

CHAPTER 1

Introduction: Dimensional Analysis: Similitude

If you have known one you have known all. (TERENCE, Phormio.)

Definition of a turbomachine

We classify as turbomachines all those devices in which energy is transferred either to, or from, a continuously flowing fluid by the *dynamic action* of one or more moving blade rows. The word *turbo* or *turbinis* is of Latin origin and implies that which spins or whirls around. Essentially, a rotating blade row, a *rotor* or an *impeller* changes the stagnation enthalpy of the fluid moving through it by either doing positive or negative work, depending upon the effect required of the machine. These enthalpy changes are intimately linked with the pressure changes occurring simultaneously in the fluid.

The definition of a turbomachine as stated above, is rather too general for the purposes of this book as it embraces *open* turbomachines such as propellers, wind turbines and unshrouded fans, all of which influence the state of a not readily quantifiable flow of a fluid. The subject *fluid mechanics, thermodynamics of turbomachinery*, therefore, is limited to machines enclosed by a closely fitting casing or shroud through which a readily measurable quantity of fluid passes in unit time. The subject of open turbomachines is covered by the classic text of Glauert (1959) or by Duncan *et al.* (1970), the elementary treatment of propellers by general fluid mechanics textbooks such as Streeter and Wylie (1979) or Massey (1979), and the important, still developing subject of wind turbines, by Freris (1990).

Two main categories of turbomachine are identified: firstly, those which *absorb* power to increase the fluid pressure or head (ducted fans, compressors and pumps); secondly, those that *produce* power by expanding fluid to a lower pressure or head (hydraulic, steam and gas turbines). Figure 1.1 shows, in a simple diagrammatic form, a selection of the many different varieties of turbomachine encountered in practice. The reason that so many different types of either pump (compressor) or turbine are in use is because of the almost infinite range of service requirements. Generally speaking, for a given set of operating requirements there is one type of pump or turbine best suited to provide optimum conditions of operation. This point is discussed more fully in the section of this chapter concerned with specific speed.

Turbomachines are further categorised according to the nature of the flow path through the passages of the rotor. When the path of the *through-flow* is wholly or mainly parallel to the axis of rotation, the device is termed an *axial flow turbomachine* (e.g.

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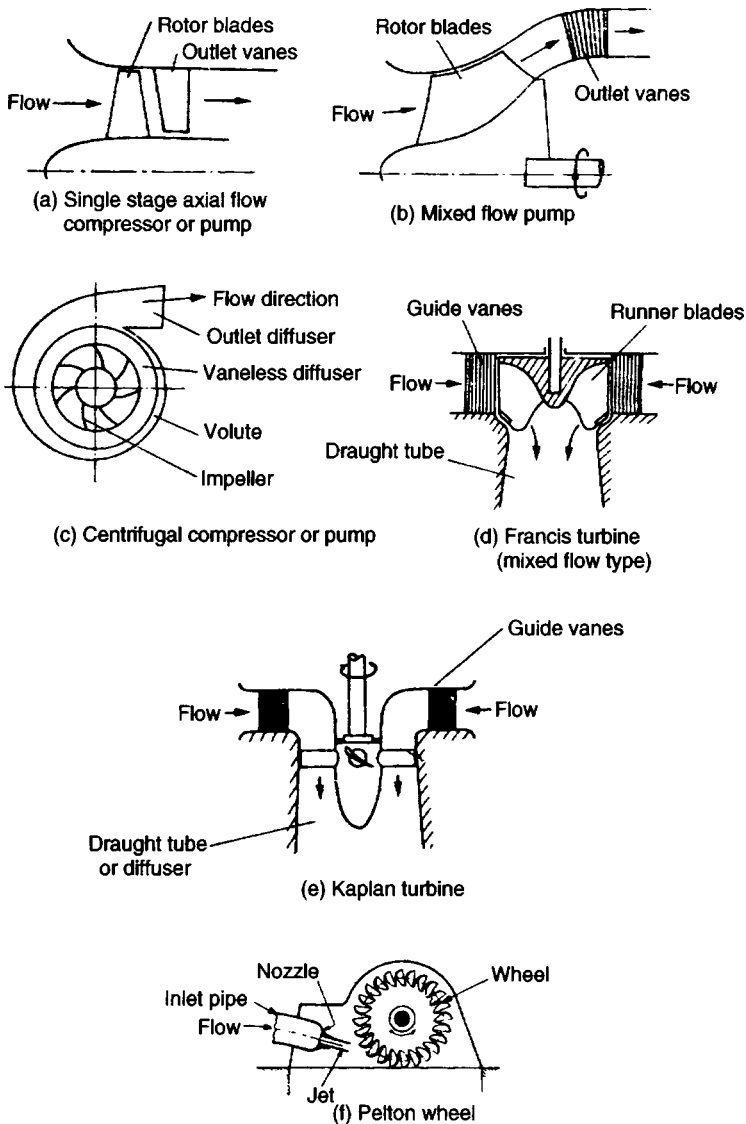


FIG. 1.1. Diagrammatic form of various types of turbomachine.

Figure 1.1(a) and (e)). When the path of the *through-flow* is wholly or mainly in a plane perpendicular to the rotation axis, the device is termed a *radial flow turbomachine* (e.g. Figure 1.1(c)). More detailed sketches of radial flow machines are given in Figures 7.1, 7.2, 8.2 and 8.3. *Mixed flow turbomachines* are widely used. The term *mixed flow* in this context refers to the direction of the through-flow at rotor outlet when both radial and axial velocity components are present in significant amounts. Figure 1.1(b) shows a mixed flow pump and Figure 1.1(d) a mixed flow hydraulic turbine.

One further category should be mentioned. All turbomachines can be classified as either *impulse* or *reaction* machines according to whether pressure changes are

absent or present respectively in the flow through the rotor. In an impulse machine all the pressure change takes place in one or more nozzles, the fluid being directed onto the rotor. The Pelton wheel, Figure 1.1(f), is an example of an impulse turbine.

The main purpose of this book is to examine, through the laws of fluid mechanics and thermodynamics, the means by which the energy transfer is achieved in the chief types of turbomachine, together with the differing behaviour of individual types in operation. Methods of analysing the flow processes differ depending upon the geometrical configuration of the machine, on whether the fluid can be regarded as incompressible or not, and whether the machine absorbs or produces work. As far as possible, a unified treatment is adopted so that machines having similar configurations and function are considered together.

Units and dimensions

The International System of Units, SI (le Système International d'Unités) is a unified self-consistent system of measurement units based on the MKS (metre–kilogram–second) system. It is a simple, logical system based upon decimal relationships between units making it easy to use. The most recent detailed description of SI has been published in 1986 by HMSO. For an explanation of the relationship between, and use of, physical quantities, units and numerical values see *Quantities, Units and Symbols*, published by The Royal Society (1975) or refer to ISO 31/0-1981.

Great Britain was the first of the English-speaking countries to begin, in the 1960s, the long process of abandoning the old Imperial System of Units in favour of the International System of Units, and was soon followed by Canada, Australia, New Zealand and South Africa. In the USA a ten year voluntary plan of conversion to SI units was commenced in 1971. In 1975 US President Ford signed the Metric Conversion Act which coordinated the metrication of units, but did so without specifying a schedule of conversion. Industries heavily involved in international trade (cars, aircraft, food and drink) have, however, been quick to change to SI for obvious economic reasons, but others have been reluctant to change.

