

Vibration in Rotating Machinery

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H. Sam Samarasekera

Sulzer Pumps (Canada), Inc.

Summary

This chapter concerns vibration in rotating machinery. Although it is impractical to totally eliminate such vibrations, it is essential that they be controlled to within acceptable limits for safe and reliable operation of such machines. The two major categories of vibration phenomena that occur in rotating machinery are forced vibration and self-excited instability. Monitoring, diagnosis and control of these vibrations requires a sound understanding of rotor dynamics in machinery. Predicting the vibration behavior of a rotating machine by analytical means has become customary in many industries. With the advent of computer technology, several computer-based programs have been developed to accurately predict the behavior of rotating machinery. Significant strides in modeling techniques have also been made over the past century to accurately represent components such as shaft sections, disks, impellers, bearings, seals, rotor dampers, and rotor–stator interactions. This has enhanced the accuracy and reliability of both analytical and computational procedures. The chapter presents useful techniques of analysis, measurement, diagnosis, and control of vibration in rotating machinery.

9.1 Introduction

Vibrations are an inherent part of all rotating machinery. Residual mass imbalance and dynamic interaction forces between the stationary and rotating components, which are practically impossible to eliminate, cause these vibrations. The challenge is to identify the source of vibration and control it to within reasonable limits. Because of economic advantages, the trend in industry has been to move towards high speed, high power, lighter and more compact machinery. This has resulted in machines operating above their first critical speeds, which was unheard of in the past. The new operating parameters have required concurrent development of vibration technology without which it is not

possible to safely and reliably operate such machinery. Industry has also come to realize that *vibration* is an essential phenomenon, which could be used to assess the performance, durability, and reliability of rotating machinery.

Engineers at different levels approach the subject of vibration in rotating machinery differently. The machinery designer has to recognize the potential sources of vibration and control them to within acceptable levels. In the past few decades, owing to the advancement in computers and modeling techniques, better understanding of the dynamics of rotating machinery, including the identification of potential sources of vibration, has been realized. This has enabled designers to accurately predict the rotordynamic behavior of machinery, allowing it to reach higher operating speeds and larger energy capacities safely and reliably.

Approaching vibration from a different perspective, the maintenance engineer uses vibration standards and guidelines to monitor the health of equipment for their timely repair and refurbishment. Reliable vibration monitoring and diagnostics techniques have moved industry into *predictive* rather than *preventive* maintenance practices, which considerably reduce plant downtimes that rely on key rotating machinery. Premature replacement of machinery components has also been minimized. The resulting financial and economic benefits provide an added incentive for the study and understanding of vibration in rotating machinery.

The vibration specialist or troubleshooter has to use his knowledge of rotordynamics and his diagnostic capabilities to solve vibration problems in rotating machinery. In most cases, it is also important to have an understanding of the interfacial dynamics of the rotating machinery with the surrounding system in order to solve a vibration problem.

From a safety and reliability standpoint, the public must be concerned with vibration in rotating machinery. Their concerns are addressed through vibration standards and guidelines. These procedures have been developed for rotating machinery by numerous organizations, both at the national and international levels. Some of these standards are industry specific and some are equipment type specific, while a number of them try to cover a wide range of rotating machinery. The objective of most of these standards is to establish and control quality, safety, durability and reliable performance of rotating machinery for the benefit of those who use or operate it.

9.1.1 History of Vibration in Rotating Machinery

Although various types of rotating machinery have been in use for many centuries, understanding of their rotordynamic behavior did not begin until 1869 (Rankine, 1869). Since that time, there has been steady growth in the development and understanding of the vibration behavior of rotating machinery. A tabulation of major historical events that have contributed to this growth is presented in [Table 9.1](#).

- All rotating machinery vibrates to some degree. For public safety and machine reliability, the vibrations have to be controlled to within acceptable limits.
- Modern trends towards more sophisticated, higher speed compact rotating machinery have contributed to the rapid development in vibration technology through a better understanding of their rotordynamics.
- Vibration technology is integrated into the areas of design, maintenance, and troubleshooting of rotating machinery.
- From a safety and reliability standpoint, the public is protected by the implementation of vibration standards and guidelines.
- The first publicly reported rotordynamic study was made in 1869.

TABLE 9.1 A Chronological Listing of Major Contributions that Have Led to the Development and Understanding of Vibration in Rotating Machinery

Year	Contributor	Description
1869	Rankine, W.J.M.	He examined the equilibrium of a frictionless, uniform shaft disturbed from its initial position. The resulting recorded article is recognized to be the first on the subject of rotor dynamics. He proposed that motion is stable below the first critical speed, is neutral or indifferent at the critical speed, and unstable above the critical speed He also developed numerical formulae for critical speeds for the cases of a shaft resting freely on a bearing at each end and for an overhanging shaft fixed in direction at one end
1883	Greenhill, A.G.	He studied the effect of end thrust and torque on the stability of a long shaft and concluded that they were both unimportant. He also obtained formulae for the cases of an unloaded shaft resting on bearings at each end and fixed in direction at each end
Circa 1890	Reynolds, O.	He extended the theory developed by Rankine and Greenhill for the case of a shaft loaded with pulleys
1893	Dunkerley, S.	He developed formulae for critical speeds for loaded shafts in terms of the diameter of the shaft, weights of pulleys, the manner in which the shaft is supported, and so on, and verified them by experiment He postulated that any degree of unbalance will excite the shaft at the critical speed to very high amplitudes and that it is possible to operate above the first critical speed. The dependence of critical speed on the moment of inertia of the rotating pulley was identified
1894	Rayleigh, J.W.S.	He developed an approximate method to calculate the natural frequency of a continuous beam with distributed mass and flexibility using the energy method
1895	DeLaval, G.	He was responsible for the first experimental demonstration that a steam turbine is capable of sustained operation above the first critical speed
1916	Timoshenko, T.	He discovered the effects of transverse shear deflection on the natural frequency of a continuous beam and applied the principle to the case of the rotating shaft
1919	Jeffcott, H.H.	He examined the effect of unbalance on the whirl amplitudes and the forces transmitted to the bearings. The case of a light uniform shaft supported freely on bearings at its ends and carrying a thin pulley of mass m at the center of the span was studied. He assumed the moment of inertia of the pulley to be negligible. Using this model, later known as the <i>Jeffcott model</i> , a comprehensive theory was developed to explain the behavior of the rotor as it passed through the critical speed The effect of damping on the whirl amplitude, a phase change of angle π as it passes through the critical speed, and the concept of synchronous rotor whirling (precession) were introduced and explained. He also recognized that with a separation margin of 10% on either side of a critical speed, the amplitude of vibration would not be excessive. He demonstrated that it is better from the vibration point of view to design the shaft with its critical speed below the working speed rather than to have a critical speed the same proportion above the working speed. Accordingly, he explained the behavior of the De Laval steam turbine and the economic advantages of operation above the critical speed
1921	Southwell, R.V. and Gough, B.S.	They found that a torque and an end thrust of constant magnitude lowers the critical speed of a rotating shaft, disproving Greenhill's earlier (1883) conclusions

(continued on next page)

TABLE 9.1 (continued)

Year	Contributor	Description
1921	Holzer, H.	He developed a numerical method to calculate torsional critical speeds and mode shapes for a multidisk rotor system
1924	Newkirk, B.L.	He observed that a rotor operating at a speed above the first critical speed can enter into high, violent whirling and the center of the rotor will precess in the forward direction at a rate equal to that of the critical speed. Unlike in the case of synchronous whirling, if the speed is increased beyond the initial whirl speed, the whirl amplitude will continue to increase, eventually leading to failure. This was the first time that it was realized that nonsynchronous unstable motion can exist in a high-speed rotor Based on experiments, he made the following key observations on nonsynchronous whirling. The amplitude and the onset speed of whirling are independent of the rotor balance. Whirling always occurs at speeds above the critical speed, and the whirl speed is always constant at the critical speed, regardless of the rotor speed. The whirl threshold speed can vary even for machines of similar construction. Whirling occurs only in built up rotors, and not in single piece constructions. Increasing the foundation flexibility, distortion or misalignment of the bearing housings, or introducing damping to the foundation or increasing the axial thrust bearing load, increased the threshold speed of whirling
1924	Kimball, A.T.	Suggested that internal friction or viscous action due to bending may cause a shaft to whirl when rotating at any speed above the first critical speed. He postulated that the nonsynchronous whirling observed by Newkirk was due to this phenomenon
1924	Newkirk, B.L.	Based on Mr Kimball's theory, he concluded that similar frictional forces are generated at the mating face between the shrunk on disk and the shaft of a built-up rotor, and the nonsynchronous whirling observed by him was due to this effect. However, he was unable to explain some of his experimental findings, in particular, the effects of bearing or foundation flexibility, damping, and misalignment
1925	Newkirk, B.L.	He experienced another form of nonsynchronous whirling, similar but different to that caused by the frictional effects of a shrink-fit disk. It occurred at rotor speeds just exceeding twice the first critical speed on shafts mounted on journal bearings. He recognized that the oil in the journal bearing was responsible for the violent motion and called it <i>oil whip</i>. The whirl speed and direction of whirling were the same as that for friction induced whirling, that is, the first critical speed in the forward direction. A theory to explain how the oil film can produce the whirling motion of a journal and to account for why it took the same direction as rotation of the shaft was proposed. However, the theory does not explain why whirling does not commence until the rotor speed reaches twice the critical speed value. The influence of foundation flexibility on the rotor stability was also found to be confusing to Newkirk. In the case of friction-induced whirl, he was able to totally eliminate the rotor instability by means of a flexibly mounted bearing. When this was tried with the journal bearings, the whirl amplitudes magnified. External damping at the bearing was found to have a favorable influence on whirl amplitudes
1925	Stodola, A.	He developed an iterative procedure to calculate the fundamental frequency of a vibrating system based on an assumed mode shape
1927	Stodola, A.	He provided an explanation and formulae for the gyroscopic moment effect on the critical speed of a rotor. He also introduced the notion of synchronous and nonsynchronous reverse precession of a rotor under specific conditions

TABLE 9.1 (continued)

Year	Contributor	Description
1933	Robertson, D.	In order to understand oil whip, he studied the stability of the ideal 360° infinitely long journal bearing, and erroneously concluded that the rotor will be unstable at all speeds and not only at speeds above twice the critical speed value
1933	Smith, D.M.	He studied the case of unsymmetrical rotors on unsymmetrical supports and obtained four different critical speed values in comparison to the single value for a symmetrical system. He also discussed the presence of additional critical speeds due to gyroscopic effects of large disks
1944	Myklestad, N.	A lumped parameter transfer matrix method to calculate natural frequencies for airplane wings was developed by him
1945	Prohl, M.	He developed a lumped parameter transfer matrix method for calculating critical speeds of flexible rotors
1953	Poritsky, H.	Using the small displacement theory, he derived a radial stiffness coefficient for the journal bearings and analyzed the rotor behavior under oil whirl conditions. He concluded that the rotor was stable below twice the critical speed and indicated that increasing the rotor or bearing flexibility will reduce the threshold speed of instability. He also proposed a stability criterion for a rotor based on the bearing and rotor stiffness
1953	Miller, D.F.	He introduced a solution to the steady-state forced vibration problem, for a beam or rotating shaft on damped, flexible end supports. The response of the rotor to an unbalance force and the damped resonance frequencies are calculated by this method
1955	Pinkus, O.	He investigated oil whirl in various journal bearing types and made the following major conclusions. The unbalance of the rotor has minimal effect on stability. The threshold of instability occurs at approximately twice the first critical speed of the rotor. In the unstable region, the whirl frequency remained constant at the first critical speed, irrespective of the shaft rotating speed. At speeds nearly equal to three times the first critical speed, whipping motion stops with a heavy shaft rotor, whereas with a light shaft rotor it does not cease. High loads, high viscosity, flexible mountings, and bearing asymmetry favor stability
1958	Lomakin, A.	The influence of the dynamic characteristics of seals on the critical speeds and stability of pump rotors were introduced by him
1958	Thomas, H.	He proposed that an eccentric turbine rotor would generate a destabilizing force due to the circumferential variation in clearance
1966	Gunter, E.J. Jr.	He combined the different theories on whirling developed by the rotor dynamist and the bearing specialist, and elegantly explained some of the conflicting experimental evidence gathered thus far. He emphasized the importance of considering the combined effects of rotor parameters and the bearing and foundation characteristics on rotor stability, and developed more comprehensive criteria for self-excited whirl instability
1969	Black, H.F.	He provided a comprehensive analysis of annular pressure seals on the vibrations of pump rotors
1970	Ruhl, R.	He introduced finite element models for flexible rotors for calculating rotor critical speeds and mode shapes. These models did not take into account gyroscopic effects and axial loading
1974	Lund, J.	A transfer matrix method to calculate damped critical speeds of a rotor taking into account the cross coupling terms as well were introduced by him
1976	Nelson, H. and McVaugh, J.	They extended the finite element model of a rotor to account for rotary inertia, gyroscopic effect, and axial loads

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TABLE 9.1 (continued)

Year	Contributor	Description
1978–1980	Benckert, H. and Wachter, J.	A method to calculate flow induced spring constants for labyrinth gas seals and the use of <i>swirl breaks</i> to reduce the destabilizing force caused by tangential velocity in labyrinth seals was introduced by them
1980	Nelson, H.	He further developed a finite element model of a rotor to include shear deflection and axial torque effects
1980	Brennen, C. et al.	They recognized the presence of substantial shroud forces, which influences the rotor dynamics of a pump
1986	Muszynska, A.	She demonstrated that oil whirl occurs at about one-half the running speed in a vertical rotor. With further increase in speed, oil whip will commence when the whirl frequency approaches the critical speed of the rotor

9.2 Vibration Basics

The vibration phenomena that manifest in rotating machinery can be divided into two major categories: *forced vibration* and *self-excited instability*. A stimulus or a source of excitation is required to initiate and sustain vibratory motion in a rotor. When the stimulus is a forcing phenomenon such as mass unbalance, it will produce forced flexural vibration in the rotor analogous to linear forced vibration response in a simple spring–mass system. On the other hand, self-excited vibration (instability) does not require a forcing phenomenon for its initiation or sustenance. A description of these phenomena is given next.

9.2.1 Forced Vibration

A rotating force vector (unbalance), a steady directional force (gravity), or a periodic force (pump impeller/diffuser interface action), will cause forced vibration in a rotating machine. The response of the rotor will depend on the nature of the forcing function and how it relates to rotor characteristics. The rotor responses to the most common excitation phenomena are examined below.

9.2.1.1 Unbalance Response — Synchronous Whirling

As an introduction to the theory on rotating machinery vibration and understanding unbalance response, it is most appropriate to examine Jeffcott’s (1919) rotor, which is a simple model that has many of the basic characteristics of more complex rotating machinery. The Jeffcott rotor represents a massless elastic shaft supported freely in bearings at its ends and carrying a disk of mass m at the center of its span. The mass center of the disk is eccentric to its geometric center by a distance e . Refer to Figure 9.1.

- C = geometric center of the disk
- β = phase angle

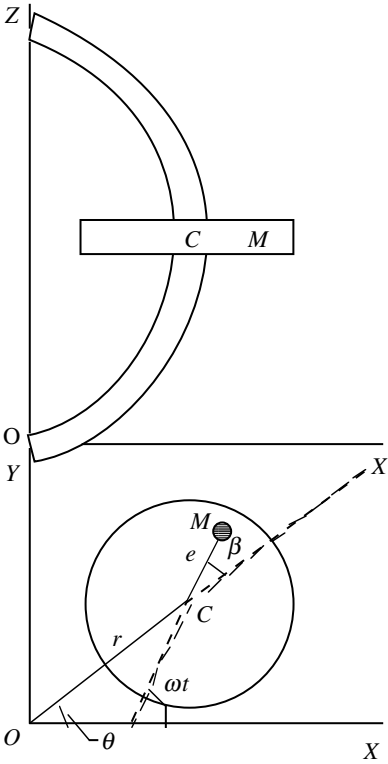


FIGURE 9.1 Jeffcott rotor.

M = mass center of the disk
 c = viscous damping coefficient of on the rotor
 O = bearing center
 r = deflection of rotor from origin
 θ = angle of precession
 k = shaft stiffness
 ω = angular velocity of the rotor = $\dot{\theta} + \dot{\beta}$

Whirling is defined as the angular velocity of rotation of the rotor geometric center (c) or the time derivative ($\dot{\theta}$) of the angle of precession (θ) (also see [Chapter 7](#)). *Synchronous whirling* is when the rate of whirling, $\dot{\theta}$, is equal to the total angular velocity, ω , of the system.

Applying Newton's Laws of motion to the rotor, the differential equations of motion in polar coordinates (r, θ) are obtained as

$$\ddot{r} + \frac{c}{m}\dot{r} + \left(\frac{k}{m} - \dot{\theta}^2\right)r = e\omega^2 \cos \beta \quad (9.1)$$

$$r\ddot{\theta} + \left(\frac{c}{m}r + 2\dot{r}\right)\dot{\theta} = e\omega^2 \sin \beta \quad (9.2)$$

For a steady-state condition, the values of r , β , $\dot{\theta}$, and ω are constant. For synchronous whirling, Equation 9.1 and Equation 9.2 reduce to

$$\left(\frac{k}{m} - \omega^2\right)r = e\omega^2 \cos \beta \quad (9.3)$$

$$\frac{c}{m}\omega r = e\omega^2 \sin \beta \quad (9.4)$$

From Equation 9.3 and Equation 9.4

$$r = \frac{e\omega^2}{\sqrt{\left(\frac{k}{m} - \omega^2\right)^2 + \left(\frac{c\omega}{m}\right)^2}} \quad (9.5)$$

$$\beta = \tan^{-1} \frac{c\omega}{m\left(\frac{k}{m} - \omega^2\right)} \quad (9.6)$$

$$F = \frac{kr}{2} = \frac{ke\omega^2}{2\sqrt{\left(\frac{k}{m} - \omega^2\right)^2 + \left(\frac{c\omega}{m}\right)^2}} \quad (9.7)$$

Using the following relationships:

$$\omega_N = \sqrt{\frac{k}{m}} \text{ — Natural frequency of rotor without damping}$$

$$c_{cr} = 2\sqrt{km} \text{ — Critical damping coefficient}$$

$$\zeta = \frac{c}{c_{cr}} \text{ — Damping ratio}$$

Equation 9.6 and Equation 9.7 are reduced to the following nondimensional form:

$$\frac{r}{e} = \frac{2F}{ke} = \frac{(\omega/\omega_N)^2}{\sqrt{(1 - (\omega/\omega_N)^2)^2 + (2\zeta\omega/\omega_N)^2}} \quad (9.8)$$

$$\beta = \tan^{-1} \frac{2\zeta(\omega/\omega_N)}{(1 - (\omega/\omega_N)^2)} \quad (9.9)$$

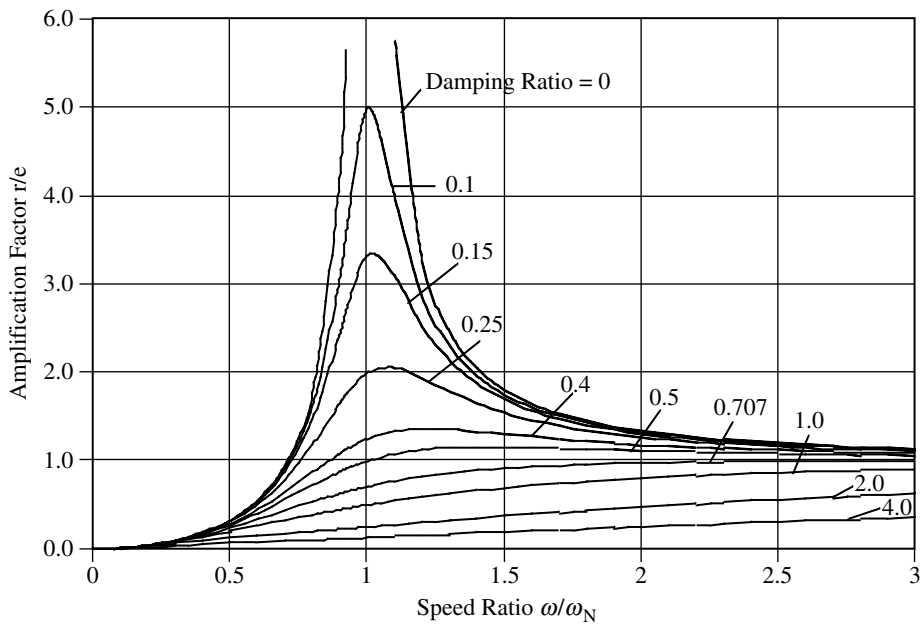


FIGURE 9.2 Jeffcott rotor response with mass eccentricity — amplification vs. speed.

Figure 9.2 is a graphical representation of the unbalance response of the rotor as a function of rotating speed, ω . Upon examination of the phase relationship, it is important to note that the phase angle, β , changes from approximately 0° at low speed to values approaching 180° at the higher speed. At ω_N , $\beta = 90^\circ$. A pictorial illustration of this phenomenon is given in Figure 9.3.

