

# fundamentals of ● rolling



**Zygmunt Wusatowski, D.Sc.**

Professor of the Polytechnical University,  
Gliwice, Poland

PERGAMON PRESS

Oxford, London, Edinburgh, New York,  
Toronto, Sydney, Paris, Braunschweig

---

WYDAWNICTWO «ŚLĄSK»

Katowice

Pergamon Press Ltd., Headington Hill Hall, Oxford  
4 & 5 Fitzroy Square, London W.1  
Pergamon Press (Scotland) Ltd., 2 & 3 Teviot Place, Edinburgh 1  
Pergamon Press Inc., Maxwell House, Fairview Park, Elmsford, New York 10523  
Pergamon of Canada Ltd., 207 Queen's Quay West, Toronto 1  
Pergamon Press (Aust.) Pty. Ltd., 19a Boundary Street, Rushcutters Bay, N.S.W. 2011, Australia  
Pergamon Press S.A.R.L., 24 rue des Écoles, Paris 5<sup>e</sup>  
Vieweg & Sohn GmbH, Burgplatz 1, Braunschweig

Copyright © 1969

WYDAWNICTWO "ŚLĄSK" KATOWICE

Revised and enlarged from the first Polish edition  
PODSTAWY WALCOWANIA  
by the author and translated by Wilhelm Gorecki, D.Sc.;  
Piotr Ochonski, M.Sc.; Teofil Hozakowski, M.A.  
Editor: Janusz Czermiński, M. Sc.

Library of Congress Catalog Card No. 68-22084

*Printed in Poland*

08 012276 0

*To my wife Ann*

## FOREWORD

ALTHOUGH rolling mill engineers in English-speaking countries can already refer to several textbooks on the subject, the appearance of the English translation of Prof. Wusatowski's book is to be warmly welcomed for several reasons. Firstly, it has been extensively revised quite recently and so is more up to date than other publications. Secondly, as Prof. Wusatowski is equally at home in West and East European literature and makes full use of this knowledge in comparing these frequently separate developments of the treatment of problems in rolling, his book affords Western readers insight into and evaluation of technology as developed in the Soviet Union and other East European countries. Thirdly, his own distinguished contributions to the subject, and that of his collaborators, particularly in the field of billet and section rolling, are dealt with comprehensively for the first time, and some of these are unique.

Conceived in the tradition of Continental textbooks, Prof. Wusatowski's book touches on all aspects of the subject, including cognate subjects, starting with physical metallurgy and ending with mill design. Having both technological and engineering topics treated in one book has its advantages, for the two are closely interrelated. Equally, the practical reader will welcome the presence of the many worked examples illustrating the use of the theory and of the numerous working diagrams, tables and nomograms as aids to his own calculations.

It is to be hoped that the book will be widely read and used and contribute to the more scientific treatment of practical problems in this important branch of technology.

J. G. WISTREICH



## PREFACE

THE theory of rolling has reached at present a stage of development where its mastery is impossible without a proper training in mathematics, and without a sufficient knowledge of the strength of materials. The work of roll designers, constructors and operators today requires a thorough knowledge of the fundamental theory and principles of rolling processes. Every operation, and almost every effort which is contrary to the basic theory and principles, can lead to errors.

For this reason I undertook the task of working out the fundamentals of rolling theory and roll pass design as simply, clearly and understandably as possible. *Fundamentals of Rolling* covers the whole field of theoretical knowledge of longitudinal rolling in a comprehensible form, but methods evolved ensure accurate practical results.

Previous publications by my collaborators and myself are listed in the Bibliography and form an important source of material for this book. All these publications have been used, up to and including the most recent.

Compared with the Polish edition, *Fundamentals of Rolling* has been extensively rewritten, and together with supplementary material has increased its size by 200 pages.

I wish to express my gratitude to two persons in England: to Mr. C. P. Birkin, the Editor of *Iron and Steel* in London, who since 1955 has been regularly publishing my articles in his periodical, thus making Polish achievements known in Great Britain, and to Dr. J. G. Wistreich from BISRA with whom I entered into collaboration a long time ago. In his critical review of the Polish edition Dr. Wistreich kindly recommended its publication in the English language.

Z. WUSATOWSKI

## CHAPTER 1

# PHENOMENA OCCURRING DURING PLASTIC WORKING OF METALS

### 1.1. Plastic Working by Rolling

Plastic working of metals is one of numerous methods permitting the manufacture of products of desired shape and size. It consists in applying compressive forces of appropriate magnitude to the metals being deformed. Industrial practice uses various plastic working techniques such as rolling, forging, pressing, stamping, extrusion or drawing.

During rolling the desired shape of metal is obtained by plastic deformation taking place between two rolls with parallel axes, revolving in opposite directions. Sometimes, instead of cylindrical rolls, conical rolls or discs set at an angle to each other are employed.

The rolling equipment consisting of one or several stands of rolls, usually the pinion stand and the drive, complete with auxiliary installations, is called the rolling mill or plant, where the whole rolling process is carried out.

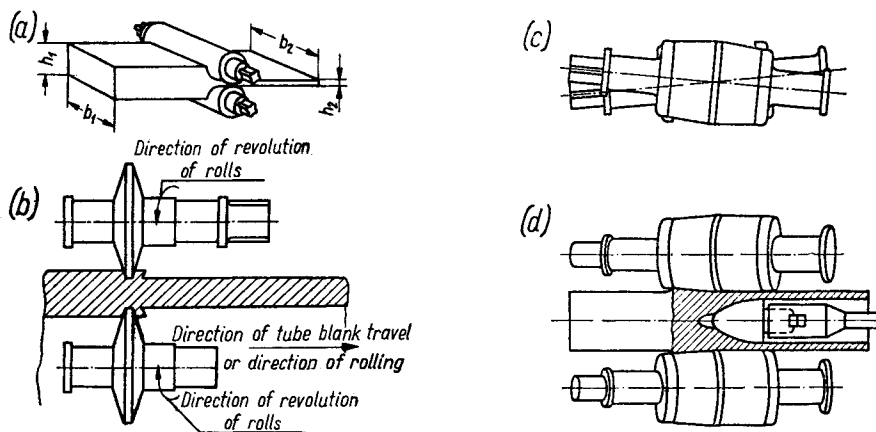


Fig. 1.1. Principal methods of rolling: (a) longitudinal rolling, (b) transverse rolling, (c) and (d) skew rolling

Three basic methods of rolling can be distinguished: longitudinal rolling (Fig. 1.1a), transverse rolling (Fig. 1.1b), and skew rolling (Figs. 1.1c and d).

During longitudinal rolling deformation takes place between the rolls with parallel axes, revolving in opposite directions. Due to friction the metal is drawn into the rolls and undergoes deformation. During this deformation the ingoing height of the stock

$h_1$  is reduced to  $h_2$ , while breadth and length of stock increase, the latter usually much more than the former. In longitudinal rolling the metal moves forward along a straight line perpendicular to the roll axes and the plastic deformation takes place mostly in this direction. This rolling method is most frequently employed and accounts for about 90 per cent of the whole rolling mill production.

Transverse and skew rolling processes are chiefly used for the manufacture of hollow bodies of revolution such as tube blanks and tubes. During transverse rolling the metal moves only about its own longitudinal axis, thus the plastic working of metal is effected principally in transverse direction of the stock. In skew rolling the non-parallel setting of rolls causes the metal not only to move about its own axis but also to proceed along it. These two movements are superimposed and produce plastic deformation of the metal along a helix line. Transverse rolling is used very seldom, e.g. to manufacture seamless tubes of very large diameters. Skew rolling, on the other hand, is commonly employed in the manufacture of tubes.

During the rolling processes, in addition to change of shape effected in a purely mechanical way, the metal undergoes structural changes, which in turn result in a variation of physical properties. Among these changes mention should be made of the following:

- (1) changes due to inhomogeneity of ingot,
- (2) changes of structure and properties, resulting from hot working of metal,
- (3) changes of structure and properties, resulting from cold working of metal (e.g. cold work, etc.).

## 1.2. Fundamental Phenomena Occurring in Plastic Working

Any solid matter occurring in nature changes its shape and dimensions when subjected to the action of external forces. This change is called deformation.

Deformation can be either elastic or plastic. The former occurs when a deformed body entirely recovers its original shape and size on removing the external load. The latter occurs when the body does not recover its shape and dimensions on removal of the external load.

Figure 1.2 shows the relation between the tensile force acting on the cross-sectional area of a metal test piece and the corresponding elongation due to tensile stress. Percentage elongations on the abscissa axis are plotted against stresses on the ordinate axis in kg per sq. mm.

Stress is defined as the force acting on unit area. Figure 1.2 permits the accurate determination of the limit of elastic deformation or elastic limit (designated by  $S$ ) and the limit of permanent or plastic deformation, defined as the yield point (designated by  $Y_p$ ). Recovery of original shape and dimensions by a metal can take place after the removal of all stresses, provided they have not exceeded the elastic limit of the metal under consideration. When the elastic limit is exceeded and then the lower  $Y_{pl}$  and the upper  $Y_{pu}$  yield points are reached the metal undergoes permanent or plastic deformation.

Permanent deformation usually takes place after the elastic deformation. Thus even after a considerable degree of deformation a partial recovery of the initial state often occurs. The total deformation or strain produced by a given system of external forces is the sum of elastic and plastic strain. Expressing the total strain in terms of extension  $\epsilon$ , it can be said that it is the sum of two values  $\epsilon_p$  and  $\epsilon_s$ , where  $\epsilon_p$  represents plastic deformation and  $\epsilon_s$  elastic deformation.

Figure 1.3 is a schematic illustration of total and permanent deformations. Under the action of external load, the total deformation is the sum of  $\epsilon_p$  and  $\epsilon_s$ . After removal of load only the permanent deformation  $\epsilon_p$  remains.

As previously stated, the main object of plastic working consists in applying compressive forces to the metal in order to obtain products of desired shape. The force to be applied should produce stresses under which the metal changes its shape

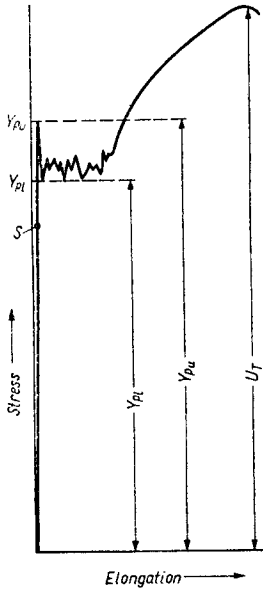


Fig. 1.2. Stress-strain diagram for steel tested in tension

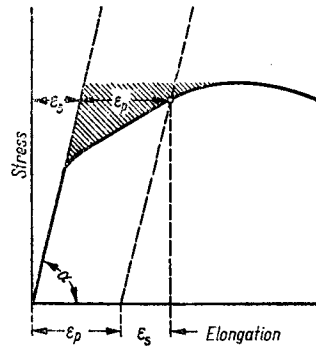


Fig. 1.3. Schematic representation of total and permanent deformations in tension

and retains that shape after removal of load. It can be concluded that the object of plastic deformation is to produce a permanent deformation of metal without destroying the cohesive forces existing between individual particles of metal.

The state in which solid matter can undergo permanent deformation without destruction of cohesive forces is defined as the plastic state.

Essentially, plastic working consists in causing relative displacement of the metal particles from one to another portion of a given work piece, depending on existing rolling conditions and the change of shape. Such a displacement of some portion of metal in plastic working should always take place without destruction of the work piece as a whole. For this reason, plastic working can be applied only to metals which exhibit a sufficient degree of plasticity i.e. those that can withstand considerable plastic strains without the destruction of the whole.

Relative displacement of metal particles in the case of plastic working of mild steel is shown by the appearance of Lüders lines in the early stages of plastic deformation, on exceeding the yield point (Fig. 1.4). The formation of these flow lines when the

work piece is compressed by a narrow punch, without friction, is represented in Fig. 1.4a. The same phenomenon occurring in cold rolling is shown in Fig. 1.4b [30].

Lüders lines in mild steel appear just after exceeding the yield point (designated by  $Y_{pu}$  in Fig. 1.2). If a work piece with polished surface is subjected to compression, the surface will become dull after a certain amount of compression and on the surface will appear flow lines, making an angle of about  $90^\circ$  (Fig. 1.4a).

The appearance of Lüder's lines on the polished surface of a test piece is to be attributed to relative displacements of metal particles, mainly on boundaries of individual

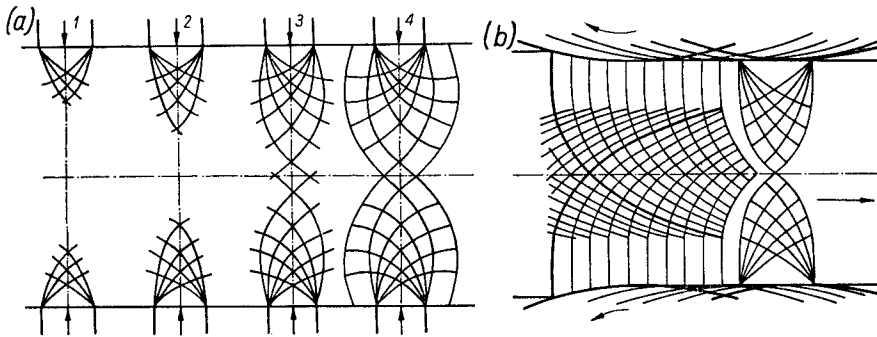


Fig. 1.4. Schematic drawing showing formation of Lüder's lines on steel surface: (a) in compression, (b) in cold rolling

crystallites. If the intercrystalline matter is weak, the dull appearance of the surface results from the formation of Lüder's lines in individual crystallites cut by the polished surface of the test piece.

The applied load produces group displacements of metal particles, relative to the magnitude of the force. Each grain shows flow lines with directions dependent on its own crystallographic orientation, which is the evidence that plastic deformation is the result of disturbances in equilibrium both on grain boundaries and in grain interior.

### 1.3. Deformation of Single Crystals and Polycrystalline Aggregates

Every metal is made up of innumerable crystals (grains) touching on grain boundaries or separated from each other by a distinct layer of intercrystalline matter. Such a layer usually contains a greater amount of impurities than the crystals themselves and that is why its temperature of fusion is lower than that of grains.

Plastic deformation of polycrystalline aggregates may be accompanied by the following phenomena:

- (1) disturbance of equilibrium on grain boundaries (intercrystalline deformation),
- (2) disturbance of equilibrium within the grains themselves,
- (3) fragmentation of grains.

In metals with a weaker intercrystalline matter as compared with that of grains, deformation takes place mainly by slip of individual grains. On the other hand, the behaviour of metals with a stronger intercrystalline matter is different in this respect. In this case deformation is caused by displacements occurring in the interior of the

grains themselves. If, however, the intercrystalline matter forms some kind of stiff skeleton, the deformation of grains can take place only after destruction of this skeleton.

Numerous experiments indicate that intercrystalline deformation (disturbance of equilibrium on grain boundaries) usually occurs at slow rates of deformation and at high temperatures, while the deformation of the grain interior takes place at greater deformation rates and at low temperatures of plastic working. Deformations in the interior of grains can be best observed by the example of single crystals. Numerous experiments have shown that the first evidence of plastic deformation of single crystals is the appearance of slip lines, which reveal the existence of slip planes in the metal. These experiments have also shown that the plastic deformation of crystals is accompanied by two phenomena;

- (1) slip along definite crystallographic planes, called displacement,
- (2) formation of twin crystals (twinning).

Slip of some portion of a crystal in relation to adjacent parts takes place along certain crystallographic planes, characteristic of the given type of crystal (Fig. 1.5a).

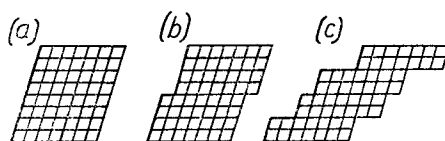


Fig. 1.5. Schematic representation of grain slip

This slip does not result in a change of crystallographic orientation and causes the slip planes to align as shown in Figs. 1.5b and c.

During formation of twin crystals some parts of a crystal rotate relative to other parts, and the displaced parts take the position which is the mirror image of the original position (Fig. 1.6).

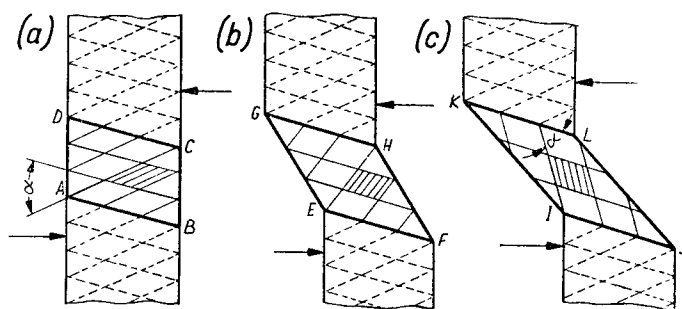


Fig. 1.6. Formation of twin crystals (schematically): (a) initial state, (b) transitional state, (c) final state

If the undeformed crystal has the form of a rhomb  $ABCD$ , the plastic deformation will result in forming two twin crystals  $EFGH$  and  $IJKL$ .

While slip is encountered during deformation of nearly all single crystals and polycrystalline aggregates of a great number of metals, twinning takes place only in certain metals and alloys.

Experiments indicate that the planes of slip correspond to definite crystallographic planes (the so-called easiest planes of slip) of a given crystal and that the grains on these planes show the weakest resistance to deformation by slip (Fig. 1.7). The orientation of the above planes in relation to the force acting on the crystal has an influence on the magnitude of stress at the yield stress of individual crystals. For example, the lowest

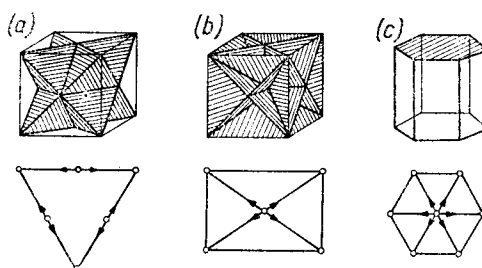


Fig. 1.7. Planes and directions of slip in crystal lattices [1]: (a) face-centred cubic lattice, (b) body-centred cubic lattice, (c) close-packed hexagonal lattice.  
Below are shown directions of slip

tensile strength will occur when the slip planes and the plane of maximum shear stress have the same direction, i.e. when they are inclined at about  $45^\circ$  to the direction of force applied, as shown in Fig. 1.8.

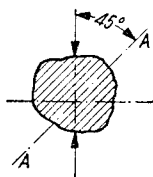


Fig. 1.8. Schematic drawing showing maximum shear stress and slip planes

Experiments have also shown that the resistance to deformation by slip varies in different directions and that this variation disappears in metals with a great number of crystals having random orientation, this being due to the compensating effect of individual crystals.

From a great number of grains with random crystallographic orientation, the first crystals to begin to deform are those where planes and directions of slip most closely approach the planes inclined at  $45^\circ$  to the direction of force. As the deformation continues, slip also occurs in other planes with a less advantageous inclination to the direction of stress.

From this point of view the plasticity of metal is considered as the ability to produce more or less slip planes. Lacking this ability the metal is brittle. The more slip planes a metal has, the greater its plasticity.

Considering the plastic deformation of polycrystalline aggregates it should be borne in mind that individual grains may be separated from each other by intercrystalline matter, constituting simultaneously a binder between the grains (Fig. 1.9). This binding

is strengthened by some kind of interlocking resulting from irregularities in shape of individual crystals (Fig. 1.10).

It is known that in a polycrystalline metal under the influence of external force, the crystals which first begin to deform, are those with slip planes inclined at  $45^\circ$  to the direction of force. As deformation continues, the deformed grains exert a pressure

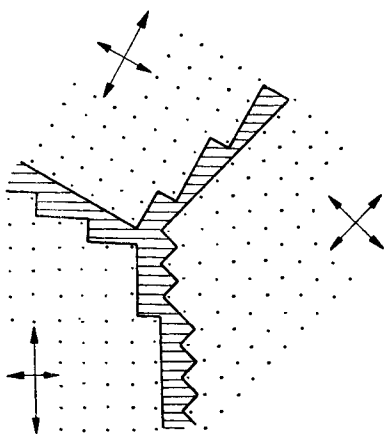


Fig. 1.9. Schematic example of cubic lattice structure and grain boundaries [3]

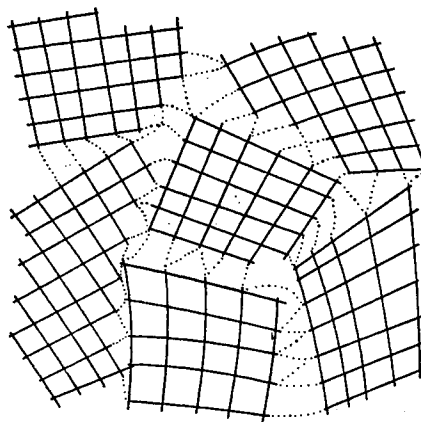


Fig. 1.10. Schematic representation of deformation of polycrystalline structures. The lattice of individual grains is elongated or elastically bowed. Indentations along the grain boundaries produce a state of stress in the adjacent grains (exaggerated) [2]

on the adjacent grains and thus produce stresses leading to plastic deformations of the latter (Fig. 1.11). In this way the number of grains in which slipping occurs will steadily grow. As soon as the plastic deformation spreads over the whole volume of metal,

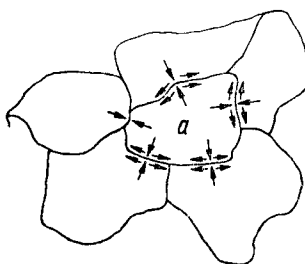


Fig. 1.11. Interaction of stresses in polycrystalline structures [3, 4]

displacements of crystal groups occur, leading to splitting or fragmentation of grains. This refinement of grains takes place along the planes of slip.

Further deformation causes the axes of all the grains to align with the direction of external forces, as for example during rolling. Due to this the grains lose their original



shape and become elongated. Grain boundaries disappear and the structure of metal becomes fibrous (Fig. 1.12). Metal with such a structure exhibits quite different properties in longitudinal and transverse directions, depending on the direction of testing.

#### 1.4. Cold Work and Strain-hardening of Metal

The term cold working of metals refers practically to working which takes place at ambient temperatures. A characteristic feature of cold working is the preservation of structure and properties obtained as a result of deformation. Thus, cold working decisively changes the properties of metal. Experiments on different grades of steel and other metals have shown that changes in mechanical properties of metal take place especially at small degrees of cold work. Figure 1.13 shows the ultimate tensile stress of several steel grades, plotted against amount of cold deformation.

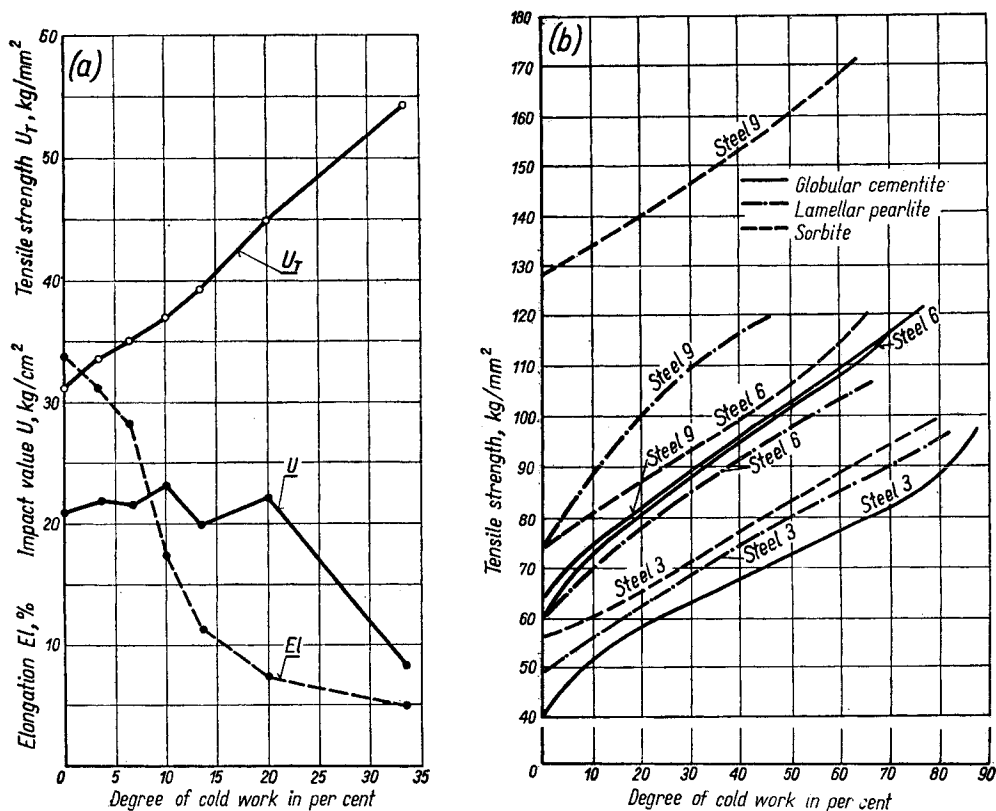


Fig. 1.13. Relation between strength characteristics of (a) mild and (b) medium-hard steels and degree of cold deformation and the initial structure: steel 3—0.29% C, steel 6—0.65% C, steel 9—0.93% C [5]

The combination of permanent changes occurring in metal due to plastic deformation below the temperature and time interval necessary to produce recrystallization, is called cold work. Diagrams in Fig. 1.13 show an increase in tensile strength. It should

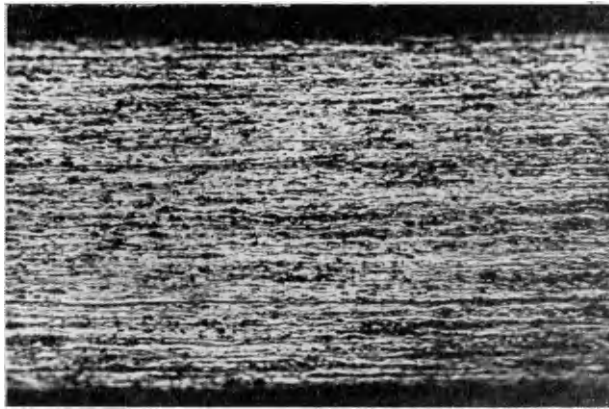


Fig. 1.12. Fibrous structure of mild steel (see Appendix 1)  
after severe plastic deformation. Degree of cold work 94%.  
Magnification 380×

be noted that the changes in strength properties for mild steel are much more appreciable than those for hard steel. Experimental data indicate that with the increase of cold work the elastic limit increases more than the ultimate tensile stress. This means that the span between elastic limit and ultimate tensile stress decreases as the extent of cold deformation increases. Due to cold plastic working the metal loses its plasticity, i.e. capacity for deformation, and becomes more and more brittle.

Increase in hardness and tensile strength as a result of plastic deformation at temperatures below the recrystallization temperature is called strain-hardening.

It can be seen from diagrams that this phenomenon proceeds at a much greater rate in the early stages of cold working. As the degree of cold work increases to about 50–60% the rate of strain-hardening decreases appreciably, and at a sufficiently high degree of cold work can reach a limiting value at which the metal loses nearly all its plasticity. Further deformation will result in rupturing of metal.

The diagram of strain-hardening is used to establish technological conditions for cold working processes. From this diagram the mechanical properties of metal at every degree of deformation can be obtained. The knowledge of these values is necessary for the calculations and analysis of the whole technological process.

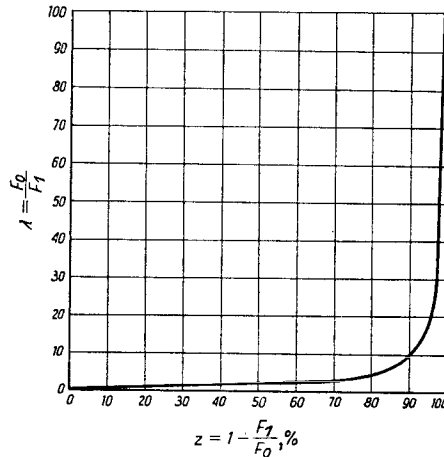


Fig. 1.14. Graphical representation of the function  $z = 1 - F_1/F_0$

The extent of cold deformation is defined as the degree of cold work. At various times doubts were raised as to the correctness of the formula (1.1), commonly used to calculate the degree of cold work  $z$

$$z = \frac{F_0 - F_1}{F_0} = 1 - \frac{F_1}{F_0} \quad (1.1)$$

where

$F_0$  = initial cross-sectional area of stock i.e. the product of  $b_0$  and  $h_0$ ,

$F_1$  = area of stock after deformation with dimensions  $b_1$  and  $h_1$ .

Essentially, this relation (1.1) has the following meaning

$$z = \frac{l_1 - l_0}{l_1} = 1 - \frac{l_0}{l_1} = \frac{F_0 - F_1}{F_0} = 1 - \frac{F_1}{F_0} \quad (1.2)$$

where

$l_0$  = length before deformation, mm,

$l_1$  = length after deformation, mm.

It follows from this relationship that the value  $z$  defines a certain mean elongation and can be applied only to round stock in which the whole cross-section is subjected to uniform deformation (Fig. 1.14).

If, however, the deformation in length is different from that in breadth, a maximum plastic deformation must take place. In these considerations this deformation is expressed as the sum of two values

$$\varphi_h = \varphi_b + \varphi_l$$

or

$$\varphi_h = \sum \varphi_i \quad (1.3)$$

Thus,  $\varphi_h$  will be greater by the sum of deformations in breadth and length.

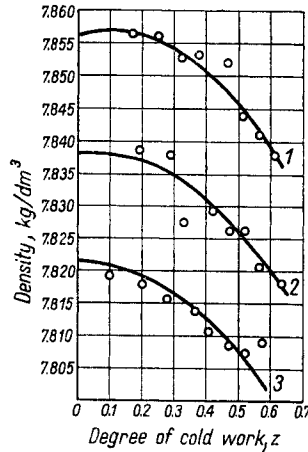


Fig. 1.15. Influence of cold work on variation of steel density [10].  
Steel grades [159]: 1 — St2x: 0.15% C, 0.45% Mn, 0.07% Si,  
2 — D55A: 0.50% C, 0.44% Mn, 0.19% Si, 3 — C85DA: 0.87% C,  
0.53% Mn, 0.23% Si

In such cases the relation (1.3) expresses a certain mean elongation of bar, smaller than the maximum deformation  $\varphi_h$ , where  $\varphi_h$  is a measure of the cold work.

From these considerations the degree of cold work can be expressed as

$$\varphi_h = \log_e \frac{h_2}{h_1} \quad \text{or} \quad \varphi_h = \log_e \frac{h_2}{h_1} 100$$

Cold plastic working also affects some of the physical properties of steel, e.g. electromagnetic properties, capacity for magnetization and demagnetization as well as specific weight.

It can be seen from Fig. 1.15 that with progressive deformation the specific weight of steel decreases and at the same time the specific volume of metal increases. However, these changes are very small and unimportant in practice.

During plastic working the metal structure undergoes considerable changes. No doubt, there is a close relation between deformation of a test piece and the shape of grains. While the grains of a test piece deformed by compression are crushed, those deformed by tension are elongated. Thus, due to cold work the grains which were equiaxial prior to deformation become non-equiaxial and align quite distinctly in the direction of the applied force.

Therefore, it is not to be expected that after such an internal deformation the metals would retain the same properties in longitudinal and transverse directions of rolling.

A metal made up of equiaxial crystals exhibits no directionality in mechanical properties and is said to be isotropic. On the other hand, anisotropic and preferential orientation of grains result in changing of mechanical properties in different directions. Such a metal is said to have an anisotropic structure (Fig. 1.12).

### 1.5. Recrystallization

Cold plastic working produces changes in different properties of metals. While ductility and magnetic permeability **decrease**, the coercive force and other properties increase in magnitude. These undesired changes can easily be removed by annealing. If, however, the extent of deformation is considerable, the plastic working of metal is for practical purposes divided into several successive operations followed by intermediate annealings.

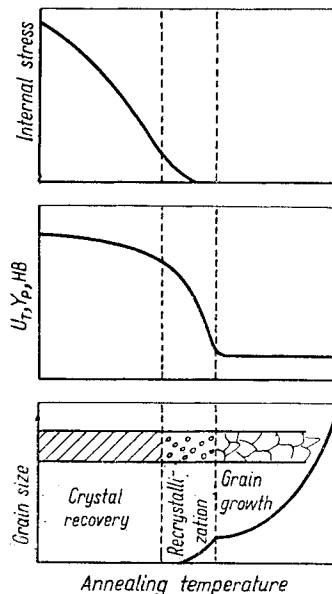


Fig. 1.16. Schematic drawing showing changes in properties of cold-worked metal during heating [9]

The number of these successive operations of plastic working depends on the initial and final dimensions of the work piece.

Reheating removes the effects of cold work partially or entirely, depending on temperature.

The cold work is the effect of disturbances in the crystal lattice of metal.

To remove these disturbances it is necessary to provide conditions which would allow the atoms\* to change their positions in the crystal lattice, i.e. a certain amount of energy in the form of heat has to be supplied. Successive deformations result in an accumulation of energy in crystal lattice. The greater this stored energy, the lower the additional heat to be supplied in order to enable the atoms to change their positions. This fact is confirmed in practice by the following statement: the greater the degree of deformation the lower the temperature at which the effects of this deformation will disappear.

As the cold-worked steel is heated, its properties such as electrical resistance, strain hardening, etc. begin to change, approaching the initial state prior to deformation, although the structure of metal after cold deformation does not reveal any perceivable changes. This phenomenon is called crystal recovery. It consists in lowering of internal stresses in the crystal lattice, especially in regions of severe deformation (Fig. 1.16).

During recrystallization the diffusion of atoms within the grains and along grain boundaries is intensified. In other words, the amount of deformation favours the progress of diffusion processes during recrystallization and contributes to compensation of differences in chemical composition of metal [7].

Krupkowski and Balicki [8] found that recrystallization does not occur suddenly, but at a rate dependent on temperature and degree of deformation. The higher the temperature and the degree of deformation, the faster is the process of recrystallization.

Increasing the degree of deformation and the time of annealing causes lowering of recrystallization temperature. During recrystallization the metal consists of a mixture of two components; the strain-hardened, non-recrystallized metal with unchanged properties and the soft recrystallized metal.

It is to be noted that individual properties are differently affected by temperature. (For example, when testing by electrical resistance methods a lower recrystallization temperature is obtained than by hardness methods). The recrystallization temperature is dependent on the degree of cold work [6].

Further reheating of metal produces the formation of recrystallization nuclei in the structure of cold-worked steel. The rapidly growing nuclei result in a complete restoration of the original structure and properties of metal prior to deformation. There are various hypotheses as to the formation of these recrystallization centres. The number of these nuclei depends on the degree of cold work. They occur in small numbers at small deformations (Fig. 1.17).

Consequently, the crystals can reach considerable dimensions before their growth is hindered by the adjacent grains. On the other hand, the number of recrystallization nuclei in a severely deformed metal is large and the resulting structure—fine-grained. The greater the amount of deformation, the lower the temperature at which the recrystallization begins. The prior deformation appreciably lowers the temperature of beginning of recrystallization.

At greater degrees of cold work the drop in temperature of the start of recrystallization is small. For pure, severely deformed metals the (absolute) temperature  $T_r$ , at which the recrystallization begins is dependent on the absolute temperature of melting,  $T_m$ . This relation has been defined by Boczwiar [1]

$$T_r \approx 0.4 T_m \quad (1.4)$$

\* Strictum sensum i.e. the ions.

Below that temperature the recrystallization proceeds extremely slowly, and above with considerable rapidity.

While the deformation causes the grains to break up and to stretch in certain directions and results in a definite crystallographic orientation, these effects are not observed in the recrystallized metal (Fig. 1.17). Thus as a result of recrystallization the metal recovers its original structure prior to deformation.

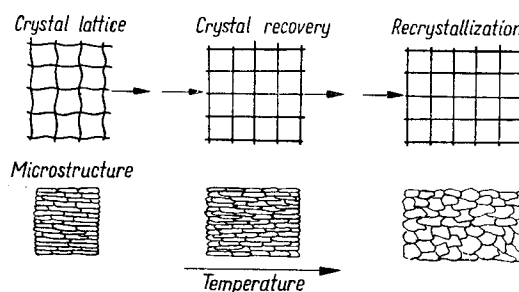


Fig. 1.17. Structure of recrystallized metal [9]

If the metal is heated for a sufficiently long time, the grains become coarser. This third phenomenon, known as grain growth, was described until now as a process quite different from recrystallization, where the formation of new grains depends exclusively on the number of recrystallization nuclei. The process of grain growth consists usually in coarsening of some grains at the expense of the others, which are partially or entirely absorbed by the former grains. For example, the same grain can absorb some portion of an adjacent grain and at the same time it can be partially absorbed by another.

The grain growth results in a decrease of the total grain boundary area of metal which consequently reaches a more stable thermodynamic state.

The recrystallization of the cold-worked metal progresses by steps. While the first step or period is characterized by the formation of nuclei for the new structure, the

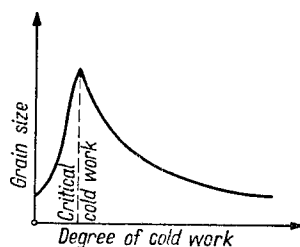


Fig. 1.18. Influence of cold work on grain size after annealing in recrystallization temperature [1]

second one (the so-called collective recrystallization) consists in growth of the new grains. The number of the newly-formed nuclei and their growth are restricted by such factors as the temperature and the degree of deformation of the metal heated for recrystallization.

The relation between the grain size, the temperature and the degree of cold work is usually represented in the form of space diagrams of recrystallization (Fig. 1.18).

These diagrams are sufficiently typical to characterize any kind of metal. Generally, there occurs a certain critical cold work for small reductions or a high value of critical grain size at small reductions.

The magnitude of critical cold work depends on the kind of metal and usually corresponds to reductions of 8 to 10 per cent. With the increase in temperature of deformation this value is shifted to the origin of the coordinate system (Fig. 1.19).

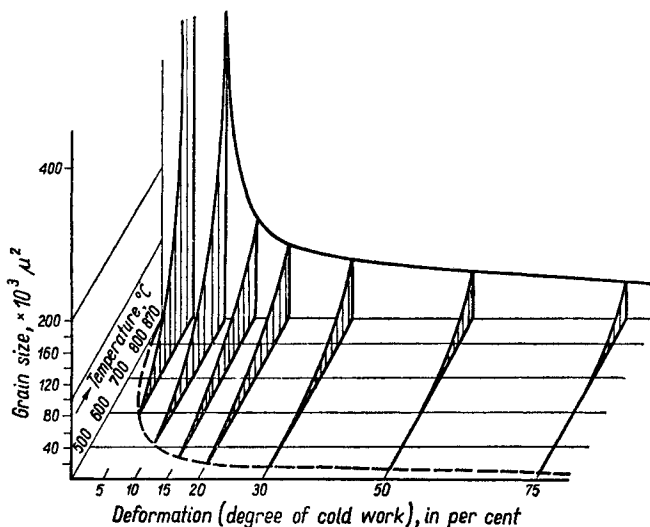


Fig. 1.19. Influence of cold work and temperature on grain size after recrystallization of mild steel [10]

In general, as the deformation prior to recrystallization increases, the dimensions of the recrystallized grains decrease. Hence, to obtain a fine-grained structure, small degrees of deformation corresponding to the critical degree of cold work, should be avoided.

Occurrence of critical cold work can be explained by the fact that in the early stages of plastic working deformation takes place exclusively due to internal crystallization processes, and consequently the intercrystalline matter does not undergo any disturbance. This facilitates contacting of grains and reconstruction of space lattice of one grain in relation to the space lattice of another grain.

The diagram of recrystallization shown in Fig. 1.19 was considered for a long time as unchanging. However, experiments have shown that for a certain steel at high degrees of deformation a second maximum value in the recrystallization diagram can be observed (Fig. 1.20). I. M. Pavlov explains this phenomenon as follows: at high degrees of deformation of about 90–95% the equalization in orientation of all the grains is such that all the grains represent as if one crystal on account of which the grains fuse in the spaces formed by disruption between the extended flakes of intercrystalline matter.

Consequently, a small number of quite coarse crystals is obtained. This process is very pronounced at a high temperature of heating and long holding time during subcritical annealing.



This difference in the bonding energy changes the surface curvature of the adjacent grains and causes the grain boundaries to displace, hence the grain size is altered.

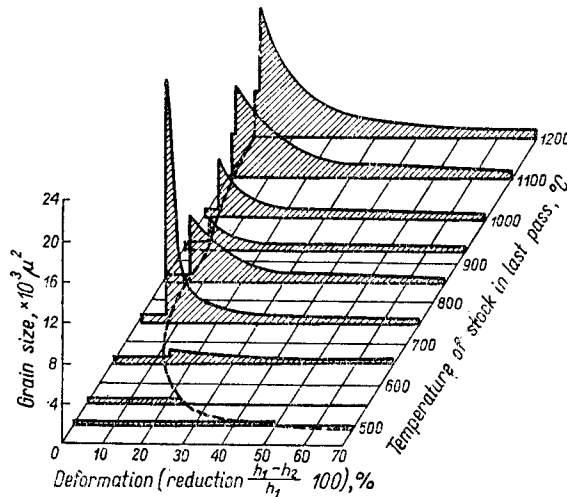


Fig. 1.20. Influence of deformation and temperature on recrystallization of mild steel after hot rolling [10]

### 1.6. Hot Working

In ordinary parlance the terms hot working and cold working are commonly used although they are imprecise for two reasons. Firstly, they may suggest that either the process of working is carried out on hot or cold metal, hence the equipment employed for this purpose is hot or cold during working. Secondly, the process is performed on metals which are plastic in either hot or cold state. The latter aspect should be taken into account to get a more accurate definition of the respective plastic working processes.

In essence, the behaviour of metal during plastic working depends on the properties of metal, i.e. on its ability to strain-harden. This property is of great importance since there are metals which do not strain-harden during plastic working at ambient temperatures, as for example lead.

The most important factor influencing the behaviour of metal during plastic working is the recrystallization range. If plastic working is performed at a higher temperature than that of recrystallization and the deformed grains have enough time to become stress-free and to recrystallize (i.e. to recover their original crystal lattice and to transform into new grains and consequently to reproduce a structure which has similar mechanical properties to those prior to deformation), the metal will not strain-harden during working. Strain-hardening cannot occur since plastic working is carried out on metal heated above the recrystallization temperature, or more accurately, beyond the conditions necessary for recrystallization, since in addition to temperature the time of holding the metal at that temperature is of importance. If, however, the plastic working is performed at a lower temperature than that of recrystallization and the time of holding is such that recrystallization cannot occur, the effects of cold working will be partially or entirely retained, resulting in a greater or smaller degree of strain-hardening of metal.

In accordance with the foregoing considerations hot plastic working is understood to mean a process which is effected under such conditions of time and temperature that recrystallization can occur. If, however, the plastic working is carried out under conditions which do not allow the metal to recrystallize, the process is defined as the cold working [11, 12].

If plastic working is performed under conditions which partially or wholly lie within the recrystallization range, but at very great rates of deformation (or cooling), the recrystallization will be incomplete. The metal will exhibit simultaneously two dissimilar structures, one composed of recrystallized grains and the other of non-recrystallized grains. This could lead to concentration of internal stresses and in consequence to fracture of metal with a lower plasticity. The process conducted under such conditions is defined by S. I. Gubkin [13] as incomplete hot plastic working.

The hot plastic working of metals is understood to mean the process of plastic deformation accompanied by recrystallization. The phenomena occurring during hot working are shown schematically in Fig. 1.21. This figure represents, according to

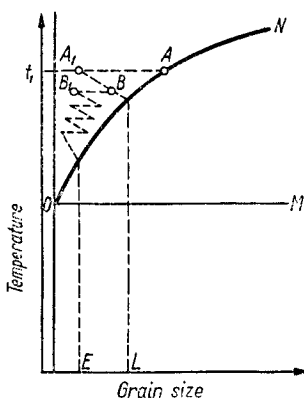


Fig. 1.21. Influence of hot working on grain size of mild steel  
(see Appendix 1) [5]

Howe [10] a general dependence of grain growth on temperature and the combined effects of temperature and plastic deformation. In this figure the temperature is plotted against the grain size.

The horizontal line  $OM$  is for the critical temperature. In the region above it a new crystal structure makes its appearance, i.e. the new grains begin to form. The growth of these newly formed grains follows the curve  $ON$ . The combined effects of temperature and plastic working on the metal structure are shown by respective dash-lines. Heating of the metal to a temperature  $t_1$  corresponds to grain size denoted by  $A$  on the curve  $ON$ . During deformation (plastic working) of this metal at the temperature  $t_1$  the grain size will diminish to the size, for example, denoted by the position  $A_1$ . After deformation effected at high temperature the grain again begins to grow according to the line  $A_1B$ . If the deformation is repeated at point  $B$  (the next pass), the grain size will again decrease according to the line  $BB_1$ . In this way, the grains become smaller during successive passes and increase in size between passes, to reach the final size given by point  $E$  after completion of plastic working.

It can be seen from the above figure that the grain size is dependent on the metal temperature at the final stage of working. It can be concluded that the higher the finishing temperature, the coarser the grain obtained after deformation. If working could be completed at the temperature corresponding to the Point *O* in Fig. 1.21, the finest grain would be obtained and consequently the metal will have a very fine structure.

The diagram shown in Fig. 1.21 is characteristic of all steels. However, experimental data show that different grades of steel do not exhibit the same coarsening of grain.

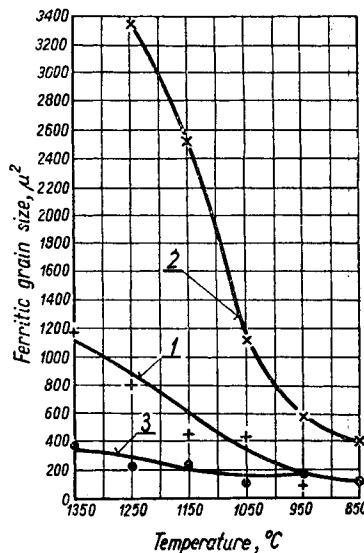


Fig. 1.22. Influence of conditions of cooling after hot working on grain size of mild steel

Figure 1.22 given by Oberhoffer [14] shows variations in grain size for a steel containing 0.1 C, 0.4 Mn, 0.02 Si, 0.012 P and 0.024 S, in per cent.

Curve 2 holds for specimens heated to various temperatures and cooled at slow rate. Curve 1 relates to specimens heated to the same temperatures but subjected to forging after heating and then to slow cooling. Curve 3 shows the grain size of the forged and rapidly cooled specimens, the finishing temperature of forging being 850°C.

The results shown in Fig. 1.22 are in good agreement with Fig. 1.21. They also show that slow cooling after forging produces a coarser grain (curve 1).

A comparison of the curves 1 and 2 indicates that the ferritic grains of steel heated to high temperatures and slowly cooled show a greater coarsening than those of steel subjected to heating and forging.

Recrystallization of steel can occur not only after plastic working at ambient temperatures but also during hot working, provided the deformation is completed below line *GOS*, i.e. below temperature  $A_{r3}$ .

To investigate this phenomenon rolling tests on 30×30 mm steel bars at various temperatures and degrees of deformation were carried out. The steel tested had the following composition, in per cent: 0.03 C, 0.03 Si, 0.08 Mn, 0.01 P and 0.012 S. The degree of deformation was expressed by per cent reduction of the rolled bar. After rolling the bars were heated for 5 minutes in a furnace and then slowly cooled in sand.

The results of these experiments are plotted in the space diagram (Fig. 1.20). The vertical axis is for the grain size and the horizontal axes for the percentage reduction and the temperature, respectively. This diagram shows a considerable coarsening of grain at certain medium values of deformation, the influence of temperature being very small.

It can be shown from Fig. 1.20 that there exists a certain critical temperature which, at constant deformation, causes the greatest coarsening of grain.

This figure has been prepared on the basis of measurements made after deformation of metal in the last pass, in accordance with the former opinion that the initial structure and conditions of rolling prevailing in preceding passes do not significantly affect the final grain size. However, later experiments showed that the grain size is to some extent dependent on the conditions prevailing during previous hot working.

Such space diagrams for different grades of steel are of great practical importance. From them it is possible to predict at what deformation and temperature the rolling process should be completed to obtain the best metal properties.

In the case shown in Fig. 1.20 it can be seen that at the final rolling temperature of 700°C and less, the amount of deformation has almost no influence on the grain size. However, for the temperature range of 750–850°C the amount of reduction in the last pass should be less than 10% or more than 25 per cent. Completion of rolling at 900–1000°C makes it possible to obtain a fine-grained structure for any arbitrary reduction.

Numerous experiments have shown that the result of recrystallization is always the same and independent of whether the recrystallization took place immediately after deformation or at appropriate temperatures of recrystallization. It should be noted, however, that on account of the structure of metal there is no distinct division between hot and cold plastic working, rather there are many intermediate cases where the metal exhibits, to a lesser or greater extent, the phenomenon of cold work.

For example, when the rolling takes place at high temperature no deformed grains in the rolled stock can be discovered since under these conditions the recrystallization is very rapid, eliminating the phenomenon of cold work. The grain obtained in this case will be equiaxial and will vary in size, depending on conditions of rolling.

When the rolling process is carried out at moderate temperatures, the process of crystallization is slower and requires more time. In this case the time necessary for complete recrystallization may be longer than that required by the metal to pass between rolls. However, the recrystallization can still occur in metal after the exit from the rolls, since the temperature of stock decreases slowly. Thus, the recrystallization process can be completed and the structure of metal will not show any traces of cold working.

Temperature has a great influence on the mechanical properties of steel. The ductility of metal at higher temperatures is considerably greater (hence the yield stress of metal is much lower) than that involved with cold working.

First, the influence of temperature on ductility of steel is considered. As previously noted, by the ductility is meant the ability of metal to undergo permanent deformations due to the action of external forces without rupturing or losing its capacity to deform plastically and without destroying the cohesive forces.

Therefore, the greater the ductility of metal, the greater the amount of deformation that can be performed on it, without causing it to fracture. On lowering the temperature to  $-40^{\circ}\text{C}$  or  $-60^{\circ}\text{C}$  many grades of steel become brittle, and lose much of their ductility.

A schematic diagram of the variations in plastic behaviour of steels for the temperature range from the absolute zero ( $-273^{\circ}\text{C}$ ) up to the melting temperature is given in Fig. 1.23 [10].

Another range of decreased ductility of metal occurs for temperatures from 250 to  $350^{\circ}\text{C}$ , the so-called blue brittleness range. With further increase in temperature the ductility of metal increases until the critical temperature is reached, when the steel undergoes phase transformation. This again causes the lowering of ductility of most steel grades. The greatest ductility of steels is in the usual hot working range, i.e. at

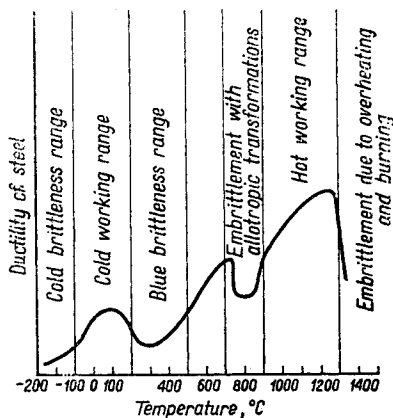


Fig. 1.23. Variations in ductility of steel, depending on temperature

temperatures from 900 to  $1200^{\circ}\text{C}$ . As the steel continues to be heated to temperatures consistent with the pasty state of metal, the ductility decreases again due to effects of overheating and burning.

Knowledge of the dependence of ductility on the temperature is necessary to determine parameters of hot plastic working and to calculate the force involved in deformation. Moreover it is necessary to know the influence of temperature on the yield stress and the compressive strength.

At high temperatures the chemical affinity of metal for oxygen increases, due to which not only iron but also other metals and alloying elements are intensely oxidized. If the metal is heated above the solidus line, the intercrystalline matter begins to melt, and oxygen can penetrate deeply into the metal, causing the crystal surfaces to oxidize.

Naturally, this must result in weakening of the intercrystalline bond and consequently of the internal structure of metal.

Burning of steel, in contrast to overheating, is the effect of chemical processes and results in complete destruction of metal which can be recovered only by remelting.

## CHAPTER 2

# FUNDAMENTAL PRINCIPLES OF PLASTIC WORKING OF METALS

### 2.1. Symbols, Formulae, and Definitions

*Change in shape of stock being rolled*

|                                                     |                                    |
|-----------------------------------------------------|------------------------------------|
| Height of stock                                     | $h$                                |
| Thickness (of plates and coatings)                  | $g$                                |
| Breadth (width)                                     | $b$                                |
| Diameter                                            | $d, D$                             |
| Length                                              | $l$                                |
| Cross-sectional area                                | $F$                                |
| Volume                                              | $V$                                |
| Initial values are designated by subscript 0, e.g.  | $n_0, F_0, g_0$                    |
| Final values are designated by subscript $n$ , e.g. | $h_n, b_n, l_n, F_n$               |
| Intermediate values, e.g.                           | $h_1, h_2 \dots h_i \dots h_{n-1}$ |

*Elongation* is defined as the increase in length of the rolled material.

|                           |                                                                      |
|---------------------------|----------------------------------------------------------------------|
| Absolute elongation       | $\Delta l = l_1 - l_0$                                               |
| Relative elongation       | $\epsilon_l = (l_1 - l_0)/l_0 = \Delta l/l_0$<br>$= (F_0 - F_1)/F_1$ |
| Natural elongation        | $\varphi_l = \log_e (l_1/l_0) = \log_e (F_0/F_1)$                    |
| Coefficient of elongation | $\lambda = l_1/l_0 = F_0/F_1$                                        |

*Draught*—a linear reduction in height of stock under the action of compressive force.

|                        |                                      |
|------------------------|--------------------------------------|
| Absolute draught       | $\Delta h = h_0 - h_1$               |
| Relative draught       | $\epsilon_h = (h_0 - h_1)/h_0$       |
| Percentage reduction   | $r = (h_0 - h_1)/h_0 \text{ } 100\%$ |
| Natural draught        | $\varphi_h = \log_e (h_1/h_0)$       |
| Coefficient of draught | $\gamma = h_1/h_0$                   |

*Spread*—an increase in breadth of the rolled material.

|                                             |                                |
|---------------------------------------------|--------------------------------|
| Absolute spread                             | $\Delta b = b_1 - b_0$         |
| Relative spread                             | $\epsilon_b = (b_1 - b_0)/b_0$ |
| Natural spread                              | $\varphi_b = \log_e (b_1/b_0)$ |
| Coefficient of spread                       | $\beta = b_1/b_0$              |
| Absolute reduction in cross-sectional area  | $\Delta F = F_0 - F_1$         |
| Relative reduction in cross-sectional area  | $U = (F_0 - F_1)/F_0$          |
| Total reduction ratio (in rolling)          | $\lambda_t = F_0/F_n$          |
| Shape factor                                | $\delta_w = b_0/h_0$           |
| Diameter factor (where $D$ = roll diameter) | $\epsilon_w = h_0/D$           |

*State of strain*

|                                                                                               |                                         |
|-----------------------------------------------------------------------------------------------|-----------------------------------------|
| Relative elongation of $dx, dy, dz$ axes of a prismatic volume unit                           | $\epsilon_x, \epsilon_y, \epsilon_z$    |
| Shear components in deformation of a volume element with<br>$dx, dy, dz$ axes                 | $\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ |
| Directions of principal strains (elongation or contraction) height—1,<br>breadth—2, length—3. |                                         |

|                                                                                                                    |                                                                              |
|--------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------|
| Principal strain (elongation or contraction)                                                                       | $\varepsilon_1, \varepsilon_2, \varepsilon_3$                                |
| Elastic strain                                                                                                     | $\varepsilon_s$                                                              |
| Plastic strain                                                                                                     | $\varepsilon_p$                                                              |
| Total strain                                                                                                       | $\varepsilon = \varepsilon_s + \varepsilon_p$                                |
| <i>State of stress</i>                                                                                             |                                                                              |
| Normal stress                                                                                                      | $\sigma$                                                                     |
| Shear stress                                                                                                       | $\tau$                                                                       |
| Stress components on faces of a volume element with dx, dy, dz<br>axes (in orthogonal coordinate system)           | $\sigma_x, \sigma_y, \sigma_z$                                               |
| Shear stress components on faces of a volume element with dx, dy, dz<br>axes (x, y, z perpendicular to each other) | $\tau_{xy}, \tau_{yz}, \tau_{zx}$                                            |
| Directions of principal stresses: 1, 2, 3.                                                                         |                                                                              |
| Principal normal stresses                                                                                          | $\sigma_1, \sigma_2, \sigma_3$                                               |
| Force, load                                                                                                        | $P$                                                                          |
| Deformation work                                                                                                   | $A, L$                                                                       |
| Specific deformation work, deformation work per unit of volume                                                     | $a$                                                                          |
| Efficiency of process (in general)                                                                                 | $\eta$                                                                       |
| Effort of material relative to yield point                                                                         | $n = \sigma_H/K$                                                             |
| <i>Characteristics relating to plastic behaviour of metals</i>                                                     |                                                                              |
| Poisson's ratio                                                                                                    | $\mu$                                                                        |
| Poisson's constant                                                                                                 | $m = 1/\mu$                                                                  |
| Elastic modulus                                                                                                    | $E$                                                                          |
| Modulus of rigidity                                                                                                | $G$                                                                          |
| Modulus of strain hardening                                                                                        | $D_u$                                                                        |
| Modulus of plasticity                                                                                              | $E_p$                                                                        |
| Yield stress of specific resistance to deformation                                                                 | $K_f$                                                                        |
| Static yield stress of stress at extremely small rates of deformation                                              | $K_{f_0}$                                                                    |
| Constrained yield stress from Huber's strength theory                                                              | $K = \eta K_f$                                                               |
| Resistance to deformation in rolling                                                                               | $K_w$                                                                        |
| Mean resistance to deformation                                                                                     | $K_{wm}$                                                                     |
| <i>Miscellaneous physical properties</i>                                                                           |                                                                              |
| Temperature, time                                                                                                  | $t$                                                                          |
| Absolute temperature                                                                                               | $T$                                                                          |
| Mass                                                                                                               | $m$                                                                          |
| Power                                                                                                              | $N$                                                                          |
| Specific weight                                                                                                    | $\gamma_c$                                                                   |
| Coefficient of static friction                                                                                     | $f_0$                                                                        |
| Coefficient of kinematic friction                                                                                  | $f$                                                                          |
| Angle of friction                                                                                                  | $\rho$                                                                       |
| <i>Notation of errors</i>                                                                                          |                                                                              |
| Measured value                                                                                                     | $W_{\text{meas}}$                                                            |
| Calculated value                                                                                                   | $W_{\text{calc}}$                                                            |
| Absolute error                                                                                                     | $\Delta_{\text{abs}} = W_{\text{meas}} - W_{\text{calc}}$                    |
| Relative error                                                                                                     | $\Delta_{\text{rel}} = (W_{\text{meas}} - W_{\text{calc}})/W_{\text{meas}}$  |
| Per cent error                                                                                                     | $\Delta_p = (W_{\text{meas}} - W_{\text{calc}})/W_{\text{meas}} \cdot 100\%$ |
| Error ratio                                                                                                        | $W_{\text{calc}}/W_{\text{meas}}$                                            |

## 2.2. Constancy of Volume and Laws of Plastic Deformation

Assumptions made in theoretical analyses of plastic deformation processes are often in contradiction to practical experience.

This applies to the so-called hypothesis of parallelepipedal deformation. According to this hypothesis a cube of metal subjected to a load changes its shape to a rectangular

prism, its angles and sides remaining orthogonal as prior to deformation (Fig. 2.1). This simplification is made to facilitate the solution of certain problems which would otherwise be difficult to solve.

The error resulting from this assumption depends on the amount of deformation, since the effective distortion of sides and angles increases with deformation. Certain conclusions result from the assumption of parallelepipedal deformation, which can be related to any plastic deformation process.

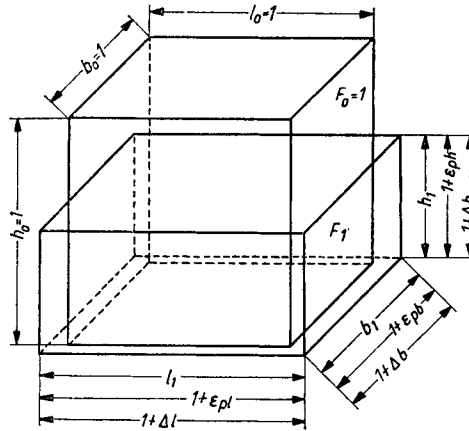


Fig. 2.1. Schematic representation of triaxial deformation of a cubic element

Neglecting the minor losses of metal due to surface oxidization, formation of scale on hot stock, and the process of closing the subcutaneous blowholes in the first passes of ingot rolling, it can be assumed that the volume of metal remains constant during successive stages of deformation, i.e.

$$V_0 = V_1 = V_2 = V_3 = \dots = V_i = \dots = V_n \quad (2.1)$$

This relation, for parallelepipedal deformation, can be expressed as

$$h_0 b_0 l_0 = h_1 b_1 l_1 = \dots = h_i b_i l_i = \dots = h_n b_n l_n \quad (2.2)$$

A more precise analysis leads to the conclusion that the constant volume condition is not perfectly complied with during the plastic deformation process.

Firstly, it is known that plastic deformation is preceded by elastic deformation, the latter sometimes reaching quite considerable values.

Secondly, structural changes occurring during cold working cause certain changes in the specific weight of metal.

Figure 1.15 shows such changes for three steels. It can be seen that they are very small. In hot working as well as filling up of the subcutaneous blowholes the metal undergoes allotropic transformations which also influence the specific weight. The shrinkage resulting from cooling of the deformed metal should also be taken into consideration.

When the change of the specific weight begins to be of importance, equation (2.1) should be corrected to the form

$$V_0 \gamma_0 = V_1 \gamma_1 = V_2 \gamma_2 = \dots = V_i \gamma_i = \dots = V_n \gamma_n$$



This equation can be also written as

$$V_i \gamma_i = \text{constant}$$

where

$V_i$  = volume of metal in successive deformations,

$\gamma_i$  = specific weight of metal in successive deformations.

Of course, these changes are very small and can usually be neglected.

The plastic strain of the cubic element shown schematically in Fig. 2.1 is considered, with initial dimensions  $h_0 = b_0 = l_0 = 1$ , which become  $h_1$ ,  $b_1$  and  $l_1$  after deformation. Expressing the plastic strain in terms of absolute displacements  $\Delta l$ ,  $\Delta b$  and  $\Delta h$ , the assumption was often made that the metal displaced by absolute draught  $\Delta h$  is equal to the sum of displacements in elongation  $\Delta l$  and lateral spread  $\Delta b$ , i.e.

$$\Delta h \simeq \Delta b + \Delta l \quad (2.3)$$

However, this assumption is erroneous, since it is not difficult to prove the contrary. From Fig. 2.1 the strain of a cubic element can be expressed in terms of its volume

$$\begin{aligned} V &= 1 \times 1 \times 1 = [(1 + \Delta h)(1 + \Delta l)(1 + \Delta b)] \\ &= 1 + \Delta h + \Delta b + \Delta l + \Delta h \Delta b + \Delta l \Delta b + \Delta h \Delta l + \Delta h \Delta b \Delta l \end{aligned} \quad (2.4)$$

Thus, if relation (2.3) is assumed, effectively the product values obtained from equation (2.4) are neglected. This can be assumed in the case of very small elastic strains where these products are very small. However for large finite deformations, when these product values are great, such omissions could result in considerable errors.

Another generally accepted method of describing plastic strains is to express it in terms of relative deformations. Using the same notations as in Fig. 2.1, the following relations are obtained [12]:

$$\left. \begin{aligned} \varepsilon_h &= \frac{1}{h_0} \int_{h_0}^{h_1} dh = \frac{h_1 - h_0}{h_0} \\ \varepsilon_b &= \frac{1}{b_0} \int_{b_0}^{b_1} db = \frac{b_1 - b_0}{b_0} \\ \varepsilon_l &= \frac{1}{l_0} \int_{l_0}^{l_1} dl = \frac{l_1 - l_0}{l_0} \end{aligned} \right\} \quad (2.5)$$

In describing plastic strains it was also assumed that the metal displaced by draught corresponded to that displaced in longitudinal and transverse directions, i.e.

$$|\varepsilon_h| \simeq |\varepsilon_b + \varepsilon_l| \quad (2.6)$$

Applying the constant volume relationship given by equation (2.1), the strain of a cubic element of metal shown in Fig. 2.1 can be expressed, using the methods previously described

$$\begin{aligned} V &= 1 \times 1 \times 1 = [(1 + \varepsilon_h)(1 + \varepsilon_b)(1 + \varepsilon_l)] \\ &= [1 + \varepsilon_h + \varepsilon_b + \varepsilon_l + \varepsilon_h \varepsilon_b + \varepsilon_h \varepsilon_l + \varepsilon_b \varepsilon_l + \varepsilon_h \varepsilon_b \varepsilon_l] \end{aligned} \quad (2.7)$$

where  $\varepsilon_h$ ,  $\varepsilon_b$ ,  $\varepsilon_l$  denote strains of individual sides of the cubic element in the direction of principal stresses.

From relation (2.7) it may be seen that for elastic deformation the individual products of strain can be neglected and equation (2.6) is valid, whereas for large and finite plastic strains this would be erroneous [11, 12].

Further methods of expressing plastic strains using coefficients of deformation will now be considered. From the constant volume relationship (2.1) represented diagrammatically in Fig. 2.1

$$V_0 = V_1 = h_0 b_0 l_0 = h_1 b_1 l_1$$

On dividing this becomes

$$\frac{V_1}{V_0} = \frac{h_1 b_1 l_1}{h_0 b_0 l_0} = \gamma \beta \lambda = 1 \quad (2.8)$$

In this relation  $\gamma$ ,  $\beta$ ,  $\lambda$  are the coefficients of deformation, i.e.  $\gamma$  = coefficient of draught,  $\beta$  = coefficient of spread, and  $\lambda$  = coefficient of elongation. Their product must be 1 if the condition of constant volume is to be satisfied.

Equation (2.8) expresses a relationship which always satisfies the constant volume condition and enables plastic strains to be described correctly.

To make practical use of equation (2.8) a nomogram shown in Fig. 2.2 has been developed. This nomogram represents not only the solution of equation (2.8) but also includes the relation

$$\log_e \gamma + \log_e \beta + \log_e \lambda = 0$$

The values of  $\gamma$  from 0.1 to 1.0 and those of  $-\log_e \gamma$  from 2.303 to 0 are plotted on the upper horizontal scales of Fig. 2.2, on the left hand vertical scale,  $\beta$  coefficients from 1 to 2.9 and  $\log_e \beta$  from 0 to 1.1 are given. Diagonal lines represent  $\lambda$  coefficients from 1.0–9.0 and the lower horizontal scale  $\log_e \lambda$  from 0 to 2.303.

This nomogram enables the rapid determination of the third coefficient of deformation when the two others are known. An example plotted on the nomogram illustrates the method of application.

For example,  $\gamma = 0.55$  and  $\beta = 1.27$  are given. On finding these values on the respective scales, the coordinate lines are drawn to intersect at point *A*. The corresponding value of  $\lambda$  will be 1.43. From the respective scales further values can also be read off:  $\log_e \beta = 0.239$  (point *B*),  $\log_e \lambda = 0.357$  (point *C*), and  $\log_e \gamma = -0.590$  (point *D*). The sum of these logarithms will be  $-0.002$ .

This diagram facilitates the selection of a third coefficient of deformation e.g.  $\lambda$ , when  $\beta$  and  $\gamma$  are known. The point at which the  $\beta$  and  $\gamma$  coordinates intersect gives the required value of  $\lambda$ .

The strain of a cubic element of metal shown in Fig. 2.1 will now be described using a further relationship

$$\begin{aligned} \int_{h_0}^{h_1} \frac{dh}{h} + \int_{b_0}^{b_1} \frac{db}{b} + \int_{l_0}^{l_1} \frac{dl}{l} &= \log_e \frac{h_1}{h_0} + \log_e \frac{b_1}{b_0} + \log_e \frac{l_1}{l_0} = \log_e \gamma + \log_e \beta + \log_e \lambda \\ &= \varphi_h + \varphi_b + \varphi_l = \varepsilon_{rh} + \varepsilon_{rb} + \varepsilon_{rl} = 0 \end{aligned} \quad (2.9)$$

In this equation  $\varphi_h$ ,  $\varphi_b$ , and  $\varphi_l$  or  $\varepsilon_{rh}$ ,  $\varepsilon_{rb}$ ,  $\varepsilon_{rl}$  represent the logarithmic (natural) strain, i.e. draught, spread and elongation, respectively.

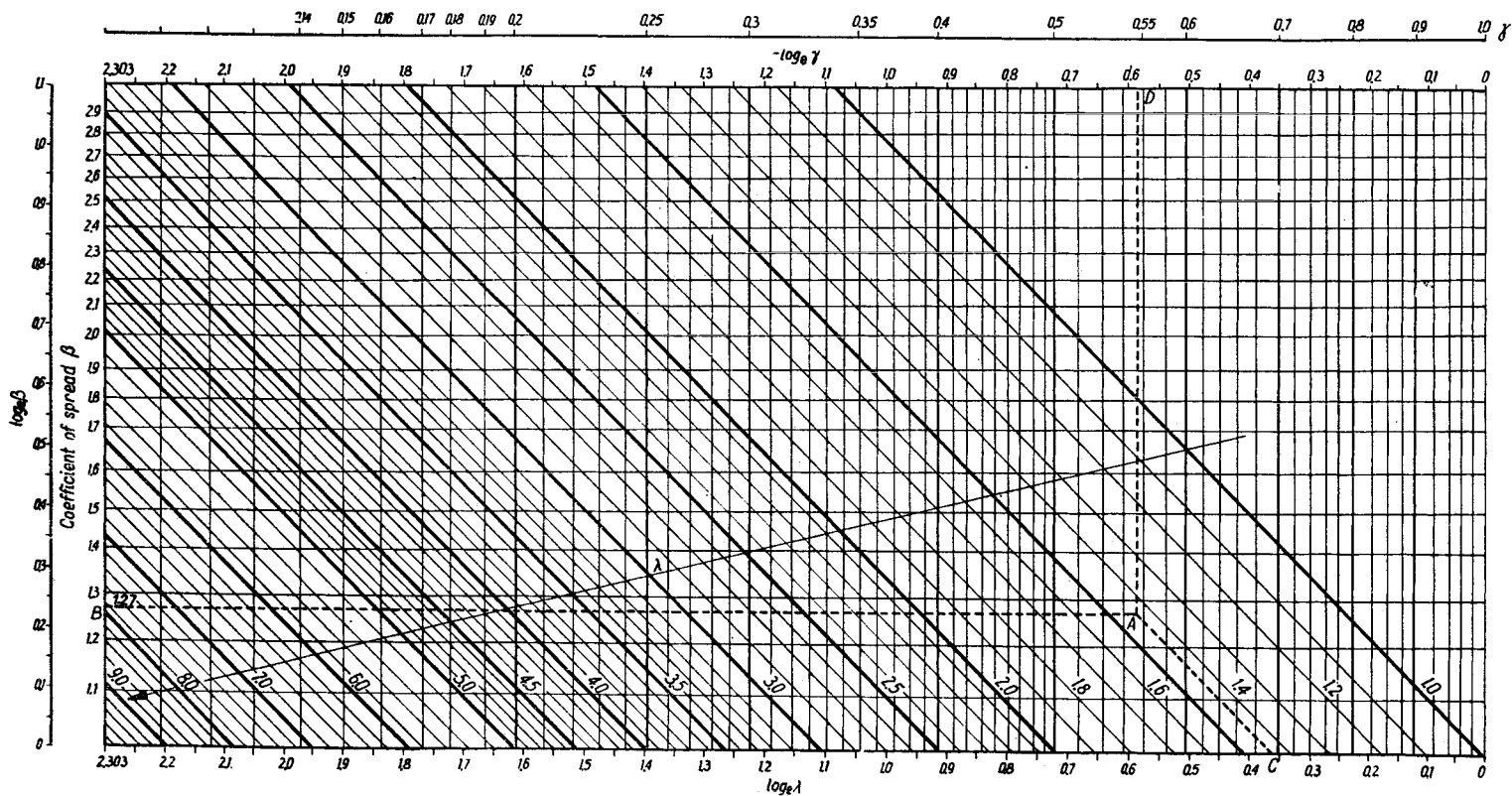


Fig. 2.2. Nomogram relating coefficients  $\beta$ ,  $\lambda$  and  $\gamma$

Equation (2.9) satisfies the constant volume relationship for the whole deformation range and thus can be used to describe correctly the plastic strains. The maximum strains are always written as

$$\varphi_h \text{ or } \varepsilon_{rh}$$

hence, the two remaining strains are given as the sums

$$|\varphi_h| = |\varphi_b + \varphi_l| \quad (2.10)$$

$$|\varepsilon_{rh}| = |\varepsilon_{rb} + \varepsilon_{rl}| \quad (2.11)$$

By taking the logarithm of equation (2.8), equation (2.9) is found directly, hence these equations are equivalent.

In processes which involve a greater number of deformations,  $n$ , the dimensions of successive sections are calculated directly from the preceding ones by using appropriate formulae and experimental data.

The areas of individual sections are calculated as follows:

$$\left. \begin{aligned} F_0 &= F_1 \lambda_1 \\ F_1 &= F_2 \lambda_2 \\ F_2 &= F_3 \lambda_3 \\ F_{n-1} &= F_n \lambda_n \end{aligned} \right\} \quad (2.12)$$

Thus, the areas of successive sections are obtained by multiplying the areas of the preceding ones by the appropriate coefficients of elongation, i.e.

$$\left. \begin{aligned} F_{n-1} &= F_n \lambda_n \\ F_{n-2} &= F_n \lambda_n \lambda_{n-1} \\ F_1 &= F_n \lambda_n \lambda_{n-1} \dots \lambda_2 \\ F_0 &= F_n \lambda_n \lambda_{n-1} \dots \lambda_2 \lambda_1 \end{aligned} \right\} \quad (2.13)$$

Hence

$$\lambda_1 \lambda_2 \lambda_3 \dots \lambda_n = \lambda_t = \frac{F_0}{F_n} \quad (2.14)$$

By assuming or calculating the mean coefficient of elongation,  $\lambda_m$  for all the deformations involved in the system, the number of the required passes,  $n$ , can be obtained from the following formulae:

$$(\lambda_m)^n = \lambda_t = \frac{F_0}{F_n} \quad (2.15)$$

$$\lambda_m = \sqrt[n]{\frac{F_0}{F_n}} = \sqrt{\lambda_t} \quad (2.16)$$

$$n = \frac{\log F_0 - \log F_n}{\log \lambda_m} = \frac{\log \lambda_t}{\log \lambda_m} \quad (2.17)$$

These formulae (2.13)–(2.17) are generally applied for plastic working processes in which a work piece undergoes several successive deformations.

When the successive sections are round in shape, the solution of equations (2.13)–(2.17) does not present any difficulties, since instead of area ratios, diameter ratios are directly obtained, e.g.

$$\lambda = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2} = \frac{d_1^2}{d_2^2} \quad (2.18)$$

For the solution of equations (2.15)–(2.17) to give the required number of passes, the nomogram shown in Fig. 2.3 [10] can be used. Joining corresponding points on

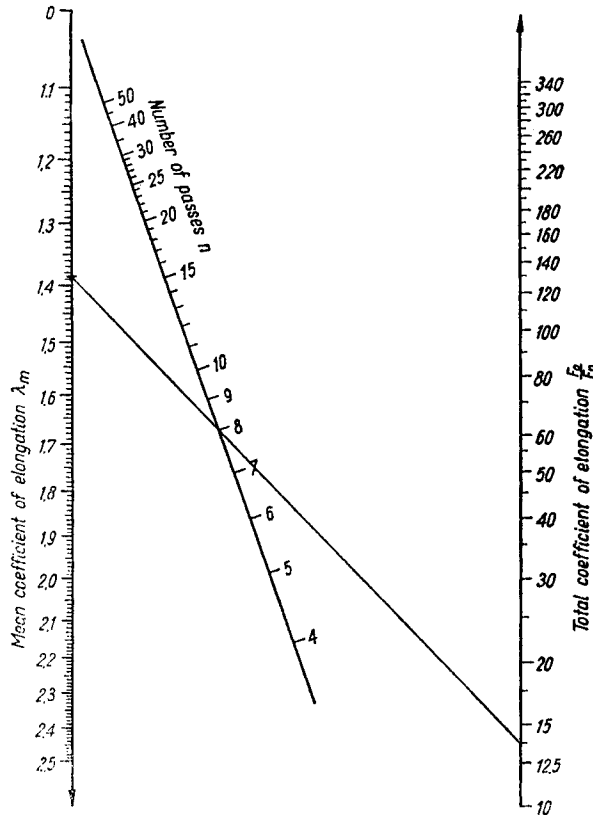


Fig. 2.3. Nomogram for determining number of passes

the  $\lambda_m$  and  $F_0/F_n$  scales with a straight line, the required number of passes is given by the point of intersection of this line with the  $n$  scale.

From the equation (2.15)

$$\lambda_t = \frac{F_0}{F_n} = \frac{h_0}{h_n} \frac{b_0}{b_n} = \frac{1}{\gamma_t \beta_t}$$

Hence, it can be concluded that the general relation (2.8)  $\gamma\beta\lambda = 1$ , also holds for multiple successive deformations, and takes the form

$$\gamma_t \beta_t \lambda_t = 1 \quad (2.19)$$

where  $\gamma_t$  and  $\beta_t$  are the coefficients of total deformation in height and breadth similarly as for elongation. Hence from relation (2.8) the following formulae are obtained

$$\gamma_t = \gamma_1 \gamma_2 \gamma_3 \dots \gamma_i \dots \gamma_n \quad (2.20)$$

$$\beta_t = \beta_1 \beta_2 \beta_3 \dots \beta_i \dots \beta_n \quad (2.21)$$

$$\lambda_t = \lambda_1 \lambda_2 \lambda_3 \dots \lambda_i \dots \lambda_n \quad (2.22)$$

$\downarrow \quad \downarrow \downarrow \downarrow \quad \downarrow \quad \downarrow$   
 $1 \quad 1 \ 1 \ 1 \quad 1 \quad 1$

Products:

It can be seen from formulae (2.20)–(2.22) that the products of respective deformation coefficients should be always equal to 1 (as indicated by arrows).

From equation (2.15), (2.16) and (2.17) the respective deformation coefficients can also be written

$$\lambda_m^n = \lambda_t \quad \text{and hence} \quad \lambda_m = \sqrt[n]{\lambda_t}$$

similarly

$$\gamma_m^n = \gamma_t \quad \gamma_m = \sqrt[n]{\gamma_t} \quad (2.23)$$

$$\beta_m^n = \beta_t \quad \beta_m = \sqrt[n]{\beta_t} \quad (2.24)$$

Since the relation (2.19) is always complied with, substituting the appropriate values from equations (2.15), (2.23) and (2.24)

$$\gamma_m^n \beta_m^n \lambda_m^n = 1 \quad (2.25)$$

and substituting further values in equation (2.25) and transposing

$$\gamma_m \beta_m \lambda_m = \sqrt[n]{\gamma_t} \sqrt[n]{\beta_t} \sqrt[n]{\lambda_t} = 1 \quad (2.26)$$

Similarly, by taking logarithms for equations (2.20), (2.21) and (2.22)

$$\log_e \gamma_t = \log_e \gamma_1 + \log_e \gamma_2 + \log_e \gamma_3 + \dots + \log_e \gamma_i + \dots + \log_e \gamma_n \quad (2.27)$$

$$\log_e \beta_t = \log_e \beta_1 + \log_e \beta_2 + \log_e \beta_3 + \dots + \log_e \beta_i + \dots + \log_e \beta_n \quad (2.28)$$

$$\log_e \lambda_t = \log_e \lambda_1 + \log_e \lambda_2 + \log_e \lambda_3 + \dots + \log_e \lambda_i + \dots + \log_e \lambda_n \quad (2.29)$$

and hence

$$\left. \begin{aligned} \log_e \gamma_1 + \log_e \beta_1 + \log_e \lambda_1 &= 0 \\ \log_e \gamma_i + \log_e \beta_i + \log_e \lambda_i &= 0 \\ \log_e \gamma_t + \log_e \beta_t + \log_e \lambda_t &= 0 \end{aligned} \right\} \quad (2.30)$$

On dividing by  $n$

$$\frac{\log_e \gamma_t}{n} + \frac{\log_e \beta_t}{n} + \frac{\log_e \lambda_t}{n} = 0 \quad (2.31)$$

and finally

$$\log_e \gamma_m + \log_e \beta_m + \log_e \lambda_m = 0 \quad (2.32)$$

Such a description of plastic strains is allowable, over the whole range of deformation as opposed to relative elongation (2.6).

TABLE 2.1  
RESULTS OBTAINED FOR A LEAD SPECIMEN ROLLED IN 6 PASSES ILLUSTRATING THE APPLICATION OF FORMULAE (2.19)–(2.26)

| Pass No. | $h_1$<br>mm | $h_2$<br>mm | $b_1$<br>mm | $b_2$<br>mm | $l_1$<br>mm | $l_2$<br>mm | $\gamma = h_2/h_1$ | $\beta = b_2/b_1$ | $\lambda = l_2/l_1$ | Product<br>$\gamma\beta\lambda$ | $\log_e \gamma$ | $\log_e \beta$ | $\log_e \lambda$ | Sum of<br>logarithms |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|--------------------|-------------------|---------------------|---------------------------------|-----------------|----------------|------------------|----------------------|
| 1        | 7.8         | 5.07        | 61.0        | 64.0        | 134.0       | 196.48      | 0.6500             | 1.0492            | 1.4663              | 0.99999                         | $\bar{1}.56921$ | 0.04803        | 0.38274          | −0.00002             |
| 2        | 5.07        | 3.0         | 64.0        | 65.0        | 196.48      | 326.94      | 0.5917             | 1.0156            | 1.6640              | 0.99995                         | $\bar{1}.47524$ | 0.01548        | 0.50922          | −0.00006             |
| 3        | 3.0         | 2.0         | 65.0        | 66.0        | 326.94      | 482.96      | 0.6667             | 1.0154            | 1.4771              | 0.99995                         | $\bar{1}.59458$ | 0.01529        | 0.39008          | −0.00005             |
| 4        | 2.0         | 1.75        | 66.0        | 66.0        | 482.96      | 551.95      | 0.8750             | 1.0000            | 1.1429              | 1.00004                         | $\bar{1}.86648$ | 0.00000        | 0.13357          | +0.00005             |
| 5        | 1.75        | 1.30        | 66.0        | 67.0        | 551.95      | 731.92      | 0.7429             | 1.0152            | 1.3261              | 1.00013                         | $\bar{1}.70280$ | 0.01509        | 0.28224          | +0.00013             |
| 6        | 1.30        | 0.75        | 67.0        | 68.0        | 731.95      | 1249.46     | 0.5769             | 1.0149            | 1.7071              | 0.99950                         | $\bar{1}.44991$ | 0.01479        | 0.53480          | −0.00050             |

$$(1) \lambda_{1,2} = \lambda_1 \lambda_2 = 2.4399$$

$$(2) \lambda_{1,2,3} = \lambda_1 \lambda_2 \lambda_3 = 3.6042$$

$$(3) \lambda_{1,2,3,4} = \lambda_1 \lambda_2 \lambda_3 \lambda_4 = 4.1191$$

$$(4) \lambda_{1,2,3,4,5} = \lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 = 5.4622$$

$$(5) \lambda_{1,2,3,4,5,6} = \lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 \lambda_6 = 9.3286$$

$$\gamma_{1,2} = \gamma_1 \gamma_2 = 0.3846$$

$$\gamma_{1,2,3} = \gamma_1 \gamma_2 \gamma_3 = 0.2564$$

$$\gamma_{1,2,3,4} = \gamma_1 \gamma_2 \gamma_3 \gamma_4 = 0.2244$$

$$\gamma_{1,2,3,4,5} = \gamma_1 \gamma_2 \gamma_3 \gamma_4 \gamma_5 = 0.1667$$

$$\gamma_{1,2,3,4,5,6} = \gamma_1 \gamma_2 \gamma_3 \gamma_4 \gamma_5 \gamma_6 = 0.0962$$

$$\log_e \gamma_t = \sum_{i=1}^{i=6} \log_e \gamma_i = \bar{3}.65820$$

$$\log_e \beta_t = \sum_{i=1}^{i=6} \log_e \beta_i = 0.10868$$

$$\log_e \lambda_t = \sum_{i=1}^{i=6} \log_e \lambda_i = 2.23265$$

$$\beta_{1,2} = \beta_1 \beta_2 = 1.0656$$

$$\beta_{1,2,3} = \beta_1 \beta_2 \beta_3 = 1.0802$$

$$\beta_{1,2,3,4} = \beta_1 \beta_2 \beta_3 \beta_4 = 1.0820$$

$$\beta_{1,2,3,4,5} = \beta_1 \beta_2 \beta_3 \beta_4 \beta_5 = 1.0983$$

$$\beta_{1,2,3,4,5,6} = \beta_1 \beta_2 \beta_3 \beta_4 \beta_5 \beta_6 = 1.1148$$

$$\gamma_m = \sqrt[6]{0.0962} = 0.6768$$

$$\beta_m = \sqrt[6]{1.115} = 1.0183$$

$$\lambda_m = \sqrt[6]{9.329} = 1.4510$$

$$\gamma_m \beta_m \lambda_m = 1.00000018$$

$$\log_e \gamma_m \frac{\log_e \gamma_t}{6} = \bar{1}.60970$$

$$\log_e \beta_m \frac{\log_e \beta_t}{6} = 0.01811 \quad \log_e \gamma_m + \log_e \beta_m + \log_e \lambda_m = -0.00008$$

$$\log_e \lambda_m \frac{\log_e \lambda_t}{6} = 0.37211$$

In this way additional relationships are obtained, which facilitate the calculation of plastic working processes. This applies especially to the situation when an additional correlation between the coefficients of deformation exists, e.g.

$$\beta = f(\gamma) \quad \lambda = \varphi(\gamma) \quad \text{or} \quad \beta = \varphi(\lambda)$$

An example of the application of formulae (2.19)–(2.26) for rolling a lead specimen is given in Table 2.1.

### 2.3. Stress-Strain Relationships and Condition of Plasticity

Theories of plastic strain are derived from the theory of elasticity, therefore the relation between theories of elasticity and plastic strain should first be examined.

A small load applied to a homogeneous and isotropic material (i.e. one having the same density and crystal structure throughout) produces a certain deformation. True metals are only approximately homogeneous and isotropic, therefore for these studies they will only be assumed as macroscopically homogeneous and isotropic.

A volume element of metal subjected to the action of external forces will be considered. For simplicity, a cubic element of metal with edges parallel to the  $x$ ,  $y$ ,  $z$  axes will be considered. On applying two tensile forces  $P$ , and  $-P$  over the cross-sectional

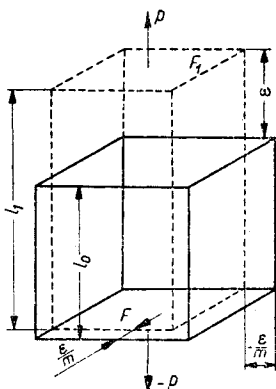


Fig. 2.4. Deformation of a cubic element [15]

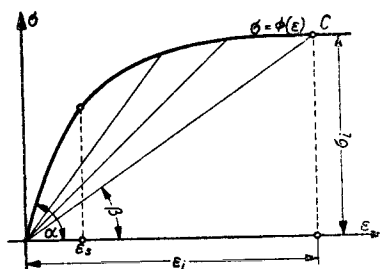


Fig. 2.5. Comparative stress-strain curve [15]

area  $F$  of this element, the normal stress produced  $\sigma = P/F$  (Fig. 2.4) will cause elongation of the specimen from the initial length  $l_0$  to  $l_1$ . The absolute elongation of the element will be

$$\Delta l = l_1 - l_0$$

and the relative elongation  $\varepsilon = \Delta l/l_0$ . If the stress is small and does not exceed the elastic limit of metal (Fig. 2.5), the relative elongation (strain) will be proportional to stress, i.e.  $\sigma = E\varepsilon$ . This is simply the expression of Hooke's law, from which the modulus of elasticity or Young's modulus

$$E = \frac{\sigma}{\varepsilon} \quad (2.33)$$

is found. This relation does not express changes of dimensions normal to the direction of stress. The generalized form of Hooke's law states that the decrease of dimensions



along the  $x$  and  $y$  axes is proportional to the stress  $\sigma$ . Hence, the ratio of the transverse strain to the longitudinal strain is also constant, and is characteristic of the given metal; i.e.

$$\frac{\varepsilon'}{\varepsilon} = \mu \quad \text{or} \quad \frac{\varepsilon}{\varepsilon'} = \frac{1}{\mu} = m \quad (2.34)$$

(Fig. 2.4), where  $\mu$  is known as Poisson's ratio and  $m$  as Poisson's number. Poisson assumed  $m = 4$ , but this value actually varies from 3 to 4, and for most metals it is about 3.3.

The above reasoning is based on the assumption of a one-dimensional state of stress.

If the elemental cube of metal is subjected to a system of 6 tensile forces  $P_x, -P_x, P_y, -P_y, P_z, -P_z$  (Fig. 2.6), acting on the faces of the cube, these forces produce in the metal normal stresses  $\sigma_x, \sigma_y$  and  $\sigma_z$ , and it is assumed that the point  $A$  is situated on the  $z$  axis.

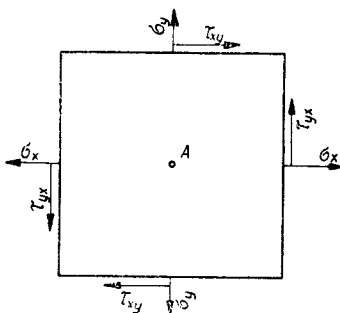


Fig. 2.6. Forces acting on faces of a volume element [15]

When the produced stresses do not exceed the elastic limit of the metal, the stress  $\sigma_x$  produces an elongation  $\sigma_x/E$  in the  $x$  direction and decrease of length in the  $y$  and  $z$  directions, the latter deformations being  $-\sigma_x/Em$  and  $-\sigma_x/Em$ , respectively.

The behaviour of metal under the action of  $\sigma_y$  and  $\sigma_z$  is exactly similar, and if their effects are added, the total strain will be

$$\left. \begin{aligned} \varepsilon_x &= \frac{1}{E} \left( \sigma_x - \frac{\sigma_y + \sigma_z}{m} \right) \\ \varepsilon_y &= \frac{1}{E} \left( \sigma_y - \frac{\sigma_x + \sigma_z}{m} \right) \\ \varepsilon_z &= \frac{1}{E} \left( \sigma_z - \frac{\sigma_x + \sigma_y}{m} \right) \end{aligned} \right\} \quad (2.35)$$

On each cross-section plane of cube, inclined to the coordinate axes system, in addition to the perpendicular stresses, shear stresses also occur (Fig. 2.6). Consequently, on cube faces normal to  $x, y, z$  directions, component shear stresses  $\tau_{xy}, \tau_{xz}, \tau_{yz}$  will occur.

Under the action of shear stresses the right angle  $\gamma$  undergoes transverse strain proportional to stress, i.e.

$$\frac{\tau}{\gamma} = G \quad (2.36)$$

where  $G$  is the modulus of elasticity in shear.

The preceding relationships for elastic deformation have been derived on the assumption that the strains of metal are very small and do not exceed the elastic limit of the material. On reaching the yield point the metal changes considerably in behaviour, the strains are permanent and irreversible.

In the plastic range, the deformed metal flows in certain directions under the action of the applied force (Fig. 2.5) and friction. The structure and properties of the metal change or do not change during the flow, depending on whether strain-hardening due to cold work occurs or does not.

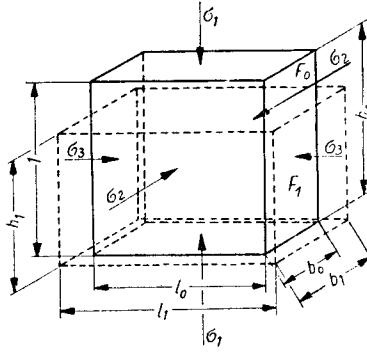


Fig. 2.7. State of stress-strain in triaxial deformation

If the plastic strain of the metal in the three co-ordinate directions is expressed as

$$\varepsilon_{px} + \varepsilon_{py} + \varepsilon_{pz} = \varepsilon_{p1} + \varepsilon_{p2} + \varepsilon_{p3}$$

then a set of equations similar to (2.35) will be obtained (Fig. 2.7)

$$\left. \begin{aligned} \varepsilon_{p1} &= \frac{1}{E_p} \left[ \sigma_1 - \frac{1}{2} (\sigma_2 + \sigma_3) \right] \\ \varepsilon_{p2} &= \frac{1}{E_p} \left[ \sigma_2 - \frac{1}{2} (\sigma_3 + \sigma_1) \right] \\ \varepsilon_{p3} &= \frac{1}{E_p} \left[ \sigma_3 - \frac{1}{2} (\sigma_1 + \sigma_2) \right] \end{aligned} \right\} \quad (2.37)$$

According to Nadai [16] the value of  $1/m$  is assumed to be equal to 0.5.

Thus, Poisson's number  $1/m$  which equals about 0.3 prior to reaching the elastic limit, increases to 0.5 on exceeding the yield point and remains at that level for the whole range of plastic deformation.

Considering the ratio  $1/E_p$ ,  $E_p$  in the range of plastic deformation corresponds to  $E$  in the range of elastic deformation and therefore  $E_p$  is called the modulus of plasticity.

As the plastic strain increases,  $E_p$  decreases (Fig. 2.5) since  $E_p$  is represented by the ratio of the stress to the corresponding strain.

In a plane inclined at an angle to the coordinate axes tangential displacements will occur, causing change of this angle. This angle is determined from Navier and Stokes formulae for the internal friction of viscous liquids [15]

$$\tau_{xy} = f \frac{\partial v}{\partial y} = f \frac{\partial \gamma_{xy}}{\partial t}$$

where

$\tau_{xy}$  = shear stress,

$f$  = coefficient of internal friction,

$v$  = velocity,

$\gamma_{xy}$  = slip or displacement,

$\partial \gamma_{xy} / \partial t$  = velocity of slip.

In a polycrystalline aggregate the slip or displacement  $\gamma_{xy}$  must be proportional to the shear stress  $\tau_{xy}$  at the point of displacement, hence

$$\tau_{xy} = \frac{E_p}{3} \gamma_{xy} \quad (2.38)$$

This formula expresses the plastic slip of the metal. In engineering calculations of the strength of metals, the ultimate allowable loading is limited by the resultant condition of stress evoked by a state of deformation. Various strength hypotheses give different choices of the limiting allowable stress in an endeavour to determine limiting conditions that will ensure for the metal an adequate reserve strength factor.

Various authors consider either the limit of elasticity, the yield point or the ultimate strength as the limiting strength value of the metal.

For non-brittle metals such as mild steel, copper, nickel etc., the yield point, and for brittle metals e.g. cast iron or hardened steel having no distinct yield point, the ultimate tensile stress is now generally accepted as the safe working limit.

For these considerations the yield point is regarded as the limiting value for safety under a condition of stress.

The principal stresses are written in order of magnitude:  $\sigma_1 > \sigma_2 > \sigma_3$ . In older hypotheses it was assumed that the maximum principal stress  $\sigma_{\max}$  caused the phenomenon of yielding, independently of the two remaining stresses. This was known as the theory of maximum stress.

A second theory stated that the stress producing the maximum strain  $\epsilon_{\max}$  was responsible for exceeding the safe limit. This was known as the maximum strain theory.

When it was found that the flow and slip of metal under load is responsible for fracture, a new theory, the so-called maximum shear stress theory was developed, where

$$\tau_{\max} = \frac{\sigma_3 - \sigma_1}{2} = \text{constant} \quad (2.39)$$

According to this theory a measure of the yield strength of the metal is the magnitude of the angle of distortion  $\gamma$  which is known to be proportional to the shear stress.

In this theory the role of the average stress  $\sigma_2$  was entirely disregarded.

The most recent theory is the deformation-energy theory evolved by M. T. Huber [17], R. von Mises and H. Hencky [18]. According to this theory the yield strength of ductile metals is measured by the elastic strain energy only.

The strain energy per unit volume of metal is given by the formula

$$A = A_v + A_f \quad (2.40)$$

where

$$A_v = \frac{1-2\mu}{6E} (\sigma_x + \sigma_y + \sigma_z)^2$$

represents the elastic strain energy associated with changes in volume, i.e. without change of shape, and

$$A_f = \frac{1+\mu}{6E} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]$$

represents the strain energy of deformation.

In simple tension or compression, when all the shear stresses and  $\sigma_y, \sigma_z = 0$ , formula (2.40) gives the critical value, where  $\sigma_x = R_{pl}$  ( $R_{pl}$  is the stress at the yield point). Therefore, the expression (2.40) reduces to

$$\frac{1+\mu}{3E} R_{pl}^2$$

Thus the limiting stress condition is defined by

$$\frac{1+\mu}{6E} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)] = \frac{1+\mu}{3E} R_{pl}^2$$

and hence

$$\frac{1}{2} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2] + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) = R_{pl}^2 \quad (2.41)$$

On replacing  $R_{pl}$  by a reduced stress  $K_{red}$  one obtains

$$K_{red} = \sqrt{\frac{1}{2} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2] + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} \quad (2.42)$$

This formula gives the value of stress for any state of stress. For a biaxial state of stress

$$K_{red} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} \quad (2.43)$$

Equation (2.42) applied to plastic working processes takes the well known form

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2R_{pl}^2 = 2K_f^2 \quad (2.44)$$

assuming  $\sigma_1 \geq \sigma_2 \geq \sigma_3$ .

In strength theories of metals  $R_{pl}$  represents the value of stress at the yield point in uniaxial tension or compression, and under conditions of yielding,  $K_f$  represents the stress at the yield point of metal for the given conditions of working.

Direct conclusions can now be drawn from the Huber criterion of yielding (2.44). Assuming a plane state of strain (Fig. 2.8) i.e.  $\varepsilon_{p2} = 0$ , the magnitude of the average stress for the plastic range of metal is obtained

$$\varepsilon_{p2} = \frac{1}{E_p} \left[ \sigma_2 - \left( \frac{\sigma_3 + \sigma_1}{2} \right) \right] = 0 \quad (2.45)$$

Hence, the average stress is calculated

$$\sigma_2 = \frac{\sigma_1 + \sigma_3}{2} \quad (2.46)$$

By substituting this expression (2.46) into equation (2.44) the first boundary condition results

$$\sigma_1 - \sigma_3 = \frac{2}{\sqrt{3}} K_f = 1.155 K_f \quad (2.47)$$

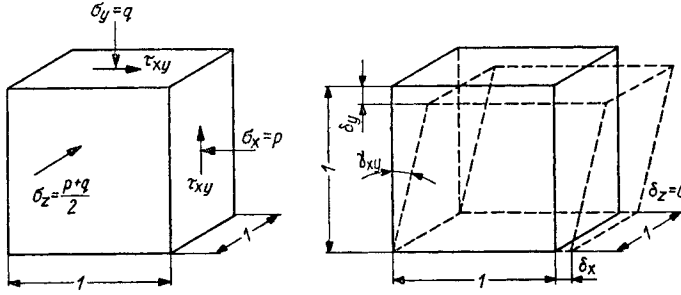


Fig. 2.8. State of stress-strain without lateral spread (plane strain) [15]

By putting into equation (2.44)

$$\sigma_2 = \sigma_3 \quad (2.48)$$

the second boundary condition is obtained

$$\sigma_1 - \sigma_3 = K_f \quad (2.49)$$

By combining equation (2.47) and (2.49)

$$\sigma_1 - \sigma_3 = \eta K_f = K \quad (2.50)$$

where  $\eta$  varies from 1 to 1.155 depending on the magnitude of  $\sigma_2$ , and  $\eta K_f = K$  is defined as the constrained yield stress. In the mathematical analysis of this problem the relation between the stress and the state of plastic strains will be sought.

When based exclusively on strains, the solution of any problem is facilitated as the strains are easy to measure directly, while the values of effective stresses must be calculated by complicated and uncertain methods. Equally, relying only on measurements of stresses is unsuitable since they are extremely difficult to carry out, and would considerably reduce the practical value of this problem. It is known that with the assumption  $\varepsilon_{p2} = 0$  (Fig. 2.8) the value of the average stress (2.46) is

$$\sigma_2 = \frac{\sigma_1 + \sigma_3}{2}$$

and the yield criterion becomes

$$\sigma_1 - \sigma_3 = 1.155 K_f$$

This is the boundary condition for biaxial strain when the elongation  $\varepsilon_{p2} = 0$ . The assumption of the condition given by formula (2.48)

$$\sigma_2 = \sigma_3 \quad \text{or} \quad \sigma_1 - \sigma_3 = K_f$$

for the plastic region of deformation is now analysed.

By putting these stresses in the equations for  $\varepsilon_{p2}$  and  $\varepsilon_{p3}$ , the following relations are obtained

$$\varepsilon_{p2} = \frac{1}{E_p} \left[ \sigma_2 - \frac{1}{2} (\sigma_3 + \sigma_1) \right] = \frac{1}{E_p} \left[ \sigma_2 - \frac{1}{2} (\sigma_2 + \sigma_1) \right] \quad (2.51)$$

$$\varepsilon_{p3} = \frac{1}{E_p} \left[ \sigma_3 - \frac{1}{2} (\sigma_1 + \sigma_2) \right] = \frac{1}{E_p} \left[ \sigma_2 - \frac{1}{2} (\sigma_1 + \sigma_2) \right] \quad (2.52)$$

Thus, it can be seen that the assumption of  $\sigma_2 = \sigma_3$  results in obtaining the next condition

$$\varepsilon_{p2} = \varepsilon_{p3} \quad (2.53)$$

In this way, the following boundary conditions for the condition of plastic yielding are obtained

$$\left. \begin{array}{lll} \text{if } \varepsilon_{p1} = 0 & \text{then } \sigma_1 - \sigma_3 = 1.155 K_f \\ \text{if } \varepsilon_{p2} = \varepsilon_{p3} & \text{then } \sigma_1 - \sigma_3 = K_f \\ \text{if } \varepsilon_{p3} = 0 & \text{then } \sigma_1 - \sigma_2 = 1.155 K_f \end{array} \right\} \quad (2.54)$$

Since it is known that  $E_p$  is a continuous function of  $\varepsilon_p$  (Fig. 2.5), the subtrahend under the radical sign changes continuously. Therefore, it can be concluded that  $\eta$  is also a continuous function.

Thus, the final condition will be

$$1.0 \leq \eta \leq 1.155 \quad (2.55)$$

for  $\varepsilon_{p2} = \varepsilon_{p3}$ , for  $\varepsilon_{p2} = 0$  or  $\varepsilon_{p3} = 0$ .

Dividing the equation (2.50) by  $K_f$  gives

$$\eta = \frac{\sigma_1 - \sigma_3}{K_f} = \eta(\mu) \quad (2.56)$$

i.e. the stress function can be expressed as by Lode [18]. To illustrate the role of the average stress  $\sigma_2$  he deduced the stress parameter  $\mu$  from the relation

$$\mu = \frac{\sigma_2 - \sigma_1 + \sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} = 2 \left( \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \right) - 1 \quad (2.57)$$

This parameter Lode [18] combined with equation (2.56) by using the criterion of yielding (2.44) in the form

$$\eta = \frac{\sigma_1 - \sigma_3}{K_f} = \frac{2}{\sqrt{3 + \mu^2}} \quad (2.58)$$

From this formula the value of  $\eta$  can be found when the stresses  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  are known.

At present it is required to derive  $\eta$ , as a function of strain and not of stress.

To define the role of the average elongation Lode introduced another parameter

$$\nu = \frac{\varepsilon_2 - \varepsilon_1 + \varepsilon_2 - \varepsilon_3}{\varepsilon_1 - \varepsilon_3} = 2 \left( \frac{\varepsilon_2 - \varepsilon_3}{\varepsilon_1 - \varepsilon_3} \right) - 1 \quad (2.59)$$

assuming

$$\varepsilon_1 \geq \varepsilon_2 \geq \varepsilon_3$$

Using the relation

$$\frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} = \frac{\sigma_2 - \sigma_3}{\varepsilon_2 - \varepsilon_3} = \frac{\sigma_1 - \sigma_3}{\varepsilon_1 - \varepsilon_3} \quad (2.60)$$

gives

$$\nu = u \quad (2.61)$$

which means that

$$\eta = \frac{\sigma_1 - \sigma_3}{K_f} = \eta(\nu) = \frac{2}{\sqrt{3 + \nu^2}} \quad (2.62)$$

Expressing strains by  $\varepsilon_1, \varepsilon_2, \varepsilon_3 \dots$  is only permissible in the case of very small strains. Where large plastic strains occur this could lead to considerable errors in calculations since expressing strains in this form would not satisfy the condition of constancy of volume.

The set of equations (2.9) for logarithmic strains, which always satisfy the constant volume principle, will now be used

$$\begin{aligned} \log_e \frac{h_1}{h_0} + \log_e \frac{b_1}{b_0} + \log_e \frac{l_1}{l_0} &= \log_e \gamma + \log_e \beta + \log_e \lambda = \varphi_h + \varphi_b + \varphi_l \\ &= \varphi_1 + \varphi_2 + \varphi_3 = \varphi_{rh} + \varphi_{rb} + \varphi_{rl} = \varepsilon_{r1} + \varepsilon_{r2} + \varepsilon_{r3} = 0 \end{aligned}$$

For this equation it must be assumed that

$$\begin{aligned} \varphi_1 &\geq \varphi_2 \geq \varphi_3 & \varepsilon_{r1} &\geq \varepsilon_{r2} \geq \varepsilon_{r3} \\ \varphi_h &\geq \varphi_l \geq \varphi_b & \varepsilon_{rh} &\geq \varepsilon_{rl} \geq \varepsilon_{rb} \end{aligned}$$

Then, the parameter  $\nu$  for logarithmic strains can be written thus (2.59)

$$\nu = \frac{2\varepsilon_{r2} - \varepsilon_{r1} - \varepsilon_{r3}}{\varepsilon_{r1} - \varepsilon_{r3}} = \frac{2\varphi_2 - \varphi_1 - \varphi_3}{\varphi_1 - \varphi_3}$$

or to define the direction of strain

$$\nu = \frac{2\varphi_l - \varphi_h - \varphi_b}{\varphi_h - \varphi_b} = \frac{2\varepsilon_{rl} - \varepsilon_{rh} - \varepsilon_{rb}}{\varepsilon_{rh} - \varepsilon_{rb}}$$

However, when coefficients of deformations  $\gamma, \beta$  and  $\lambda$  are used to describe the strains, there is no doubt that the constant volume relation (2.8) will be satisfied, since

$$\gamma\beta\lambda = 1$$

The analysis made by the author [19] shows that there are only seven possible simple deformation patterns, as shown in Fig. 2.9.

Plane (biaxial) strain (Fig. 2.9a) occurs for  $\gamma = 1/\lambda$  or  $\gamma = 1/\beta$ ; in this case  $\eta = 1.155$  as is known.

Three further compression patterns are shown in Figs. 2.9b, c, and d. In Fig. 2.9b the compression is non-uniform, i.e.  $\gamma > \lambda > \beta$ . The scheme in Fig. 2.9c is for uniform deformation, where  $\beta = \lambda$ . In both these cases  $\eta = 1.0$ . In the next deformation pattern shown in Fig. 2.9d the spread is greater than elongation, thus  $\gamma > \beta > \lambda$ .

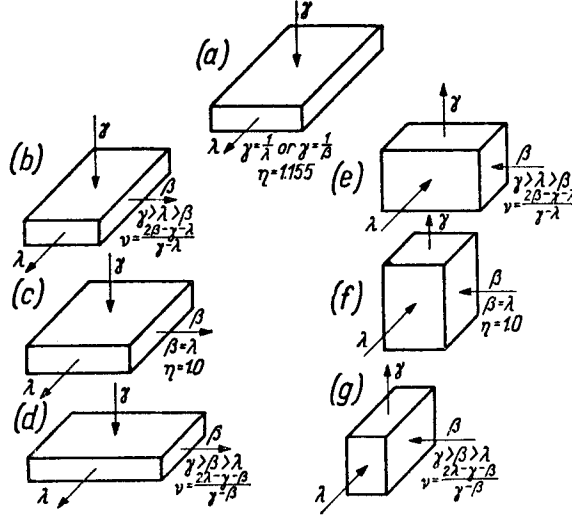


Fig. 2.9. Possible cases of plastic deformation in compression and tension [19]

Therefore, this case will be assumed for further considerations.

Figures 2.9 e, f, g illustrate similar cases of triaxial tensile strains. Figure 2.9e shows non-uniform tension when  $\gamma > \lambda > \beta$ , Fig. 2.9f—uniform tension when  $\lambda = \beta$ , for which  $\eta = 1.0$ , and Fig. 2.9g shows non-uniform tension at  $\gamma > \beta > \lambda$ .

The values of these coefficients can vary as follows:

| (a) in compression | (b) in tension   |                                                                   |
|--------------------|------------------|-------------------------------------------------------------------|
| $\gamma \leq 1$    | $\gamma \geq 1$  | $\left. \begin{array}{l} \\ \\ \end{array} \right\} \quad (2.63)$ |
| $\beta \geq 1$     | $\beta \leq 1$   |                                                                   |
| $\lambda \geq 1$   | $\lambda \leq 1$ |                                                                   |

From the above considerations, two relationships valid for the whole region of plastic deformation can be stated, i.e. the known relation (2.9) assuming that  $\gamma > \lambda > \beta$ , and

$$(1 + \varepsilon_{p1})(1 + \varepsilon_{p2})(1 + \varepsilon_{p3}) = \gamma\beta\lambda = 1 \quad (2.64)$$

From equation (2.64) it follows that the respective elongations can be written as follows:

$$\left. \begin{array}{l} \varepsilon_{p1} = \gamma - 1 \\ \varepsilon_{p2} = \beta - 1 \\ \varepsilon_{p3} = \lambda - 1 \end{array} \right\} \quad (2.65)$$



Substituting these values, the general expression for  $\nu$  can be found

$$\nu = \frac{2(\beta-1) - (\gamma-1) - (\lambda-1)}{(\gamma-1) - (\lambda-1)} = \frac{2\beta - \gamma - \lambda}{\gamma - \lambda} = \frac{2(\beta - \lambda)}{\gamma - \lambda} - 1 \quad (2.66)$$

Applying this equation for the various types of plastic deformation as shown in Fig. 2.9, further relationships can be found depending on the actual values of coefficients of deformation:

(1) *for the case of compression*

— non-uniform compression where  $\gamma > \lambda > \beta$  as in Fig. 2.9b

$$\nu = \frac{2\beta - \gamma - \lambda}{\gamma - \lambda}$$

as in equation (2.66) and for

— non-uniform compression where  $\gamma > \beta > \lambda$  as in Fig. 2.9d

$$\nu = \frac{2\lambda - \gamma - \beta}{\gamma - \beta} \quad (2.67)$$

(2) *for the case of tension*

— non-uniform tension where  $\gamma > \lambda > \beta$  as in Fig. 2.9e

$$\nu = \frac{2\beta - \gamma - \lambda}{\gamma - \lambda}$$

as in equation (2.66) and for

— non-uniform tension where  $\gamma > \beta > \lambda$  as in Fig. 2.9g

$$\nu = \frac{2\lambda - \gamma - \beta}{\gamma - \beta}$$

as in equation (2.67).

From Fig. 2.9, together with formulae (2.66) and (2.67),  $\nu$  can be determined for every possible case of plastic deformation in tension or compression.

Thus, two relations (2.66) and (2.67) for compression and tension are found:  
for  $\gamma > \lambda > \beta$

$$\nu = \frac{2\beta - \gamma - \lambda}{\gamma - \lambda}$$

or for  $\gamma > \beta > \lambda$

$$\nu = \frac{2\lambda - \gamma - \beta}{\gamma - \beta}$$

To analyse thoroughly formulae (2.62)–(2.66), the author used an example, in which the coefficient of draught  $\gamma$  was assumed to be constant at 0.68 and the coefficient of spread increased from the lowest value of 1.002 to almost that of  $\lambda$ .

The values of  $\eta$  calculated by using the above equations are given in Table 2.2. It can be seen that the calculated values are in good agreement with those predicted theoretically.

In calculating the values given in Table 2.2 an unexpected result was noted, i.e. that correct values of  $\eta$  are obtained, as a rule, when the coefficients  $\beta$  and  $\lambda$  are reasonably large, whereas for very small values of  $\beta$  between 1.0–1.05 an irregular run of values for  $\eta_a$  is obtained. Further analysis showed that for small values of  $\beta$ ,  $\eta$  shows

TABLE 2.2  
COMPARISON OF RESULTS CALCULATED FROM THE DERIVED EQUATION (2.68) AND (2.71)–(2.75)

| Coefficients of deformation |         |           | Values of indexes |          |
|-----------------------------|---------|-----------|-------------------|----------|
| $\gamma$                    | $\beta$ | $\lambda$ | $\eta_a$          | $\eta_b$ |
| 0.68                        | 1.0     | 1.4705    | 1.155             | 1.155    |
| 0.68                        | 1.002   | 1.4676    | 1.15468           | 1.15467  |
| 0.68                        | 1.005   | 1.4632    | 1.15462           | 1.15454  |
| 0.68                        | 1.01    | 1.456     | 1.15438           | 1.15408  |
| 0.68                        | 1.03    | 1.4277    | 1.15186           | 1.14944  |
| 0.68                        | 1.05    | 1.4005    | 1.14669           | 1.14073  |
| 0.68                        | 1.0974  | 1.340     | 1.14144           | 1.10830  |
| 0.68                        | 1.1579  | 1.27      | 1.08715           | 1.05327  |
| 0.68                        | 1.1764  | 1.25      | 1.06145           | 1.03533  |
| 0.68                        | 1.1956  | 1.23      | 1.03068           | 1.01661  |
| 0.68                        | 1.2103  | 1.215     | 1.00438           | 1.00222  |
| 0.68                        | 1.21264 | 1.21264   | 1.000             | 1.000    |

inexplicable variations. For this reason, the author suggests the following empirical formula for calculating  $\nu$  for  $\beta$  between 1.0–1.05

$$\nu = \left( \frac{2\beta - \gamma\lambda}{\gamma\lambda} \right) - 1 \quad (2.68)$$

The values of  $\eta_a$  given in Table 2.2 have been calculated from this formula. However a value of  $\eta = 1.155$  can be assumed for  $\beta = 1.0$ .

It can be seen from Table 2.2 that such an assumption results in obtaining a minimal error even in the most exact scientific context.

The application of formula (2.68) for large values of  $\beta$  is not advisable, as this would lead to incorrect results. Since in calculating  $\eta_a$  for very small values of  $\beta$  for Table 2.2 from equations (2.66) and (2.68), these irregularities were met with, the author attempted to obtain  $\eta$  in yet another way. Considering an actual case of plastic strain

$$\gamma_{\text{act}}\beta_{\text{act}}\lambda_{\text{act}} = 1 \quad \text{and} \quad \lambda_{\text{act}} \geq \beta_{\text{act}}$$

The following boundary conditions are determined

$$\left. \begin{aligned} \beta_{11m} &= \frac{1}{\gamma_{\text{act}}} & \text{at} & \lambda_{\text{act}} = 1 \\ \beta_m &= \sqrt{\frac{1}{\gamma_{\text{act}}}} & \text{at} & \lambda_m = \beta_m \end{aligned} \right\} \quad (2.69)$$

or

$$\left. \begin{aligned} \lambda_{11m} &= \frac{1}{\gamma_{\text{act}}} & \text{at} & \beta_{\text{act}} = 1 \\ \lambda_m &= \sqrt{\frac{1}{\gamma_{\text{act}}}} & \text{at} & \beta_m = \lambda_m \end{aligned} \right\} \quad (2.70)$$

The equation of the curve shown in Fig. 2.10 for the variable  $x$ , based on the experiments of Lode [18] will now be derived. However, the variable  $x$  is itself a function. If, to simplify the problem, a parabolic form of the curve is assumed, the equation of  $\eta$  will be

$$\eta = \frac{2}{\sqrt{3+x^2}} \quad (2.71)$$

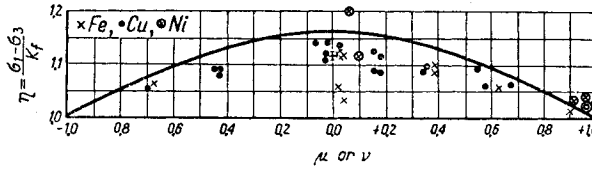


Fig. 2.10. Relation between the values of  $\eta$  and  $\mu$  or  $\nu$  for iron, copper and nickel [18]

where  $x$  is a function of the deformation coefficients, as given by one of the following relationships

$$x = \frac{\lambda_{\text{act}} - \lambda_m}{\lambda_{\text{lim}} - \lambda_m} - 1, \quad \text{for } \lambda_{\text{act}} > \lambda_m, \quad \lambda_{\text{lim}} = 1/\gamma_{\text{act}} \quad (2.72)$$

$$x = \frac{\beta_{\text{act}} - \beta_m}{\beta_{\text{lim}} - \beta_m} - 1, \quad \text{for } \beta_{\text{act}} > \beta_m, \quad \beta_{\text{lim}} = 1/\gamma_{\text{act}} \quad (2.73)$$

or

$$x = \frac{\lambda_{\text{act}} - \lambda_m}{\lambda_m - \lambda_{\text{lim}}} - 1, \quad \text{for } \lambda_{\text{act}} < \lambda_m, \quad \lambda_{\text{lim}} = 1 \quad (2.74)$$

and

$$x = \frac{\beta_{\text{act}} - \beta_m}{\beta_m - \beta_{\text{lim}}} - 1, \quad \text{for } \beta_{\text{act}} < \beta_m, \quad \beta_{\text{lim}} = 1 \quad (2.75)$$

For  $\beta_m$  and  $\lambda_m$  the values from equations (2.69) and (2.70) are taken

$$\beta_m = \sqrt{\frac{1}{\gamma_{\text{act}}}} \quad \lambda_m = \sqrt{\frac{1}{\gamma_{\text{act}}}}$$

The values of  $\eta_b$  in Table 2.2 are calculated from equations (2.71)–(2.75).

These considerations suggest the necessity of revising the conclusions arising from the deformation–energy theory, such that the condition of plasticity would take the general form (2.50)

$$\sigma_1 - \sigma_3 = \eta K_f$$

where  $\eta$  varies from 1–1.155 depending on the magnitude of  $\varepsilon_2$  or  $\sigma_2$ . The shear stress is given by the relation

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \quad \text{when} \quad \frac{\eta}{2} K_f = 0.5 - 0.577 K_f \quad (2.76)$$

depending also on the magnitude of transverse strain.

*Example*

Consider a case in which

$$h_2/h_1 = \gamma_{\text{act}} = 0.68$$

$$b_2/b_1 = \beta_{\text{act}} = 1.0974$$

$$l_2/l_1 = \lambda_{\text{act}} = 1.340$$

so that  $\gamma > \lambda > \beta$  i.e. compression as in Fig. 2.9b.

Using formula (2.66) the value of  $\nu$  is calculated

$$\nu = \frac{2\beta - \gamma - \lambda}{\gamma - \lambda}$$

Substituting

$$\nu = \frac{2 \times 1.097 - 0.68 - 1.34}{0.68 - 1.34} = -0.2636$$

Putting this value into formula (2.62)

$$\eta = \frac{2}{\sqrt{3 + \nu^2}} = \frac{2}{\sqrt{3 + (-0.2636)^2}} = \frac{2}{\sqrt{3.0695}} = 1.1416$$

This calculated value of  $\eta$  is as expected, since one coefficient, i.e.  $\beta$  is almost equal to 1.0. This is the value at which a plane state of strain exists, for which

$$\eta = \frac{2}{\sqrt{3}} = 1.155$$

## 2.4. Yield Stress

The relations (2.44)

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2K_f^2$$

and (2.50)

$$\sigma_1 - \sigma_3 = \eta K_f = K$$

(where  $K$  = constrained yield stress for the given method of working) are known as the criteria of yielding.  $K_f$  represents the stress at the yield point of metal under the uniaxial state of stress and is called the yield stress.

The basic factors which affect the magnitude of the yield stress are the temperature and rate of deformation. Their influence is different in cold and hot working. In cold working the metal undergoes strain-hardening and the yield stress increases with the amount of cold work (Fig. 1.13), but the rate of increase falls off as the degree of cold work increases. Finally a degree of cold work is reached, beyond which no increase in strain-hardening can be observed.

During the hot working of metals the phenomena of strain-hardening and recrystallization take place simultaneously; therefore the effects of strain-hardening are not apparent. Thus, assuming that the working temperature is constant, it can be concluded that the yield stress should also remain constant.

The determination of the yield stress at temperatures above 600–700°C presents some difficulty. In this case the stress-strain curve does not show any great change of direction on reaching the yield stress, therefore the assumption is made that at these

working temperatures the yield stress differs very little from the ultimate tensile stress, and this value is defined as the constrained yield stress,  $\eta K_f$ .

For every analysis of yield stress the problem of velocity of deformation (Fig. 2.11) must be solved. This figure is a modification of Fig. 2.9 and presents the mode of deformation in compression and tension, respectively.

In Figure 2.11 the directions of deformation velocities are given according to the work of S. Koncewicz [181] for the appropriate coefficients of deformation.

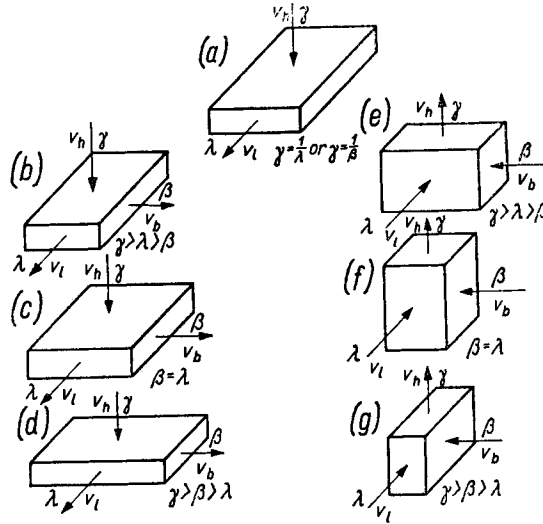


Fig. 2.11. Schematic representation of deformation in compression and tension

The instantaneous velocities of relative deformation can be resolved in three mutually perpendicular directions, e.g. corresponding to the height, breadth and length of the deformed metal (Fig. 2.11). In this case the rate of deformation can be considered with respect to draught, spread and elongation.

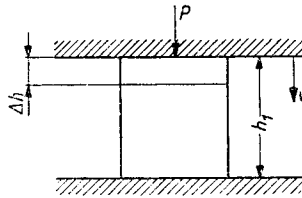


Fig. 2.12. Schematic representation of deformation rate [20]

According to Fig. 2.11 the rate of deformation can be written as

$$v_h = \frac{dh}{dt} \quad v_b = \frac{db}{dt} \quad v_l = \frac{dl}{dt} \quad (2.77)$$

or considering only relative deformation

$$\varepsilon'_h = \frac{v_h}{h}, \quad \varepsilon'_b = \frac{v_b}{b}, \quad \varepsilon'_l = \frac{v_l}{l} \quad (2.78)$$

where  $v_h$ ,  $v_b$  and  $v_l$  are components of deformation velocity in the directions of height, breadth and length, respectively. From the constant volume relationship

$$v_h F_h + v_b F_b + v_l F_l = 0$$

and

$$F_h = \frac{V}{h} \quad F_b = \frac{V}{b} \quad \text{and} \quad F_l = \frac{V}{l}$$

hence

$$\frac{v_h}{h} + \frac{v_b}{b} + \frac{v_l}{l} = 0 \quad (2.79)$$

or

$$\varepsilon'_h + \varepsilon'_b + \varepsilon'_l = 0 \quad (2.80)$$

Thus, it can be concluded that the algebraic sum of the velocity components of the relative strain in the three principal directions is equal to zero.

Since in compression the stock always decreases in height and increases in breadth and length, the assumption can be made that the maximum velocity of relative strain is the draught velocity, therefore

$$|\varepsilon'_h| = |\varepsilon'_b| + |\varepsilon'_l|$$

In practice, it is more convenient to use the average velocities of the relative strain which in the general case, can be calculated from the relationship

$$\bar{\varepsilon}' = \frac{1}{t} \int_0^t \varepsilon' dt \quad (2.81)$$

Taking into account relation (2.78) and integrating the results within appropriate limits, it can be written

$$\bar{\varepsilon}'_h = \frac{1}{t} \log_e \frac{h_2}{h_1} \quad \bar{\varepsilon}'_b = \frac{1}{t} \log_e \frac{b_2}{b_1} \quad \bar{\varepsilon}'_l = \frac{1}{t} \log_e \frac{l_2}{l_1} \quad (2.82)$$

where:  $\bar{\varepsilon}'_h$ ,  $\bar{\varepsilon}'_b$ , and  $\bar{\varepsilon}'_l$  are the average strain rates in appropriate directions. It is easy to show that these values are at the same time the components of the mean strain rate.

Since the component strains change simultaneously, it is easy to prove that

$$\bar{\varepsilon}'_h + \bar{\varepsilon}'_b + \bar{\varepsilon}'_l = 0 \quad (2.83)$$

i.e. the algebraic sum of average strain rates is equal to zero.

This can be written

$$|\bar{\varepsilon}'_h| = |\bar{\varepsilon}'_b| + |\bar{\varepsilon}'_l|$$

where

$$q = |\bar{\varepsilon}'_h| = \frac{1}{t} \left| \log_e \frac{h_1}{h_2} \right| \quad (2.84)$$

can be assumed as the maximum value of the average strain rate. This symbol  $q$  will be used throughout to indicate the mean strain rate.

The strain rate, described by the intensity of deformation, also has a considerable influence on the yield stress. The rate of strain is defined as the magnitude of deformation per unit time.

According to Geleji [20], Unksov [21] and Trink [22], for small deformations the strain rate of a cubic element with initial height  $h_1$  which decreases to final height  $h_2$  in time  $t$  can be expressed by the relation (Fig. 2.12)

$$dq = \frac{d\varepsilon}{dt} = \frac{dh}{h} \frac{1}{dt} = \frac{1}{h} v \quad (2.85)$$

Integrating this equation for a finite deformation

$$q = \frac{\varepsilon h}{t} = \frac{h_1 - h_2}{h_1} \frac{1}{t} = \frac{\Delta h}{h_1 t} \quad (2.86)$$

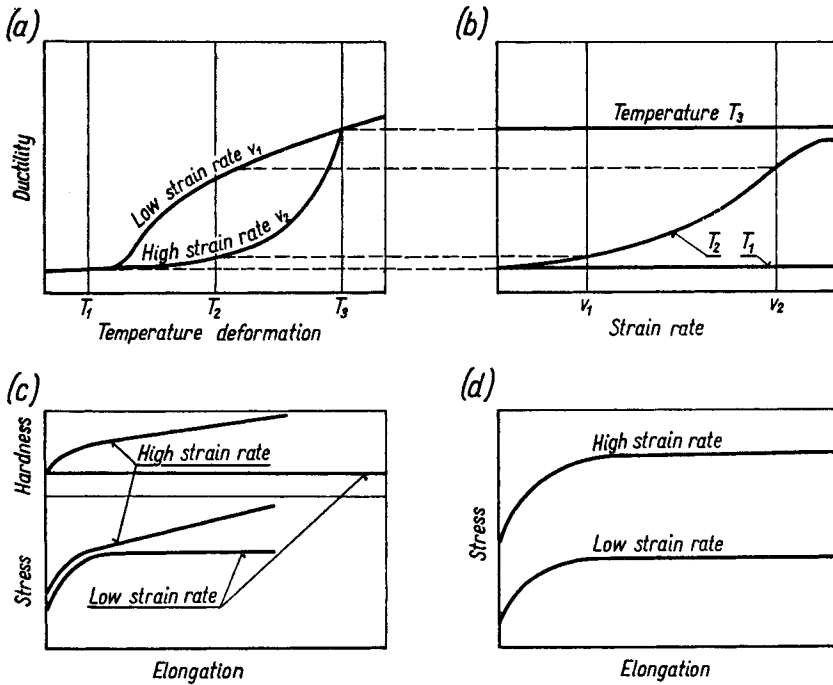


Fig. 2.13. Influence of deformation rate at working temperatures

From the previous considerations it is known that the relative strain  $\varepsilon_h$  does not precisely define the deformation, and therefore it would be more correct to express equation (2.85) in terms of natural values, i.e.

$$dq = \frac{d\varepsilon_{rh}}{dt} \quad (2.87)$$

which on integrating gives

$$q = \log_e \frac{h_2}{h_1} \frac{1}{t} \quad (2.88)$$

It should be noted that for rolling  $t$  is expressed as

$$t = \frac{\alpha 60^\circ}{360^\circ n} = \frac{\alpha}{6n} \text{ sec}$$

where

$\alpha$  = angle of bite, in degrees,

$n$  = rpm of rolls.

Considering the influence of temperature on strain rate, the following temperature ranges can be distinguished in metal working processes [23]:

(1) The lowest temperature range.

Brittleness can occur on applying large strain rates to metals which are ductile at low strain rates (Fig. 2.13a and b).

(2) The cold working range.

The influence of strain rate is very small, but the mechanical factors introduced due to increase of rate of strain can be of importance (Fig. 2.14a and b).

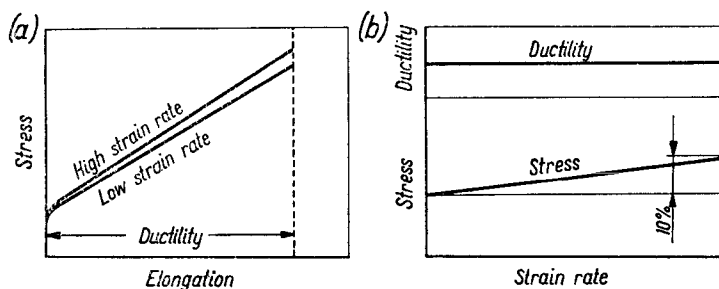


Fig. 2.14. Influence of deformation rate on ductility and yield stress of metal in cold working [23]

(3) Higher temperature range.

While the phenomena of cold work occur at high strain rates, hot working takes place at low rates (Fig. 2.13c).

(4) Hot working range.

The strain rate was found to have a great influence in the hot working range. Brittleness can occur by low strain rates in the lower temperature range (Fig. 2.13d). It can be seen from Fig. 2.13a and b that many metals show brittleness at high strain rates within narrow limits of low temperatures, and behave as ductile metals at low strain rates. For example, at  $T_2$  this influence is of importance only in particular cases. A transition from ductile to brittle condition was found to occur only in a few metals, such as zinc and magnesium, near the ambient temperature  $T_1$ .

In such cases either the temperature of plastic working should be increased a little, or the working carried out at a low strain rate.

In the cold working range the influence of the strain rate is of almost no importance (Fig. 2.14a and b). The direct influence of strain rate shows itself only as a small increase in the yield stress of metal, but this is of no practical importance, since the differences in stress magnitudes are insignificant (Fig. 2.14a).

In the temperature range between hot and cold working the influence of deformation rate becomes more noticeable (Fig. 2.13c).

At high deformation rates the metal behaves similarly as in cold working, i.e. it undergoes strain-hardening. However, at slow deformation rates the metal behaves



as if it were hot-worked, it is perfectly plastic and shows no increase in strain-hardening (no increase in stress). The higher the strain rate, the higher is the lower limit of temperature of the hot working range.

This temperature range is not used for the proper working of metals, but is of use for investigations of metal properties at critical creep temperatures.

In the effective hot working range, the strain rate has an important influence, shown chiefly in the increase of the yield stress (Fig. 2.13d). If the strain rate is doubled, the increase in stress is about 10–20%.

At very high temperatures, as the strain rate increases the heat has no time to be dissipated and remains in the metal, causing a rise in temperature. This results in lowering the temperature of fusion of metal at high strain rates, and hence the permissible temperature range for hot working becomes narrower, therefore low strain rates are sometimes necessary in forging or rolling.

For both low and high strain rates these phenomena take place according to the laws of statics, for the established working conditions.

Thus the basic factor which decides the degree of plastic deformation of metal is the yield stress,  $K_f$ , which is dependent on the condition of metal at the moment of deformation, viz.:

- (a) deformation under conditions insufficient for recrystallization, cold-working—the yield stress depends on:
  - (i) kind of metal,
  - (ii) amount of cold work or initial strain-hardening,
  - (iii) strain rate to a small degree.
- (b) deformation above the temperature of recrystallization—the yield stress depends on:
  - (i) kind of metal,
  - (ii) strain rate,
  - (iii) temperature of working.

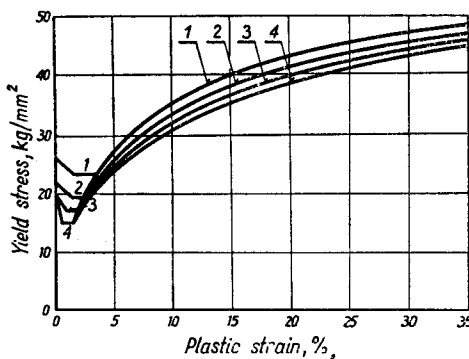


Fig. 2.15. Yield stress curves of mild steel for different rates of deformation [24]: 1 — about 1.25% per sec, 2 — about 0.2% per sec, 3 — about 0.25% per sec, 4 — 0% per sec

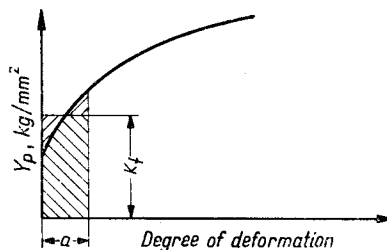


Fig. 2.16. Method of determining the average yield stress with strain-hardening [20]  $a$ —denotes the decrease of height expressed as:  $[(h_1 - h_2)/h_1]100\%$  or  $h_2/h_1$  or  $\log_e (h_2/h_1)$

The yield stress of metal at the given rate of deformation can be determined with an accuracy sufficient for practical purposes, by using the following formulae and diagrams.

