

15

Hydrodynamic Bearings Under Dynamic Conditions

15.1 INTRODUCTION

Hydrodynamic journal bearings under steady conditions were discussed in the previous chapters. The equations for pressure wave and load capacity were limited to a constant, steady load and constant speed. Under steady conditions of constant load and speed, the journal center is at a stationary point defined by a constant eccentricity and attitude angle. Under dynamic conditions, however, such as oscillating load or variable speed, the journal center moves relative to the bearing.

Under harmonic oscillations such as sinusoidal load, the journal center moves in a *trajectory* that repeats itself during each cycle. This type of trajectory is referred to as *journal center locus*. In practice, bearings in machines are always subjected to some dynamic conditions. In rotating machinery, there are always vibrations due to the shaft imbalance. The machine is a dynamic system that has a spectrum of vibration frequencies. Vibrations in a machine result in small oscillating forces (inertial forces) on the bearings at various frequencies, which are superimposed on the main, steady load. If the magnitude of the dynamic forces is very small in comparison to the main, steady force, the dynamic forces are disregarded.

However, there are many important cases where the dynamic bearing performance is important and must be analyzed. For example, the effects of

bearing whirl near the critical speeds of the shaft can result in bearing failure. Dynamic analysis must always be performed in critical applications where bearing failure is expensive, such as the high cost of loss of production in generators or steam turbines or where there are safety considerations. In these cases, it is important for the design engineer to perform a dynamic analysis in order to predict undesired dynamic effects and prevent them by appropriate design.

In many machines the load is not steady. For example, the bearings in car engines are subjected to a cycle of a variable force that results from the combustion and inertial forces in the engine. There are many variable-speed machines that involve unsteady bearing performance, and even machines that operate at steady conditions are subjected to dynamic conditions during start-up and stopping.

In fact, most bearing failures result from an unexpected dynamic effect, such as a large vibration or severe disturbances. Engineers can improve the resistance of hydrodynamic bearings to unexpected dynamic effects by comparing the dynamic response of various bearing designs to scenarios of possible disturbances. An example of a unique design that improves the dynamic response is included in [Chapter 18](#).

15.2 ANALYSIS OF SHORT BEARINGS UNDER DYNAMIC CONDITIONS

The following is a dynamic analysis of a short bearing. Short bearings are widely used in many applications under dynamic conditions, including car engines. The dynamic analysis of a short bearing is relatively simple because the bearing load can be expressed by a closed-form equation, as shown in [Chapter 7](#). This analysis can be extended to a finite-length bearing, but the computations are more complex because the load capacity at each step must be determined by a numerical procedure.

The objective of a dynamic analysis is to solve for the trajectory of the journal center. The analysis involves the derivation of a set of differential equations and their solution by a finite-difference method with the aid of a computer program.

Dubois and Ocvirk (1953) solved the pressure distribution and load capacity of a short journal bearing under a steady load (see Chapter 7). This analysis is extended here to include unsteady conditions where the journal center, O_1 , has an arbitrary velocity. It is shown in [Fig. 15-1](#) that the journal center velocity is described by two components, $de/dt = C(d\varepsilon/dt)$ and $e(d\phi/dt)$, in the radial and tangential directions, respectively. The two assumptions of Dubois and Ocvirk for a steady short bearing are maintained here for dynamic conditions.

