

# Vibration Control

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Nader Jalili

Clemson University

Ebrahim Esmailzadeh

University of Ontario

## Summary

*The fundamental principles of vibration-control systems are formulated in this chapter (also see [Chapter 7](#)). There are many important areas directly or indirectly related to the main theme of the chapter. These include practical implementation of vibration-control systems, nonlinear control schemes, actual hardware implementation, actuator bandwidth requirements, reliability, and cost. Furthermore, in the process of designing a vibration-control system, in practice, several critical criteria must be considered. These include weight, size, shape, center-of-gravity, types of dynamic disturbances, allowable system response, ambient environment, and service life. Keeping these in mind, general design steps and procedures for vibration-control systems are provided.*

## 5.1 Introduction

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The problem of reducing the level of vibration in constructions and structures arises in various branches of engineering, technology, and industry. In most of today's mechatronic systems, a number of possible devices such as reaction or momentum wheels, rotating devices, and electric motors are essential to the system's operation and performance. These devices, however, can also be sources of detrimental vibrations that may significantly influence the mission performance, effectiveness, and accuracy of operation. Therefore, there is a need for vibration control. Several techniques are utilized either to limit or alter the vibration response characteristics of such systems. During recent years, there has been considerable interest in the practical implementation of these vibration-control systems. This chapter presents the basic theoretical concepts for vibration-control systems design and implementation, followed by an overview of recent developments and control techniques in this subject. Some related practical developments in variable structure control (VSC), as well as piezoelectric vibration control of flexible structures, are also provided, followed by a summary of design steps and procedures for vibration-control systems. A further treatment of the subject is found in Chapter 7.

5.1.1 Vibration Isolation vs. Vibration Absorption

In vibration isolation, either the source of vibration is isolated from the system of concern (also called “force transmissibility”; see Figure 5.1a), or the device is protected from vibration of its point of attachment (also called “displacement transmissibility”, see Figure 5.1b). Unlike the isolator, a vibration absorber consists of a secondary system (usually mass–spring–damper trio) added to the primary device to protect it from vibrating (see Figure 5.1c). By properly selecting absorber mass, stiffness, and damping, the vibration of the primary system can be minimized (Inman, 1994).

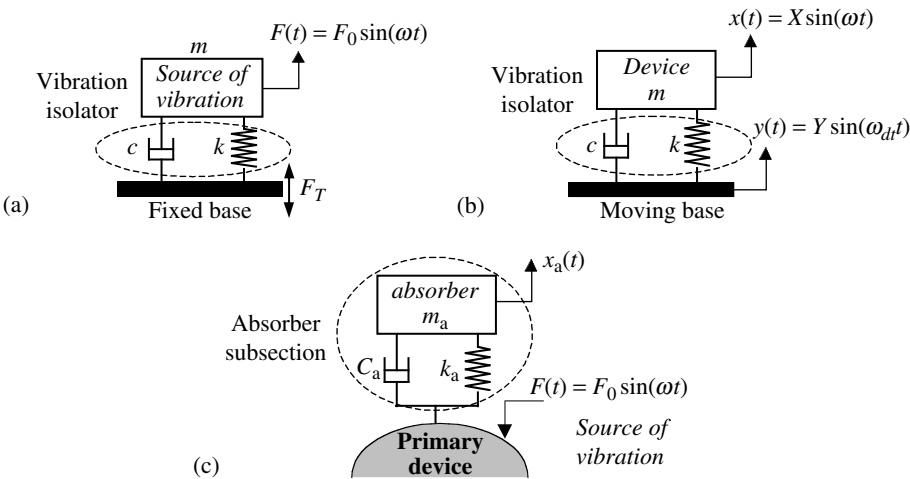
5.1.2 Vibration Absorption vs. Vibration Control

In vibration-control schemes, the driving forces or torques applied to the system are altered in order to regulate or track a desired trajectory while simultaneously suppressing the vibrational transients in the system. This control problem is rather challenging since it must achieve the motion tracking objectives while stabilizing the transient vibrations in the system. Several control methods have been developed for such applications: optimal control (Sinha, 1998); finite element approach (Bayo, 1987); model reference adaptive control (Ge et al., 1997); adaptive nonlinear boundary control (Yuh, 1987); and several other techniques including VSC methods (Chalhoub and Ulsoy, 1987; de Querioz et al., 1999; de Querioz et al., 2000).

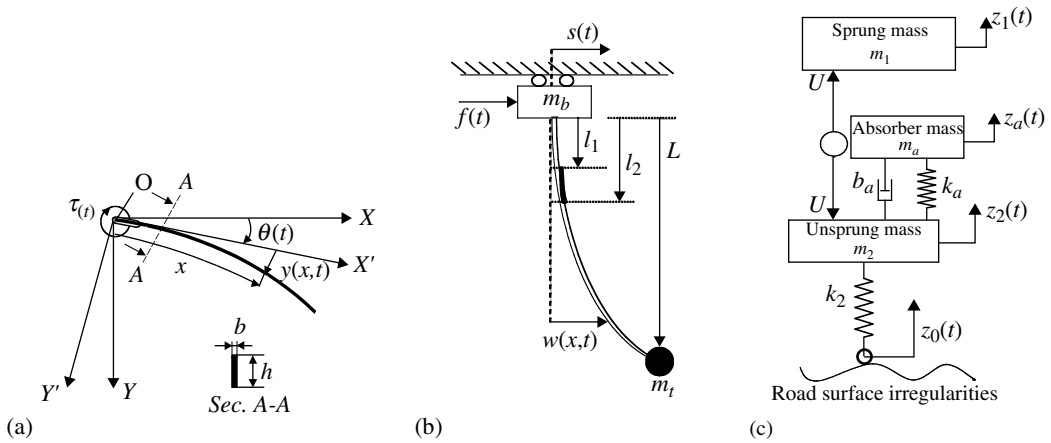
As discussed before, in vibration-absorber systems, a secondary system is added in order to mimic the vibratory energy from the point of interest (attachment) and transfer it into other components or dissipate it into heat. Figure 5.2 demonstrates a comparative schematic of vibration control (both single-input control and multi-input configurations) on translating and rotating flexible beams, which could represent many industrial robot manipulators as well as vibration absorber applications for automotive suspension systems.

5.1.3 Classifications of Vibration-Control Systems

Passive, active, and semiactive (SA) are referred to, in the literature, as the three most commonly used classifications of vibration-control systems, either as isolators or absorbers (see Figure 5.3; Sun et al., 1995). A vibration-control system is said to be active, passive, or SA depending on the amount of



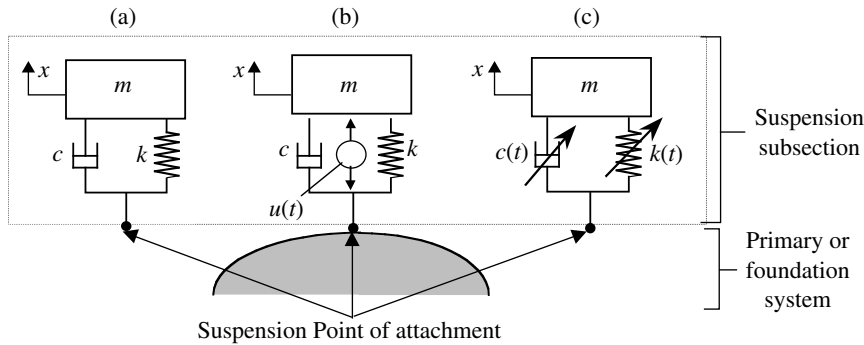
**FIGURE 5.1** Schematic of (a) force transmissibility for foundation isolation; (b) displacement transmissibility for protecting device from vibration of the base and (c) application of vibration absorber for suppressing primary system vibration.



**FIGURE 5.2** A comparative schematic of vibration-control systems: (a) single-input simultaneous tracking and vibration control; (b) multi-input tracking and vibration control and (c) a two-DoF vehicle model with dynamic vibration absorber.

external power required for the vibration-control system to perform its function. A passive vibration control consists of a resilient member (stiffness) and an energy dissipater (damper) either to absorb vibratory energy or to load the transmission path of the disturbing vibration (Korenev and Reznikov, 1993; Figure 5.3a). This type of vibration-control system performs best within the frequency region of its highest sensitivity. For wideband excitation frequency, its performance can be improved considerably by optimizing the system parameters (Puksand, 1975; Warburton and Ayorinde, 1980; Esmailzadeh and Jalili, 1998a). However, this improvement is achieved at the cost of lowering narrowband suppression characteristics.

The passive vibration control has significant limitations in structural applications where broadband disturbances of highly uncertain nature are encountered. In order to compensate for these limitations, active vibration-control systems are utilized. With an additional active force introduced as a part of absorber subsection,  $u(t)$  (Figure 5.3b), the system is controlled using different algorithms to make it more responsive to source of disturbances (Soong and Constantinou, 1994; Olgac and Holm-Hansen, 1995; Sun et al., 1995; Margolis, 1998). The SA vibration-control system, a combination of active and passive treatment, is intended to reduce the amount of external power necessary to achieve the desired performance characteristics (Lee-Glauser et al., 1997; Jalili, 2000; Jalili and Esmailzadeh, 2002), see Figure 5.3c.



**FIGURE 5.3** A typical primary structure equipped with three versions of suspension systems: (a) passive; (b) active and (c) SA configurations.

### 5.1.4 Performance Characteristics of Vibration-Control Systems

In the design of a vibration-control system, it often occurs that the system is required to operate over a wideband load and frequency range that is impossible to meet with a single choice of required stiffness and damping. If the desired response characteristics cannot be obtained, an active vibration-control system may provide an attractive alternative vibration control for such broadband disturbances. However, active vibration-control systems suffer from control-induced instability in addition to the large control effort requirement. This is a serious concern, which prevents them from the common usage in most industrial applications. On the other hand, passive systems are often hampered by a phenomenon known as “detuning.” Detuning implies that the passive system is no longer effective in suppressing the vibration it was designed for. This occurs due to one of the following reasons: (1) the vibration-control system may deteriorate and its structural parameters can be far from the original nominal design, (2) the structural parameters of the primary device itself may alter, or (3) the excitation frequency or the nature of disturbance may change over time.

A semiactive (also known as adaptive-passive) vibration-control system addresses these limitations by effectively integrating a tuning control scheme with tunable passive devices. For this, active force generators are replaced by modulated variable compartments such as variable rate damper and stiffness (see Figure 5.3c; Hrovat et al., 1988; Nemir et al., 1994; Franchek et al., 1995). These variable components are referred to as “tunable parameters” of the suspension system, which are retailored *via* a tuning control, thus resulting in semiactively inducing optimal operation. Much attention is being paid to these systems because of their low energy requirement and cost. Recent advances in smart materials, and adjustable dampers and absorbers have significantly contributed to applicability of these systems (Garcia et al., 1992; Wang et al., 1996; Shaw, 1998).

## 5.2 Vibration-Control Systems Concept

### 5.2.1 Introduction

With a history of almost a century (Frahm, 1911), the dynamic vibration absorber has proven to be a useful vibration-suppression device, widely used in hundreds of diverse applications. It is elastically attached to the vibrating body to alleviate detrimental oscillations from its point of attachment (see Figure 5.3). This section overviews the conceptual design and theoretical background of three types of vibration-control systems, namely the passive, active and SA configurations, along with some related practical implementations.

### 5.2.2 Passive Vibration Control

The underlying proposition in all vibration control or absorber systems is to adjust properly the absorber parameters such that the system becomes absorbent of the vibratory energy within the frequency interval of interest. In order to explain the underlying concept, a single-degree-of-freedom (single-DoF) primary system with a single-DoF absorber attachment is considered (Figure 5.4). The governing dynamics is expressed as

$$\begin{aligned} m_a \ddot{x}_a(t) + c_a \dot{x}_a(t) + k_a x_a(t) \\ = c_a \dot{x}_p(t) + k_a x_p(t) \end{aligned} \quad (5.1)$$

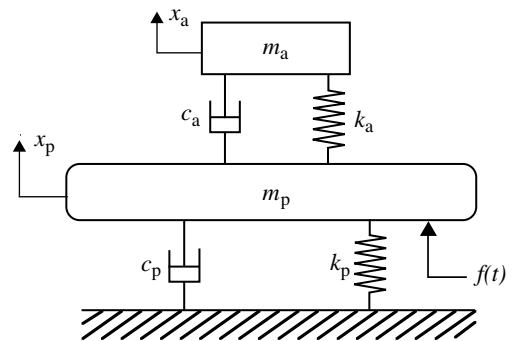


FIGURE 5.4 Application of a passive absorber to single-DoF primary system.

$$m_p \ddot{x}_p(t) + (c_p + c_a) \dot{x}_p(t) + (k_p + k_a) x_p(t) - c_a \dot{x}_a(t) - k_a x_a(t) = f(t) \quad (5.2)$$

where  $x_p(t)$  and  $x_a(t)$  are the respective primary and absorber displacements,  $f(t)$  is the external force, and the rest of the parameters including absorber stiffness,  $k_a$ , and damping,  $c_a$ , are defined as per Figure 5.4. The transfer function between the excitation force and primary system displacement in the Laplace domain is then written as

$$TF(s) = \frac{X_p(s)}{F(s)} = \left\{ \frac{m_a s^2 + c_a s + k_a}{H(s)} \right\} \quad (5.3)$$

where

$$H(s) = \{m_p s^2 + (c_p + c_a)s + k_p + k_a\}(m_a s^2 + c_a s + k_a) - (c_a s + k_a)^2 \quad (5.4)$$

and  $X_a(s)$ ,  $X_p(s)$ , and  $F(s)$  are the Laplace transformations of  $x_a(t)$ ,  $x_p(t)$ , and  $f(t)$ , respectively.

### 5.2.2.1 Harmonic Excitation

When excitation is tonal, the absorber is generally tuned at the disturbance frequency. For this case, the steady-state displacement of the system due to harmonic excitation can be expressed as

$$\left| \frac{X_p(j\omega)}{F(j\omega)} \right| = \left| \frac{k_a - m_a \omega^2 + j c_a \omega}{H(j\omega)} \right| \quad (5.5)$$

where  $\omega$  is the disturbance frequency and  $j = \sqrt{-1}$ . An appropriate parameter tuning scheme can then be selected to minimize the vibration of primary system subject to external disturbance,  $f(t)$ .

For complete vibration attenuation, the steady state,  $|X_p(j\omega)|$ , must equal zero. Consequently, from Equation 5.5, the ideal stiffness and damping of absorber are selected as

$$k_a = m_a \omega^2, \quad c_a = 0 \quad (5.6)$$

Notice that this tuned condition is only a function of absorber elements ( $m_a$ ,  $k_a$ , and  $c_a$ ). That is, the absorber tuning does not need information from the primary system and hence its design is stand alone. For tonal application, theoretically, zero damping in the absorber subsection results in improved performance. In practice, however, the damping is incorporated in order to maintain a reasonable trade-off between the absorber mass and its displacement. Hence, the design effort for this class of application is focused on having precise tuning of the absorber to the disturbance frequency and controlling the damping to an appropriate level. Referring to Snowdon (1968), it can be proven that the absorber, in the presence of damping, can be most favorably tuned and damped if adjustable stiffness and damping are selected as

$$k_{\text{opt}} = \frac{m_a m_p^2 \omega^2}{(m_a + m_p)^2}, \quad c_{\text{opt}} = m_a \sqrt{\frac{3k_{\text{opt}}}{2(m_a + m_p)}} \quad (5.7)$$

### 5.2.2.2 Broadband Excitation

In broadband vibration control, the absorber subsection is generally designed to add damping to and change the resonant characteristics of the primary structure in order to dissipate vibrational energy maximally over a range of frequencies. The objective of the absorber design is, therefore, to adjust the *absorber parameters* to minimize the peak magnitude of the frequency transfer function ( $FTF(\omega) = |TF(s)|_{s=j\omega}$ ) over the absorber parameters vector  $\mathbf{p} = \{c_a \ k_a\}^T$ . That is, we seek  $\mathbf{p}$  to

$$\min_{\mathbf{p}} \left\{ \max_{\omega_{\min} \leq \omega \leq \omega_{\max}} \{|FTF(\omega)|\} \right\} \quad (5.8)$$

Alternatively, one may select the mean square displacement response (MSDR) of the primary system for vibration-suppression performance. That is, the absorber parameters vector,  $\mathbf{p}$ , is selected such that

the MSDR

$$E\{\{\tilde{x}_p\}^2\} = \int_0^\infty \{FTF(\omega)\}^2 S(\omega) d\omega \quad (5.9)$$

is minimized over a desired wideband frequency range.  $S(\omega)$  is the power spectral density of the excitation force,  $f(t)$ , and FTF was defined earlier.

This optimization is subjected to some constraints in  $\mathbf{p}$  space, where only positive elements are acceptable. Once the optimal absorber suspension properties,  $c_a$  and  $k_a$ , are determined, they can be implemented using adjustment mechanisms on the spring and the damper elements. This is viewed as a SA adjustment procedure as it adds no energy to the dynamic structure. The conceptual devices for such adjustable suspension elements and SA treatment will be discussed later in Section 5.2.5.

