

Concepts and parameters associated with cylindrical gears and cylindrical gear pairs with involute teeth

DIN 3960

Begriffe und Bestimmungsgrößen für Stirnräder (Zylinderräder) und Stirnradpaare (Zylinderradpaare) mit Evolventenverzahnung

As it is current practice in standards published by the International Organization for Standardization (ISO), the comma has been used throughout as a decimal marker.

Compared with the October 1976 edition the numbering system in this standard has been changed. The new equation numbers are numbered continuously within the main clauses and the first digit represents the number of the respective main clause in which they appear.

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0 Other relevant standards

DIN 867	Standard basic rack tooth profile of cylindrical gears with involute teeth for general and heavy engineering
DIN 868	General concepts and parameters for gears, gear pairs and gear trains
DIN 1301 Part 1	Units; names, symbols
DIN 1315	Angles; concepts, units
DIN 3961	Tolerances for cylindrical gear teeth; bases
DIN 3967	System of gear fits; backlash, tooth thickness deviations, tooth thickness tolerances; bases
DIN 3972	Standard basic rack tooth profiles of gear-cutting tools for involute tooth systems according to DIN 867
DIN 3992	Addendum modification of external cylindrical gears
DIN 3993	Geometrical design of cylindrical internal involute gear pairs
DIN 3999	Symbols for gear teeth
DIN 7182 Part 1	Tolerances and fits; fundamental concepts
DIN 58 400	Standard basic rack tooth profile for cylindrical gears with involute teeth for fine mechanics
DIN 58 412	Standard basic rack tooth profile for gear tools for fine mechanics; involute gears according to DIN 58 400 and DIN 867

1 Symbols, terms, units

1.1 Symbols and terms

This standard uses the following symbols and terms:

a	centre distance
a_d	reference centre distance
a''	centre distance for double-flank engagement
b	facewidth
b_M	contact line overlap or width of gauge head for measuring base tangent length
c	bottom clearance
c''	bottom clearance coefficient
d	reference diameter
d_a	tip diameter
d_{aM}	measured value of tip diameter of overcut cylindrical gears
d_b	base diameter
d_f	root diameter
d_n	virtual reference diameter
d_v	V-circle diameter
d_w	pitch diameter
d_y	Y-circle diameter
d_K	diameter of circle through centre of ball
d_M	diameter of a measuring circle (at point of contact with measuring instrument)
d_N	Usable diameter
d_{Na}	usable tip diameter
d_{Nf}	usable root diameter
d_{NE}	usable root diameter at generated wheel (usable flank)
e	spacewidth on the reference cylinder
e_a	spacewidth on the tip cylinder
e_b	base spacewidth (on the base cylinder)
e_f	spacewidth on the root cylinder

e_v	spacewidth on the V-cylinder
e_y	spacewidth on the Y-cylinder
f	individual error
f_b	base circle error
f_c	eccentricity
f_f	profile form error
f_{fw}	profile waviness
f_t	tangential tooth-to-tooth composite error
f_r	radial tooth-to-tooth composite error
f_p	adjacent pitch error
f_{pe}	normal pitch error
f_{px}	axial pitch error
f_{pz}	lead error
f_p'	individual working error
f_r'	tip circle radial run-out
f_z	difference between adjacent pitches
f_{EI}	generant form error
f_{IE}	generant angle error
$f_{\alpha E}$	profile angle error
$f_{\alpha p}$	tooth alignment error
f_{α}	pressure angle error
f_{β}	helix angle error
$f_{\beta t}$	longitudinal form error
$f_{\beta w}$	longitudinal form waviness
$f_{\beta p}$	deviation error of axes
F_{β}	inclination error of axes
l	path of contact
l_r	length of recess path
l_a	length of approach path
l_c	length of path of contact (total)
l_{ca}	length of addendum path of contact
l_{cd}	length of dedendum path of contact
l_{cy}	distance of a point Y from pitch point C
l_p	overlap length
h	tooth depth (between tip line and root line)
h_a	addendum
h_f	dedendum
h_w	working depth
h_y	tooth depth of standard basic rack tooth profile
$\bar{h}_n$	chordal height $\bar{s}_n$
$\bar{h}_c$	constant chord height $\bar{s}_c$
i	transmission ratio
inv	(to be read as: involute) involute function
j	backlash
j_n	normal backlash
j_r	radial backlash
j_c	circumferential backlash
k	tip diameter modification coefficient
k	number of teeth spanned when measuring base tangent length
k	number of teeth or pitches in a span
m	module
m_b	basic module
n	number of revolutions (rotational frequency)
n_a	number of revolutions (rotational frequency) of driving gear
n_b	number of revolutions (rotational frequency) of driven gear
p	pitch on the reference cylinder
p_b	pitch on the base cylinder
p_n	normal base pitch
p_s	pitch span
p_v	pitch on the V-cylinder

p_x	axial pitch	$F_{p\ z/8}$	cumulative circular pitch error over a sector of $z/8$ pitches
p_y	pitch on the Y-cylinder	F'_p	cumulative working pitch error
p_z	lead	F_r	radial run-out of a tooth system
r	reference circle radius	F_{rK}	radial run-out at rear of gear
r_a	tip radius	F_{rV}	radial run-out at front of gear
r_b	base radius	F''_r	working radial run-out
r_f	root radius	F_E	total generant error
r_v	radius of the V-circle	F_β	total alignment error
r_w	radius of the working pitch circle	F_o	wobble
r_y	radius of the Y-circle	K_g	sliding factor
s	tooth thickness on the reference cylinder	K_{ga}	sliding factor at tooth tip
s_a	tooth thickness on the tip cylinder	K_{gf}	sliding factor at tooth root
s_b	base thickness (on the base cylinder)	L	distance between points of measurement
s_v	tooth thickness on the V-cylinder	L_a	working length from involute origin to tooth tip
s_w	tooth thickness on the pitch cylinder	L_f	working length from involute origin to tooth root
s_y	tooth thickness on the Y-cylinder	L_y	working length to point Y
$\bar{s}$	normal chordal tooth thickness	L_E	generator test range
$\bar{s}_c$	constant chord	L_G	bearing centre distance on a gear axis
u	gear ratio	L_α	profile test range
v	linear speed	L_β	tooth trace test range
v_g	sliding speed	M_{dK}	dimension over balls
v_{ga}	sliding speed at the addendum	M_{dR}	dimension over pins
v_{gf}	sliding speed at the dedendum	M_{rK}	radial single-ball dimension
v_t	circumferential speed, tangential speed	M_{rR}	radial single-pin dimension
x	addendum modification coefficient	P	contact point (e.g. between test ball and tooth flank)
$x_{\min}$	addendum modification coefficient at undercut limit	R	range of errors
x_E	generating addendum modification coefficient	R_i	range of errors for backlash
x_{Em}	mean generating addendum modification coefficient	R_p	range of pitch errors
x''	addendum modification coefficient for double-flank engagement	R_s	range of tooth thickness errors
y	centre distance modification coefficient	$R_{\bar{s}}$	range of normal chordal tooth thickness errors
z	number of teeth	R_{Md}	range of errors for dimension over balls or pins
z_a	number of teeth of driving gear	R_{Mr}	range of errors for radial single-ball or single-pin dimension
z_b	number of teeth of driven gear	R_W	range of base tangent length errors
z_{nx}	virtual number of teeth for addendum modification calculations	T	tolerance
z_{nM}	virtual number of teeth for ball or roller dimensions	T_a	centre distance tolerance
z_{nW}	virtual number of teeth for base tangent length calculations	T_a''	tolerance on centre distance for double-flank engagement
A	deviation	T_{da}	tip diameter tolerance for overcut cylindrical gears
A_a	deviation of centre distance	T_s	tooth thickness tolerance
A_a''	deviation of centre distance for double-flank engagement	$T_{\bar{s}}$	tolerance on normal chordal tooth thickness
A_{da}	tip circle diameter deviation for overcut cylindrical gears	T_{Md}	tolerance on dimension over balls or pins
A_s	tooth thickness deviation (on the reference cylinder)	T_{Mr}	tolerance on radial single-ball or single-pin dimension
A_{sy}	tooth thickness deviation on Y-cylinder	T_W	base tangent length tolerance
$A_{\bar{s}}$	deviation of normal chordal tooth thickness	W_k	base tangent length over k measured teeth or measured spacewidths
A_{Md}	deviation of dimension over balls or pins	Y	any point on a tooth flank
A_{Mr}	deviation of radial single-ball or single-pin dimension	α	pressure angle at the reference cylinder
A_W	base tangent length deviation	α_a	pressure angle at the tip cylinder
A	starting point of meshing	α_v	pressure angle at the V-cylinder
B	starting point of single-tooth contact on driving gear	α_w	pressure angle at the pitch cylinder
C	pitch point	α_{wt}	working transverse pressure angle
D_M	ball or pin diameter	α_y	pressure angle at the Y-cylinder
D	end point of single-tooth contact on driving gear	α_K	pressure angle at circle through centre of ball
E	end point of meshing	α_M	pressure angle at measuring circle
F	composite and cumulative errors	α''	pressure angle for double-flank engagement with master gear
F_f	total profile error	β	helix angle at reference cylinder
F'_f	tangential composite error	β_b	base helix angle
F''_f	radial composite error	β_v	helix angle at V-cylinder
F_p	cumulative pitch error	β_w	helix angle at pitch cylinder
F_{pk}	cumulative circular pitch error over k pitches	β_y	helix angle at Y-cylinder

β_M	helix angle at measuring circle
γ	lead angle at reference cylinder
γ_b	base lead angle
ϵ	contact ratio
ϵ_α	transverse contact ratio
ϵ_β	overlap ratio
ϵ_γ	total contact ratio
ζ	specific sliding
ζ_f	specific sliding at end point of path of contact
η	spacewidth half angle at reference circle
η_b	base spacewidth half angle
η_f	spacewidth half angle at root circle
η_v	spacewidth half angle at V-circle
η_w	spacewidth half angle at pitch circle
η_y	spacewidth half angle at Y-circle
ξ	rolling angle of the involute
ξ_a	rolling angle of the involute at tooth tip end
ξ_f	rolling angle of the involute at tooth root end
ξ_y	rolling angle of the involute at point Y
ρ	radius of curvature, rounding radius
ρ_{an}	tooth crest rounding radius in normal section of cylindrical gear
ρ_{a0}	tooth crest rounding radius on tool
ρ_y	radius of curvature of involute at point Y
τ	angular pitch
φ	angle at the centre, contact angle
φ_c	angle at the centre between maximum values for radial run-out F_{rv} and F_{rh}
φ_α	transverse angle of transmission
φ_β	overlap angle
φ_γ	total angle of transmission
ψ	tooth thickness half angle at reference circle
ψ_a	tooth thickness half angle at tip circle
ψ_b	base thickness half angle
ψ_n	virtual tooth thickness half angle
ψ_v	tooth thickness half angle at V-circle
ψ_w	tooth thickness half angle at pitch circle
ψ_y	tooth thickness half angle at Y-circle
ω	angular velocity
ω_a	angular velocity of driving gear
ω_b	angular velocity of driven gear
Δ	difference
ΔW	length difference in base tangent length measurement
$\Delta\varphi$	angular difference
Δx	sum of addendum modification coefficients
Σz	sum of numbers of teeth

For further symbols for gear teeth see DIN 3999.

In addition, the following standards also deal with symbols:

DIN 1302	Mathematical signs and symbols,
DIN 1304	General symbols for use in formulae,
DIN 1313	Physical quantities and equations; concepts, methods of writing,
DIN 66 030	(at present at draft stage) Representations for names of units to be used in systems with limited graphic character sets.

1.2 Additional subscripts

The symbols indicated in subclause 1.1 may be supplemented by the following additional subscripts:

a	for the driving gear
b	for the driven gear
e	for the upper limit of a quantity

i	for the lower limit of a quantity
l	for "left-hand" or "in the direction of a left-hand helix"
m	for a mean value
n	for quantities in a normal section
r	for "right-hand" or "in the direction of a right-hand helix"
t	for quantities in a transverse section or in the tangential direction
x	for quantities in an axial section or in the direction of the gear axis
L	for designating left flanks or a master gear
M	for designating a measured value
P	for quantities of the standard basic rack tooth profile
R	for designating right flanks
max	for a maximum value
min	for a minimum value
zul	for designating a permissible limiting value
(per)	
0	for quantities associated with the generating tool
1	for quantities associated with the smaller gear of a gear pairing
2	for quantities associated with the larger gear of a gear pairing
*	for designating a factor with which a quantity is expressed in fractions or multiples of the normal module (e.g. $c = c^* \cdot m$ or $h_{ap} = h_{ap}^* \cdot m_n$) or of the number of teeth (e.g. z_{nx}^* or z_{nm}^* or z_{nw}^*), or for designating a change factor (see clause 5).

1.3 Units

The following standards or guidelines must be observed:

DIN 1301 Part 1	Units; unit names, unit symbols
DIN 1315	Angles; concepts, units
VDE/VDI 2605	Circular pitches and plane angles; basic concepts for angular dimensions, angle measurements, angle standards and their errors.

Accordingly it is convenient to state the quantities dealt with in this standard in the following units:

modules and all lengths in millimetres (mm),
deviations in and tolerances on length in micrometres (μm),
angles whereby size is denoted in degrees ($^\circ$)

(e.g. pressure angle, helix angle, rolling angle); for calculations with computers it is advantageous to use decimal subdivision of the degree;

angles used in equations in radians (rad)

(e.g. angular pitch τ , tooth thickness half angle ψ , spacewidth half angle η , overlap angle φ);

angular deviations in milliradians (mrad) or in microradians (μrad)

(see subclauses 6.3.1.4, 6.3.1.6, 6.3.2.4, 6.3.2.5);

angular velocities in radians per second (rad/s);

number of revolutions (frequency of rotation) in one per second (1/s)

(instead of the unit 1/min hitherto mainly used, the new system of units adopts the unit 1/s, see DIN 1301).

For angular units the following relationships apply:

$$\begin{aligned}
 2 \pi \text{ rad} &= 360^\circ \\
 1 \text{ rad} &= 1 \frac{\text{mm}}{\text{mm}} \\
 1 \text{ mrad} &= 1 \frac{\text{mm}}{\text{m}} = 1 \frac{\mu\text{m}}{\text{mm}}
 \end{aligned}$$

β_M	helix angle at measuring circle
γ	lead angle at reference cylinder
γ_b	base lead angle
c	contact ratio
c_α	transverse contact ratio
c_β	overlap ratio
c_γ	total contact ratio
ζ	specific sliding
ζ_l	specific sliding at end point of path of contact
η	spacewidth half angle at reference circle
η_b	base spacewidth half angle
η_l	spacewidth half angle at root circle
η_v	spacewidth half angle at V-circle
η_w	spacewidth half angle at pitch circle
η_y	spacewidth half angle at Y-circle
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ρ_{a0}	tooth crest rounding radius on tool
ρ_y	radius of curvature of involute at point Y
τ	angular pitch
φ	angle at the centre, contact angle
φ_c	angle at the centre between maximum values for radial run-out F_{rV} and F_{rH}
φ_α	transverse angle of transmission
φ_β	overlap angle
φ_γ	total angle of transmission
ψ	tooth thickness half angle at reference circle
ψ_a	tooth thickness half angle at tip circle
ψ_b	base thickness half angle
ψ_n	virtual tooth thickness half angle
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ω	angular velocity
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Δ	difference
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zul (per)	for designating a permissible limiting value
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1	for quantities associated with the smaller gear of a gear pairing
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modules and all lengths in millimetres (mm),
deviations in and tolerances on length in micrometres (μm),
angles whereby size is denoted in degrees ($^\circ$)
(e.g. pressure angle, helix angle, rolling angle); for calculations with computers it is advantageous to use decimal subdivision of the degree;

angles used in equations in radians (rad)
(e.g. angular pitch τ , tooth thickness half angle ψ , spacewidth half angle η , overlap angle φ);

angular deviations in milliradians (mrad) or in microradians (μrad)

(see subclauses 6.3.1.4, 6.3.1.6, 6.3.2.4, 6.3.2.5);

angular velocities in radians per second (rad/s);

number of revolutions (frequency of rotation) in one per second (1/s)

(instead of the unit 1/min hitherto mainly used, the new system of units adopts the unit 1/s, see DIN 1301).

For angular units the following relationships apply:

$$\begin{aligned}
 2\pi \text{ rad} &= 360^\circ \\
 1 \text{ rad} &= 1 \frac{\text{mm}}{\text{mm}} \\
 1 \text{ mrad} &= 1 \frac{\text{mm}}{\text{m}} = 1 \frac{\mu\text{m}}{\text{mm}}
 \end{aligned}$$

2.2.3 Sections through cylindrical gear teeth

2.2.3.1 Transverse section

The sectioning of cylindrical gear teeth on a plane perpendicular to the gear axis yields a transverse section. A transverse section of a rack is its intersection with a plane perpendicular to the axis of the cylindrical gear mated with the rack.

Quantities in the transverse section are denoted by the subscript *t*.

2.2.3.2 Normal section

The sectioning of involute helical gear teeth by a surface disposed perpendicular to the tooth traces of the involute helicoids yields a normal section. The surface of the normal section is curved three-dimensionally. In the case of a spur gear the normal section and transverse section coincide. A normal section of a rack is its intersection with a plane perpendicular to the tooth traces of the teeth.

Quantities in the normal section are denoted by the subscript *n*.

The subscripts *n* and *t* are not applicable to spur gear teeth.

2.2.3.3 Axial section

The sectioning of cylindrical gear teeth by a plane containing the gear axis yields an axial section. An axial section of a rack is its intersection with a plane perpendicular to the rack datum plane (see subclause 2.2.7) which contains the axis of the cylindrical gear mated with the rack.

Quantities in the axial section are denoted by the subscript *x*.

Axial sections and the stating of gear tooth quantities in such sections are not meaningful where spur gear teeth are concerned.

2.2.4 Gear tooth profile, flank profile

A gear tooth profile results as the line of intersection of the gear teeth with a plane.

A flank profile is the line of intersection of a tooth flank with a plane. Right flank profiles and left flank profiles are to be distinguished where necessary.

2.2.4.1 Transverse profile

The transverse profile is the gear tooth profile lying in a transverse section.

The transverse profile of spur gear teeth and the transverse profile of the gear mating with such teeth lie in the same plane.

2.2.4.2 Normal profile, virtual spur gear teeth

Because of the curved normal section it is only possible in the case of a spur gear with involute helical teeth to indicate an approximate normal profile lying in a tangential plane or in an osculating plane to the normal section.

Geometrical studies are therefore often based on virtual spur gear teeth lying in the tangential or osculating plane, their datum line being a circle of curvature on the normal section of the reference surface of the helical teeth (see subclauses 2.7.1.1 and 2.7.2).

2.2.4.3 Axial profile

The axial profile is the profile of the gear teeth lying in an axial section.

2.2.5 Tooth traces

The tooth traces are lines of intersection of the right and left flanks with a cylinder the axis of which coincides with the gear axis. Hence, right tooth traces and left tooth traces are to be distinguished.

The reference tooth trace (reference cylinder tooth trace) is the line of intersection of the flank with the reference cylinder (see subclause 2.2.7). The base tooth trace is the

line of intersection of the — possibly imagined as extended — involute flank with the base cylinder (see subclause 2.2.8); the base tooth traces are the helices of the involute helicoids (see subclause 2.3.1). The tip tooth trace (root tooth trace) is the line of intersection of the — possibly imagined as extended — involute flank with the tip cylinder (root cylinder).

The tooth traces are helices in the case of helical gear teeth and straight lines in the case of spur gear teeth.

2.2.6 Modules $m_n, m_t, m_x; m, m_b$

The module *m* of the standard basic rack tooth profile is the normal module (module in the normal section) m_n of the gear teeth.

In a transverse section the transverse module m_t is found as

$$m_t = \frac{m_n}{\cos \beta} \quad (2.1)$$

For a helical gear the axial module m_x in an axial section is found as

$$m_x = \frac{m_n}{\sin |\beta|} = \frac{m_n}{\cos \gamma} = \frac{m_t}{\tan |\beta|} \quad (2.2)$$

For a spur gear $\beta = 0$ and the module is *m* ($m_n = m_t = m$).

The basic module m_b is

$$m_b = m_n / \sqrt{\tan^2 \alpha_n + \cos^2 \beta} \quad (2.3)$$

2.2.7 Reference cylinder, reference circle; reference diameter *d*

The reference cylinder is the reference surface for the gear teeth. Its axis coincides with the locating axis of the gear (gear axis). Hence in the case of a rack the reference plane is the rack datum plane. Quantities on the reference cylinder are stated without subscript.

The reference circle is the intersection of the reference cylinder with a plane of transverse section. The reference diameter *d* is determined by

$$d = z \cdot m_t = \frac{z \cdot m_n}{\cos \beta} \quad (2.4)$$

Note: In the case of an internal gear the reference diameter is a negative value, see subclause 2.1.

2.2.8 Base cylinder, base circle; base diameter d_b

The base cylinder is that cylinder coaxial with the reference cylinder that is determinative for the generation of the involute surfaces (involute helicoids). Quantities associated with the base cylinder are denoted by the subscript *b*.

The base circle is the intersection of the base cylinder with a plane of transverse section; the involutes of the base circle form the usable parts of the tooth profiles. The base diameter d_b is given by

$$\begin{aligned} d_b &= d \cdot \cos \alpha_t = z \cdot m_t \cdot \cos \alpha_t = \frac{z \cdot m_n \cdot \cos \alpha_t}{\cos \beta} \\ &= z \cdot m_n / \sqrt{\tan^2 \alpha_n + \cos^2 \beta} = z \cdot m_b \end{aligned} \quad (2.5)$$

Note: In the case of an internal gear the base diameter is a negative value, see subclause 2.1.

2.3 Involute surface (involute helicoid)

2.3.1 Generator of an involute surface

In developing the envelope of the base cylinder, an envelope line of the base cylinder describes an involute surface of a spur gear; it is the generator of the involute surface.

A straight line inclined to the envelope line in the developed envelope surface is the generator of an involute surface (involute helicoid) of a helical gear having its origin on the base cylinder in the helix, see figure 2.

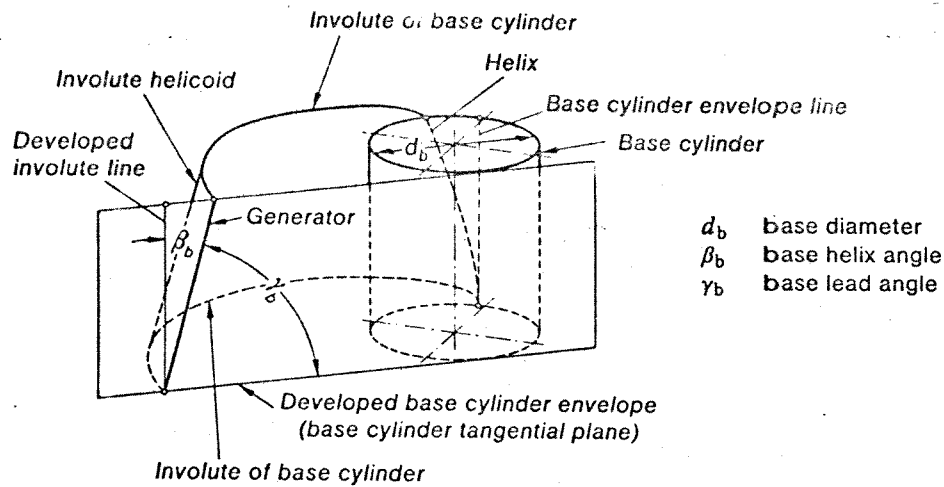


Figure 2. Base cylinder with involute helicoid and generator

2.3.2 Base helix angle β_b , base lead angle γ_b

The acute angle in the angled base cylinder envelope between the generator and an envelope line is the base helix angle β_b , see figure 2 and equations (2.30) to (2.32). The complement angle of the base helix angle β_b is the base lead angle γ_b :

$$|\gamma_b| = 90^\circ - |\beta_b| \quad (2.6)$$

Both angles have the same sign.

With the development of the base cylinder envelope first in the clockwise direction, involute surfaces curved in opposite directions result and from these the left and right flanks of the gear teeth are derived.

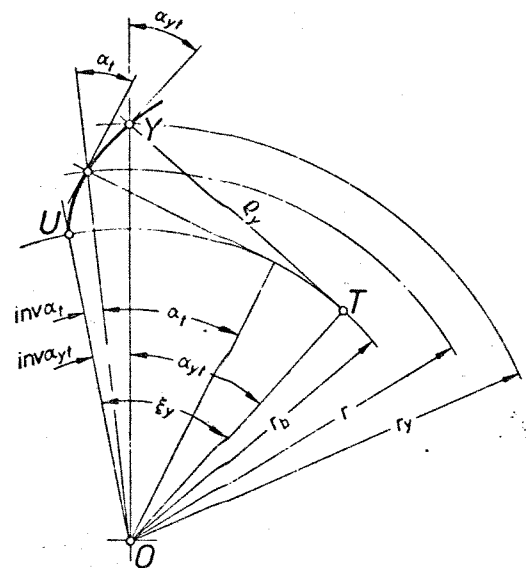
2.3.3 Contact line

The two base cylinders of a gear pair have two common tangential planes which intersect in the instantaneous axis. These are the planes of action of the gear teeth. Each plane of action cuts the associated tooth flanks (flank and mating flank) in the contact line corresponding to the working position. Each contact line is the common generator of flank and mating flank; its prolongations are tangents to the base cylinder. On rotation of the base cylinders about their axes and simultaneous winding up and unwinding of the tangential planes the contact lines travel through their zone of action, see subclause 3.4.

2.3.4 Transverse pressure angle at a point α_{yt} , transverse pressure angle α_t

The involute (always lying in a transverse section) is inclined at the arbitrary point Y by the transverse pressure angle at a point α_{yt} relative to the radius passing through Y, see figure 3.

The angle of inclination at the point of intersection of the involute with the reference circle is the transverse pressure angle α_t .



T	point of contact of tangent with base circle
U	origin of involute
Y	arbitrary point on involute
r	reference radius
r_b	base circle radius
r_y	Y-circle radius
α_t	transverse pressure angle
α_{yt}	transverse pressure angle at point Y
ξ_y	rolling angle between U and T
ρ_y	radius of curvature at point Y = working length L_y
$inv \alpha_t$	involute function for α_t
$inv \alpha_{yt}$	involute function for α_{yt} = angle at the centre between U and Y

Figure 3. Parameters relating to an involute

2.3.5 Normal pressure angle at a point α_{yn} , normal pressure angle α_n

In the normal section through an involute helicoid the tangent to this surface at an arbitrary point Y is inclined to the radius through Y by the normal pressure angle at a point α_{yn} . The corresponding angle of inclination at the reference cylinder is the normal pressure angle α_n ; this is equal to the pressure angle α_p of the standard basic rack tooth profile.

$$\cos \alpha_t = \frac{r_b}{r} = \frac{d_b}{d} = \frac{\cos \beta}{\sqrt{\tan^2 \alpha_n + \cos^2 \beta}} \quad (2.7)$$

$$= \frac{\cos \alpha_n}{\sqrt{1 + \sin^2 \alpha_n \cdot \tan^2 \beta}}$$

$$\cos \alpha_{yt} = \frac{r_b}{r_y} = \frac{d_b}{d_y} = \frac{d}{d_y} \cdot \cos \alpha_t \quad (2.8)$$

See also equations (2.9) and (2.10).

$$\tan \alpha_n = \tan \alpha_t \cdot \cos \beta \quad (2.9)$$

$$\tan \alpha_{yt} = \tan \alpha_{yt} \cdot \cos \beta_y \quad (2.10)$$

For a spur gear $\beta = 0$ and $\alpha_n = \alpha_t = \alpha$, also $\alpha_{yt} = \alpha_{yt} = \alpha_y$

2.3.6 Rolling angle ζ of the involute

The angle of the centre defined by the origin U of the involute and the contact point T of the tangent from point Y to the base circle is the rolling angle ζ_y of the involute, see figure 3. The base circle arc UT is equal to the tangent portion YT, hence

$$\zeta_y = \tan \alpha_{yt} \quad (2.11)$$

2.3.7 Radius of curvature ρ of the involute, working length L_y

The tangent portion YT is the radius of curvature ρ_y of the involute at point Y and at the same time the working length L_y belonging to point Y, i.e. the developed base circle arc from the involute origin U. In the triangle OTY it is the side opposite the transverse pressure angle α_{yt} at the centre of the circle O, see figure 3.

$$\rho_y = r_b \cdot \zeta_y = r_b \cdot \tan \alpha_{yt} = \frac{z}{|z|} \cdot \sqrt{r_y^2 - r_b^2} \quad (2.12)$$

Note: In the case of internal gear the radius of curvature is a negative value, see subclause 2.1.

