

10

Regenerative Chatter in Machine Tools

10.1	Introduction	10-1
10.2	Chatter in Turning Operations	10-3
	Example 1	
10.3	Chatter in Face-Milling Operations	10-9
	Example 2	
10.4	Time-Domain Simulation	10-14
	Example 3 • Example 4	
10.5	Chatter Detection	10-18
	Example 5	
10.6	Chatter Suppression	10-20
	Spindle-Speed Selection • Example 6 • Feed and Depth-of-Cut Selection • Spindle-Speed Variation • Example 7	
10.7	Case Study	10-24

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Summary

Regenerative chatter, a result of unstable interactions between machining forces and structural deflections, is a great limitation in machining operations. This chapter describes the modeling, analysis, simulation, detection, and control of regenerative chatter in machining operations, and, in particular, turning and face milling. An analytical method is applied to calculate the limiting stable depth-of-cut and corresponding spindle speeds to generate stability lobe diagrams. The method is applied to both turning and face-milling operations. Time-domain simulation is described and applied to turning and face-milling operations. Methods for chatter detection are presented and experimental results from a face-milling operation are given. Chatter suppression techniques, namely spindle-speed selection, feed selection, depth-of-cut selection, and spindle-speed variation, are presented and two simulations of a turning operation are used to illustrate the spindle-speed selection and spindle-speed variation techniques. Finally, a case study of a face-milling operation is presented. The nomenclature used in the presentation is listed at the end of the chapter.

10.1 Introduction

Regenerative chatter is a major limitation in machining operations. This phenomenon is a result of an unstable interaction between the machining forces and the structural deflections. The forces generated when the cutting tool and part come into contact produce significant structural deflections. These structural deflections modulate the chip thickness that, in turn, changes the machining forces. For certain cutting conditions, this closed-loop, self-excited system becomes unstable and regenerative chatter occurs. Regenerative chatter may result in excessive machining forces and tool wear, tool failure, and scrap parts due to unacceptable surface finish, thus severely decreasing operation productivity and part quality.

A typical chatter stability chart, the so-called stability lobe diagram, is shown in Figure 10.1. If the process parameters are above the stability borderline, chatter will occur, and if the process parameters are below the stability borderline, chatter will not occur. The asymptotic stability borderline is the depth-of-cut below which stable machining is guaranteed regardless of the spindle speed. The lobed nature of the stability borderline allows stable pockets to form; thus, at specific ranges of spindle speeds, the depth-of-cut may be substantially increased beyond the asymptotic stability limit. These pockets become smaller as the spindle speed decreases. The stability borderline is “pulled up” for low spindle speeds due to process damping (i.e., the back side of the tool rubbing on the part surface). If accurate models of the structural components and the cutting process are available, the stability lobe diagram may be used to plan chatter-free machining operations.

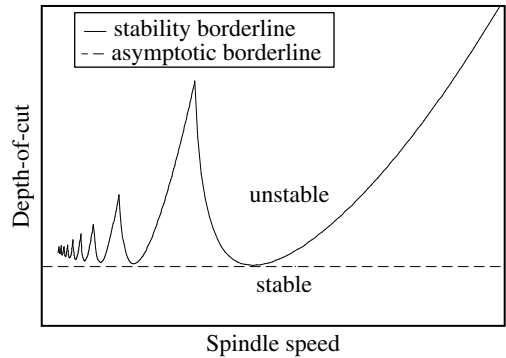


FIGURE 10.1 Stability lobe diagram.

The analysis of regenerative chatter as the interaction between the cutting forces and structural vibrations was established by Tobias (1965) and Koenigsberger and Tlustý (1971). Merritt (1965) used systems theory to determine stability and construct the stability lobe diagram by generating specialized plots from the harmonic solutions of the system's characteristic equation. Chatter analysis reveals a natural delay in the system leading many researchers to use Nyquist techniques to generate stability lobe diagrams (Minis et al., 1990a, 1990b; Lee and Liu, 1991a, 1991b; Minis and Yanushevsky, 1993). A set of process parameters is selected and the characteristic equation is formed. The Nyquist criterion is applied to determine if the system for this process parameter set is stable. The depth-of-cut is adjusted and the procedure is repeated until the critical depth-of-cut is determined. Another chatter analysis technique capable of generating stability lobe diagrams analytically for linear systems has recently been introduced (Altintas and Budak, 1995; Budak and Altintas, 1998a, 1998b). This technique is utilized in this chapter.

The theoretical analysis of regenerative chatter laid the foundation for developing techniques to automatically detect its occurrence and to automatically suppress it. Since there is a dominant chatter frequency, which is near a structural frequency, that occurs when chatter develops, most monitoring techniques analyze the frequency of a process variable, and chatter is detected when significant energy is present near a structural frequency. Most automatic chatter suppression routines either adjust the spindle speed to be in a pocket of the stability lobe diagram or vary the spindle speed to bring the current and previous tooth passes into phase. While automatic monitoring and control of regenerative chatter shows great promise, it has been mostly limited to laboratory applications. Therefore, commercial tools are not currently available.

While regenerative chatter in turning and face-milling operations is discussed in this chapter, this phenomenon is not limited to these specific manufacturing operations. Other machining operations for which chatter has been analyzed include end milling (Budak and Altintas, 1998a, 1998b), grinding (Inasaki et al., 2001), drilling (Tarng and Li, 1994), and so on. Also, the regenerative chatter phenomenon occurs in other manufacturing operations, most notably in rolling (Yun et al., 1998; Tlustý, 2000).

Section 10.2 and Section 10.3 present an analytical method to examine regenerative chatter in turning and face-milling operations, respectively. Section 10.4 discusses a numerical technique known as time domain simulation that may be used to analyze regenerative chatter for nonlinear systems. The subject of chatter detection is presented in Section 10.5, then methods to perform chatter suppression are discussed and illustrated in Section 10.6. Section 10.7 presents a case study of a face-milling operation.

10.2 Chatter in Turning Operations

A schematic of a turning operation is shown in Figure 10.2. The part structure is assumed to be perfectly rigid, while the cutting-tool structure is capable of vibrations in the longitudinal (i.e., the z) direction only. The machining force in the longitudinal direction is

$$F(t) = Pd f(t) \quad (10.1)$$

The depth-of-cut is assumed to be constant; however, the feed, and hence the machining force, is time-varying due to structural vibrations. It is assumed here that the machining force does not explicitly depend upon the cutting speed.

The feed is the chip thickness in the longitudinal direction. The nominal feed is the distance the tool advances relative to the part each spindle revolution and is constant once the tool fully engages the part. However, the cutting tool vibrates, leaving an undulated surface on the part and, thus, modulates the feed. The instantaneous feed is

$$f(t) = f_{\text{nom}} + \Delta z(t) = f_{\text{nom}} + z(t) - z(t - T) \quad (10.2)$$

The term f_{nom} is the nominal feed, also known as the static feed. The term $\Delta z(t)$ is the feed due to the cutting-tool vibrations and is known as the dynamic feed. The parameter T is the spindle-rotation period. The structural vibration, $z(t)$, known as the inner modulation, is the cutting-tool vibration at the current time. The delayed structural vibration $z(t - T)$, known as the outer modulation, is the cutting-tool vibration as of when the part was at the current angular during the previous spindle rotation. The modulation in feed due to structural vibrations is illustrated in Figure 10.3.

Inserting Equation 10.2 into Equation 10.1:

$$F(t) = Pd f_{\text{nom}} + Pd \Delta z(t) = F_{\text{nom}} + \Delta F(t) \quad (10.3)$$

The force $F_{\text{nom}} = Pd f_{\text{nom}}$ is due to the nominal chip thickness and does not vary since the depth-of-cut and nominal feed are constant. The force $\Delta F(t) = Pd \Delta z(t)$ is due to changes in the nominal feed caused by structural vibrations.

The structural vibrations are related to the machining force by

$$z(s) = -g(s)F(s) \quad (10.4)$$

where $g(s)$ is the transfer function relating the structural vibrations to the machining forces. Since $F(t) - F(t - T) = \Delta F(t) - \Delta F(t - T)$, the structural vibrations are related to the machining

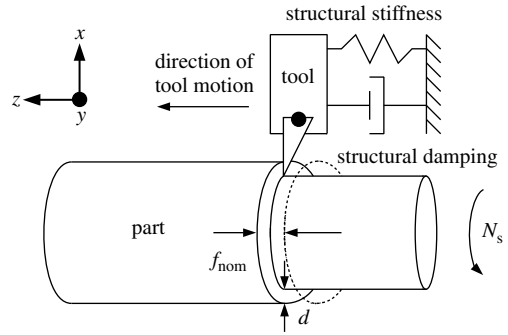


FIGURE 10.2 Turning operation schematic: current pass (solid line) and previous pass (dotted line).

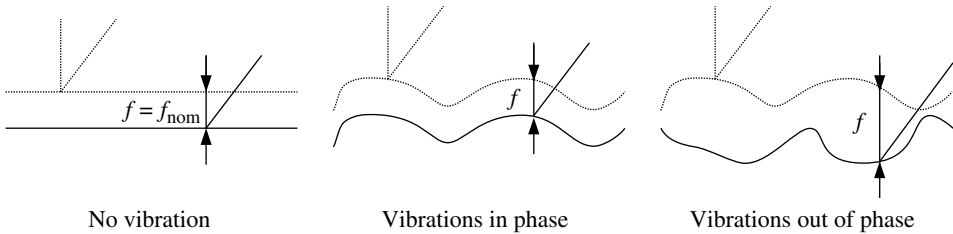


FIGURE 10.3 Modulation in feed due to structural vibrations in a turning operation: current pass (solid line) and previous pass (dotted line).

forces by

$$\Delta z(s) = -(1 - e^{-sT})g(s)\Delta F(s) \quad (10.5)$$

Substituting for Δz in Equation 10.5 and rearranging:

$$\Delta F(s)\{1 + Pd[1 - e^{-sT}]g(s)\} = 0 \quad (10.6)$$

Equation 10.6 is now solved, based on the method presented by Budak and Altintas (1998a, 1998b), to determine the stability lobe diagram. Assuming the steady-state solution is a harmonic function at a single chatter frequency ω_c , Equation 10.6 becomes

$$\Delta F(j\omega_c)e^{j\omega_c T}\{1 + Pd[1 - e^{-j\omega_c T}]g(j\omega_c)\} = 0 \quad (10.7)$$

where $j^2 = -1$. For nontrivial solutions of Equation 10.7, the following eigenvalue problem is derived:

$$\det\{1 + Pd[1 - e^{-j\omega_c T}]g(j\omega_c)\} = 0 \quad (10.8)$$

Since the structural dynamics are one-dimensional, Equation 10.8 reduces to

$$1 + Pd[1 - e^{-j\omega_c T}]g(j\omega_c) = 0 \quad (10.9)$$

The parameter Λ is defined as

$$\Lambda = Pd[1 - e^{-j\omega_c T}] = \Lambda_R + j\Lambda_I \quad (10.10)$$

Using the Euler identity $1 - e^{-j\omega_c T} = 1 - \cos(\omega_c T) + j \sin(\omega_c T)$, the limiting stable depth-of-cut is

$$d_{\lim} = \left(\frac{1}{P}\right) \frac{\Lambda_R + j\Lambda_I}{1 - \cos(\omega_c T) + j \sin(\omega_c T)} \quad (10.11)$$

Equation 10.11 is rewritten as

$$d_{\lim} = \left(\frac{1}{2P}\right) \left\{ \frac{\Lambda_R[1 - \cos(\omega_c T)] + \Lambda_I \sin(\omega_c T)}{1 - \cos(\omega_c T)} + j \frac{\Lambda_R \sin(\omega_c T) + \Lambda_I[1 - \cos(\omega_c T)]}{1 - \cos(\omega_c T)} \right\} \quad (10.12)$$

Since the limiting depth-of-cut must be a real number:

$$\Lambda_R \sin(\omega_c T) + \Lambda_I[1 - \cos(\omega_c T)] = 0 \quad (10.13)$$

The parameter κ is defined as

$$\kappa = \frac{\Lambda_I}{\Lambda_R} = \frac{\sin(\omega_c T)}{1 - \cos(\omega_c T)} \quad (10.14)$$