SI has now become established as the only system of units used for teaching engineering in colleges, schools and universities in most industrialised countries throughout the world. The Imperial System was derived arbitrarily and has no consistent numerical base, making it confusing and difficult to learn. In this book all numerical problems involving units are performed in metric units as this is more convenient than attempting to use a mixture of the two systems. However, it is recognised that some problems exist as a result of the conversion to SI units. One of these is that many valuable papers and texts written prior to 1969 contain data in the old system of units and would need converting to SI units. A brief summary of the conversion factors between the more frequently used Imperial units and SI units is given in Appendix 1 of this book.

Some SI units

The SI basic units used in fluid mechanics and thermodynamics are the *metre* (m), *kilogram* (kg), *second* (s) and *thermodynamic temperature* (K). All the other units used in this book are derived from these basic units. The *unit of force* is the

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newton (N), defined as that force which, when applied to a mass of 1 kilogram, gives an acceleration to the mass of 1 m/s^2 . The recommended *unit of pressure* is the *pascal* (Pa) which is the pressure produced by a force of 1 newton uniformly distributed over an area of 1 square metre. Several other units of pressure are in widespread use, however, foremost of these being the *bar*. Much basic data concerning properties of substances (steam and gas tables, charts, etc.) have been prepared in SI units with pressure given in bars and it is acknowledged that this alternative unit of pressure will continue to be used for some time as a matter of expediency. It is noted that 1 bar equals 10^5 Pa (i.e. 10^5 N/m^2), roughly the pressure of the atmosphere at sea level, and is perhaps an inconveniently large unit for pressure in the field of turbomachinery anyway! In this book the convenient size of the *kilopascal* (kPa) is found to be the most useful multiple of the recommended unit and is extensively used in most calculations and examples.

In SI the units of all forms of energy are the same as for work. The *unit of energy* is the *joule* (J) which is the work done when a force of 1 newton is displaced through a distance of 1 metre in the direction of the force, e.g. kinetic energy ($\frac{1}{2}mc^2$) has the dimensions $\text{kg} \times \text{m}^2/\text{s}^2$; however, $1 \text{ kg} = 1 \text{ N s}^2/\text{m}$ from the definition of the newton given above. Hence, the units of kinetic energy must be $\text{Nm} = \text{J}$ upon substituting dimensions.

The *watt* (W) is the *unit of power*; when 1 watt is applied for 1 second to a system the input of energy to that system is 1 joule (i.e. 1 J).

The *hertz* (Hz) is the number of repetitions of a regular occurrence in 1 second. Instead of writing c/s for cycles/sec, Hz is used instead.

The *unit of thermodynamic temperature* is the *kelvin* (K), written without the $^\circ$ sign, and is the fraction $1/273.16$ of the thermodynamic temperature of the triple point of water. The degree celsius ($^\circ\text{C}$) is equal to the unit kelvin. Zero on the celsius scale is the temperature of the ice point (273.15 K). Specific heat capacity, or simply specific heat, is expressed as J/kg K or as $\text{J/kg}^\circ\text{C}$.

Dynamic viscosity, dimensions $ML^{-1}T^{-1}$, has the SI units of pascal seconds, i.e.

$$\frac{M}{LT} \equiv \frac{\text{kg}}{\text{m.s}} = \frac{\text{N.s}^2}{\text{m}^2\text{s}} = \text{Pa s}.$$

Hydraulic engineers find it convenient to express pressure in terms of *head* of a liquid. The static pressure at any point in a liquid at rest is, relative to the pressure acting on the free surface, proportional to the vertical distance of the free surface above that point. The head H is simply the height of a column of the liquid which can be supported by this pressure. If ρ is the mass density (kg/m^3) and g the local gravitational acceleration (m/s^2), then the static pressure p (relative to atmospheric pressure) is $p = \rho gH$, where H is in metres and p is in pascals (or N/m^2). This is left for the student to verify as a simple exercise.

Dimensional analysis and performance laws

The widest comprehension of the general behaviour of all turbomachines is, without doubt, obtained from *dimensional analysis*. This is the formal procedure whereby the group of variables representing some physical situation is reduced

into a smaller number of dimensionless groups. When the number of independent variables is not too great, dimensional analysis enables experimental relations between variables to be found with the greatest economy of effort. Dimensional analysis applied to turbomachines has two further important uses: (a) prediction of a prototype's performance from tests conducted on a scale model (similitude); (b) determination of the most suitable type of machine, on the basis of maximum efficiency, for a specified range of head, speed and flow rate. Several methods of constructing non-dimensional groups have been described by Douglas *et al.* (1995) and by Shames (1992) among other authors. The subject of dimensional analysis was made simple and much more interesting by Edward Taylor (1974) in his comprehensive account of the subject. It is assumed here that the basic techniques of forming non-dimensional groups have already been acquired by the student.

Adopting the simple approach of elementary thermodynamics, an imaginary envelope (called a *control surface*) of fixed shape, position and orientation is drawn around the turbomachine (Figure 1.2). Across this boundary, fluid flows steadily, entering at station 1 and leaving at station 2. As well as the flow of fluid there is a flow of work across the control surface, transmitted by the shaft either to, or from, the machine. For the present all details of the flow within the machine can be ignored and only externally observed features such as shaft speed, flow rate, torque and change in fluid properties across the machine need be considered. To be specific, let the turbomachine be a *pump* (although the analysis could apply to other classes of turbomachine) driven by an electric motor. The speed of rotation N , can be adjusted by altering the current to the motor; the volume flow rate Q , can be *independently* adjusted by means of a throttle valve. For fixed values of the set Q and N , all other variables such as torque τ , head H , are thereby established. The choice of Q and N as *control variables* is clearly arbitrary and any other pair of independent variables such as τ and H could equally well have been chosen. The important point to recognise is, that there are for this pump, *two* control variables.

If the fluid flowing is changed for another of different density ρ , and viscosity μ , the performance of the machine will be affected. Note, also, that for a turbomachine handling compressible fluids, other *fluid properties* are important and are discussed later.

So far we have considered only one particular turbomachine, namely a pump of a given size. To extend the range of this discussion, the effect of the *geometric*

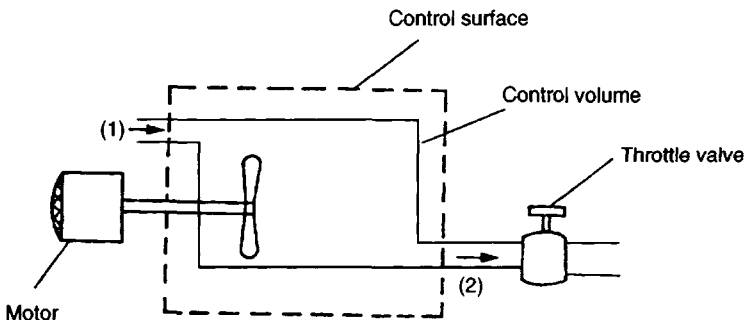


FIG. 1.2. Turbomachine considered as a control volume.

variables on the performance must now be included. The size of machine is characterised by the impeller diameter D , and the shape can be expressed by a number of length ratios, l_1/D , l_2/D , etc.

