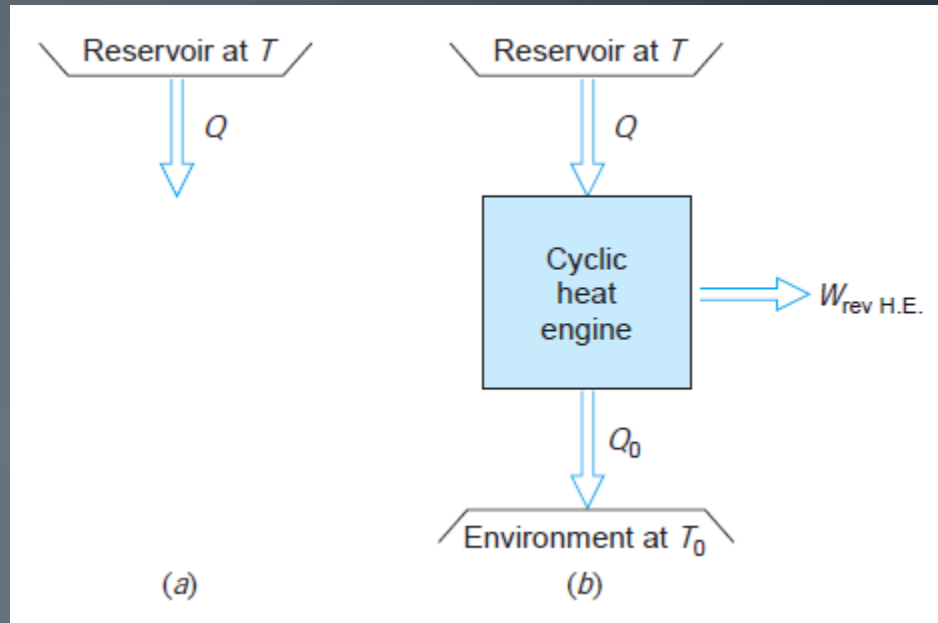




IRREVERSIBILITY AND AVAILABILITY

- What is the **ultimate potential** for producing work?

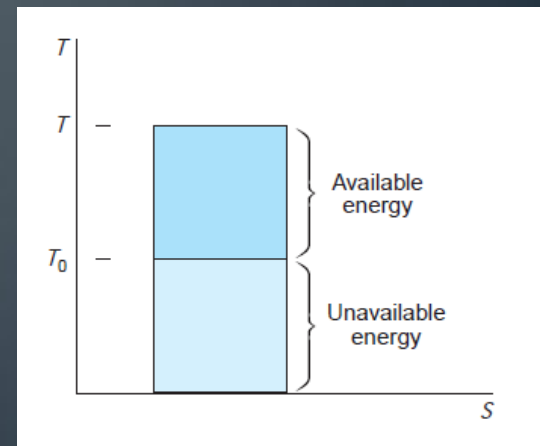
AVAILABLE ENERGY, REVERSIBLE WORK, AND IRREVERSIBILITY



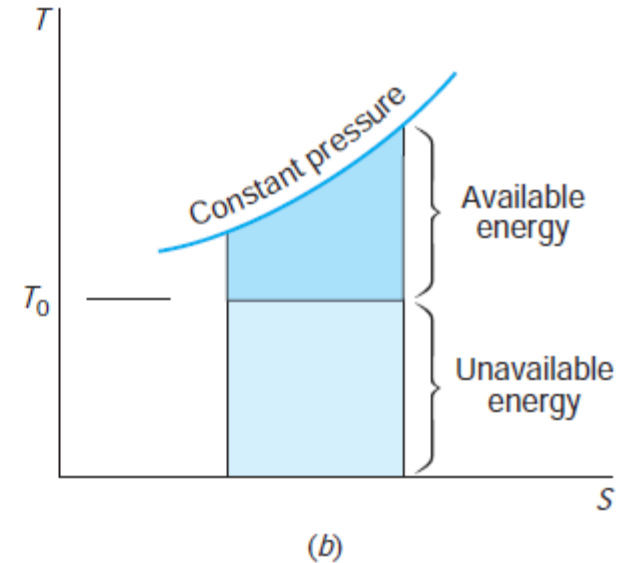
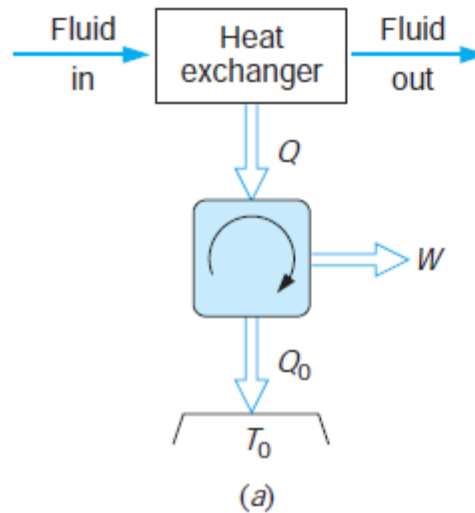
$$W_{\text{rev H.E.}} = Q - Q_0$$

$$\frac{Q}{T} = \frac{Q_0}{T_0}$$

$$W_{\text{rev H.E.}} = Q \left(1 - \frac{T_0}{T} \right)$$



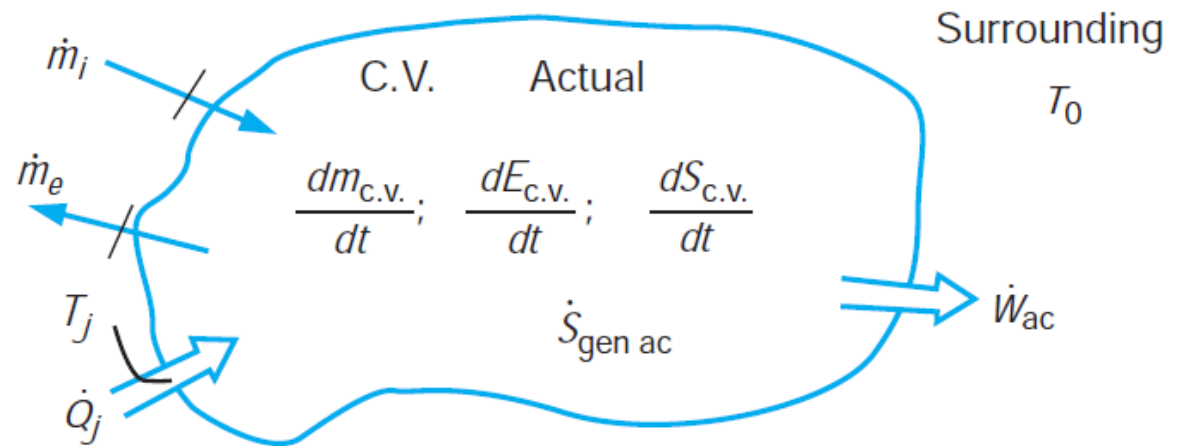
AVAILABLE ENERGY, REVERSIBLE WORK, AND IRREVERSIBILITY



$$\Delta S = \int \frac{\delta Q_{\text{rev}}}{T} = \frac{Q_0}{T_0}$$

$$W_{\text{rev H.E.}} = Q - T_0 \Delta S$$

AVAILABLE ENERGY, REVERSIBLE WORK, AND IRREVERSIBILITY



$$\frac{dm_{c.v.}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e$$

$$\frac{dE_{c.v.}}{dt} = \sum \dot{Q}_j + \sum \dot{m}_i h_{tot i} - \sum \dot{m}_e h_{tot e} - \dot{W}_{c.v. ac}$$

$$\frac{dS_{c.v.}}{dt} = \sum \frac{\dot{Q}_j}{T_j} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{gen ac}$$

AVAILABLE
ENERGY,
REVERSIBLE
WORK,
AND
IRREVERSIBILITY

Actual C.V. terms

Ideal C.V. terms

$$\dot{S}_{\text{gen ac}} = \frac{\dot{Q}_0^{\text{rev}}}{T_0}$$

$$-\dot{W}_{\text{c.v. ac}} = \dot{Q}_0^{\text{rev}} - \dot{W}^{\text{rev}}$$

$$\dot{Q}_0^{\text{rev}} = T_0 \dot{S}_{\text{gen ac}}$$

$$\dot{W}^{\text{rev}} = \dot{W}_{\text{c.v. ac}} + \dot{Q}_0^{\text{rev}}$$

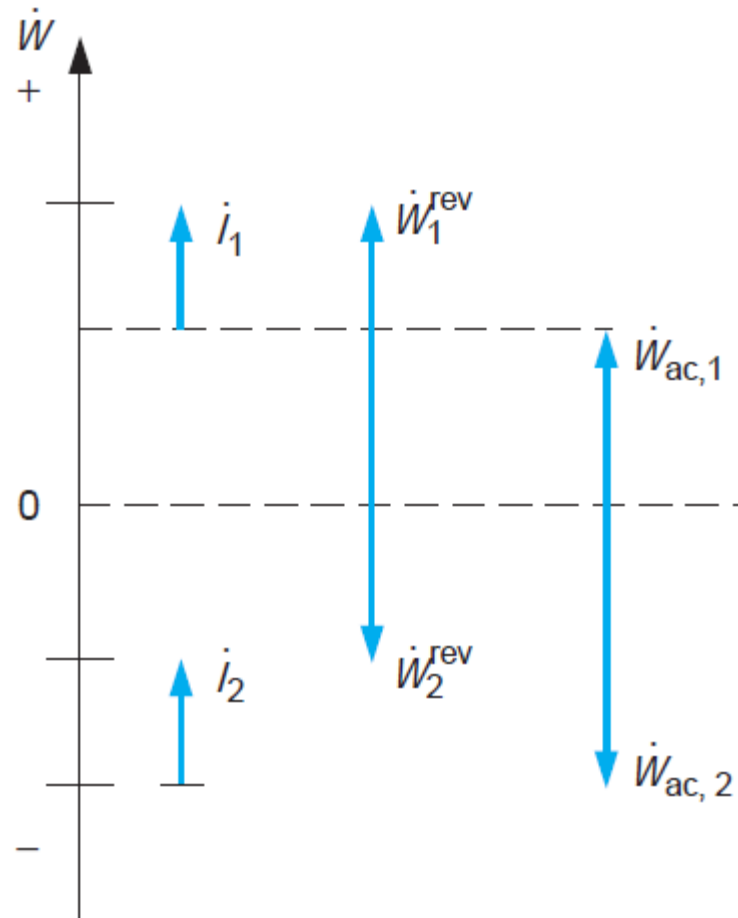
AVAILABLE
ENERGY,
REVERSIBLE
WORK,
AND
IRREVERSIBILITY

$$\begin{aligned}\dot{W}^{\text{rev}} &= \dot{W}_{\text{c.v. ac}} + \dot{Q}_0^{\text{rev}} \\ &= \sum \dot{Q}_j + \sum \dot{m}_i h_{\text{tot } i} - \sum \dot{m}_e h_{\text{tot } e} - \frac{dE_{\text{c.v.}}}{dt} \\ &\quad + T_0 \left[\frac{dS_{\text{c.v.}}}{dt} - \sum \frac{\dot{Q}_j}{T_j} - \sum \dot{m}_i s_i + \sum \dot{m}_e s_e \right]\end{aligned}$$

$$\begin{aligned}\dot{W}^{\text{rev}} &= \sum \left(1 - \frac{T_0}{T_j} \right) \dot{Q}_j \\ &\quad + \sum \dot{m}_i (h_{\text{tot } i} - T_0 s_i) - \sum \dot{m}_e (h_{\text{tot } e} - T_0 s_e) \\ &\quad - \left[\frac{dE_{\text{c.v.}}}{dt} - T_0 \frac{dS_{\text{c.v.}}}{dt} \right]\end{aligned}$$

$$\dot{I} = \dot{W}^{\text{rev}} - \dot{W}_{\text{c.v. ac}}$$

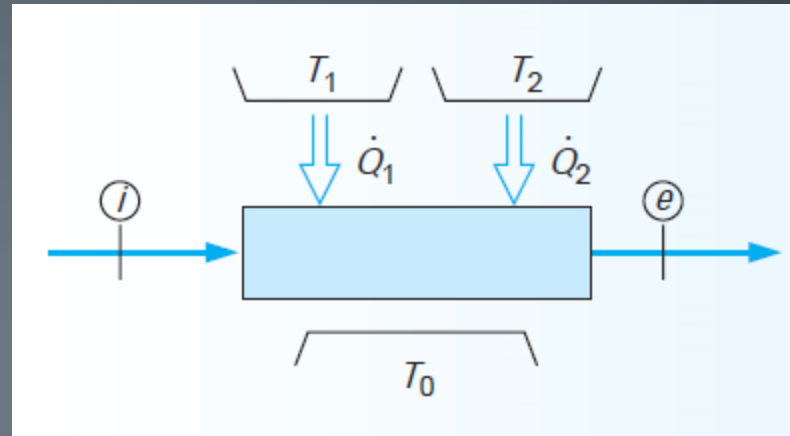
AVAILABLE ENERGY, REVERSIBLE WORK, AND IRREVERSIBILITY



$$\dot{I} = \dot{W}^{\text{rev}} - \dot{W}_{\text{c.v. ac}} = \dot{Q}_0^{\text{rev}} = T_0 \dot{S}_{\text{gen ac}}$$

The Steady-State Process

AVAILABLE
ENERGY,
REVERSIBLE
WORK,
AND
IRREVERSIBILITY



$$w^{\text{rev}} = \dot{W}^{\text{rev}} / \dot{m} = \sum \left(1 - \frac{T_0}{T_j} \right) q_j + (h_{\text{tot } i} - T_0 s_i) - (h_{\text{tot } e} - T_0 s_e)$$

$$\begin{aligned} i = \dot{I} / \dot{m} &= w^{\text{rev}} - w_{\text{c.v. ac}} = q_0^{\text{rev}} = T_0 s_{\text{gen ac}} \\ &= T_0 \left[s_e - s_i - \sum \frac{q_j}{T_j} \right] \end{aligned}$$

The Control Mass Process

AVAILABLE
ENERGY,
REVERSIBLE
WORK,
AND
IRREVERSIBILITY

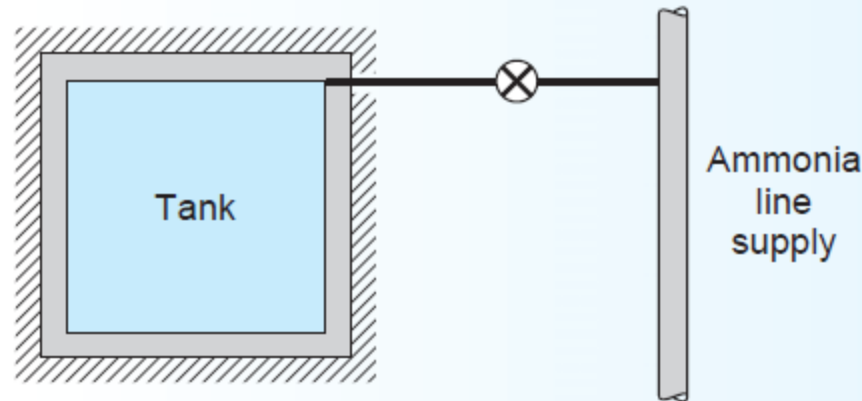
$$\dot{W}^{\text{rev}} = \sum \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \left[\frac{dE_{\text{c.v.}}}{dt} - T_0 \frac{dS_{\text{c.v.}}}{dt} \right]$$

$${}_1W_2^{\text{rev}} = \sum \left(1 - \frac{T_0}{T_j}\right) {}_1Q_{2j} - [E_2 - E_1 - T_0(S_2 - S_1)]$$

$$\begin{aligned} {}_1I_2 &= {}_1W_2^{\text{rev}} - {}_1W_{2\text{ac}} = T_0 {}_1S_{2\text{genac}} \\ &= T_0(S_2 - S_1) - \sum \frac{T_0}{T_j} {}_1Q_{2j} \end{aligned}$$

The Transient Process

AVAILABLE
ENERGY,
REVERSIBLE
WORK,
AND
IRREVERSIBILITY



$${}_1W_2^{\text{rev}} = \sum \left(1 - \frac{T_0}{T_j}\right) {}_1Q_{2j} + \sum m_i(h_{\text{tot } i} - T_0 s_i) - \sum m_e(h_{\text{tot } e} - T_0 s_e) \\ - [m_2 e_2 - m_1 e_1 - T_0(m_2 s_2 - m_1 s_1)]$$

$${}_1I_2 = {}_1W_2^{\text{rev}} - {}_1W_{2\text{ac}} = T_0 {}_1S_{2\text{gen ac}}$$

$$= T_0 \left[(m_2 s_2 - m_1 s_1) + \sum m_e s_e - \sum m_i s_i - \sum \frac{1}{T_j} {}_1Q_{2j} \right]$$

AVAILABILITY AND SECOND- LAW EFFICIENCY

- What is the **maximum reversible work** that can be done by a given mass in a given state?

$$\dot{\Phi}_q = \sum \left(1 - \frac{T_0}{T_j} \right) \dot{Q}_j$$

$$\begin{aligned}\psi &= (h_{\text{tot}} - T_0 s) - (h_{\text{tot } 0} - T_0 s_0) \\ &= (h - T_0 s + \frac{1}{2} \mathbf{V}^2 + gZ) - (h_0 - T_0 s_0 + gZ_0)\end{aligned}$$

$$\begin{aligned}\psi_i - \psi_e &= [(h_{\text{tot } i} - T_0 s_i) - (h_0 - T_0 s_0 + gZ_0)] \\ &\quad - [(h_{\text{tot } e} - T_0 s_e) - (h_0 - T_0 s_0 + gZ_0)] \\ &= (h_{\text{tot } i} - T_0 s_i) - (h_{\text{tot } e} - T_0 s_e)\end{aligned}$$

AVAILABILITY AND SECOND- LAW EFFICIENCY

$$\dot{W}_{\text{surr}} = P_0 \dot{V}$$

$$\begin{aligned}\dot{W}_{\text{avail}}^{\text{max}} &= \dot{W}_{\text{storage}}^{\text{rev}} - \dot{W}_{\text{surr}} \\ &= -\left[\frac{dE_{\text{c.v.}}}{dt} - T_0 \frac{dS_{\text{c.v.}}}{dt}\right] - P_0 \dot{V}\end{aligned}$$

$$\begin{aligned}\Phi &= -[E_0 - E - T_0(S_0 - S) + P_0(V_0 - V)] \\ &= (E - T_0 S) - (E_0 - T_0 S_0) + P_0(V - V_0) \\ \dot{\Phi}_{\text{c.v.}} &= \frac{dE_{\text{c.v.}}}{dt} - T_0 \frac{dS_{\text{c.v.}}}{dt} + P_0 \dot{V}\end{aligned}$$

AVAILABILITY AND SECOND- LAW EFFICIENCY

$$\phi = (e - T_0 s + P_0 v) - (e_0 - T_0 s_0 + P_0 v_0)$$

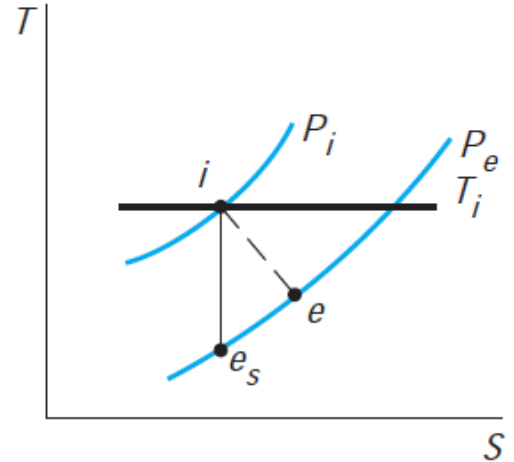
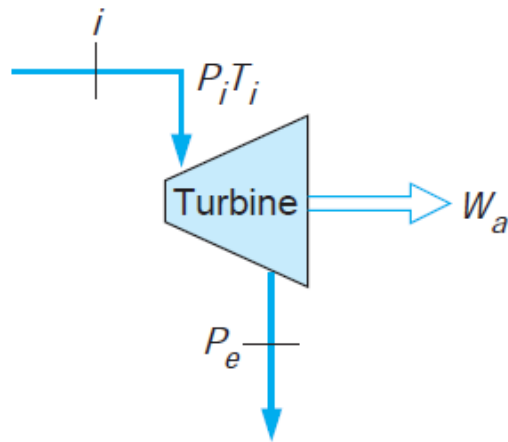
$$\phi_2 - \phi_1 = (e_2 - T_0 s_2 + P_0 v_2) - (e_1 - T_0 s_1 + P_0 v_1)$$

$$\dot{W}^{\text{rev}} = \dot{\Phi}_q + \sum \dot{m}_i \psi_i - \sum \dot{m}_e \psi_e - \dot{\Phi}_{\text{c.v.}} + P_0 \dot{V}$$

$$\dot{W}_{\text{c.v. ac}} = \dot{W}^{\text{rev}} - \dot{Q}_0^{\text{rev}} = \dot{W}^{\text{rev}} - \dot{I}$$

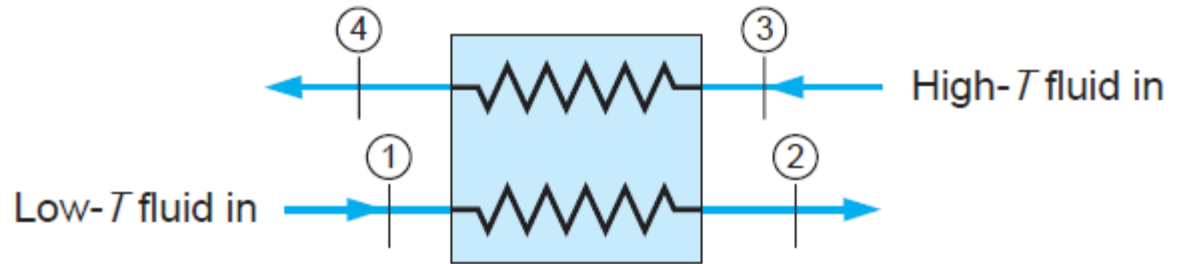
Second-law efficiency

AVAILABILITY AND SECOND- LAW EFFICIENCY



$$\eta_{\text{2nd law}} = \frac{W_a}{\psi_i - \psi_e}$$

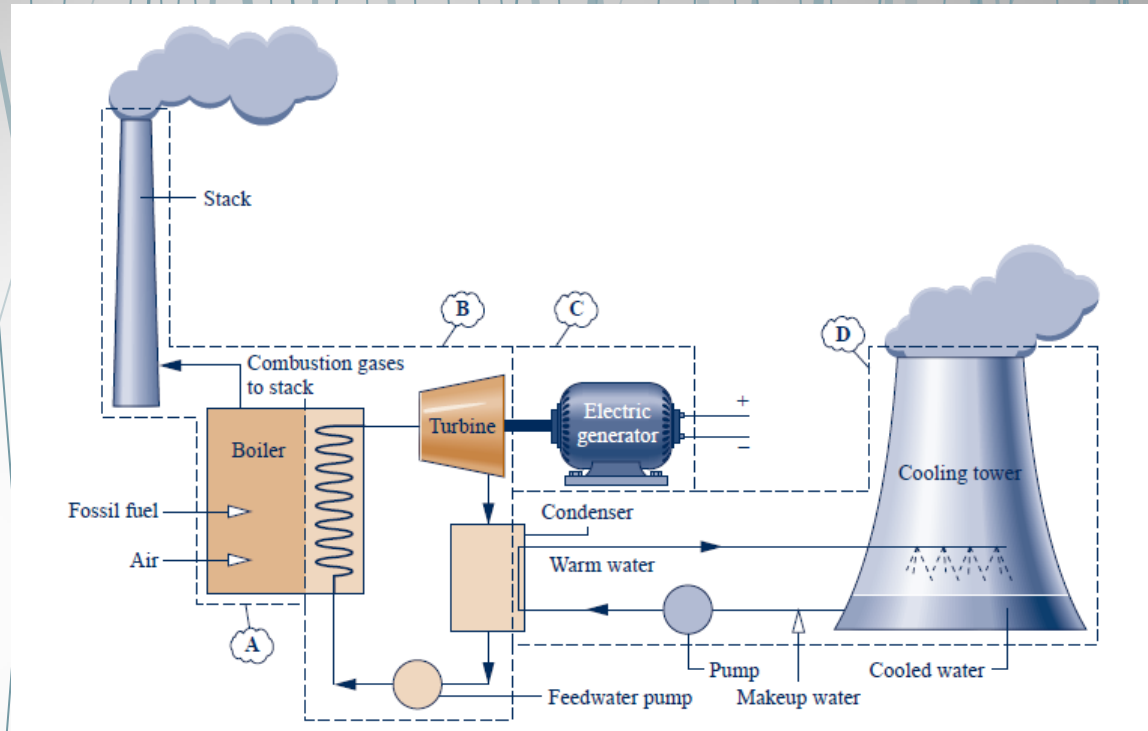
AVAILABILITY AND SECOND- LAW EFFICIENCY



$$\eta_{\text{2nd law}} = \frac{\dot{m}_1(\psi_2 - \psi_1)}{\dot{m}_3(\psi_3 - \psi_4)}$$

$$\dot{W}_{\text{c.v.}} = \dot{\Phi}_{\text{source}} - \dot{I}_{\text{c.v.}} = \dot{\Phi}_{\text{source}} - T\dot{S}_{\text{gen c.v.}}$$

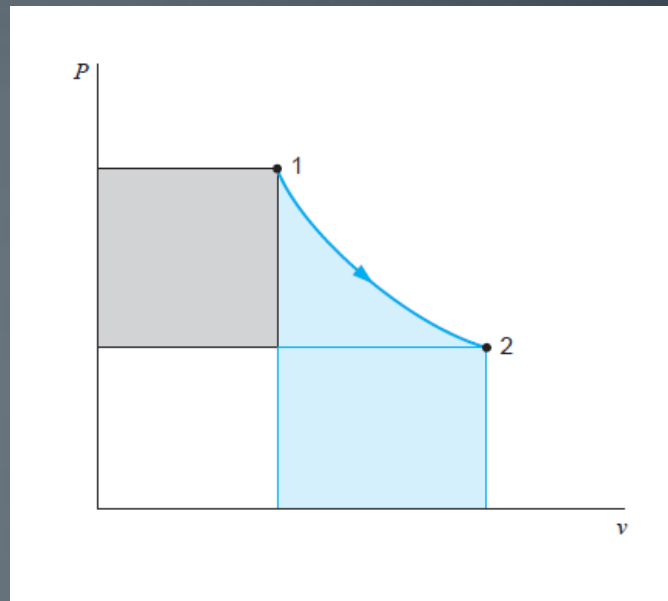
$$\eta_{\text{2nd law}} = \frac{\dot{\Phi}_{\text{wanted}}}{\dot{\Phi}_{\text{source}}} = \frac{\dot{\Phi}_{\text{source}} - \dot{I}_{\text{c.v.}}}{\dot{\Phi}_{\text{source}}}$$



POWER AND REFRIGERATION SYSTEMS

INTRODUCTION TO POWER SYSTEMS

- Comparison of shaft work and boundary movement work.



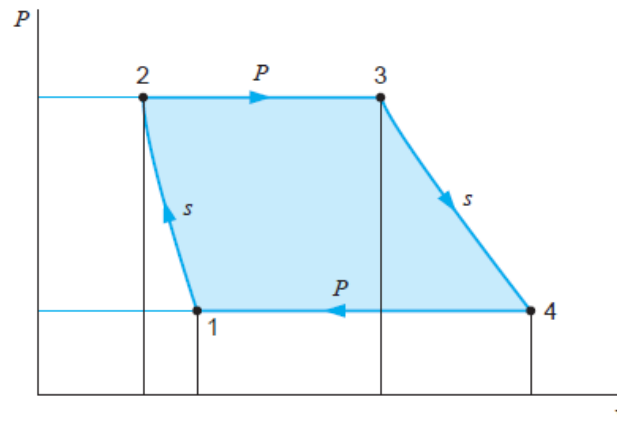
- For a reversible steady-state process involving negligible kinetic and potential energy changes, the shaft work per unit mass is given by

$$w = - \int v dP$$

INTRODUCTION TO POWER SYSTEMS

- For a reversible process involving a simple compressible substance, the boundary movement work per unit mass is given by

$$w = \int P dv$$

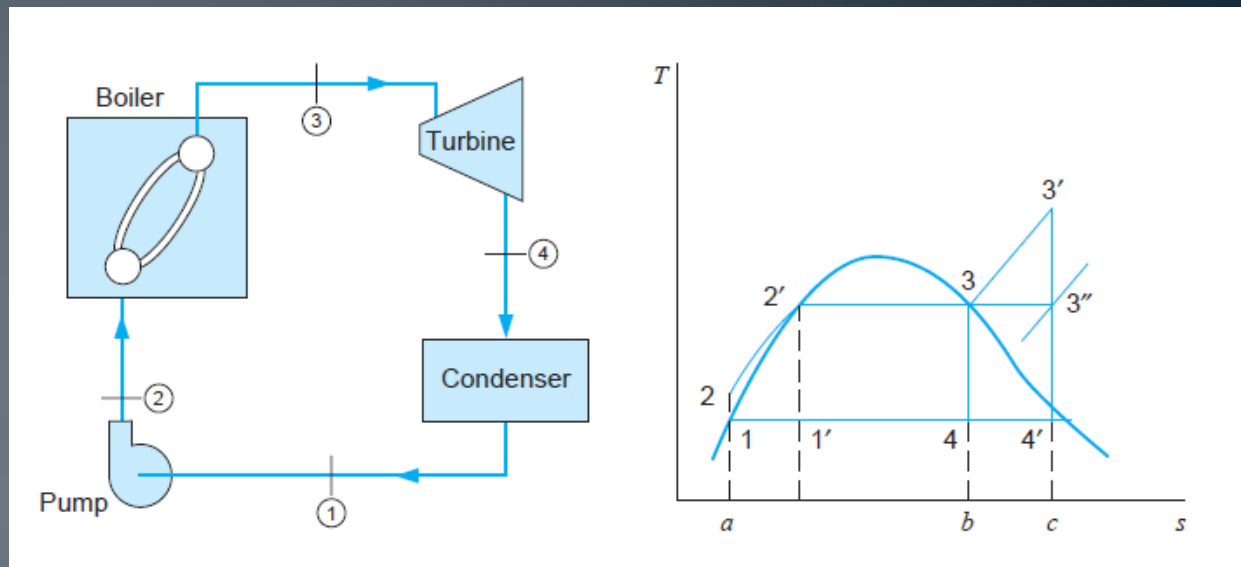


$$W_{\text{net}} = - \int_1^2 v dP + 0 - \int_3^4 v dP + 0 = - \int_1^2 v dP + \int_4^3 v dP$$

$$W_{\text{net}} = \int_1^2 P dv + \int_2^3 P dv + \int_3^4 P dv + \int_4^1 P dv$$

THE RANKINE CYCLE

- Simple steam power plant that operates on the Rankine cycle.

