

3

Experimental Techniques in Damping

3.1	Electronic Considerations	3-2
	Sensor Linearity • Frequency Issues • Data Acquisition	
3.2	Data Processing.....	3-3
	Language Type • Integration Technique • Fourier Transform	
3.3	Sensor Choices	3-7
	Direct Measurement • Indirect Measurement	
3.4	Damping Examples.....	3-8
	Case 1: Vibrating Bar — Linear with Significant Noise •	
	Case 2: Vibrating Reed — Example of Nonlinear Damping •	
	Case 3: Seismometer • Case 4: Rod Pendulum with Photogate	
	Sensor • Case 5: Rod Pendulum Influenced by Material under	
	the Knife-Edge • Hard Materials with Low Q • Anisotropic	
	Internal Friction	
3.5	Driven Oscillators with Damping.....	3-19
	MUL Apparatus • Driven Harmonic Oscillator	
3.6	Oscillator with Multiple Nonlinearities.....	3-21
3.7	Multiple Modes of Vibration.....	3-24
	The System • Some Experimental Results • Short Time	
	Fourier Transform • Nonlinear Effects — Mode Mixing	
3.8	Internal Friction as Source of Mechanical Noise.....	3-28
3.9	Viscous Damping — Need for Caution	3-29
3.10	Air Influence	3-31
	Brass and Solder Rod Pendula	

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Summary

This chapter is a continuation of [Chapter 1](#) and [Chapter 2](#), and is concerned with practical experimental techniques for measuring damping. It begins with a discussion of the requirements placed on electronics. After demonstrating the importance of sensor linearity using a computer simulation, the issues of data acquisition and processing are addressed. The power of the Fast Fourier Transform is illustrated, not just for spectral analysis, but also (in “short time” form) for measuring the damping of each component when a system oscillates with multiple modes. Various sensor types are discussed in relation to their advantages and disadvantages for specific applications through the treatment of seven different systems studied in free decay. These seven cases differ with respect to factors such as (i) eigenfrequency, (ii) material type, and (iii) method of estimating the logarithmic decrement, and thus the Q of the decay. In the case of some solids, damping is shown to result largely from defects in the structures. A powerful test for nonlinear damping is demonstrated: simply looking at a graph of Q to see whether it changes with time. Two examples of driven oscillators are given. The first being very nearly linear, and the second being highly nonlinear, due to an anharmonic restoring force involving magnets plus several simultaneously acting damping mechanisms. The nonlinear system is used to illustrate difficulties in interpretation that can arise in driven systems due to

phenomena such as frequency and/or amplitude jumps involving hysteresis. Then an illustration is given of how elastic-type nonlinearities may couple with damping-type nonlinearities in order to determine which modes of a complex system survive during the transient approach to steady state. Mechanical noise, another important feature of nonlinear damping, is also examined. The magnitude of the $1/f$ character in an evacuated pendulum is shown to decrease with time, as the oscillator is allowed to stabilize against creep. The final sections address the common misconception that viscous air friction is the most important form of mechanical oscillator damping. Cases are chosen to demonstrate that (i) internal friction is nearly always also important, if not the most important, and moreover, that (ii) fluid damping is rarely simple — involving the density as well as the viscosity of the fluid. It is shown that damping has a complicated frequency dependence, as opposed to the simple (overly idealized) form predicted by common theory.

3.1 Electronic Considerations

3.1.1 Sensor Linearity

The importance of sensor linearity is often overlooked. It is naively assumed that one can simply employ a lookup table to provide calibration corrections. This assumption can result in serious misinterpretations of spectral data, especially in a multimode system. A classic example of artifacts (nonreal signals) that result from a nonlinear sensor is to be found in the ear. The phenomenon, known as aural harmonics, is well known to musicians and figures in the use of “fortissimo” and “pianissimo” in orchestral music. In this chapter we describe how the artifacts mentioned in [Chapter 2](#) are generated. Figure 3.1 illustrates differences according to the nature of the nonlinearity.

The only “real” signals in Figure 3.1 are at frequencies f_1 and f_2 . The number and type of other “unreal” (artifact) signals depends on the type of nonlinearity. The sensor response for the left graph (quadratic) is of the form $V = ax + bx^2$, whereas for the right graph $V = ax + bx^2 + cx^3$. The influence of terms other than $V = ax$ (ideal, linear output voltage) was generated by (i) simulating the pair of harmonic signals, (ii) inputting these signals to each simulated sensor, respectively, and (iii) performing a Fast Fourier Transform (FFT) on the output.

Although it is possible to understand mathematically the various artifacts using trigonometric identities, the phenomenon is much easier to demonstrate with a computer. For Figure 3.1, all

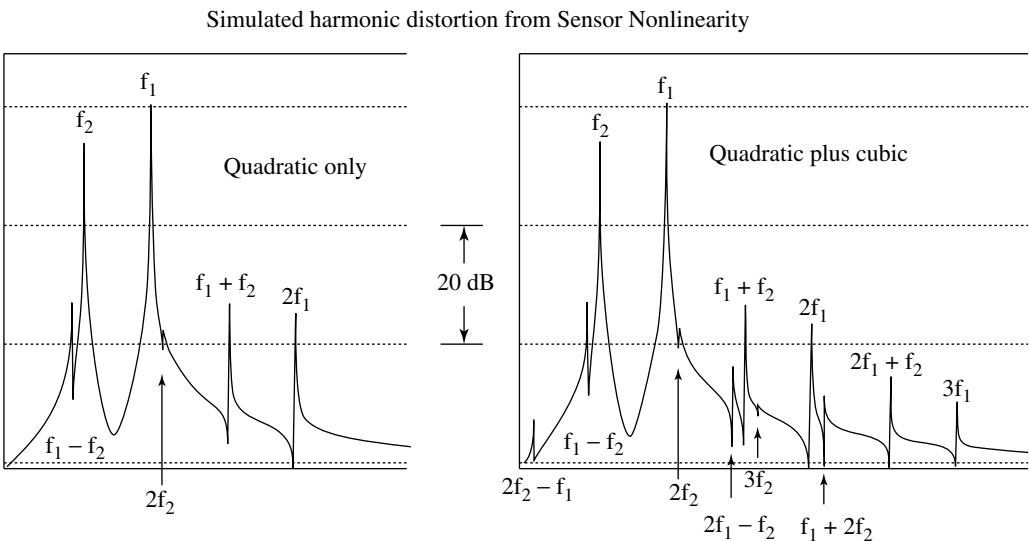


FIGURE 3.1 Spectral illustration of sum and difference artifact frequencies according to nonlinear sensor type.

numerical operations were performed with code written by the author using QuickBasic. It was used to (i) simulate the harmonic signal that was written to a data file, after which it was (ii) read by the FFT algorithm based on the details supplied in *Numerical Recipes* (Press et al., 1986).

3.1.2 Frequency Issues

The choice of a sensor depends largely on the frequencies to be measured. For higher frequencies an excellent instrument for data collection is a digital (storage) oscilloscope, where a microphone can often be directly connected to the instrument. At lower frequencies, a serial-port analog-to-digital converter (ADC) is generally adequate and user-friendly. Examples of each will be provided. The majority of examples considered in this chapter involve low frequencies, where the eigenmode is typically described not in terms of frequency but rather the period (reciprocal of frequency).

3.1.3 Data Acquisition

In the absence of sophisticated data collection and analysis tools, the true character of damping is not readily discovered. Proper characterization is important, since a crude estimate of the damping, based on a single parameter (such as the viscous linear model), may be inappropriate if the oscillator is driven at places (either frequency or amplitude) other than where the parameter was measured. Some of the examples from the experiment that follows were selected to demonstrate the importance of nonlinearity. The probability that an oscillator, selected at random, might have a Q that varies in time is proving to be more significant than anticipated. Were it not for dramatic improvements in numerical-type technology, this improved understanding of damping would not have been possible.

As with computer technology in general over the last decade, ADCs have become much more powerful. The Dataq model 700, for example, is superior (at lower frequencies) to many of the “plug-in” boards of the previous generation that were several times more expensive. The Dataq ADC operates through the USB port (Windows 98 and later), has 16-bit resolution and the software support is excellent. Especially useful for the present purposes are its ability to (i) easily perform data compression with which to view long records, (ii) quickly compute an FFT according to different, useful options, and (iii) easily output files to a spreadsheet.

3.2 Data Processing

3.2.1 Language Type

The author’s experience with software began with early computers and even included the loading of the Fortran compiler of a PDP-11 using punch-tape. He has programmed computers (or hardware-specific processors) with (i) machine code, (ii) assembly language, (iii) Fortran, and (iv) Basic, and he has acquired a rudimentary knowledge of Pascal and C++. The drudgery of machine coding was a factor in his quest to better understand the Fourier transform (Peters, 1992, 2003a, 2003b, 2003c, 2003d). His philosophy with regard to numerical methods is similar to his view of hardware: choose the simplest package (lowest level of sophistication) consistent with the desired results for the problem at hand. The reader may be surprised to learn that QuickBasic (which some have modernized to Visual Basic for Windows) is his favorite language. Nearly all simulation results presented in this chapter were generated with the DOS version of QuickBasic.

3.2.2 Integration Technique

Too few have discovered the powerful integration scheme in which Cromer (1981) modified the unstable Euler algorithm. The difference between the two methods involves the sequencing (order) of updates to the state vectors in the discrete approximation of the integrals. The method was called the “last point

approximation” (LPA) by Cromer, whereas the Euler technique would be called the “first point approximation” according to this nomenclature. The LPA was discovered by a high school student working for Cromer. She was attempting to simulate planetary motion with the Euler method and accidentally coded the LPA. The author first used the LPA to do intercept analyses for the U.S. antisatellite program — computing, among other things, orbital ephemerides. More recently he has used it in place of Runge Kutta techniques to do all kinds of mechanical system simulations, including nonlinear types with several DoF. A physics theorist at Texas Technical University, Professor Thomas Gibson, now regularly uses the LPA as part of the graduate-level course which he teaches in numerical methods.

3.2.3 Fourier Transform

With the Cooley–Tukey improvement to make it fast, the Fourier transform has become a tool of major software importance. Just as the integrated circuit dramatically changed hardware development, the FFT has had a profound influence on the evolution of scientific code.

Whereas many recognize the value of the FFT for viewing “raw” spectral data, few have discovered other powerful tools based in the FFT. For example, autocorrelation is unrivaled in its ability to uncover low-frequency signals of fairly short duration that are corrupted by noise. The number of cycles is not great enough (Heisenberg effect) for a well-defined line to be observed in the FFT by itself. Through the Wiener–Khinchin theorem, the autocorrelation overcomes this limitation. It is computed by multiplying the transform by its conjugate and then taking the inverse transform. The author has used this technique to study free oscillations of the earth (Peters, 2004).

3.2.3.1 Short Time Fourier Transform

A powerful software tool is one in which the Fourier transform is not computed over the entire length of a record. Instead, the record is subdivided (usually with some degree of overlap between adjacent subsections), and the FFT is computed for each subsection. Because the data are generally of the temporal (rather than spatial) type, the technique is called the short time Fourier transform (STFT). For equivalent processing, where the independent variable has units of meters rather than seconds (as in optics applications), the technique could just as well be called the short space Fourier transform.

The STFT is especially useful when waveforms are not pure harmonic, as from a single-degree-of-freedom (single-DoF) oscillator. For systems with multiple modes, whether they derive from eigenmodes as recognized by most, or from mechanical noise as recognized by a few (generated as part of the internal friction of load bearing members); the STFT is a powerful means for isolating and thus determining the temporal history of individual spectral components.

The most common form of the STFT is the canned programs that are a part of software packages such as LabVIEW. With the Dataq software it is easy to accomplish the same thing manually, since one can readily step in time from place to place of a stored record, computing the FFT at any position. The intensity at a given position is obtained by clicking on the displayed spectral line of interest, which provides the value either in dB (Dataq version) or in volts. The amplitude history in dB of the line is thus obtained (equally spaced-in-time values) with a simple click of the mouse. In this way, the free decay of a single component of the system can be readily extracted from the total system response. For the present purposes, the individual intensities were copied by hand to paper and later typed into a spreadsheet for plotting. The process is not laborious, since the number of necessary points is typically less than two dozen.

An example of a manually generated STFT is provided in [Figure 3.2](#). Unlike the methodology described above (operating on experimental data residing in a Dataq folder), the record of Figure 3.2 was generated by computer and written to an output file. The data correspond to three superposed, exponentially damped sinusoids.

From the upper graph (time record), it is not clear how the individual components are changing with time. The triplet of components becomes obvious in the frequency domain (lower left), and when the

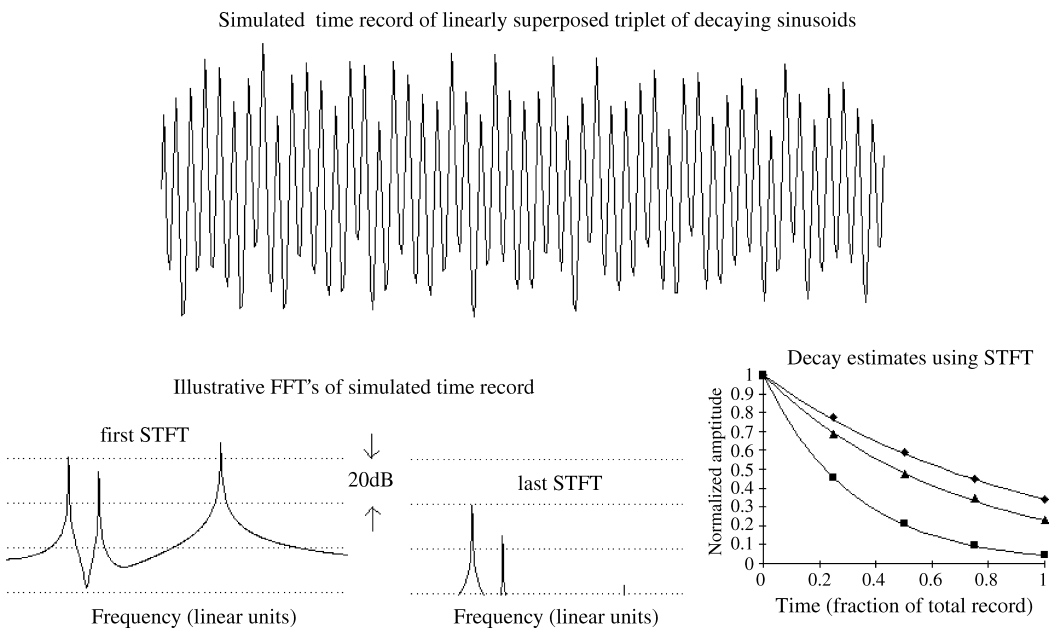


FIGURE 3.2 Example of separating the time decay of superposed components using the STFT.

intensities of each line are plotted *versus* time (lower right) the exponential character of each becomes visible. To test the viability of this method, the individual damping parameters were evaluated from a given STFT graph and found to be in excellent agreement with the values supplied to the simulation.

