

19

Design of Sound Insulation

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Summary

This chapter presents useful theory and design procedures for sound insulation. Related concepts and representations of transmission loss, the transmission coefficient, and impedance are given. Analysis and design procedures for sound insulation structures such as single and multiple panels and walls with sound absorption material are presented. Practical applications for the design of sound insulation components and systems are described.

19.1 Theory of Sound Insulation

19.1.1 Expressions of Sound Insulation [1]

19.1.1.1 Transmission Coefficient

Let us denote by I_i the acoustic energy incident on a wall per unit area and unit time. Some energy is dissipated in the wall, and, apart from the energy that is reflected by the wall, the rest is transmitted through the wall. Using I_t to denote the transmitted acoustic energy, the transmission coefficient of the wall is defined as

$$\tau = \frac{I_t}{I_i} \quad (19.1)$$

19.1.1.2 Transmission Loss

As an expression for sound insulation performance, we may use transmission loss (TL), which is defined as (also see [Chapter 17](#) and [Chapter 18](#))

$$TL = 10 \log \left(\frac{1}{\tau} \right) = 10 \log \left(\frac{I_i}{I_t} \right) \quad (19.2)$$

19.1.2 Transmission Loss of a Single Wall

Consider a plane sound wave incident on a impermeable infinite plate at angle θ , which is placed in a uniform air space as shown in [Figure 19.1](#). The sound pressure of the incident, reflected, and transmitted

waves, denoted by p_i , p_r , and p_t , respectively, are given by

$$\begin{aligned} p_i &= P_i e^{j\omega t - jk(x \cos \theta + y \sin \theta)} \\ p_r &= P_r e^{j\omega t - jk(-x \cos \theta + y \sin \theta)} \\ p_t &= P_t e^{j\omega t - jk(x \cos \theta + y \sin \theta)} \end{aligned} \quad (19.3)$$

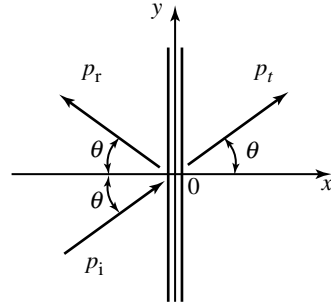


FIGURE 19.1 Plane sound wave incidence on an infinite plate.

where P_i , P_r , and P_t are the sound pressure amplitudes of incident, reflected, and transmitted waves, respectively; ω is angular frequency; k is the wave number of the sound wave; c is the speed of sound, respectively in the air. Assuming that the plate is sufficiently thin compared with the wavelength of the incident sound wave, the vibration velocities on the incident and transmitted surfaces of the plate are equal. Then vibration velocity, u , of the plate in the x direction is equal to the particle velocity of the incident and transmitted sound waves, and we obtain relations

$$u = -\frac{1}{j\omega\rho} \frac{\partial(p_i + p_r)}{\partial x} = -\frac{1}{j\omega\rho} \frac{\partial p_t}{\partial x} \quad (19.4)$$

$$\frac{p_i + p_r - p_t}{u} = Z_m \quad (19.5)$$

where ρ is the air density and Z_m is the *mechanical impedance* of the plate per unit area. From these equations, the transmission coefficient, τ_θ , and then the transmission loss, TL_θ , at the incident angle, θ , are obtained according to

$$TL_\theta = 10 \log \frac{1}{\tau_\theta} = 10 \log \left| \frac{p_i}{p_t} \right|^2 = 10 \log \left| 1 + \frac{Z_m \cos \theta}{2\rho c} \right|^2 \quad (19.6)$$

19.1.2.1 Coincidence Effect

Consider the vibration of the plate in the x - y plane shown in Figure 19.1. Denoting by m the surface density, and by B the bending stiffness per unit length of the plate, the equation of motion of the plate is given by

$$m \frac{\partial^2 \xi}{\partial t^2} + B(1 + j\eta) \frac{\partial^4 \xi}{\partial y^4} = p_i + p_r - p_t, \quad B = \frac{Eh^3}{12(1 - \nu^2)} \cong \frac{Eh^3}{12} \quad (19.7)$$

where

ξ = displacement in the x direction

E = Young's modulus of the plate

h = thickness of the plate

η = loss factor of the plate

ν = Poisson's ratio of the plate

The plane sound wave of angular frequency, ω , and of incidence angle, θ , causes a bending wave in the plate where displacement is assumed to be $\xi = \xi_0 e^{j(\omega t - k_1 y)}$, as a solution of Equation 19.7. Hence, the mechanical impedance per unit area is obtained:

$$Z_m = \frac{p_i + p_r - p_t}{\partial \xi / \partial t} = \eta \frac{Bk_1^4}{\omega} + j \left(\omega m - \frac{Bk_1^4}{\omega} \right) \quad (19.8)$$

where $k_1 = k \sin \theta$ ($k = \omega/c$) is the wave number of the bending wave in y direction caused by the incident sound wave. Propagation speed of the forced bending wave, c_1 , and a free bending wave of

the plate, c_B , are given by

$$c_1 = \omega/k_1, \quad c_B = \left(\frac{\omega^2 B}{m} \right)^{1/4} \quad (19.9)$$

Equation 19.9 reduces Equation 19.8 to

$$Z_m = \eta \omega m \left(\frac{c_B}{c_1} \right)^4 + j \omega m \left[1 - \left(\frac{c_B}{c_1} \right)^4 \right] \quad (19.10)$$

When the speed of forced bending wave, c_1 , and the speed of free bending wave, c_B , are equal in Equation 19.10, the imaginary part of Z_m becomes 0, and a form of “resonance” occurs. Then the transmission loss decreases rapidly. This phenomenon is called the *coincidence effect*, and the resonant frequency dependent on the incident angle is given by

$$f = \frac{c^2}{2\pi \sin^2 \theta} \sqrt{\frac{m}{B}} \quad (19.11)$$

The minimum of the resonant frequency is called coincidence critical frequency, or critical frequency for short, and it reduces to

$$f_c = \frac{c^2}{2\pi} \sqrt{\frac{m}{B}} \cong \frac{c^2}{1.8 c_L h} \quad (19.12)$$

where $c_L = \sqrt{E/\rho_p}$ is the speed of longitudinal wave in the plate, and ρ_p denotes the density of the plate.

Let us show the relations of the critical frequency and the plate thickness of typical material of sound insulation shown in Figure 19.2. Using the relation $c_B/c_1 = \sqrt{f/f_c} \sin \theta$, Equation 19.10

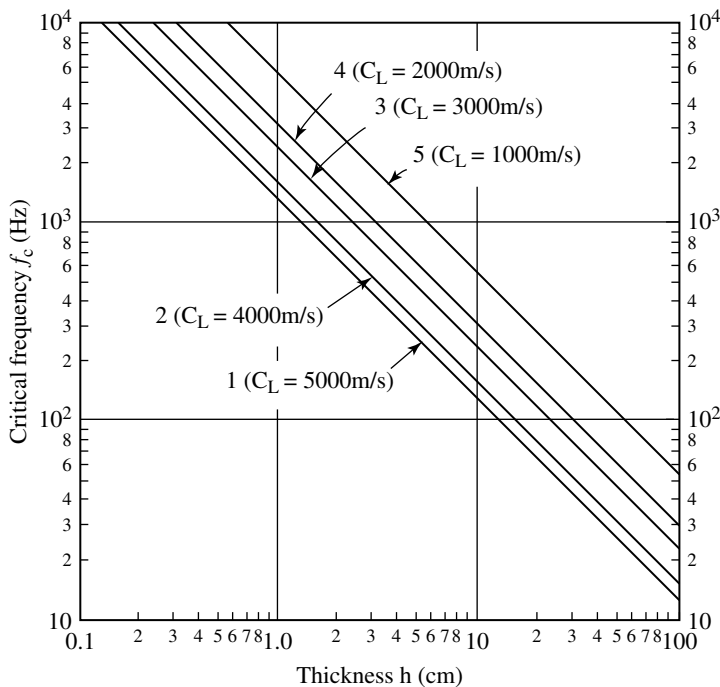


FIGURE 19.2 Critical frequency vs. plate thickness of typical sound insulation materials: (1) aluminum, steel, or glass; (2) hardboard or copper; (3) dense concrete, plywood, or brick; (4) gypsum board; (5) lead or light weight concrete. (Source: Beranek, L.L. 1988. *Noise and Vibration Control*, INCE/USA. With permission.)

becomes

$$Z_m = \eta \omega m \left(\frac{f}{f_c} \right)^2 \sin^4 \theta + j \omega m \left[1 - \left(\frac{f}{f_c} \right)^2 \sin^4 \theta \right] \quad (19.13)$$

19.1.2.2 Mass Law of Transmission Loss

When $f \ll f_c$, Equation 19.13 becomes $Z_m \cong j \omega m$. Then, the transmission loss depends on the incident angle, the frequency and the surface density of the plate. This is called the mass law of transmission loss.