2.3.8 Involute function $\text{inv } \alpha$

The angular difference $\zeta - \alpha_t$ is termed the involute function of angle α_t and is denoted by $\text{inv } \alpha_t$ (to be read as: involute α_t), see figure 3.

$$\text{inv } \alpha_{yt} = \zeta_y - \alpha_{yt} = \tan \alpha_{yt} - \alpha_{yt} \quad (2.13)$$

Note: It has previously been customary to use the symbol $\text{ev } \alpha$ for the involute function. In conformity with ISO 701-1976 however it is recommended that in future the symbol $\text{inv } \alpha$ should be used.

2.3.9 Lead p_z

The lead p_z (of an involute helicoid, of a tooth flank) is the portion of an envelope line of a cylinder concentric with the gear axis between two successive turns of an involute helicoid (of a tooth flank), see figure 4. The lead is independent of the diameter of the cylinder.

$$p_z = \frac{|z| \cdot m_n \cdot \pi}{\sin |\beta|} = \frac{|z| \cdot m_t \cdot \pi}{\tan |\beta|} = |z| \cdot p_x \quad (2.14)$$

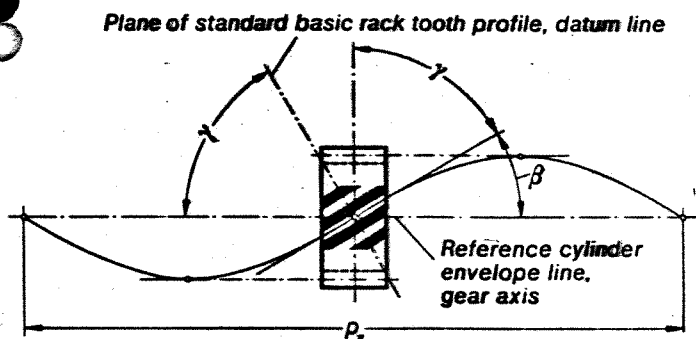


Figure 4. Helical gear: lead p_z , helix angle β , lead angle γ

2.4 Angular pitch and pitches

2.4.1 Angular pitch τ

The angular pitch τ is that angle lying in a transverse section which results from the dividing of the complete periphery of a circle into z equal parts.

$$\tau = \frac{2 \cdot \pi}{z} \quad \text{in radians} \quad (2.15)$$

$$\tau = \frac{360}{z} \quad \text{in degrees} \quad (2.16)$$

Note: In the case of an internal gear the angular pitch is a negative value, see subclause 2.1.

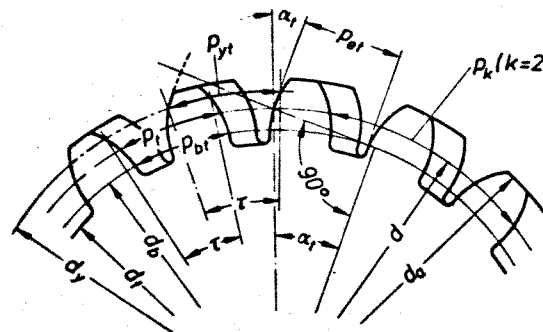


Figure 5. Helical gear: diameter, angular pitch, pitches

2.4.2 Pitches on the reference cylinder

2.4.2.1 Transverse pitch (pitch) p_t

The transverse pitch (pitch for short) p_t (p in the case of a spur gear) is the length of the reference circle arc between two successive right or left flanks, see figure 5.

$$p_t = r \cdot \tau = \frac{d \cdot \pi}{z} = m_t \cdot \pi = \frac{m_n \cdot \pi}{\cos \beta} \quad (2.17)$$

2.4.2.2 Normal pitch p_n

The normal pitch p_n is the length of the helix arc between two successive right or left flanks on the reference cylinder in the normal section of the gear teeth.

$$p_n = p_t \cdot \cos \beta = m_n \cdot \pi \quad (2.18)$$

2.4.2.3 Pitch span p_k

The pitch span p_k is the reference circle arc between two right or left flanks separated from each other by k reference circle pitches such that $1 < k < z$, see figure 5. It is the sum of k reference circle pitches.

$$p_k = k \cdot p_t \quad (2.19)$$

2.4.3 Pitches on the Y-cylinder

On a cylinder with the arbitrary diameter d_y (Y-cylinder) the following pitches are present:

2.4.3.1 Y-circle pitch p_{yt}

The Y-circle pitch p_{yt} (p_y in the case of a spur gear) is the length of the arc with diameter d_y between two successive right or left flanks, see figure 5.

$$p_{yt} = r_y \cdot \tau = \frac{d_y \cdot \pi}{z} = \frac{d_y}{d} \cdot p_t \quad (2.20)$$

2.4.3.2 Y-cylinder normal pitch p_{yn}

The Y-cylinder normal pitch p_{yn} is the length of the helix arc on the Y-cylinder between two successive right or left flanks in the normal section of the gear teeth.

$$p_{yn} = p_{yt} \cdot \cos \beta_y \quad (2.21)$$

2.4.4 Pitches on the V-cylinder

On the cylinder with diameter d_v (see subclause 2.5.6.3) the following pitches are present:

2.4.4.1 V-circle pitch p_{vt}

The V-circle pitch p_{vt} (p_v in the case of a spur gear) is the length of the arc with diameter d_v between two successive right or left flanks.

$$p_{vt} = r_v \cdot \tau = \frac{d_v \cdot \pi}{z} = \frac{d_v}{d} \cdot p_t \quad (2.22)$$

2.4.4.2 V-cylinder normal pitch p_{vn}

The V-cylinder p_{vn} is the length of the helix arc on the V-cylinder between two successive right or left flanks in the normal section of the gear teeth.

$$p_{vn} = p_{vt} \cdot \cos \beta_v \quad (2.23)$$

2.4.5 Pitches on the base cylinder**2.4.5.1 Transverse base pitch p_{bt}**

The transverse base pitch p_{bt} (p_b in the case of a spur gear) is the length of the base circle arc between the points of origin of two successive right or left flanks, see figure 5.

$$p_{bt} = r_b \cdot \tau = \frac{d_b \cdot \pi}{z} = \frac{d_b}{d} \cdot p_t = p_t \cdot \cos \alpha_t \quad (2.24)$$

2.4.5.2 Base cylinder normal pitch p_{bn}

The base cylinder normal pitch p_{bn} is the length of the helix arc on the base cylinder between the helices of two successive right or left flanks in the normal section of the gear teeth.

$$p_{bn} = p_{bt} \cdot \cos \beta_b = p_n \cdot \cos \alpha_n \quad (2.25)$$

2.4.6 Normal base pitches p_e

The distances between two successive right or left flanks in the plane of action are the normal base pitches p_e .

2.4.6.1 Transverse pitch on path of contact p_{et}

The transverse pitch on path of contact p_{et} (normal base pitch p_e in the case of a spur gear) is the distance between two parallel tangents in a plane of transverse section which contact two successive equal-handed flanks.

In the transverse section of the gear teeth the transverse pitch on path of contact is the length of the transverse line of action between two successive equal-handed flanks, see figure 5.

The transverse pitch on path of contact is equal to the transverse base pitch.

$$p_{et} = p_t \cdot \cos \alpha_t = p_{bt} \quad (2.26)$$

2.4.6.2 Normal base pitch p_{en}

The normal base pitch p_{en} is the distance between two parallel tangential planes which contact two successive equal-handed flanks in the active area of the flanks. It is equal to the base cylinder normal pitch.

$$p_{en} = p_n \cdot \cos \alpha_n = p_{bn} \quad (2.27)$$

2.4.7 Axial pitch p_x

The axial pitch p_x is the portion of an envelope line of a cylinder concentric with the gear axis between two successive right or left flanks, see figure 6. The axial pitch is

independent of the diameter of the cylinder and is $\frac{1}{|z|}$ times the lead.

$$p_x = m_x \cdot \pi = \frac{m_n \cdot \pi}{\sin |\beta|} = \frac{m_t \cdot \pi}{\tan |\beta|} = \frac{p_z}{|z|} \quad (2.28)$$

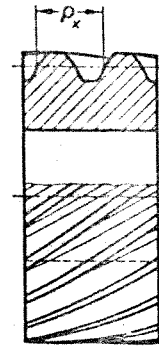


Figure 6. Axial pitch p_x of a helical gear

2.5 Concepts and parameters associated with gear teeth arising from the position of the standard basic rack tooth profile relative to the reference cylinder

The datum line can

- cross the gear axis with the lead angle γ
- have a distance of $x \cdot m_n$ from the reference cylinder.

The following parameters for gear teeth thus arise.

2.5.1 Lead angle γ , helix angle β

The lead angle γ is the acute angle at which the datum line crosses the gear axis, see figure 4. It is also the acute angle between a tangent to a reference helix and the plane perpendicular to the gear axis through the tangent contact point.

The helix angle β is the acute angle between a tangent to a reference helix and the reference cylinder envelope line through the tangent contact point.

$$|\beta| = 90^\circ - |\gamma| \quad (2.29)$$

The lead angle and helix angle have the same sign. For spur gears $\gamma = 90^\circ$ and $\beta = 0^\circ$. The helix angle β_R of right flanks may differ from the helix angle β_L of left flanks.

The relationship between β and the base helix angle β_b is found from

$$\tan \beta_b = \tan \beta \cdot \cos \alpha_t \quad (2.30)$$

$$\sin \beta_b = \sin \beta \cdot \cos \alpha_n \quad (2.31)$$

$$\begin{aligned} \cos \beta_b &= \cos \beta \cdot \frac{\cos \alpha_n}{\cos \alpha_t} = \frac{\sin \alpha_n}{\sin \alpha_t} = \frac{\sin \alpha_{yn}}{\sin \alpha_{yt}} \\ &= \cos \alpha_n \cdot \sqrt{\tan^2 \alpha_n + \cos^2 \beta} \end{aligned} \quad (2.32)$$

On a cylinder with diameter d_y the helix angle β_y is found from

$$\begin{aligned} \tan \beta_y &= \tan \beta \cdot \frac{d_y}{d} = \tan \beta \cdot \frac{\cos \alpha_t}{\cos \alpha_{yt}} \\ &= \tan \beta_b \cdot \frac{d_y}{d_b} = \frac{\tan \beta_b}{\cos \alpha_{yt}} \end{aligned} \quad (2.33)$$

$$\sin \beta_y = \sin \beta \cdot \frac{\cos \alpha_n}{\cos \alpha_{yn}} = \frac{\sin \beta_b}{\cos \alpha_{yn}} \quad (2.34)$$

$$\cos \beta_y = \frac{\tan \alpha_{yn}}{\tan \alpha_{yt}} = \frac{\cos \alpha_{yt} \cdot \cos \beta_b}{\cos \alpha_{yn}} \quad (2.35)$$

2.5.2 Flank direction; right-handed; left-handed

In the case of external and internal gear teeth the flank direction is right-handed (left-handed) if the helical tooth trace corresponds to a right-hand helix (left-hand helix).

2.5.3 Sign of the helix angle

If signs are needed for the helix angle (e.g. for evaluating the nominal dimensions of a cylindrical gear pair) the following applies:

For an external gear a helix angle belonging to a right-handed flank direction counts as positive, whilst a helix angle belonging to a left-handed flank direction counts as negative. For an internal gear the signs are reversed. The sum of the helix angles of a pair of cylindrical gears is therefore zero.

Note: In production documents only the absolute values of the helix angles are used with additional indication of the hand (r for right-handed, l for left-handed).

2.5.4 Addendum modification; addendum modification coefficient x and sign of addendum modification

The addendum modification of ~~single~~ gear teeth is the displacement of the datum line from the reference cylinder. The magnitude of the addendum modification is expressed by the addendum modification coefficient x in parts of the normal module:

$$\text{addendum modification} = x \cdot m_n$$

(for spur gears: addendum modification = $x \cdot m$).

An addendum modification is

positive if the datum line is displaced from the reference circle towards the tip circle as a result, the tooth thickness in the reference circle is greater than for zero addendum modification, see figure 7

negative if the datum line is displaced from the reference circle towards the root circle as a result, the tooth thickness in the reference circle is smaller than with zero addendum modification, see figure 8.

Note: Where internal gears are concerned, this standard reverses the definition of the sign of the addendum modification compared with DIN 3960, August 1960 edition, i.e. for internal gears $x \cdot m_n$ according to this standard equals $-x \cdot m_n$ DIN 3960 (August 1960 edition), see subclause 2.1

2.5.5 V-gear, zero gear

A cylindrical gear with addendum modification is termed a V-gear (V_{plus} gear if addendum modification is positive, V_{minus} gear if addendum modification is negative). A cylindrical gear with zero addendum modification is termed a zero gear.

2.5.6 Diameters of gear teeth, angles at V-cylinder

The position of the standard basic tooth profile relative to the reference cylinder gives rise to the following cylindrical surfaces and diameters in respect of the gear teeth.

Note: In the case of an internal gear the diameters and radii are negative values, see subclause 2.1.

2.5.6.1 Tip cylinder, tip circle, tip diameter d_a

The tip cylinder is the cylindrical envelope surface at the tips of a gear tooth system; a transverse section yields the tip circle.

Quantities related to the tip cylinder are denoted by the subscript a.

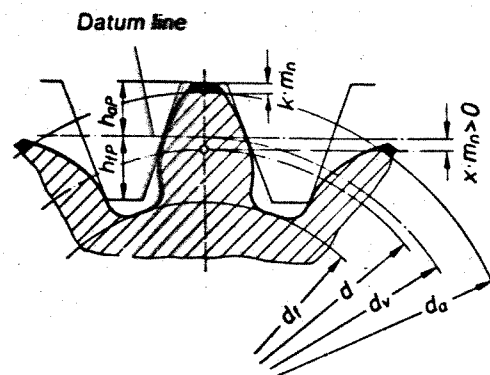


Figure 7. Positive addendum modification on an external gear

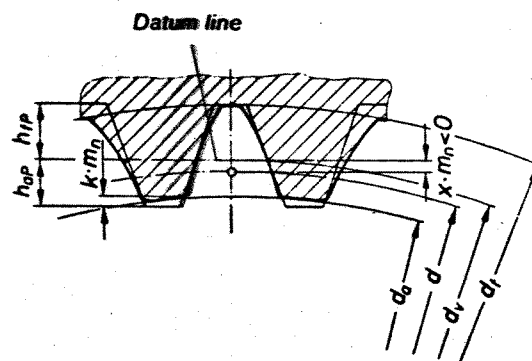


Figure 8. Negative addendum modification on an internal gear

The tip diameter d_a amounts in the case of gear 1 (see figure 7) to

$$\begin{aligned} d_{a1} &= 2 \cdot a - d_{f1} - 2 \cdot c \\ &= d_1 + 2 \cdot x_1 \cdot m_n + 2 \cdot h_{ap} + 2 \cdot k \cdot m_n \quad (2.36) \end{aligned}$$

and in the case of gear 2 (see figure 7 and figure 8)

$$\begin{aligned} d_{a2} &= 2 \cdot a - d_{f2} - 2 \cdot c \\ &= d_2 + 2 \cdot x_2 \cdot m_n + 2 \cdot h_{ap} + 2 \cdot k \cdot m_n \quad (2.37) \end{aligned}$$

Note: For definitions of the quantities a , c and k see subclauses 3.2.3, 3.2.7 and 3.3.6 as well as equation (2.47).

If the tip diameters are dimensioned according to these equations, the bottom clearance will be $c \cdot 0,167 \cdot m_n$ or $c = 0,25 \cdot m_n$, depending on the standard basic rack tooth profile of the cutting tool. For details see DIN 3972 or DIN 58412.

For an external gear the relationship should be $d_a \geq d_b + 2 \cdot m_n$. For external gears this sets a lower limit for the addendum modification coefficient x , see subclause 2.6.4.

For an internal gear the relationship must always be $|d_a| > |d_b|$. For internal gears this sets an upper limit for the addendum modification coefficient, see subclause 2.6.4.

2.5.6.2 Root cylinder, root circle; root diameter d_f

The root cylinder is the cylindrical envelope surface at the bottom of the tooth spaces of a gear tooth system; a transverse section yields the root circle.

Quantities related to the root cylinder are denoted by the subscript f.

The root diameter d_f amounts in the case of gear 1 (see figure 7) to

$$d_{f1} = d_1 - 2 \cdot h_f = d_1 + 2 \cdot x_1 \cdot m_n - 2 \cdot h_{fp} \quad (2.38)$$

and in case of gear 2 (see figure 7 and figure 8) to

$$d_{f2} = d_2 - 2 \cdot h_f = d_2 + 2 \cdot x_2 \cdot m_n - 2 \cdot h_{fp} \quad (2.39)$$

Note: If the tooth thickness deviations A_{st} are produced by deeper infeed of the gear cutting tool, then the following is found

$$d_f = d - 2 \cdot h_f + A_{st} \cdot \cot \alpha_t \quad (2.40)$$

In the case of ground and shaved cylindrical gears the tool addendum h_{a0} is to be substituted for h_{fp} in equations (2.38) and (2.39) and the addendum modification coefficient of the pre-cut teeth for x .

2.5.6.3 V-circle diameter d_v , V-cylinder

The V-cylinder contacts the datum line, see figure 7 and figure 8. Its diameter (V-circle diameter) is

$$\begin{aligned} d_v &= d + 2 \cdot x \cdot m_n \\ &= d \left(1 + 2 \cdot \frac{x}{z} \cdot \cos \beta \right) \end{aligned} \quad (2.41)$$

Quantities related to the V-cylinder are denoted by the subscript v.

2.5.6.4 Pressure angle α_v , helix angle β_v

The following angles are present where the V-cylinder intersects an involute tooth flank:

helix angle β_v

$$\begin{aligned} \tan \beta_v &= \frac{z + 2 \cdot x \cdot \cos \beta}{z} \cdot \tan \beta \\ &= \tan \beta + 2 \cdot \frac{x}{z} \cdot \sin \beta = \tan \beta \cdot \frac{d_v}{d} \end{aligned} \quad (2.42)$$

transverse pressure angle at a point α_{vt}

$$\begin{aligned} \cos \alpha_{vt} &= \frac{z}{z + 2 \cdot x \cdot \cos \beta} \cdot \cos \alpha_t \\ &= \frac{\cos \alpha_t}{1 + 2 \cdot \frac{x}{z} \cdot \cos \beta} \end{aligned} \quad (2.43)$$

normal pressure angle at a point α_{vn}

$$\begin{aligned} \tan \alpha_{vn} &= \tan \alpha_{vt} \cdot \cos \beta_v \\ \cos \alpha_{vn} &= \frac{\cos \alpha_n \cdot \cos \beta}{\cos \beta_v \cdot \left(1 + 2 \cdot \frac{x}{z} \cdot \cos \beta \right)} \\ &= \frac{d}{d_v} \cdot \frac{\cos \alpha_n \cdot \cos \beta}{\cos \beta_v} \end{aligned} \quad (2.44) \quad (2.45)$$

2.5.7 Gear teeth heights

Note: The working depth resulting from meshing with a mating gear is defined in subclause 3.2.6.

2.5.7.1 Tooth depth h

The tooth depth h of cylindrical gear teeth results from the tooth depth h_p of the standard basic rack tooth profile and the addendum alteration $k \cdot m_n$, see subclause 3.3.6.

$$h = h_p + k \cdot m_n \quad (2.46)$$

2.5.7.2 Addendum h_a , dedendum h_f

The addendum h_a and the dedendum h_f of a cylindrical gear are stated on the basis of the reference circle:

$$h_a = h_{ap} + x \cdot m_n + k \cdot m_n \quad (2.47)$$

$$h_f = h_{fp} - x \cdot m_n \quad (2.48)$$

2.5.8 Tooth thicknesses, spacewidths

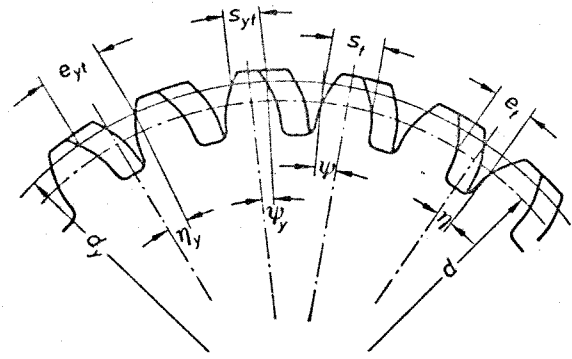


Figure 9. Helical gear: tooth thicknesses, spacewidths and their half angles

2.5.8.1 Transverse tooth thicknesses s_t , s_{yt} , s_{vt} and s_{bt}

The transverse tooth thickness s_t (tooth thickness s in the case of a spur gear) is the length of the reference circle arc between the two flanks of a tooth.

$$\begin{aligned} s_t &= \frac{p_t}{2} + 2 \cdot x \cdot m_n \cdot \tan \alpha_t \\ &= m_t \cdot \left(\frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha_n \right) = \frac{s_n}{\cos \beta} \end{aligned} \quad (2.49)$$

The transverse tooth thickness s_{yt} (tooth thickness s_y in the case of a spur gear) is the length of the circular arc of diameter d_y between the two flanks of a tooth.

$$\begin{aligned} s_{yt} &= d_y \cdot \left(\frac{s_t}{d} + \operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{yt} \right) \\ &= d_y \cdot \left(\frac{\pi + 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} + \operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{yt} \right) \end{aligned} \quad (2.50)$$

The transverse tooth thickness s_{vt} (tooth thickness s_v in the case of a spur gear) is the length of the V-circle arc between the two flanks of a tooth.

$$\begin{aligned} s_{vt} &= d_v \cdot \left(\frac{s_t}{d} + \operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{vt} \right) \\ &= d_v \cdot \left(\frac{\pi + 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} + \operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{vt} \right) \end{aligned} \quad (2.51)$$

The base tooth thickness s_{bt} (s_b in the case of a spur gear) is the base circle arc between the points of origin of the involutes of a tooth.

$$s_{bt} = d_b \cdot \left(\frac{\pi + 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} + \operatorname{inv} \alpha_t \right) \quad (2.52)$$

2.5.8.2 Tooth thickness half angles

ψ , ψ_y , ψ_v and ψ_b

Angles at the centre in a transverse section which are enclosed by radii bounding the tooth thicknesses s_t , s_{yt} , s_{vt} or s_{bt} are tooth thickness angles. The corresponding tooth thickness half angles are:

$$\psi = \frac{s_t}{d} = \frac{\pi + 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} \quad (2.53)$$

$$\psi_y = \frac{s_{yt}}{d_y} = \psi + \operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{yt} \quad (2.54)$$

$$\psi_v = \frac{s_{vt}}{d_v} = \psi + \operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{vt} \quad (2.55)$$

The base tooth thickness half angle ψ_b is

$$\psi_b = \frac{s_{bt}}{d_b} = \psi + \text{inv } \alpha_t \quad (2.55)$$

Note: In the case of an internal gear the tooth thickness half angles are negative values; see subclause 2.1.

2.5.8.3 Spacewidths e_t , e_{yt} , e_{vt} and e_{bt}

The spacewidth e_t (e in the case of a spur gear) is the length of the reference circle arc between the tooth flanks enclosing a tooth space. The tooth thickness s_t and spacewidth e_t together yield the transverse pitch p_t .

$$s_t + e_t = p_t \quad (2.57)$$

$$\begin{aligned} e_t &= \frac{p_t}{2} - 2 \cdot x \cdot m_n \cdot \tan \alpha_t \\ &= m_t \cdot \left(\frac{\pi}{2} - 2 \cdot x \cdot \tan \alpha_n \right) \end{aligned} \quad (2.58)$$

The spacewidth e_{yt} (e_y in the case of a spur gear) is the length of a circular arc of diameter d_y between the tooth flanks enclosing a tooth space. The tooth thickness s_{yt} and spacewidth e_{yt} together yield the Y-circle pitch p_{yt} .

$$s_{yt} + e_{yt} = p_{yt} \quad (2.59)$$

$$\begin{aligned} e_{yt} &= d_y \cdot \left(\frac{e_t}{d} - \text{inv } \alpha_t + \text{inv } \alpha_{yt} \right) \\ &= d_y \cdot \left(\frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} - \text{inv } \alpha_t + \text{inv } \alpha_{yt} \right) \end{aligned} \quad (2.60)$$

The spacewidth e_{vt} (e_v in the case of a spur gear) is the length of the V-circle arc between the two flanks enclosing a tooth space. The tooth thickness s_{vt} and spacewidth e_{vt} together yield the V-circle pitch p_{vt} .

$$s_{vt} + e_{vt} = p_{vt} \quad (2.61)$$

$$\begin{aligned} e_{vt} &= d_v \cdot \left(\frac{e_t}{d} - \text{inv } \alpha_t + \text{inv } \alpha_{vt} \right) \\ &= d_v \cdot \left(\frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} - \text{inv } \alpha_t + \text{inv } \alpha_{vt} \right) \end{aligned} \quad (2.62)$$

The base spacewidth e_{bt} (e_b in the case of a spur gear) is the base circle arc between the points of origin of the two involutes enclosing a tooth space. The base tooth thickness s_{bt} and base spacewidth e_{bt} together yield the base circle pitch p_{bt} .

$$s_{bt} + e_{bt} = p_{bt} \quad (2.63)$$

$$e_{bt} = d_b \cdot \left(\frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} - \text{inv } \alpha_t \right) \quad (2.64)$$

2.5.8.4 Spacewidth half angles η , η_y , η_v and η_b

Angles at the centre in a transverse section which are enclosed by the radii bounding the spacewidths e_t , e_{yt} , e_{vt} or e_{bt} are spacewidth angles. The corresponding spacewidth half angles are:

$$\eta = \frac{e_t}{d} = \frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} \quad (2.65)$$

$$\eta_y = \frac{e_{yt}}{d_y} = \eta - \text{inv } \alpha_t + \text{inv } \alpha_{yt} \quad (2.66)$$

$$\eta_v = \frac{e_{vt}}{d_v} = \eta - \text{inv } \alpha_t + \text{inv } \alpha_{vt} \quad (2.67)$$

The base spacewidth half angle η_b is

$$\eta_b = \frac{e_{bt}}{d_b} = \eta - \text{inv } \alpha_t \quad (2.68)$$

Note: In the case of an internal gear the spacewidth half angles are negative values; see subclause 2.1.

Reference: For spacewidth half angles at deviation of tooth thickness A_s see equation (4.3) in subclause 4.1.4.

