

CHAPTER 3

Two-dimensional Cascades

Let us first understand the facts and then we may seek the causes. (ARISTOTLE.)

Introduction

The operation of any turbomachine is directly dependent upon changes in the working fluid's angular momentum as it crosses individual blade rows. A deeper insight of turbomachinery mechanics may be gained from consideration of the flow changes and forces exerted within these individual blade rows. In this chapter the flow past two-dimensional blade cascades is examined.

A review of the many different types of cascade tunnel, which includes low-speed, high-speed, intermittent blowdown and suction tunnels, etc. is given by Sieverding (1985). The range of Mach number in axial-flow turbomachines can be considered to extend from $M = 0.2$ to 2.5 (of course, if we also include fans then the lower end of the range is very low). Two main types of cascade tunnel are:

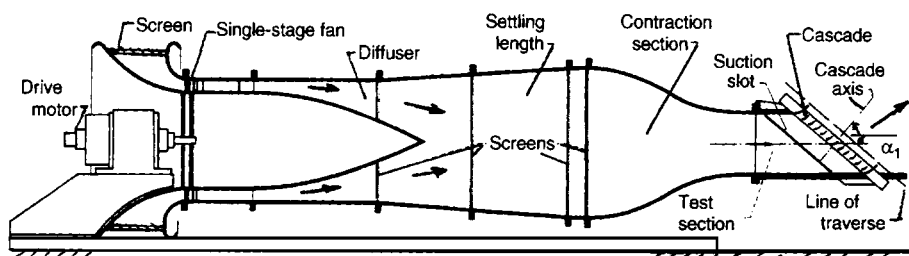
- (1) low-speed, operating in the range 20–60 m/s; and
- (2) high-speed, for the compressible flow range of testing.

A typical low-speed, continuous running, cascade tunnel is shown in Figure 3.1(a). The linear cascade of blades comprises a number of identical blades, equally spaced and parallel to one another. A suction slot is situated on the ceiling of the tunnel just before the cascade to allow the controlled removal of the tunnel boundary layer. Carefully controlled suction is usually provided on the tunnel sidewalls immediately upstream of the cascade so that two-dimensional, constant axial velocity flow can be achieved.

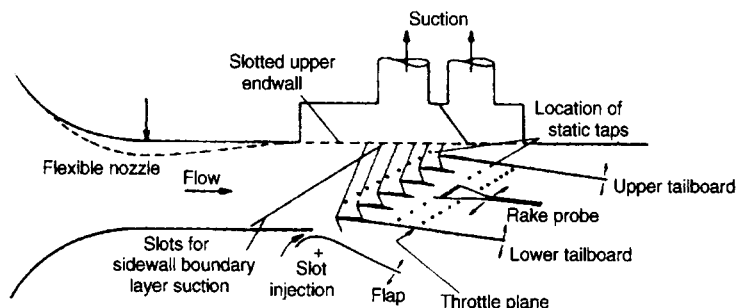
Figure 3.1b shows the test section of a cascade facility for transonic and moderate supersonic inlet velocities. The upper wall is slotted and equipped for suction, allowing operation in the transonic regime. The flexible section of the upper wall allows for a change of geometry so that a convergent–divergent nozzle is formed, thus allowing the flow to expand supersonically upstream of the cascade.

To obtain truly two-dimensional flow would require a cascade of infinite extent. Of necessity cascades must be limited in size, and careful design is needed to ensure that at least the central regions (where flow measurements are made) operate with approximately two-dimensional flow.

For axial flow machines of high hub-tip ratio, radial velocities are negligible and, to a close approximation, the flow may be described as two-dimensional. The flow in a cascade is then a reasonable model of the flow in the machine. With lower hub-tip radius ratios, the blades of a turbomachine will normally have an appreciable amount



(a)



(b)

FIG. 3.1. Compressor cascade wind tunnels. (a) Conventional low-speed, continuous running cascade tunnel (adapted from Carter *et al.* 1950). (b) Transonic/supersonic cascade tunnel (adapted from Sieverding 1985).

of twist along their length, the amount depending upon the sort of “vortex design” chosen (see Chapter 6). However, data obtained from two-dimensional cascades can still be of value to a designer requiring the performance at discrete blade sections of such blade rows.

Cascade nomenclature

A cascade blade profile can be conceived as a curved *camber line* upon which a *profile thickness distribution* is symmetrically superimposed. Referring to Figure 3.2 the camber line $y(x)$ and profile thickness $t(x)$ are shown as functions of the distance x along the *blade chord* l . In British practice the shape of the camber line is usually either a circular arc or a parabolic arc defined by the maximum camber b located at distance a from the leading edge of the blade. The profile thickness distribution may be that of a standard aerofoil section but, more usually, is one of the sections specifically developed by the various research establishments for compressor or turbine applications. Blade camber and thickness distributions are generally presented as tables of y/l and t/l against x/l . Some examples of these tables are quoted by Horlock (1958, 1966). Summarising, the useful parameters for describing a cascade blade are: camber line shape, b/l , a/l , type of thickness distribution and maximum thickness to chord ratio, $t_{\max}/l$.

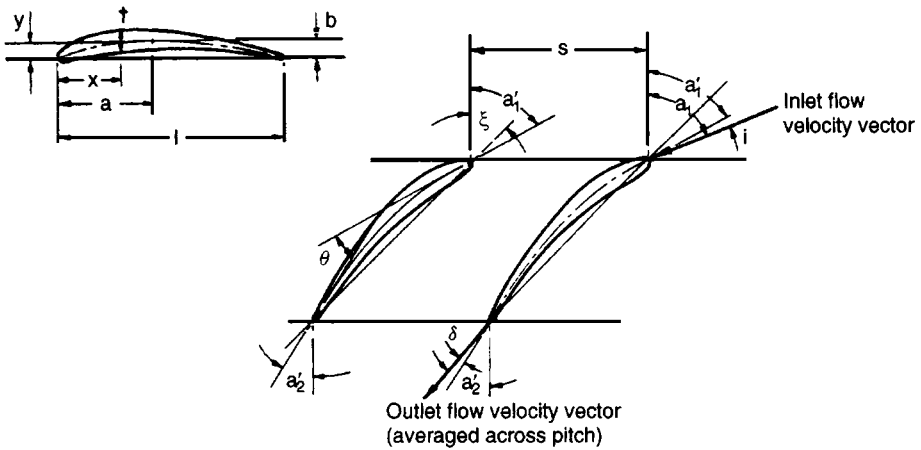


FIG. 3.2. Compressor cascade and blade notation.

With the blades arranged in cascade, two important additional geometric variables which define the cascade are the *space-chord ratio* s/l and the *stagger angle* ξ , which is the angle between the chord line and a reference direction *perpendicular to the cascade front*. Throughout the remainder of this book, all fluid and blade angles are referred to this perpendicular so as to avoid the needless complication arising from the use of other reference directions. However, custom dies hard; in steam turbine practice, blade and flow angles are conventionally measured from the *tangential* direction (i.e. parallel to the cascade front). Despite this, it is better to avoid ambiguity of meaning by adopting the single reference direction already given.

The blades angles at entry to and at exit from the cascade are denoted by α'_1 and α'_2 respectively. A most useful blade parameter is the *camber angle* θ which is the change in angle of the camber line between the leading and trailing edges and equals $\alpha'_1 - \alpha'_2$ in the notation of Figure 3.2. For circular arc camber lines the stagger angle is $\xi = \frac{1}{2}(\alpha'_1 + \alpha'_2)$. For parabolic arc camber lines of low camber (i.e. small b/l) as used in some compressor cascades, the inlet and outlet blade angles are

$$\alpha'_1 = \xi + \tan^{-1} \frac{b/l}{(a/l)^2} \quad \alpha'_2 = \xi - \tan^{-1} \frac{b/l}{(1 - a/l)^2}$$

the equation approximating for the parabolic arc being $Y = X\{A(X - 1) + BY\}$ where $X = x/l$, $Y = y/l$. A , B are two arbitrary constants which can be solved with the conditions that at $x = a$, $y = b$ and $dy/dx = 0$. The exact general equation of a parabolic arc camber line which has been used in the design of highly cambered turbine blades is dealt with by Dunham (1974).

Analysis of cascade forces

The fluid approaches the cascade from far upstream with velocity c_1 at an angle α_1 and leaves far downstream of the cascade with velocity c_2 at an angle α_2 . In the following analysis the fluid is assumed to be incompressible and the flow to be

steady. The assumption of steady flow is valid for an isolated cascade row but, in a turbomachine, relative motion between successive blade rows gives rise to unsteady flow effects. As regards the assumption of incompressible flow, the majority of cascade tests are conducted at fairly low Mach numbers (e.g. 0.3 on compressor cascades) when compressibility effects are negligible. Various techniques are available for correlating incompressible and compressible cascades; a brief review is given by Csanady (1964).

A portion of an isolated blade cascade (for a compressor) is shown in Figure 3.3. The forces X and Y are exerted by unit depth of blade upon the fluid, exactly equal and opposite to the forces exerted by the fluid upon unit depth of blade. A control surface is drawn with end boundaries far upstream and downstream of the cascade and with side boundaries coinciding with the median stream lines.

Applying the principle of continuity to a unit depth of span and noting the assumption of incompressibility, yields

$$c_1 \cos \alpha_1 = c_2 \cos \alpha_2 = c_x. \quad (3.1)$$

The momentum equation applied in the x and y directions with constant axial velocity gives,

$$X = (p_2 - p_1)s, \quad (3.2)$$

$$Y = \rho s c_x (c_{y1} - c_{y2}), \quad (3.3)$$

or

$$Y = \rho s c_x^2 (\tan \alpha_1 - \tan \alpha_2) \quad (3.3a)$$

Equations (3.1) and (3.3) are completely valid for a flow incurring total pressure losses in the cascade.

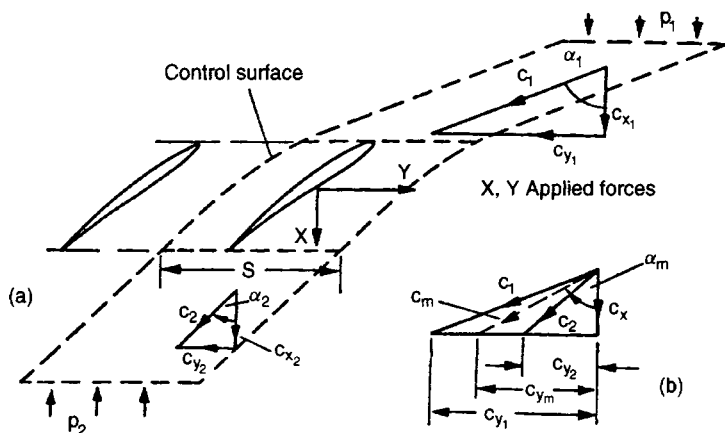


FIG. 3.3. Forces and velocities in a blade cascade.

Energy losses

A real fluid crossing the cascade experiences a loss in total pressure Δp_0 due to skin friction and related effects. Thus

$$\frac{\Delta p_0}{\rho} = \frac{p_1 - p_2}{\rho} + \frac{1}{2}(c_1^2 - c_2^2). \quad (3.4)$$

Noting that $c_1^2 - c_2^2 = (c_{y1}^2 + c_x^2) - (c_{y2}^2 + c_x^2) = (c_{y1} + c_{y2})(c_{y1} - c_{y2})$, substitute eqns. (3.2) and (3.3) into eqn. (3.4) to derive the relation,

$$\frac{\Delta p_0}{\rho} = \frac{1}{\rho s}(-X + Y \tan \alpha_m), \quad (3.5)$$

where

$$\tan \alpha_m = \frac{1}{2}(\tan \alpha_1 + \tan \alpha_2). \quad (3.6)$$

A non-dimensional form of eqn. (3.5) is often useful in presenting the results of cascade tests. Several forms of total pressure-loss coefficient can be defined of which the most popular are,

$$\zeta = \Delta p_0 / (\frac{1}{2} \rho c_x^2) \quad (3.7a)$$

and

$$\bar{\omega} = \Delta p_0 / (\frac{1}{2} \rho c_1^2). \quad (3.7b)$$

Using again the same reference parameter, a pressure rise coefficient C_p and a tangential force coefficient C_f may be defined

$$C_p = \frac{p_2 - p_1}{\frac{1}{2} \rho c_x^2} = \frac{X}{\frac{1}{2} \rho s c_x^2}, \quad (3.8)$$

$$C_f = \frac{Y}{\frac{1}{2} \rho s c_x^2} = 2(\tan \alpha_1 - \tan \alpha_2), \quad (3.9)$$

using eqns. (3.2) and (3.3a).

Substituting these coefficients into eqn. (3.5) to give, after some rearrangement,

$$C_p = C_f \tan \alpha_m - \zeta. \quad (3.10)$$

Lift and drag

A mean velocity c_m is defined as

$$c_m = c_x / \cos \alpha_m, \quad (3.11)$$

where α_m is itself defined by eqn. (3.6). Considering unit depth of a cascade blade, a lift force L acts in a direction perpendicular to c_m and a drag force D in a direction parallel to c_m . Figure 3.4 shows L and D as the reaction forces exerted by the blade upon the fluid.