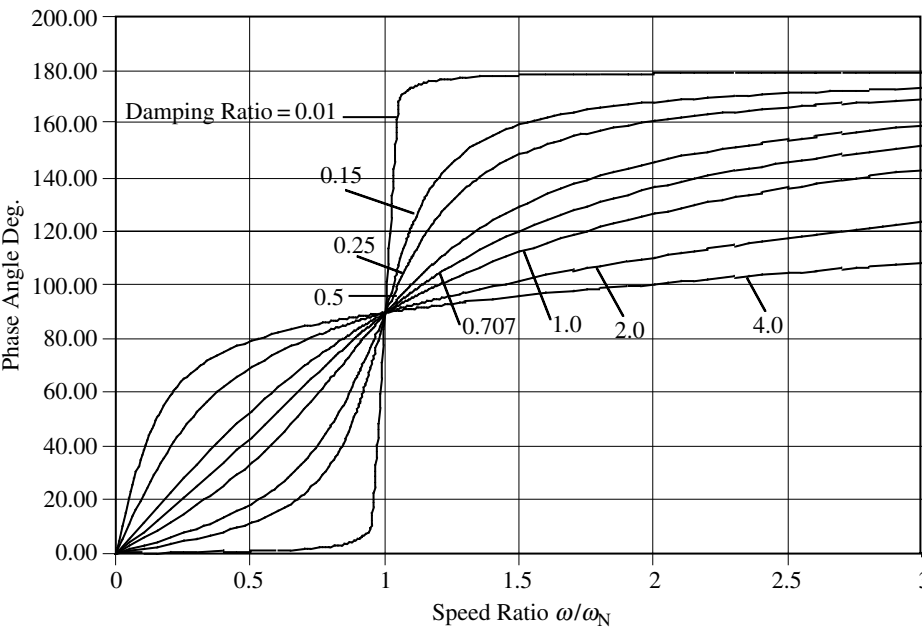


FIGURE 9.3 Jeffcott rotor response to unbalance — phase angle vs. speed.

For the case of zero damping, when $\omega = \omega_N$, the rotor deflection and the bearing forces are unbounded. For all other cases, the rotor deflection and the bearing forces are bounded, and their amplitude depends on the damping ratio. If a shaft is quickly accelerated through its critical speed to a higher working speed, then there may not be enough time for large rotor deflection to take place. At high speeds, $\omega \gg \omega_N$, the amplitude of the rotor deflection decreases and approaches the value e , the eccentricity of the rotor.

The critical speed, ω_{cr} , of a rotor in the general case, is the speed at which the rotor deflection amplitude or the force amplitude transmitted to the bearings is a maximum.

This implies that, at $\omega = \omega_{cr}$

$$\frac{dr}{d\omega} = \frac{dF}{d\omega} = 0$$

Using Equation 9.8, the following relationship between the natural frequency of the rotor and its critical speed is derived:

$$\omega_{cr} = \frac{\omega_N}{\sqrt{1 - 2\zeta^2}} \tag{9.10}$$

From Equation 9.10, it is evident that the critical speed of a rotor is not a fixed value and is dependent on the degree of rotor damping. When $\zeta = 1/\sqrt{2}$, the system is said to be critically damped.

It is important to note that rotor response to unbalance (or imbalance) is recognizable and controllable. The amplitude of the force transmitted to the bearing can be reduced by operation at speeds above the critical speed, reducing unbalance, increasing viscous damping, and avoiding operation close to critical speeds.

9.2.1.2 Shaft Bow

A rotor with a bent shaft will behave in a similar manner to a rotor with an eccentric mass (Ehrich, 1999). At high rotor speeds ($\omega \gg \omega_{cr}$), the shaft will tend to correct the bow as illustrated in Figure 9.4. When shaft bow is combined with mass eccentricity, unique behavior patterns are produced depending on the phase angle between the bow and the eccentric mass (Childs, 1993).

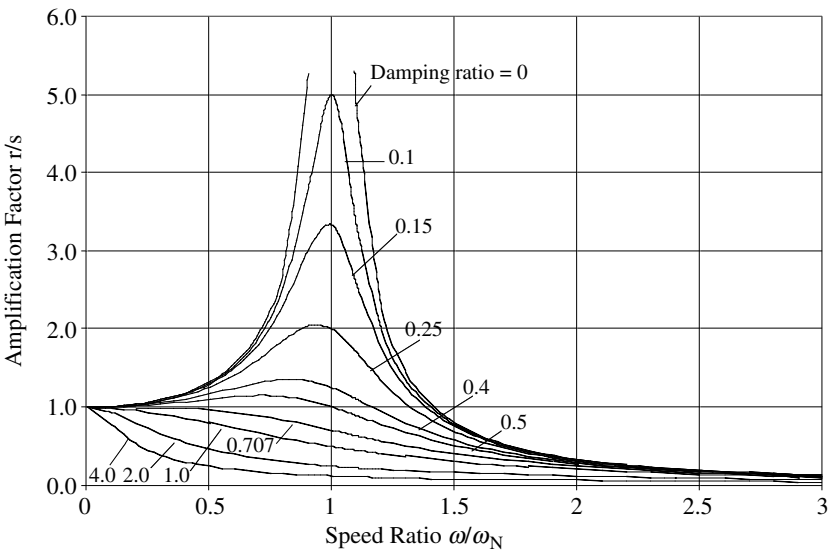


FIGURE 9.4 Jeffcott rotor response with shaft bow — amplification vs. speed.

9.2.1.3 Gravity Critical

A special case of synchronous whirling may occur in certain types of horizontal rotors due to the gravitational force. It is a secondary critical speed commonly called the *gravity critical*, which can occur in a very heavy lightly damped rotor. The critical speed will occur at approximately half the natural frequency of the rotor and its amplitudes of deflection at the critical speed are bounded and approximately twice the static deflection of the rotor (Gunter, 1966).

9.2.1.4 The Influence of Rotor Inertia and Gyroscopic Action

The effect of rotor inertia is ignored in the Jeffcott model. However, in practice, it is recognized that rotor inertia and gyroscopic action has an influence on the natural frequencies, critical speeds, and unbalance response of the rotor, including reverse whirling. In the case of the natural frequency of the rotor (zero speed), the diametral or rotary inertia provides an additional natural frequency associated with the rotational degree of freedom (DoF). Also, the inertia effect lowers the first natural frequency (Childs, 1993). In the rotating case, the effect of inertia generates both forward and reverse whirling critical speeds (Childs, 1993). These forward whirling critical speeds tend to be higher (stiffening effect) and the reverse whirling critical speed lower than the natural frequency of the rotor. At the forward critical speeds, large amplitude whirling motion due to imbalance occurs, whereas the reverse critical speeds are insensitive to imbalance of the rotor.

9.2.1.5 Rotor Housing Response across an Annular Clearance

If the rotor deflection due to imbalance exceeds the uniform annular gap, continuous contact would occur between the rotor and stator resulting in coupled motion between the rotor and stator (Childs, 1993). For low contact frictional forces, synchronous forward whirling driven by the imbalance forces will occur. If the contact friction force is large enough to prevent slipping between the rotor and stator, reverse whirling will take place. For the case of synchronous forward whirling in a certain range of running speeds, instability will occur due to engagement between the rotor and stator (Black, 1968). The zones of instability depend on the coupled natural frequency of the rotor and stator and the degree of rotor deflection with respect to the annular gap.

9.2.1.6 Effect of Nonlinearity and Asymmetry on Forced Vibration Response

The foregoing analysis has assumed that stiffness and damping are linear and symmetric and the resulting forces are proportional to the deflection and velocity of the rotor. However, in reality, rotating machinery components have inherent nonlinearities and asymmetries that can have a profound influence on their rotordynamic behavior. At large amplitudes of motion, stiffness and damping coefficients become nonlinear and result in modifying the response amplitude and critical speeds of the rotor. Nonlinearity in the support stiffness will introduce considerable distortion to the otherwise simple harmonic vibration behavior of a purely linear system. The stiffness and damping coefficients of the bearings and their supports are asymmetric in most cases, in particular in horizontal machines. As a result, the forced vibratory responses in the two principal directions are different and can behave independent of each other. Each principal direction will display a critical speed unique to itself. Ehrich (1999) has presented a discussion on how nonlinearity and asymmetry of stator systems influence forced vibration response.

The influence of rotor stiffness asymmetry and inertia asymmetry on rotor stability is discussed in Section 9.2.3.

9.2.2 Self-Excited Vibration

Instability (nonsynchronous whirling) is a self-induced excitation phenomenon, sometimes described as *sustained transient motion*, that can occur in rotating machinery. At the inception of instability, the rotor deflection will continue to build up with increase in speed, whereas in the case of a critical speed resonance, the amplitude of the deflection reaches a maximum value and then decreases. If the rotor speed is increased above the instability threshold speed, the large amplitudes of motion will normally

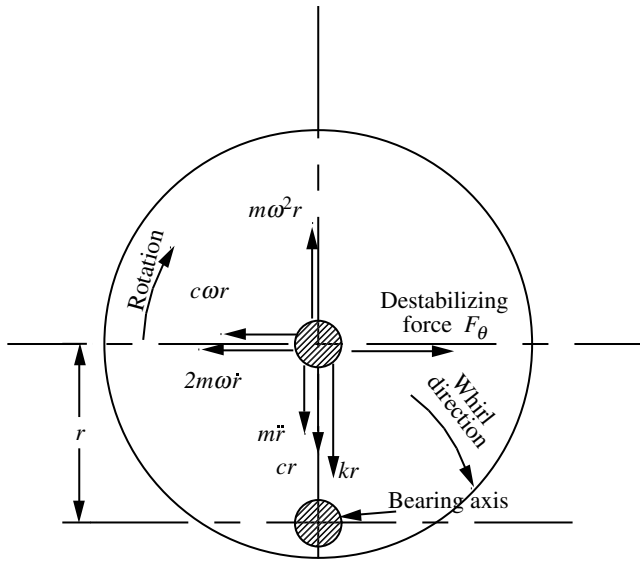


FIGURE 9.5 Rotor instability — general case.

result in damage to the machine. Unlike forced flexural vibration, rotor instability is self-induced and does not require a sustained forcing phenomenon to initiate or maintain the motion. It is known to occur only in machines operating at speeds well above the critical speeds of the rotor. Furthermore, the rotor whirling speed is different to the rotor speed (nonsynchronous whirling), and it is identical to the critical speed irrespective of the rotor speed.

In general, the rotor instability is associated with the existence of a tangential force vector, F_θ , acting at right angles to the deflection vector and directly opposing the damping force vector, as illustrated in Figure 9.5. The nature of F_θ is such that its magnitude increases proportionately with the rotor deflection. At the point where F_θ equals the external damping force, rotor instability will commence due to the nullification of the stabilizing force. This will produce a whirling motion of ever increasing amplitude. Several phenomena inherent in the rotor system that generates such tangential force vectors have been identified and are discussed below. Rotordynamists believe that there still remain more such phenomena to be discovered.

9.2.2.1 Internal Friction Damping

This type of instability was first experienced in the early 1920s in blast furnace compressors made by the General Electric Company. These machines were subject to occasional fits of violent vibration. Newkirk (1924) carried out a series of experiments to understand the unusual behavior of these machines. Based on the *internal friction theory* of Kimball (1924), Newkirk (1924) concluded that the interfacial friction damping forces at the disk shaft interface caused the subsynchronous whirling.

In order to understand the internal friction damping phenomena let us examine the shaft stresses in the whirling Jeffcott rotor (Figure 9.1). Figure 9.6 is a cross section of the shaft disk interface. Owing to its deflection, all of the fibers in the right half of the cross section are in tension, T_e , and those in the left half are in compression, C_e . These fiber stresses tend to straighten the shaft and produce a restoring force, F_r , which opposes the centrifugal force, $m\theta r$. Furthermore, a set of frictional forces are generated at the shaft disk interface due to stretching and compression of the fibers. The fibers in the bottom lower half will be stretched and are under frictional tension, T_f , and those in the upper half are being compressed under frictional compression C_f . Similarly to the reaction force, F_r , produced from right to left by T_e and C_e , a reaction force, F_θ , from bottom to top will be produced by the frictional stresses T_f and C_f . The disturbing force, F_θ , is in the same direction as the whirling motion and as a result will increase as

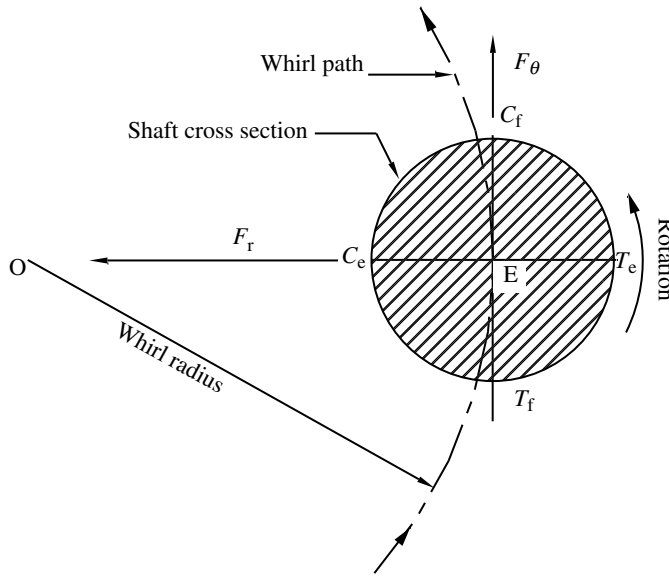


FIGURE 9.6 Internal friction damping forces acting on a rotor.

the whirling motion increases. The force, F_θ , will oppose the external damping force, $c\dot{\theta}r$. At the threshold of instability, the two forces nullify each other. It is also known that the frequency of whirling, $\dot{\theta}$, at the threshold of instability equals the natural frequency, ω_n , of the rotor. Mathematically it can be expressed as follows:

$$F_\theta = c_i r (\omega - \dot{\theta}) \quad (9.11)$$

where c_i is the rotor internal damping coefficient:

$$c\dot{\theta}r = c_i r (\omega - \dot{\theta}) \quad (9.12)$$

$$\dot{\theta} = \omega_N \quad (9.13)$$

Equation 9.12 and Equation 9.13 yield the following relationship between the threshold speed of instability, the first critical speed, and the damping factors (both internal and external):

$$\frac{\omega}{\omega_N} = 1 + \frac{c}{c_i} \quad (9.14)$$

9.2.2.2 Tip Clearance Excitation (Alford's Force, Steam Whirl)

Thomas (1958) investigated the instability of steam turbines and suggested that nonsymmetric radial clearances caused by an eccentric rotor could result in destabilizing forces, and called them *clearance excitation forces*. Subsequently Alford (1965) discovered a similar phenomenon in aircraft gas turbines and, as a result, the destabilizing forces are sometimes referred to as *Alford forces* in North America.

The destabilizing force is created as a result of the variation in the gap between the blade tip and the stator. When the gap decreases, the leakage decreases and consequently the efficiency increases, resulting in a torque higher than the average torque produced by a uniform gap. When the gap increases, there is a corresponding decrease in the torque relative to the average. The variation in torque produced by the eccentricity results in a tangential force, which is normal to the radial deflection and is in the direction of the whirling motion as shown in Figure 9.7. Furthermore, it has been illustrated that the magnitude of the resulting force increases proportionately with the increase in rotor deflection, that is, the decrease

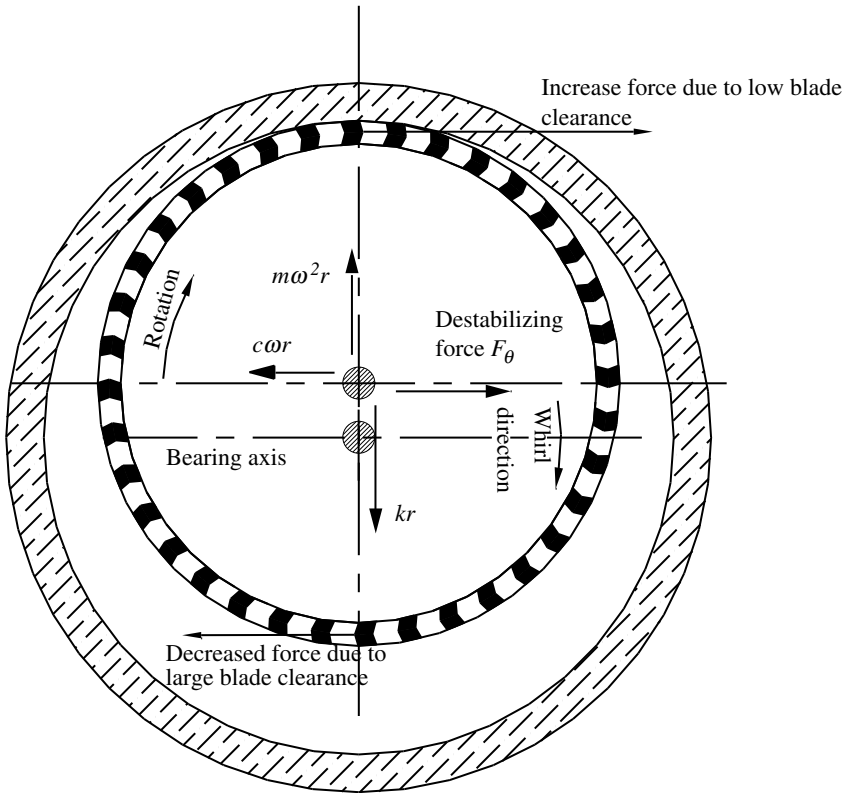


FIGURE 9.7 Tip clearance excitation (Alford forces).

in the gap. The resulting force is a destabilizing force that will oppose the external damping force, and at some point when they balance each other, rotor instability will occur. A detailed analysis of the tip clearance forces has been made by Urlichs (1977).

9.2.2.3 Impeller Diffuser Excitation Forces

Instability experienced in centrifugal compressors provides a strong suspicion that impeller–diffuser interaction phenomena are involved in the development of destabilizing forces. However, to date, no satisfactory destabilizing mechanism or source involving impellers has been identified. In the last two decades there have been several studies related to rotordynamic forces arising from shrouded centrifugal pump impellers, but very little work has been done on compressor impellers. The destabilizing force arising from the impeller–diffuser/volute interaction of a pump has been determined to be relatively small (Jery et al., 1984; Bolleter et al., 1985; Adkins and Brennen, 1986; Ohashi et al., 1986). The major portion of the destabilizing force is known to be generated in the narrow gap region between the casing and shroud of the impeller (Childs, 1986; Bolleter et al., 1989; Baskharone et al., 1994; Moore and Palazzolo, 2001). In the case of centrifugal compressors, an empirical method to determine stability of multistage machines has been proposed (Kirk and Donald, 1983). The stability maps proposed by them for flow through and back-to-back centrifugal compressors are shown in [Figure 9.8](#) and [Figure 9.9](#).

9.2.2.4 Propeller Whirl

Propeller whirl (Taylor and Browne, 1938; Houbolt and Reed, 1961) is another form of instability which occurs in aircraft rotors when there is a mismatch in the angular velocity vector of the propeller and the linear velocity vector of the aircraft. This angular mismatch results in the generation of a moment whose vector has a component of significant magnitude, which contributes to the instability of the

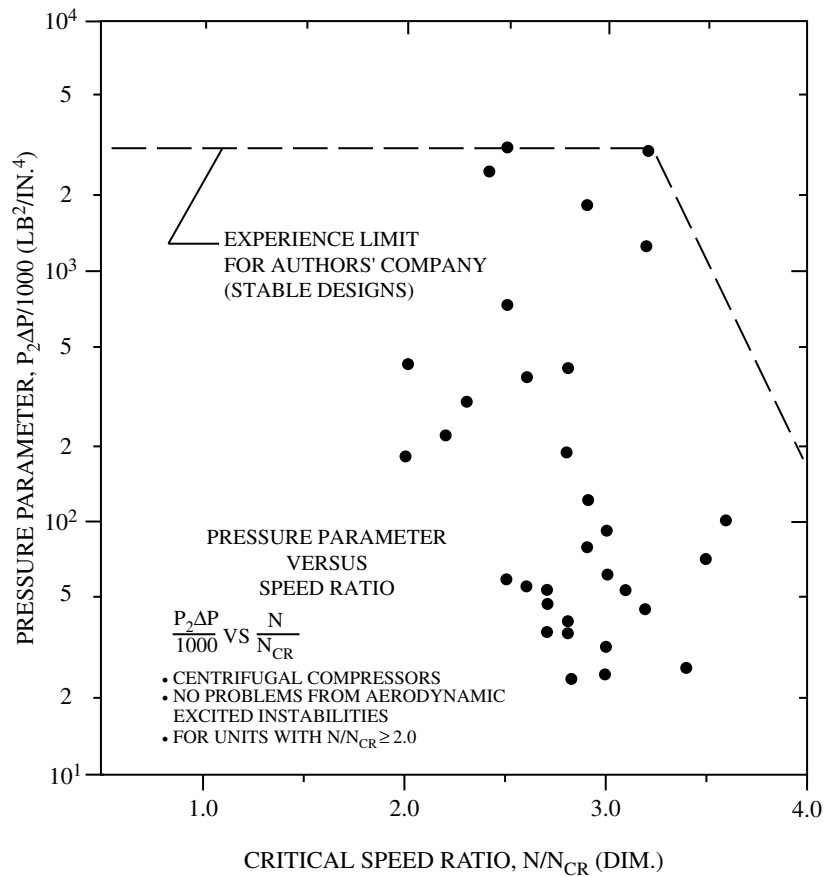


FIGURE 9.8 Proposed stability map for flow through centrifugal compressors. (Source: Rotor Dynamical Instability, 1983. With permission.)

propeller (Vance, 1988). Its magnitude is proportional to both the angular mismatch and the linear speed of the aircraft. With increasing speed, the magnitude of the destabilizing moment will exceed the rotor viscous damping moment and result in propeller instability (refer to Figure 9.5). Since the propeller is supported only from one end, the whirling motion is conical and is found to be in the reverse direction to propeller rotation. The instability is sensitive to the velocity and density of the air and not a function of the torque of the machine.

9.2.2.5 Fluid Trapped in a Hollow Rotor

Wolf (1968) has demonstrated that trapped fluid inside a hollow rotor can produce a force component tangential to the whirl orbit due to viscous drag forces. Under subsynchronous whirling speeds, this force component acts in the same direction of whirling motion and its magnitude is proportional to the rotor deflection. With reference to Figure 9.5, this force has all the markings of a destabilizing force, which can produce instability in the rotor. The threshold speed of instability is reached when the whirling speed equals the first critical speed of the rotor. It has been shown (Ehrich, 1999) that, at the threshold of instability, the rotor speed is less than twice the first critical speed. This results in a ratio of whirl speed to rotor speed in the range of 0.5 to 1.0.