The velocities on the right-hand side of this equation are in Fig. 5-2. If the bearing is stationary and the shaft rotates, the fluid film boundary conditions on the bearing surface are

$$U_1 = 0, \quad V_1 = 0 \quad (15-2)$$

Under dynamic conditions, the velocity on the journal surface is a vector summation of the velocity of the journal center, O_1 , and the journal surface velocity relative to that center. The journal center has radial and tangential velocity components, as shown in Fig. 15-1. The components are $d\varepsilon/dt = C(d\varepsilon/dt)$ in the radial direction and $e(d\phi/dt)$ in the tangential direction. Summation of the velocity components of O_1 with that of the journal surface relative to O_1 results in the following components, U_2 and V_2 (see Fig. 5-2 for the direction of the components):

$$U_2 = \omega R + \frac{de}{dt} \sin \theta - e \frac{d\phi}{dt} \cos \theta \quad (15-3)$$

$$V_2 = \omega R \frac{dh}{dx} + \frac{de}{dt} \cos \theta + e \frac{d\phi}{dt} \sin \theta \quad (15-4)$$

Here, h is the fluid film thickness around a journal bearing, given by the equation

$$h = C(1 + \varepsilon \cos \theta) \quad (15-5)$$

After substitution of U_2 and V_2 as well as U_1 and V_1 in the right-hand side of the Reynolds equation, Eq. (15-1), the pressure distribution can be derived in a similar way to that of a steady short bearing. The load components are obtained by integrating the pressure in the converging clearance only ($0 < \theta < \pi$) as follows:

$$W_x = -2R \int_0^\pi \int_0^{L/2} p \cos \theta d\theta dz \quad (15-6)$$

$$W_y = 2R \int_0^\pi \int_0^{L/2} p \sin \theta d\theta dz \quad (15-7)$$

Converting into dimensionless terms, the equations for the two load capacity components (in the X and Y directions as shown in Fig. 15-1) become

$$\overline{W}_x = -0.5\varepsilon J_{12}|\overline{U}| + \varepsilon\dot{\phi}J_{12} + \dot{\varepsilon}J_{22} \quad (15-8)$$

$$\overline{W}_y = +0.5\varepsilon J_{11}\overline{U} - \varepsilon\dot{\phi}J_{11} - \dot{\varepsilon}J_{12} \quad (15-9)$$

Here, the dimensionless load capacity and velocity are

$$\overline{W} = \frac{C^2}{\mu U_0 L^3} W \quad \text{and} \quad \overline{U} = \frac{U}{U_0} \quad (15-10)$$

where $U = \omega R$ is the time-variable velocity of the journal surface and U_0 is a reference constant velocity used for normalizing the velocity. The integrals J_{ij} and their solutions are given in Eq. (7-13).

Under steady conditions, the external force, F , is equal to the bearing load capacity, W . However, under dynamic conditions, the resultant vector of the two forces accelerates the journal mass according to Newton's second law:

$$\vec{F} - \vec{W} = m\vec{a} \quad (15-11)$$

Here, m is the mass of the journal, $\vec{a}$ is the acceleration vector of the journal center, $\vec{F}$ is the external load, and $\vec{W}$ is the hydrodynamic load capacity. Under general dynamic conditions, $\vec{F}$ and $\vec{W}$ are not necessarily in the same direction, and both can be a function of time. In order to convert Eq. (15-11) to dimensionless terms, the following dimensionless variables are defined:

$$\overline{m} = \frac{C^3 U_0}{\mu L^3 R^2} m, \quad \overline{F} = \frac{C^2}{\mu U_0 L^3} F \quad (15-12)$$

Dividing Eq. (15-11) into two components in the directions of $\overline{F}_x$ and $\overline{F}_y$ (along X and Y but in opposite directions) and substituting the acceleration components in the radial and tangential directions in polar coordinates, the following two equations are obtained:

$$\overline{F}_x - \overline{W}_x = \overline{m}\ddot{\varepsilon} - \overline{m}\varepsilon\dot{\phi}^2 \quad (15-13)$$

$$\overline{F}_y - \overline{W}_y = -\overline{m}\varepsilon\ddot{\phi} - 2\overline{m}\dot{\varepsilon}\dot{\phi} \quad (15-14)$$

