

Helicopter Rotor Tuning

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Summary

Before a helicopter leaves the plant, its rotors need to be tuned so that the helicopter vibration meets the required specifications during different flight regimes. For this, three different adjustments can be made to each rotor blade in response to the magnitude and phase of vibration. In this chapter (also see [Chapter 7](#)), the basic concepts for determining the blade adjustments are discussed, and three methods with fundamentally different approaches are described. A neural network-based method is described, which trains a feedforward network as the inverse model of the effect of the blade adjustments on helicopter vibrations, and uses the inverse model to determine the blade adjustments. Another is a probability-based method that maximizes the likelihood of success of the selected blade adjustments based on a stochastic model of the probability densities of the vibration components. The third method is an adaptive method that uses an interval model to represent the range of effect of blade adjustments on helicopter vibration, so as to cope with the nonlinear and stochastic nature of aircraft vibration. This method includes the a priori knowledge of the process by defining the initial coefficients of the interval model according to sensitivity coefficients between the blade adjustments and helicopter vibration, but then transforms these coefficients into intervals and updates them after each tuning iteration, to improve the model estimation accuracy. The details of rotor tuning are described through a case study, which demonstrates the application of the adaptive method.

6.1 Introduction

Helicopter rotor tuning (track and balance) is the process of adjusting the rotor blades so as to reduce the aircraft vibration and the spread of rotors. Rotor tuning as applied to Sikorsky's Black Hawk (H-60) helicopters is performed as follows. For initial measurements, the aircraft is flown through six different regimes, during which measurements of rotor track and vibration (balance) are recorded. Rotor track is measured by optical sensors, which detect the vertical position of the blades. Vibration is measured at the frequency of once per blade revolution (per rev) by two accelerometers, A and B, attached to the sides of the cockpit (see [Figure 6.1](#), detail B). The vibration data are vectorially combined into two components: $A + B$, representing the vertical vibration of the aircraft, and $A - B$, representing its roll vibration. A sample of peak vibration levels for the six flight regimes, as well as the peak angular positions relative to a reference blade, are given in [Table 6.1](#), along with a sample of track data.

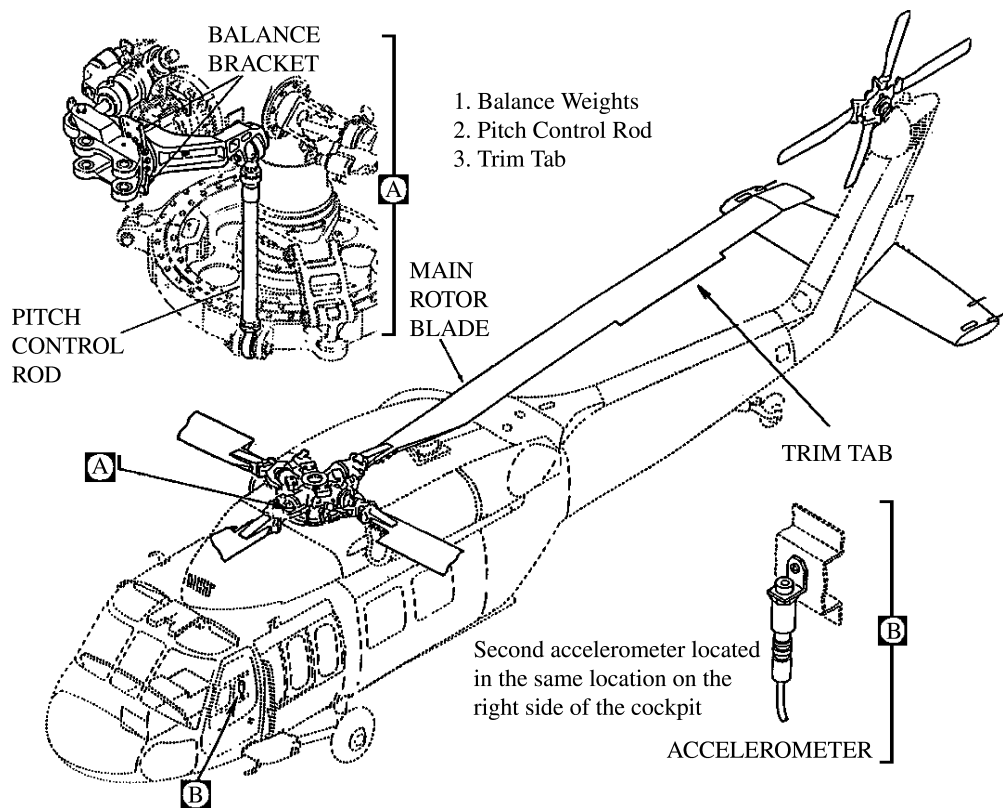


FIGURE 6.1 Illustration of the position of accelerometers A and B on the aircraft, and the rotor blade adjustments (push rod, trim tab, and hub weights).

TABLE 6.1 Typical Track and Balance Data Recorded during a Flight

Flight Regime	Vibration			
	A + B		A - B	
	Magnitude (ips)	Phase (°)	Magnitude (ips)	Phase (°)
fpm	0.19	332	0.38	272
hov	0.07	247	0.10	217
80	0.02	86	0.04	236
120	0.04	28	0.04	333
145	0.02	104	0.07	162
vh	0.10	312	0.12	211
Track (mm)				
Blade #				
	1	2	3	4
fpm	-2	3	1	-2
hov	-1	3	0	-2
80	1	11	1	-13
120	2	13	-1	-14
145	5	18	-3	-20
vh	2	13	-1	-14

The six flight regimes in Table 6.1 are: ground (fpm), hover (hov), 80 knots (80), 120 knots (120), 145 knots (145), and maximum horizontal speed (vh). The track data indicate the vertical position of each blade relative to a mean position.