The limiting stable depth-of-cut is solved explicitly as

$$d_{\lim} = \frac{\Lambda_R}{2P}(1 + \kappa^2) \quad (10.15)$$

Note that Λ_R must be positive for $d_{\lim}$ to be positive. From Equation 10.9, the parameter Λ is

$$\Lambda = -\frac{1}{g(j\omega_c)} \quad (10.16)$$

Equation 10.16 is used to determine Λ_R and Λ_I , and these values are used to solve for $d_{\lim}$. Next, the spindle speed at which the limiting depth-of-cut occurs is determined. The trivial solution to Equation 10.14 is

$$\omega_c T = 0 + 2l\pi, \quad l = 0, 1, 2, \dots \quad (10.17)$$

The quantity $\omega_c T$ may be interpreted as the number of vibration cycles during a spindle rotation. The trivial solution indicates that the successive vibrations are in phase (i.e., there is no regeneration). The nontrivial solution to Equation 10.14 is

$$\cos(\omega_c T) = \frac{\kappa^2 - 1}{\kappa^2 + 1} \quad (10.18)$$

and may be rewritten as

$$\omega_c T = \varepsilon + 2l\pi, \quad l = 0, 1, 2, \dots \quad (10.19)$$

where

$$\varepsilon = \cos^{-1} \left(\frac{\kappa^2 - 1}{\kappa^2 + 1} \right) \quad (10.20)$$

The parameter ε is the fraction of the vibration cycles during a spindle rotation. The angle of Λ in the complex plane is

$$\varphi = \tan^{-1} \left(\frac{\Lambda_I}{\Lambda_R} \right) = \tan^{-1}(\kappa) \quad (10.21)$$

Substituting $\kappa = \tan(\varphi)$ into Equation 10.18 yields

$$\cos(\omega_c T) = -\cos(2\varphi) \quad (10.22)$$

A solution to Equation 10.22 is

$$\omega_c T = \pi - 2\varphi + 2l\pi, \quad l = 0, 1, 2, \dots \quad (10.23)$$

Comparing Equation 10.19 and Equation 10.23, it is seen that the fraction of vibration cycles is $\varepsilon = \pi - 2\varphi$. Since $0 \leq \varepsilon \leq 2\pi$, one must ensure that $-\pi/2 \leq \varphi \leq \pi/2$ when computing φ . For example, if Equation 10.21 is solved using a four-quadrant inverse tangent function whose solution is bounded between $-\pi$ and π , then $-\pi/2 \leq \varphi \leq \pi/2$ since Λ_R is positive. For milling applications, it will be seen that Λ_R must be negative; therefore, the following conditions must be enforced to ensure $0 \leq \varepsilon \leq 2\pi$:

$$\begin{aligned} \text{if } \Lambda_I < 0 \quad \text{then } \varphi &\rightarrow \varphi + \pi \\ \text{if } \Lambda_I > 0 \quad \text{then } \varphi &\rightarrow \varphi - \pi \end{aligned} \quad (10.24)$$

The spindle speed is

$$N_s = \frac{60}{T} = \frac{60\omega_c}{\varepsilon + 2l\pi}, \quad l = 0, 1, 2, \dots \quad (10.25)$$

To construct a stability lobe diagram, the following steps are implemented:

1. Select a chatter frequency (ω_c) near a dominant structural frequency.
2. Calculate Λ_R and Λ_I using Equation 10.16.
3. Calculate d_{lim} using Equation 10.15.
4. Select a stability lobe number (l) and calculate N_s using Equation 10.25. The point (N_s, d_{lim}) is the point on the stability lobe diagram corresponding to the chatter frequency, ω_c , and the stability lobe number, l .
5. Repeat Step 4 for the desired number of stability lobes. The result is a vector of spindle speeds, $\vec{N}_s = \{N_{s_1} \ N_{s_2} \ \dots \ N_{s_n}\}$. Each point $\{(N_{s_1}, d_{\text{lim}}) \ (N_{s_2}, d_{\text{lim}}) \ \dots \ (N_{s_n}, d_{\text{lim}})\}$ corresponds to a different stability lobe, and all of the points correspond to the chatter frequency ω_c .
6. Select another chatter frequency and repeat Steps 2 to 5. In this manner, the stability lobe diagram is constructed. The smaller the difference between successive chatter frequencies, the greater the resolution of the stability lobe diagram. In general, the lobes will overlap. In this case, the

minimum limiting depth-of-cut is the smallest depth-of-cut. If the lobes do not overlap, then the range of chatter frequencies must be increased.

10.2.1 Example 1

The feed force for a turning operation is given by Equation 10.1 and the structural dynamics are given by Equation 10.26. An analytical expression for the limiting depth-of-cut and corresponding spindle speed for a given chatter frequency and stability lobe number is developed. The stability lobe diagram is plotted for $P = 0.6 \text{ kN/mm}^2$, $\omega_n = 600 \text{ Hz}$, $\zeta = 0.2$, and $k = 12 \text{ kN/mm}$. The stability lobe diagram is compared to stability lobe diagrams for $\zeta = 0.1, 0.3$, and 0.4 . The stability lobe diagram is then compared with stability lobe diagrams for $\omega_n = 500, 700$, and 800 Hz . The first ten lobes are included for all stability lobe diagrams:

$$\ddot{z}(t) + 2\zeta\omega_n\dot{z}(t) + \omega_n^2 z(t) = -\frac{\omega_n^2}{k} F(t) \quad (10.26)$$

The parameter Λ is

$$\Lambda = \frac{-1}{g(j\omega_c)} = \Lambda_R + j\Lambda_I = \frac{k}{\omega_n^2} (\omega_c^2 - \omega_n^2) - j\frac{k}{\omega_n^2} (2\zeta\omega_c\omega_n) \quad (10.27)$$

The limiting depth-of-cut is

$$d_{\text{lim}} = \frac{k(\omega_c^2 - \omega_n^2)}{2K\omega_n^2} \left[1 + \frac{4\zeta^2 \omega_c^2 \omega_n^2}{(\omega_c^2 - \omega_n^2)^2} \right] \quad (10.28)$$

The spindle speed is

$$N_s = \frac{60\omega_c}{\pi - 2 \tan^{-1} \left(\frac{-2\zeta\omega_c\omega_n}{\omega_c^2 - \omega_n^2} \right) + 2l\pi}, \quad l = 0, 1, 2, \dots \quad (10.29)$$

where l is the stability lobe number. Note that the chatter frequency must be greater than the structural natural frequency for the limiting depth-of-cut to be positive. The first ten lobes of the

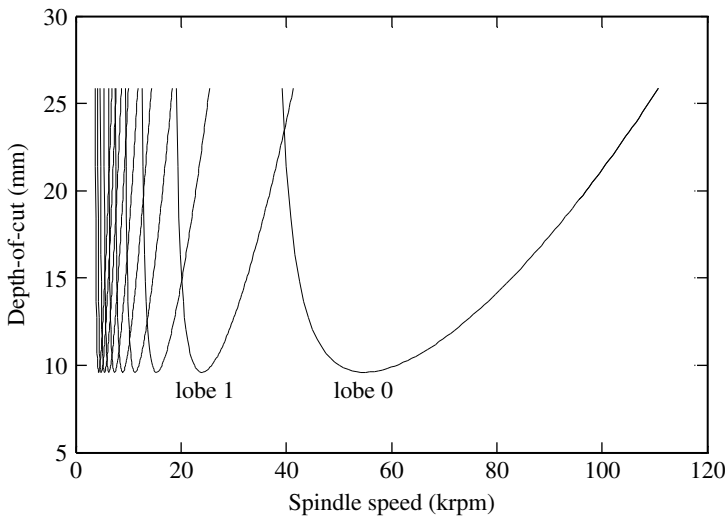


FIGURE 10.4 Unprocessed stability lobe diagram for Example 1.

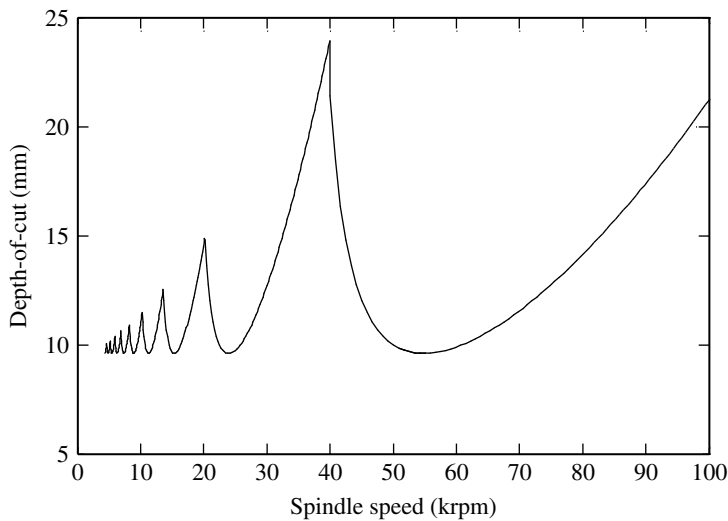


FIGURE 10.5 Processed stability lobe diagram for Example 1.

stability lobe diagram are plotted in Figure 10.4 and Figure 10.5. In Figure 10.4, the entire solution for each of the ten stability lobes is shown. The largest stability lobe is the zeroth lobe on the right. The lobe number increases from right to left on the stability lobe diagram and successive lobes become closer together. The stability lobe diagram is processed in Figure 10.5 such that the minimum depth-of-cut is selected at each spindle speed showing the true stability borderline.

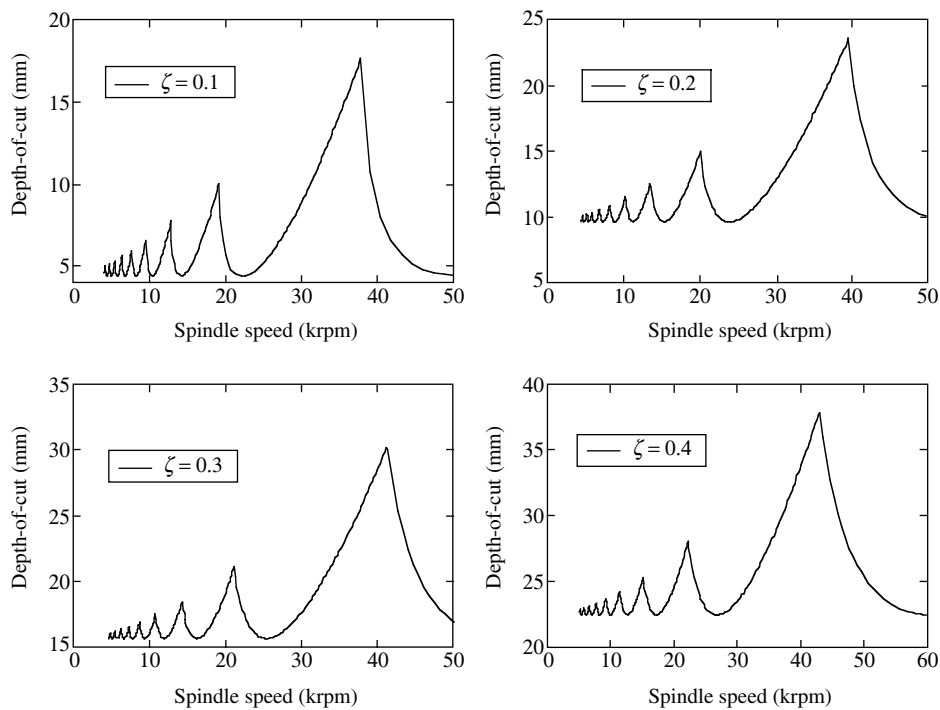


FIGURE 10.6 Stability lobe diagrams for Example 1 with $\zeta = 0.1, 0.2, 0.3$, and 0.4 .

