

Fluid-Induced Vibration

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Seon M. Han

Texas Tech University

Summary

This chapter gives an overview on the subject of fluid-induced vibration in an ocean environment. The main objective is to show how the fluid forces on an offshore structure due to current and random waves are modeled. The chapter is divided into three sections. The first section describes the ocean environment, especially the currents and random waves. The second section is dedicated to obtaining fluid forces utilizing the results from the first section and the third section gives some examples to show how the results from the first two sections can be used in practice. In the first section, the concept of spectral density is introduced. For a given spectrum, methods to obtain a sample time series are given. In the second section, the forces that the fluid can exert on a body are discussed. The regimes in which inertia, drag, or diffraction forces are dominant are shown in terms of the ratio of the wave height to the structural diameter and the ratio of the structural diameter to the wavelength. The Morison equation is extended to the case of a moving inclined cylinder. The Morison equation requires the use of experimentally determined fluid coefficients such as added mass, inertia, and drag coefficients. Plots of these fluid coefficients for various values of the fluid parameters are reproduced here. The vortex shedding force is discussed briefly. In the third section, four examples are given to show how fluid forces affect the static and dynamics of ocean structures, how the significant wave height can be chosen to represent the condition in a certain area for a long time, and how the time series can be constructed from a given spectrum. Finally, the available numerical codes for modeling slender flexible bodies in fluids are listed.

11.1 Description of the Ocean Environment

In modeling offshore structures, one needs to account for the forces exerted by the surrounding fluid. In-depth studies are given in Kinsman (1965), Sarpkaya and Isaacson (1981), Wilson (1984), Chakrabarti (1987), and Faltnsen (1993). The vibration characteristics of a structure can be significantly altered when it is surrounded by water. For example, damping by the fluid (or the added mass) lowers the natural

frequency of vibration. When considering the dynamics of an offshore structure, one must also consider the forces due to the surrounding fluid. The two important sources of fluid motion are ocean waves and ocean currents.

Most steady large currents are generated by the drag of the wind passing over the surface of the water, and they are confined to a region near the ocean surface. Tidal currents are generated by the gravitational attraction of the sun and the moon, and they are most significant near coasts. The ultimate source of the ocean circulation is the uneven radiation heating of the Earth by the Sun.

Isaacson (1988) suggested an empirical formula for the current velocity in the horizontal direction as a function of depth:

$$U_c(x) = (U_{\text{tide}}(d) + U_{\text{circulation}}(d)) \left(\frac{x}{d} \right)^{1/7} + U_{\text{drift}}(d) \left(\frac{x - d + d_0}{d_0} \right) \quad (11.1)$$

where U_{drift} is the wind-induced drift current, U_{tide} is the tidal current, $U_{\text{circulation}}$ is the low-frequency long-term circulation, x is the vertical distance measured from the ocean bottom, d is the depth of the water, and d_0 is the smaller of the depth of the thermocline and 50 m. The value of U_{tide} is obtained from tide tables, and U_{drift} is about 3% of the 10 min mean wind velocity at 10 m above the sea level.

It should be noted that these currents evolve slowly compared with the time scales of engineering interests. Therefore, they can be treated as a quasisteady phenomenon. Waves, on the other hand, cannot be treated as a steady phenomenon. The underlying physics that govern wave dynamics are too complex and, therefore, waves must be modeled stochastically. The subsequent section discusses the concept of the spectral density, available ocean wave spectral densities, a method to obtain the spectral density from wave time histories, methods to obtain a sample time history from a spectral density, the short-term and long-term statistics, and a method to obtain fluid velocities and accelerations from wave elevation using linear wave theory.

11.1.1 Spectral Density

Here, we will consider only surface gravity waves. Let us first consider a regular wave in order to familiarize ourselves with the terms that are used to describe a wave. The wave surface elevation is denoted as $\eta(x, t)$ and can be written as $\eta(x, t) = A \cos(kx - \omega t)$, where k is the wave number, and ω is the angular frequency. Figure 11.1 shows the surface elevation at two time instances ($t = 0$ and $t = \tau$) and the surface elevation at a fixed location ($x = 0$). A is the amplitude, H is the wave height or the distance between the maximum and minimum wave elevation or twice the amplitude, and T is the period given by $T = 2\pi/\omega$.

In practice, waves are not regular. Figure 11.2 shows a schematic time history of an irregular wave surface elevation. The wave height and frequency are not easy to find. Therefore, we rely on a statistical description for the wave elevation such as the wave spectral density. The spectral density tells us how the energy of the system is distributed among frequencies. The random surface elevation $\eta(t)$ can be thought of as a summation of regular waves with different frequencies. The surface elevation $\eta(t)$ is related to its Fourier transform $X(\omega)$ by

$$\eta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \exp(-i\omega t) d\omega$$

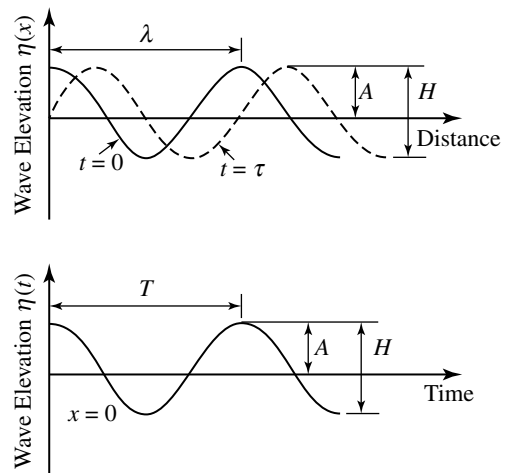


FIGURE 11.1 Regular wave.

Suppose that the energy of the system is proportional to $\eta^2(t)$ so that we can write the energy as

$$E = \frac{1}{2} C \eta^2(t)$$

where C is the proportionality constant.

Let us assume that the expected value of the energy is given by

$$E\{E\} = \frac{1}{2} C E\{\eta^2(t)\}$$

where $E\{\eta^2(t)\}$ is the mean square of $\eta(t)$. If $\eta(t)$ is an *ergodic process*, then the mean square of $\eta(t)$ can be approximated by the time average over a long period of time:

$$E\{\eta^2(t)\} = \lim_{T_s \rightarrow +\infty} \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \eta^2(t) dt = \lim_{T_s \rightarrow +\infty} \frac{1}{T_s} \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega \quad (11.2)$$

where we have used Parseval's theorem

$$\int_{-\infty}^{\infty} \eta^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega \quad (11.3)$$

where

$$|X(\omega)|^2 = X(\omega)X^*(\omega), \quad X(\omega) = \int_{-\infty}^{\infty} \eta(t) \exp(-i\omega t) dt, \quad X^*(\omega) = \int_{-\infty}^{\infty} \eta(t) \exp(i\omega t) dt$$

We define the power spectral density (or simply the spectrum) as

$$S_{\eta\eta}(\omega) \equiv \frac{1}{2\pi T_s} |X(\omega)|^2 \quad (11.4)$$

so that $E\{\eta^2(t)\}$ is given by

$$E\{\eta^2(t)\} = \int_{-\infty}^{\infty} S_{\eta\eta}(\omega) d\omega \quad (11.5)$$

For a zero-mean process, $E\{\eta^2(t)\}$ is also the variance σ_η^2 . The spectral density has units of $\eta^2 t$. Where η is the wave elevation, the spectral density has a unit of $m^2 \text{ sec}$.

It can also be shown that $S_{\eta\eta}(\omega)$ is related to the autocorrelation function, $R(\tau)$, by the Wiener–Khinchine relations (Wiener, 1930; Khinchine, 1934):

$$S_{\eta\eta}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{\eta\eta}(\tau) \exp(-i\omega\tau) d\tau, \quad R_{\eta\eta}(\tau) = \int_{-\infty}^{\infty} S_{\eta\eta}(\omega) \exp(i\omega\tau) d\omega \quad (11.6)$$

It should be noted that, in some textbooks, the factor $1/2\pi$ appears in the second equation instead of the first. Figure 11.3 shows some important pairs of $S_{\eta\eta}(\omega)$ and $R_{\eta\eta}(\tau)$.

There are a few properties of the spectral density that readers should become familiar with. The first property is that the spectral density function of a real-valued stationary process is both real and symmetric. That is, $S_{\eta\eta}(\omega) = S_{\eta\eta}(-\omega)$ (Equation 11.4). Secondly, the area under the spectral density is equal to $E\{\eta^2(t)\}$ (Equation 11.5) and is also equal to $R_{\eta\eta}(0) = \sigma_\eta^2 - \mu_\eta^2$, where σ_η^2 is the variance and μ_η^2 is the mean of $\eta(t)$. In most cases, we only consider a zero-mean process so that the area under the spectral density is just σ_η^2 . If the process does not have a zero mean, the mean can be subtracted from it so that the process has a zero mean.

For ocean applications, a one-sided spectrum in terms of cycles per second (cps) or hertz is often used. We will denote the one-sided spectrum with a superscript “o”. The one-sided spectrum can be obtained from the two-sided spectrum by

$$S_{\eta\eta}^o(\omega) = 2S_{\eta\eta}(\omega), \quad \omega \geq 0$$

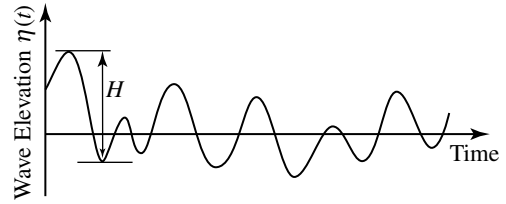


FIGURE 11.2 Time history of random wave.

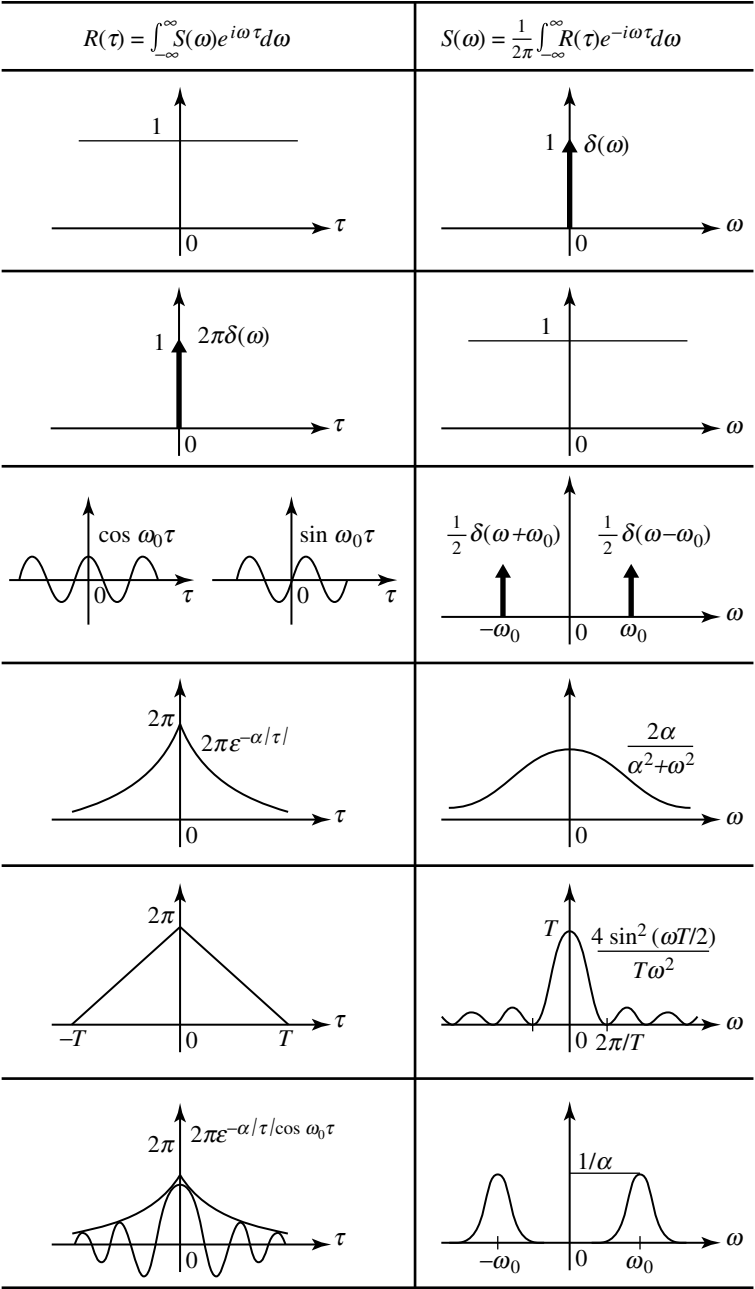


FIGURE 11.3 Relationship between the autocorrelation function and the power spectral density.