3.2.3.2 Example Use of the STFT

The majority of the examples given in this chapter avoid low-energy oscillations as space does not allow mesoanelastic regimes to be treated at length. The following case was chosen to illustrate (i) the importance of the Portevin–LeChatelier (PLC) effect on damping (Portevin and Le Chatelier, 1923), and (ii) the power of the STFT in eliminating the influence of clutter. The STFT benefit was expected, since the difference between a noisy record and a multimode case like the previous example is in the number of modes. The PLC effect is significant for high-energy internal friction dissipation even though the jumps for which the effect is known are not obvious at these energies. Evidently, this is a consequence of the large number of events, for which the average effect is a fairly smooth decay.

The scales for the two graphs of Figure 3.3 are different by nearly three orders of magnitude, with the level of the sensor output for each case being indicated (mid-range values). Two features are evident from a direct visual inspection: (i) the change from a smooth to a jerky decay in going from high to low levels, and (ii) a 4% increase in the frequency of oscillation at the lower energy. The latter is recognizable from the vertical lines that have been added (every fifth peak). One of the more interesting (and surprising to most) features of the lower graph is that phase of the oscillation is not significantly altered as the result of mean position jumps.

The following information is provided for those skeptical of the comments concerning the lower trace of Figure 3.3. From the study of hundreds of low-level decays in different mechanical oscillators, the author has become confident that the jumps shown are not sensor (or other electronic) artifacts. Prejudice against this conclusion has been considerable over the last 14 years, in spite of the fact that a similar phenomenon was noted (and accepted) in magnetic materials many years ago, i.e., the Barkhausen effect (Barkhausen, 1919). Unfortunately, the related phenomenon in mechanical systems (PLC effect) is hardly known among physicists.

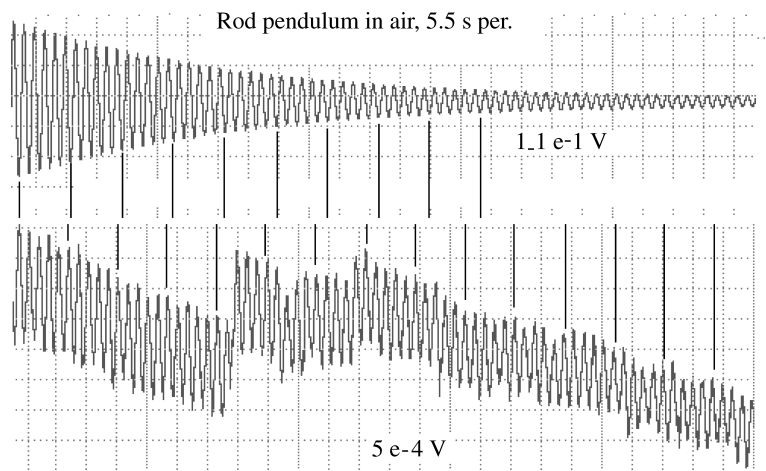


FIGURE 3.3 Illustration of the character changes in decay in going from high to low energies of a pendulum.

The value of the STFT for the study of data such as that of Figure 3.3 is illustrated in Figure 3.4. Whereas Figure 3.3 showed only the first and the last portions of a long record, here the STFT was applied to the entire record using 27 different FFTs.

As seen in the upper left curve, there is a sharp decrease in the damping in STFT No. 5. Thus, the intensity was replotted in each of the intervals from 0 to 5 and 5 to 27. Both intervals show a near perfect linear trendline fit, indicating exponential decay. Thus the Q was found to quickly change from 78 to 340 at an energy level in the region of 10^{-10} J. Although air damping was a factor in the early part of the record, it is not thought to be capable of causing the rapid change in Q that was observed. A similar sharp

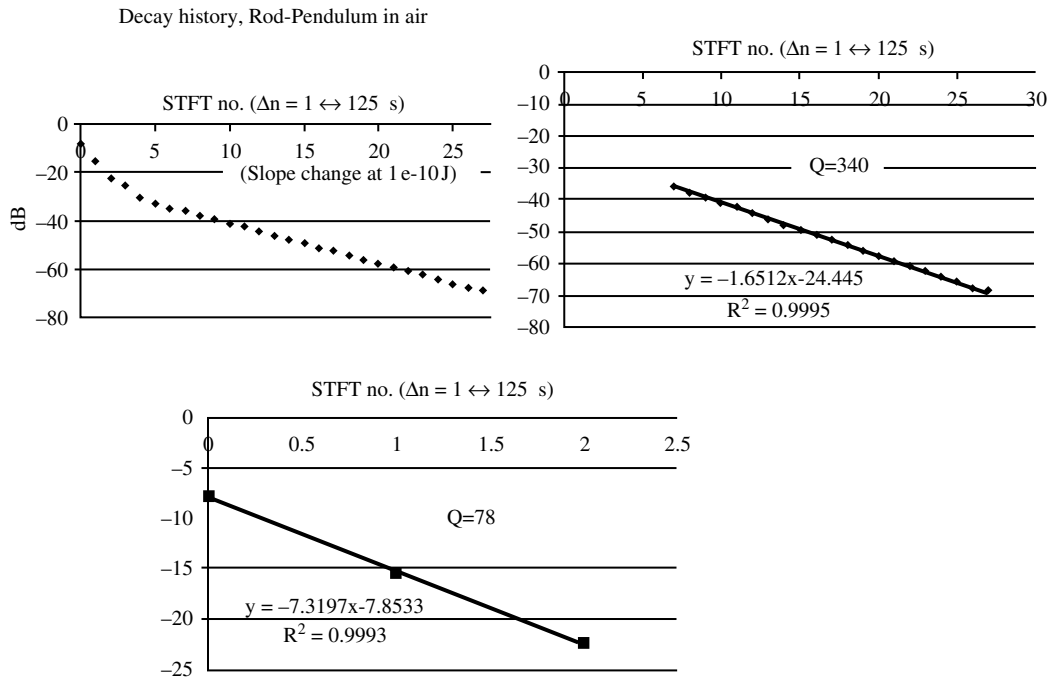


FIGURE 3.4 Example of the use of STFT analysis applied to a dataset, the first and last portions of which are shown in Figure 3.3.

change in the damping of this same pendulum was seen at roughly 10^{-11} J with the pendulum swinging in a high vacuum. The slope change was equally rapid for the vacuum case, but the change in Q was from 120 to 210. It is not known why the damping at low-level energy in a vacuum would have a lower Q than in air. Perhaps the difference derives from a different placement of the knife-edges on the silicon flats. Some of the damping of this pendulum is the result of the knife-edges being fabricated from brass rather than a harder metal such as carbon steel.

3.3 Sensor Choices

Box 3.1 shows some representative sensors for damping measurements. The list is far from exhaustive; for a detailed description of each (plus discussion of other types), the reader is referred to Fraden (1996). Of the transducer types indicated, position sensors are generally the most versatile; but the present chapter also provides examples of the use of (i) velocity, (ii) microphone, and (iii) photogate measurements.

In addition to the need for linearity (discussed in Section 3.1 above), the ideal sensor will be noninvasive. In reality, it is not possible to perform a measurement that does not at some level perturb the system under study. The least perturbative types of direct measurement are optical and electrical — capacitive, followed by inductive.

Some of the advantages and disadvantages of the devices indicated in Box 3.1 are provided below.

Box 3.1

SOME SENSOR TYPES

Representative Sensors for Damping Measurements					
Position	Velocity	Pressure	Time Interval	Acceleration	Force/Strain
Capacitive	Faraday law (electromagnetic)	Microphone	Photogate	Accelerometer	Strain gauge
LVDT		Pressure gauge			
Optical		Capacitive			
Encoder		Optoelectronic			
Shadow		Piezoresistive			
Potentiometric					

3.3.1 Direct Measurement

3.3.1.1 Position Sensors

The inductive linear variable differential transformer (LVDT) is a sensor that is commonly used in engineering applications. Thus, it has been a natural choice for many position-sensing purposes; but it is both more invasive and noisier than capacitive sensors. Wielandt (2001) notes the following concerning the advantage of capacitive over inductive sensors: “Their sensitivity is ... typically a hundred times better than that of the inductive type.”

Optical encoders are also readily available and have been used extensively. Because of their digital nature, based in a finite number of elements, their low-level resolution is poor compared to capacitive devices.

Optical sensing by shadow means is easy to employ — for example, using a solar cell of the type discussed later. The method is afflicted, however, by (i) an offset voltage, and (ii) the degrading influence of background light.

Potentiometers are very easy to use, but compared to other position sensors they are extremely invasive because of Coulomb friction in the slider and also the bearings that support it.

3.3.1.2 Velocity Sensor

The most important velocity sensor is that which functions on the basis of Faraday's law. Using a magnet and a coil, an electromagnetic force is generated in the wire of the coil when it experiences a changing magnetic flux. Prevalent in seismometers before the advent of broadband (feedback) instruments, its primary shortcoming is poor sensitivity at low frequencies.

3.3.1.3 Time Interval

Photogates have become the primary means for kinematic studies in introductory physics laboratories. Combined with compact, user-friendly timers, it is possible to measure both period and velocity. As illustrated later, they can be easily used to measure damping in slowly oscillating systems, but only in a limited amplitude range.

3.3.2 Indirect Measurement

In the cases of (i) pressure, (ii) acceleration, and (iii) force/stress sensing, the measurement is an indirect one. Consider, for example, the Ruchhardt experiment to measure the ratio of heat capacities of a gas (discussed in [Chapter 2](#)). The oscillation of the piston could be measured in several different ways. For instance, direct position sensing could be accomplished by attaching a small electrode to the piston and allowing it to move between stationary capacitor plates. Alternatively, a “flag” on the piston could be used to interrupt the light beam of a photogate. Depending on constraints, however, the easiest method might be an indirect measurement in which a pressure sensor monitors the gas through a catheter communicating with the cylinder of the apparatus.

Accelerometers can sometimes be connected directly to an oscillator, but only if the mass of the instrument is very small compared to the system being studied. As with the measurement of velocity, their sensitivity at low frequency is very poor (the second derivative of position yielding a response that is proportional to frequency-squared).

Strain gauges are easy to employ but also lack sensitivity (compared to position measurement), since they communicate with a very small portion of the oscillating sample (if noninvasive).

3.4 Damping Examples

3.4.1 Case 1: Vibrating Bar — Linear with Significant Noise

The simplest means to measure the Q of an oscillator whose frequency is in the range of the human ear is to use a microphone connected to a digital oscilloscope. In this case, the microphone was an inexpensive dynamic type and the oscilloscope was a Tektronix TDS 3054. A better choice, had it been available, would be an electret microphone. The ring-down of a xylophone bar, following a strong (sharp) hammer strike, is shown in [Figure 3.5](#).

The voltage *versus* time of the microphone output was saved to memory in the oscilloscope, from which the digital record was output to a floppy disk, using the CSV format. Data from the disk were read into columns A and B of an Excel spreadsheet using “Open file.” An envelope fit was then performed on the turning points by placement of trial and error data into column C, using “autofill.” A separate graph was generated for each value of the constant b in the expression “ $= 0.04 * \exp(-b * A1)$ ” typed into Cell C1. (The lower turning points were obtained by typing “ $= -b1$ ” into Cell D1 and using autofill.

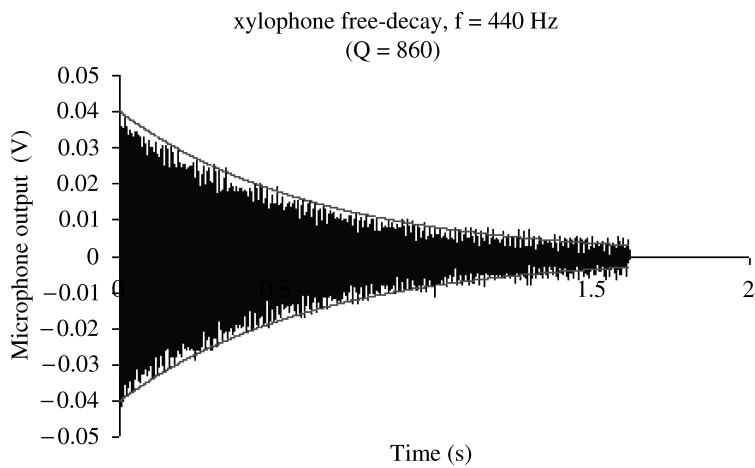


FIGURE 3.5 Free-decay record of a vibrating bar.

(Additional details concerning the use of Excel in this manner will be provided in the discussion of seismometer damping that follows.)

Although optimizing algorithms could be generated to perform such a fit (with a probable slight increase in accuracy), this visual technique is preferred here as it is more understandable, user-friendly, and its performance is proven. The total time required in Excel to generate Figure 3.5 using a 2K record (2048 points) is typically only a few minutes with a modern Pentium computer.*

Once a satisfactory fit was obtained ($b = 1.6$ in Figure 3.5), the Q was estimated using

$$Q = \frac{\pi}{\Delta} = \frac{\pi f}{b} \tag{3.1}$$

There is a fair amount of electronic noise in Figure 3.5 because the microphone was connected directly to the oscilloscope. The smallest bandwidth of the oscilloscope, at 20 MHz, causes a large amount of Johnson (white) noise. Narrowing the bandpass by means of a preamplifier would improve the quality of the data dramatically. Such is typically true of signal to noise ratio (SNR) improvement by tailoring the electronics to the need.

3.4.2 Case 2: Vibrating Reed — Example of Nonlinear Damping

To illustrate another sensing technique, the system shown in Figure 3.6 was used.

A 90° twist was given to a hacksaw blade after heating with a torch and quenching. One end was clamped to the vertical post shown and a piece of cardboard was taped to the other end. An incandescent lamp is placed above the cardboard, which vibrates horizontally, and a solar panel below the cardboard is used as a sensor. The solar panel in this case is a commercial unit that comes with a cigarette lighter plug for charging automobile batteries. The output from the panel goes to the Tektronix digital scope also in the picture.