Mass law of normal incidence represents the transmission loss at the incident angle $\theta = 0$, as given by

$$TL_0 = 10 \log \left[1 + \left(\frac{\omega m}{2\rho c} \right)^2 \right] \quad (19.14)$$

For $\omega m \gg 2\rho c$, it becomes

$$TL_0 \cong 10 \log \left(\frac{\omega m}{2\rho c} \right)^2 = 20 \log mf - 42.5; \text{ for air} \quad (19.15)$$

Mass law of random incidence represents the transmission loss at the angle averaged over a range of θ from 0 to 90° , which is realized for perfectly diffused sound field. We have

$$TL_r = 10 \log(1/\tau_r) \cong TL_0 - 10 \log(0.23 TL_0) \quad (19.16)$$

where the random incident transmission coefficient, τ_r , is defined as

$$\tau_r = \int_0^{\pi/2} \tau_\theta \cos \theta \sin \theta d\theta / \int_0^{\pi/2} \cos \theta \sin \theta d\theta \quad (19.17)$$

An approximation for Equation 19.16, as given below, is generally used for a practical use and this is often useful.

$$TL_r = 18 \log mf - 44 \quad (19.18)$$

Mass law of field incidence represents the transmission loss at the angle averaged over a range of θ from 0 to about 78° , which is said to agree with actual sound field. We have

$$TL_f = TL_0 - 5 \quad (19.19)$$

The three types of transmission loss presented above are compared in [Figure 19.3](#).

19.1.2.3 Stiffness Law of Transmission Loss [2]

The plate described above is assumed to be infinite. However, an actual plate is always supported by some structures at its boundaries and the plate size is finite. Transmission loss of a finite plate is considered to be related to the nature of excitation of vibration in the plate, for example, sound wave incidence, modes of vibration and characteristics of sound radiation. Therefore, the governing relationships become very complex. However, in the following frequency range, it is known that the transmission loss conforms to the mass law

$$\frac{c}{2a} < f \ll f_c \quad (19.20)$$

where a is length of shorter edge for rectangular plate.

When $f < c/2a$, the whole plate is excited in phase, and stiffness effects from the supports of its edges will appear. If we denote the equivalent stiffness of the plate as K and assume a loss factor of 0,

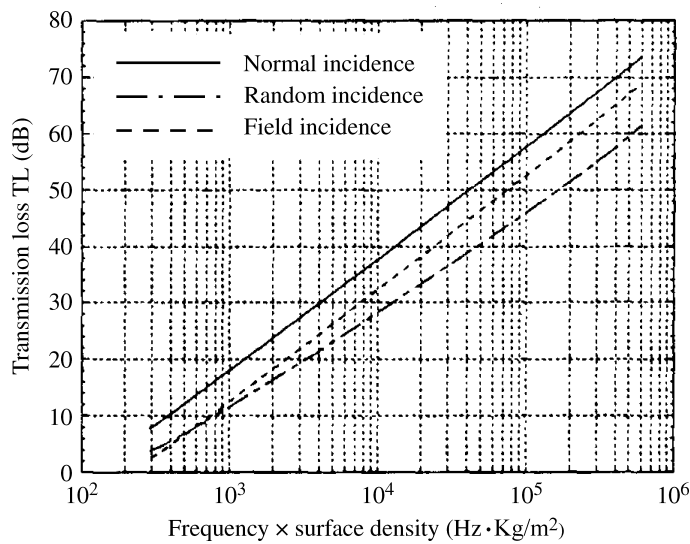


FIGURE 19.3 Theoretical transmission loss based on mass law.

the mechanical impedance of the plate is obtained by using Equation 19.8; thus

$$Z_m = j\left(\omega m - \frac{K}{\omega}\right) \tag{19.21}$$

The frequency at $Z_m = 0$ corresponds to the first mode natural frequency, f_{11} , of the plate and consequently, the equivalent stiffness of rectangular plate with simple edge-support is given by

$$f_{11} = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \equiv \frac{\pi}{2} \sqrt{\frac{B}{m}} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 \right] \tag{19.22}$$

Then,

$$K = B\pi^4 \left(\frac{1}{a^2} + \frac{1}{b^2} \right)^2$$

where a and b are the length of the short and long edges for the rectangular plate, respectively. When $f \ll f_{11}$ is assumed in Equation 19.21, the mass term can be neglected, and from Equation 19.6 the normal incidence transmission loss, TL_{S0} , is given by

$$\begin{aligned} TL_{S0} &= 10 \log \left| 1 - j \frac{K}{2\omega\rho c} \right|^2 \\ &= 10 \log \left[1 + \left(\frac{K}{2\omega\rho c} \right)^2 \right] \\ &\cong 20 \log(K/f) - 74.5 \end{aligned} \tag{19.23}$$

This is called the stiffness law of the transmission loss, and it shows a 6 dB decay per octave.

The characteristics mentioned above for single wall transmission loss are shown in Figure 19.4 and summarized below.

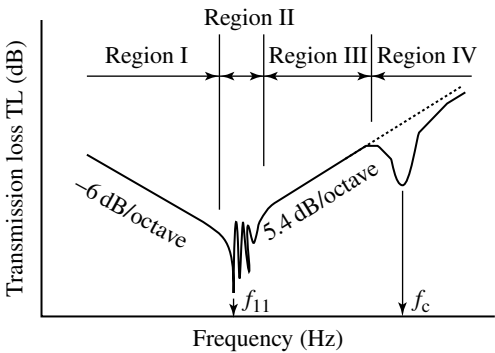


FIGURE 19.4 Transmission loss characteristics of a single wall.

1. Region I ($f \ll f_{11}$). Transmission loss is controlled by the stiffness of the panel:

$$TL = TL_0 - 40 \log\left(\frac{f}{f_{11}}\right) \quad (19.24)$$

2. Region II ($f \approx f_{11}$). Transmission loss is controlled by the lower-mode natural frequencies of the panel, and the estimation becomes very complex.
3. Region III ($f_{11} \ll f \leq f_c/2$). Transmission loss is controlled by the mass (surface density) of the panel:

$$TL = 18 \log mf - 44 \quad (19.25)$$

4. Region IV ($f > f_c/2$). Transmission loss is controlled by the mass and the damping of the panel, and it is reduced by coincidence effects.

For $f_c/2 < f \leq f_c$:

TL is represented by a straight line connecting the value at $f = f_c/2$ of Equation 19.25 and the value at $f = f_c$ of Equation 19.26.

For $f > f_c$:

$$TL = TL_0 + 10 \log\left(\frac{2\eta}{\pi} \frac{f}{f_c}\right) \quad (19.26)$$

19.1.3 Transmission Loss of Multiple Panels

To realize sound insulation of high performance, we often use a double wall or a multiple panel composed of insulation materials like steel plates and absorbing materials like fiber-glass. In this subsection, transmission loss of a multiple panel is described [3].

19.1.3.1 Calculation Method

Consider a multiple panel of infinite lateral extent as shown in Figure 19.5, which is composed of n acoustic elements, each element consisting of three basic materials, an impermeable plate, air space, and an absorption layer. Furthermore, consider a plane wave incident on the left-hand side surface of the n th element at angle θ . Let the sound pressure of the incident wave be p_i , and of the reflected wave be p_r , and the wave be propagating through the structure, and then radiating from the right-hand side of the first element as a plane wave of pressure p_t into a free field at transmission angle θ .

