

6

Long Hydrodynamic Journal Bearings

6.1 INTRODUCTION

A hydrodynamic journal bearing is shown in [Fig. 6-1](#). The journal is rotating inside the bore of a sleeve with a thin clearance. Fluid lubricant is continuously supplied into the clearance. If the journal speed is sufficiently high, pressure builds up in the fluid film that completely separates the rubbing surfaces.

Hydrodynamic journal bearings are widely used in machinery, particularly in motor vehicle engines and high-speed turbines. The sleeve is mounted in a housing that can be a part of the frame of a machine. For successful operation, the bearing requires high-precision machining. For most applications, the mating surfaces of the journal and the internal bore of the bearing are carefully made with precise dimensions and a good surface finish.

The radial clearance, C , between the bearing bore of radius R_1 and of the journal radius, R , is $C = R_1 - R$ (see [Fig. 6-1](#)). In most practical cases, the clearance, C , is very small relative to the journal radius, R . The following order of magnitude is applicable for most design purposes:

$$\frac{C}{R} \approx 10^{-3} \quad (6-1)$$

A long hydrodynamic bearing is where the bearing length, L , is long in comparison to the journal radius ($L \gg R$). Under load, the center of the journal,

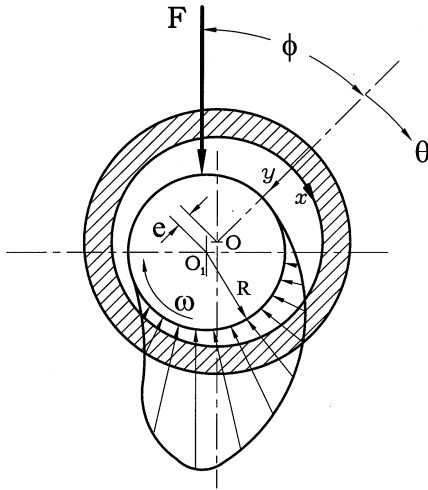


FIG. 6-1 Hydrodynamic journal bearing (clearance exaggerated).

O_1 , is displaced in the radial direction relative to the bearing center, O , as indicated in Fig. 6-1. The distance $O - O_1$ is the eccentricity, e , and the dimensionless eccentricity ratio, ϵ , is defined as

$$\epsilon = \frac{e}{C} \quad (6-2)$$

If the journal is concentric to the bearing bore, there is a uniform clearance around the bearing. But due to the eccentricity, there is a variable-thickness clearance around the bearing. The variable clearance, h , is equal to the lubricant film thickness. The clearance is converging (h decreases) along the region from $\theta = 0$ to $\theta = \pi$, where the clearance has its minimum thickness, h_n . After that, the clearance is diverging (h increases) along the region from $\theta = \pi$ to $\theta = 2\pi$, where the clearance has a maximum thickness, h_m . The clearance, h , is a function of the coordinate θ in the direction of the rotation of the journal. The coordinate θ is measured from the point of maximum clearance on the centerline $O - O_1$. An approximate equation for the thin clearance in a journal bearing is

$$h(\theta) = C(1 + \epsilon \cos \theta) \quad (6-3)$$

The minimum and maximum clearances are h_n and h_m , respectively, along the symmetry line $O - O_1$. They are derived from Eq. (6-3) as follows:

$$h_n = C(1 - \epsilon) \quad \text{and} \quad h_m = C(1 + \epsilon) \quad (6-4)$$

Whenever ϵ approaches 1 (one), h_n approaches zero, and there is an undesirable contact between the journal and bearing bore surfaces, resulting in severe wear,

particularly in a high-speed journal bearing. Therefore, for proper design, the bearing must operate at eccentricity ratios well below 1, to allow adequate minimum film thickness to separate the sliding surfaces. Most journal bearings operate under steady conditions in the range from $\varepsilon = 0.6$ to $\varepsilon = 0.8$. However, the most important design consideration is to make sure the minimum film thickness, h_n , will be much higher than the size of surface asperities or the level of journal vibrations during operation.

6.2 REYNOLDS EQUATION FOR A JOURNAL BEARING

In Chapter 4, the hydrodynamic equations of a long journal bearing were derived from first principles. In this chapter, the hydrodynamic equations are derived from the Reynolds equation. The advantage of the present approach is that it can apply to a wider range of problems, such as bearings under dynamic conditions.

Let us recall that the general Reynolds equation for a Newtonian incompressible thin fluid film is

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6(U_1 - U_2) \frac{\partial h}{\partial x} + 12(V_2 - V_1) \quad (6-5)$$

Here, U_1 and U_2 are velocity components, in the x direction, of the lower and upper sliding surfaces, respectively (fluid film boundaries), while the velocities components V_1 and V_2 are of the lower and upper boundaries, respectively, in the y direction (see Fig. 5-2). The difference in normal velocity ($V_1 - V_2$) is of relative motion (squeeze-film action) of the surfaces toward each other.

For most journal bearings, only the journal is rotating and the sleeve is stationary, $U_1 = 0$, $V_1 = 0$ (as shown in Fig. 6-1). The second fluid film boundary is at the journal surface that has a velocity $U = \omega R$. However, the velocity U is not parallel to the x direction (the x direction is along the bearing surface).

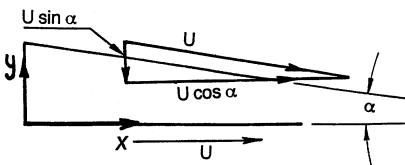


FIG. 6-2 Velocity components of the fluid film boundaries.

Therefore, it has two components (as shown in Fig. 6-2), U_2 and V_2 , in the x and y directions, respectively:

$$\begin{aligned} U_2 &= U \cos \alpha \\ V_2 &= -U \sin \alpha \end{aligned} \quad (6-6)$$

Here, the slope α is between the bearing and journal surfaces. In a journal bearing, the slope α is very small; therefore, the following approximations can be applied:

$$\begin{aligned} U_2 &= U \cos \alpha \approx U \\ V_2 &= -U \sin \alpha \approx -U \tan \alpha \end{aligned} \quad (6-7)$$

The slope α can be expressed in terms of the function of the clearance, h :

$$\tan \alpha = -\frac{\partial h}{\partial x} \quad (6-8)$$

The normal component V_2 becomes

$$V_2 \approx U \frac{\partial h}{\partial x} \quad (6-9)$$

After substituting Eqs. (6.7) and (6.9) into the right-hand side of the Reynolds equation, it becomes

$$6(U_1 - U_2) \frac{\partial h}{\partial x} + 12(V_2 - V_1) = 6(0 - U) \frac{\partial h}{\partial x} + 12U \frac{\partial h}{\partial x} = 6U \frac{\partial h}{\partial x} \quad (6-10)$$

The Reynolds equation for a Newtonian incompressible fluid reduces to the following final equation:

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6U \frac{\partial h}{\partial x} \quad (6-11)$$

For an infinitely long bearing, $\partial p / \partial z \cong 0$; therefore, the second term on the left-hand side of Eq. (6-11) can be omitted, and the Reynolds equation reduces to the following simplified one-dimensional equation:

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) = 6U \frac{\partial h}{\partial x} \quad (6-12)$$

6.3 JOURNAL BEARING WITH ROTATING SLEEVE

There are many practical applications where the sleeve is rotating as well as the journal. In that case, the right-hand side of the Reynolds equation is not the same as for a common stationary bearing. Let us consider an example, as shown in Fig.

6-3, where the sleeve and journal are rolling together like internal friction pulleys. The rolling is similar to that in a cylindrical rolling bearing. The internal sleeve and journal surface are rolling together without slip, and both have the same tangential velocity $\omega_j R = \omega_b R_1 = U$.

