CHAPTER 5

MASS TRANSFER

MASS transfer by either molecular diffusion or convection is the transport of one component of a mixture relative to the motion of the mixture and is the result of a **concentration gradient**. In an air-conditioning process, water vapor is added or removed from the air by a simultaneous transfer of heat and mass (water vapor) between the airstream and a wetted surface. The wetted surface can be water droplets in an air washer, wetted slats of a cooling tower, condensate on the surface of a dehumidifying coil, surface presented by a spray of liquid absorbent, or wetted surfaces of an evaporative condenser. The performance of equipment with these phenomena must be calculated carefully because of the simultaneous heat and mass transfer.

This chapter addresses the principles of mass transfer and provides methods of solving a simultaneous heat and mass transfer problem involving air and water vapor. Emphasis is on air-conditioning processes involving mass transfer. The formulations presented can help in analyzing the performance of specific equipment. For a discussion on the performance of air washers, cooling coils, evaporative condensers, and cooling towers, see Chapters 19, 21, 35, and 36, respectively, of the *ASHRAE Handbook—Systems and Equipment*.

This chapter is divided into (1) the principles of molecular diffusion, (2) a discussion on the convection of mass, and (3) simultaneous heat and mass transfer and its application to specific equipment.

MOLECULAR DIFFUSION

Most mass transfer problems can be analyzed by considering the diffusion of a gas into a second gas, a liquid, or a solid. In this chapter, the diffusing or dilute component is designated as component B, and the other component as component A. For example, when water vapor diffuses into air, the water vapor is component B and dry air is component A. Properties with subscripts A or B are local properties of that component. Properties without subscripts are local properties of the mixture.

The primary mechanism of mass diffusion at ordinary temperature and pressure conditions is **molecular diffusion**, a result of density gradient. In a binary gas mixture, the presence of a concentration gradient causes transport of matter by molecular diffusion; that is, because of random molecular motion, gas B diffuses through the mixture of gases A and B in a direction that reduces the concentration gradient.

Fick's Law

The basic equation for molecular diffusion is Fick's law. Expressing the concentration of component B of a binary mixture of components A and B in terms of the mass fraction ρ_B/ρ or mole fraction C_B/C, Fick's law is

$$J_B = -\rho D_v \frac{d(\rho_B/\rho)}{di} = -J_A \tag{1a}$$

$$J_B^* = -CD_v \frac{d(C_B/C)}{di} = -J_A^* \tag{1b}$$

where $\rho = \rho_A + \rho_B$ and $C = C_A + C_B$.

The minus sign indicates that the concentration gradient is negative in the direction of diffusion. The proportionality factor D_v is the **mass diffusivity** or the **diffusion coefficient**. The total mass flux $\dot{m}_B''$ and molar flux $\dot{m}_B''^*$ are due to the average velocity of the mixture plus the diffusive flux:

$$\dot{m}_B'' = \rho_B v - \rho D_v \frac{d(\rho_B/\rho)}{dy} \tag{2a}$$

$$\dot{m}_B''^* = C_B v^* - CD_v \frac{d(C_B/C)}{dy} \tag{2b}$$

where v is the mass average velocity of the mixture and v^* is the molar average velocity.

Bird et al. (1960) present an analysis of Equations (1a) and (1b). Equations (1a) and (1b) are equivalent forms of Fick's law. The equation used depends on the problem and individual preference. This chapter emphasizes mass analysis rather than molar analysis. However, all results can be converted to the molar form using the relation $C_B \equiv \rho_B/M_B$.

Fick's Law for Dilute Mixtures

In many mass diffusion problems, component B is dilute; the density of component B is small compared to the density of the mixture. In this case, Equation (1a) can be written as

$$J_B = -D_v \frac{d\rho_B}{dy} \tag{3}$$

when $\rho_B << \rho$ and $\rho_A \approx \rho$.

Equation (3) can be used without significant error for water vapor diffusing through air at atmospheric pressure and a temperature less than 80°F. In this case, $\rho_B < 0.02\rho$, where ρ_B is the density of water vapor and ρ is the density of moist air (air and water vapor mixture). The error in J_B caused by replacing $\rho[d(\rho_B/\rho)/dy]$ with $d\rho_B/dy$ is less than 2%. At temperatures below 140°F where $\rho_B < 0.10\rho$, Equation (3) can still be used if errors in J_B as great as 10% are tolerable.

Fick's Law for Mass Diffusion Through Solids or Stagnant Fluids (Stationary Media)

Fick's law can be simplified for cases of dilute mass diffusion in solids, stagnant liquids, or stagnant gases. In these cases, $\rho_B \ll \rho$ and $v \approx 0$, which yields the following approximate result:

$$\dot{m}_B'' = J_B = -D_v \frac{d\rho_B}{dy} \tag{4}$$

The preparation of this chapter is assigned to TC 1.3, Heat Transfer and Fluid Flow.

Fick's Law for Ideal Gases with Negligible Temperature Gradient

For cases of dilute mass diffusion, Fick's law can be written in terms of partial pressure gradient instead of concentration gradient. When gas B can be approximated as ideal,

$$p_B = \frac{\rho_B R_u T}{M_B} = C_B R_u T \tag{5}$$

and when the gradient in T is small, Equation (3) can be written as

$$J_B = -\left(\frac{M_B D_v}{R_u T}\right)\frac{dp_B}{dy} \tag{6a}$$

or

$$J_B^* = -\left(\frac{D_v}{R_u T}\right)\frac{dp_B}{dy} \tag{6b}$$

If v ≈ 0, Equation (4) may be written as

$$\dot{m}_B'' = J_B = -\left(\frac{M_B D_v}{R_u T}\right)\frac{dp_B}{dy} \tag{7a}$$

or

$$\dot{m}_B''^* = J_B^* = -\left(\frac{D_v}{R_u T}\right)\frac{dp_B}{dy} \tag{7b}$$

The partial pressure gradient formulation for mass transfer analysis has been used extensively; this is unfortunate because the pressure formulation [Equations (6) and (7)] applies only to cases where one component is dilute, the fluid closely approximates an ideal gas, and the temperature gradient has a negligible effect. The density (or concentration) gradient formulation expressed in Equations (1) through (4) is more general and can be applied to a wider range of mass transfer problems, including cases where neither component is dilute [Equation (1)]. The gases need not be ideal, nor the temperature gradient negligible. Consequently, this chapter emphasizes the density formulation.

Diffusion Coefficient

For a binary mixture, the diffusion coefficient D_v is a function of temperature, pressure, and composition. Experimental measurements of D_v for most binary mixtures are limited in range and accuracy. Table 1 gives a few experimental values for diffusion of some gases in air. For more detailed tables, see the section on Bibliography at the end of this chapter.

In the absence of data, use equations developed from (1) theory or (2) theory with constants adjusted from limited experimental data. For binary gas mixtures at low pressure, D_v is inversely proportional to pressure, increases with increasing temperature, and is almost independent of composition for a given gas pair. Bird et al. (1960) present the following equation, developed from kinetic theory and corresponding states arguments, for estimating D_v at pressures less than $0.1p_{c\,min}$:

Table 1 Mass Diffusivities for Gases in Air[a]

Gas	D_v, ft²/h
Ammonia	1.08
Benzene	0.34
Carbon dioxide	0.64
Ethanol	0.46
Hydrogen	1.60
Oxygen	0.80
Water vapor	0.99

[a]Gases at 77°F and 14.696 psi.

$$D_v = a\left(\frac{T}{\sqrt{T_{cA}+T_{cB}}}\right)^b \sqrt{\frac{1}{M_A}+\frac{1}{M_B}} \times \frac{(p_{cA}p_{cB})^{1/3}(T_{cA}T_{cB})^{5/12}}{p} \tag{8}$$

where

D_v = diffusion coefficient, ft²/h
a = constant, dimensionless
b = constant, dimensionless
T = absolute temperature, K
p = pressure, psi
M = molecular weight, lb_m/lb mol

The subscripts cA and cB refer to the critical states of the two gases. Analysis of experimental data gives the following values of the constants a and b:

For nonpolar gas pairs,

$$a = 7.266 \times 10^{-3} \text{ and } b = 1.823$$

For water vapor with a nonpolar gas,

$$a = 9.635 \times 10^{-3} \text{ and } b = 2.334$$

A **nonpolar gas** is one for which the intermolecular forces are independent of the relative orientation of molecules, depending only on the separation distance from each other. Air, composed of nonpolar gases O_2 and N_2, is nonpolar.

Equation (8) is stated to agree with experimental data at atmospheric pressure to within about 8% (Bird et al. 1960).

The mass diffusivity D_v for binary mixtures at low pressure is predictable within about 10% by kinetic theory (Reid et al. 1987).

$$D_v = 0.0072\frac{T^{1.5}}{p(\sigma_{AB})^2\Omega_{D,AB}}\sqrt{\frac{1}{M_A}+\frac{1}{M_B}} \tag{9}$$

where

σ_{AB} = characteristic molecular diameter, nm
$\Omega_{D,AB}$ = temperature function, dimensionless

D_v is in ft²/h, p in atmospheres, and T in kelvins. If the gas molecules of A and B are considered rigid spheres having diameters σ_A and σ_B [and $\sigma_{AB} = (\sigma_A/2) + (\sigma_B/2)$], all expressed in nanometers, the dimensionless function $\Omega_{D,AB}$ equals unity. More realistic models for the molecules having intermolecular forces of attraction and repulsion lead to values of $\Omega_{D,AB}$ that are functions of temperature. Reid et al. (1987) present tabulations of this quantity. These results show that D_v increases as the 2.0 power of T at low temperatures and as the 1.65 power of T at very high temperatures.

The diffusion coefficient of moist air has been calculated for Equation (8) using a simplified intermolecular potential field function for water vapor and air (Mason and Monchick 1965).

The following is an empirical equation for mass diffusivity of water vapor in air up to 2000°F (Sherwood and Pigford 1952):

$$D_v = \frac{0.00215}{p}\left(\frac{T^{2.5}}{T+441}\right) \tag{10}$$

where D_v is in ft²/h, p in psi, and T in °R.

