

Structure and Equipment Isolation

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Summary

In this chapter, a brief review will be given of the concept of isolation for suppressing the vibrations in structures and equipment subjected either to harmonic or seismic ground excitations. The mechanism of various isolation devices, including the elastomeric bearing, sliding bearing, resilient-friction base isolator, and Electricite de France system, will first be described in Section 4.2, together with their mathematical models. In Section 4.3, a closed form solution will be derived for the dynamic response of a structure–equipment system isolated by bearings of the elastomeric type, subjected to harmonic motions. Such a solution enables us to interpret the various behaviors of the structure and equipment under excitation. The elastomeric bearings can help increase the fundamental period of the structure, thereby, reducing the accelerations transmitted to the superstructure. In Section 4.4 and Section 4.5, the seismic behavior of a structure–equipment system isolated by a sliding support, with and without resilient force, will be studied using a state-space incremental-integration approach. With the introduction of a frictional sliding interface, the motion of the structure–equipment system will be uncoupled from the ground excitation, and the influence of the latter will be mitigated. The residual base displacement caused by the sliding isolator can be reduced

through inclusion of a resilient mechanism in the isolator. Nevertheless, the resilient mechanism can make the system more sensitive to the low-frequency components of excitation. In Section 4.6, issues related to design of base isolators will be discussed, along with the concepts underlying some design codes and guidelines. The notation used is listed at the end of the chapter.

4.1 Introduction

Conventionally, structural designers are concerned about the safety of buildings, bridges, and other civil engineering structures that are subjected to earthquakes. The recent history of earthquakes reveals that strong earthquakes, such as the 1994 Northridge earthquake (U.S.A.), 1995 Kobe earthquake (Japan), and 1999 Chi-Chi earthquake (Taiwan), can cause some badly designed structures or buildings to fail or collapse, and also cause some well-designed structures to malfunction due to the damage or failure of the equipment housed in the structure or building. Both the failures of structures and equipment, also known as *structural* and *nonstructural* failures, respectively, can cause serious harm to the residents or personnel working in a building. For the case where the equipment is part of a key service system, such as in hospitals, power stations, telecommunication centers, high-precision factories, and the like, the lives and economic losses resulting from the malfunctioning of the equipment can be tremendous. Thus, the maintenance of the safety of structures and attached equipment during a strong earthquake is a subject of high interest in earthquake engineering. In this regard, *base isolation* has been proved to be an effective means for protecting the structures and attached equipment, which is made possible through reduction of the seismic forces transmitted from the ground to the superstructure (Yang et al., 2002).

For light secondary systems mounted on heavier primary systems, it was concluded that the response of the light secondary system, that is, the equipment, is affected by four major dynamic characteristics in earthquakes (Igusa and Der Kiureghian, 1985a, 1985b, 1985c; Yang and Huang, 1993). The first issue is *tuning*, which means that the natural frequency of the equipment is coincident with that of the structure. Such an effect may amplify the response of the equipment due to the fact that the light secondary system behaves as if it were a vibration absorber of the heavier primary system. The second issue is *interaction*, which is related to the feedback effect between the motions of the primary and secondary systems. Ignoring the feedback effect of interaction may result in an overestimation of the true response of the combined system. The third issue is *non-classical damping*, which may occur when the damping properties of the two systems are drastically different, such that the natural frequencies and mode shapes of the combined system can only be expressed in terms of complex numbers. Under such a circumstance, the conventional response spectrum analysis, based on modal superposition, becomes inapplicable. The last issue is *spatial coupling*, which relates to the effect of multiple support motions when the secondary system of interest is mounted at multiple locations. By considering the inelastic effect, Igusa (1990) proposed an equivalent linearization technique for investigating the response characteristics of an inelastic primary–secondary system with two degrees of freedom (DoF) under random vibrations. His results indicated that the existence of small nonlinearity is helpful for reducing the coupling system responses. With the concept of equivalent linearization, Huang et al. (1994) explored the response and reliability of a linear secondary system mounted on a yielding primary structure under white-noise excitations. It was concluded that the response of the secondary system could be reduced by increasing the equipment damping or by locating equipment at higher levels of the primary structure.

Owing to the fact that the mass and stiffness of a secondary system are much smaller than those of the primary structure, the interaction effect of the combined system, as well as the ill-conditioning in system matrices, may take place when one performs the dynamic analysis. To deal with this problem, some researchers chose to evaluate the response of the secondary systems from the floor motions. To avoid solving large eigenvalue problems and to account for the interaction between the building and equipment components, Villaverde (1986) applied the response spectrum technique to the analysis of a combined building–equipment system, by which the maximum response of light equipment mounted on the building under the earthquake is expressed in terms of the natural frequencies and mode shapes of

the building and equipment. To take into account the equipment–structure interactions, Suarez and Singh (1989) proposed an analytical scheme for computing the dynamic characteristics of the combined system, using the modal properties to compute the floor spectra. Lai and Soong (1990) presented a statistical energy analysis technique for evaluating the response of coupling primary–secondary structural systems, based on the concept of power-balance equation, that is, the power input to the primary system is equal to the dissipated energy of the primary system plus the transferred energy to the secondary system. Using a mean-square condensation procedure, Chen and Soong (1994) considered the effect of interaction by calculating the multi-DoF response of a primary–secondary system under random excitations. Later on, Chen and Soong (1996) derived an exact solution for the mean-square response of a structure–equipment system under dynamic loads, indicating that there exists an optimal damping ratio for reducing the vibration of equipment attached to the primary structure. Gupta and coworkers investigated the response of a secondary system with multiple supports on a primary structure subjected to earthquakes, taking into account the interaction effect between the equipment and structure (Dey and Gupta, 1998, 1999; Chaudhuri and Gupta, 2002). Their results indicated that when the soil–structure interaction (SSI) is taken into account, the response of the equipment–structure system will be affected by the SSI, unless a very stiff soil condition is considered.

On the other hand, a number of research works have been conducted by implementing isolation systems at the base of the equipment–structure system, aiming to reduce the earthquake forces transmitted from the ground. Based on a theoretical and experimental investigation, Kelly and Tsai (1985) showed that seismic protection can be achieved effectively for lightweight equipment mounted on an isolated structure installed with elastic bearings at the base. A hybrid isolation system with base-isolated floors was proposed by Inaudi and Kelly (1993), for the protection of highly sensitive devices mounted on a structure subjected to support motions. Considering the effects of torsion and translation, Yang and Huang (1998) studied the seismic response of light equipment items mounted on torsional buildings supported by elastic bearings. Their results indicated that the response of an equipment–structure system can be effectively reduced through installation of base isolators, and that there exists an optimal location for mounting the equipment. Juhn et al. (1992) presented a series of experimental results for the secondary systems mounted on a sliding base-isolated structure. They concluded that the acceleration response of the secondary system may be amplified when the input motions are composed of low-frequency vibrations. In this case, the sliding bearings are not considered to be an effective isolation device, which implies that the base-isolated structure is not suitable for a construction site with soft soil.

Concerning the use of sliding bearings (supports) as base isolators, Lu and Yang (1997) investigated the response of an equipment item attached to a sliding primary structure under earthquake excitations. Their results showed that the response of the equipment can be effectively reduced through the installation of a sliding support at the structural base, in comparison with that of a structure with fixed base. To overcome the discontinuous nature of the sliding and nonsliding phases of a structural system with sliding base, a fictitious spring model was proposed by Yang and coworkers for simulating the mechanism of sliding and nonsliding (Yang et al., 1990, 2000; Yang and Chen, 1999). Such a model will be described in a later section of this chapter. Agrawal (2000) adopted the same fictitious spring model in studying the response of an equipment item mounted on a torsionally coupled structure with sliding support. His results indicated that sliding supports could effectively reduce the equipment response, compared to that of a fixed-base structure. However, in the tuning region, where the natural frequency of the equipment coincides with the fundamental frequency of the structure, the equipment response may be adversely amplified due to the increase in eccentricity of the torsionally coupled structure.

The problem of building isolation has recently received more attention than ever from researchers and engineers, due to the construction of high-precision factories worldwide. More and more stringent requirements have been employed in this regard for removing the ambient or man-made vibrations (Rivin, 1995; Steinberg, 2000). To allow sensitive electronic equipment to operate in a harsh environment, Veprik and Babitsky (2000) proposed an optimization procedure for the design of vibration isolators aimed at minimizing the response of the internal components of electronic equipment. As for the protection of high-tech equipment from micro- or ambient

vibrations, Yang and Agrawal (2000) showed that passive hybrid floor isolation systems are more effective in mitigating the equipment response than passive or hybrid base isolation systems. Xu and coworkers studied the response of a batch of high-tech equipment mounted on a hybrid platform, which in turn is mounted on a building floor (Xu et al., 2003; Yang et al., 2003). Both their theoretical and experimental studies showed that the hybrid platform, which is composed of leaf springs, oil dampers, and an electron-magnetic actuator with velocity feedback control, is more effective in mitigating the velocity response of the high-tech equipment than the passive platform.

The objective of this chapter is to give an overview on the seismic behavior of various base isolators. The organization of this chapter can be summarized as follows. In Section 4.2, the mechanisms of various seismic isolators that are currently in use are introduced and explained. In Section 4.3, a structure–equipment system isolated by bearings of the elastomeric type is modeled by a three-DoF system composed of a spring and dashpot unit, for which a closed-form solution is obtained for the dynamic response of the isolated system subjected to harmonic earthquakes; remarks on the dynamic response of the system components are also made. In Section 4.4 and Section 4.5, the seismic behaviors of a structure–equipment system isolated by a sliding support, with and without resilient capability, will be investigated. Also presented are numerical methods based on the incremental-integration procedure for the analysis of structural systems with sliding-type isolators.

4.2 Mechanisms of Base-Isolated Systems

Figure 4.1 shows a simplified model for a structural system subjected to a support motion.

For this single-DoF system, the equation of motion can be written as

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{x}_g \quad (4.1)$$

where m denotes the mass, c the damping, k the stiffness, x the displacement of the system, and $\ddot{x}_g$ the ground acceleration. By assuming the system to be linearly elastic, the response $x(t)$ can be obtained using Duhamel's integral, as

$$x(t) = \frac{1}{\Omega_d} \int_0^t \ddot{x}_g(\tau) e^{-\zeta\Omega(t-\tau)} \sin \Omega_d(t-\tau) d\tau \quad (4.2)$$

where the natural angular frequency, Ω , damped natural frequency, Ω_d , and damping ratio, ζ , of the system are defined as follows:

$$\Omega = \sqrt{\frac{k}{m}} \quad (4.3a)$$

$$\Omega_d = \Omega \sqrt{1 - \zeta^2} \quad (4.3b)$$

$$\zeta = \frac{c}{2m\Omega} \quad (4.3c)$$

Correspondingly, the natural period, T , and damped period, T_d , of the structure are

$$T = \frac{2\pi}{\Omega} = 2\pi \sqrt{\frac{m}{k}} \quad (4.4a)$$

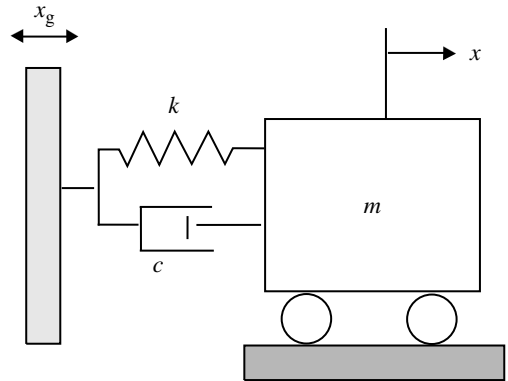


FIGURE 4.1 Model of a single-DoF system.

$$T_d = \frac{2\pi}{\Omega_d} = \frac{T}{\sqrt{1 - \zeta^2}} \quad (4.4b)$$

For a given support acceleration, $\ddot{x}_g$, the displacement, x , and acceleration, $\ddot{x}$, of the single-DoF system can be related to the natural period, T , and damping ratio, ζ , of the system. Thus, for a specific earthquake, by first selecting a damping ratio, ζ , and using Equation 4.2, one can compute the peak displacement x , for a structure with a period of vibration, T , with given values of m , c , and k . Repeating the above procedure for a wide range of periods, T , while keeping the damping ratio, ζ , constant, one can obtain response curves similar to those shown in Figure 4.2. By varying the damping ratio, ζ , one can construct the *displacement response spectra* and *pseudo-acceleration response spectra* for all single-DoF structures under a given earthquake, as schematically shown in Figures 4.2 and 4.3, respectively.

A general impression that is gained from Figure 4.2 and Figure 4.3 is that a structure with a shorter natural period has less displacement response when subjected to an earthquake, but it also has a larger acceleration response. Specifically, let us consider a structure of a constant damping ratio, ζ , with its period increased from T_1 to T_2 . As can be observed from the figures, the displacement of the structure increases from D_1 to D_2 , while the acceleration decreases from A_1 to A_2 . Such a feature is known as the *period shift* effect. On the other hand, by increasing damping ratio of the structure, the displacement of the structure decreases significantly, as can be seen from Figure 4.2. The same is also true with the acceleration response, as can be seen from Figure 4.3. Moreover, a larger damping ratio also makes the structure less sensitive to the variation in ground vibration characteristics, as indicated by the smoother response curves for structures having higher damping ratios, in both figures. From the aforementioned two response spectra, one observes that the philosophy of base isolation is to lengthen the vibration period of the structure to be protected, using base isolators of some kind, by which the earthquake force transmitted to the structure can be greatly reduced. In the meantime, some additional damping must be introduced on the base isolators in order to control the relative displacements across the base isolators with tolerable limits.

To fulfill the function of lengthening the period of vibration of the structure to be protected, the base isolators that are inserted between the structure and its foundation must be flexible in the horizontal direction, but stiff enough in the vertical direction so as to carry the heavy loads transmitted from the superstructure. With such devices, the natural period of vibration of the structure will be significantly lengthened and shifted away from the dominant frequency range of the expected earthquakes. The following is a summary of the fundamental features of four types of isolators frequently used in engineering practice.

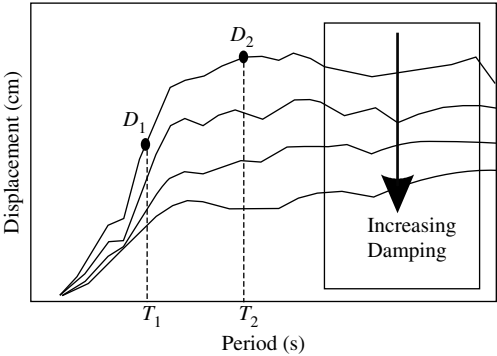


FIGURE 4.2 Schematic of displacement response spectra.

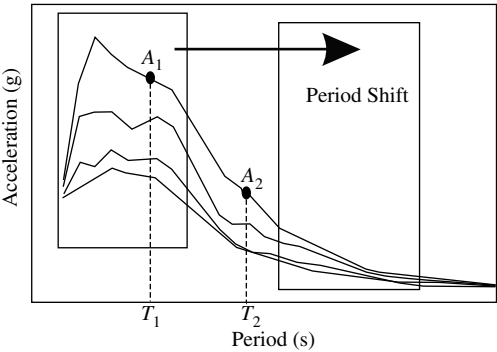


FIGURE 4.3 Schematic of pseudo-acceleration response spectra.

4.2.1 Elastomeric Isolation System

Elastomeric bearing is the type of base isolator most commonly known to researchers and engineers working on base isolation. It is usually composed of alternating layers of steel and hard rubber and, for this reason, it is also known as the *laminated rubber bearing*. This type of bearing is stiff enough to sustain the vertical loads, yet flexible under the lateral forces. The ability to deform horizontally enables the bearing to reduce significantly the structural base shear transmitted from the ground. While the major function of elastomeric bearings is to reduce the transmission of shear forces to the superstructure by lengthening the vibration period of the entire system, they must also provide sufficient rigidity under vertical loads. Let us consider a structure installed with elastomeric bearings, which is subjected to a support acceleration, $\ddot{x}_g$, as in Figure 4.4. By representing the isolated structure as a single-DoF system, based on the assumption that the superstructure is rigid in comparison with the stiffness of the elastic bearings, the equation of motion for the entire system can be written as

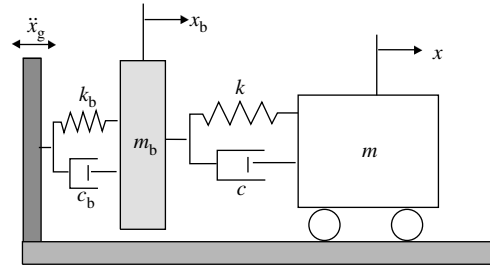


FIGURE 4.4 Model for base-isolation systems with elastic bearing.

$$\begin{bmatrix} m & 0 \\ 0 & m_b \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{x}_b \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c + c_b \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{x}_b \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k + k_b \end{bmatrix} \begin{Bmatrix} x \\ x_b \end{Bmatrix} = - \begin{Bmatrix} m\ddot{x}_g \\ m_b\ddot{x}_g \end{Bmatrix} \quad (4.5)$$

where m , c , and k denote the mass, damping, and stiffness of the superstructure, respectively, m_b , c_b , and k_b denote the mass, damping, and stiffness of the base raft, respectively, and x and x_b denote the displacements of the superstructure and the base, respectively.

In reality, the reduction in the seismic forces transmitted to a superstructure through the installation of laminated rubber bearings is achieved at the expense of large relative displacements across the bearings. If substantial damping can be introduced into the bearings or the isolation system, then the problem of large displacements can be alleviated. It is for this reason that the laminated rubber bearing with a central lead plug inserted has been devised (Yang et al., 2002). To simulate the dynamic properties of the lead-rubber bearing (LRB) system, an equivalent linearized system has been proposed, for which the equation of motion is

$$\begin{bmatrix} m & 0 \\ 0 & m_b \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{x}_b \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c + c_{eq} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{x}_b \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k + k_{eq} \end{bmatrix} \begin{Bmatrix} x \\ x_b \end{Bmatrix} = - \begin{Bmatrix} m\ddot{x}_g \\ m_b\ddot{x}_g \end{Bmatrix} \quad (4.6)$$

where c_{eq} and k_{eq} respectively represent the equivalent linearized damping and stiffness coefficients of the LRB system. The dynamic behavior of a structure-equipment system isolated by elastomeric bearings with linearized damping and stiffness coefficients, when subjected to harmonic and earthquake excitations, will be investigated analytically and numerically, respectively, in Section 4.3.

4.2.2 Sliding Isolation System

Another means for increasing the horizontal flexibility of a base-isolated structure is to insert a *sliding* or *friction surface* between the foundation and the base of the structure. The shear force transmitted to the superstructure through the sliding interface is limited by the static frictional force, which equals the product of the coefficient of friction and the weight of the superstructure. The coefficient of friction is usually kept as low as is practical. However, it must be high enough to provide a frictional force that can sustain strong winds and minor earthquakes without sliding. Since the sliding system has no dominant

natural period, it is generally frequency-independent when the structure is subjected to earthquakes with a wideband frequency content. As mentioned previously, when a sliding structure is subjected to a ground motion, transitions may occur repeatedly between the sliding and nonsliding phases. To take into account such a phase transition, Yang et al. (1990) proposed the use of a fictitious spring between the structural base raft and the underlying ground to simulate the static–dynamic frictional force of the sliding device. With reference to Figure 4.5, the equation of motion for the structure with sliding base can be written as follows:

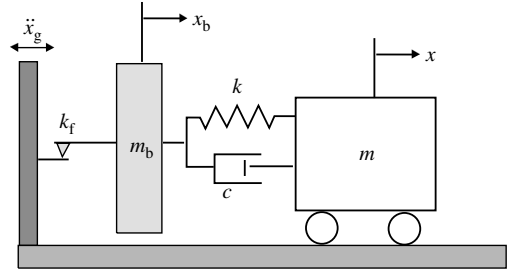


FIGURE 4.5 Model for base-isolation systems with sliding support.

$$\begin{bmatrix} m & 0 \\ 0 & m_b \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{x}_b \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{x}_b \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x \\ x_b \end{Bmatrix} + \begin{Bmatrix} 0 \\ f_r \end{Bmatrix} = - \begin{Bmatrix} m\ddot{x}_g \\ m_b\ddot{x}_g \end{Bmatrix} \quad (4.7)$$

where k_f is the stiffness of the fictitious spring and the frictional force, f_r , can be represented as

$$f_r = \begin{cases} k_f(x_b - x_{b0}) & \text{for non-sliding phase,} \\ \pm \mu(m + m_b)g & \text{for sliding phase} \end{cases} \quad (4.8)$$

with x_{b0} indicating the initial elongation of the fictitious spring in the current nonsliding phase, μ the coefficient of friction, and g the acceleration of gravity. The fictitious spring concept will be incorporated in the analysis of sliding structures in Section 4.4 of this chapter, when considering both harmonic and seismic excitations.

4.2.3 Sliding Isolation System with Resilient Mechanism

One particular problem with a sliding structure is the occurrence of residual displacements after earthquakes. To remedy such a drawback, the sliding surface is often made concave, so as to provide a recentering mechanism for the isolated structures. This is the idea behind the *friction pendulum system* (FPS), shown in Figure 4.6, which utilizes a spherical concave surface to produce a recentering force for the superstructure under excitations. To guarantee that a sliding structure can return to its original position, other mechanisms, such as high-tension springs and elastomeric bearings, can be used as an auxiliary system for providing the restoring forces. Previously, the sliding isolation systems have been successfully applied in the protection of important structures, such as nuclear power plants, emergency fire water tanks, large chemical storage tanks, and so on, from the damaging actions of severe earthquakes.

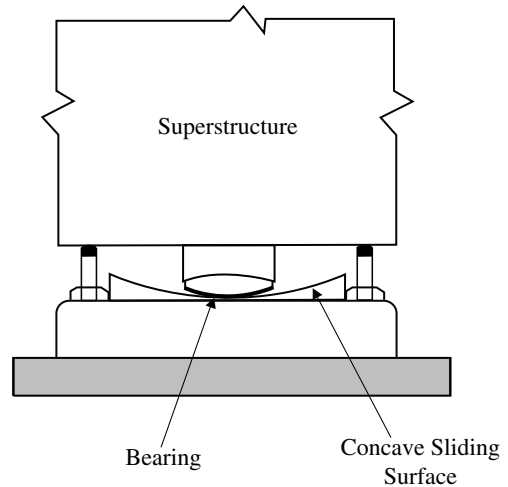


FIGURE 4.6 Friction pendulum system.

To improve the performance of sliding isolators under strong earthquakes, Mostaghel (1984) and Mostaghel and Khodavardian (1987) proposed the *resilient-friction base isolator* (RFBI) for

controlling the transmission of shear force to the superstructures, while keeping the residual displacements within an allowable level. The RFBI device is basically made of a central rubber core and Teflon-coated steel plates, and offers a friction resistance for keeping the system in the nonsliding mode under wind excitations and small earthquakes, and a restoring force by the rubber ingredient for limiting the maximum sliding displacements. The equation of motion for a structure installed with RFBI, as shown in Figure 4.7, can be written as

$$\begin{bmatrix} m & 0 \\ 0 & m_b \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{x}_b \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c + c_b \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{x}_b \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k + k_b \end{bmatrix} \begin{Bmatrix} x \\ x_b \end{Bmatrix} + \begin{Bmatrix} 0 \\ f_r \end{Bmatrix} = - \begin{Bmatrix} m\ddot{x}_g \\ m_b\ddot{x}_g \end{Bmatrix} \quad (4.9)$$

The interfacial frictional force, f_r , existing in the RFBI and appearing in Equation 4.9 serves as the outlet for energy dissipation. The behavior of a structure–equipment system supported by sliding isolators with resilient mechanism subjected to both harmonic and earthquake excitations will be investigated in Section 4.5.