In the analysis, we append the subscript k ($k = 1, 2, \dots, n$) to the physical parameters of the k th element, and “2” and “1” to the left- and right-hand side values of these parameters, respectively, as shown in Figure 19.5. The ratio of the sound pressure at the incident surface, p_{n2} , to the incident wave, p_i , is given by

$$\frac{p_{n2}}{p_i} = \frac{p_i + p_r}{p_i} = \frac{2Z_{n2}}{Z_{n2} + \rho c / \cos \theta} \quad (19.27)$$

where Z_{n2} is the acoustic impedance of the left-hand side normal to the surface of the n th element and $\rho c / \cos \theta$ is the acoustic impedance normal to the surface, which is equal to the radiation impedance of the

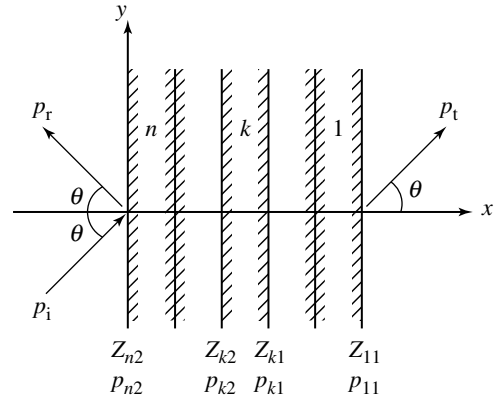


FIGURE 19.5 Calculation model of n -element multiple panel.

first element, Z_{11} , shown in Figure 19.5. Using the usual condition of pressure matching at each interface, we can write the expression for the oblique incidence transmission coefficient as

$$\tau(\theta) = \left| \frac{p_t}{p_i} \right|^2 = \left| \frac{p_{11}}{p_i} \right|^2 = \left| \frac{p_{n2}}{p_i} \right|^2 \cdot \left| \frac{p_{n1}}{p_{n2}} \dots \frac{p_{k1}}{p_{k2}} \dots \frac{p_{11}}{p_{12}} \right|^2 \quad (19.28)$$

Hence, we obtain the following expression for the random incidence transmission loss:

$$TL = 10 \log \left(\frac{\int_0^{\theta_1} \cos \theta \sin \theta d\theta}{\int_0^{\theta_1} \tau(\theta) \cos \theta \sin \theta d\theta} \right) \quad (19.29)$$

where θ_1 is the limiting angle above which no sound is assumed to be received, and it varies between 78° and 85° .

If we know Z_{n2} in Equation 19.27 and the pressure ratio across each of the single elements in Equation 19.28, we can calculate the TL using Equation 19.29. We can obtain Z_{n2} by using the conditions of impedance matching at each interface from the rightmost to the leftmost element in order, if we know the impedance relations across each of the single elements.

Now, we present the pressure ratios and the acoustic impedance relations across three basic elements.

19.1.3.2 Impermeable Plate

Consider the vibration of an infinite impermeable plate of thickness, h , induced by the sound pressure difference on each side of the plate, as illustrated in Figure 19.6. In this case, the particle velocity on both sides of the plate must be the same as the plate vibration velocity. Then, from Equation 19.8, the following expressions are obtained:

$$Z_2 = Z_1 + Z_m \quad (19.30)$$

$$\frac{p_2}{p_1} = \frac{Z_2}{Z_1} \quad (19.31)$$

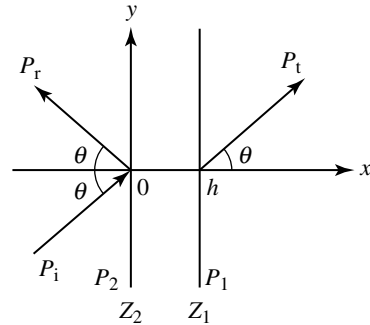


FIGURE 19.6 Excitation of infinite plate by a plane sound wave.

where p_2, p_1 are the sound pressure at the incident surface $x = 0$ and at the transmitted surface $x = h$, respectively, Z_m is the mechanical impedance of the plate, and Z_2, Z_1 are the acoustic impedance normal to the incident surface at $x = 0$ and the transmitted one at $x = h$, respectively.

19.1.3.3 Sound Absorbing Material

For a sound absorbing material layer of thickness d and infinite lateral extent, consider a plane wave incident at an angle θ to the normal, as shown in Figure 19.7. Deriving the wave equation in the sound absorbing material and applying the continuity conditions of the sound pressure across the surface at $x = 0$ and $x = d$, with some mathematical manipulation we get the following results:

$$Z_2 = \frac{\gamma Z_0}{q} \coth(qd + \varphi) \quad (19.32)$$

$$\frac{p_2}{p_1} = \frac{\cosh(qd + \varphi)}{\cosh \varphi} \quad (19.33)$$

$$q = \gamma \sqrt{1 + \left(\frac{k}{\gamma}\right)^2 \sin^2 \theta}, \quad \varphi = \coth^{-1} \left(\frac{q Z_1}{\gamma Z_0} \right) \quad (19.34)$$

where γ is a propagation constant and Z_0 is a characteristic impedance of a homogeneous, isotropic absorbing material.

If porous material is used as the absorbing material, the following relations are applicable for γ and Z_0 [4]:

$$Z_0 = R + jX$$

$$R/\rho c = 1 + 0.0571(\rho f/R_1)^{-0.754} \quad (19.35)$$

$$X/\rho c = -0.0870(\rho f/R_1)^{-0.732}$$

$$\gamma = \alpha + j\beta$$

$$\alpha/k = 0.189(\rho f/R_1)^{-0.595} \quad (19.36)$$

$$\beta/k = 1 + 0.0978(\rho f/R_1)^{-0.700}$$

$$(0.01 \leq \rho f/R_1 \leq 1)$$

where ρ is the air density, f is the frequency, and R_1 is the flow resistivity, respectively. Specifically, note that R_1 is defined as the flow resistance of the porous absorbing material per unit thickness. With data measured with a measuring tube of flow resistance, we can write

$$R_1 = \frac{\Delta p}{l \cdot u} \quad (19.37)$$

where Δp is pressure difference between the inlet and the outlet of the absorbing material in the tube, u is the mean flow velocity in the tube, and l is the thickness of the absorbing material. It is known that the flow resistivity of porous absorbing material such as fiber-glass or rock wool is related to the bulk density, as shown in Figure 19.8.

19.1.3.4 Air Space

For an air space, $Z_0 = \rho c$ and $\gamma = jk$. Hence, Equation 19.32 to Equation 19.34 reduce to

$$Z_2 = \frac{\rho c}{\cos \theta} \coth(jkd \cos \theta + \delta) \quad (19.38)$$

$$\frac{p_2}{p_1} = \frac{\cosh(jkd \cos \theta + \delta)}{\cosh \delta} \quad (19.39)$$

$$\delta = \coth^{-1} \left(\frac{Z_1 \cos \theta}{\rho c} \right) \quad (19.40)$$

19.1.3.5 Double Wall [2]

Applying the theory formulated above, we can easily obtain the transmission loss of a double wall composed of the three elements: impermeable plate, air space, and impermeable plate, as shown in Figure 19.9. Assume that the two impermeable plates have the same surface density, m , and the mechanical impedance of the plates is $j\omega m$. Then, we can obtain following equations for element

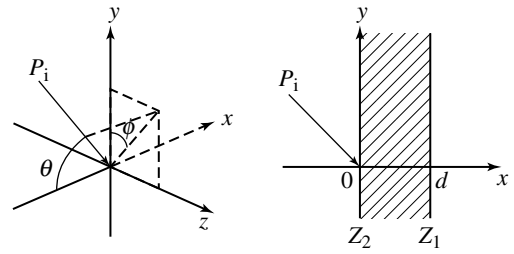


FIGURE 19.7 Schematic relation of sound wave directions.