Diffusion of One Gas Through a Second Stagnant Gas

Figure 1 shows diffusion of one gas through a second stagnant gas. Water vapor diffuses from the liquid surface into surrounding stationary air. It is assumed that local equilibrium exists through the gas mixture, that the gases are ideal, and that the Gibbs-Dalton law is valid, which implies that the temperature gradient has a negligible effect. Diffusion of water vapor is due to concentration gradient and is given by Equation (6a). There is a continuous gas phase, so the mixture pressure p is constant, and the Gibbs-Dalton law yields

$$p_A + p_B = p = \text{constant} \tag{11a}$$

or

$$\frac{\rho_A}{M_A} + \frac{\rho_B}{M_B} = \frac{p}{R_u T} = \text{constant} \tag{11b}$$

The partial pressure gradient of the water vapor causes a partial pressure gradient of the air such that

$$\frac{dp_A}{dy} = -\frac{dp_B}{dy}$$

or

$$\left(\frac{1}{M_A}\right)\frac{d\rho_A}{dy} = -\left(\frac{1}{M_B}\right)\frac{d\rho_B}{dy} \tag{12}$$

Air, then, diffuses toward the liquid water interface. Because it cannot be absorbed there, a bulk velocity v of the gas mixture is established in a direction away from the liquid surface, so that the net transport of air is zero (i.e., the air is stagnant):

$$\dot{m}''_A = -D_v \frac{d\rho_A}{dy} + \rho_A v = 0 \tag{13}$$

The bulk velocity v transports not only air but also water vapor away from the interface. Therefore, the total rate of water vapor diffusion is

$$\dot{m}''_B = -D_v \frac{d\rho_B}{dy} + \rho_B v \tag{14}$$

Substituting for the velocity v from Equation (13) and using Equations (11b) and (12) gives

$$\dot{m}''_B = \left(\frac{D_v M_B p}{\rho_A R_u T}\right)\frac{d\rho_A}{dy} \tag{15}$$

Integration yields

$$\dot{m}''_B = \frac{D_v M_B p}{R_u T}\left[\frac{\ln(\rho_{AL}/\rho_{A0})}{y_L - y_0}\right] \tag{16a}$$

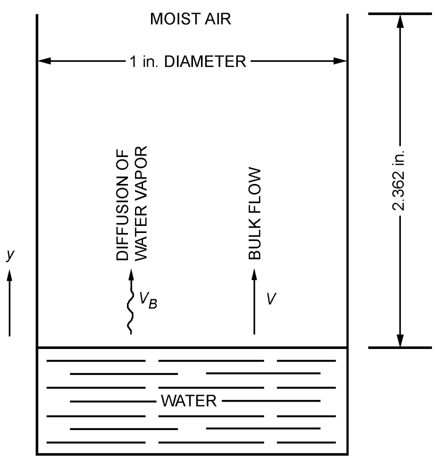

Fig. 1 Diffusion of Water Vapor Through Stagnant Air

or

$$\dot{m}''_B = -D_v P_{Am}\left(\frac{\rho_{BL} - \rho_{B0}}{y_L - y_0}\right) \tag{16b}$$

where

$$P_{Am} \equiv \frac{p}{p_{AL}}\rho_{AL}\left[\frac{\ln(\rho_{AL}/\rho_{A0})}{\rho_{AL} - \rho_{A0}}\right] \tag{17}$$

P_{Am} is the logarithmic mean density factor of the stagnant air. The pressure distribution for this type of diffusion is illustrated in Figure 2. **Stagnant** refers to the net behavior of the air; it does not move because the bulk flow exactly offsets diffusion. The term P_{Am} in Equation (16b) approximately equals unity for dilute mixtures such as water vapor in air at near atmospheric conditions. This condition makes it possible to simplify Equations (16) and implies that in the case of dilute mixtures, the partial pressure distribution curves in Figure 2 are straight lines.

Example 1. A vertical tube of 1 in. diameter is partially filled with water so that the distance from the water surface to the open end of the tube is 2.362 in., as shown in Figure 1. Perfectly dried air is blown over the open tube end, and the complete system is at a constant temperature of 59°F. In 200 h of steady operation, 0.00474 lb of water evaporates from the tube. The total pressure of the system is 14.696 psia (1 atm). Using these data, (a) calculate the mass diffusivity of water vapor in air, and (b) compare this experimental result with that from Equation (10).

Solution:

(a) The mass diffusion flux of water vapor from the water surface is

$$\dot{m}_B = 0.00474/200 = 0.0000237 \text{ lb/h}$$

The cross-sectional area of a 1 in. diameter tube is $\pi(0.5)^2/144 = 0.005454$ ft^2. Therefore, $\dot{m}''_B = 0.004345$ lb/ft^2·h. The partial densities are determined with the aid of the psychrometric tables.

$$\rho_{BL} = 0; \quad \rho_{B0} = 0.000801 \text{ lb/ft}^3$$

$$\rho_{AL} = 0.0765 \text{ lb/ft}^3; \quad \rho_{A0} = 0.0752 \text{ lb/ft}^3$$

Because $p = p_{AL} = 1$ atm, the logarithmic mean density factor [Equation (17)] is

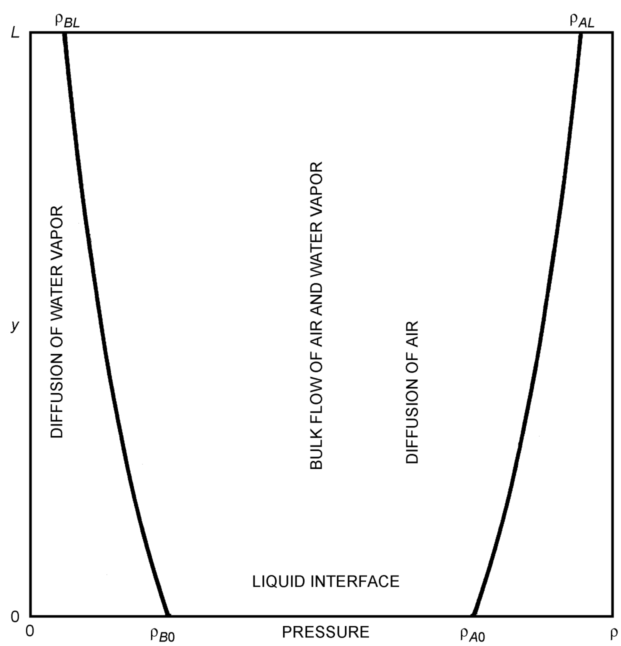

Fig. 2 Pressure Profiles for Diffusion of Water Vapor Through Stagnant Air

$$P_{Am} = 0.0765\left[\frac{\ln(0.0765/0.0752)}{0.0765 - 0.0752}\right] = 1.009$$

The mass diffusivity is now computed from Equation (16b) as

$$D_v = \frac{-\dot{m}''_B(y_L - y_0)}{P_{Am}(\rho_{BL} - \rho_{B0})} = \frac{-(0.004345)(2.362)}{(1.009)(0 - 0.000801)(12)}$$

$$= 1.058 \text{ ft}^2/\text{h}$$

(b) By Equation (10), with p = 14.696 psi and T = 59 + 460 = 519°R,

$$D_v = \frac{0.00215}{14.696}\left(\frac{519^{2.5}}{519 + 441}\right) = 0.935 \text{ ft}^2/\text{h}$$

Neglecting the correction factor P_{Am} for this example gives a difference of less than 1% between the calculated experimental and empirically predicted values of D_v.

Equimolar Counterdiffusion

Figure 3 shows two large chambers, both containing an ideal gas mixture of two components A and B (e.g., air and water vapor) at the same total pressure p and temperature T. The two chambers are connected by a duct of length L and cross-sectional area A_{cs}. Partial pressure p_B is higher in the left chamber, and partial pressure p_A is higher in the right chamber. The partial pressure differences cause component B to migrate to the right and component A to migrate to the left.

At steady state, the molar flows of A and B must be equal, but in the opposite direction, or

$$\dot{m}''^*_A + \dot{m}''^*_B = 0 \quad (18)$$

because the total molar concentration C must stay the same in both chambers if p and T remain constant. Since the molar fluxes are the same in both directions, the molar average velocity v^* = 0. Thus, Equation (7b) can be used to calculate the molar flux of B (or A):

$$\dot{m}''^*_B = \frac{-D_v}{R_u T}\frac{dp_B}{dy} \quad (19)$$

or

$$\dot{m}^*_B = \frac{A_{cs}D_v}{R_u T}\frac{p_{B0} - p_{BL}}{L} \quad (20)$$

or

$$\dot{m}_B = \frac{M_B A_{cs} D_v}{R_u T}\frac{p_{B0} - p_{BL}}{L} \quad (21)$$

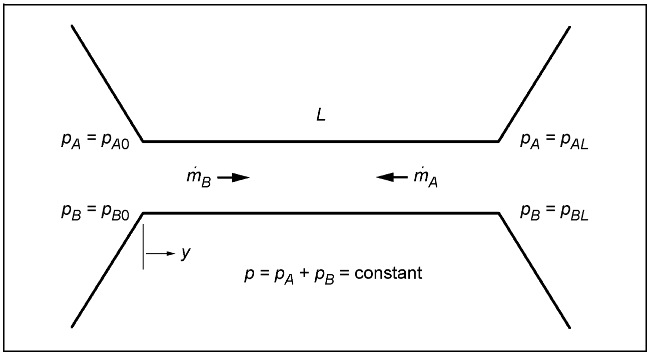

Fig. 3 Equimolar Counterdiffusion

Example 2. One large room is maintained at 70°F (530°R), 14.7 psia, 80% rh. A 60 ft long duct with cross-sectional area of 1.5 ft^2 connects the room to another large room at 70°F, 14.7 psia, 10% rh. What is the rate of water vapor diffusion between the two rooms?

Solution: Let air be component A and water vapor be component B. Equation (21) can be used to calculate the mass flow of water vapor B. Equation (10) can be used to calculate the diffusivity.

$$D_v = \frac{0.00215}{14.7}\left(\frac{530^{2.5}}{530 + 441}\right) = 0.964 \text{ ft}^2/\text{h}$$

From a psychrometric table (Table 3, Chapter 6), the saturated vapor pressure at 70°F is 0.363 psi. The vapor pressure difference $p_{B0} - p_{BL}$ is

$$p_{B0} - p_{BL} = (0.8 - 0.1)0.363 \text{ psi} = 0.254 \text{ psi}$$

Then, Equation (21) gives

$$\dot{m}_b = 18\,\frac{\text{lb}_\text{m}}{\text{lb mol}} \times \frac{\text{lb mol·°R}}{1545 \text{ ft·lb}} \times \frac{144 \text{ in}^2}{\text{ft}^2} \times \frac{1}{530\text{°R}} \times 1.5 \text{ ft}^2$$

$$\times\, 0.964 \text{ ft}^2/\text{h} \times \frac{0.254 \text{ psi}}{60 \text{ ft}} = 1.94 \times 10^{-5} \text{ lb}_\text{m}/\text{h}$$

Molecular Diffusion in Liquids and Solids

Because of the greater density, diffusion is slower in liquids than in gases. No satisfactory molecular theories have been developed for calculating diffusion coefficients. The limited measured values of D_v show that, unlike for gas mixtures at low pressures, the diffusion coefficient for liquids varies appreciably with concentration.