Unlike other sensing schemes described in this chapter, the solar panel output is not bipolar but instead has a constant voltage offset corresponding to the equilibrium position of the reed.

The frequency of oscillation is too low to operate the oscilloscope with a.c. coupling. Thus, it is important to make the d.c. offset as small as possible. This was accomplished by shielding nonactive parts

*A disclaimer is in order at this point. Present comments by the author should not be interpreted as an endorsement of Microsoft products in general. Although QuickBasic and Excel have both proven unusually beneficial to the work described in this chapter, they are the only software packages marketed by the company to have received a strong endorsement from the author.

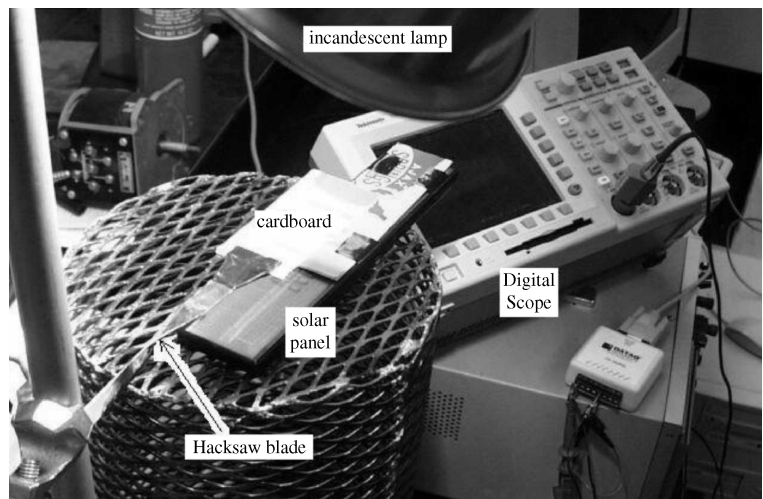


FIGURE 3.6 Setup for measurement of vibrating reed free-decay.

of the solar cell from the lamp. Although the d.c. offset could be removed with a voltage bucking battery, this was found to introduce unacceptable noise spikes. With the offset, the gain of the electronics was limited by the amount of vertical position shift allowed by the scope. The results of this study are illustrated in Figure 3.7.

The nonzero value of b (0.018) in Figure 3.7 indicates the presence of amplitude-dependent damping (refer to Equation 2.61, Chapter 2). The nonlinear damping in this case probably derives from the air, rather than internal friction of the hacksaw blade. Its presence causes the Q of the system to increase with time.

The Q of the system is calculated from the expression

$$Q = \frac{\pi}{(b + a\gamma)\tau} \tag{3.2}$$

where τ is the period of oscillation. At the start of the record ($\gamma = 0.12$) the Q is 390 and it approaches 2700 as the amplitude approaches zero.

3.4.3 Case 3: Seismometer

Since the ability of a seismometer to detect tremors is proportional to the square of the period of the instrument, they require a good low-frequency sensor. The most common sensor for the latest generation commercial instruments is a half-bridge (differential) capacitive type. Because of the greater sensitivity and linearity of the full-bridge symmetric differential capacitive (SDC) sensor mentioned in Chapter 2, it is well suited to these applications, being easy to employ. (The full-bridge character is described in a TEL-Atomic tutorial (Peters, 2002).) A significant advantage of the SDC symmetry (equivalent electrically to the inductive LVDT) is its relative insensitivity to construction imperfections, such as roughness of surface and nonparallelism of electrodes. Thus, construction can be done crudely without serious degradation of performance. For example, electrodes of the first prototype of the SDC sensor were fabricated from sheet

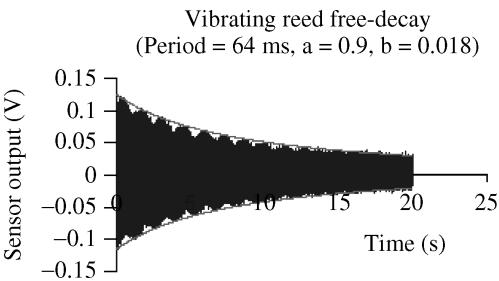


FIGURE 3.7 Vibrating reed decay with amplitude-dependent damping.

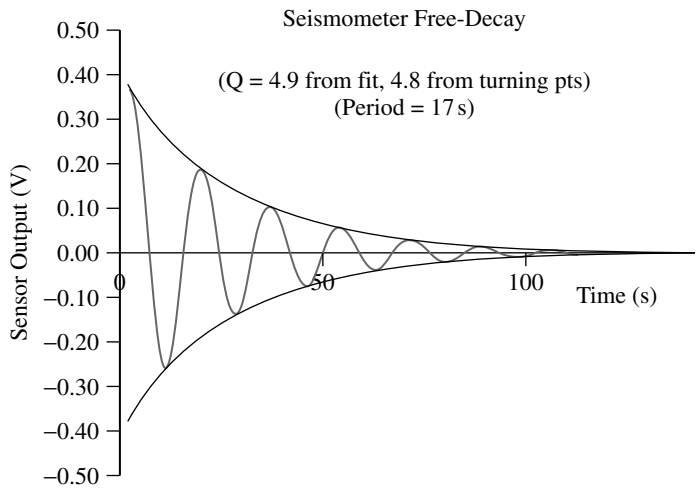


FIGURE 3.8 Free-decay record of the Sprengnether vertical seismometer, period 17 sec.

copper that was cut with shears and subsequently flattened by hammer on a hard plane surface. This stands in stark contrast to the optically polished surfaces necessary for the use of some sensors.

The Sprengnether vertical seismometer discussed at several points in Chapter 2 was studied in a configuration for which $Q = 4.9$, as determined by an external $990\ \Omega$ resistor to provide induced-current damping. The resistor was connected across the coil that is part of the original equipment and which moves (with the mass of the instrument) in the field of a stationary magnet, i.e., a Faraday’s law (velocity) detector. Excitation to initiate the free-decay study was accomplished by applying an alternating (square wave) current to the coil, reversing the direction of the current at each turning point of the motion of the mass. The fundamental (Fourier series) of a square-drive generated this way is shifted 90° from the mass motion, corresponding therefore to resonance. After cessation of the drive, data as shown in Figure 3.8 were collected with a Dataq DI-700 ADC (16-bit).

The graph in Figure 3.8 was generated with Excel after the Dataq record was saved to floppy disc as an *.dat (CSV) file. It was imported to Excel using “open file” with “comma delimiter.” Once in Excel, these data were shifted one place to the right (from the default A column to the B column) to accommodate computer generation of a time-data column. The column of time values was generated according to the sample rate, the value of which is by default saved to the data file. To generate the time column, a 0 was placed in the first row, n corresponding to the start of data. Dropping down one row in the A column, “ $= A_n + 1/(\text{sample rate})$ ” was typed, to increment the time. Then the lower right hand corner “small solid square” of the box containing this time was grabbed and held with the left button of the mouse to autofill all the way to the last time point of the data. The computer-generated exponentials, which correspond to the turning points, were obtained by generating two additional columns. These were obtained by placing the cursor at a row corresponding to the time A_n (in column C) and then typing “ $= A_0 * \exp(-(\omega/2/Q) * A_n)$.” The value of A_0 is obvious from the data and a first estimate for Q can be quickly obtained from about a dozen turning points (read with the Dataq software before the data are ever saved).

The technique is illustrated in Table 3.1. For example, in the case of Figure 3.8, $Q = 4.8$ from the 13 turning points. Thus the argument of the exponential was set to 0.0385. Using autofill, the columnar (upper) exponential was then quickly produced. Then a second (adjacent) column D was generated in similar manner, by taking the negative of the last point and then autofilling to the top row. (When one autofills downward, the rate with which Excel traverses the rows increases exponentially after the last row of data has been passed; thus, it is much easier to fill upwards rather than downwards.)

Once the pair of exponentials being fitted to the data have been graphed, along with the data, it is simple to adjust the curves by varying the argument (in this case, small changes around 0.0385) until a

TABLE 3.1 Estimation of Q from the Turning Points

Use of Excel to estimate logarithmic decrement from turning points of the motion		
Free-decay of Cavendish balance		
1 2 3 4 5 6 7 8 9 10 11 12 13 -0.252 0.247 -0.185 -0.194 -0.133 -0.155 -0.095 0.122 -0.062 0.107 -0.037 0.088 0.024		
$-2*\ln(1-(A1-A13)/(A1-A2+A3-A4+A5-A6+A7-A8+A9-A10+A11-A12))$	→	0.292266
$-2*\ln(1-(A2-A12)/(A2-A3+A4-A5+A6-A7+A8-A9+A10-A11))$	→	0.272743
(A15+A16)/2 = mean		
(final estimate from computerized fit = 0.271)		

A	B	C
Turning Pt (V)		
3	0.3643	
4	-0.2607	
5	0.1913	
6	-0.1343	
7	0.1029	
8	-0.0687	
9	0.0549	
10	-0.0371	
11	0.0298	
12	-0.0194	
13	0.0149	
14	-0.0109	
15	0.0071	
	0.6554	0.651
	0.6532	

Free-decay of Seismometer

1st set

$B15 = -2*\ln(1-(A3-A15)/(A3-A4+A5-A6+A7-A8+A9-A10+A11-A12+A13-A14))$

2nd set

$C15 = -2*\ln(1-(A4-A14)/(A4-A5+A6-A7+A8-A9+A10-A11+A12-A13))$

average yields Q = 4.8 (π/0.653)

(final estimate from computerized fit = 4.9)

good fit is obtained (using autofill each time). The fit is rapid and accurate when there are not too many parameters to vary, since the eye is well suited to this operation. Upon obtaining the best-fit by this means, the preliminary value of $Q = 4.8$ was altered to the final value of $Q = 4.9$.

Note that in Table 3.1 a new Excel worksheet was employed (column A no longer the time as in the discussion above). The voltages corresponding to the turning points (maximum and minimum) were each read by placing the cursor at an extremum and manually recording the value displayed in turn by the Dataq software. These values were then typed into Excel, as opposed to the “file open” method for importing large datasets, i.e., as used to generate [Figure 3.7](#).

3.4.4 Case 4: Rod Pendulum with Photogate Sensor

One of the simplest ways to measure damping at larger levels is to use a photogate of the type common to general education physics laboratories. The infrared beam of the photogate is tripped by a “flag” attached

to the oscillator. The rod pendulum pictured in Figure 3.9 was studied by this means. Figure 3.10 shows a closeup of the flag which is attached to the top of the pendulum and which passes through the photogate during oscillation.

As seen in the figures, various parts of the pendulum are clamped to a vertical steel rod. Both upper and lower masses are made of lead, each with a pair of holes drilled in it — one to pass the brass rod of the pendulum through and the other (after tapping) to hold a thumbscrew for securing the mass at different vertical positions on the rod.

This mechanical oscillator is a compound pendulum; the period can be made long as the knife-edge (also clamped to the rod) approaches the center of mass. At long periods, the instrument is not very responsive to external accelerations of the supporting frame; but it is sensitive to internal structural changes. Low-frequency instability is encountered as the upper parts of the instrument experience creep, particularly in the materials just above the knife-edge. The upper pendulum has similarities to an inverted pendulum, except that it is rigidly connected to the lower pendulum, and causes the oscillator to eventually exhibit double-well (Duffing) characteristics. This happens at larger amplitudes as the period is increased toward really long times. The tendency toward mesoanelastic complexity depends on the dimensions of the rod. As expected because of the well-known engineering properties of rods and tubes, a large diameter, thin-wall tube will behave differently from a solid rod made from the same amount of material (same total mass). This will be true if the tube does not experience localized (sharp) deformation prone to creasing.

Damping measurements with a photogate require that the time required for the flag to pass through the beam be fairly small — thus larger amplitudes of motion are required than with other sensors. Of course really large motion would result in a period increase, consistent with long-understood pendulum dynamics. For amplitudes within the acceptable range (which in practice is not overly restrictive), the velocity of the pendulum as it passes through the equilibrium position is inversely proportional to the time

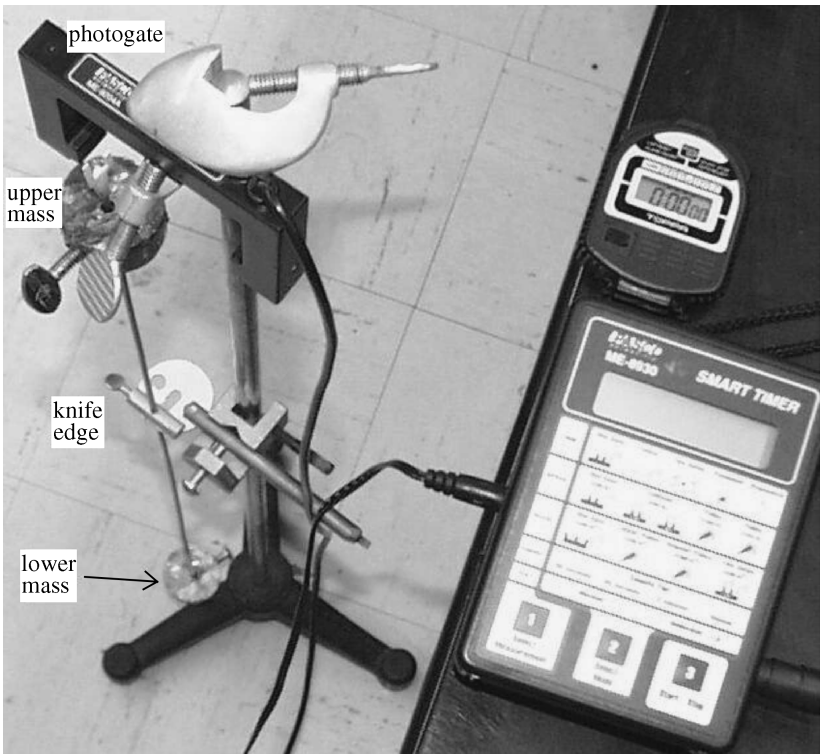


FIGURE 3.9 Rod pendulum in which damping measurements are made with a photogate.