### 5.2.2.3 Example Case Study

To better recognize the effectiveness of the dynamic vibration absorber over the passive and optimum passive absorber settings, a simple example case is presented. For the simple system shown in Figure 5.4, the following nominal structural parameters (marked by an overscore) are taken:

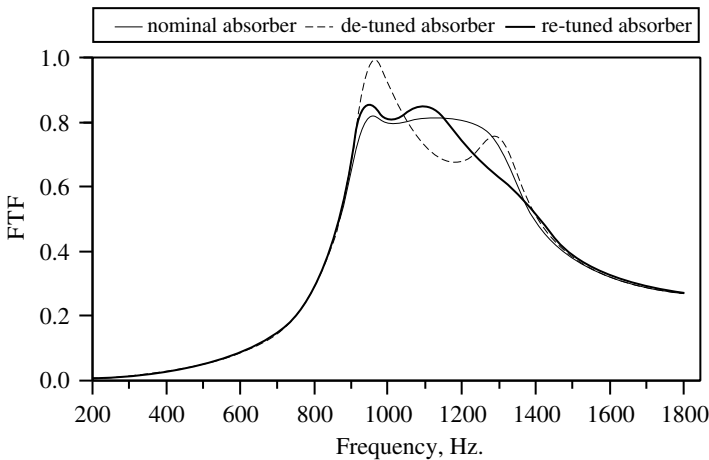
$$\begin{aligned} \bar{m}_p &= 5.77 \text{ kg}, & \bar{k}_p &= 251.132 \times 10^6 \text{ N/m}, & \bar{c}_p &= 197.92 \text{ kg/sec} \\ \bar{m}_a &= 0.227 \text{ kg}, & \bar{k}_a &= 9.81 \times 10^6 \text{ N/m}, & \bar{c}_a &= 355.6 \text{ kg/sec} \end{aligned} \quad (5.10)$$

These are from an actual test setting, which is optimal by design (Olgac and Jalili, 1999). That is, the peak of the FTF is minimized (see thin lines in Figure 5.5). When the primary stiffness and damping increase 5% (for instance during the operation), the FTF of the primary system deteriorates considerably (the dashed line in Figure 5.5), and the absorber is no longer an optimum one for the present primary. When the absorber is optimized based on optimization problem 8, the retuned setting is reached as

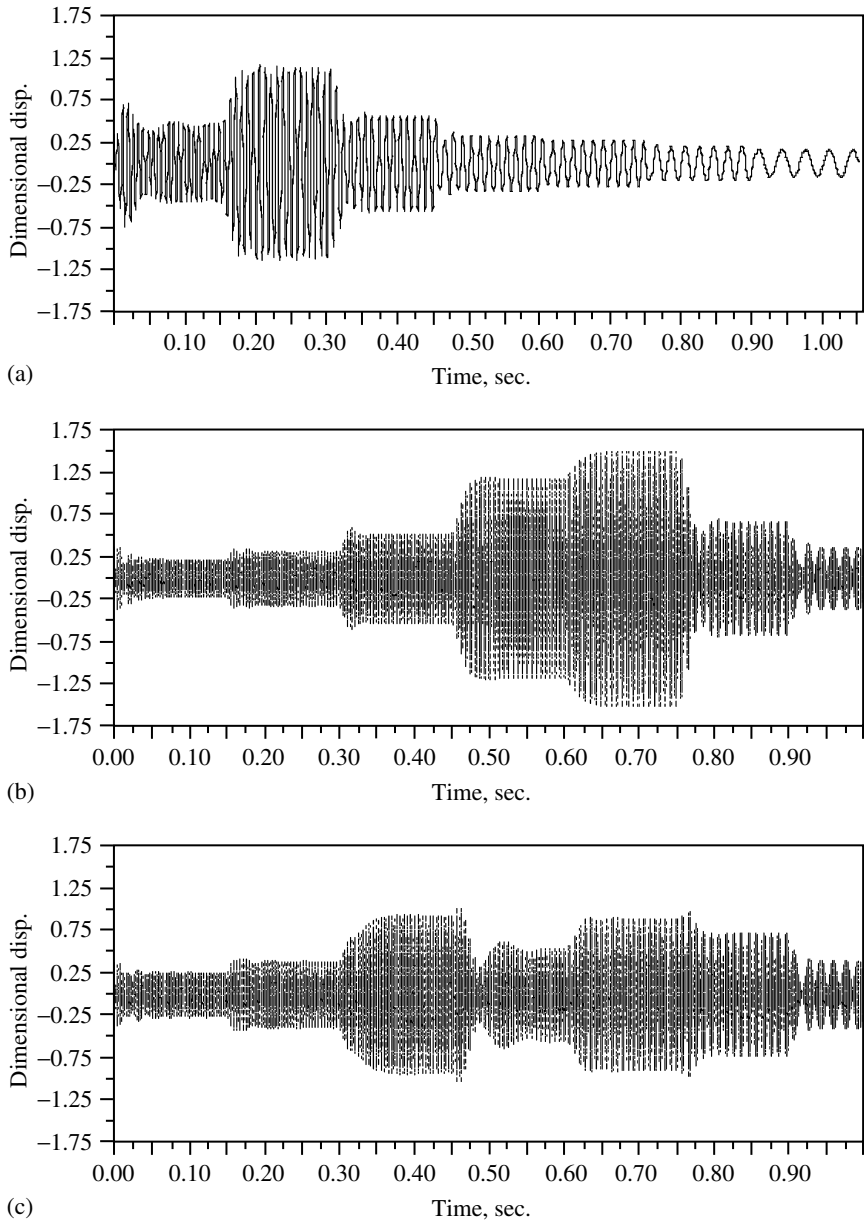
$$k_a = 10.29 \times 10^6 \text{ N/m}, \quad c_a = 364.2 \text{ kg/sec} \quad (5.11)$$

which yields a much better frequency response (see dark line in Figure 5.5).

The vibration absorber effectiveness is better demonstrated at different frequencies by frequency sweep test. For this, the excitation amplitude is kept fixed at unity and its frequency changes every 0.15 sec from



**FIGURE 5.5** Frequency transfer functions (FTFs) for nominal absorber (thin-solid line), detuned absorber (thin-dotted line), and retuned absorber (thick-solid line) settings. (Source: From Jalili, N. and Olgac, N., *AIAA J. Guidance Control Dyn.*, 23, 961–970, 2000a. With permission.)



**FIGURE 5.6** Frequency sweep each 0.15 with frequency change of 1860, 1880, 1900, 1920, 1930, 1950, and 1970 Hz: (a) nominally tuned absorber settings; (b) detuned absorber settings and (c) retuned absorber settings. (Source: From Jalili, N. and Olgac, N., *AIAA J. Guidance Control Dyn.*, 23, 961–970, 2000a. With permission.)

1860 to 1970 Hz. The primary responses with nominally tuned, with detuned, and with retuned absorber settings are given in Figure 5.6a–c, respectively.

### 5.2.3 Active Vibration Control

As discussed, passive absorption utilizes resistive or reactive devices either to absorb vibrational energy or load the transmission path of the disturbing vibration (Korennev and Reznikov, 1993; see [Figure 5.7](#), top). Even with optimum absorber parameters (Warburton and Ayorinde, 1980;

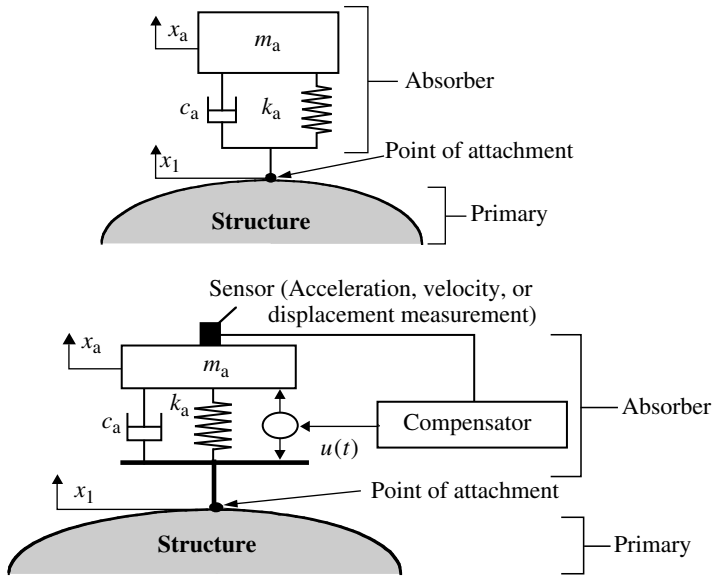


FIGURE 5.7 A general primary structure with passive (top) and active (bottom) absorber settings.

Esmailzadeh and Jalili, 1998a), the passive absorption has significant limitations in structural applications where broadband disturbances of highly uncertain nature are encountered.

In order to compensate for these limitations, active vibration-suppression schemes are utilized. With an additional active force,  $u(t)$  (Figure 5.7, bottom), the absorber is controlled using different algorithms to make it more responsive to primary disturbances (Sun et al., 1995; Margolis, 1998; Jalili and Olgac, 1999). One novel implementation of the tuned vibration absorbers is the active resonator absorber (ARA) (Knowles et al., 2001b). The concept of the ARA is closely related to the concept of the delayed resonator (Olgac and Holm-Hansen, 1994; Olgac, 1995). Using a simple position (or velocity or acceleration) feedback control within the absorber subsection, the delayed resonator enforces that the dominant characteristic roots of the absorber subsection be on the imaginary axis, hence leading to resonance. Once the ARA becomes resonant, it creates perfect vibration absorption at this frequency. The conceptual design and implementation issues of such active vibration-control systems, along with their practical applications, are discussed in Section 5.3.

## 5.2.4 Semiactive Vibration Control

Semiactive (SA) vibration-control systems can achieve the majority of the performance characteristics of fully active systems, thus allowing for a wide class of applications. The idea of SA suspension is very simple: to replace active force generators with continually adjustable elements which can vary and/or shift the rate of the energy dissipation in response to instantaneous condition of motion (Jalili, 2002).

## 5.2.5 Adjustable Vibration-Control Elements

Adjustable vibration-control elements are typically comprised of variable rate damper and stiffness. Significant efforts have been devoted to the development and implementation of such devices for a variety of applications. Examples of such devices include electro-rheological (ER) (Petek, 1992; Wang et al., 1994; Choi, 1999), magneto-rheological (MR) (Spencer et al., 1998; Kim and Jeon, 2000) fluid dampers, and variable orifice dampers (Sun and Parker, 1993), controllable friction braces (Dowell and Cherry, 1994), and variable stiffness and inertia devices (Walsh and Lamnacus, 1992;

Nemir et al., 1994; Franchek et al., 1995; Abe and Igusa, 1996). The conceptual devices for such adjustable properties are briefly reviewed in this section.

### 5.2.5.1 Variable Rate Dampers

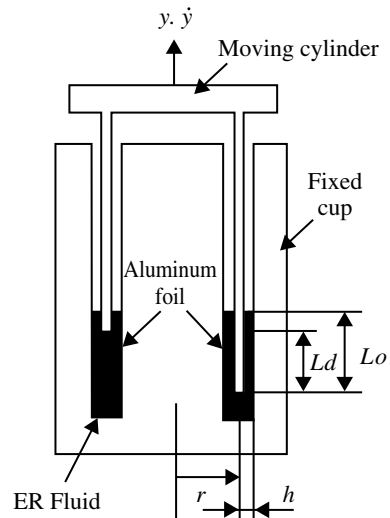
A common and very effective way to reduce transient and steady-state vibration is to change the amount of damping in the SA vibration-control system. Considerable design work on SA damping was done in the 1960s to the 1980s (Crosby and Karnopp, 1973; Karnopp et al., 1974) for vibration control of civil structures such as buildings and bridges (Hrovat et al., 1983) and for reducing machine tool oscillations (Tanaka and Kikushima, 1992). Since then, SA dampers have been utilized in diverse applications ranging from trains (Stribersky et al., 1998) and other off-road vehicles (Horton and Crolla, 1986) to military tanks (Miller and Nobles, 1988). During recent years, there has been considerable interest in the SA concept in the industry for improvement and refinements of the concept (Karnopp, 1990; Emura et al., 1994). Recent advances in smart materials have led to the development of new SA dampers, which are widely used in different applications.

In view of these SA dampers, ER and MR fluids probably serve as the best potential hardware alternatives for the more conventional variable-orifice hydraulic dampers (Sturk et al., 1995). From a practical standpoint, the MR concept appears more promising for suspension, since it can operate, for instance, on vehicle battery voltage, whereas the ER damper is based on high-voltage electric fields. Owing to their importance in today's SA damper technology, we briefly review the operation and fundamental principles of SA dampers here.

#### 5.2.5.1.1 Electro-Rheological Fluid Dampers

ER fluids are materials that undergo significant instantaneous reversible changes in material characteristics when subjected to electric potentials (Figure 5.8). The most significant change is associated with complex shear moduli of the material, and hence ER fluids can be usefully exploited in SA absorbers where variable-rate dampers are utilized. Originally, the idea of applying an ER damper to vibration control was initiated in automobile suspensions, followed by other applications (Austin, 1993; Petek et al., 1995).

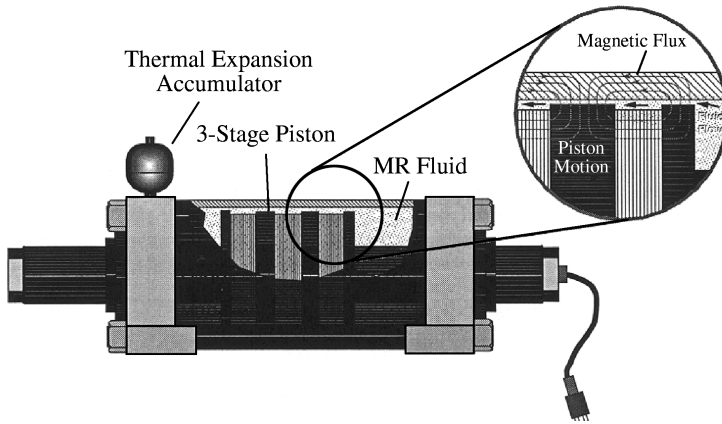
The flow motions of an ER fluid-based damper can be classified by shear mode, flow mode, and squeeze mode. However, the rheological property of ER fluid is evaluated in the shear mode (Choi, 1999). As a result, the ER fluid damper provides an adaptive viscous and frictional damping for use in SA system (Dimarogonas-Andrew and Kollias, 1993; Wang et al., 1994).



**FIGURE 5.8** A schematic configuration of an ER damper. (Source: From Choi, S.B., *ASME J. Dyn. Syst. Meas. Control*, 121, 134–138, 1999. With permission.)

#### 5.2.5.1.2 Magneto-Rheological Fluid Dampers

MR fluids are the magnetic analogies of ER fluids and typically consist of micron-sized, magnetically polarizable particles dispersed in a carrier medium such as mineral or silicon oil. When a magnetic field is applied, particle chains form and the fluid becomes a semisolid, exhibiting plastic behavior similar to that of ER fluids (Figure 5.9). Transition to rheological equilibrium can be achieved in a few milliseconds, providing devices with high bandwidth (Spencer et al., 1998; Kim and Jeon, 2000).



**FIGURE 5.9** A schematic configuration of an MR damper. (Source: From Spencer, B.F. et al., *Proc. 2nd World Conf. on Structural Control*, 1998. With permission.)

### 5.2.5.2 Variable-Rate Spring Elements

In contrast to variable dampers, studies of SA springs or time-varying stiffness have also been geared for vibration-isolation applications (Hubard and Marolis, 1976), for structural controls and for vibration attenuation (Sun et al., 1995 and references therein). The variable stiffness is a promising practical complement to SA damping, since, based on the discussion in Section 5.2, both the absorber damping and stiffness should change to adapt optimally to different conditions. Clearly, the absorber stiffness has a significant influence on optimum operation (and even more compared to the damping element; Jalili and Olgac, 2000b).

Unlike the variable rate damper, changing the effective stiffness requires high energy (Walsh and Lamnacus, 1992). Semiactive or low-power implementation of variable stiffness techniques suffers from limited frequency range, complex implementation, high cost, and so on. (Nemir et al., 1994; Franchek et al., 1995). Therefore, in practice, both absorber damping and stiffness are concurrently adjusted to reduce the required energy.

#### 5.2.5.2.1 Variable-Rate Stiffness (Direct Methods)

The primary objective is to directly change the spring stiffness to optimize a vibration-suppression characteristic such as the one given in Equation 5.8 or Equation 5.9. Different techniques can be utilized ranging from traditional variable leaf spring to smart spring utilizing magnetostrictive materials. A tunable stiffness vibration absorber was utilized for a four-DoF building (Figure 5.10), where a spring is threaded through a collar plate and attached to the absorber mass from one side and to the driving gear from the other side (Franchek et al., 1995). Thus, the effective number of coils,  $N$ , can be changed resulting in a variable spring stiffness,  $k_a$ :

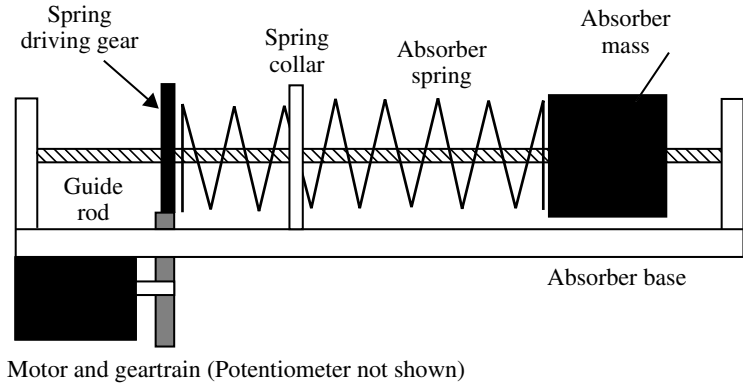
$$k_a = \frac{d^4 G}{8D^3 N} \quad (5.12)$$

where  $d$  is the spring wire diameter,  $D$  is the spring diameter, and  $G$  is modulus of shear rigidity.

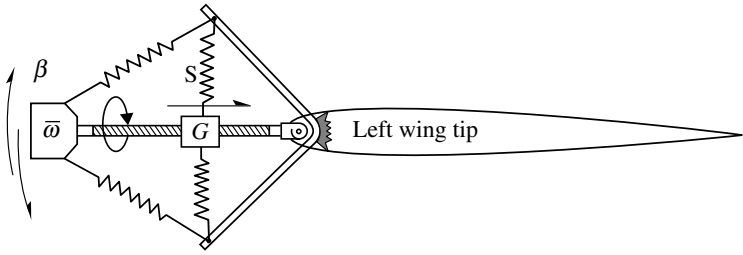
#### 5.2.5.2.2 Variable-Rate Effective Stiffness (Indirect Methods)

In most SA applications, directly changing the stiffness might not be always possible or may require large amount of control effort. For such cases, alternatives methods are utilized to change the effective tuning ratio ( $\tau = \sqrt{k_a/m_a}/\omega_{\text{primary}}$ ), thus resulting in a tunable resonant frequency.

In Liu et al. (2000), a SA flutter-suppression scheme was proposed using differential changes of external store stiffness. As shown in Figure 5.11, the motor drives the guide screw to rotate with slide block,  $G$ , moving along it, thus changing the restoring moment and resulting in a change of store

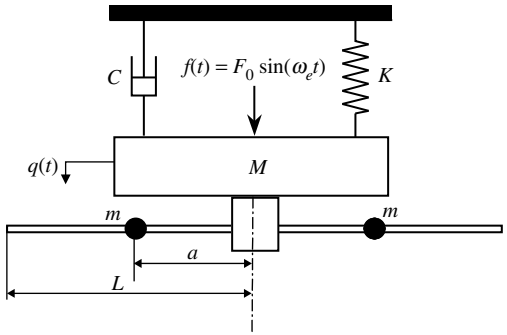


**FIGURE 5.10** The application of a variable-stiffness vibration absorber to a four-DoF building. (Source: From Franchek, M.A. et al., *J. Sound Vib.*, 189, 565–585, 1995. With permission.)



**FIGURE 5.11** A SA flutter control using adjustable pitching stiffness. (Source: From Liu, H.J. et al., *J. Sound Vib.*, 229, 199–205, 2000. With permission.)

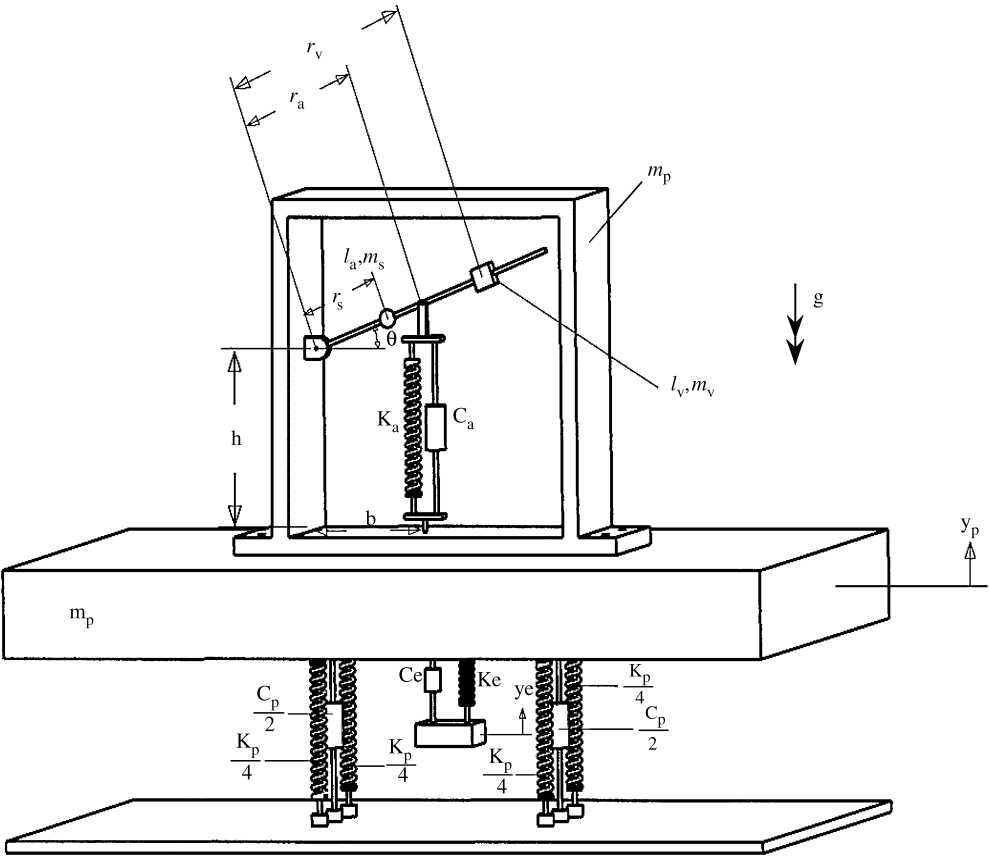
pitching stiffness. Using a double-ended cantilever beam carrying intermediate lumped masses, a SA vibration absorber was recently introduced (Jalili and Esmailzadeh, 2002), where positions of moving masses are adjustable (see Figure 5.12). Figure 5.13 shows an SA absorber with an adjustable effective inertia mechanism (Jalili et al., 2001; Jalili and Fallahi, 2002). The SA absorber consists of a rod carrying a moving block and a spring and damper, which are mounted on a casing. The position of the moving block,  $r_v$ , on the rod is adjustable which provides a tunable resonant frequency.