Reasoning largely from analogy to the case of one-dimensional diffusion in gases and employing Fick's law as expressed by Equation (4) gives

$$\dot{m}''_B = D_v\left(\frac{\rho_{B1} - \rho_{B2}}{y_2 - y_1}\right) \quad (22)$$

Equation (22) expresses the steady-state diffusion of the solute B through the solvent A in terms of the molal concentration difference of the solute at two locations separated by the distance $\Delta y = y_2 - y_1$. Bird et al. (1960), Hirschfelder et al. (1954), Sherwood and Pigford (1952), Reid and Sherwood (1966), Treybal (1980), and Eckert and Drake (1972) provide equations and tables for evaluating D_v. Hirschfelder et al. (1954) provide comprehensive treatment of the molecular developments.

Diffusion through a solid when the solute is dissolved to form a homogeneous solid solution is known as **structure-insensitive diffusion** (Treybal 1980). This solid diffusion closely parallels diffusion through fluids, and Equation (22) can be applied to one-dimensional steady-state problems. Values of mass diffusivity are generally lower than they are for liquids and vary with temperature.

The diffusion of a gas mixture through a porous medium is common (e.g., the diffusion of an air-vapor mixture through porous insulation). The vapor diffuses through the air along the tortuous narrow passages within the porous medium. The mass flux is a function of the vapor pressure gradient and diffusivity as indicated in Equation (7a). It is also a function of the structure of the pathways within the porous medium and is therefore called **structure-sensitive diffusion**. All of these factors are taken into account in the following version of Equation (7a):

$$\dot{m}''_B = -\bar{\mu}\,\frac{dp_B}{dy} \quad (23)$$

where $\bar{\mu}$ is called the permeability of the porous medium. Chapter 23 presents this topic in more depth.

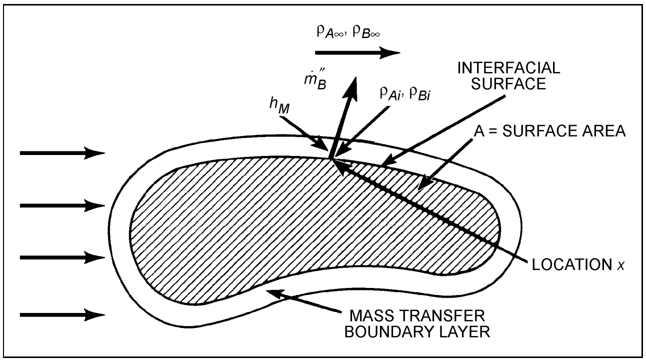

Fig. 4 Nomenclature for Convective Mass Transfer from External Surface at Location x Where Surface is Impermeable to Gas A

CONVECTION OF MASS

Convection of mass involves the mass transfer mechanisms of molecular diffusion and bulk fluid motion. Fluid motion in the region adjacent to a mass transfer surface may be laminar or turbulent, depending on geometry and flow conditions.

Mass Transfer Coefficient

Convective mass transfer is analogous to convective heat transfer where geometry and boundary conditions are similar. The analogy holds for both laminar and turbulent flows and applies to both external and internal flow problems.

Mass Transfer Coefficients for External Flows. Most external convective mass transfer problems can be solved with an appropriate formulation that relates the mass transfer flux (to or from an interfacial surface) to the concentration difference across the boundary layer illustrated in Figure 4. This formulation gives rise to the convective mass transfer coefficient, defined as

$$h_M \equiv \frac{\dot{m}''_B}{\rho_{Bi} - \rho_{B\infty}} \tag{24}$$

where

h_M = local external mass transfer coefficient, ft/h
$\dot{m}''_B$ = mass flux of gas B from surface, $lb_m/ft^2 \cdot h$
ρ_{Bi} = density of gas B at interface (saturation density), lb_m/ft^3
$\rho_{B\infty}$ = density of component B outside boundary layer, lb_m/ft^3

If ρ_{Bi} and $\rho_{B\infty}$ are constant over the entire interfacial surface, the mass transfer rate from the surface can be expressed as

$$\dot{m}''_B = \bar{h}_M(\rho_{Bi} - \rho_{B\infty}) \tag{25}$$

where $\bar{h}_M$ is the average mass transfer coefficient, defined as

$$\bar{h}_M \equiv \frac{1}{A}\int_A h_m \, dA \tag{26}$$

Mass Transfer Coefficients for Internal Flows. Most internal convective mass transfer problems, such as those that occur in channels or in the cores of dehumidification coils, can be solved if an appropriate expression is available to relate the mass transfer flux (to or from the interfacial surface) to the difference between the concentration at the surface and the bulk concentration in the channel, as illustrated in Figure 5. This formulation leads to the definition of the mass transfer coefficient for internal flows:

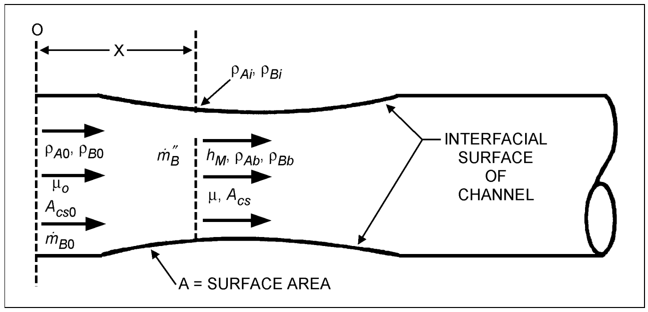

Fig. 5 Nomenclature for Convective Mass Transfer from Internal Surface Impermeable to Gas A

$$h_M \equiv \frac{\dot{m}''_B}{\rho_{Bi} - \rho_{Bb}} \tag{27}$$

where

h_M = internal mass transfer coefficient, ft/h
$\dot{m}''_B$ = mass flux of gas B at interfacial surface, $lb_m/ft^2 \cdot h$
ρ_{Bi} = density of gas B at interfacial surface, lb_m/ft^3
$\rho_{Bb} \equiv (1/\bar{u}_B A_{cs}) \int_{A_{cs}} u_B \rho_B \, dA_{cs}$ = bulk density of gas B at location x
$\bar{u}_B \equiv (1/A_{cs}) \int_A u_B \, dA_{cs}$ = average velocity of gas B at location x, fpm
A_{cs} = cross-sectional area of channel at station x, ft^2
u_B = velocity of component B in x direction, fpm
ρ_B = density distribution of component B at station x, lb_m/ft^3

Often, it is easier to obtain the bulk density of gas B from

$$\rho_{Bb} = \frac{\dot{m}_{Bo} + \int_A \dot{m}''_B \, dA}{\bar{u}_B A_{cs}} \tag{28}$$

where

$\dot{m}_{Bo}$ = mass flow rate of component B at station $x = 0$, lb_m/h
A = interfacial area of channel between station $x = 0$ and station $x = x$, ft^2

Equation (28) can be derived from the preceding definitions. The major problem is the determination of $\bar{u}_B$. If, however, the analysis is restricted to cases where B is dilute and concentration gradients of B in the x direction are negligibly small, $\bar{u}_B \approx \bar{u}$. Component B is swept along in the x direction with an average velocity equal to the average velocity of the dilute mixture.

Analogy Between Convective Heat and Mass Transfer

Most expressions for the convective mass transfer coefficient h_M are determined from expressions for the convective heat transfer coefficient h.

For problems in internal and external flow where mass transfer occurs at the convective surface and where component B is dilute, it is shown by Bird et al. (1960) and Incropera and DeWitt (1996) that the Nusselt and Sherwood numbers are defined as follows:

$$\text{Nu} = f(X, Y, Z, \text{Pr}, \text{Re}) \tag{29}$$

$$\text{Sh} = f(X, Y, Z, \text{Sc}, \text{Re}) \tag{30}$$

and

$$\overline{\text{Nu}} = g(\text{Pr}, \text{Re}) \tag{31}$$

$$\overline{\text{Sh}} = g(\text{Sc}, \text{Re}) \tag{32}$$

where the function f is the same in Equations (29) and (30), and the function g is the same in Equations (31) and (32). The quantities Pr and Sc are dimensionless Prandtl and Schmidt numbers, respectively, as defined in the section on Symbols. The primary restrictions on the analogy are that the surface shapes are the same and that the temperature boundary conditions are analogous to the density distribution boundary conditions for component B when cast in dimensionless form. Several primary factors prevent the analogy from being perfect. In some cases, the Nusselt number was derived for smooth surfaces. Many mass transfer problems involve wavy, droplet-like, or roughened surfaces. Many Nusselt number relations are obtained for constant temperature surfaces. Sometimes ρ_{Bi} is not constant over the entire surface because of varying saturation conditions and the possibility of surface dryout.

In all mass transfer problems, there is some blowing or suction at the surface because of the condensation, evaporation, or transpiration of component B. In most cases, this blowing/suction phenomenon has little effect on the Sherwood number, but the analogy should be examined closely if $v_i/u_\infty > 0.01$ or $v_i/\bar{u} > 0.01$, especially if the Reynolds number is large.

Example 3. Use the analogy expressed in Equations (31) and (32) to solve the following problem. An expression for heat transfer from a constant temperature flat plate in laminar flow is

$$\overline{\text{Nu}}_L = 0.664\text{Pr}^{1/3}\text{Re}_L^{1/2} \quad (33)$$

Sc = 0.35, $D_v = 3.87 \times 10^{-4}$ ft²/s, and Pr = 0.708 for the given conditions; determine the mass transfer rate and temperature of the water-wetted flat plate in Figure 6 using the heat/mass transfer analogy.