Incompressible fluid analysis

The performance of a turbomachine can now be expressed in terms of the control variables, geometric variables and fluid properties. For the hydraulic pump it is convenient to regard the net energy transfer gH , the efficiency η , and power supplied P , as dependent variables and to write the three functional relationships as

$$gH = f_1 \left(Q, N, D, \rho, \mu, \frac{l_1}{D}, \frac{l_2}{D}, \dots \right), \quad (1.1a)$$

$$\eta = f_2 \left(Q, N, D, \rho, \mu, \frac{l_1}{D}, \frac{l_2}{D}, \dots \right), \quad (1.1b)$$

$$P = f_3 \left(Q, N, D, \rho, \mu, \frac{l_1}{D}, \frac{l_2}{D}, \dots \right), \quad (1.1c)$$

By the procedure of dimensional analysis using the three primary dimensions, mass, length and time, or alternatively, using three of the independent variables we can form the dimensionless groups. The latter, more direct procedure, requires that the variables selected, ρ , N , D , do not of themselves form a dimensionless group. The selection of ρ , N , D as common factors avoids the appearance of special fluid terms (e.g. μ , Q) in more than one group and allows gH , η and P to be made explicit. Hence the three relationships reduce to the following easily verified forms.

Energy transfer coefficient, sometimes called head coefficient

$$\psi = \frac{gH}{(ND)^2} = f_4 \left(\frac{Q}{ND^3}, \frac{\rho ND^2}{\mu}, \frac{l_1}{D}, \frac{l_2}{D}, \dots \right), \quad (1.2a)$$

$$\eta = f_5 \left(\frac{Q}{ND^3}, \frac{\rho ND^2}{\mu}, \frac{l_1}{D}, \frac{l_2}{D}, \dots \right). \quad (1.2b)$$

Power coefficient

$$\hat{P} = \frac{P}{\rho N^3 D^5} = f_6 \left(\frac{Q}{ND^3}, \frac{\rho ND^2}{\mu}, \frac{l_1}{D}, \frac{l_2}{D}, \dots \right). \quad (1.2c)$$

The non-dimensional group $Q/(ND^3)$ is a volumetric flow coefficient and $\rho ND^2/\mu$ is a form of Reynolds number, Re . In axial flow turbomachines, an alternative to $Q/(ND^3)$ which is frequently used is the velocity (or flow) coefficient $\phi = c_x/U$ where U is blade tip speed and c_x the average axial velocity. Since

$$Q = c_x \times \text{flow area} \propto c_x D^2$$

and $U \propto ND$.

then

$$\frac{Q}{ND^3} \propto \frac{c_x}{U}.$$

Because of the large number of independent groups of variables on the right-hand side of eqns. (1.2), those relationships are virtually worthless unless certain terms can be discarded. In a family of *geometrically similar* machines l_1/D , l_2/D are constant and may be eliminated forthwith. The kinematic viscosity, $\nu = \mu/\rho$ is very small in turbomachines handling water and, although speed, expressed by ND , is low the Reynolds number is correspondingly high. Experiments confirm that effects of Reynolds number on the performance are small and may be ignored in a first approximation. The functional relationships for geometrically similar hydraulic turbomachines are then,

$$\psi = f_4[Q/(ND^3)] \quad (1.3a)$$

$$\eta = f_5[Q/(ND^3)] \quad (1.3b)$$

$$\hat{P} = f_6[Q/(ND^3)]. \quad (1.3c)$$

This is as far as the reasoning of dimensional analysis alone can be taken; the actual form of the functions f_4 , f_5 and f_6 must be ascertained by experiment.

One relation between ψ , ϕ , η and $\hat{P}$ may be immediately stated. For a pump the *net hydraulic power*, P_N equals ρQgH which is the minimum shaft power required in the absence of all losses. No real process of power conversion is free of losses and the actual shaft power P must be larger than P_N . We define pump efficiency (more precise definitions of efficiency are stated in Chapter 2) $\eta = P_N/P = \rho QgH/P$. Therefore

$$P = \frac{1}{\eta} \left(\frac{Q}{ND^3} \right) \frac{gH}{(ND)^2} \rho N^3 D^5. \quad (1.4)$$

Thus f_6 may be derived from f_4 and f_5 since $\hat{P} = \phi\psi/\eta$. For a turbine the net hydraulic power P_N supplied is greater than the actual shaft power delivered by the machine and the efficiency $\eta = P/P_N$. This can be rewritten as $\hat{P} = \eta\phi\psi$ by reasoning similar to the above considerations.

Performance characteristics

The operating condition of a turbomachine will be *dynamically similar* at two different rotational speeds if all fluid velocities at *corresponding points* within the machine are in the same direction and proportional to the blade speed. If two points, one on each of two different head-flow characteristics, represent dynamically similar operation of the machine, then the non-dimensional groups of the variables involved, ignoring Reynolds number effects, may be expected to have the same numerical value for both points. On this basis, non-dimensional presentation of performance data has the important practical advantage of collapsing into virtually a single curve, results that would otherwise require a multiplicity of curves if plotted dimensionally.

Evidence in support of the foregoing assertion is provided in Figure 1.3 which shows experimental results obtained by the author (at the University of Liverpool) on a simple centrifugal laboratory pump. Within the normal operating range of this pump, $0.03 < Q/(ND^3) < 0.06$, very little systematic scatter is apparent which

might be associated with a Reynolds number effect, for the range of speeds $2500 \leq N \leq 5000$ rev/min. For smaller flows, $Q/(ND^3) < 0.025$, the flow became unsteady and the manometer readings of uncertain accuracy but, nevertheless, dynamically similar conditions still appear to hold true. Examining the results at high flow rates one is struck by a marked systematic deviation away from the “single-curve” law at increasing speed. This effect is due to *cavitation*, a high speed phenomenon of hydraulic machines caused by the release of vapour bubbles at low pressures, which is discussed later in this chapter. It will be clear at this stage that under cavitating flow conditions, dynamical similarity is not possible.

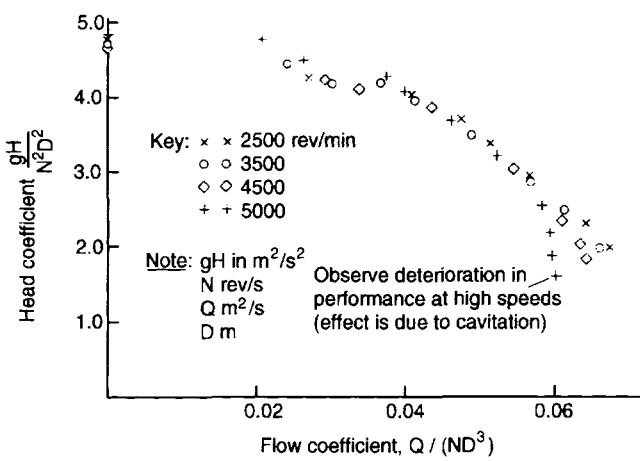


FIG. 1.3. Dimensionless head-volume characteristic of a centrifugal pump.

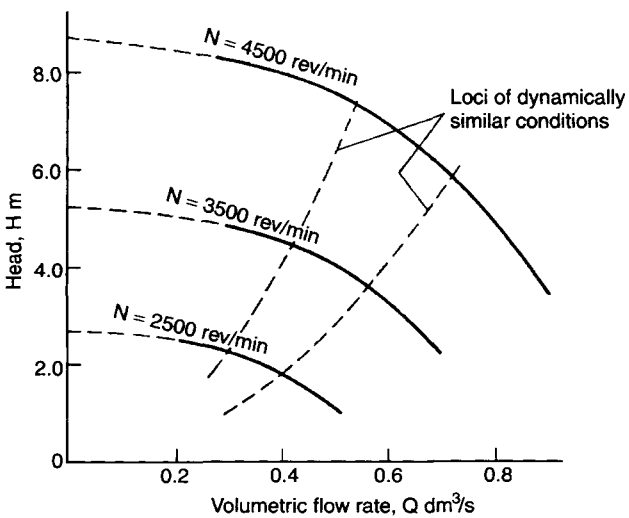


FIG. 1.4. Extrapolation of characteristic curves for dynamically similar conditions at $N = 3500$ rev/min.