2.5.8.5 Normal tooth thicknesses s_n , s_{yn} , s_{vn} and s_{bn}

The normal tooth thicknesses are the tooth thicknesses in a normal section of the gear teeth; they are the lengths of the helical arcs on the respective cylinders between the tooth flanks of a tooth. They result from the standard basic rack tooth profile and the addendum modification.

$$\begin{aligned} s_n &= s_t \cdot \cos \beta = \frac{p_n}{2} + 2 \cdot x \cdot m_n \cdot \tan \alpha_n \\ &= m_n \cdot \left(\frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha_n \right) \end{aligned} \quad (2.69)$$

$$s_{yn} = s_{yt} \cdot \cos \beta_y \quad (2.70)$$

$$s_{vn} = s_{vt} \cdot \cos \beta_v \quad (2.71)$$

$$s_{bn} = s_{bt} \cdot \cos \beta_b \quad (2.72)$$

2.5.8.6 Normal spacewidths e_n , e_{yn} , e_{vn} and e_{bn}

The normal spacewidths are the spacewidths in a normal section of the gear teeth; they are the lengths of the helical arcs on the respective cylinders between the two tooth flanks enclosing a tooth space. They result from the standard basic rack tooth profile and the addendum modification:

$$\begin{aligned} e_n &= e_t \cdot \cos \beta = \frac{p_n}{2} - 2 \cdot x \cdot m_n \cdot \tan \alpha_n \\ &= m_n \cdot \left(\frac{\pi}{2} - 2 \cdot x \cdot \tan \alpha_n \right) \end{aligned} \quad (2.73)$$

$$e_{yn} = e_{yt} \cdot \cos \beta_y \quad (2.74)$$

$$e_{vn} = e_{vt} \cdot \cos \beta_v \quad (2.75)$$

$$e_{bn} = e_{bt} \cdot \cos \beta_b \quad (2.76)$$

2.6 Geometrical limits for involute gear teeth

2.6.1 Undercut on cylindrical gears with external teeth

When the teeth of an external gear are being produced undercutting of the tooth flanks occurs if the path of the corner of the cutter tip cuts into the involute portion of the tooth root flank of the workpiece during the rolling action.

By suitable choice of gearcutting data, particularly as regards addendum modification, helix angle, addenda and pressure angle, undercut can be avoided. For an external gear with standard basic rack profile according to DIN 867 produced by a rack cutting tool the following relationship exists between the cutting data:

$$x_{\min} = -\frac{z \cdot \sin^2 \alpha_t}{2 \cdot \cos \beta} + \frac{h_{a0} - \rho_{a0} \cdot (1 - \sin \alpha_n)}{m_n} \quad (2.77)$$

where h_{a0} is the addendum and ρ_{a0} is the tip corner rounding radius of the tool. In the case of an external gear with standard basic rack tooth profile according to DIN 867, $\rho_{a0} \cdot (1 - \sin \alpha_n)$ in equation (2.77) must be replaced by the bottom clearance c .

The value $x_{\min}$ calculated in this way can usually be further reduced by a small amount (not exceeding 0,17) if a small degree of undercut, which generally does not have any harmful effects, is acceptable.

2.6.2 Pointing limit and minimum tooth thickness at the tip circle of an external gear

The teeth of an external gear become pointed at the tip circle if

$$\psi_a = \frac{s_{at}}{d_a} = \psi + \text{inv } \alpha_t - \text{inv } \alpha_{at} = 0 \quad (2.78)$$

The pointing limit resulting in this way is considered in subclause 2.6.4.

At the tip cylinder the tooth thickness s_{an} should not have a value less than $0,2 \cdot m_n$. This provides an upper limit for the addendum modification of an external gear which is suitable for practical application, see subclause 2.6.4.

The undercut and pointing limit set limits in the downward direction to the number of teeth which it is practicable to cut in an external gear. For spur gears and the standard basic rack tooth profile according to DIN 867 the following apply

$z_{\min} = 8$, taking $x = +0,57$ as the pointing limit,

$z_{\min} = 9$, taking $x = +0,45$ for $s_{an} = 0,2 \cdot m_n$,

see figure 10.

If a small amount of undercut is accepted, the following is still feasible

$z_{\min} = 7$, taking $x = +0,41$

2.6.3 Pointing limit and minimum spacewidth at the root circle of an internal gear

The tooth space of an internal gear becomes pointed at the root circle if

$$\eta_l = \frac{e_{fl}}{d_f} = \eta - \text{inv } \alpha_t + \text{inv } \alpha_{fl} = 0 \quad (2.79)$$

The resulting pointing limit is considered in subclause 2.6.4.

At the root circle of the internal gear the spacewidth e_{fn} should have a value not less than $0,2 \cdot m$. This provides an upper limit for the addendum modification of an internal gear which is suitable for practical application, see subclause 2.6.4.

The tip circle diameter limit and the pointing limit for the tooth spaces limit the feasible number of teeth for an internal gear in the downward direction. For spur gearing and the standard basic rack tooth profile according to DIN 867 the following applies

$|z|_{\min} = 16$ with $x = -0,52$ for pointing limit

$|z|_{\min} = 21$ with $x = -0,38$ for $e_{fn} = 0,2 \cdot m$,

see figure 10.

Other limits arising from mating reasons are dealt with in DIN 3993.

2.6.4 Range of feasible involute gear teeth

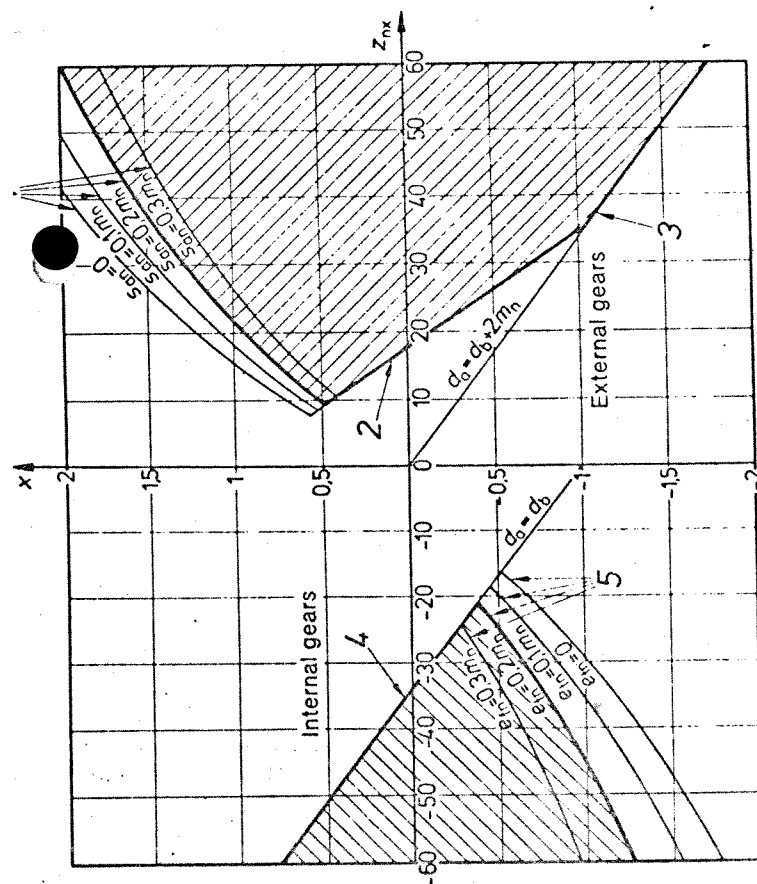
The feasibility range for involute gear teeth which is imposed by the limits according to subclauses 2.6.1 to 2.6.3 is indicated in figure 10 for the standard basic rack tooth profile according to DIN 867 and in figure 11 for the standard basic rack tooth profile according to DIN 58400. The relationships for helical gear teeth are shown here for virtual spur gear teeth which, for a standard basic rack tooth profile according to DIN 867 or DIN 58400, are determined by the addendum modification coefficient x of the helical gear teeth and by the virtual tooth number

$$z_{nx} = \frac{z}{\cos^2 \beta_b \cdot \cos \beta} = z \cdot z_{nx}^* \quad (2.80)$$

For the virtual tooth number factor z_{nx}^* see table 1. This relationship between the numbers of teeth is so chosen that for a helical gear tooth system and the corresponding virtual spur gear tooth system the same value $x_{\min}$ is yielded by equation (2.77). In figure 10 and figure 11 the addendum modifications x which are feasible each time are plotted against the virtual tooth numbers z_{nx} .

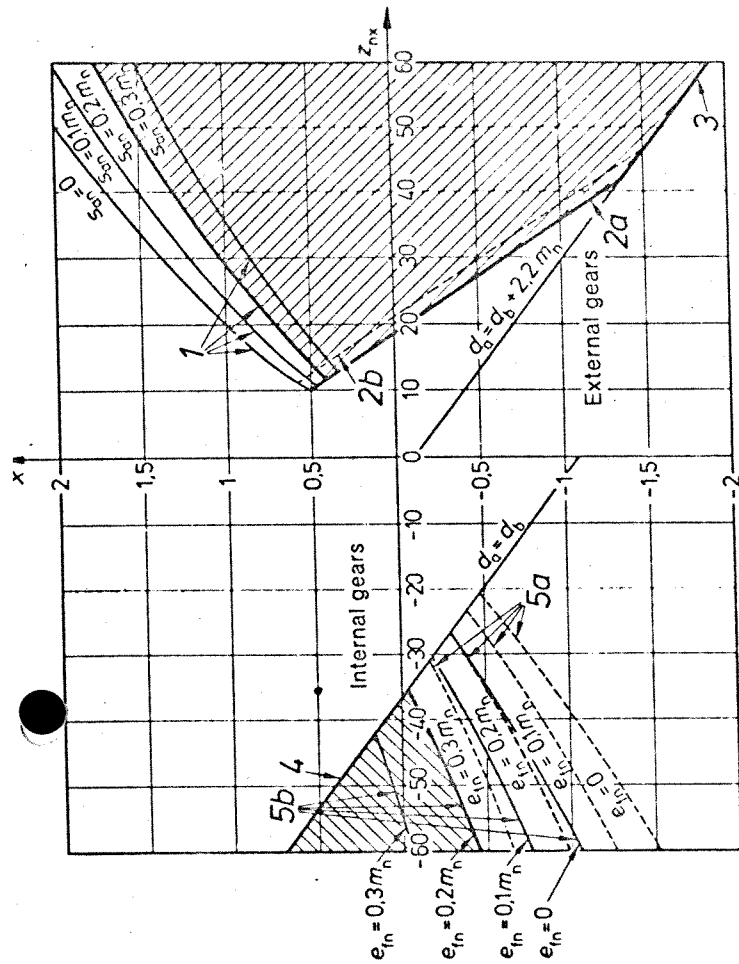
Table 1. Virtual tooth number factor $z_{nx}^* = \frac{1}{\cos^2 \beta_b \cdot \cos \beta}$ as a function of helix angle β for $\alpha_n = 20^\circ$

β in degrees	z_{nx}^*	β in degrees	z_{nx}^*	β in degrees	z_{nx}^*	β in degrees	z_{nx}^*	β in degrees	z_{nx}^*
1	1,0004	11	1,0526	21	1,2082	31	1,5235	41	2,1373
2	1,0017	12	1,0629	22	1,2311	32	1,5680	42	2,2255
3	1,0038	13	1,0743	23	1,2556	33	1,6155	43	2,3203
4	1,0068	14	1,0868	24	1,2819	34	1,6663	44	2,4223
5	1,0106	15	1,1004	25	1,3100	35	1,7206	45	2,5322
6	1,0153	16	1,1151	26	1,3400	36	1,7787		
7	1,0209	17	1,1311	27	1,3720	37	1,8409		
8	1,0274	18	1,1483	28	1,4063	38	1,9074		
9	1,0348	19	1,1668	29	1,4428	39	1,9788		
10	1,0432	20	1,1868	30	1,4818	40	2,0553		



- Curves 1 Limiting values for external gears due to pointing limit $s_{an} = 0$ or minimum crest width $s_{an} = (0.1 \dots 0.3) \cdot m_n$
- Line 2 Limiting values for external gears due to undercut at root flanks
- Line 3 Limiting values for external gears due to minimum tip circle diameter $d_a = d_b + 2 \cdot m_n$
- Line 4 Limiting values for internal gears due to condition that the amount of the tip circle diameter shall not be smaller than the amount of the base circle diameter
- Curves 5 Limiting values for internal gears with $c = 0.25 \cdot m_n$ due to pointing limit $e_{fn} = 0$ or minimum root spacewidth $e_{fn} = (0.1 \dots 0.3) \cdot m_n$

Figure 10. Range of feasible involute gear teeth with standard basic rack tooth profile according to DIN 867
Relationship between virtual number of teeth z_{nx} and addendum modification coefficient x for tooth numbers up to $z_{nx} = \pm 60$
For internal gears see also DIN 3993
In the case of cylindrical gears with deviations in tooth thickness x_E replaces x



- Curves 1 Limiting values for external gears due to pointing limit $s_{an} = 0$ or minimum crest width $s_{an} = (0.1 \dots 0.3) \cdot m_n$
- Line 2a Limiting values for external gears due to undercut at root flanks
- Line 2b Limiting values for "toleranced minimum number of teeth" according to DIN 58405 (active flank on tool extended by 10 percent)
- Line 3 Limiting values for external gears due to minimum tip circle diameter $d_a = d_b + 2.2 \cdot m_n$
- Line 4 Limiting values for internal gears due to condition that the amount of the tip circle diameter shall not be smaller than the amount of the base circle diameter
- Curves 5a Limiting values for internal gears with $c = 0.25 \cdot m_n$ due to pointing limit $e_{fn} = 0$ or minimum root spacewidth $e_{fn} = (0.1 \dots 0.3) \cdot m_n$
- Curves 5b Same as curves 5a, but limiting values for internal gears with $c = 0.4 \cdot m_n$

Figure 11. Range of feasible involute gear teeth with standard basic rack tooth profile according to DIN 58400 (without considering the 15° face on the bottom of the tooth)
Relationship between virtual number of teeth z_{nx} and addendum modification coefficient x for tooth numbers up to $z_{nx} = \pm 60$
For internal gears see also DIN 3993
In the case of cylindrical gears with deviations in tooth thickness x_E replaces x

Note: Curves 5 in figures 10 and 11 are based on the (not realizable) assumption that the involute faces go right through to the bottom of the tooth; furthermore, the 15° faze at the bottom of the tooth of the standard basic rack profile in the case of curves 5a and 5b in figure 11 has not been taken into account. It is therefore generally advisable or necessary to carry out an investigation according to DIN 3993 in the case of internal gears.

For an external gear the limits of the range are set by the pointing limit or the minimum crest width according to subclause 2.6.2 (curves 1), by the undercut according to subclause 2.6.1 (line 2) and by the minimum tip circle diameter according to subclause 2.5.6.1 (line 3).

For an internal gear the limits of the range are set by the limiting value of the tip circle diameter according to subclause 2.5.6.1 (line 4) and by the pointing limit or minimum spacewidth according to subclause 2.6.3 (curves 5). These limits are however not always achievable in practice due to mating reasons, see DIN 3993.

2.7 Test dimensions for tooth thickness

The tooth thickness is a circular or helical arc and as such is not directly measurable. For checking tooth thickness therefore indirect measuring methods are used and from the values obtained in this way the tooth thickness is found by calculation.

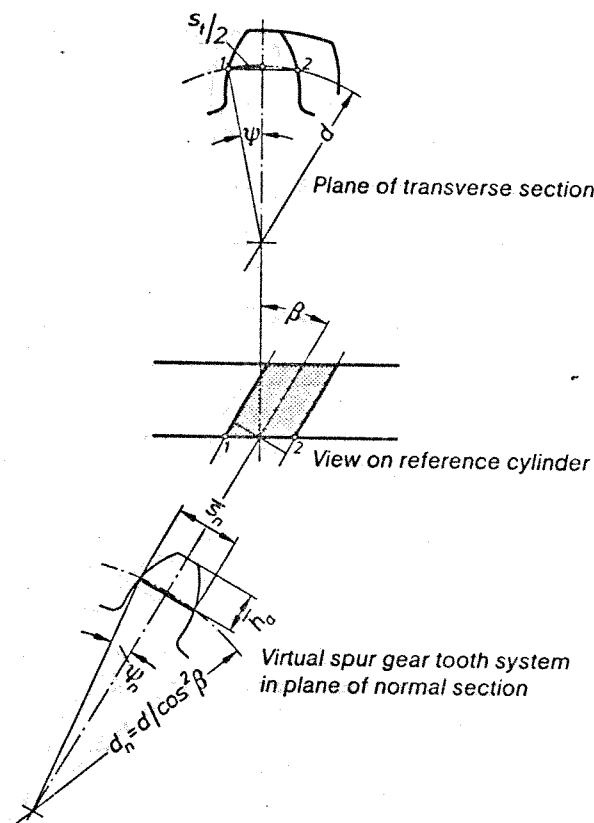


Figure 12. Normal chordal tooth thickness $\bar{s}_n$ and height $\bar{h}_a$ above the chord of an external helical gear derived from the virtual spur gear tooth system at the curvature ellipse in the normal section

2.7.1 Normal chordal tooth thicknesses

2.7.1.1 Normal chordal tooth thicknesses $\bar{s}_n$ and $\bar{s}_{yn}$, heights $\bar{h}_a$ and $\bar{h}_y$ above the chords

The normal chordal tooth thickness $\bar{s}_n$ or $\bar{s}_{yn}$ is the shortest distance between the tooth traces of a tooth on the reference cylinder and the Y-cylinder respectively. It is the only tooth thickness quantity directly measurable on a tooth.

For helical gear teeth the normal chordal tooth thickness at the reference cylinder can be calculated with adequate accuracy from a virtual spur gear tooth system defined in the area of the reference circle by the virtual reference

circle diameter $d_n = \frac{d}{\cos^2 \beta}$ (circle of curvature of the ellipse in the normal section) and the virtual tooth thickness half angle $\psi_n = \psi \cdot \cos^3 \beta$, see figure 12.

On the reference cylinder the following applies

$$\bar{s}_n = d_n \cdot \sin \psi_n = \frac{d \cdot \sin (\psi \cdot \cos^3 \beta)}{\cos^2 \beta} \quad (2.81)$$

Correspondingly the normal chordal tooth thickness on the Y-cylinder is found as

$$\bar{s}_{yn} = \frac{d_y \cdot \sin (\psi_y \cdot \cos^3 \beta_y)}{\cos^2 \beta_y}, \quad (2.82)$$

and on the V-cylinder as

$$\bar{s}_{vn} = \frac{d_v \cdot \sin (\psi_v \cdot \cos^3 \beta_v)}{\cos^2 \beta_v} \quad (2.83)$$

By series expansion of the sin value and putting ψ terms of 4th and higher order equal to zero the following approximation equations are obtained from equations (2.81) to (2.83)

$$\bar{s}_n \approx s_n \cdot \left(1 - \frac{1}{6} \cdot \psi^2 \cdot \cos^6 \beta\right) \quad (2.84)$$

$$\bar{s}_{yn} \approx s_{yn} \cdot \left(1 - \frac{1}{6} \cdot \psi_y^2 \cdot \cos^6 \beta_y\right) \quad (2.85)$$

$$\bar{s}_{vn} \approx s_{vn} \cdot \left(1 - \frac{1}{6} \cdot \psi_v^2 \cdot \cos^6 \beta_v\right) \quad (2.86)$$

For nearly all cases arising in practice, these give the numerical values with adequate accuracy.

The height $\bar{h}_a$ above the chord $\bar{s}_n$ (or $\bar{h}_y$ above $\bar{s}_{yn}$ or $\bar{h}_v$ above $\bar{s}_{vn}$) is

$$\bar{h}_a \approx h_a + \frac{1}{4} \cdot d \cdot \psi^2 \cdot \cos^4 \beta \quad (2.87)$$

$$\bar{h}_y \approx \frac{d_a - d_y}{2} + \frac{1}{4} \cdot d_y \cdot \psi_y^2 \cdot \cos^4 \beta_y \quad (2.88)$$

$$\bar{h}_v \approx h_a - x \cdot m_n + \frac{1}{4} \cdot d_v \cdot \psi_v^2 \cdot \cos^4 \beta_v \quad (2.89)$$

For a spur gear with $\beta = 0$, $\beta_y = 0$ and $\beta_v = 0$ equations (2.81) to (2.83) hold good exactly and equations (2.84) to (2.89) with good approximation.

Note: For an internal gear the values for d and ψ are negative, see subclauses 2.1, 2.5.6 and 2.5.8.2. Hence according to equations (2.81) to (2.86) the values for the normal chordal tooth thickness are positive. In equations (2.87) to (2.89) the last term is negative for an internal gear.

2.7.1.2 Constant chord $\bar{s}_c$ of a spur gear

At distance $\bar{h}_c$ from the crown of the tip circle the teeth of a spur gear have a normal chordal tooth thickness $\bar{s}_c$ the length of which is dependent only on the module, the pressure angle and the addendum modification, but on the other hand is independent of the number of teeth. All gears of the same standard basic rack tooth profile and with the same addendum modification thus have the same normal chordal tooth thickness $\bar{s}_c$ at the same point $\bar{h}_c$, see figure 13 and figure 14. For this reason it is termed the constant chord.

$$\bar{s}_c = s \cdot \cos^2 \alpha$$

$$= m \cdot \cos^2 \alpha \cdot \left(\frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha \right) \quad (2.90)$$

$$\bar{h}_c = a_a - \frac{s}{2} \cdot \cos \alpha \cdot \sin \alpha \quad (2.91)$$

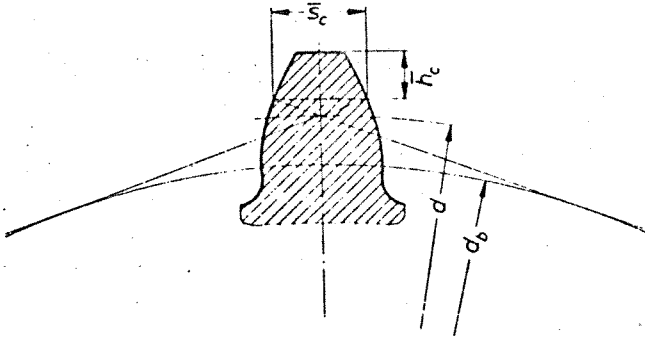


Figure 13. Constant chord $\bar{s}_c$ and height $\bar{h}_c$ above the constant chord of an external spur gear

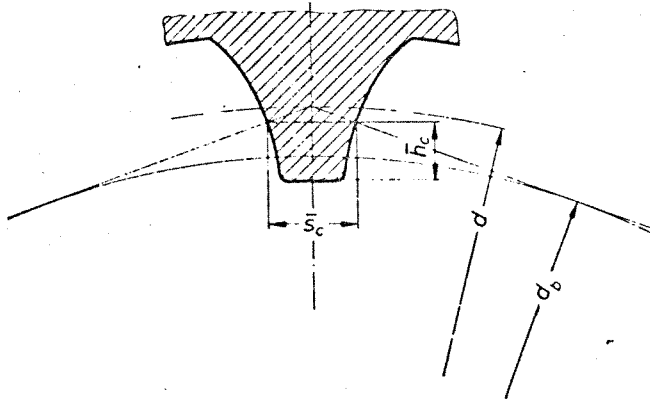


Figure 14. Constant chord $\bar{s}_c$ and height $\bar{h}_c$ above the constant chord of an internal spur gear

2.7.2 Base tangent length W_k

The base tangent length W_k of an external gear is the distance, measured over k teeth, or over k tooth spaces in the case of internal gear, between two parallel planes contacting a right flank and a left flank respectively in the involute part of the tooth flanks, see figure 15 and figure 16. In the case of internal gears the base tangent length is only measurable on gears of the spur type.

The base tangent length is not referred to the gear axis and is therefore independent of any eccentricity of the gear teeth.

The number of teeth spanned (measured number of tooth spaces) k must be chosen so that the measuring planes contact the tooth flanks at a point close to half the tooth depth, i.e. in the vicinity of the V-cylinder. For contact in the V-cylinder the following relationship applies

$$k = \frac{z}{\pi} \cdot \left(\frac{\tan \alpha_{vt}}{\cos^2 \beta_b} - 2 \cdot \frac{x}{z} \cdot \tan \alpha_n - \text{inv } \alpha_t \right) + \frac{z}{2 \cdot |z|} \quad (2.92)$$

with α_{vt} according to equation (2.43) and $\cos \beta_b$ according to equation (2.32).

As an approximation k is found from the virtual spur gear tooth system as

$$k \approx \frac{z_{nW}}{\pi} \cdot \left(\tan \alpha_{vn} - 2 \cdot \frac{x}{z_{nW}} \cdot \tan \alpha_n - \text{inv } \alpha_n \right) + \frac{z}{2 \cdot |z|} \quad (2.93)$$

or with less good approximation from the following expression

$$k \approx z_{nW} \cdot \frac{\alpha_{vn}}{180} + \frac{z}{2 \cdot |z|} \quad (2.94)$$

using the virtual tooth number z_{nW} according to equation (2.97) or according to table 2 and α_{vn} in degrees from equations (2.44) or (2.45).

For k a fractional number is generally obtained and for the base tangent length calculation this must be rounded up to the next whole number. A value $k < 1,5$ shall be rounded to 2. With $k = 1$ it would be necessary to measure near the base cylinder and such measurements are unsatisfactory and can be avoided in the majority of cases, see figure 17 and figure 18.

As a result of rounding the number of teeth spanned the measuring elements no longer make contact with the tooth flanks on the V-cylinder, but instead (when symmetrically positioning the measuring elements) on or near the measuring circle diameter

$$d_M = \sqrt{d_b^2 + (W_k \cdot \cos \beta_b)^2} \cdot \frac{z}{|z|} \quad (2.95)$$

Note: In the case of an internal gear d_M is a negative value.

The distance between the contact point of the measuring element and the tip circle is $0,5 \cdot (d_a - d_M)$. If this value is negative the (theoretical) contact point lies outside the tooth system. A different number of teeth spanned (measured number of spaces) must be chosen.

In the case of an external gear with symmetrical contact of the measuring elements it is generally permissible for the contact points on the tooth flanks to lie up to $0,7 \cdot m_n$ outside the V-cylinder or up to $0,5 \cdot m_n$ inside the V-cylinder, this means that the following should apply

$$(d_v + 1,4 \cdot m_n) \geq d_M \geq (d_v - m_n).$$

Consequently, for measuring the base tangent length, different numbers of teeth spanned k may be applicable, see figure 15 and figure 17.

In the case of an internal (spur) gear with symmetrical contact of the measuring elements it is generally permissible for the contact points on the tooth flanks to lie up to $0,5 \cdot m$ outside the V-circle or up to $0,5 \cdot m$ inside the V-circle, this means that the following should apply

$$|(d_v + m_n)| \leq |d_M| \leq |(d_v - m_n)|.$$

Consequently, for measuring the base tangent length, different numbers of teeth spanned k may be applicable, see figure 16 and figure 18.

Using the whole-number value for k yields the base tangent length as

$$W_k = m_n \cdot \cos \alpha_n \cdot \left[\left(k - \frac{z}{2 \cdot |z|} \right) \cdot \pi + z \cdot \text{inv } \alpha_t \right] + 2 \cdot x \cdot m_n \cdot \sin \alpha_n \quad (2.96)$$

Note: Because of the negative tooth numbers the values of k and W_k are negative for internal gears, see sub-clause 2.1.