Solution: To solve the problem, properties should be evaluated at film conditions. However, since the plate temperature and the interfacial water vapor density are not known, a first estimate will be obtained assuming the plate t_{i1} to be at 77°F. The plate Reynolds number is

$$\text{Re}_{L1} = \frac{\rho u_\infty L}{\mu} = \frac{(0.0728\ \text{lb/ft}^3)(1970\ \text{fpm})(0.328\ \text{ft})}{(1.32 \times 10^{-5}\text{lb/ft}\cdot\text{s})(60\ \text{s/min})} = 59{,}340$$

The plate is entirely in laminar flow, since the transitional Reynolds number is about 5×10^5. Using the mass transfer analogy given by Equations (31) and (32), Equation (33) yields

$$\overline{\text{Sh}}_{L1} = 0.664\ \text{Sc}^{1/3}\text{Re}_L^{1/2}$$
$$= 0.664(0.35)^{1/3}(59{,}340)^{1/2} = 114$$

From the definition of the Sherwood number,

$$\bar{h}_{M1} = \overline{\text{Sh}}_{L1}D_v/L = (114)(3.87 \times 10^{-4}\ \text{ft}^2/\text{s})/0.328\ \text{ft} = 0.135\ \text{fps}$$

The psychrometric tables give a humidity ratio W of 0.0121 at 77°F and 60% rh. Therefore,

$$\rho_{B\infty} = 0.0121\rho_{A\infty} = (0.0121)(0.0728\ \text{lb/ft}^3) = 0.000881\ \text{lb/ft}^3$$

Psychrometric tables give the saturation density for water at 77°F as

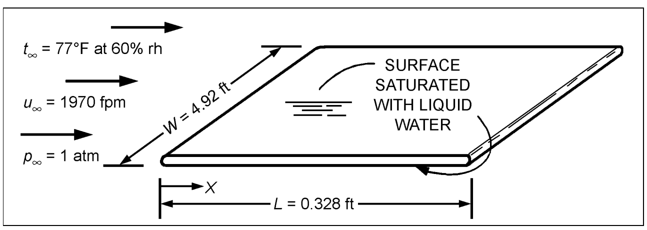

Fig. 6 Water-Saturated Flat Plate in Flowing Airstream

$$\rho_{Bi1} = 0.02017\rho_{A\infty} = (0.02017)(0.0728\ \text{lb/ft}^3) = 0.001468\ \text{lb/ft}^3$$

Therefore, the mass transfer rate from the double-sided plate is

$$\dot{m}_{B1} = \bar{h}_{M1}A(\rho_{Bi} - \rho_{B\infty})$$
$$= (0.135\ \text{fps})(0.328\ \text{ft} \times 4.92\ \text{ft} \times 2)(0.001468 - 0.000881\ \text{lb/ft}^3)$$
$$= 0.000256\ \text{lb/s}$$

This mass rate, transformed from the liquid state to the vapor state, requires the following heat rate to the plate to maintain the evaporation:

$$q_{i1} = \dot{m}_{B1}h_{fg} = (0.000256\ \text{lb/s})(1050\ \text{Btu/lb}) = 0.269\ \text{Btu/s}$$

To obtain a second estimate of the wetted plate temperature in this type of problem, the following criteria are used. Calculate the t_i necessary to provide a heat rate of q_{i1}. If this temperature t_{iq1} is above the dew-point temperature t_{id}, set the second estimate at $t_{i2} = (t_{iq1} + t_{i1})/2$. If t_{iq1} is below the dew-point temperature, set $t_{i2} = (t_{id} + t_{i1})/2$. For this problem, the dew point is t_{id} = 57°F.

Obtaining the second estimate of the plate temperature requires an approximate value of the heat transfer coefficient.

$$\overline{\text{Nu}}_{L1} = 0.664\text{Pr}^{1/3}\text{Re}_L^{1/2} = 0.664(0.708)^{1/3}(59{,}340)^{1/2}$$
$$= 144.2$$

From the definition of the Nusselt number,

$$\bar{h}_1 = \overline{\text{Nu}}_{L1}k/L = (144.2)(0.0151\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})/(0.328\ \text{ft})$$
$$= 6.64\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

Therefore, the second estimate for the plate temperature is

$$t_{iq1} = t_\infty - q_{i1}/(\bar{h}_1 A)$$
$$= 77^\circ\text{F} - [(0.269\ \text{Btu/s})(3600\ \text{s/h})/(6.64\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})]$$
$$(2 \times 0.328\ \text{ft} \times 4.92\ \text{ft}) = 77^\circ\text{F} - 45^\circ\text{F} = 32^\circ\text{F}$$

This temperature is below the dew-point temperature; therefore,

$$t_{i2} = (57^\circ\text{F} + 77^\circ\text{F})/2 = 67^\circ\text{F}$$

The second estimate of the film temperature is

$$t_{f2} = (t_{i2} + t_\infty)/2 = (67^\circ\text{F} + 77^\circ\text{F})/2 = 72^\circ\text{F}$$

The next iteration on the solution is as follows:

$$\text{Re}_{L2} = 61{,}010$$
$$\overline{\text{Sh}}_{L2} = 0.664(0.393)^{1/3}(61{,}010)^{1/2} = 120$$
$$\bar{h}_{M2} = (120)(3.63 \times 10^{-4})/0.328 = 0.133\ \text{fps}$$

The free stream density of the water vapor has been evaluated. The density of the water vapor at the plate surface is the saturation density at 67°F.

$$\rho_{Bi2} = (0.01374)(0.0739\ \text{lb/ft}^3) = 0.00101\ \text{lb/ft}^3$$
$$A = 2 \times 4.92 \times 0.328 = 3.228\ \text{ft}^2$$
$$\dot{m}_{B2} = (0.133\ \text{fps})(3.228\ \text{ft}^2)(0.00101\ \text{lb/ft}^3 - 0.000881\ \text{lb/ft}^3)$$
$$= 0.0000554\ \text{lb/s}$$
$$q_{i2} = (0.0000554)(1056) = 0.0585\ \text{Btu/s}$$
$$\overline{\text{Nu}}_{L2} = 0.664(0.709)^{1/3}(61{,}010)^{1/2} = 146$$
$$\bar{h}_2 = (146)(0.01493)/0.328 = 6.65\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$
$$t_{iq2} = 77^\circ\text{F} - (0.0585)(3600)/(6.65 \times 3.228) = 67.2^\circ\text{F}$$

This temperature is above the dew-point temperature; therefore,

$$t_{i3} = (t_{i2} + t_{iq2})/2 = (67 + 67.2)/2 = 67.1°\text{F}$$

This is approximately the same result as that obtained in the previous iteration. Therefore, the problem solution is

$$t_i = 67°\text{F}$$
$$\dot{m}_B = 0.199 \text{ lb/h}$$

The kind of similarity between heat and mass transfer that results in Equation (29) through Equation (32) can also be shown to exist between heat and momentum transfer. Chilton and Colburn (1934) used this similarity to relate Nusselt number to friction factor by the analogy

$$j_H = \frac{\text{Nu}}{\text{Re Pr}^{(1-n)}} = \text{St Pr}^n = \frac{f}{2} \tag{34}$$

where $n = 2/3$, St = Nu/(Re Pr) is the Stanton number, and j_H is the Chilton-Colburn j-factor for heat transfer. Substituting Sh for Nu and Sc for Pr in Equations (31) and (32) gives the Chilton-Colburn j-factor for mass transfer, j_D:

$$j_D = \frac{\text{Sh}}{\text{Re Sc}^{(1-n)}} = \text{St}_m\text{Sc}^n = \frac{f}{2} \tag{35}$$

where $\text{St}_m = \text{Sh}P_{AM}/(\text{Re Sc})$ is the Stanton number for mass transfer. Equations (34) and (35) are called the **Chilton-Colburn *j*-factor analogy**.

The power of the Chilton-Colburn j-factor analogy is represented in Figures 7 through 10. Figure 7 plots various experimental values of j_D from a flat plate with flow parallel to the plate surface. The solid line, which represents the data to near perfection, is actually $f/2$ from Blasius' solution of laminar flow on a flat plate (left-hand portion of the solid line) and Goldstein's solution for a turbulent boundary layer (right-hand portion). The right-hand portion of the solid line also represents McAdams' (1954) correlation of turbulent flow heat transfer coefficient for a flat plate.

A **wetted-wall column** is a vertical tube in which a thin liquid film adheres to the tube surface and exchanges mass by evaporation or absorption with a gas flowing through the tube. Figure 8 illustrates typical data on vaporization in wetted-wall columns, plotted as j_D versus Re. The spread of the points with variation in $\mu/\rho D_v$ results from Gilliland's finding of an exponent of 0.56, not 2/3, representing the effect of the Schmidt number. Gilliland's equation can be written as follows:

$$j_D = 0.023\text{Re}^{-0.17}\left(\frac{\mu}{\rho D_v}\right)^{-0.56} \tag{36}$$

Similarly, McAdams' (1954) equation for heat transfer in pipes can be expressed as

$$j_H = 0.023\text{Re}^{-0.20}\left(\frac{c_p\mu}{k}\right)^{-0.7} \tag{37}$$

This is represented by the dash-dot curve in Figure 8, which falls below the mass transfer data. The curve $f/2$ representing friction in smooth tubes is the upper, solid curve.

Data for the evaporation of liquids from single cylinders into gas streams flowing transversely to the cylinders' axes are shown in Figure 9. Although the dash-dot line on Figure 9 represents the data, it is actually taken from McAdams (1954) as representative of a large collection of data on heat transfer to single cylinders placed transverse to airstreams. To compare these data with friction, it is necessary to distinguish between total drag and skin friction. Since the analogies are based on skin friction, the normal pressure drag must be subtracted from the measured total drag. At Re = 1000, the skin friction is 12.6% of the total drag; at Re = 31,600, it is only 1.9%. Consequently, the values of $f/2$ at a high Reynolds number, obtained by the difference, are subject to considerable error.

In Figure 10, data on the evaporation of water into air for single spheres are presented. The solid line, which best represents these data, agrees with the dashed line representing McAdams' correlation for heat transfer to spheres. These results cannot be compared with friction or momentum transfer because total drag has not been allocated to skin friction and normal pressure drag. Application of these data to air-water contacting devices such as air washers and spray cooling towers is well substantiated.