The two-sided spectrum in terms of ω can be transformed to the spectrum in terms of f (where $\omega = 2\pi f$) by

$$S_{\eta\eta}(f) = 2\pi S_{\eta\eta}(\omega), \quad f, \omega \geq 0$$

Then, the two-sided spectrum in terms of ω can be transformed to the one-sided spectrum in terms of cps (or hertz) by

$$S_{\eta\eta}^o(f) = 4\pi S_{\eta\eta}(\omega), \quad f, \omega \geq 0$$

It should be noted that the spectral density that we have defined here is the *amplitude half-spectrum*. The amplitude, height, and height double spectra are related to the amplitude half-spectrum by

$$S^A(\omega) = 2S(\omega), \quad S^H(\omega) = 8S(\omega), \quad S^{2H}(\omega) = 16S(\omega)$$

11.1.2 Ocean Wave Spectral Densities

In this section, we will discuss spectral density models to describe a random sea. An excellent review of existing spectral density models is given in Chapter 4 of Chakrabarti (1987).

The ocean wave spectrum models are semiempirical formulas. That is, they are derived mathematically but the formulation requires one or more experimentally determined parameters. The accuracy of the spectrum depends significantly on the choice of these parameters.

In formulating spectral densities, the parameters that influence the spectrum are *fetch limitations, decaying vs. developing seas, water depth, current, and swell*. The fetch is the distance over which a wind blows in a wave-generating phase. Fetch limitation refers to the limitation on the distance due to some physical boundaries so that full wave development is prohibited. In a developing sea, the sea has not yet reached its stationary state under a stationary wind. In contrast, a wind has blown for a sufficient time in a fully developed sea, and the sea has reached its stationary state. In a decaying sea, the wind has dropped off from its stationary value. Swell is the wave motion caused by a distant storm and persists even after the storm has died down or moved away.

The Pierson–Moskowitz (P–M) spectrum (Pierson and Moskowitz, 1964) is the most extensively used spectrum for representing a fully developed sea. It is a one-parameter model in which the sea severity can be specified in terms of the wind velocity. The P–M spectrum is given by

$$S_{\eta\eta}^o(f) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} \exp\left(-0.74 \left(\frac{g}{U_{w,19.5 \text{ m}}}\right)^4 \omega^{-4}\right)$$

where g is the gravitational constant and $U_{w,19.5 \text{ m}}$ is the wind speed at a height of 19.5 m above the still water. The P–M spectrum is also called the wind-speed spectrum because it requires wind data. It can also be written in terms of the modal frequency ω_m as

$$S_{\eta\eta}^o(f) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} \exp\left(-1.25 \left(\frac{\omega_m}{\omega}\right)^4\right) \quad (11.7)$$

Note that the modal frequency is the frequency at which the spectrum is the maximum.

In some cases, it may be more convenient to express the spectrum in terms of significant wave height rather than the wind speed or modal frequency. For a narrowband Gaussian process¹, the significant wave height is related to the standard deviation by $H_s = 4\sigma_\eta$. The standard deviation is the square root of the area under the spectral density, $\int_{-\infty}^{\infty} S_{\eta\eta}(\omega) d\omega = \sigma_\eta^2$. Then, the spectrum can be written as

$$S_{\eta\eta}^o(f) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} \exp\left(-\frac{0.0324 g^2}{H_s^2} \omega^{-4}\right) \quad (11.8)$$

and the peak frequency and the significant wave height are related by

$$\omega_m = 0.4 \sqrt{g/H_s} \quad (11.9)$$

The P–M spectrum is applicable for deep water, unidirectional seas, fully developed and local-wind-generated seas with unlimited fetch, and was developed for the North Atlantic. The effect of swell is not accounted for. Although it was developed for the North Atlantic, the spectrum is valid for other locations. However, the limitation that the sea is fully developed may be too restrictive because it cannot model the

¹See Section 11.1.5 for details.

effect of waves generated at a distance. Therefore, we consider a two-parameter spectrum, such as the Bretschneider spectrum, in order to model a sea that is not fully developed as well as a fully developed sea.

The Bretschneider spectrum (Bretschneider, 1959, 1969) is a two-parameter spectrum in which both the sea severity and the state of development can be specified. The Bretschneider spectrum is given by

$$S_{\eta\eta}^o(f) = 0.169 \frac{\omega_s^4}{\omega^5} H_s^2 \exp\left(-0.675\left(\frac{\omega_s}{\omega}\right)^4\right)$$

where $\omega_s = 2\pi/T_s$ and T_s is the significant period. The sea severity can be specified by H_s and the state of development can be specified by ω_s . It can be shown that the relationship $\omega_s = 1.167\omega_m$ (equivalent to $\omega_s = 1.46/\sqrt{H_s}$) renders the Bretschneider spectrum and the P-M spectrum equivalent. Figure 11.4 shows the Bretschneider spectra for $H_s = 4$ m. When $\omega_s = 0.731$ rad/sec, the P-M and the Bretschneider spectra are identical. It should be noted that the developing sea will have a slightly higher modal frequency than the fully developed sea, and can be described by ω_s greater than $1.46/\sqrt{H_s}$.

Other two-parameter spectral densities that are often used are the International Ship Structures Congress (ISSC) and the International Towing Tank Conference (ITTC) spectra. The ISSC spectrum is written in terms of the significant wave height and the mean frequency, where the mean frequency is given by

$$\bar{\omega} = \sqrt{\frac{\int_0^\infty \omega S(\omega) d\omega}{\int_0^\infty S(\omega) d\omega}} = 1.30\omega_m$$

Thus, the ISSC spectrum is given by

$$S_{\eta\eta}^o(f) = 0.111 \frac{\bar{\omega}^4}{\omega^5} H_s^2 \exp\left(-0.444\left(\frac{\bar{\omega}}{\omega}\right)^4\right)$$

The ITTC spectrum is based on the significant wave height and the zero crossing frequency and is given by

$$S_{\eta\eta}^o(f) = 0.0795 \frac{\omega_z^4}{\omega^5} H_s^2 \exp\left(-0.318\left(\frac{\omega_z}{\omega}\right)^4\right)$$

where the zero crossing frequency, ω_z , is given by

$$\omega_z = \sqrt{\frac{\int_0^\infty \omega^2 S(\omega) d\omega}{\int_0^\infty S(\omega) d\omega}} = 1.41\omega_m$$

The Bretschneider, ITTC, and ISSC spectra are called two-parameter spectra, and they can be written as

$$S_{\eta\eta}^o(f) = \frac{A}{4} \frac{\bar{\omega}^4}{\omega^5} H_s^2 \exp\left(-A\left(\frac{\bar{\omega}}{\omega}\right)^4\right)$$

with A and $\bar{\omega}$ given in Table 11.1.

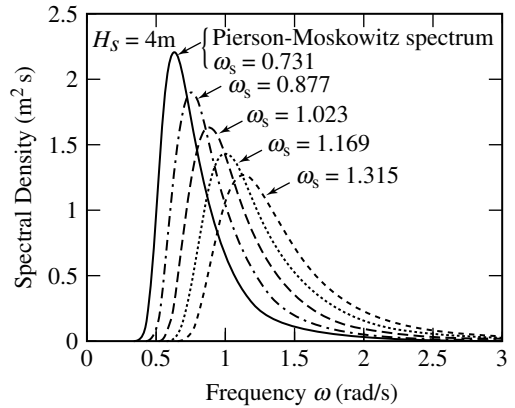


FIGURE 11.4 Bretschneider spectrum with various values of ω_s .

TABLE 11.1 Two-Parameter Spectrum Models
 $S_{\eta\eta}^o(\omega) = (A/4)H_s^2\bar{\omega}^4/\omega^5 \exp(-A(\omega/\bar{\omega})^{-4})$

Model	A	$\bar{\omega}$
Bretschneider	0.675	ω_s
ITTC	0.318	ω_z
ISSC	0.4427	$\bar{\omega}$

The spectra that we have discussed so far do not allow us to generate spectra with two peaks to represent local or distant storms or to specify the sharpness of the peaks. The Ochi–Hubble (O–H) spectrum (Ochi and Hubble, 1976) is a six-parameter spectrum with the form:

$$S_{\eta\eta}^o(\omega) = \frac{1}{4} \sum_{i=1}^2 \frac{((4\lambda_i + 1)\omega_{mi}^4/4)^{\lambda_i}}{\Gamma(\lambda_i)} \frac{H_{si}^2}{\omega^{4\lambda_i+1}} \exp\left(-\left(\frac{4\lambda_i + 1}{4}\right)\left(\frac{\omega_{mi}}{\omega}\right)^4\right)$$

where $\Gamma(\lambda_i)$ is the Gamma function, H_{s1} , ω_{m1} , and λ_1 are the significant wave height, modal frequency, and shape factor for the lower frequency components, respectively, and H_{s2} , ω_{m2} , and λ_2 are those for the higher frequency component. Assuming that the entire spectrum is that of a narrow band, the equivalent significant wave height is given by

$$H_s = \sqrt{H_{s1}^2 + H_{s2}^2}$$

For $\lambda_1 = 1$ and $\lambda_2 = 0$, the spectrum reduces to the P–M spectrum. With the assumption that the entire spectrum is narrowband, the value of λ_1 is much higher than λ_2 . The O–H spectrum represents unidirectional seas with unlimited fetch. The sea severity and the state of development can be specified by H_{si} and ω_{mi} , respectively. In addition, λ_i can be selected to control the frequency width of the spectrum. For example, a small λ_i (wider frequency range) describes a developing sea, and a large λ_i (narrower frequency range) describes a swell condition. Figure 11.5 shows the O–H spectrum with $\lambda_1 = 2.72$, $\omega_{m1} = 0.626$ rad/sec, $H_{s1} = 3.35$ m, $\lambda_2 = 2.72$, $\omega_{m2} = 1.25$ rad/sec, and $H_{s2} = 2.19$ m.

Finally, another spectrum that is commonly used is the Joint North Sea Wave Project (JONSWAP) spectrum developed by Hasselmann et al. (1973). It is a fetch-limited spectrum because the growth over a limited fetch is taken into account. The attenuation in shallow water is also taken into account. The JONSWAP spectrum is written as

$$S_{\eta\eta}^o(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left(-1.25\left(\frac{\omega_m}{\omega}\right)^4\right) \gamma^{\exp(-(\omega-\omega_m)/2\tau^2\omega_m^2)}$$

where γ is the peakedness parameter and τ is the shape parameter. The peakedness parameter γ is the ratio of the maximum spectral energy to the maximum spectral energy of the corresponding P–M spectrum. That is, when $\gamma = 7$, the peak spectral energy is seven times that of the P–M spectrum.

$$\gamma = \begin{cases} 7.0 & \text{for very peaked data} \\ 3.3 & \text{for mean of selected JONSWAP data} \\ 1.0 & \text{for P–M spectrum} \end{cases}$$
$$\tau = \begin{cases} 0.07 & \text{for } \omega \leq \omega_m \\ 0.09 & \text{for } \omega > \omega_m \end{cases}$$
$$\alpha = 0.076(\bar{X})^{-0.22} \text{ or } 0.0081 \text{ if fetch independent}$$
$$\bar{X} = gX/U_w^2$$

X = fetch length (nautical miles)

U_w = wind speed (knots)

$$\omega_m = 2\pi \times 3.5(g/U_w)\bar{X}^{-0.33}$$

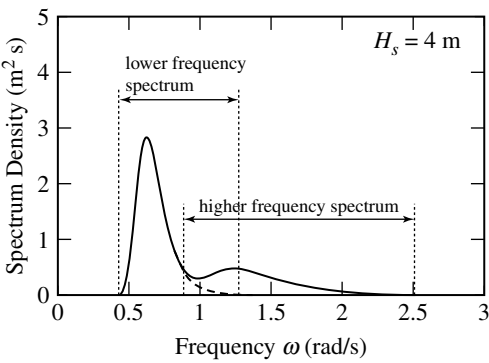


FIGURE 11.5 Ochi–Hubble spectrum.

Figure 11.6 shows the JONSWAP spectrum when $\alpha = 0.0081$ and $\omega_m = 0.626$ rad/sec for three peakedness parameters.

11.1.3 Approximation of Spectral Density from Time Series

From the time history of the wave elevation, the spectral density function can be obtained by two methods.

The first method is to use the autocorrelation function $R_{\eta\eta}(\tau)$, which is related to the spectral density function $S_{\eta\eta}(\omega)$ by the Wiener–Khinchine relations (Equation 11.6).

The autocorrelation $R_{\eta\eta}(\tau)$ is the expected value of $\eta(t)\eta(t+\tau)$ or $R_{\eta\eta}(\tau) = E\{\eta(t)\eta(t+\tau)\}$, where t is an arbitrary time and τ is the time lag. For a weakly stationary process, the autocorrelation is a function of the time lag only.

