

CHAPTER 2

FLUID FLOW

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FLOWING fluids in heating, ventilating, air-conditioning, and refrigeration systems transfer heat and mass. This chapter introduces the basics of fluid mechanics that are related to HVAC processes, reviews pertinent flow processes, and presents a general discussion of single-phase fluid flow analysis.

FLUID PROPERTIES

Fluids differ from solids in their reaction to shearing. When placed under shear stress, a solid deforms only a finite amount, whereas a fluid deforms continuously for as long as the shear is applied. Both liquids and gases are fluids. Although liquids and gases differ strongly in the nature of molecular actions, their primary mechanical differences are in the degree of compressibility and liquid formation of a free surface (interface).

Fluid motion can usually be described by one of several simplified modes of action or models. The simplest is the ideal-fluid model, which assumes no resistance to shearing. Ideal flow analysis is well developed (Baker 1983, Schlichting 1979, Streeter and Wylie 1979), and when properly interpreted is valid for a wide range of applications. Nevertheless, the effects of viscous action may need to be considered. Most fluids in HVAC applications can be treated as Newtonian, where the rate of deformation is directly proportional to the shearing stress. Turbulence complicates fluid behavior, and viscosity influences the nature of the turbulent flow.

Density

The density ρ of a fluid is its mass per unit volume. The densities of air and water at standard indoor conditions of 20°C and 101.325 kPa (sea level atmospheric pressure) are

$$\rho_{\text{water}} = 998 \text{ kg/m}^3$$

$$\rho_{\text{air}} = 1.20 \text{ kg/m}^3$$

Viscosity

Viscosity is the resistance of adjacent fluid layers to shear. For shearing between two parallel plates, each of area A and separated by distance Y , the tangential force F per unit area required to slide one plate with velocity V parallel to the other is proportional to V/Y :

$$F/A = \mu(V/Y)$$

where the proportionality factor μ is the **absolute viscosity** or **dynamic viscosity** of the fluid. The ratio of the tangential force F to area A is the **shearing stress** τ , and V/Y is the **lateral velocity gradient** (Figure 1A). In complex flows, velocity and shear stress may vary across the flow field; this is expressed by the following differential equation:

The preparation of this chapter is assigned to TC 1.3, Heat Transfer and Fluid Flow.

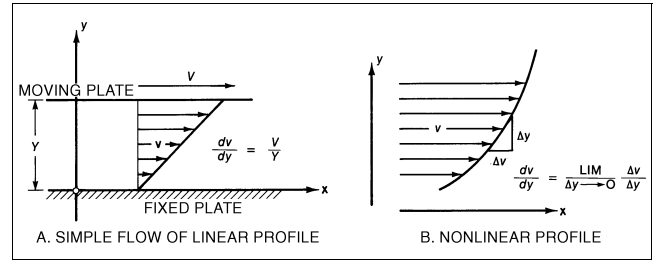


Fig. 1 Velocity Profiles and Gradients in Shear Flows

$$\tau = \mu \frac{dv}{dy} \quad (1)$$

The velocity gradient associated with viscous shear for a simple case involving flow velocity in the x direction but of varying magnitude in the y direction is illustrated in Figure 1B.

Absolute viscosity μ depends primarily on temperature. For gases (except near the critical point), viscosity increases with the square root of the absolute temperature, as predicted by the kinetic theory. Liquid viscosity decreases with increasing temperature. Viscosities of various fluids are given in Chapter 38.

Absolute viscosity has dimensions of force $\cdot$ time/length². At standard indoor conditions, the absolute viscosities of water and dry air are

$$\mu_{\text{water}} = 1.0 \text{ mN}\cdot\text{s/m}^2$$

$$\mu_{\text{air}} = 18 \text{ }\mu\text{N}\cdot\text{s/m}^2$$

In fluid dynamics, **kinematic viscosity** ν is the ratio of absolute viscosity to density:

$$\nu = \mu/\rho$$

At standard indoor conditions, the kinematic viscosities of water and dry air are

$$\nu_{\text{water}} = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\nu_{\text{air}} = 16 \times 10^{-6} \text{ m}^2/\text{s}$$

BASIC RELATIONS OF FLUID DYNAMICS

This section considers homogeneous, constant-property, incompressible fluids and introduces fluid dynamic considerations used in most analyses.

Continuity

Conservation of matter applied to fluid flow in a conduit requires that

$$\int \rho v \, dA = \text{constant}$$

where

v = velocity normal to the differential area dA

ρ = fluid density

Both ρ and v may vary over the cross section A of the conduit. If both ρ and v are constant over the cross-sectional area normal to the flow, then

$$\dot{m} = \rho VA = \text{constant} \quad (2a)$$

where $\dot{m}$ is the mass flow rate across the area normal to the flow. When flow is effectively incompressible, $\rho = \text{constant}$; in pipeline and duct flow analyses, the average velocity is then $V = (1/A) \int v \, dA$. The continuity relation is

$$Q = AV = \text{constant} \quad (2b)$$

where Q is the volumetric flow rate. Except when branches occur, Q is the same at all sections along the conduit.

For the ideal-fluid model, flow patterns around bodies (or in conduit section changes) result from displacement effects. An obstruction in a fluid stream, such as a strut in a flow or a bump on the conduit wall, pushes the flow smoothly out of the way, so that behind the obstruction, the flow becomes uniform again. The effect of fluid inertia (density) appears only in pressure changes.

Pressure Variation Across Flow

Pressure variation in fluid flow is important and can be easily measured. Variation across streamlines involves fluid rotation (vorticity). Lateral pressure variation across streamlines is given by the following relation (Bober and Kenyon 1980, Olson 1980, Robertson 1965):

$$\frac{\partial}{\partial r} \left(\frac{p}{\rho} + gz \right) = \frac{v^2}{r} \quad (3)$$

where

r = radius of curvature of the streamline

z = elevation

This relation explains the pressure difference found between the inside and outside walls of a bend and near other regions of conduit section change. It also states that pressure variation is hydrostatic ($p + \rho gz = \text{constant}$) across any conduit where streamlines are parallel.

Bernoulli Equation and Pressure Variation along Flow

A basic tool of fluid flow analysis is the Bernoulli relation, which involves the principle of energy conservation along a streamline. Generally, the Bernoulli equation is not applicable across streamlines. The first law of thermodynamics can be applied to mechanical flow energies (kinetic and potential) and thermal energies: heat is a form of energy and energy is conserved.

The change in energy content ΔE per unit mass of flowing material is a result from the work W done on the system plus the heat Q absorbed:

$$\Delta E = W + Q$$

Fluid energy is composed of kinetic, potential (due to elevation z), and internal (u) energies. Per unit mass of fluid, the above energy change relation between two sections of the system is

$$\Delta \left(\frac{v^2}{2} + gz + u \right) = E_M - \Delta \left(\frac{p}{\rho} \right) + Q$$

where the work terms are (1) the external work E_M from a fluid machine (E_M is positive for a pump or blower) and (2) the pressure or flow work p/ρ . Rearranging, the energy equation can be written as the **generalized Bernoulli equation**:

$$\Delta \left(\frac{v^2}{2} + gz + \frac{p}{\rho} \right) + \Delta u = E_M + Q \quad (4)$$

The term in parentheses in Equation (4) is the **Bernoulli constant**:

$$\frac{p}{\rho} + \frac{v^2}{2} + gz = B \quad (5a)$$

In cases with no viscous action and no work interaction, B is constant; more generally its change (or lack thereof) is considered in applying the Bernoulli equation. The terms making up B are fluid energies (pressure, kinetic, and potential) per mass rate of fluid flow. Alternative forms of this relation are obtained through multiplication by ρ or division by g :

$$p + \frac{\rho v^2}{2} + \rho gz = \rho B \quad (5b)$$

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{B}{g} \quad (5c)$$

The first form involves energies per volume flow rate, or pressures; the second involves energies per mass flow rate, or heads. In gas flow analysis, Equation (5b) is often used with the ρgz term dropped as negligible. Equation (5a) should be used when density variations occur. For liquid flows, Equation (5c) is commonly used. Identical results are obtained with the three forms if the units are consistent and the fluids are homogeneous.

Many systems of pipes or ducts and pumps or blowers can be considered as one-dimensional flow. The Bernoulli equation is then considered as velocity and pressure vary along the conduit. Analysis is adequate in terms of the section-average velocity V of Equation (2a) or (2b). In the Bernoulli relation [Equations (4) and (5)], v is replaced by V , and variation across streamlines can be ignored; the whole conduit is now taken as one streamline. Two- and three-dimensional details of local flow occurrences are still significant, but their effect is combined and accounted for in factors.

The kinetic energy term of the Bernoulli constant B is expressed as $\alpha V^2/2$, where the **kinetic energy factor** ($\alpha > 1$) expresses the ratio of the true kinetic energy of the velocity profile to that of the mean flow velocity.

For laminar flow in a wide rectangular channel, $\alpha = 1.54$, and for laminar flow in a pipe, $\alpha = 2.0$. For turbulent flow in a duct $\alpha \approx 1$.

Heat transfer Q may often be ignored. The change of mechanical energy into internal energy Δu may be expressed as E_L . Flow analysis involves the change in the Bernoulli constant ($\Delta B = B_2 - B_1$) between stations 1 and 2 along the conduit, and the Bernoulli equation can be expressed as

$$\left(\frac{p}{\rho} + \alpha \frac{V^2}{2} + gz\right)_1 + E_M = \left(\frac{p}{\rho} + \alpha \frac{V^2}{2} + gz\right)_2 + E_L \quad (6a)$$

or, dividing by g , in the form as

$$\left(\frac{p}{\rho g} + \alpha \frac{V^2}{2g} + z\right)_1 + H_M = \left(\frac{p}{\rho g} + \alpha \frac{V^2}{2g} + z\right)_2 + H_L \quad (6b)$$

The factors E_M and E_L are defined as positive, where $gH_M = E_M$ represents energy added to the conduit flow by pumps or blowers, and $gH_L = E_L$ represents energy dissipated, that is, converted into heat as mechanically nonrecoverable energy. A turbine or fluid motor thus has a negative H_M or E_M . For conduit systems with branches involving inflow or outflow, the total energies must be treated, and analysis is in terms of mB and not B .