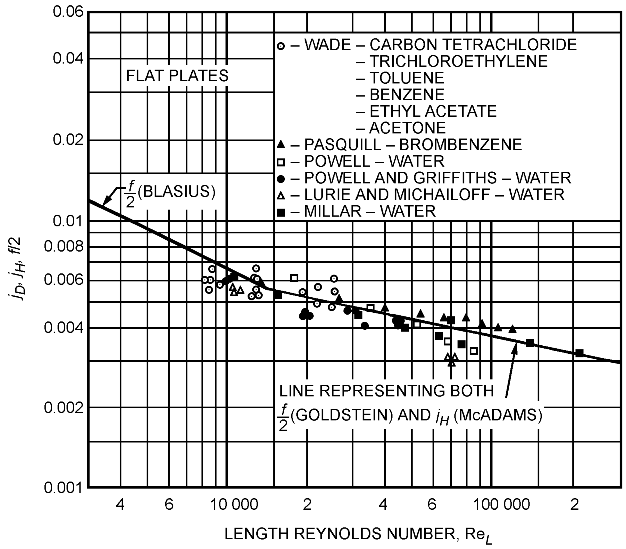

Fig. 7 Mass Transfer from Flat Plate

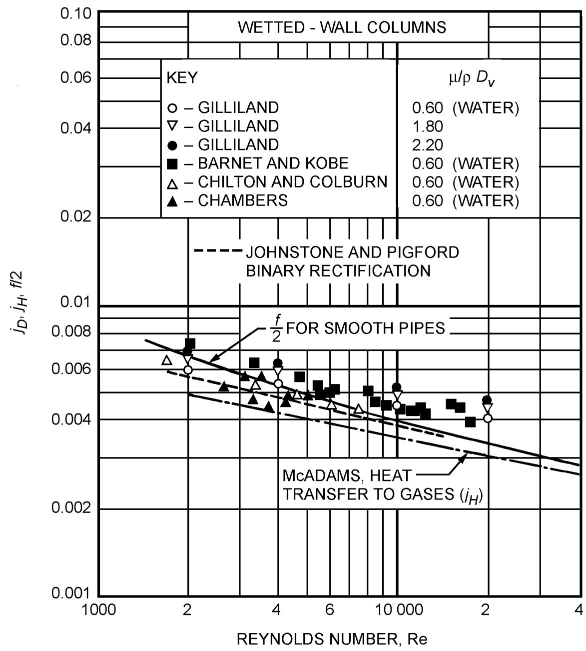

Fig. 8 Vaporization and Absorption in Wetted-Wall Column

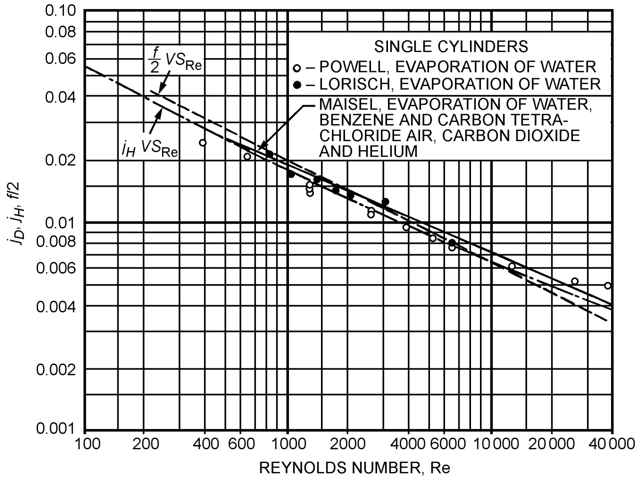

Fig. 9 Mass Transfer from Single Cylinders in Crossflow

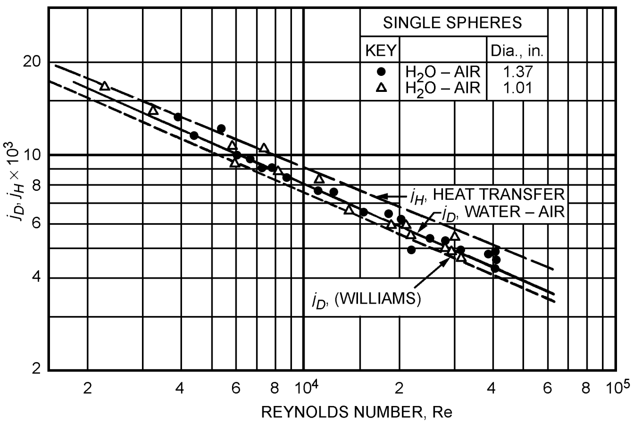

Fig. 10 Mass Transfer from Single Spheres

When the temperature of the heat exchanger surface in contact with moist air is below the dew-point temperature of the air, vapor condensation occurs. Typically, the air dry-bulb temperature and humidity ratio both decrease as the air flows through the exchanger. Therefore, sensible and latent heat transfer occur simultaneously. This process is similar to one that occurs in a spray dehumidifier and can be analyzed using the same procedure; however, this is not generally done.

Cooling coil analysis and design are complicated by the problem of determining transport coefficients h, h_M, and f. It would be convenient if heat transfer and friction data for dry heating coils could be used with the Colburn analogy to obtain the mass transfer coefficients. However, this approach is not always reliable, and work by Guillory and McQuiston (1973) and Helmer (1974) shows that the analogy is not consistently true. Figure 11 shows j-factors for a simple parallel plate exchanger for different surface conditions with sensible heat transfer. Mass transfer j-factors and the friction factors exhibit the same behavior. Dry surface j-factors fall below those obtained under dehumidifying conditions with the surface wet. At low Reynolds numbers, the boundary layer grows quickly; the droplets are soon covered and have little effect on the flow field. As the Reynolds number is increased, the boundary layer becomes thin and more of the total flow field is exposed to the droplets. The roughness caused by the droplets induces mixing and larger j-factors. The data in Figure 11 cannot be applied to all surfaces because the length of the flow channel is also an important variable. However, the water collecting on the surface is mainly responsible for breakdown of the

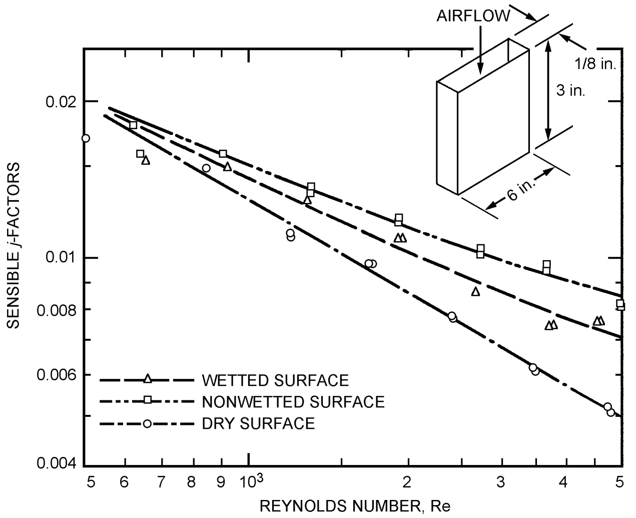

Fig. 11 Sensible Heat Transfer j-Factors for Parallel Plate Exchanger

j-factor analogy. The j-factor analogy is approximately true when the surface conditions are identical. Under some conditions, it is possible to obtain a film of condensate on the surface instead of droplets. Guillory and McQuiston (1973) and Helmer (1974) related dry sensible j- and f-factors to those for wetted dehumidifying surfaces.

The equality of j_H, j_D, and $f/2$ for certain streamline shapes at low mass transfer rates has experimental verification. For flow past bluff objects, j_H and j_D are much smaller than $f/2$, based on total pressure drag. The heat and mass transfer, however, still relate in a useful way by equating j_H and j_D.

Example 4. Using solid cylinders of volatile solids (e.g., naphthalene, camphor, dichlorobenzene) with airflow normal to these cylinders, Bedingfield and Drew (1950) found that the ratio between the heat and mass transfer coefficients could be closely correlated by the following relation:

$$\frac{h}{\rho h_M} = (0.294\ \text{Btu/lb}_\text{m}\cdot{}^\circ\text{F})\left(\frac{\mu}{\rho D_v}\right)^{0.56}$$

For completely dry air at 70°F flowing at a velocity of 31 fps over a wet-bulb thermometer of diameter d = 0.300 in., determine the heat and mass transfer coefficients from Figure 9 and compare their ratio with the Bedingfield-Drew relation.

Solution: For dry air at 70°F and standard pressure, $\rho = 0.075\ \text{lb}_\text{m}/\text{ft}^3$, $\mu = 0.044\ \text{lb}_\text{m}/\text{h}\cdot\text{ft}$, $k = 0.0149\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$, and $c_p = 0.240\ \text{Btu/lb}_\text{m}\cdot{}^\circ\text{F}$. From Equation (10), $D_v = 0.973\ \text{ft}^2/\text{h}$. Therefore,

$$\text{Re}_{da} = \rho u_\infty d/\mu = 0.0749 \times 31 \times 3600 \times 0.300/(12 \times 0.044) = 4750$$

$$\text{Pr} = c_p\mu/k = 0.240 \times 0.044/0.0149 = 0.709$$

$$\text{Sc} = \mu/\rho D_v = 0.044/(0.0749 \times 0.973) = 0.604$$

From Figure 9 at Re_{da} = 4750, read j_H = 0.0088, j_D = 0.0099. From Equations (34) and (35),

$$h = j_H\rho c_p u_\infty/(\text{Pr})^{2/3}$$

$$= 0.0088 \times 0.0749 \times 0.240 \times 31 \times 3600/(0.709)^{2/3}$$

$$= 22.2\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$h_M = j_D u_\infty/(\text{Sc})^{2/3} = 0.0099 \times 31 \times 3600/(0.604)^{2/3}$$

$$= 1550\ \text{ft/h}$$

$$h/\rho h_M = 22.2/(0.0749 \times 1550) = 0.191\ \text{Btu/lb}_\text{m}\cdot{}^\circ\text{F}$$

From the Bedingfield-Drew relation,

$$h/\rho h_M = 0.294(0.604)^{0.56} = 0.222 \text{ Btu/lb}_m \cdot °\text{F}$$

Equations (34) and (35) are call the Reynolds analogy when Pr = Sc = 1. This suggests that $h/\rho h_M = c_p$ = 0.240 Btu/$\text{lb}_m \cdot$°F. This close agreement is because the ratio Sc/Pr is 0.604/0.709 or 0.85, so that the exponent of these numbers has little effect on the ratio of the transfer coefficients.

The extensive developments for calculating heat transfer coefficients can be applied to calculate mass transfer coefficients under similar geometrical and flow conditions using the *j*-factor analogy. For example, Table 6 of Chapter 3 lists equations for calculating heat transfer coefficients for flow inside and normal to pipes. Each equation can be used for mass transfer coefficient calculations by equating j_H and j_D and imposing the same restriction to each stated in Table 6 of Chapter 3. Similarly, mass transfer experiments often replace corresponding heat transfer experiments with complex geometries where exact boundary conditions are difficult to model (Sparrow and Ohadi 1987a, 1987b).

The *j*-factor analogy is useful only at low mass transfer rates. As the rate of mass transfer increases, the movement of matter normal to the transfer surface increases the convective velocity. For example, if a gas is blown from many small holes in a flat plate placed parallel to an airstream, the boundary layer thickens, and resistance to both mass and heat transfer increases with increasing blowing rate. Heat transfer data are usually collected at zero or, at least, insignificant mass transfer rates. Therefore, if such data are to be valid for a mass transfer process, the mass transfer rate (i.e., the blowing) must be low.

The *j*-factor relationship $j_H = j_D$ can still be valid at high mass transfer rates, but neither j_H nor j_D can be represented by data at zero mass transfer conditions. Eckert and Drake (1972) and Chapter 24 of Bird et al. (1960) have detailed information on high mass transfer rates.