If in equation (2.96) the value $z \cdot \text{inv } \alpha_t$ is replaced by $z_{nW} \cdot \text{inv } \alpha_n$ the resulting virtual tooth number for base tangent length calculations is found as

$$z_{nW} = z \cdot \frac{\text{inv } \alpha_t}{\text{inv } \alpha_n} = z \cdot z_{nW}^* \quad (2.97)$$

For the virtual tooth number factor z_{nW}^* see table 2. Tables of base tangent lengths for spur gears are therefore applicable also to helical gears if the virtual tooth number z_{nW} is used instead of the number of teeth z . Since z_{nW} is generally a fractional value it is necessary in such cases to find the base tangent length of a helical gear by interpolation.

In the case of external helical gears the contact lines must have adequate overlap b_M and the measuring surfaces used must have at least the same width so that the length of the diagonal formed by the base tangent length W_k and the contact line overlap b_M is greater than W_k by an appreciable value ΔW , see figure 19.

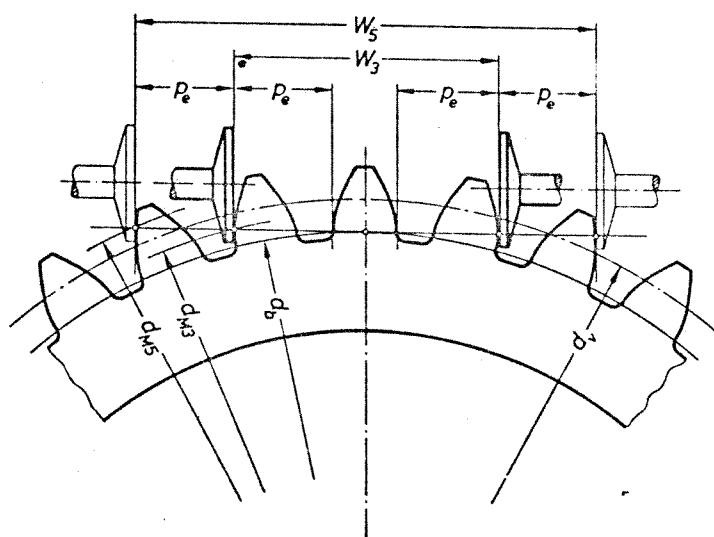
In this way measuring errors resulting from a diagonal position of the contact points or by measuring over the edges of the tooth ends are avoided.

The minimum represented by $b_M > 1,2 + 0,018 \cdot W_k$ in mm should not be under-run; if the tooth edges have been chamfered, a value $b_M > 2,0 + 0,03 \cdot W_k$ in mm is recommended. For measuring the base tangent length in this case a minimum facewidth of

$$b \geq W_k \cdot \sin |\beta_b| + b_M \cdot \cos \beta_b \quad (2.98)$$

is necessary. The relationship between the length difference ΔW , the contact line overlap or measuring element width b_M and the base tangent length W_k is shown in figure 20.

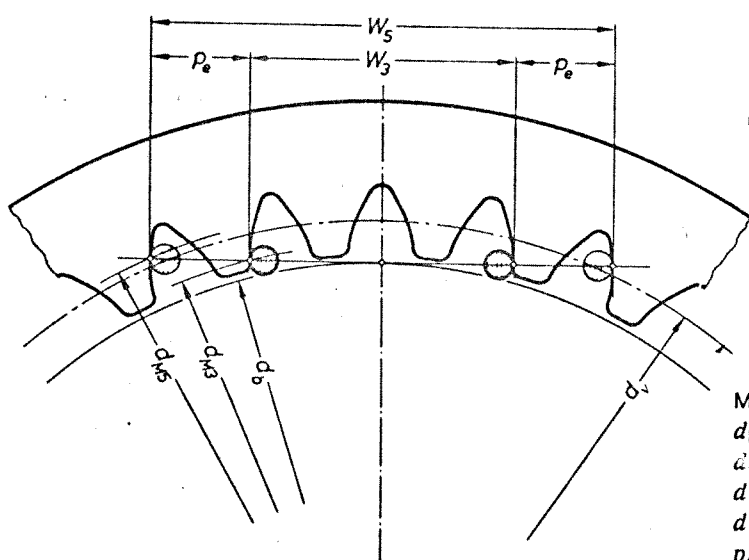
When V-heads are used, see figure 19, the overlap $2 \cdot b_M$ is necessary for a given size ΔW . If the overlap is small or known it is recommended that the V-heads should be brought into contact at the overlap mid-point from the end of the tooth.



Measurement by using disc-shaped measuring elements

- d_b base circle diameter
- d_v V-circle diameter
- d_{M3} measuring circle diameter for base tangent length W_3
- d_{M5} measuring circle diameter for base tangent length W_5
- p_c normal base pitch

Figure 15. Base tangent length W_3 (over $k = 3$ teeth) and base tangent length W_5 (over $k = 5$ teeth) on an external spur gear



Measurement by means of pins

- d_b base circle diameter
- d_v V-circle diameter
- d_{M3} measuring circle diameter for base tangent length W_3
- d_{M5} measuring circle diameter for base tangent length W_5
- p_c normal base pitch

Figure 16. Base tangent length W_3 (over $k = 3$ tooth spaces) and base tangent length W_5 (over $k = 5$ tooth spaces) on an internal spur gear

Table 2. Virtual tooth number factor $z_{nW}^* = \text{inv } \alpha_t / \text{inv } \alpha_n$ as a function of helix angle β for $\alpha_n = 20^\circ$

β in de- grees	z_{nW}^*	β in de- grees	z_{nW}^*	β in de- grees	z_{nW}^*	β in de- grees	z_{nW}^*	β in de- grees	z_{nW}^*
1	1,0004	11	1,0544	21	1,2162	31	1,5480	41	2,2078
2	1,0017	12	1,0651	22	1,2491	32	1,5952	42	2,3040
3	1,0039	13	1,0769	23	1,2658	33	1,6457	43	2,4078
4	1,0070	14	1,0898	24	1,2933	34	1,6999	44	2,5200
5	1,0109	15	1,1039	25	1,3227	35	1,7579	45	2,6414
6	1,0158	16	1,1192	26	1,3543	36	1,8201		
7	1,0216	17	1,1358	27	1,3880	37	1,8869		
8	1,0283	18	1,1538	28	1,4240	38	1,9586		
9	1,0360	19	1,1731	29	1,4626	39	2,0356		
10	1,0447	20	1,1938	30	1,5038	40	2,1185		

Table 3. Virtual tooth number factor $z_{nM}^* = 1/\cos^{3,3} \beta$ as a function of helix angle β

β in de- grees	z_{nM}^*	β in de- grees	z_{nM}^*	β in de- grees	z_{nM}^*	β in de- grees	z_{nM}^*	β in de- grees	z_{nM}^*
1	1,0005	11	1,0631	21	1,2546	31	1,6630	41	2,5312
2	1,0020	12	1,0756	22	1,2834	32	1,7227	42	2,6635
3	1,0045	13	1,0895	23	1,3144	33	1,7871	43	2,8079
4	1,0081	14	1,1046	24	1,3477	34	1,8565	44	2,9656
5	1,0127	15	1,1212	25	1,3835	35	1,9315	45	3,1383
6	1,0183	16	1,1393	26	1,4221	36	2,0125		
7	1,0250	17	1,1589	27	1,4635	37	2,1001		
8	1,0328	18	1,1801	28	1,5080	38	2,1950		
9	1,0417	19	1,2031	29	1,5560	39	2,2980		
10	1,0518	20	1,2279	30	1,6075	40	2,4097		

2.7.3 Radial test dimensions for tooth thickness

2.7.3.1 Radial single-ball dimension M_{rK}

The radial single-ball dimension M_{rK} is the distance between the gear axis and

in the case of an external gear the outermost point,
in the case of an internal gear the innermost point

of a measuring ball of diameter D_M in contact with both tooth flanks in a tooth space, see figure 21 and figure 22. The points of contact P_R and P_L between the measuring ball and the tooth flanks shall lie on the V-cylinder or near it.

The corresponding measuring ball diameters D_M must be chosen according to DIN 3977 (at present at draft stage); for $m_n = 1\text{ mm}$ they can be found from figure 23. Since the measuring balls only need to contact the tooth flanks in the vicinity of the V-cylinder, use can be made of available balls having diameters differing somewhat from the nomogram values or calculated values.

In the case of contact in the V-cylinder the measuring ball diameters are

$$D_M \approx z_{nM} \cdot m_n \cdot \cos \alpha_n \cdot (\tan \alpha_{KnM} - \tan \alpha_{vnM}), \quad (2.99)$$

the virtual number of teeth z_{nM} for dimensions over balls or pins being

$$z_{nM} = \frac{z}{\cos^{3,3} \beta} = z \cdot z_{nM}^* \quad (2.100)$$

For factor z_{nM}^* of the virtual number of teeth see table 3.

The angle α_{vnM} must be calculated according to

$$\cos \alpha_{vnM} = \frac{z_{nM} \cdot \cos \alpha_n}{z_{nM} + 2 \cdot x}, \quad (2.101)$$

the angle α_{KnM} according to

$$\alpha_{KnM} = \tan \alpha_{vnM} - \text{inv } \alpha_n + \frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z_{nM}} \quad (2.102)$$

The measuring balls according to equation (2.99) contact the tooth flanks of spur gears exactly on the V-circle (in the case of gears with tooth thickness deviations on the generating V-circle if it is calculated with x_E according to equation (2.114) instead of x); they contact the helical gear flanks close to the V-cylinder. The contact points may be checked by equation (2.106).

Note: In the case of an internal gear the tooth numbers z and z_{nW} and the bracketed expression in equation (2.99) are negative. For the angles α and the measuring ball diameter D_M the values are always positive.

If the measuring ball diameter D_M is known the pressure angle at a point α_{Kt} in the transverse section on the circle through the centre of the ball is found from

$$\begin{aligned} \text{inv } \alpha_{Kt} &= \frac{D_M}{z \cdot m_n \cdot \cos \alpha_n} - \eta + \text{inv } \alpha_t \\ &= \frac{D_M}{d_b \cdot \cos \beta_b} - \eta_b \end{aligned} \quad (2.103)$$

The diameter d_K of the circle on which the centre of the measuring ball lies is found as

$$d_K = d \cdot \frac{\cos \alpha_t}{\cos \alpha_{Kt}} = \frac{d_b}{\cos \alpha_{Kt}} \quad (2.104)$$

The radial single-ball dimension is

$$M_{rK} = \frac{1}{2} \cdot (d_K + D_M) \quad (2.105)$$

The diameter d_M of the cylinder on which the contact points P_L and P_R between the measuring ball and the two tooth flanks lie is found as

$$d_M = \frac{d_b}{\cos \alpha_M}, \quad (2.106)$$

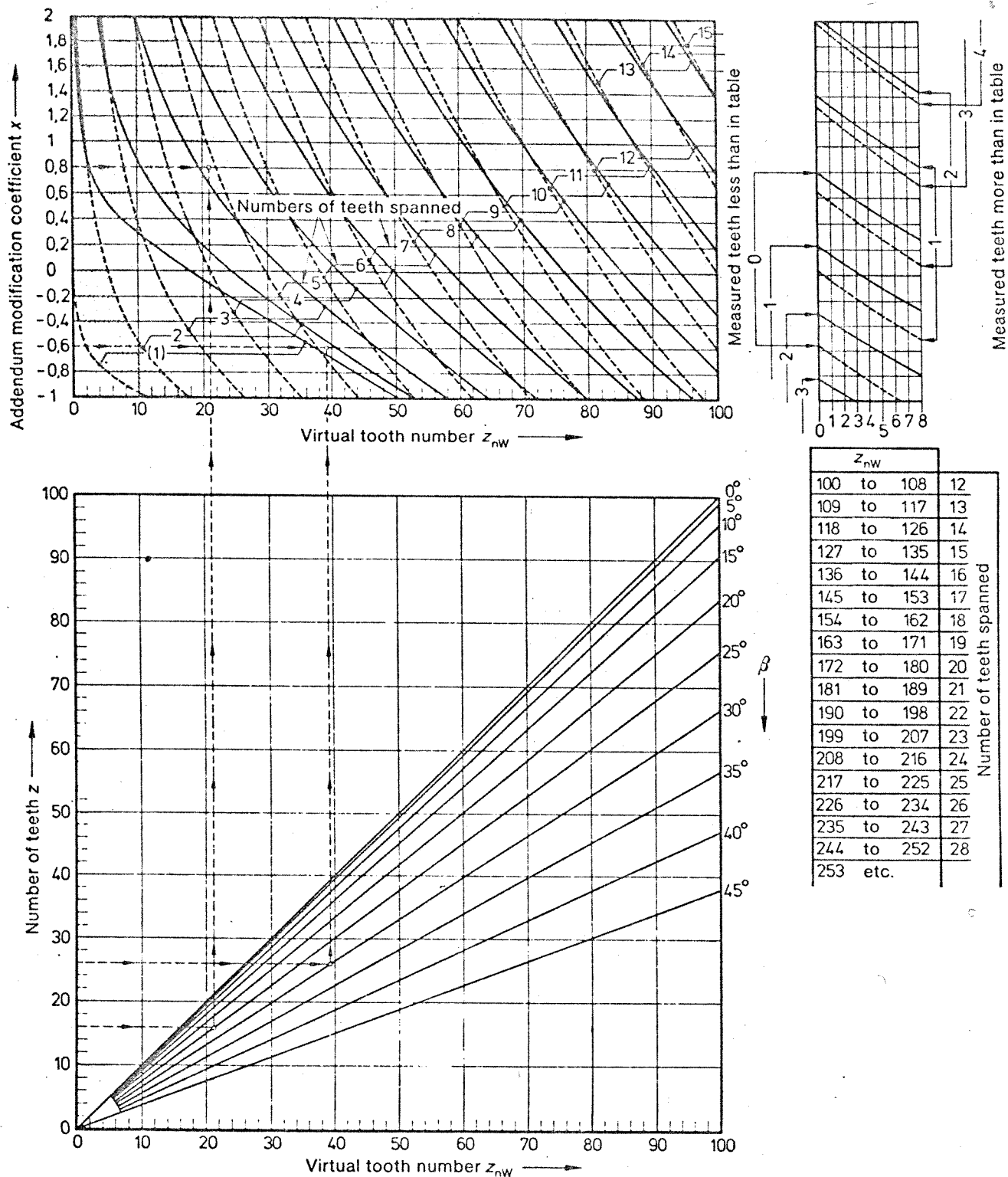
the pressure angle α_M on the circle having diameter d_M being calculated as

$$\tan \alpha_M = \tan \alpha_{Kt} - \frac{D_M}{d_b} \cdot \cos \beta_b \quad (2.107)$$

Note: In the case of an internal gear z , η and η_b as well as d , d_b and d_K are negative, see subclause 2.1. Hence negative values are also obtained for M_{rK} and d_M . The measuring ball diameter D_M and the pressure angles α_K and α_M are always positive.

2.7.3.2 Radial single-pin dimension M_{rP}

In the case of spur gear teeth and external helical gear teeth measuring pins of diameter D_M can also be used instead of measuring balls. Equations (2.99) to (2.107) apply equally to the radial single-pin dimension M_{rP} .



In the lower part of the figure the virtual tooth number z_{nw} according to equation (2.97) or table 2 is found from the number of teeth z in the gear and the helix angle β . Example: $z = 26$ and $\beta = 30^\circ$ gives $z_{nw} = 26 \cdot 1,5038 = 39,10$.

The upper part of the figure shows on the left the relationship between z_{nw} , the addendum modification coefficient x and the number of teeth spanned k for $z_{nw} \leq 100$.

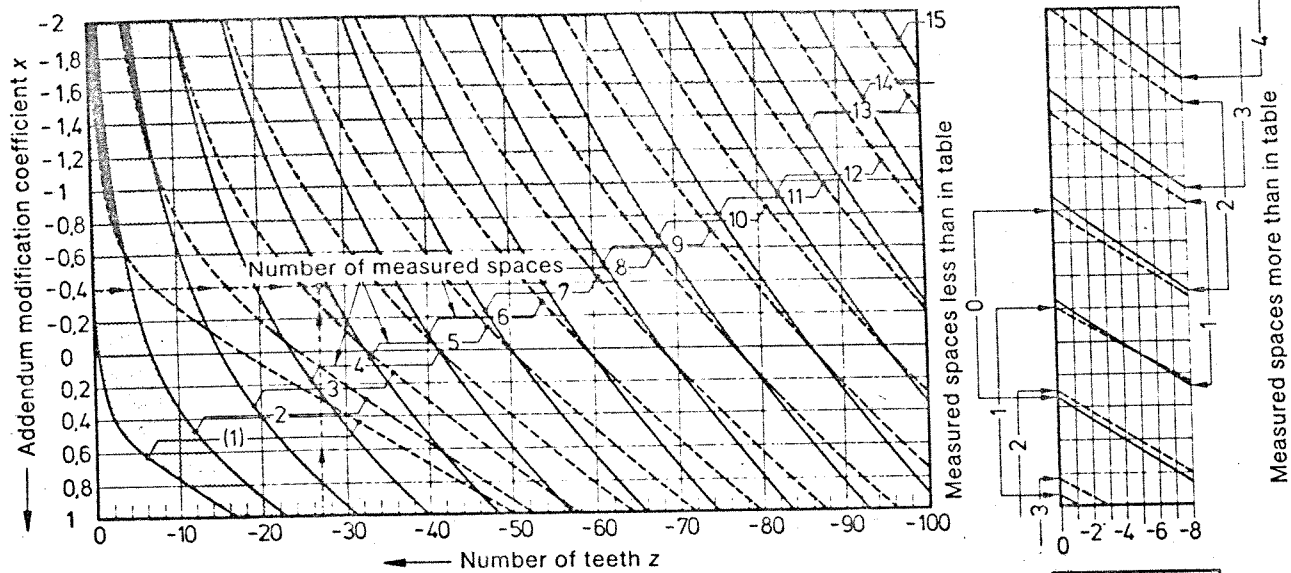
The solid-line curves correspond to the condition $d_M = d_v - 2 \cdot 0,5 \cdot m_n$ and the broken-line curves to the condition $d_M = d_v + 2 \cdot 0,7 \cdot m_n$. The boundaries of the tooth systems feasible according to subclause 2.6 are not indicated here, see figures 10 and 11.

The areas covered by a given number of teeth spanned overlap in large parts of the diagram, so that here several numbers of teeth spanned are feasible for a single virtual number of teeth and a single addendum modification coefficient. Example: $z_{nw} = 39,1$ and $x = -0,6$ lies in the area of the numbers of teeth spanned $k = 2, 3, 4$ and 5. Only in the shaded portions is measurement with only one number of teeth spanned possible. Example: $z_{nw} = 21$ and $x = +0,8$ lies in the range $k = 4$.

For $z_{nw} > 100$ the upper right-hand part of the figure is applicable, being designed for virtual tooth-number groups 100 ... 108, 109 ... 117 etc., rising by 9 each time. From the corresponding table a tabulated value is found for the number of teeth spanned which, depending on the addendum modification coefficient, either yields the value of k direct, or which must be either decreased or increased by 1, 2, 3 or 4 measured teeth corresponding to the ranges in the upper part of the figure in order to yield the number of teeth spanned k . In this part of the figure also the ranges overlap.

Example: $z_{nw} = 142$ (tabulated value: 16) and $x = +0,9$ yields the possible values $k = 16 + 1 = 17$ or $k = 16 + 2 = 18$ or $k = 16 + 3 = 19$.

Figure 17. Nomogram for determining the number of teeth spanned k for external gears with $\alpha_n = 20^\circ$



The left-hand part of the figure contains the relationship between the number of teeth z , the addendum modification coefficient x and the number of measured spaces k for $|z| \leq 100$.

The solid-line curves correspond to the condition that the contact points of the measuring elements are $0,5 \cdot m$ outside the V-circle, whilst the broken-line curves correspond to the condition that the contact points of the measuring elements are $0,5 \cdot m$ inside the V-circle.

The limits of the tooth systems feasible according to subclause 2.6 are not indicated here, see figures 10 and 11.

The areas covered by a given measured number of spaces overlap in large parts of the diagram, so that here several measured numbers of spaces are feasible for a single number of teeth and a single addendum modification coefficient. Example: $z = -48$ and $x = +0,9$ lie in the range of the measured numbers of spaces 2, 3, 4 and 5. Only in the shaded portions is measurement with only one measured number of spaces possible. For example: $z = -27$ and $x = -0,4$ lie in the range $k = 4$.

For $|z| > 100$ the right-hand part of the figure is applicable. For explanation see figure 17.

z		Number of measured spaces
-100 to -108	12	
-109 to -117	13	
-118 to -126	14	
-127 to -135	15	
-136 to -144	16	
-145 to -153	17	
-154 to -162	18	
-163 to -171	19	
-172 to -180	20	
-181 to -189	21	
-190 to -198	22	
-199 to -207	23	
-208 to -216	24	
-217 to -225	25	
-226 to -234	26	
-235 to -243	27	
-244 to -252	28	
253 etc.		

Figure 18. Nomogram for determining the measured number of spaces k for internal spur gears with $\alpha = 20^\circ$

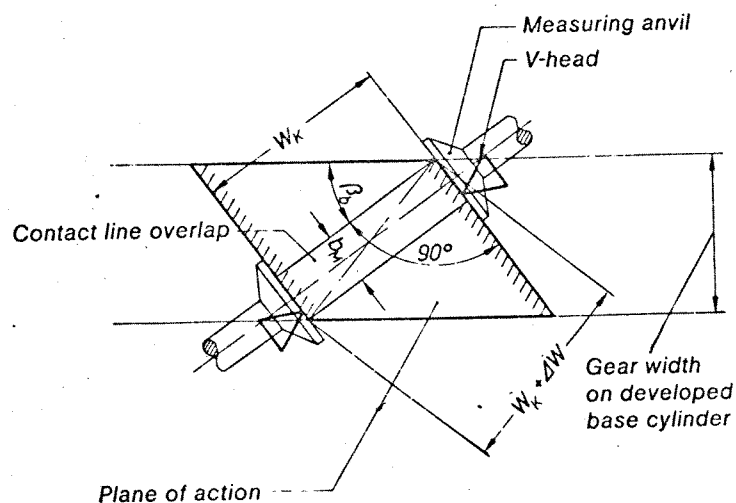
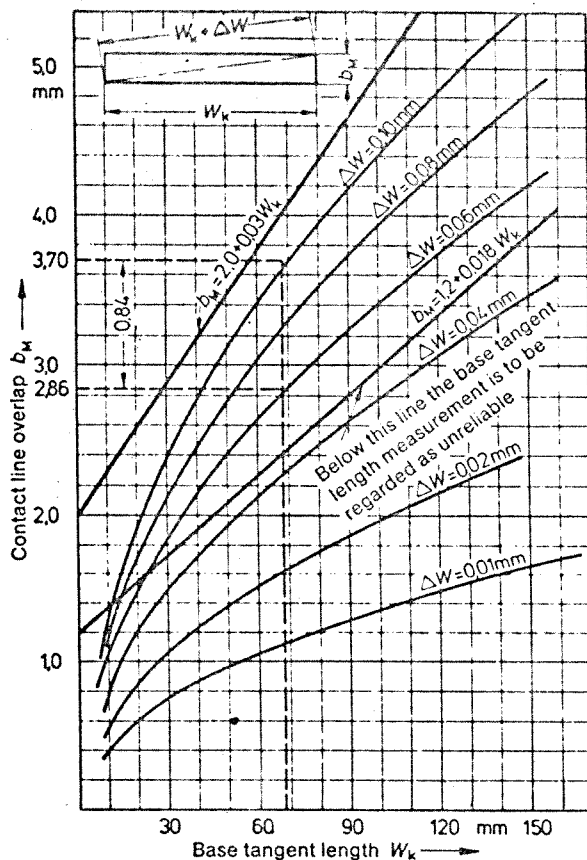
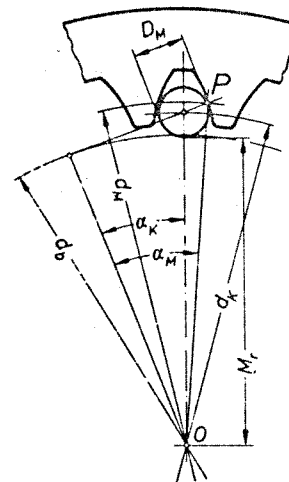


Figure 19. Contact line overlap b_M in base tangent length measurement



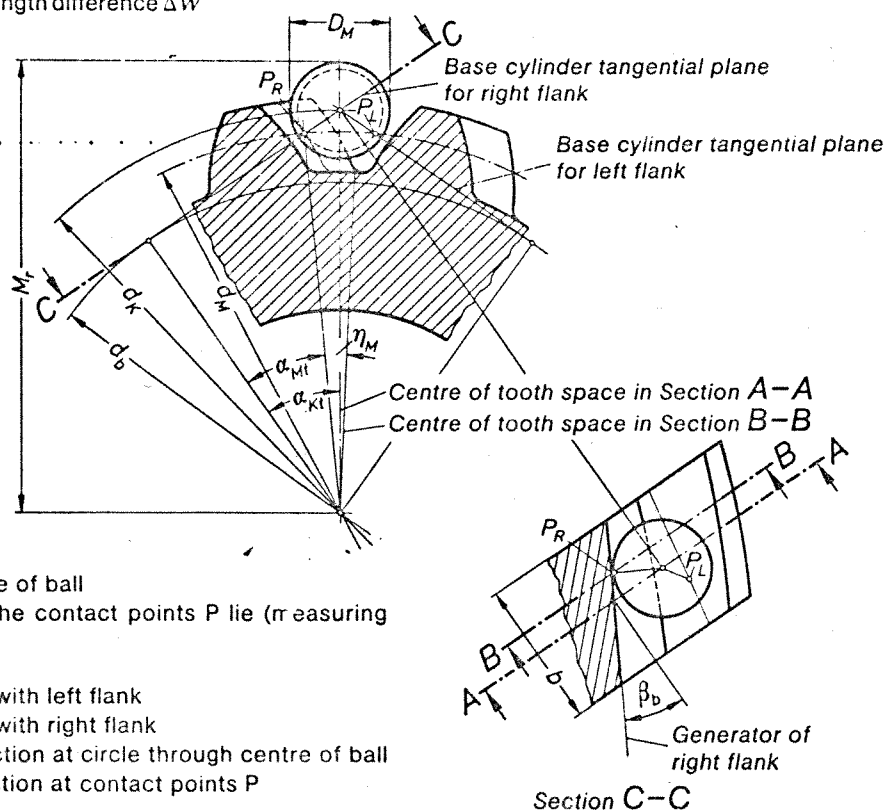
Example: For $W_k = 69,548$ mm and $b_M = 3,7$ mm the diagonal is longer than W_k by about 0,10 mm. If the overlap b_M is reduced, for example by edge chamfering of the flanks and facewidth deviation by 0,84 mm, then the diagonal is still longer than W_k by 0,06 mm. In this case the measurement can still be regarded as adequately reliable.

Figure 20. Relationship between base tangent length W_k , contact line overlap b_M and length difference ΔW



For meaning of symbols see figure 21.