The minus signs in Eq. 15-14 are minus because $\overline{F}_y$ is in the opposite direction to the acceleration. Here, the dimensionless time is defined as $\overline{t} = \omega t$, and the dimensionless time derivatives are

$$\dot{\varepsilon} = \frac{1}{\omega} \frac{d\varepsilon}{dt}, \quad \dot{\phi} = \frac{1}{\omega} \frac{d\phi}{dt} \quad (15-15)$$

Substituting the values of the components of the load capacity, W_x and W_y , from Eqs. (15-8) and (15-9) into Eqs. (15-13) and (15-14), the following two differential equations for the journal center motion are obtained:

$$\overline{F}(t) \cos(\phi - \pi) = -0.5\varepsilon J_{12} |\overline{U}| + J_{12} \varepsilon \dot{\phi} + J_{22} \dot{\varepsilon} + \overline{m}\ddot{\varepsilon} - \overline{m}\varepsilon\dot{\phi}^2 \quad (15-16)$$

$$\overline{F}(t) \sin(\phi - \pi) = 0.5\varepsilon J_{11} \overline{U} - J_{11} \varepsilon \dot{\phi} - J_{12} \dot{\varepsilon} - \overline{m}\varepsilon\ddot{\phi} - 2\overline{m}\dot{\varepsilon}\dot{\phi} \quad (15-17)$$

Here, $\bar{F}(t)$ is a time-dependent dimensionless force acting on the bearing. The force (magnitude and direction) is a function of time. In the two equations, ε is the eccentricity ratio, ϕ is the attitude angle, and $\bar{m}$ is dimensionless mass, defined by Eq. (15-12). The definition of the integrals J_{ij} and their solution are in [Chapter 7](#).

Equations (15-16) and (15-17) are two differential equations required for the solutions of the two time-dependent functions ε and ϕ . The variables ε and ϕ represent the motion of the shaft center, O_1 , with time, *in polar coordinates*. The solution of the two equations as a function of time is finally presented as a plot of the trajectory of the journal center. If there are steady-state oscillations, such as sinusoidal force, after the initial transient, the trajectory becomes a closed locus that repeats itself each load cycle. A repeated trajectory is referred to as a *journal center locus*.

15.3 JOURNAL CENTER TRAJECTORY

The integration of Eqs. (15-16) and (15-17) is performed by finite differences with the aid of a computer program. Later, a computer graphics program is used to plot the journal center motion. The plot of the time variables ε and ϕ , in polar coordinates, represents the trajectory of the journal center motion relative to the bearing. The eccentricity ratio ε is a radial coordinate and ϕ is an angular coordinate.

Under harmonic conditions, such as sinusoidal load, the trajectory is a closed loop, referred to as a *locus*. Under harmonic oscillations of the load, there is initially a transient trajectory; and after a short time, a steady state is reached where the locus repeats itself during each cycle.

In heavily loaded bearings, the locus can approach the circle $\varepsilon = 1$, where there is a contact between the journal surface and the sleeve. The results allow comparison of various bearing designs. The design that results in a locus with a lower value of maximum eccentricity ratio ε is preferable, because it would resist more effectively any unexpected dynamic disturbances.

15.4 SOLUTION OF JOURNAL MOTION BY FINITE-DIFFERENCE METHOD

Equations (15-16) and (15-17) are the two differential equations that are solved for the function of ε versus ϕ . The two equations contain first- and second-order time derivatives and can be solved by a finite-difference procedure. The equations are not linear because the acceleration terms contain second-power time derivatives. Similar equations are widely used in dynamics and control, and commercial

software is available for numerical solution. However, the reader will find it beneficial to solve the equations by himself or herself, using a computer and any programming language that he or she prefers. The following is a demonstration of a solution by a simple finite-difference method.