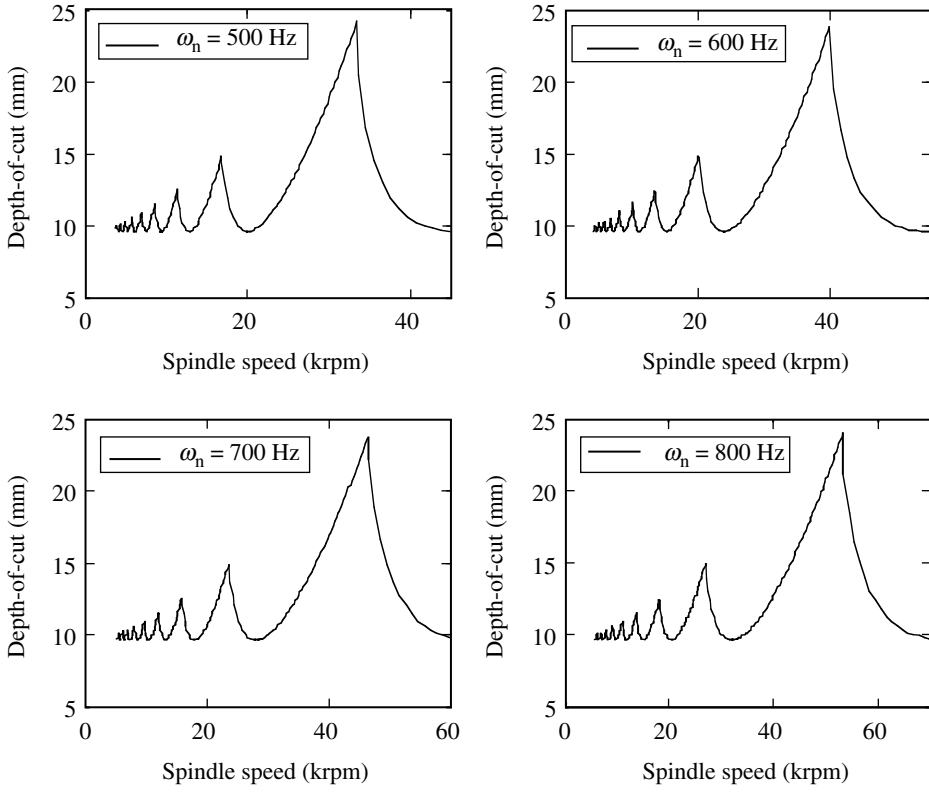


FIGURE 10.7 Stability lobe diagrams for Example 1 with $\omega_n = 500, 600, 700$, and 800 Hz.

In [Figure 10.6](#), the effect of the structural damping ratio is illustrated: as the structural damping ratio increases, the lobes shift slightly to the left and the asymptotic stability boundary shifts up dramatically. The effect of the structural natural frequency is illustrated in [Figure 10.7](#): as the structural natural frequency increases, the lobes shift to the right but the magnitude remains the same.

This section presented an analytical method to generate stability lobe diagrams for turning operations. The limiting depth-of-cut in a turning operation is given by

$$d_{\text{lim}} = \frac{A_R}{2P}(1 + \kappa^2)$$

where A_R is the real part of $-1/g(j\omega_c)$, $g(j\omega_c)$ is the structural transfer function evaluated at the chatter frequency, ω_c , $\kappa = A_I/A_R$, and A_I is the imaginary part of $-1/g(j\omega_c)$. The corresponding spindle speed is

$$N_s = \frac{60\omega_c}{\varepsilon + 2l\pi}$$

where $l = 0, 1, 2, \dots$ is the stability lobe number and

$$\varepsilon = \cos^{-1}\left(\frac{\kappa^2 - 1}{\kappa^2 + 1}\right)$$

10.3 Chatter in Face-Milling Operations

A schematic of a face-milling operation is shown in Figure 10.8. In milling operations, multiple teeth may be in contact with the part simultaneously, the feed naturally varies as a function of the tooth angle even when structural vibrations are not present, and each tooth enters and leaves contact with the part every spindle revolution. The depth-of-cut is the chip thickness in the z direction and is assumed to be constant, since the machine tool and part structures are typically much stiffer in the z direction than in the x and y directions.

The instantaneous feed of the i th tooth, illustrated in Figure 10.9, is

$$f_i(t) = f_t \cos[\theta_i(t)] + \Delta x(t) \cos[\theta_i(t)] + \Delta y(t) \sin[\theta_i(t)] \tag{10.30}$$

where

$$\Delta x(t) = \{x_t(t) - x_t(t - T_t)\} - \{x_p(t) - x_p(t - T_t)\} \tag{10.31}$$

$$\Delta y(t) = \{y_t(t) - y_t(t - T_t)\} - \{y_p(t) - y_p(t - T_t)\} \tag{10.32}$$

The term $f_t \cos[\theta_i(t)]$ in Equation 10.30 represents the feed due to the distance the part advances relative to the cutting tool each tooth rotation and is known as the static feed. The terms $\Delta x(t) \cos[\theta_i(t)]$

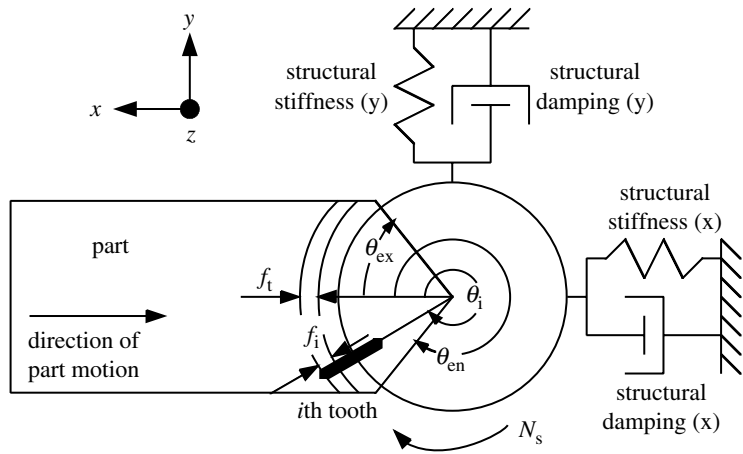


FIGURE 10.8 Face milling operation schematic: current pass (solid line), previous pass (dotted line), and depth-of-cut in z direction.

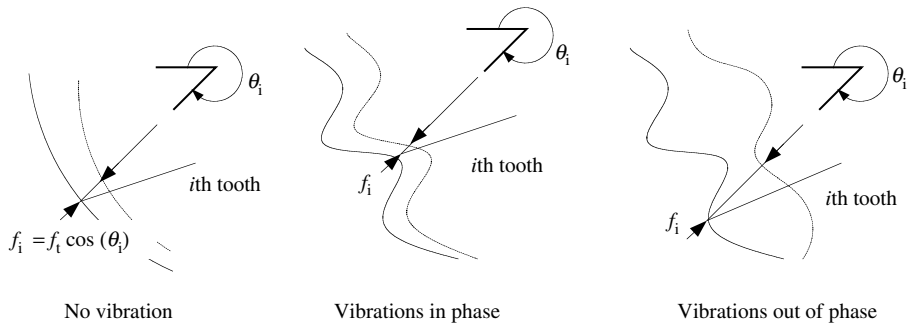


FIGURE 10.9 Modulation in feed due to structural vibrations in a face-milling operation: current pass (solid line) and previous pass (dotted line).

and $\Delta y(t) \sin[\theta_i(t)]$ in Equation 10.30 represent the feed due to tool and part vibrations in the x and y directions, respectively, at the tooth angle $\theta_i(t)$, and are known as the dynamic feed.

The machining forces in the x and y directions, respectively, are

$$\begin{aligned} F_x(t) = & df_t \sum_{i=1}^{N_t} \left\{ -P_T \cos(\psi_r) \cos^2[\theta_i(t)] + P_C \cos[\theta_i(t)] \sin[\theta_i(t)] \right\} \sigma[\theta_i(t)] \\ & + d\Delta x(t) \sum_{i=1}^{N_t} \left\{ -P_T \cos(\psi_r) \cos^2[\theta_i(t)] + P_C \cos[\theta_i(t)] \sin[\theta_i(t)] \right\} \sigma[\theta_i(t)] \\ & + d\Delta y(t) \sum_{i=1}^{N_t} \left\{ -P_T \cos(\psi_r) \sin[\theta_i(t)] \cos[\theta_i(t)] + P_C \sin^2[\theta_i(t)] \right\} \sigma[\theta_i(t)] \end{aligned} \quad (10.33)$$

$$\begin{aligned} F_y(t) = & df_t \sum_{i=1}^{N_t} \left\{ -P_T \cos(\psi_r) \cos[\theta_i(t)] \sin[\theta_i(t)] - P_C \cos^2[\theta_i(t)] \right\} \sigma[\theta_i(t)] \\ & + d\Delta x(t) \sum_{i=1}^{N_t} \left\{ -P_T \cos(\psi_r) \cos[\theta_i(t)] \sin[\theta_i(t)] - P_C \cos^2[\theta_i(t)] \right\} \sigma[\theta_i(t)] \\ & + d\Delta y(t) \sum_{i=1}^{N_t} \left\{ -P_T \cos(\psi_r) \sin^2[\theta_i(t)] - P_C \cos[\theta_i(t)] \sin[\theta_i(t)] \right\} \sigma[\theta_i(t)] \end{aligned} \quad (10.34)$$

where

$$\sigma[\theta_i(t)] = \begin{cases} 1 & \text{if } \theta_{\text{en}} \leq \theta_i(t) \leq \theta_{\text{ex}} \\ 0 & \text{if } \theta_{\text{en}} > \theta_i(t) > \theta_{\text{ex}} \end{cases} \quad (10.35)$$

The function $\sigma[\theta_i(t)]$ determines if the i th tooth is in contact with the part at the tooth angle, $\theta_i(t)$. The first terms in Equation 10.33 and Equation 10.34 are the machining forces acting on the tool in the x and y directions, respectively, due to the static feed. The second terms in Equation 10.33 and Equation 10.34 are the machining forces acting on the tool in the x and y directions, respectively, due to the dynamic feed resulting from structural vibrations in the x direction. The third terms in Equation 10.33 and Equation 10.34 are the machining forces acting on the tool in the x and y directions, respectively, due to the dynamic feed resulting from structural vibrations in the y direction.

The dynamic portion of the face milling force process model may be written compactly as

$$\begin{bmatrix} \Delta F_x(t) \\ \Delta F_y(t) \end{bmatrix} = dA(t) \begin{bmatrix} \Delta x(t) \\ \Delta y(t) \end{bmatrix} = d \begin{bmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{bmatrix} \begin{bmatrix} \Delta x(t) \\ \Delta y(t) \end{bmatrix} \quad (10.36)$$

where

$$A_{11}(t) = \sum_{i=1}^{N_t} \left\{ -P_T \cos(\psi_r) \cos^2[\theta_i(t)] + P_C \cos[\theta_i(t)] \sin[\theta_i(t)] \right\} \sigma[\theta_i(t)] \quad (10.37)$$

$$A_{12}(t) = \sum_{i=1}^{N_t} \left\{ -P_T \cos(\psi_r) \sin[\theta_i(t)] \cos[\theta_i(t)] + P_C \sin^2[\theta_i(t)] \right\} \sigma[\theta_i(t)] \quad (10.38)$$

$$A_{21}(t) = \sum_{i=1}^{N_t} \left\{ -P_T \cos(\psi_r) \cos[\theta_i(t)] \sin[\theta_i(t)] - P_C \cos^2[\theta_i(t)] \right\} \sigma[\theta_i(t)] \quad (10.39)$$

$$A_{22}(t) = \sum_{i=1}^{N_t} \left\{ -P_T \cos(\psi_r) \sin^2[\theta_i(t)] - P_C \cos[\theta_i(t)] \sin[\theta_i(t)] \right\} \sigma[\theta_i(t)] \quad (10.40)$$