9.2.2.6 Dry Friction Rubs

In Section 9.2.1.5, a dry friction rub situation was identified, where slipping was prevented between the rotor and stator under contact conditions. The contact was made possible by the deflection of the rotor

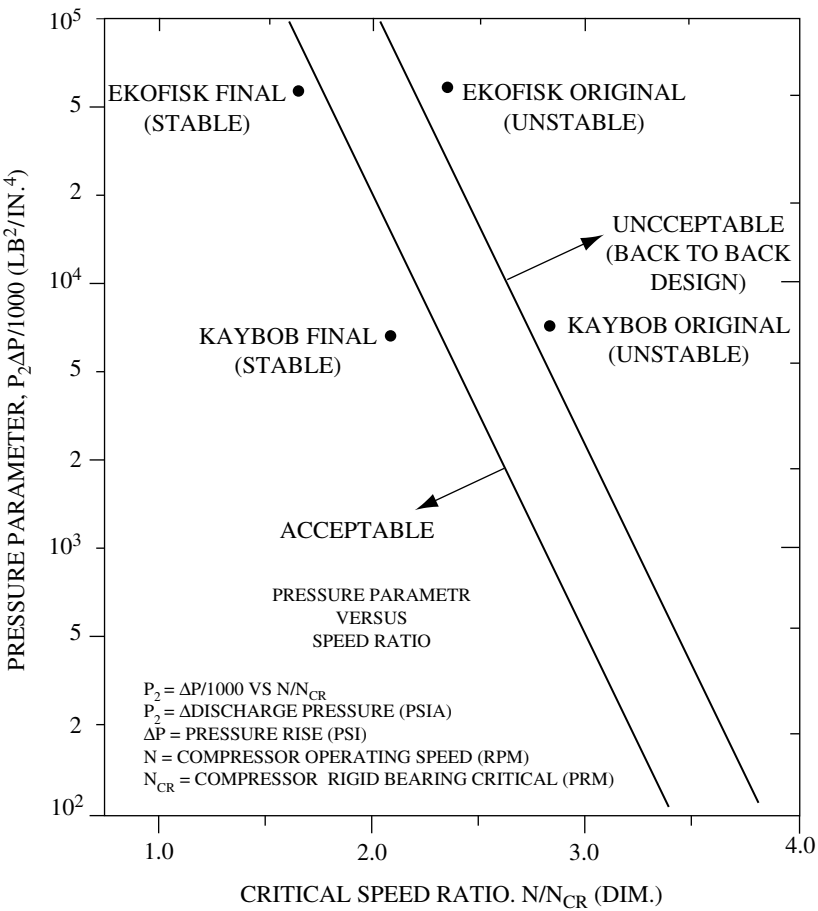


FIGURE 9.9 Proposed stability map for back-to-back centrifugal compressors. (Source: Rotor Dynamical Instability, 1983. With permission.)

due to unbalance forces. When contact is made between the rotor and stator, Coulomb friction produces a tangential force in the direction opposite to shaft rotation. Since the frictional force prevents slipping, whirling in the reverse direction to rotation occurs. The whirling speed is equal to $r\omega/C$, where r is the radius of the rotor, C the radial gap, and ω the speed of the rotor. Since the frictional force is in the same direction of whirling, it will cause the magnitude of whirling to increase, resulting in further increase of the frictional force. When the magnitude of this force exceeds the viscous damping force, rotor instability will occur. Another possibility is for the dry friction whirling speed to approach the coupled natural frequency of the rotor and stator, in which case unstable motion termed *dry friction whipping* takes place (Ehrich, 1999).

In addition to the case described above, dry friction rubs can occur in journal bearings, seals, wear rings, or any situation where a small clearance between a rotor and stator exists. The inadvertent closure of the clearance due to unbalance or lack of proper lubrication can initiate dry friction rub induced instability in these cases as well.

9.2.2.7 Torque Whirl/Load Torque

When the rotor disk axis is not aligned with the bearing axis, as in the case with an overhung rotor, Vance (1988) has shown that nonsynchronous whirling (torque whirl) can occur as a result of the misalignment between the load torque and the driving torque. His findings are based on the analysis of a simple

rotor model. It appears that torque whirl instability can occur only in the case of long slender shafts with high-load torque values. The practical implications of this theory are still to be fully explored.

9.2.2.8 Oil Whirl/Whip

Newkirk and Taylor (1925) first experienced shaft whipping due to oil action in journal bearings during their investigation into internal friction induced whirling of rotors. They found that under certain conditions, a rotor mounted on journal bearings whipped when the rotor was running at any speed above double the critical speed; the whirling motion was in the forward direction and its speed matched the critical speed of the rotor. He provided a qualitative explanation of the phenomenon based on the fact that the oil film rotates at half the velocity of the shaft due to friction drag. Hence, for rotational speeds near twice the critical speed, the oil film provides the stimulus as its speed matches the critical speed value resulting in large displacements and whipping. Others have also drawn similar conclusions based on the suggestion of an oil wedge rotating at half speed, or rotating fluid force fields at half shaft speed. However, the foregoing fails to explain why oil whip persists at speeds greater than twice the critical speed. Ehrich (1999) has also provided a qualitative explanation for oil whirl based on the general theory of rotor instability.

Although a comprehensive explanation of the physical phenomena of oil whirl is still outstanding, numerous analytical models to identify where it could be encountered have been suggested. Gunter (1966) has analytically demonstrated that the instability in a rotor supported on journal bearings can be attributed to the cross coupling bearing coefficients. As a result, most of the research on oil whirl instability has narrowed to accurate estimation of bearing cross coupling coefficients.

9.2.2.9 Influence of Bearings and Supports on Rotor Instability

The results of the Jeffcott model can be easily adopted to include bearing stiffness and bearing support stiffness effects, provided they are both linear and circumferentially symmetric (isotropic). For this particular case, the rotor stiffness, k , is the equivalent stiffness resulting from the series connection of the shaft, bearings, and support stiffness. The resulting values for ω_N and ω_{cr} will be less than those for the simply supported Jeffcott model. This will result in lowering the threshold speed of instability.

If the bearing stiffness or the bearing-support stiffness is not symmetric (orthotropic) then it can be shown (Childs, 1993) that the threshold speed of instability is increased and the maximum amplitude of deflection of the rotor is reduced in comparison to the case with symmetric bearings.

The effect of damping at the bearings or at the bearing-support is very similar to the influence of stiffness. It reduces the amplitude of the synchronous rotor response at the critical speed, and elevates the threshold speed of instability. However, there is a limit to the amount of damping that can be applied. Excessive damping causes a reduction in stability (Childs, 1993).

The mass of the bearings plays a significant role on rotor stability. If the bearing mass is significantly larger than the rotor mass, the threshold speed of instability is lowered.

9.2.3 Parametric Instability

The instability phenomena described in Section 9.2.2 can be represented by linear differential equations where the system parameters such as mass, inertia, stiffness, damping, and natural frequency are assumed to be constants. There is another subcategory of self-excited motion, referred to as *parametric instability*, since it is induced by the periodic variation of the system parameters such as inertia, mass, and stiffness. A discussion of the more common forms of this phenomenon follows.

9.2.3.1 Shaft Stiffness Asymmetry

If the shaft of a rotor contains a sufficient level of stiffness asymmetry in the two principal axis of flexure, rotor instability could occur. Smith (1933) investigated the rotor behavior under unsymmetrical flexibility of the bearing supports and unsymmetrical transverse flexibility of the shaft, taking into consideration the damping effects as well. The following conclusions were derived based on his investigation:

In the presence of stiffness asymmetry, the onset speed of internal-friction induced instability is lowered.

When there is no external damping, the rotor becomes unstable at all speeds between the two undamped natural frequencies in the two orthogonal directions. However, if external damping is significant, parametric instability may be eliminated.

Within the unstable range, the whirling motion is in the forward direction and is synchronous with the shaft speed. Further, unlike in the case of internal-friction induced instability; it is theoretically possible to run through parametric instability. This makes parametric instability quite similar to the case of unbalance response.

When the asymmetric rotor is acted upon by a transverse disturbing steady force such as gravitational force, the rotor whirls at twice the speed of the shaft. This motion exhibits a resonant increase in amplitude at a speed that is approximately half the mean of the two natural frequencies.

9.2.3.2 Rotor Inertia Asymmetry

Crandall and Brosens (1961) analyzed the parametric excitation of a rotor with nonsymmetrical principal moments of inertia. Their results indicate that the rotor behavior is very similar to the case of rotors with stiffness asymmetry described in Section 9.2.3.1, and parametric instability over similar speed ranges occurs.

9.2.3.3 Pulsating Torque

Constant torque acting on a rotor is known to lower its critical speeds because they effectively reduce the rotor's lateral stiffness. A pulsating torque introduces lateral vibrations and instabilities into a rotor. When a combination of a pulsating and constant torque is applied to a rotor, it will induce unstable lateral vibrations in a specific range of rotor speeds and certain combinations of torque amplitudes. In the region of unstable lateral motion, the whirling speed of the rotor will coincide with the first critical speed of the rotor regardless of the rotor speed or the frequency of the pulsating torque. At rotor speeds outside the unstable region, the whirling speed of the rotor will be coincident with the pulsating torque frequency.

9.2.3.4 Pulsating Longitudinal Loads

Pulsating axial forces on a shaft that are in the order of magnitude of the buckling load will effectively cause a periodic variation in its lateral stiffness. This will result in a proportionate reduction of the lateral natural frequency of the shaft. Therefore, pulsating axial loads are capable of inducing parametric instability in a shaft for both the rotating and the stationary cases.

9.2.3.5 Nonsymmetric Clearance Effects

Bentley (1974) recognized that large subsynchronous whirling can occur in rotating machinery due to certain types of nonsymmetric clearance conditions. One such condition is when a rotor's whirling motion causes rubbing with a stationary surface over a portion of the rotor orbit. This effectively results in an increase in the rotor stiffness during the contact portion of the orbit, producing a periodic variation in rotor stiffness during each cycle. Another situation that produces cyclic variation in rotor stiffness can occur in the case of a rotor supported on antifriction bearings mounted with a clearance fit to the housing. The cyclic variation of the effective rotor stiffness produces a subsynchronous whirling motion at exactly half the rotational speed. Occasionally, whirling at one third and one fourth the running speed has also been observed by Childs (1993). Instability will occur when the whirling speed corresponds to a critical speed of the rotor.

9.2.4 Torsional Vibration

Rotating machinery with rotors that have relatively large moments of inertia are susceptible to torsional vibration problems. Torsional vibration is an oscillatory angular motion that is superimposed on the steady rotational motion of the rotor. In practice it can easily go undetected, as standard vibration

monitoring equipment is not geared to measure torsional vibration. Special equipment must be used to detect torsional vibration in rotating machinery. Since the introduction of electric motor variable frequency drives (VFD), the incidence of field torsional vibration problems has increased. This has been attributed to the inherent torsional excitation forces present in current designs of VFDs. An added complication when using VFDs, as compared with using fixed speed drives, is the requirement of eliminating torsional natural frequencies over a wider speed range. Large synchronous electric motors are known to produce a large pulsating torque at a frequency that changes from twice the line frequency at the start, down to zero at the synchronous operating speed. In this case, any torsional natural frequency between zero and twice the line frequency may be subject to excitation. Most industries have come to recognize that torsional vibration is a potential hazard, and therefore, needs to be investigated at the design stage of rotating machinery. Several standard design specifications now require that torsional analysis is part of the design procedure.

The standard design practice in modeling the system for torsional analysis has been to calculate the undamped eigenvalues of the rotor as a free body in space. This practice is acceptable for most types of rotating machinery since the torsional stiffness and damping of bearings is insignificant. Also, in most cases, the torsional damping of the rotors itself is extremely low. Although the absence of damping is favorable from an analysis point of view, it makes it extremely difficult to eliminate a torsional vibration problem when encountered. This deficiency has been partly addressed with the introduction of several new lines of couplings that have a significant degree of torsional damping. Although not commonly used in rotating machinery, several torsional dampers such as the Lanchester damper have been developed for use on reciprocating machines. Dampers of similar design could be developed for use in rotating machinery to solve torsional vibration problems.

The torsional critical speeds of a simple rotor with one or two DoFs can be calculated by analytical methods. Numerical methods are used to calculate critical speeds and mode shapes of more complex systems with higher DoF. The Holzer numerical method (Holzer, 1921) described in Section 9.3.1.11 is one of the common methods used for these analyses.

- The two major categories of vibration phenomena that occur in rotating machinery are forced vibration and self-excited instability.
- Parametric instability is a special case of self-excited instability where some of the normally constant parameters vary, influencing rotor motion.
- Torsional vibrations are similar to lateral vibrations but occur in the planes perpendicular to the shaft axis.

9.3 Rotordynamic Analysis

Rotordynamic analysis is a part of current design procedures for rotating machinery that is carried out to predict the vibration behavior during operation of the machine. Potential problems are identified and eliminated by analytical means well before the manufacturing of components is begun. Furthermore, when a machine in operation displays unusual vibration behavior, analytical means are employed to study, identify, and help resolve the problem. In order for the analysis to be useful, it must be accurate and cost-effective.

During the last 100 years, several analytical procedures have been developed to understand the vibration behavior of rotating machinery. Some of these techniques are of historical interest only, and their usefulness in practical systems is very limited. With the advent of computer technology and advanced modeling techniques, several computer-based procedures have been developed to predict the vibration behavior of rotating machinery quite precisely. Of the most commonly used procedures, two are based on the lumped-parameter model where the distributed elastic and inertial properties are

represented as a collection of rigid bodies connected by massless elastic beam elements. These two procedures are the transfer matrix formulations introduced by Myklestad (1944) and Prohl (1945), and the direct stiffness matrix formulations proposed by Biezeno and Grammel (1954). Ruhl and Booker (1972) introduced the third commonly used method, based on the finite element analysis (FEA) model in which the rotor is represented as an assemblage of elements with distributed elastic and inertial properties. Several of the well-known procedures are discussed next, some of them for historical significance.

9.3.1 Analysis Methods

9.3.1.1 Rankine's Numerical Method

Rankine (1869) proposed that, for a shaft of a given length, diameter, and material, there is a limit of speed, and for a shaft of a given diameter and material, turning at a given speed, there is a limit of length, below which centrifugal whirling is impossible. The limits of length and speed depend on the way the shaft is supported. The critical speed of the shaft is given by the following equation:

$$\omega = \frac{k(Hg)^{1/2}}{b^2} \quad (9.15)$$

where

ω = critical speed in rad/sec

k = radius of gyration of the cross section of the shaft

g = acceleration due to gravity

H = modulus of elasticity expressed in units of height of itself ($H = E/\rho$)

E = Young's modulus

ρ = density of the material

l = shaft length

$b = l/\pi$ for a simply supported shaft

$b = l/0.595\pi$ for an overhanging shaft

9.3.1.2 Greenhill's Formulae

Greenhill (1883) introduced the following differential equation of motion for a uniform shaft slightly deformed from straightness by centrifugal whirling:

$$\frac{d^4 y}{dx^4} - \frac{m\omega^2}{gEak^2} y = 0 \quad (9.16)$$

The general solution to Equation 9.16 is given by

$$y = B \cosh \mu x + A \cos \mu x \quad (9.17)$$

where

$$\mu^4 = m\omega^2/gEak^2$$

m = weight of the shaft per unit length

ω = rotational speed

a = cross-sectional area of the shaft

The constants A and B depend on the boundary conditions at the support locations.

9.3.1.3 Reynolds' Equations

Reynolds extended the differential equation of motion for a uniform rotating shaft (Equation 9.16) to include shafts loaded with pulleys (disks) and for multispan rotors (Dunkerly, 1894).

At a bearing support, the difference in shear force must equal the bearing load P :

$$\frac{dM_r}{dx} - \frac{dM_l}{dx} = P \quad (9.18)$$

where M_r and M_l are bending moments in the right (r) and left (l) sides of the load.

At a load consisting of a revolving weight, W , the above equation becomes

$$\frac{dM_r}{dx} - \frac{dM_l}{dx} = \frac{W}{g} \omega^2 y \quad (9.19)$$

A further equation may be obtained by considering the *centrifugal couple* (gyroscopic moment) as given by

$$M_r - M_l = \omega^2 I' \frac{dy}{dx} \quad (9.20)$$

where I' = moment of inertia of the pulley.

The solution to Equation 9.20 is given by

$$y = A \cosh mx + B \sinh mx + C \cos mx + D \sin mx \quad (9.21)$$

The values of A , B , C , and D will depend on the boundary conditions between any two singular points.

9.3.1.4 Dunkerley Method

When considering the effects of the pulleys and the shaft together, the formulae derived by Reynolds were found to be limited for practical purposes. Dunkerly (1894) proposed an empirical method to consider the effects of the shaft and each of the pulleys separately, and then combine them using the following formula to obtain the critical speed of the rotor:

$$\frac{1}{\omega_c^2} = \frac{1}{\omega_s^2} + \sum_{i=1}^n \frac{1}{\omega_i^2} \quad (9.22)$$

where

ω_c = critical speed of the rotor

ω_s = critical speed of the shaft alone

ω_i = critical speed of the i th disk on a weightless shaft

In the case of the unloaded shaft, the critical speed, ω_s , is given by the following formula

$$\left(\frac{m\omega_s^2}{gEI} \right)^{1/4} l = a \quad (9.23)$$

where

I = sectional inertia of shaft

l = length of the span

a = a coefficient dependent on the manner of support of the shaft

The critical speed of the rotor ω_i with a single disk of weight W_i on a weightless shaft is given by

$$\omega_i = \theta \left(\frac{gEI}{W_i c^3} \right)^{1/2} \quad (9.24)$$

where

c = distance of disk from nearest support

θ = a coefficient dependant on the manner in which the shaft is supported, the position of the disk within the span and the dimensions of the disk

9.3.1.5 Rayleigh Method

The Rayleigh method is based on the premise that, when a system vibrates at its natural frequency, the maximum potential energy stored in the elastic components is equal to the maximum kinetic energy stored in the masses (Rayleigh, 1945).

The first natural frequency of a vibrating uniform beam is given by the following equation:

$$\omega^2 = \frac{EI \int_0^l \left(\frac{d^2 y}{dx^2} \right)^2 dx}{\mu \int_0^l y^2 dx} \quad (9.25)$$

The Rayleigh formula for a lumped mass system is

$$\omega^2 = \frac{\sum_{i=1}^n m_i y_i}{\sum_{i=1}^n m_i y_i^2} \quad (9.26)$$

where

ω = first natural frequency

y = deflection of the beam

x = distance along x -axis

l = length of the beam

m_i = i th lumped mass

y_i = static deflection of i th mass

The accuracy of the Rayleigh method depends upon the selection of a suitable deflection curve that approximates the fundamental mode shape. If the assumed curve represents the true mode shape, then the correct fundamental natural frequency will result. All deviations from the true mode shape will yield frequencies that are higher than the correct value.

9.3.1.6 Ritz Method

The Ritz method is an improvement on the Rayleigh method (Timoshenko et al., 1974) where the mode shape is represented by several orthogonal functions with unknown coefficients that satisfy the boundary conditions. The orthogonal functions are represented by a series of functions, $\Phi_i(x)$, where i varies from 1 to n . The mode shape is represented by the following expression:

$$y = \sum_{i=1}^n a_i \Phi_i(x) \quad (9.27)$$

In order for the coefficients a_i in the above equation to yield minimum values when substituted in the energy balance equation proposed by Rayleigh, the following expression needs to be satisfied:

$$\frac{\partial}{\partial a_i} \frac{\int_0^l \left(\frac{d^2 y}{dx^2} \right)^2 dx}{\int_0^l y^2 dx} = 0 \quad (9.28)$$

From Equation 9.25 and Equation 9.28, we find

$$\frac{\partial}{\partial a_i} \int_0^l \left[\left(\frac{d^2 y}{dx^2} \right)^2 - \frac{\omega^2 \mu}{EI} y^2 \right] dx = 0 \quad (9.29)$$

Substituting Equation 9.27 for y in Equation 9.29 and performing the mathematical operations, a system of linear equations in a_i is obtained. The number of such equations will be equal to n . These equations will yield solutions different from zero only if the determinant of the coefficients of a_i is equal to zero. This condition yields the frequency equation, from which the frequency of each mode can be derived.

9.3.1.7 Stodola–Vianello Method

The Stodola–Vianello method is a numerical iterative process (Timoshenko et al., 1974) that can be used to calculate the natural frequencies and mode shapes of vibrating systems. An approximate mode shape is first assumed and by successive iterations it is refined until convergence is obtained to the desired level of accuracy. This method can be used to refine the assumed mode shape when using the Rayleigh formulae or in the more general case of the matrix iteration process illustrated below.

Using Newton's Second law, the equations of motion for a multi-DoF system in matrix notation are

$$\{Y\} = \omega_i^2 [A][m]\{Y\} \quad (9.30)$$

where

$$[A] = [K]^{-1} \quad (9.31)$$

$[A]$ is the flexibility matrix

$[m]$ is the mass matrix

$[K]$ is the stiffness matrix

To start the iterative process a trial vector, $\{Y\}_1$, representing the mode shape is substituted to both sides of Equation 9.30 and solve for the natural frequency, ω_i . For this reason, let the product of $[A]$, $[m]$, and $\{Y\}_1$ be $\{Y\}'_2$. The first approximation for ω_i may be obtained by dividing any one of the elements on $\{Y\}_1$ by $\{Y\}'_2$ (Note that, if $\{Y\}_1$ was the true mode shape, then the ratio for all such elements will be equal.) The vector $\{Y\}'_2$ is then normalized by dividing all the elements by the first element to produce $\{Y\}_2$. The vector $\{Y\}_2$ is premultiplied by $[m]$ and $[A]$ to produce $\{Y\}'_3$. Once again the ratio of corresponding elements of $\{Y\}'_3$ and $\{Y\}_2$ are compared for equality.