4.2.4 Electricite de France System

To limit effectively the acceleration of base-isolated structures and internal secondary systems, such as those of nuclear power plants, when subjected to strong earthquakes, the *Electricite de France* (EDF) system was proposed by Gueraud et al. (1985). The design concept of an EDF system is to arrange the elastomeric bearing and sliding device at the base of a structure in series. For low-level ground motions, the EDF system will behave as an elastomeric bearing and return to the original position after support motions, while for strong earthquakes, the EDF system will behave as a sliding device. The EDF system may have a residual displacement after some major earthquakes. Because of the sliding mechanism of the EDF system, the maximum horizontal acceleration of the superstructure is kept within a certain range (Gueraud et al., 1985; Park et al., 2002), while the shear force transmitted to the superstructure through the frictional interface is smaller than the static frictional force. For the mathematical model shown for the EDF system in Figure 4.8, the equations of motion for the nonsliding and sliding phases can be written as

(a) nonsliding phase:

$$\begin{Bmatrix} m\ddot{x} \\ m_b\ddot{x}_b \\ 0 \end{Bmatrix} + \begin{bmatrix} c & -c & 0 \\ -c & c & 0 \\ 0 & 0 & c_{EDF} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{x}_b \\ \dot{x}_{EDF} \end{Bmatrix} + \begin{bmatrix} k & -k & 0 \\ -k & k + k_f & -k_f \\ 0 & -k_f & k_f + k_{EDF} \end{bmatrix} \begin{Bmatrix} x \\ x_b \\ x_{EDF} \end{Bmatrix} = \begin{Bmatrix} -m\ddot{x}_g \\ -m_b\ddot{x}_g \\ 0 \end{Bmatrix} \quad (4.10a)$$

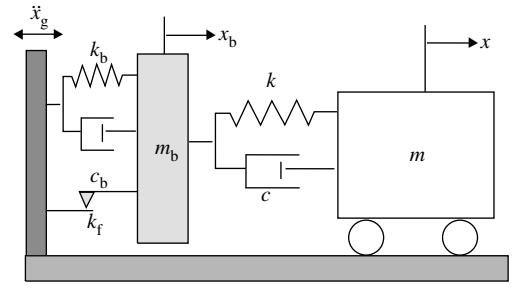


FIGURE 4.7 Model for base-isolation systems with RFBI device.

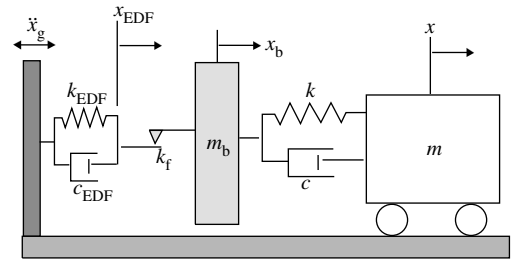


FIGURE 4.8 Model for base-isolation systems with EDF device.

(b) sliding phase:

$$\begin{aligned} \begin{Bmatrix} m\ddot{x} \\ m_b\ddot{x}_b \\ 0 \end{Bmatrix} + \begin{bmatrix} c & -c & 0 \\ -c & c & 0 \\ 0 & 0 & c_{\text{EDF}} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{x}_b \\ \dot{x}_{\text{EDF}} \end{Bmatrix} + \begin{bmatrix} k & -k & 0 \\ -k & k & 0 \\ 0 & 0 & k_{\text{EDF}} \end{bmatrix} \begin{Bmatrix} x \\ x_b \\ x_{\text{EDF}} \end{Bmatrix} \\ = \begin{Bmatrix} -m\ddot{x}_g \\ -m_b\ddot{x}_g \mp \mu(m + m_b)g \\ \pm \mu(m + m_b)g \end{Bmatrix} \end{aligned} \quad (4.10b)$$

where c_{EDF} and k_{EDF} , respectively, denote the damping and stiffness of the EDF system, and x_{EDF} denotes the displacement of the system.

4.2.5 Concluding Remarks

To mitigate the transmission of earthquake forces to a structure, and the potentially earthquake-induced damage to the equipment attached to the structure, base isolation is an effective structural design philosophy. With the installation of base isolators, the natural period of vibration of the structure will be significantly lengthened and shifted away from the dominant frequency range of the expected earthquakes. In accordance, the earthquake force transmitted to the structure can be significantly reduced. In this section, the mechanisms of four types of base isolator frequently used in engineering practice are introduced. Since the base isolators, such as the elastomeric bearings or sliding isolations, have relatively flexible stiffness in the horizontal direction, the occurrence of residual displacements after earthquakes may cause certain problems on the structure to be protected. To remedy such a drawback and to further guarantee that a base-isolated structure can return to its original position, the RFBI is implemented for controlling the transmission of shear force to the superstructure, while keeping the residual displacement within an allowable level. On the other hand, to limit the acceleration level of internal secondary systems housed in a base-isolated structure under strong earthquakes, such as those of the nuclear power plants, the EDF system can be used as an alternative device for base isolation, even though some residual displacements may be induced after the earthquakes.

4.3 Structure–Equipment Systems with Elastomeric Bearings

Owing to the stringent requirements for normal functioning of high-tech facilities, such as printed circuit boards, semiconductor factories, and sensitive medical devices, the need to suppress excessive vibrations in sensitive structure–equipment systems has become an issue of great concern to structural designers. Besides, these high-tech facilities may suffer significant damages during a major earthquake. Using elastomeric isolation systems to reduce the earthquake forces transmitted from the ground is one of the most popular ways adopted by structural designers. In this section, the performance of elastomeric bearings in protecting structure–equipment systems against horizontal ground motions will be investigated.

4.3.1 Formulation of Base Isolation Systems with Elastic Bearing

By modeling the structure, internal equipment and the base of an isolated structure–equipment system as a lumped mass system, one can construct the mathematical model for the structure–equipment isolation system supported by an elastic bearing in [Figure 4.9](#). The following is the

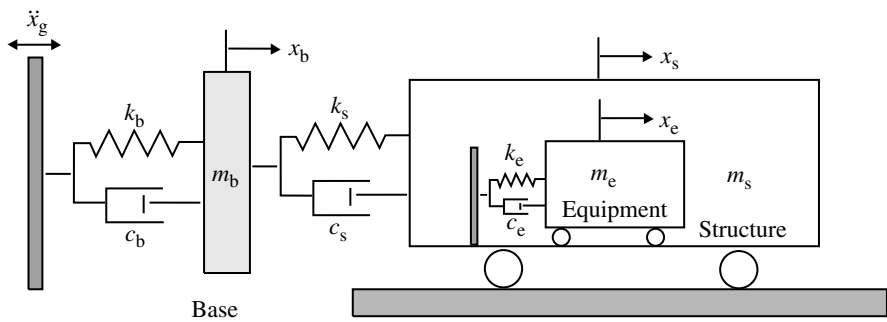


FIGURE 4.9 Model of a structure–equipment isolation system with elastic bearing.

equation of motion for the base-isolated structure–equipment system when it is subjected to a ground acceleration, $\ddot{x}_g$:

$$\begin{aligned} &\begin{Bmatrix} m_e \ddot{x}_e \\ m_s \ddot{x}_s \\ m_b \ddot{x}_b \end{Bmatrix} + \begin{bmatrix} c_e & -c_e & 0 \\ -c_e & c_s + c_e & -c_s \\ 0 & -c_s & c_s + c_b \end{bmatrix} \begin{Bmatrix} \dot{x}_e \\ \dot{x}_s \\ \dot{x}_b \end{Bmatrix} + \begin{bmatrix} k_e & -k_e & 0 \\ -k_e & k_s + k_e & -k_s \\ 0 & -k_s & k_s + k_b \end{bmatrix} \begin{Bmatrix} x_e \\ x_s \\ x_b \end{Bmatrix} \\ &= - \begin{Bmatrix} m_e \\ m_s \\ m_b \end{Bmatrix} \ddot{x}_g \end{aligned} \tag{4.11}$$

where m represents the mass, c the damping coefficient, and k the stiffness of the system. Also, the subscripts ‘e’, ‘s’, and ‘b’ are associated with the DoF of the equipment, structure, and base, respectively. The notations employed in Figure 4.9 have been defined in Table 4.1. It should be mentioned that the elastic bearing stiffness, k_b , appearing in Equation 4.11, is a parameter relating to the boundary conditions of the system considered here. A small value of k_b relative to the structural stiffness, k_s , means that the system is isolated by a set of soft bearings. In contrast, a large value of k_b means that the structure is rigidly supported.

TABLE 4.1 Definition of Symbols

Symbol	Definition
c_e, c_s	Damping coefficients of equipment and superstructure
k_e, k_s	Stiffness of equipment and superstructure
k_b	Stiffness of elastic bearing or resilient stiffness of isolation system
m_e, m_s, m_b	Masses of equipment, superstructure and base mat
$x_e(t), x_s(t), x_b(t)$	Relative-to-the-ground displacements of equipment, superstructure and base mat
$\ddot{x}_g(t)$	Ground acceleration
μ	Frictional coefficient of sliding isolation system
ω_b	Frequency of isolation system
ω_e, ω_s	Frequencies of equipment and superstructure
ω_g	Frequency of ground excitation
ζ_e, ζ_s	Damping ratios of equipment and structure

4.3.2 Free Vibration Analysis

By neglecting the damping and forcing terms in Equation 4.11, the equation of motion for free vibration can be written as

$$\begin{Bmatrix} m_e \ddot{x}_e \\ m_s \ddot{x}_s \\ m_b \ddot{x}_b \end{Bmatrix} + \begin{bmatrix} k_e & -k_e & 0 \\ -k_e & k_s + k_e & -k_s \\ 0 & -k_s & k_s + k_b \end{bmatrix} \begin{Bmatrix} x_e \\ x_s \\ x_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (4.12)$$

By solving the preceding equation, one can obtain the natural frequencies and vibration modes of the structure–equipment system with elastic bearings. As for the present problem, the horizontal stiffness of the elastic bearing is designed to be quite low compared with that of the superstructure. It follows that the superstructure in its entirety behaves essentially as a *rigid body* for the fundamental vibration mode shape of the combined system, which implies that the displacement responses for the equipment, structure, and base under free vibration can be approximately taken as the same, that is, $x_e = x_s = x_b = x$. By introducing such a condition into Equation 4.12, the equation of motion for the equivalent single-DoF base-isolated system can be written as

$$(m_e + m_s + m_b)\ddot{x} + k_b x = 0 \quad (4.13)$$

Equation 4.13 indicates that the fundamental frequency, ω_1 , of the base-isolated system can be approximated by $\omega_1 \approx \sqrt{k_b/(m_e + m_s + m_b)}$. Further, if the condition of fixed base is considered, that is, by letting the responses of the base be equal to zero, $x_b = \ddot{x}_b = 0$, the structure–equipment isolation system will be reduced to the case of a fixed-base system, such that the equation of motion becomes

$$\begin{Bmatrix} m_e \ddot{x}_e \\ m_s \ddot{x}_s \end{Bmatrix} + \begin{bmatrix} k_e & -k_e \\ -k_e & k_s + k_e \end{bmatrix} \begin{Bmatrix} x_e \\ x_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (4.14)$$

as is well known.

4.3.3 Dynamics of Structure–Equipment Isolation Systems to Harmonic Excitations

The advantage of a closed-form solution is that it allows us to examine the key parameters involved in the problem considered. This is what will be sought herein. For the case of a harmonic ground excitation, x_g , with amplitude, X_g , that is, with $x_g = X_g e^{i\omega t}$, one may derive from Equation 4.11 the following:

$$\begin{Bmatrix} m_e \ddot{x}_e \\ m_s \ddot{x}_s \\ m_b \ddot{x}_b \end{Bmatrix} + \begin{bmatrix} k_e & -k_e & 0 \\ -k_e & k_s + k_e & -k_s \\ 0 & -k_s & k_s + k_b \end{bmatrix} \begin{Bmatrix} x_e \\ x_s \\ x_b \end{Bmatrix} = \begin{Bmatrix} m_e \\ m_s \\ m_b \end{Bmatrix} X_g \omega^2 e^{i\omega t} \quad (4.15)$$

Correspondingly, the steady-state responses of the system can be expressed as

$$\begin{Bmatrix} x_e \\ x_s \\ x_b \end{Bmatrix} = \begin{Bmatrix} X_e \\ X_s \\ X_b \end{Bmatrix} e^{i\omega t} \quad (4.16)$$

where (X_e , X_s , and X_b) represent the amplitudes of the equipment, structure, and base, respectively.

Substituting Equation 4.16 into Equation 4.15 yields

$$\begin{bmatrix} k_e - m_e \omega^2 & -k_e & 0 \\ -k_e & k_s + k_e - m_s \omega^2 & -k_s \\ 0 & -k_s & k_s + k_b - m_b \omega^2 \end{bmatrix} \begin{Bmatrix} X_e \\ X_s \\ X_b \end{Bmatrix} = \begin{Bmatrix} m_e \\ m_s \\ m_b \end{Bmatrix} X_g \omega^2 \quad (4.17)$$

from which the amplitudes (X_e , X_s , and X_b) for the system can be solved as follows:

$$X_e = \frac{X_s + X_g f_e^2}{1 - f_e^2} \quad (4.18a)$$

$$X_b = \frac{X_s + \varepsilon_b f_s^2 X_g}{1 + k_b/k_s - \varepsilon_b f_s^2} \quad (4.18b)$$

$$X_s = \frac{\left[m_s + \frac{m_e}{1 - f_e^2} + \frac{m_b}{1 + k_b/k_s - \varepsilon_b f_s^2} \right] X_g \omega^2}{\frac{k_b - m_b \omega^2}{1 + k_b/k_s - \varepsilon_b f_s^2} - \left(m_s + \frac{m_e}{1 - f_e^2} \right) \omega^2} \quad (4.18c)$$

where the amplitudes of the equipment and base, that is, X_e and X_b , have been expressed in terms of the amplitude of the base, X_s . The parameters in Equation 4.18 are defined as

$$f_e = \omega/\omega_e \quad (4.19a)$$

$$f_s = \omega/\omega_s \quad (4.19b)$$

$$\varepsilon_b = m_b/m_s \quad (4.19c)$$

$$\omega_e = \sqrt{k_e/m_e} \quad (4.19d)$$

$$\omega_s = \sqrt{k_s/m_s} \quad (4.19e)$$

Finally, the state-steady absolute acceleration responses of the structure, equipment, and base can be expressed in terms of the ground acceleration $\ddot{x}_g$ as

$$a_s = \ddot{x}_s + \ddot{x}_g = -(X_s + X_g) \omega^2 e^{i\omega t} = \frac{k_b \ddot{x}_g}{D(\omega)} \quad (4.20a)$$

$$a_e = \ddot{x}_e + \ddot{x}_g = -\frac{(X_s + X_g) \omega^2 e^{i\omega t}}{1 - f_e^2} = \frac{k_b \ddot{x}_g}{(1 - f_e^2) D(\omega)} \quad (4.20b)$$

$$a_b = \ddot{x}_b + \ddot{x}_g = \frac{-(X_s + X_g + (k_b/k_s) X_g) \omega^2 e^{i\omega t}}{1 + k_b/k_s - \varepsilon_b f_s^2} = \frac{(D^{-1}(\omega) + k_s^{-1}) k_b \ddot{x}_g}{1 + k_b/k_s - \varepsilon_b f_s^2} \quad (4.20c)$$

$$D(\omega) = (k_b - m_b \omega^2) - (1 + k_b/k_s - \varepsilon_b f_s^2) \left(m_s + \frac{m_e}{1 - f_e^2} \right) \omega^2 \quad (4.20d)$$

As can be seen, the acceleration response of each component in the structure–equipment system depends mainly on the stiffness of the elastic bearing, k_b . In particular, the use of a smaller bearing stiffness, k_b , can result in significant reduction of the shear forces transmitted to the superstructure, as indicated by Equation 4.20a. This explains why an elastic bearing can be effectively used as an isolator for reducing the base shear of the structure–equipment system. In contrast, if the bearing stiffness, k_b , is made to be infinitely large, that is, by letting $k_b \rightarrow \infty$, the acceleration responses in Equation 4.20, reduce to

$$a_s = \frac{(1 - f_e^2) \ddot{x}_g}{(1 - f_e^2)(1 - f_s^2) - \varepsilon_e f_s^2} \quad (4.21a)$$

$$a_e = \frac{\ddot{x}_g}{(1 - f_e^2)(1 - f_s^2) - \varepsilon_e f_s^2} \quad (4.21b)$$

$$a_b = \ddot{x}_g \quad (4.21c)$$

with the use of L'Hospital's Rule, where $\varepsilon_e = m_e/m_s$. As can be seen from Equation 4.21c, the acceleration of the structural base is equal to the ground acceleration. Clearly, the present problem has been reduced to a two-DoF system with a rigid base, for which the solutions have been given in Equation 4.21a and Equation 4.21b.

Some important high-tech facilities, such as semiconductor factories and medical devices, are very sensitive to vibrations, especially to those caused by resonance. To consider the effect of resonance, we shall let the ground excitation frequency, ω , coincide with the equipment frequency, ω_e , that is, by letting $f_e = 1$ or $\omega_e = \omega$. For this case, the acceleration responses of the system in Equation 4.20 reduce to

$$a_s = 0 \quad (4.22a)$$

$$a_e = \frac{-k_b \ddot{x}_g}{k_e [1 + (k_b - m_b \omega_e^2)/k_s]} \quad (4.22b)$$

$$a_b = \frac{k_b \ddot{x}_g}{k_s + k_b - m_b \omega_e^2} \quad (4.22c)$$

Because of the coincidence of the ground excitation frequency with the equipment frequency, the equipment behaves like a vibration absorber of the structure. For this reason, the response of the equipment is greatly amplified, as implied by Equation 4.22b, while the response of structure is completely suppressed, as indicated by Equation 4.22a. Moreover, if the frequency of the equipment is equal to the fundamental frequency of the structure–equipment isolation system, that is, $\omega_e (\approx \omega_1) = \sqrt{k_b/(m_e + m_s + m_b)}$, then the responses of the system in Equation 4.22 can further be expressed as follows:

$$a_s = 0 \quad (4.23a)$$

$$a_e = \frac{-k_b \ddot{x}_g}{k_e [1 + (m_s + m_e) \omega_e^2/k_s]} \quad (4.23b)$$

$$a_b = \frac{k_b \ddot{x}_g}{k_s + (m_s + m_e) \omega_e^2} \quad (4.23c)$$

Since the equipment mass is generally much smaller than the structural mass, the preceding equation can be further reduced to

$$a_s = 0 \quad (4.24a)$$

$$a_e = \frac{-k_b \ddot{x}_g}{k_e (1 + \omega_e^2/\omega_s^2)} \quad (4.24b)$$

$$a_b = \frac{k_b \ddot{x}_g}{k_s (1 + \omega_e^2/\omega_s^2)} \quad (4.24c)$$

As indicated by Equation 4.24b, the acceleration response of the equipment depends on the stiffness ratio, k_b/k_e , of the base to the equipment.

For the resonance condition of $\omega_e = \omega$, mentioned previously, let us consider the case when the structural frequency is equal to the equipment frequency, that is, $\omega_e = \omega_s$. For this case, the responses of the system in Equation 4.22 reduce to

$$a_s \approx 0 \quad (4.25a)$$

$$a_e \approx \frac{-k_b \ddot{x}_g}{\varepsilon_e [k_s (1 - \varepsilon_b) + k_b]} \quad (4.25b)$$

$$a_b = \frac{k_b \ddot{x}_g}{k_s (1 - \varepsilon_b) + k_b} \quad (4.25c)$$

which indicates that the acceleration response of the equipment may be greatly amplified, as implied by the relatively small mass ratio $\varepsilon_e (= m_e/m_s)$ and large stiffness ratio, k_b/k_s , in Equation 4.25b. Such a phenomenon has been referred to as the *tuning of equipment*.

On the other hand, when the excitation frequency, ω , coincides with the fundamental frequency, ω_1 , of the isolated system, that is, $\omega (\approx \omega_1) = \sqrt{k_b/(m_e + m_s + m_b)}$, resonant response may be induced on the structure–equipment isolation system. Considering that the first priority in design of high-tech

equipment is to reduce the vibrations of the equipment, rather than the structure, by comparing the denominators in Equation 4.20a and Equation 4.20b, one may assume that the condition $|1 - f_e^2| \geq 1$ or $f_e = \omega/\omega_e \geq \sqrt{2}$ remains satisfied for a good design, which is equivalent to

$$\frac{\omega_e}{\omega_s} \leq \sqrt{\frac{k_b/k_s}{2[1 + (m_b + m_e)/m_s]}}$$

(4.26)

Since the fundamental frequency, ω_s , of a base-isolated structure is generally low in practice, the horizontal stiffness of the equipment attached to the structure must be designed to be soft enough such that Equation 4.26 can be satisfied. Certainly, this is one of the guidelines to be obeyed in the design of equipment for the sake of vibration reduction.

4.3.4 Illustrative Example

The forgoing formulations have been made by neglecting the damping of the structural system and by assuming the ground motion to be of the harmonic type. In practice, there is always some damping with the structural system, while the ground motion may be random in nature. To deal with such problems, the only recourse is to use numerical methods that are readily available. In this section, the Newmark β method, proposed by Newmark (1959), with $\gamma = 1/2$ and $\beta = 1/4$, will be adopted for solving the second-order differential equation presented in Equation 4.11, which has the advantage of being numerically stable.

The example considered is the structure–equipment system isolated by elastomeric bearings, shown in Figure 4.9, with the data given in Table 4.2. As can be seen, the equipment has a frequency equal to five times the structural frequency, that is, $\omega_e = 5\omega_s$ ($= 8.34$ Hz). The 1940 El Centro earthquake (NS component) with a peak ground acceleration (PGA) of 341.55 gal is adopted as the ground excitation, as given in Figure 4.10. By an eigenvalue analysis, the natural frequencies solved for the base-isolated system are 2.46, 21.41, and 52.74 rad/sec. Because of the installation of elastic bearings on the structure–equipment system, the fundamental frequency of the system decreases significantly and is approximately equal to $\omega_1 \approx \sqrt{k_b(m_e + m_s + m_b)} = 2.51$ rad/sec, according to Section 4.3.2. From this example, one observes that the use of a single-DoF system to model a base-isolated system can give a generally good result for the first frequency of vibration.

As can be seen from Figure 4.11, for the structural acceleration of the system, the main-shock response of the fixed-base structure has been effectively eliminated due to installation of the elastic bearings. However, as indicated by Figure 4.12, because of the installation of soft bearings, the base displacement response of the isolated system is much larger and decays much slowly, even after the main shocks.