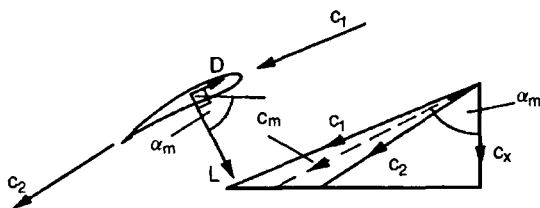


FIG. 3.4. Lift and drag forces exerted by a cascade blade (of unit span) upon the fluid.

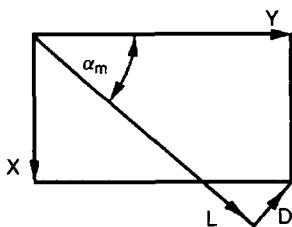


FIG. 3.5. Axial and tangential forces exerted by unit span of a blade upon the fluid.

Experimental data are often presented in terms of lift and drag when the data may be of greater use in the form of tangential force and total pressure loss. The lift and drag forces can be resolved in terms of the axial and tangential forces. Referring to Figure 3.5,

$$L = X \sin \alpha_m + Y \cos \alpha_m, \quad (3.12)$$

$$D = Y \sin \alpha_m - X \cos \alpha_m. \quad (3.13)$$

From eqn. (3.5)

$$D = \cos \alpha_m (Y \tan \alpha_m - X) = s \Delta p_0 \cos \alpha_m. \quad (3.14)$$

Rearranging eqn. (3.14) for X and substituting into eqn. (3.12) gives,

$$\begin{aligned} L &= (Y \tan \alpha_m - s \Delta p_0) \sin \alpha_m + Y \cos \alpha_m \\ &= Y \sec \alpha_m - s \Delta p_0 \sin \alpha_m \\ &= \rho s c_x^2 (\tan \alpha_1 - \tan \alpha_2) \sec \alpha_m - s \Delta p_0 \sin \alpha_m, \end{aligned} \quad (3.15)$$

after using eqn. (3.9).

Lift and drag coefficients may be introduced as

$$C_L = \frac{L}{\frac{1}{2} \rho c_m^2 l}, \quad (3.16a)$$

$$C_D = \frac{D}{\frac{1}{2} \rho c_m^2 l}. \quad (3.16b)$$

Using eqn. (3.14) together with eqn. (3.7),

$$C_D = \frac{s \Delta p_0 \cos \alpha_m}{\frac{1}{2} \rho c_m^2 l} = \zeta \frac{s}{l} \cos^3 \alpha_m. \quad (3.17)$$

With eqn. (3.15)

$$\begin{aligned} C_L &= \frac{\rho s c_x^2 (\tan \alpha_1 - \tan \alpha_2) \sec \alpha_m - s \Delta p_0 \sin \alpha_m}{\frac{1}{2} \rho c_m^2 l} \\ &= 2 \frac{s}{l} \cos \alpha_m (\tan \alpha_1 - \tan \alpha_2) - C_D \tan \alpha_m. \end{aligned} \quad (3.18)$$

Alternatively, employing eqns. (3.9) and (3.17),

$$C_L = \frac{s}{l} \cos \alpha_m \left(C_f - \zeta \frac{\sin 2\alpha_m}{2} \right). \quad (3.19)$$

Within the normal range of operation in a cascade, values of C_D are very much less than C_L . As α_m is unlikely to exceed 60 deg, the quantity $C_D \tan \alpha_m$ in eqn. (3.18) can be dropped, resulting in the approximation,

$$\frac{L}{D} = \frac{C_L}{C_D} \doteq \frac{2 \sec^2 \alpha_m}{\zeta} (\tan \alpha_1 - \tan \alpha_2) = \frac{C_f}{\zeta} \sec^2 \alpha_m. \quad (3.20)$$

Circulation and lift

The lift of a single isolated aerofoil for the ideal case when $D = 0$ is given by the Kutta–Joukowski theorem

$$L = \rho \Gamma c, \quad (3.21)$$

where c is the relative velocity between the aerofoil and the fluid at infinity and Γ is the circulation about the aerofoil. This theorem is of fundamental importance in the development of the theory of aerofoils (for further information see Glauert (1959).

In the absence of total pressure losses, the lift force per unit span of a blade in cascade, using eqn. (3.15), is

$$\begin{aligned} L &= \rho s c_x^2 (\tan \alpha_1 - \tan \alpha_2) \sec \alpha_m \\ &= \rho s c_m (c_{y1} - c_{y2}). \end{aligned} \quad (3.22)$$

Now the *circulation* is the contour integral of velocity around a closed curve. For the cascade blade the circulation is

$$\Gamma = s(c_{y1} - c_{y2}). \quad (3.23)$$

Combining eqns. (3.22) and (3.23),

$$L = \rho \Gamma c_m. \quad (3.24)$$

As the spacing between the cascade blades is increased without limit (i.e. $s \rightarrow \infty$), the inlet and outlet velocities to the cascade, c_1 and c_2 , becomes equal in

magnitude and direction. Thus $c_1 = c_2 = c$ and eqn. (3.24) becomes identical with the Kutta–Joukowski theorem obtained for an isolated aerofoil.

Efficiency of a compressor cascade

The efficiency η_D of a compressor blade cascade can be defined in the same way as diffuser efficiency; this is the ratio of the actual static pressure rise in the cascade to the maximum possible theoretical pressure rise (i.e. with $\Delta p_0 = 0$). Thus,

$$\begin{aligned}\eta_D &= \frac{p_2 - p_1}{\frac{1}{2}\rho(c_1^2 - c_2^2)} \\ &= 1 - \frac{\Delta p_0}{\rho c_x^2 \tan \alpha_m (\tan \alpha_1 - \tan \alpha_2)}.\end{aligned}$$

Inserting eqns. (3.7) and (3.9) into the above equation,

$$\eta_D = 1 - \frac{\zeta}{C_f \tan \alpha_m}. \quad (3.25)$$

Equation (3.20) can be written as $\zeta/C_f \doteq (\sec^2 \alpha_m) C_D/C_L$ which when substituted into eqn. (3.25) gives

$$\eta_D = 1 - \frac{2C_D}{C_L \sin 2\alpha_m}. \quad (3.26)$$

Assuming a constant lift–drag ratio, eqn. (3.26) can be differentiated with respect to α_m to give the optimum mean flow angle for maximum efficiency. Thus,

$$\frac{\partial \eta_D}{\partial \alpha_m} = \frac{4C_D \cos 2\alpha_m}{C_L \sin^2 2\alpha_m} = 0,$$

so that

$$\alpha_{m \text{ opt}} = 45 \text{ deg},$$

therefore

$$\eta_{D \text{ max}} = 1 - \frac{2C_D}{C_L}. \quad (3.27)$$

This simple analysis suggests that maximum efficiency of a compressor cascade is obtained when the mean flow angle is 45 deg, but ignores changes in the ratio C_D/C_L with varying α_m . Howell (1945) calculated the effect of having a specified variation of C_D/C_L upon cascade efficiency, comparing it with the case when C_D/C_L is constant. Figure 3.6 shows the results of this calculation as well as the variation of C_D/C_L with α_m . The graph shows that $\eta_{D \text{ max}}$ is at an optimum angle only a little less than 45 deg but that the curve is rather flat for a rather wide change in α_m . Howell suggested that value of α_m *rather less* than the optimum could well be chosen with little sacrifice in efficiency, and with some benefit with regard to power–weight ratio of compressors. In Howell's calculations, the drag is an estimate based on cascade

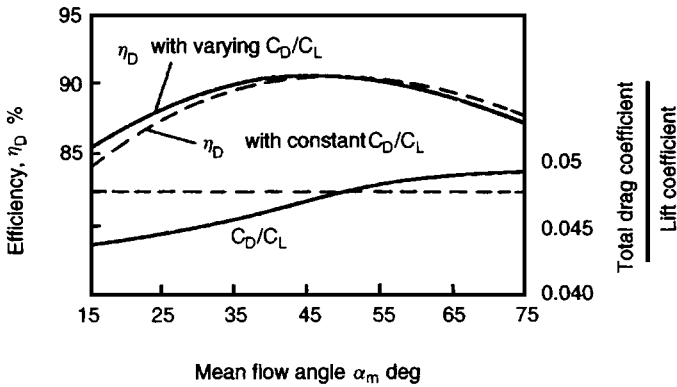


FIG. 3.6. Efficiency variation with average flow angle (adapted from Howell 1945).

experimental data together with an allowance for wall boundary-layer losses and “secondary-flow” losses.

Performance of two-dimensional cascades

From the relationships developed earlier in this chapter it is apparent that the effects of a cascade may be completely deduced if the flow angles at inlet and outlet together with the pressure loss coefficient are known. However, for a given cascade only one of these quantities may be arbitrarily specified, the other two being fixed by the cascade geometry and, to a lesser extent, by the Mach number and Reynolds number of the flow. For a given family of geometrically similar cascades the performance may be expressed functionally as,

$$\zeta, a_2 = (a_1, M_1, Re), \quad (3.28)$$

where ζ is the pressure loss coefficient, eqn. (3.7), M_1 is the inlet Mach number $= c_1/(\gamma RT_1)^{1/2}$, Re is the inlet Reynolds number $= \rho_1 c_1 l / \mu$ based on blade chord length.

Despite numerous attempts it has not been found possible to determine, accurately, cascade performance characteristics by theoretical means alone and the experimental method still remains the most reliable technique. An account of the theoretical approach to the problem lies outside the scope of this book, however, a useful summary of the subject is given by Horlock (1958).

The cascade wind tunnel

The basis of much turbomachinery research and development derives from the cascade wind tunnel, e.g. Figure 3.1 (or one of its numerous variants), and a brief description of the basic aerodynamic design is given below. A more complete description of the cascade tunnel is given by Carter *et al.* (1950) including many of the research techniques developed.

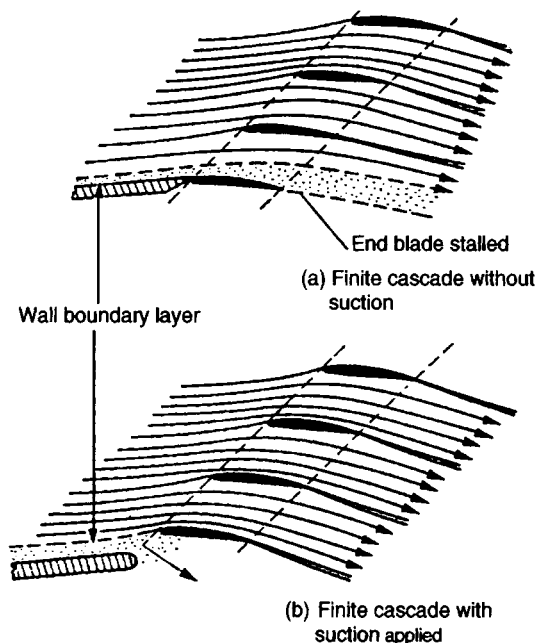


FIG. 3.7. Streamline flow through cascades (adapted from Carter *et al.* 1950).

In a well-designed cascade tunnel it is most important that the flow near the central region of the cascade blades (where the flow measurements are made) is approximately two-dimensional. This effect could be achieved by employing a large number of long blades, but an excessive amount of power would be required to operate the tunnel. With a tunnel of more reasonable size, aerodynamic difficulties become apparent and arise from the tunnel wall boundary layers interacting with the blades. In particular, and as illustrated in Figure 3.7a, the tunnel wall boundary layer mingles with the end blade boundary layer and, as a consequence, this blade stalls resulting in a non-uniform flow field.

Stalling of the end blade may be delayed by applying a controlled amount of suction to a slit just upstream of the blade, and sufficient to remove the tunnel wall boundary layer (Figure 3.7b). Without such boundary-layer removal the effects of flow interference can be quite pronounced. They become most pronounced near the cascade “stalling point” (defined later) when any small disturbance of the upstream flow field precipitates stall on blades adjacent to the end blade. Instability of this type has been observed in compressor cascades and can affect every blade of the cascade. It is usually characterised by regular, periodic “cells” of stall crossing rapidly from blade to blade; the term *propagating stall* is often applied to the phenomenon. Some discussion of the mechanism of propagating stall is given in Chapter 6.

The boundary layers on the walls to which the blade roots are attached, generate *secondary vorticity* in passing through the blades which may produce substantial *secondary flows*. The mechanism of this phenomenon has been discussed at some length by Carter (1948), Horlock (1958) and many others and a brief explanation is included in Chapter 6.

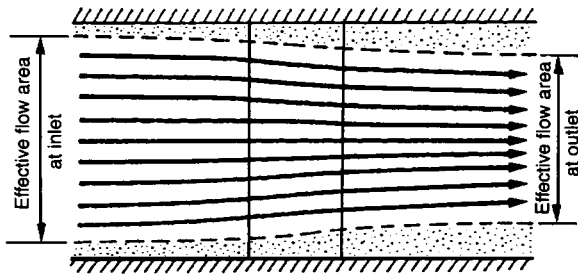


FIG. 3.8. Contraction of streamlines due to boundary layer thickening (adapted from Carter *et al.* 1950).