Lewis Relation

Heat and mass transfer coefficients are satisfactorily related at the same Reynolds number by equating the Chilton-Colburn *j*-factors. Comparing Equations (34) and (35) gives

$$\text{St Pr}^n = \frac{f}{2} = \text{St}_m \text{Sc}^n$$

Inserting the definitions of St, Pr, St_m, and Sc gives

$$\frac{h}{\rho c_p \bar{u}}\left(\frac{c_p\mu}{k}\right)^{2/3} = \frac{h_M P_{Am}}{\bar{u}}\left(\frac{\mu}{\rho D_v}\right)^{2/3}$$

or

$$\frac{h}{h_M \rho c_p} = P_{Am}\left[\frac{(\mu/\rho D_v)}{(c_p\mu/k)}\right]^{2/3}$$

$$= P_{Am}(\alpha/D_v)^{2/3} \tag{38}$$

The quantity α/D_v is the **Lewis number** Le. Its magnitude expresses relative rates of propagation of energy and mass within a system. It is fairly insensitive to temperature variation. For air and water vapor mixtures, the ratio is (0.60/0.71) or 0.845, and $(0.845)^{2/3}$ is 0.894. At low diffusion rates, where the heat-mass transfer analogy is valid, P_{Am} is essentially unity. Therefore, for air and water vapor mixtures,

$$\frac{h}{h_M \rho c_p} \approx 1 \tag{39}$$

The ratio of the heat transfer coefficient to the mass transfer coefficient is equal to the specific heat per unit volume of the mixture at constant pressure. This relation [Equation (39)] is usually called the Lewis relation and is nearly true for air and water vapor at low mass transfer rates. It is generally not true for other gas mixtures because the ratio Le of thermal to vapor diffusivity can differ from unity. The agreement between wet-bulb temperature and adiabatic saturation temperature is a direct result of the nearness of the Lewis number to unity for air and water vapor.

The Lewis relation is valid in turbulent flow whether or not α/D_v equals 1 because eddy diffusion in turbulent flow involves the same mixing action for heat exchange as for mass exchange, and this action overwhelms any molecular diffusion. Deviations from the Lewis relation are, therefore, due to a laminar boundary layer or a laminar sublayer and buffer zone where molecular transport phenomena are the controlling factors.

SIMULTANEOUS HEAT AND MASS TRANSFER BETWEEN WATER-WETTED SURFACES AND AIR

A simplified method used to solve simultaneous heat and mass transfer problems was developed using the Lewis relation, and it gives satisfactory results for most air-conditioning processes. Extrapolation to very high mass transfer rates, where the simple heat-mass transfer analogy is not valid, will lead to erroneous results.

Enthalpy Potential

The water vapor concentration in the air is the humidity ratio *W*, defined as

$$W \equiv \frac{\rho_B}{\rho_A} \tag{40}$$

A mass transfer coefficient is defined using *W* as the driving potential:

$$\dot{m}_B'' = K_M(W_i - W_\infty) \tag{41}$$

where the coefficient K_M is in $\text{lb}_m/\text{h}\cdot\text{ft}^2$. For dilute mixtures, $\rho_{Ai} \cong \rho_{A\infty}$; that is, the partial mass density of dry air changes by only a small percentage between interface and free stream conditions. Therefore,

$$\dot{m}_B'' = \frac{K_M}{\rho_{Am}}(\rho_{Bi} - \rho_\infty) \tag{42}$$

where ρ_{Am} = mean density of dry air, lb_m/ft^3. Comparing Equation (42) with Equation (24) shows that

$$h_M = \frac{K_M}{\rho_{Am}} \tag{43}$$

The **humid specific heat** c_{pm} of the airstream is, by definition (Mason and Monchick 1965),

$$c_{pm} = (1 + W_\infty)c_p \tag{44a}$$

or

$$c_{pm} = \left(\frac{\rho}{\rho_{A\infty}}\right)c_p \tag{44b}$$

where c_{pm} is in $\text{Btu/lb}_{da}\cdot$°F.

Substituting from Equations (43) and (44b) into Equation (39) gives

$$\frac{h\rho_{Am}}{K_M \rho_{A\infty} c_{pm}} = 1 \approx \frac{h}{K_M c_{pm}} \quad (45)$$

since $\rho_{Am} \cong \rho_{A\infty}$ because of the small change in dry-air density. Using a mass transfer coefficient with the humidity ratio as the driving force, the Lewis relation becomes ratio of heat to mass transfer coefficient equals humid specific heat.

For the plate humidifier illustrated in Figure 6, the total heat transfer from liquid to interface is

$$q'' = q''_A + \dot{m}''_B h_{fg} \quad (46)$$

Using the definitions of the heat and mass transfer coefficients gives

$$q'' = h(t_i - t_\infty) + K_M(W_i - W_\infty)h_{fg} \quad (47)$$

Assuming Equation (45) is valid gives

$$q'' = K_M\left[c_{pm}(t_i - t_\infty) + (W_i - W_\infty)h_{fg}\right] \quad (48)$$

The enthalpy of the air is approximately

$$h = c_{pa}t + Wh_s \quad (49)$$

The enthalpy h_s of the water vapor can be expressed by the ideal gas law as

$$h_s = c_{ps}(t - t_o) + h_{fgo} \quad (50)$$

where the base of enthalpy is taken as saturated water at temperature t_o. Choosing t_o = 0°F to correspond with the base of the dry-air enthalpy gives

$$h = (c_{pa} + Wc_{ps})t + Wh_{fgo} = c_{pm}t + Wh_{fgo} \quad (51)$$

If small changes in the latent heat of vaporization of water with temperature are neglected when comparing Equations (49) and (51), the total heat transfer can be written as

$$q'' = K_M(h_i - h_\infty) \quad (52)$$

Where the driving potential for heat transfer is temperature difference and the driving potential for mass transfer is mass concentration or partial pressure, the driving potential for simultaneous transfer of heat and mass in an air water-vapor mixture is, to a close approximation, enthalpy.

Basic Equations for Direct-Contact Equipment

Air-conditioning equipment can be classified as (1) having direct contact between air and water used as a cooling or heating fluid or (2) having the heating or cooling fluid separated from the airstream by a solid wall. Examples of the former are air washers and cooling towers; an example of the latter is a direct-expansion refrigerant (or water) cooling and dehumidifying coil. In both cases, the airstream is in contact with a water surface. Direct contact implies contact directly with the cooling (or heating) fluid. In the dehumidifying coil, the contact with the condensate removed from the airstream is direct, but it is indirect with the refrigerant flowing inside the tubes of the coil. These two cases are treated separately because the surface areas of direct-contact equipment cannot be evaluated.

For the direct-contact spray chamber air washer of cross-sectional area A_{cs} and length l (Figure 12), the steady mass flow rate of dry air per unit cross-sectional area is

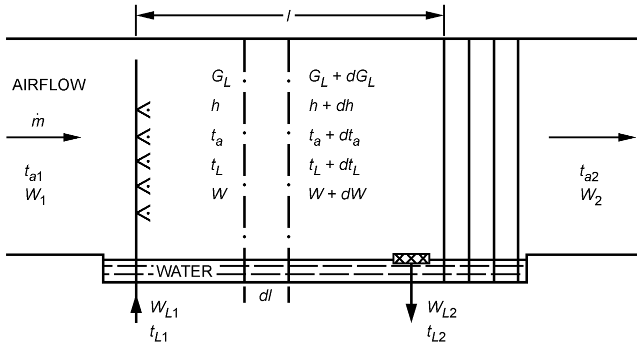

Fig. 12 Air Washer Spray Chamber

$$\dot{m}_a / A_{cs} = G_a \quad (53)$$

and the corresponding mass flux of water flowing parallel with the air is

$$\dot{m}_L / A_{cs} = G_L \quad (54)$$

where

$\dot{m}_a$ = mass flow rate of air, lb/h
G_a = mass flux or flow rate per unit cross-sectional area for air, lb/h·ft²
$\dot{m}_L$ = mass flow rate of liquid, lb/h
G_L = mass flux or flow rate per unit cross-sectional area for liquid, lb/h·ft²

Because water is evaporating or condensing, G_L changes by an amount dG_L in a differential length dl of the chamber. Similar changes occur in temperature, humidity ratio, enthalpy, and other properties.

Because evaluating the true surface area in direct-contact equipment is difficult, it is common to work on a unit volume basis. If a_H and a_M are the area of heat transfer and mass transfer surface per unit of chamber volume, respectively, the total surface areas for heat and mass transfer are

$$A_H = a_H A_{cs} l \quad \text{and} \quad A_M = a_M A_{cs} l \quad (55)$$

The basic equations for the process occurring in the differential length dl can be written for

1. *Mass transfer*

$$-dG_L = G_a dW = K_M a_M (W_i - W) dl \quad (56)$$

That is, the water evaporated, the moisture increase of the air, and the mass transfer rate are all equal.

2. *Heat transfer to air*

$$G_a c_{pm} dt_a = h_a a_H (t_i - t_a) dl \quad (57)$$

3. *Total energy transfer to air*

$$G_a(c_{pm} dt_a + h_{fgo} dW) = [K_M a_M (W_i - W) h_{fg} + h_a a_H (t_i - t_a)]\, dl \quad (58)$$

Assuming $a_H = a_M$ and Le = 1, and neglecting small variations in h_{fg}, Equation (58) reduces to

$$G_a dh = K_M a_M (h_i - h)\, dl \quad (59)$$

The heat and mass transfer areas of spray chambers are assumed to be identical ($a_H = a_M$). Where packing materials, such as wood slats or Raschig rings, are used, the two areas may be considerably different because the packing may not be wet uniformly. The validity of the Lewis relation was discussed previously. It is not necessary to account for the small changes in latent heat h_{fg} after making the two previous assumptions.

4. *Energy balance*

$$G_a dh = \pm G_L c_L dt_L \tag{60}$$

A minus sign refers to parallel flow of air and water; a plus sign refers to counterflow (water flow in the opposite direction from airflow).

The water flow rate changes between inlet and outlet as a result of the mass transfer. For exact energy balance, the term ($c_L t_L dG_L$) should be added to the right side of Equation (60). The percentage change in G_L is quite small in usual applications of air-conditioning equipment and, therefore, can be ignored.

5. *Heat transfer to water*

$$\pm G_L c_L dt_L = h_L a_H (t_L - t_i) dl \tag{61}$$

Equations (56) to (61) are the basic relations for solution of simultaneous heat and mass transfer processes in direct-contact air-conditioning equipment.