This procedure is repeated until the mode shape and the associated frequency is determined to the desired level of accuracy. In the above iteration procedure, the mode shape converges to the one corresponding to the lowest natural frequency. If the stiffness matrix had been used instead of the flexibility matrix, then convergence at the highest natural frequency is obtained. After the first mode of vibration is determined, it is removed from the system matrices by the use of a sweeping matrix so that higher modes can be obtained. This procedure is repeated until all the desired mode shapes and natural frequencies are determined.

9.3.1.8 Myklestad–Prohl Method (Transfer Matrix Method)

The Myklestad–Prohl transfer matrix formulation (Myklestad, 1944; Prohl, 1945) is commonly used to analyze lumped parameter models of rotating machinery. The distributed elastic and inertial properties of the rotor are represented as a collection of rigid bodies connected by massless elastic beam elements as illustrated in Figure 9.10. This method is best suited to calculate critical speeds and mode shapes of rotors neglecting the effects of viscous damping. The Myklestad–Prohl procedure can also be adopted to perform synchronous response and stability analysis, including for the effects of damping.

In order to demonstrate the transfer matrix procedure, an axisymmetric rotor is analyzed to determine its undamped critical speeds and mode shapes. Referring to Figure 9.10, the rotor is divided into n nodes, and each node is connected to the adjacent node by a massless elastic beam with uniform cross-sectional properties. The mass of components such as disks, impellers, and so on, together with the mass of the adjacent portion of the shaft, is lumped at the nodes. The Myklestad–Prohl method is based on the solution of the Bernoulli–Euler equation and the variables of interest are displacement (y), slope (θ), moment (M), and shear (V). The development of the following procedure follows Childs (1993).

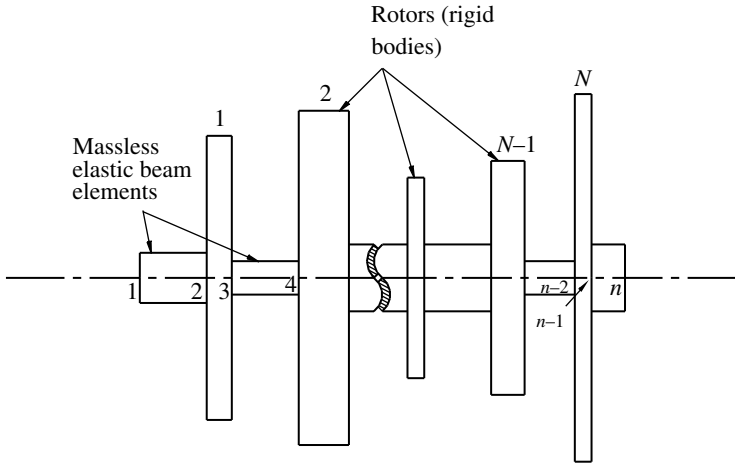


FIGURE 9.10 Lumped-parameter model of rotor.

At a typical nodal point (n), the variables on the left-hand side (l) are related to the variables on the right-hand side (r) by the following relationship:

$$\begin{Bmatrix} y_n^r \\ \theta_n^r \\ M_n^r \\ V_n^r \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -J_n \omega^2 & 1 & 0 \\ -m_n \omega^2 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} y_n^l \\ \theta_n^l \\ M_n^l \\ V_n^l \end{Bmatrix} \quad (9.32)$$

For the purpose of abbreviation:

$$(Q)_n^T = (y_n \quad \theta_n \quad M_n \quad V_n) \quad (9.33)$$

and Equation 9.32 can be written in a more compact form as follows:

$$(Q)_n^r = [T_{mn}](Q)_n^l \quad (9.34)$$

where $[T_{mn}]$ represents the transfer mass matrix at node n .

At a massless beam section, connecting node n to node $n+1$ the transfer matrix is given by

$$\begin{Bmatrix} y_{n+1}^l \\ \theta_{n+1}^l \\ M_{n+1}^l \\ V_{n+1}^l \end{Bmatrix} = \begin{bmatrix} 1 & l_n & \frac{l_n^2}{2EI_n} & -\frac{l_n^3}{6EI_n} \\ 0 & 1 & \frac{l_n}{EI_n} & \frac{l_n^2}{2EI_n} \\ 0 & 0 & 1 & -l_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} y_n^r \\ \theta_n^r \\ M_n^r \\ V_n^r \end{Bmatrix} \quad (9.35)$$

Equation 9.35 may be written in a more abbreviated form as

$$(Q)_{n+1}^l = [T_{bn}](Q)_n^r \quad (9.36)$$

where $[T_{bn}]$ represents the beam element transfer matrix connecting node n to node $n+1$.

From Equation 9.34 and Equation 9.36, we obtain the combined transfer matrix for nodes n and $n+1$:

$$(Q)_{n+1}^l = [T_{bn}][T_{mn}](Q)_n^l = [T_n](Q)_n^l \quad (9.37)$$

Starting with node one, successive matrix multiplications are carried out until node $n + 1$ is reached. The last node ($n + 1$) is a dummy node with the beam length, l , equal to zero, and the mass and inertias also equal to zero. This makes the nodal parameters on the left-hand side of node $n + 1$ equal to those on the right-hand side of node n . The result is as follows:

$$(Q)_n^r = [T_n][T_{n-1}] \cdots [T_1](Q)_1^l \quad \text{or} \quad (Q)_n^r = [T](Q)_1^l \quad (9.38)$$

The matrix $[T]$ is a function of the rotational speed, ω . The Myklestad–Prohl method uses a trial and error solution to determine the values of ω which satisfy the boundary conditions and Equation 9.38 simultaneously. It is not necessary to store and multiply all the matrices together. The transfer matrix procedure is used to proceed from one end to the other without having to store all the nodal matrices. In all cases, two boundary conditions each are known at the two ends of the shaft, and the frequencies that satisfy these boundary conditions are the critical speeds of the rotor. Once the critical speeds are calculated, the corresponding mode shapes can also be determined using the transfer matrix procedure. It should be noted that other types of elements, such as elastic supports, flexible couplings, and so on, could also be introduced very conveniently.

9.3.1.9 Direct Stiffness Method

The direct stiffness method uses a lumped-parameter formulation to evaluate the dynamic characteristics of a flexible rotor. The general differential equation of motion that characterizes its behavior (less the damping and gyroscopic forces) is as follows:

$$\begin{bmatrix} [m] & 0 \\ 0 & [J] \end{bmatrix} \begin{Bmatrix} (\ddot{Y}) \\ (\ddot{\theta}) \end{Bmatrix} + [K] \begin{Bmatrix} (Y) \\ (\theta) \end{Bmatrix} = \begin{Bmatrix} (F) \\ (T) \end{Bmatrix} \quad (9.39)$$

where $[m]$ and $[J]$ are diagonal matrices which contains the nodal masses, m_i , and nodal moments of inertia, J_i , respectively. The stiffness matrix, $[K]$, contains the internal stiffness terms of the beam elements as well as any external spring stiffness at the supports. The vectors (F) and (T) represent external forces and moments acting on the system, respectively.

The stiffness matrix for a typical beam element based on the Bernoulli–Euler equations is as follows (Childs, 1993):

$$[K^i] = \frac{2EI_i}{l_i^3} \begin{bmatrix} 6 & 3l_i & -6 & 3l_i \\ 3l_i & 2l_i^2 & -3l_i & l_i^2 \\ -6 & -3l_i & 6 & -3l_i \\ 3l_i & l_i^2 & -3l_i & 2l_i^2 \end{bmatrix} \quad (9.40)$$

The overall stiffness matrix, $[K]$, has to be assembled by combining the individual component matrices in a systematic manner. The following procedure illustrates the process.

The stiffness matrix of the i th beam element in matrix notation is

$$[K^i] = [k_{j,k}^i] \quad (9.41)$$

where j and k vary from $(2i - 1)$ to $(2i + 2)$.

To form the overall stiffness matrix, the elements with the same subscripts of adjacent beam elements are added over n beam elements as given by the following equation:

$$[K] = [K_{j,k}] = \sum_{i=1}^n \sum_{j=2i-1}^{2i+2} \sum_{k=2i-1}^{2i+2} k_{j,k}^i \quad (9.42)$$

Once the inertia matrix and the stiffness matrix for the entire system are assembled, the eigenvalues and eigenvectors can be evaluated by solving the following homogeneous equation derived

from Equation 9.39:

$$[M](\ddot{Y}) + [K](Y) = 0 \quad (9.43)$$

There are numerous analysis procedures (Meirovitch, 1986) for the solution of Equation 9.43 that yield the eigenvalues and eigenvectors of the system. The method of choice will depend on the complexity and nature of the inertia and stiffness matrices. Perhaps the most widely known is the matrix iteration using the power method in conjunction with the sweeping technique. However, this method is not necessarily the most efficient, particularly for higher-order systems. The Jacobi's method, which uses matrix iteration to diagonalize a matrix by successive rotations, is more commonly used owing to its higher efficiency. Details of these techniques are given in the text by Meirovitch (1986).

When the damping matrix and the gyroscopic matrix is also included in Equation 9.39, the direct stiffness method can be used to calculate damped critical speeds, forced rotor response, and instability of the rotor in addition to the eigenvalues using similar methods of solution.

9.3.1.10 The Finite Element Analysis Method

The basis of the FEA method is to provide formulation for complex and irregular systems that can utilize the automation capabilities of computers (also see Chapter 9). The FEA method considers a rotordynamic system as an assemblage of discrete elements, where every such element has distributed and continuous properties, namely, the consistent representation of both mass and stiffness as distributed parameters. As illustrated in Section 9.3.1.9, the lumped-parameter method uses a consistent stiffness matrix equation (Equation 9.40) in its formulation, and therefore, the identical procedure can be adopted for the finite element method as well. For the distributed mass representation of an element, Archer (1963) procedure, which is based on the assumption that the mass distribution is proportional to the elastic distribution similar to the Rayleigh–Ritz formulation, is utilized. The resulting mass matrix is as follows:

$$[m^i] = \frac{m^i l_i}{420} \begin{bmatrix} 156 & 22l_i & 54 & -13l_i \\ 22l_i & 4l_i^2 & 13l_i & -3l_i^2 \\ 54 & 13l_i & 156 & -22l_i \\ -13l_i & -3l_i^2 & -22l_i & 4l_i^2 \end{bmatrix} \quad (9.44)$$

The overall stiffness matrix, $[K]$, for the entire system is assembled by combining the individual component matrices in a systematic manner according to Equation 9.41 and Equation 9.42. The overall mass matrix can also be assembled in precisely the same manner, as given by Equation 9.45 and Equation 9.46.

The mass matrix of the i th beam element in matrix notation can be represented as

$$[m^i] = [m_{j,k}^i] \quad (9.45)$$

where j and k varies from $(2i - 1)$ to $(2i + 2)$:

$$[M] = [M_{j,k}] = \sum_{i=1}^n \sum_{j=2i-1}^{2i+2} \sum_{k=2i-1}^{2i+2} m_{j,k}^i \quad (9.46)$$

Once the mass matrix and the stiffness matrix for the entire system are assembled, Equation 9.43 that describes the free vibration of the complete system can be solved. The solution methods of the eigenvalue problem, which can be utilized, are the same as those used for the direct stiffness method illustrated in Section 9.3.1.9 above. Details of the FEA methods are given in Ruhl and Booker (1972).

9.3.1.11 Torsional Analysis (Holzer Method)

The development of torsional analysis methods have gone through a similar evolutionary process to lateral vibration methods. Holzer (1921) first introduced the lumped-parameter numerical method to calculate torsional natural frequencies of a multi-DoF system. Even to-date, this is the most commonly used method because of its simplicity and reasonable degree of accuracy. The Holzer method is a transfer matrix formulation that uses a lumped parameter model similar to that used in the Myklestad–Phrol method described in Section 9.3.1.8. The only difference is that the transfer matrices represented by Equation 9.32 and Equation 9.35 are replaced by the equations

$$\begin{Bmatrix} \theta \\ T \end{Bmatrix}_n^r = \begin{bmatrix} 1 & 0 \\ -\omega^2 J & 1 \end{bmatrix}_n \begin{Bmatrix} \theta \\ T \end{Bmatrix}_n^l \quad (9.47)$$

$$\begin{Bmatrix} \theta \\ T \end{Bmatrix}_{n+1}^l = \begin{bmatrix} 1 & \frac{1}{k} \\ 0 & 1 \end{bmatrix}_{n+1} \begin{Bmatrix} \theta \\ T \end{Bmatrix}_n^r \quad (9.48)$$

Starting with node one, successive matrix multiplications are carried out until node $n + 1$ is reached. The result can be represented by Equation 9.38. The matrix $[T]$ is a function of the rotational speed, ω . In all cases, one boundary condition at each end of the rotor is known. A trial-and-error solution to determine the values of ω which satisfy the boundary conditions and Equation 9.38 are simultaneously determined. These values are the torsional critical speeds of the rotor. Once the critical speeds are calculated, the corresponding torsional mode shapes can also be determined using the transfer matrix procedure.

In the case of branched systems and geared systems, particular attention has to be paid to the relative rotational speeds of the components. The rule is quite simple: multiply all stiffness and inertias of the geared shaft by N^2 , where N is the speed ratio of the geared shaft to the reference shaft.

Other methods such as the distributed mass matrix method, direct stiffness method, and finite element method can also be used to determine torsional critical speeds of rotors. These procedures are very similar to those for lateral critical speed analysis.

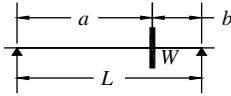
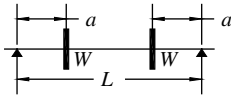
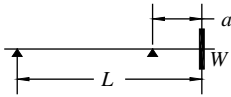
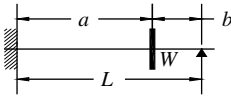
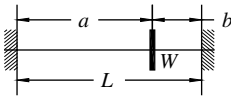
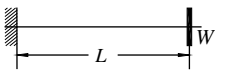
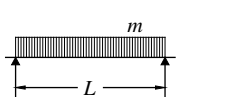
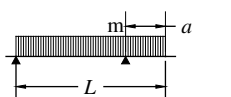
9.3.2 Modeling

The design and analysis of rotordynamic systems require the development of models that simulate the behavior of the physical system. In the past, the critical speed of the rotor was considered to be the main criterion for stable operation. Today, stable, well-damped rotordynamic response to the exciting forces within a machine is considered to be a necessary condition for high reliability. The accuracy and reliability of the results greatly depends on the credibility of the system model and its adaptability to the analytical procedure. Even the most accurate and efficient analytical method cannot produce good results from a bad model. The methods that are commonly used to model shaft sections and disks and other such elements attached to shafts have been discussed in the previous sections. Useful formulae for calculating critical speeds of simple systems are given in Table 9.2. Models to represent bearings, rotor dampers, seals, and rotor–stator interactions are discussed in the following sections.

9.3.2.1 Journal Bearings

Journal bearings were used in rotating machinery for a long time before their dynamic characteristics were fully understood. Considerable effort has been expended in the last few decades to understand and develop techniques for their accurate representation in rotordynamic analysis. A variety of bearing types with improved characteristics have been developed over the years. Figure 9.11 shows the most commonly used types in rotating machinery. Hagg and Sankey (1958) were amongst the first to provide dynamic stiffness and damping coefficients for a number of these bearing types. However, these coefficients are considered incomplete as cross-coupling terms were not considered. Soon after, there was a flurry of activity related to the analysis of journal bearings; Sternlicht (1959), Warner (1963),

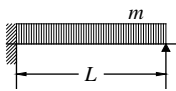
TABLE 9.2 Useful Formulas in Vibration Analysis and Design

Rankine formula	$\omega = \frac{k(Hg)^{1/2}}{b^2}$	Note: This formula is of historical interest only and has limited practical value
Greenhill formula	$\frac{d^4y}{dx^4} - \frac{m\omega^2}{gEak^2}y = 0$	
Dunkerly equation	$\frac{1}{\omega_c^2} = \frac{1}{\omega_s^2} + \sum_{i=1}^n \frac{1}{\omega_i^2}$	
The above equation reduces to		
	$\omega_c^2 = \frac{g}{\sum y_{\text{stat}}}$	
Formulas for natural frequency calculation (Blevins, 2001; Gorman, 1975)		$\omega_c = \frac{1}{ab} \sqrt{\frac{3EI}{W}}$
		$\omega_c = \frac{1}{a} \sqrt{\frac{6EI}{W(3L - 4a)}}$
		$\omega_c = \frac{1}{a} \sqrt{\frac{3EI}{WL}}$
		$\omega_c = \left(\frac{12EI L^3}{W a^3 b^2 (3L + b)} \right)^{1/2}$
		$\omega_c = \left(\frac{3L^3 EI}{W a^3 b^3} \right)^{1/2}$
		$\omega_c = \left(\frac{3EI}{WL^3} \right)^{1/2}$
		$\omega_c = \left(\frac{98EI}{mL^4} \right)^{1/2}$
		$\omega_c = \left(\left(\frac{64a}{9L} \right)^2 - \left(\frac{6a}{L} \right) + \frac{16}{3} \right) \left(\frac{EI}{mL^4} \right)^{1/2},$ $\frac{a}{L} \geq 0.25$

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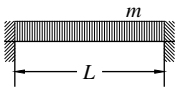
TABLE 9.2 (continued)

Formulas for torsional
natural frequency
calculation

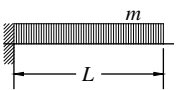

$$\omega_c = \left(\frac{237EI}{mL^4} \right)^{1/2}$$

$$\omega_c = \left(-74.7 \left(\frac{a}{L} \right)^2 + 22.1 \left(\frac{a}{L} \right) + 3.14 \right)^2 \left(\frac{EI}{mL^4} \right)^{1/2},$$

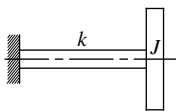
$$\frac{a}{L} < 0.25$$



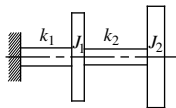
$$\omega_c = \left(\frac{502EI}{mL^4} \right)^{1/2}$$



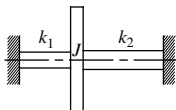
$$\omega_c = \left(\frac{12.4EI}{mL^4} \right)^{1/2}$$



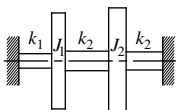
$$\omega_c = \left(\frac{k}{J} \right)^{1/2}$$



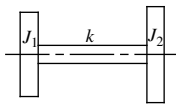
$$\omega_c = \frac{1}{\sqrt{2}} \left[\frac{k_1 + k_2}{J_1} + \frac{k_2}{J_2} \mp \left\{ \left(\frac{k_1 + k_2}{J_1} + \frac{k_2}{J_2} \right)^2 - \frac{4k_1k_2}{J_1J_2} \right\}^{1/2} \right]^{1/2}$$



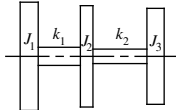
$$\omega_c = \left(\frac{k_1 + k_2}{J} \right)^{1/2}$$



$$\omega_c = \frac{1}{\sqrt{2}} \left[\frac{k_1 + k_2}{J_1} + \frac{k_2 + k_3}{J_2} \mp \left\{ \left(\frac{k_1 + k_2}{J_1} + \frac{k_2 + k_3}{J_2} \right)^2 - \frac{4(k_1k_2 + k_2k_3 + k_1k_3)}{J_1J_2} \right\}^{1/2} \right]^{1/2}$$



$$\omega_c = \left(\frac{k}{J_1} + \frac{k}{J_2} \right)^{1/2}$$



$$\omega_c = \frac{1}{\sqrt{2}} \left[\frac{k_1}{J_1} + \frac{k_1 + k_2}{J_2} + \frac{k_2}{J_3} \mp \left\{ \left(\frac{k_1}{J_1} + \frac{k_1 + k_2}{J_2} + \frac{k_2}{J_3} \right)^2 - \frac{4k_1k_2(J_1 + J_2 + J_3)}{J_1J_2J_3} \right\}^{1/2} \right]^{1/2}$$

Rayleigh equations

$$\omega^2 = \frac{EI \int_0^l \left(\frac{d^2y}{dx^2} \right)^2 dx}{\mu \int_0^l y^2 dx}$$

TABLE 9.2 (continued)

