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Source of Noise

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Summary

In this chapter, a mathematical description of sound radiation is briefly presented accompanied by an introduction to sound sources, monopole, and dipole. The modeling of a simple source of noise is discussed, introducing Green's function. As an example of simple sound radiation, the sound field generated by a source embedded in a plane surface is described. Finally, an estimation of noise-source sound power is presented by introducing the power conversion factor of actual machinery.

16.1 Introduction

A careful examination of noise measurement data reveals that there exists a very wide variety of noise sources. Each source is peculiar to its generation mechanism, which can be any of a wide range of phenomena including fluid mechanics and the vibration of structures. However, in analysis, sources are normally simplified to rather simple and typical models in their generation mechanism.

The vibration of a solid body, which may be in contact with the fluid medium, generates sound waves or vibratory forces acting directly on a fluid, will result in the emission of acoustic energy in the medium. In the next section, an expression for an idealized sound source is introduced. We will assume that the fluid medium outside the source region is initially uniform and at rest. Also, we will concentrate on wave propagation in an infinite medium.

Generally, acoustic waves sensed as a sound represent a very small energy density in the medium. Only a very small fraction of the mechanical energy of a source body is converted into acoustic energy. The conversion factor, defined as the ratio of sound power to the mechanical power of the source, is in the order of 10^{-7} to 10^{-5} . Some examples of estimated sound power conversion factor are given for typical common noise sources.

16.2 Radiation of Sound

16.2.1 Point Source

16.2.1.1 Simple Source: Spherical Wave by a Monopole

Propagation of sound pressure wave $p(x, y, z, t)$ is described by the following partial differential equation for a medium where a field point is expressed by orthogonal coordinate system O -xyz, as shown

in Figure 16.1(a):

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0, \quad (16.1)$$

$$\nabla^2 \equiv \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$$

where c denotes the velocity of sound propagation. If the source region is compact and the generating motion has no preferred direction, it will produce a wave, which spreads spherically outwards. As the medium is assumed infinite in extent, the wave-form will depend on the distance, r , from the center of the source. The wave equation in this case is

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right) - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \quad (16.2)$$

When a monopole source of angular frequency ω is assumed, the simplest solution for the outward propagating waveform is expressed as

$$p(r, t) = p_\omega(\mathbf{r}) e^{-i\omega t},$$

$$p_\omega(\mathbf{r}) = \frac{-\omega \rho}{4\pi r} S_\omega e^{ikr}, \quad k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \quad (16.3)$$

where ρ denotes the density of the medium and k denotes the wave number. Here, p_ω is used to denote the sinusoidal component of the sound pressure with angular frequency, ω . The subscript ω on a variable typically indicates the sinusoidal component of a variable, but the variables related to sound energy and sound power, such as w , I , W , do not have the subscript ω even if they mean the sinusoidal component. In this case, the source of sound is taken as a “pulsating globe” of radius a and radial velocity U_ω on the surface. Therefore, the flow outward from the origin, S_ω , is related to U_ω as follows, and as shown in Figure 16.1(a):

$$S_\omega = 4\pi a^2 U_\omega \quad (16.4)$$

We should note that, while a pulsating globe with a finite radius a is assumed as the physical sound source, the sound field by a monopole with infinitesimal small size and finite magnitude, as expressed mathematically in Equation 16.3, is used.

The other quantities related to the spherical wave are described next [1]:

$$\left\{ \begin{array}{ll} u_{r\omega} = \frac{-1}{4\pi r^2} (ikr - 1) S_\omega e^{ik(r-ct)}; & \text{radial velocity} \\ w = \rho \left(\frac{1}{4\pi r^2} \right)^2 |S_\omega|^2 \left[(kr)^2 + \frac{1}{2} \right]; & \text{energy density} \\ I = \rho c \left(\frac{k}{4\pi r} \right)^2 |S_\omega|^2 = \frac{|p|^2}{\rho c}; & \text{energy flux intensity} \\ W = (4\pi r^2) I = \rho c \frac{\pi}{\lambda^2} |S_\omega|^2 = \frac{\rho \omega^2}{4\pi c} |S_\omega|^2; & \text{total power} \end{array} \right. \quad (16.5)$$

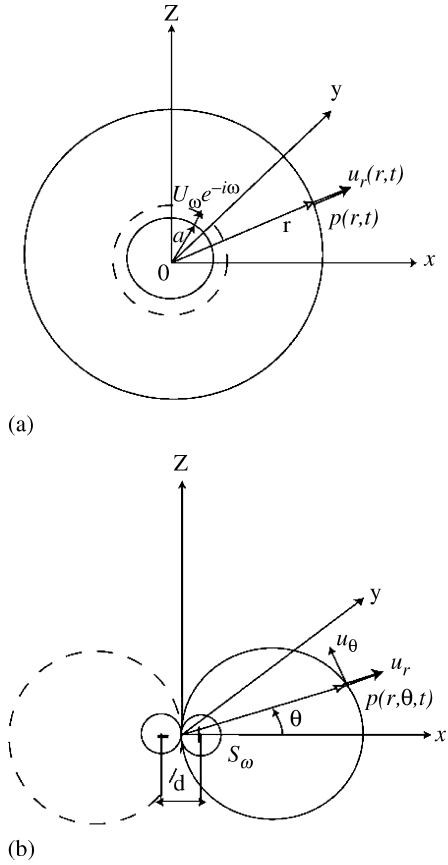


FIGURE 16.1 Directivity of monopole and dipole, (a) Spherical sound field by a monopole; (b) Axisymmetric sound field by a dipole.