In Figure 4.13 the acceleration response of the equipment for the isolated and fixed-base cases are compared. As can be seen, the main-shock response of the fixed-base structure has been effectively suppressed through the installation of the elastic bearings. Furthermore, the equipment response appears to be almost identical to the structure response shown in Figure 4.11, due to the fact that the equipment

TABLE 4.2 System Parameters Used in Simulation (Section 4.3.4)

Equipment		Superstructure		Isolation System	
Parameter	Value	Parameter	Value	Parameter	Value
Mass m_e	3 t ($= m_s/100$)	Mass m_s	300 t	Mass m_b	100 t ($= m_s/3$)
Horizontal stiffness k_e	8258 kN/m	Horizontal stiffness k_s	33,030 kN/m	Horizontal stiffness k_b	2546 kN/m
Damping	15.74 kN m/s	Damping	314.79 kN m/s	Damping	50.46 kN m/s
Frequency $\omega_e = \sqrt{k_e/m_e}$	52.47 rad/sec	Frequency $\omega_s = \sqrt{k_s/m_s}$	10.46 rad/sec	Frequency $\omega_b = \sqrt{k_b/m_b}$	5.05 rad/sec

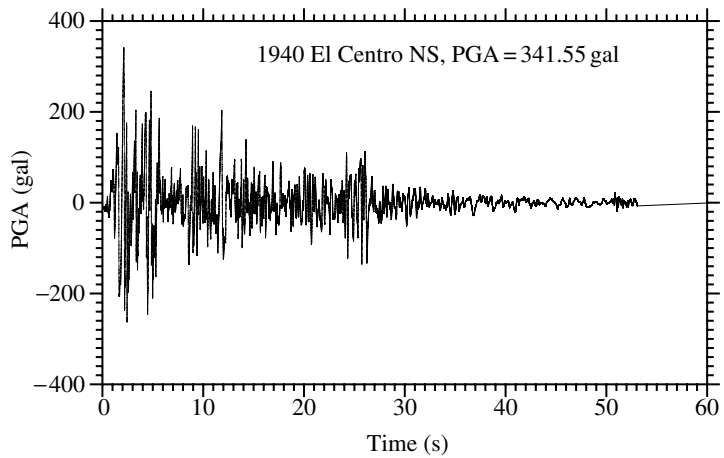


FIGURE 4.10 Waveform of 1940 El Centro earthquake (NS component).

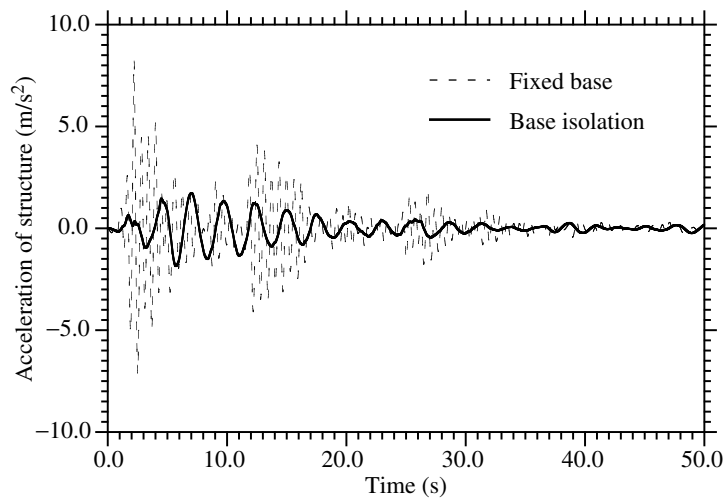


FIGURE 4.11 Comparison of structural accelerations.

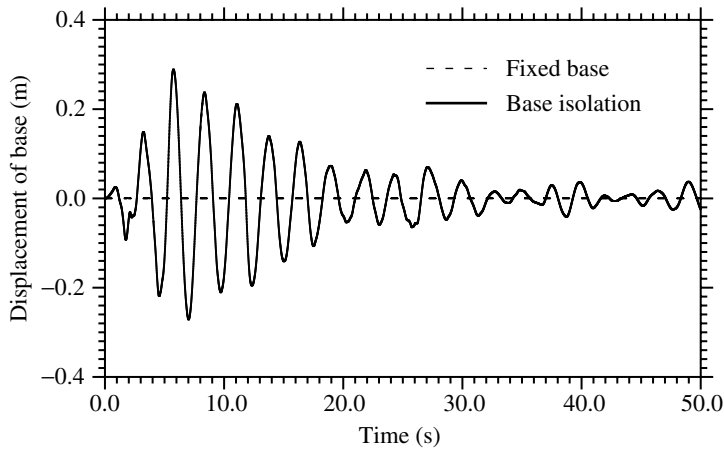


FIGURE 4.12 Comparison of base displacements.

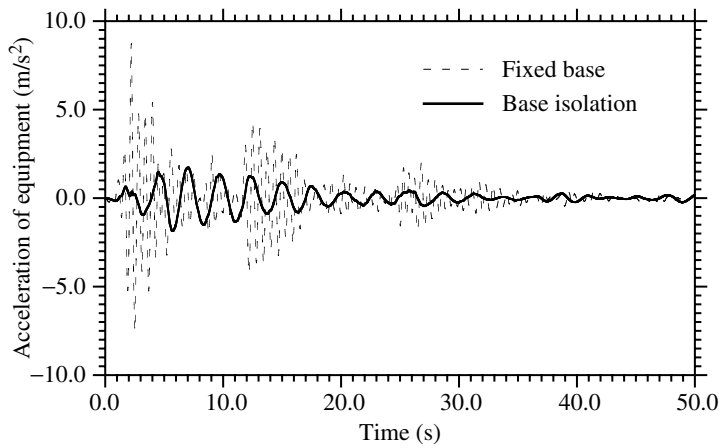


FIGURE 4.13 Comparison of equipment accelerations.

is rigidly attached to the structure, as implied by the relatively higher frequency of the equipment. As for the present example, the effectiveness of the elastic bearings in reducing the equipment response is ascertained.

For high-tech equipment, engineers may be concerned about the effect on equipment tuning induced by external excitations, such as earthquakes and traffic-induced vibrations. To investigate this effect on the structure–equipment isolation system considered, the maximum equipment acceleration was plotted as a function of the frequency ratio, ω_e/ω_s , in Figure 4.14. As can be seen, the response of the equipment is greatly amplified when it has a frequency close to the fundamental frequency of the structure–equipment isolation system, that is, when $\omega_e = 2.51$ rad/sec or $\omega_e/\omega_s = 0.24$. To avoid such a situation, it is suggested that isolators be mounted at both the structure base and equipment base. From Figure 4.14, one observes that the use of a small horizontal stiffness for the equipment will generally lead to greater equipment response due to tuning effect. However, as the stiffness of the equipment is further reduced, the equipment will reach another isolation state, in which the equipment response will be substantially suppressed, as indicated by the region with relatively small values of ω_e/ω_s . The margin for such a frequency ratio of ω_e/ω_s can be obtained by substituting the parameters in Table 4.2 into

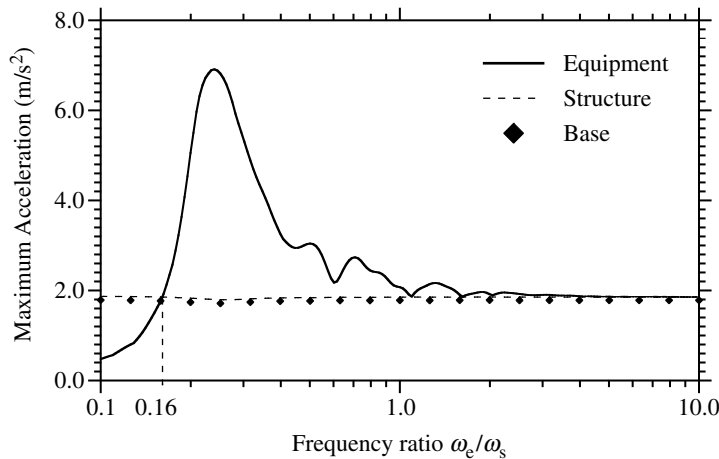


FIGURE 4.14 Maximum accelerations of structure–equipment isolation systems.

Equation 4.26, which yields a critical ratio of $\omega_e/\omega_s = 0.168$, very close to the value of 0.16 marked in Figure 4.14.

4.3.5 Concluding Remarks

This section investigates the dynamic response of a mathematical model of a structure–equipment system isolated by elastomeric bearings and subjected to ground excitations. Based on the closed-form solution of a structure–equipment isolation system subjected to harmonic support motions, one observes that the coincidence of the ground excitation frequency with the equipment frequency will make the equipment behave like a vibration absorber of the structure, of which the acceleration response will be greatly amplified. For the case that the first priority in design is to reduce the vibration of the equipment rather than that of the structure, Equation 4.26 provides a guideline for the design of equipment, which has been verified in the numerical example.

4.4 Sliding Isolation Systems

Sliding isolation can be an effective means for the seismic protection of structural systems. By implementing sliding isolators under the base mat of a structure, the transmission of ground excitation to the structure can be greatly reduced. Currently, applications of sliding isolation systems can be found elsewhere (Naeim and Kelly, 1999). A *sliding isolator* usually consists of a slider with frictional surfaces. For this reason, it is also referred to as a *friction isolator*. When subjected to an earthquake, the slider will slide along the frictional contact surfaces whenever the horizontal seismic force exceeds the maximum frictional force of the support, which, by Coulomb's theory, is equal to the normal contact force multiplied by the static (or dynamic) coefficient of friction of the sliding surfaces. Because of this, the seismic force transmitted to the superstructure is generally less than the maximum frictional force of the isolator. Obviously, the maximum frictional force is an important parameter for the design of a sliding isolation system, because it decides when the system starts to slide and how large the shear force is to be transmitted to the superstructure.

The motion of a sliding structure consists of two different states, namely, the sliding state and the stick (or nonsliding) state. At any instant of motion, the structure can only belong to one of the two states. Although in each state the sliding structure can be modeled as a linear system, the governing equations of motion for the two states are different. As a result, the overall behavior of the sliding structure is nonlinear. Such nonlinearity has resulted in the occurrence of subharmonic resonance in the frequency response of a sliding structure (Mostaghel et al., 1983; Westermo and Udawadia, 1983), making the dynamic response much more complicated. In some applications, a sliding isolation system has been designed with an automatic recentering mechanism (or resilient mechanism), so that the structure can slide back to its original position after the earthquake (Mokha et al., 1991). This type of sliding systems has been called the *resilient sliding isolation system*, which will be investigated in Section 4.5. The implementation of a recentering mechanism offers some advantages, but will inevitably introduce some disadvantages, as will be discussed in Section 4.5.

The purpose of this section is to investigate the seismic behavior of a sliding isolated structure and also the behavior of an equipment item mounted on the structure. No consideration will be made for the recentering mechanism. The nonlinear dynamic equation for a structure with an underneath friction element is first formulated. Next, two numerical approaches will be presented for solving the nonlinear equation, the *shear balance method* and *fictitious spring method*. Finally, using some assumed data, the harmonic response and seismic behavior of a sliding structure, together with the equipment mounted on it, will be presented. In this section, the frictional coefficient of the sliding system is assumed to be of the Coulomb type, that is, a time-invariant constant. For simplification, no distinction will be made between the static and dynamic frictional coefficients, or between the dynamic and maximum static frictional force.

4.4.1 Mathematical Modeling and Formulation

4.4.1.1 Equation of Motion

A sliding isolated structure with an attached equipment item, as schematically shown in Figure 4.15, can be represented as a mass–spring–dashpot system of three DoF, as shown in Figure 4.16, for which the notations employed have been defined in Table 4.1. When the structural system is excited by an earthquake, the equation of motion can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}\mathbf{L}_1\ddot{x}_g(t) + \mathbf{L}_2f(t) \quad (4.27)$$

where the vector $\mathbf{x}$ denotes the dynamic responses of the whole structural system

$$\mathbf{x}(t) = \begin{Bmatrix} x_e(t) \\ x_s(t) \\ x_b(t) \end{Bmatrix} \quad (4.28)$$

The mass, damping, and stiffness matrices in Equation 4.27 are defined as

$$\mathbf{M} = \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c_e & -c_e & 0 \\ -c_e & c_e + c_s & -c_s \\ 0 & -c_s & c_s \end{bmatrix}, \quad (4.29)$$

$$\mathbf{K} = \begin{bmatrix} k_e & -k_e & 0 \\ -k_e & k_e + k_s & -k_s \\ 0 & -k_s & k_s \end{bmatrix}$$

and the force distribution vectors as

$$\mathbf{L}_1 = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}, \quad \mathbf{L}_2 = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \quad (4.30)$$

Note that in Equation 4.27, the isolator frictional force, $f(t)$, which is not a constant, is moved to the right-hand side of the equation. This nonlinear force requires a special treatment in an analysis procedure, as will be explained later on.

For a systematic treatment, the above equation of motion can be further written in a state space form as shown below (Meirovitch, 1990):

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{E}\ddot{x}_g(t) + \mathbf{B}f(t) \quad (4.31)$$

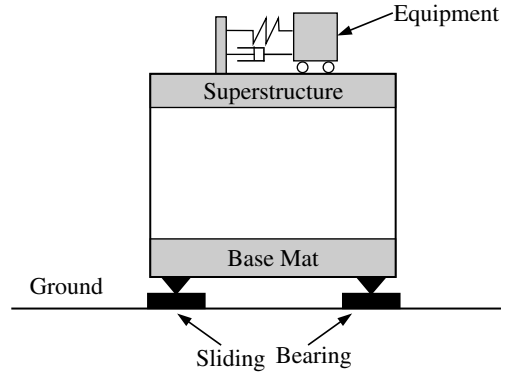


FIGURE 4.15 Schematic for an isolated structure–equipment system with sliding bearing.

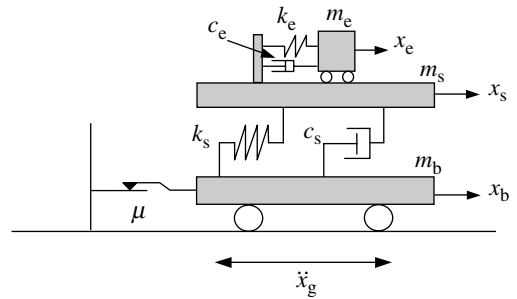


FIGURE 4.16 Model for an isolated structure–equipment system with sliding support.

where the state vector $\mathbf{z}(t)$ and the system matrix $\mathbf{A}$ are defined as

$$\mathbf{z}(t) = \begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{x}(t) \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{K} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \quad (4.32)$$

and the excitation and friction distribution vectors as

$$\mathbf{E} = \begin{bmatrix} -\mathbf{L}_1 \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{M}^{-1}\mathbf{L}_2 \\ \mathbf{0} \end{bmatrix} \quad (4.33)$$

4.4.1.2 Conditions for Stick and Sliding States

As mentioned above, the motion of a sliding structure at any instant has two possible states, namely, the stick (or nonsliding) and sliding states. The following are the conditions that must be satisfied by the sliding structure:

- (1) In stick state

$$|f(t)| < f_{\max} = \mu W \quad (4.34a)$$

$$\dot{x}_b(t) = 0 \quad (4.34b)$$

where μ is the coefficient of friction and W is the total weight of the structure. According to the preceding equations, the frictional force in the stick state is an unknown with a magnitude less than the maximum frictional force, $f_{\max}$, which equals the product of μ and W , while the sliding velocity of the structure is simply zero. Whenever the frictional force satisfies Equation 4.34a, the sliding system remains in the stick state, otherwise it changes into the sliding state.

- (2) In sliding state

$$f(t) = -\text{sgn}(\dot{x}_b(t))f_{\max} = -\text{sgn}(\dot{x}_b(t))\mu W \quad (4.35a)$$

$$\dot{x}_b(t) \neq 0 \quad (4.35b)$$

where the function $\text{sgn}(x)$ denotes the sign of the variable x . According to Equation 4.35a and Equation 4.35b, the frictional force in the sliding state has a magnitude equal to the maximum frictional force, but directed in a sense opposite to that of the sliding velocity. On the other hand, the sliding velocity of the isolator remains as an unknown.

4.4.2 Methods for Numerical Analysis

Two numerical methods commonly used for the analysis of sliding isolated structural systems, the shear balance method and fictitious spring method, will be introduced in this section. By employing the discrete-time state-space formula, both methods can be cast in an incremental form that is suitable for the analysis of sliding systems with multiple DoF.

4.4.2.1 Shear Balance Method

Consider the state-space equation, Equation 4.31, and assume that both the ground acceleration and frictional force vary linearly within each time interval, as shown in Figure 4.17. Equation 4.31 may be written in the following incremental form (Meirovitch, 1990)

$$\mathbf{z}[k+1] = \mathbf{A}_d\mathbf{z}[k] + \mathbf{E}_0\ddot{x}_g[k] + \mathbf{E}_1\ddot{x}_g[k+1] + \mathbf{B}_0f[k] + \mathbf{B}_1f[k+1] \quad (4.36)$$

where the symbol $x[k]$ denotes that the variable x is evaluated at the k th time step. The other coefficient

matrices in equation 4.36 are defined as

$$\mathbf{A}_d = e^{\mathbf{A}\Delta t} = \sum_{i=0}^{\infty} \frac{\Delta t^i}{i!} \mathbf{A}^i \quad (4.37)$$

$$\mathbf{B}_0 = \left[(\mathbf{A})^{-1} \mathbf{A}_d + \frac{1}{\Delta t} (\mathbf{A})^{-2} (\mathbf{I} - \mathbf{A}_d) \right] \mathbf{B} \quad (4.38a)$$

$$\mathbf{B}_1 = \left[-(\mathbf{A})^{-1} + \frac{1}{\Delta t} (\mathbf{A})^{-2} (\mathbf{A}_d - \mathbf{I}) \right] \mathbf{B} \quad (4.38b)$$

where Δt denotes the size of the time step considered for analysis. The matrices $\mathbf{E}_0$ and $\mathbf{E}_1$ can be computed in a way similar to $\mathbf{B}_0$ and $\mathbf{B}_1$, except that the matrix $\mathbf{B}$ in Equation 4.38a and Equation 4.38b should be replaced by the matrix $\mathbf{E}$. In some applications, the system matrix $\mathbf{A}$ may not be invertible, that is, $\mathbf{A}^{-1}$ does not always exist. If this is the case, one may compute $\mathbf{B}_0$ and $\mathbf{B}_1$ (and similarly $\mathbf{E}_0$ and $\mathbf{E}_1$) by the following formulas:

$$\mathbf{B}_0 = \mathbf{A}_d^* \mathbf{B}, \quad \mathbf{B}_1 = (\hat{\mathbf{A}}_d - \mathbf{A}_d^*) \mathbf{B} \quad (4.39)$$

where

$$\mathbf{A}_d^* = \sum_{i=0}^{\infty} \frac{(\Delta t)^{i+1}}{i!(i+2)} \mathbf{A}^i, \quad \hat{\mathbf{A}}_d = \sum_{i=0}^{\infty} \frac{(\Delta t)^{i+1}}{(i+1)!} \mathbf{A}^i \quad (4.40)$$

Note that, on the right-hand side of Equation 4.36, the only unknown at the k th time step is the frictional force $f[k+1]$; therefore, $f[k+1]$ must be determined before the next time step response $\mathbf{z}[k+1]$ is computed. Wang et al. (1998) proposed the shear balance method for computing the frictional force $f[k+1]$. By this method, the sliding structure is first assumed to be in the stick state at the $(k+1)$ th step, for which the condition given in Equation 4.34b must be satisfied

$$\dot{\mathbf{x}}_b[k+1] = \mathbf{D}\mathbf{z}[k+1] = 0 \quad (4.41)$$

where $\mathbf{D}$ is a relation matrix, equal to $\mathbf{D} = [0 \ 0 \ 1 \ 0 \ 0 \ 0]$ for the model shown in Figure 4.16. Substituting $\mathbf{z}[k+1]$ in Equation 4.36 into Equation 4.41, one may solve for the *estimated frictional force* at the $(k+1)$ th time step as

$$\bar{f}[k+1] = -(\mathbf{D}\mathbf{B}_1)^{-1} \mathbf{D}(\mathbf{A}_d \mathbf{z}[k] + \mathbf{B}_0 f[k] + \mathbf{E}_0 \ddot{x}_g[k] + \mathbf{E}_1 \ddot{x}_g[k+1]) \quad (4.42)$$

where $\bar{f}[k+1]$ with an overbar signifies that the frictional force is an estimate obtained by assuming the sliding structure to be at the stick state. Such a value may not be the actual one if the system is not in the stick state. The physical meaning for $\bar{f}[k+1]$ is that it represents the *balanced shear force* required at the $(k+1)$ th time step for the structure to remain in the stick state. Therefore, the sign of $\bar{f}[k+1]$ indicates the direction of the resistant force provided by the isolation system. In spite of the fact that $\bar{f}[k+1]$ may not be the actual frictional force, it plays an important role for determining the actual state (stick or sliding) and the actual frictional force of the sliding isolated structure, as will be described below based on Equation 4.34a and Equation 4.35a.

- (1) The system is in the “stick state” if $|\bar{f}[k+1]| < f_{\max}$ and the frictional force is

$$f[k+1] = \bar{f}[k+1] \quad (4.43)$$

- (2) The system is in the “sliding state” if $|\bar{f}[k+1]| \geq f_{\max}$ and the frictional force is

$$f[k+1] = \text{sgn}(\bar{f}[k+1]) f_{\max} \quad (4.44)$$

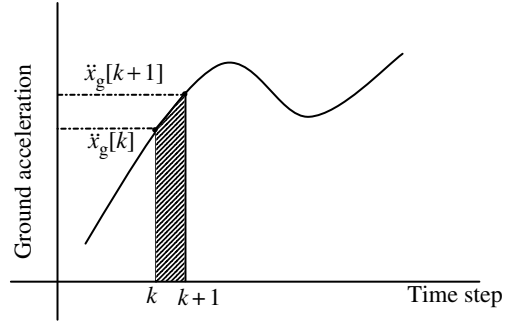


FIGURE 4.17 Force integration scheme with linear interpolation.

As can be seen, the term $-\text{sgn}(\dot{x}_b[k+1])$ in Equation 4.35a is replaced by $\text{sgn}(\tilde{f}[k+1])$ in Equation 4.44. Such a replacement is justified since the sign of $\tilde{f}[k+1]$ indicates the direction of the resistant force at the $(k+1)$ th time step. Once the correct frictional force, $f[k+1]$, is determined by using either Equation 4.43 or Equation 4.44, it can be substituted into Equation 4.36 to obtain the response $\mathbf{z}[k+1]$ for the next time step. The computational flow-chart for the shear balance method has been given in Figure 4.18.