The principle of the finite-difference solution method is the replacement of the time derivatives by the following finite-difference equations (for simplifying the finite difference procedure, $\bar{F}$, $\bar{m}$ and $\bar{t}$ are renamed F , m and t):

$$\dot{\phi}_n = \frac{\phi_{n+1} - \phi_{n-1}}{2 \Delta t}; \quad \dot{\varepsilon}_n = \frac{\varepsilon_{n+1} - \varepsilon_{n-1}}{2 \Delta t} \quad (15-18)$$

and the second time derivatives are

$$\ddot{\phi}_n = \frac{\phi_{n+1} - 2\phi_n + \phi_{n-1}}{\Delta t^2}; \quad \ddot{\varepsilon}_n = \frac{\varepsilon_{n+1} - 2\varepsilon_n + \varepsilon_{n-1}}{\Delta t^2} \quad (15-19)$$

For the nonlinear terms (the last term in the two equations), the equation can be linearized by using the following backward difference equations:

$$\dot{\phi}_n = \frac{\phi_n - \phi_{n-1}}{\Delta t} \quad (15-20)$$

By substituting the foregoing finite-element terms for the time-derivative terms, the two unknowns ε_{n+1} and ϕ_{n+1} can be solved as two unknowns in two regular linear equations.

After substitution, the differential equations become

$$\begin{aligned} F_x + \frac{1}{2} \varepsilon_n J_{12} = & \varepsilon_n J_{12} \left(\frac{\phi_{n+1} - \phi_{n-1}}{2 \Delta t} \right) + J_{22} \left(\frac{\varepsilon_{n+1} - \varepsilon_{n-1}}{2 \Delta t} \right) \\ & + m \left(\frac{\varepsilon_{n+1} - 2\varepsilon_n + \varepsilon_{n-1}}{\Delta t^2} \right) - m \varepsilon_n \left(\frac{\phi_n - \phi_{n-1}}{\Delta t} \right)^2 \end{aligned} \quad (15-21)$$

$$\begin{aligned} F_y - \frac{1}{2} \varepsilon_n J_{11} = & -\varepsilon_n J_{11} \left(\frac{\phi_{n+1} - \phi_{n-1}}{2 \Delta t} \right) - J_{12} \left(\frac{\varepsilon_{n+1} - \varepsilon_{n-1}}{2 \Delta t} \right) \\ & - m \varepsilon_n \left(\frac{\phi_{n+1} - 2\phi_n + \phi_{n-1}}{\Delta t^2} \right) - 2m \left(\frac{\varepsilon_{n+1} - \varepsilon_{n-1}}{2 \Delta t} \right) \\ & \times \left(\frac{\phi_n - \phi_{n-1}}{\Delta t} \right) \end{aligned} \quad (15-22)$$

Here, F_x and F_y are the external load components in the X and Y directions, respectively. Under dynamic conditions, the load components vary with time:

$$F_x = F(t)(\cos \phi - \pi) \quad (15-23)$$

$$F_y = F(t)(\sin \phi - \pi) \quad (15-24)$$

Equations (15-21) and (15-22) can be rearranged as two linear equations in terms of ε_{n+1} and ϕ_{n+1} as follows:

$$\text{Rearranging Eq. (15-21): } A = B\varepsilon_{n+1} + C\phi_{n+1} \quad (15-25)$$

$$\text{Rearranging Eq. (15-22): } P = R\varepsilon_{n+1} + Q\phi_{n+1} \quad (15-26)$$

In the following equations, F and m are dimensionless terms (the bar is omitted for simplification). The values of the coefficients of the unknown variables [in Eqs. (15-25) and (15-26)] are

$$A = F_x + \frac{\varepsilon_n J_{12}}{2} + \frac{\varepsilon_n J_{12} \phi_{n-1}}{2 \Delta t} + \frac{J_{22} \varepsilon_{n-1}}{2 \Delta t} + \frac{2m\varepsilon_n}{\Delta t^2} - \frac{m\varepsilon_{n-1}}{\Delta t^2} + m\varepsilon_n \left(\frac{\phi_n - \phi_{n-1}}{\Delta t} \right)^2 \quad (15-27)$$