Assuming that the process is ergodic, the autocorrelation function for a given time history of length T_s can be approximated as

$$\hat{R}_{\eta\eta}(\tau) = \lim_{T_s \rightarrow \infty} \frac{1}{T_s - \tau} \int_0^{T_s - \tau} \eta(t)\eta(t+\tau)dt \quad \text{for } 0 < \tau < T_s$$

Note that the superscript $\wedge$ is used to emphasize that the variable is an approximation based on a sample time history of length T_s . The spectral density is then obtained by taking the Fourier cosine transform of $\hat{R}_{\eta\eta}(\tau)$,

$$\hat{S}_{\eta\eta}(\omega) = \frac{1}{\pi} \int_0^{T_s} \hat{R}_{\eta\eta}(\tau) \cos \omega\tau d\tau \quad (11.10)$$

The second method for obtaining the spectral density function is to use the relationship between the spectral density and the Fourier transform of the time series. They are related by

$$\hat{S}_{\eta\eta}(\omega) = \lim_{T_s \rightarrow \infty} \frac{1}{2\pi T_s} |\hat{X}(\omega)\hat{X}^*(\omega)| \quad (11.11)$$

where $\hat{X}(\omega)$ is given by

$$\hat{X}(\omega) = \int_0^{T_s} \eta(t) \exp(-i\omega t)dt$$

and $\hat{X}^*(\omega)$ is the complex conjugate given by

$$\hat{X}^*(\omega) = \int_0^{T_s} \eta(t) \exp(i\omega t)dt$$

In order to obtain the Fourier transforms of the time series, the discrete Fourier transform (DFT) or the fast Fourier transform (FFT) procedure can be used. For detailed descriptions of how this is done, see Appendix 1 in [Tucker \(1991\)](#). Nowadays, spectral analysis is almost always carried out via FFTs because it is easier to use and faster than the formal method via correlation function.

It should be noted that the length of the sample time history only needs to be long enough so that the limits converge. Taking a longer sample will not improve the accuracy of the estimate. Instead, one should take many samples or break one long sample into many parts. For n samples, the spectral densities

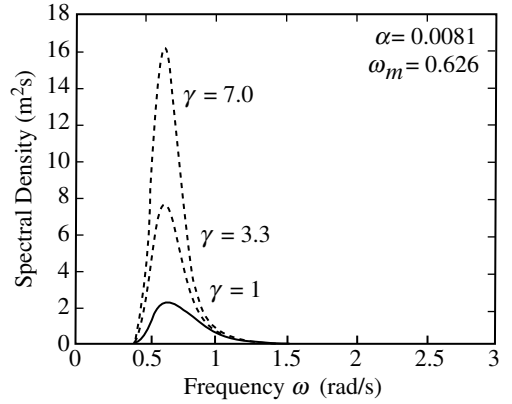


FIGURE 11.6 JONSWAP spectrum for $\gamma = 1.0, 3.3$, and 7.0 .

are obtained for each sample time history using either Equation 11.10 or Equation 11.11, and they are averaged to give the estimate.

The determination of the spectral density from wave records depends on the details of the procedure such as the length of the record, sampling interval, degree and type of filtering and smoothing, and time discretization.

11.1.4 Generation of Time Series from a Spectral Density

In a nonlinear analysis, the structural response is found by a numerical integration in time. Therefore, one needs to convert the wave elevation spectrum into an equivalent time history. The wave elevation can be represented as a sum of many sinusoidal functions with different angular frequencies and random phase angles. That is, we write $\eta(t)$ as

$$\eta(t) = \sum_{i=1}^N \cos(\omega_i t - \varphi_i) \sqrt{2S_{\eta\eta}(\omega_i)\Delta\omega_i} \quad (11.12)$$

where φ_i is a uniform random number between 0 and 2π , ω_i are discrete sampling frequencies, $\Delta\omega_i = \omega_i - \omega_{i-1}$, and N is the number of partitions. Recall that the area under the spectrum is equal to the variance, σ_η^2 . The incremental area under the spectrum, $S_{\eta\eta}(\omega_i)\Delta\omega_i$, can be denoted as σ_i^2 such that the sum of all the incremental area equals the variance of the wave elevation or $\sigma_\eta^2 = \sum_{i=1}^N \sigma_i^2$. The time history can be written as

$$\eta(t) = \sum_{i=1}^N \cos(\omega_i t - \varphi_i) \sqrt{2}\sigma_i$$

The sampling frequencies, ω_i , can be chosen at equal intervals such that $\omega_i = i\omega_1$. However, the time history will then have the lowest frequency of ω_1 and will have a period of $T = 2\pi/\omega_1$. In order to avoid this unwanted periodicity, Borgman (1969) suggested that the frequencies are chosen so that the area under the spectrum curve for each interval is equal or $\sigma_i^2 = \sigma^2 = \sigma_\eta^2/N$. The time history is written as

$$\eta(t) = \sqrt{\frac{2}{N}}\sigma_\eta \sum_{i=1}^N \cos(\bar{\omega}_i t - \varphi_i) \quad (11.13)$$

where $\bar{\omega}_i = (\omega_i + \omega_{i-1})/2$. The discrete frequencies, ω_i , are chosen such that the area between the interval $0 < \omega < \omega_i$ is equal to i/N of the total area under the curve between the interval $0 < \omega < \omega_N$ or

$$\int_0^{\omega_i} S_{\eta\eta}(\omega) d\omega = \frac{i}{N} \int_0^{\omega_N} S_{\eta\eta}(\omega) d\omega \quad \text{for } i = 1, \dots, N$$

where it is assumed that the area under the spectrum beyond ω_N is negligible. If $\eta(t)$ is a narrowband Gaussian process, the standard deviation can be replaced by $\sigma_\eta = H_s/4$, and the time history can be written as

$$\eta(t) = \frac{H_s}{4} \sqrt{\frac{2}{N}} \sum_{i=1}^N \cos(\bar{\omega}_i t - \varphi_i)$$

Shinozuka (1972) proposed that the sampling frequencies, $\bar{\omega}_i$, in Equation 11.13 should be randomly chosen according to the density function, $f(\omega) \equiv S_{\eta\eta}^o(\omega)/\sigma_\eta^2$. This is equivalent to performing an integration using the Monte Carlo method. The random frequencies ω distributed according to $f(\omega)$ can be obtained from uniformly distributed random numbers, x , by $\omega = F^{-1}(x)$, where $F(\omega)$ is the cumulative distribution of $f(\omega)$.

The random frequencies obtained this way are used in Equation 11.13 to generate a sample time series. It should be noted that many sample time histories should be obtained and averaged to synthesize a time history for use in numerical simulations.

11.1.5 Short-Term Statistics

In discussing wave statistics, we often use the term *significant wave* to describe an irregular sea surface. The significant wave is not a physical wave that can be seen but rather a statistical description of random waves. The concept of significant wave height was first introduced by Sverdrup and Munk (1947) as the average height of the highest one third of all waves. Usually, ships co-operate in programs to find sea statistics by reporting a rough estimate of the storm severity in terms of an observed wave height.

This observed wave height is consistently very close to the significant wave height.

Stationarity and ergodicity are two assumptions that are made in describing short-term waves statistics. These assumptions are valid only for “short” time intervals — approximately two hours or the duration of a storm — but not for weeks or years. The wave elevation is assumed to be weakly stationary so that its autocorrelation is a function of time lag only. As a result, the mean and the variance are constant, and the spectral density is invariant with time. Therefore, the significant wave height and the significant wave period are constant when we consider short-term statistics. In this case, the individual wave height and wave period are the stochastic variables. We then need to determine certain statistics for the analysis and design of offshore structures when we consider short time intervals.

Consider a sample time history of a zero-mean random process, as shown in Figure 11.7. The questions that we ask are how often is a certain level (e.g., z in the figure) exceeded, and how are the maxima distributed? Likewise, we can ask when we can expect to see that a certain level is exceeded for the first time, and what are the values of the peaks of a random process? The first question is important when a structure may fail due to a one-time excessive load, and the second question is important when a structure may fail due to cyclic loads.

It is found that the rate at which a random process $X(t)$ crosses Z with a positive slope (zero up-crossing) may be calculated from

$$\nu_{z^+} = \int_0^{\infty} \nu f_{X\dot{X}}(z, \nu) d\nu$$

where $f_{X\dot{X}}(x, \dot{x})$ is the joint probability density function of X and $\dot{X}(t)$. The expected time of the first up-crossing is then the inverse of the crossing rate or

$$E\{T\} = 1/\nu_{z^+}$$

The probability density function of the maxima, A , can be calculated from

$$f_A(a) = \frac{\int_{-\infty}^0 -\omega f_{X\dot{X}\ddot{X}}(a, 0, \omega) d\omega}{\int_{-\infty}^0 -\omega f_{\dot{X}\ddot{X}}(0, \omega) d\omega}$$

where $f_{X\dot{X}\ddot{X}}(x, \dot{x}, \ddot{x})$ is the joint probability density function of X , $\dot{X}$, and $\ddot{X}$.

If $X(t)$ is a Gaussian process, then we can write the joint probability density functions as

$$f_{X\dot{X}}(x, \dot{x}) = \frac{1}{2\pi\sigma_X\sigma_{\dot{X}}} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma_X}\right)^2 - \frac{1}{2}\left(\frac{\dot{x}}{\sigma_{\dot{X}}}\right)^2\right], \quad -\infty < x < \infty, \quad -\infty < \dot{x} < \infty$$

and

$$f_{X\dot{X}\ddot{X}}(x, \dot{x}, \ddot{x}) = \frac{1}{(2\pi)^{3/2}|M|^{1/2}} \exp\left[-\frac{1}{2}(\{x\} - \{\mu_X\})^T [M]^{-1} (\{x\} - \{\mu_X\})\right]$$

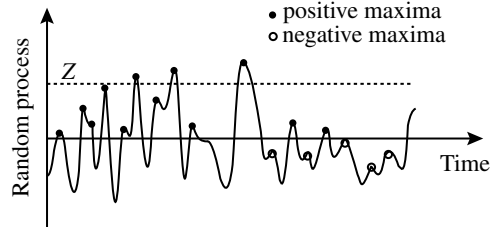


FIGURE 11.7 A sample time history.

where

$$[M] = \begin{bmatrix} \sigma_X^2 & 0 & \sigma_{\dot{X}}^2 \\ 0 & \sigma_{\dot{X}}^2 & 0 \\ \sigma_{\dot{X}}^2 & 0 & \sigma_{\ddot{X}}^2 \end{bmatrix} \quad \text{and} \quad \{x\} - \{\mu_X\} = \begin{bmatrix} x - \mu_X \\ \dot{x} - \mu_{\dot{X}} \\ \ddot{x} - \mu_{\ddot{X}} \end{bmatrix}$$

Then, for a stationary Gaussian process, the up-crossing rate is given by

$$\begin{aligned} \nu_z^+ &= \int_0^\infty f_{\dot{X}\ddot{X}}(Z, \dot{x}) \dot{x} \, d\dot{x} = \frac{1}{2\pi\sigma_X\sigma_{\dot{X}}} \exp\left[-\frac{1}{2}\left(\frac{Z}{\sigma_X}\right)^2\right] \int_0^\infty \exp\left[-\frac{1}{2}\left(\frac{\dot{x}}{\sigma_{\dot{X}}}\right)^2\right] \dot{x} \, d\dot{x} \\ &= \frac{\sigma_{\dot{X}}}{2\pi\sigma_X} \exp\left[-\frac{1}{2}\left(\frac{Z}{\sigma_X}\right)^2\right] \end{aligned} \quad (11.14)$$

and the probability density function of maxima is given by the Rice density function (Rice, 1954)

$$f_A(a) = \frac{\sqrt{1-\alpha^2}}{\sqrt{2\pi}\sigma_\eta} \exp\left[-\frac{1}{2}\frac{a^2}{\sigma_\eta^2(1-\alpha^2)}\right] + a\frac{\alpha}{\sigma_\eta^2} \Phi\left(\frac{a\alpha}{\sigma_\eta\sqrt{\alpha^2-1}}\right) \exp\left(-\frac{1}{2}\frac{a^2}{\sigma_\eta^2}\right) \quad \text{for } -\infty < a < \infty$$

where $\Phi(x)$ is the cumulative distribution function of standard normal random variable

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp(-z^2/2) dz$$

and α is the irregularity factor equivalent to the ratio of the number of zero up-crossings (number of times that $\eta[t]$ crosses zero with a positive slope) to the number of peaks. α ranges from 0 to 1, and it is also equal to

$$\alpha = \frac{\sigma_{\dot{\eta}}^2}{\sigma_\eta^2 \sigma_{\ddot{\eta}}^2}$$

If $X(t)$ is a broadband process, $\alpha = 0$ and the Rice distribution is reduced to the Gaussian probability density function given by

$$f_A(a) = \frac{1}{\sqrt{2\pi}\sigma_\eta} \exp\left[-\frac{1}{2}\frac{a^2}{\sigma_\eta^2}\right] \quad \text{for } -\infty < a < \infty$$

If $X(t)$ is a narrowband process, it is guaranteed that it will have a peak whenever $\eta(t)$ crosses its mean. In this case, the irregularity factor is close to unity, and the Rice distribution is reduced to the Rayleigh probability density function given by

$$f_A(a) = \frac{a}{\sigma_\eta^2} \exp\left[-\frac{1}{2}\frac{a^2}{\sigma_\eta^2}\right] \quad \text{for } 0 < a < \infty$$

In other words, the amplitudes of a narrowband stationary Gaussian process are distributed according to the Rayleigh distribution.