These coefficients modulate the instantaneous feed as the tooth angular displacement changes. The summation from $i = 1$ to N_t represents the contribution to this modulation for each of the N_t teeth. Note the matrix $A(t)$ is time-varying and periodic with the tooth-passing period, T_t . For chatter analysis, the matrix $A(t)$ is typically expanded in a Fourier series using the zeroth term (Minis and Yanushevsky, 1993; Budak and Altintas, 1998a). The zeroth term of the Fourier expansion of the force process matrix $A(t)$ is

$$A^0 = \frac{N_t}{2\pi} \begin{bmatrix} A_{11}^0 & A_{12}^0 \\ A_{21}^0 & A_{22}^0 \end{bmatrix} \quad (10.41)$$

where

$$A_{11}^0 = \frac{1}{2} \left[-P_T \cos(\psi_r) \left\{ \theta + \frac{1}{2} \sin(2\theta) \right\} + P_C \sin^2(\theta) \right]_{\theta=\theta_{en}}^{\theta=\theta_{ex}} \quad (10.42)$$

$$A_{12}^0 = \frac{1}{2} \left[-P_T \cos(\psi_r) \sin^2(\theta) + P_C \left\{ \theta - \frac{1}{2} \sin(2\theta) \right\} \right]_{\theta=\theta_{en}}^{\theta=\theta_{ex}} \quad (10.43)$$

$$A_{21}^0 = \frac{1}{2} \left[-P_T \cos(\psi_r) \sin^2(\theta) - P_C \left\{ \theta + \frac{1}{2} \sin(2\theta) \right\} \right]_{\theta=\theta_{en}}^{\theta=\theta_{ex}} \quad (10.44)$$

$$A_{22}^0 = \frac{1}{2} \left[-P_T \cos(\psi_r) \left\{ \theta - \frac{1}{2} \sin(2\theta) \right\} - P_C \sin^2(\theta) \right]_{\theta=\theta_{en}}^{\theta=\theta_{ex}} \quad (10.45)$$

The dynamic force process is now approximated by the linear, time-invariant relationship:

$$\begin{bmatrix} \Delta F_x(t) \\ \Delta F_y(t) \end{bmatrix} = dA^0 \begin{bmatrix} \Delta x(t) \\ \Delta y(t) \end{bmatrix} \quad (10.46)$$

The tool and part vibrations, respectively, are related to the machining forces by

$$\begin{bmatrix} x_t(s) \\ y_t(s) \end{bmatrix} = G_t(s) \begin{bmatrix} F_x(s) \\ F_y(s) \end{bmatrix} = \begin{bmatrix} G_{t11}(s) & G_{t12}(s) \\ G_{t21}(s) & G_{t22}(s) \end{bmatrix} \begin{bmatrix} F_x(s) \\ F_y(s) \end{bmatrix} \quad (10.47)$$

$$\begin{bmatrix} x_p(s) \\ y_p(s) \end{bmatrix} = -G_p(s) \begin{bmatrix} F_x(s) \\ F_y(s) \end{bmatrix} = - \begin{bmatrix} G_{p11}(s) & G_{p12}(s) \\ G_{p21}(s) & G_{p22}(s) \end{bmatrix} \begin{bmatrix} F_x(s) \\ F_y(s) \end{bmatrix} \quad (10.48)$$

where $G_t(s)$ and $G_p(s)$ are the transfer functions relating the tool structural and part structural vibrations, respectively, to the machining forces. The negative sign in Equation 10.48 is due to the fact that the forces acting on the part are equal in magnitude and opposite in direction to the machining forces given in Equation 10.33 and Equation 10.34. Since

$$\begin{bmatrix} F_x(t) \\ F_y(t) \end{bmatrix} - \begin{bmatrix} F_x(t - T_t) \\ F_y(t - T_t) \end{bmatrix} = \begin{bmatrix} \Delta F_x(t) \\ \Delta F_y(t) \end{bmatrix} - \begin{bmatrix} \Delta F_x(t - T_t) \\ \Delta F_y(t - T_t) \end{bmatrix}$$

the structural vibrations can be related to the machining forces by

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = (1 - e^{-sT_t}) [G_t(s) + G_p(s)] \begin{bmatrix} \Delta F_x(s) \\ \Delta F_y(s) \end{bmatrix} \quad (10.49)$$

The machine tool and part vibrations are assumed to occur at a chatter frequency, ω_c , when a marginally stable depth-of-cut is taken. Assuming the steady-state solution is a harmonic function at a chatter

frequency, ω_c , and substituting for the structural vibrations, Equation 10.49 becomes

$$\begin{bmatrix} \Delta F_x \\ \Delta F_y \end{bmatrix} e^{j\omega_c t} = \frac{dN_t}{2\pi} (1 - e^{j\omega_c T_t}) G^0(j\omega_c) \begin{bmatrix} \Delta F_x \\ \Delta F_y \end{bmatrix} e^{j\omega_c t} \quad (10.50)$$

where the matrix G^0 is

$$G^0(j\omega_c) = \frac{2\pi}{N_t} A^0 [G_t(j\omega_c) + G_p(j\omega_c)] \quad (10.51)$$

Equation 10.50 is now solved based on the method presented by Budak and Altintas (1998a, 1998b) to determine the stability lobe diagram. The characteristic equation of Equation 10.50 is

$$\det \left[I_2 - \frac{dN_t}{2\pi} (1 - e^{j\omega_c T_t}) G^0(j\omega_c) \right] = 0 \quad (10.52)$$

where I_2 is the 2×2 identity matrix. The solution of Equation 10.52 yields the limiting stable depth-of-cut. The inverse of the eigenvalue of G^0 is defined as

$$\Lambda(j\omega_c) = \Lambda_R(j\omega_c) + j\Lambda_I(j\omega_c) = -\frac{dN_t}{2\pi} (1 - e^{j\omega_c T_t}) \quad (10.53)$$

Expanding the exponential term in Equation 10.53 and noting that the depth-of-cut must be a real number, the limiting stable depth-of-cut may be written as

$$d_{\lim} = -\frac{\pi \Lambda_R}{N_t} (1 + \kappa^2) \quad (10.54)$$

where the parameter κ is defined by the transcendental equation

$$\kappa = \frac{\Lambda_I}{\Lambda_R} = \frac{\sin(\omega_c T_t)}{1 - \cos(\omega_c T_t)} \quad (10.55)$$

Equation 10.55 is solved for the tooth-passing period of the l th stability lobe and the tooth-passing period is related to the spindle speed to yield

$$N_s = \frac{60\omega_c}{N_t[\pi - 2\varphi + 2l\pi]}, \quad l = 0, 1, 2, \dots \quad (10.56)$$

where, again, $\varphi = \tan^{-1}(\kappa)$. A chatter frequency is selected and the limiting stable depth-of-cut is calculated from Equation 10.54 corresponding to the spindle speed on the l th lobe as given by Equation 10.56.

10.3.1 Example 2

The cutting and thrust pressures in a face-milling operation are given by $P_C = 2.0 \text{ kN/mm}^2$ and $P_T = 0.8 \text{ kN/mm}^2$, respectively, and the lead angle is 45° . The part is assumed to be perfectly rigid and the tool structural dynamics for the x and y directions are given by Equation 10.57 and Equation 10.58, respectively. The nominal parameters are $\theta_{\text{en}} = -45^\circ$, $\theta_{\text{ex}} = 45^\circ$, $N_t = 4$, $k_x = 14 \text{ kN/mm}$, $k_y = 17 \text{ kN/mm}$, $\zeta_x = 0.15$, $\zeta_y = 0.1$, $\omega_x = 3000 \text{ rad/s}$, and $\omega_y = 4000 \text{ rad/s}$. Stability lobe diagrams are generated for the nominal parameters and $N_t = 1, 2$, and 8 teeth. Next, stability lobe diagrams are generated for the nominal parameters and $\theta_{\text{ex}} = 30^\circ, 60^\circ$, and 75° . The first 15 lobes are included for all stability lobe diagrams.

$$\ddot{x}_t(t) + 2\zeta_x \omega_x \dot{x}_t(t) + \omega_x^2 x_t(t) = \frac{\omega_x^2}{k_x} F_x(t) \quad (10.57)$$

$$\ddot{y}_t(t) + 2\zeta_y \omega_y \dot{y}_t(t) + \omega_y^2 y_t(t) = \frac{\omega_y^2}{k_y} F_y(t) \quad (10.58)$$

The effect of the number of teeth is illustrated in Figure 10.10: as the number of teeth increases, the lobes shift to the left and the asymptotic stability borderline decreases. In Figure 10.11, the effect of the exit angle is illustrated: as the exit angle increases, the asymptotic stability borderline decreases.

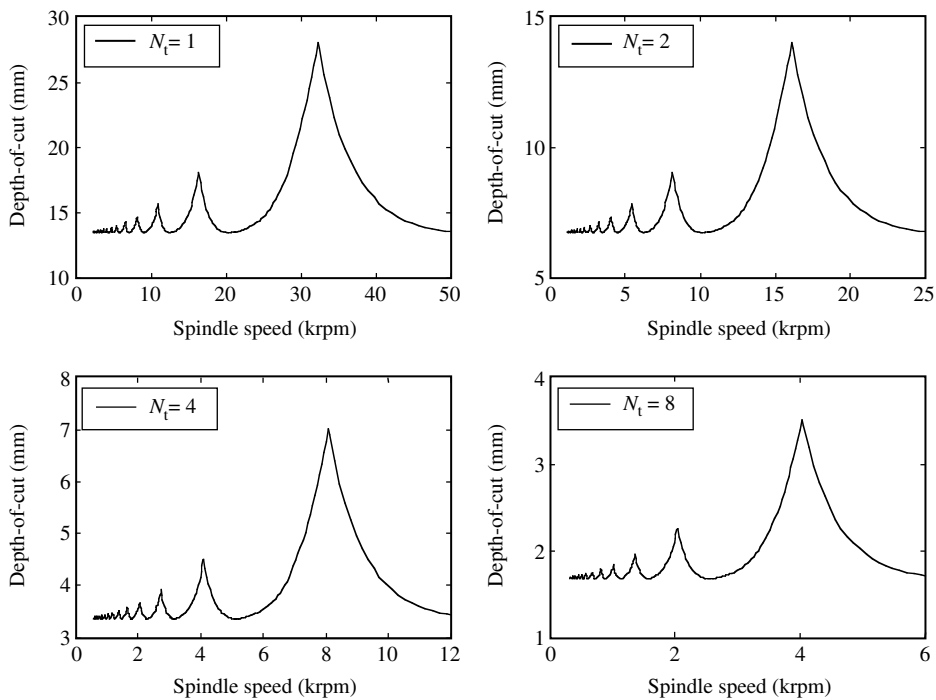


FIGURE 10.10 Stability lobe diagrams for Example 2, with $N_t = 1, 2, 4$, and 8 .

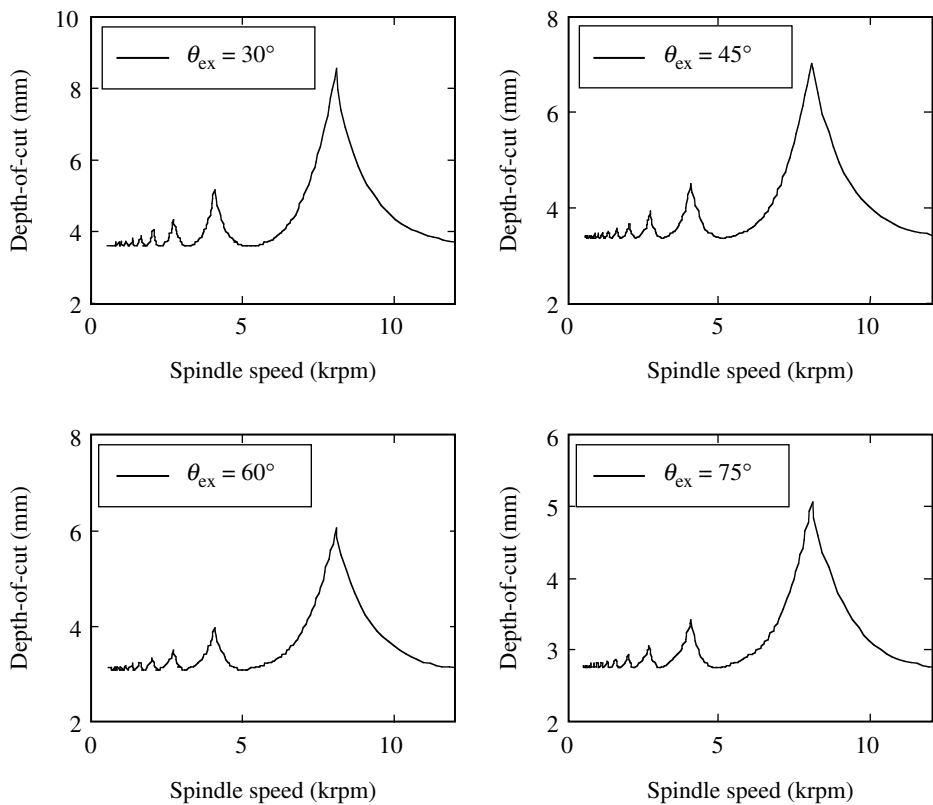


FIGURE 10.11 Stability lobe diagrams for Example 2, with $\theta_{ex} = 30^\circ, 45^\circ, 60^\circ$, and 75° .