To facilitate the use of these relations in equipment design or performance, three other equations can be extracted from the above set. Combining Equations (59), (60), and (61) gives

$$\frac{h - h_i}{t_L - t_i} = -\frac{h_L a_H}{K_M a_M} = -\frac{h_L}{K_M} \tag{62}$$

Equation (62) relates the enthalpy potential for the total heat transfer through the gas film to the temperature potential for this same transfer through the liquid film. Physical reasoning leads to the conclusion that this ratio is proportional to the ratio of gas film resistance ($1/K_M$) to liquid film resistance ($1/h_L$). Combining Equations (57), (59), and (45) gives

$$\frac{dh}{dt_a} = \frac{h - h_i}{t_a - t_i} \tag{63}$$

Similarly, combining Equations (56), (57), and (45) gives

$$\frac{dW}{dt_a} = \frac{W - W_i}{t_a - t_i} \tag{64}$$

Equation (64) indicates that at any cross section in the spray chamber, the instantaneous slope of the air path dW/dt_a on a psychrometric chart is determined by a straight line connecting the air state with the interface saturation state at that cross section. In Figure 13, state 1 represents the state of the air entering the parallel flow air washer chamber of Figure 12. The washer is operating as a heating and humidifying apparatus so that the interface saturation state of the water at air inlet is the state designated 1_i. Therefore, the initial slope of the air path is along a line directed from state 1 to state 1_i. As the air is heated, the water cools and the interface temperature drops. Corresponding air states and interface saturation states are indicated by the letters *a*, *b*, *c*, and *d* in Figure 13. In each instance, the air path is directed toward the associated interface state. The interface states are derived from Equations (60) and (62). Equation (60) describes how the air enthalpy changes with water temperature; Equation (62) describes how the interface saturation state

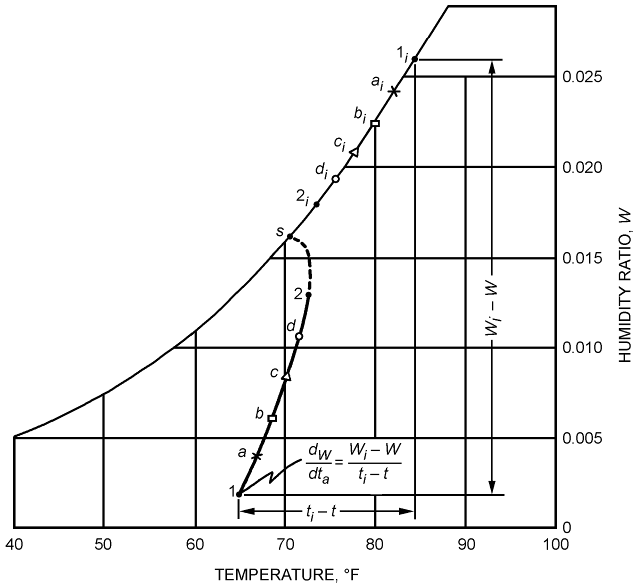

Fig. 13 Air Washer Humidification Process on Psychrometric Chart

changes to accommodate this change in air and water conditions. The solution for the interface state on the normal psychrometric chart of Figure 13 can be determined either by trial and error from Equations (60) and (62) or by a complex graphical procedure (Kusuda 1957).

Air Washers

Air washers are direct-contact apparatus used to (1) simultaneously change the temperature and humidity content of air passing through the chamber and (2) remove air contaminants such as dust and odors. Adiabatic spray washers, which have no external heating or chilling source, are used to cool and humidify air. Chilled spray air washers have an external chiller to cool and dehumidify air. Heated spray air washers, whose external heating source provides additional energy for evaporation of water, are used to humidify and possibly heat air.

Example 5. A parallel flow air washer with the following design conditions is to be designed (see Figure 12).

Water temperature at inlet t_{L1} = 95°F
Water temperature at outlet t_{L2} = 75°F
Air temperature at inlet t_{a1} = 65°F
Air wet-bulb at inlet t'_{a1} = 45°F
Air mass flow rate per unit area G_a = 1200 lb/h·ft^2
Spray ratio G_L/G_a = 0.70
Air heat transfer coefficient per cubic foot of chamber volume $h_a a_H$ = 72 Btu/h·°F·ft^3
Liquid heat transfer coefficient per cubic foot of chamber volume $h_L a_H$ = 900 Btu/h·°F·ft^3
Air volumetric flow rate Q = 6500 cfm

Solution: The air mass flow rate $\dot{m}_a$ = 6500 × 0.075 = 490 lb/min; the required spray chamber cross-sectional area is, then, $A_{cs} = \dot{m}_a / G_a$ = 490 × 60/1200 = 24.5 ft^2. The mass transfer coefficient is given by the Lewis relation [Equation (45)] as

$$K_M a_M = (h_a a_H)/c_{pm} = 72/0.24 = 300 \text{ lb/h}\cdot\text{ft}^3$$

Figure 14 shows the enthalpy-temperature psychrometric chart with the graphical solution for the interface states and the air path through the washer spray chamber. The solution proceeds as follows:

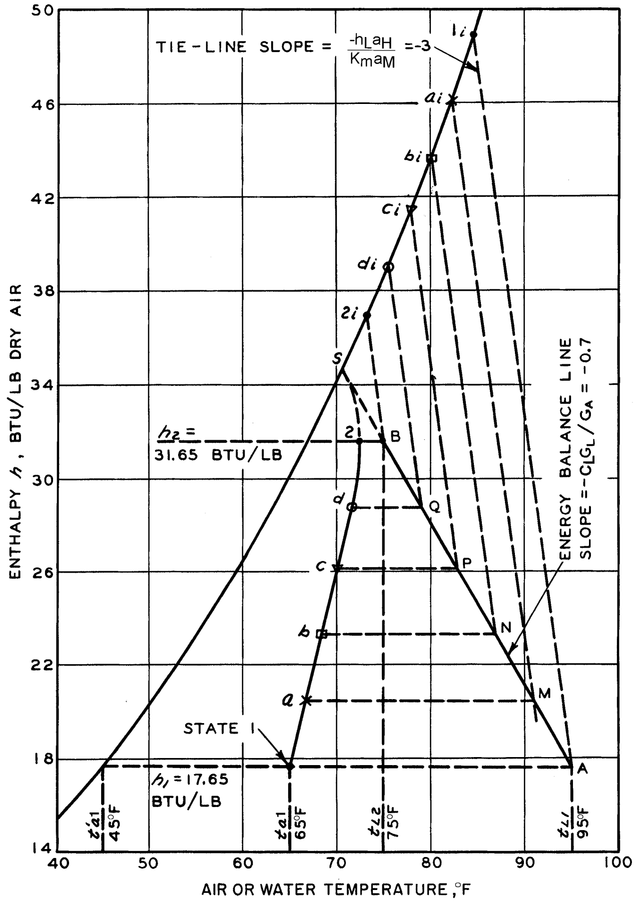

Fig. 14 Graphical Solution for Air-State Path in Parallel Flow Air Washer

1. Enter bottom of chart with t'_{a1} of 45°F, and follow up to saturation curve to establish air enthalpy h_1 of 17.65 Btu/lb. Extend this enthalpy line to intersect initial air temperature t_{a1} of 65°F (state 1 of air) and initial water temperature t_{L1} of 95°F at point A. (Note that the temperature scale is used for both air and water temperatures.)
2. Through point A, construct the *energy balance* line A-B with a slope of

$$\frac{dh}{dt_L} = -\frac{c_L G_L}{G_a} = -0.7$$

Point B is determined by intersection with the leaving water temperature t_{L2} = 75°F. The negative slope here is a consequence of the parallel flow, which results in the air-water mixture's approaching, but not reaching, the common saturation state *s*. (The line A-B has no physical significance in representing any *air state* on the psychrometric chart. It is merely a construction line in the graphical solution.)
3. Through point A, construct the *tie-line* A-1_i having a slope of

$$\frac{h - h_i}{t_L - t_i} = -\frac{h_L a_H}{K_M a_M} = -\frac{900}{300} = -3$$

The intersection of this line with the saturation curve gives the initial interface state 1_i at the chamber inlet. (Note how the energy balance line and tie-line, representing Equations (60) and (62), combine for a simple graphical solution on Figure 14 for the interface state.)
4. The initial slope of the air path can now be constructed, according to Equation (63), drawing line 1-*a* toward the initial interface state 1_i. (The length of the line 1-*a* will depend on the degree of accuracy required in the solution and the rate at which the slope of the air path is changing.)

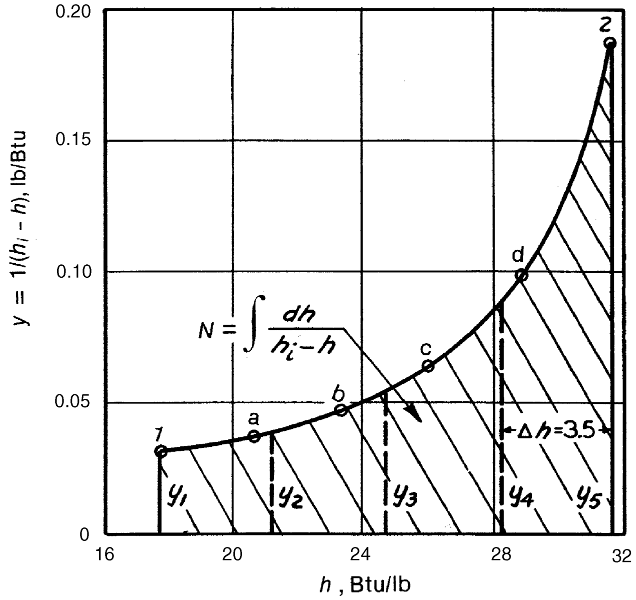

Fig. 15 Graphical Solution of $\int dh/(h_i - h)$

5. Construct the horizontal line *a*-M locating the point M on the energy-balance line. Draw a new tie-line (slope of −3 as before) from M to a_i locating interface state a_i. Continue the air path from *a* to *b* by directing it toward the new interface state a_i. (Note that the change in slope of the air path from 1-*a* to *a*-*b* is quite small, justifying the path incremental lengths used.)
6. Continue in the manner of step 5 until point 2, the final state of the air leaving the chamber, is reached. In this example, six steps are used in the graphical construction with the following results:

State	1	*a*	*b*	*c*	*d*	2
t_L	95.0	91.0	87.0	83.0	79.0	75.0
h	17.65	20.45	23.25	26.05	28.85	31.65
t_i	84.5	82.3	80.1	77.8	75.6	73.2
h_i	49.00	46.25	43.80	41.50	39.10	37.00
t_a	65.0	66.8	68.5	70.0	71.4	72.4

The final state of the air leaving the washer is t_{a2} = 72.4°F and h_2 = 31.65 Btu/lb (wet-bulb temperature t'_{a2} = 67°F).

7. The final step involves calculating the required length of the spray chamber. From Equation (59),

$$l = \frac{G_a}{K_M a_M} \int_1^2 \frac{dh}{(h_i - h)}$$

The integral is evaluated graphically by plotting $1/(h_i - h)$ versus h as shown in Figure 15. Any satisfactory graphical method can be used to evaluate the area under the curve. Simpson's rule with four equal increments of Δh equal to 3.5 gives

$$N = \int_1^2 \frac{dh}{(h_i - h)} \approx (\Delta h/3)(y_1 + 4y_2 + 2y_3 + 4y_4 + y_5)$$

$$N = (3.5/3)[0.0319 + (4 \times 0.0400) + (2 \times 0.0553) + (4 \times 0.0865) + 0.1870] = 0.975$$

The design length is, therefore, l = (1200/300)(0.975) = 3.9 ft.