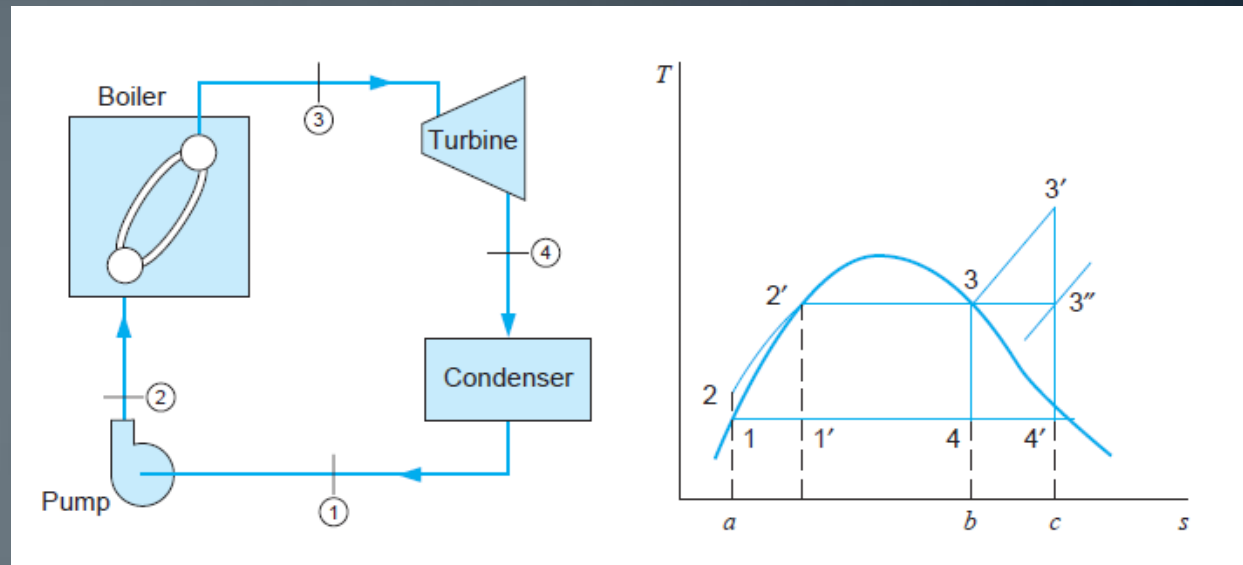


- 1–2:** Reversible adiabatic pumping process in the pump
- 2–3:** Constant-pressure transfer of heat in the boiler
- 3–4:** Reversible adiabatic expansion in the turbine (or other prime mover such as a steamengine)
- 4–1:** Constant-pressure transfer of heat in the condenser

$$\eta_{th} = \frac{w_{net}}{q_H} = \frac{\text{area } 1-2-2'-3-4-1}{\text{area } a-2-2'-3-b-a}$$

THE RANKINE CYCLE

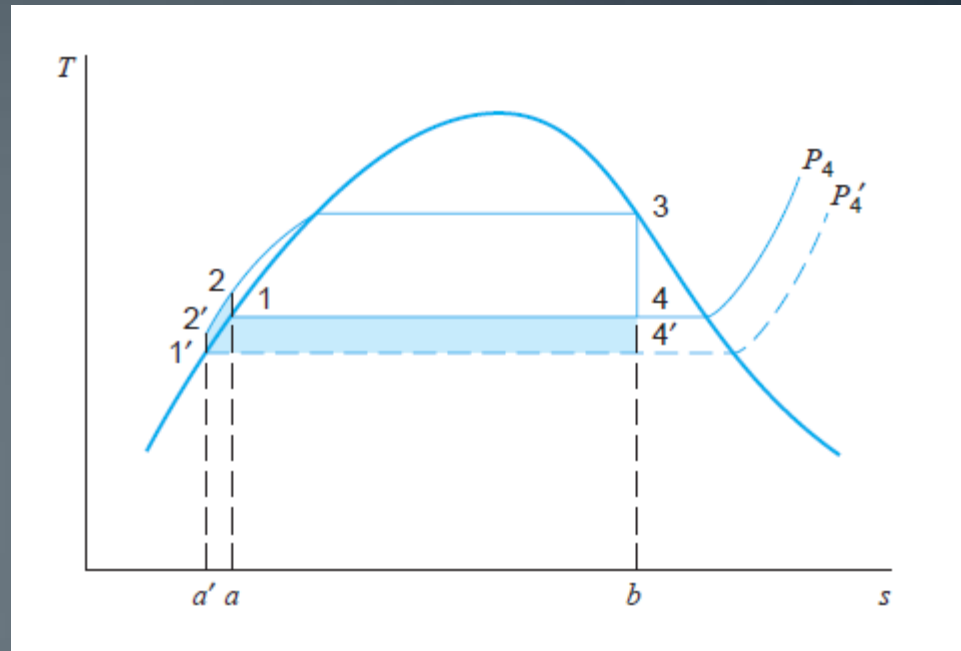
- Why not select the Carnot cycle $1'-2'-3'-4'-1'$?



- At least two reasons can be given. The first reason concerns the pumping process.
- The second reason concerns superheating the vapor.

EFFECT OF PRESSURE AND TEMPERATURE ON THE RANKINE CYCLE

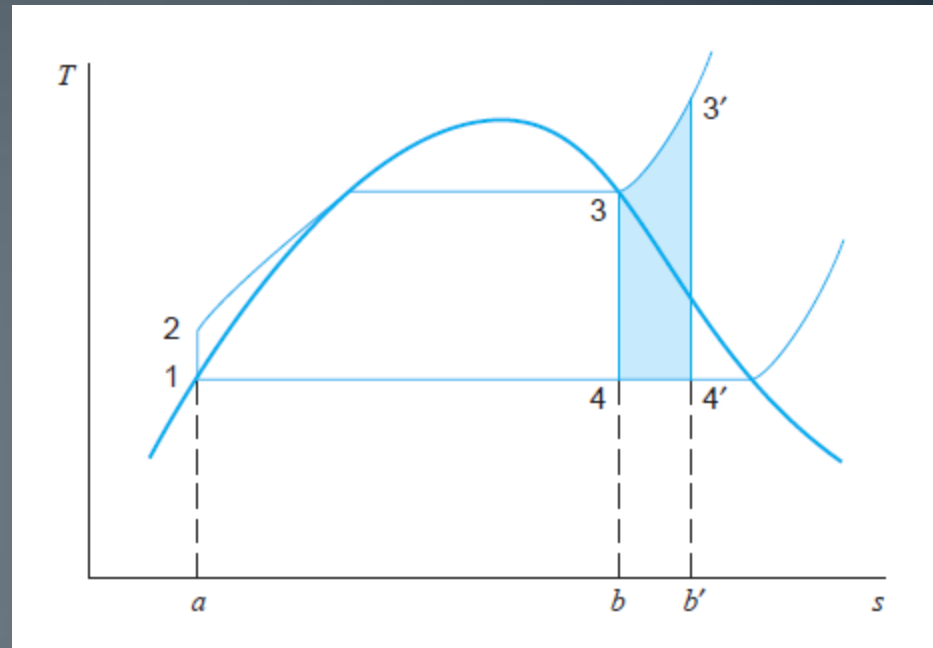
- Effect of exhaust pressure on Rankine-cycle efficiency.



- The net work is increased by area 1-4-4'-1-2-2'-1 (shown by the shading).
- The heat transferred to the steam is increased by area $a-2-2'-a-a$.
- Since these two areas are approximately equal, the net result is an increase in cycle efficiency.

EFFECT OF PRESSURE AND TEMPERATURE ON THE RANKINE CYCLE

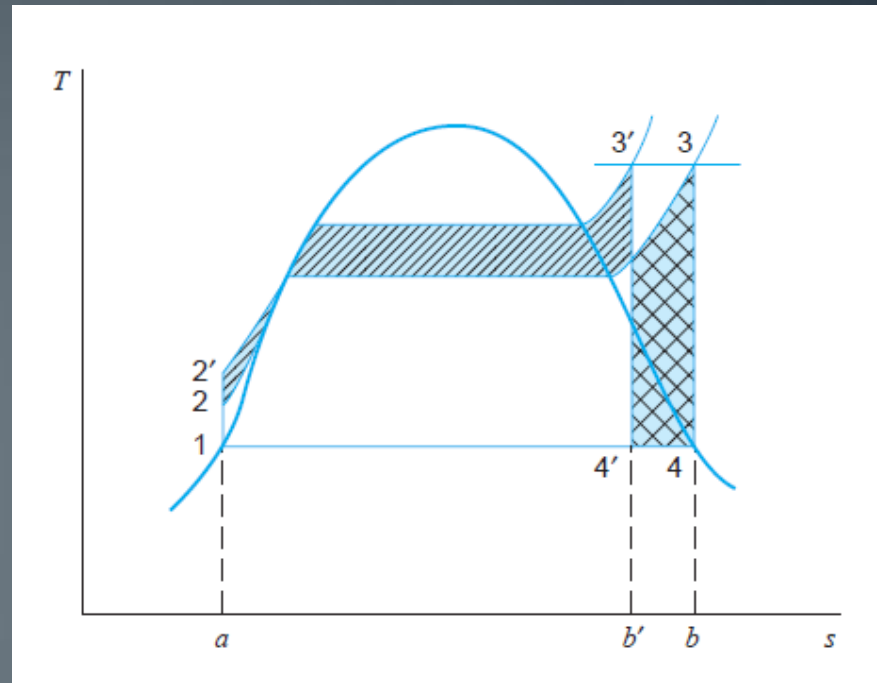
- Effect of superheating on Rankine-cycle efficiency.



- The work is increased by area 3-3'-4'-4-3, and the heat transferred in the boiler is increased by area 3-3'-b-b'-3.
- Since the ratio of these two areas is greater than the ratio of net work to heat supplied for the rest of the cycle, superheating the steam increases the Rankine-cycle efficiency.

EFFECT OF PRESSURE AND TEMPERATURE ON THE RANKINE CYCLE

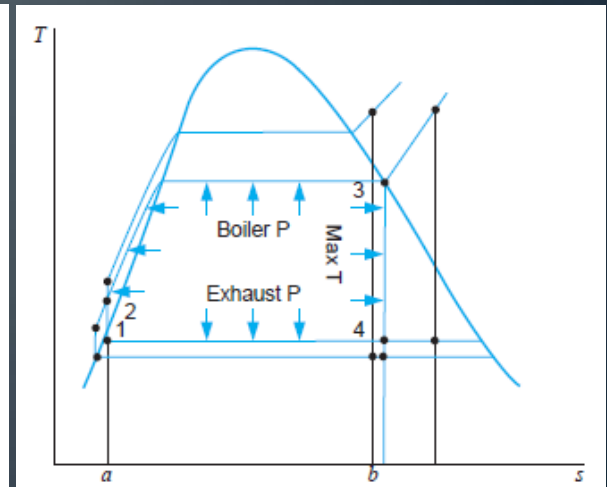
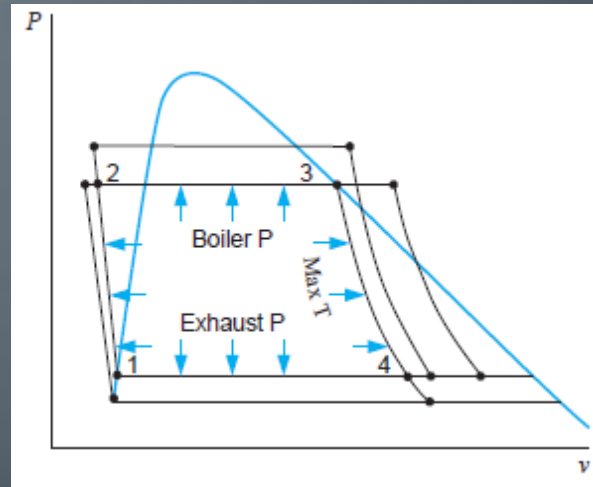
- Effect of boiler pressure on Rankine-cycle efficiency.



- The heat rejected decreases by area $b-4-4'-b$.
- The net work increases by the amount of the single cross-hatching and decreases by the amount of the double cross-hatching. Therefore, the net work tends to remain the same, but the heat rejected decreases, and hence the Rankine-cycle efficiency increases with an increase in maximum pressure.

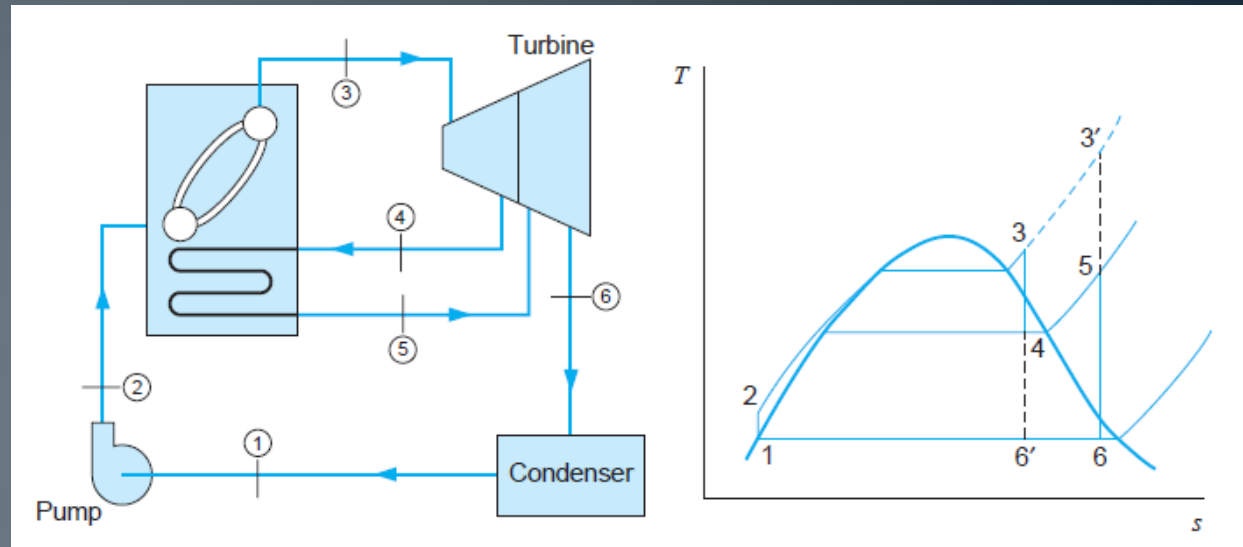
EFFECT OF PRESSURE AND TEMPERATURE ON THE RANKINE CYCLE

- The net work and the efficiency of the Rankine cycle can be increased by **lowering the condenser pressure**, by **increasing the pressure during heat addition**, and by **superheating the steam**.
- The quality of the steam leaving the turbine is **increased** by *superheating the steam* and **decreased** by *lowering the exhaust pressure* and by *increasing the pressure during heat addition*.



THE REHEAT CYCLE

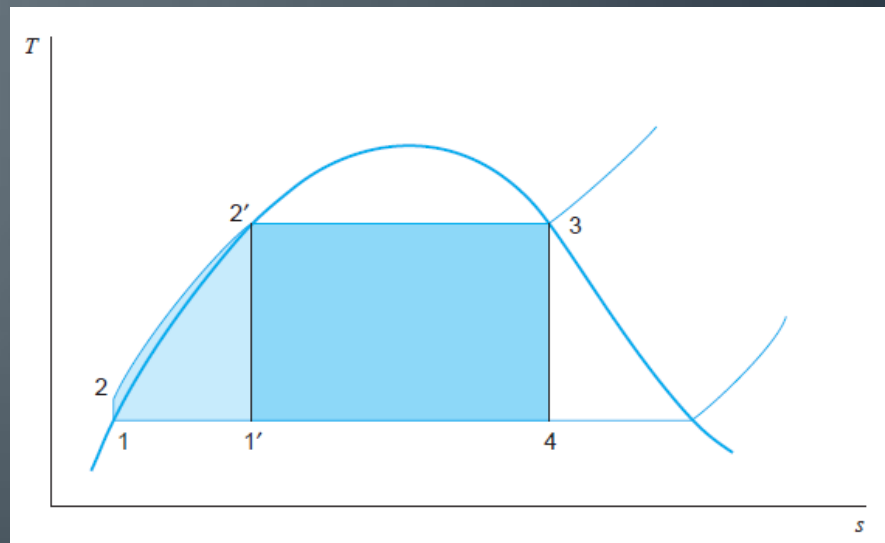
- The **reheat cycle** has been developed to take advantage of the increased efficiency with higher pressures and yet avoid excessive moisture in the low-pressure stages of the turbine.



- there is **very little gain in efficiency** from reheating the steam, because the average temperature at which heat is supplied is not greatly changed.
- The chief advantage is in **decreasing** to a safe value the **moisture content** in the low-pressure stages of the turbine.
- If metals could be found that would enable us to superheat the steam to 3', the simple Rankine cycle would be more efficient than the reheat cycle.

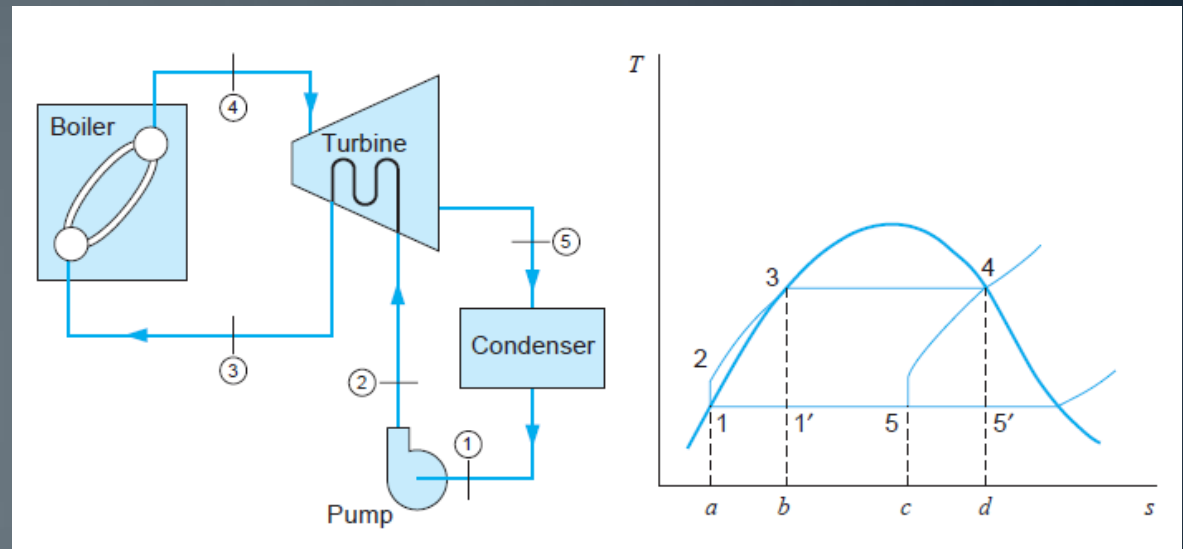
THE REGENERATIVE CYCLE

- The process between states 2 and 2' causes the average temperature at which heat is supplied in the Rankine cycle to be **lower** than in the Carnot cycle 1–2–3–4–1. Consequently, the efficiency of the Rankine cycle is lower than that of the corresponding Carnot cycle.
- In the regenerative cycle the working fluid enters the boiler at some state **between 2 and 2'**; consequently, the average temperature at which heat is supplied is **higher**.



THE REGENERATIVE CYCLE

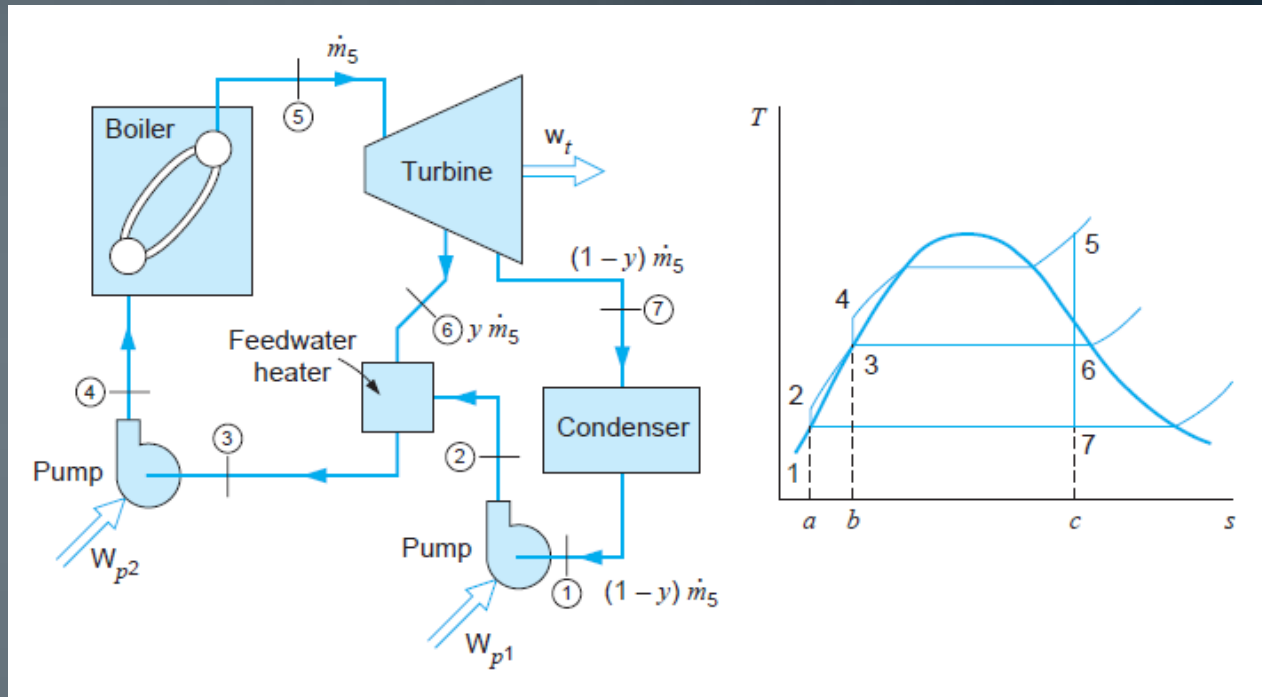
- The efficiency of this idealized regenerative cycle is exactly **equal** to the efficiency of the Carnot cycle with the same heat supply and heat rejection temperatures.



- This idealized regenerative cycle is **impractical**.
- First, it would be impossible to effect the necessary **heat transfer** from the vapor in the turbine to the liquid feedwater.
- Furthermore, the **moisture content** of the vapor leaving the turbine **increases** considerably as a result of the heat transfer.

THE REGENERATIVE CYCLE

- The practical regenerative cycle extracts some of the vapor after it has partially expanded in the turbine and uses **feedwater** heaters.



$$\dot{m}_2 + \dot{m}_6 = \dot{m}_3$$

$$y = \dot{m}_6 / \dot{m}_5$$

$$\dot{m}_7 = (1 - y)\dot{m}_5 = \dot{m}_1 = \dot{m}_2$$

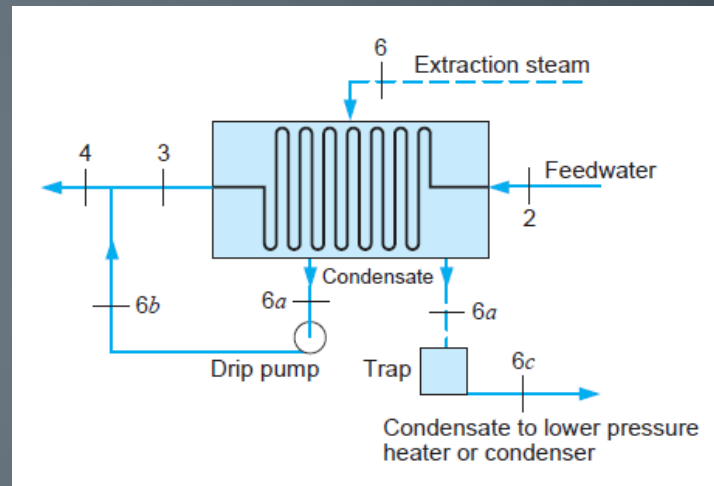
$$\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_3 h_3$$

$$(1 - y)\dot{m}_5 h_2 + y\dot{m}_5 h_6 = \dot{m}_5 h_3$$

$$y = \frac{h_3 - h_2}{h_6 - h_2}$$

THE REGENERATIVE CYCLE

- Another frequently used type of feedwater heater, known as a **closed** heater, is one in which the steam and feedwater do not mix.
- The condensate may be pumped into the feedwater line, or it may be removed through a **trap** to a lower-pressure heater or to the condenser.



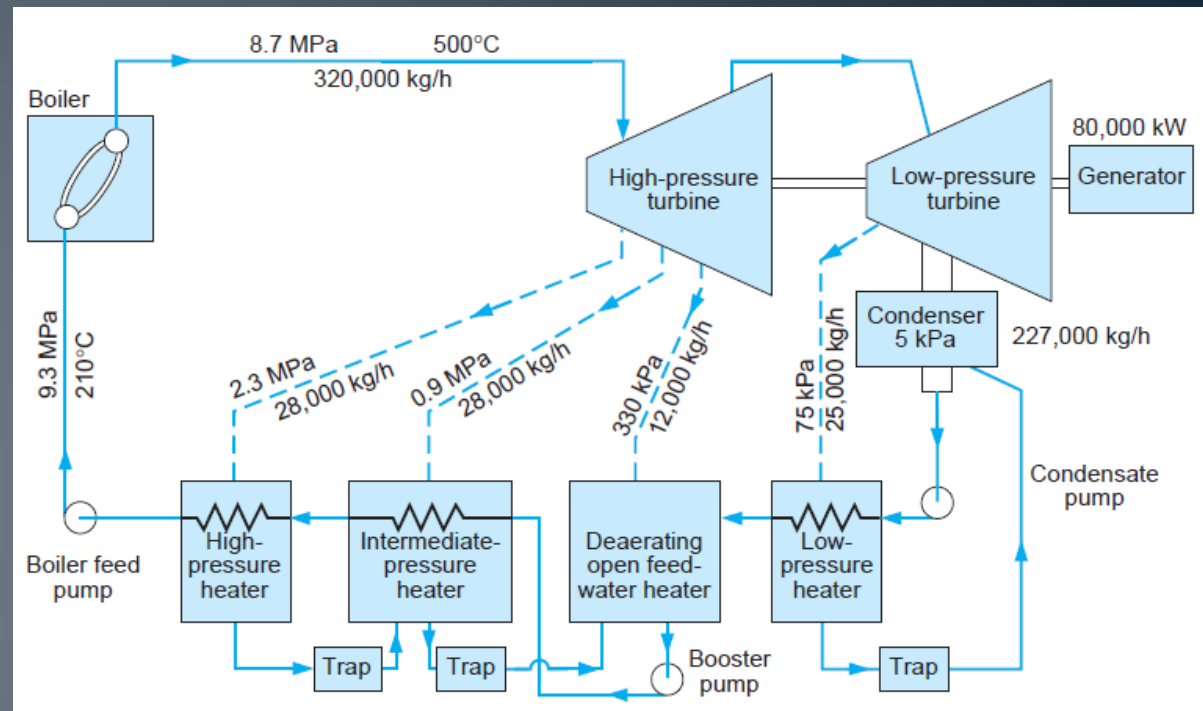
$$\dot{m}_4 = \dot{m}_3 = \dot{m}_2 = \dot{m}_5; \quad \dot{m}_6 = y\dot{m}_5 = \dot{m}_{6a} = \dot{m}_{6c}$$

$$\dot{m}_5 h_2 + y\dot{m}_5 h_6 = \dot{m}_5 h_3 + y\dot{m}_5 h_{6a}$$

$$y = \frac{h_3 - h_2}{h_6 - h_{6a}}$$

THE REGENERATIVE CYCLE

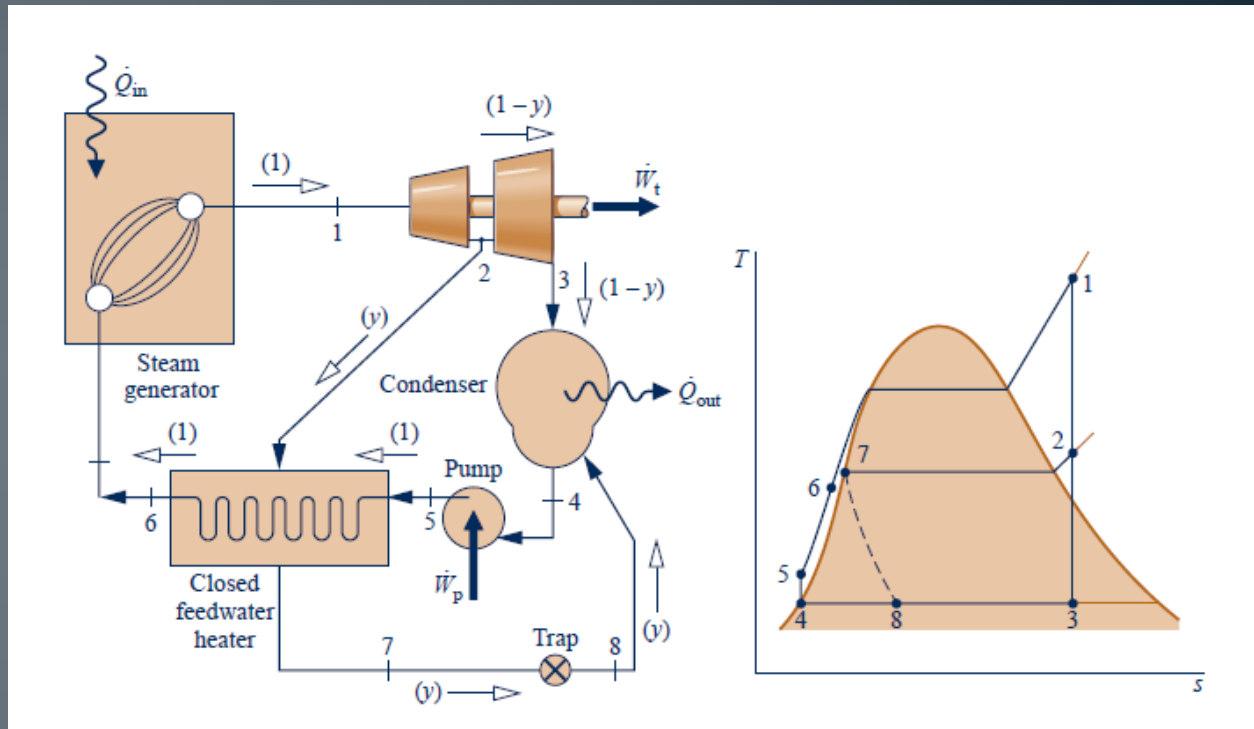
- A typical arrangement of the main components in an actual power plant.



- One open feedwater heater is a **deaerating feedwater heater**; this heater has the dual purpose of **heating** and **removing the air** from the feedwater.
- Unless the air is removed, excessive corrosion occurs in the boiler.

THE REGENERATIVE CYCLE

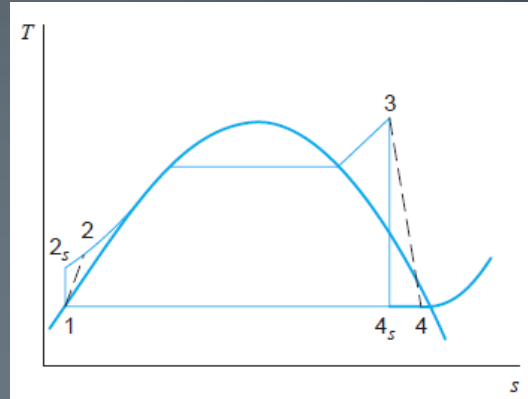
- Regenerative vapor power cycle with one closed feedwater heater.