	$\omega^2 = \frac{\sum_{i=1}^n m_i y_i}{\sum_{i=1}^n m_i y_i^2}$	
Ritz method	$y = \sum_{i=1}^n a_i \Phi_i(x)$ $\frac{\partial}{\partial a_i} \frac{\int_0^l \left(\frac{d^2 y}{dx^2} \right)^2 dx}{\int_0^l y^2 dx} = 0$ $\frac{\partial}{\partial a_i} \int_0^l \left[\left(\frac{d^2 y}{dx^2} \right)^2 - \frac{\omega^2 \mu}{EI} y^2 \right] dx = 0$	
Stodola–Vianello method	$\{Y\} = \omega_r^2 [A] [m] \{Y\}, [A] = [K]^{-1}$	
Transfer matrix — Myklestad–Phrool method	$\begin{Bmatrix} y_{n+1}^1 \\ \theta_{n+1}^1 \\ M_{n+1}^1 \\ V_{n+1}^1 \end{Bmatrix} = \begin{bmatrix} 1 & l_n & \frac{l_n^2}{2EI_n} & -\frac{l_n^3}{6EI_n} \\ 0 & 1 & \frac{l_n}{EI_n} & \frac{l_n^2}{2EI_n} \\ 0 & 0 & 1 & -l_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} y_n^r \\ \theta_n^r \\ M_n^r \\ V_n^r \end{Bmatrix}$	
Stiffness matrix for a beam element	$[K^i] = \frac{2EI_i}{l_i^3} \begin{bmatrix} 6 & 3l_i & -6 & 3l_i \\ 3l_i & 2l_i^2 & -3l_i & l_i^2 \\ -6 & -3l_i & 6 & -3l_i \\ 3l_i & l_i^2 & -3l_i & 2l_i^2 \end{bmatrix}$	
Mass matrix for a beam element	$[m^i] = \frac{m^i l_i}{420} \begin{bmatrix} 156 & 22l_i & 54 & -13l_i \\ 22l_i & 4l_i^2 & 13l_i & -3l_i^2 \\ 54 & 13l_i & 156 & -22l_i \\ -13l_i & -3l_i^2 & -22l_i & 4l_i^2 \end{bmatrix}$	
Squeeze-film damper coefficients	$k = \frac{24R^3 L \mu \omega \varepsilon}{C_i^3 (2 + \varepsilon^2)(1 - \varepsilon^2)}$ $c = \frac{12\pi R^3 L \mu}{C_i^3 (2 + \varepsilon^2)(1 - \varepsilon^2)^{1/2}}$ $k = \frac{2RL^3 \mu \omega \varepsilon}{C_i^3 (1 - \varepsilon^2)^2}$ $c = \frac{\pi RL^3 \mu}{2C_i^3 (1 - \varepsilon^2)^{3/2}}$	
Unbalance sensitivity	$SF = \frac{a}{U} M$	
Rolling element bearing defect frequencies	$f_{\text{bor}} = \frac{ND}{60d} \left[1 - \left(\frac{d}{D} \cos \theta \right)^2 \right]$	$N = \text{rotational speed (rpm)},$ $D = \text{rolling element pitch diameter}$

(continued on next page)

TABLE 9.2 (continued)

	$f_{ir} = \frac{Nn}{120} \left(1 + \frac{d}{D} \cos \theta \right)$	d = rolling element diameter, N = number of rolling elements
	$f_{or} = \frac{Nn}{120} \left(1 - \frac{d}{D} \cos \theta \right)$	θ = contact angle with respect to axis, bor = ball or roller defect
	$f_c = \frac{N}{120} \left(1 - \frac{d}{D} \cos \theta \right)$	ir = inner race defect, or = outer race defect, c = cage defect
Lomakin formula for radial stiffness for a close clearance bushing	$k = \frac{\pi}{8} (1 + s) \lambda \mu^4 \left(\frac{l}{b_m} \right)^2 \Delta p D$	
	$\mu^2 = \frac{1}{1 + s + (\lambda l / 2 b_m)}$	
	b_m = radial clearance, l = length of bushing,	D = diameter, Δp = differential pressure across bushing,
	ζ = inlet loss coefficient,	λ = friction coefficient

Lund (1964), Lund (1965), Glienicke (1966), Orcutt (1967), Lund (1968), Someya et al. (1988), and several others provided complete bearing coefficients, including cross-coupling terms, for several bearing types. This information is considered to be a valuable resource for those engaged in rotordynamic analysis. The general form of the rotordynamic model for a journal bearing resulting from the above contributions is given by the following equation:

$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = - \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} - \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{Y} \end{Bmatrix}$$

(9.49)

Since the dawn of the digital computer era, several computer codes have been developed to analyze all aspects of journal bearings, including stiffness and damping coefficients. Many of these codes have been developed by equipment manufactures and research centers for their exclusive use. Several commercially available software codes popularized in North America are given in Table 9.3. Although bearing coefficients given in the form of charts and tables from the earlier studies are still in use, computer-based codes are growing in popularity.

9.3.2.2 Rolling Element Bearings

Rolling element bearings are used in numerous types of rotating machinery which are required to be compact, manage high loads, and have low heat rejection and simple lubrication systems. Unlike journal bearings, their load-carrying capacity is not speed-dependent and as a result is capable of full load capacity down to zero speed. Some of these salient features make rolling element bearings very attractive to many industries.

From a rotordynamic standpoint, rolling element bearings are modeled as linear spring elements with direct spring coefficients only. The damping terms are insignificant and as a result do not attenuate rotor deflections at critical speeds. A typical rolling element bearing is represented by the following equation:

$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = - \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix}$$

(9.50)

The absence of cross-coupling stiffness and damping terms signifies that bearing induced rotor instability will not occur. Although, for convenience of analysis, the spring stiffness is considered linear, its

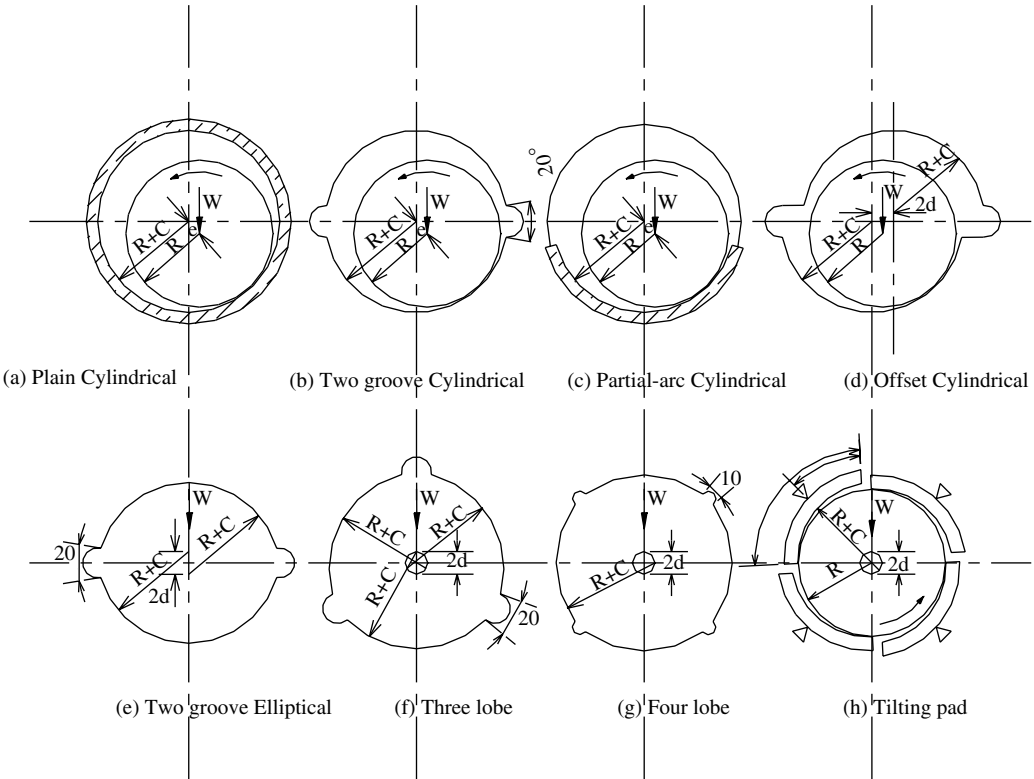


FIGURE 9.11 Common types of journal bearings.

true behavior can be quite the opposite, leading to some calculation inaccuracies. The nonlinearities are most significant where the bearings have no preload and some internal clearance in the bearings exists. Preloaded bearings with little or no internal clearance behave quite linearly. Jones (1960), Harris (1991), and Kramer (1993) have analyzed the bearing stiffness coefficients for the common types of rolling element bearings, and this data can be utilized for rotordynamic study of rotating machinery.

9.3.2.3 Squeeze-Film Dampers

Squeeze-film dampers are used to introduce damping capacity to a rolling element bearing or, in the case of journal bearings, to provide additional damping and stiffness to eliminate rotor instability problems. Squeeze-film dampers have come into prominence through the modern aircraft gas turbine industry where the bearing of choice is the rolling element bearing. In the mid-1970s, several designs were introduced to add damping capacity and predictable stiffness to the rolling element bearings. In its basic form, a squeeze-film damper is very similar to a nonrotating cylindrical journal bearing where the outer race of the rolling element bearing forms the journal as illustrated in Figure 9.12. The addition of end seals (to control leakage) and centering springs are modifications that have been introduced to enhance its performance. The interactive force at a bearing using a squeeze-film damper can be represented by the following equation:

$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = - \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} - \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{Y} \end{Bmatrix} \quad (9.51)$$

The stiffness and damping coefficients for the squeeze-film dampers have been derived (Ehrich, 1999), from the solution of the Reynolds' equation for the case of a nonrotating journal bearing. For dampers with end seals, the long journal bearing theory is used to generate the following stiffness and damping

TABLE 9.3 Rotordynamic Analysis Software

Name of Software	Type of Analysis	Supplier
CAD20	Lateral critical speeds of flexible rotors	CADENSE Programs, Foster Miller Technologies Inc., Albany, NY, USA
CAD21	Unbalance response of flexible rotors	
CAD21a	Response of flexible rotors to nonsynchronous sinusoidal excitation	
CAD22	Torsional critical speeds and response of geared systems	
CAD24	Transient torsional critical speeds of geared system	Dipartimento di Meccanica, Politecnico di Torino, Torino, Italy
CAD25	Dynamic stability of flexible rotors	
CAD25a	Transient response of flexible rotors	
CAD26	Lateral critical speeds of multilevel rotors	
CAD27	Unbalance response of multilevel rotors	
CAD30	Dynamic coefficients of liquid lubricated journal bearings	
CAD30a	Dynamic coefficients of ball bearings	
CAD31	Dynamic coefficients of liquid lubricated tilting pad journal bearings	
CAD32	Dynamic coefficients of liquid lubricated axial-groove and single pad journal bearings	
CAD34a	Performance of tilting pad thrust bearings	
CAD34b	Performance of tapered-land thrust bearings	
CAD36	Dynamic coefficients of liquid lubricated pressure dam journal bearings	
CAD38	Dynamic coefficients of liquid lubricated deep-pocket hydrostatic journal bearings	
CAD40	Dynamic coefficients of gas lubricated journal bearings	
CAD41	Dynamic coefficients of gas lubricated tilting pad journal bearings	
CAD42	Dynamic coefficients of gas lubricated spiral groove journal bearings	
CAD42i	Dynamic coefficients of liquid lubricated spiral groove journal bearings	
FEATURE	Rotor bearing system analysis	
COJOUR	Analysis of journal bearings	
DYNROT	A program designed to perform a complete study of the rotordynamic behavior of rotors. It is capable of linear, nonlinear and torsional analysis of rotors	AGILE SOFTWARE CONCEPTS NREC White River Junction, VT, USA
DyRoBeS	Comprehensive rotordynamic analysis software for lateral and torsional analysis, including bearing analysis of rotor-bearing systems	
RotorLab	A software package for agile modeling of rotor systems, bearings, and seals. It combines the tasks of design, modeling, analysis, post processing, and data management into a consistent user interface	
DAMBRG2	Coefficients and rigid rotor stability information for two-lobe isoviscous bearings with a pressure dam in only one pad	ROMAC—Rotating Machinery and Controls Laboratory, University of Virginia, Charlottesville, VA, USA
HYDROB	Predicts the steady state and dynamic operating characteristics of hybrid journal bearings	
PDAM2D	This program can analyze stiffness and damping coefficients, and the rigid rotor stability threshold of multipad pressure dam bearings	
SQFDAMP	Determines stiffness and damping coefficients for short and long squeeze-film bearings with and without fluid film cavitation	
THBRG	Dynamic coefficients of multilobe journal bearings with incompressible fluid	

TABLE 9.3 (continued)

Name of Software	Type of Analysis	Supplier
THPAD	Dynamic coefficients of tilting pad journal bearings with incompressible fluid	Rotordynamics Laboratory, Texas A&M University, College Station, TX, USA
THRUST	Predicts the steady-state operating characteristics of tilting-pad and fixed geometry fluid-film thrust bearings	
CRTSP2	Undamped lateral critical speeds of dual-level rotor systems	
MODFR2	Undamped lateral critical speeds of single or dual-level rotor systems	
TWIST2	Undamped torsional critical speeds and mode shapes of rotor systems	
FRESP2	Predicts the modal frequency forced response of dual rotor systems with a flexible substructure	
RESP2V3	Nonplanar synchronous unbalance response of dual-level multimass flexible rotors	
HCOMB	Dynamic coefficients of straight-through honeycomb seals with a compressible gas	
LABY3	Dynamic coefficients for straight-through and uniform interlocking type labyrinth seals with a compressible fluid	
SEAL2	Stiffness and damping coefficients for plain and grooved seals with incompressible turbulent axial flow	
SEAL3	Stiffness, damping and mass coefficients of both plain and circumferentially grooved seals	
TURSEAL	Stiffness and damping coefficients of turbulent flow annular seals or water lubricated bearings	
FSTB3	Stability, damped critical speeds, and whirl mode shapes of multispool rotor systems	
ROTSTB	Stability, damped critical speeds, and whirl mode shapes of single spool rotor systems	
COTRAN	Nonlinear time transient analysis of multilevel rotors with substructure	
TORTRAN3	Transient torsional rotor response	
hydrosealt	Stiffness and damping coefficients, and threshold speed of instability of cylindrical-pad journal bearings and pad-hydrostatic bearings of arbitrary arc lengths and preloads	
hydroflext	Stiffness and damping coefficients, and threshold speed of instability of a variety of bearing and seal types	
hydrotran	Predicts the transient force response of a rigid rotor supported on fluid film bearings	
hydrojet	Force coefficients for a variety of hybrid bearing and seal types handling process fluids	
hydroTRC	Stiffness and damping coefficients for a variety of bearing and seal types and for different types of fluids	
hseal2p	Stiffness and damping coefficients of seals that operate under two-phase flow conditions	
fembear	Stiffness and damping coefficients of cylindrical and fixed arc pad hydrostatic and hydrodynamic bearings for laminar and isothermal flow conditions	
sdfdem	Damping force coefficients of finite length squeeze-film dampers executing circular centered motion	
sdflexs	Instantaneous fluid film forces for arbitrary journal motions and circular centered orbits in multiple pad integral squeeze-film dampers	
hsealm	Stiffness and damping coefficients of cylindrical annular pressure seals	

(continued on next page)

TABLE 9.3 (continued)

Name of Software	Type of Analysis	Supplier
lubsealn	Stiffness and damping coefficients of single-land and multiple-land high pressure oil seal rings and cylindrical journal bearings	
ROTECH	Lateral rotordynamic analysis for critical speeds; unbalance response, linear stability and nonlinear transient response of rotors. Also includes a torsional rotordynamic analysis program	ROTECH Engineering Services, Delmont, PA, USA
ROTOR-E	A comprehensive software package for lateral rotordynamic analysis of rotating equipment	Engineering Dynamics Inc., San Antonio, TX, USA
ROTORINSA	A software package devoted to the prediction of the steady-state lateral dynamic behavior of rotors	Laboratoire de Mecanique des Structures, LMST INSA Lyon, Lyon, France
TURBINE-PAK	A software package for rotordynamic analysis of nonlinear multibearing rotor-bearing-foundation systems	Scientific Engineering Research, Mt Best, Vic., Australia.
TURBINE-PAK NONLINEAR	Designed to study transient responses of rotor-bearing-foundation systems, including the loss of stability of the system	
XLrotor	A complete suite of analysis tools for rotating machinery dynamics. Handles both lateral and torsional analysis of rotors. Also includes codes for calculating coefficients for fluid film and antifriction bearings	Rotating Machinery Analysis Inc., Austin, TX, USA
XLTRC	A suite of codes for executing a complete lateral rotordynamic analysis of rotating machinery	The Turbomachinery Laboratory, Texas A&M University, College Station, TX, USA
XLAnSeal	Force and moment coefficients for annular turbulent seals in the laminar, turbulent, and transition flow regimes	
XLcGrv	Coefficients for centered grooved-stator, turbulent flow, annular pump seals	
XLlaby	Stiffness and damping coefficients for tooth-on-rotor or tooth-on-stator gas labyrinth seals	
XLIsotSL	Coefficients for smooth rotor/honeycomb stator annular seals	
XLlubGT	Coefficients for high-pressure oil bushing seals of compressors or smooth pump seals in the laminar flow regime	
XLJrnl	Stiffness and damping coefficients for fixed-arc and tilting-pad bearings	
HLHydPad	Stiffness and damping coefficients for hydrostatic and hybrid journal-pad bearings in the laminar flow regime	
XLTFPBrg	Stiffness and damping coefficients for fixed-arc, tilting-pad and flexure-pivot hydrostatic bearings	
XLPresDm	Stiffness and damping coefficients for multilobed, rigid-pad arc bearings with preload and pressure-dam bearings with relief tracks	
XLBalBrg	Stiffness coefficients for ball bearings	
XLlSFD	Damping and mass coefficients for locally sealed squeeze-film dampers	
XLlSFD	Damping and mass coefficients for open ended squeeze-film dampers	
XLlSFDfEM	Damping coefficients for squeeze-film dampers with various types of end seals	
XLPlmPLR	Stiffness, damping, and mass matrices for centrifugal pump impellers	
XLWachel	Destabilizing cross-coupled force coefficients for impellers of centrifugal compressors	
XLClrEx	Destabilizing cross-coupled stiffness coefficients for unshrouded turbines	

coefficients:

$$k = \frac{24R^3 L \mu \omega \varepsilon}{C_r^3 (2 + \varepsilon^2)(1 - \varepsilon^2)} \quad (9.52)$$

$$c = \frac{12\pi R^3 L \mu}{C_r^3 (2 + \varepsilon^2)(1 - \varepsilon^2)^{1/2}} \quad (9.53)$$

where

R = the damper radius

ω = whirl speed

L = length of damper

μ = viscosity of oil

C_r = the radial clearance

ε = eccentricity ratio (orbit radius/ C_r)

Similarly, for a damper without end seals, the short journal bearing theory yields the following stiffness and damping coefficients:

$$k = \frac{2RL^3 \mu \omega \varepsilon}{C_r^3 (1 - \varepsilon^2)^2} \quad (9.54)$$

$$c = \frac{\pi RL^3 \mu}{2C_r^3 (1 - \varepsilon^2)^{3/2}} \quad (9.55)$$

Although the above equations, based on the Reynolds' equation, have been proposed to predict damper characteristics, the experimental evidence does not validate these equations. Therefore, these equations should be used with caution for practical purposes.

9.3.2.4 Annular Seals

Annular seals are primarily used in pumps, compressors, gas turbines, and steam turbines to minimize leakage and thereby improve the volumetric efficiency of the machine. In addition to their basic function, they also play a vital role in the rotordynamics of the machine, especially in multistage machines, providing stiffness and damping and thereby enhancing high-speed operational capability. In fact, in the last few decades, most of the development work on seals has focused on understanding and improving their dynamic vibration characteristics rather than improving their efficiency in sealing.

Lomakin (1958) was the first to publish on the restoring forces in smooth annular clearances in pumps. However, it was more than a decade later that Black (1968) provided the major initial impetus for the understanding and development of seals. Childs (1993) provided an excellent compendium of the research work conducted in the area of seals. His book also provides the most comprehensive coverage of the subject of seal dynamics.

In the present context, seals are handled in the same manner as the stiffness and damping characteristics of journal bearings with some degree of modifications. In particular, fluid inertia effects are included, and it is assumed that the center of the shaft orbit is the same as the center of the stationary seal ring. Assuming rotational symmetry the reaction force-seal motion model can be represented by the following equation:

$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = - \begin{bmatrix} k & k_c \\ -k_c & k \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} - \begin{bmatrix} c & c_c \\ -c_c & c \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{Y} \end{Bmatrix} - \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{X} \\ \ddot{Y} \end{Bmatrix} \quad (9.56)$$

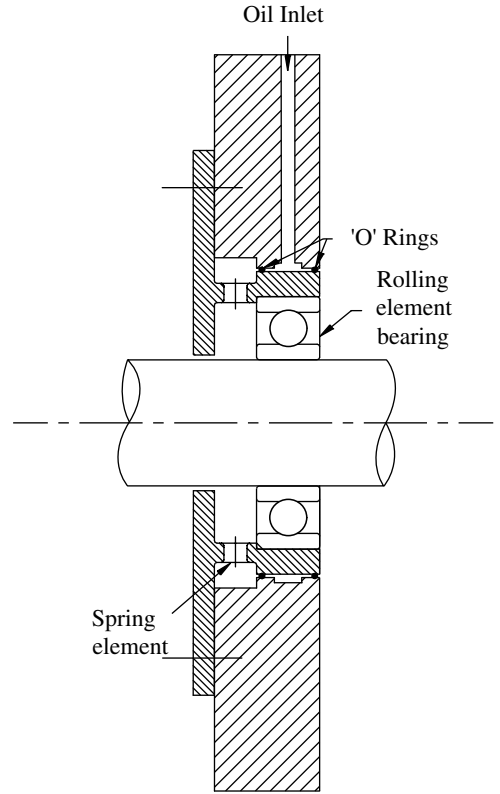


FIGURE 9.12 A squeeze-film damper.

An added complexity is the predominance of turbulent flow in annular seals. This invalidates the use of Reynolds' equation for the derivation of seal coefficients. The highest degree of accuracy can be obtained by the direct solution of the Navier–Stokes and continuity equations. However, at the present moment, such methods are considered to be excessively costly and impractical. As a result, two practical semiempirical methods have been developed to derive seal coefficients. In the first approach, the semiempirical turbulent model is directly substituted in the Navier–Stokes equation and a numerical technique is used for its solution. The second, most commonly used technique uses a bulk flow model together with control volume formulations, namely, the continuity equation and momentum equation, to obtain the desired results. For a detailed discussion of these methods, solution techniques, the influence of various physical parameters on the coefficients, and an excellent compilation of computational and experimental results, the publication by Childs (1993) is recommended.