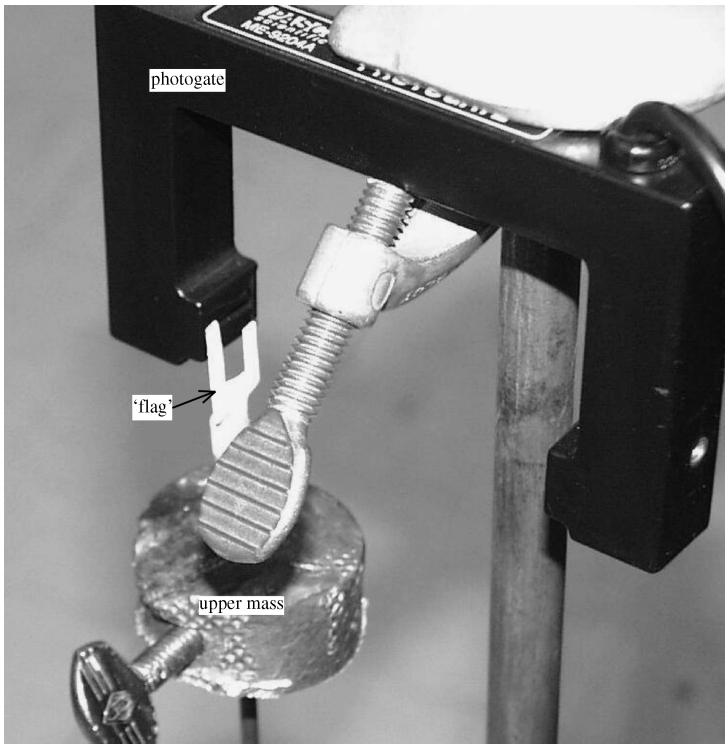


FIGURE 3.10 Top of the pendulum showing the upper mass and “flag” for tripping the photogate.

interval between interrupts of the photogate beam by the two vertical arms of the flag. If the period does not change with amplitude, then there is also an inverse relationship between the gate time and the amplitude. A plot of the inverse of these times *versus* cycle number is, for the constraints indicated, a reasonable approximation of the turning points of the free-decay.

In this case the single gate time interval measurements were made by a Pasco Smart Timer. It is a user-friendly instrument that also permits the period of the pendulum to be accurately measured by the flag (by using two different lengths of the flag arms). For experiments of this type, it may prove more convenient to measure the period with a stopwatch (infrequently as compared to the velocity). The sequentially increasing time intervals are read manually from the Smart Timer and recorded by hand, once per cycle. Of course, to do so requires that the period be long enough to permit these operations. The recorded values are conveniently analyzed by typing to a spreadsheet, which is then used to graph damping curves such as shown in Figure 3.11.

A pure exponential fit is not appropriate to the decay of Figure 3.11, in which the upper mass had been removed and a business card taped to the bottom of the pendulum to cause turbulent air damping (period near 1 sec). The fit shown, however, involving both linear and quadratic dampings, is seen to be quite reasonable. As in the case of the vibrating reed discussed earlier, this system is adequately described by the nonlinear damping equation 2.61 given in Chapter 2. For the data of Figure 3.11, $Q = 25$ initially and increases to 70 at the end of the record.

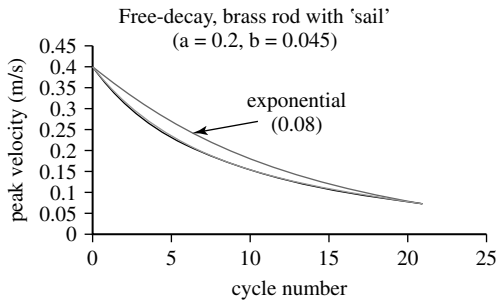


FIGURE 3.11 Free-decay of a pendulum as determined by photogate measurements.

Without the business card, and with the upper mass in place, the decay of this pendulum was found with the photogate measurement technique to be exponential, as expected for viscous damping with periods of about 5 sec. At periods in excess of about 10 sec, however, internal friction of the rod becomes more important than air damping. Although the decay is then still exponential at larger levels, the frequency dependence is not the same as required by linear air damping.

3.4.5 Case 5: Rod Pendulum Influenced by Material under the Knife-Edge

The data in Figure 3.11 were collected with the knife-edges resting on hard ceramic alumina flats. When supported by other materials, the damping of a rod pendulum can be influenced by anelastic flexure other than that of the rod. Hardness of the material does not guarantee low damping, as will be seen in the following examples. The data that follows were collected with a different pendulum, depicted in Figure 3.12.

The sensor in this case was an SDC unit, connected to the computer through the Dataq DI-700 A/D converter. The upper and lower masses are each approximately 1 kg and their separation distance on the aluminum hunting arrow from which the pendulum was fabricated was about 70 cm.

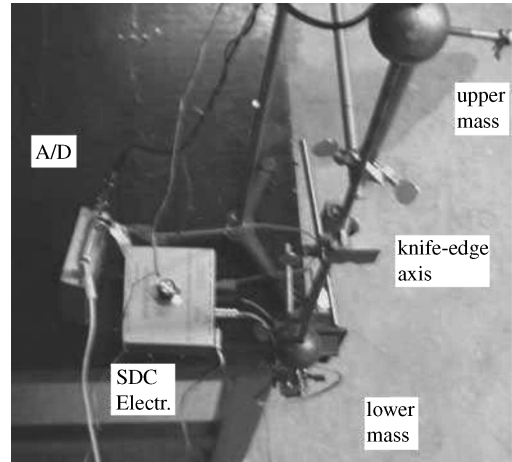


FIGURE 3.12 Long period rod pendulum used to study the influence of different materials under the knife-edge.

3.4.5.1 Lithium Fluoride Samples

The samples used to collect the data in Figure 3.13 were identical pairs, except that one pair had been irradiated with a huge dose of gamma rays. The resulting changes to the structure of the crystal are responsible not only for color centers as noted in the photograph in Figure 3.14, but also a dramatic change in the internal friction. It is clear from Figure 3.13 that internal friction in the LiF is the dominant source of damping of the rod pendulum that was used (Peters, 2003a, 2003b, 2003c, 2003d).

Lithium fluoride is used in thermoluminescent film badges (radiation monitors). When exposed to energetic radiation, atoms are “knocked” from their crystal lattice sites into metastable states corresponding to interstitial positions of the lattice. Upon ramping the temperature of the sample in an oven fitted with a photomultiplier tube, jumps from the metastable state are accompanied by the release of photons. The amount of light so generated is a measure of the dose that was received by the crystal. Because light flashes are observed with rather small changes in the temperature, it is reasonable to expect that mechanical strains might also cause a significant change to the defect state of such crystals. This postulate is confirmed by the data in Figure 3.13, which show a dramatic difference in the decay character of the pure (clear) crystals (bottom figure) and those which were extensively damaged by gammas (top figure).

In both of the decays in Figure 3.13 there is significant nonlinear damping, as evidenced in the early portions of each of the two records. The top case is nearly pure Coulombic, and the bottom case is partially amplitude dependent. This is revealed from estimates of the Q , shown in Figure 3.15.

The Q values in Figure 3.15 were computed from successive triplet-values of the turning points of the motion, read directly from the decay pattern displayed on the monitor by the Dataq software. The equation used is

$$Q = \frac{\pi}{-2 \ln[1 - (\theta_n - \theta_{n+2})/(\theta_n - \theta_{n+1})]}, \quad n = 0, 1, 2, \dots \quad (3.3)$$

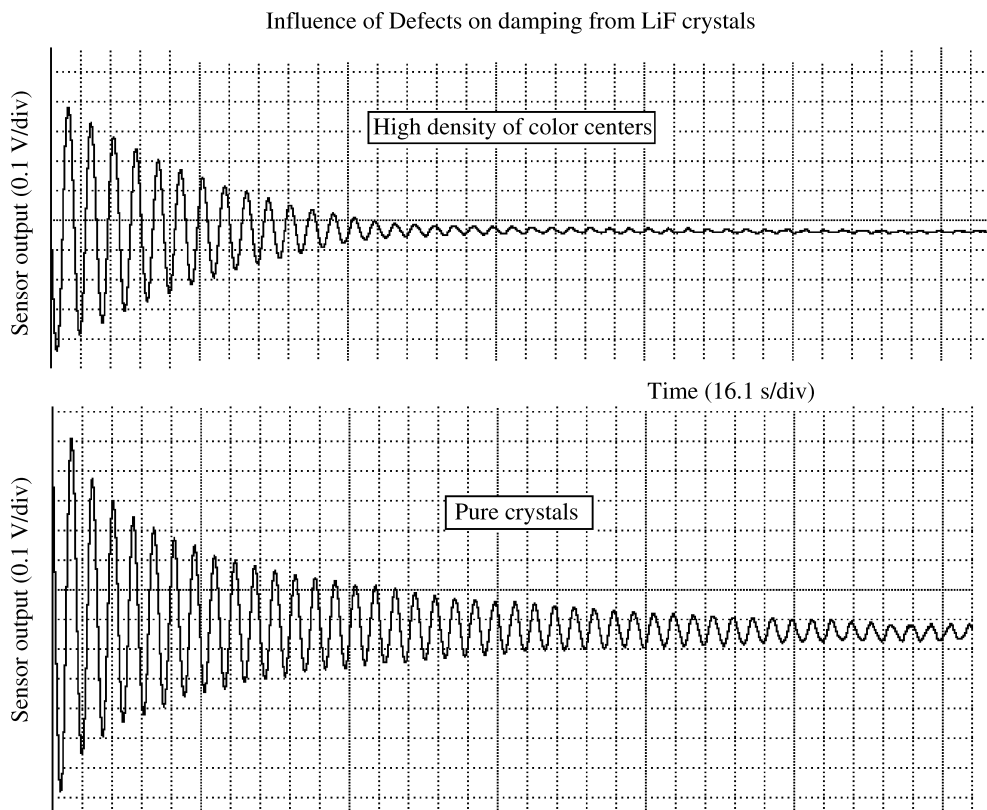


FIGURE 3.13 Illustration of damping difference according to specimen type under the knife-edge.

3.4.6 Hard Materials with Low Q

It is commonly (and mistakenly) thought that hard materials must necessarily also have low damping. The following two examples show that this is not necessarily so. Even though cast iron is very hard, it is also quite dissipative, which makes it an ideal material for engine blocks. Figure 3.16 shows a decay curve for the steel knife-edges of the pendulum resting on cast iron samples.

At the start of the record the damping with cast iron is nearly twice as great as that of steel-on-sapphire or steel-on-silicon, where the Q was found to be of the order of 80. This large damping measurement is consistent with the known excellent properties of cast iron for use in engine blocks, although the frequencies for such applications are much higher.

Figure 3.17 is another very hard material which has large damping — the ceramic piezoelectric wafer formed from lead, zirconium, and titanium (PZT), which by means of a mechanical impulse is commonly used to generate an electric spark to ignite a gas grill. The secular decline of Q based on the short temporal record indicates Coulomb damping. It is consistent with the nearly straight-line turning points for the early part of the long-term record, also shown. The long-term record

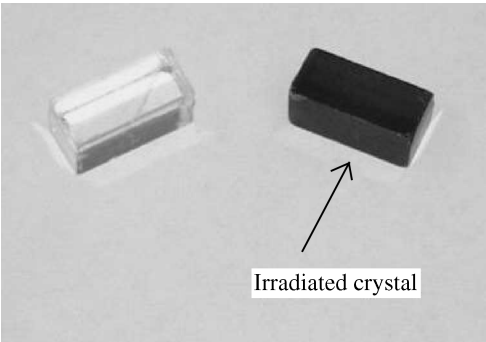


FIGURE 3.14 Photograph of LiF single crystals used to obtain the data in Figure 3.13.

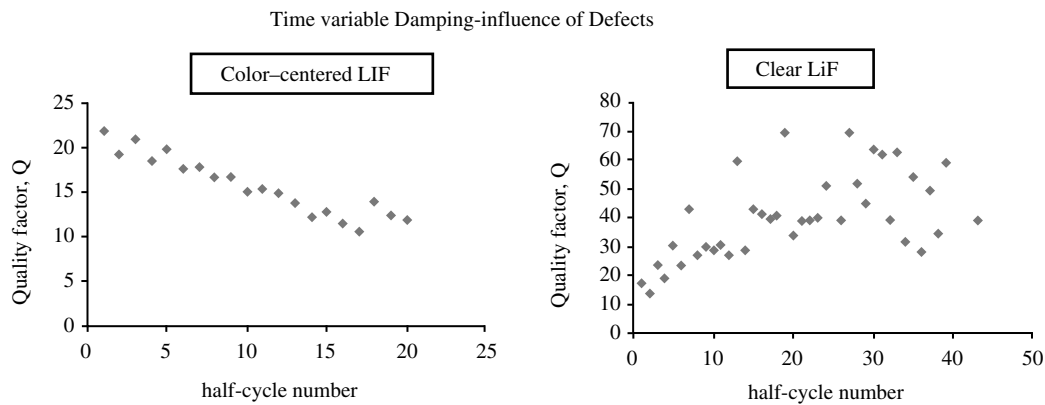


FIGURE 3.15 Temporal dependence of the Q , LiF crystal experiments.

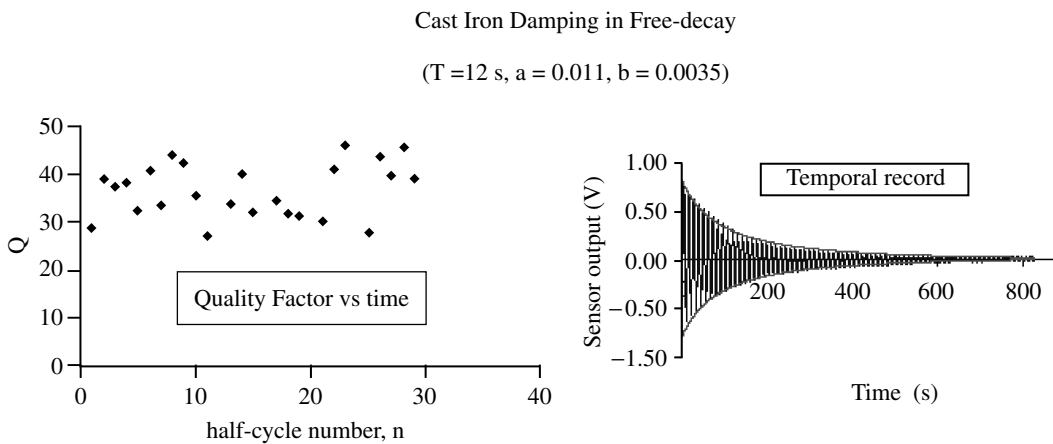


FIGURE 3.16 Data collected using cast iron samples.

is labeled as anomalous because it does not appear to be consistent with several simultaneously acting dissipation mechanisms. Instead, the strong Coulomb damping seen early on seems to disappear later, once the amplitude has dropped below a particular level. This suggests activation processes of a quantal type. It would be interesting to study the PZT wafers in a different pendulum configuration, and not operating “open-circuit” as in the present case, but rather with different resistors connected between the top and bottom of the wafers.