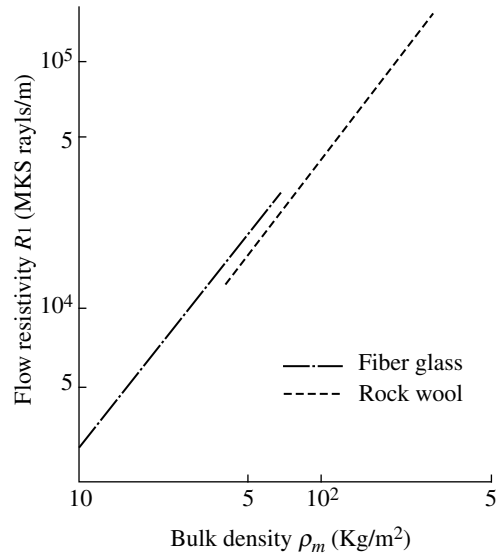


FIGURE 19.8 Flow resistivity vs. bulk density for porous, sound absorbing materials.

one and element three:

$$Z_{12} = Z_{11} + Z_m = \frac{\rho c}{\cos \theta} + j\omega m, \quad (19.41)$$

$$\frac{p_{12}}{p_{11}} = \frac{Z_{12}}{Z_{11}} = 1 + j \frac{\omega m \cos \theta}{\rho c}$$

$$Z_{32} = Z_{31} + Z_m = Z_{22} + j\omega m, \quad (19.42)$$

$$\frac{p_{32}}{p_{31}} = \frac{Z_{32}}{Z_{22}} = 1 + j \frac{\omega m}{Z_{22}}$$

For element two:

$$Z_{22} = \frac{\rho c}{\cos \theta} \coth(jkd \cos \theta + \delta'), \quad (19.43)$$

$$\frac{p_{22}}{p_{21}} = \frac{\cosh(jkd \cos \theta + \delta')}{\cosh \delta'}$$

where, by applying impedance matching

conditions at the interface of element one and element two, the following definition is introduced:

$$\delta' = \coth^{-1} \left(\frac{Z_{21} \cos \theta}{\rho c} \right) = \coth^{-1} \left(\frac{Z_{12} \cos \theta}{\rho c} \right) = \coth^{-1} \left(1 + j \frac{\omega m \cos \theta}{\rho c} \right) \quad (19.44)$$

In this case, Equation 19.27 reduces to

$$\frac{p_i}{p_{32}} = \frac{Z_{32} + \rho c / \cos \theta}{2Z_{32}} = \frac{Z_{22} + j\omega m + \rho c / \cos \theta}{2(Z_{22} + j\omega m)} \quad (19.45)$$

Substituting Equation 19.41 to Equation 19.45 into Equation 19.28, we obtain the transmission loss of the double wall:

$$TL_\theta = 10 \log[1/\tau(\theta)] = 10 \log[1 + 4a^2 \cos^2 \theta (\cos \beta - a \cos \theta \sin \beta)^2] \quad (19.46)$$

$$a = \omega m / 2\rho c, \quad \beta = kd \cos \theta$$

In Equation 19.46, the transmission loss is zero, and full passage (i.e., “all-pass” in the filter terminology) of sound occurs when the following equation holds:

$$\cos \beta - a \cos \theta \sin \beta = 0 \quad (19.47)$$

When $\beta \ll 1$ ($kd \ll 1$), the frequency of full passage for normal incidence is given by

$$f_r = \frac{1}{2\pi} \sqrt{\frac{2\rho c^2}{md}} \quad (19.48)$$

This is the natural frequency of a vibrating system consisting of two masses, m , connected by a spring of spring constant, $\rho c^2/d$.

When $\beta \gg 1$ ($kd \gg 1$), the solution of Equation 19.47 for β is $\beta \cong n\pi$, and the frequency of all passage for normal incidence is given by

$$f_n = \frac{nc}{2d} \quad (n = 1, 2, 3, \dots) \quad (19.49)$$

These are the acoustic resonant frequencies of the air space d .

Characteristics of the transmission loss given by Equation 19.46, in case of normal incidence ($\theta = 0$), are as follows:

$$1. f < f_r (\beta < \sqrt{2\rho d/m})$$

$$TL \cong 10 \log(4a^2) = TL_0 + 6 \quad (19.50)$$

This is equal to the transmission loss of a single wall of surface density $2m$.

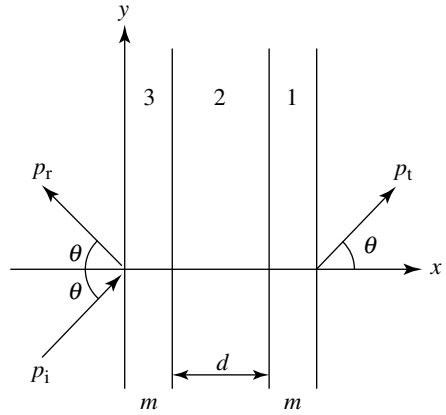


FIGURE 19.9 Calculation model of a double wall with air space.

$$2. f_r \leq f < f_1/\pi \left(\sqrt{2\rho d/m} \leq \beta < 1 \right)$$

$$TL \cong 10 \log(4a^4 \beta^2) = 2TL_0 + 20 \log(2kd) \quad (19.51)$$

This transmission loss indicates an 18 dB increase per octave.

$$3. f = (2n - 1)c/4d (\beta = n\pi - \pi/2)$$

$$TL \cong 10 \log(4a^4) = 2TL_0 + 6 \quad (19.52)$$

A straight line connecting the transmission losses at these frequencies in Figure 19.10 indicates a 12 dB increase per octave. When the two impermeable plates have different surface densities, m_1 and m_2 , Equation 19.41 to Equation 19.52 reduce to

$$1. f < f_r (\beta < \sqrt{2\rho d/m})$$

$$TL = 20 \log[\omega(m_1 + m_2)/2\rho c] \quad (19.53)$$

$$2. f_r \leq f < f_1/\pi (\sqrt{2\rho d/m} \leq \beta < 1)$$

$$TL = TL_1 + TL_2 + 20 \log(2kd) \quad (19.54)$$

$$3. f = (2n - 1)c/4d (\beta = n\pi - \pi/2)$$

$$TL = TL_1 + TL_2 + 6 \quad (19.55)$$

In these equations, TL_1 and TL_2 are the transmission losses of each plate, which are given by Equation 19.15.

The transmission loss of a double wall, as mentioned above, is shown schematically in Figure 19.10. An actual double wall, however, is finite in size and the air space forms a closed acoustic field, which

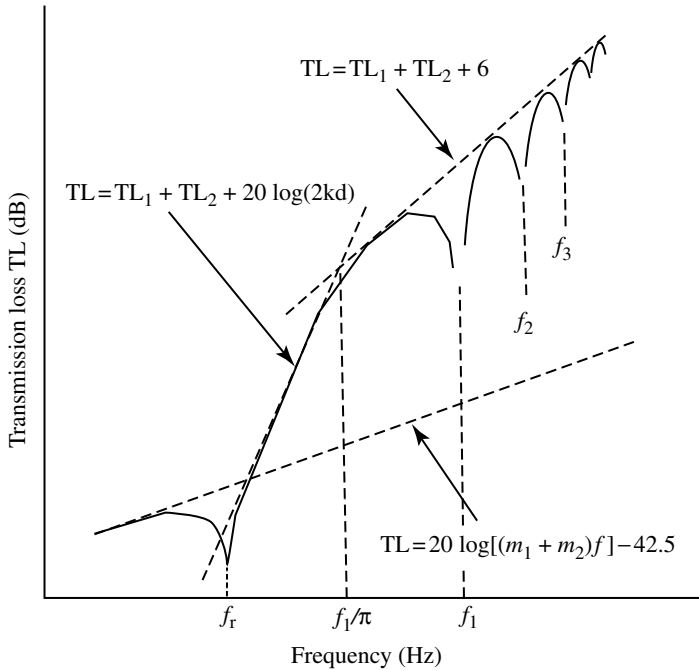


FIGURE 19.10 Transmission loss of a double wall with air space.

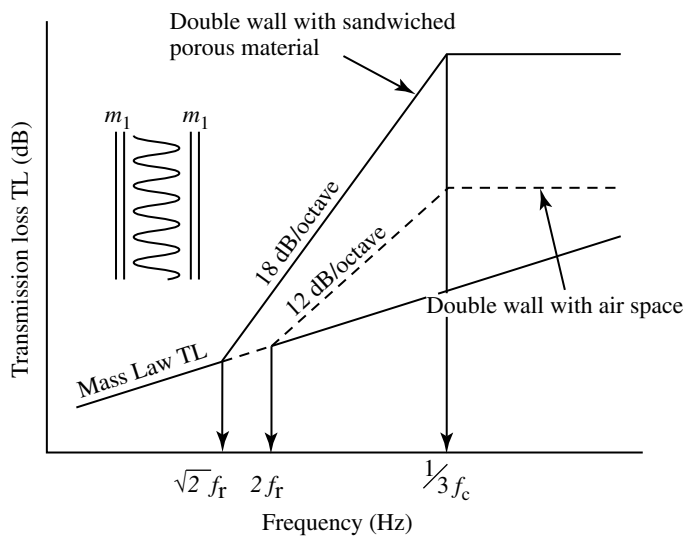


FIGURE 19.11 Design chart for estimating the transmission loss of a double wall with sandwiched porous material or air space.

makes the transmission loss deviate from the theoretical value. Figure 19.11 gives a design chart of an actual double wall, which is based on theory and experiments.