The tangential velocities of the film boundaries, in the x direction, of the two surfaces are

$$\begin{aligned} U_1 &= U \\ U_2 &= U \cos \alpha \approx U \end{aligned} \quad (6-13)$$

The normal velocity components, in the y direction, of the film boundaries (the journal and sleeve surfaces) are

$$\begin{aligned} V_1 &= 0 \\ V_2 &\approx U \frac{\partial h}{\partial x} \end{aligned} \quad (6-14)$$

The right-hand side of the Reynolds equation becomes

$$6(U_1 - U_2) \frac{\partial h}{\partial x} + 12(V_2 - V_1) = 6(U - U) \frac{\partial h}{\partial x} + 12(U \frac{\partial h}{\partial x} - 0) = 12U \frac{\partial h}{\partial x} \quad (6-15)$$

For the rolling action, the Reynolds equation for a journal bearing with a rolling sleeve is given by

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 12U \frac{\partial h}{\partial x} \quad (6-16)$$

The right-hand side of Eq. (6-16) indicates that the rolling action will result in a doubling of the pressure wave of a common journal bearing of identical geometry as well as a doubling its load capacity. The physical explanation is that the fluid is squeezed faster by the rolling action.

6.4 COMBINED ROLLING AND SLIDING

In many important applications, such as gears, there is a combination of rolling and sliding of two cylindrical surfaces. Also, there are several unique applications where a hydrodynamic journal bearing operates in a combined rolling-and-sliding mode. A combined bearing is shown in Fig. 6-3, where the journal surface velocity is $R\omega_j$ while the sleeve inner surface velocity is $R_1\omega_b$.

The coefficient ξ is the ratio of the rolling and sliding velocity. In terms of the velocities of the two surfaces, the ratio is,

$$\xi = \frac{R_1 \omega_b}{R \omega_j} \quad (6-17)$$

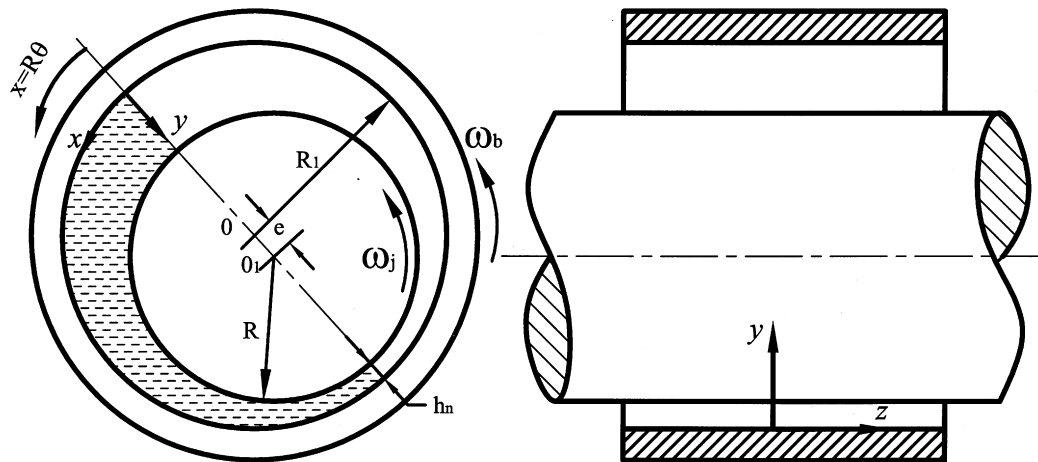


FIG. 6.3 Journal bearing with a rotating sleeve.

The journal surface velocity is $U = R\omega_j$, and the sleeve surface velocity is the product ξU , where ξ is the rolling-to-sliding ratio. The common journal bearing has a pure sliding and $\xi = 0$, while in pure rolling, $\xi = 1$. For all other combinations, $0 < \xi < 1$.

The tangential velocities (in the x direction) of the fluid–film boundaries of the two surfaces are

$$\begin{aligned} U_1 &= R_1\omega_b = \xi R\omega_j \\ U_2 &= R\omega_j \cos \alpha \approx R\omega_j \end{aligned} \quad (6-18)$$

The normal components, in the y direction, of the velocity of the fluid film boundaries (journal and sleeve surfaces) are

$$\begin{aligned} V_1 &= 0 \\ V_2 &= R\omega_j \frac{\partial h}{\partial x} \end{aligned} \quad (6-19)$$

For the general case of a combined rolling and sliding, the expression on the right-hand side of Reynolds equation is obtained by substituting the preceding components of the surface velocity:

$$\begin{aligned} 6(U_1 - U_2) \frac{\partial h}{\partial x} + 12(V_2 - V_1) &= 6(\xi U - U) \frac{\partial h}{\partial x} + 12(U \frac{\partial h}{\partial x} - 0) \\ &= 6U(1 + \xi) \frac{\partial h}{\partial x} \end{aligned} \quad (6-20)$$

Here, $U = R\omega_j$ is the journal surface velocity. Finally, the Reynolds equation for a combined rolling and sliding of a journal bearing is as follows:

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6R\omega_j(1 + \xi) \frac{\partial h}{\partial x} \quad (6-21a)$$

For $\xi = 0$ and $\xi = 1$, the right-hand-side of the Reynolds equation is in agreement with the previous derivations for pure sliding and pure rolling, respectively. The right-hand side of Eq. (6-21a) indicates that pure rolling action doubles the pressure wave in comparison to a pure sliding.

Equation (6-21a) is often written in the basic form:

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6R(\omega_j + \omega_b) \frac{\partial h}{\partial x} \quad (6-21b)$$

In journal bearings, the difference between the journal radius and the bearing radius is small and we can assume that $R_1 \approx R$.

6.5 PRESSURE WAVE IN A LONG JOURNAL BEARING

For a common long journal bearing with a stationary sleeve, the pressure wave is derived by a double integration of Eq. (6.12). After the first integration, the following explicit expression for the pressure gradient is obtained:

$$\frac{dp}{dx} = 6U\mu \frac{h + C_1}{h^3} \quad (6-22)$$

Here, C_1 is a constant of integration. In this equation, a regular derivative replaces the partial one, because in a long bearing, the pressure is a function of one variable, x , only. The constant, C_1 , can be replaced by h_0 , which is the film thickness at the point of peak pressure. At the point of peak pressure,

$$\frac{dp}{dx} = 0 \quad \text{at} \quad h = h_0 \quad (6-23)$$

Substituting condition (6-23), in Eq. (6-22) results in $C_1 = -h_0$, and Eq. (6-22) becomes

$$\frac{dp}{dx} = 6U\mu \frac{h - h_0}{h^3} \quad (6-24)$$

Equation (6-24) has one unknown, h_0 , which is determined later from additional information about the pressure wave.

The expression for the pressure distribution (pressure wave) around a journal bearing, along the x direction, is derived by integration of Eq. (6-24), and there will be an additional unknown—the constant of integration. By using the two boundary conditions of the pressure wave, we solve the two unknowns, h_0 , and the second integration constant.

The pressures at the start and at the end of the pressure wave are usually used as boundary conditions. However, in certain cases the locations of the start and the end of the pressure wave are not obvious. For example, the fluid film of a practical journal bearing involves a fluid cavitation, and other boundary conditions of the pressure wave are used for solving the two unknowns. These boundary conditions are discussed in this chapter. The solution method of two unknowns for these boundary conditions is more complex and requires computer iterations.