**FIGURE 5.12** A typical primary system equipped with the double-ended cantilever absorber with adjustable tuning ratio through moving masses,  $m$ . (Source: From Jalili, N. and Esmailzadeh, E., *J. Multi-Body Dyn.*, 216, 223–235, 2002. With permission.)

### 5.2.5.3 Other Variable-Rate Elements

Recent advances in smart materials have led to the development of new SA vibration-control systems using indirect influence on the suspension elements. Wang et al. used a SA piezoelectric network (1996) for structural-vibration control. The variable resistance and inductance in an external RL circuit are used as real-time adaptable control parameters. Another class of adjustable suspensions is the so-called hybrid treatment (Fujita et al., 1991). The hybrid design has two modes, an active mode and a passive mode. With its aim of lowering the



**FIGURE 5.13** Schematic of the adjustable effective inertia vibration absorber. (Source: From Jalili, N. et al., *Int. J. Model. Simulat.*, 21, 148–154, 2001. With permission.)

control effort, relatively small vibrations are reduced in active mode, while passive mode is used for large oscillations. Analogous to hybrid treatment, the semiautomated approach combines SA and active suspensions to benefit from the advantages of individual schemes while eliminating their shortfalls (Jalili, 2000). By altering the adjustable structural properties (in the SA unit) and control parameters (in the active unit), a search is conducted to minimize an objective function subject to certain constraints, which may reflect performance characteristics.

### 5.3 Vibration-Control Systems Design and Implementation

#### 5.3.1 Introduction

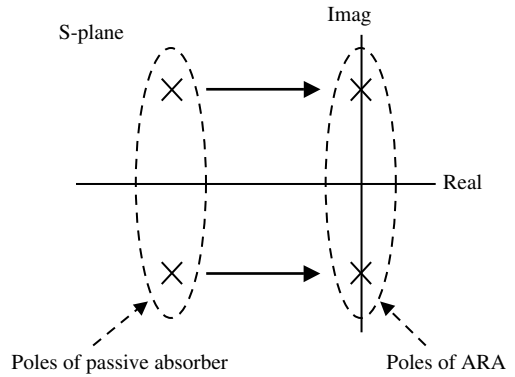
This section provides the basic fundamental concepts for vibration-control systems design and implementation. These systems are classified into two categories: vibration absorbers and vibration-control systems. Some related practical developments in ARAs and piezoelectric vibration control of flexible structures are also provided.

#### 5.3.2 Vibration Absorbers

Undesirable vibrations of flexible structures have been effectively reduced using a variety of dynamic vibration absorbers. The active absorption concept offers a wideband of vibration-attenuation

frequencies as well as real-time tunability as two major advantages. It is clear that the active control could be a destabilizing factor for the combined system, and therefore, the stability of the combined system (i.e., the primary and the absorber subsystems) must be assessed.

An actively tuned vibration-absorber scheme utilizing a resonator generation mechanism forms the underlying concept here. For this, a stable primary system (see Figure 5.7, top, for instance) is forced into a marginally stable one through the addition of a controlled force in the active unit (see Figure 5.7, bottom). The conceptual design for generating such resonance condition is demonstrated in Figure 5.14, where the system's dominant characteristic roots (poles) are moved and placed on the imaginary axis. The absorber then becomes a resonator capable of mimicking the vibratory energy from the primary system at the point of attachment. Although there seem to be many ways to generate such resonance, only two widely accepted practical vibration-absorber resonators are discussed next.



**FIGURE 5.14** Schematic of the active resonator absorber concept through placing the poles of the characteristic equation on the imaginary axis.

### 5.3.2.1 Delayed-Resonator Vibration Absorbers

A recent active vibration-absorption strategy, the delayed resonator (DR), is considered to be the first type of active vibration absorber when operated on a flexible beam (Olgac and Jalili, 1998; Olgac and Jalili, 1999). The DR vibration absorber offers some attractive features in eliminating tonal vibrations from the objects to which it is attached (Olgac and Holm-Hansen, 1994; Olgac, 1995; Renzulli et al., 1999), some of which are real-time tunability, the stand-alone nature of the actively controlled absorber, and the simplicity of the implementation. Additionally, this single-DoF absorber can also be tuned to handle multiple frequencies of vibration (Olgac et al., 1996). It is particularly important that the combined system, that is, the primary structure and the absorber together, is asymptotically stable when the DR is implemented on a flexible beam.

#### 5.3.2.1.1 An Overview of the Delayed-Resonator Concept

An overview of the DR is presented here to help the reader. The equation of motion governing the absorber dynamics alone is

$$m_a \ddot{x}_a(t) + c_a \dot{x}_a(t) + k_a x_a(t) - u(t) = 0, \quad u(t) = g \ddot{x}_a(t - \tau) \quad (5.13)$$

where  $u(t)$  represents the delayed acceleration feedback. The Laplace domain transformation of this equation yields the characteristics equation

$$m_a s^2 + c_a s + k_a - g s^2 e^{-\tau s} = 0 \quad (5.14)$$

Without feedback ( $g = 0$ ), this structure is dissipative with two characteristic roots (poles) on the left half of the complex plane. For  $g$  and  $\tau > 0$ , however, these two finite stable roots are supplemented by infinitely many additional finite roots. Note that these characteristic roots (poles) of Equation 5.14 are discretely located (say at  $s = a + j\omega$ ), and the following relation holds:

$$g = \frac{|m_a s^2 + c_a s + k_a|}{|s^2|} e^{\tau a} \quad (5.15)$$

where  $|\cdot|$  denotes the magnitude of the argument.

Using Equation 5.15, the following observation can be made:

- For  $g = 0$ : there are two finite stable poles and all the remaining poles are at  $a = -\infty$ .
- For  $g = +\infty$ : there are two poles at  $s = 0$ , and the rest are at  $a = +\infty$ .

Considering these and taking into account the continuity of the root loci for a given time delay,  $\tau$ , and as  $g$  varies from 0 to  $\infty$ , it is obvious that the roots of Equation 5.14 move from the stable left half to the unstable right half of the complex plane. For a certain critical gain,  $g_c$ , one pair of poles reaches the imaginary axis. At this operating point, the DR becomes a perfect resonator and the imaginary characteristic roots are  $s = \pm j\omega_c$ , where  $\omega_c$  is the resonant frequency and  $j = \sqrt{-1}$ . The subscript “c” implies the crossing of the root loci on the imaginary axis. The control parameters of concern,  $g_c$  and  $\tau_c$ , can be found by substituting the desired  $s = \pm j\omega_c$  into Equation 5.14 as

$$g_c = \frac{1}{\omega_c^2} \sqrt{(c_a \omega_c)^2 + (m_a \omega_c - k_a)^2}, \quad \tau_c = \frac{1}{\omega_c} \left\{ \tan^{-1} \left[ \frac{c_a \omega_c}{m_a \omega_c^2 - k_a} \right] + 2(\ell - 1)\pi \right\}, \quad \ell = 1, 2, \dots \quad (5.16)$$

When these  $g_c$  and  $\tau_c$  are used, the DR structure mimics a resonator at frequency  $\omega_c$ . In turn, this resonator forms an *ideal absorber* of the tonal vibration at  $\omega_c$ . The objective of the control, therefore, is to maintain the DR absorber at this marginally stable point. On the DR stability, further discussions can be found in Olgac and Holm-Hansen (1994) and Olgac et al. (1997).

### 5.3.2.1.2 Vibration-Absorber Application on Flexible Beams

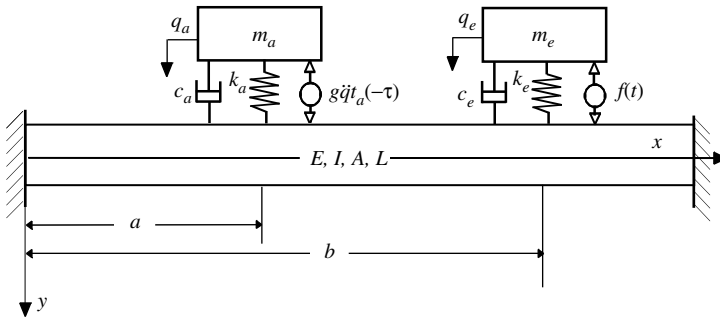
We consider a general beam as the primary system with absorber attached to it and subjected to a harmonic force excitation, as shown in Figure 5.15. The point excitation is located at  $b$ , and the absorber is placed at  $a$ . A uniform cross section is considered for the beam and Euler–Bernoulli assumptions are made. The beam parameters are all assumed to be constant and uniform. The elastic deformation from the undeformed natural axis of the beam is denoted by  $y(x, t)$  and, in the derivations that follow, the dot ( $\dot{\cdot}$ ) and prime ( $'$ ) symbols indicate a partial derivative with respect to the time variable,  $t$ , and position variable  $x$ , respectively.

Under these assumptions, the kinetic energy of the system can be written as

$$T = \frac{1}{2} \rho \int_0^L \left( \frac{\partial y}{\partial t} \right)^2 dx + \frac{1}{2} m_a \dot{q}_a^2 + \frac{1}{2} m_e \dot{q}_e^2 \quad (5.17)$$

The potential energy of this system using linear strain is given by

$$U = \frac{1}{2} EI \int_0^L \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx + \frac{1}{2} k_a \{y(a, t) - q_a\}^2 + \frac{1}{2} k_e \{y(b, t) - q_e\}^2 \quad (5.18)$$



**FIGURE 5.15** Beam–absorber–exciter system configuration. (Source: From Olgac, N. and Jalili, N., *J. Sound Vib.*, 218, 307–331, 1998. With permission.)

The equations of motion may now be derived by applying Hamilton's Principle. However, to facilitate the stability analysis, we resort to an assumed-mode expansion and Lagrange's equations. Specifically,  $y$  is written as a finite sum "Galerkin approximation":

$$y(x, t) = \sum_{i=1}^n \Phi_i(x) q_{bi}(t) \quad (5.19)$$

The orthogonality conditions between these mode shapes can also be derived as (Meirovitch, 1986)

$$\int_0^L \rho \Phi_i(x) \Phi_j(x) dx = N_i \delta_{ij}, \quad \int_0^L EI \Phi_i''(x) \Phi_j''(x) dx = S_i \delta_{ij} \quad (5.20)$$

where  $i, j = 1, 2, \dots, n$ ,  $\delta_{ij}$  is the Kronecker delta, and  $N_i$  and  $S_i$  are defined by setting  $i = j$  in Equation 5.20.

The feedback of the absorber, the actuator excitation force, and the damping dissipating forces in both the absorber and the exciter are considered as non-conservative forces in Lagrange's formulation. Consequently, the equations of motion are derived.

Absorber dynamics is governed by

$$m_a \ddot{q}_a(t) + c_a \left\{ \dot{q}_a(t) - \sum_{i=1}^n \Phi_i(a) \dot{q}_{bi}(t) \right\} + k_a \left\{ q_a(t) - \sum_{i=1}^n \Phi_i(a) q_{bi}(t) \right\} - g \ddot{q}_a(t - \tau) = 0 \quad (5.21)$$

The exciter is given by

$$m_e \ddot{q}_e(t) + c_e \left\{ \dot{q}_e(t) - \sum_{i=1}^n \Phi_i(b) \dot{q}_{bi}(t) \right\} + k_e \left\{ q_e(t) - \sum_{i=1}^n \Phi_i(b) q_{bi}(t) \right\} = -f(t) \quad (5.22)$$

Finally, the beam is represented by

$$\begin{aligned} & N_i \ddot{q}_{bi}(t) + S_i q_{bi}(t) + c_a \left\{ \sum_{i=1}^n \Phi_i(a) \dot{q}_{bi}(t) - \dot{q}_a(t) \right\} \Phi_i(a) + c_e \left\{ \sum_{i=1}^n \Phi_i(b) \dot{q}_{bi}(t) - \dot{q}_e(t) \right\} \Phi_i(b) \\ & + k_a \left\{ \sum_{i=1}^n \Phi_i(a) q_{bi}(t) - q_a(t) \right\} \Phi_i(a) + k_e \left\{ \sum_{i=1}^n \Phi_i(b) q_{bi}(t) - q_e(t) \right\} \Phi_i(b) + g \Phi_i(a) \ddot{q}_a(t - \tau) \\ & = f(t) \Phi_i(b), \quad i = 1, 2, \dots, n \end{aligned} \quad (5.23)$$

Equation 5.21 to Equation 5.23 form a system of  $n + 2$  second-order coupled differential equations.

By proper selection of the feedback gain, the absorber can be tuned to the desired resonant frequency,  $\omega_c$ . This condition, in turn, forces the beam to be motionless at  $a$ , when the beam is excited by a tonal force at frequency  $\omega_c$ . This conclusion is reached by taking the Laplace transform of Equation 5.21 and using feedback control law for the absorber. In short,

$$Y(a, s) = \sum_{i=1}^n \Phi_i(a) Q_{bi}(s) = 0 \quad (5.24)$$

where  $Y(a, s) = \mathfrak{I}\{y(a, t)\}$ ,  $Q_a(s) = \mathfrak{I}\{q_a(t)\}$  and  $Q_{bi}(s) = \mathfrak{I}\{q_{bi}(t)\}$ . Equation 5.24 can be rewritten in time domain as

$$y(a, t) = \sum_{i=1}^n \Phi_i(a) q_{bi}(t) = 0 \quad (5.25)$$

which indicates that the steady-state vibration of the point of attachment of the absorber is eliminated. Hence, the absorber mimics a resonator at the frequency of excitation and absorbs all the vibratory energy at the point of attachment.

### 5.3.2.1.3 Stability of the Combined System

In the preceding section, we have derived the equations of motion for the beam–exciter–absorber system in its most general form. As stated before, inclusion of the feedback control for active absorption is, indeed, an invitation to instability. This topic is treated next.

The Laplace domain representation of the combined system takes the form (Olgac and Jalili, 1998)

$$\mathbf{A}(s)\mathbf{Q}(s) = \mathbf{F}(s) \quad (5.26)$$

where

$$\mathbf{Q}(s) = \begin{Bmatrix} Q_a(s) \\ Q_e(s) \\ Q_{b1}(s) \\ \vdots \\ Q_{bn}(s) \end{Bmatrix}_{(n+2) \times 1}, \quad \mathbf{F}(s) = \begin{Bmatrix} 0 \\ -F(s) \\ 0 \\ \vdots \\ 0 \end{Bmatrix}_{(n+2) \times 1},$$

$$\mathbf{A}(s) = \begin{pmatrix} m_a s^2 + c_a s + k_a - g s^2 e^{-\tau s} & 0 & -\Phi_1(a)(c_a s + k_a) & \cdots & -\Phi_n(a)(c_a s + k_a) \\ 0 & m_e s^2 + c_e s + k_e & -\Phi_1(b)(c_e s + k_e) & \cdots & -\Phi_n(b)(c_e s + k_e) \\ m_a \Phi_1(a) s^2 & m_e \Phi_1(b) s^2 & N_1 s^2 + c s + S_1(1 + j\delta) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_a \Phi_n(a) s^2 & m_e \Phi_n(b) s^2 & 0 & \cdots & N_n s^2 + c s + S_n(1 + j\delta) \end{pmatrix} \quad (5.27)$$

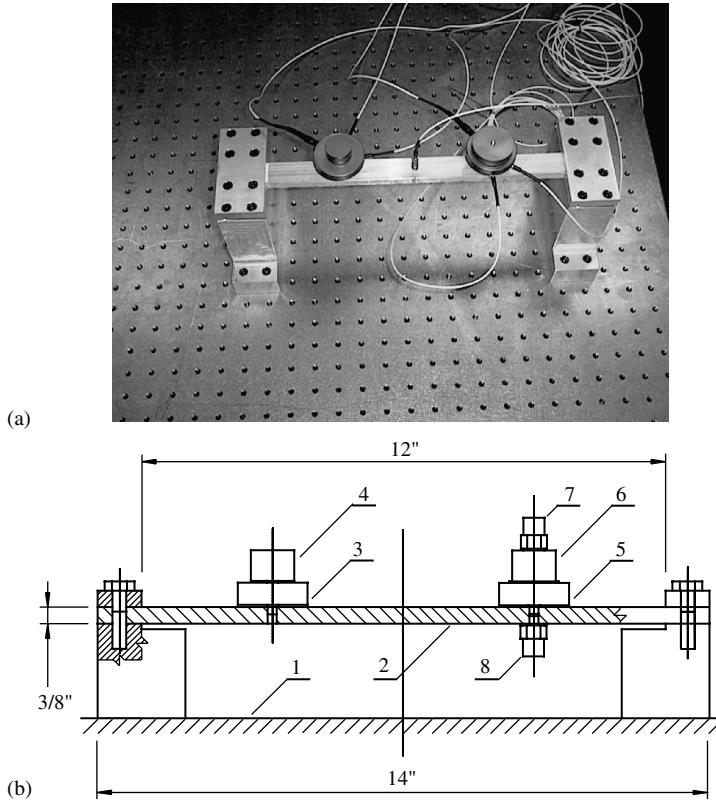
In order to assess the combined system stability, the roots of the characteristic equation,  $\det(\mathbf{A}(s)) = 0$  are analyzed. The presence of feedback (transcendental delay term for this absorber) in the characteristic equations complicates this effort. The root locus plot observation can be applied to the entire system. It is typical that increasing feedback gain causes instability as the roots move from left to right in the complex plane. This picture also yields the frequency range for stable operation of the combined system (Olgac and Jalili, 1998).

### 5.3.2.1.4 Experimental Setting and Results

The experimental setup used to verify the findings is shown in Figure 5.16. The primary structure is a 3/8' in.  $\times$  1' in.  $\times$  12' in. steel beam (2) clamped at both ends to a granite bed (1). A piezoelectric actuator with a reaction mass (3 and 4) is used to generate the periodic disturbance on the beam. A similar actuator-mass setup constitutes the DR absorber (5 and 6). The two setups are located symmetrically at one quarter of the length along the beam from the center. The feedback signal used to implement the DR is obtained from the accelerometer (7) mounted on the reaction mass of the absorber structure. The other accelerometer (8) attached to the beam is present only to monitor the vibrations of the beam and to evaluate the performance of the DR absorber in suppressing them. The control is applied *via* a fast data acquisition card using a sampling of 10 kHz.