The non-dimensional results shown in Figure 1.3 have, of course, been obtained for a particular pump. They would also be approximately valid for a range of *different* pump sizes so long as all these pumps are geometrically similar and cavitation is absent. Thus, neglecting any change in performance due to change in Reynolds number, the dynamically similar results in Figure 1.3 can be applied to predicting the dimensional performance of a given pump for a series of required speeds. Figure 1.4 shows such a dimensional presentation. It will be clear from the above discussion that the locus of dynamically similar points in the H - Q field lies on a parabola since H varies as N^2 and Q varies as N .

Variable geometry turbomachines

The efficiency of a fixed geometry machine, ignoring Reynolds number effects, is a unique function of flow coefficient. Such a dependence is shown by line (b) in Figure 1.5. Clearly, off-design operation of such a machine is grossly inefficient and designers sometimes resort to a *variable geometry* machine in order to obtain a better match with changing flow conditions. Figure 1.6 shows a sectional sketch of a mixed-flow pump in which the impeller vane angles may be varied *during* pump operation. (A similar arrangement is used in Kaplan turbines, Figure 1.1.) Movement of the vanes is implemented by cams driven from a servomotor. In some very large installations involving many thousands of kilowatts and where operating

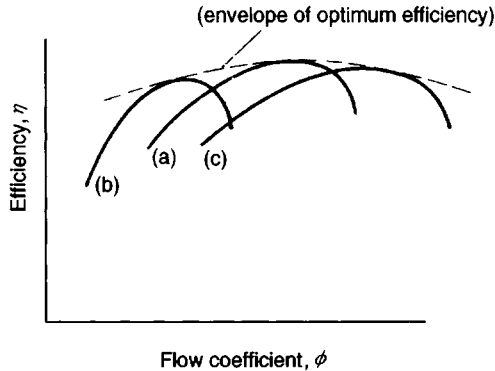


FIG. 1.5. Different efficiency curves for a given machine obtained with various blade settings.

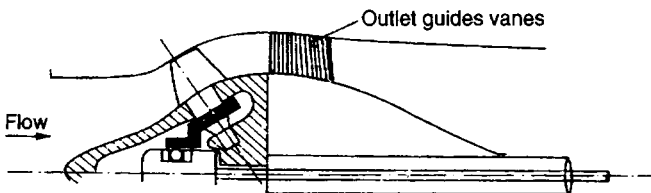


FIG. 1.6. Mixed-flow pump incorporating mechanism for adjusting blade setting.

conditions fluctuate, sophisticated systems of control may incorporate an electronic computer.

The lines (a) and (c) in Figure 1.5 show the efficiency curves at other blade settings. Each of these curves represents, in a sense, a different constant geometry machine. For such a variable geometry pump the desired operating line intersects the points of maximum efficiency of each of these curves.

Introducing the additional variable β into eqn. (1.3) to represent the setting of the vanes, we can write

$$\psi = f_1(\phi, \beta); \eta = f_2(\phi, \beta). \quad (1.5)$$

Alternatively, with $\beta = f_3(\phi, \eta) = f_4(\phi, \psi)$, β can be eliminated to give a new functional dependence

$$\eta = f_5(\phi, \psi) = f_5\left(\frac{Q}{ND^3}, \frac{gH}{N^2D^2}\right) \quad (1.6)$$

Thus, efficiency in a variable geometry pump is a function of both flow coefficient and energy transfer coefficient.

Specific speed

The pump or hydraulic turbine designer is often faced with the basic problem of deciding what type of turbomachine will be the best choice for a given duty. Usually the designer will be provided with some preliminary design data such as the head H , the volume flow rate Q and the rotational speed N when a pump design is under consideration. When a turbine preliminary design is being considered the parameters normally specified are the shaft power P , the head at turbine entry H and the rotational speed N . A non-dimensional parameter called the *specific speed*, N_s , referred to and conceptualised as the *shape number*, is often used to facilitate the choice of the most appropriate machine. This new parameter is derived from the non-dimensional groups defined in eqn. (1.3) in such a way that the characteristic diameter D of the turbomachine is eliminated. The value of N_s gives the designer a guide to the type of machine that will provide the normal requirement of high efficiency at the design condition.

For any one hydraulic turbomachine *with fixed geometry* there is a unique relationship between efficiency and flow coefficient if Reynolds number effects are negligible and cavitation absent. As is suggested by any one of the curves in Figure 1.5, the efficiency rises to a maximum value as the flow coefficient is increased and then gradually falls with further increase in ϕ . This optimum efficiency $\eta = \eta_{\max}$, is used to identify a unique value $\phi = \phi_1$ and corresponding unique values of $\psi = \psi_1$ and $\hat{P} = \hat{P}_1$. Thus,

$$\frac{Q}{ND^3} = \phi_1 = \text{constant}, \quad (1.7a)$$

$$\frac{gH}{N^2D^2} = \psi_1 = \text{constant}, \quad (1.7b)$$

$$\frac{P}{\rho N^3 D^5} = \hat{P}_1 = \text{constant}. \quad (1.7c)$$

It is a simple matter to combine any pair of these expressions in such a way as to eliminate the diameter. For a pump the customary way of eliminating D is to divide $\phi_1^{1/2}$ by $\psi_1^{3/4}$. Thus

$$N_s = \frac{\phi_1^{1/2}}{\psi_1^{3/4}} = \frac{NQ^{1/2}}{(gH)^{3/4}}, \quad (1.8)$$

where N_s is called the *specific speed*. The term specific speed is justified to the extent that N_s is directly proportional to N . In the case of a turbine the *power specific speed* N_{sp} , is more useful and is defined by,

$$N_{sp} = \frac{\hat{P}_1^{1/2}}{\psi_1^{5/4}} = \frac{N(P/\rho)^{1/2}}{(gH)^{5/4}} \quad (1.9)$$

Both eqns. (1.8) and (1.9) are *dimensionless*. It is always safer and less confusing to calculate specific speed in one or other of these forms rather than dropping the factors g and ρ which would make the equations *dimensional* and any values of specific speed obtained using them would then depend upon the choice of the units employed. The dimensionless form of N_s (and N_{sp}) is the only one used in this book. Another point arises from the fact that the rotational speed, N , is expressed in the units of revolutions per unit of time so that although N_s is dimensionless, numerical values of specific speed need to be thought of as revs. Alternative versions of eqns. (1.8) and (1.9) in radians are also in common use and are written

$$\Omega_s = \frac{\Omega Q^{1/2}}{(gH)^{3/4}}, \quad (1.8a)$$

$$\Omega_{sp} = \frac{\Omega \sqrt{P/\rho}}{(gH)^{5/4}}. \quad (1.9a)$$

There is a simple connection between N_s and N_{sp} (and between Ω_s and Ω_{sp}). By dividing eqn. (1.9) by eqn. (1.8) we obtain

$$\frac{N_{sp}}{N_s} = \frac{N(P/\rho)^{1/2}}{(gH)^{5/4}} \frac{(gH)^{3/4}}{NQ^{1/2}} = \left(\frac{P}{\rho g Q H} \right)^{1/2}.$$