Figure 11.8 shows the Rice distribution for various values of α . Note that the Rice distribution includes both positive and negative maxima except when $\alpha = 1$, in which case all the maxima are positive. The positive maxima are the local maxima that occur above the mean of $X(t)$, and the negative maxima are the local maxima that occur below the mean, as shown in Figure 11.7. In some cases, the negative maxima may not mean

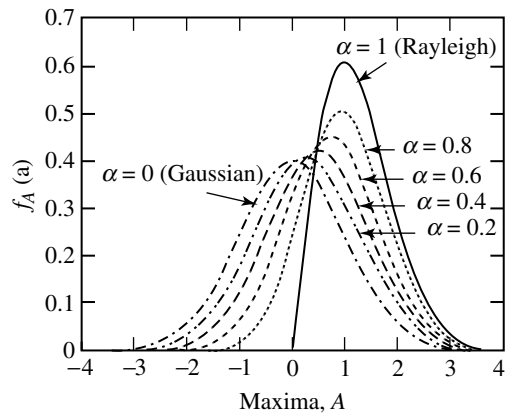


FIGURE 11.8 Rice distribution for maxima.

much physically. In those cases, we can use the truncated Rice distribution, where only the positive portion of $f_A(a)$ is used. $f_A(a)$ is normalized by the area under the probability density for positive maxima (Longuet-Higgins, 1952; Ochi, 1973):

$$f_A^{\text{trunc}}(a) = \frac{f_A(a)}{\int_0^\infty f_A(a) da}, \quad a \geq 0$$

The truncated Rice distribution is shown in Figure 11.9.

If $X(t)$ is the wave elevation, its maxima, A , are the amplitudes of the wave elevation. The wave height, $H = 2A$, is then distributed according to

$$f_H(h) = f_A(H/2) \frac{dA}{dH} = \frac{h}{4\sigma_\eta^2} \exp\left[-\frac{1}{2} \frac{h^2}{4\sigma_\eta^2}\right] \quad \text{for } 0 < h < \infty$$

For any given wave, the probability that the height is less than h (the cumulative distribution) is

$$F_H(h) = 1 - \exp\left[-\frac{1}{2} \frac{h^2}{4\sigma_\eta^2}\right] \quad \text{for } 0 < h < \infty$$

If $\eta(t)$ is a stationary narrowband process so that the peaks are distributed according to the Rayleigh distribution, we find that the root-mean-square wave height, $\sqrt{E\{H^2\}}$, is given by

$$\sqrt{E\{H^2\}} = \int_0^\infty h^2 f_H(h) dh = 2\sqrt{2}\sigma_\eta$$

In addition, it can be shown that the average and the significant wave heights are given by

$$H_0 \equiv E\{H\} = \sqrt{2\pi}\sigma_\eta, \quad H_s \equiv E\{H_{1/3}\} = 4\sigma_\eta \quad (11.15)$$

where $E\{H_{1/3}\}$ means that it is the expectation of the highest one third of the waves.

11.1.6 Long-Term Statistics

Because offshore structures are designed for long life spans, we must also consider long-term wave statistics. Previously, when we considered the short-term statistics, the significant wave height and spectrum were assumed to be invariant with time. This assumption is valid only over time periods of days at most. For longer time periods, the significant wave height has its own statistics and is a random variable.

When one uses short-term statistics to describe long-term events, improbable events seem unjustifiably probable. For example, let us consider the probability that the wave height, distributed according to the Rayleigh distribution, exceeds a certain extreme value. Let us assume that the mean period of this wave is 10 sec and the probability that the height of any given wave is greater than 300 ft is 10^{-10} . The value is small and the occurrence of a 300 ft wave seems improbable. However, the probability that the height will exceed 300 ft at least once in 10 years (3×10^8 sec) is given by

$$1 - (1 - 10^{-10})^{3 \times 10^8 / 10} = 0.997$$

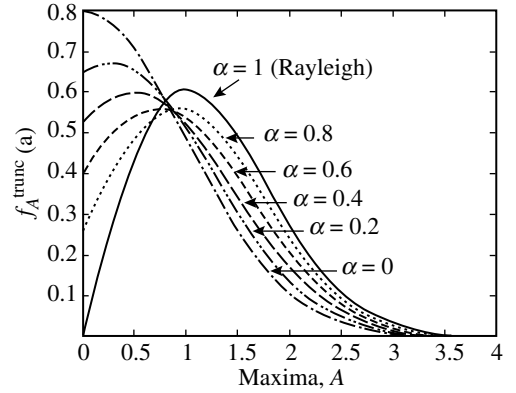


FIGURE 11.9 Truncated Rice distribution.

Thus, the statistical description states that it is almost certain that the wave height will exceed 300 ft at least once in 10 years. This prediction is a shortcoming of the short-term statistics since waves of this magnitude do not arrive at this probability.

In order to compute the probability that a wave height will exceed a certain extreme value, we require statistics for these extreme events. The actual maximum amplitude in a sequence of random amplitudes is a random variable itself. It has a probability distribution with mean value, standard deviation and other statistical properties. In fact, the distributions of these maximum values are called the extreme value distributions (EVDs). Gumbel (1958) obtained three methods of extrapolation known as three asymptotes. They are the Gumbel, Fretchet, and Weibull distributions. We will discuss the Gumbel and Weibull distributions in the next section. For the moment, we will discuss the concept of the N -year storm.

In long-term statistics, we often speak of an N -year storm. It means that, for any given year, the probability that we will have an N -year storm is

$$p = \frac{1}{N}$$

It follows that the probability that we will have m storms in n years is given by

$$\Pr\{mN\text{-year storms in } n \text{ years}\} = {}_n P_m \left(\frac{1}{N}\right)^m \left(1 - \frac{1}{N}\right)^{n-m}$$

where ${}_n P_m$ is the permutation given by

$${}_n P_m = \frac{n!}{(n-m)!}$$

The probability that we will have at least one N -year storm in n years is

$$\Pr\{\text{at least one } N\text{-year storms in } n \text{ years}\} = 1 - \left(1 - \frac{1}{N}\right)^n$$

For a large N , the probability can be approximated as $1 - \exp(-n/N)$. It should be noted that the probability that we will have exactly one N -year storm in N years is not one, but

$$\Pr\{\text{one } N\text{-year storm in } N \text{ years}\} = \left(1 - \frac{1}{N}\right)^{N-1}$$

As $N \rightarrow \infty$, we find that

$$P = 1/e \approx 0.3679$$

The probability that we will have at least one N -year storm in N years is

$$\Pr\{\text{at least one } N\text{-year storms in } n \text{ years}\} = 1 - \left(1 - \frac{1}{N}\right)^N$$

As $N \rightarrow \infty$, we find that

$$P = 1 - 1/e \approx 0.6321 \quad (11.16)$$

11.1.6.1 Weibull Distribution

The Weibull distribution fits probabilities of extremes quite satisfactorily. In long-term statistics, the significant wave height follows the Weibull distribution closely. The probability density and the cumulative distribution are given by

$$f(h) = \frac{m}{\beta} \left(\frac{h-\gamma}{\beta}\right)^{m-1} \exp\left(-\left(\frac{h-\gamma}{\beta}\right)^m\right), \quad F(h) = 1 - \exp\left(-\left(\frac{h-\gamma}{\beta}\right)^m\right) \text{ for } \gamma < h \quad (11.17)$$

where m is called the shape parameter. Manipulating the cumulative distribution, we can write

$$\ln(-\ln\{1 - F(h)\}) = m\{\ln(h - \gamma) - \ln(\beta)\}$$

where the left-hand side is known from data. If we let $y = \ln(-\ln\{1 - F(h)\})$ and $x = \ln(h - \gamma)$, y is a straight line with slope of m and a y -intercept of $-m \ln \beta$:

$$y = mx - m \ln \beta$$

Suppose we have significant wave height data over a long period of time, and our goal is to find the Weibull parameters, γ , β , and m that best fit the distribution of the significant wave heights. These parameters can be determined by the least-squares method or using the Weibull paper. Using the latter method, we first guess γ so that the discrete points (x, y) or $(\ln(h - \gamma), \ln(-\ln\{1 - F(h)\}))$ form a straight line. The slope of this line is m , and the value of y when the line intersects the y axis is $-m \ln \beta$. This method will be illustrated in Section 11.3.3.

The Gumbel distribution is given by

$$\begin{aligned} f(h) &= \alpha \exp(-\alpha(h - \beta)) \exp\{\exp[-\alpha(h - \beta)]\} \\ F(h) &= \exp\{-\exp[-\alpha(h - \beta)]\} \quad \text{for } -\infty < h < \infty \end{aligned} \quad (11.18)$$

When $\ln[-\ln F(h)]$ is plotted against h , the result is a line with a slope of $-\alpha$ and y intercept of $\alpha\beta$.

Another distribution that may be used is the lognormal distribution given by (Jasper, 1954)

$$f(h) = \frac{1}{\sqrt{2\pi}\sigma h} \exp\left[-\frac{1}{2} \frac{(\ln h - \mu)^2}{\sigma^2}\right], \quad F(h) = \Phi\left(\frac{\ln h - \mu}{\sigma}\right) \quad \text{for } 0 \leq h \quad (11.19)$$

11.1.6.2 Wave Velocities via Linear Wave Theory

The wave velocities that correspond to the wave elevation given in Equation 11.12 can be obtained by *linear wave theory*. Linear wave theory, also called *airy wave theory*, *sinusoidal wave theory*, and *small-amplitude theory*, is the simplest wave theory. It is also the most important wave theory because it forms the basis for the probabilistic spectral description of waves.

Linear wave theory assumes that the wave height is small compared with the wavelength and wave depth. In addition, fluid particles are assumed to follow a circular orbit. The readers should refer to [Kinsman \(1965\)](#) and [LeMehaute \(1976\)](#) for detailed descriptions.

In linear wave theory, the surface elevation is given by

$$\eta(y, t) = A \cos(\omega t - ky) \quad (11.20)$$

which is a plane wave traveling to the right in Figure 11.10. Linear wave theory relates this sinusoidal surface elevation to the wave velocities given by

$$w_y(x, y, t) = A\omega \frac{\cosh kx}{\sinh kd} \cos(\omega t - ky), \quad w_y(x, y, t) = A\omega \frac{\sinh kx}{\sinh kd} \sin(\omega t - ky) \quad (11.21)$$

where k , ω , and A are wave number, angular frequency, and amplitude of a surface wave, respectively. The velocities vary with time, horizontal coordinate y , and depth x measured from the ocean floor. The wave velocities are sinusoidal in y and t , but exponentially decrease with the distance from the surface.

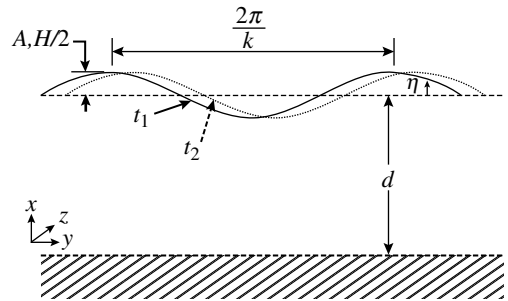


FIGURE 11.10 A schematic of a simple sinusoidal wave shown at two different times.

The frequency ω is related to the wave number k by the dispersion relation given by

$$\omega^2 = gk \tanh kd$$

where d is the water depth. For deep water, $\tanh kd$ approaches unity and the frequency is given by

$$\lim_{d \rightarrow \infty} \omega^2 = gk$$

For the surface elevation given in Equation 11.12, the surface elevation and the wave velocities are given by

$$\begin{aligned} \eta(y, t) &= \sum_{i=1}^N \cos(\omega_i t - k_i y - \varphi_i) \sqrt{2S_{\eta\eta}(\omega_i) \Delta\omega_i} \\ w_x(x, y, t) &= \sum_{i=1}^N \omega_i \frac{\cosh k_i x}{\sinh k_i d} \cos(\omega_i t - k_i y - \varphi_i) \sqrt{2S_{\eta\eta}(\omega_i) \Delta\omega_i} \\ w_y(x, y, t) &= \sum_{i=1}^N \omega_i \frac{\sinh k_i x}{\sinh k_i d} \sin(\omega_i t - k_i y - \varphi_i) \sqrt{2S_{\eta\eta}(\omega_i) \Delta\omega_i} \end{aligned} \quad (11.22)$$

The wave accelerations can be obtained by differentiating the wave velocities with respect to time. Sample time histories of the wave velocity and acceleration can be obtained using either Borgman's or Shinozuka's method.