The numerical values for this beam–absorber–exciter setup are taken as

- *Beam*:  $E = 210$  GPa,  $\rho = 1.8895$  kg/m
- *Absorber*:  $m_a = 0.183$  kg,  $k_a = 10,130$  kN/m,  $c_a = 62.25$  N sec/m,  $a = L/4$
- *Exciter*:  $m_e = 0.173$  kg,  $k_e = 6426$  kN/m,  $c_e = 3.2$  N sec/m,  $b = 3L/4$



**FIGURE 5.16** (a) Experimental structure and (b) schematic depiction of the setup. (Source: From Olgac, N. and Jalili, N., *J. Sound Vib.*, 218, 307–331, 1998. With permission.)

### 5.3.2.1.5 Dynamic Simulation and Comparison with Experiments

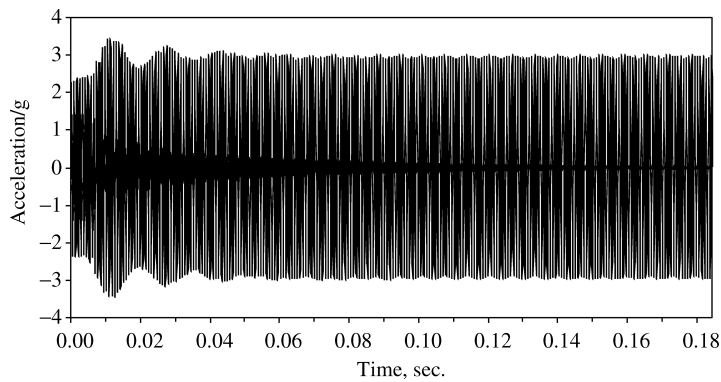
For the experimental set up at hand, the natural frequencies are measured for the first two natural modes,  $\omega_1$  and  $\omega_2$ . These frequencies are obtained much more precisely than those of higher-order natural modes. Table 5.1 offers a comparison between the experimental (*real*) and analytical (*ideal*) clamped–clamped beam natural frequencies.

The discrepancies arrive from two sources: first, the experimental frequencies are structurally damped natural frequencies, and second, they reflect the effect of partially clamped BCs. The theoretical frequencies, on the other hand, are evaluated for an undamped ideal clamped–clamped beam.

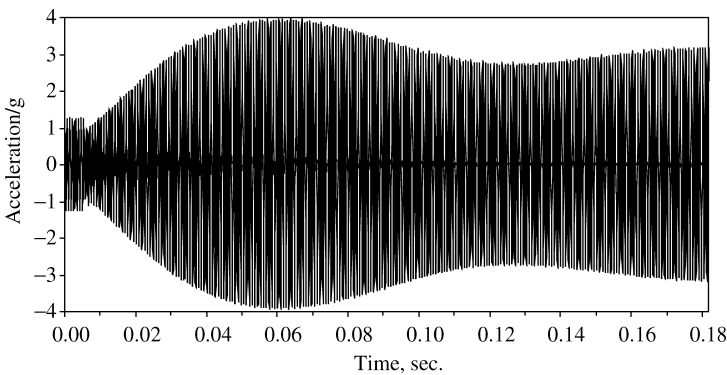
After observing the effect of the number of modes used on the beam deformation, a minimum of three natural modes are taken into account. We then compare the simulated time response vs. the experimental results of vibration suppression. Figure 5.17 shows a test with the excitation frequency  $\omega_c = 1270$  Hz. The corresponding theoretical control parameters are  $g_{c \text{ theory}} = 0.0252$  kg and  $\tau_{c \text{ theory}} = 0.8269$  msec. The experimental control parameters for this frequency are found to be  $g_{c \text{ exp.}} = 0.0273$  kg and  $\tau_{c \text{ exp.}} = 0.82$  msec. The exciter disturbs the beam for 5 msec, then the DR tuning is triggered. The acceleration of the beam at the point of attachment decays exponentially. For all intents and purposes, the suppression takes effect in approximately 200 ms. These results match very closely with the experimental data, Figure 5.18. The only noticeable difference is in the frequency content of the exponential decay. This property is dictated by the dominant poles of the combined system. The imaginary part, however, is smaller in the analytical study. This difference is a nuance that does not affect the earlier observations.

**TABLE 5.1** Comparison between Experimental and Theoretical Beam Natural Frequencies (Hz)

| Natural Modes | Peak Frequencies (Experimental) | Natural Frequencies (Clamped–Clamped) |
|---------------|---------------------------------|---------------------------------------|
| First mode    | 466.4                           | 545.5                                 |
| Second mode   | 1269.2                          | 1506.3                                |



**FIGURE 5.17** Beam and absorber response to 1270 Hz disturbance, analytical. (Source: From Olgac, N. and Jalili, N., *J. Sound Vib.*, 218, 307–331, 1998. With permission.)

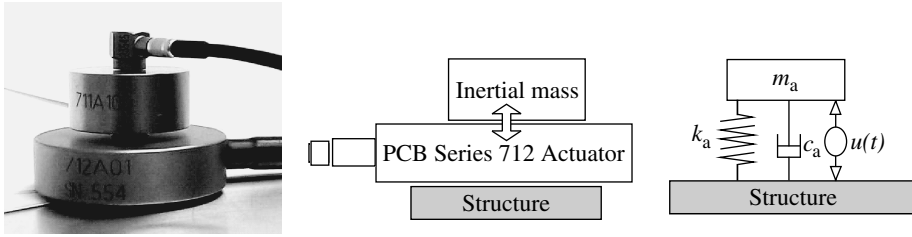


**FIGURE 5.18** Beam and absorber response to 1270 Hz disturbance, experimental. (Source: From Olgac, N. and Jalili, N., *J. Sound Vib.*, 218, 307–331, 1998. With permission.)

5.3.2.2 Active Resonator Vibration Absorbers

One novel implementation of the tuned vibration absorbers is the ARA (Knowles et al., 2001b). The concept of the ARA is closely related to the concept of the delayed resonator (Olgac and Holm-Hansen, 1994; Olgac and Jalili, 1999). Using a simple position (or velocity or acceleration) feedback control within the absorber section, it enforces the dominant characteristic roots of the absorber subsection to be on the imaginary axis, and hence leading to resonance. Once the ARA becomes resonant, it creates perfect vibration absorption at this frequency.

A very important component of any active vibration absorber is the actuator unit. Recent advances in smart materials have led to the development of advanced actuators using piezoelectric ceramics, shape memory alloys, and magnetostrictive materials (Garcia et al., 1992; Shaw, 1998). Over the past two decades, piezoelectric ceramics have been utilized as potential replacements for conventional transducers.



**FIGURE 5.19** A PCB series 712 PZT inertial actuator (left), schematic of operation (middle), and a simple single-DoF mathematical model (right). (Active Vibration Control Instrumentation, A Division of PCB Piezotronics, Inc., [www.pcb.com](http://www.pcb.com).)

These materials are compounds of lead zirconate–titanate (PZT). The PZT properties can be optimized to suit specific applications by appropriate adjustment of the zirconate–titanate ratio. Specifically, a piezoelectric inertial actuator is an efficient and inexpensive solution for active structural vibration control. As shown in Figure 5.19, it applies a point force to the structure to which it is attached.

#### 5.3.2.2.1 An Overview of PZT Inertial Actuators

PZT inertial actuators are most commonly made out of two parallel piezoelectric plates. If voltage is applied, one of the plates expands as the other one contracts, hence producing displacement that is proportional to the input voltage. The resonance of such an actuator can be adjusted by the size of the inertial mass (see Figure 5.19). Increasing the size of the inertial mass will lower the resonant frequency and decreasing the mass will increase it. The resonant frequency,  $f_r$ , can be expressed as

$$f_r = \frac{1}{2\pi} \sqrt{\frac{k_a}{m_a}} \quad (5.28)$$

where  $k_a$  is the effective stiffness of the actuator,  $m_a$ , is defined as

$$m_a = m_{\text{epzt}} + m_{\text{inertial}} + m_{\text{acc}} \quad (5.29)$$

The PZT effective mass is  $m_{\text{epzt}}$ ,  $m_{\text{inertial}}$  is the inertial mass, and  $m_{\text{acc}}$  is the accelerometer mass. Using a simple single-DoF system (see Figure 5.19), the parameters of the PZT inertial actuators can be experimentally determined (Knowles et al., 2001a). This “parameter identification” problem is an inverse problem. We refer interested readers to Banks and Ito (1988) and Banks and Kunisch (1989) for a general introduction to parameter estimation or inverse problems governed by differential equations.

#### 5.3.2.2.2 Active Resonator Absorber Concept

The concept of ARA is closely related to that of the DR (Olgac, 1995; Olgac and Holm-Hansen, 1995; Olgac and Jalili, 1998). Instead of a compensator, the DR uses a simple delayed position (or velocity, or acceleration) feedback control within the absorber subsection for the mentioned “sensitization.”

In contrast to that of the DR absorber, the characteristic equation of the proposed control scheme is rational in nature and is hence easier to implement when closed-loop stability of the system is concerned. Similar to the DR absorber, the proposed ARA requires only one signal from the absorber mass, absolute or relative to the point of attachment (see Figure 5.7 bottom). After the signal is processed through a compensator, an additional force is produced, for instance, by a PZT inertial actuator. If the compensator parameters are properly set, the absorber should behave as an ideal resonator at one or even more frequencies. As a result, the resonator will absorb vibratory energy from the primary mass at given frequencies. The frequency to be absorbed can be tuned in real time. Moreover, if the controller or the actuator fails, the ARA will still function as a passive absorber, and thus it is inherently fail-safe. A similar vibration absorption methodology is given by Filipović and Schröder (1999) for linear systems. The ARA, however, is not confined to the linear regime.

For the case of linear assumption for the PZT actuator, the dynamics of the ARA (Figure 5.7, bottom) can be expressed as

$$m_a \ddot{x}_a(t) + c_a \dot{x}_a(t) + k_a x_a(t) - u(t) = c_a \dot{x}_1(t) + k_a x_1(t) \quad (5.30)$$

where  $x_1(t)$  and  $x_a(t)$  are the respective primary (at the absorber point of attachment) and absorber mass displacements. The mass,  $m_a$ , is given by Equation 5.29 and the control,  $u(t)$ , is designed to produce designated resonance frequencies within the ARA.

The objective of the feedback control,  $u(t)$ , is to convert the dissipative structure (Figure 5.7, top) into a conservative or marginally stable one (Figure 5.7, bottom) with a designated resonant frequency,  $\omega_c$ . In other words, the control aims the placement of dominant poles at  $\pm j\omega_c$  for the combined system, where  $j = \sqrt{-1}$  (see Figure 5.14). As a result, the ARA becomes marginally stable at particular frequencies in the determined frequency range. Using simple position (or velocity or acceleration) feedback within the absorber section (i.e.,  $U(s) = \bar{U}(s)X_a(s)$ ), the corresponding dynamics of the ARA, given by Equation 5.30, in the Laplace domain become

$$(m_a s^2 + c_a s + k_a)X_a(s) - \bar{U}(s)X_a(s) = C(s)X_a(s) = (c_a s + k_a)X_1(s) \quad (5.31)$$

The compensator transfer function,  $\bar{U}(s)$ , is then selected such that the primary system displacement at the absorber point of attachment is forced to be zero; that is

$$C(s) = (m_a s^2 + c_a s + k_a) - \bar{U}(s) = 0 \quad (5.32)$$

The parameters of the compensator are determined through introducing resonance conditions to the absorber characteristic equation,  $C(s)$ ; that is, the equations  $\text{Re}\{C(j\omega_i)\} = 0$  and  $\text{Im}\{C(j\omega_i)\} = 0$  are simultaneously solved, where  $i = 1, 2, \dots, l$  and  $l$  is the number of frequencies to be absorbed. Using additional compensator parameters, the stable frequency range or other properties can be adjusted in real time.

Consider the case where  $U(s)$  is taken as a proportional compensator with a single time constant based on the acceleration of the ARA, given by

$$U(s) = \bar{U}(s)X_a(s), \quad \text{where } \bar{U}(s) = \frac{g s^2}{1 + T s} \quad (5.33)$$

Then, in the time domain, the control force,  $u(t)$ , can be obtained from

$$u(t) = \frac{g}{T} \int_0^t e^{-(t-\tau)/T} \ddot{x}_a(\tau) d\tau \quad (5.34)$$

To achieve ideal resonator behavior, two dominant roots of Equation 5.32 are placed on the imaginary axis at the desired crossing frequency,  $\omega_c$ . Substituting  $s = \pm j\omega_c$  into Equation 5.32 and solving for the control parameters,  $g_c$  and  $T_c$ , one can obtain

$$g_c = m_a \left( \frac{c_a^2}{m_a^2 \left( \omega^2 - \frac{k_a}{m_a} \right)} - \frac{k_a}{m_a \omega^2} + 1 \right), \quad T_c = \frac{c_a \sqrt{k_a/m_a}}{\sqrt{m_a k_a} \left( \omega^2 - \frac{k_a}{m_a} \right)}, \quad \text{for } \omega = \omega_c \quad (5.35)$$

The control parameters,  $g_c$  and  $T_c$ , are based on the physical properties of the ARA (i.e.,  $c_a$ ,  $k_a$ , and  $m_a$ ) as well as the frequency of the disturbance,  $\omega$ , illustrating that the ARA does not require any information from the primary system to which it is attached. However, when the physical properties of the ARA are not known within a high degree of certainty, a method to autotune the control parameters must be considered. The stability assurance of such autotuning proposition will bring primary system parameters into the derivations, and hence the primary system cannot be totally decoupled. This issue will be discussed later in the chapter.

5.3.2.2.3 Application of ARA to Structural-Vibration Control

In order to demonstrate the effectiveness of the proposed ARA, a simple single-DoF primary system subjected to tonal force excitations is considered. As shown in Figure 5.20, two PZT inertial actuators are used for both the primary (model 712-A01) and the absorber (model 712-A02) subsections. Each system consists of passive elements (spring stiffness and damping properties of the PZT materials) and active compartment with the physical parameters listed in Table 5.2. The top actuator acts as the ARA with the controlled force,  $u(t)$ , while the bottom one represents the primary system subjected to the force excitation,  $f(t)$ .

The governing dynamics for the combined system can be expressed as

$$m_a \ddot{x}_a(t) + c_a \dot{x}_a(t) + k_a x_a(t) - u(t) = c_a \dot{x}_1(t) + k_a x_1(t) \tag{5.36}$$

$$m_1 \ddot{x}_1(t) + (c_1 + c_a) \dot{x}_1(t) + (k_1 + k_a) x_1(t) - \{c_a \dot{x}_a(t) + k_a x_a(t) - u(t)\} = f(t) \tag{5.37}$$

where  $x_1(t)$  and  $x_a(t)$  are the respective primary and absorber displacements.

5.3.2.2.4 Stability Analysis and Parameter Sensitivity

The sufficient and necessary condition for asymptotic stability is that all roots of the characteristic equation have negative real parts. For the linear system, Equation 5.36 and Equation 5.37, when utilizing controller (Equation 5.34), the characteristic equation of the combined system (Figure 5.20, right) can be determined and the stability region for compensator parameters,  $g$  and  $T$ , can be obtained using the Routh–Hurwitz method.

5.3.2.2.5 Autotuning Proposition

When using the proposed ARA configuration in real applications where the physical properties are not known or vary over time, the compensator parameters,  $g$  and  $T$ , only provide partial vibration suppression. In order to remedy this, a need exists for an autotuning method to adjust the compensator

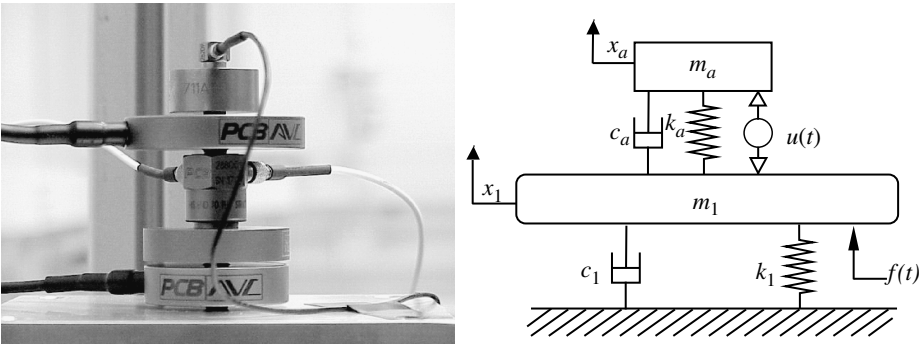


FIGURE 5.20 Implementation of the ARA concept using two PZT actuators (left) and its mathematical model (right).

TABLE 5.2 Experimentally Determined Parameters of PCB Series 712 PZT Inertial Actuators

| PZT System Parameters                     | PCB Model 712-A01 | PCB Model 712-A02 |
|-------------------------------------------|-------------------|-------------------|
| Effective mass, $m_{\text{ePZT}}$ (gr)    | 7.20              | 12.14             |
| Inertial mass, $m_{\text{inertial}}$ (gr) | 100.00            | 200.00            |
| Stiffness, $k_a$ (kN/m)                   | 3814.9            | 401.5             |
| Damping, $c_a$ (Ns/m)                     | 79.49             | 11.48             |

parameters,  $g$  and  $T$ , by some quantities,  $\Delta g$  and  $\Delta T$ , respectively (Jalili and Olgac, 1998b; Jalili and Olgac, 2000a). For the case of the linear compensator with a single time constant, given by Equation 5.33, the transfer function between primary displacement,  $X_1(s)$ , and absorber displacement,  $X_a(s)$ , can be obtained as

$$G(s) = \frac{X_1(s)}{X_a(s)} = \frac{m_a s^2 + c_a s + k_a - \frac{g s^2}{1 + Ts}}{c_a s + k_a} \quad (5.38)$$

The transfer function can be rewritten in the frequency domain for  $s = j\omega$  as

$$G(j\omega) = \frac{X_1(j\omega)}{X_a(j\omega)} = \frac{-m_a \omega^2 + c_a \omega j + k_a + \frac{g \omega^2}{1 + T \omega j}}{c_a \omega j + k_a} \quad (5.39)$$

where  $G(j\omega)$  can be obtained in real time by convolution of accelerometer readings (Renzulli et al., 1999) or other methods (Jalili and Olgac, 2000a). Following a similar procedure as is utilized in Renzulli et al. (1999), the numerator of the transfer function (Equation 5.39) must approach zero in order to suppress primary system vibration. This is accomplished by setting

$$G(j\omega) + \Delta G(j\omega) = 0 \quad (5.40)$$

where  $G(j\omega)$  is the real-time transfer function and  $\Delta G(j\omega)$  can be written as a variational form of Equation 5.39 as

$$\Delta G(j\omega) = \frac{\partial G}{\partial g} \Delta g + \frac{\partial G}{\partial T} \Delta T + \text{higher order terms} \quad (5.41)$$

Since the estimated physical parameters of the absorber (i.e.,  $c_a$ ,  $k_a$ , and  $m_a$ ) are within the vicinity of the actual parameters,  $\Delta g$  and  $\Delta T$  should be small quantities and the higher-order terms of Equation 5.41 can be neglected. Using Equation 5.40 and Equation 5.41 and neglecting higher-order terms, we have

$$\begin{aligned} \Delta g &= \text{Re}[G(j\omega)] \left[ \frac{2Tc_a \omega^2 - k_a + k_a T^2 \omega^2}{\omega^2} \right] + \text{Im}[G(j\omega)] \left[ \frac{c_a - T^2 \omega^2 c_a + 2k_a T}{\omega^2} \right], \\ \Delta T &= \text{Re}[G(j\omega)] \left[ \frac{c_a - T^2 \omega^2 c_a + 2k_a T}{g \omega^2} \right] + \text{Im}[G(j\omega)] \left[ \frac{k_a - 2Tc_a \omega^2 - k_a T^2 \omega^2}{g \omega^3} \right] + \frac{T}{g} \Delta g \end{aligned} \quad (5.42)$$