In order to bring track and one per rev vibration within specification, three types of adjustments can be made to the rotor system: pitch control rod adjustments, trim tab adjustments, and balance weight adjustments (see Figure 6.1). Pitch control rods can be extended or contracted by a certain number of notches to alter the pitch of the rotor blades. Positive push rod adjustments indicate extension. Trim tabs, which are adjustable surfaces on the trailing edge of the rotor blades, affect the aerodynamic pitch moment of the air foils and consequently their vibration characteristics. Tab adjustments are measured in thousandths of an inch, with positive and negative changes representing upward and downward tabbing, respectively. Finally, balance weights can be either added to or removed from the rotor hub to tune vibrations through changes in the blade mass. Balance weights are measured in ounces, with positive adjustments representing the addition of weight. In the case of the Sikorsky H-60 helicopter, which has four main rotor blades, a total of 12 adjustments can be made to tune the rotors (i.e., three adjustments per blade). Among them, balance weights primarily affect the ground vibration, so they are not commonly used for in-flight tuning. Furthermore, since the symmetry of rotor blades in four-bladed aircraft produces identical effects for adjustments to opposite blades, the combined form of blade adjustments to opposite blade pairs can be used as inputs. Accordingly, the input vector can be defined as

$$\Delta \mathbf{x} = [\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4]^T \tag{6.1}$$

where Δx_1 and Δx_3 denote the combined (condensed) trim tab adjustments (ΔT) to blade combinations one/three and two/four, respectively, and Δx_2 and Δx_4 represent the combined pitch control rod adjustments (ΔP) to blade combinations one/three and two/four, respectively. The relationships between the combined and individual adjustments are in the form:

$$\Delta x_1 = \Delta T_3 - \Delta T_1 \tag{6.2}$$

$$\Delta x_2 = \Delta P_3 - \Delta P_1 \tag{6.3}$$

$$\Delta x_3 = \Delta T_4 - \Delta T_2 \tag{6.4}$$

$$\Delta x_4 = \Delta P_4 - \Delta P_2 \tag{6.5}$$

Ideally, identical adjustments made to any two aircraft with different tail numbers should result in identical changes in vibration. In reality, however, significant inconsistencies in vibration changes may be present for identical adjustments to different tail numbers. This is perhaps due more to nonuniformity of flight conditions from weather or error in implementing the blade adjustments than factors such as dissimilarities between aircraft and rotor blades.

Virtually all of the current systems of rotor track and balance rely on the strategy shown in Figure 6.2, whereby the measurements of the flight just completed are used as the basis of search for the new blade adjustments. The search for blade adjustments is guided by the “process model” (see Figure 6.2), which represents the relationship between vibration changes and blade adjustments. A difficulty of rotor tuning is the excess of equations compared to degrees of freedom (four inputs to control 24 outputs), which translates into one-to-many mapping. Another difficulty is caused by the high level of noise present in the vibration measurements.

The traditional approach to rotor tuning uses linear relationships to define the process model

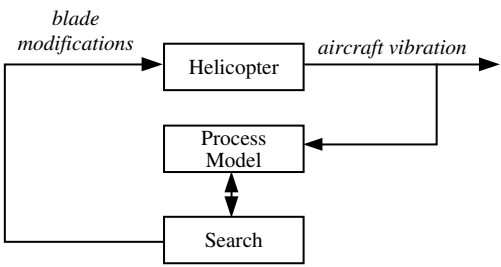


FIGURE 6.2 Tuning strategy of the current methods.

and uses model inversion to streamline the search. The drawback of the traditional approach, therefore, is its neglect of the potential nonlinearity of track and balance, and the vibration noise, as well as its limited capacity to produce comprehensive solutions to facilitate model inversion due to its consideration of the most extreme vibration components. In an attempt to include the potential nonlinearity of the process, Taitel et al. (1995) trained a set of neural networks with actual track and balance data to map vibration measurements to blade adjustments as well as to evaluate the goodness of the solution. In effect, they developed an inverse model based on the solutions available in the historical track and balance data, and provided a forward model to evaluate the solution. The potential advantage of this method is that it can interpolate among the historical solutions to address potential nonlinearity and vibration noise. Its disadvantages are that it is only applicable to helicopters with extensive track and balance history, and that its solutions are constrained by those contained in the historical data.

Another deviation from the traditional approach is introduced by Ventres and Hayden (2000), who define the relationships between blade adjustments and vibration in frequency domain, and provide an extension of these relationships to higher order vibrations. They use an optimization method to search for the adjustments to reduce per rev vibration as well as higher-order vibrations. Accordingly, this approach has the capacity to provide a comprehensive solution, but it too neglects the potential nonlinearity between the blade adjustments and aircraft vibration as well as the noise in the measurements.

The most recent solutions to rotor tuning are those by Wang et al. (2005a, 2005b), which are designed to address both the stochastics of vibration and the potential nonlinearity of the tuning process. In the first solution, which is a probability-based method, the underlying model comprises two components: a deterministic component and a probability component. The method relies on the probability model to estimate the likelihood of the measured vibration satisfying the specifications and to search for blade adjustments that will maximize this likelihood. The likelihood measures in the probability model are computed according to the probability distribution of vibration derived from historical track and balance data. The second solution is an adaptive method that uses an interval model to cope with the potential nonlinearity of the process and to account for vibration noise. This method, which also incorporates learning to provide adaptation to the rotor tuning process, initializes the coefficients of the interval model according to the sensitivity coefficients between the blade adjustments and helicopter vibration. However, it modifies these coefficients after the first iteration to better represent the vibration measurements acquired. This method takes into account vibration data from all of the flight regimes during the search for the appropriate blade adjustments; therefore, it has the capacity to provide comprehensive solutions. The remainder of this chapter describes three of the methods discussed above to provide a representation of various solutions proposed for rotor tuning, followed by a case study to demonstrate the application of the adaptive method.

6.2 Neural Network-Based Tuning

As mentioned earlier, rotor tuning in four-bladed aircraft is performed by first specifying a condensed set of adjustments to reduce vibrations, and then expanding these adjustments into a detailed set to satisfy the track requirements. This same strategy is implemented in the system of neural networks shown in [Figure 6.3](#) (Taitel et al., 1995). The first network in this system, called the selection net, determines the condensed blade adjustments (output) that will bring about a given change in vibration (input). To eliminate vibration, the negatives of the vibration measurements from the flight are utilized as inputs to this network. The validity of the condensed adjustments is then checked by predicting their effect on vibration via the condensed simulation net. Theoretically, these simulated vibration changes should be the negative of the vibration measurements from the aircraft so that their summation will be zero. However, owing to the inexactness of the neural network models and noise, the resultant vibration will most likely not equal zero. In cases where the resultant vibration is not within specifications (usually less than 0.20 inches per second [ips]), the condensed adjustments may be refined by feeding the resultant