When real-fluid effects of viscosity or turbulence are included, the continuity relation in Equation (2b) is not changed, but V must be evaluated from the integral of the velocity profile, using time-averaged local velocities.

In fluid flow past fixed boundaries, the velocity at the boundary is zero and shear stresses are produced. The equations of motion then become complex and exact solutions are difficult to find, except in simple cases.

Laminar Flow

For steady, fully developed laminar flow in a parallel-walled conduit, the shear stress τ varies linearly with distance y from the centerline. For a wide rectangular channel,

$$\tau = \left(\frac{y}{b}\right) \tau_w = \mu \frac{dv}{dy}$$

where

$$\begin{aligned} \tau_w &= \text{wall shear stress} = b \, (dp/ds) \\ 2b &= \text{wall spacing} \\ s &= \text{flow direction} \end{aligned}$$

Because the velocity is zero at the wall ($y = b$), the integrated result is

$$v = \left(\frac{b^2 - y^2}{2\mu}\right) \frac{dp}{ds}$$

This is the **Poiseuille-flow parabolic velocity profile** for a wide rectangular channel. The average velocity V is two-thirds the maximum velocity (at $y = 0$), and the longitudinal pressure drop in terms of conduit flow velocity is

$$\frac{dp}{ds} = -\left(\frac{3\mu V}{b^2}\right) \quad (7)$$

The parabolic velocity profile can also be derived for the axisymmetric conduit (pipe) of radius R but with a different constant. The average velocity is then half the maximum, and the pressure drop relation is

$$\frac{dp}{ds} = -\left(\frac{8\mu V}{R^2}\right) \quad (8)$$

Turbulence

Fluid flows are generally turbulent, involving random perturbations or fluctuations of the flow (velocity and pressure), characterized by an extensive hierarchy of scales or frequencies (Robertson 1963). Flow disturbances that are not random, but have some degree of periodicity, such as the oscillating vortex trail behind bodies,

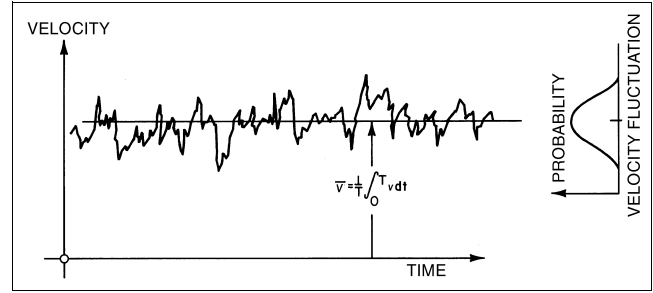


Fig. 2 Velocity Fluctuation at Point in Turbulent Flow

have been erroneously identified as turbulence. Only flows involving random perturbations without any order or periodicity are turbulent; the velocity in such a flow varies with time or locale of measurement (Figure 2).

Turbulence can be quantified by statistical factors. Thus, the velocity most often used in velocity profiles is the temporal average velocity $\bar{v}$, and the strength of the turbulence is characterized by the root-mean-square of the instantaneous variation in velocity about this mean. The effects of turbulence cause the fluid to diffuse momentum, heat, and mass very rapidly across the flow.

The **Reynolds number** Re , a dimensionless quantity, gives the relative ratio of inertial to viscous forces:

$$Re = VL/\nu$$

where

$$\begin{aligned} L &= \text{characteristic length} \\ \nu &= \text{kinematic viscosity} \end{aligned}$$

In flow through round pipes and tubes, the characteristic length is the diameter D . Generally, laminar flow in pipes can be expected if the Reynolds number, which is based on the pipe diameter, is less than about 2300. Fully turbulent flow exists when $Re_D > 10,000$. Between 2300 and 10,000, the flow is in a transition state and predictions are unreliable. In other geometries, different criteria for the Reynolds number exist.

BASIC FLOW PROCESSES

Wall Friction

At the boundary of real-fluid flow, the relative tangential velocity at the fluid surface is zero. Sometimes in turbulent flow studies, velocity at the wall may appear finite, implying a fluid slip at the wall. However, this is not the case; the difficulty is in velocity measurement (Goldstein 1938). Zero wall velocity leads to a high shear stress near the wall boundary and a slowing down of adjacent fluid layers. A velocity profile develops near a wall, with the velocity increasing from zero at the wall to an exterior value within a finite lateral distance.

Laminar and turbulent flow differ significantly in their velocity profiles. Turbulent flow profiles are flat compared to the more pointed profiles of laminar flow (Figure 3). Near the wall, velocities of the turbulent profile must drop to zero more rapidly than those of the laminar profile, so the shear stress and friction are much greater in the turbulent flow case. Fully developed conduit flow may be characterized by the **pipe factor**, which is the ratio of average to maximum (centerline) velocity. Viscous velocity profiles result in pipe factors of 0.667 and 0.50 for wide rectangular and axisymmetric conduits. Figure 4 indicates much higher values for rectangular and circular conduits for turbulent flow. Due to the flat velocity profiles, the kinetic energy factor α in Equation (6) ranges from 1.01 to 1.10 for fully developed turbulent pipe flow.

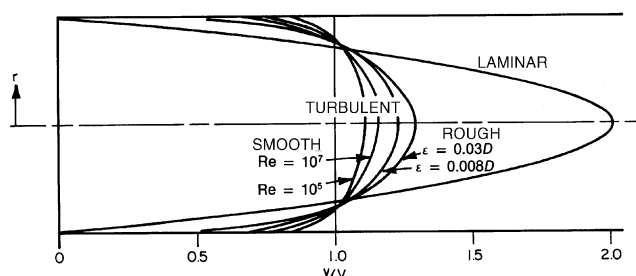


Fig. 3 Velocity Profiles of Flow in Pipes

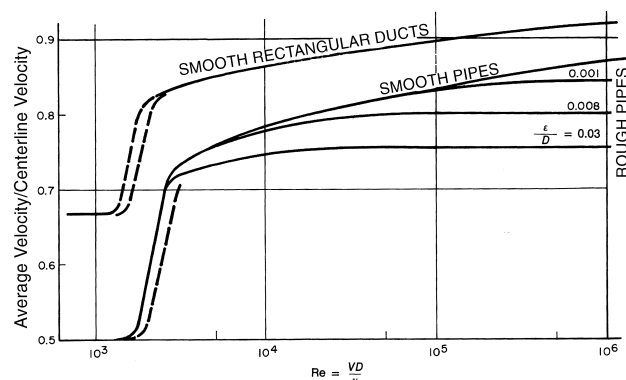


Fig. 4 Pipe Factor for Flow in Conduits

Boundary Layer

In most flows, the friction of a bounding wall on the fluid flow is evidenced by a boundary layer. For flow around bodies, this layer (which is quite thin relative to distances in the flow direction) encompasses all viscous or turbulent actions, causing the velocity in it to vary rapidly from zero at the wall to that of the outer flow at its edge. Boundary layers are generally laminar near the start of their formation but may become turbulent downstream of the transition point (Figure 5). For conduit flows, spacing between adjacent walls is generally small compared with distances in the flow direction. As a result, layers from the walls meet at the centerline to fill the conduit.

A significant boundary-layer occurrence exists in a pipeline or conduit following a well-rounded entrance (Figure 5). Layers grow from the walls until they meet at the center of the pipe. Near the start of the straight conduit, the layer is very thin (and laminar in all probability), so the uniform velocity core outside has a velocity only slightly greater than the average velocity. As the layer grows in thickness, the slower velocity near the wall requires a velocity increase in the uniform core to satisfy continuity. As the flow proceeds, the wall layers grow (and the centerline velocity increases) until they join, after an entrance length L_e . Application of the Bernoulli relation of Equation (5) to the core flow indicates a decrease in pressure along the layer. Ross (1956) shows that although the entrance length L_e is many diameters, the length in which the pressure drop significantly exceeds those for fully developed flow is on the order of 10 diameters for turbulent flow in smooth pipes.

In more general boundary-layer flows, as with wall layer development in a diffuser or for the layer developing along the surface of a strut or turning vane, pressure gradient effects can be severe and may even lead to separation. The development of a layer in an adverse-pressure gradient situation (velocity v_1 at edge $y = \delta$ of layer decreasing in flow direction) with separation is shown in Figure 6. Downstream from the separation point, fluid backflows near the wall. Separation is due to frictional velocity (thus local kinetic

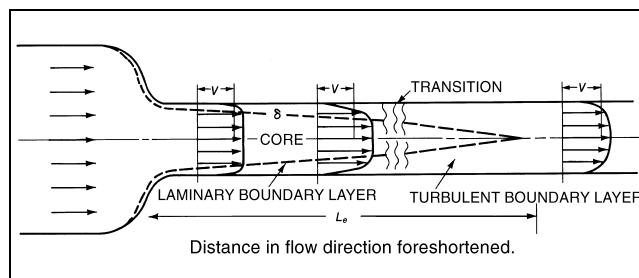


Fig. 5 Flow in Conduit Entrance Region

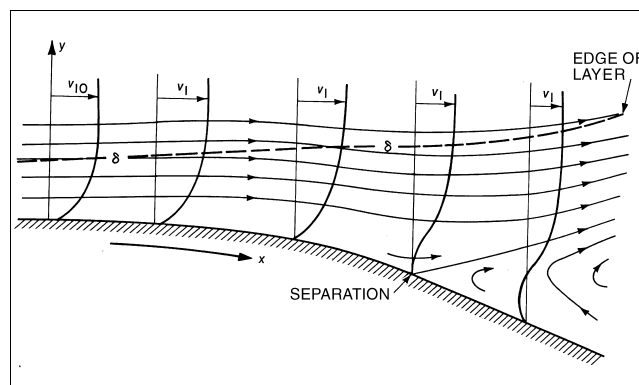


Fig. 6 Boundary Layer Flow to Separation

energy) reduction near the wall. Flow near the wall no longer has energy to move into the higher pressure imposed by the decrease in v_1 at the edge of the layer. The locale of this separation is difficult to predict, especially for the turbulent boundary layer. Analyses verify the experimental observation that a turbulent boundary layer is less subject to separation than a laminar one because of its greater kinetic energy.