3.4.7 Anisotropic Internal Friction

With Polaroid material (H sheet) placed under the knife-edges it was found that the damping depends on the direction of the long-chain polymeric molecules. The direction of the molecules in a sample is readily determined by looking through the Polaroid at reflected light from a polished floor. When the reflection occurs close to the Brewster angle, only the horizontal component of the electric field is significant in the reflected light for unpolarized incident light. The direction of the molecules is thus determined by rotating the sample until the minimum of level of light is found. When this occurs the molecular chains are situated horizontally.

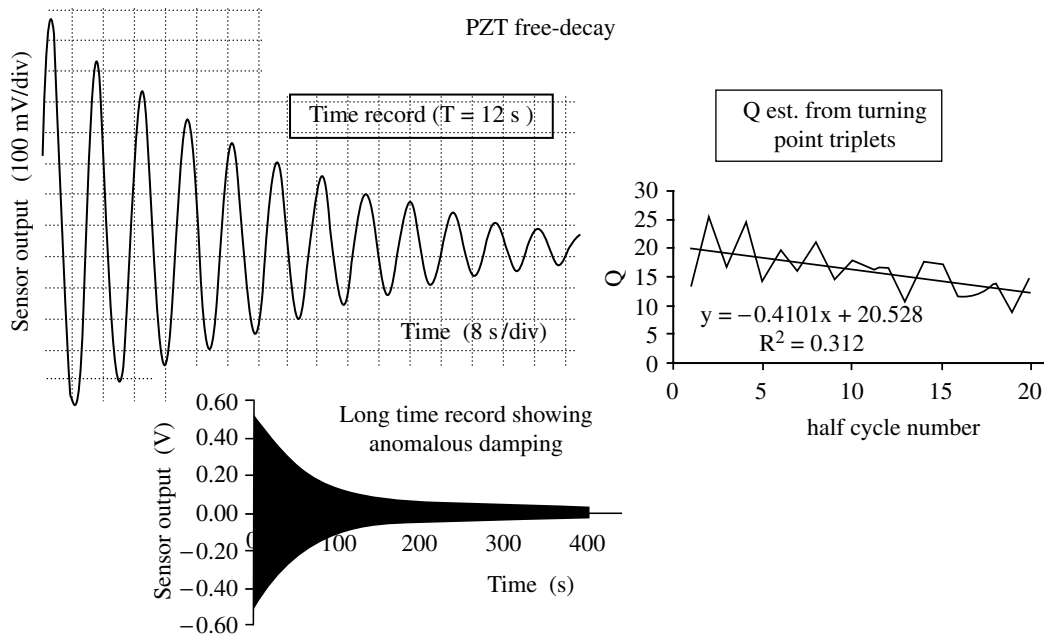


FIGURE 3.17 Data from an experiment involving PZT ceramic wafers.

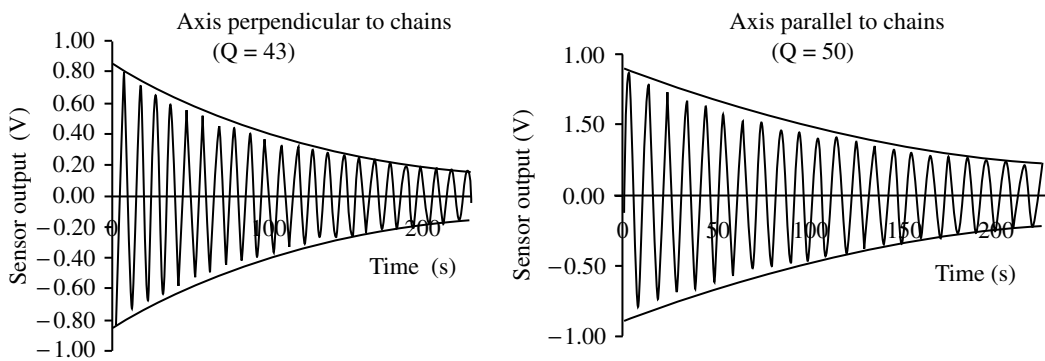


FIGURE 3.18 Free-decay curves showing anisotropy of the internal friction in polaroid material.

It was reasoned that the molecular properties of Polaroid might result in mechanical as well as optical anisotropies. This postulate proved to be true, as shown in Figure 3.18.

When oscillating on silicon at a period of 10 sec, previous studies have found that the instrument decays consistently with a Q of 80 (uncertainty 3%). In the present study, half a dozen free-decay records were obtained for (i) edges parallel to the chains and (ii) edges perpendicular to the chains. The average Q of oscillation was estimated at 50 for the parallel case and 43 for the perpendicular case. Reproducibility proved slightly better for the parallel case (4%) as compared to the perpendicular case (5%). Additional details are documented elsewhere (Peters, 2003a, 2003b, 2003c, 2003d).

3.4.7.1 Summary, Free-Decay Q Estimation

All of the techniques so far described are methods based on free-decay, which is especially important for nonlinear systems. With linear systems it is also possible to use steady-state methods, as noted in Box 3.2 (last column; de Silva, 2006). Box 3.2 summarizes the techniques used in the present chapter to estimate the logarithmic decrement, βT , from which $Q = \pi/(\beta T)$.

Box 3.2

METHODS FOR QUANTIFYING DAMPING

Damping (Q Estimation) Techniques ($Q = \pi/\beta T$, $T = \text{Period}$)					
Logarithmic Decrement (Full-Cycle, N)	Turning Points (Half-Cycle, n)	Nonlinear Fit to Envelope	Time τ to $1/e$ ($0.3679x_0$)	Short Time Fourier Transform	Bandwidth, Magnification Factor, Hysteresis Loop, Step-Response
$\beta T = \frac{1}{N} \ln \frac{x_0}{x_N}$	$\beta T = -2 \ln \left[\frac{1 - (x_{n-1} - x_{n+1})}{(x_{n-1} - x_{n+1})} \right]$	$-\dot{y} = ay^2 + by + c$	$\beta T = T/\tau$	$-\beta T = \frac{T \ln 10}{20 [\text{slope}(\text{dB/s})]}$	de Silva (2006), Chapter 7

The best method is to use a full nonlinear fit; the worst is to measure the time to $1/e$. The expression in Box 3.2 for the logarithmic decrement, using the STFT, is equivalent to

$$Q = 27.29 \frac{f}{\left| \frac{\text{dB}}{\text{s}} \right|}$$

(3.4)

where f is the frequency in Hz and the STFT slope is specified in dB per s.

3.5 Driven Oscillators with Damping

This chapter has been mainly concerned with oscillators in free-decay. It is also possible to make quantitative predictions from measurements at steady state. Confidence in predictions, however, depends on the nature of the damping. Such data are of limited value for most nonlinear systems, unless supplemented with free-decay data.

3.5.1 MUL Apparatus

Some of the techniques applicable to driven systems are illustrated by the multipurpose undergraduate laboratory (MUL) apparatus shown in Figure 3.19, that has been used by students in the physics department at Mercer University.

For the purpose of measuring the Lorenz force (basis for defining the current unit, the ampere) a constant current is supplied through the posts to the pivoted-on-points brass wire on which a weight, W , is shown hanging on one of the horizontal arms of the wire. Current enters the wire through one post *via* the banana plug inserted into a drilled hole. It thereafter travels through the lower (invisible) shorter

straight segment of the wire located between the poles of the drive magnet; and it finally exits through the banana plug on the opposite post. When carrying a current, the force on the wire from the part inside the magnet causes vertical deflection, the direction up or down being determined by the direction of the current. This results in a rotation about the pivot points (indented tops of the posts). The position is measured by the capacitive sensor, S (one of several variants of the SDC patent).

The sensitivity of this current balance depends on the location of the center of mass of the oscillatory wire, which is determined in part by the position of the rare earth magnet, M, which hangs from a steel nut on the threaded part of the heavier brass rod having a 90° bend. The upper end of this threaded rod is held by a plexiglass member that also holds the ends of the oscillatory wire.

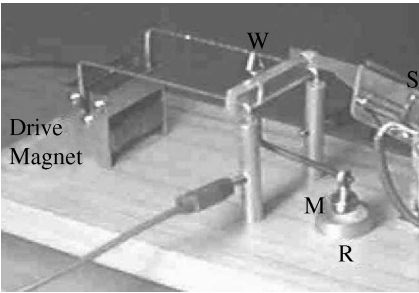


FIGURE 3.19 Apparatus for studying resonance and the Lorentz force law.

3.5.2 Driven Harmonic Oscillator

The MUL becomes a driven harmonic oscillator when the excitation current is a.c. rather than the d.c. used for the Lorentz force study. The damping is determined primarily by eddy currents in the aluminum ring, R, that lies on the wooden base underneath and in close proximity to magnet M.

The apparatus is useful for studying both free-decay and driven oscillation. Engineering students Brandon R. Bowden and James D. Sipe have programmed LabVIEW to generate both free-decay curves and resonances.

An example Lorentzian (resonance response) is given in Figure 3.20. (Additional information is found in a laboratory writeup (Peters, 1998).)

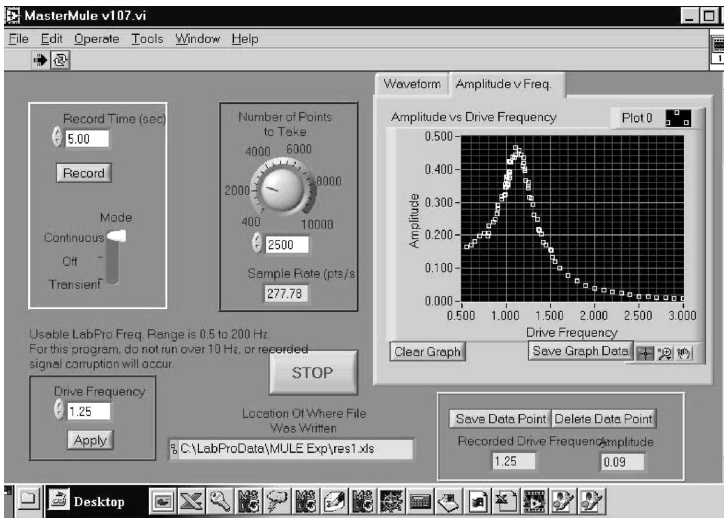


FIGURE 3.20 Screens from the LabVIEW program used with the MUL to study both transient and resonance phenomena.

3.6 Oscillator with Multiple Nonlinearities

An oscillator can have significant nonlinearities of both the elastic and damping types. An example is the mechanical system pictured in Figure 3.21.

The instrument is a modified extensometer that was sold by TEL-Atomic and which was designed around the SDC sensor to measure Young's modulus and thermal expansion coefficients. The wire sample normally used with the instrument (along with a hollow power resistor that fits in the black clamp) has been removed, and two rare earth magnets have been employed. One magnet is superglued to the bottom of the pan where the weights are normally placed, as shown in Figure 3.22; and the other magnet is attached to the bottom of the inductor that is sitting on the top of the oscillator (cased instrument, Pasco) used for drive. The pair of magnets are positioned in close proximity so as to repel each other, thus supporting the mass of the moveable arm of the extensometer.

The study of nonlinear systems requires a linear sensor; i.e., any nonlinear contributions from the detector must be negligible. Figure 3.23 shows the calibration results for the instrument and its linear response for the range of amplitudes used in the study.

The potential energy of this oscillator was assumed to have the following form

$$U(x) = \frac{b}{x^n} + cx \quad (3.5)$$

and the parameters were estimated by measuring x as small masses were placed on the pan. A linear regression fit to a log-log plot (using the sensor calibration constant of 550 V/m) yielded $b = 1.02 \times 10^{-5}$, $n = 1.526$, and $c = 0.304$ (system international units). Anharmonicity of the

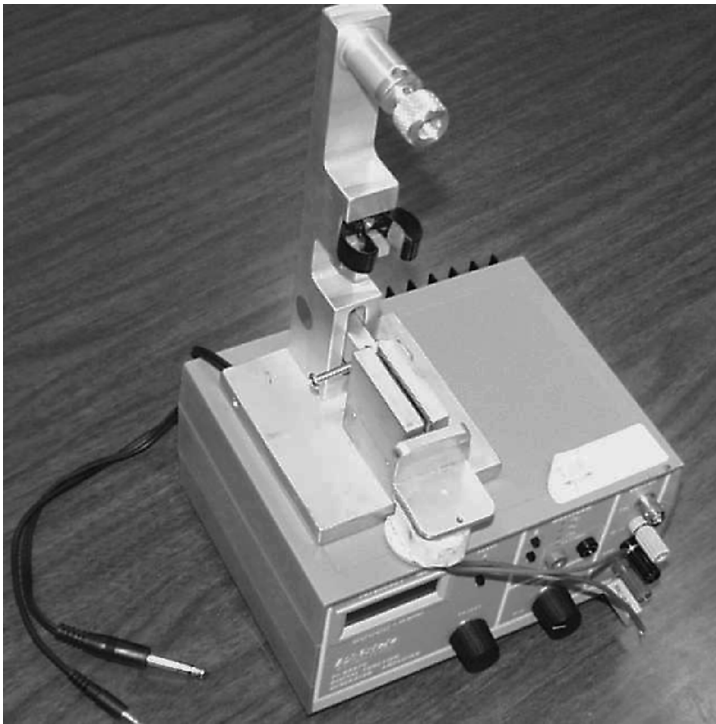


FIGURE 3.21 Mechanical oscillator with multiple nonlinearities.

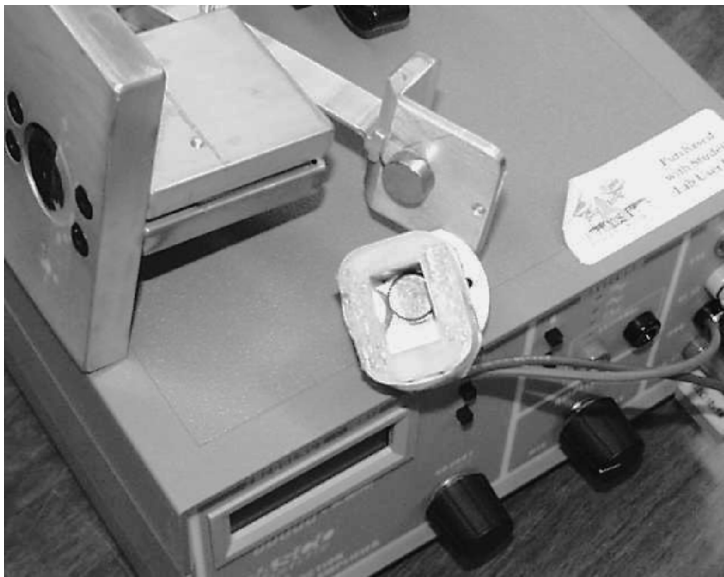


FIGURE 3.22 Closeup picture of the oscillator in [Figure 3.21](#) (nonoperational configuration), showing placement of the rare earth magnets.

potential is readily apparent in the plot shown in [Figure 3.24](#), with the force of restoration being greater in compression (x decreasing) than it is in extension. This feature is reminiscent of interatomic potentials, with anharmonicity being responsible for thermal expansion.