Replacing x by an angular coordinate θ , we get

$$x = R\theta \quad (6-25)$$

Equation (6-24) takes the form

$$\frac{dp}{d\theta} = 6UR\mu \frac{h - h_0}{h^3} \quad (6-26)$$

For the integration of Eq. (6-26), the boundary condition at the start of the pressure wave is required. The pressure wave starts at $\theta = 0$, and the magnitude of pressure at $\theta = 0$ is p_0 . The pressure, p_0 , can be very close to atmospheric pressure, or much higher if the oil is fed into the journal bearing by an external pump.

The film thickness, h , as a function of θ for a journal bearing is given in Eq. (6-3), $h(\theta) = C(1 + \varepsilon \cos \theta)$. After substitution of this expression into Eq. (6-26), the pressure wave is given by

$$\frac{C^2}{6\mu UR} (p - p_0) = \int_0^\theta \frac{d\theta}{(1 + \varepsilon \cos \theta)^2} - \frac{h_0}{C} \int_0^\theta \frac{d\theta}{(1 + \varepsilon \cos \theta)^3} \quad (6-27)$$

The pressure, p_0 , is determined by the oil supply pressure (inlet pressure). In a common journal bearing, the oil is supplied through a hole in the sleeve, at $\theta = 0$. The oil can be supplied by gravitation from an oil container or by a high-pressure pump. In the first case, p_0 is only slightly above atmospheric pressure and can be approximated as $p_0 = 0$. However, if an external pump supplies the oil, the pump pressure (at the bearing inlet point) determines the value of p_0 . In industry, there are often central oil circulation systems that provide oil under pressure for the lubrication of many bearings.

The two integrals on the right-hand side of Eq. (6-27) are functions of the eccentricity ratio, ε . These integrals can be solved by numerical or analytical integration. Sommerfeld (1904) analytically solved these integrals for a full (360°) journal bearing. Analytical solutions of these two integrals are included in integral tables of most calculus textbooks. For a full bearing, it is possible to solve for the load capacity even in cases where p_0 can't be determined. This is because p_0 is an extra constant hydrostatic pressure around the bearing, and, similar to atmospheric pressure, it does not contribute to the load capacity.

In a journal bearing, the pressure that is predicted by integrating Eq. (6-27) increases (above the inlet pressure, p_0) in the region of converging clearance, $0 < \theta < \pi$. However, in the region of a diverging clearance, $\pi < \theta < 2\pi$, the pressure wave reduces below p_0 . If the oil is fed at atmospheric pressure, the pressure wave that is predicted by integration of Eq. (6-27) is negative in the divergent region, $\pi < \theta < 2\pi$. This analytical solution is not always valid, because a negative pressure would result in a "fluid cavitation." The analysis may predict negative pressures below the absolute zero pressure, and that, of course, is physically impossible.

A continuous fluid film cannot be maintained at low negative pressure (relative to the atmospheric pressure) due to fluid cavitation. This effect occurs whenever the pressure reduces below the vapor pressure of the oil, resulting in an oil film rupture. At low pressures, the boiling process can take place at room temperature; this phenomenon is referred to as *fluid cavitation*. Moreover, at low pressure, the oil releases its dissolved air, and the oil foams in many tiny air bubbles. Antifoaming agents are usually added into the oil to minimize this effect.

As a result of cavitation and foaming, the fluid is not continuous in the divergent clearance region, and the actual pressure wave cannot be predicted anymore by Eq. (6-27). In fact, the pressure wave is maintained only in the converging region, while the pressure in most of the diverging region is close to ambient pressure. Cole and Hughes (1956) conducted experiments using a transparent sleeve. Their photographs show clearly that in the divergent region, the fluid film ruptures into filaments separated by air and lubricant vapor.

Under light loads or if the supply pressure, p_0 , is high, the minimum predicted pressure in the divergent clearance region is not low enough to generate a fluid cavitation. In such cases, Eq. (6-27) can be applied around the complete journal bearing. The following solution referred to as the *Sommerfeld solution*, is limited to cases where there is a full fluid film around the bearing.

6.6 SOMMERFELD SOLUTION OF THE PRESSURE WAVE

Sommerfeld (1904) solved Eq. (6-27) for the pressure wave and load capacity of a full hydrodynamic journal bearing (360°) where a fluid film is maintained around the bearing without any cavitation. This example is of a special interest because this was the first analytical solution of a hydrodynamic journal bearing based on the Reynolds equation. In practice, a full hydrodynamic lubrication around the bearing is maintained whenever at least one of the following two conditions are met:

- a. The feed pressure, p_0 , (from an external oil pump), into the bearing is quite high in order to maintain positive pressures around the bearing and thus prevent cavitation.
- b. The journal bearing is lightly loaded. In this case, the minimum pressure is above the critical value of cavitation.

Sommerfeld assumed a periodic pressure wave around the bearing; namely, the pressure is the same at $\theta = 0$ and $\theta = 2\pi$:

$$P_{(\theta=0)} = P_{(\theta=2\pi)} \quad (6-28)$$

The unknown, h_0 , that represents the film thickness at the point of a peak pressure can be solved from Eq. (6-27) and the Sommerfeld boundary condition in Eq. (6-28). After substituting $p - p_0 = 0$, at $\theta = 2\pi$, Eq. (6-27) yields

$$\int_0^{2\pi} \frac{d\theta}{(1 + \varepsilon \cos \theta)^2} - \frac{h_0}{C} \int_0^{2\pi} \frac{d\theta}{(1 + \varepsilon \cos \theta)^3} = 0 \quad (6-29)$$

This equation can be solved for the unknown, h_0 . The following substitutions for the values of the integrals can simplify the analysis of hydrodynamic journal bearings:

$$J_n = \int_0^{2\pi} \frac{d\theta}{(1 + \varepsilon \cos \theta)^n} \quad (6-30)$$

$$I_n = \int_0^{2\pi} \frac{\cos \theta d\theta}{(1 + \varepsilon \cos \theta)^n} \quad (6-31)$$

Equation (6-29) is solved for the unknown, h_0 , in terms of the integrals J_n :

$$\frac{h_0}{C} = \frac{J_2}{J_3} \quad (6-32)$$

Here, the solutions for the integrals J_n are:

$$J_1 = \frac{2\pi}{(1 - \varepsilon^2)^{1/2}} \quad (6-33)$$

$$J_2 = \frac{2\pi}{(1 - \varepsilon^2)^{3/2}} \quad (6-34)$$

$$J_3 = \left(1 + \frac{1}{2}\varepsilon^2\right) \frac{2\pi}{(1 - \varepsilon^2)^{5/2}} \quad (6-35)$$

$$J_4 = \left(1 + \frac{3}{2}\varepsilon^2\right) \frac{2\pi}{(1 - \varepsilon^2)^{7/2}} \quad (6-36)$$

The solutions of the integrals I_n are required later for the derivation of the expression for the load capacity. The integrals I_n can be obtained from J_n by the following equation:

$$I_n = \frac{J_{n-1} - J_n}{\varepsilon} \quad (6-37)$$

Sommerfeld solved for the integrals in Eq. (6-27), and obtained the following equation for the pressure wave around an infinitely long journal bearing with a full film around the journal bearing:

$$p - p_0 = \frac{6\mu UR}{C^2} \frac{\varepsilon(2 + \varepsilon \cos \theta) \sin \theta}{(2 + \varepsilon^2)(1 + \varepsilon \cos \theta)^2} \quad (6-38)$$

The curves in Fig. 6-4 are dimensionless pressure waves, relative to the inlet pressure, for various eccentricity ratios, ϵ . The pressure wave is an antisymmetrical function on both sides of $\theta = \pi$. The curves indicate that the peak pressure considerably increases with the eccentricity ratio, ϵ . According to Eq. (6-38), the peak pressure approaches infinity when ϵ approaches 1. However, this is not possible in practice because the surface asperities prevent a complete contact between the sliding surfaces.