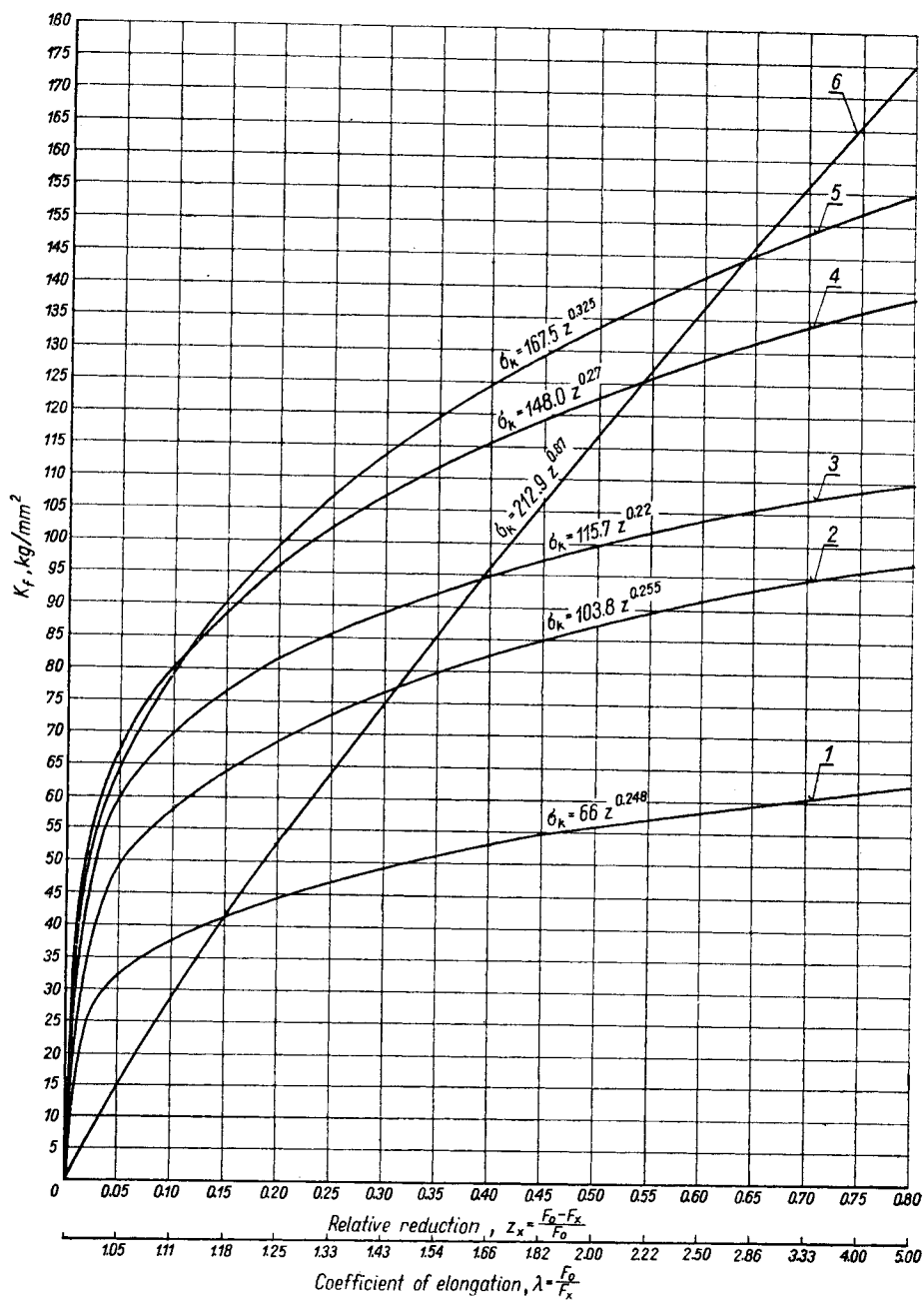


Fig. 2.17. Strain-hardening curves for steel according to Krupkowski [176, 177, 178];  
 1 — carbon steel 0.12% C, 2 — C30 steel 0.33% C, 3 — T45 steel 0.42% C, 4 — N steel  
 0.9% C, 5 — T55 steel 0.55% C, 6 — 18-8 steel

Figure 2.15 [24] shows the relation between the yield stress and the deformation in cold working of mild steel\* at the strain rates ranging from 0 to 1.25% per sec.

It can be seen that the influence of strain rate is very small in this case, and amounts to about 10%.

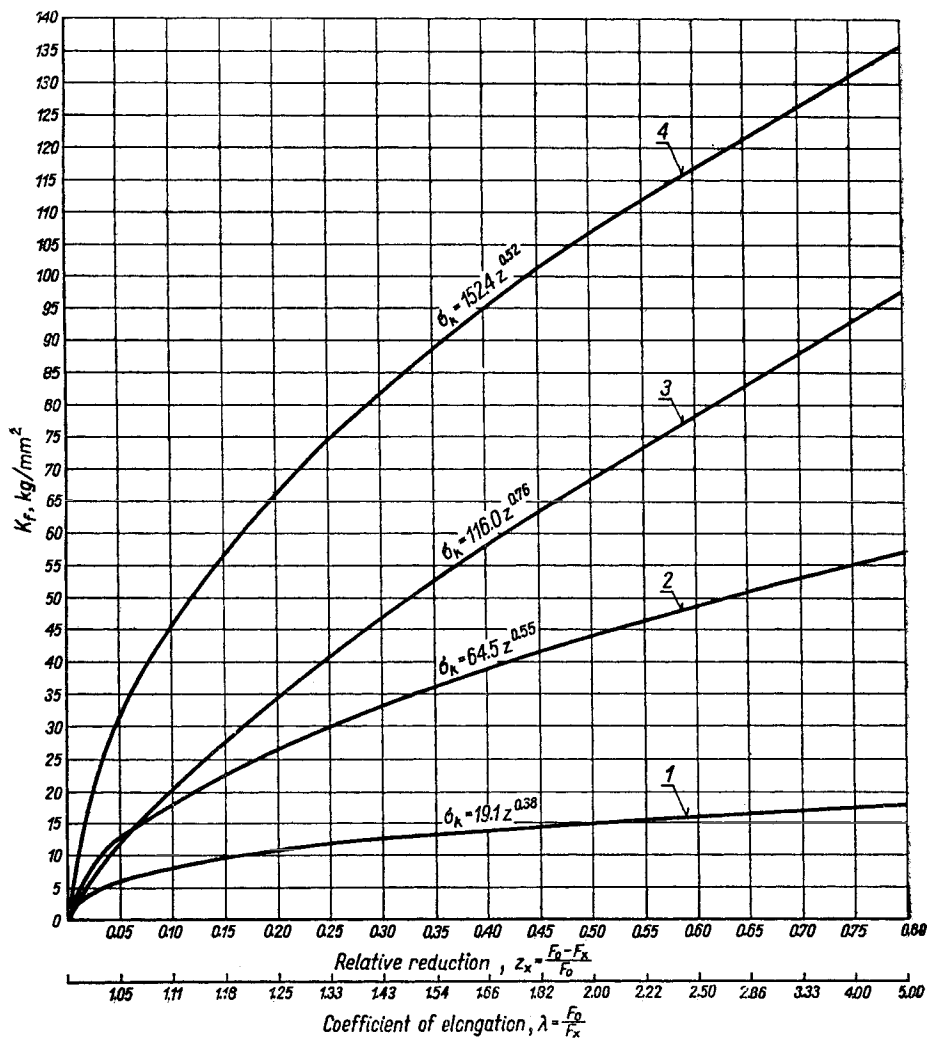


Fig. 2.18. Strain-hardening curves for various non-ferrous metals obtained from the equations derived by Krupkowski [176, 177, 178]; 1 — aluminium, 2 — copper, 3 — brass, 4 — Monel alloy

In applying the curves of the yield stress with strain-hardening, mean values are used, reading from the curve the geometrical mean at the beginning and at the end of deformation (Fig. 2.16).

For rolling a special method of deriving the mean value should be used, which will be discussed in the following chapters. A. Krupkowski derived strain hardening formulae for calculating the yield stress [176, 177, 178] (Figs. 2.17 and 2.18).

\* See Appendix 1.

This formula has the general form

$$\sigma = k z_i^m \tag{2.89}$$

where

$$z_i = z_1 + (1 - z_1)z \tag{2.90}$$

The index  $z_1$  takes values close to zero—as may be seen in Table 2.3.

The formula (2.89) can be simplified to the form

$$K_f = k z^m \tag{2.91}$$

where  $K_f$  = yield stress of the cold metal.

$$z = \frac{F_0 - F_1}{F_0} = 1 - \frac{F_1}{F_0} = 1 - \frac{1}{\lambda}$$

The values of  $k$  and  $m$  depend on the tendency of metal to strain-harden and have been determined experimentally, see Table 2.4.

TABLE 2.3  
VALUES OF INDEX  $z_1$

| Value of $z_1$ | Material |         |
|----------------|----------|---------|
| 0.0            | C12      | 0.12% C |
| 0.005          | C30      | 0.33% C |
| 0.0082         | T45      | 0.45% C |
| 0.0            | T55      | 0.55% C |
| 0.037          | Steel    | 0.9% C  |
| 0.098          | Steel    | 18/8    |
| 0.0057         | Cu       |         |

TABLE 2.4  
VALUES OF INDICES  $k$  AND  $m$

| Material    | $k$   | $m$   |
|-------------|-------|-------|
| Al          | 19.1  | 0.38  |
| Cu          | 64.5  | 0.55  |
| Ms (brass)  | 116.0 | 0.76  |
| Monel alloy | 152.4 | 0.52  |
| 0.12% C     | 66.0  | 0.248 |
| T45—0.42% C | 115.7 | 0.22  |
| T55—0.55% C | 167.5 | 0.325 |
| C30—0.33% C | 103.8 | 0.255 |
| N—0.9% C    | 148.0 | 0.270 |
| 18/8        | 219.5 | 0.87  |

Table 2.5 gives the chemical analyses of the steels tested. Using the data from Table 2.4 and formula (2.91), the strain hardening curves for yield stress of steels and other metals have been obtained (Figs. 2.17 and 2.18). The yield stress is given here as a function of degree of cold work,  $z$ .

TABLE 2.5  
CHEMICAL ANALYSIS OF TESTED STEELS

| Grade of steel                             | Contents, % |      |      |       |       |       |      |      |
|--------------------------------------------|-------------|------|------|-------|-------|-------|------|------|
|                                            | C           | Si   | Mn   | P     | S     | Cr    | Ni   | Mo   |
| C12 (annealed steel)                       | 0.10        | 0.28 | 0.43 | 0.033 | 0.048 | —     | —    | —    |
| C30 (annealed steel)                       | 0.33        | 0.30 | 0.56 | 0.032 | 0.025 | —     | —    | —    |
| T45 (annealed steel)                       | 0.42        | 0.34 | 0.60 | 0.034 | 0.014 | —     | —    | —    |
| T55 (annealed steel)                       | 0.55        | 0.27 | 0.62 | 0.019 | 0.022 | —     | —    | —    |
| N (annealed steel)                         | 0.90        | 0.29 | 0.24 | 0.010 | 0.010 | 0.17  | 0.32 | —    |
| A (hardened steel<br>quenched from 1050°C) | 0.07        | 0.35 | 0.89 | 0.034 | 0.003 | 18.30 | 9.50 | 1.08 |

In practical calculations the increase in strain-hardening must be taken into account and a mean yield stress can be found as the arithmetical mean (Fig. 2.16)

$$K_{fm} = \frac{K_{f1} + K_{f2}}{2} \quad (2.92)$$

It is more accurate to calculate the parabolic mean

$$K_{fm} = \frac{K_{f1} + 2K_{f2}}{3} \quad (2.93)$$

These values are calculated for the initial and final stages of strain-hardening of metal.

Good results are also obtained using Ford's formula for calculating the degree of average cold work,  $z_m$  [36]

$$z_m = 0.4z_1 + 0.6z_2 \quad (2.94)$$

where

$z_1$  = initial strain-hardening,

$z_2$  = final strain-hardening.

For the first stage of deformation after annealing  $z_1$  is assumed to be equal to 0. In cold rolling  $z$  can be replaced by  $h$  in equation (2.94), provided  $b$  is constant.

The average value of the constrained yield stress is obtained as the product of  $K_{fm}$  and  $\eta$

$$K_m = \eta K_{fm} \quad (2.95)$$

For  $\beta = 1$  or  $\varepsilon_2 = 0$ , then  $\eta = 1.155$  and hence  $K_m = 1.155 K_{fm}$ .

For variable strains and stresses  $\eta$  is obtained from the relation (2.62)

$$\eta = \frac{2}{\sqrt{3+\mu^2}} = \frac{2}{\sqrt{3+\nu^2}}$$

In equation (2.62), the coefficient  $\mu$  is calculated

$$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$$

It is more convenient to calculate in terms of the deformation coefficients where  $\gamma > \lambda > \beta$  (Fig. 2.9)

$$\nu = \frac{2\beta - \gamma - \lambda}{\gamma - \lambda}$$

or for  $\gamma > \beta > \lambda$

$$\nu = \frac{2\lambda - \gamma - \beta}{\gamma - \beta}$$

For working above the temperature of recrystallization (hot working) the influence of deformation rate on the yield stress is considerable.

When the working temperature is only slightly higher than the temperature of recrystallization, the speed of recrystallization is usually so slow that the initial structure of metal cannot be reconstructed. This results in partial strain-hardening of metal, similar to that occurring in cold working.

A. L. Nadai [16] and Manjoine carried out a series of investigations illustrating the influence of strain rate at different temperatures on the behaviour of test pieces in tension. They determined the relation between yield stress and the strain rate and temperature. The influence of friction was neglected.

Figure 2.19 shows the yield stress curve of mild steel\* for temperatures ranging from 20 to 1200°C. These curves are not so flat as in the preceding plots. Those relating

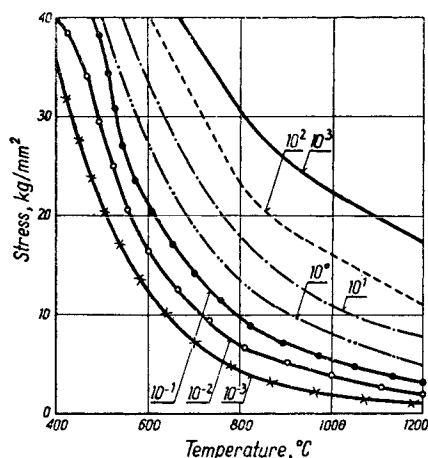


Fig. 2.19. Yield stress of mild steel at different temperatures and deformation rates [16, 10]

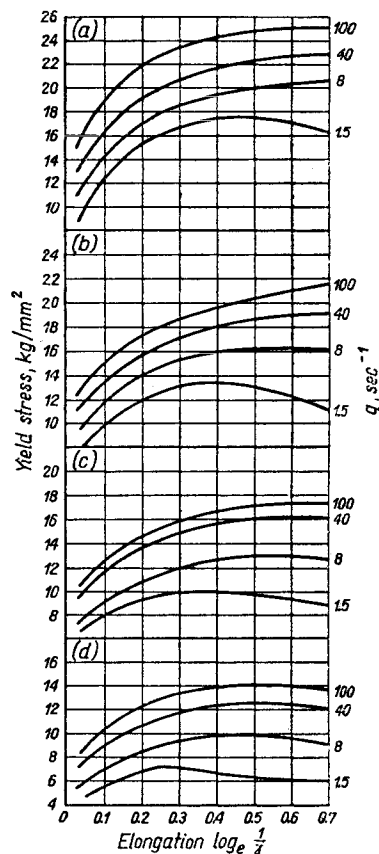


Fig. 2.20. Variation of yield stress of different steels with method of deformation, rate of deformation and temperature: (a) 900°C, (b) 1000°C, (c) 1100°C, (d) 1200°C, according to Cook [25] (see Table 2.6), for mild steel

to temperatures 400 and 500°C show considerable deviations. These curves have been redrawn by the author from the original stress-strain diagram of Nadai [16].

The strain rates normally used in rolling vary from 1 to 10³ sec⁻¹.

Figure 2.20 gives the yield stress curve for mild steel with 0.15% C, 0.12% Si, 0.68% Mn. These are the results of measurements published by P. M. Cook [25]. The

\* See Appendix 1.

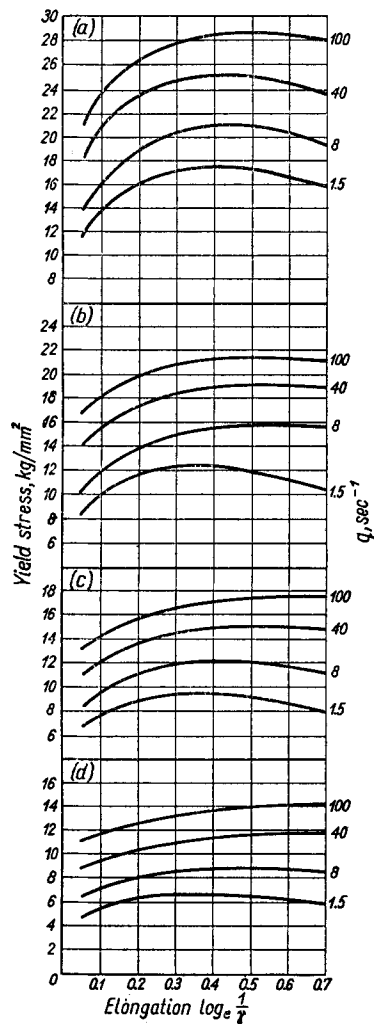


Fig. 2.21. As in Fig. 2.20 for medium carbon steel

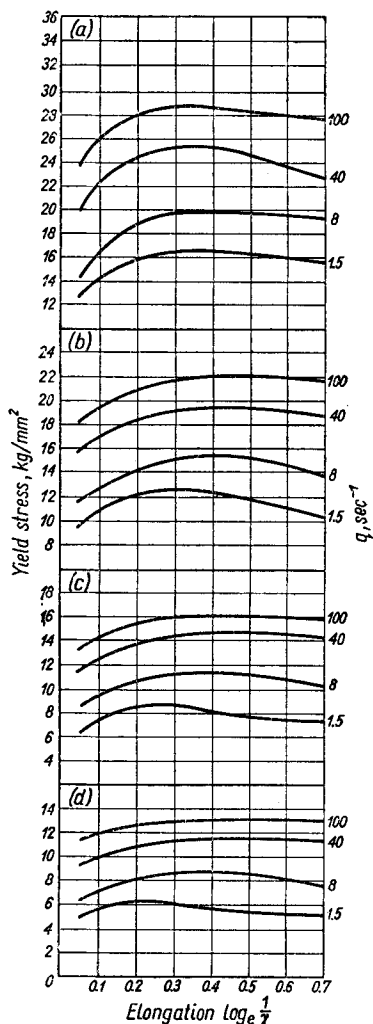


Fig. 2.22. As in Fig. 2.20 for high carbon steel

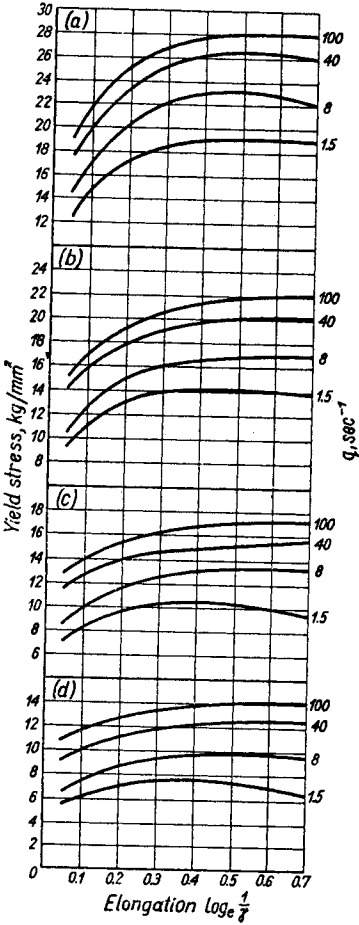


Fig. 2.23. As in Fig. 2.20 for manganese-molybdenum steel

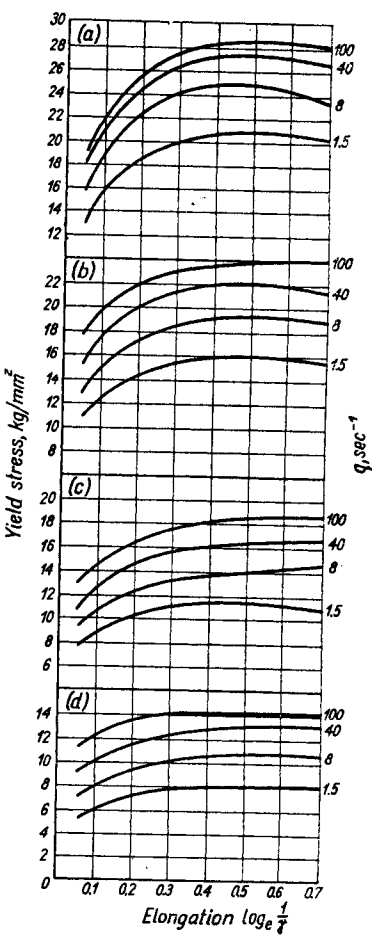


Fig. 2.24. As in Fig. 2.20 for chrome-nickel-molybdenum steel



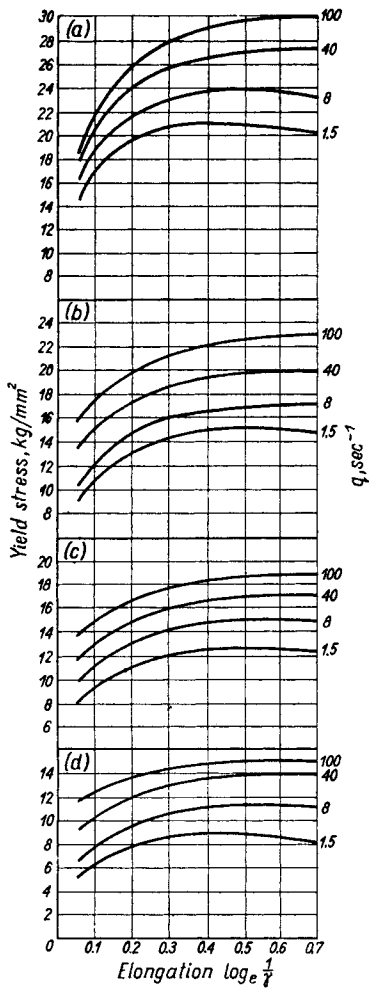


Fig. 2.25. As in Fig. 2.20 for chrome-molybdenum steel

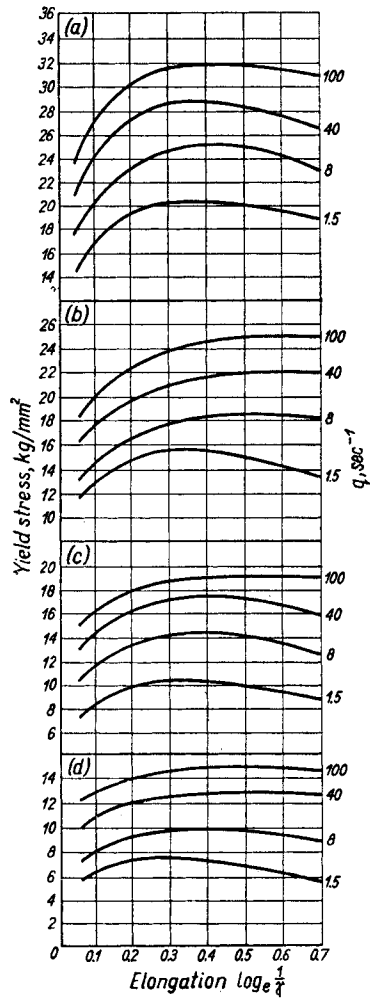


Fig. 2.26. As in Fig. 2.20 for silicon-manganese steel

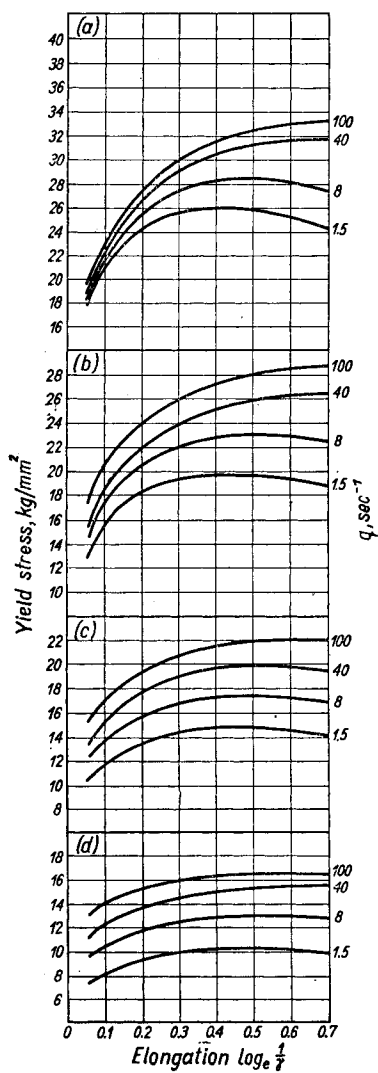


Fig. 2.27. As in Fig. 2.20 for 18-8 steel

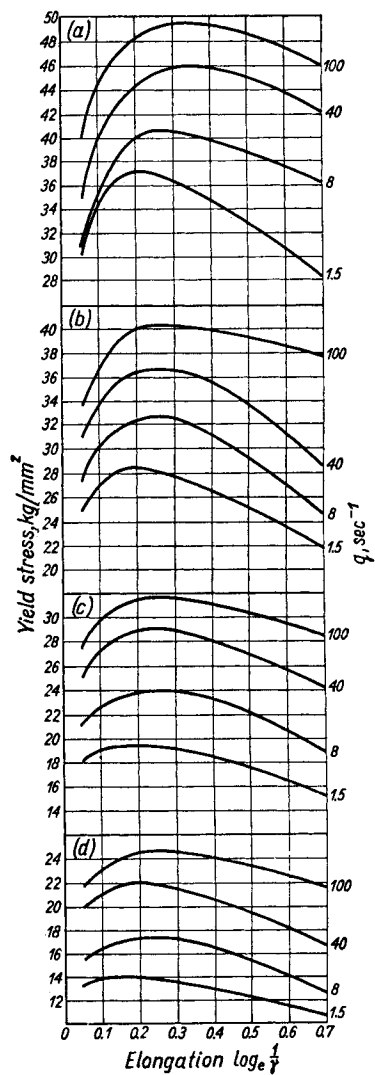


Fig. 2.28. As in Fig. 2.20 for SW18 steel

yield stress is related to the effective elongation  $\log_e(1/\gamma)$  for the following temperature ranges: (a) 900°C, (b) 1000°C, (c) 1100°C and (d) 1200°C.

Further yield stress curves for other carbon and alloy steels are shown in Figs. 2.21–2.28, according to P. M. Cook [25]. Chemical compositions of the tested steels are given in Table 2.6. In each figure the yield stress is plotted against the effective elongation  $\log_e(1/\gamma)$  at 900°C, 1000°C, 1100°C and 1200°C and strain rates of 1.5, 8, 40 and 100 sec<sup>-1</sup>.

TABLE 2.6  
CHEMICAL ANALYSIS OF CARBON AND ALLOY STEELS

| Steel type                 | Contents, % |      |      |       |       |       |      |      |       |      |
|----------------------------|-------------|------|------|-------|-------|-------|------|------|-------|------|
|                            | C           | Si   | Mn   | S     | P     | Cr    | Ni   | Mo   | W     | V    |
| Mild steel                 | 0.15        | 0.12 | 0.68 | 0.034 | 0.025 | —     | —    | —    | —     | —    |
| Medium carbon steel        | 0.56        | 0.26 | 0.28 | 0.014 | 0.013 | 0.12  | 0.09 | —    | —     | —    |
| High carbon steel          | 1.00        | 0.19 | 0.17 | 0.027 | 0.023 | 0.10  | 0.09 | —    | —     | —    |
| Manganese-molybdenum steel | 0.35        | 0.27 | 1.49 | 0.041 | 0.037 | 0.03  | 0.11 | 0.28 | —     | —    |
| Cr–Ni–Mo steel             | 0.35        | 0.27 | 0.66 | 0.023 | 0.029 | 0.59  | 2.45 | 0.59 | —     | —    |
| Cr–Mo steel                | 0.26        | 0.35 | 0.57 | 0.009 | 0.023 | 3.03  | 0.29 | 0.49 | —     | —    |
| Si–Mn steel                | 0.61        | 1.58 | 0.94 | 0.038 | 0.035 | 0.12  | 0.27 | 0.06 | —     | —    |
| 18–8 steel                 | 0.07        | 0.43 | 0.48 | —     | —     | 18.60 | 7.70 | —    | —     | —    |
| SW18 steel                 | 0.80        | 0.28 | 0.32 | —     | —     | 4.30  | 0.18 | 0.55 | 18.40 | 1.54 |

The yield stress of carbon steels under hot working conditions can be calculated, without taking into account the rate of deformation, i.e. under assumption of nearly static conditions of deformation, by using the formula given by S. Ekelund [10].

$$K_f = (14 - 0.01t)(1.4 + C + \text{Mn} + 0.3 \text{ Cr}) \text{ kg/mm}^2 \quad (2.96)$$

where

$t$  = temperature to which the steel is heated (above 700°C),

$C$  = carbon content of steel, %,

$\text{Mn}$  = manganese content of steel, % (max. 1%),

$\text{Cr}$  = chromium content of steel, %.

A. Geleji [20] states that the values calculated from the formula (2.96) are too high as compared with those obtained from measurements, and therefore he recommends that the following formula be used for calculating  $K_f$  of carbon steels with tensile strength up to 60 kg/mm<sup>2</sup>, for the temperature range from 800 to 1300°C

$$K_f = 0.015(1400 - t) \text{ kg/mm}^2 \quad (2.97)$$

For many carbon steel grades with  $C < 0.6\%$ ,  $\text{Si} < 0.5\%$ ,  $\text{Mn} < 0.8\%$  A. Geleji [20] derived a curve shown in Fig. 2.29 in which the yield stress is plotted versus the temperature. This diagram shows the yield stress curve obtained for low strain rates.

In the light of the preceding considerations of the conditions of plasticity it becomes clear that the yield stress  $K_f$  is affected by the method of working metal. Thus  $K_f$  must be multiplied by  $\eta$  giving  $K = \eta K_f$ —the constrained yield stress for given conditions of working, where  $\eta$  is calculated from equation (2.62) and the required value of  $\nu$

from (2.66) or (2.67) (Fig. 2.9). Only in cases where  $\beta = \lambda$  the value of  $\eta$  should be assumed as  $= 1$ , and in plastic deformation with  $\beta = 1$  or  $\lambda = 1$ , the value of  $\eta$  should be assumed as equal to 1.155.

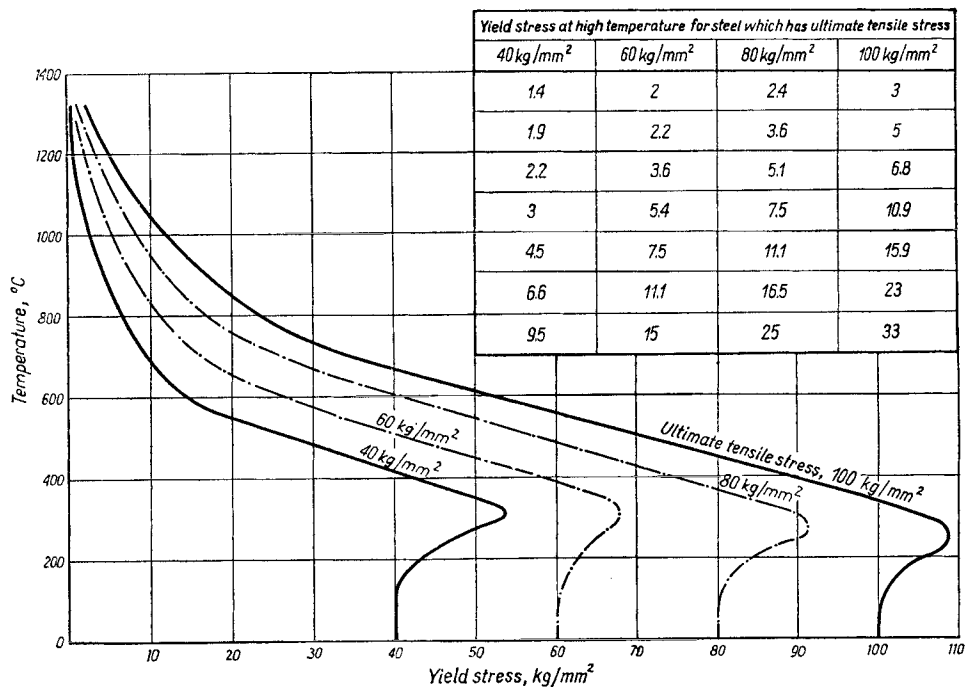


Fig. 2.29. Variation of yield stress of mild carbon steels with temperature [20]

It should also be borne in mind that in hot rolling the relation

$$K_m = \frac{1}{\alpha} \int_0^{\alpha} K d\varphi \quad (2.98)$$

will also be obtained, depending on the varying conditions of rolling.

## 2.5. Resistance of Metal to Deformation

It is assumed that an ideal smoothness of working tools and work piece is obtained, so that the influence of friction can be entirely neglected (Fig. 2.30).

In this case, the action of the external force  $P$  gives rise to uniformly distributed vertical forces over the whole cross-section, and the specimen itself can deform freely in the lateral direction. No frictional force on the surface or horizontal stresses occur.

In this case, if the stress produced by the force  $P$  reaches the constrained yield stress of metal,  $\eta K_f$  (formulae (2.58) and (2.62)) the specimen will deform permanently and uniformly but the angles of planes and faces remain unchanged. Such ideal con-

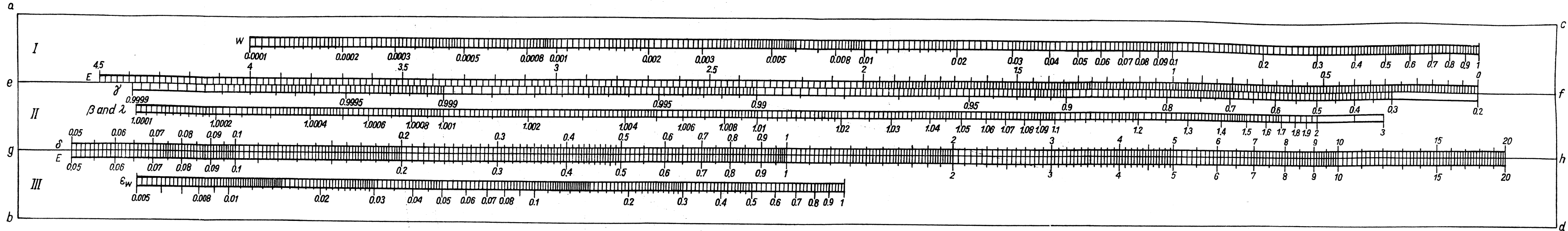


Fig. 7.37. Wojtylak's slide rule for Z. Wusatowski's formula [10]

ditions never exist in practice, and inevitably friction always occurs on the surface. These frictional forces upset the uniform stress distribution over the whole section of the specimen.

Frictional forces on the surface of specimen (horizontal arrows in Fig. 2.30) restrict the lateral flow of metal, and hence the pressure of the tool must overcome not only the constrained yield stress  $\eta K_f$  but also the frictional forces.

Considering any point on the surface of the specimen, it is clear that the nearer it is to the neutral axis of the specimen, the greater is the increase in stress. At the neutral

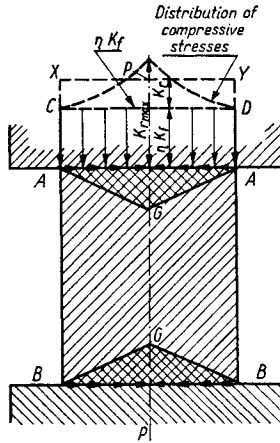


Fig. 2.30. The distribution of compressive stresses [10]

axis of the specimen the frictional forces attain their highest value and decrease to zero at the sides, as indicated by the area  $CPD$  in Fig. 2.30.

These frictional forces are the cause of non-uniform stress distribution and disturbances in deformation of metal.

Whenever two solid bodies move over each other the phenomenon of friction occurs, i.e. a force resisting the relative movement of the bodies. If a resultant force  $P$  (Fig. 2.31) acts on the base of a rigid body, then to move the body a certain force is required to overcome the frictional resistance between the base of the body and the plane on which it rests. This force must be at least

$$T = fP \quad (2.99)$$

where  $f$  = coefficient of friction.

In these considerations it is assumed that the surfaces of the contacting bodies are flat i.e. that both bodies are sufficiently rigid. Actually both the body and the tool undergo a certain deformation, which is usually elastic and locally also plastic. In this case the tool can often deform elastically and the metal plastically.

The coefficient of friction is not a constant value as was formerly supposed, but depends on several factors and essentially on whether the bodies in contact are dry or well lubricated. The coefficient of dry friction  $f$  increases with the force  $P$  and decreases with the decrease of this force (Fig. 2.32). In a state of relative rest the coefficient of static friction,  $f_0$ , reaches its highest value. Conversely, with ideal lubrication the coefficient of friction,  $f$ , decreases as the velocity of the body increases. Thus, the coefficient of static friction,  $f_0$ , is greater than the coefficient of kinetic friction.

The basic assumptions associated with the phenomenon of friction can be stated as follows:

Dry friction:

$f$  increases with force  $P$ ,

$f$  is almost independent of relative velocity,  $v$ ,

$f$  depends on surface roughness of both bodies; the coefficient of static friction,  $f_0$ , is greater than the coefficient of kinetic friction.

Friction under conditions of ideal lubrication:

$f$  is independent of  $P$ ,

$f$  decreases with increasing  $v$ ,

$f$  is almost independent of minor roughnesses and tends to zero with increasing  $v$ .

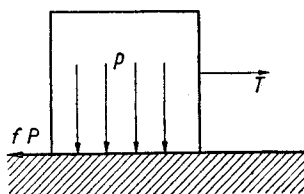


Fig. 2.31. Schematic representation of static friction

In practice, the friction condition occurring in machines is an intermediate case between these two extremes. Therefore the coefficients of friction used and quoted represent either mean values or extreme values for the two different conditions of friction.

Formula (2.99) gives a simplified concept of the phenomenon of friction. Practically it should be interpreted according to Chertavskikh [184] as follows: if there is molecular friction between the surfaces of the bodies in contact, formula (2.99) takes the form

$$T = f(P + N_0) \quad (2.100)$$

where  $N_0$  = force of molecular friction, i.e. the molecular bond between the surfaces of the two bodies.

With the increase of the effective contact area the term  $N_0$  increases considerably

$$N_0 = P_0 F_0 \quad (2.101)$$

where

$P_0$  = force of molecular bond between two bodies per unit area of contact,

$F_0$  = effective contact area.

Hence, equation (2.100) becomes

$$T = f(P + P_0 F_0) \quad (2.102)$$

Thus, the action of frictional forces in accordance with the molecular-mechanical theory, can be expressed as

$$T = \sum T_{mol} + \sum T_{mech} \quad (2.103)$$

If a hard body exerts a pressure on a soft body, the former penetrates the surface of the latter and causes deformation. In this case the frictional force must be determined differently, as

$$T = Q + S \quad (2.104)$$

where

$Q$  = force required to overcome the metallic bond between the contact surfaces,

$S$  = force required to enable the hard metal to penetrate the soft one.

The influence of friction between the metal and the rolls will be discussed more fully later.

In all the older theories of deformation, the coefficient of friction was assumed constant along the surface of contact of metal and tool.

In deriving the newer theories of deformation a constant value of friction coefficient is still assumed, but a distinction is made between the slipping friction at the surface of contact between the metal and the tool, and the sticking of metal to the tool occurring in certain areas of contact.

Other factors should also be taken into account. In hot working of metals these factors include: the formation of scale, the presence of a thin layer of water or water vapour on the surface of contact between the metal and the tool, and modifications of the roll surface due to wear.

In cold working, the additional influence of lubrication should be taken into account, and the possibility of the lubricant being pressed into the contact surface by a high load.

Orowan [35 and 29] carried out numerous experiments to determine the value of the variable coefficient  $f$  in slipping under large pressures. These tests were effected on a special apparatus which was built into the tensile testing machine to obtain the

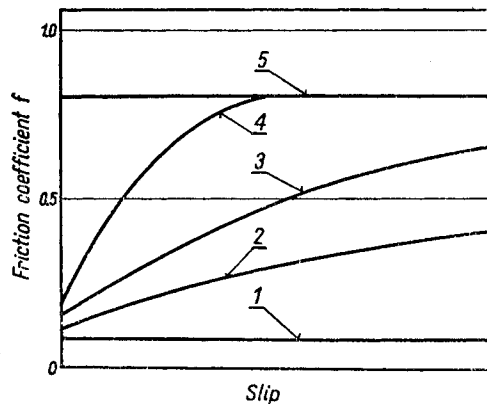


Fig. 2.32. Variation of coefficient of friction with various testing conditions according to Orowan [29 and 35]; 1 — lubricated surfaces and low pressures, 2 — lubricated surfaces and high pressures, 3 — matt surfaces and low pressures, 4 — matt surfaces and high pressures, 5 — surfaces without lubricant and high pressures

necessary loads. Certain elements of this apparatus represented the roll surface and others the metal. The conditions necessary to obtain the cylindrical shape of rolls and the appropriate arc of contact were also provided. Tests were carried out using tools with (a) polished, (b) dry and (c) lubricated surfaces.

The results illustrating the relation between the magnitude of friction coefficients and the surface condition are given in Fig. 2.32. In applying these results to plastic working processes it should be noted that those for polished surfaces correspond to cold working without lubricant, and the results for lubricated surfaces correspond to cold working processes with lubrication. The results for dry surfaces



correspond to conditions of hot working. The coefficients of friction used practically for the plastic working processes of different metals and alloys are given in Table 2.7, according to Gubkin [1]. The frictional force (frictional resistance) is calculated from equation (2.99). The magnitude of this force depends on the pressure  $P$  exerted by the metal on the working tool or vice versa, and on the coefficient of friction,  $f$ . The rougher

TABLE 2.7  
COEFFICIENTS OF FRICTION OBTAINED IN PLASTIC WORKING OF METALS AND ALLOYS  
ACCORDING TO GUBKIN [1]

| Different temperatures<br>of deformed metal | Friction coefficients of:       |      |                                         |      |                                         |      |                                              |      |                                                          |      |
|---------------------------------------------|---------------------------------|------|-----------------------------------------|------|-----------------------------------------|------|----------------------------------------------|------|----------------------------------------------------------|------|
|                                             | steel<br>and<br>alloy<br>steels |      | aluminium<br>and<br>aluminium<br>alloys |      | magnesium<br>and<br>magnesium<br>alloys |      | heavy<br>non-ferrous<br>metals and<br>alloys |      | nickel<br>and<br>heat resisting<br>non-ferrous<br>alloys |      |
|                                             | Rate of deformation, m/sec      |      |                                         |      |                                         |      |                                              |      |                                                          |      |
|                                             | < 1                             | > 1  | < 1                                     | > 1  | < 1                                     | > 1  | < 1                                          | > 1  | < 1                                                      | > 1  |
| (0.8–0.95) $T_m$                            | 0.40                            | 0.35 | 0.50                                    | 0.48 | 0.40                                    | 0.35 | 0.32                                         | 0.30 | 0.28                                                     | 0.25 |
| (0.5–0.8) $T_m$                             | 0.45                            | 0.40 | 0.48                                    | 0.45 | 0.38                                    | 0.32 | 0.34                                         | 0.32 | 0.26                                                     | 0.25 |
| (0.3–0.5) $T_m$                             | 0.35                            | 0.30 | 0.35                                    | 0.30 | 0.32                                    | 0.24 | 0.26                                         | 0.24 | 0.24                                                     | 0.20 |

Friction coefficient for deformation without heating and with lubrication 0.06–0.12 depending on the lubricant and surface condition.

$T_m$  — melting point of metal.

the surfaces, the higher is the coefficient of friction and hence the resistance of metal to the pressure of punch or roll. The magnitude of  $f$  is also dependent on the speed of deformation, which is assumed to be constant. The frictional forces produced by the punch on the planes  $A-A$  and  $B-B$  (Fig. 2.30) act concentrically towards the axis of the specimen. At the centre, where the static friction reaches its highest values a change of direction occurs, as indicated by the horizontal arrows in Fig. 2.30. The frictional resistance causes a change in stress distribution, i.e. instead of a uniform stress distribution  $ACDA$ , with the value corresponding to the constrained yield stress,  $K = \eta K_f$ , the stress distribution is non-uniform and of the form of the previous rectangle  $ACDA$  plus an area bounded by the curves of two hyperbolas meeting at the centre of specimen (Fig. 2.30) giving a final stress distribution  $ACPDA$ .

Due to the action of frictional forces (resistance to flow) a region of restricted capability of deformation arises, just below the surface of specimen. Since the frictional resistance reaches its highest value at the centre of specimen, the region of restricted capability of deformation here reaches its greatest depth (point  $G$ ).

Thus, two cones of revolution  $AGA$ , with restricted ability to deform are generated (cross-hatched in Figs. 2.30 and 2.33). These cones are called the regions of restricted capability of plastic deformation. At the same time the side faces of the freely deforming portion of the specimen become rounded and take a barrel-like shape.

The extent of these regions depends on the external friction, the pressure of the working tool and the shape of the work piece (specimen).

When thin specimens and high tool pressures are employed, the cones of restricted capability of deformation can touch each other as shown in Fig. 2.33. In this case the resistance to deformation rapidly increases as only very small portions can deform freely.

Thus, the external friction results not only in non-uniform stress distribution, but also in formation of regions with varying capability of deformation. Consequently, the metal undergoes non-uniform deformation, resulting in barreling of the cylindrical specimen (Fig. 2.33). The line, or in the most recent investigations, the plane or the

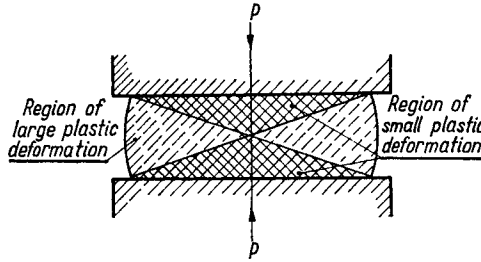


Fig. 2.33. Regions of plastic deformation in a specimen with restricted capability of deformation in compression [10]

region where the slip friction disappears and becomes static friction, is called the neutral region. This region is characterized by lack of relative motion between tool and metal ( $PP$  in Fig. 2.30).

The pressure distribution pattern  $ACPDA$  shown in Fig. 2.30 is composed of two portions. The lower portion  $ACDA$  represents the forces required to overcome the constrained yield stress  $K = \eta K_f$ , and the upper portion  $CPD$  those necessary to overcome the frictional resistance between the tool and the metal, i.e. the so called resistance to flow,  $K_r$ .

This can be expressed mathematically

$$K_w = \eta K_f + K_r \quad (2.105)$$

where

$K_w$  = resistance of metal to deformation,

$K = \eta K_f$  = constrained yield stress,

$K_r$  = resistance of metal to flow.

For every case of plastic working where friction occurs, equation (2.105) is valid, with  $\eta = 1.0$  for uniform deformation of metal, i.e.  $\epsilon_{r2} = \epsilon_{r3}$  or  $\beta = \lambda$ .

The coefficient  $\eta$  varies from 1.0 to 1.155 depending on the method of deformation, i.e. depending chiefly on the value of  $\epsilon_{r2}$  or  $\beta$ . If  $\epsilon_{r3} = 0$  or  $\beta = 1$ , i.e. if there is no lateral spread of metal, then  $\eta$  reaches its highest value of 1.155.

In all intermediate cases the accurate value of  $\eta$  is calculated from equations (2.62)–(2.76) derived by the author [19]. It should be remembered that the relation (2.105) expresses the condition of yielding, known already from formula (2.50)

$$\sigma_1 - \sigma_3 = \eta R_{pl} = \eta K_f$$

or using the other notation

$$\sigma_1 = K_w$$

$$\sigma_3 = K_r$$

In these formulae,  $K_f$ , the yield stress of metal, is chosen depending on the conditions of deformation, as given in the preceding chapter.

The value of  $\sigma_3 = K_f$  (resistance of metal to flow), is calculated differently for individual methods of working, e.g. stamping, forging, drawing, rolling, extruding, etc.

In the preceding considerations it was assumed that the metal of the compressed specimen can flow freely under conditions of slip friction along the planes  $A-A$  or  $B-B$  as shown in Fig. 2.30. In more recent theories it is assumed that slipping and sticking occur simultaneously.

Unksov [21, 27 and 28] and Orowan [29] state that the boundary where slipping disappears and the metal begins to stick to the tool, depends only on the magnitude of the frictional resistance

$$\tau = f\sigma \quad (2.106)$$

where

$\tau$  = static friction,

$\sigma$  = normal stress in metal, i.e. pressure exerted by the tool at the given point of contact area.

If the product  $f\sigma$  is smaller than the constrained yield stress in shear at a given point of contact area  $(\eta/2) K_f$ , the metal will slip along the surface. On the other hand, if  $f\sigma$  is equal to or greater than the constrained yield stress in shear, sticking of metal to the tool occurs, i.e.

$$\tau = f\sigma \leq \frac{\eta}{2} K_f \quad (2.107)$$

In the light of these consideration, the concepts of shear stress and the constrained yield stress in compression, formerly expressed by formula (2.76)

$$\tau = \frac{\sigma_1 - \sigma_3}{2} = \frac{1}{\sqrt{3}} K_f = 0.577 K_f$$

should also be modified.

It is known now that  $\eta$  is not a constant but a variable and can be determined from the relation

$$\tau = \frac{\eta}{2} K_f = 0.5-0.577 K_f$$

depending on the mode of deformation of metal, i.e. on the coefficients  $\beta$  and  $\lambda$ . Therefore it can be concluded that the mode of deformation can increase or decrease the area of sticking, which was not known up till now.

On reaching the constrained yield stress in shear, further variations of  $f\sigma$ , and of the coefficient of friction,  $f$ , are unimportant, since they cannot exceed that limit.

Thus, from the point on the contact area where the metal begins to stick to the tool, the coefficient of friction  $f$  becomes theoretically constant (Fig. 2.32). Therefore after this point it is not necessary to determine the value of  $f$ .

However experiments were carried out to determine the value of  $f$  for the area where slip friction occurs. It was found that it depends on the kind of metals in contact, the condition of their surfaces, temperature, pressure and the relative velocity.

Other factors should also be taken into account in hot working, e.g. the formation of scale, the presence of water or vapour films on the metal and tool surfaces and the deterioration of roll surface.

Due to the varying magnitude of stresses (Fig. 2.30) they are not used for practical calculations. For this reason the pressure of metal on the tool is calculated practically from the formula

$$P = F_d K_{wm} \quad (2.108)$$

where

$P$  = pressure of tool on metal or vice versa, kg,

$F_d$  = projection of contact area between metal and tool on the plane normal to the direction of the force, mm<sup>2</sup>,

$K_{wm}$  = the mean resistance of metal to deformation, kg/mm<sup>2</sup>.

The corresponding stress diagrams are plotted in Fig. 2.30. The rectangle  $ACDA$  represents the constant value of the constrained yield stress,  $\eta K_f$ , provided there is no work-hardening of metal. Two hyperbolas meeting on  $P$  determine the area  $CPD$  corresponding to the value of  $K_r$ .

The variable  $K_w$  is determined by the area  $ACPDA$ . The magnitude of  $K_w$  changes at each point of the  $CPD$  curve. Thus, the mean resistance to deformation  $K_{wm}$  will be represented by the rectangle  $AXYA$ , equal in area to  $ACPDA$ .

This simplified mean value  $K_{wm}$  is usually applied for all practical calculations from formula (2.108).

In practice, first the highest mean constrained yield stress  $\eta K_{f,m}$  is determined, then the mean resistance to flow  $K_{r,m}$  is added, and the mean resistance to deformation,  $K_{wm}$  is obtained. Alternatively,  $K_{wm}$  is found directly by some other method, most frequently by using diagrams already worked out for the given process and conditions of deformation. These methods will be discussed in detail for every practical method of calculating roll pressure.

## 2.6. Work of Plastic Deformation

The work done in deformation during plastic working is equal to the work of external forces [30], which have to overcome the reaction of metal.

For finite strains a somewhat different reasoning will be applied. A cubic element of metal is assumed deformed by the action of a vertical force, and the work required for deformation is then calculated.

The first to solve this problem correctly and derive appropriate equations was Fink [30].

Figure 2.34 shows schematically the process of deformation. Under the action of a perpendicular force  $P$ , the height of the element decreases from the initial value  $h_0$  to  $h_1$  and the initial area  $F_0 = b_0 l_0$  changes to  $F_1 = b_1 l_1$ , passing through several intermediate stages of deformation where the area  $F = bl$ .

The volume of the cubic element remains constant during plastic deformation [10]

$$V_0 = F_0 h_0 = Fh = F_1 h_1 = V_1$$

It is assumed that there is no friction between the punch and the metal and the constrained yield stress  $\eta K_f$  is constant. Then, neglecting the frictional losses, the work

necessary to produce the plastic deformation of a volume element of metal with height  $dh$  is determined by the greatest true deformation of the cubic element

$$dA_0 = P dh = \eta K_f F dh = \eta K_f \frac{V}{h} dh$$

and the theoretical work necessary to deform the rectangular element will be

$$A_0 = \eta K_f V \int_{h_1}^{h_0} \frac{dh}{h} = \eta K_f V \log_e \frac{h_0}{h_1} = \eta K_f V \log_e \frac{1}{\gamma} \quad (2.109)$$

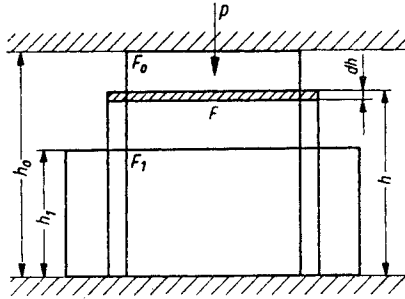


Fig. 2.34. Schematic representation of parallelepipedal deformation [31]

The theoretical work of deformation given by this equation is defined by three expressions:

- $\eta K_f$  = constrained yield stress under given conditions of plastic deformation,
- $V$  = volume of the stock,
- $\varphi_h = \log_e(h_0/h_1) = \log_e(1/\gamma)$ , natural logarithm of the reciprocal of the draught coefficient.

The total theoretical work of deformation is given in kilogram-metres or ton-metres. If the assumption is made that the width of stock is constant, the deformation will result entirely in elongation; or with assumed constant length of stock deformation will result in spread. With variable width and length of stock some part of the displaced metal will give increased length.

The deformation of a bar under tensile load, from initial length  $l_0$  to  $l_1$  is exactly similar. The work done in displacing the metal volume over a very short distance  $dl$  can be expressed analogously

$$dA_0 = P dl = \eta K_f F dl = \eta K_f \frac{V}{l} dl$$

Integrating within limits  $l_0$ – $l_1$ , the work of plastic deformation, neglecting frictional losses, will be

$$A_0 = \eta K_f V \int_{l_0}^{l_1} \frac{dl}{l} = \eta K_f V \log_e \frac{l_1}{l_0} = \eta K_f V \log_e \lambda \quad (2.110)$$

In this case it was assumed that the width of bar remained constant, i.e. there was no lateral spreading of metal. Thus, equation (2.110) applies to the cases where there

is no change of width, and the total elongation takes place at the expense of loss in height.

Taking logarithms of equation (2.8)  $\gamma\beta\lambda = 1$  gives [12]

$$\log_e \gamma + \log_e \beta + \log_e \lambda = 0$$

Since it is known that

$$\varphi_h = \sum \varphi_i$$

then for  $\gamma < 1$  and  $\beta$  and  $\lambda > 1$

$$-\log_e \gamma = \log_e \frac{1}{\gamma} = \log_e \beta + \log_e \lambda \quad (2.111)$$

Hence, it can be concluded that the largest deformation  $1/\gamma$  is a measure of the total deformation of metal, and also determines the work involved in this process.

Thus, in determining the theoretical work of deformation (in compression), the relation

$$\log_e \frac{1}{\gamma} = \log_e \beta + \log_e \lambda$$

expresses both elongation and spread of metal.

Dividing equation (2.109) by the volume  $V$ , the effective work related to unit volume of metal is found

$$a_v = \eta K_f \log_e \frac{1}{\gamma} \quad (2.112)$$

$$a_0 = \eta K_f \log_e \lambda \quad (2.113)$$

In general, if  $d\varphi$  denotes any arbitrary plastic deformation  $dh/h$ ,  $dl/l$  or  $dF/F$  and  $\eta K_f$  is not constant and depends on  $\varphi$ , then the integral  $\int \eta K_f d\varphi$  represents the area below the curve  $\eta K_f = f(\varphi)$  (Fig. 2.35), which is the stress-strain curve.

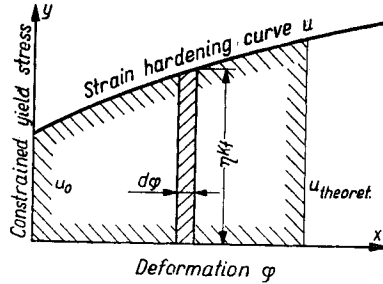


Fig. 2.35. Relation between constrained yield stress and strain [31]

If it is assumed that the form of the stress-strain curve in particular ranges of deformation is rectilinear then

$$A_0 = V \eta K_{fm} \int_0^\varphi d\varphi \quad (2.114)$$

where  $K_{fm}$  is the arithmetical mean i.e.

$$\eta K_{fm} = \frac{\eta K_f(\varphi_1) + \eta K_f(\varphi_2)}{2} \quad (2.115)$$

The theoretical work of plastic deformation in shear given by these equations is the minimum work which must be done, since it expresses only the plastic strain of metal.

In practice, due to friction between the metal and the working tool at the surface, and the displacements in the interior of metal occurring during plastic deformation, the actual work of deformation always exceeds the theoretical work. In theory, it is usually assumed that a cubic element of metal undergoes parallelepipedal deformation during plastic working, i.e. all the edges and faces of the element remain rectilinear during the whole process. In fact this is not true, as is well known [29]. Both the edges and faces become curved or otherwise distorted. For this reason, not all portions of the work piece undergo the same amount of plastic deformation.

These different degrees of deformation cause different plastic displacements of metal, which in turn cause additional relative flow of metal and displacements in the interior of the work piece.

Therefore, the resistance to deformation  $K_w$  represents the sum of the constrained yield stress  $\eta K_f$  and the resistance to flow,  $K_r$ . This applies even for the case when lateral spreading occurs i.e.  $K_w = \eta K_f + K_r$ .

If there is no spreading,  $K_w = 1.15 K_f + K_r$ .

The formula for calculating the effective work of deformation will take the form

$$A_w = (\eta K_f + K_r) V \log_e \frac{h_0}{h_1} = K_w V \log_e \frac{h_0}{h_1} = A_0 + A_r \quad (2.116)$$

where

$A_0$  = theoretical work of deformation,

$A_r$  = additional work of deformation required to overcome the frictional resistance.

Considering the problem of theoretical work of deformation neglecting frictional losses, it makes no difference if this work is done in one or several passes. For practical operation this may cause certain differences depending on the method of working, the variations of friction conditions, and most of all the magnitude of  $K_w$ .

For practical calculations of the work of plastic deformation the mean value of resistance to deformation,  $K_{wm}$ , should be used, taken from diagrams plotted from experimental results for various materials, temperatures, deformation rates, reductions, friction etc.

Considering the variation of deformation work over a short distance, or the work done when strain-hardening due to cold work occurs, a linear increase of  $K_{wm}$  can be assumed. Then

$$K_{wm} = \frac{K_{w1} + K_{w2}}{2} \quad (2.117)$$

where  $K_{w1}$  and  $K_{w2}$  denote resistance to deformation at the beginning and at the end of deformation. This simplification is made to avoid graphical or other complicated calculation methods.

The detailed procedures for particular cases will be given in the following chapters.

## CHAPTER 3

### 3.1. Basic Concepts and Symbols in Rolling

Figures 1.1a, b and c represent schematically longitudinal, transverse, and skew rolling processes. Skew and transverse rolling methods are used in tube manufacture. In this chapter the case of longitudinal rolling (Fig. 3.1) will be discussed more fully. The basic symbols relating to this rolling process are the same as those given in Chapter 2.

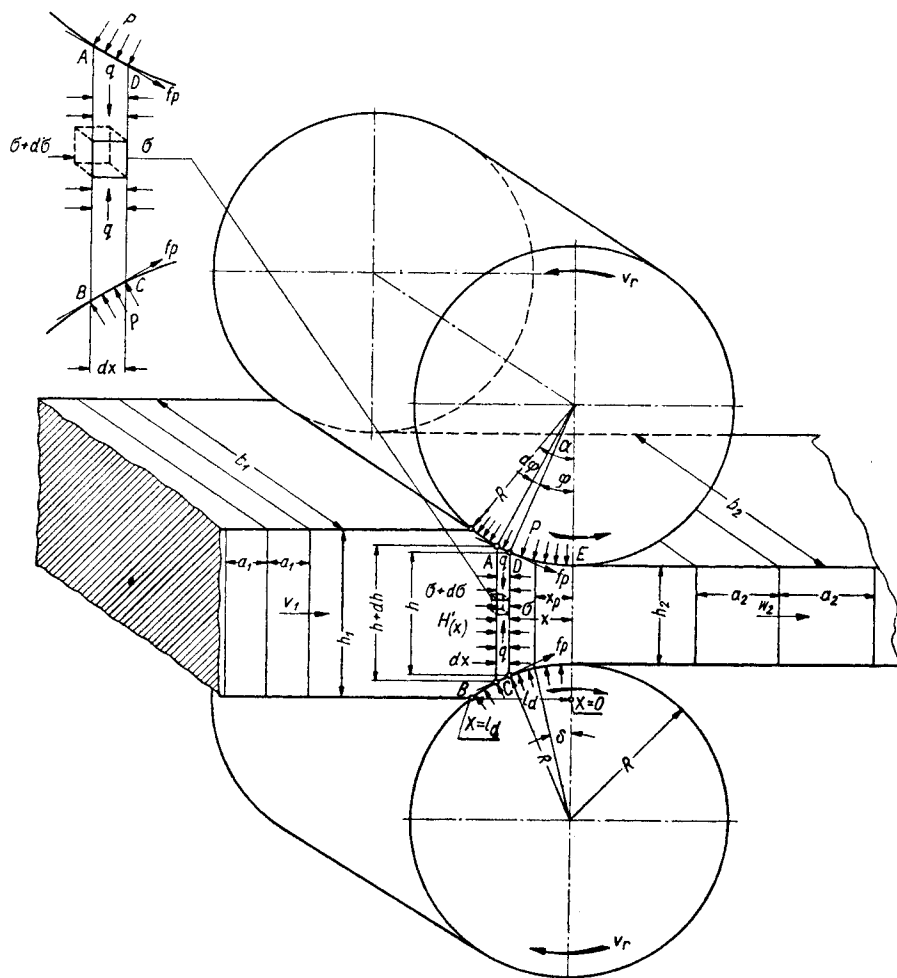


**Fig. 3.1. Schematic representation of flat rolling [29]**



The height  $h$  of the rolled stock is measured normally to the roll axis. The breadth  $b$  is measured parallel to the roll axis. The dimension of metal in the direction of rolling is denoted as the length  $l$  (Fig. 3.2).

In the successive stages of rolling, the dimensions of the rolled bar are expressed as follows:

$V_0 = h_0 b_0 l_0$  — initial values of volume, height, breadth and length of stock, when rolling starts (Fig. 3.2);

$F_0 = h_0 b_0$  — initial cross-sectional area of stock;

$V_1 = h_1 b_1 l_1$  — volume, height, breadth and length of stock after first pass;

$F_1 = h_1 b_1$  — cross-sectional area of stock after first pass;

$V_n = h_n b_n l_n$  — volume, height, breadth and length of stock after  $n$  passes ( $n$  denotes either the final pass or the number of passes);

$F_n = h_n b_n$  — cross-sectional area of stock after  $n$  passes.

The increase in length of stock after a pass in rolling is usually greater than the increase in breadth. The above notations are given for rectangular or flat sections, i.e. squares, flat bars, band steel or strip rolled between the plain cylindrical horizontal rolls (Figs. 3.1 and 3.2).

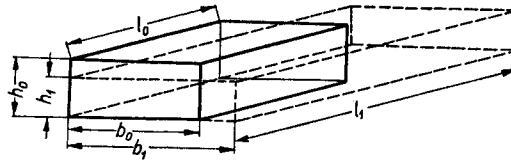


Fig. 3.2. Deformation in rolling [32]

For rolling of non-rectangular sections such as bars, shapes, rails, etc., an additional term, the mean height of stock, is introduced [32] (Figs. 3.3a and b).

This mean height of stock is expressed as

$$h_m = \frac{F}{b} \quad (3.1)$$

i.e. it is calculated by dividing the cross-sectional area  $F$  by the maximum breadth  $\bar{b}$  of the filled section. Figure 3.3a illustrates the methods of determining the mean height of stock in rolling regular sections such as squares, ovals, gothic squares, diamonds etc., having two axes of symmetry. The method of determining mean height for non-rectangular profiles having only one axis of symmetry is shown in Fig. 3.3b. A similar method of determining  $h_m$  is employed for profiles having no axis of symmetry.

The concept of mean height has been introduced to maintain the principle of constancy of volume.

For sections with

$$h_{0m} = \frac{F_0}{b_0} \quad h_{1m} = \frac{F_1}{b_1} \quad h_{nm} = \frac{F_n}{b_n} \quad (\text{Figs. 3.3a and b})$$

the following relationships are obtained from the condition of constancy of volume [47].

$$\begin{aligned} V_0 &= F_0 l_0 = h_{0m} b_0 l_0 \\ &= V_1 = F_1 l_1 = h_{1m} b_1 l_1 \\ &= V_n = F_n l_n = h_{nm} b_n l_n \end{aligned} \quad (3.2)$$

On dividing these relations

$$\frac{h_{2m} b_2 l_2}{h_{1m} b_1 l_1} = \gamma_m \beta \lambda = 1 \quad (3.3)$$

where

$$\frac{h_{2m}}{h_{1m}} = \frac{F_2}{b_2} \frac{b_1}{F_1} = \gamma_m \quad (3.4)$$

denotes the mean coefficient of draught, and the mean absolute draught is given by

$$h_{1m} - h_{2m} = \frac{F_1}{b_1} - \frac{F_2}{b_2} = \Delta h_m \quad (3.5)$$

$$\frac{h_{1m} - h_{2m}}{h_{1m}} = \varepsilon_m \text{ — mean relative draught}$$

$$\frac{h_{1m} - h_{2m}}{h_{1m}} 100\% = G_m 100\% \text{ — mean percentage draught}$$

where, according to Fig. 3.3a, the terms  $h_{1m}$ ,  $h_{2m}$  and  $h_{3m}$  are equal to  $h_i$  and  $h_i = F_i/b_i$ .

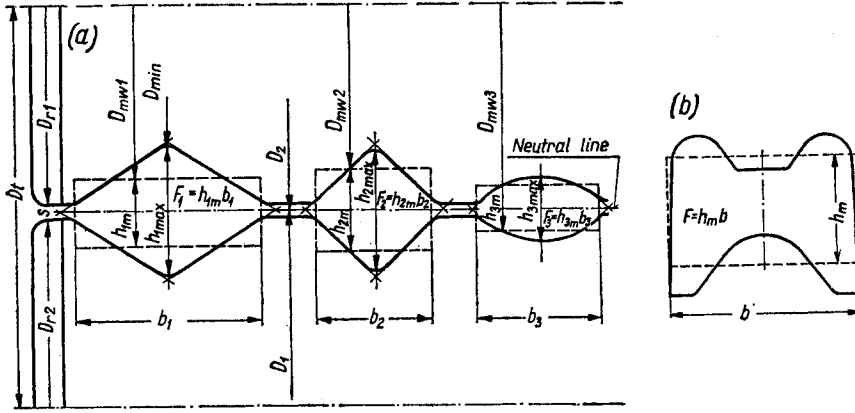


Fig. 3.3. Methods of determining the average height of passes [10]: (a) — three successive regular passes for rolling bars, (b) — pass for rolling a section

In rolling non-rectangular sections, the term maximum draught is sometimes used (Fig. 3.3b):

$$\frac{h_{2\max}}{h_{1\min}} = \gamma_{\max} \text{ — maximum coefficient of draught,}$$

$$h_{1\max} - h_{2\min} = \Delta h_{\max} \text{ — maximum absolute draught,}$$

$$\frac{h_{1\max} - h_{2\min}}{h_{1\max}} = \varepsilon_{\max} \text{ — maximum relative draught, and}$$

$$\frac{h_{1\max} - h_{2\min}}{h_{1\max}} 100\% = G_{\max} 100\% \text{ — maximum percentage draught.}$$

In rolling, the coefficient of elongation is also expressed as follows (Fig.3.1):

$$\lambda = \frac{F_1}{F_2} = \frac{h_{1m} b_1}{h_{2m} b_2} = \frac{l_2}{l_1} = \frac{w_2}{v_1} \quad (3.6)$$

where

$v_1$  = entry speed,

$w_2$  = exit speed.

This method of calculation is convenient since to find the area, height or breadth of pass it is sufficient to multiply or to divide by appropriate coefficients of elongation, draught and spread.

If the applied reduction or the coefficients of elongation in rolling are known, each can be easily calculated from the other using the following formula:

$$\lambda = \frac{1}{1-U} = \frac{100}{100-U\%} \quad (3.7)$$

where

$U$  = relative reduction,

$U\%$  = percentage reduction

$$\left. \begin{aligned} U &= 1 - 1/\lambda \\ U\% &= 100 - 100/\lambda \end{aligned} \right\} \quad (3.8)$$

Table 3.1 and Figure 3.4 give numerically and graphically the relation between coefficient of elongation  $\lambda$  and percentage reduction  $U\%$ .

#### Example 1

A bloom with the following dimensions is rolled:

$h_1 = 200$  mm,  $b_1 = 250$  mm,  $l_1 = 2000$  mm.

After the first pass the dimensions are:

$h_2 = 150$  mm,  $b_2 = 262$  mm,  $l_2 = 2545$  mm.

Area and volume before pass

$$F_1 = h_1 b_1 = 200 \times 250 = 50,000 \text{ mm}^2$$

$$V_1 = h_1 b_1 l_1 = 200 \times 250 \times 2000 = 100,000,000 \text{ mm}^3$$

Area and volume after first pass

$$F_2 = h_2 b_2 = 150 \times 262 = 39,300 \text{ mm}^2$$

$$V_2 = h_2 b_2 l_2 = 150 \times 262 \times 2545 = 100,000,000 \text{ mm}^3$$

The coefficients of deformation are calculated as follows:

Coefficient of draught

$$\frac{h_2}{h_1} = \gamma = \frac{150}{200} = 0.750$$

Coefficient of spread

$$\frac{b_2}{b_1} = \beta = \frac{262}{250} = 1.048$$

Coefficient of elongation

$$\frac{l_2}{l_1} = \lambda = \frac{2545}{2000} = 1.2725$$

Checking from formula (2.8)

$$\frac{h_2}{h_1} \frac{b_2}{b_1} \frac{l_2}{l_1} = \frac{150}{200} \times \frac{262}{250} \times \frac{2545}{2000} = \gamma \beta \lambda = 0.750 \times 1.048 \times 1.2725 = 1.0000$$

i.e. the calculation is correct.

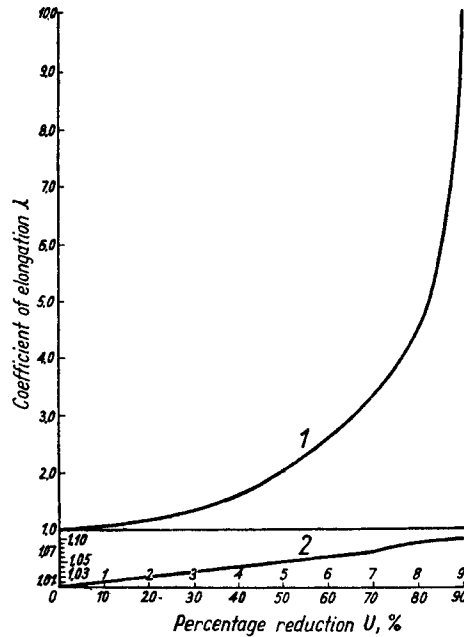


Fig. 3.4. Relation between coefficient of elongation and percentage reduction  $U, \%$  [10];  
 1 — curve for reductions from 0 to 90%, 2 — enlarged section of curve for reductions from 0 to 9%

TABLE 3.1  
 RELATION BETWEEN  $U, \%$ , AND THE COEFFICIENT OF ELONGATION,  $\lambda$

| $U, \%$ | $\lambda$ | $U, \%$ | $\lambda$ | $U, \%$ | $\lambda$ | $U, \%$ | $\lambda$ |
|---------|-----------|---------|-----------|---------|-----------|---------|-----------|
| 1       | 1.01      | 26      | 1.35      | 51      | 2.04      | 76      | 4.16      |
| 2       | 1.02      | 27      | 1.37      | 52      | 2.08      | 77      | 4.35      |
| 3       | 1.03      | 28      | 1.39      | 53      | 2.12      | 78      | 4.54      |
| 4       | 1.04      | 29      | 1.41      | 54      | 2.17      | 79      | 4.76      |
| 5       | 1.05      | 30      | 1.43      | 55      | 2.22      | 80      | 5.00      |
| 6       | 1.06      | 31      | 1.45      | 56      | 2.27      | 81      | 5.26      |
| 7       | 1.07      | 32      | 1.47      | 57      | 2.32      | 82      | 5.56      |
| 8       | 1.09      | 33      | 1.49      | 58      | 2.38      | 83      | 5.88      |
| 9       | 1.10      | 34      | 1.51      | 59      | 2.44      | 84      | 6.25      |
| 10      | 1.11      | 35      | 1.54      | 60      | 2.50      | 85      | 6.67      |
| 11      | 1.12      | 36      | 1.56      | 61      | 2.56      | 86      | 7.14      |
| 12      | 1.13      | 37      | 1.59      | 62      | 2.63      | 87      | 7.70      |
| 13      | 1.15      | 38      | 1.61      | 63      | 2.70      | 88      | 8.82      |
| 14      | 1.16      | 39      | 1.64      | 64      | 2.78      | 89      | 9.10      |
| 15      | 1.18      | 40      | 1.67      | 65      | 2.86      | 90      | 10.00     |
| 16      | 1.19      | 41      | 1.69      | 66      | 2.94      | 91      | 11.10     |
| 17      | 1.20      | 42      | 1.72      | 67      | 3.03      | 92      | 12.50     |
| 18      | 1.22      | 43      | 1.75      | 68      | 3.12      | 93      | 14.28     |
| 19      | 1.23      | 44      | 1.78      | 69      | 3.22      | 94      | 16.66     |
| 20      | 1.25      | 45      | 1.82      | 70      | 3.33      | 95      | 20.00     |
| 21      | 1.27      | 46      | 1.86      | 71      | 3.44      | 96      | 25.00     |
| 22      | 1.28      | 47      | 1.88      | 72      | 3.57      | 97      | 33.35     |
| 23      | 1.30      | 48      | 1.92      | 73      | 3.70      | 98      | 50.00     |
| 24      | 1.32      | 49      | 1.96      | 74      | 3.84      | 99      | 100.00    |
| 25      | 1.33      | 50      | 2.00      | 75      | 4.00      |         |           |

Further parameters can now be calculated:

absolute draught

$$\Delta h = h_1 - h_2 = 200 - 150 = 50 \text{ mm}$$

relative draught

$$\varepsilon_h = \frac{h_1 - h_2}{h_1} = \frac{200 - 150}{200} = \frac{50}{200} = 0.25$$

percentage draught

$$G_h\% = \frac{h_1 - h_2}{h_1} 100\% = \frac{200 - 150}{200} 100\% = 25\%$$

absolute spread

$$\Delta b = b_2 - b_1 = 262 - 250 = 12 \text{ mm}$$

absolute elongation

$$\Delta l = l_2 - l_1 = 2545 - 2000 = 545 \text{ mm}$$

absolute reduction

$$\Delta F = F_1 - F_2 = 50,000 - 39,300 = 10,700 \text{ mm}^2$$

relative reduction

$$U = \frac{F_1 - F_2}{F_1} = \frac{50,000 - 39,300}{50,000} = 0.214$$

percentage reduction

$$U\% = \frac{F_1 - F_2}{F_1} 100\% = \frac{50,000 - 39,300}{50,000} 100\% = 21.4\%$$

The relative reduction can be calculated in terms of the coefficient of elongation and vice versa from formulae (3.7) and (3.8)

$$U = \frac{F_1 - F_2}{F_1} = 1 - \frac{1}{\lambda} = 1 - \frac{1}{1.2725} = 1 - 0.786 = 0.214$$

$$\lambda = \frac{l_2}{l_1} = \frac{F_1}{F_2} = \frac{1}{1 - U} = \frac{2545}{2000} = \frac{50,000}{39,300} = \frac{1}{1 - 0.214} = 1.2725$$

### Example 2

This example is given to illustrate the use of maximum and mean draught and the difference between them. Three passes of a breaking-down system square-oval-square are considered. The ingoing profile is a square with  $h_1 = b_1 = 17.7 \text{ mm}$ , and  $F_1 = 314 \text{ mm}^2$ . This square is entered flat into the horizontal oval pass. The outgoing oval has dimensions:  $h_2 = 10 \text{ mm}$ ,  $b_2 = 30 \text{ mm}$  and  $F_2 = 200 \text{ mm}^2$ . This oval on turning through  $90^\circ$  enters into the square pass, as a diagonal pass. The oval dimensions after turning are:  $h_2 = 30 \text{ mm}$ ,  $b_2 = 10 \text{ mm}$ ,  $F_2 = 200 \text{ mm}^2$ . The outgoing square with sides  $a_3 = 12 \text{ mm}$  has the following dimensions:

$$h_3 = a_3 \times 1.414 = 17.0 \text{ mm}$$

$$b_3 = a_3 \times 1.414 = 17.0 \text{ mm}$$

$$F_3 = a_3 a_3 = \frac{h_3 b_3}{2} = 144 \text{ mm}^2$$

### Pass I

The maximum absolute draught

$$\Delta h_{\max} = h_{1\max} - h_{2\min} = 17.7 - 10.0 = 7.7 \text{ mm}$$

The maximum percentage draught

$$G_{\max} = \frac{h_{1\max} - h_{2\min}}{h_{1\max}} 100\% = \frac{17.7 - 10.0}{17.7} 100\% = 43.5\%$$

The maximum coefficient of draught

$$\gamma_{\max} = \frac{h_{2\min}}{h_{1\max}} = \frac{10.0}{17.7} = 0.565$$

The mean absolute draught

$$\Delta h_m = \frac{F_1}{b_1} - \frac{F_2}{b_2} = \frac{314}{17.7} - \frac{200}{30} = 17.7 - 6.7 = 11.0 \text{ mm}$$

The mean percentage draught

$$G_m = \frac{h_{1m} - h_{2m}}{h_{1m}} 100\% = \frac{17.7 - 6.7}{17.7} 100\% = 62.1\%$$

The mean coefficient of draught

$$\gamma_m = \frac{h_{2m}}{h_{1m}} = \frac{6.7}{17.7} = 0.379$$

From these calculations it may be seen that the values of the mean draught are greater than those calculated from maximum dimensions.

*Pass II*

The oval enters on edge into the diagonal square pass.

Maximum absolute draught

$$\Delta h_{\max} = h_{2\max} - h_{3\min} = 30 - 17.0 = 13.0 \text{ mm}$$

Maximum percentage draught

$$G_{\max} = \frac{h_{2\max} - h_{3\min}}{h_{2\max}} 100\% = \frac{30 - 17.0}{30} 100\% = 43.3\%$$

Maximum coefficient of draught

$$\gamma_{\max} = \frac{h_{3\min}}{h_{2\max}} = \frac{17.0}{30} = 0.566$$

Mean absolute draught

$$\Delta h_m = \frac{F_2}{b_2} - \frac{F_3}{b_3} = \frac{200}{10} - \frac{144}{17.0} = 20.0 - 8.5 = 11.5 \text{ mm}$$

Mean percentage draught

$$G_m = \frac{h_{2m} - h_{3m}}{h_{2m}} 100\% = \frac{20 - 8.5}{20} 100\% = 57.5\%$$

Mean draught coefficient

$$\gamma_m = \frac{h_{3m}}{h_{2m}} = \frac{8.5}{20.0} = 0.425$$

Here again mean values are greater than those calculated from maximum dimensions.

### 3.2. Geometrical and Trigonometrical Relations in Rolling

When a bar with initial thickness  $h_1$  enters the rolls, the edge of the bar touches the roll at a point through which passes one arm of the angle having its apex on the roll axis, and the other arm in the plane passing through the roll axes (Fig. 3.5a). The included angle  $\alpha$  is called the angle of bite.

The height of bar leaving the rolls is  $h_2$ ;  $l_d$  denotes the projected arc of contact between rolls and metal,  $c$  the chord of the arc of contact, and  $h$  the height of bar in the rolls at the distance  $x$  from the exit side of rolls, corresponding to a rolling angle  $\varphi$ . The rolling angle is an angle  $\varphi$ , less than or equal to the angle of bite, having its apex at the roll centre and arms passing through point  $A$  the considered point defined by  $x$  and  $B$ , the point of exit from rolls (Fig. 3.5a).

If both rolls have the same diameter  $D$ , as in Fig. 3.5a, the difference between the ingoing and outgoing thicknesses, i.e. the absolute draught, is

$$\Delta h = h_1 - h_2$$

or for one roll

$$\frac{\Delta h}{2} = \frac{h_1 - h_2}{2}$$

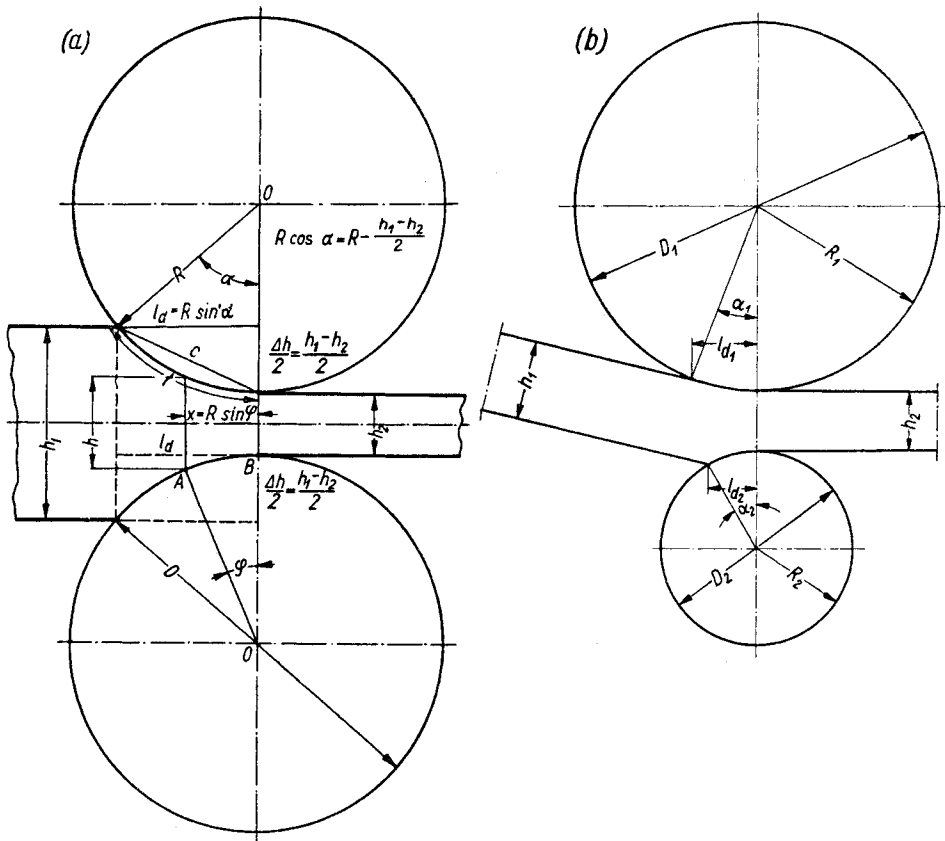


Fig. 3.5. Geometrical and trigonometrical relationships in rolling [10]: (a) rolls of the same diameter, (b) rolls of different diameters

Hence the basic formulae for rolling with rolls of the same diameter can be derived

$$R \cos \alpha = R - \frac{h_1 - h_2}{2}$$

and transposing

$$1 - \cos \alpha = \frac{h_1 - h_2}{2R}$$

Hence the formula for calculating the angle of bite is found

$$\cos \alpha = 1 - \frac{h_1 - h_2}{2R} = 1 - \frac{\Delta h}{D} \quad (3.9)$$

From this formula either the ingoing thickness

$$h_1 = h_2 + D(1 - \cos \alpha) \quad (3.10)$$

the outgoing thickness

$$h_2 = h_1 - D(1 - \cos \alpha) \quad (3.11)$$

or the absolute draught

$$\Delta h = D(1 - \cos \alpha) \quad (3.12)$$

can be calculated.

The projected arc of contact between the metal and the rolls is calculated from the geometrical relationship

$$l_d^2 = R^2 - \left( R - \frac{h_1 - h_2}{2} \right)^2$$

hence

$$l_d = \sqrt{R^2 - \left( R - \frac{\Delta h}{2} \right)^2} = \sqrt{\Delta h R - \frac{(\Delta h)^2}{4}} \quad (3.13)$$

Equation (3.13) may be assumed without great error in a simplified form

$$l_d \approx \sqrt{R \Delta h} \quad (3.14)$$

This simplification is allowable for small bite angles not exceeding  $20^\circ$ . The error when  $\Delta h < 0.08 R$  is less than 1% [30].

The inaccuracy involved in omitting the expression  $(\Delta h)^2/4$  under the root sign, is geometrically equivalent to assuming the chord  $c$  for  $l_d$ , since

$$c^2 = \left( \frac{\Delta h}{2} \right)^2 + \left( \sqrt{\Delta h R - \frac{(\Delta h)^2}{4}} \right)^2 = \Delta h R$$

and hence

$$c = \sqrt{\Delta h R} \quad (3.15)$$

which is of the same form as (3.14).

The angle of bite  $\alpha$  can be also calculated from

$$l_d = R \sin \alpha \quad (3.16)$$

hence

$$\sin \alpha = \frac{l_d}{R} = \frac{\sqrt{\Delta h R - (\Delta h)^2/4}}{R} \approx \frac{\sqrt{R \Delta h}}{R} = \sqrt{\frac{\Delta h}{R}} \quad (3.17)$$

or

$$\tan \alpha = \frac{l_d}{R - \Delta h/2} = \sqrt{\frac{\Delta h R - (\Delta h)^2/4}{R - \Delta h/2}} \approx \frac{\sqrt{R \Delta h}}{R - \Delta h/2} \quad (3.18)$$

For very small angles it can be assumed that  $\sin \alpha$  is equal to the arc, then the angle of bite can be expressed in radians

$$\alpha \approx \sqrt{\frac{\Delta h}{R}} \quad (3.19)$$



The values of  $\alpha$  in radians, as obtained from this formula, can be converted into degrees.

The arc of contact between the rolls and the metal will be calculated from the formula

$$t = \frac{2R\pi\alpha}{360} = \frac{\pi R\alpha}{180} \quad (3.20)$$

Similar formulae can be deduced for the rolling angle  $\varphi$  and thickness  $h$

$$\cos \varphi = 1 - \frac{h-h_2}{D} \quad (3.21)$$

hence

$$h = h_2 + D(1 - \cos \varphi) \quad (3.22)$$

The angle  $\varphi$  is calculated from the relationship

$$\sin \varphi = \frac{x}{R} \quad (3.23)$$

or assuming that sine is equal to the arc (allowable for very small angles)

$$\varphi \approx \frac{x}{R} \quad (3.24)$$

When the rolls have different diameters (Fig. 3.5b) two different cases can occur (formulae derived for flat sections are correct for all other sections) [32].

(1) The rolled stock is not supported. In this case the decisive angle of bite is that of the smaller roll. The larger roll must adapt to these conditions by lowering its angle of contact. This can be expressed

$$R_1 + h_2 + R_2 = R_1 \cos \alpha_1 + h_1 + R_2 \cos \alpha_2$$

Hence

$$\begin{aligned} \Delta h &= h_1 - h_2 = R_1 - R_1 \cos \alpha_1 + R_2 - R_2 \cos \alpha_2 \\ \Delta h &= h_1 - h_2 = R_1(1 - \cos \alpha_1) + R_2(1 - \cos \alpha_2) \end{aligned} \quad (3.25)$$

Hence it follows that the projected length of arc of contact of one roll should be equal to that of the other, i.e.  $R_1 \sin \alpha_1 = R_2 \sin \alpha_2$ , and hence

$$\sin \alpha_1 = \frac{R_2}{R_1} \sin \alpha_2 \quad (3.26)$$

(2) The rolled stock is suitably supported on the side of the thinner roll along a length  $a$ , and the thicker roll can take advantage of its greater length of bite  $l_d$  (Fig. 3.6), where

$$a = R_1 \sin \alpha_1 - R_2 \sin \alpha_2$$

In these formulae the mean calculated roll radius can be substituted

$$R_m = \frac{2R_1R_2}{R_1 + R_2} = \frac{D_1D_2}{D_1 + D_2} \quad (3.27)$$



Each value is the area of the live groove or the cross-sectional area of the bar before or after the pass, divided by its maximum breadth.

For  $D$  the mean working diameter  $D_{mw}$  is substituted, where (Fig. 3.3)

$$D_{mw} = D_t - h_{2m} \quad (3.30)$$

and

$D_t$  = theoretical roll diameter i.e. the distance between the roll axes together with roll gap  $s$ ,

$h_{2m}$  = the mean outgoing thickness of the bar.

$D_t$  can be also calculated if the roll diameters  $D_{r1}$  and  $D_{r2}$  (Fig. 3.3a) are known. These diameters are usually measured at the first collar counting from the largest groove side

$$D_t = \frac{D_{r1}}{2} + \frac{D_{r2}}{2} + s \quad (3.31)$$

On substituting this expression in equation (3.30)

$$D_{mw} = D_{r1} + s - h_{2m} \quad (3.32)$$

is found.

In the above formulae  $s$  is the roll gap measured between the extreme collars of the two rolls during the pass.

If there is a considerable difference between  $D_{r1}$  and  $D_{r2}$ , the mean roll diameter

$$D_{rm} = \frac{D_{r1} + D_{r2}}{2} \quad (3.33)$$

is substituted in equation (3.31) and (3.32) and hence

$$D_{mw} = D_{rm} - h_{2m} + s \quad (3.34)$$

is obtained.

From the nomogram shown in Fig. 3.7 the angle of contact can be determined from equation (3.9). The vertical axes of this nomogram give the values of absolute draught  $h_1 - h_2$  and of roll diameters, and the oblique line gives the values of the contact angle. The angle of contact depends on  $h_1 - h_2$  and the roll diameter  $D$ . The bite angle  $\alpha$  is determined as follows. The value of absolute draught on  $(h_1 - h_2)$  axis is joined by a straight line with the roll diameter  $D$  given on the other vertical axis. This line cuts the oblique line at a point which determines the magnitude of the bite angle.

Two examples of calculation of angle of contact are given.

#### Example 1

A slab  $h_0 b_0 = 200 \times 250$  mm is to be rolled down to  $h_1 = 150$  mm by using steel rolls with  $D = 700$  mm.

The angle of contact is calculated from formula (3.9)

$$\cos \alpha = 1 - \frac{h_1 - h_2}{D} = 1 - \frac{200 - 150}{700} = 1 - 0.0715 = 0.9285$$

hence

$$\alpha = 21^\circ 47'$$

From equation (3.17)

$$\sin \alpha \approx \sqrt{\frac{\Delta h}{R}} = \sqrt{\frac{50}{350}} = \sqrt{0.143} = 0.3780$$

hence

$$\alpha = 22^{\circ}13'$$

Finally, from the nomogram in Fig. 3.7 for  $D = 700$  mm and  $\Delta h = 50$  mm

$$\alpha = 21^{\circ}40'$$
 is obtained

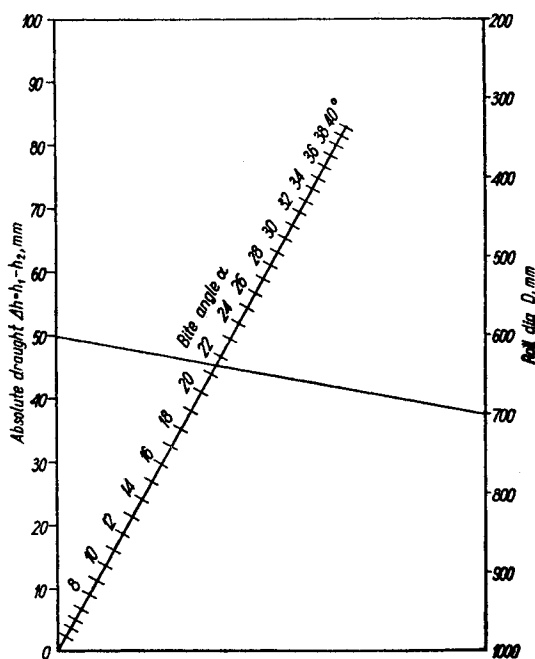


Fig. 3.7. Nomogram for determining bite angle [33]

### Example 2

A square with  $h_1 = b_1 = 17.7$  mm,  $F_1 = 314$  mm<sup>2</sup> is entered flat into a horizontal oval as shown in Fig. 3.8. The dimensions of the outgoing oval are:  $h_2 = 10$  mm,  $b_2 = 30$  mm,  $F_2 = 200$  mm<sup>2</sup>,  $D_t = 300$  mm. The angle of bite is calculated from the highest values, i.e. from equation (3.28)

$$\cos \alpha_{\max} = 1 - \frac{h_{1\max} - h_{2\min}}{D_{\min}} = 1 - \frac{17.7 - 10}{290} = 0.9735$$

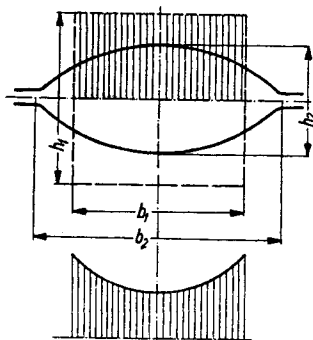


Fig. 3.8. A square entering into oval pass

where

$$D_{\min} = D_t - h_{2\min} = 300 - 10 = 290 \text{ mm}$$

hence

$$\alpha_{\max} = 13^\circ 14'$$

The mean value of angle of bite is calculated from equation (3.29)

$$\cos \alpha_m = 1 - \frac{h_{1m} - h_{2m}}{D_{mw}} = 1 - \frac{17.7 - 6.7}{293.3} = 0.9625$$

where

$$\Delta h_m = h_{1m} - h_{2m} = \frac{F_1}{b_1} - \frac{F_2}{b_2} = \frac{314}{17.7} - \frac{200}{30} = 17.7 - 6.7 = 11 \text{ mm}$$

$$D_{mw} = D_t - h_{2m} = 300 - 6.7 = 293.3 \text{ mm}$$

hence

$$\alpha_m = 15^\circ 45'$$

This shows that the angle of bite calculated from average parameters is greater than that from maximum parameters.

### 3.3. Flow of Metal in Rolling

The first attempt to investigate metal flow during rolling was made by A. Hollenberg in 1883. His method consisted in drilling transverse holes in the bar to be rolled and in inserting rivets to fill them [5].

A specimen prepared in this way was first heated and then rolled between the rolls, then sectioned and inspected. From the curvature of the rivets the degree of deformation of the specimen was determined.

This technique was improved by N. Metz [34] who used screws instead of rivets, and also cut a rectangular network on the 4 sides of the bar. From the deformation of this network after rolling, the degree and mode of deformation of the surface layer was determined. N. Metz carried out rolling tests at 1050°C using specimens from certain steel grades, with square, rectangular and also other cross-sections. Figure 3.9 shows a 30×30 mm specimen after rolling at 1100°C between rolls with diameters 250 and 262 mm, with a draught of 33%. Prior to rolling a rectangular network was cut on specimen faces.

After rolling, the straight lines on the face of the bar curved in a direction opposite to the rolling direction.

The flow of metal under various conditions of rolling will now be considered.

If it is assumed that slipping friction occurs throughout the roll gap, the deformation pattern should be similar to that obtained in parallelepipedal deformation. Actually the metal undergoes various distortions, thus the deformation pattern obtained will be similar to, but not identical with the ideal one. The greater the influence of friction on metal surface, the more pronounced are the distortions it causes, and the metal is forced to move together with the roll surface.

Deformations of this nature were investigated by E. Orowan [35] who used specially prepared plasticine bars (Figs. 3.10a and b) for this purpose. The multi-coloured plasticine bars, 70 mm wide and 25 mm thick, were rolled between wooden rolls of 73 mm dia. and with a draught of 13.5 mm, the rolling being interrupted before the bar had passed completely through the rolls. The vertical plasticine laminae shown in Fig. 3.10a became

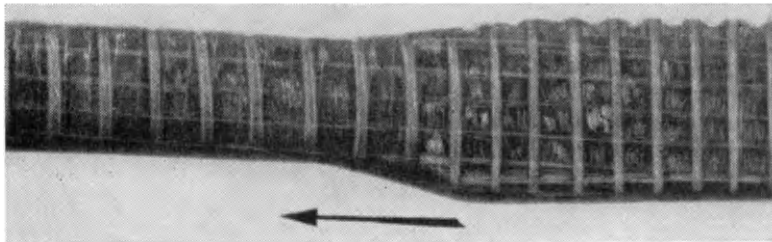


Fig. 3.9. Half section of steel specimen rolled at about 1000°C. The arrow indicates the rolling direction

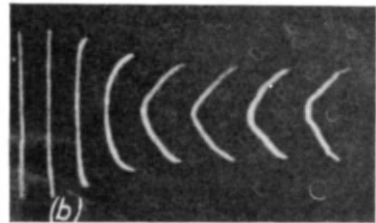
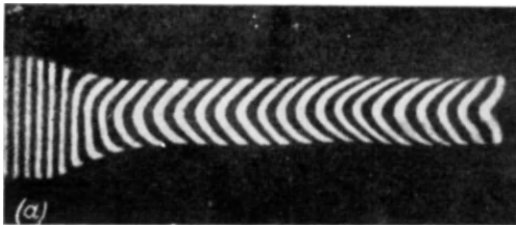


Fig. 3.10. Plasticine bars rolled between wooden rolls 73 mm diameter. Bars — 60–70 mm wide [35]: (a) for  $h_0 = 25$  mm, (b) for  $h_1 = 13.5$  mm

curved in the direction of roll revolution under the action of frictional forces. The coefficient of friction in this case was about 1.0, a value very much higher than that occurring normally in rolling. After rolling the bars were cut in two along their longitudinal centre line.

These experiments show that the plasticine laminae become curved in a direction opposed to that of rolling. In these tests the distance between the laminae was nearly constant along the arc of contact, showing that very little slip takes place at the contact surface. Examination of these specimens shows that near the plane of entry the deformation is localized near the surface of the bar, and that as the material passes between the rolls, the deformation penetrates gradually deeper into the bar.

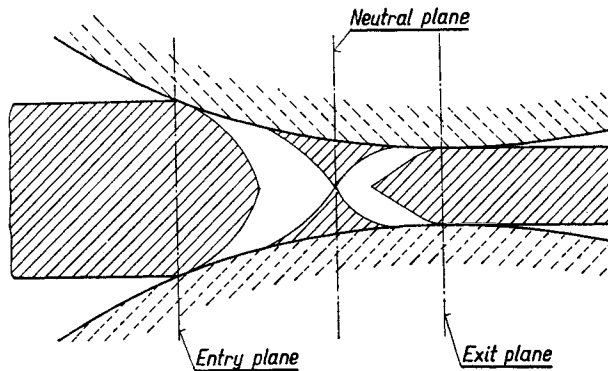


Fig. 3.11. Schematic representation of non-uniform deformation of metal between rolls. Areas of limited plasticity are shown shaded [36]

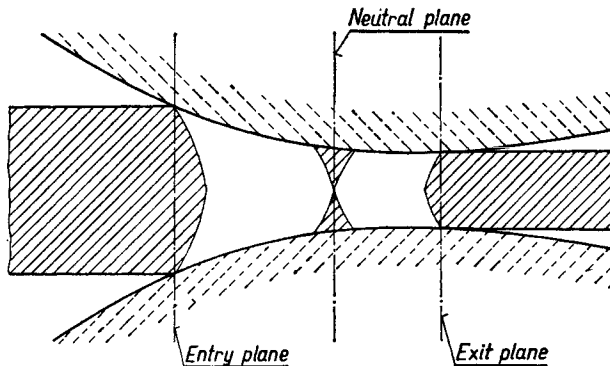


Fig. 3.12. Non-uniform deformation of metal, as in Fig. 3.11, but with small neutral zone [36]

Such a deformation scheme indicates that there are regions of limited plasticity near the plane of entry, as shown diagrammatically in Fig. 3.11. However, regions of plastic deformation, approaching conditions at the neutral plane, cover the major portion of the arc of contact. Fig. 3.12 shows a case in which the region of restricted deformation in the neutral plane is small and consequently it shows a negligible plastic deformation [36].

H. Ford made an analysis of results obtained by Orowan [36] and found that the Orowan experiments can be related to conditions existing in hot rolling when considerable friction and sticking of metal to the rolls occur. In cold rolling, when smooth,

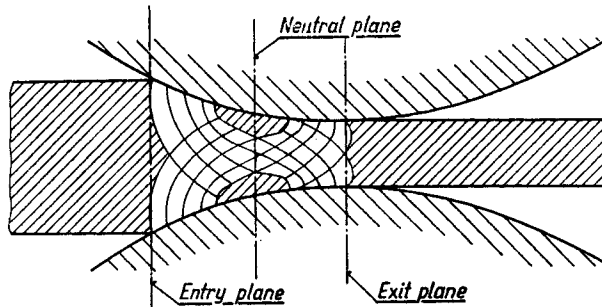


Fig. 3.13. Hypothetical deformation of metal in roll gap during cold rolling with a low coefficient of friction

well lubricated rolls are used, the phenomenon of sticking seems to be eliminated, but with non-parallelepipedal deformation slipping appears over the whole length of arc of contact, as shown diagrammatically in Fig. 3.13. If there were no influence of friction on metal surface, the deformation would be closer to parallelepipedal with all the edges remaining straight, parallel and vertical. The greater the influence of friction, the greater is the deformation of the bar; especially the side faces of the bar restricted by the roll barrel become bellied as shown in Fig. 2.25.

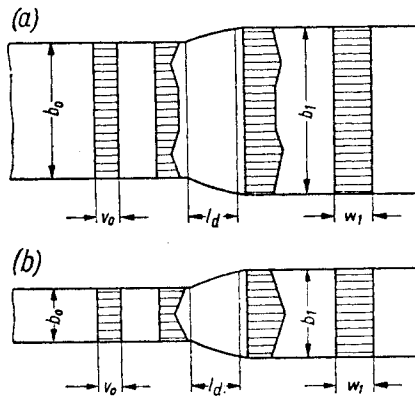


Fig. 3.14. Velocity of flow in rolling a narrow and wide flat: (a) flat bar, (b) square bar

Figure 3.14 represents the speed of metal flow in cold rolling of a narrow flat and of a wide one.

### 3.4. Interdependence of Draught, Elongation and Spread of Metal in Hot Rolling

The increase in breadth of the bar during rolling is called spread.

In hot rolling of wide strip or sheet spread is negligible. This is due to the fact that the metal is compressed by cylindrical rolls, and hence the effective frictional



resistance in the direction of rolling is smaller than that in transverse direction, thus encouraging the flow of metal in the direction of rolling.

Figures 3.15a and b show the projected contact areas obtained in rolling a narrow and a wide bar, respectively [32]. Particles of metal between two points denoted by 4 can displace more easily in the direction 10 than in the direction 11 and 9, because in the direction of rolling they have to overcome a small surface friction between the bar and the rolls. The same applies but to a somewhat lesser extent, to particles of metal near points 3 and 5.

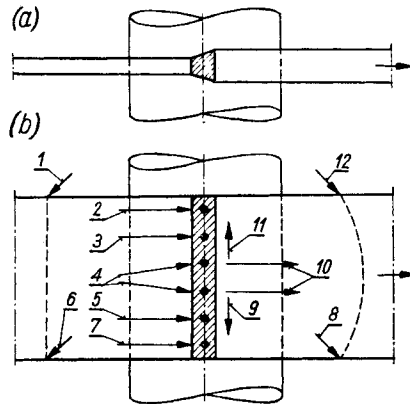


Fig. 3.15. Projected contact area in rolling: (a) narrow bar, (b) wide bar

For particles of metal near points 2 and 7 the frictional resistance is not greater in the direction 9 and 11 than in the direction 10. These particles could spread freely if they were not bound by the central particles 3-4-5 which force them to elongate. Thus, the elongation of the sides of the bar is due not to the compressive effect of the rolls but to their bond with the central particles of the bar.

At the same time the sides inhibit the lateral flow of the central part of the bar, so that only elongation occurs, while the central portions of the bar exert a pull on the sides, thus inhibiting their tendency to spread. The straight line 1-6 marked on the bar becomes curved after rolling in the form of the curve shown as 8-12.

In rolling narrow bars the lateral spreading of metal may be much greater since the lateral friction is not greater than that inhibiting the flow of metal particles in the longitudinal direction.

Little spread occurs when the projection of the contact area is short in the direction of rolling and wide transversely to that direction.

Up till now the principal dimensions of a groove were calculated as follows:

(1) groove depth was found from formulae for relative or percentage draught, where the magnitude of the draught used resulted from the chosen roll pass design system;

(2) groove width was determined from known formulae for lateral spread.

The amount of spread depends on many factors, such as the draught, temperature, rolling speed, initial shape of metal, type of steel, ratio of bar diameter to roll diameter, condition of roll and metal surface, etc. Many investigators attempted to consider all these factors simultaneously and derive a general formula for spread.

The formulae given here relate to free spread, i.e. unrestricted by the side faces of the pass. Siebel's formula has the form [30]

$$\Delta b = k \frac{\Delta h}{h_0} l_d = k \frac{\Delta h}{h_0} \sqrt{R \Delta h} \quad (3.35)$$

where

$R$  = roll radius, mm,

$k$  = coefficient: 0.36 for copper, 0.45 for aluminium, 0.33 for lead and 0.35 for steel.

Better results are obtained from Tafel and Sedlaczek's formulae [37] which especially in the unsimplified form are in good agreement with the practical results for narrow strip

$$\Delta b = \frac{\Delta h b_1 \sqrt{b_1 R}}{c (b_1^2 + h_1 h_2)} \quad (3.36)$$

where  $c = 3.0$  for mild steel, 0.75 for aluminium alloys, 1.45 for magnesium alloys at  $v = 3$  m/sec

For other rolling speeds Sedlaczek's formula gives the following values:

| $v, \text{ m/sec}$ | 0    | 0.5  | 1.5  | 3.0 | 5.0 | 7.5 | 10   | 15   |
|--------------------|------|------|------|-----|-----|-----|------|------|
| $c_1$              | 1.62 | 1.37 | 1.15 | 1.0 | 0.9 | 0.8 | 0.76 | 0.69 |

where the constant  $c$  in (3.36) is multiplied by  $c_1$  for appropriate rolling speeds.

S. Koncewicz compared the values obtained from different formulae with measured values and found that of the simple formulae, good results are given by Zolotnikov's formula [38] in the form

$$\Delta b = 0.66 \frac{\Delta h}{h_1 + h_2} l_d \quad (3.37)$$

where

$$l_d \approx \sqrt{R \Delta h}$$

This formula is similar to (3.35).

If the coefficient of friction  $f$  is properly chosen, Bakhtinov's formula [38] can be used, where

$$\Delta b = 1.15 \frac{\Delta h}{2 h_1} \left( l_d - \frac{\Delta h}{2 f} \right) \quad (3.38)$$

From recently published literature Tselikov's formula may be quoted [151] in the form

$$\Delta b = c \Delta h \left( 2 \sqrt{\frac{R}{\Delta h} - \frac{1}{f}} \right) \frac{1 - \varepsilon_h}{\varepsilon_h^2} \left[ (1 - \varepsilon_h) \log_e \frac{1}{1 - \varepsilon_h} - \frac{\varepsilon_h}{1 - \varepsilon_h} \left( 1 - \frac{3}{2} \varepsilon_h \right) \right] \quad (3.39)$$

In this equation  $c$  is a factor dependent on the ratio of breadth of specimen to arc of contact, i.e.

$$\frac{b_1}{\sqrt{R \Delta h}}$$

and varies from 0.5 to 1.0 (Fig. 3.16).

For practical application equation (3.39) can be reduced to the following form:

$$\Delta b = c \Delta h \left( 2 \sqrt{\frac{R}{\Delta h} - \frac{1}{f}} \right) \varphi \left( \frac{\Delta h}{h_1} \right) \quad (3.40)$$

where

$$\varphi\left(\frac{\Delta h}{h_1}\right) = \varphi(\varepsilon_h) = \frac{1-\varepsilon_h}{\varepsilon_h^2} \left[ (1-\varepsilon_h) \log_e \frac{1}{1-\varepsilon_h} - \frac{1}{1-\varepsilon_h} \left(1 - \frac{3}{2}\varepsilon_h\right) \right] \quad (3.41)$$

The relation between  $\varphi(\varepsilon_h)$  and  $\varepsilon_h$  is shown in Fig. 3.17 where the equation (3.41) is plotted in a simplified form as

$$\varphi(\varepsilon_h) = 0.138 \varepsilon_h^2 + 0.328 \varepsilon_h \quad (3.42)$$

This equation can be used for practical calculations.

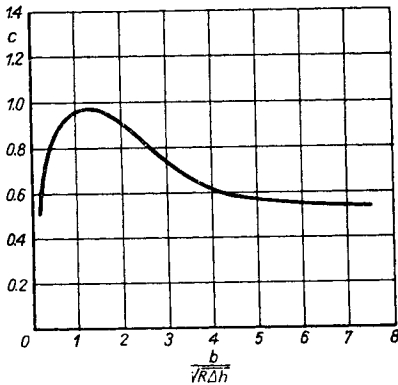


Fig. 3.16. Variation of coefficient  $c$  with ratio of initial width to projected arc of contact [151]

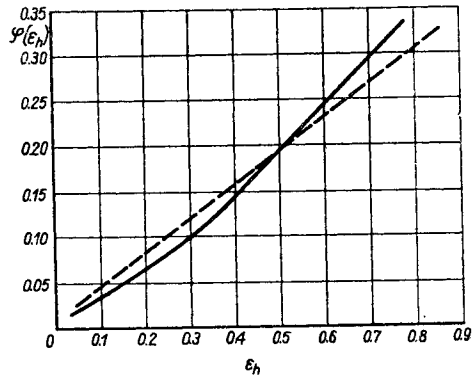


Fig. 3.17. Variation of  $\varphi(\varepsilon_h)$  with the relative draught,  $\varepsilon_h$ , [151]

Formulae already published for the determination of spread in hot rolling of flat steel bars did not give sufficient accuracy for practical rolling conditions, or due to their complicated form (e.g. Ekelund's formula) their practical application was questionable. Furthermore, these formulae only allowed of very approximate determination of the coefficient of elongation. This was due to the fact that the phenomena of spread and elongation were considered independently of each other, as certain absolute or relative values dependent on draught and other factors, and the direct correlation between coefficients of draught, elongation and spread was not taken into account.

These three phenomena, draught, elongation and spread, all occurring in every plastic working process and thus in rolling, are interdependent and closely related to each other.

Any compression applied to the metal in rolling causes it to elongate in the direction of rolling and to spread in the transverse direction. The factors which govern the ratio of elongation to spread are: the degree of draught applied to the metal, the initial shape of stock, the roll diameter, the chemical composition of metal, the rolling temperature and also the influence of friction between roll and metal, and the rolling speed. During rolling of metal not on the smooth barrel but in the grooves, the flow of metal is also affected by the shape of the rolled bar and the pass contour.

In considering this problem, first of all the effect was investigated of initial shape of stock and of roll diameter on draught, elongation and spread of mild steel for practical rolling temperature ranges and suitable sections.

Next the influence of temperature, rolling speed, chemical composition of steel, type of rolls and the condition of their surface [32, 39] was also taken into account.

During rolling the volume of metal for practical purposes remains constant, i.e. as in formula (2.1)  $V_1 = V_2 = \dots = V_i = \dots = V_n$  from which follows the constant volume condition (2.8).

$$\gamma\beta\lambda = 1$$

By definition the coefficient of draught

$$\gamma = \frac{h_2}{h_1} = \left(1 - \frac{\Delta h}{h_1}\right)$$

and is always greater than zero and less than 1, since it is equal to 1 when  $\Delta h/h_1 = 0$ , i.e. when there is no draught and no rolling takes place.

The coefficient of elongation

$$\lambda = \frac{F_1}{F_2} = \frac{l_2}{l_1} = \left(1 + \frac{\Delta l}{l_1}\right)$$

which is always greater than 1 since the denominator is always smaller than the numerator. If  $F_2 = F_1$  or  $l_2 = l_1$  then there is no rolling since no reduction or elongation takes place.

The coefficient of spread

$$\beta = \frac{b_2}{b_1} = \left(1 + \frac{\Delta b}{b_1}\right)$$

is usually greater than 1 since  $b_2 > b_1$ . However, it can occur that  $\beta$  will take values from 0 to 1 i.e. either  $b_2 = b_1$  and no spread takes place, or  $b_2 < b_1$ , i.e. the bar rolled becomes narrower.

In a continuous rolling mill it may occur that due to differences in exit rolling speed of individual roll stands the bar is subjected to tension. A similar situation occurs in cold rolling mills where coilers are used to produce tension in the metal. Such cases have been omitted in further considerations.

It is known that during hot rolling, under limiting conditions  $\beta$  and  $\lambda$  can take values equal to 1. Then equation (2.8) takes the form

$$\beta = 1/\gamma \quad \text{for } \lambda = 1 \quad (3.43)$$

$$\lambda = 1/\gamma \quad \text{for } \beta = 1 \quad (3.44)$$

These equations have a hyperbolic form.

Spread is now considered relative to its dependence on the two most important factors, i.e. initial shape of the rolled stock and roll diameter. From Fig. 3.15a and b it can be seen that there is little difference between the breadth of a thin flat bar before and after rolling. A square bar, however spreads considerably in the rolls. A test bar will spread less when rolled with small diameter rolls than with rolls of large diameter, but its elongation in the direction of rolling will be correspondingly higher. To take into account the initial shape of the rolled stock, a form factor

$$\delta_w = \frac{b_1}{h_1}$$

and a roll factor or thickness ratio

$$\varepsilon_w = \frac{h_1}{D}$$

have been introduced.

These considerations apply only to plain sections, i.e. square and rectangular bars rolled between plain cylindrical rolls (see Fig. 3.1). The question of complex sections will be discussed later.

The solution of this problem is found in the form of a final equation [32]

$$\beta = \gamma^{-W} \quad (3.45)$$

where

$$-W = -10^{-1.269 \varepsilon_w^{0.56} \delta_w} \quad (3.46)$$

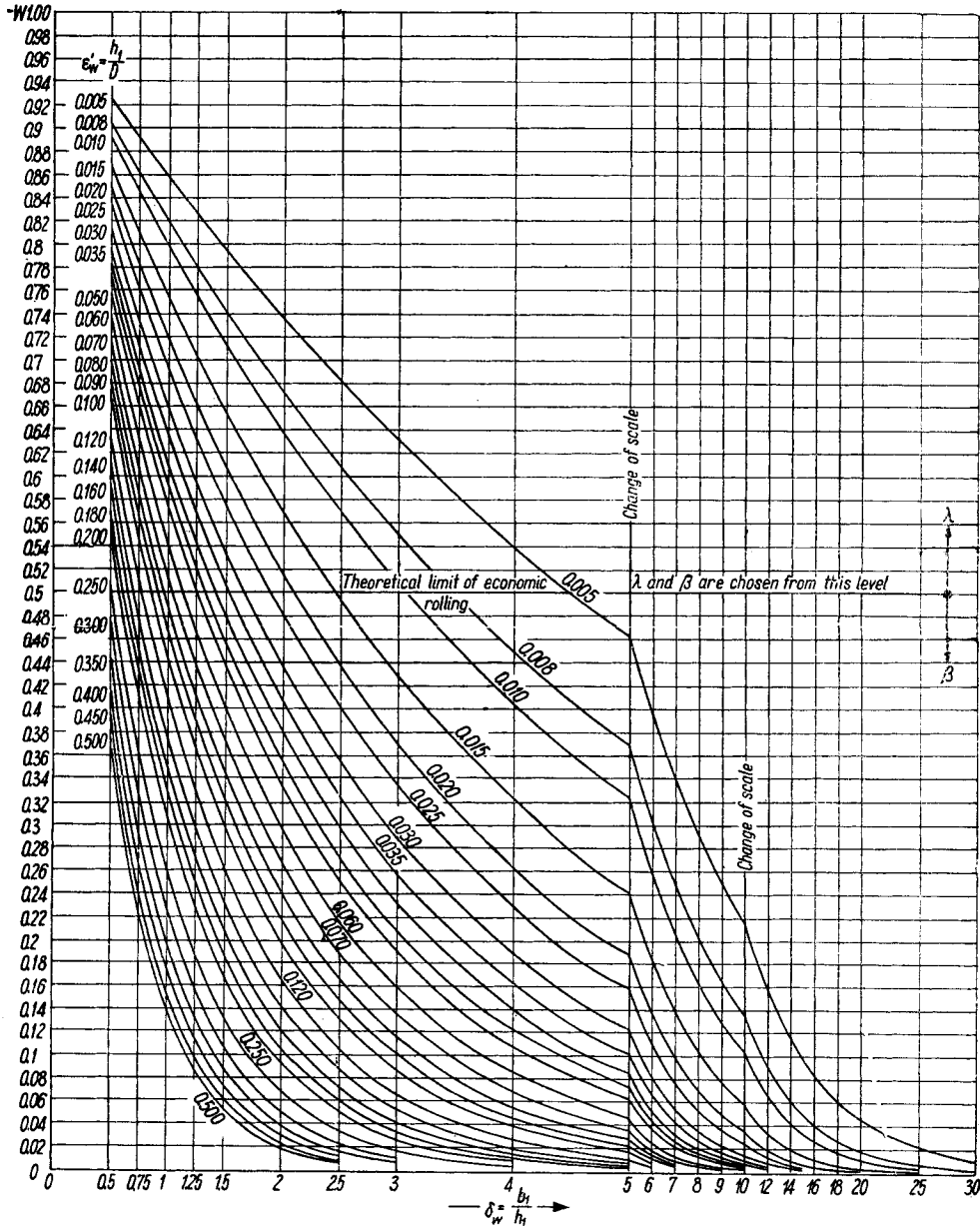


Fig. 3.18. Relation between  $W$  in the function  $\gamma^{-W}$  and  $\delta_w$  for different values of  $\varepsilon_w$ .

Substituting from  $\gamma\beta\lambda = 1$  into this equation gives

$$\lambda = \gamma^{-(1-W)} \quad (3.47)$$

or combining (3.45) and (3.47) a useful formula is found from which  $\beta$  can be determined from known  $\lambda$

$$\beta = \lambda^{W/(1-W)} \quad (3.48)$$

The coefficients of spread and elongation are so closely related in any given pass that by calculating  $-W$  for spread, the elongation is determined simultaneously.

To facilitate the application of the somewhat complicated formula (3.45), values of  $-W$  for various values of  $\delta_w$  and  $\varepsilon_w$  have been calculated.

Figure 3.18 shows a nomogram from which values of  $-W$  can be read off or interpolated. To simplify practical calculations, two further diagrams Figs. 3.19 and 3.20 are given, from which the coefficients

$$\beta = \gamma^{-W}$$

$$\lambda = \gamma^{-(1-W)}$$

or

$$\beta = \lambda^{W/(1-W)}$$

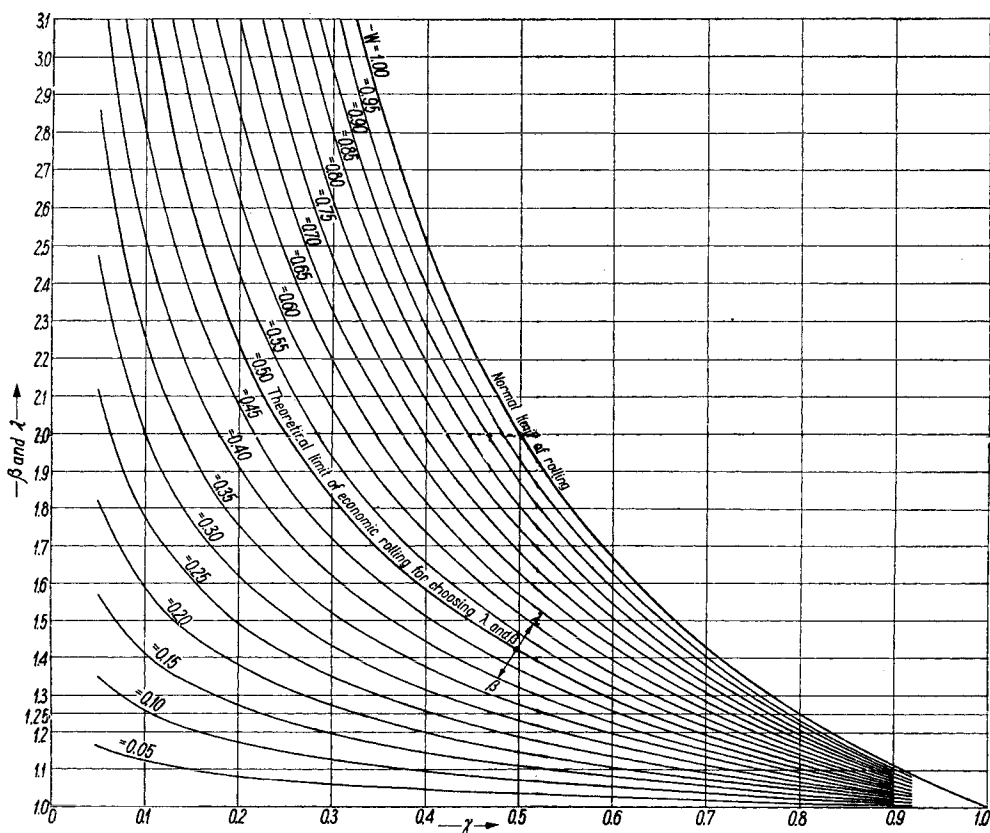


Fig. 3.19. Relation between  $\beta$ ,  $\lambda$  and  $\gamma$  for various values of the exponent  $-W$  for the whole rolling range

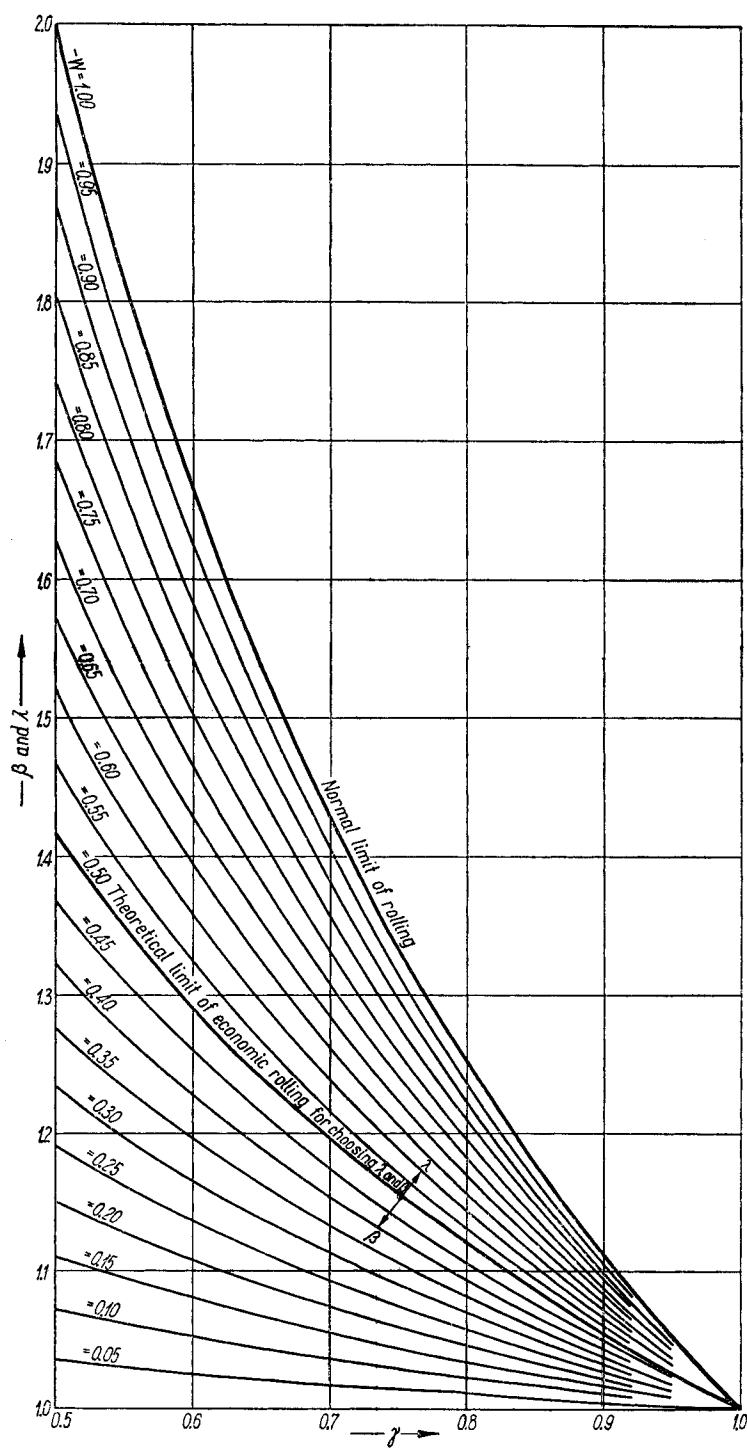


Fig. 3.20. Relation between  $\beta$ ,  $\lambda$  and  $\gamma$  for  $\gamma$  0.5–1.0 for various values of  $-W$ . Enlarged portion of Fig. 3.19 for the most often used range

can be read off for known values of  $-W$  and  $\gamma$ . Intermediate values can be found by interpolation. Practically therefore the calculation is confined to calculating the coefficients

$$\gamma = \frac{h_2}{h_1} \quad \delta_w = \frac{b_1}{h_1} \quad \varepsilon_w = \frac{h_1}{D}$$

With known  $\delta_w$  and  $\varepsilon_w$ ,  $-W$  is read off from Fig. 3.18. From Figs. 3.19 and 3.20 values of  $\beta$  and  $\lambda$  are found. Thus, all the required values are found.

The nomograms shown in Figs. 3.18, 3.19 and 3.20 apply only to cases where the reduction in height is uniformly distributed over the whole cross-sectional area, as in rolling of square or rectangular bars between plain rolls.

It is known that  $\beta$  and  $\lambda$  are closely related. Part of the metal displaced by draught causes spread and part causes elongation. If there is no spread, the whole volume of metal displaced by draught goes to provide elongation. In this case  $\lambda = 1/\gamma$ , that is a boundary condition which can be regarded as ideal for every roll pass design. In the opposite case where there is no elongation, the whole volume of metal displaced by draught goes to provide spread, and the boundary condition is expressed as  $\beta = 1/\gamma$ .

Between these two boundary conditions there are many intermediate cases in which  $\beta$  and  $\lambda$  take values varying from 1 to  $1/\gamma$ .

As long as  $\lambda > \beta$ , the deformation results chiefly in elongation, and the roll pass design and hence the rolling process can be regarded as economical.

If  $\beta > \lambda$ , the deformation results chiefly in spread of metal, which is undesirable from the economic aspect for rolling and roll pass design.

The case in which  $\beta = \lambda$ , i.e. the plastic deformation results in equal spread and elongation is called the theoretical limit of economic rolling. In practice there are cases in which the metal is rolled under unfavourable conditions where  $\beta > \lambda$ , e.g. in rod mills.

The theoretical limit of economic rolling occurs at  $-W = -0.5$ , since then

$$\beta = \gamma^{-0.5}$$

$$\lambda = \gamma^{-(1-0.5)} = \gamma^{-0.5}$$

i.e.  $\beta = \lambda$ , corresponding to equal deformation coefficients and equal deformations.

If  $\delta_w$  and  $\varepsilon_w$  are known, the rolling conditions can be chosen with reference to Fig. 3.18 such that the theoretical limit of economic rolling is not exceeded, which is indicated by the value of  $-W$ . This limit is also plotted in Figs. 3.19 and 3.20, to assist in the choice of appropriate values of  $\beta$  and  $\lambda$ .

To give an indication of maximum values taken by  $\beta$  and  $\lambda$  in a standard rolling process, limiting values are tabulated in Table 3.2.

#### Example

A flat is rolled from  $h_1 = 27.5$  mm and  $b_1 = 115.4$  mm down to  $h_2 = 16.7$  mm between rolls of 598 mm diameter. It is required to find  $\beta$  and  $\lambda$

$$\gamma = \frac{h_2}{h_1} = \frac{16.7}{27.5} = 0.607$$

$$\delta_w = \frac{b_1}{h_1} = \frac{115.4}{27.5} = 4.20$$

$$\varepsilon_w = \frac{h_1}{D} = \frac{27.5}{598.0} = 0.046$$



The value of  $-W$  is taken from Fig. 3.18 where for  $\delta_w = 4.20$  and  $\varepsilon_w = 0.046$ ,  $-W = 0.110$  can be read off;  $\beta$  is then found from Fig. 3.20. where it can be read off as 1.054

$$b_2 = b_1 \times 1.054 = 115.4 \times 1.054 = 121.63 \text{ mm}$$

$$-(1-W) = -(1-0.110) = -0.890$$

$\lambda$  is taken from Fig. 3.20, where the value 1.566 is read off. Checking

$$\gamma \beta \lambda = 0.607 \times 1.054 \times 1.566 = 1.0019$$

TABLE 3.2  
COEFFICIENTS  $\beta$  AND  $\lambda$  FOR NORMAL ROLLING CONDITIONS

| $\gamma$ | $\beta_m = \lambda_m = \sqrt{1/\gamma}$ | $\beta = \lambda = 1/\gamma$ | $\gamma$ | $\beta_m = \lambda_m = \sqrt{1/\gamma}$ | $\beta = \lambda = 1/\gamma$ |
|----------|-----------------------------------------|------------------------------|----------|-----------------------------------------|------------------------------|
| 0        |                                         |                              |          |                                         |                              |
| 0.05     | 4.4721                                  | 20.0000                      | 0.55     | 1.3484                                  | 1.8181                       |
| 0.10     | 3.1623                                  | 10.0000                      | 0.60     | 1.2910                                  | 1.6667                       |
| 0.15     | 2.5820                                  | 6.6667                       | 0.65     | 1.2403                                  | 1.5384                       |
| 0.20     | 2.2361                                  | 5.0000                       | 0.70     | 1.1952                                  | 1.4285                       |
| 0.25     | 2.0000                                  | 4.0000                       | 0.75     | 1.1547                                  | 1.3333                       |
| 0.30     | 1.8257                                  | 3.3333                       | 0.80     | 1.1180                                  | 1.2500                       |
| 0.35     | 1.6903                                  | 2.8571                       | 0.85     | 1.0846                                  | 1.1764                       |
| 0.40     | 1.5811                                  | 2.5000                       | 0.90     | 1.0541                                  | 1.1111                       |
| 0.45     | 1.4907                                  | 2.2222                       | 0.95     | 1.0261                                  | 1.0526                       |
| 0.50     | 1.4142                                  | 2.0000                       | 1.00     | 1.0000                                  | 1.0000                       |

To obtain a greater accuracy from equations (3.45) to (3.47), corrections are introduced for practical temperature of rolling, rolling speeds from 0.4 to 17 m/sec, taking into account condition of roll surface and different types and qualities of steel. Then the formula for the coefficient of spread will take the form, [32, 39]

$$\beta' = a c d f \gamma^{-W} \quad (3.49)$$

and for the coefficient of elongation

$$\lambda' = \frac{1}{a} \frac{1}{c} \frac{1}{d} \frac{1}{f} \gamma^{-(1-W)} \quad (3.50)$$

where

- $a$  = correction factor depending on practical temperature of rolling of steel,
- $c$  = correction factor depending on rolling speed,
- $d$  = correction factor depending on grade of steel,
- $f$  = correction factor depending on type of rolls and the condition of their surfaces.

The correction factor  $a$  for practical rolling temperature is taken equal to 1.005 for the range 750–900°C, at 950°C and above  $a$  is assumed equal to 1.000 [32], [39].

The correction factor  $c$  for rolling speeds from 0.4 to 17 m/sec can be obtained from the following formula relative to  $\gamma$  [39]

$$c = (0.002958 + 0.00341 \gamma) v + 1.07168 - 0.10431 \gamma \quad (3.51)$$

where

- $\gamma$  = coefficient of draught,
- $v$  = rolling speed, m/sec.

From the nomogram in Fig. 3.21 the values of  $c$  can be easily obtained.

The correction factor  $d$  depending on steel grade is taken from Table 3.3. The correction factor  $f$ , depending on surface and material of rolls is taken to be 1.020 for cast iron and rough steel rolls, 1.000 for chilled and smooth steel rolls and 0.980 for ground steel rolls [39].