Figure 22. Radial single-ball dimension M_r on an internal spur gear



- d_b base cylinder diameter
- d_k diameter of circle through centre of ball
- d_M diameter of cylinder on which the contact points P lie (measuring circle diameter)
- D_M measuring ball diameter
- P_L contact point of measuring ball with left flank
- P_R contact point of measuring ball with right flank
- α_{Kt} pressure angle in transverse section at circle through centre of ball
- α_{Mt} pressure angle in transverse section at contact points P

Figure 21. Radial single-ball dimension M_r in a helical external gear

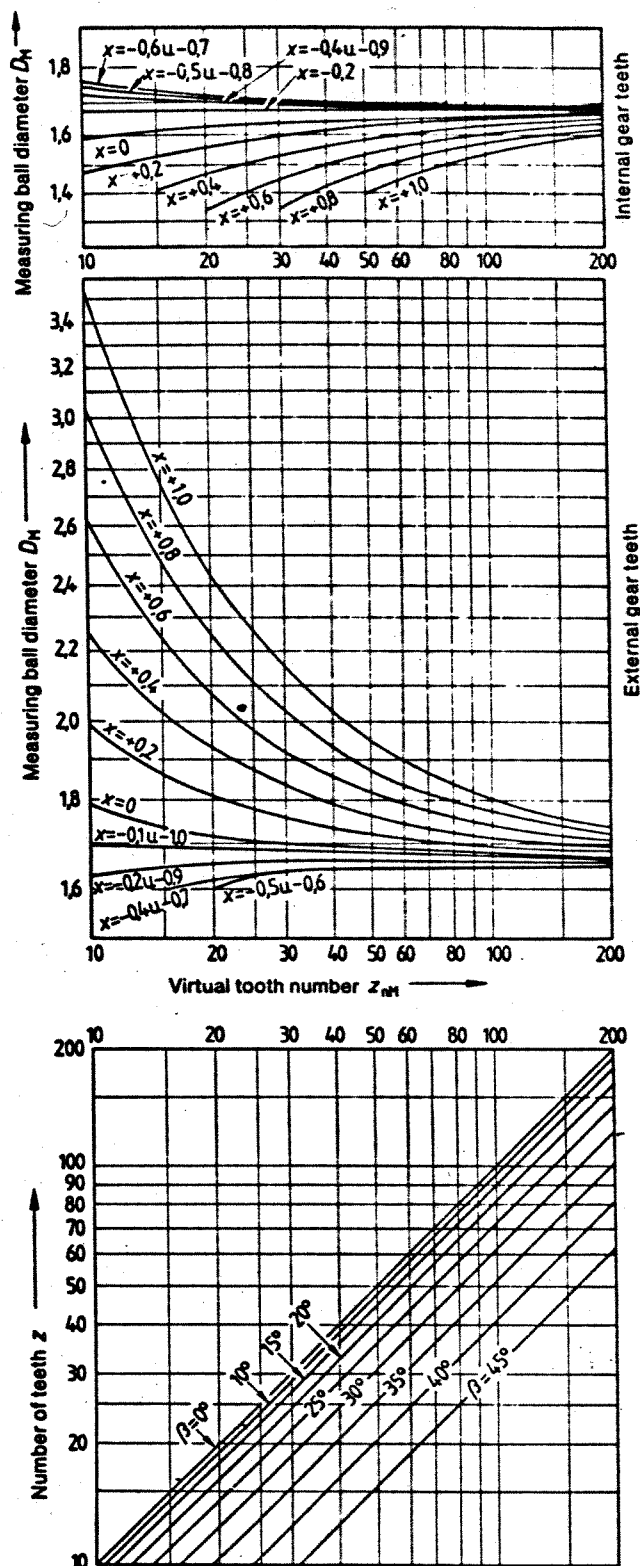


Figure 23. Nomogram for determining the measuring ball diameter D_M for radial single-ball or diametral two-ball measurements for $\alpha_n = 20^\circ$ and $m_n = 1\text{ mm}$
In the case of cylindrical gears with tooth thickness deviations x_E replaces x

2.7.4 Diametral test dimensions for tooth thickness

2.7.4.1 Dimension over balls M_{dK}

In the case of an external gear the dimension over balls M_{dK} is the largest external dimension over two balls whilst in the case of an internal gear it is the smallest internal dimension between two balls having diameter D_M and in contact with the flanks in two tooth spaces at the maximum possible separation from each other on the gear, see figures 24 to 26. The two balls must be in the same plane perpendicular to the gear axis; the measuring element faces (making contact externally or internally) must be held parallel to the gear teeth axis.

Note: The dimension over balls is not referred to the gear axis and hence is independent of any eccentricity of the gear teeth.

For an even number of teeth, see figure 24, the following applies

$$M_{dK} = d_K + D_M, \quad (2.108)$$

for an odd number of teeth, see figure 25 and figure 26, the following applies

$$M_{dK} = d_K \cdot \cos \frac{\pi}{2 \cdot z} + D_M \quad (2.109)$$

For choice or calculation of the measuring ball diameters D_M see subclause 2.7.3.1.

Note: In the case of an internal gear the value found for M_{dK} is negative, see subclause 2.1.

2.7.4.2 Dimension over pins M_{dR}

External gears with spur or helical teeth and internal gears with spur teeth can also be measured with pins instead of with balls. Equations (2.99) to (2.104) and (2.106) to (2.109) apply equally to the dimension over pins M_{dR} .

2.7.4.2.1 Dimension over pins on spur gear teeth

For spur gear teeth equations (2.108) or (2.109) apply. For positioning the measuring pins it is not possible to state a common plane perpendicular to the gear teeth axis. The measuring elements need only be swivelled sideways to find the maximum value; this is the actual size.

2.7.4.2.2 Dimension over pins on external helical gear teeth with even number of teeth

Equation (2.108) applies in this case. The measurement is more straightforward even than for spur gear teeth because the actual size is not a maximum value; instead, as the measuring elements are swivelled in the axial plane the actual size is a minimum to which the measuring pins automatically adjust themselves in all positions owing to the parallel measuring element faces. Swivelling sideways does not alter the measured value (disregarding form deviations on the tooth flanks), see figure 27. This simply moves the measuring pins on the same helical path in the tooth spaces.

Note: In the case of an internal gear pair u is a negative value, see subclause 2.1.

3.2.2 Transmission ratio i

The transmission ratio i of a gear pair is the ratio of the angular speed (rotational speed) of the driving gear (subscript a) to that of the driven gear (subscript b).

$$i = \frac{\omega_a}{\omega_b} = \frac{n_a}{n_b} = -\frac{z_b}{z_a} \quad (3.2)$$

In the case of an external gear pair the two cylindrical gears rotate in opposite directions, i.e. their angular speeds or rotational speeds have opposite signs; the transmission ratio is negative. In the case of an internal gear the two cylindrical gears have the same direction of rotation, i.e. their angular speeds or rotational speeds have the same sign and the transmission ratio is positive.

If it is necessary to make a distinction, ratios such that $|i| > 1$ are said to be "speed reducing ratios" whilst ratios such that $|i| < 1$ are said to be "speed increasing ratios".

3.2.3 Line of centres, centre distance a

In a transverse section the line of centres of a gear pair is the straight line joining the gear centres of the cylindrical gears mated with each other. The centre distance a is the distance between the gear axes.

Note: In the case of an internal gear pair the centre distance is taken as negative, see subclause 2.1.

3.2.4 Pitch cylinders, pitch circles; pitch diameter d_w

In the case of a cylindrical gear pair the pitch cylinders (pitch circles) are those cylinders (circles) about the gear axis which have the same velocities. The pitch circles divide the centre distance in the ratio of the tooth numbers.

The pitch circles established during the operation of a cylindrical gear pair (cylindrical gear pair in a gear unit) are termed working pitch circles. The pitch circles established by a generating cutter during the generating of a tooth system are termed generating pitch circles.

$$\frac{1}{2} \cdot (d_{w1} + d_{w2}) = a \quad (3.3)$$

$$\begin{aligned} d_{w1} &= \frac{2 \cdot z_1}{z_1 + z_2} \cdot a = \frac{2 \cdot a}{u + 1} \\ &= d_1 \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}} = \frac{d_{b1}}{\cos \alpha_{wt}} \end{aligned} \quad (3.4)$$

$$\begin{aligned} d_{w2} &= \frac{2 \cdot z_2}{z_1 + z_2} \cdot a = \frac{2 \cdot a \cdot u}{u + 1} \\ &= d_2 \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}} = \frac{d_{b2}}{\cos \alpha_{wt}} \end{aligned} \quad (3.5)$$

Note: In the case of an internal gear pair d_{w2} is a negative value, see subclause 2.1.

3.2.5 Working transverse pressure angle α_{wt} or α''

The working transverse pressure angle α_{wt} is that pressure angle whose vertex lies on the pitch circle (working pitch circle). It is calculated from

$$\cos \alpha_{wt} = \frac{d_{b1}}{d_{w1}} = \frac{d_{b2}}{d_{w2}} = \frac{(z_1 + z_2) \cdot m_t}{2 \cdot a} \cdot \cos \alpha_t \quad (3.6)$$

or

$$\text{inv } \alpha_{wt} = \text{inv } \alpha_t + 2 \cdot \frac{x_1 + x_2}{z_1 + z_2} \cdot \tan \alpha_n \quad (3.7)$$

The values $\text{inv } \alpha_{wt}$ and $\text{inv } \alpha_t$ can be found from tables of the involute function or according to equation (2.13).

In the double-flank working test with a master gear the working transverse pressure angle is denoted by α'' , see subclause 2.7.5.

3.2.6 Working depth h_w

The working depth h_w of a gear pair is the distance apart of the tip circles of the two cylindrical gears on the line of centres, see figure 30.

$$h_w = \frac{d_{a1} + d_{a2}}{2} - a \quad (3.8)$$

3.2.7 Bottom clearance c , bottom clearance factor c^*

The bottom clearance c is the distance by which the tip circle of a gear is separated from the root circle of its mating gear, see figure 30. It is equal to the difference between the tooth depth h and the working depth h_w .

$$c = h - h_w = c^* \cdot m_n \quad (3.9)$$

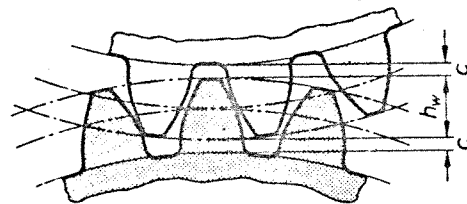


Figure 30. Working depth h_w and bottom clearance c of a gear pair

3.3 Calculation quantities and factors for mating gears

3.3.1 Reference centre distance a_d

The reference centre distance a_d is the sum of the reference circle radii of the two gears.

$$\begin{aligned} a_d &= \frac{d_1 + d_2}{2} = m_t \cdot \frac{z_1 + z_2}{2} \\ &= \frac{m_n \cdot (z_1 + z_2)}{2 \cdot \cos \beta} \end{aligned} \quad (3.10)$$

Note: In the case of an internal gear pair a_d is a negative value, see subclause 2.1.

3.3.2 Centre distance modification $y \cdot m_n$, centre distance modification coefficient y

The algebraic difference between the centre distance a and the reference centre distance a_d is the centre distance modification $y \cdot m_n$ which is expressed by the centre distance modification coefficient y in fractions of the normal module.

$$y \cdot m_n = a - a_d \quad (3.11)$$

$$y = \frac{z_1 + z_2}{2 \cdot \cos \beta} \cdot \left(\frac{\cos \alpha_t}{\cos \alpha_{wt}} - 1 \right) \quad (3.12)$$

3.3.3 Centre distance modification coefficient y and sum of addendum modification coefficients Σx

Between the centre distance modification coefficient y and the sum of the addendum modification coefficients $\Sigma x = x_1 + x_2$ the following relationship exists

$$y \cdot (\operatorname{inv} \alpha_{wt} - \operatorname{inv} \alpha_t) = \Sigma x \cdot \left(\frac{\cos \alpha_t}{\cos \alpha_{wt}} - 1 \right) \cdot \tan \alpha_t \quad (3.13)$$

3.3.4 Calculation of the sum of the addendum modification coefficients Σx

If the centre distance a , the numbers of teeth z_1 and z_2 , the normal module m_n , the pressure angle α_n and the helix angle β are known, then the working transverse pressure angle α_{wt} arising in the mating gears is given by equation (3.6). In accordance with the previous equations of this subclause the sum of the addendum modifications is then found from

$$\Sigma x = \frac{(z_1 + z_2) \cdot (\operatorname{inv} \alpha_{wt} - \operatorname{inv} \alpha_t)}{2 \cdot \tan \alpha_n} \quad (3.14)$$

The way in which $\Sigma x = x_1 + x_2$ is distributed between the two gears is governed by the permissible stressing of the teeth or by other specified dimensions of the gear teeth, e.g. root diameter, see DIN 3992 and DIN 3993.

3.3.5 Calculation of the centre distance a

If the normal module m_n , the pressure angle α_n , the helix angle β , the numbers of teeth z_1 and z_2 and the addendum modification coefficients x_1 and x_2 of the two spur gears are known, then the centre distance of the gear pair is found as

$$a = a_d \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}} = \frac{m_n \cdot (z_1 + z_2)}{2 \cdot \cos \beta} \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}} \quad (3.15)$$

3.3.6 Addendum alteration $k \cdot m_n$, addendum alteration factor k

It is sometimes necessary for the addenda to be altered to suit the mating conditions and the specified minimum bottom clearance. If the bottom clearance c corresponding to the standard basic rack tooth profile of the gears is to be retained then the necessary addendum alteration $k \cdot m_n$ and the addendum alteration factor k , respectively, are found from

$$k \cdot m_n = a - a_d - m_n \cdot \Sigma x \quad (3.16)$$

$$k = y - \Sigma x \quad (3.17)$$

The addendum alterations calculated in this way are obtained with correct sign, that is to say:

negative values in the case of external gear pairs, so that the tip diameters become smaller,

positive values in the case of internal gear pairs, so that the value of the tip diameter of the internal gear becomes smaller whilst the diameter of the pinion tip circle becomes larger.

The calculated values are often so small that they are cancelled out by the deeper feed of the cutting tool which is necessary for producing the backlash and by the negative root diameter deviations additionally produced by standard pinion type cutters, so that the remaining effective bottom clearance is altered to only a slight extent (or within permissible limits).

In the case of internal gear pairs it is to be noted that the addendum alterations which are always positive in this case can usually not be realized because the special engagement and manufacturing conditions of internal gear pairs limit the usable addenda of the internal gear and pinion. For details see DIN 3993.

3.4 Tooth engagement

For the investigation of tooth engagement the working pitch circles and the corresponding transverse profile are the criteria.

3.4.1 Point of contact

In a gear pair a tooth flank and its mating flank contact each other on the contact line, see subclause 2.3.3. A point of contact of a flank profile is the point at which it makes contact with the mating flank in any particular working position.

At each contact point in the transverse section the normal to the point of contact (erected perpendicularly on the flank and passing in all cases through the pitch point C) together with the tangent to the pitch circles in the pitch point makes the working transverse pressure angle α_{wt} . In the case of involute gear teeth the normal to the point of contact and the path of contact coincide, so that the pressure angle is the same for all flank points.

3.4.2 Plane of action, line of action

The plane of action is the geometrical locus of all points of contact of an involute flank with the mating flank of the mating gear. It contacts the base cylinder and hence is parallel to the gear axis and is distant from it by the radius of the base cylinder.

Each gear pair has two planes of action, one for the right flanks and one for the left flanks, see subclause 2.3.3. The two planes intersect in the instantaneous axis which is parallel to the two gear axes and the angle enclosed by the two planes is twice the working transverse pressure angle α_{wt} .

The lines of action of an involute gear tooth system are the intersections of the planes of contact with transverse sections. They are the geometrical locus of all possible points of contact of the flank profiles in the transverse section of a gear pair and at the same time the projection of all possible contact lines of the tooth flanks in the direction of the instantaneous axis.

In the transverse section of the gear teeth, a line of action in the case of external gears is the common internal tangent, and in the case of internal gears the common external tangent, to the base circles of the mating cylindrical gears. It contacts the two base circles at points T_1 and T_2 , see figure 31 and figure 32. In both figures pinion 1 is assumed to be the driver, that is to say the ratio is speed-reducing.

It is necessary to distinguish between the right flank line of action and the left flank line of action.

3.4.2.1 Special points on the line of action

Special points on the line of action are as follows (see figure 31 and figure 32)

A starting point of engagement.

This is the point at which the line of action intersects the tip circle of the driven gear.

B internal individual point of contact on the driving gear, external individual point of contact on the driven gear. This is the point on the line of action which is distant by one normal base pitch from point E.

C pitch point

- D external individual point of contact on the driving gear, internal individual point of contact on the driven gear. This is the point on the line of action which is distant by one normal base pitch from point A.
- E finishing point of engagement. This is the point at which the line of action intersects the tip circle of the driving gear.
- T₁ point of contact between the line of action and the base circle of gear 1.
- T₂ point of contact between the line of action and the base circle of gear 2.

3.4.2.2 Curvature radii of the tooth flanks

The following segments of the line of action give rise to the curvature radii of the tooth flanks which are the criteria for the contact ratio and motion conditions of a gear pair (see figure 33 and figure 34):

$$\begin{aligned}\overline{T_1C} &= \rho_{C1} = \frac{1}{2} \cdot \sqrt{d_{a1}^2 - d_{b1}^2} \\ &= \frac{1}{2} \cdot d_{b1} \cdot \tan \alpha_{wt}\end{aligned}\quad (3.18)$$

$$\overline{T_2A} = \rho_{A2} = \frac{1}{2} \cdot \frac{z_2}{|z_2|} \cdot \sqrt{d_{a2}^2 - d_{b2}^2} \quad (3.19)$$

$$\overline{T_1E} = \rho_{E1} = \frac{1}{2} \cdot \sqrt{d_{a1}^2 - d_{b1}^2} \quad (3.20)$$

$$\overline{T_1B} = \rho_{B1} = \rho_{E1} - p_{ct} \quad (3.21)$$

$$\overline{T_2D} = \rho_{D2} = \rho_{A2} - p_{ct} \quad (3.22)$$

$$\begin{aligned}\overline{T_1T_2} &= \rho_{C1} + \rho_{C2} = a \cdot \sin \alpha_{wt} \\ &= \rho_{A1} + \rho_{A2} = \rho_{E1} + \rho_{E2}\end{aligned}\quad (3.23)$$

Equations (3.19) to (3.22) apply to the case when gear 1 is the driving gear and gear 2 the driven gear. In the opposite case A and E as well as B and D are to be interchanged in figures 31 to 34 and in equations (3.19) to (3.22).

Note: The values obtained for the curvature radii of an internal gear and for the segment $\overline{T_1T_2}$ of an internal gear pair are negative, see subclauses 2.1 and 2.5.7.

3.4.3 Zone of action, length of path of contact

The zone of action (length of path of contact in the transverse section) is that part of the plane of action (line of action in the transverse section) which comes into use during the operation of a gear pair. In the case of speed-reducing ratios the length of the path of contact is terminated at its starting point A by the tip circle of gear 2 and at its finishing point E by the tip circle of gear 1, see figure 31 to figure 34.

In the extreme case the length of the path of contact may extend as far as the base circles. If the tip circle of the generating cutter cuts the line of action outside the points T₁ or T₂ the result is undercut and the length of the path of contact is shortened accordingly. If the tip circle of the mating gear cuts the line of action outside the segment $\overline{T_1T_2}$ in the case of external gear teeth, or inside the segment $\overline{T_1T_2}$ in the case of internal gear teeth, then meshing difficulties will arise.

The length g_a of the path of contact of two mating cylindrical gears is given by:

$$g_a = \frac{1}{2} \left[\sqrt{d_{a1}^2 - d_{b1}^2} + \frac{z_2}{|z_2|} \cdot \sqrt{d_{a2}^2 - d_{b2}^2} - (d_{b1} + d_{b2}) \cdot \tan \alpha_{wt} \right] \quad (3.24)$$

Note: z_2 , d_{a2} and d_{b2} are negative in the case of an internal gear, see subclause 2.1.

The length of the path of contact when a cylindrical gear (subscript 1) is mated with a rack (standard basic rack tooth profile according to DIN 867) is given by

$$g_a = \frac{1}{2} \left(\sqrt{d_{a1}^2 - d_{b1}^2} - d_{b1} \cdot \tan \alpha_t \right) + \frac{m_n \cdot (1 - x_1)}{\sin \alpha_t} \quad (3.25)$$

The length of the path of contact g_a is divided by the pitch point C into the length of approach path g_f (portion of the length of path of contact at the root flank of the driving gear between the tip circle of the driven gear and the pitch point) and the length of recess path g_a (portion of the length of path of contact at the tip flank of the driving gear between the pitch point and the tip circle of the driving gear, see figure 31 and figure 32. These portions of the length of path of contact are also termed the tip length of the path of contact and root length of the path of contact of the gear.

For the case where gear 1 is the driving gear and gear 2 the driven gear, the following applies:

The length of approach path is equal to the length of dedendum path of contact of gear 1 which is equal to the length of addendum path of contact of gear 2

$$\begin{aligned}g_f &= \overline{AC} = \rho_{A2} - \rho_{C2} \\ &= \frac{1}{2} \cdot \left(\frac{z_2}{|z_2|} \cdot \sqrt{d_{a2}^2 - d_{b2}^2} - d_{b2} \cdot \tan \alpha_{wt} \right)\end{aligned}\quad (3.26)$$

The length of recess path is equal to the length of addendum path of contact of gear 1 which is equal to the length of dedendum path of contact of gear 2

$$\begin{aligned}g_a &= \overline{CE} = \rho_{E1} - \rho_{C1} \\ &= \frac{1}{2} \cdot \left(\sqrt{d_{a1}^2 - d_{b1}^2} - d_{b1} \cdot \tan \alpha_{wt} \right)\end{aligned}\quad (3.27)$$

For the opposite case (gear 2 driving, gear 1 driven) g_a and g_f as well as the subscripts A and E for the curvature radii are to be interchanged in equations (3.26) and (3.27) (see subclause 3.4.2.2).

Note: In the case of an internal gear the number of teeth, the diameters and the curvature radii are negative values, see subclauses 2.1 and 2.5.7.

Note: The length of the path of contact can be shortened by tooth tip chamfering or radiusing or by tip relief. In such a case the usable tip diameter d_{Na1} or d_{Na2} obtained from the production specifications is to be substituted for d_{a1} and d_{a2} , respectively, in equations (3.19) and (3.20) as well as in (3.24) to (3.27), see subclause 3.4.4.

3.4.4 Usable diameter d_N : active and usable ranges of the tooth flanks

The usable circles enclose the active ranges of the tooth flanks which are effective when the gears are operating, see figure 31 and figure 32. The starting point A of the path

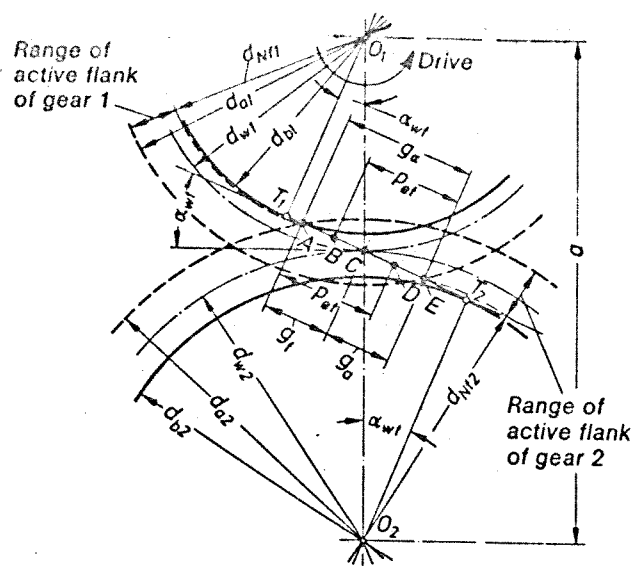


Figure 31. Length of path of contact and active flank ranges of an external gear pair with speed-reducing ratio

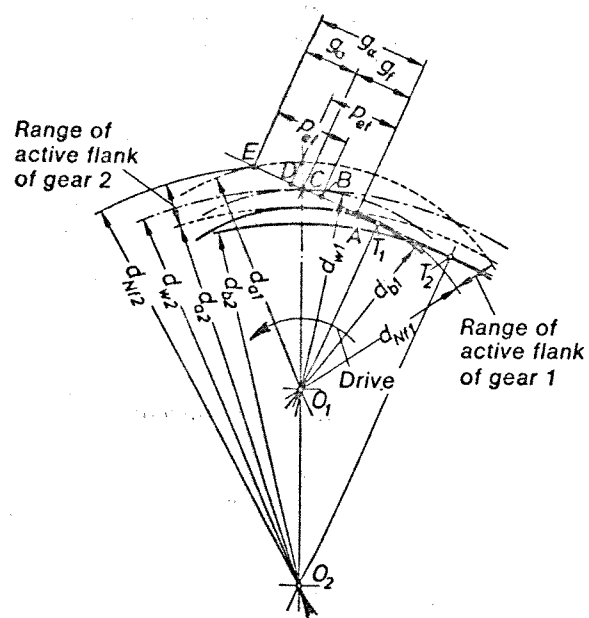


Figure 32. Length of path of contact and active flank range of an internal gear pair with speed-reducing ratio

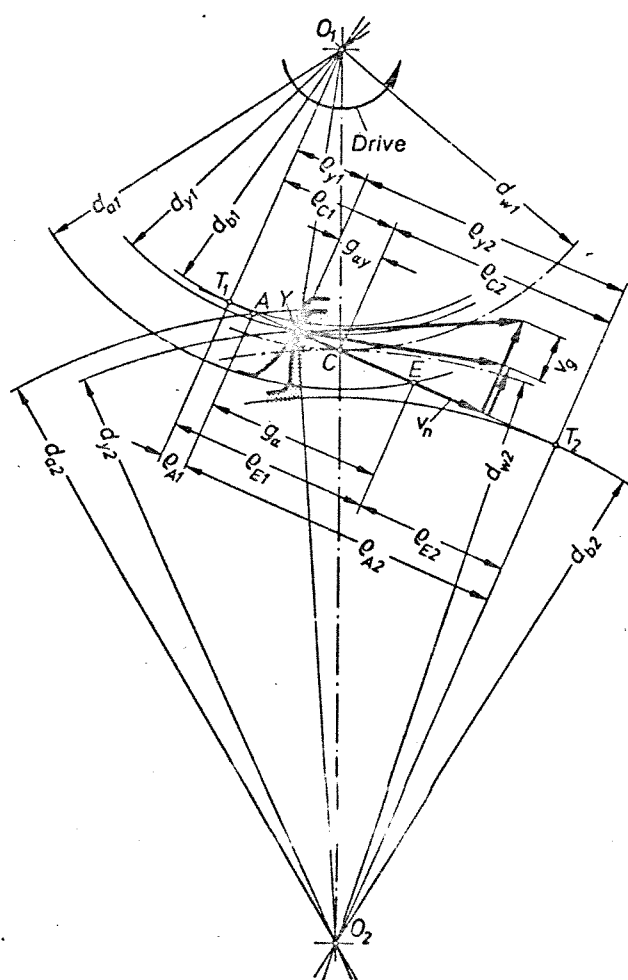


Figure 33. Curvature radii ρ of tooth flanks and sliding speed v_g at point of contact Y of an external gear pair with speed-reducing ratio

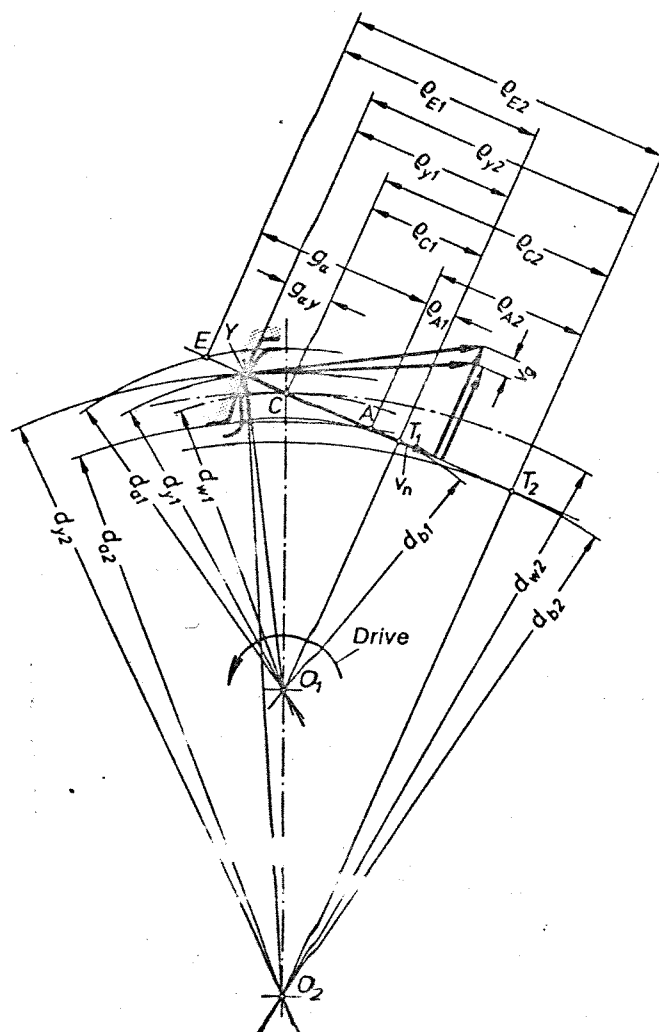


Figure 34. Curvature radii ρ of tooth flanks and sliding speed v_g at point of contact Y of an internal gear pair with speed-reducing ratio

of contact determines the usable root circle of gear 1 with the usable root diameter d_{Nf1} , whilst the finishing point E of the path of contact determines the usable root circle of gear 2 with the usable root diameter d_{Nf2} . The usable tip diameters d_{Na1} and d_{Na2} are governed by the production specifications of the gear and in general are practically equal to the tip diameters d_{a1} and d_{a2} , respectively. For example, with tooth tip radiusing ρ_{an} in the normal section the following applies

$$d_{Na} \approx d_a - 2 \cdot \rho_{an} \cdot (1 - \sin \alpha_{an}) \quad (3.28)$$

Hence for a gear pair

$$d_{Nf1} = \sqrt{\left(2 \cdot a \cdot \sin \alpha_{wt} - \frac{z_2}{|z_2|} \cdot \sqrt{d_{Na2}^2 - d_{b2}^2}\right)^2 + d_{b1}^2} \quad (3.29)$$

$$d_{Nf2} = \frac{z_2}{|z_2|} \times \sqrt{\left(2 \cdot a \cdot \sin \alpha_{wt} - \sqrt{d_{Na1}^2 - d_{b1}^2}\right)^2 + d_{b2}^2} \quad (3.30)$$

Note: For an internal gear pair the centre distance a and the diameters of the internal gear are negative, see sub-clause 2.1.