19.1.4 Transmission Loss of Double Wall with Sound Bridge [5]

In the previously presented theory, each plate of the multiple panel is considered to be structurally independent. In actual multiple panels, such as partitions of a building or sound insulation laggings of a duct, however, each plate is connected with steel sections, stud bolts, and the like, which are called sound bridges. This is illustrated in Figure 19.12.

The sound pressure of the transmitted wave through a double wall with sound bridges is given by the summation of radiated sound pressure from the vibration of the transmitted side plate excited by the sound in the air space and that mechanically excited by the sound bridges.

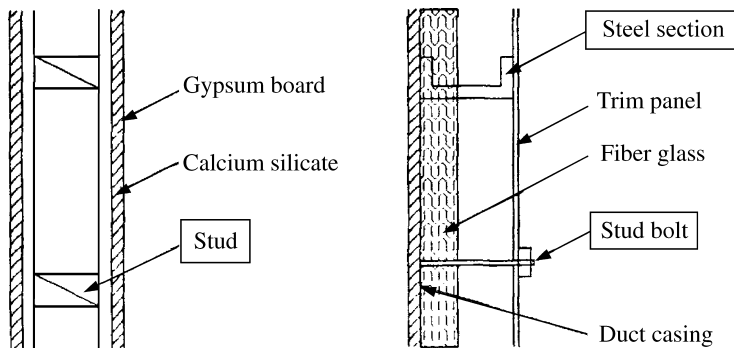


FIGURE 19.12 Examples of actual double wall with sound bridges.

The acoustic power radiated from the area, S , of an infinite plate excited by sound pressure is given by

$$W_p = \rho c S v_2^2 \quad (19.56)$$

where v_2^2 is the space averaged mean square vibration velocity over the plate. The acoustic power radiated from the plate mechanically excited by a point force or a line force is

$$W_B = \rho c \chi v^2 (f \ll f_c) \quad (19.57)$$

$$\chi = \begin{cases} \frac{8}{\pi^3} \lambda_c^2 & (\text{point force excitation}) \\ \frac{2}{\pi} l \lambda_c & (\text{line force excitation}) \end{cases} \quad (19.58)$$

where v^2 is the mean square vibration velocity of the plate at the excitation point, $\lambda_c = c/f_c$ is the wavelength of the bending wave at the critical frequency, and l is the length of the line force. By comparing Equation 19.56 and Equation 19.57, it is noted that χ is the effective area of the acoustic power radiated from the infinite plate excited by the point or line force. Acoustic power, W_B , is the power radiated from a small area near the excitation point, because a free bending wave propagating in an infinite plate can radiate little sound when $f < f_c$.

From the equations given above, the total acoustic power radiated from the transmitted side plate is obtained as

$$W_T = W_p + W_B = \rho c S v_2^2 \left[1 + \frac{n\chi}{S} \left(\frac{v}{v_2} \right)^2 \right] \quad (19.59)$$

where n is the number of excitation forces applied to the area, S . Then, transmission loss TL_T of the double wall with sound bridges is given by

$$TL_T = 10 \log \left(\frac{W_I}{W_T} \right) = 10 \log \left(\frac{W_I}{W_p} \cdot \frac{W_p}{W_T} \right) = TL - TL_B \quad (19.60)$$

where W_I is the acoustic power incident on the double wall, TL is the transmission loss of the double wall without a sound bridge, and TL_B denotes the transmission loss reduction by the sound bridges, and is given by

$$TL_B = 10 \log \left(\frac{W_T}{W_p} \right) = 10 \log \left[1 + \frac{n\chi}{S} \left(\frac{v}{v_2} \right)^2 \right] \quad (19.61)$$

We assume the following:

1. The vibration velocity of the incident side plate is not affected by the sound bridges.
2. The vibration velocity of the transmitted side plate at the excitation points (connecting points) is equal to the velocity v_1 of the incident side plate, and consequently, the next equation holds

$$\frac{v}{v_2} \cong \frac{v_1}{v_2}$$

With these assumptions, we apply the method presented in section 19.1.3, to determine v/v_2 as

$$\frac{v}{v_2} \cong \frac{v_1}{v_2} = \frac{\omega^2 m_2 d}{\rho c^2} \quad (f_r < f < f_1/\pi) = \frac{\omega m_2}{\rho c} \quad (f > f_1/\pi) \quad (19.62)$$

Using Equation 19.60 and Equation 19.53, we can obtain the increase in transmission loss, ΔTL , from the transmission loss of mass law based on the total mass of the double wall, as presented below.

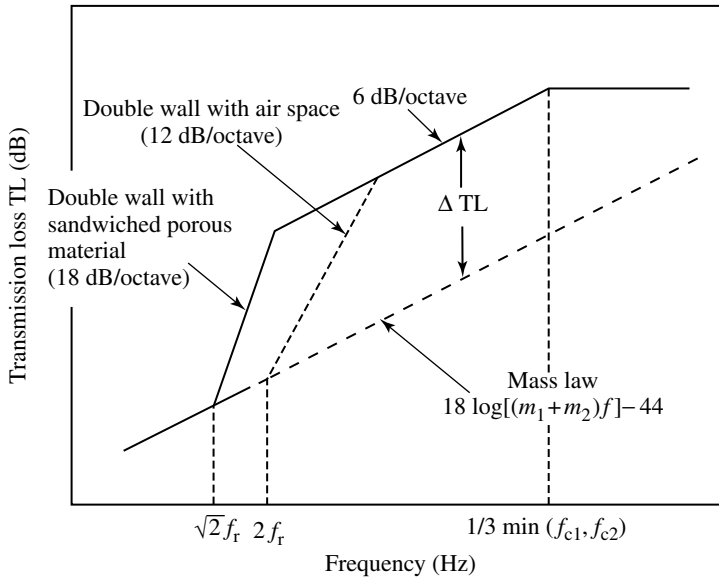


FIGURE 19.13 Design chart for estimating the transmission loss of a double wall with sound bridges.

1. Point connection

$$\begin{aligned}\Delta TL &= TL - TL_B - 20 \log \left[\frac{\omega(m_1 + m_2)}{2\rho c} \right] \\ &= 20 \log(ef_c) + 20 \log \left(\frac{m_1}{m_1 + m_2} \right) + 10 \log \left(\frac{\pi^3}{8c^2} \right)\end{aligned}\quad (19.63)$$

2. Line connection

$$\begin{aligned}\Delta TL &= TL - TL_B - 20 \log \left[\frac{\omega(m_1 + m_2)}{2\rho c} \right] \\ &= 10 \log(bf_c) + 20 \log \left(\frac{m_1}{m_1 + m_2} \right) + 10 \log \left(\frac{\pi}{2c} \right)\end{aligned}\quad (19.64)$$

where $e = \sqrt{S/n}$ is the distance between point forces and $b = S/nl$ is the distance between line forces.

Figure 19.13 presents a practical and useful design chart of the transmission loss for a double wall with sound bridges, which is based on Figure 19.11 and Equation 19.63 and 19.64.

19.2 Application of Sound Insulation

19.2.1 Acoustic Enclosure

Performance of an enclosure may be represented by the insertion loss (IL), which is the difference of acoustic power level before and after installation of the enclosure. When we assume a noise source and also an enclosure with one-dimensional model as shown in Figure 19.14a, the insertion loss through frequency is shown in Figure 19.14b. It is divided into the following four regions:

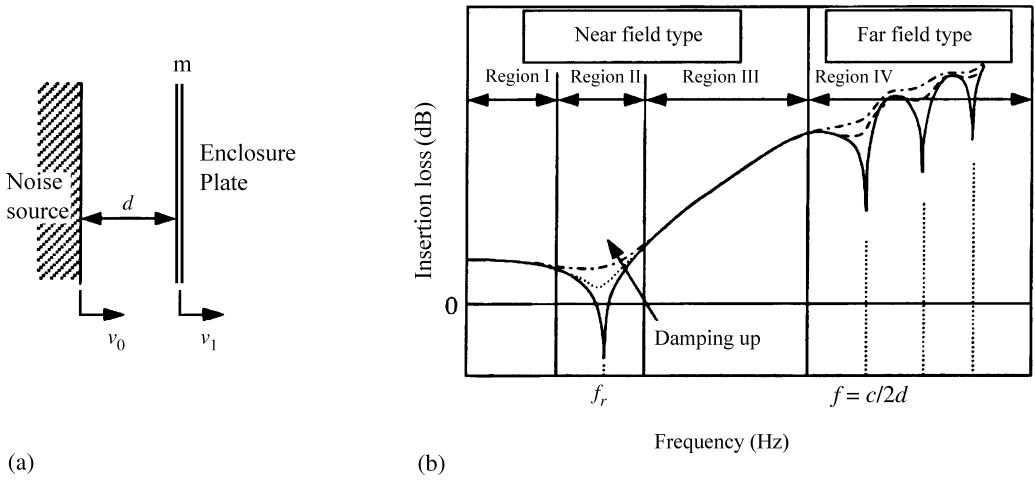


FIGURE 19.14 One-dimensional model for calculating the insertion loss characteristics of an acoustic enclosure: (a) One-dimensional model; (b) insertion loss of an enclosure.