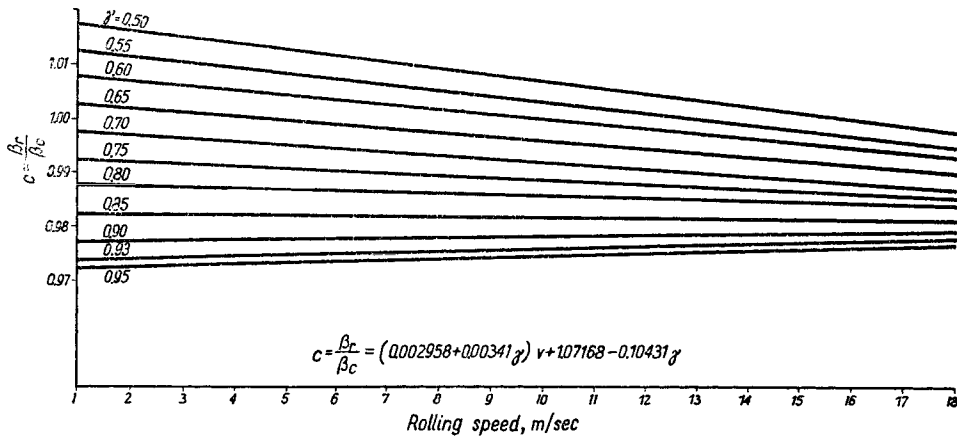


Fig. 3.21. Variation of correction factor  $c$  with rolling speed of different values for draught coefficient

Formulae (3.49) to (3.51) have been derived for average rolling conditions i.e. average temperature, rolling speed and the condition of roll surface. The introduction of correction factors  $a$ ,  $c$ ,  $d$  and  $f$  gives useful results only when deviations from average rolling conditions are marked, e.g. rolling temperatures too low, rolling speeds higher or lower than average, properties of the rolled steels different from those of mild steel, roll surfaces rougher or smoother than rolls commonly used. Under average rolling

TABLE 3.3  
CHEMICAL ANALYSIS OF STEELS

| Steel grade No. | Composition, % |        |      |      |      |      | Correction factor<br>$d = \beta_{act}/\beta_{calc}$ | Designation of steel                 |
|-----------------|----------------|--------|------|------|------|------|-----------------------------------------------------|--------------------------------------|
|                 | C              | Si     | Mn   | Ni   | Cr   | W    |                                                     |                                      |
| 1               | 0.06           | traces | 0.22 | —    | —    | —    | 1.00000                                             | Thomas steel                         |
| 2               | 0.20           | 0.20   | 0.50 | —    | —    | —    | 1.02026                                             | C. 25.61 steel                       |
| 3               | 0.30           | 0.25   | 0.50 | —    | —    | —    | 1.02338                                             | C. 35.61 steel                       |
| 5               | 1.04           | 0.30   | 0.45 | —    | —    | —    | 1.00734                                             | } tool steel                         |
| 7               | 1.25           | 0.20   | 0.25 | —    | —    | —    | 1.01454                                             |                                      |
| 8               | 0.35           | 0.50   | 0.60 | —    | —    | —    | 1.01636                                             | } manganese steel for heat treatment |
| 10              | 1.00           | 0.30   | 1.5  | —    | —    | —    | 1.01066                                             |                                      |
| 11              | 0.50           | 1.70   | 0.70 | —    | —    | —    | 1.01410                                             | spring steel                         |
| 12              | 0.50           | 0.40   | 24.0 | —    | —    | —    | 0.99741                                             | } abrasion resistant                 |
| 13              | 1.20           | 0.35   | 13.0 | —    | —    | —    | 1.00887                                             |                                      |
| 21              | 0.06           | 0.20   | 0.25 | 3.50 | 0.40 | —    | 1.01034                                             | } manganese steel carburizing steel  |
| 24              | 1.30           | 0.25   | 0.30 | —    | 0.50 | 1.80 | 1.00902                                             |                                      |
| 27              | 0.40           | 1.90   | 0.60 | 2.00 | 0.30 | —    | 1.02719                                             | } alloy tool steel                   |

conditions it is advisable to use equations (3.45) to (3.47) which give perfectly satisfactory results.

To obtain good results, all the correction factors have to be used together since they have been derived from the influence of rolling speed on the coefficient of spread  $\beta$ . None of them should be used without taking into account the correction for rolling speed, with the exception of  $d$  [39].

#### Example

A flat bar from 0.25% C steel is to be rolled at 850°C between chilled cast iron rolls with the speed of 10 m/sec. The correction factors can be found: the correction factor for temperature is  $a = 1.005$ , the correction factor for rolling speed (from Fig. 3.19) is  $c = 0.999$  for  $\gamma = 0.607$ , the correction factor  $d$  for steel composition is taken from Table 3.3 where  $d = 1.020$  for  $C = 0.25$ , the correction factor for chilled rolls  $f = 1.000$ .

Thus, the corrected coefficient of spread from the previous example

$$\beta' = 1.054 a c d f = 1.054 \times 1.005 \times 0.999 \times 1.020 \times 1.000 = 1.0794$$

and the corrected coefficient of elongation

$$\lambda' = 1.566 \frac{1}{a} \frac{1}{c} \frac{1}{d} \frac{1}{f} = 1.566 \frac{1}{1.005} \times \frac{1}{0.999} \times \frac{1}{1.020} \times \frac{1}{1.000} = 1.5292$$

Hence

$$b_2 = b_1 \beta' = 115.4 \times 1.0794 = 124.6 \text{ mm}$$

checking

$$\gamma \beta' \lambda' = 0.607 \times 1.0794 \times 1.5294 = 1.002$$

This method of calculation using correction factors given in Table 3.3 does not give satisfactory results for alloy steels. On the basis of numerous experiments a different result was found giving the final form of equation (3.45) for spread of alloy steels, [40, 41]

$$\beta_{\text{act}} = d \gamma^{-w} \quad (3.52)$$

The value of the correction factor  $d$  is taken from the appropriate nomograms depending on the rolling conditions for the given steel. The value of  $\gamma^{-w}$  is calculated similarly as for plain steel. To find the coefficient of elongation in rolling alloy steels, the reciprocal of  $d$  is used

$$\lambda = \frac{1}{d} \gamma^{(w-1)} \quad (3.53)$$

The correction factor  $d$  must be used only when rolling steels for which a diagram has been evolved allowing for coefficient of draught, the roll diameter and rolling temperature.

By taking the correction factor  $d$  from the nomograms, the introduction of correction factors for temperature  $a$ , rolling speed  $c$  and condition of roll surface  $f$  becomes unnecessary.

The values of  $d$  for the 13 steels given in Table 3.4 have been determined from rolling tests carried out on 50×50×500 mm bars between rolls with diameters 265, 400 and 414 mm [40]. The results show the value of  $d$  plotted against coefficient of draught in Figs. 3.22a, b and c against rolling temperature in Figs. 3.23a, b and c.

Calculations of results show that it is not possible to use a constant (mean) value of  $d$  for a given steel, without taking into account draught and rolling temperature.

For example, in rolling No. 5 acid resisting steel at a constant temperature with a speed  $v = 1.25\text{--}1.4$  m/sec and a varying coefficient of draught rising from 0.68 to 0.85, for an average temperature of  $975^\circ\text{C}$ , the values  $\gamma = 0.674$ ,  $\beta_{\text{act}} = 1.153$  and  $d = 1.022$  (Fig. 3.22a) were found.

TABLE 3.4  
SELECTED STEEL GRADES

| No. | Steel grade | Composition, % |      |      |       |       |       |       |      |      |
|-----|-------------|----------------|------|------|-------|-------|-------|-------|------|------|
|     |             | C              | Mn   | Si   | P     | S     | Cr    | Ni    | V    | Ti   |
| 1   | TP2         | 0.38           | 0.60 | 0.30 | 0.020 | 0.018 | 0.64  | 1.55  |      |      |
| 2   | PS2         | 0.47           | 0.73 | 1.69 | 0.050 | 0.026 |       |       |      |      |
| 3   | T55         | 0.56           | 0.66 | 0.30 | 0.021 | 0.030 |       |       |      |      |
| 4   | KC12        | 0.11           | 0.53 | 0.21 | 0.015 | 0.029 | 11.58 | 0.39  |      | 0.26 |
| 5   | KP2         | 0.12           | 0.53 | 0.56 | 0.010 | 0.020 | 18.23 | 8.85  |      | 0.50 |
| 6   | KC6         | 1.39           | 0.50 | 0.21 | 0.014 | 0.016 | 1.39  | 0.23  |      |      |
| 7   | PCS         | 0.50           | 0.40 | 0.82 | 0.018 | 0.008 | 1.05  | 0.42  |      |      |
| 8   | TMS1        | 0.36           | 1.00 | 1.34 | 0.022 | 0.012 | 0.33  | 0.99  |      |      |
| 9   | TC4         | 1.07           | 0.34 | 0.28 | 0.021 | 0.010 | 1.44  | 0.23  |      |      |
| 10  | T85         | 0.80           | 0.53 | 0.30 | 0.021 | 0.020 | 0.19  |       |      |      |
| 11  | KNS         | 0.17           | 2.10 | 1.02 | 0.007 | 0.012 | 25.00 | 18.37 | 0.24 |      |
| 12  | PCV         | 0.48           | 0.69 | 0.28 | 0.017 | 0.013 | 1.05  | 0.28  | 0.20 |      |
| 13  | M34         | 0.12           | 0.29 | —    | 0.017 | 0.033 |       |       |      |      |

Lowering the value of  $\gamma$  to 0.843, the coefficient  $\beta_{\text{act}}$  became 1.05 and the correction factor  $d = 0.994$ .

The correction factor  $d$  decreases gradually for this steel but this is not the case when rolling several other steels. This is caused probably by the variation of the deformation resistance, depending on the chemical composition of the steel and the friction conditions on the roll surface.

In rolling this same steel No. 5 under different conditions, i.e. at  $\gamma = 0.735 =$  constant, between rolls of fixed diameter, at constant rolling speed and different rolling temperatures, the following values of  $d$  were found:

$$t_a = 1040^\circ\text{C} \quad d = 1.018$$

$$t_b = 960^\circ\text{C} \quad d = 1.015$$

$$t_c = 830^\circ\text{C} \quad d = 1.012$$

For this same steel rolled at constant temperature of  $975^\circ\text{C}$ , between rolls of fixed diameter, at a constant rolling speed and different coefficients of draught, the correction factor  $d$  had the following values:

$$\gamma_1 = 0.668 \quad d = 1.022$$

$$\gamma_2 = 0.736 \quad d = 1.012$$

$$\gamma_3 = 0.796 \quad d = 1.0025$$

$$\gamma_4 = 0.841 \quad d = 0.996$$

From the comparison of the above results it can be concluded that the influence of temperature in rolling steel No. 5 is less important than that of draught.

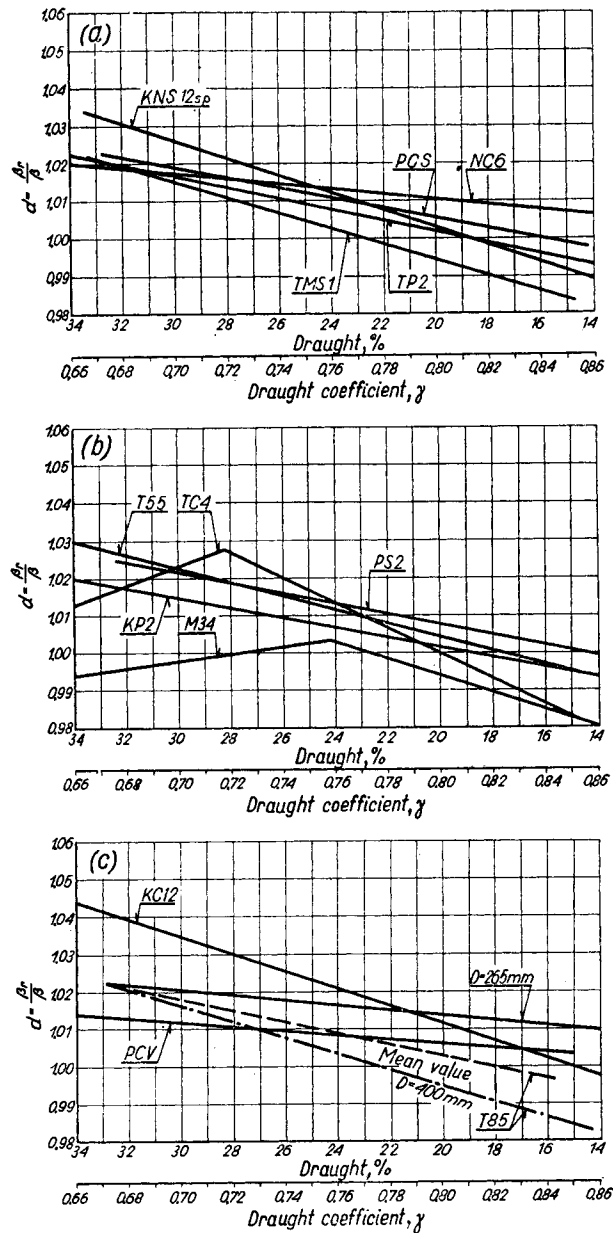


Fig. 3.22. Variation of correction factor  $d$  with draught coefficient for different steels rolled at constant temperature: (a) steel TP2, TMS1, KNS12sp, PCS, NC6 (b) steel T55, KP2, M34, PS2, TC4 (c) steel KCI2, PCV, T85

From Fig. 3.22 it can be seen that with increase of coefficient of draught,  $d$  decreases, following a straight line. Only the curves for steels No. 13 and 9 show clearly marked break-off points and a maximum value. However, no clear-cut influence of roll diameter was found, with the exception of No. 10 steel, for which separate curves have been plotted for roll diameter  $D = 268$  and  $400$  mm.

On the other hand, the influence of rolling temperature is more varied, as can be seen from Figs. 3.23a, b and c.

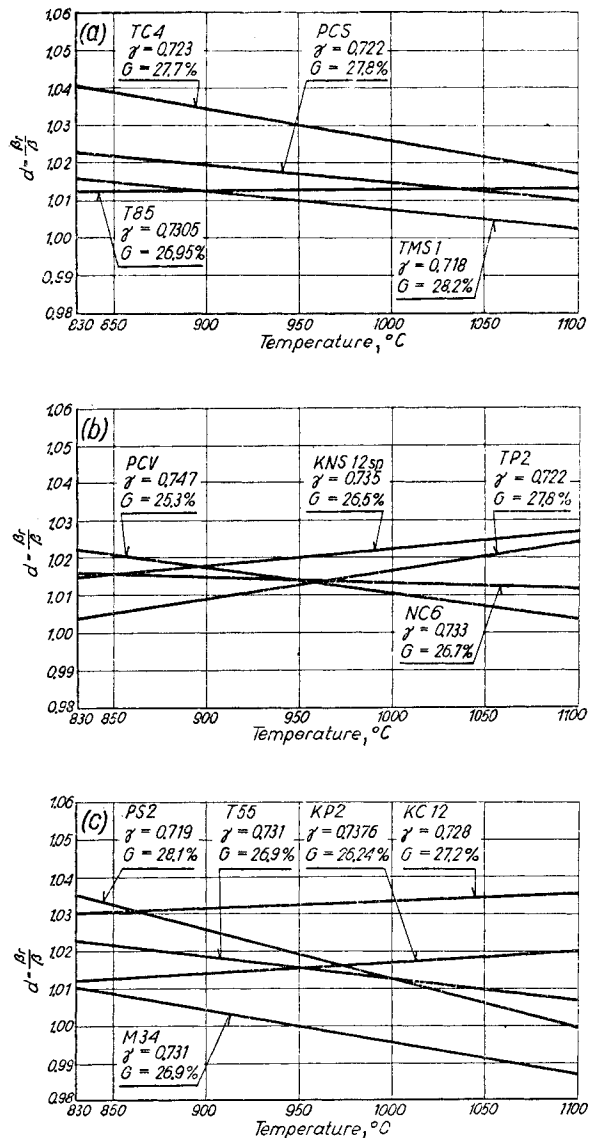


Fig. 3.23. Variation of correction factor  $d$  with temperature for different steels rolled with constant draught coefficient: (a) steel T85, PCS, TC4, TMS1 (b) steel PCV, KNS12sp, NC6, TP2 (c) steel KP2, T55, PS2, KC12, M34

For certain steels the value of  $d$  decreases with increase of rolling temperature, for other steels  $d$  increases, and finally there are also steels, as for example steel No. 10, for which the factor  $d$  is virtually unaffected by change of rolling temperature.

TABLE 3.5  
SELECTED STEEL GRADES

| No. | Steel grade | Composition, % |      |      |       |       |       |      |
|-----|-------------|----------------|------|------|-------|-------|-------|------|
|     |             | C              | Mn   | Si   | P     | S     | Cr    | Ni   |
| 1   | SzCh15      | 0.97           | 0.34 | 0.26 | 0.018 | 0.011 | 1.41  | 0.25 |
| 2   | KC3         | 0.19           | 0.28 | 0.38 | 0.021 | 0.022 | 12.18 | 0.13 |
| 3   | 18HnWA      | 0.15           | 0.44 | 0.32 | 0.018 | 0.011 | 1.27  | 3.73 |
| 4   | H25T        | 0.11           | 0.44 | 1.16 | 0.023 | 0.012 | 25.33 | 0.12 |
| 5   | 35HGSA      | 0.30           | 1.03 | 1.13 | 0.024 | 0.006 | 0.89  | 0.28 |
| 6   | KNR1        | 0.11           | 0.52 | 1.42 | 0.023 | 0.018 | 17.01 | 7.01 |
| 7   | KP2         | 0.13           | 0.51 | 0.57 | 0.023 | 0.022 | 17.59 | 6.63 |

Various further experiments [41] gave similar results. Experiments were carried out on 7 grades of alloy steel, the chemical composition of which is given in Table 3.5. Three series of experiments were made, using three form factors:

|     | Form factors             | Bar dimensions               |
|-----|--------------------------|------------------------------|
| (1) | $\delta_w = b_1/h_1 = 1$ | $30 \times 30 \times 500$ mm |
| (2) | $\delta_w = b_1/h_1 = 2$ | $20 \times 40 \times 500$ mm |
| (3) | $\delta_w = b_1/h_1 = 5$ | $16 \times 80 \times 500$ mm |

Rolling tests were carried out on a 2-high rolling mill with 308 mm dia. plain-body rolls, the rolling speed being constant  $v = 0.6\text{--}0.8$  m/sec. For all the steels, three rolling temperature ranges were laid down: low, medium and high.

The lowest temperature at which hot rolling can still take place was chosen as the low rolling temperature range  $t_a = 850\text{--}950^\circ\text{C}$ .

The temperature normally used in rolling the given steel was taken as the mean temperature range  $t_b = 950\text{--}1100^\circ\text{C}$ .

The upper limit of temperature permissible in hot rolling was chosen as the high temperature range  $t_c = 1100\text{--}1200^\circ\text{C}$ .

The correction factors found, i.e.  $d$  for spread (Figs. 3.24 to 3.27) or its reciprocal for coefficients of elongation, can be used for rolling steel in cases complying with the conditions laid down for these experiments.

In practice, the mean values of  $d$  corresponding to certain temperature ranges and the most commonly used reductions can be used, with other conditions unaltered.

However, the mean values of  $d$  for a given steel cannot be determined without taking into account the remaining factors.

The degree of draught, chemical compositions of steel, form factor and roll factor have a deciding influence on the magnitude of the coefficient of spread, the influence of rolling temperature being of minor importance.

In determining the correction factor  $d$  only the chemical composition of steel was taken into account. This factor probably also includes the effect of the structure of steel.

It has been shown that increase of spread is most noticeable with the addition of nickel to the steel, or of a high proportion of chrome.

For the determination of spread and elongation of mild steel in cold rolling [175], mathematical relationships between the coefficients of spread and elongation depending on the coefficient of draught, were established by the known and tried method for hot rolling.

The rolling tests were carried out on 0.1%C mild steel, normalized and pickled in sulphuric acid solution [175].

To obtain  $\delta_w = b_1/h_1$  varying from 1 to 20, specimens with the following dimensions were used:

|           |                |          |                 |
|-----------|----------------|----------|-----------------|
| 10×10 mm, | $\delta_w = 1$ | 3×36 mm, | $\delta_w = 12$ |
| 5×20 mm,  | $\delta_w = 4$ | 3×48 mm, | $\delta_w = 16$ |
| 5×40 mm,  | $\delta_w = 8$ | 3×60 mm, | $\delta_w = 20$ |

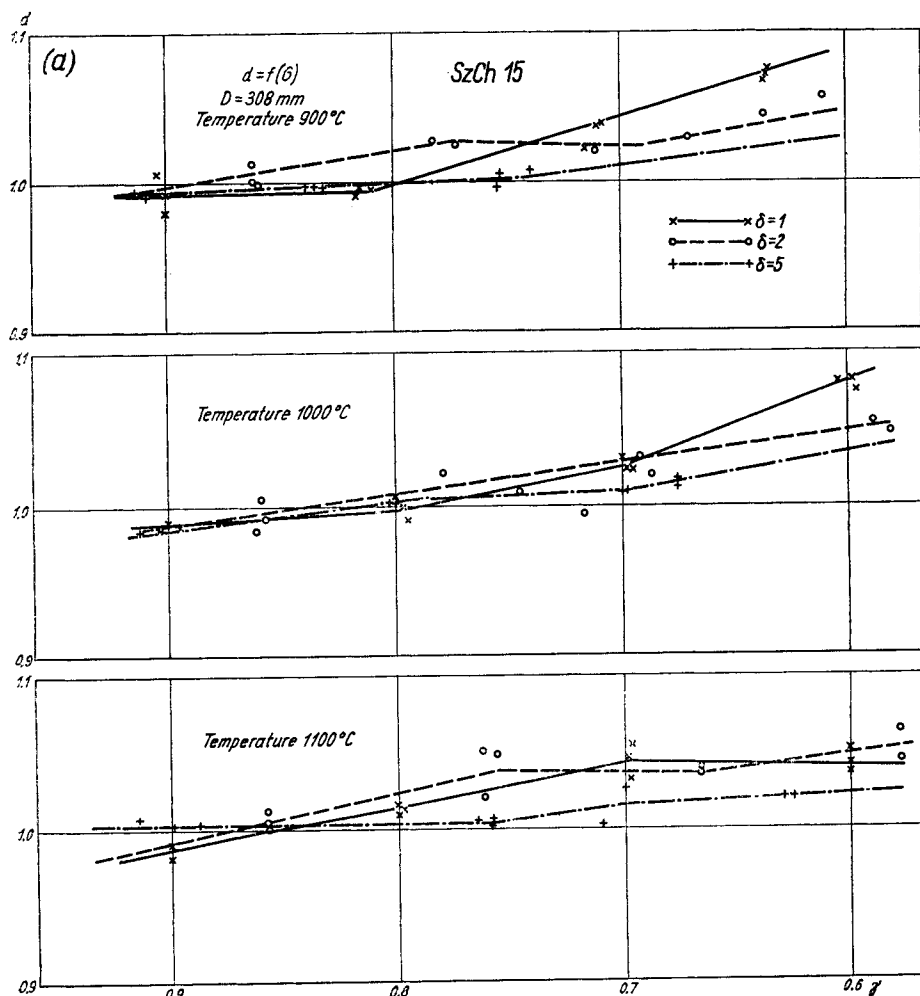


Fig. 3.24. Variation of correction factor  $d$  for SzCh15 steel:  
(a) with draught



The draughts taken were 10–70%, corresponding to coefficients of draught of 0.9–0.3.

The rolling tests were carried out on a 2-high rolling mill with smooth ground rolls of diameter 240 mm, barrel length 450 mm, without lubrication, the rolling speed being about 0.5 m/sec.

The final form of the formula for spread was found to be

$$\beta = \gamma^{-W_e} = \gamma^{-10^{-13.112 \epsilon_w^{1.126} \delta_w}} \quad (3.54)$$

and

$$\lambda = \gamma^{W_e - 1}$$

where

$$W = -10^{-13.112 \epsilon_w^{1.126} \delta_w} \quad (3.55)$$

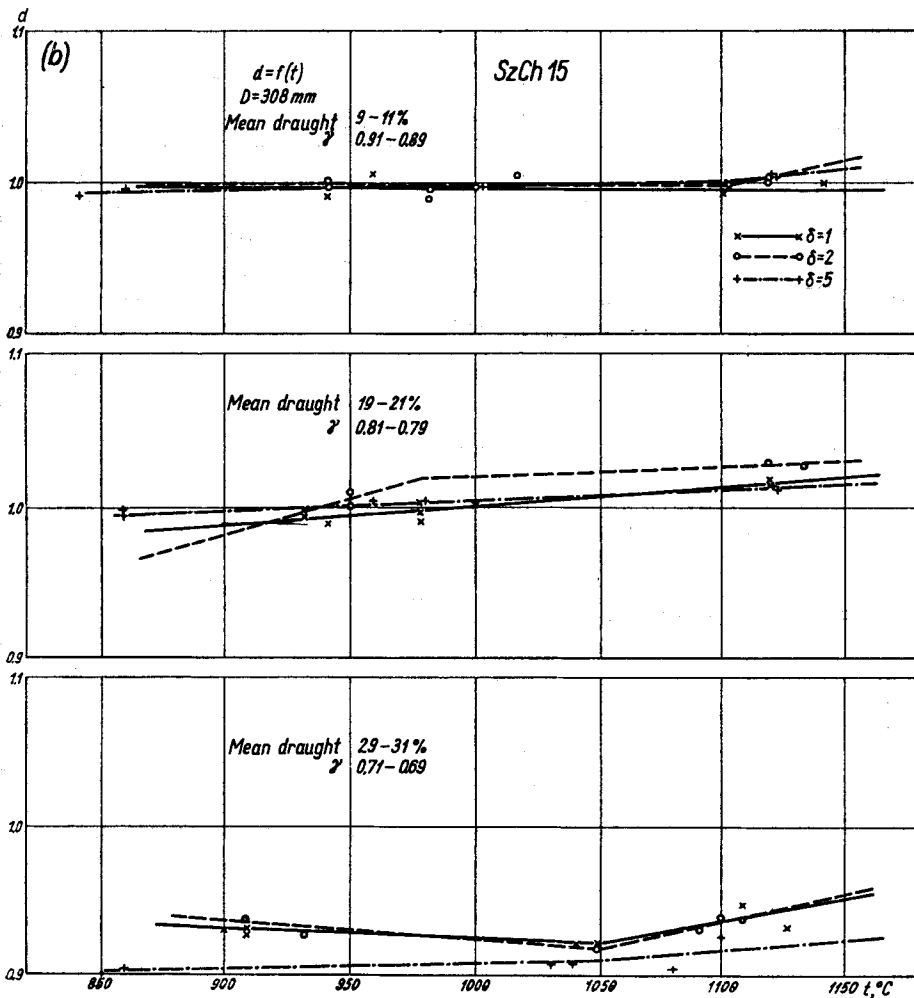


Fig. 3.24 (cont.). Variation of correction factor  $d$  for SzCh15 steel:  
 (b) with temperature

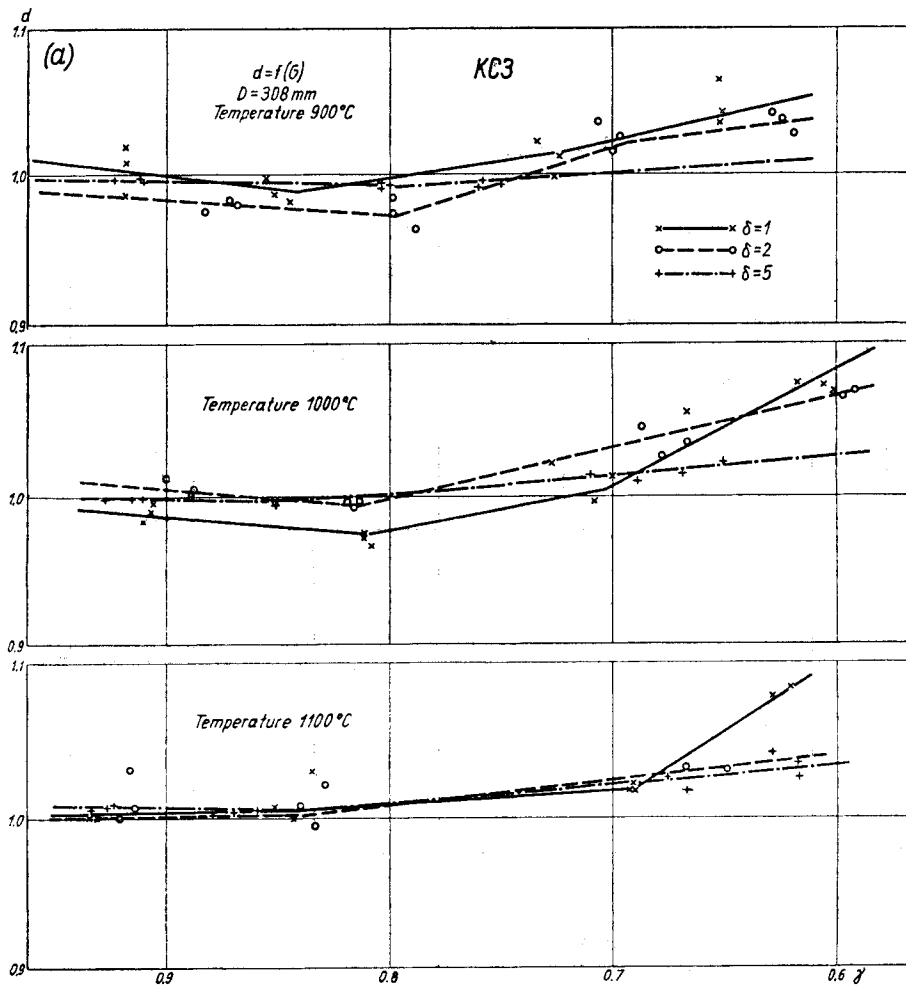


Fig. 3.25. Variation of correction factor  $d$  for KC3 steel: (a) with draught

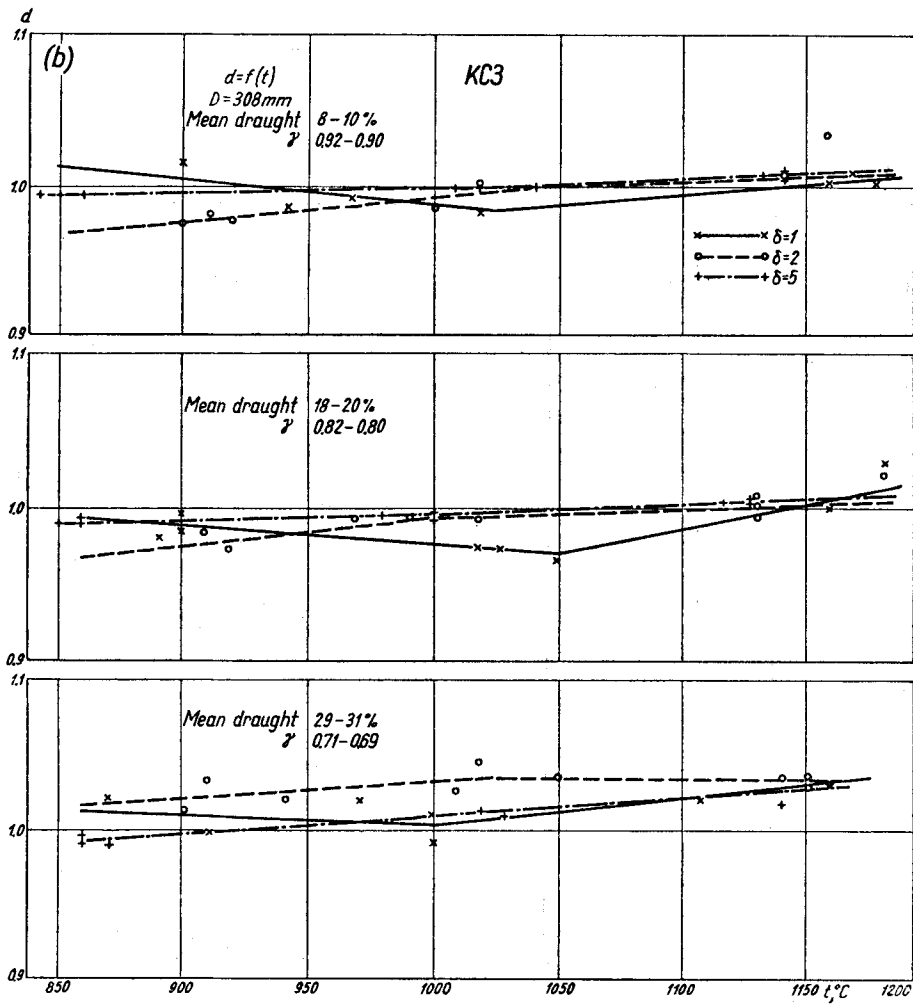


Fig. 3.25 (cont.). Variation of correction factor  $d$  for KC3 steel: (b) with temperature

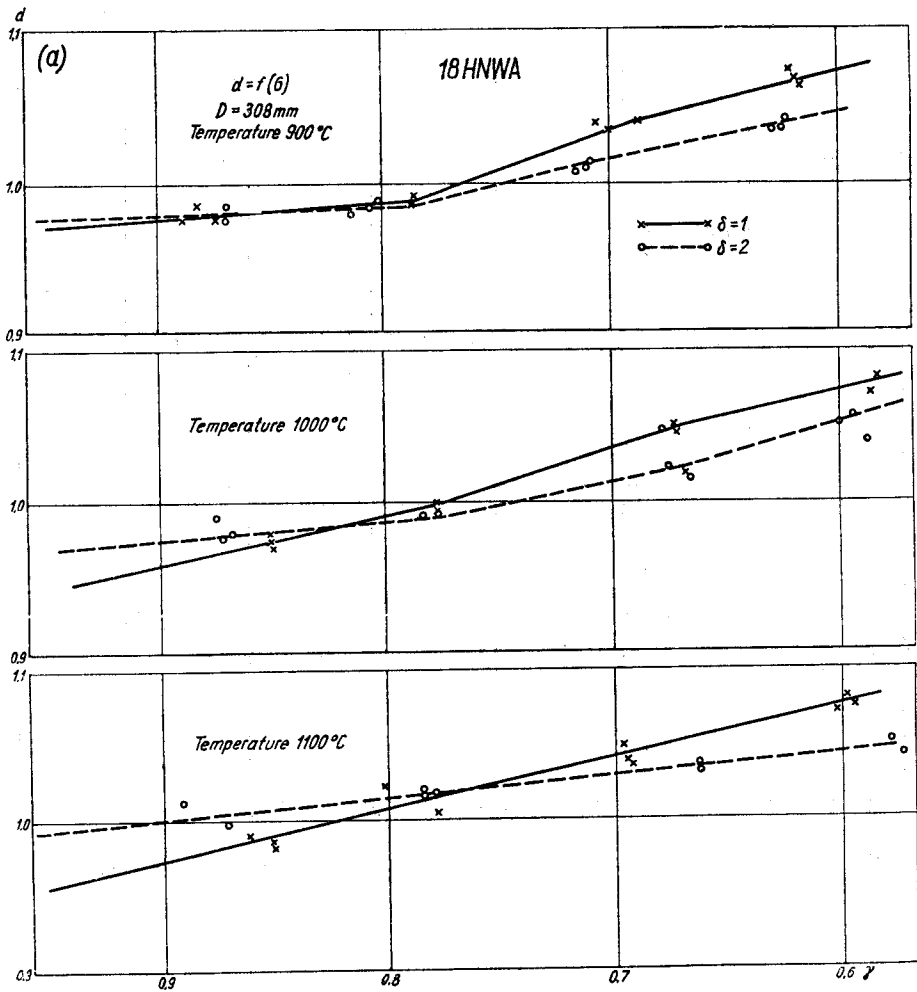


Fig. 3.26. Variation of correction factor  $d$  for 18HNWA steel: (a) with draught

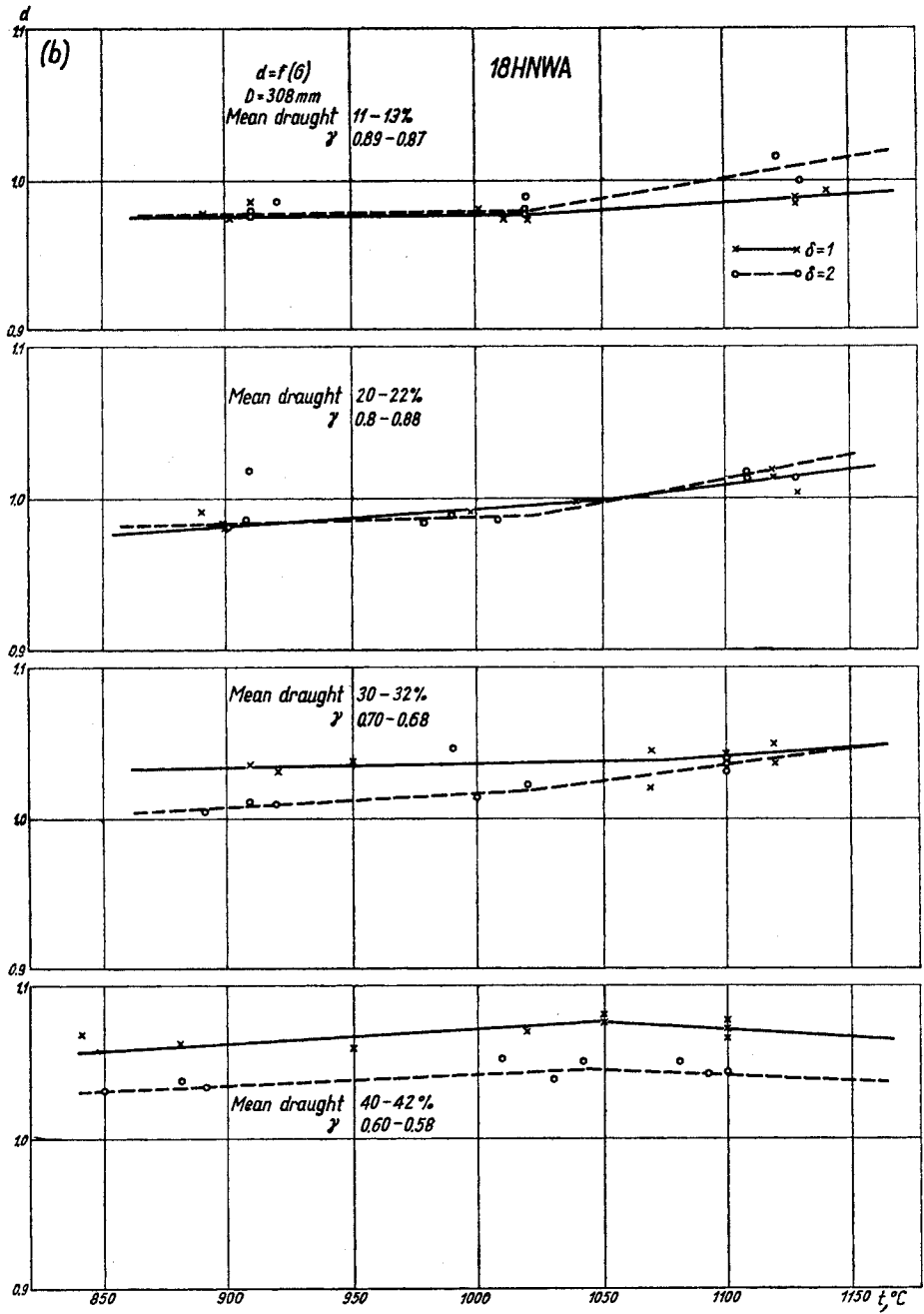


Fig. 3.26 (cont.). Variation of correction factor  $d$  for 18HNA steel: (b) with temperature

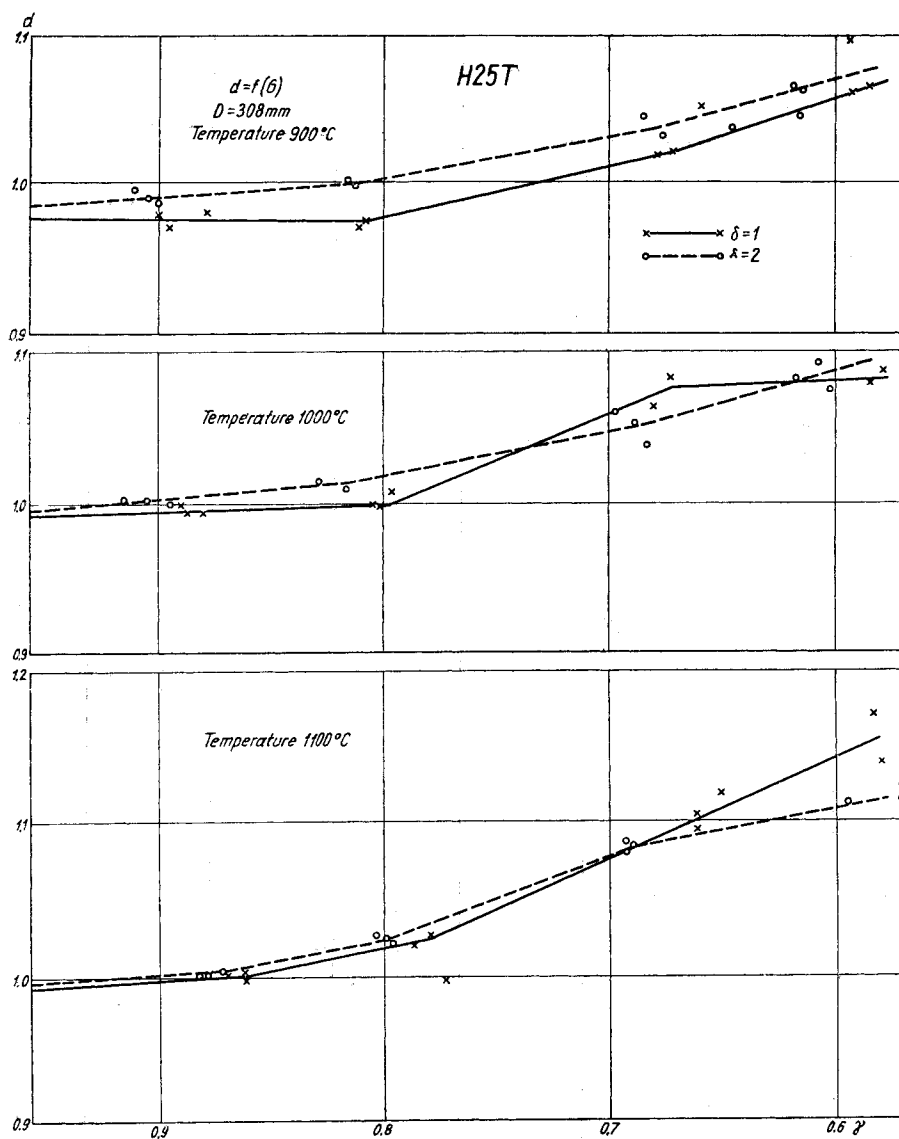


Fig. 3.27. Variation of the correction factor  $d$  with draught for H25T steel

Comparison of results shows that the spread of mild steel in cold rolling without lubrication does not differ appreciably from spread on hot rolling of the same steel, with the exception of sections with small form factors  $\delta_w$ , varying from 1 to 4. This can be seen from Fig. 3.28 which represents the relation between the exponent  $-W$  for hot rolling, from equation (3.46), and the exponent  $-W_c$  for cold rolling without lubrication, from equation (3.55).

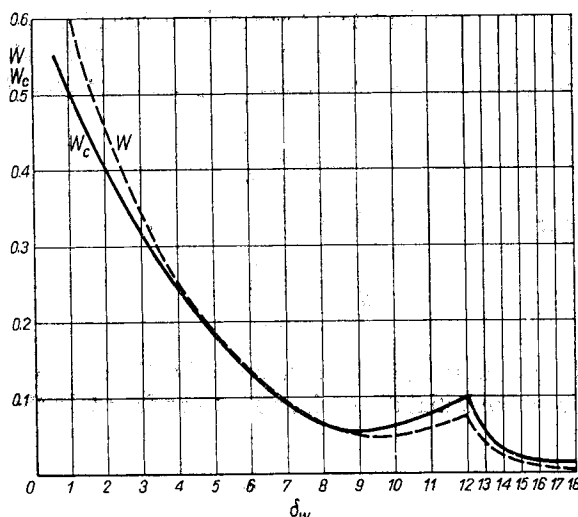


Fig. 3.28. Comparison of exponents  $W$  for hot rolling and  $W_c$  for cold rolling

From Fig. 3.28 it can be seen that for  $\delta_w = 4-9$  and  $14-18$  the differences between  $-W$  and  $-W_c$  are small. Thus within these limits formulae (3.45) and (3.46) for hot rolling can also be used for calculating coefficients of spread and elongation in cold rolling without lubrication.

### 3.5. Calculation of Draught, Elongation and Spread in Symmetrical Passes During Rolling of Bars

This problem can be solved [42] by substituting the mean coefficient of draught,  $\gamma_m$ , and the mean working diameter of rolls in equations (3.45)–(3.50) and using nomograms from Fig. 3.18 to Fig. 3.20. This applies to rolling of rounds, ovals, diamonds, hexagons, squares, etc. (Fig. 3.3).

Substituting mean values equation (3.45) will take the form

$$\beta = \gamma_m^{-W_m} \quad (3.56)$$

where

$$-W_{wm} = -10 - 1.269 \varepsilon_{wm}^{0.556} \delta_{wm} \quad (3.57)$$

$\varepsilon_{wm} = h_{1m}/D_{mw}$  = mean roll factor,

$\delta_{wm} = b_1/h_{1m}$  = mean form factor,

$D_{mw}$  = mean effective roll diameter (3.30).

Equation (3.47) will take the form

$$\lambda = \gamma_m^{-(1-W_m)} \quad (3.58)$$

In formula (3.48) it is sufficient to substitute the mean value of the exponent  $W_m$ , giving

$$\beta = \lambda^{W_m/(1-W_m)} \quad (3.59)$$

Flat rectangular bars can be calculated directly from equations (3.45)–(3.48), but the calculation of regular sections by this method is impossible as in calculating the average height from equation (3.1) the unknown value of  $b_2$  has to be substituted in the relation

$$h_{2m} = \frac{F_2}{b_2}$$

This value  $h_{2m}$  occurs in formula (3.45)

$$\beta = \left( \frac{h_{2m}}{h_{1m}} \right)^{-W_m} \quad (3.60)$$

In roll pass design the situation is reversed. The dimensions of stock after deformation are known and the initial dimensions are to be found. In this case the term  $b_1$  in the expression

$$h_{1m} = \frac{F_1}{b_1}$$

will be unknown.

The values of  $h_{1m}$  or  $h_{2m}$  in equation (3.57) are required to calculate

$$\varepsilon_{wm} = \frac{h_1}{D_{mw}}$$

where from equation (3.30)

$$D_{mw} = D_r - h_{2m}$$

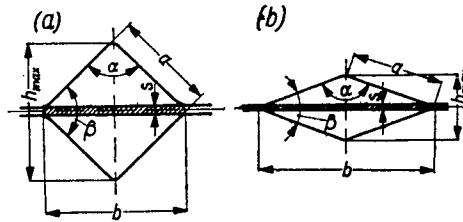


Fig. 3.29. Area of roll passes allowing for roll clearance,  $s$  (a) square, (b) diamond

Clearly  $h_{2m}$  or  $h_{1m}$  must be calculated first, since otherwise none of those equations can be solved. Therefore, a correct method of calculating the mean height of the pass is a problem of major importance. To solve this problem Z. and R. Wusatowski



developed an original method [42] to determine the average height  $h_m$  of a given section and the ratio of  $h_m/h_{\max}$  (Fig. 3.3)

$$h_m = m h_{\max} \quad (3.61)$$

where

$h_{\max}$  = maximum height of section,

$m$  = coefficient determined for different sections.

For a square or diamond placed diagonally (Fig. 3.3) the common value of  $m = h_m/h_{\max}$  can be determined.

If a pass of an exact geometrical form is turned on the roll barrel, its area will increase during rolling by an amount equal to the area of a rectangle of width  $b$  and height equal to the roll gap  $s$ , shown hatched in Fig. 3.29 [42].

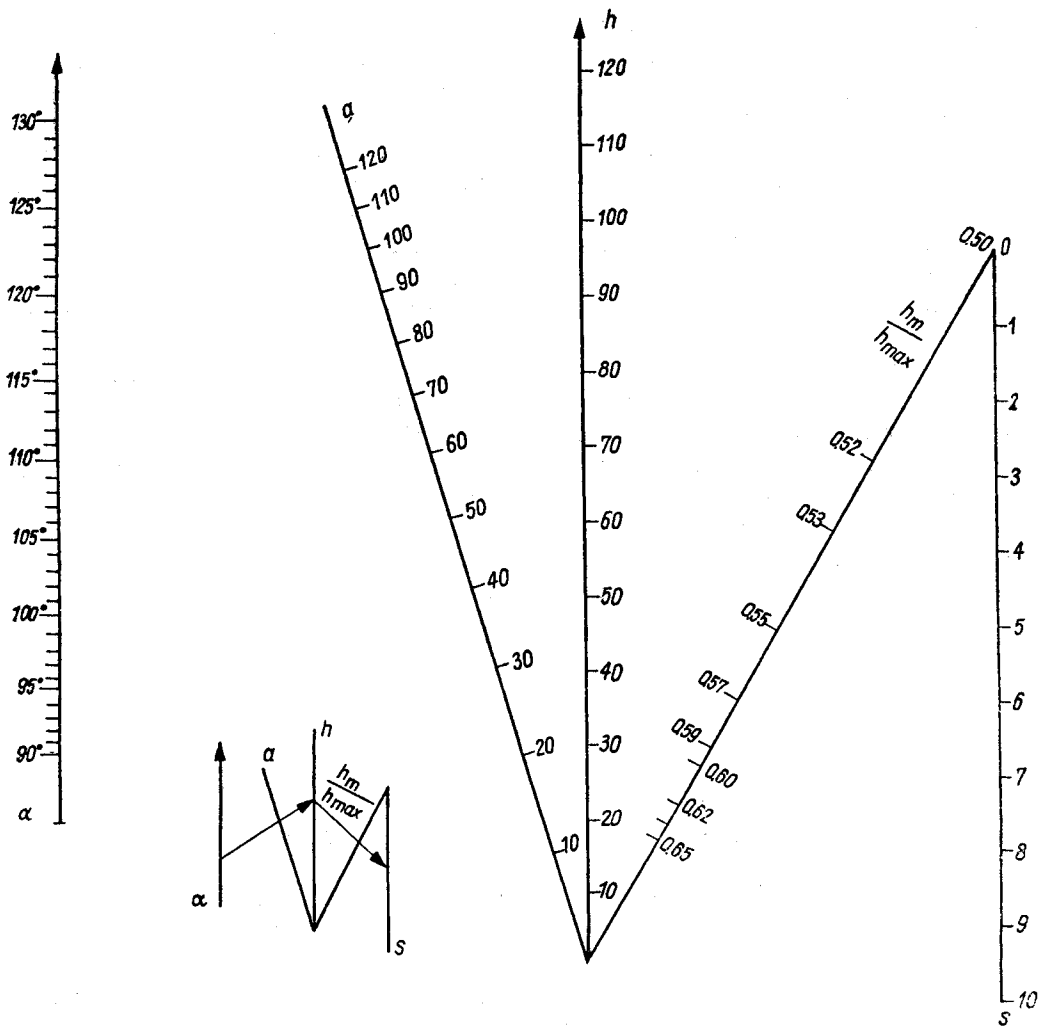


Fig. 3.30. Nomogram for calculating ratio  $h_m/h_{\max}$  for square and diamond passes relative to apex angle  $\alpha$ , side  $a$  and roll clearance,  $s$

Taking this increase into account, the formula becomes

$$m = \frac{h_m}{h_{\max}} = \frac{a \cos \frac{\alpha}{2} + s}{2 a \cos \frac{\alpha}{2} + s} \quad (3.62)$$

This equation expresses the relation between  $m$  and the roll clearance  $s$ , assuming that the pass is of an exact geometrical form.

For this reason the length of the pass side is taken always in the groove and not the length to which the metal fills the pass.

Since the practical application of equation (3.62) presents difficulties, a corresponding nomogram has been evolved (Fig. 3.30).

Knowing the apex angle  $\alpha$ , the side of a square or diamond  $a$ , and the roll clearance  $s$ , the required value of  $h_m/h_{\max}$  can easily be determined from this nomogram.

The procedure is shown schematically at the bottom of the figure. On the left hand scale a point corresponding to the angle  $\alpha$  of the square or diamond, is joined to a point on the  $a$  scale corresponding to the side length, and this line is produced to cut the  $h$  scale giving an auxiliary value  $h$ . The  $s$  scale represents the total roll clearance. Joining the value  $h$  to the appropriate value on the  $s$  scale, the required value of  $h_m/h_{\max}$  is found at the point of intersection with the  $h_m/h_{\max}$  scale.

The values calculated from equation (3.62) or taken from the nomogram relate to purely geometrical figures (e.g. squares, diamonds), while the grooves turned on the roll barrel are always distorted to a certain degree. Therefore these values should be increased by 0.02 for squares and diamonds with rounded sides.

When introducing the square flat into the pass it should be noticed that the maximum height of the bar differs from its mean height. The corner radii of the square can have various values, depending on the size of the section.

In these cases the value of  $m$  varies from 0.96 to 0.99 depending on corner radii and roll clearance. Since the roll clearance in rolling small squares is small, in general the value of  $h_m/h_{\max}$  can be assumed 0.99. As the size of the square increases this value can be gradually lowered to 0.96.

The basic shapes of ovals most often used are shown in Figs. 3.31a, b, c and d.

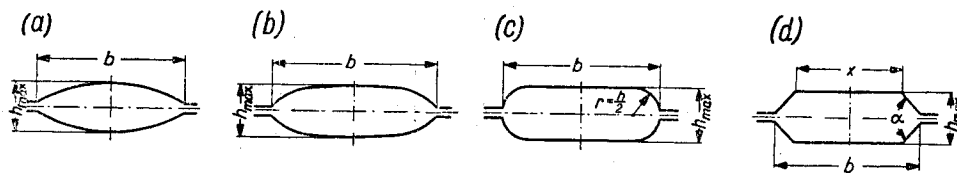


Fig. 3.31. Different types of ovals: (a) common, (b) elliptical, (c) flat, (d) hexagonal

*Calculation of a simple oval.* In this case  $h_m/h_{\max}$  depends on the size of oval, the breadth to height ratio and the roll setting, which can be expressed by the equation

$$\frac{h_m}{h_{\max}} = \frac{F/b + s}{h + s} \quad (3.63)$$

For practical calculations the nomogram shown in Fig. 3.32, calculated from equation (3.63) can be used. The  $b/h$  and  $s/h$  ratios are calculated, and the corresponding

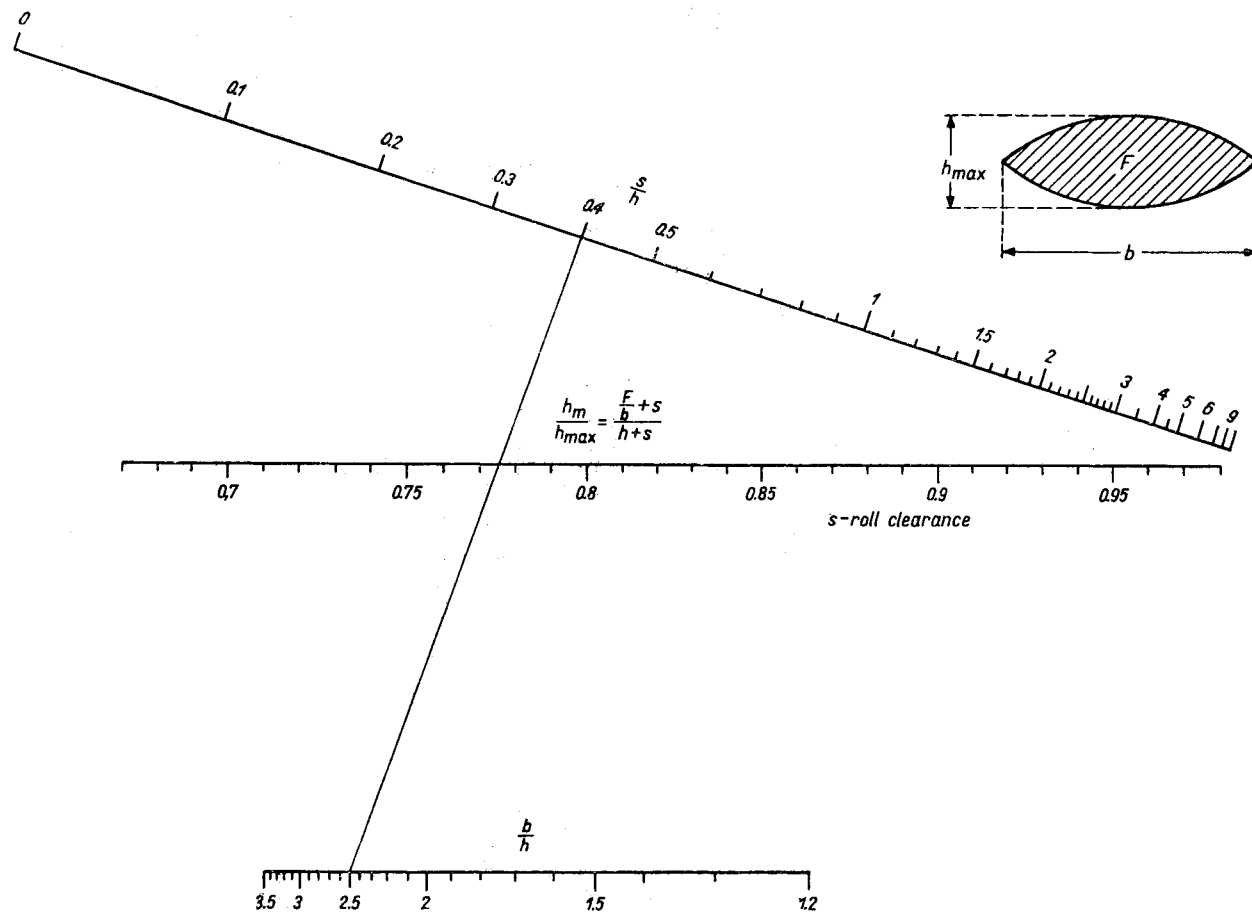


Fig. 3.32. Nomogram for determining ratio  $h_m/h_{max}$  for common ovals relative to  $s$ ,  $h_1$  and  $b$

points are joined by a straight line [42]. The point of intersection with the  $h_m/h_{max}$  scale gives the required value of  $m$ . The values of  $m$  found from equation (3.63) or taken from nomogram (Fig. 3.32), can only be used in the case when an oval is in-

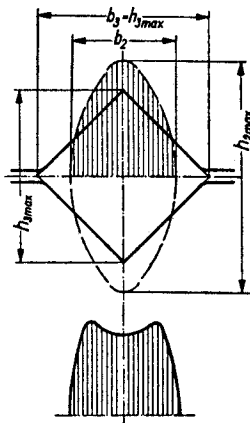


Fig. 3.33. An oval entering the square pass laid on the diagonal

troduced into a square pass, with the major axis vertical (Fig. 3.33). When a square is rolled in an oval pass, the major axis being horizontal, the corresponding value of  $m$  must always be increased by 0.05.

To improve accuracy in calculating the area of simple ovals, two nomograms have been prepared, one for ovals of width 7 mm–35 mm (Fig. 3.34) and the other for ovals of width 35 mm–120 mm (Fig. 3.35).

The points corresponding to calculated values of  $b/h$  and  $b$  for the given oval are joined and the area of the ovals is read off at the point of intersection with the  $F$  axis.

*Calculation of an elliptical oval* (Fig. 3.31b). The ratio is expressed by the equation

$$\frac{h_m}{h_{max}} = \frac{0.785 h + s}{h + s} \quad (3.64)$$

In this case the  $h_m/h_{max}$  ratio depends only on the height of oval and the roll setting. To find this value the nomogram shown in Fig. 3.36 has been worked out. Unfortunately, it was not possible to describe this relation by a simpler method. To determine  $h_m/h_{max}$  ratio (with known  $h$  and  $s$ ), the point corresponding to value  $s$  is joined to the value of  $h$  on the left hand  $h$  scale, and the point of intersection with the  $h_{max}$  scale gives  $h_{max}$ . Then point  $s$  is joined to  $h$  on the right hand  $h$  scale, and the point of intersection of this line produced with the  $A$  scale, gives a point  $A$ . Joining  $A$  and  $h_{max}$ , the intersection with the  $h_m/h_{max}$  scale gives  $h_m/h_{max}$ . This procedure is illustrated in the sketch in the figure.

*Calculation of a flat oval* (Fig. 3.31c). The required value is obtained from the equation

$$m = \frac{h_m}{h_{max}} = \frac{h + s - 0.215(h^2/b)}{h + s} \quad (3.65)$$

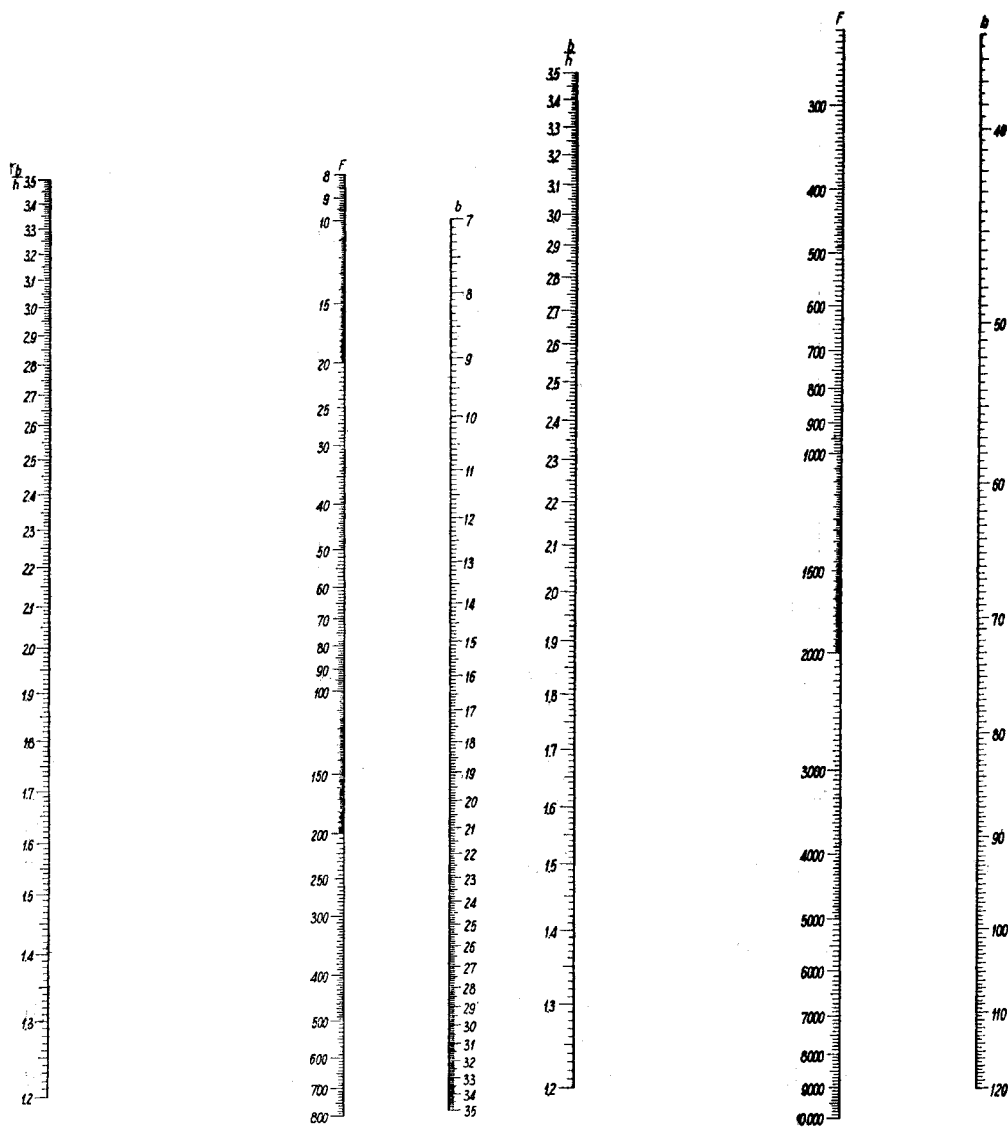


Fig. 3.34. Nomogram for determining the areas of common ovals, of width 7–35 mm in mm<sup>2</sup>

Fig. 3.35. Nomogram for determining the areas of common ovals, of width 35–120 mm in mm<sup>2</sup>

From this equation it follows that  $h_m/h_{max}$  ratio is dependent on the height, the roll setting, and on the breadth to height ratio  $b/h$ . To determine the ratio  $h_m/h_{max}$  more easily the nomogram shown in Fig. 3.37 has been worked out. With known values

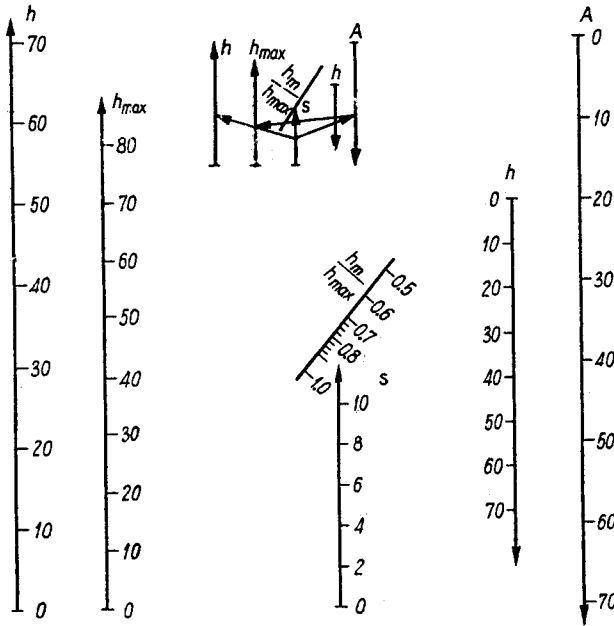


Fig. 3.36. Nomogram for determining ratio  $h_m/h_{max}$  for elliptical ovals relative to  $h$  and  $s$

of  $h$ ,  $b/h$  and  $s$  (roll setting), first  $(h+s)$  is calculated and the appropriate point on the  $(h+s)$  scale is joined to the point on the  $b/h$  scale corresponding to the value of  $b/h$ , and this line is produced to cut scale  $A$ . The point  $A$  is joined to the point on the  $h$  scale corresponding to value  $h$ , and where this line cuts the diagonal scale  $h_m/h_{max}$ , gives the required value.

*Calculation of hexagonal oval* (Fig. 3.31d). The relation derived for  $m$  is

$$m = \frac{h_m}{h_{max}} = \frac{h+s-0.5(h^2/b)}{h+s} \quad (3.66)$$

shown graphically in Fig. 3.38. This nomogram can be used similarly to that for the oval shown in Fig. 3.36:

- (1)  $h$  on the left hand  $h$  scale is joined to  $s$  on the  $s$  scale, and  $h_{max}$  is read off on the  $h_{max}$  scale,
- (2)  $h$  on the right hand  $h$  scale is joined to  $b/h$ , and the line produced to intersect the  $A$  scale.
- (3) Point  $A$  is joined to  $h_{max}$ , and the required value is read off on the  $h_m/h_{max}$  scale. This procedure is illustrated in the sketch shown in Fig. 3.38.

The values of  $m$  calculated from the derived equations or determined from the nomograms shown in Figs. 3.36–3.38 are for the passes. The  $h_m/h_{max}$  ratio and hence

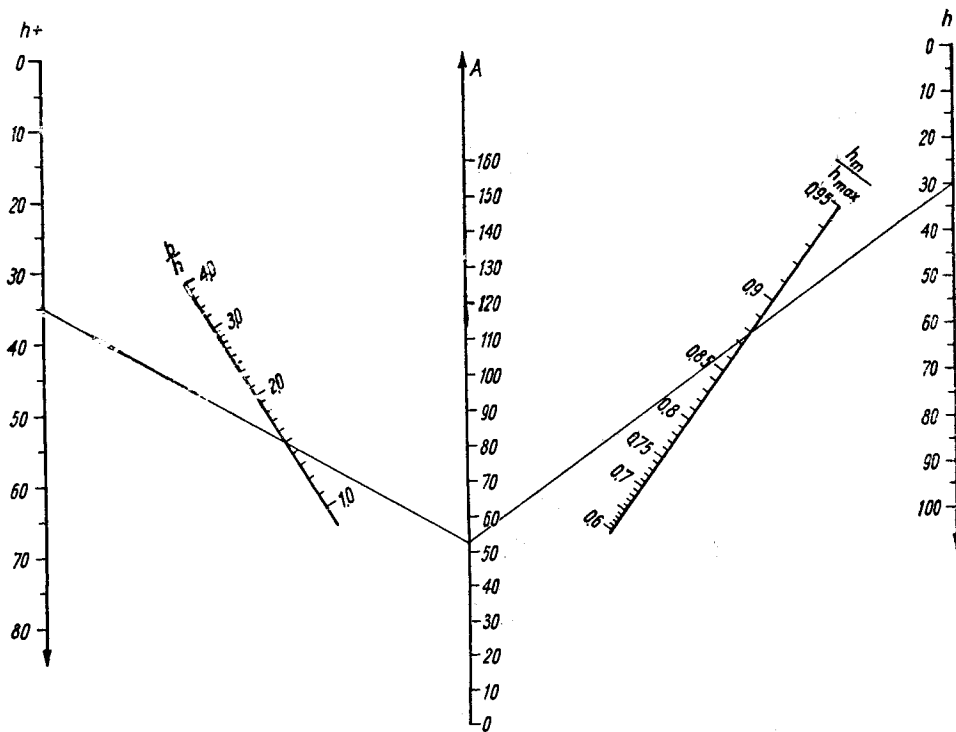


Fig. 3.37. Nomogram for determining ratio  $h_m/h_{max}$  for flattened ovals relative to  $h$  and  $s$

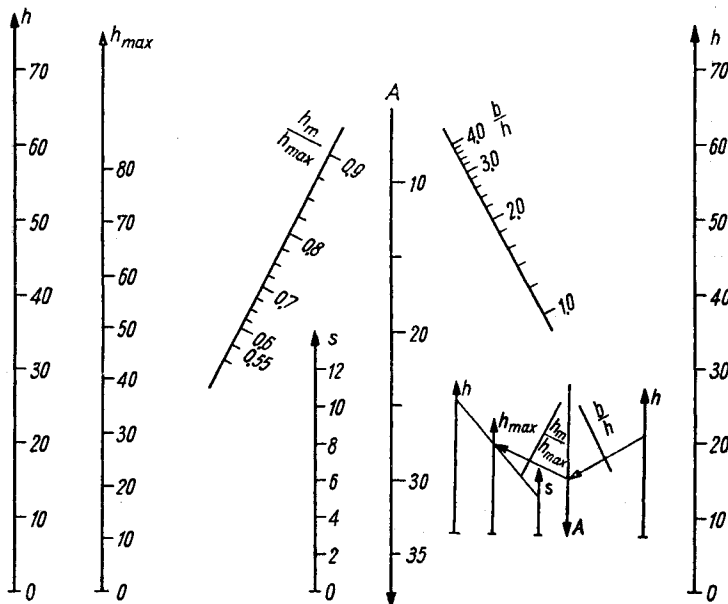


Fig. 3.38. Nomogram for determining ratio  $h_m/h_{max}$  for hexagonal oval relative to  $b$ ,  $h$  and  $s$

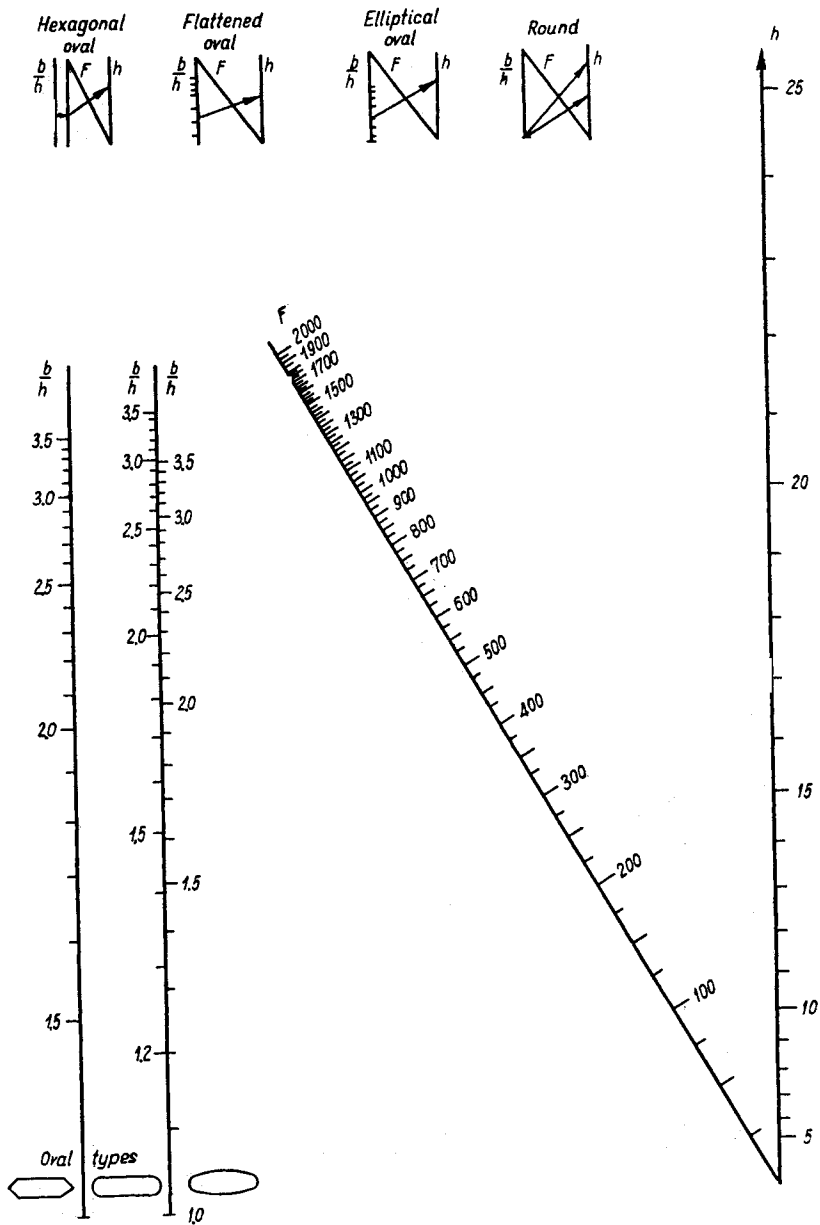


Fig. 3.39. Nomogram for determining the area (in mm<sup>2</sup>) of elliptical, flattened and hexagonal ovals,  $h = 5-25$  mm



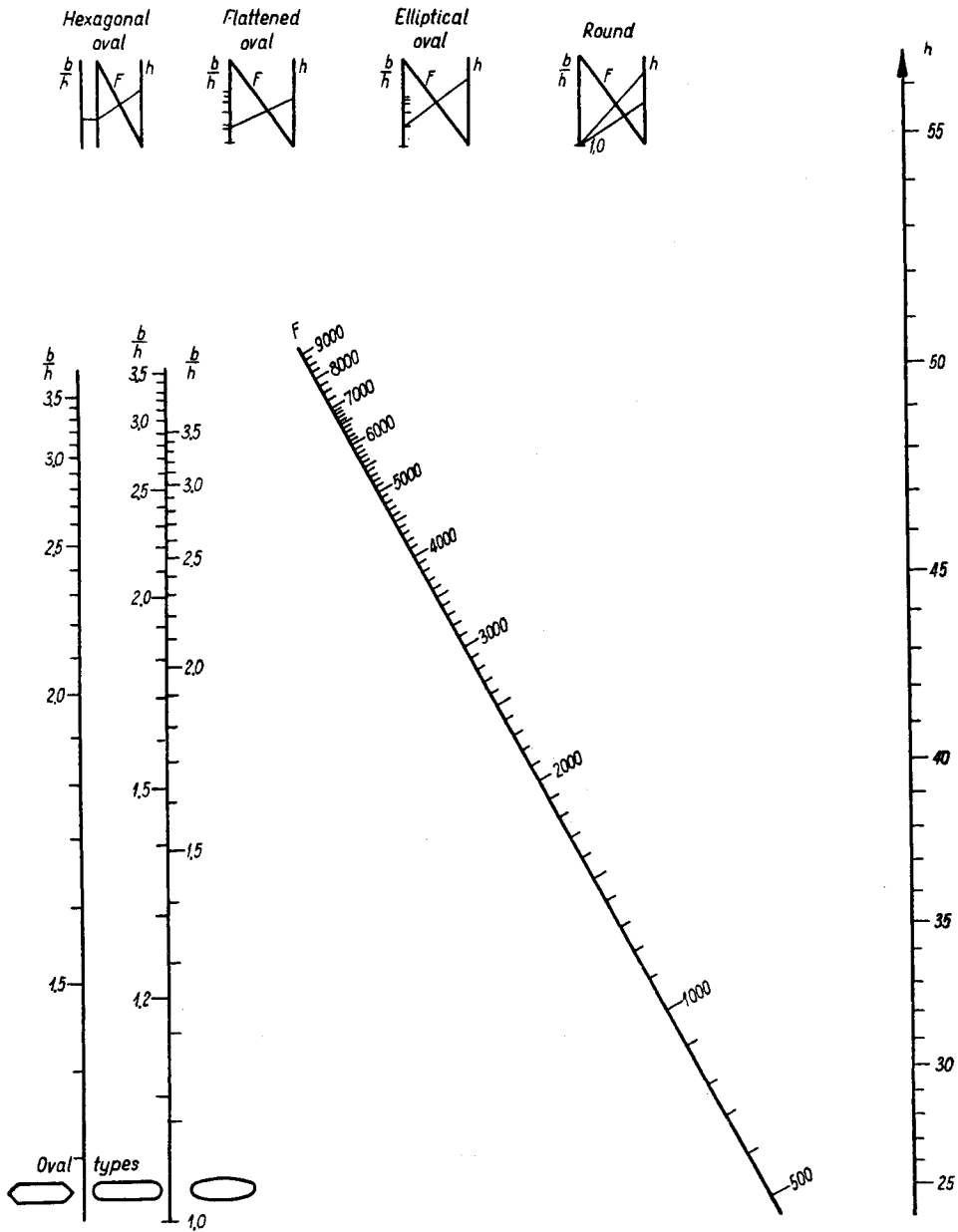


Fig. 3.40. Nomogram for determining area (in mm<sup>2</sup>) of elliptical, flattened and hexagonal ovals,  $h = 25-55$  mm

the average height of oval or square entering into the oval is calculated for  $s = 0$ , i.e. without clearance between rolls.

In practice the cross-sectional areas of various ovals are also required. Since calculation from equations is onerous, appropriate nomograms shown in Figs. 3.39 and 3.40 have been devised. From these nomograms the areas of elliptical, flat and hexagonal ovals can be read off quickly. For greater accuracy in reading off the scale values two separate nomograms have been prepared, one for  $h = 5\text{--}25$  mm (Fig. 3.39) and the other for  $h = 25\text{--}55$  mm (Fig. 3.40).

Each nomogram has a scale for height  $h$ , area  $F$  and three for  $b/h$  ratio. Depending on the type of oval, the  $b/h$  value on the appropriate  $b/h$  scale is joined to the  $h$  value on the  $h$  scale, and the area is given by the point of intersection with the  $F$  scale, as shown schematically in both nomograms.

These nomograms can also be used to find the area of a round section. If  $b/h$  is assumed equal to 1, then  $b = h = d$ , i.e. a circle is obtained.

Joining point 1 on the  $b/h$  scale for elliptical ovals with the appropriate value of  $h = d$  the area of the circle is read off at the point of intersection with the  $F$  scale.

*Calculation of  $h_m/h_{\max}$  ratio for a gothic square pass.* The value of  $m = h_m/h_{\max}$  with roll clearance  $s$  is calculated from

$$m = \frac{h_m}{h_{\max}} = \frac{F/b+s}{h+s} \quad (3.67)$$

which can be read from the nomogram shown in Fig. 3.41. The procedure for determining the average height is similar to that given in Fig. 3.32 from the ratio  $s/h$  and  $b/h$  (example given in the figure).

Figure 3.42 shows a hexagon rolled in two different positions a and b. The ratio of  $m = h_m/h_{\max}$  for hexagon in both positions a and b is 0.75. The ratio of  $h_m/h_{\max}$  for a round pass as shown in Fig. 3.43 is 0.785.

The procedure for using the nomograms shown in Figs. 3.30–3.38 and 3.41, is as follows. It is assumed that  $h_{1\max}$ ,  $h_{2\max}$ ,  $h_{1m}$ ,  $b_1$ ,  $D_{1m}$  are known. From equations or nomograms the ratio  $m = h_{2m}/h_{2\max}$  is found and from this the value of  $h_{2m}$  is calculated. Then the mean value of coefficient of draught

$$\gamma_m = \frac{h_{2m}}{h_{1m}}$$

is determined, where

$$h_{1m} = \frac{F_1}{b_1} \quad \text{and} \quad \frac{b_1}{h_{1m}} = \delta_{wm} \quad \frac{h_{1m}}{D_m} = \varepsilon_{wm}$$

and  $D_m = D_t - h_{2m}$  (Fig. 3.3a).

In roll pass design the procedure will be reversed, since the value of  $h_{1m}$  must be found.

On determining the above values,  $-W$  is calculated next and then the coefficients  $\beta$  and  $\lambda$  from equations (3.45)–(3.48) or the nomograms shown in Figs. 3.18–3.20.

#### Example

A sharp cornered square of dimensions  $h_1 = 17.7$  mm,  $b_1 = 17.7$  mm,  $F_1 = 314$  mm<sup>2</sup> enters a standard oval on the flat with  $h_{2\max} = 10$  mm (Fig. 3.8).

The roll diameter  $D_r = 300$  mm

$$h_{1m} = \frac{F_1}{b_1} = \frac{314}{17.7} = 17.7 \text{ mm}$$

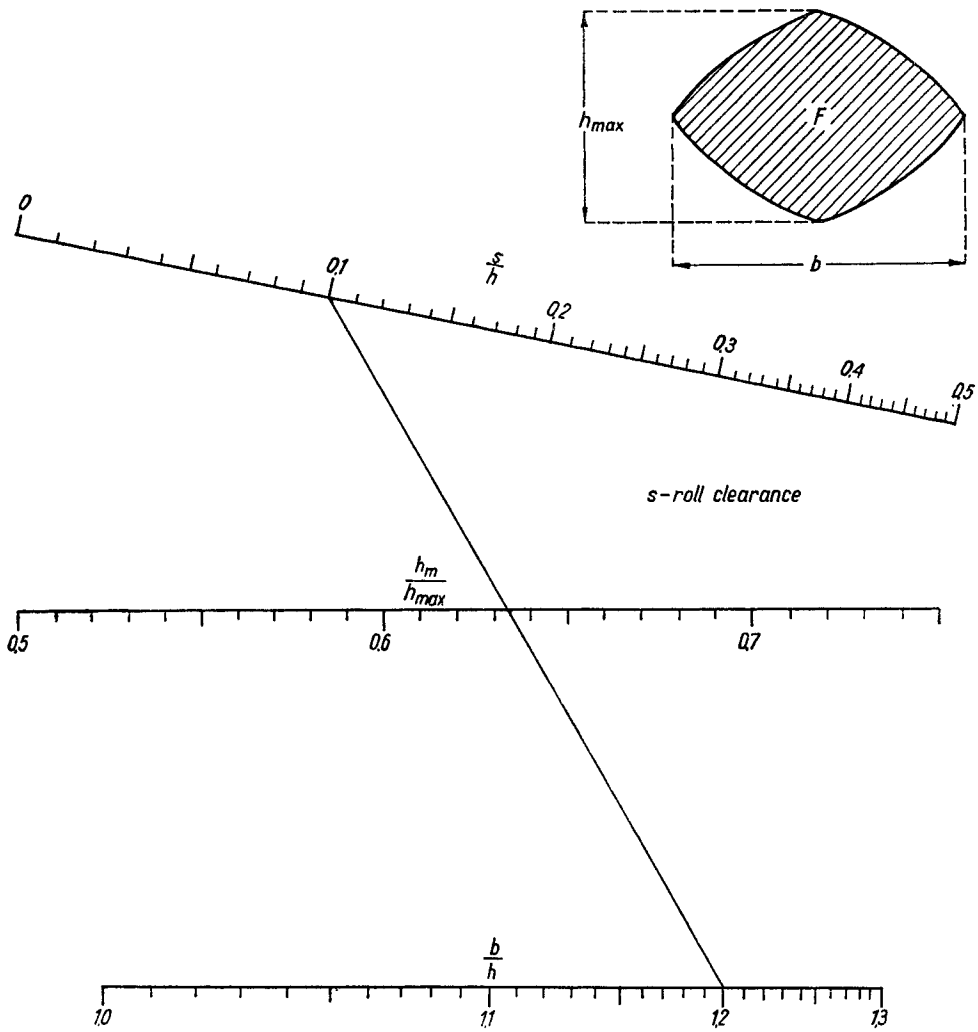


Fig. 3.41. Nomogram for determining ratio  $h_m/h_{max}$  for gothic sections relative to  $b$ ,  $h$  and  $s$

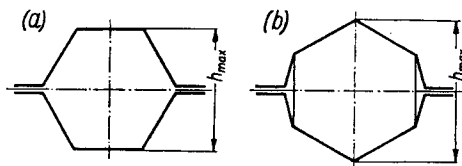


Fig. 3.42. Hexagonal pass: (a) flat-down position, (b) point-down position

Assuming a value of  $b/h \approx 3.0$  the value of  $m$  is found from the nomogram in Fig. 3.32

$$m = \frac{h_{2m}}{h_{2\max}} = 0.685$$

$$h_{2m} = 0.685 h_{2\max} = 0.685 \times 10 = 6.85 \text{ mm}$$

$$\gamma_m = \frac{h_{2m}}{h_{1m}} = \frac{6.85}{17.7} = 0.387$$

$$\delta_{wm} = \frac{b_1}{h_{1m}} = \frac{17.7}{17.7} = 1$$

$$\varepsilon_{1wm} = \frac{h_{1m}}{D_{mw}} = \frac{17.7}{293.15} = 0.060$$

where  $D_{mw} = D_t - h_{2m} = 300 - 6.85 = 293.15 \text{ mm}$ .

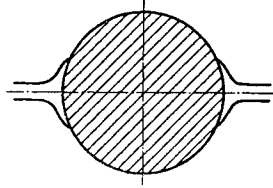


Fig. 3.43. Round pass

From Fig. 3.18 for  $\delta_{wm} = 1$  and  $\varepsilon_{1wm} = 0.060$  the value of  $-W = 0.54$  is found. For  $-W = 0.54$  and  $\gamma_m = 0.387$  from Fig. 3.19 the value of  $\beta = 1.67$  is found. The width of the oval will be  $b_2 = b_1 \beta = 17.7 \times 1.67 = 29.6 \text{ mm}$ . The actual value of  $b_2$ , as measured after rolling, was 30 mm, the difference being 0.4 mm, and the coefficient of elongation

$$\lambda = \frac{1}{\gamma_m \beta} = \frac{1}{0.387 \times 1.67} = 1.55$$

$$F_2 = \frac{F_1}{\lambda} = \frac{314}{1.55} = 202.2 \text{ mm}^2$$

### 3.6. Mechanism of Bite and Friction in Rolling

Depending on the conditions under which the metal is introduced into the roll gap two situations can occur:

- (1) the metal is gripped by the rolls and pulled along into the roll gap (Fig. 3.44):
- (2) the metal slips over the roll surface (Fig. 3.45).

The factors which decide the behaviour of metal in the roll gap are the magnitude of the angle of bite, and the ratio of this angle to the friction angle.

The size of the frictional force depends on the condition of the surfaces in contact (the rougher the surfaces the greater the frictional force) and on the velocity of slip and the roll pressure.

By the third law of dynamics, which states that every action has an equal and opposite reaction, metal introduced between the rolls exerts a pressure  $P$  on the rolls at the point of contact, and the rolls act on the metal with an equal and opposite reactive force. This force  $P$  passes through the point of contact and the roll centre (Fig. 3.44). Simultaneously, due to the difference in speeds of metal and rolls, frictional forces  $T$  arise (Fig. 3.44), acting tangentially to roll surface and normal to force  $P$ .

When the rolls have the same diameter, it is sufficient to investigate conditions arising on one roll.

The forces arising in rolling are represented by vectors, and resolved into components:  $P_v$  = vertical component of roll pressure  $P$ , due to which the height of stock

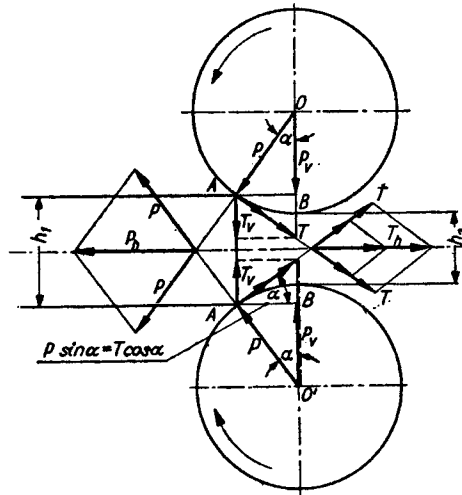


Fig. 3.44. Distribution of forces acting at the instant of bite of metal by the rolls. Roll bite is possible

decreases,  $P_h$  = horizontal component of  $P$ , tending to eject the metal from the rolls. The frictional force  $T$  resolves into a vertical component  $T_v$ , which compresses the metal and a horizontal component  $T_h$ , which tends to draw the metal into the roll gap.

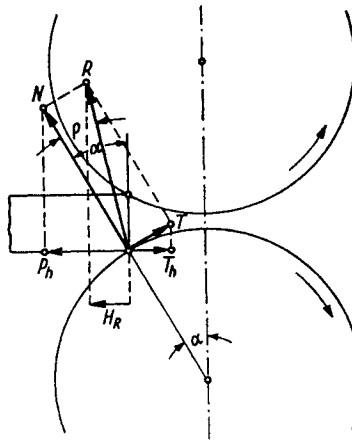


Fig. 3.45. Distribution of forces at the instant of bite. Roll bite is impossible

If a series of rolling tests is carried out on bars of the same initial height and with the roll gap decreasing in successive tests (Fig. 3.46), the bite angle  $\alpha$  will increase and

consequently the horizontal component force  $P_h$  will increase. At the same time the vertical component of friction  $T_v$  will decrease, so that there is an increasing tendency to eject the metal from the rolls, and at a certain value of the angle  $\alpha$ , free rolling becomes impossible. The maximum angle  $\alpha$  at which free rolling can take place, i.e. without using force to push the metal into the rolls, is called the maximum angle of bite.

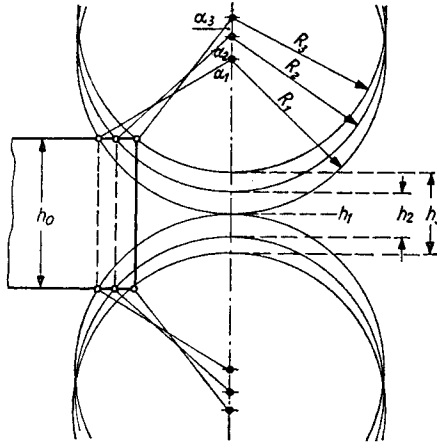


Fig. 3.46. Decrease of angle of bite as the roll gap increases

The maximum angle of bite varies within a wide range of values, depending on surface condition of rolls, metal, temperature and speed of rolling, etc. A mathematical relationship between the angle of bite and the angle of friction must now be deduced.

The frictional force is

$$T = fP \quad (3.68)$$

where

$f$  = coefficient of friction:  $f = \tan \rho$ ,

$\rho$  = angle of friction.

Resolving  $P$  and  $T$  along the horizontal axis of the rectangular coordinate system (the direction of rolling being taken as the positive direction) two component forces are obtained (Fig. 3.44):

horizontal component of  $P$   $P_h = -P \sin \alpha$

horizontal component of  $T$   $T_h = T \cos \alpha$

For a condition of equilibrium the algebraic sum of horizontal forces pulling and rejecting the metal must be equal to 0, or considering forces acting on the roll

$$P_h + T_h = 0$$

hence

$$P \sin \alpha = T \cos \alpha$$

and since

$$T = fP$$

then  $P \sin \alpha = fP \cos \alpha$

Hence

$$f = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha \quad (3.69)$$

The equilibrium of horizontal forces occurs when the coefficient of friction equals the tangent of the angle of bite. On the other hand, it is known that the coefficient of friction  $f$  is equal to the tangent of the angle of friction, thus

$$f = \tan \alpha_{\max} = \tan \rho \quad (3.70)$$

and hence

$$\alpha_{\max} = \rho \quad (3.71)$$

or taking into account equation (3.18)

$$f = \tan \alpha_{\max} = \frac{\sqrt{R \Delta h_{\max}}}{\left(R - \frac{\Delta h_{\max}}{2}\right)} \quad (3.72)$$

where  $\Delta h_{\max}$  is the largest draught at which the rolls bite the metal.

From the preceding considerations the following conclusions can be drawn:

(1) The rolls bite the metal when the angle of contact is less than or equal to the angle of friction, i.e. when

$$\alpha \leq \rho \quad (3.73)$$

(2) Free rolling takes place within the limits

$$0 < \alpha \leq \rho \quad (3.74)$$

(3) Using the maximum value of angle of bite the coefficient of friction can be calculated by using equation (3.72). During rolling the angle of the resultant  $P$  varies and its value is about  $\alpha/2$  when the roll gap is completely filled (Fig. 3.47).

Hence the fourth conclusion can be drawn:

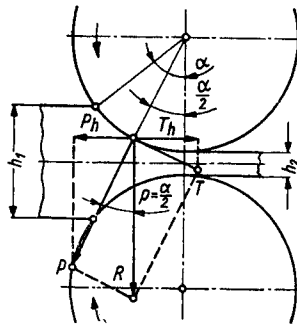


Fig. 3.47. Diagram illustrating theoretical equality  $\rho = \alpha/2$  and  $T_h = P_h$  [22]

(4) When the roll gap is filled by the metal, the angle of contact can be reduced to  $\alpha/2$ , and in spite of this rolling will continue, i.e.

$$\alpha/2 \leq \rho$$

Thus, after biting the metal and filling the roll gap, free rolling can take place within the theoretical limits

$$0 < \alpha \leq 2\rho \quad (3.75)$$

For rectangular sections the angle of bite is calculated from equations (3.9), (3.17) and (3.18).

The relation (3.72) can be also represented graphically (Fig. 3.48). From this figure either the coefficient of friction,  $f$ , or the permissible absolute draught,  $\Delta h$ , can be determined.

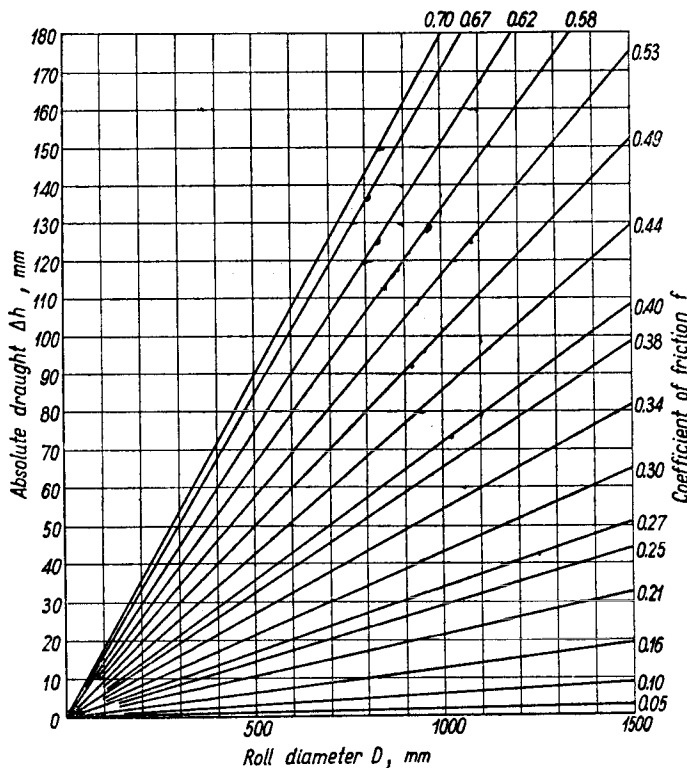


Fig. 3.48. Influence of absolute draught and roll diameter on the coefficient of friction in hot rolling [10]

Figure 3.49 represents the relations expressed by equations (3.72) and (3.9), from which permissible values of  $\Delta h/D$  and coefficients of friction can be found.

Figure 3.50 represents more clearly the conditions occurring during rolling when the roll gap is completely filled by metal [34].

The frictional forces change their direction at the plane denoted by  $F-F$ , called the neutral plane.

If rolling takes place, the frictional forces assisting rolling defined by the angle  $\rho$  in the sector  $a-b$ , should be greater than the sum of the frictional forces hindering rolling, in the sector  $b-c$  and the horizontal component of the roll pressure. Therefore, the resultant force can be defined in this way for the filled roll gap, by the condition  $\rho > \alpha - \delta$ . This result is confirmed by recent investigations.

Tests carried out by Chekmariev and Firsov [44] give practical values of the angle of friction varying from 1.5 to 1.7, thus the practical condition for rolling (3.75) will take the form

$$0 \leq \alpha \leq (1.5 \text{ to } 1.7)\rho \quad (3.76)$$



When the rolls have different diameters, as shown in Fig. 3.5b, then the condition for the angle of contact can be found assuming

$$T_{hR_1} + T_{hR_2} = 0$$

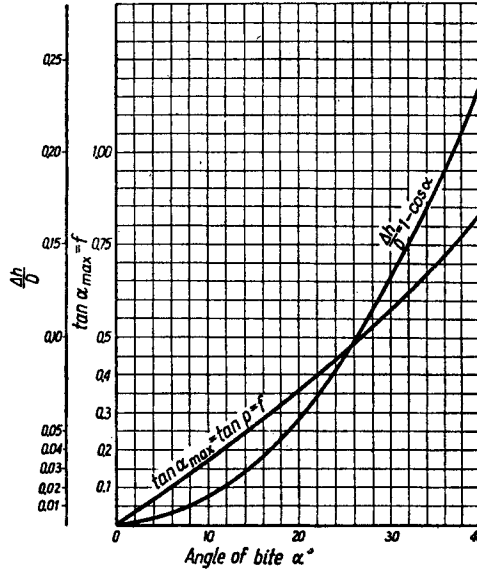


Fig. 3.49. Ratio  $\Delta h/D$  for various angles of bite and coefficient of friction  $f$ , calculated from the equation  $\tan \alpha_{\max} = \tan \rho = f$  [32]

or

$$\tan(\rho - \alpha_1) + \tan(\rho - \alpha_2) = 0$$

hence

$$(\rho - \alpha_1) + (\rho - \alpha_2) = 0$$

i.e.

$$\alpha_1 + \alpha_2 = 2\rho \quad (3.77)$$

Practical rolling conditions must be taken into account in these relation, thus from equation (3.76)

$$\alpha_1 + \alpha_2 \leq (1.5 \text{ to } 1.7)\rho \quad (3.78)$$

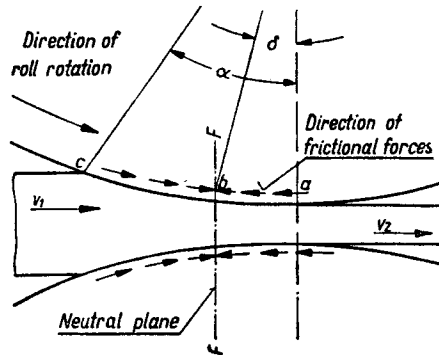


Fig. 3.50. Distribution of frictional forces in the roll gap

From equation (3.26)

$$\alpha_1 = \alpha_2 \frac{R_2}{R_1} \quad \text{or} \quad \alpha_2 = \alpha_1 \frac{R_1}{R_2}$$

and hence

$$\alpha_1 = \frac{(1.5 \text{ to } 1.7)\rho}{1 + R_1/R_2} \quad (3.79)$$

$$\alpha_2 = \frac{(1.5 \text{ to } 1.7)\rho}{1 + R_2/R_1} \quad (3.80)$$

Permissible values of angle of bite for rolling of steels and other metals as obtained from rolling practice, are given in Table 3.6 and 3.7.

Tiagunov [45] gives an interesting example of the decrease of permissible angle of bite with increasing rolling speed. For smooth rolls and rolling speed

$$v_r \leq 1.6 \text{ m/sec}$$

then

$$\alpha = 25.5^\circ - 2v_r \quad (3.81)$$

where  $v_r$  = peripheral speed of rolls, m/sec.

TABLE 3.6  
MAXIMUM ANGLES OF CONTACT USED IN ROLLING STEELS

| Rolls and lubricant                                              | Maximum angle of bite<br>$\alpha^\circ$ | Coefficient of friction<br>$f$ | $\Delta h_{\max}/D$ |
|------------------------------------------------------------------|-----------------------------------------|--------------------------------|---------------------|
| <i>Cold rolling</i>                                              |                                         |                                |                     |
| Smooth ground rolls lubricated with mineral oil                  | 3-4                                     | 0.052-0.070                    | 1/700-1/420         |
| Chromium steel rolls, medium ground, lubricated with mineral oil | 6-7                                     | 0.105-0.123                    | 1/180-1/120         |
| Dry rough rolls                                                  | up to 8                                 | 0.150                          | 1/97                |
| <i>Hot rolling</i>                                               |                                         |                                |                     |
| Smooth ground rolls                                              | 12-15                                   | 0.212-0.268                    | 1/46-1/29           |
| Plate mill rolls                                                 | 15-22                                   | 0.268-0.404                    | 1/29-1/14           |
| Smooth rolls of small section mills                              | 22-24                                   | 0.404-0.445                    | 1/14-1/12           |
| Rectangular grooves for rolling flats                            | 24-25                                   | 0.445-0.466                    | 1/12-1/11           |
| Box passes                                                       | 28-30                                   | 0.532-0.577                    | 1/8.5-1/7.5         |
| Box passes with ragging                                          | 28-34                                   | 0.532-0.675                    | 1/8.5-1/6           |
| Plain rolls                                                      |                                         |                                |                     |
| (1) according to Geuze                                           | 20°30'                                  | 0.414                          | 1/14                |
| (2) according to Tafel                                           | 24                                      | 0.445                          | 1/12                |
| Ragged rolls                                                     |                                         |                                |                     |
| (1) according to Hirt                                            | 30                                      | 0.577                          | 1/7.5               |
| (2) according to Tafel                                           | 34                                      | 0.675                          | 1/6                 |
| Continuous mills                                                 | 27-30                                   | 0.509-0.577                    | 1/9-1/7.5           |

TABLE 3.7  
MAXIMUM ANGLES OF BITE FOR ROLLING SHEET AND STRIP

| Material of stock and condition of roll surface | Maximum angle of contact, $\alpha$ degrees | Coefficient of friction, $f$ | $\Delta h/D$ | Remarks                                         |
|-------------------------------------------------|--------------------------------------------|------------------------------|--------------|-------------------------------------------------|
| <i>Cold rolling</i>                             |                                            |                              |              |                                                 |
| Mild steel, smooth ground rolls                 | 3-4                                        | 0.05-0.07                    | 1/700-1/420  | Mineral oil lubricant                           |
| Chromium steel medium rolls ground              |                                            |                              |              |                                                 |
| (1) brass                                       | 3-4                                        | 0.05-0.07                    | 1/700-1/420  |                                                 |
| (2) aluminium                                   | 4-5                                        | 0.08-0.09                    | 1/420-1/260  |                                                 |
| (3) copper                                      | 6-7                                        | 0.10-0.13                    | 1/180-1/120  |                                                 |
| (4) steel                                       | 6-7                                        | 0.10-0.13                    | 1/180-1/120  | Kerosene lubricant                              |
| (5) zinc                                        | 7-8                                        | 0.12-0.15                    | 1/140-1/97   |                                                 |
| <i>Hot rolling</i>                              |                                            |                              |              |                                                 |
| Medium ground rolls                             |                                            |                              |              |                                                 |
| (1) aluminium                                   | 9-13                                       | —                            | 1/81-1/38    | Paraffin lubricant. Temperature range 400-500°C |
| (2) aluminium                                   | 13-15                                      | —                            | 1/38-1/29    | Temperature range 400-500°C                     |
| (3) brass                                       | 12-15                                      | —                            | 1/46-1/29    | Temperature range 750-850°C                     |
| (4) copper                                      | 15-18                                      | —                            | 1/29-1/20    |                                                 |

TABLE 3.8  
MAXIMUM ANGLES OF CONTACT FROM TIAGUNOV

| Rolls        | Maximum angles of contact (in degrees) for rolling speed, m/sec |      |      |      |      |      |      |      |
|--------------|-----------------------------------------------------------------|------|------|------|------|------|------|------|
|              | 0                                                               | 0.5  | 1.0  | 1.5  | 2.0  | 2.5  | 3.0  | 3.5  |
| Smooth rolls | 25.5                                                            | 24.5 | 23.5 | 22.5 | 19.5 | 16.0 | —    | 9    |
| Edged passes | 29.0                                                            | 27.5 | 26.0 | 24.5 | 21.0 | 17.0 | 12.0 | —    |
| Ragged rolls | 33.0                                                            | 32.0 | 31.0 | 30.0 | 28.0 | 26.0 | 24.0 | 21.0 |

These values for other types of roll surface are given in Table 3.8.

During rolling both the frictional forces and the coefficient of friction vary along the arc of contact.

Korolev [46] published the first attempts to calculate the variation of frictional resistance, in which he differentiated the values of roll pressure distribution along the arc of contact.

Since the frictional forces or their variation along the arc of contact in the roll gap cannot be calculated accurately, certain mean values for the whole roll gap are

substituted. These values depend on the temperature of rolling stock and the condition of roll surface.

At first two basic methods of determining the average coefficient of friction in rolling were used, one based on the maximum angle of bite and the other on the maximum value of forward slip.

In both these methods it was assumed that the coefficient of friction  $f$  is constant along the arc of contact.

In the first method, measuring the maximum angle of bite, the friction coefficient  $f$  is determined only for the moment when the outer edges of the bar contact the roll surface. The value obtained relates only to the point of bite and to a certain slip velocity produced by revolution of rolls. Therefore this value cannot express the mean coefficient of friction  $f$  along the arc of contact, since the slip is maximum in the plane of bite, then decreases to zero at the neutral plane and increases towards the plane of exit after change of direction of slip. The rolling load also is not uniform. The maximum angle of bite depends also on the force with which the metal is pushed into the rolls. Although this force is small, it affects the coefficient of friction  $f$ , which also causes scatter of the resulting values.

In the rolling of sheet, narrow band and strip, high rolling loads and flattening of rolls make the application of the method based on measuring the maximum angle of bite quite unsuitable.

In determining  $f$  by the method based on the maximum value of forward slip, it is necessary to combine formulae for forward slip with those for no-slip angle. In doing so, many erroneous assumptions are made, e.g.:

- (a) that the roll force is constant along the arc of contact,
- (b) that the plastic deformation of metal is parallelepipedal,
- (c) that metal does not spread in rolling,
- (d) that the coefficient of friction is constant over the whole length of arc of contact.

This method seems to be highly doubtful, since the measurement of forward slip on the surface of the bar does not represent the true situation due to entirely different type of metal flow on the surface and in the interior of the bar.

The method of measuring the roll force and the front tension when the rolls slip on the bar, as given by Pavlov [47], allows the coefficient of friction  $f$  to be measured only in certain particular cases, i.e. when the neutral plane coincides with the exit plane and the metal slips on the rolls. This condition does not represent standard rolling conditions. However, by this method the influence of high rolling loads and considerable rolling speeds on the slip of metal during rolling can be demonstrated. This method is suitable for cold rolling, though high roll forces can press the lubricant out of the rolls, causing a situation of "dry" friction.

Bland and Ford [36] gave better methods of determining the mean coefficient of friction in cold rolling. These methods were tested in practice and the results compared with those obtained from calculations based on rolling theory. The results obtained were in good general agreement with theoretical assumptions.

One of these methods is similar to that given by Pavlov. The principle of this method is represented in Fig. 3.51. Under standard conditions of rolling the frictional forces change their direction at the neutral plane. However, if large back tension or heavy draught is applied, the neutral plane shifts towards the exit of the rolls and the metal will slip on the roll surface. For this condition the coefficient of friction can be calculated from the rolling load, the magnitude of back tension and torque.

The authors reject straight away the method based on measuring the maximum angle of bite, due to uncertainty in determining the true values of  $h_1$  and  $h_2$ .

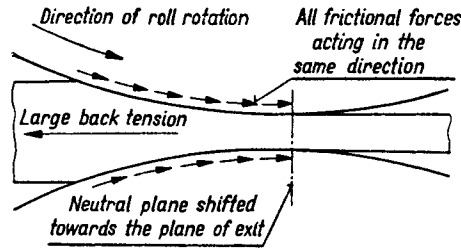


Fig. 3.51. Influence of large back tension in rolling [36]

In their opinion it is much better to make use of the measured values of roll load and to calculate the coefficient of friction from one of the existing formulae for no-slip angle, assuming that friction at the plane of exit is zero.

The distribution of roll force along the arc of contact is given by the equation

$$P = \int_0^{x=l_d} \sigma dx \quad (3.82)$$

where

$\sigma$  = normal stress produced by radial pressure of rolls, as calculated from the accepted theories of rolling,

$l_d$  = projection of arc of contact.

From the measured values the average coefficient of friction  $f_m$  can be derived.

For practical calculations the following formulae are most frequently used.

In hot rolling of steel Ekelund's empirical formula [34] is used, expressing the coefficient of friction relative to the temperature of rolling:

(1) for cast iron and rough steel rolls

$$f = 1.05 - 0.0005 t$$

(2) for chilled and smooth steel rolls

$$f = 0.8(1.05 - 0.0005 t)$$

(3) for ground steel rolls (from Siebel)

$$f = 0.55(1.05 - 0.0005 t)$$

(3.83)

where  $t$  = rolling temperature, °C.

Figure 3.52 gives the coefficients of friction calculated from (3.83).

Bachtinov [147] proposes a modification to Ekelund's formulae to allow for the influence of rolling speed:

$$f = aK_1(1.05 - 0.0005 t) \quad (3.84)$$

where  $a$  = depends on roll quality (3.83),

$K_1$  = depends on rolling speed  $v_r$ :

| $v_r$ , m/sec | up to 2 | 3   | 4   | 5    | 6    | 7   | 8    | 9    | 10   | 12   | 14   | 16   | 18   | 20 to 22 |
|---------------|---------|-----|-----|------|------|-----|------|------|------|------|------|------|------|----------|
| $K_1$         | 1       | 0.9 | 0.8 | 0.72 | 0.66 | 0.6 | 0.57 | 0.55 | 0.52 | 0.47 | 0.45 | 0.43 | 0.42 | 0.41     |

Figure 3.53 after Tafel [37] and Schneider represents the graphical relation between the angle of bite and the peripheral speed of rolls in hot rolling. It applies to rolling

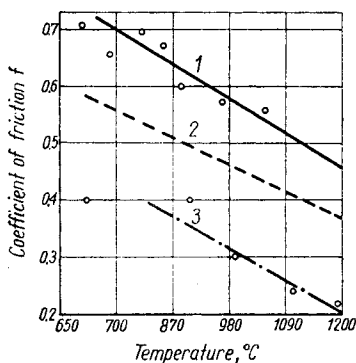


Fig. 3.52. Variation of coefficient of friction in hot rolling [10]: 1—cast iron and rough steel rolls (from Ekelund), 2—steel and chilled smooth rolls (from Ekelund), 3—smooth ground steel rolls (from Siebel)

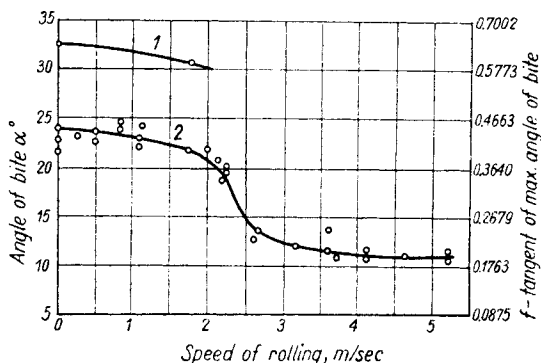


Fig. 3.53. Variation of angle of bite with rolling speed for hot rolling of mild steel in blooming mills: 1—rough rolls, 2—smooth rolls

of mild steel at 1200°C on rough and smooth rolls, respectively. By substituting  $f = \tan \alpha_{\max}$ , the required value of coefficient of friction relative to the peripheral speed of rolls can be read off.

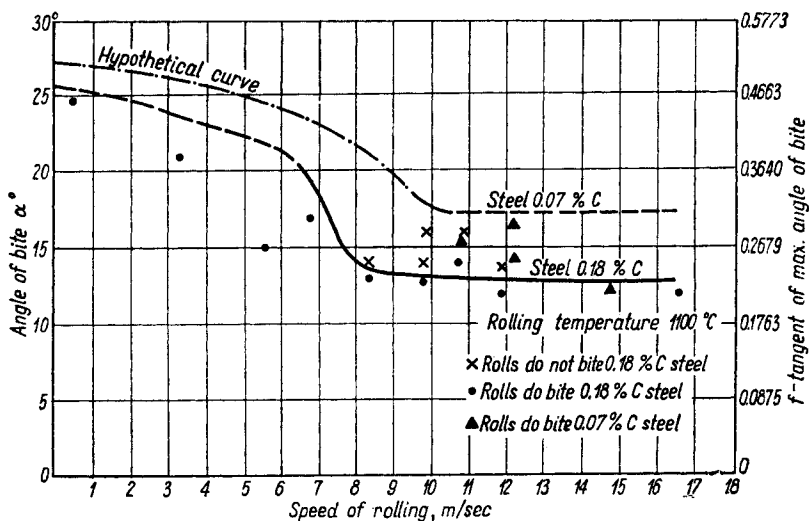


Fig. 3.54. Variation of angle of bite with rolling speed in grooved rolls

Z. Wusatowski and R. Wusatowski [39] carried out similar rolling tests on mild steel bars with dimensions  $65 \times 65$  to  $20 \times 20$  mm and carbon contents 0.07% and 0.18% at 1100°C with a wide range of rolling speeds.

The experimental results, represented in Fig. 3.54, show that the change of slope occurs at a much higher rolling speed than in the results obtained by Tafel and Schneider.

In general it can be accepted that the coefficient of friction in hot rolling varies from 0.25 to 0.7, the values from 0.55 to 0.7 being obtained only in rolling on ragged rolls.

Pavlov and Kuprin [47] carried out extensive research in the problem of deriving the coefficient of friction for hot rolling steel. For chemical analysis of the tested steels see Table 3.9.

TABLE 3.9  
CHEMICAL ANALYSIS OF TESTED STEELS

| Steel grade | Composition, % |      |      |       |       |      |       |      |
|-------------|----------------|------|------|-------|-------|------|-------|------|
|             | C              | Mn   | Si   | S     | P     | Ni   | Cr    | Cu   |
| 30HGSA      | 0.21           | 0.96 | 1.12 | 0.028 | 0.013 | 8.50 | —     | —    |
| H18N9       | 0.12           | 0.85 | 0.37 | 0.012 | 0.018 | 8.50 | 17.98 | —    |
| EJ94        | 0.79           | 13.8 | 0.65 | 0.025 | 0.03  | 3.20 | 0.20  | —    |
| St10        | 0.12           | 0.43 | 0.27 | 0.032 | 0.016 | 0.13 | 0.12  | —    |
| St20        | 0.17           | 0.40 | 0.35 | 0.040 | 0.014 | 0.12 | 0.10  | 0.20 |
| St40        | 0.43           | 0.67 | 0.33 | 0.027 | 0.020 | 0.16 | 0.14  | 0.20 |
| A12         | 0.12           | 0.69 | 0.24 | 0.184 | 0.018 | —    | —     | —    |
| A20         | 0.21           | 0.65 | 0.21 | 0.154 | 0.016 | —    | —     | —    |
| A40         | 0.43           | 1.32 | 0.29 | 0.246 | 0.026 | —    | —     | —    |

Figure 3.55 gives the relation between coefficient of friction and rolling speed, and Fig. 3.56 the relation between coefficient of friction and temperature for rolling speeds varying from 0 to 3.5 m/sec.

Figures 3.56–3.59 give coefficients of friction relative to temperature and rolling speed for five different steels.

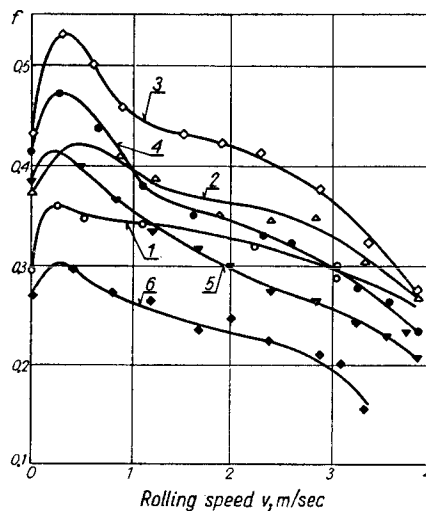


Fig. 3.55. Variation of coefficient of friction with rolling speed for St10 steel at temperature:  
1 — 735°C, 2 — 820°C, 3 — 902°C, 4 — 1020°C, 5 — 1100°C, 6 — 1212°C

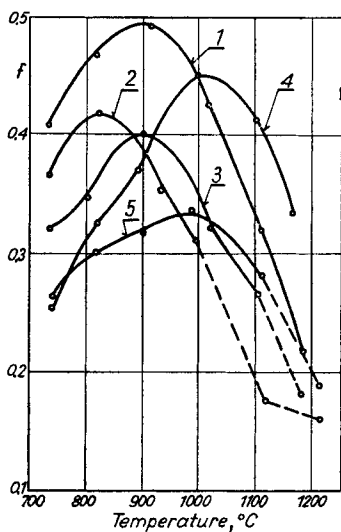


Fig. 3.56. Variation of friction coefficient with temperature for different steels [47]. The velocity of slip — 0.5 m/sec. 1 — St20, 2 — A20, 3 — 30HGSA, 4 — H18N9, 5 — EJ94

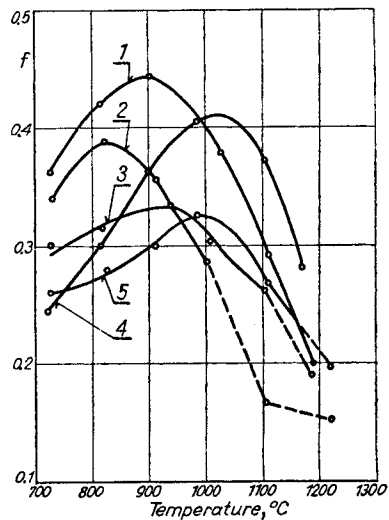


Fig. 3.57. Variation of friction coefficient with temperature for different steels. Velocity of slip — 1.0 m/sec. Steels as for Fig. 3.56

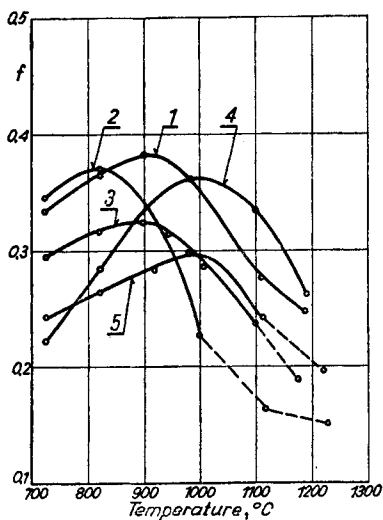


Fig. 3.58. Variation of friction coefficient with temperature for different steels. Velocity of slip 2.0 m/sec. Steels as for Fig. 3.56

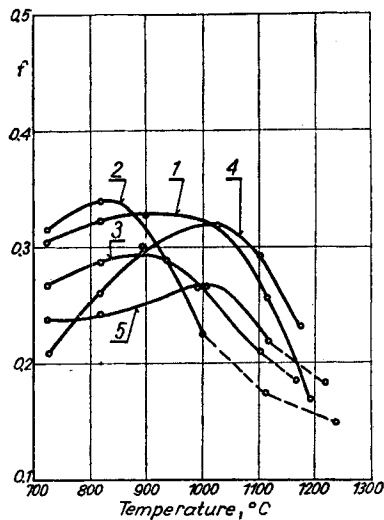


Fig. 3.59. Variation of friction coefficient with temperature for different steels. Velocity of slip 3.0 m/sec. Steels as for Fig. 3.56



*Example*

A  $200 \times 250$  mm bloom is rolled at  $1000^\circ\text{C}$  on 700 mm dia. steel rolls,  $\Delta h = 50$  mm.

(1) Coefficient of friction is found from equation (3.83) for cast iron and rough steel rolls

$$f = 1.05 - 0.0005 t = 1.05 - 0.0005 \times 1000 = 0.55$$

for chilled and smooth steel rolls

$$f = 0.8(1.05 - 0.0005 t) = 0.8(1.05 - 0.0005 \times 1000) = 0.44$$

for ground steel rolls

$$f = 0.55(1.05 - 0.0005 t) = 0.55(1.05 - 0.0005 \times 1000) = 0.30$$

(2) Coefficient of friction is calculated from equation (3.72) using the condition

$$f = \tan \rho = \tan \alpha_{\max}$$

$$\tan \alpha_{\max} = \frac{\sqrt{R\Delta h}}{R - \Delta h/2} = \frac{\sqrt{350 \times 50}}{350 - 50/2} = \frac{17,500}{325} = 0.408$$

$$f = \tan \rho = \tan \alpha_{\max} = 0.408$$

$$\alpha_{\max} = 21^\circ 50'$$

From Fig. 3.48 for  $D = 700$  mm and  $\Delta h = 50$  mm the coefficient of friction  $f = 0.4$ , a value very close to those previously found, i.e.  $f = 0.44$  (for smooth steel rolls) and  $f = 0.408$  from equation (3.72).

The effect of rolling conditions on the magnitude of coefficient of friction in cold rolling of steel has also been investigated.

Pomp and Lueg [48] determined experimentally the influence of shape of stock and condition of roll surface on the coefficient of friction using the method of maximum forward slip, in cold rolling of steel strip. The results are given in Table 3.10. Table 3.11 gives values of  $f$  for other metals, roll surfaces and lubricants.

From Table 3.10 the lowest and highest values of  $f$  in cold rolling of steel are about 0.07 and 0.15. From other sources [49] the lowest value of the coefficient of friction

TABLE 3.10  
INFLUENCE OF STOCK MATERIAL AND CONDITION OF ROLL SURFACE  
ON COEFFICIENT OF FRICTION IN COLD ROLLING OF STEEL

| Material of stock, material and surface of rolls                                                                            | Coefficient of friction $f$ | Test conditions                                                                                                                                                   |
|-----------------------------------------------------------------------------------------------------------------------------|-----------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Chemical analysis of steel<br>0.02% C, 0.30% Mn<br>0.17% C, 0.72% Mn<br>0.37% C, 0.81% Mn, 0.25% Si                         | 0.073<br>0.089<br>0.11      | Strip rolled with smooth dry chromium steel rolls. Roll diameter about 46 mm, 10 rpm, strip dimensions $2 \times 30 \times 600$ mm                                |
| Roll material<br>Chilled cast iron<br>Chromium steel                                                                        | 0.078<br>0.087              | Carbon steel strip 0.17% C, 0.72% Mn on rolls with polished surface. The strip was bright annealed. Lubrication with oil emulsion. Roll dia. about 180 mm, 10 rpm |
| Condition of roll surface<br>(a) smooth (polished) lubricated rolls<br>(b) smooth dry rolls<br>(c) rough sand-blasted rolls | 0.07<br>0.09–0.11<br>0.15   | Strips $2 \times 30$ mm of 0.17% C steel. Chromium steel roll 36 rpm                                                                                              |

TABLE 3.11  
 VARIATION OF COEFFICIENT OF FRICTION WITH TYPE OF LUBRICANT FOR VARIOUS  
 METALS IN COLD ROLLING [10]

| Material and condition<br>of roll surface     | Type of lubricant           |                  |          |       |                |
|-----------------------------------------------|-----------------------------|------------------|----------|-------|----------------|
|                                               | without<br>lubri-<br>cation | molten<br>tallow | kerosene | wax   | machine<br>oil |
| Pickled aluminium, ground rolls               | 0.212                       | 0.112            | 0.174    | 0.09  | —              |
| Pickled aluminium, metal<br>sticking to rolls | 0.247                       | 0.124            | 0.254    | 0.215 | —              |
| Pickled brass, ground rolls                   | 0.109                       | 0.095            | 0.119    | —     | 0.095          |
| Pickled brass, metal sticking to<br>rolls     | 0.116                       | 0.099            | 0.129    | —     | 0.126          |
| Non-pickled copper, ground rolls              | 0.123                       | 0.101            | —        | —     | —              |
| Non-pickled copper with copper<br>oxide layer | 0.141                       | 0.108            | —        | —     | —              |

is 0.04, and this lowest value occurs with high peripheral speed of rolls in cold rolling with oil emulsion as lubricant (Table 3.12).

To establish the change of influence of friction with change of rolling speed Sims and Arthur [50] carried out a series of tests on an experimental rolling mill in Sheffield. They rolled mild carbon steel and copper strips, varying the rolling speeds from 0.05–1.6 m/sec and with draughts up to 30%.

TABLE 3.12  
 VARIATION OF COEFFICIENT OF FRICTION WITH TYPE OF LUBRICANT IN COLD ROLLING  
 OF VARIOUS METALS (FROM ROKOTIAN) [10]

| Material of rolled<br>stock | Coefficient of friction |                  |                           |
|-----------------------------|-------------------------|------------------|---------------------------|
|                             | without<br>lubrication  | with<br>kerosene | with light<br>machine oil |
| Steel                       | 0.20–0.30               | 0.15–0.17        | 0.10–0.13                 |
| Copper                      | 0.20–0.25               | 0.13–0.15        | 0.10–0.13                 |
| Aluminium                   | 0.20–0.30               | 0.10–0.15        | 0.08–0.09                 |
| Brass                       | 0.20–0.25               | 0.10–0.15        | 0.08                      |
| Zinc                        | 0.25–0.30               | 0.12–0.15        | 0.09                      |

In these experiments three types of lubricants were used: ordinary rolling oil, a suspension of dried magnesium oxide with No. 80 colloidal graphite in spirit, and dry lubricants i.e. colloidal talc and colloidal vermiculite.

Rolling tests carried out at constant rolling speed and with constant roll force showed that the influence of these lubricants on strip thickness varies (Fig. 3.60).

It can be seen from this figure that the oils used in these experiments had a negative effect, resulting in a considerable decrease in strip thickness, whereas graphite caused an increase in strip thickness, or maintained a constant thickness. Talc and vermiculite had an effect similar to that for graphite. In rolling without lubricant on polished rolls the coefficient of friction decreased from 0.05 at 0.05 m/sec to 0.04 at 1.5 m/sec. Under the same conditions it decreased from an initial value of 0.80 to 0.048.

An interesting and important relationship between drop of coefficient of friction and increase of rolling speed is reported in American publications [182] (Fig. 3.61).

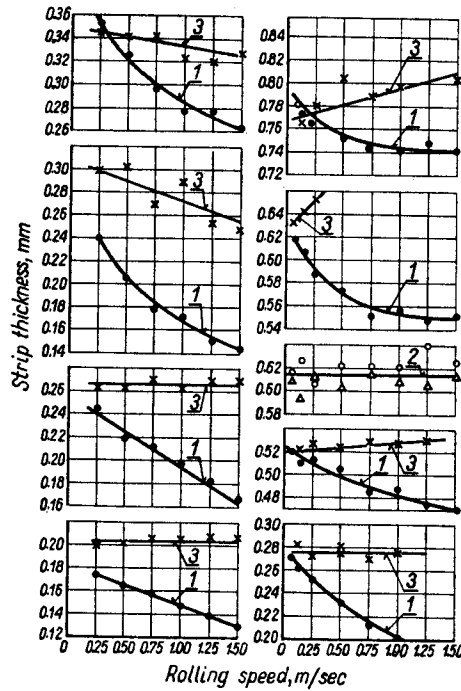


Fig. 3.60. Effect of rolling speed on thickness of copper strip rolled with different lubricants [50]: 1 — oil, 2 — talc or vermiculite, 3 — oil with graphite

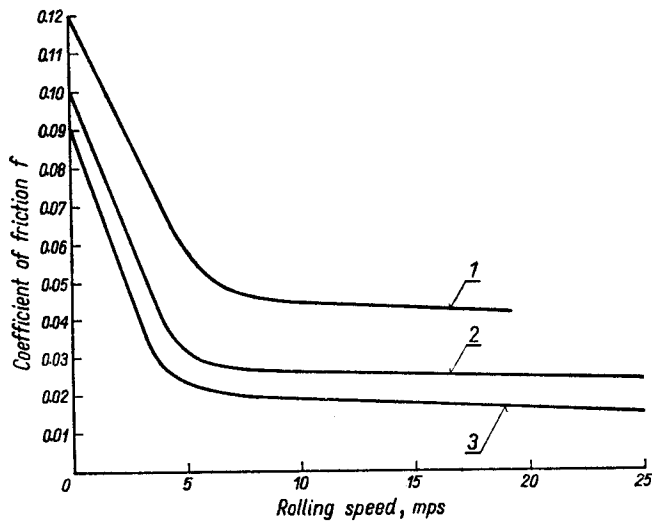


Fig. 3.61. Variation of coefficient of friction with rolling speed and type of lubricant: 1 — oil emulsion, 2 — palm oil emulsion, 3 — palm oil

This figure shows values of  $f$  from 0.12 to 0.02 obtained with palm oil and water as lubricant. It can be seen that a rapid decrease of the coefficient of friction occurs at rolling speeds starting from 6 m/sec. At higher rolling speeds no great changes occur. It should be remembered that in this case the decrease in coefficient of friction causes a lowering of roll force with increase of rolling speed.

### 3.7. Calculation of Rolling Temperature

In drawing up a rolling programme for an existing or newly designed mill there is always the problem of correct determination of rolling temperature. The roll pass designer when designing the roll passes, their sequence and layout in the roll stands, must also determine the rolling temperatures.

In practice, the temperature of stock on removing from the heating furnace is usually known. Knowing the time of travel from the furnace to the first roll stand, then the temperature of metal in the first pass can be determined or measured directly. The temperature of stock in the following passes is assumed from experience, as lack of time and the position of the roll guides prevents accurate measurement.

The factors affecting the cooling of stock during rolling include heat losses by radiation, convection and conduction, temperature increase due to partial conversion of deformation work into heat and heat losses resulting from scale formation and descaling.

If most of these factors are disregarded and only the radiation loss is considered, heat loss can be calculated from Stefan-Boltzmann's relation [51]

$$Q_s = \varepsilon C_s \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_{am}}{100} \right)^4 \right] z F \quad (3.85)$$

where

$Q_s$  = quantity of heat lost in radiation, kcal,

$\varepsilon \approx 0.8$  = emissivity of different rolled products,

$C_s = 4.96$  = radiation constant of black body, kcal/m<sup>2</sup> h °K<sup>4</sup>,

$z$  = radiation time, hrs,

$F$  = radiation surface of product, m<sup>2</sup>,

$T_1$  = absolute temperature of the radiating product at the beginning of radiation time  $z$ ,

$T_{am}$  = absolute ambient temperature (273 + 20 = 293°K).

At the ambient temperature of 20°C the expression

$$\left( \frac{T_{am}}{100} \right)^4 \approx 74$$

The heat lost by radiation should amount to

$$Q_s = G c \Delta t \quad (3.86)$$

where

$G$  = weight of rolled stock, kg,

$c$  = average specific heat (for steel above 800°C)  $c = 0.166$  kcal/kg °C,

$\Delta t$  = difference in metal temperature at the beginning and at the end of radiation time,  $z$ , or  $T_1 - T_2 = \Delta t$ ,

hence

$$\Delta t = \frac{Q_s}{G_s}$$

Substituting equation (3.86) into equation (3.85) gives for steel

$$\Delta t = \frac{4.0 z F}{G \times 0.166} \left[ \left( \frac{T_1}{100} \right)^4 - 74 \right] ^\circ \text{C} \quad (3.87)$$

In this way the drop of temperature in each pass can be calculated, if only the radiation loss is taken into account. Knowing  $T_1$  as the initial temperature, from equation (3.87) the required value of  $T_2$  can be calculated.

Then the temperature change after each pass can be calculated when the cross-section area  $F$  and the pass time  $z$  are known.

To simplify the application of equations (3.85)–(3.87), H. Neumann [52] has derived a nomogram shown in Fig. 3.62. Part *A* in this figure represents dimensions of stock, part *B* the radiation equation (3.85) and the quantity of heat lost by radiation (3.86), part *C* the weight of product and part *D* the time factor.

To use this nomogram values of length of bar and its perimeter must be known (part *A*) from which the area of the radiating product is determined.

From point 2 in part *A*, giving dimensions of bar, a vertical line 2–3 is drawn, to cut the appropriate temperature line in part *B*. Next a horizontal line 3–4 cutting the line corresponding to weight of bar in part *C* and a vertical line 4–5 to cut the line giving time of radiation in part *D* are drawn. From point 5 a straight line perpendicular to the  $\Delta t$  axis gives the difference in temperature, in  $^\circ\text{C}$ .

The numerical example given below illustrates the method of determining the drop in temperature of a rolled product from the nomogram in Fig. 3.62.

#### Example

To determine the drop in temperature of rolled stock from the nomogram in Fig. 3.62, where the bar is 11 m long, perimeter 18 cm, weight 300 kg. The initial temperature  $t_1 = 1000^\circ\text{C}$ , cooling time—10 second.

On the “perimeter of bar” scale (part *A* of nomogram) the stock perimeter of 18 cm is denoted by the number 1. From here a horizontal line is drawn to cut the diagonal line *II* corresponding to “length of bar” at point 2. From here a vertical is drawn to cut the diagonal in part *B* representing initial temperature of  $1000^\circ\text{C}$  at point 3. From 3 a horizontal line is drawn to cut the line giving the weight of bar (300 kg), in part *C* at point 4. From 4 a vertical line is drawn to the appropriate “time of radiation” diagonal in part *D*, giving point 5. For a cooling time of 10 seconds a temperature drop of  $12.5^\circ\text{C}$  is found on  $\Delta t$  scale, point 6.

Final temperature of bar will be  $1000 - 12.5 = 987.5^\circ\text{C}$ .