The method used in Example 5 can also be used to predict the performance of existing direct-contact equipment and can determine the transfer coefficients when performance data from test runs are available. By knowing the water and air temperatures entering and leaving the chamber and the spray ratio, it is possible, by trial and error, to determine the proper slope of the tie-line necessary to achieve the measured final air state. The tie-line slope gives the ratio $h_L a_H / K_M a_M$; $K_M a_M$ is found from the integral relationship in Example 5 from the known chamber length l.

Additional descriptions of air spray washers and general performance criteria are given in Chapter 19 of the *ASHRAE Handbook—Systems and Equipment*.

Cooling Towers

A cooling tower is a direct-contact heat exchanger in which waste heat picked up by the cooling water from a refrigerator, air conditioner, or industrial process is transferred to atmospheric air by cooling the water. Cooling is achieved by breaking up the water flow to provide a large water surface for air, moving by natural or forced convection through the tower, to contact the water. Cooling towers may be counterflow, crossflow, or a combination of both.

The temperature of the water leaving the tower and the packing depth needed to achieve the desired leaving water temperature are of primary interest for design. Therefore, the mass and energy balance equations are based on an overall coefficient K, which is based on (1) the enthalpy driving force due to h at the bulk water temperature and (2) neglecting the film resistance. Combining Equations (59) and (60) and using the parameters described above yields

$$G_L c_L \, dt = K_M a_M (h_i - h) dl = G_a \, dh$$

$$= \frac{K_a \, dV (h' - h_a)}{A_{cs}} \quad (65)$$

or

$$\frac{K_a V}{\dot{m}_L} = \int_{t_1}^{t_2} \frac{c_L \, dt}{(h' - h_a)} \quad (66)$$

Chapter 36 of the *ASHRAE Handbook—Systems and Equipment* covers cooling tower design in detail.

Cooling and Dehumidifying Coils

When water vapor is condensed out of an airstream onto an extended surface (finned) cooling coil, the simultaneous heat and mass transfer problem can be solved by the same procedure set forth for direct-contact equipment. The basic equations are the same, except that the true surface area of the coil A is known and the problem does not have to be solved on a unit volume basis. Therefore, if in Equations (56), (57), and (59) $a_M dl$ or $a_H dl$ is replaced by dA/A_{cs}, these equations become the basic heat, mass, and total energy transfer equations for indirect-contact equipment such as dehumidifying coils. The energy balance shown by Equation (60) remains unchanged. The heat transfer from the interface to the refrigerant now encounters the combined resistances of the condensate film ($R_L = 1/h_L$); the metal wall and fins, if any (R_m); and the refrigerant film ($R_r = A/h_r A_r$). If this combined resistance is designated as $R_i = R_L + R_m + R_r = 1/U_i$, Equation (61) becomes, for a coil dehumidifier,

$$\pm \dot{m}_L c_L \, dt_L = U_i (t_L - t_i) dA \quad (67)$$

(plus sign for counterflow, minus sign for parallel flow).

The tie-line slope is then

$$\frac{h - h_i}{t_L - t_i} = \mp \frac{U_i}{K_M} \quad (68)$$

Figure 16 illustrates the graphical solution on a psychrometric chart for the air path through a dehumidifying coil with a constant refrigerant temperature. Because the tie-line slope is infinite in this case, the energy balance line is vertical. The corresponding interface states and air states are denoted by the same letter symbols, and the solution follows the same procedure as in Example 5.

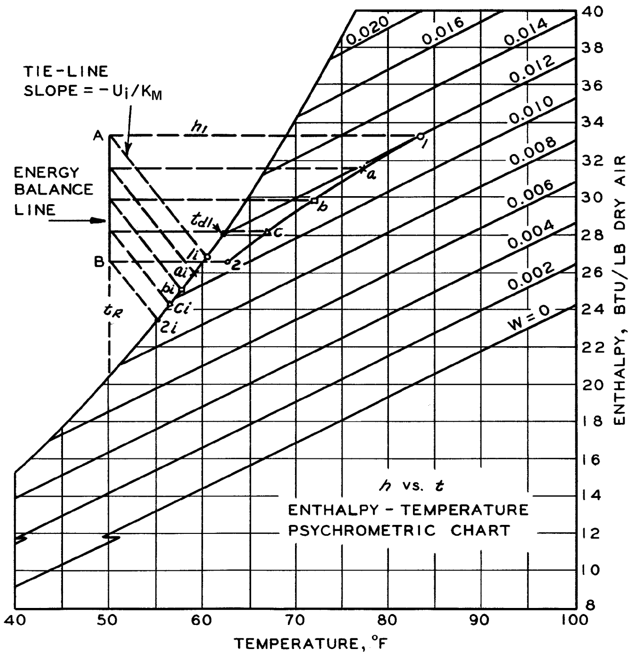

Fig. 16 Graphical Solution for Air-State Path in Dehumidifying Coil with Constant Refrigerant Temperature

If the problem is to determine the required coil surface area for a given performance, the area is computed by the following relation:

$$A = \frac{\dot{m}_a}{K_M} \int_1^2 \frac{dh}{(h_i - h)} \quad (69)$$

This graphical solution on the psychrometric chart automatically determines whether any part of the coil is dry. Thus, in the example illustrated in Figure 16, the entering air at state 1 initially encounters an interface saturation state 1_i, clearly below its dew-point temperature t_{d1}, so the coil immediately becomes wet. Had the graphical technique resulted in an initial interface state above the dew-point temperature of the entering air, the coil would be initially dry. The air would then follow a constant humidity ratio line (the sloping W = constant lines on the chart) until the interface state reached the air dew-point temperature.

Mizushina et al. (1959) developed this method not only for water vapor and air, but also for other vapor-gas mixtures. Chapter 21 of the *ASHRAE Handbook—Systems and Equipment* shows another related method, based on ARI *Standard* 410, of determining air-cooling and dehumidifying coil performance.

SYMBOLS

a = constant, dimensionless; or surface area per unit volume, ft²/ft³
A = surface area, ft²
A_{cs} = cross-sectional area, ft²
b = exponent, dimensionless
c_L = specific heat of liquid, Btu/lb·°F
c_p = specific heat at constant pressure, Btu/lb·°F
c_{pm} = specific heat of moist air at constant pressure, Btu/lb$_{da}$·°F
C = molal concentration of solute in solvent, lb mol/ft²
d = diameter, ft
D_v = diffusion coefficient (mass diffusivity), ft²/h
f = Fanning friction factor, dimensionless
g_c = gravitational constant, ft·lb$_m$/h²·lb$_f$
G = mass flux, flow rate per unit of cross-sectional area, lb$_m$/h·ft²
h = enthalpy, Btu/lb; or heat transfer coefficient, Btu/h·ft²·°F

h_{fg} = enthalpy of vaporization, Btu/lb$_m$
h_M = mass transfer coefficient, ft/h
j_D = Colburn mass transfer group = Sh/(Re·Sc$^{1/3}$), dimensionless
j_H = Colburn heat transfer group = Nu/(Re·Pr$^{1/3}$), dimensionless
J = diffusive mass flux, lb$_m$/h·ft^2
J^* = diffusive molar flux, lb mol/h·ft^2
k = thermal conductivity, Btu/h·ft·°F
K_M = mass transfer coefficient, lb/h·ft^2
l = length, ft
L = characteristic length, ft
L/G = liquid-to-air mass flow ratio
Le = Lewis number = α/D_v, dimensionless
$\dot{m}$ = rate of mass transfer, lb/h
$\dot{m}''$ = mass flux, lb/h·ft^2
$\dot{m}''^*$ = molar flux, lb mol/h·ft^2
M = molecular weight, lb$_m$/lb mol
Nu = Nusselt number = hL/k, dimensionless
p = pressure, atmospheres or psi
P_{Am} = logarithmic mean density factor
Pr = Prandtl number = $c_p\mu/k$, dimensionless
q = rate of heat transfer, Btu/h
q'' = heat flux per unit area, Btu/h·ft^2
Q = volumetric flow rate, cfm
R_i = combined thermal resistance, ft^2·°F·h/Btu
R_L = thermal resistance of condensate film, ft^2·°F·h/Btu
R_m = thermal resistance across metal wall and fins, ft^2·°F·h/Btu
R_r = thermal resistance of refrigerant film, ft^2·°F·h/Btu
R_u = universal gas constant = 1545 lb$_f$·ft/lb mol·°R
Re = Reynolds number = $\rho uL/\mu$, dimensionless
Sc = Schmidt number = $\mu/\rho D_v$, dimensionless
Sh = Sherwood number = $h_M L/D_v$, dimensionless
St = Stanton number = $h/\rho c_p \bar{u}$, dimensionless
St$_m$ = mass transfer Stanton number = $h_M P_{Am}/\bar{u}$, dimensionless
t = temperature, °F
T = absolute temperature, °R
u = velocity in x direction, fpm
U_i = overall conductance from refrigerant to air-water interface for dehumidifying coil, Btu/h·ft^2·°F
v = velocity in y direction, fpm
v_i = velocity normal to mass transfer surface for component i, ft/h
V = fluid stream velocity, fpm
W = humidity ratio, lb of water vapor per lb of dry air, lb(w)/lb(da)
x, y, z = coordinate direction, ft
X, Y, Z = coordinate direction, dimensionless
α = thermal diffusivity = $k/\rho c_p$, ft^2/h
ε_D = eddy mass diffusivity, ft^2/h
θ = dimensionless time parameter
μ = absolute (dynamic) viscosity, lb$_m$/ft·h
$\bar{\mu}$ = permeability, grains·in/h·ft^2·in. Hg
ν = kinematic viscosity, ft^2/h
ρ = mass density or concentration, lb$_m$/ft^3
σ = characteristic molecular diameter, nm
τ = time
τ_i = shear stress in the x-y coordinate plane, lb$_f$/ft^2
ω = mass fraction, lb/lb
$\Omega_{D,AB}$ = temperature function in Equation (9)

Subscripts

a = air property
Am = logarithmic mean
A = gas component of binary mixture
B = the more dilute gas component of binary mixture
c = critical state
da = dry air property or air-side transfer quantity
H = heat transfer quantity
i = air-water interface value
L = liquid
m = mean value or metal
M = mass transfer quantity
o = property evaluated at 0°F
s = water vapor property or transport quantity
w = water vapor
∞ = property of main fluid stream

Superscripts

$*$ = on molar basis
$\bar{}$ = average value
$'$ = wet bulb

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