From the definition of hydraulic efficiency, for a pump we obtain:

$$\frac{N_{sp}}{N_s} = \frac{\Omega_{sp}}{\Omega_s} = \frac{1}{\sqrt{\eta}}, \quad (1.9b)$$

and, for a turbine we obtain:

$$\frac{N_{sp}}{N_s} = \frac{\Omega_{sp}}{\Omega_s} = \sqrt{\eta}. \quad (1.9c)$$

Remembering that specific speed, as defined above, is at the point of maximum efficiency of a turbomachine, it becomes a parameter of great importance in selecting the type of machine required for a given duty. The maximum efficiency condition *replaces* the condition of geometric similarity, so that any alteration in specific

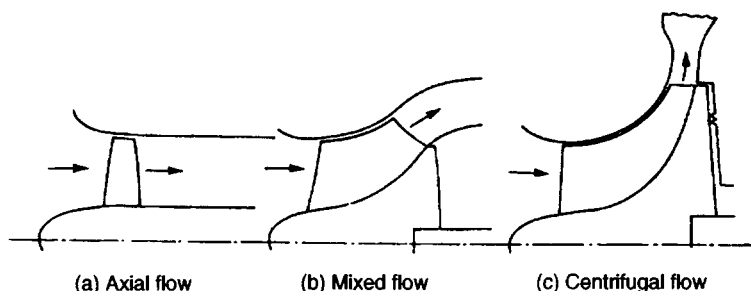


FIG. 1.7. Range of pump impellers of equal inlet area.

speed implies that the machine design changes. Broadly speaking, each different class of machine has its optimum efficiency within its own fairly narrow range of specific speed.

For a pump, eqn. (1.8) indicates, for constant speed N , that N_s is increased by an increase in Q and decreased by an increase in H . From eqn. (1.7b) it is observed that H , at a constant speed N , increased with impeller diameter D . Consequently, to increase N_s the entry area must be made large and/or the maximum impeller diameter small. Figure 1.7 shows a range of pump impellers varying from the axial-flow type, through mixed flow to a centrifugal- or radial-flow type. The size of each inlet is such that they all handle the same volume flow Q . Likewise, the head developed by each impeller (of different diameter D) is made equal by adjusting the speed of rotation N . Since Q and H are constant, then N_s varies with N alone. The most noticeable feature of this comparison is the large change in size with specific speed. Since a higher specific speed implies a smaller machine, for reasons of economy, it is desirable to select the *highest possible specific speed* consistent with good efficiency.

Cavitation

In selecting a hydraulic turbomachine for a given head H and capacity Q , it is clear from the definition of specific speed, eqn. (1.8), that the highest possible value of N_s should be chosen because of the resulting reduction in size, weight and cost. On this basis a turbomachine could be made extremely small were it not for the corresponding increase in the fluid velocities. For machines handling liquids the lower limit of size is dictated by the phenomenon of *cavitation*.

Cavitation is the boiling of a liquid at normal temperature when the static pressure is made sufficiently low. It may occur at the entry to pumps or at the exit from hydraulic turbines in the vicinity of the moving blades. The dynamic action of the blades causes the static pressure to reduce locally in a region which is already normally below atmospheric pressure and cavitation can commence. The phenomenon is accentuated by the presence of dissolved gases which are released with a reduction in pressure.

For the purpose of illustration consider a centrifugal pump operating at constant speed and capacity. By steadily reducing the inlet pressure head a point is reached

when streams of small vapour bubbles appear within the liquid and close to solid surfaces. This is called *cavitation inception* and commences in the regions of lowest pressure. These bubbles are swept into regions of higher pressure where they collapse. This condensation occurs suddenly, the liquid surrounding the bubbles either hitting the walls or adjacent liquid. The pressure wave produced by bubble collapse (with a magnitude of the order 400 MPa) momentarily raises the pressure level in the vicinity and the action ceases. The cycle then repeats itself and the frequency may be as high as 25 kHz (Shepherd 1956). The repeated action of bubbles collapsing near solid surfaces leads to the well-known cavitation erosion.

The collapse of vapour cavities generates noise over a wide range of frequencies – up to 1 MHz has been measured (Pearsall 1972) i.e. so-called “white noise”. Apparently it is the collapsing smaller bubbles which cause the higher frequency noise and the larger cavities the lower frequency noise. Noise measurement can be used as a means of detecting cavitation (Pearsall 1966/7). Pearsall and McNulty (1968) have shown experimentally that there is a relationship between cavitation noise levels and erosion damage on cylinders and concludes that a technique could be developed for predicting the occurrence of erosion.

Up to this point no detectable deterioration in performance has occurred. However, with further reduction in inlet pressure, the bubbles increase both in size and number, coalescing into pockets of vapour which affects the whole field of flow. This growth of vapour cavities is usually accompanied by a sharp drop in pump performance as shown conclusively in Figure 1.3 (for the 5000 rev/min test data). It may seem surprising to learn that with this large change in bubble size, the solid surfaces are much less likely to be damaged than at inception of cavitation. The avoidance of cavitation inception in conventionally designed machines can be regarded as one of the essential tasks of both pump and turbine designers. However, in certain recent specialised applications pumps have been designed to operate under *supercavitating* conditions. Under these conditions large size vapour bubbles are formed but, bubble collapse takes place *downstream* of the impeller blades. An example of the specialised application of a supercavitating pump is the fuel pumps of rocket engines for space vehicles where size and mass must be kept low at all costs. Pearsall (1966) has shown that the supercavitating principle is most suitable for axial flow pumps of high specific speed and has suggested a design technique using methods similar to those employed for conventional pumps.

Pearsall (1966) was one of the first to show that operating in the supercavitating regime was practicable for axial flow pumps and he proposed a design technique to enable this mode of operation to be used. A detailed description was later published (Pearsall 1973), and the cavitation performance was claimed to be much better than that of conventional pumps. Some further details are given in Chapter 7 of this book.

Cavitation limits

In theory cavitation commences in a liquid when the static pressure is reduced to the vapour pressure corresponding to the liquid’s temperature. However, in practice, the physical state of the liquid will determine the pressure at which cavitation starts (Pearsall 1972). Dissolved gases come out of solution as the pressure is reduced forming gas cavities at pressures in excess of the vapour pressure. Vapour cavitation requires the presence of nuclei – submicroscopic gas bubbles or solid non-wetted

particles – in sufficient numbers. It is an interesting fact that in the absence of such nuclei a liquid can withstand negative pressures (i.e. *tensile stresses*)! Perhaps the earliest demonstration of this phenomenon was that performed by Osborne Reynolds (1882) before a learned society. He showed how a column of mercury more than twice the height of the barometer could be (and was) supported by the internal cohesion (stress) of the liquid. More recently Ryley (1980) devised a simple centrifugal apparatus for students to test the tensile strength of both plain, untreated tap water in comparison with water that had been filtered and then de-aerated by boiling. Young (1989) gives an extensive literature list covering many aspects of cavitation including the tensile strength of liquids. At room temperature the theoretical tensile strength of water is quoted as being as high as 1000 atm (100 MPa)! Special pre-treatment (i.e. rigorous filtration and pre-pressurization) of the liquid is required to obtain this state. In general the liquids flowing through turbomachines will contain some dust and dissolved gases and under these conditions negative pressure do not arise.

A useful parameter is the available suction head at entry to a pump or at exit from a turbine. This is usually referred to as the *net positive suction head*, NPSH, defined as

$$H_s = (p_o - p_v)/(\rho g) \quad (1.10)$$

where p_o and p_v are the absolute stagnation and vapour pressures, respectively, at pump inlet or at turbine outlet.