A. Geleji [20] derived a different equation for calculating temperature during rolling. The final temperature is obtained from the formula

$$t_{rd} = t_1 e^{-\alpha R} \quad (3.88)$$

with the cooling time divided into 30 sec intervals,  
where

$t_1$  = initial temperature,

$t_{rd}$  = required temperature.

$$\alpha = 0.036 \left( \frac{t - 800}{500} \right) + 0.02 \quad (3.89)$$

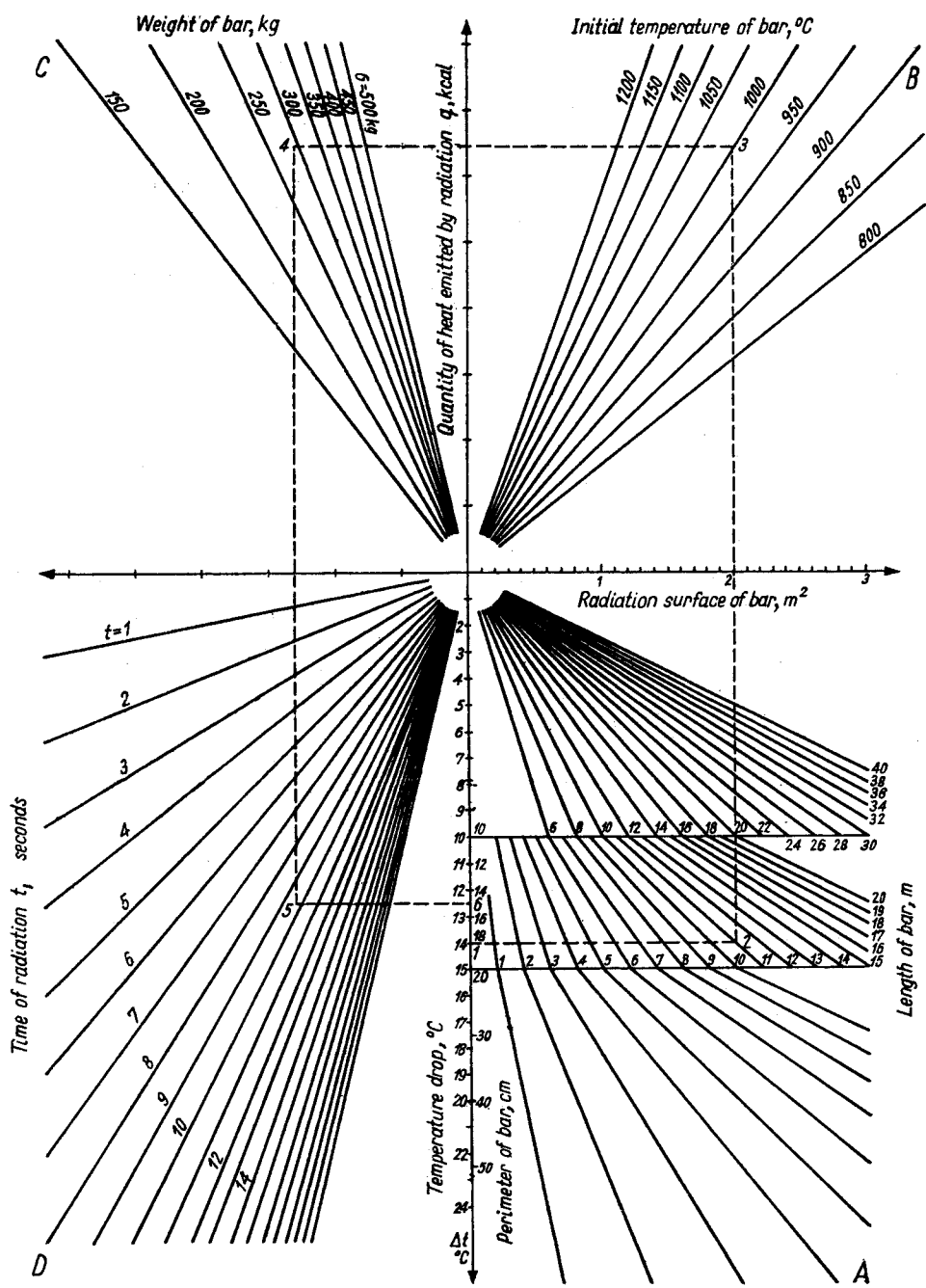


Fig. 3.62. Nomogram for determining the radiation loss in rolling bars [52]

where for  $t$  an approximate temperature of stock should be substituted

$$R = \frac{1}{Gc} \left[ Q_1 z + 0.5(Q_2 - Q_1) \frac{z^2}{z_g} \right] \quad (3.90)$$

where

$Q_1$  = initial radiation surface of stock =  $U_1 l_1$  perimeter  $\times$  length,

$Q_2$  = radiation surface of stock after pass =  $U_2 l_2$  perimeter  $\times$  length,

$z$  = time interval of 30 sec,

$z_g$  = total time of rolling, sec.

Geleji calculated the heat losses of stock due to convection from the formula

$$\Delta t_d = \frac{2l_d b(t_p - t_r) z_c \varphi}{Gc} \quad ^\circ\text{C} \quad (3.91)$$

where

$t_p$  = temperature of stock,

$t_r$  = roll temperature,

$z_c$  = duration of pass,

$\varphi$  = 5000–1000, kcal/m<sup>2</sup>  $^\circ\text{C}$  h.

Since the work of deformation is converted into heat, the heat gain of stock will be

$$\Delta t_\alpha = \frac{1}{A} \frac{L}{Gc} = \frac{1}{427} = \frac{FK_{wm}v}{Gc} t_p \quad ^\circ\text{C} \quad (3.92)$$

where

$$v = \frac{(l_2 - l_1)}{z_g} = \text{speed of rolls, m/sec,}$$

$$F = \frac{2G}{b_1 \gamma_c} + 2l_1 b_1,$$

$K_{wm}$  = mean resistance to deformation.

To obtain the effective drop in temperature of stock, all these factors have to be taken into account.

#### Example (from Geleji)

From a 130  $\times$  130 mm bloom of 118 kg weight a mine rail of 4.6 kg/m weight has to be rolled.

|                                 |                              |
|---------------------------------|------------------------------|
| Initial temperature of bloom    | $t_1 = 1200^\circ\text{C}$   |
| weight of bloom                 | $G = 118 \text{ kg}$         |
| initial length                  | $l_1 = 0.89 \text{ m}$       |
| initial cross-section           | $h_1 = b_1 = 0.13 \text{ m}$ |
| length after rolling            | $l_2 = 25.5 \text{ m}$       |
| initial section area            | $F_1 = 169 \text{ cm}^2$     |
| final section area              | $F_2 = 5.9 \text{ cm}^2$     |
| radiation surface area of bloom | $Q_1 = 0.462 \text{ m}^2$    |
| radiation surface area of rail  | $Q_2 = 4.25 \text{ m}^2$     |
| total rolling time              | $z_g = 119.76 \text{ sec}$   |

$$Q_1 z = 0.462 \times 30 = 13.86 \text{ m}^2/\text{sec}$$

$$Q_2 - Q_1 = 4.25 - 0.462 = 3.79 \text{ m}^2$$

$$\frac{z^2}{z_g} = \frac{(30)^2}{119.76} = 7.5$$

$$Gc = 118 \times 0.16 = 19 \text{ kcal}/^\circ\text{C}$$

then from equation (3.90)

$$R = \frac{1}{19} (13.86 + 0.5 \times 3.79 \times 7.5) = 1.47$$

and hence from equation (3.88)

$$t_{rd} = t_1 e^{-\alpha \times 1.47}$$

assuming

$t_{rd} = t_1$  multiplied by 30, 60, 90 and 120, °C

$$t_1 = 1200^\circ\text{C}$$

$$t_{30} = 1200 e^{-0.047 \times 1.47} = 1122^\circ\text{C}$$

$$t_{60} = 1122 e^{-0.041 \times 1.47} = 1060^\circ\text{C}$$

$$t_{90} = 1060 e^{-0.037 \times 1.47} = 1005^\circ\text{C}$$

$$t_{120} = 1005 e^{-0.033 \times 1.47} = 960^\circ\text{C}$$

The work required to roll this mine rail is 930,000 kgm, corresponding to 2180 kcal. This amount of heat raises the temperature of the rail by 115°C. The heat lost by convection amounts to 2420 kcal, which corresponds to a drop in temperature of the rolled rail of 128°C.

The temperature drop is shown in Fig. 3.63 [20].

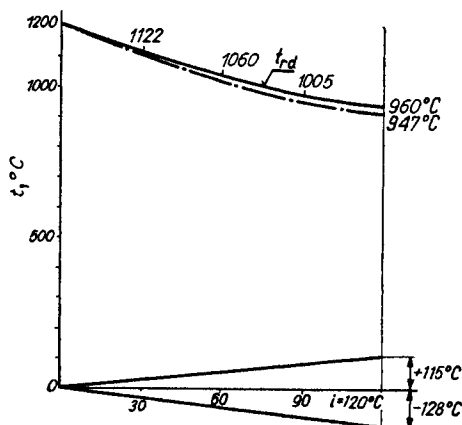


Fig. 3.63. Example of calculation of temperature of stock during rolling

The temperature of metal in successive passes and the total drop in temperature during one rolling cycle when rolling sheet can be calculated using the method published by Tiagunov [53].

Tiagunov's formulae are empirical formulae based on those used in thermodynamics. By analysing a very great number of experimental results collected in Soviet metallurgical plants he succeeded in deducing the most important constants, and verified them in actual practice.

The drop in temperature of sheet during rolling is due to:

- (1) radiation loss from sheet surface,
- (2) loss of heat by convection to the surrounding air,
- (3) loss of heat by conduction due to sticking of sheet to the rolls, the table rollers or to contact with the cooling water.

Apart from these heat losses there is also a certain heat gain during rolling due to heat generated by the deformation of metal.

By making certain modification to formulae used in thermodynamics, Tiagunov derived appropriate working formulae for each type of heat loss during rolling of sheet.



The resultant value of heat lost by radiation, convection and conduction minus the heat generated due to the deformation of metal, will give the drop in temperature of sheet in one pass. Tiagunov's formula for calculating the drop in temperature is

$$\Delta t = \frac{(T/100)^2}{4.5} \frac{z}{h} = K \frac{z}{h} \quad (3.93)$$

where

$T$  = absolute temperature of sheet in a pass, in °K,

$z$  = cooling time, sec,

$h$  = thickness of sheet in given pass, mm.

Tiagunov took 1400°C as the theoretical upper limit of temperature and 400°C as the lower limit of temperature of sheet during rolling. Taking 400°C as the lower limit of temperature, Tiagunov puts forwards the hypothesis that at that rolling temperature there is no heat loss.

In equation (3.93), called by Tiagunov the quadratic equation of temperature drop, the expression

$$\frac{(T/100)^2}{4.5} \quad (3.94)$$

is denoted by the symbol  $K$ .

For the temperature range 1400–400°C this expression can be replaced, with sufficient accuracy for practical purposes, by the expression

$$K' = 0.625 t - 25 = \frac{t - 400}{16} \quad (3.95)$$

This expression allows the quadratic equation of temperature drop to be simplified to the following form

$$\Delta t = \frac{t_n - 400}{16} \frac{z}{h} \quad (3.96)$$

where  $t_n$  = temperature of sheet in the given pass, in °C.

This equation Tiagunov called the linear equation of temperature drop.

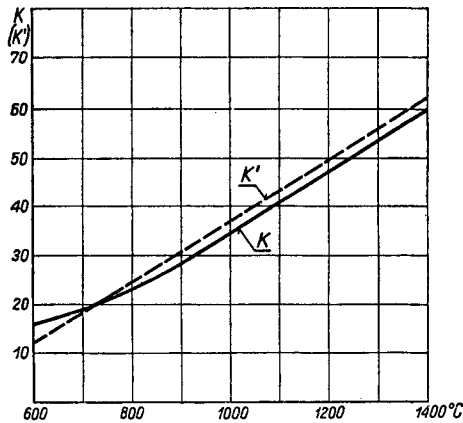


Fig. 3.64. Specific temperature drop  $K$  and  $K'$  calculated from Tiagunov's linear equation

The symbols  $K$  and  $K'$  in these equations are called the specific temperature drop [53]. Their values are plotted in Fig. 3.64.

Sheet bar, 7.1 mm thick, is to be rolled down to 5 mm in the first pass at 850°C. The reduction scheme is shown in Table 3.13.

TABLE 3.13  
DRAUGHT SEQUENCE

| Specification | Rolling sheet bars down to pack |       |       |       |       | Pack rolling two sheet bars together |     |     |
|---------------|---------------------------------|-------|-------|-------|-------|--------------------------------------|-----|-----|
|               | $h_0$                           | $h_1$ | $h_2$ | $h_3$ | $h_4$ |                                      |     |     |
| No. of pass   | 0                               | 1     | 2     | 3     | 4     | 0                                    | 1   | 2   |
| Thickness, mm | 7.1                             | 5.0   | 3.7   | 2.9   | 2.5   | 5.0                                  | 3.9 | 3.3 |

The average duration of each pass is 4 seconds. Since the temperature of metal in the first pass is known, 850°C, the drop of temperature between the first and the second pass is calculated from the linear equation (3.96)

$$\Delta t_1 = t_1 - t_2 = \frac{t_1 - 400}{16} \frac{z}{h_1} = \frac{850 - 400}{16} \frac{4}{5} = 22.5 \approx 23^\circ\text{C}$$

The temperature in the second pass is

$$t_2 = t_1 - \Delta t_1 = 850 - 23 = 827^\circ\text{C}$$

The drop in temperature before the third pass will be

$$\Delta t_2 = t_2 - t_3 = \frac{t_2 - 400}{16} \frac{z}{h_2} = \frac{827 - 400}{16} \frac{4}{3.7} = 29^\circ\text{C}$$

The temperature in the third pass will be

$$t_3 = t_2 - \Delta t_2 = 827 - 29 = 798^\circ\text{C}$$

The temperature drop in the following passes is calculated similarly and the resultant temperatures in individual passes are given in Table 3.14.

TABLE 3.14  
TEMPERATURES IN SUCCESSIVE PASSES

| No. of pass     | 1   | 2   | 3   | 4   | 1   | 2   |
|-----------------|-----|-----|-----|-----|-----|-----|
| Temperature, °C | 850 | 827 | 798 | 764 | 746 | 724 |

Table 3.15 gives a comparison of measured temperatures of sheet in individual passes and those calculated from Tiagunov's formulae in a Soviet steelworks [53].

In practice, it is often necessary to determine in advance the total temperature loss during the whole rolling cycle for a given sheet gauge.

Tiagunov introduced an auxiliary concept, the so-called summary temperature drop in a given pass. This drop is defined as the difference between a constant arbitrary temperature  $t$  (corresponding to the temperature of stock) and the temperature in a given pass,  $t_n$ .

The summary temperature drop in a pass is expressed by Tiagunov as

$$\Delta t_n = t_0 - t_{n+1}$$

TABLE 3.15  
TEMPERATURE DROP IN ROLLING OF SHEET

| Operation          | No. of pass | Rolled sheets<br>3 × 730 × 1160 mm |            |                         |                         | Rolled sheets<br>2.5 × 735 × 1180 mm |            |                         |                         | Rolled sheets<br>2 × 685 × 1400 mm |            |                         |                         |
|--------------------|-------------|------------------------------------|------------|-------------------------|-------------------------|--------------------------------------|------------|-------------------------|-------------------------|------------------------------------|------------|-------------------------|-------------------------|
|                    |             | $h_1$<br>mm                        | $z$<br>sec | $t_{\text{meas}}$<br>°C | $t_{\text{calc}}$<br>°C | $h_1$<br>mm                          | $z$<br>sec | $t_{\text{meas}}$<br>°C | $t_{\text{calc}}$<br>°C | $h_1$<br>mm                        | $z$<br>sec | $t_{\text{meas}}$<br>°C | $t_{\text{calc}}$<br>°C |
| Rolling sheet bars | 0           | 21.5                               | —          | —                       | —                       | 17.5                                 | —          | —                       | —                       | 18.1                               | —          | —                       | —                       |
|                    | 1           | 17.4                               | 4.5        | 850                     | 850                     | 13.4                                 | 5          | 840                     | 840                     | 14.1                               | 4.5        | 850                     | 850                     |
|                    | 2           | 14.1                               | 4.5        | —                       | 843                     | 10.0                                 | 5          | —                       | 830                     | 11.0                               | 4.5        | —                       | 841                     |
|                    | 3           | 11.4                               | 4.5        | 830                     | 834                     | 7.8                                  | 5          | —                       | 816                     | 8.6                                | 4.5        | 830                     | 830                     |
|                    | 4           | 9.25                               | 4.5        | —                       | 823                     | 5.9                                  | 5          | 790                     | 798                     | 6.7                                | 4.5        | —                       | 815                     |
|                    | 5           | 5.75                               | 4.5        | 800                     | 810                     | 4.5                                  | 5          | —                       | 784                     | 5.3                                | 4.5        | 800                     | 796                     |
|                    | 6           | 6.1                                | 4.5        | —                       | 788                     | 3.5                                  | 5          | 740                     | 743                     | 4.7                                | 4.5        | —                       | 772                     |
|                    | 7           | 4.95                               | 4.5        | —                       | 767                     | —                                    | —          | —                       | —                       | 3.2                                | 4.5        | 740                     | 741                     |
|                    | 8           | 3.95                               | 4.5        | 750                     | 742                     | —                                    | —          | —                       | —                       | —                                  | —          | —                       | —                       |
| Reheating          | —           | —                                  | —          | —                       | —                       | —                                    | —          | —                       | —                       | —                                  | —          | —                       | —                       |
| Rolling of packs   | 1           | 3.48                               | 4.5        | 860                     | 850                     | 3.0                                  | 5          | 850                     | 850                     | 2.8                                | 6          | 850                     | 810                     |
|                    | 2           | 3.07                               | —          | 800                     | 800                     | 2.8                                  | 5          | —                       | 803                     | 2.4                                | 6          | —                       | 790                     |
|                    | 3           | —                                  | —          | —                       | —                       | 2.5                                  | —          | 760                     | 753                     | 2.1                                | 6          | 710                     | 720                     |

On a basis of experimental results, Tiagunov confirmed the correctness of his hypothesis with the following statement: "Under standard rolling conditions the product of the summary temperature drop in a given pass and the corresponding sheet thickness in that pass at a given initial temperature  $t_1$ , is constant".

The Tiagunov hypothesis is expressed by the relation

$$\Delta t_n h_n = K'$$

hence

$$\Delta t_n = \frac{K'}{h_n}$$

The constant  $K'$  is expressed in degree-millimetres. To calculate the temperature drop during rolling it is necessary to use the formula for  $K'$  derived by Tiagunov

$$K' = \frac{t_1 - 400}{16} \frac{\lambda_m}{\lambda_m - 1} z \quad (3.97)$$

where

$t_1$  = initial rolling temperature,

$\lambda_m$  = mean coefficient of elongation, found from equation (2.16),

$z$  = average duration of pass.

The temperature in the  $n$ -th pass is obtained from the equation

$$t_n = t_0 - \Delta t_{n-1}$$

Since the Tiagunov hypothesis states the relation

$$\Delta t_{n-1} h_{n-1} = K'$$

hence

$$\Delta t_{n-1} = \frac{K'}{h_{n-1}}$$

Thus the final formula for calculating the temperature of stock in the  $n$ -th pass is

$$t_n = t_0 - \frac{K'}{h_{n-1}} \quad (3.98)$$

In practice, it is often necessary to calculate temperatures in individual passes, when the drop of temperature in all the passes is known. In such cases the initial and the final rolling temperatures  $t_1$  and  $t_f$  are known, and thus  $K'$  can be calculated

$$K' = \frac{t_1 - t_f}{h_0 - h_{n-1}} h_0 h_{n-1} \quad (3.99)$$

#### Example

A  $3 \times 1400 \times 8400$  mm sheet is to be rolled in 8 passes from a sheet bar 15 mm thick and 340 kg in weight. The initial temperature of stock in the first pass  $t_1$ , is  $1007^\circ\text{C}$ . The rolling time for 9 passes (one planishing pass) is 40 seconds. The final rolling temperature  $t_f$ , is required.

The mean coefficient of elongation  $\lambda_m$  is calculated from equation (2.16)

$$\lambda_m = \sqrt[n]{\lambda_t} = \sqrt[8]{\frac{15}{3}} = 1.223$$

The mean duration of one pass is

$$z = \frac{40}{9} = 4.5 \text{ sec}$$

The constant  $K'$  will be (3.97)

$$K' = \frac{t_1 - 400}{16} \frac{\lambda_m}{\lambda_m - 1} z = \frac{1007 - 400}{16} \frac{1.223}{1.223 - 1} 4.5 = 940 \text{ deg. C/mm}$$

The drop in temperature from  $t_1$  to  $t_f$  (equation 3.98)

$$\Delta t = t_1 - t_f = t_0 - \frac{K}{h_0} = t_0 + \frac{K'}{h_{n-1}} = \frac{K'}{h_{n-1}} - \frac{K'}{h_0} = \frac{940}{3} - \frac{940}{15} = 250^\circ\text{C}$$

The final temperature of rolling  $t_f$  will be

$$t_f = t_1 - \Delta t = 1007 - 250 = 757^\circ\text{C}$$

### 3.8. Calculation of Rolling Speed

Rolled stock enters the gap with a speed less than the peripheral roll speed, as has been shown in the chapter dealing with the mechanism of bite. On the other hand, by means of a simple experiment it can be shown that the exit speed of stock is greater than the peripheral speed of rolls [54]. If transverse rags are cut in the roll, then after rolling of any stock it can be seen that the distance between impressions on the stock are greater than those on the roll, i.e. in unit time a greater length of rolled stock leaves the rolls than enters. Therefore, it can be concluded, that beginning at some point in the roll gap the speed of the rolled stock is greater than the peripheral speed of rolls.

Since on the entry side the stock passes through with a speed less than the horizontal component of the peripheral speed of rolls, there must be a place in the roll gap at which this horizontal component of peripheral speed is equal to the speed of rolled stock (Fig. 3.65).

This place is called the neutral line or plane.  $\delta$  denotes the neutral angle, i.e. the angle determining the position of the neutral line relative to roll axis (Figs. 3.50 and 3.65). This relation can be expressed mathematically as

$$v_N = v_r \cos \delta \quad (3.100)$$

where

$v_N$  = speed of rolled stock at neutral plane,

$v_r$  = peripheral speed of rolls.

$$v_r = \frac{\pi D_w n}{60} \text{ m/sec} \quad (3.101)$$

$D_w$  = working roll diameter, or  $D_{mw}$  = mean working roll diameter (formulae (3.30) and (3.32)).

$n$  = roll revolutions, rpm.

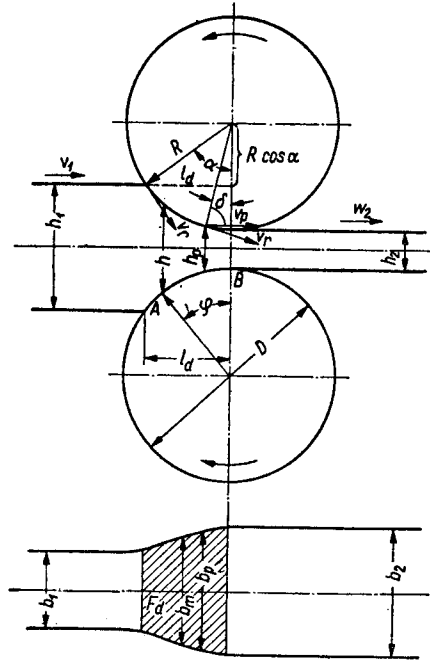


Fig. 3.65. Diagram of longitudinal rolling process

The peripheral speed of rolls can be determined graphically from the nomogram shown in Fig. 3.66 instead of using formula (3.101). The point on the diameter scale corresponding to roll diameter  $D$  in m (e.g. 0.32 m) is joined to the point on the rpm scale (e.g. 295 rpm), and the intersection of this line with the speed scale gives the required peripheral speed of rolls (in this case 5.0 m/sec).

At the neutral plane another phenomenon occurs, the frictional resistance forces change their direction (see direction of arrows in Fig. 3.50).

The entry speed of stock is denoted as  $v_1$  (m/sec), and the exit speed as  $w_2$  (m/sec) (Fig. 3.65). As the speed of stock and the horizontal component of peripheral speed

of rolls are equal at the neutral plane only, slip occurs between the stock and rolls at every other point.

At the plane of exit the speed of stock is greater than the peripheral speed of rolls. This difference of speed is called the forward slip. Mathematically the forward slip is expressed as

$$s_f = \frac{w_2 - v_r}{v_r} = \frac{w_2}{v_r} - 1 \tag{3.102}$$

or as a percentage

$$s_f = \frac{w_2 - w_r}{v_r} 100\% \tag{3.103}$$

or as a coefficient of forward slip [55]

$$s = \frac{w_2}{v_r} \tag{3.104}$$

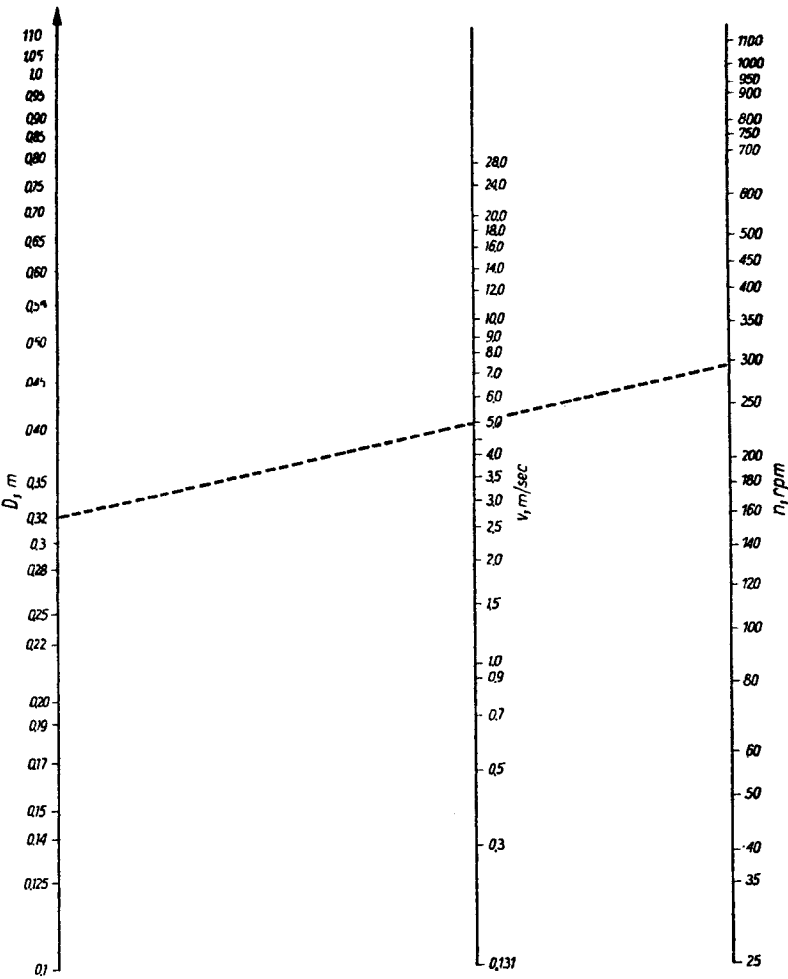


Fig. 3.66. Nomogram for determination of peripheral speed of rolls [39]

Backward slip at the entry side of the roll gap is defined as difference between the horizontal component of peripheral speed of rolls and speed of entry of the stock, relative to the horizontal component of peripheral speed of rolls, i.e.

$$s_b = \frac{v_r \cos \alpha - v_1}{v_r \cos \alpha} \quad (3.105)$$

When neglecting spread, the value of coefficient of forward slip can be found from Fink's formula

$$\frac{w_2}{v_r} = \frac{D(1 - \cos \delta) + h_2}{h_2} \cos \delta \quad (3.106)$$

where  $\delta$  = neutral angle.

Hence

$$s = \frac{w_2}{v_r} - 1 = \frac{[D(1 - \cos \delta) + h_2] \cos \delta}{h_2} - 1 \quad (3.107)$$

Transforming and simplifying formula (3.107) gives an approximate formula for forward slip

$$s \approx \frac{D\delta^2}{2h_2} = \delta^2 \frac{R}{h_2} \quad (3.108)$$

At the point of bite the entry speed of stock  $v_1$  is smaller than the horizontal component of peripheral speed of rolls  $v_r \cos \alpha$  at this point, i.e.  $v_r \cos \alpha > v_1$ ; at the exit side the speed of stock  $w_2$  is greater than the peripheral speed of rolls, i.e.  $w_2 > v_r$ .

Expressing these results with a single inequality, along the arc of contact

$$v_1 < v_r \cos \alpha_r < v \cos \delta = v_A < v_r < w_2 \quad (3.109)$$

The neutral plane (or neutral zone) in the roll gap is the place away from which metal flow inside the rolled stock occurs as will be explained in Chapter 4, Figs. 4.15 and 4.16. This metal flow is caused by compression of the rolled stock, i.e. displacement of part of the volume of stock due to draught.

Metal flows from the neutral zone in the direction of rolling and sideways. The horizontal speeds are additive, thus in the direction of rolling (towards the plane of exit) the stock moves with a speed which is the sum of rolling speed and speed due to metal flow.

Between the neutral zone and the plane of entry, speed is decreasing: the resultant speed is equal to the speed of rolls minus speed of metal flow. Also towards the plane of entry the horizontal component of speed of rolls decreases with the value  $v_r \cos \varphi$  up to the limiting value of  $v_r \cos \alpha$ , where  $\alpha$  is the bite angle. Therefore at every point in the roll gap there is a forward or backward slip relative to the horizontal component of peripheral speed of rolls.

There will be no forward and backward slip in the case where total deformation of the stock due to draught is in spreading. This is a very rare case in rolling practice, occurring only in special roll pass designs, or when the rolled stock sticks to rolls along the total arc of contact, i.e. there is no slip between stock and rolls.

From the law of constant volume, two basic relations for metal passing through the roll gap are obtained (Fig. 3.65):

$$(1) h_1 b_1 l_1 = h_x b_x l_x = h_N b_N l_N = h_2 b_2 l_2 \quad (3.110)$$

(2) Equations of speed of metal flow.

Assuming the time of stock transference through the roll gap to be equal to  $t$  seconds, then dividing formula (3.110) by  $t$  gives  $V_0$  the volume/second of metal passing through the roll gap, i.e.

$$\frac{V_i}{t_i} = V_s = \frac{F_1 l_1}{t_1} = \frac{F_2 l_2}{t_2} = \frac{F_3 l_3}{t_3} = \dots = \frac{F_n l_n}{t_n} \quad (3.111)$$

When considering one pass only, it can be concluded, that the time of entry of the whole length of stock into the rolls is equal to the total time of exit from rolls, as the rolling time during pass is constant, i.e.  $t_1 = t_2$ .

Hence

$$\frac{l_1}{t_1} = v_1 = \text{mean entry speed of stock into rolls,} \\ \text{corresponding to cross-section } F_1 \quad (3.112)$$

$$\frac{l_2}{t_2} = w_2 = \text{mean exit speed of stock from rolls,} \\ \text{corresponding to cross-section } F_2 \quad (3.113)$$

Substituting these equations into formula (3.111)

$$V_s = F_1 v_1 = F_2 w_2 \quad (3.114)$$

As from (2.15)

$$\lambda = \frac{l_2}{l_1} = \frac{F_1}{F_2}$$

then after combining with equation (3.6)

$$\lambda = \frac{F_1}{F_2} = \frac{l_2}{l_1} = \frac{w_2}{v_1}$$

and

$$w_2 = \lambda v_1 \quad (3.115)$$

Combining with (3.111), (3.112), (3.113) and (3.114) equation (3.110) can be expressed in terms of speed of metal flow in the roll gap, as

$$h_1 b_1 v_1 = h_x b_x v_x = h_N b_N v_N = h_2 b_2 v_2 \quad (3.116)$$

where index  $x$  indicates any given cross-section in the roll gap, and  $N$  indicates the neutral plane. The length  $l_x$  (or  $l_N$ ) represents the length which would be achieved by stock of a cross-section of  $x$  (or  $N$ ). The value  $v_N$  expresses the speed of metal flow at the neutral plane, with known peripheral speed of rolls  $v_r$  and neutral angle.

From equations (3.116) and (3.110) further equations for the exit side can be derived [55]:

$$\frac{h_2 b_2}{h_N b_N} = \frac{v_N}{w_2} = \frac{l_N}{l_2} \quad (3.117)$$

and similarly for the entry side

$$\frac{h_1 b_1}{h_N b_N} = \frac{v_1}{v_N} = \frac{l_1}{l_N} \quad (3.118)$$

The thickness at neutral plane can be determined from the following equation

$$h_N = D_w(1 - \cos \delta) + h_2 \quad (3.119)$$

where  $D_w$  = working diameter of roll.



If it can be assumed that  $b_2 \simeq b_N$  then equations (3.117) and (3.118) can be solved, giving

$$w_2 = \frac{h_N}{h_2} v_N \quad (3.120)$$

$$v_1 = \frac{h_N}{h_1} \frac{b_2}{b_1} v_N = \frac{h_N}{h_1} v_N \beta \quad (3.121)$$

where the value of  $v_N$  is calculated from equation (3.100),  $h_N$  from equation (3.119), and the coefficient of spread  $\beta = b_2/b_1$  from known spread formulae (3.45) or (3.49).

If considerable spread occurs and if strictly correct solutions are required,  $b_N = b_2$  cannot be assumed. In this case, in equation (3.117) for the exit side there are three unknowns:  $b_N$ ,  $w_2$ ,  $l_N$ , and for the entry side (3.118):  $b_N$ ,  $l_N$ , and  $v_1$ , therefore they cannot be solved directly. First equation (3.121) may be solved, assuming that when rolling a flat on a smooth roll barrel the coefficient of spread can be found from initial values of  $b_1$ ,  $h_1$  and  $D_w$  [32], as a function of coefficient of draught

$$\gamma = h_2/h_1$$

It is assumed that a flat of initial width of  $b_1$  after rolling has a width  $b_2$  (Fig. 3.65). Therefore, knowing initial values and the thickness at neutral plane  $h_N$  from equation (3.119), then

$$\gamma_N = h_N/h_1$$

i.e. the coefficient of draught at neutral plane can be determined. Similarly

$$\beta_N = b_N/b_1 = \gamma_N^{-W} \quad (3.122)$$

Similarly, for the coefficient of elongation at neutral plane

$$\lambda_N = \gamma_N^{W-1} \quad (3.123)$$

or combining (3.122) with (3.123)

$$\beta_N = \lambda_N^{W/(1-W)} \quad (3.124)$$

The exponent  $W$  is calculated from initial conditions of constant values

$$\delta_w = b_1/h_1 \quad \text{and} \quad \epsilon_w = h_1/D_w$$

Using these equations, equation (3.121) can now be solved as follows:

The value of  $h_N$  is calculated from (3.119),  $v_N$  from (3.100), and  $b_N$  from (3.122). Then equation (3.121) can be expressed as

$$\gamma_N \beta_N \frac{v_N}{v_1} = 1 \quad (3.125)$$

from which the required value of initial speed  $v_1$  can be found

$$v_1 = v_N \gamma_N \beta_N \quad (3.126)$$

When  $v_1$  is known, equation (3.126) can be easily solved by substituting equation (3.115)

$$\lambda = \frac{w_2}{v_1}$$

Hence

$$w_2 = v_N \gamma_N \beta_N \lambda \quad (3.127)$$

In this way it is possible to solve this problem and determine the required values of  $v_1$  and  $w_2$ , without the methods previously used for calculation of forward slip. It

is known that these methods with Fink's assumption  $b_2 \approx b_N$  gave only an approximate solution.

The ratio of the volume of metal displaced as elongation, to that displaced as spread, i.e. the changes to the initial length and width  $l_1$  and  $b_1$ , is expressed as  $\lambda/\beta$ . To investigate these changes in the roll gap, the coefficients  $\beta_x$  and  $\lambda_x$  are considered, corresponding to a variable cross-section  $x$  in the roll gap, defined by the rolling angle  $\varphi_x$  (Fig. 3.67).

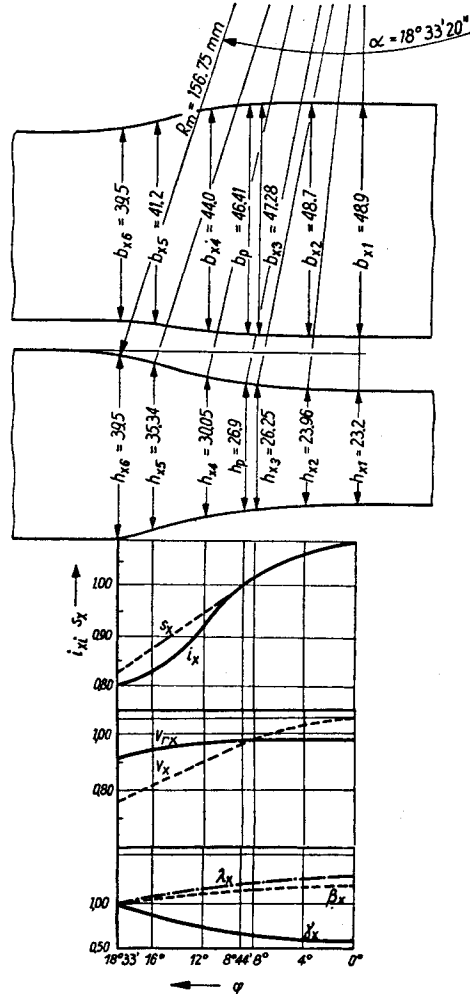


Fig. 3.67. Specimen rolled with given variations of deformation coefficients and rolling speed along the arc of contact

The value of  $\beta_x$ , i.e. the ratio  $b_x/b_1$  in the roll gap, as a function of coefficients of draught

$$\gamma_x = \frac{h_x}{h_1}$$

can be determined as follows.

From the initial values of coefficients

$$\delta_w = \frac{b_1}{h_1} \quad \text{and} \quad \varepsilon_w = \frac{h_1}{D_w}$$

then

$$\beta_x = \gamma_x^{-W} \quad (3.128)$$

or

$$\beta_x = \lambda_x^{W/(1-W)} \quad (3.129)$$

and also

$$\lambda_x = \gamma_x^{W-1} \quad (3.130)$$

In the expression for coefficient of draught

$$\gamma_x = \frac{h_x}{h_1}$$

the value of  $h_x$  is calculated similarly as in equation (3.22)

$$h_x = D_w(1 - \cos \varphi_x) + h_2 \quad (3.131)$$

In this equation  $\varphi_x$  denotes the angle corresponding to the position of the required cross-section  $x$ .

Then the required ratio

$$\frac{\lambda_x}{\beta_x} = \frac{\gamma_x^{W-1}}{\gamma_x^{-W}} = \gamma_x^{2W-1} \quad (3.132)$$

From equations (3.128) and (3.129) it can also be seen that changes in value of  $\beta_x$  and  $\lambda_x$  depend only on simultaneous variation of  $\gamma_x$ , since the exponent  $-W$  depends only on constant parameters, i.e.

$$\delta_w = \frac{b_1}{h_1} \quad \text{and} \quad \varepsilon_w = \frac{h_1}{D_w}$$

Therefore during any hot rolling process the exponent  $-W$  is constant, whereas values of  $\beta_x$  and  $\lambda_x$  depend on variations of  $\gamma_x$  only.

Using the derived formulae and equations, metal flow in the roll gap, and the mean speed of metal flow at a given point, can be readily determined.

Considering a cross-section positioned at a distance  $x$  from the plane of exit, with a rolling angle of  $\varphi_x$  (Fig. 3.65), the metal flow at this cross-section is defined by three coefficients of deformation

$$\gamma_x = \frac{h_x}{h_1}$$

where  $h_x$  is determined from equation (3.131), and coefficients of spread and elongation found from formulae (3.128) and (3.130)

$$\beta_x = \gamma_x^{-W} \quad \text{and} \quad \lambda_x = \gamma_x^{W-1}$$

In this way it is possible to find the values of all three coefficients of deformation for any arbitrary cross-section in the roll gap, and thus to determine the metal flow. To find the speed of metal flow at the same cross-section, the horizontal component of peripheral speed of rolls for this cross-section must be calculated from the relation

$$v_{rx} = v_r \cos \varphi_x \quad (3.133)$$

In this way the speed of metal flow at cross-section  $x$  can be determined independently of whether it is positioned in the zone of forward or backward slip.

For determining the mean speed  $v_x$ , the derived relations can be used, if the value of  $v_1$  can be calculated from equation (3.126). Then from equation (3.116)

$$\frac{h_x b_x v_x}{h_1 b_1 v_1} = \gamma_x \beta_x \frac{v_x}{v_1} = 1$$

Hence

$$v_x = \frac{v_1}{\gamma_x \beta_x} \quad (3.134)$$

Substituting (3.126) into (3.124)

$$v_x = \frac{v_N \gamma_N \beta_N}{\gamma_x \beta_x} \quad (3.135)$$

From these derived equations (3.134) and (3.135), the mean speed of metal flow at a given cross-section can be calculated, independently of whether it is positioned in the zone of forward or backward slip.

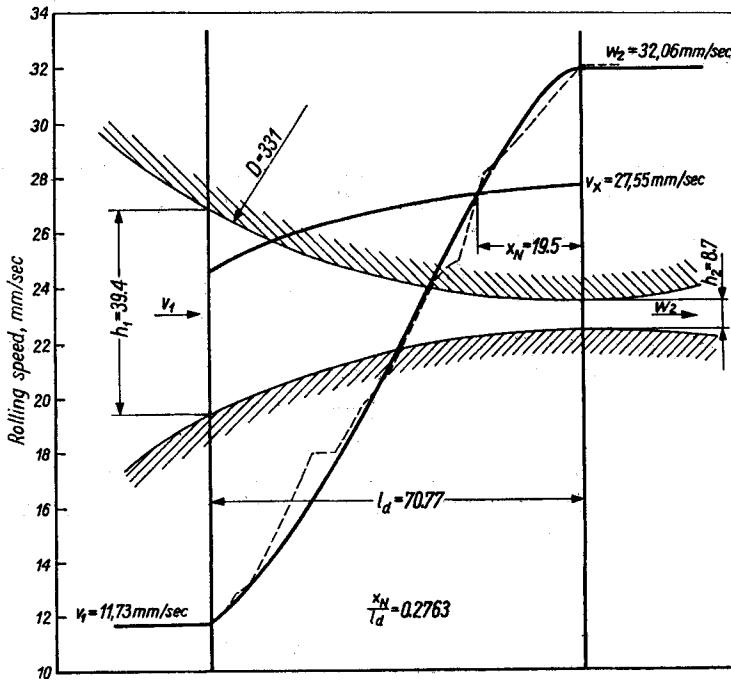


Fig. 3.68. Measured and calculated curves of distribution of speed of stock  $v_x$  in the roll gap for rolling 40 mm square at 1000°C, for  $\gamma = 0.2148$ , from Koncewicz [181]

The distribution of speed of  $v_x$  along the arc of contact, determined from equation

$$v_x = w_2 \gamma_x^{W-1} \quad (3.136)$$

for given conditions of rolling, is given in Fig. 3.68 according to Koncewicz [181].

Instead of the previously used method of determination of forward or backward slip from equations (3.102)–(3.105) it is proposed by the author [55] to relate these

values to the horizontal component of peripheral speed of rolls  $v_{rx}$  by means of a relation analogous to equation (3.104)

$$s = v_x/v_{rx} \quad (3.137)$$

where, as in (3.133)

$$v_{rx} = v_r \cos \varphi_x$$

By means of equation (3.137) the mean speed of stock relative to the rolls can be determined at any point along the arc of contact, as also at planes of entry and exit.

In the case of sticking this slip speed will represent the internal mean displacement of metal particles at any considered cross-section (Fig. 4.11 in Chapter 4).

As a second coefficient of forward or backward slip it is proposed by the author [55] to relate the speed of metal flow  $v_x$  to the speed at neutral plane  $v_N$ . This factor represents variations of flow speed in the stock itself, independent of peripheral speed of rolls

$$i = \frac{v_x}{v_N} \quad (3.138)$$

Expressing forward slip as speed ratios, as in equations (3.137) and (3.138) the true magnitude of mean speed of metal flow  $v_x$  can be found. In the zone of forward slip these values are greater than 1.0, at the neutral plane equal to 1.0, and at the zone of backward slip less than 1.0. Therefore, the coefficients of forward slip give a simple way of representing changes of speed in the roll gap.

To investigate the forward slip, a specimen of about 1.5 m in length was rolled and the results analysed. This specimen was of mild steel, rolled at a speed of 1 m/sec—on rolls of 313.5 mm diameter. The rolling temperature was 1050°C.

The results were analysed as follows. The bite angle of specimen 1 was calculated, as being equal to  $\alpha = 18^\circ 33' 20''$ . Then the arc of contact was divided into some segments, corresponding to values of rolling angle  $\varphi_x$  (Table 3.16 and Fig. 3.67)

$$0^\circ, 4^\circ, 8^\circ, 12^\circ, 16^\circ \text{ and } 18^\circ 33' 20''$$

From equation (3.16) the length of arc of contact

$$l_d = R \sin \alpha$$

was calculated.

Then the selected cross-sections were calculated and for each value of angle  $\varphi_x$

$$l_{dx} = R \sin \alpha_x$$

was calculated.

Next

$$\gamma_x = \frac{h_x}{h_1}$$

and finally from equation (3.129) the coefficient of spread

$$\beta_x = \gamma_x^{-w}$$

for the constant values  $\varepsilon_w$  and  $\delta_w$ , were calculated.

As a check on errors in calculation, all changes in rolled stock were drawn for each specimen on a scale of 1:1, as shown in Fig. 3.67 for specimen 1, so that calculated values could be compared with experimental ones. These results show variations of calculated coefficient of spread  $\beta_x$  in the roll gap.

TABLE 3.16  
VARIATION OF COEFFICIENTS OF DEFORMATION AND OF SPEED ALONG ARC OF CONTACT, MEASURED FROM SPECIMEN NO. 1

Data  $h_1 = 39.5$  mm       $\alpha = 18^\circ 33' 20''$        $n = 60$  rev/min  
 $h_2 = 23.2$  mm       $\delta = 8^\circ 48' 20''$        $D = 313.5$  mm  
 $b_1 = 39.5$  mm       $\delta_w = 1.00$        $v_r = 0.98439$  m/sec  
 $b_2 = 48.7$  mm       $\varepsilon_w = 0.126$

| $\varphi_x$                                 | $\gamma_x$ | $\beta_{xact}$ | $\lambda_{xact}$ | $\beta_{xact}$ | $\lambda_x$ | $v_{rx}$<br>m/sec | $v_x$<br>m/sec | $i_x = v_x/v_N$ | $s_x = v_x/v_{rx}$ |
|---------------------------------------------|------------|----------------|------------------|----------------|-------------|-------------------|----------------|-----------------|--------------------|
| $\varphi_{x1} = 0^\circ$                    | 0.587      | 1.238          | 1.376            | 1.236          | 1.378       | 0.98439           | 1.0628         | 1.0925          | 1.0796             |
| $\varphi_{x2} = 4^\circ$                    | 0.607      | 1.233          | 1.336            | 1.219          | 1.351       | 0.98199           | 1.0421         | 1.0712          | 1.0612             |
| $\varphi_{x3} = 8^\circ$                    | 0.664      | 1.196          | 1.258            | 1.176          | 1.281       | 0.97481           | 0.9875         | 1.0151          | 1.01301            |
| $\delta = 8^\circ 48' 20''$                 | 0.681      | 1.175          | 1.250            | 1.164          | 1.261       | 0.97278           | 0.97278        | 1.0000          | 1.0000             |
| $\varphi_{x4} = 12^\circ$                   | 0.761      | 1.114          | 1.179            | 1.114          | 1.179       | 0.96288           | 0.9096         | 0.93505         | 0.94466            |
| $\varphi_{x5} = 16^\circ$                   | 0.895      | 1.043          | 1.0712           | 1.044          | 1.070       | 0.94625           | 0.8253         | 0.8484          | 0.87217            |
| $\varphi_{x6} = \alpha = 18^\circ 33' 20''$ | 1.000      | 1.000          | 1.000            | 1.000          | 1.000       | 0.93321           | 0.77111        | 0.7929          | 0.82629            |

Then the true values of  $h_x$  and  $b_{x\text{act}}$  at the same cross-section  $x$  were measured as accurately as possible and the coefficient

$$\beta_{x\text{act}} = \frac{b_{x\text{act}}}{b_1}$$

calculated, and the error of calculated and experimental values compared.

To facilitate these measurements, large specimens were rolled on rolls of large diameter, using high draughts.

If considerable bulging of specimens at the centre occurs, as in Fig. 3.67, to the extent that the assumption  $b_{\text{max}} = b_{\text{act}}$  would involve serious deviation from the law of constant volume, then this bulging must be taken into account and the mean height  $h_m$  calculated from known formulae.

From (3.1)

$$h_{mx} = \frac{F_x}{b_{x\text{max}}}$$

and mean coefficient of draught

$$\gamma_m = \frac{h_{mx}}{h_1}$$

i.e. similarly as for calculation of spread of symmetrical bars.

These bulgings are shaped like half a flat oval positioned at each side of specimen, and therefore two such areas were added to cross-section  $F_x$

$$F_{\text{ov}} = \frac{2}{3} h_x b_{x\text{ov}} \quad (3.139)$$

where the value of  $h_x$  is determined as usual, and

$$b_{x\text{ov}} = b_{x\text{max}} - b_{x\text{min}}$$

Hence

$$F_t = F_x + F_{\text{ov}} \quad (3.140)$$

and

$$h_m = \frac{F_t}{b_{x\text{max}}} \quad (3.141)$$

Next a full analysis of specimen *I* was carried out, recalculating coefficients of deformation and variation of speed in given cross-sections, using the method proposed [55] (Table 3.16). From known values of  $\gamma_x$  and  $\beta_x$

$$\lambda_x = \frac{1}{\gamma_x \beta_x}$$

and

$$\gamma_{\text{act}} = \frac{1}{\lambda_{\text{act}} \beta_{\text{act}}}$$

were calculated.

The horizontal component of peripheral speed of rolls  $v_{rx}$  was found from equation (3.133). From equation (3.126) speed

$$v_1 = v_N \gamma_N \beta_N$$

was calculated, using the author's method (next section 3.81) for determination of neutral angle  $\delta$  with spread [56]. Then from (3.100) the value of

$$v_N = v_r \cos \delta$$

was found, and from (3.119)

$$h_N = D_w(1 - \cos \delta) + h_2$$

and hence

$$\gamma_N = \frac{h_N}{h_2}$$

could be found.

Then from (3.122) the last coefficient

$$\beta_N = \gamma_N^w$$

was calculated.

Then it was possible to calculate the variable speed of stock in the roll gap, from equation (3.134)

$$v_r = \frac{v_1}{\gamma_x \beta_x}$$

The coefficient giving the value of slip between stock and rolls relative to peripheral speed of rolls (3.137)

$$s_x = \frac{v_x}{v_{rx}}$$

and the coefficient of backward and forward slip relative to neutral plane (3.138)

$$i_x = v_x / v_N$$

were also calculated.

These values given and calculated for the variable cross-section of  $x$  are tabulated in Table 3.16, and shown in Fig. 3.67. At the neutral plane the coefficients  $s_x$  and  $i_x$  become equal to 1. These values increase in the zone of forward slip, and decrease in the zone of backward slip.

The analysis of the influence of variation of coefficients  $\beta$  and  $\lambda$  on speed of metal flow for the same case of hot rolling would be of interest. Such considerations could only be theoretical, as these conditions do not exist in practice, since given rolling conditions determine the nature of metal flow exactly.

TABLE  
VARIATION OF COEFFICIENTS OF DEFORMATION AND  
MEASURED FROM

| No | $\beta$                   | $\gamma$ | $\lambda$ | $\beta_N$                   | $\gamma_N$ | $\lambda_N$ |
|----|---------------------------|----------|-----------|-----------------------------|------------|-------------|
| a  | 1.0                       | 0.581    | 1.721     | 1.0                         | 0.676      | 1.479       |
| b  | 1.251                     | 0.581    | 1.376     | 1.114                       | 0.676      | 1.293       |
| c  | $\sqrt{1/\gamma} = 1.312$ | 0.581    | 1.312     | $\sqrt{1/\gamma_N} = 2.216$ | 0.676      | 1.216       |
| d  | 1.376                     | 0.581    | 1.251     | 1.293                       | 0.676      | 1.114       |
| e  | $1/\gamma = 1.721$        | 0.581    | 1.0       | $1/\gamma_N = 1.479$        | 0.676      | 1.0         |



Variation of coefficients of deformation and speed can occur only with simultaneous variation of general rolling conditions resulting in absence of basic values for comparison purposes.

For comparative purposes the following rolling cases are investigated [56]:

$$\left. \begin{array}{l} \text{(a)} \quad \beta = 1 \\ \text{(b)} \quad \beta < \lambda \\ \text{(c)} \quad \beta = \lambda = \sqrt{1/\gamma} \\ \text{(d)} \quad \beta > \lambda \\ \text{(e)} \quad \beta = 1/\gamma \end{array} \right\} \quad (3.142)$$

i.e. the coefficient of spread varies from the minimum to maximum limiting values.

This analysis will be done for relations (3.126)

$$v_1 = v_N \gamma_N \beta_N$$

and (3.115)

$$w_2 = v_1 \lambda$$

For this analysis specimen 2 was chosen, since it showed large absolute spread (Table 3.17). The rolling data for this specimen were as follows:

| <i>measured values</i> |           | <i>calculated values</i> |                                         |
|------------------------|-----------|--------------------------|-----------------------------------------|
| $h_1$                  | = 50.6 mm | $\alpha$                 | = $21^\circ 11' 30''$ (equation (3.9))  |
| $h_2$                  | = 29.4 mm | $\delta$                 | = $10^\circ 4' 11''$ (equation (3.150)) |
| $b_1$                  | = 51.0 mm | $v_r$                    | = 0.98439 m/sec (equation (3.101))      |
| $b_2$                  | = 63.8 mm | $v_r$                    | = $\cos \alpha = 0.91782$ m/sec         |
| $D_{mw}$               |           | = 313.5 mm               |                                         |

The rolling temperature was  $1000^\circ\text{C}$ .

For actual rolling of this specimen

$$\gamma = 0.581 \quad \beta = 1.251 \quad \lambda = 1.371$$

Calculating from equation (3.100) the speed at neutral plane is

$$v_N = v_r \cos \delta = 0.96921 \text{ m/sec}$$

Coefficient of draught at neutral plane is then calculated

$$\gamma_N = h_N/h_1 = 0.676$$

where  $h_N$  is determined from (3.119).

Knowing the value of  $\gamma_N$ , the coefficient of spread at neutral plane is determined from formula (3.122)

$$\beta_N = b_N/b_1 = \gamma_N^w = 1.144$$

### 3.17

#### OF SPEED ALONG ARC OF CONTACT, SPECIMEN No. 2

| $v_N$<br>m/sec | $v_1 = v_N \gamma_N \beta_N$ | $w_2 = v_1 \lambda$<br>m/sec | $v_1/v_N$ | $w_2/v_N$ | $v_1/v_r \alpha$ | $w_2/v_r$ |
|----------------|------------------------------|------------------------------|-----------|-----------|------------------|-----------|
| 0.96921        | 0.65518                      | 1.12756                      | 0.67599   | 1.1633    | 0.71384          | 1.1454    |
| 0.96921        | 0.74952                      | 1.03134                      | 0.77333   | 1.0641    | 0.81663          | 1.0476    |
| 0.96921        | 0.79683                      | 1.04534                      | 0.82214   | 1.0785    | 0.86817          | 1.0619    |
| 0.96921        | 0.84715                      | 1.05978                      | 0.87406   | 1.0934    | 0.92300          | 1.0765    |
| 0.96921        | 0.96921                      | 0.96921                      | 1.0       | 1.0       | 1.05599          | 1.98457   |

For comparison purposes and equation (3.142) five following cases were chosen:

- |                                         |                      |
|-----------------------------------------|----------------------|
| (a) $\beta = 1,$                        | $\lambda = 1/\gamma$ |
| (b) $\beta = 1.251,$                    | $\lambda = 1.371$    |
| (c) $\beta = \lambda = \sqrt{1/\gamma}$ |                      |
| (d) $\beta = 1.371,$                    | $\lambda = 1.251$    |
| (e) $\beta = 1/\gamma,$                 | $\lambda = 1$        |

Results of calculation are given in Table 3.17.

Analysing equation (3.126)

$$v_1 = v_N \gamma_N \beta_N$$

the following relations are obtained for the three main cases (3.142):

- |                                                                   |                                     |
|-------------------------------------------------------------------|-------------------------------------|
| (a) $v_1 = v_N \gamma_N$                                          | since $\beta_N = 1$                 |
| (c) $v_1 = v_N \gamma_N \beta_N = v_N \gamma_N \sqrt{1/\gamma_N}$ | since $\beta_N = \sqrt{1/\gamma_N}$ |
| (e) $v_1 = v_N$                                                   | since $\beta_N = 1/\gamma_N$        |

Hence if  $\lambda = 1$ ,  $w_2 = v_N$ , i.e. for case (e) the relation

$$v_1 = v_N = w_2 \quad (3.143)$$

is obtained.

This leads to the interesting conclusion that for  $\lambda = 1$  the speed of stock is constant throughout the roll gap, and equal to  $v_N$ . However in this case very great differences occur between speed of stock and peripheral speed of rolls, especially at the plane of entry and exit. This fact is confirmed by results of calculations given in Table 3.17, and particularly by the ratios

$$v_1/v_N, \quad w_2/v_N, \quad v_1/v_{ra}, \quad w_2/v_r$$

It can be seen from the last column, that for very great spread, the exit speed of stock can be equal to peripheral speed of rolls, and sometimes it becomes less than peripheral speed of rolls. In such a case backward slip occurs in the exit zone, and forward slip in the entry zone, i.e. contrary to accepted theories of rolling. However, it should be taken into account that values given in Table 3.17 are not real, but calculated ones.

### 3.8.1. NEUTRAL ANGLE AND FORWARD SLIP WHEN ROLLING WITH SPREAD

Considering the process of rolling with spread [56] (Figs. 3.2 and 3.65), and taking into account the principle of constant volume, the following equations can be obtained (formula (3.116)):

$$h_1 b_1 v_1 = h_x b_x v_x = h_N b_N v_N = h_2 b_2 v_2$$

From this equation further formulae can be derived for entry side (3.126) and (3.115)

$$v_1 = v_N \gamma_N \beta_N = \frac{w_2}{\lambda}$$

where

$$\gamma_N = \frac{h_N}{h_1} \quad \beta_N = \frac{b_N}{b_1}$$

From the principle of constant volume

$$\gamma_N \beta_N = \frac{1}{\lambda_N} \quad (3.144)$$

Substituting (3.144) into (3.125)

$$\frac{v_N}{\lambda_N} = \frac{w_2}{\lambda} = v_1 \quad (3.145)$$

where  $v_N$  = speed of stock at neutral plane, expressed as horizontal component of peripheral speed of rolls  $v_r$  from (3.100), and

$$v_N = v_r \cos \delta$$

Substituting (3.100) into (3.145)

$$\frac{\cos \delta}{\lambda_N} = \frac{w_2}{v_N \lambda} \quad (3.146)$$

$\lambda_N$  can be expressed as from (3.123) in the form

$$\lambda = \gamma_N^{W-1}$$

where the factor  $W$  for hot rolling depends on constant initial parameters.

$$\delta_w = \frac{b_1}{h_1} \quad \text{and} \quad \varepsilon_w = \frac{h_1}{D}$$

Substituting (3.123) into (3.146)

$$\frac{\cos \delta}{\gamma_N^{W-1}} = \frac{w_2}{\lambda v_r} \quad (3.147)$$

As it is known from Fig. 3.65 and equation (3.119)

$$\gamma_N = \frac{h_N}{h_1} = \frac{D - D \cos \delta + h_2}{h_1} \quad (3.148)$$

Substituting (3.148) into (3.147) gives the relation

$$\frac{\cos \delta}{\left( \frac{D - D \cos \delta + h_2}{h_1} \right)^{W-1}} = \frac{w_2}{\lambda v_r} \quad (3.149)$$

where  $\delta$  is given in a rather complicated form. Values of angle  $\delta$  calculated from this equation should be very close to true values, since no simplifications have been used.

To find  $\delta$  a nomogram has been plotted, as it is impossible to solve equation (3.149) by any normal method.

Substituting

$$\frac{w_2}{v_r} = s, \quad \frac{h_1}{D} = \varepsilon_w, \quad \frac{h_2}{h_1} = \gamma, \quad \lambda = \gamma^{W-1}$$

formula (3.149) can be expressed as

$$\left( \frac{s}{\cos \delta} \right)^{1/(1-W)} = \frac{1 - \cos \delta}{\varepsilon_w \gamma} + 1 \quad (3.150)$$

For calculation of forward slip, the form given by Koncewicz [181] can also be used

$$s = \frac{w_2}{v_r} = \left[ 1 + \frac{D}{h_2} (1 - \cos \delta) \right]^{1-W} \cos \delta - 1 \quad (3.151)$$

To determine  $(s/\cos \delta)^{1/(1-W)} = y$  a diagram has been plotted for a range of variations

$$1 \leq s \leq 1.20$$

$$0.9397 \leq \cos \delta \leq 1$$

$$0.05 \leq W \leq 0.9$$

For  $W = 0$ ,  $y = s/\cos \delta$ , and for  $W \rightarrow 1$ ,  $\lim y = \infty$ .

The diagram for finding  $y$  consists of two parts (Fig. 3.69). In part *A* expression  $s/\cos \delta$  is calculated, and in *B*,  $(s/\cos \delta)^{1/(1-W)}$ .

As  $y$  denotes one side of equation (3.151)

$$y = 1 + \frac{1 - \cos \delta}{\varepsilon_w \gamma} \quad (3.152)$$

Straight lines in part *D* of the nomogram give  $(1 - \cos \delta)/\varepsilon_w \gamma$  for any value of  $\varepsilon_w \gamma$ . Part *C* of the nomogram joins points from  $(1 - \cos \delta)/\varepsilon_w \gamma$  scale of part *D* to points greater by 1 on the  $(s/\cos \delta)^{1/(1-W)}$  scale in part *B*.

From this nomogram the required neutral angle can be determined using the method of successive approximations. An initial value of  $\delta$  is assumed, estimated as accurately as possible from diagrams giving  $\delta/\alpha$ . On part *A* the value  $\cos \delta$  is found, corresponding to the assumed value of  $\delta$ , then knowing  $s$  the value  $s/\cos \delta$  is read off from the  $s/\cos \delta$  scale. Knowing this value and exponent  $W$ , the value of  $(s/\cos \delta)^{1/(1-W)}$  can be read off on the appropriate scale of part *B*. This value is then reduced by 1. For a known value of  $\varepsilon_w \gamma$ ,  $1 - \cos \delta$  can be read off from the scale in part *D*. If this value of  $\cos \delta$  corresponds to the assumed value, then this is the required solution of equation (3.151). If there is a difference, the operation should be repeated using the method of successive approximations, until a closed diagram is achieved.

#### Example

The following initial values are given

$$\alpha^\circ = 6^\circ 51', \quad \gamma = \frac{h_2}{h_1} = 0.861, \quad \delta_w = \frac{b_1}{h_1} = 1.007,$$

$$\varepsilon_w = \frac{h_1}{D} = 0.0517, \quad s = \frac{w_2}{v_r} = 1.022$$

From these values  $W = 0.57$  and  $\gamma \varepsilon_w = 0.0445$  are calculated. For an assumed value of  $\delta = 4^\circ$  on the  $\cos \delta$  scale of Fig. 3.69 (part *A*),  $\cos \delta$  is read off as  $\cos 4^\circ = 0.9975$  (point 1). Then for  $s = 1.022$ , (point 2) the value  $s/\cos \delta = 1.0245$  is read off from the appropriate scale (point 3). Knowing  $s/\cos \delta$  and the calculated value  $W = 0.57$  (point 4) is determined, then  $(s/\cos \delta)_{\text{exp.}}^{1/(1-W)} = 1.06$  can be read off (point 5). This value is then decreased by 1, i.e. down to 0.06, moving along the oblique line to scale  $(1 - \cos \delta)/\varepsilon_w \gamma$  (point 6). From point 6 a straight line is drawn to cut the line representing the calculated value of  $\varepsilon_w \gamma = 0.0445$  (point 7), and from this point a vertical line is drawn to cut the  $1 - \cos \delta$  scale at the value 0.0024 (point 8), corresponding to  $\cos \delta = 0.9976$ , i.e.  $\delta = 4^\circ$ .

This value corresponds to the assumed one, and is therefore the correct solution of equation (3.150).

If the values found do not close the diagram, this operation should be repeated until a closed diagram is achieved.

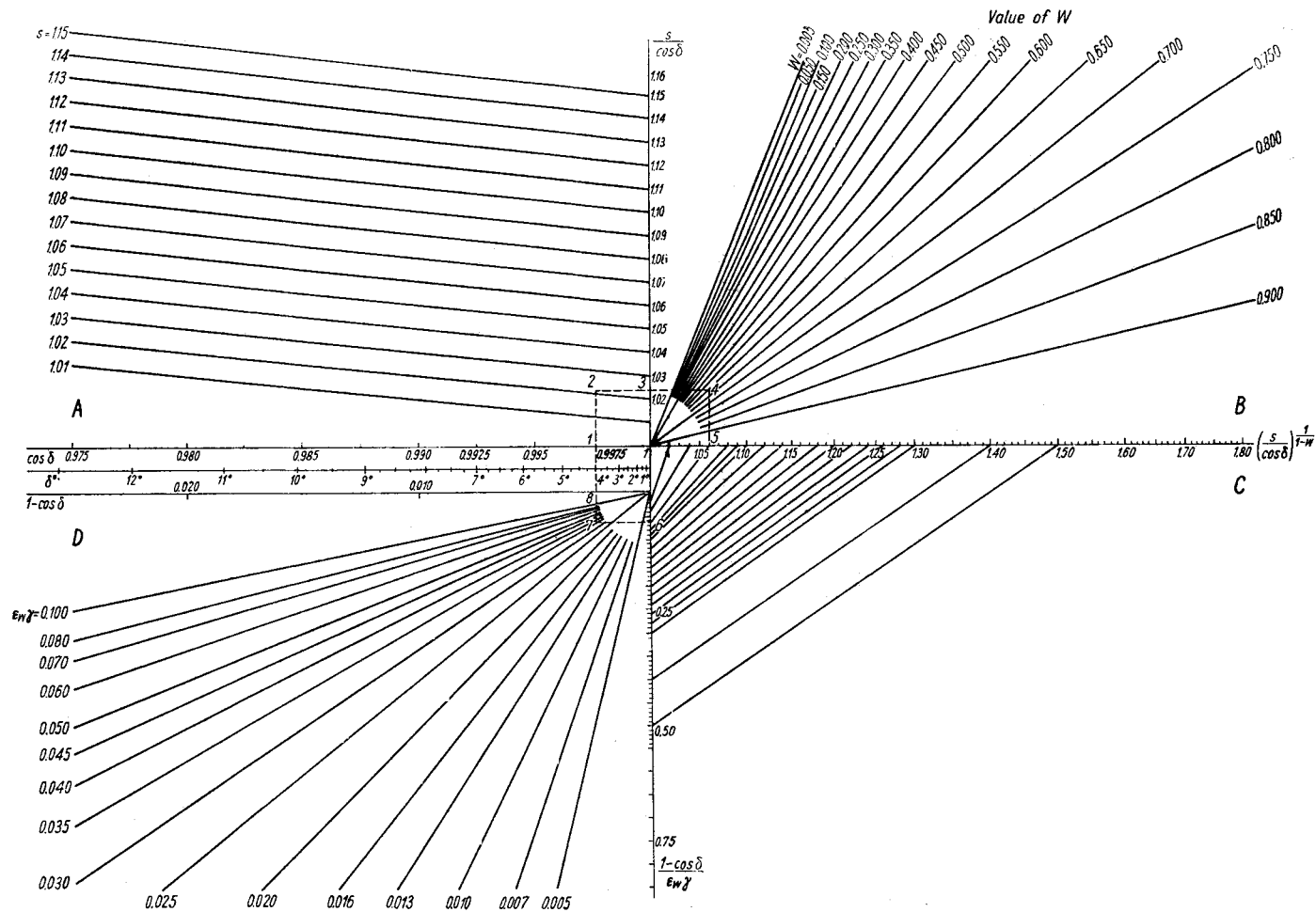


Fig. 3.69. Nomogram for determination of neutral angle

To use the diagram shown in Fig. 3.69 it is necessary to know measured values of forward slip relative to draught, temperature, spread and ratio  $\delta_w = b_1/h_1$ .

Investigations were carried out on a 2-high reversible mill at the Institute of Ferrous Metallurgy, Gliwice. The following types of rolls were used:

- (a) forged steel rolls of rough surface and diameter  $D = 308$  mm,
- (b) cast iron rolls of diameter  $D = 382$  mm,
- (c) forged steel rolls of smooth surface and diameter  $D = 305$  mm.

The rolled stock was heated in a four-chamber gas heated furnace.

For exact determination of forward slip oscillograph measurements were used [56], and also the method of the marks on rolls.

The following equation is used to determine approximate forward slip

$$s = \frac{w_2}{v_r} = \frac{l_p}{a_0} \quad (3.153)$$

where

$a_0$  = distance between cuts on rolls, measured on roll circumference,

$l_p$  = distance between cuts on metal at rolling temperature.

In hot rolling the effect of contraction must be allowed for, as in the equation

$$l_p = l'_p(1 + \beta \Delta t) \quad (3.154)$$

where

$l'_p$  = measured distance between cuts on metal after cooling to ambient temperature,

$\beta$  = mean coefficient of expansion of metal,

$\Delta t$  = difference between rolling and ambient temperatures.

Specimens of mild steel of the following chemical analysis were rolled (%):

| C    | Mn   | Si   | P     | S     |   |
|------|------|------|-------|-------|---|
| 0.19 | 0.34 | 0.10 | 0.022 | 0.031 | % |

The cross-sections of specimens rolled were (mm):  $10 \times 100$ ,  $20 \times 100$ ,  $20 \times 40$  and  $20 \times 20$ . The length of all specimens was about 600 mm. Each section was rolled at four levels of temperature and various draughts. Four theoretical temperatures were assumed: 900, 1000, 1100, 1200°C, and five values of percentage draught:

| Percentage draught, % | Coefficient of draught $\gamma$ |
|-----------------------|---------------------------------|
| 10                    | 0.9                             |
| 20                    | 0.8                             |
| 30                    | 0.7                             |
| 40                    | 0.6                             |
| 50                    | 0.5                             |

Obviously, during rolling it was impossible accurately to maintain the proposed values of temperature and draught. Tests were carried out in such a way that a set of specimens of the same dimensions were heated to the same temperature, and rolled with successive draughts.

The rolling temperature was measured by means of an optical pyrometer, when the specimen was in the roll gap.

Measurement of forward slip by means of marks on rolls always involves some errors, due to deformation and often twisting of specimens after rolling, and at high temperatures this method becomes almost impossible to use due to formation of scale.

Width and thickness of specimens before and after rolling were measured at two points, and for calculations the mean value of these measurements was used. Two values of forward slip were found for each specimen, one from measuring the speed on the oscillograph, and the second from marks on the rolls, making marks on specimens during rolling.

When comparing all results it could be seen that values of forward slip determined directly from measurement of speeds is slightly greater than that determined from measured distance between marks on specimens and marks on rolls, for all specimens.

For plotting variation of forward slip against draught and spread, values found directly from speed measurements have been used. This choice was based on the assumption that measured values of speed are probably more accurate, as with the second method errors can be made both when measuring distances between marks on rolls, and marks on specimens. Large errors can also arise due to incorrect allowance for contraction of stock after rolling.

Forward slip as a function of draught and spread at 900°C, is shown in Fig. 3.70. Forward slip increases with increasing draught, with a corresponding decrease in coefficient of draught. The greater the form factor  $\delta_w = b_1/h_1$ , the smaller is the spread and the greater the forward slip, for a given value of draught. An exception for this temperature are flats  $\delta_w = b_1/h_1 = 5$  which show greater forward slip than flats of  $\delta_w = b_1/h_1 = 10$  within the range of coefficients of draught of 0.9 to 0.8, corresponding to smaller spread.

The increase in forward slip with increasing draught (decreasing coefficient of draught) varies also for particular values of form factor  $\delta_w = b_1/h_1$ , e.g. for form factors of  $\delta_w = 10$  and  $\delta_w = 2$ , and for the coefficient of draught of 0.79, there is a more rapid increase in forward slip than for higher values of draught. Therefore, an increase in coefficient of draught above 0.79 corresponds to a greater increase in forward slip than for values below 0.79. There is a slower increase in forward slip for form factor  $\delta_w = 5$  and  $\delta_w = 1$  than for  $\delta_w = 10$ , within the range of coefficient of draught of 0.9 to 0.8. Increase in forward slip with increasing draught (decreasing coefficient  $\gamma$ ) is varied, depending on form factor  $\delta_w = b_1/h_1$ .

Forward slip as a function of draught and spread at 1000°C, is given in Fig. 3.71.

At a temperature of 1100°C (Fig. 3.72) there is a similar relation between forward slip and coefficient of draught, as for preceding temperature values. When rolling flats of form factor  $\delta_w = 5$ , the value of forward slip is greater than for form factor of 10, within the range of coefficient of draught 0.68 to 0.9, similarly as at 1000°C.

Also for flats of form factors of  $\delta_w = 2$  greater values of forward slip were obtained than for  $\delta_w = 10$ , but only within the range of coefficient of draught of 0.9 to 0.84.

There is a more rapid increase in forward slip for form factor of  $\delta_w = 10$ , than for  $\delta_w = 5$ , over the whole range of coefficient of draught. Curves of forward slip for form factors of  $\delta_w = 5$ ,  $\delta_w = 2$ , and  $\delta_w = 1$  at 1200°C (Fig. 3.73) have a similar form, as greater values of form factors correspond to smaller spread and greater forward slip, for a given coefficient of draught. For form factor  $\delta_w = 10$  a rapid increase in forward slip is obtained over the whole range of coefficient of draught.

As for previous temperatures, at 1200°C the value of forward slip at  $\delta_w = 5$  is greater than for  $\delta_w = 10$  up to a certain limit of coefficient of draught. The limiting value is  $\gamma = 0.84$ .

Values of forward slip given in Figs. 3.70–3.73 were used to determine the neutral angle  $\delta$ , from formula (3.149), as shown in the nomogram given in Fig. 3.69.

Values of  $\delta$  are found relative to bite angle  $\alpha$ , giving relative dimensionless values  $\delta/\alpha$ , as the value of  $\delta$  depends on size of rolled specimen, and for small specimens is smaller than for large ones.

Values of  $\delta/\alpha$  have been plotted against the same measured values of  $\gamma$  and  $\beta$ , as for forward slip.

The ratio  $\delta/\alpha$  as a function of draught and spread is shown in Figs. 3.74–3.77.

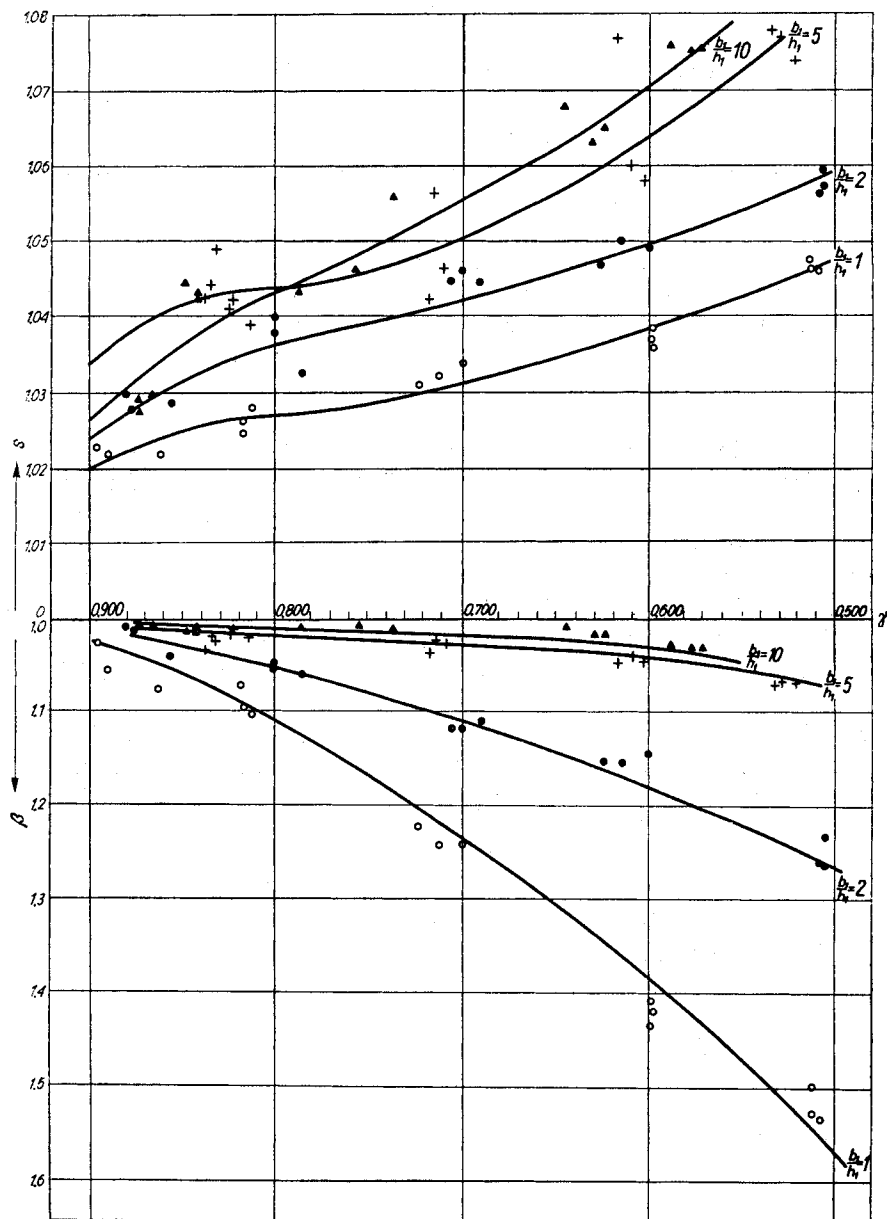


Fig. 3.70. Forward slip and spread as a function of  $\gamma$  at 900°C [56]



The ratio  $\delta/\alpha$  as a function of coefficient of spread and draught for form factor of  $\delta_w = 1$ , is shown in Fig. 3.74. An increase in coefficient of spread causes a decrease in ratio  $\delta/\alpha$  for all temperatures. It can be seen from the diagram, that in the draught coefficient range  $\gamma = 0.7-0.9$ , for different temperatures the ratio  $\delta/\alpha$  shows considerable variations. However  $\beta$  shows only small changes for different temperatures.

For form factor  $\delta_w = 1$  there are very small differences between values of ratio of  $\delta/\alpha$  for temperatures 900 and 1000°C, as also for 1100 and 1200°C.

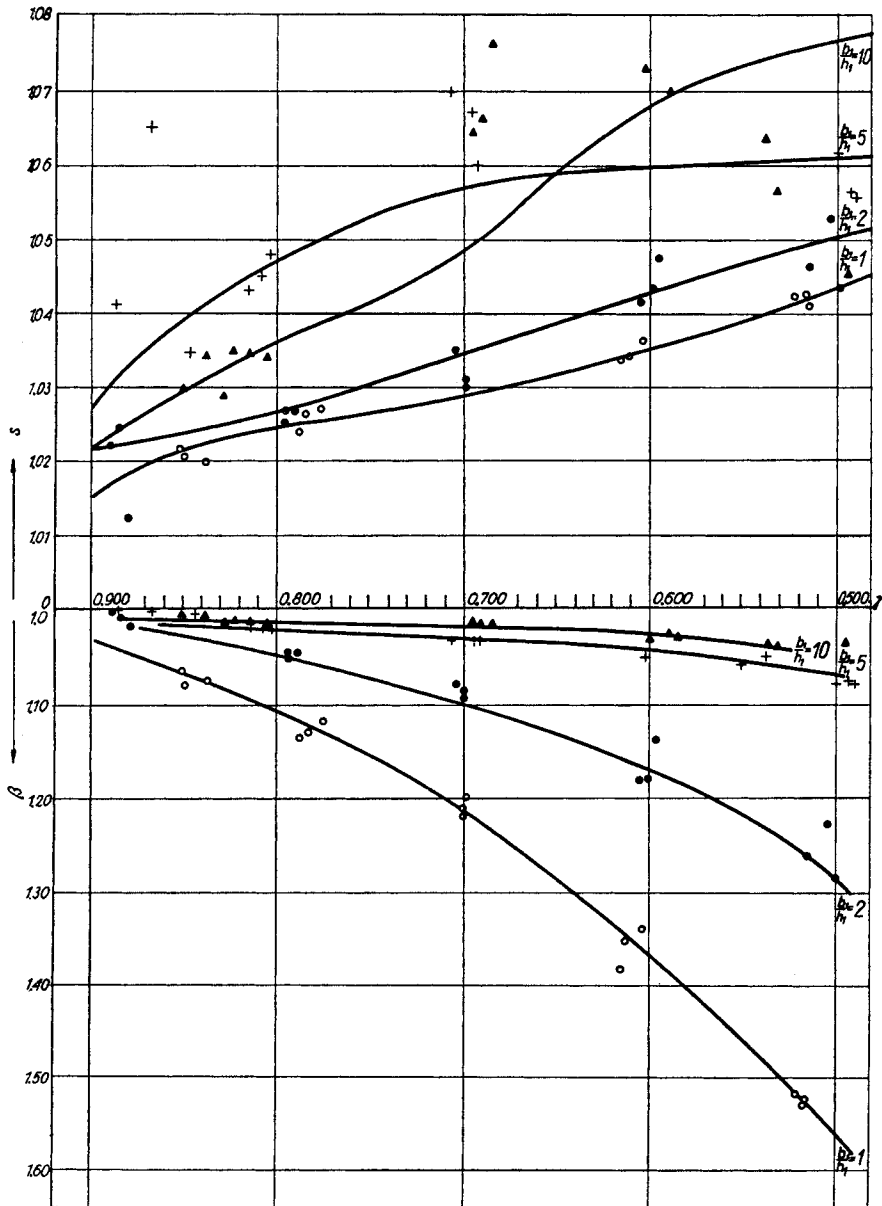


Fig. 3.71. Forward slip and spread as a function of  $\gamma$  at 1000°C [56]

The ratio  $\delta/\alpha$  as a function of coefficient of draught and spread for  $\delta_w = b_1/h_1 = 2$ , is given in Fig. 3.75. The value of ratio  $\delta/\alpha$  decreases with increasing coefficient of draught, similarly as for previous conditions. The relation of coefficient of draught and ratio  $\delta/\alpha$  follows a similar curve at all temperatures.

It can be seen from the diagram that at temperatures 900 and 1100°C, for minor variations of  $\beta$  in the range  $\beta \approx 1.02$ –1.08, the values of ratio  $\delta/\alpha$  show considerable variations, the higher values being found for temperature 900°C.

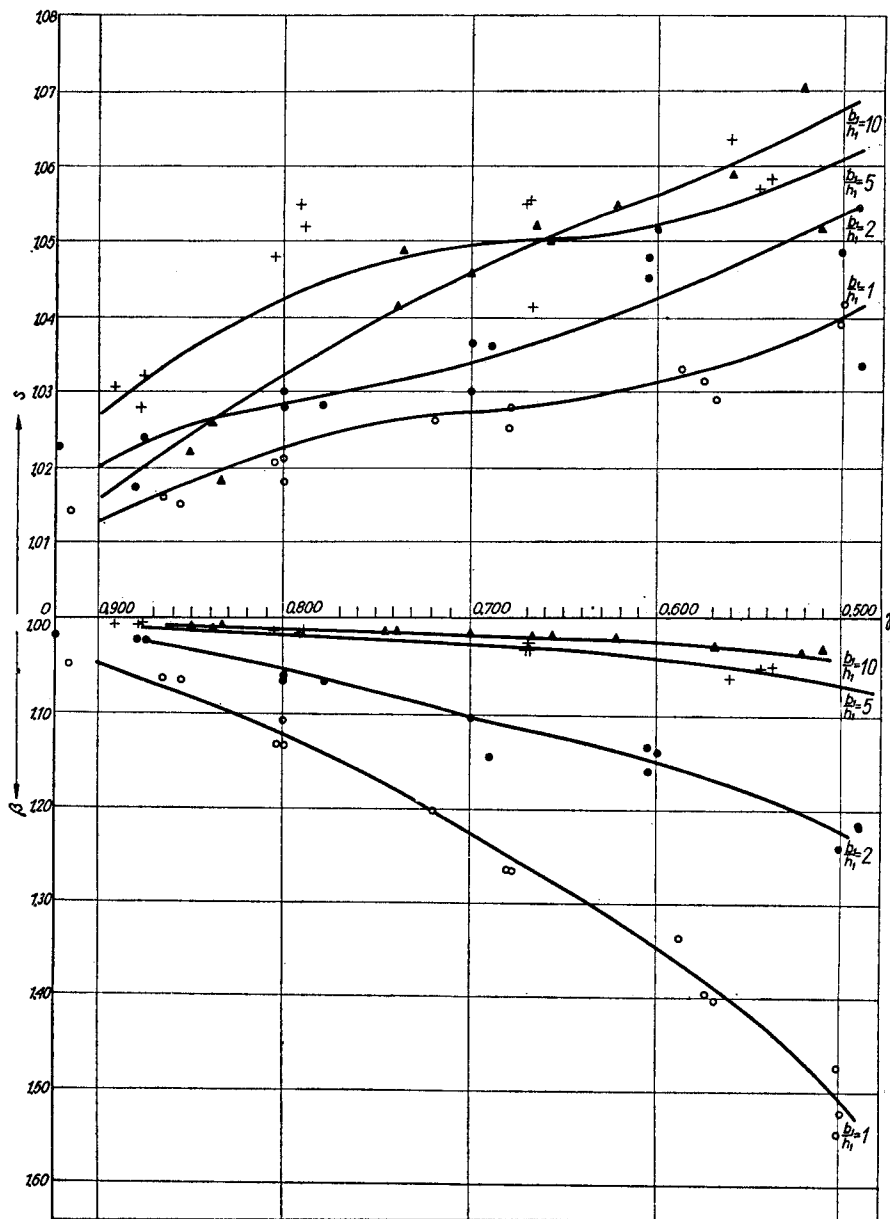


Fig. 3.72. Forward slip and spread as a function of  $\gamma$  at 1100°C [56]

There are only insignificant differences between values of ratio  $\delta/\alpha$  over the whole range of draught coefficients at given temperatures, except for the draught range  $\gamma = 0.600\text{--}0.900$  and temperature  $900^\circ\text{C}$ . The same is true for coefficient of spread  $\beta$ , except in the draught range  $\gamma = 0.500\text{--}0.720$  at  $1100^\circ\text{C}$ .

The ratio  $\delta/\alpha$  as a function of coefficient of draught and spread for form factor  $\delta_w = 5$ , is given in Fig. 3.76. Ratio  $\delta/\alpha$  decreases with increasing coefficient of spread throughout the whole range of draught coefficients.

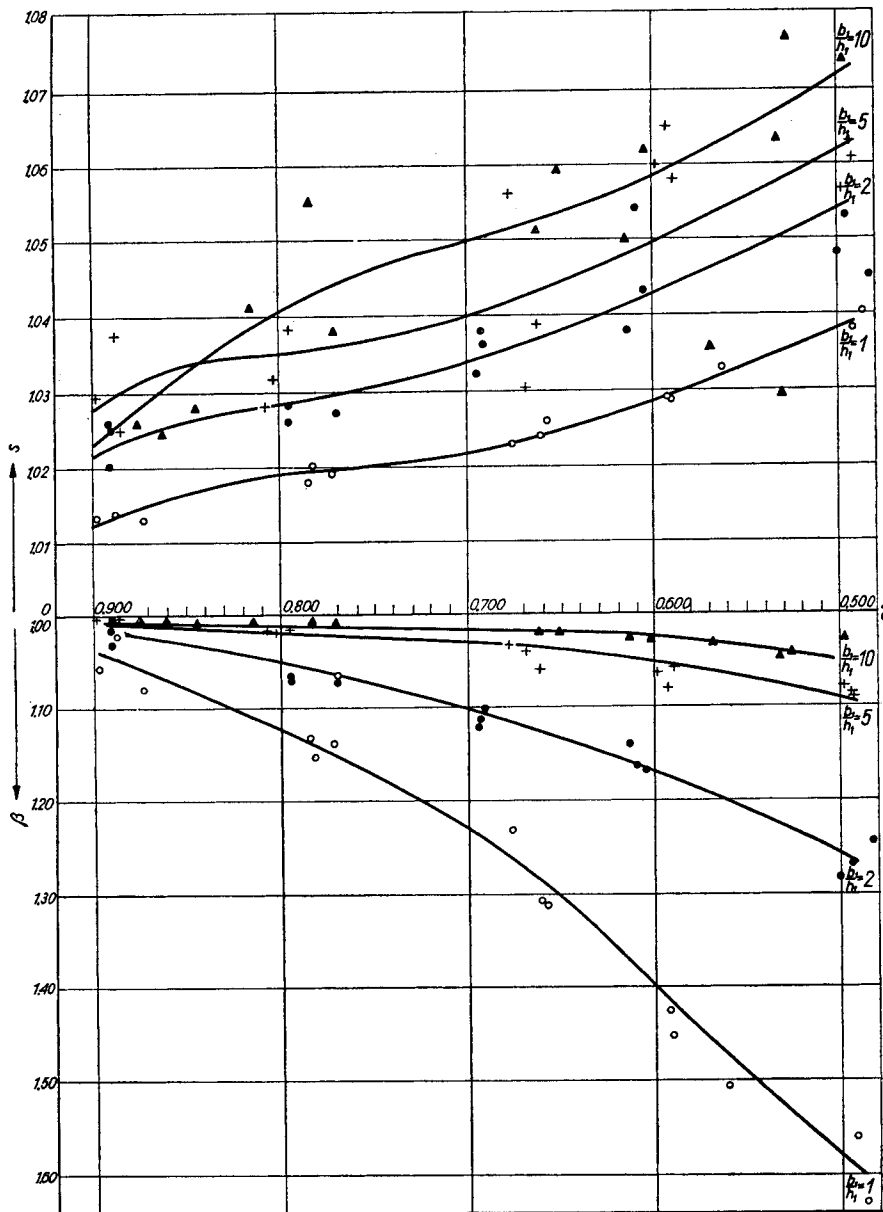


Fig. 3.73. Forward slip and spread as a function of  $\gamma$  at  $1200^\circ\text{C}$  [56]

From Fig. 3.77 it can be seen that at 1200°C and given value of coefficient of draught,  $\delta/\alpha$  and  $\beta$  take smaller values in some parts of the diagram, than for the other temperatures. The relation between ratio  $\delta/\alpha$  and coefficient of spread is similar for all the temperatures, i.e. increase in spread results in decrease of ratio  $\delta/\alpha$ .

For form factor  $\delta_w = 10$  there is a decrease in ratio  $\delta/\alpha$  with increasing draught, and resulting increase in coefficient of spread, as can be seen from Fig. 3.77. The ratio

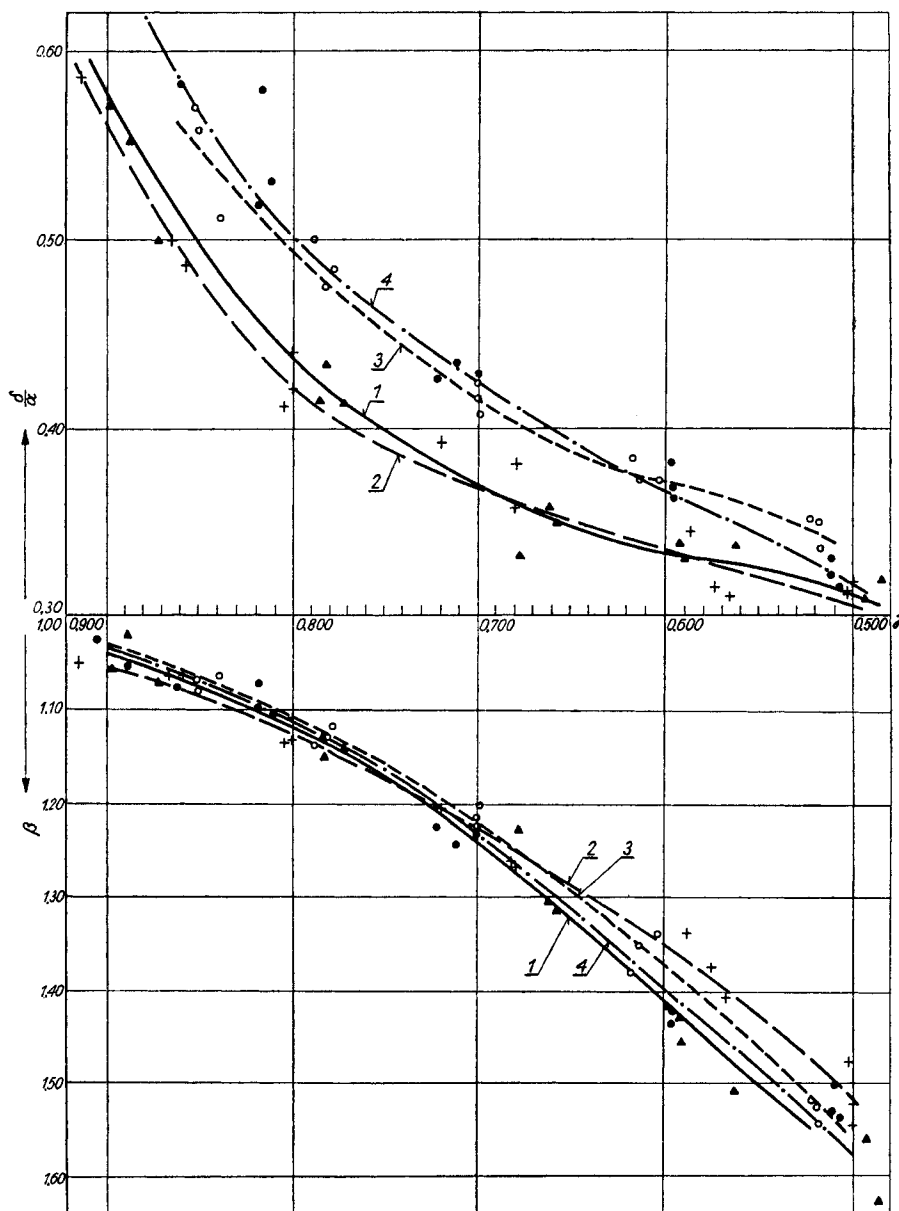


Fig. 3.74. The ratio of  $\delta/\alpha$  as a function of  $\gamma$  and  $\beta$  for  $b_1/h_1 = 1$ . Temperature: 1 — 1200°C, 2 — 1100°C, 3 — 1000°C, 4 — 900°C

$\delta/\alpha$  also depends on temperature, decreasing for nearly all values of  $\gamma$  and  $\beta$  with increasing temperature.

The practical value of Figs. 3.74–3.77 is that for a given value of coefficient of draught and spread, and given temperature, the ratio  $\delta/\alpha$  can be determined, and hence forward slip, and finally entry and exit speeds of stock can be determined.

Due to lack of values of neutral angle  $\delta$  obtained directly from measurement of pressure distribution along the roll gap, the accuracy could not be confirmed by comparison. At present such measurements can be carried out in laboratories.

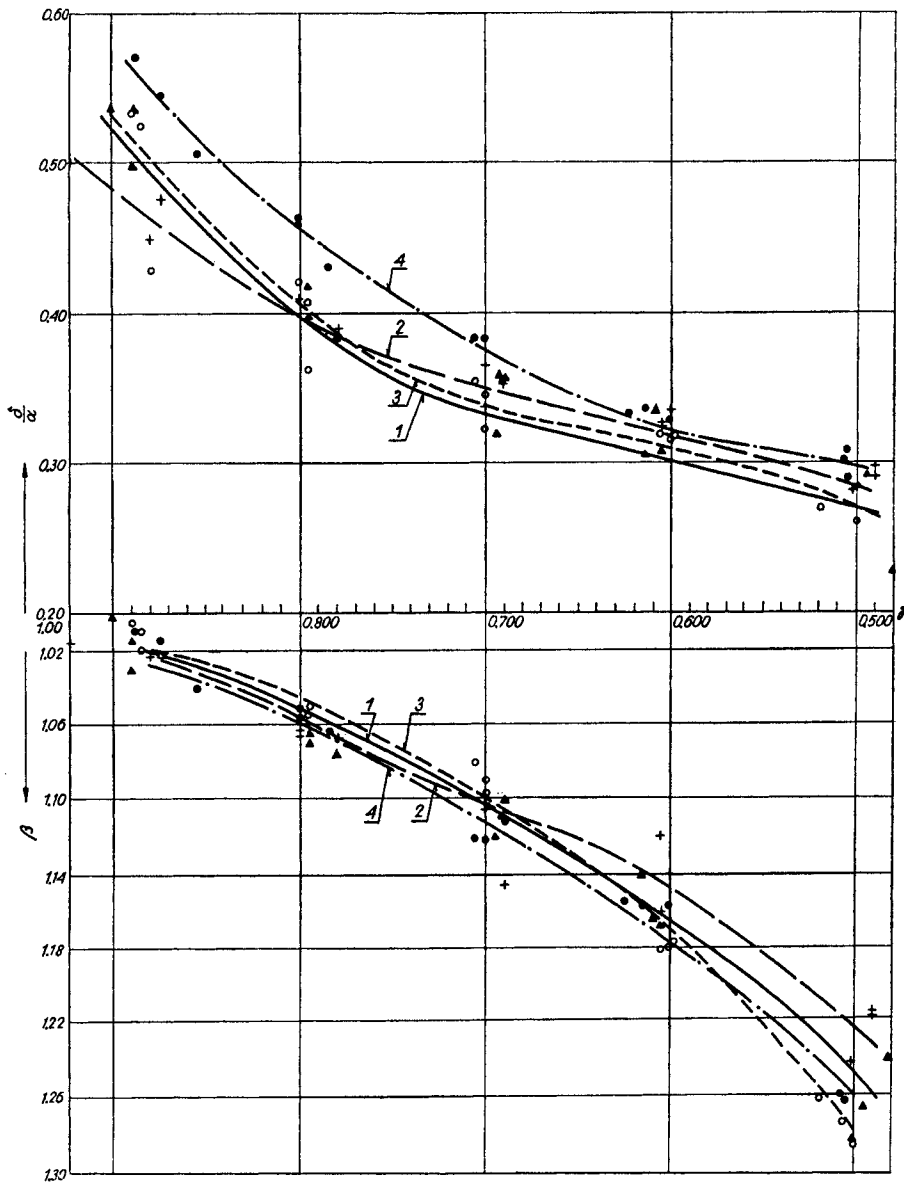


Fig. 3.75. The ratio of  $\delta/\alpha$  as a function of  $\gamma$  and  $\beta$  for  $b_1/h_1 = 2$ . Temperature: 1 — 1200°C, 2 — 1100°C, 3 — 1000°C, 4 — 900°C

A formula for determination of neutral angle while allowing for spread has been derived by Koncewicz [181].

It is assumed that roll pressure in the roll gap is symmetrical with respect to two planes, passing through the axis of rolled stock and the main axes of inertia of its cross-section. Therefore, only one condition of equilibrium remains, namely the equilibrium condition of the projection on the rolling axis, from which the neutral angle can be calculated. Assuming slip along the whole arc of contact between rolls and stock,

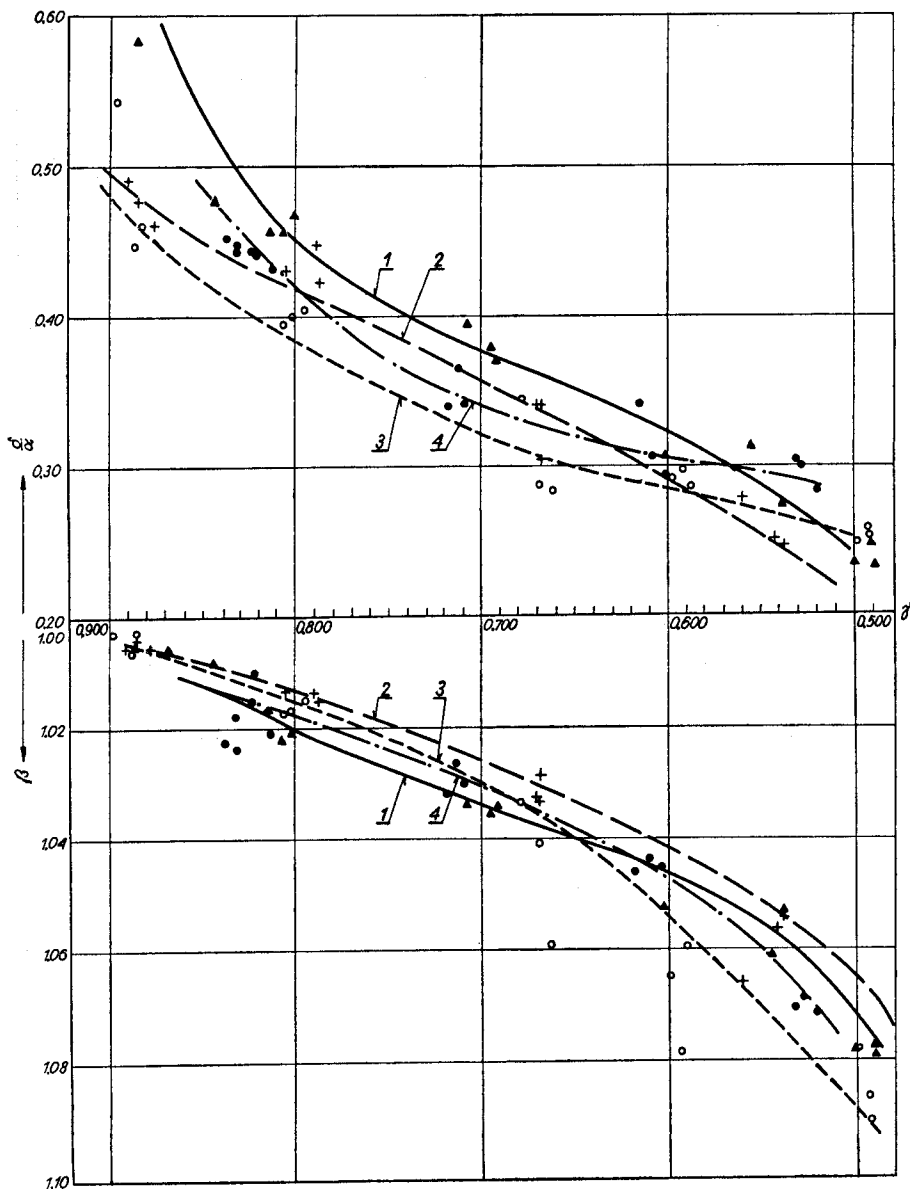


Fig. 3.76. The ratio  $\delta/\alpha$  as a function of  $\gamma$  and  $\beta$  for  $b_1/h_1 = 5$ . Temperature: 1 — 1200°C, 2 — 1100°C, 3 — 1000°C, 4 — 900°C

the roll gap can be divided into two zones: zone of forward slip and zone of backward slip (Fig. 3.78).

The horizontal component of elementary force due to roll pressure in the zone of backward slip ( $\alpha \geq \varphi \leq \delta$ ) is equal to (Fig. 3.78)

$$dH_1 = p_\varphi R d\varphi b_\varphi (\sin \varphi - f \cos \varphi) \quad (3.155)$$

and for the zone of forward slip ( $\delta \geq \varphi \geq 0$ )

$$dH_2 = p_\varphi R d\varphi b_\varphi (\sin \varphi + f \cos \varphi) \quad (3.156)$$

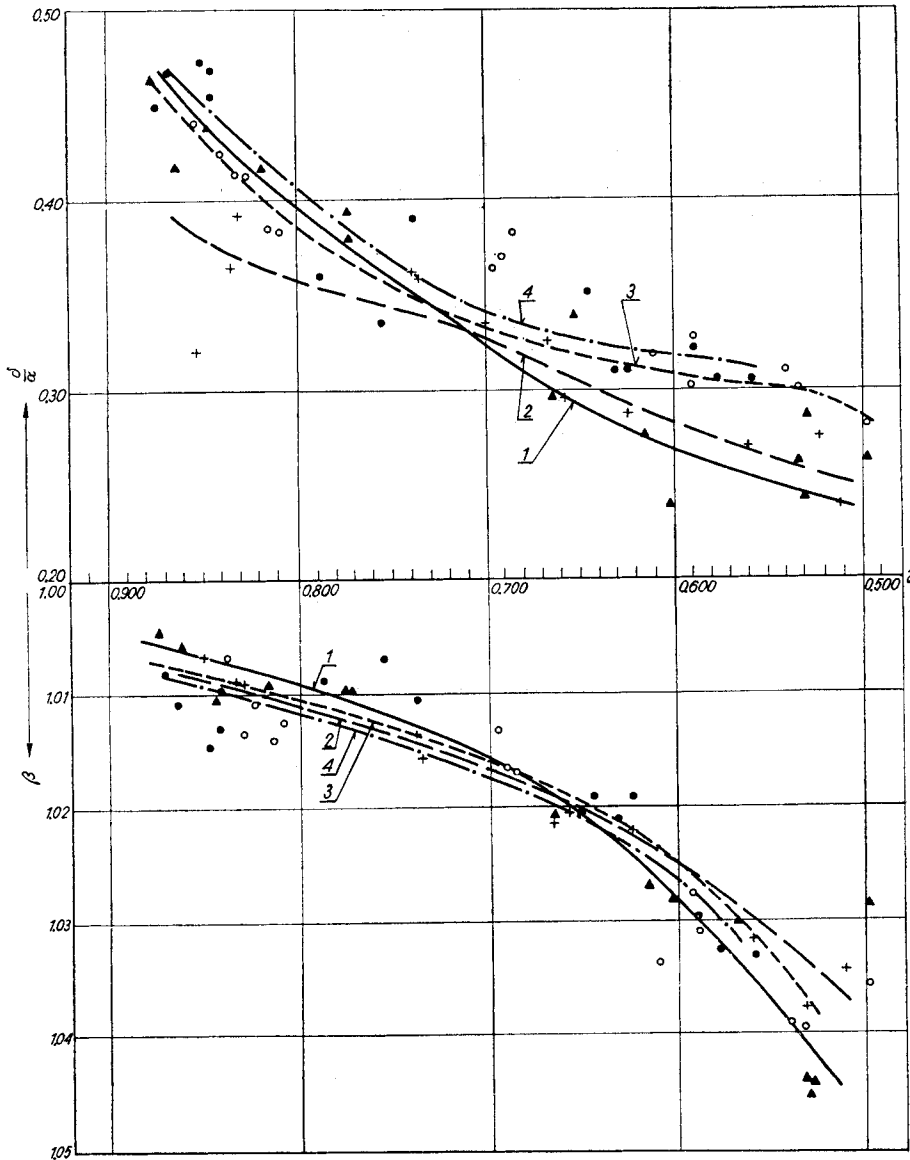


Fig. 3.77. The ratio of  $\delta/\alpha$  as a function of  $\gamma$  and  $\beta$  for  $b_1/h_1 = 10$ . Temperature: 1 — 1200°C, 2 — 1100°C, 3 — 1000°C, 4 — 900°C

When rolling without front and back tension, the condition of equilibrium can be expressed as

$$\left. \begin{aligned} \int_{\alpha}^{\delta} dH_1 + \int_{\delta}^0 dH_2 &= 0 \\ \int_0^{\delta} dH_2 + \int_{\delta}^{\alpha} dH_1 &= 0 \end{aligned} \right\} \quad (3.157)$$

or

After substituting (3.155) and (3.156) into (3.157)

$$\int_0^{\delta} p_{\varphi} R b_{\varphi} (\sin \varphi + f \cos \varphi) d\varphi + \int_{\delta}^{\alpha} p_{\varphi} R b_{\varphi} (\sin \varphi - f \cos \varphi) d\varphi = 0 \quad (3.158)$$

In this equation it has been assumed that the vertical roll pressure is independent of the coordinate in the direction of width  $b$  of the stock. Due to symmetrical loading this assumption results in only an insignificant error.

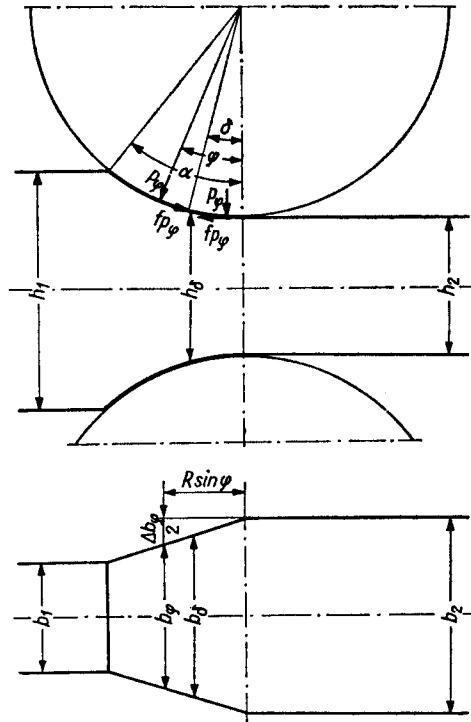


Fig. 3.78. Rolling conditions assuming linear variation of width, from Koncewicz [181]

For simplicity it has been assumed that the roll pressure is constant along the whole length of arc of contact and equal to the mean resistance to deformation.

Hence, equation (3.158) can be simplified to:

$$\int_0^{\delta} b_{\varphi} (\sin \varphi + f \cos \varphi) d\varphi + \int_{\delta}^{\alpha} b_{\varphi} (\sin \varphi - f \cos \varphi) d\varphi = 0 \quad (3.159)$$



and after simple transformations

$$\int_0^{\alpha} b_{\varphi} \sin \varphi \, d\varphi + f \left( \int_0^{\delta} b_{\varphi} \cos \varphi \, d\varphi - \int_{\delta}^{\alpha} b_{\varphi} \cos \varphi \, d\varphi \right) = 0 \quad (3.160)$$

where a constant value of coefficient of friction along the whole length of contact between roll and stock, has been assumed.

To solve equation (3.160), the function  $b = f(\varphi)$  should be determined.

A linear variation of width of the rolled stock along roll gap has been assumed (Fig. 3.78), hence

$$b_{\varphi} = b_2 - \Delta b \frac{\sin \varphi}{\sin \alpha} \quad (3.161)$$

where values of  $b_2$  and  $\Delta b$  can be determined by any of the formulae for calculation of spread, or obtained directly from measurement of appropriate rolled specimens.

After substituting (3.161) into (3.159)

$$\begin{aligned} & \int_0^{\alpha} \left( b_2 - \frac{\Delta b}{\sin \alpha} \sin \varphi \right) \sin \varphi \, d\varphi \\ & + f \int_0^{\delta} \left( b_2 - \frac{\Delta b}{\sin \alpha} \sin \varphi \right) \cos \varphi \, d\varphi - f \int_{\delta}^{\alpha} \left( b_2 - \frac{\Delta b}{\sin \alpha} \sin \varphi \right) \cos \varphi \, d\varphi = 0 \end{aligned} \quad (3.162)$$

After simple transformations

$$\begin{aligned} & \int_0^{\alpha} \left[ \beta - (\beta - 1) \frac{\sin \varphi}{\sin \alpha} \right] \sin \varphi \, d\varphi \\ & + f \int_0^{\delta} \left[ \beta - (\beta - 1) \frac{\sin \varphi}{\sin \alpha} \right] \cos \varphi \, d\varphi - f \int_{\delta}^{\alpha} \left[ \beta - (\beta - 1) \frac{\sin \varphi}{\sin \alpha} \right] \cos \varphi \, d\varphi = 0 \end{aligned} \quad (3.163)$$

After integrating and re-forming, equation (3.163) can be written as

$$\begin{aligned} & \sin^2 \delta - \frac{2\beta}{\beta - 1} \sin \alpha \sin \delta - \frac{\beta}{\beta - 1} \frac{\sin \alpha}{f} (1 - \cos \alpha - f \sin \alpha) \\ & + \frac{\sin \alpha}{2f} \left( \frac{\alpha}{\sin \alpha} - \cos \alpha - f \sin \alpha \right) = 0 \end{aligned} \quad (3.164)$$

Putting

$$A = \frac{2\beta}{\beta - 1} \sin \alpha$$

and

$$B = \frac{\beta}{\beta - 1} \frac{\sin \alpha}{f} (1 - \cos \alpha - f \sin \alpha) + \frac{\sin \alpha}{2f} \left( \frac{\alpha}{\sin \alpha} - \cos \alpha - f \sin \alpha \right)$$

equation (3.164) can be written as

$$\sin^2 \delta - A \sin \delta - B = 0 \quad (3.165)$$

and hence

$$\sin \delta = \frac{A}{2} \pm \sqrt{\frac{A^2}{4} + B} \quad (3.166)$$

Neglecting the irrational root for normal rolling conditions and substituting for  $A$  and  $B$ , the following equation for  $\sin \delta$  is obtained

$$\sin \delta = \frac{\beta}{\beta-1} \sin \alpha - \sqrt{\left(\frac{\beta}{\beta-1}\right)^2 \sin^2 \alpha + \frac{\beta}{\beta-1} \frac{\sin \alpha}{f} (1 - \cos \alpha - f \sin \alpha) - \frac{\sin \alpha}{2f} \left(\frac{\alpha}{\sin \alpha} - \cos \alpha - f \sin \alpha\right)} \quad (3.167)$$

or transforming

$$\sin \delta = \frac{\beta}{\beta-1} \sin \alpha \times \left\{ 1 - \sqrt{1 + \frac{\beta-1}{\beta} \frac{1}{f \sin \alpha} \left[ 1 - \cos \alpha - f \sin \alpha - \frac{\beta-1}{2\beta} \left( \frac{\alpha}{\sin \alpha} - \cos \alpha - f \sin \alpha \right) \right]} \right\} \quad (3.168)$$

Equation (3.168) is the final formula for calculation of neutral angle, using the additional relation

$$\delta = \arcsin \sin \delta \quad (3.169)$$

This derived formula (3.168) for calculation of neutral angle is of rather a complicated form, and its solution presents some difficulties. It can be considerably simplified, assuming an approximate value under the root sign [181]

$$\frac{\alpha}{\sin \alpha} \approx 1$$

Taking this into account

$$\sin \delta = \frac{\beta}{\beta-1} \sin \alpha \left\{ 1 - \frac{1}{\beta} \sqrt{1 + \frac{\beta^2-1}{2} \left( 1 + \frac{1-\cos \alpha}{f \sin \alpha} \right)} \right\} \quad (3.170)$$

This is the final simplified form of formula for more accurate calculation of neutral angle during rolling with spread.

Formula (3.170) can be further simplified assuming for small values of bite angle

$$\sin \alpha \approx \alpha, \quad 1 - \cos \alpha \approx \frac{\alpha^2}{2} \quad \text{and} \quad \sin \delta \approx \delta$$

Hence

$$\delta = \frac{\beta}{\beta-1} \alpha \left\{ 1 - \frac{1}{\beta} \sqrt{1 + \frac{\beta^2-1}{2} \left( 1 + \frac{\alpha}{2f} \right)} \right\} \quad (3.171)$$

To facilitate calculations with these derived formulae, a nomogram has been worked out, as given in Fig. 3.79. Dotted lines show how this nomogram can be used [181].

During tests for measurement of forward slip several parameters were measured [181]:

- (1) initial dimensions of specimen before heating,
- (2) dimensions of specimen after rolling,
- (3) temperature of the heating furnace,
- (4) temperature of each specimen after leaving the rolls,
- (5) peripheral speed of rolls,
- (6) exit speed of the rolled stock.

Apart from these measurements, templates of initial stock and of each specimen after rolling were prepared, and true cross-sectional areas were measured from them.

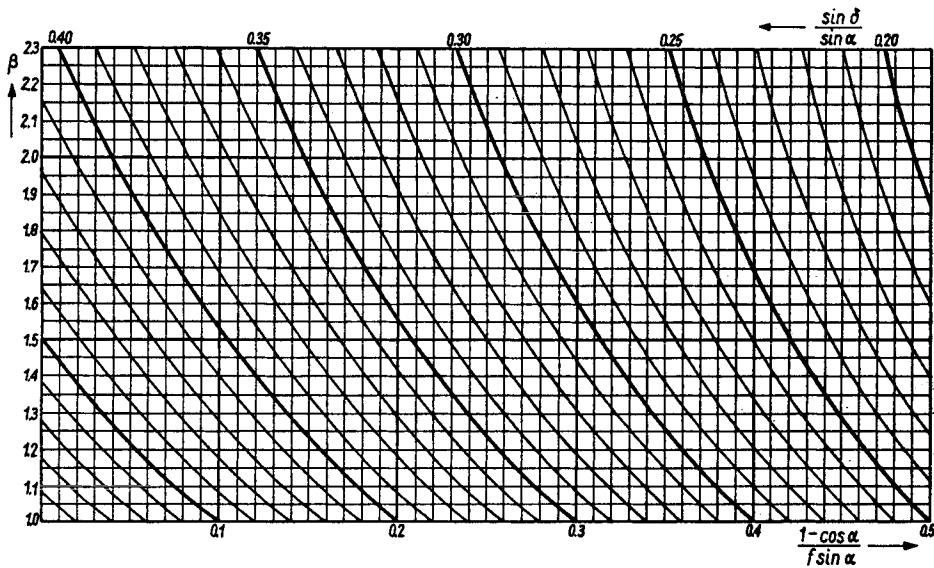


Fig. 3.79. Ratio  $m = \sin \delta / \sin \alpha$  as a function of  $(1 - \cos \alpha) / f \sin \alpha$ , from formula (3.170)

For measuring the exit speed of stock a special device was used, shown schematically in Fig. 3.80. It consists of a housing 1, inserted in the guides 2 perpendicular to roll axis. A movable slide 3 is mounted in the housing, and during operation is pressed down on the rolled stock by means of spring 4 and double-arm lever 5. The slide 3 is coupled to the rolled stock, and therefore housing 1 moves with a speed equal to that

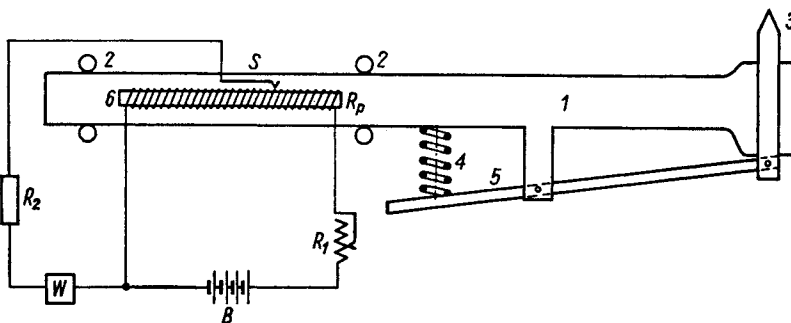


Fig. 3.80. Scheme of device for measuring exit speed of the rolled stock from rolls

of the stock. A potentiometer  $\delta$  is fitted to the housing, and is fed from a battery  $B$  through an adjustable resistance  $R_1$ . Movement of the potentiometer relative to the fixed slide contact  $S$ , gives a change in tension at recorder terminal  $W$ , causing a deflection of the indicator pen. The recorder indications are proportional to the change of setting of potentiometer. The two extreme settings of potentiometer (i.e. length  $l_p$ ) correspond to the two extreme indications of the recorder pointer  $l_r$ . For a constant feed of paper of  $v_i$ , recording deflections of recorder pen, the speed of motion of the potentiometer can be calculated from relation

$$v_p = v_i \frac{l_p}{l_r} \tan \alpha \quad (3.172)$$

where  $\alpha$  is the slope of the curve on the recorder tape.

As this device was made specially for these tests, it was calibrated on an Amsler horizontal tensile strength testing machine for a constant speed of travel of potentiometer. The constant speed of travel was achieved by means of the lead-screw of the testing machine, driven by a synchronous motor.

An example of calibration curve is shown in Fig. 3.81 [181].

Tests were carried out on specimens of MSt3-steel with the following analysis: 0.14–0.22% C, 0.40–0.65% Mn, 0.15–0.30% Si, 0.05% P, and 0.05% S.

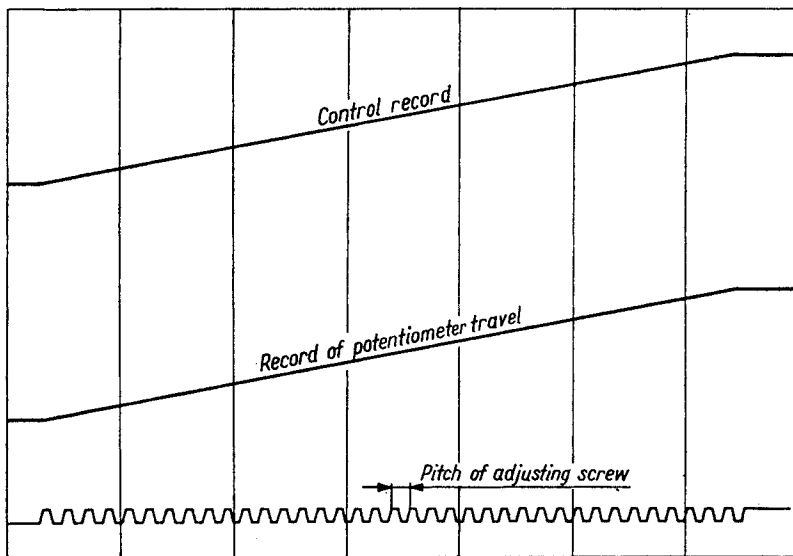


Fig. 3.81. Calibration curve for speed measurement device

Two basic groups of specimen dimensions were used, i.e.  $\delta_w = 1$ , and  $\delta_w = 5$ .

Trials were carried out for the range of normally used values of draught, i.e. up to  $\varepsilon_h = 50\%$ . The following draughts were used: 10, 20, 30, 40 and 50%, and only for one group of specimens ( $40 \times 40$  mm,  $t = 1000^\circ\text{C}$ ) this range was enlarged up to 60, 70 and 80%.

Based on errors of measured peripheral speed of rolls  $v_r$ , and exit speed  $w_2$ , it can be assumed that the error of measured coefficient of forward slip  $s$  should not exceed  $\pm 1.5\%$  (average value  $\pm 1.28\%$ ).

Therefore, values of coefficient of forward slip should vary from 0.985 to 1.015 of the most probable values. Results of measurements are shown in diagrams (Figs. 3.82–3.90). Curves can be drawn through the measured points, shown as full lines on these figures, and the probable scatter is indicated by dotted lines.

In certain cases measured values lay outside the assumed area of scatter. This can be explained by greater error of given measurements, variation of rolling conditions, e.g. by lapping into the specimen a layer of scale, insufficiently cleaned before the pass.

For the whole range of draughts used, forward slip increases with increasing percentage draught. This increase is more rapid for small final thicknesses of rolled stock.

It was also found that with increasing temperature there is initially a decrease in forward slip, followed by a rapid increase (Figs. 3.82, 3.83, and 3.84); when rolling flats the influence of temperature on forward slip is considerably less than for squares [181].

For calculation of forward slip formulae by the following have been considered: Fink, Drezden, Bachtinov, Pavlov, Vinogradov and modified formula of Wusatowski, for which the neutral angle was calculated by the author's formula. Of these formulae, only those of Fink and Drezden do not take into account spread and its influence on forward slip. However in all of these formulae the neutral angle is calculated neglecting the influence of spread.

For comparison, calculated and measured values of coefficient of forward slip are shown in diagrams (Figs. 3.85–3.90) [181].

Full lines show the mean measured values and broken lines show the probable limit of error of measured values, thus indicating the scatter of measured values. Values of forward slip, calculated from Fink's and Drezden's formulae for rolling squares, lie inside this area of scatter for only a small range of draught. Marked differences between values calculated from these formulae and measured ones, occur at a rolling temperature of about 1000°C. When rolling flats these differences are considerably less marked.

Values obtained from Pavlov's and Vinogradov's formulae are considerably too small, when rolling squares within normally used range of temperature and draught

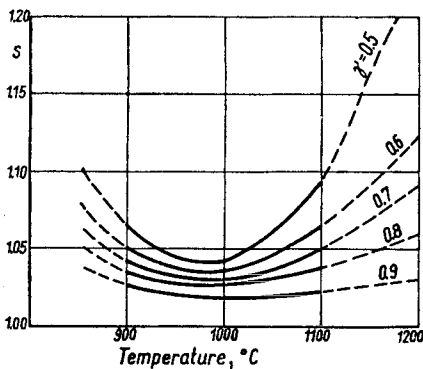


Fig. 3.82. Coefficient of forward slip as a function of temperature and draught when rolling 40×40 mm square

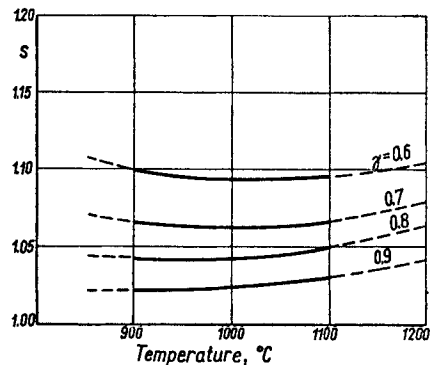


Fig. 3.83. Coefficient of forward slip as a function of temperature and draught, for rolling 100×20 mm flat

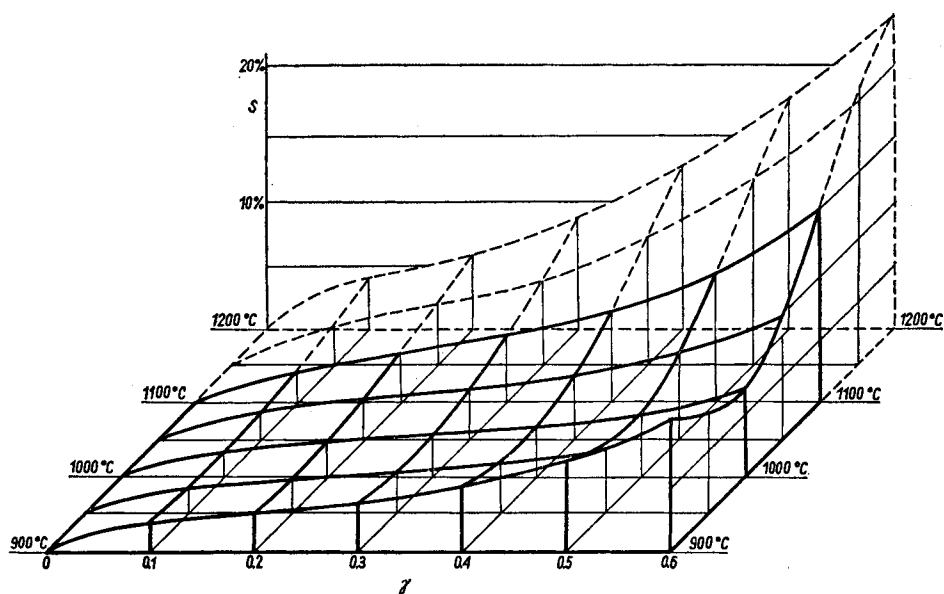


Fig. 3.84. Coefficient of forward slip as a function of temperature and draught, for rolling  $40 \times 40$  mm square

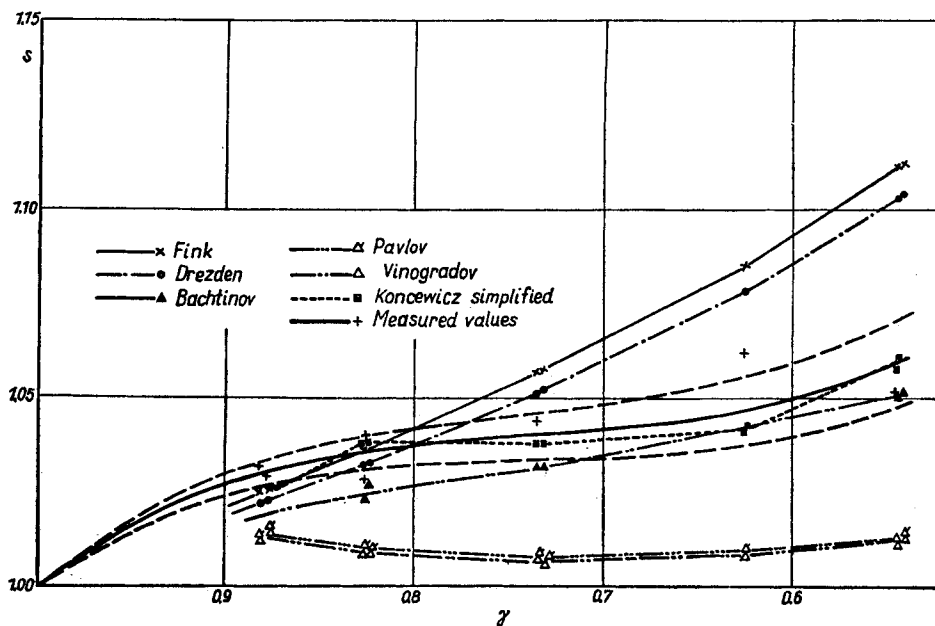


Fig. 3.85. Coefficient of forward slip as a function of draught, measured values and calculated from various authors for rolling  $40 \times 40$  mm square;  $t = 900^\circ\text{C}$

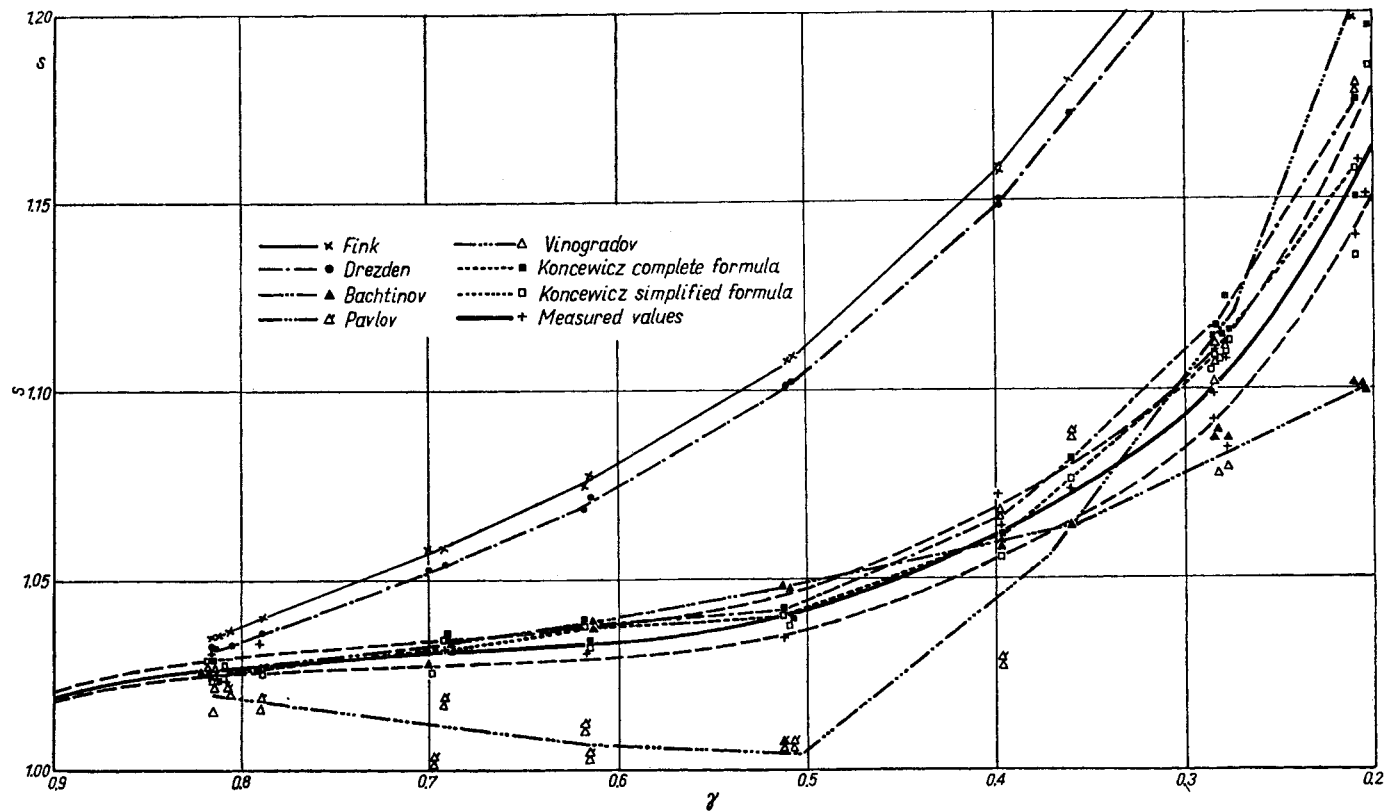


Fig. 3.86. Coefficient of forward slip as a function of draught, measured values and calculated from various authors, for rolling  $40 \times 40$  mm square;  $t = 1000^\circ\text{C}$

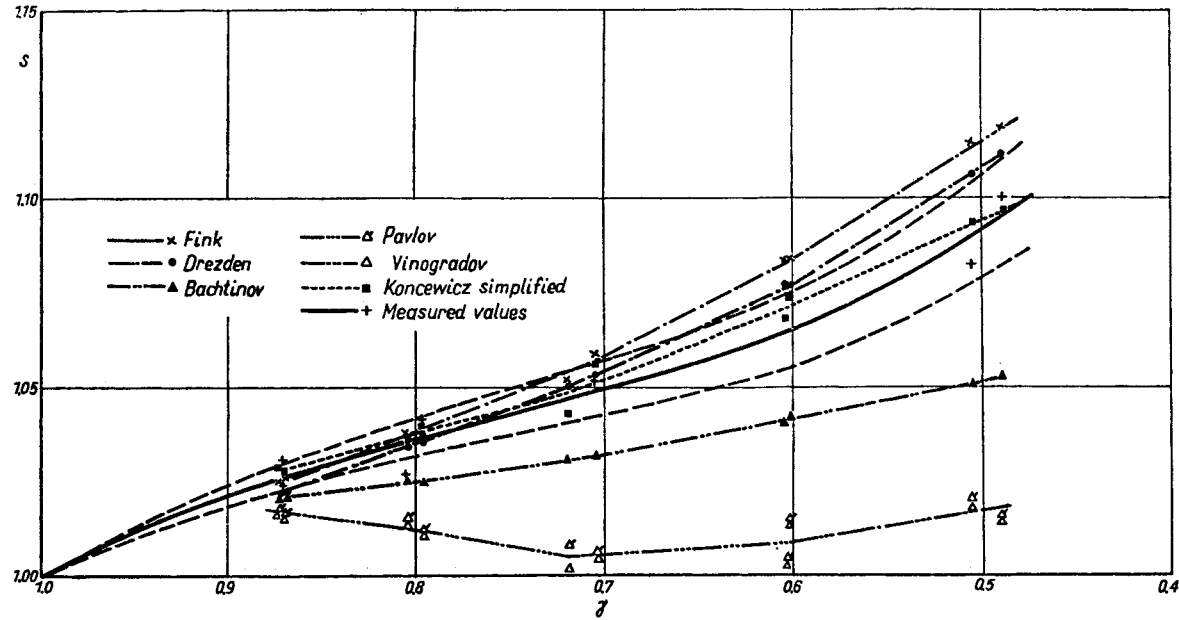


Fig. 3.87. Coefficient of forward slip as a function of draught, measured values and calculated from various authors, for rolling  $40 \times 40$  mm square;  $t = 1100^\circ\text{C}$



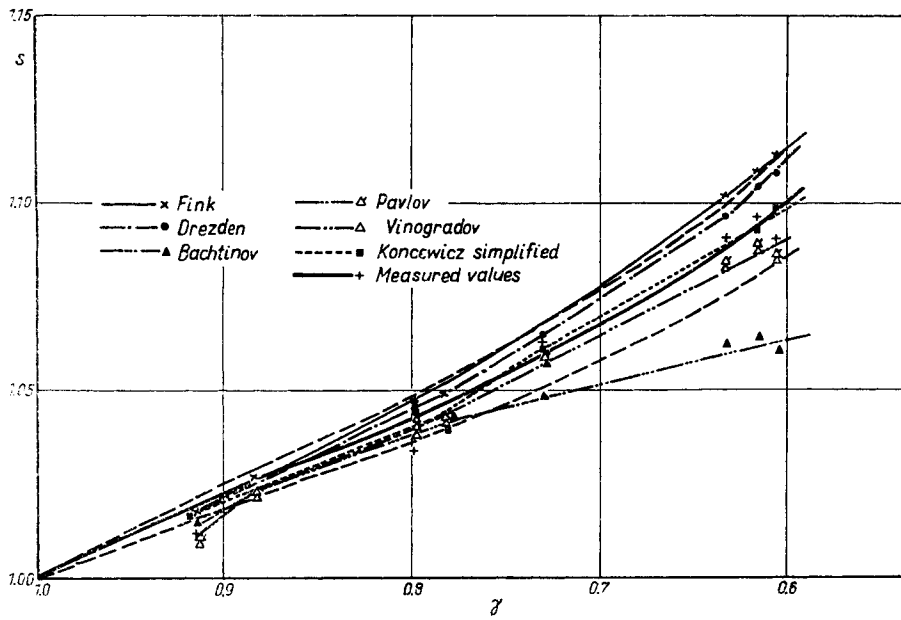


Fig. 3.88. Coefficient of forward slip as a function of draught, measured values and calculated from various authors, for rolling  $100 \times 20$  mm flat;  $t = 900^\circ\text{C}$

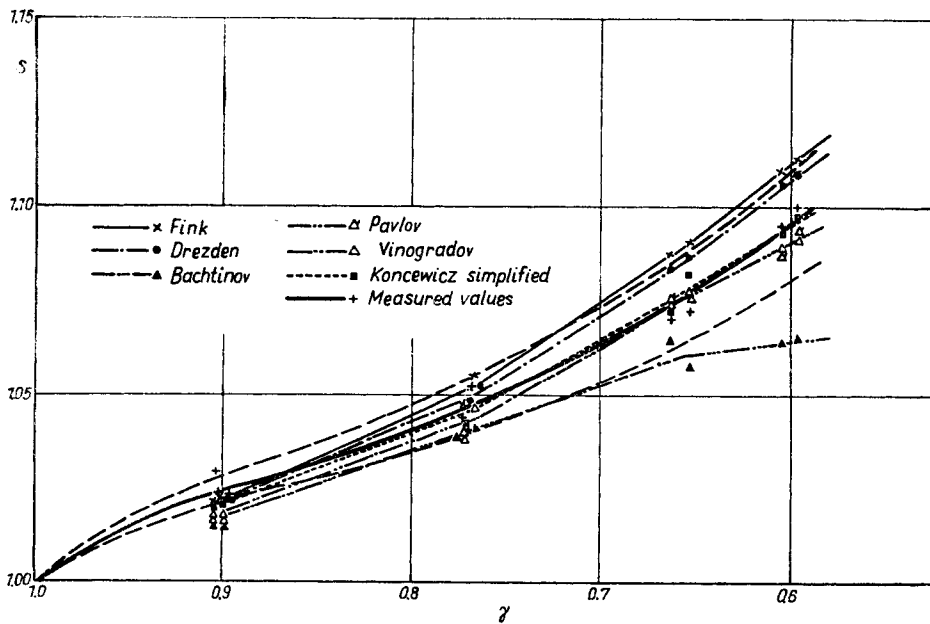


Fig. 3.89. Coefficient of forward slip as a function of draught, measured values and calculated from various authors, for rolling  $100 \times 20$  mm flat;  $t = 1000^\circ\text{C}$

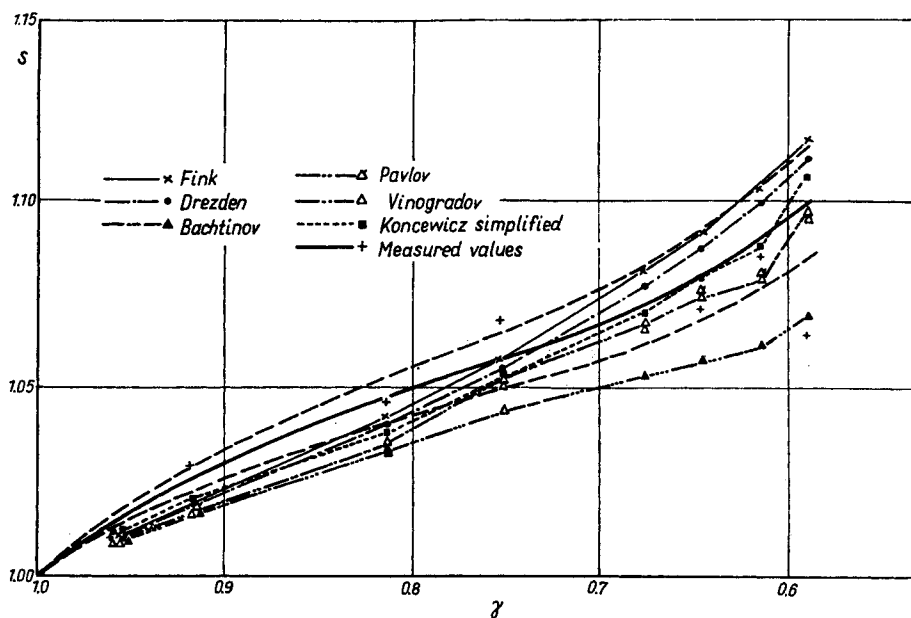


Fig. 3.90. Coefficient of forward slip as a function of draught, measured values and calculated from various authors, for rolling  $100 \times 20$  mm flat;  $t = 1100^\circ\text{C}$

(up to 60%). When rolling flats (small spread) values obtained from these formulae approach measured ones. In this case the curve for calculated values is within the scatter field.