Flow Patterns with Separation

In technical applications, flow with separation is common and often accepted if it is too expensive to avoid. Flow separation may be geometric or dynamic. Dynamic separation is shown in Figure 6. Geometric separation (Figures 7 and 8) results when a fluid stream passes over a very sharp corner, as with an orifice; the fluid generally leaves the corner irrespective of how much its velocity has been reduced by friction.

For geometric separation in orifice flow (Figure 7), the outer streamlines separate from the sharp corners and, because of fluid inertia, contract to a section smaller than the orifice opening, the **vena contracta**, with a limiting area of about six-tenths of the orifice opening. After the vena contracta, the fluid stream expands rather slowly through turbulent or laminar interaction with the fluid along its sides. Outside the jet, fluid velocity is small compared to that in the jet. Turbulence helps spread out the jet, increases the losses, and brings the velocity distribution back to a more uniform profile. Finally, at a considerable distance downstream, the velocity profile returns to the fully developed flow of Figure 3.

Other geometric separations (Figure 8) occur at a sharp entrance to a conduit, at an inclined plate or damper in a conduit, and at a sudden expansion. For these, a vena contracta can be identified; for sudden expansion, its area is that of the upstream contraction. Ideal-fluid theory, using free streamlines, provides insight and predicts contraction coefficients for valves, orifices, and vanes (Robertson 1965). These geometric flow separations are large loss-producing devices. To expand a flow efficiently or to have an entrance with

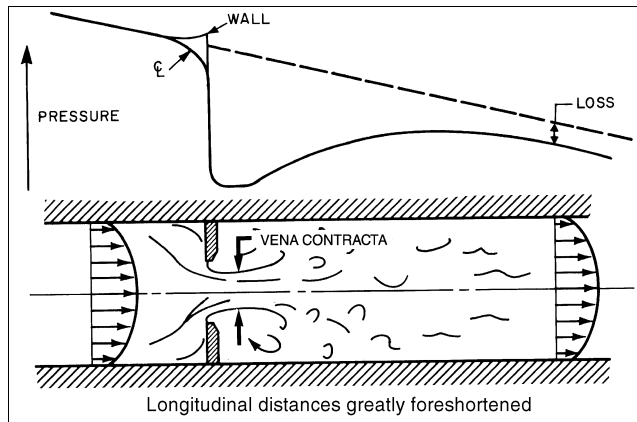


Fig. 7 Geometric Separation, Flow Development, and Loss in Flow Through Orifice

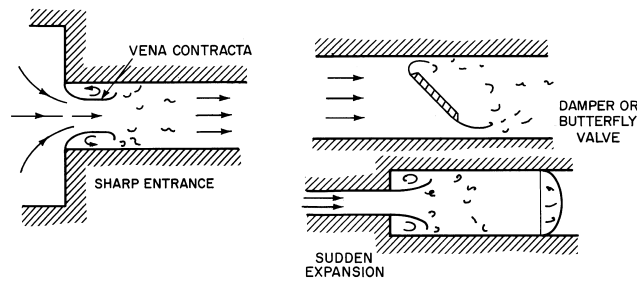


Fig. 8 Examples of Geometric Separation Encountered in Flows in Conduits

minimum losses, the device should be designed with gradual contours, a diffuser, or a rounded entrance.

Flow devices with gradual contours are subject to separation that is more difficult to predict, because it involves the dynamics of boundary layer growth under an adverse pressure gradient rather than flow over a sharp corner. In a diffuser, which is used to reduce the loss in expansion, it is possible to expand the fluid some distance at a gentle angle without difficulty (particularly if the boundary layer is turbulent). Eventually, separation may occur (Figure 9), which is frequently asymmetrical because of irregularities. Downstream flow involves flow reversal (backflow) and excess losses exist. Such separation is termed **stall** (Kline 1959). Larger area expansions may use splitters that divide the diffuser into smaller divisions less likely to have separations (Moore and Kline 1958). Another technique for controlling separation is to bleed some low-velocity fluid near the wall (Furuya et al. 1976). Alternatively, Heskestad (1965, 1970) shows that suction at the corner of a sudden expansion has a strong positive effect on geometric separation.

Drag Forces on Bodies or Struts

Bodies in moving fluid streams are subjected to appreciable fluid forces or drag. Conventionally expressed in coefficient form, drag forces on bodies can be expressed as

$$D = C_D \rho A V^2 / 2 \quad (9)$$

where A is the projected (normal to flow) area of the body. The **drag coefficient** C_D depends on the body's shape and angularity and on the Reynolds number of the relative flow in terms of the body's characteristic dimension.

For Reynolds numbers of 10^3 to above 10^5 , the C_D of most bodies is constant due to flow separation, but above 10^5 , the C_D of

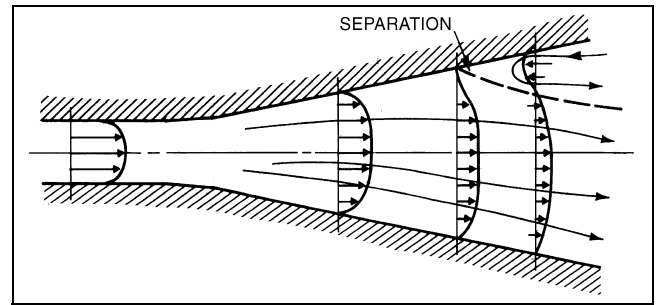


Fig. 9 Separation in Flow in Diffuser

Table 1 Drag Coefficients

Body Shape	$10^3 < Re < 2 \times 10^5$	$Re > 3 \times 10^5$
Sphere	0.36 to 0.47	~0.1
Disk	1.12	1.12
Streamlined strut	0.1 to 0.3	< 0.1
Circular cylinder	1.0 to 1.1	0.35
Elongated rectangular strut	1.0 to 1.2	1.0 to 1.2
Square strut	~2.0	~2.0

rounded bodies drops suddenly as the surface boundary layer undergoes transition to turbulence. Typical C_D values are given in Table 1; Hoerner (1965) gives expanded values.

For a strut crossing a conduit, the contribution to the loss of Equation (6b) is

$$H_L = C_D \left(\frac{A}{A_c} \right) \left(\frac{V^2}{2g} \right) \quad (10)$$

where

A_c = conduit cross-sectional area
 A = area of the strut facing the flow

Cavitation

Liquid flow with gas- or vapor-filled pockets can occur if the absolute pressure is reduced to vapor pressure or less. In this case, a cavity or series of cavities forms, because liquids are rarely pure enough to withstand any tensile stressing or pressures less than vapor pressure for any length of time (John and Haberman 1980, Knapp et al. 1970, Robertson and Wislicenus 1969). Robertson and Wislicenus (1969) indicate significant occurrences in various technical fields, chiefly in hydraulic equipment and turbomachines.

Initial evidence of cavitation is the collapse noise of many small bubbles that appear initially as they are carried by the flow into regions of higher pressure. The noise is not deleterious and serves as a warning of the occurrence. As flow velocity further increases or pressure decreases, the severity of cavitation increases. More bubbles appear and may join to form large fixed cavities. The space they occupy becomes large enough to modify the flow pattern and alter performance of the flow device. Collapse of the cavities on or near solid boundaries becomes so frequent that the cumulative impact in time results in damage in the form of cavitation erosion of the surface or excessive vibration. As a result, pumps can lose efficiency or their parts may erode locally. Control valves may be noisy or seriously damaged by cavitation.

Cavitation in orifice and valve flow is indicated in Figure 10. With high upstream pressure and a low flow rate, no cavitation occurs. As pressure is reduced or flow rate increased, the minimum pressure in the flow (in the shear layer leaving the edge of the orifice) eventually approaches vapor pressure. Turbulence in this layer causes fluctuating pressures below the mean (as in vortex cores) and small bubble-like cavities. These are carried downstream into the

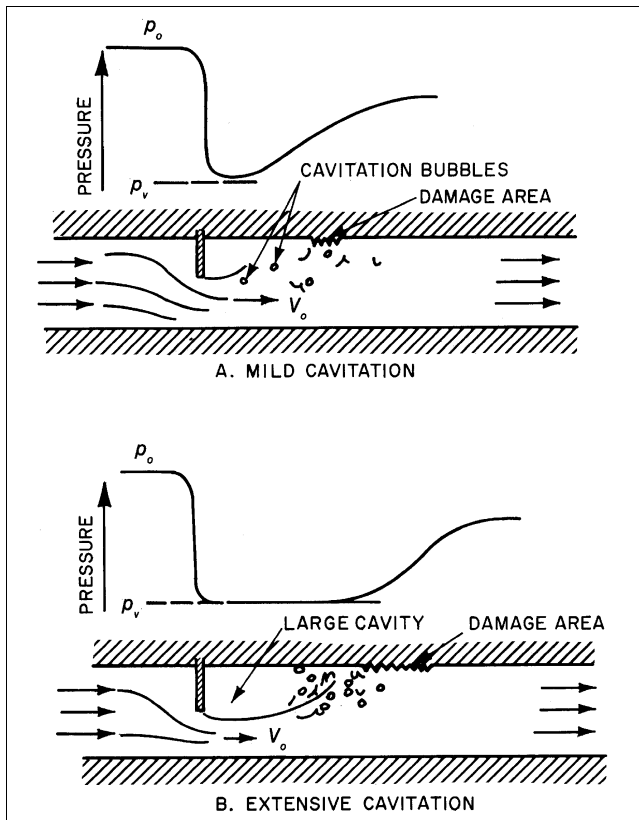


Fig. 10 Cavitation in Flows in Orifice or Valve

region of pressure regain where they collapse, either in the fluid or on the wall (Figure 10A). As the pressure is reduced, more vapor- or gas-filled bubbles result and coalesce into larger ones. Eventually, a single large cavity results that collapses further downstream (Figure 10B). The region of wall damage is then as many as 20 diameters downstream from the valve or orifice plate.