4.4.2.2 Fictitious Spring Method

The fictitious spring method was first proposed by Yang et al. (1990) for the analysis of a sliding structure. Later, Lu and Yang (1997) reformulated the method into a state-space form for the analysis of equipment

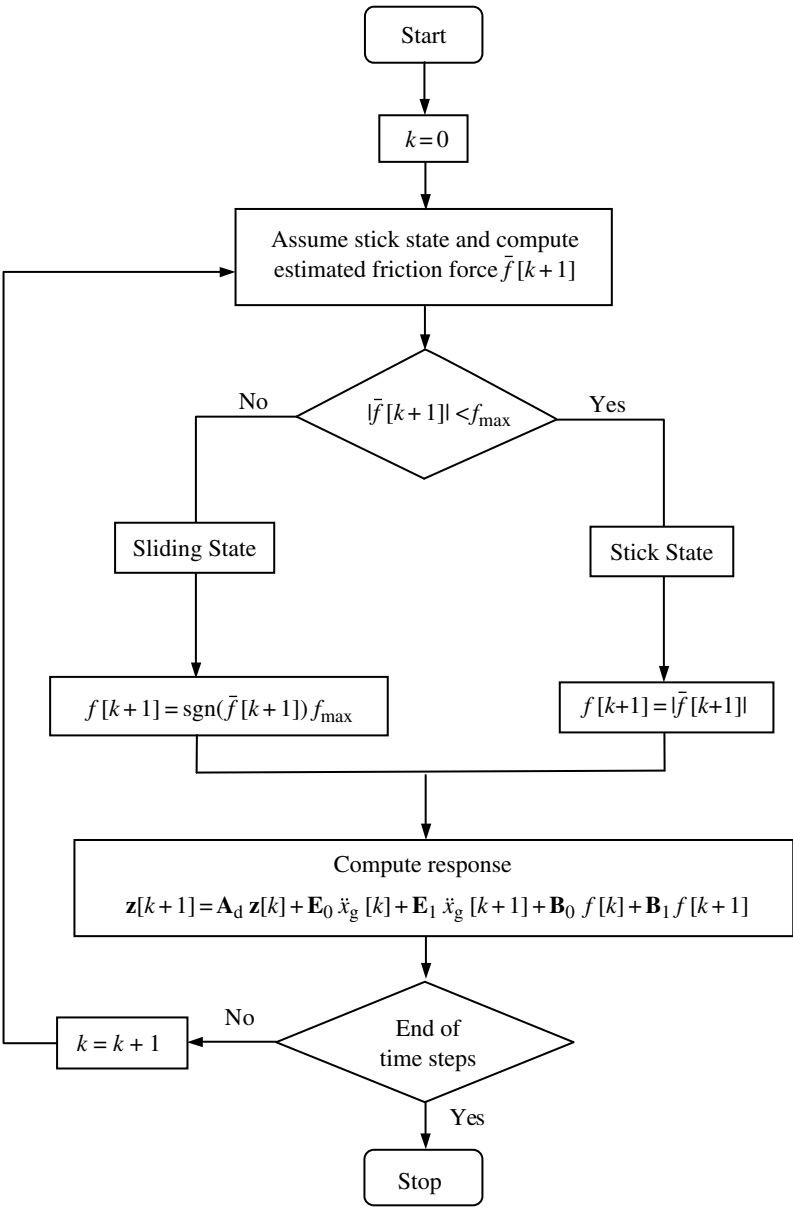


FIGURE 4.18 Computational flow-chart for shear balance method.

mounted on a sliding structure. By this method, a fictitious spring, k_f , is introduced between the base mat and the ground, as in Figure 4.19, to represent the mechanism of sliding or friction. The stiffness, k_f , of the fictitious spring is taken as zero for the sliding state and as a very large value for the stick state. With the introduction of the fictitious spring, the stiffness matrix, $\mathbf{K}$, in Equation 4.29 should be modified as follows:

$$\mathbf{K} = \mathbf{K}(k_f) = \begin{bmatrix} k_e & -k_e & 0 \\ -k_e & k_e + k_s & -k_s \\ 0 & -k_s & k_s + k_f \end{bmatrix} \quad (4.45)$$

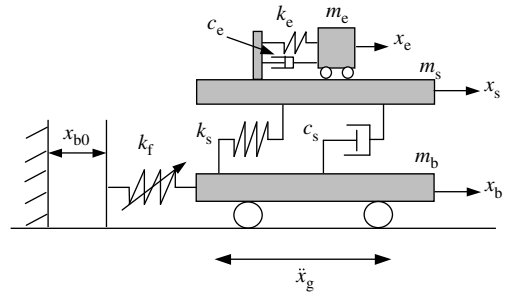


FIGURE 4.19 Model for isolated structures with fictitious spring.

Accordingly, the state-space dynamic equation, Equation 4.31, should be modified as

$$\dot{\mathbf{z}}(t) = \mathbf{A}(k_f)\mathbf{z}(t) + \mathbf{E}\ddot{x}_g(t) + \mathbf{B}\tilde{f}(t) \quad (4.46)$$

where

$$\mathbf{A} = \mathbf{A}(k_f) = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{K}(k_f) \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \quad (4.47)$$

Depending on the current state of the sliding system, the fictitious stiffness, k_f , and the modified friction term, $\tilde{f}(t)$, in Equation 4.46 may take one of the following two sets of values:

(1) In the stick state

$$k_f = \alpha k_s, \quad \tilde{f}(t) = k_f x_{b0} \quad (4.48)$$

(2) In the sliding state

$$k_f = 0, \quad \tilde{f}(t) = -\text{sgn}(\dot{x}_b(t))\mu W \quad (4.49)$$

In Equation 4.48, the symbol α represents a constant of very large value, and x_{b0} the initial elongation of the fictitious spring in the current stick state (computation of x_{b0} will be explained later). Note that the modified friction term, $\tilde{f}(t)$, may not be the actual frictional force. The actual frictional force can be determined as follows:

(1) In the stick state

$$f(t) = k_f(x_b(t) - x_{b0}) \quad (4.50)$$

(2) In the sliding state

$$f(t) = \tilde{f}(t) \quad (4.51)$$

According to Equation 4.50 and Equation 4.51, the actual frictional force of the isolation system in the stick state is equal to the internal force of the fictitious spring, while in the sliding state it is equal to the modified frictional force, $\tilde{f}(t)$. The frictional force computed from the preceding two equations should obey the conditions given in Equation 4.34a and Equation 4.35a as well.

With the conditions imposed for the stick and sliding states in Equation 4.48 and Equation 4.49, respectively, the equation of motion in Equation 4.46 actually represents two different sets of equations.

Specifically, Equation 4.46 and Equation 4.48 collectively describe the motion of the structure in the stick state, while Equation 4.46 and Equation 4.49 represent the motion of the structure in the sliding state. Owing to the fact that a sliding system may switch between the two states at certain instants, the behavior of the entire system should undoubtedly be regarded as a nonlinear one. Nevertheless, within each particular state, the behavior of the system as represented either by Equation 4.46 and Equation 4.48 or Equation 4.46 and Equation 4.49 is a linear one.

In the following, a numerical solution scheme based on the concept of fictitious spring will be introduced. Let Δt denote a time increment, which is usually taken as a very small value, and assume that the ground excitation and frictional force are constant within each time increment, Δt (see Figure 4.20). Accordingly, the discrete-time solution of Equation 4.46 can be rewritten in an incremental form (Meirovitch, 1990) as

$$\mathbf{z}[k+1] = \mathbf{A}_d \mathbf{z}[k] + \mathbf{E}_d \ddot{x}_g[k] + \mathbf{B}_d \tilde{f}[k] \quad (4.52)$$

where

$$\mathbf{A}_d = \mathbf{A}_d(k_f) = e^{\mathbf{A}(k_f)\Delta t} = \sum_{i=0}^{\infty} \frac{\Delta t^i}{i!} \mathbf{A}(k_f)^i \quad (4.53)$$

$$\mathbf{E}_d = \mathbf{E}_d(k_f) = \mathbf{A}(k_f)^{-1} (\mathbf{A}_d(k_f) - \mathbf{I}) \mathbf{E} \quad (4.54a)$$

$$\mathbf{B}_d = \mathbf{B}_d(k_f) = \mathbf{A}(k_f)^{-1} (\mathbf{A}_d(k_f) - \mathbf{I}) \mathbf{B} \quad (4.54b)$$

For the case where the system matrix $\mathbf{A}$ is invertible, $\mathbf{B}_d$ and $\mathbf{E}_d$ may be computed instead using the following formulas:

$$\mathbf{E}_d = \mathbf{E}_d(k_f) = \left[\sum_{i=0}^{\infty} \frac{\Delta t^i}{i!} \mathbf{A}(k_f)^{i-1} \right] \mathbf{E} \quad (4.55)$$

$$\mathbf{B}_d = \mathbf{B}_d(k_f) = \left[\sum_{i=0}^{\infty} \frac{\Delta t^i}{i!} \mathbf{A}(k_f)^{i-1} \right] \mathbf{B} \quad (4.56)$$

Equation 4.52 is the solution of the sliding system given in incremental form, because the response, $\mathbf{z}[k+1]$, can be computed from the solution of the previous step, $\mathbf{z}[k]$. Note that, in Equation 4.53 and Equation 4.54, the coefficient matrices $\mathbf{A}_d$, $\mathbf{E}_d$, and $\mathbf{B}_d$ have two possible sets of values, as the fictitious spring constant, k_f , may take different values for the sliding and stick states. Nevertheless, once the time step size, Δt , is chosen, the coefficient matrices $\mathbf{A}_d$, $\mathbf{E}_d$, and $\mathbf{B}_d$, remain constant for each state. As such, they need only be calculated once at the beginning of the incremental procedure. The computational flow-chart for the fictitious spring method described above has been given in Figure 4.21.

The dynamic equation and its discrete-time solution for the sliding structure in the two states have been presented above. In the following, we shall describe how to determine the *transition time* for the sliding structure to switch from one state to the other. Once the transition time is determined, the original step size should be scaled down accordingly to reflect the transition point (Yang et al., 1990).

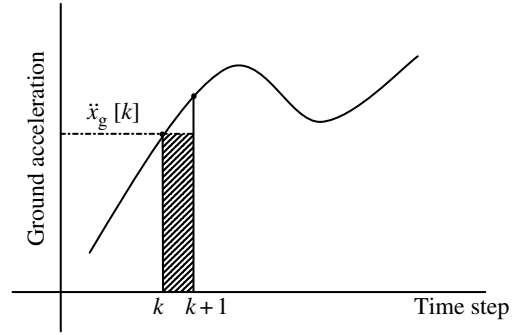


FIGURE 4.20 Constant force integration scheme.

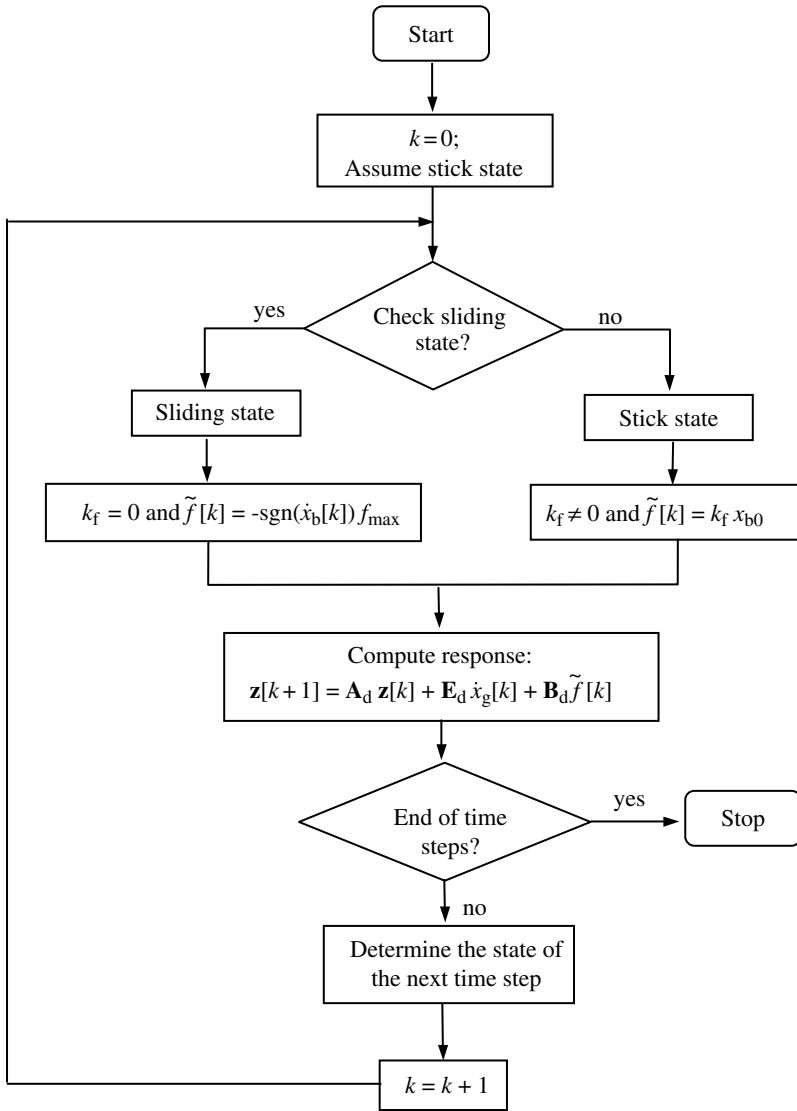


FIGURE 4.21 Computational flow-chart for fictitious spring method.

Transition from stick to sliding state. As stated in Equation 4.34a, the condition for a sliding isolated structure to remain in the stick state is that the static frictional force under the base mat be less than its maximum value, $f_{\max}$. Once the frictional force exceeds this maximum value, the system starts to slide. With the fictitious spring method, the static frictional force is computed as the internal force of the fictitious spring, as in Equation 4.50. Based on the above considerations and Equation 4.50, the condition for the sliding system to transfer from the stick to the sliding state is

$$|f(t_0)| = |k_f(x_b(t_0) - x_{b0})| = f_{\max} = \mu W \quad (4.57)$$

where t_0 denotes the transition time at which the structure starts to slide and k_f is the spring constant given in Equation 4.48 for the stick state. Because a very large value has been used for the fictitious spring constant, k_f , in the stick state, the deviation in base displacement due to spring elongation is very small in this state, which can just be neglected. In practice, the transition time, t_0 may not occur precisely

at the discrete points of time considered. It is likely that the spring force is less than $f_{\max}$ at the current time step, say, the k th step, but exceeds $f_{\max}$ at the following time step. If this is the case, numerical methods such as the bisection method should be employed to locate the transition time, t_0 within the time interval $(k\Delta t, (k + 1)\Delta t)$ considered based on Equation 4.57.

Transition from sliding state to stick state. The structure in the sliding state may return to the stick state whenever the following two conditions are satisfied. (1) The relative velocity of the base mat to the ground reaches zero, that is, $\dot{\xi}_b(t_0) = 0$ where t_0 is the transition time; (2) the estimated static frictional force, denoted by $\tilde{f}(t_0)$, is less than the maximum static frictional force, that is, $\tilde{f}(t_0) < f_{\max}$. Here, the estimated static frictional force, $\tilde{f}(t_0)$, is defined as the shear force required to balance the motion of the superstructure if the system is assumed to be in the stick state, similar to the one given in Equation 4.42 for $\tilde{f}[k + 1]$. By letting the relative velocity and relative acceleration between the base mat and the ground be equal to zero, that is, $\ddot{\xi}_b(t_0) = \dot{\xi}_b(t_0) = 0$, the estimated frictional force can be calculated from the free-body diagram of the base mat as

$$\tilde{f}(t_0) = m_b \ddot{x}_g(t_0) - k_s(x_s(t_0) - x_b(t_0)) - c_s \dot{x}_s(t_0)$$

(4.58)

For the sliding structure to transfer from the sliding to the stick state, both the aforementioned conditions must be satisfied simultaneously. Once the structure enters the stick state, the term $f(t_0)$ should be set equal to $\tilde{f}(t_0)$ and used as the initial frictional force. For the sake of equilibrium, the initial base mat displacement, x_{b0} , should be computed as

$$x_{b0} = x_b(t_0) - (\tilde{f}(t_0)/k_f)$$

(4.59)

where the value of k_f is the one given for the stick state in Equation 4.48.

Concerning the two conditions mentioned above, it may happen that only the first condition, $\dot{\xi}_b = 0$ is satisfied, while the computed $\tilde{f}(t_0)$ is still larger than $f_{\max}$. If this is the case, the sliding system should not be regarded as a transition to the stick state. Rather, the situation should be regarded as an indication for reversing the direction of sliding in the next time step. Correspondingly, the frictional force, $f(t_0)$, should be set equal to the sliding frictional force, rather than the estimated one, $\tilde{f}(t_0)$.

4.4.3 Simulation Results for Sliding Isolated Systems

4.4.3.1 Numerical Model and Ground Excitations

In this section, the dynamic behavior of the sliding isolated structure–equipment system shown in Figure 4.16 will be analyzed using the shear balance method. Although the sliding structure and equipment considered are both of single-DoF, there exists no difficulty for use of the method to solve problems with multi-DoF systems. In Table 4.3, the material properties adopted for the present model have been listed, which are intended to simulate a small, five-story, reinforced concrete frame. For the present purposes, two types of ground excitation are considered, namely, harmonic and earthquake excitations. The harmonic excitation is considered primarily for studying the frequency response of the sliding system, while the earthquake excitation is considered for the effect of earthquake intensity. For the

TABLE 4.3 System Parameters Used in Simulation (Section 4.4.3)

Equipment		Superstructure		Isolation System	
Parameter	Value	Parameter	Value	Parameter	Value
Mass m_e	3 t (= $m_s/100$)	Mass m_s	300 t	Mass m_b	100 t (= $m_s/3$)
Frequency ω_e	$5\omega_s$ or a variable	Frequency ω_s	1.67 Hz	Frictional coefficient μ	0.05, 0.1, 0.25
Damping ratio ζ_e	5%	Damping ratio ζ_s	5%	—	—

harmonic excitation, a sinusoidal ground acceleration of the following form is adopted:

$$\ddot{x}_g(t) = 0.5g \sin \omega_g t \quad (4.60)$$

where ω_g denotes the excitation frequency and g is the gravitational acceleration. For the earthquake excitation, the 1940 El Centro earthquake (NS component) is considered, for which the waveform has been given in Figure 4.10. The PGA level of the earthquake will be adjusted for reasons of research. Concerning the effectiveness of response reduction, three quantities are chosen as the indices, namely, the base displacement (base drift), structural acceleration, and equipment acceleration. For all the figures shown in Section 4.4.3 below, the symbol μ in the legend is used to denote the frictional coefficient of the sliding isolation system and the word “fixed” denotes the response of the corresponding fixed-base structure.

4.4.3.2 Harmonic Response of Structure

Time history. For the isolated system subjected to a harmonic excitation of $\omega_g = 1$ Hz, the base displacements computed for different coefficients of friction were plotted in Figure 4.22 and Figure 4.23. As can be seen, the base displacements quickly reach the steady-state response within the first few cycles. Meanwhile, the use of a smaller frictional coefficient results in a larger permanent displacement before the steady state is reached. From the structural accelerations plotted in Figure 4.24, one observes that, for a sliding structure with a smaller coefficient of friction, the steady-state response is achieved in a faster way, accompanied by a larger reduction on structural acceleration. Of interest in Figure 4.24 is that the response of the fixed-base case shows a clear period of 1 sec, while in the sliding case, the response is contaminated by high-frequency signals caused by the sliding-stick transitions.

Hysteretic behavior. In order to understand the mechanical characteristics of a nonlinear device used for vibration control, it is common to present a diagram showing the force–deformation relation of the device, also referred to as the *hysteretic diagram* (Soong and Dargush, 1997). Figure 4.25 shows the hysteresis loops of the sliding isolation system (the sliding layer) for $\mu = 0.1$ and 0.25 , when the system is subjected to a harmonic excitation of $\omega_g = 1$ Hz. In the figure, the horizontal and vertical axes, respectively, represent the base displacement and shear force, that is, the frictional force, under the mat. Just like many other frictional elements or devices, the shape of the hysteresis loop of a sliding bearing is rectangular. The height of the rectangle is equal to the maximum frictional force that depends on the coefficient of friction, while the width of the hysteresis loop is determined by the base-sliding displacement. As the coefficient of friction decreases, the height of the loop decreases, while the width increases. The total area of the hysteresis loop is equivalent to the portion of the energy dissipated by the sliding bearing.

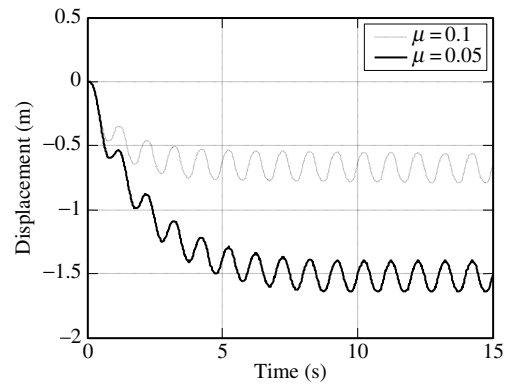


FIGURE 4.22 Comparison of base displacements ($\omega_g = 1$ Hz; $\omega_e = 5\omega_g$).

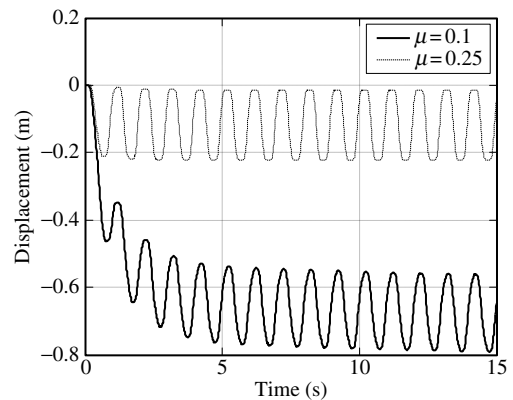


FIGURE 4.23 Comparison of base displacements ($\omega_g = 1$ Hz; $\omega_e = 5\omega_g$).

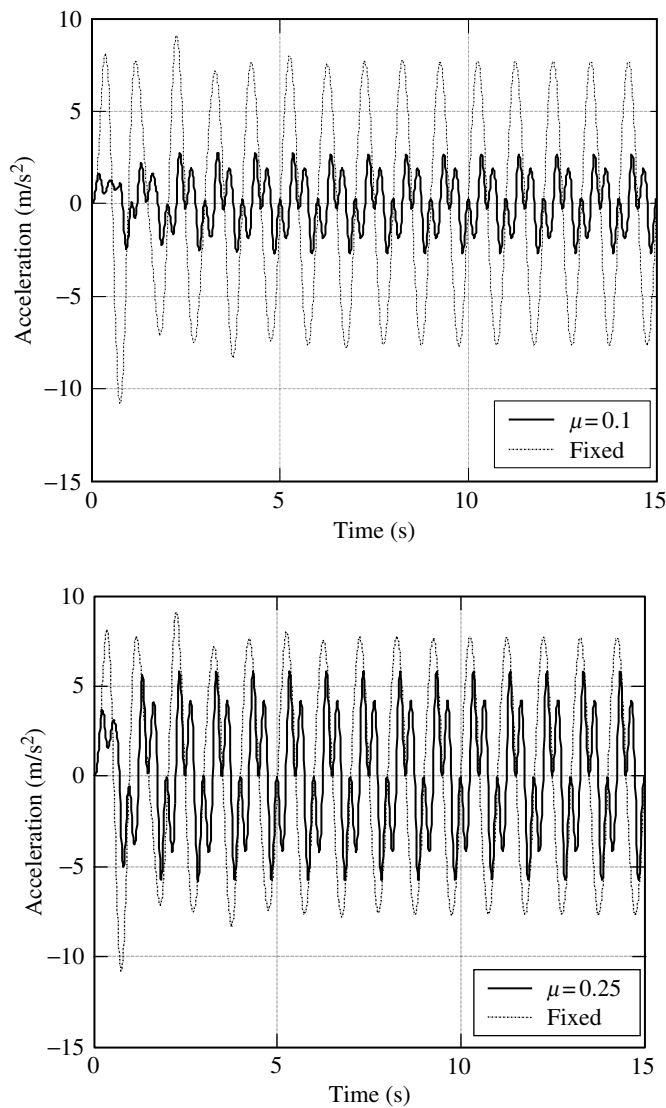


FIGURE 4.24 Comparison of structural accelerations ($\omega_g = 1$ Hz; $\omega_e = 5\omega_s$).