Relatively good results are found from Bachtinov's formula when rolling squares at temperatures 900 and  $1000^\circ\text{C}$ , however at  $1100^\circ\text{C}$  values of forward slip considerably lower than measured ones are found. For draughts greater than 25% too small values are obtained from Bachtinov's formula, when rolling flats.

Values from formulae proposed by the author (dotted line) are within the field of scatter for almost the whole range of measurements, except two points obtained for rolling squares at  $900^\circ\text{C}$ , and when rolling flats at  $1100^\circ\text{C}$  and draught of 25%. As in this last case all calculated curves lie below the field of scatter, it is probable that the run of the calculated curve has been influenced by two points with greater error than the permissible one calculated from the analysis of errors [180].

### 3.8.2. NEUTRAL ANGLE IN HOT AND COLD ROLLING WITHOUT SPREAD

The choice of a suitable formula for determination of neutral angle for a given condition of rolling is of basic importance for all calculations resulting from the theory of rolling, since a correct value of neutral angle is necessary for all other calculations, i.e. forward slip, backward slip, and exit and entry speed [54, 57].

Early formulae for neutral angle were derived with the following assumptions, both for hot and cold rolling:

- (1) width of stock  $b_x$  is constant,
- (2) radial roll pressure along the arc of contact between roll and stock, is constant,
- (3) coefficient of friction during rolling is constant,

(4) slip friction between stock and roll is taken into account, but possible sticking between roll and stock is neglected,

(5) roll flattening in cold rolling is not taken into account.

Results calculated from early derived formulae differ considerably from true values, and the more given rolling conditions vary from those in points (1) to (5) the greater differences are obtained.

When discussing these formulae, the unsimplified form is given, and then brought to a form suitable for comparison (trigonometric form).

The combined formulae of Pavlov [54] and Ekelund [57] give

$$\sin \delta = \frac{\sin \alpha}{2} - \frac{1 - \cos \alpha}{2f} \quad (3.173)$$

where

$\alpha$  = bite angle,

$\delta$  = neutral angle,

$f$  = coefficient of friction.

Tselikov's formula [59] is

$$\sin \delta = \frac{\Delta h}{2R} \left( \sqrt{\frac{R}{\Delta h} - \frac{1}{4}} - \frac{1}{2f} \right) \quad (3.174)$$

If

$$\sin \delta = \frac{\sqrt{R\Delta h - \Delta h^2/4}}{2R} - \frac{\Delta h}{4Rf} \quad (3.175)$$

and

$$\frac{\Delta h}{D} = 1 - \cos \alpha \quad \sin \alpha = \frac{l_d}{R}$$

then after substitution formula (3.173) is obtained

$$\sin \delta = \frac{\sin \alpha}{2} - \frac{1 - \cos \alpha}{2f}$$

From these transformations it can be concluded formulae (3.173) and (3.174) are identities, which can be transformed to the final form (3.173).

For analysis of these formulae, and determination of values achieved by them for  $\alpha = \varphi = f$ , i.e. for bite angle equal to coefficient of friction, or for  $f$  equal to maximum bite angle, all formulae are transformed into sine form, and as small angles are assumed,  $\sin \alpha \approx \alpha$ .

Assuming  $\sin \alpha = \alpha = f$ , then

$$\delta = \frac{f}{2} - \frac{(f/2)^2}{f} = \frac{f}{2} - \frac{f^2}{4f} = \frac{f}{4} \quad (3.176)$$

Assuming  $f = \alpha$ , gives

$$\delta = \frac{\alpha}{4}$$

More accurate formulae for calculation of neutral angle for all conditions of hot and cold rolling of flats, with assumed constant width and slip between roll and stock, have been derived by E. Orowan [35].

Similarly, using Orowan's method a formula for calculation of neutral angle has been derived by the author [54], assuming non-uniform deformation and slip

over the whole length of contact, i.e. the frictional resistance at stock-roll boundary equals  $\tau = f\sigma$ . These formulae are only valid for cold rolling with values of  $f$  up to a maximum of 0.2. The following auxiliary functions were derived by Orowan for such rolling conditions:

(a) from the plane of exit to the neutral plane

$$z^+ = \frac{h}{h_2} e^{fH} \quad (3.177)$$

(b) from the plane of entry to the neutral plane

$$z^- = \frac{h}{h_2} e^{f(H_1-H)} \quad (3.178)$$

Assuming these values are equal to each other at neutral plane

$$z^+ = z^-$$

hence

$$H_N = \frac{H_1}{2} - \frac{1}{2f} \log_e \frac{h_1}{h_2} \quad (3.179)$$

The value of  $H_1$  of auxiliary function for plane of entry is calculated from the formula derived by Orowan [35]

$$H_1 = \frac{2(1+h_2/D)}{\sqrt{\frac{h_2}{D}(2+h_2/D)}} \arctan \left[ \sqrt{\frac{2+h_2/D}{h_2/D}} \tan \frac{\alpha_1}{2} \right] - \alpha_1 \quad (3.180)$$

The value of bite angle  $\alpha_1$  should be substituted in radians.

$D$  = roll diameter.

Substituting value  $\delta$  in formula (3.180), the value of function  $H$  at neutral plane is

$$H_N = \frac{2(1+h_2/D)}{\sqrt{\frac{h_2}{D}(2+h_2/D)}} \arctan \left( \sqrt{\frac{2+h_2/D}{h_2/D}} \tan \frac{\delta}{2} \right) - \delta \quad (3.181)$$

Introducing known initial values, and some small simplifications [54] then

$$\delta = \frac{H_N h_2}{D} \quad (3.182)$$

The calculation of neutral angle is then simple, the value of  $H_N$ , calculated from equations (3.179) and (3.180) is substituted into (3.182), and multiplied by ratio of thickness after pass to roll diameter, and the value found is transformed from radians into degrees.

Hence to find angle  $\delta$ , the following operations are necessary:

(a) from equation (3.180)  $H_1$  is found, knowing bite angle  $\alpha$  and ratio  $h_2/D$ ,

(b)  $H_1$  is substituted into equation (3.179), giving  $H_N$ ,

(c) knowing  $H_N$  equation (3.182) can be solved, (simplified solution) giving  $\delta$ .

Substituting equation (3.179) directly into (3.182), after transforming the formula for  $\delta$  becomes

$$\delta \approx \frac{\alpha}{2} - \frac{h_2}{D} \frac{1}{2f} \log_e \frac{h_1}{h_2}$$

This formula can be used for calculation of neutral angle when rolling with slip.

When using the non-simplified form the method is similar. The value of function  $H_1$  is calculated from equation (3.180), after substituting the bite angle  $\alpha_1$  in radians. Instead of laborious calculations with formula (3.180), results obtained by Orowan [35] can be used (Figs. 3.91, 3.92, 3.93), and for given values of  $\alpha$  and  $h_2/D$  the function  $H_1$  can be read off from diagrams.

In Figures 3.91, 3.92 and 3.93 values of angle are intentionally denoted as  $\varphi$  to stress the fact that any rolling angle from 0 to  $\alpha$  can be substituted. The value of  $H_1$  obtained corresponds to the given angle, i.e.  $H_\varphi$  for angle  $\varphi$  is found.

This value of  $H_1$  is substituted into equation (3.179), giving  $H_N$ . Knowing  $H_N$  and the ratio  $h_2/D$ , the value of angle  $\delta$  can be read off from Orowan's diagrams.

Although for the case of sticking between stock and rolls Orowan's formula can only be solved graphically, integrating point by point [25], however after the assumption of certain simplifications it can be solved as follows. Using the differential equations given by Cook and Larke (Fig. 3.1):

for the entry side

$$\frac{H_\varphi}{h} = D \int_{\varphi_1}^{\varphi} K_f m' \frac{d\varphi}{h} \quad (3.183)$$

for the exit side

$$\frac{H_\varphi}{h} = D \int_{\varphi_2}^{\varphi} K_f m \frac{d\varphi}{h} \quad (3.184)$$

where

$H_\varphi$  = value of variable horizontal roll force in given cross-section of rolled stock,

$h$  = variable thickness of stock along length of contact,

$\varphi$  = rolling angle.

Values of  $m$  are determined as follows:

for the entry side

$$m' = \left[ 0.785 + \frac{1}{2} \left( \frac{1}{\varphi} - \frac{1}{\tan \varphi} \right) \right] \sin \varphi - \frac{1}{2} \cos \varphi \approx 0.785 \sin \varphi + \frac{1}{2} \quad (3.185)$$

for the exit side

$$m = \left[ 0.785 - \frac{1}{2} \left( \frac{1}{\varphi} - \frac{1}{\tan \varphi} \right) \right] \sin \varphi + \frac{1}{2} \cos \varphi \approx 0.785 \sin \varphi + \frac{1}{2} \quad (3.186)$$

The coefficient of 0.785 in these formulae takes into account non-uniform deformation of rolled stock [35].

For solving these equations, the constrained yield stress  $\eta K_f$  is assumed equal to the mean constrained yield stress of stock  $\eta K_{fm}$ , as without this assumption a numerical solution is impossible.

As a second assumption, the width of stock  $b$  during the pass is made constant.

In this way, solutions are obtained for the entry and exit side, expressing variation of horizontal force  $H_\varphi$  as a function of given thickness  $h$  of rolled stock along the arc of contact.

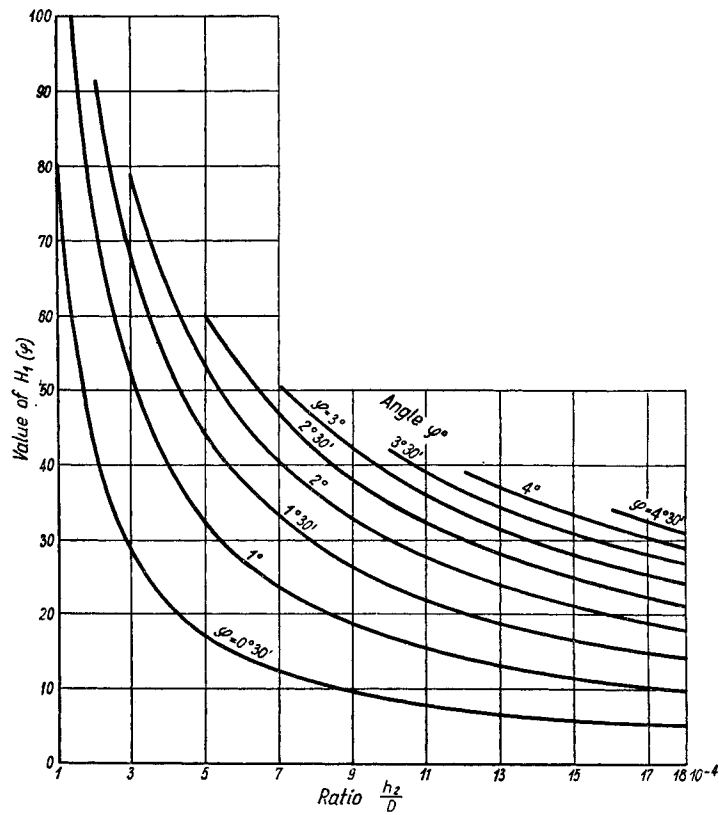


Fig. 3.91. Relation between  $H_1$  and  $h_2/D$  at  $\delta \leq 4^\circ 35'$  [35]

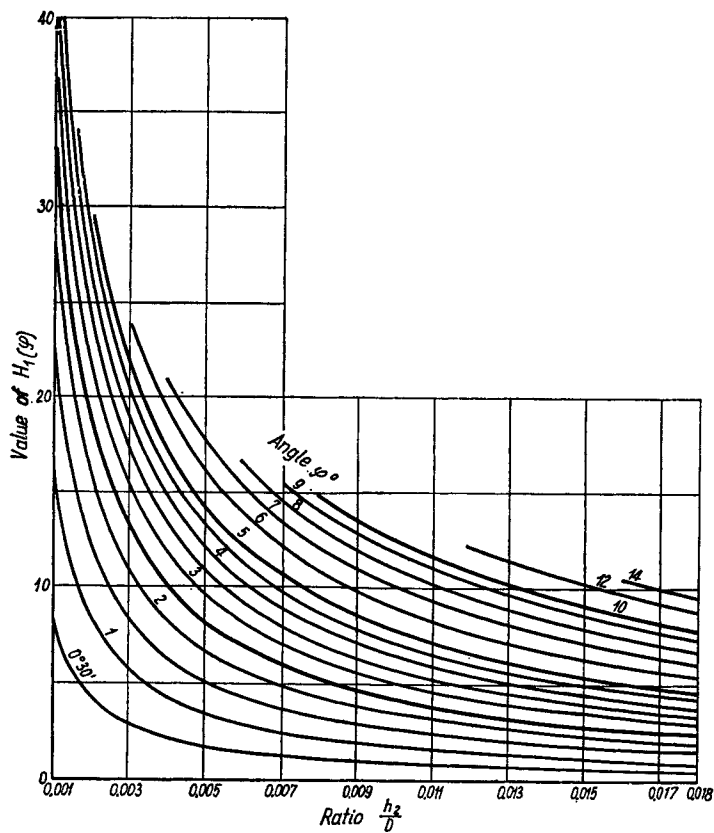


Fig. 3.92. Relation between  $H_1$  and  $h_2/D$  at  $\delta \leq 14^\circ$  [35]

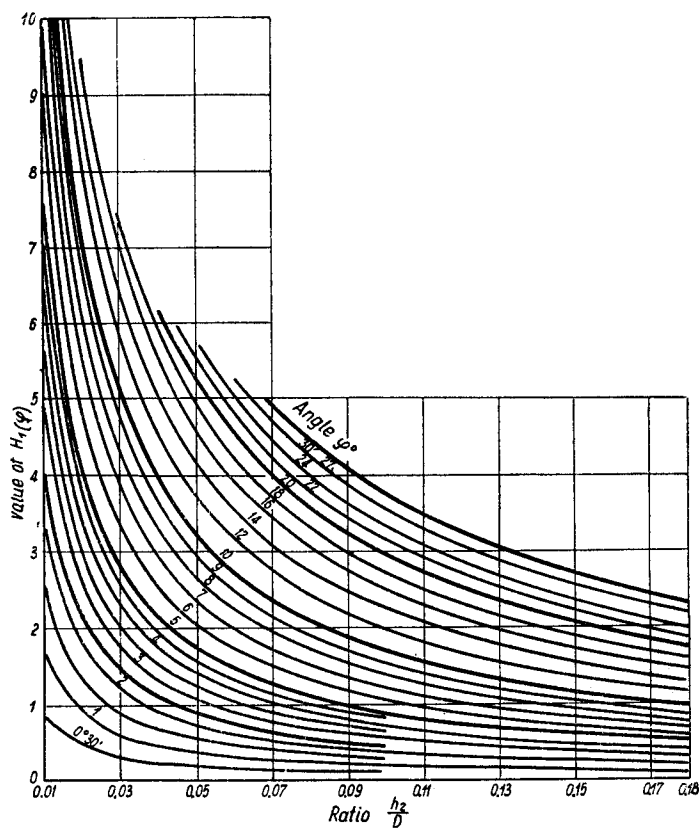


Fig. 3.93. Relation between  $H_1$  and  $h_2/D$  at  $\delta \leq 30^\circ$  [35]



For the entry side

$$\frac{H_\varphi}{h} = D \int_{\varphi_1}^{\varphi} K_{fm} m' \frac{d\varphi}{h}$$

For the exit side

$$\frac{H_\varphi}{h} = D \int_{\varphi_2}^{\varphi} K_{fm} m \frac{d\varphi}{h}$$

At the neutral plane horizontal forces should be in equilibrium, as the process is a continuous one and at any point along the arc of contact the forces acting upon an element of stock are in equilibrium.

Hence

$$\frac{H_\varphi}{h} (\text{entry}) = \frac{H_\varphi}{h} (\text{exit})$$

The value of  $K_{fm}$  can be disregarded, as it occurs in all expressions of this equation i.e.  $K_{fm}$  does not influence the position of neutral angle.

After ordering and solving the equation [54]

$$\tan \frac{\delta}{2} = \frac{1}{a} \tan \left[ \frac{0.785 a}{a^2 - 1} \log_e \frac{1}{a^2 - (a^2 - 1) \cos^2 \frac{\alpha}{2}} + \frac{1}{2} \arctan \left( a \tan \frac{\alpha}{2} \right) \right] \quad (3.187)$$

where

$$a = \sqrt{\frac{2D}{h_2}} + 1$$

By means of formula (3.187) it is possible to calculate the position of neutral angle in the case of sticking of stock to rolls along almost the whole length of contact, for rolling without spread.

Special nomograms have been worked out (Figs. 3.94 and 3.95) for giving the neutral angle  $\delta$  directly in degrees, from equation (3.187), without laborious calculations. Fig. 3.94 gives the angle  $\delta$  as a function of ratio  $D/h_2$ , and bite angles up to  $15^\circ$ , and Fig. 3.95 the same relation for values of bite angle from  $15$  to  $40^\circ$  [54].

It is only necessary to calculate the appropriate value of ratio  $D/h_2$ , and on this curve for the given values of bite angle for the pass, to read off the neutral angle  $\delta$  directly in degrees and minutes.

As the boundaries between slipping and sticking are unknown, for practical calculations it can be assumed with close approximation that slip occurs between stock and rolls along the whole arc of contact in cold rolling, while in hot rolling slip would occur when using chilled smooth ground rolls, i.e. for small values of coefficient of friction [54]. According to Orowan-Pascoe [60] sticking occurs for large coefficients of friction of 0.4–0.5.

To illustrate the variation of these relations for conditions of hot rolling, a theoretical example has been calculated for rolling steel strip of initial thickness 2.0 mm to a final one of 1.0 mm in all cases at a constant coefficient of friction  $f = 0.40$ , but

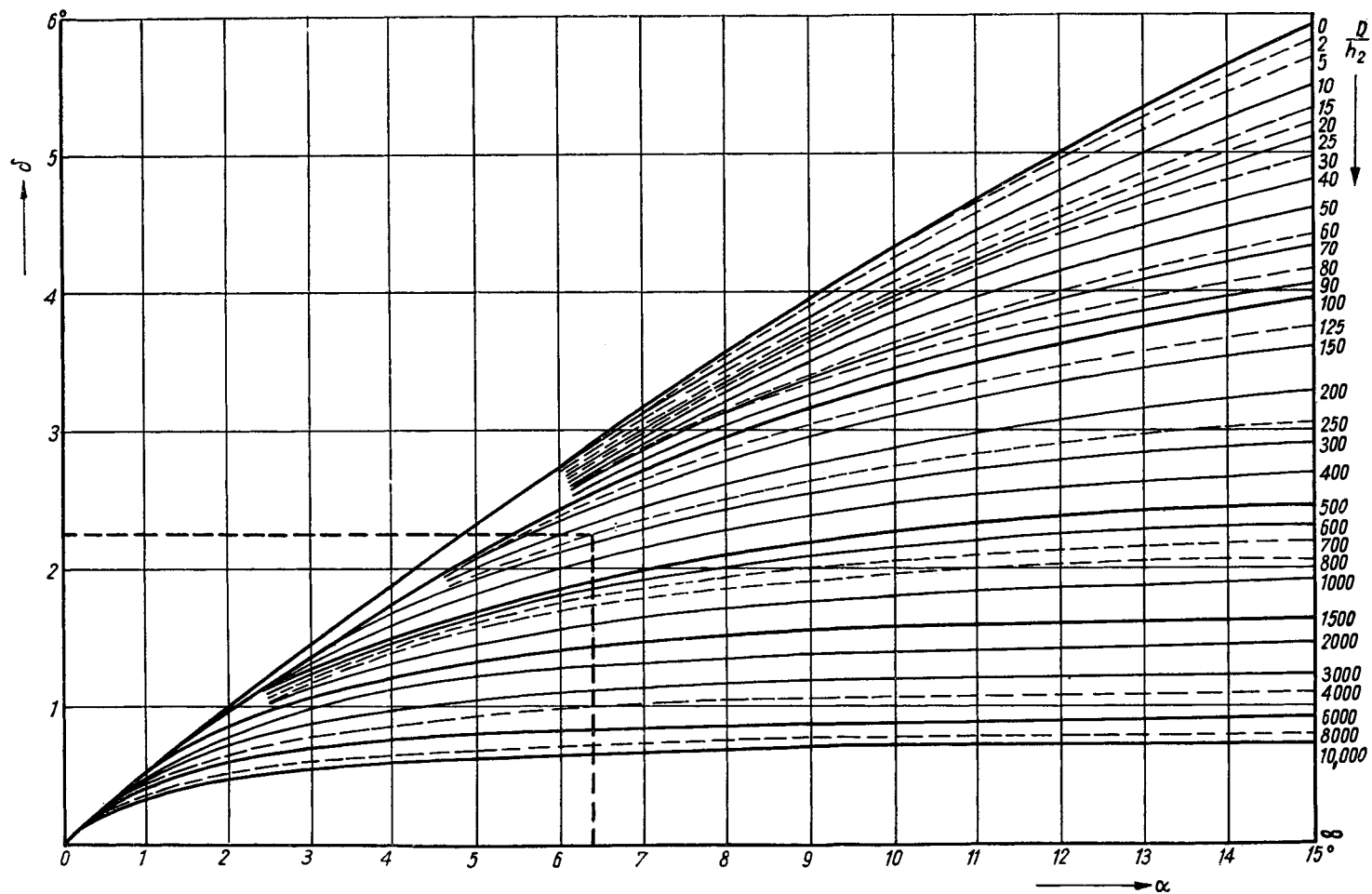


Fig. 3.94. Diagram for determination of  $\delta$  from formula (3.187) for a bite angle  $\alpha$  varying within 0 and  $15^\circ$  [10]

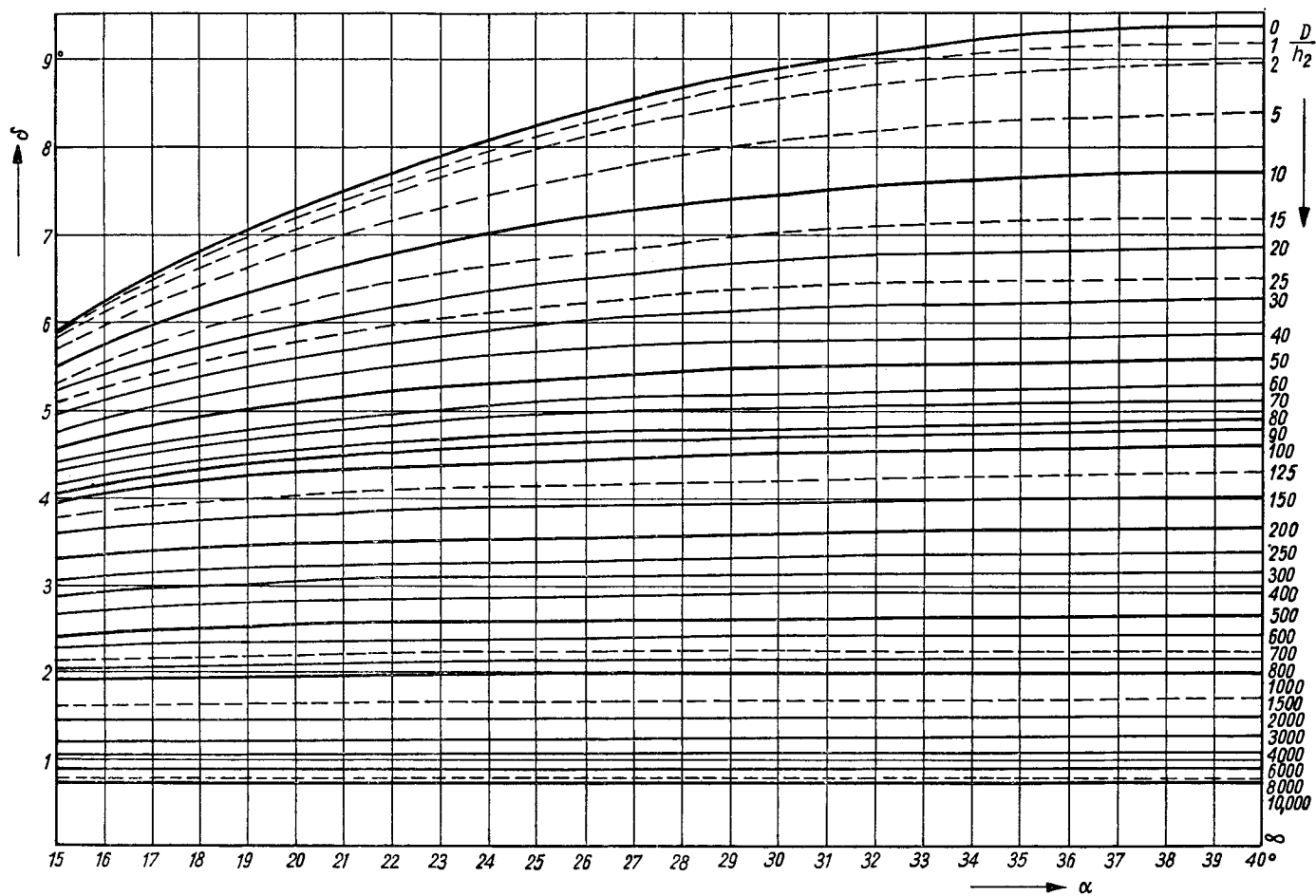


Fig. 3.95. Diagram for determination of  $\delta$  from formula (3.187) for a bite angle  $\alpha$  varying within 15° and 40° [10]

with varying bite angles for rolls of 10, 50, 100, 300 and 500 mm diameter (Fig. 3.96). It can be seen from the curves that with increasing bite angle ratio  $\delta/\alpha$  decreases, and this decrease is almost independent of the method of calculation used.

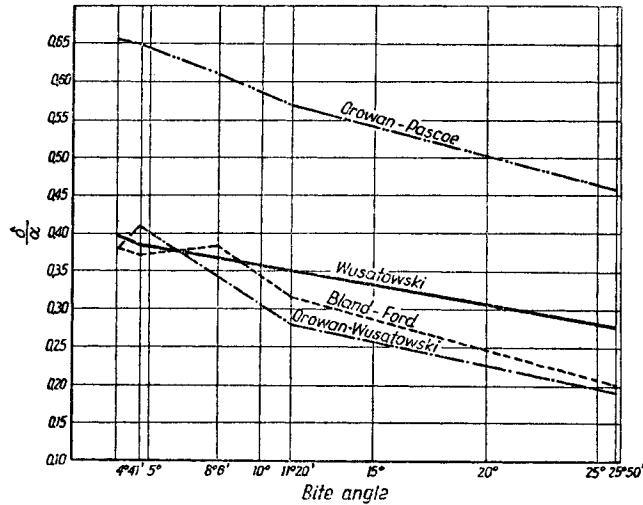


Fig. 3.96. Relation between  $\delta/\alpha$  and  $\alpha$  for rolling narrow steel strip on rolls of diameter 10 to 500 mm

The methods of Bland-Ford and Orowan-Wusatowski show only very slight differences.

Comparison of theoretical values and those determined experimentally shows a certain error, due to simplified initial assumptions.

For rolling with front and back tension, Tselikov's formula is used for determination of neutral angle [59]

$$\sin \delta = \frac{h_2}{\sqrt{R \Delta h}} \left( \frac{h_n}{h_2} - 1 \right) \quad (3.188)$$

where

$$\frac{h_n}{h_2} = \left[ \frac{1 + \sqrt{1 + (\xi_1 \delta - 1)(\xi_2 \delta + 1) \left( \frac{h_1}{h_2} \right)^\delta}}{\xi_2 \delta + 1} \right]^{1/\delta} \quad (3.189)$$

In these equations

$$\xi_1 = 1 - \frac{\sigma_1}{1.15 K_f}, \quad \xi_2 = 1 - \frac{\sigma_2}{1.15 K_f}, \quad \delta = 2f \sqrt{\frac{R}{\Delta h}} \quad (3.190)$$

$K_f$  = yield stress in normal tensile test (yield stress will be discussed in chapter of the roll forces),

$\sigma_1, \sigma_2$  = stresses produced by front and back tension,

$f$  = coefficient of friction between stock and rolls.

From these considerations and results, the following recommendations for choice of formulae for neutral angle are given.

(1) For cold rolling of narrow and wide strip without tension, and for sheet, first of all, Orowan-Wusatowski's method is the most accurate (equations (3.179), (3.181) and (3.182)).

When speed without special accuracy is required, Pavlov-Ekelund's formula can be used (equation (3.173)). It should be noted that when using diagrams in Figs. 3.91, 3.92 and 3.93 for Orowan-Wusatowski's method, no more effort is required than for Pavlov-Ekelund's method.

If it is found that the coefficient of friction in cold rolling exceeds 0.2, formula (3.187) should be used, as sticking of stock to roll surfaces should be expected. Such conditions are very rare in cold rolling.

(2) For hot rolling of strip and sheet, sticking between stock and rolls should be expected. There is no spread for  $b_1/h_1 > 20$ , and good results can be obtained using equation (3.187). This equation can also be used for smaller values of  $b_1/h_1$ , e.g. 8–20, but it should be taken into account that slightly lower values are obtained, which should be increased according to the actual spread. For still smaller values of  $b_1/h_1$  i.e. when rolling flats and squares, the diagram in Fig. 3.69 is recommended.

If when rolling narrow strip coefficients of friction are very small, Orowan-Wusatowski's method gives the best results, corresponding to conditions of slip between stock and rolls in the roll gap.

(3) For hot rolling of bars and shapes, good results can only be obtained from the author's method (Fig. 3.69), since it is the only one to allow for spread, but mean values  $h_{2m}$ ,  $D_m$  must be substituted.

#### Example 1

The neutral angle is to be calculated for cold rolling of annealed aluminium strip. Conditions of rolling:  $h_1 = 2.0$  mm,  $h_2 = 1.0$  mm,  $D = 180$  mm, coefficient of friction  $f = 0.14$ , bite angle  $\alpha = 6^\circ 02'$ . Orowan-Wusatowski's method is used (Figs. 3.91–3.93).

For  $\alpha = 6^\circ 02'$  and  $h_2/D = 0.005555$ ,  $H_\phi = H_1 = 15.0$  can be read off from Fig. 3.92. This value is then substituted into equation (3.179) to determine  $H_N$

$$H_N = \frac{H_1}{2} - \frac{1}{2f} \log_e \frac{h_1}{h_2} = \frac{15.0}{2} - \frac{1}{2 \times 0.14} \log_e \frac{2.0}{1.0} = 7.5 - 2.47552 = 5.02$$

For known values of  $H_1$  and  $h_2/D$ , the neutral angle of  $\delta = 1^\circ 46'$  can be read off from Fig. 3.92.

Calculating directly, from equation (3.182)

$$\delta = \frac{H_N h_2}{D} = \frac{5.02 \times 1.0}{180} = 0.028 \text{ radians}$$

i.e.

$$\delta = 1^\circ 36'$$

Calculating the same example using Pavlov-Ekelund's method (3.173)

$$\sin \delta = \frac{\sin \alpha}{2} - \frac{1 - \cos \alpha}{2f}$$

$$\sin \delta = \frac{0.10511}{2} - \frac{1 - 0.0055556}{2 \times 0.14} = 0.052555 - 0.0198413 = 0.033142$$

Hence

$$\delta = 1^\circ 54'$$

In this case, there is close agreement between values from the simplified formula and exactly determined values.

*Example 2*

Rolling speed is to be calculated for a bloom of cross-section  $b_1 h_1 = 250 \times 200$  mm rolled on smooth roll barrel with free spread. Rolling conditions: final cross-section  $b_2 h_2 = 262 \times 150$  mm, cast iron rolls of  $D_w = 700$  mm,  $n = 85$  rpm, temperature of stock =  $1000^\circ\text{C}$ .

The coefficient of elongation is

$$\lambda_1 = \frac{b_1 h_1}{b_2 h_2} = \frac{250 \times 200}{262 \times 150} = 1.2723$$

(a) From formula (3.101) the peripheral speed of roll is

$$v_r = \frac{\pi D_w n}{60} = \frac{\pi \times 0.700 \times 85}{60} = 3.114 \text{ m/sec}$$

(b) From formula (3.83), the coefficient of friction for cast iron rolls is

$$f = 1.05 - 0.0005 t = 1.05 - 0.50 = 0.55$$

(c) From formula (3.9) the bite angle  $\alpha$  is given as

$$\cos \alpha = 1 - \frac{\Delta h}{D_w} = 1 - \frac{200 - 150}{700} = 1 - 0.0715 = 0.9285$$

hence

$$\alpha = 21^\circ 47'$$

(d) The neutral angle, calculated allowing for spread from Fig. 3.69, is

$$\delta = 7^\circ 13'$$

(e) From equation (3.126) the entry speed allowing for spread is

$$v_1 = v_N \gamma_N \beta_N$$

where from (3.100)

$$v_N = v_r \cos \delta = 3.114 \times \cos 7^\circ 13' = 3.114 \times 0.9923 = 3.090 \text{ m/sec}$$

$$\gamma_N = \frac{h_N}{h_1}$$

From (3.119)

$$h_N = D_w (1 - \cos \delta) + h_2$$

$$D_w = 700 \text{ mm}, \quad h_2 = 150 \text{ mm}, \quad h_1 = 200 \text{ mm},$$

hence

$$h_N = 700(1 - 0.9923) + 150 = 155.39 \text{ mm}$$

$$\gamma_N = \frac{155.39}{200} = 0.7769$$

$$\delta_w = \frac{b_1}{h_1} = \frac{250}{200} = 1.25, \quad \epsilon_w = \frac{h_1}{D} = \frac{200}{700} = 0.2857$$

$$\beta_N = \gamma_N^{-w} = 1.0418 \text{ from (3.122)}$$

$$v_1 = 3.090 \times 0.7769 \times 1.0418 = 2.506 \text{ m/sec}$$

(f) The exit speed is calculated from formula (3.115)

$$w_2 = v_1 \lambda = 2.506 \times 1.2725 = 3.189 \text{ m/sec}$$

(g) The forward slip is calculated from formula (3.104)

$$s = \frac{w_2}{v_r} = \frac{3.189}{3.114} = 1.0238$$

**Example 3**

A steel flat,  $b_1 = 300$  mm wide, is rolled with very small spread from initial thickness  $h_1 = 9.275$  mm to a final one of  $h_2 = 3.5$  mm on rolls  $D = 900$  mm in diameter. The bite angle is  $\alpha = 6^\circ 26'$ , and rolling temperature  $1000^\circ\text{C}$ , coefficient of friction  $f = 0.30$ .

It is suggested that this example be solved using the diagram in Fig. 3.94.

Given data:

$$\frac{D}{h_2} = \frac{900}{3.50} = 257.1 \quad \text{and} \quad \alpha = 6^\circ 26'$$

After interpolation a value of  $\delta = 2^\circ 15'$  is read off.

**3.8.3. CALCULATION OF ROLLING SPEED OF BARS AND SECTIONS**

The rolling speed of all simple symmetrical sections (e.g. as in Fig. 3.3) for which the mean thickness can be determined directly, can be calculated by a modified method, based on formula (3.101) substituting the mean peripheral speed of rolls

$$v_{rm} = \frac{\pi D_{mw} n}{60} \quad (3.191)$$

In this equation  $D_{mw}$  from formula (3.30) is

$$D_{mw} = D_t - h_{2m}$$

where  $D_t$  is the distance between the axes of rolls, and  $h_{2m}$  is the mean thickness of outgoing stock. For a known value of  $v_{rm}$ , the coefficient of forward slip is calculated from formula (3.104)

$$s_m = \frac{w_{2m}}{v_{rm}} \quad (3.192)$$

The value of  $s_m$  can be determined for mean values using formulae for calculation of neutral angle, and taking into account the coefficient of spread. Mean values of angles and lengths must be used, when calculating  $s_m$ .

After calculating  $v_{1m}$  from formula (3.126), the value of  $w_{2m}$  is obtained from (3.115), where  $\lambda$  is the elongation coefficient for the given pass. In this way all data necessary for roll pass design or rolling can be calculated.

The problem of calculation of rolling speed for bars and shapes is limited to determination of the mean working diameter in the pass, as for calculating position of neutral plane and forward slip a suitable method is used appropriate to existing conditions of rolling.

When rolling flats, the mean working diameter can be determined as follows

$$D_{mw} = D_t - s \quad (3.193)$$

$$D_{mw} = D_t - h_2 \quad (3.194)$$

Formulae (3.191), (3.193) and (3.194) relate to bars with simple cross-sections without horizontal elements in the groove, i.e. diamonds, rounds etc. For other bars and sections corresponding relations will be derived.

In formulae (3.193) and (3.194)

$D_t$  = theoretical roll diameter (Fig. 3.1 and Fig. 3.3a),

$s$  = roll clearance,

$h_2$  = thickness of outgoing stock.

The calculation of rolling speed of bars and shapes is limited to determination of the mean working radius of the groove and pass.

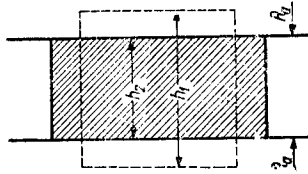


Fig. 3.97. Deformation of stock rolled between top and bottom wall of the pass only [54]

Two cases will first be considered:

(1) A bar,  $h_1$  in thickness, is rolled in a rectangular pass with free spread to a thickness of  $h_2$  (Fig. 3.97). For working radii of rolls of  $R_a$ , the exit speed of stock from formulae (3.101) and (3.192) is

$$w_{2m} = \frac{\pi R_a n}{30} s_m \quad (3.195)$$

where

$s_m$  = mean forward slip,

$n$  = rpm.

In the case of unequal working radii  $R_a \neq R_b$ , the stock leaves the roll gap with a mean speed, determined for the mean working radius of rolls

$$R_m = \frac{R_a + R_b}{2} \quad (3.196)$$

Hence

$$w_{2m} = \frac{\pi R_m n}{30} s_m \quad (3.197)$$

If one roll of different diameter is not driven (e.g. the centre roll in 3-high and Lauth mills), then the exit speed of stock is a function of the diameter of the driven roll. This speed is determined as for rolls of equal diameters.

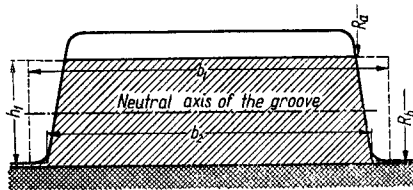


Fig. 3.98. Deformation of stock rolled between side-walls of the pass only [54]

(2) An exceptional case sometimes met in rolling practice is when a bar of a width of  $b_1$  is drawn into a deeper pass of width  $b_2$ , where there is no vertical compression and the height  $h_1$  undergoes only small variations (Fig. 3.98). Fig. 3.98 shows a certain kind of horizontal edging pass, in which only the width is changed, almost without change in height of rolled stock. In this case the rolling speed of stock depends only



on the radii  $R_a$  and  $R_b$ , and hence on  $R_m = (R_a + R_b)/2$ , as pressure is uniformly distributed along side walls of the pass. Therefore, in general it is possible to calculate the mean rolling speed using the mean working radius of the groove of the pass (equation (3.196)), if the influence of the horizontal straight part of pass, parallel to the neutra axis of rolls, can be neglected.

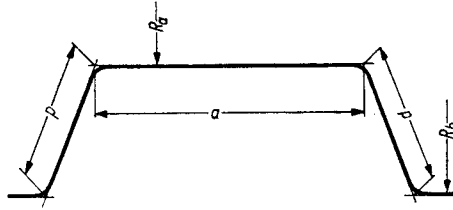


Fig. 3.99. Pass groove consisting of one straight sector and two diagonal sectors

However, if the groove of the pass is of the shape shown in Fig. 3.99, i.e. it consists of a horizontal sector  $a$ , and two diagonal ones  $p$ , then the speed distribution in the groove depends on the ratio  $a/p$ , as at sector  $a$  the speed depends on the working radius  $R_a$ , and at sectors  $p$  on the mean working diameter

$$R_m = \frac{R_a + R_b}{2}$$

In this case the rolling speed will depend on the mean working radius calculated as follows

$$R_m = \frac{a R_a + 2 \left( \frac{R_a + R_b}{2} \right) p}{a + 2p} \quad (3.198)$$

This equation is correct, as when rolling the stock on the smooth roll barrel with free spreading,  $p$  becomes equal to 0, and  $R_m = R_a$ , as in Fig. 3.97, while for passes without horizontal sectors  $p = 0$ , and  $R_m = (R_a + R_b)/2$ .

Therefore, for these rolling cases the simplified method of speed calculation can be used, substituting the mean height of pass  $h_m = F/b$ .

Hence, this method can be used for all pass designs of regular sections, except rolling of flats in box passes.

Methods of determining the rolling speed directly from the mean height have been discussed previously.

A schematic pass as shown in Fig. 3.100 is assumed, consisting of horizontal sectors  $a$  and diagonal ones  $p$ , where horizontal sectors are considered to be those parallel to the roll axis, and all other ones are considered as diagonal.

For the top groove the mean working radius will be

$$R_{mu} = \frac{a_2 R_2 + a_3 R_3 + \left( \frac{R_1 + R_2}{2} \right) p_1 + \left( \frac{R_2 + R_3}{2} \right) p_2 + \left( \frac{R_3 + R_4}{2} \right) p_3}{a_2 + a_3 + p_1 + p_2 + p_3} \quad (3.199)$$

and similarly for the bottom groove

$$R_{ml} = \frac{a_I R_I + a_{II} R_{II} + a_{III} R_{III} + \left( \frac{R_I + R_{II}}{2} \right) p_I + \left( \frac{R_{II} + R_{III}}{2} \right) p_{II}}{a_I + a_{II} + a_{III} + p_I + p_{II}} \quad (3.200)$$

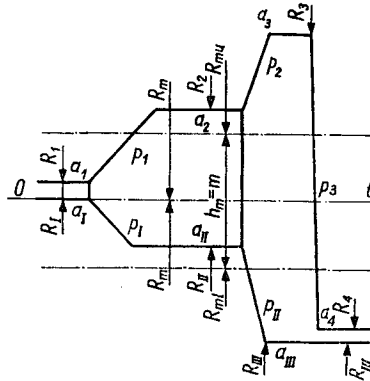


Fig. 3.100. Irregular pass consisting of horizontal, vertical and diagonal sectors

Hence, the mean working radius for the pass is

$$R_m = \frac{R_{mu} + R_{ml}}{2} \quad (3.201)$$

In general, equations (3.199) and (3.200) can be expressed as sums

$$R_{mw} = \frac{\sum a_i R_i + \sum \left( \frac{R_i + R_{i+1}}{2} \right) p_i}{\sum (a_i + p_i)} \quad (3.202)$$

Formulae (3.198)–(3.202) are valid only for the assumption that roll pressure is uniformly distributed in every part of the pass.

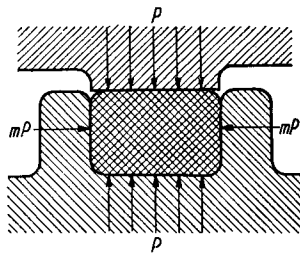


Fig. 3.101. Compression of stock with limited spread [22]

A rolling case with non-uniformly distributed pressure in various parts of the pass will be considered next. If the section shown in Fig. 3.101 is compressed by a force  $P$ , then the stock acts on the side walls of the pass with a force  $mP$ , where  $m$  is a coefficient. The lower limit of this coefficient is zero, i.e. for all passes of width greater

than that of the stock after pass, i.e. if sides of stock do not, or almost do not, touch side walls of the pass. The lateral forces  $mP$  become maximum for completely suppressed spread, i.e. the value of  $mP$  varies with the degree of spread suppression.

Thus maximum values correspond to  $\beta = 1$ , i.e. completely suppressed spread, and minimum values to free spread of stock.

The correct value of  $m$  should be determined from a relation between partly suppressed spread and temperature, but data are available for only a very small range, and approximate values must be taken for calculation.

Extensive research has been carried out by Z. Wusatowski and J. Ludyga [141] to investigate rolling in box passes.

Diagrams include values of coefficients:

$a'$  = coefficient of increase in roll force,

$a''$  = coefficient of increase in mean resistance to deformation,

$a'''$  = coefficient of increase in roll torque.

These diagrams show each of the coefficients  $a'$ ,  $a''$ ,  $a'''$  as a function of draught at temperatures of 900, 1000 and 1100°C (see Chapter 4, Section 4.3).

Values of  $m$  can be found from Siebel's results [6], worked out for roll force but applicable also in this case. These values are given in Table 3.18 as values of coefficient of loss  $m$  in open and closed passes.

TABLE 3.18  
VALUES OF COEFFICIENT OF LOSS  $m$  AT SIDE WALLS OF PASS FOR DIFFERENT TYPES OF PASS

| Types of pass          | Coefficient $m$ | Type of pass             | Coefficient $m$ |
|------------------------|-----------------|--------------------------|-----------------|
| Closed passes:         |                 | Open passes:             |                 |
| flat                   | 0.2–0.3         | square                   | 0.1–0.2         |
| for narrow strip       | 0.2–0.3         | gothic square            | 0.2–0.3         |
| roughing forming pass  | 0.3–0.5         | diamond                  | 0.2–0.3         |
| finishing forming pass | 0.5–1.0         | oval (from square)       | 0.2–0.3         |
|                        |                 | roughing pass for shapes | 0.2–0.5         |

Unfortunately, these values show a large scatter and it was not possible to determine values of coefficient of  $m$  in relation to true rolling conditions.

For non-uniformly distributed forces in the pass and a constant coefficient of friction  $f$ , there are frictional forces of  $T = fP$  in straight and compressed sectors of the pass, and in diagonal ones where spreading occurs, a force  $T_1 = mfP$ , where  $m$  is a function of degree of spread restriction.

Assuming a groove as in Fig. 3.99 with a force  $P$  acting on horizontal sectors  $a$ , and a force  $mP$  on sides of the pass  $p$ , and that the product  $Pf$  is constant, then formula (3.198) becomes

$$R_m = \frac{aR_a + 2mp \left( \frac{R_a + R_b}{2} \right)}{a + 2pm} \quad (3.203)$$

From this it can be concluded that when the influence of side walls of the pass is less than that of the horizontal sectors, the mean working diameter of pass approaches the horizontal sector of pass.

This problem cannot be treated so simply for compound passes, as draught and displacement of metal particles from point to point occurs. However, for any compound pass an analysis can be carried out dividing roll forces relative to draught, also spread of particular parts of the pass and displacement of particles from point to point can be calculated.

For a constant coefficient of  $m$  for the whole pass, equations (3.199) and (3.200) take the following forms (Fig. 3.100):

$$R_{mu} = \frac{a_1 R_1 + a_2 R_2 + a_3 R_3 + a_4 R_4 + [(R_1 + R_2)p_1 + (R_2 + R_3)p_2 + (R_3 + R_4)p_3]m/2}{a_1 + a_2 + a_3 + a_4 + m(p_1 + p_2 + p_3)} \quad (3.204)$$

$$R_{ml} = \frac{a_I R_I + a_{II} R_{II} + a_{III} R_{III} + [(R_I + R_{II})p_I + (R_{II} + R_{III})p_{II}]m/2}{a_I + a_{II} + a_{III} + m(p_I + p_{II})} \quad (3.205)$$

For different values of coefficient of  $m$  at particular parts of the pass the following formulae should be used:

$$R_{mu} = \frac{a_1 R_1 + a_2 R_2 + a_3 R_3 + a_4 R_4 + [(R_1 + R_2)p_1 m_1 + (R_2 + R_3)p_2 m_2 + (R_3 + R_4)p_3 m_3]/2}{a_1 + a_2 + a_3 + a_4 + m_1 p_1 + m_2 p_2 + m_3 p_3} \quad (3.206)$$

$$R_{ml} = \frac{a_I R_I + a_{II} R_{II} + a_{III} R_{III} + [(R_I + R_{II})p_I m_I + (R_{II} + R_{III})p_{II} m_{II}]/2}{a_I + a_{II} + a_{III} + p_I m_I + p_{II} m_{II}} \quad (3.207)$$

#### Example 1

The mean working radius for the roughing section shown in Fig. 3.102 is required.

Applying equation (3.199) for the top part of the pass, the numerator is

$$\begin{aligned} 10.7 \times 145.1 + 25.4 \times 186.4 + 26.2 \left( \frac{186.4 + 210.2}{2} \right) + 53.8 \left( \frac{210.2 + 193.0}{2} \right) \\ + 47.3 \left( \frac{193.0 + 145.1}{2} \right) + 133.4 \left( \frac{145.1 + 275.7}{2} \right) = 58392.09 \text{ mm}^2 \end{aligned}$$

and the denominator is

$$10.7 + 25.4 + 26.2 + 53.8 + 47.3 + 133.4 = 296.8 \text{ mm}$$

$$R_{mu} = \frac{58392.09}{296.8} = 196.74 \text{ mm}$$

Similarly for the lower part of pass,  $R_{ml}$  can be determined from formula (3.200).

The numerator is

$$\begin{aligned} 23.5 \times 171.8 + 62.4 \left( \frac{171.8 + 233.8}{2} \right) + 25.8 \left( \frac{171.8 + 198}{2} \right) \\ + 58.1 \left( \frac{198 + 212}{2} \right) + 75.2 \left( \frac{212 + 144.5}{2} \right) = 46777.34 \text{ mm}^2 \end{aligned}$$

and the denominator

$$23.5 + 62.4 + 25.8 + 58.1 + 75.2 = 245 \text{ mm}$$

Hence

$$R_{ml} = \frac{46,777.34}{245} = 190.93 \text{ mm}$$

Knowing  $R_{mu} = 196.74 \text{ mm}$  and  $R_{ml} = 190.93 \text{ mm}$ , the mean working diameter of pass is calculated from equation (3.201)

$$R_m = \frac{R_{mu} + R_{ml}}{2} = \frac{190.93 + 196.74}{2} = 193.83 \text{ mm}$$

It should be emphasized that formula (3.201) should only be used where draughts are uniformly distributed in the whole pass.

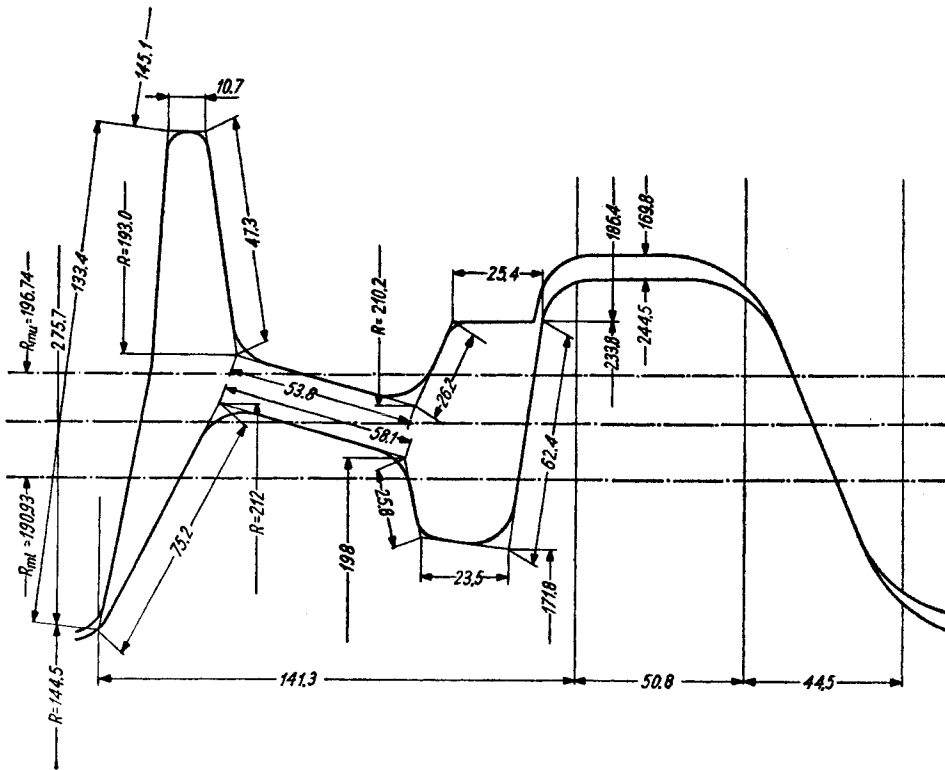


Fig. 3.102. Example of calculation of mean working radius, assuming uniform pressure distribution in the pass [54]

In this way the correct line of mean speed of the groove is determined by means of an equation. If the pass is positioned at the pitch line of rolls, i.e. on the line of mean speed, differences in speed between top and bottom half of the pass are reduced to a minimum. Better accuracy can be achieved when taking into account coefficient  $m$ .

#### Example 2

A square of  $b_1 = h_1 = 17.7 \text{ mm}$  ( $F_1 = 314 \text{ mm}^2$ ) is entered into a flat oval of  $h_2 = 10 \text{ mm}$ ,  $b_2 = 30 \text{ mm}$ ,  $F_2 = 200 \text{ mm}^2$  (Fig. 3.8). The roll diameter  $D_r = 300 \text{ mm}$ ,  $n = 400 \text{ rpm}$ . Chilled rolls are used, with rolling temperature  $900^\circ\text{C}$ .

The coefficient of elongation is

$$\lambda_1 = \frac{314}{200} = 1.570$$

(1) From formula (3.5) the mean absolute draught is

$$\Delta h_m = \frac{F_1}{b_1} - \frac{F_2}{b_2} = \frac{314}{17.7} - \frac{200}{30} = 17.7 - 6.7 = 11.0 \text{ mm}$$

(2) From equation (3.30), the mean working diameter of roll is

$$D_{mw} = D_t - h_{2m} = 300 - \frac{200}{30} = 293.3 \text{ mm}$$

$$R_{mw} = \frac{293.3}{2} = 146.6 \text{ mm}$$

(3) From equation (3.191), the mean peripheral speed of rolls is

$$v_{rm} = \frac{\pi D_{mw} n}{60} = \frac{\pi \times 293.3 \times 0.400}{60} = 6.139 \text{ m/sec}$$

(4) From formula (3.29), the mean bite angle is

$$\cos \alpha_m = 1 - \frac{\Delta h_m}{D_{mw}} = 1 - \frac{17.7 - 6.7}{293.3} = 0.9625$$

(5) The coefficient of friction  $f$  is found from equation (3.83)

$$f = 0.8(1.05 - 0.0005 t) = 0.8(1.05 - 0.0005 \times 900) = 0.48$$

(6) The neutral angle calculated from diagram in Fig. 3.69 allowing for spread is  $\delta = 6^\circ 40'$

(7) The entry speed from equation (3.126) is

$$v_1 = v_N \gamma_N \beta_N$$

where

$$v_N = v_{rm} \cos \delta \quad v_{rm} = 6.139 \text{ m/sec}, \quad \delta = 6^\circ 40', \quad \cos \delta = 0.99347$$

$$v_N = 6.139 \times 0.99347 = 6.097 \text{ m/sec}$$

$$\gamma_N = h_N / h_{1m}, \quad h_N = D_{mw}(1 - \cos \delta) + h_{2m}, \quad D_{mw} = 293.3 \text{ mm},$$

$$h_N = 293.3(1 - 0.99347) + 6.7 = 8.64 \text{ mm},$$

$$h_{1m} = 17.7 \text{ mm}$$

$$\gamma_N = \frac{8.64}{17.7} = 0.4881$$

$$\beta_N = \gamma_N^{-W} \quad \text{from equation (3.122)}$$

$$\delta_w = \frac{b_1}{h_{1m}} = \frac{17.7}{17.7} = 1.0 \quad \varepsilon_w = \frac{h_1}{D_{mw}} = \frac{17.7}{293.3} = 0.0603$$

$$\beta_N = 1.478$$

$$v_{1m} = v_N \gamma_N \beta_N = 6.097 \times 0.4881 \times 1.478 = 4.398 \text{ m/sec}$$

(8) The exit speed from equation (3.115) is

$$w_{2m} = v_{1m} \lambda = 4.398 \times 1.570 = 6.905 \text{ m/sec}$$

(9) The coefficient of forward slip from equation (3.104) is

$$s = \frac{w_{2m}}{v_{rm}} = \frac{6.905}{6.139} = 1.1248$$

## CHAPTER 4

# ROLL PRESSURE, TORQUE, WORK, AND POWER IN ROLLING

### 4.1. Roll Pressure and Load

During rolling the material deforms plastically. To determine the roll pressure causing this deformation, it is necessary to know exactly the stresses arising during this process (Fig. 3.1).

The deformation process is classified as hot or cold working according to whether or not work hardening occurs over the whole section. There is no work hardening during hot rolling, due to recrystallization.

During cold compression of a smooth specimen between ideally smooth platens the same stress-strain diagram is obtained as in the tensile test, but only up to the limit of uniform elongation, and the occurring stresses are uniformly distributed throughout the cross-section.

A similar phenomenon occurs during cold rolling. The diagram of assumed plastic deformation of a metal deformed between two ideally smooth rolls [36] is shown in Fig. 4.1. Using a 2 to 3% draught the yield stress of the metal being deformed is exceeded, and increasing the reduction in the roll gap the work hardening increases too, as shown on the curve of constrained yield stress.

The deformation during rolling is very similar to the case of simple compression by a narrow punch, as long as only elongation is considered, entirely neglecting the spread (the limiting condition is that  $b_1/h_1 \geq 20$ , where  $b_1$  = initial width and  $h_1$  = initial height of the deformed metal). Such a condition exists in rolling of wide strip and sheet, where the projected arc of contact is small and the roll action may be with close approximation compared with that of a punch [32].

Friction causes non-uniform stress distribution during rolling. The frictional forces arising in the roll gap act in two directions opposite to each other. The zone in which the slipping friction forces disappear and static friction arises, is called the neutral zone.

The true roll pressure diagram (Fig. 4.2) consists of two parts [10]. The lower part *ADGEC*, the same as in Fig. 2.22, shows the work hardening curve of the metal during ideal (frictionless) plastic deformation, i.e. the curve of roll pressure necessary to overcome the constrained yield stress  $\eta K_f$ . The upper part *DFEGD* shows the roll pressure necessary to overcome the additional constraint caused by the friction forces between the surface of metal being rolled and roll, i.e. the so called resistance to flow in rolling  $K_r$ .

Expressing this mathematically, [19] (2.105)

$$K_w = \eta K_f + K_r$$

where

$K_w$  = resistance to deformation in rolling,

$\eta K_f$  = constrained yield stress (yield stress in compression or frictionless rolling),

$K_r$  = resistance to friction in rolling.

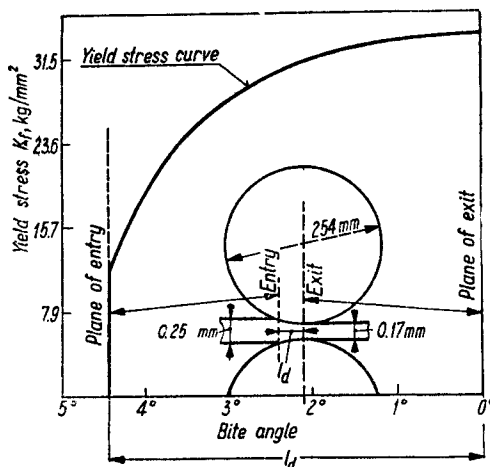


Fig. 4.1. Constrained yield stress distribution along the roll gap due to work hardening in cold rolling between ideally smooth rolls

The coefficient  $\eta = 1.0$  for uniform deformation (Fig. 2.9), i.e. if  $\lambda = \beta$ . Generally the coefficient varies between 1.0 and 1.155 depending on the conditions of deformation, i.e. the  $\beta$  value. For  $\beta = 1$ , i.e. if the metal being rolled does not spread, the maximum value of  $\eta = 1.155$  is obtained. For all other intermediate cases the coefficient  $\eta$  may be calculated from formulae (2.62) to (2.76), derived by the author [19].

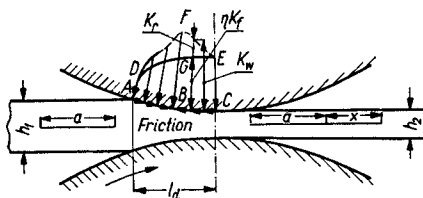


Fig. 4.2. Roll pressure distribution, i.e. the resistance to deformation  $K_w$  in the roll gap along the arc of contact

To simplify the calculations, for practical purposes the value of  $\eta K_f$  is often assumed constant along the whole projected arc of contact  $l_d$ , which is not strictly true. During hot rolling the value of  $\eta K_f$  decreases slightly from the entry toward the exit plane due to the decrease of rate of deformation, and during cold rolling the value of  $\eta K_f$  increases due to work hardening of the metal.

The diagram shown in Fig. 4.2 is shown once more on an enlarged scale in Fig. 4.3 where the arc of contact  $AC$  between the roll and metal being rolled has been replaced by the length  $l_d$ , i.e. the projected arc of contact on the rolling direction. The values



of  $\eta K_f$  and  $K_w$  have been plotted against the angular co-ordinate of arc of contact in degrees. Since the angles used are small, this simplification of the true conditions, i.e. replacement of the arc of contact by its projection is permissible in practice and introduces no large errors in calculations.

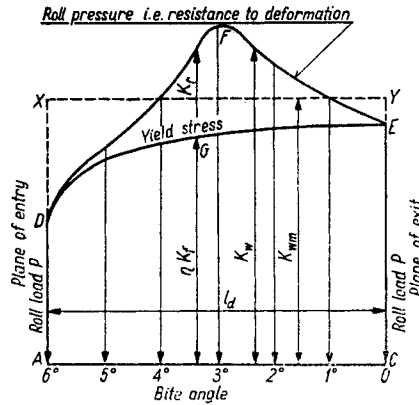


Fig. 4.3. Roll pressure distribution, i.e. the resistance to deformation, and yield stress along the arc of contact, related to the horizontal axis. Yield stress  $\eta K_f$  is represented by the area  $ADGEC$ , whereas mean resistance to deformation  $K_{wm}$  — by the rectangle  $AXYC$ , equal in area to  $ADFEC$

The influence of some further rolling factors on the magnitude of resistance to deformation and therefore on the roll forces will be examined next.

Figure 4.4 shows schematically and in a rather simplified form the roll pressure distribution along the roll gap in cold rolling [10]. It will be seen, that the constrained yield stress increases (from  $\eta K_{f1}$  at the plane of entry up to  $\eta K_{f2}$  at the exit) due to work hardening.

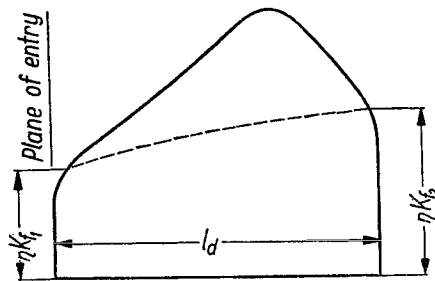


**Fig. 4.4. Influence of work hardening on roll pressure distribution**

The influence of the coefficient of friction  $f$  on the resistance to deformation is shown in Fig. 4.5. As  $f_1 > f_2 > f_3$  it will be seen that each increase of the coefficient of friction causes a rapid increase of the resistance to flow  $K_r$ . This phenomenon must be taken into account in rolling, since changes in frictional resistance  $K_r$  influence the resistance to deformation  $K_w$ .

Increase of the roll diameter influences the resistance to friction  $K_r$  and the resistance to deformation  $K_w$  in an exactly similar way. The change of resistance to deformation

over the roll gap for three different roll diameters is shown schematically in Fig. 4.6 where  $D_1 > D_2 > D_3$ . Each increase of roll diameter causes an increase in length of arc of contact  $l_d$  and hence an increase of rolling force. It may be concluded from Fig. 4.6, that rolling should be carried out with roll diameters as small as possible. A further advantage is a greater elongation of the rolled stock.

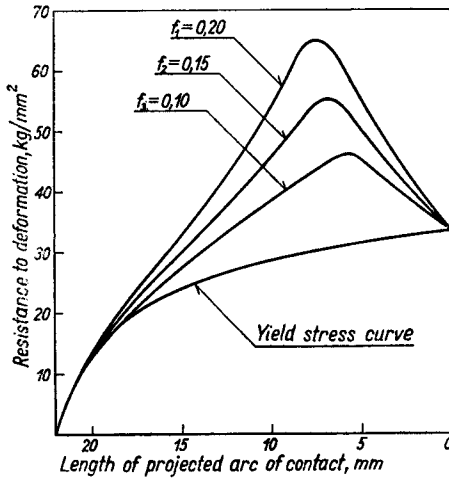


Fig. 4.5. Influence of different coefficients of friction  $f_1 > f_2 > f_3$  on the magnitude of resistance to deformation [62]

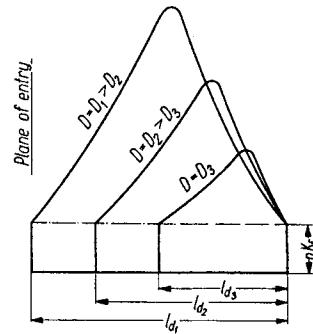


Fig. 4.6. Influence of different roll diameters  $D_1 > D_2 > D_3$  on the magnitude of resistance to deformation

If at the entry side the rolled stock is under the action of a force  $Q_1$  acting opposite to the rolling direction, called back tension, then the stress  $\sigma_1$  due to  $Q_1$  reduces the constrained yield stress  $\eta K_f$  to  $\eta K_f - \sigma_1$  (Fig. 4.7a). Therefore the initial resistance to deformation decreases also. Tension acting along the rolling direction, called front tension, has a similar effect. The initial constrained yield stress  $\eta K_f$  at the plane of exit decreases to  $\eta K_f - \sigma_2$  due to the front tension  $\sigma_2$ , reducing also the resistance to deformation (from the values shown by curve 1 down to these of curve 2 in Fig. 4.7b).

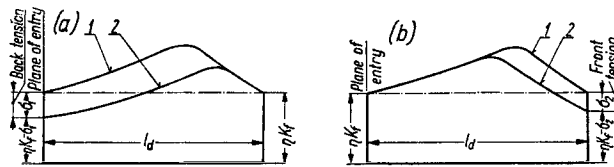


Fig. 4.7. Influence of: (a) back tension on the magnitude of resistance to deformation; 1 — resistance to deformation without back tension, 2 — resistance to deformation with back tension; (b) front tension on the distribution of the resistance to deformation; 1 — resistance to deformation without front tension; 2 — resistance to deformation with front tension

If the resulting distribution of roll pressure without tension is represented as  $\overline{ADFECA}$  (Fig. 4.8), then after applying front tension the diagram takes the form  $\overline{ADNLCA}$ , where  $EL$  represents the value of front tension. If only back tension is

applied the diagram takes the form *AMOECA*, where *DM* represents the value of stress caused by back tension. If front tension and back tension are applied simultaneously, the resultant diagram will be lowered and take the form *AMKLCA*.

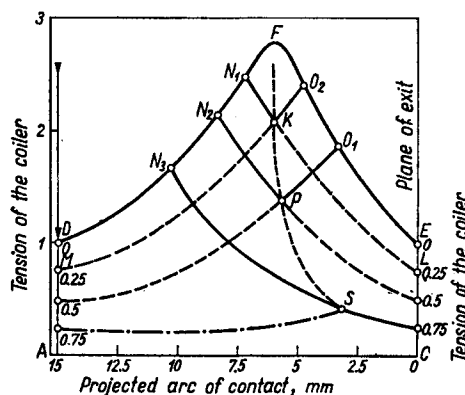


Fig. 4.8. Roll pressure distribution with simultaneously applied front and back tension [62]

In rolling theories up to now it has been assumed that slipping friction occurs along the whole surface of contact  $F_d$  between rolls and rolled stock (Fig. 3.50). The only exception is the neutral plane (or neutral zone), where the direction of friction forces changes and there is therefore no relative motion between roll surface and surface of the rolled stock. However investigations have shown that slip between rolls and surface of the rolled stock may occur only as long as the relation  $fK_{wx} < (\eta/2)K_f$  is fulfilled.  $K_{wx}$  indicates the momentary value of the variable resistance to deformation at the considered point  $x$  in the length of contact  $l_d$  (Fig. 3.1). Slip may occur as long as the varying friction force  $T_x = fK_{wx}$  is smaller than the constrained yield stress in shear  $(\eta/2)K_f$ . If the friction stress exceeds the yield stress in shear of the rolled stock, then this will stick to the roll surface.

For practical purposes, especially in hot rolling when there are high values of  $f$ , one has to assume the existence of slipping and sticking zones whose extent depends on the magnitude of roll forces and the coefficient of friction  $f$ .

Therefore in rolling with smooth rolls (as in Fig. 3.1) larger or smaller sticking zones occur and slipping zones move towards the planes of entry and exit of the rolls. This is caused by the increase of resistance to deformation  $K_w$  near the neutral plane. The variation of the extent of zones of sticking with changes of resistance to deformation is shown schematically in Fig. 4.9 [59].

Our investigations [66] show a somewhat different picture of deformation in rolling than those of Tselikov [59].

As is seen from Fig. 4.9, the initial zone of plastic disturbance ahead of hot rolling proper increases with the degree of deformation, causing an initial reduction in height of the rolled stock ahead of the roll entry [66].

In Figure 4.9 one assumes in the case of hot rolling the existence of zones of sticking, slipping, and disturbance at exit. However in cold rolling (Fig. 4.10) zones of plastic deformation with slipping, of elastic disturbance, a stress-free zone at entry, and a final work hardened zone arise [66].

A. Korolev gives the following formulae for the length of sticking zone during rolling without work hardening expressing

$$\delta' = 2f \frac{l_d}{\Delta h} \approx \frac{2f}{a} \quad (4.1)$$

one obtains

$$\frac{l_{st}}{l_d} = \frac{1}{\varepsilon_h} \left[ \left( \frac{2f-a}{1-a} \right)^{1/\delta'} - (1-\varepsilon_h) \left( \frac{1+a}{2f+a} \right)^{1/\delta'} \right] \quad (4.2)$$

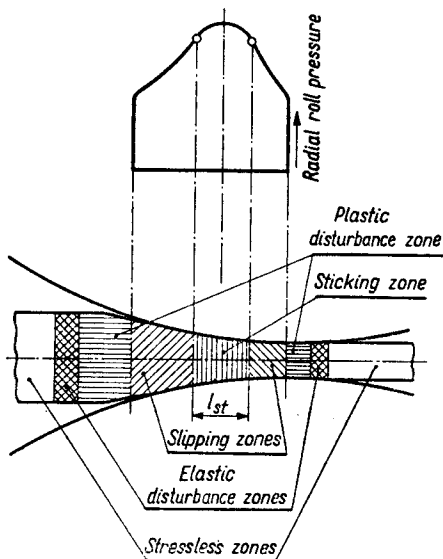


Fig. 4.9. Schematic representation of conditions arising in hot rolling

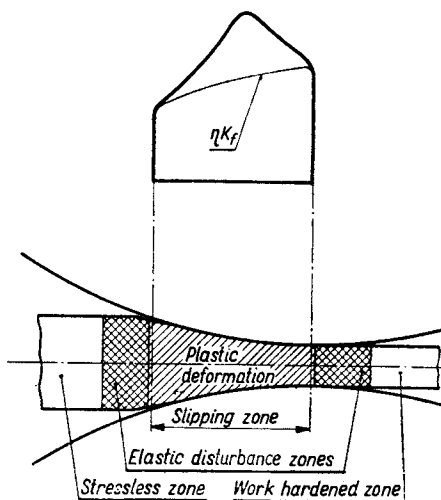


Fig. 4.10. Schematic representation of conditions arising in cold rolling without tension

Analysing the formulae (4.1) and (4.2) an important condition may be obtained, namely if during rolling  $f = f_{\max} = 0.5$ , then the ratio  $l_{st}/l_d = 1$ , i.e. that in rolling with maximum coefficient of friction the sticking zone extends along the whole arc of contact.

Tselikov considers the speed conditions in rolling along the arc of contact [59], as being directly connected with the building up of a sticking zone. He assumes, basing his assumption on the definition of the neutral plane, that in that plane all particles of the rolled stock of a cross-sectional height of  $2y_p$  and width of  $b = 1$  (Fig. 4.11), have the same horizontal speed (3.109)

$$v_p = v_r \cos \delta$$

where

$v_r$  = peripheral speed of rolls, m/sec,

$\delta$  = neutral angle.

Subsequently Tselikov considers the speed distribution over the cross-sectional height of the rolled stock for two other arbitrarily chosen planes  $A-A$  and  $B-B$  positioned in the sticking zone, one on each side of the neutral plane. The heights of the rolled stock in the considered planes are denoted as  $2y_A$  and  $2y_B$ .

Owing to the disappearance of slip between rolls and rolled stock within the sticking zone all particles in contact with the rolls along the whole zone will have the same speed as the peripheral speed of rolls. Therefore it follows that the horizontal speed component of the rolled stock at points  $A$  and  $B$  will be

$$v_A = v_r \cos \varphi_A \quad (4.3)$$

$$v_B = v_r \cos \varphi_B \quad (4.4)$$

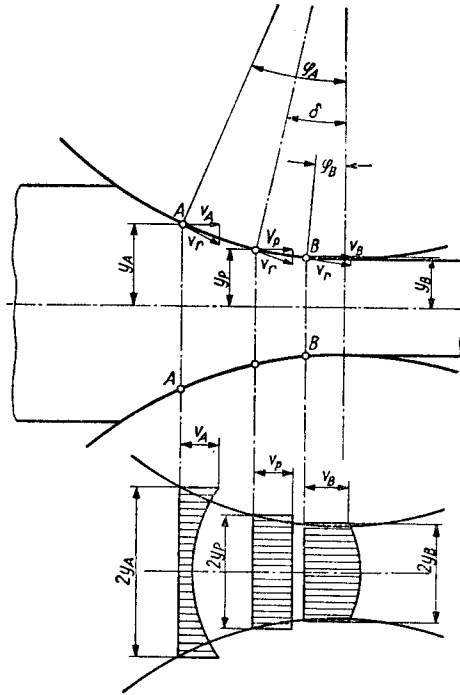


Fig. 4.11. Speed distribution of metal flow along its cross-section, when sticking occurs [59]

where  $\varphi_A$  and  $\varphi_B$  indicate rolling angles at points  $A$  and  $B$ . The volume per second of the rolled stock passing through any cross-section, i.e. also the planes  $A-A$ ,  $B-B$  and the neutral plane, should be constant

$$V_s = 2y_p v_r \cos \delta \quad (4.5)$$

Tselikov states that for points on the roll circumference the condition of uniform speed is not fulfilled, as

$$2y_A v_r \cos \varphi_A > 2y_p v_r \cos \delta > 2y_B v_r \cos \varphi_B \quad (4.6)$$

Therefore in cross-sections  $A-A$  and  $B-B$  unequal speeds of the rolled stock should occur, and the mean rolling speed of the metal in cross-section  $A-A$  should be

$$v_{Am} < v_A \quad (4.7)$$

and in cross-section  $B-B$

$$v_{Bm} > v_B \quad (4.8)$$

The non-uniform speed distribution in cross-sections  $A-A$  and  $B-B$  is shown in the lower diagram of Fig. 4.11. The areas under these speed curves must be equal to each other, as they represent the volume per second passing a given cross-section.

It is difficult to determine how these phenomena manifest themselves in section rolling. The sticking zones remain, of course, near to the neutral plane, similarly as in flat rolling. However the true course of the neutral plane is exceedingly difficult to derive, because the stresses in the cross-section are non-uniform due to varying draughts and by metal flow from one part of the section to another, and also because the coefficient of friction varies. In the plane of entry and exit slipping friction occurs. In the sticking zones, metal deformation and flow can only occur through internal displacement, and the overcoming of internal resistances and friction.

Slip theories in rolling have been derived based on the following simplified assumptions [29] (Figs. 3.1 and 4.10):

(1) Plane and perpendicular cross-sections  $a_1$  of the initial stock (Fig. 3.1) remain also plane and perpendicular after rolling ( $a_2$ ). Such deformation is called a parallelepiped or homogeneous one, and follows from the assumption that metal slips at every point along the plane of contact, except in the neutral plane.

(2) The rolled stock does not spread in rolling, i.e.  $b_1 = b = b_2$ .

(3) The coefficient of friction between roll and surface of rolled stock is constant at every point along the arc of contact.

(4) The constrained yield stress  $\eta K_f$  is constant along the arc of contact. Therefore it is assumed that the metal is not affected by work hardening in cold rolling, nor by variable rate of reduction in hot rolling. This assumption also implies that the temperature of the rolled stock does not vary during a pass.

(5) The rolled metal is homogeneous. Also its elastic deformation is neglected, in view of the considerably larger plastic deformation.

(6) Rolls are rigid and are not deformed during rolling.

(7) The geometrical formulation of basic parameters, such as roll diameter, width and height of the rolled stock, draught etc., must conform to real values. These values may easily be measured, but the measurement of the distribution of constrained yield stress and variation of coefficient of friction between roll and stock along the arc of contact, present considerable difficulties.

In Figure 3.1 all relations are shown as they have hitherto been presented in "classic" theories of homogeneous plastic deformation. An element  $ABCD$  of a width of  $dx$  is under the action of the components of the normal pressure  $p$  caused by the radial roll pressure, and by the tangential stress  $\tau$  on the roll circumference which is perpendicular to the radial pressure, and is caused by friction between roll and stock, where

$$\tau = fp$$

This stress is acting on every cubic element in the zone  $ABCD$ .

Every cubic element is under the action of compressive stresses: the vertical  $q$  and the horizontal one  $\sigma$ .

As it has been assumed that no lateral spread occurs, a third compressive stress

$$\frac{\sigma + q}{2}$$

acts laterally.

Therefore each cubic element is under the action of three compressive stresses (Fig. 3.1).

#### 4.1.1. MEASUREMENTS OF PRESSURE DISTRIBUTION ALONG THE ARC OF CONTACT IN HOT AND COLD ROLLING

Investigations and measurements were carried out in the years 1947 to 1949 by Smith, Scott and Sylwestrowicz [63] on a laboratory 2-high rolling mill with a roll diameter of 160 mm and a constant rolling speed of 15 m/min. Measurements of roll pressure distribution along the arc of contact were carried out for hot and cold rolling.

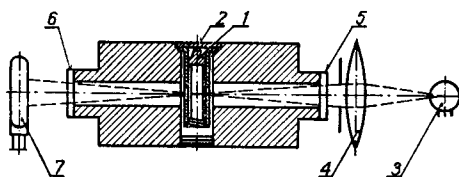


Fig. 4.12. Roll set of the laboratory rolling mill [63]: 1 — piezoelectric dynamometer, 2 — radial pin, 3 — source of light, 4 — lens, 5, 6 — light polarizers, 7 — photoelectric cell recording the "off balance" deflections

The lower roll was specially constructed to contain a piezoelectric dynamometer. It was fitted with a tungsten radial pin, which transmitted the pressure to the dynamometer (Fig. 4.12).

The authors state that a device eliminating the effects of unequal heating of its components makes it possible to carry out measurements with the greatest accuracy. Signals from the dynamometer were transmitted to simple resistance amplifiers, and then to an oscillograph, where they were photographed with simultaneous recording of the distance covered by the rolls.

The experimental rolling mill was also fitted with measuring devices for the control of dynamometer indications and its calibration.

When cold rolling, measurements of back tension were also carried out, by fitting to the end of a strip a second one of known properties. Whilst the first strip was passing through the rolls the second one was passing through a flat die of known size and pressure (Fig. 4.13).

The back tension was determined from the elongation of a cylindrical mild-steel rod with four strain gauges attached, whose readings were photographed with an oscilloscope.

A very simple and ingenious method was applied for the determination of the constrained yield stress  $\eta K_f$  relative to the current work hardening due to cold rolling. It consisted of stopping the rolls with the rolled stock still between them, lifting the upper roll as fast as possible, and measuring the variations of hardness along that part of the strip which bore the impression of the arc of contact (using a Vickers hardness testing machine). The results obtained agreed with the calculated ones. The methods of calculation will be discussed in the next chapter.

The coefficient of friction was determined by measuring the bite angle on specimens machined to exact dimensions. Its value for dry rolls was determined as  $0.1 \pm 0.01$  for reductions of 5 and 10%, and  $0.12 \pm 0.01$  for reductions higher than 20%.

The effect of the protrusion of recording pin was tested before measurements began, and then the conditions for which the effect of roll flattening begins were determined. To obtain this value two consecutive recordings of pressure distribution

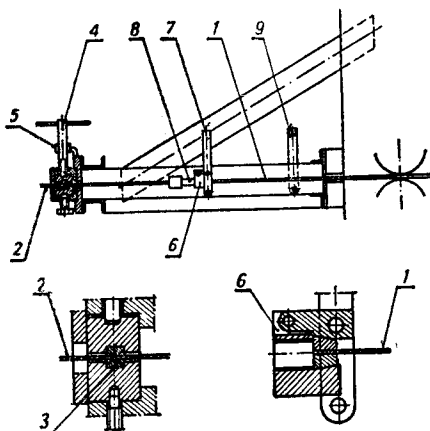


Fig. 4.13. Device for application and measurement of back tension 1 — rolled strip, 2 — strip under tension, 3 — flat die, 4 — screw changing the reduction, 5 — calibrated scale for the screw, 6 — unit for coupling the two strips, 7 — vertical bar, which interrupts rolling when coming into contact with horizontal rods 9, 8 — cylindrical rod measuring back tension, 9 — horizontal rods

were always made. The first recording, carried out in the normal manner with the pin making contact with the rolled strip, includes the pressure on the pin. The second recording was made on a part of the strip in which a longitudinal groove 2 mm wide had been cut; in this case the pressure recording pin does not come into contact with the strip, and the recorded pressure is due solely to the elastic deformation of the rolls. The difference between these two recordings gives the pressure on the pin. This method was always used for reductions higher than 20%.

For these experiments, reductions of 5 to 50% were used, but back tension was applied only for reductions greater than 20%. Fully annealed copper strip, 40 mm wide and 2 mm thick, was used; the measuring pin always made contact with the centre of the strip. For each measurement both the rolls and rolled stock were cleaned and made as free of oil as possible.

Each parameter was measured five times; the results show the mean value of these five measurements.

The negatives of the original oscillogram photographs were enlarged in a photographic enlarger and were projected on to a sheet of graph paper, the curves being redrawn directly on the paper. For reductions below 20% the curve for roll deformation was a horizontal straight line.

For reductions above 20% each test gave three pressure distribution curves: (a) without back tension, (b) with back tension, and (c) due to elastic deformation of rolls.

The differences in the ordinates of curves (a) and (c), and (b) and (c) determine the measured roll pressure distribution.

The curves obtained were corrected for inertia of electrical measuring apparatus and for the final size of the pin.



The time intervals were obtained from the known speed of the rolls, and from the calculations for change of radius due to flattening of the rolls in each case.

Figure 4.14 shows an example of such measurements in cold rolling. On all figures the curve of measured values (continuous curves), the curve for values calculated according to Orowan [35], and the curve of  $1.15 K_f$ , have been plotted. The last curve begins from the point  $x$  and runs up to the point  $y$ ; these points determine the beginning and end of plastic deformation of the rolled stock.

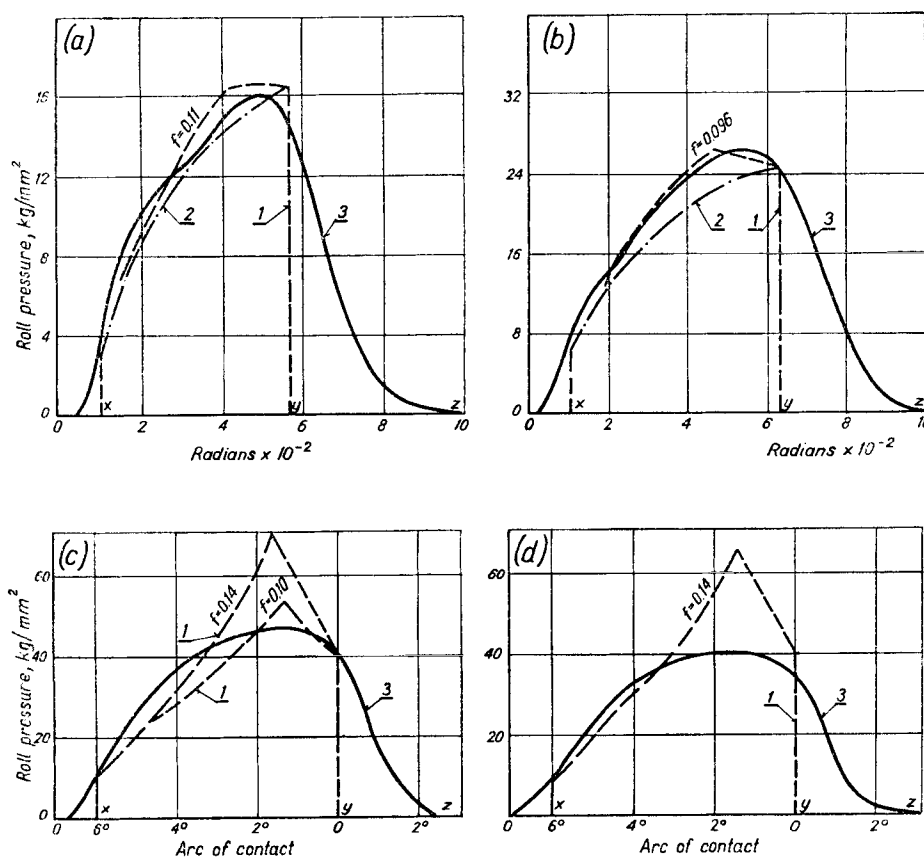


Fig. 4.14. Curves of roll pressure distribution in cold rolling of copper [63]: Draught: (a) 5.5%, (b) 10%, (c) 41% without back tension, (d) 44% with applied back tension; 1 — Orowan's curve, 2 — curve of  $1.15 K_f$ , 3 — corrected curve of measured values

The theoretical curve of roll pressure distribution has been calculated assuming several successive coefficients of friction.

From the results plotted in Fig. 4.14 it can be seen that for small reductions of 5 to 10% and  $f = 0.096$  or  $0.11$  very good agreement was obtained between measured and calculated values.

For greater reductions, coefficient of friction values of  $0.12$  to  $0.14$  give the nearest agreement to the experimental curves, probably because towards the end of experiments, the roll surface near the pin was becoming locally rather rougher than the remainder.

For all cases a good agreement between measured values and those calculated according to Orowan [35] was confirmed. In agreement with theory, the roll pressure

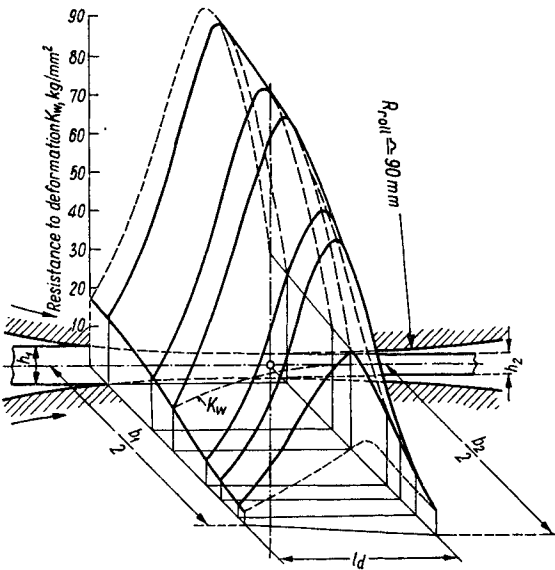


Fig. 4.15. True roll pressure distribution in roll gap in cold rolling of aluminium with rough rolls. The diagram shows only a half width of rolled stock, from the centre to  $b/2$  [65]

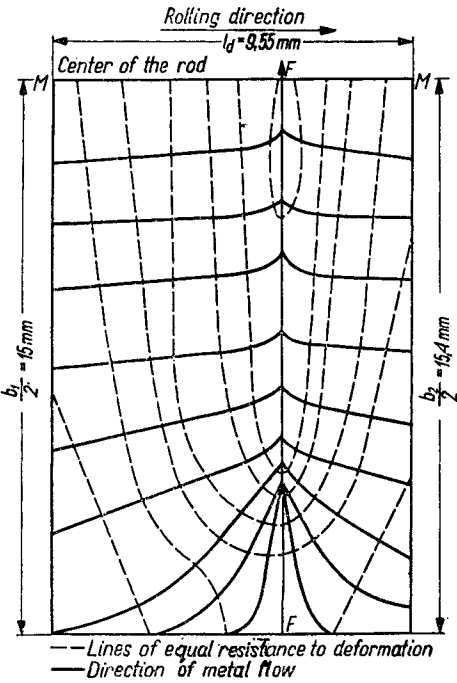


Fig. 4.16. Directions of metal flow lines (Fig. 4.15) projected from Fig. 4.15 onto a horizontal plane [65]

decreases when tension applied to the metal increases, and the neutral plane moves towards the roll exit.

Maximum differences between measured and calculated results arise in the neutral plane due to sharp peaks occurring in the calculations.

Figure 4.15 shows other results from investigations of Lueg and Siebel for rolling of aluminium [65].

Friction causes further variation of roll pressure in the direction perpendicular to the direction of rolling. It may be seen from Fig. 4.15, showing a spatial roll pressure distribution, that maximum pressure occurs in the centre of the specimen width, and gradually decreases towards the edges.

Projecting the solid figures from Fig. 4.15 onto a horizontal plane, the picture of metal flow may be obtained (Fig. 4.16). This figure shows phenomena occurring in the half width of the rolled stock, from the longitudinal axis sideways. Lines of equal resistance to deformation have been drawn onto this figure, as also directions of metal flow from the neutral plane, indicated as *FF*.

Results of measurements of roll pressure distribution along the arc of contact have been published also by Korolev [46], carried out on a 2-high mill with a roll diameter of 248.5 mm.

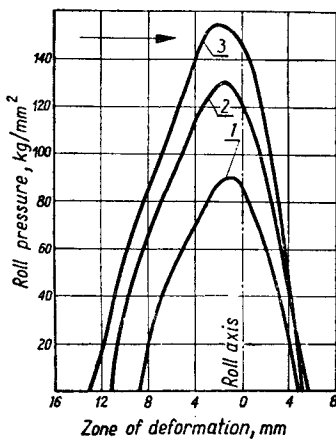


Fig. 4.17. Roll pressure distribution in cold rolling of 1010 steel strip quality, 1.9 mm thick and 30 mm wide [46]. Draught  $\varepsilon_h$ : 1 — 0.24, 2 — 0.37, 3 — 0.58

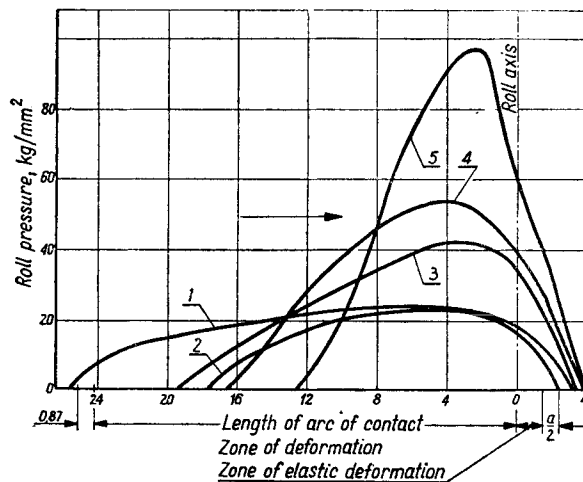


Fig. 4.18. Roll pressure distribution in hot rolling of St3-steel [46]

| Curve                   | 1    | 2    | 3    | 4    | 5    |
|-------------------------|------|------|------|------|------|
| Draught $\varepsilon_h$ | 0.28 | 0.28 | 0.54 | 0.46 | 0.43 |
| $l_d$ , mm              | 25   | 16.5 | 18.3 | 15.7 | 11.2 |

Figure 4.17 shows his result when rolling steel strip of 1010-quality\*, 1.9 mm thick and 30 mm wide, using relative draught  $\varepsilon_h$ .

Figure 4.18 shows curves of roll pressure distribution when hot rolling of St3-steel\* [46] with various thicknesses and draughts, carried out by the same author.

\* See Appendix 1.

Figures 4.17 and 4.18 show values recalculated from oscillograph records, taking into account the error due to recording pin thickness.

#### 4.1.2. DETERMINATION OF ROLL FORCES IN HOT ROLLING

The theoretical roll pressure distribution along the arc of contact, or along the projected length of contact, according to Figs. 3.1 and 4.1 may be expressed as an integral

$$P = \int_0^{\varphi=\alpha} R \sigma d\varphi = \int_0^{x=l_d} \sigma dx \quad (4.9)$$

where

$R$  = roll radius,

$\varphi$  = rolling angle,

$\alpha$  = bite angle,

$\sigma$  = vertical pressure caused by radial roll pressure.

Inserting instead of  $\sigma$  the expression (2.105) one obtains

$$P = \int_0^{\alpha} R K_{wx} d\varphi = \int_0^{l_d} K_{wx} dx \quad (4.10)$$

To solve this equation (4.10) it is necessary to use a graphical method, or a laborious point by point calculation. This is sometimes done if very exact results are desired. For practical purposes such a method takes too much time.

Generally the following method for calculation of roll force has been successful (Figs. 3.3 and 3.64) in rolling of flat cross-sections (square, flat), break-downs or sheets, on smooth flat roll barrels of equal diameter [92]

$$P = F_d K_{wm}$$

where

$P$  = roll separating force, kg,

$F_d$  = projected area of contact between roll and stock, mm<sup>2</sup>,

$K_{wm}$  = mean resistance to deformation, kg/mm<sup>2</sup>, (Fig. 4.3),

and also

$$F_d = l_d b_m \quad (4.11)$$

The mean width  $b_m$  may be determined assuming the curve of spread as a straight line, thus

$$b_m = \frac{b_1 + b_2}{2} \quad (4.12)$$

$b_1$  = initial width of the rolled stock, mm,

$b_2$  = width of the rolled stock after pass, mm,

$l_d$  = projected arc of contact between roll and stock, mm.

The mean width of the rolled stock may be determined more exactly, assuming that the curve of spread is a parabola. Then

$$b_m = \frac{b_1 + 2b_2}{3} \quad (4.13)$$

The curve of spread may be determined most exactly from the equation (3.128)

$$b_x = b_1 \beta_x = b_1 \gamma_x^{-w}$$

After graphical plotting  $b_m$  may be determined.

If the two rolls are of equal diameters (e.g. 2-high), then the equation (3.13)

$$l_d = \sqrt{R \Delta h - \frac{\Delta h^2}{4}} \approx \sqrt{R \Delta h}$$

is used, where

$\Delta h$  = absolute draught,

$R$  = active roll radius.

Instead of the simplified formula,  $l_d$  may be calculated from

$$l_d = R \sin \alpha$$

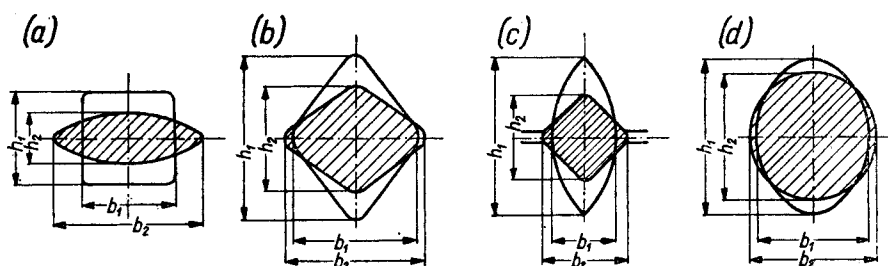


Fig. 4.19. Absolute draught  $\Delta h$  for different stretching passes: (a) for  $\Delta h = h_1 - 0.7 h_2$ , (b) for  $\Delta h = 0.6 (h_1 - h_2)$ , (c) for  $\Delta h = 0.7 h_1 - 0.6 h_2$ , (d) for  $\Delta h = 0.85 h_1 - 0.79 h_2$

If the rolls are of unequal diameters (3-high Lauth mill), for top or bottom draught, the mean radius is inserted into the formula (3.27)

$$R_m = \frac{2R_1 R_2}{R_1 + R_2} = \frac{D_1 D_2}{D_1 + D_2}$$

where

$R_1, D_1$  = active radius or diameter of one roll, mm,

$R_2, D_2$  = active radius or diameter of the second roll, mm.

As a result one obtains

$$l_{dm} \approx \sqrt{R_m \Delta h} \approx \sqrt{\frac{2R_1 R_2}{R_1 + R_2} \Delta h} \quad (4.14)$$

This formula has been derived assuming that roll separating forces, and also areas of contact on both rolls are equal to each other.

When rolling bars and some sections (except square and flat sections rolled on the smooth and horizontal part of the roll barrel), it is necessary to insert into formulae mean heights and mean draughts (Fig. 3.3a, b).

Then one calculates

$$F_{dm} = l_{dm} b_m$$

where

$$l_{dm} \approx \sqrt{R_m \Delta h_m} \quad (4.15)$$

In this formula

$$\Delta h_m = h_{1m} - h_{2m} = \frac{F_1}{b_1} - \frac{F_2}{b_2}$$

and

$$R_m = \frac{D_m}{2} = \frac{D_1 - h_{2m}}{2}$$

where  $h_{2m}$  denotes the mean height of the emerging stock.

For practical purposes, equation (3.61) may be used

$$h_m = m h_{\max}$$

where

$h_{\max}$  = maximum height of the section,

$m$  = a coefficient, which has been determined for respective types of sections.

To apply equation (3.61), values given in Table 4.1 may be used.

TABLE 4.1  
MEAN HEIGHT OF THE PASS  $h_m$  FOR DIFFERENT PASS FORMS  
( $h_m = m h_{\max}$ )

| Type of pass                    | Coefficient, $m$                             | Remarks                                |
|---------------------------------|----------------------------------------------|----------------------------------------|
| Gothic                          | 0.55–0.6                                     | Flat position                          |
| Square with rounded edges       | 0.97–0.99                                    |                                        |
| Square with sharp edges         | 0.51                                         |                                        |
| Diamond with sharp edges        | 0.51                                         |                                        |
| Square with rounded edges       | 0.56–0.58                                    | Diagonal position                      |
| Diamond with rounded edges      | 0.56–0.58                                    |                                        |
| Flat oval (depending on $h/b$ ) | 0.67–0.75                                    |                                        |
| Elliptic oval                   | 0.785–0.82                                   |                                        |
| Rounded oval                    | 0.80–0.94                                    | Entering into oval<br>Comes out square |
| Pyramid oval                    | 0.55–0.88                                    |                                        |
| Oval coming out flat            | above values should be<br>increased by +0.05 | Diagonal position                      |
| Hexagon                         | 0.75                                         |                                        |
| Round                           | 0.785                                        |                                        |

More exact values of  $m$  may be obtained using the given nomograms for different sections (Figs. 3.30–3.41).

In some difficult cases, if the method of computation fails (e.g. when rolling sections), the graphical method of Trinks may be used for determination of  $F_d$  [22].

As shown in Fig. 4.20, the outlines of the pass and of the entering section are drawn superimposed. If the reduction is not symmetrical about the centre line of the pass, the outlines should be adjusted so that the total reduction is evenly divided between the two rolls. If the sections are not symmetrical, or if the roll diameters are decidedly unequal, the projected areas of contact between the stock and bottom roll and between the stock and the upper roll, may be quite different and both areas must be drawn. In the example, however, the sections are symmetrical about both axes, and the roll diameters are approximately equal.

In Figure 4.20 a vertical line is drawn at any convenient location, such as the line 1-24 in the centre of the pass, which meets the centre line of the roll at point 1. A horizontal line such as 3-9 is drawn perpendicular to 1-24 from any convenient point such

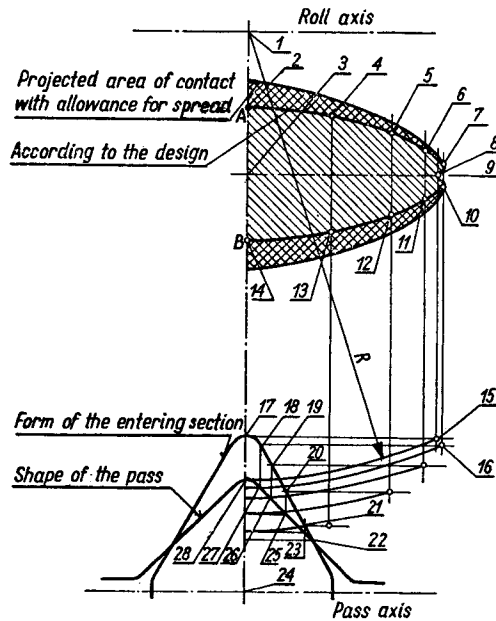


Fig. 4.20. Constructing the projected area of contact in a diamond pass [22]

as 3. This line serves as the lateral axis line of the projected area of contact. Now several vertical lines such as 17-28, 18-27, 19-26 etc., are drawn through points on the outlines of the entering section and of the pass. From the intersection of each of these lines with the pass outline, a horizontal line is drawn to the vertical 1-24. From point 17, a horizontal line of indefinite length is drawn. Using point 1 as a centre, an arc of a circle of radius 1-28 is drawn, intersecting the horizontal from 17 at point 15. The distance from the vertical 1-24 to point 15 represents the length of contact at point 28 of the pass, and because point 28 lies in the centre of the pass laterally, the length 17-15 should be transferred to line 3-9. A vertical line from 15, intersecting line 3-9 at 8, locates 8 as the first point on the boundary of the projected area of contact.

The next point may be found similarly. A horizontal line of indefinite length is next drawn from 18. From the intersection of the vertical 1-24 with the horizontal through point 27, and with point 1 as a centre, an arc of a circle is drawn to intersect the horizontal through point 18 at point 16. As before, the distance from the vertical 1-24 to point 16 represents the length of contact at point 27, and a vertical line from point 16 onto line 3-9 determines the location of point 10 on the boundary of the projected area of contact.

With dividers the distance from point 27 to the vertical centre line of the pass is measured, and point 10 is located at an equal distance from the centre line 3-9 on the vertical drawn through 16. As the sections are symmetrical, the point 7 can be located on the same vertical line, at the same distance from, but on the opposite side of the centre line 3-9. This process is repeated giving points 11-6, 12-5, 13-4, etc., on the boundary of the projected area of contact.

At point *A* it is apparent that contact ceases. From that point outward, the entering section is smaller than the pass; consequently, points 14 and 2 can be located on the vertical 1-24 as *AB*, and at the proper distance from the centre line 3-9. This determination of the projected area of contact is based on the assumption that no spreading occurs. It is apparent that in the early part of the pass, for instance when contact occurs only at points 28 and 27, any spreading which occurs will alter the shape of the entering section (hatched part), increasing the draught at points of later contact beyond point *A* (cross-hatched part). The area of contact is therefore dependent upon the spreading.

It is necessary now to discuss another problem, namely the second factor of equation (2.105) i.e. the resistance to deformation  $K_w$  (Fig. 4.3).

As  $K_w$  varies along the arc of contact between roll and stock, for calculations the mean value  $K_{wm}$  is used (Fig. 4.3, area *AXYC*). This value may be calculated in such a way, or determined from curves obtained in hot rolling of different metals, with different temperatures, rolling speeds and draughts.

Generally the mean resistance to deformation may be expressed as follows:

$$K_{wm} = \eta K_{fm} + K_{rm} \quad (4.16)$$

where

$\eta K_{fm}$  = mean constrained yield stress (the method of its determination will be discussed in detail in later chapters). The  $\eta$ -value varies from 1 to 1.155; it is calculated from formulae (2.62) to (2.76).

$K_{rm}$  = mean resistance to flow (will be discussed later together with respective methods).

Some methods omit separate determination of  $\eta K_{fm}$  and  $K_{rm}$ , calculating either the value of  $K_{wm}$ , or directly the roll separating force *P*.

To determine the value of yield stress  $K_f$  in hot rolling, first of all it is necessary to calculate the rate of deformation from formulae (2.85) and (2.86), with the greatest possible accuracy, and then transfer it to the rolling processes.

It is apparent from Figs. 4.21 and 4.22 that the strain rate in rolling is a maximum at the plane of entry, and decreases continuously toward the roll exit.

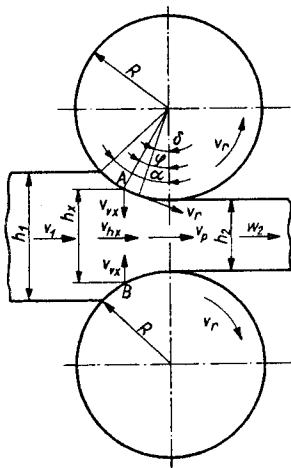


Fig. 4.21. Phenomena arising during slipping [125]

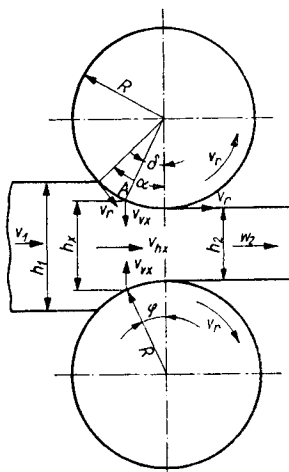


Fig. 4.22. Phenomena arising during sticking [125]



Therefore when analysing the phenomena arising in the roll gap, it is necessary to determine the local strain rate, while for the total pass, the mean strain rate is determined.

For this purpose appropriate formulae have been derived [125] for the case of slipping (Fig. 4.21) and sticking (Fig. 4.22).

Inside the slipping zone the strain rate is related to the height  $h_x$ . It results from Fig. 3.67 and 4.21

$$q_{sx} = \frac{2v_{hx} \tan \varphi}{h_x} = \frac{2v_1 \tan \varphi}{\gamma_x \beta_x h_x} \quad (4.17)$$

This equation determines the distribution of strain rate along the roll gap. Then for the total deformation from  $h_1$  to  $h_2$ , a mean value is determined solving the integral

$$\int_0^\alpha q \, d\varphi \quad (4.18)$$

i.e. after integration

$$q_s = \frac{v_1 \tan \alpha}{\gamma \beta h_1}$$

Since  $\lambda = 1/\gamma\beta$  and  $w_2 = v_1 \lambda$ , one obtains finally

$$q_s = \frac{\lambda v_1 \tan \alpha}{h_1} = \frac{w_2 \tan \alpha}{h_1} \quad (4.19)$$

For the case of sticking one obtains [125] (Fig. 4.22)

$$q_{stx} = \frac{2v_r \sin \varphi}{h_x} \quad (4.20)$$

Using this relation it is possible to determine the strain rate at any point of the roll gap. For the total reduction from  $h_1$  to  $h_2$ , one obtains a similar formula for calculation of the mean strain rate, when sticking occurs between roll and stock

$$q_{st} = \frac{v_r \sin \alpha}{h_1} \quad (4.21)$$

#### Example

The following initial values are given:

|                                          |                                            |
|------------------------------------------|--------------------------------------------|
| $h_1 = 76 \text{ mm}$                    | $D = 570 \text{ mm}$                       |
| $h_2 = 60.8 \text{ mm}$                  | $v_r = \pi D n / 60 = 2450 \text{ mm/sec}$ |
| $b_1 = 131 \text{ mm}$                   | $\alpha = 13^\circ 16'$                    |
| $b_2 = 136.5 \text{ mm}$                 | $s = 1.0434$                               |
| $\Delta h = h_1 - h_2 = 15.2 \text{ mm}$ | $w_2 = s v_r = 2556.55 \text{ mm/sec}$     |

The strain rate may be determined from formula (4.19),

$$q_s = \frac{w_2 \tan \alpha}{h_1} = \frac{2556.55 \times 0.2357}{76} = 7.91 \text{ sec}^{-1}$$

From the approximate equation (3.18) one obtains

$$\tan \alpha \approx \frac{\sqrt{R \Delta h}}{R - \Delta h / 2} = \frac{\sqrt{285 \times 15.2}}{285 - 15.2 / 2} = 0.2372$$

Inserting this value into equation (4.19), one obtains

$$q_s = \frac{w_2}{h_1} \tan \alpha = \frac{2556.55}{76} 0.2372 = 7.98 \text{ sec}^{-1}$$

As may be seen, the values obtained are very close.

For sticking, formula (4.22) has been used

$$q_{st} = \frac{v_r \sin \alpha}{h_1} = \frac{2450 \times 0.2295}{76} = 7.40 \text{ sec}^{-1}$$

From the approximate formula (3.17)

$$\begin{aligned} \sin \alpha &\approx \sqrt{\frac{\Delta h}{R}} \\ q_{st} &\approx \frac{v_r}{h_1} \sqrt{\frac{\Delta h}{R}} \\ q_{st} &= \frac{2450}{76} \sqrt{\frac{15.2}{285}} = 7.46 \text{ sec}^{-1} \end{aligned} \quad (4.22)$$

Also for this case very close values were obtained. Recalculating the same case based on formula (2.86) for mean strain rate, one obtains

$$\frac{\Delta h}{h_1} \frac{1}{t} = 0.2 \frac{1}{0.027} = 7.43 \text{ sec}^{-1}$$

Calculating the time  $t$  from  $l_d/v_r$  or  $l/v_r$ , gives the same value of 0.0269 sec.

This value also is very close to those obtained by formulae (4.21) and (4.22).

To simplify the calculation of mean strain rate, three nomograms have been worked out by the author (Figs. 4.23a, b, c).

They are valid for different ranges of thickness ratio  $R/h_2$ , namely Fig. 4.23a for  $R/h_2 = 0$  to 10, Fig. 4.23b for  $R/h_2 = 2.5$  to 50, and Fig. 4.23c for  $R/h_2 = 50$  to 500. These nomograms were based on the formula

$$q = \frac{\pi}{30} \frac{R}{h_1} \sqrt{\frac{h_1}{R} (1-\gamma)}$$

The following example explains how to use this nomogram.

#### Example

To calculate the mean strain rate, when rolling a flat bar of mild steel of C 0.15%, with a rolling temperature of 1100°C. The following data are given:  $h_1 = 5$  mm,  $h_2 = 4$  mm,  $b_1 = b_2 = 250$  mm, roll radius  $R = 200$  mm, and  $n = 75$  rpm.

The initial values of relative draught and thickness ratio are

$$\varepsilon_h = \frac{5-4}{5} = 0.2 \quad \text{and} \quad \frac{R}{h_2} = \frac{200}{4} = 50$$

Using these values one obtains from Fig. 4.23b a mean strain rate  $q = 0.33$ , for a number of revolutions  $n = 1$ . For  $n = 75$  rpm, therefore,  $q_m = 0.33 \times 75 = 24.75 \text{ sec}^{-1}$ .

In hot rolling to calculate roll separating forces, it is important to determine the mean constrained yield stress  $\eta K_f$ . This value may be determined by means of yield stress curves for hot rolling, published by P.M. Cook [25] for alloy and carbon steels, of chemical compositions given in Table 2.6. In Figs. 2.20–2.28, the variation of yield stress is shown in relation to effective elongation  $\log_e 1/\gamma$  for temperatures of 900, 1000, 1100 and 1200°C, and strain rate of 1.5; 8; 40, and 100  $\text{sec}^{-1}$ . The intermediate values may be obtained by interpolation.

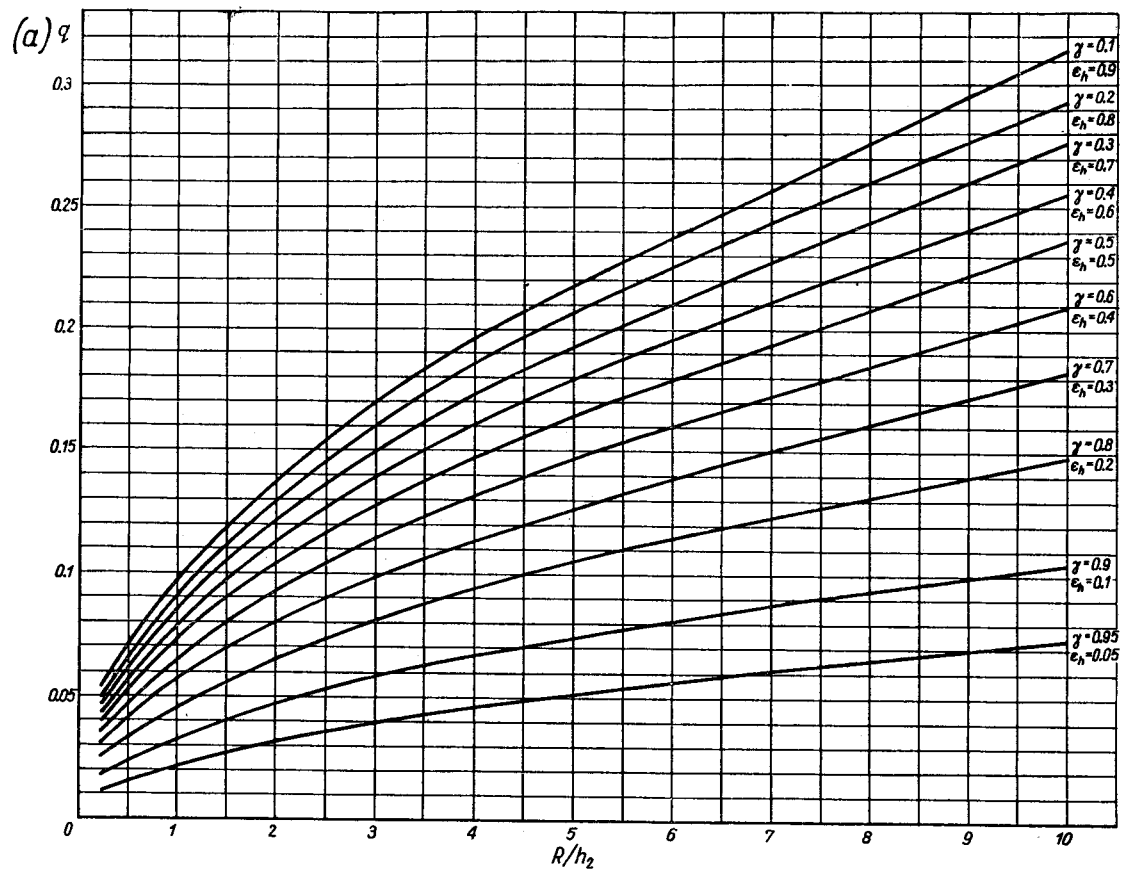


Fig. 4.23. Diagram for determination of strain rate  $q$  related to thickness ratio  $R/h_2$  and reduction  $\epsilon_h$ . Ratio  $R/h_2$ :  
(a) 0 to 10

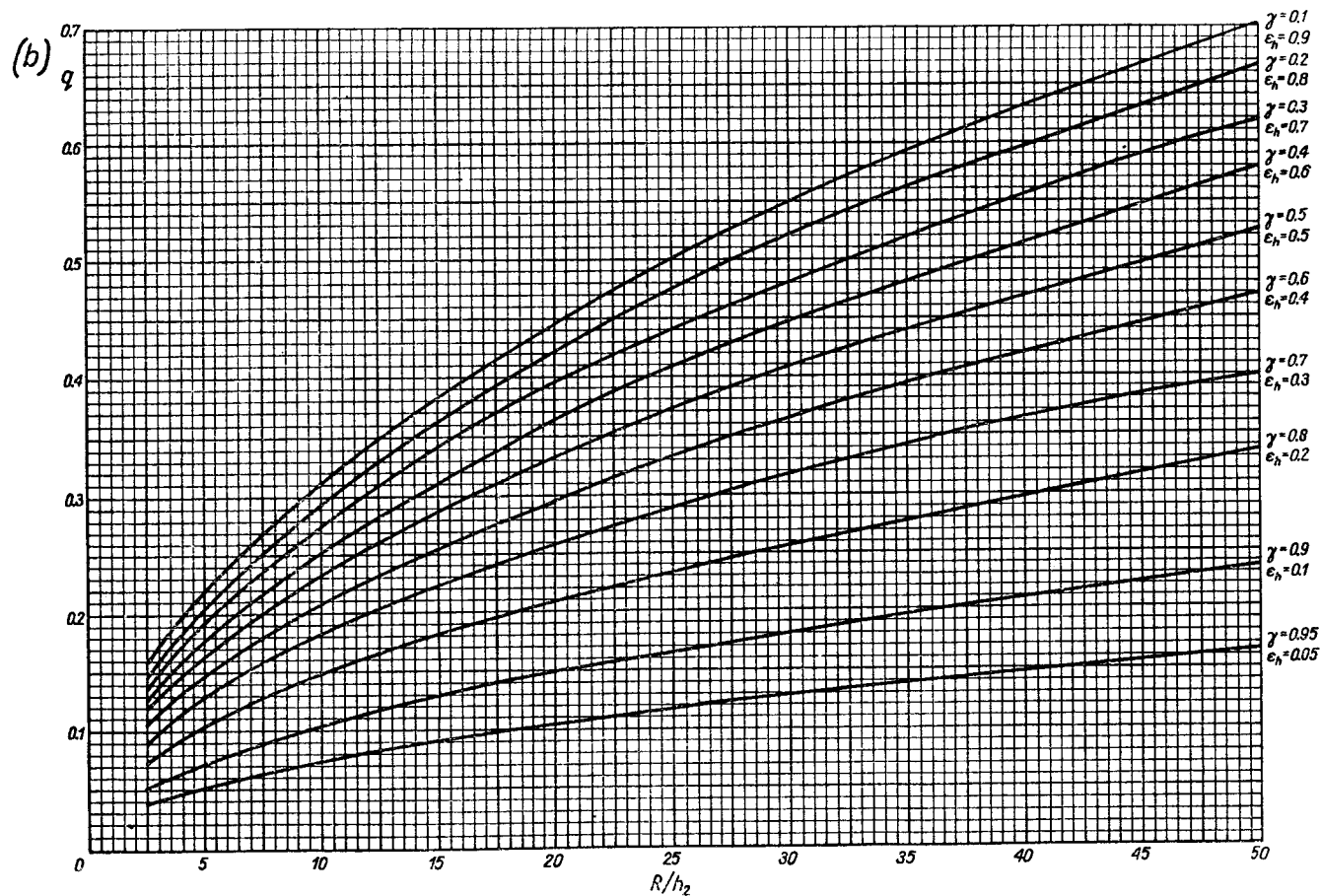


Fig. 4.23 (cont.). Diagram for determination of strain rate  $q$  related to thickness ratio  $R/h_2$  and reduction  $\epsilon_h$ . Ratio  $R/h_2$ :  
(b) 2.5 to 50

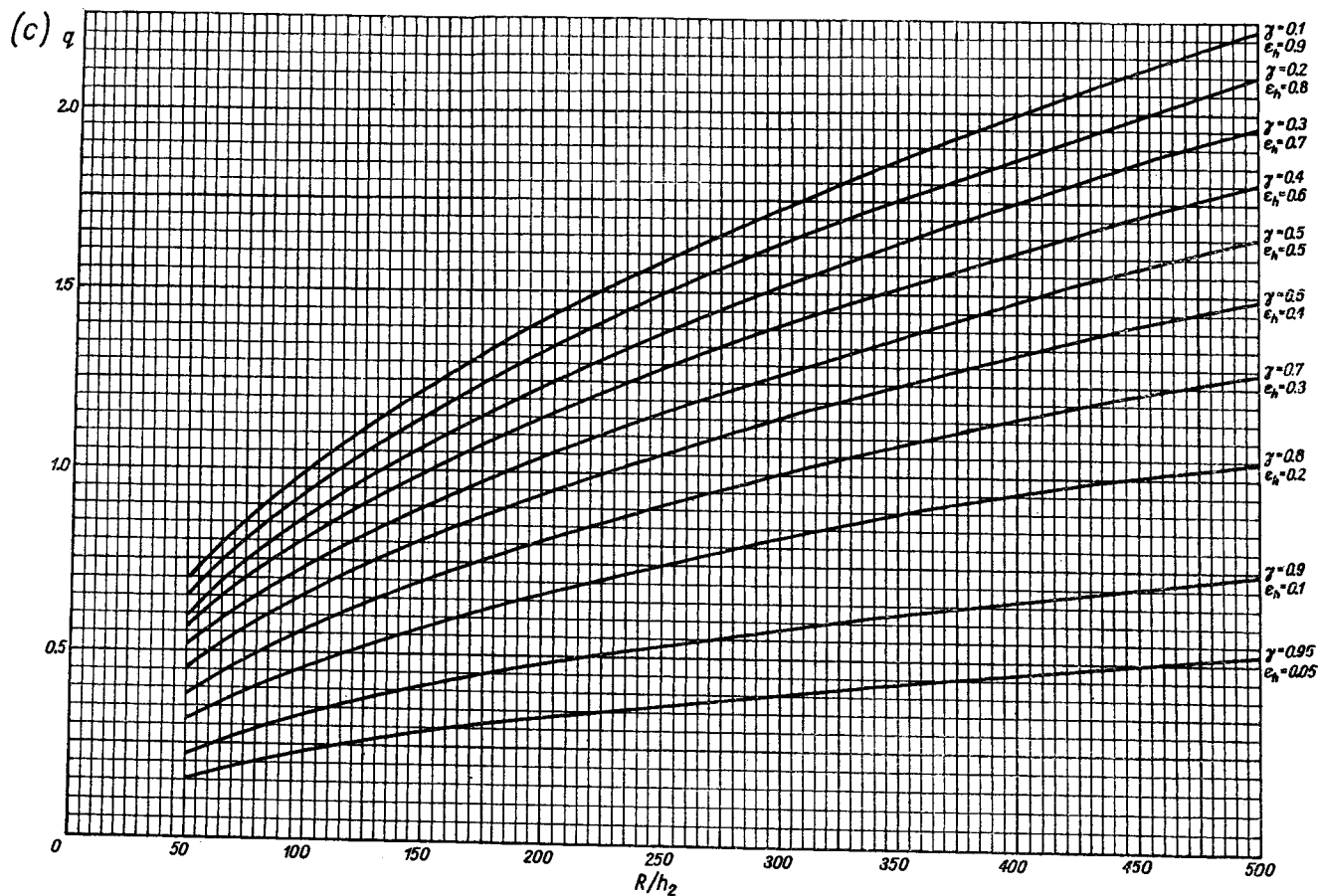


Fig. 4.23 (cont.). Diagram for determination of strain rate  $q$  related to thickness ratio  $R/h_2$  and reduction  $\epsilon_h$ . Ratio  $R/h_2$ : (c) 50 to 500

These diagrams may be applied as follows. In hot rolling, the mean yield stress, taking into account the continuously changing rolling conditions in the roll gap, may be expressed as

$$K_{fm} = \frac{1}{\alpha} \int_0^\alpha K d\varphi = \frac{1}{\alpha} \int_0^\alpha \eta K_f d\varphi \quad (4.23)$$

Usually the values of  $K_f$  determined from Figs. 2.20–2.28 must be recalculated for the constrained yield stress, multiplying through by the coefficient  $\eta$

$$K = \eta K_f$$

The coefficient  $\eta$  may be determined from equation (2.62), and  $\nu$  from equations (2.66) and (2.67). If  $\beta = \lambda$ , then  $\eta = 1$ . If  $\beta = 1$ , or  $\lambda = 1$ , the value of  $\eta = 1.155$  is to be used.

The graphical solution of integral (4.23) is illustrated by an example given by E.C. Larke [62]. By this method the mean constrained yield stress over the roll gap in hot rolling may be determined very exactly, taking into account the varying strain rate between plane of entry and exit. This method is based on measurements carried out by Alder and Phillips [62] for mild carbon steel.

Figures 4.24a and b show the run of yield stress for low-carbon steel, with a carbon content of 0.17%, relative to reduction and strain rate, for 930 and 1000°C.

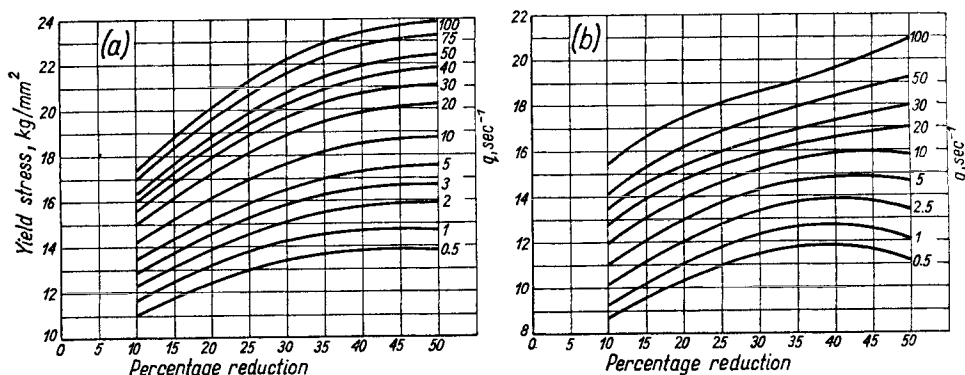


Fig. 4.24. Yield stress of a low-carbon steel with 0.17% C for different temperatures and rates of deformation [25]: (a) 930°C (b) 1000°C

In the example of Larke, sheet strip is rolled from the initial thickness of 4.83 mm, to a final one of 3.30 mm, with a temperature of 980°C, roll diameter of 685.80 mm, roll speed of 5.21 m/sec, and strain rate  $q = 89 \text{ sec}^{-1}$ .

The value of  $K_f$  for temperature 980°C and strain rate  $89 \text{ sec}^{-1}$  can readily be obtained by linear interpolation as follows: for strain rates 75 and  $100 \text{ sec}^{-1}$  in Fig. 4.24a, and 50 and  $100 \text{ sec}^{-1}$  in Fig. 4.24b, read off the value of the yield stress corresponding to reductions of 10, 20, 30, 40 and 50%. Plot these values as in Fig. 4.25. On each diagram draw a vertical line from the point along the horizontal scale, where the strain rate is  $89 \text{ sec}^{-1}$ . Note the values of the yield stress where the vertical lines intercept the various sloping lines, i.e. the point marked by circles in Figs. 4.25a and b. Plot these values as shown in Fig. 4.26. From the point corresponding to the specified temperature, i.e. 980°C, draw a vertical line as shown in Fig. 4.26. Note the values of the yield stress where the vertical line intercepts the various sloping lines.

The yield stress values marked as circles in this diagram are then plotted against the respective percentage reductions of 10 to 50%, and the yield stress curve is obtained (Fig. 4.27, curve C-D). For reductions below 10 per cent this curve has been extrapolated, determining for  $\varepsilon_h = 0$  the value

$$A = 0 \text{ kg/mm}^2$$

This curve, which specially refers to the required strain rate of  $89 \text{ sec}^{-1}$  and the specified rolling temperature of  $980^\circ\text{C}$  is the basic curve needed to compute the yield stress (Fig. 4.27).

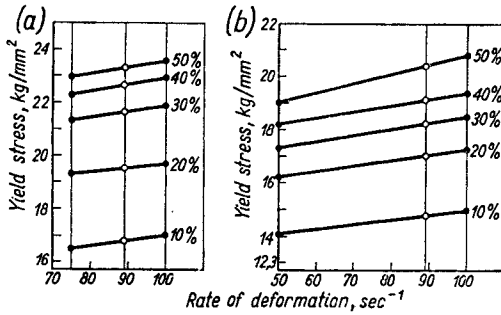


Fig. 4.25. Linear interpolation for determination of the basic yield stress curve for a rate of deformation of  $89 \text{ sec}^{-1}$  [62]: (a) low-carbon steel of 0.17% C for  $900^\circ\text{C}$  (b) the same for  $1000^\circ\text{C}$

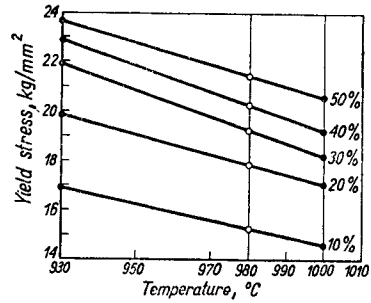


Fig. 4.26. Linear interpolation for determination of the basic yield stress curve at  $980^\circ\text{C}$  and rate of deformation  $89 \text{ sec}^{-1}$  for a low-carbon steel of 0.17% C

To determine the true curve for given values, it is necessary to calculate local reductions for chosen parts of arc of contact, using the formula

$$G_{pr} = G_t - \frac{100D}{h_1} (1 - \cos \varphi) \quad (4.24)$$

where

$G_{pr}$  = progressive reduction,

$G_t$  = total percentage reduction obtained in the pass.

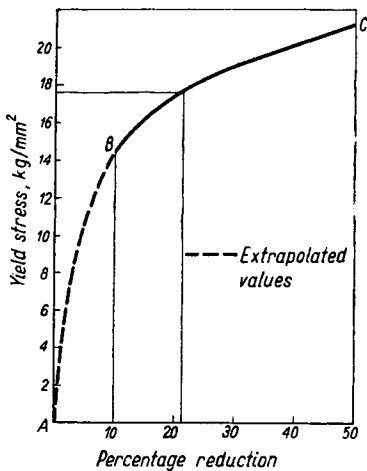


Fig. 4.27. Basic yield stress curve of a 0.17% C low-carbon steel at  $980^\circ\text{C}$  and rate of deformation of  $89 \text{ sec}^{-1}$

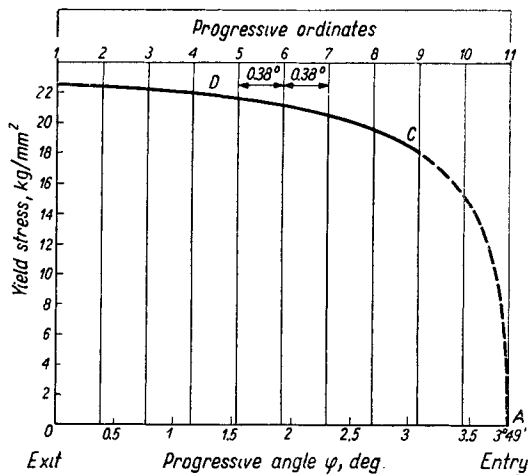


Fig. 4.28. Distribution of the basic yield stress curve along the bite angle

Substituting into formula (4.24) the following values:  $h_1 = 4.83$  mm,  $D = 685.80$  mm, and  $G_t = 31.6\%$ , as appropriate values for angle into which the arc of contact is divided, one obtains the values of corresponding reduction given in Table 4.2.