This section presented an analytical method to generate stability lobe diagrams for face-milling operations. The limiting depth-of-cut in a face-milling operation is given by

$$d_{\text{lim}} = -\frac{\pi \Lambda_R}{N_t}(1 + \kappa^2)$$

where N_t is the number of teeth, Λ_R is the inverse eigenvalue of $(2\pi/N_t)A^0[G_t(j\omega_c) + G_p(j\omega_c)]$, A^0 is the zeroth term of the Fourier expansion of the force process matrix, $G_t(j\omega_c)$ and $G_p(j\omega_c)$ are the transfer functions relating the tool structural and part structural vibrations, respectively, to the machining forces evaluated at the chatter frequency ω_c , $\kappa = \Lambda_I/\Lambda_R$, and Λ_I is the imaginary part of $(2\pi/N_t)A^0[G_t(j\omega_c) + G_p(j\omega_c)]$. The corresponding spindle speed is

$$N_s = \frac{60\omega_c}{N_t[\pi - 2 \tan^{-1}(\kappa) + 2l\pi]}$$

where $l = 0, 1, 2, \dots$ is the stability lobe number.

10.4 Time-Domain Simulation

Time-domain simulation (Tlusty and Ismail, 1981, 1983; Tlusty, 1986; Tsai et al., 1990; Lee and Liu, 1991a, 1991b; Smith and Tlusty, 1993; Elbestawi et al., 1994; Tarng and Li, 1994; Weck et al., 1994) is an alternative method for determining regenerative chatter. In a time-domain simulation, the machining forces and structural vibrations are simulated in the time domain for a specific set of process parameters and the resulting signals (i.e., forces and displacements) are examined to determine if chatter is present. The analyses presented above for the turning and face-milling operations assume that the tool always maintains contact with the part and that the cutting and thrust pressures are independent of the process parameters. Further, the face milling analysis approximated the time-varying force process matrix, $A(t)$, by the zeroth term of its Fourier expansion. With time domain simulations, nonlinear effects may be directly incorporated into the simulation; thus, more accurate stability prediction is possible. The disadvantage of time-domain simulations is the extreme computational cost that is required. For a specific spindle speed, several simulations must be conducted at different depths-of-cut; thus, the stability boundary for that spindle speed is determined iteratively. This procedure is repeated for a range of spindle speeds to construct a complete stability lobe diagram.

For turning operations, the machining force is calculated using Equation 10.1, the feed is calculated using Equation 10.2, and the tool displacement is calculated using Equation 10.4. For face-milling operations, the feed is calculated using Equation 10.30, the machining forces in the x and y directions are calculated using Equation 10.33 to Equation 10.35, and the tool and part displacements, respectively, are calculated using Equation 10.47 and Equation 10.48. To calculate the machining forces in the face-milling operation, the angular displacement of each tooth is required. The angular displacement of the i th tooth is

$$\theta_i(t) = \frac{2\pi}{60} N_s t + \frac{2\pi}{N_t}(i - 1) \quad (10.59)$$

The feed and force equations are static, while the structural displacement equations are dynamic and must be solved via a numerical integration technique. A sufficiently small time step must be utilized in the numerical integrations to account for the small system time constants associated with the large structural frequencies.

10.4.1 Example 3

The feed force for a turning operation is given by $F(t) = 0.6df^{0.7}(t)$. The structural dynamics are given by Equation 10.26 with the following parameters: $\omega_n = 600$ Hz, $\zeta = 0.2$, and $k = 12$ kN/mm.

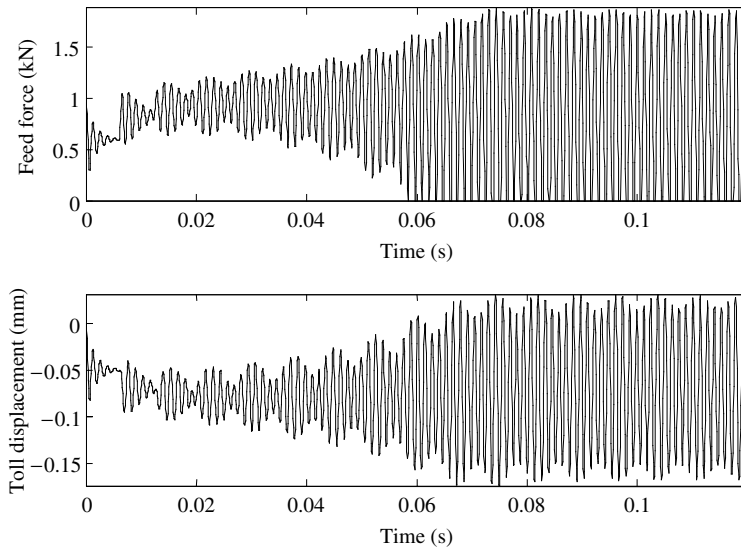


FIGURE 10.12 Time-domain simulations for Example 3 with $f_{\text{nom}} = 0.1$ mm.

A time-domain simulation of the system including the effect of the tool disengaging from the part is constructed, and simulations for $N_s = 10,000$ rpm, $d = 8$ mm, and $f_{\text{nom}} = 0.1$ mm are conducted. The simulation is repeated for $f_{\text{nom}} = 0.2$ mm. For both simulations, the time history of the feed force and the tool displacement are plotted.

To account for the phenomenon of the tool disengaging from the part, the feed in Equation 10.2 must be modified as follows:

$$f(t) = \begin{cases} f_{\text{nom}} + z(t) - f_p(t - T) & \text{if } f_{\text{nom}} + z(t) - f_p(t - T) \geq 0 \\ 0 & \text{if } f_{\text{nom}} + z(t) - f_p(t - T) < 0 \end{cases} \quad (10.60)$$

where

$$f_p(t) = \begin{cases} z(t) & \text{if } f_{\text{nom}} + z(t) - f_p(t - T) \geq 0 \\ -f_{\text{nom}} + f_p(t - T) & \text{if } f_{\text{nom}} + z(t) - f_p(t - T) < 0 \end{cases} \quad (10.61)$$

If the feed at the current time is calculated to be negative, then the cutting tool has disengaged from the part and the feed is zero. The term $f_p(t)$ accounts for feed due to structural vibrations at the previous spindle rotation, even when the cutting tool disengages from the part. The results for $f_{\text{nom}} = 0.1$ mm and $f_{\text{nom}} = 0.2$ mm are shown in Figure 10.12 and Figure 10.13, respectively. As the nominal feed is increased, chatter is suppressed.

10.4.2 Example 4

The cutting and thrust forces in a face-milling operation are given by $F_C(t) = 1.4df^{0.6}(t)$ and $F_T(t) = 0.4df^{0.8}(t)$, respectively. The lead angle is 45° , the entry angle is -60° ; the exit angle is 60° , the number of teeth is four, and the feed per tooth is $f_t = 0.15$ mm. The part is assumed to be perfectly rigid, and tool structural dynamics for the x and y directions are given by Equation 10.58 and Equation 10.59, respectively. A time-domain simulation is developed to determine the limiting stable depth-of-cut for spindle speeds of 1000 and 32,000 rpm. For both spindle speeds, the system is simulated for a depth-of-cut 10% below the limiting stable depth-of-cut and for a depth-of-cut 10% above the limiting stable depth-of-cut. The cutting force, thrust force, x tool displacement, and y tool displacement are plotted.

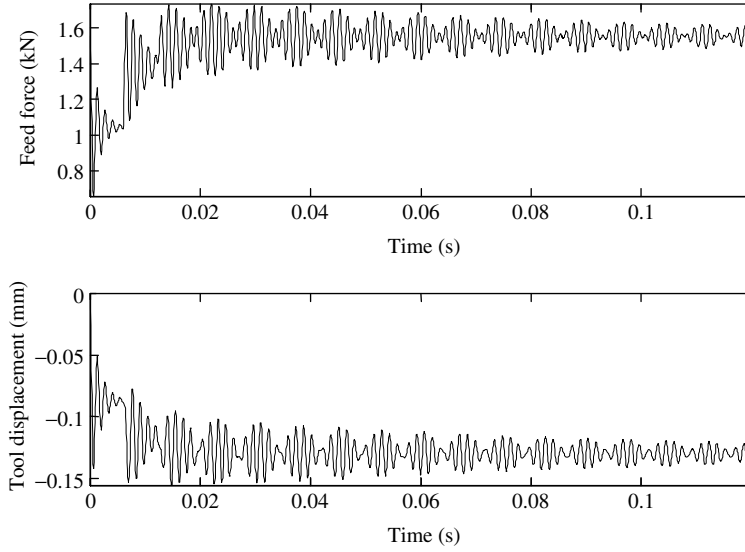


FIGURE 10.13 Time-domain simulations for Example 3 with $f_{nom} = 0.2$ mm.

The nonlinear effect of tooth disengagement is included:

$$\ddot{x}_t(t) + 2(0.15)(3000)\dot{x}_t(t) + 3000^2 x_t(t) = \frac{3000^2}{15} F_x(t) \quad (10.62)$$

$$\ddot{y}_t(t) + 2(0.1)(4000)\dot{y}_t(t) + 4000^2 y_t(t) = \frac{4000^2}{17} F_y(t) \quad (10.63)$$

To account for the phenomenon of the tool disengaging from the part, the feed in Equation 10.30 must be modified as follows:

$$f_i(t) = \begin{cases} f_{ci}(t) - f_{pi}(t - T_i) & \text{if } f_{ci}(t) \geq 0 \\ 0 & \text{if } f_{ci}(t) < 0 \end{cases} \quad (10.64)$$

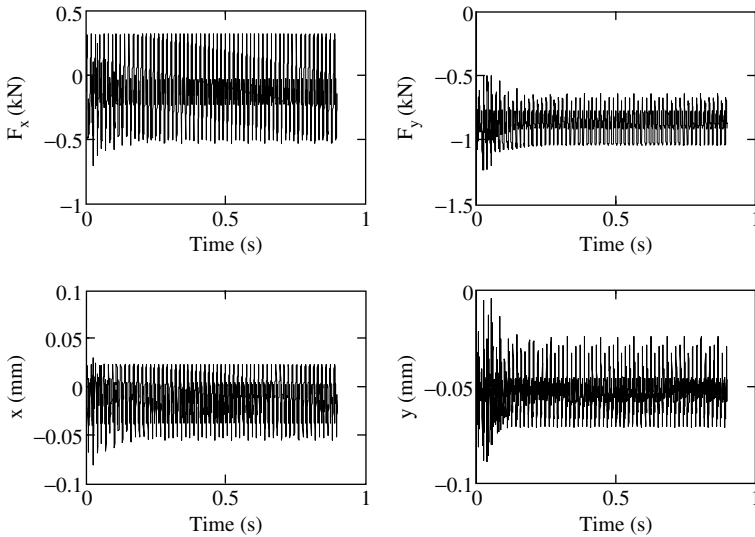


FIGURE 10.14 Time-domain simulation for Example 4 with $N_s = 1000$ rpm and $d = 2.025$ mm.