For special cases arising with internal gear pairs see DIN 3993.

During the production of a gear tooth system by the generating method the usable circles corresponding to the transverse pressure angle at generation and the cutter tip diameter come into action. These usable circles enclose the usable ranges of tooth flanks.

During tooth production on an external gear with a rack cutter or a hobbing cutter without protuberance (addendum h_{a0} , tip rounding radius ρ_{a0}), the usable root diameter d_{NfE} on the generated cylindrical gear is as follows if undercutting does not occur:

$$d_{NfE} = \sqrt{[d \cdot \sin \alpha_t - 2 \cdot (h_{a0} - x_E \cdot m_n - \rho_{a0} \cdot (1 - \sin \alpha_n)) / \sin \alpha_t]^2 + d_b^2} \quad (3.31)$$

The addendum modification factor x_E at generation must be determined taking into account the tooth thickness deviation A_s and a grinding or shaving allowance, if required. In case of an allowance of this kind d_{NfE} is the diameter usable during preliminary treatment.

During shaping the usable diameter d_{NfE} is found according to equation (3.30), the corresponding quantities of the pinion type cutter with subscript 0 replacing the quantities with subscript 1. Furthermore, the relationship specified in DIN 3993 must be observed in the case of internal gears.

3.4.5 Transverse angle of transmission φ_α , transverse contact ratio ε_α

The transverse angle of transmission φ_α of a gear is the centre angle through which it rotates with its mating profile from start to finish of engagement of a flank profile, see figure 35. The transverse angle of transmission of pinion and gear is given as follows:

$$\varphi_{\alpha 1} = \frac{g_\alpha}{r_{b1}} = u \cdot \varphi_{\alpha 2} \quad (3.32)$$

$$\varphi_{\alpha 2} = \frac{g_\alpha}{r_{b2}} = \frac{\varphi_{\alpha 1}}{u} \quad (3.33)$$

The transverse contact ratio ε_α is the ratio of the transverse angle of transmission φ_α to the angular pitch τ or the ratio of the length of path of contact to the transverse normal base pitch.

$$\varepsilon_\alpha = \frac{\varphi_{\alpha 1}}{\tau_1} = \frac{\varphi_{\alpha 2}}{\tau_2} = \frac{g_\alpha}{p_{\alpha t}} = \frac{g_t + g_a}{p_{\alpha t}} \quad (3.34)$$

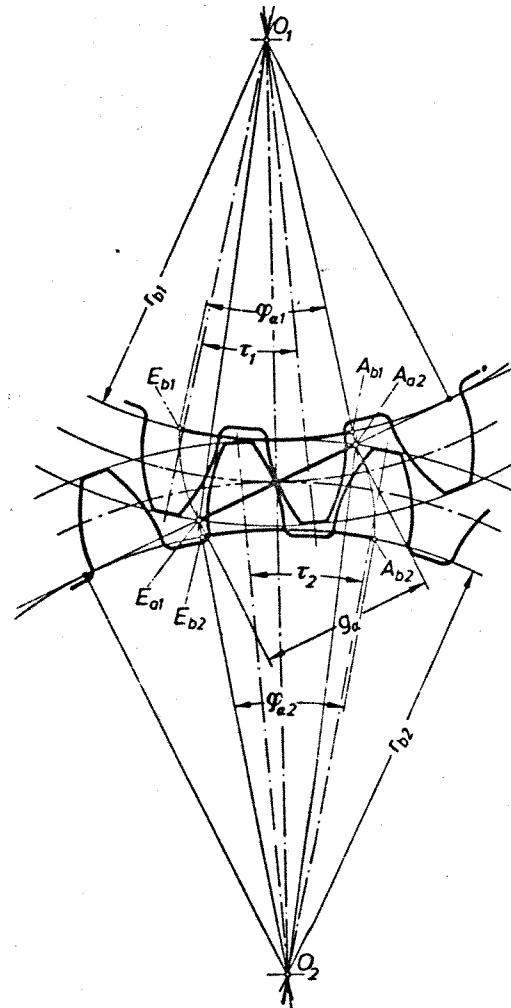


Figure 35. Transverse angle of transmission φ_α of an external gear pair

3.4.6 Overlap angle φ_β , overlap ratio ε_β

The overlap angle φ_β is the angle between the two axial planes enclosing the end points of a tooth trace, see figure 36.

$$\varphi_{\beta 1} = \frac{b \cdot \tan |\beta|}{r_1} = \frac{2 \cdot b \cdot \sin |\beta|}{m_n \cdot z_1} = u \cdot \varphi_{\beta 2} \quad (3.35)$$

$$\varphi_{\beta 2} = \frac{b \cdot \tan |\beta|}{r_2} = \frac{2 \cdot b \cdot \sin |\beta|}{m_n \cdot z_2} = \frac{\varphi_{\beta 1}}{u} \quad (3.36)$$

The overlap ratio ε_β is the ratio of the overlap angle φ_β to the angular pitch τ or the ratio of the facewidth b to the axial pitch p_x .

$$\varepsilon_\beta = \frac{\varphi_{\beta 1}}{\tau_1} = \frac{\varphi_{\beta 2}}{\tau_2} = \frac{b}{p_x} = \frac{b \cdot \sin |\beta|}{m_n \cdot \pi} \quad (3.37)$$

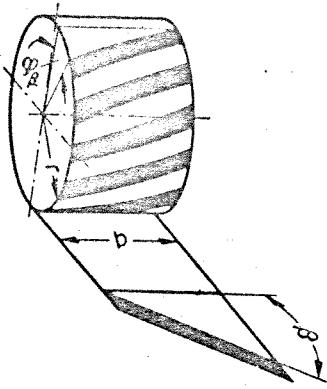


Figure 36. Overlap angle φ_β of a cylindrical gear

3.4.7 Overlap length g_β

The overlap length g_β of a helical gear is the reference circle arc belonging to the overlap angle φ_β .

$$g_\beta = r \cdot \varphi_\beta = b \cdot \tan |\beta| \quad (3.38)$$

3.4.8 Total angle of transmission φ_γ , total contact ratio ε_γ

The total angle of transmission φ_γ is the angle at the centre of a gear through which it rotates from start to finish of contact of one of its flanks with the flank mated with it. It is equal to the sum of the transverse angle of transmission and the overlap angle.

$$\varphi_{\gamma 1} = \varphi_{\alpha 1} + \varphi_{\beta 1} = u \cdot \varphi_{\gamma 2} \quad (3.39)$$

$$\varphi_{\gamma 2} = \varphi_{\alpha 2} + \varphi_{\beta 2} = \frac{\varphi_{\gamma 1}}{u} \quad (3.40)$$

The total contact ratio ε_γ is the ratio of the total angle of transmission to the angular pitch. It is equal to the sum of the transverse contact ratio and the overlap ratio.

$$\varepsilon_\gamma = \frac{\varphi_{\gamma 1}}{\tau_1} = \frac{\varphi_{\gamma 2}}{\tau_2} = \varepsilon_\alpha + \varepsilon_\beta \quad (3.41)$$

3.5 Sliding conditions at the tooth flanks

3.5.1 Sliding speed v_g

At a point of contact of two cylindrical gears in engagement the sliding speed v_g is the difference of the speeds of the two transverse profiles prevailing in the direction of the common tangent.

At the point of contact Y, see figure 33 and figure 34, the two transverse profiles have the normal speed $v_n = \frac{1}{2} \cdot \omega_1 \cdot d_{b1}$. Because of the similarity of the speed triangles with the triangles YO_1T_1 and YO_2T_2 the sliding speed is found in conjunction with the curvature radii ϱ_{y1} and ϱ_{y2} (see equation (2.12)) as

$$v_g = \pm \omega_1 \cdot \left(\frac{\varrho_{y2}}{u} - \varrho_{y1} \right) \quad (3.42)$$

According to figure 33 and figure 34 the distance g_{ay} between Y and C is

$$g_{ay} = \pm (\varrho_{c1} - \varrho_{y1}) = \mp (\varrho_{c2} - \varrho_{y2}) \quad (3.43)$$

hence

$$v_g = \pm \omega_1 \cdot g_{ay} \cdot \left(1 + \frac{1}{u} \right) \quad (3.44)$$

Note: g_{ay} always counts as positive. Since u is positive for an external gear pair and negative for an internal gear pair it usually follows that the sliding speed is greater for external gear teeth than for internal gear teeth.

The sliding speed is proportional to the distance g_{ay} and at the pitch point is equal to zero. It reaches its maximum values at the point of root or tip contact,

$$v_{gf} = \pm \omega_1 \cdot g_f \cdot \left(1 + \frac{1}{u} \right) \quad (3.45)$$

$$v_{ga} = \pm \omega_1 \cdot g_a \cdot \left(1 + \frac{1}{u} \right) \quad (3.46)$$

g_f and g_a being found according to equations (3.26) and (3.27), respectively.

3.5.2 Sliding factor K_g

The sliding factor K_g is the ratio of the sliding speed v_g to the velocity v_t of the pitch circles:

$$K_g = \frac{v_g}{v_t} = \frac{2 \cdot g_{ay}}{d_{w1}} \cdot \left(1 + \frac{1}{u} \right) \quad (3.47)$$

The maximum values for K_g are attained at the end points A and E of the path of contact

$$\text{at A: } K_{gf} = \frac{2 \cdot g_f}{d_{w1}} \cdot \left(1 + \frac{1}{u} \right) \quad (3.48)$$

$$\text{at E: } K_{ga} = \frac{2 \cdot g_a}{d_{w1}} \cdot \left(1 + \frac{1}{u} \right) \quad (3.49)$$

Note: In the case of an internal gear pair u is taken as negative, see subclause 3.2.1.

The variation of the sliding factor along the line of action is shown in figure 37 for an external gear pair and in figure 38 for an internal gear pair, taking $u = \pm 2$ as the example. For judging addendum modifications the lines K_{g1} and K_{g2} are continued beyond points A and E.

3.5.3 Specific sliding ζ

The specific sliding ζ is the ratio of the sliding speed to the speed of a transverse profile in the direction of the tangent to the profile, see figure 33 and figure 34.

Equation (3.42) yields

$$\zeta_1 = 1 - \frac{\varrho_{y2}}{u \cdot \varrho_{y1}} \quad (3.50)$$

$$\zeta_2 = 1 - \frac{u \cdot \varrho_{y1}}{\varrho_{y2}} \quad (3.51)$$

The maximum values of ζ are reached at the end points A and E of the path of contact

$$\text{at A: } \zeta_{f1} = 1 - \frac{\varrho_{A2}}{u \cdot \varrho_{A1}} \quad (3.52)$$

$$\text{at E: } \zeta_{f2} = 1 - \frac{u \cdot \varrho_{E1}}{\varrho_{E2}} \quad (3.53)$$

using the curvature radii ϱ_A and ϱ_E according to subclause 3.4.2.2.

Note: In the case of an internal gear pair u is taken as negative, see subclause 3.2.1.

In the variation of the specific sliding along the path of contact is shown in figure 37 for an external gear pair and in figure 38 for an internal gear pair using the example of $u = \pm 2$. For judging addendum modifications the curves of specific sliding are continued beyond points A and E.

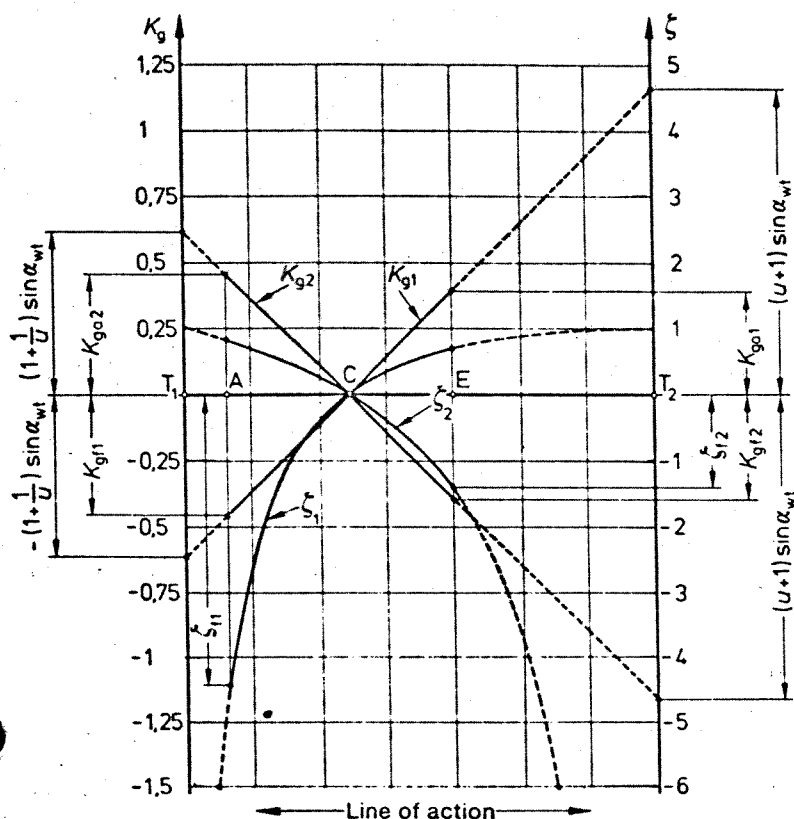


Figure 37. Sliding factor K_g and specific sliding ζ as a function of the normal base pitch for an external gear pair with $u = +2$

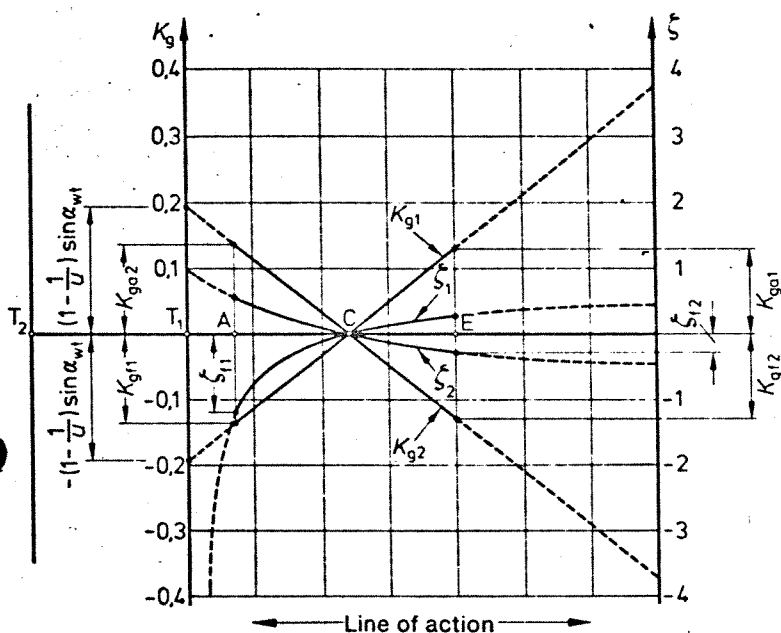


Figure 38. Sliding factor K_g and specific sliding ζ as a function of the normal base pitch for an internal gear pair with $u = -2$

4 Deviations and tolerances for cylindrical gear teeth

For the mating of two gears it is necessary to have deviations from the zero-play condition in order to achieve the necessary backlash. The minimum deviations depend on the backlash-reducing effects of the gear teeth and of the housing and as well on the operating conditions and the influence exerted by the design, whilst the maximum deviations depend additionally on the gearcutting tolerances. These influences are mostly independent of the helix angle. Therefore the deviations and tolerances on tooth thickness

and its test dimensions are referred to the normal section (see DIN 3967), likewise the change factors.

All definitions and equations in this clause hold good for zero-deviation gears.

4.1 Deviations A of tooth thickness and its test dimensions

4.1.1 Deviations of tooth thickness A_s

To achieve backlash the teeth of external gears and of internal gears must have negative tooth thickness deviations. The tooth thickness deviation A_s is the difference

between an actual size and the nominal size of the normal tooth thickness s_n on the reference cylinder. The largest permissible tooth thickness of a gear is given by the upper tooth thickness A_{se} and the smallest permissible tooth thickness by the lower tooth thickness deviation A_{si} .

In the case of cylindrical gears with addendum modification measurements near the middle of the tooth depth (i.e. near the V-cylinder) are to be preferred to measurements on the reference cylinder. The tooth thickness deviations on a Y-cylinder are found from

$$A_{sy} = A_s \cdot \frac{d_y \cdot \cos \beta_y}{d \cdot \cos \beta} \approx A_s \cdot \left(1 + 2 \cdot \frac{x}{z_{nW}} \right), \quad (4.1)$$

the last part of this equation being applicable to the tooth thickness deviations on the V-cylinder ($d_y = d_v$).

For calculations in the transverse section the tooth thickness deviations A_s have to be converted into the corresponding transverse section deviations A_{st} :

$$A_{st} = \frac{A_s}{\cos \beta} \quad (4.2)$$

For calculating the deviations according to subclause 4.1.3 to 4.1.7 it is advisable to calculate first with the average tooth thickness deviation $A_{sm} = \frac{1}{2} \cdot (A_{se} + A_{si})$ and then to determine the limits with the aid of the change factors according to clause 5.

4.1.2 Deviations $A_{\bar{s}}$ of the normal chordal tooth thicknesses

The deviation $A_{\bar{s}}$ is the difference between an actual size and the nominal size of the normal chordal tooth thickness $\bar{s}_n$. For the normal chordal tooth thicknesses $\bar{s}_{yn}$ at diameter d_y the deviations alter accordingly, see subclause 4.1.1.

The deviations $A_{\bar{s}}$ and $A_{\bar{s}y}$ are generally only insignificantly different from the deviations A_s and A_{sy} , respectively, of the tooth thicknesses. They can be equated with them except for very small numbers of teeth, see subclause 5.1.

4.1.3 Base tangent length deviations A_W

The base tangent length deviation A_W is the difference between an actual size and the nominal size of the base tangent length W_k . In the case of external gears negative deviations make the actual size of the base tangent length smaller than the nominal size for zero-backlash engagement, whilst in the case of internal gears negative deviations make the absolute value of the actual size larger than the absolute value of the nominal size.

The upper base tangent length deviation is denoted by A_{We} and the lower by A_{Wi} .

It is convenient first to calculate the mean base tangent length according to equation (2.96) using for x the mean generating addendum modification coefficient x_{Em} which is obtained from equation (2.114) with the mean tooth thickness deviation A_{sm} . Then by using the change factor according to subclause 5.2 the limits are found.

4.1.4 Deviations A_{Md} of dimension over balls or pins

The deviation A_{Md} of the dimension over balls or pins M_d of tooth thickness is the difference between an actual size and the nominal size of the dimension over balls or pins. In the case of external gears negative deviations make the actual size smaller than the nominal size for zero-backlash engagement, whilst in the case of internal gears negative deviations make the absolute value of the actual size larger than the absolute value of the nominal size. The permissible

maximum value is denoted by A_{Mde} and the permissible minimum value by A_{Mdi} .

The relationship between an actual size of the dimension over balls or pins and the corresponding tooth thickness deviation is found from equations (2.104) and (2.108) or (2.109) on substituting in equation (2.103) the value

$$\eta = \frac{e_t - A_{st}}{d} = \frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} - \frac{A_s}{z \cdot m_n} \quad (4.3)$$

for the spacewidth half angle η .

It is convenient to calculate first the mean dimension over balls or pins according to equation (2.108) or (2.109), the value of η for equation (2.103) being calculated according to equation (4.3) using the mean tooth thickness deviation A_{stm} (from equation (4.2) with A_{sm}). The limits are then found with the aid of the change factors according to subclause 5.3.

4.1.5 Deviations A_{Mr} of radial single-ball or single-pin dimension

For the deviations A_{Mr} the definitions corresponding to subclause 4.1.4 apply.

$$A_{Mr} \approx \frac{1}{2} \cdot \frac{\cos \alpha_t}{\sin \alpha_{Kt}} \cdot \frac{A_s}{\cos \beta} \quad (4.4)$$

4.1.6 Deviations $A_{a''}$ of centre distance for double-flank engagement a'' with master gear

Deviation $A_{a''}$ of the centre distance a'' for double-flank engagement with a master gear is the difference between an actual size and the nominal size of the centre distance a'' , see subclause 2.7.5. If in equation (2.113) the upper tooth thickness deviation A_{se} is substituted first, and following this the lower tooth thickness deviation A_{si} , then from equations (2.111) and (2.112) the permissible limits for the double-flank engagement a'' and hence the upper deviation $A_{a''e}$ and the lower deviation $A_{a''i}$ can be found.

It is convenient first to calculate the mean centre distance for double-flank engagement according to equation (2.111), the mean value a''_m being calculated according to equation (2.112) using the mean working addendum modification coefficient x'_{lm} (from equation (2.113) with A_{sm}). The limits are then found with the aid of the change factor according to subclause 5.5.

4.1.7 Tip diameter deviation A_{da} with overcut cylindrical gears

The tip diameter deviation A_{da} with overcut cylindrical gears is the difference between the actual size (measured value d_{aM}) and the nominal size of the tip diameter, see subclause 2.7.6.

The upper tip diameter deviation is denoted by A_{dae} and the lower deviation by A_{dai} . The permissible limits are found according to equations (2.114) and (2.115) by substituting the upper or lower deviations A_{se} or A_{si} respectively.

It is convenient first to calculate the mean tip diameter according to equation (2.115), for which purpose the value substituted for x_E is the mean generating addendum modification coefficient which is found from equation (2.114) using the mean tooth thickness deviation A_{sm} . The limits are then found with the aid of the change factor according to subclause 5.6.

4.2 Tolerances T

The tolerance is the difference between the permissible maximum value (upper deviation) and the permissible minimum value (lower deviation) of a quantity. The deviations A_e and A_i referred to in this subclause denote the permissible deviations.

Corresponding to the quantities considered in subclause 4.1, the tolerances are distinguished according to

subclauses 4.2.1 to 4.2.7 and for this purpose the appropriate quantities are to be inserted in the general definition. The range of tooth thickness errors and the deviations of other gear quantities (e.g. radial run-out) affect the various test dimensions for tooth thickness differently. Hence the conversion of a tooth thickness tolerance according to subclauses 4.1.3 to 4.1.7 into the corresponding tolerance on a test dimension for tooth thickness does not automatically yield the correct values. Instead, additional correction values arising from the characteristic features of the various measuring methods have to be considered. For details see DIN 3967.

4.2.1 Tooth thickness tolerance T_s

$$T_s = A_{sc} - A_{si} \quad (4.5)$$

4.2.2 Tolerance $T_{\bar{s}}$ on normal chordal tooth thickness

$$T_{\bar{s}} = A_{\bar{s}c} - A_{\bar{s}i} \quad (4.6)$$

4.2.3 Base tangent length tolerance T_W

$$T_W = A_{Wc} - A_{Wi} \quad (4.7)$$

4.2.4 Tolerance T_{M_d} on dimension over balls or pins

$$T_{M_d} = A_{M_dc} - A_{M_di} \quad (4.8)$$

4.2.5 Tolerance T_{M_r} on radial single-ball or single-pin dimension

$$T_{M_r} = A_{M_rc} - A_{M_ri} \quad (4.9)$$

4.2.6 Tolerance $T_{a''}$ on centre distance for double-flank engagement with master gear

$$T_{a''} = A_{a''c} - A_{a''i} \quad (4.10)$$

4.2.7 Tip diameter tolerance T_{da} with overcut cylindrical gears

$$T_{da} = A_{dac} - A_{dai} \quad (4.11)$$

5 Change factors A^*

For use in gear production it has been found expedient to calculate the ratios between the deviations $A_{\bar{s}}$, A_W , A_{M_d} or A_{M_r} , respectively, $A_{a''}$ and A_{da} on the one hand, and the tooth thickness deviation A_s on the other and to define them notionally as change factors. With such change factors it is easy and convenient to compare also the deviation of $\bar{s}_n$, W_k , M_d or M_r and a'' with one another. The change factors are referred to the normal section because the deviations and tolerances also are defined for the normal section, see DIN 3967.

The deviation A mentioned in this subclause denote the permissible deviations.

5.1 Change factor $A_{\bar{s}}^*$ of normal chordal tooth thickness

The change factor $A_{\bar{s}}^*$ for the normal chordal tooth thickness $\bar{s}_v$ existing on the V-cylinder is given by

$$A_{\bar{s}}^* = \frac{A_{\bar{s}v}}{A_s} \approx \frac{d_v \cdot \cos \beta_v}{d \cdot \cos \beta} \cdot \left(\cos \psi_v - \sin \psi_v \cdot \tan(\alpha_{vt} - \psi_v) \right) \quad (5.1)$$

where β_v is to be calculated according to equation (2.42), α_{vt} according to equation (2.43) and ψ_v according to equation (2.55).