9.3.2.5 Impeller–Diffuser/Volute Interface

It is widely known that the flow fields within certain types of rotating machinery can significantly influence its vibration behavior. Thomas (1958) recognized and explained the presence of destabilizing clearance excitation forces in axial flow steam turbines. Black (1974) was the first to suggest that centrifugal pump impellers could also develop destabilizing forces. The nature of these forces and their influence on rotor instability has been explained in Section 9.2.2.2 and Section 9.2.2.3 of this chapter. The impeller–diffuser/volute forces assuming rotational symmetry can generally be modeled by an equation of the following form:

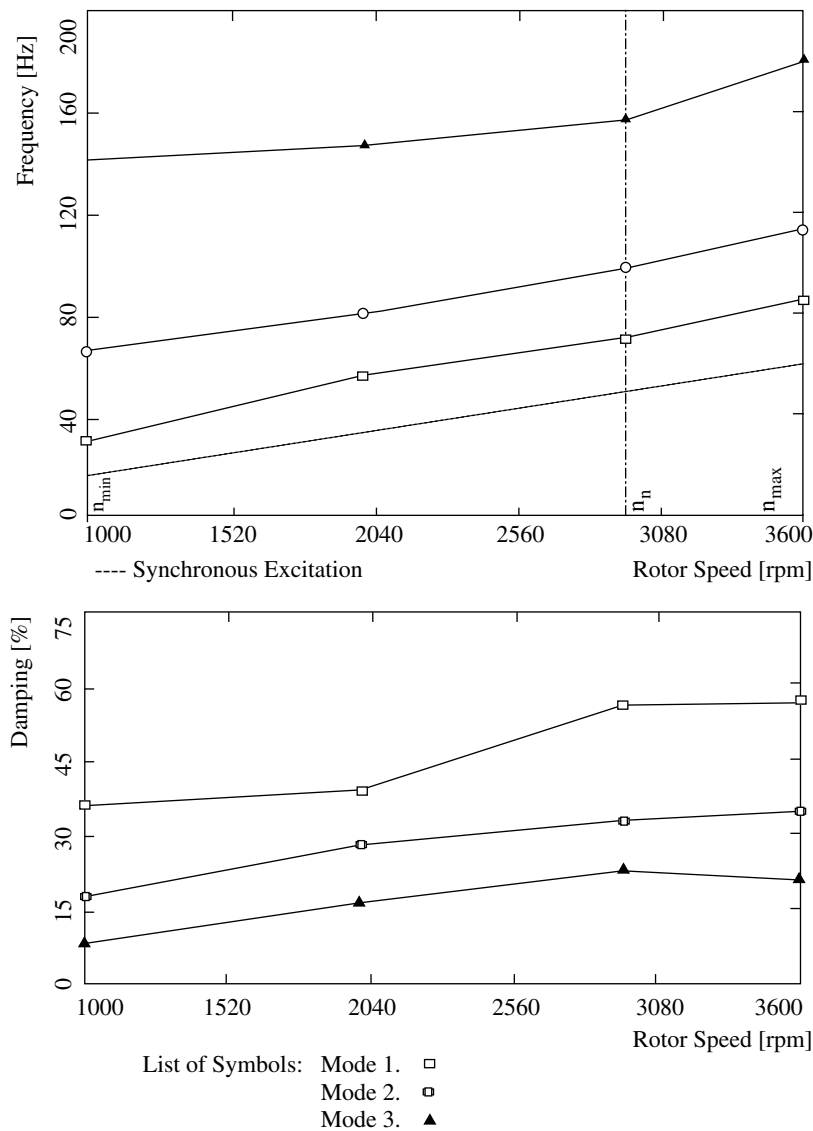
$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = - \begin{bmatrix} k & k_c \\ -k_c & k \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} - \begin{bmatrix} c & c_c \\ -c_c & c \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{Y} \end{Bmatrix} - \begin{bmatrix} m & m_c \\ -m_c & m \end{bmatrix} \begin{Bmatrix} \ddot{X} \\ \ddot{Y} \end{Bmatrix} \quad (9.57)$$

For analytical procedures for the derivation of impeller interaction coefficients and a comparison of experimental data, the work by Childs (1993) is recommended. It is well recognized that a considerable amount of work still needs to be done towards understanding the complex nature of impeller–diffuser/volute interactive forces, especially at off-design conditions.

9.3.3 Design

Since the real machine is not available for tests, at the preliminary design stage it is a common practice to develop an accurate mathematical model of the machine to predict its dynamic behavior in operation. It is also prudent to understand and estimate how the machine will interact with its operating environment and how the environment could influence the operation of the machine. A suitable model of the rotor can be developed using the techniques described in Section 9.3.2, and the rotordynamic characteristics of the machine can be analyzed using one of the methods described in Section 9.3.1, above. Based on these methods, numerous computer-based rotordynamic analysis programs have been developed. A listing of the most widely known computer programs in North America is given in Table 9.3. The objectives of the analysis are to predict the critical speeds, excitation frequencies, the amplitudes of deflection, and the magnitude of the forces of the rotor within its full operating range. In certain situations, evaluation of the energy content of the excitation may also be required.

Once the mathematical model is developed, the eigenvalues of the rotor and the mode shapes can be determined. The results can then be presented in the form of a Campbell diagram, where the eigenvalues along with the excitation frequencies are plotted as a function of rotor speed. Critical speeds occur at the speeds corresponding to the points of intersection of the excitation frequency lines and the eigenvalue lines. The Campbell diagram presentation (Figure 9.13) of the results is very useful since the influence of key parameters such as stiffness, damping, clearances (new and worn conditions), and so on can all be shown on the same diagram. A critical speed, although present, may be of little consequence if it is associated with sufficient damping. As illustrated in Figure 9.4, when the damping ratio $\zeta \geq 0.707$, the system is critically damped and above this level of damping there is no amplification of the rotor deflection. At or near a critical speed the amplification factor is $\approx 1/2\zeta$. Using this estimated value,



Pump Stat: New
Analysis..... 807281012. FEED PUMP

FIGURE 9.13 Campbell diagram for a multistage pump.

depending on the internal clearances of the machine, it is possible to assess if the rotor can pass through a critical speed without causing damage to the components. An amplification factor of 2.5 or below is a typical acceptance limit for centrifugal pumps, even for continuous operation at or near it. However, if the amplification factor exceeds the acceptable limits or the critical speeds are too close to the continuous operating speed, then design modifications have to be made to change the critical speed values. At the design stage, it is considered good practice to ensure that the critical speeds are not within $\pm 10\%$ of the continuous operating speed; these limits are sometimes referred to as *separation margins*. The mode shapes of the rotor are important from the standpoint of identifying where the maximum deflections occur. It also provides a good guide for assessing design modifications to improve damping or reduce sensitivity to unbalance forces.

The eigen analysis only provides relative deflections of the rotor. In order to estimate true deflections, a forced response analysis has to be made. Forcing functions of estimated magnitude are applied at selected locations to determine the resulting deflections at specific points on the rotor. This type of analysis is typically carried out for synchronous excitation forces only. The nature of the forcing function depends on the type of the machine; mechanical unbalance is common to all types of machines, whereas hydraulic unbalance is relevant to centrifugal pumps and electrical unbalance to electric motors. The challenge, of course, is to determine the magnitudes, directions, and locations of the forces to apply and how the resulting rotor response should be judged. Of course, these criteria are machine type-dependent and not necessarily applicable to all types of rotating machinery. An example of how forced response analysis on centrifugal pumps is evaluated is given below (Bolleter et al., 1992):

1. Maximum amplification factors and required separation margins are defined by specifications; example as shown in API 610, 8th edn., 1995.
2. Excitation forces are defined and the response is judged relative to admissible shaft vibration limits, and relative to clearances.
3. Apply unbalance forces of such a magnitude that maximum permissible vibration limits at the vibration probe locations are reached, and then evaluate if the deflections exceed the minimum clearances in the machine.
4. Apply an unbalance force of arbitrary magnitude and determine the resulting response at the same or another location, and calculate the sensitivity factor (SF) using the following formula:

$$SF = \frac{a}{U} M \quad (9.58)$$

where

a = rotor deflection

U = unbalance force

M = rotor mass

The sensitivity factor should then be compared with experimental base values of similar machines for acceptance. The rotor responses to the applied forces can be further analyzed to extract other parameters of interest, such as phase angles and force magnitudes at the bearings, in order to evaluate the design.

In order to optimize the rotating machine design in terms of placement of critical speeds and control of deflections and forces, a parameter sensitivity coefficients analysis (Lund, 1979; Rajan et al., 1986; Rajan et al. 1987) may be carried out. For speed and convenience of analysis, the optimization routine can be automated.

- Rotordynamic analysis is a part of the current rotating machinery design practice used to predict their vibration behavior.
- The most current rotordynamic analytical procedures are computer-based and are derived from the lump-parameter model or the transfer matrix method.
- In the lumped-parameter model method, the distributed elastic and inertial properties of the rotor are represented as a collection of rigid bodies connected by massless elastic beams.
- In the transfer matrix method, commonly called the FEA method, the rotor is represented as an assembly of elements with distributed elastic and inertial properties.
- Accurate modeling and representation of rotor components is vital to the accuracy and reliability of analysis results. As a result, significant advancement in modeling shaft sections, disks, impellers, bearings, seals, rotor dampers, and rotor–stator interactions have been made.

9.4 Vibration Measurement and Techniques

9.4.1 Units of Measurement

The most commonly used units of measurement of vibration are as follows:

- Displacement (peak to peak): millimeter (mm) or micron (10^{-6} m) in metric units, and *thou* or *mil* (0.001 in) in imperial units
- Velocity (peak, RMS or “true peak”): mm/sec or m/sec in metric units, and in./sec in imperial units
- Acceleration (RMS): *g* or m/sec² in metric units, and *g* or in./sec² in imperial units
- Frequency: hertz (Hz) or cycles/min (cpm) in both systems of units
- Phase angle: degrees in both systems of units

9.4.2 Measured Parameters and Methods

Under steady-state conditions, the vibration from a rotating machine is a periodic signal of a complex waveform. During unstable operation or upset conditions, the signal may become random in nature. In certain types of machines, transient signals that are nonperiodic could also be present due to internal impacts and damping. Based on the simple spring–mass system and the mathematical Fourier analysis procedure, all periodic complex waveforms can be reduced to the sum of a series of sinusoidal functions. In the case of random signals, averaging techniques are used to reduce them to periodic signals for convenience of analysis.

A quantitative assessment of the vibration can be made in terms of the amplitude, velocity, acceleration, or the magnitude of force of the motion. Other key parameters such as frequency, phase angle, and the time-varying nature of the signal are important in fully characterizing it. Because the true signal is not purely sinusoidal, it is important to identify its magnitude as, *peak*, *peak-to-peak* or *root mean square* (RMS). The preferred parameter of measurement varies throughout the industry and depends on the nature, complexity, and type of machine, and the purpose for which it is measured. A general classification of measured parameters and techniques based on industry and rotating machine type is given in [Table 9.4](#).

- The displacement amplitude, velocity, acceleration, frequency, and phase angle of the vibration signal are some of the parameters that can be used to assess the condition of a rotating machine.
- The vibration signals generated by a typical rotating machine is complex in nature and, therefore, requires various mathematical analysis procedures and signal averaging techniques to reduce them to simple and interpretable forms.

9.5 Vibration Control and Diagnostics

9.5.1 Standards and Guidelines

Given the fact that some degree of vibration is always present in rotating machinery, some means of judging “how much is too much” has to be established so that vibrations can be controlled within reasonable limits. When such a judgment is not based on a scientific method, there is room for speculation and it will depend on the one making the decision, the manufacturer, end user, the governing authority, and so on. Since vibrations are a key indication parameter of the performance of rotating machinery, different interest groups monitor it for a variety of purposes. It can be used as a measure of

TABLE 9.4 Measurement Parameters and Techniques

Measurement/Technique	Description	When/Where Used
Acceleration, RMS	When high frequency or force of the vibration is of interest	Gear boxes, rolling element bearings, gas/steam turbines. Mainly used by defense and aerospace industry
Bode/Nyquist plot	Plot of displacement amplitude and phase angle vs. speed	Observe critical speeds and instability in machines using journal bearings
Cepstrum analysis	Inverse Fourier transform of logarithmic power spectrum	To detect families of harmonics and sidebands in gearboxes, rolling element bearings, and electric motors
Condition monitoring	Analysis of signals generated by the machine to determine its condition on a continuous or periodic basis	On critical equipment where predictive maintenance programs are used. Can reduce equipment redundancy
<i>Displacement peak-to-peak</i>		
(a) Absolute	Absolute displacement amplitude of rotor vibration	Where the rotor mass is very much larger than the stator mass. Large motors, generators, and fans
(b) Relative	Relative displacement amplitude of rotor vibration	Commonly used on machines with journal bearings or in close clearance seals
(c) External	Absolute displacement amplitude of stator component vibration	Low speed machines (less than 1000 rpm)
Modal analysis	To measure the vibration response of a structure to an applied force. The force can be periodic or an impact force	To determine the modal mass, stiffness and damping properties of a structure. Also used to measure structural natural frequencies
Orbit analysis	The path of the shaft centerline motion during rotation	Diagnostics of machines using journal bearings. Provides a picture of the motion of the journal in the bearing
Polar plots	A polar graph of amplitude versus phase at various machine speeds	Similar to Bode plots, can be used to detect critical speeds and instability. Modal properties can also be extracted from polar plots
Phase angle	Phase angle of vibration signal	Useful in balancing, diagnosing critical speeds, and misalignment problems
<i>Rolling element bearing analysis</i>		
(a) Acceleration	Measure the amplitude of all pass and discrete frequency accelerations	When damage has progressed to generate audible noise amplitudes increase. Useful from 5–5 kHz
(b) Shock pulse method	A high frequency resonance technique tuned to the detector natural frequency	Early detection of failure, measures ultrasonic noise. Proprietary technique
(c) Envelope technique	Bearing defects cause periodic impacts, which make bearing components resonate. Demodulation and enveloping techniques are used to detect the impact (fault) frequencies	Used for early failure detection (ultrasonic noise) as well as for advanced stages of damage (audible noise)
(d) Spike energy method	Measure the broadband acceleration over the 5–45 kHz range	Used for early failure detection (ultrasonic noise) as well as for advanced stages of damage (audible noise)
(e) Kurtosis method	The normalized fourth moment of the probability distribution of acceleration over the 2–80 kHz range	Used for early failure detection (ultrasonic noise) as well as for advanced stages of damage (audible noise)
Run-up, run-down analysis (waterfall/cascade plots)	Three-dimensional plot of frequency or time spectrum vs. time or speed	Used for diagnosing a variety of vibration problems. Helpful in analyzing transient signals

TABLE 9.4 (continued)

Measurement/Technique	Description	When/Where Used
Spectrum analysis	Plot of amplitude vs. frequency of vibration	Used for diagnostics, to determine frequency, harmonics, side bands, beats, transfer functions, etc., and to control of vibrations levels at discreet frequencies
Trend analysis	Vibration data collected periodically over an extended time domain	Useful in predictive maintenance programs in assessing machine conditions
Time averaging	Averaging of time records using triggering at the same point of the waveform of a repetitive signal	Used in the analysis of faulty gearboxes. Can reduce asynchronous components in the signal and improve signal-to-noise ratio
Time-domain analysis	Plot of amplitude versus time	To observe amplitude modulation, beats, impacts, transients, and phase angle. Very useful in diagnostics
Velocity-peak or RMS	Velocity amplitude of vibration signal	Parameter most commonly used in many industries to monitor vibrations. Peak readings relate to peak stress levels and rms to energy of vibrations

quality and workmanship or the common basis for acceptance between the user and manufacturer. From a safety point of view, the operator can establish *normal*, *alarm*, and *shutdown levels* based on vibration limits. Vibration levels are also used in making maintenance decisions in rotating machinery.

Rathbone (1939) was the first to publish guidelines for vibration limits for machinery, based on his experience as an insurance agent. Since that time, numerous individuals, organizations, and governing bodies have developed a variety of guidelines and standards for vibration levels in rotating machinery. A listing of the more commonly used guidelines and standards is given in [Table 9.5](#). It should be recognized that these are experience-based standards, and therefore, will grow and develop with technology.

ISO 10816 Part 1 to Part 5 is a comprehensive set of standards that has been developed for the evaluation of vibration of rotating machinery by measuring the vibration response on nonrotating, structural components such as bearing housings. Vibration measuring points as specified by these standards are shown in [Figure 9.14](#). In a similar vein, ISO 7919 Part 1 to Part 5 has been developed for the evaluation of vibration by measuring the vibration on rotating shafts. These standards cover the most widely used types of rotating machinery and they relate to both operational monitoring and acceptance testing of equipment. [Table 9.6](#) and [Table 9.7](#) are derived from these standards and are presented as a general guideline for vibration limits of rotating machinery. For specific details, including the limitations of the standards, the reader is advised to refer to the relevant sections of [ISO 10816](#) and [ISO 7919](#) Standards.

9.5.2 Vibration Cause Identification

Vibrations are an inherent part of all rotating machinery. Vibration can be due to many causes: improper design, practical manufacturing limits, poor installation, the effect of system environment, component deterioration, operation outside of design limits, or a combination of the above. At times, finding the exact cause of vibration can be quite a challenge, as several of the causes have similar symptoms. [Table 9.8](#) is a list of the more commonly known causes of vibration in rotating machinery and their symptoms.

9.5.3 Vibration Analysis — Case Study

In the past, it was common practice to operate centrifugal pumps at a fixed speed and attain required flow changes by means of throttling. This forces the pump to operate at low efficiency conditions

TABLE 9.5 Vibration Guidelines and Standards

Year	Author/ Organization	Reference Number	Title/Description
2002	AGMA	ANSI/AGMA 6000-B96	Specification for Measurement of Linear Vibration on Gear Units
2003	API	ANSI/API std 541-2003	Form-Wound Squirrel-Cage Induction Motors 500 hp and Larger
1997	API	API STD 546, second edition	Brushless Synchronous Machines, 500 kVA and Larger
2004	API	API STD 610/ISO 13709, ninth edition	Centrifugal Pumps for Petroleum, Petrochemical and Natural Gas Industries
1997	API	API STD 611, fourth edition	General Purpose Steam Turbines for Petroleum, Chemical and Gas Industry Services
2005	API	API STD 612/ISO 10437, sixth edition	Petroleum, Petrochemical and Natural Gas Industries – Steam Turbines – Special-Purpose Applications
2003	API	API STD 613, fifth edition	Special Purpose Gear Units for Petroleum, Chemical and Gas Industry Services
1998	API	API STD 616, fourth edition	Gas Turbines for the Petroleum, Chemical, and Gas Industry Services
2002	API	API STD 617, seventh edition	Axial and Centrifugal Compressors and Expander-compressors for Petroleum, Chemical and Gas Industry Services
2000	API	API STD 670, fourth edition	Mechanical Protection Systems
2004	API	API STD 672, fourth edition	Packaged Integrally Geared, Centrifugal Air Compressors for Petroleum, Chemical, and Gas Industry Services
2001	API	API STD 673, second edition	Special Purpose Fans
1997	API	API STD 677, second edition	General Purpose Gear Units for Petroleum, Chemical, and Gas Industry Services
1996	API	API STD 681, first edition	Liquid Ring Vacuum Pumps for Petroleum, Chemical, and Gas Industry Services
2000	API	API STD 685, first edition	Sealless Centrifugal Pumps for Petroleum, Heavy-Duty Chemical, and Gas Industry Services
1965	BDS	BDS 5626-65	Measurement of Vibration on Electrical Rotating Machines
1964	Blake, M.P.	Hydrocarbon Processing, January 1964	New Vibration Standards for Maintenance
1963	CAGI		In-Service Standards for Centrifugal Compressors
1975	CAGI		Standard for Centrifugal Air Compressors
1971	CSN	CSN 011410	Permitted Limits for Unbalanced Solid Machine Elements
1968	Dresser		General Guidelines for Vibration on Clark Centrifugal Compressors
1966	Industrial Gosstandart	GOST 12379-66	Measurement of Vibration on Electrical Rotating Machines
2002	HI	ANSI/HI 9.6.4	Centrifugal and Vertical Pumps — Vibration Measurement and Allowable Values
1996	IEC	IEC 60034-14	Rotating Electrical Machines, Part 14: Mechanical Vibrations of Certain Machines with Shaft Heights 56 mm and Higher — Measurement, Evaluation and Limits of Vibration
1964	IRD	IRD #305D	General Machinery Vibration Severity Chart
1995–2001	ISO		Mechanical Vibration — Evaluation of Machine Vibration by Measurements on Nonrotating Parts:
		ISO 10816-1:1995	Part 1: General Guidelines
		ISO 10816-2:2001	Part 2: Land-Based Steam Turbines and Generators in Excess of 50 MW with Normal Operating Speeds of 1500, 1800, 3000 and 3600 rpm
		ISO 10816-3:1998	Part 3: Industrial Machines with Nominal Power above 15 kW and Nominal Speeds between 120 and 15,000 rpm when Measured <i>In Situ</i>
		ISO 10816-4:1998	Part 4: Gas Turbine Driven Sets Excluding Aircraft Derivations

TABLE 9.5 (continued)