In the above expressions,  $g$  and  $T$  are the current compensator parameters given by Equation 5.35,  $c_a$ ,  $k_a$ , and  $m_a$  are the estimated absorber parameters,  $\omega$  is the absorber base excitation frequency, and  $G(j\omega)$  is the transfer function obtained in real time. That is, the retuned control parameters,  $g$  and  $T$ , are determined as follows

$$g_{\text{new}} = g_{\text{current}} + \Delta g \quad \text{and} \quad T_{\text{new}} = T_{\text{current}} + \Delta T \quad (5.43)$$

where  $\Delta g$  and  $\Delta T$  are those given by Equation 5.42. After compensator parameters,  $g$  and  $T$ , are adjusted by Equation 5.43, the process can be repeated until  $|G(j\omega)|$  falls within the desired level of tolerance.  $G(j\omega)$  can be determined in real time as shown in Liu et al. (1997) by

$$G(j\omega) = |G(j\omega)| e^{(j\phi(j\omega))} \quad (5.44)$$

where the magnitude and phase are determined assuming that the absorber and primary displacements are harmonic functions of time given by

$$x_a(t) = X_a \sin(\omega t + \phi_a), \quad x_1(t) = X_1 \sin(\omega t + \phi_1) \quad (5.45)$$

With the magnitudes and phase angles of Equation 5.44, the transfer function can be determined from Equation 5.44 and the following:

$$|G(j\omega)| = \frac{X_1}{X_a}, \quad \phi(j\omega) = \phi_1 - \phi_a$$

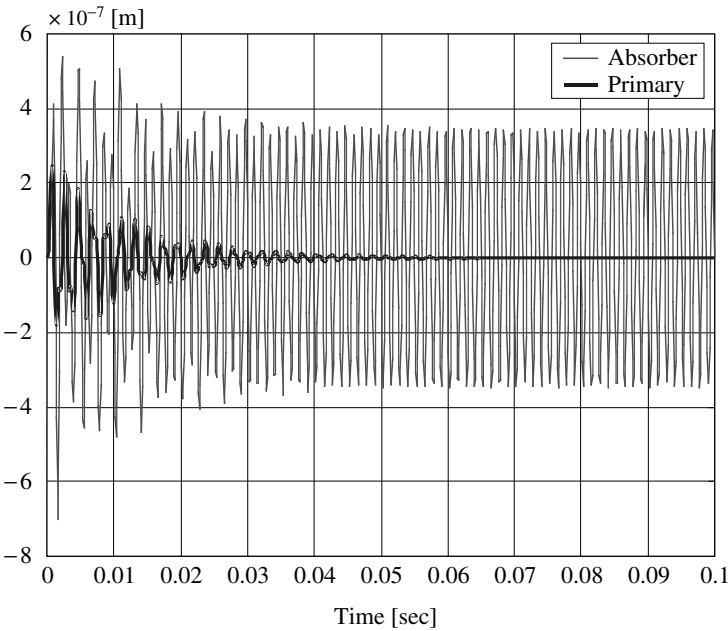
**5.3.2.2.6 Numerical Simulations and Discussions**

To illustrate the feasibility of the proposed absorption methodology, an example case study is presented. The ARA control law is the proportional compensator with a single time constant as given in Equation 5.34. The primary system is subjected to a harmonic excitation with unit amplitude and a frequency of 800 Hz. The ARA and primary system parameters are taken as those given in Table 5.2. The simulation was done using Matlab/Simulink® and the results for the primary system and the absorber displacements are given in Figure 5.21.

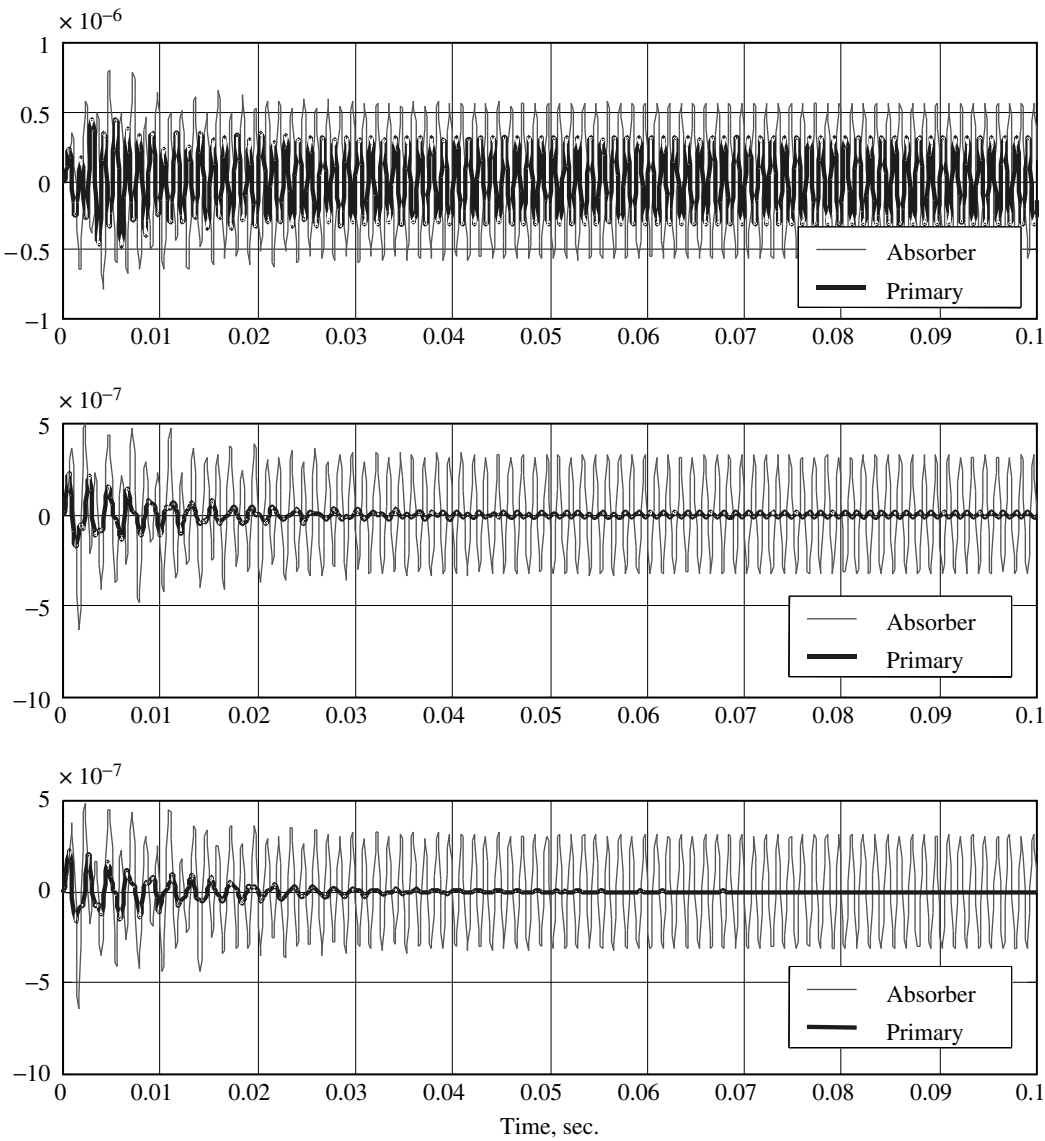
As seen, vibrations are completely suppressed in the primary subsection after approximately 0.05 sec, at which the absorber acts as a marginally stable resonator. For this case, all physical parameters are assumed to be known exactly. However, in practice these parameters are not known exactly and may vary with time, so the case with estimated system parameters must be considered.

To demonstrate the feasibility of the proposed autotuning method, the nominal system parameters ( $m_a, m_1, k_a, k_1, c_a, c_1$ ) were fictitiously perturbed by 10% (i.e., representing the actual values) in the simulation. However, the nominal values of  $m_a, m_1, k_a, k_1, c_a$ , and  $c_1$  were used for calculation of the compensator parameters,  $g$  and  $T$ . The results of the simulation using nominal parameters are given in Figure 5.22. From Figure 5.22, top, the effect of parameter variation is shown as steady-state oscillations of the primary structure. This undesirable response will undoubtedly be encountered when the experiment is implemented. Thus, an autotuning procedure is needed.

The result of the first autotuning iteration is given in Figure 5.22, middle, where the control parameters,  $g$  and  $T$ , are adjusted based on Equation 5.43. One can see tremendous improvement in the primary system response with only one iteration (see Figure 5.22, middle). A second iteration is



**FIGURE 5.21** Primary system and absorber displacements subjected to 800 Hz harmonic disturbance.



**FIGURE 5.22** System responses (displacement, m) for (a) nominal absorber parameters; (b) after first autotuning procedure and (c) after second autotuning procedure.

performed, as shown in Figure 5.22, bottom. The response closely resembles that from Figure 5.21, where all system parameters are assumed to be known exactly.

### 5.3.3 Vibration-Control Systems

As discussed, in vibration-control schemes, the control inputs to the systems are altered in order to regulate or track a desired trajectory while simultaneously suppressing the vibrational transients in the system. This control problem is rather challenging since it must achieve the motion-tracking objectives while stabilizing the transient vibrations in the system. This section provides two recent control methods developed for the regulation and tracking of flexible beams. The experimental implementations are also discussed. The first control method is a single-input vibration-control system discussed in

Section 5.3.3.1, while the second application utilizes a secondary input control in addition to the primary control input to improve the vibrational characteristics of the system (see Section 5.3.3.2).

### 5.3.3.1 Variable Structure Control of Rotating Flexible Beams

The vibration-control problem of a flexible manipulator consists in achieving the motion-tracking objectives while stabilizing the transient vibrations in the arm. Several control methods have been developed for flexible arms (Skaar and Tucker, 1986; Bayo, 1987; Yuh, 1987; Sinha, 1988; Lou, 1993; Ge et al., 1997; de Querioz et al., 1999). Most of these methods concentrate on a model-based controller design, and some of these may not be easy to implement due to the uncertainties in the design model, large variations of the loads, ignored high frequency dynamics, and high order of the designed controllers. In view of these methods, VSC is particularly attractive due to its simplicity of implementation and its robustness to parameter uncertainties (Yeung and Chen, 1989; Singh et al., 1994; Jalili et al., 1997).

#### 5.3.3.1.1 Mathematical Modeling

As shown in Figure 5.23, one end of the arm is free and the other end is rigidly attached to a vertical gear shaft, driven by a DC motor. A uniform cross section is considered for the arm, and we make the Euler–Bernoulli assumptions. The control torque,  $\tau$ , acting on the output shaft, is normal to the plane of motion. Viscous frictions and the ever-present unmodeled dynamics of the motor compartment are to be compensated *via* a perturbation estimation process, as explained later in the text. Since the dynamic system considered here has been utilized in literature quite often, we present only the resulting partial differential equation (PDE) of the system and refer interested readers to [Junkins and Kim](#) (1993) and [Luo et al.](#) (1999) for detailed derivations.

The system is governed by

$$I_h \ddot{\theta}(t) + \rho \int_0^L x \ddot{z}(x, t) dx = \tau \quad (5.46)$$

$$\rho \ddot{z}(x, t) + EI z''''(x, t) = 0 \quad (5.47)$$

with the corresponding boundary conditions

$$z(0, t) = 0, \quad z'(0, t) = \theta(t), \quad z''(L, t) = 0, \quad z'''(L, t) = 0 \quad (5.48)$$

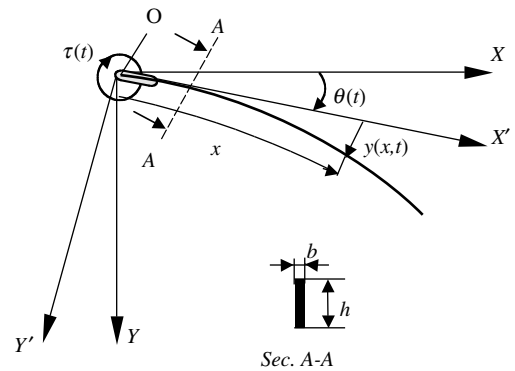
where  $\rho$  is the arm's linear mass density,  $L$  is the arm length,  $E$  is Young's modulus of elasticity,  $I$  is the cross-sectional moment of inertia,  $I_h$  is the equivalent mass moment of inertia at the root end of the arm,  $I_t = I_h + \rho L^3/3$  is the total inertia, and the global variable  $z$  is defined as

$$z(x, t) = x\theta(t) + y(x, t) \quad (5.49)$$

Clearly, the arm vibration equation (Equation 5.47) is a homogeneous PDE but the boundary conditions (Equation 5.48) are nonhomogeneous. Therefore, the closed form solution is very tedious to obtain, if not impossible. Using the application of VSC, these equations and their associated boundary conditions can be converted to a homogeneous boundary value problem, as discussed next.

#### 5.3.3.1.2 Variable Structure Controller

The controller objective is to track the arm angular displacement from an initial angle,  $\theta_d = \theta(0)$ , to zero position,  $\theta(t \rightarrow \infty) = 0$ , while minimizing the flexible arm oscillations. To achieve the control



**FIGURE 5.23** Flexible arm in the horizontal plane and kinematics of deformation. (Source: From Jalili, N., *ASME J. Dyn. Syst. Meas. Control*, 123, 712–719, 2001. With permission.)

insensitivity against modeling uncertainties, the nonlinear control routine of sliding mode control with an additional perturbation estimation (SMCPE) compartment is adopted here (Elmali and Olgac, 1992; Jalili and Olgac, 1998a). The SMCPE method, presented in Elmali and Olgac (1992), has many attractive features, but it suffers from the disadvantages associated with the truncated-model-base controllers. On the other hand, the infinite-dimensional distributed (IDD)-base controller design, proposed in Zhu et al. (1997), has practical limitations due to its measurement requirements in addition to the complex control law.

Initiating from the idea of the IDD-base controller, we present a new controller design approach in which an online perturbation estimation mechanism is introduced and integrated with the controller to relax the measurement requirements and simplify the control implementation. As utilized in Zhu et al. (1997), for the tip-vibration suppression, it is further required that the sliding surface enable the transformation of nonhomogeneous boundary conditions (Equation 5.48) to homogeneous ones. To satisfy vibration suppression and robustness requirements simultaneously, the sliding hyperplane is selected as a combination of tracking (regulation) error and arm flexible vibration as

$$s = \dot{w} + \sigma w \quad (5.50)$$

where  $\sigma > 0$  is a control parameter and

$$w = \theta(t) + \frac{\mu}{L} z(L, t) \quad (5.51)$$

with the scalar,  $\mu$ , being selected later. When  $\mu = 0$ , controller (Equation 5.50) reduces to a sliding variable for rigid-link manipulators (Jalili and Olgac, 1998a; Yeung and Chen, 1988). The motivation for such sliding a variable is to provide a suitable boundary condition for solving the beam Equation 5.47, as will be discussed next and is detailed in Jalili (2001).

For the system described by Equation 5.46 to Equation 5.48, if the variable structure controller is given by

$$\tau = \psi_{\text{est}} + \frac{I_t}{1 + \mu} \left( -k \operatorname{sgn}(s) - Ps - \frac{\mu}{L} \ddot{y}(L, t) - \sigma(1 + \mu)\dot{\theta} - \frac{\sigma\mu}{L} \dot{y}(L, t) \right) \quad (5.52)$$

where  $\psi_{\text{est}}$  is an estimate of the beam flexibility effect

$$\psi = \rho \int_0^L x \ddot{y}(x, t) dx \quad (5.53)$$

$k$  and  $P$  are positive scalars  $k \geq 1 + \mu/I_t |\psi - \psi_{\text{est}}|$ ,  $-1.2 < \mu < -0.45$ ,  $\mu \neq -1$  and  $\operatorname{sgn}(\cdot)$  represents the standard signum function, then, the system's motion will first reach the sliding mode  $s = 0$  in a finite time, and consequently converge to the equilibrium position  $w(x, t) = 0$  exponentially with a time-constant  $1/\sigma$  (Jalili, 2001).

### 5.3.3.1.3 Controller Implementation

In the preceding section, it was shown that by properly selecting control variable,  $\mu$ , the motion exponentially converges to  $w = 0$  with a time-constant  $1/\sigma$ , while the arm stops in a finite time. Although the discontinuous nature of the controller introduces a robustifying mechanism, we have made the scheme more insensitive to parametric variations and unmodeled dynamics by reducing the required measurements and hence easier control implementation. The remaining measurements and ever-present modeling imperfection effects have all been estimated through an online estimation process. As stated before, in order to simplify the control implementation and reduce the measurement effort, the effect of all uncertainties, including flexibility effect ( $\int_0^L x \ddot{y}(x, t) dx$ ) and the ever-present unmodeled dynamics, is gathered into a single quantity named perturbation,  $\psi$ , as given by Equation 5.53. Noting Equation 5.46, the perturbation term can be expressed as

$$\psi = \tau - I_t \ddot{\theta}(t) \quad (5.54)$$

which requires the yet-unknown control feedback  $\tau$ . In order to resolve this dilemma of causality, the current value of control torque,  $\tau$ , is replaced by the most recent control,  $\tau(t - \delta)$ , where  $\delta$  is the small time-step used for the loop closure. This replacement is justifiable in practice since such an algorithm is implemented on a digital computer and the sampling speed is high enough to claim this. Also, in the absence of measurement noise,  $\ddot{\theta}(t) \cong \ddot{\theta}_{\text{cal}}(t) = [\dot{\theta}(t) - \dot{\theta}(t - \delta)]/\delta$ .

In practice and in the presence of measurement noise, appropriate filtering may be considered and combined with these approximate derivatives. This technique is referred to as “switched derivatives”. This backward differences are shown to be effective when  $\delta$  is selected to be small enough and the controller is run on a fast DSP (Cannon and Schmitz, 1984). Also,  $\ddot{y}(L, t)$  can be obtained by attaching an accelerometer at arm-tip position. All the required signals are, therefore, measurable by currently available sensor facilities and the controller is thus realizable in practice. Although these signals may be quite inaccurate, it should be pointed out that the signals, either measurements or estimations, need not be known very accurately, since robust sliding control can be achieved if  $k$  is chosen to be large enough to cover the error existing in the measurement/signal estimation (Yeung and Chen, 1989).