We should note that the first term in the parentheses, kr , is negligible in the region where $|kr| \ll 1$ is valid. Hence,

$$w = \frac{\rho}{2} (|S_\omega|/4\pi r^2)^2 = \frac{\rho}{2} u_{r\omega}^2 = \left(\frac{\lambda}{\omega}\right) \frac{I}{4\pi r^2}$$

is reduced. Conversely, when $|kr| \gg 1$ is valid, then $w = \rho(k/4\pi r)^2 |S_\omega|^2 = I/c$ is valid.

16.2.1.2 Simple Source: Plane Wave by an Alternating Piston

Another example of simple sound wave is generated in the one-dimensional field of fluid medium, as shown in Figure 16.1. Let us set the coordinate x along the axis of wave propagation, for example, the axis of duct with a constant cross-sectional area. Then the wave equation is

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \quad (16.6)$$

The solution for a periodic source is given by

$$p(x, t) = p_\omega(x) e^{-i\omega t}, \quad p_\omega(x) = \rho c u_\omega e^{ikx} \quad (16.7)$$

This is known as a *plane wave*, which is generated by the piston motion at the origin, the velocity of which is expressed by

$$u(t) = u_\omega(x) e^{-i\omega t} \quad (16.8)$$

The other quantities related with the plane wave are given below:

$$\begin{cases} u_\omega(x) = u_\omega e^{ikx}; & \text{particle velocity} \\ w = I = \rho c |u_\omega|^2 = |p_\omega|^2 / \rho c^2; & \text{energy density, sound intensity} \\ W = SI \text{ (} S; \text{ cross-sectional area);} & \text{total power} \end{cases} \quad (16.9)$$

A plane wave is generated in very limited situations, but its utility is rather wide since the sound wave propagating through a duct or duct-like space with a gradually varying cross section is approximated as the plane wave. Network theory is applied to the sound wave propagating through a branch and junction by using the description of a plane wave.

16.2.1.3 Dipole and Multipoles and Their Sound Field

Let us return to the three-dimensional sound field. The second simple solution to Equation 16.1 is the “dipole” sound field. Suppose that a pair of monopoles, close together, opposite in sign, and equal in magnitude, S_ω , are located along the x -axis as shown in Figure 16.1(b). Since only a preferred direction is assigned along the x -axis, the sound field is axisymmetric as represented by

$$p(r, \theta, t) = -k^2 D_\omega \frac{\rho c \cos \theta}{4\pi r} \left(1 + \frac{i}{kr}\right) e^{-i(\omega t - kr)} \quad (16.10)$$

D_ω is defined by

$$D_\omega = S_\omega d \quad (16.11)$$

where d denotes the separation of the monopoles as shown in Figure 16.1(b). Mathematically, d tends to zero, keeping D_ω finite, and the preferred axis is the x -axis. Physically, a sound field is commonly realized by a pair of monopoles with a finite separation that is short compared with the wavelength, λ , as illustrated next with realistic examples.

The characteristic quantities relating with dipole sound field are described below [1]:

$$\left\{ \begin{array}{ll} u_{r\omega} = -\frac{k^2 D_\omega \cos \theta}{4\pi r} \left(1 + \frac{2i}{kr} - \frac{2}{k^2 r^2}\right) e^{-i\omega t + kr}; & \text{radial velocity} \\ u_{\theta\omega} = i\frac{k^2 D_\omega \sin \theta}{4\pi r^2} \left(1 + \frac{i}{kr}\right) e^{-i\omega t + kr}; & \text{peripheral velocity} \\ w = \rho \left(\frac{k^2 |D_\omega|}{4\pi r}\right)^2 \left[\cos^2 \theta + \frac{1}{2} \left(\frac{1}{kr}\right)^2 + \frac{1}{2} \left(\frac{1}{kr}\right)^4 (1 + 3 \cos^2 \theta)\right]; & \text{energy density} \\ I_r = \rho c \left(\frac{k^2 |D_\omega|}{4\pi r}\right)^2 \cos^2 \theta, \quad I_\theta = 0; & \text{sound intensity} \\ W = \frac{\rho \omega^4}{12\pi c^3} |D_\omega|^2; & \text{total power} \end{array} \right. \quad (16.12)$$

We should add the following notes on the dipole sound field.

When $|kr| \gg 1$ is assumed, the second term in parentheses in Equation 16.10 is negligible. Then the directivity for $p(r, \theta, t)$ is expressed by $\cos \theta$. A similar directivity is found on I_r and on $u_{r\omega}$, $u_{\theta\omega}$, and w with the assumption $|kr| \gg 1$.