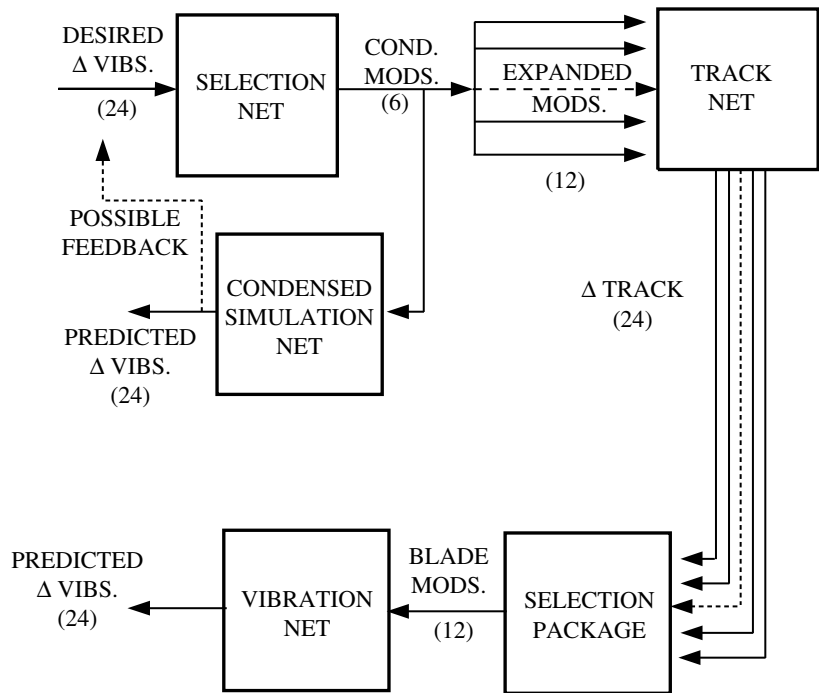


FIGURE 6.3 Schematic of the rotor tuning system. The numbers inside parentheses represent the number of inputs or outputs of individual nets.

vibration back into the selection net. This feedback is depicted by the dashed feedback line in Figure 6.3. It should be noted that the condensed simulation net may also serve as a diagnostic tool by indicating behavior out of the norm. For example, an aircraft with vibrations significantly different from those predicted by this network may suffer from defective components.

Just as with the traditional approach, once the condensed solution has been specified, it needs to be expanded into a detailed form to satisfy the rotor track requirements. As previously mentioned, the condensed set of adjustments may be viewed as the constraint on detailed adjustments so as to ensure that the vibration solution is not compromised for track. Each one of these detailed sets of adjustments is a candidate for the final rotor tuning solution, and it is left to the track net and the selection package to determine which set of detailed adjustments provides the best tracking performance. For selection purposes, the track net simulates the changes in track due to a candidate set of detailed adjustments, and then adds these changes to the initial track measurements from the flight to estimate the resultant track. The set of detailed adjustments that yields the smallest estimated track (i.e., smallest maximum blade spread) is selected as the solution to the rotor tuning problem. The selected set of detailed adjustments is then checked via the vibration net, which, similar to the condensed simulation net, serves as an independent evaluator of the selected adjustments.

6.3 Probability-Based Tuning

The noted contribution of this method is its introduction of the likelihood of success as a criterion in the search for the blade adjustments (Wang et al., 2005). This method speculates the effectiveness of various adjustment sets in reducing the vibration and selects the set with the maximum probability of producing acceptable vibration. The concept of this method is explained in the context of a simple example. If the measured vibration from the current flight is

denoted by $V_j(k - 1)$ and the estimated vibration change according to the model is represented by $\Delta \hat{V}_j(k) = f(\Delta x)$ as a function of the blade adjustments, Δx , then the predicted vibration of the next flight, $\hat{V}_j(k)$, can be defined as

$$\hat{V}_j(k) = V_j(k - 1) + \Delta \hat{V}_j(k) \tag{6.6}$$

$$V_j(k) = \hat{V}_j(k) + \hat{e}_j(k) \tag{6.7}$$

where $V_j(k)$ denotes the measured vibration for the next flight. In rotor tuning, the adjustments are selected according to the predicted vibration, $\hat{V}_j(k)$, whereas the objective is defined in terms of the measured vibration. The inclusion of the probability model here is to account for the inevitable uncertainty in the actual position of the measured vibration. According to Equation 6.7, the mean value of the measured vibration is equal to the value of the predicted vibration plus the mean value of the prediction error. However, since the predicted vibration is a deterministic entity, the probability distribution of the measured vibration is the same as that of the prediction error. Accordingly, whereas the nominal value of the measured vibration can be controlled by the blade adjustment, its optimal position within the specification region should be determined according to its probability distribution. For a case where the prediction error, $\hat{e}_j(k)$, is zero-mean and normally distributed, as illustrated in Figure 6.4, placing the predicted vibration at the center of the specification range will be synonymous with maximizing the probability that the measured vibration will be within the range. The likelihood of success of blade adjustments can therefore be measured by the area under the probability density function of prediction error located within the specification region. The blade adjustment set that produces the highest likelihood will be the preferred adjustment.

The main difficulty with rotor tuning, however, is the limited number of DoFs, which precludes perfect positioning of the predicted vibration. This point is illustrated in Figure 6.5 for a case where two vibration components are to be positioned at the center of the specification region with only one adjustment. If one assumes that the effect of adjustment, Δx , on the change in the two vibration components, $\Delta \hat{V}_j(k)$, can be represented by a linear model, as

$$\Delta \hat{V}_j(k) = a_{ij} \Delta x$$

then the position of the predicted vibration components will be constrained to the line L in Figure 6.5. As illustrated in this figure, since it will be impossible to place the predicted vibration components at the center, a compromised position needs to be selected. In this method, the best compromised position for the predicted vibration is that which renders the largest probability of satisfying the specifications for the measured vibration. This position, for the two-component vibration example, is one that maximizes $P_r[(V_1, V_2) \in S] = \int_{(V_1, V_2) \in S} P(V_1, V_2) dV_1 dV_2$. The above formulation indicates that the placement of the predicted vibration requires knowledge of

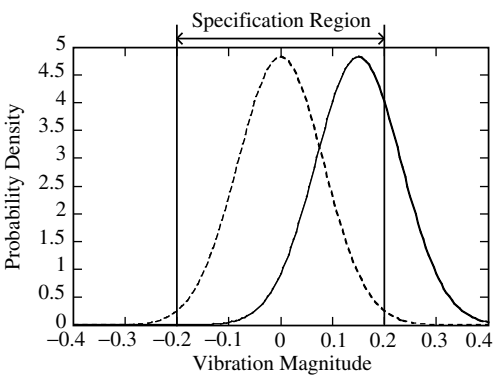


FIGURE 6.4 Illustration of improved placement of the predicted vibration within the specification range.