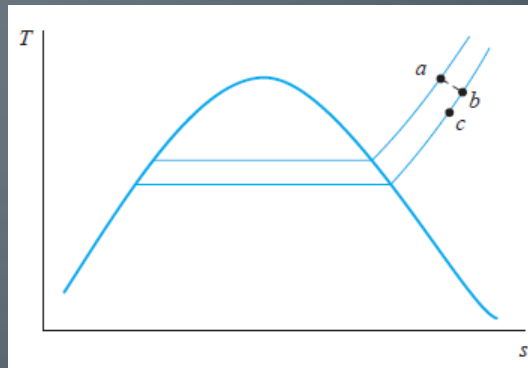
- The expansion from state 7 to state 8 through the trap is **irreversible**, so it is shown by a dashed line on the T - s diagram.

DEVIATION OF ACTUAL CYCLES FROM IDEAL CYCLES

- Turbine Losses
- Pump Losses

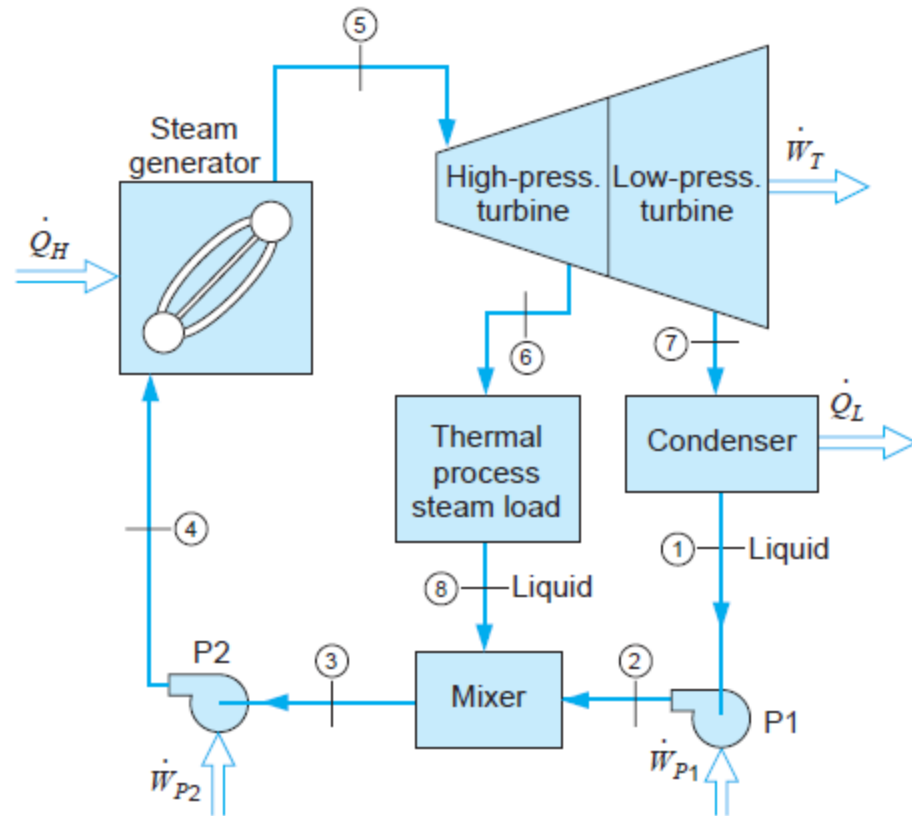


- Piping Losses



- Condenser Losses

COGENERATION



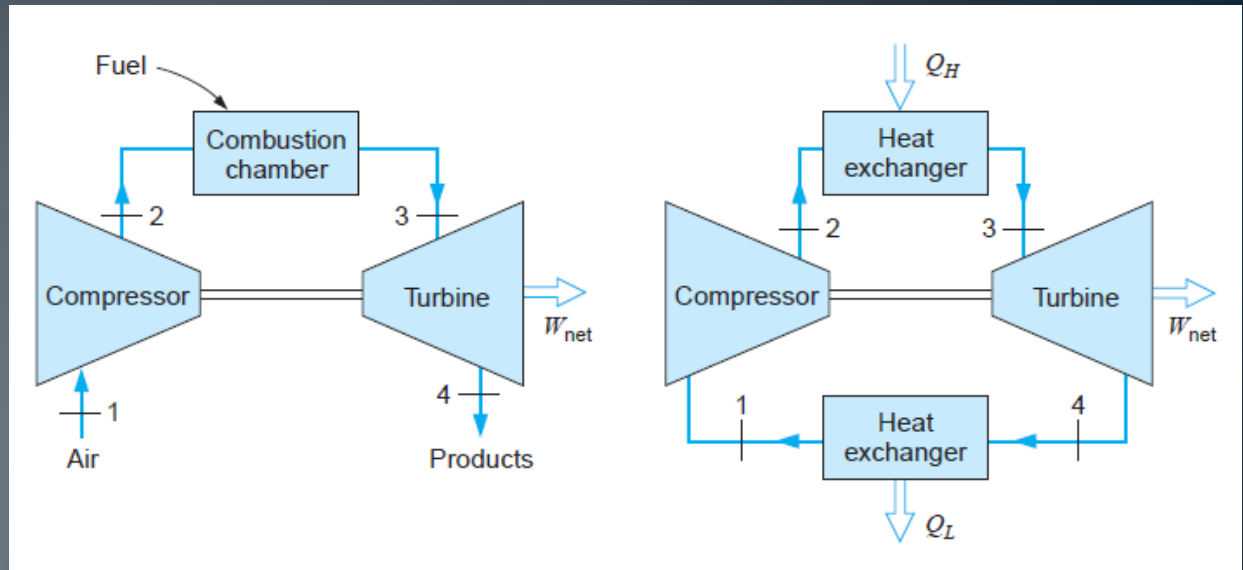
AIR-STANDARD POWER CYCLES

- many work producing devices (engines) utilize a working fluid that is always a **gas**. The spark-ignition automotive engine is a familiar example, as are the diesel engine and the conventional gas turbine.
- In all these engines there is a **change** in the composition of the working fluid, because during combustion it changes from **air and fuel** to **combustion products**.
- For this reason, these engines are called ***internal-combustion engines***. In contrast, the steam power plant may be called an ***external-combustion engine***, because heat is transferred from the products of combustion to the working fluid.

AIR-STANDARD POWER CYCLES

- Because the working fluid does not go through a complete thermodynamic cycle in the engine, the internal-combustion engine operates on the so-called **open cycle**.
- However, for analyzing internal-combustion engines, it is advantageous to devise **closed cycles** that closely approximate the open cycles.
- 1. A **fixed mass** of air is the working fluid throughout the entire cycle, and the air is always an **ideal gas**. Thus, there is **no inlet process** or **exhaust process**.
- 2. The **combustion process** is replaced by a process **transferring heat** from an external source.
- 3. The cycle is completed by **heat transfer to the surroundings** (in contrast to the exhaust and intake process of an actual engine).
- 4. All processes are internally **reversible**.
- 5. An additional assumption is often made that air has a **constant specific heat**, evaluated at 300 K, called cold air properties, recognizing that this is not the most accurate model.

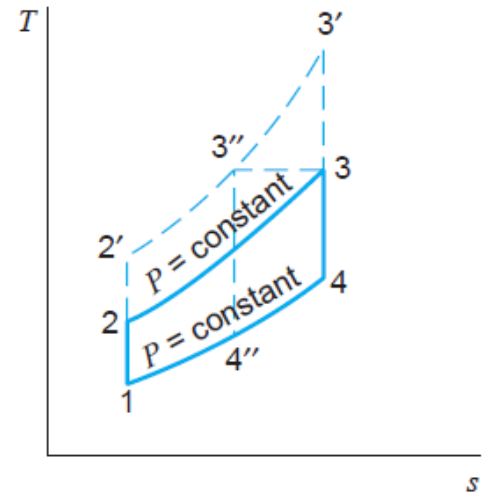
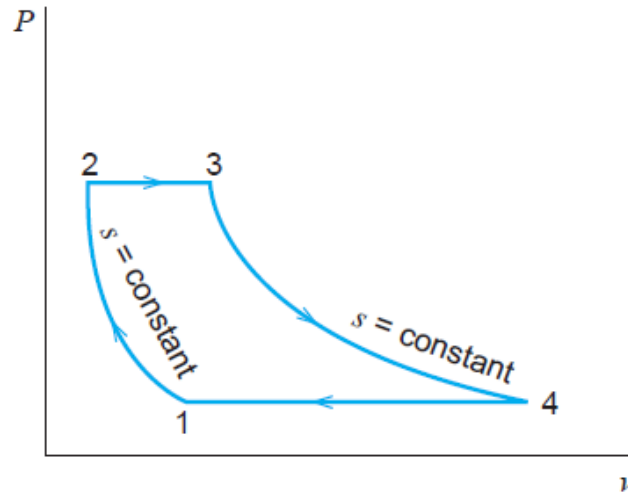
THE BRAYTON CYCLE



- The air-standard Brayton cycle is the ideal cycle for the simple **gas turbine**.

$$\eta_{th} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

THE BRAYTON CYCLE



$$\frac{P_3}{P_4} = \frac{P_2}{P_1}$$

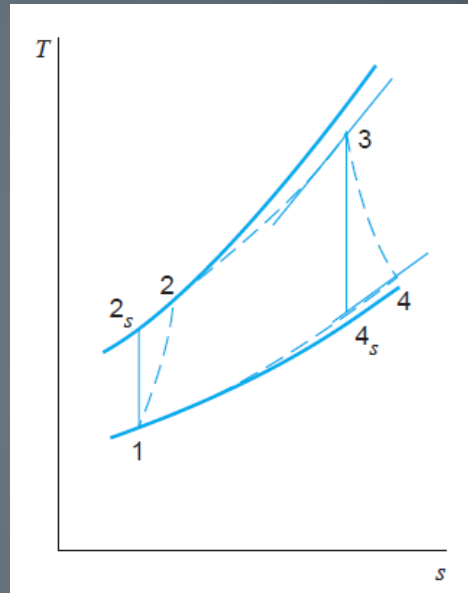
$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{k/(k-1)} = \frac{P_3}{P_4} = \left(\frac{T_3}{T_4} \right)^{k/(k-1)}$$

$$\frac{T_3}{T_4} = \frac{T_2}{T_1} \therefore \frac{T_3}{T_2} = \frac{T_4}{T_1} \quad \text{and} \quad \frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$$

$$\eta_{\text{th}} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{(P_2/P_1)^{(k-1)/k}}$$

THE BRAYTON CYCLE

- The actual gas-turbine engine differs from the ideal cycle primarily because of **irreversibilities** in the compressor and turbine, and because of **pressure drop** in the flow passages and combustion chamber.



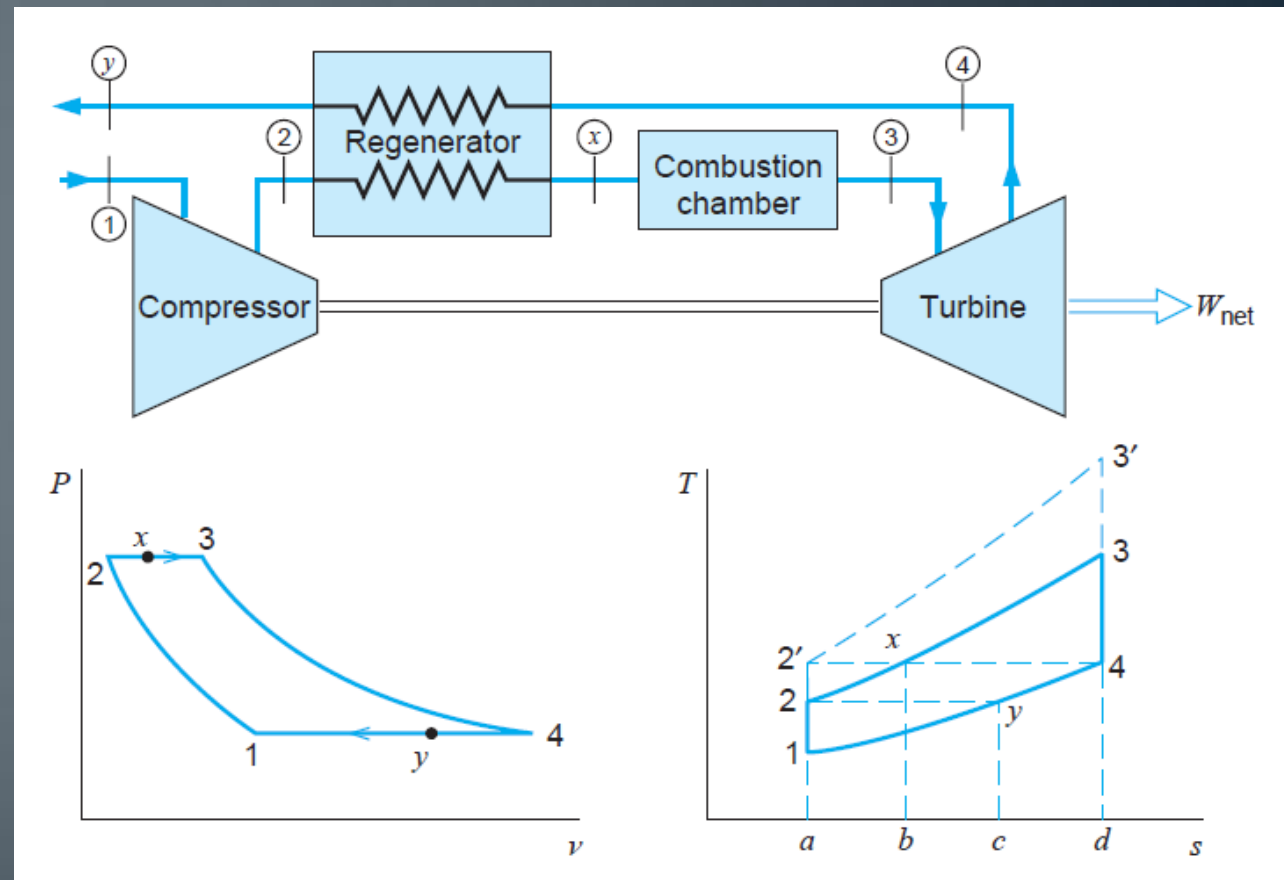
$$\eta_{\text{comp}} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$\eta_{\text{turb}} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

- Another important feature of the Brayton cycle is the large amount of compressor work (also called **back work**) compared to turbine work. Thus, the compressor might require 40 to 80% of the output of the turbine.

- The efficiency of the gas-turbine cycle may be improved by introducing a **regenerator**.

THE SIMPLE GAS-TURBINE CYCLE WITH A REGENERATOR



THE SIMPLE GAS- TURBINE CYCLE WITH A REGENERATOR

- The efficiency of the ideal gas-turbine cycle with a regenerator is found as follows

$$\eta_{th} = \frac{W_{net}}{q_H} = \frac{W_t - W_c}{q_H}$$
$$q_H = C_p(T_3 - T_x)$$
$$W_t = C_p(T_3 - T_4)$$

- But for an ideal regenerator, $T_4 = T_x$, and therefore $q_H = w_t$. Consequently,

$$\eta_{th} = 1 - \frac{W_c}{W_t} = 1 - \frac{C_p(T_2 - T_1)}{C_p(T_3 - T_4)}$$
$$= 1 - \frac{T_1(T_2/T_1 - 1)}{T_3(1 - T_4/T_3)} = 1 - \frac{T_1[(P_2/P_1)^{(k-1)/k} - 1]}{T_3[1 - (P_1/P_2)^{(k-1)/k}]}$$
$$\eta_{th} = 1 - \frac{T_1}{T_3} \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = 1 - \frac{T_2}{T_3}$$

- Thus, for the ideal cycle with regeneration, the thermal efficiency depends not only on the **pressure ratio** but also on the **ratio of the minimum to the maximum temperature**.

THE SIMPLE GAS- TURBINE CYCLE WITH A REGENERATOR

- The efficiency of the ideal gas-turbine cycle with a regenerator is found as follows

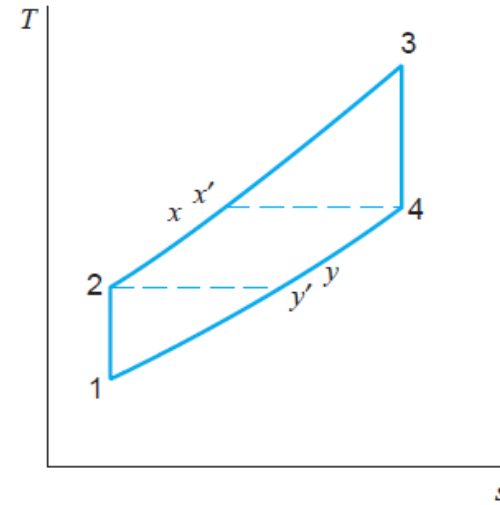
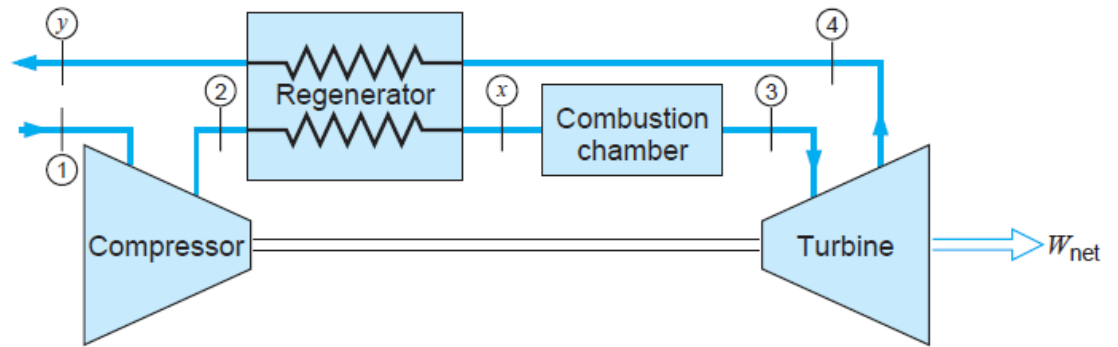
$$\eta_{th} = \frac{W_{net}}{q_H} = \frac{W_t - W_c}{q_H}$$
$$q_H = C_p(T_3 - T_x)$$
$$W_t = C_p(T_3 - T_4)$$

- But for an ideal regenerator, $T_4 = T_x$, and therefore $q_H = w_t$. Consequently,

$$\eta_{th} = 1 - \frac{W_c}{W_t} = 1 - \frac{C_p(T_2 - T_1)}{C_p(T_3 - T_4)}$$
$$= 1 - \frac{T_1(T_2/T_1 - 1)}{T_3(1 - T_4/T_3)} = 1 - \frac{T_1[(P_2/P_1)^{(k-1)/k} - 1]}{T_3[1 - (P_1/P_2)^{(k-1)/k}]}$$
$$\eta_{th} = 1 - \frac{T_1}{T_3} \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = 1 - \frac{T_2}{T_3}$$

- Thus, for the ideal cycle with regeneration, the thermal efficiency depends not only on the **pressure ratio** but also on the **ratio of the minimum to the maximum temperature**.
- We note that, in contrast to the Brayton cycle, the efficiency **decreases** with an increase in pressure ratio.

THE SIMPLE GAS-TURBINE CYCLE WITH A REGENERATOR

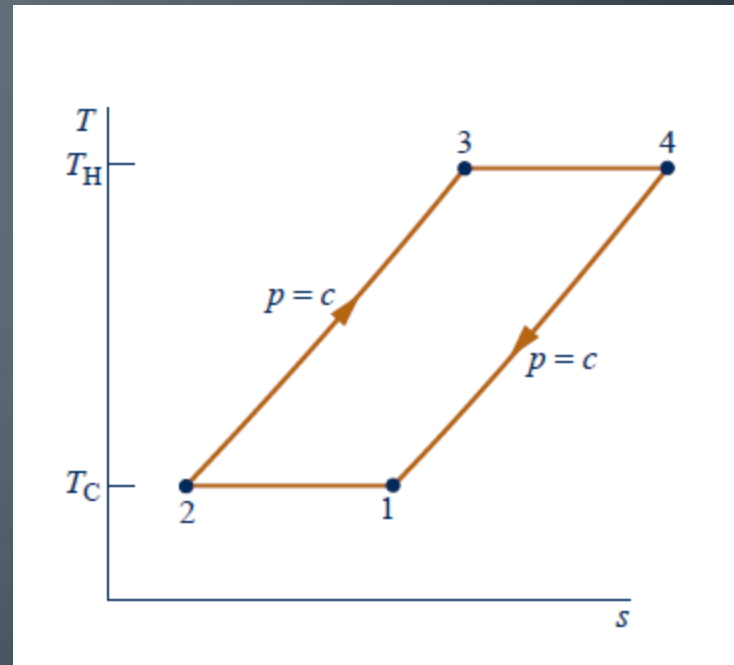


- The **regenerator efficiency** is defined by

$$\eta_{reg} = \frac{h_x - h_2}{h'_x - h_2}$$

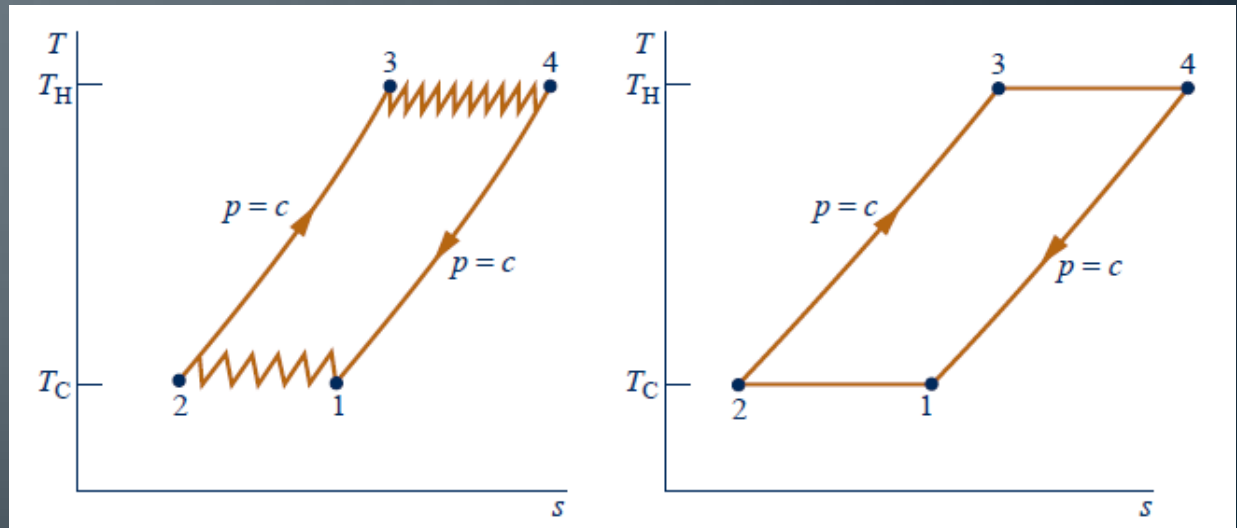
GAS-TURBINE POWER CYCLE CONFIGURATIONS

- It is found that the **isothermal process** would be preferable to the **adiabatic process** in both the compressor and turbine.
- The resulting cycle, called the **Ericsson cycle**, consists of two reversible, constant-pressure processes and two reversible, constant-temperature processes.

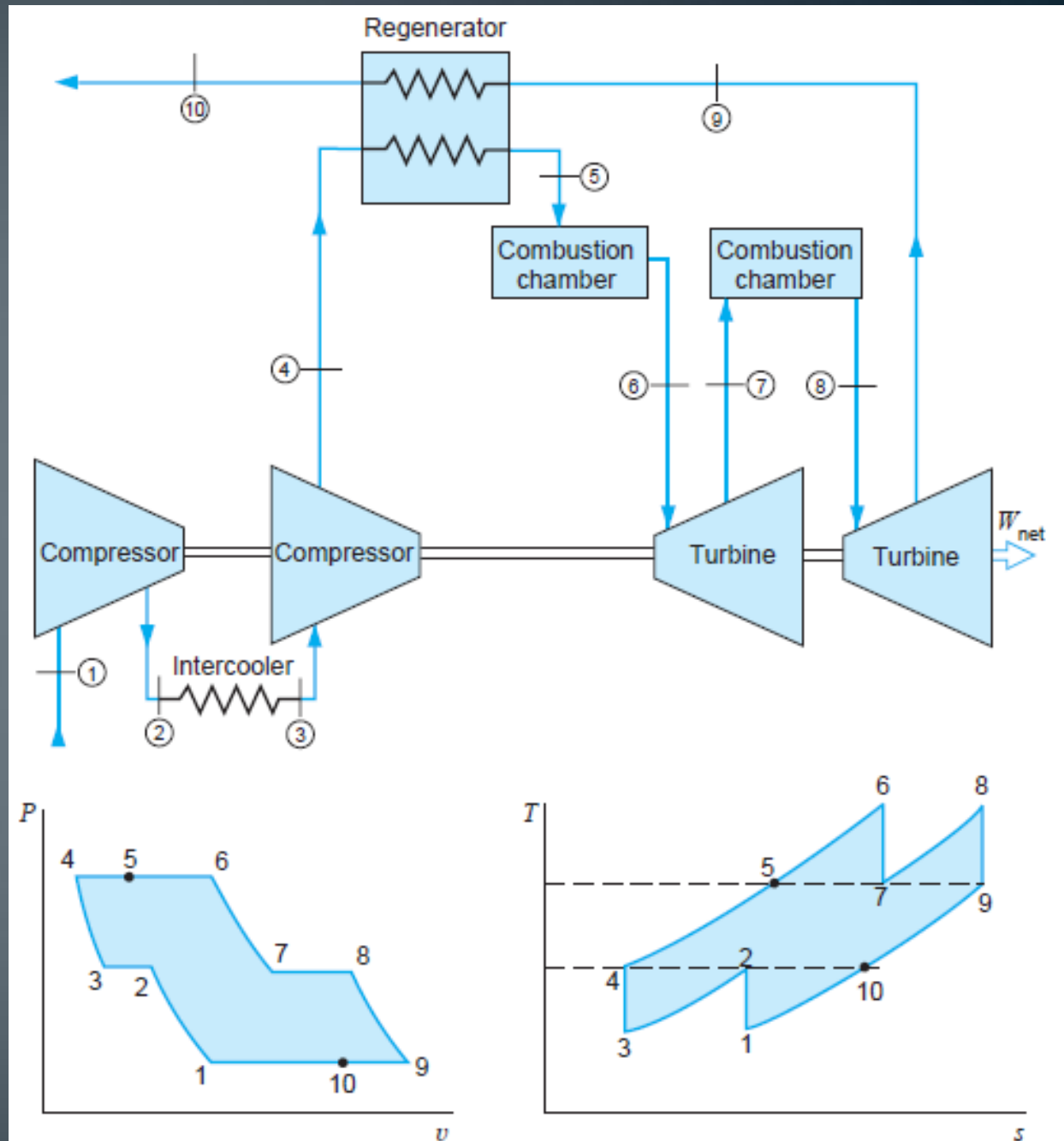


GAS-TURBINE POWER CYCLE CONFIGURATIONS

- The reason the actual gas turbine does not attempt to emulate this cycle rather than the Brayton cycle is that the compressor and turbine processes are both **high-flow-rate processes** involving work-related devices in which it is not **practical** to attempt to transfer large quantities of heat.
- There is a **modification** of the Brayton/gas turbine cycle that tends to change its performance in the direction of the Ericsson cycle. This modification is to use **multiple stages of compression** with **intercooling** and **multiple stages of expansion** with **reheat**.