Frequency response. Figure 4.26 shows the maximum structural acceleration with respect to the excitation frequency for four different frictional coefficients, $\mu = 0.05, 0.1, 0.25$ and ∞ (for the fixed-base case). Here, the maximum acceleration means the steady-state acceleration response. The following observations can be made from Figure 4.26: (1) Compared with the fixed-base case, the use of a smaller frictional coefficient can reduce the structural acceleration for the frequency range considered. (2) The sliding mechanism can effectively suppress the main resonant response, associated with the natural frequency of 1.67 Hz of the superstructure system. (3) As the coefficient of friction, μ , decreases from ∞ to 0.05, the main resonant frequency associated with the structural natural frequency drifts from the fixed-base frequency of 1.67 Hz toward a higher value. (4) For the sliding cases of $\mu = 0.05, 0.1, 0.25$, there exist some minor peaks in the range of lower excitation frequencies, besides the main resonant peak. Such a phenomenon is called the *subharmonic resonance*. For a large frictional coefficient, say, with $\mu = 0.25$, the subharmonic resonant response may be even larger than that of the main resonance.

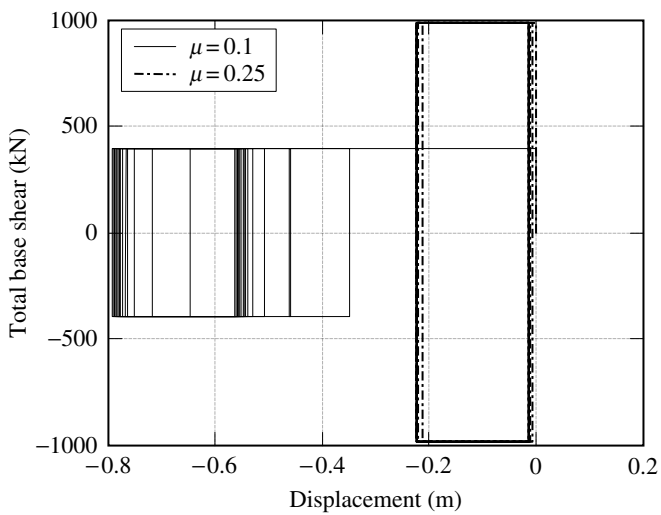


FIGURE 4.25 Hysteresis loops of a sliding bearing ($\omega_g = 1$ Hz; $\omega_e = 5\omega_s$).

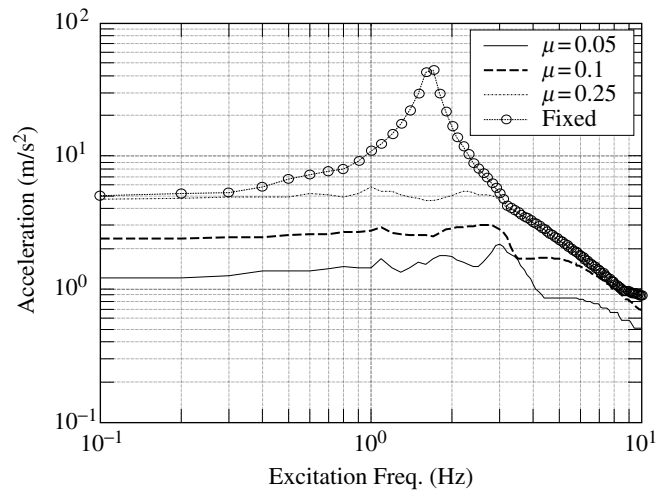


FIGURE 4.26 Maximum structural acceleration vs. ground excitation frequency ($\omega_e = 5\omega_s$).

Figure 4.27 shows the frequency responses of the maximum base displacement for various frictional coefficients. The following are observed: (1) The larger the frictional coefficient, the smaller the base drift is. (2) The base displacement has very large magnitudes in the lower excitation frequencies and decreases monotonically as the excitation frequency increases. (3) The extremely large base drift exhibited in the lower excitation frequency range is due to the initial permanent displacement observed in Figure 4.22.

4.4.3.3 Harmonic Response of Equipment

Time history. Consider an equipment item of a natural frequency equal to five times of the structural frequency, that is, $\omega_e = 5\omega_s$. For the case of a harmonic excitation of $\omega_g = 1$ Hz, the accelerations solved for the equipment mounted on the structure with $\mu = 0.1$ and 0.25 , along with the fixed-base case, have

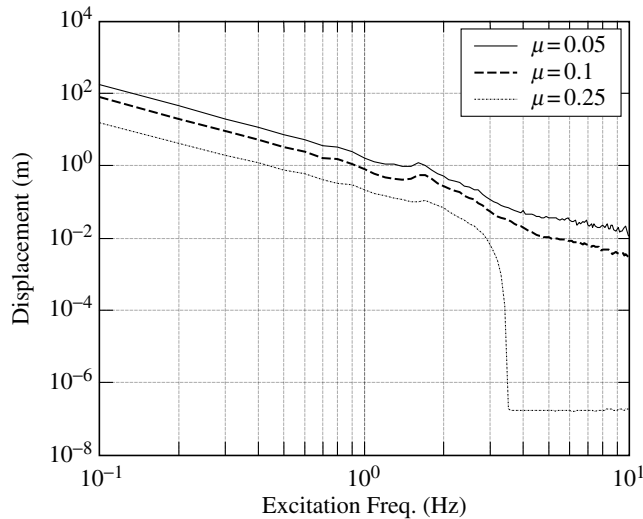


FIGURE 4.27 Maximum base displacement vs. ground excitation frequency ($\omega_e = 5\omega_s$).

been plotted in Figure 4.28. As can be seen, the equipment quickly reaches the steady state within a few cycles of oscillation. The equipment response is effectively suppressed for the case with a smaller frictional coefficient. Additionally, the waveforms shown in Figure 4.28 for the equipment appear to be marginally higher than those of the primary structure shown in Figure 4.24, which can be attributed to the use of a relatively stiff equipment, that is, with $\omega_e = 5\omega_s$.

Frequency responses. For an equipment item of the frequency $\omega_e = 5\omega_s$ ($= 8.34$ Hz), the maximum acceleration response has been plotted as a function of the excitation frequency in Figure 4.29. By comparing Figure 4.29 with Figure 4.26, one observes that the frequency response curves of the equipment and primary structure are generally similar, except that a secondary resonant peak occurs around the equipment natural frequency of 8.34 Hz in Figure 4.29. The other observations from Figure 4.29 are as follows: (1) In comparison with the fixed-base case, the sliding isolation alleviates both the structural and equipment resonant peaks around the frequencies of 1.67 and 8.34 Hz, respectively. However, the level of alleviation is more apparent for the former than for the latter. (2) The equipment also exhibits the same subharmonic resonance behavior as that of the primary structure, in terms of the resonance peaks and frequencies. (3) As the frictional coefficient μ decreases from ∞ (for the fixed-base case) to 0.05, the main resonant frequency associated with the structure drifts toward a higher value. However, the resonant frequency associated with the equipment remains the same.

Effect of equipment tuning. The effect of equipment tuning refers to the case when the equipment frequency is tuned to the structural frequency, that is, $\omega_e = \omega_s$. Figure 4.30 shows the frequency response of the equipment when the equipment tuning occurs. Compared with Figure 4.29, this figure shows the following: (1) When equipment tuning occurs, the sliding isolation system can still mitigate the main resonant peak of the equipment, but the effectiveness of mitigation is drastically reduced. (2) Although the subharmonic resonance can still be observed, the relevant frequencies of the equipment are different from those of the primary structure. (3) The frequency of the maximum resonant response remains equal to the tuned equipment's natural frequency of 1.67 Hz, regardless of the change in the frictional coefficient, μ , from ∞ to 0.05.

4.4.3.4 Earthquake Response of Structure

Time history. For the isolated system subjected to the El Centro earthquake with a PGA of 0.5g, the structural acceleration and base displacement of the sliding system have been plotted in Figure 4.31 and Figure 4.32, respectively, together with the response for the fixed-base case in Figure 4.31. As can be

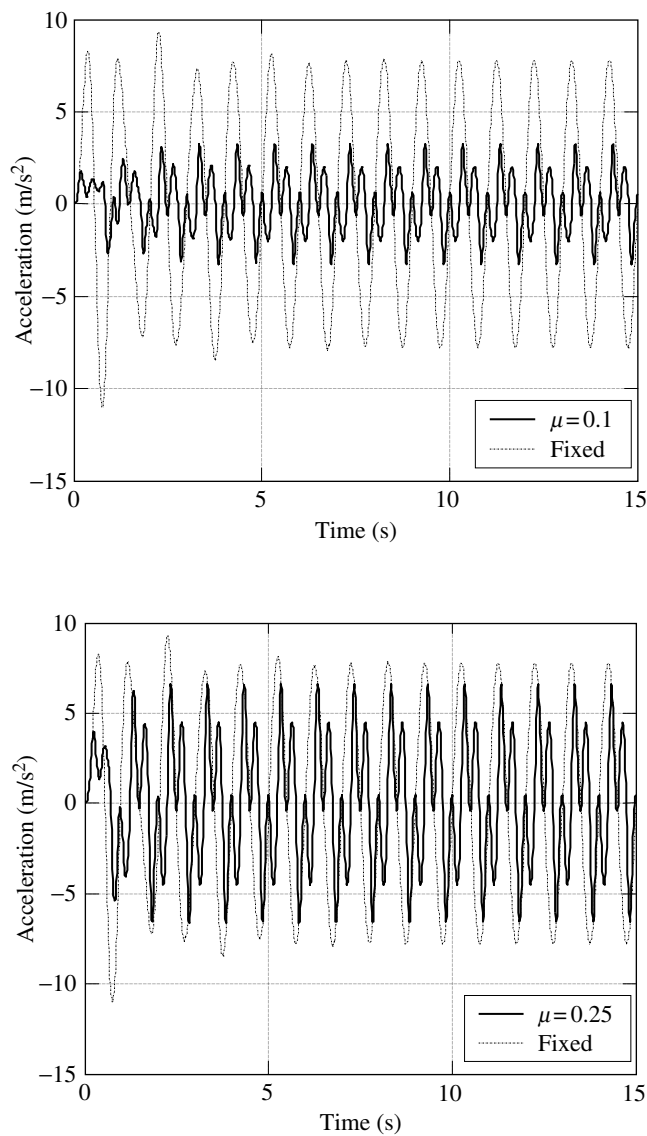


FIGURE 4.28 Comparison of equipment accelerations ($\omega_g = 1$ Hz; $\omega_e = 5\omega_s$).

seen from Figure 4.31, the main-shock response occurring between 0 and 10 sec for the fixed-base structure has been effectively suppressed by the sliding isolators with $\mu = 0.1$ and 0.25 . About 80 and 60% of the maximum structural acceleration have been suppressed by the isolators with $\mu = 0.1$ and 0.25 , respectively. On the other hand, Figure 4.32 demonstrates that the better suppression effect for the case with $\mu = 0.1$ is achieved at the expense of a larger base displacement. It is interesting to note that the horizontal segments in the curves of Figure 4.32 actually represent the stick state of the sliding system, which is useful for unveiling the sliding-stick mechanism involved.

Effect of earthquake intensity. The maximum structural acceleration and base displacement vs. the PGA have been plotted in Figure 4.33 and Figure 4.34, respectively. As can be seen from Figure 4.33, the maximum structural acceleration for the fixed-base case is proportional to the earthquake intensity, while in all the sliding cases it remains essentially as a constant after the PGA reaches a certain level. In other words, the reduction in structural maximum response and the efficiency of isolation have increased

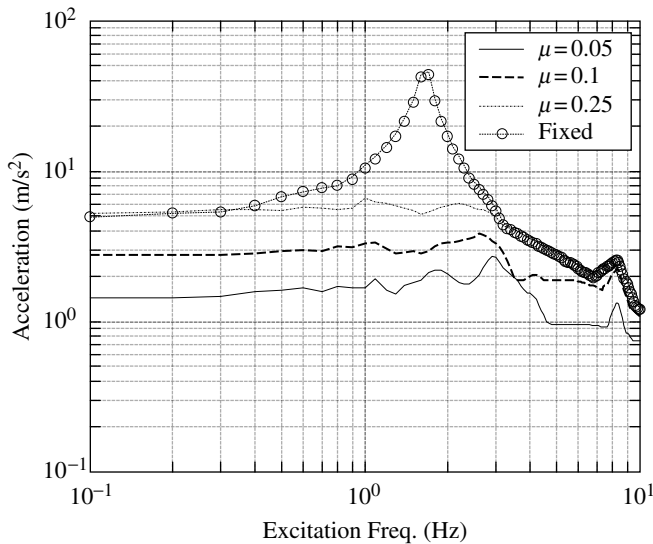


FIGURE 4.29 Maximum equipment acceleration vs. ground excitation frequency ($\omega_e = 5\omega_s$).

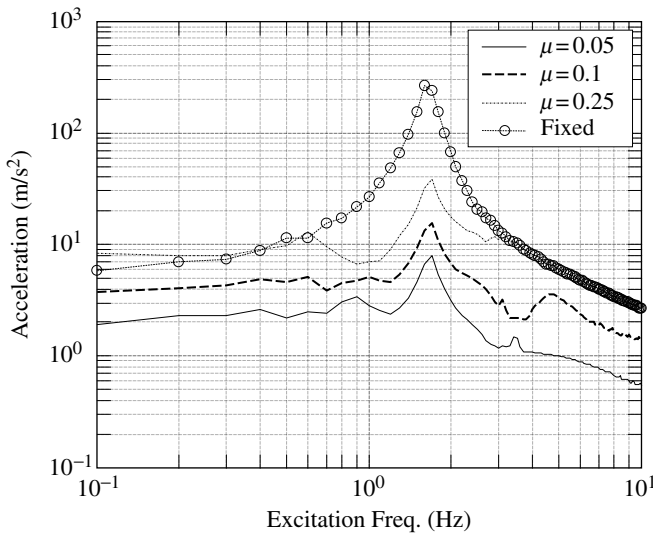


FIGURE 4.30 Maximum equipment acceleration vs. ground excitation frequency for equipment tuning ($\omega_e = \omega_s$).

with the increase in earthquake intensity. For example, for the case of $\mu = 0.1$, Figure 4.33 shows that the structural acceleration is reduced by around 60% at $\text{PGA} = 0.2g$, while it is reduced by more than 90% at $\text{PGA} = 1.0g$. However, as indicated by Figure 4.34, the above reduction in structural response has been achieved at the expense of increased base displacements. For the same PGA level, a sliding system with a smaller frictional coefficient has a better effect of vibration reduction, but this is accompanied by a larger base displacement.

Residual base displacement. The residual base displacement is defined as the permanent base displacement of the structure after it stops vibrating. This quantity is important in the study of sliding structures. Figure 4.35 shows the residual base displacement as a function of the earthquake PGA level. A first look at the figure reveals that no clear relation exists between the earthquake intensity and residual

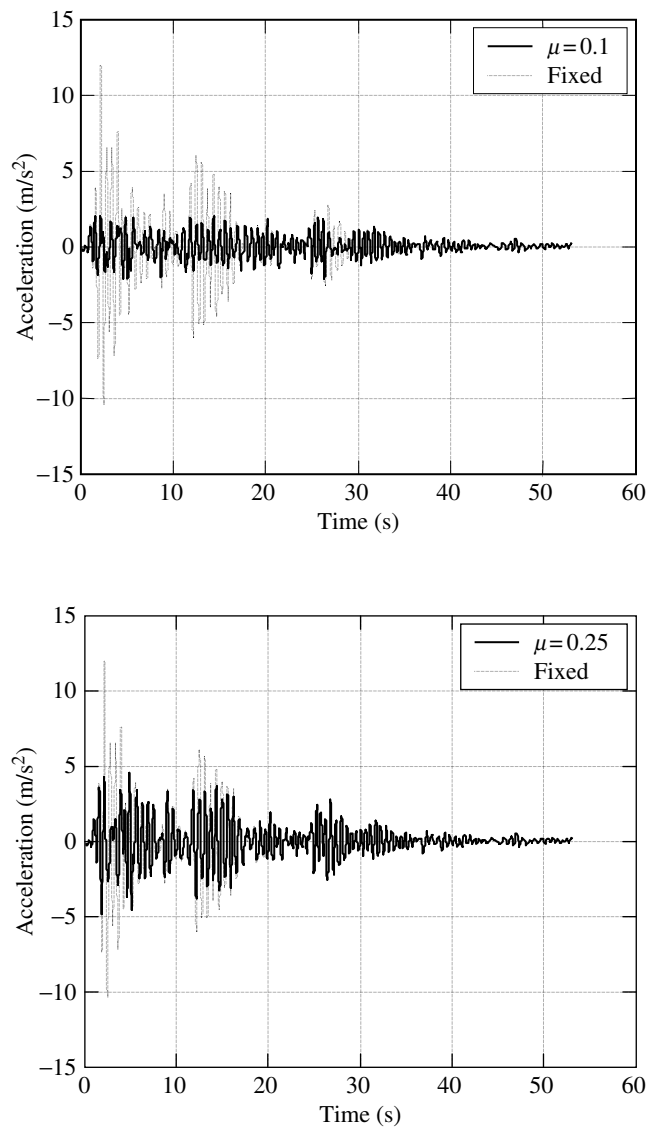


FIGURE 4.31 Comparison of structural accelerations ($\omega_e = 5\omega_s$, PGA = 0.5g).

displacement, because a larger PGA may lead to a smaller residual base displacement. Nevertheless, after taking the average of the residual displacements over the PGA range of 0.1 to 1g, we obtain $x_{res} = 0.083, 0.084, 0.084$ m for the case of $\mu = 0.05, 0.1, 0.25$, respectively. These values indicate that a smaller frictional coefficient leads to a larger residual base displacement in general. However, when the frictional coefficient, μ , approaches zero, the residual displacement approaches a constant equal to the permanent ground displacement.

4.4.3.5 Earthquake Response of Equipment

Time history. Consider an equipment item with a natural frequency equal to five times the structural frequency, that is, $\omega_e = 5\omega_s$ ($= 8.34$ Hz). For the isolated system subjected to the El Centro earthquake

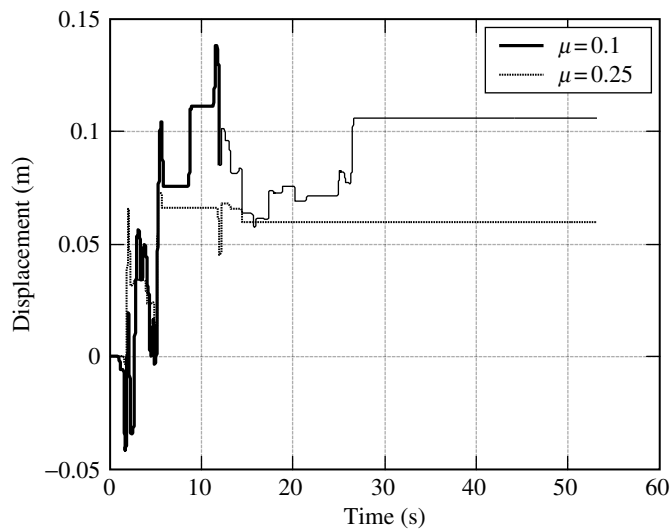


FIGURE 4.32 Comparison of base displacements ($\omega_e = 5\omega_s$, PGA = 0.5g).

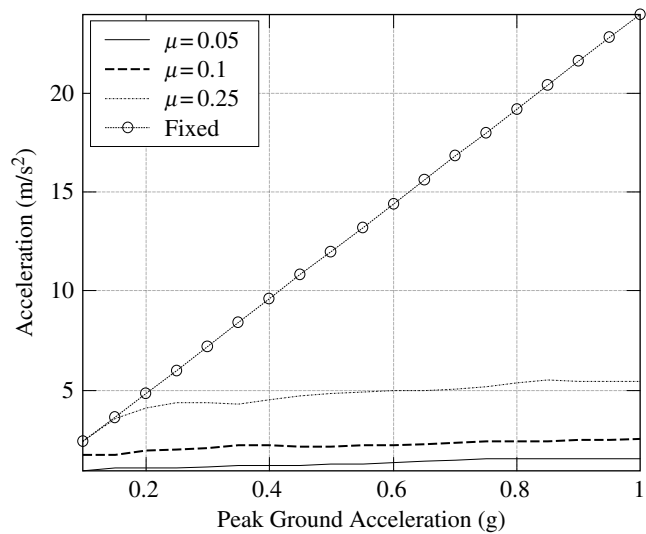


FIGURE 4.33 Maximum structural acceleration vs. PGA ($\omega_e = 5\omega_s$).

with PGA = 0.5g, the time histories computed for the equipment acceleration for the cases with $\mu = 0.1$ and 0.25, along with the fixed-base case, have been plotted in Figure 4.36a and b. As can be seen, the main-shock response of the fixed-base structure occurring for the first 10 sec has been effectively suppressed through installation of the sliding isolator with $\mu = 0.1$ and 0.25. A higher level of reduction can be achieved if a smaller frictional coefficient is chosen.

Effect of earthquake intensity. Figure 4.37 shows the maximum equipment acceleration as a function of the PGA level. Because of the use of a relatively stiff equipment ($\omega_e = 5\omega_s$), the curves shown in Figure 4.37 are similar to those for the primary structure in Figure 4.33, but with slightly higher values. Therefore, the observations made previously for Figure 4.33 are applicable to Figure 4.37. The maximum response of equipment items with other frequencies will be discussed below.

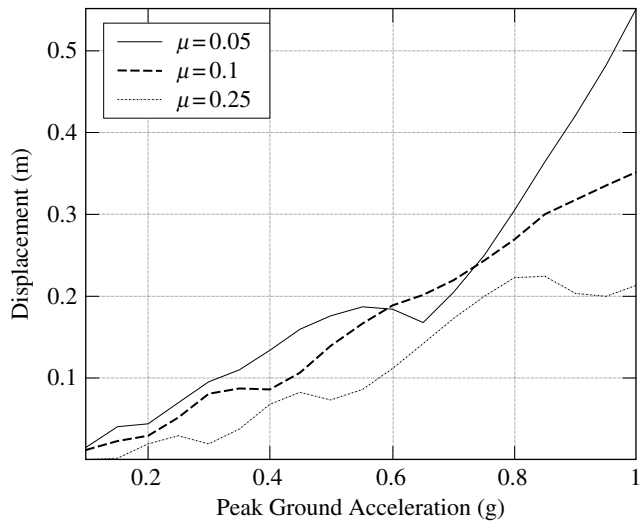


FIGURE 4.34 Maximum base displacement vs. PGA ($\omega_c = 5\omega_s$).

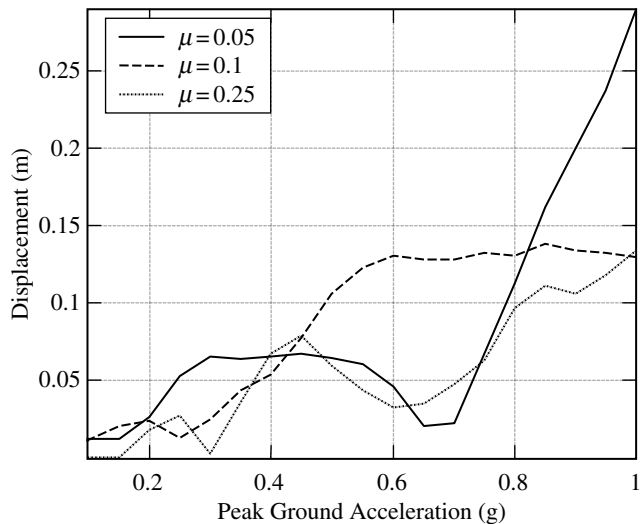


FIGURE 4.35 Residual base displacement vs. PGA ($\omega_c = 5\omega_s$).