To take into account the effects of cavitation, the performance laws of a hydraulic turbomachine should include the additional independent variable H_s . Ignoring the effects of Reynolds number, the performance laws of a constant geometry hydraulic turbomachine are then dependent on two groups of variable. Thus, the efficiency,

$$\eta = f(\phi, N_{ss}) \quad (1.11)$$

where the *suction specific speed* $N_{ss} = NQ^{1/2}/(gH_s)^{3/4}$, determines the effect of cavitation, and $\phi = Q/(ND^3)$, as before.

It is known from experiment that cavitation inception occurs for an almost constant value of N_{ss} for all pumps (and, separately, for all turbines) designed to resist cavitation. This is because the blade sections at the inlet to these pumps are broadly similar (likewise, the exit blade sections of turbines are similar) and it is the *shape* of the low pressure passages which influences the onset of cavitation.

Using the alternative definition of suction specific speed $\Omega_{ss} = \Omega Q^{1/2}/(gH_s)^{1/2}$, where Ω is the rotational speed in rad/s, Q is the volume flow in m^3/s and gH_s , is in m^2/s^2 , it has been shown empirically (Wislicehus 1947) that

$$\Omega_{ss} \simeq 3.0 \text{ (rad)} \quad (1.12a)$$

for pumps, and

$$\Omega_{ss} \simeq 4.0 \text{ (rad)} \quad (1.12b)$$

for turbines.

Pearsall (1973) described a supercavitating pump with a cavitation performance much better than that of conventional pumps. For this pump suction specific speeds, Ω_{ss}

up to 9.0 were readily obtained and, it was claimed, even better values might be possible, but at the cost of reduced head and efficiency. It is likely that supercavitating pumps will be increasingly used in the search for higher speeds, smaller sizes and lower costs.

Compressible gas flow relations

Stagnation properties

In turbomachines handling compressible fluids, large changes in flow velocity occur across the stages as a result of pressure changes caused by the expansion or compression processes. For any point in the flow it is convenient to combine the energy terms together. The enthalpy, h , and the kinetic energy, $\frac{1}{2}c^2$ are combined and the result is called the *stagnation enthalpy*,

$$h_0 = h + \frac{1}{2}c^2.$$

The stagnation enthalpy is constant in a flow process that does not involve a work transfer or a heat transfer even though irreversible processes may be present. In Figure 1.8, point 1 represents the actual or static state of a fluid in an enthalpy–entropy diagram with enthalpy, h_1 at pressure p_1 and entropy s_1 . The fluid velocity is c_1 . The stagnation state is represented by the point 01 brought about by an irreversible deceleration. For a reversible deceleration the stagnation point would be at point 01s and the state change would be called *isentropic*.

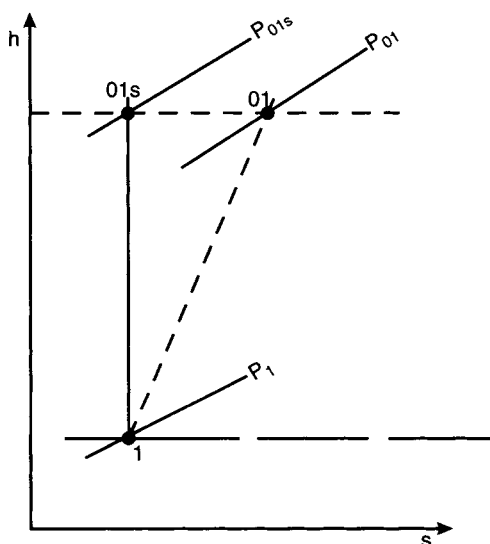


FIG. 1.8. The static state (point 1), the stagnation (point 01) and the isentropic stagnation (point 01s) of a fluid.

Stagnation temperature and pressure

If the fluid is a perfect gas, then $h = C_p T$, where $C_p = \gamma R / (\gamma - 1)$, so that the stagnation temperature can be defined as

$$T_0 = T + \frac{1}{2} c^2 / C_p,$$

$$\frac{T_0}{T} = 1 + \frac{1}{2} (\gamma - 1) \frac{c^2}{\gamma R T} = 1 + \frac{1}{2} (\gamma - 1) M^2, \quad (1.13a)$$

where the *Mach number*, $M = c/a = c/\sqrt{\gamma R T}$.

The Gibb's relation, derived from the second law of thermodynamics (see Chapter 2), is

$$T ds = dh - \frac{1}{\rho} dp.$$

If the flow is brought to rest both adiabatically and isentropically (i.e. $ds = 0$), then, using the above Gibb's relation,

$$dh = C_p dT = \frac{dp}{\rho} RT$$

so that

$$\frac{dp}{p} = \frac{C_p}{R} \frac{dT}{T} = \frac{\gamma}{\gamma - 1} \frac{dT}{T}.$$

Integrating, we obtain

$$\ln p = \ln \text{constant} + \frac{\gamma}{\gamma - 1} \ln T,$$

and so,

$$\frac{p_0}{p} = \left(\frac{T_0}{T} \right)^{\gamma/(\gamma-1)} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/(\gamma-1)} \quad (1.13b)$$

From the gas law density, $\rho = p/(RT)$, we obtain $\rho_0/\rho = (p_0/p)(T/T_0)$ and hence,

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T} \right)^{1/(\gamma-1)} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{1/(\gamma-1)}. \quad (1.13c)$$

Compressible fluid analysis

The application of dimensional analysis to compressible fluids increases, not unexpectedly, the complexity of the functional relationships obtained in comparison with those already found for incompressible fluids. Even if the fluid is regarded as a perfect gas, in addition to the previously used fluid properties, two further characteristics are required; these are a_{01} , the stagnation speed of sound at entry to

the machine and γ , the ratio of specific heats C_p/C_v . In the following analysis the compressible fluids under discussion are either perfect gases, or else, dry vapours approximating in behaviour to a perfect gas.

Another choice of variables is usually preferred when appreciable density changes occur across the machine. Instead of volume flow rate Q , the mass flow rate $\dot{m}$ is used; likewise for the head change H , the isentropic *stagnation enthalpy* change Δh_{0s} is employed.

The choice of this last variable is a significant one for, in an ideal and adiabatic process, Δh_{0s} is equal to the work done by unit mass of fluid. This will be discussed still further in Chapter 2. Since heat transfer from the casings of turbomachines is, in general, of negligible magnitude compared with the flux of energy through the machine, temperature on its own may be safely excluded as a fluid variable. However, temperature is an easily observable characteristic and, for a perfect gas, can be easily introduced at the last by means of the equation of state, $p/\rho = RT$, where $R = R_0/m = C_p - C_v$, m being the molecular weight of the gas and $R_0 = 8.314 \text{ kJ/(kg mol K)}$ is the *Universal gas constant*.

The performance parameters Δh_{0s} , η and P for a turbomachine handling a compressible flow, are expressed functionally as:

$$\Delta h_{0s}, \eta, P = f(\mu, N, D, \dot{m}, \rho_{01}, a_{01}, \gamma). \quad (1.14a)$$

Because ρ_0 and a_0 change through a turbomachine, values of these fluid variables are selected at inlet, denoted by subscript 1. Equation (1.14a) express *three* separate functional relationships, each of which consists of eight variables. Again, selecting ρ_{01} , N , D as common factors each of these three relationships may be reduced to five dimensionless groups,

$$\frac{\Delta h_{0s}}{N^2 D^2}, \eta, \frac{P}{\rho_{01} N^3 D^5} = f \left\{ \frac{\dot{m}}{\rho_{01} N D^3} \frac{\rho_{01} N D^2}{\mu}, \frac{N D}{a_{01}}, \gamma \right\}. \quad (1.14b)$$

Alternatively, the flow coefficient $\phi = \dot{m}/(\rho_{01} N D^3)$ can be written as $\phi = \dot{m}/(\rho_{01} a_{01} D^2)$. As ND is proportional to blade speed, the group ND/a_{01} is regarded as a *blade Mach number*.