Sensitivity of a device to cavitation occurrence is measured by the **cavitation index** or **cavitation number**, which is the ratio of the available pressure above vapor pressure to the dynamic pressure of the reference flow:

$$\sigma = \frac{2(p_o - p_v)}{\rho V_o^2} \quad (11)$$

where p_v is the vapor pressure, and the subscript o refers to appropriate reference conditions. Valve analyses use such an index in order to determine when cavitation will affect the discharge coefficient (Ball 1957). With flow-metering devices such as orifices, venturis, and flow nozzles, there is little cavitation, because it occurs mostly downstream of the flow regions involved in establishing the metering action.

The detrimental effects of cavitation can be avoided by operating the liquid-flow device at high enough pressures. When this is not possible, the flow must be changed or the device must be built to withstand cavitation effects. Some materials or surface coatings are more resistant to cavitation erosion than others, but none is immune. Surface contours can be designed to delay the onset of cavitation.

Nonisothermal Effects

When appreciable temperature variations exist, the primary fluid properties (density and viscosity) are no longer constant, as usually

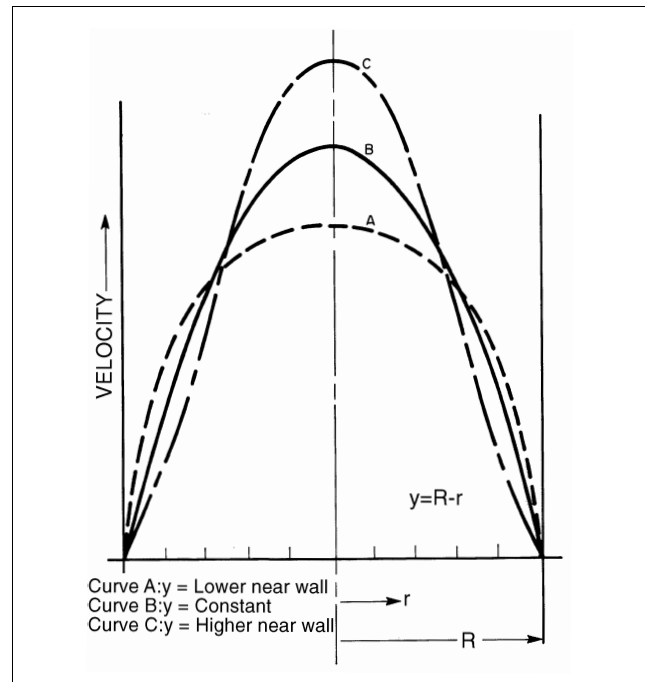


Fig. 11 Effect of Viscosity Variation on Velocity Profile of Laminar Flow in Pipe

assumed, but vary across or along the flow. The Bernoulli equation in the form of Equations (5a) through (5c) must be used, because volumetric flow is not constant. With gas flows, the thermodynamic process involved must be considered. In general, this is assessed in applying Equation (5a), written in the following form:

$$\int \frac{dp}{\rho} + \frac{V^2}{2} + gz = B \quad (12)$$

Effects of viscosity variations also appear. With nonisothermal laminar flow, the parabolic velocity profile (Figure 3) is no longer valid. For gases, viscosity increases as the square root of absolute temperature, and for liquids, it decreases with increasing temperature. This results in opposite effects.

For fully developed pipe flow, the linear variation in shear stress from the wall value τ_w to zero at the centerline is independent of the temperature gradient. In the section on Laminar Flow, τ is defined as $\tau = (y/b)\tau_w$, where y is the distance from the centerline and $2b$ is the wall spacing. For pipe radius $R = D/2$ and distance from the wall $y = R - r$ (see Figure 11), then $\tau = \tau_w (R - y)/R$. Then, solving Equation (1) for the change in velocity gives

$$dv = \left[\frac{\tau_w (R - y)}{R\mu} \right] dy = - \left(\frac{\tau_w}{R\mu} \right) r dr \quad (13)$$

When the fluid has a lower viscosity near the wall than at the center (due to external heating of liquid or cooling of gas via heat transfer through the pipe wall), the velocity gradient is steeper near the wall and flatter near the center, so the profile is generally flattened. When liquid is cooled or gas is heated, the velocity profile becomes more pointed for laminar flow (Figure 11). Calculations were made for such flows of gases and liquid metals in pipes (Deissler 1951). Occurrences in turbulent flow are less apparent. If enough heating is applied to gaseous flows, the viscosity increase can cause reversion to laminar flow.

Buoyancy effects and gradual approach of the fluid temperature to equilibrium with that outside the pipe can cause considerable variation in the velocity profile along the conduit. Thus, Colborne and Drobitch (1966) found the pipe factor for upward vertical flow of hot air at a Reynolds number less than 2000 reduced to about 0.6 at 40 diameters from the entrance, then increased to about 0.8 at 210 diameters, and finally decreased to the isothermal value of 0.5 at the end of 320 diameters.

Compressibility

All fluids are compressible to some degree; their density depends on the pressure. Steady liquid flow may ordinarily be treated as incompressible, and incompressible flow analysis is satisfactory for gases and vapors at velocities below about 20 to 40 m/s, except in long conduits.

For liquids in pipelines, if flow is suddenly stopped, a severe pressure surge or water hammer is produced that travels along the pipe at the speed of sound in the liquid. This pressure surge alternately compresses and decompresses the liquid. For steady gas flows in long conduits, a decrease in pressure along the conduit can reduce the density of the gas significantly enough to cause the velocity to increase. If the conduit is long enough, velocities approaching the speed of sound are possible at the discharge end, and the Mach number (the ratio of the flow velocity to the speed of sound) must be considered.

Some compressible flows occur without heat gain or loss (adiabatically). If there is no friction (conversion of flow mechanical energy into internal energy), the process is reversible as well. Such a reversible adiabatic process is called isentropic, and follows the relationship

$$\begin{aligned} p/\rho^k &= \text{constant} \\ k &= c_p/c_v \end{aligned}$$

where k , the ratio of specific heats at constant pressure and volume, has a value of 1.4 for air and diatomic gases.

The Bernoulli equation of steady flow, Equation (12), as an integral of the ideal-fluid equation of motion along a streamline, then becomes

$$\int \frac{dp}{\rho} + \frac{V^2}{2} = \text{constant} \quad (14)$$

where, as in most compressible flow analyses, the elevation terms involving z are insignificant and are dropped.

For a frictionless adiabatic process, the pressure term has the form

$$\int_1^2 \frac{dp}{\rho} = \frac{k}{k-1} \left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} \right) \quad (15)$$

Then, between stations 1 and 2 for the isentropic process,

$$\frac{p_1}{\rho_1} \left(\frac{k}{k-1} \right) \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right] + \frac{V_2^2 - V_1^2}{2} = 0 \quad (16)$$

Equation (16) replaces the Bernoulli equation for compressible flows and may be applied to the stagnation point at the front of a body. With this point as station 2 and the upstream reference flow ahead of the influence of the body as station 1, $V_2 = 0$. Solving Equation (16) for p_2 gives

$$p_s = p_2 = p_1 \left[1 + \left(\frac{k-1}{2} \right) \frac{\rho_1 V_1^2}{k p_1} \right]^{k/(k-1)} \quad (17)$$

where p_s is the stagnation pressure.

Because $k\rho/V$ is the square of the acoustic velocity a and the Mach number $M = V/a$, the stagnation pressure relation becomes

$$p_s = p_1 \left[1 + \left(\frac{k-1}{2} \right) M_1^2 \right]^{k/(k-1)} \quad (18)$$

For Mach numbers less than one,

$$p_s = p_1 + \frac{\rho_1 V_1^2}{2} \left[1 + \frac{M_1}{4} + \left(\frac{2-k}{24} \right) M_1^4 + \dots \right] \quad (19)$$

When $M = 0$, Equation (19) reduces to the incompressible flow result obtained from Equation (5a). Appreciable differences appear when the Mach number of the approaching flow exceeds 0.2. Thus a pitot tube in air is influenced by compressibility at velocities over about 66 m/s.

Flows through a converging conduit, as in a flow nozzle, venturi, or orifice meter, also may be considered isentropic. Velocity at the upstream station 1 is negligible. From Equation (16), velocity at the downstream station is

$$V_2 = \sqrt{\frac{2k}{k-1} \left(\frac{p_1}{\rho_1} \right) \left[1 - \left(\frac{p_2}{p_1} \right)^{(k-1)/k} \right]} \quad (20)$$

The mass flow rate is

$$\begin{aligned} \dot{m} &= V_2 A_2 \rho_2 = \\ &A_2 \sqrt{\frac{2k}{k-1} (p_1 \rho_1) \left[\left(\frac{p_2}{p_1} \right)^{2/k} - \left(\frac{p_2}{p_1} \right)^{(k+1)/k} \right]} \end{aligned} \quad (21)$$

The corresponding incompressible flow relation is

$$\dot{m}_{in} = A_2 \rho \sqrt{2 \Delta p / \rho} = A_2 \sqrt{2 \rho (p_1 - p_2)} \quad (22)$$

The compressibility effect is often accounted for in the **expansion factor Y** :

$$\dot{m} = Y \dot{m}_{in} = A_2 Y \sqrt{2 \rho (p_1 - p_2)} \quad (23)$$

Y is 1.00 for the incompressible case. For air ($k = 1.4$), a Y value of 0.95 is reached with orifices at $p_2/p_1 = 0.83$ and with venturis at about 0.90, when these devices are of relatively small diameter (D_2/D_1 less than 0.5).