In a compressor cascade the rapid increase in pressure across the blades causes a marked thickening of the wall boundary layers and produces an effective contraction of the flow, as depicted in Figure 3.8. A *contraction coefficient*, used as a measure of the boundary-layer growth through the cascade, is defined by $\rho_1 c_1 \cos a_1 / (\rho_2 c_2 \cos a_2)$. Carter *et al.* (1950) quotes values of 0.9 for a good tunnel dropping to 0.8 in normal high-speed tunnels and even less in bad cases. These are values for compressor cascades; with turbine cascades slightly higher values can be expected.

Because of the contraction of the main through-flow, the theoretical pressure rise across a compressor cascade, even allowing for losses, is never achieved. This will be evident since a contraction (in a subsonic flow) accelerates the fluid, which is in conflict with the diffuser action of the cascade.

To counteract these effects it is customary (in Great Britain) to use *at least* seven blades in a compressor cascade, each blade having a minimum aspect ratio (blade span—chord length) of 3. With seven blades, suction is desirable in a compressor cascade but it is not usual in a turbine cascade. In the United States much lower aspect ratios are commonly employed in compressor cascade testing, the technique being the almost complete removal of tunnel wall boundary layers from all four walls using a combination of suction slots and perforated end walls to which suction is applied.

Cascade test results

The basic cascade performance data in low-speed flows are obtained from measurements of total pressure, flow angle and velocity taken across one or more complete pitches of the cascade, the plane of measurement being about half a chord downstream of the trailing edge plane. The literature on instrumentation is very extensive as are the various measurement techniques employed and the student is referred to the works of Horlock (1958), Bryer and Pankhurst (1971), Sieverding (1975, 1985). The publication by Bryer and Pankhurst for deriving air speed and flow direction is particularly instructive and recommended, containing as it does details of the design and construction of various instruments used in cascade tunnel measurements as well as their general principles and performance details.

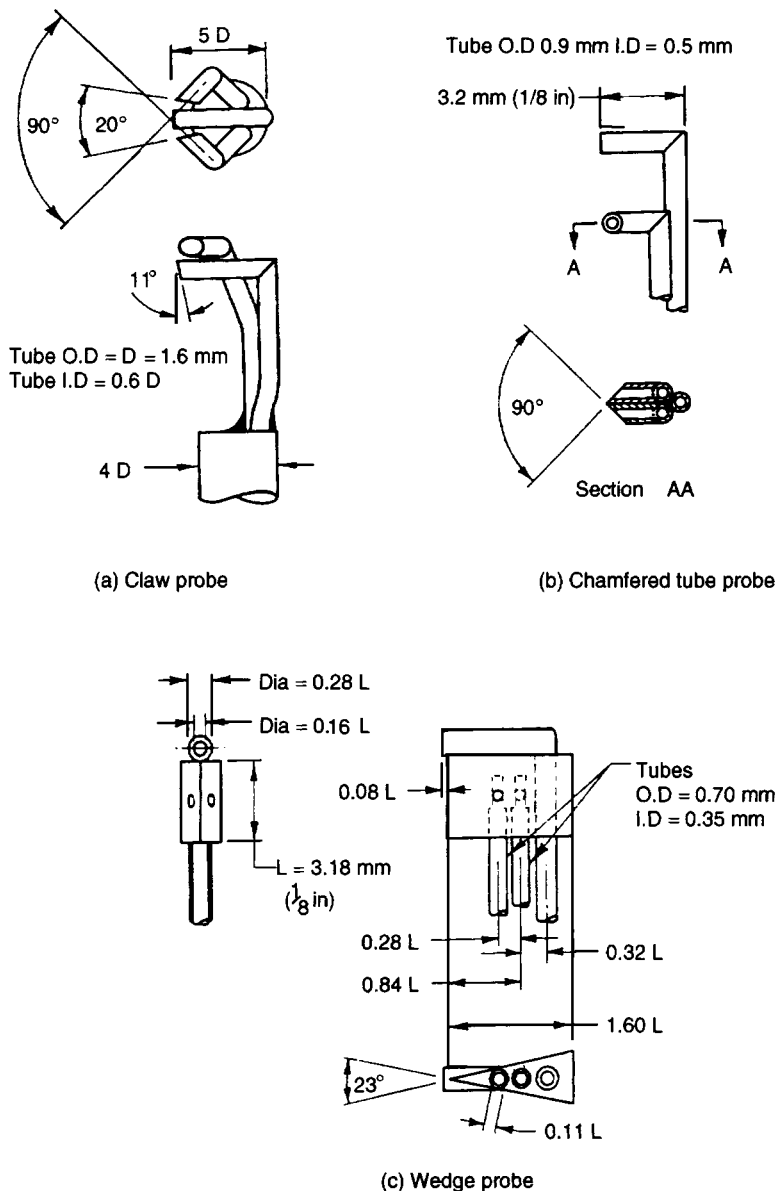


FIG. 3.9. Some combination pressure probes (adapted from Bryer and Pankhurst 1971):
 (a) claw probe; (b) chamfered tube probe; (c) wedge probe.

Some representative combination pressure probes are shown in Figure 3.9. These types are frequently used for pitchwise traversing across blade cascades but, because of their small size, they are also used for interstage (radial) flow traversing in compressors. For the measurement of flow direction in conditions of severe transverse total pressure gradients, as would be experienced during the measurement of

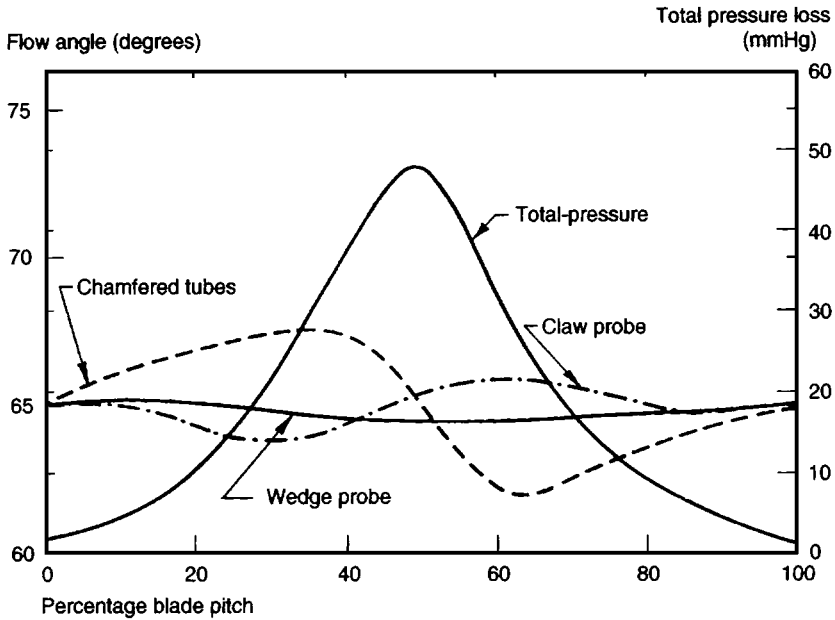


FIG. 3.10. Apparent flow angle variation measured by three different combination probes traversed across a transverse pressure (adapted from Bryer and Pankhurst 1971).

blade cascade flows, quite substantial errors in the measurement of flow direction do arise. Figure 3.10 indicates the *apparent* flow angle variation measured by these same three types of pressure probe when traversed across a transverse gradient of total pressure caused by a compressor stator blade. It is clear that the wedge probe is the least affected by the total pressure gradient. An investigation by Dixon (1978) did confirm that all pressure probe instruments are subject to this type of directional error when traversed across a total pressure variation such as a blade wake.

An extensive bibliography on all types of measurement in fluid flow is given by Dowden (1972). Figure 3.11 shows a typical cascade test result from a traverse across 2 blade pitches taken by Todd (1947) at an inlet Mach number of 0.6. It is observed that a total pressure deficit occurs across the blade row arising from the fluid friction on the blades. The fluid deflection is not uniform and is a maximum at each blade trailing edge on the pressure side of the blades. From such test results, average values of total pressure loss and fluid outlet angle are found (usually on a mass flow basis). The use of terms like “total pressure loss” and “fluid outlet angle” in the subsequent discussion will signify these *average* values.

Similar tests performed for a range of fluid inlet angles, at the same inlet Mach number M_1 and Reynolds number Re , enables the complete performance of the cascade to be determined (at that M_1 and Re). So as to minimise the amount of testing required, much cascade work is performed at low inlet velocities, but at a Reynolds number greater than the “critical” value. This critical Reynolds number Re_c is approximately 2×10^5 based on inlet velocity and blade chord. With $Re > Re_c$, total pressure losses and fluid deflections are only slightly dependent on changes

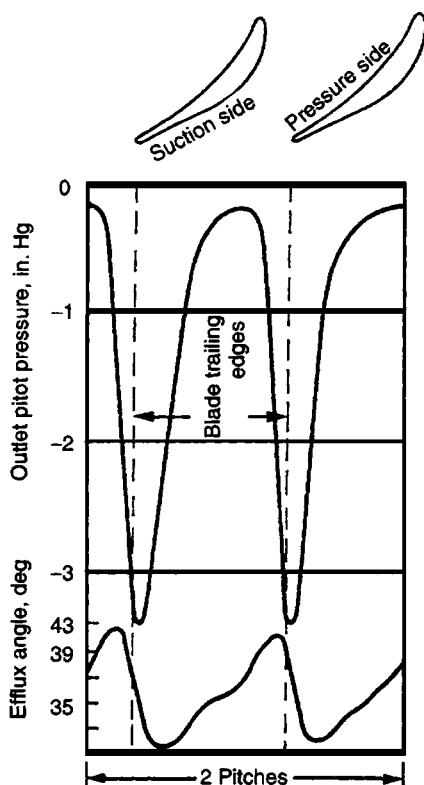


FIG. 3.11. A sample plot of inlet and outlet stagnation pressures and fluid outlet angle (adapted from Todd 1947).

in *Re*. Mach number effects are negligible when $M_1 < 0.3$. Thus, the performance laws, eqn. (3.28), for this flow simplify to,

$$\zeta, a_2 = f(a_1). \quad (3.28a)$$

There is a fundamental difference between the flows in turbine cascades and those in compressor cascades which needs emphasising. A fluid flowing through a channel in which the mean pressure is falling (mean flow is accelerating) experiences a relatively small total pressure loss in contrast with the mean flow through a channel in which the pressure is rising (diffusing flow) when losses may be high. This characteristic difference in flow is reflected in turbine cascades by a wide range of low loss performance and in compressor cascades by a rather narrow range.

Compressor cascade performance

A typical set of low-speed compressor cascade results (Howell 1942) for a blade cascade of specified geometry, is shown in Figure 3.12. These results are presented in the form of a pressure loss coefficient $\Delta p_0 / (\frac{1}{2} \rho c_1^2)$ and fluid deflection $\epsilon = a_1 - a_2$ against incidence $i = a_1 - a'_1$ (refer to Figure. 3.2 for nomenclature). Note

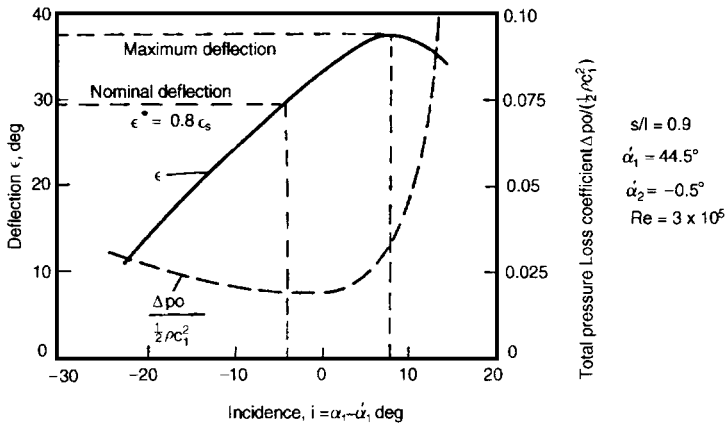


FIG. 3.12. Compressor cascade characteristics (Howell 1942). (By courtesy of the Controller of H.M.S.O., Crown copyright reserved).

that from eqn. (3.7), $\Delta p_0 / (\frac{1}{2} \rho c_1^2) = \zeta \cos^2 \alpha_1$. There is a pronounced increase in total pressure loss as the incidence rises beyond a certain value and the cascade is *stalled* in this region. The precise incidence at which stalling occurs is difficult to define and a *stall point* is arbitrarily specified as the incidence at which the total pressure loss is *twice* the minimum loss in total pressure. Physically, stall is characterised (at positive incidence) by the flow separating from the suction side of the blade surfaces. With decreasing incidence, total pressure losses again rise and a “negative incidence” stall point can also be defined as above. The *working range* is conventionally defined as the incidence range between these two limits at which the losses are twice the minimum loss. Accurate knowledge of the extent of the working range, obtained from two-dimensional cascade tests, is of great importance when attempting to assess the suitability of blading for changing conditions of operation. A *reference incidence* angle can be most conveniently defined either at the mid-point of the working range or, less precisely, at the minimum loss condition. These two conditions do not necessarily give the same reference incidence.