A pair of dipoles produces a *quadrupole*, a pair of quadrupoles produce an *octopole*, and so on. These are called *multipole* in general. Out of multipoles, the quadrupole is common in representing a sound field generated by mixing fluid flow, especially jet flow. More details on multipoles are found in Ref. [1].

16.2.2 Sources of Finite Volume

16.2.2.1 Description of Sound Field by Green's Function

In order to describe the sound field from distributed sources, source terms are introduced to the right side of Equation 16.1. The partial differential equation with source term is derived from the equation system representing the dynamics of fluid flow with periodic motion at angular frequency ω :

$$\nabla^2 p_\omega + k^2 p_\omega = -m_\omega + \text{div } \mathbf{F}_\omega \quad (16.13)$$

where p_ω denotes the acoustic pressure amplitude according to $p(x, y, z, t) = p_\omega(x, y, z) e^{-i\omega t}$, m_ω denotes the effective monopole source density expressed by $-i\omega \rho s_\omega$, s_ω denotes the generalization of the point source strength, S_ω , of Equation 16.4 for a distributed source, and $\mathbf{F}_\omega$ denotes the vector representation of point force-density in the fluid. Introduction of Green's function, g_ω , of angular frequency, ω , satisfying the following equation is useful in general:

$$\nabla^2 g_\omega + k^2 g_\omega = -\delta(x - x_0)\delta(y - y_0)\delta(z - z_0)e^{i\omega t_0} \quad (16.14)$$

Here, $\delta(z)$ denotes the Dirac impulse (delta) function of the variable z ; the coordinate (x_0, y_0, z_0) denotes the position of unit source, r_0 , with periodic angular velocity, ω , and t_0 denotes the time pertaining to the source. The solution of Equation 16.14 is

$$g_\omega(\mathbf{r}, \mathbf{r}_0) = \frac{1}{4\pi R} e^{ikR}, \quad |\mathbf{r} - \mathbf{r}_0| = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \quad (16.15)$$

where $\mathbf{r}$ denotes the position vector of sound field with coordinates (x, y, z) . The solution of Equation 16.13 is described by using $g_\omega(\mathbf{r}, \mathbf{r}_0)$ as follows:

$$p_\omega(\mathbf{r}) = \iiint [m_\omega(\mathbf{r}_0) - \text{div } \mathbf{F}_\omega(\mathbf{r}_0)] g_\omega(\mathbf{r}, \mathbf{r}_0) dx_0 dy_0 dz_0 \quad (16.16)$$

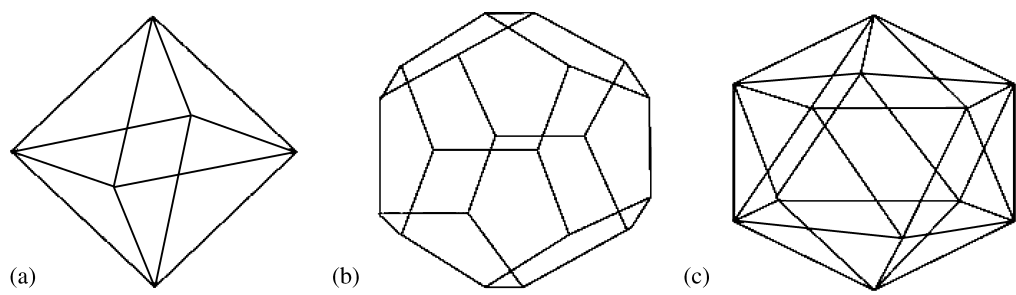


FIGURE 16.2 Practical monopole sources.

16.2.2.2 Radiation from Vibrating Small Body

A pulsating globe, as the simplest sound source, is rarely realized in the real world. The approximated monopole source required in most measurements is an eight-sided polyhedron, as depicted in Figure 16.2, where a loud speaker is installed on each of the surfaces. The sound field radiated by a thus approximated source is almost the same as that by a pulsating globe at the far field, where $kr \gg 1$ is valid.

In this case, the sound field is represented by Equation 16.16, assuming that $\mathbf{F}_\omega(\mathbf{r}_0) = 0$, $m_\omega(\mathbf{r}_0) = -i\omega\rho U_\omega\delta(n)$, and $dx_0dy_0dz_0 = dn dS$, where the source $m_\omega(\mathbf{r}_0)$ is distributed on a thin layer of thickness, dn , on the spherical surface element, dS . Substitution of these relationships into Equation 16.16 yields the formula

$$\begin{aligned} p_\omega(\mathbf{r}) &= - \iint_S \frac{i\omega\rho U_\omega}{4\pi R} e^{ikR} dS \\ &= - \frac{i\omega\rho}{4\pi R_0} (4\pi a^2) U_\omega e^{ikR_0} \end{aligned} \quad (16.17)$$

where $\iint_S dS$ means integration on the approximately spherical surface, and then $R \equiv |\mathbf{r} - \mathbf{r}_0| \cong R_0 = \sqrt{x^2 + y^2 + z^2}$ is applied. The final reduction of Equation 16.17 gives the same expression as is deduced from Equation 16.3 and Equation 16.4.