6.7 JOURNAL BEARING LOAD CAPACITY

Figure 6-5 shows the load capacity, W , of a journal bearing and its two components, W_x and W_y . The direction of W_x is along the bearing symmetry line $O-O_1$. This direction is inclined at an attitude angle, ϕ , from the direction of the external force and load capacity, W . In Fig. 6-5 the external force is in a vertical direction. The direction of the second component, W_y is normal to the W_x direction.

The elementary load capacity, dW , acts in the direction normal to the journal surface. It is the product of the fluid pressure, p , and an elementary area,

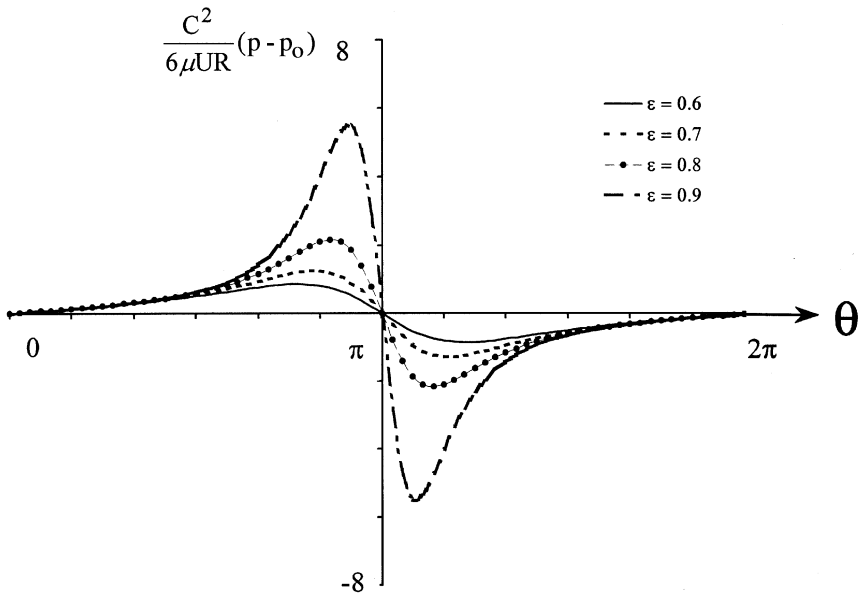


FIG. 6-4 Pressure waves in an infinitely long, full bearing according to the Sommerfeld solution.

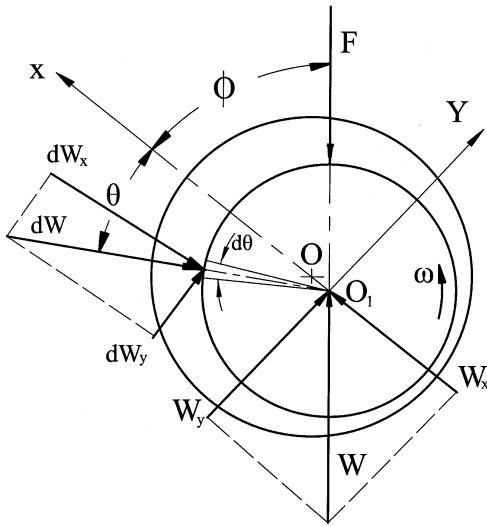


FIG. 6-5 Hydrodynamic bearing force components.

$dA = LR d\theta$, of the journal surface bounded by a small journal angle, $d\theta$. An elementary fluid force ($dW = p dA$) is given by,

$$dW = pLR d\theta \quad (6-39)$$

The pressure is acting in the direction normal to the journal surface, and dW is a radial elementary force vector directed toward the journal center, as shown in Fig. 6-5.

In a plane-slider, the pressure is acting in one direction and the load capacity has been derived by integration of the pressure wave. However, in a journal bearing, the direction of the pressure varies around the bearing, and simple summation of the elementary forces, dW , will not yield the resultant force. In order to allow summation, the elementary force, dW , is divided into two components, dW_x and dW_y , in the directions of W_x and W_y , respectively. By using force components, it is possible to have a summation (by integration) of each component around the bearing. The elementary force components are:

$$dW_x = -pLR \cos \theta d\theta \quad (6-40)$$

$$dW_y = pLR \sin \theta d\theta \quad (6-41)$$

The two components of the load capacity, W_x and W_y , are in the X and Y directions, respectively, as indicated in Fig. 6-5. Note the negative sign in Eq. (6-40), since dW_x is opposite to the W_x direction. The load components are:

$$W_x = -LR \int_0^{2\pi} p \cos \theta \, d\theta \quad (6-42)$$

$$W_y = LR \int_0^{2\pi} p \sin \theta \, d\theta \quad (6-43)$$

The attitude angle ϕ in Fig. 6-5 is determined by the ratio of the force components:

$$\tan \phi = \frac{W_y}{W_x} \quad (6-44)$$

6.8 LOAD CAPACITY BASED ON SOMMERFELD CONDITIONS

The load capacity components can be solved by integration of Eqs. (6-42 and (6-43), where p is substituted from Eq. (6-38). However, the derivation can be simplified if the load capacity components are derived directly from the basic Eq. (6-24) of the pressure gradient. In this way, there is no need to integrate the complex Eq. (6-38) of the pressure wave. This can be accomplished by employing the following identity for product derivation:

$$(uv)' = uv' + vu' \quad (6-45)$$

Integrating and rearranging Eq. (6-45) results in

$$\int uv' = uv - \int u'v \quad (6-46)$$

In order to simplify the integration of Eq. (6-42) for the load capacity component W_x , the substitutions $u = p$ and $v' = \cos \theta$ are made. This substitution allows the use of the product rule in Eq. (6-46), and the integral in Eq. (6-42) results in the following terms:

$$\int p \cos \theta \, d\theta = p \sin \theta - \int \frac{dp}{d\theta} \sin \theta \, d\theta \quad (6-47)$$

In a similar way, for the load capacity component, W_y , in Eq. (6-43), the substitutions $u = p$ and $v' = \sin \theta$ result in

$$\int p \sin \theta \, d\theta = -p \cos \theta + \int \frac{dp}{d\theta} \cos \theta \, d\theta \quad (6-48)$$

Equations (6-47) and (6-48) indicate that the load capacity components in Eqs. (6-42) and (6-43) can be solved directly from the pressure gradient. By using this method, it is not necessary to solve for the pressure wave in order to find the load capacity components (it offers the considerable simplification of one simple integration instead of a complex double integration). The first term, on the right-hand side in Eqs. (6-47) and (6-48) is zero, when integrated around a full bearing, because the pressure, p , is the same at $\theta = 0$ and $\theta = 2\pi$.

Integration of the last term in Eq. (6-47), in the boundaries $\theta = 0$ and $\theta = 2\pi$, indicates that the load component, W_x , is zero. This is because it is an integration of the antisymmetrical function around the bearing (the function is antisymmetric on the two sides of the centerline $O-O_1$, which cancel each other). Therefore:

$$W_x = 0 \quad (6-49)$$

Integration of Eq. (6-43) with the aid of identity (6-48), and using the value of h_0 in Eq. 6-32 results in

$$W_y = \frac{6\mu UR^2 L}{C^2} \left(I_2 - \frac{J_2}{J_3} I_3 \right) \quad (6-50)$$

Substituting for the values of I_n and J_n as a function of ε yields the following expression for the load capacity component, W_y . The other component is $W_x = 0$; therefore, for the Sommerfeld conditions, W_y is equal to the total load capacity, $W = W_y$:

$$W = \frac{12\pi\mu UR^2 L}{C^2} \frac{\varepsilon}{(2 + \varepsilon^2)(1 - \varepsilon^2)^{1/2}} \quad (6-51)$$

The attitude angle, ϕ , is derived from Eq. (6-44). For Sommerfeld's conditions, $W_x = 0$ and $\tan \phi \rightarrow \infty$; therefore,

$$\phi = \frac{\pi}{2} \quad (6-52)$$

Equation (6-52) indicates that in this case, the symmetry line $O-O_1$ is normal to the direction of the load capacity W .