As p_2/p_1 decreases, the flow rate increases, but more slowly than for the incompressible case because of the nearly linear decrease in Y . However, the downstream velocity reaches the local acoustic value and the discharge levels off at a value fixed by upstream pressure and density at the critical ratio:

$$\left. \frac{p_2}{p_1} \right|_c = \left(\frac{2}{k+1} \right)^{k/(k-1)} = 0.53 \text{ for air} \quad (24)$$

At higher pressure ratios than critical, **choking** (no increase in flow with decrease in downstream pressure) occurs and is used in some

flow control devices to avoid flow dependence on downstream conditions.

FLOW ANALYSIS

Fluid flow analysis is used to correlate pressure changes with flow rates and the nature of the conduit. For a given pipeline, either the pressure drop for a certain flow rate, or the flow rate for a certain pressure difference between the ends of the conduit, is needed. Flow analysis ultimately involves comparing a pump or blower to a conduit piping system for evaluating the expected flow rate.

Generalized Bernoulli Equation

Internal energy differences are generally small and usually the only significant effect of heat transfer is to change the density ρ . For gas or vapor flows, use the generalized Bernoulli equation in the pressure-over-density form of Equation (6a), allowing for the thermodynamic process in the pressure-density relation:

$$\int_1^2 \frac{dp}{\rho} + \alpha_1 \frac{V_1^2}{2} + E_M = \alpha_2 \frac{V_2^2}{2} + E_L \quad (25a)$$

The elevation changes involving z are often negligible and are dropped. The pressure form of Equation (5b) is generally unacceptable when appreciable density variations occur, because the volumetric flow rate differs at the two stations. This is particularly serious in friction-loss evaluations where the density usually varies over considerable lengths of conduit (Benedict and Carlucci 1966). When the flow is essentially incompressible, Equation (25a) is satisfactory.

Example 1. Specify the blower to produce an isothermal airflow of 200 L/s through a ducting system (Figure 12). Accounting for intake and fitting losses, the equivalent conduit lengths are 18 and 50 m and the flow is isothermal. The pressure at the inlet (station 1) and following the discharge (station 4), where the velocity is zero, is the same. The frictional losses H_L are evaluated as 7.5 m of air between stations 1 and 2, and 72.3 m between stations 3 and 4.

Solution: The following form of the generalized Bernoulli relation is used in place of Equation (25a), which also could be used:

$$(p_1/\rho_1 g) + \alpha_1 (V_1^2/2g) + z_1 + H_M = (p_2/\rho_2 g) + \alpha_2 (V_2^2/2g) + z_2 + H_L \quad (25b)$$

The term $V_1^2/2g$ can be calculated as follows:

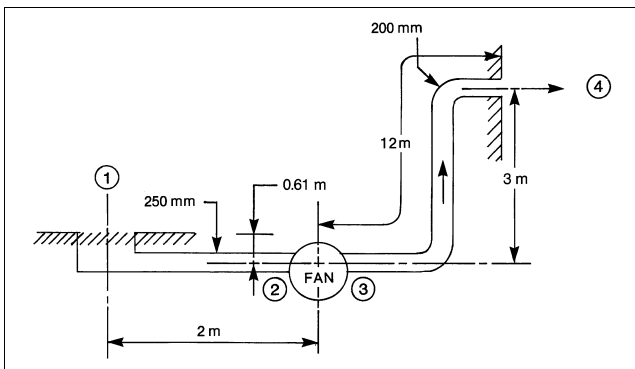


Fig. 12 Blower and Duct System for Example 1

$$A_1 = \pi \left(\frac{D}{2} \right)^2 = \pi \left(\frac{0.250}{2} \right)^2 = 0.0491 \text{ m}^2$$

$$V_1 = Q/A_1 = \frac{0.200}{0.0491} = 4.07 \text{ m/s}$$

$$V_1^2/2g = (4.07)^2/2(9.8) = 0.846 \text{ m}$$

The term $V_2^2/2g$ can be calculated in a similar manner.

In Equation (25b), H_M is evaluated by applying the relation between any two points on opposite sides of the blower. Because conditions at stations 1 and 4 are known, they are used, and the location-specifying subscripts on the right side of Equation (25b) are changed to 4. Note that $p_1 = p_4 = p$, $\rho_1 = \rho_4 = \rho$, and $V_1 = V_4 = 0$. Thus,

$$(p/\rho g) + 0 + 0.61 + H_M = (p/\rho g) + 0 + 3 + (7.5 + 72.3)$$

so $H_M = 82.2$ m of air. For standard air ($\rho = 1.20 \text{ kg/m}^3$), this corresponds to 970 Pa.

The pressure difference measured across the blower (between stations 2 and 3), is often taken as the H_M . It can be obtained by calculating the static pressure at stations 2 and 3. Applying Equation (25b) successively between stations 1 and 2 and between 3 and 4 gives

$$(p_1/\rho g) + 0 + 0.61 + 0 = (p_2/\rho g) + (1.06 \times 0.846) + 0 + 7.5$$

$$(p_3/\rho g) + (1.03 \times 2.07) + 0 + 0 = (p_4/\rho g) + 0 + 3 + 72.3$$

where α just ahead of the blower is taken as 1.06, and just after the blower as 1.03; the latter value is uncertain because of possible uneven discharge from the blower. Static pressures p_1 and p_4 may be taken as zero gage. Thus,

$$p_2/\rho g = -7.8 \text{ m of air}$$

$$p_3/\rho g = 73.2 \text{ m of air}$$

The difference between these two numbers is 81 m, which is *not* the H_M calculated after Equation (25b) as 82.2 m. The apparent discrepancy results from ignoring the velocity at stations 2 and 3. Actually, H_M is the following:

$$\begin{aligned} H_M &= (p_3/\rho g) + \alpha_3 (V_3^2/2g) - [(p_2/\rho g) + \alpha_2 (V_2^2/2g)] \\ &= 73.2 + (1.03 \times 2.07) - [-7.8 + (1.06 \times 0.846)] \\ &= 75.3 - (-6.9) = 82.2 \text{ m} \end{aligned}$$

The required blower energy is the same, no matter how it is evaluated. It is the specific energy added to the system by the machine. Only when the conduit size and velocity profiles on both sides of the machine are the same is E_M or H_M simply found from $\Delta p = p_3 - p_2$.

Conduit Friction

The loss term E_L or H_L of Equation (6a) or (6b) accounts for friction caused by conduit-wall shearing stresses and losses from conduit-section changes. H_L is the loss of energy per unit weight (J/N) of flowing fluid.

In real-fluid flow, a frictional shear occurs at bounding walls, gradually influencing the flow further away from the boundary. A lateral velocity profile is produced and flow energy is converted into heat (fluid internal energy), which is generally unrecoverable (a loss). This loss in fully developed conduit flow is evaluated through the **Darcy-Weisbach equation**:

$$(H_L)_f = f \left(\frac{L}{D} \right) \left(\frac{V^2}{2g} \right) \quad (26)$$

where L is the length of conduit of diameter D and f is the **friction factor**. Sometimes a numerically different relation is used with the **Fanning friction factor** (one-quarter of f). The value of f is nearly constant for turbulent flow, varying only from about 0.01 to 0.05.

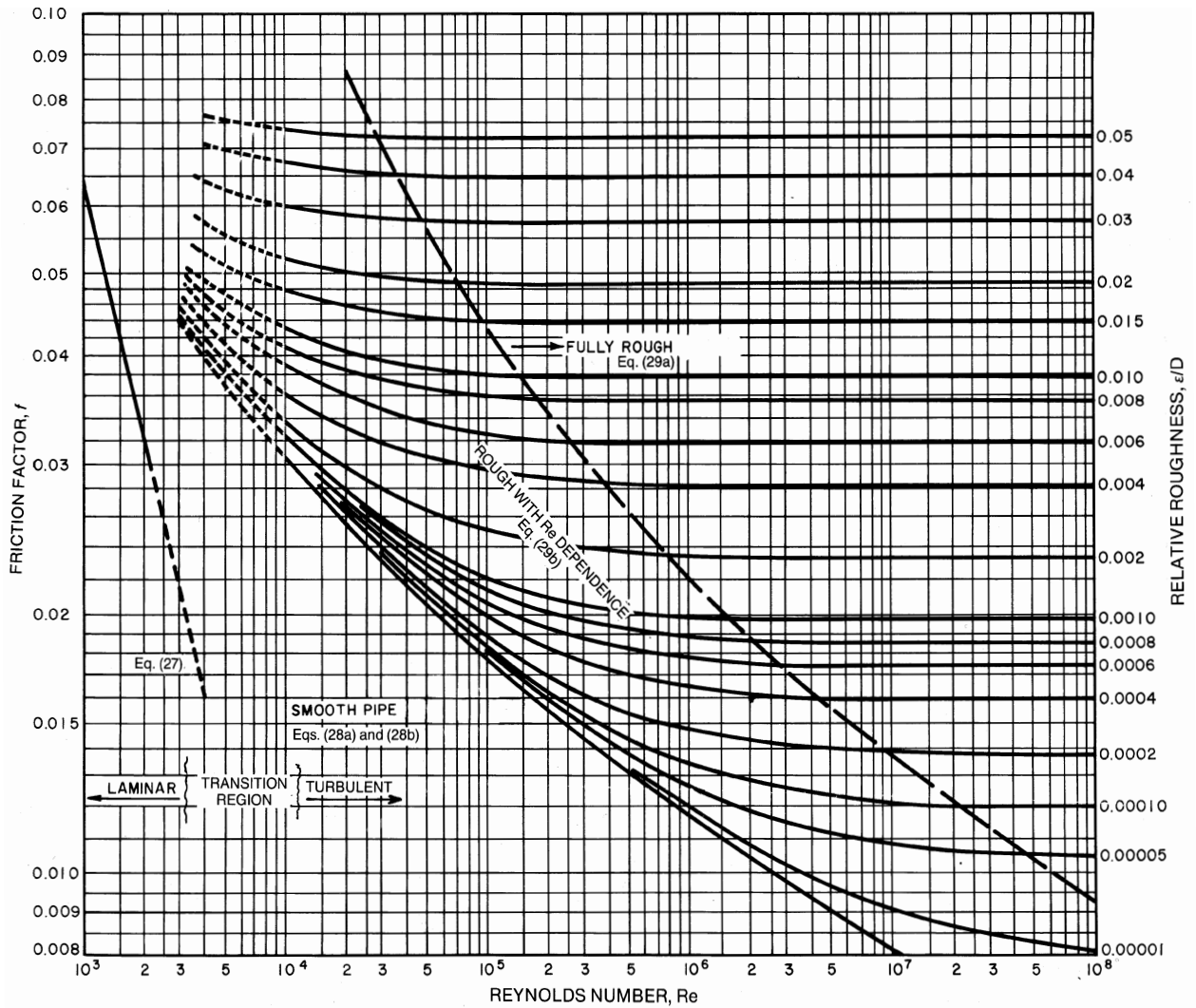


Fig. 13 Relation Between Friction Factor and Reynolds Number
(Moody 1944)

For fully developed laminar-viscous flow in a pipe, the loss is evaluated from Equation (8) as follows:

$$(H_L)_f = \frac{L}{\rho g} \left(\frac{8\mu V}{R^2} \right) = \frac{32L\nu V}{D^2 g} = \frac{64}{VD/\nu} \left(\frac{L}{D} \right) \left(\frac{V^2}{2g} \right) \quad (27)$$

where $Re = VD/\nu$ and $f = 64/Re$. Thus, for laminar flow, the friction factor varies inversely with the Reynolds number.