From such cascade test results the *profile losses* through compressor blading of the same geometry may be estimated. To these losses estimates of the annulus skin friction losses and other secondary losses must be added, and from which the efficiency of the compressor blade row may be determined. Howell (1945) suggested that these losses could be estimated using the following drag coefficients. For the annulus walls loss,

$$C_{Da} = 0.02s/H \quad (3.29a)$$

and for the so-called “secondary” loss,

$$C_{Ds} = 0.018C_L^2 \quad (3.29b)$$

where s , H are the blade pitch and blade length respectively, and C_L the blade lift coefficient. Calculations of this type were made by Howell and others to estimate the efficiency of a complete compressor stage. A worked example to illustrate the

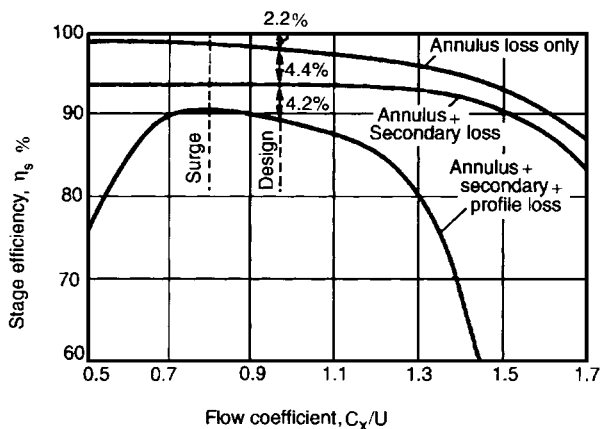


FIG. 3.13. Losses in a compressor stage (Howell 1945). (Courtesy of the Institution of Mechanical Engineers).

details of the method is given in Chapter 5. Figure 3.13 shows the variation of stage efficiency with flow coefficient and it is of particular interest to note the relative magnitude of the profile losses in comparison with the overall losses, especially at the design point.

Cascade performance data to be easily used, are best presented in some condensed form. Several methods of empirically correlating low-speed performance data have been developed in Great Britain. Howell's correlation (1942) relates the performance of a cascade to its performance at a "nominal" condition defined at 80% of the stalling deflection. Carter (1950) has referred performance to an optimum incidence given by the highest lift-drag ratio of the cascade. In the United States, the National Advisory Committee for Aeronautics (NACA), now called the National Aeronautics and Space Administration (NASA), systematically tested whole families of different cascade geometries, in particular, the widely used NACA 65 Series (Herrig *et al.* 1957). The data on the NACA 65 Series has been usefully summarised by Felix (1957) where the performance of a fixed geometry cascade can be more readily found. A concise summary is also given by Horlock (1958).

Turbine cascade performance

Figure 3.14 shows results obtained by Ainley (1948) from two sets of turbine cascade blades, impulse and "reaction". The term "reaction" is used here to denote, in a qualitative sense, that the fluid accelerates through the blade row and thus experiences a *pressure drop* during its passage. There is no pressure change across an impulse blade row. The performance is expressed in the form $\lambda = \Delta p_o / (p_{o2} - p_2)$ and α_2 against incidence.

From these results it is observed that:

(a) the reaction blades have a much wider range of low loss performance than the impulse blades, a result to be expected as the blade boundary layers are subjected to a favourable pressure gradient,

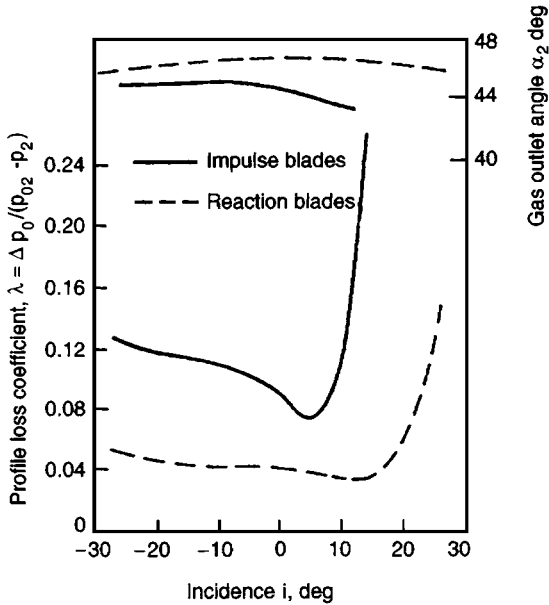


FIG. 3.14. Variation in profile loss with incidence for typical turbine blades (adapted from Ainley 1948).

(b) the fluid outlet angle α_2 remains relatively constant over the whole range of incidence in contrast with the compressor cascade results.

For turbine cascade blades, a method of correlation is given by Ainley and Mathieson (1951) which enables the performance of a gas turbine to be predicted with an estimated tolerance of within 2% on peak efficiency. In Chapter 4 a rather different approach, using a method attributed to Soderberg, is outlined. While being possibly slightly less accurate than Ainley's correlation, Soderberg's method employs fewer parameters and is rather easier to apply.

Compressor cascade correlations

Many experimental investigations have confirmed that the efficient performance of compressor cascade blades is limited by the growth and separation of the blade surface boundary layers. One of the aims of cascade research is to establish the generalised loss characteristics and stall limits of conventional blades. This task is made difficult because of the large number of factors which can influence the growth of the blade surface boundary layers, viz. surface velocity distribution, blade Reynolds number, inlet Mach number, free-stream turbulence and unsteadiness, and surface roughness. From the analysis of experimental data several correlation methods have been evolved which enable the first-order behaviour of the blade losses and limiting fluid deflection to be predicted with sufficient accuracy for engineering purposes.

LIEBLEIN. The correlation of Lieblein (1959), NASA (1965) is based on the experimental observation that a large amount of velocity diffusion on blade surfaces

tends to produce thick boundary layers and eventual flow separation. Lieblein states the general hypothesis that in the region of minimum loss, the wake thickness and consequently the magnitude of the loss in total pressure, is proportional to the diffusion in velocity on the suction-surface of the blade in that region. The hypothesis is based on the consideration that the boundary layer on the suction-surface of conventional compressor blades contributes the largest share of the blade wake. Therefore, the suction-surface velocity distribution becomes the main factor in determining the total pressure loss.

Figure 3.15 shows a typical velocity distribution derived from surface pressure measurements on a compressor cascade blade in the region of minimum loss. The diffusion in velocity may be expressed as the ratio of maximum suction-surface

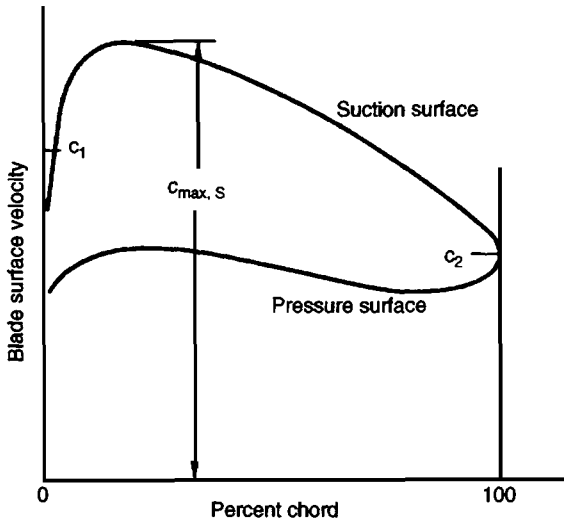


FIG. 3.15. Compressor cascade blade surface velocity distribution.

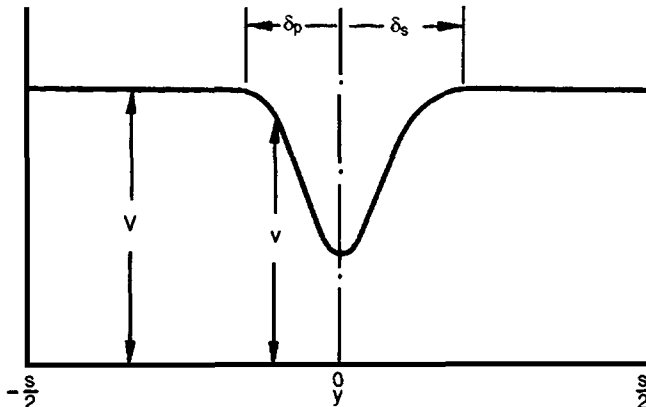


FIG. 3.16. Model variation in velocity in a plane normal to axial direction.

velocity to outlet velocity, $c_{\max,s}/c_2$. Lieblein found a correlation between the diffusion ratio $c_{\max,s}/c_2$ and the wake momentum-thickness to chord ratio, θ_2/l at the reference incidence (mid-point of working range) for American NACA 65-(A₁₀) and British C.4 circular-arc blades. The wake momentum-thickness, with the parameters of the flow model in Figure 3.16 is defined as

$$\theta_2 = \int_{\delta_p}^{\delta_s} \frac{v}{V} \left(1 - \frac{v}{V}\right) dy. \quad (3.30)$$

The Lieblein correlation, with his data points removed for clarity, is closely fitted by the mean curve in Figure 3.17. This curve represents the equation

$$\frac{\theta_2}{l} = 0.004 / \left\{ 1 - 1.17 \ln \left(\frac{c_{\max,s}}{c_2} \right) \right\} \quad (3.31)$$

which may be more convenient to use in calculating results. It will be noticed that for the limiting case when $(\theta_2/l) \rightarrow \infty$, the corresponding *upper* limit for the diffusion ratio $c_{\max,s}/c_2$ is 2.35. The *practical* limit of efficient operation would correspond to a diffusion ratio of between 1.9 and 2.0.

Losses are usually expressed in terms of the stagnation pressure loss coefficient $\bar{\omega} = \Delta p_0 / (\frac{1}{2} \rho c_1^2)$ or $\zeta = \Delta p_0 / (\frac{1}{2} \rho c_x^2)$ as well as the drag coefficient C_D . Lieblein and Roudebush (1956) have demonstrated the simplified relationship between momentum-thickness ratio and total pressure loss coefficient, valid for unstalled blades,

$$\bar{\omega} = 2 \left(\frac{\theta_2}{l} \right) \left(\frac{l}{s} \right) \frac{\cos^2 \alpha_1}{\cos^3 \alpha_2}. \quad (3.32)$$

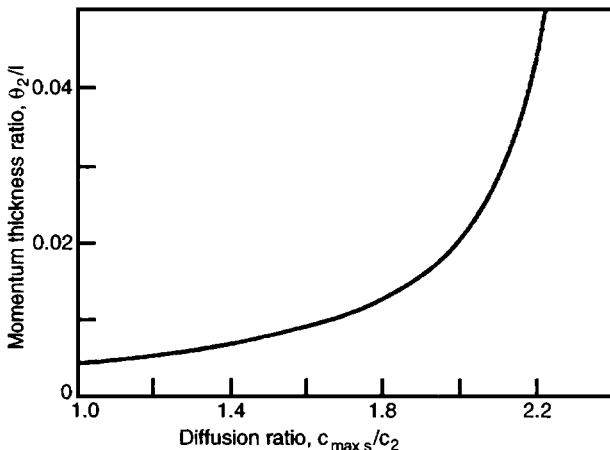


FIG. 3.17. Mean variation of wake momentum-thickness/chord ratio with suction-surface diffusion ratio at reference incidence condition for NACA 65 - (C₁₀A₁₀)10 blades and British C.4 circular-arc blades (adapted from Lieblein (1959)).

Combining this relation with eqns. (3.7) and (3.17) the following useful results can be obtained:

$$C_D = \bar{\omega} \left(\frac{s}{l} \right) \frac{\cos^3 \alpha_m}{\cos^2 \alpha_1} = 2 \left(\frac{\theta_2}{l} \right) \left(\frac{\cos \alpha_m}{\cos \alpha_2} \right)^3 = \zeta \left(\frac{s}{l} \right) \cos^3 \alpha_m. \quad (3.33)$$

The correlation given above *assumes* a knowledge of suction-surface velocities in order that total pressure loss and stall limits can be estimated. As this data may be unavailable it is necessary to establish an *equivalent diffusion ratio*, approximately equal to $c_{\max,s}/c_2$, that can be easily calculated from the inlet and outlet conditions of the cascade. An empirical correlation was established by Lieblein (1959) between a circulation parameter defined by $f(\Gamma) = \Gamma \cos \alpha_1 / (lc_1)$ and $c_{\max,s}/c_1$ at the reference incidence, where the ideal circulation $\Gamma = s(c_{y1} - c_{y2})$, using eqn. (3.23). The correlation obtained is the simple *linear* relation.

$$c_{\max,s}/c_1 = 1.12 + 0.61 f(\Gamma) \quad (3.34)$$

which applies to both NACA 65-(A₁₀) and C.4 circular arc blades. Hence, the equivalent diffusion ratio, after substituting for Γ and simplifying, is

$$D_{eq} = \frac{c_{\max,s}}{c_2} = \frac{\cos \alpha_2}{\cos \alpha_1} \left\{ 1.12 + 0.61 \left(\frac{s}{l} \right) \cos^2 \alpha_1 (\tan \alpha_1 - \tan \alpha_2) \right\} \quad (3.35)$$

At incidence angles greater than reference incidence, Lieblein found that the following correlation was adequate:

$$D_{eq} = \frac{\cos \alpha_2}{\cos \alpha_1} \left\{ 1.12 + k(i - i_{ref})^{1.43} + 0.61 \left(\frac{s}{l} \right) \cos^2 \alpha_1 (\tan \alpha_1 - \tan \alpha_2) \right\} \quad (3.36)$$

where $k = 0.0117$ for the NACA 65-(A₁₀) blades and $k = 0.007$ for the C.4 circular arc blades.