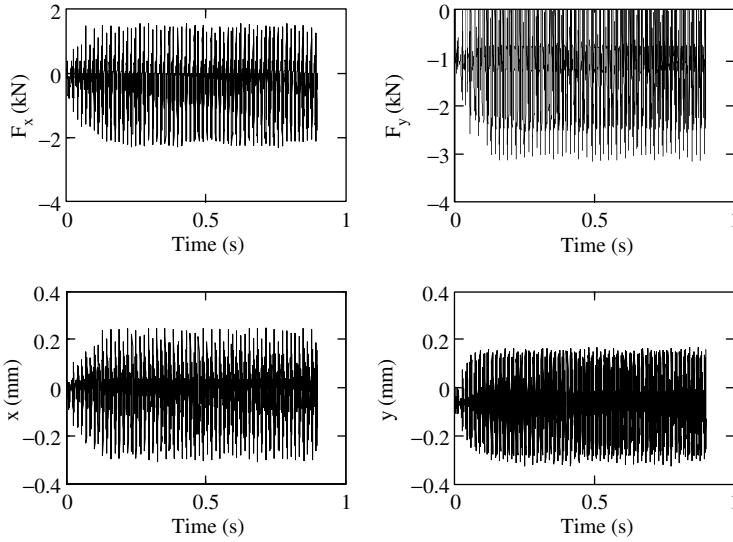


FIGURE 10.15 Time-domain simulation for Example 4 with $N_s = 1000$ rpm and $d = 2.475$ mm.

where

$$f_{ci}(t) = f_t \cos[\theta_i(t)] + \{x_t(t) - x_p(t)\} \cos[\theta_i(t)] + \{y_t(t) - y_p(t)\} \sin[\theta_i(t)] \quad (10.65)$$

$$f_{pi}(t) = \begin{cases} \{x_t(t - T_i) - x_p(t - T_i)\} \cos[\theta_i(t)] + \{y_t(t - T_i) - y_p(t - T_i)\} \sin[\theta_i(t)] & \text{if } f_{ci}(t) \geq 0 \\ -f_t \cos[\theta_i(t)] + f_{pi}(t - T_i) & \text{if } f_{ci}(t) < 0 \end{cases} \quad (10.66)$$

Note that $\theta_i(t) = \theta_{i+1}(t - T_i)$ and $i - 1 \rightarrow N_i$ if $i = 1$. The term $f_{pi}(t)$ represents the contribution to the instantaneous feed when the previous tooth was at the same angular location as the i th tooth. If the tooth and part are in contact, this contribution is due to the tool and part vibrations. If the tooth and part are not in contact, this contribution is the previous contribution added to the static portion and the instantaneous feed is set to zero. Through time-domain simulations, the limiting stable depth-of-cut for

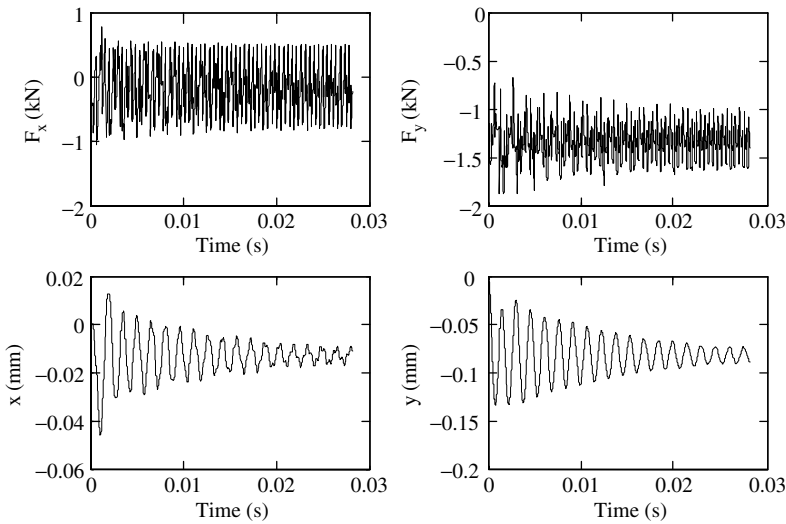


FIGURE 10.16 Time-domain simulation for Example 4 with $N_s = 32,000$ rpm and $d = 3.105$ mm.

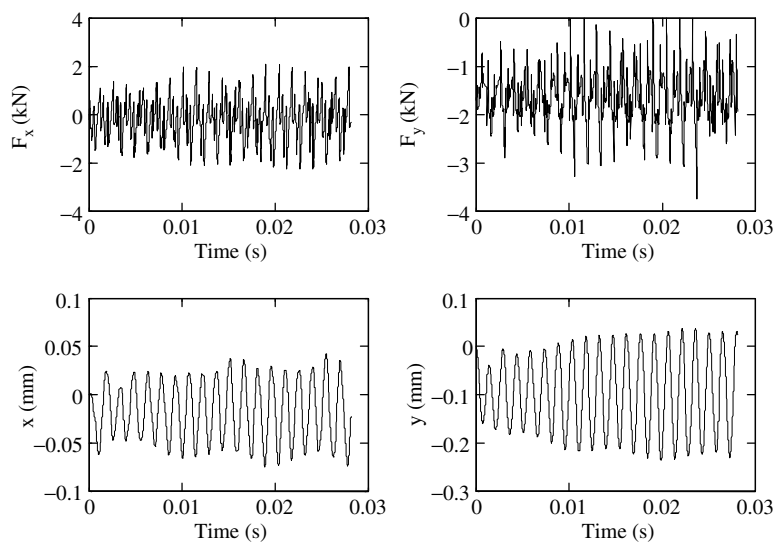


FIGURE 10.17 Time-domain simulation for Example 4 with $N_s = 32,000$ rpm and $d = 3.795$ mm.

$N_s = 1000$ rpm is found to be 2.25 mm and the limiting depth-of-cut for $N_s = 32,000$ rpm is found to be 3.45 mm. The results are shown in Figure 10.14 to Figure 10.17. The system is stable in Figure 10.14 and Figure 10.16, while instability is evidenced in Figure 10.15 and Figure 10.17 by the force in the y direction saturating at 0 kN.

This section presented the technique of time-domain simulation as an alternative means to analyze regenerative chatter. Time-domain simulations are the direct numerical simulations of the force process and structural vibrations. A process parameter is changed iteratively from simulation to simulation to determine the critical value at which chatter occurs. A sufficiently small time step must be utilized in the numerical integrations to account for the small system time constants associated with the large structural frequencies.

10.5 Chatter Detection

Regenerative chatter is easily detected by an operator due to the loud, high-pitched noise it produces and the distinctive “chatter marks” it leaves on the part surface. However, automatic detection is required for intelligent manufacturing (Cho and Ehmann, 1988; Delio et al., 1992). At the onset of chatter, process signals (e.g., force, vibration) contain significant energy at the chatter frequency. It is a well-known fact that the chatter frequency will be close to a dominant structural frequency. The most common method to detect the presence of chatter is to threshold the frequency signal of a process signal. To analyze the frequency content of a signal, a Fourier transform, or fast Fourier transform, is performed. If the frequency content of the resulting signal near a dominant chatter frequency is above a threshold value, then chatter is determined to be present. It should be noted that machining process signals also contain significant energy at the tooth-passing frequency. If the dominant structural frequencies and tooth-passing frequency are sufficiently separated, then the tooth-passing frequency may be ignored when determining the presence of chatter. If the dominant structural frequencies and tooth-passing frequency are close, then the signal must be filtered at the tooth-passing frequency using a notch filter. Also, forced vibrations, such as those resulting from the impact between the cutting tool and part, must not be allowed to falsely trigger the chatter

detection algorithm. These thresholding algorithms all suffer from the lack of an analytical method of selecting a threshold value. This value is typically selected empirically and will not be valid over a wide range of cutting conditions and machining operations.

10.5.1 Example 5

An experimental face-milling operation, a complete description of which is given in Landers (1997), is conducted with a spindle speed of 1500 rpm and a tool with four teeth. The dominant structural frequencies are 334, 414, 653, and 716 Hz. The machining force F_z is sampled at a frequency of 2000 Hz, and the time-domain signal is transformed into the frequency domain via a Fourier transform using 80 points (i.e., one spindle revolution). The power spectral density of the force signal is shown for depths-of-cut of 1.0 and 1.5 mm in Figure 10.18 and Figure 10.19, respectively. In Figure 10.18, there is significant energy at 100 Hz, which is the tooth-passing frequency. There is also significant energy at 750 Hz due to structural vibrations; however, the system did not chatter, as evidenced by the lack of chatter marks on the part and a high-pitched sound during machining. In Figure 10.19, there is significant energy at 665 Hz as well as 100 Hz. Chatter was evidenced by the chatter marks left on the part surface and the high-pitched sound during machining. The results demonstrate that the chatter frequency is 665 Hz, which is near the dominant structural frequency of 653 Hz. Note that the power spectral density at the frequency of 0 Hz is ignored in Figure 10.18 and Figure 10.19. This component is stronger than the components at all other frequencies since the machining force F_z fluctuates about a static, nonzero value. In this application, a thresholding algorithm may ignore the low frequencies where the tooth-passing frequency is strong; however, if the operation were to be performed at a higher spindle speed, say 7500 rpm, or the number of teeth were increased from 4 to 20, the tooth-passing frequency would be 500 Hz, close to the structural frequencies. In this case, the force signal would have to be filtered at the tooth-passing frequency.

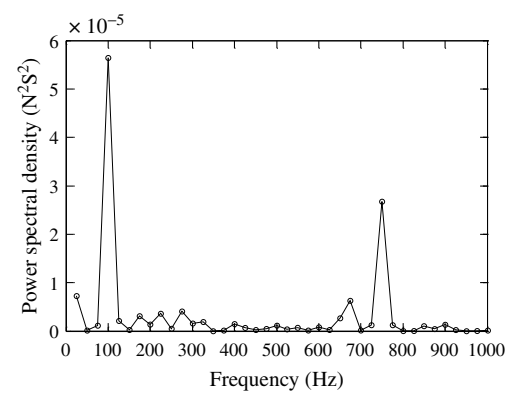


FIGURE 10.18 Power spectral density of F_z in a face-milling operation with $d = 1.0$ mm.

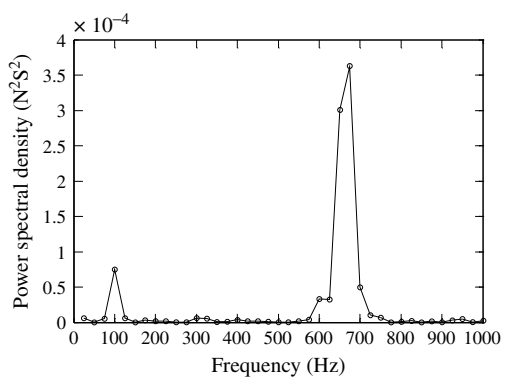


FIGURE 10.19 Power spectral density of F_z in a face-milling operation with $d = 1.5$ mm.

This section presented techniques to detect the occurrence of regenerative chatter. The phenomenon of regenerative chatter is easily detected by an operator due to the loud, high-pitched noise it produces and the distinctive “chatter marks” it leaves on the part surface. The most common method to detect the presence of chatter is to threshold the frequency signal of a process signal. In this case, one must be careful to separate out the spindle rotation and tooth-passing frequencies.

10.6 Chatter Suppression

Most machining process plans are derived from handbooks or from a database. Since these plans do not consider the physical machine that will be used, chatter-free operations cannot be guaranteed. Thus, multiple iterations, where the feed or spindle speed are adjusted using the operator's experience, are typically required. The tool position may also be adjusted (e.g., the depth-of-cut may be decreased) to suppress chatter and, while this is guaranteed to be effective due to the presence of the asymptotic stability borderline, this approach is typically not employed since part program must be rewritten to add multiple passes, thereby drastically decreasing productivity. The stability lobe diagram can be used as a tool to plan chatter-free machining operations and productivity can be greatly increased by selecting the process parameters to lie in a pocket between two lobes. A cutting tool design methodology (Altintas et al., 1999) has also been proposed for milling tools where the pitch is slightly adjusted such that the teeth are not evenly spaced. The variable pitch has the effect of changing the phase difference between successive teeth vibrations and, if designed properly, will suppress chatter. These design techniques are very sensitive to parameter variations and model uncertainty, and may not be used reliably for a large range of operating conditions. This section will describe methods for automatic chatter suppression.