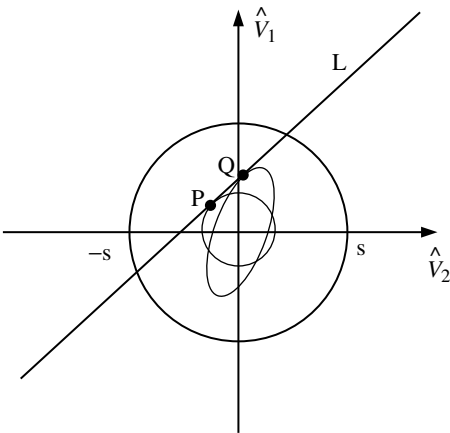


FIGURE 6.5 Restricted placement of vibration components within the specification region for a two-dimensional case.

the joint probability density function, $p(V_1, V_2)$, of the vibration components. In the ideal case of independent vibration components with equal probability distributions, the loci of the points with equal probabilities $P_r[(V_1, \dots, V_n) \in S]$ are surfaces of hyperspheres. Such ideal loci for the two-component vibration example of Figure 6.5 are circles centered at the origin (see Figure 6.5), which lead to point P as the best compromised position closest on line L to the center of the specification circle. Point P, however, does not represent the best position if the two vibration components are dependent or have unequal distributions. The loci of equal probabilities for this more general case are elliptical, as also shown in Figure 6.5, indicating point Q as the best position on line L for placing the predicted vibration. The inadequacy of the DoFs illustrated here is exacerbated in rotor tuning, where 24 correlated vibration components need to be positioned within the specification region using only four condensed blade adjustments. For the 24-component vector of measured vibration $\mathbf{V}(k) = [V_{c1}(k), V_{s1}(k), \dots, V_{c12}(k), V_{s12}(k)]^T$, where V_c and V_s represent the cosine and sine components of each vibration measurement, respectively, the joint probability density function of measured vibration for the k th flight, $\mathbf{V}(k)$, can be characterized as an N -dimensional Gaussian function:

$$p(\mathbf{V}(k)) = \frac{1}{(2\pi)^{N/2} |\Phi|^{1/2}} \exp \left[-\frac{1}{2} \hat{\mathbf{e}}(k)^T \Phi^{-1} \hat{\mathbf{e}}(k) \right] \quad (6.8)$$

$$\hat{\mathbf{e}}(k) = \mathbf{V}(k) - \mathbf{V}(k-1) - C\Delta\mathbf{x}(k) \quad (6.9)$$

where Φ represents the covariance matrix of the prediction error. Now, if $\Gamma = \{|\mathbf{V}_j| = \sqrt{V_{cj}^2 + V_{sj}^2} \leq \alpha, j = 1, \dots, 12\}$ denotes the specification region in 24-dimensional Euclidean space, the blade adjustments, $\Delta\mathbf{x}^*$, can be selected such that the probability that the measured vibration is within the acceptable range is maximized (see also Table 6.2). Formally,

$$\Delta\mathbf{x}^* = \arg_{\Delta\mathbf{x}} \max \left[\Pr(\mathbf{V}(k) \in \Gamma) = \int_{\Gamma} p(\mathbf{V}(k)) d\mathbf{V}(k) \right] \quad (6.10)$$

TABLE 6.2 Summary of Probability-Based Tuning

For the input vector:

$$\Delta\mathbf{x} = [\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4]^T$$

where Δx_1 and Δx_3 denote the combined trim tab adjustments to blade combinations one to three and two to four, respectively, and Δx_2 and Δx_4 represent the combined pitch control rod adjustments to blade combinations one to three and two to four, respectively, the blade adjustments, $\Delta\mathbf{x}$, can be selected such that the probability that the measured vibration is within the acceptable range is maximized. Formally,

$$\Delta\mathbf{x}^* = \arg_{\Delta\mathbf{x}} \max [\Pr(\mathbf{V}(k) \in \Gamma) = \int_{\Gamma} p(\mathbf{V}(k)) d\mathbf{V}(k)]$$

where $\Pr(\mathbf{V}(k))$ denotes the probability of the measured vibration, Γ denotes the specification region in 24-dimensional Euclidean space, and $p(\mathbf{V}(k))$ represents the joint probability density of the measured vibration for the k th flight characterized as an N -dimensional Gaussian function:

$$p(\mathbf{V}(k)) = \frac{1}{(2\pi)^{N/2} |\Phi|^{1/2}} \exp \left[-\frac{1}{2} \hat{\mathbf{e}}(k)^T \Phi^{-1} \hat{\mathbf{e}}(k) \right]$$

with

$$\hat{\mathbf{e}}(k) = \mathbf{V}(k) - \mathbf{V}(k-1) - C\Delta\mathbf{x}(k)$$

representing the predicted error in vibration.

6.4 Adaptive Tuning

The schematic of this method is shown in Figure 6.6 (Wang et al., 2005). As in the other methods, it uses a process model as the basis of search for the appropriate blade adjustments, but instead of using a linear model, it uses an interval model to accommodate process nonlinearity and measurement noise. According to this model, the feasible region of the process is estimated first, to include the adjustments that will result in acceptable vibration estimates. This feasible region is then used to search for the blade adjustments that will minimize the modeled vibration. If the application of these adjustments does not result in satisfactory vibration, the interval model will be updated to better estimate the feasible region and improve the choice of blade adjustments for the next flight. Important parameters of adaptive tuning are summarized in Table 6.3.

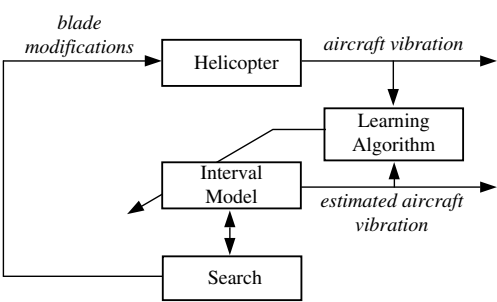


FIGURE 6.6 The strategy of the proposed tuning method.

6.4.1 The Interval Model

In order to account for the stochastics and nonlinearity of vibration, an interval model (Moore, 1979) is defined to represent the range of aircraft vibration caused by blade adjustments. The interval model used here has the form:

$$\Delta\tilde{y}_j = \sum_{i=1}^n \tilde{C}_{ji} \Delta x_i, \quad j = 1, \dots, m \tag{6.11}$$

where each coefficient is defined as an interval:

$$\tilde{C}_{ji} = [C_{Lji}, C_{Uji}]$$

In the above model, the variables with the two-sided arrow, $\leftrightarrow$, denote intervalled variables, C_{Lji} and C_{Uji} represent, respectively, the current values of the lower and upper bounds of the sensitivity coefficients between each input, Δx_i , and output, $\Delta\tilde{y}_j$. The interval $\Delta\tilde{y}_j$ denotes the estimated range of change of the j th output caused by the change to the current inputs, $\Delta x_1, \dots, \Delta x_n$.