With turbulent flow, friction loss depends not only on flow conditions, as characterized by the Reynolds number, but also on the nature of the conduit wall surface. For smooth conduit walls, empirical correlations give

$$f = \frac{0.3164}{Re^{0.25}} \quad \text{for } Re < 10^5 \quad (28a)$$

$$f = 0.0032 + \frac{0.221}{Re^{0.237}} \quad \text{for } 10^5 < Re < 3 \times 10^6 \quad (28b)$$

Generally, f also depends on the wall roughness ϵ . The variation is complex and best expressed in chart form (Moody 1944) as shown

in Figure 13. Inspection indicates that, for high Reynolds numbers and relative roughness, the friction factor becomes independent of the Reynolds number in a fully-rough flow regime. Then

$$\frac{1}{\sqrt{f}} = 1.14 + 2 \log (D/\epsilon) \quad (29a)$$

Values of f between the values for smooth tubes and those for the fully-rough regime are represented by **Colebrook's natural roughness function**:

$$\frac{1}{\sqrt{f}} = 1.14 + 2 \log (D/\epsilon) - 2 \log \left[1 + \frac{9.3}{Re(\epsilon/D)\sqrt{f}} \right] \quad (29b)$$

A transition region appears in Figure 13 for Reynolds numbers between 2000 and 10 000. Below this critical condition, for smooth walls, Equation (27) is used to determine f ; above the critical condition, Equation (28b) is used. For rough walls, Figure 13 or Equation (29b) must be used to assess the friction factor in turbulent flow. To do this, the roughness height ϵ , which may increase with conduit use or aging, must be evaluated from the conduit surface (Table 2).

Table 2 Effective Roughness of Conduit Surfaces

Material	ϵ , μm
Commercially smooth brass, lead, copper, or plastic pipe	1.52
Steel and wrought iron	46
Galvanized iron or steel	152
Cast iron	259

Although the preceding discussion has focused on circular pipes and ducts, air ducts are often rectangular in cross section. The equivalent circular conduit corresponding to the noncircular conduit must be found before [Figure 13](#) or Equations (28) or (29) can be used. Based on turbulent flow concepts, the **equivalent diameter** is determined by

$$D_{eq} = 4A/P_w \quad (30)$$

where

A = flow area

P_w = wetted perimeter of the cross section

For turbulent flow, D_{eq} is substituted for D in Equation (26) and the Reynolds number definition in Equation (27). Noncircular duct friction can be evaluated to within 5% for all except very extreme cross sections. A more refined method for finding the equivalent circular duct diameter is given in [Chapter 34](#). With laminar flow, the loss predictions may be off by a factor as large as two.

Section Change Effects and Losses

Valve and section changes (contractions, expansions and diffusers, elbows or bends, tees), as well as entrances, distort the fully developed velocity profiles ([Figure 3](#)) and introduce extra flow losses (dissipated as heat) into pipelines or duct systems. Valves produce such extra losses to control flow rate. In contractions and expansions, flow separation as shown in [Figures 8](#) and [9](#) causes the extra loss. The loss at rounded entrances develops as the flow accelerates to higher velocities. The resulting higher velocity near the wall leads to wall shear stresses greater than those of fully developed flow ([Figure 5](#)). In flow around bends, the velocity increases along the inner wall near the start of the bend. This increased velocity creates a secondary motion, which is a double helical vortex pattern of flow downstream from the bend. In all these devices, the disturbance produced locally is converted into turbulence and appears as a loss in the downstream region.

The return of disturbed flow to a fully developed velocity profile is quite slow. Ito (1962) showed that the secondary motion following a bend takes up to 100 diameters of conduit to die out but the pressure gradient settles out after 50 diameters.

With laminar flow following a rounded entrance, the entrance length depends on the Reynolds number:

$$L_e/D \approx 0.06 \text{ Re} \quad (31)$$

At $\text{Re} = 2000$, a length of 120 diameters is needed to establish the parabolic profile. The pressure gradient reaches the developed value of Equation (26) much sooner. The extra drop is $1.2V^2/2g$; the change in profile from uniform to parabolic results in a drop of $1.0V^2/2g$ (since $\alpha = 2.0$), and the rest is due to excess friction. With turbulent flow, 80 to 100 diameters following the rounded entrance are needed for the velocity profile to become fully developed, but the friction loss per unit length reaches a value close to that of the fully developed flow value more quickly. After six diameters, the loss rate at a Reynolds number of 10^5 is only 14% above that of fully developed flow in the same length, while at 10^7 , it is only 10% higher (Robertson 1963). For a sharp entrance, the flow separation ([Figure 8](#)) causes a greater disturbance, but fully developed flow is achieved in about half the length required for a rounded entrance. With

sudden expansion, the pressure change settles out in about eight times the diameter change ($D_2 = D_1$), while the velocity profile takes at least a 50% greater distance to return to fully developed pipe flow (Lipstein 1962).

These disturbance effects are assumed compressed (in the flow direction) into a point, and the losses are treated as locally occurring. Such a loss is related to the velocity by the **fitting loss coefficient** K :

$$\text{Loss of section} = K \left(\frac{V^2}{2g} \right) \quad (32)$$

[Chapter 35](#) and the *Pipe Friction Manual* (Hydraulic Institute 1961) have information for pipe applications. [Chapter 34](#) gives information for airflow. The same type of fitting in pipes and ducts may give a different loss, because flow disturbances are controlled by the detailed geometry of the fitting. The elbow of a small pipe may be a threaded fitting that differs from a bend in a circular duct. For 90 screw-fitting elbows, K is about 0.8 (Ito 1962), whereas smooth flanged elbows have a K as low as 0.2 at the optimum curvature.

[Table 3](#) gives a list of fitting loss coefficients. These values indicate the losses, but there is considerable variance. Expansion flows, such as from one conduit size to another or at the exit into a room or reservoir, are not included. For such occurrences, the **Borda loss prediction** (from impulse-momentum considerations) is appropriate:

$$\text{Loss at expansion} = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g} \left(1 - \frac{A_1}{A_2} \right)^2 \quad (33)$$

Such expansion loss is reduced by avoiding or delaying separation using a gradual diffuser ([Figure 9](#)). For a diffuser of about 7° total angle, the loss is minimal, about one-sixth that given by Equation (33). The diffuser loss for total angles above 45° to 60° exceeds that of the sudden expansion, depending somewhat on the diameter ratio of the expansion. Optimum design of diffusers involves many factors; excellent performance can be achieved in short diffusers with splitter vanes or suction. Turning vanes in miter bends produce the least disturbance and loss for elbows; with careful design, the loss coefficient can be reduced to as low as 0.1.

For losses in smooth elbows, Ito (1962) found a Reynolds number effect (K slowly decreasing with increasing Re) and a minimum loss at a bend curvature (bend radius to diameter ratio) of 2.5. At this optimum curvature, a 45° turn had 63%, and a 180° turn approximately 120%, of the loss of a 90° bend. The loss does not vary linearly with the turning angle because secondary motion occurs.

Use of coefficient K presumes its independence of the Reynolds number. Crane Co. (1976) found a variation with the Reynolds number similar to that of the friction factor; Kittridge and Rowley (1957) observed it only with laminar flow. Assuming that K varies with Re similarly to f , it is convenient to represent fitting losses as adding to the effective length of uniform conduit. The **effective length** of a fitting is then

$$L_{eff}/D = K/f_{ref} \quad (34)$$

where f_{ref} is an appropriate reference value of the friction factor. Deissler (1951) uses 0.028, and the air duct values in [Chapter 34](#) are based on an f_{ref} of about 0.02. For rough conduits, appreciable errors can occur if the relative roughness does not correspond to that used when f_{ref} was fixed. It is unlikely that the fitting losses involving separation are affected by pipe roughness. The effective length method for fitting loss evaluation is still useful.

When a conduit contains a number of section changes or fittings, the values of K are added to the fL/D friction loss, or the L_{eff}/D of the fittings are added to the conduit length L/D for evaluating the total loss H_L . This assumes that each fitting loss is fully developed and its disturbance fully smoothed out before the next section

change. Such an assumption is frequently wrong, and the total loss can be overestimated. For elbow flows, the total loss of adjacent bends may be over or underestimated. The secondary flow pattern following an elbow is such that when one follows another, perhaps in a different plane, the secondary flow of the second elbow may reinforce or partially cancel that of the first. Moving the second elbow a few diameters can reduce the total loss (from more than twice the amount) to less than the loss from one elbow. Screens or perforated plates can be used for smoothing velocity profiles (Wile 1947) and flow spreading. Their effectiveness and loss coefficients depend on their amount of open area (Baines and Peterson 1951).