5.3.3.1.4 Numerical Simulations

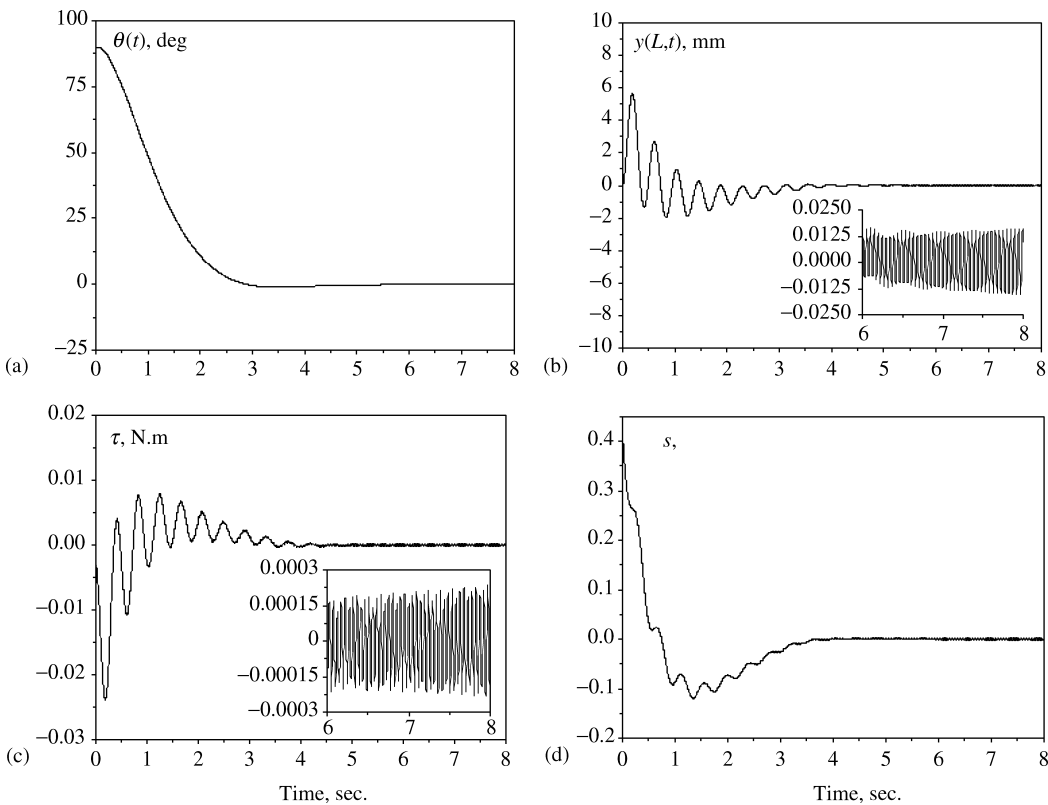
In order to show the effectiveness of the proposed controller, a lightweight flexible arm is considered ( $h \gg b$  in Figure 5.23). For numerical results, we consider  $\theta_d = \theta(0) = \pi/2$  for the initial arm base angle, with zero initial conditions for the rest of the state variables. The system parameters are listed in Table 5.3. Utilizing assumed mode model (AMM), the arm vibration, Equation 5.47, is truncated to three modes and used in the simulations. It should be noted that the controller law, Equation 5.52, is based on the original infinite dimensional equation, and this truncation is utilized only for simulation purposes.

We take the controller parameter  $\mu = -0.66$ ,  $P = 7.0$ ,  $k = 5$ ,  $\varepsilon = 0.01$  and  $\sigma = 0.8$ . In practice,  $\sigma$  is selected for maximum tracking accuracy taking into account unmodeled dynamics and actuator hardware limitations (Moura et al., 1997). Although such restrictions do not exist in simulations (i.e., with ideal actuators, high sampling frequencies and perfect measurements), this selection of  $\sigma$  was decided based on actual experiment conditions.

The sampling rate for the simulations is  $\delta = 0.0005$  sec, while data are recorded at the rate of only 0.002 sec for plotting purposes. The system responses to the proposed control scheme are shown in Figure 5.24. The arm-base angular position reaches the desired position,  $\theta = 0$ , in approximately 4 to 5 sec, which is in agreement with the approximate settling time of  $t_s = 4/\sigma$  (Figure 5.24a). As soon as the system reaches the sliding mode layer,  $|s| < \varepsilon$  (Figure 5.24d), the tip vibrations stop (Figure 5.24b), which demonstrates the feasibility of the proposed control technique. The control torque exhibits some residual vibration, as shown in Figure 5.24c. This residual oscillation is expected since the system

TABLE 5.3 System Parameters Used in Numerical Simulations and Experimental Setup for Rotating Arm

| Properties                  | Symbol | Value             | Unit              |
|-----------------------------|--------|-------------------|-------------------|
| Arm Young’s modulus         | $E$    | $207 \times 10^9$ | N/m <sup>2</sup>  |
| Arm thickness               | $b$    | 0.0008            | m                 |
| Arm height                  | $h$    | 0.02              | m                 |
| Arm length                  | $L$    | 0.45              | m                 |
| Arm linear mass density     | $\rho$ | 0.06/ $L$         | kg/m              |
| Total arm base inertia      | $I_h$  | 0.002             | kg m <sup>2</sup> |
| Gearbox ratio               | $N$    | 14:1              | —                 |
| Light source mass           | —      | 0.05              | kg                |
| Position sensor sensitivity | —      | 0.39              | V/cm              |
| Motor back EMF constant     | $K_b$  | 0.0077            | V/rad/sec         |
| Motor torque constant       | $K_t$  | 0.0077            | N m/A             |
| Armature resistance         | $R_a$  | 2.6               | $\Omega$          |
| Armature inductance         | $L_a$  | 0.18              | mH                |
| Encoder resolution          | —      | 0.087             | Deg/count         |



**FIGURE 5.24** Analytical system responses to controller with inclusion of arm flexibility, that is,  $\mu = -0.66$ : (a) arm angular position; (b) arm-tip deflection; (c) control torque and (d) sliding variable sec. (Source: From Jalili, N., *ASME J. Dyn. Syst. Meas. Control*, 123, 712–719, 2001. With permission.)

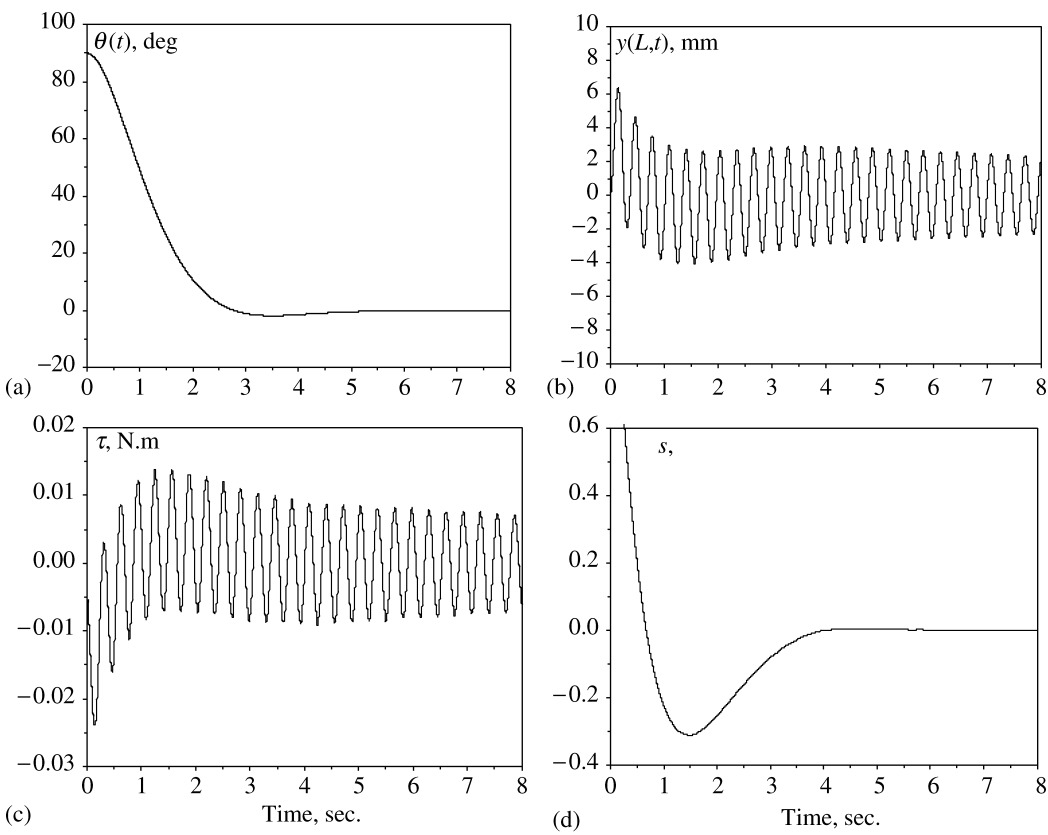
motion is not forced to stay on  $s = 0$  surface (instead it is forced to stay on  $|s| < \varepsilon$ ) when saturation function is used. The sliding variable  $s$  is also depicted in Figure 5.24d. To demonstrate better the feature of the controller, the system responses are displayed when  $\mu = 0$  (Figure 5.25). As discussed,  $\mu = 0$  corresponds to the sliding variable for the rigid link. The undesirable oscillations at the arm tip are evident (see Figure 5.25b and c).

**5.3.3.1.5 Control Experiments**

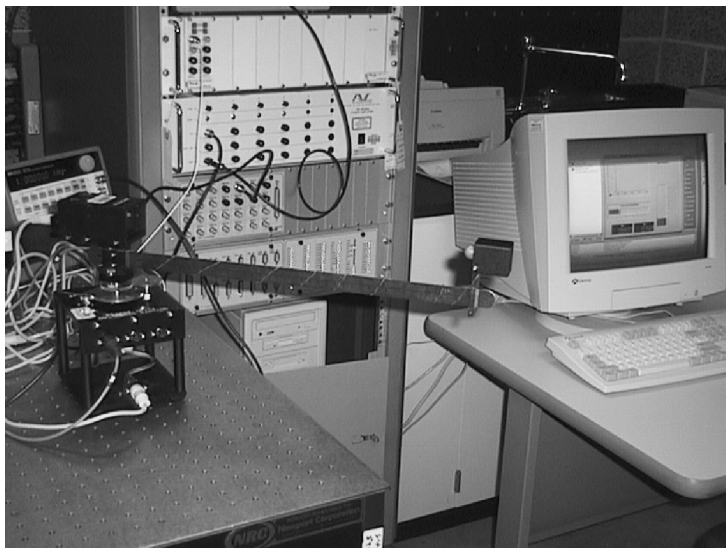
In order to demonstrate better the effectiveness of the controller, an experimental setup is constructed and used to verify the numerical results and concepts discussed in the preceding sections. The experimental setup is shown in Figure 5.26. The arm is a slender beam made of stainless steel, with the same dimensions as used in the simulations. The experimental setup parameters are listed in Table 5.3. One end of the arm is clamped to a solid clamping fixture, which is driven by a high-quality DC servomotor. The motor drives a built-in gearbox ( $N = 14:1$ ) whose output drives an antibacklash gear. The antibacklash gear, which is equipped with a precision encoder, is utilized to measure the arm base angle as well as to eliminate the backlash. For tip deflection, a light source is attached to the tip of the arm, which is detected by a camera mounted on the rotating base.

The DC motor can be modeled as a standard armature circuit; that is, the applied voltage,  $v$ , to the DC motor is

$$v = R_a i_a + L_a \frac{di_a}{dt} + K_b \dot{\theta}_m \tag{5.55}$$



**FIGURE 5.25** Analytical system responses to controller without inclusion of arm flexibility, that is,  $\mu = 0$ : (a) arm angular position; (b) arm-tip deflection; (c) control torque and (d) sliding variable sec. (Source: From Jalili, N., *ASME J. Dyn. Syst. Meas. Control*, 123, 712–719, 2001. With permission.)



**FIGURE 5.26** The experimental device and setup configuration. (Source: From Jalili, N., *ASME J. Dyn. Syst. Meas. Control*, 123, 712–719, 2001. With permission.)

where  $R_a$  is the armature resistance,  $L_a$  is the armature inductance,  $i_a$  is the armature current,  $K_b$  is the back-EMF (electro-motive-force) constant, and  $\theta_m$  is the motor shaft position. The motor torque,  $\tau_m$  from the motor shaft with the torque constant,  $K_t$ , can be written as

$$\tau_m = K_t i_a \quad (5.56)$$

The motor dynamics thus become

$$I_e \ddot{\theta}_m + C_v \dot{\theta}_m + \tau_a = \tau_m = K_t i_a \quad (5.57)$$

where  $C_v$  is the equivalent damping constant of the motor, and  $I_e = I_m + I_L/N^2$  is the equivalent inertia load including motor inertia,  $I_m$ , and gearbox, clamping frame and camera inertia,  $I_L$ . The available torque from the motor shaft for the arm is  $\tau_a$ .

Utilizing the gearbox from the motor shaft to the output shaft and ignoring the motor electric time constant,  $(L_a/R_a)$ , one can relate the servomotor input voltage to the applied torque (acting on the arm) as

$$\tau = \frac{NK_t}{R_a} v - \left( C_v + \frac{K_t K_b}{R_a} \right) N^2 \dot{\theta} - I_h \ddot{\theta} \quad (5.58)$$

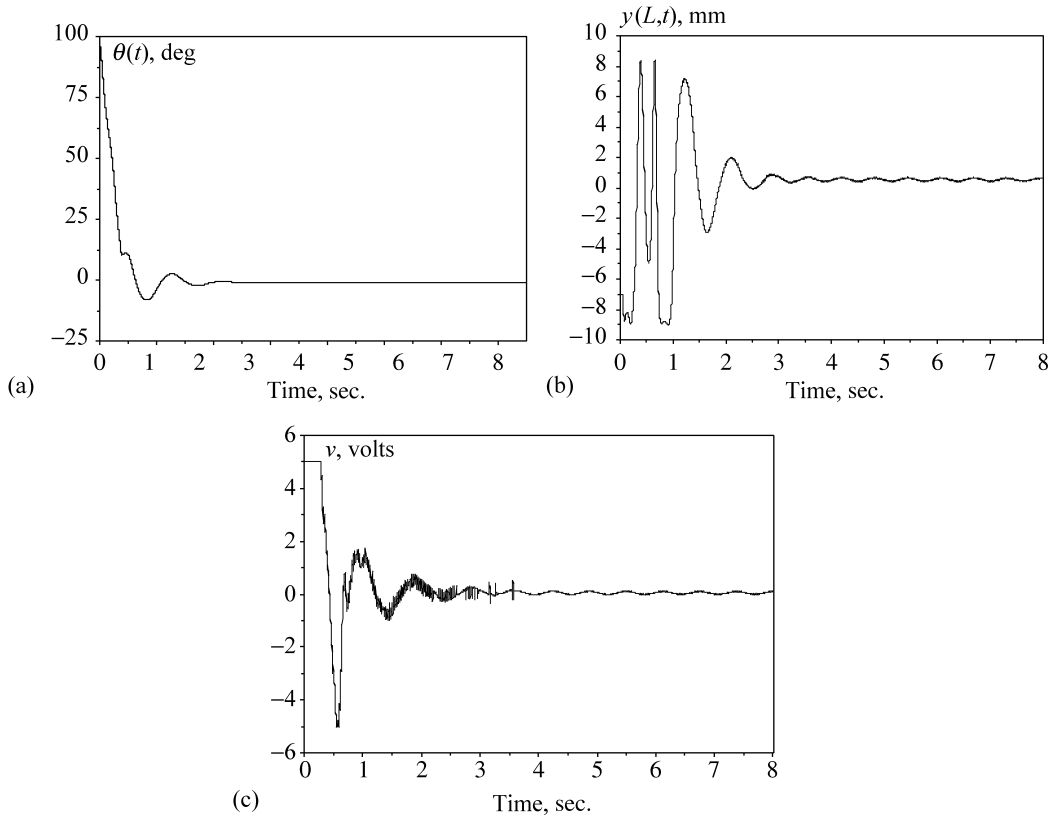
where  $I_h = N^2 I_e$  is the equivalent inertia of the arm base used in the derivation of governing equations. By substituting this torque into the control law, the reference input voltage,  $V$ , is obtained for experiment.

The control torque is applied *via* a digital signal processor (DSP) with sampling rate of 10 kHz, while data are recorded at the rate 500 Hz (for plotting purposes only). The DSP runs the control routine in a single-input–single-output mode as a free standing CPU. Most of the computations and hardware commands are done on the DSP card. For this setup, a dedicated 500 MHz Pentium III serves as the host PC, and a state-of-the-art dSPACE® DS1103 PPC controller board equipped with a Motorola Power PC 604e at 333 MHz, 16 channels ADC, 12 channels DAC, as microprocessor.

The experimental system responses are shown in Figure 5.27 and Figure 5.28 for similar cases discussed in the numerical simulation section. Figure 5.27 represents the system responses when controller (Equation 5.52) utilizes the flexible arm (i.e.,  $\mu = -0.66$ ). As seen, the arm base reaches the desired position (Figure 5.27a), while tip deflection is simultaneously stopped (Figure 5.27b). The good correspondence between analytical results (Figure 5.24) and experimental findings (Figure 5.27) is noticeable from a vibration suppression characteristics point of view. It should be noted that the controller is based on the original governing equations, with arm-base angular position and tip deflection measurements only. The unmodeled dynamics, such as payload effect (owing to the light source at the tip, see Table 5.3) and viscous friction (at the root end of the arm), are being compensated through the proposed online perturbation estimation routine. This, in turn, demonstrates the capability of the proposed control scheme when considerable deviations between model and plant are encountered. The only noticeable difference is the fast decaying response as shown in Figure 5.27b and c. This clearly indicates the high friction at the motor, which was not considered in the simulations (Figure 24b and c). Similar responses are obtained when the controller is designed based on the rigid link only, that is,  $\mu = 0$ . The system responses are displayed in Figure 5.28. Similarly, the undesirable arm-tip oscillations are obvious. The overall agreement between simulations (Figure 24 and Figure 25) and the experiment (Figure 27 and Figure 28) is one of the critical contributions of this work.

### 5.3.3.2 Observer-Based Piezoelectric Vibration Control of Translating Flexible Beams

Many industrial robots, especially those widely used in automatic manufacturing assembly lines, are Cartesian types (Ge et al., 1998). A flexible Cartesian robot can be modeled as a flexible cantilever beam with a translational base support. Traditionally, a PD control strategy is used to regulate the movement of the robot arm. In lightweight robots, the base movement will cause undesirable vibrations at the arm tip because of the flexibility distributed along the arm. In order to eliminate



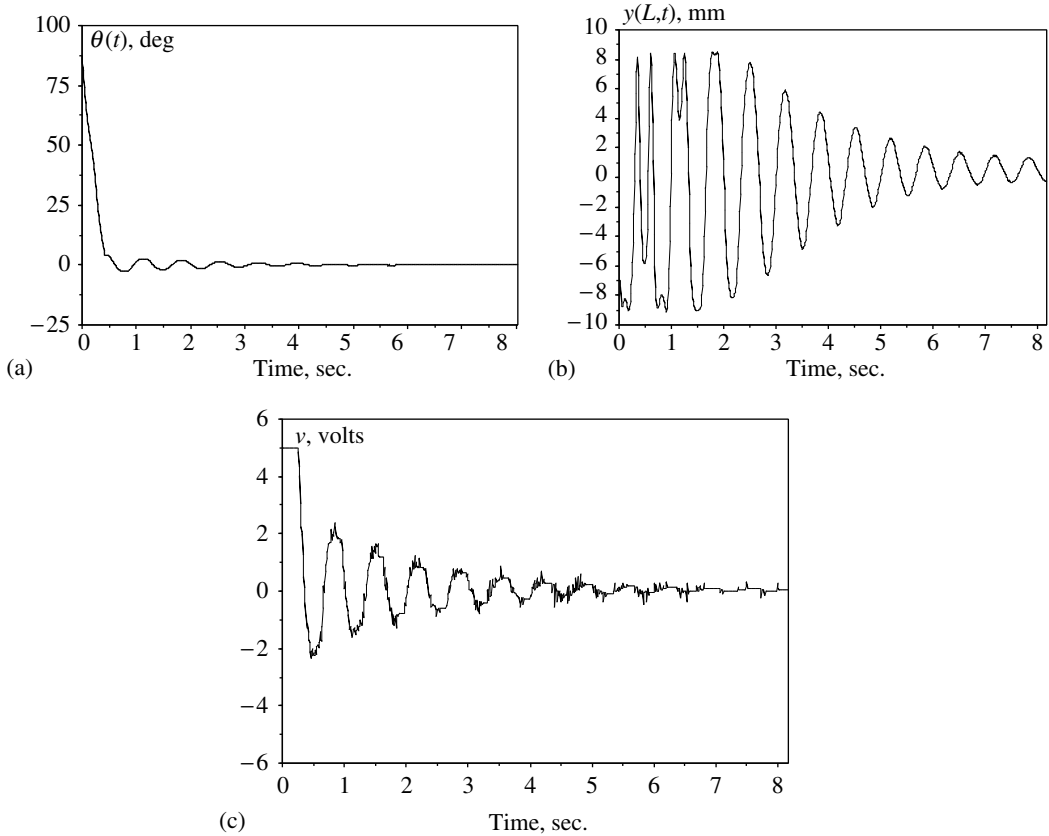
**FIGURE 5.27** Experimental system responses to controller with inclusion of arm flexibility, that is,  $\mu = -0.66$ : (a) arm angular position; (b) arm-tip deflection; (c) control voltage applied to DC servomotor. (Source: From Jalili, N., *ASME J. Dyn. Syst. Meas. Control*, 123, 712–719, 2001. With permission.)

such vibrations, the PD controller must be upgraded with additional compensating terms. In order to improve further the vibration suppression performance, which is a requirement for the high-precision manufacturing market, a second controller, such as a piezoelectric (PZT) patch actuator attached on the surface of the arm, can be utilized (Oueini et al., 1998; Ge et al., 1999; Jalili et al., 2002).