The sound field generated by a monopole source is illustrated in Figure 16.1. This field is unchanged with the presence of a rigid plane AB in Figure 16.3. It is clear, on account of symmetry, that the presence of the rigid plane does not alter the sound field in any way, because only the tangential component of particle velocity is induced on the plane. This utilization of the symmetry and construction of the semi-infinite field is applied to the expression of the sound field generated by the baffled structure, as shown in Figure 16.4.

Let us imagine a sound field generated by an oscillating small body in an infinite medium, as shown in Figure 16.4(a). Since the oscillation is caused by an external force, the sound source is modeled by the force $-\mathbf{F}_\omega(\mathbf{r}_0)$. Therefore, the sound field is described by Equation 16.16, assuming $m_\omega(\mathbf{r}_0) = 0$. After applying a law of vector analysis, such as $g_\omega(\mathbf{r}, \mathbf{r}_0) \text{div } \mathbf{F}_\omega(\mathbf{r}_0) + \text{grad}_0 g_\omega + \text{div}_0(g_\omega \mathbf{F}_\omega)$, the sound field is

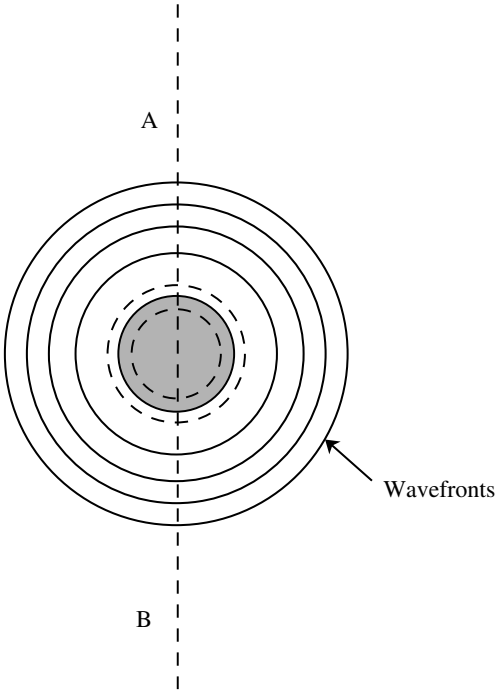


FIGURE 16.3 Field of a pulsating spherical source.

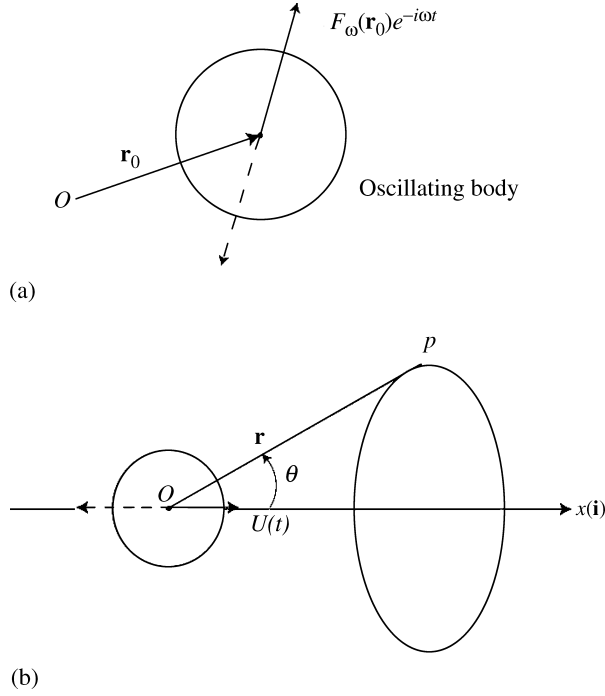


FIGURE 16.4 Dipole field generated by an oscillating small body: (a) oscillating small body; (b) oscillating sphere along x -axis.

represented by

$$p_{\omega}(\mathbf{r}) = \iiint \mathbf{F}_{\omega}(\mathbf{r}_0) \cdot \nabla_0 g_{\omega}(\mathbf{r}, \mathbf{r}_0) dx_0 dy_0 dz_0 \quad (16.18)$$

where $\iiint \text{div}_0(g_{\omega} \mathbf{F}_{\omega}) dx_0 dy_0 dz_0 = \iint_S g_{\omega} \mathbf{F}_{\omega} dS_n(x_0, y_0, z_0) = 0$ is applied. Assume that a small sphere of radius a_0 is oscillating along the x -axis with angular frequency ω . Instead of an oscillating sphere with velocity $U_t = U_{\omega} e^{-i\omega t}$, the sound field is generated by the concentrated body force, $F(t) = -m(d/dt)U(t) = i\omega m U_{\omega} e^{-i\omega t}$, (m is the mass of the sphere), at the origin, $\mathbf{r}_0 = 0$. Then $\mathbf{F}_{\omega}(\mathbf{r}_0)$ is expressed by $F_{\omega}(\mathbf{r}_0) = (F_{x\omega} \mathbf{i}) \delta(x_0) \delta(y_0) \delta(z_0)$; $F_{x\omega} = i m \omega U_{\omega}$. This approximate reduction is appropriate when $ka_0 \ll 1$ is valid. Substituting the approximation, Equation 16.18 is rewritten as