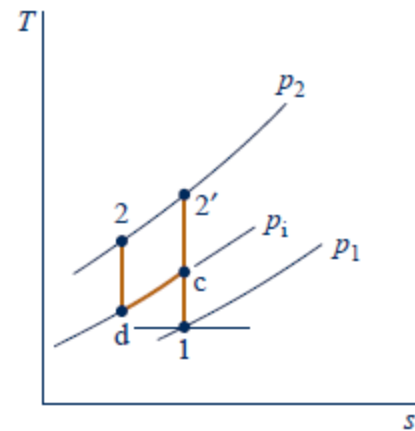
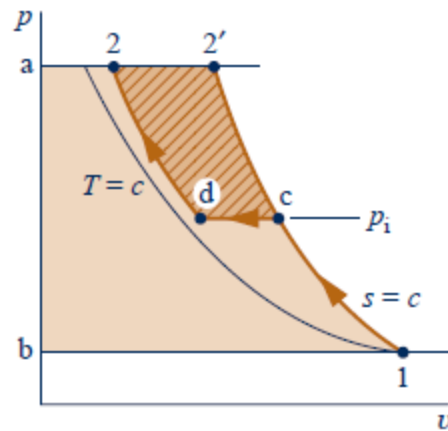
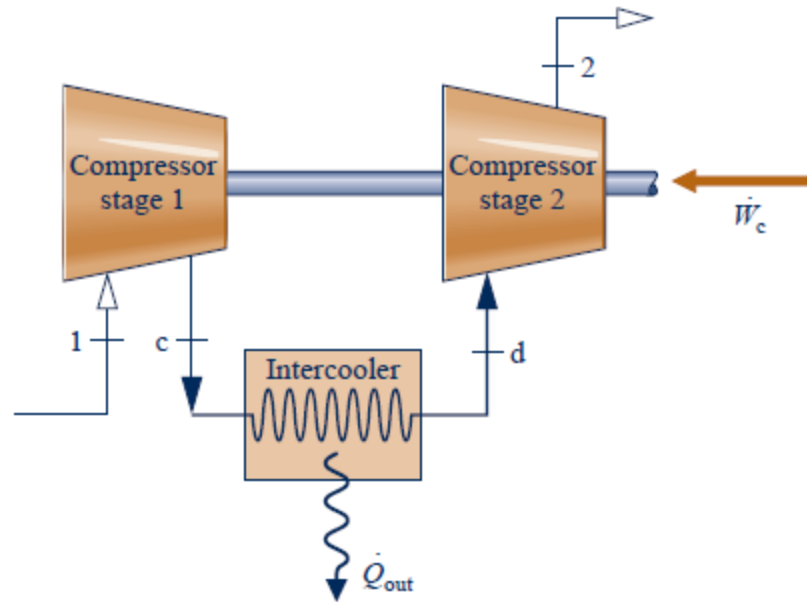


GAS-TURBINE POWER CYCLE CONFIGURATIONS



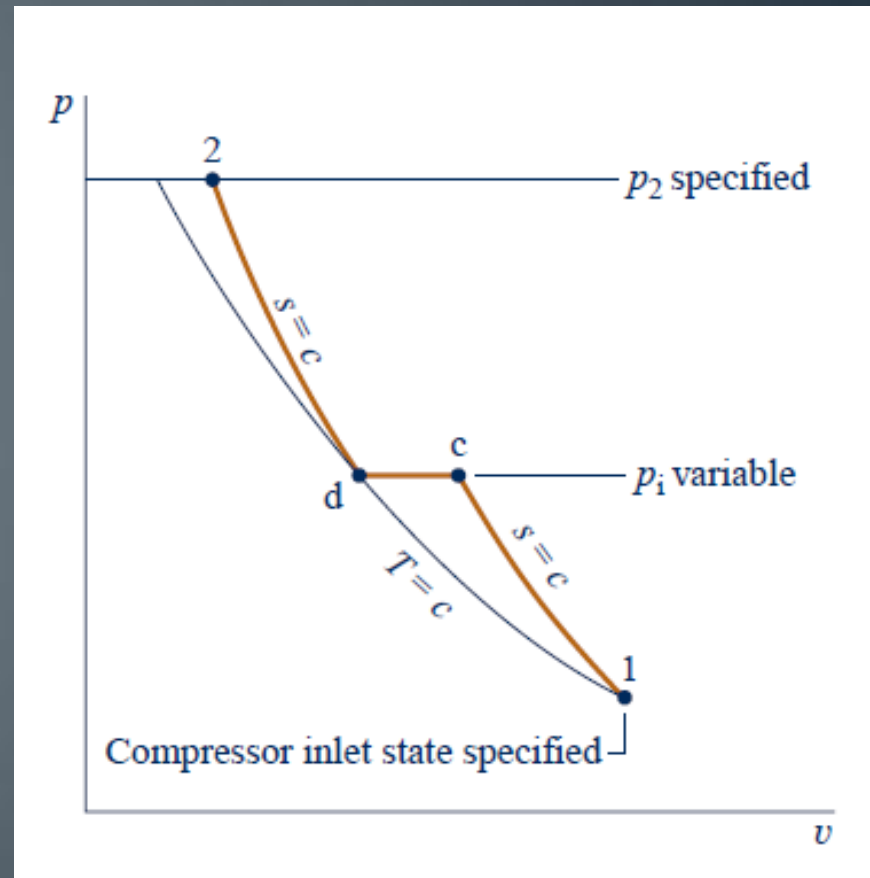
- Compression with Intercooling

GAS-TURBINE POWER CYCLE CONFIGURATIONS



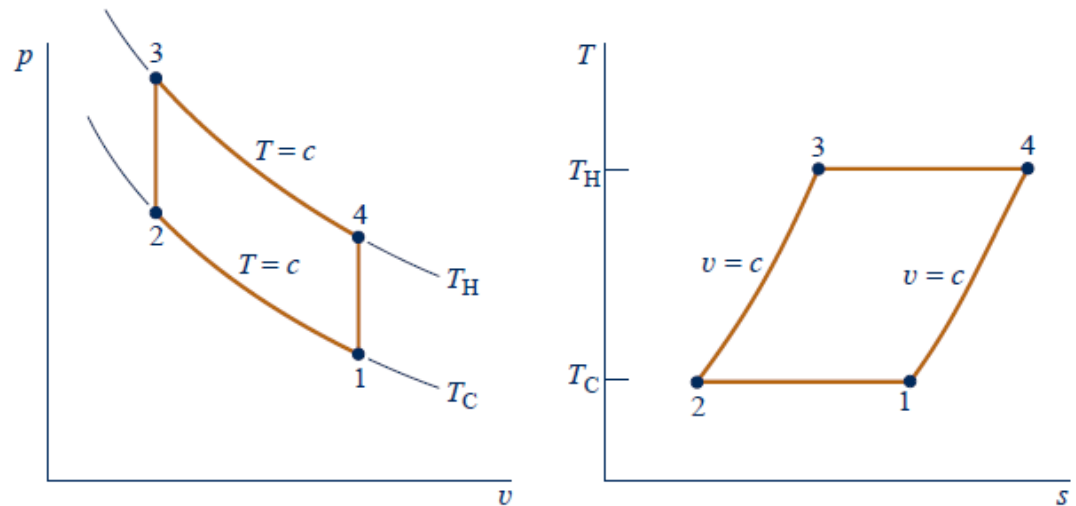
GAS-TURBINE POWER CYCLE CONFIGURATIONS

- **Determining Intercooler Pressure for Minimum Compressor Work**

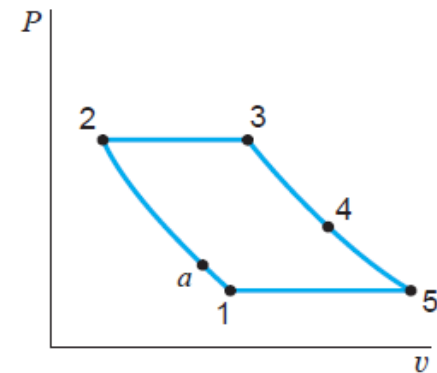
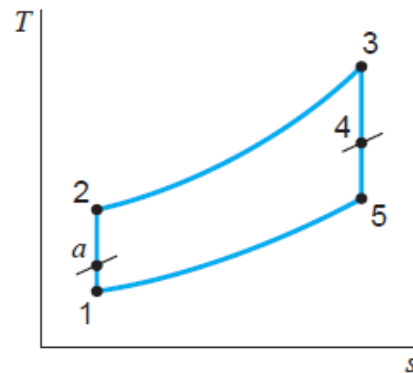
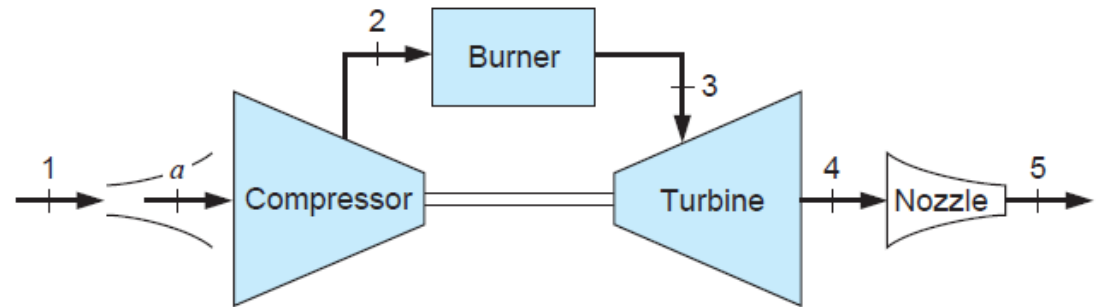
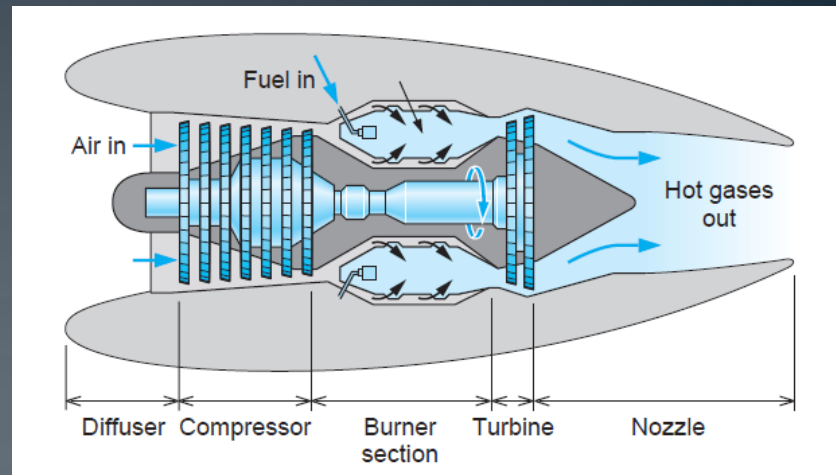


GAS-TURBINE POWER CYCLE CONFIGURATIONS

- Another cycle that employs a regenerator is the **Stirling cycle**.
- The cycle consists of four internally reversible processes in series: isothermal compression from state 1 to state 2, constant-volume heating from state 2 to state 3, isothermal expansion from state 3 to state 4, and constant-volume cooling from state 4 to state 1 to complete the cycle.

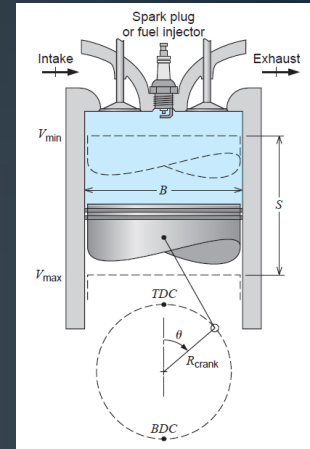
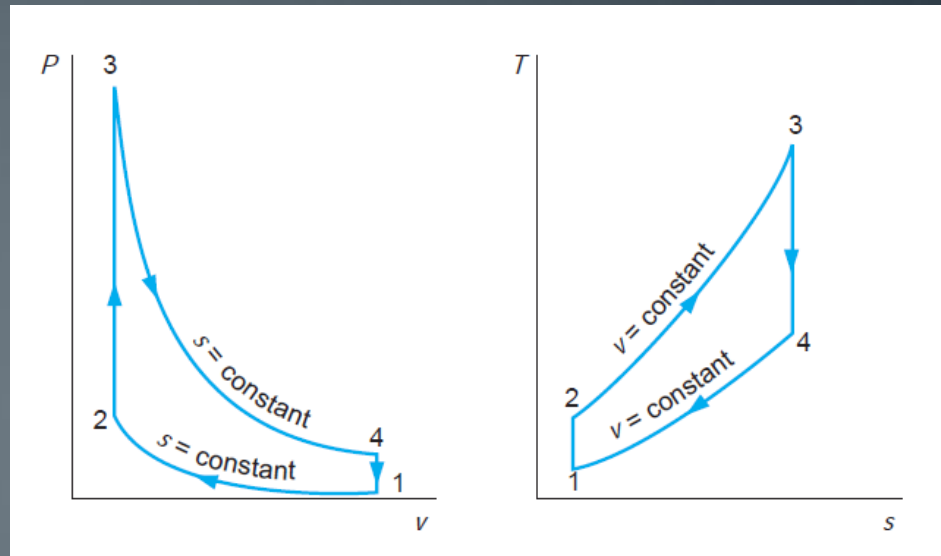


THE AIR-STANDARD CYCLE FOR JET PROPULSION



THE OTTO CYCLE

- The air-standard **Otto cycle** is an ideal cycle that approximates a spark-ignition internal combustion engine.



- Process 1–2 is an isentropic compression of the air as the piston moves from bottom dead center (BDC) to top dead center (TDC). Heat is then added at constant volume while the piston is momentarily at rest at TDC. (This process corresponds to the ignition of the fuel–air mixture by the spark and the subsequent burning in the actual engine.) Process 3–4 is an isentropic expansion, and process 4–1 is the rejection of heat from the air while the piston is at BDC.

THE OTTO CYCLE

- The thermal efficiency of this cycle is found as follows, assuming constant specific heat of air:

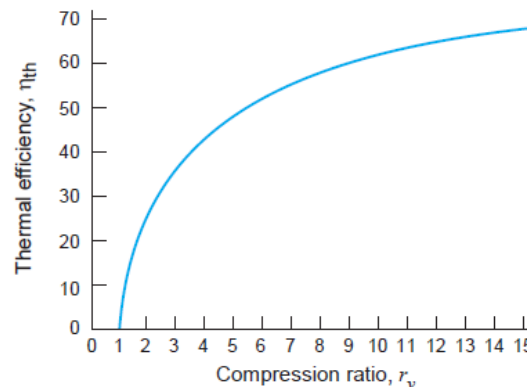
$$\eta_{th} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{mC_v(T_4 - T_1)}{mC_v(T_3 - T_2)} \\ = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} = \left(\frac{V_4}{V_3}\right)^{k-1} = \frac{T_3}{T_4}$$

$$\frac{T_3}{T_2} = \frac{T_4}{T_1}$$

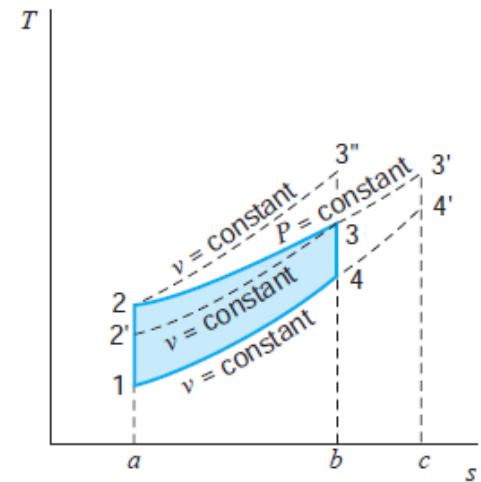
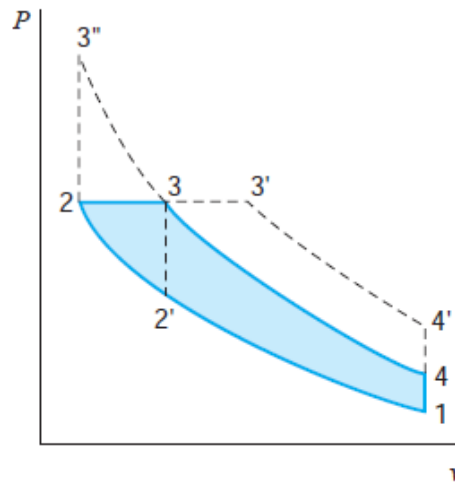
$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - (r_v)^{1-k} = 1 - \frac{1}{r_v^{k-1}}$$

$$r_v = \text{compression ratio} = \frac{V_1}{V_2} = \frac{V_4}{V_3}$$



THE DIESEL CYCLE

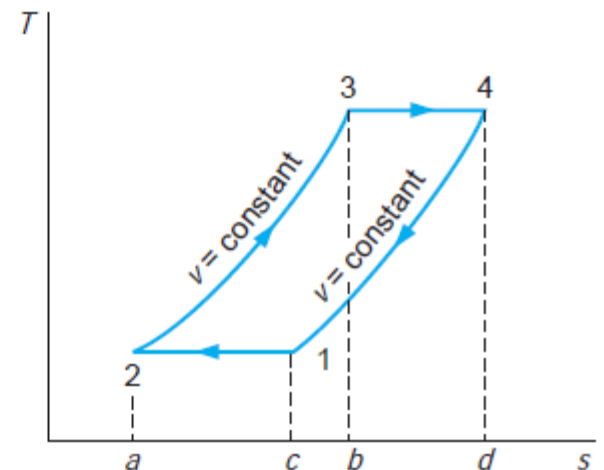
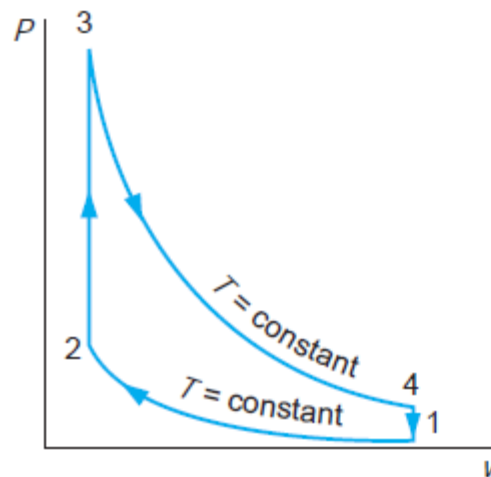
- The air-standard **Diesel cycle** is shown below. This is the ideal cycle for the diesel engine.



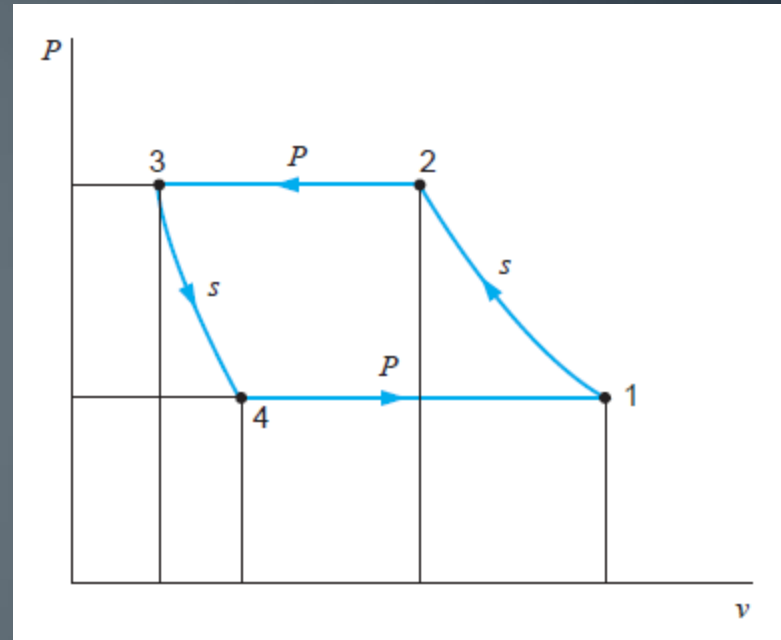
$$\eta_{th} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{kT_2(T_3/T_2 - 1)}$$

THE STIRLING CYCLE

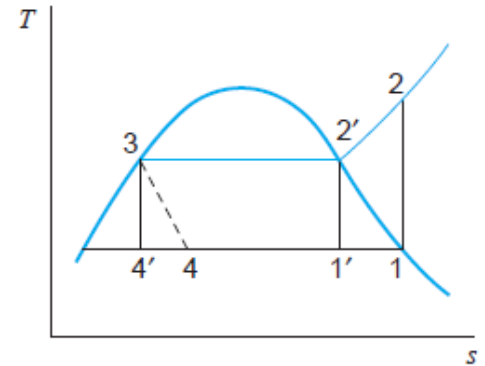
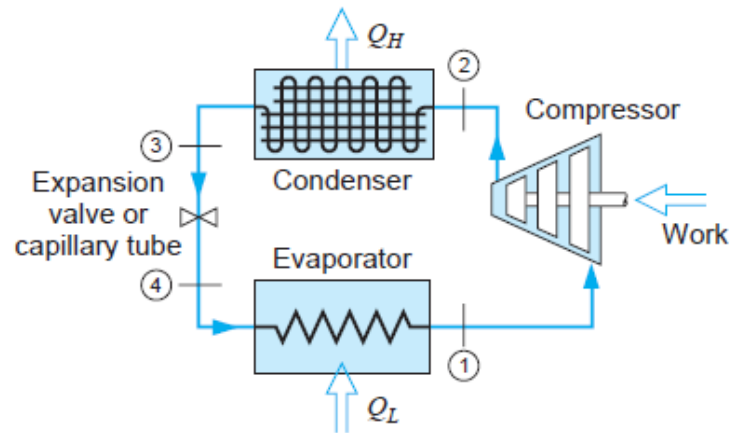
- Another air-standard power cycle to be discussed is the **Stirling cycle**.
- Stirling-cycle engines have been developed in recent years as **external combustion engines** with regeneration.
- Since all heat is supplied and rejected isothermally, the efficiency of this cycle equals the efficiency of a Carnot cycle operating between the same temperatures.



INTRODUCTION TO REFRIGERATION SYSTEMS



THE VAPOR-COMPRESSION REFRIGERATION CYCLE



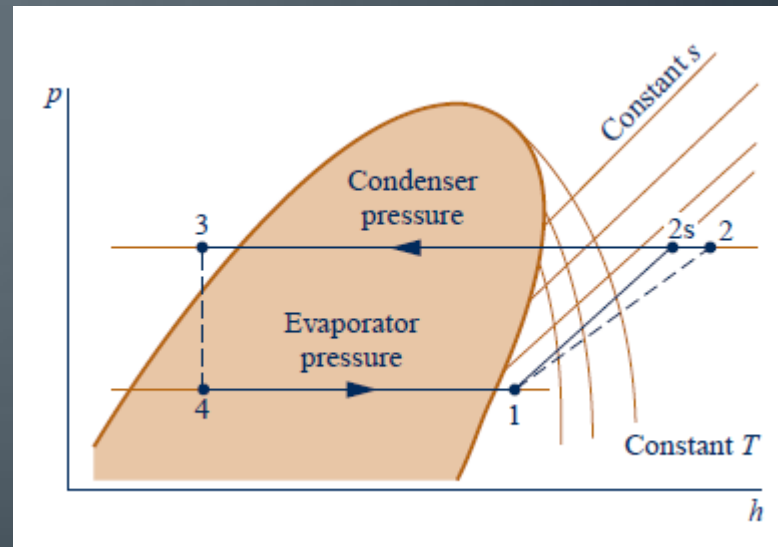
$$\beta = \frac{q_L}{w_c}$$

$$\beta' = \frac{q_H}{w_c}$$

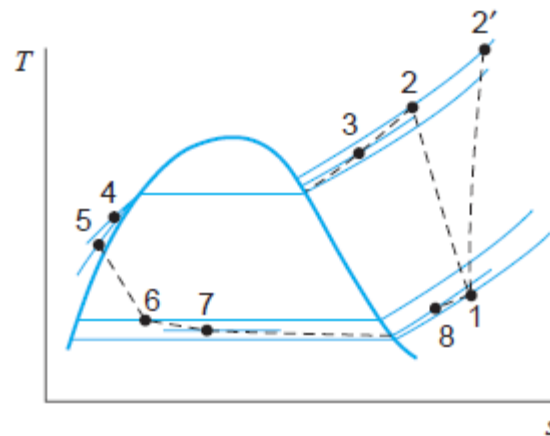
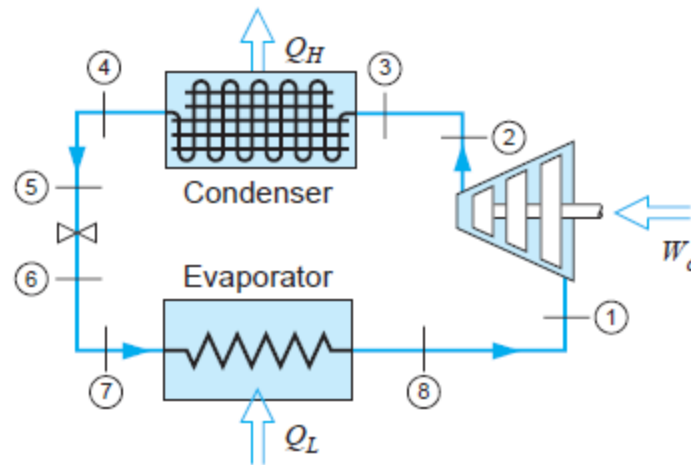
THE VAPOR-COMPRESSION REFRIGERATION CYCLE

- A much larger number of working fluids (refrigerants) are utilized in vapor-compression refrigeration systems than in vapor power cycles.

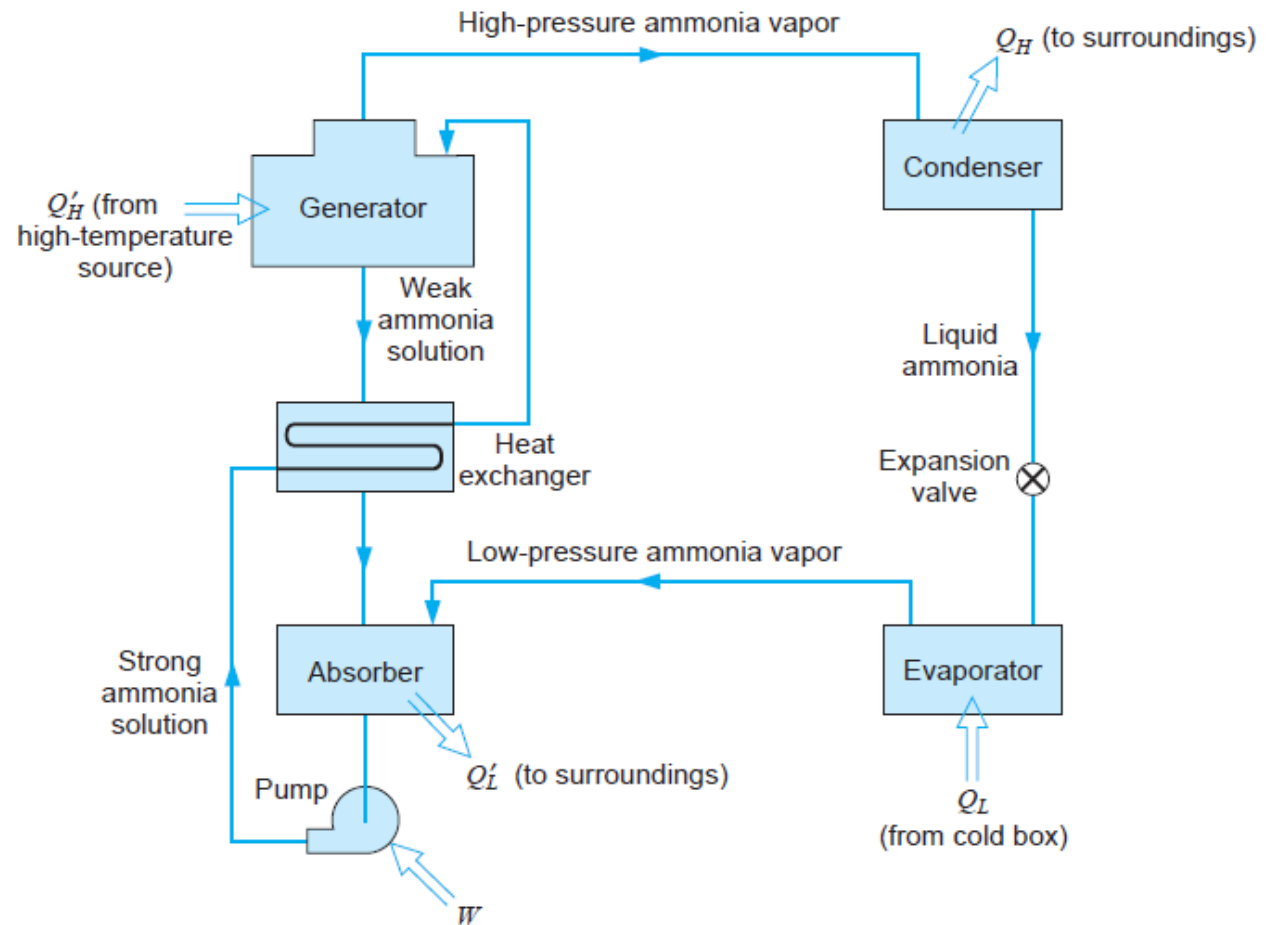
Old refrigerant	R-11	R-12	R-13	R-22	R-502	R-503
Alternative refrigerant	R-123	R-134a	R-23 (low T)	NH ₃	R-404a	R-23 (low T)
	R-245fa	R-152a	CO ₂	R-410a	R-407a	CO ₂
		R-401a	R-170 (ethane)		R-507a	



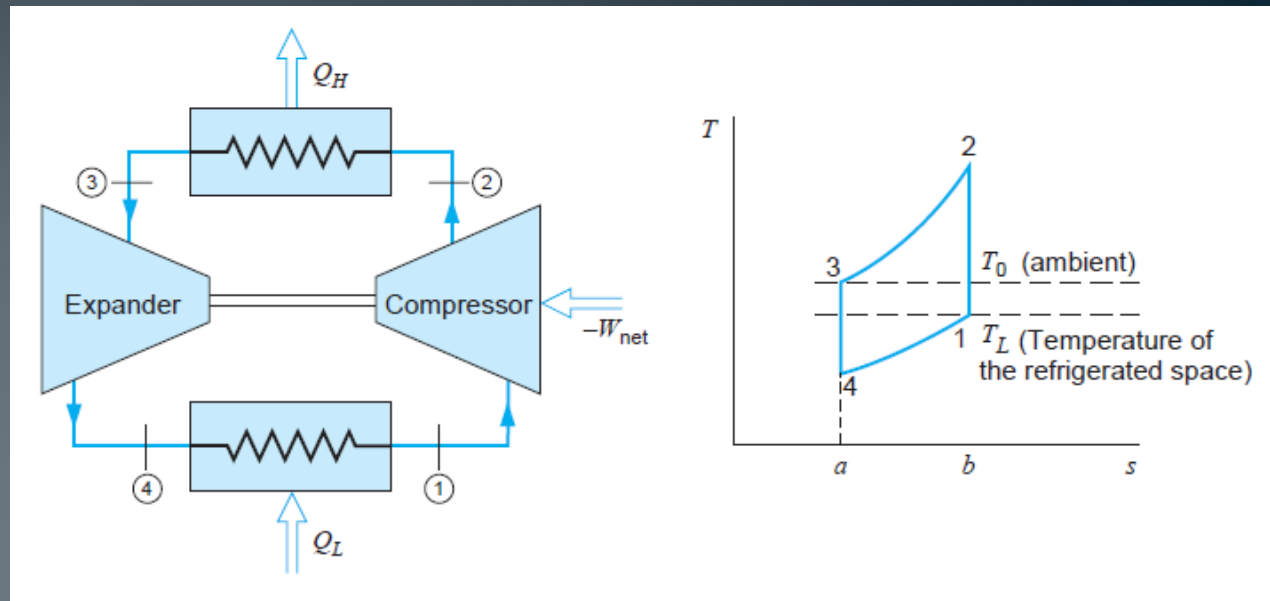
DEVIATION OF
THE ACTUAL
VAPOR-
COMPRESSION
REFRIGERATION
CYCLE FROM THE
IDEAL CYCLE



THE AMMONIA ABSORPTION REFRIGERATION CYCLE



THE AIR-STANDARD REFRIGERATION CYCLE

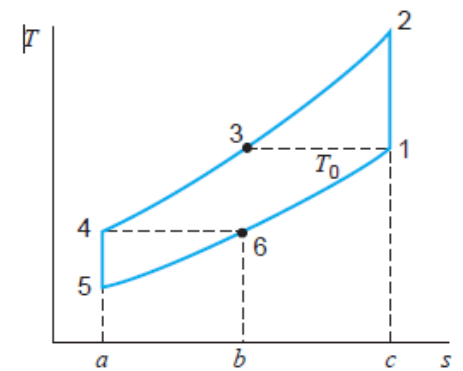
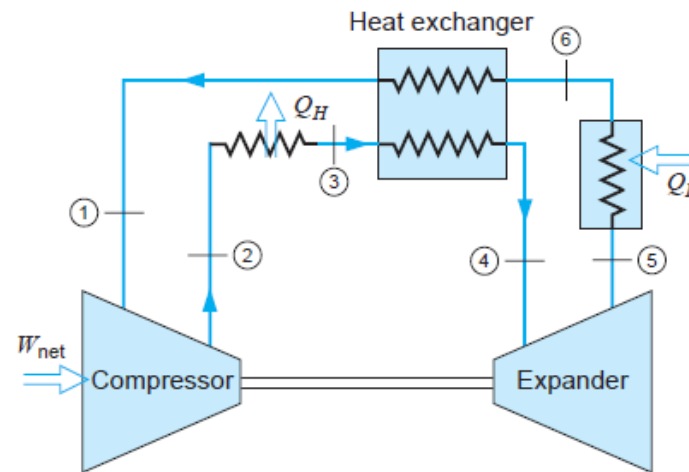
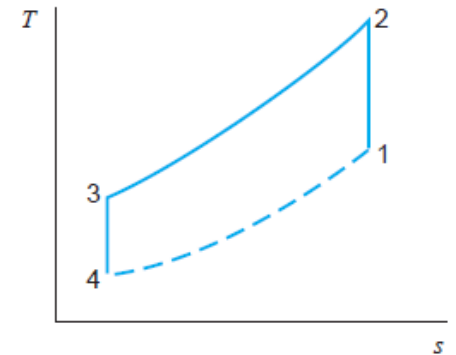
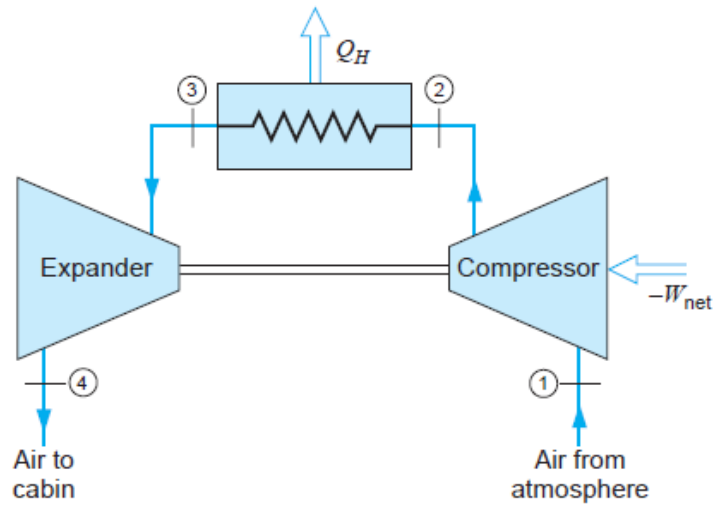


$$\beta = \frac{q_L}{w_{net}} = \frac{q_L}{w_C - w_E} = \frac{h_1 - h_4}{h_2 - h_1 - (h_3 - h_4)} \approx \frac{C_P(T_1 - T_4)}{C_P(T_2 - T_1) - C_P(T_3 - T_4)}$$