10.6.1 Spindle-Speed Selection

For the stability lobe diagram generated from a system modeled as having a one-dimensional structure, it is seen that the maximum depths-of-cut are located at the tooth-passing frequencies (i.e., the number of teeth multiplied by the spindle speed) corresponding to the dominant structural frequency and integer fractions thereof. If the dominant structural frequency is known, it may be used as an aid in selecting spindle speeds; however, the structural dynamics are often unknown and may be determined only through costly testing. Further, structural dynamics change drastically over time.

It is known, however, that during chatter, the dominant frequency seen in the cutting-process output is close to a dominant structural frequency. This fact is used in Smith and Delio (1992) to suppress chatter automatically. The following steps are taken:

1. Implement a chatter detection routine to determine the presence of chatter.
2. If chatter is detected, determine the chatter frequency, ω_c . This will be the frequency at which the process signal has the greatest energy.
3. Set the new spindle speed to be $N_s = \omega_c / [N_t(N + 1)]$, where N is the smallest positive integer such that the new spindle speed does not violate the maximum spindle speed constraint.
4. Repeat Steps 1 to 3 until the chatter has been suppressed.

The equation $N_s = \omega_c / [N_t(N + 1)]$ may be interpreted as selecting the tooth-passing frequency, or an integer fraction thereof, corresponding to the approximate dominant structural frequency. Note that if the depth-of-cut is too large and the maximum spindle speed is too small, this technique will not be effective and the feed or depth-of-cut must be adjusted, or the spindle speed must be continuously varied.

10.6.2 Example 6

The feed force for a turning operation is given by Equation 10.1, and the structural dynamics are given by Equation 10.26. The system parameters are $P = 0.75 \text{ kN/mm}^2$, $f_{\text{nom}} = 0.1 \text{ mm}$, $\omega_n = 750 \text{ Hz}$, $\zeta = 0.1$, and $k = 15 \text{ kN/mm}$. The depth-of-cut is 5 mm. The spindle speed that should be selected to suppress chatter if the chatter frequency is 725 Hz, when the spindle speed is not constrained, is determined. The spindle speed that should be selected to suppress chatter if the maximum spindle speed is 15,000 rpm is also determined. The system is simulated for a spindle speed of 10,000 rpm for ten spindle revolutions and then for ten spindle revolutions for the spindle

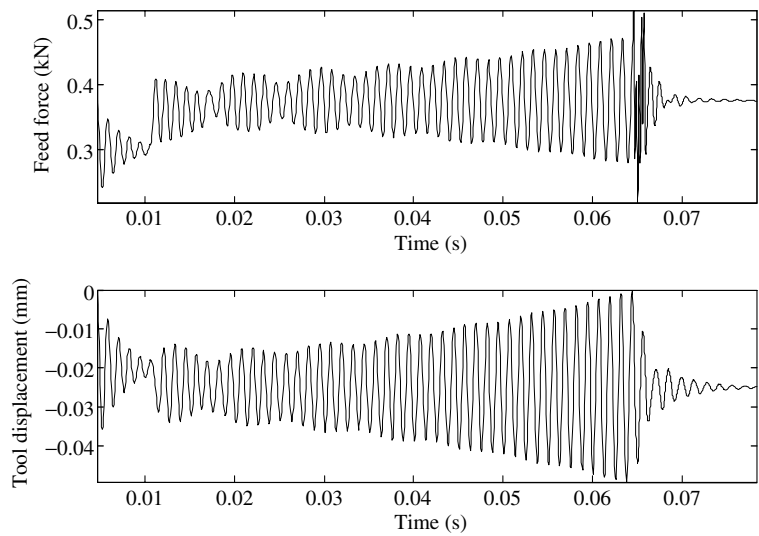


FIGURE 10.20 Time-domain simulations using spindle speed selection with $N_s = 43,500$ rpm.

speed calculated when the spindle speed is not constrained. The simulation is then repeated for the spindle speed calculated when the spindle speed is constrained. Feed force and tool displacement are plotted for both cases.

For a chatter frequency of 725 Hz, the optimal spindle speed is $60(725) = 43,500$ rpm. Other possible spindle speeds are $43,500/2 = 21,750$ rpm, $43,500/3 = 14,500$ rpm, $43,500/4 = 10,875$ rpm, and so on. Therefore, when the maximum spindle speed is 15,000 rpm, a spindle speed of 14,500 rpm is used. The time domain simulations are in Figure 10.20 and Figure 10.21. The results illustrate that a depth-of-cut of 5.3 mm is stable at 43,500 rpm, but not at 14,500 rpm. Therefore, if the spindle speed is limited to 15,000 rpm, spindle-speed selection may not be used to suppress the chatter present in the machining operation.

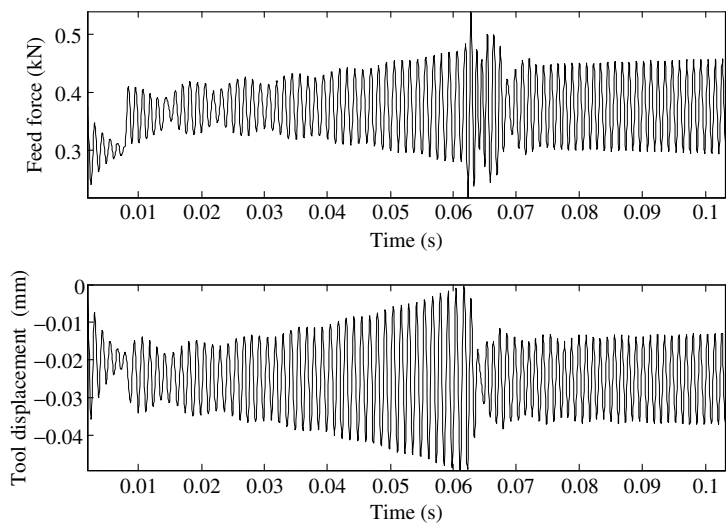


FIGURE 10.21 Time-domain simulations using spindle speed selection with $N_s = 14,500$ rpm.

10.6.3 Feed and Depth-of-Cut Selection

When chatter occurs, operators will sometimes increase the feedrate via the feedrate override button on the machine tool control panel. This has the effect of increasing the feed, assuming the spindle speed remains constant. When linear chatter analysis techniques are employed, the force process gains are linearized about the nominal feed, and stability does not appear to be affected by the nominal feed. However, the stability results are only valid for a small region about the nominal feed. It is well known that there is a nonlinear relationship between the machining forces and the feed of the form $F = P(f)df$. The pressure can be expressed in the form $P(f) = Kf^\alpha$ where $\alpha < 0$; thus, the pressure decreases as the feed increases. Since the stable depth-of-cut is inversely proportional to the pressure, the stability limit will increase as the feed increases, assuming the spindle speed remains constant. An illustration of this phenomenon was shown in Example 3: when the feed was increased from 0.1 to 0.2 mm, chatter was suppressed. While increasing the feed can suppress chatter, the sensitivity of chatter to feed is limited and other adverse phenomenon, such as tooth chippage, may occur.

Another method to suppress chatter is to decrease the depth-of-cut (Weck et al., 1975). This method is guaranteed to work as evidenced by stability lobe diagrams. However, this method is typically not preferred as it dramatically decreases operation productivity by increasing the total number of tool passes that are required to complete the operation.

10.6.4 Spindle-Speed Variation

Spindle speed variation (SSV) is another technique that has shown the ability to suppress chatter (Inamura and Sata, 1974; Lin et al., 1990). The spindle speed is varied about some nominal value, typically in a sinusoidal manner. Although SSV is a promising technique, the theory required to guide the designer in the selection of suitable amplitudes and frequencies is in its infancy (Radulescu et al., 1997a, 1997b; Sastry et al., 2002). Also, in some cases, SSV may create chatter that would not occur when using a constant spindle speed.

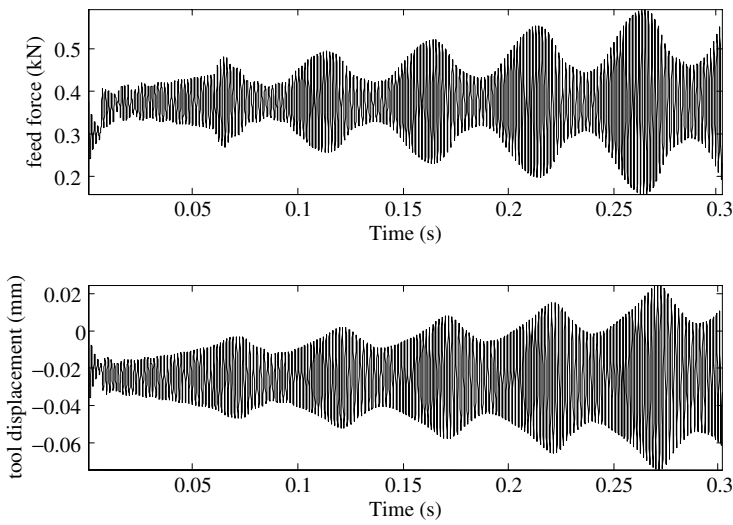


FIGURE 10.22 Time-domain simulations using spindle-speed variation with $A = 0.1$ and $\Omega = 20$ Hz.

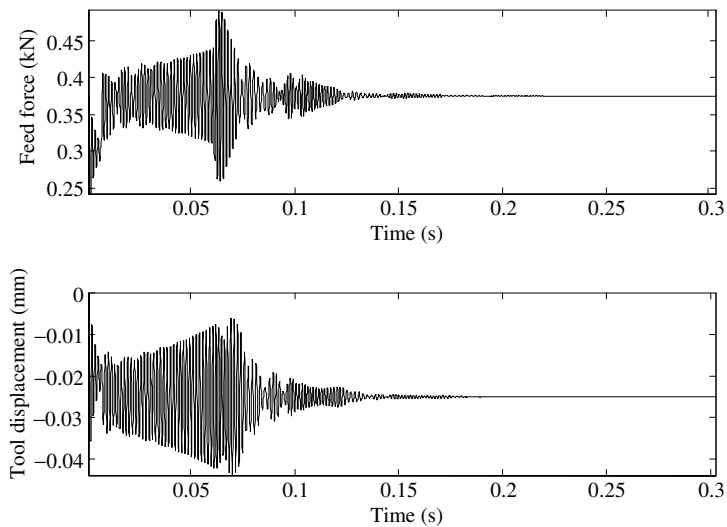


FIGURE 10.23 Time-domain simulations using spindle-speed variation with $A = 0.25$ and $\Omega = 20$ Hz.

10.6.5 Example 7

The feed force for a turning operation is given by Equation 10.1 and the structural dynamics are given by Equation 10.26. The system parameters are $P = 0.75 \text{ kN/mm}^2$, $f_{\text{nom}} = 0.1 \text{ mm}$, $\omega_n = 750 \text{ Hz}$, $\zeta = 0.1$, and $k = 15 \text{ kN/mm}$. The depth-of-cut is 5 mm. The system is simulated for a nominal spindle speed of $N_{\text{nom}} = 10,000 \text{ rpm}$ for 10 spindle revolutions and then for 30 spindle revolutions for the spindle speed calculated from Equation 10.67 for the following three cases: $A = 0.1$ and $\Omega = 20 \text{ Hz}$, $A = 0.25$ and $\Omega = 20 \text{ Hz}$, and $A = 0.25$ and $\Omega = 160 \text{ Hz}$. Feed force and tool displacement are plotted for all three cases.

$$N_s(t) = N_{\text{nom}}[1 + A \sin(\Omega t)] \tag{10.67}$$

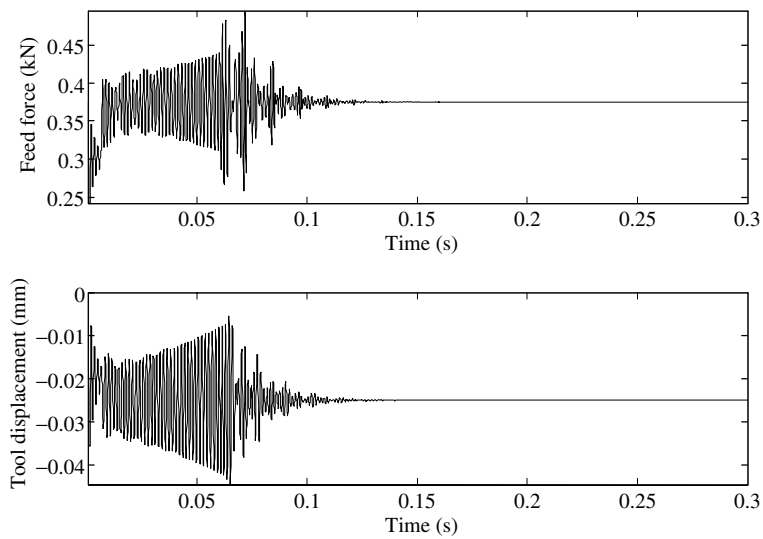


FIGURE 10.24 Time-domain simulations using spindle-speed variation with $A = 0.25$ and $\Omega = 160$ Hz.