TABLE 4.2  
DATA FOR PLOTTING MEAN YIELD STRESS FOR A 0.17 PER CENT CARBON STEEL STRIP,  
HOT ROLLED AT 980°C

Initial thickness  $h_1 = 4.83$  mm, reduction 31.6%. Roll speed 5.4 m/sec. Roll diameter 685.8 mm

| Angle $\varphi$<br>degrees | Corresponding<br>reduction<br>% | Yield stress $K_f$<br>kg/mm <sup>2</sup> | Constrained yield stress<br>1.155 $K_f$<br>kg/mm <sup>2</sup> |
|----------------------------|---------------------------------|------------------------------------------|---------------------------------------------------------------|
| 0                          | 31.6                            | 19.28                                    | 22.27                                                         |
| 0.5                        | 31.1                            | 19.20                                    | 22.18                                                         |
| 1                          | 29.4                            | 19.09                                    | 22.05                                                         |
| 1.5                        | 26.7                            | 18.76                                    | 21.67                                                         |
| 2                          | 22.9                            | 18.22                                    | 21.04                                                         |
| 2.5                        | 18.1                            | 17.52                                    | 20.24                                                         |
| 3                          | 12.1                            | 16.03                                    | 18.51                                                         |
| 3.5                        | 5.1                             | 12.08                                    | 13.95                                                         |
| 3.8                        | 0                               | 0                                        | 0                                                             |

The bite angle is  $3^\circ 49'$ , and for this value the arc of contact has been divided into angles of  $0.5^\circ$ . Corresponding values of yield stress read from Fig. 4.27 multiplied by 1.155 for plane deformation (as  $\beta \approx 1$  was assumed), are included in Table 4.2.

Then values shown in Table 4.2 were plotted against the bite angle, and after determining appropriate heights, were collated in Table 4.3. The mean constrained yield stress obtained from the  $ACD$ -curve, was 18.42 kg/mm<sup>2</sup>. The values of mean constrained yield stress were calculated from Fig. 4.28, using the equation

$$K_m = \frac{1}{10} \left( \frac{K_1}{2} + K_2 + K_3 + K_4 + K_5 + K_6 + K_7 + K_8 + K_9 + K_{10} + \frac{K_{11}}{2} \right) \tag{4.25}$$

TABLE 4.3  
DATA FOR DETERMINING MEAN YIELD STRESS IN PLANE STRAIN  
(For details see head of previous table)

| Ordinate<br>No. | Yield stress in plane strain<br>kg/mm <sup>2</sup> | Ordinate<br>No. | Yield stress in plane strain<br>kg/mm <sup>2</sup> |
|-----------------|----------------------------------------------------|-----------------|----------------------------------------------------|
| 1               | 22.27                                              | 7               | 20.66                                              |
| 2               | 22.24                                              | 8               | 19.77                                              |
| 3               | 22.13                                              | 9               | 18.35                                              |
| 4               | 21.97                                              | 10              | 15.15                                              |
| 5               | 21.65                                              | 11              | 0                                                  |
| 6               | 21.18                                              |                 |                                                    |

It may be seen from the example quoted that this method is very laborious, and hence the following simplified method for calculation of the mean yield stress relative to the mean reduction (Fig. 4.29), has been proposed by Larke [62]. The mean height of the area  $abc$  is

$$x = \frac{\text{area } abc}{ac}$$

which, since  $ac = R \sin \alpha$



becomes

$$x = \frac{R(\alpha - \sin \alpha \cos \alpha)}{2 \sin \alpha}$$

As the total percentage reduction  $\varepsilon_h$  may be obtained from the formula

$$\varepsilon_h = \frac{h_1 - h_2}{h_1} 100$$

Larke determined the mean reduction, after transformation, as

$$\varepsilon_{hm} = 0.67 \varepsilon_h$$

To obtain the mean constrained yield stress, one reads the values directly from diagrams in Fig. 4.24a and b, for a calculated value of  $\varepsilon_{hm}$ . The read off value should be then multiplied by 1.155 for plane deformation, and by appropriate values of  $\eta$  for other cases.

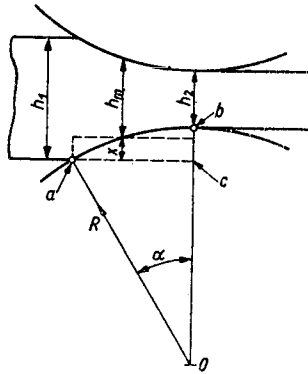


Fig. 4.29. Determination of the mean reduction [62]

For the example just discussed,  $\varepsilon_h = 31.6\%$ , hence

$$\varepsilon_{hm} = 31.6 \times 0.67 = 21.2 \text{ per cent}$$

Directly from Fig. 4.27 one obtains

$$K = 17.6 \times 1.155 = 20.76 \text{ kg/mm}^2$$

The mean value obtained by the more exact method amounts to 18.4 kg/mm<sup>2</sup>, i.e. 7% less.

## 4.2. Calculation of Roll Force in Hot Rolling

There are many formulae to determine the roll separating force in hot rolling. All of them have been analysed and tested [68, 69, 70], and the best chosen. The selected formulae are presented in a form most suitable for practical use.

### 4.2.1. TSELIKOV'S METHOD OF CALCULATION OF ROLL FORCE [70]

The formula for calculation of roll force (method I) published by Tselikov [59] is as follows:

$$P = F_4 K \frac{2h_2}{\Delta h(\delta' - 1)} \left( \frac{h_n}{h_2} \right) \left[ \left( \frac{h_n}{h_2} \right)^{\delta'} - 1 \right] \quad (4.26)$$

where

$F_d = b_m l_d =$  projected area of contact,  $\text{mm}^2$ ,

$b_m = (b_1 + b_2)/2 =$  mean width of the rolled stock,  $\text{mm}$ ,

$l_d = \sqrt{R \Delta h} =$  projected arc of contact,  $\text{mm}$ ,

$\delta' = f(2l_d/\Delta h) = f\sqrt{2D/\Delta h}$ ,

$\Delta h = h_1 - h_2 =$  absolute draught,  $\text{mm}$ ,

$D =$  active roll diameter,  $\text{mm}$ ,

$f =$  coefficient of friction,

$h_n =$  height at the neutral plane,  $\text{mm}$ ,

$K = 1.155 K_f$ ,

$K_f =$  yield stress, read off from Tselikov's diagrams (Fig. 4.30) for various carbon steels and rolling temperatures,  $\text{kg/mm}^2$ ,

The coefficient of friction  $f$  in hot rolling, included in Tselikov's formula, should be calculated for steel from Ekelund's equation (3.83).

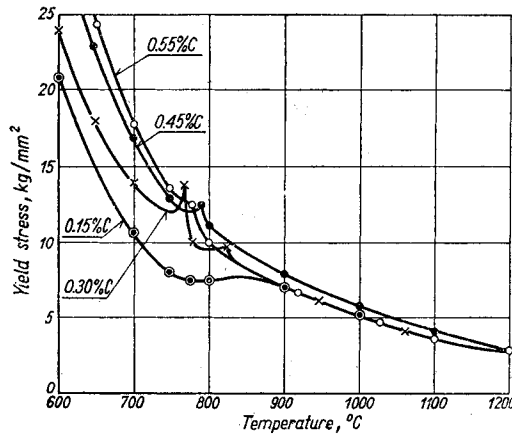


Fig. 4.30. Yield stress of carbon steels related to temperature

For determining the relation  $h_n/h_2$  Tselikov derived the following formula:

$$\frac{h_n}{h_2} = \left[ \frac{1 + \sqrt{1 + (\delta'^2 - 1) \left( \frac{h_1}{h_2} \right)^{\delta'}}}{\delta' + 1} \right]^{1/\delta'} \quad (4.27)$$

As calculation with formulae (4.26) and (4.27) is very laborious, they have been transformed by Tselikov [46] into the following form:

$$P = F_d \frac{2K}{\varepsilon_h \delta'} \left[ \left( \frac{1}{1 - \varepsilon_h} \right)^{(\delta' - 1)/2} - \left( 1 - \frac{\varepsilon_h}{2} \right) \right] \quad (4.28)$$

where

$\varepsilon_h =$  relative draught,

$\delta' =$  coefficient as in equation (4.26).

To make the solution of equation (4.28) easier, Tselikov gives a diagram, from which the ratio  $K_{wm}/K$  relative to draught and coefficient  $\delta'$  may be read off directly.

From here onwards the graphical method shown in Fig. 4.31 will be called the second Tselikov method for determining roll force.

The formula (4.28) has been then further simplified by Tselikov, and brought to the following form:

$$P = F_d \frac{K(2-\varepsilon_h)}{\varepsilon_h \delta'} \left[ \left( \frac{1}{1-\varepsilon_h} \right)^{\delta'/2} - 1 \right] \quad (4.29)$$

All symbols used are the same as for equation (4.28). This formula will be called the third Tselikov method.

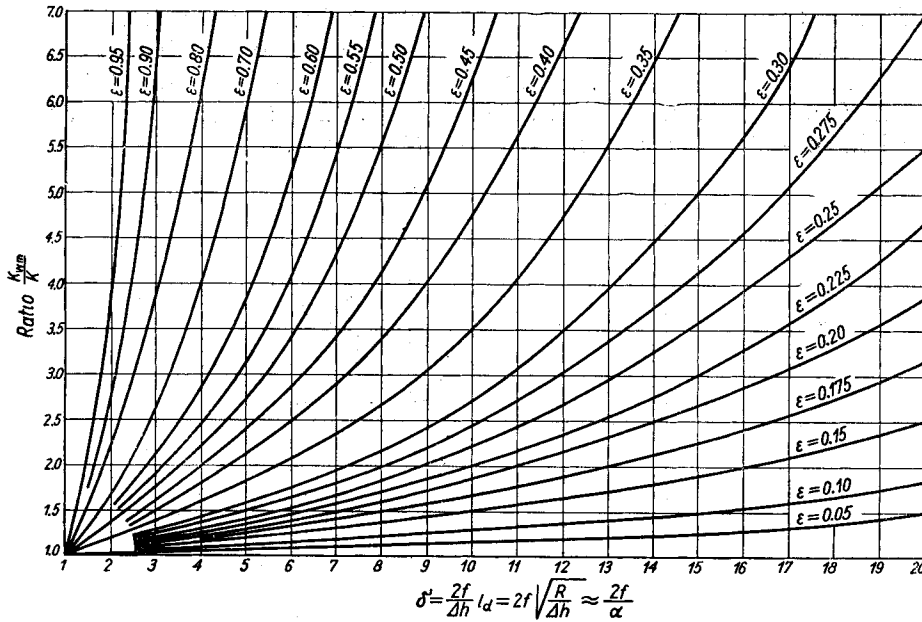


Fig. 4.31. Diagram for determination of the ratio  $K_{wm}/K$  according to Tselikov's second method [71]

The values of  $K$  and  $f$  shown in formulae (4.29) and (4.28), should be calculated or estimated in the same way as for equation (4.26).

#### 4.2.2. GOLOVIN-TIAGUNOV'S METHOD OF CALCULATION OF ROLL FORCE [70]

According to this method the constrained yield stress of the material at higher temperatures is found from the formula

$$p_0 = K_t Y_P \quad (4.30)$$

where for a temperature range above  $(t_m - 575^\circ\text{C})$ , the coefficient  $K_t$  may be obtained from the relation

$$K_t = \frac{t_m - 75 - t}{1500} \quad (4.31)$$

and for a range below ( $t_m - 575^\circ\text{C}$ ), from the equation

$$K_t = \left( \frac{t_m - t}{100} \right)^2 \quad (4.32)$$

where

$K_t$  = coefficient of the constrained yield stress, depending on the temperature,

$t$  = rolling temperature,  $^\circ\text{C}$ ,

$t_m$  = temperature of fusion,  $^\circ\text{C}$ ,

$Y_p$  = constrained yield stress at  $20^\circ\text{C}$  (Fig. 4.34).

It may be concluded from formula (4.31) that in the temperature range of  $t_m - 575 = 700$  to  $900^\circ\text{C}$ , the  $K_t$  value obtained from equation (4.31) goes over to that obtained from formula (4.32). The range of application of both formulae is limited by the temperature for which identical values of  $K_t$  for both equations are obtained. For a steel of  $t_m = 1300^\circ\text{C}$ , a minimum value of the temperature of  $1300 - 575 = 725^\circ\text{C}$ , determines the limit of validity of formula (4.31). If  $t_m = 1500^\circ\text{C}$ , then the formula is valid down to a minimum rolling temperature of  $1500 - 575 = 925^\circ\text{C}$ .

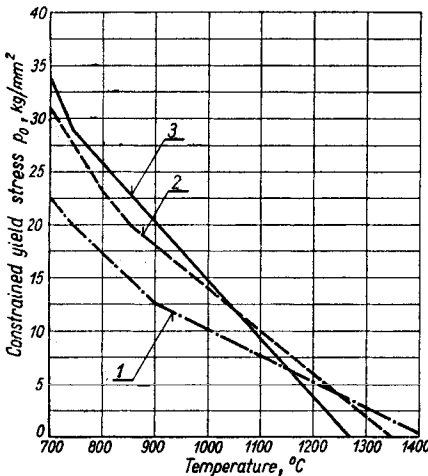


Fig. 4.32. Constrained yield stress from Golovin-Tiagunov method. Carbon steel compositions [71]: 1—0.1% C, 2—0.5% C, 3—0.9% C

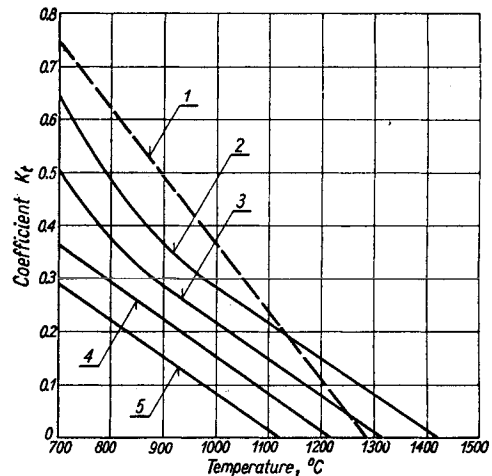


Fig. 4.33. Diagram for determination of the  $K_t$ -coefficient by the Golovin-Tiagunov and Samarin method [71]: 1— $K_t$  according to Samarin, 2—for  $t_m = 1500^\circ\text{C}$ , 3—for  $t_m = 1400^\circ\text{C}$ , 4—for  $t_m = 1300^\circ\text{C}$ , 5—for  $t_m = 1200^\circ\text{C}$

For practical purposes, of the two values obtained from formulae (4.31) and (4.32), the greater one is always chosen. Figure 4.32 shows values of  $p_0$  for certain carbon steels, in relation to temperature. Figure 4.33 shows the distribution of  $K_t$ -values for steels of different melting points, related to temperature. This method may be used, of course, also for alloy and other quality steels. Using the Golovin-Tiagunov's method of calculation, maximum values of constrained yield stress are obtained.

A. Golovin takes into account the resistance to flow, due to frictional forces, in the following simplified formula:

$$K_r = 1 + f \left( \frac{l_d}{h_m} - 1 \right) = f \left( \frac{2\sqrt{\Delta h R}}{h_1 + h_2} - 1 \right) \quad (4.33)$$

where  $f$  = coefficient of friction, equals  $1/3$  according to Golovin, and for  $l_d/h_m \leq 1$  the value of  $K_r = 1$  should be substituted.

Taking into account these relations, the resistance to deformation in the method of Golovin-Tiagunov is obtained, as

$$K_{wm} = \left[ 1 + f \left( \frac{l_d}{h_m} - 1 \right) \right] K_r Y_P \quad (4.34)$$

where  $K_{wm}$  = mean resistance to deformation.

For determining the values of  $t_m$  and  $Y_P$  for various carbon steels, the diagram shown in Fig. 4.34 may be used.

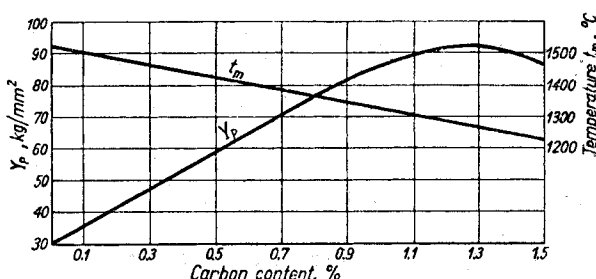


Fig. 4.34. Diagram for determination of yield stress and melting point for carbon steels [71]

Roll force is calculated as  $P = K_{wm} F_d$ , where  $K_{wm}$  is determined from formula (4.34).

#### 4.2.3. SAMARIN'S METHOD OF CALCULATION OF ROLL FORCE [70]

A. Samarin's formula has been derived from analysis of practically obtained roll forces, measured on a plate mill. Strain-gauges have been used, and the derived formula is of the following form:

$$K_{wm} = K_t Y_P = (30 - 0.023)t - 0.0055 Y_P \quad (4.35)$$

where

$Y_P$  = constrained yield stress for steel at 20°C, kg/mm²,

$t$  = rolling temperature, °C.

Samarin's formula does not take into account the influence of external friction, i.e. for a rolling temperature of 1375°C independently of the steel quality, it becomes a simplified Tiagunov formula. For a steel with  $t_m = 1375^\circ\text{C}$  a value of  $K_{wm}$  twice that of  $p_0$  in Tiagunov's formula, is obtained. This coefficient of 2 takes into account, to some extent, the influence of external friction.

For steel of lower or higher melting temperature, average values from Samarin's formula, in comparison with those found by multiplying  $p_0$  by 2 in Tiagunov's equation, are determined.

Values of  $K_f$  for Samarin's method may be obtained from Fig. 4.33, and those of  $Y_P$  from Fig. 4.34.

#### 4.2.4. EKElund's METHOD OF CALCULATION OF ROLL FORCE [10]

For hot rolling the following formula for calculation of roll force has been derived by Ekelund

$$P = b_m \sqrt{R(h_1 - h_2)} \left[ 1 + \frac{1.6f \sqrt{R(h_1 - h_2)} - 1.2(h_1 - h_2)}{h_1 + h_2} \right] \left( K_{f0} + \frac{2\eta v_m \sqrt{\frac{h_1 - h_2}{R}}}{h_1 + h_2} \right) \quad (4.36)$$

This formula may be defined as follows: roll force  $P$  equals the projected area of contact, multiplied by  $(1 + \text{function of friction})$  and by  $(\text{statical yield stress} + \text{coefficient of plasticity multiplied by function of strain rate})$ .

In this formula:

$f$  = coefficient of friction, calculated by means of formula (3.83),

$t$  = rolling temperature, °C (starting from 700°C),

$K_{f0}$  = yield stress, calculated by means of formula

$$K_{f0} = (14 - 0.01t)(1.4 + C + \text{Mn} + 0.3\text{Cr}) \text{ kg/mm}^2$$

where

$C$  = carbon content of steel, %,

$\text{Mn}$  = manganese content of steel, % (max. 1%),

$\text{Cr}$  = chromium content of steel, %,

$\eta$  = coefficient of plasticity of the rolled stock (starting from 800°C)

$$\eta = 0.01(14 - 0.01t) \text{ kg sec/mm}^2$$

$v_m$  = mean rolling speed in the pass, which is equal to the peripheral speed ( $v_m = v_r$ ), when rolling on smooth roll barrels mm/sec. This formula is valid for a speed up to 7 m/sec.

When rolling with speeds higher than 6 m/sec SKF recommend the following corrections  $c$  for the coefficient of plasticity  $\eta$ :

| $v$           | $c$  |
|---------------|------|
| up to 6 m/sec | 1.0  |
| 6–10          | 0.8  |
| 10–15         | 0.65 |
| 15–20         | 0.6  |

In these cases the corrected value of  $\eta' = c\eta$  should be substituted for  $\eta$  in Ekelund's formula.

Formula (4.36) is valid for sheet, strip and flats. When rolling in passes, instead of  $h_1$  and  $h_2$ , mean heights of the pass, calculated from Table 4.1 or Figs. 3.30 to 3.40 are used.

The Ekelund formula for calculation of resistance to deformation of the rolled stock is based upon the assumption that yield stress and internal friction decrease in direct proportion to the increase of temperature. According to Ekelund's formula the yield stress would be equal to zero, if the rolling temperature were to increase up to 1400°C, i.e. the rolled stock would begin to melt, which is obviously erroneous.

However despite disadvantages, the Ekelund formula gives results very closely approximating to the true ones. This method is suitable for calculation of resistance to deformation on blooming mills, cogging mills, and plate and sheet mills, in which cases it generally gives good results.

#### 4.2.5. OROWAN-PASCOE METHOD OF CALCULATION OF ROLL FORCE [10, 60]

As theoretical formulae derived by Orowan are very complicated, he proposed together with Pascoe [60] an approximate method for calculating the roll force in hot rolling. Good results are obtained for this method, if the coefficient of friction  $f$  is greater than 0.3; if the coefficient  $f$  lies between 0.3 to 0.4, results of medium accuracy are obtained, but very inaccurate for  $f = 0.2$  to 0.3. The product  $l_d f$  should not exceed very much the value of  $h_m$ , calculated from formula (4.38), for conditions of rolling of the stock. Due to these limitations the method considered should not be applied for rolling of thin and wide sheet and strip; however, good results may be obtained for heavy, medium and light-section rolling mills. Orowan proposes the following method for determination of geometrical relations:

length of projected arc of contact

$$l_d = \sqrt{R \Delta h} \quad (4.37)$$

relative draught

$$\varepsilon_h = \frac{h_1 - h_2}{h_1}$$

mean height in the roll gap

$$h_m = \frac{2h_2 + h_1}{3} \quad (4.38)$$

mean width

$$b_m = \frac{2b_2 + b_1}{3}$$

Substituting into the last equation the coefficient of spread  $\beta = b_2/b_1$ , then

$$b_m = \frac{b_1(2\beta + 1)}{3}$$

Orowan and Pascoe assume the constrained yield stress at the plane of entry and of exit, as  $0.8 K$ , due to unequal strain distribution in the roll gap, where  $K = 1.155 K_f$ .

Orowan and Pascoe recommend three different formulae for calculation of roll force, depending upon the initial dimensions of the rolled stock.

(1) If the width of the rolled stock is 6 to 8 times greater than the mean height, then

$$P = K b_1 l_d \left( 0.8 + \frac{l_d}{4 h_2} \right) \quad (4.39)$$

(2) If the width of the rolled stock is more than 1.5 to 2 times greater than the mean height, then

$$P = K b_m l_d \left[ 0.8 + 0.25 \frac{l_d}{h_2} - \frac{h_m h_2}{1.5 b_m l_d} \left( \frac{l_d}{2 h_2} - 0.2 \right)^3 \right] \quad (4.40)$$

(3) If the width of the rolled stock is 1.5 to 2 times smaller than the mean height, then

$$P = Kb_m l_d \quad (4.41)$$

where  $K = 1.155 K_f$ .

Deriving these formulae Orowan has assumed that the distance between neutral plane and plane of exit is  $0.5 l_d$ .

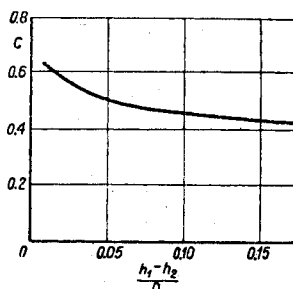


Fig. 4.35. Relation between  $c$ -coefficient and ratio  $(h_1 - h_2)/D$

If more exact results are required, then the coefficient  $c$  should be inserted into these formulae, to take into account the true distance between neutral plane and plane of exit. Its value relative to  $(h_1 - h_2)/D$  is shown in Fig. 4.35. Then the Orowan-Pascoc formulae for calculation of roll force take the following form

$$P = Kb_1 l_d \left( 0.8 + \frac{c l_d}{2 h_2} \right) \quad (4.42)$$

$$P = Kb_m l_d \left[ 0.8 + \frac{c l_d}{2 h_2} - \frac{h_m h_2}{3 b_m c l_d} \left( \frac{c l_d}{h_2} - 0.2 \right)^3 \right] \quad (4.43)$$

Formula (4.41) does not change.

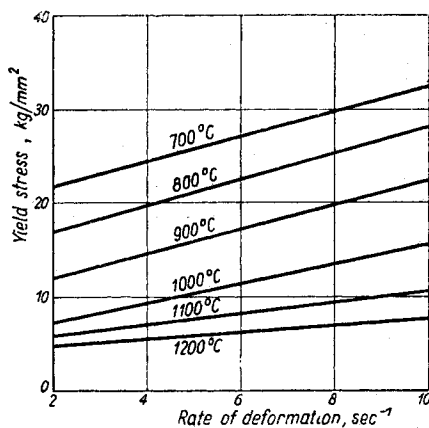


Fig. 4.36. Yield stress of carbon steel (0.28% C) related to rate of deformation and temperature, from Orowan-Pascoc [60]

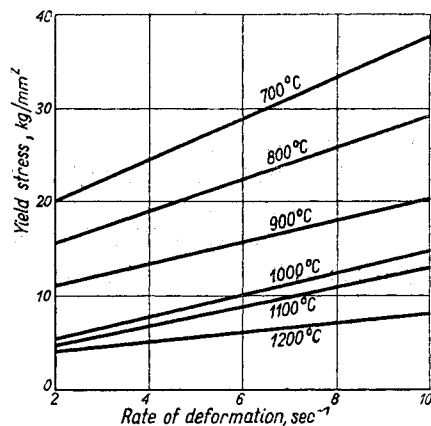


Fig. 4.37. Yield stress of carbon steel (0.45% C) related to rate of deformation and temperature, from Orowan-Pascoc [60]



Calculation of roll force from Orowan's formula gives good agreement with those obtained from practical measurements, if  $K_f$  is read from diagrams shown in Fig. 4.36 and 4.37. These curves have been plotted based on investigations carried out by Pomp and Lueg for 0.28 and 0.45% C-steel, relative to strain rate and temperature of rolling. The coefficient  $c$  should be taken from Fig. 4.35. The  $K_f$ -values show a linear relation in both figures, and do not curve for higher strain rates, as opposed to results obtained by Trinks. To determine the strain rate, diagrams shown in Figs. 4.23a-c, are recommended.

#### 4.2.6. GELEJI'S METHOD OF CALCULATION OF ROLL FORCE [20]

Geleji worked out his method from results of investigations and measurements of roll force, carried out by Puppe, Siebel, Emicke and Lucas. To confirm the correctness of his method, Geleji has compared calculated results with those obtained from measurements. Generally this method is valid for rectangular cross-sections only, but it may also be applied with sufficient accuracy for calculation of roll force in breaking-down passes. The derivation of the formula is based on the hypothesis of maximum shearing stresses.

Assuming  $\sigma_1 = K_w$ ; and  $\sigma_2 = K_f$ , then

$$K_w = K_f + \sigma_3 \quad (4.44)$$

According to Geleji the resistance to flow  $\sigma_3$  depends upon normal pressure acting on the roll, coefficient of friction, and stresses acting along the roll direction, due to slipping of both surfaces. This relation has been expressed as follows:

$$\sigma_3 = c K_f f \frac{l_d}{h_m} \sqrt[n]{v_r} \quad (4.45)$$

where

$K_f$  = 0.015(1400 -  $t$ ), yield stress, kg/mm<sup>2</sup>. This formula is valid for carbon steels of a tensile strength up to 60 kg/mm<sup>2</sup>, and a temperature range of 800 to 1300°C,

$h_m$  =  $(h_1 + h_2)/2$  mean height,

$v_r$  = peripheral rolling speed, m/sec,

$t$  = rolling temperature, °C,

$f$  = coefficient of friction between roll and rolled stock, depending upon the surface quality of rolls, which may be calculated from equation (3.84) derived by Bachtinov,

$l_d$  = length of projected arc of contact,  $l_d = \sqrt{R \Delta h}$ ,

$c, n$  = coefficients determined from results of test measurements.

The coefficient  $c$  may be also calculated from formulae, namely relative to the ratio  $l_d/h_m$ :

if  $0.25 \leq l_d/h_m \leq 1$ , then

$$c_{0.25/1} = 17.0 \left( \frac{l_d}{h_m} \right)^2 - 29.85 \left( \frac{l_d}{h_m} \right) + 18.3 \quad (4.46)$$

if  $1 \leq l_d/h_m \leq 3$ , then

$$c_{1/3} = 0.8 \left( \frac{l_d}{h_m} \right)^2 - 4.9 \left( \frac{l_d}{h_m} \right) + 9.6 \quad (4.47)$$

if  $3 \leq l_d/h_m \leq 13$ , then

$$c_{3/13} = 0.013 \left( \frac{l_d}{h_m} \right)^2 - 0.29 \left( \frac{l_d}{h_m} \right) + 2.86 \quad (4.48)$$

For practical purposes the coefficient  $c$  should be determined from Fig. 4.38, and the exponent  $n = 4$ .

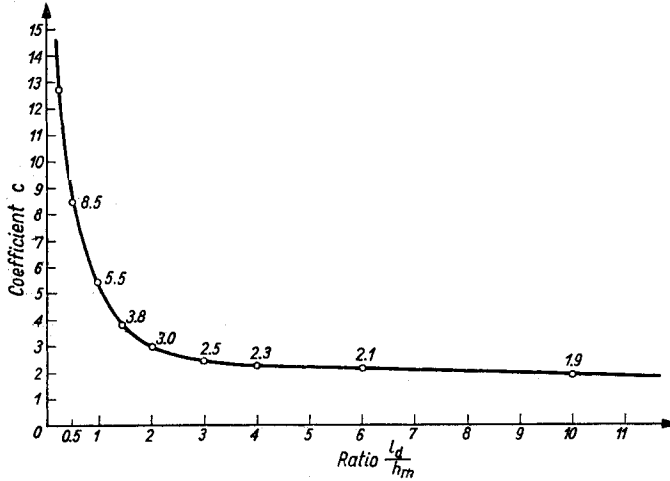


Fig. 4.38.  $c$ -coefficient related to  $l_d/h_m$  from Geleji [20]. Mean resistance to deformation  $K_{wm} = K_f(1 + cf(l_d/h_m)^4 \sqrt{v_r})$ ; for steel rolls,  $f = (1.05 - 0.0005 t)K_1$ ; for hardened rolls,  $f = (0.92 - 0.0005 t)K_1$ ; for hardened and ground steel rolls,  $f = (0.82 - 0.0005 t)K_1$ ;  $t$  = rolling temperature, °C,  $K_1$  see formula (3.84)

Substituting formula (4.45) into (4.44), and taking into account the coefficients, according to Geleji the following formula for determination of mean resistance to deformation is obtained

$$K_{wm} = K_f \left( 1 + cf \frac{l_d}{h_m} \sqrt[4]{v_r} \right) \quad (4.49)$$

Then the formula for calculation of roll force takes the form

$$P = F_d K_f \left( 1 + cf \frac{l_d}{h_m} \sqrt[4]{v_r} \right)$$

These formulae may be also applied for calculation of roll force during cold rolling, substituting the radius of flattened arc of contact in the  $l_d$  formula (3.13).

#### 4.2.7. COOK AND MCCRUM'S METHOD OF CALCULATION OF ROLL FORCE [67]

The graphical method for determination of roll force and torque worked out by Cook and McCrum, is based on the formula and relations derived by R. B. Sims [147]. Also measurements for determination of yield stress for steel, carried out on a Cam Plastometer by Cook [25], have been reproduced, as shown in Figs. 2.20–2.28.

The Sims' formulae for calculation of roll force and torque during flat hot rolling are

$$P = \sqrt{R\Delta h} k_p Q_p \quad (4.50)$$

and

$$M_w = 2RR'k_g Q_g \quad (4.51)$$

where

$Q_p$  and  $Q_g$  are dimensionless parameters depending on  $\varepsilon_h$  and the ratio  $R/h_2$ ,  $k_p$  and  $k_g$  are mean values of constrained yield stress in the roll gap, for roll force and torque, derived from the mean yield stress  $k_{fm}$ .

The mean constrained yield stresses in the roll gap are obtained from equations

$$k_p = \frac{1}{\alpha} \int_0^\alpha K d\varphi \quad (4.52)$$

and

$$k_g = \frac{1}{\varepsilon_h} \int_0^\alpha K d\varepsilon_x \quad (4.53)$$

where, as known,  $K = \eta K_f$  and  $\varepsilon_x = (h_1 - h_2)/h_1$  should be substituted.

These different expressions for the mean constrained yield stresses depend on the method of calculation proposed by Sims [147].

For determination of roll force and torque Cook and McCrum worked out diagrams for hot flat rolling, which are based upon the yield stress curves mentioned already, published by P. M. Cook (Figs. 2.20–2.28), and the two equations (4.50) and (4.51).

The graphical method for determination of roll force and torque is based on the two following formulae:

$$P' = R'C_p I_p \quad (4.54)$$

$$M_w = 2RR'C_g I_g \quad (4.55)$$

The factors shown in formulae (4.54) and (4.55) have the values

$$C_p = Q_p \sqrt{\frac{h_2}{R} \frac{\varepsilon_h}{1+\varepsilon_h}} \quad (4.56)$$

$$C_g = Q_g \sqrt{\frac{1-\varepsilon_h}{1+\varepsilon_h}} \quad (4.57)$$

The meaning of values of  $Q_g$  and  $Q_p$  has been already explained at the beginning of this chapter

$$I_p = k_p \sqrt{\frac{1+\varepsilon_h}{1-\varepsilon_h}} \quad (4.58)$$

and

$$I_g = k_g \sqrt{\frac{1+\varepsilon_h}{1-\varepsilon_h}} \quad (4.59)$$

The constrained yield stresses  $k_p$  and  $k_g$  are defined by formulae (4.52) and (4.53).

$R$  denotes the radius of undeformed roll, and  $R'$  the radius of deformed arc of contact, due to elastic deformation of the roll barrel. It may be calculated from formula (4.150). In hot rolling no error is made, if  $R' = R$  is substituted.

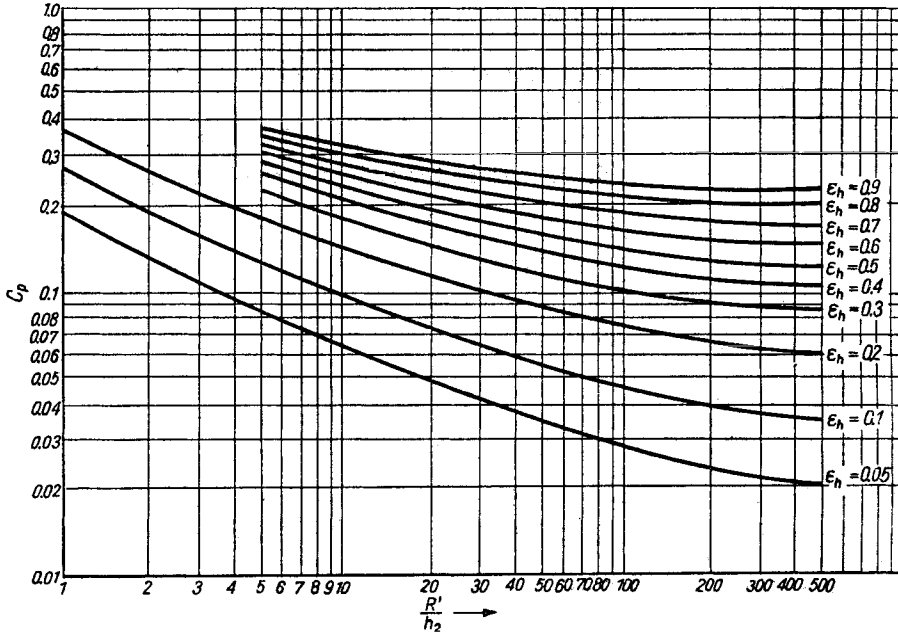


Fig. 4.39.  $C_p$  functions from Cook and McCrum [67]

The geometrical functions  $C_p$  and  $C_g$ , according to Cook and McCrum [67], are plotted against  $R'/h_2$  in Fig. 4.39 and 4.40.

The curves have been calculated from formulae (4.56) and (4.57), whereas the range of  $C_p$  shown in Fig. 3.39 was enlarged to include  $\epsilon_h = 0.05$  to 0.9, i.e. for draught coefficients  $\gamma = 0.95$  to 0.1.

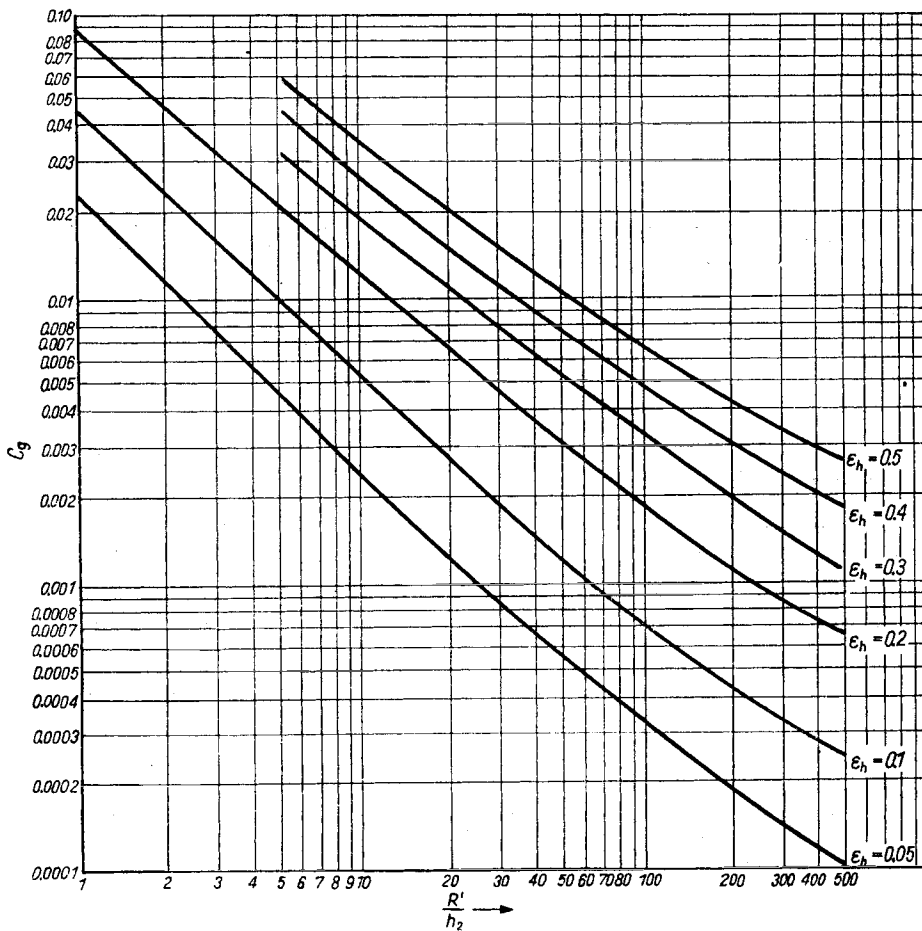
The function  $I_p$  for a low-carbon steel for temperatures of 900, 1000, 1100 and 1200°C, is shown in Figs. 4.41a–d. The function  $I_g$  for the same rolling conditions and the same composition of the rolled stock, is shown in Figs. 4.42a–d. The composition of the rolled stock is the same as that for which the yield stress curve has been drawn in Fig. 2.20a.

The diagrams published by Cook and McCrum for determination of roll force and torque in hot flat rolling, provide the simplest method for calculating the forces arising in flat passes [67]. This method, published by BISRA in 1958, will be further explained by the following example.

#### Example

Consider the rolling of a strip where width  $b_1 = b_2 = 250$  mm, from  $h_1 = 5$  mm to  $h_2 = 4$  mm between  $R = 200$  mm rolls at 75 rpm. The material is a low-carbon steel at 1100°C. The magnitude of roll force and torque is to be calculated.

First of all, the relative draught  $\epsilon_h = (5-4)/5 = 0.2$ , and the two thickness ratios should be calculated ( $R/h_1 = 200/5 = 40$ ,  $R/h_2 = 200/4 = 50$ ).

Fig. 4.40.  $C_g$  functions from Cook and McCrum [67]

The strain rate  $q$  may be obtained from Figs. 4.23a-c. For  $\epsilon_h = 0.2$  and  $R/h_1 = 40$  from Fig. 4.23b, a strain rate of  $q = 0.3 \text{ sec}^{-1}$  per roll revolution, is obtained. For roll revolutions of 75 rpm the mean strain rate amounts to  $q_m = 0.3 \times 75 = 22.5 \text{ sec}^{-1}$ . Roll force and torque per mm width of the stock are obtained from formulae (4.54) and (4.55), as

$$P' = R' C_p I_p \text{ kg/mm width}$$

$$M_w = 2 R R' C_g I_g \text{ kgmm/mm width}$$

or, as for hot rolling  $R' = R$  may be substituted

$$M_w = 2 R^2 C_g I_g$$

The values of  $C_p$  and  $C_g$  for  $\epsilon_h = 0.2$  and  $R/h_2 = 50$  should be read off from Figs. 4.39 and 4.40, and value  $C_p = 0.089$  and  $C_g = 0.0032$ .

From Figs. 4.41 and 4.42 for a low-carbon steel at  $1100^\circ\text{C}$ , a mean strain rate  $q_m = 22.5 \text{ sec}^{-1}$ , and  $\epsilon_h = 0.2$ , values of  $I_p = 16.4 \text{ kg/mm}^2$  and  $I_g = 15.1 \text{ kg/mm}^2$  can be found.

Therefore from the above formula:

$$\text{a roll force of } P' = 200 \times 0.089 \times 16.4 = 292 \text{ kg/mm width}$$

$$\text{and a roll torque of } M_w = 2 \times 200^2 \times 0.0032 \times 15.1 = 3865.6 \text{ kgmm/mm width}$$

are obtained.

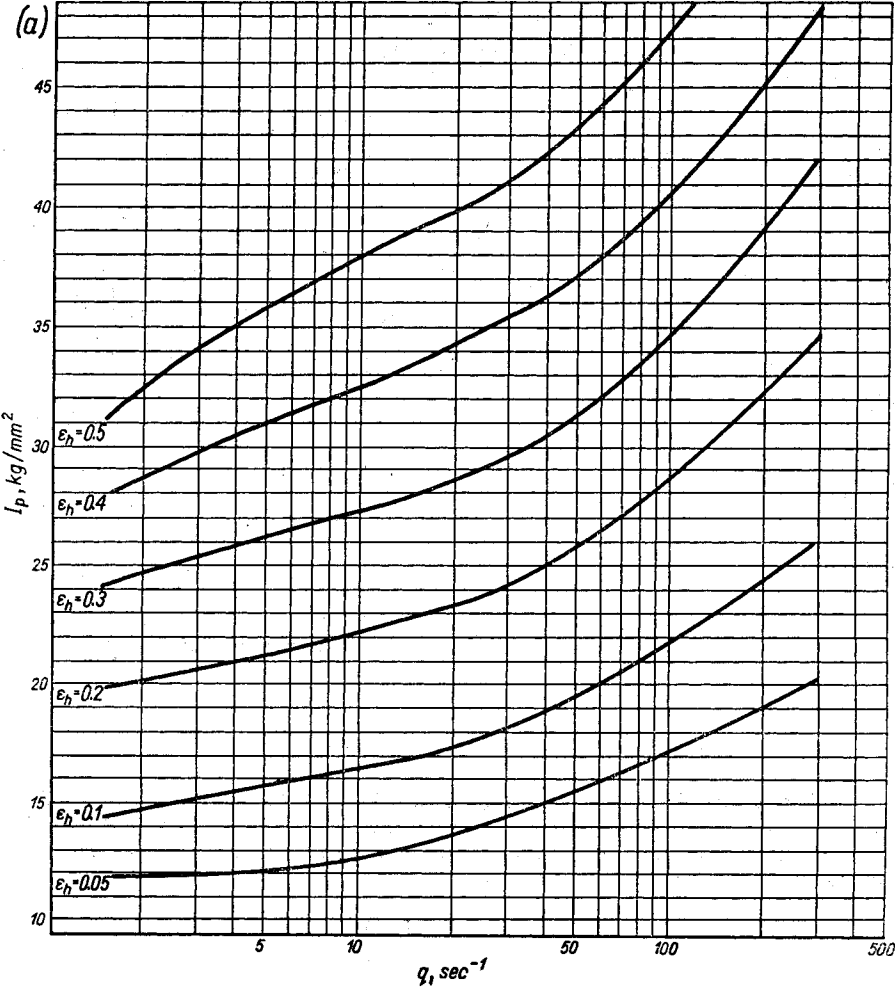


Fig. 4.41.  $I_p$  functions from Cook and McCrum [67] for a low-carbon steel shown in Fig. 2.20a: (a)  $900^\circ\text{C}$

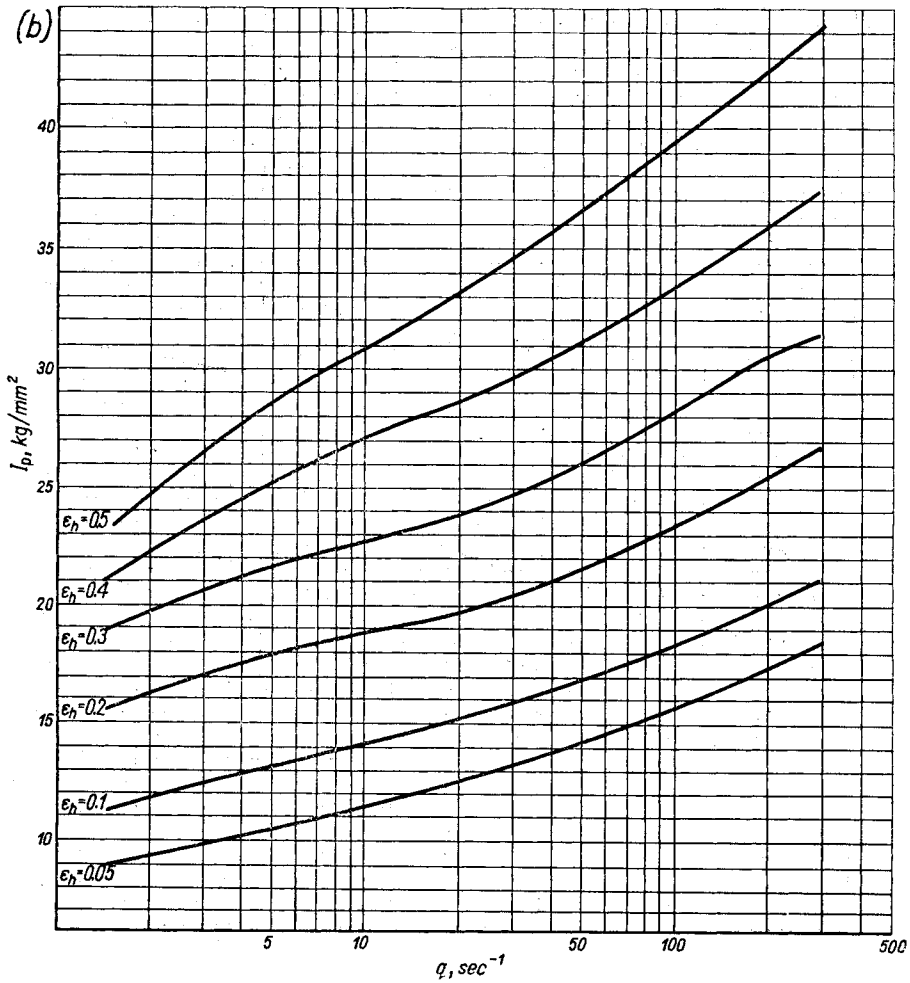


Fig. 4.41 (cont.).  $I_p$  functions from Cook and McCrum [67] for a low-carbon steel shown in Fig. 2.20a: (b) 1000°C

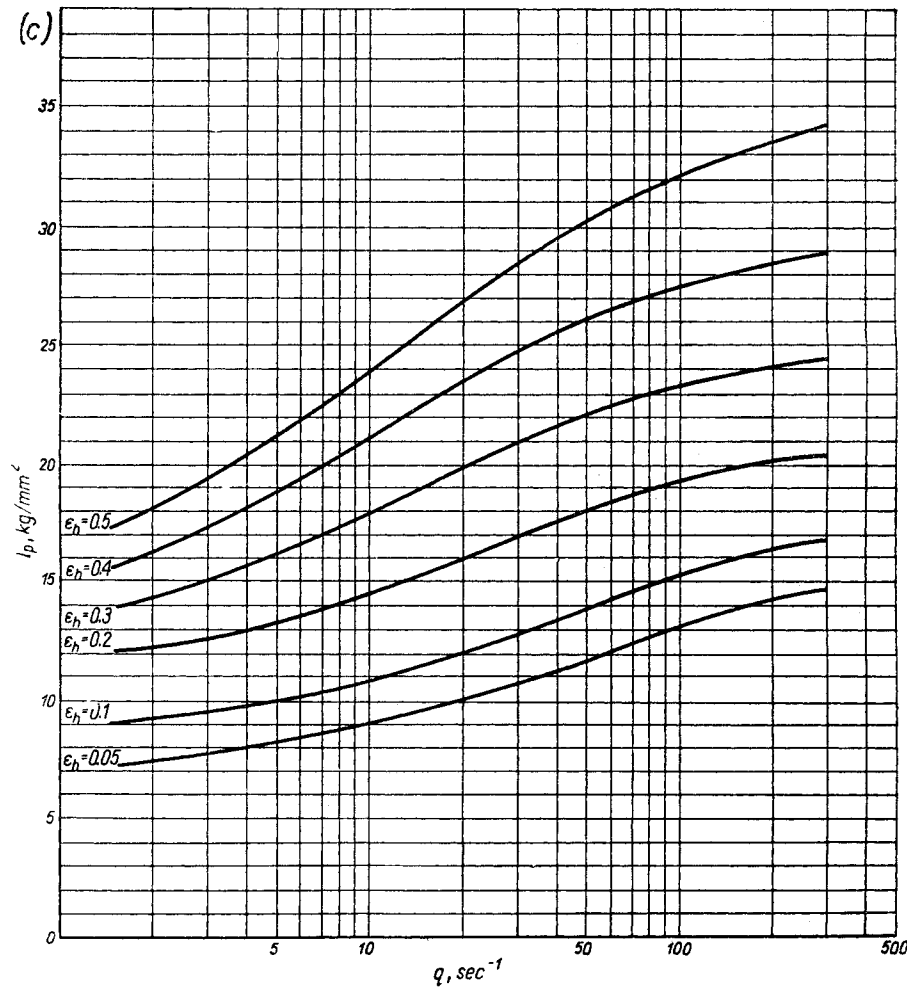


Fig. 4.41 (cont.).  $I_p$  functions from Cook and McCrum [67] for a low-carbon steel shown in Fig. 2.20a: (c)  $1100^\circ\text{C}$



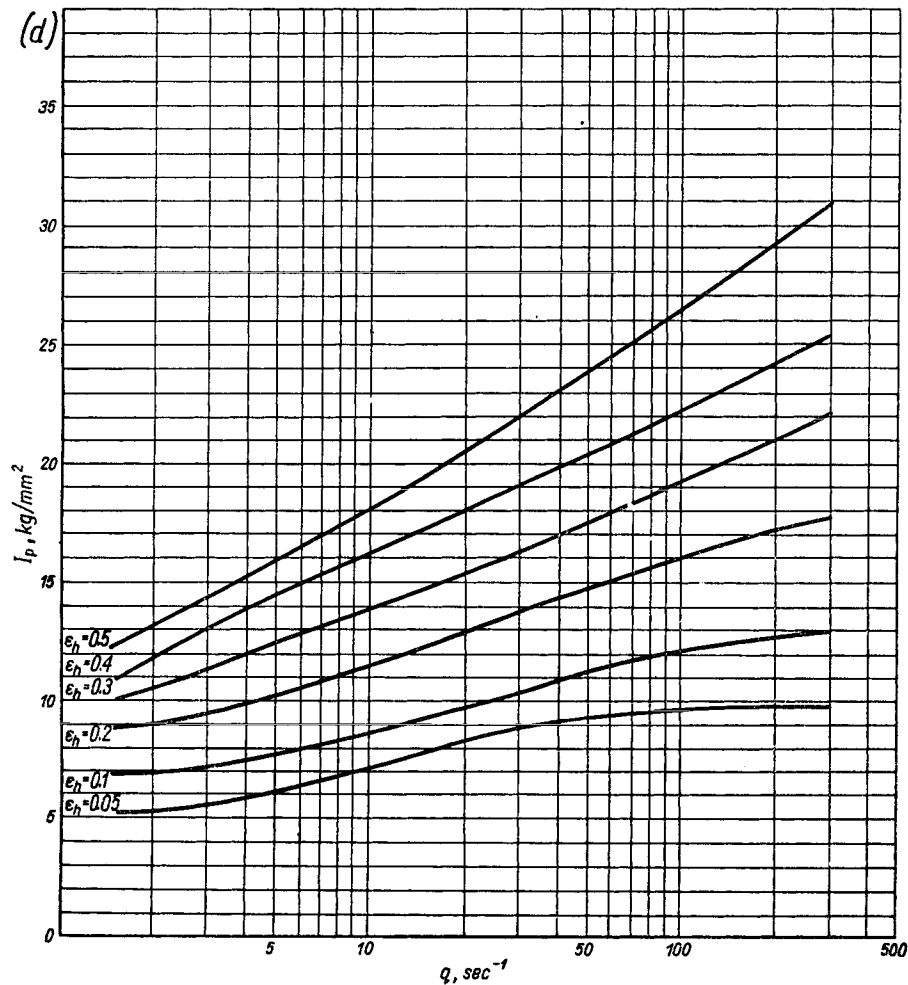


Fig. 4.41 (cont.).  $I_p$  functions from Cook and McCrum [67] for a low-carbon steel shown in Fig. 2.20a: (d) 1200°C

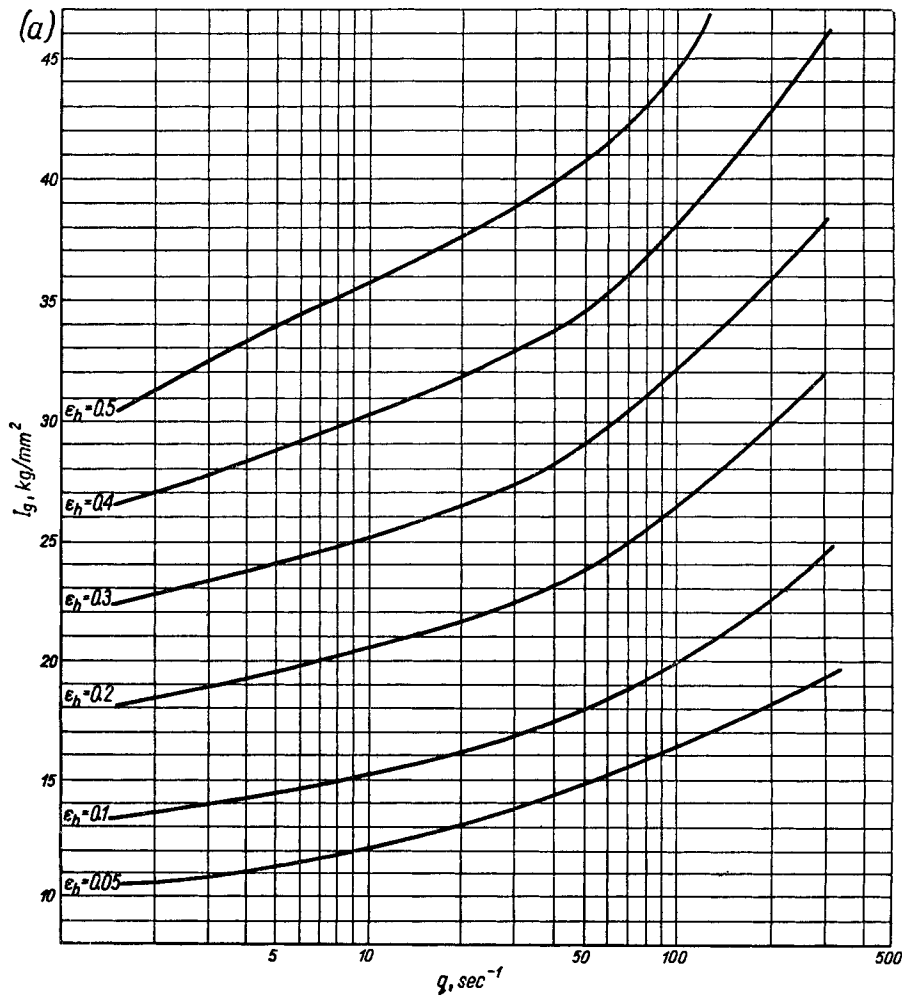


Fig. 4.42.  $I_g$  functions from Cook and McCrum [67] for a low-carbon steel shown in Fig. 2.20a: (a) 900°C

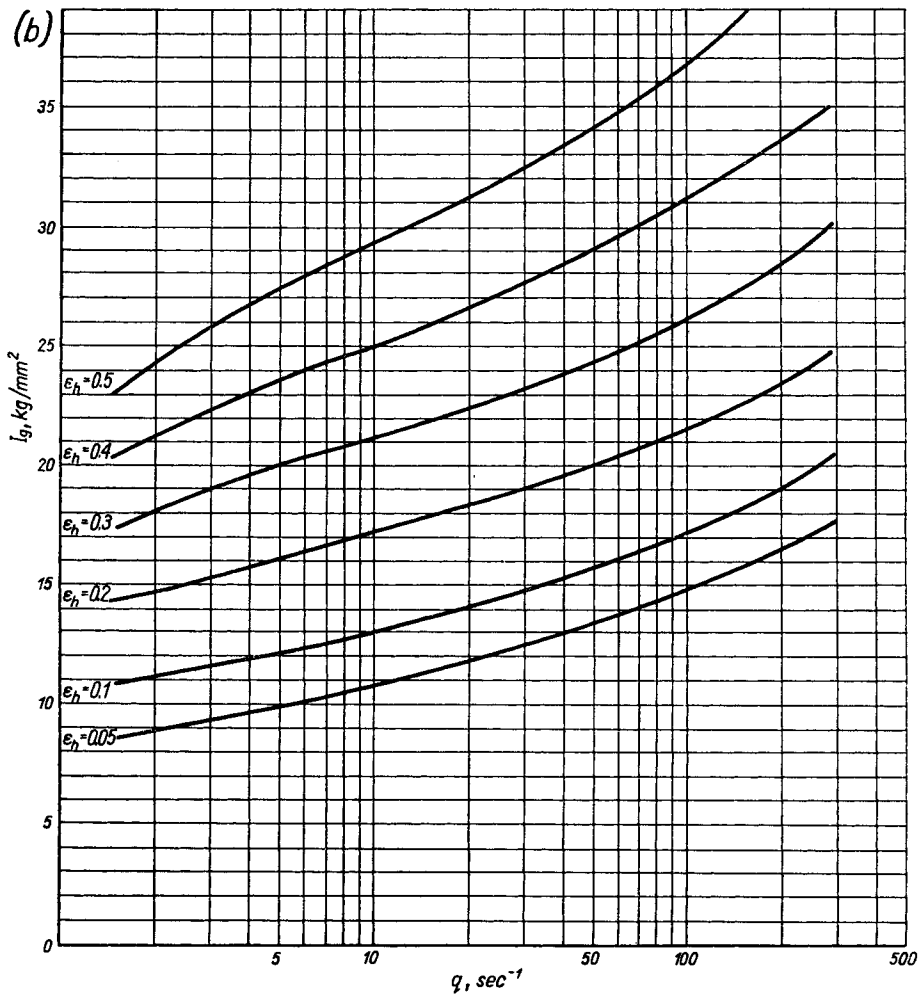


Fig. 4.42 (cont.).  $I_g$  functions from Cook and McCrum [67] for a low-carbon steel shown in Fig. 2.20a: (b)  $1000^\circ\text{C}$

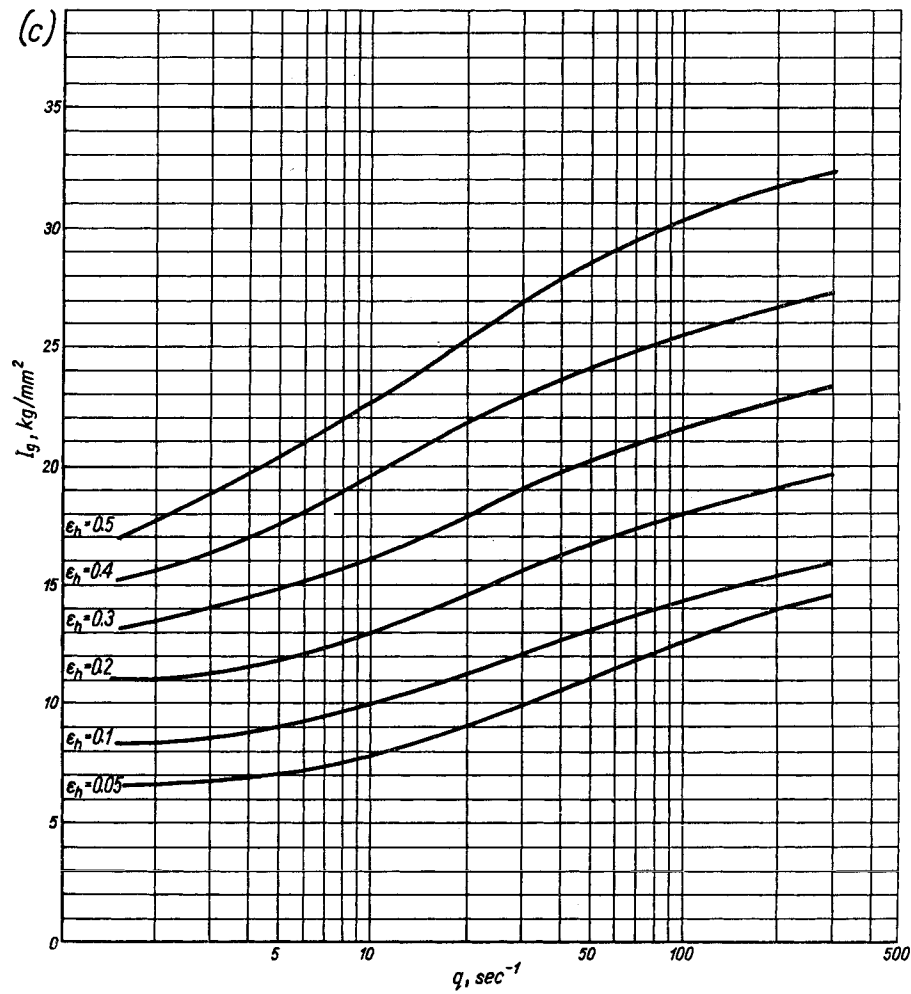


Fig. 4.42 (cont.).  $I_g$  functions from Cook and McCrum [67] for a low-carbon steel shown in Fig. 2.20a: (c)  $1100^\circ\text{C}$

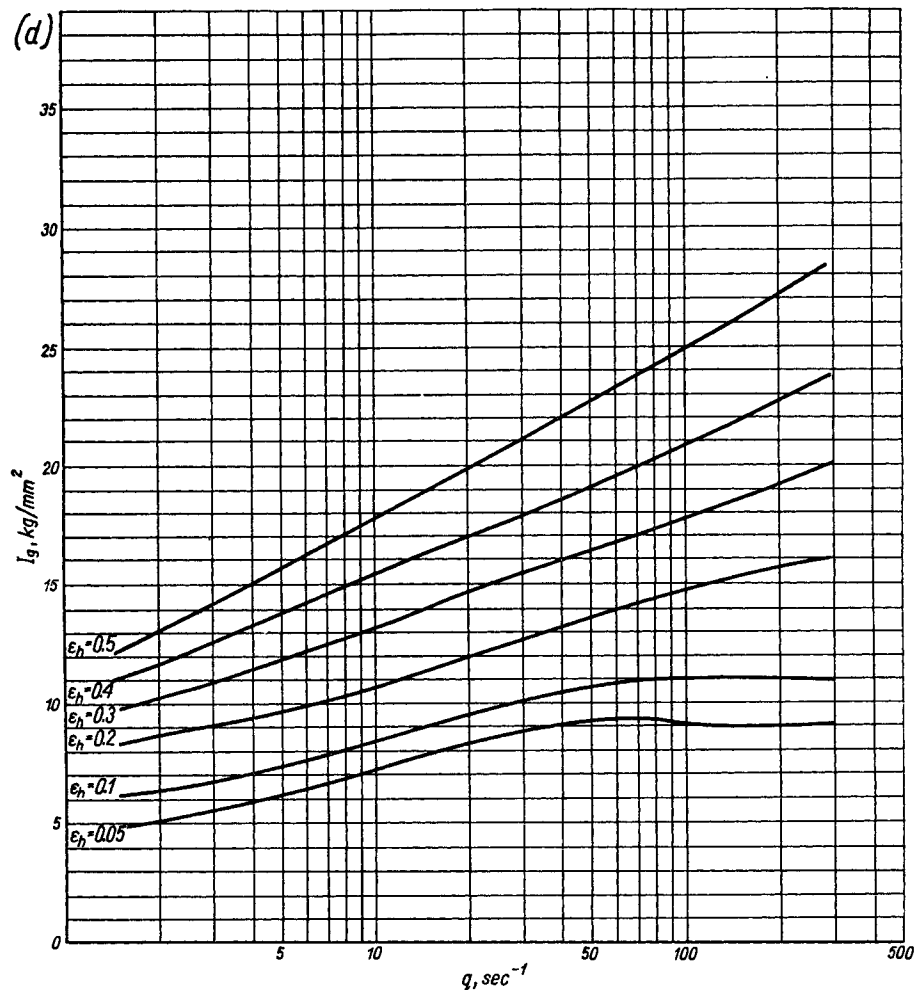


Fig. 4.42 (cont.).  $I_g$  functions from Cook and McCrum [67] for a low-carbon steel shown in Fig. 2.20a: (d)  $1200^\circ\text{C}$

Therefore the total roll force and torque amount to:

$$P = 292 \times 250 = 73,000 \text{ kg} = 73 \text{ tons}$$

and

$$M_w = 3865.6 \times 250 = 966,400 \text{ kgmm} = 0.9664 \text{ tm}$$

4.2.8. SIEBEL'S METHOD OF CALCULATION OF ROLL FORCE [61]

It is known that the resistance to deformation of steel depends on its chemical composition, rolling temperature, and conditions of deformation. None of the formulae discussed up to now provide a method for calculation of resistance to deformation with perfect accuracy, since certain simplifications have been used in each of them. Therefore the most reasonable method seems to be the determination of mean resistance to deformation in rolling, based on results of investigations being exactly equivalent to rolling conditions.

Applying Siebel's method, his diagrams of resistance to deformation (Fig. 4.43) plotted against rolling temperature and thickness ratio  $h_2/D$ , are used. These diagrams may be used only for similar rolling conditions, and in particular it is important, that thickness ratio  $h_2/D$ , relative draught  $\Delta h/h_1$ , and relative speed  $v_r/D$ , should be approximate as given in the diagrams.

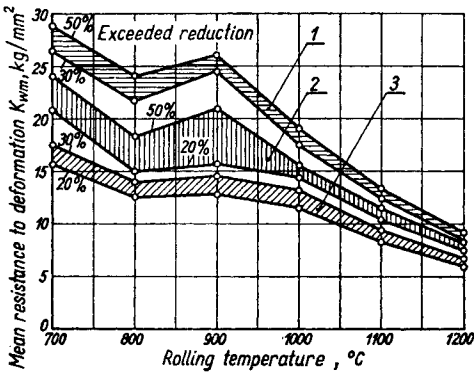


Fig. 4.43. Mean resistance to deformation plotted against rolling temperature and thickness ratio ( $h_2/D$ ) 100% 1—2.8%, 2—5.6%, 3—11.0% [61]

TABLE 4.4  
CHEMICAL ANALYSIS OF STEELS

| Steel | Content, % |      |      |       |       |      |      |      |      |
|-------|------------|------|------|-------|-------|------|------|------|------|
|       | C          | Si   | Mn   | P     | S     | Cr   | Ni   | W    | Al   |
| A     | 0.11       | 0.22 | 0.50 | 0.020 | 0.018 | —    | —    | —    | —    |
| B     | 0.88       | 0.18 | 0.63 | 0.014 | 0.016 | —    | —    | —    | —    |
| C     | 0.06       | 1.19 | 0.29 | 0.010 | 0.002 | 22.5 | 0.14 | —    | 2.25 |
| D     | 0.11       | 0.63 | 0.64 | 0.015 | 0.026 | 18.4 | 9.1  | —    | —    |
| E     | 0.14       | 1.90 | 0.09 | 0.015 | 0.010 | 25.0 | 20.5 | —    | —    |
| F     | 0.47       | 1.98 | 0.85 | 0.015 | 0.010 | 15.4 | 13.1 | 1.95 | —    |

Siebel [61] adapted for his method values of the mean resistance to deformation for six alloy steel qualities, determined by Pomp and Weddige. The chemical analysis of these steel qualities is shown in Table 4.4, and values of mean resistance to deformation in Fig. 4.44, plotted against relative draught and temperature.

The influence of rolling speed is not yet sufficiently investigated, but it is known that greater relative speeds increase the resistance to deformation.

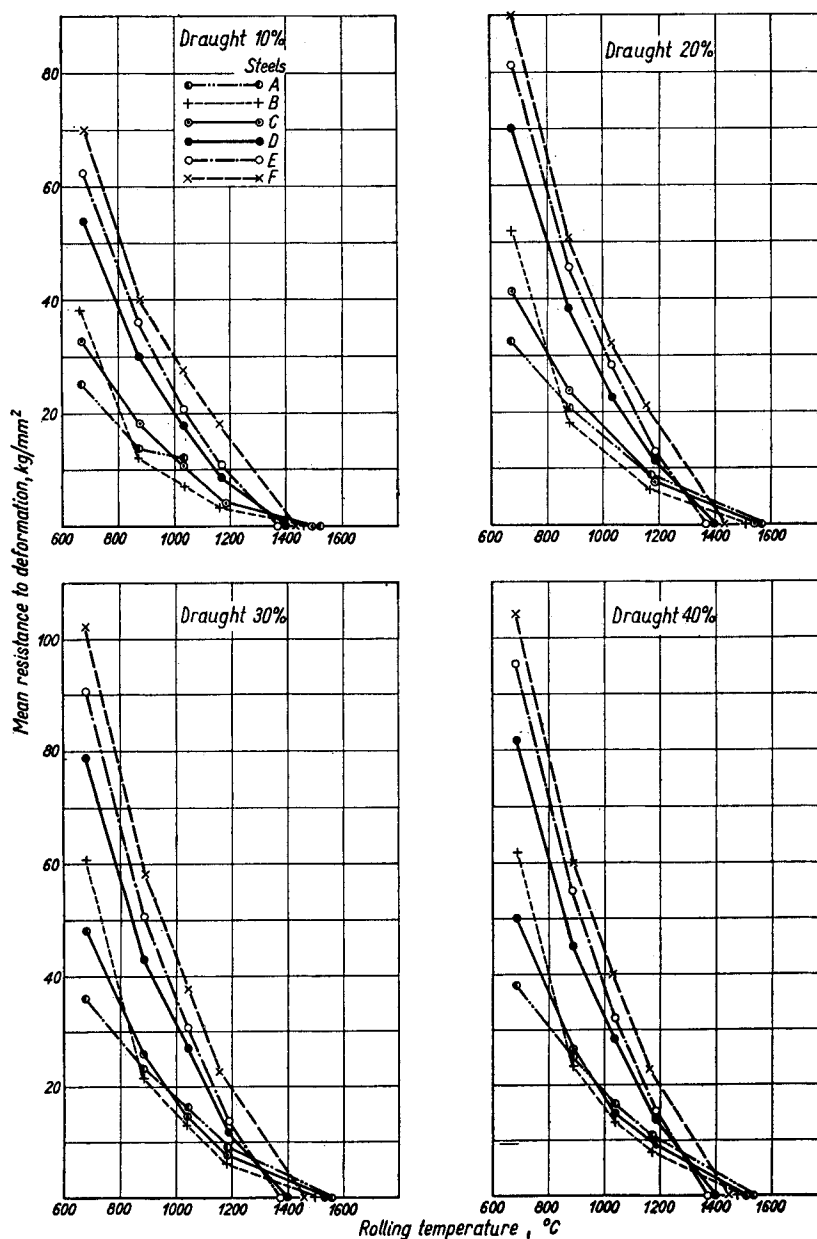


Fig. 4.44. Mean resistance to deformation for alloy steels of a composition shown in Table 4.4, used in Siebel's method, based on measurements of Pomp and Weddige

The value of mean resistance to deformation  $K_{wm}$  read off from Fig. 4.43 should be multiplied by coefficient  $a_1$ , read off from Table 4.5 for the appropriate relative rolling speed  $v_r/D$ .

TABLE 4.5  
VALUES OF COEFFICIENT  $a_1$  PLOTTED AGAINST  
RELATIVE ROLLING SPEED [62]

| Relative rolling speed $v_r/D$<br>m/sec m | Value of<br>$a_1$ |
|-------------------------------------------|-------------------|
| 1- 2                                      | 1.0               |
| 2- 5                                      | 1.1               |
| 5-10                                      | 1.2               |
| 10-20                                     | 1.35              |
| 20-50                                     | 1.5               |

The values of mean resistance to deformation obtained during measurements on plain roll barrels may be utilized also for rolling in shaped passes, taking into account that due to additional friction between stock and sides of the pass the mean resistance

TABLE 4.6  
VALUES OF COEFFICIENT OF ADDITIONAL FRICTION OF PASSES  $a_2$ , IN RELATION TO PASS  
FORM [61] AND SIMILAR VALUES CORRECTED BY INVESTIGATIONS OF G. ZOUHAR [157]

| Pass form                | Coefficient $a_2$ |
|--------------------------|-------------------|
| <i>Before correction</i> |                   |
| Closed passes:           |                   |
| flat                     | 1.2-1.3           |
| for strip                | 1.2-1.3           |
| rough forming pass       | 1.3-1.5           |
| finishing forming pass   | 1.5-2.0           |
| Open passes:             |                   |
| square                   | 1.1-1.2           |
| gothic pass              | 1.2-1.3           |
| diamond                  | 1.2-1.3           |
| oval from square         | 1.2-1.3           |
| square from oval         | 1.2-1.3           |
| rough forming pass       | 1.2-1.5           |
| <i>After correction</i>  |                   |
| Closed passes            | as above          |
| Open passes:             |                   |
| square                   | 1.1               |
| gothic pass              | 1.2-1.3           |
| diamond into square      | 1.1               |
| square into diamond      | 1.1-1.2           |
| square into oval         | 1.2-1.3           |
| oval into square         | 1.1-1.2           |
| round into oval          | 1.2-1.3           |
| oval into round          | 1.1-1.2           |
| rough forming pass       | 1.2-1.5           |
| diamond                  | 1.1               |



to deformation increases about 10 to 50% for open passes, and 20 to 100% when rolling in closed passes. The frictional losses given by Siebel are shown as a coefficient  $a_2$  in Table 4.6.

Therefore the mean resistance to deformation according to Siebel may be obtained from formula

$$K_{wm} = K_w a_1 a_2 \quad (4.60)$$

Figure 4.45 shows the mean resistance to deformation for rolling of different sections, such as channels, beams, angles, obtained by Siebel and Lueg, based on measurements by Puppe. The field of dispersion between the two curves is valid for low-carbon steel, rolling temperature between 1050 and 1200°C, and various values of thickness ratio  $h_{2m}/D_m$ . For practical purposes it is necessary to determine the percentage ratio

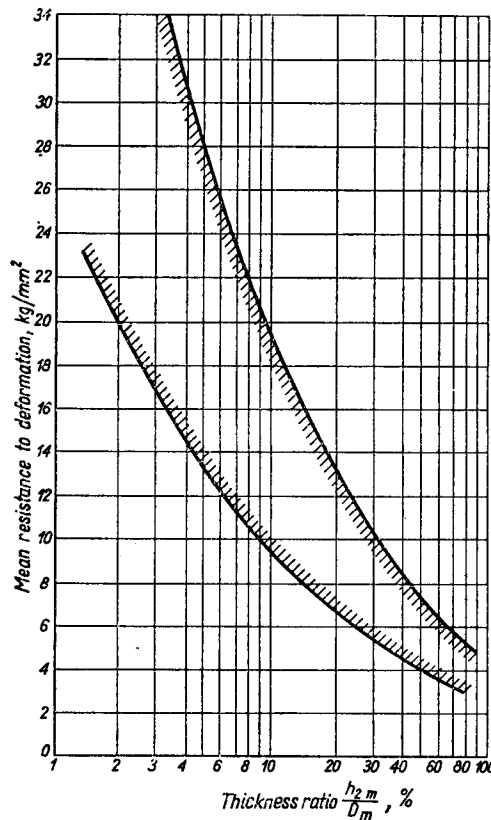


Fig. 4.45. Curves of mean resistance to deformation plotted against thickness ratio ( $h_{2m}/D_m$ ) 100% [61]

$h_{2m}/D_m$  for a given pass, and then read off the value of  $K_{wm}$ . For steels of greater hardness and those rolled at lower temperatures, and also if spreading is greatly restricted, values from the upper curve should be chosen. For opposite conditions values from the lower curve should be chosen and for intermediate conditions intermediate values of mean resistance to deformation.

Siebel confirmed furthermore that values of  $K_{wm}$  determined from Fig. 4.45 may be applied also to blooming mills, i.e. to box passes.

Since there are numerous formulae for calculating the roll force in hot rolling, therefore it becomes necessary to compare results obtained from various methods of calculation with those measured practically. The author together with S. Bala compared the methods of Ekelund, Siebel, Trinks, Orowan-Pascoe, Geleji, and Tselikov [68].

Based upon the results obtained in the work [68], the best methods have been chosen, namely methods of Ekelund and Siebel for rolling of plates, of Ekelund, Siebel and Tselikov for rolling of sheet, and Ekelund, Siebel and Orowan-Pascoe for rolling in shaped passes.

Similarly, methods for calculating the roll force derived by Soviet authors [70] have been analysed, namely three formulae by Tselikov I, II and III, as also those by Gubkin, Golovin-Tiagunov, Golovin-Shvejkin, and Samarin.

This analysis showed that there are only inessential differences between Tselikov's formulae I and III, and therefore for comparison Tselikov's formula III was chosen. The best agreement with measured values has been obtained for the methods of Golovin-Tiagunov and Samarin.

It then becomes necessary to compare these best methods as selected on the basis of [68, 70]. It is clear that particular methods may be compared only for the same conditions [68], i.e. results of measurements published by Tiagunov for rolling of plates  $7 \times 1400 \times 9000$  mm in 26 passes,  $0.35 \times 750 \times 1640$  mm sheets in 19 passes,  $2 \times 685 \times 1400$  mm sheets in 10 passes, furthermore the results obtained by Puppe for 2550 kg ingots on a blooming mill, omitted by Siebel for a light-section rolling mill, and by SKF for a wire and rod mill [68].

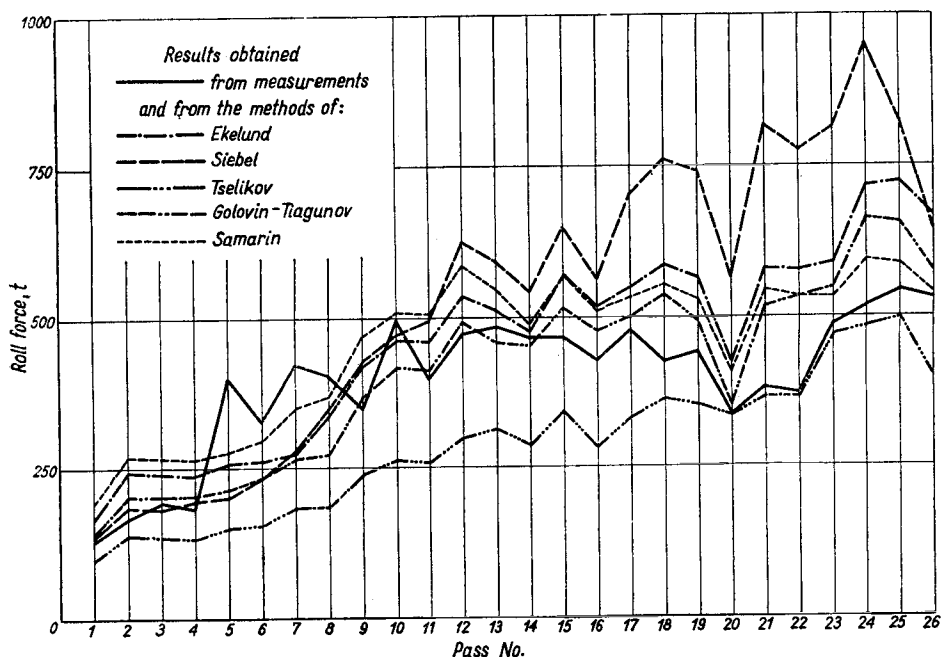


Fig. 4.46. Comparison of measured and calculated roll forces arising during rolling of  $7 \times 1400 \times 9600$  mm plates

The scale for roll force has been enlarged for all diagrams [69], since the scales used in previous investigations [68, 70] do not sufficiently show up the differences between roll force values obtained.

Comparing the calculated results shown in Fig. 4.46 with those measured for rolling of plates, it may be confirmed that with Siebel's method too high, with Tselikov's too small values are obtained. Therefore these methods are unsuitable for calculation of roll force. Also values obtained with Samarin's method are mostly too large, except in passes No. 5 and 8. Roll forces calculated by this method are greater than those obtained by Ekelund and Golovin-Tiagunov, except in the last few passes. Therefore for rolling of plates the best agreement with true conditions is obtained by means of the formulae of Ekelund and Golovin-Tiagunov. However this comparison is insufficient to determine the universally best method of calculation.

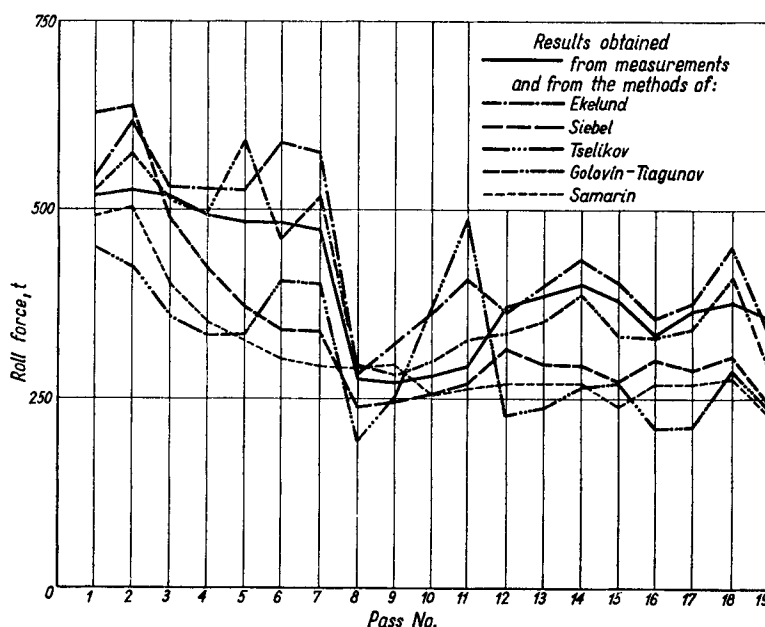


Fig. 4.47. Comparison of measured and calculated roll forces arising during rolling of  $0.33 \times 750 \times 1640$  mm sheets

Figure 4.47 shows the comparison of measured and calculated results for rolling of sheets. Values obtained for roll forces calculated according to Samarin, Siebel and Tselikov are mostly too small. It is difficult, however, to decide between the two remaining formulae, i.e. Golovin-Tiagunov and Ekelund. The Golovin-Tiagunov method seems to be rather more suitable for the case considered.

It may be seen from curves shown in Fig. 4.48 that for the case considered the methods of Samarin and Siebel also give roll forces that are too small. The Golovin-Tiagunov method also gives too small values in this case, as opposed to values shown in Fig. 4.47. Therefore there remain the methods of Tselikov and Ekelund, of which the second seems to give closer agreement with measured values.

Comparing finally the results shown in Fig. 4.49 for rolling in shaped passes, where the additional method of Orowan-Pascoe is included, from Fig. 4.49a it appears that

for rolling on blooming mills such small values are obtained from the Tselikov, Ekelund, and Golovin-Tiagunov methods, that they cannot be recommended for calculation of roll forces. Better agreement has been obtained with the formulae of Orowan-Pascoe and Samarin, though these values are also too small. Only the Siebel method gives values corresponding to the true ones for the case considered.

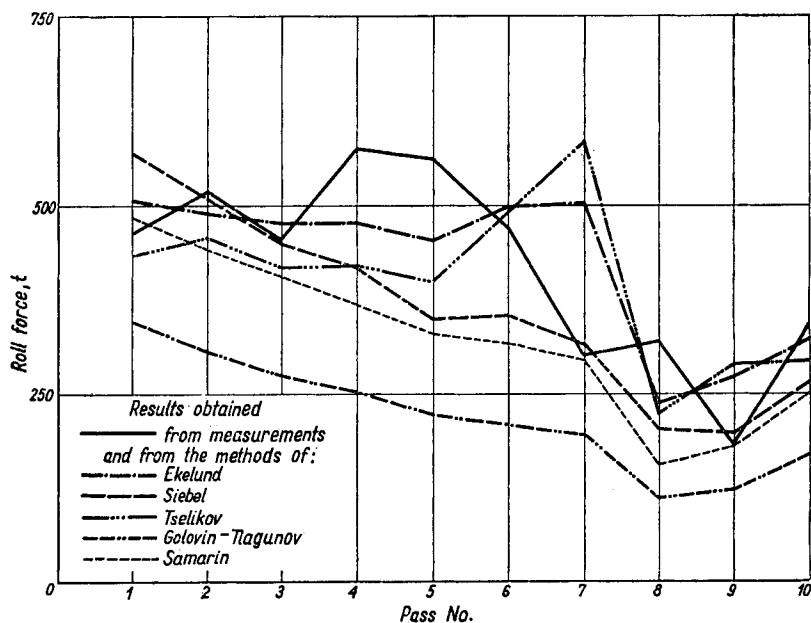


Fig. 4.48. Comparison of measured and calculated roll forces arising during rolling of  $2 \times 625 \times 1400$  mm sheets

Figure 4.49b shows that for rolling mills of heavy products values considerably too low are obtained from formulae of Tselikov, Ekelund, Orowan-Pascoe. Better results are obtained from formulae of Golovin-Tiagunov, and Samarin, though they are also too low in comparison with the measured ones. For this case also the most suitable method seems to be that of Siebel.

Figure 4.49c shows that for a small rolling stand too low roll forces are obtained from the formulae of Tselikov, Ekelund, and Samarin. Better agreement is obtained for formulae of Orowan-Pascoe, and Golovin-Tiagunov; however, these values also are too low. For this case again the best results have been obtained from Siebel's method.

Investigations up to now confirm that for calculation of roll forces arising during rolling in shaped passes, for big and small stands, only Siebel's method leads to useful results. This is due to the fact that only this method takes into account the influence of additional friction between stock and groove sides (Table 4.6).

From Fig. 4.49d it may be seen that roll forces calculated according to Tselikov and Samarin are too low, and those of Golovin-Tiagunov, Ekelund and Siebel are too high. Hence only the Orowan-Pascoe method remains, though this also gives too low roll forces and it is necessary to apply certain corrections.

It has been shown that in section rolling only Siebel's method gives results close to the measured ones, due to taking into account the additional friction between stock

and groove side, and other methods give too low values. Therefore the author tried to take into account the influence of additional friction, in the manner of Siebel, for those methods where specially low values had been obtained (Table 4.6). Roll forces recalculated on this way, i.e. a. according to Ekelund-Siebel, b. according to Orowan-Pascoe-Siebel, c. according to Golovin-Tiagunov-Siebel, and d. according to Samarin-Siebel, are plotted in Fig. 4.49.

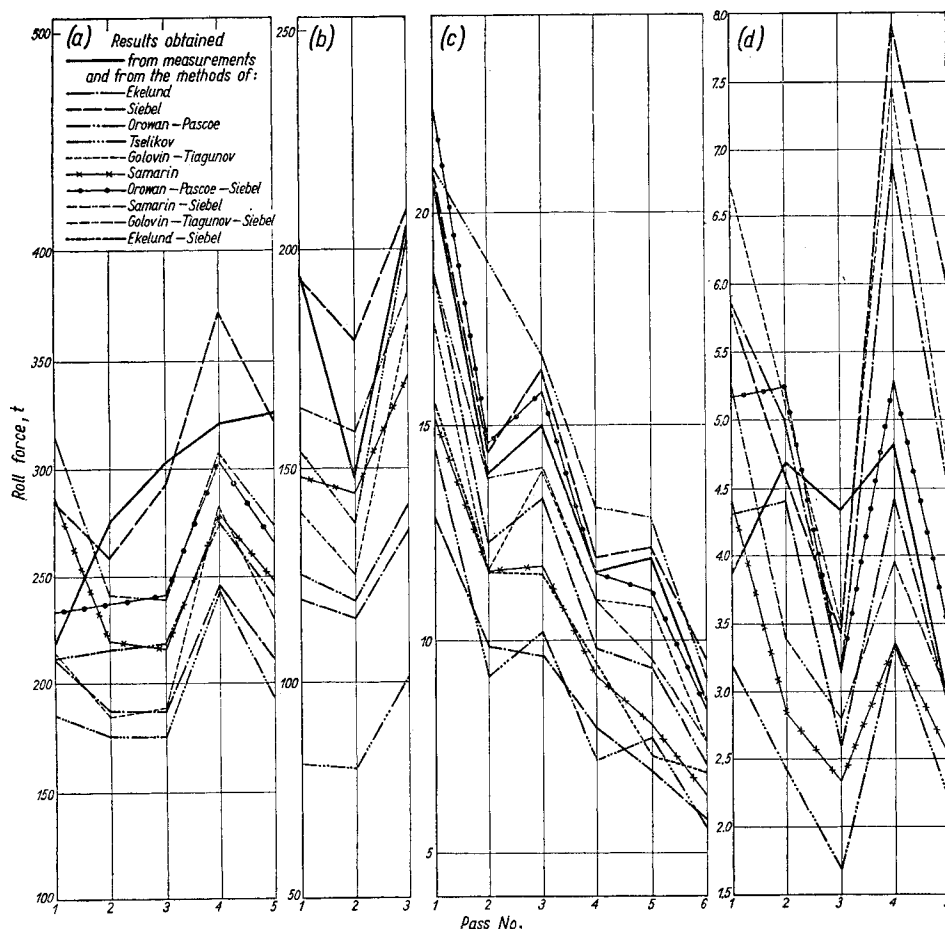


Fig. 4.49. Comparison of measured and calculated roll forces arising during rolling on: (a) a blooming mill (b) a plate mill (c) a small stand, (d) a rod and bar mill

This method results in a considerable improvement of calculated values, though not all formulae are suitable for practical application. For rolling on blooming mills the formulae of Orowan-Pascoe-Siebel, and Samarin-Siebel, are recommended (Fig. 4.49a), although slightly too small values are obtained; and methods of Golovin-Tiagunov-Siebel, and Ekelund-Siebel, have been omitted due to considerably too low results.

Comparison of roll forces shown in Fig. 4.49b allow us to confirm that for plate mills good agreement with measured values is obtained with the Samarin-Siebel formula.

Roll forces calculated from the formulae of Golovin-Tiagunov-Siebel are slightly too low, whereas those of Ekelund-Siebel and Orowan-Pascoe-Siebel are unsuitable, giving values considerably too small.

For small rolling stands as in Fig. 4.49c, formulae of Golovin-Tiagunov-Siebel, and Orowan-Pascoe-Siebel give very good results. Useful but slightly too low results are obtained from the Samarin-Siebel method. Considerably too low values are obtained once more with the Ekelund-Siebel formula. For rolling conditions shown in Fig. 4.49d the Orowan-Pascoe-Siebel method is suitable, the Samarin-Siebel method gives roll forces which are slightly lower than measured ones. The Golovin-Tiagunov-Siebel and Ekelund-Siebel methods however, give too great roll forces.

Also for rolling in shaped passes for each set of conditions there are several methods of calculation, which agree sufficiently well with measured results, and which may be successfully recommended for practical purposes.

In this way it is possible to overcome difficulties immediately connected with the use of Siebel's method, i.e. the lack of diagrams for different materials, and the difficulty of extra- or interpolating values from diagrams, if rolling conditions are different from those shown on curves.

To give values independent of the projected area of contact, the mean resistance to deformation has been plotted against draught and constant temperatures in °C, i.e. as  $K_{wm} = P/F_d$ .

Measurements have been carried out in established rolling conditions, using the draughts from 5 up to 90 per cent, rolling speed of 0.2 m/sec, roll diameter of 330 mm, to enable a better presentation, and a critical estimation of the problems considered. The chosen temperature range was 900, 1000, 1100, and 1200°C.

The same shapes of specimen have been rolled, i.e. 40 mm square with  $\delta_w = 1$  ( $\delta_w$ —form factor), 100×20 mm flat with  $\delta_w = 5$ . For flats with  $\delta_w = 10$  only a few specimens have been rolled at 1100°C.

After initial evaluation of different methods of calculation, for the final analysis as the best ones two Soviet methods of Tselikov and Golovin-Tiagunov, the Swedish of Ekelund, the Hungarian of Geleji (from 1957), and the English one of Orowan-Pascoe, were chosen.

Applying formulae of Orowan-Pascoe to our calculations, values of yield stress from the original publication of Orowan-Pascoe were used. These values come from earlier measurements of Pomp and Lueg, carried out before the last war.

Also Orowan's formulae have been taken into consideration, which were solved by Sims, and presented in the form of graphs by Cook and McCrum.

Figure 4.50 shows values for measured mean resistance to deformation for draughts varying between 10 and 90% during rolling of 40×40 mm square specimens of  $\delta_w = 1$ , at 900°C. It may be confirmed from this figure that curves of measured values run below those of calculated data for percentage draughts up to 70%. For the range of extremely high percentage draughts only three methods, namely: of Ekelund, Orowan-Pascoe and Golovin-Tiagunov, give results lower than calculated ones.

For the range of percentage draughts up to 75%, two methods, i.e. those of Golovin-Tiagunov and Tselikov, give slightly too high values, which may be accepted for calculation. The Orowan-Pascoe and Ekelund methods give considerably too high values, though results approach measured values for extremely high percentage draughts. Curves for values calculated from formulae of Cook-McCrum and Geleji (1957) run too high throughout all the range investigated.

Figure 4.51 shows values of mean resistance to deformation plotted against percentage draught for a range from 20 up to 80%, for a  $100 \times 20$  mm flat specimen of  $\delta_w = 5$ , which was rolled at  $900^\circ\text{C}$ . It may be seen from this figure that values obtained from Tselikov's formula are close to measured ones, throughout the whole range of draughts. Curves calculated from the Golovin-Tiagunov formula run too low throughout the whole range of draughts, whereas values obtained from Ekelund's formula are slightly too high for the range up to about 50% draught, and considerably too low for extremely high draughts. Curves calculated according to Orowan-Pascoe, Cook-McCrum and Geleji (1957) run too high throughout the whole range of draughts investigated.

Figure 4.52 shows measured values of mean resistance to deformation for square specimens of  $\delta_w = 1$ , rolled at  $1000^\circ\text{C}$ . It may be seen from this figure that throughout the whole range of draughts the calculated values run above the measured ones. The closest approximation to the measured curves are curves calculated from the Orowan-Pascoe, and Golovin-Tiagunov formulae, but those of Tselikov and Ekelund show considerable deviations. The Cook-McCrum and Geleji (1957) methods give very high differences throughout the whole considered range of draughts.

Figure 4.53 shows values of mean resistance to deformation plotted against percentage draught from 10% up to 80% for flat specimens of  $\delta_w = 5$ , rolled at  $1000^\circ\text{C}$ . It may be seen from this figure that values calculated from Golovin-Tiagunov and Tselikov seem to approach to measured ones. In this case Ekelund's curve gives considerable differences for draughts up to about 60%, and then too low values, in comparison with measured ones. Curves calculated according to Orowan-Pascoe, Cook-McCrum and Geleji (1957) run considerably too high above the measured ones.

Figure 4.54 shows values of measured mean resistance to deformation plotted against percentage draught for a range of 10% up to 90%, for  $40 \times 40$  mm square specimens of  $\delta_w = 1$ , rolled at  $1100^\circ\text{C}$ . It may be seen from this figure that these curves give the lowest values of all the calculated values.

Relatively small differences were obtained for formulae of Orowan-Pascoe, Tselikov and Ekelund. Large differences in comparison with measured values, are obtained from the Golovin-Tiagunov, Geleji (1957), and Cook-McCrum formulae. This figure demonstrates that there are very high differences between calculated and measured values during rolling of square specimens, and calculated values are always too high.

The following Fig. 4.55 shows values of measured mean resistance to deformation plotted against percentage draught for a range up to 90%, for  $100 \times 20$  mm flat specimens of  $\delta_w = 5$ , rolled at  $1100^\circ\text{C}$ . It may be seen from this figure that calculated and measured values approach each other. For the range of small draughts best results are obtained from formulae of Tselikov, Golovin-Tiagunov and Ekelund, though these values are slightly too high. For high draughts, too low values are obtained from these same methods. It was interesting to note that in this case the Orowan-Pascoe, Cook-McCrum and Geleji methods give the worst results. Comparing directly Fig. 4.54 and 4.55 it can be seen that for square and narrow flat specimens the effect of form factor has not been taken into account.

Better agreement occurs for specimens of  $\delta_w = 5$  or greater (for form factors the mean values should be substituted).

Throughout this range the nearest agreement between measured and calculated values is obtained.

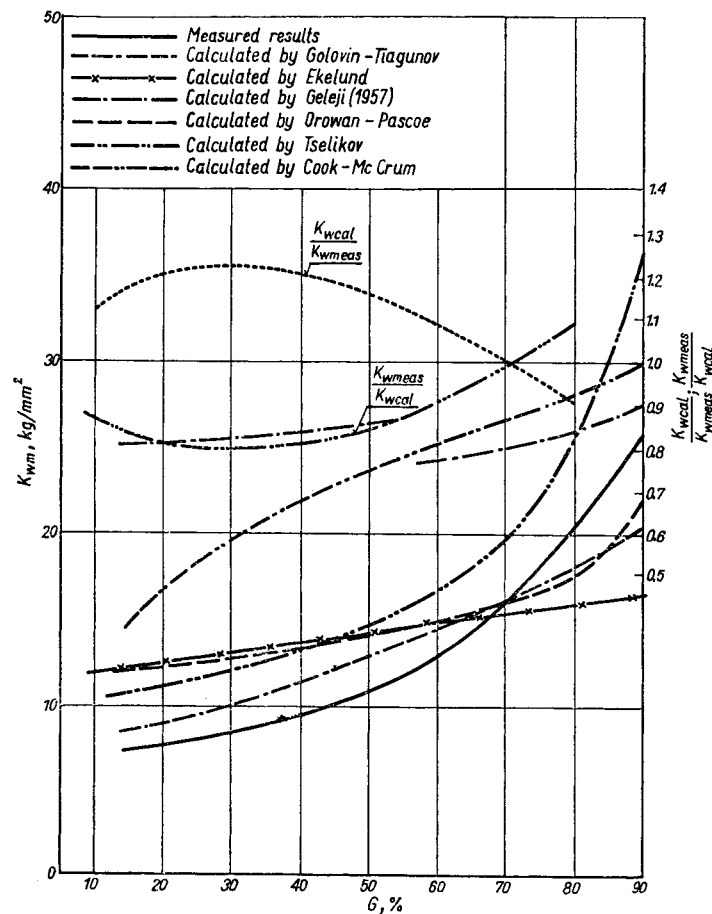


Fig. 4.50. Comparison of measured and calculated mean resistance to deformation  $K_{wm}$  for a 40 mm square at 900°C

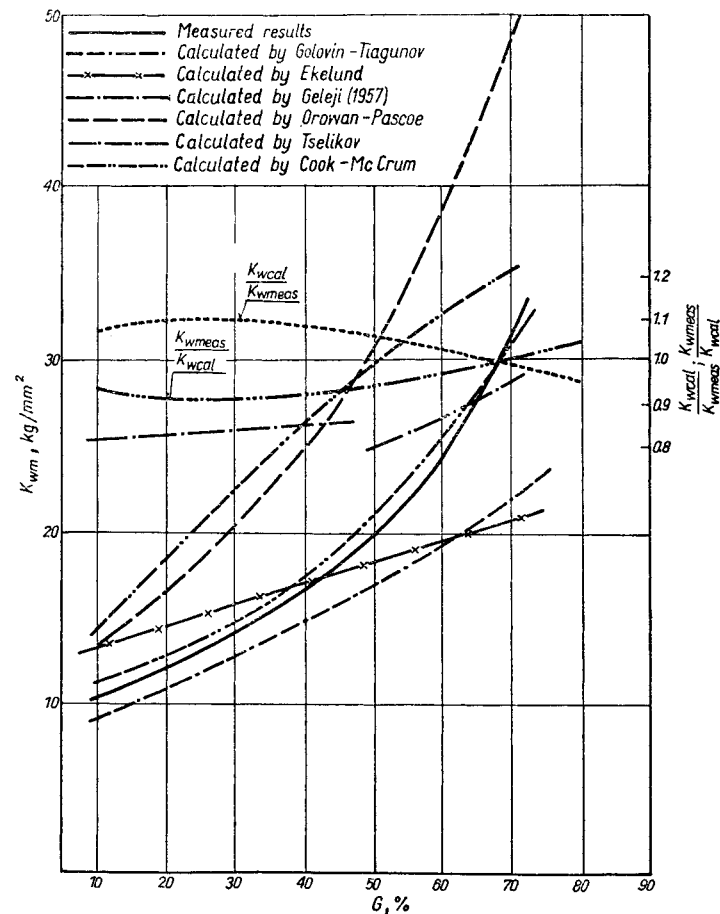


Fig. 4.51. Comparison of measured and calculated mean resistance to deformation  $K_{wm}$  for a 100 × 20 mm specimen at 900°C



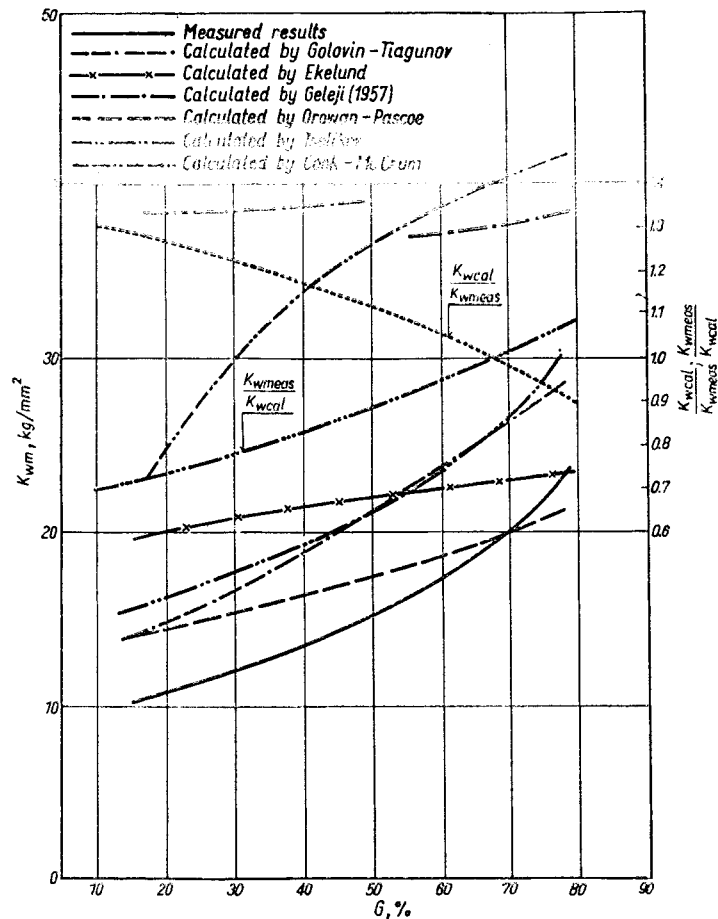


Fig. 4.52. Comparison of experimental and calculated mean resistance to deformation  $K_{wm}$  for a 40 mm square specimen, rolled at 1000°C

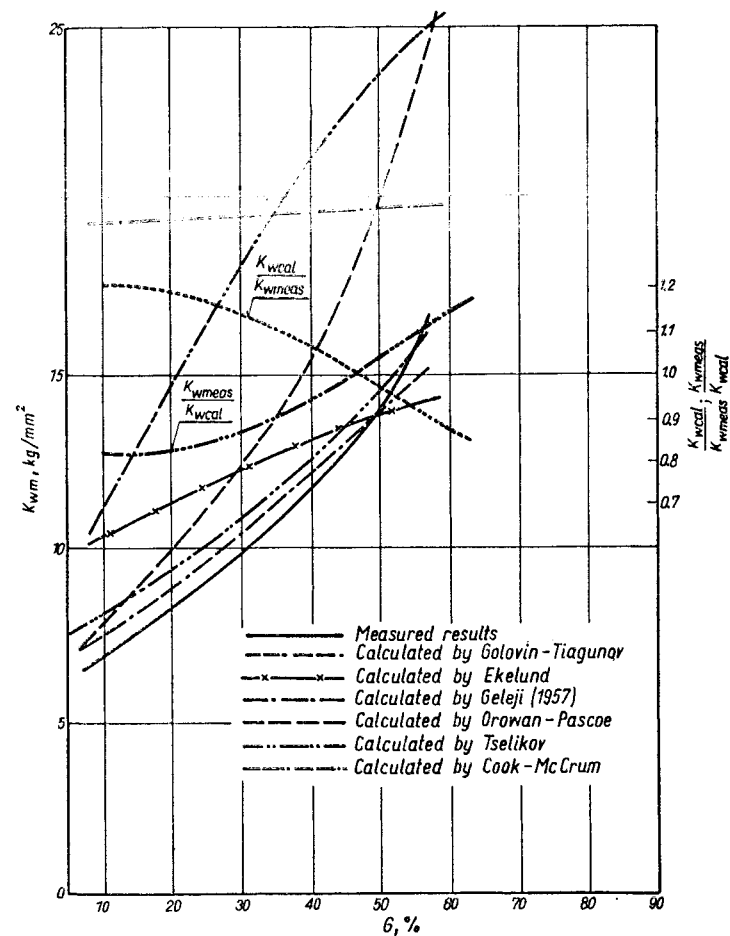


Fig. 4.53. Comparison of experimental and calculated mean resistance to deformation  $K_{wm}$  for a 100×20 mm flat specimen, rolled at 1000°C

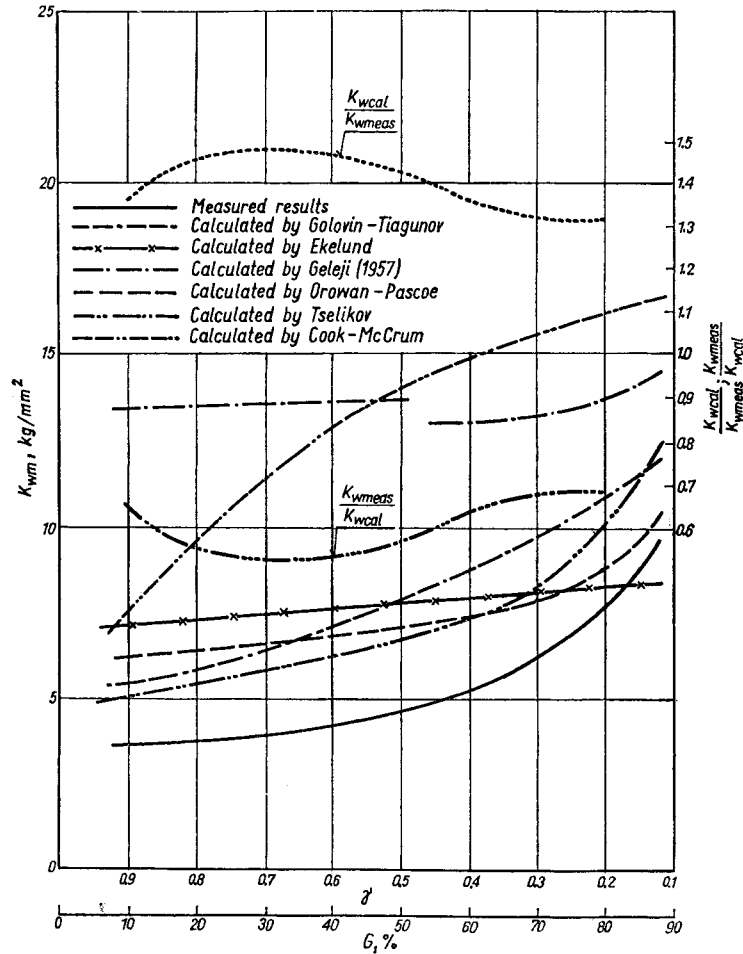


Fig. 4.54. Comparison of experimental and calculated mean resistance to deformation  $K_{wm}$  for a 40 mm square specimen, rolled at 1100°C

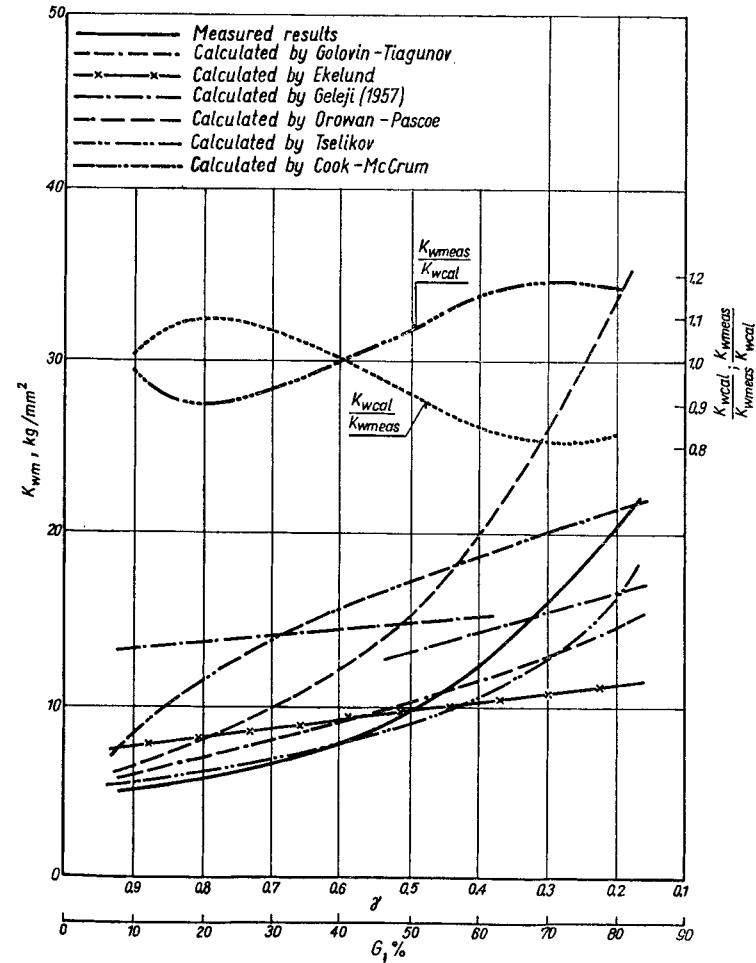


Fig. 4.55. Comparison of experimental and calculated mean resistance to deformation  $K_{wm}$  for a 100×20 mm flat specimen, rolled at 1100°C

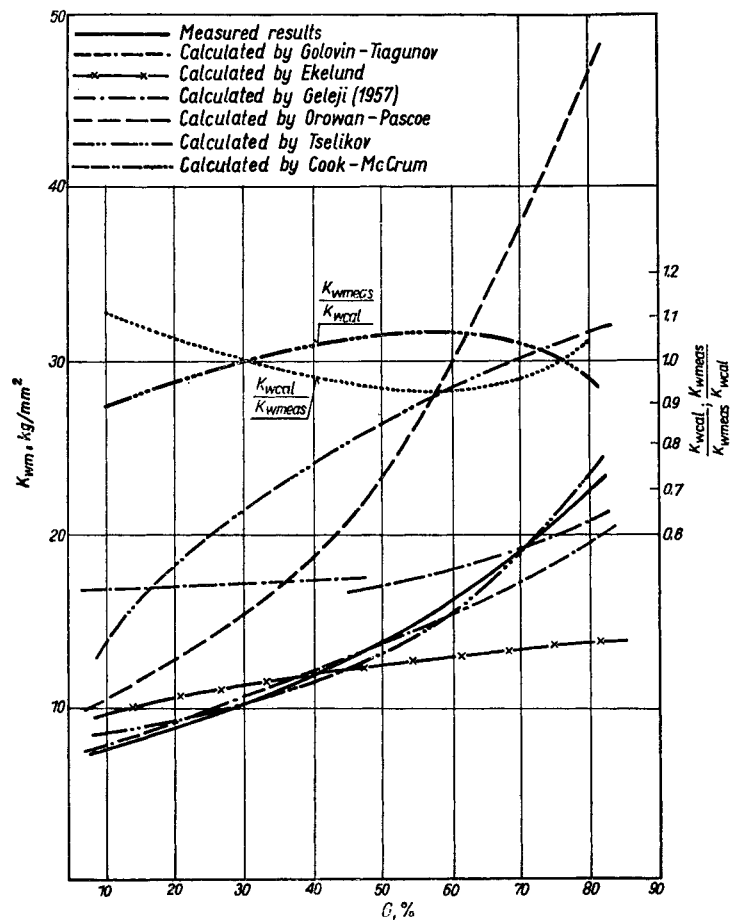


Fig. 4.56. Comparison of experimental and calculated mean resistance to deformation  $K_{wm}$  for a 40 mm square specimen, rolled at 1200°C

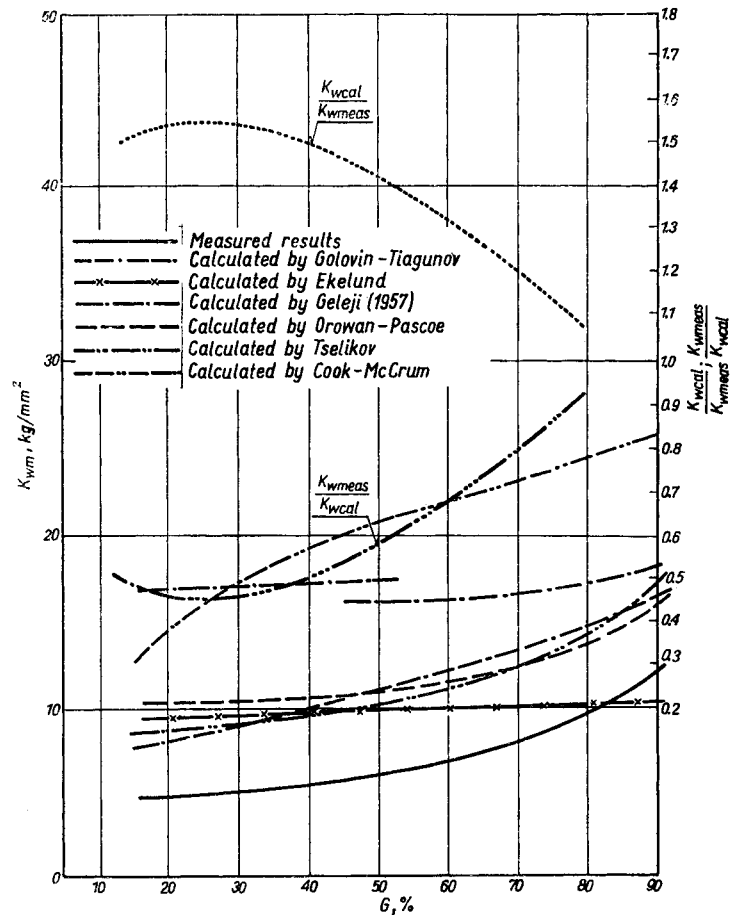


Fig. 4.57. Comparison of experimental and calculated mean resistance to deformation  $K_{wm}$  for a 100x20 mm flat specimen, rolled at 1200°C

Figure 4.56 shows values of mean resistance to deformation plotted against percentage draught for a range of 15 to 90%, measured for square specimens rolled at 1200°C. All curves for calculated values run above the measured ones. The methods of Tselikov, Golovin-Tiagunov, Ekelund and Orowan-Pascoe give closer agreement with measured values, than those calculated from formulae of Geleji (1957) and Cook-McCrum, which are too high throughout the whole range of draughts considered.

The following Fig. 4.57 shows values of mean resistance to deformation plotted against percentage draught for a range of 10 to 80%, for flat specimens of  $\delta_w = 5$ , rolled at 1200°C. For values calculated according to Tselikov, Golovin-Tiagunov a very close approximation to measured results is shown, though for small draughts the values are slightly too low.

Curves calculated from Cook-McCrum, Geleji (1957) and Orowan-Pascoe run too high above the measured one, throughout the whole range of draughts considered.

When calculating the mean resistance to deformation and roll separating force for small squares, it is necessary to decrease the calculated result so that values approximate to measured ones, throughout the whole range of draughts considered. Measured values are always lower than any of the calculated ones. The nearest agreement is obtained from formulae of Tselikov, and Orowan-Pascoe throughout the whole range of draughts considered. This fact is demonstrated by ratios of  $K_{w\text{ cal}}/K_{w\text{ meas}}$  or their reciprocal  $K_{w\text{ meas}}/K_{w\text{ cal}}$ , plotted in Figs. 4.50, 4.52, 4.54, and 4.56. The plotted curves relate always to the best method for the considered temperature. It may be confirmed from all these figures that the differences between the plotted ratios are very great and increase with increase of temperature, from 900°C in Fig. 4.50 up to 1200°C in Fig. 4.56.

When calculating for flat specimens with form factors of  $\delta_w = 5$  or greater, calculated values approach those obtained by measuring. The best results have been obtained from the Tselikov, Golovin-Tiagunov and Ekelund formulae. Up to about 50% draught values slightly too high relative to measured ones are obtained, however in the range of extremely high draughts calculated values are considerably lower than measured ones.

Similarly, the ratios of  $K_{w\text{ cal}}/K_{w\text{ meas}}$ , and their reciprocal  $K_{w\text{ meas}}/K_{w\text{ cal}}$  have been plotted in Figs. 4.51, 4.53, 4.55 and 4.57. It may be seen from these figures that in this case differences between these ratios are considerably less, and they are apparently not affected by rolling temperature.

To explain this problem better, and specially the influence of the form factor, diagrams shown in Figs. 4.58 and 4.59 have been plotted.

Figure 4.58 shows the probable run of measured values of rolling force plotted against percentage draught, for different form factors from minimum values for squares of  $\delta_w = 1$ , up to maximum ones of  $\delta_w = 500$  to 3000 for sheets.

For squares it was necessary to draw two curves, due to considerable differences between values obtained for small and large squares. Undoubtedly here a second factor becomes important, i.e. the ratio  $h/R$ , which is the decisive factor in this case. The curves calculated from the last theoretical methods run between the measured curves for very wide flats and plates. The course of this curve is drawn with a thick line. It is apparent that all other rolling parameters, such as roll radius, rolling speed and temperature, have been assumed constant.

Similarly, Fig. 4.59 shows total roll torques plotted against percentage draught, for the whole range of form factors from unity for squares up to maximum ones obtained

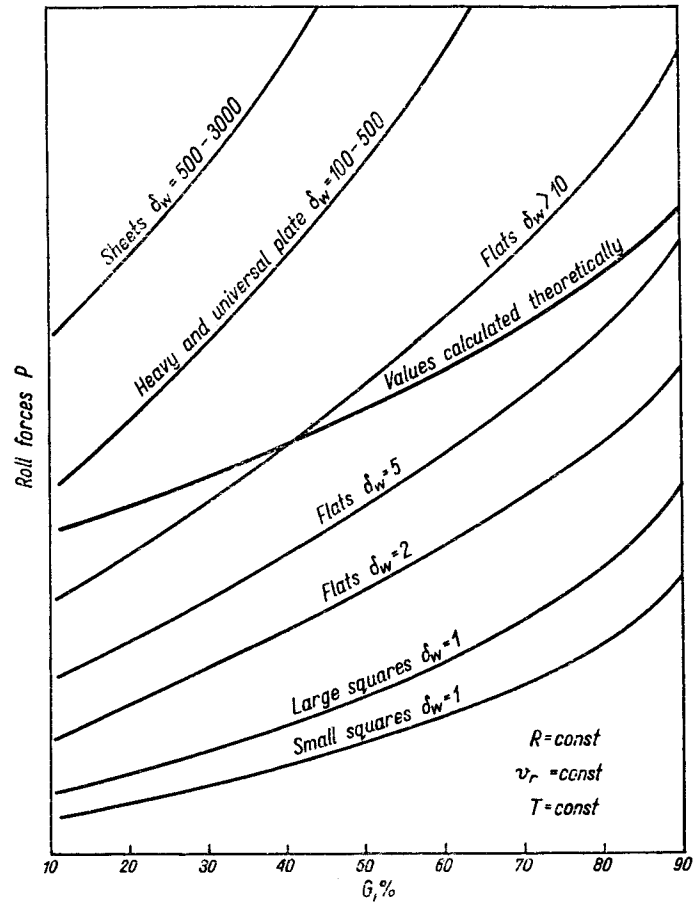


Fig. 4.58. Schematic diagram of roll forces for different form factors  $\delta_w$

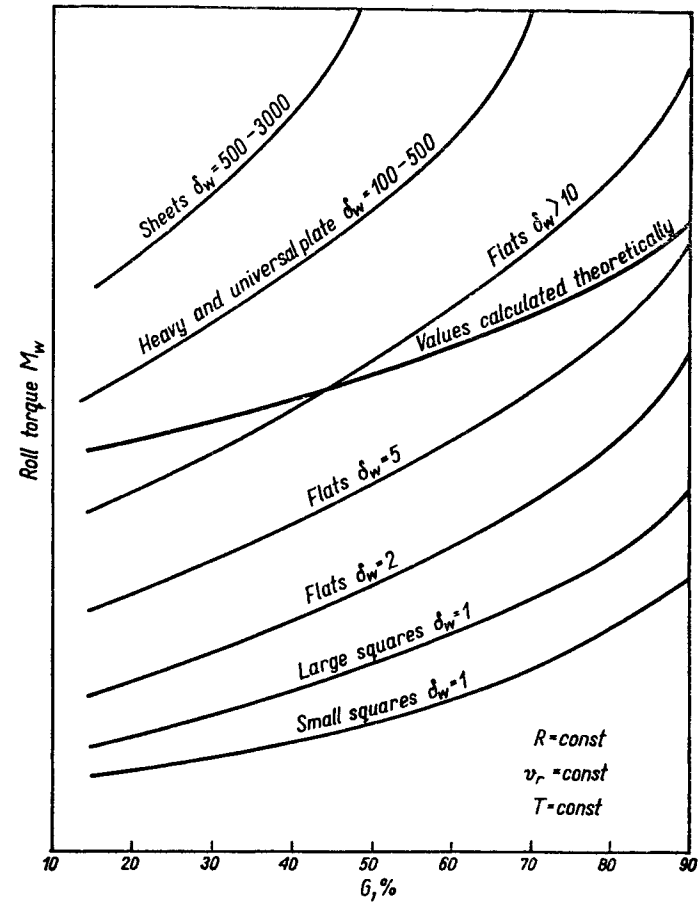


Fig. 4.59. Schematic diagram of rolling torque for different form factors  $\delta_w$

for sheets. The similarity between curves of torque and roll force is explained by the direct relation between torque and roll force.

#### 4.3. Work, Torque, and Power in Rolling

The standard formulae for calculation of work of deformation have been checked by Pavlov [80], who transformed them into the three following basic forms:

$$A_w = K_w V \frac{h_1 - h_2}{h_1} \quad (4.61)$$

$$A_w = K_w V \frac{h_1 - h_2}{h_2} \quad (4.62)$$

and

$$A_w = K_w V \log_e \frac{h_1}{h_2} \quad (4.63)$$

Pavlov shows that the work of deformation calculated from formula (4.61) corresponds to the work done in deformation for the entry speed of the rolled stock  $v_1$  into the roll gap. Using formula (4.62), the work of deformation corresponding to the speed of exit  $v_2$ , is obtained. Correct values are obtained from formula (4.63), which corresponds to the mean rolling speed.

Omitting the friction between roll surface and rolled stock and also friction of roll neck bearings, the theoretical work of deformation may be calculated from formula (2.109)

$$A_0 = \eta K_f V \log_e \frac{h_1}{h_2}$$

where  $V$  = volume of the rolled stock.

Instead of  $\eta K_f$  the mean resistance to deformation  $K_{wm}$  may be substituted. The actual work done in deformation is

$$A_w = A_0 + A_r$$

where

$A_0$  = theoretical work of deformation,

$A_r$  = additional work of deformation due to frictional resistance during rolling.

Calculating the work of deformation in rolling from Fink's formula, the following results are obtained:

(a) when rolling with spreading of the rolled stock, i.e. if  $b_1 \leq b_2$  (formula 4.63)

$$A_w = K_{wm} V \log_e \frac{h_1}{h_2} = K_{wm} V \log_e \frac{1}{\gamma}$$

(b) when rolling without spread, i.e. if  $b_1 = b_2$ , i.e.  $l_2/l_1 = h_1/h_2$

$$A_w = K_{wm} V \log_e \frac{l_2}{l_1} = K_{wm} V \log_e \lambda \quad (4.64)$$

where

$K_{wm}$  = mean resistance to deformation in the roll gap,

$l_1, l_2$  = length of the rolled stock before and after pass.

From these formulae the work of deformation in rolling is proportional to the deformed volume, which in turn is equal to the product of true volume and the true strain

$$V_v = V \log_e \frac{h_1}{h_2} \quad \text{or} \quad V_v = V \log_e \frac{l_2}{l_1} \quad (4.65)$$

Another method of determining the work of deformation in rolling, is to calculate the roll torque and from this to calculate the work.

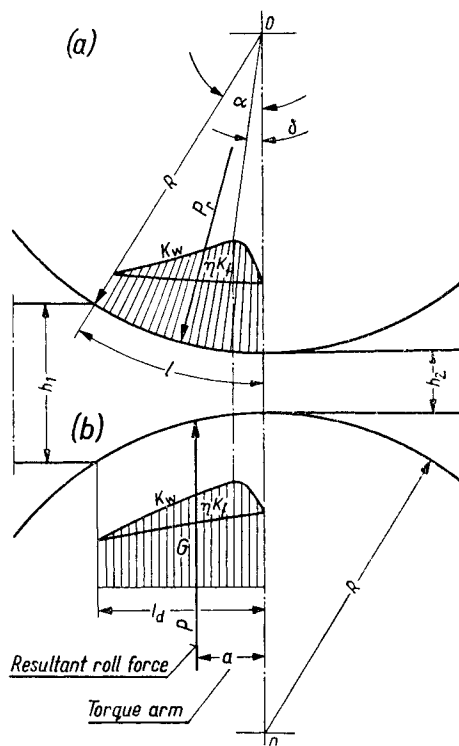


Fig. 4.60. Resultant roll force and torque arm relative to: (a) arc of contact, (b) projected arc of contact [81], [82]

Figures 4.60a and b show the case of stock of constant width  $b$  rolled between two smooth rolls with centres  $O-O$ , with an absolute draught of  $h_1 - h_2$  with no external forces acting. Roll diameters are equal, and have equal circumferential speeds. If  $b = 1$ , then the shaded area in Fig. 4.60b represents the roll force  $P = K_{wm} l_d$ . The resultant  $P$  of the roll force passes through the centre of gravity of the diagram of roll pressure distribution on the arc of contact, i.e. point  $G$  in Fig. 4.60b. On this assumption  $P$  is acting parallel to the rolling plane, which passes through  $O-O$  the centres of the rolls. The distance between the resultant  $P$  and roll axis  $O-O$  gives the lever arm  $a$ , from which the torque acting upon the roll can be calculated.

According to Tselikov [59], this definition of the lever arm  $a$ , perpendicular to the resultant  $P$  and the axis  $O-O$  (Fig. 4.60b), is only correct if the following conditions are fulfilled:

(a) both rolls are driven, and have equal diameters and circumferential speeds,

(b) the mechanical properties of the rolled stock do not vary along the roll gap,  
 (c) the metal flows uniformly along the roll gap, and apart from the roll forces, no external forces act upon it.

Figure 4.60a gives a truer value of torque, it shows a non-uniform distribution of roll pressure along the contact arc, and its radial resultant  $P_r$ . The error introduced into torque calculations by assuming that  $P \approx P_r$  is, according to Emicke, not more than 1% [81], [82].

From Fig. 4.60b the torque required to drive one roll is  $Pa$ , hence the total torque required to drive both rolls is

$$M_w = 2Pa \quad (4.66)$$

This is a simplification, as the calculation of roll torque is not based on the true pressure distribution along the contact arc as shown in Fig. 4.60a, but on its projection on a horizontal plane (Fig. 4.60b).

Denoting the distance of a volume element from the plane of exit as  $x$ , (Fig. 3.1) and the vertical component of the normal pressure as  $q$ , then  $q dx$  represents the vertical force acting upon this element of width  $b$  and thickness  $dx$ . The torque due to this force about the roll axis is then equal to the product  $q dx x$ , therefore the total torque due to these vertical components is given by

$$M_w = 2b \int_0^{l_a} q x dx \quad (4.67)$$

where  $b$  is assumed constant. After simplification and integration formula (4.66) is obtained from formula (4.67).

The experiments of Emicke [81, 82, 83], show that this method of determination of roll torque is permissible for practical calculations. However for exact scientific investigations, e.g. [7] and Juretzek [84], this method gives insufficient accuracy, due to neglect of true roll pressure distribution along the arc of contact, and also of slipping and sticking.

Calculation of roll torque from true roll pressure distribution as shown in Fig. 3.1 gives the following results according to Orowan [35]. Denoting the friction force on unit area of contact between roll and rolled stock by  $\tau$ , and assuming  $b = 1$ , then upon an element of area of the length of  $R d\varphi$  a frictional force of  $\tau R d\varphi$  is acting. Its torque related to the roll axis equals  $R\tau R d\varphi = R^2\tau d\varphi$ . Summing up all frictional forces acting circumferentially along the contact arc, and taking into account the change of frictional forces at the neutral plane, then the torque relative to the roll axis is

$$M_w = 2R^2 \left( -\int_0^{\delta} \tau d\varphi + \int_{\delta}^{\alpha} \tau d\varphi \right) \quad (4.68)$$

If roll and rolled stock slip relatively to each other, then  $\tau = pf$ , and formula (4.68) takes the form

$$M_w = f \frac{D^2}{2} \left( -\int_0^{\delta} p d\varphi + \int_{\delta}^{\alpha} p d\varphi \right) \quad (4.69)$$



If sticking occurs, then  $\tau = (1/2)\eta K_f$  and formula (4.69) becomes

$$M_w = \eta R^2 - \left( - \int_0^\delta K_f d\varphi + \int_\delta^\alpha K_f d\varphi \right) \quad (4.70)$$

Assuming mean constant yield stress  $K_{fm}$  then

$$M_w = \eta K_{fm} R^2 (\alpha - 2\delta) \quad (4.71)$$

The rolling torque obtained from formula (4.66) represents only the pure torque of deformation. Friction losses in roll neck bearings or torques due to applied front or back tension have been neglected.

Calculating the roll force according to formulae (2.108) and (4.11)

$$P = K_{wm} b \sqrt{R \Delta h}$$

and substituting this value into formula (4.66), then

$$M_w = 2Pa = 2K_{wm} b \sqrt{R \Delta h} a \quad (4.72)$$

is obtained.

For calculating the rolling work, first of all the work necessary to complete one revolution of one roll is expressed

$$M_w = Pa \text{ (pure deformation torque)}$$

If the applied torque  $M_w$  turns the roll through an angle  $\varphi$ , the work done in rotating the roll is  $A_w = Pa\varphi$ , and the work done in one full revolution where  $\varphi = 2\pi$ , is

$$2\pi M_w = 2\pi Pa$$

Therefore, for one revolution of both rolls

$$A_w = 4\pi Pa \quad (4.73)$$

is obtained. The angle of  $\varphi$  may be approximated to the ratio  $l_2/R$ , and expressed in radians.

Assuming a constant width of the rolled stock, then

$$\frac{h_1}{h_2} = \frac{l_2}{l_1} = \frac{v_2}{v_1} \quad \text{and} \quad l_2 = \frac{h_1 l_1}{h_2}$$

and then the following form of the formula of rolling work is obtained

$$A_w \approx 2Pc \sqrt{R \Delta h} \frac{l_2}{R} \quad (4.74)$$

Where the lever arm  $a$  is expressed as a fraction  $c$  of the projected arc of contact  $l_d$  (Fig. 4.60b), as

$$a = c \sqrt{R \Delta h} \quad (4.75)$$

according to formulae (4.72) and (4.75) the following formula of rolling torque is obtained

$$M_w = 2K_{wm} b \sqrt{R \Delta h} c \sqrt{R \Delta h} = 2K_{wm} bc R \Delta h \quad (4.76)$$

This torque is for both rolls, and only one half of this value is acting upon one roll. If spreading occurs during rolling, i.e.  $b_2 > b_1$ , then into all formulae for calculation of roll torque instead of  $b$  the mean width  $b_m$  should be substituted. This value may be calculated from formula (4.13).

For hot rolling the relation

$$a \approx \frac{l_d}{2} \approx \frac{1}{2} \sqrt{R \Delta h} \quad \text{i.e. } c = 1/2 \quad (4.77)$$

is often applied with great approximation, therefore the formula (4.76) for calculation of roll torque becomes in a simplified form

$$M_w \approx K_{wm} b R \Delta h \quad (4.78)$$

The correct choice of the value of lever arm  $a$  for calculating the roll torque is of great importance. If the range of variation of  $a$  is known then the error introduced by assuming  $a$  constant may be estimated.

For practical purposes the following formula for calculation of lever arm has been recommended by Siebel [61]:

$$a = \frac{l_d}{2} \frac{1}{\sqrt{\lambda}} (1+s) \quad (4.79)$$

where

$\lambda$  = elongation coefficient,

$s = (w_2 - v_r)/v_r$  = forward slip.