11.1.7 Summary

In this section, the concept of spectral density is introduced. It is then shown how the concept is used to describe the ocean wave heights. The spectral density or spectrum is related to the autocorrelation function by the Wiener–Khinchine relations:

$$\begin{aligned} S_{\eta\eta}(\omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{\eta\eta}(\tau) \exp(-i\omega\tau) d\tau \\ R_{\eta\eta}(\tau) &= \int_{-\infty}^{\infty} S_{\eta\eta}(\omega) \exp(-i\omega\tau) d\omega \end{aligned}$$

In addition, the spectral density function of a real-valued stationary process is also real and symmetric, or

$$S_{\eta\eta}(\omega) = S_{\eta\eta}(-\omega) \quad (11.23)$$

and the area under the spectral density is given by

$$\int_{-\infty}^{\infty} S_{\eta\eta}(\omega) d\omega = R_{\eta\eta}(0) = \sigma_{\eta}^2 - \mu_{\eta}^2 \quad (11.24)$$

If the spectral density is given only for $\omega \geq 0$, then this one-sided spectrum is related to the two-sided spectrum by

$$S_{\eta\eta}^o(\omega) = 2S_{\eta\eta}(\omega), \quad \omega \geq 0$$

When the frequency is given in Hertz instead of in rad/sec, the spectra are related by

$$S_{\eta\eta}(f) = 2\pi S_{\eta\eta}(\omega), \quad \omega \geq 0$$

The spectra that are often used to describe wave heights are the P–M, Bretschneider, ITTC, ISSC, O–H, and JONSWAP spectra. The most widely used spectrum is the P–M spectrum, which is a single parameter spectrum. The P–M spectrum is applicable for deep water, unidirectional seas, fully developed and local-wind-generated sea with unlimited fetch, and was originally developed for the North Atlantic. The single parameter for this spectrum can be expressed as the wind velocity at 19.5 m above sea level or the significant wave height that specifies the sea severity. When it is written

in terms of the significant wave height, it is given by

$$S_{\eta\eta}^o(f) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} \exp\left(-\frac{0.0324 g^2}{H_s^2} \omega^{-4}\right)$$

Bretschneider, ITTC, and ISSC spectra are two-parameter spectra in which the state of development as well as the sea severity can be specified. The O-H spectrum is a six-parameter spectrum that allows us to represent local and distant storm effects and to specify sharpness of the peaks as well as to specify the sea severity and the state of development. The JONSWAP spectrum allows us to account for growth over a limited fetch.

For a given ocean spectrum, a sample time history can be obtained by

$$\eta(t) = \sqrt{\frac{2}{N}} \sigma_\eta \sum_{i=1}^N \cos(\bar{\omega}_i t - \varphi_i)$$

In Borgman's method, the sampling frequencies $\bar{\omega}_i = (\omega_i + \omega_{i-1})/2$ are chosen so that the area between ω_{i-1} and ω_i are equal. In Shinozuka's method, the sampling frequencies are chosen randomly. The traditional method of choosing the sampling frequency at even intervals is not recommended.

When a relatively short interval of time is considered, for example, about two hours or the duration of a storm, it can be assumed that the spectrum and its statistics are invariant with time. In this case, the distribution of local maxima or peaks of a stationary Gaussian process is described by the Rice distribution. The Rice distribution can be reduced to the Rayleigh distribution when the process is narrowband and to the Gaussian distribution when the process is broadband. In the long-term statistics, the spectrum and its statistics may vary with time. In this case, the term “*N*-year storm” is often used to indicate the sea severity, and the significant wave heights closely follow the Weibull distribution.

Finally, the wave velocities and accelerations are related to the wave velocities using linear wave theory.

11.2 Fluid Forces

The following is a list of several types of forces that the fluid can exert on a body:

1. *Drag force.* This is due to the pressure difference between the downstream and upstream flow region. It can be thought of as the force required to hold a body stationary in a fluid of constant velocity. The drag force is proportional to the square of the velocity of the fluid relative to the structure.
2. *Inertia force.* This is the force exerted by the fluid while it accelerates and decelerates as it passes the structure. It is also the force required to hold a rigid structure in a uniformly accelerating flow, and it is proportional to the fluid acceleration. The concept of the inertia force in an inviscid flow was first formulated by Lamb (1945).
3. *Added mass.* As the body accelerates or decelerates in a stationary fluid, the body carries a certain amount of the surrounding fluid along with it. This entrained fluid is called the *added, apparent, or virtual mass*. In order to accelerate the body, additional force is required to accelerate or decelerate the added mass.
4. *Diffraction force.* This is due to the scattering of an incident wave on the surface of the structure. It is important when the body is large compared with the wavelength of the incident wave.
5. *Froude-Kryloff force.* This is the pressure force on the structure due to the incident wave, assuming that the structure does not exist and does not interfere with the incident wave.
6. *Lift force.* This is due to nonsymmetrical separation of the fluid or due to vortices that are shed in a nonsymmetrical way. The component of the force perpendicular to the flow direction is the lift force.

7. *Wave slamming force.* This is due to a single occasional wave with a particularly high amplitude and energy, and it may be important at the free surface. Sarpkaya and Isaacson (1981) reviewed the research on slamming of water against circular cylinders. Miller (1977, 1980) found that the peak wave slamming force on a rigidly held horizontal circular cylinder is proportional to the square of the horizontal water particle velocity.

11.2.1 Wave Force Regime

Previously, we discussed various types of forces caused by waves and currents. In some cases, one type of force may be dominant. Hogben (1976) gave a literature review of the fluid force in various regimes. The load regime of importance can be demonstrated for the case of a vertical cylinder in Figure 11.11 in terms of H/D and $\pi D/\lambda$, where H is the wave height, D is the cylinder diameter, and λ is the wavelength. When linear wave theory is used, H/D is related to the Keulegan–Carpenter number by

$$K = \pi H/D$$

The Keulegan–Carpenter number gives a measure of the importance of drag force relative to the inertia force. The term $\pi D/\lambda$ is called the diffraction parameter, and it determines the importance of the diffraction effect. As H/D increases, the drag force becomes more important and the inertia force becomes less important. As $\pi D/\lambda$ increases, the diffraction force becomes important.

Using linear wave theory, the maximum drag force to the maximum inertia force can be written as

$$\frac{f_{\text{drag}}}{f_{\text{inertia}}} = \frac{1}{2\pi} \frac{H}{D} = \frac{K}{2\pi^2}$$

From the last relation, we find that the drag force is 5% of the inertial force when $H/D = 0.314$. The Morison equation may be used for $D/\lambda < 0.2$ and $f_{\text{drag}}/f_{\text{inertia}} > 0.1$ or thereabouts. It should be noted that Figure 11.11 is valid only near the surface. The drag force is predominant for a cylinder that extends from the bottom to the near surface, so that the Morison equation may be used.

For example, consider a fixed jacket platform with legs with a diameter of 10 m and bracings with a diameter of 0.8 m. For a 10-year storm with $\lambda = 100$ m and $H = 8$ m, the ratios H/D and D/λ for the leg are 0.8 and 0.1, respectively. Similarly, the ratios H/D and D/λ for the bracings are 10 and 0.08, respectively. Figure 11.11 shows that the inertia force is dominant for the legs, and both inertia and the drag forces are important for the bracings.

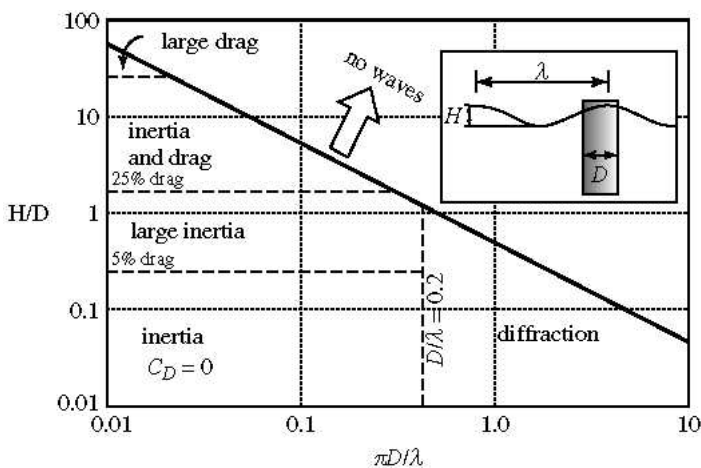


FIGURE 11.11 Load regimes near surface.

11.2.2 Wave Forces on Small Structures — Morison Equation

The added mass, M_A , can be written as

$$M_A = C_A M_{\text{disp}}$$

where C_A is called the *added mass coefficient* and M_{disp} is the mass of the fluid displaced by the structure. For a cylinder with a diameter, D , and height, h , the displaced fluid mass is $\pi D^2 h/4$. It should be noted that the added mass is a tensor quantity. That is, we can speak of the added mass force in the x_i direction due to the acceleration of the body in the x_j direction, denoted as M_{ij}^A . M_{ij}^A is symmetric so that the added mass force in the x_i direction due to the acceleration in the x_j direction is equal to the added mass force in the x_j direction due to the acceleration in the x_i direction. The off-diagonal terms are not zero if the cross-section is not symmetric.

Similarly, the inertia force can be written as

$$F_M = C_M M_{\text{disp}} \dot{w} \quad (11.25)$$

where the proportionality constant, C_M , is called the *inertia coefficient*.

It should be noted that the added mass and the inertia effects are often neglected for a body vibrating in air since the displaced air mass is negligible.

The drag force is proportional to the square of the fluid velocity, w , the density of the fluid, ρ , and the area of the body projected onto the plane perpendicular to the flow direction, A_f ,

$$F_D = \frac{1}{2} C_D \rho A_f w |w|$$

where C_D is the *drag coefficient*. The absolute value sign is used to ensure that the drag force always acts in the direction of the flow. For a cylinder with a diameter D and height h , the projected area A_f is Dh .

For a body with nonzero velocity, the drag force is given by

$$F_D = \frac{1}{2} C_D \rho A_f (w - v) |w - v| \quad (11.26)$$

where $w - v$ is the velocity of the fluid relative to the body.

Morison et al. (1950) combined the inertia and drag terms (Equation 11.25 and Equation 11.26) so that the fluid force on a body is given by

$$f = \frac{1}{2} C_D \rho A_f w |w| + C_M M_{\text{disp}} \dot{w}$$

For a cylinder, the fluid force per unit length can be written as

$$f = \frac{1}{2} C_D \rho D w |w| + C_M \rho \pi \frac{D^2}{4} \dot{w}$$

For a moving cylinder with velocity v , the Morison force is given by

$$f = \frac{1}{2} C_D \rho D (w - v) |w - v| + C_M \rho \pi \frac{D^2}{4} \dot{w}$$

11.2.2.1 Inclined Cylinder

Let us now consider the inclined cylinder shown in [Figure 11.13](#). The direction of the flow makes an angle of θ with the cylinder. Often, only the fluid force in the normal direction is considered. The normal component is given by

$$f^n = \frac{1}{2} C_D \rho D (w^n - v^n) |w^n - v^n| + C_M \rho \pi \frac{D^2}{4} \dot{w}^n \quad (11.27)$$

where the superscript is used for the normal component. The term, $w^n - v^n$, is the normal component of the relative velocity of the fluid with respect to the structure. Suppose that fluid is flowing to the right,

and the cylinder is also moving to the right, as shown in Figure 11.12. The normal components of the fluid and cylinder velocities are

$$w^n = |w| \cos \theta, \quad v^n = |v| \cos \theta$$

In three dimensions, it may be difficult to picture what the normal component should be. Here, we can find the normal component using the formula

$$(w^n - v^n)\vec{n} = \vec{t} \times (\vec{w} - \vec{v}) \times \vec{t} \quad (11.28)$$

where $\vec{t}$ is the unit vector tangent to the cylinder and $\vec{n}$ is the unit vector normal to the cylinder.

Note that the normal direction depends on the direction of the flow as well as the inclination of the cylinder.

In some cases, the tangential drag force may be included, and it can be written as

$$f^t = \frac{1}{2} C_T \rho D (w^t - v^t) |w^t - v^t| \quad (11.29)$$

where C_T is the tangential drag coefficient. Note that C_T is usually a very small number.

The normal component of the fluid force is more dominant than the tangential component. It may seem strange that the fluid force does not act in the direction of the fluid motion. Instead, the force is predominantly in the normal direction defined by Equation 11.28. In Section 11.3.1, we will demonstrate what this means by considering a towing cable.

11.2.2.1.1 Determination of Fluid Coefficients

The drag, inertia, and added mass coefficients must be obtained by experiment. However, for a long cylinder, C_M approaches its theoretical limiting value (uniformly accelerated inviscid flow) of 2, and C_A approaches unity (Lamb, 1945; Wilson, 1984). In reality, the inertia and drag coefficients are functions of at least three parameters (Wilson, 1984):

$$\begin{aligned} C_M &= C_M(\text{Re}, K, \text{cylinder roughness}) \\ C_D &= C_D(\text{Re}, K, \text{cylinder roughness}) \end{aligned}$$

where Re is the Reynolds number and K is the Keulegan–Carpenter number given by

$$\text{Re} \equiv \frac{\rho_f U D}{\mu}, \quad K \equiv \frac{U T}{D} \quad (11.30)$$

where ρ_f is the density of the fluid, U is the free stream velocity, D is the diameter of the structure, μ is the dynamic or absolute viscosity, and T is the wave period.

Sarpkaya looked at the variation of these hydrodynamic coefficients extensively and obtained the plots shown in Figure 11.13 to Figure 11.15 (Sarpkaya, 1976; Sarpkaya et al., 1977). Figure 11.13 shows the inertia and drag coefficients for a smooth cylinder as a function of K for various values of Re and the reduced frequency β , defined by $\beta = \text{Re}/K$. From this figure, we find that for low Re and β , the inertial coefficient decreases and the drag coefficient increases at about $10 < K < 15$. It is found that the drop and the increase in these coefficients are due to shedding vortices, which also exert forces perpendicular to the structure and the flow.