Because of the elastic nonlinearity, the mean position depends on the amplitude of the oscillation, as is evident in the free-decay curve in [Figure 3.25](#).

The damping of this oscillator was also found to be nonlinear, as seen in [Figure 3.26](#).

The oscillator exhibits hysteresis when driven at larger amplitudes, as shown in [Figure 3.27](#), where it can be seen that the location of an amplitude jump depends on which way the oscillator is adjusted, either up or down in frequency. Such jumps (well known with oscillators with nonlinear elasticity) stand in stark contrast with the behavior of a linear oscillator, as can be seen by comparing [Figure 3.27](#) with the screen picture in [Figure 3.20](#).

A surprise from this study involves the frequency of oscillation. In general, the oscillator did not entrain to the drive. Moreover, the preferred frequencies were not necessarily the same as the free-decay frequency of 6.01 Hz. Some of the frequencies (measured with power spectra) are indicated in

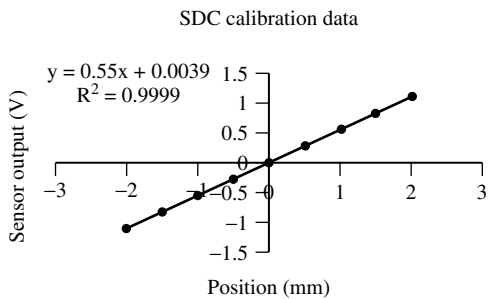


FIGURE 3.23 Calibration data for the sensor used with the oscillator having multiple nonlinearities.

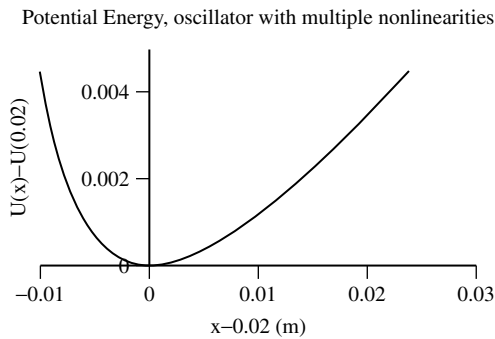


FIGURE 3.24 Plot of the potential energy function of the oscillator.

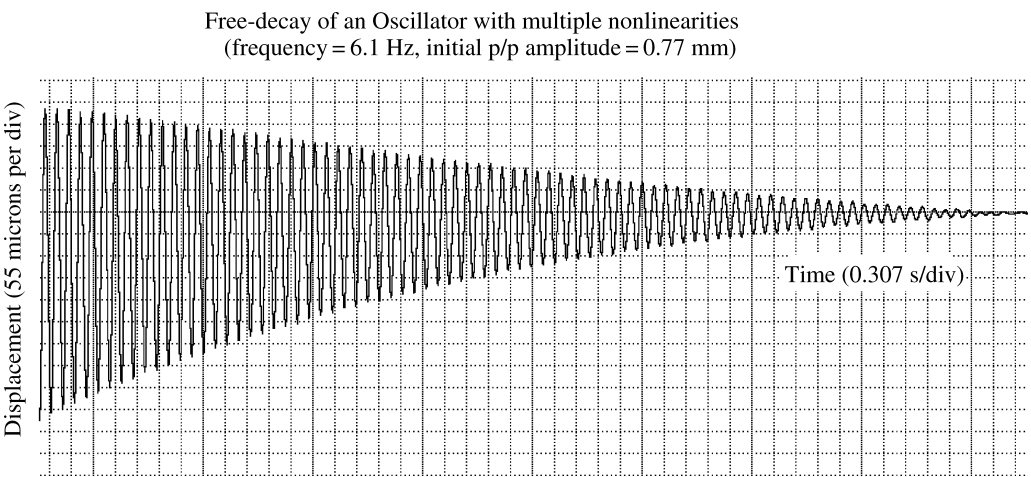


FIGURE 3.25 Free-decay curve showing the mean position shift as a function of oscillator amplitude. (Decreasing sensor voltage corresponds to increasing x .) The frequency of oscillation is 6.01 Hz.

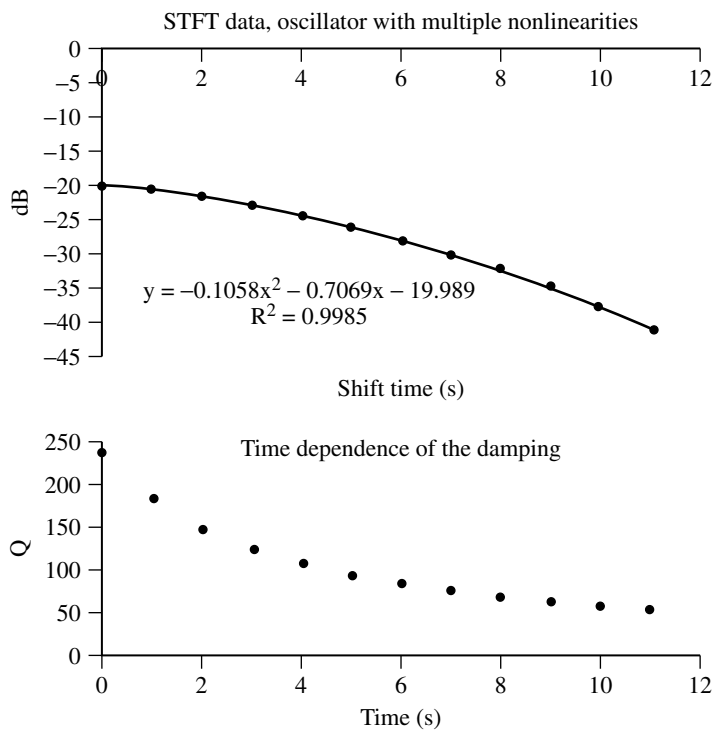


FIGURE 3.26 Free-decay character as determined using the short time Fourier transform.

[Figure 3.27](#). The 3% frequency jumps observed in [Figure 3.27](#) (in going from 5.46 Hz to 6.20 Hz) are not real but rather artifacts of the finite resolution of the 1024 point transforms that were employed.

[Figure 3.27](#) demonstrates why nonlinear damping measurements should be done in free-decay. [Figure 3.25](#) demonstrates how exponential fits make no sense for some oscillators. Fortunately, the STFT can be used to determine the amplitude dependence of the Q .

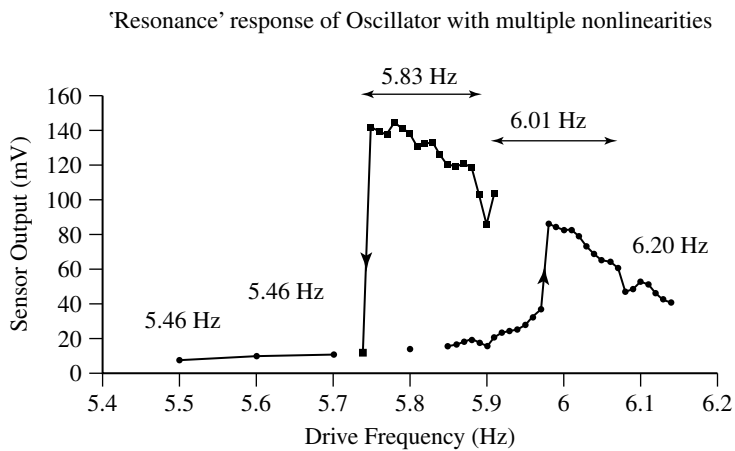


FIGURE 3.27 Resonance response (steady state) of the driven oscillator.

3.7 Multiple Modes of Vibration

3.7.1 The System

In engineering, multimode oscillations are common. Many, if not most, cases have mode mixing features even though they may in some cases be too small to be readily observed. The importance of nonlinearity to these problems is not widely appreciated, so a case to illustrate salient features is provided here. Free-decay records were obtained with an oscillator in the form of a vertically oriented (hanging) tungsten wire, of length 24 cm and diameter 0.31 mm. It was clamped at the top end, and at the bottom a rectangular plate was attached that was 11.3 cm long, 1.3 cm wide, and 0.8 mm thick. The plate was cut from double-sided copper circuit board. The board was positioned between the stationary plates of a capacitive sensor, as shown in Figure 3.28.

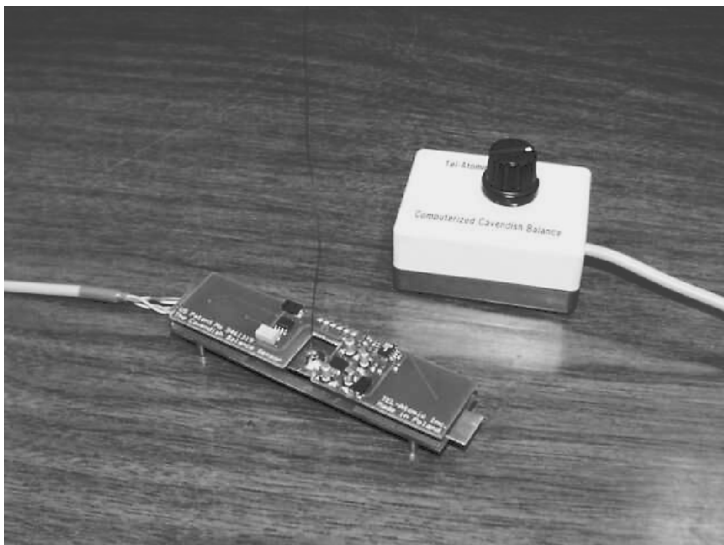


FIGURE 3.28 Photograph of the detector used to monitor the multimode oscillator.

For the picture, the apparatus was disassembled and the plate allowed to rest on the top of the bottom electrode set. Operationally, the plate was positioned midway between the upper and lower static electrode sets (separation distance of 4 mm); and there was no mechanical contact during oscillation. As can be seen, the top of the circuit board containing the upper electrode set contains more than a dozen electronic components; these are of the surface mount technology type. The detector is of the SDC type and this particular embodiment is manufactured in Poland for TEL-Atomic Inc., Jackson, MI, for use in the Computerized Cavendish Balance.

As can be seen in the picture, the wire was rather kinked instead of straight, which is expected to be a significant source of nonlinearity. For this reason, not to mention that it is very difficult to make larger diameter tungsten wires reasonably straight, no serious attempt was undertaken to remove the kinks.

3.7.2 Some Experimental Results

An example decay record generated with this apparatus is illustrated in Figure 3.29.

3.7.3 Short Time Fourier Transform

When multiple modes are present in a decay, as in Figure 3.29, it is not possible to readily estimate Q for all of the various modes using time data. The decays can be estimated using the FFT, in a technique called the short time Fourier transform, which is built-in to various software packages related to acquisition systems, such as LabVIEW (see Appendix 15A). With the versatile software supplied with the Dataq A/D converter, it is straightforward to employ an equivalent manual technique. Using the number of points to define the FFT a value (always a power of 2 total) that is substantially smaller than the number of points in the record, a manual scan is performed in which one simply increments from start to finish, calculating a separate FFT at each position in time along the way. As an illustration of this powerful tool, Figure 3.30 shows spectra corresponding to the start and the finish of the data in Figure 3.29.

All the modes decay in time, and the rate of decay is especially large for those modes that correspond to sum and difference frequencies of the primary modes at 1.19 and 2.19 Hz. Table 3.2 gives the spectral intensities in dB for the two times considered. Where the rows are blank for the end of record case, the values were insignificantly small.

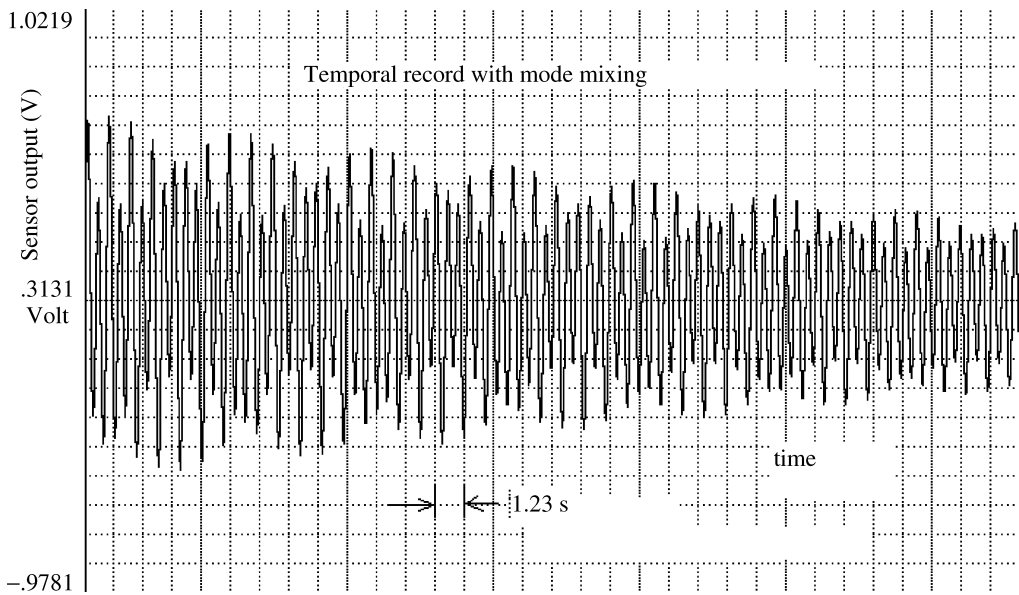


FIGURE 3.29 Example free-decay of a multimode wire oscillator.