Year	Author/ Organization	Reference Number	Title/Description
2002	ISO	ISO 10816-5: 2000	Part 5: Machine Sets in Hydraulic Power Generating and Pumping Plants
			Mechanical Vibration — Vibration of Active Magnetic Bearing Equipped Rotating Machinery
		ISO 14839-1: 2002	Part 1: Vocabulary
1996–2001	ISO	ISO/CD 14839-2:2004	Part 2: Evaluation of Vibration
			Mechanical Vibrations of Nonreciprocating Machines — Measurement on Rotating Shafts and Evaluation Criteria
		ISO 7919-1: 1996	Part 1 (1996): General Guidelines
		ISO 7919-2: 2001	Part 2 (2001): Land-Based Steam Turbines and Generators in Excess of 50 MW with Normal Operating Speeds of 1500, 1800, 3000 and 3600 rpm
		ISO 7919-3: 1996	Part 3 (1996): Coupled Industrial Machines
		ISO 7919-4: 1996	Part 4 (1996): Gas Turbine Sets
		ISO 7919-5: 1997	Part 5 (1997): Machine Sets in Hydraulic Power Generating and Pumping Plants
1993	ISO	ISO 8579-2	Acceptance Code for Gears, Part 2: Determination of Mechanical Vibration of Gear Units During Acceptance Testing
2004	ISO		Rolling Bearings — Measuring Methods for Vibration
		ISO 15242-1:2004	Part 1: Fundamentals
		ISO 15242-2:2004	Part 2: Radial Ball Bearings with Cylindrical Bore and Outside surface
		ISO/CD 15242-3	Part 3: Spherical and Taper Radial Roller Bearings with Cylindrical Bore and Outside Diameter
1959	Kruglov, N.V.	Teplonerg, 8 (85), 1959	Turbomachine Vibration Standards
1967	Maten, S	Hydrocarbon Processing, January 1967	New Vibration Velocity Standards
1983	McHugh, J.D.	J. Lub. Tech., Trans. ASME, 1983, 105	Estimating the Severity of Shaft Vibration within Fluid Film Journal Bearings
1974	MIL	MIL-STD-167-1	Mechanical Vibration of Shipboard Equipment, Type I: Environmental, Type II: Internally Excited
2003	NEMA	NEMA MG 1-2003	Motors and Generators, Part 7 — Mechanical Vibration — Measurement, Evaluation and Limits
1991	NEMA	NEMA SM 23-1991	Steam Turbines for Mechanical Drive Service
1991	NEMA	NEMA SM 24-1991	Land Based Steam Turbine Generator Sets 0 to 33,000 kW
1965	PKN	PN-65/E-04255	Measurement of Vibration of Electrical Rotating Machines
1939	Rathbone, T.C.	Power Plant Engineering, November 1939	Vibration Tolerances
1964	VDI	VDI 2056	Evaluation Criteria for Mechanical Vibrations in Machines
1982	VDI	VDI 2059 P1	Shaft Vibrations of Turbosets Principles for Measurement and Evaluation
1990	VDI	VDI 2059 P2	Shaft Vibrations of Steam Turbosets for Power Station Measurement and Evaluation
1985	VDI	VDI 2059 P3	Shaft Vibrations of Industrial Turbosets Measurement and Evaluation
1981	VDI	VDI 2059 P4	Shaft Vibrations of Gas Turbosets Measurement and Evaluation
1982	VDI	VDI 2059 P5	Shaft Vibrations of Hydraulic Machinesets Measurement and Evaluation
1949	Yates, H.G.	Trans. N.E. Coast Inst. Engrs Ship Builders, Vol. 65, 1949	Vibration Diagnosis of Marine Geared Turbines

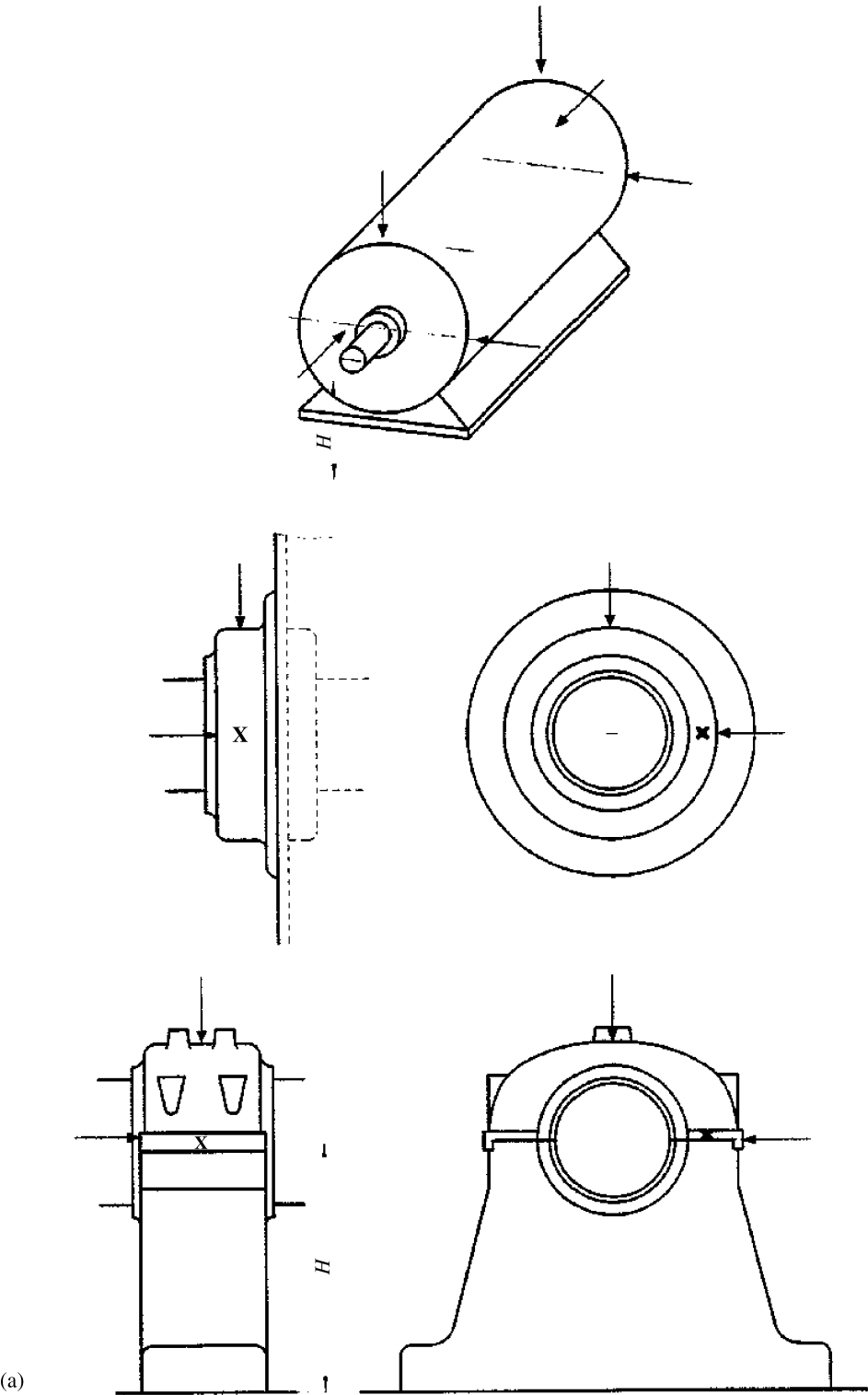


FIGURE 9.14 (a) Measuring points; (b) measuring points for vertical machine sets. (Source: ISO 10816-3, 1998-05-15. With permission.)

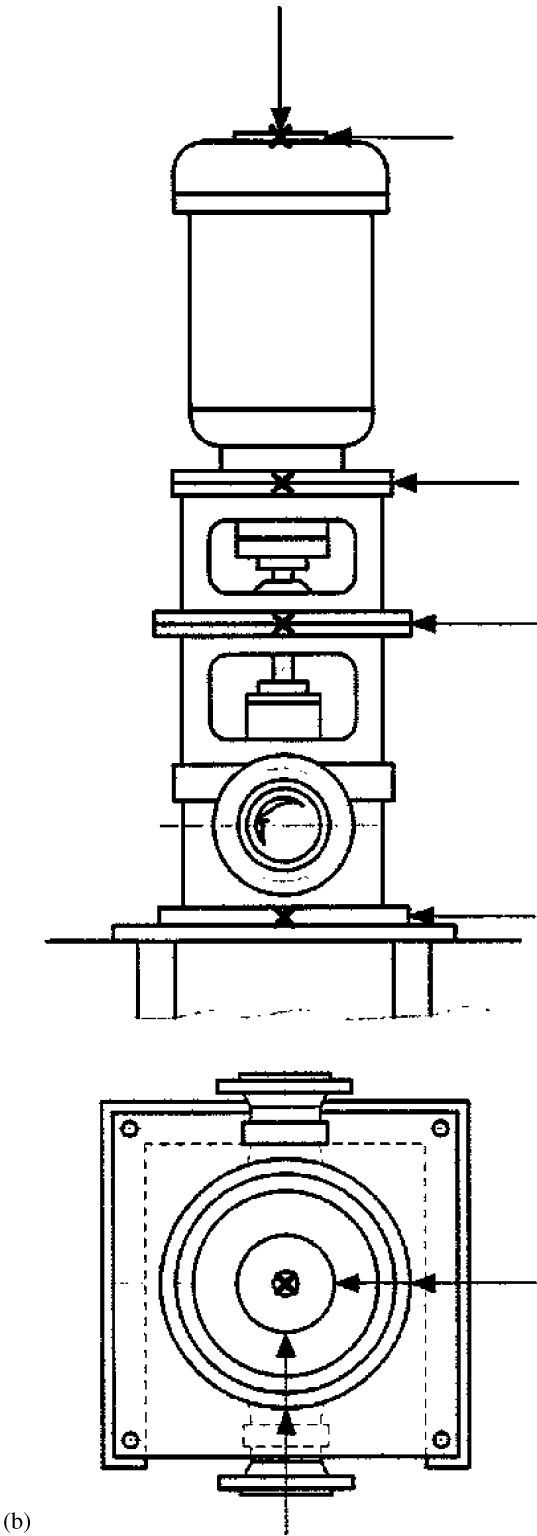


FIGURE 9.14 (continued)

TABLE 9.6A Acceptable Vibration Levels for Rotating Machinery Measured on Nonrotating Parts

Machinery Type	Power Level	Speed Range (rpm)	Applicable Vibration Level	
			Rigid Support	Flexible Support
Steam turbines	$15 \leq P \leq 300 \text{ kW}$	$120 \leq N \leq 15,000$	V1 and D3	V3 and D7
	$300 \text{ kW} \leq P \leq 50 \text{ MW}$	$120 \leq N \leq 15,000$	V3 and D5	V6 and D8
	$P > 50 \text{ MW}$	$N < 1,500$ or $N > 3,600$	V3 and D5	V6 and D8
	$P > 50 \text{ MW}$	$N = 1,500$ or $1,800$	V5	V5
	$P > 50 \text{ MW}$	$N = 3,000$ or $3,600$	V7	V7
Gas turbines	$15 \leq P \leq 300 \text{ kW}$	$120 \leq N \leq 15,000$	V1 and D3	V3 and D7
	$300 \text{ kW} \leq P \leq 3 \text{ MW}$	$120 \leq N \leq 15,000$	V3 and D5	V6 and D8
	$P > 3 \text{ MW}$	$3,000 \leq N \leq 20,000$	V8	V8
Hydraulic turbines and pump turbine	Horizontal machines			
	$P > 1 \text{ MW}$	$60 \leq N \leq 300$	N/A	V4
	$P > 1 \text{ MW}$	$300 < N \leq 1,800$	V2 and D6	N/A
Vertical machines	$P > 1 \text{ MW}$	$60 < N \leq 1,800$	V2 and D6	N/A
	$P > 1 \text{ MW}$	$60 < N \leq 1,000$	V2 and D6	V4 and D9
Centrifugal pumps				
Separate driver	$P > 15 \text{ kW}$	$120 \leq N \leq 15,000$	V3 and D2	V6 and D4
Integral driver	$P > 15 \text{ kW}$	$120 \leq N \leq 15,000$	V1 and D1	V3 and D2
Electric motors				
Shaft height $H \geq 315 \text{ mm}$	$P > 15 \text{ kW}$	$120 \leq N \leq 15,000$	V3 and D5	V6 and D8
Shaft height $160 \leq H < 315 \text{ mm}$	$P > 15 \text{ kW}$	$120 \leq N \leq 15,000$	V1 and D3	V3 and D7
Generators, excluding those used in hydraulic power generation	$15 \leq P \leq 300 \text{ kW}$	$120 \leq N \leq 15,000$	V1 and D3	V3 and D7
	$300 \text{ kW} \leq P \leq 50 \text{ MW}$	$120 \leq N \leq 15,000$	V3 and D5	V6 and D8
	$P > 50 \text{ MW}$	$N < 1,500$ or $N > 3,600$	V3 and D5	V6 and D8
	$P > 50 \text{ MW}$	$N = 1,500$ or $1,800$	V5	V5
	$P > 50 \text{ MW}$	$N = 3,000$ or $3,600$	V7	V7
Generators and motors used in hydraulic power generation	Horizontal machines			
	$P > 1 \text{ MW}$	$60 \leq N \leq 300$	N/A	V4
	$P > 1 \text{ MW}$	$300 < N \leq 1,800$	V2 and D6	N/A
Vertical machines	$P > 1 \text{ MW}$	$60 < N \leq 1,800$	V2 and D6	N/A
	$P > 1 \text{ MW}$	$60 < N \leq 1,000$	V2 and D6	V4 and D9
Compressors, rotary, blowers, and fans	$15 \leq P \leq 300 \text{ kW}$	$120 \leq N \leq 15,000$	V1 and D3	V3 and D7
	$300 \text{ kW} \leq P \leq 50 \text{ MW}$	$120 \leq N \leq 15,000$	V3 and D5	V6 and D8

resulting in wasted energy and premature failure of components due to high vibration. The current practice to obtain flow changes in the pump is by means of speed change. This eliminates flow throttling and allows the pump to operate close to its best efficiency point, where energy is not wasted and vibrations are a minimum. However, as illustrated below, variable speed operation of a pump-motor set over a wide speed range could pose several challenging problems.

TABLE 9.6B Maximum Vibration Velocity Limits for Different Levels (mm/sec, RMS)

Vibration Level	Zone A	Zone B	Zone C	Alarm	Trip
V1	1.4	2.8	4.5	3.5	5.6
V2	1.6	2.5	4.0	3.1	5.0
V3	2.3	4.5	7.1	5.6	8.9
V4	2.5	4.0	6.4	5.0	8.0
V5	2.8	5.3	8.5	6.6	10.6
V6	3.5	7.1	11.0	8.9	13.8
V7	3.8	7.5	11.8	9.4	14.8
V8	4.5	9.3	14.7	11.6	18.4

TABLE 9.6C Maximum Vibration Displacement Limits for Different Levels (μm, RMS)

Vibration Level	Zone A	Zone B	Zone C	Alarm	Trip
D1	11	22	36	28	45
D2	18	36	56	45	70
D3	22	45	71	56	89
D4	28	56	90	70	113
D5	29	57	90	71	113
D6	30	50	80	63	100
D7	37	71	113	89	141
D8	45	90	140	113	175
D9	65	100	160	125	200

Zone A: Newly commissioned machines should fall within this zone. *Zone B:* Machines with vibrations within this zone are considered acceptable for long-term operation. *Zone C:* Machines with vibrations within this zone are normally considered unsatisfactory for long-term operation. Such a machine may be operated for a short period in this condition. *Alarm:* The values chosen will normally be set relative to a baseline value determined from experience. However, it is recommended that the alarm value shall not exceed those given herein. *Trip:* The values will generally relate to the mechanical integrity of the machine. They will generally be the same for all machines with similar design. It is recommended that the trip value shall not exceed those given herein. *Notes:* (1) The measured vibration is broadband, and the frequency range will depend on the type of machine being considered. A range from 2 to 1000 Hz is typical except for in high-speed machines, >10,000 rpm, where the upper limit should at least be six times the rotational frequency. (2) It is common practice to evaluate rotating machinery based on the broadband RMS vibration velocity, since it can be related to the vibration energy levels. However, other quantities such as vibration displacement or acceleration may be preferred. Especially low speed machines can have unacceptably large vibration displacements when the 1 × rpm component is dominant. Therefore, where specified, both the velocity and displacement criteria are met. (3) Since typical vibration waveforms measured on rotating machinery are complex in nature, there is no simple relationship between broadband velocity, displacement, and acceleration. (4) Vibration measurements shall be taken on bearing support housings, or other structural components, which adequately respond to the dynamic forces of the machine. Recommended locations for bearing housings are shown in Figure 9.14. (5) For certain types of machines, the axial vibration limits may differ from those for radial directions. Also, within the same machine set, in particular hydraulic power-generating sets, the applicable level may differ from bearing to bearing depending on its classification as a rigid or flexible support. (6) Above vibration limits apply to steady-state/normal operating conditions of the machine. If the vibration levels are sensitive to the operational conditions, then evaluation of the machine for operating conditions outside steady-state conditions will have to be based on different criteria. (7) The vibration limits specified herein should not be used to assess the condition of rolling element type bearings although it encompasses machines that may have these types of bearings. (8) It must be recognized that the vibration measurement on nonrotating parts alone does not form the only basis for judging the condition of a machine. In certain types of machines, it is common practice also to judge the vibration based on measurements taken on rotating shafts. (9) A support may be considered as rigid in a specific direction only if its natural frequency in that direction exceeds the main excitation frequency by at least 25%, otherwise it is considered to be flexible. In some cases, a support may be rigid in one direction and flexible in another. (10) In the case of hydraulic machine sets, major differences in radial bearing support arrangement can occur. For evaluation of the support type it is recommended that the reader refer to ISO 10816-5.

Description. The following case study is taken from a petroleum pipeline pump application where pump-motor sets with VFDs were installed in a new pipeline starting in Alberta, Canada and terminating in Minnesota, USA. VFDs are frequently used in the pipeline industry to power high horsepower pumps to eliminate power wasted by throttling, reduce inrush current at motor startup, and to provide greater operating flexibility. However, variable speed operation can cause vibration problems in the pump, motor, and the couplings that are not normally experienced with fixed speed pumps. Unexpected high torsional and lateral vibrations were experienced with these pumps and motors at certain operating speeds. A rotor torsional resonance, motor housing resonance, acoustic resonance in the internals of the pump, and discharge piping were identified to be the causes of the high vibration in the pump-motor set. Details on diagnosing the problems and the corrective measures taken to resolve them are given below:

Pump type: The pump was a centrifugal, two-stage, double volute horizontal pump, with six vane impellers, normally designed to operate at a fixed speed. Generation of pressure pulsations at the vane passing frequencies of 6 × and 12 × rotational speed is normally expected.

TABLE 9.7A Acceptable Vibration Levels for Rotating Machinery, Measured on Rotating Shafts

Machinery Type	Power Level	Speed Range (RPM)	Applicable Vibration Level	
			Relative Displacement	Absolute Displacement
Steam turbines	$P \leq 50 \text{ MW}$	$1,000 \leq N \leq 30,000$	D8	—
	$P > 50 \text{ MW}$	$N = 1,500$	D5	D7
	$P > 50 \text{ MW}$	$N = 1,800$	D4	D6
	$P > 50 \text{ MW}$	$N = 3,000$	D2	D5
	$P > 50 \text{ MW}$	$N = 3,600$	D1	D3
Gas turbines	$P > 3 \text{ MW}$	$3,000 \leq N \leq 30,000$	D8	—
	$P \leq 3 \text{ MW}$	$1,000 \leq N \leq 30,000$	D8	—
Hydraulic turbines and pumps used in hydraulic power generation and pumping plants	$P > 1 \text{ MW}$	$60 \leq N \leq 1,800$	D9	D9
Centrifugal pumps	All	$1,000 \leq N \leq 30,000$	D8	—
Electric motors	All	$1,000 \leq N \leq 30,000$	D8	—
Generators, excluding those used in hydraulic power generation	$P \leq 50 \text{ MW}$	$1,000 \leq N \leq 30,000$	D8	—
	$P > 50 \text{ MW}$	$N = 1,500$	D5	D7
	$P > 50 \text{ MW}$	$N = 1,800$	D4	D6
	$P > 50 \text{ MW}$	$N = 3,000$	D2	D5
	$P > 50 \text{ MW}$	$N = 3,600$	D1	D3
Generators and motors used in hydraulic power generation	$P > 1 \text{ MW}$	$60 \leq N \leq 1,000$	D9	D9
	$P > 1 \text{ MW}$	$1,000 < N \leq 1,800$	D8	—
Compressors, rotary, blowers, and fans	All	$1,000 \leq N \leq 30,000$	D8	—

Motor: The motor was a 3000 hp, two pole horizontal induction motor, designed to operate at 3600 rpm.

Supply: The supply was a VFD of the current source inverter type. These drives are known to generate an oscillatory torque at $6 \times$ and $12 \times$ the operating frequency.

Coupling: Flexible disc type coupling with a spacer was used. These couplings have very little torsional damping capacity.

Speed range: The speed range was from 1440 rpm (24 Hz) to 3900 rpm (65 Hz).

Reference: Refer to Figure 9.15 to Figure 9.18.

As for the resonance at second and third torsional critical speeds (Figure 9.15) the second and third torsional modes are excited when the $6 \times$ component of rotational speed corresponds to the critical speeds of 92 and 268 Hz, respectively. The $6 \times$ rpm torsional excitation is caused by the pressure pulsations in the pump. The waterfall plot (Figure 9.15a) was taken during a run down of the set with the power to the motor turned off.

A similar plot taken during run up of the motor (Figure 9.15b) shows excitations at the same frequencies but having different amplitude. Since both the pump and motor generate $6 \times$ excitation, it suggests a phase difference between the excitation torques.

It is important to note that the conventional vibration monitoring devices cannot detect the torsional resonance problem. The only indication of a problem was the unusual chattering noise emitted by the coupling. Special techniques to measure dynamic torque using strain gauges had to be used to detect the torsional vibrations.

As for the motor housing resonance (Figure 9.16), the $2 \times$ rotational speed vibration of the motor is dominant and peaks at 118 Hz corresponding to a natural frequency of the motor frame. In the waterfall plot, the natural frequencies corresponds to excitations that are parallel to the axis. Excitation at harmonics, including the $6 \times$ component, is present but is not dominant.