1. Region I ($f < f_r$) is controlled by the stiffness of the enclosure plate and air space.
2. Region II ($f \cong f_r$) is the resonance region for a vibrating system consisting of the mass, the stiffness of the enclosure plate, and the capacitance of air space.
3. Region III ($f_r < f < c/2d$) is controlled by the mass of the enclosure plate.
4. Region IV ($f > c/2d$) is controlled by the diffused sound field. This region cannot be represented by a one-dimensional model.

19.2.1.1 Near Field Type [4]

When the distance between the noise source and the enclosure is less than half of a wavelength of the emitted sound from the source, insertion loss corresponds to the characteristics of the regions I to III, and we can represent them with a one-dimensional acoustic model, as shown in Figure 19.14a.

Consider an infinite flat enclosure plate with distance d from a noise source of plane sound wave, as shown in Figure 19.14a. The insertion loss of the plate is given by

$$IL = 10 \log \left(\frac{v_0}{v_1} \right)^2 = 10 \log \left[1 - \frac{2 \sin \theta (X \cos \theta - R \sin \theta)}{\rho c} + \frac{\sin^2 \theta (X^2 + R^2)}{\rho^2 c^2} \right] \quad (19.65)$$

$$\theta = kd = \omega d/c, \quad R = \eta \omega m, \quad X = (\omega m - K/\omega) = \omega m [1 - (\omega_{11}/\omega)^2], \quad \omega_{11} = \sqrt{K/m}$$

where m and K are the density and equivalent stiffness of the plate per unit area, respectively, and η is the loss factor of the plate. If the enclosure is a rectangular plate of size $a \times b$ and is simply supported at its edges, the equivalent stiffness is given by Equation 19.22, and ω_{11} is the natural (angular) frequency of the first mode.

In Equation 19.65, the conditions in which the brackets of the right-hand side are equal to zero or $IL = -\infty$ are satisfied by following frequencies:

- (1) $\theta \ll \pi$

$$f_r = \frac{1}{2\pi} \sqrt{\frac{K + \rho c^2/d}{m}} = \frac{1}{2\pi} \sqrt{\omega_{11}^2 + \frac{\rho c^2}{md}} \quad (19.66)$$

This is the natural frequency of vibration of the one-degree-of-freedom (one-DoF) system determined by the stiffness of the plate, the spring constant of the air space, and the surface density of the plate, as shown in Figure 19.15.

$$(2) \theta = n\pi \quad (n = 1, 2, \dots)$$

$$f_n = \frac{nc}{2d} \quad (n = 1, 2, \dots) \quad (19.67)$$

These are the resonant frequencies of the air space.

The frequency characteristics of the IL given by Equation 19.65 are shown in Figure 19.16, where the normal incidence transmission losses are shown by broken lines, as a reference. Equation 19.65 is approximated by

$$1. f < f_r$$

$$IL \cong 10 \log \left(1 + \frac{2Kd}{\rho c^2} \right) \quad (19.68)$$

$$2. f_r \leq f < f_1$$

$$IL \cong 20 \log(mdf^2) + 20 \log \left(\frac{4\pi^2}{\rho c^2} \right) \\ = 20 \log(mdf^2) - 71 \quad (19.69)$$

19.2.1.2 Far Field-Type (Absorption Type) Enclosure [1]

When the distance between the noise source and the enclosure is larger than half of a wavelength of the emitted sound, insertion loss may be represented by the characteristics of Region IV, and it can be analyzed using the theory of room or hall acoustics.

Consider the enclosure shown in Figure 19.17, with a noise source of power level L_{W0} .

From the theory of room acoustics, the average sound pressure level, L_{p0} , on the inner surface of the enclosure plate is obtained as the sum of the direct and reverberant sound pressures:

$$L_{p0} = L_{W0} + 10 \log \left(\frac{1}{S} + \frac{4}{R} \right) \\ = L_{W0} - 10 \log S + 10 \log \left(1 + \frac{4S}{R} \right) \quad (19.70)$$

$$R = \frac{\bar{\alpha}S}{(1 - \bar{\alpha})} \quad (19.71)$$

where S is the inner surface area of the enclosure and $\bar{\alpha}$ is the average absorption coefficient on the inner surface of the enclosure. In Equation 19.70, the first and the second terms of the right-hand side represent the influence of the direct sound field, and the third term represents the buildup caused by the covering.

When we use S_p for the area of the enclosure plate and $S_O (= S - S_p)$ for the area of the enclosure opening, and assume diffusing condition for the sound field in the enclosure, acoustic power levels

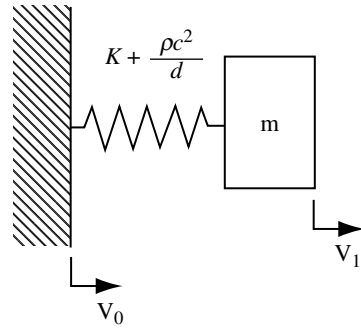
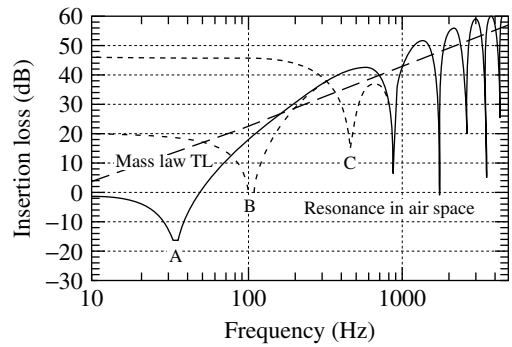


FIGURE 19.15 One-degree-of-freedom vibrating system.



$$m = 16 \text{ Kg/m}^2, d = 0.19 \text{ m}$$

$$\text{Curve A: } f_{11} = 0 \text{ Hz, } \eta \cong 0.033 \text{ (at 33 Hz)}$$

$$\text{Curve B: } f_{11} = 100 \text{ Hz, } \eta \cong 0.033 \text{ (at 105 Hz)}$$

$$\text{Curve C: } f_{11} = 475 \text{ Hz, } \eta \cong 0.033 \text{ (at 475 Hz)}$$

FIGURE 19.16 Example of theoretical insertion loss of a near field-type enclosure.

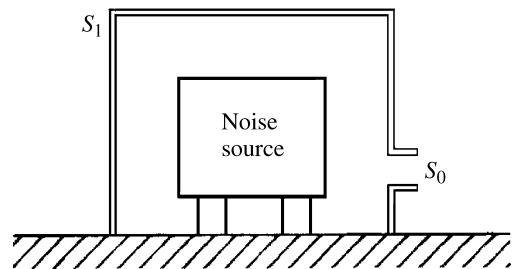


FIGURE 19.17 Calculation model of a far field type enclosure.

radiated from the plate and the opening are given, respectively, by

$$L_{WP} = L_{P0} - (TL + 6) + 10 \log S_p, \quad (19.72)$$

$$L_{W0} = L_{P0} - 6 + 10 \log S_O$$

where TL is the random incident transmission loss of the enclosure plate. Then, the insertion loss of the enclosure is

$$IL = L_{W0} - 10 \log(10^{L_{WP}/10} + 10^{L_{W0}/10}) \quad (19.73)$$

In the design of an acoustic enclosure, special attention should be paid to the following points:

1. *Buildup* increases the sound pressure in the enclosure and also the power level radiated from the enclosure. We must treat the inner surface of the enclosure with sound absorbing materials to reduce the absorption coefficient and to decrease the buildup.
2. *Opening* radiates more acoustic power than the enclosure plate by TL, as is clear from Equation 19.72. It is desirable to make the opening as small as possible within the range given by

$$\frac{S_O}{S_p} \leq 10^{-TL/10} = \tau \quad (19.74)$$

This equation means that the acoustic power from the opening is less than that from the enclosure plate. If the relation in Equation 19.74 cannot be satisfied because of ventilation requirements, and so on, some type of silencers should be provided at the opening, as shown in Figure 19.18.