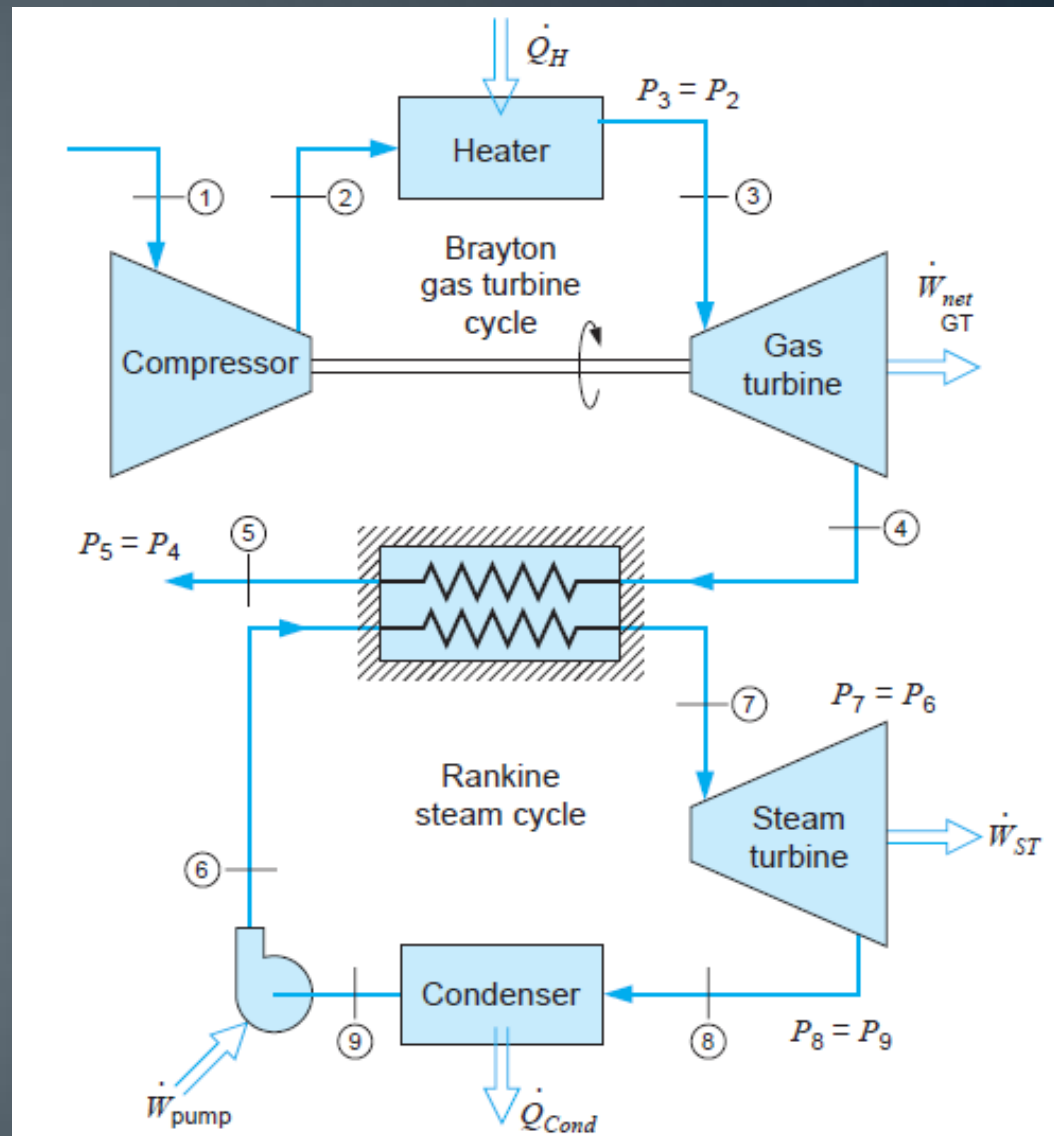
$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{k/(k-1)} = \frac{P_3}{P_4} = \left(\frac{T_3}{T_4} \right)^{k/(k-1)}$$

$$\begin{aligned} \beta &= \frac{T_1 - T_4}{T_2 - T_1 - T_3 + T_4} = \frac{1}{\frac{T_2}{T_1} \frac{1 - T_3/T_2}{1 - T_4/T_1} - 1} = \frac{1}{\frac{T_2}{T_1} - 1} \\ &= \frac{1}{r_p^{(k-1)/k} - 1} \end{aligned}$$

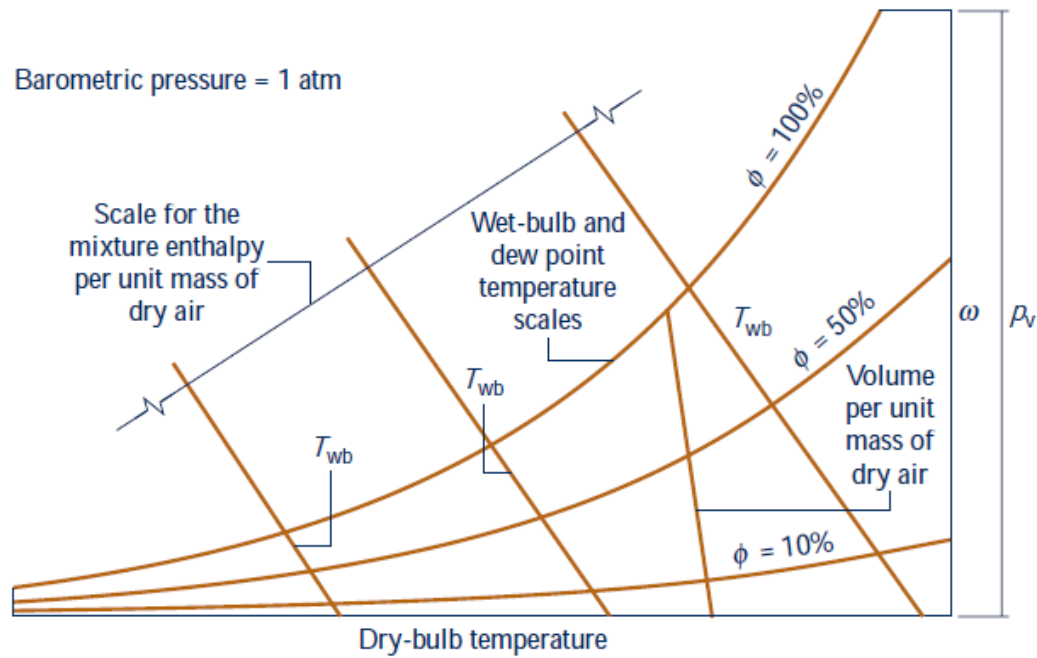
THE AIR-STANDARD REFRIGERATION CYCLE



COMBINED-CYCLE POWER AND REFRIGERATION SYSTEMS



Barometric pressure = 1 atm



GAS MIXTURES

GENERAL CONSIDERATIONS AND MIXTURES OF IDEAL GASES

- Let us consider a general mixture of N components, each a pure substance, so the total mass and the total number of moles are

$$m_{\text{tot}} = m_1 + m_2 + \cdots + m_N = \sum m_i$$
$$n_{\text{tot}} = n_1 + n_2 + \cdots + n_N = \sum n_i$$

- The mixture is usually described by a **mass fraction** (concentration)

$$c_i = \frac{m_i}{m_{\text{tot}}}$$

- or a **mole fraction** for each component as

$$y_i = \frac{n_i}{n_{\text{tot}}}$$

GENERAL
CONSIDERATIONS
AND
MIXTURES OF
IDEAL GASES

- We may then convert from a mole basis to a mass basis as

$$c_i = \frac{m_i}{m_{\text{tot}}} = \frac{n_i M_i}{\sum n_j M_j} = \frac{n_i M_i / n_{\text{tot}}}{\sum n_j M_j / n_{\text{tot}}} = \frac{y_i M_i}{\sum y_j M_j}$$

- and from a mass basis to a mole basis as

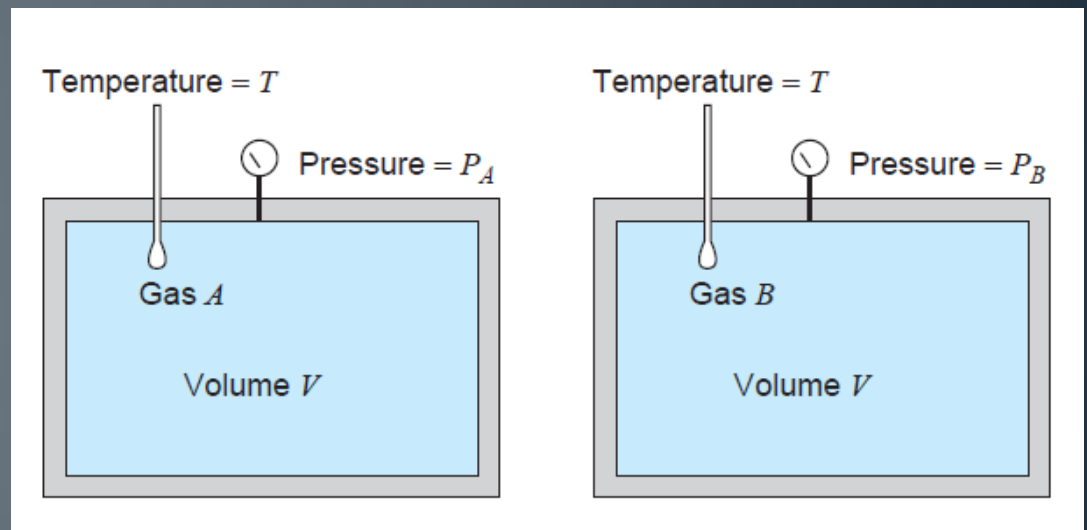
$$y_i = \frac{n_i}{n_{\text{tot}}} = \frac{m_i / M_i}{\sum m_j / M_j} = \frac{m_i / (M_i m_{\text{tot}})}{\sum m_j / (M_j m_{\text{tot}})} = \frac{c_i / M_i}{\sum c_j / M_j}$$

- The molecular mass for the mixture becomes

$$M_{\text{mix}} = \frac{m_{\text{tot}}}{n_{\text{tot}}} = \frac{\sum n_i M_i}{n_{\text{tot}}} = \sum y_i M_i$$

Dalton Model

- For the **Dalton model** of gas mixtures, the properties of each component of the mixture are considered as though each component exists separately and independently at the temperature and volume of the mixture.



- We further assume that both the gas mixture and the separated components behave according to **the ideal gas model**.

Dalton Model

- It is more convenient to use a mole basis, since the gas constant is then the universal gas constant for each component and also for the mixture.

$$PV = n\bar{R}T$$
$$n = n_A + n_B$$

- and for the components

$$P_A V = n_A \bar{R}T$$
$$P_B V = n_B \bar{R}T$$

- On substituting, we have

$$n = n_A + n_B$$
$$\frac{PV}{\bar{R}T} = \frac{P_A V}{\bar{R}T} + \frac{P_B V}{\bar{R}T}$$

$$P = P_A + P_B$$

$$P_A = y_A P, \quad P_B = y_B P$$

- where P_A and P_B are referred to as **partial pressures**.

Dalton Model

- Since for ideal gases the internal energy and enthalpy are functions only of temperature, it follows that for a mixture of components A and B, on a mass basis,

$$\begin{aligned}U &= mu = m_A u_A + m_B u_B \\&= m(c_A u_A + c_B u_B)\end{aligned}$$

$$\begin{aligned}H &= mh = m_A h_A + m_B h_B \\&= m(c_A h_A + c_B h_B)\end{aligned}$$

- The ideal-gas mixture equation of state on a mass basis is

$$PV = mR_{\text{mix}}T$$

$$R_{\text{mix}} = \frac{1}{m} \left(\frac{PV}{T} \right) = \frac{1}{m} (n\bar{R}) = \bar{R}/M_{\text{mix}}$$

Dalton Model

- Alternatively,

$$\begin{aligned}R_{\text{mix}} &= \frac{1}{m} (n_A \bar{R} + n_B \bar{R}) \\&= \frac{1}{m} (m_A R_A + m_B R_B) \\&= c_A R_A + c_B R_B\end{aligned}$$

- The entropy of an ideal-gas mixture is expressed as

$$\begin{aligned}S &= ms = m_A s_A + m_B s_B \\&= m(c_A s_A + c_B s_B)\end{aligned}$$

$$s_i = s_{0i} + C_{p0i} \ln\left(\frac{T}{T_0}\right) - R_i \ln\left(\frac{y_i P}{P_0}\right)$$

$$(s_2 - s_1)_i = C_{p0i} \ln \frac{T_2}{T_1} - R_i \ln \frac{P_2}{P_1}$$

$$\Delta S_{\text{mix}} = -\bar{R} \sum_k n_k \ln y_k$$

Dalton Model

- One special case that arises frequently involves an ideal-gas mixture undergoing a process in which there is no change in composition.

$$\begin{aligned}u_2 - u_1 &= c_A C_{v0\ A}(T_2 - T_1) + c_B C_{v0\ B}(T_2 - T_1) \\&= C_{v0\ \text{mix}}(T_2 - T_1)\end{aligned}$$

$$C_{v0\ \text{mix}} = c_A C_{v0\ A} + c_B C_{v0\ B}$$

$$\begin{aligned}h_2 - h_1 &= c_A C_{p0\ A}(T_2 - T_1) + c_B C_{p0\ B}(T_2 - T_1) \\&= C_{p0\ \text{mix}}(T_2 - T_1)\end{aligned}$$

$$C_{p0\ \text{mix}} = c_A C_{p0\ A} + c_B C_{p0\ B}$$

$$s_2 - s_1 = c_A(s_2 - s_1)_A + c_B(s_2 - s_1)_B$$

$$= c_A C_{p0\ A} \ln \frac{T_2}{T_1} - c_A R_A \ln \frac{P_2}{P_1} + c_B C_{p0\ B} \ln \frac{T_2}{T_1} - c_B R_B \ln \frac{P_2}{P_1}$$

$$= C_{p0\ \text{mix}} \ln \frac{T_2}{T_1} - R_{\text{mix}} \ln \frac{P_2}{P_1}$$

Dalton Model

- We can treat a mixture similarly to a pure substance once the mixture properties are found from the composition and the component properties.
- The isentropic process where s is constant leads to the power relation between temperature and pressure.

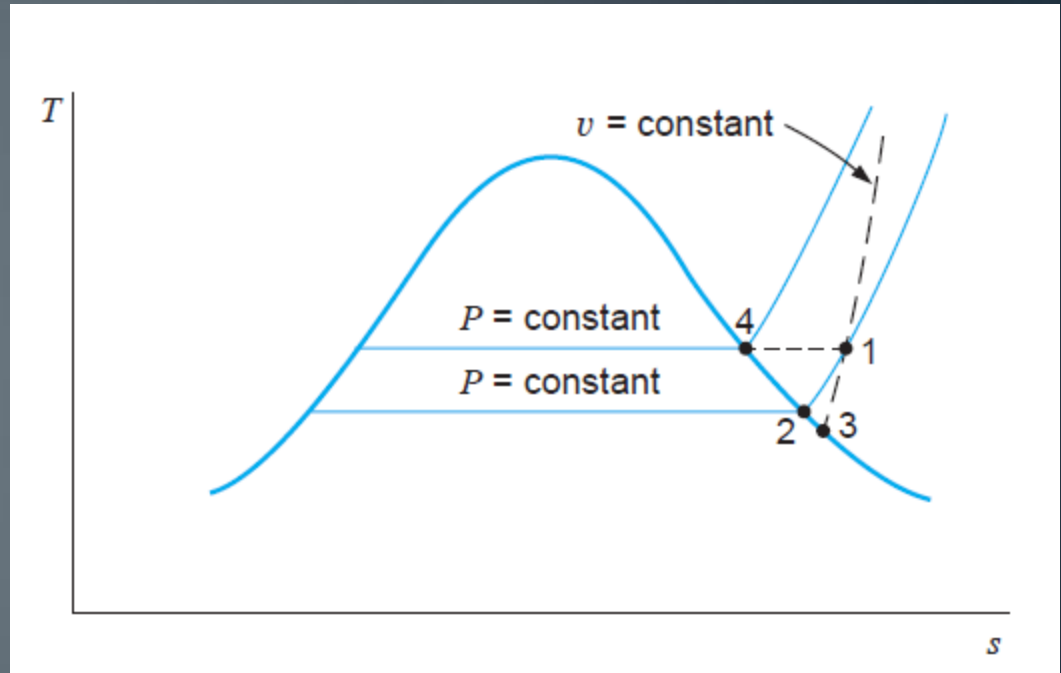
$$k = k_{\text{mix}} = \frac{C_{p \text{ mix}}}{C_{v \text{ mix}}} = \frac{C_{p \text{ mix}}}{C_{p \text{ mix}} - R_{\text{mix}}}$$

A SIMPLIFIED
MODEL OF A
MIXTURE
INVOLVING
GASES AND A
VAPOR

- Let us now consider a simplification of the problem involving a mixture of ideal gases that is in contact with a solid or liquid phase of one of the components.
- 1. The solid or liquid phase contains **no dissolved gases**.
- 2. The gaseous phase can be treated as a mixture of **ideal gases**.
- 3. When the mixture and the condensed phase are at a given pressure and temperature, the equilibrium between the condensed phase and its vapor is **not influenced** by the presence of the other component.

A SIMPLIFIED MODEL OF A MIXTURE INVOLVING GASES AND A VAPOR

- The **dew point** of a gas–vapor mixture is the temperature at which the vapor condenses or solidifies when it is cooled at constant pressure.



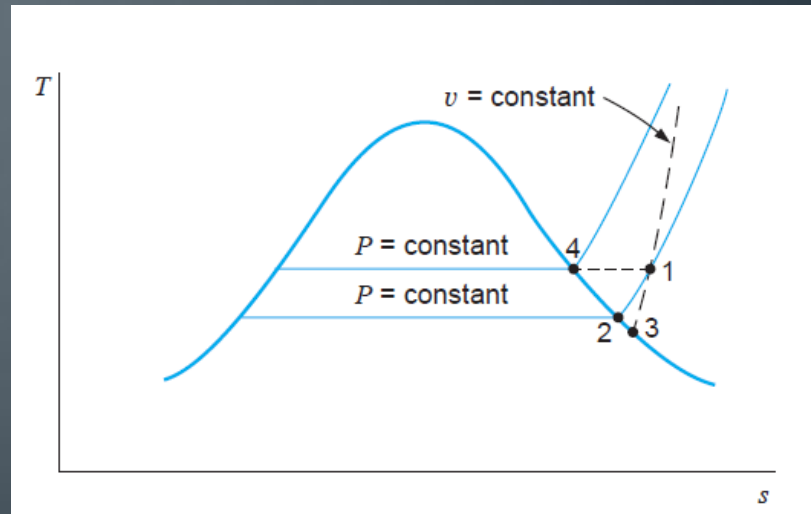
- If the vapor is at the saturation pressure and temperature, the mixture is referred to as a **saturated mixture**, and for an air–water vapor mixture, the term **saturated air** is used.

A SIMPLIFIED MODEL OF A MIXTURE INVOLVING GASES AND A VAPOR

- The **relative humidity** ϕ is defined as the ratio of the mole fraction of the vapor in the mixture to the mole fraction of vapor in a saturated mixture at the same temperature and total pressure. Since the vapor is considered an ideal gas, the definition reduces to the ratio of the partial pressure of the vapor as it exists in the mixture, P_v , to the saturation pressure of the vapor at the same temperature, P_g :

$$\phi = \frac{P_v}{P_g}$$

$$\phi = \frac{P_1}{P_4}$$



A SIMPLIFIED
MODEL OF A
MIXTURE
INVOLVING
GASES AND A
VAPOR

- The **humidity ratio** ω of an air–water vapor mixture is defined as the ratio of the mass of water vapor m_v to the mass of dry air m_a . The term **dry air** is used to emphasize that this refers only to air and not to the water vapor. The terms **specific humidity** or **absolute humidity** are used synonymously with *humidity ratio*.

$$\omega = \frac{m_v}{m_a}$$

$$m_v = \frac{P_v V}{R_v T} = \frac{P_v V M_v}{\bar{R} T}, \quad m_a = \frac{P_a V}{R_a T} = \frac{P_a V M_a}{\bar{R} T}$$

$$\omega = \frac{P_v V / R_v T}{P_a V / R_a T} = \frac{R_a P_v}{R_v P_a} = \frac{M_v P_v}{M_a P_a}$$

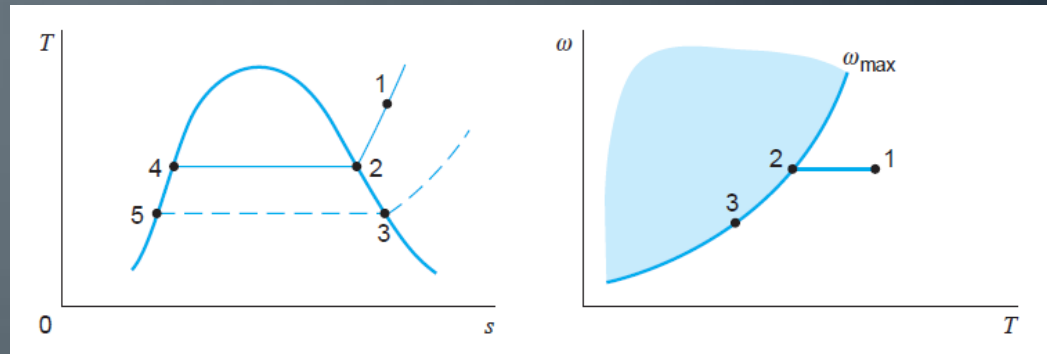
$$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P_{\text{tot}} - P_v}$$

A SIMPLIFIED MODEL OF A MIXTURE INVOLVING GASES AND A VAPOR

- The **degree of saturation** is defined as the ratio of the actual humidity ratio to the humidity ratio of a saturated mixture at the same temperature and total pressure. This refers to the maximum amount of water that can be contained in moist air.

$$\omega = 0.622 \frac{\phi P_g}{P_{\text{tot}} - \phi P_g} \leq \omega_{\text{max}} = 0.622 \frac{P_g}{P_{\text{tot}} - P_g}$$

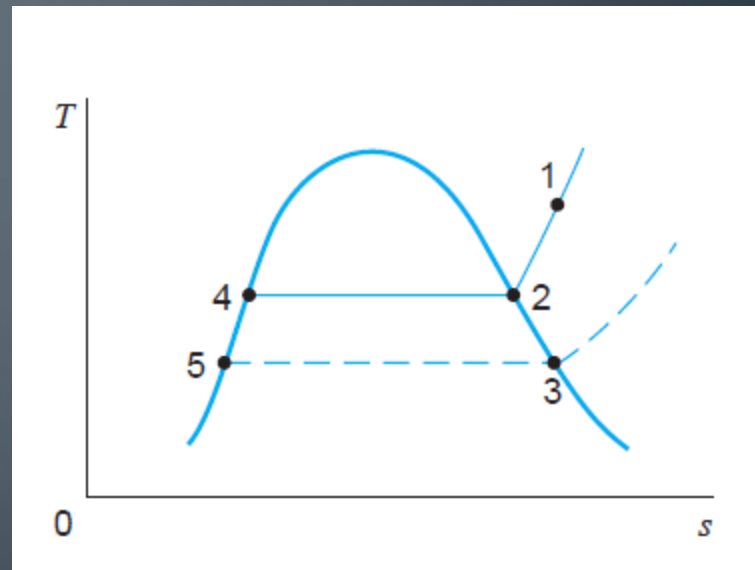
- The maximum humidity ratio corresponds to a relative humidity of 100%. the function has an asymptote at a temperature where $P_g = P_{\text{tot}}$, which is 100°C for atmospheric pressure.



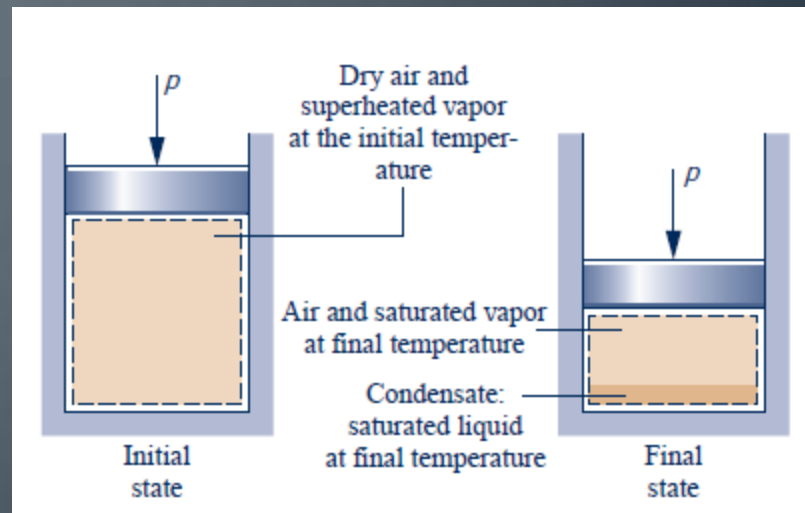
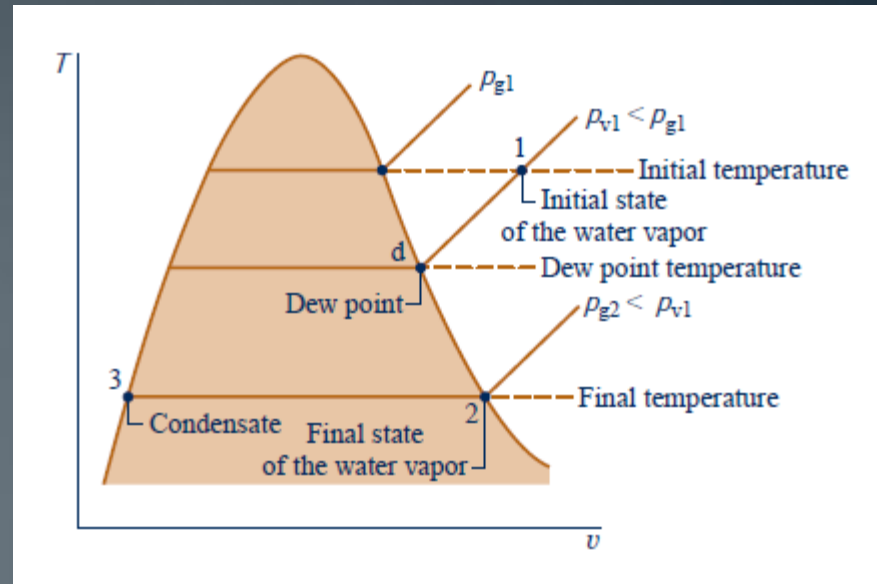
- The shaded regions are states not permissible, as the water vapor pressure would be larger than the saturation pressure.

A SIMPLIFIED MODEL OF A MIXTURE INVOLVING GASES AND A VAPOR

- In a cooling process at constant total pressure, the partial pressure of the vapor remains constant until the dew point is reached at state 2. Further cooling lowers the maximum possible humidity ratio, and some of the vapor condenses. The vapor that remains in the mixture is always saturated, and the liquid or solid is in equilibrium with it. For example, when the temperature is reduced to T_3 , the vapor in the mixture is at state 3, and its partial pressure is P_g at T_3 and the liquid is at state 5 in equilibrium with the vapor.

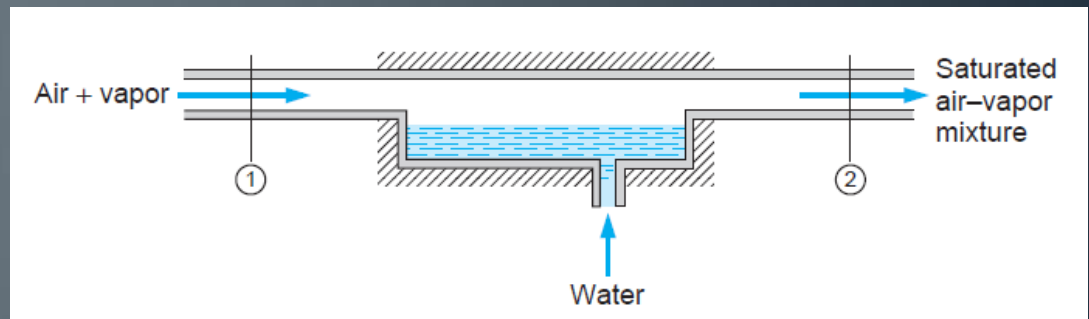


A SIMPLIFIED MODEL OF A MIXTURE INVOLVING GASES AND A VAPOR



THE ADIABATIC SATURATION PROCESS

- An important process for an air–water vapor mixture is the adiabatic saturation process. In this process, an air–vapor mixture comes in contact with a body of water in a well-insulated duct. If the initial humidity is less than 100%, some of the water will evaporate and the temperature of the air–vapor mixture will decrease. If the mixture leaving the duct is saturated and if the process is adiabatic, the temperature of the mixture on leaving is known as the **adiabatic saturation temperature**.



- For this to take place as a steady-state process, makeup water at the adiabatic saturation temperature is added at the same rate at which water is evaporated. The pressure is assumed to be constant.