Figure 11.14 and Figure 11.15 show the inertia and drag coefficients for a rough cylinder, whose roughness is measured by k/D . Figure 11.14a shows a drop in the drag coefficient for Re between 10^4 and 10^5 , and this is called the “drag crisis.” For a larger Re , the drag coefficient stays constant. As the surface becomes rougher, the drop occurs at lower Re and the drag coefficients for the larger Re increases.

Figure 11.14 to Figure 11.16 can be used to obtain proper values of the drag and inertia coefficients for fluid with known Re , Keulegan–Carpenter number, and cylinder roughness.

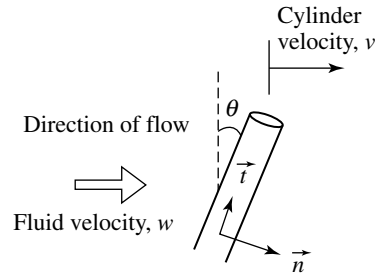


FIGURE 11.12 Inclined cylinder.

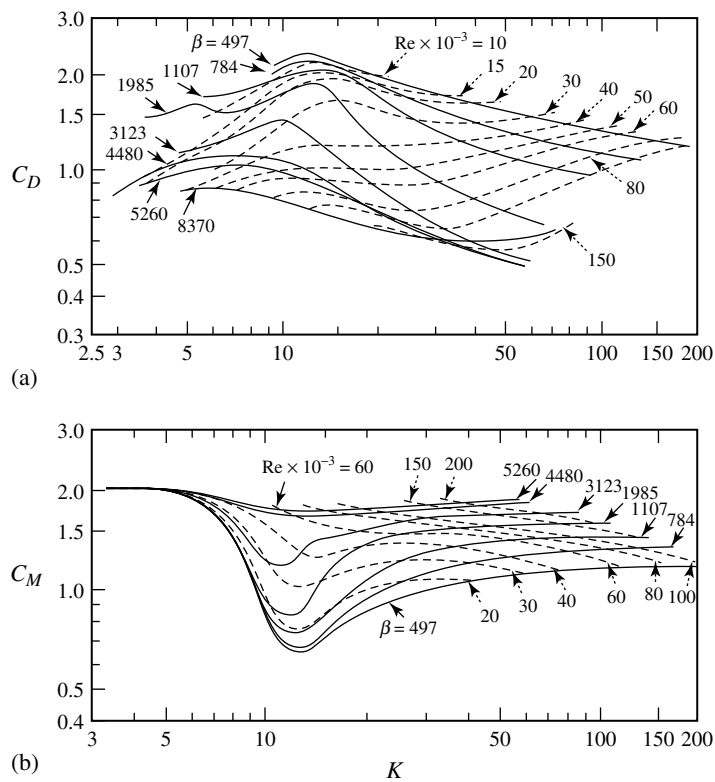


FIGURE 11.13 Drag and inertia coefficients as functions of K for various values of Re and β . (Source: Sarpkaya, 1976, *Proceedings of the Eighth Offshore Technology Conference*. With permission.)

11.2.3 Vortex-Induced Vibration

When the flow passes around a fixed cylinder, for a very low Re ($0 < Re < 4$), the flow separates and reunites smoothly. When the Re is between 4 and 40, eddies are formed and are attached to the downstream side of cylinder. They are stable and there is no oscillation in the flow. For a flow with a Reynolds number greater than about 40, the fluid near the cylinder starts to oscillate due to shedding vortices. These shedding vortices exert an oscillatory force on the cylinder in the direction perpendicular to both the flow and the structure. The frequency of oscillation is related to the nondimensionalized parameter, the Strouhal number, defined by

$$St = \frac{f_v D}{U} \tag{11.31}$$

where f_v is the frequency of oscillation, U is the steady velocity of the flow, and D is the diameter of the cylinder. For circular cylinders, the Strouhal number stays roughly at 0.22 for laminar flow ($10^3 < Re < 2 \times 10^5$) and 0.3 for turbulent flow (Patel, 1989).

The lift force due to these shedding vortices can be written as

$$f_L = \frac{1}{2} C_L \rho A_f U^2 \cos 2\pi f_v t \tag{11.32}$$

where C_L is the lift coefficient, which is also a function of Re , K , and the surface roughness. The experimental data of the lift coefficients show considerable scatter with typical values ranging from 0.25 to 1. For smooth cylinders, the lift coefficient approaches about 0.25 as Re and K increase.

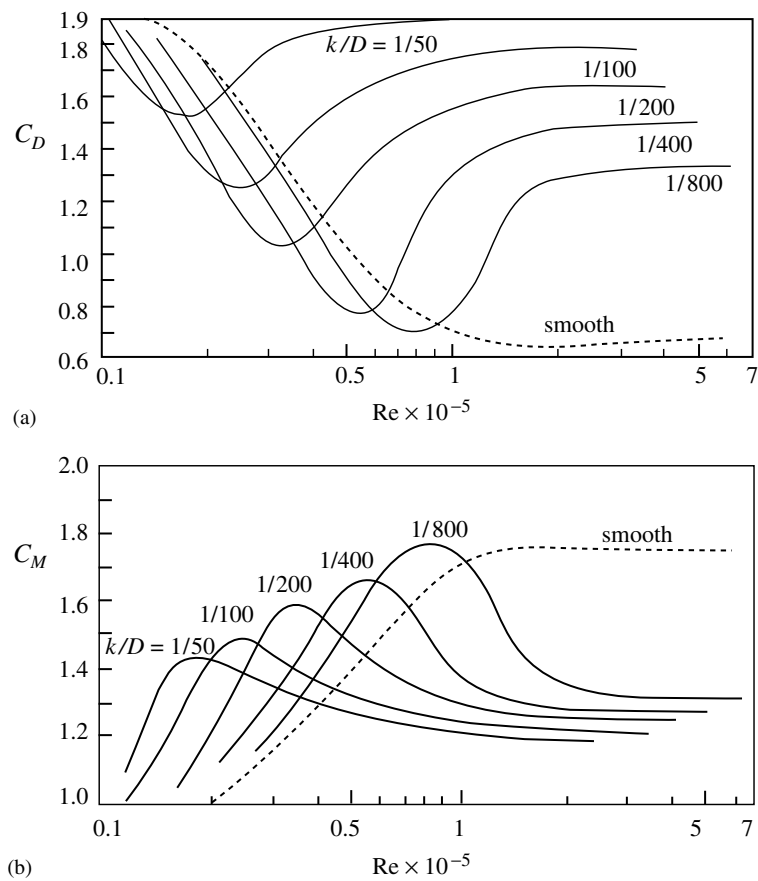


FIGURE 11.14 Drag and inertia coefficients for a rough cylinder as functions of Re for various values of cylinder roughness (as measured by k/D) for $K = 20$. (Source: Sarpkaya et al., 1977, *Proceedings of the Ninth Offshore Technology Conference*. With permission.)

It should be noted that the vortex forces are not generally correlated on the entire cylinder length. That is, the phase of the vortex shedding forces varies over the length. The correlation length — the length over which vortex shedding is synchronized — for a stationary cylinder is about three to seven diameters for laminar flow. If sectional forces are randomly phased, the net effect will be small. The total force on a cylinder of length L will be only a fraction of Lf_l . This fraction is called the joint acceptance and depends on the ratio of the correlation length to the total length.

When the flow passes by a cylinder that is free to vibrate, the shedding frequency is also controlled by the movement of the cylinder. When the shedding frequency is close to the first natural frequency of the cylinder (± 25 to 30% of the natural frequency [Sarpkaya and Isaacson, 1981]), the cylinder takes control of the vortex shedding. The vortices will shed at the natural frequency instead of at the frequency determined by the Strouhal number. This is called lock-in or synchronization, which is a result of nonlinear interaction between the oscillation of the body and the action of the fluid. Figure 11.16 shows the shedding frequency, as a function of flow velocity in the presence of a structure. f_1 and f_2 are the natural frequencies of the structure.

The amplitude of the structural response and the range of the fluid velocity over which the lock-in phenomenon persists are functions of a reduced damping parameter — the ratio of the damping force to the exciting force (Vandiver, 1985, 1993). If the reduced damping parameter is small, the lock-in can persist over a greater range of flow velocity.

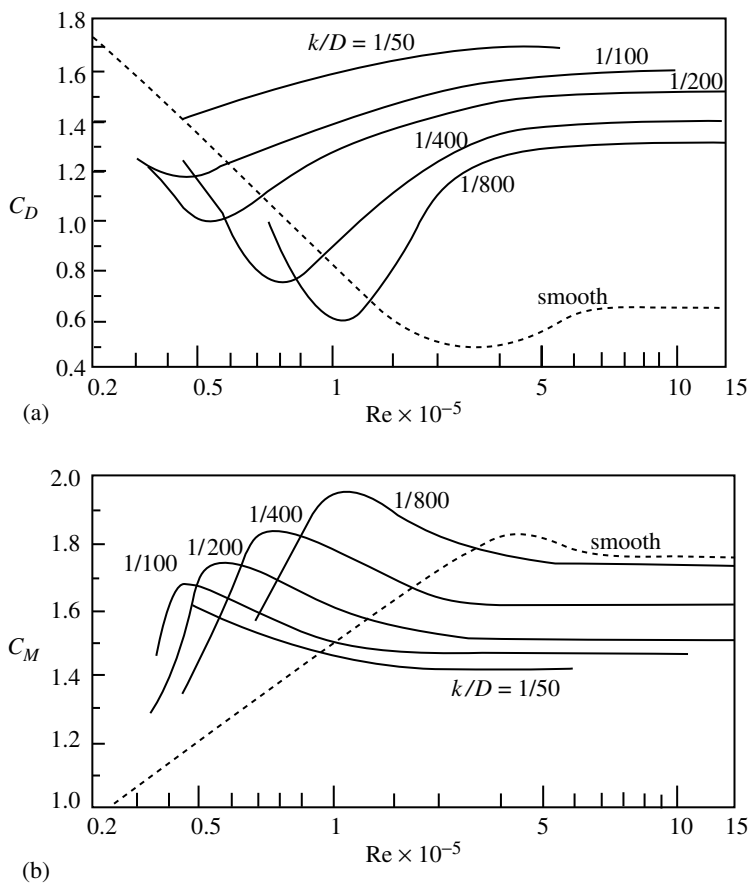


FIGURE 11.15 Drag and inertia coefficients for a rough cylinder as functions of Re for various values of cylinder roughness (as measured by k/D) for $K = 60$. (Source: Sarpkaya et al., 1977, *Proceedings of the Ninth Offshore Technology Conference*. With permission.)

The existing models for vortex-induced oscillation for a rigid cylinder include single-degree-of-freedom models and coupled models. The single-DoF models assume that the effect of vortex shedding is an external forcing function, which is not affected by the motion of the body. The coupled models assume that the equations that govern the motion of the structure and the lift coefficients are coupled so that the fluid and the structure affect each other (Billah, 1989).

11.2.4 Summary

Some of the fluid forces are discussed briefly, and the regimes where inertia, drag, and diffraction forces are important are shown as functions of the ratio of the structural diameter to the wave

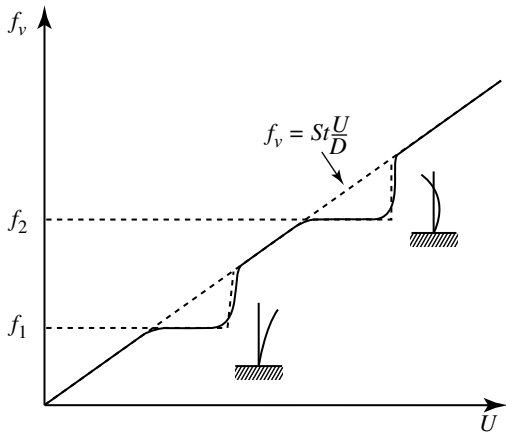


FIGURE 11.16 An example of fluid elastic resonance.

length, D/λ , and the ratio of the wave height to the structural diameter, H/D . The wave forces on small structures are modeled by the Morison equation, and it is valid for $D/\lambda < 0.2$ and $H/D > 0.63$ or thereabouts. The Morison equation includes the effects of added mass, inertia, and drag. The added mass is simply

$$M_A = C_A M_{\text{disp}}$$

For a cylinder with transverse velocity, v , the normal and the tangential components of the drag and the inertia forces are given by

$$\begin{aligned} f^n &= \frac{1}{2} C_D \rho D (w^n - v^n) |w^n - v^n| + C_M \rho \pi \frac{D^2}{4} \dot{w}^n \\ f^t &= \frac{1}{2} C_T \rho D (w^t - v^t) |w^t - v^t| \end{aligned}$$

The fluid coefficients are at least functions of three parameters: the Reynolds number, the Keulegan–Carpenter number, and the cylinder roughness. The plots of these coefficients are reproduced in [Figure 11.13](#) to [Figure 11.15](#).

The frequency of the lift force that is exerted by shedding vortices is closely related to the Strouhal number given by

$$St = \frac{f_v D}{U}$$

The lift force due to these shedding vortices can be written as

$$f_L = \frac{1}{2} C_L \rho A_f U^2 \cos 2\pi f_v t$$

If the structure is free to vibrate, then lock-in or synchronization may occur when the shedding frequency is close to the structure's natural frequency. The structure takes control of the vortex shedding. Many nonlinear models are available to capture this phenomenon.