Note: The change factor $A_{\bar{s}}^*$ is only significant for small numbers of teeth and is normally put equal to 1.

5.2 Change factor A_W^* of base tangent length

The change factor A_W^* of the base tangent length W_k is

$$A_W^* = \frac{A_W}{A_s} = \cos \alpha_n \quad (5.2)$$

5.3 Change factor $A_{M_d}^*$ of dimension over balls or pins

The change factor $A_{M_d}^*$ of the diametral dimension M_d is given by the following for even numbers of teeth and for odd numbers of teeth in the case of external helical gears when measuring with pins

$$A_{M_d}^* = \frac{A_{M_d}}{A_s} \approx \frac{\cos \alpha_t}{\sin \alpha_{kt} \cdot \cos \beta} \quad (5.3)$$

and for odd numbers of teeth (for helical gear teeth however only when measuring with balls) by

$$A_{M_d}^* = \frac{A_{M_d}}{A_s} \approx \frac{\cos \alpha_t}{\sin \alpha_{kt} \cdot \cos \beta} \cdot \cos \frac{\pi}{2 \cdot z} \quad (5.4)$$

This approximation formula becomes more and more inaccurate as the deviation becomes larger. For small deviations of the gear teeth from the zero-play condition however it is sufficiently exact. Where relatively large deviations are concerned it is appropriate to calculate the change factor from a tooth thickness corresponding to the mean deviation of the tolerance zone.

5.4 Change factor $A_{M_r}^*$ of radial single-ball or single-pin dimension

The change factor $A_{M_r}^*$ of the radial single-ball or single-pin dimension M_r is half the size of the change factor according to equation (5.3) for both even and odd numbers of teeth.

5.5 Change factor $A_{a''}^*$ of centre distance for double-flank engagement a'' with master gear

The change factor $A_{a''}^*$ of the centre distance for double-flank engagement a'' with master gear is

$$A_{a''}^* = \frac{A_{a''}}{A_s} \approx \frac{1}{2} \cdot \frac{\cos \alpha_t}{\sin a'' \cdot \cos \beta} \quad (5.5)$$

This approximation formula becomes more and more inaccurate as the deviation becomes larger. For small deviations of the gear teeth from the zero-play condition however it is sufficiently exact. Where relatively large deviations are concerned it is appropriate to calculate the change factor from a tooth thickness corresponding to the mean deviation of the tolerance zone. The angle a'' is the working transverse pressure angle when engaging with the master gear, see subclause 3.2.5.

5.6 Change factor A_{da}^* for tip diameter of overcut cylindrical gears

The change factor for overcut gears made by a zero-deviation and zero addendum modification cutter (e.g. a hob) is

$$A_{da}^* = \frac{A_{da}}{A_s} = \cot \alpha_n \quad (5.6)$$

In the case of cylindrical gears of the overcut type produced with a pinion type cutter it is generally not possible to convert with the constant change factor according to equation (5.6). With gear teeth of this kind it is necessary to take account of the total number of teeth and the sum of the addendum modification coefficients of the generating gear pair, see subclause 2.7.6.

6 Deviations of individual parameters of cylindrical gear teeth

The definitions of this clause are concerned with the deviations of individual parameters of cylindrical gear teeth which are referred to the zero-deviation reference cylinder, see subclause 2.2.7.

Individual deviations f are those deviations from their nominal values which are exhibited by the various parameters of the gear teeth, such as pitch, profile shape, base diameter, pressure angle, tooth traces and helix angle. A total deviation F contains the joint effects of several individual deviations.

The permissible maximum values of the individual deviations, see DIN 3961 to DIN 3964, are denoted where necessary by the additional subscript „per“.

For general information on deviations, see DIN 7182 Part 1.

6.1 Circular pitch deviations

Circular pitch deviations, or pitch deviations in brief, are measured on the reference circle or on a measuring circle as close as possible to the reference circle and concentric with the gear axis. The difference between the measuring circle diameter d_M and the reference diameter d affects the measured values of the deviations by the factor d_M/d ; generally speaking these effects are negligibly small.

The measured values are also influenced by the effects of any eccentricity of the gear teeth and by profile deviations.

For a graphical representation of circular pitch deviations see figure 39.

6.1.1 Adjacent pitch errors f_p

An adjacent pitch error f_p is the difference between the actual size of an individual transverse pitch and the nominal size p_t .

On a gear having z teeth there are z adjacent pitch errors of the right flanks and equally as many of the left flanks. The deviations f_p arise as differences between the individual measured values and the mean of all z measured values.

6.1.2 Cumulative circular pitch errors F_{pk}

A cumulative circular pitch error F_{pk} is the deviation of the actual size of a pitch span over k individual pitches from its nominal size. It is also found — provided the measuring errors are sufficiently small — as the algebraic sum of the k adjacent pitch errors contained in the span:

$$F_{pk} = \sum_k f_p \quad (6.1)$$

If the deviations of all pitch spans are measured from a given reference tooth flank as the starting point, or are calculated from the adjacent pitch errors f_p according to equation (6.1) and then plotted versus the corresponding tooth numbers, the result obtained are the cumulative circular pitch errors according to figure 39 b.

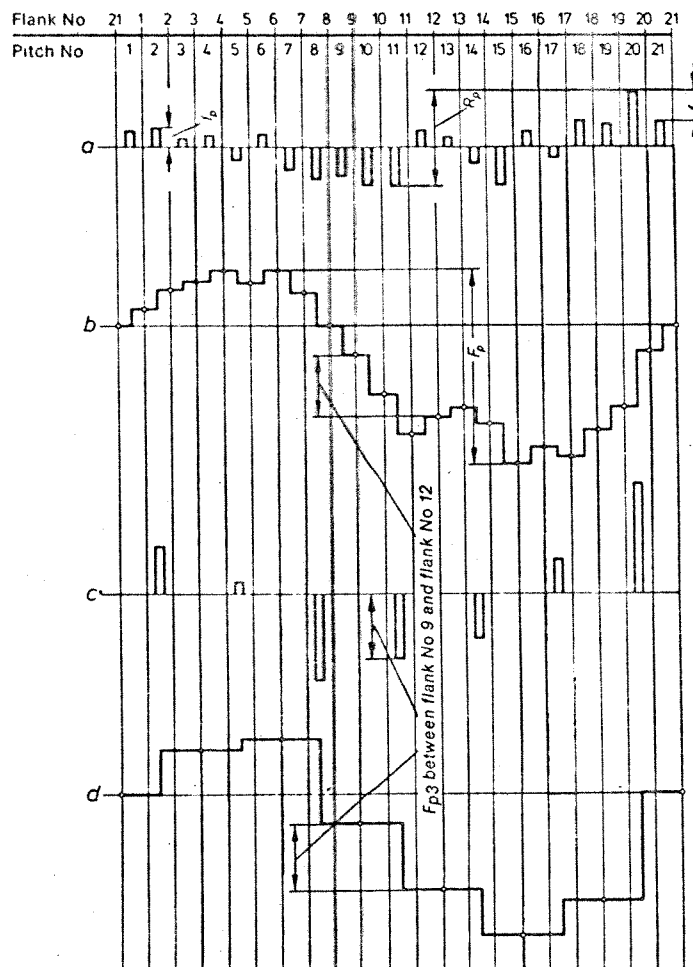
The values plotted in figure 39 b represent the position deviations of the tooth flanks on the reference circle in relation to the notional zero-deviation circular pitch with z uniformly distributed pitch points one of which coincides with the reference tooth flank. From the series of these values all z individual pitch deviations and all cumulative circular pitch errors can be found as differences.

6.1.3 Cumulative circular pitch error $F_{pz/8}$

A cumulative circular pitch error $F_{pz/8}$ is the cumulative circular pitch error existing over a sector of (approximately) $1/8$ of the gear periphery ($k \approx z/8$).

6.1.4 Cumulative pitch error F_p

The maximum cumulative circular pitch error on the gear is termed the cumulative pitch error F_p . It is stated without sign and is found from the cumulative pitch errors as the difference between the algebraically largest and the algebraically smallest value.



a) adjacent pitch errors f_p shown as vertical blocks between the flank numbers

R_p range of pitch errors

f_u difference between adjacent pitches (in this case between pitches Nos 20 and 21, i.e. on flank No 20)

b) cumulative circular pitch errors referred to flank No 21, shown as a stepped-line diagram

F_p cumulative pitch error

c) cumulative circular pitch errors over a sector of $k = 3$ individual pitches each time, shown as vertical blocks in mid-span

d) cumulative pitch-span deviations calculated from the span deviations of figure c, referred to flank No 21, shown as a stepped-line diagram (with correspondingly large step width)

Figure 39. Graphical representation of circular pitch deviations (example: $z = 21$)

6.1.5 Range of pitch errors R_p

The range of pitch errors R_p is the difference between the largest and the smallest actual size of the transverse pitches p_t of the right or left flanks of a gear.

The range of pitch errors R_p can be determined directly from the measured values of a circular pitch measurement without knowledge of the adjacent pitch errors f_p .

6.1.6 Difference between adjacent pitches f_u

A difference between adjacent pitches f_u is the difference between the actual sizes of two successive transverse pitches of the right or left flanks of a gear.

Differences between adjacent pitches are found direct from circular pitch measurements as the differences of the measured values of pairs of adjacent pitches.

6.2 Normal base pitch errors f_{pc}

A normal base pitch error f_{pc} is the difference between the actual size and the nominal size of a normal base pitch p_c . Errors of the transverse base pitches are denoted by f_{pct} and errors of the normal base pitches by f_{pcn} .

The results of the base pitch measurements are independent of any eccentricity of the gear teeth. Base pitch deviations may derive both from irregularity of the circular pitch as well as from form and position deviations of the two tooth flanks. As a result of irregular profile or tooth trace configuration they may differ at different points on the same tooth flanks.

For connection between base pitch deviations and transverse pressure angle deviation or base circle deviation, see subclause 6.3.1.6.

6.3 Flank deviations

Flank deviations are the deviations existing within the flank test area of the gear by which the tooth flanks depart from the involute helicoids of the nominal base cylinder when taking into account the desired deviations (e.g. crowning).

Flank deviations on a cylindrical gear can be determined

- a) in a plane of transverse section (deviations of the transverse profile, or of the profile for short)
- b) on the reference cylinder or some other coaxial cylinder (deviations of the tooth trace)
- c) in a plane tangential to the base cylinder (deviations of the generator).

The records of flank deviations obtained with the aid of flank testing machines are the flank test diagrams. In the test diagram produced by the majority of flank testing machines the nominal involute, the nominal tooth trace and the nominal generator appear as straight lines, see figure 40. From the flank test diagram (test diagram curve) numerical values of the actual deviations on the gear are derived by the use of relationships indicated below.

The flank test range generally comprises the range of the usable flank (height and width of the flank). Alternatively, it may be specially agreed for acceptance test purposes on gears.

6.3.1 Deviations of transverse profile

It is only in transverse sections that involute gear teeth exhibit involutes of their base circle. Deviations of the tooth profiles from their nominal profile are therefore measured only in transverse sections. The nominal profiles are formed by involutes of the nominal base circle with allow-

ance made for intended deviations from the involute form (e.g. profile modification).

For this measurement it is usual to employ measuring machines which guide the stylus according to the involute generating law during the measurement and trace the tooth flank along a transverse profile. When the direction of stylus motion is at right angles to the flank the measured values must be converted to deviations in the transverse

section by multiplying by $\frac{1}{\cos \beta_b}$.

Note: In the test diagram traced by the measuring machine the measured values are read off at right angles to the direction of chart feed, that is to say in the case of a test diagram according to figure 40 in the direction at right angles to the line AA which corresponds to the recording of a nominal involute. For evaluation of the test diagram the equalizing line BB is drawn as the actual involute. This is calculated if necessary by the "method of the least sum of the squares of the errors". An inclined position relative to the line AA indicates the presence of a deviation from the nominal base diameter or from the nominal pressure angle.

The profile test range must generally be taken as the length of path of contact when the gear is meshed with its mating gear (or if the latter is unknown, with the rack). The profile test range L_a is conveniently defined by stating the working length L_f for the root and L_a for the tip or by the corresponding working angles ζ_f and ζ_a .

The following are distinguished:

- Total profile error F_t
- Profile form error f_t
- Profile angle error $f_{H\alpha}$
- Profile waviness f_{tw}

6.3.1.1 Total profile deviation F_t

The total profile error F_t of a tooth flank is the distance between the two nominal profiles which enclose and touch the tooth flanks within the profile test range.

Note: In a test diagram according to figure 40 the total profile error F_t is the distance measured at right angles to the direction of chart feed between the line AA and the further line A'A' parallel to it which are drawn through the outermost points of the test pattern in the direction of chart feed within the profile test range. The desired deviations from the involute form are allowed for by appropriate deviations of the lines AA and A'A' from straightness.

6.3.1.2 Profile form error f_t

The profile form error f_t of a tooth flank is the distance between the two involutes of the base circle which — allowing for desired deviations from the involute form — enclose and touch the actual profile within the profile test range. The profile form error also comprises the wave depth of the profile waviness, see subclause 6.3.1.5.

Note: In a test diagram according to figure 40 the profile form error f_t is the distance measured at right angles to the direction of chart feed between the parallel lines B'B' and B''B'' parallel to the averaging actual involute BB, which lines contact the test diagram within the profile test range.

6.3.1.3 Profile angle error $f_{H\alpha}$

The profile angle error $f_{H\alpha}$ is the distance between the two nominal profiles which cut the involutes of the actual base circle at the starting and finishing points, respectively, of the profile test range. The profile angle error $f_{H\alpha}$ is usually

	Profile	Tooth trace	Generator
①	Total profile error F_f	Total alignment error F_β	Total generant error F_E
②	Profile angle error $f_{H\alpha}$	Tooth alignment error $f_{H\beta}$	Generant angle error f_{HE}
③	Profile form error f_f	Longitudinal form error $f_{\beta f}$	Generant form error f_{Ef}
Test range	Profile test range L_α	Tooth trace test range L_β	Generator test range L_E
BB	Averaging actual involute	Averaging actual tooth trace	Averaging actual generator
AA, A'A'	Nominal profiles which envelope the actual flank	Nominal tooth traces	Nominal generator
B'B', B''B''	Actual involutes which envelope the actual flank	Actual helices	Actual generator
C'C', C''C''	Nominal profiles which cut the actual generators or tooth traces at the starting and finishing point, respectively, of the test range	Nominal tooth traces	Nominal generator

If f_{pe} is inserted in μm and the module m in mm, then f_a is found in mrad (see subclause 1.3).

In the case of helical gears it is usual to measure p_{en} ; for this purpose it is necessary to replace α by α_n , m by m_n , p_{eM} by p_{eMn} and f_{pe} by f_{pen} in the above equations.

Similarly the following relationship exists between the base circle error f_b and the normal pitch error f_{pc}

$$f_b = \frac{d_b}{m \cdot \pi \cdot \cos \alpha} \cdot f_{pc} = \frac{z}{\pi} \cdot f_{pc} \quad (6.6)$$

The equations hold good exactly for helical gear teeth if the transverse section values are used, otherwise only approximately. If the calculations made with equations (6.2) to (6.6) are to relate not only to a single flank but instead to several flanks or all the flanks of a gear, then the corresponding average values (e.g. over 4 or all flanks of the gear) must be used.

When the average is formed over the entire periphery of the gear the influence of eccentricity of the tooth system is eliminated. Otherwise the eccentricity can be determined by the values of f_b on the two (opposite) flanks having the algebraically largest or smallest value of f_{Ha} or f_{pc} .

6.3.2 Deviations of tooth traces

A tooth trace is the intersection of the tooth flank with a cylinder, usually the reference cylinder, see subclause 2.2.5. Deviations of the tooth traces from their nominal form are measured in successive transverse sections tangentially to the base cylinder. The nominal tooth traces are formed by helices (of the reference cylinder) with the nominal lead, allowance being made for desired variations (e.g. crowning).

The measurement is made by the use of machines which simulate a nominal helix (with infinitely large lead in the case of spur gear teeth) and which cause the stylus to explore the tooth flank along a tooth trace.

Note: In the diagram drawn by the test machine the measured values are read off at right angles to the direction of chart feed; hence in a diagram according to figure 40, at right angles to the straight line AA which corresponds to the plot of a nominal tooth trace. For evaluation of the diagram the equalizing line BB is drawn in as an averaging actual tooth trace, if necessary by using the "method of the least sum of the squares of the errors". An inclined position relative to the line AA indicates the presence of a deviation from the nominal lead p_z or from the nominal helix angle β .

It is usual to specify the entire facewidth as the tooth trace test range L_β .

The following are distinguished:

- Total alignment error F_β
- Longitudinal form error $f_{\beta f}$
- Tooth alignment error $f_{H\beta}$
- Longitudinal form waviness $f_{\beta w}$

6.3.2.1 Total alignment error F_β

The total alignment error F_β of a tooth flank is the distance between the two nominal tooth traces which enclose and touch the flanks within the tooth trace test range.

Note: In the test diagram according to figure 40 the total alignment error F_β is the distance measured at right angles to the direction of chart feed between the line AA and the

line A'A' parallel to it which are drawn in the direction of chart feed within the tooth trace test range through the outermost points of the diagram. Desired deviations from the helical form are allowed for by appropriate deviations of the lines AA and A'A' from straightness.

6.3.2.2 Longitudinal form error $f_{\beta f}$

The longitudinal form error $f_{\beta f}$ of a tooth flank is the distance between the two helices with the actual lead which — with allowance made for desired deviations from the helical form — enclose and touch the actual tooth trace within the tooth trace test range. The longitudinal form error $f_{\beta f}$ also includes the wave-depth of the longitudinal form waviness, see subclause 6.3.2.6.

Note: In a test diagram according to figure 40 the longitudinal form error $f_{\beta f}$ is the distance measured at right angles to the direction of chart feed between the lines B'B' and B''B'' which are drawn parallel to the averaging actual tooth trace BB and which contact the test diagram within the tooth trace test range.

6.3.2.3 Tooth alignment error $f_{H\beta}$

The tooth alignment error $f_{H\beta}$ is the distance in a transverse plane between the two nominal tooth traces which intersect the helix with actual lead at the starting and finishing points, respectively, of the tooth trace test range. The tooth alignment error $f_{H\beta}$ is usually stated in μm as a linear dimension assigned to the tooth trace test range L_β , see DIN 3961.

For all the definitions and calculations concerning the gear pair a tooth alignment error in the case of an external gear is termed positive if its helix is right-handed relative to a helix with the nominal lead; it is termed negative if its helix is left-handed relative to a helix with the nominal lead. In the case of an internal gear the opposite signs apply. Hence, when cylindrical gears are mated deviations of equal magnitude but opposite sign cancel out.

Note: In contrast to this in gear production and for measurement techniques tooth alignment errors which result in a higher value (absolute value) of the actual helix angle compared with the nominal helix angle are termed positive ($F_{H\beta}$ positive); tooth alignment errors which result in a smaller value of the actual helix angle are termed negative ($F_{H\beta}$ negative). In the case of spur gear teeth tooth alignment errors are always termed positive, the letters "r" (i.e. "in the sense of a right-hand helix") or "l" (i.e. "in the sense of a left-hand helix") are added indicating the direction of the deviation.

Two different definitions for the signs of the tooth alignment errors are found from the two preceding paragraphs. Therefore the active direction of the deviations must be unambiguously specified in all the production documents, test certificates etc.

A tooth alignment error is caused by a deviation of the helix angle or by non-parallelism of the gear-cutting axis and axis of rotation (wobble) which manifests itself on the tooth individually in the same way as the former.

Note: In a test diagram according to figure 40 the tooth alignment error $f_{H\beta}$ is the distance measured at right angles to the direction of chart feed between the lines C'C' and C''C'' which run parallel to the line AA and intersect the line BB at the end of the tooth trace test range.

6.3.2.4 Helix angle error f_β

A tooth alignment error $f_{H\beta}$ can if necessary be converted into the corresponding helix angle error f_β (difference between the actual helix angle and the nominal helix angle).

Approximately it is

$$f_\beta \approx \frac{f_{H\beta}}{L_\beta} \cdot \cos^2 \beta \quad (6.7)$$

If $f_{H\beta}$ is inserted in μm and L_β in mm, then f_β is found in mrad (see subclause 1.3).

6.3.2.5 Axial pitch error f_{px} , lead error f_{pz}

A helix angle error f_β can be converted into the corresponding axial pitch error f_{px} and into the corresponding lead error f_{pz} :

$$f_{px} = -f_\beta \cdot \frac{p_x}{\sin |\beta| \cdot \cos \beta} \quad (6.8)$$

also

$$f_{pz} = -f_\beta \cdot \frac{p_z}{\sin |\beta| \cdot \cos \beta} \quad (6.9)$$

If p_x and p_z are inserted in mm and f_β in mrad, then f_{px} and f_{pz} are found in μm . In the case of spur gearing conversion is pointless since $p_x = \infty$ and $p_z = \infty$.

6.3.2.6 Longitudinal form waviness f_{pw}

Longitudinal form waviness is a longitudinal form error repeating cyclically over the facewidth. It is characterized by the wave-depth and wave-length.

6.3.3 Deviations of the generator

A generator is the line of intersection of the tooth flank with a plane developed from the base cylinder, see subclause 2.3.1. Deviations of the generator from its nominal form are measured in successive transverse sections tangentially to the base cylinder. The nominal generators are straight lines or lines containing straight lines and intended deviations from straightness.

The measurement is made by machines which guide the stylus in accordance with the law of generation of the involute helicoid in a plane tangential to the base cylinder and parallel to the direction of stylus motion, and explore a tooth flank along a line of contact.

Note: In the diagram drawn by the testing machine the measured values are read off at right angles to the direction of chart feed; hence in a test diagram according to figure 40, at right angles to the straight line AA which corresponds to the plot of a nominal generator. For evaluation of the test diagram the equalizing line BB is drawn in as an averaging actual generator, if necessary by using the "method of the least sum of the squares of the errors". An inclined position relative to the line AA indicates the presence of a deviation from the base helix angle β_b or from the base diameter d_b .

It is usual to specify the length of the longest line of contact as the generator test range L_E . On a helical tooth flank the lines of contact are of unequal length, see figure 41.

6.3.3.1 Total generant error F_E

The total generant error F_E is the distance between the two nominal generators which enclose and touch the tooth flanks within the generator test range.

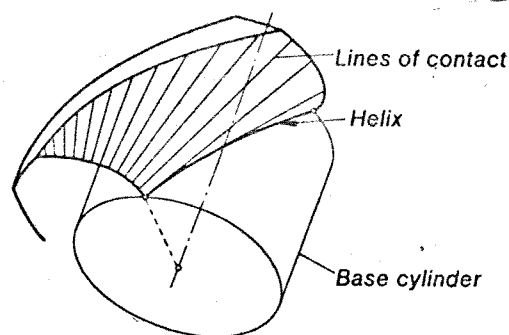


Figure 41. Contact lines on a helical tooth flank

The following are distinguished:

- Total generant error F_E
- Generant form error f_{Ef}
- Generant angle error f_{HE} .

Note: In a test diagram according to figure 40 the total generant error F_E is the distance measured at right angles to the direction of chart feed between the line AA and the line A'A' parallel to it, which are drawn through the outermost points of the test diagram within the generator test range in the direction of chart feed. Intended variations from the generator form are allowed for by suitable deviations the lines AA and A'A' from straightness.

6.3.3.2 Generant form error f_{Ef}

The generant form error f_{Ef} is the distance between the two generators having the actual lead which — taking account of intended deviations from straightness — enclose and touch the actual generator within the generator test range.

Note: In a test diagram according to figure 40 the generant form error f_{Ef} is the distance measured at right angles to the direction of chart feed between the two lines B'B' and B''B'' drawn parallel to the averaging actual generator BB which contact the test diagram within the generator test range.

6.3.3.3 Generant angle error f_{HE}

The generant angle error f_{HE} is the distance between the two nominal generators which cut the averaging actual generator at the starting and finishing points, respectively, of the generator test range. It is usually stated in μm as a linear dimension allocated to the generator test range L_E .

Note: In a test diagram according to figure 40 the generant angle error f_{HE} is the distance measured at right angles to the direction of chart feed between the lines C'C' and C''C'' drawn parallel to the line AA which cut the line BB at the starting and finishing points, respectively, of the generator test range.

6.4 Radial run-out

6.4.1 Radial run-out F_r of a tooth system

The radial run-out F_r of a tooth system is the difference in radial position of a measuring element (ball or cylinder or V-head) which is inserted successively into all tooth spaces and which contacts the tooth flanks near the V-circle, the gear being rotatably mounted for this purpose in its guidance axis. F_r denotes the largest difference between the measured values occurring at the gear periphery, see also subclause 6.6.3.

A radial run-out F_r is caused mainly by eccentricity of the gear teeth in the measuring plane at right angles to the gear axis and by irregularity of the spacewidth resulting from pitch deviations of the right and left flanks.

6.4.2 Radial run-out of tip circle f_r

In the case of overcut cylindrical gears (see subclause 2.7.6) a radial run-out of the tooth system may be determined by measuring the radial run-out of the tip circle f_r . This is found by moving the gear as the largest difference of position of a pressure foot pin radially movable and being in contact with the tip cylinder.

6.5 Position deviation of gearcutting axis

A position deviation of the gearcutting axis exists if the axis of the reference cylinder of the gear teeth (gearcutting axis) and the guidance axis of the gear fail to coincide when measurements are being made or in the operating condition (e.g. as a result of a position deviation during the clamping of the gear blank for cutting). In the general case of such a position deviation the two axes intersect; their position relative to one another is characterized by the distance between the two axes in the middle of the facewidth (eccentricity of the gear teeth) and by the angle of intersection (cutting wobble).

With, generally speaking, adequately good approximation a position deviation of the gearcutting axis can be determined by measuring the radial run-out F_{rV} and F_{rR} in two transverse planes (near the front V and the rear R of the gear with separation L). From the course of the radial run-out the averaging curves and their fundamental are determined for each periphery of the gear. The double amplitudes of the fundamental are denoted by $2 \cdot f_{eV}$ or $2 \cdot f_{eR}$, and the centre angle between the maximum values of the fundamental by φ_e .

6.5.1 Eccentricity f_e

The eccentricity f_e of the gear teeth is equal to the distance between the gearcutting axis and the guidance axis of the gear in the middle of the facewidth. It is found as

$$f_e = \frac{1}{2} \cdot \sqrt{f_{eV}^2 + f_{eR}^2 + 2 \cdot f_{eV} \cdot f_{eR} \cdot \cos \varphi_e} \quad (6.10)$$

6.5.2 Wobble F_0

The wobble F_0 of the gear teeth is equal to the angle of intersection between the gearcutting axis and the guidance axis of the gear. It is found as

$$F_0 \approx \tan F_0 = \frac{1}{L} \cdot \sqrt{f_{eV}^2 + f_{eR}^2 - 2 \cdot f_{eV} \cdot f_{eR} \cdot \cos \varphi_e} \quad (6.11)$$

When $\varphi_e = 0$ and $f_{eV} \neq f_{eR}$ these two axes intersect at a point which is usually situated a long way outside the gear body, whilst with $\varphi_e = \pi$ they intersect within the gear body. With $\varphi_e = 0$ and $f_{eV} = f_{eR}$ the axes are parallel to one another. With $\varphi_e \neq 0$ and $\varphi_e \neq \pi$ the axes are skewed relative to one another (they cross one another).