In this section, an observer-based control strategy is presented for regulating the arm motion (Liu et al., 2002). The base motion is controlled utilizing an electrodynamic shaker, while a piezoelectric (PZT) patch actuator is bonded on the surface of the flexible beam for suppressing residual arm vibrations. The control objective here is to regulate the arm base movement, while simultaneously suppressing the vibration transients in the arm. To achieve this, a simple PD control strategy is selected for the regulation of the movement of the base, and a Lyapunov-based controller is selected for the PZT voltage signal. The selection of the proposed energy-based Lyapunov function naturally results in velocity-related signals, which are not physically measurable (Dadfarnia et al., 2003). To remedy this, a reduced-order observer is designed to estimate the velocity related signals. For this, the control structure is designed based on the truncated two-mode beam model.

### 5.3.3.2.1 Mathematical Modeling

For the purpose of model development, we consider a uniform flexible cantilever beam with a PZT actuator bonded on its top surface. As shown in Figure 5.29, one end of the beam is clamped into



**FIGURE 5.28** Experimental system responses to controller without inclusion of arm flexibility, that is,  $\mu = 0$ : (a) arm angular position; (b) arm-tip deflection; (c) control voltage applied to DC servomotor. (Source: From Jalili, N., *ASME J. Dyn. Syst. Meas. Control*, 123, 712–719, 2001. With permission.)

a moving base with the mass of  $m_b$ , and a tip mass,  $m_t$ , is attached to the free end of the beam. The beam has total thickness  $t_b$ , and length  $L$ , while the piezoelectric film possesses thickness and length  $t_b$  and  $(l_2 - l_1)$ , respectively. We assume that the PZT and the beam have the same width,  $b$ . The PZT actuator is perfectly bonded on the beam at distance  $l_1$  measured from the beam support. The force,  $f(t)$ , acting on the base and the input voltage,  $v(t)$ , applied to the PZT actuator are the only external effects.

To establish a coordinate system for the beam, the  $x$ -axis is taken in the longitudinal direction and the  $z$ -axis is specified in the transverse direction of the beam with midplane of the beam to be  $z = 0$ , as shown in Figure 5.30. This coordinate is fixed to the base.

The fundamental relations for the piezoelectric materials are given as (Ikeda, 1990)

$$\mathbf{F} = \mathbf{c}\mathbf{S} - \mathbf{h}\mathbf{D} \tag{5.59}$$

$$\mathbf{E} = -\mathbf{h}^T\mathbf{S} + \boldsymbol{\beta}\mathbf{D} \tag{5.60}$$

where  $\mathbf{F} \in \mathfrak{R}^6$  is the stress vector,  $\mathbf{S} \in \mathfrak{R}^6$  is the strain vector,  $\mathbf{c} \in \mathfrak{R}^{6 \times 6}$  is the symmetric matrix of elastic stiffness coefficients,  $\mathbf{h} \in \mathfrak{R}^{6 \times 3}$  is the coupling coefficients matrix,  $\mathbf{D} \in \mathfrak{R}^3$  is the electrical displacement vector,  $\mathbf{E} \in \mathfrak{R}^3$  is the electrical field vector, and  $\boldsymbol{\beta} \in \mathfrak{R}^{3 \times 3}$  is the symmetric matrix of impermeittivity coefficients.

An energy method is used to derive the equations of motion. Neglecting the electrical kinetic energy, the total kinetic energy of the system is expressed as (Liu et al., 2002; Dadfarnia et al., 2004)

$$\begin{aligned}
 E_k &= \frac{1}{2} m_b \dot{s}(t)^2 + \frac{1}{2} b \int_0^{l_1} \rho_b t_b (\dot{s}(t) + \dot{w}(x, t))^2 dx \\
 &+ \frac{1}{2} b \int_{l_1}^{l_2} (\rho_b t_b + \rho_p t_p) (\dot{s}(t) + \dot{w}(x, t))^2 dx \\
 &+ \frac{1}{2} b \int_{l_2}^L \rho_b t_b (\dot{s}(t) + \dot{w}(x, t))^2 dx \\
 &+ \frac{1}{2} m_t (\dot{s}(t) + \dot{w}(L, t))^2 \\
 &= \frac{1}{2} m_b \dot{s}(t)^2 + \frac{1}{2} \int_0^L \rho(x) (\dot{s}(t) + \dot{w}(x, t))^2 dx \\
 &+ \frac{1}{2} m_t (\dot{s}(t) + \dot{w}(L, t))^2 \quad (5.61)
 \end{aligned}$$

where

$$\begin{aligned}
 \rho(x) &= [\rho_b t_b + G(x) \rho_p t_p] b \\
 G(x) &= H(x - l_1) - H(x - l_2) \quad (5.62)
 \end{aligned}$$

and  $H(x)$  is the Heaviside function,  $\rho_b$  and  $\rho_p$  are the respective beam and PZT volumetric densities. Neglecting the effect of gravity due to planar motion and the higher-order terms of quadratic in  $w'$  (Esmailzadeh and Jalili, 1998b), the total potential energy of the system can be expressed as

$$\begin{aligned}
 E_p &= \frac{1}{2} b \int_0^{l_1} \int_{-t_b/2}^{t_b/2} \mathbf{F}^T \mathbf{S} dy dx + \frac{1}{2} b \int_{l_1}^{l_2} \int_{-t_b/2}^{t_b/2} \mathbf{F}^T \mathbf{S} dy dx + \frac{1}{2} b \int_{l_1}^{l_2} \int_{t_b/2}^{(t_b/2)+t_p} [\mathbf{F}^T \mathbf{S} + \mathbf{E}^T \mathbf{D}] dy dx \\
 &+ \frac{1}{2} b \int_{l_2}^L \int_{-t_b/2}^{t_b/2} \mathbf{F}^T \mathbf{S} dy dx \\
 &= \frac{1}{2} \int_0^L c(x) \left[ \frac{\partial^2 w(x, t)}{\partial x^2} \right]^2 dx + h_l D_y(t) \int_{l_1}^{l_2} \frac{\partial^2 w(x, t)}{\partial x^2} dx + \frac{1}{2} \beta_l (l_2 - l_1) D_y(t)^2 \quad (5.63)
 \end{aligned}$$

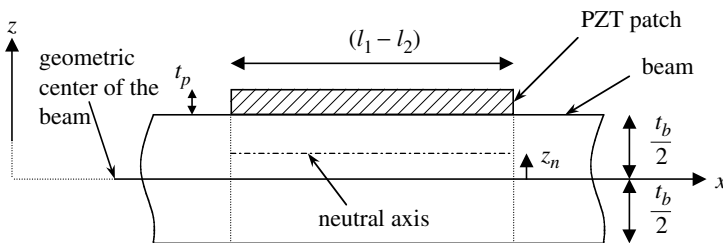


FIGURE 5.30 Coordinate system.

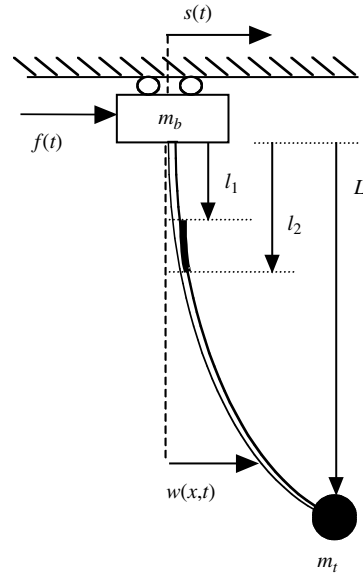


FIGURE 5.29 Schematic of the SCARA/Cartesian robot (last link).

where

$$c(x) = \frac{b}{3} \left\{ \left( \frac{c_{11}^b t_b^3}{4} \right) + G(x) \left\{ 3c_{11}^b t_b z_n^2 + c_{11}^p \left( t_p^3 + 3t_p \left( \frac{t_b}{2} - z_n \right)^2 + 3t_p^2 \left( \frac{t_b}{2} - z_n \right) \right) \right\} \right\} \quad (5.64)$$

$$h_l = h_{12} t_p b (t_p + t_b - 2z_n)/2, \quad \beta_l = \beta_{22} b t_p$$

and

$$z_n = \frac{c_{11}^p t_p (t_p + t_b)}{c_{11}^b t_b + c_{11}^p t_p}$$

The beam and PZT stiffnesses are  $c_{11}^b$  and  $c_{11}^p$ , respectively.

Using the AMM for the beam vibration analysis, the beam deflection can be written as

$$w(x, t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t), \quad P(x, t) = s(t) + w(x, t) \quad (5.65)$$

The equations of motion can now be obtained using the Lagrangian approach

$$\left[ m_b + m_t + \int_0^L \rho(x) dx \right] \ddot{s}(t) + \sum_{j=1}^{\infty} m_j \ddot{q}_j(t) = f(t) \quad (5.66a)$$

$$m_i \ddot{s}(t) + m_{di} \ddot{q}_i(t) + \omega_i^2 m_{di} q_i(t) + h_l (\phi'_i(l_2) - \phi'_i(l_1)) D_y(t) = 0 \quad (5.66b)$$

$$h_l \sum_{j=1}^{\infty} \{ (\phi'_j(l_2) - \phi'_j(l_1)) q_j(t) \} + \beta_l (l_2 - l_1) D_y(t) = b(l_2 - l_1) v(t) \quad (5.66c)$$

where

$$m_{dj} = \int_0^L \rho(x) \phi_j^2(x) dx + m_t \phi_j^2(L), \quad m_j = \int_0^L \rho(x) \phi_j(x) dx + m_t \phi_j(L) \quad (5.67)$$

Calculating  $D_y(t)$  from Equation 5.66b and substituting into Equation 5.66c results in

$$\begin{aligned} & m_i \ddot{s}(t) + m_{di} \ddot{q}_i(t) + \omega_i^2 m_{di} q_i(t) - \frac{h_l^2 (\phi'_i(l_2) - \phi'_i(l_1))}{\beta_l (l_2 - l_1)} \sum_{j=1}^{\infty} \{ (\phi'_j(l_2) - \phi'_j(l_1)) q_j(t) \} \\ & = - \frac{h_l b (\phi'_i(l_2) - \phi'_i(l_1))}{\beta_l} v(t), \quad i = 1, 2, \dots \end{aligned} \quad (5.68)$$

which will be used to derive the controller, as discussed next.

### 5.3.3.2.2 Derivation of the Controller

Utilizing Equation 5.66a and Equation 5.68, the truncated two-mode beam with PZT model reduces to

$$\left[ m_b + m_t + \int_0^L \rho(x) dx \right] \ddot{s}(t) + m_1 \ddot{q}_1(t) + m_2 \ddot{q}_2(t) = f(t) \quad (5.69a)$$

$$\begin{aligned} & m_1 \ddot{s}(t) + m_{d1} \ddot{q}_1(t) + \omega_1^2 m_{d1} q_1(t) - \frac{h_l^2 (\phi'_1(l_2) - \phi'_1(l_1))}{\beta_l (l_2 - l_1)} \\ & \quad \times \{ (\phi'_1(l_2) - \phi'_1(l_1)) q_1(t) + (\phi'_2(l_2) - \phi'_2(l_1)) q_2(t) \} \\ & = - \frac{h_l b (\phi'_1(l_2) - \phi'_1(l_1))}{\beta_l} v(t) \end{aligned} \quad (5.69b)$$

$$\begin{aligned}
& m_2 \ddot{s}(t) + m_{d2} \ddot{q}_2(t) + \omega_2^2 m_{d2} q_2(t) - \frac{h_l^2 (\phi_2'(l_2) - \phi_2'(l_1))}{\beta_l (l_2 - l_1)} \\
& \quad \times \{(\phi_1'(l_2) - \phi_1'(l_1)) q_1(t) + (\phi_2'(l_2) - \phi_2'(l_1)) q_2(t)\} \\
& = - \frac{h_l b (\phi_2'(l_2) - \phi_2'(l_1))}{\beta_l} v(t)
\end{aligned} \tag{5.69c}$$

The equations in Equation 5.69 can be written in the following more compact form

$$\mathbf{M} \ddot{\Delta} + \mathbf{K} \Delta = \mathbf{F}_e \tag{5.70}$$

where

$$\mathbf{M} = \begin{bmatrix} \psi & m_1 & m_2 \\ m_1 & m_{d1} & 0 \\ m_2 & 0 & m_{d2} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_{11} & k_{12} \\ 0 & k_{12} & k_{22} \end{bmatrix}, \quad \mathbf{F}_e = \begin{Bmatrix} f(t) \\ \epsilon_1 v(t) \\ \epsilon_2 v(t) \end{Bmatrix}, \quad \Delta = \begin{Bmatrix} s(t) \\ q_1(t) \\ q_2(t) \end{Bmatrix} \tag{5.71}$$

and

$$\begin{aligned}
\psi &= m_b + m_t + \int_0^L \rho(x) dx, \quad \epsilon_1 = -\frac{h_l b}{\beta_l} (\phi_1'(l_2) - \phi_1'(l_1)), \quad \epsilon_2 = -\frac{h_l b}{\beta_l} (\phi_2'(l_2) - \phi_2'(l_1)), \\
k_{11} &= \omega_1^2 m_{d1} - \frac{h_l^2}{\beta_l (l_2 - l_1)} (\phi_1'(l_2) - \phi_1'(l_1))^2, \\
k_{12} &= -\frac{h_l^2}{\beta_l (l_2 - l_1)} (\phi_1'(l_2) - \phi_1'(l_1)) (\phi_2'(l_2) - \phi_2'(l_1)), \\
k_{22} &= \omega_2^2 m_{d2} - \frac{h_l^2}{\beta_l (l_2 - l_1)} (\phi_2'(l_2) - \phi_2'(l_1))^2
\end{aligned} \tag{5.72}$$

For the system described by Equation 5.70, if the control laws for the arm base force and PZT voltage generated moment are selected as

$$f(t) = -k_p \Delta s - k_d \dot{s}(t) \tag{5.73}$$

$$v(t) = -k_v (\epsilon_1 \dot{q}_1(t) + \epsilon_2 \dot{q}_2(t)) \tag{5.74}$$

where  $k_p$  and  $k_d$  are positive control gains,  $\Delta s = s(t) - s_d$ ,  $s_d$  is the desired set-point position, and  $k_v > 0$  is the voltage control gain, then the closed-loop system will be stable, and in addition

$$\lim_{t \rightarrow \infty} \{q_1(t), q_2(t), \Delta s\} = 0$$

See [Dadfarnia et al. \(2004\)](#) for a detailed proof.

### 5.3.3.2.3 Controller Implementation

The control input,  $v(t)$ , requires the information from the velocity-related signals,  $\dot{q}_1(t)$  and  $\dot{q}_2(t)$ , which are usually not measurable. Sun and Mills (1999) solved the problem by integrating the acceleration signals measured by the accelerometers. However, such controller structure may result in unstable closed-loop system in some cases. In this paper, a reduced-order observer is designed to estimate the velocity signals,  $\dot{q}_1$  and  $\dot{q}_2$ . For this, we utilize three available signals: base displacement,  $s(t)$ , arm-tip deflection,  $P(L, t)$ , and beam root strain,  $\epsilon(0, t)$ ; that is

$$y_1 = s(t) = x_1 \tag{5.75a}$$

$$y_2 = P(L, t) = x_1 + \phi_1(L) x_2 + \phi_2(L) x_3 \tag{5.75b}$$

$$y_3 = \epsilon(0, t) = \frac{t_b}{2} (\phi_1''(0) x_2 + \phi_2''(0) x_3) \tag{5.75c}$$

It can be seen that the first three states can be obtained by

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \mathbf{C}_1^{-1} \mathbf{y} \quad (5.76)$$

Since this system is observable, we can design a reduced-order observer to estimate the velocity-related state signals. Defining  $\mathbf{X}_1 = [x_1 \ x_2 \ x_3]^T$  and  $\mathbf{X}_2 = [x_4 \ x_5 \ x_6]^T$ , the estimated value for  $\mathbf{X}_2$  can be designed as

$$\hat{\mathbf{X}}_2 = \mathbf{L}_r \mathbf{y} + \hat{\mathbf{z}} \quad (5.77)$$

$$\dot{\hat{\mathbf{z}}} = \mathbf{F}\hat{\mathbf{z}} + \mathbf{G}\mathbf{y} + \mathbf{H}\mathbf{u} \quad (5.78)$$

where  $\mathbf{L}_r \in R^{3 \times 3}$ ,  $\mathbf{F} \in R^{3 \times 3}$ ,  $\mathbf{G} \in R^{3 \times 3}$ , and  $\mathbf{H} \in R^{3 \times 2}$  will be determined by the observer pole placement. Defining the estimation error as

$$\mathbf{e}_2 = \mathbf{X}_2 - \hat{\mathbf{X}}_2 \quad (5.79)$$

the derivative of the estimation error becomes

$$\dot{\mathbf{e}}_2 = \dot{\mathbf{X}}_2 - \dot{\hat{\mathbf{X}}}_2 \quad (5.80)$$

Substituting the state-space equations of the system (Equation 5.77 and Equation 5.78) into Equation 5.80 and simplifying, we obtain

$$\dot{\mathbf{e}}_2 = \mathbf{F}\mathbf{e}_2 + (\mathbf{A}_{21} - \mathbf{L}_r \mathbf{C}_1 \mathbf{A}_{11} - \mathbf{G}\mathbf{C}_1 + \mathbf{F}\mathbf{L}_r \mathbf{C}_1) \mathbf{X}_1 + (\mathbf{A}_{22} - \mathbf{L}_r \mathbf{C}_1 \mathbf{A}_{12} - \mathbf{F}) \mathbf{X}_2 + (\mathbf{B}_2 - \mathbf{L}_r \mathbf{C}_1 \mathbf{B}_1 - \mathbf{H}) \mathbf{u} \quad (5.81)$$

In order to force the estimation error,  $\mathbf{e}_2$ , to go to zero, matrix  $\mathbf{F}$  should be selected to be Hurwitz and the following relations must be satisfied (Liu et al., 2002):

$$\mathbf{F} = \mathbf{A}_{22} - \mathbf{L}_r \mathbf{C}_1 \mathbf{A}_{12} \quad (5.82)$$

$$\mathbf{H} = \mathbf{B}_2 - \mathbf{L}_r \mathbf{C}_1 \mathbf{B}_1 \quad (5.83)$$

$$\mathbf{G} = (\mathbf{A}_{21} - \mathbf{L}_r \mathbf{C}_1 \mathbf{A}_{11} + \mathbf{F}\mathbf{L}_r \mathbf{C}_1) \mathbf{C}_1^{-1} \quad (5.84)$$

The matrix  $\mathbf{F}$  can be chosen by the desired observer pole placement requirement. Once  $\mathbf{F}$  is known,  $\mathbf{L}_r$ ,  $\mathbf{H}$ , and  $\mathbf{G}$  can be determined utilizing Equation 5.82, to Equation 5.84, respectively. The velocity variables,  $\hat{\mathbf{X}}_2$ , can now be estimated by Equation 5.77 and Equation 5.78.