Compressible Conduit Flow

When friction loss is included, as it must be except for a very short conduit, the incompressible flow analysis previously considered applies until the pressure drop exceeds about 10% of the initial pressure. The possibility of sonic velocities at the end of relatively long conduits limits the amount of pressure reduction achieved. For an inlet Mach number of 0.2, the discharge pressure can be reduced to about 0.2 of the initial pressure; for an inflow at $M = 0.5$, the discharge pressure cannot be less than about $0.45p_1$ in the adiabatic case and about $0.6p_1$ in isothermal flow.

Analysis of such conduit flow must treat density change, as evaluated from the continuity relation in Equation (2), with the frictional occurrences evaluated from wall roughness and Reynolds number correlations of incompressible flow (Binder 1944). In evaluating valve and fitting losses, consider the reduction in K caused by compressibility (Benedict and Carlucci 1966). Although the analysis differs significantly, isothermal and adiabatic flows involve essentially the same pressure variation along the conduit, up to the limiting conditions.

Control Valve Characterization

Control valves are characterized by a discharge coefficient C_d . As long as the Reynolds number is greater than 250, the orifice equation holds for liquids:

$$Q = C_d A_o \sqrt{2\Delta P / \rho} \quad (35)$$

where

A_o = area of orifice opening
 P = absolute pressure

The discharge coefficient is about 0.63 for sharp-edged configurations and 0.8 to 0.9 for chamfered or rounded configurations. For gas flows at pressure ratios below the choking critical [Equation (24)], the mass rate of flow is

$$\dot{m} = C_d A_o C_1 \left(\frac{P_u}{\sqrt{T_u}} \right) \sqrt{\frac{P_d}{P_u}} \sqrt{1 - \left(\frac{P_d}{P_u} \right)^{(k-1)/k}} \quad (36)$$

where

$C_1 = \sqrt{2k/R(k-1)}$
 k = ratio of specific heats at constant pressure and volume
 R = gas constant
 T = absolute temperature
 u, d = subscripts referring to upstream and downstream positions

Incompressible Flow in Systems

Flow devices must be evaluated in terms of their interaction with other elements of the system, for example, the action of valves in modifying flow rate and in matching the flow-producing device (pump or blower) with the system loss. Analysis is via the general Bernoulli equation and the loss evaluations noted previously.

A valve regulates or stops the flow of fluid by throttling. The change in flow is not proportional to the change in area of the valve opening. Figures 14 and 15 indicate the nonlinear action of valves in controlling flow. Figure 14 shows a flow in a pipe discharging water from a tank that is controlled by a gate valve. The fitting loss coefficient K values are those of Table 3; the friction factor f is 0.027. The degree of control also depends on the conduit L/D ratio. For a relatively long conduit, the valve must be nearly closed before its high K value becomes a significant portion of the loss. Figure 15 shows a control damper (essentially a butterfly valve) in a duct discharging air from a plenum held at constant pressure. With a long duct, the damper does not affect the flow rate until it is about one-quarter closed. Duct length has little effect when the damper is more than half closed. The damper closes the duct totally at the 90° position ($K = \infty$).

Flow in a system (pump or blower and conduit with fittings) involves interaction between the characteristics of the flow-producing device (pump or blower) and the loss characteristics of the pipeline or duct system. Often the devices are centrifugal, in which case the pressure produced decreases as the flow increases, except for the lowest flow rates. System pressure required to overcome losses increases roughly as the square of the flow rate. The flow rate of a given system is that where the two curves of pressure versus flow rate intersect (point 1 in Figure 16). When a control valve (or

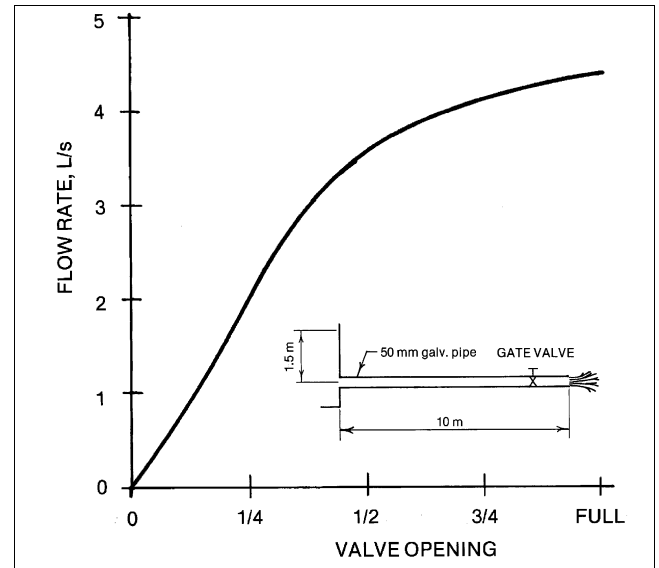


Fig. 14 Valve Action in Pipeline

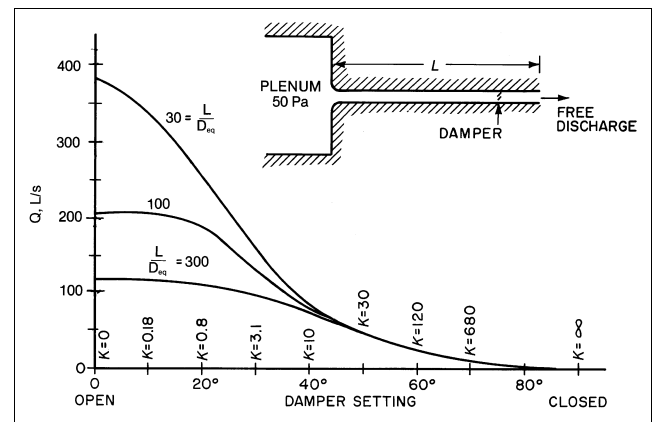


Fig. 15 Effect of Duct Length on Damper Action

Table 3 Fitting Loss Coefficients of Turbulent Flow

Fitting	Geometry	$K = \frac{\Delta p / \rho g}{V^2 / 2g}$
Entrance	Sharp	0.50
	Well-rounded	0.05
Contraction	Sharp ($D_2/D_1 = 0.5$)	0.38
90° Elbow	Miter	1.3
	Short radius	0.90
	Long radius	0.60
	Miter with turning vanes	0.2
Globe valve	Open	10
Angle valve	Open	5
Gate valve	Open	0.19 to 0.22
	75% open	1.10
	50% open	3.6
	25% open	28.8
Any valve	Closed	∞
Tee	Straight through flow	0.5
	Flow through branch	1.8

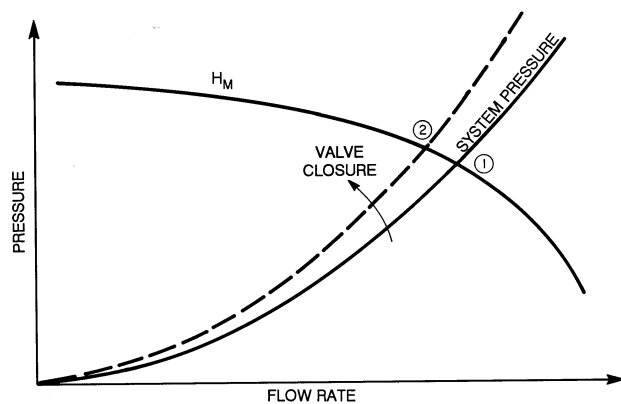


Fig. 16 Matching of Pump or Blower to System Characteristics

damper) is partially closed, it increases the losses and reduces the flow (point 2 in Figure 16). For cases of constant pressure, the flow decrease due to valving is not as great as that indicated in Figures 14 and 15.

Flow Measurement

The general principles noted (the continuity and Bernoulli equations) are basic to most fluid-metering devices. Chapter 14 has further details.

The pressure difference between the stagnation point (total pressure) and that in the ambient fluid stream (static pressure) is used to give a point velocity measurement. The flow rate in a conduit is measured by placing a pitot device at various locations in the cross section and spatially integrating the velocity profile found. A single point measurement may be used for approximate flow rate evaluation. When the flow is fully developed, the pipe-factor information of Figure 4 can be used to estimate the flow rate from a centerline measurement. Measurements can be made in one of two modes. With the pitot-static tube, the ambient (static) pressure is found from pressure taps along the side of the forward-facing portion of the tube. When this portion is not long and slender, static pressure indication will be low and velocity indication high; as a result, a tube coefficient less than unity must be used. For parallel conduit flow, wall piezometers (taps) may take the ambient pressure, and the pitot tube indicates the impact (total pressure).

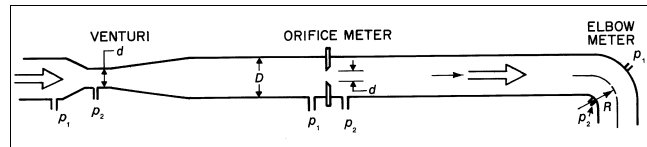


Fig. 17 Differential Pressure Flowmeters

The venturi meter, flow nozzle, and orifice meter are flow rate metering devices based on the pressure change associated with relatively sudden changes in conduit section area (Figure 17). The elbow meter (also shown in Figure 17) is another differential pressure flowmeter. The flow nozzle is similar to the venturi in action, but does not have the downstream diffuser. For all these, the flow rate is proportional to the square root of the pressure difference resulting from fluid flow. With the area change devices (venturi, flow nozzle, and orifice meter), a theoretical flow rate relation is found by applying the Bernoulli and continuity equations in Equations (6) and (2) between stations 1 and 2:

$$Q_{theor} = \frac{\pi d^2}{4} \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} \quad (37)$$

where $\beta = d/D$ = ratio of throat (or orifice) diameter to conduit diameter.