3. *Structure-borne noise*, which is caused by the vibration propagating from the base of the machine (noise source) to the enclosure plate, significantly decreases the insertion loss of the enclosure. In this case, some means of noise/vibration suppression should be provided, for example, the following:
 - Place supporting structures of the enclosure at the points of lowest vibration level, and the vibrations of the machine should be prevented from propagating to the enclosure plate, using vibration isolation materials.
 - Add damping materials to the enclosure plate so as to reduce the vibration level of the plate.

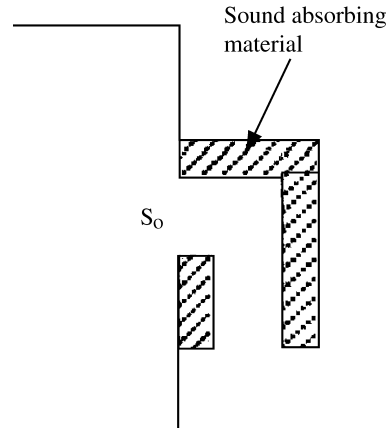


FIGURE 19.18 Example of a silencer at the opening.

19.2.2 Sound Insulation Lagging

In electric power plants and chemical plants, for example, piping for high-pressure water or steam, and ducts for air or gas flow form major noise sources. For controlling these noise sources, we usually use sound insulation laggings, which cover the noise sources with heavy and impermeable plates or sheets with sound absorbing materials, as shown in Figure 19.19.

19.2.2.1 Pipe Lagging [2]

Approximate the cylindrical piping and pipe lagging with a one-dimensional model as shown in Figure 19.20. The insertion loss of one-layered lagging approximated by a one-DoF system is given by Equation 19.69 in the frequency region $f_r \leq f < f_1$, as mentioned before. It is not practical, however, to directly apply Equation 19.69 to actual laggings, and we approximate the insertion loss of actual laggings by

$$IL = a \log(mdf^2) + b$$

where a and b are constants.

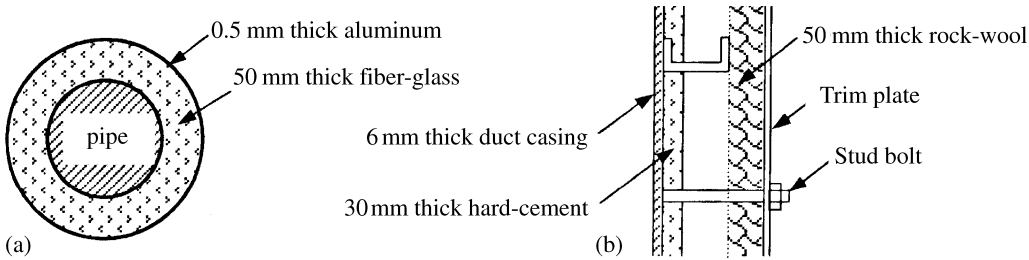


FIGURE 19.19 Examples of typical sound insulation laggings: (a) pipe lagging, (b) duct lagging.

By taking mdf^2 as the horizontal axis and plotting the insertion loss data from laboratory tests and field tests (the vertical coordinates), we obtain Figure 19.21. Apply regression analysis to the data in Figure 19.21 to obtain the insertion loss of one layered lagging as

$$IL = 11.7 \log(mdf^2) - 43.3 \quad (5 \times 10^3 \leq mdf^2 \leq 10^8) \tag{19.75}$$

Applying the same method to double layered lagging, approximated by a two-DoF vibrating system, we get Figure 19.22, and the insertion loss

$$IL = 6.9 \log(m_1 m_2 d_1 d_2 f^4) - 40.3 \quad (10^6 \leq m_1 m_2 d_1 d_2 f^4 \leq 10^{15}) \tag{19.76}$$

where the subscripts “1” and “2” denote the first layer and the second layer, respectively.

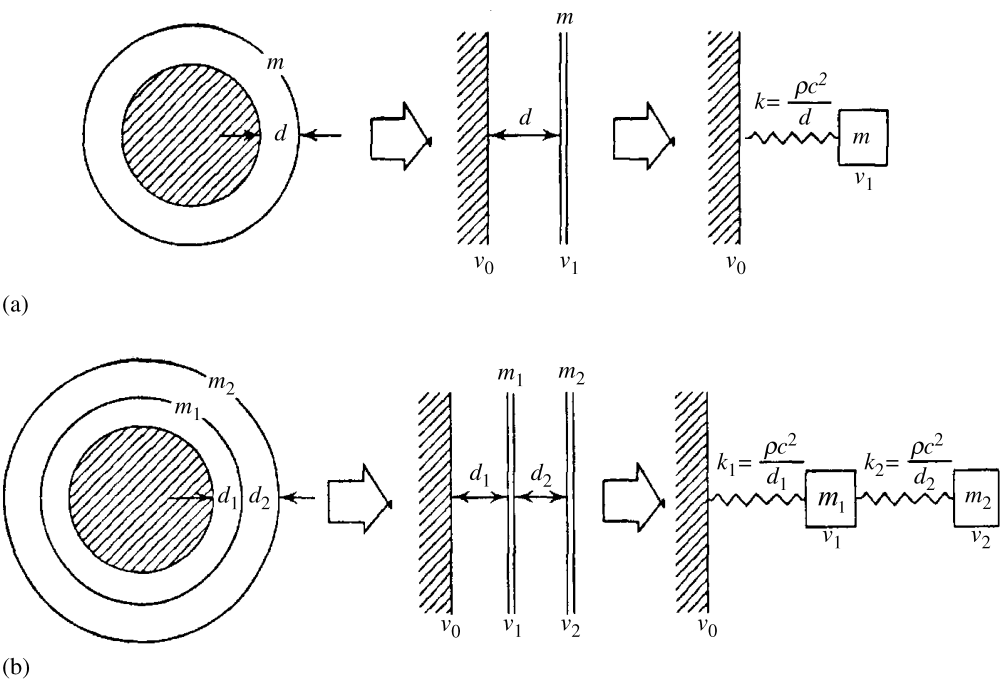


FIGURE 19.20 Examples of pipe laggings and calculation model: (a) one-layered lagging; (b) double-layered lagging.

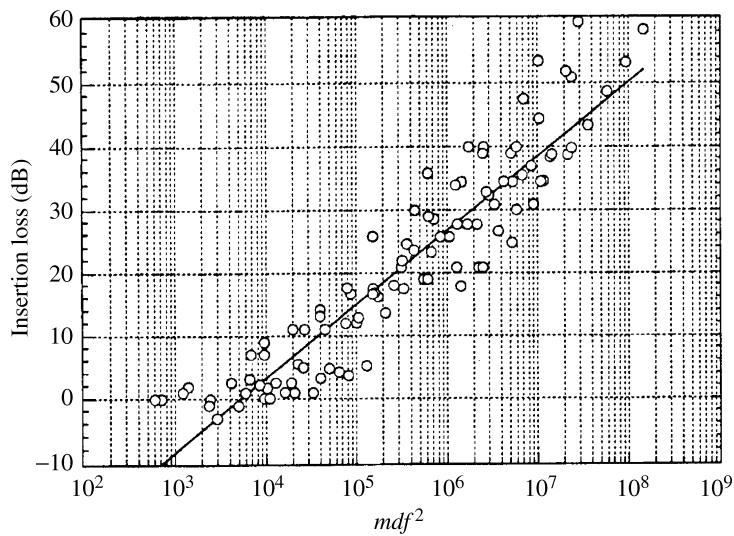


FIGURE 19.21 Measured insertion loss data obtained from laboratory and field tests for one-layered laggings and regression analysis.