6.9 FRICTION IN A LONG JOURNAL BEARING

The bearing friction force, F_f , is the viscous resistance force to the rotation of the journal due to high shear rates in the fluid film. This force is acting in the tangential direction of the journal surface and results in a resistance torque to the

rotation of the journal. The friction force is defined as the ratio of the friction torque, T_f , to the journal radius, R :

$$F_f = \frac{T_f}{R} \quad (6-53)$$

The force is derived by integration of the shear stresses over the area of the journal surface, at $y = h$, around the bearing. The shear stress distribution at the journal surface (shear at the wall, τ_w) around the bearing, is derived from the velocity gradient, as follows:

$$\tau_w = \mu \left. \frac{du}{dy} \right|_{(y=h)} \quad (6-54)$$

The friction force is obtained by integration:

$$F_f = \int_A \tau_{(y=h)} dA \quad (6-55)$$

After substituting $dA = RL d\theta$ in Eq. (6-55), the friction force becomes

$$F_f = \mu RL \int_0^{2\pi} \tau_{(y=h)} d\theta \quad (6-56)$$

Substitution of the value of the shear stress, Eq. (6-56) becomes

$$F_f = \mu URL \int_0^{2\pi} \left(\frac{4}{h} - \frac{3h_0}{h^2} \right) d\theta \quad (6-57)$$

If we apply the integral definitions in Eq. (6-30), the expression for the friction force becomes

$$F_f = \frac{\mu RL}{C} \left(4J_1 - 3 \frac{J_2^2}{J_3} \right) \quad (6-58)$$

The integrals J_n are functions of the eccentricity ratio. Substituting the solution of the integrals, J_n , in Eqs. (6.33) to (6.37) results in the following expression for the friction force:

$$F_f = \frac{\mu URL}{C} \frac{4\pi(1 + 2\varepsilon^2)}{(2 + \varepsilon^2)(1 - \varepsilon^2)^{1/2}} \quad (6-59)$$

Let us recall that the bearing friction coefficient, f , is defined as

$$f = \frac{F_f}{W} \quad (6-60)$$

Substitution of the values of the friction force and load in Eq. (6-60) results in a relatively simple expression for the coefficient of friction of a long hydrodynamic journal bearing:

$$f = \frac{C}{R} \frac{1 + 2\varepsilon^2}{3\varepsilon} \quad (6-61)$$

Comment: The preceding equation for the friction force in the fluid film is based on the shear at $y = h$. The viscous friction force around the bearing bore surface, at $y = 0$, is not equal to that around the journal surface, at $y = h$. The viscous friction torque on the journal surface is unequal to that on the bore surface because the external load is eccentric to the bore center, and it is an additional torque. However, the friction torque on the journal surface is the actual total resistance to the journal rotation, and it is used for calculating the friction energy losses in the bearing.

6.10 POWER LOSS ON VISCOUS FRICTION

The energy loss, per unit of time (power loss) $\dot{E}_f$, is determined from the friction torque, or friction force, by the following equations:

$$\dot{E}_f = T_f \omega = F_f U \quad (6-62)$$

where ω (rad/s) is the angular velocity of the journal. Substituting Eq. (6-59) into Eq. (6-62) yields

$$\dot{E}_f = \frac{\mu U^2 R L}{C} \frac{4\pi(1 + 2\varepsilon^2)}{(2 + \varepsilon^2)(1 - \varepsilon^2)^{1/2}} \quad (6-63)$$

The friction energy losses are dissipated in the lubricant as heat. Knowledge of the amount of friction energy that is dissipated in the bearing is very important for ensuring that the lubricant does not overheat. The heat must be transferred from the bearing by adequate circulation of lubricant through the bearing as well as by conduction of heat from the fluid film through the sleeve and journal.

6.11 SOMMERFELD NUMBER

Equation (6-51) is the expression for the bearing load capacity in a long bearing operating at steady conditions with the Sommerfeld boundary conditions for the pressure wave. This result was obtained for a full film bearing without any cavitation around the bearing. In most practical cases, this is not a realistic expression for the load capacity, since there is fluid cavitation in the diverging clearance region of negative pressure. The Sommerfeld solution for the load capacity has been improved by applying a more accurate analysis with realistic

boundary conditions of the pressure wave. This solution requires iterations performed with the aid of a computer.

In order to simplify the design of hydrodynamic journal bearings, the realistic results for the load capacity are provided in dimensionless form in tables or graphs. For this purpose, a widely used dimensionless number is the Sommerfeld number, S .

Equation (6-51) can be converted to dimensionless form if all the variables with dimensions are placed on the left-hand side of the equations and the dimensionless function of ε on the right-hand side. Also, it is the tradition in this discipline to have the Sommerfeld dimensionless group as a function of journal speed, n , in revolutions per second, and the average bearing pressure, P , according to the following substitutions:

$$U = 2\pi Rn \quad (6-64)$$

$$W = 2RLP \quad (6-65)$$

By substituting Eqs. (6.64) and (6.65) into Eq. (6.51), we obtain the following dimensionless form of the Sommerfeld number for an infinitely long journal bearing where cavitation is disregarded:

$$S = \frac{\mu n}{P} \left(\frac{R}{C} \right)^2 = \frac{(2 + \varepsilon^2)(1 - \varepsilon^2)^{1/2}}{12\pi^2 \varepsilon} \quad (6-66)$$

The dimensionless Sommerfeld number is a function of ε only. For an infinitely long bearing and the Sommerfeld boundary conditions, the number S can be calculated via Eq. (6-66). However, for design purposes, we use the realistic conditions for the pressure wave. The values of S for various ratios of length and diameter, L/D , have been computed; the results are available in [Chapter 8](#) in the form of graphs and tables for design purposes.

6.12 PRACTICAL PRESSURE BOUNDARY CONDITIONS

The previous discussion indicates that for most practical applications in machinery the pressures are very high and cavitation occurs in the diverging region of the clearance. For such cases, the Sommerfeld boundary conditions do not apply and the pressure distribution can be solved for more realistic conditions. In an actual journal bearing, there is no full pressure wave around the complete bearing, and there is a positive pressure wave between θ_1 and θ_2 . Outside this region, there is cavitation and low negative pressure that can be ignored for the purpose of calculating the load capacity. The value of θ_2 is unknown, and an additional condition must be applied for the solution. The boundary condition commonly used is that the pressure gradient is zero at the end of the pressure wave, at θ_2 .

This is equivalent to the assumption that there is no flow, u , in the x direction at θ_2 . When the lubricant is supplied at θ_1 at a pressure p_0 , the following boundary conditions can be applied:

$$\begin{aligned} p &= p_0 & \text{at } \theta &= \theta_1 \\ \frac{dp}{d\theta} &= 0 & \text{at } \theta &= \theta_2 \\ p &= 0 & \text{at } \theta &= \theta_2 \end{aligned} \quad (6-67)$$

In most cases, the feed pressure is close to ambient pressure at $\theta = 0$, and the first boundary condition is $p = 0$ at $\theta = 0$. The pressure wave with the foregoing boundary conditions is shown in Fig. 6-6.

The location of the end of the pressure wave, θ_2 , is solved by iterations. The solution is performed by guessing a value for θ_2 that is larger than 180° , then integrating Eq. (6-27) at the boundaries from 0 to θ_2 . The solution is obtained when the pressure at θ_2 is very close to zero.