#### 5.3.3.2.4 Numerical Simulations

In order to show the effectiveness of the controller, the flexible beam structure in Figure 5.29 is considered with the PZT actuator attached on the beam surface. The system parameters are listed in Table 5.4.

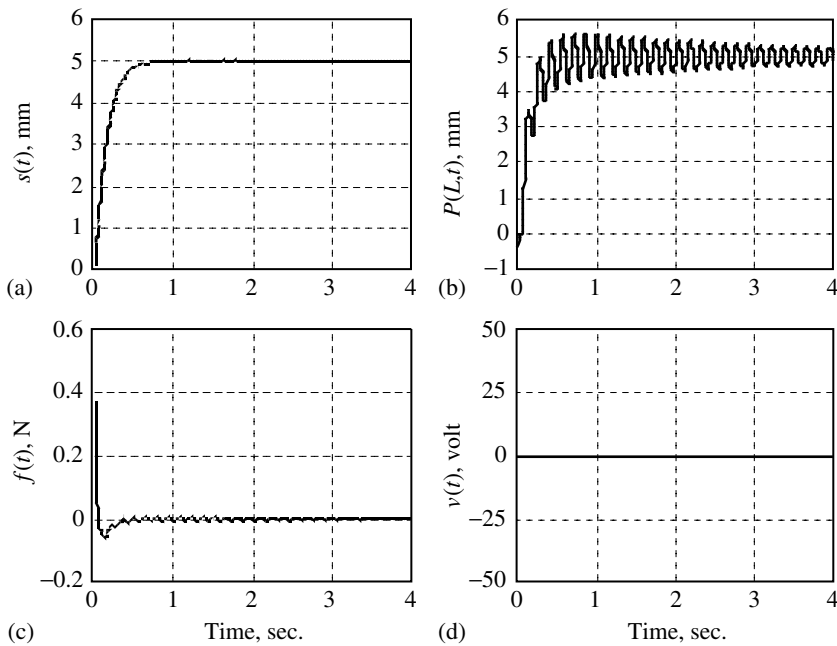
First, we consider the beam without PZT control. We take the PD control gains to be  $k_p = 120$  and  $k_d = 20$ . Figure 5.31 shows the results for the beam without PZT control (i.e., with only PD force control for the base movement). To investigate the effect of PZT controller on the beam vibration, we consider the voltage control gain to be  $k_v = 2 \times 10^7$ . The system responses to the proposed controller with a piezoelectric actuator based on the two-mode model are shown in Figure 5.32. The comparison between the tip displacement, from Figure 5.31 and Figure 5.32, shows that the beam vibration can be suppressed significantly utilizing the PZT actuator.

#### 5.3.3.2.5 Control Experiments

In order to demonstrate better the effectiveness of the controller, an experimental setup is constructed and used to verify the numerical results. The experimental apparatus consists of a flexible beam with a PZT actuator and strain sensor attachments, as well as data acquisition,

**TABLE 5.4** System Parameters Used in Numerical Simulations and Experimental Setup for Translational Beam

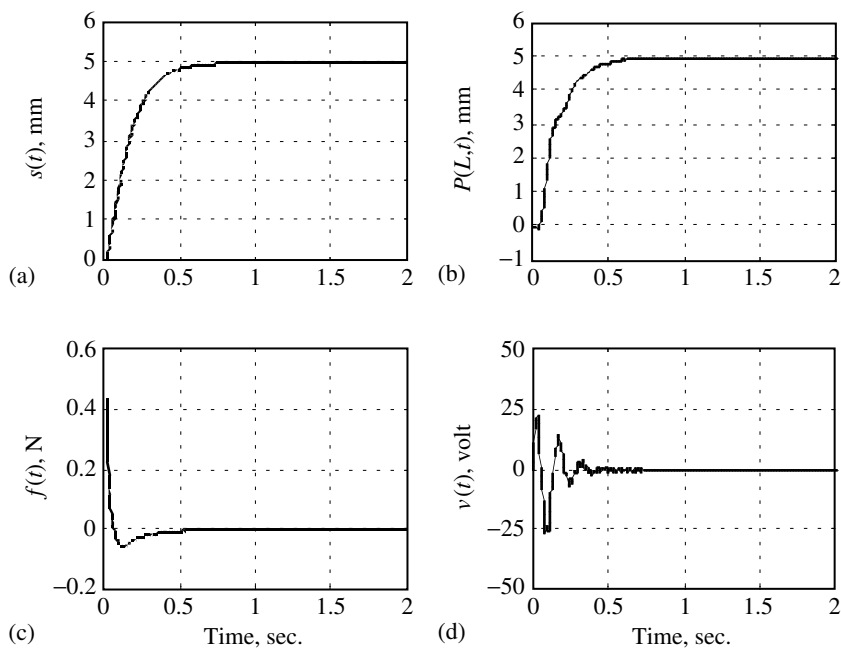
| Properties              | Symbol       | Value               | Unit              |
|-------------------------|--------------|---------------------|-------------------|
| Beam Young's modulus    | $c_{11}^b$   | $69 \times 10^9$    | N/m <sup>2</sup>  |
| Beam thickness          | $t_b$        | 0.8125              | mm                |
| Beam and PZT width      | $b$          | 20                  | mm                |
| Beam length             | $L$          | 300                 | mm                |
| Beam volumetric density | $\rho_b$     | 3960.0              | kg/m <sup>3</sup> |
| PZT Young's modulus     | $c_{11}^p$   | $66.47 \times 10^9$ | N/m <sup>2</sup>  |
| PZT coupling parameter  | $h_{12}$     | $5 \times 10^8$     | V/m               |
| PZT impermittivity      | $\beta_{22}$ | $4.55 \times 10^7$  | m/F               |
| PZT thickness           | $t_p$        | 0.2032              | mm                |
| PZT length              | $l_2 - l_1$  | 33.655              | mm                |
| PZT position on beam    | $l_1$        | 44.64               | mm                |
| PZT volumetric density  | $\rho_p$     | 7750.0              | kg/m <sup>3</sup> |
| Base mass               | $m_b$        | 0.455               | kg                |
| Tip mass                | $m_t$        | 0                   | kg                |



**FIGURE 5.31** Numerical simulations for the case without PZT control: (a) base motion; (b) tip displacement; (c) control force and (d) PZT voltage.

amplifier, signal conditioner and the control software. As shown in Figure 5.33, the plant consists of a flexible aluminum beam with a strain sensor and a PZT patch actuator bound on each side of the beam surface. One end of the beam is clamped to the base with a solid clamping fixture, which is driven by a shaker. The shaker is connected to the arm base by a connecting rod. The experimental setup parameters are listed in Table 5.4.

Figure 5.34 shows the high-level control block diagram of the experiment, where the shaker provides the input control force to the base and the PZT applies a controlled moment on the beam. Two laser sensors measure the position of the base and the beam-tip displacement. A strain-gauge sensor, which is attached near the base of the beam, is utilized for the dynamic strain measurement. These three signals



**FIGURE 5.32** Numerical simulations for the case with PZT control: (a) base motion; (b) tip displacement; (c) control force and (d) PZT voltage.

are fed back to the computer through the ISA MultiQ data acquisition card. The remaining required signals for the controller (Equation 5.66) are determined as explained in the preceding section. The data acquisition and control algorithms are implemented on an AMD Athlon 1100 MHz PC running under the RT-Linux operating system. The Matlab/Simulink environment and Real Time Linux Target are used to implement the controller.

The experimental results for both cases (i.e., without PZT and with PZT control) are depicted in Figure 5.35 and Figure 5.36, respectively. The results demonstrate that with PZT control, the arm vibration is eliminated in less than 1 sec, while the arm vibration lasts for more than 6 sec when PZT control is not used. The experimental results are in agreement with the simulation results except for some differences at the beginning of the motion. The slight overshoot and discrepancies at the beginning of the motion are due to the limitations of the experiment (e.g., the shaker saturation limitation) and unmodeled dynamics in the modeling (e.g., the friction modeling). However, it is still apparent that the PZT voltage control can substantially suppress the arm vibration despite such limitations and modeling imperfections.

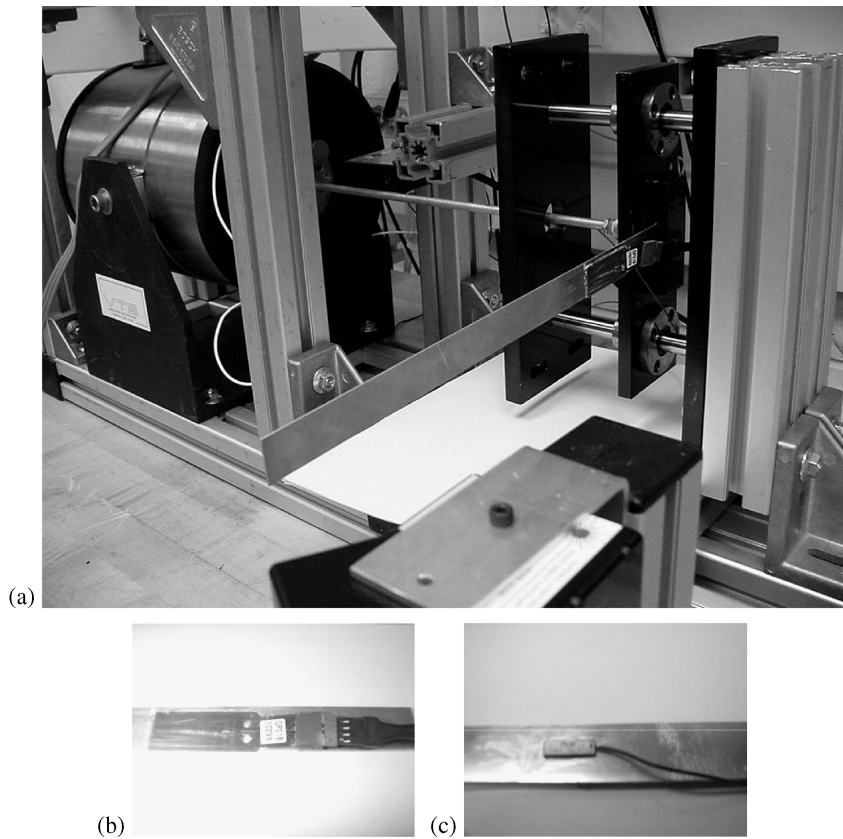
## 5.4 Practical Considerations and Related Topics

### 5.4.1 Summary of Vibration-Control Design Steps and Procedures

In order to select a suitable vibration-control system, especially a vibration isolator, a number of factors must be considered.

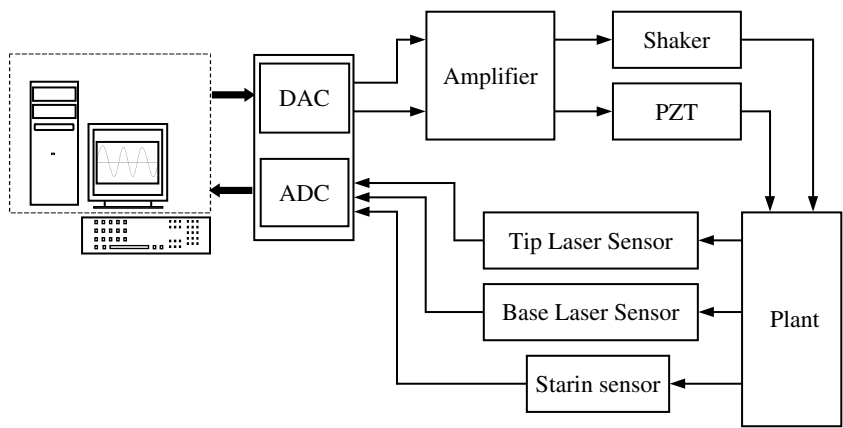
#### 5.4.1.1 Static Deflection

The static deflection of the vibration-control system under the deadweight of the load determines to a considerable extent the type of the material to be used in the isolator. Organic materials, such as rubber



**FIGURE 5.33** The experimental setup: (a) the whole system; (b) PZT actuator, ACX model No. QP21B; (c) dynamic strain sensor (attached on the other side of the beam), model No. PCB 740B02.

and cork, are capable of sustaining very large strains provided they are applied momentarily. However, if large strains remain for an appreciable period of time, they tend to drift or creep. On the other hand, metal springs undergo permanent deformation if the stress exceeds the yield stress of the material, but show minimal drift or creep when the stress is maintained below the yield stress.



**FIGURE 5.34** High-level control-block diagram.

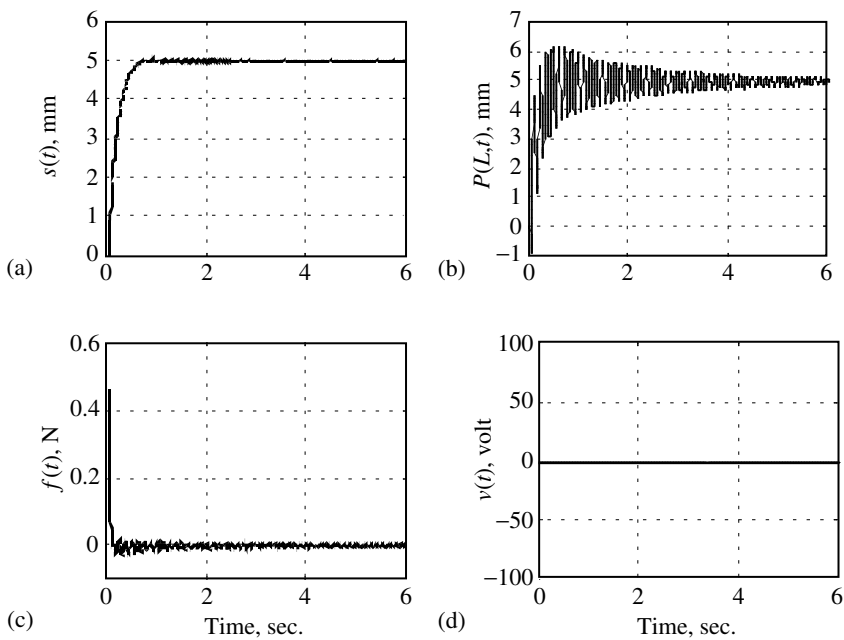


FIGURE 5.35 Experimental results for the case without PZT control: (a) base motion; (b) tip displacement; (c) control force and (d) PZT voltage.

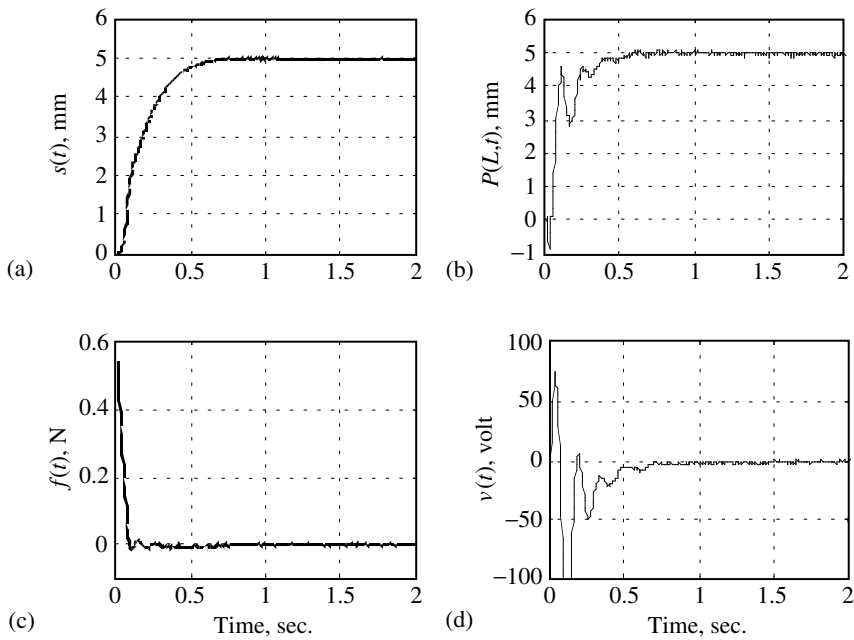


FIGURE 5.36 Experimental results for the case with PZT control: (a) base motion; (b) tip displacement; (c) control force and (d) PZT voltage.

5.4.1.2 Stiffness in Lateral Directions

Resilient materials strained in compression are most useful when the load is relatively large and the static deflection is small. Such applications are difficult to design for a small load, unless the required

static deflection is small. Otherwise, the small area and great thickness tend to cause a condition of instability. To a considerable extent, this limitation can be overcome by using sponge rubber, a material of lower modulus. In general, when the load is small, it is preferable to use rubber springs that carry the load in shear.

#### **5.4.1.3 Environmental Conditions**

It is highly common for vibration-control systems to be subjected to harsh environmental conditions. Especially in military applications, extreme ambient temperatures are encountered in addition to exposure to substances like ozone, rocket fuels, and so on. Organic materials are usually more susceptible to these environmental conditions than metal materials. However, owing to the superior mechanical properties of organic materials, such as lighter weight, smaller size, greater damping, and the ability to store large amounts of energy under shock, organic materials that are capable of withstanding the harsh conditions are being developed.

#### **5.4.1.4 Damping Characteristics**

In most of the vibration-control applications, the excitations cover a wide range of frequencies and may have random properties requiring the vibration-control systems to possess adequate damping. Elastomers possess very good damping properties when compared with metal springs, and they also eliminate the trouble of standing waves that occurs at high frequencies. If a metal spring is used in vibration-control applications requiring isolation of vibration at high frequencies, it is common to employ rubber pads in series with the metal spring, which also results in the damping of vibrations due to the addition of damping material.

#### **5.4.1.5 Weight and Space Limitations**

The amount of load-carrying resilient material is determined by the quantity of energy to be stored. In most of the cases, the vibration amplitude tends to be small relative to the static deflection, and the amount of material may be calculated by equating the energy stored in the material to the work done on the vibration control system.

#### **5.4.1.6 Dynamic Stiffness**

In the case of organic materials like rubber, the natural frequency calculated using the stiffness determined from a static-force deflection test of the spring is almost invariably lower than that experienced during vibration; that is, the dynamic modulus is greater than static modulus. The ratio between the dynamic and static modulus is generally between one and two. In many vibration-control applications, it is not feasible to mount the equipment directly upon the vibration-control system (isolator). Instead, a heavy, rigid block, usually made of concrete or heavy steel, supported by the isolator is employed.

### **5.4.2 Future Trends and Developments**

During recent years, there has been considerable interest in the design and implementation of a variety of vibration-control systems. Recent developments in multivariable control design methodology and microprocessor implementation of modern control algorithms have opened a new era for the design of externally controlled passive systems for use in such systems: fuzzy reasoning (Yoshimura, 1998); adaptive algorithms (Venhovens, 1994); observer design (Hedrick et al., 1994); and many others.

Observing these developments combined with the substantial ongoing theoretical advances in the areas of adaptive and nonlinear controls (Astrom and Wittenmark, 1989; Alleyne and Hedrick, 1995), it is expected that the future will bring applications of these techniques in advanced vibration-control system design. For practical implementation, however, it is preferable to simplify these strategies, thus leading to simpler software implementations. Suboptimal policy neglecting some performance requirements can serve as an example of such simplifications.

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