As for the pump vibrations at the vane passing frequency (Figure 9.17), the $6 \times$ vane passing frequency is dominant at all operating speeds. It peaks at 238 Hz, possibly due to an acoustic resonance in

TABLE 9.7B Maximum Vibration Displacement S_{p-p} Limits (μm) Peak-to-Peak Limits for Different Levels

	Zone A	Zone B	Zone C
D1	75	150	240
D2	80	165	260
D3	90	180	290
D4	90	185	290
D5	100	200	320
D6	110	220	350
D7	120	240	385
D8	$4800/\sqrt{n}$	$9000/\sqrt{n}$	$13,200/\sqrt{n}$
D9	$10^{(2.3381-0.0704 \log n)}$	$10^{(2.5599-0.0704 \log n)}$	$10^{(2.8609-0.0704 \log n)}$

Zone A: Newly commissioned machines should fall within this zone. *Zone B:* Machines with vibrations within this zone are considered acceptable for long-term operation. *Zone C:* Machines with vibrations within this zone are normally considered unsatisfactory for long-term operation. Such a machine may be operated for a short period in this condition. *Alarm:* The values chosen will normally be set relative to a baseline value determined from experience. However, it is recommended that the alarm value shall not exceed those given herein. *Trip:* The values will generally relate to the mechanical integrity of the machine. They will generally be the same for all machines with similar design. It is recommended that the trip value shall not exceed those given herein. *Notes:* (1) The measured vibration is broadband and is shaft vibration displacement peak to peak. Where applicable, vibration limits for both absolute and relative radial shaft vibrations are given in certain cases. (2) Relative displacement is the vibratory displacement between the shaft and an appropriate structural component such as the bearing housing. Absolute displacement is the vibratory displacement of the shaft with reference to an inertial frame of reference. (3) Relative measurements are carried out with a noncontacting transducer. Absolute readings are obtained by one of the following methods: by a shaft riding probe on which a seismic transducer is mounted so that it measures absolute shaft displacement directly, or with the combination of a noncontacting transducer which measures relative shaft displacement and a seismic transducer which measures support vibration. Their conditioned outputs are vectorially added to provide a measure of the absolute shaft motion. (4) The vibration evaluation criteria are dependent upon a variety of factors and the criteria adopted will vary for different types of machines. Some of these factors are the bearing type, clearance, and diameter. The adopted criteria have to be compared with the bearing diametral clearance (C) and adjusted to suit. Typical values are: Zone A $\leq 0.4C$; Zone B $\leq 0.6C$; and Zone C $\leq 0.7C$. (5) Above vibration limits apply to steady state/normal operating conditions of the machine. If the vibration levels are sensitive to the operational conditions then evaluation of the machine for operating conditions outside steady-state conditions will have to be based on different criteria. (6) It is recommended that vibration readings at each location be made with a pair of transducers and that the transducers are mounted perpendicular to the shaft axis and they are at an angle of 90° to one another. The vibration limits apply to each measured direction. (7) The mechanical and electrical run-out at each measurement location must be assessed and should be $<25\%$ of the allowable limit or $6 \mu\text{m}$, whichever is greater. (8) It must be recognized that the vibration measurement on rotating shafts does not form the only basis for judging the condition of a machine. In certain types of machine, it is common practice also to judge the vibration based on measurements taken on nonrotating parts. (9) ALARM levels should be set relative to a baseline value determined from experience for the measurement position, direction and type of machine. It must provide a warning that a defined value, which is significantly above the baseline value, has been reached. The maximum ALARM setting should be $\leq 0.75C$. (10) The TRIP values should be based on protecting the mechanical integrity of the machine. Consideration of damage to bearings is typical; therefore, maximum TRIP setting should be $\leq 0.9C$.

the discharge pipe. It does not seem to correspond to a structural natural frequency due to the absence of excitations at 238 Hz at all speeds.

As for the acoustic resonance in the pump cross-over pipe (Figure 9.18), dynamic pressure pulsation measurements made on the pump cross-over pipe from the first stage discharge to the second stage suction show an acoustic resonance at 540 Hz. The consistent presence of some excitation at 540 Hz at all speeds confirms that it is an acoustic natural frequency of the cross-over pipe. When the $6 \times \text{rpm}$ pressure pulsation frequency coincides with the acoustic natural frequency, a resonance condition occurs and the magnitude of the pressure pulsation increases by almost a factor of 30.

As for corrective action, for a pump that has to operate over a wide speed range, totally eliminating the coincidence of all the frequencies of exciting forces with the system natural frequencies is impractical. Therefore, the system has to be designed such that the resulting magnitudes of the forces are controlled to within tolerable levels so that safe and reliable operation can take place. This can be accomplished by a

TABLE 9.8 Vibration Cause Identification

Cause	Dominant Frequency	Spectrum, Time Domain, Orbit Shape	Characteristics, Corrections, Comments
Mass unbalance	$1 \times$	High $1 \times$ with much lower harmonics; circular or elliptic orbits	Corrected by shop or field balancing
Shaft bow	$1 \times$	Run down plot shows decrease of vibration at critical speed	The shaft has to be straightened using an acceptable method
Misalignment	$1 \times$ and $2 \times$	Equally high $1 \times$ and $2 \times$, figure 8 orbits	Realign at operating conditions; loads causing misalignment, such as nozzle loads, may have to be reduced
Worn journal bearings	$1 \times$, $1/2 \times$	Equally high $1 \times$ and $1/2 \times$	Difficult to balance
Gravity critical	$2 \times$	Run down plot will show excitation at $1/2$ critical speed	Can be corrected by balancing
Asymmetric shaft	$2 \times$	Run down plot will show excitation at $1/2$ critical speed	Typically occurs on multistage machines when all the keyways lie in the same plane; correct by staggering them
Shaft crack	$1 \times$ and $2 \times$	High $1 \times$ and run down plots may show excitation at $1/2$ critical speed	Confirmation and detection of location of the crack may require NDE techniques
Loose components	$1 \times$ and higher orders plus fractional subharmonics	High $1 \times$ with lower level orders and fractional subharmonics	Shimming and peening may be used as temporary methods to fix the problem
Coupling lockup	$1 \times$ and $2 \times$	Equally high $1 \times$ and $2 \times$, figure 8 orbits	Stop starts may change vibration pattern
Thermal instability	$1 \times$	High $1 \times$ varies with temperature. Phase angle may change	Proper prewarming or compromise balancing can correct the problem
Oil whirl	$< 1/2 \times$, typically $0.35 \times$ to $0.47 \times$	Run-up plot will show $1/2 \times$ increasing and locking into fixed value $< 1/2 \times$	Temporary problem may be caused by excess clearances, oil viscosity, or unloading of the bearing; if it is a design problem, correct by changing to tilting pad bearings
Internal rubs	$1/4 \times$, $1/3 \times$, $1/2 \times$, $2 \times$, $3 \times$, $4 \times$, etc.	Run down plots may show decreasing amplitudes and disappearance; loops in orbits	May get progressively worse; galling between contact surfaces or heat build-up may cause seizure and shaft failure
Trapped fluids in rotor	$0.8 \times$ to $0.9 \times$	Time domain signal will show beating	Balancing the rotor may reduce the vibration
Defective rolling element bearings	At bearing defect frequency	Peaks at defect frequencies in spectrum	Shock pulse measurements can also be used to detect problem
Damaged gears	Gear mesh frequency	High peaks at gear mesh frequency with side bands. Time domain may also show pulses	To determine exact nature of damage further analysis may be required

TABLE 9.8 (continued)

Cause	Dominant Frequency	Spectrum, Time Domain, Orbit Shape	Characteristics, Corrections, Comments
Electric motor problems	1 × (line frequency), 2 × (line frequency)	High peaks at 1 × and 2 × line frequency with side bands; disappears when power to motor is turned off	In the case of two pole motors, it can be confused with mechanical causes as the rotational speed is the same as line frequency
Casing distortion	1 ×	High 1 ×, may change with time	Caused by high nozzle loads, casing not free to expand, soft foot or foundation distortion
Piping forces	1 ×, 2 ×	Equally high 1 × and 2 ×	Causes misalignment between bearings or between coupled equipment
Rotor and bearing critical	1 ×	High 1 ×, on rundown plot 1 × decreases rapidly, may also show a large phase angle change	More common in machines originally designed for fixed speed operation, later converted to variable speed operation
Structural resonance	1 ×, 2 ×	High 1 × and some 2 ×; can be easily identified on run down plot	Increase or decrease stiffness of structure or add or remove mass to change natural frequency
Rotor hysteresis	0.65 × to 0.85 ×	Spectrum will show high magnitudes at 0.65 × to 0.85 ×	Occurs in built up rotors with transitional fits
Hydraulic causes	1 × (vane pass frequency), 2 × (vane pass frequency)	High 1 × and 2 × vane pass frequency	Common in centrifugal pumps due to flow recirculation or inadequate gap between impeller and casing

direct reduction of the exciting force or by means of increased damping. Based on these guidelines, the following modifications were proposed to correct the problem:

1. Torsional resonance
 - Use an electrometric type coupling that has a high degree of torsional damping to reduce the magnitude of the torsional excitation forces such that the torsional stresses within the rotors are within acceptable limits.
 - Since both the pump and VFD generate excitation at 6 × rpm, their effects could be compounding one another. Introducing either five vane or seven vane impellers into the pump will eliminate this possibility.
 - Additional filters could be introduced into the VFD to reduce the 6 × and 12 × component periodic torsional excitation.
 - Consider not operating (lock out) the pump within ±10% of the frequency at which torsional resonance occurs.
2. Motor housing resonance at 2 ×
 - Although the 2 × vibration is dominant, its magnitude is within tolerable levels. The fact that some 2 × vibration is also present in the pump indicates that the 2 × vibration is perhaps caused by misalignment between the pump and the motor. This can be corrected by proper

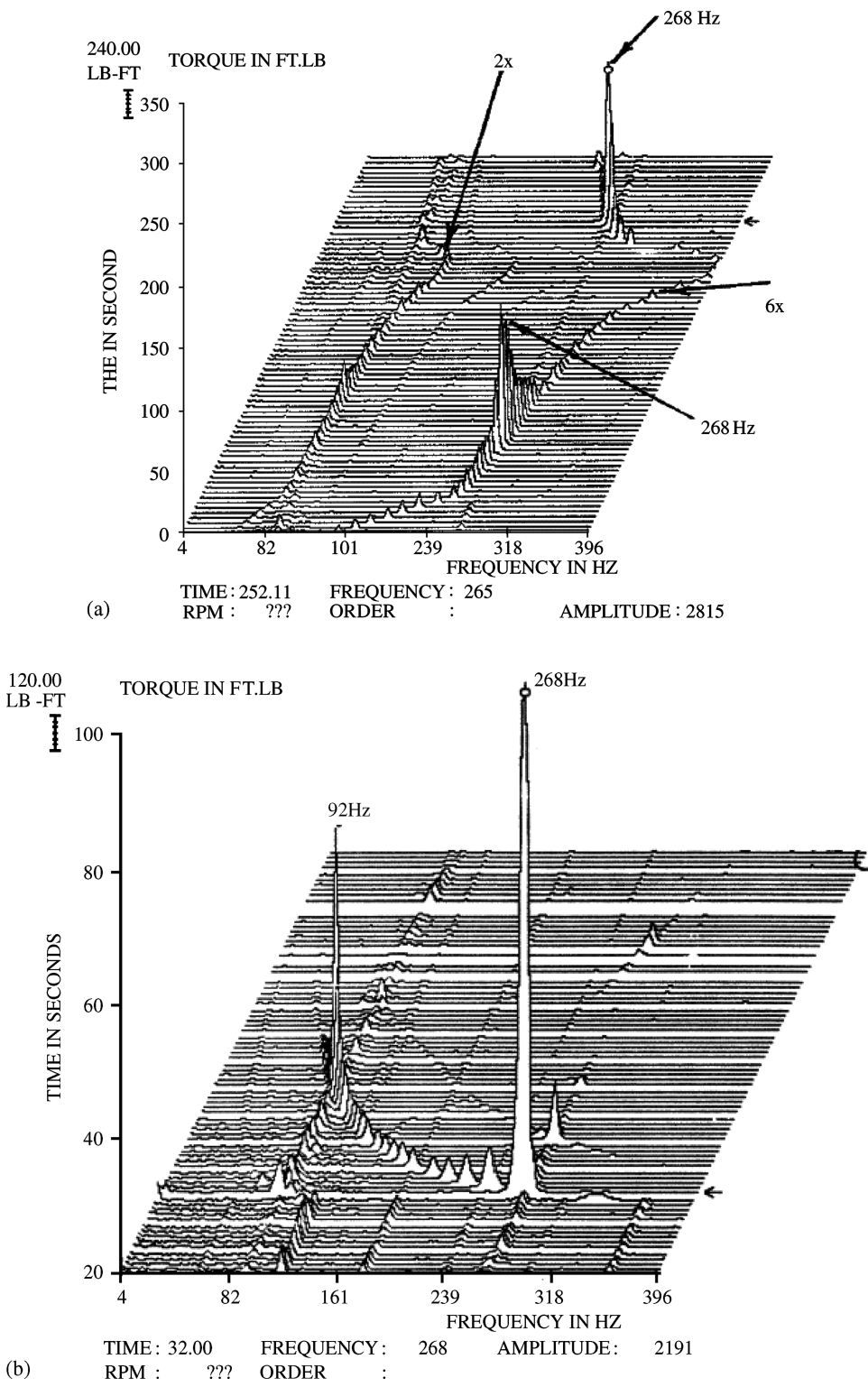


FIGURE 9.15 (a) Torsional resonance run-up and run-down plots; (b) torsional resonance run-down plot. (Source: Private communice, Insight Engineering Services Ltd. Alta., Canada. With permission.)

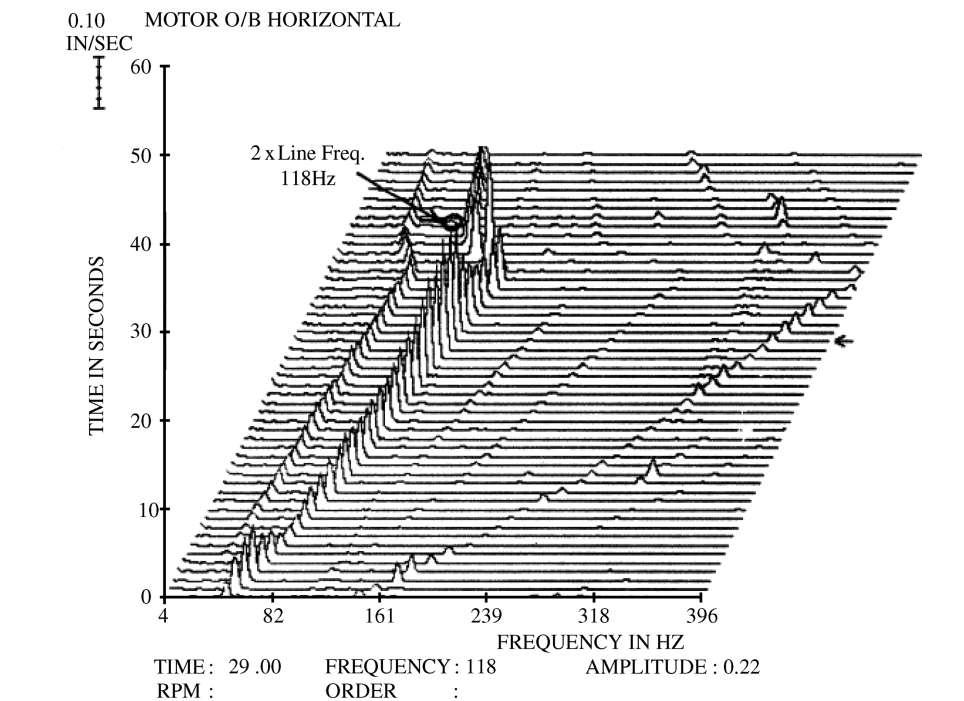


FIGURE 9.16 Motor frame resonance. (Source: Private communique, Insight Engineering Services Ltd., Alta., Canada. With permission.)

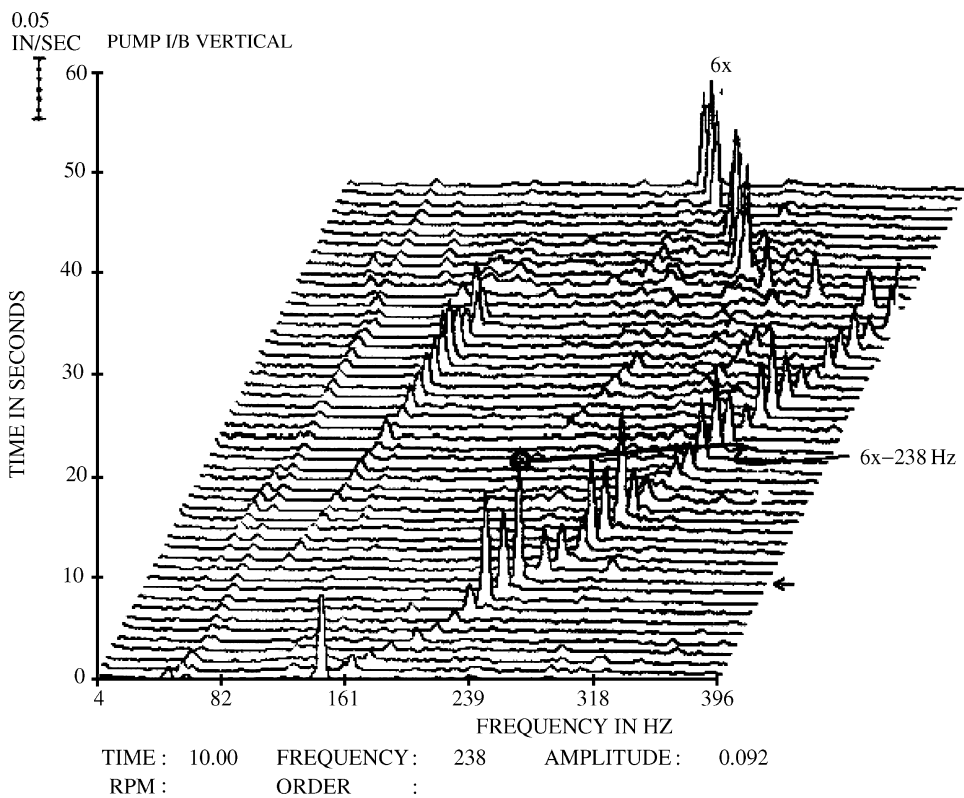


FIGURE 9.17 Pump bearing housing resonance. (Source: Private communique, Insight Engineering Services Ltd., Alta., Canada. With permission.)

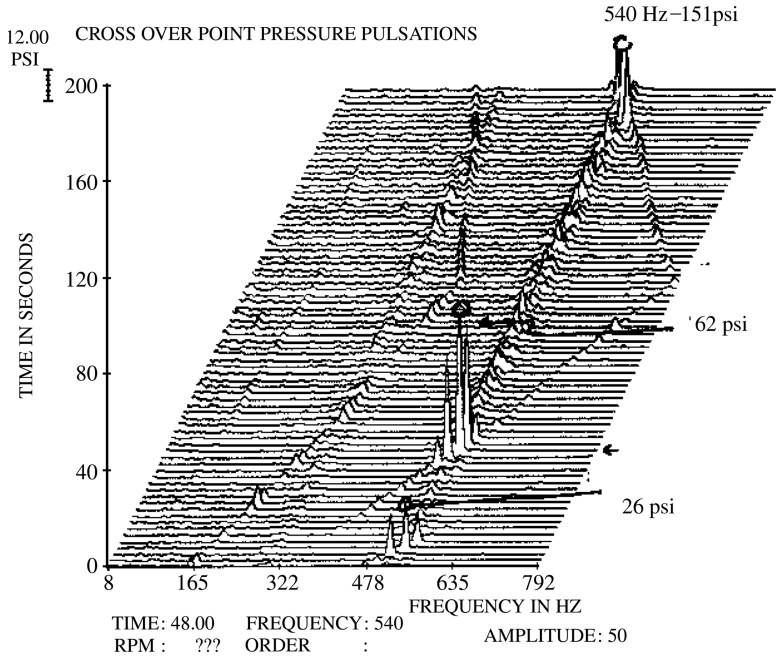


FIGURE 9.18 Pump cross-over pipe acoustic resonance. (Source: Private communique, Insight Engineering Services Ltd., Alta., Canada. With permission.)

alignment and thus reducing the $2 \times$ excitation forces. In some cases, due to an unequal air gap between the rotor and stator of the motor, the motor could generate the $2 \times$ vibration. Under such conditions, accurate centering of the motor bearings will generally correct the problem.

- 3. Pump vibrations at vane passing frequency
 - Generally, high vibrations at vane passing frequency are caused by pressure pulsations generated at the discharge of the impeller. There are several hydraulic modifications that can be made to the pump to reduce the amplitude of these pulsations that occur at vane passing frequency. The most common method is to increase the gap between the impeller discharge vanes and diffuser/volute. Also, changing the ratio of the number of impeller vanes to diffuser/volute vanes can help in reducing vane passing frequency pressure pulsations and the resulting vibration.
- 4. Acoustic resonance in the pump cross-over pipe
 - Once the pump is constructed, it is not possible to change the acoustic natural frequency of the cross-over pipe. However, the excitation force, pressure pulsations generated at the impeller discharge, can be reduced by the methods outlined above.

- The root cause of a vibration problem in a rotating machine can be determined by careful study and analysis of the vibration signals.
 - Industrial and international vibration standards and guidelines have been developed to ensure safe and reliable operation of rotating machinery.
 - Equipment manufacturers, users, insurance companies, and public interest groups use vibration standards to control vibration to within acceptable levels.

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