For a machine handling a perfect gas a different set of functional relationships is often more useful. These may be found either by selecting the appropriate variables for a perfect gas and working through again from first principles or, by means of some rather straightforward transformations, rewriting eqn. (1.14b) to give more suitable groups. The latter procedure is preferred here as it provides a useful exercise.

As a concrete example consider an adiabatic compressor handling a perfect gas. The isentropic stagnation enthalpy rise can now be written $C_p(T_{02s} - T_{01})$ for the perfect gas. This compression process is illustrated in Figure 1.9a where the stagnation state point changes at constant entropy between the stagnation pressures p_{01} and p_{02} . The equivalent process for a turbine is shown in Figure 1.9b. Using the adiabatic isentropic relationship $p/\rho^\gamma = \text{constant}$, together with $p/\rho = RT$, the expression

$$\frac{T_{02s}}{T_{01}} = \left(\frac{p_{02}}{p_{01}} \right)^{(\gamma-1)/\gamma}$$

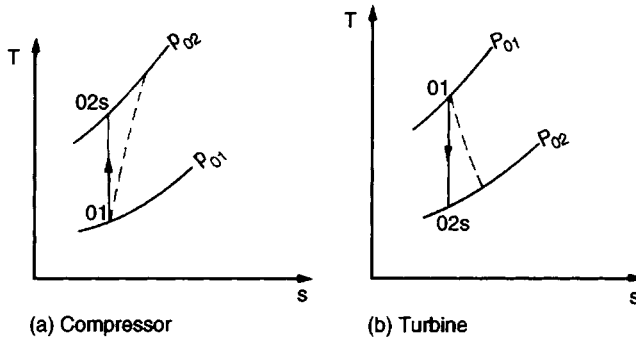


FIG. 1.9. The ideal adiabatic change in stagnation conditions across a turbomachine.

is obtained. Hence $\Delta h_{0s} = C_p T_{01} [(p_{02}/p_{01})^{(\gamma-1)/\gamma} - 1]$. Since $C_p = \gamma R/(\gamma - 1)$ and $a_{01}^2 = \gamma R T_{01}$, then

$$\Delta h_{0s}/a_{01}^2 \propto f(p_{02}/p_{01}).$$

The flow coefficient can now be more conveniently expressed as

$$\frac{\dot{m}}{\rho_{01} a_{01} D^2} = \frac{\dot{m} R T_{01}}{p_{01} \sqrt{(\gamma R T_{01})} D^2} = \frac{\dot{m} \sqrt{(R T_{01})}}{D^2 p_{01} \sqrt{\gamma}}.$$

As $\dot{m} \equiv \rho_{01} D^2 (ND)$, the power coefficient may be written

$$\hat{P} = \frac{P}{\rho_{01} N^3 D^5} = \frac{\dot{m} C_p \Delta T_0}{\{\rho_{01} D^2 (ND)\} (ND)^2} = \frac{C_p \Delta T_0}{(ND)^2} \equiv \frac{\Delta T_0}{T_{01}}.$$

Collecting together all these newly formed non-dimensional groups and inserting them in eqn. (1.14b) gives

$$\frac{p_{02}}{p_{01}}, \eta, \frac{\Delta T_0}{T_{01}} = f \left\{ \frac{\dot{m} \sqrt{(R T_{01})}}{D^2 p_{01}}, \frac{ND}{\sqrt{(R T_{01})}}, Re, \gamma \right\}. \quad (1.15)$$

The justification for dropping γ from a number of these groups is simply that it already appears separately as an independent variable.

For a machine of a specific size and handling a single gas it has become customary, in industry at least, to delete γ , R , and D from eqn. (1.15) and similar expressions. If, in addition, the machine operates at high Reynolds numbers (or over a small speed range), Re can also be dropped. Under these conditions eqn. (1.15) becomes

$$\frac{p_{02}}{p_{01}} \eta, \frac{\Delta T_0}{T_{01}} = f \left\{ \frac{\dot{m} \sqrt{T_{01}}}{p_{01}}, \frac{N}{\sqrt{T_{01}}} \right\}. \quad (1.16)$$

Note that by omitting the diameter D and gas constant R , the independent variables in eqn. (1.16) are no longer dimensionless.

Figures 1.10 and 1.11 represent typical performance maps obtained from compressor and turbine test results. In both figures the pressure ratio across the whole

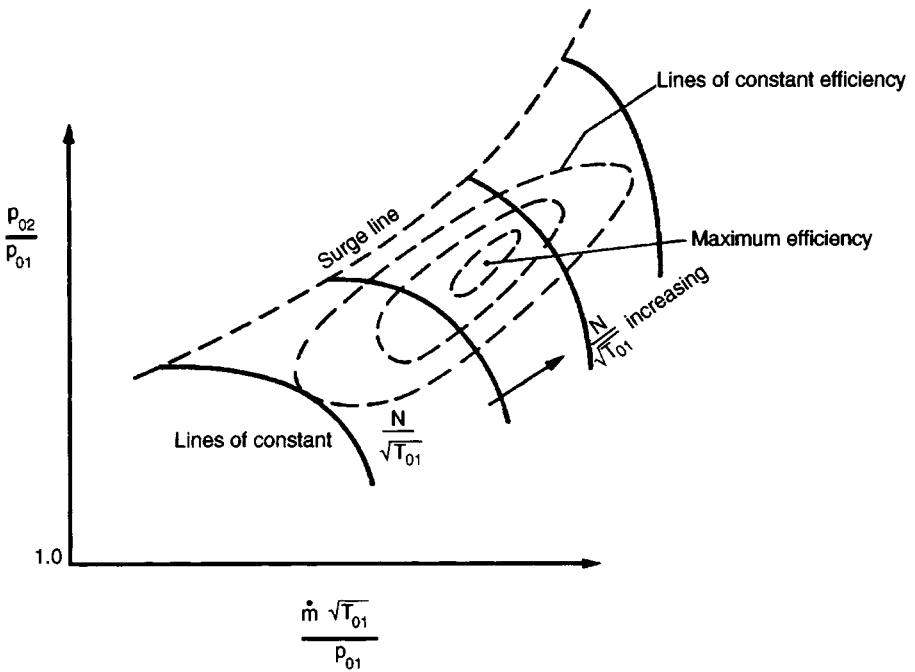


FIG. 1.10. Overall characteristic of a compressor.