The expressions given above are still very widely used as a means of estimating total pressure loss and the unstalled range of operation of blades commonly employed in subsonic axial compressors. The method has been modified and extended by Swann to include the additional losses caused by shock waves in transonic compressors. The discussion of transonic compressors is outside the scope of this text and is not included.

HOWELL. The low-speed correlation of Howell (1942) has been widely used by designers of axial compressors and is based on a nominal condition such that the deflection ε^* is 80% of the stalling deflection, ε_s (Figure 3.12). Choosing $\varepsilon^* = 0.8\varepsilon_s$ as the *design condition* represents a compromise between the ultraconservative and the overoptimistic! Howell found that the nominal deflections of various compressor cascades are, primarily, a function of the space-chord ratio s/l , the nominal fluid outlet angle α_2^* and the Reynolds number Re

$$\varepsilon^* = f(s/l, \alpha_2^*, Re). \quad (3.37)$$

It is important to note that the correlation (which is really a correlation of stalling deflection, $\varepsilon_s = 1.25\varepsilon^*$) is virtually independent of blade camber θ in the normal range of choice of this parameter ($20^\circ < \theta < 40^\circ$). Figure 3.18 shows the variation of

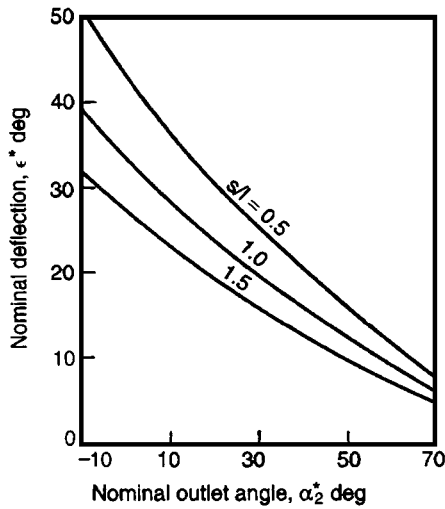


FIG. 3.18. Variation of nominal deflection with nominal outlet angle for several space/chord ratios (adapted from Howell 1945).

ϵ^* found by Howell (1945) against α_2^* for several space–chord ratios. The dependence on Reynolds number is small for $Re > 3 \times 10^5$, based on blade chord.

An approximating formula to the data given in Figure 3.18, which was quoted by Howell and frequently found to be useful in preliminary performance estimation, is the tangent-difference rule:

$$\tan \alpha_1^* - \tan \alpha_2^* = \frac{1.55}{1 + 1.5s/l} \quad (3.38)$$

which is applicable in the range $0 \leq \alpha_2^* \leq 40^\circ$.

Fluid deviation

The *difference* between the fluid and blade inlet angles at cascade inlet is under the arbitrary control of the designer. At cascade outlet however, the difference between the fluid and blade angles, called the *deviation* δ , is a function of blade camber, blade shape, space–chord ratio and stagger angle. Referring to Figure 3.2, the deviation $\delta = \alpha_2 - \alpha_2'$ is drawn as positive; almost without exception it is in such a direction that the deflection of the fluid is reduced. The deviation may be of considerable magnitude and it is important that an accurate estimate is made of it. Re-examining Figure 3.11 again, it will be observed that the fluid receives its maximum guidance on the pressure side of the cascade channel and that this diminishes almost linearly towards the suction side of the channel.

Howell used an empirical rule to relate nominal deviation δ^* to the camber and space–chord ratio,

$$\delta^* = m\theta(s/l)^n, \quad (3.39)$$

where $n \doteq \frac{1}{2}$ for compressor cascades and $n \doteq 1$ for compressor *inlet guide vanes*. The value of m depends upon the shape of the camber line and the blade setting.

For a compressor cascade (i.e. diffusing flow),

$$m = 0.23(2a/l)^2 + \alpha_2^*/500, \quad (3.40a)$$

where α is the distance of maximum camber from the leading edge. For the inlet guide vanes, which are essentially *turbine* nozzles (i.e. accelerating flow),

$$m = \text{constant} = 0.19 \quad (3.40b)$$

EXAMPLE 3.1. A compressor cascade has a space–chord ratio of unity and blade inlet and outlet angles of 50 deg and 20 deg respectively. If the blade camber line is a circular arc (i.e. $a/l = 50\%$) and the cascade is designed to operate at Howell's nominal condition, determine the fluid deflection, incidence and ideal lift coefficient at the design point.

Solution. The camber, $\theta = \alpha'_1 - \alpha'_2 = 30$ deg. As a first approximation put $\alpha_2^* = 20$ deg in eqn. (3.40) to give $m = 0.27$ and, using eqn. (3.39), $\delta^* = 0.27 \times 30 = 8.1$ deg. As a better approximation put $\alpha_2^* = 28.1$ deg in eqn. (3.40) giving $m = 0.2862$ and $\delta^* = 8.6$ deg. Thus, $\alpha_2^* = 28.6$ deg is sufficiently accurate.

From Figure 3.16, with $s/l = 1.0$ and $\alpha_2^* = 28.6$ deg obtain $\varepsilon^* = \alpha_1^* - \alpha_2^* = 21$ deg. Hence $\alpha_1^* = 49.6$ deg and the nominal incidence $i^* = \alpha_1^* - \alpha'_1 = -0.4$ deg.

The *ideal* lift coefficient is found by setting $C_D = 0$ in eqn. (3.18),

$$C_L = 2(s/l) \cos \alpha_m (\tan \alpha_1 - \tan \alpha_2).$$

Putting $\alpha_1 = \alpha_1^*$, $\alpha_2 = \alpha_2^*$ and noting $\tan \alpha_m^* = \frac{1}{2}(\tan \alpha_1^* + \tan \alpha_2^*)$ obtain $\alpha_m^* = 40.75$ deg and $C_L^* = 2(1.172 - 0.545)0.758 \div 0.95$.

In conclusion it will be noted that the estimated deviation is one of the most important quantities for design purposes, as small errors in it are reflected in large changes in deflection and thus, in predicted performance.

Off-design performance

To obtain the performance of a given cascade at conditions removed from the design point, generalised performance curves of Howell (1942) shown in Figure 3.19 may be used. If the nominal deflection ε^* and nominal incidence i^* are known the off-design performance (deflection, total pressure loss coefficient) of the cascade at any other incidence is readily calculated.

EXAMPLE 3.2. In the previous exercise, with a cascade of $s/l = 1.0$, $\alpha'_1 = 50$ deg and $\alpha'_2 = 20$ deg the nominal conditions were $\varepsilon^* = 21$ deg and $i^* = -0.4$ deg.

Determine the off-design performance of this cascade at an incidence $i = 3.8$ deg.

Solution. Referring to Figure 3.19 and with $(i - i^*)/\varepsilon^* = 0.2$ obtain $C_D \div 0.017$, $\varepsilon/\varepsilon^* = 1.15$. Thus, the off-design deflection, $\varepsilon = 24.1$ deg.

From eqn. (3.17), the total pressure loss coefficient is,

$$\zeta = \Delta p_0 / \left(\frac{1}{2} \rho c_x^2 \right) = C_D / [(s/l) \cos^3 \alpha_m].$$

Now $\alpha_1 = \alpha'_1 + i = 53.8$ deg, also $\alpha_2 = \alpha_1 - \varepsilon = 29.7$ deg, therefore,

$$\alpha_m = \tan^{-1} \left\{ \frac{1}{2} (\tan \alpha_1 + \tan \alpha_2) \right\} = \tan^{-1} \{0.969\} = 44.1 \text{ deg},$$

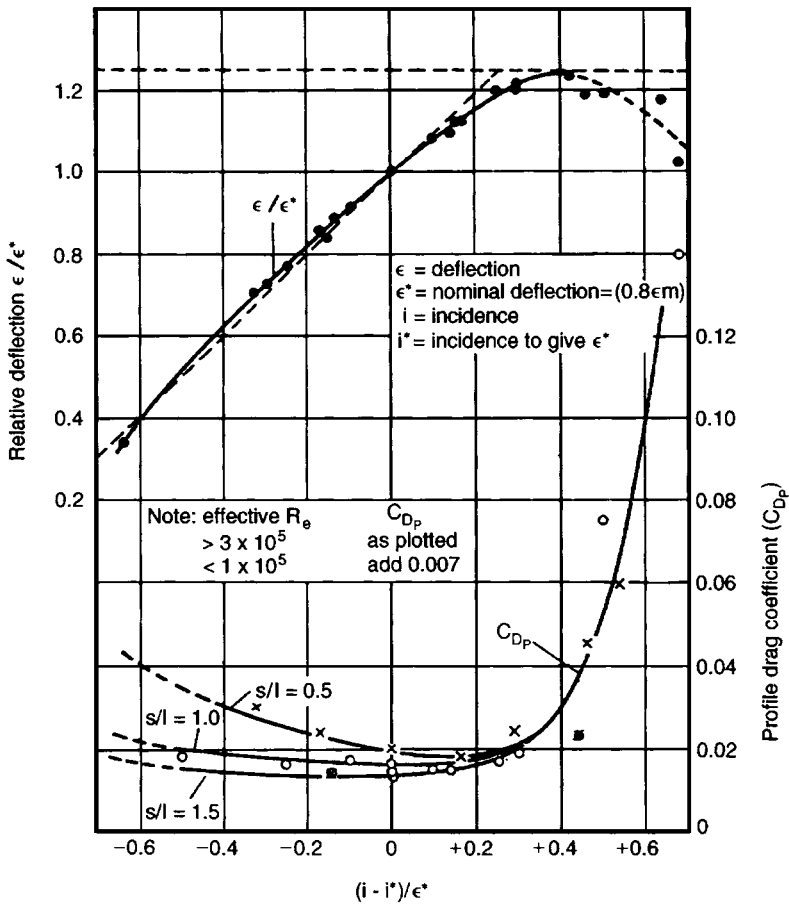


FIG. 3.19. The off-design performance of a compressor cascade (Howell 1942). (By courtesy of the Controller of H.M.S.O., Crown copyright reserved).

hence

$$\zeta = 0.017/0.719^3 = 0.0458.$$

The tangential lift force coefficient, eqn. (3.9), is

$$C_f = (p_2 - p_1)/(\frac{1}{2}\rho c_x^2) = 2(\tan \alpha_1 - \tan \alpha_2) = 1.596.$$

The diffuser efficiency, eqn. (3.25), is

$$\eta_D = 1 - \zeta/(C_f \tan \alpha_m) = 1 - 0.0458/(1.596 \times 0.969) = 97\%.$$

It is worth nothing, from the representative data contained in the above exercise, that the validity of the approximation in eqn. (3.20) is amply justified.

Howell's correlation, clearly, is a simple and fairly direct method of assessing the performance of a given cascade for a range of inlet flow angles. The data can also be used for solving the more complex *inverse problem*, namely, the selection

of a suitable cascade geometry when the fluid deflection is given. For this case, if the previous method of a nominal design condition is used, mechanically unsuitable space-chord ratios are a possibility. The space-chord ratio may, however, be determined to some extent by the mechanical layout of the compressor, the design incidence then only fortuitously coinciding with the nominal incidence. The design incidence is therefore somewhat arbitrary and some designers, ignoring nominal design conditions, may select an incidence best suited to the operating conditions under which the compressor will run. For instance, a negative design incidence may be chosen so that at reduced flow rates a *positive* incidence condition is approached.

Mach number effects

High-speed cascade characteristics are similar to those at low speed until the *critical* Mach number M_c is reached, after which the performance declines. Figure 3.20, taken from Howell (1942) illustrates for a particular cascade tested at varying Mach number and fixed incidence, the drastic decline in pressure rise coefficient up to the *maximum* Mach number at entry M_m , when the cascade is fully choked. When the cascade is choked, no further increase in mass flow through the cascade is possible. The definition of inlet critical Mach number is less precise, but one fairly satisfactory definition (Horlock 1958) is that the maximum *local* Mach number in the cascade has reached unity.

Howell attempted to correlate the decrease in both efficiency and deflection in the range of inlet Mach numbers, $M_c \leq M \leq M_m$ and these are shown in Figure 3.21. By employing this correlation, curves similar to that in Figure 3.20 may be found for each incidence.