The actual flow rate through the device can differ because the approach flow kinetic energy factor α deviates from unity and because of small losses. More significantly, the jet contraction of orifice flow is neglected in deriving Equation (37), to the extent that it can reduce the effective flow area by a factor of 0.6. The effect of all these factors can be combined into the discharge coefficient C_d :

$$Q = C_d Q_{theor} = C_d \left(\frac{\pi d^2}{4} \right) \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} \quad (38)$$

Sometimes an alternate coefficient is used of the form

$$\frac{C_d}{\sqrt{1-\beta^4}}$$

For compressible fluid metering, the expansion factor Y as described by Equation (23) must be included, and the mass flow rate is

$$\dot{m} = C_d Y \rho Q_{theor} = C_d Y \left(\frac{\pi d^2}{4} \right) \sqrt{\frac{2\rho\Delta p}{1-\beta^4}} \quad (39)$$

Values of Y depend primarily on the pressure ratio p_2/p_1 , and also on the metering device and k value of the particular gas.

The general mode of variation in C_d for orifices and venturis is indicated in Figure 18 as a function of Reynolds number and, to a lesser extent, diameter ratio β . For Reynolds numbers less than 10, the coefficient varies as $\sqrt{\text{Re}}$.

The elbow meter employs the pressure difference between inside and outside the bend as the metering signal (Murdock et al. 1964). A momentum analysis gives the flow rate as

$$Q_{theor} = \frac{\pi D^2}{4} \sqrt{\frac{R}{2D} \left(\frac{2\Delta p}{\rho} \right)} \quad (40)$$

where R is the radius of curvature of the bend. Again, a discharge coefficient C_d is needed; as in Figure 18, this drops off for the lower Reynolds numbers (below 10^5). These devices are calibrated in pipes

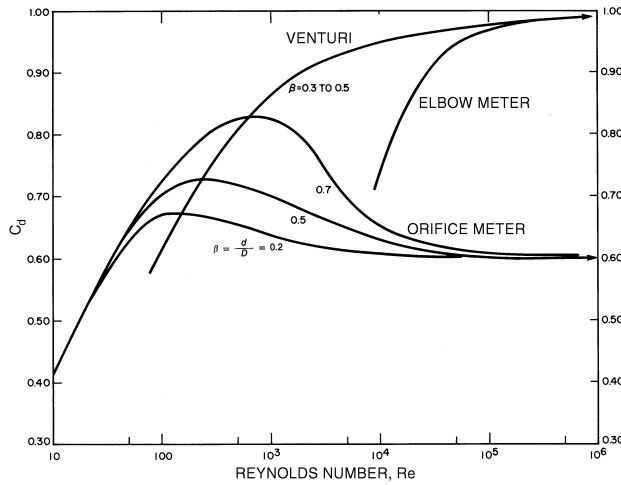


Fig. 18 Flowmeter Coefficients

with fully developed velocity profiles, so they must be located far enough downstream of sections that modify the approach velocity.

Unsteady Flow

Conduit flows are not always steady. In a compressible fluid, the acoustic velocity is usually high and the conduit length is rather short, so the time of signal travel is negligibly small. Even in the incompressible approximation, system response is not instantaneous. If a pressure difference Δp is applied between the conduit ends, the fluid mass must be accelerated and wall friction overcome, so a finite time passes before the steady flow rate corresponding to the pressure drop is achieved.

The time it takes for an incompressible fluid in a horizontal constant-area conduit of length L to achieve steady flow may be estimated by using the unsteady flow equation of motion with wall friction effects included. On the quasi-steady assumption, friction is given by Equation (26); also by continuity, V is constant along the conduit. The occurrences are characterized by the relation

$$\frac{dV}{d\theta} + \left(\frac{1}{\rho}\right) \frac{dp}{ds} + \frac{fV^2}{2D} = 0 \quad (41)$$

where

θ = time

s = distance in the flow direction

Since a certain Δp is applied over the conduit length L ,

$$\frac{dV}{d\theta} = \frac{\Delta p}{\rho L} - \frac{fV^2}{2D} \quad (42)$$

For laminar flow, f is given by Equation (27), and

$$\frac{dV}{d\theta} = \frac{\Delta p}{\rho L} - \frac{32\mu V}{\rho D^2} = A - BV \quad (43)$$

Equation (43) can be rearranged and integrated to yield the time to reach a certain velocity:

$$\theta = \int d\theta = \int \frac{dV}{A - BV} = -\frac{1}{B} \ln(A - BV) \quad (44)$$

and

$$V = \frac{\Delta p}{L} \left(\frac{D^2}{32\mu} \right) \left[1 - \frac{\rho L}{\Delta p} \exp\left(\frac{-32\mu V \theta}{D^2} \right) \right] \quad (45a)$$

For long times ($\theta \rightarrow \infty$), this indicates steady velocity as

$$V_{\infty} = \frac{\Delta p}{L} \left(\frac{D^2}{32\mu} \right) = \frac{\Delta p}{L} \left(\frac{R^2}{8\mu} \right) \quad (45b)$$

as by Equation (8). Then, Equation (45a) becomes

$$V = V_{\infty} \left[1 - \frac{\rho L}{\Delta p} \exp\left(\frac{-f_{\infty} V_{\infty} \theta}{2D} \right) \right] \quad (46)$$

where

$$f_{\infty} = \frac{64\mu}{V_{\infty} D}$$

The general nature of velocity development for starting-up flow is derived by more complex techniques; however, the temporal variation is as given above. For shutdown flow (steady flow with $\Delta p = 0$ at $\theta > 0$), the flow decays exponentially as $e^{-\theta}$.

Turbulent flow analysis of Equation (41) also must be based on the quasi-steady approximation, with less justification. Daily et al. (1956) indicate that the frictional resistance is slightly greater than the steady-state result for accelerating flows, but appreciably less for decelerating flows. If the friction factor is approximated as constant,

$$\frac{dV}{d\theta} = \frac{\Delta p}{\rho L} - \frac{fV^2}{2D} = A - BV^2$$

and, for the accelerating flow,

$$\theta = \frac{1}{\sqrt{AB}} \tanh^{-1} \left(V \sqrt{\frac{B}{A}} \right)$$

or

$$V = \sqrt{\frac{A}{B}} \tanh(\theta \sqrt{AB})$$

Because the hyperbolic tangent is zero when the independent variable is zero and unity when the variable is infinity, the initial ($V = 0$ at $\theta = 0$) and final conditions are verified. Thus, for long times ($\theta \rightarrow \infty$),

$$V_{\infty} = \sqrt{\frac{A}{B}} = \sqrt{\frac{\Delta p / \rho L}{f_{\infty} / 2D}} = \sqrt{\frac{\Delta p (2D)}{\rho L (f_{\infty})}}$$

which is in accord with Equation (26) when f is constant (the flow regime is the fully rough one of Figure 13). The temporal velocity variation is then

$$V = V_{\infty} \tanh(f_{\infty} V_{\infty} \theta / 2D) \quad (47)$$

In Figure 19, the turbulent velocity start-up result is compared with the laminar one in Figure 19, where initially the turbulent is steeper but of the same general form, increasing rapidly at the start but reaching V_{∞} asymptotically.

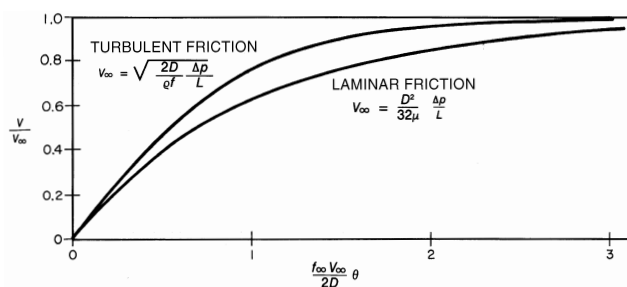


Fig. 19 Temporal Increase in Velocity Following Sudden Application of Pressure

NOISE FROM FLUID FLOW

Noise in flowing fluids results from unsteady flow fields and can be at discrete frequencies or broadly distributed over the audible range. With liquid flow, cavitation results in noise through the collapse of vapor bubbles. The noise in pumps or fittings (such as valves) can be a rattling or sharp hissing sound. It is easily eliminated by raising the system pressure. With severe cavitation, the resulting unsteady flow can produce indirect noise from induced vibration of adjacent parts. See [Chapter 47 of the ASHRAE Handbook—Applications](#) for more information on sound control.

The disturbed laminar flow behind cylinders can be an oscillating motion. The shedding frequency f of these vortices is characterized by a Strouhal number $St = fd/V$ of about 0.21 for a circular cylinder of diameter d , over a considerable range of Reynolds numbers. This oscillating flow can be a powerful noise source, particularly when f is close to the natural frequency of the cylinder or some nearby structural member so that resonance occurs. With cylinders of another shape, such as impeller blades of a pump or blower, the characterizing Strouhal number involves the trailing edge thickness of the member. The strength of the vortex wake, with its resulting vibrations and noise potential, can be reduced by breaking up the flow with downstream splitter plates or boundary-layer trip devices (wires) on the cylinder surface.

Noise produced in pipes and ducts, especially from valves and fittings, is associated with the loss through such elements. The sound pressure of noise in water pipe flow increases linearly with the pressure loss; the broad-band noise increases, but only in the lower frequency range. Fitting-produced noise levels also increase with fitting loss (even without cavitation) and significantly exceed noise levels of the pipe flow. The relation between noise and loss is not surprising because both involve excessive flow perturbations. A valve's pressure-flow characteristics and structural elasticity may be such that for some operating point it oscillates, perhaps in resonance with part of the piping system, to produce excessive noise. A change in the operating point conditions or details of the valve geometry can result in significant noise reduction.

Pumps and blowers are strong potential noise sources. Turbo-machinery noise is associated with blade-flow occurrences. Broad-band noise appears from vortex and turbulence interaction with walls and is primarily a function of the operating point of the machine. For blowers, it has a minimum at the peak efficiency point (Groff et al. 1967). Narrow-band noise also appears at the blade-crossing frequency and its harmonics. Such noise can be very annoying because it stands out from the background. To reduce this noise, increase clearances between impeller and housing, and space impeller blades unevenly around the circumference.

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