11.3 Examples

Four examples are given in this section. The first example illustrates the roles of the normal and the tangential components of the drag force in the static configuration of a towing cable. The second example shows how the equation of motion of an articulated tower can be formulated in the presence of surrounding fluid. The third example shows how to choose a single significant wave height to represent a certain condition from significant wave height data over a long period of time. The final example shows how to reconstruct time series data from a given spectrum.

11.3.1 Static Configuration of a Towing Cable

For the purpose of ocean surveillance, oceanographic or geographic measurements, or ocean exploration, marine cables with instrument packages or Remotely Operated Vehicles are often towed behind ships or submarines. For example, the goal of the VENTS program by the National Oceanic and Atmospheric Administration (NOAA) is to conduct research on the impacts and consequences of submarine volcanoes and hydrothermal venting on the global ocean. In attempts to locate and map the distributions of hydrothermal plumes in the Mid-Ocean Ridge system, an instrument package called a CTD (Conductivity, Temperature and Depth Sensors) is towed behind a ship.

Let us consider a cable and a body towed behind a ship at a constant velocity with no current as shown in Figure 11.17. What kind of shape will the cable take? What will be the distance between the ship and the towed body?

We immediately recognize that this is equivalent to having a stationary ship with a steady current in the opposite direction. The equation of motion is given by

$$\sum \vec{F} = m\vec{a}(s, t) = \vec{0} = \frac{\partial}{\partial s}(T\vec{t}) + f^n\vec{n} + f^t\vec{t} + mg\vec{k}$$

where m is the mass of the cable per unit length, $\vec{a}(s, t)$ is the acceleration of the cable, s is the coordinate along the cable, T is the tension which is a function of s , $(\vec{t}, \vec{n}, \vec{b})$ is the set of unit vectors of the curvilinear coordinate system, $\vec{k}$ is the unit vector downward in the direction of gravity, g is the gravitational acceleration, f^n is the normal drag force, and f^t is the tangential drag force. The added mass and the inertial terms are zero because the fluid acceleration and the cable acceleration are zero. The normal and tangential drag forces are given in Equation 11.27 and Equation 11.29. In our case, they are given by

$$f^n = C_D \rho \frac{D}{2} U^2 \cos^2 \theta, \quad f^t = -C_T \rho \frac{D}{2} U^2 \sin^2 \theta$$

The corresponding scalar equations are given by

$$\begin{aligned} \frac{dT}{ds} - C_T \rho \frac{D}{2} U^2 \sin^2 \theta - mg \cos \theta &= 0 \\ -T \frac{d\theta}{ds} + C_D \rho \frac{D}{2} U^2 \cos^2 \theta - mg \sin \theta &= 0 \end{aligned} \quad (11.33)$$

where θ is the angle that the tangential vector makes with the vertical and measured positive clockwise. Note that we have used $\partial \vec{t} / \partial s = (-\partial \theta / \partial s) \vec{n}$ and $\vec{k} = -\cos \theta \vec{t} - \sin \theta \vec{n}$. Equation 11.33 shows that the tangential components of the external forces act to increase the tension, while the normal components cause the towline to bend. Because the normal component of the drag force is much larger than the tangential component, most of the fluid force is used to turn the cable.

From the force diagram (in Figure 11.17), the angle that the cable makes with the vertical where it is connected to the towed body is given by

$$T(0) \cos \theta(0) = W, \quad T(0) \sin \theta(0) = \text{Drag}$$

Once we know the weight and the drag force on the towed body, the tension and the angle at $s = 0$ can be found. If the drag is negligible compared with the weight, then the cable must be near vertical and the tension must be equal to the weight of the towed body at $s = 0$:

$$T(0) \approx W \quad \text{and} \quad \theta(0) \approx 0$$

For now, let us assume that this is the case. Then, with these initial conditions, the system of ordinary differential equations (Equation 11.33) can be solved numerically for $T(s)$ and $\theta(s)$. For example,

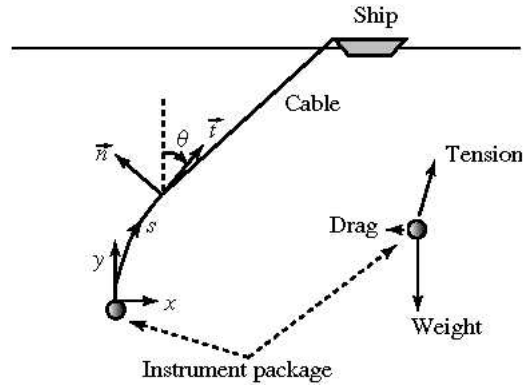


FIGURE 11.17 Towed system in equilibrium and the forces acting on the towed body.

even very simple finite difference equations will work. A set of equations

$$T_{i+1} = T_i + \left(mg \cos \theta_i - C_T \rho \frac{D}{2} U^2 \sin^2 \theta_i \right) \Delta s \quad (11.34)$$

$$\theta_{i+1} = \theta_i - \left(mg \sin \theta_i + C_D \rho \frac{D}{2} U^2 \cos^2 \theta_i \right) \Delta s / T_i \quad (11.35)$$

where $T_i = T(i\Delta s)$, are used here, and it works very well for $\Delta s = 0.05$.

The Cartesian coordinates, x and y , are related to θ by

$$\frac{dx}{ds} = \sin \theta \quad \text{and} \quad \frac{dy}{ds} = \cos \theta$$

and can also be obtained by integrating them numerically.

Figure 11.18 shows the results when $mg = 1.5$ N/m, $C_D \rho D U^2 / 2 = 10$ N/m, $C_T \rho D U^2 / 2 = 0.1$ N/m, $W = 100$ N, and the cable is 100 m long. Care is taken so that the ship is located at $x = 0$ and $y = 0$.

It is interesting to note that θ approaches a critical value, and the shape gradually becomes linear toward the ship. Mathematically, $d\theta/ds$ becomes zero. This is when the drag force is completely balanced by the normal component of the cable weight. The angle at which this occurs, θ_{cr} , can be obtained from the second governing equation and

$$mg \sin \theta_{cr} = -f^n, \quad \frac{\sin \theta_{cr}}{\cos \theta_{cr}} = C_D \rho \frac{D}{2} U^2 \frac{1}{mg}$$

In our case, $\theta_{cr} = 1.184$ rad, and this value agrees with Figure 11.18.

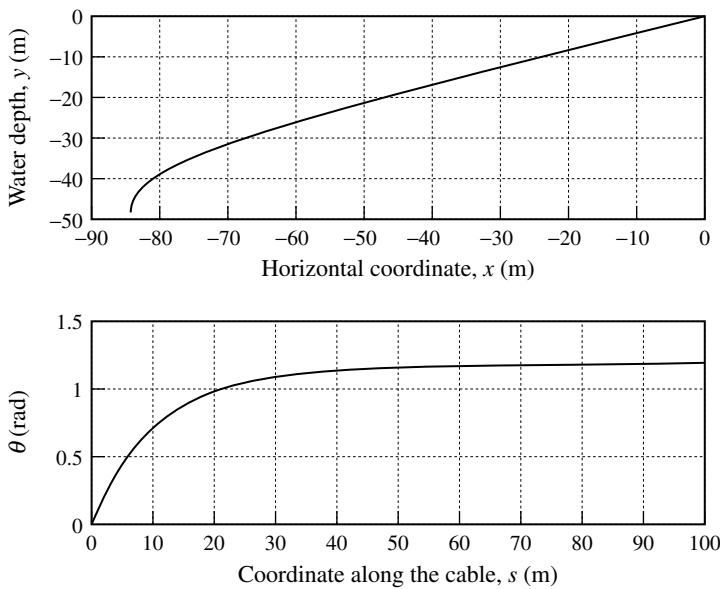


FIGURE 11.18 The equilibrium configuration of a towed cable and the angle that the cable makes with the vertical when $mg = 1.5$ N/m, $C_D \rho D U^2 / 2 = 10$ N/m, $C_T \rho D U^2 / 2 = 0.1$ N/m, and $W = 100$ N.

11.3.2 Fluid Forces on an Articulated Tower

Offshore structures are used in the oil industry as exploratory, production, oil storage, and oil landing facilities. They are designed to be self-supporting and sufficiently stable for offshore activities such as drilling and production of oil. An articulated tower as seen in Figure 11.19 is an example of an offshore platform that consists of a base, shaft, universal joint that connects the base and the shaft, ballast chamber, buoyancy chamber, and deck. The ballast chambers provide the extra weight so that the tower's bottom stays on the ocean floor, and the buoyancy chamber adds the necessary buoyancy so that the tower does not fall.

An articulated tower can be effectively modeled as a rigid inverted pendulum, where the deck is modeled as a point mass, the shaft as a uniform rigid bar, and the buoyancy chamber by a point buoyancy. In two dimensions, motion of the tower can be described with a single DoF (Chakrabarti and Cottor, 1979; Bar-Avi, 1996). The equation of motion in terms of the tower's deflection angle is obtained by summing the moment about the point O in Figure 11.20 and is given by

$$I \frac{d^2 \theta}{dt^2} = \sum M_O = mg \frac{L}{2} \sin \theta + MgL \sin \theta - Bl \sin \theta + \int_0^L f^n x dx$$

where I is the mass moment of inertia about the point O given by $I = mL^2/3 + ML^2$, m is the mass of the shaft, g is the gravitational acceleration, L is the length of the shaft, M is the point mass at the top, B is the buoyancy provided by the buoyancy chamber, l is its moment arm, f^n is the normal fluid force per unit length, and x is the coordinate along the shaft from O.

The fluid force per unit length in the normal direction is given by

$$f^n = C_D \rho \frac{D}{2} (w^n - v^n) |w^n - v^n| + C_M \rho \pi \frac{D^2}{4} \dot{w}^n - C_A \rho \pi \frac{D^2}{4} a^n$$

where the last term is the force in the normal direction due to the added mass. v^n and a^n are the velocity and the acceleration of the body in the normal direction and are given by

$$v^n = x \frac{d\theta}{dt} \quad \text{and} \quad a^n = x \frac{d^2 \theta}{dt^2}$$

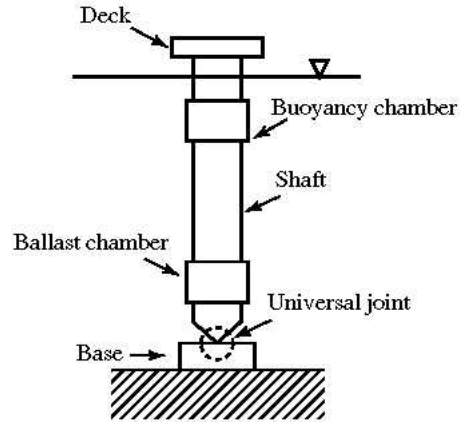


FIGURE 11.19 Schematic of an articulated tower.

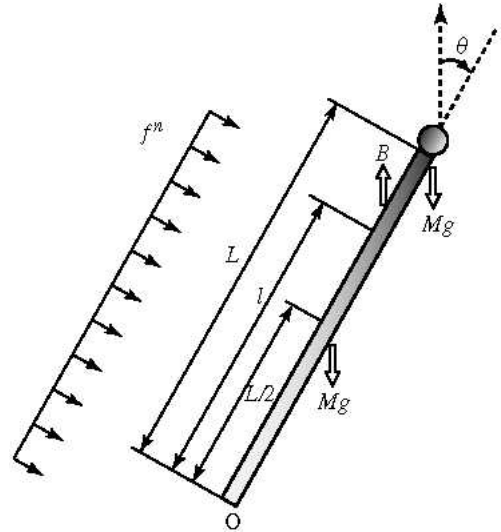


FIGURE 11.20 Free-body diagram.

If we assume that the surrounding fluid is stationary, then the normal velocity and the acceleration of the fluid (w and $\dot{w}$) are zeros. Thus, the moment due to the fluid force is given by

$$\begin{aligned}\int_0^L f^n x \, dx &= \int_0^L \left(-C_D \rho \frac{D}{2} x^2 \left(\frac{d\theta}{dt} \right)^2 \text{sign} \left(\frac{d\theta}{dt} \right) + C_A \rho \pi \frac{D^2}{4} x \frac{d^2 \theta}{dt^2} \right) x \, dx \\ &= -C_D \rho \frac{D}{2} \frac{L^4}{4} \left(\frac{d\theta}{dt} \right)^2 \text{sign} \left(\frac{d\theta}{dt} \right) + C_A \rho \pi \frac{D^2}{4} \frac{L^3}{3} \frac{d^2 \theta}{dt^2}\end{aligned}$$

and the equation of motion is given by

$$\left(m \frac{L^2}{3} + ML^2 + C_A \rho \pi \frac{D^2}{4} \frac{L^3}{3} \right) \frac{d^2 \theta}{dt^2} = \left(mg \frac{L}{2} + MgL - Bl_b \right) \sin \theta - C_D \rho \frac{D}{2} \frac{L^4}{4} \text{sign} \left(\frac{d\theta}{dt} \right) \left(\frac{d\theta}{dt} \right)^2$$

Note that the normal fluid drag force adds directly to the restoring moment in the case of a rigid bar. The equation of motion can be solved numerically once the initial conditions ($\theta[0]$ and $d\theta/dt[0]$) are given.