Effect of equipment tuning. In order to study the equipment tuning effect, the maximum equipment acceleration has been plotted as a function of the equipment frequency in Figure 4.38. As can be seen, for all the values of μ considered, the equipment response is amplified when the equipment frequency moves close to the structural frequency of 1.67 Hz for the fixed-base case. Note that, since the resonant frequency of a sliding structure shifts to a higher value as the frictional coefficient decreases (see Figure 4.26), the frequency for which the most severe tuning effect occurs in Figure 4.38 also shifts from 1.67 Hz to a higher value as μ decreases. Nevertheless, it is concluded that, by choosing a smaller μ , the amplification of the equipment response due to tuning effect can be effectively suppressed.

4.4.4 Concluding Remarks

The dynamic behavior of a sliding isolated structural system with an attached equipment item was investigated in this section. A sliding isolated structure is classified as a nonlinear dynamic system, as the

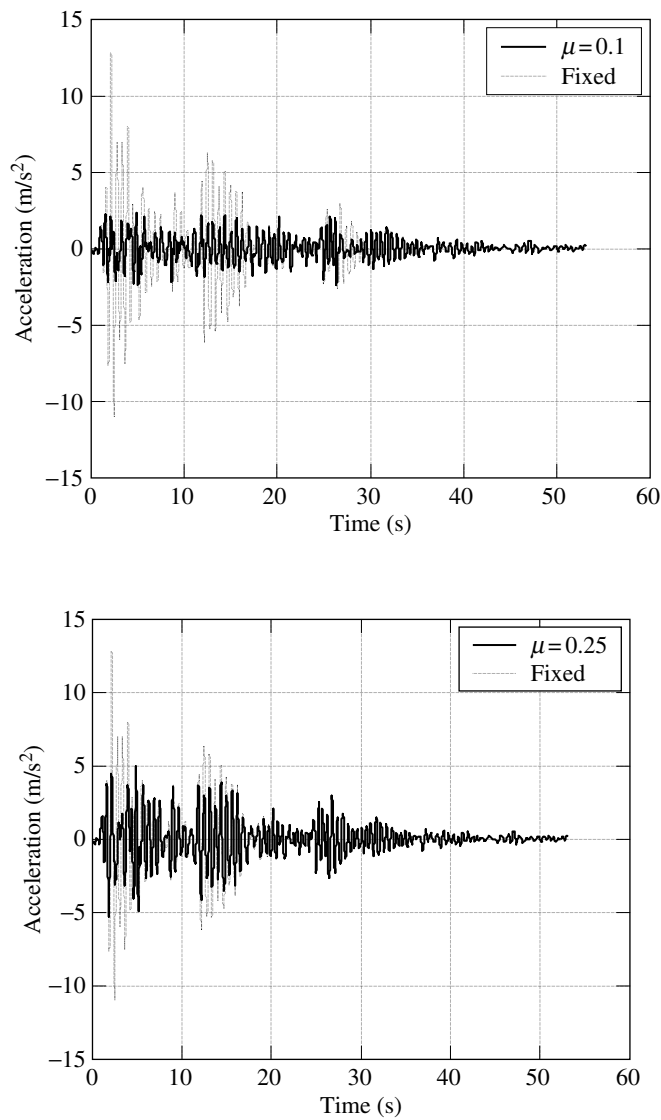


FIGURE 4.36 Comparison of equipment accelerations ($\omega_e = 5\omega_s$, $\text{PGA} = 0.5g$).

frictional forces induced on the sliding surface do not remain constant. To deal with such nonlinear systems, two analysis methods were formulated, the shear balance method and the fictitious spring method, both of which were presented in an incremental form that is suitable for direct implementation. Through the selection of a sliding isolated structure–equipment model, the responses of the structure and equipment subjected to both harmonic and earthquake excitations were analyzed. For the case of harmonic excitation, the results showed that the resonant responses of both the structure and attached equipment can be effectively suppressed, which remains good even when the equipment frequency is tuned to the structural frequency. For the case of seismic excitation, the results indicated that the level of reduction on the structural and equipment responses increases as the PGA level of the earthquake increases. Moreover, a sliding system with a smaller frictional coefficient has a higher isolation efficiency, at the expense of a larger base displacement.

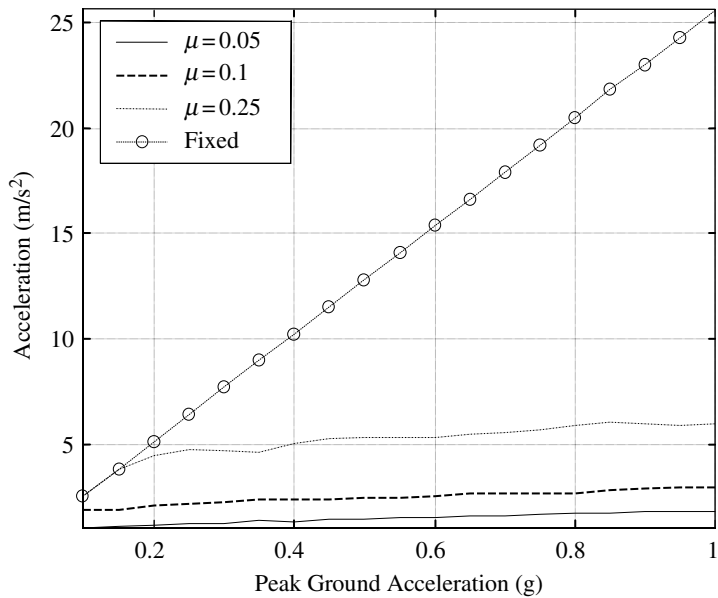


FIGURE 4.37 Maximum equipment acceleration vs. PGA ($\omega_e = 5\omega_s$).

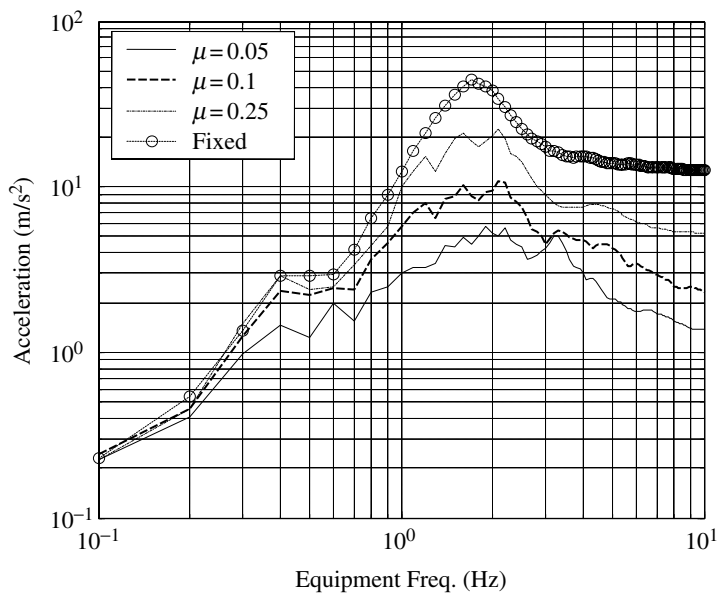


FIGURE 4.38 Maximum equipment acceleration vs. equipment frequency (PGA = 0.5g).

4.5 Sliding Isolation Systems with Resilient Mechanism

In Section 4.4, the relevant equations of analysis have been presented for a sliding isolated structural system, with no consideration made for the resilient (or recentering) mechanism. Because of this, rather large residual displacements may occur on the sliding isolation system, as have been numerically illustrated. If the concept of sliding isolators is to be applied to a real structure, it is important that the residual base displacements be controlled within certain limits, since they are not tolerable for some

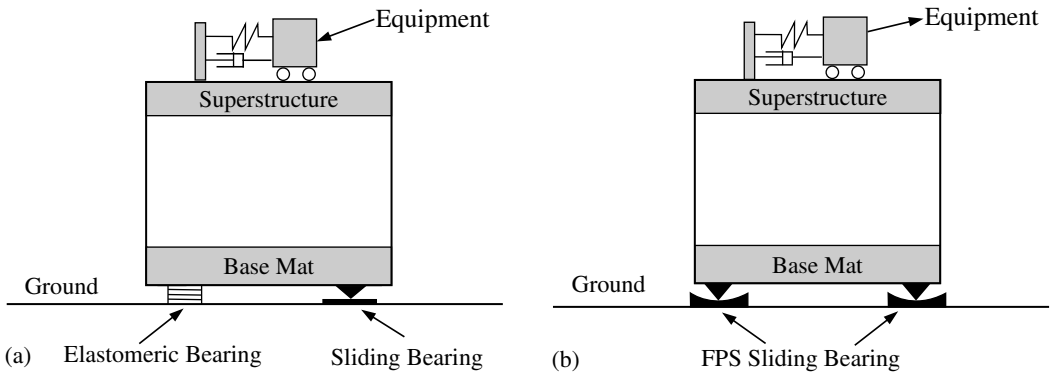


FIGURE 4.39 Schematic for a structure–equipment system isolated by a sliding bearing with resilience capability: (a) combined isolation system; (b) friction pendulum system (FPS).

engineering applications. For example, residual base displacements may distort the networking of water and power lines, change the space between the isolated structure and adjacent buildings, and widen the gaps at building entrances. Therefore, in practice, a sliding isolation system is usually enhanced through inclusion of mechanisms that can provide some resilient force. However, for certain ground motions, the added resilient force may also present some negative effects not readily transparent to structural designers, as will be illustrated in the numerical studies later on.

As shown in Figure 4.39, there are at least two ways of implementing the resilient mechanism in a sliding isolation system. Figure 4.39a shows an isolation system that combines the elastomeric bearings with sliding bearings (Chalhoub and Kelly, 1990), in which the elastomeric bearings are used to provide the resilient force, and the sliding bearings to uncouple the structural system from the ground motion. On the other hand, the resilient mechanism can also be incorporated into each single sliding bearing, in a way similar to that in the RFBI described in Section 4.2.3 (Mostaghel and Khodaverdian, 1987) or the FPS shown in Figure 4.39b (Mokha et al., 1991). The FPS isolation system has been implemented in many existing buildings and bridges. A typical FPS bearing consists a spherical sliding surface and a slider, which usually has a smooth coating of very low friction. When an FPS device is implemented under a structure, the slider will slide on the spherical surface during an earthquake, and the gravitational load of the structure, together with the curved sliding surface, will provide the resilient force for the system to return to its original position. The resilient stiffness of an FPS bearing depends on the radius of curvature of the sliding surface and the structural weight carried by the bearing.

This section is aimed at investigating the behavior of a structure–equipment system isolated by a sliding system with resilient device. For convenience of discussion, a system with resilient device will be referred to as the *resilient sliding isolation* (RSI), and a sliding system without resilient device, as the one studied previously, as the *pure sliding isolation* (PSI).

4.5.1 Mathematical Modeling and Formulation

Both the RSI systems shown in Figure 4.39 can be represented by the mathematical model given in Figure 4.40, for which the symbols used have been defined in Table 4.1. The RSI model shown in Figure 4.40 differs from the PSI model shown Figure 4.16 in that a linear spring of stiffness, k_b , is added to simulate the resilient force of the isolator. Obviously, an RSI model can be considered as the composition of a friction element and a spring element in parallel. Owing to addition of resilient stiffness, the number of vibration frequencies of the system is increased by one. The newly introduced frequency, which depends on the resilient stiffness, is called the *isolation frequency*, which can be approximated by

$$\omega_b = \sqrt{k_b/(W/g)} \quad (4.61)$$

where W is the total weight of the isolated structure–equipment system. The isolation frequency commonly used in design is between 0.33 and 0.5 Hz, which implies a period of 2 to 3 sec (Naeim and Kelly, 1999).

For the type of combined sliding system shown in Figure 4.39(a), the actual value of resilient stiffness, k_b , is decided by the total horizontal stiffness of the elastomeric bearings implemented. On the other hand, for the FPS shown in Figure 4.39(b), the resilient stiffness, k_b , is approximated by the following equation for small isolator displacements:

$$k_b = \frac{W}{R} \quad (4.62)$$

where R denotes the radius of curvature of the sliding surface. By substituting Equation 4.62 into Equation 4.61, one can verify that the isolation period of an FPS is equal to the oscillation period of a pendulum; that is

$$T_b = \frac{2\pi}{\omega_b} = \frac{2\pi}{\sqrt{k_b/(W/g)}} = 2\pi\sqrt{R/g} \quad (4.63)$$

When the sliding isolated system of Figure 4.40 is excited by a ground motion, its equation of motion may be written in exactly the same form as that of Equation 4.27; that is

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}\mathbf{L}_1\ddot{x}_g(t) + \mathbf{L}_2f(t) \quad (4.64)$$

All the variables used in the preceding equation are the same as those defined in Equation 4.28 to Equation 4.30, except that the stiffness matrix, $\mathbf{K}$, should be modified as

$$\mathbf{K} = \begin{bmatrix} k_e & -k_e & 0 \\ -k_e & k_e + k_s & -k_s \\ 0 & -k_s & k_s + k_b \end{bmatrix} \quad (4.65)$$

Note that, in Equation 4.64, the frictional force, $f(t)$, which does not remain constant, is placed on the right-hand side, while the resilient stiffness, k_b , which remains constant, is absorbed by the stiffness matrix, $\mathbf{K}$, as in Equation 4.65. However, if the total shear force, $s(t)$ of the isolation system is of interest, it should be computed as the summation of the frictional force and resilient force (see Figure 4.40); that is

$$s(t) = k_b x_b(t) + f(t) \quad (4.66)$$

The equation of motion as given in Equation 4.64 can be recast in the following form of the first-order state-space equation:

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{E}\ddot{x}_g(t) + \mathbf{B}f(t) \quad (4.67)$$

The definitions of the matrices $\mathbf{z}(t)$, $\mathbf{E}$, $\mathbf{A}$, and $\mathbf{B}$ are the same as those defined in Equation 4.32 and Equation 4.33, except that the system matrix, $\mathbf{A}$, should be modified to account for the addition of the resilient stiffness k_b in the stiffness matrix, $\mathbf{K}$.

4.5.2 Methods for Numerical Analysis

If one compares the equation of motion for the RSI system in Equation 4.67 with that for the PSI system in Equation 4.31, one will conclude that the only source of nonlinearity in both equations comes from

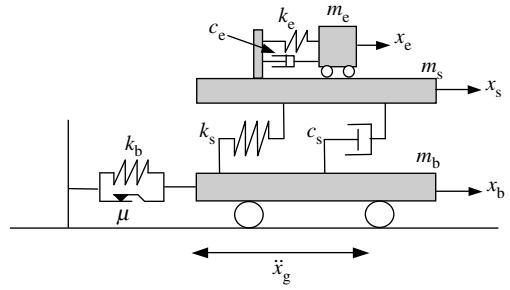


FIGURE 4.40 Model for a structure–equipment system isolated by a sliding bearing with resilience capability.

the same term, namely, the frictional force, $f(t)$. As a result, the two methods of solution mentioned in Section 4.4.2, the shear balance method (Wang et al., 1998) and fictitious spring method (Yang et al., 1990), remain valid for the analysis of the RSI systems, with no modification required. Moreover, owing to inclusion of the resilient stiffness, k_b , in the structural stiffness matrix, $\mathbf{K}$, the system matrix, $\mathbf{A}$, becomes nonsingular and invertible. This introduces some advantage in computation of relevant coefficient matrices, including the $\mathbf{B}_0$ and $\mathbf{B}_1$ matrices in Equation 4.38a and Equation 4.38b. In Section 4.5.3, the shear balance method will be employed to simulate the response of an RSI system.

4.5.3 Simulation Results for Sliding Isolation with Resilient Mechanism

4.5.3.1 Numerical Model and Ground Excitations

In this section, the dynamic behavior of a sliding system represented by the model shown in Figure 4.40 will be investigated. The data adopted in the analysis for the equipment, structure, and the isolator have been listed in Table 4.4. To facilitate comparison, some of the data are selected to be the same as those in Table 4.3. In particular, the isolation frequency chosen is $\omega_b = 0.4$ Hz, falling in the common range of 0.33 to 0.5 Hz. Again, two types of ground excitations are considered, namely, the harmonic and earthquake excitations. For the harmonic excitation, a waveform of ground acceleration identical to the one given in Equation 4.60 is used. And for the earthquake excitation, the 1940 El Centro earthquake with different levels of PGA will be used, of which the acceleration waveform has been given in Figure 4.10. The harmonic excitation is adopted mainly for studying the frequency response of the sliding isolated system, while the earthquake excitation is for studying the effect of earthquake intensity. The dynamic responses computed for the RSI system, including the structure and equipment, will be presented, with emphasis placed on comparison with the PSI system of the same parameters. Similar to what was done in Section 4.4.3, the symbol μ will be used to denote the frictional coefficient of the sliding isolation system in all figures, and the word “fixed” denotes the fixed-base structure.

4.5.3.2 Harmonic Response of Structure

Time history. Consider an RSI system subjected to a harmonic excitation of $\omega_g = 1$ Hz. The base displacement and structural acceleration of the RSI system have been plotted in Figure 4.41 and Figure 4.42, respectively. Clearly, both the base displacement and structural acceleration of the RSI system reach their steady-state harmonic responses in the first few cycles. Moreover, a smaller sliding frictional coefficient ($\mu = 0.1$) is more effective for suppressing the structural acceleration, as indicated by Figure 4.42. However, this is achieved only at the expense of a larger base displacement, as indicated by Figure 4.41. By comparing the result for the RSI system in Figure 4.41 with those for the PSI system in Figure 4.23, the effect of resilient mechanism in eliminating the permanent base displacement for the case with a small frictional coefficient of $\mu = 0.1$ can be clearly appreciated. In spite of the large difference in base displacement, the structural accelerations for the RSI and PSI systems shown in Figure 4.42 and Figure 4.24, respectively, appear to be quite similar, when interpreted in terms of the waveform and response amplitude. This implies that, for the harmonic excitation considered, the resilient mechanism in RSI has little influence on the isolation effectiveness.

Hysteretic behavior. In Figure 4.43, the hysteresis loops for RSI systems with $\mu = 0.1$ and 0.25 subjected to a harmonic excitation of $\omega_g = 1$ Hz have been plotted, in which the vertical axis represents

TABLE 4.4 System Parameters Used in Simulation (Section 4.5.3)

Equipment		Superstructure		Isolation System	
Parameter	Value	Parameter	Value	Parameter	Value
Mass m_e	3 t (= $m_s/100$)	Mass m_s	300 t	Mass m_b	100 t (= $m_s/3$)
Frequency ω_e	$5\omega_e$ or a variable	Frequency ω_s	1.67 Hz	Frictional coefficient μ	0.05, 0.1, 0.25
Damping ratio ζ_e	5%	Damping ratio ζ_s	5%	Isolation frequency ω_b	0.4 Hz

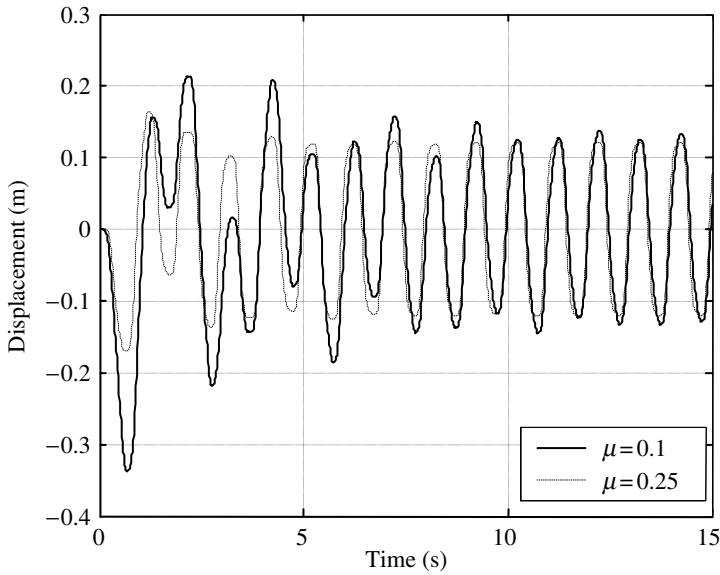


FIGURE 4.41 Comparison of base displacements ($\omega_g = 1$ Hz, $\omega_e = 5\omega_s$).

the total shear, $s(t)$, of the bearing computed from Equation 4.66. It is interesting to note that, for an RSI system, the hysteresis loop is a parallelogram, of which the slope of the inclined upper and lower sides exactly represents the total resilient stiffness, k_b , and the height and width, respectively, are decided by the maximum frictional force and maximum base displacement. As the frictional coefficient decreases, the height of the parallelogram decreases, but the width increases. The total area of the hysteresis loop represents the portion of energy dissipated by the RSI system. Noteworthy is the fact that, when the resilient stiffness, k_b , reduces to zero, the hysteresis parallelogram reduces to a square as well, identical to the one shown in Figure 4.25 for the PSI system.

Frequency responses. The maximum accelerations of the steady-state response of the structure for four different frictional coefficients, that is, $\mu = 0.05, 0.1, 0.25$ and ∞ (fixed-base), have been plotted in Figure 4.44. From this figure, the following observations can be made: (1) Compared with the fixed-base case, the resonant peak occurring around the structural frequency, ω_s , of 1.67 Hz was effectively suppressed by the RSI system, but a resonance of higher amplitude was induced in the lower frequency range (with frequencies lower than 0.6 Hz for the case studied). A further investigation reveals that the newly induced resonance is associated with the isolation frequency, ω_b , of 0.4 Hz. Such an observation remains valid for all values of frictional coefficients, μ . (2) The use of a lower frictional coefficient, μ , will result in a smaller response in the high-frequency range for the RSI system, for example, with frequencies higher than 0.6 Hz, but a larger response for the low-frequency range. (3) Although both the RSI and PSI systems can effectively remove the resonant peak around the structural frequency of 1.67 Hz, the RSI system has the side effect of creating a low-frequency resonant peak at the isolation frequency, ω_b . This implies that the RSI system is more sensitive to the excitation frequency.