TABLE 6.3 Summary of Adaptive Tuning

In adaptive tuning, each vibration component is defined as

$$\Delta\tilde{y}_j = \sum_{i=1}^n \tilde{C}_{ji} \Delta x_i, \quad j = 1, \dots, m$$

where each coefficient is defined as an interval:

$$\tilde{C}_{ji} = [C_{Lji}, C_{Uji}]$$

with C_{Lji} and C_{Uji} representing, respectively, the current values of the lower and upper bounds of the sensitivity coefficients between each input, Δx_i , and output, $\Delta\tilde{y}_j$. The blade adjustments are then sought by minimizing the objective function:

$$S = \frac{\sum_{e=1}^{N_e} \text{Distance}(x_c, x_e)}{(\prod_{s=1}^{N_s} \text{Distance}(x_c, x_s))^{1/N_s}}$$

where x_c represents a candidate set of blade adjustments within the feasible region, x_e represents any set of blade adjustments within the selection region, x_s denotes each of the previously selected blade adjustments, and N_e and N_s represent the number of the estimated feasible blade adjustments and the previously selected blade adjustments, respectively.

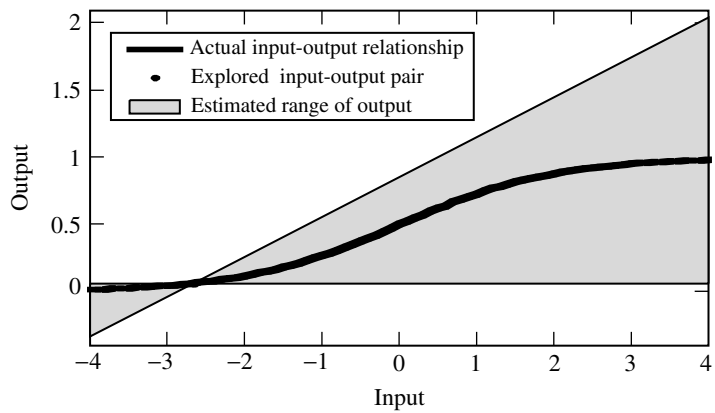


FIGURE 6.7 Estimated range of output by the interval model using one reference input.

The fit provided by the interval model for a mildly nonlinear input/output relationship is illustrated in Figure 6.7, where the output range is estimated relative to one explored input.¹ According to Equation 6.11, the estimated range of the output becomes larger, and therefore less accurate, as the potential input is selected farther from the current input (producing a large Δx_i). This potential drawback of the interval model is considerably reduced when multiple inputs have been explored so that the interval model can take advantage of several inputs for estimating the output range. The estimated output, $\tilde{y}_j$, at a potential input, x_i , may be computed relative to any set of previously explored inputs, yielding different estimates of $\tilde{y}_j$ (due to different values of Δx_i). In order to cope with the multiplicity of estimates, $\tilde{y}_j$ is defined as the common range among all of the $\tilde{y}_j$ estimates (Yang, 2000). The estimation of $\tilde{y}_j$ using this commonality rule is illustrated in Figure 6.8, which indicates that using this estimation approach enables representation of the system nonlinearities in a piecewise fashion. It can be shown that the lack of commonality between the estimated ranges of output will cause a part of the input–output relationship to not be represented by the interval model. In such cases, however, the lack of compliance between the interval model and the *input–output* relationship can be corrected by adaptation of the coefficient intervals through learning.

6.4.2 Estimation of Feasible Region

The feasible region comprises all sets of blade adjustments that will reduce the aircraft vibration within specifications. The feasible region is estimated here by comparing the individually estimated $\tilde{y}_j$ values with their corresponding constraints, so as to decide whether the corresponding blade adjustments belong to the feasible region. In this method, even when the interval $\tilde{y}_j$ partly overlaps the vibration constraint, the corresponding blade adjustments are included in the estimated feasible region. The above procedure of estimating the feasible region based on individual outputs is then extended to multiple outputs by forming the conjunction of the estimated feasible regions from each output.

6.4.3 Selection of Blade Adjustments

The blade adjustments provide the coordinates of the feasible region, therefore, they need to provide a balanced coverage of the input space. As such, blade adjustment selection becomes synonymous with maximizing the distance of the selected blade adjustments from the previous blade adjustments, as well as

¹An explored input represents an input for which the exact value of the output is available. In rotor tuning, an explored input would denote a blade adjustment that has been applied to the helicopter, and for which the corresponding vibration changes have been measured.

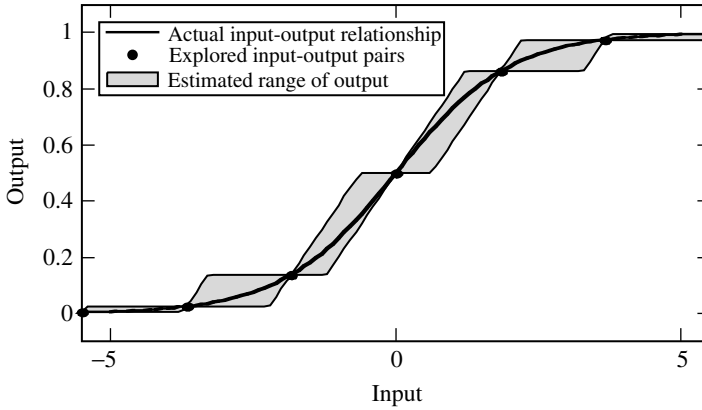


FIGURE 6.8 Estimated range of output by the interval model using seven reference inputs.

bringing them closer to the center of the feasible region. This objective can be pursued by minimizing the following objective function:

$$S = \frac{\sum_{e=1}^{N_e} \text{Distance}(x_c, x_e)}{\left(\prod_{s=1}^{N_s} \text{Distance}(x_c, x_s)\right)^{1/N_s}} \quad (6.12)$$

where x_c represents a candidate set of blade adjustments within the feasible region, x_e represents any set of blade adjustments within the selection region, x_s denotes each of the previously selected blade adjustments, and N_e and N_s represent, respectively, the numbers of the estimated feasible blade adjustments and the previously selected blade adjustments. Note that when the candidate set, x_c , is close to the previously selected blade adjustments, $(\prod_{s=1}^{N_s} \text{Distance}(x_c, x_s))^{1/N_s}$ becomes small, and when the candidate set of blade adjustments, x_c , is far from the center of the feasible region, the value of $\sum_{e=1}^{N_e} \text{Distance}(x_c, x_e)$ becomes large. By minimizing S , the candidate blade adjustments are selected such that the above extremes are avoided.