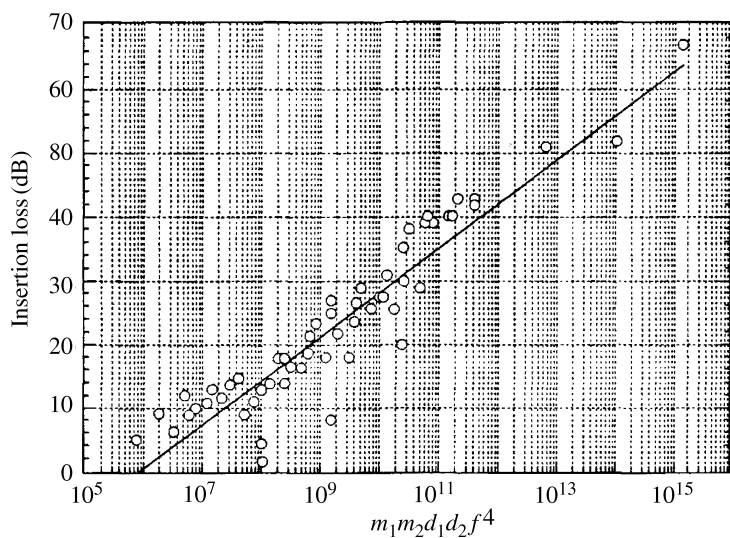


FIGURE 19.22 Measured data of insertion loss obtained from laboratory and field tests for double-layered laggings and regression analysis.

19.2.2.2 Duct Lagging [4]

Various types of the duct laggings are used according to the need, for example, as shown in Figure 19.19. A simpler and more practical approach is to place a thin plate on the duct casing through absorbing materials, as shown in Figure 19.23. In this case, assuming that vibration of the duct casing is not affected by the placed plate, the insertion loss of the duct lagging is obtained by following equations, which can be deduced from the method used in the transmission loss of double wall with sound bridges.

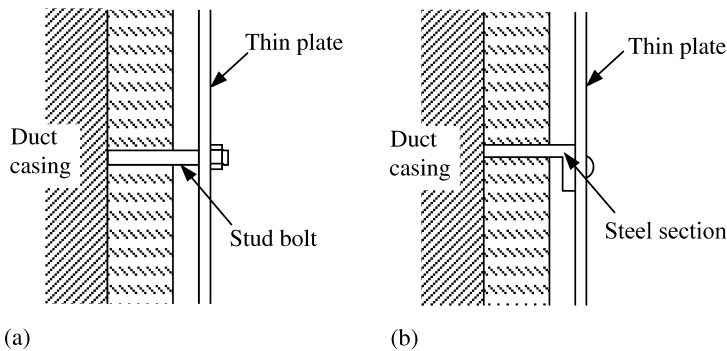


FIGURE 19.23 Examples of duct laggings and connection types of thin plate to the duct casing: (a) point connection; (b) line connection.

1. Point connection

$$IL = -10 \log \left[\beta^2 \frac{8}{\pi^3} n_p \frac{c^2}{f_c^2} + \left(\frac{f_r}{f} \right)^4 \right] + 10 \log \sigma \tag{19.77}$$

2. Line connection

$$IL = -10 \log \left[0.64 n_L \frac{c}{f_c} + \left(\frac{f_r}{f} \right)^4 \right] + 10 \log \sigma \tag{19.78}$$

where

β = vibration isolation factor of the flexible support ($\beta = 1$ for rigid support)

n_p = number of attachment points per unit area

n_L = number of studs per unit length

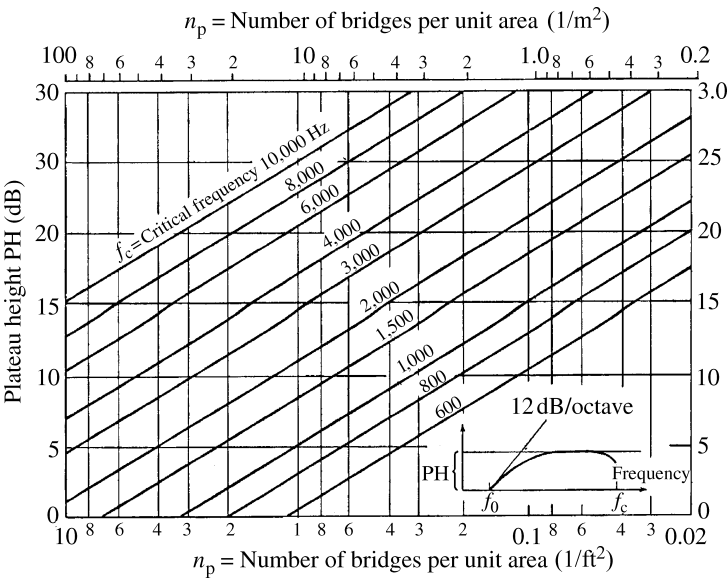


FIGURE 19.24 Plateau height for point connection as a function of the number of bridges per unit area [4]. (Source: Beranek, L. L. 1988. *Noise and Vibration Control*, INCE/USA. With permission.)

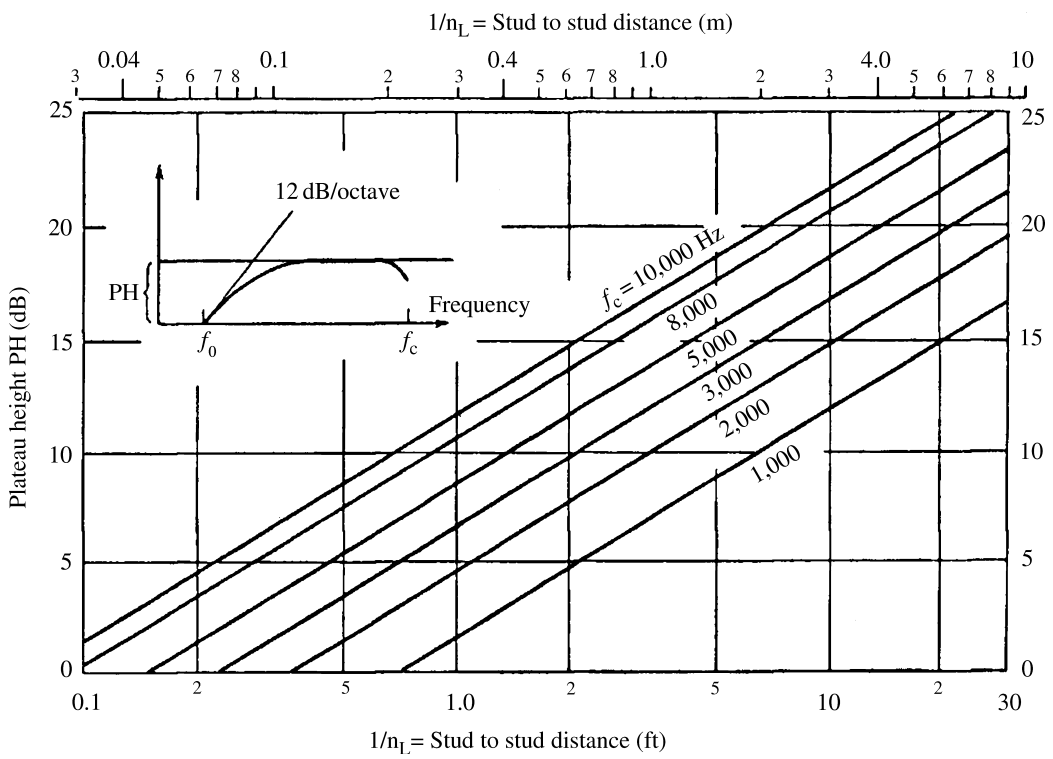


FIGURE 19.25 Plateau height for line connection as a function of the number of studs per unit length [4]. (Source: Beranek, L. L. 1988. *Noise and Vibration Control*, INCE/USA. With permission.)

f_c = critical frequency of the thin plate given by Equation 19.12
 f_r = resonant frequency given by Equation 19.66
 σ = sound radiation efficiency of the thin plate

Knowing the critical frequency, f_c , and the resonant frequency, f_r , we can obtain the insertion loss from the charts given in Figure 19.24 and Figure 19.25 instead of using Equation 19.77 and Equation 19.78. In Figure 19.24 and Figure 19.25, it is assumed that $\sigma = 1$. Note from Equation 19.77 and Equation 19.78 that we must consider the following measures to obtain a higher insertion loss.

- 1. Make the distance between attachment points or studs as large as possible (decrease n_p and n_L).
- 2. Make the air space as large as possible (decrease f_r).

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