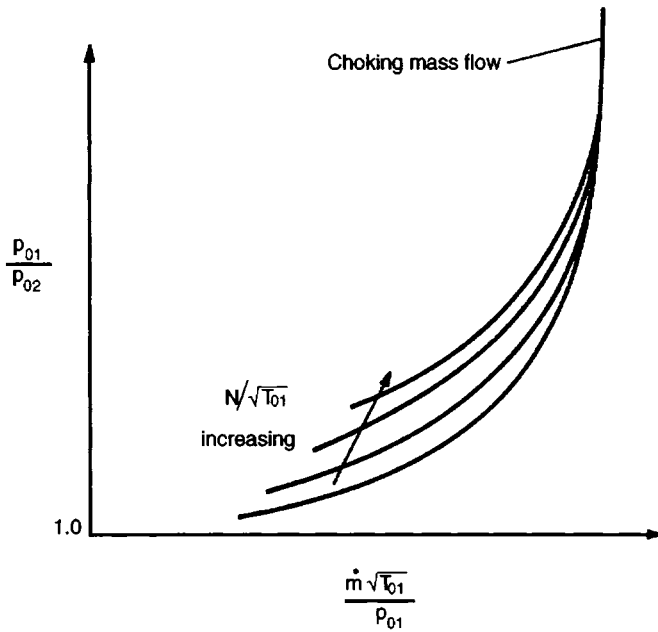


FIG. 1.11. Overall characteristic of a turbine.

machine is plotted as a function of $\dot{m}(\sqrt{T_{01}})/p_{01}$ for fixed values of $N/(\sqrt{T_{01}})$, this being a customary method of presentation. Notice that for both machines subscript 1 is used to denote conditions as inlet. One of the most striking features of these performance characteristics is the rather weak dependence of the turbine performance upon $N/\sqrt{T_{01}}$ contrasting with the strong dependence shown by the compressor on this parameter.

For the compressor, efficient operation at constant $N/\sqrt{T_{01}}$ lies to the right of the line marked “surge”. A discussion of the phenomenon of surge is included in Chapter 5; in brief, for multistage compressors it commences approximately at the point (for constant $N/\sqrt{T_{01}}$) where the pressure ratio flattens out to its maximum value. The surge line denotes the limit of *stable operation* of a compressor, unstable operation being characterised by a severe oscillation of the mass flow rate through the machine. The choked regions of both the compressor and turbine characteristics may be recognised by the vertical portions of the constant speed lines. No further increase in $\dot{m}(\sqrt{T_{01}})/p_{01}$ is possible since the Mach number across some section of the machine has reached unity and the flow is said to be *choked*.

The inherent unsteadiness of the flow within turbomachines

A fact often ignored by turbomachinery designers, or even unknown to students, is that turbomachines can only work the way they do because of unsteady flow effects taking place within them. The fluid dynamic phenomena that are associated with the unsteady flow in turbomachines has been examined by Greitzer (1986) in a discourse which was intended to be an introduction to the subject but actually extended far beyond the technical level of this book! Basically Greitzer, and others before him, in considering the fluid mechanical process taking place on a fluid particle in an isentropic flow, deduced that *stagnation enthalpy of the particle can change only if the flow is unsteady*. Dean (1959) appears to have been the first to record that without an *unsteady flow* inside a turbomachine, no work transfer can take place. Paradoxically, both at the inlet to and outlet from the machine the conditions are such that the flow can be considered as steady.

A physical situation considered by Greitzer is the axial compressor rotor as depicted in Figure 1.12a. The pressure field associated with the blades is such that the pressure increases from the suction surface (S) to the pressure surface (P). This pressure field moves with the blades and, to an observer situated at the point * (in the absolute frame of reference), a pressure that varies with time would be recorded, as shown in Figure 1.12b. Thus, fluid particles passing through the rotor would experience a positive pressure increase with time (i.e. $\partial p / \partial t > 0$). From this fact it can then be shown that the stagnation enthalpy of the fluid particle also increases because of the unsteadiness of the flow, i.e.

$$\frac{Dh_0}{Dt} = \frac{1}{\rho} \frac{\partial p}{\partial t},$$

where D/Dt is the rate of change following the fluid particle.

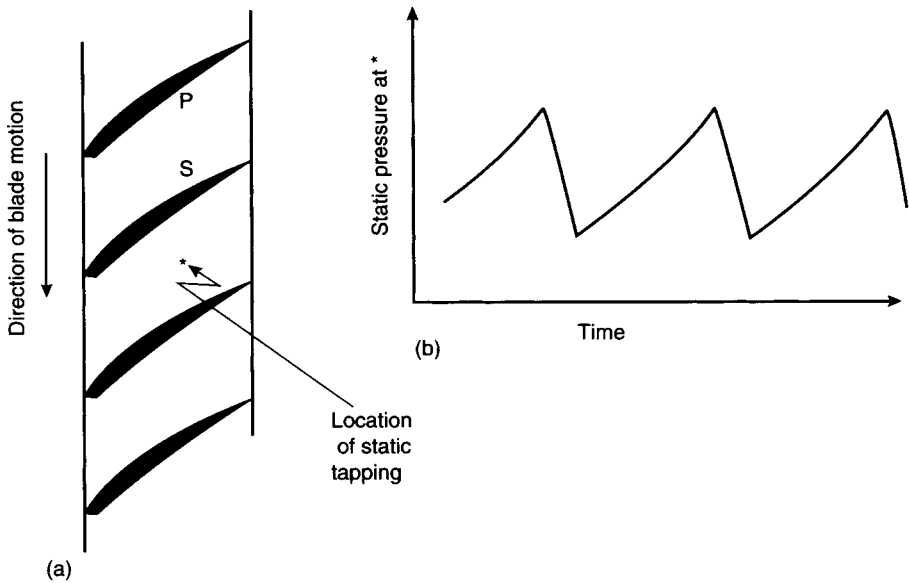


FIG. 1.12. Measuring unsteady pressure field of an axial compressor rotor. (a) Pressure is measured at point * on the casing. (b) Fluctuating pressure measured at point *.

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Problems

1. A fan operating at 1750 rev/min at a volume flow rate of $4.25 \text{ m}^3/\text{s}$ develops a head of 153 mm measured on a water-filled U-tube manometer. It is required to build a larger, geometrically similar fan which will deliver the same head at the same efficiency as the existing fan, but at a speed of 1440 rev/min. Calculate the volume flow rate of the larger fan.

2. An axial flow fan 1.83 m diameter is designed to run at a speed of 1400 rev/min with an average axial air velocity of 12.2 m/s. A quarter scale model has been built to obtain a check on the design and the rotational speed of the model fan is 4200 rev/min. Determine the axial air velocity of the model so that dynamical similarity with the full-scale fan is preserved. The effects of Reynolds number change may be neglected.

A sufficiently large pressure vessel becomes available in which the complete model can be placed and tested under conditions of complete similarity. The viscosity of the air is independent of pressure and the temperature is maintained constant. At what pressure must the model be tested?

3. A water turbine is to be designed to produce 27 MW when running at 93.7 rev/min under a head of 16.5 m. A model turbine with an output of 37.5 kW is to be tested under dynamically similar conditions with a head of 4.9 m. Calculate the model speed and scale ratio. Assuming a model efficiency of 88%, estimate the volume flow rate through the model.

It is estimated that the force on the thrust bearing of the full-size machine will be 7.0 GN. For what thrust must the model bearing be designed?

4. Derive the non-dimensional groups that are normally used in the testing of gas turbines and compressors.

A compressor has been designed for normal atmospheric conditions (101.3 kPa and 15°C). In order to economise on the power required it is being tested with a throttle in the entry duct to reduce the entry pressure. The characteristic curve for its normal design speed of 4000 rev/min is being obtained on a day when the ambient temperature is 20°C . At what speed should the compressor be run? At the point on the characteristic curve at which the mass flow would normally be 58 kg/s the entry pressure is 55 kPa. Calculate the actual rate of mass flow during the test.

Describe, with the aid of sketches, the relationship between geometry and specific speed for pumps.