$$p_{\omega}(\mathbf{r}) \cong F_{x\omega} \frac{\partial}{\partial x_0} g_{\omega}(\mathbf{r}, \mathbf{r}_0, \theta) = -k^2 D_{\omega} \frac{\rho c}{4\pi r} \cos \theta \left(1 + \frac{i}{kr}\right) e^{ikr} \quad (16.19a)$$

$$D_{\omega} = \frac{i}{k\rho c} F_{x\omega} \quad (16.19b)$$

As the sound field is axisymmetric about x -axis, the sound pressure, $p_{\omega}(\mathbf{r}, \theta)$, depends only on $\mathbf{r}$ and θ , where $\mathbf{r}$ and θ are defined in Figure 16.4(b). The expression is applicable to the sound field generated by an oscillating small body in a free space.

16.2.3 Radiation from a Plane Surface

16.2.3.1 Radiation from a Small Body in Infinite Plane Surface

The introduction of an infinite rigid plane surface to the sound field, as shown in Figure 16.3, simplifies the formulation of the sound field generated by an oscillating body adjacent to a large plane.

The configuration discussed in this section relates to a source in the presence of an infinite plane barrier, so that the medium is confined to one side of the plane.

By taking the effect of the image caused by the rigid plane surface, the Green's function, $g_\omega(\mathbf{r}, \mathbf{r}_0)$, in this case simplifies as given below, to what is called Rayleigh's formula [2]:

$$g_\omega(\mathbf{r}, \mathbf{r}_0) = \frac{1}{2\pi R} e^{ikR}, \quad R = |\mathbf{r} - \mathbf{r}_0| \quad (16.20)$$

where $\mathbf{r}_0$ denotes the projection position of the source on the surface. Therefore, Equation 16.16 is rewritten as follows:

$$p_\omega(\mathbf{r}) = \iint \int \frac{e^{ikR}}{2\pi R} [m_\omega(\mathbf{r}_0) - \text{div } \mathbf{F}_\omega(\mathbf{r}_0)] dx_0 dy_0 dz_0 \quad (16.21)$$

16.2.3.2 Radiation from a Circular Piston

Let us consider the sound field generated by a rigid circular piston of radius a mounted flush with the surface of an infinite baffle and vibrating with simple harmonic motion of angular frequency, ω . The solution of this example is applicable to a number of related problems, including the radiation from the open end of a flanged organ pipe.

The coordinate system is shown in Figure 16.5, where the infinite baffle and the piston are placed in the Oxy -plane. Note that the observation point, P , is denoted by $\mathbf{r} = \overrightarrow{OP}$, while the source point, Q , is denoted by $\mathbf{r}_0 = \overrightarrow{OQ}$.

Since sound is observed only in the semi-infinite plane where $z > 0$ is valid, only $m_\omega(\mathbf{r}_0)$ remains nonzero in Equation 16.21. As the velocity of the piston is denoted by $U_\omega e^{-i\omega t}$ along the z -axis, $m_\omega(\mathbf{r}_0) = -i\omega\rho U_\omega \delta(z_0)$, is distributed only on the circular piston. Finally, Equation 16.21 is rewritten as

$$p_\omega(\mathbf{r}) = \frac{-i\omega\rho U_\omega}{2\pi} \iint_S \frac{e^{ikR}}{R} dx_0 dy_0 \quad (16.22)$$

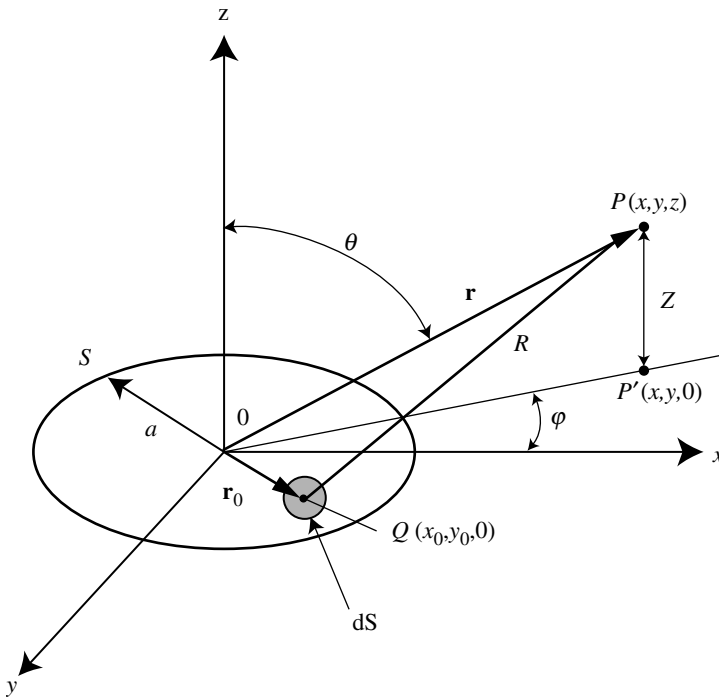


FIGURE 16.5 The rigid circular piston and the coordinate system.