THE ADIABATIC SATURATION PROCESS

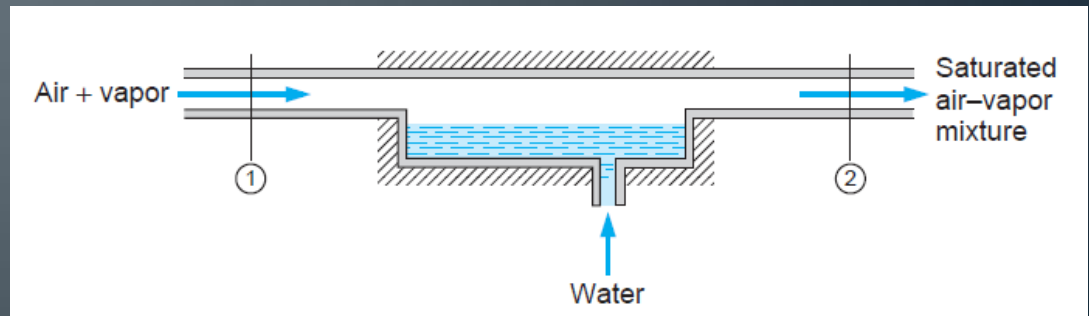
- Considering the adiabatic saturation process to be a steady-state process, and neglecting changes in kinetic and potential energy, the first law reduces to

$$h_{a1} + \omega_1 h_{v1} + (\omega_2 - \omega_1) h_{f2} = h_{a2} + \omega_2 h_{v2}$$

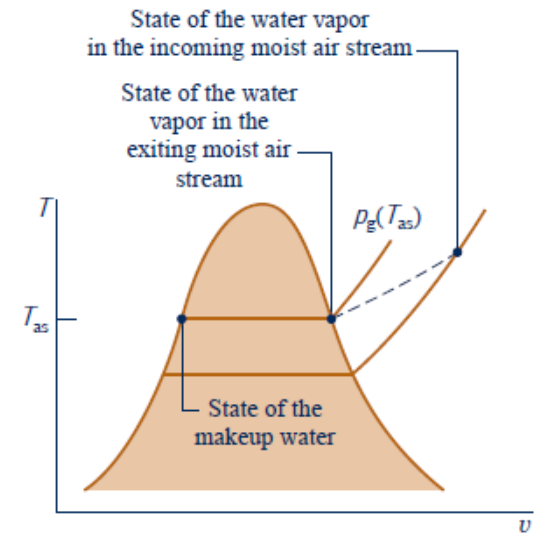
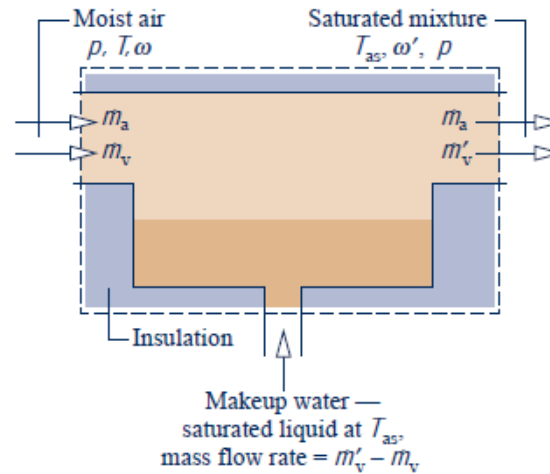
$$\omega_1 (h_{v1} - h_{f2}) = C_{pa} (T_2 - T_1) + \omega_2 (h_{v2} - h_{f2})$$

$$\omega_1 (h_{v1} - h_{f2}) = C_{pa} (T_2 - T_1) + \omega_2 h_{fg2}$$

- The most significant point to be made about the adiabatic saturation process is that the adiabatic saturation temperature is a function of the **pressure**, **temperature**, and **relative humidity** of the entering air–vapor mixture and of the exit pressure.



THE ADIABATIC SATURATION PROCESS



$$H = H_a + H_v = m_a h_a + m_v h_v$$

$$\frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = h_a + \omega h_v$$

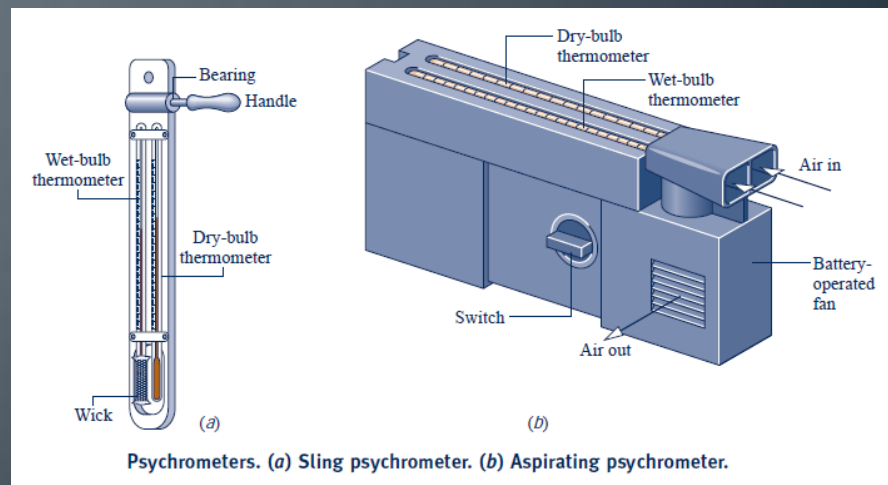
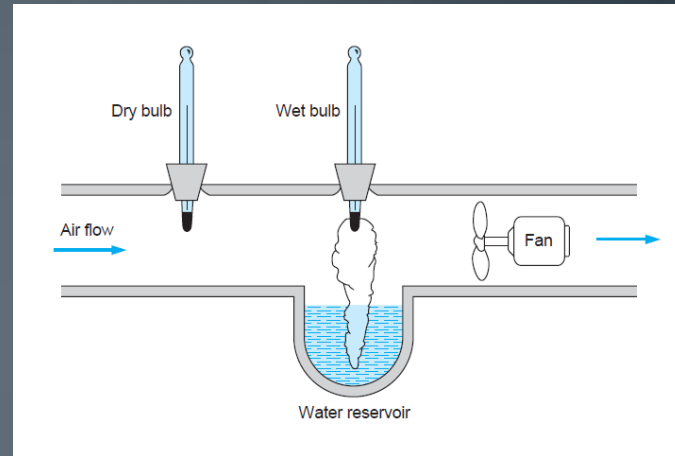
$$h_v \approx h_g(T)$$

$$(h_a + \omega h_g)_{\text{moist air entering}} + [(\omega' - \omega) h_f]_{\text{makeup water}} = (h_a + \omega' h_g)_{\text{moist air exiting}}$$

$$\omega = \frac{h_a(T_{as}) - h_a(T) + \omega' [h_g(T_{as}) - h_f(T_{as})]}{h_g(T) - h_f(T_{as})}$$

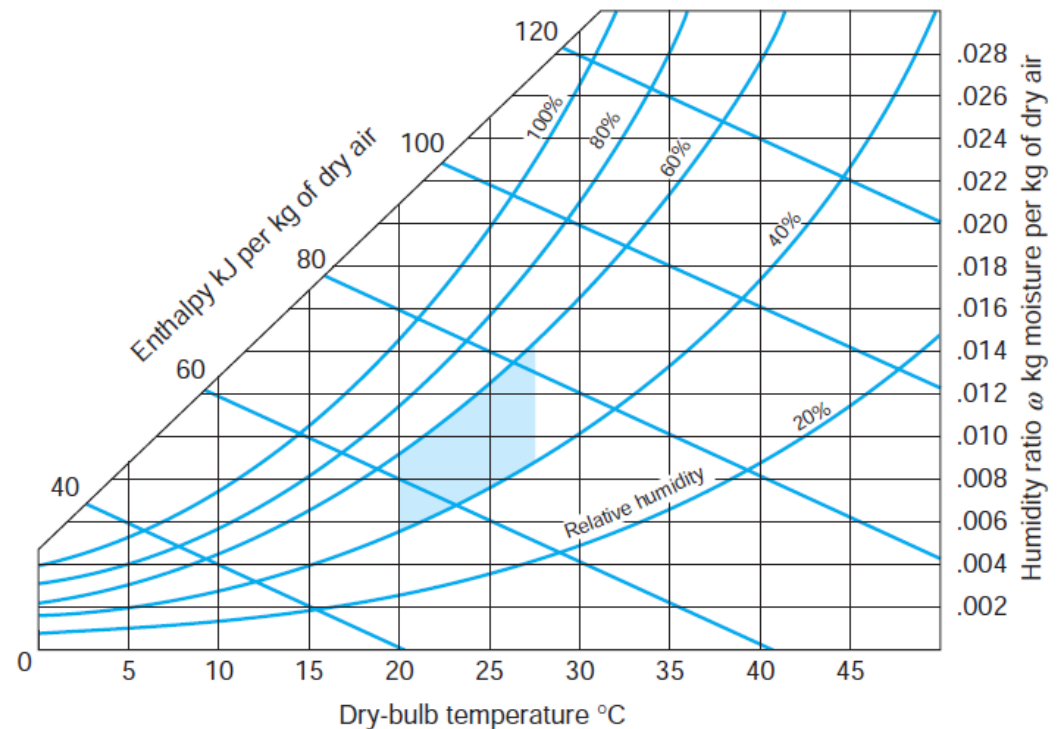
WET-BULB AND DRY-BULB TEMPERATURES AND THE PSYCHROMETRIC CHART

- The humidity of air–water vapor mixtures has traditionally been measured with a device called a psychrometer, which uses the flow of air past **wet-bulb** and **dry-bulb** thermometers.



WET-BULB AND DRY-BULB TEMPERATURES AND THE PSYCHROMETRIC CHART

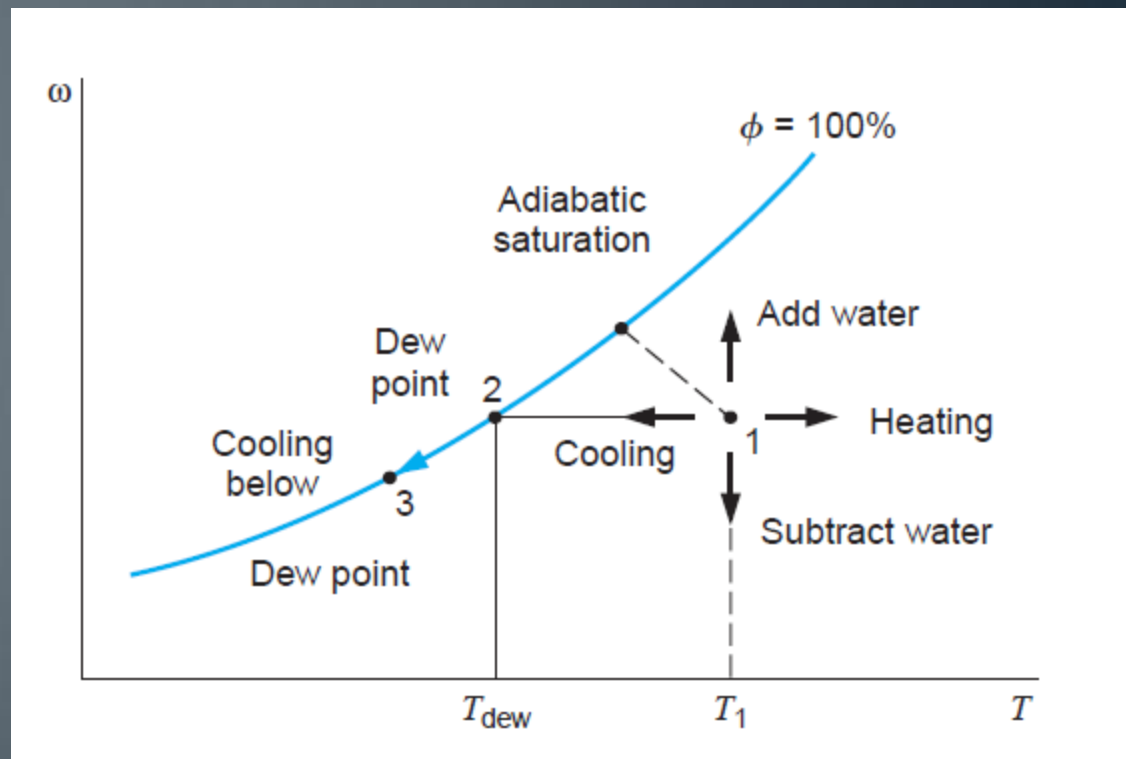
- Properties of air–water vapor mixtures are given in graphical form on psychrometric charts.



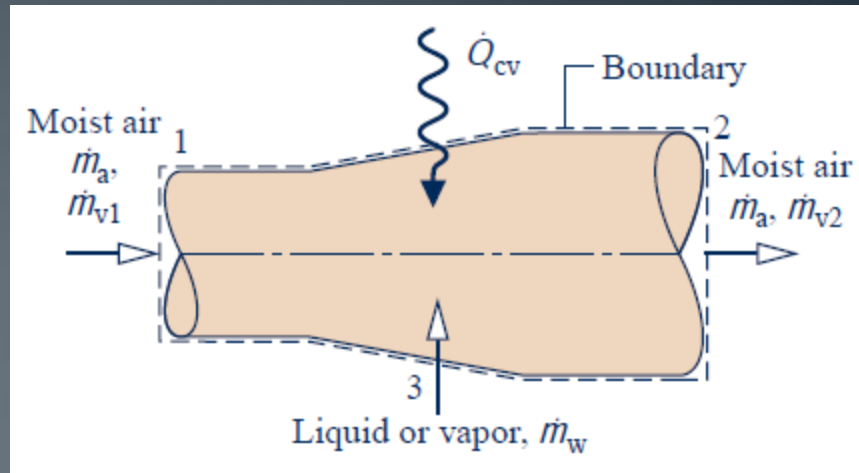
$$\tilde{h} \equiv h_a - h_a(-20^\circ\text{C}) + \omega h_v$$

WET-BULB
AND DRY-BULB
TEMPERATURES
AND
THE
PSYCHROMETRIC
CHART

- The direction in which various processes proceed for an air–water vapor mixture is shown below



APPLYING MASS AND ENERGY BALANCES TO AIR CONDITIONING SYSTEMS



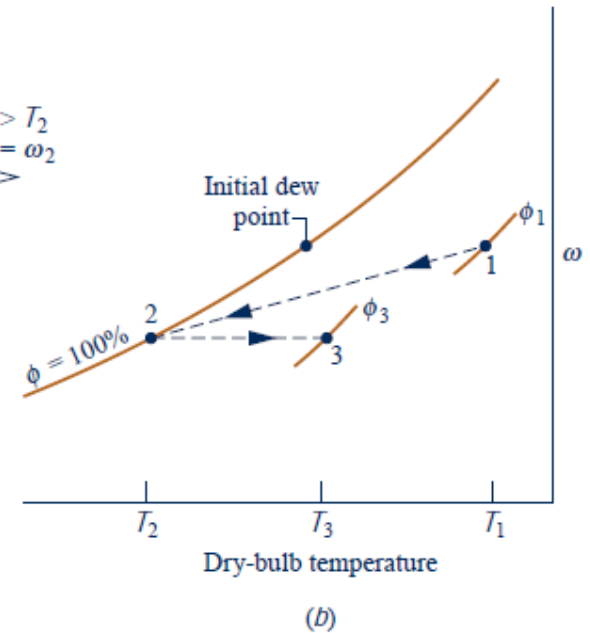
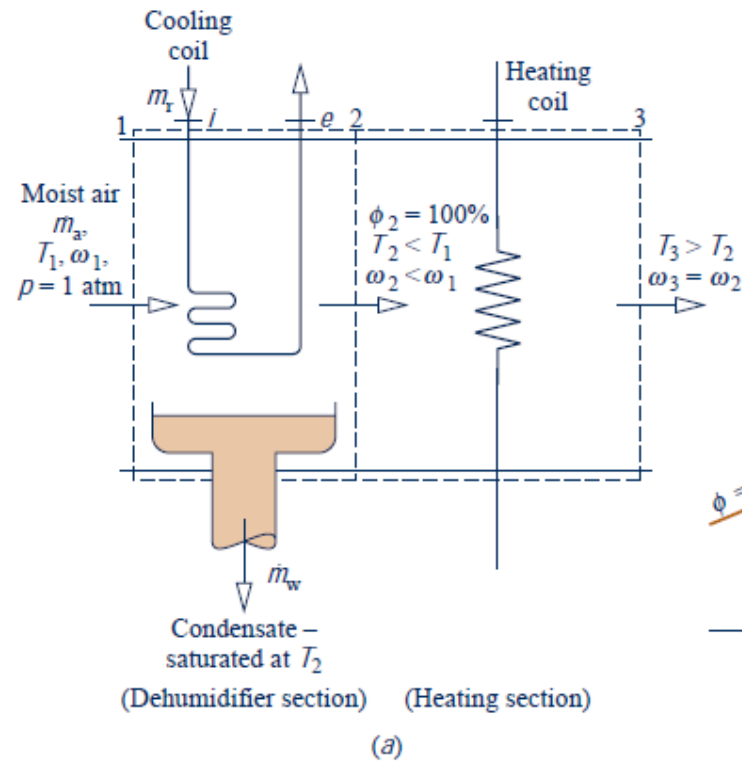
- **Mass Balance**

$$\dot{m}_w = \dot{m}_a(\omega_2 - \omega_1)$$

- **Energy Balance**

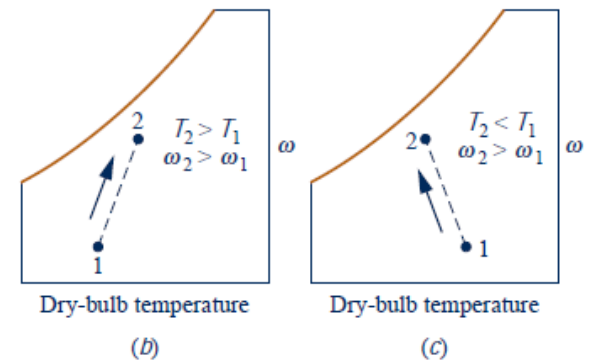
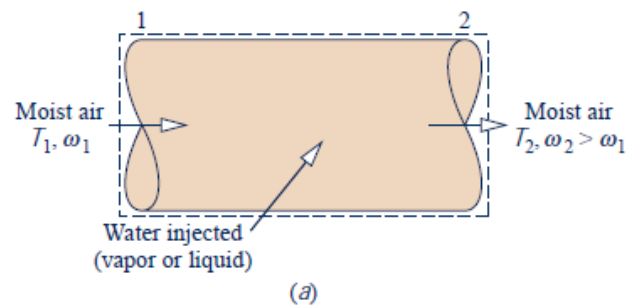
$$0 = \dot{Q}_{cv} + \dot{m}_a(h_{a1} + \omega_1 h_{g1}) + \dot{m}_w h_w - \dot{m}_a(h_{a2} + \omega_2 h_{g2})$$

DEHUMIDIFICATION



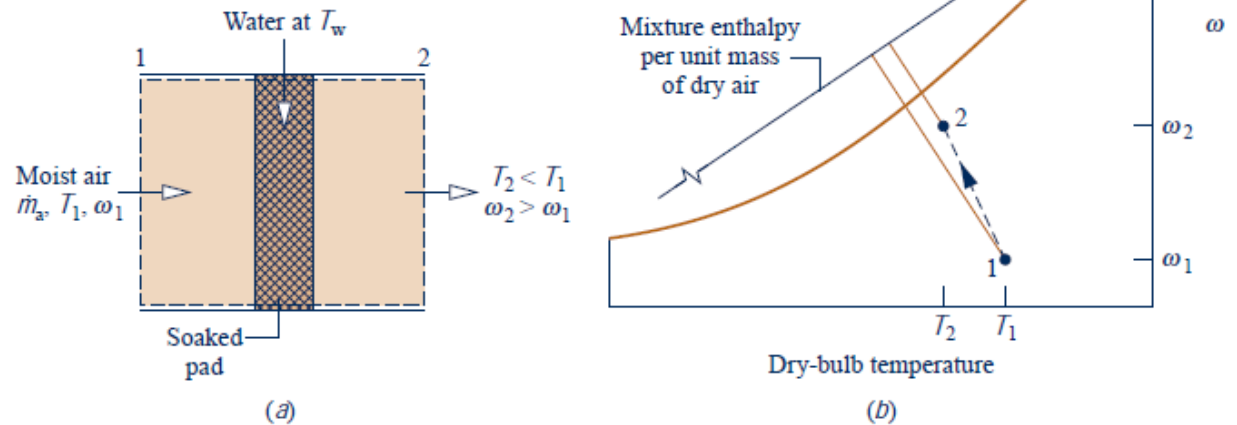
Dehumidification. (a) Equipment schematic. (b) Psychrometric chart representation.

HUMIDIFICATION



Humidification. (a) Control volume. (b) Steam injected. (c) Liquid injected.

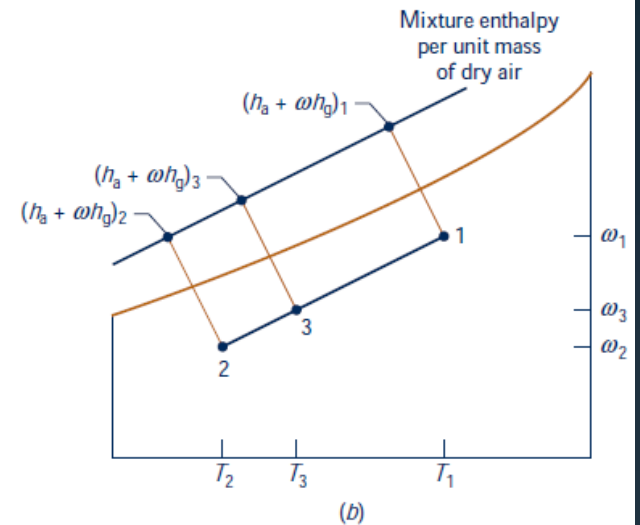
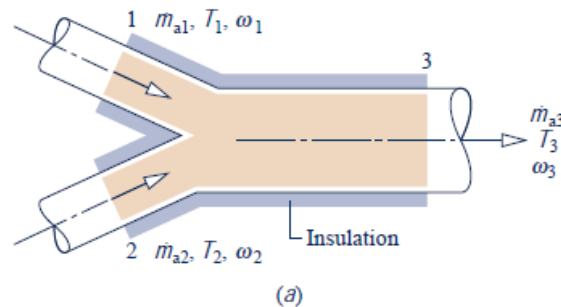
EVAPORATIVE COOLING



Evaporative cooling. (a) Equipment schematic. (b) Psychrometric chart representation.

$$(h_{a2} + \omega_2 h_{g2}) = \frac{(\omega_2 - \omega_1) h_f}{1} + (h_{a1} + \omega_1 h_{g1})$$

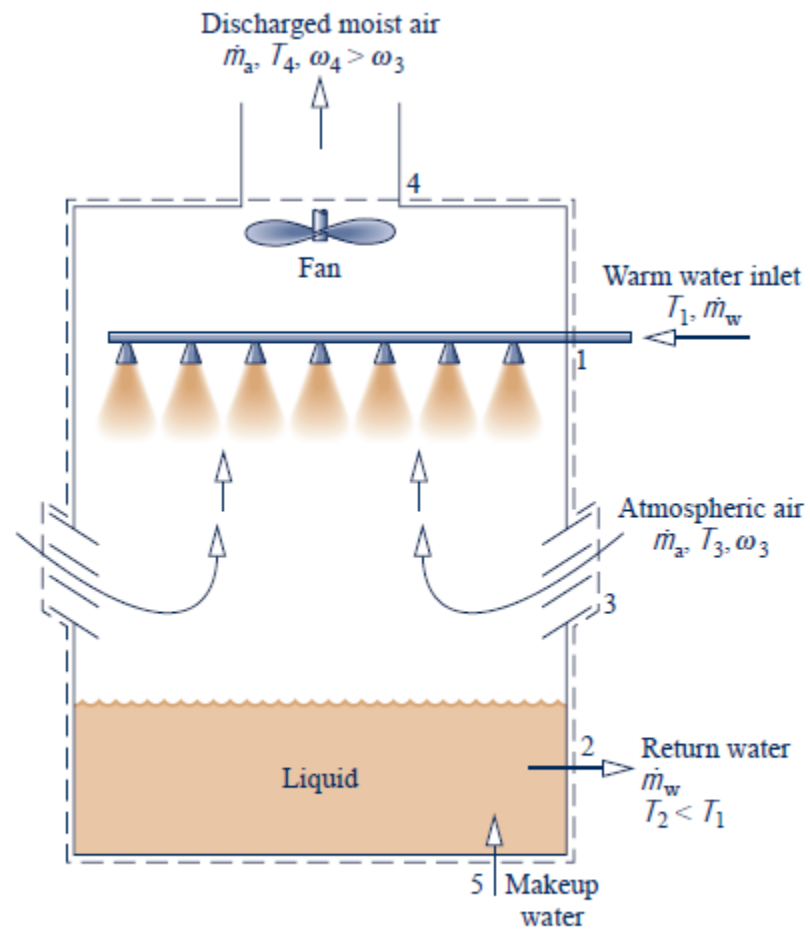
ADIABATIC MIXING OF TWO AIR STREAM



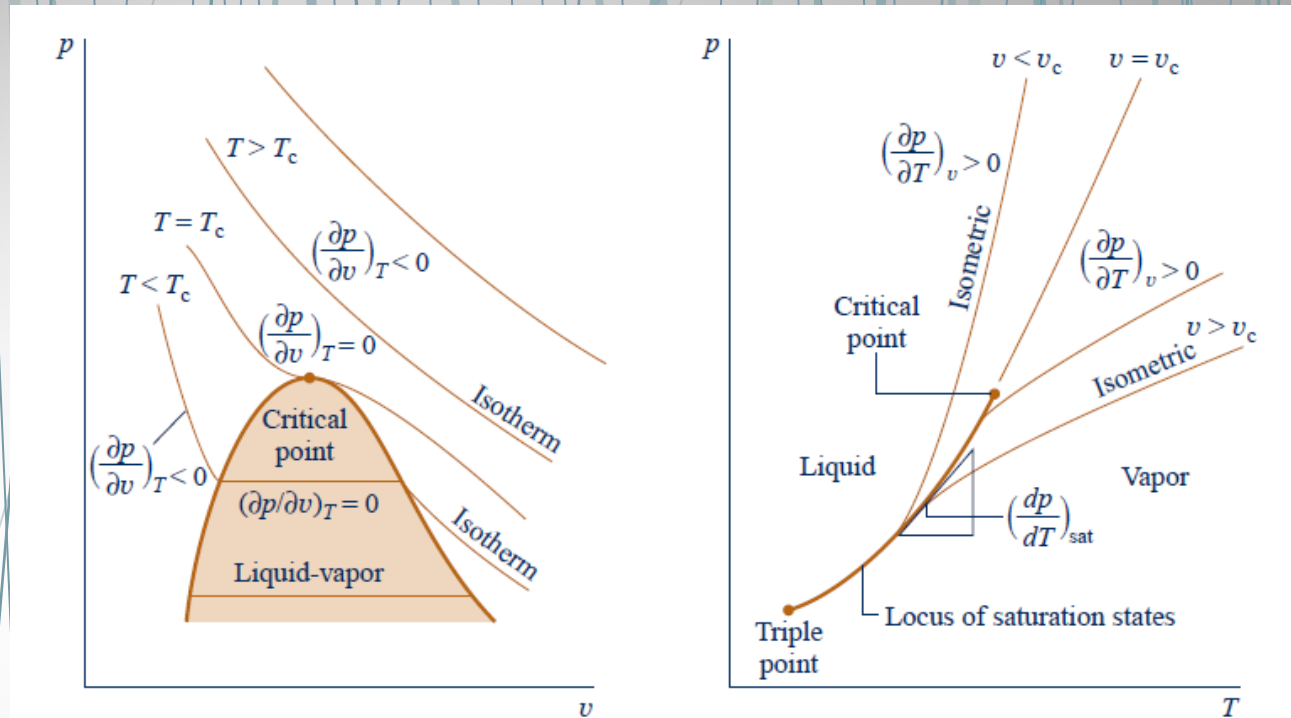
Adiabatic mixing of two moist air streams. (a) Equipment representation.
(b) Psychrometric chart representation.

$$\frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3} = \frac{(h_{a3} + \omega_3 h_{g3}) - (h_{a2} + \omega_2 h_{g2})}{(h_{a1} + \omega_1 h_{g1}) - (h_{a3} + \omega_3 h_{g3})}$$

COOLING TOWERS



Schematic of a cooling tower.



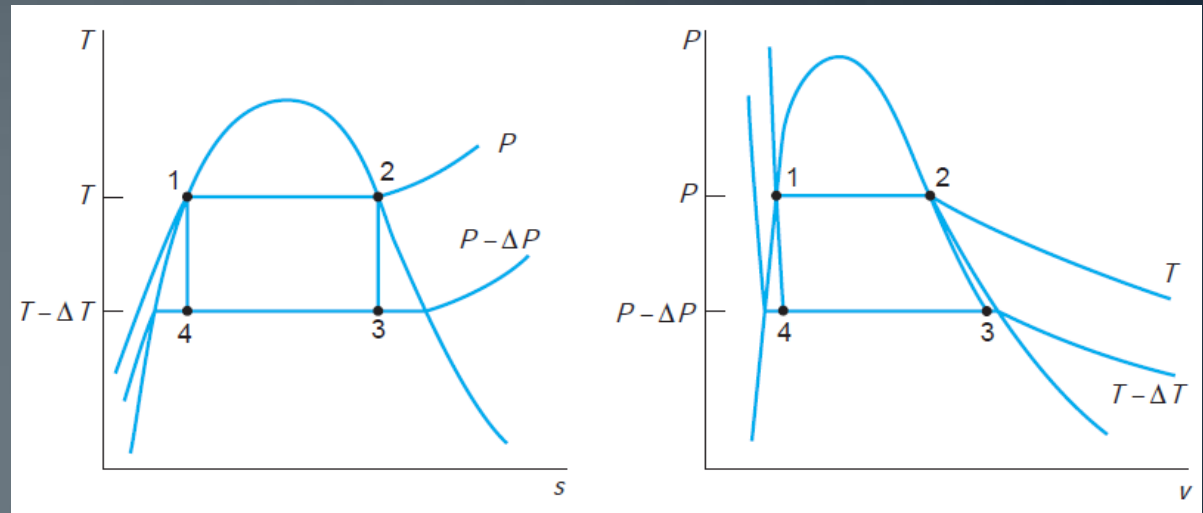
THERMODYNAMIC RELATIONS

THERMODYNAMIC RELATIONS

- Which thermodynamic properties can be experimentally measured?
- Some of the properties, such as internal energy and entropy, cannot be measured directly and must be calculated from other experimental data.
- If we carefully consider all these thermodynamic properties, we conclude that there are only four that can be directly measured: pressure, temperature, volume, and mass.
- How can values of the thermodynamic properties that cannot be measured be determined from experimental data on those properties that can be measured?
- How can values of the thermodynamic properties that cannot be measured be determined from experimental data on those properties that can be measured?

- Consider a Carnot-cycle heat engine operating across a small temperature difference.