6.4.4 Learning

Although an interval model defined according to the sensitivity coefficients may provide a suitable initial basis for tuning, it may not be the most representative of the rotor tuning process. As such, it may not be able to carry the search process to the end. A noted feature of the proposed method is its learning capability, which enables it to refine its knowledge base. To this end, the coefficients of the model are updated by considering new values for each of the upper and lower limits of individual coefficients. The objective is to make the range of the coefficients as small as possible while making sure that the interval model envelopes the acquired *input–output* data. The learning problem can be defined as

$$\text{Minimize } E = \sum_{m=1}^{K-1} \sum_{k>m}^K \{[y_L(m, k) - y(k)]^2 + [y_U(m, k) - y(k)]^2\} \quad (6.13)$$

subject to

$$y_U(m, k) \geq y(k) \quad (6.14)$$

$$y_L(m, k) \leq y(k) \quad (6.15)$$

$$C_{Ui} - \gamma \geq C_{Li} \quad (6.16)$$

where K represents the total number of sample points collected so far, $y_L(m, k)$ and $y_U(m, k)$ represent, respectively, the lower and upper limits of the estimated output range at the k th sample point relative to

the m th sample point, $y(k)$ denotes the actual output value at the k th sample point, and C_{Ui} and C_{Li} represent the upper and lower limits of the i th coefficient interval, respectively. The parameter γ is a small positive number to control the range of the coefficients.

Most of the approaches that can be potentially used for adapting the coefficient intervals, such as gradient descent (Ishibuchi et al., 1993) or nonlinear programming, cannot be applied to rotor tuning due to their demand for rich training data and their impartiality to the initial value of the coefficients representing the *a priori* knowledge of the process. As an alternative, a learning algorithm is devised here to cope with the scarcity of track and balance data while staying true to the initial values of the coefficients. In this algorithm, the coefficients of the interval model, initially set pointwise at the sensitivity coefficients, are adapted after each flight in two steps: enlargement and shrinkage. First, the vibration measurements from all of the flights completed for the present tail number are matched against the estimated output ranges from the current interval model. If any of the measurements do not fit the upper or lower limits of the estimates, the coefficient intervals are enlarged in small steps, iteratively, and the output ranges are re-estimated at each iteration using the updated interval model. The enlargement of the coefficient intervals stops when the estimated output ranges include all of the measurements. At this point, even though the updated interval model provides a fit for the *input-output* data, it may be overcompensated. In order to rectify this situation, the coefficient intervals are shrunk individually by selecting new candidates for their upper and lower limits.

The shrinkage–enlargement learning algorithm has the form:

$$\Delta C_{Li} = -\eta \delta_L \Delta x_i(m, k) \quad (6.17)$$

$$\Delta C_{Ui} = -\eta \delta_U \Delta x_i(m, k) \quad (6.18)$$

where, during the enlargement phase, δ_L and δ_U are defined as

$$\delta_L = \begin{cases} \Delta y_L & \text{If } \Delta x_i(m, k) > 0 \text{ and } \Delta y_L > 0 \\ \Delta y_U & \text{If } \Delta x_i(m, k) < 0 \text{ and } \Delta y_U < 0 \\ 0 & \text{otherwise} \end{cases} \quad (6.19)$$

$$\delta_U = \begin{cases} \Delta y_U & \text{If } \Delta x_i(m, k) > 0 \text{ and } \Delta y_U < 0 \\ \Delta y_L & \text{If } \Delta x_i(m, k) < 0 \text{ and } \Delta y_L > 0 \\ 0 & \text{otherwise} \end{cases} \quad (6.20)$$

and during the shrinkage phase, they are defined as

$$\delta_L = \begin{cases} \Delta y_L & \text{If } \Delta x_i(m, k) > 0 \text{ and } \Delta y_L < 0 \\ \Delta y_U & \text{If } \Delta x_i(m, k) < 0 \text{ and } \Delta y_U > 0 \\ 0 & \text{otherwise} \end{cases} \quad (6.21)$$

$$\delta_U = \begin{cases} \Delta y_U & \text{If } \Delta x_i(m, k) > 0 \text{ and } \Delta y_U > 0 \\ \Delta y_L & \text{If } \Delta x_i(m, k) < 0 \text{ and } \Delta y_L < 0 \\ 0 & \text{otherwise} \end{cases} \quad (6.22)$$

with

$$\Delta x_i(m, k) = x_i(k) - x_i(m) \quad (6.23)$$

$$\Delta y_L = y_L(m, k) - y(k) \quad (6.24)$$

$$\Delta y_U = y_U(m, k) - y(k) \quad (6.25)$$

This procedure is repeated for each coefficient interval in an iterative fashion until the objective function E (Equation 6.13) is minimized. The minimization of E ensures limited adaptation of the coefficient intervals within the smallest possible range.

At the beginning of tuning, the limited number of *input–output* data available for learning will not provide a comprehensive representation of the process. Therefore, the coefficient intervals should not be shrunk drastically until enough *input–output* data have become available. For this, the length of each coefficient interval $[C_{Li}, C_{Ui}]$ is constrained by the minimal interval length for each tuning iteration as

$$\min L = \{C_{Ui}(0) - C_{Li}(0)\}(1 - \beta)^n \quad (6.26)$$

where $\beta \in [0, 1]$ controls the shrinkage rate of the coefficient interval, and n denotes the number of tuning iterations. The coefficient interval cannot be shrunk when $\beta = 0$ and can be shrunk without limit when $\beta = 1$. Usually, β is selected closer to 0.

6.5 Case Study

The utility of the Interval Model (IM) method is demonstrated in application to Black Hawks. Ideally, the performance of the proposed method should be evaluated side by side against that of the traditional method. However, such an evaluation would require tuning the aircraft with one method, undoing changes, and tuning the aircraft with another. Since such testing is prohibitively costly and infeasible, a compromised approach of evaluating the method in simulation is utilized. A process simulation model is therefore used to represent the block “helicopter” in Figure 6.6, with the block “forward model” represented by an interval model.

6.5.1 Simulation Model

Considering the potential nonlinearity of the effect of blade adjustments on the helicopter vibration and the high level of noise present in vibration measurements, multilayer neural networks offer the most suitable framework for modeling. A series of neural networks were trained with historical balance data to represent the relationships between vibration changes and blade adjustments, and the stochastic aspects of vibration were represented by the addition of random numbers to the outputs of the networks.