One of the principal aims of high-speed cascade testing is to obtain data for determining the values of M_c and M_m . Howell (1945) indicates how, for a typical cascade, M_c and M_m vary with incidence (Figure 3.22)

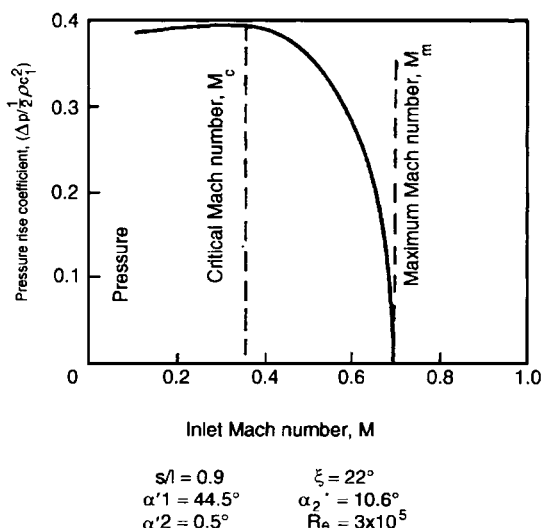


FIG. 3.20. Variation of cascade pressure rise coefficient with inlet Mach number (Howell 1942). (By courtesy of the Controller of H.M.S.O., Crown copyright reserved).

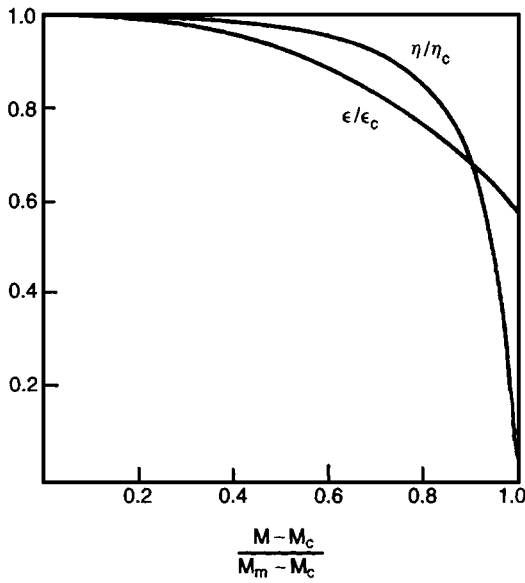


FIG. 3.21. Variation of efficiency and deflection with Mach number (adapted from Howell 1942).

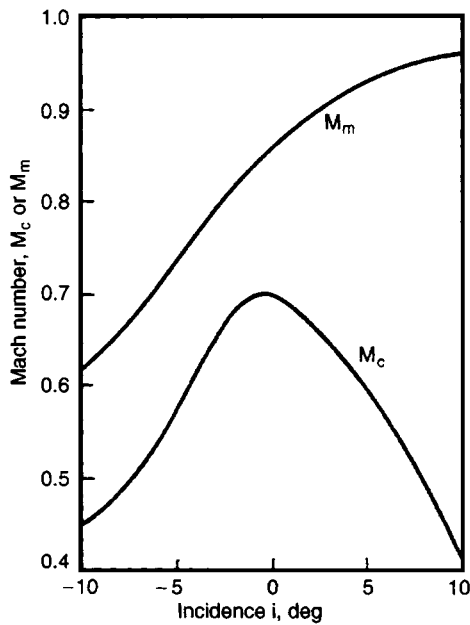


FIG. 3.22. Dependence of critical and maximum Mach numbers upon incidence (Howell 1945). (By courtesy of the Institution of Mechanical Engineers).

Fan blade design (McKenzie)

The cascade tests and design methods evolved by Howell, Carter and others, which were described earlier, established the basis of British axial compressor design. However, a number of empirical factors had to be introduced into the methods in order to correlate actual compressor performance with the performance predicted from cascade data. The system has been in use for many years and has been gradually modified and improved during this time.

McKenzie (1980) has described work done at Rolls-Royce to further develop the correlation of cascade and compressor performance. The work was done on a low-speed four-stage compressor with 50 per cent reaction blading of constant section. The compressor hub to tip radius ratio was 0.8 and a large number of combinations of stagger and camber was tested.

McKenzie pointed out that the deviation rule originated by Howell (1945), i.e. eqns. (3.39) and (3.40a) with $n = 0.5$, was developed from cascade tests performed *without* sidewall suction. Earlier in this chapter it was explained that the consequent thickening of the sidewall boundary layers caused a contraction of the main through-flow (Figure 3.8), resulting in a reduced static pressure rise across the cascade and an increased air deflection. Rolls-Royce conducted a series of tests on C5 profiles with circular arc camber lines using a number of wall suction slots to control the axial velocity ratio (AVR). The deviation angles at mid-span with an AVR of unity were found to be significantly greater than those given by eqn. (3.39).

From cascade tests McKenzie derived the following rule for the deviation angle:

$$\delta = (1.1 + 0.31\theta)(s/l)^{1/3} \quad (3.41)$$

where δ and θ are in degrees. From the results a relationship between the blade stagger angle ξ and the vector mean flow angle α_m was obtained:

$$\tan \xi = \tan \alpha_m - 0.213, \quad (3.42)$$

where $\tan \alpha_m$ is defined by eqn. (3.6). The significance of eqn. (3.42) is, that if the air inlet and outlet angles (α_1 and α_2 respectively) are specified, then the stagger angle *for maximum efficiency* can be determined, assuming that a C5 profile (or a similar profile such as C4) on a circular arc camber line is being considered. Of course, the camber angle θ and the pitch/chord ratio s/l still need to be determined.

In a subsequent paper McKenzie (1988) gave a graph of efficiency in terms of Cp_i and s/l , which was an improved presentation of the correlation given in his earlier paper. The ideal static pressure rise coefficient is defined as

$$Cp_i = 1 - (c_2/c_1)^2. \quad (3.43)$$

McKenzie's efficiency correlation is shown in Figure 3.23, where the ridge line of optimum efficiency is given by

$$s/l = 9 \times (0.567 - Cp_i) \quad (3.44)$$

EXAMPLE 3.3. At the midspan of a proposed fan stator blade the inlet and outlet air angles are to be $\alpha_1 = 58^\circ$ and $\alpha_2 = 44^\circ$. Using the data and correlation of McKenzie, determine a suitable blade camber and space-chord ratio.

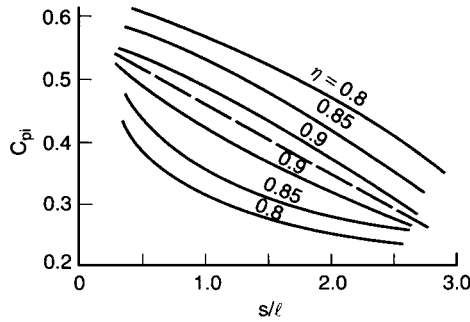


FIG. 3.23. Efficiency correlation (adapted from McKenzie 1988).

Solution. From eqn. (3.6) the vector mean flow angle is found,

$$\tan \alpha_m = \frac{1}{2}(\tan \alpha_1 + \tan \alpha_2) = 1.2830.$$

From eqn. (3.42) we get the stagger angle,

$$\tan \xi = \tan \alpha_m - 0.213 = 1.0700.$$

Thus, $\alpha_m = 52.066^\circ$ and $\xi = 46.937^\circ$.

From eqn. (3.43), assuming that $AVR = 1.0$, we find

$$C p_i = 1 - \left(\frac{\cos \alpha_1}{\cos \alpha_2} \right)^2 = 0.4573.$$

Using the optimum efficiency correlation, eqn. (3.44),

$$s/l = 9 \times (0.567 - 0.4573),$$

$$\therefore s/l = \underline{0.9872}.$$

To determine the blade camber we combine

$$\delta = \alpha_2 - \alpha'_2 = \alpha_2 - \xi - \theta/2$$

with eqn. (3.41), to get

$$\theta = \frac{\xi - \alpha_2 + 1.1(s/l)^{1/3}}{0.5 - 0.31(s/l)^{1/3}} = \frac{46.937 - 44 + 1.1 \times 0.9957}{0.5 - 0.31 \times 0.9957}$$

$$\therefore \theta = \underline{21.08^\circ}.$$

According to McKenzie the correlation gives, for high stagger designs, peak efficiency conditions well removed from stall and is in good agreement with earlier fan blade design methods.

Turbine cascade correlation (Ainley)

Ainley and Mathieson (1951) published a method of estimating the performance of an axial flow turbine and the method has been widely used ever since. In essence

the total pressure loss and gas efflux angle for each row of a turbine stage is determined at a single *reference* diameter and under a wide range of inlet conditions. This reference diameter was taken as the arithmetic mean of the rotor and stator rows inner and outer diameters. Dunham and Came (1970) gathered together details of several improvements to the method of Ainley and Mathieson which gave better performance prediction for *small* turbines than did the original method. When the blading is *competently* designed the revised method appears to give reliable predictions of efficiency to within 2% over a wide range of designs, sizes and operating conditions.

Total pressure loss correlations

The overall total pressure loss is composed of three parts, viz. (i) profile loss, (ii) secondary loss, and (iii) tip clearance loss.

(i) A profile loss coefficient is defined as the loss in stagnation pressure across the blade row or cascade, divided by the difference between stagnation and static pressures at blade outlet; i.e.

$$Y_p = \frac{p_{01} - p_{02}}{p_{02} - p_2}. \quad (3.45)$$

In the Ainley method, profile loss is determined initially at zero incidence ($i = 0$). At any other incidence the profile loss ratio $Y_p/Y_{p(i=0)}$ is assumed to be defined by a unique function of the incidence ratio i/i_s (Figure 3.24), where i_s is the stalling incidence. This is defined as the incidence at which $Y_p/Y_{p(i=0)} = 2.0$.

Ainley and Mathieson correlated the profile losses of turbine blade rows against space/chord ratio s/l , fluid outlet angle α_2 , blade maximum thickness/chord ratio t/l

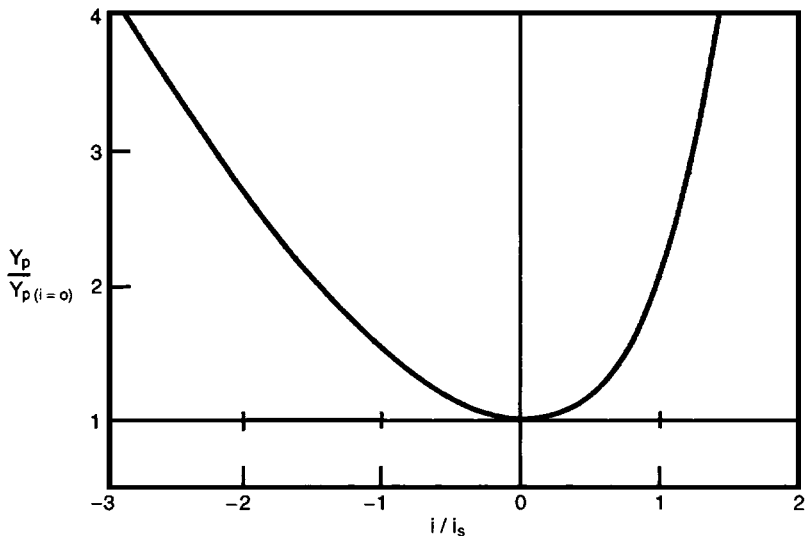


FIG. 3.24. Variation of profile loss with incidence for typical turbine blading (adapted from Ainley and Mathieson 1951).

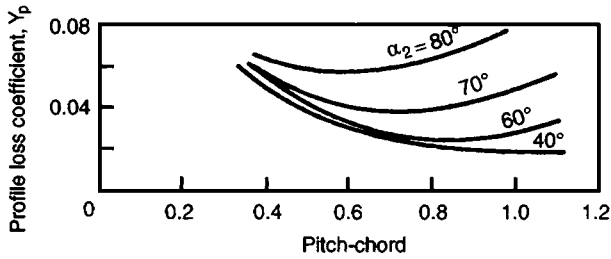
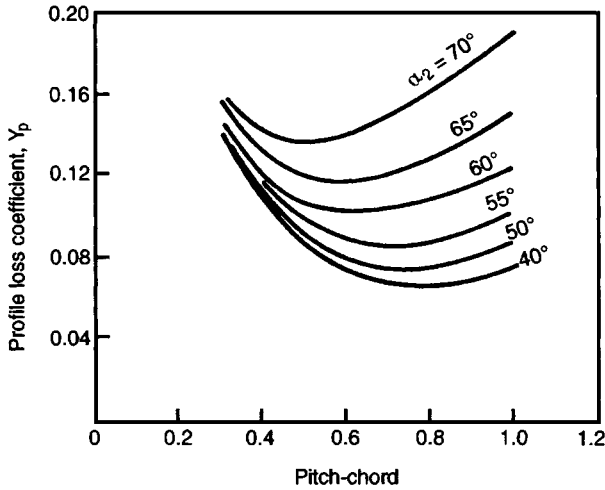
(a) Nozzle blades, $\alpha_1 = 0$ (b) Impulse blades, $\alpha_1 = \alpha_2$ (t/l = 0.2 Re = 2×10^5 ; M < 0.6)

FIG. 3.25. Profile loss coefficients of turbine nozzle and impulse blades at zero incidence ($t/l = 20\%$; $Re = 2 \times 10^5$; $M < 0.6$) (adapted from Ainley and Mathieson 1951).

and blade inlet angle. The variation of $Y_{p(i=0)}$ against s/l is shown in Figure 3.25 for nozzles and impulse blading at various flow outlet angles. The sign convention used for flow angles in a turbine cascade is indicated in Figure 3.27. For other types of blading intermediate between nozzle blades and impulse blades the following expression is employed:

$$Y_{p(i=0)} = \left\{ Y_{p(\alpha_1=0)} + \left(\frac{\alpha_1}{\alpha_2} \right)^2 [Y_{p(\alpha_1=\alpha_2)} - Y_{p(\alpha_1=0)}] \right\} \left(\frac{t/l}{0.2} \right)^{\alpha_1/\alpha_2} \quad (3.46)$$

where all the Y_p 's are taken at the same space/chord ratio and flow outlet angle. If rotor blades are being considered, put $3\beta_2$ for α_1 and β_3 for α_2 . Equation (3.46) includes a correction for the effect of thickness-chord ratio and is valid in the range $0.15 \leq t/l \leq 0.25$. If the actual blade has a t/l greater or less than the limits quoted, Ainley recommends that the loss should be taken as equal to a blade having

t/l either 0.25 or 0.15. By substituting $\alpha_1 = \alpha_2$ and $t/l = 0.2$ in eqn. (3.46), the zero incidence loss coefficient for the impulse blades $Y_{p(\alpha_1=\alpha_2)}$ given in Figure 3.25 is recovered. Similarly, with $\alpha_1 = 0$ at $t/l = 0.2$ in eqn. (3.46) gives $Y_{p(\alpha_1=0)}$ of Figure 3.25.