Figure 6-6 indicates that the angle θ_2 and the angle of the peak pressure are symmetrical on both sides of $\theta = \pi$; therefore both have an equal film thickness h (or clearance thickness). The constant, h_0 , representing the film thickness at the maximum pressure is

$$h_0 = C(1 + \cos \theta_2) \quad (6-68)$$

After the integration is completed, the previous guess for θ_2 is corrected until a satisfactory solution is obtained (the pressure at θ_2 is very close to zero). Preparation of a small computer program to solve for θ_2 by iterations is recommended as a beneficial exercise for the reader. After h_0 is solved the pressure wave can be plotted.

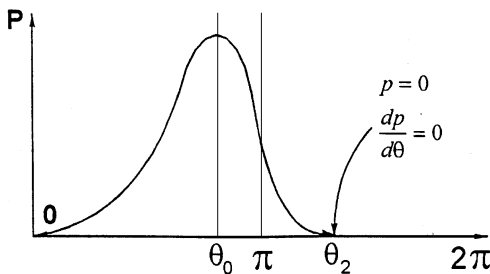


FIG. 6-6 Pressure wave plot under realistic boundary conditions.

Example Problem 6-1

Ice Sled

Two smooth cylindrical sections support a sled as shown in Fig. 6-7. The sled is running over ice on a thin layer of water film. The total load (weight of the sled and the person) is 1000 N. This load is acting at equal distances between the two blades. The sled velocity is 15 km/h, the radius of the blade is 30 cm, and its width is $L = 90$ cm. The viscosity of water $\mu = 1.792 \times 10^{-3}$ N-s/m².

- Find the pressure distribution at the entrance region of the fluid film under the ski blade as it runs over the ice. Derive and plot the pressure wave in dimensionless form.
- Find the expression for the load capacity of one blade.
- Find the minimum film thickness ($h_n = h_{\min}$) of the thin water layer.

Solution

a. Pressure Distribution and Pressure Wave

For $h_n/R \ll 1$, the pressure wave is generated only near the minimum-film region, where $x \ll R$, or $x/R \ll 1$.

For a small value of x/R , the equation for the clearance, between the quarter-cylinder and the ice, $h = h(x)$, can be approximated by a parabolic wedge. The following expression is obtained by expanding the equation for the variable clearance, $h(x)$, which is equal to the fluid-film thickness, into a Taylor series and truncating powers higher than x^2 (see Chapter 4).

If the minimum thickness of the film is $h_n = h_{\min}$, the equation for the variable clearance becomes

$$h(x) = h_n + \frac{x^2}{2R}$$

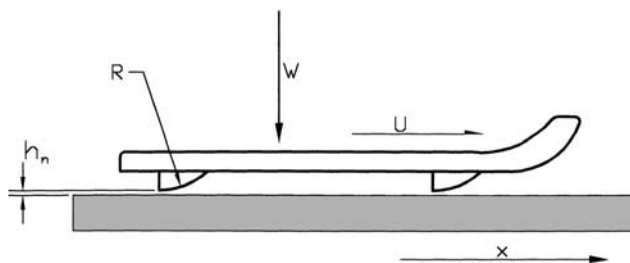


FIG. 6-7 Sled made of two quarter-cylinder.

For dimensionless analysis, the clearance function is written as

$$h(x) = h_n \left(1 + \frac{x^2}{2Rh_n} \right)$$

The width, L (in the direction normal to the sled speed), is very large in comparison to the film length in the x direction. Therefore, it can be considered an infinitely long bearing, and the pressure gradient is

$$\frac{dp}{dx} = 6\mu U \frac{h - h_0}{h^3}$$

The unknown, h_0 , is the fluid film thickness at the point of peak pressure, where $dp/dx = 0$.

Conversion to Dimensionless Terms. The conversion to dimensionless terms is similar to that presented in [Chapter 4](#). The length, x , is normalized by $\sqrt{2Rh_n}$, and the dimensionless terms are defined as

$$\bar{x} = \frac{x}{\sqrt{2Rh_n}} \quad \text{and} \quad \bar{h} = \frac{h}{h_n}$$

The dimensionless clearance gets a simple form:

$$\bar{h}(\bar{x}) = 1 + \bar{x}^2$$

Using the preceding substitutions, the pressure gradient equation takes the form

$$\frac{dp}{dx} = \frac{6\mu U}{h_n^2} \frac{\bar{h} - \bar{h}_0}{(1 + \bar{x}^2)^3}$$

Here the unknown, h_0 , is replaced by

$$\bar{h}_0 = 1 + \bar{x}_0^2$$

The unknown, h_0 , is replaced by unknown x_0 , which describes the location of the peak pressure. Using the foregoing substitutions, the pressure gradient equation is reduced to

$$\frac{dp}{dx} = \frac{6\mu U}{h_n^2} \frac{\bar{x}^2 - \bar{x}_0^2}{(1 + \bar{x}^2)^3}$$

Converting dx into dimensionless form produces

$$dx = \sqrt{2Rh_n} d\bar{x}$$

The dimensionless differential equation for the pressure takes the form

$$\frac{h_n^2}{\sqrt{2Rh_n} 6\mu U} dp = \frac{\bar{x}^2 - \bar{x}_0^2}{(1 + \bar{x}^2)^3} d\bar{x}$$

Here, the dimensionless pressure is

$$\bar{p} = \frac{h_n^2}{6U\mu\sqrt{2Rh_n}}p$$

The final equation for integration of the dimensionless pressure is

$$\bar{p} = \frac{h_n^2}{\sqrt{2Rh_n}} \frac{1}{6\mu U} \int_0^p dp = \int_\infty^x \frac{\bar{x}^2 - \bar{x}_0^2}{(1 + \bar{x}^2)^3} d\bar{x} + p_0$$

Here p_0 is a constant of integration, which is atmospheric pressure far from the minimum clearance. In this equation, p_0 and x_0 are two unknowns that can be solved for by the following boundary conditions of the pressure wave:

$$\begin{aligned} \text{at } x = \infty, \quad p &= 0 \\ \text{at } x = 0, \quad p &= 0 \end{aligned}$$

The first boundary condition, $p = 0$ at $x = \infty$, yields $p_0 = 0$. The second unknown, x_0 , is solved for by iterations (trial and error). The value of x_0 is varied until the second boundary condition of the pressure wave, $p = 0$ at $x = 0$, is satisfied.

Numerical Solution by Iterations. For numerical integration, the boundary $\bar{x} = \infty$ is replaced by a relatively large finite dimensionless value where pressure is small and can be disregarded, such as $\bar{x} = 4$.

The numerical solution involves iterations for solving for x_0 . Each one of the iterations involves integration in the boundaries from 4 to 0. The solution for x_0 is obtained when the pressure at $x = 0$ is sufficiently close to zero. Using trial and error, we select each time a value for x_0 and integrate. This is repeated until the value of the dimensionless pressure is nearly zero at $x = 0$. The solution of the dimensionless pressure wave is plotted in Fig. 6-8. The maximum dimensionless pressure occurs at a dimensionless distance $\bar{x} = 0.55$, and the maximum dimensionless pressure is $\bar{p} = 0.109$.

$$\text{Dimensionless Pressure, } \bar{p} = \frac{h_n^2}{\sqrt{2Rh_n}} \frac{1}{6\mu U} p$$

Discussion of Numerical Iterations. The method of iteration is often referred to as the *shooting method*. In order to run iteration, we guess a certain value of $\bar{x}_0$, and, using this value, we integrate and attempt to hit the target point $\bar{x} = 0$, $\bar{p} = 0$.