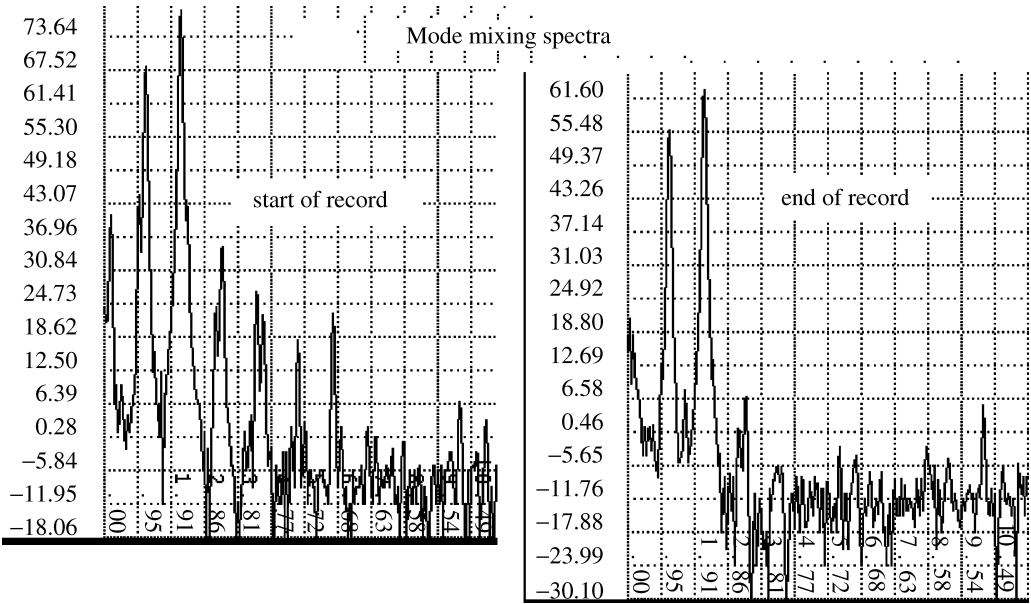


FIGURE 3.30 Beginning and end spectra corresponding to the temporal data from Figure 3.29. Ordinate values are spectral intensity in dB, abscissa values are frequency in Hz (linear scale).

The decibel values in the table are referenced to the bit-size (16 corresponding to 65536) of the ADC. In terms of the sensor output voltage, V , it is defined by Dataq as:

$$\text{dB} = 20 \log_{10}(32,768 \times V/\text{FS}) \tag{3.6}$$

where FS is the full-scale voltage as determined by the gain setting.

Elsewhere in this chapter, the decibel is calculated with a different reference. For example, for an FFT spectral line having real and imaginary components R and I , respectively (voltage based), the intensity in dB is calculated using

$$\text{dB} = 20 \log_{10} \left[\sqrt{R^2 + I^2} / \left(\frac{n}{2} \right) \right] \tag{3.7}$$

where n is the number of points in the FFT. This is convenient for determining noise levels. For example, from later graphs showing electronics noise, the floor of the SDC sensor is found to be of the order of -120 dB, corresponding to a microvolt. The position resolution defined by this noise level is about 500 nm, i.e., the wavelength of visible light.

TABLE 3.2 Spectral Intensities for Some of the Lines Shown in Figure 3.30

Frequency (Hz)	Start of Record (dB)	End of Record (dB)
2.19	78.3	63.0
1.19	68.1	55.6
1.00	44.6	
0.19	40.8	
3.38	35.0	6.7
4.34	26.7	
4.53	22.4	
6.53	22.9	
5.53	17.8	

Of the two primary modes of this kinked-wire case study, the higher frequency (2.19 Hz) is the twisting mode and the lower frequency (1.19 Hz) is the swinging mode. The swinging mode is a little higher frequency than that which would result if the wire were completely flexible, yielding a near simple pendulum (1.02 Hz for 24 cm length). The swinging mode is two dimensional (pendulum equivalent called conical), but the sensor only responds (first order) to motion perpendicular to the long axis of the electrodes. It should also be noted that this motion is attenuated, relative to the twisting response, because of the mechanical common-mode rejection feature discussed in Chapter 2.

The manual STFT was used on the data that generated Figure 3.29 to estimate the decay history of three different modes — both of the primary ones (twist and swing) and also the mode whose frequency is the difference between the frequencies of the primaries, i.e., 1 Hz. Figure 3.31 shows the results, where a Hanning window was used, and the total number of points in the record permitted five equally time-spaced FFTs, when working with a 1024 point transform.

Although the decay of the twisting mode is seen to be reasonably exponential, there was large beating between the modes (readily observed in Figure 3.29). Beating alone would not yield a mix signal whose frequency is 1.0 Hz. However, beating in a linear system can cause amplitude variations in the weaker swinging mode.

3.7.4 Nonlinear Effects — Mode Mixing

At least two signals in the spectra are the result of nonlinearity, i.e., the lines corresponding to the sum and difference of the frequencies of the primary pair — at 3.38 and 1.00 Hz, respectively. If the system of oscillator and detector were completely linear, then no such sum and difference cases would be possible. It is also to be noted that these mixtures are not the result of sensor nonlinearity, which as noted previously one must be careful to avoid.

The amplitude of a mix signal was expected to approximately obey the following relation:

A_m \propto A_1 A_2 (3.8)

To test this premise, the STFT was used to estimate the amplitudes of each of the three components indicated in Equation 3.8. The amplitudes were all normalized, relative to the starting value for each case, and the results used to generate the graphs in Figure 3.32.

The amplitude of oscillation for a given mode, at the time of the transform, is found by using the peak value in dB of the intensity of the spectral line for that mode, according to

A \propto 10^{dB/20} (3.9)

where the factor of 20 is used since the spectral intensities were calculated in terms of voltages. Although calibration constants (in V/m and V/rad) could be used to express the amplitude in meters or in radians, corresponding to the mode, nothing is gained by doing so for the present purposes.

The mixing index for these cases is defined by the expression

index = \frac{A_m}{\sqrt{A_1 A_2}} (3.10)

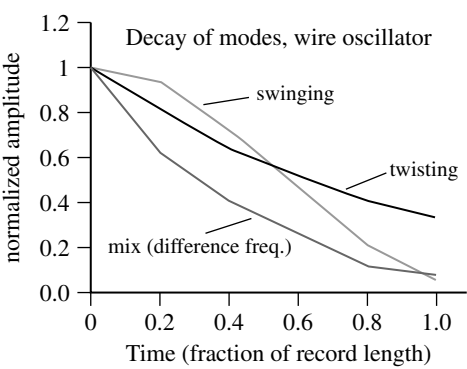


FIGURE 3.31 Decay of modes of the wire oscillator, determined by the manual STFT.

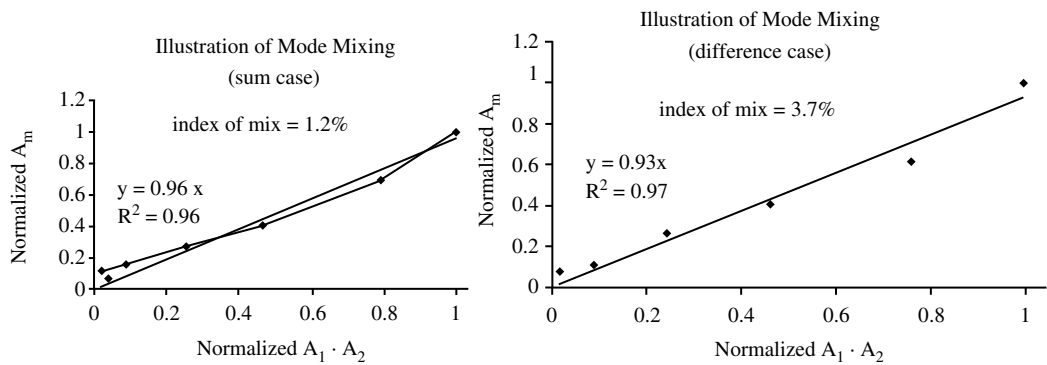


FIGURE 3.32 Evidence in support of nonlinear mixing according to Equation 3.8.

which is similar to expressions encountered in optics. It can be seen that the sum and difference frequencies are approximated reasonably well by theoretical expectation.

3.8 Internal Friction as Source of Mechanical Noise

Chapter 2 claims that internal friction is responsible not only for damping but also for significant mechanical noise of $1/f$ type. Figure 3.33 is provided in support of that claim.

The pendulum in these experiments (lead spheres near the ends of an aluminum tube with a pair of steel-points for the axis) was operated in a high vacuum to eliminate the influence of air. The electronics

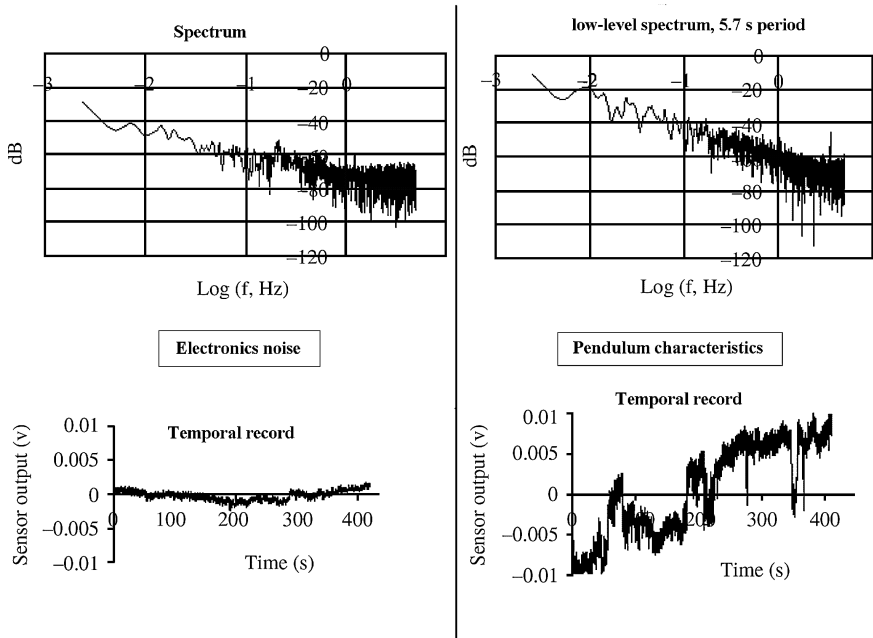


FIGURE 3.33 Power spectrum and associated temporal record showing mechanical $1/f$ noise (right pair). For reference, electronics noise is also provided (left pair).

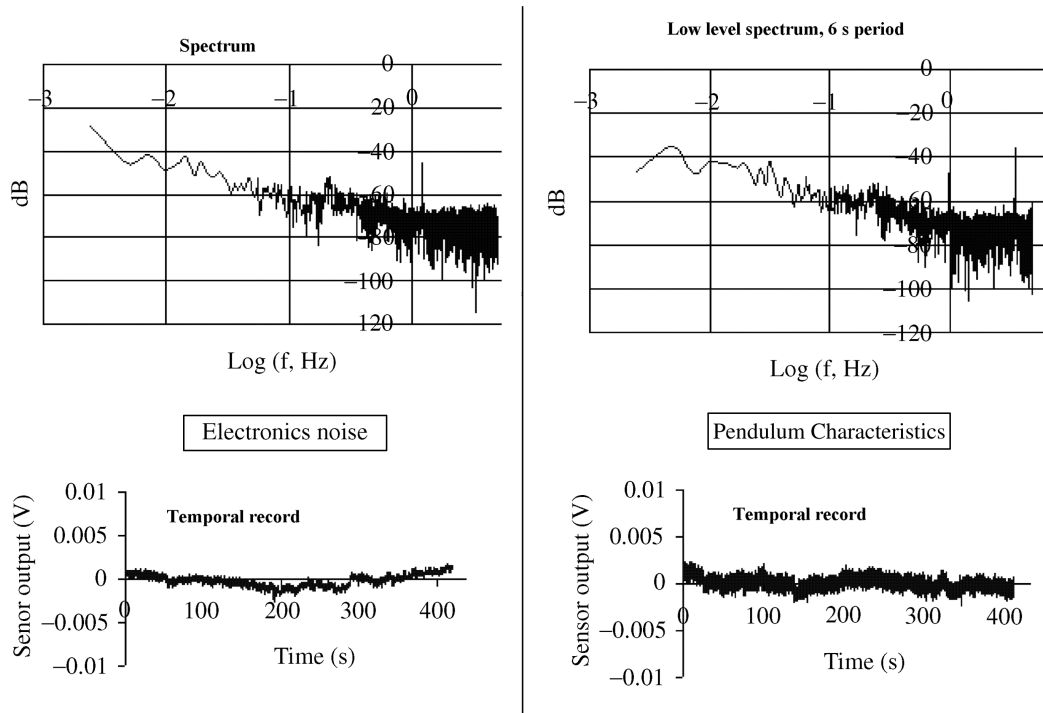


FIGURE 3.34 Same as Figure 3.33, but after the pendulum had stabilized.

noise was obtained by removing the top mass and measuring the motion after oscillation reached a minimum (frequency approximately 1 Hz); some pendulous mode remains because of pump noise transmitted through the vacuum hose. Similarly, the pump vibrations excite a vibratory mode in the long-period pendulum (sharp spectral line at 3.5 Hz, right spectrum). In this vibratory mode the lead masses move in the same direction relative to the stationary axis (similar to the bending mode of the carbon dioxide molecule). It is interesting to note that there is no coherent oscillation to be seen above noise corresponding to the period of 5.7 sec. The mechanical noise is seen to include bistability, which is not uncommon for this type of system before hardening, following a significant force disturbance. The data in Figure 3.33 were collected after replacing the upper mass, which had been removed to measure the electronics noise, and after pumping to the operating pressure (below 5 $\mu\text{m Hg}$).

The mechanical noise is seen to be $1/f$ for $f < 1.5$ Hz, which is where electronics begins to contribute noticeably. Everywhere below 1 Hz, the electronics noise is an order of magnitude smaller than the mechanical noise.

After the pendulum had stabilized overnight and been allowed to oscillate through a number of free-decays (initialization by tilting the chamber), the data shown in Figure 3.34 was collected.

It can be seen that the mechanical noise has mostly settled out, leaving the remnant electronics noise.

3.9 Viscous Damping — Need for Caution

Recent experiments have shown important subtleties of viscous damping (Peters, 2003a, 2003b, 2003c, 2003d). It is true that the dissipation at a specified frequency can be adequately modeled by simply multiplying the velocity term in the differential equation by a coefficient. It is not proper, however, to call

this coefficient a constant, since the damping coefficient is frequency dependent and also involves the density as well as the viscosity of the fluid in which oscillation takes place.