The equation of motion can be simplified if we assume that the angle of rotation θ is small. More specifically, if we assume that θ^2 is negligible when compared with 1, then we find that²

$$\sin \theta \approx \theta$$

The equation of motion can be simplified to

$$\left(m \frac{L^2}{3} + ML^2 + C_A \rho \pi \frac{D^2}{4} \frac{L^3}{3} \right) \frac{d^2 \theta}{dt^2} - \left(mg \frac{L}{2} + MgL - Bl_b \right) \theta + C_D \rho \frac{D}{2} \frac{L^4}{4} \text{sign} \left(\frac{d\theta}{dt} \right) \left(\frac{d\theta}{dt} \right)^2 = 0$$

which resembles the equation for a linear oscillator with a nonlinear damping term. Note that the system becomes unstable when the stiffness term (the coefficient of θ) becomes negative. This occurs when the buoyancy is not sufficient or

$$B < \frac{1}{l_b} \left(mg \frac{L}{2} + MgL \right)$$

11.3.3 Distribution of Significant Wave Heights — Weibull and Gumbel Distributions

The National Buoy Data Center (NBDC) run by NOAA collects ocean data such as wind, current, wave, pressure, and temperature data in various locations and the records are made public. Let us say that we are to design an articulated tower (in Section 11.3.2) in one of these locations where the data are available. The first task is to characterize the environment. Using all of the information that is collected is inefficient and impractical. Instead, we are interested in choosing a single number that can represent typical and extreme situations such as 10- and 50-year storms. For now, let us only consider random waves. We are then interested in finding the significant wave heights representing 10- and 50-year storms.

From NBDC data for a buoy outside Monterey Bay, the number of occurrences for ranges of significant wave heights is constructed in Table 11.2. The measurements were taken every hour for about 12 years. We first construct the corresponding Weibull distribution using the method described in Section 11.1.6. We first guess γ so that a pair of $\ln(-\ln\{1 - F(h)\})$ and $\ln(h - \gamma)$ form a

²This is called the small angle assumption.

TABLE 11.2 Number of Occurrences of Various Sea States

Significant Wave Height, h (m)	Number of Occurrences	Sum
< 1	2,367	2,367
1–2	46,353	48,720
2–3	3,4285	83,005
3–4	1,3181	96,186
4–5	3,813	99,999
5–6	716	100,715
6–7	145	100,860
7–8	32	100,892
8–9	8	100,900
9–10	2	100,902
Total	100,902	

straight line. Figure 11.21 shows that the pair yields nearly a straight line when $\gamma \approx 0.84$. The slope and the y intercept of this line are 1.6 and -0.78 , respectively. The Weibull parameters are then $m = 1.6$ and $\beta = 1.6$.

Similarly, we can find the corresponding Gumbel probability density function by plotting pairs of $(h, \ln(-\ln\{F(h)\}))$ to form a line. For the data shown in Table 11.2, the line has a slope of -1.52 and y intercept of 2.84 so that $\alpha = -1.52$ and $\beta = 1.87$.

Figure 11.22 shows the Weibull probability density and the cumulative distribution (Equation 11.17) in solid lines, the Gumbel probability density and the cumulative distribution in dotted lines (Equation 11.18), and the discrete probability density and the cumulative distribution derived from Table 11.2 in symbols.

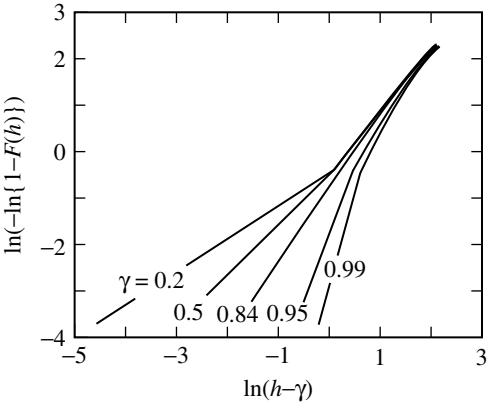


FIGURE 11.21 Plots of $(\ln(h - \gamma), \ln[-\ln\{1 - F(h)\}])$ for various values of γ .

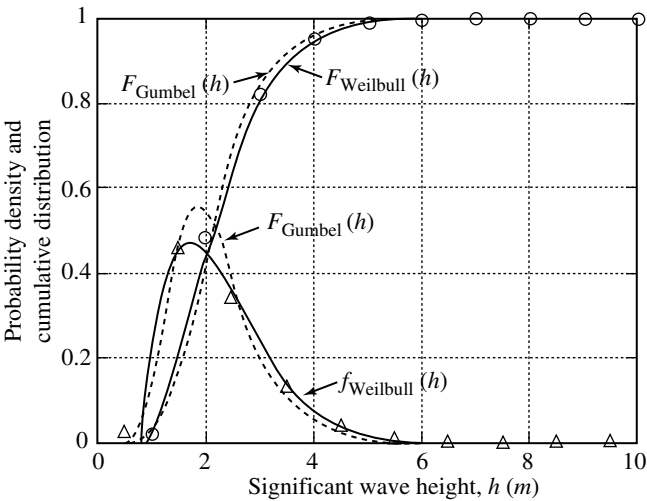


FIGURE 11.22 Weibull approximations of the probability density and cumulative distribution of significant wave heights measured in the outer Monterey Bay area. The symbols are the values given in Table 11.2.

TABLE 11.3 Comparison of Representative Significant Wave Heights for Long-Term Predictions from Gumbel and Weibull Distributions

	5-Year (m)	10-Year (m)	50-Year (m)
Weibull	7.84	8.15	8.79
Gumbel	8.83	9.33	10.4

The next step is to find a significant wave height that can represent an N -year storm, h_N . The probability that we will not have an N -year storm in any given year is $1 - 1/N$ and is equivalent to the probability that the significant wave height will not exceed h_N in the same year. The probability that $h < h_N$ in a single measurement is $F(h_N)$, and the probability that $h < h_N$ in every measurement taken in a year is $F(h_N)^{24 \times 365}$. Then, we have

$$1 - \frac{1}{N} = F(h_N)^{24 \times 365}$$

Table 11.3 shows significant wave heights that represent 5-, 10-, and 50-year storms obtained using the Weibull and Gumbel distributions.

The Gumbel probability distribution gives higher significant wave heights. For this particular set of data, the Weibull distribution seems to fit the data better (Figure 11.22), and the Weibull distribution is the most often used distribution in the offshore industry.

11.3.4 Reconstructing Time Series for a Given Significant Wave Height

Previously, we found significant wave heights that could represent 5-, 10-, and 25-year storms for a given site. Recall that the significant wave height can entirely characterize the Pierson–Moskowitz spectra. Once the spectral density is determined, a sample time history of the wave profile, $\eta(t)$, can be determined using either Borgman’s or Shinozuka’s method (Section 11.1.4). Here, Shinozuka’s method is used to generate the random wave elevations.

Let us first find the random frequencies distributed according to $S_{\eta\eta}^o(\omega)/\sigma_\eta^2$. The P–M spectrum in terms of the significant wave height is given by Equation 11.8.

$$S_{\eta\eta}^o(\omega) = 0.7795\omega^{-5} \exp\left(-\frac{3.118}{H_s^2}\omega^{-4}\right)$$

The variance is given by

$$\sigma_\eta^2 = \int_0^\infty S_{\eta\eta}^o(\omega)d\omega = \frac{H_s^2}{16}$$

The probability density and the cumulative distribution functions are given by

$$f(\omega) = S_{\eta\eta}^o(\omega)/\sigma_\eta^2 = \frac{12.472}{H_s^2}\omega^{-5} \exp\left(-\frac{3.118}{H_s^2}\omega^{-4}\right), \quad F(\omega) = 1 - \exp\left(-\frac{3.118}{H_s^2}\omega^{-4}\right)$$

The inverse of the cumulative distribution function is given by

$$F^{-1}(x) = \left(-\frac{H_s^2}{3.118}\ln(1-x)\right)^{-1/4}$$

The random frequencies distributed according to $f(\omega)$ can be obtained from uniformly distributed random numbers x from 0 and 1. Table 11.4 shows uniform random numbers between

TABLE 11.4 Generation of Random Frequencies Distributed According to $f(\omega)$ from Uniform Random Numbers

Uniform Random Numbers $0 < x < 1$	Random Frequencies ω Distributed According to $f(\omega)$
0.950	$(-19.713 \ln[1 - 0.950])^{-1/4} = 0.360$
0.231	$(-19.713 \ln[1 - 0.231])^{-1/4} = 0.662$
0.606	$(-19.713 \ln[1 - 0.606])^{-1/4} = 0.483$
$\vdots$	$\vdots$

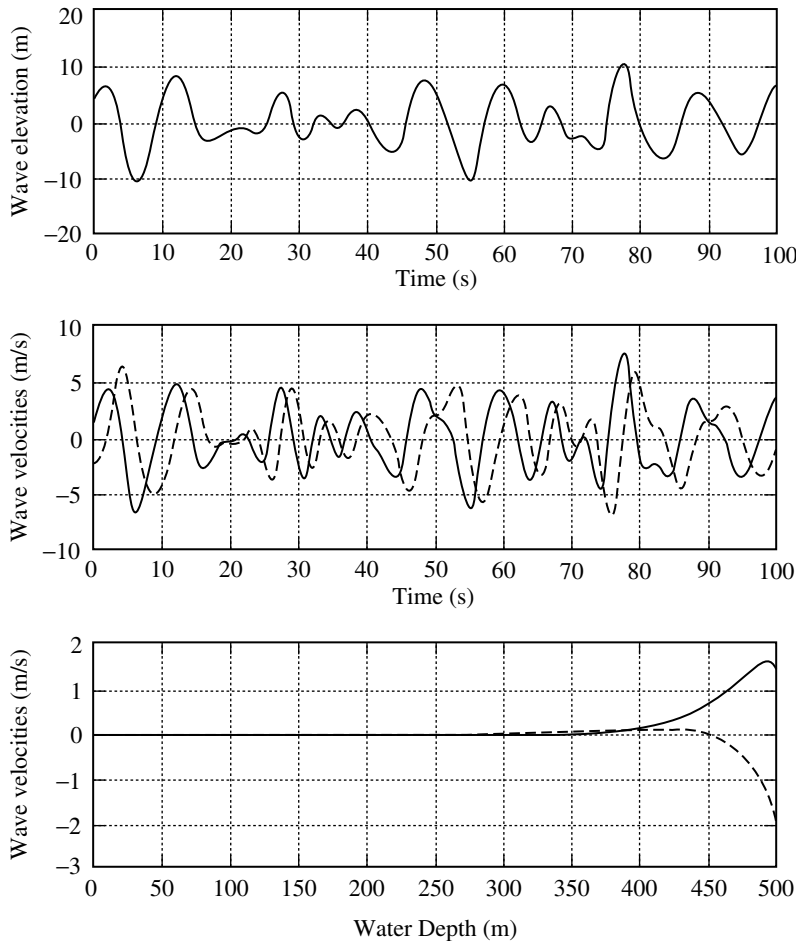


FIGURE 11.23 Wave elevation and velocities.

0 and 1 and the random frequencies distributed according to $f(\omega)^3$. The significant wave height of 7.84 m is used.

We can obtain 100 in this way, and the wave elevation is also obtained using Equation 11.13. The random phase φ_i is obtained by multiplying uniform random numbers (different from the ones used to generate the random frequencies) by 2π .

Figure 11.23 shows the surface elevation as a function of time, the corresponding wave velocities at the water surface (Section 11.1.7) as functions of time, and the wave velocities at $t = 0$ as functions of the water depth. Note that the wave velocities decay with depth.

³The uniform random numbers can be generated by the MATLAB[®] rand function.

11.3.5 Available Numerical Codes

Many numerical codes are available for modeling the dynamics of slender structures such as risers, tether, umbilicals, and mooring lines. The first example in this section was solved by a numerical code, WHOI Cable, developed at Woods Hole Oceanographic Institution. WHOI Cable is a time-domain program that can be used for analyzing the dynamics of towed and moored cable systems in both two and three dimensions. It takes into account bending and torsion as well as extension.

Comparative studies investigating flexible risers were carried out by ISSC Committee V7 from computer programs developed by 11 different institutions in the period between 1988 and 1991, and the results were reported by Larsen (1992). More recently, Brown and Mavrakos (1999) conducted a comparative study on the dynamic analysis of suspended wire and chain mooring lines and reported results from 15 different numerical codes. The participants included engineering consultancies, and academic and research institutions involved in marine technology. Some of the time-domain programs that were included in the comparative study are MODEX by Chalmers University of Technology, FLEXAN-C by Institute Francais du Petrole, DYWFLX95 by MARIN, R.FLEX by MARINTEK, CABLEDYN by National Technical University of Athens, DMOOR by Noble Denton Consultancy Services Ltd, V.ORCAFLEX by Orcina Ltd Consulting Engineers, ANFLEX by Petrobras SA, TDMOOR-DYN by University College London, FLEXRISER by Zentech International. Some of these programs are available to academic institutions and government laboratories at no cost.

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