For the purpose of illustrating the shooting method, three iterations are shown in Fig. 6-9. The iteration for $\bar{x}_0 = 0.25$ results in a pressure that is too high at $\bar{x} = 0$; for $\bar{x}_0 = 0.85$, the pressure is too low at $\bar{x} = 0$. When the final iteration of $\bar{x}_0 = 0.55$ is made, the target point $\bar{x} = 0$, $\bar{p} = 0$ is reached with sufficient accuracy. The solution requires a small computer program.

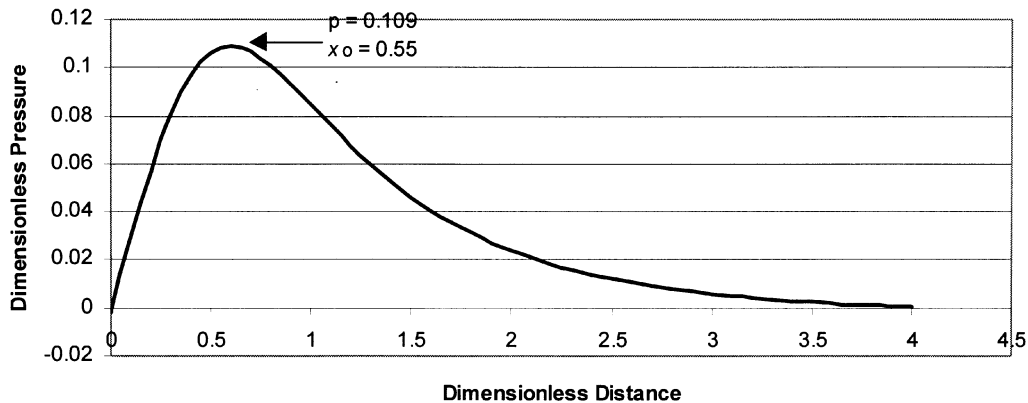


FIG. 6-8 Dimensionless pressure wave.

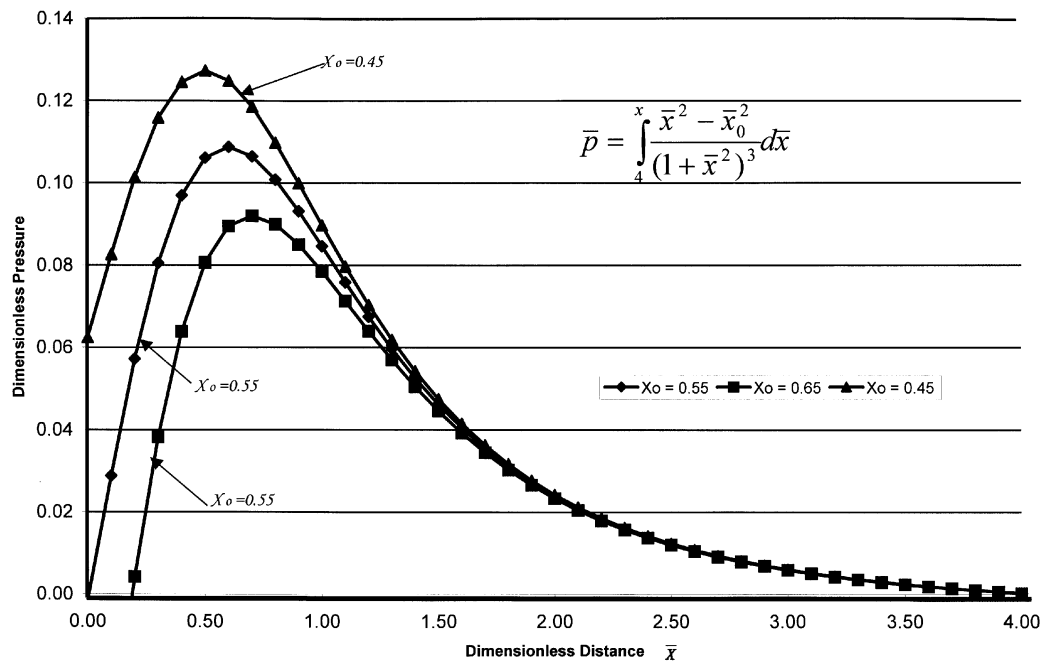


FIG. 6-9 Solution by iterations.

b. Load Capacity

The load capacity for one cylindrical section is solved by the equation

$$W = L \int_{(A)} p \, dx$$

Converting to dimensionless terms, the equation for the load capacity becomes

$$W = L \frac{\sqrt{2Rh_n}}{h_n^2} 6\mu U \sqrt{2Rh_n} \int_{(A)} \bar{p} \, d\bar{x}$$

For the practical boundaries of the pressure wave, the equation takes the form

$$W = L \frac{2R}{h_n} 6\mu U \int_4^0 \bar{p} \, d\bar{x}$$

c. Minimum Film Thickness

Solving for h_n , the minimum thickness between the ice and sled blade is

$$h_n = \frac{12LR\mu U}{W} \int_4^0 \bar{p} \, d\bar{x}$$

Numerical integration is performed based on the previous results of the dimensionless pressure in [Fig. 6-8](#). The result for the load capacity is obtained by numerical integration, in the boundaries 4 to 0:

$$\int_4^0 \bar{p} \, d\bar{x} \approx \sum_4^0 \bar{p}_i \Delta x_i = 0.147$$

The minimum film thickness is

$$h_n = \frac{12 \times 0.9 \, \text{m} \times 0.3 \, \text{m} \times 1.792 \times 10^{-3} (\text{N}\cdot\text{s}/\text{m}^2) \times 4.2 \, \text{m/s}}{500 \, \text{N}} (0.147)$$

$$h_n = 7.27 \times 10^{-6} \, \text{m} = 7.27 \times 10^{-3} \, \text{mm}$$

The result is: $h_n = 7.27 \, \mu\text{m}$.

Example Problem 6-2

Cylinder on a Flat Plate

A combination of a long cylinder of radius R and a flat plate surface are shown in Fig. 5-4. The cylinder rotates and slides on a plane (there is a combination of rolling and sliding), such as in the case of gears where there is a theoretical line contact with a combination of rolling and sliding. However, due to the hydrodynamic action, there is a small minimum clearance, h_n . The viscosity, μ , of the lubricant is constant, and the surfaces of the cylinder and flat plate are rigid.

Assume practical pressure boundary conditions [Eqs. (6-67)] and solve the pressure wave (use numerical iterations). Plot the dimensionless pressure-wave for various rolling and sliding ratios ξ .

Solution

In Chapter 4, the equation for the pressure gradient was derived. The following is the integration for the pressure wave. The clearance between a cylinder of radius R and a flat plate is discussed in Chapter 4; see Eq. (4-33). For a fluid film near the minimum clearance, the approximation for the clearance is

$$h(x) = h_{\min} + \frac{x^2}{2R}$$

The case of rolling and sliding is similar to that of Eq. (6-20), (see Section 6.4). The Reynolds equation is in the form

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6U(1 + \xi) \frac{\partial h}{\partial x}$$

Here, the coefficient ξ is the ratio of rolling and sliding. In terms of the velocities of the two surfaces, the ratio is

$$\xi = \frac{\omega R}{U}$$

The fluid film is much wider in the z direction in comparison to the length in the x direction. Therefore, the pressure gradient in the axial direction can be neglected in comparison to that in the x direction. The Reynolds equation is simplified to the form

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) = 6U(1 + \xi) \frac{\partial h}{\partial x}$$