Assuming $a \ll r$, the following approximation can be made:

$$R \cong r - (x_0 \cos \varphi + y_0 \sin \varphi) \sin \theta, \quad (16.23)$$

$$\frac{1}{R} \cong \frac{1}{r}$$

where φ and θ denote the angles defining the observation point, P , as shown in Figure 16.5. By changing from Cartesian coordinates, x_0, y_0 , to polar coordinates, ρ, φ_0 , such that $x_0 = \rho \cos \varphi_0$, $y_0 = \rho \sin \varphi_0$ in the integral above, we rewrite Equation 16.22 as

$$p_\omega(\mathbf{r}) = -\frac{i\omega\rho U_\omega}{2\pi R} e^{ikr} \int_0^{2\pi} d\varphi_0 \times \int_0^a \exp[-k\rho \cos(\varphi_0 - \varphi) \sin \theta] \rho d\rho \quad (16.24)$$

The integration is performed by introducing the Bessel function of the first order, $J_1(z)$ as follows:

$$p_\omega(\mathbf{r}) = -\frac{i\omega\rho e^{ikr}}{2\pi r} f_\omega(\theta), \quad (16.25)$$

$$f_\omega(\theta) = \pi a^2 U_\omega \left[\frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right]$$

Note that dependency on φ has disappeared in the integration process, which follows from the axisymmetry of the sound field. The corresponding intensity at $\mathbf{r}$ is given by

$$I_r(\theta) \cong \frac{|p_\omega|^2}{\rho c} = \frac{\rho c U_\omega^2 a^2}{4r^2} (ka)^2 \left[\frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right]^2 \quad (16.26)$$

Figure 16.6 illustrates the dependency of I_r on θ for two cases of ka . Note that, for the smaller $ka = 2\pi a/\lambda$, I_r is almost independent on θ , which is similar to the dependence of the monopole.

16.2.3.3 Radiation from a Rectangular Plate

An normal velocity distribution, $u_\omega(x_0, y_0)$, is prescribed over a baffled planar radiator located in the plane $z_0 = 0$ in the region $-L_x \leq x_0 \leq L_x$, $-L_y \leq y_0 \leq L_y$, as shown in Figure 16.7.

In this case, the sound pressure field is represented by the following equation, similar to the previous section:

$$p_\omega(\mathbf{r}) = \frac{-i\omega\rho}{2\pi} \int_{-L_y}^{L_y} dy_0 \int_{-L_x}^{L_x} \frac{U_\omega(x_0, y_0) e^{ikR}}{R} dx_0 \quad (16.27)$$

Assuming $2L_x, 2L_y \ll r$, the same approximation in as Equation 16.23 is acceptable. Therefore, $p_\omega(\mathbf{r}) = p_\omega(r, \theta, \phi)$ takes the form [2]:

$$p_\omega(r, \theta, \phi) = \frac{-i\omega\rho e^{ikr}}{2\pi r} \int_{-L_y}^{L_y} dy_0 \int_{-L_x}^{L_x} U_\omega(x_0, y_0) \exp[-ik \sin \theta (x_0 \cos \phi + y_0 \sin \phi)] dx_0 \quad (16.28)$$

The result of the above integration, assuming $U_\omega(x_0, y_0) = U_\omega = \text{const}$, which means the rigid rectangular piston oscillates with amplitude U_ω along the z -axis, is

$$p_\omega(r, \theta, \phi) = \frac{i\omega\rho U_\omega}{2\pi r} (4L_x L_y) e^{ikr} S(kL_x \sin \theta \cos \phi) S(kL_y \sin \theta \sin \phi) \quad (16.29a)$$

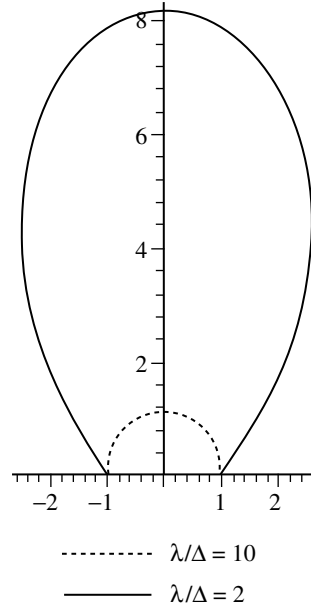


FIGURE 16.6 Directivity of sound intensity generated by an oscillating piston.