A feature of the losses given in Figure 3.25 is that, compared with the impulse blades, the nozzle blades have a much lower loss coefficient. This trend confirms the results shown in Figure 3.14, that flow in which the mean pressure is falling has a lower loss coefficient than a flow in which the mean pressure is constant or increasing.

(ii) The secondary losses arise from complex three-dimensional flows set up as a result of the end wall boundary layers passing through the cascade. There is substantial evidence that the end wall boundary layers are convected inwards along the suction-surface of the blades as the main flow passes through the blade row, resulting in a serious mal-distribution of the flow, with losses in stagnation pressure often a significant fraction of the total loss. Ainley found that secondary losses could be represented by

$$C_{Ds} = \lambda C_L^2 / (s/l) \quad (3.47)$$

where λ is parameter which is a function of the flow acceleration through the blade row. From eqn. (3.17), together with the definition of Y , eqn. (3.45) for incompressible flow, $C_D = Y(s/l) \cos^3 \alpha_m / \cos^2 \alpha_2$, hence

$$Y_s = \frac{C_{Ds} \cos^2 \alpha_2}{(s/l) \cos^3 \alpha_m} = \lambda \left(\frac{C_L}{s/l} \right)^2 \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m} = \lambda Z \quad (3.48)$$

where Z is the blade aerodynamic loading coefficient. Dunham (1970) subsequently found that this equation was not correct for blades of low aspect ratio, as in small turbines. He modified Ainley's result to include a better correlation with aspect ratio and at the same time simplified the flow acceleration parameter. The correlation, given by Dunham and Came (1970), is

$$Y_s = 0.0334 \left(\frac{l}{H} \right) \left(\frac{\cos \alpha_2}{\cos \alpha_1'} \right) Z \quad (3.49)$$

and this represents a significant improvement in the prediction of secondary losses using Ainley's method.

Recently, more advanced methods of predicting losses in turbine blade rows have been suggested which take into account the *thickness* of the entering boundary layers on the annulus walls. Came (1973) measured the secondary flow losses on *one* end wall of several turbine cascades for various thicknesses of inlet boundary layer. He correlated his own results, and those of several other investigators, and obtained a modified form of Dunham's earlier result, viz.,

$$Y_s = \left(0.25 Y_1 \frac{\cos^2 \alpha_1}{\cos^2 \alpha_2} + 0.009 \frac{l}{H} \right) \left(\frac{\cos \alpha_2}{\cos \alpha_1'} \right) Z - Y_1 \quad (3.50)$$

which is the net secondary loss coefficient for *one* end wall only and where Y_1 is a mass-averaged inlet boundary layer total pressure loss coefficient. It is evident that the increased accuracy obtained by use of eqn. (3.50) requires the additional

effort of calculating the wall boundary layer development. In *initial* calculations of performance it is probably sufficient to use the earlier result of Dunham and Came, eqn. (3.49), to achieve a reasonably accurate result.

(iii) The tip clearance loss coefficient Y_k depends upon the blade loading Z and the size and nature of the clearance gap k . Dunham and Came presented an amended version of Ainley's original result for Y_k :

$$Y_k = B \left(\frac{l}{H} \right) \left(\frac{k}{l} \right)^{0.78} Z \quad (3.51)$$

where $B = 0.5$ for a plain tip clearance, 0.25 for shrouded tips.

Reynolds number correction

Ainley and Mathieson (1951) obtained their data for a mean Reynolds number of 2×10^5 based on the mean chord and exit flow conditions from the turbine state. They recommended for lower Reynolds numbers, down to 5×10^4 , that a correction be made to stage efficiency according to the rough rule:

$$(1 - \eta_{it}) \propto Re^{-1/5}.$$

Dunham and Came (1970) gave an optional correction which is applied directly to the sum of the profile and secondary loss coefficients for a blade row using the Reynolds number *appropriate* to that row. The rule is:

$$Y_p + Y_s \propto Re^{-1/5}.$$

Flow outlet angle from a turbine cascade

It was pointed out by Ainley (1948) that the method of defining deviation angle as adopted in several well-known compressor cascade correlations had proved to be impracticable for turbine blade cascade. In order to predict fluid outlet angle α_2 , steam turbine designers had made much use of the simple empirical rule that

$$\alpha_2 = \cos^{-1} \Theta/s \quad (3.52a)$$

where Θ is the opening at the throat, depicted in Figure 3.26, and s is the pitch. This widely used rule gives a very good approximation to measured pitchwise averaged flow angles when the outlet Mach number is at or close to unity. However, at low Mach numbers substantial variations have been found between the rule and observed flow angles. Ainley and Mathieson (1951) recommended that for low outlet Mach numbers $0 < M_2 \leq 0.5$, the following rule be used:

$$\alpha_2 = f(\cos^{-1} \Theta/s) + 4s/e \text{ (deg)} \quad (3.52b)$$

where $f(\cos^{-1} \Theta/s) = -11.15 + 1.154 \cos^{-1} \Theta/s$ and $e = j^2/(8z)$ is the mean radius of curvature of the blade suction surface between the throat and the trailing edge. At a gas outlet Mach number of unity Ainley and Mathieson assumed, for a turbine blade row, that

$$\alpha_2 = \cos^{-1} A_t/A_{n2} \quad (3.52c)$$

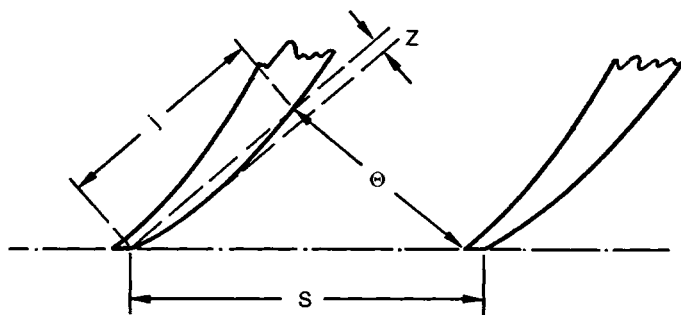


FIG. 3.26. Details near turbine cascade exit showing “throat” and suction-surface curvature parameters.

where A_t is the passage throat area and A_{n2} is the annulus area in the reference plane downstream of the blades. If the annulus walls at the ends of the cascade are not flared then eqn. (3.52c) is the same as eqn. (3.52a). Between $M_2 = 0.5$ and $M_2 = 1.0$ a linear variation of α_2 can be reasonably assumed in the absence of any other data.

Comparison of the profile loss in a cascade and in a turbine stage

The aerodynamic efficiency of an axial-flow turbine is significantly less than that predicted from measurements made on equivalent cascades operating under steady flow conditions. The importance of flow unsteadiness originating from the wakes of a preceding blade row was studied by Lopatitskii *et al.* (1969) who reported that the rotor blade profile loss was (depending on blade geometry and Reynolds number) between two and four times greater than that for an equivalent cascade operating with the same flow. Hodson (1984) made an experimental investigation of the rotor to stator interaction using a large-scale, low-speed turbine, comparing the results with those of a rectilinear cascade of identical geometry. Both tunnels were operated at a Reynolds number of 3.15×10^5 . Hodson reported that the turbine rotor midspan profile loss was approximately 50 per cent higher than that of the rectilinear cascade. Measurements of the shear stress showed that as a stator wake is convected through a rotor blade passage, the *laminar* boundary layer on the suction surface undergoes transition in the vicinity of the wake. The 50 per cent increase in profile loss was caused by the time-dependent transitional nature of the boundary layers. The loss increase was largely independent of spacing between the rotor and the stator.

In a turbine stage the interaction between the two rows can be split into two parts: (a) the effects of the potential flow; and (b) the effects due to wake interactions. The effects of the potential influence extend upstream and downstream and decay exponentially with a length scale typically of the order of the blade chord or pitch. Some aspects of these decay effects are studied in Chapter 6 under the heading “Actuator Disc Approach”. In contrast, blade wakes are convected downstream of

the blade row with very little mixing with the mainstream flow. The wakes tend to persist even where the blade rows of a turbomachine are very widely spaced.

A designer usually assumes that the blade rows of an axial-flow turbomachine are sufficiently far apart that the flow is steady in both the stationary and rotating frames of reference. The flow in a real machine, however, is unsteady both as a result of the relative motion of the blade wakes between the blade rows and the potential influence. In modern turbomachines, the spacing between the blade rows is typically of the order of $1/4$ to $1/2$ of a blade chord. As attempts are made to make turbomachines more compact and blade loadings are increased, then the levels of unsteadiness will increase.

The earlier Russian results showed that the losses due to flow unsteadiness were greater in turbomachines of high reaction and low Reynolds number. With such designs, a larger proportion of the blade suction surface would have a laminar boundary layer and would then exhibit a correspondingly greater profile loss as a result of the wake-induced boundary layer transition.

Optimum space–chord ratio of turbine blades (Zweifel)

It is worth pondering a little upon the effect of the space–chord ratio in turbine blade rows as this is a factor strongly affecting efficiency. Now if the spacing between blades is made small, the fluid then tends to receive the maximum amount of guidance from the blades, but the friction losses will be very large. On the other hand, with the same blades spaced well apart, friction losses are small but, because of poor fluid guidance, the losses resulting from flow separation are high. These considerations led Zweifel (1945) to formulate his criterion for the optimum space–chord ratio of blading having large deflection angles. Essentially, *Zweifel's criterion* is simply that the ratio (ψ_T) of the actual to an “ideal” tangential blade loading, has a certain constant value for minimum losses. The tangential blade loads are obtained from the real and ideal pressure distributions on both blade surfaces, as described below.

Figure 3.27 indicates a typical pressure distribution around one blade in a turbine cascade, curves P and S corresponding to the pressure (or concave) side and suction (convex) side respectively. The pressures are projected parallel to the cascade front so that the area enclosed between the curves S and P represents the *actual tangential blade load per unit span*,

$$Y = \rho s c_x (c_{y2} + c_{y1}), \quad (3.53)$$

cf. eqn. (3.3) for a compressor cascade.

It is instructive to examine the pressures along the blade surfaces. Assuming incompressible flow the static inlet pressure is $p_1 = p_0 - \frac{1}{2}\rho c_1^2$; if losses are also ignored the outlet static pressure $p_2 = p_0 - \frac{1}{2}\rho c_2^2$. The pressure on the P side remains high at first (p_0 being the maximum, attained only at the stagnation point), then falls sharply to p_2 . On the S side there is a rapid decrease in static pressure from the leading edge, but it may even rise again towards the trailing edge. The closer the blade spacing s the smaller the load Y becomes (eqn. (3.53)). Conversely, wide spacing implies an increased load with pressure rising on the P side and falling

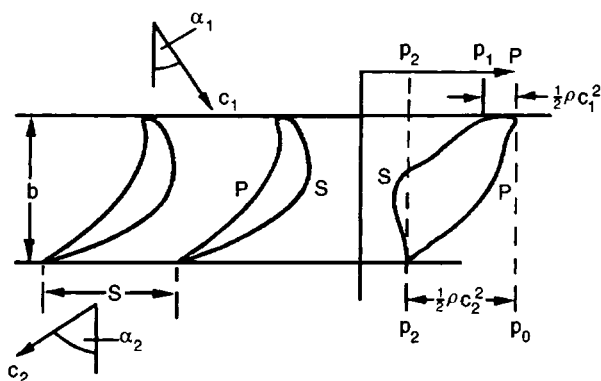


FIG. 3.27. Pressure distribution around a turbine cascade blade (after Zweifel 1945).

on the *S* side. Now, whereas the static pressure can never rise above p_0 on the *P* surface, very low pressures are possible, at least in theory on the *S* surface. However, the pressure rise towards the trailing edge is limited in practice if flow separation is to be avoided, which implies that the load carried by the blade is restricted.