As the forward slip seldom exceeds 5 per cent, the formula (4.79) may often be replaced by its approximation

$$a \approx \frac{l_d}{2} \frac{1}{\sqrt{\lambda}} \quad (4.80)$$

Calculating the lever arm of roll force by another method, if (4.63) and (4.74) are compared then [31]

$$A_w = K_{wm} V \log_e \frac{1}{\gamma} = 2 P a \frac{l_2}{R}$$

from which

$$a = \frac{l_d}{2} \frac{b_2}{b_m} \frac{\gamma}{1-\gamma} \log_e \frac{1}{\gamma} \quad (4.81)$$

The spread of the rolled stock has been taken into account in this formula, and a corresponding value of mean width substituted.

If spread is not taken into account, then since  $b_2 \approx b_m$  the formula simplifies to

$$a \approx \frac{l_d}{2} \frac{\gamma}{1-\gamma} \log_e \frac{1}{\gamma} \quad (4.82)$$

Very different values for the lever arm  $a$  for varying rolling conditions have been determined by various authors, e.g. L.R. Underwood [34] who published the following values:

for ingots, slabs and blooms, for which the specific roll pressure along

the arc of contact is approximately constant

for rolling of light sections

for roughing stands of continuous hot strip mills

for remaining stands of continuous hot strip mills

$$a = 0.5 l_d$$

$$a = 0.45-0.42 l_d$$

$$a = 0.48 l_d$$

$$a = 0.39 l_d$$

According to Tselikov [59] for hot rolling of sections:

for round cross-sections

for closed passes

for strip cold rolling

$$a = 0.6 l_d$$

$$a = 0.7 l_d$$

$$a = 0.38-0.42 l_d$$

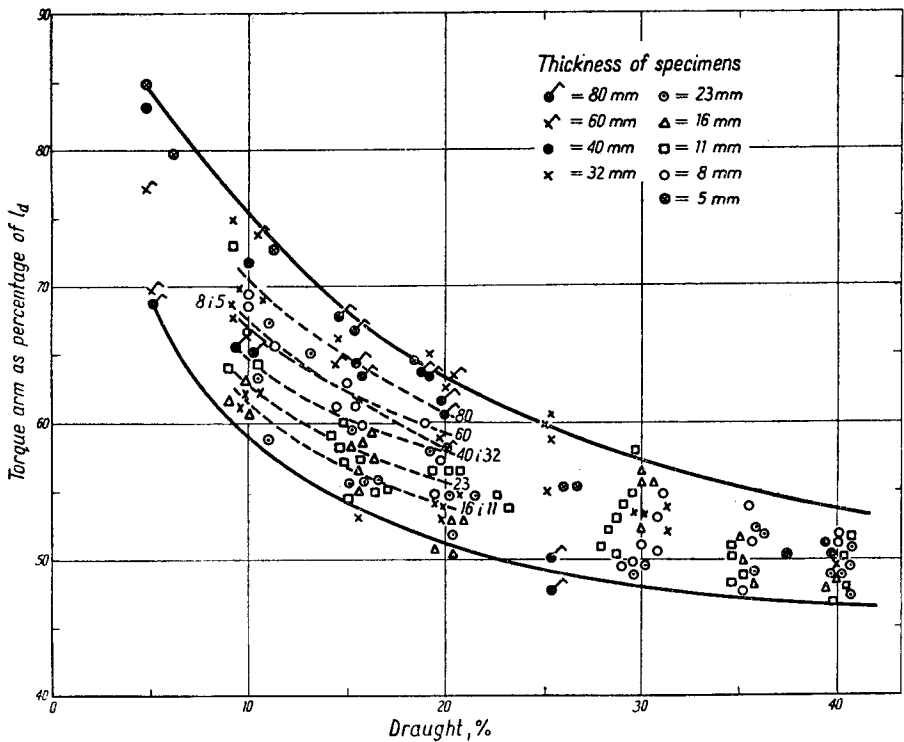


Fig. 4.61. Ratio of torque arm to length of projected arc of contact  $a/l_d$  (%), plotted against percentage draught and various initial thickness of specimens during hot rolling of St60 steel (see Appendix 1) [81]

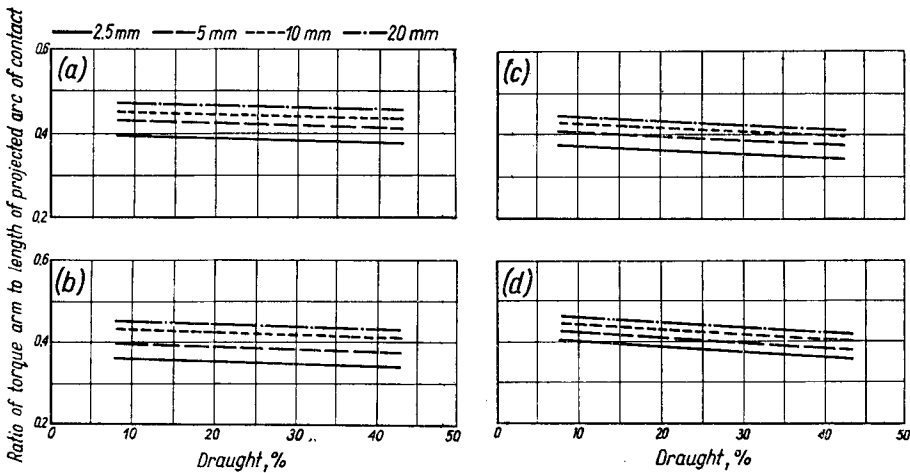


Fig. 4.62. Ratio of torque arm to length of projected arc of contact for hot rolling of flats from 0.1% low-carbon steel at rolling temperatures of: (a) 1100°C, (b) 1000°C, (c) 900°C, (d) 800°C;  $b_m = 85.5$  mm,  $D = 205$  mm,  $n = 150$  rpm

With these assumptions the maximum error may be  $\pm 33\%$ . Results of investigations on rolling of aluminium, its alloys and steels (Fig. 4.61) were published by Emicke and Lucas [83]; roll forces and roll torques on the spindle were measured. It was confirmed that for flat rolling on a smooth barrel the lever arm may be greater, equal to or smaller than  $l_d/2$  depending upon rolling conditions.

Figure 4.62 shows the ratio of lever arm to length of arc of contact  $a/l_d$  when rolling a 0.1 per cent low-carbon steel plotted against % draught, for hot flat rolling and different initial thicknesses, carried out by G. Wallquist [85]. In this figure the value of  $a$  varies in the range  $0.4-0.35 l_d$ . For alloy steels ratios of  $a/l_d = 0.28-0.56$  have been obtained by Wallquist.

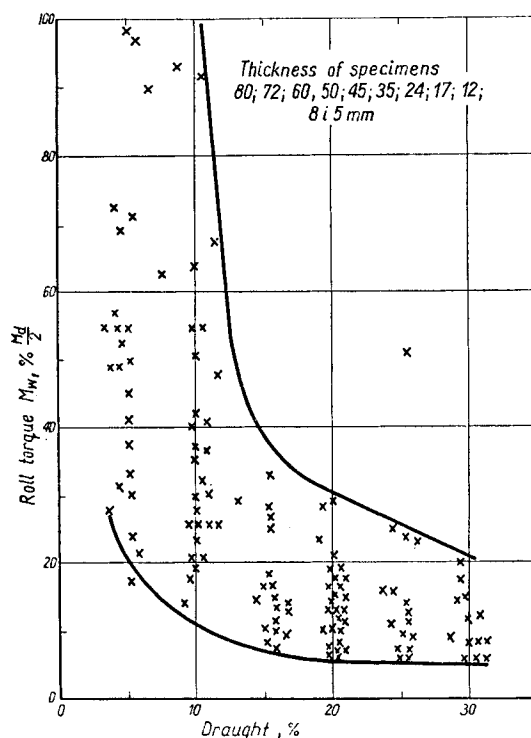


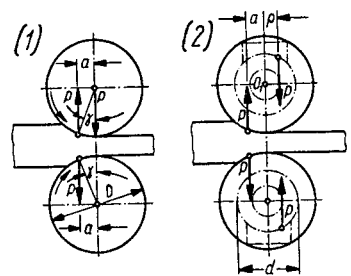
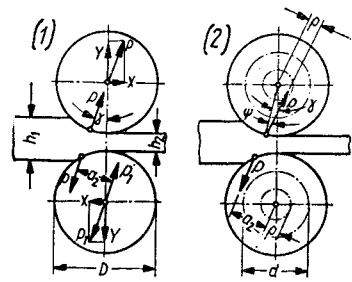
Fig. 4.63. Variation of roll torque  $M_w$  as percentage of half value of total rolling torque  $M_d/2$ , plotted against percentage draught in hot rolling of St60 steel at  $1080^\circ\text{C}$

Figure 4.63 shows results obtained by G. Zouhar [86] for hot rolling of steel St60\*, carried out on the laboratory mill of the Mechanical Engineering Institute of the Mining Academy, Freiberg. The rolling temperature reached about  $1100-1200^\circ\text{C}$ , the thickness of specimens was from 5 up to 80 mm, and width 80–100 mm. Figure 4.61 shows ratios of lever arm to length of arc of contact relative to draught and thickness of specimens.

This work also confirms the fact already found by Emicke and Lucas, that the total rolling torque is not equally distributed between the rolls. Depending upon the draught applied in a given pass, one or the other roll may transmit more than the half values of total roll torque. It has been confirmed by Zouhar (Fig. 4.63) that for

\* See Appendix 1.

TABLE 4.7  
CALCULATION OF THE RESULTANT ROLL FORCE FOR VARIOUS ROLLING CONDITIONS

| Rolling conditions                                                                                                                                                                                                                                                                                                                                        | Direction of forces acting on roll                                                  | Formulae for roll torque $M_w$                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Simple rolling case, rolls have equal diameters and peripheral speeds. Rolled stock is homogeneous with respect to mechanical properties and moves continuously through the roll gap, and is under the action of roll pressure only.</p> <p>Case (1) Not allowing for bearing friction losses</p> <p>Case (2) Allowing for bearing friction losses</p> |   | <p>(1) <math>M_{w1} = Pa = PR \sin \gamma</math><br/> <math>\approx P 0.5 \sqrt{R \Delta h}</math>;</p> <p>(2) <math>M_{w2} = P(a + \rho) = P \left( R \sin \gamma + \frac{d f_1}{2} \right)</math></p> <p><math>\Delta h</math> = absolute draught,<br/> <math>d</math> = roll neck diameter,<br/> <math>f_1</math> = coefficient of friction of bearings,<br/> <math>\rho</math> = lever arm for calculating the frictional torque on the roll neck</p> |
| <p>One roll driven (e.g. the bottom one), second is a drag roll driven by friction between rolled stock and roll surface.</p> <p>All other conditions as in simple rolling case.</p> <p>Case (1) Not allowing for bearing friction losses</p> <p>Case (2) Allowing for bearing friction losses</p>                                                        |  | <p>(1) <math>M_{w1} = Pa_2 = P(D + h_2) \sin \gamma</math><br/> <math>P = P_1</math></p> <p>(2) <math>M_{w2} = P(a_2 + \rho) = P(D + h_2) \sin(\gamma + \psi)</math><br/>         where <math>\sin \psi = (d/D)f_1</math></p>                                                                                                                                                                                                                             |



percentage draughts between 20 and 30% this unequal distribution of roll torque may be up to 35%, i.e. the torque acting on one roll may exceed by up to 35% the half value of the total roll torque. For smaller draughts this value reaches considerably greater values, as may be seen from Fig. 4.63. These considerations are of great importance both for rolling practice, and for rolling mill design.

The conditions shown in Fig. 4.60 are rarely fulfilled in practice. Thus the resultant roll force is seldom parallel to the plane passing through the rolls axes. Methods for determining the direction and value of the resultant roll force for the most frequently occurring conditions are compared in Table 4.7 [87].

Values of lever arm  $a$  or angle  $\gamma$  occurring in the various formulae shown in Table 4.7, are determined from the distance between point of action of the resultant roll force on the roll surface and the plane passing through the roll axes. The point of action is defined as the centre of gravity of the roll pressure distribution curve along the contact arc, as shown on the schedules in Fig. 4.7.

The following relations between angle of inclination of the roll force  $\gamma$  and the bite angle  $\alpha$  may be assumed with sufficient accuracy for practical calculations:

(a) for hot rolling

$$\gamma \approx 0.5 \alpha \quad (4.83)$$

(b) for cold rolling due to displacement of the point of action of the roll force towards the plane of roll axis

$$\gamma \approx 0.35-0.45 \alpha \quad (4.84)$$

For the simple rolling case, i.e. if the roll force acts parallel to the plane of roll axes

$$\frac{\gamma}{\alpha} = \frac{a}{l_d} \quad (4.85)$$

where

$a$  = lever arm of the resultant roll force about the roll axis,

$l_d$  = length of projected arc of contact.

#### 4.3.1. INFLUENCE OF ADDITIONAL FRICTION AND DEFORMATION WHEN ROLLING IN BOX PASSES [109]

Extensive research has been carried out by Z. Wusatowski and J. Ludyga [141] to investigate conditions when rolling in box passes, as mentioned in the previous chapter.

An examination of the published data shows that only Siebel's method [61] gives indications on how the additional friction in the passes may be calculated.

Siebel's correction factor,  $a_2$ , given in Table 4.6, gives values which take directly into consideration the increase in resistance to deformation due to the additional friction.

Siebel's final equation for the mean resistance to deformation is (4.60)

$$K_{wm} = K_w a_1 a_2$$

where

$a_1$  = rolling speed factor (Table 4.5),

$a_2$  = additional friction factor (Table 4.6).

It must be pointed out, however, that Siebel used a number of diagrams which are of only limited application. The values given in Table 4.6 also show great scatter and no indication of accuracy is given. Their application is therefore problematical.

The purpose of these investigations was to determine the effect of additional friction on roll force and torque when rolling mild steel bars in box passes.

The basic factor which is assumed to govern the friction at the side wall of the pass, is the limitation of spread.

The calculated area of spread ranging from the completely suppressed  $\beta$  to the free  $\beta$  was divided into two or three ranges. When spread is calculated in this way, the increase in the coefficient of friction depends on the permissible percentage draught and on the additional quantity of the rolled stock which is reduced due to restricted spread.

In other words, the friction at the side walls reaches a maximum if spread is prevented.

It is very difficult to make an accurate determination of the coefficient of spread in individual passes, and in practice it is not possible because of the different values of true spread depending on the applied draughts. The coefficient of spread was therefore calculated for individual passes as follows (the coefficient of spread  $\beta$  was determined from formula (3.45)):

Initial cross-section  $30 \times 30$  mm square (Fig. 4.64):

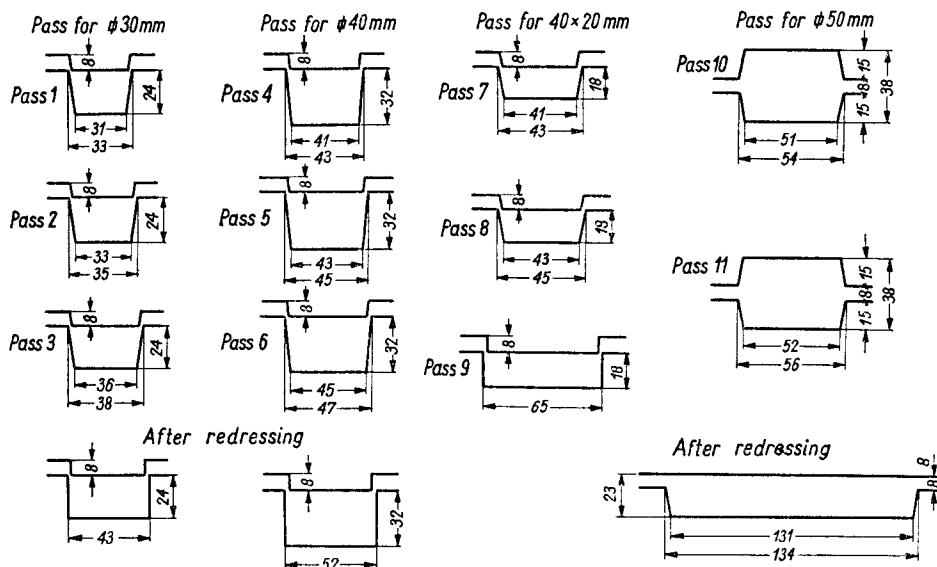


Fig. 4.64. Shape of passes used in tests

for pass No.1 the range of the coefficient  $\beta_{calc} = 1.03-1.06$

for pass No.2 the range of the coefficient  $\beta_{calc} = 1.07-1.13$

for pass No.3 the coefficient of free spread  $\beta_{calc} = 1.32$

Initial cross-section  $40 \times 40$  mm square (Fig. 4.64):

for pass No.4 the range of the coefficient  $\beta_{calc} = 1.03-1.05$

for pass No.5 the range of the coefficient  $\beta_{calc} = 1.06-1.10$

for pass No.6 the coefficient of free spread  $\beta_{calc} = 1.28$



Initial cross-section  $40 \times 20$  mm (Fig. 4.64):

- for pass No.7 the range of the coefficient  $\beta_{calc} = 1.02-1.05$
- for pass No.8 the range of the coefficient  $\beta_{calc} = 1.03-1.10$
- for pass No.9 the coefficient of free spread  $\beta_{calc} = 1.28$

Initial cross-section  $50 \times 50$  mm (Fig. 4.64):

- for pass No.10 the range of the coefficient  $\beta_{calc} = 1.03-1.05$
- for pass No.11 the range of the coefficient  $\beta_{calc} = 1.06-1.13$
- for pass No.12 the coefficient of free spread  $\beta_{calc} = 1.28$

These values are constant and independent of the variable factors which are chosen as follows:

- (a) temperature: about 900, 1000 and 1100°C,
- (b) percentage draught: about 10, 20, 30, 40 (%),
- (c) size of test sample:  $30 \times 30$ ,  $40 \times 40$ ,  $50 \times 50$ , and  $40 \times 20$  mm

Test pieces were rolled in passes considerably wider than the width of the rolled stock, under conditions of free spread, so that comparable results could be obtained. The roll index  $\epsilon_w$  and the adjustment of the rolls were the same in both rolling experiments. For squares  $30 \times 30$  and  $40 \times 40$  mm the comparable test pieces were rolled in passes 3 and 6 after the passes were widened (Fig. 4.64).

The comparable specimens of the  $40 \times 20$  mm flat were rolled in pass 9, and for  $50 \times 50$  mm square the comparable specimens were rolled in pass 10 (Fig. 4.64).

The use of a whole series of passes within one rolling system facilitated a considerable variation in the top pressure. The maximum distance from the basic setting was 8 mm and the minimum 1 mm. The height of the pass was changed by varying the position of the top and bottom roll. The following top roll pressures were measured:

- initial cross-section 30 mm square, top roll pressure  $D_t - D_b = 40$  mm
- initial cross-section 40 mm square, top roll pressure  $D_t - D_b = 60$  mm
- initial cross-section 50 mm square, top roll pressure  $D_t - D_b = 10$  mm
- initial cross-section  $40 \times 20$  mm flat, top roll pressure  $D_t - D_b = 30$  mm

Therefore the ratio of the torque of the top roll to that of the total torque is

$$\frac{M_t}{M_{tot}} = 0.8-1.0$$

The total torque which is required for the work was therefore transmitted almost completely by the top roll. The torque of the bottom roll had a negative or a very small positive value depending on the absolute draught. When the value of the torque transmitted was negative, the bottom roll exerted a retarding torque on the rolling process.

The top roll was turned for the second series of tests so as to avoid this situation. After it had been turned the roll pressure was then changed to:

- initial cross-section 30 mm square, bottom roll pressure  $D_b - D_t = 4$  mm
- initial cross-section 40 mm square, top roll pressure  $D_t - D_b = 16$  mm
- initial cross-section  $40 \times 20$  mm flat, top roll pressure  $D_t - D_b = 10$  mm

The effect of friction at the side walls on the roll force was obtained from equation (4.86)

$$a' = \frac{P_t}{P_0} \quad (4.86)$$

where

- $P_t$  = the roll force with restricted spread and
- $P_0$  = the roll force with free spread.

The factor allowing for increase in the resistance to deformation as a function of the friction at the side walls, was obtained from equation (4.87)

$$a'' = \frac{K_{wt}}{K_{w0}} \quad (4.87)$$

where

$K_{wt}$  = mean resistance to deformation with restricted spread,

$K_{w0}$  = mean resistance to deformation with free spread.

The factor  $a'''$  allowing for the effect of friction on the torque when rolling in closed passes, was obtained by using equation (4.88)

$$a''' = \frac{M_{wt}}{M_{w0}} \quad (4.88)$$

where

$M_{wt}$  = roll torque with restricted spread,

$M_{w0}$  = roll torque with free spread.

Diagrams have been worked out which give a number of factors and their relative values, so that experimental and calculated results can be utilized. Mean curves were drawn through the range of scatter.

In Figures 4.65–4.67 the average values of the total roll force are plotted as a function of percentage draught at constant spread and constant temperature.

Figures 4.65a–c give the roll force when rolling squares  $30 \times 30$  mm and  $40 \times 40$  mm at temperatures of 900, 1000 and 1100°C.

The curves for 900°C are almost straight lines up to a percentage draught of 30%, which shows that the increase in roll force is almost directly proportional to the draught (Fig. 4.65a), and is almost independent of the coefficient of spread. At a percentage draught of 10–20% the roll force with restricted spread is about 1.3 times greater than that with completely free spread. Above 30% the slope of the curve becomes less steep. It varies and depends on the coefficient of spread and the initial thickness of the rolled sample. It can be assumed that at a percentage draught of over 30%, when the pass is completely filled, the roll force increase due to additional friction will not be proportional to the percentage draught. Comparing the curves for the squares  $30 \times 30$  mm and  $40 \times 40$  mm, it can be seen that the value of the roll force depends on the initial thickness of the rolled sample. The curves for free spread conditions are almost straight lines within the range of percentage draught of 10 to 30%. An increase to over 30% causes the slope of the curves to decrease towards lower values. Fig. 4.65b gives results for squares  $30 \times 30$  mm and  $40 \times 40$  mm rolled at a temperature of 1000°C. The results are similar to those already discussed except for the fact that the roll force at this temperature is 20% lower than shown in Fig. 4.65a. The roll force at a temperature of 1100°C is about 12% lower than shown by the curves obtained at 1000°C (Fig. 4.65c). Two changes of slope are shown in the curves obtained for squares  $30 \times 30$  mm and  $40 \times 40$  mm at 1100°C, the first occurring at 18–20% draught, increasing, and the second at 26–31%, decreasing. The individual points for various conditions of spread lie closely together at this rolling temperature.

Figures 4.66a–c give results obtained with squares  $50 \times 50$  mm at 900, 1000 and 1100°C, and the curves show characteristics approximately similar to the preceding curves, due to the fact that the pass was similarly designed. In this case, the pass was partly open with a seam being formed at a greater percentage draught, and giving rise to greater additional friction on the side walls of the pass. The difference in roll force

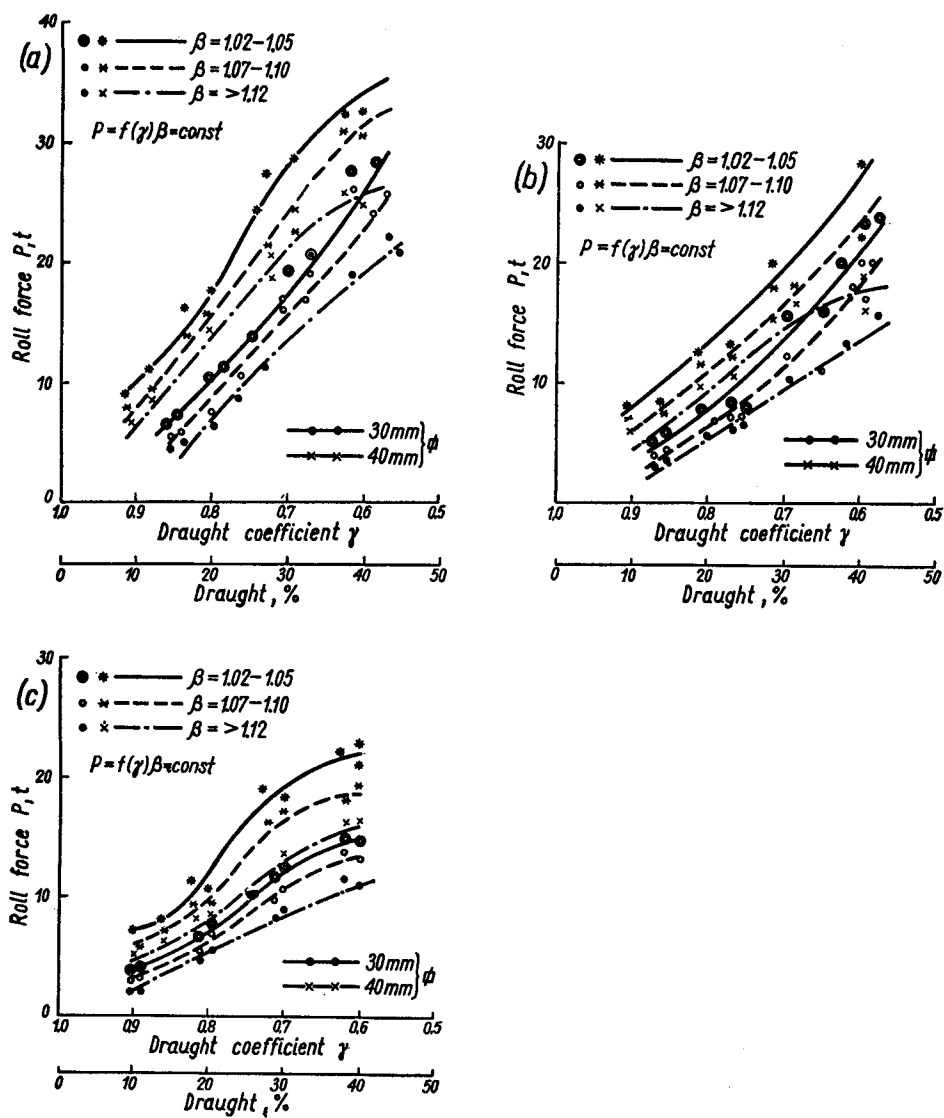


Fig. 4.65. Roll force plotted against percentage draught and coefficient of spread: (a) 900°C, (b) 1000°C, (c) 1100°C. Initial cross-sections 30 and 40 mm square

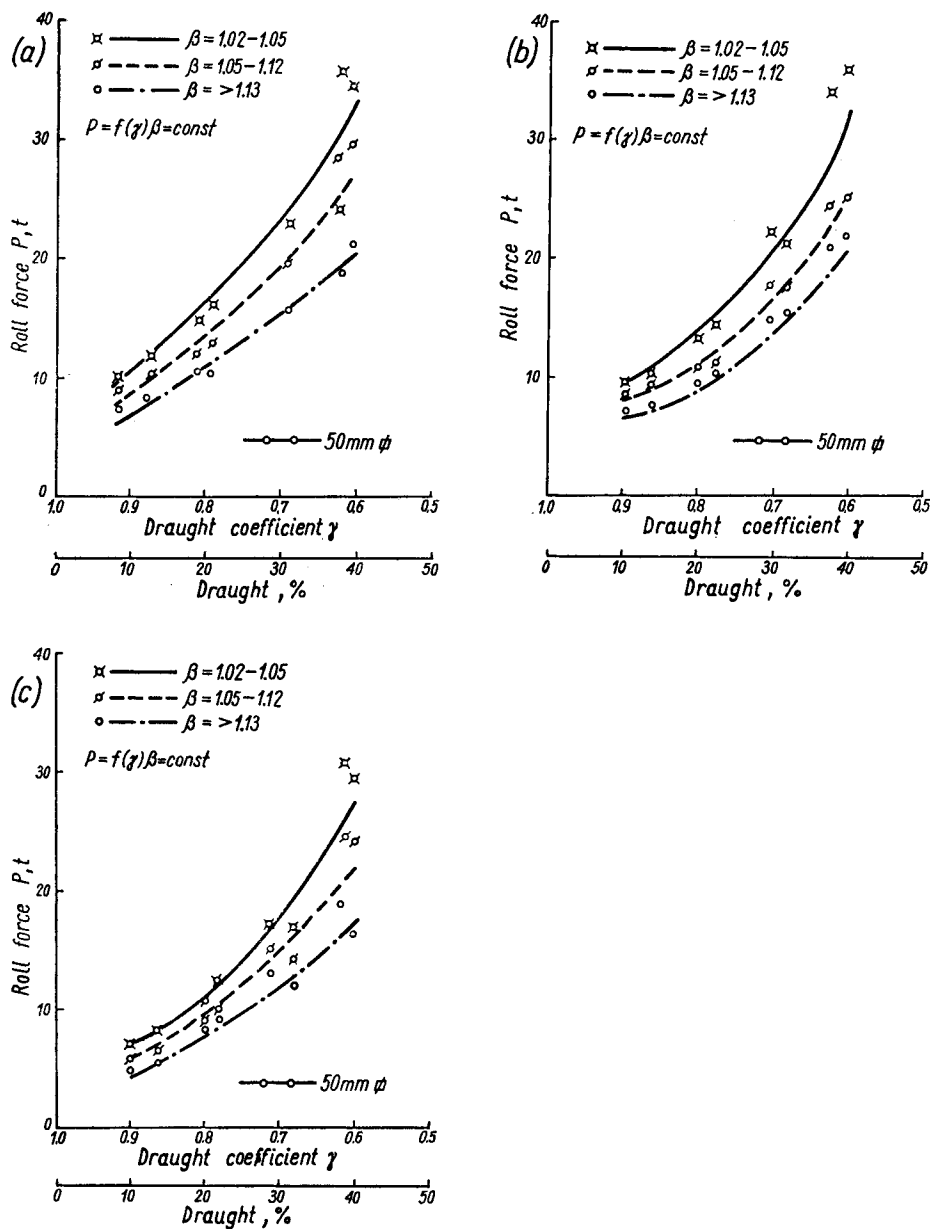


Fig. 4.66. Roll force plotted against percentage draught and coefficient of spread: (a) 900°C, (b) 1000°C, (c) 1100°C. Initial cross-section 50 mm square

is about 20–30% greater, depending on the coefficient of restricted spread. The plots (Figs. 4.67a–c) show similar types of curves for the flat  $40 \times 20$  mm rolled at 900, 1000 and 1100°C.

Figures 4.68a–c show the values of the mean resistance to deformation as a function of the rate of deformation at the three temperatures of 900, 1000 and 1100°C. The full curves represent maximum values when rolling with restricted spread. The curve representing minimum values of mean resistance to deformation was obtained when rolling with free spread, and the broken curve gives values with the coefficient of spread of  $1.06 < \beta < 1.1$ .

The large scatter observed is due to the large deformation when rolling samples of various initial thicknesses. The scatter of points is dependent on the conditions of spread at the constant rate of deformation.

The field limited by the curves varies with the rolling temperature. It is largest for a temperature of 900°C and smallest for a temperature of 1100°C. The individual points are distributed depending on the draught, rolling temperature and conditions of spread. At a temperature of 900°C, the values of rate of deformation are evenly distributed along the centre curve. At 1000°C and under similar conditions, the values are lower. At 1100°C, however, the values move more towards the bottom curve. The distance between the individual points for various conditions of spread is 15–22%, depending on the temperature. Values of resistance to deformation vary by between 35 and 55% between the top and bottom line. The variation is greatest at a rolling temperature of 900°C at the maximum rate of deformation. However, at 1100°C the greatest variation was measured at the smallest rate of deformation. It can be therefore concluded that the increase in additional friction varies inversely as the resistance to deformation and directly as the temperature.

Variations in the resistance to deformation at constant percentage draught and rolling temperature are represented in Figs. 4.69a–c for various spread conditions. For restricted spread the mean resistance to deformation increases considerably, especially for large draughts and low rolling temperatures.

Total rolling torque is plotted as a function of percentage draught at constant rolling temperature and under various conditions of spread in Figs. 4.70 and 4.71. Comparing the results with those obtained for the roll force as a function of draught at constant rolling temperature and known conditions of spread, it can be seen that the values lie along similar curves. This is not surprising if it is considered that the total torque, as a product of roll force and lever arm must vary directly as each of its factors. Due to the fact that the lever arm of the torque varies within small limits, the torque reflects in the first instance the effect of the roll force. Figs. 4.70a–c give values of the total torque for squares  $30 \times 30$  mm and  $40 \times 40$  mm, and the curves are almost straight lines for rolling with restricted spread. When rolling with free spread, the values of total torque lie along a straight line following the increase in draught, the line having a slight change of slope when the draught exceeds 28%. The slope becomes greater with increase in rolling temperature. The torque is about 1.35 times greater with restricted spread than with free spread. The square  $30 \times 30$  mm gave the same values as the square  $40 \times 40$  mm. Values obtained at 1000 and 1100°C (Figs. 4.70a and c) show a similar form to these for roll force at these temperatures. The values of roll torque for restricted spread are 1.2–1.7 times greater than those for free spread.

Values of the torque for the square  $50 \times 50$  mm and the flat bar  $40 \times 20$  mm are given as a function of temperature in Figs. 4.71a–c.

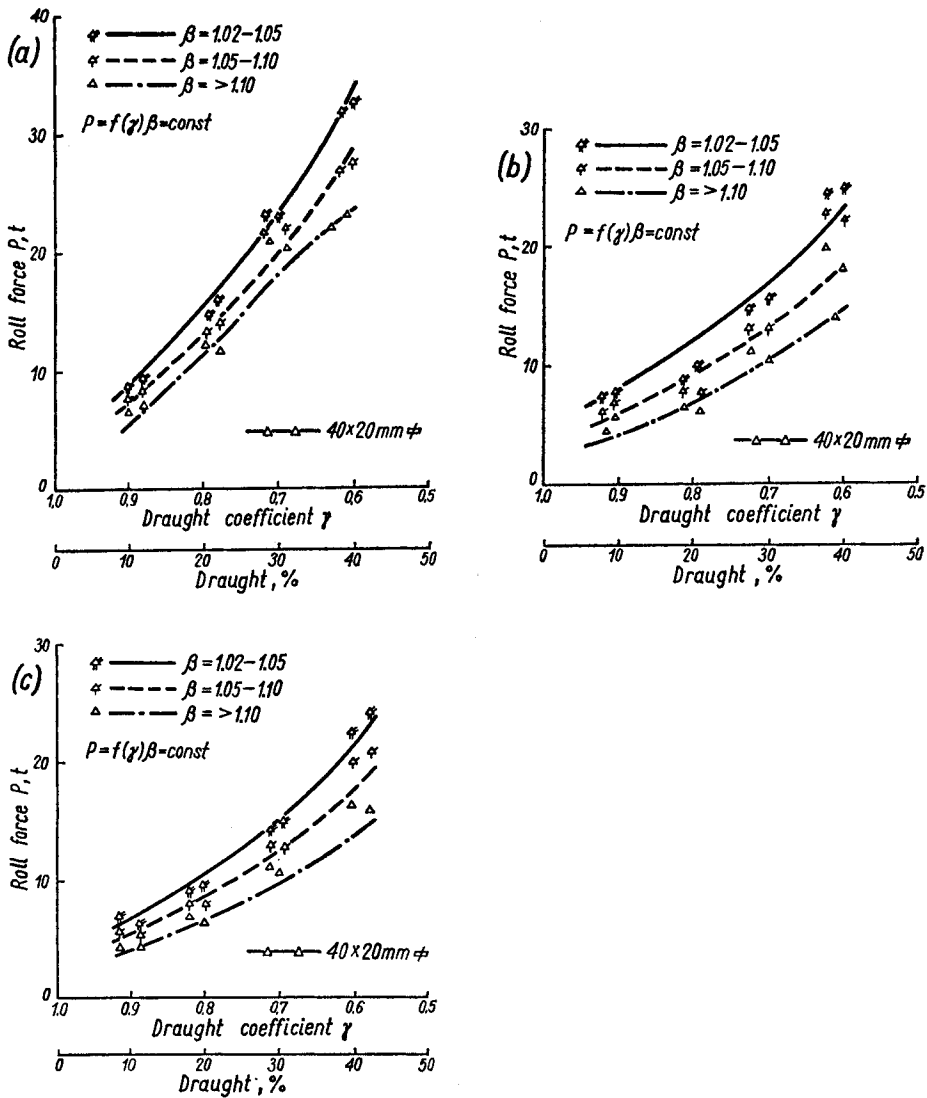


Fig. 4.67. Roll force plotted against percentage draught and coefficient of spread: (a) 900°C, (b) 1000°C, (c) 1100°C. Initial cross-section  $40 \times 20 \text{ mm}$

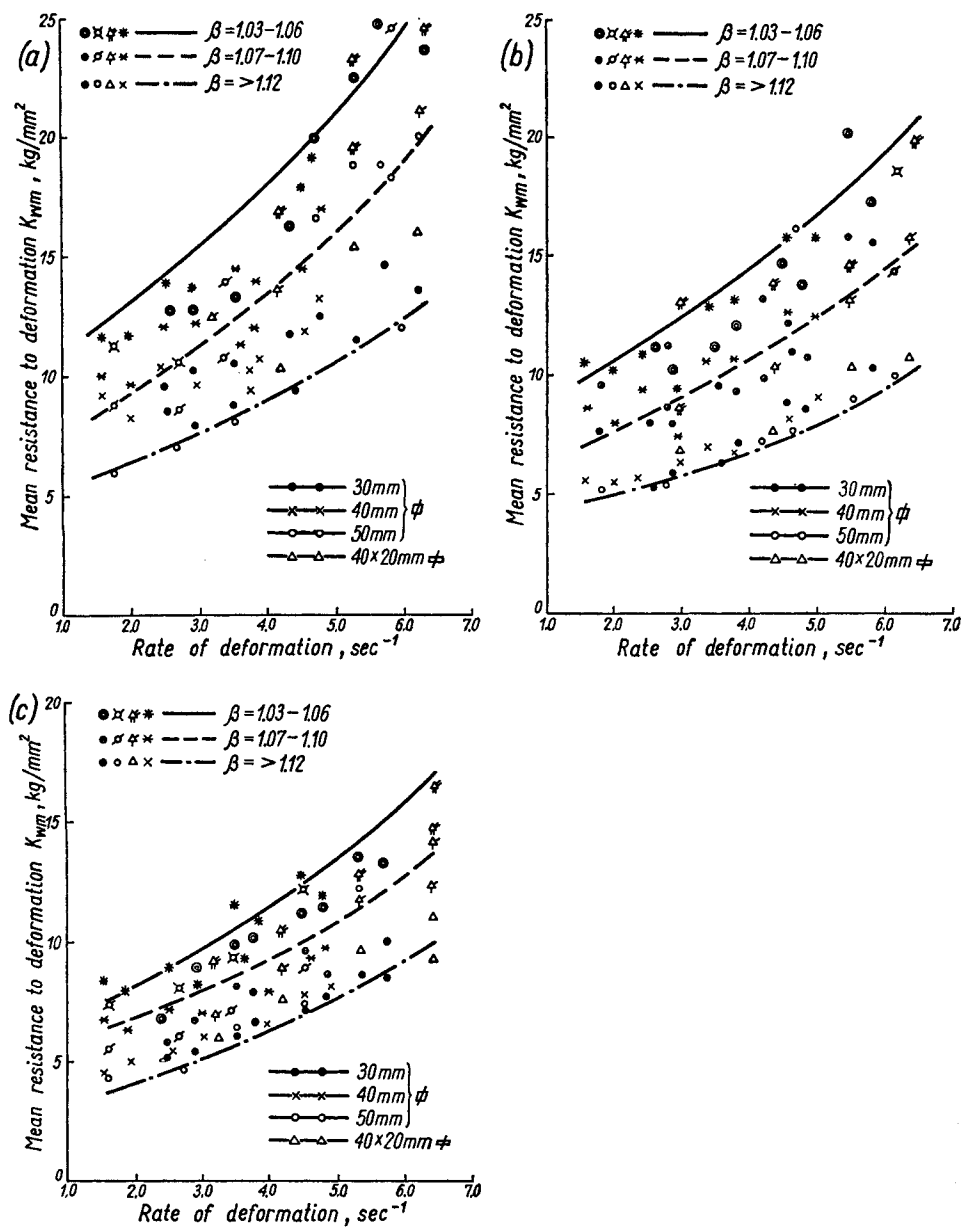


Fig. 4.68. Mean resistance to deformation for rolling in box passes with various coefficients of spread as a function of mean rate of deformation: (a) 900°C, (b) 1000°C, (c) 1100°C

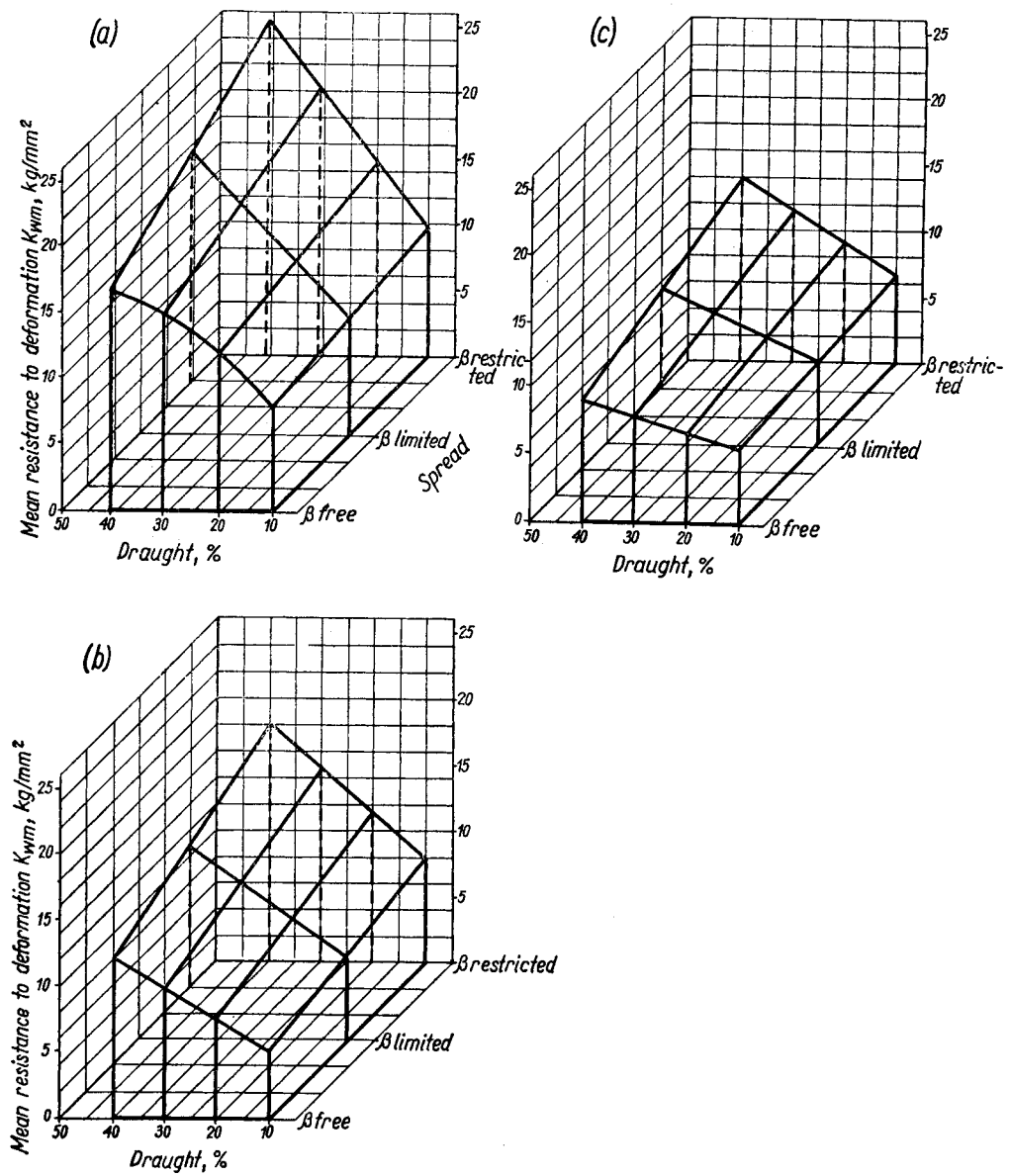


Fig. 4.69. Mean resistance to deformation for rolling in box passes with various coefficients of spread as a function of percentage draught: (a) 900°C, (b) 1000°C, (c) 1100°C



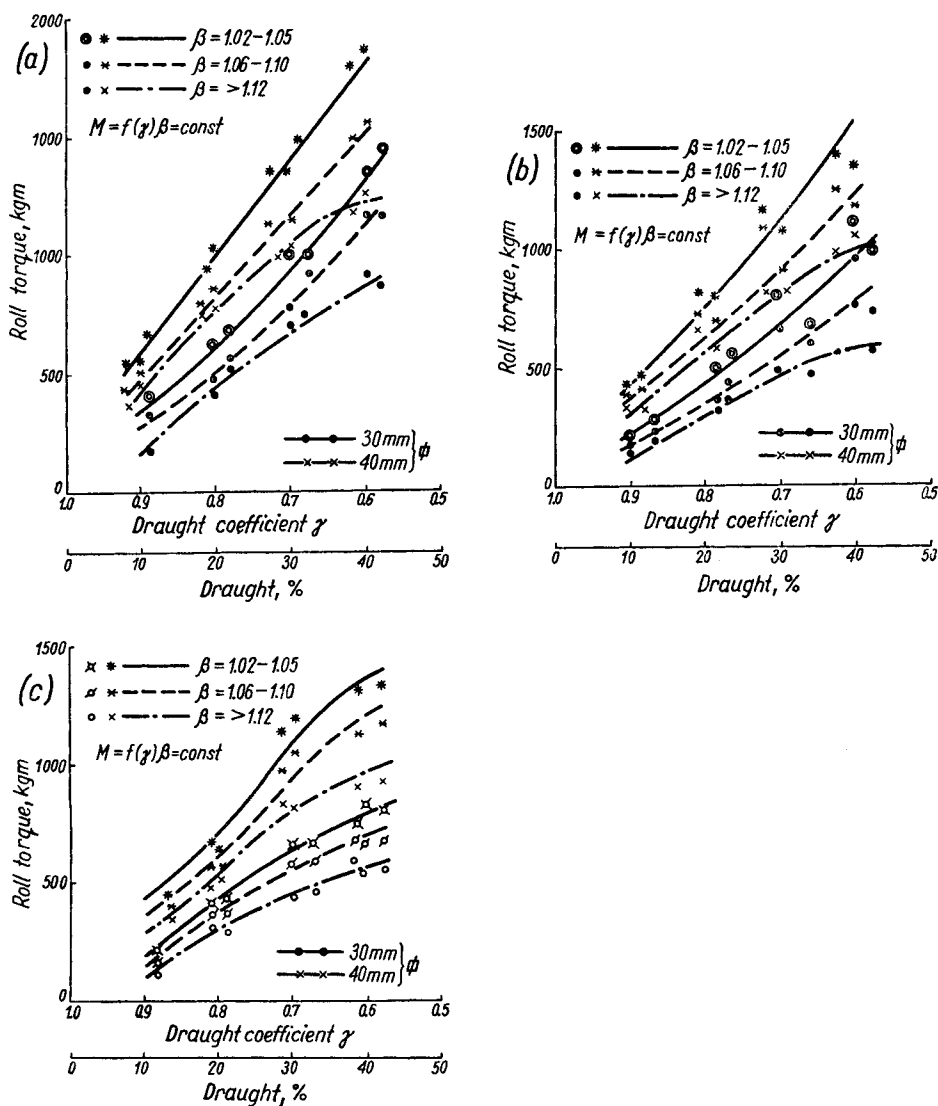


Fig. 4.70. Rolling torque as a function of percentage draught and coefficient of spread  $\beta$ :  
 (a) 900°C, (b) 1000°C, (c) 1100°C. Initial cross-sections 30 and 40 mm square

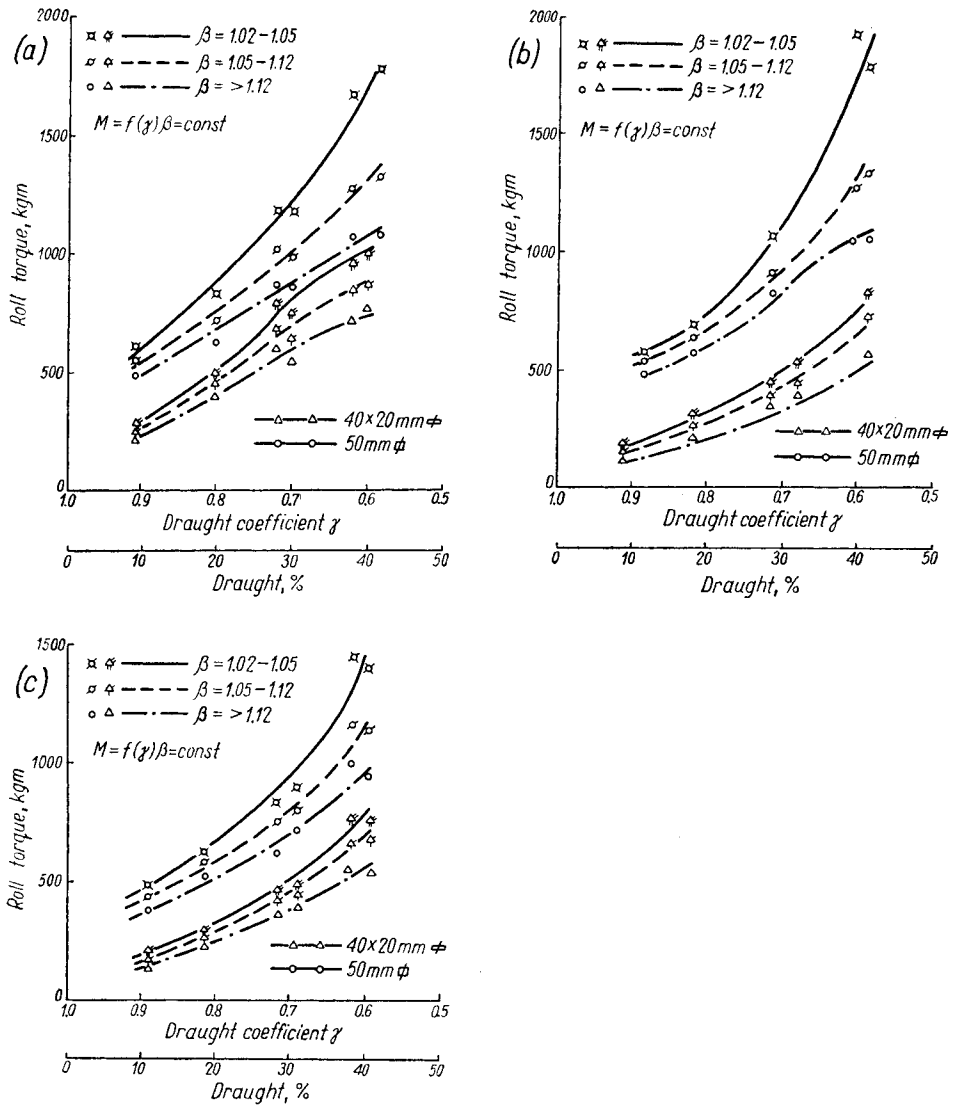


Fig. 4.71. Total rolling torque as a function of percentage draught and coefficient of spread: (a) 900°C, (b) 1000°C, (c) 1100°C. Initial cross-section 50 mm square and 400 × 20 mm flat

The slopes of the curves for limited spread are similar at all temperatures, but vary for free spread depending on the rolling temperature.

The slopes of torque curves for flat bars are similar to those for roll forces at various rolling temperatures (Fig. 4.67).

Considering the behaviour of roll torque and force, it can be seen that the curves are all similar to each other up to a draught of 30% regardless of rolling conditions. Above 30% draught, however, the curves differ and values depend on temperature, conditions of spread and type of pass. The slope of the curves obtained with restricted spread may be explained by the fact that the increase in force due to additional friction in completely filled passes is not proportional to the steadily increasing draught. On the other hand, the steeper slope of the curves obtained with free spread can be explained as being due to lack of force, of additional friction and also speed, which have a marked effect on the type of curve above 30% draught.

Diagrams have been worked out on the basis of the tests which show the effect of additional friction on:

- (a) the roll force,
- (b) the mean resistance to deformation, and
- (c) the total rolling torque.

Having obtained values for these factors under various conditions of rolling, it was possible to plot results against percentage draught at the three rolling temperatures. Bounding lines were drawn through maximum and minimum values thus establishing limiting conditions corresponding to the test conditions. In this case the upper curve shows the threshold values of spread and corresponds to

$$1.02 < \beta < 1.03$$

The lower curve shows the threshold values of spread and corresponds to

$$1.06 < \beta < 1.20$$

These sharply defined assumptions do not accurately determine the conditions which are met with in individual passes. New relations have therefore been determined, based on the principle of constant volume

$$\gamma \beta \lambda = 1$$

Therefore for limited spread

$$\beta_{lim} = 1, \quad \lambda_{lim} = \frac{1}{\gamma} \quad (4.89)$$

This condition is hardly ever found to occur in practice. A correction factor is, therefore, applied which takes into account elongation and spread factors depending on the draught according to the principle of constant volume. Using the principle of constant volume, the following equation can be derived giving the position of the top and bottom boundary curve

$$\gamma \beta \lambda C = 1 \quad (4.90)$$

where  $C$  is a correction factor.

Therefore with limited spread (4.89) we have

$$\beta_{lim} = \frac{1}{C}, \quad \lambda_{lim} = \frac{1}{\gamma} C \quad (4.91)$$

The value of  $C$  is defined as follows

$$C = \frac{(1 + \beta_f) - \beta_r}{\beta_f} \quad (4.92)$$

where

$\beta_f$  = coefficient with free spread,

$\beta_r$  = coefficient with restricted spread.

The reciprocal factor  $C' = 1/C$ , given in the following diagrams, is calculated as

$$C' = \frac{1}{C} = \frac{\beta_f}{(1 + \beta_f) - \beta_r} \quad (4.93)$$

$C'$  is a factor which, depending on conditions of spread, controls the coefficient of additional friction within the range outlined by the top and bottom boundary curves; it varies similarly to the resistance to spread.

Finally, graphs have been worked out which for the above conditions give the effect of additional friction on:

- (a) the roll force,
- (b) the mean resistance to deformation,
- (c) the total rolling torque.

Figures 4.72a–c give values of the correction factor for determination of roll force as a function of additional friction at temperatures of 900, 1000 and 1100°C.

The lateral force  $P_t$  due to friction on the sides of pass can be defined by index  $a'$ , given on the ordinate where (4.86)

$$a' = \frac{P_t}{P_0}$$

The lower limit of index  $a'$  is 1.0 in all cases where spread is free. If spread is restricted,  $a'$  reaches a maximum. A change in temperature results in a slight change of  $a'$ . A change in draught however, shows that the value of  $a'$  follows the bounding lines.

The lower limit of the index in the range from 900–1100°C, and draughts of 8–41%, lies between 5 and 25%. This means that within this range of draught, the forces acting upon the side walls are at most 25% of all vertically acting forces. The maximum values of index  $a'$  lie within limits of 5 and 55% for the same draught and temperatures. In other words, when spread is restricted, the forces due to friction at the side of the pass can reach 55% of the vertical forces. It can be assumed on the average, that for closed passes, the side pressure is about 40% of the vertical forces.

It is known that the resistance to deformation depends on the chemical composition of the steel, the rolling temperature and the conditions of deformation. The preceding calculations for resistance to deformation did not take into account the increase in  $K_{wm}$  due to friction on the sides of the pass; this can be calculated by using the method of pressure measurements in rolling mills, or comparative measurements when rolling with plain cylindrical rolls. Diagrams are presented in Figs. 4.73a–c which give results obtained by means of the second method and show clearly the effect of resistance to deformation. The values given in Figs. 4.73a–c have been obtained with mild steel in the temperature range of 900–1100°C.

To discover the maximum divergence, the mean resistances to deformation in separate passes have been obtained, plotted on graphs, and points of constant initial thicknesses linked. Then the range has been bounded by curves.

Figure 4.73a gives values of  $K_{wm}$  at 900°C. Bounding curves have been drawn for maximum and minimum values. The value of  $a''$  at a constant draught depends on the conditions of spread, which have been determined for limiting values of  $C$ . In general, the maximum values correspond to rolling conditions with fully suppressed spread,

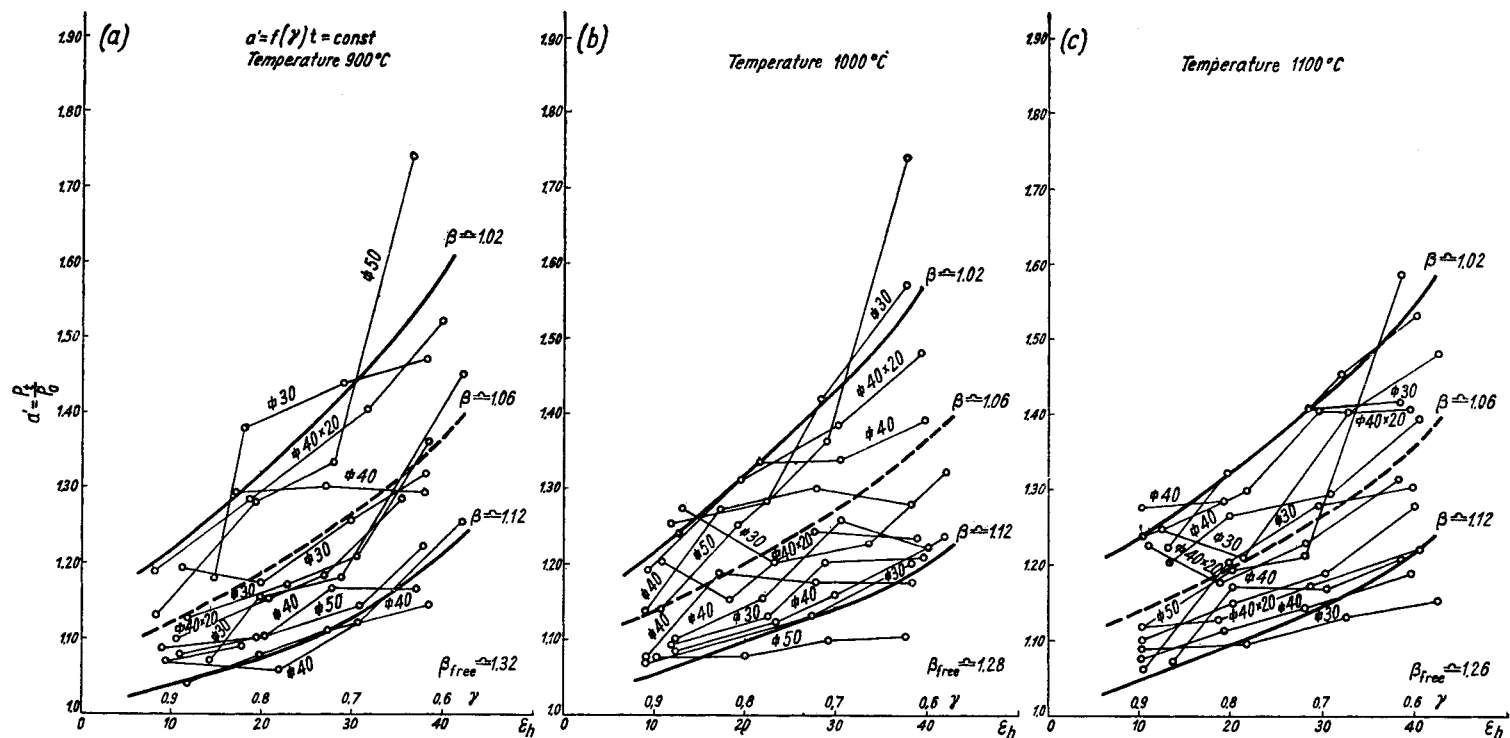


Fig. 4.72. Factor  $a'$  for calculation of roll force in box passes as a function of percentage draught and spread. The upper shadowed curve is valid for  $C' = 1.02$  and the lower boundary curve for  $C' = 1.20$ : (a) 900°C, (b) 1000°C, (c) 1100°C

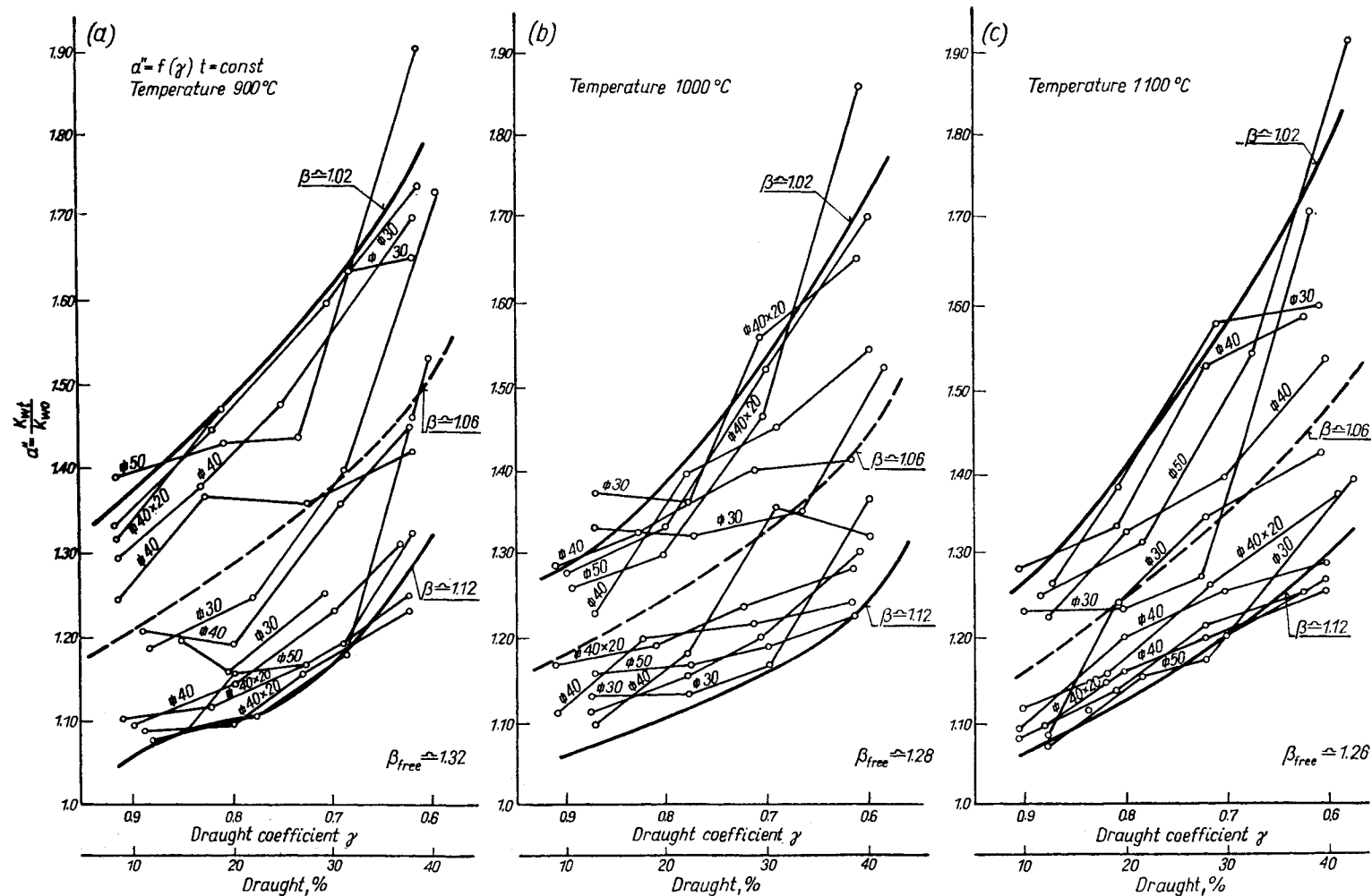


Fig. 4.73. Factor  $\alpha'$  for calculation of resistance to deformation in box passes as a function of percentage draught and spread. The upper shadowed boundary curve is valid for  $C' = 1.02$  and the lower curve for  $C' = 1.20$ : (a) 900°C, (b) 1000°C, (c) 1100°C

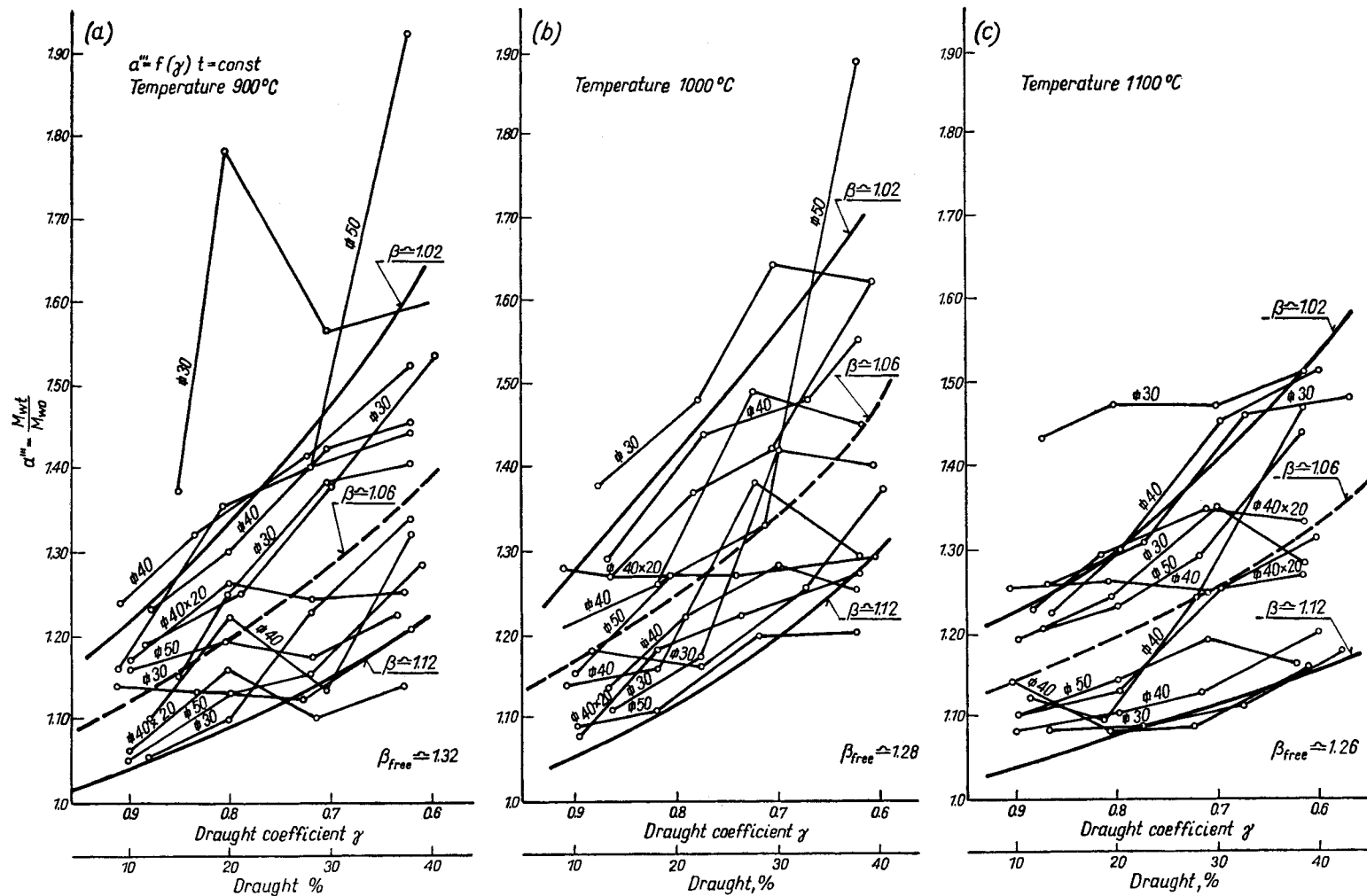


Fig. 4.74. Factor  $a'''$  for calculation of roll torque in box passes as a function of percentage draught and spread. The upper shadowed boundary curve is valid for  $C' = 1.02$  and the lower curve for  $C' = 1.20$ : (a) 900°C, (b) 1000°C, (c) 1100°C

and the minimum values to free spread. Intermediate values correspond to intermediate conditions. A change in rolling temperature alters the limiting field. It is larger when rolling at 900°C (Fig. 4.73a), and smaller at 1000°C (Fig. 4.73b), decreasing further when rolling at 1100°C (Fig. 4.73c). In both cases the maximum values decrease at a greater rate than the minimum ones.

The maximum percentage values of these factors at 900°C lie between 38 and 85%. At 1000°C the values of  $a''$  vary between 8 and 73%, and at 1100°C between 20 and 70%.

Figures 4.74a–c give values of the total torque as a function of draught at temperatures of 900, 1000 and 1100°C. It can be seen that the values of  $a'''$  for roll torque and force do not vary greatly from each other.

A change in temperature results in only a small variation as was observed for  $a'$ . Minimum values of  $a'''$ , within the temperature investigated and at a draught of 8–41%, lie within the limits of 4–30%. The maximum values of  $a'''$ , for the same draught and temperature range, lie within limits of 18–60%. For the top and bottom bounding curves similar conditions are valid as for previous diagrams. Mean values have also been plotted.

The diagrams give information on the effect of additional friction on

- (a) the roll force,
- b) the mean resistance to deformation, and
- (c) the total torque,

and can, therefore, be applied in industry. Above all, the true degree of resistance to spread must be determined. The conditions of spread, outlined by the area, are given by  $1.02 < \beta < 1.20$ . Upper limit values correspond to fully suppressed spread and low rolling temperatures, and the lower limit values to the opposite conditions. The correction factor  $C'$  given in equation (4.93) was applied when calculating mean values.

The coefficient  $\beta_f$  for free spread was obtained by using Wusatowski's equation (3.45). The coefficient  $\beta_f$  can also be calculated or determined when the size of the pass is known.

After calculation of the correction factor  $C'$  the resistance to spread within the limiting conditions can be determined as in equation (4.89). Before multiplying by the coefficient of draught, elongation and spread, it must be checked whether the principle of constant volume is fulfilled according to equation (4.90).

Values of  $a'$ ,  $a''$ , and  $a'''$  are then found by interpolation. The effect of additional friction under the various conditions of spread can, therefore, be determined.

#### 4.3.2. INFLUENCE OF ADDITIONAL FRICTION AND DEFORMATION WHEN ROLLING IN BREAKING-DOWN PASSES [157]

Until now no data was available for determining the influence of additional friction and internal metal flow during rolling in breaking-down passes. In this field only investigations carried out by G. Zouhar [157], are known. In these investigations roll forces and torques have been measured for rolling in breaking-down systems diamond-square, diamond-diamond, oval-square-oval, and oval-round-oval.

For these tests carbon steels of varying carbon content, some low and high-alloy steels, and also copper and aluminium, have been used. The chemical composition and size of specimens are given in Table 4.8 [157]. The pass layout for rolls of different diameters, cross-sectional dimensions of specimens, and the number of passes, are



given in Table 4.9 [157]. Finally Table 4.10 gives values of initial rolling temperatures and roll speeds [157]. Values calculated from experimental results were tabulated and compared, and plotted on diagrams [157].

TABLE 4.8  
COMPOSITION OF MATERIALS USED FOR INVESTIGATIONS ON BREAKING-DOWN SYSTEMS [157]

| No. | Metal          | Chemical analysis, % |      |      |      | Dimensions and condition                      |
|-----|----------------|----------------------|------|------|------|-----------------------------------------------|
|     |                | C                    | Si   | Mn   | Cr   |                                               |
| 1   | MSt3b          | 0.17                 | 0.17 | 0.51 | —    | 50 □ and 50 Ø rolled                          |
| 2   | CK45           | 0.48                 | 0.29 | 0.74 | 0.06 | 50 □ and 50 Ø rolled                          |
| 3   | MK73           | 0.70                 | 0.19 | 0.46 | 0.09 | 50 □ rolled                                   |
| 4   | St7            | 0.47                 | 0.22 | 0.86 | 0.13 | 50 Ø rolled                                   |
| 5   | 16MnCr5        | 0.18                 | 0.35 | 1.14 | 0.87 | 50 □ and 50 Ø rolled                          |
| 6   | 100Cr6         | 1.00                 | 0.28 | 0.30 | 1.48 | 50 □ and 50 Ø rolled                          |
| 7   | 18Mn5          | 0.15                 | 0.52 | 1.36 | 0.04 | 50 □ and 50 Ø rolled                          |
| 8   | 65Si7          | 0.72                 | 1.60 | 0.76 | 0.15 | 50 □ rolled                                   |
| 9   | 55SiMn7        | 0.60                 | 1.60 | 0.72 | 0.27 | 50 Ø rolled and turned from 55 Ø              |
| 10  | Copper         | —                    | —    | —    | —    | 50 □ and 50 Ø bright drawn                    |
| 11  | Pure aluminium | —                    | —    | —    | —    | 50 □ and 50 Ø<br>48 □ chill-cast and machined |

TABLE 4.9  
PASS LAYOUT AND APPROXIMATE CROSS-SECTIONAL DIMENSIONS OF SPECIMENS

| Pass No. | Square-oval |           | Round-oval        | Square-diamond  | Diamond-diamond  |                          |
|----------|-------------|-----------|-------------------|-----------------|------------------|--------------------------|
| 0        | 50 □        |           | 50 Ø              | 50 □            | 50 □             | Roll dia. 445 and 456 mm |
| 1        | 61.5/25     | 61.5/26   | 60/31.3           | 75/44           | 71/52            |                          |
| 2        | 30 □        | 31.8 □    | 37.6 Ø            | 39 □            | 56/46            |                          |
| 3        | 49.5/15     | 47.5/17.2 | 49/22             | 56/34           | 53/41            |                          |
| 4        | 20.3 □      | 22.8 □    | 28.5 Ø            | 30.5 □          | 47.5/35.5        |                          |
| 5        | 36.5/10.4   | 35/12.7   | 39.5/16.8         | 46.5/27         | 44/31            | Roll dia. 360 mm         |
| 6        | 15.2 □      | 17.5 □    | 22.4 Ø            | 24 □            | 36.5/27.5        |                          |
| 7        | 23/10.5     | 28.5/10.3 | 30/13.4           | 39.20           | 33/24.5          |                          |
| 8        | 11.6 □      | 12.7 □    | 18 Ø              | 18 □            | 30.5/21.5        |                          |
| 9        | 17.8/7.3    | 20.7/7.6  | 26/10.7<br>14.4 Ø | 29/15<br>14.2 □ | 26.5/19<br>24/17 |                          |
| 10       | 8.7 □       | 9.7 □     | 19.3/9.7          | 24.2/11.4       | 22/15.5          | Roll dia. 280 mm         |
| 11       | 14.8/5.7    | 14.1/6.9  | 11.9 Ø            | 10.5 □          | 20/13.7          |                          |
| 12       | 6.9 □       | 8.2 □     |                   |                 |                  |                          |
| 13       | —           |           | 16.9/7.7          | 17.5/10.7       | 17.4/13.5        |                          |
| 14       | —           |           | 9.2 Ø             | 9.2 □           | 17/10.9          |                          |
| 15       | —           |           | 13.9/6.3          | —               | 15.7/9.4         |                          |
| 16       | —           |           | 7.7 Ø             | —               | 13.7/8.5         |                          |
| 17       | —           |           | 11/5.6            | —               | 6.7 □            |                          |
| 18       | —           |           | 6.6 Ø             | —               |                  |                          |

TABLE 4.10  
INITIAL ROLLING TEMPERATURES FOR VARIOUS BREAKING-DOWN SYSTEMS

| Pass sequence   | Temperature of furnace (°C) for rolls |                          |                           |
|-----------------|---------------------------------------|--------------------------|---------------------------|
|                 | Ø 445 and 456 mm<br>$n = 65$ rpm      | Ø 360 mm<br>$n = 80$ rpm | Ø 380 mm<br>$n = 100$ rpm |
| Square-oval     | 1260<br>1200                          | 1050<br>970              | 950<br>900                |
| Round-oval      | 1260<br>1200<br>1140                  | 1070<br>1020<br>970      | 930<br>900                |
| Square-diamond  |                                       | 1040<br>980              |                           |
| Diamond-diamond |                                       | 1080<br>1020             |                           |

Figure 4.75 gives the results of the breaking-down system round-oval. Figures 4.76 and 4.77 give results for both the breaking-down systems square-oval. Results obtained for the breaking-down system square-diamond are shown in Fig. 4.78, and finally those for the breaking-down system diamond-diamond, in Fig. 4.79.

Each columnar diagram consists of:

- the finishing rolling temperatures, °C,
- the roll forces  $P$ , t,
- the roll torques  $M_w$ , kgm,
- the mean resistance to deformation  $K_{wm}$ , kg/mm<sup>2</sup>,
- the specific roll work  $a_w$ , kgmm/mm<sup>3</sup>.

On the abscissa of Figs. 4.75–4.79 numbers of passes of the breaking-down systems investigated are plotted. The diameter of the roll in which the respective passes are cut may be seen in Table 4.9 and 4.10.

The values of mean resistance to deformation when rolling in pass No. 3 and 4 of the breaking-down system round-oval, for various finishing rolling temperatures, are given in Fig. 4.80. Steels MSt3b, St7, 100Cr6 (acc. to SES), copper and aluminium, have been used in the experiments. It may be seen from this diagram that the steel quality has little influence on the results. The resistance to deformation in the oval pass is about 10–15% greater than in the succeeding round pass

$$K_{w3} \approx 1.1-1.15K_{w4}$$

Results of similar tests in the third and fourth pass of the breaking-down system square-diamond are plotted in Fig. 4.81. Once more MSt3b, St7 and 100Cr6 steel qualities have been used. However, in this case small variations in the materials used were observed. The resistance to deformation in pass No. 3, for a pass square into diamond lies 20–28% higher than in pass No. 4 for diamond into square, i.e.

$$K_{w3} \approx 1.2-1.28K_{w4}$$

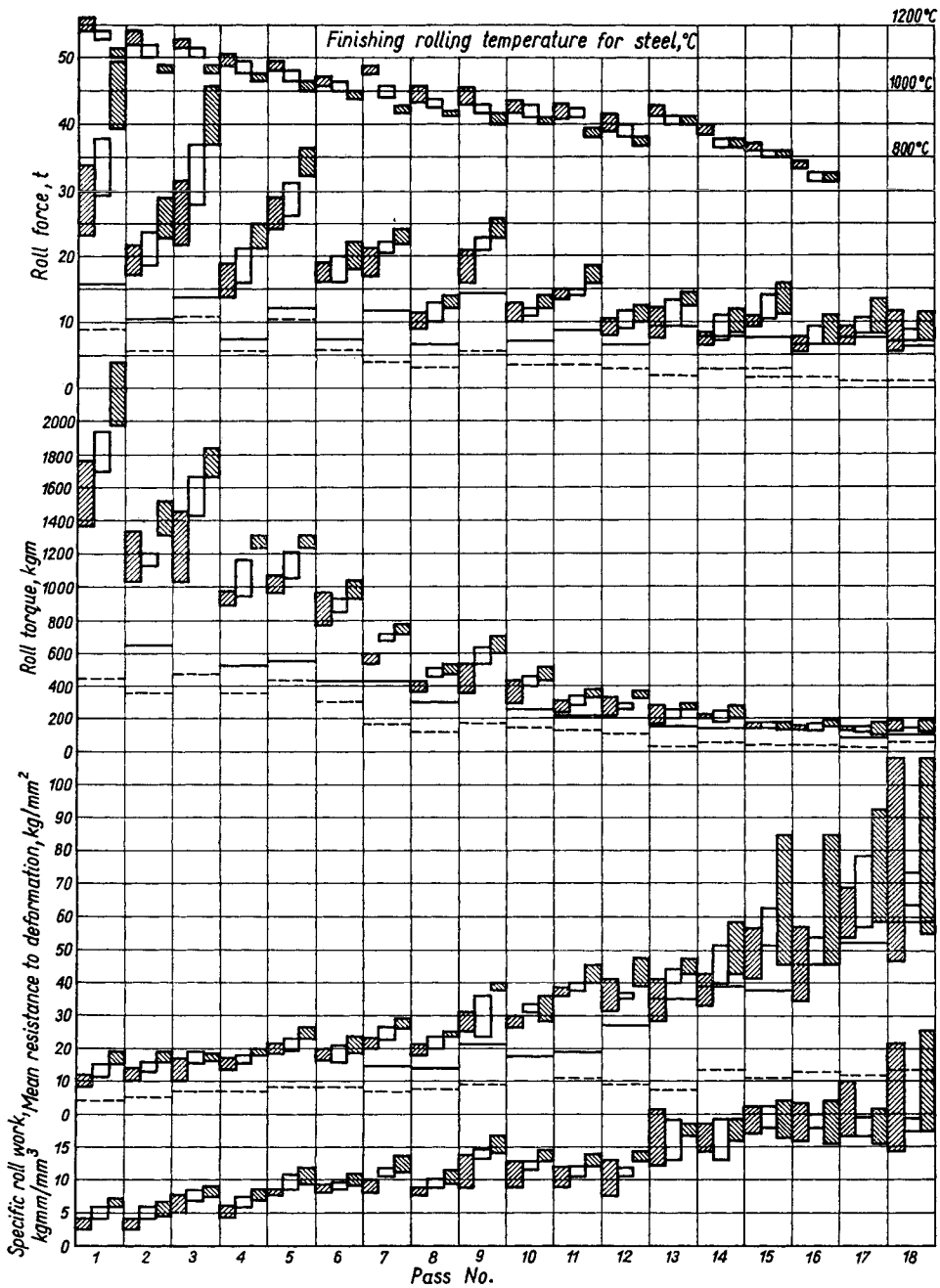


Fig. 4.75. Finishing rolling temperature, roll force, mean resistance to deformation, roll torque and specific roll work for the breaking-down pass system round-oval

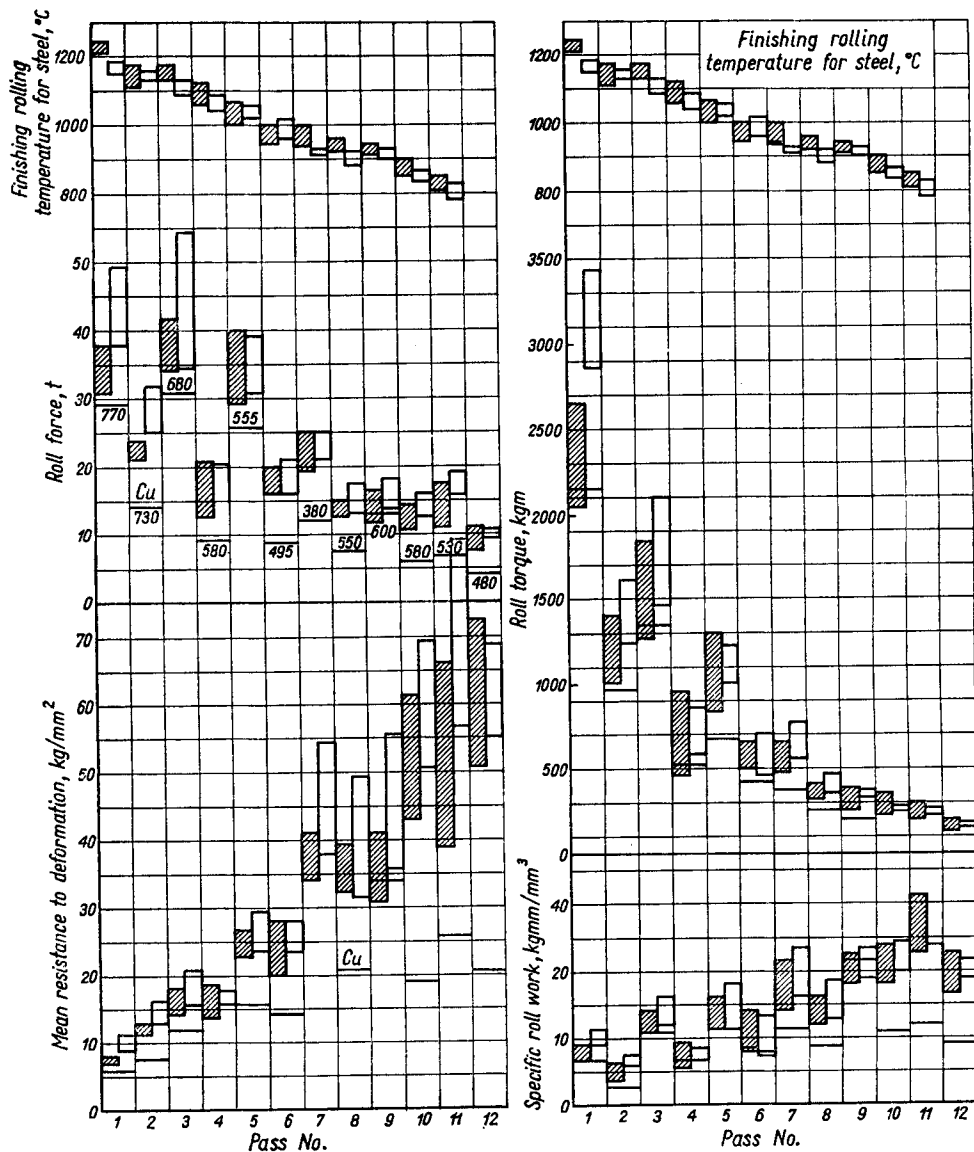


Fig. 4.76. Finishing rolling temperature, roll force, torque and specific roll work for the breaking-down pass system square-oval (with great elongation)

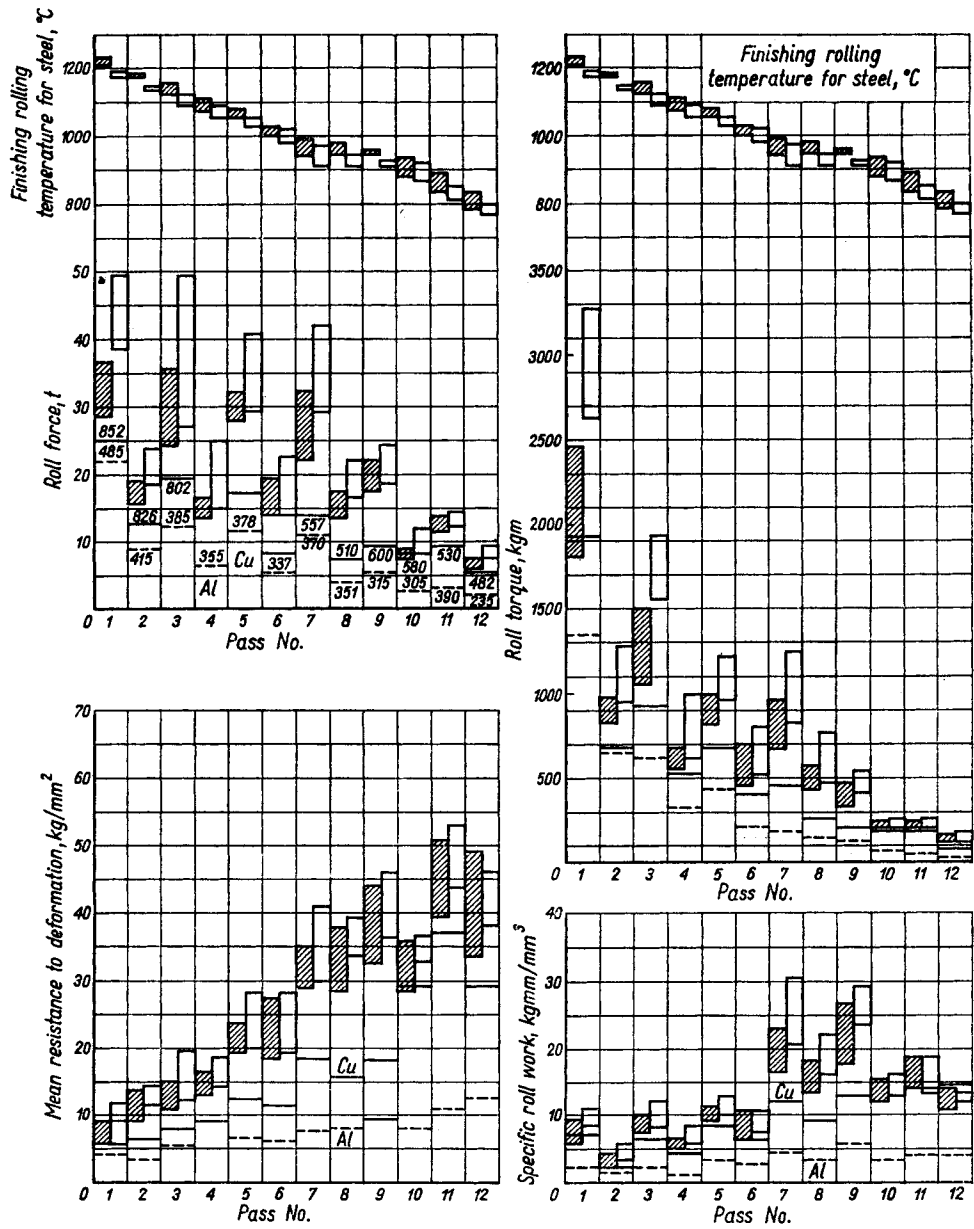


Fig. 4.77. Finishing rolling temperature, roll force, torque and specific roll work for the breaking-down pass system square-oval (with small elongation)

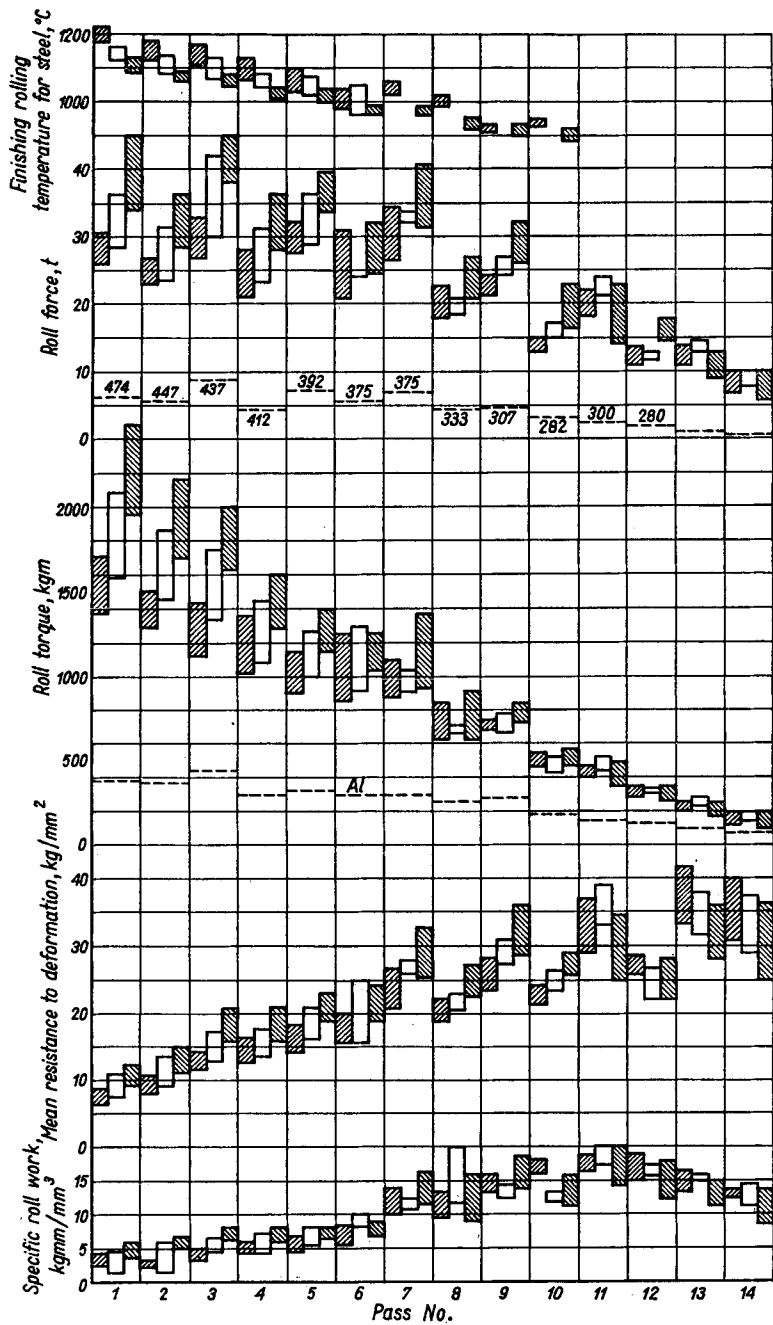


Fig. 4.78. Finishing rolling temperature, roll force, torque and specific roll work for the breaking-down pass system square-diamond

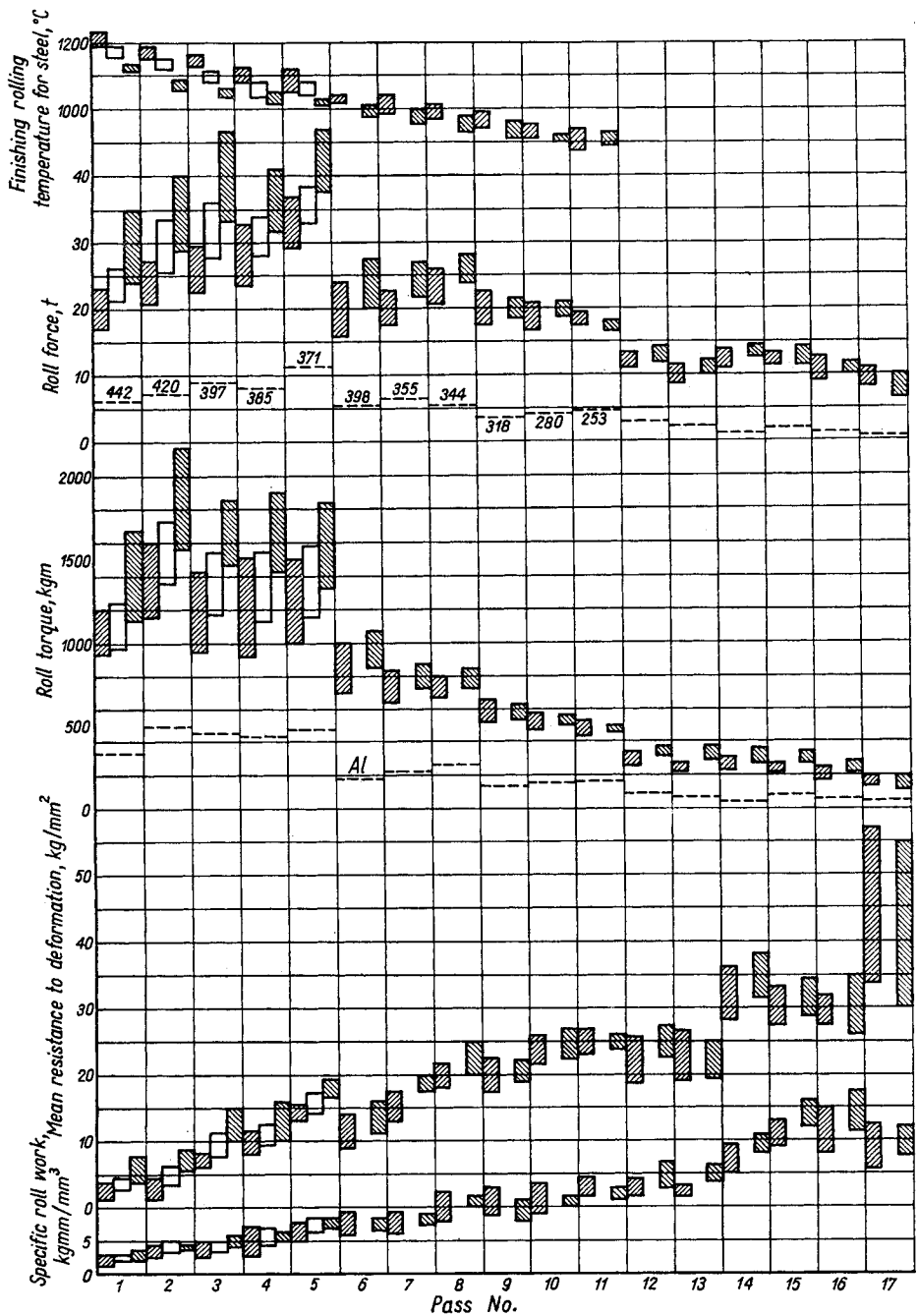


Fig. 4.79. Finishing rolling temperature, roll force, torque and specific roll work for the breaking-down pass system diamond-diamond

Furthermore, comparative experiments have been carried out in the third and fourth pass of the breaking-down system diamond–diamond using the same steel qualities. Figure 4.82 shows that differences occur for various steel qualities. The influence of the pass form is relatively small, i.e.

$$K_{w3} \approx 1.0\text{--}1.05K_{w4}$$

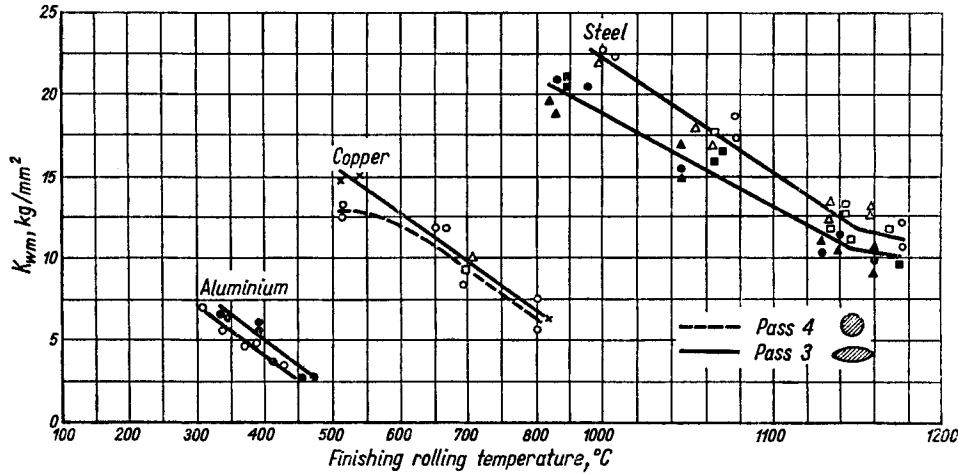


Fig. 4.80. Mean resistance to deformation in passes 3 and 4 (round-oval) as a function of the finishing rolling temperature

Different conditions occur in the breaking-down sequence square-oval. Figures 4.83 and 4.84 give results of rolling experiments in oval pass No. 3 for initial cross-sections of 30 and 33  $\text{mm}^2$ , and various rolling temperatures of the MSt3b low-carbon

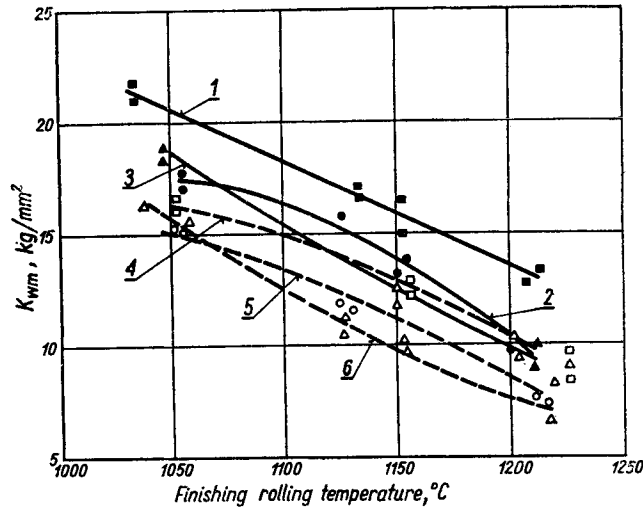


Fig. 4.81. Mean resistance to deformation in passes 3 and 4 (square-diamond) as a function of the finishing rolling temperature for steels in pass 3: 1 — MSt3b, 2 — MK73, 3 — 100Cr6; in pass 4: 4 — MSt3b, 5 — MK73, 6 — 100Cr6 (symbols of German steels — see Table 4.8).



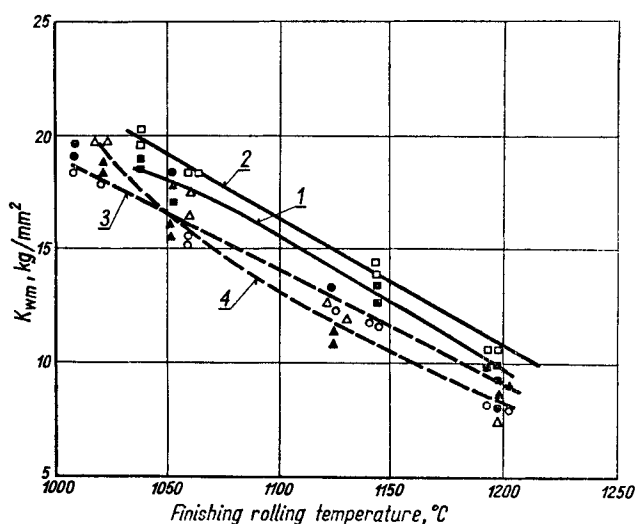


Fig. 4.82. Mean resistance to deformation in passes 3 and 4 (diamond-diamond) as a function of the finishing rolling temperature for steels in pass 3: 1 — MSt3b; in pass 4: 2 — MSt3b, 3 — MK73, 4 — 100Cr6 (German symbols — Table 4.8).

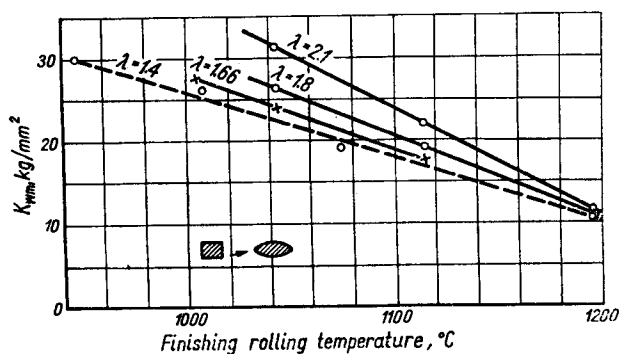


Fig. 4.83. Mean resistance to deformation in pass 3 (square-oval) for MSt3b low-carbon steel, as a function of the finishing rolling temperature for various values of coefficient of elongation (Table 4.8)

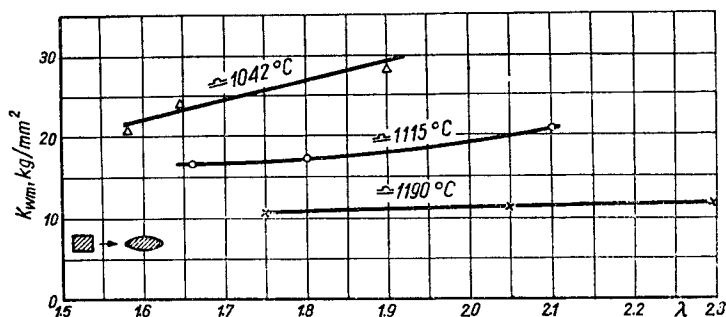


Fig. 4.84. Mean resistance to deformation in pass 3 (square-oval) for MSt3b low-carbon steel as a function of the coefficient of elongation for various finishing rolling temperatures

steel. It can be seen from Fig. 4.84 that the influence of the breaking-down factor at high rolling temperatures, is insignificant. Figure 4.85 shows the resistance to deformation arising during rolling of oval bars of different cross-section of finishing squares

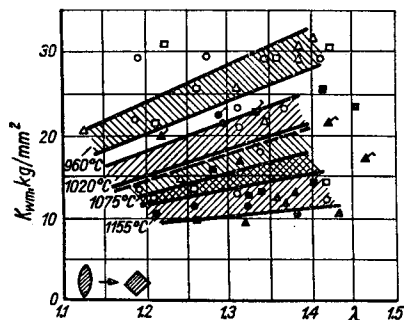


Fig. 4.85. Mean resistance to deformation in pass 4 (square-oval) for MST3b low-carbon steel, as a function of the coefficient of elongation for various finishing rolling temperatures

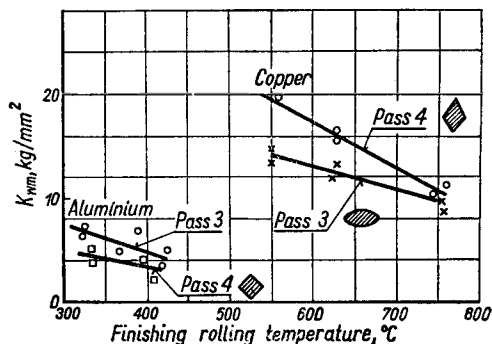


Fig. 4.86. Mean resistance to deformation for copper and aluminium in passes 3 and 4 (square-oval)

of side 20.5 and 22.5 mm at various rolling temperatures. Also for this diagram the decreasing influence of the breaking-down factor with increasing rolling temperature can be seen. Results obtained for copper and aluminium are given in Fig. 4.86.

TABLE 4.11  
RESISTANCE TO DEFORMATION FOR PASS SEQUENCE SQUARE-OVAL AND  
OVAL-SQUARE FOR 0.17 PER CENT LOW-CARBON STEEL  
(MSt3b acc. to SES, Table 4.8)

| $\lambda$ | Pass sequence    | $K_w$<br>kg/mm <sup>2</sup> | Finishing rolling<br>temperature, °C |
|-----------|------------------|-----------------------------|--------------------------------------|
| 1.8       | Square into oval | 10.0                        | 1190                                 |
|           |                  | 17.0                        | 1115                                 |
|           |                  | 26.5                        | 1042                                 |
| 1.4       | Oval into square | 15.0                        | 1155                                 |
|           |                  | 18.5                        | 1075                                 |
|           |                  | 24.5                        | 1020                                 |
|           |                  | 30.0                        | 942                                  |

Table 4.11 gives values of resistance to deformation of the MSt3b low-carbon steel at various rolling temperatures of the breaking-down sequence square-oval-square. The coefficients of elongation were  $\lambda = 1.8$  for the sequence square into oval, and  $\lambda = 1.4$  for oval into square. The values of resistance to deformation arising during rolling of copper and aluminium in the same passes show smaller differences, than for mild steel, with constant elongation factors. The resistance to deformation  $K_{w4}$  of copper shows a different behaviour from those of aluminium and steel.

During these experiments measurements for calculation of rolling torque have also been made. Figure 4.87 gives the relation of the ratio of lever arm to the projected length of contact  $a/l_d$ , as a function of pass sequence and rolling temperature. In the

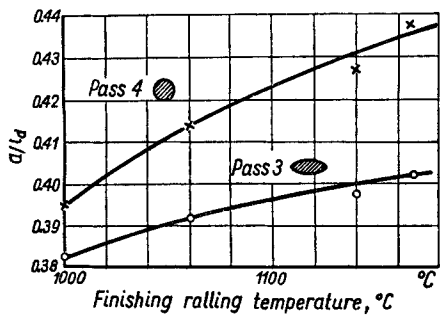


Fig. 4.87. Ratio of lever arm to length of arc of contact  $a/l_d$  for passes 3 and 4 of the round-oval sequence, as a function of finishing rolling temperature

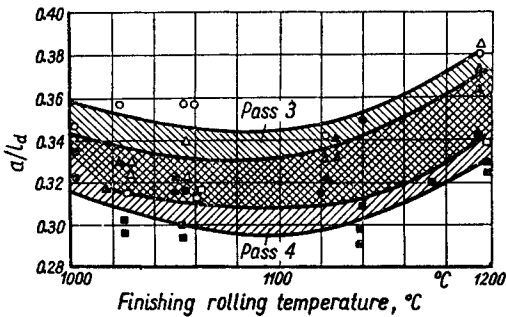


Fig. 4.88. Ratio of lever arm to length of arc of contact  $a/l_d$  for passes 3 and 4 of the diamond-diamond sequence, as a function of finishing rolling temperature

breaking-down system it is approximately 0.4 and decreases with rolling temperature. The projected length of contact  $l_d$  has been related to the pass groove. The roll torque is determined as

$$M_w = 2 Pa$$

The rolling temperature does not influence the breaking-down system diamond-diamond, as shown in Fig. 4.88 for results of rolling in passes No. 3 and 4. For the passes considered, the ratio of lever arm to the projected length of contact lies considerably below 0.4.

Similar conditions occur also for the breaking-down sequence square-diamond, as shown in Fig. 4.89.

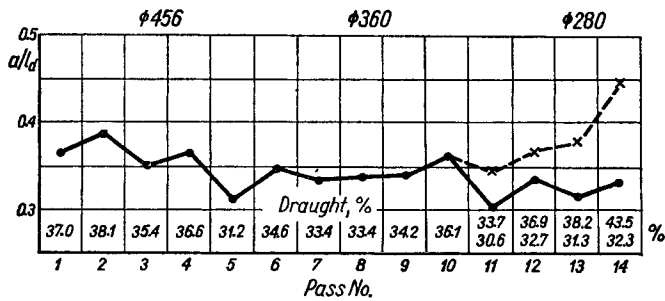


Fig. 4.89. Ratio of lever arm to length of arc of contact  $a/l_d$  for the breaking-down pass system square-diamond

Values of the ratio of the lever arm to the projected length of contact are shown in Figs. 4.90 and 4.91. They show the influence of the breaking-down factor and geometrical conditions, giving values of 0.3–0.48.

Plotting the ratios of  $\varphi_h = \log_e h_1/h_2$  to  $\varphi_\lambda = \log_e F_1/F_2$  against number of passes for the investigated breaking-down sequences, Fig. 4.92 is obtained. The distribution of the ratio of  $\varphi_h/\varphi_\lambda$  shows obviously an increase in elongation with increasing number of passes, though a significant influence of breaking-down systems may be confirmed.

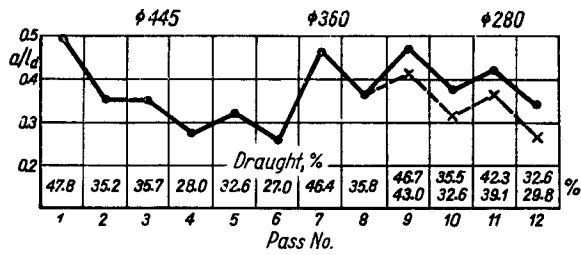


Fig. 4.90. Ratio of lever arm to length of arc of contact  $a/l_d$  for the breaking-down pass system square-oval (with great elongation)

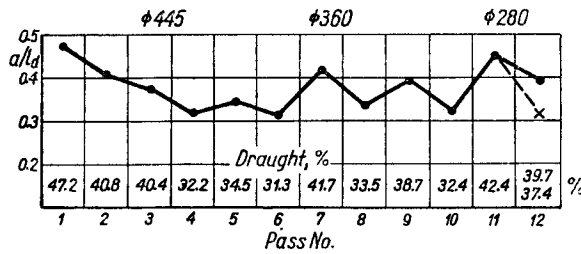


Fig. 4.91. Ratio of lever arm to length of arc of contact  $a/l_d$  for the breaking-down pass system square-oval (with small elongation)

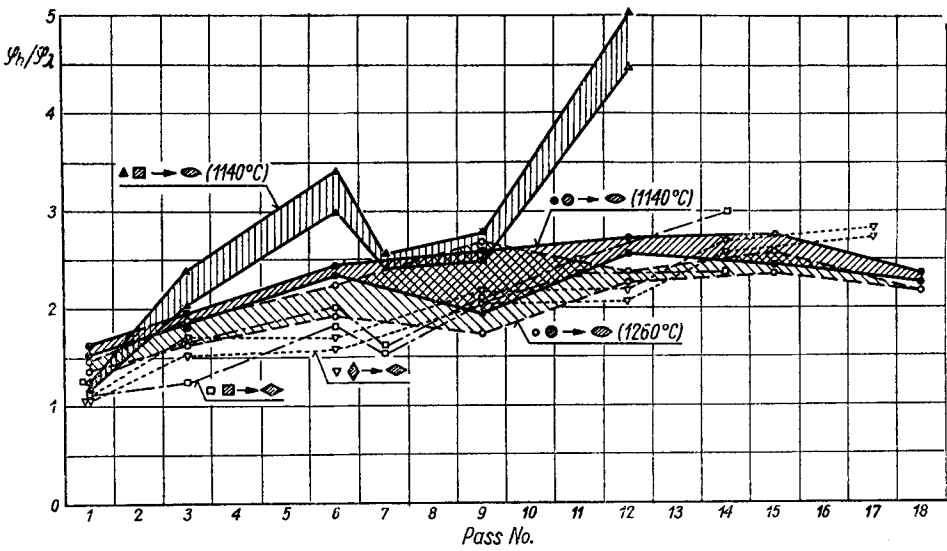


Fig. 4.92. Ratio of breaking-down pass factors  $\varphi_h/\varphi_\lambda$  of the investigated breaking-down pass systems

Results of experiments for determining the influence of the strain rate on the resistance to deformation during rolling in breaking-down systems, are given as an example for the sequence square–diamond in Table 4.12 [157]. MSt3b steel quality has been rolled in passes No. 3 and 4 at various temperatures, and at different rates of deformation. It can be seen from the values that within the investigated range, the influence of strain rate must be taken into account when calculating roll forces.

TABLE 4.12  
INFLUENCE OF ROLLING SPEED ON RESISTANCE TO DEFORMATION FOR BREAKING-DOWN PASS  
SEQUENCE SQUARE–DIAMOND–SQUARE  
(furnace temperature 1200°C)

| Pass No. | $n$<br>rpm | $\varphi$<br>sec <sup>-1</sup> | Steel              | Finishing rolling temperature, °C | $K_w$<br>kg/mm <sup>2</sup> |
|----------|------------|--------------------------------|--------------------|-----------------------------------|-----------------------------|
| 3        | 18.7       | 2.97                           | MSt3b<br>(0.17% C) | 1175                              | 10.8                        |
| 3        | 18.6       |                                |                    | 1161                              | 10.5                        |
| 3        | 142.0      |                                |                    | —                                 | 13.6                        |
| 3        | 142.0      |                                |                    | —                                 | 13.7                        |
| 4        | 18.0       | 2.67                           | MSt3b<br>(0.17% C) | 1149                              | 13.2                        |
| 4        | 18.8       |                                |                    | —                                 | 13.8                        |
| 4        | 140.0      |                                |                    | 1170                              | 16.0                        |
| 4        | 143.0      |                                |                    | —                                 | 14.8                        |
| 3        | 19.0       | 2.67                           | MSt3b<br>(0.17% C) | —                                 | 11.4                        |
| 3        | 19.0       |                                |                    | 1109                              | 12.0                        |
| 3        | 142.0      |                                |                    | 1136                              | 17.0                        |
| 3        | 142.0      |                                |                    | —                                 | 17.3                        |
| 4        | 18.8       | 2.69                           | MSt3b<br>(0.17% C) | 1086                              | 10.8                        |
| 4        | 19.4       |                                |                    | 1100                              | 11.3                        |
| 4        | 140.0      |                                |                    | 1140                              | 14.3                        |
| 4        | 140.0      |                                |                    | 1122                              | 15.2                        |

Finally experimentally determined values of resistance to deformation for the breaking-down sequences square–oval and square–diamond have been compared with calculated values according to Siebel's equation (4.60) in Figs. 4.93–4.95. The full lines denote measured values, and the broken lines those calculated according to Siebel. Though the general run of curves gives sufficient agreement, the absolute values differ considerably. Based on this investigation values of coefficient of additional friction  $a_2$ , given in Table 4.6, should be corrected as follows [157]:

|                 |                         |                |                         |
|-----------------|-------------------------|----------------|-------------------------|
| square–square   | $a_2 = 1.1$             | square–diamond | $a_2 = 1.1$             |
| diamond–diamond | $a_2 = 1.1$             | oval–square    | $a_2 = 1.1\text{--}1.2$ |
| square–oval     | $a_2 = 1.2\text{--}1.3$ |                |                         |

To obtain a comparison of the preceding results with the behaviour of some high-alloy steels during rolling in breaking-down systems, the steel qualities given in Table 4.13 have been rolled in the breaking-down sequences square–diamond and round–oval, measuring once more roll forces and torques. The initial rolling temperatures were between 1050 and 1090°C. As initial blanks billets 50 mm square have been used.

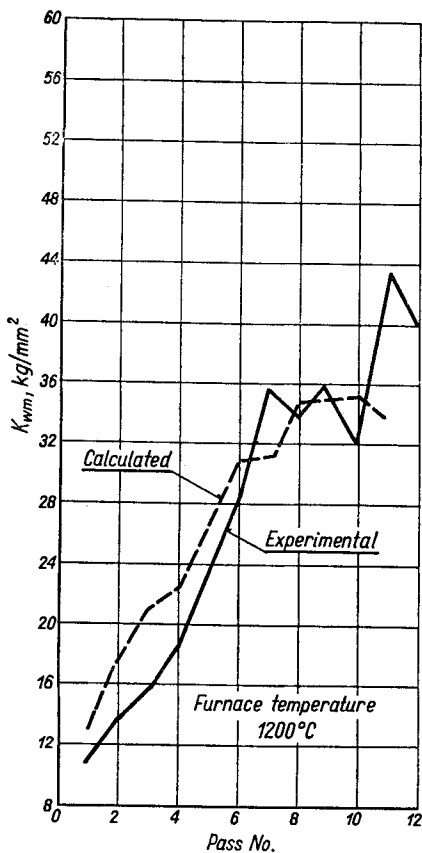


Fig. 4.93. Experimental and calculated values of mean resistance to deformation for the MSt3b low-carbon steel (square-oval, with small elongation)

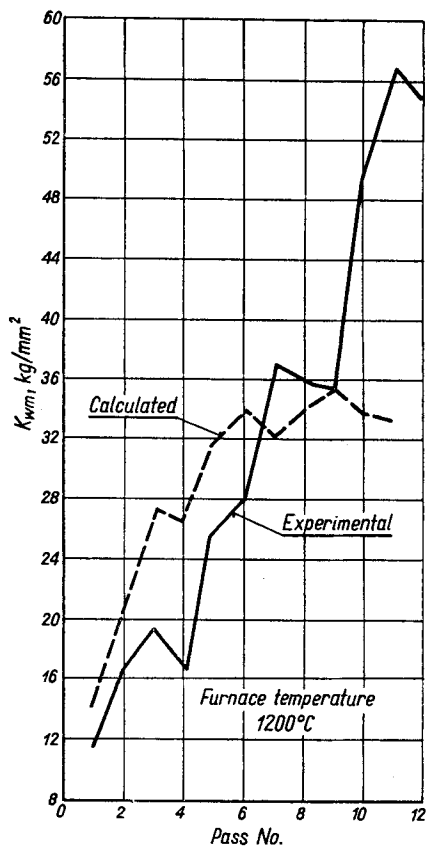


Fig. 4.94. Experimental and calculated values of mean resistance to deformation for the MSt3b low-carbon steel (square-oval, with great elongation)

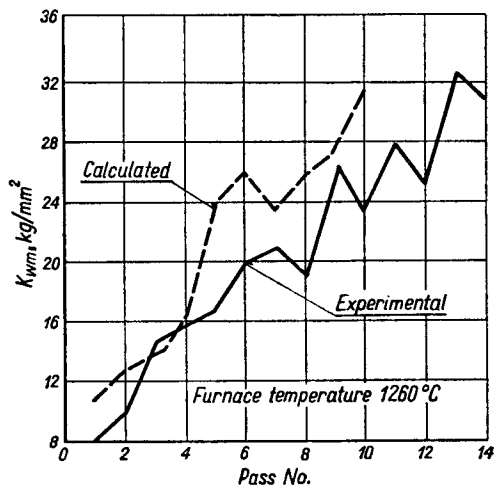


Fig. 4.95. Experimental and calculated values of mean resistance to deformation for the MSt3b low-carbon steel (square-diamond)

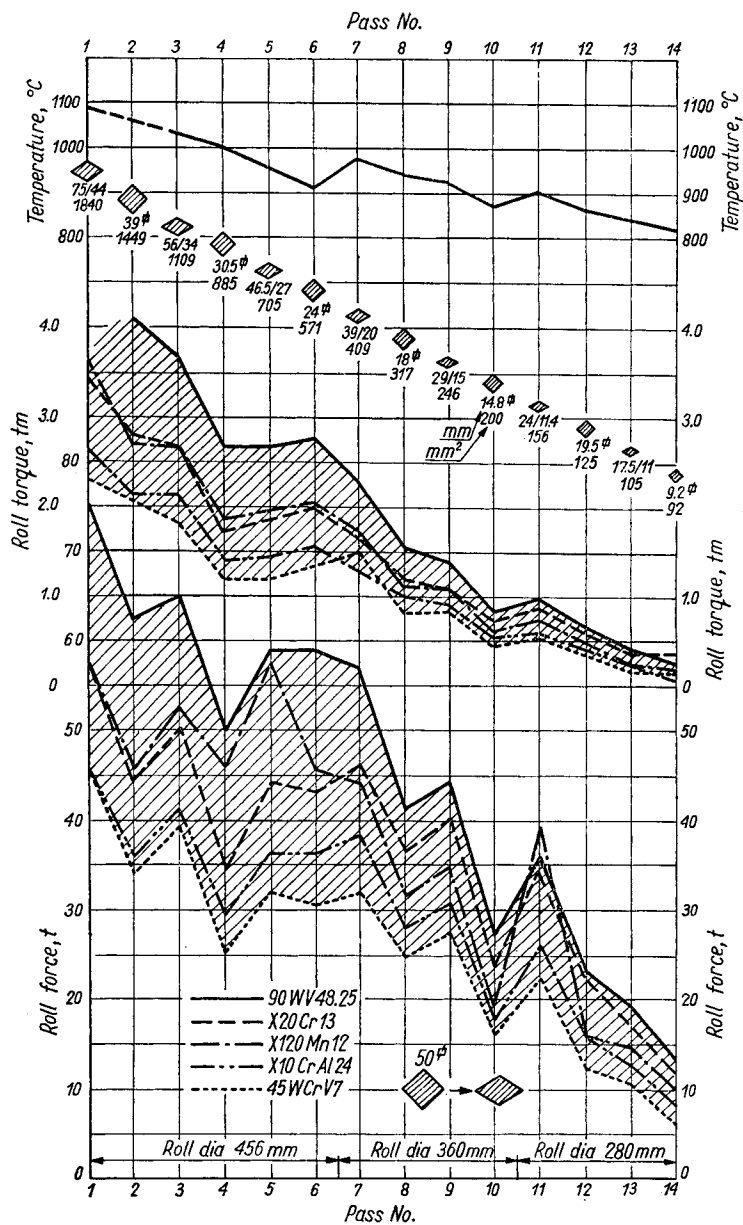


Fig. 4.96. Finishing rolling temperature, roll force and torque for rolling of high-alloy steels given in Table 4.13, in the breaking-down pass sequence square-diamond [158]

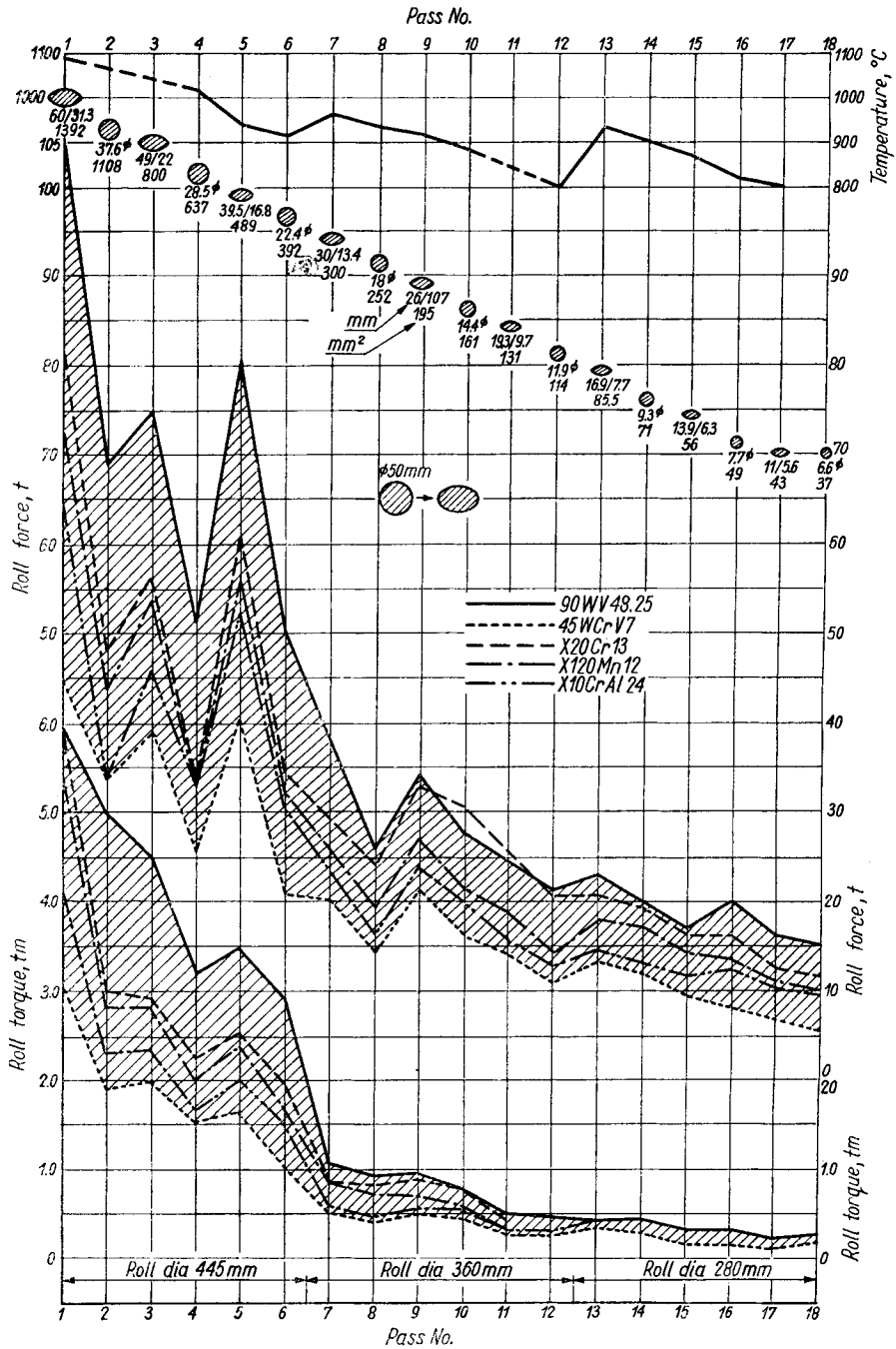


Fig. 4.97. Finishing rolling temperature, roll force and torque for rolling of high-alloy steels given in Table 4.13 in the breaking-down pass system round-oval [158]



TABLE 4.13  
CHEMICAL ANALYSIS OF ROLLED STEELS

| No. | Steel type from SES           | Chemical analysis, % |              |              |              |              |              |      |
|-----|-------------------------------|----------------------|--------------|--------------|--------------|--------------|--------------|------|
|     |                               | C                    | Si           | Mn           | Cr           | V            | W            | Al   |
| 1   | 45WCrV7 for high temperatures | 0.40 to 0.50         | 0.80 to 1.00 | 0.20 to 0.40 | 0.90 to 1.20 | 0.15 to 0.20 | 1.70 to 2.00 | —    |
| 2   | 90WV48.25 high speed steel    | 0.85                 | to 0.25      | to 0.25      | 4.10         | 2.30         | 11.8         | —    |
| 3   | X10CrAl24 stainless steel     | to 0.12              | to 1.30      | 0.20         | 23.0         | —            | —            | 1.30 |
| 4   | X20Cr13 stainless steel       | 0.17                 | 0.30         | 0.20         | 12.50        | —            | —            | —    |
| 5   | X120Mn12 manganese steel      | 1.10                 | 0.30         | 12.0         | —            | —            | —            | —    |

Initial billet: 50 mm  $\Phi$ ; condition: rolled, for all steels and stretching systems.

The results obtained are shown in Figs. 4.96 and 4.97 [157]. It can be seen that when rolling these steels considerable differences and significantly greater roll forces occur, than in the cases previously discussed.

#### 4.3.3. BASIC FORMULAE FOR CALCULATION OF ROLL POWER

The formula for determination of the true work of deformation during rolling is

$$A_w = K_w b_2 h_2 l_2 \log_e \frac{h_1}{h_2}$$

When this is known, it is easy to calculate the true power. Since by definition power equals the work done in unit time, the power  $N_w$  required for a pass is equal to  $A_w$  divided by the pass time  $t$ .

As the exit speed of the rolled stock from the roll gap is  $w_2 = l_2/t$  the following formula for the pure power of deformation is obtained

$$N_w = K_w b_2 h_2 w_2 \log_e \frac{h_1}{h_2} \quad (4.94)$$

or expressing in kW

$$N_w = \frac{1}{102} K_w b_2 h_2 w_2 \log_e \frac{h_1}{h_2} \quad (4.95)$$

The torque acting on both rolls is (4.66)

$$M_w = 2Pa$$

and if the corresponding angle of rotation is calculated in radians  $\varphi = l_2/R$ , then the pure rolling work is determined as

$$A_w = 2Pa\varphi = 2Pa \frac{l_2}{R} \quad (4.96)$$

The power  $N_w$  required for driving both rolls is

$$N_w = M_w \omega \quad (4.97)$$

where

$$\omega = \frac{v_r}{R} = \frac{2\pi n}{60} \quad (4.98)$$

$\omega$  = angular speed of rolls,  $\text{sec}^{-1}$ ,

$n$  = number of revolutions, rpm,

$v_r$  = peripheral speed, m/sec.

Expressing the power in kW, then from (4.97)

$$N_w = \frac{M_w \omega}{102} = \frac{2\pi n M_w}{60 \times 102} \quad (4.99)$$

To obtain the hourly energy consumption per ton of the rolled stock, it is necessary to divide the energy consumption per hour by the weight of stock rolled in the same time, and then

$$A'_w = \frac{3600 N_w}{G} \quad (4.100)$$

The quantity of the stock rolled per hour is to be obtained from

$$G = b_2 h_2 l_2 \rho = b_2 h_2 \rho w_2 3600 \quad (4.101)$$

where  $\rho$  = density of rolled stock ( $\text{t/m}^3$ ) or ( $\text{kg/dm}^3$ ).

Substituting equation (4.101) into (4.100), inserting the value of  $N_w$  from formula (4.99) and  $M_w$  from (4.78), then after simplification the following formula is obtained:

$$A'_w = K_{wm} \frac{\Delta h}{h_2} \frac{1}{\rho} \quad (4.102)$$

Expressing this formula in HPh or kWh

$$A'_w = \frac{K_{wm}}{75} \frac{\Delta h}{h_2} \frac{1}{\rho} \quad (4.103)$$

or

$$A'_w = \frac{K_{wm}}{102} \frac{\Delta h}{h_2} \frac{1}{\rho} \quad (4.104)$$

The total driving torque necessary on the motor shaft, consists of following component torques [10], [31]

$$M_m = M_w + M_f + M_i \pm M_d \quad (4.105)$$

where

$M_w$  = rolling torque proper transferred to the motor shaft, i.e. torque required to overcome the resistance to deformation of the rolled stock in the roll gap, and the consequent frictional resistance between roll surface and rolled stock;

$M_f$  = torque necessary to overcome additional frictional forces arising during the pass in the roll bearings, pinion, and other driving elements of the stand, without idling torque;

$M_i$  = idling torque, i.e. torque necessary to overcome frictional forces along the drive;

$M_d$  = dynamic torque necessary to overcome inertia forces arising due to varying roll speeds.

The first three values represent a constant load on the drive, and their sum forms the so called static torque, which the driving motor of each stand must be able to provide. The rolling torque is the greatest component of the total torque; only in rare cases, e.g. for driving of 2-high sheet mills, the additional friction torque becomes the greater component.

Of these torques, together comprising the static torque, only the rolling torque is useful, and components due to friction between stock and rolls represent unavoidable losses, inherent in every rolling process.  $M_f$  and  $M_i$  represent avoidable losses due to imperfections of design and manufacture of the roll stand.

The ratio of rolling torque proper to the required total torque on the motor shaft, is called rolling efficiency coefficient

$$\eta_w = \frac{M_w}{M_w + M_f + M_i + M_d} \quad (4.106)$$

Depending on the rolling method and design of the rolling stand and to a large extent on the design of roll neck bearing, the value of this coefficient varies within a wide range, having values

$$\eta_w = 0.3-0.9$$

The additional friction torque is equal to the torque required to overcome the frictional forces occurring during rolling plus friction arising in bearings, and also mechanical losses in gears. Idling losses are not included.

The most important component of the additional friction torque is the torque due to frictional losses in the bearings of the roll neck. The frictional torque for four bearings and two rolls is

$$M_{f1} = P d f_1 \quad (4.107)$$

where

- $P$  = bearing load, normally equal to the roll force (except for 4-high mills),
- $d$  = roll neck diameter, for normal roll layout two driven rolls of equal diameter are assumed,
- $f_1$  = coefficient of friction of the roll bearings.

From this relation it follows that the frictional torque of one roll neck, assuming the roll force acting at the centre of the barrel, equals

$$M'_{f1} = f_1 \frac{P}{2} \frac{d}{2}$$

If the roll force does not act at the centre of the roll barrel, giving an unequal load distribution on the bearings, the corresponding bearing load should be substituted for  $P/2$ .

The coefficient of friction of bearings  $f_1$  depends on the design and working conditions of bearings. Approximate values can be assumed for  $f_1$  [47, 87]:

- |                                                    |             |
|----------------------------------------------------|-------------|
| (a) Sliding bearings with bronze or babbitt bushes |             |
| for hot rolling mills                              | 0.07-0.1    |
| for cold rolling mills                             | 0.05-0.07   |
| for mills with forced lubrication                  | about 0.003 |
| (b) Sliding bearings with plastic bushes           | 0.01-0.03   |
| (c) Roller bearings                                | about 0.003 |

From the formula for calculation of the additional frictional torque of the bearings, the efficiency  $\eta_n$  related to the roll neck can be determined, as the ratio of the roll torque necessary for plastic deformation of the rolled stock, to the driving torque on the roll neck, i.e. the efficiency  $\eta_n$  may be expressed as follows:

$$\eta_n = \frac{2Pa}{2Pa + Pdf_1} = \frac{2a}{2a + df_1} \quad (4.108)$$

This efficiency is not dependent on the roll force, but only on the roll neck diameter, the coefficient of friction of bearings, and the lever arm of the roll torque.

Substituting  $a = c\sqrt{R\Delta h}$  into formula (4.108) then

$$\eta_n = \frac{1}{1 + \frac{f_1 d}{2c\sqrt{R\Delta h}}} \quad (4.109)$$

For certain values of  $f_1$ , the values  $R$  and  $d$  are constant, in which case the value of  $\eta_n$  depends only on the absolute draught  $\Delta h$ .