Some engineers have known about the history term, which is most simply treated in the case of a sphere executing simple harmonic motion. The friction force acting on the sphere in this case can be reasonably approximated by

$$f_{\text{harmonic}} = 6\pi\eta a \left(1 + C_H \frac{a}{\delta}\right) v, \quad \delta = \sqrt{\frac{2\eta}{\omega\rho}}, \quad (C_H \rightarrow 1 \text{ as } v \rightarrow 0) \quad (3.11)$$

where ω is the angular frequency of oscillation, a is the radius of the sphere; and for the fluid, η and ρ are its viscosity and density, respectively. Only in the limit of zero frequency does the damping reduce to the form that one expects on the basis of Stokes' law of viscous friction (steady flow).

Using Equation 3.11 in the equation of motion for a pendulum yields for the Q

$$Q_v = \frac{I\omega}{6\pi\eta a \left(1 + \frac{a}{\delta}\right) L^2}, \quad \delta = \sqrt{\frac{2\eta}{\omega\rho}} \quad (3.12)$$

where I is the moment of inertia, and L is the distance from the axis to the center of the sphere. Typically, the ratio a/δ is significantly greater than unity so that the damping is governed by the surface area of the sphere rather than by its radius. These complexities of viscous damping are summarized in Box 3.3.

Reasonable experimental validation of the estimate for Q was provided, as demonstrated in Figure 3.35.

The instrument in this case was a compound pendulum in which a mass was located above the axis of rotation, as well as the usual situation of mass below the axis. The water damping was provided through a small sphere at the bottom of the pendulum, immersed in water held by a rectangular container.

If the history term in Equation 3.12 is ignored there can be huge errors. For example, in the case of water damping, the damping can be underestimated by 1000 to 3000%, as shown in Figure 3.36.

At low frequencies, it is also important to correct for the influence of hysteretic damping of the pendulum. Figure 3.37 shows the large errors that occur when one fails to do so.

For some cases, buoyancy and added mass of the fluid are also quite significant to the frequency of oscillation, as shown in Figure 3.38.

Box 3.3

COMPLEXITIES OF VISCOUS DAMPING

Friction force is not a function only of viscosity η ; it also depends on density ρ and angular frequency ω

$$f_{\text{harmonic}} = 6\pi\eta a \left(1 + C_H \frac{a}{\delta}\right) v, \quad \delta = \sqrt{\frac{2\eta}{\omega\rho}}, \quad (C_H \rightarrow 1 \text{ as } v \rightarrow 0)$$

resulting in a complicated frequency dependence for the Q of viscous damping

$$Q_v = \frac{I\omega}{6\pi\eta a \left(1 + \frac{a}{\delta}\right) L^2}, \quad \delta = \sqrt{\frac{2\eta}{\omega\rho}}$$

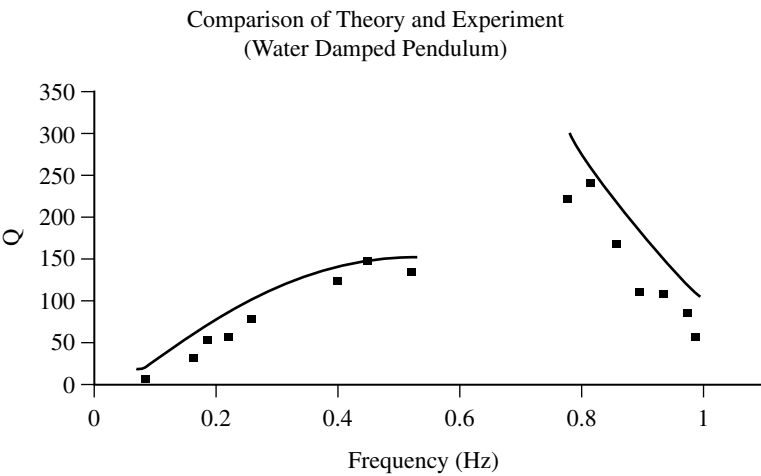


FIGURE 3.35 Comparison of theory and experiment for a pendulum damped by water.

3.10 Air Influence

As seen from Figure 3.37, low-frequency motions are likely to be influenced more by internal friction than by any fluids that interact with the oscillator. The most important fluid is of course air, and a true delineation between external and internal effects requires that the oscillator be studied in a high vacuum. It is not enough to just remove most of the air, since the viscosity of gases is surprisingly constant until the mean free path between collisions becomes a significant fraction of chamber dimensions.

Theoretically, it is possible to roughly estimate air influence, although only in the simplest of geometries, such as a sphere. In such cases, Equation 3.11 could be used (with accounts for the history term, using appropriate values for the viscosity and density). It is also possible in some cases to estimate air influence experimentally, as in the example that follows.

3.10.1 Brass and Solder Rod Pendula

Because of its malleability, the internal friction of solder (lead–tin alloy) is large, compared to that of much harder brass. A pendulum of each material was studied, both having a length of about 50 cm and a

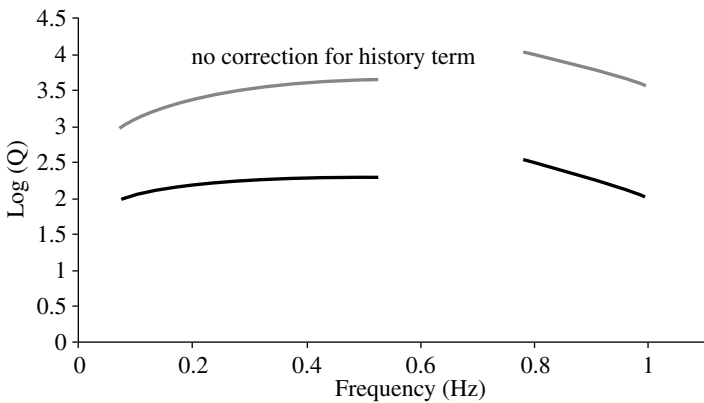


FIGURE 3.36 Illustration of how huge errors can occur in damping estimates if one ignores the history term.

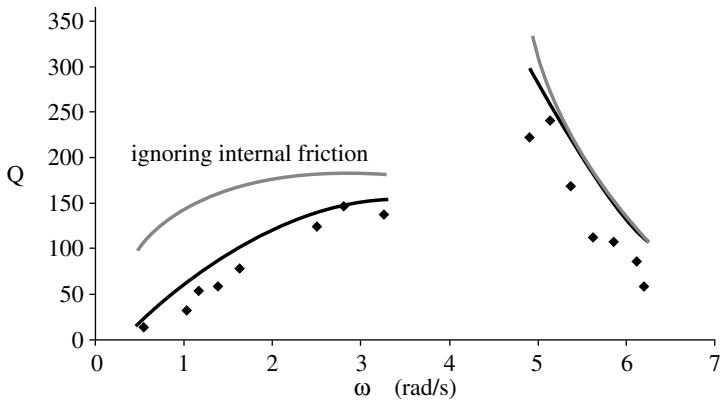


FIGURE 3.37 Illustration of significant low-frequency errors that result from a failure to recognize the hysteretic damping component of the pendulum.

diameter of about 3 mm. The technique used was the photogate method described in Section 3.4.4 (Case 4 above). Unlike the previous study, no lead masses were clamped on the rod — but it used the same adjustable knife-edge.

Figure 3.39 clearly shows that the internal friction for the solder pendulum is much greater than that of the brass pendulum.

A nonlinear fit was generated for each decay curve, from which the history of the quality factor was graphed as a function of velocity amplitude, as shown in Figure 3.40.

Consider the pair of brass curves in Figure 3.40. The large difference in Q at 10 cm/sec (387 compared to 266) is in stark contrast with their near equality at 50 cm/sec. This is primarily a consequence of air drag that is quadratic in the velocity at the larger amplitude. It is more important to brass than to solder because of the small internal friction of the brass.

From the large difference in internal friction of the two materials, a first order correction for air influence on the solder pendulum is to simply subtract $1/Q$ of the brass from $1/Q$ (raw data) of the solder, to yield the reciprocal Q (corrected) due to internal friction of the solder. This has been done in Figure 3.41.

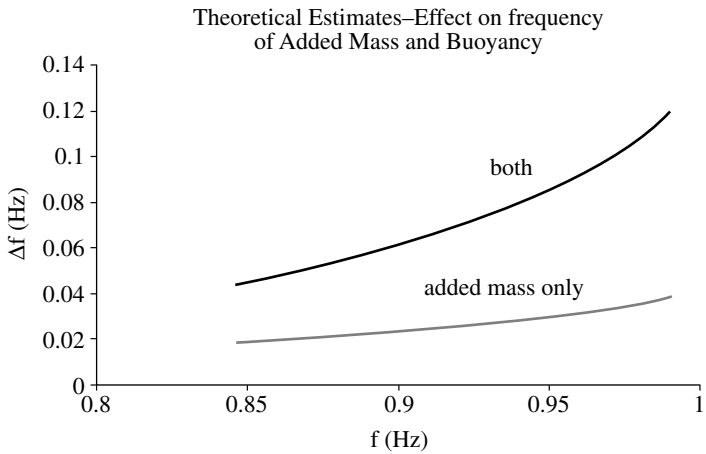


FIGURE 3.38 Example of how fluid properties influence the frequency as well as damping of an oscillator.

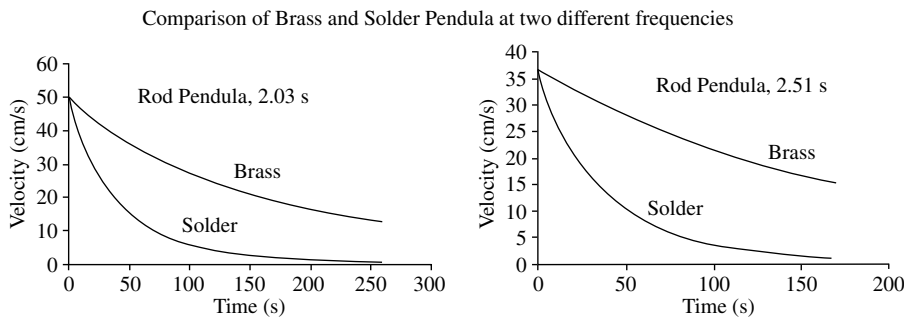


FIGURE 3.39 Free-decay curves for brass and solder pendula at two different frequencies, showing the larger internal friction of solder. The velocity is that of the peak value (amplitude) at the top of the pendulum, approx. 22 cm above the axis.

From [Figure 3.41](#) it can be seen that the internal friction damping is not simply hysteretic (constant Q); rather it is a function of amplitude. It can also be seen, from the close proximity of the solid and dashed curves, that the air influence on the solder pendulum is much less than that of the internal friction. By contrast, air influence is of comparable magnitude to the internal friction in the case of the brass pendulum (or even larger, at large amplitude).

A minimum of two frequencies was considered for the study, since the frequency variation of the damping is different for external and internal frictions. (Note: although the period is a function of amplitude, the amount of nonisochronism is small compared to the damping changes and is ignored here.) The periods were matched for the two pendula at each of 2.03 and 2.51 sec. For hysteretic-only (internal friction) damping, the Q at the shorter period should in theory be 1.53 times that of the longer period, for both brass and solder. If the damping were viscous only, the factor should be 1.24. In the case of solder at 10 cm/sec (corrected), the ratio is $1.66 = 131/71$, and for brass it is $1.46 = 387/266$. Although the ratio for solder is greater than the expected 1.53, the difference is within experimental

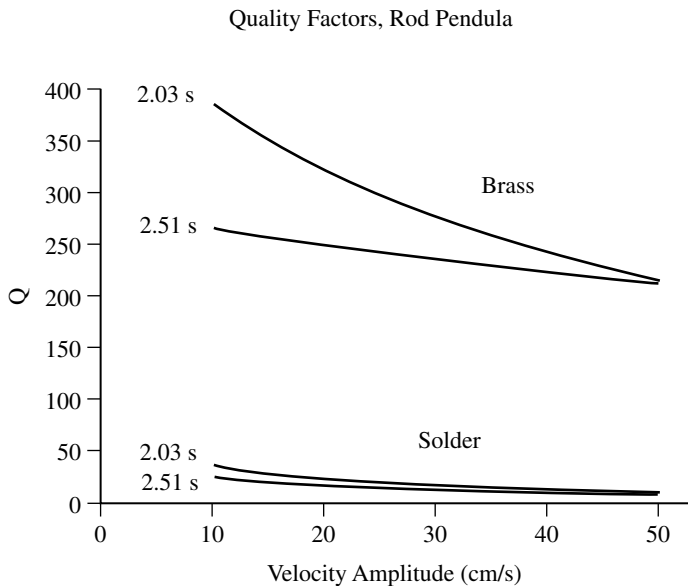


FIGURE 3.40 Illustration of amplitude-dependent damping in a rod pendulum made of (i) brass and (ii) solder. The two different matched periods of oscillation are indicated in seconds.

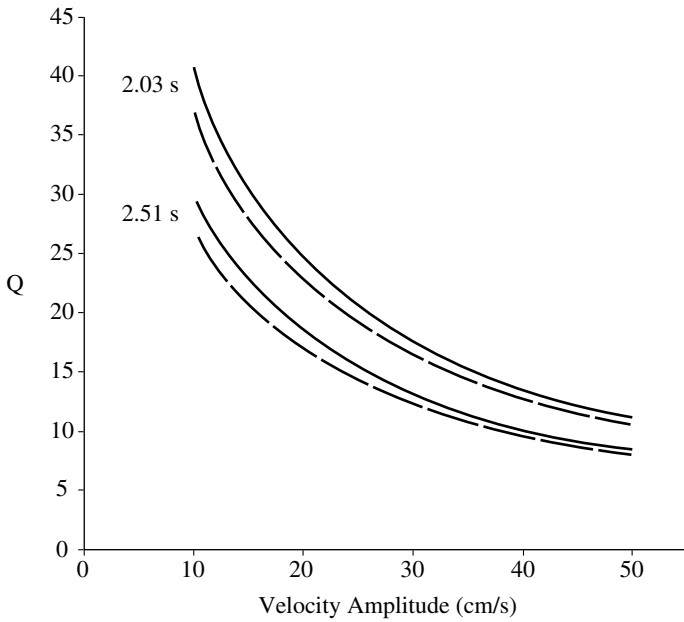


FIGURE 3.41 Amplitude dependence of the estimated quality factor due only to internal friction in the solder pendulum. Dashed lines show the Q before correction for air damping.

uncertainty for individual Q values, which from other, more detailed experiments were in the neighborhood of 5 to 10%.

The ratio for brass (1.46) is between 1.24 and 1.53, as expected, because of the comparable influence of air and internal friction.

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