A total of 102 sets of vibration data were used to train and test the neural networks. The inputs to these networks were the combined blade adjustments of push rods and trim tabs to opposite blade pairs, and their outputs were the resulting vibration changes between two consecutive flights. Since the vibration data are vector quantities that are represented by both magnitude and phase components (see Table 6.1), the vibration data were transformed into Cartesian coordinates, so that each vector element would denote the change in the cosine or sine component of the $A + B$ or $A - B$ vibration of each of the six flight regimes (see Table 6.1). In this study, each neural network model consisted of four inputs and one output, so a total of 24 networks were trained to represent all of the vibration components. Alternatively, all of the vibration measurements may be represented by one neural network, but such a network is more difficult to train. Formally, the outputs of the neural networks, which represent the cosine and sine components of the vibration at different regimes, $v_{sj}(k)$ and $v_{cj}(k)$, respectively, are defined as

$$\hat{V}_{sj}(k) = V_{sj}(k - 1) + \Delta V_{sj}(k) + R_{sj}(k) \quad (6.27)$$

$$\hat{V}_{cj}(k) = V_{cj}(k - 1) + \Delta V_{cj}(k) + R_{cj}(k) \quad (6.28)$$

$$\Delta V_{sj}(k) = F_{sj}(\Delta x) \quad (6.29)$$

$$\Delta V_{cj}(k) = F_{cj}(\Delta x) \quad (6.30)$$

$$\hat{V}_j(k) = \sqrt{\hat{V}_{sj}(k)^2 + \hat{V}_{cj}(k)^2} \quad (6.31)$$

where the input vector $\Delta \mathbf{x} = \{\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4\}$ denotes the set of combined blade adjustments, each of the functionals, F_{sj} and F_{cj} , represent the change in vibration between two consecutive flights as represented by a neural network, and $R_{sj}(k)$ and $R_{cj}(k)$ denote random numbers added to the outputs of the networks to account for measurement noise. Each of the networks consisted of two hidden layers, with four and eight processing elements in the first and second layers, respectively. To avoid overtraining, the 102 sets of data were divided into two equal subsets, one set to train the network and the other to test its performance. The random numbers, R_{cj} and R_{sj} , were generated according to the Gaussian distribution $N(\mu, \sigma^2)$, with the mean μ and variance σ^2 defined as

$$\hat{\mu} = \frac{1}{M} \sum_{i=1}^M e_i \quad (6.32)$$

$$\hat{\sigma}^2 = \frac{1}{M-1} \sum_{i=1}^M (e_i - \hat{\mu})^2 \quad (6.33)$$

In the above formulation, M represents the total number of data sets and e_i denotes the difference between the measured and expected value of vibration, defined as

$$e_j(k) = V_j(k) - V_j(k-1) - \Delta V_j(k) \quad (6.34)$$

A sample of estimated vibration changes generated by the neural network model is compared side by side with the actual vibration changes in Figure 6.9. The results indicate close agreement between the predicted and actual vibration changes.

6.5.2 Interval Modeling

In application to the Black Hawks, a total of 24 interval models need to be constructed to approximate the changes in the cosine and sine components of the $A + B$ and $A - B$ vibrations at each of the six flight regimes. The interval models have the form

$$\vec{V}_{cj}(k) = V_{cj}(k-1) + \sum_{i=1}^4 \vec{C}_{cji}(k-1) \Delta x_i(k) \quad (6.35)$$

$$\vec{V}_{sj}(k) = V_{sj}(k-1) + \sum_{i=1}^4 \vec{C}_{sji}(k-1) \Delta x_i(k) \quad (6.36)$$

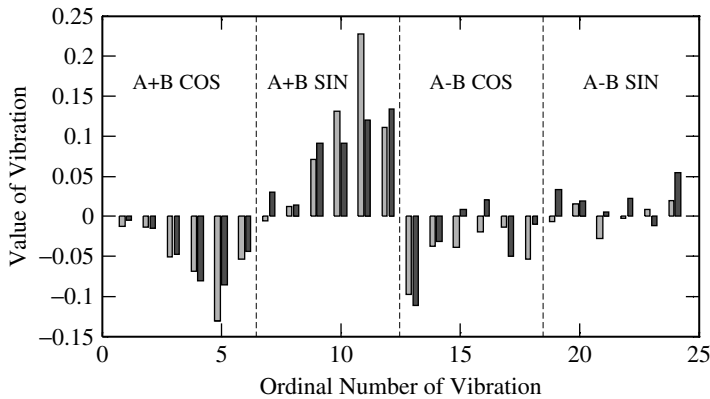


FIGURE 6.9 A sample set of simulated vibration changes shown side by side with the actual vibration changes.

$$\vec{V}_j(k) = \sqrt{\vec{V}_{sj}^2(k) + \vec{V}_{cj}^2(k)}, \quad i = 1, \dots, 4 \text{ and } j = 1, \dots, 12 \quad (6.37)$$

where the $\vec{V}_{cj}(k)$ and $\vec{V}_{sj}(k)$ represent, respectively, the estimated cosine and sine components of $A + B$ or $A - B$ vibration at each of the six flight regimes, $\vec{V}_j(k)$ denotes the magnitude of the vibration, and Δx_i are the same as those in Equation 6.2 to Equation 6.5. For this study, the feasible region was defined to include all of the blade adjustments associated with vibration estimates that satisfied the specification: $\max\{\min(\vec{V}_1), \dots, \min(\vec{V}_{12})\} \leq 0.2$. The above specification ensures that the lower limit of the estimated vibration range of the largest vibration component will be less than 0.2 ips (an industry standard). The selection of the lower limit here is to ensure that the feasible region is as large as possible, so as not to eliminate any potentially good candidate blade adjustments. The computation of the feasible region was based on the range $[-0.015, 0.015]$ for push rods and $[-0.035, 0.035]$ for trim tabs, within which 20,000 random sets of blade adjustments were evaluated for their feasibility. The blade adjustments associated with vibration ranges satisfying the specification were included in the feasible region.

As noted earlier, the proposed method uses the feasible region as the basis of search for the blade adjustments. For this study, the blade adjustments set that produced the smallest value for the objective function S (Equation 6.12) was selected to be applied to the helicopter. It should be noted that, given the stringent constraints on the vibration components, there were cases where the search algorithm could not find any feasible blade adjustments that would satisfy all of the constraints. In such cases, the set of blade adjustments that produced the smallest lower limit of the maximum estimated vibration was used as a compromised solution.

The interval model was updated after each tuning iteration. For shrinkage–enlargement learning, the parameter β in Equation 6.26 was set to 1 and γ to 0, so that the coefficient intervals could be shrunk without limits. Learning was performed separately for each tail number to customize the interval model to individual tail numbers; that is, the interval model was set to the sensitivity coefficients for each tail number and was adapted after the first tuning iteration. Accordingly, the interval model was actually a pointwise model for the first iteration and took the form of an interval model thereafter.

6.5.3 Performance Evaluation

The interval model (IM) method was tested on 39 tail numbers, for which actual track and balance data were available from the field. For each tail number, the IM method was applied iteratively until either the simulated vibrations were within their specifications, or an upper limit of five process iterations had been reached.

Since the stochastic aspects of vibration measurements impose randomness on the rotor tuning process, rotor tuning solutions cannot be evaluated by deterministic measures. This calls for the creation of performance measures that account for uncertainty. One such measure that assesses tuning efficiency is the *average tuning iteration number* (ATIN) which represents the average number of iterations taken for tuning each tail number. The number of flights used by the IM method for the 39 tail numbers is included in Table 6.4 along with those actually performed in the field. The results indicate that the IM method requires a smaller ATIN relative to that actually performed.