The time-domain simulations are in Figure 10.22 to Figure 10.24 for the respective cases. The results illustrate that SSV may be utilized to suppress chatter; however, the amplitude and frequency of the spindle speed vibration must be carefully chosen.

This section presented several techniques to suppress regenerative chatter. The three major techniques to suppress chatter are spindle-speed selection, feed selection, and SSV. In spindle-speed selection, the spindle speed is adjusted to be a multiple of the chatter frequency to place the spindle speed in a pocket of the stability lobe diagram. In feed selection, the feed is increased to suppress chatter. In SSV, the spindle speed is varied in a sinusoidal manner to decrease the phase difference between the current and previous tooth passes.

10.7 Case Study

A case study of regenerative chatter for a face-milling operation is now presented. Further details are presented in Landers (1997). The machine tool is a three-axis vertical milling machine (Figure 10.25). Each axis has a linear encoder with a resolution of $10\text{ }\mu\text{m}$ mounted on it. The axis motors (186 W) drive pulleys that rotate leadscrews and provide motion to the linear axes. The spindle (2240 W) drives the face mill (Carbology R/L220.13-02.00-12, 50 mm diameter). The tool holds four carbide inserts (Carbology SEAN 42AFTN-M14 HX, 45° lead angle). The part is 6061 aluminum. The spindle is run open-loop. A Kistler 9293 piezoelectric three-component dynamometer was utilized for force process modeling and chatter detection. The x and y channels have a natural frequency of 4.5 kHz, rigidity of $0.7\text{ kN}/\mu\text{m}$, and range of -20 to 20 kN . The z channel has a natural frequency of 5 kHz, rigidity of $7\text{ kN}/\mu\text{m}$, and range of -100 to 200 kN . A Bently Nevada 3000 Series Type 190 proximity transducer was utilized to measure the static stiffnesses of the structural components. The sensor gain is 8 V/mm , the response is flat to 10 kHz, and the range is 1.02 mm . A Kistler Quartz Model #802A accelerometer (resonant frequency 36.7 kHz) was utilized to measure the dynamic characteristics of the structural components.

The cutting and thrust pressures, respectively, are

$$P_C = 0.29f^{-0.25}d^{-0.13}\left(\frac{V}{1000}\right)^{-0.72} \quad (10.68)$$

$$P_T = 0.16f^{-0.40}d^{-0.41}\left(\frac{V}{1000}\right)^{-0.58} \quad (10.69)$$

The transfer function matrices, respectively, between the tool structure and machining forces, and the part structure and machining forces are modeled as

$$\begin{bmatrix} x_t(s) \\ y_t(s) \end{bmatrix} = \begin{bmatrix} \frac{(4500^2/14)}{s^2 + 2(0.07)(4500)s + 4500^2} & 0 \\ 0 & \frac{(4100^2/14)}{s^2 + 2(0.11)(4100)s + 4100^2} \end{bmatrix} \begin{bmatrix} F_x(s) \\ F_y(s) \end{bmatrix} \quad (10.70)$$

$$\begin{bmatrix} x_p(s) \\ y_p(s) \end{bmatrix} = - \begin{bmatrix} \frac{(2600^2/9.5)}{s^2 + 2(0.09)(2600)s + 2600^2} & 0 \\ 0 & \frac{(2100^2/9.5)}{s^2 + 2(0.22)(2100)s + 2100^2} \end{bmatrix} \begin{bmatrix} F_x(s) \\ F_y(s) \end{bmatrix} \quad (10.71)$$

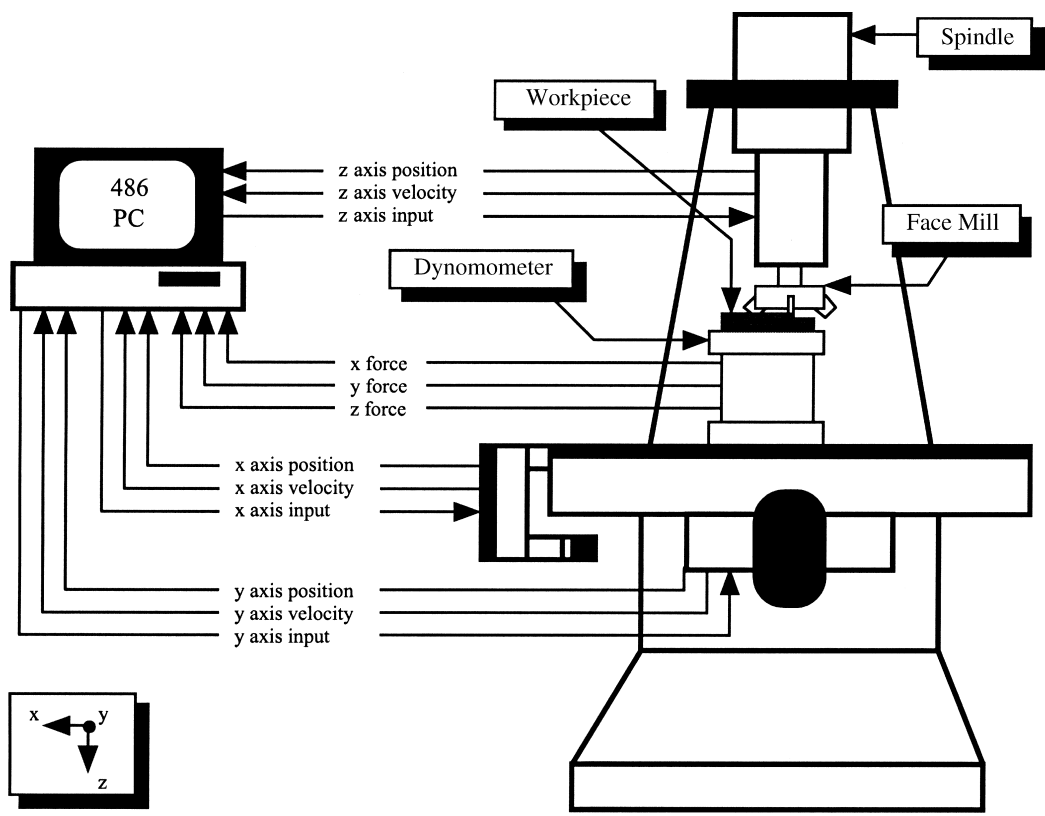


FIGURE 10.25 Three-axis vertical machine tool schematic.

Using the machining force and structural models, a stability lobe diagram was constructed using time domain simulations. Experimental data were collected by adjusting the depth-of-cut in increments of 0.1 mm until chatter occurred. The time-domain simulations and the experimental data are plotted in Figure 10.26. The cutting conditions were $f_t = 0.10$ mm/tooth, $N_t = 4$ teeth, $\theta_{en} = -90^\circ$, and $\theta_{ex} = 90^\circ$. The chatter detection methodology for this system was described in Example 5.

Spindle-speed adjustment is typically a more productive chatter suppression option. However, the machine tool in this case study is not equipped with automatic spindle speed control and, thus, the depth-of-cut is adjusted to suppress chatter.

When chatter is detected, the chatter suppressor rewrites the part program to accommodate one additional tool pass (Figure 10.27). Therefore, the new operation depth-of-cut is

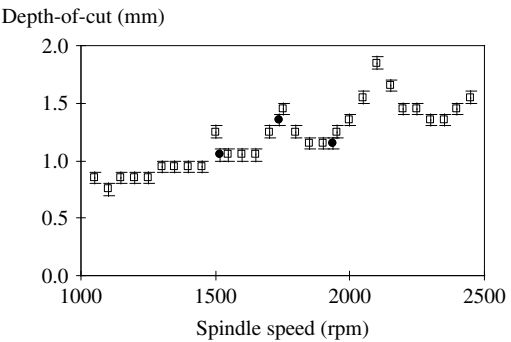


FIGURE 10.26 Stability lobe diagram for a face-milling operation — time-domain simulations (empty boxes) and experimental points (filled circles).

$$d_n = \frac{d_p}{1 + N_c} \tag{10.72}$$

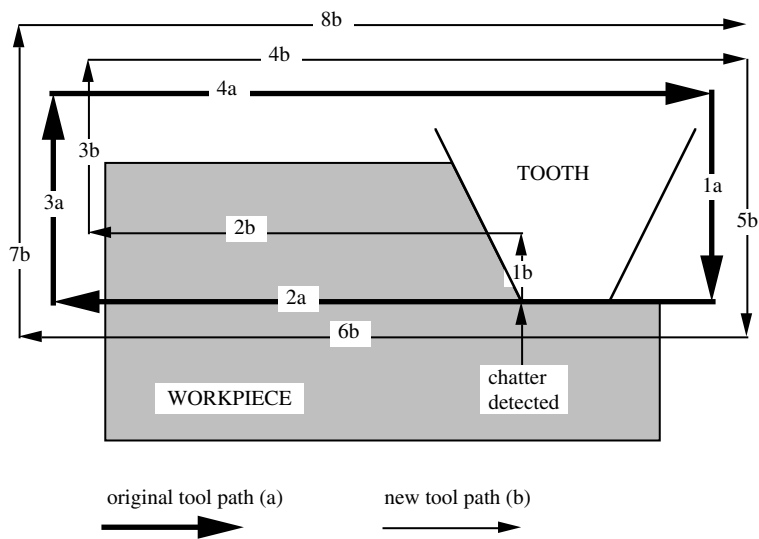


FIGURE 10.27 Original tool path (a) is rewritten when chatter occurs. New tool path (b) contains an additional tool pass.

where d_p is the previous operation depth-of-cut and N_c is the number of times the chatter suppression routine has been invoked. The new value may be well below the stability limit; however, making all passes an equal depth-of-cut provides a good balance between productivity and the search for a stable depth-of-cut. Results of this controller are presented in Landers and Ulsoy (1998, 2001).

Nomenclature

Symbol	Quantity	Symbol	Quantity
d	depth-of-cut (mm)	T	spindle rotational period (s)
d_{lim}	limiting stable depth-of-cut (mm)	T_t	tooth rotational period (s)
f	feed (mm)	x_p	part structural displacement in x direction (mm)
f_t	feed per tooth (mm)	x_t	cutting tool structural displacement in x direction (mm)
F	machining force (kN)	y_p	part structural displacement in y direction (mm)
F_C	cutting force (kN)	y_t	cutting tool structural displacement in y direction (mm)
F_T	thrust force (kN)	z	cutting tool structural displacement in z direction (mm)
F_x	force acting on cutting tool in x direction (kN)	θ	tooth angle (rad)
F_y	force acting on cutting tool in y direction (kN)	θ_{en}	tooth entry angle (rad)
F_z	force acting on cutting tool in z direction (kN)	θ_{ex}	tooth exit angle (rad)
k	structural stiffness (kN/mm)	ω_c	chatter frequency (rad/sec)
N_s	spindle speed (rpm)	ω_n	structural natural frequency (rad/sec)
N_t	number of teeth	ψ_r	lead angle (rad)
P	machining pressure (kN/mm ²)	ζ	structural damping ratio
P_C	cutting pressure (kN/mm ²)		
P_T	thrust pressure (kN/mm ²)		
t	time (sec)		

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