The frequency responses of the maximum base displacement for the RSI system with various frictional coefficients have been plotted in Figure 4.45. When compared with the results for the PSI system in Figure 4.27, it is clear that the resilient mechanism of the RSI system considerably reduces the base displacement in the nonresonant excitation range, but it also amplifies the base displacement in the region when the excitation frequency is close to the isolation frequency, ω_b . From Figure 4.44 and Figure 4.45, we observe that both the structural acceleration and base displacement of an RSI system may resonate at the isolation frequency, which is usually designed to be less than 0.5 Hz. This implies that an RSI system may be ineffective or unsafe for a ground motion with enriched low-frequency

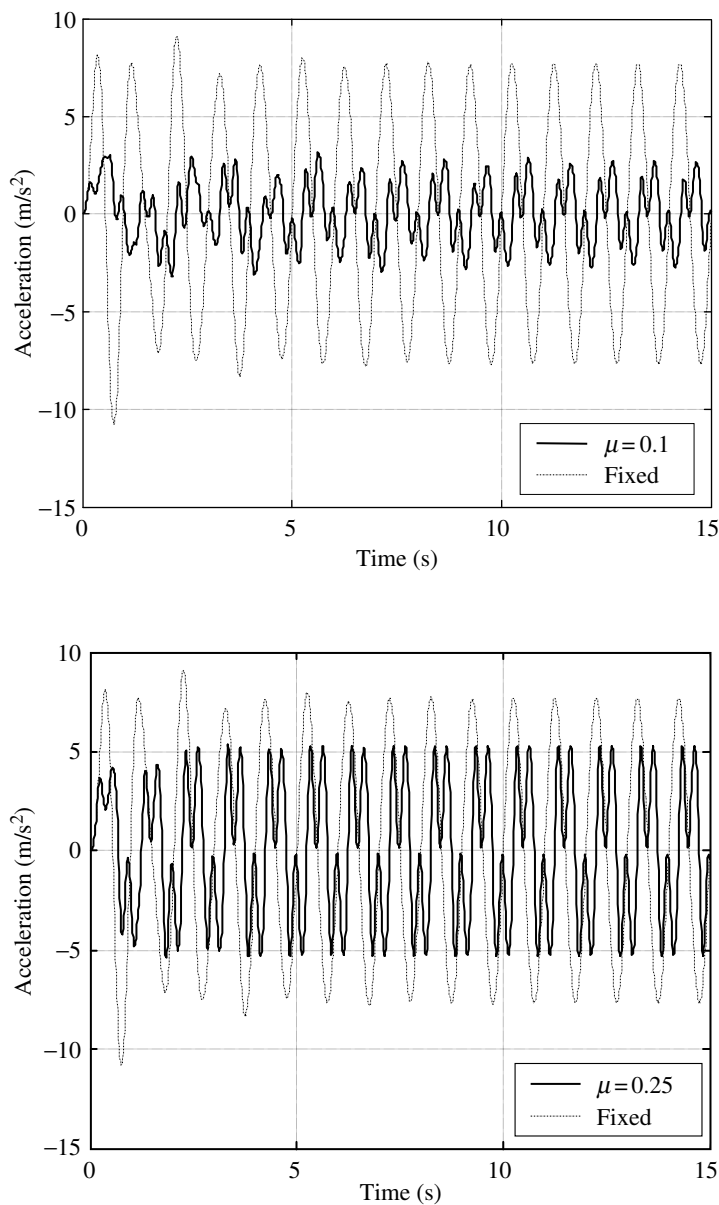


FIGURE 4.42 Comparison of structural accelerations ($\omega_g = 1$ Hz, $\omega_e = 5\omega_s$).

(long-period) vibrations, such as the case with a near-fault earthquake containing a long-period, pulse-like waveform (Jangid and Kelly, 2001; Lu et al., 2003). Structural designers should be aware of such a side effect when designing an RSI system.

4.5.3.3 Harmonic Response of Equipment

Time history. Consider an equipment item with a frequency of $\omega_e = 5\omega_s$ ($= 8.34$ Hz), attached to the RSI system. The harmonic acceleration responses of the equipment for the case with $\mu = 0.1$ and 0.25 have been plotted in Figure 4.46a and b, respectively, together with those for the fixed-base case. As can be seen, the equipment acceleration has been effectively suppressed by the RSI with the smaller frictional coefficient ($\mu = 0.1$). Moreover, the acceleration waveforms shown in Figure 4.46 are similar to those of

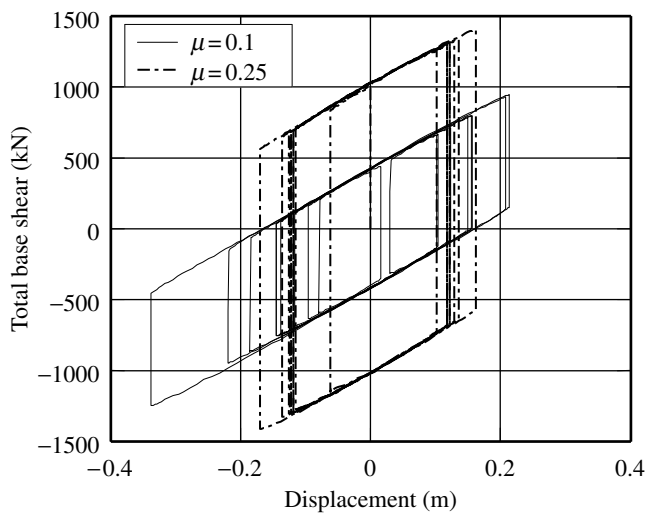


FIGURE 4.43 Hysteresis loops of a sliding bearing with resilience capability ($\omega_g = 1$ Hz, $\omega_e = 5\omega_s$).

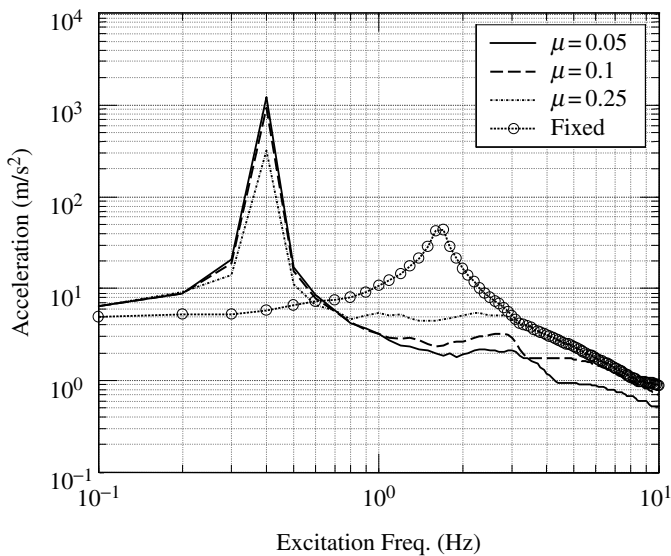


FIGURE 4.44 Maximum structural acceleration vs. ground excitation frequency ($\omega_e = 5\omega_s$).

the PSI system shown in Figure 4.28. This implies that for the given excitation, the behavior of the equipment was not altered by introduction of the resilient mechanism in the RSI system.

Frequency responses. Figure 4.47 shows the acceleration frequency response curve of the attached equipment with a frequency of $\omega_e = 5\omega_s$ (8.34 Hz). A comparison of Figure 4.47 with Figure 4.44 indicates that the frequency responses of the equipment and primary structure are generally similar, except that a resonant peak associated with the equipment frequency around 8.34 Hz appears in Figure 4.47. Owing to such a similarity, the observations made previously for Figure 4.44 are applicable to Figure 4.47 for the attached equipment.

Effect of equipment tuning. Figure 4.48 shows the frequency response of the attached equipment for the case when the equipment frequency is tuned to the structural frequency, that is, with $\omega_e = \omega_s$. Similar to

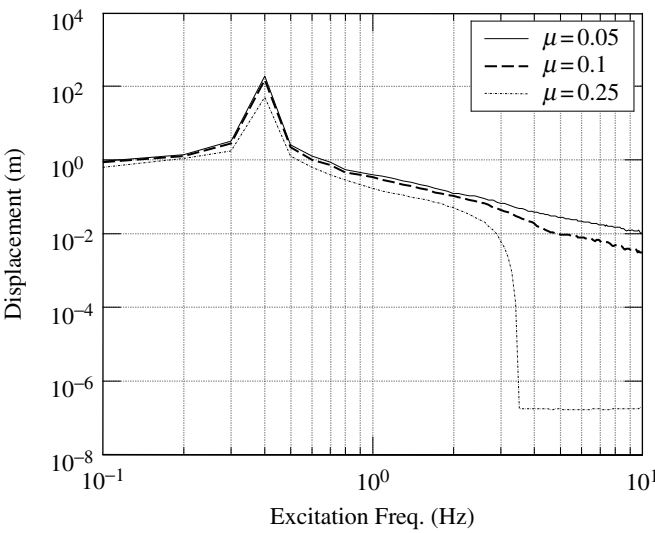


FIGURE 4.45 Maximum base displacement vs. ground excitation frequency ($\omega_e = 5\omega_s$).

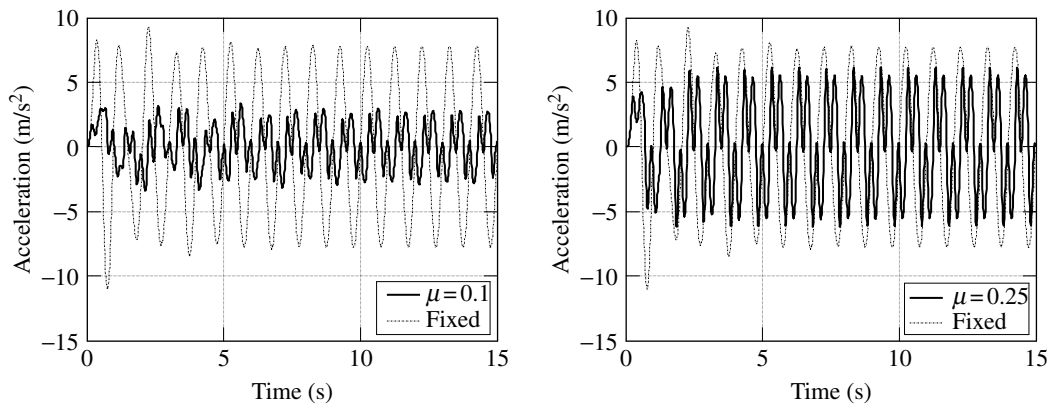


FIGURE 4.46 Comparison of equipment accelerations ($\omega_g = 1$ Hz, $\omega_e = 5\omega_s$).

the structural frequency response shown in Figure 4.44, the equipment attached to the RSI system also resonates at the isolation frequency ω_b of 0.4 Hz. Such a resonance does not occur for the equipment attached to the PSI system (see Figure 4.30). Through comparison of the tuned case in Figure 4.48 with the detuned case in Figure 4.47, the following observations can be made: (1) Even when the equipment tuning occurs, an RSI system mitigates the equipment's resonant peak associated with the structural frequency at 1.67 Hz, although the effectiveness of isolation has been reduced. (2) The tuning effect has no influence on the resonant response associated with the isolation frequency of 0.4 Hz.

4.5.3.4 Earthquake Response of Structure

Time history. For an RSI system subjected to the El Centro earthquake with $PGA = 0.5g$, the structural acceleration and base displacement have been shown in Figure 4.49 and Equation 4.50, respectively. By comparing Figure 4.49 with Figure 4.31 for the corresponding PSI system, one observes that the structural accelerations of the RSI and PSI systems are generally similar, in terms of the response waveform and the response magnitude. Both systems reduce the maximum structural acceleration quite effectively, for example, by about 80% for $\mu = 0.1$. However, significant difference does exist between the

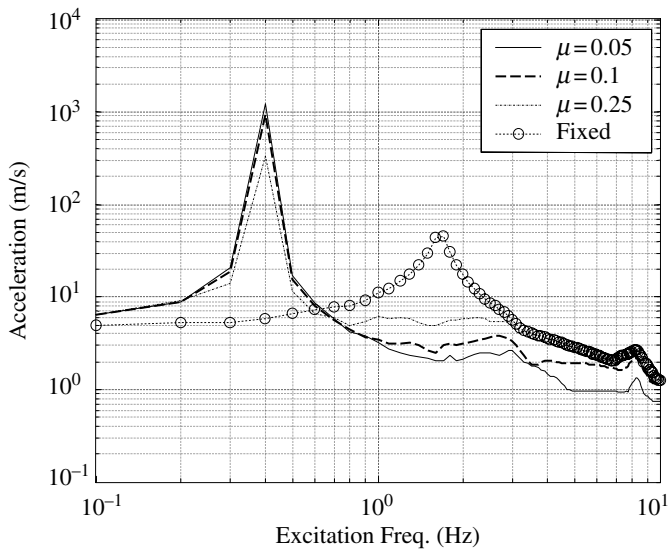


FIGURE 4.47 Maximum equipment acceleration vs. ground excitation frequency ($\omega_e = 5\omega_s$).

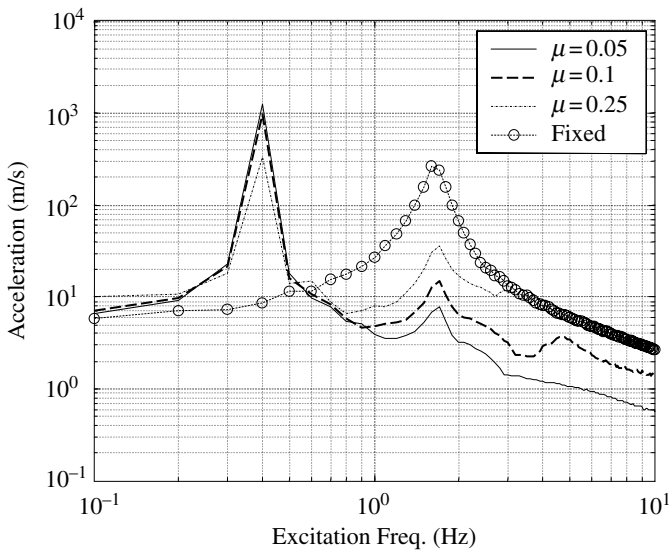


FIGURE 4.48 Maximum equipment acceleration vs. ground excitation frequency under tuning condition ($\omega_e = 5\omega_s$).

base displacements for the RSI system in Figure 4.50 and those for the PSI system in Figure 4.32. For example, for $\mu = 0.1$, the maximum base displacement experienced by the RSI system has been reduced by about 30%, while the residual base displacement has been reduced by about 70%, as can be seen by comparing Figure 4.50 with Figure 4.32. This implies that the resilient mechanism of the RSI system plays an important role in reducing the maximum and residual base displacements, especially the latter.

In spite of the observations made above, one should not forget that the frequency content of one earthquake may be different from another. As was demonstrated in Figure 4.44 and Figure 4.45, an RSI system is generally sensitive to low-frequency excitations and may resonate at the isolation frequency.

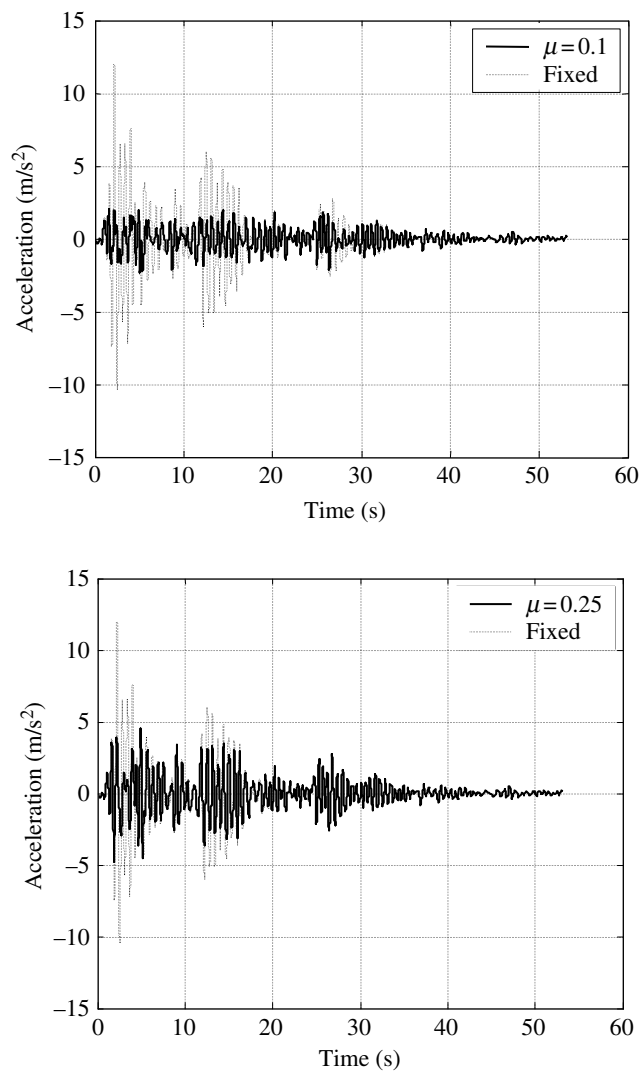


FIGURE 4.49 Comparison of structural accelerations ($\omega_c = 5\omega_s$, PGA = 0.5g).

Therefore, if the RSI system is subjected to an earthquake containing more low-frequency components, unlike the El Centro earthquake, it is likely that the maximum structural responses induced exceed those of the PSI system.

Effect of earthquake intensity. The maximum structural acceleration and base displacement of the RSI system have been plotted with respect to the PGA in Figure 4.51 and Figure 4.52, respectively. These figures indicate that as the earthquake intensity increases from 0.1 to 1g, the structural acceleration is reduced by an increasing amount by the RSI system, while the maximum base displacement also increases. By comparing Figure 4.51 and Figure 4.52 with Figure 4.33 and Figure 4.34 for the PSI system, one observes that both the RSI and PSI systems perform equally well for the El Centro earthquake, although the PSI system induces a slightly larger base displacement. On the other hand, unlike the response for the PSI system, the use of a smaller frictional coefficient for the RSI system does not always lead to a lower structural acceleration, as can be verified by comparing the responses for $\mu = 0.1$ and 0.05 with a PGA greater than 0.8g in Figure 4.33 and Figure 4.51. This can be attributed to the large resilient force induced by the large base displacement under higher PGA levels.

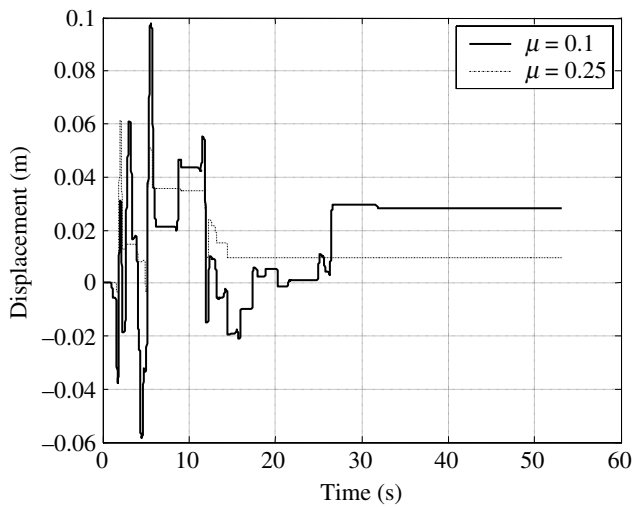


FIGURE 4.50 Comparison of base displacements ($\omega_c = 5\omega_s$, PGA = 0.5g).

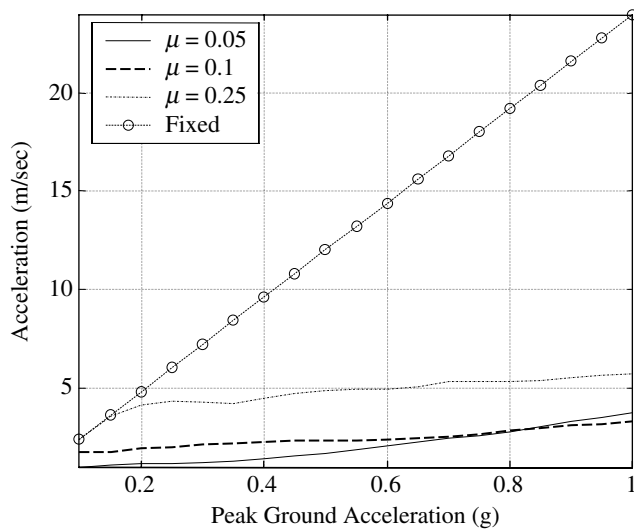


FIGURE 4.51 Maximum structural acceleration vs. PGA ($\omega_c = 5\omega_s$).

Residual base displacement. Figure 4.53 shows the residual base displacement of the RSI system vs. the PGA of the earthquake. For a given μ , it is difficult to establish a relation between the earthquake intensity and residual displacement, because a larger PGA may result in a smaller residual base displacement in some cases. However, if one takes the average of residual displacements over the PGA range from 0.1 to 1g, the following can be computed: $x_{res} = 0.0065, 0.011$, and 0.014 m for $\mu = 0.05, 0.1$, and 0.25 , respectively. These values indicate that a smaller frictional coefficient leads to a smaller residual base displacement, which can be attributed to the fact that for a SRI system with a smaller coefficient of friction, it is easier for the resilient mechanism to return the structure to its initial position after an earthquake. On the other hand, a comparison of Figure 4.53 with Figure 4.35 for the PSI system indicates that for the same value of μ , the residual displacement was reduced substantially by the RSI system. This is certainly an advantage offered by the resilient mechanism of the RSI system.

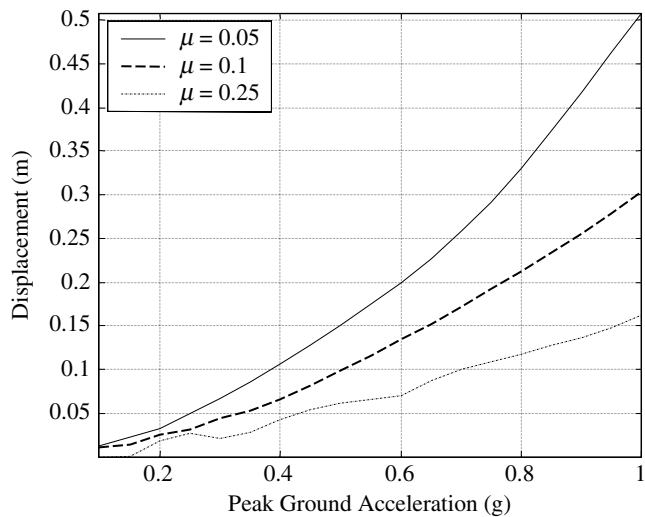


FIGURE 4.52 Maximum base displacement vs. PGA ($\omega_c = 5\omega_s$).

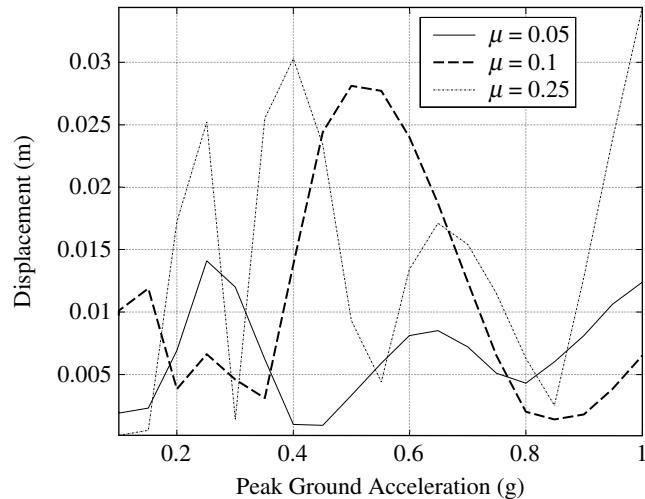


FIGURE 4.53 Residual base displacement vs. PGA ($\omega_c = 5\omega_s$).

4.5.3.5 Earthquake Response of Equipment

Time history. Let us consider an equipment item of natural frequency equal to five times the structural frequency, that is, $\omega_c = 5\omega_s$ ($= 8.34$ Hz). The acceleration responses of the equipment mounted on the RSI system that were subjected to the El Centro earthquake with a PGA of 0.5g for $\mu = 0.1$ and 0.25 have been plotted in Figure 4.54a and b, respectively, along with those for the fixed-base cases. As can be seen, the main-shock response of the equipment appearing during the first 10 sec for the fixed-base system was effectively suppressed by the RSI system with $\mu = 0.1$ or 0.25. The level of reduction is more pronounced for the case with a smaller frictional coefficient, that is, with $\mu = 0.1$. By comparing Figure 4.54 with Figure 4.36 for the PSI system, one concludes that the effect of the resilient mechanism of the RSI system on the equipment response is insignificant for the earthquake and equipment frequency considered.