THE CLAPEYRON EQUATION



$$q_H = T s_{fg}; \quad q_L = (T - \Delta T) s_{fg}$$

$$W_{NET} = q_H - q_L = \Delta T s_{fg}$$

THE CLAPEYRON EQUATION

$$W = - \int v \, dP$$

$$\begin{aligned} W_{NET} &= 0 - \int_2^3 v \, dP + 0 - \int_4^1 v \, dP \\ &\approx - \left(\frac{v_2 + v_3}{2} \right) (P - \Delta P - P) - \left(\frac{v_1 + v_4}{2} \right) (P - P + \Delta P) \\ &\approx \Delta P \left[\left(\frac{v_2 + v_3}{2} \right) - \left(\frac{v_1 + v_4}{2} \right) \right] \end{aligned}$$

$$\frac{\Delta P}{\Delta T} \approx \frac{s_{fg}}{\left(\frac{v_2 + v_3}{2} \right) - \left(\frac{v_1 + v_4}{2} \right)}$$

$$\lim_{\Delta T \rightarrow 0} \frac{\Delta P}{\Delta T} = \frac{dP_{\text{sat}}}{dT} = \frac{s_{fg}}{v_{fg}}$$

THE CLAPEYRON EQUATION

- Since the heat addition process 1 – 2 is at constant pressure as well as constant temperature,

$$q_H = h_{fg} = T s_{fg}$$

$$\frac{dP_{\text{sat}}}{dT} = \frac{s_{fg}}{v_{fg}} = \frac{h_{fg}}{T v_{fg}}$$

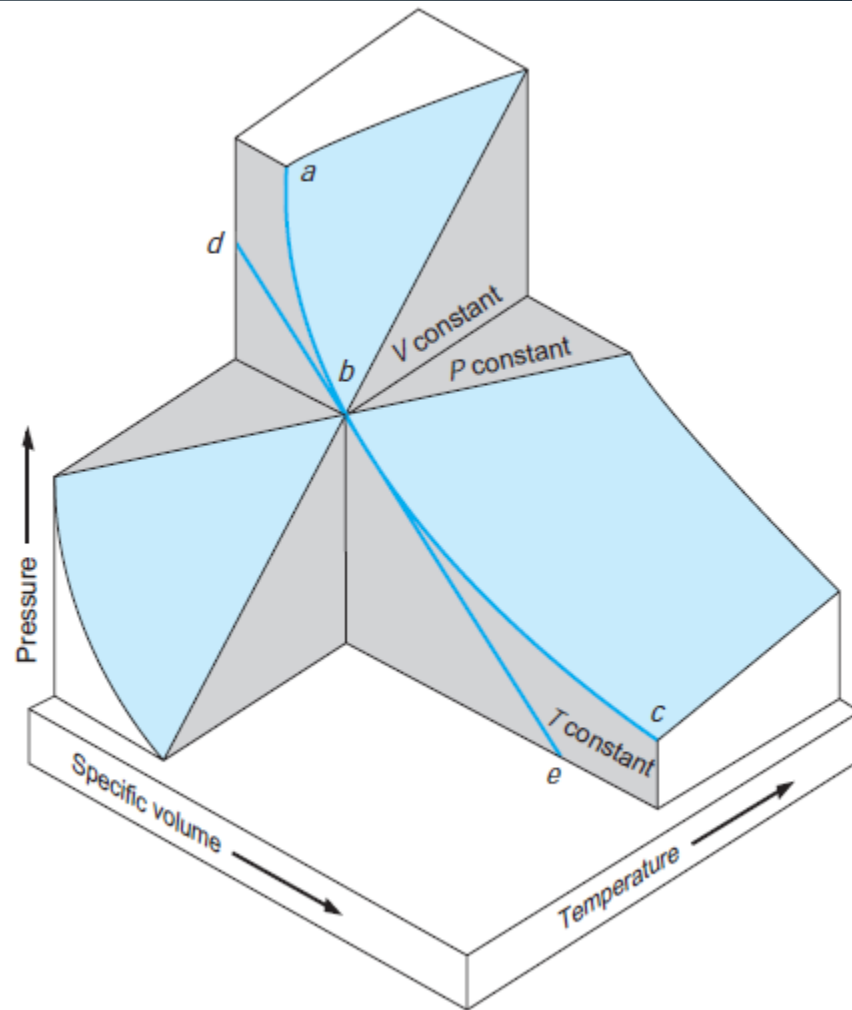
- which is called the **Clapeyron** equation,

$$\frac{dP_{\text{fus}}}{dT} = \frac{s_{if}}{v_{if}} = \frac{h_{if}}{T v_{if}}$$

$$\frac{dP_{\text{sub}}}{dT} = \frac{s_{ig}}{v_{ig}} = \frac{h_{ig}}{T v_{ig}}$$

$$\frac{dP_{\text{sat}}}{dT} = \frac{h_{fg}}{T v_{fg}} = \frac{h_{fg} P_{\text{sat}}}{RT^2}$$

MATHEMATICAL RELATIONS FOR A HOMOGENEOUS PHASE



MATHEMATICAL
RELATIONS FOR
A HOMOGENEOUS
PHASE

- Consider a variable (thermodynamic property) that is a continuous function of x and y .

$$z = f(x, y)$$

$$dz = \left(\frac{\partial z}{\partial x} \right)_y dx + \left(\frac{\partial z}{\partial y} \right)_x dy$$

$$dz = M dx + N dy$$

$$M = \left(\frac{\partial z}{\partial x} \right)_y$$

= partial derivative of z with respect to x (the variable y being held constant)

$$N = \left(\frac{\partial z}{\partial y} \right)_x$$

= partial derivative of z with respect to y (the variable x being held constant)

MATHEMATICAL
RELATIONS FOR
A HOMOGENEOUS
PHASE

- If x , y , and z are all point functions (that is, quantities that depend only on the state and are independent of the path), the differentials are exact differentials.

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial^2 z}{\partial x \partial y}$$

$$\left(\frac{\partial N}{\partial x}\right)_y = \frac{\partial^2 z}{\partial y \partial x}$$

MATHEMATICAL
RELATIONS FOR
A HOMOGENEOUS
PHASE

$$du = T ds - P dv$$

$$dh = T ds + v dP$$

$$u = u(s, v), \quad h = h(s, P)$$

- Helmholtz function A

$$A = U - TS, \quad a = u - Ts$$

$$\begin{aligned} da &= du - T ds - s dT \\ &= -s dT - P dv \end{aligned}$$

- Gibbs function G

$$G = H - TS, \quad g = h - Ts$$

$$\begin{aligned} dg &= dh - T ds - s dT \\ &= -s dT + v dP \end{aligned}$$

- Maxwell relations

THE MAXWELL RELATIONS

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

THE MAXWELL RELATIONS

$$\left(\frac{\partial u}{\partial s}\right)_v = T,$$

$$\left(\frac{\partial u}{\partial v}\right)_s = -P$$

$$\left(\frac{\partial h}{\partial s}\right)_P = T,$$

$$\left(\frac{\partial h}{\partial P}\right)_s = v$$

$$\left(\frac{\partial a}{\partial v}\right)_T = -P,$$

$$\left(\frac{\partial a}{\partial T}\right)_v = -s$$

$$\left(\frac{\partial g}{\partial P}\right)_T = v,$$

$$\left(\frac{\partial g}{\partial T}\right)_P = -s$$

Summary of Property Relations from Exact Differentials

Basic relations:

from $u = u(s, v)$

$$T = \left(\frac{\partial u}{\partial s} \right)_v \quad (11.24)$$

$$-p = \left(\frac{\partial u}{\partial v} \right)_s \quad (11.25)$$

from $h = h(s, p)$

$$T = \left(\frac{\partial h}{\partial s} \right)_p$$

$$v = \left(\frac{\partial h}{\partial p} \right)_s$$

from $\psi = \psi(v, T)$

$$-p = \left(\frac{\partial \psi}{\partial v} \right)_T \quad (11.28)$$

$$-s = \left(\frac{\partial \psi}{\partial T} \right)_v \quad (11.29)$$

from $g = g(T, p)$

$$v = \left(\frac{\partial g}{\partial p} \right)_T$$

$$-s = \left(\frac{\partial g}{\partial T} \right)_p$$

Maxwell relations:

$$\left(\frac{\partial T}{\partial v} \right)_s = - \left(\frac{\partial p}{\partial s} \right)_v \quad (11.32)$$

$$\left(\frac{\partial T}{\partial p} \right)_s = \left(\frac{\partial v}{\partial s} \right)_p \quad (11.33)$$

$$\left(\frac{\partial p}{\partial T} \right)_v = \left(\frac{\partial s}{\partial v} \right)_T$$

$$\left(\frac{\partial v}{\partial T} \right)_p = - \left(\frac{\partial s}{\partial p} \right)_T$$

Additional relations:

$$\left(\frac{\partial u}{\partial s} \right)_v = \left(\frac{\partial h}{\partial s} \right)_p$$

$$\left(\frac{\partial h}{\partial p} \right)_s = \left(\frac{\partial g}{\partial p} \right)_T$$

$$\left(\frac{\partial u}{\partial v} \right)_s = \left(\frac{\partial \psi}{\partial v} \right)_T$$

$$\left(\frac{\partial \psi}{\partial T} \right)_v = \left(\frac{\partial g}{\partial T} \right)_p$$

THERMODYNAMIC
RELATIONS
INVOLVING
ENTHALPY,
INTERNAL ENERGY,
AND ENTROPY

$$C_p \equiv \left(\frac{\partial h}{\partial T} \right)_p$$

$$T ds = dh - v dP$$

$$C_p = \left(\frac{\partial h}{\partial T} \right)_p = T \left(\frac{\partial s}{\partial T} \right)_p$$

$$C_v \equiv \left(\frac{\partial u}{\partial T} \right)_v$$

$$T ds = du + P dv$$

$$C_v = \left(\frac{\partial u}{\partial T} \right)_v = T \left(\frac{\partial s}{\partial T} \right)_v$$

THERMODYNAMIC
RELATIONS
INVOLVING
ENTHALPY,
INTERNAL ENERGY,
AND ENTROPY

$$h = h(T, P)$$

$$dh = \left(\frac{\partial h}{\partial T} \right)_P dT + \left(\frac{\partial h}{\partial P} \right)_T dP$$

$$T ds = dh - v dP$$

$$\left(\frac{\partial h}{\partial P} \right)_T = v + T \left(\frac{\partial s}{\partial P} \right)_T$$

$$\left(\frac{\partial h}{\partial P} \right)_T = v - T \left(\frac{\partial v}{\partial T} \right)_P$$

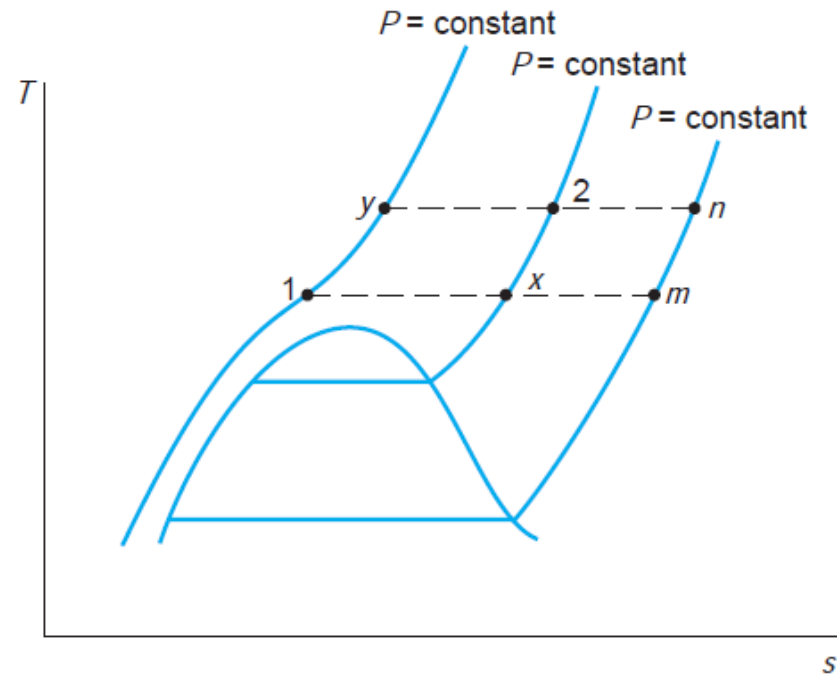
$$dh = C_p dT + \left[v - T \left(\frac{\partial v}{\partial T} \right)_P \right] dP$$

THERMODYNAMIC
RELATIONS
INVOLVING
ENTHALPY,
INTERNAL ENERGY,
AND ENTROPY

$$dh_p = C_p dT_p$$

$$dh_T = \left[v - T \left(\frac{\partial v}{\partial T} \right)_P \right] dP_T$$

$$h_2 - h_1 = \int_1^2 C_p dT + \int_1^2 \left[v - T \left(\frac{\partial v}{\partial T} \right)_P \right] dP$$



THERMODYNAMIC
RELATIONS
INVOLVING
ENTHALPY,
INTERNAL ENERGY,
AND ENTROPY

$$u = f(T, v)$$

$$du = \left(\frac{\partial u}{\partial T} \right)_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv$$

$$T ds = du + P dv$$

$$\left(\frac{\partial u}{\partial v} \right)_T = T \left(\frac{\partial s}{\partial v} \right)_T - P$$

$$\left(\frac{\partial u}{\partial v} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_v - P$$

$$du = C_v dT + \left[T \left(\frac{\partial P}{\partial T} \right)_v - P \right] dv$$

$$du_v = C_v dT_v$$

$$du_T = \left[T \left(\frac{\partial P}{\partial T} \right)_v - P \right] dv_T$$

THERMODYNAMIC
RELATIONS
INVOLVING
ENTHALPY,
INTERNAL ENERGY,
AND ENTROPY

$$s = s(T, P)$$

$$ds = \left(\frac{\partial s}{\partial T} \right)_P dT + \left(\frac{\partial s}{\partial P} \right)_T dP$$

$$ds = C_p \frac{dT}{T} - \left(\frac{\partial v}{\partial T} \right)_P dP$$

$$s_2 - s_1 = \int_1^2 C_p \frac{dT}{T} - \int_1^2 \left(\frac{\partial v}{\partial T} \right)_P dP$$

$$(s_2 - s_1)_P = \int_1^2 C_p \frac{dT_P}{T}$$

$$(s_2 - s_1)_T = - \int_1^2 \left(\frac{\partial v}{\partial T} \right)_P dP$$

THERMODYNAMIC
RELATIONS
INVOLVING
ENTHALPY,
INTERNAL ENERGY,
AND ENTROPY

$$s = s(T, v)$$

$$ds = \left(\frac{\partial s}{\partial T} \right)_v dT + \left(\frac{\partial s}{\partial v} \right)_T dv$$

$$ds = C_v \frac{dT}{T} + \left(\frac{\partial P}{\partial T} \right)_v dv$$

$$s_2 - s_1 = \int_1^2 C_v \frac{dT}{T} + \int_1^2 \left(\frac{\partial P}{\partial T} \right)_v dv$$

VOLUME
EXPANSIVITY AND
ISOTHERMAL AND
ADIABATIC
COMPRESSIBILITY

- Coefficient of linear expansion

$$\delta_T = \frac{1}{L} \left(\frac{\delta L}{\delta T} \right)_P$$

- Volume expansivity

$$\alpha_P \equiv \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P = 3\delta_T$$

- Isothermal compressibility

$$\beta_T \equiv -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T = -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_T$$

- Adiabatic compressibility

$$\beta_s \equiv -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_s$$

- Velocity of sound

$$c^2 = \left(\frac{\partial P}{\partial \rho} \right)_s$$

14.35 The Joule–Thomson coefficient μ_J is a measure of the direction and magnitude of the temperature change with pressure in a throttling process. For any three properties x, y, z , use the mathematical relation

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

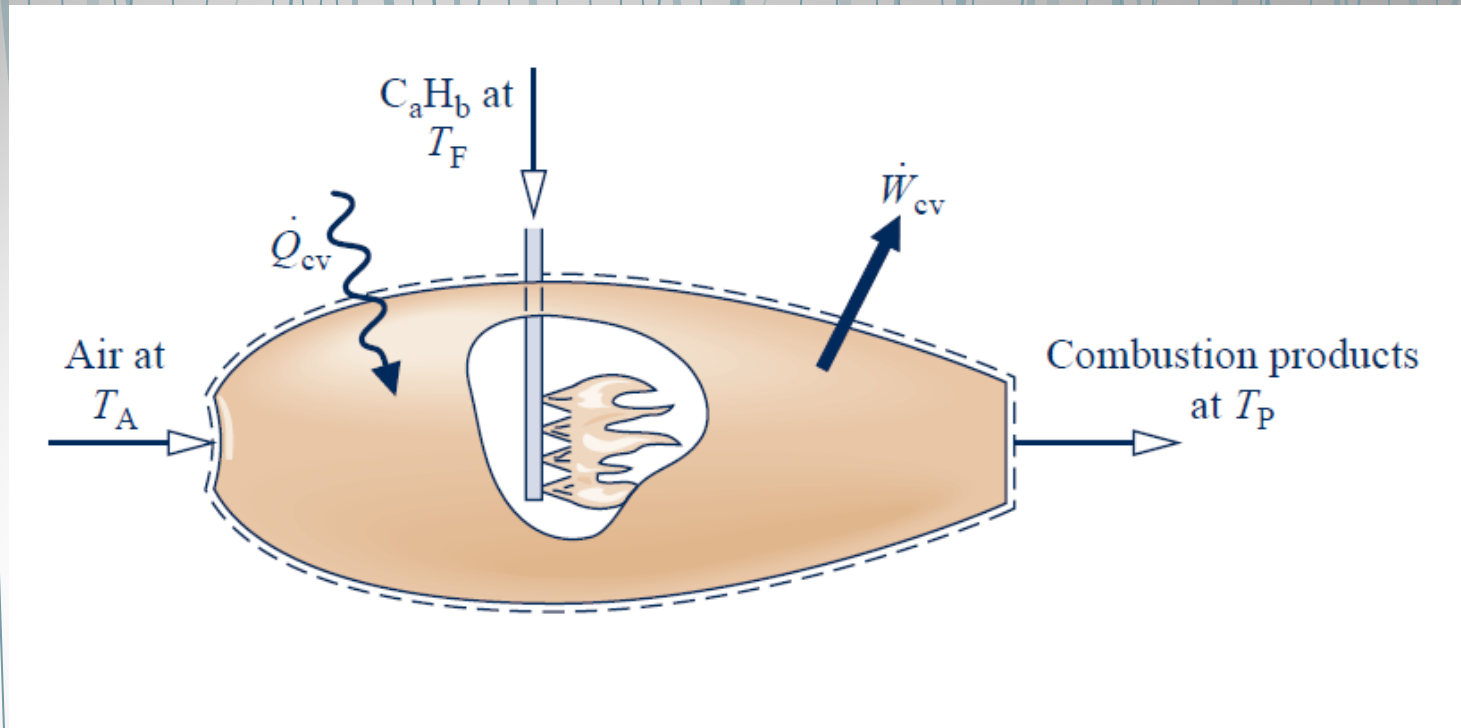
to show the following relations for the Joule–Thomson coefficient:

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T\left(\frac{\partial V}{\partial T}\right)_P - V}{C_P} = \frac{RT^2}{PC_P} \left(\frac{\partial Z}{\partial T}\right)_P$$

$$c_p - c_v = -T \left(\frac{\partial v}{\partial T} \right)_p^2 \left(\frac{\partial p}{\partial v} \right)_T$$

$$k = \frac{c_p}{c_v} = \left(\frac{\partial v}{\partial p} \right)_T \left(\frac{\partial p}{\partial v} \right)_s$$

$$c = \sqrt{-k v^2 (\partial p / \partial v)_T}$$



CHEMICAL REACTIONS

THE COMBUSTION PROCESS

- The combustion process consists of the oxidation of constituents in the fuel that are capable of being oxidized and can therefore be represented by a chemical equation.



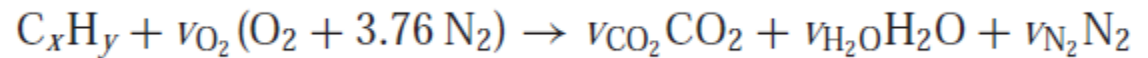
- The assumption that air is 21.0% oxygen and 79.0% nitrogen by volume leads to the conclusion that for each mole of oxygen, $79.0/21.0 = 3.76$ moles of nitrogen are involved.



- The minimum amount of air that supplies sufficient oxygen for the **complete combustion** of all the carbon, hydrogen, and any other elements in the fuel that may oxidize is called the **theoretical air**.

THE COMBUSTION PROCESS

- When complete combustion is achieved with theoretical air, the products contain no oxygen. A general combustion reaction with a hydrocarbon fuel and air is thus written



- with the coefficients to the substances called **stoichiometric coefficients**. The balance of atoms yields the theoretical amount of air as

$$\begin{aligned}\text{C:} \quad & \nu_{\text{CO}_2} = x \\ \text{H:} \quad & 2\nu_{\text{H}_2\text{O}} = y \\ \text{N}_2: \quad & \nu_{\text{N}_2} = 3.76 \times \nu_{\text{O}_2} \\ \text{O}_2: \quad & \nu_{\text{O}_2} = \nu_{\text{CO}_2} + \nu_{\text{H}_2\text{O}}/2 = x + y/4\end{aligned}$$

- and the total number of moles of air for 1 mole of fuel becomes

$$n_{\text{air}} = \nu_{\text{O}_2} \times 4.76 = 4.76(x + y/4)$$

THE COMBUSTION PROCESS

- Two important parameters often used to express the ratio of fuel and air are the **air–fuel ratio** (designated AF) and its reciprocal, the **fuel–air ratio** (designated FA).

$$AF_{\text{mass}} = \frac{m_{\text{air}}}{m_{\text{fuel}}}$$

$$AF_{\text{mole}} = \frac{n_{\text{air}}}{n_{\text{fuel}}}$$

- They are related through the molecular masses as

$$AF_{\text{mass}} = \frac{m_{\text{air}}}{m_{\text{fuel}}} = \frac{n_{\text{air}} M_{\text{air}}}{n_{\text{fuel}} M_{\text{fuel}}} = AF_{\text{mole}} \frac{M_{\text{air}}}{M_{\text{fuel}}}$$

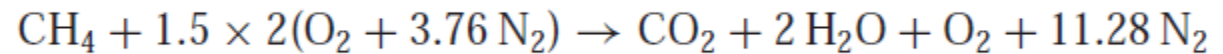
- and a subscript s is used to indicate the ratio for 100% theoretical air, also called a stoichiometric mixture. In an actual combustion process, an amount of air is expressed as a fraction of the theoretical amount, called **percent theoretical air**.

THE COMBUSTION PROCESS

- A similar ratio named the **equivalence ratio** equals the actual fuel–air ratio divided by the theoretical fuel–air ratio as

$$\Phi = FA/FA_s = AF_s/AF$$

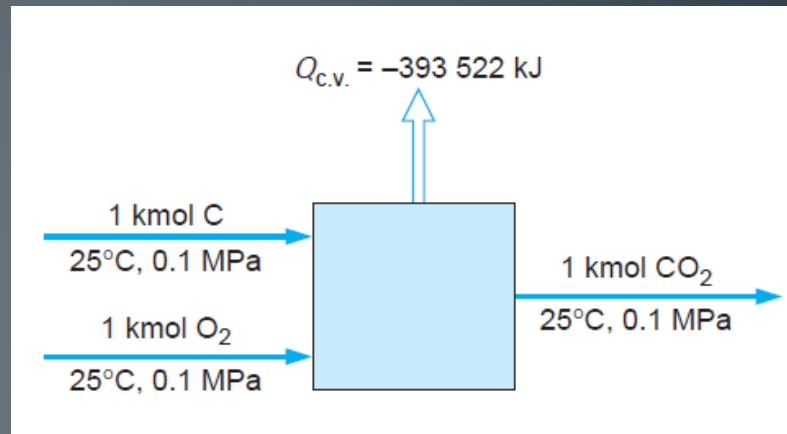
- Thus, 150% theoretical air means that the air actually supplied is 1.5 times the theoretical air and the equivalence ratio is 2/3. The complete combustion of methane with 150% theoretical air is written



- The amount of air actually supplied may also be expressed in terms of percent excess air. The excess air is the amount of air supplied over and above the theoretical air. Thus, 150% theoretical air is equivalent to 50% excess air.

ENTHALPY OF FORMATION

- Consider the simple steady-state combustion process shown below. The carbon dioxide (ideal gas) formed by the reaction leaves the chamber at the reference state, 25°C and 0.1 MPa.



- The chemical reaction can be written

$$Q_{c.v} + H_R = H_P$$

- Applying the first law to this process, we have

$$Q_{c.v.} + \sum_R n_i \bar{h}_i = \sum_P n_e \bar{h}_e$$

ENTHALPY OF FORMATION

- Suppose, however, that we assign the value of zero to the enthalpy of all the elements at the reference state. In this case, the enthalpy of the reactants is zero, and

$$Q_{\text{c.v.}} = H_p = -393\,522 \text{ kJ/kmol}$$

- The enthalpy of (hypothetical) ideal-gas carbon dioxide at 25°C, 0.1 MPa pressure (with reference to this arbitrary base in which the enthalpy of the elements is chosen to be zero), is called the **enthalpy of formation**.

$$\bar{h}_f^0 = -393\,522 \text{ kJ/kmol}$$

- The enthalpy of carbon dioxide in any other state, relative to this base in which the enthalpy of the elements is zero, would be found by adding the change of enthalpy between ideal gas at 25°C, 0.1 MPa, and the given state to the enthalpy of formation.

$$\bar{h}_{T,P} = (\bar{h}_f^0)_{298,0.1 \text{ MPa}} + (\Delta \bar{h})_{298,0.1 \text{ MPa} \rightarrow T,P}$$

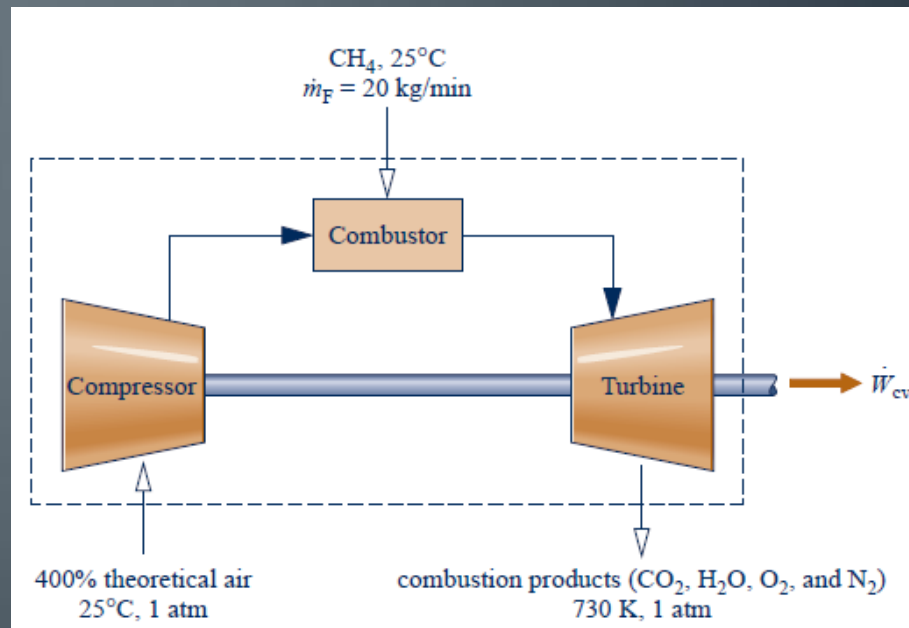
FIRST-LAW ANALYSIS OF REACTING SYSTEMS

- We can write the first law for a steady-state, steady-flow process in the form

$$Q_{c.v.} + H_R = W_{c.v.} + H_P$$

- or

$$Q_{c.v.} + \sum_R n_i \bar{h}_i = W_{c.v.} + \sum_P n_e \bar{h}_e$$



ENTHALPY AND INTERNAL ENERGY OF COMBUSTION; HEAT OF REACTION

- The **enthalpy of combustion** is defined as the difference between the enthalpy of the products and the enthalpy of the reactants when complete combustion occurs at a given temperature and pressure.

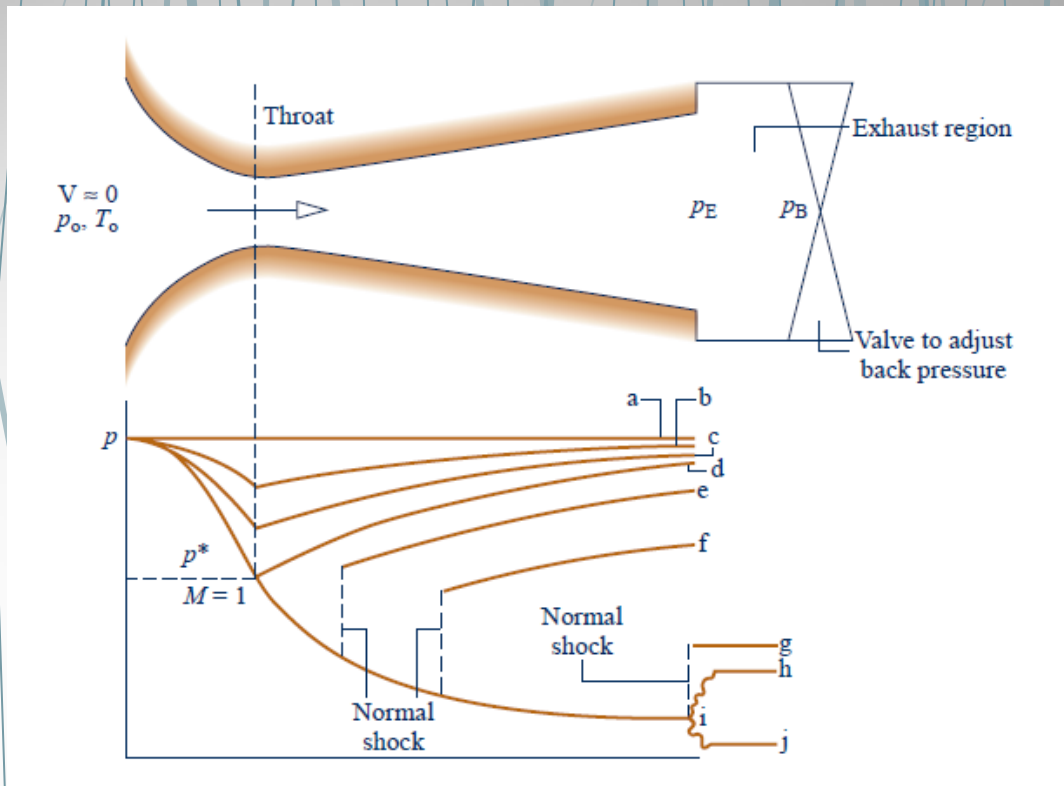
$$\bar{h}_{RP} = H_P - H_R$$

$$\bar{h}_{RP} = \sum_P e_n (\bar{h}_f^0 + \Delta \bar{h})_e - \sum_R n_i (\bar{h}_f^0 + \Delta \bar{h})_i$$

- Frequently the term **heating value** or **heat of reaction** is used. This represents the heat transferred from the chamber during combustion or reaction at constant temperature. In the case of a constant pressure or steady-flow process, we conclude from the first law of thermodynamics that it is equal to the negative of the enthalpy of combustion.
- The **higher heating value** is the heat transfer with liquid water in the products, and the **lower heating value** is the heat transfer with vapor water in the products.

ADIABATIC FLAME TEMPERATURE

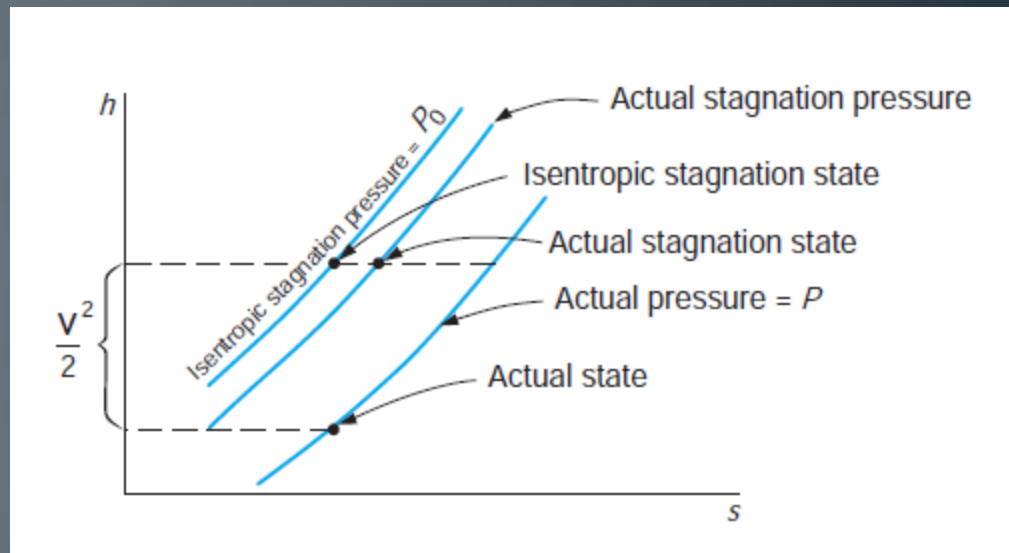
- Consider a given combustion process that takes place adiabatically and with no work or changes in kinetic or potential energy involved. For such a process the temperature of the products is referred to as the **adiabatic flame temperature**.
- With the assumptions of no work and no changes in kinetic or potential energy, this is the **maximum temperature** that can be achieved for the given reactants because any heat transfer from the reacting substances and any incomplete combustion would tend to lower the temperature of the products.
- For a given fuel and given pressure and temperature of the reactants, the maximum adiabatic flame temperature that can be achieved is with a **stoichiometric mixture**.
- The adiabatic
- flame temperature can be controlled by the amount of **excess air** that is used.