$$B = \frac{J_{22}}{2 \Delta t} + \frac{m}{\Delta t^2} \quad (15-28)$$

$$C = \frac{\varepsilon_n J_{12}}{2 \Delta t} \quad (15-29)$$

$$P = F_y - \frac{\varepsilon_n J_{11}}{2} - \frac{\varepsilon_n J_{11} \phi_{n-1}}{2 \Delta t} - \frac{J_{12} \varepsilon_{n-1}}{2 \Delta t} - \frac{2m\varepsilon_n \phi_n}{\Delta t^2} + \frac{m\varepsilon_n \phi_{n-1}}{\Delta t^2} - \frac{m\varepsilon_{n-1} \phi_n}{\Delta t^2} + \frac{m\varepsilon_{n-1} \phi_{n-1}}{\Delta t^2} \quad (15-30)$$

$$Q = -\frac{\varepsilon_n J_{11}}{2 \Delta t} - \frac{m\varepsilon_n}{\Delta t^2} \quad (15-31)$$

$$R = -\frac{J_{12}}{2 \Delta t} - \frac{m}{\Delta t^2} \phi_n + \frac{m\phi_{n-1}}{\Delta t^2} \quad (15-32)$$

The numerical solution of the two equations for the two unknowns becomes

$$\varepsilon_{n+1} = \frac{AQ - PC}{BQ - RC} \quad (15-33)$$

$$\phi_{n+1} = \frac{AR - PB}{CR - QB} \quad (15-34)$$

The last two equations make it possible to march from the initial conditions and find ε_{n+1} and ϕ_{n+1} from any previous values, in dimensionless time intervals of $\Delta \bar{t} = \omega \Delta t$.

For a steady-state solution such as periodic load, the first two initial values of ε and ϕ can be selected arbitrarily. The integration of the equations must be conducted over sufficient cycles until the initial transient solution decays and a periodic steady-state solution is reached, i.e., when the periodic ε and ϕ will repeat at each cycle.

The following example is a solution for the locus of a short hydrodynamic bearing loaded by a sinusoidal force that is superimposed on a constant vertical load. The example compares the locus of a Newtonian and a viscoelastic fluid. The load is according to the equation

$$F(t) = 800 + 800 \sin 2\omega t \quad (15-35)$$

In this equation, ω is the journal angular speed. This means that the frequency of the oscillating load is twice that of the journal rotation. The direction of the load is constant, but its magnitude is a sinusoidal function. The dimensionless load is according to the definition in Eq. (15-12).

The dimensionless mass is $\bar{m} = 100$ and the journal velocity is constant. The resulting steady-state locus is shown in Fig. 15-2 by the full line for a Newtonian fluid. The dotted line is for a viscoelastic lubricant under identical

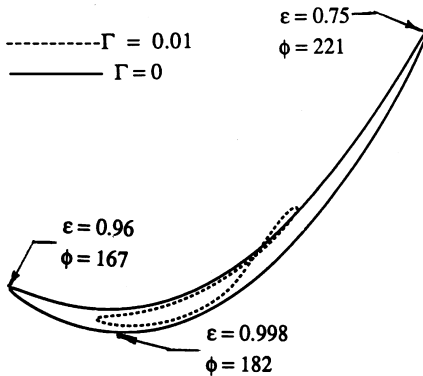


FIG. 15-2 Locus of the journal center for the load $F_t = 800 + 800 \sin 2\omega t$ and journal mass $\bar{m} = 100$.

conditions (see Chapt. 19). The viscoelastic lubricant is according to the Maxwell model in [Chapter 2](#) [Eq. (2-9)]. The dimensionless viscoelastic parameter Γ is

$$\Gamma = \lambda\omega \quad (15-36)$$

where λ is the relaxation time of the fluid and ω is the constant angular speed of the shaft. In this case, the result is dependent on the ratio of the load oscillation frequency, ω_1 , and the shaft angular speed, ω_1/ω .