To give some idea of blade load capacity, the real pressure distribution is compared with an ideal pressure distribution giving a maximum load Y_{id} without risk of fluid separation on the *S* surface. Upon reflection, one sees that these conditions for the ideal load are fulfilled by p_0 acting over the *whole* *P* surface and p_2 acting over the *whole* *S* surface. With this ideal pressure distribution (which cannot, of course, be realised), the tangential load per unit span is,

$$Y_{id} = \frac{1}{2} \rho c_2^2 b \quad (3.54)$$

and, therefore,

$$\psi_T = Y/Y_{id} = 2(s/b) \cos^2 \alpha_2 (\tan \alpha_1 + \tan \alpha_2) \quad (3.55)$$

after combining eqns. (3.53) and (3.54) together with angles defined by the geometry of Figure 3.27.

Zweifel found from a number of experiments on turbine cascades that for minimum losses the value of ψ_T was approximately 0.8. Thus, for specified inlet and outlet angles the optimum space–chord ratio can be estimated. However, according to Horlock (1966), Zweifel's criterion predicts optimum space–chord ratio for the data of Ainley *only for outlet angles of 60 to 70 deg*. At other outlet angles it does not give an accurate estimate of optimum space–chord ratio.

References

- Ainley, D. G. (1948). Performance of axial flow turbines. *Proc. Instn. Mech. Engrs.*, **159**.
 Ainley, D. G. and Mathieson, G. C. R. (1951). A method of performance estimation for axial flow turbines. *ARC. R. and M.* 2974.

- Bryer, D. W. and Pankhurst, R. C. (1971). *Pressure-probe Methods for Determining Wind Speed and Flow Direction*. National Physical Laboratory, HMSO.
- Came, P. M. (1973). Secondary loss measurements in a cascade of turbine blades. *Proc. Instn. Mech. Engrs.* Conference Publication 3.
- Carter, A. D. S. (1950). Low-speed performance of related aerofoils in cascade. ARC. Current Paper, No. 29.
- Carter, A. D. S., Andrews, S. J. and Shaw, H. (1950). Some fluid dynamic research techniques. *Proc. Instn. Mech. Engrs.*, **163**.
- Carter, A. D. S. (1948). Three-dimensional flow theories for axial compressors and turbines. *Proc. Instn. Mech. Engrs.*, **159**.
- Csanady, G. T. (1964). *Theory of Turbomachines*. McGraw-Hill, New York.
- Dixon, S. L. (1978). Measurement of flow direction in a shear flow. *J. Physics E: Scientific Instruments*, **2**, 31–4.
- Dowden, R. R. (1972). *Fluid Flow Measurement – a Bibliography*. BHRA.
- Dunham, J. (1970). A review of cascade data on secondary losses in turbines. *J. Mech. Eng. Sci.*, **12**.
- Dunham, J. (1974). A parametric method of turbine blade profile design. *Am. Soc. Mech. Engrs.* Paper 74-GT-119.
- Dunham, J. and Came, P. (1970). Improvements to the Ainley-Mathieson method of turbine performance prediction. *Trans. Am. Soc. Mech. Engrs.*, Series A, **92**.
- Felix, A. R. (1957). Summary of 65-Series compressor blade low-speed cascade data by use of the carpet-plotting technique. *NACA T.N.* 3913.
- Glauert, H. (1959). *Aerofoil and Airscrew Theory*. (2nd edn). Cambridge University Press.
- Hay, N., Metcalfe, R. and Reizes, J. A. (1978). A simple method for the selection of axial fan blade profiles. *Proc. Instn Mech Engrs.*, **192**, (25) 269–75.
- Herrig, L. J., Emery, J. C. and Erwin, J. R. (1957). Systematic two-dimensional cascade tests of NACA 65-Series compressor blades at low speeds. *NACA T.N.* 3916.
- Hodson, H. P. (1984). Boundary layer and loss measurements on the rotor of an axial-flow turbine. *J. Eng. for Gas Turbines and Power. Trans Am. Soc. Mech. Engrs.*, **106**, 391–9.
- Horlock, J. H. (1958). *Axial Flow Compressors*. Butterworths. (1973 reprint with supplemental material, Huntington, New York: Krieger).
- Horlock, J. H. (1966). *Axial-flow Turbines*. Butterworths. (1973 reprint with corrections, Huntington, New York: Krieger).
- Howell, A. R. (1942). The present basis of axial flow compressor design: Part I, Cascade theory and performance. *ARC R and M.* 2095.
- Howell, A. R. (1945). Design of axial compressors. *Proc. Instn. Mech. Engrs.*, **153**.
- Howell, A. R. (1945). Fluid dynamics of axial compressors. *Proc. Instn. Mech. Engrs.*, **153**.
- Lieblein, S. (1959). Loss and stall analysis of compressor cascades. *Trans. Am. Soc. Mech. Engrs.*, Series D, **81**.
- Lieblein, S. and Roubush, W. H. (1956). Theoretical loss relation for low-speed 2D cascade flow. *NACA T.N.* 3662.
- Lopatitskii, A. O. *et al.* (1969). Energy losses in the transient state of an incident flow on the moving blades of turbine stages. *Energomashinostroenie*, **15**.
- McKenzie, A. B. (1980). The design of axial compressor blading based on tests of a low speed compressor. *Proc. Instn. Mech. Engrs.*, **194**, 6.
- McKenzie, A. B. (1988). The selection of fan blade geometry for optimum efficiency. *Proc. Instn. Mech. Engrs.*, **202**, A1, 39–44.

- National Aeronautics and Space Administration, (1965). Aerodynamic design of axial-flow compressors. NASA SP 36.
- Sieverding, C. H. (1975). Pressure probe measurements in cascades. In *Modern Methods of Testing Rotating Components of Turbomachines*, AGARDograph 207.
- Sieverding, C. H. (1985). Aerodynamic development of axial turbomachinery blading. In *Thermodynamics and Fluid Mechanics of Turbomachinery*, Vol. 1 (A.S. Ücer, P. Stow and Ch. Hirsch, eds) pp. 513–65. Martinus Nijhoff.
- Swann, W. C. (1961). A practical method of predicting transonic compressor performance. *Trans. Am. Soc. Mech. Engrs.*, Series A, **83**.
- Todd, K. W. (1947). Practical aspects of cascade wind tunnel research. *Proc. Instn. Mech. Engrs.*, **157**.
- Zweifel, O. (1945). The spacing of turbomachine blading, especially with large angular deflection. *Brown Boveri Rev.*, **32**, 12.

Problems

1. Experimental compressor cascade results suggest that the stalling lift coefficient of a cascade blade may be expressed as

$$C_L \left(\frac{c_1}{c_2} \right)^3 = 2.2$$

where c_1 and c_2 are the entry and exit velocities. Find the stalling inlet angle for a compressor cascade of space–chord ratio unity if the outlet air angle is 30 deg.

2. Show, for a turbine cascade, using the angle notation of Figure 3.27, that the lift coefficient is

$$C_L = 2(s/l)(\tan \alpha_1 + \tan \alpha_2) \cos \alpha_m + C_D \tan \alpha_m$$

where $\tan \alpha_m = \frac{1}{2}(\tan \alpha_2 - \tan \alpha_1)$ and $C_D = \text{Drag}/(\frac{1}{2}\rho c_m^2 l)$.

A cascade of turbine nozzle vanes has a blade inlet angle $\alpha'_1 = 0$ deg, a blade outlet angle α'_2 of 65.5 deg, a chord length l of 45 mm and an axial chord b of 32 mm. The flow entering the blades is to have zero incidence and an estimate of the deviation angle based upon similar cascades is that δ will be about 1.5 deg at low outlet Mach number. If the blade load ratio ψ_T defined by eqn. (3.55) is to be 0.85, estimate a suitable space–chord ratio for the cascade.

Determine the drag and lift coefficients for the cascade given that the profile loss coefficient

$$\lambda = \Delta p_0 / (\frac{1}{2}\rho c_2^2) = 0.035.$$

3. A compressor cascade is to be designed for the following conditions:

Nominal fluid outlet angle	α'_2	=	30 deg
Cascade camber angle	θ	=	30 deg
Pitch/chord ratio	s/l	=	1.0
Circular arc camberline	a/l	=	0.5

Using Howell's curves and his formula for nominal deviation, determine the nominal incidence, the actual deviation for an incidence of +2.7 deg and the approximate lift coefficient at this incidence.

4. A compressor cascade is built with blades of circular arc camber line, a space/chord ratio of 1.1 and blade angles of 48 and 21 deg at inlet and outlet. Test data taken from the cascade shows that at zero incidence ($i = 0$) the deviation $\delta = 8.2$ deg and the total

pressure loss coefficient $\bar{\omega} = \Delta p_0 / (\frac{1}{2} \rho c_1^2) = 0.015$. At positive incidence over a limited range ($0 \leq i \leq 6^\circ$) the variation of both δ and $\bar{\omega}$ for this particular cascade can be represented with sufficient accuracy by linear approximations, viz.

$$\frac{d\delta}{di} = 0.06, \quad \frac{d\bar{\omega}}{di} = 0.001$$

where i is in degrees.

For a flow incidence of 5.0 deg determine

- (i) the flow angles at inlet and outlet;
- (ii) the diffuser efficiency of the cascade;
- (iii) the static pressure rise of air with a velocity 50 m/s normal to the plane of the cascade.

Assume density of air is 1.2 kg/m^3 .

5. (a) A cascade of compressor blades is to be designed to give an outlet air angle α_2 of 30 deg for an inlet air angle α_1 of 50 deg measured from the normal to the plane of the cascade. The blades are to have a *parabolic arc* camber line with $a/l = 0.4$ (i.e. the fractional distance along the chord to the point of maximum camber). Determine the space/chord ratio and blade outlet angle if the cascade is to operate at zero incidence and nominal conditions. You may assume the linear approximation for nominal deflection of Howell's cascade correlation:

$$\epsilon^* = (16 - 0.2\alpha_2^*)(3 - s/l) \text{ deg}$$

as well as the formula for nominal deviation:

$$\delta^* = \left[0.23 \left(\frac{2a}{l} \right)^2 + \frac{\alpha_2^*}{500} \right] \theta \sqrt{\frac{s}{l}} \text{ deg.}$$

(b) The space/chord ratio is now changed to 0.8, but the blade angles remain as they are in part (a) above. Determine the lift coefficient when the incidence of the flow is 2.0 deg. Assume that there is a linear relationship between ϵ/ϵ^* and $(i - i^*)/\epsilon^*$ over a limited region, viz. at $(i - i^*)/\epsilon^* = 0.2$, $\epsilon/\epsilon^* = 1.15$ and at $i = i^*$, $\epsilon/\epsilon^* = 1$. In this region take $C_D = 0.02$.

6. (a) Show that the pressure rise coefficient $C_p = \Delta p / (\frac{1}{2} \rho c_1^2)$ of a compressor cascade is related to the diffuser efficiency η_D and the total pressure loss coefficient ζ by the following expressions:

$$C_p = \eta_D (1 - \sec^2 \alpha_2 / \sec^2 \alpha_1) = 1 - (\sec^2 \alpha_2 + \zeta) / \sec^2 \alpha_1$$

where $\eta_D = \Delta p / \{ \frac{1}{2} \rho (c_1^2 - c_2^2) \}$

$$\zeta = \Delta p_0 / (\frac{1}{2} \rho c_x^2)$$

α_1, α_2 = flow angles at cascade inlet and outlet.

(b) Determine a suitable *maximum* inlet flow angle of a compressor cascade having a space/chord ratio 0.8 and $\alpha_2 = 30$ deg when the diffusion factor D is to be limited to 0.6. The definition of diffusion factor which should be used is the early Lieblein formula (1956),

$$D = \left(1 - \frac{\cos \alpha_1}{\cos \alpha_2} \right) + \left(\frac{s}{l} \right) \frac{\cos \alpha_1}{2} (\tan \alpha_1 - \tan \alpha_2).$$

(c) The stagnation pressure loss derived from flow measurements on the above cascade is 149 Pa when the inlet velocity c_1 is 100 m/s at an air density ρ of 1.2 kg/m^3 . Determine the values of

- (i) pressure rise;
- (ii) diffuser efficiency;
- (iii) drag and lift coefficients.

7 (a) A set of circular arc fan blades, camber $\theta = 8$ deg, are to be tested in a cascade wind tunnel at a space/chord ratio, $s/l = 1.5$, with a stagger angle $\xi = 68$ deg. Using McKenzie's method of correlation and assuming optimum conditions at an axial velocity ratio of unity, obtain values for the air inlet and outlet angles.

(b) Assuming the values of the derived air angles are correct and that the cascade has an effective lift/drag ratio of 18, determine

- (i) the coefficient of lift of the blades;
- (ii) the efficiency of the cascade (treating it as a diffuser).