The preceding equation is converted into dimensionless terms (see Example Problem 6-1):

$$\bar{x} = \frac{x}{\sqrt{2Rh_n}}, \quad \bar{h} = \frac{h}{h_n}, \quad \text{and} \quad \bar{h}_0 = \frac{h_0}{h_n}$$

$$\frac{dp}{dx} = \frac{6\mu U(1 + \xi)}{h_n^2} \frac{\bar{x}^2 - \bar{x}_0^2}{(1 + \bar{x}^2)^3}$$

Converting the pressure gradient to dimensionless form yields

$$\frac{h_n^2}{\sqrt{2Rh_n}6\mu U} dp = (1 + \xi) \frac{\bar{x}^2 - \bar{x}_0^2}{(1 + \bar{x}^2)^3} d\bar{x}$$

The left hand side of the equation is the dimensionless pressure:

$$\bar{p} = (1 + \xi) \frac{h_n^2}{\sqrt{2RH_n}6\mu U} \int_0^p dp = (1 + \xi) \int_{-\infty}^x \frac{\bar{x}^2 - \bar{x}_0^2}{(1 + \bar{x}^2)^3} d\bar{x} + p_0$$

Here, p_0 is a constant of integration, which is atmospheric pressure far from the minimum clearance. In this equation, p_0 and x_0 are two unknowns that can be solved for by the practical boundary conditions of the pressure wave; compare to Eqs. (6-67):

$$p = p_0 \quad \text{at} \quad x = x_1$$

$$\frac{dp}{dx} = 0 \quad \text{at} \quad x = x_2$$

$$p = 0 \quad \text{at} \quad x = x_2$$

Atmospheric pressure is zero, and the first boundary condition results in $p_0 = 0$. The location of the end of the pressure wave, x_2 , is solved by iterations. The solution is performed by guessing a value for $x = x_2$; then x_0 is taken as x_2 , because at that point the pressure gradient is zero.

The solution requires iterations in order to find $x_0 = x_2$, which satisfies the boundary conditions. For each iteration, integration is performed in the boundaries from 0 to x_2 , and the solution is obtained when the pressure at x_2 is very close to zero.

For numerical integration, the boundary $\bar{x}_1$, where the pressure is zero, is taken as a small value, such as $\bar{x}_1 = -4$. The solution is presented in Fig. 6-10. The curves indicate that the pressure wave is higher for higher rolling ratios. This means that the rolling plays a stronger role in hydrodynamic pressure generation in comparison to sliding.

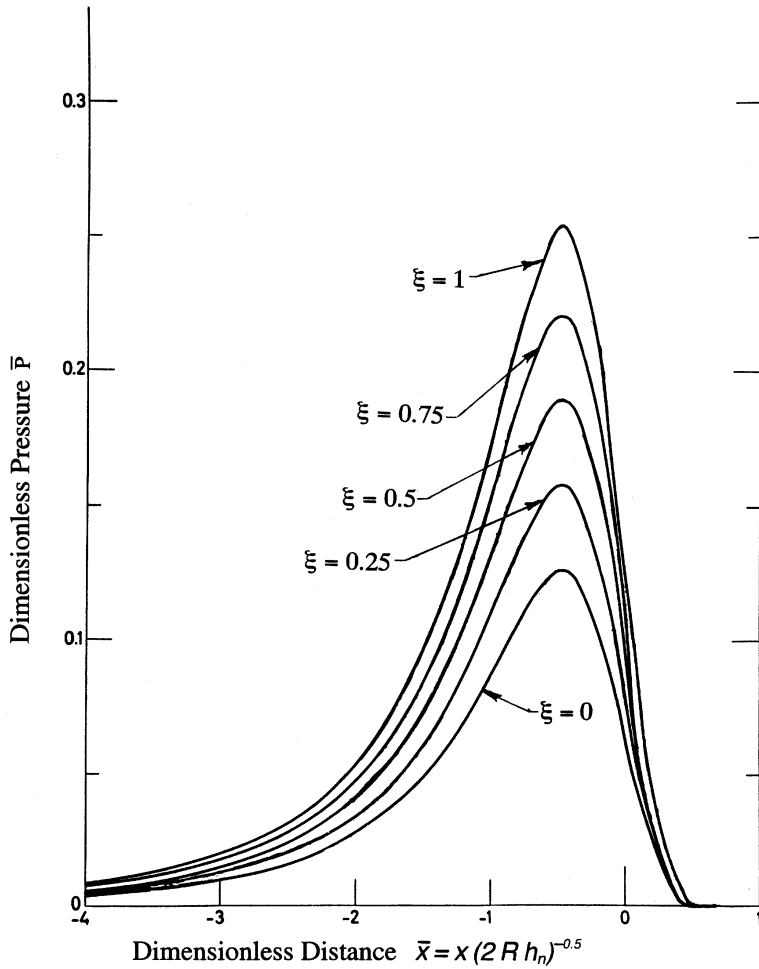


FIG. 6-10 Pressure wave along a fluid film between a cylinder and a flat plate for various rolling-to-sliding ratios.

Problems

- 6-1 A journal bearing is fed a by high-pressure external pump. The pump pressure is sufficient to avoid cavitation. The bearing length $L = 2D$. The diameter $D = 100$ mm, the shaft speed is 6000 RPM, and the clearance ratio is $C/R = 0.001$.

Assume that the infinitely-long-bearing analysis can be approximated for this bearing, and find the maximum load capacity for lubricant SAE 10 at average fluid film temperature of 80°C , if the maximum allowed eccentricity ratio $\varepsilon = 0.7$.

- 6-2 A flat plate slides on a lubricated cylinder as shown in Fig. 4-7. The cylinder radius is R , the lubricant viscosity is μ , and the minimum clearance between the stationary cylinder and plate is h_n . The elastic deformation of the cylinder and plate is negligible.
1. Apply numerical iterations, and plot the dimensionless pressure wave. Assume practical boundary conditions of the pressure, according to Eq. 6.67.
 2. Find the expression for the load capacity by numerical integration.
- 6-3 In problem 6-2, the cylinder diameter is 250 mm, the plate slides at $U = 0.5$ m/s, and the minimum clearance is $1\text{ }\mu\text{m}$ (0.001 mm). The lubricant viscosity is constant, $\mu = 10^{-4}$ N-s/m². Find the hydrodynamic load capacity.
- 6-4 Oil is fed into a journal bearing by a pump. The supply pressure is sufficiently high to avoid cavitation. The bearing operates at an eccentricity ratio of $\varepsilon = 0.85$, and the shaft speed is 60 RPM. The bearing length is $L = 3D$, the journal diameter is $D = 80$ mm, and the clearance ratio is $C/R = 0.002$. Assume that the pressure is constant along the bearing axis and there is no axial flow (long-bearing theory).
- a. Find the maximum load capacity for a lubricant SAE 20 operating at an average fluid film temperature of 60°C.
 - b. Find the bearing angle θ where there is a peak pressure.
 - c. What is the minimum supply pressure from the pump in order to avoid cavitation and to have only positive pressure around the bearing?
- 6-5 An air bearing operates inside a pressure vessel that has sufficiently high pressure to avoid cavitation in the bearing. The average viscosity of the air inside the bearing is $\mu = 2 \times 10^{-4}$ N-s/m². The bearing operates at an eccentricity ratio of $\varepsilon = 0.85$. The bearing length is $L = 2D$, the journal diameter is $D = 30$ mm, and the clearance ratio is $C/R = 8 \times 10^{-4}$. Assume that the pressure is constant along the bearing axis and there is no axial flow (long-bearing theory).
- a. Find the journal speed in RPM that is required for a bearing load capacity of 200 N. Find the bearing angle θ where there is a peak pressure.
 - b. What is the minimum ambient pressure around the bearing (inside the pressure vessel) in order to avoid cavitation and to have only positive pressure around the bearing?