Effect of earthquake intensity. Figure 4.55 shows the maximum equipment acceleration vs. the PGA of the earthquake. This figure illustrates that for all values of the frictional coefficient, μ , considered, an

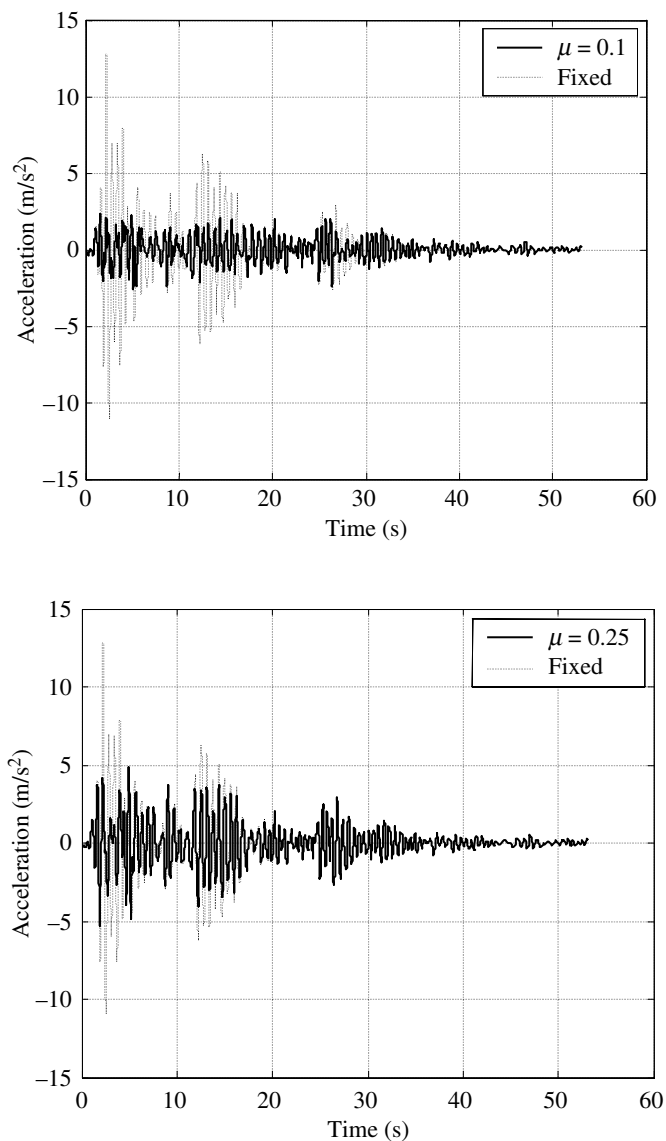


FIGURE 4.54 Comparison of equipment accelerations ($\omega_e = 5\omega_s$, PGA = 0.5g).

increasing amount of reduction can be achieved by the RSI system as the earthquake intensity increases from 0.1 to 1g. Because relatively stiff equipment (i.e., with $\omega_e = 5\omega_s$) was assumed in the simulation, the curves shown in Figure 4.55 are similar to those of Figure 4.51 for the primary structure. Therefore, the observations made previously for Figure 4.51 apply here. The maximum response of equipment items with other natural frequencies will be discussed below. Moreover, a comparison of Figure 4.55 with Figure 4.37 (for the PSI system) reveals that the resilient mechanism can have some minor effect on the equipment response, but only when a smaller frictional coefficient (i.e., $\mu = 0.05$ or 0.1) is used and when the PGA level is high.

Effect of equipment tuning. In order to study the effect of equipment tuning, the maximum acceleration of the equipment has been plotted in Figure 4.56 for equipment frequencies ranging from 0.1 to 10 Hz.

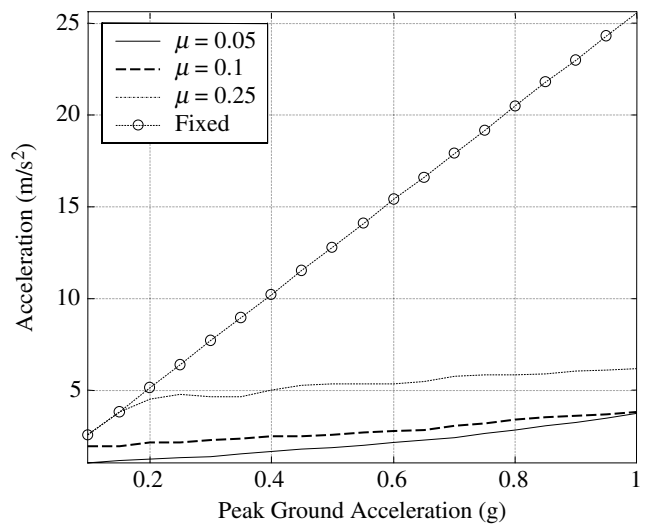


FIGURE 4.55 Maximum equipment acceleration vs. PGA ($\omega_e = 5\omega_s$).

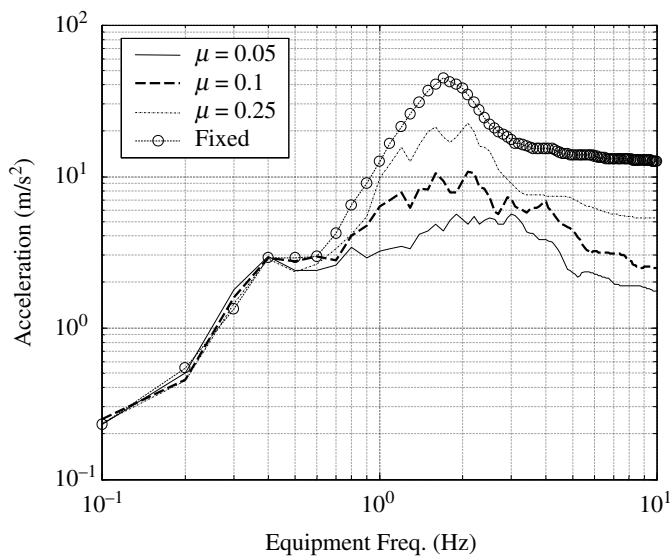


FIGURE 4.56 Maximum equipment acceleration vs. equipment frequency (PGA = 0.5g).

As can be seen, for all the values of μ considered, the equipment response is amplified when the equipment frequency is close to the structural frequency, ω_s , of 1.67 Hz, which means that the tuning effect tends to enlarge the equipment response. However, the use of a smaller μ can help in reducing the amplification of the equipment response resulting from the tuning effect. Finally, a comparison between Figure 4.56 and Figure 4.38 (for the PSI system) shows that the two diagrams are quite similar for an equipment item with a frequency higher than 1 Hz, but are different for that with a lower frequency. This implies that for the earthquake considered, the resilient mechanism of the RSI system has little effect on the response of the equipment with a higher stiffness.

4.5.4 Concluding Remarks

In this section, the behavior of a structure–equipment system isolated by an RSI system under both the harmonic and earthquake excitations has been investigated. Both the responses of the structure and equipment were studied, with special attention given to the effect of the resilient mechanism that characterizes an RSI system. The numerical results demonstrated that when subjected to a harmonic excitation, an RSI system is able to effectively suppress the resonant peaks associated with the structural frequency for both the structure and equipment, but it may also induce some resonant response near the isolation frequency due to the presence of resilient stiffness. Therefore, an RSI system is more sensitive to the frequency content of the ground excitation than a PSI system, especially to excitations of low-frequency components. As for the earthquake responses, the numerical results showed that the resilient mechanism of an RSI system can considerably reduce the residual base displacement. The resilient mechanism has a minor effect on the acceleration response of the structure and equipment, as long as no resonance is induced by the RSI system at the isolation frequency. By and large, both the RSI and PSI systems can be used as effective devices for reducing the acceleration responses of a structure and equipment.

4.6 Issues Related to Seismic Isolation Design

4.6.1 Design Methods

Having been developing for over 30 years, the technology of seismic isolation has matured. Many earthquake-prone countries, including the U.S., Japan, New Zealand, Taiwan, China, and European countries, have developed their own design codes, regulations, or guidelines (Fujita, 1998; Kelly, 1998; Martelli and Forni, 1998). Although most of the codes were developed based on the theory of structural dynamics, the design details outlined in the codes vary from one country to another. While a comprehensive explanation of the various design codes is not the purpose of this section, a brief overview of the concept underlying the design codes will be given. For more details, interested readers should refer to each code or to the books by [Naeim and Kelly](#) (1999) or [Skinner et al.](#) (1993). The design concept introduced herein is based on the series of Uniform Building Code (UBC, 1994, 1997).

Given the fact that base isolation devices are diverse, most design codes or regulations have been written in such a way as not to be specific with respect to the isolation systems. For instance, in the UBC (1997), no particular isolation system is identified as being acceptable; rather, it requires that every isolation system is stable for required displacement, has properties that do not degrade under repeated cyclic loadings, and provides increasing resistance with increasing displacement.

The design methods for base isolation can be classified as static analysis and dynamic analysis. The static analysis is applicable for stiff and regular buildings (in vertical and horizontal directions) that are constructed on soil of a relatively stiff condition. On the other hand, dynamic analysis is usually required for isolation systems with an irregular or long-period superstructure, or constructed on relatively soft soils. For a sophisticated design case, static analysis may be used in the preliminary design phase in order to draft or initiate the isolation design parameters, while dynamic analysis is employed in the final design phase for tuning or finalizing the design details of the isolation system. For simple design cases, static analysis alone is considered sufficient.

4.6.2 Static Analysis

For static analysis, a number of formulas have been specified in the design codes, so that engineers can easily calculate the following design parameters (shown in the design sequence): maximum isolator displacement, D ; isolator total shear, V_b ; total base shear, V_s , of superstructure; and seismic load, F_i , applied on each floor. These formulas were usually derived based on a simplified isolation model,

assuming the isolation system can be linearized (even though most isolation systems are nonlinear) and the superstructure can be modeled as a rigid block. Such a simplified model is considered reasonable, since the displacements of an isolated structure are concentrated at the isolation level, which implies that the superstructure behaves as a rigid block. Based on such a model, only the first vibration mode with the superstructure treated as a rigid body has been considered in deriving the formulas. This explains why static analysis is suitable only for rigid and regular structures.

4.6.2.1 Computation of Maximum Isolator Displacement

An isolation design by static analysis usually starts with the calculation of the maximum isolator displacement, D , which depends on several factors:

$$D = D(Z, N, S, T_{ef}, \zeta_{ef}) \quad (4.68)$$

where Z denotes the earthquake zone factor, N the near-fault factor, S the soil condition factor, T_{ef} the effective isolation period, and ζ_{ef} the effective isolation damping. For example, in the UBC (1994), the formula derived from the constant-velocity spectra over the period range of 1.0 to 3.0 sec has been given in the following form:

$$D = \frac{0.25ZNST_{ef}}{B} \quad (4.69)$$

where B is the damping factor, given as

$$B = B(\zeta_{ef}) \approx 0.25(1 - \ln \zeta_{ef}) \quad (4.70)$$

In the above equations, the factors Z , N , and S depend on conditions of the construction site of the isolated structure; however, the factors T_{ef} and ζ_{ef} depend solely on the properties of the chosen isolation system. The factors T_{ef} and ζ_{ef} are called the “effective” period and damping of the isolation system, because they are frequently obtained by linearizing a nonlinear isolation system. The way to linearize an isolation system will be explained below, along with the formulas for computing T_{ef} and ζ_{ef} . Suppose that for a nonlinear isolation system, the force-displacement relation (hysteresis loop) obtained from a component test is shown in Figure 4.57. The effective stiffness of this isolation system can be computed by

$$K_{ef} = \frac{F^+ - F^-}{2D} \quad (4.71)$$

where F^+ and F^- , respectively, denote the largest positive and negative forces in the test. After the linearized stiffness is obtained from Equation 4.71, the corresponding effective quantities T_{ef} and ζ_{ef} can be computed from the dynamic theory for a single DoF oscillation system; that is

$$T_{ef} = 2\pi \sqrt{\frac{W}{K_{ef}g}} \quad (4.72)$$

$$\zeta_{ef} = \frac{1}{2\pi} \left(\frac{A}{K_{ef}D^2} \right) \quad (4.73)$$

where W is the structural weight, g the gravitational acceleration, and A the total area enclosed by the hysteresis loop in Figure 4.57.

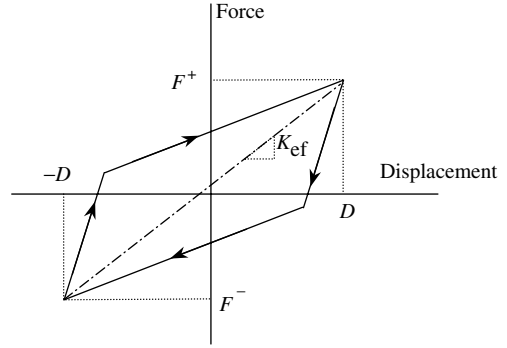


FIGURE 4.57 Typical force-displacement diagram for an isolation system.

4.6.2.2 Computation of Maximum Isolator Shear

After the maximum isolator displacement, D , is obtained, the maximum isolator shear, V_b , can be estimated by the following formula:

$$V_b = K_{ef}D \quad (4.74)$$

Obviously, the above equation represents an equivalent static force exerted on the isolation system, when the system is displaced by an amount, D . In some design codes, V_b has also been referred to as the design force beneath the isolation system.

4.6.2.3 Computation of Total Base Shear

The total base shear, V_s , of the superstructure can be given as

$$V_s = \frac{K_{ef}D}{R_l} \quad (4.75)$$

where R_l is a reduction factor (ductility factor) to account for structural ductility, which will be developed when the structure is subjected to an earthquake with intensity above the design level. In some codes, V_s has also been referred to as the design force above the isolation system.

4.6.2.4 Computation of Shear Force for Each Floor

Having computed the above total base shear, V_s , a formula is employed to distribute this total shear to each floor of the isolated structure. For instance, in the, UBC (1997), the shear force, F_i , exerted on each floor is computed by

$$F_i = V_s \frac{h_i w_i}{\sum_{j=1}^n h_j w_j} \quad (4.76)$$

where n denotes the number of floors, w_i the weight of the i th floor, and h_i the height of the i th floor above the isolation level. Note that the sum of F_i ($i = 1$ to n) must be equal to V_s .

The general procedure for static analysis was illustrated in [Figure 4.58](#). Once the design parameters, D , V_b , V_s , and F_i , are all determined according to the code, they can be used in the detailed design of structural elements as well as of isolator elements. Nevertheless, in most applications, because the test data of the isolation system may not be available in the beginning of design, the values of K_{ef} , T_{ef} , and ζ_{ef} , which are required in computing D , are not known to the designer. If this is the case, the design can begin with assumed values of K_{ef} , T_{ef} , and ζ_{ef} , which may be obtained from experience or previous test data on similar isolators. After the preliminary design is completed, prototype isolators will be fabricated and tested. The actual values of K_{ef} , T_{ef} , and ζ_{ef} , obtained from the tests will be used in the aforementioned code formulas to update the design parameters D , V_b , V_s . Moreover, one observes from Equation 4.71 that the linearized isolator stiffness, K_{ef} , is a function of the design parameter, D , itself, and so are T_{ef} , and ζ_{ef} , obtained from Equation 4.72 and 4.73. In order to obtain K_{ef} , as well as T_{ef} , and ζ_{ef} , an initial guess of D is required at the beginning of design. As a result, the design procedure may have to be repeated iteratively until the difference between the final value of D and the value D_0 computed in the last iteration is less than a preset tolerance. Such an iterative process is illustrated in [Figure 4.58](#).

4.6.3 Dynamic Analysis

The dynamic analysis may be carried out in one of the two forms: response spectrum analysis and time-history analysis. Response spectrum analysis usually involves application of the concepts of response spectrum and modal superposition, and so on. Since these concepts primarily come from the dynamics of linear systems, the response spectrum analysis is only suitable for isolation systems with linear

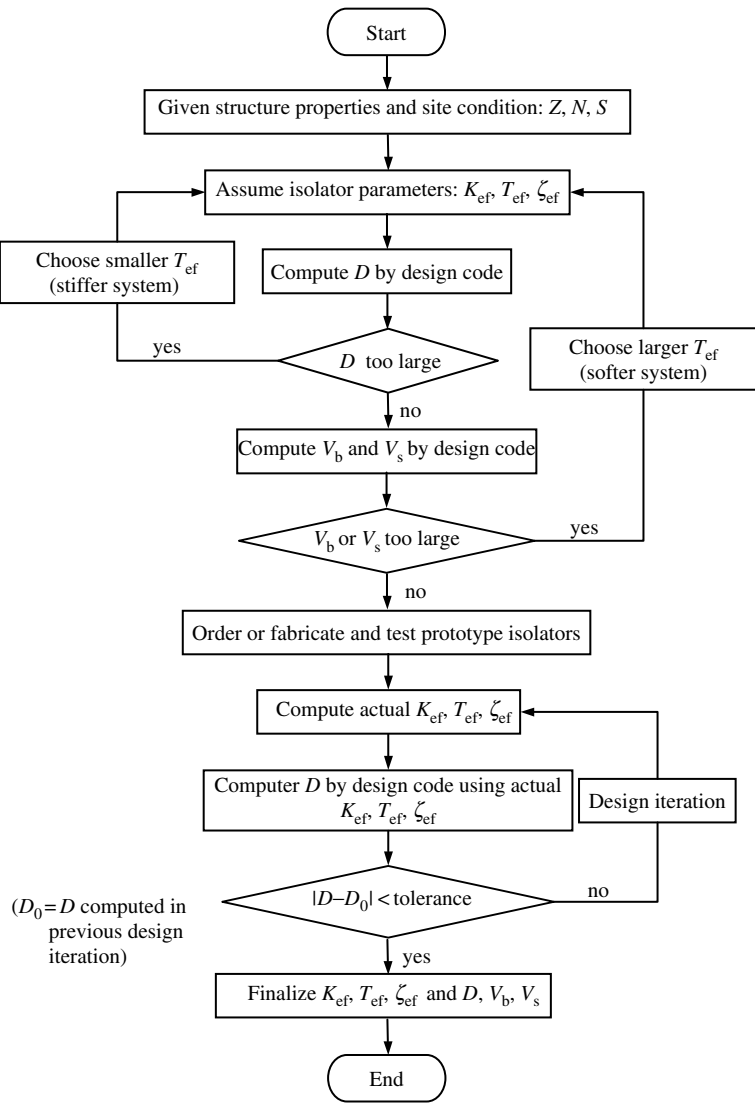


FIGURE 4.58 Flow chart of static analysis.

properties. For the case when the isolation system or the superstructure appears to be highly nonlinear, a time-history analysis is generally required.

Because dynamic analysis depends generally on the usage of computer programs, relatively few formulas have been given in the dynamic analysis sections of design codes. Nevertheless, for a successful time-history analysis, the designer must prepare the following three basic elements: (1) a set of representative input ground motions, (2) accurate mathematic models for isolators and superstructures, and (3) a computer program that is capable of performing the nonlinear time-history analysis. These three elements are explained below.

4.6.3.1 Input Ground Motions

The response of an isolated system depends greatly on the chosen input ground motions, which are usually expressed in the form of ground accelerations. Each ground motion is called one record event.

The chosen events must be representative of the site conditions and soil characteristics. Design codes usually specify the minimum number of events required for analysis. Each ground motion event must be scaled so that all events are compatible with each other and also with the code specified target spectrum. In the UBC (1997), the scaling factor for each event is obtained in response spectra, and then applied to the time domain of the record data. In particular, site specific ground motions are required in the UBC for the following cases: (1) an isolated structure located on a soft soil, (2) an isolated structure located within certain distance (e.g., 10 km) of an active fault, (3) an isolated structure with very long period of vibration (e.g., greater than 3 sec).

4.6.3.2 Mathematic Models

Before any time-history analysis can be carried out, a mathematic model that can accurately reflect the mechanical behavior of the isolation system and the superstructure must be constructed. If the isolation system is nonlinear, the nonlinear parameters must be identified so that the constructed mathematic model can correctly describe the hysteretic behavior of the isolation system. In many cases, the isolation system is assumed to be nonlinear, but the superstructure linear. Establishing an accurate mathematic model is curial for obtaining reliable results in a time-history analysis.

4.6.3.3 Computer Programs

In practice, the task of time-history analysis is executed through the use of a computer program. The mathematic model properties mentioned above will be input to the program for analysis. The computer program selected should be capable of simulating the three-dimensional behavior of structures with selected nonlinear elements. To serve the purpose of isolation design and analysis, several structural analysis programs running on the platform of personal computers have been developed for easy access. Some of the widely used programs include (but are not limited to): ETABS (ETABS, 2004), SAP-2000 Nonlinear (SAP, 2000), and 3D-BASIS (Nagarajaiah et al., 1993). Most of these programs provide a set of imbedded mathematic models for the widely used isolator elements with linear or nonlinear parameters. The designers using these programs can easily build up the mathematic model for the isolated structure considered, specify the parameters of the isolator elements selected, and execute a nonlinear time-history analysis on a personal computer.

4.6.4 Concluding Remarks

In this section, the design concept of seismic isolation for structures was briefly reviewed. The design methods can be based either on static or dynamic analysis. The fundamental issues that should be considered in each design method were highlighted, along with some relevant formulas for computing the relevant parameters. It is believed that, with the concepts and procedures presented in this section, the readers should have a general knowledge of the procedure for base isolation design of structures and equipment.

Acknowledgments

The authors are indebted to the graduate student, Cheng-Yan Wu, at the Department of Construction Engineering, National Kaohsiung First University of Science and Technology, for preparing some of the graphs presented in this chapter.

Nomenclature

Symbol	Quantity	Symbol	Quantity
c, c_s	damping coefficients of superstructure	V_b	total shear force of isolation system
c_e	damping coefficients of equipment	V_s	total base shear of superstructure
c_{EDF}	damping of the EDF system	w_i	weight of the i th floor
D	maximum isolator displacement	W	total weight of superstructure
f_r	interfacial frictional force	$x, x_s(t)$	relative-to-the-ground displacements of superstructure
F_i	seismic load applied on the i th floor	$x_b(t)$	relative-to-the-ground displacements of base mat
k, k_s	stiffness of superstructure	$x_e(t)$	relative-to-the-ground displacements of equipment
k_b	stiffness of isolation system	$\ddot{x}_g(t)$	ground acceleration
k_e	stiffness of equipment	Z	zone factor
k_{EDF}	stiffness of the EDF system	μ	frictional coefficient of sliding isolation system
k_f	stiffness of the fictitious spring in sliding isolation	ω_b	frequency of isolation system
K_{ef}	effective stiffness of isolation system	ω_e	frequencies of equipment
m, m_s	mass of superstructure	ω_g	frequency of ground excitation
m_b	mass of base mat	ζ_e	damping ratios of equipment
m_e	mass of equipment	ζ_{ef}	effective damping ratios of isolation system
n	number of building stories	ζ, ζ_s	damping ratios of superstructure
N	near fault factor	Ω_d	damped natural frequency
R_I	ductility factor	Ω, ω_s	frequencies of superstructure
S	soil factor		
T	natural period of superstructure		
T_d	damped period of superstructure		
T_{ef}	effective isolation period		

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