6.6 Range of errors R

A range of errors (span) R is the difference between the largest and smallest measured value of measured quantities of like kind within a series of measurements. On a gear tooth system ranges of errors are found as a result of deviations in quantities measured at different points on the gear teeth (e.g. for pitches see subclause 6.1.5).

In the different measuring methods the ranges of tooth thickness errors and the deviations in other quantities of the gear teeth (e.g. radial run-out) manifest themselves differently. The ranges of errors of different measured quantities can therefore not be readily converted one to another, e.g. by means of change factors. For allocating ranges of tooth thickness errors to the ranges of errors of another measured quantity it is necessary to take into account the special features of the measuring method used, see DIN 3967.

(For range of pitch errors R_p see subclause 6.1.5.)

6.6.1 Range of tooth thickness errors R_s , range of normal chordal tooth thickness errors R_{Σ}

The range of tooth thickness errors R_s is the difference between the largest and smallest tooth thickness s of a gear.

$$R_s = s_{\max} - s_{\min} \quad (6.12)$$

The corresponding range of normal chordal tooth thickness errors R_{Σ} can generally be equated with the range of tooth thickness errors R_s .

6.6.2 Range of base tangent length errors R_W

The range of base tangent length errors R_W is the difference between the largest and smallest base tangent length W_k of a gear.

$$R_W = W_{k \max} - W_{k \min} \quad (6.13)$$

6.6.3 Range of errors R_{Mr} for the radial single-ball or single-pin dimension

The difference between the largest and smallest measured value of the radial single-ball or single-pin dimension M_r of a gear is equal to the radial run-out F_r , see subclause 6.4.

6.6.4 Range of errors R_{Md} for the dimension over balls or pins

The range of errors R_{Md} is the difference between the largest and smallest measured value of the dimension over balls or pins M_d of a gear.

$$R_{Md} = M_{d \max} - M_{d \min} \quad (6.14)$$

6.6.5 Range of errors for the centre distance for double-flank engagement

The difference between the largest and smallest measured value of the centre distance for double-flank engagement a'' on a gear is equal to the radial composite error F_r'' , see subclause 7.2.1.

6.7 Contact pattern

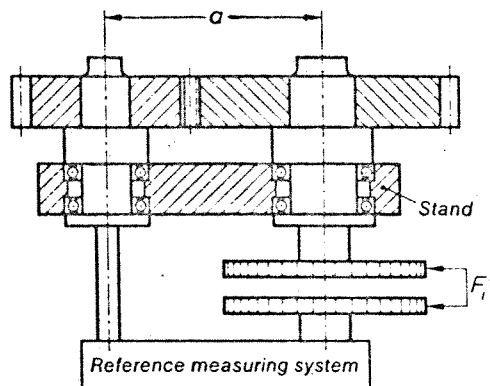
As a result of gearcutting deviations, deviations in the gear positions and operating factors a tooth flank will not be contacted by the mating flanks in all points of its active range during the meshing action. The contact pattern denotes that area of a tooth flank in which contact with the mating flanks takes place.

7 Composite and cumulative errors

In the working test gear teeth are mated with counter gear teeth and the combined effects of their individual geometrical deviations (individual errors) on the working action are determined as composite and cumulative errors. These can be assigned to one of the gears (the test gear) if the gear used as the mating gear is a master gear with deviations which are negligibly small compared with the deviations of the test gear. It is common to use master gears with a quality at least three grades higher than the specified quality of the gear under test.

If the deviations of the mating gear are not negligibly small (e.g. when the working test is performed with two transmission gears) the composite and cumulative errors can only be assigned jointly to the gear pair.

See VDI/VDE Code of practice 2608 "Single-flank and double-flank working test on spur and helical gears with involute profile".



Centre distance a set to a fixed value.

Single-flank engagement of the right flanks or left flanks through torque constraint.

Tangential composite errors = relative deviations of angle of rotation compared with the corresponding zero-deviation angular settings brought about by a reference measuring system.

Figure 42. Measuring arrangement for the single-flank working test

7.1 Single-flank working test

In the single-flank working test two gears are meshed with one another at the prescribed centre distance so that either the right flanks or the left flanks remain in continuous engagement (single-flank engagement), see figure 42. The tangential opposite error of the right flanks of a gear are generally different from those of the left flanks of the same gear.

From a given starting position the deviations occurring in the angle of rotation, that is to say the deviations in the angular positions of the gear compared with the theoretical positions governed by the positions of the mating gear and the ratio of the numbers of teeth, are measured. This requires the use of a reference measuring system (reference gear unit) in which the zero-deviation angular positions (theoretical positions) are realized. The deviations are generally stated as a distance along the circumference of a test circle, e.g. the reference circle or the base circle; alternatively, they may also be stated in angular measure (e.g. in seconds or in milliradians or microradians).

7.1.1 Tangential composite error F_t'

The tangential composite error F_t' is the range of errors of the actual angular positions compared with the theoretical angular positions. It is found as the difference (sum of the absolute amounts) of the largest leading and the largest lagging angular position deviation compared with a given starting position within one revolution of the test gear, see figure 43.

In a circular test diagram the tangential composite error F_t' is the difference between the largest and the smallest distance of the recorded test trace from the axis of rotation of the test diagram chart. In the case of a strip type test

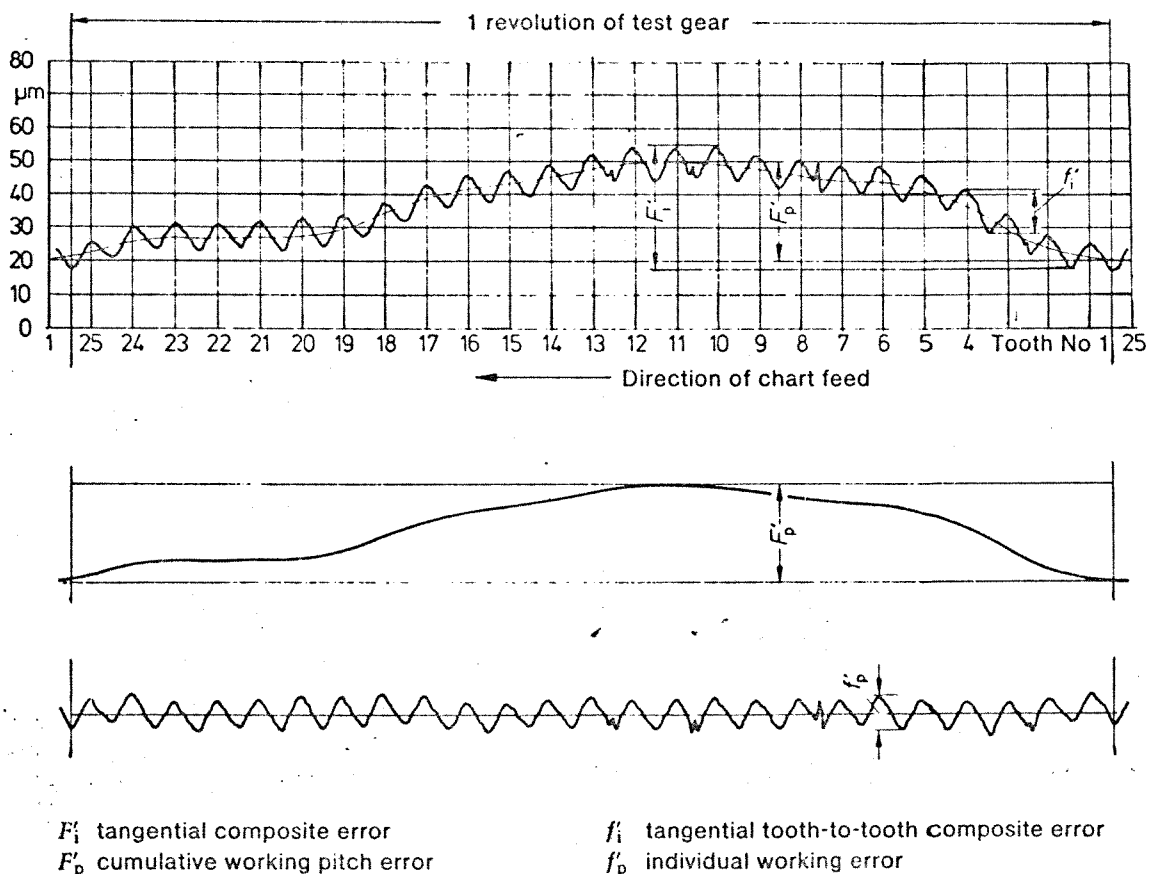


Figure 43. Strip type single-flank working diagrams of a gear

diagram the tangential composite error F'_t is the difference between the largest and the smallest ordinate of the test trace.

7.1.2 Cumulative working pitch error F'_p

The cumulative working pitch error F'_p is the long-undulation component in the test diagram of the tangential composite errors. This component can be obtained from the test diagram produced during the single-flank working test by drawing in an "averaging line" with which the short-undulation components are suppressed. The "averaging line" has an essentially sinusoidal characteristic of one cycle per gear periphery, see figure 43. The cumulative working pitch error F'_p is the difference (determined radially or in the ordinate direction) between the highest and the lowest point of the averaging line.

7.1.3 Individual working error f'_p

Individual working errors are the short-undulation components in the test diagram of the tangential composite errors; they result from the differences between the recorded test trace and the "averaging line" according to subclause 7.1.2. The periodicity of the short-undulation components per gear periphery agrees in many cases with the number of teeth of the test gear. These components may however also contain the effects of undulations in the profile or longitudinal form errors.

The individual working error f'_p is the difference between the highest and the lowest point in the trace of the short-undulation components, see figure 43. The cumulative working pitch error F'_p and the individual working error f'_p together yield approximately the tangential composite error F'_t .

7.1.4 Tangential tooth-to-tooth composite error f'_t

The tangential tooth-to-tooth composite error f'_t is the largest difference which occurs in the angular position deviations within an angle of rotation corresponding to the duration of a tooth engagement, see figure 43.

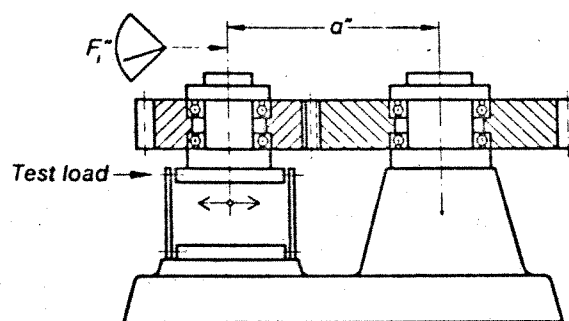
7.1.5 Transmission deviation of a multi-stage gear mating

The transmission deviation of a multi-stage gear mating is the angular position deviation of the shaft at one end of the gear train from its theoretical position of the shaft at the other end of the gear train and the ratio of the numbers of teeth. It is referred to the shaft at one end of the gear train. It contains the tangential composite errors of the individual gear pairs, the effects of load in the different stages of the gearing and the effects of inaccuracies of mounting, and it is composed of two components. One component derives from deviations in the parameters from their theoretical values whilst the other results from elastic deformations of the transmission elements and their bearings.

7.2 Double-flank working test

In the double-flank working test two gears are meshed with each other with zero backlash and at the same time a force acting coaxially with the line of centres is applied to keep a left flank and a right flank of the gears simultaneously in engagement (double-flank engagement), see figure 44. The alterations in centre distance occurring under these conditions are measured.

The centre distance established in the double-flank working test is denoted by a'' and the working transverse pressure angle by α'' , see also subclause 2.7.5.

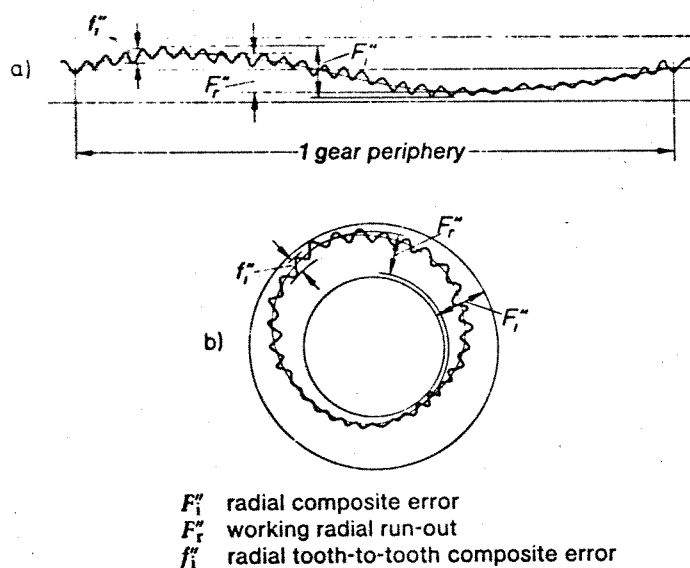


The centre distance a'' is established through the action of a load acting coaxially with the line of centres and varies with the rotation of gear and mating gear.

Double-flank engagement.

Radial composite errors = Centre distance alterations.

Figure 44. Test arrangement for double-flank working test



a) strip chart b) circular chart

Figure 45. Double-flank working diagrams

7.2.1 Radial composite error F''_r

The radial composite error F''_r is the range of errors of the working centre distance a'' , in other words it is the difference between the largest and the smallest working centre distance within a single revolution of the test gear, see figure 45.

7.2.2 Working radial run-out F'_r

The working radial run-out F'_r is the long-undulation component in the test diagram of the radial composite errors. This component is obtained from the test diagram yielded by the double-flank working test by drawing in the "averaging line" whereby the short-undulation components (the periodicity of which per gear periphery usually agrees with the number of teeth of the test gear) are suppressed, see figure 45. The working radial run-out F'_r is then the distance between the highest and the lowest point of the "averaging line".

7.2.3 Radial tooth-to-tooth composite error f''_r

The radial tooth-to-tooth composite error f''_r is the largest difference in the working centre distance which occurs within an angle of rotation corresponding to the duration of one tooth engagement, see figure 45.

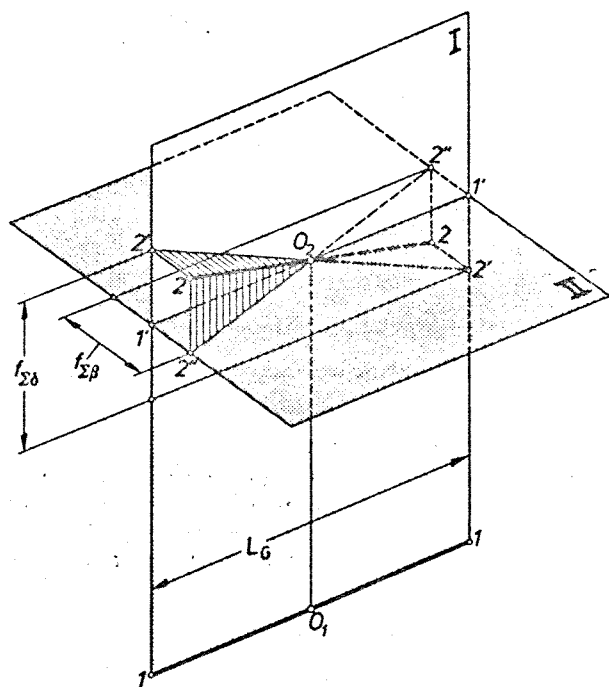
8 Deviations of the axial positions of a cylindrical gear pair

The axes of a cylindrical gear pair may exhibit variations from parallelism and a deviation from the nominal centre distance. These quantities are stated as variations or as a deviation of the axial position of gear 2 relative to the reference gear axis 1. When the two cylindrical gears are mounted in the housing different variations or a different deviation may occur compared with those obtained when mounted in a test machine.

8.1 Deviations from parallelism

The deviations of the axial positions of a cylindrical gear pair from parallelism are stated in the following manner, see figure 46. Gear axis 1 (reference axis) and point O_2 , which bisects the distance L_G between the bearing centres of gear axis 2 determine the plane of centres I. The perpendicular dropped from O_2 to axis 1 lies in the plane of centres I and meets axis 1 at point O_1 . The plane II which also passes through point O_2 is at right angles to plane I and parallel to gear axis 1. Planes I and II intersect one another in the straight line 1'-1' which is parallel to gear axis 1.

In the event of deviations from parallelism gear axis 2 (which also passes through O_2) makes an angle with the parallel line 1'-1'. By projecting gear axis 2 on to planes I and II this angle can be resolved into two components; the two components are generally stated as portions of lines referred to the distance between the bearing centres, see the permissible deviations in DIN 3964.



- 1-1 gear axis 1
- 2-2 gear axis 2
- O_2 point of bisection of gear axis 2
- O_1 meeting point of perpendicular from O_2 with gear axis 1
- I plane of centres of gear axes
- II plane perpendicular to plane of centres and parallel to 1-1
- 1'-1' line of intersection of planes I and II
- 2'-2' projection of 2-2 onto plane I
- 2''-2'' projection of 2-2 onto plane II

Figure 46. Deviations of axial positions from parallelism

8.1.1 Inclination error of axes $f_{\Sigma\delta}$

The inclination error of axes is the deviation of gear axis 2 from parallelism to gear axis 1 in the plane of centres I. In plane I of figure 46 $f_{\Sigma\delta}$ is the vertical distance to the line of section 1'-1' of the bearing centres of gear axis 2 projected on to the plane of centres.

8.1.2 Deviation error of axes $f_{\Sigma\beta}$

The deviation error of axes is the deviation of gear axis 2 from parallelism to gear axis 1 in the plane II perpendicular to the plane of centres.

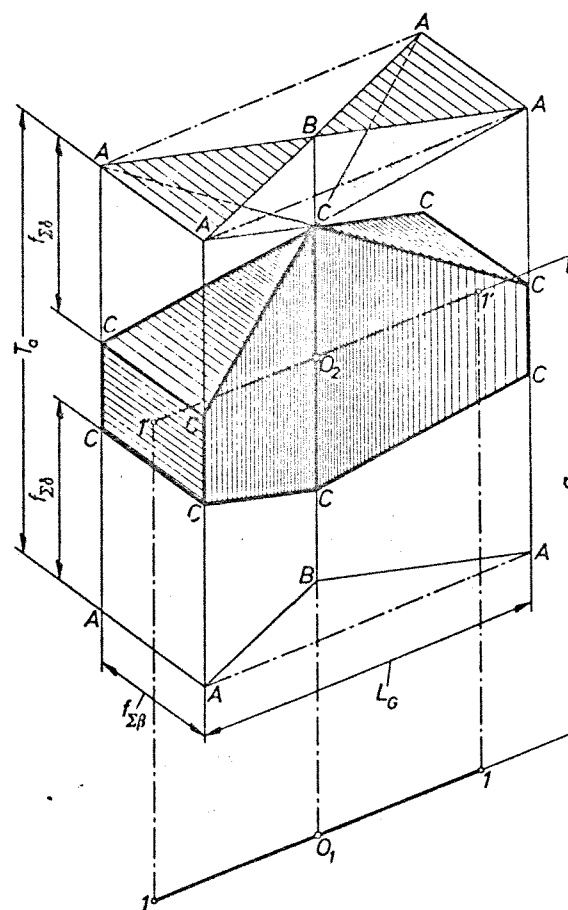
In plane II in figure 46 $f_{\Sigma\beta}$ is the vertical distance to the line of section 1'-1' of the bearing centres of gear axis 2 projected on to plane II.

8.2 Deviations and tolerance for position of gear axes

8.2.1 Centre distance deviations A_a

The centre distance deviation A_a is the difference between the actual centre distance of the gear pair and the nominal size.

To avoid unduly large deviations of the axial positions from the nominal sizes in the case of multi-stage gear pairings, and to enable the same production specifications to be used for both external gear pairs and internal gear pairs, the permissible limiting values of the centre distance are specified symmetrically to the nominal size, so that the permissible centre distance deviations have the sign $\pm$.



- A-A-A enveloping right parallelepiped
- B-B-B double-cuneiform tolerance space determined by T_a and $f_{\Sigma\beta}$
- C-C-C tolerance restricted from above and below by $f_{\Sigma\delta}$

Figure 47. Tolerance space for position of gear axis 2 in relation to gear axis 1

8.2.2 Centre distance tolerances T_a

The centre distance tolerance T_a is the difference between the upper centre distance deviation A_{ac} and the lower centre distance deviation A_{ai} .

$$T_a = A_{ac} - A_{ai} \quad (8.1)$$

8.2.3 Tolerance space for position of gear axes

The tolerance space for gear axis 2, which is aligned with gear axis 1 (reference axis) and is not referred to the housing, is given by the positions of gear axis 2 relative to gear axis 1 which are possible as a result variations or deviations. The tolerance space is enclosed by a right parallelepiped the longitudinal axis 1'-1' of which is parallel to gear axis 1 and is distant from it by the nominal centre distance a , see figure 47. The right parallelepiped has length L_G , height T_a in the plane of centres I (see figure 46) and width $f_{\Sigma\beta}$ in plane II (see figure 46). In figure 47 the right parallelepiped is denoted by the points A.

Form the centre distance tolerance T_a and the permissible deviation error of axes $f_{\Sigma\beta}$ therefore a tolerance space according to figure 47 is obtained which has the shape of a double wedge (denoted by points A and B) the knife-edges B-B of which face one another and are coincident with the perpendicular O_1O_2 (see figure 46). Through the existing (as maximum permissible) inclination error of axes $f_{\Sigma\delta}$ the tolerance space is additionally restricted. In the vertical direction wedge-shaped elements are lost at the ends of the tolerance space, these having in each case the height $f_{\Sigma\delta}$ and an end face of the double wedge, whilst at the opposite end face of the double wedge they finish at a knife-edge. This tolerance space thus has the form of a bevelled double wedge the knife-edges of which have the height $T_a - f_{\Sigma\delta}$ and the end faces the height $T_a - 2 \cdot f_{\Sigma\delta}$. This tolerance space is denoted in figure 47 by the points C. It is the minimum tolerance space for full utilization of the permissible inclination error of axes. With smaller inclination error of axes the tolerance space is correspondingly larger; with $f_{\Sigma\delta} = 0$ it is equal to the double wedge ABA.

The tolerance spaces of further gear axes are to be formed similarly. Their planes of centres contain the reference axis and lie within their angular tolerance.

9 Backlash j

On zero-deviation cylindrical gears the backlash is the distance, stated in a given direction, between the non-working flanks of the teeth of a gear pair when their working flanks are in contact.

The size of the backlash depends on the deviations of the tooth thicknesses of the two gear tooth systems and of the centre distance, as well as on deviations of profile form, tooth trace, pitch and concentricity and on temperature differences between the gear teeth and the housing, and possibly also on effects resulting from swelling of the materials used and elastic deformation. See DIN 3967.

9.1 Circumferential backlash j_t

The circumferential backlash j_t is the length of the pitch circle arc through which each of the two gears can be rotated, whilst the other is held stationary, from the point where the right flanks are in contact to the point where the left flanks are in contact. Its magnitude is represented in the transverse section.

9.2 Normal backlash j_n

The normal backlash j_n is the shortest distance between the non-working flanks of the teeth of a gear pair when their working flanks are in contact.

$$j_n = j_t \cdot \cos \alpha_n \cdot \cos \beta = j_t \cdot \cos \alpha_t \cdot \cos \beta_n \quad (9.1)$$

9.3 Radial backlash j_r

The radial backlash j_r is the difference in the centre distance between the working condition and that of zero-backlash engagement.

$$j_r = \frac{j_t}{2 \cdot \tan \alpha_{wt}} \quad (9.2)$$

9.4 Range of errors for backlash R_j

The range of errors for backlash R_j (circumferential range of errors for backlash R_{jt} , normal range of errors for backlash R_{jn}) is the difference between the largest and the smallest backlash occurring in the case of a gear pair in mounted state.

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Further standards and documents

Supplement 1 to

DIN 3960	Concepts and parameters for cylindrical gears and cylindrical gear pairs; classification of the equations
DIN 3962 Part 1	Tolerances for cylindrical gear teeth; tolerances for deviations of individual parameters
DIN 3962 Part 2	Tolerances for cylindrical gear teeth; tolerances for tooth trace deviations
DIN 3962 Part 3	Tolerances for cylindrical gear teeth; tolerances for pitch-span deviations
DIN 3963	Tolerances for cylindrical gear teeth; tolerances for working deviations
DIN 3964	Deviations of shaft centre distances and shaft position tolerances of casings for cylindrical gears
DIN 3977	(at present at draft stage) Diameter of measuring element for test dimension of the tooth thickness on cylindrical gears
DIN 3998 Part 1	Designations on gears and gear pairs; general concepts
DIN 3998 Part 2	Designations on gears and gear pairs; cylindrical gears and gear pairs
Supplement 1 to	
DIN 3998	Designations on gears and gear pairs; alphabetical index of equivalent terms
DIN 8000	Parameters and errors of hobs for involute cylindrical gears; fundamental concepts
DIN 58405 Part 1	Cylindrical gear drives for fine mechanics; scope, classification
VDE/VDI 2605	Circular pitches and plane angles; basic concepts for angular dimensions, angle measurements, angle standards and their errors
VDI/VDE 2608	Single-flank and double-flank working test on spur and helical gears with involute profile
VDI 3336	Production of cylindrical gear teeth with involute profile, cutting methods

Explanations

This standard contains terminological principles for manufacturers and users of gear transmissions and for other standards on tooth systems.

Compared with DIN 3960, August 1960 edition, concepts and parameters have been fully revised and new concepts required by the state of the art have been included. Certain obsolete terms such as "limiting number of teeth" and "shaft angle error", have been omitted. The symbols and designations have been brought into line with the basic standards DIN 3998 and DIN 3999 and have thus at the same time been made conform to international practice.

Compared with DIN 3960, October 1976 edition, some printer's errors and mistakes (e.g. in the caption of figure 17, in figure 35 and in subclause 4.1.6) have been corrected and desirable supplements added; in subclause 6.3.2.3 reference was made to the use of the symbols of tooth alignment errors used in gear measuring technique differing from the specifications in this standard. Subclause 2.5.6.3 has been divided into 2.5.6.3 and 2.5.6.4, subclause 9.4 has been newly added. In subclause 2.7.3.1 the virtual tooth number z_{nM} determined empirically has been introduced which yields better approximation values for D_M with a large β than z_{nW} in the case of helix gear teeth; the corresponding equations have been simplified. They hold good exactly for spur tooth systems and approximately for helix tooth systems.

Compared with the October 1976 edition the number and order of the equations have been changed because equations have been either added or transposed. To avoid errors

when using the previous edition and this revised edition simultaneously, the numbering system has now been changed: the new equation numbers include the numbers of the main clauses in which they occur.

The grouping now adopted in DIN 3960 is as follows:

Symbols, terms and units are covered in clause 1.

In clauses 2 and 3 the definitions and parameters for zero-deviation and zero-deviation cylindrical gear teeth and for zero-backlash mating of zero-deviation cylindrical gears are presented.

Clauses 4 and 5 describe the deviations and tolerances and the change factors for zero-deviation cylindrical gear teeth.

The deviations of individual parameters relating to cylindrical gear teeth are dealt with in clause 6, the composite and cumulative errors in clause 7, the deviations of axial positions of a gear pair in clause 8 and backlash in clause 9.

This standard is concerned with cylindrical gears with involute teeth. Definitions and parameters for bevel gears and hypoid gears as well as for cylindrical worm gear transmissions will be dealt with in DIN 3971 and in DIN 3975, respectively. General definitions and parameters of overall coverage for gears, gear pairs and gear transmissions are grouped in DIN 863.

The composition of symbols is explained in DIN 3999. The terms collected in ISO Recommendation R 1122 - 1969 are listed in DIN 3998 Part 1 and Part 2 in German together with the corresponding English and French terms in so far as the nomenclature concerns this standard.