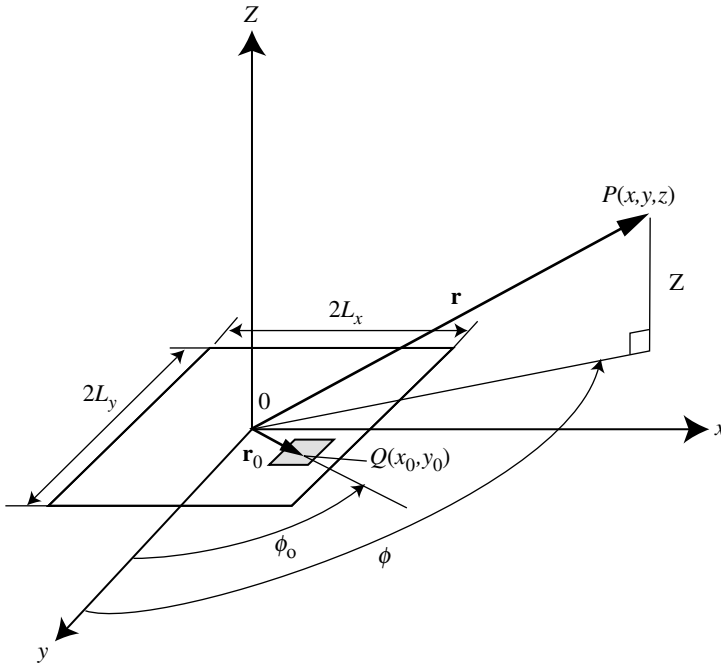


FIGURE 16.7 A flexural rectangular plate and the coordinate system.

where the function $S(z)$ is defined by

$$S(z) = \frac{\sin z}{z} \quad (16.29b)$$

Now, let us discuss the case of flexural plate, which has the same lengths, $2L_x$ and $2L_y$, but $U_\omega(x_0, y_0)$ represents the flexural vibration mode. Flexural mode patterns of rectangular panels take the general form of contiguous regions of roughly equal area and shape, which vary alternately in vibration phase, and are separated by nodal lines of zero vibration.

For simply supported edges, the normal vibration velocity distribution is

$$U_\omega(x_0, y_0) = U_{mn} \sin\left(\frac{m\pi x}{2L_x}\right) \sin\left(\frac{n\pi y}{2L_y}\right), \quad (0 \leq x \leq 2L_x, 0 \leq y \leq 2L_y) \quad (16.30)$$

Note that the coordinate system $O-x_0y_0z_0$ shifts its origin from the center point of the panel to the edge at the leftmost and frontmost point in Figure 16.7.

The approximation represented by Equation 16.23 is valid as well in this case. The result of integration is rather simple as given below [3]:

$$p_\omega(r, \theta, \phi) = \frac{-i\omega\rho U_{mn} e^{kr}}{2\pi r} \frac{4L_x L_y}{mn\pi^2} \left[\frac{(-1)^m e^{i\alpha} - 1}{(\alpha/m\pi)^2 - 1} \right] \left[\frac{(-1)^n e^{i\beta} - 1}{(\beta/n\pi)^2 - 1} \right] \quad (16.31)$$

where α and β are defined by

$$\alpha = 2kL_x \sin \theta \cos \phi, \quad \beta = 2kL_y \sin \theta \sin \phi \quad (16.32)$$

The corresponding intensity, $I_r(\theta, \phi)$, at $\mathbf{r}$ is expressed by

$$I_r(r, \theta, \phi) = \frac{|p_\omega|^2}{\rho c} = 4\rho c |U_{mn}|^2 \left(\frac{4kL_x L_y}{\pi^3 r m n} \right)^2 \left\{ \frac{\cos\left(\frac{\alpha}{2}\right) \cos\left(\frac{\beta}{2}\right)}{[(\alpha/m\pi)^2 - 1][(\beta/n\pi)^2 - 1]} \right\} \quad (16.33)$$

where $\cos(\alpha/2)$ is used when m is an odd integer, and $\sin(\beta/2)$ is used when m is an even integer; $\cos(\beta/2)$ is used for even n and $\sin(\beta/2)$ for odd n .

16.2.4 Estimation of Noise-Source Sound Power

16.2.4.1 Power Conversion Factor of Machinery

It is often necessary to estimate the expected sound power that a particular machine might introduce into an environment [4]. One way where such an estimate may be approached for a particular class of machine is by means of the sound power conversion factor, η_n . This factor is defined as

$$\eta_n = \frac{P}{P_m}$$

(16.34)

where P = sound power of the machine (W), and P_m = power of the machine (W). This relationship is valid for both mechanical and electrical machinery. The conversion factors for some common noise sources are given in Table 16.1.

Example 16.1

Estimate the sound power level of a typical 1-kW electric motor that operates at 1200 rpm.