Another potentially significant aspect of the IM method is its adaptation capability, which enables it to transform a pointwise model into an IM, and to subsequently update it after the first iteration. Adaptation capability, however, may not be as significant in rotor tuning, which offers limited possibility for training. In order to evaluate the significance of learning in the performance of the IM method, the results in Table 6.4 were reproduced in Table 6.5 with the learning feature turned off. The ATINs indicate that with learning, the IM method requires fewer iterations for tuning each tail number, despite the small number of iterations taken to tune each tail number. This, in turn, indicates that the interval model enhances the performance of the IM method, since without learning, the model remains pointwise at the sensitivity coefficients. However, perhaps an equally interesting set of results in Table 6.5 are those indicating that even without learning, the IM method requires fewer iterations than actually

TABLE 6.4 The Number of Tuning Iterations Required by the Interval Model Method and Those Applied in the Field

Tail # (39)	Number of Tuning Iterations	
	Actual	IM Method
176	1	1
178	1	1
179	3	2
180	1	1
184	2	1
⋮	⋮	⋮
260	4	1
⋮	⋮	⋮
861	1	1
Total	71	48
ATIN	1.82	1.23

TABLE 6.5 The Number of Tuning Iterations Required by the IM Method (with and without Learning) along with Those Actually Applied in the Field

Tail # (39)	Tuning Iteration Number		
	Actual	IM Method	
		With Learning	Without Learning
185	3	2	3
186	3	2	3
208	2	2	3
245	3	2	3
260	4	2	3
802	3	2	3
822	3	2	3
⋮	⋮	⋮	⋮
Total	71	48	62
ATIN	1.82	1.23	1.59

performed in the field. Given that the adjustments associated with both sets of results were selected from the same model (i.e., sensitivity coefficients), the better performance of the IM method can only be attributed to its more effective search strategy that leads to more comprehensive solutions.

A preferred aspect of a system of rotor tuning is its ability to tune the aircraft within one iteration. This aspect of the method was evaluated by checking the number of tail numbers tuned within one iteration. For these results, in order to eliminate the difference between the simulation model and the helicopter, only the vibration estimates from simulation were used to evaluate the suitability of the adjustments. The results of this study are shown in Table 6.6, where the tail numbers tuned within one iteration are shown by a $\surd$ and those requiring more than one iteration are denoted by $\times$. The results indicate that the IM method satisfies this more stringent criterion better than the actual adjustments, further validating the claim that the IM method benefits from a more effective search engine.

Owing to the randomness of the vibration measurements, repeated applications of an adjustment set may lead to slightly different vibration measurements. This, in turn, may cause a variance in the number of iterations produced by adjustments when the resulting vibration is close to the specified threshold. It would be beneficial, therefore, to devise a measure for the probability of success of adjustments.

TABLE 6.6 Tally of the Tail Numbers Tuned within One Iteration According to Simulated Vibration

Tail # (39)	Tuned within One Iteration	
	Actual	IM Method
176	✓	✓
178	×	✓
179	✓	✓
⋮	⋮	⋮
822	×	✓
858	×	✓
859	✓	✓
861	✓	✓
Total	19	30

The empirical measure, the acceptability index (AI), is defined here as

$$AI = \frac{1}{N} \sum_{l=1}^N s_l$$

(6.38)

to denote the percentage of times an adjustment set will result in the vibration satisfying the specification. In the above equation, N represents the total number of flights simulated to represent the repeated applications of the same adjustment set, and

$$s_l = \begin{cases} 1 & \text{if vibration of the } l\text{th simulation flight is acceptable} \\ 0 & \text{if vibration of the } l\text{th simulation flight is unacceptable} \end{cases}$$

The AIs computed for both the actual and selected adjustments at the first iteration are included in Table 6.7. The results indicate that the IM method provides adjustments with a higher probability of success as judged by the acceptability of vibration estimates from the simulation model. These results, which indicate that the selected adjustments from the IM method can more consistently tune the rotors within one iteration, imply the better positioning of the adjustments within the feasible region.

TABLE 6.7 The Values of Acceptability Index (Trial Mode) Computed for Both the Actual and Selected Adjustments at the First Flight

Tail # (39)	Acceptability Index	
	Actual	IM Method
176	0.92	0.87
178	0	0.54
179	0.61	0.52
⋮	⋮	⋮
260	0.40	0.89
261	0.09	0.95
263	0.18	0.12
⋮	⋮	⋮
822	0.00	0.64
857	0.62	0.55
858	0.64	0.93
859	0.94	0.67
861	0.96	0.74
Average	0.581	0.724

TABLE 6.8 Comparison of the First Iteration Solutions of IM Method and Actual Solutions from Sikorsky’s Production Line with the Cumulative Acceptable Adjustments

Tail #	Modifications		
	Actual Iteration 1 Modifications	CAM	IM Iteration 1 Modifications
801	6, −4, −10, 11	2, −4, −4, 14	3, −5, −6, 12
802	5, 2, 0, 0	9, 0, −10, 10	8, −2, −5, 10
822	6, 0, −20, 0	10, −4, −23, 13	8, −4, −22, 6
858	7, 0, −14, 0	9, −2, −10, 3	9, −2, −15, 4

Another evaluation basis for the adjustments can be established by comparing them to the actual cumulative adjustments performed in the field. The cumulative adjustment set, $\sum x$, can be defined as

$$\sum x = \sum_{k=1}^N \Delta x_k$$

(6.39)

where N represents the total number of tuning iterations performed in the field for the tail number and Δx_k denotes the adjustments applied at the k th iteration. A sample of actual first iteration adjustments, actual cumulative adjustments, and first iteration adjustments from the IM method is shown in Table 6.8. The results indicate that the adjustments from the IM method are closer to the actual cumulative adjustments than are the actual first iteration adjustments. Although the cumulative adjustments may not be the most desirable ones for the aircraft, they represent an acceptable set that has been proven in the field. The closeness of the IM method’s solutions to the actual cumulative adjustments further validates its effectiveness.

6.6 Conclusion

A logical feature for future rotor tuning systems will be the capability to adjust the blades during the flight. For this, these systems will need to have the capability to learn from their mistakes. They will also need to be able to monitor the condition of the rotor system in-flight, so they will stop modifying the blade parameters when more drastic actions are necessary for saving the aircraft. As such, these systems will need to be used with strong operator interaction to prevent implementation of inappropriate adjustments, and must have the ability to explain the recommended adjustments to the operator.

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