COMPRESSIBLE FLOW

STAGNATION PROPERTIES

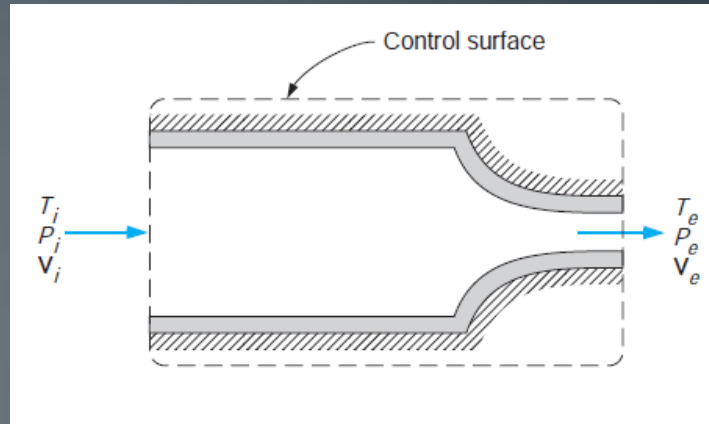
- The isentropic stagnation state is the state a flowing fluid would attain if it underwent a reversible adiabatic deceleration to zero velocity.



$$h + \frac{V^2}{2} = h_0$$

ADIABATIC, ONE-
DIMENSIONAL,
STEADY-STATE
FLOW OF AN
INCOMPRESSIBLE
FLUID THROUGH
A NOZZLE

- Consider the nozzle shown below, and assume an adiabatic, one dimensional, steady-state process of an incompressible fluid.



$$\dot{m}_e = \dot{m}_i = \rho A_i \mathbf{V}_i = \rho A_e \mathbf{V}_e$$

$$h_e - h_i + \frac{\mathbf{V}_e^2 - \mathbf{V}_i^2}{2} + (Z_e - Z_i)g = 0$$

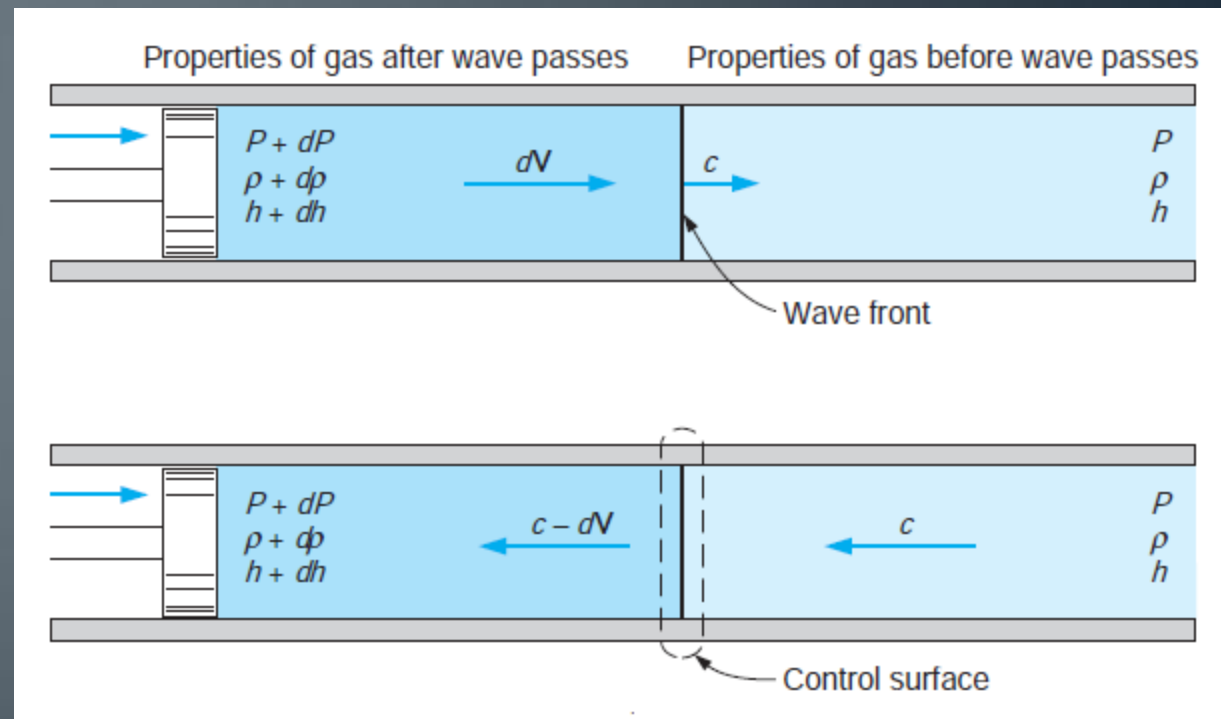
$$T ds = dh - v dP$$

$$h_e - h_i = \int_i^e v dP$$

$$v(P_e - P_i) + \frac{\mathbf{V}_e^2 - \mathbf{V}_i^2}{2} + (Z_e - Z_i)g = 0$$

VELOCITY OF SOUND IN AN IDEAL GAS

- When a pressure disturbance occurs in a compressible fluid, the disturbance travels with a velocity that depends on the state of the fluid. A sound wave is a very small pressure disturbance.



VELOCITY OF SOUND IN AN IDEAL GAS

$$h + \frac{c^2}{2} = (h + dh) + \frac{(c - d\mathbf{V})^2}{2}$$

$$dh - c d\mathbf{V} = 0$$

$$\rho A c = (\rho + d\rho) A (c - d\mathbf{V})$$

$$c d\rho - \rho d\mathbf{V} = 0$$

$$T ds = dh - \frac{dP}{\rho}$$

$$\frac{dP}{\rho} - c d\mathbf{V} = 0$$

$$\frac{dP}{d\rho} = c^2$$

$$\left(\frac{\partial P}{\partial \rho} \right)_s = c^2$$

VELOCITY OF SOUND IN AN IDEAL GAS

$$\left(\frac{\partial P}{\partial \rho}\right)_s = c^2$$

$$\frac{dP}{\rho} - k \frac{d\rho}{\rho} = 0$$

$$\left(\frac{\partial P}{\partial \rho}\right)_s = \frac{kP}{\rho}$$

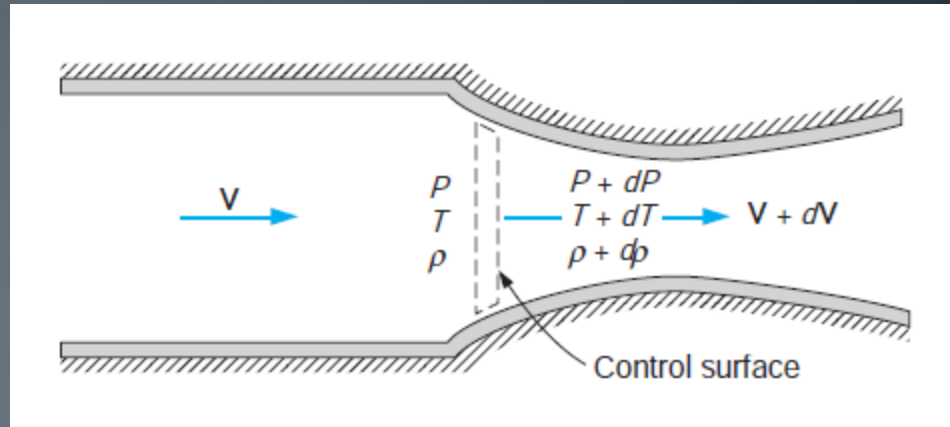
$$c^2 = \frac{kP}{\rho}$$

$$\frac{P}{\rho} = RT$$

$$c^2 = kRT$$

REVERSIBLE,
ADIABATIC, ONE-
DIMENSIONAL
FLOW OF AN
IDEAL GAS
THROUGH A
NOZZLE

- A nozzle or diffuser with both a converging and diverging section is shown below. The minimum cross-sectional area is called the **throat**.



- First law:

$$dh + \mathbf{V} d\mathbf{V} = 0$$

- Property relation:

$$T ds = dh - \frac{dP}{\rho} = 0$$

- Continuity equation:

$$\rho A \mathbf{V} = \dot{m} = \text{constant}$$
$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{d\mathbf{V}}{\mathbf{V}} = 0$$

REVERSIBLE,
ADIABATIC, ONE-
DIMENSIONAL
FLOW OF AN
IDEAL GAS
THROUGH A
NOZZLE

$$dh = \frac{dP}{\rho} = -\mathbf{V} d\mathbf{V}$$

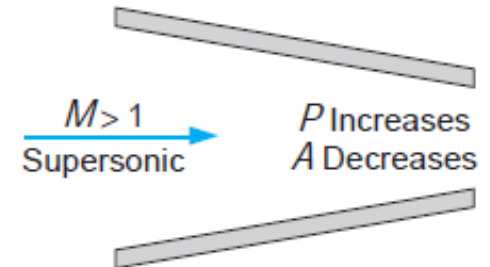
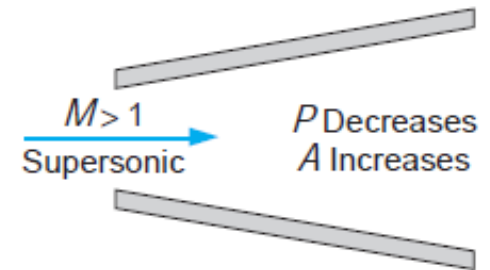
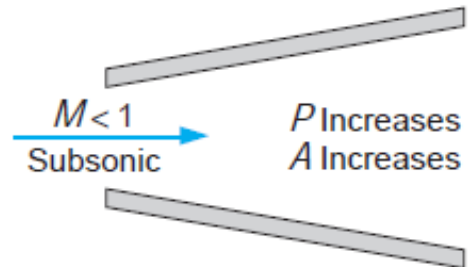
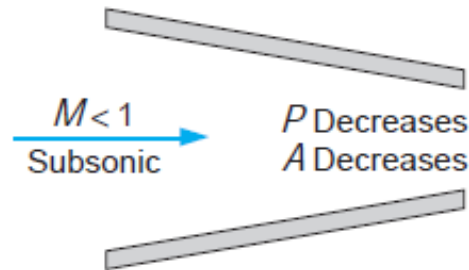
$$d\mathbf{V} = -\frac{1}{\rho\mathbf{V}} dP$$

$$\begin{aligned}\frac{dA}{A} &= \left(-\frac{d\rho}{\rho} - \frac{d\mathbf{V}}{\mathbf{V}} \right) = -\frac{d\rho}{\rho} \left(\frac{dP}{d\rho} \right) + \frac{1}{\rho\mathbf{V}^2} dP \\ &= \frac{-dP}{\rho} \left(\frac{d\rho}{dP} - \frac{1}{\mathbf{V}^2} \right) = \frac{dP}{\rho} \left(-\frac{1}{(dP/d\rho)} + \frac{1}{\mathbf{V}^2} \right)\end{aligned}$$

$$\frac{dP}{d\rho} = c^2 = \frac{\mathbf{V}^2}{M^2}$$

$$\frac{dA}{A} = \frac{dP}{\rho\mathbf{V}^2} (1 - M^2)$$

REVERSIBLE,
ADIABATIC, ONE-
DIMENSIONAL
FLOW OF AN
IDEAL GAS
THROUGH A
NOZZLE



REVERSIBLE,
ADIABATIC, ONE-
DIMENSIONAL
FLOW OF AN
IDEAL GAS
THROUGH A
NOZZLE

$$h + \frac{\mathbf{v}^2}{2} = h_0$$

$$\mathbf{v}^2 = 2C_{p0}(T_0 - T) = 2\frac{kRT}{k-1}\left(\frac{T_0}{T} - 1\right)$$

$$c^2 = kRT$$

$$\mathbf{v}^2 = \frac{2c^2}{k-1}\left(\frac{T_0}{T} - 1\right)$$

$$\frac{\mathbf{v}^2}{c^2} = M^2 = \frac{2}{k-1}\left(\frac{T_0}{T} - 1\right)$$

$$\frac{T_0}{T} = 1 + \frac{(k-1)}{2}M^2$$

$$\left(\frac{T_0}{T}\right)^{k/(k-1)} = \frac{P_0}{P} \quad \left(\frac{T_0}{T}\right)^{1/(k-1)} = \frac{\rho_0}{\rho}$$

$$\frac{P_0}{P} = \left[1 + \frac{(k-1)}{2}M^2\right]^{k/(k-1)}$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{(k-1)}{2}M^2\right]^{1/(k-1)}$$

REVERSIBLE,
ADIABATIC, ONE-
DIMENSIONAL
FLOW OF AN
IDEAL GAS
THROUGH A
NOZZLE

- These properties at the throat of a nozzle when $M = 1$ are frequently referred to as critical pressure, critical temperature, and critical density.

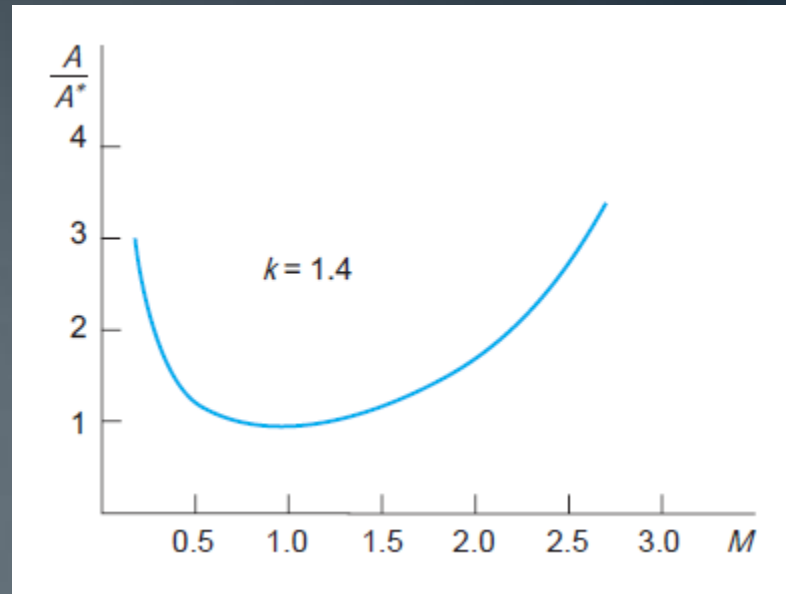
$$\begin{aligned}\frac{T^*}{T_0} &= \frac{2}{k+1} \\ \frac{P^*}{P_0} &= \left(\frac{2}{k+1}\right)^{k/(k-1)} \\ \frac{\rho^*}{\rho_0} &= \left(\frac{2}{k+1}\right)^{1/(k-1)}\end{aligned}$$

MASS RATE OF
FLOW OF AN
IDEAL GAS
THROUGH AN
ISENTROPIC
NOZZLE

$$\begin{aligned}\frac{\dot{m}}{A} &= \rho \mathbf{V} = \frac{P \mathbf{V}}{RT} \sqrt{\frac{k T_0}{k T_0}} \\ &= \frac{P \mathbf{V}}{\sqrt{k R T}} \sqrt{\frac{k}{R}} \sqrt{\frac{T_0}{T}} \sqrt{\frac{1}{T_0}} \\ &= \frac{P M}{\sqrt{T_0}} \sqrt{\frac{k}{R}} \sqrt{1 + \frac{k-1}{2} M^2}\end{aligned}$$

$$\frac{\dot{m}}{A} = \frac{P_0}{\sqrt{T_0}} \sqrt{\frac{k}{R}} \times \frac{M}{\left(1 + \frac{k-1}{2} M^2\right)^{(k+1)/2(k-1)}}$$

MASS RATE OF FLOW OF AN IDEAL GAS THROUGH AN ISENTROPIC NOZZLE

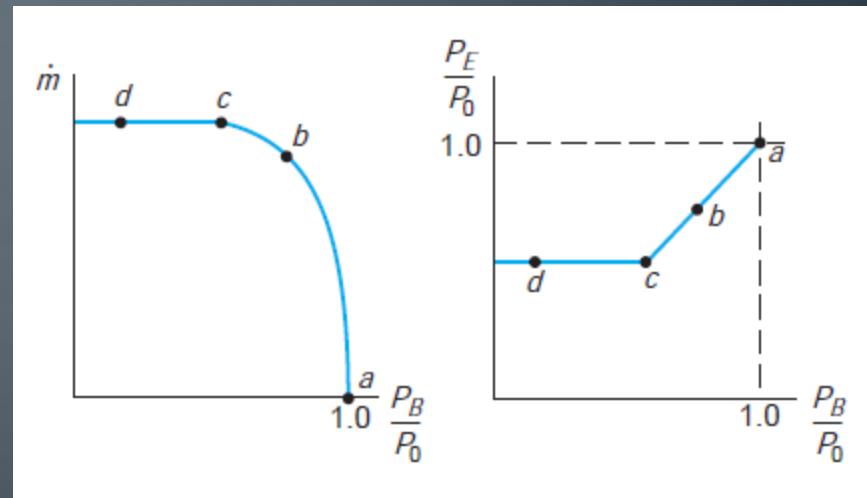
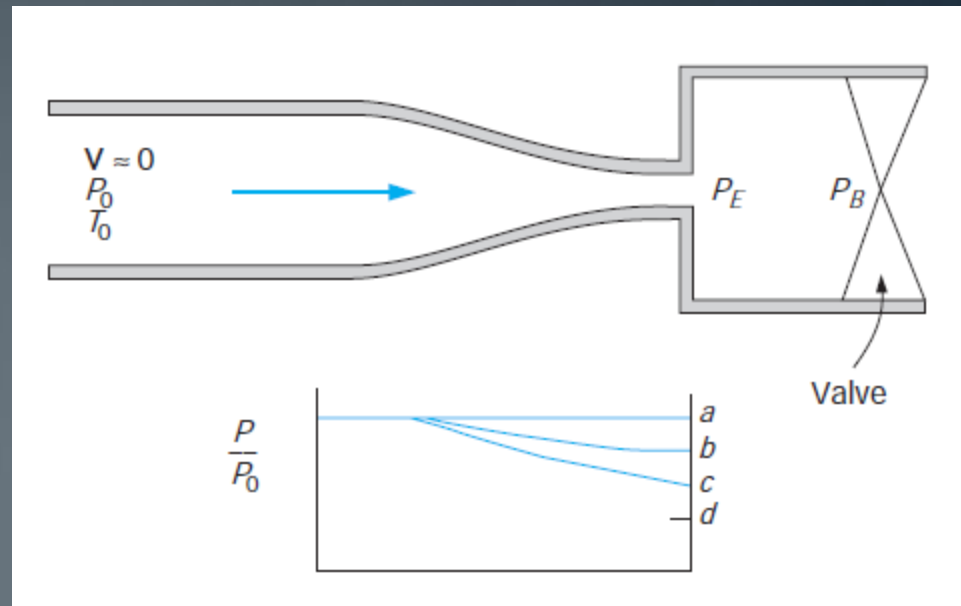


$$\frac{\dot{m}}{A} = \frac{P_0}{\sqrt{T_0}} \sqrt{\frac{k}{R}} \times \frac{M}{\left(1 + \frac{k-1}{2} M^2\right)^{(k+1)/2(k-1)}}$$

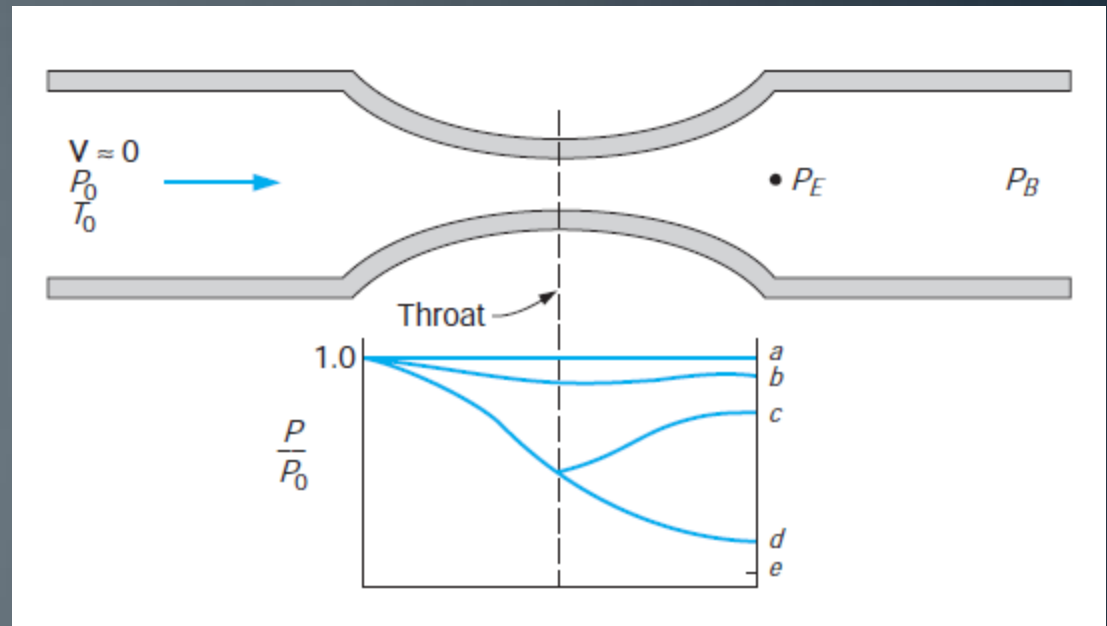
$$\frac{\dot{m}}{A^*} = \frac{P_0}{\sqrt{T_0}} \sqrt{\frac{k}{R}} \times \frac{1}{\left(\frac{k+1}{2}\right)^{(k+1)/2(k-1)}}$$

$$\frac{A}{A^*} = \frac{1}{M} \left[\left(\frac{2}{k+1} \right) \left(1 + \frac{k-1}{2} M^2 \right) \right]^{(k+1)/2(k-1)}$$

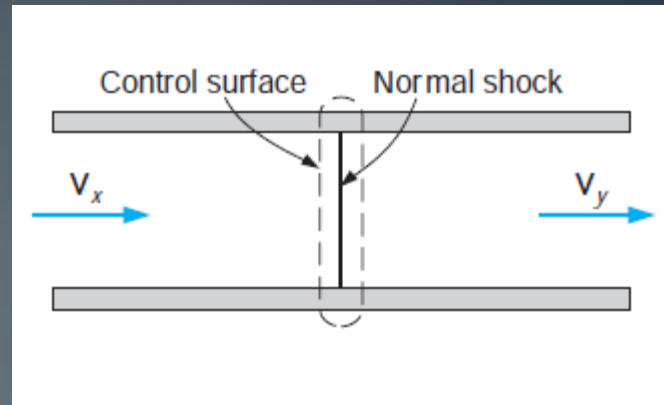
MASS RATE OF FLOW OF AN IDEAL GAS THROUGH AN ISENTROPIC NOZZLE



MASS RATE OF FLOW OF AN IDEAL GAS THROUGH AN ISENTROPIC NOZZLE



NORMAL SHOCK IN AN IDEAL GAS FLOWING THROUGH A NOZZLE



$$h_x + \frac{\mathbf{V}_x^2}{2} = h_y + \frac{\mathbf{V}_y^2}{2} = h_{0x} = h_{0y}$$

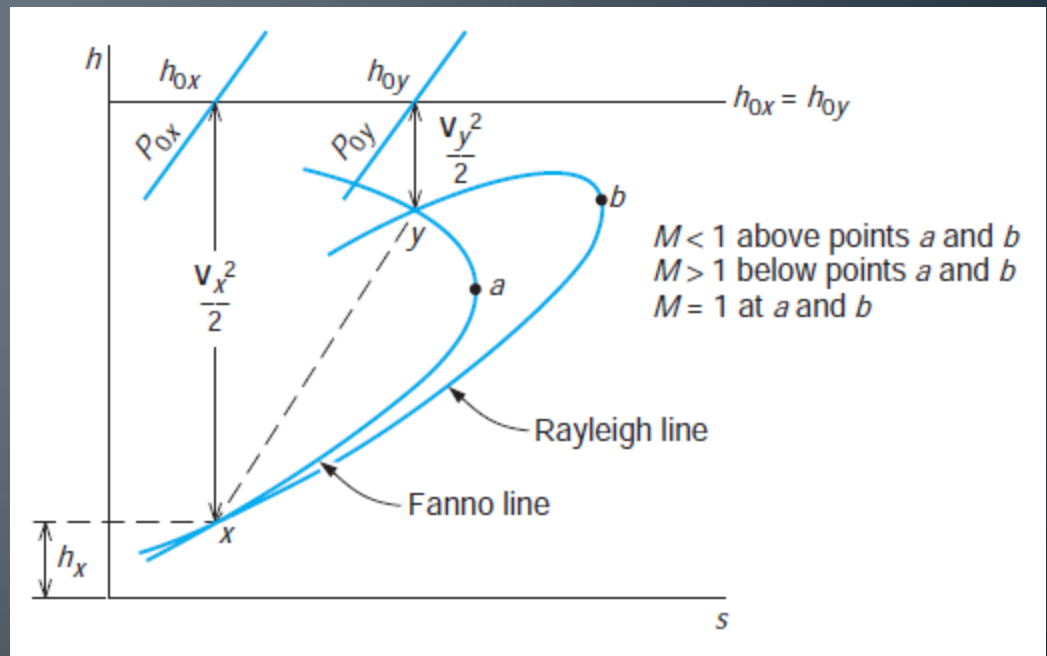
$$\frac{\dot{m}}{A} = \rho_x \mathbf{V}_x = \rho_y \mathbf{V}_y$$

$$A(P_x - P_y) = \dot{m}(\mathbf{V}_y - \mathbf{V}_x)$$

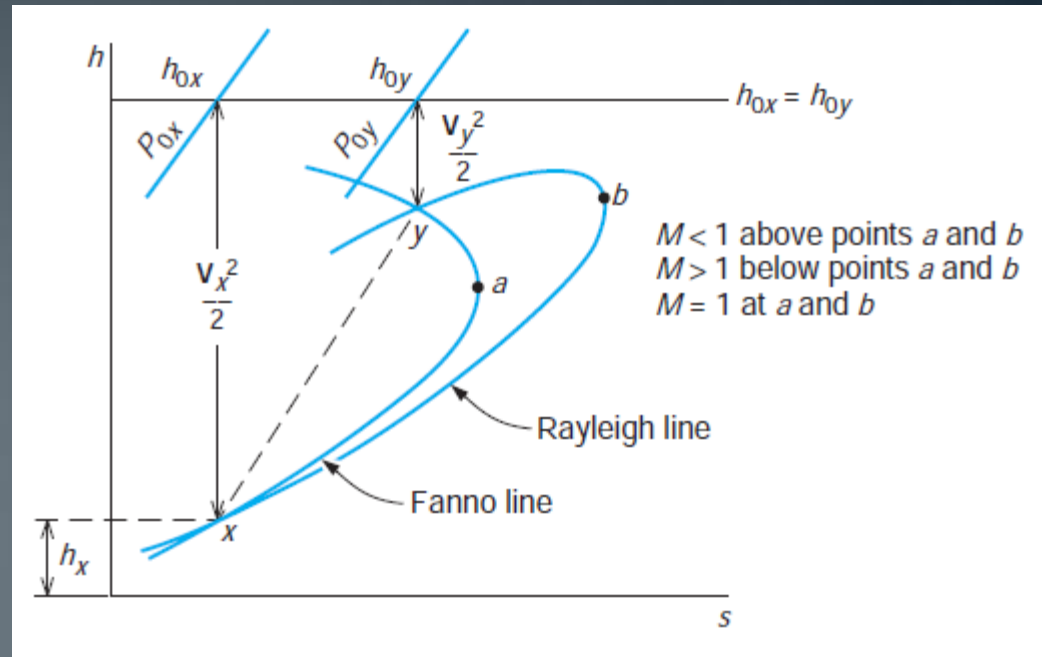
$$s_y - s_x = s_{\text{gen}} \geq 0$$

NORMAL SHOCK IN AN IDEAL GAS FLOWING THROUGH A NOZZLE

- The energy and continuity equations can be combined to give an equation that when plotted on the h - s diagram is called the **Fanno line**.
- The momentum and continuity equations can be combined to give an equation the plot of which on the h - s diagram is known as the **Rayleigh line**.



NORMAL SHOCK IN AN IDEAL GAS FLOWING THROUGH A NOZZLE



$$T_{0x} = T_{0y}$$

$$\frac{T_{0x}}{T_x} = 1 + \frac{k-1}{2} M_x^2 \quad \frac{T_{0y}}{T_y} = 1 + \frac{k-1}{2} M_y^2$$

$$\frac{T_y}{T_x} = \frac{1 + \frac{k-1}{2} M_x^2}{1 + \frac{k-1}{2} M_y^2}$$

NORMAL SHOCK IN AN IDEAL GAS FLOWING THROUGH A NOZZLE

- Combining the energy equations and the continuity equation, gives the equation of the **Fanno line**.

$$\frac{T_y}{T_x} = \frac{1 + \frac{k-1}{2} M_x^2}{1 + \frac{k-1}{2} M_y^2}$$

$$\rho_x \mathbf{V}_x = \rho_y \mathbf{V}_y$$

$$\begin{aligned} \rho_x &= \frac{P_x}{RT_x} & \rho_y &= \frac{P_y}{RT_y} \\ \frac{T_y}{T_x} &= \frac{P_y \mathbf{V}_y}{P_x \mathbf{V}_x} = \frac{P_y M_y c_y}{P_x M_x c_x} = \frac{P_y M_y \sqrt{T_y}}{P_x M_x \sqrt{T_x}} \\ &= \left(\frac{P_y}{P_x} \right)^2 \left(\frac{M_y}{M_x} \right)^2 \end{aligned}$$

$$\frac{P_y}{P_x} = \frac{M_x \sqrt{1 + \frac{k-1}{2} M_x^2}}{M_y \sqrt{1 + \frac{k-1}{2} M_y^2}}$$

NORMAL SHOCK IN AN IDEAL GAS FLOWING THROUGH A NOZZLE

- The momentum and continuity equations can be combined as follows to give the equation of the **Rayleigh line**.

$$P_x - P_y = \frac{\dot{m}}{A} (\mathbf{V}_y - \mathbf{V}_x) = \rho_y \mathbf{V}_y^2 - \rho_x \mathbf{V}_x^2$$

$$P_x + \rho_x \mathbf{V}_x^2 = P_y + \rho_y \mathbf{V}_y^2$$

$$P_x + \rho_x M_x^2 c_x^2 = P_y + \rho_y M_y^2 c_y^2$$

$$P_x + \frac{P_x M_x^2}{RT_x} (kRT_x) = P_y + \frac{P_y M_y^2}{RT_y} (kRT_y)$$

$$P_x(1 + kM_x^2) = P_y(1 + kM_y^2)$$

$$\frac{P_y}{P_x} = \frac{1 + kM_x^2}{1 + kM_y^2}$$

$$M_y^2 = \frac{M_x^2 + \frac{2}{k-1}}{\frac{2k}{k-1} M_x^2 - 1}$$