Solution

From Table 16.1, we find that for typical electric motors, $\eta_n = 1 \times 10^{-7}$. Thus, using Equation 16.34, we obtain

$$P = \eta_n P_m = (1 \times 10^{-7}) \times 1000 = 10^{-4} \text{ (W)}$$

as the total sound power of motor. Then using Equation 37.7, L_W is given by

$$L_W = 10 \log \left(\frac{10^{-4}}{10^{-12}} \right) = 80 \text{ (dB)}$$

16.2.4.2 Fan Noise

We are familiar with noise nuisance caused by a domestic ventilating fan. The mechanical power of the fan is expressed by

$$P_m = p_T Q$$

(16.35)

where p_T denotes the total pressure rise through the fan and Q denotes the volumetric flow rate. According to the law of sixth power of flow velocity deduced by the aeroacoustics theory, sound power of the fan is proportional to $p_{T^{2.5}} Q$. Therefore, the specific ratio k_T defined below is more useful than η_n for the fan:

$$k_T = \frac{P_m}{p_{T^{2.5}} Q}$$

(16.36)

TABLE 16.1 Estimated Sound Power Conversion Factors for Common Noise Sources^a

Noise Source	Conversion Factor		
	Low	Midrange	High
Compressor, air (1–100 hp)	3×10^{-7}	5.3×10^{-7}	1×10^{-6}
Gear trains	1.5×10^{-8}	5×10^{-7}	1.5×10^{-6}
Loud speakers	3×10^{-2}	5×10^{-2}	1×10^{-1}
Motors, diesel	2×10^{-7}	5×10^{-7}	2.5×10^{-6}
Motors, electric (1200 rpm)	1×10^{-8}	1×10^{-7}	3×10^{-7}
Pumps, over 1600 rpm	3.5×10^{-6}	1.4×10^{-5}	5×10^{-5}
Pumps, under 1600 rpm	1.1×10^{-6}	4.4×10^{-6}	1.6×10^{-5}
Turbines, gas	2×10^{-6}	5×10^{-6}	5×10^{-5}

^a Total sound power for the four octave bands from 500 to 4000 Hz.

Source: Irwin, J. D. and Graf, E. R. 1979. *Industrial Noise and Vibration Control*, Prentice Hall, Englewood Cliffs, NJ.

TABLE 16.2 Specific Fan Noise Level for Low-Pressure Fans

Type	K_T (dB)
Axial	− 87.5 to − 70.7
Centrifugal/cirroco	− 87.5 to − 85.7
Centrifugal/radial	− 98.7 to − 84.7
Centrifugal/turbo	− 104.7 to − 89.7
Cross-flow	− 76.7 to − 68.7

Furthermore, usually the specific fan noise level, K_T , as given below is more useful than k_T .

$$K_T = L_W - 10 \log(p_T^{2.5} Q) \text{dB} \tag{16.37}$$

where p_T denotes the total pressure rise in mmAg, Q denotes the flow rate in m^3/min , L_W denotes the total sound power level in dB, and K_T represents the radiation efficiency of the fan noise. This efficiency varies with the type of the fan and with the flow rate when the model is assigned. In particular, K_T will be the lowest at the flow rate at which the aerodynamic power efficiency is the highest. Therefore, we will have the best advantage on the sound environment when the fan is operated at the highest efficiency. For convenience of design, the L_W is often evaluated by A-weighted total sound level (dB-A) not by linear total level (dB). Table 16.2 gives the specific fan noise level evaluated with dB-A for five types of the low-pressure fans.

Example 16.2

Consider a ventilating axial-flow fan, the specifications of which are: $p_T = 10$ mmAg, $Q = 30 \text{ m}^3/\text{min}$, $D = 30$ cm (diameter of the duct containing fan rotor). Estimate the directional distribution of intensity $I_r(\theta)$, assuming $r = 3$ m and $f = 150$ Hz for the main component of the fan noise, and $K_T = -79.0$ dB-A from Table 16.2. The fan noise is assumed to be radiated as a plane sound wave at the mouth of the duct.

Solution

First, we modify the K_T in dB-A to that of linear scale. From the frequency response for A-weighting network shown in Figure 2.5, for $f = 150$ Hz, the modification is found as $\Delta K_T = 15$ dB. Then, the modified specific fan noise level is $K_T = -79.0 + 15.0 = -64.0$ dB. By using Equation 16.37, $L_W = K_T + 25 \log p_T + 10 \log Q = -64 + 25 + 15 = -24$ dB is obtained. This means the emitted total sound power is $W = 10^{-24/10} = 3.98 \times 10^{-3}$ W. Since we assume a plane sound wave at the mouth of the duct for the fan noise, we can use the sound radiation model of a circular piston with $a = 15$ cm for the radiated sound wave. In our case $k_a = 2\pi a/(cf) = 0.415$, or $\lambda/a = 15.1$. A monopole model will be valid for the directional distribution of intensity from Figure 16.6. Therefore, $I_r = W/2\pi r^2 = 0.704 \times 10^{-4} \text{ W/m}^2$ or $|p|^2 = I_r \rho c = 2.92 \times 10^{-2} (\text{Pa}^2)$. This is the same as the sound pressure level $L_p = 10 \log |p|^2/p_{\text{ref}} = 78.6$ dB or $L_p(A) = L_p - 15 = 63.6$ dB-A.

References

1. Morse, P.M. and Ingard, K.U. 1986. *Theoretical Acoustics*, Princeton University Press, Princeton, NJ, chap. 7.

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4. Irwin, J.D. and Graf, E.R. 1979. *Industrial Noise and Vibration Control*, Prentice Hall, Englewood Cliffs, NJ, chap. 5.