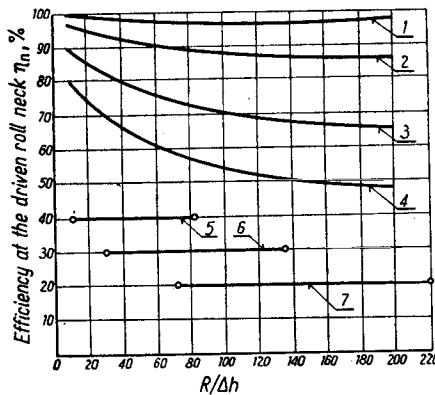


Fig. 4.98. Efficiency at the driven roll neck for various rolling mills and types of bearings as a function of the ratio  $R/\Delta h$ ; 1 — roller bearings with continuous lubrication  $f_1 = 0.0025$ ; 2 — anti-friction bearings with forced lubrication  $f_1 = 0.015$ ; 3 — bronze-slide bearings with forced lubrication  $f_1 = 0.05$ ; 4 — bronze-slide bearings with grease lubrication; 5 — blooming mill; 6 — billet and roughing trains; 7 — plate and sheet mill

The percentage value of  $\eta_n$  is plotted in Fig. 4.98 as a function of the ratio  $R/\Delta h$  for various types of bearings. For simplicity the value of lever arm  $a = l_d/2$  is assumed. The values of coefficient of bearing friction have been assumed based on experience from various stands. It can be seen from this diagram that the coefficient of bearing friction has a considerable influence on the energy losses due to friction.

The next largest component of the additional friction torque is the frictional torque loss in driving gears, i.e. in the pinion stand and toothed gear. This torque may be determined from efficiencies of individual gears of the drive, as follows:

$$M_{f2} = (M_w + M_{f1}) \left( \frac{1}{\eta_1 \eta_2 \dots \eta_n} - 1 \right) \quad (4.110)$$

where

$M_{f2}$  = torque loss due to gear friction, relative to the roll axis,  
 $\eta_i$  = efficiency of individual gears positioned between motor and roll neck,  
 i.e.  $\eta_1$  — pinion gear,  $\eta_2$  — first gear,  $\eta_n$  —  $n$ -th gear,  
 $M_w$  and  $M_{f1}$  = roll and bearing friction torque, relative to the roll axis. These values  
 should be calculated relative to the main motor shaft taking into  
 account the gear ratio of every intermediate gear between motor and  
 rolls.

Values of efficiencies for various transmissions are given below: [59]

|                       |           |
|-----------------------|-----------|
| single-reduction gear | 0.95–0.98 |
| belt transmission     | 0.8 –0.9  |
| cable transmission    | 0.8 –0.85 |

The total torque of additional friction about the main motor axis is

$$M_f = \left( \frac{M_w + M_{f1}}{i\eta_1\eta_2 \dots \eta_n} \right) - M_w \quad (4.111)$$

where  $i$  = gear ratio between driving motor and rolls = motor revs/roll revs.

For a rolling stand of unpowered back-up rolls, the expression in brackets should be divided by the gear ratio between working and back-up roll  $i_r = D_b/D_w$ .

Therefore it follows from equation (4.111) that the sum of rolling and additional friction torques relative to the roll axis is

$$M_w + M_f = \frac{1}{\eta_1\eta_2 \dots \eta_n} (M_w + M_{f1}) \quad (4.112)$$

Substituting for the lever arm of the roll force  $a = l_d/2$  and for the reciprocal of efficiency of a single-reduction gear  $1/\eta_1 = 1.05$  and for a double-reduction gear  $(1/\eta_1)(1/\eta_2) = 1.10$ , then the following simplified relation is found for equation (4.112)

$$M_w + M_f = (1.05\text{--}1.10) P(\sqrt{R\Delta h} + df_1) \quad (4.113)$$

Finally, substituting the expression for roll force, the following equation is found

$$M_w + M_f = (1.05\text{--}1.10) K_{wm} b_m \sqrt{R\Delta h} (\sqrt{R\Delta h} + df_1) \quad (4.114)$$

Therefore, using equations (4.113) and (4.114) the sum of the rolling torque and additional friction torque, for given assumptions may be easily determined.

The idling torque, i.e. torque required to maintain the stand in motion during breaks can be calculated from the mass of rotating elements and the radius of bearing necks, since the coefficients of bearing friction are known. As the rotating parts of a rolling mill, i.e. work rolls, couplings, spindles, coupling boxes, pinions, flywheel and other elements, have varying masses, rotational speeds and consequently also tangential speeds, bearing diameters and values of coefficient of bearing friction, therefore the idling torque can be expressed as [31]

$$M_i = \sum_{i=1}^{i=n} \frac{G_i f_i d_i}{2i_i} \quad (4.115)$$

where

- $G_i$  = mass of the given element (appropriate to the bearing load),
- $f_i$  = coefficient of bearing friction of the considered bearing,
- $d_i$  = bearing neck diameter,
- $i_i$  = total gear ratio from the motor to the part considered = motor revolutions / number of revolutions of the part considered.

When calculating the idling torque for hot sheet mills, cold rolling mills, and for some other cases, however, the initial compression of rolls should be taken into account. In such cases the bearing load due to initial compression may be so large that it equals the roll force. The resulting additional torque due to bearing friction can be determined using equation (4.107), substituting for  $P$  the initial compression force. Then, during the pass the additional friction torque should be correspondingly decreased, as the roll force acts against the initial compression force.

To determine the total idling torque of the mill (including flywheel), the air friction torque of the flywheel must also be considered. The loss of power due to air friction is often calculated by use of Backer's empirical formula [88]

$$N_a = v^{2.5} D^2 (1 + 5b) 10^{-5} \times 0.736 \text{ kW} \quad (4.116)$$

where

- $v$  = circumferential speed of the flywheel,
- $D$  = flywheel diameter,
- $b$  = width of the flywheel rim.

*Dynamic torque.* For some rolling mills, e.g. plate mills, blooming mills, heavy reversible mills, the rolls are accelerated immediately after biting the rolled stock. In this case, when determining the torque acting on the motor shaft, the acceleration torques of all rotating parts of the mill should also be taken into consideration, due to increase of the required motor torque during this period of time. When braking the rolls before the pass end, the motor power consumption decreases, since the accumulated energy, due to inertia of rotating elements assists the motor (Fig. 4.99) [31].

When a body of a mass of  $G$  and diameter  $D$ , turns about its axis with a uniform angular acceleration of  $d^2\varphi/dt^2$  (radians/sec<sup>2</sup>), then the dynamic torque required to produce this acceleration is

$$M_d = \frac{G}{4g} D^2 \frac{d^2\varphi}{dt^2}$$

where  $g = 9.81 \text{ m/sec}^2$ .

If  $G_1, G_2, G_3 \dots$  are the masses of rotating parts of inertial diameters of  $D_1, D_2, D_3 \dots$  with the same angular acceleration  $d^2\varphi/dt^2$ , the total acceleration torque is

$$M_d = \frac{1}{4g} (G_1 D_1^2 + G_2 D_2^2 + G_3 D_3^2 + \dots) \frac{d^2\varphi}{dt^2} \quad (4.117)$$

If the individual masses have varying accelerations and hence varying angular speeds, then the following equation is valid

$$M_d = \frac{G_1 D_1^2}{4g} \frac{d^2\varphi_1}{dt^2} + \frac{G_2 D_2^2}{4g} \frac{d^2\varphi_2}{dt^2} + \frac{G_3 D_3^2}{4g} \frac{d^2\varphi_3}{dt^2} + \dots \quad (4.118)$$

When a rotating body under the action of a uniform angular acceleration, increases its number of revolutions from  $n_1$  to  $n_2$  per unit of time, then

$$\frac{d^2\varphi}{dt^2} = \frac{1}{t} \left( \frac{d\varphi_2}{dt} - \frac{d\varphi_1}{dt} \right) = \frac{2\pi}{60t} (n_2 - n_1)$$

where

$$\frac{d\varphi_1}{dt} = \frac{2\pi n_1}{60} \quad \text{and} \quad \frac{d\varphi_2}{dt} = \frac{2\pi n_2}{60} \quad \text{rad/sec}$$

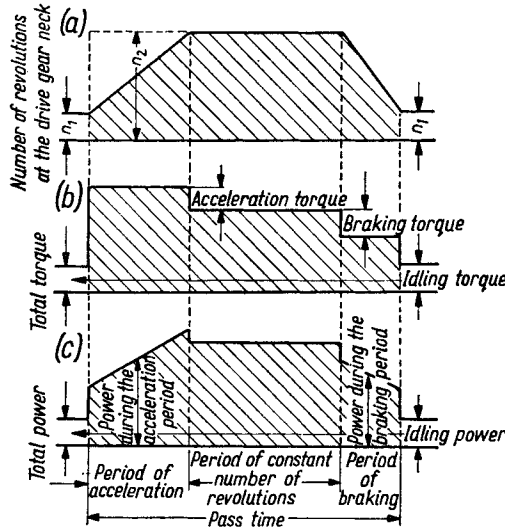


Fig. 4.99 Diagram of rotational speed, roll torque and power during one pass: (a) speed, (b) torque, (c) power [31]

Differentiating with respect to  $n$  and substituting in the torque equation gives

$$M_d = I \frac{2\pi}{60} \frac{dn}{dt} = \frac{GD^2}{4g} \frac{2\pi}{60} \frac{dn}{dt} = \frac{GD^2}{375} \frac{dn}{dt} \quad (4.119)$$

where

$dn/dt$  = angular acceleration,

$I$  = moment of inertia of the rotating body, about the axis of rotation.

For blooming mills of older types the angular acceleration during starting is usually 20 rpm/sec, and deceleration during braking 40 rpm/sec. In newer plants these values have been increased to 40 rpm/sec for the accelerating period, and 60 and even more rpm/sec for the braking period.

Replacing the differential in equation (4.119)  $dn/dt$  by the actual change in number of revolutions in a period of time of  $t$ , then dynamic torque is calculated from the following formula

$$M_d = \frac{GD^2}{375} \left( \frac{n_2 - n_1}{t} \right) \quad \text{kgm} \quad (4.120)$$

where

$GD^2$  = moment of inertia of all rotating elements up to the motor coupling,

$n_1$  = minimum revolutions per min (at the beginning of acceleration period),

$n_2$  = maximum revolutions per min (at the end of acceleration period).

For braking,  $n_1$  and  $n_2$  will be in the opposite sense.

For a roll of radius of gyration  $R$  and mass  $G$  rotating about its longitudinal axis, the following relation is valid

$$GD^2 = 4GR^2$$

If  $\Sigma 4GR^2$  denotes the sum of all accelerated masses, then equation (4.120) may be expressed in the form

$$M_d = \frac{\Sigma G}{375} 4R^2 \frac{(n_2 - n_1)}{t} \quad \text{kgm} \quad (4.121)$$

The energy required to accelerate these elements can be calculated similarly, thus

$$E = \frac{GD^2}{2 \times 4g} \left( \frac{2\pi}{60} \right)^2 (n_2^2 - n_1^2) \quad (4.122)$$

where

$G$  = mass of roll, kg,

$D$  = roll diameter, m,

$n_1$  and  $n_2$  = initial and final rpm.

The driving power required to accelerate the masses is calculated from the torque  $M_d$

$$N_d = \frac{2\pi n}{60 \times 102} M_d \quad \text{kW} \quad (4.123)$$

The acceleration work, due to dynamic torques, and also values for idling power requirements for various rolling mills are compared in Table 4.14.

*Torque on the motor shaft.* The total torque acting upon the motor shaft is calculated relative to the coupling between motor and gears. From equation (4.105) this is

(a) during the acceleration period

$$M_m = M_d + M_w + M_f + M_i \quad (4.124)$$

TABLE  
WORK OF ACCELERATION DUE TO DYNAMIC TORQUE, AND IDLING

| Mill layout       | Type of stands | Number of stands | Roll diameter<br>mm                                                                                                                                                                                                                 | Barrel length<br>mm |
|-------------------|----------------|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|
| Blooming mill     | 2-high         | 1                | 1150                                                                                                                                                                                                                                | 2800                |
| Cogging mill      | 3-high         | 2                | 650                                                                                                                                                                                                                                 | 1700                |
| Cogging mill      | 3-high         | 2                | 780                                                                                                                                                                                                                                 | 2100                |
| Section mill      | 3-high         | 2                | 650                                                                                                                                                                                                                                 | 1850                |
| Section mill      | 3-high         | 2                | 550                                                                                                                                                                                                                                 | 1500                |
| Intermediate mill | 3-high         | 2                | 600                                                                                                                                                                                                                                 | 1800                |
| Intermediate mill | double 2-high  | 5                | 360                                                                                                                                                                                                                                 | 1000                |
| Roughing mill     | 3-high         | 2                | 500                                                                                                                                                                                                                                 | 1500                |
| Roughing mill     | double 2-high  | 2                | 280                                                                                                                                                                                                                                 | 630                 |
| Rod mill          | 2-high         | 22               | —                                                                                                                                                                                                                                   | —                   |
| Plate mill        | Lauth 3-high   | 1                | <div style="display: flex; align-items: center;"> <div style="font-size: 2em; margin-right: 5px;">{</div> <div style="display: flex; flex-direction: column; gap: 5px;"> <div>775</div> <div>675</div> <div>775</div> </div> </div> | —                   |
| Sheet mill        | 2-high         | 4                | 630                                                                                                                                                                                                                                 | 760                 |



(b) during the braking period

$$M_m = -M_d + M_w + M_f + M_i \quad (4.125)$$

(c) during the steady running of the motor

$$M_m = M_w + M_f + M_i \quad (4.126)$$

For blooming, slabbing and reversible mills, where acceleration occurs, the following torques may be distinguished (Fig. 4.99):

(a) torque during the accelerating period, before bite

$$M_a = +M_d + M_i \quad (4.127)$$

(b) torque during the accelerating period, after bite

$$M_b = +M_d + M_w + M_i + M_f \quad (4.128)$$

(c) torque for rolling with constant speed

$$M_c = M_w + M_f + M_i \quad (4.129)$$

(d) torque during the braking period, rolled stock still between the rolls

$$M_d = M_w + M_f + M_i - M_d \quad (4.130)$$

(e) torque after rolled stock has left the rolls, but before reversing

$$M_e = M_i - M_d \quad (4.131)$$

Figure 4.99 gives the relation between number of revolutions, torque and driving power, during one pass. It is assumed that the rolls before bite have a constant rotational speed  $n_1$ , and after bite this increases to  $n_2$ . Figure 4.99a shows diagrammatically the rotational speed variation during the pass. The corresponding torque variations are shown in Fig. 4.99b. Figure 4.99c shows variations of power required in kW in this same pass. These diagrams are only schematic and indicate instantaneous changes

#### 4.14

##### MOTOR POWER, FOR VARIOUS ROLLING MILLS [87]

| Number of revolutions during idling rpm | Type of drive | Motor power kW | Accelerating work due to dynamic torque kWh | Motor response time min | Motor power idling kW |
|-----------------------------------------|---------------|----------------|---------------------------------------------|-------------------------|-----------------------|
| 40-60                                   | Ilgner set    | 5680           | 153                                         | 6'15"                   | 250-400               |
| —                                       | Normal drive  | 1250           | 55-72                                       | 5-7                     | 243                   |
| —                                       | Normal drive  | 885            | 33-44                                       | 4-5                     | 264                   |
| 125                                     | AC motor      | 1325           | 35-53                                       | 3                       | 290-440               |
| 168                                     | AC motor      | 1325           | 35-53                                       | 3                       | 290-440               |
| 118                                     | AC motor      | 825            | 40                                          | —                       | 210-327               |
| 230-250                                 | Normal drive  | 1030           | —                                           | —                       | 105-118               |
| 142                                     | AC motor      | 515            | 22-25                                       | —                       | 136-197               |
| 380-425                                 | Normal drive  | 880            | 25                                          | —                       | 59-80                 |
| —                                       | —             | 3125           | —                                           | —                       | 474                   |
| 75                                      | AC motor      | 1100           | 24                                          | —                       | 120                   |
| 34                                      | AC motor      | 400            | —                                           | —                       | 105                   |

and endings of the several periods. In actual practice these changes take place rapidly but not, of course, instantaneously and thus certain transitional periods occur. The maximum values of torque and power occur immediately before the end of the period of acceleration.

**Static load diagram.** When calculating the power of the driving motor of a given mill it is not only necessary to know the load in each pass, but also the temporary distribution of the load, i.e. the so called loading diagram. To draw this diagram it is necessary to calculate the static load torque on the main motor shaft plotted against the time, for the total period of rolling of a given section, taking into account pass and break times (Fig. 4.100) [87].

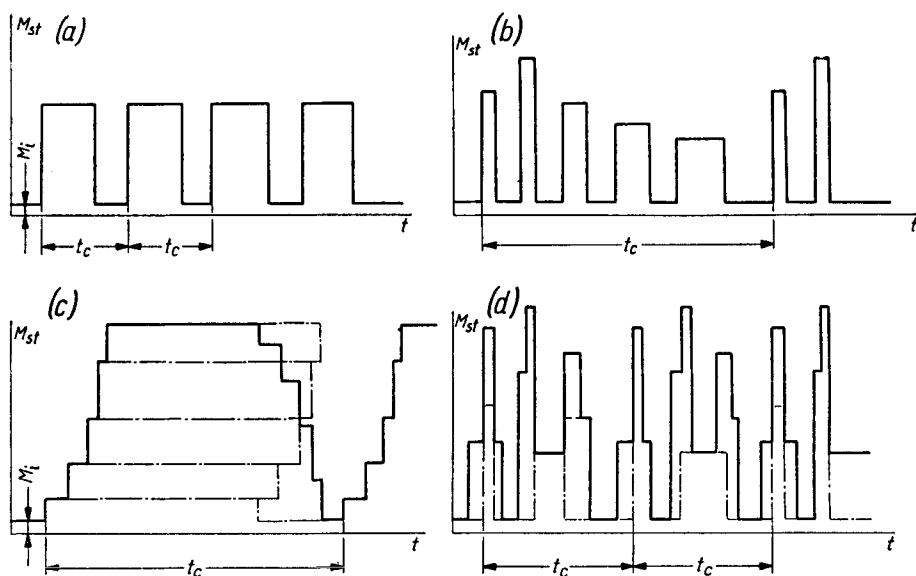


Fig. 4.100. Diagrams of static roll torque for various rolling conditions: (a) continuous mill of individually driven stands, (b) single stand mill for rolling of bars in five passes, (c) continuous mill of five stands group-driven, for single-strand rolling, (d) two or multi-strand rolling in five stands [10]

From equation (4.105) the static torque is

$$M_{st} = M_w + M_f + M_i$$

The pass times can be calculated knowing the length of the rolled stock and rolling speed since

$$t = \frac{l_2}{w_2} = \frac{l_1}{v_1}$$

where

- $l_1$  = initial length before pass,
- $l_2$  = final length after pass,
- $w_2$  = exit speed,
- $v_1$  = entry speed.

The break times between individual passes of 1.5–6 sec may be calculated or assumed from knowledge of time required for the essential operations between passes. These operations include laying the rolled stock on the roller table, turning-over, shifting to another pass or stand, moving up or down of the top roll, reversing etc.

The total time for one cycle of operations  $t_c$ , shown in the loading diagram, is the time between biting of one stock and biting of the next.

For single-stand mills the time  $t_c$  is called the rolling cycle which repeats continually as long as the type of the rolled stock and the rolling temperature are constant. Figure 4.100 gives typical static torque loading diagrams for different mills. The heights of rectangles shown in Figs. 4.100a and b correspond to one pass, and in Figs. 4.100c and d to two or more simultaneous passes. For the second case the loading diagram is obtained by superposing rectangles for corresponding passes. The heights of these rectangles correspond to the static torque, their width to the pass time, and the distance between them to the break time between two passes.

*Inertia energy of flywheel.* Some non-reversible stands or trains operating with constant number of revolutions with relatively short and heavy passes, are equipped with flywheels. For light passes the inertia energy of rotating masses of the drive are sufficient, and a flywheel is not necessary.

During the pass the motor speed decreases, and the flywheel gives back a part of the accumulated energy to the drive. Decrease of motor loading in such a case, can only take place by means of a drop of the motor speed.

Calculations to determine dimensions of the flywheel are almost identical with those for determining the effect of dynamic torques. If the acceleration  $dn/dt$  of the flywheel is negative, i.e. if there is a braking effect, then

$$M_m = M_{st} - \frac{GD^2}{375} \frac{dn}{dt} \quad (4.132)$$

For the static torque the expression  $M_{st} = M_w + M_f + M_i$ , should be substituted. The required dynamic torque of the flywheel  $GD^2$  must be determined from the flywheel kinetic energy, based on the known mill design and number of revolutions.

According to Tselikov [87] the kinetic energy of a flywheel is

$$E = I \frac{\omega^2}{2} \quad \text{kgm} \quad (4.133)$$

and the dynamic torque  $GD^2$  and inertial torque, are related by

$$GD^2 = 4gl = 4Gr^2 \quad \text{kgm} \quad (4.134)$$

For a drop in number of revolutions from  $n_1$  to  $n_2$ , the following kinetic energy is given back by the flywheel

$$E = \frac{I}{2} \left[ \left( \frac{\pi n_1}{30} \right)^2 - \left( \frac{\pi n_2}{30} \right)^2 \right] \quad (4.135)$$

Hence from equations (4.133) and (4.134)

$$GD^2 = \frac{2 \times 4gE}{\left( \frac{\pi n}{30} \right)^2} = 7200 \frac{E}{n^2} \quad \text{kgm}^2 \quad (4.136)$$

Expressing the drop in revolutions of the flywheel relative to the initial number of revolutions, the following equation is obtained

$$GD^2 = 540 \frac{E_1}{n^2} \quad \text{tm}^2 \quad (4.137)$$

where

- $G$  = flywheel mass,
- $\omega = 2\pi n/60$  = angular speed of the flywheel,
- $I$  = moment of inertia of the flywheel,
- $D$  = diameter of gyration of the wheel,
- $g$  = acceleration due to gravity i.e.  $9.81 \text{ m/sec}^2$ ,
- $r$  = radius of gyration of the flywheel,
- $n$  = rpm of the flywheel,
- $E$  = kinetic energy of the flywheel, kgm,
- $E_1$  = kinetic energy of the flywheel, kW/sec.

In this way it is possible to determine the size of the flywheel, taking into account all the rotating parts of the mill. The higher the number of revolutions  $n$ , the lower is the required weight of flywheel since the kinetic energy is proportional to  $n^2$ . Therefore, it is usually mounted on the motor shaft. If a gear is positioned between motor and stand then usually two flywheels are placed on the shaft, to obtain symmetrical loading. The kinetic energy of the flywheel may be calculated only when determining simultaneously the power of the driving motor.

The kinetic energy  $E_1$  given back by the flywheel can be calculated approximately as follows [87]:

$$E_1 = \frac{N_n t}{2 s_n} \frac{1}{\log_e \frac{M_{st} - M_0}{M_{st} - k M_n}} \quad (4.138)$$

where

- $N_n$  = nominal motor power, kW,
- $t$  = pass time,
- $s_n = (n_0 - n_n)/n_0$  = slip (unloaded 0.1 or 10%, fully loaded 0.15 or 15%),
- $n_0$  = idling speed, rpm,
- $n_n$  = rated motor speed, rpm,
- $k$  = overloading coefficient (for 50% overloading  $k = 1.5-2$ ),
- $M_n$  = rated torque of the driving motor,
- $M_{st}$  = static torque acting upon the motor shaft,
- $M_0$  = torque given by the motor at the beginning or end of pass.

The value of  $E_1$  should be calculated in kW/sec for each pass. As the values of rated power  $N_n$  and  $s_n$  are constant, then to determine the pass for which the maximum  $E_1$ -value occurs, the maximum value of the term

$$\frac{1}{\log_e \frac{M_{st} - M_0}{M_{st} - k M_n}}$$

should be calculated.

The rated power of the driving motor can be determined by various methods:

- (a) from tables, based on designer's data,
- (b) from statistical data,
- (c) from the equation  $N_n = (1.2-1.3) N_m$ , where the mean power is determined from equation

$$N_m = \frac{3600(A_2 - A_1)}{t} G \quad (4.139)$$

The values of  $A_2$  and  $A_1$  are taken for respective rolling mill designs from diagrams in units of kWh/t or HPh/t. The coefficient 1.2-1.3 takes into account the heating of the driving motor.

The kinetic energy  $E_1$  can be determined from following equations:

$E_1/N_n = 10-25$  for section mills and Lauth's 3-high mills (without slip control),

$E_1/N_n = 10-15$  for section mills and Lauth's 3-high mills (with slip control),

$E_1/N_n = 10-82$  for 2-high sheet mills.

From these values, the torque  $GD^2$  may be determined from equation (4.137).

*Example of calculation according to Filasiewicz [88]:*

Reading off from curve 1 in Fig. 4.101 the difference  $A = A_2 - A_1 = 34$  kWh, then for a stand with an output of 18.5 t/h, the mean power of

$$N_m = 18.5 \times 34.0 = 630 \text{ kW}$$

When taking into account the idling power  $N_i = 37$  kW, then the value of  $N_m$  increases to  $N_m = 630 + 37 = 667$  kW.

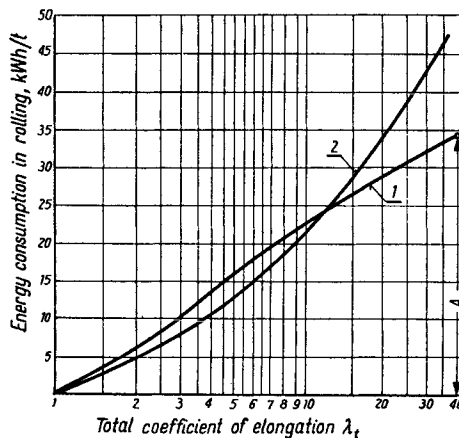


Fig. 4.101. Energy consumption as a function of total coefficient of elongation during rolling of steel strip: 1 — strip of  $30 \times 2.2$  mm rolled in 13 passes from a billet  $60 \times 60$  mm in a semi-continuous train of 270 mm roll dia.; 2 — strip of  $30 \times 1.47$  mm rolled in 10 passes from a billet of  $51 \times 51 \times 9000$  mm in a semi-continuous train of 250 mm roll dia. [87]

Choosing the coefficient of motor heating as 1.2, then the following rated power of the driving motor is obtained

$$N_n = 1.2 \times 667 = 800 \text{ kW}$$

If for a motor speed of  $n = 720$  rpm a ratio of  $E_1/N_n = 20$  is chosen, then

$$E_1 = 20 \times 800 = 16,000 \text{ kW/sec}$$

hence

$$GD^2 = 540 \frac{E_1}{n^2} = 540 \frac{16,000}{720^2} = 16.7 \text{ tm}^2$$

is obtained.

When  $GD^2$  is known the dimensions of the flywheel can be calculated.

#### 4.3.4. CALCULATION OF ROLLING WORK AND DRIVE POWER FROM ENERGY AND POWER CONSUMPTION DIAGRAMS

Another method for determination of rolling work and drive power is the use of diagrams, measuring the power output of the main motor, the energy consumption is plotted as a function of draught or elongation coefficient, taking into consideration the rolling speed of the succeeding passes. The work or energy consumption is given in HPh/t or kWh/t of the rolled stock, and the power in HP or kW, as a function of coefficient of elongation [31, 87].

Calculation of energy consumption and power by means of diagrams is mostly used when rolling irregular sections, where a direct calculation of the roll force is more complicated than for sheets, strip and other flat products, due to difficulties in determination of projected area of contact, and of resistance to deformation.

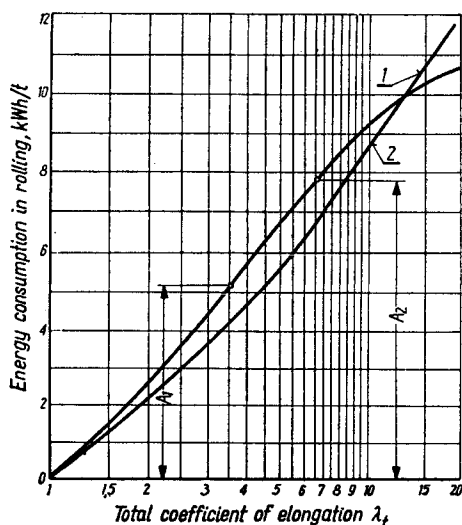


Fig. 4.102. Energy consumption as a function of the total coefficient of elongation during rolling of ingots: 1 — ingot  $652 \times 652$  mm and 6.41 t, rolled in 24 passes down to blooms of  $170 \times 170$  mm in a blooming mill of  $D = 1100$  mm; 2 — ingot of  $635 \times 762$  mm rolled to billets in a blooming mill of roll diameter  $D = 1000$  mm [87]

A diagram of this kind is shown in Fig. 4.102 [87], where the energy consumption per ton of the rolled stock is given as a function of the coefficient of total elongation, i.e. the ratio of increased length to initial length. The energy consumption or the rolling work for a given pass is always shown as a difference of the ordinates  $A_1$  and  $A_2$ . Values of ordinates appropriate to the total elongation coefficient should be read off from the

diagram. The total elongation coefficient is found by dividing the increased length by the initial length  $\lambda = l_2/l_1$ . Therefore

$$A_{\text{pass}} = A_w + A_f = A_2 - A_1 \quad (4.140)$$

If diagrams are used, in which energy or work is expressed in other units, e.g. in kWh or tm, the read off values must be recalculated. To determine the work or energy consumption for a given pass in kgm, equation (4.140) should be written in the form of

$$A_{\text{pass}} = 102 \times 3600 (A_2 - A_1) G \quad (4.141)$$

if  $A_2$  and  $A_1$  are in kWh/t units, or

$$A_{\text{pass}} = 75 \times 3600 (A_2 - A_1) G \quad (4.142)$$

if  $A_2$  and  $A_1$  are in HPh/t units.  $G$  denotes the rolled tonnage in t/h (4.101).

The rolling work shown in Figs. 4.101 and 4.102 includes not only the pure work of deformation necessary to overcome the deformation resistance of the rolled stock, and the work required to overcome the resistance to flow between rolls and stock, i.e. the rolling work  $A_w$ , but also the frictional work in roll neck bearings and all of the driving elements. Idling work is not included. All of the curves of energy consumption have been drawn omitting the idling losses. Therefore, the zero ordinate on the diagram is not a true zero, but represents the idling losses, which must be taken into account. Calculating torque from the formulae for work (4.74) and (4.141), using diagrams in kWh/t units

$$M_w + M_f = \frac{102 \times 3600 (A_2 - A_1) D}{2 l_2} G \quad (4.143)$$

As the tonnage of the rolled stock is equal to  $G = b_2 h_2 l_2 \rho$ , then for this case

$$M_w + M_f = 184 (A_2 - A_1) \rho F D \quad \text{tm} \quad (4.144)$$

For values in HPh/t of the rolled stock

$$M_w + M_f = 135 (A_2 - A_1) \rho F D \quad \text{tm} \quad (4.145)$$

where

$\rho$  = density of the stock, t/m<sup>3</sup>,

$F$  = cross-sectional area of the rolled stock after pass, m<sup>2</sup>,

$D$  = effective roll diameter, m.

Substituting for steel density the value of 7.8 t/m<sup>3</sup> in equations (4.144) and (4.145) then for kWh/t of the rolled stock

$$M_w + M_f = 1436 (A_2 - A_1) F D \quad \text{tm} \quad (4.146)$$

and for HPh/t of the rolled stock

$$M_w + M_f = 1050 (A_2 - A_1) F D \quad \text{tm} \quad (4.147)$$

This torque corresponds to a power of

$$N_w + N_f = \frac{3600 (A_2 - A_1)}{t} G \quad (4.148)$$

(for diagrams in kWh/t of the rolled stock), where

$G$  = weight of the rolled stock (4.101), t,

$t$  = time of passing of stock between rolls, the so-called pass time, sec.

This equation is valid for kWh/t or HPh/t diagrams, but results are valid in kW or HP.

When using these diagrams for energy consumption or average resistance to deformation, they should be chosen so as to conform as closely as possible to the given rolling conditions as regards:

- the material of the rolled stock,
- the design of the rolling mill or train,
- initial and final dimensions, and shape of cross-section,
- the value of effective roll diameter,
- the rolling temperatures in the first pass and the finishing pass, and the temperature distribution for the intermediate passes,
- the draught or elongation in each pass,
- the rolling speed or peripheral speed of rolls.

In all these cases the coefficient of total elongation should be calculated relative to the initial cross-section, being the cross-section of the billet leaving the furnace in the case of hot rolling, and the cross-section of the blank in the case of cold rolling.

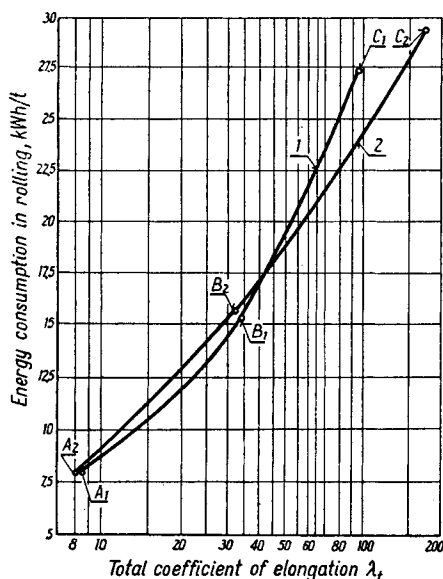


Fig. 4.103. Energy consumption as a function of the coefficient of total elongation during rolling of  $200 \times 200$  mm blooms, immediately after heating, in a continuous mill consisting of two 6-stand trains: 1 — finishing cross-section  $100 \times 100$  mm at a roll diameter of  $D = 630$  mm (sector  $A_1B_1$  of the curve), and finishing cross-section  $60 \times 60$  mm at a roll diameter of  $D = 450$  mm (sector  $B_1C_1$  of the curve); 2 — finishing cross-section of  $102 \times 102$  mm at a roll diameter of  $535$  mm (sector  $A_2B_2$  of the curve), and finishing cross-section of  $44 \times 44$  mm at a roll diameter of  $450$  mm (sector  $B_2C_2$  of the curve) [87]

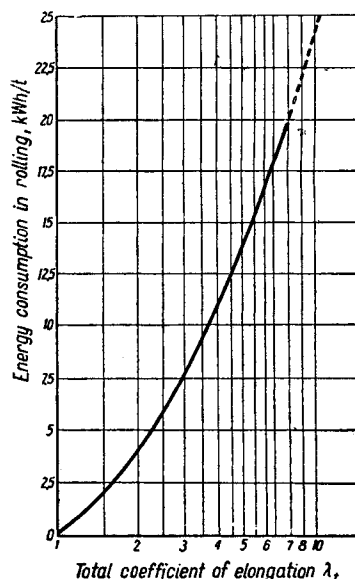


Fig. 4.104. Energy consumption as a function of the coefficient of total elongation during rolling of rails of a total weight of  $46.3$  kg/m from  $203 \times 229$  mm blooms at a roll diameter of  $D = 760$  mm [87]

Figures 4.103–4.107 show typical diagrams of energy consumption for rolling of steel under various conditions, according to Tselikov [87].



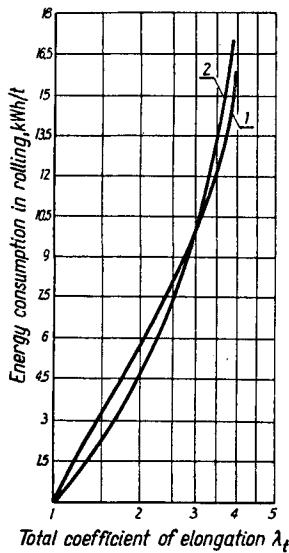


Fig. 4.105. Energy consumption as a function of the coefficient of total elongation during rolling of: 1 — sleepers No. 18 and 2 — sleepers No. 16 from billets of  $150 \times 150 \times 4800$  mm with roll diameter of  $D = 500$  mm [87]

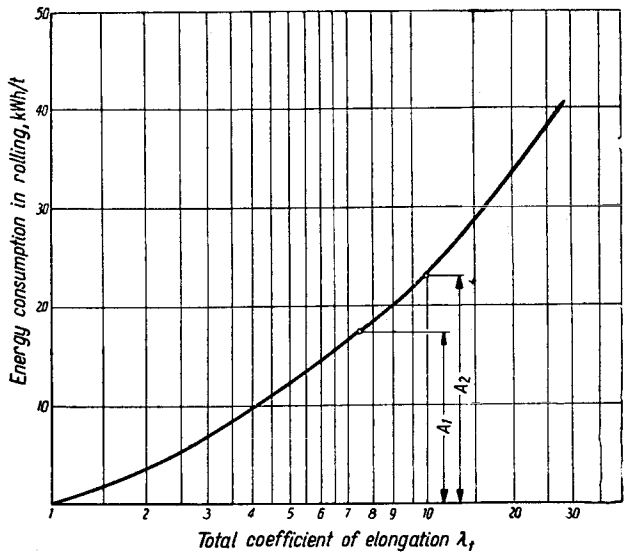


Fig. 4.106. Energy consumption as a function of the coefficient of total elongation during rolling of angles  $75 \times 75 \times 8.2$  mm in 9 passes from  $100 \times 100$  mm billets of 120 kg weight at a roll diameter of  $D = 355$  mm

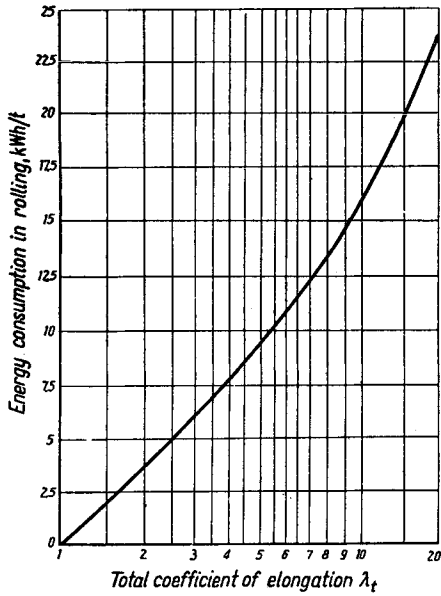


Fig. 4.107. Energy consumption as a function of the coefficient of total elongation during rolling of 25 mm dia. steel rounds from  $100 \times 100$  mm billets [87]

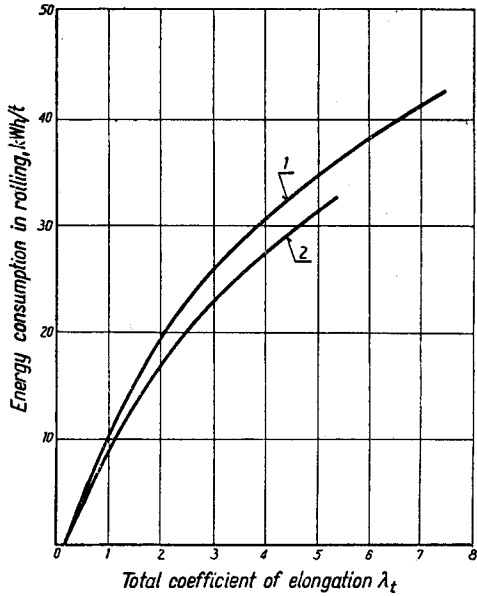


Fig. 4.108. Energy consumption as a function of the coefficient of total elongation during rolling of strip on a 10-stand continuous mill of roll barrel width of 1420 mm from 115 mm thick slabs

A continuous hot strip mill consists of a scale breaker, four stands of the roughing train, followed by an edger acting as a scale breaker, and then by a six-stand finishing train. Figure 4.108 gives the energy consumption for two different cases when rolling on this mill. The first stand of the roughing train is a  $914 \times 1422$  mm 2-high mill with force-lubricated bearings, and the next three stands are 4-high mills of 635 mm work roll diameter, 1245 mm back-up roll diameter and 1472 mm barrel length. All of the bearings are force-lubricated anti-friction bearings. The following six stands of the finishing train are also 4-high stands of 610 mm work roll diameter and 1425 mm back-up roll diameter. The barrel length of the rolls, operating in force-lubricated bearings, is 1472 mm. The finishing roll speed is from 300–600 m/min. All stands are individually driven by 3000 HP motors.

The curves drawn in Fig. 4.108 show the energy consumption in kWh/t as a function of the total elongation coefficient of the strip (Fig. 4.108). Curve 1 is for rolling of 115 mm thick slabs to strips of 1.45–1.90 mm thickness, at an initial temperature of 1230°C and rolling speed of 7.4–9.2 m/sec. The finishing rolling temperatures varied from 1010–1065°C. Curve 2 is for rolling of 2.7 mm thick strips from 114 mm thick slabs at a rolling speed of 7 m/sec.

Besides rolling work Figs. 4.101–4.108 include all frictional losses, except idling work. If a wide range of such diagrams is available, corresponding to various rolling conditions, then for any given case a suitable curve can be chosen.

Figure 4.109 gives the energy consumption in tm per ton of rolled stock for six different billets, as a function of the total elongation coefficient. The first part of each curve is for blooming mills and the second part is for billet mills. These curves have been drawn by the Siemens company, based on experience in German rolling mills. For comparison, this diagram includes the curve of theoretical energy consumption, when only yield stress is taken into account. This curve relates to steel at a constant temperature of 1140°C [31].

From Figs. 4.110–4.112 the energy and power consumption for rolling of beams (Fig. 4.110), channels (Fig. 4.111) and angles (Fig. 4.112) may be determined. These curves give values for various sectional dimensions relative to output in t/h, at a constant rolling temperature of 1125°C.

Rolling conditions are given in more detail in Table 4.15. Beams were rolled in a heavy-section train, the channels in the medium-sized mill No. I, and the angles in the medium-sized mill No. II. The point of intersection of the ordinate of energy consumption and the power curve, determines the power required for idle running.

### Example

To test if a given rolling mill is capable of rolling 4t slabs to plates.

Assuming for these calculations the permissible overloading of the motor due to torque as about 100% nominal loading then it becomes  $2.0 \times 32.5 = 65$  tm, and assuming the cut-out torque as 2.2 times the nominal torque it becomes  $2.2 \times 32.5 = 71.5$  tm.

Rolling temperature and decrease of motor revolutions are taken from previous measurements. The idling power is neglected [89].

Based upon these assumptions, Table 4.16 has been calculated, and a speed of 55 rpm was taken for determining the pass time. The break time between two passes has been assumed as 5 sec, and between two succeeding slabs as 120 sec, resulting in a sum of all break times of 245 sec. The maximum theoretical output of the train calculated for such assumptions is 32 t/h.

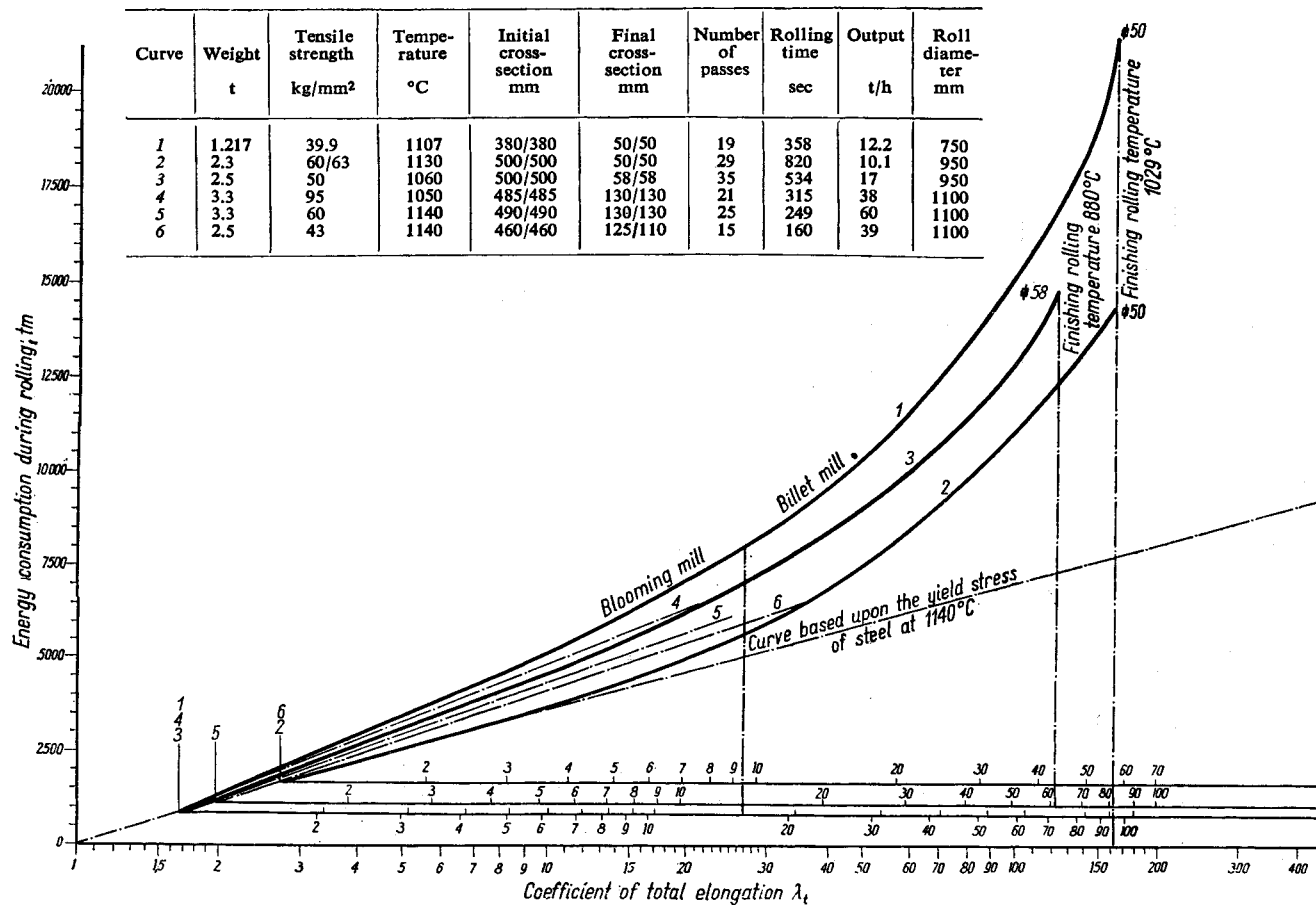


Fig. 4.109. Energy consumption in tm as a function of the coefficient of total elongation for six different rolling mills compared with the work consumption required to overcome the yield stress of steel at a temperature of 1140°C

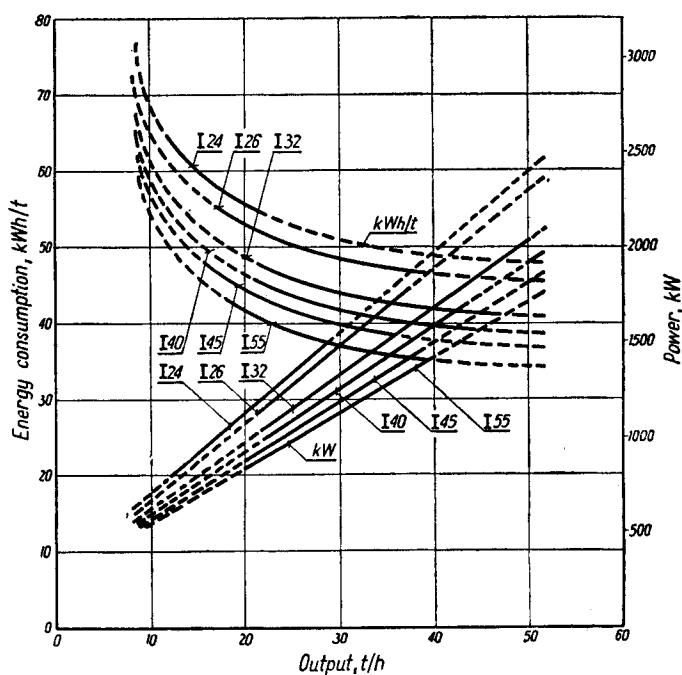


Fig. 4.110. Energy and power consumption during rolling of various beams (Table 4.15)

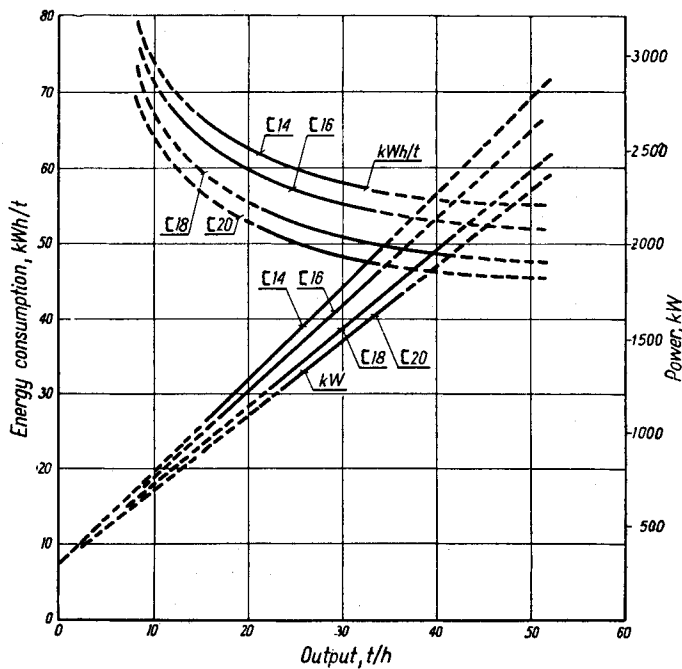


Fig. 4.111. Energy and power consumption during rolling of channels (Table 4.15)

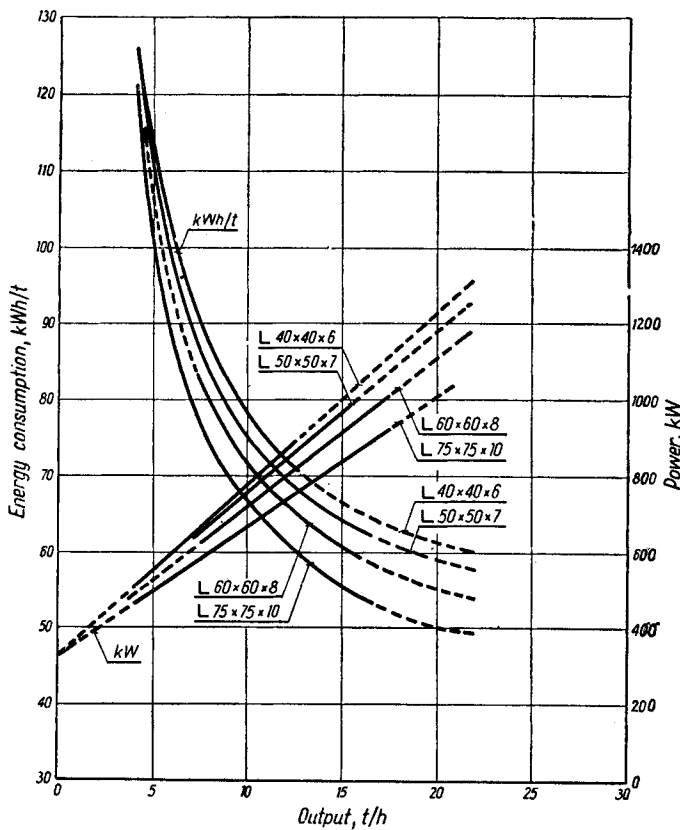


Fig. 4.112. Energy and power consumption during rolling of angles (Table 4.15)

TABLE 4.15  
ROLLING CONDITIONS FOR ENERGY AND POWER-CONSUMPTION CURVES  
SHOWN IN FIGS. 4.110, 4.111 and 4.112

| Characteristics             | Data for mill types                     |                                       |                              |                              |
|-----------------------------|-----------------------------------------|---------------------------------------|------------------------------|------------------------------|
|                             | roughing<br>reversing<br>2-high<br>mill | intermediate<br>3-high train<br>No. I | intermediate train<br>No. II |                              |
|                             |                                         |                                       | 3-high<br>roughing<br>stand  | 3-high<br>finishing<br>stand |
| Number of stands            | 4                                       | 2                                     | 1                            | 1                            |
| Roll diameter $D$ , mm      | 900                                     | 710                                   | 650                          | 500                          |
| Roll neck diameter $d$ , mm | 500                                     | 410                                   | 360                          | 290                          |
| Barrel length $L$ , mm      | 2500                                    | 2100                                  | 1800                         | 1500                         |
| Roll speed, rpm             | 180                                     | 150                                   | 105                          | 163                          |
| Dimension of billets, mm    | 125×145                                 | 125×145                               | 120×130                      | 70×70                        |
| Dimension of blooms, mm     | 510×240                                 | 175×175                               | 175×175                      | 130×130                      |
| Type of drive               | Ilgner set                              | AC motor                              | AC motor                     | AC motor                     |
| Drive power, kW             | 2×3000                                  | —                                     | 500                          | 1300                         |
| Motor speed, rpm            | 0–180                                   | —                                     | 105                          | 163                          |

TABLE 4.16  
ROLLING SCHEDULE FOR 2300×3600×6 mm SHEETS

| Pass No. | $b_2$<br>mm | $h_1$<br>mm | $h_2$<br>mm | $t$<br>°C | $P$<br>t | Pass time<br>sec | $M_w$<br>tm | $M_f$<br>tm | $M_i$<br>tm | $M_s$<br>tm | $M_d$<br>tm | Decrease of motor speed rpm |
|----------|-------------|-------------|-------------|-----------|----------|------------------|-------------|-------------|-------------|-------------|-------------|-----------------------------|
| 1        | 1010        | 150         | 140         | 1200      | 277      | 0.15             | 16.82       | 13.65       |             |             | 13.02       | 69.7                        |
| 2        | 1021        | 140         | 130         | 1195      | 289.7    | 0.15             | 17.56       | 14.26       |             |             | 14.37       | 69.6                        |
| 3        | 1031        | 130         | 120         | 1190      | 302.2    | 0.16             | 18.37       | 14.91       |             |             | 15.83       | 69.5                        |
| 4        | 1042        | 120         | 110         | 1180      | 323      | 0.18             | 19.34       | 15.91       |             |             | 17.80       | 69.4                        |
| 5        | 1052        | 110         | 100         | 1170      | 348      | 0.19             | 20.87       | 17.13       |             |             | 20.55       | 66.3                        |
| 6        | 1063        | 100         | 92          | 1160      | 327.5    | 0.21             | 17.67       | 16.63       |             |             | 15.25       | 69.4                        |
| 7        | 1073        | 92          | 84          | 1150      | 348.2    | 0.23             | 18.70       | 17.05       |             |             | 18.30       | 69.3                        |
| 8        | 1084        | 84          | 77          | 1140      | 346      | 0.25             | 17.47       | 16.93       |             |             | 16.95       | 69.3                        |
| 9        | 1094        | 77          | 70          | 1130      | 368      | 0.27             | 18.52       | 17.96       |             |             | 19.03       | 69.2                        |
| 10       | 1105        | 70          | 63          | 1120      | 391.5    | 0.30             | 19.57       | 19.21       |             |             | 21.33       | 69.1                        |
| 11       | 1115        | 63          | 57          | 1110      | 383.5    | 0.32             | 17.82       | 18.66       |             |             | 19.03       | 69.0                        |
| 12       | 1126        | 57          | 51          | 1100      | 408      | 0.36             | 18.87       | 19.89       |             |             | 21.31       | 68.8                        |
| 13       | 1136        | 51          | 46          | 1085      | 400      | 0.39             | 18.52       | 19.48       |             |             | 20.55       | 68.6                        |
| 14       | 1147        | 46          | 41          | 1070      | 435      | 0.44             | 18.32       | 21.08       |             |             | 21.95       | 68.4                        |
| 15       | 1157        | 41          | 37          | 1055      | 413      | 0.48             | 15.70       | 19.96       |             |             | 18.21       | 68.6                        |
| 16       | 1168        | 37          | 33          | 1040      | 448      | 0.54             | 16.86       | 21.68       |             |             | 21.09       | 68.1                        |
| 17       | 1178        | 33          | 30          | 1025      | 408      | 0.59             | 13.44       | 19.61       |             |             | 15.60       | 68.5                        |
| 18       | 1189        | 30          | 27          | 1010      | 442      | 0.64             | 14.48       | 21.27       | 3.00        | 20.45       | 18.30       | 68.0                        |
| 19       | 1200        | 27          | 24.5        | 995       | 426      | 0.70             | 12.82       | 20.43       |             |             | 15.80       | 68.2                        |
| 20       | 1210        | 24.5        | 22          | 980       | 461      | 0.78             | 13.72       | 22.10       |             |             | 18.37       | 67.7                        |
| 21       | 2540        | 22          | 20          | 965       | 903      | 0.41             | 24.15       | 43.25       |             |             | 49.95       | 66.7                        |
| 22       | 2540        | 20          | 18.5        | 950       | 802.5    | 0.45             | 18.78       | 38.26       |             |             | 39.59       | 67.1                        |
| 23       | 2540        | 18.5        | 17          | 935       | 855      | 0.49             | 19.92       | 40.78       |             |             | 43.25       | 66.6                        |
| 24       | 2540        | 17          | 15.5        | 920       | 913      | 0.53             | 21.20       | 43.55       |             |             | 47.30       | 65.9                        |
| 25       | 2540        | 15.5        | 14          | 905       | 978      | 0.60             | 22.60       | 46.68       |             |             | 51.83       | 65.0                        |
| 26       | 2540        | 14          | 13          | 890       | 798      | 0.64             | 15.30       | 37.95       |             |             | 35.80       | 66.4                        |
| 27       | 2540        | 13          | 12          | 875       | 848      | 0.70             | 16.17       | 39.33       |             |             | 39.05       | 65.6                        |
| 28       | 2540        | 12          | 11          | 860       | 906      | 0.76             | 17.22       | 43.02       |             |             | 42.79       | 64.7                        |
| 29       | 2540        | 11          | 10          | 845       | 970      | 0.84             | 18.34       | 46.06       |             |             | 46.95       | 63.6                        |
| 30       | 2540        | 10          | 9           | 830       | 1046     | 0.95             | 19.71       | 49.74       |             |             | 52.00       | 62.1                        |
| 31       | 2540        | 9           | 8.25        | 810       | 976      | 1.03             | 16.10       | 46.30       |             |             | 44.95       | 62.5                        |
| 32       | 2540        | 8.25        | 7.5         | 790       | 1010     | 1.14             | 16.57       | 47.83       |             |             | 46.95       | 61.3                        |
| 33       | 2540        | 7.5         | 7           | 770       | 813      | 1.21             | 11.02       | 38.43       |             |             | 32.00       | 63.7                        |
| 34       | 2540        | 7           | 6.5         | 750       | 871      | 1.31             | 11.77       | 41.23       |             |             | 35.55       | 62.5                        |
| 35       | 2540        | 6.5         | 6           | 730       | 936      | 1.43             | 12.60       | 44.30       |             |             | 39.45       | 60.8                        |

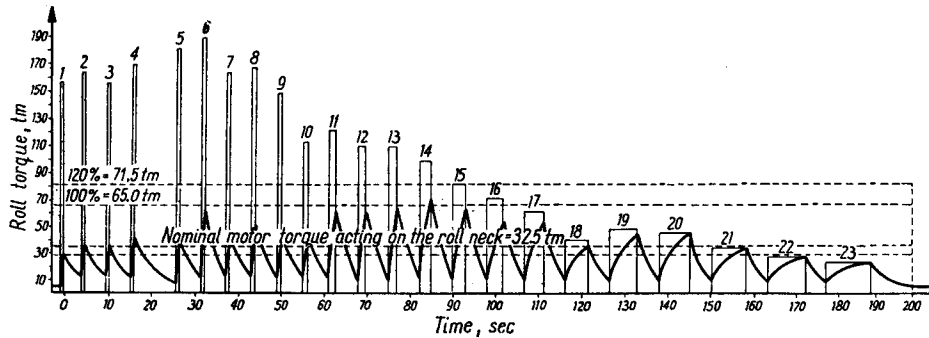


Fig. 4.113. Roll torque-time diagram for rolling of 4 t slabs to 15×2000 mm plates

Elektrical energy taken from the grid without auxiliary power units was taken as 20 kWh/t of the rolled stock, assuming rather better heating of the initial stock.

These calculations, and the diagram in Fig. 4.113 show clearly that under existing conditions, it is impossible to roll such slabs, since the main motor would break down due to overloading several times greater than the permissible loading [89].

#### 4.4. Calculation of Roll Force in Cold Rolling

It is more difficult to calculate roll forces in cold rolling than in hot rolling, due to work-hardening of the rolled stock, resulting in increase of yield stress, and when  $\eta K_{fm} \geq 50 \text{ kg/mm}^2$  the rolls are flattened, i.e. the projected length of arc of contact is increased from  $l_d$  to  $l'_d$ , as shown in Fig. 4.114.

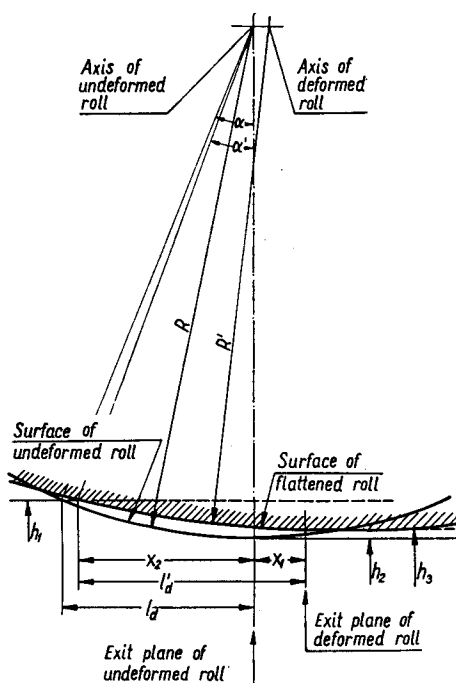


Fig. 4.114. Roll flattening [10]:  $R'$  — radius of the roll flattened by elastic deformation,  $R$  — radius of the undeformed roll,  $h_1$  — initial height,  $h_2$  — final height of the rolled stock with undeformed roll,  $h_3$  — final height of the rolled stock with deformed roll

Due to roll flattening the projected length of arc of contact is increased at both ends in comparison with the value for undeformed rolls. The line of entry of stock is shifted backwards, and the line of exit is shifted in the rolling direction by an amount  $x_1$  measured from the plane of the roll centres. The direct result of roll flattening is an increase of projected arc of contact and decrease of draught. The indirect result is the changing of resistance to deformation due to increased projected length of contact and hence increased projected area of contact between roll and stock.

Hitchcock [49] gives the following formula for calculation of the projected length of contact  $l'_d$  of a deformed roll

$$l'_d = \sqrt{\left[ \frac{8R(1-\mu^2)K_{wm}}{\pi E} \right]^2 + (h_1 - h_2)R + \frac{8R(1-\mu^2)K_{wm}}{\pi E}} \quad (4.149)$$

where

$E$  = Young's modulus of elasticity for the material of the rolls,

$\mu$  = Poisson's number for the material of the rolls,

$R$  = radius of the undeformed roll.

For a rigid roll, i.e. when Young's modulus is infinitely large, equation (4.149) simplifies to the known formula for projected length of arc of contact  $l'_d = \sqrt{R\Delta h}$ .

Formula (4.149) was derived by Hitchcock on the following assumptions:

(a) the pressure distribution between roll and stock is of approximately elliptic form,

(b) the arc of contact between roll and stock is approximately an arc of a circle.

Equation (4.149) was simplified by Ford, who gave a formula for flattened radius  $R$ , assuming the roll to be still of cylindrical form in the flattened place, but of greater diameter [10, 71, 72].

Then

$$R' = R \left( 1 + \frac{2cP}{b_m \Delta h} \right) \quad (4.150)$$

where

$R$  = radius of the undeformed roll, mm,

$R'$  = radius of the deformed roll, mm,

$P$  = total roll force, kg,

$b_m$  = mean width of the rolled stock, mm,

$\Delta h$  = absolute draught, mm,

$c$  = coefficient equal to  $1.06 \times 10^{-4}$  mm<sup>2</sup>/kg for steel rolls, and  $2.06 \times 10^{-4}$  mm<sup>2</sup>/kg for cast-iron rolls.

When  $R'$  is known, then the length of flattened arc of contact is obtained from the formula

$$l'_d \approx \sqrt{R' \Delta h}$$

F. S. Larke [62] gives a nomograph for the solution of formula (4.150) in form of

$$\frac{R'}{R} = 1 + \frac{2cP}{b_m \Delta h}$$

This relation is shown in Fig. 4.115. On the abscissa the ratio  $R'/R$ , and on the ordinate the expression  $P/b_m \Delta h$  have been plotted. The three curves are for various values of coefficient  $c$ , i.e. for steel, chilled cast iron and cast-iron rolls.

The determination of the mean yield stress of the rolled material in the roll gap, is of great importance for cold rolling. The mean yield stress is independent of roll flattening, therefore it can be determined as for an undeformed roll. For example, the mean yield stress will be determined for the first pass of the reduction schedule given in Table 4.17 [72].



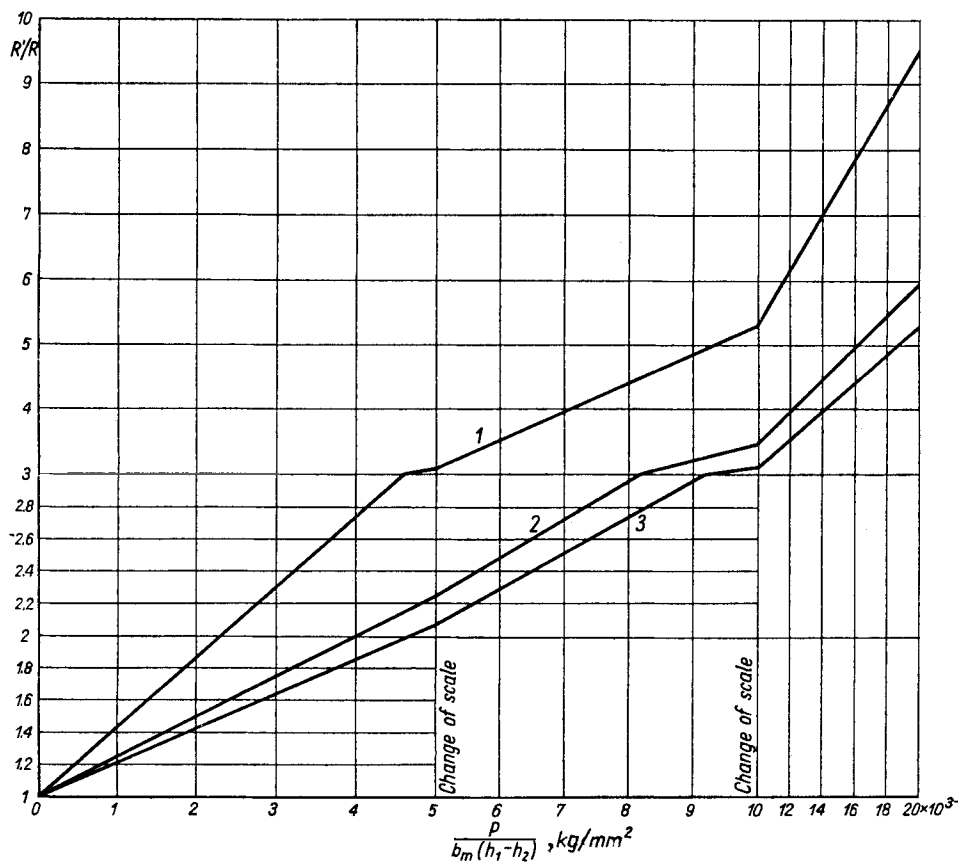


Fig. 4.115. Nomogram to determine ratio  $R'/R$  for deformed rolls: 1 — cast-iron rolls; 2 — chilled rolls; 3 — steel rolls

TABLE 4.17

H. FORD'S EXPERIMENTAL RESULTS FOR STEEL STRIP 0.2% C AND 0.89% Mn  
2-high rolling mill with roll diameter  $D = 254$  mm; strip  $2.54 \times 7.62$  mm

| Pass No. | Initial thickness $h_1$<br>mm | Final thickness $h_2$<br>mm | Percentage draught $\Delta h/h_1$<br>% | Total draught<br>% | Rate of rolling<br>t/h | Coefficient of friction $f$ | Roll peripheral speed $v_r$<br>m/sec | Measured roll force $P$<br>t |
|----------|-------------------------------|-----------------------------|----------------------------------------|--------------------|------------------------|-----------------------------|--------------------------------------|------------------------------|
| 1        | 2.54                          | 2.03                        | 20.1                                   | 20.1               | 0.869                  | 0.05                        | 0.203                                | 45.72                        |
| 2        | 2.03                          | 1.61                        | 20.7                                   | 36.6               | 0.689                  | 0.05                        | 0.203                                | 54.66                        |
| 3        | 1.61                          | 1.29                        | 19.9                                   | 49.2               | 0.552                  | 0.05                        | 0.203                                | 58.42                        |
| 4        | 1.29                          | 1.02                        | 20.9                                   | 59.8               | 0.436                  | 0.05                        | 0.203                                | 60.96                        |
| 5        | 1.02                          | 0.81                        | 20.6                                   | 68.1               | 0.347                  | 0.05                        | 0.203                                | 61.16                        |
| 6        | 0.81                          | 0.55                        | 32.1                                   | 78.3               | 0.235                  | 0.05                        | 0.203                                | 98.04                        |

For the first pass the bite angle can be determined from equation (3.17). For bite angles up to  $6^\circ$  formula (3.19) may be used, hence

$$\alpha \cong \sqrt{\frac{h_1 - h_2}{R}} \cong \sqrt{\frac{2.54 - 2.03}{127}} = 0.0633 \text{ radians}$$

The bite angle is divided into four parts each of 0.0158 radians giving five points along the arc of contact, corresponding to the following angles:

$$\begin{aligned}\varphi_1 &= 0 \\ \varphi_2 &= \alpha/4 = 0.0158 \text{ radians} \\ \varphi_3 &= 2 \alpha/4 = 0.0316 \text{ radians} \\ \varphi_4 &= 3 \alpha/4 = 0.0474 \text{ radians} \\ \varphi_5 &= 4 \alpha/4 = 0.0633 \text{ radians}\end{aligned}$$

For each point on the arc of contact defined by these angles, the following thicknesses of the roll gap  $h_{\varphi_i}$  between plane of entry and exit, are obtained from the formula

$$h_{\varphi_i} = h_1 + \varphi_i^2 R \quad (4.151)$$

$$\begin{aligned}h_{\varphi_1} &= 2.03 \text{ mm}, & h_{\varphi_2} &= 2.06 \text{ mm}, \\ h_{\varphi_3} &= 2.157 \text{ mm}, & h_{\varphi_4} &= 2.315 \text{ mm}, \\ h_{\varphi_5} &= 2.54 \text{ mm}\end{aligned}$$

Then for each of these points the corresponding percentage draught was calculated from

$$\varepsilon_{hi} = \frac{h_1 - h_{\varphi_i}}{h_1} 100\%, \text{ i.e.}$$

$$\varepsilon_{h1} = 20.1\%, \quad \varepsilon_{h2} = 18.9\%, \quad \varepsilon_{h3} = 15.08\%, \quad \varepsilon_{h4} = 8.86\%, \quad \varepsilon_{h5} = 0\%.$$

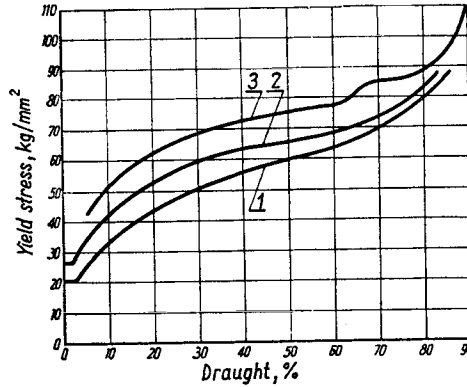


Fig. 4.116. Work-hardening curves for three steels: 1 — Armco iron (see Appendix 1), 2 — carbon steel of 0.11% C, 3 — carbon steel of 0.2% C

To determine the required mean yield stress  $K_{fm}$ , the yield stress curve for a 0.2% low-carbon steel, published by Polakowski [75] was used. Figure 4.116 gives the distribution of yield stress  $K_f$  as a function of percentage reduction for steels of various carbon contents. Curve 3 corresponds to a steel of 0.2% C content. From Fig. 4.116, the following yield stresses corresponding to the given draughts  $\varepsilon_{hi}$ , can be read off:

$$\begin{aligned}\varepsilon_{h1} = 20.1\% & \quad -K_{f1} = 61.75 \text{ kg/mm}^2 \\ \varepsilon_{h2} = 18.9\% & \quad -K_{f2} = 61.00 \text{ kg/mm}^2 \\ \varepsilon_{h3} = 15.08\% & \quad -K_{f3} = 58.3 \text{ kg/mm}^2 \\ \varepsilon_{h4} = 8.86\% & \quad -K_{f4} = 50.5 \text{ kg/mm}^2 \\ \varepsilon_{h5} = 0\% & \quad -K_{f5} = 23.8 \text{ kg/mm}^2\end{aligned}$$

From these values, the mean yield stress can be determined from the equation

$$K_{fm} = \frac{1}{4} \left( \frac{K_{f1}}{2} + K_{f2} + K_{f3} + K_{f4} + \frac{K_{f5}}{2} \right) \quad (4.152)$$

and for the first pass of Table 4.17 the value of

$$K_{fm} = 53.14 \text{ kg/mm}^2$$

is found.

For comparison the different values necessary to determine the mean yield stress are plotted once more in Table 4.18.

TABLE 4.18  
CALCULATION OF MEAN YIELD STRESS OF A LOW-CARBON STEEL 0.2% C FOR THE  
FIRST PASS OF TABLE 4.17

| Angular co-ordinate of the roll gap                                        | 1     | 2      | 3      | 4      | 5      |
|----------------------------------------------------------------------------|-------|--------|--------|--------|--------|
| Angular co-ordinate $\varphi_i$ , radians                                  | 0     | 0.0158 | 0.0316 | 0.0474 | 0.0633 |
| Product $\varphi_i^2 R$                                                    | 0     | 0.0317 | 0.1268 | 0.2853 | 0.5089 |
| Thickness $h_{\varphi_i} = h_1 + \varphi_i^2 R$ , mm                       | 2.03  | 2.06   | 2.157  | 2.315  | 2.54   |
| Percentage draught $\varepsilon_n = \frac{h_1 - h_{\varphi_i}}{h_1} 100\%$ | 20.1  | 18.9   | 15.08  | 8.86   | 0      |
| Field stress $K_f$ , kg/mm <sup>2</sup> (Fig. 4.116, curve 3)              | 61.75 | 61.0   | 58.3   | 50.5   | 23.8   |

As there is no spread in cold rolling, the coefficient  $\eta = 1.155$ , and therefore the mean yield stress in plane strain  $K_m = 1.155 K_{fm}$ . Therefore:

$$K_m = 1.155 K_{fm} = 1.155 \times 53.14 = 61.11 \text{ kg/mm}^2$$

In the same way the values of mean yield stress in plane strain for other passes given in Table 4.17 were determined. For other steels, the corresponding stress-strain curves in Fig. 4.116 should be chosen.

As this method of calculation is very laborious, other methods have been developed to simplify the determination of the mean yield stress in plane strain, also using the stress-strain curves (Fig. 4.116), and the following procedure is used:

(a) calculation of the mean percentage reduction [36]:

$$R_m = 0.4 \varepsilon_{h1} + 0.6 \varepsilon_{h2} \quad (4.153)$$

$\varepsilon_{h1}$  = total percentage reduction before the pass,

$\varepsilon_{h2}$  = total percentage reduction after the pass,

(b) for this value of mean percentage reduction  $R_m$  the value of mean yield stress should be read off from stress-strain curves shown in Fig. 4.116,

(c) the value found should be multiplied by coefficient of  $\eta$  to determine  $K_m = \eta K_{fm}$ .

This method is only approximate in comparison with the previous one.

#### Example

In the first pass  $\varepsilon_{h1} = 0$ , due to delivery of the rolled stock in annealed condition. The percentage reduction  $\varepsilon_{h2} = 18\%$ . It is required to find the mean yield stress in plane strain.

From formula (4.153)

$$R_m = 0.4 \varepsilon_{h1} + 0.6 \varepsilon_{h2} = 0.4 \times 0 + 0.6 \times 18 = 10.8\%$$

From Fig. 4.116 the corresponding mean yield stress for this value is  $K_{fm} = 54.0 \text{ kg/mm}^2$  for 0.2% C steel (curve 3).

Since the coefficient  $\eta = 1.155$ , due to suppressed spread during cold rolling, therefore the mean yield stress in plane strain becomes

$$K_m = 1.155 K_{fm} = 1.155 \times 54 = 62.4 \text{ kg/mm}^2$$

Comparing the values of mean yield stress in plane strain calculated by the approximate method and the accurate method, only small differences are observed.

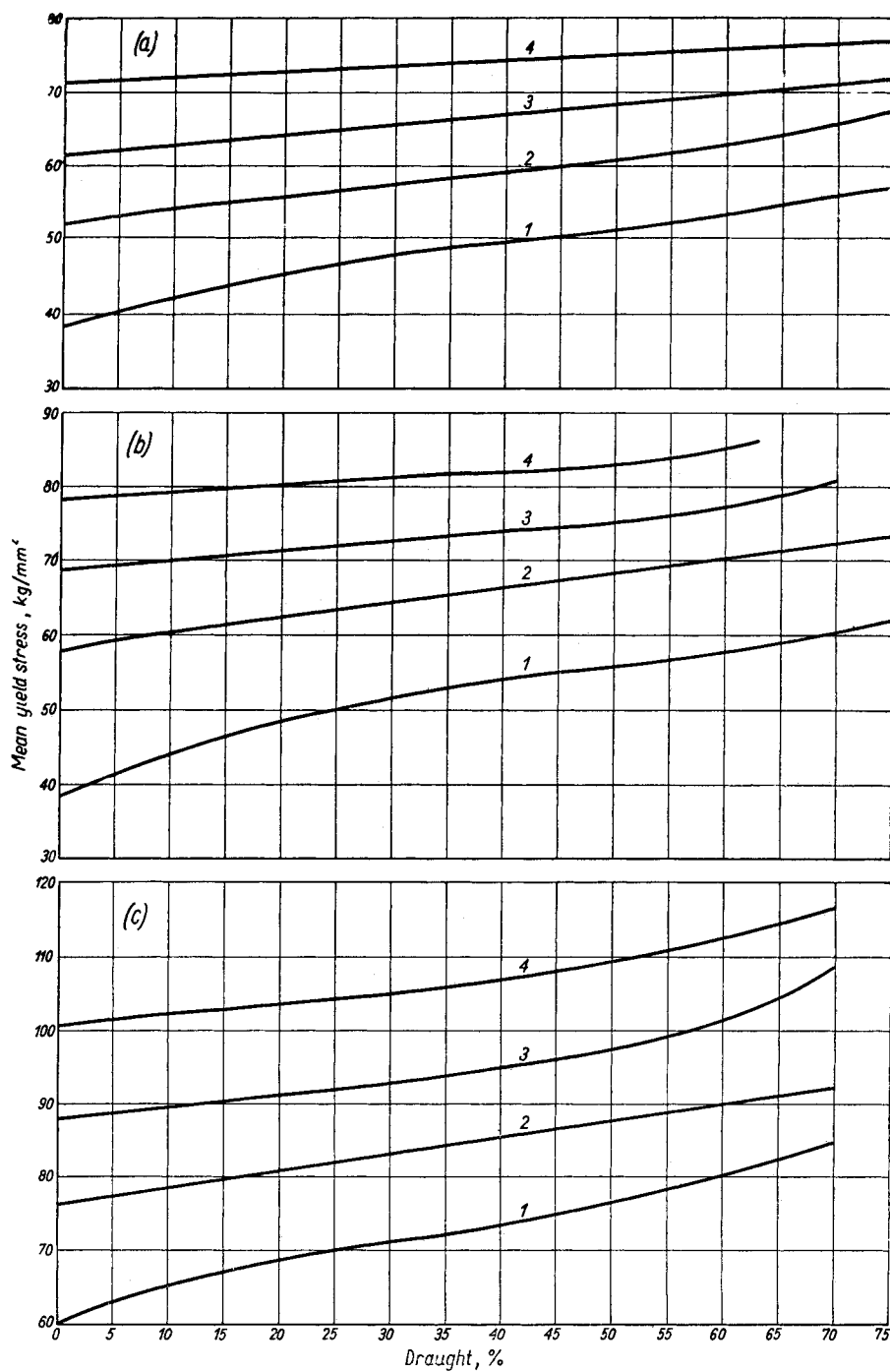


Fig. 4.117. Mean yield stress for three steels: (a) 0.1% C, (b) 0.29% C, (c) 0.63% C, as a function of draught and initial deformation. Initial deformation: 1—0%, 2—20%, 3—40%, 4—60%

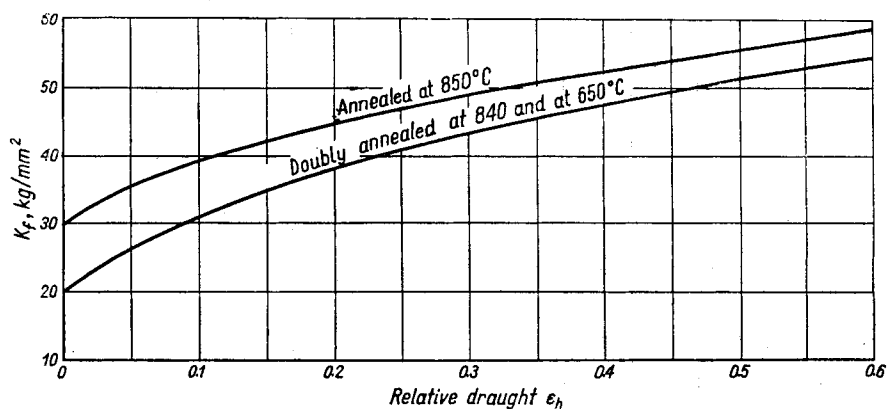


Fig. 4.118. Yield stress of Armco iron after different heat treatments (Table 4.19) [154]

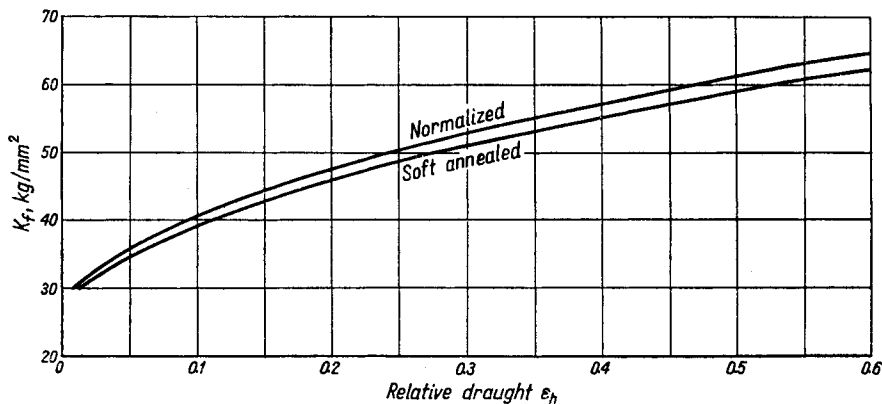


Fig. 4.119. Yield stress of normalized and annealed St37 steel (Table 4.19)

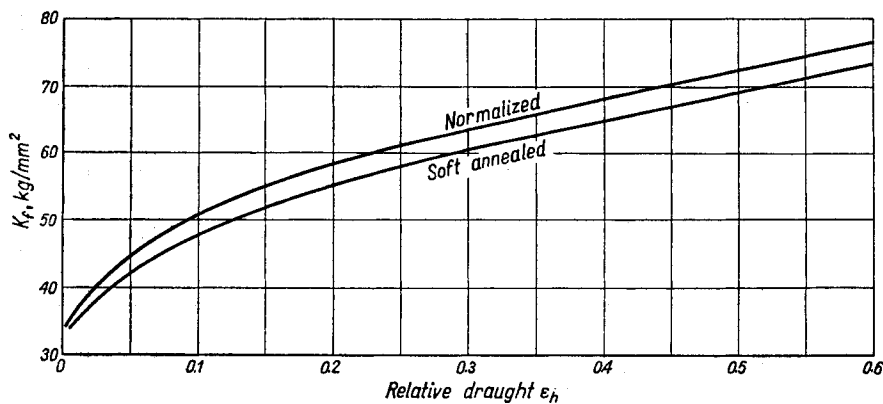


Fig. 4.120. Yield stress of normalized and annealed St42 steel (Table 4.19)

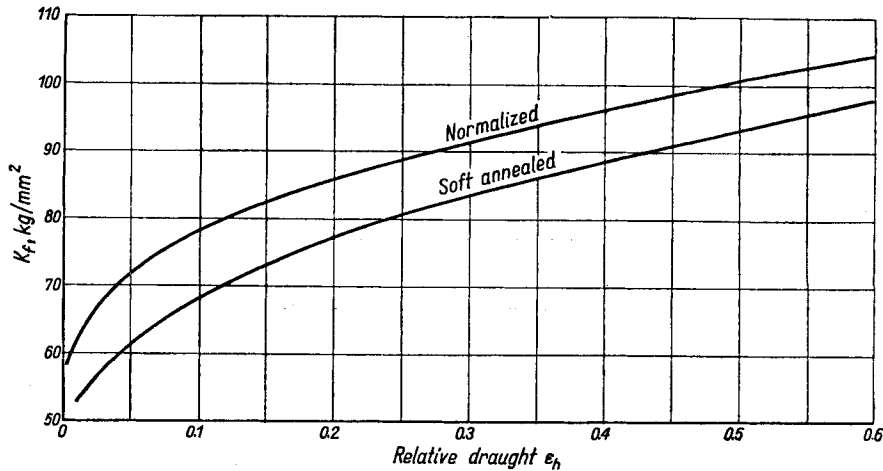


Fig. 4.121. Yield stress of normalized and annealed St70 steel (Table 4.19)

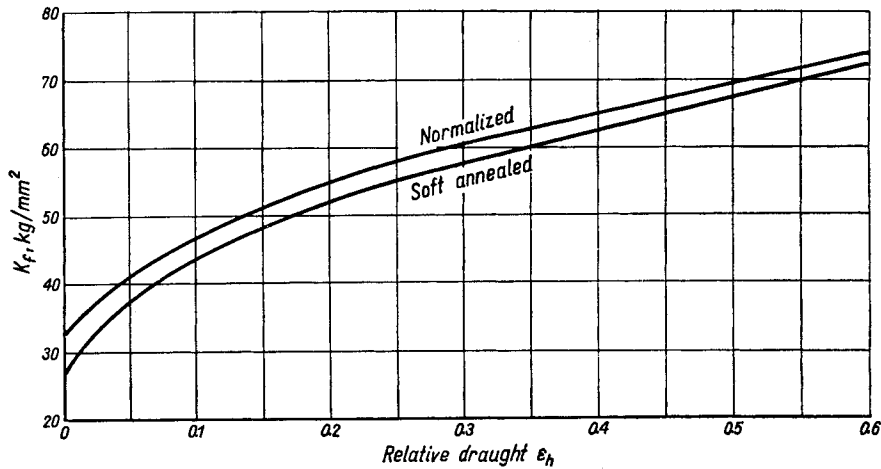


Fig. 4.122. Yield stress of normalized and annealed C10 steel (Table 4.19)

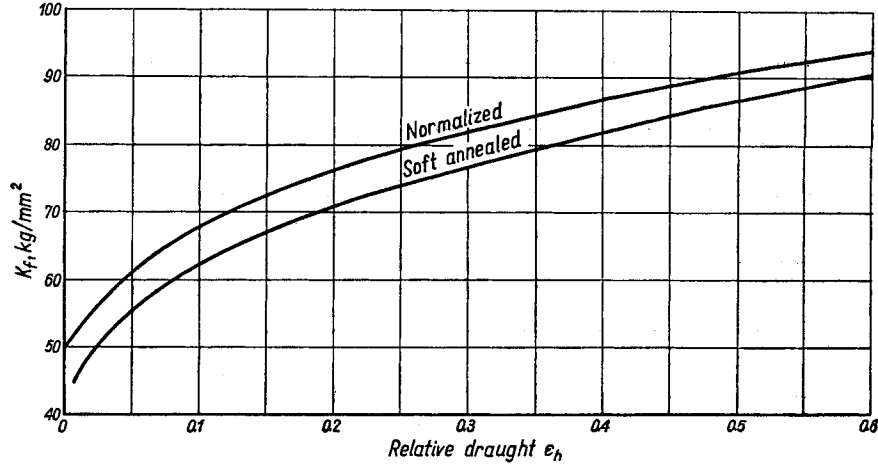


Fig. 4.123. Yield stress of normalized and annealed C35 steel (Table 4.19)

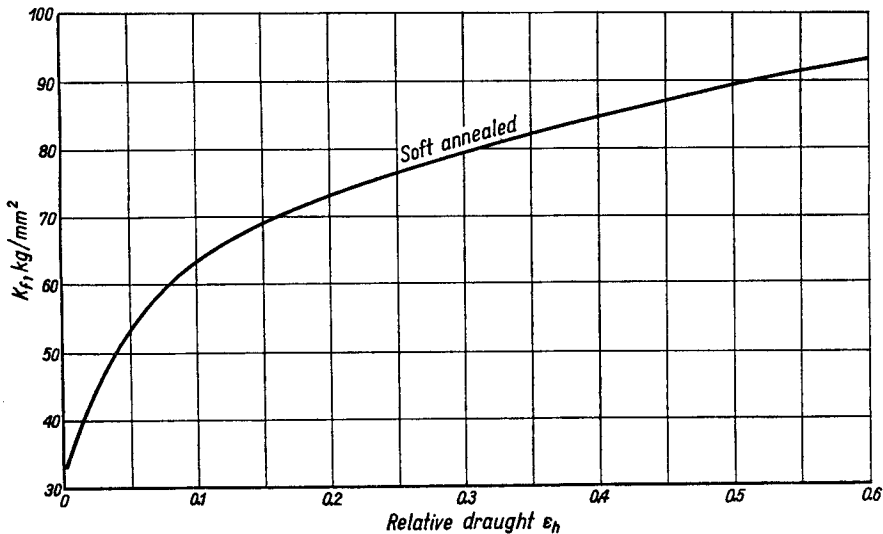


Fig. 4.124. Yield stress of annealed C100W1 steel (Table 4.19)

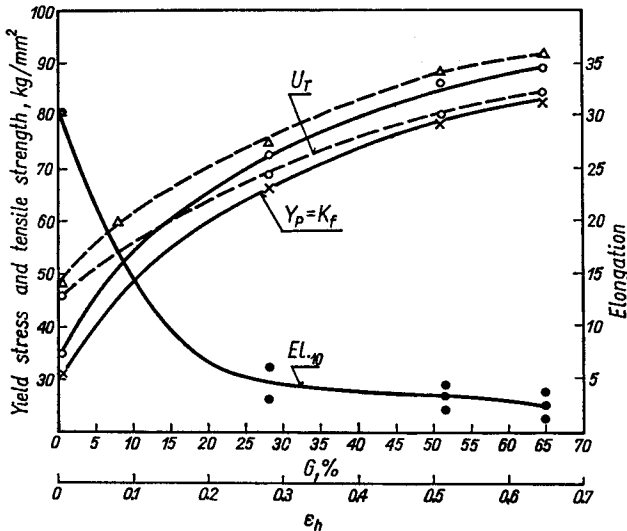


Fig. 4.125. Tensile strength, yield point and elongation of steel 10GL as a function of deformation

TABLE 4.19  
CHEMICAL COMPOSITION OF STEELS

| Steel type<br>(from SES) | Composition, % |      |       |      |       |        | Figure |
|--------------------------|----------------|------|-------|------|-------|--------|--------|
|                          | C              | Mn   | Si    | Ni   | Cr    | N      |        |
| Armco                    | 0.03           | 0.03 | 0.002 | —    | —     | —      | 4.118  |
| St37                     | 0.08           | 0.36 | 0.06  | —    | —     | 5.007  | 4.119  |
| St42                     | 0.22           | 0.44 | —     | —    | —     | 0.005  | 4.120  |
| St70                     | 0.51           | 0.55 | 0.28  | —    | —     | 0.0062 | 4.121  |
| C10                      | 0.13           | 0.31 | 0.23  | —    | —     | 0.004  | 4.122  |
| C35                      | 0.36           | 0.69 | 0.27  | —    | —     | 0.004  | 4.123  |
| C100W1                   | 1.05           | 0.21 | 0.16  | —    | —     | —      | 4.124  |
| 10GL                     | 0.18           | 1.38 | 0.20  | 0.3  | 0.3   | —      | 4.125  |
| 30HGSA                   | 0.32           | 0.98 | 1.03  | 0.17 | 0.95  | —      | 4.126  |
| 1H18N9                   | 0.10           | 1.80 | 0.60  | 9.00 | 19.00 | —      | 4.127  |

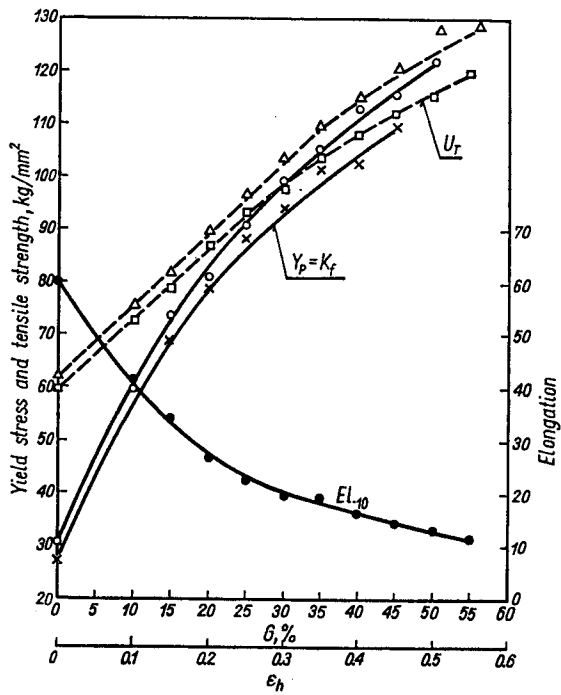


Fig. 4.126. Tensile strength, yield point and elongation of 30HGSA steel as a function of deformation (Table 4.19)

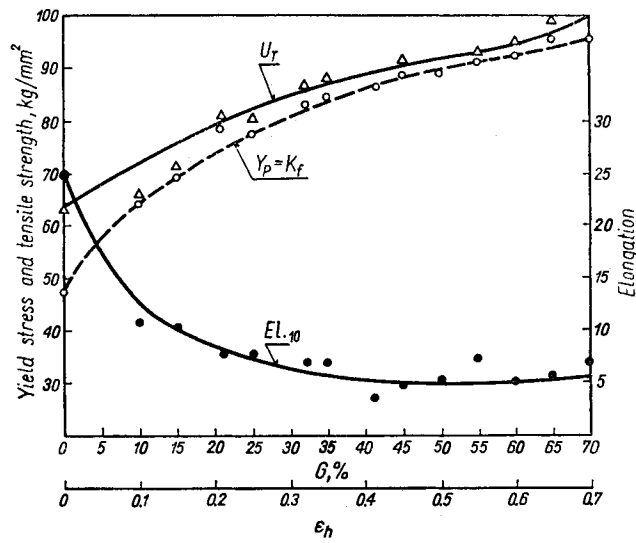


Fig. 4.127. Tensile strength, yield point and elongation of 1H18N9 steel as a function of deformation (Table 4.19)



The speed and relative accuracy of this method (4.153) make it most suitable for practical use.

Besides the stress-strain curves shown in Figs. 4.116 and 4.117, curves have been published by VDI-Arbeitsblätter [154]. From these, curves for the following steels were chosen: Armco iron, St37, St42, St70, C10, C35 and C100W1. These curves are shown in Figs. 4.118–4.124. The structure has been taken into account, and curves for the normalized as well as for the soft annealed condition, are given. The chemical composition of the steels is given in Table 4.19.

For high-alloy steels results of Soviet investigations [158] are shown on stress-strain curves for 10GL, 30HGSA and 1H18N9 steels in Figs. 4.125–4.127. The yield stress in plane strain is denoted as  $K_f$ .

#### 4.4.1. TSELIKOV'S METHOD OF CALCULATION OF ROLL FORCE IN COLD ROLLING

The roll force formula derived by Tselikov is based on the theory of dry friction between roll surface and stock, assuming that the arc of contact is equal to the chord joining the points of entry and exit of metal from the rolls. The total rolling force is obtained by integration of forces acting upon a volume element in the roll gap. Equations and method of calculation are identical with those for hot rolling. Therefore, the roll force can be determined using equations (4.26), (4.27), (4.28) and Fig. 4.31. To determine the mean resistance to deformation, the mean yield stress in plane strain must be calculated from the formula

$$K_m = 1.155K_{fm} = \int_0^\alpha \frac{1.155K_f df}{\alpha} \quad (4.154)$$

Measured and calculated roll forces are very close, but the error in first passes equals  $\pm 10\%$  [72]. For final passes at lower roll peripheral speeds, greater errors of up to  $-27\%$  are found. In these cases considerable flattening of the rolls occurs [72]. For increasing roll peripheral speed  $v$ , the calculated roll force values exceed those of measured ones, due to the fact that Tselikov's formula does not take into account the influence of roll speed on the roll force [72], [73].

#### 4.4.2. EKEKUND'S METHOD OF CALCULATION OF ROLL FORCE IN COLD ROLLING

The formula derived by Ekelund serves for calculation of roll forces in hot rolling. Formula (4.36) has been simplified by Bland and Ford [36] after a precise analysis, and transformed for cold rolling into the following form [10], [15]

$$P = bK_{fm}\sqrt{R'(h_1-h_2)} \left[ 1 + \frac{1.6f\sqrt{R'(h_1-h_2)} - 1.2(h_1-h_2)}{h_1+h_2} \right] \quad (4.155)$$

In this equation the value of  $K_{fm}$  is calculated from formulae (4.152 and 4.153) as the mean value of the yield stress in plane strain in the roll gap. The roll radius  $R'$  is found from equation (4.150).

As in Tselikov's formula the results of calculation depend considerably on roll flattening; the influence of roll peripheral speed  $v$ , is not taken into account. Therefore, for the first passes, i.e. for small rolling speeds, the calculated values of roll force are

almost equal to or slightly less than measured values, the error varying from  $+1$  to  $-6.5\%$ . Due to greater roll flattening in succeeding passes the difference between calculated and measured values increases to about  $-30\%$  [72, 73].

#### 4.4.3. SIEBEL'S METHOD OF CALCULATION OF ROLL FORCE IN COLD ROLLING

For determining the roll force by Siebel's method, curves for yield stress shown in Figs. 4.117a-c can be used. These curves based on experiments carried out by Siebel for straight carbon steels 0.10% C, 0.20% C and 0.63% C, and various coefficients of initial cold-working, show mean yield stress as the arithmetic mean of values before and after pass. From these curves the value of  $K_{fm}$  may be read off for a known percentage reduction in the given pass and initial deformation.

For annealed strip the initial deformation equals zero. In the  $n$ -th pass this deformation is

$$A_i = \frac{h_1 - h_{n-1}}{h_1} 100\% \quad (4.156)$$

where

$h_1$  = initial thickness of the strip before the first pass,

$h_{n-1}$  = initial thickness of the work-hardened strip before the  $n$ -th pass, without intermediate annealing.

The value of mean resistance to deformation  $K_{wm}$  is found by dividing  $K_{fm}$  by the rolling efficiency coefficient  $\eta_F$ . Values of  $\eta_F$  for various ranges of percentage reduction and  $h_2/D$  ratios are given in Table 4.20.

TABLE 4.20  
EFFICIENCY OF DEFORMATION,  $\eta_F$ , IN PER CENT FOR COLD ROLLING,  
RELATIVE TO PERCENTAGE DRAUGHT AND RATIO OF THICKNESS AFTER  
PASS  $h_2$  TO ROLL DIAMETER  $D$

| $h_2/D$<br>% | Coefficient $\eta_F$ for percentage draught $(\Delta h/h_1) 100\%$ |        |           |
|--------------|--------------------------------------------------------------------|--------|-----------|
|              | less than 20%                                                      | 20-40% | above 40% |
| 5            | 70-90                                                              | 75-90  | —         |
| 2-5          | 60-80                                                              | 65-85  | 70-90     |
| 1-2          | 50-70                                                              | 55-75  | 60-80     |
| 0.5-1        | 40-60                                                              | 45-65  | 50-70     |
| 0.2-0.5      | 30-50                                                              | 35-55  | 40-60     |

By Siebel's method roll forces can be quickly and simply calculated as follows:

(a) for the given pass first of all the initial total reduction should be calculated, to determine the initial conditions of the rolled stock. As the pass reduction is known, the value of the mean yield stress  $K_{fm}$  may be read off from Figs. 4.117a, b, c.

(b) from Table 4.20 the appropriate rolling efficiency coefficient  $\eta_F$  can be found, hence the mean resistance to deformation

$$K_{wm} = \frac{K_{fm}}{\eta_F} 100 \quad (4.157)$$

(c) roll force  $P$  can then be found

$$P = K_{wm} F_d$$

The projected area of contact  $F_d$  is calculated without taking into account possible roll flattening.

Roll forces determined in this way not taking into account roll flattening are generally greater than those obtained by measurement; however in no case does the error exceed 30%. The average error varies from 5–20%. Only in the last passes is the calculated roll force lower than the measured value, from experimental results given in Table 4.17, due to difficulty in reading off from the extrapolated  $K_{fm}$ -curves [72, 73].

Siebel states that the roll flattening should be taken into account for values of yield stress exceeding 50 kg/mm<sup>2</sup> due to work-hardening of the rolled stock [61]. Since for almost every pass  $K_{fm}$  exceeded 50 kg/mm<sup>2</sup>, these values were recalculated taking into account the elastic roll flattening. Values much higher than measured ones were found, the error for large roll force and considerable roll flattening being as much as 357%. Hence Siebel's proposition is incorrect, and for calculations it is better to use the undeformed roll [72, 73].

Good results are obtained using Siebel's method for rigid rolls and as it is quick and straightforward, it is suitable for practical purposes. Differences between values of calculated and measured roll forces are found, but as calculated values usually exceed true ones, their use for design calculations gives a greater safety reserve factor.

Difficulty in the correct choice of the deformation efficiency coefficient  $\eta_F$  is a disadvantage of this method.

#### 4.4.4. CALCULATION OF ROLL FORCE IN COLD ROLLING FROM THE MEAN RESISTANCE TO DEFORMATION [15, 31]

This method is based on diagrams which have been plotted for cold rolling experiments of different materials at various rolling conditions. The mean resistance to deformation in each pass was found by measuring roll force, and dividing by the projected area of contact, assuming undeformed rolls. Hence

$$K_{wm} = \frac{P}{b_m \sqrt{R(h_1 - h_2)}} \quad (4.158)$$

For practical applications such diagrams are very suitable, for a given case the value of  $K_{wm}$  is read off, and multiplied by the projected area of contact  $F_d$  giving the roll force  $P$  as in equation (2.108). Diagrams must be chosen to fit actual rolling conditions. Among the best known curves for determining mean resistance to deformation of steel, relative to the initial deformation and the thickness ratio  $h_2/R$ , are those published by SKF [76, 77]. Curves shown in Fig. 4.128 are denoted by fractions, where the numerator indicates the initial deformation and the denominator the pass reduction, e.g. 0/10 indicates steel in a soft annealed condition rolled with 10% draught. If the steel considered is then rolled using 20% draught per pass, then for this pass the curve 10/20 is valid. It may be seen from these curves that the mean resistance to deformation  $K_{wm}$  increases considerably with increased roll radius, i.e. for diminished thickness ratio. The curves shown in Fig. 4.128 may be interpolated, but only with difficulty.

To apply this diagram, first the reduction per pass, the total reduction, and the thickness ratio  $h_2/R$ , should be calculated, and then from Fig. 4.128 the value of  $K_{wm}$  should be read off. Multiplying this value by the projected area of contact the roll force is obtained.

Values of mean resistance to deformation in cold rolling of steels of different chemical compositions, are given in Fig. 4.129, as a function of the mean total percentage draught. Test data are given in Table 4.21. Values of mean percentage reduction plotted on the abscissa denote the arithmetical mean of the total percentage reduction

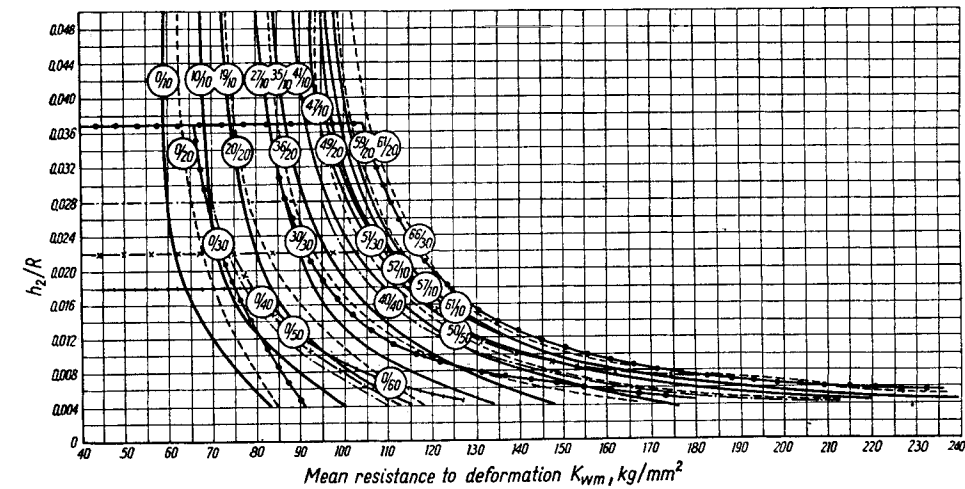


Fig. 4.128. SKF curves mean for resistance to deformation during cold rolling of mild steel [77]

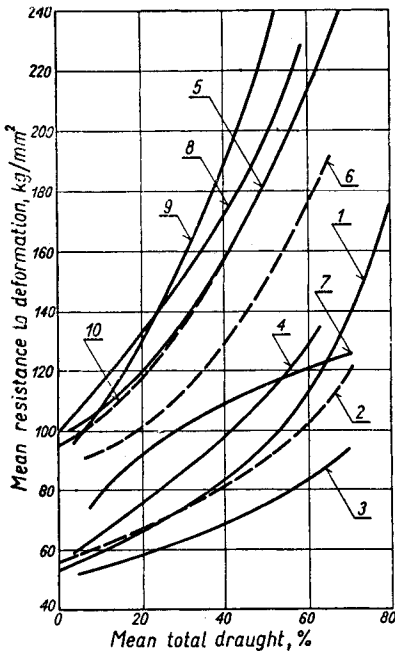


Fig. 4.129. Mean resistance to deformation during cold rolling of annealed specimens of carbon and alloy steels, as a function of mean total draught (see Table 4.21)

of stock in initial condition after annealing and percentage reduction in the given pass, e.g. if initial work-hardening is 20%, and the pass reduction 30%, then in Fig. 4.129 as the mean reduction the value of  $(20+30)/2 = 25\%$  is plotted. If the calculation of

TABLE 4.21  
ROLLING CONDITION FOR FIG. 4.129

| Curve | Material        | Chemical analysis<br>%           | Initial thickness<br>mm | Width<br>mm    | Roll diameter<br>mm | Rolling speed<br>m/sec | Lubrication                    | Source                            |
|-------|-----------------|----------------------------------|-------------------------|----------------|---------------------|------------------------|--------------------------------|-----------------------------------|
| 1     | Mild steel      | —                                | 2.5                     | 114.3          | 152.4               | 1.12                   | oil emulsion:<br>up to 60% oil | Davies                            |
| 2     | Mild steel      | C 0.1<br>Mn 0.45                 | 1.9<br>2                | 190.5<br>57.15 | 180                 | 0.356                  | over 60% oil<br>dry            | Siebel and<br>Pomp                |
| 3     | Mild steel      | C 0.1<br>Mn 0.45                 | 2                       | 57.15          | 180                 | 0.356                  | oil                            | Siebel and<br>Pomp                |
| 4     | Mild steel      | C 0.15                           | 3.48                    | 66             | 211                 | 0.44                   | oil                            | SKF                               |
| 5     | Carbon steel    | C 0.6                            | 3.48                    | 66             | 211                 | 0.44                   | oil                            | SKF                               |
| 6     | Carbon steel    | C 0.63<br>Mn 0.62                | 2                       | 57.15          | 180                 | 0.356                  | dry                            | Siebel and<br>Pomp                |
| 7     | Carbon steel    | C 0.63<br>Mn 0.62                | 2                       | 57.15          | 180                 | 0.356                  | oil                            | Siebel and<br>Pomp                |
| 8     | Carbon steel    | C 1.00                           | 3.48                    | 66             | 211                 | 0.44                   | oil                            | SKF                               |
| 9     | Stainless steel | C 0.15<br>Mn 0.5<br>Cr 16.5–19.5 | 1.52                    | 31.7<br>140    | 343                 | 0.9                    | oil                            | Timken<br>Roller<br>Bearing<br>Co |
| 10    | Stainless steel | as 9+<br>Ni 7–10                 |                         |                |                     |                        |                                |                                   |

roll force is based upon a 0.15% C carbon steel as for curve 4 in Fig. 4.129, then lower values are found than for the 0.2% C curve published by SKF. However, for some passes, specially for higher rolling speeds, in spite of this fact calculated values exceed measured values by as much as 20%. The error varies from  $-10$  to  $+20\%$ . Using the SKF diagram the difference between calculated and measured values becomes slightly greater.

All curves shown in Fig. 4.129 are for successive passes, starting from annealed condition of the steel with thickness ratio  $h_1/R$  for the first pass, given in Table 4.21. This diagram can be recalculated for other thickness ratios  $h_1/R$ . The given values of resistance to deformation are for cold rolling without back tension assuming rigid undeformed rolls. The following example shows the marked influence of roll diameter on roll force.

#### Example

Roll forces arising during rolling of strip in 2-high, 3-high and 4-high mills are compared and the influence of roll diameter considered [5]. The test steel strip is of 1000 mm width and 2 mm thickness. The mean resistance to deformation is 60 kg/mm<sup>2</sup>, and the permissible roll force 657 t. It is required

to find the permissible absolute draught for a 2-high mill of 600 mm roll diameter, a 3-high Lauth mill of 300 and 600 mm roll diameters, and a 4-high mill of 300 mm work roll diameter.  
From the formula

$$P \leq K_{wm} b \sqrt{R \Delta h}$$

for 2-high mill:  $657 = 60 \times 1000 \sqrt{300(2-h_1)}$

hence  $h_1 = 1.6$  mm and  $\Delta h = 0.4$  mm,  $\varepsilon_h = 20\%$

for 3-high Lauth:  $657 = 60 \times 1000 \sqrt{\frac{2 \times 300 \times 150}{300+150} (2-h_1)}$

hence  $h_1 = 1.4$  mm,  $\Delta h = 0.6$  mm, and  $\varepsilon_h = 30\%$

for 4-high mill:  $657 = 60 \times 1000 \sqrt{150(2-h_1)}$

hence  $h_1 = 1.2$  mm,  $\Delta h = 0.8$  mm, and  $\varepsilon_h = 40\%$

The results obtained show that for the same rolling conditions, with a 4-high mill of small roll diameter, draughts twice as large as for a 2-high can be used.

#### 4.4.5. CALCULATION OF ROLL FORCE IN COLD ROLLING OF STRIP WITH APPLIED FRONT AND BACK TENSION

If the coiler acts upon the strip leaving the rolls with a tension of  $Q_2$ , and the decoiler acts before rolling with a tension  $Q_1$  directed opposite to the rolling direction, then a stress  $\sigma_2$  [kg/mm<sup>2</sup>], due to front tension, and  $\sigma_1$  [kg/mm<sup>2</sup>], due to back tension arises in the strip (Fig. 4.130).

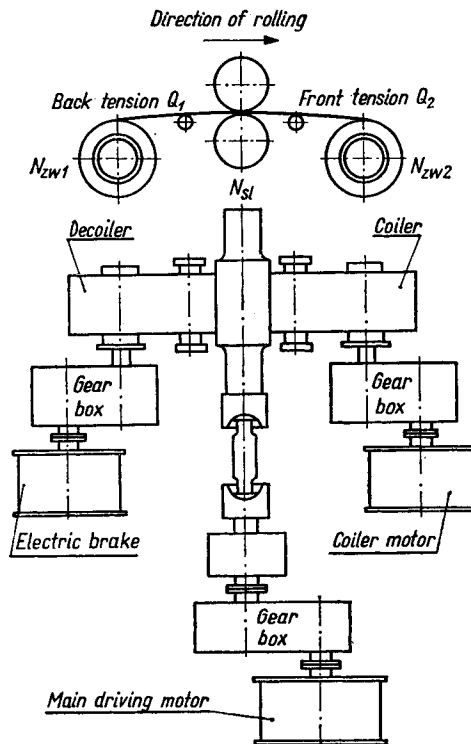


Fig. 4.130. Layout of a reversing cold rolling mill with coiler and decoiler

For a strip width of  $b$ , the tension forces are given by

$$Q_1 = \sigma_1 b h_1 \quad (4.159)$$

and

$$Q_2 = \sigma_2 b h_2 \quad (4.160)$$

The influence of front and back tension on the resistance to deformation has already been discussed, and is shown schematically in Fig. 4.8. The curve  $DGE$  in Fig. 4.3 shows the distribution of the yield stress in plane strain in the roll gap. The value of this stress increases from the plane of entry towards the plane of exit, due to work-hardening of the rolled stock. If any front or back-tension is applied, the distribution of the resistance to deformation along the roll gap shown in Figs. 4.3 and 4.8, is obtained, when adding the resistance to flow to the yield stress in plane strain and determining in this way the curve  $DFE$ . It may be seen from Fig. 4.8 that the resistance to deformation, and hence roll force, decreases with applied front and back tension.

Front tension acts in the rolling direction and assists the main driving motor, but back tension acts against the rolling direction, having therefore a braking effect on the main driving motor. When front and back tension are applied simultaneously, then in the plane of entry the yield stress in plane strain decreases by an amount represented by  $DM$ , and in the plane of exit by an amount represented by  $EL$ . Resistance to deformation at each point along the roll gap decreases and therefore roll force too.

The influence of strip tension can also be determined by means of calculation. By Tselikov's method the total roll force can be calculated with applied front and back tension, if the distribution of unit roll pressure  $p_x$  in the roll gap is known. Assuming that stresses caused by front and back tension, and also the pressure distribution along the roll gap are known, then the roll force is given by

$$P = \frac{b_1 + b_2}{2} \int_0^{l_d} p_x dx = b_m \int_0^{l_d} p_x dx \quad (4.161)$$

An approximate solution to equation (4.161) is given by Tselikov [152], based on the known differential equation

$$\frac{dp_x}{dx} - \frac{k'_f}{y} \frac{dy}{dx} \pm \frac{f}{y} p_x = 0 \quad (4.162)$$

where

$k'_f$  = yield stress in plane strain equals to  $K = 1.15 K_f$ ,

$f$  = coefficient of friction between roll and stock in the gap.

As strip tension is applied in cold rolling only, when the bite angle is less than  $\alpha = 4^\circ$ , then Tselikov uses the following simplification for the zone of forward slip (Fig. 4.131). Denoting as

$$\xi_1 = \frac{k'_{f1} - \sigma_1}{k'_{f1}} \quad \text{and} \quad \xi_2 = \frac{k'_{f2} - \sigma_2}{k'_{f2}} \quad \text{and} \quad m = \frac{2fl_d}{h_1 + h_2}$$

then the following formula for calculation of roll force in cold rolling with applied tension is found

$$P = \frac{b_m l_d \xi_2 k'_{f2}}{m} \left[ \left( \frac{\xi_1 k'_{f1}}{\xi_2 k'_{f2}} \right)^{h_1/(h_1 + h_2)} e^m + \frac{\xi_1 k'_{f1}}{\xi_2 k'_{f2}} \left( \frac{h_1}{h_1 + h_2} - \frac{h_2}{h_1 + h_2} \right) \right] \quad (4.163)$$

Dividing formula (4.163) through by the projected area of contact  $F_d = b_m l_d$ , then the mean roll pressure in the roll gap is found, equal to the mean resistance to deformation of the rolled stock

$$p_m = K_{wm} = \frac{\xi_2 k'_{f2}}{m} \left[ \left( \frac{\xi_1 k'_{f1}}{\xi_2 k'_{f2}} \right)^{h_1/(h_1+h_2)} e^m + \frac{\xi_1 k'_{f1}}{\xi_2 k'_{f2}} \left( \frac{h_1}{h_1+h_2} - \frac{h_2}{h_1+h_2} \right) \right] \quad (4.164)$$

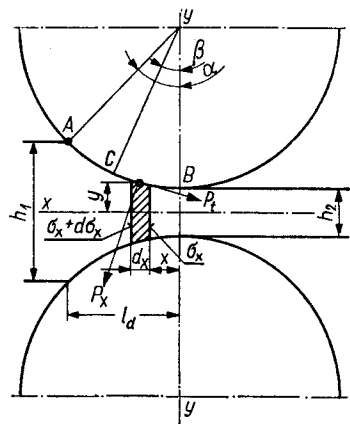


Fig. 4.131. Schematic figure for determining roll force during cold rolling with applied tension

According to Tselikov this equation is also applicable to cold rolling without tension, i.e. to the case  $\xi_1 = \xi_2 = 1$ , for calculating roll force. Assuming further

$$k'_{fm} = \frac{k'_{f1} + k'_{f2}}{2} \quad \text{and} \quad h_m = \frac{h_1 + h_2}{2}$$

then the following simplified formula is found:

$$K_{wm} = \frac{k'_{fm}}{m} (e^m - 1) \quad (4.165)$$

where  $m$  = denotes the same value as before.

Results of measurements given in Table 4.22, and those calculated from formula (4.163) are compared in Table 4.23. It is assumed that rolls are elastically flattened.

TABLE 4.22

EXPERIMENTAL VALUES FOR COLD ROLLING OF 0.2% C STEEL STRIP, OBTAINED BY H. FORD [36]  
Three-stand 4-high tandem mill of  $D = 254$  mm work roll diameter, with front and back tension. Initial strip dimensions  $2.54 \times 254$  mm, coefficient of friction  $f = 0.09$

| Pass No. | $h_1$ | $h_2$ | $\Delta h/h_1$ | $G_t$ | $\sigma_0$         | $\sigma_1$         | $v_r$ | $P$   | $M$    | $N_w$   | Torque acting on the roll neck | Total torque | Motor Power |
|----------|-------|-------|----------------|-------|--------------------|--------------------|-------|-------|--------|---------|--------------------------------|--------------|-------------|
|          | mm    | mm    | %              | %     | kg/mm <sup>2</sup> | kg/mm <sup>2</sup> | m/sec | t     | kgm    | kgm/sec | kgm                            | kgm          | kW          |
| 1        | 2.54  | 1.4   | 45             | 45    | 0                  | 16.75              | 3.2   | 224.5 | 1589.7 | 42,750  | 106.6                          | 1696.3       | 526.2       |
| 2        | 1.4   | 0.98  | 30             | 61.4  | 15.75              | 28.35              | 4.6   | 147.3 | 753.6  | 29,812  | 69.9                           | 823.5        | 330.6       |
| 3        | 0.98  | 0.88  | 10             | 65.4  | 28.35              | 31.5               | 5.1   | 77.2  | 214.2  | 9,900   | 36.7                           | 250.9        | 144.0       |



TABLE 4.23  
COMPARISON OF CALCULATED AND EXPERIMENTAL ROLL FORCES IN COLD ROLLING

| No. | Method of calculation and error                   | Pass No. |       |       |
|-----|---------------------------------------------------|----------|-------|-------|
|     |                                                   | 1        | 2     | 3     |
| 1   | Measured roll force, t                            | 224.5    | 147.3 | 77.2  |
| 2   | Trinks's graphical method worked out by Keller, t | 255.2    | 186.2 | 100.9 |
| 3   | Error, %                                          | +13.7    | +26.4 | +30.6 |
| 4   | Korolev-Nikolayevski, t                           | 225.2    | 207.8 | 95.9  |
| 5   | Error, %                                          | +0.3     | +41.0 | +24.2 |
| 6   | Tselikov, t                                       | 150.5    | 204.8 | 98.8  |
| 7   | Error, %                                          | -33.0    | +39.0 | +27.9 |
| 8   | Graphical method of Bland, Ford and Ellis, t      | 269.7    | 199.2 | 97.21 |
| 9   | Error, %                                          | +20.1    | +35.2 | +25.9 |

A comparison of results of measurement given in Table 4.22 or 4.23 and those calculated by Tselikov's method shows considerable divergence between calculation and experiment. The percentage error in the first, second, and third passes equals -33%, +39% and +27.9%, respectively.

Moreover, Tselikov's formula cannot be applied for  $K_{f1} < \sigma_1$ , if  $K_{f2} > \sigma_2$ , or if  $K_{f2} < \sigma_2$  for  $K_{f1} > \sigma_1$ . For such cases the expression

$$\left( \frac{\xi_1 k'_{f1}}{\xi_2 k'_{f1}} \right)^{h_1/(h_1+h_2)}$$

gives unreal results.

In pass No. 2 and 3 (Tables 4.22 and 4.23) bite angles are smaller than  $4^\circ$ . The calculated roll forces exceed measured ones by approximately 30-40%. Thus these results contradict Tselikov's statement that for small bite angles below  $4^\circ$  good agreement is found between calculation and experiment.

Also it should be remembered that Tselikov's formula and also that of Trinks, does not take into account the influence of rolling speed  $v_r$ .

#### 4.4.6. BLAND, FORD AND ELLIS'S GRAPHICAL METHOD FOR CALCULATION OF ROLL FORCE

This method [147] is based on the rolling theory of Orowan, which has been considered in detail by Z. Wusatowski [29].

Roll force per mm width of the rolled stock is found from equation

$$P' = R' \int_0^\delta p_x^+ d\varphi + R' \int_\delta^\alpha p_x^- d\varphi \quad (4.166)$$

$\delta$  = neutral angle which may be determined as follows:

$$\delta = \sqrt{\frac{h_2}{R'}} \tan \left( \sqrt{\frac{h_2}{R'}} \frac{H_p}{2} \right) \quad (4.167)$$

where

$$H_p = \frac{H_{\max}}{2} - \frac{1}{2f} \log_e \frac{h_1}{h_2} \left( \frac{1 - \sigma_2/k'_{f2}}{1 - \sigma_1/k'_{f1}} \right) \quad (4.168)$$

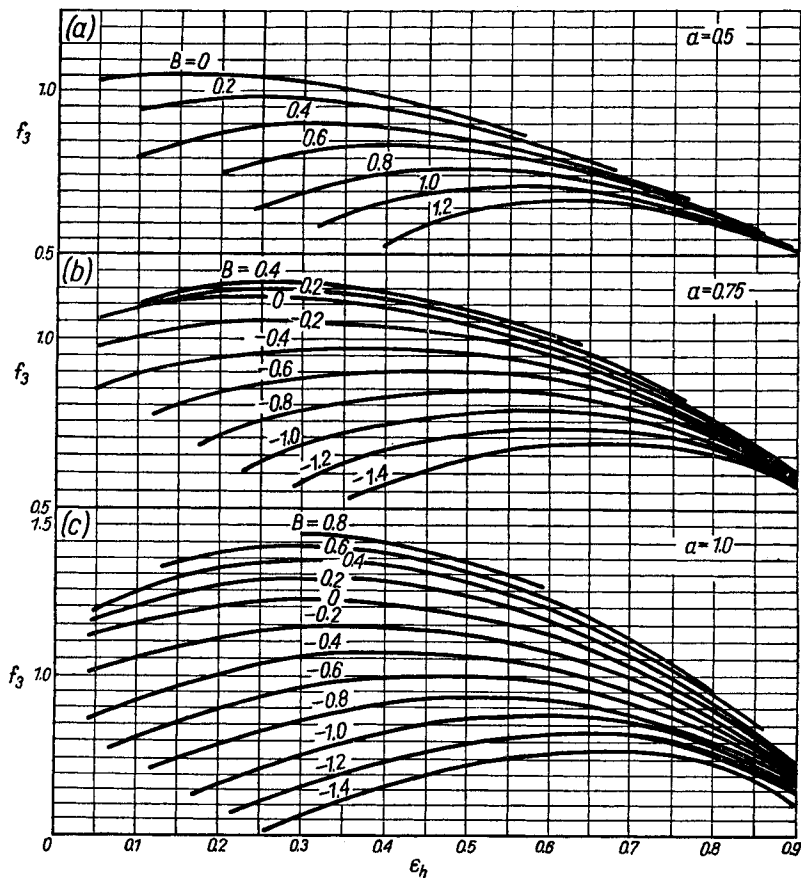
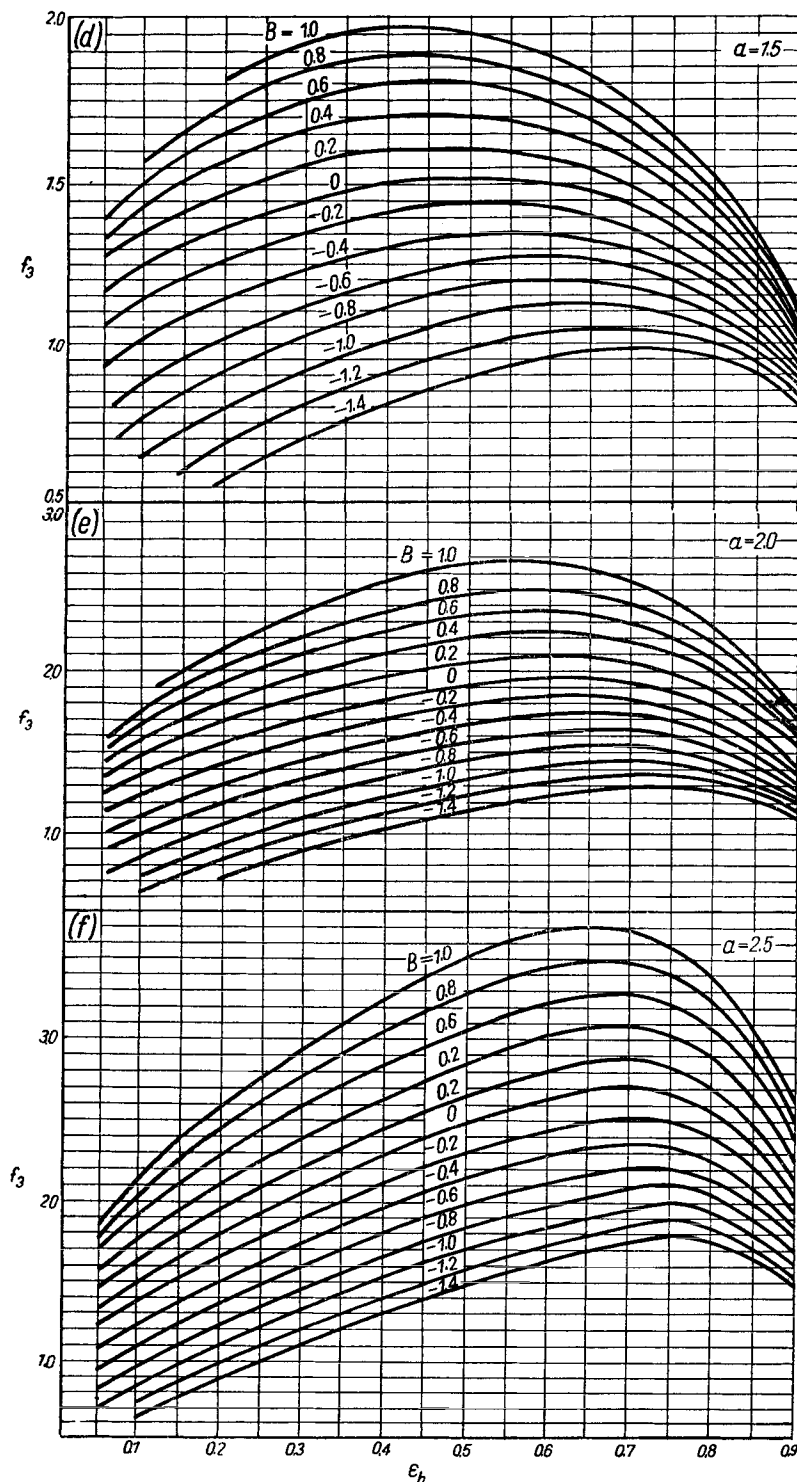


Fig. 4.132. Function  $f_3(\alpha, \epsilon_h, B)$  for calculation of roll force in cold rolling [147]

Fig. 4.132 (cont.). Function  $f_3(\alpha, \epsilon_h, B)$  for calculation of roll force in cold rolling [147]

Considerable difficulties arise when calculating the roll force  $P'$  per unit of width using equation (4.166), due to the variation of yield stress along the roll gap.

A graphical method for calculation of roll force has been published by Ford, Bland and Ellis [147], using a constant mean value of yield stress during the pass equal to

$$K_{fm} = \frac{\int_0^\alpha k'_f d\varphi}{\alpha}$$

Hence, the roll force per unit width is determined as

$$P' = K_{fm} \sqrt{R'(h_1 - h_2)} \left( 1 - \frac{\sigma_1}{K_{fm}} \right) f_3(a, \varepsilon_h, B) \quad (4.169)$$

where

$P'$  = roll force per unit width, kg/mm width,

$K_{fm}$  = mean yield stress, kg/mm<sup>2</sup>,

$f_3(a, \varepsilon_h, B)$  = non-dimensional roll force function found from curves given in Figs. 4.132a-f.

The function  $f_3$  is shown in Fig. 4.132 as a function of relative draught  $\varepsilon_h$  and two other factors:

$$a = f \sqrt{\frac{R'}{h_2}} \quad \text{and} \quad B = \log_e \frac{1 - \sigma_2/K_{fm}}{1 - \sigma_1/K_{fm}}$$

#### Example

It is required to calculate the roll force during the first pass in Table 4.22 and compare with results of measurements.

The flattened roll radius is found from the relation

$$R' = \frac{l_d'^2}{\Delta h}$$

The length of flattened arc of contact  $l_d'$  can be found from the Trinks-Keller method considered next.

For the first pass  $R' = 152.8$  mm and  $K_{fm} = 72.92$  kg/mm<sup>2</sup>. First values of  $a$  and  $B$  should be calculated

$$a = f \sqrt{\frac{R'}{h_2}} = 0.09 \sqrt{\frac{152.8}{1.4}} = 0.9405$$

$$B = \log_e \frac{1 - \sigma_2/K_{fm}}{1 - \sigma_1/K_{fm}} = \log_e \frac{1 - 15.75/72.92}{1 - 0/72.92} = -0.2431$$

Then from Fig. 4.132 the value of  $f_3$  for  $a = 0.9405$ ,  $B = -0.2431$ , and pass reduction  $\varepsilon_h = 0.45$  can be read off. Usually interpolation is necessary, as in the following example:

(a) From nomogram for  $f_3$  for a value of  $a = 0.75$  (Fig. 4.132b)

|                |               |
|----------------|---------------|
| for $B = -0.2$ | $f_3 = 1.03$  |
| for $B = -0.4$ | $f_3 = 0.975$ |
| difference     | $= 0.055$     |

Therefore for  $B = -0.2431$  the value of  $f_3$  is

$$f_3 = 0.975 + \frac{0.055}{0.20} (0.4 - 0.2431) = 1.0181$$

(b) From nomogram for  $f_3$  for a value of  $a = 1.0$  (Fig. 4.132c)

$$\begin{array}{rcl} \text{for } B = -0.2 & f_3 = 1.145 \\ \text{for } B = -0.4 & f_3 = 1.075 \\ \text{difference} & = 0.070 \end{array}$$

Therefore for  $B = -0.2431$  the value of  $f_3$  is

$$f_3 = 1.075 + \frac{0.070}{0.20} (0.40 - 0.2431) = 1.1299$$

(c) Both values of  $f_3$  should be now interpolated for  $a = 0.9405$

$$f_3 = 1.0181 + \frac{1.1299 - 1.0181}{0.25} (0.9405 - 0.75) = 1.1033$$

(d) The roll force is then obtained from

$$\begin{aligned} P' &= K_{fm} \sqrt{R'(h_1 - h_2)} \left( 1 - \frac{\sigma_1}{K_{fm}} \right) f_3 \\ &= 72.92 \times 13.2 \times 1 \times 1.1033 = 1062 \text{ kg/mm} \\ P &= P' b = 1.062 \times 254 = 269.74 \text{ t} \end{aligned}$$

Thus all three passes in Table 4.22 are calculated and results obtained are compared in Table 4.23. The agreement between calculation and experiment is insufficient. Calculated values of roll force are as much as 20% lower or 30% higher than measured values, which is an inadmissibly large error.

#### 4.4.7. TRINKS'S GRAPHICAL METHOD FOR DETERMINATION OF ROLL FORCE IN KELLER'S FORM

This method of calculation takes into account work hardening of the rolled stock, strip tension, and also roll flattening. The best way to explain this method is to consider an example of calculation of the first pass in Table 4.22.

##### Example

(a) The absolute draught  $\Delta h = h_1 - h_2 = 1.14$  mm. This value corresponds to a projected length of contact for undeformed roll  $l_d \approx \sqrt{R \Delta h} \approx \sqrt{127 \times 1.14} = 12.0$  mm ( $R = 127$  mm—undeformed roll radius).

(b) If the back tension  $\sigma_1$  [kg/mm<sup>2</sup>] is known, and the front tension of  $\sigma_2$  [kg/mm<sup>2</sup>] caused by the decoiler, the value of mean strip tension  $\sigma_m$  is

$$\sigma_1 = 0 \quad \text{and} \quad \sigma_2 = 15.75 \text{ kg/mm}^2$$

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = 7.9 \text{ kg/mm}^2$$

(c) The mean yield stress in plane strain in this pass can be determined from Fig. 4.116 (curve I) for a mean reduction of  $R_m = 0.4 \varepsilon_{h1} + 0.6 \varepsilon_{h2}$  from Ford's equation (4.153) [72], [73]

$$K_m = \eta K_{fm} = 72.92 \text{ kg/mm}^2$$

From this value of mean yield stress in plane strain the yield stress  $K_{fx}$  can be found as follows:

$$K_{fx} = K_m - \sigma_m = 72.92 - 7.9 = 65.02 \text{ kg/mm}^2$$

(d) The calculated length of contact  $l_d = 12 \text{ mm}$  is first of all assumed to be increased due to roll flattening up to  $l'_d = 13.2 \text{ mm}$ .

(e) Next  $\frac{l'_d}{\Delta h} f$  and  $\frac{\Delta h}{h_1}$  can be calculated

$$\frac{l'_d}{\Delta h} f = \frac{13.2}{1.14} 0.09 = 1.04 \quad \text{and} \quad \frac{\Delta h}{h_1} = 0.449$$

For these values the ratio of

$$\frac{K_{wm}}{K_{fm}} = 1.17$$

is read off from Fig. 4.133.

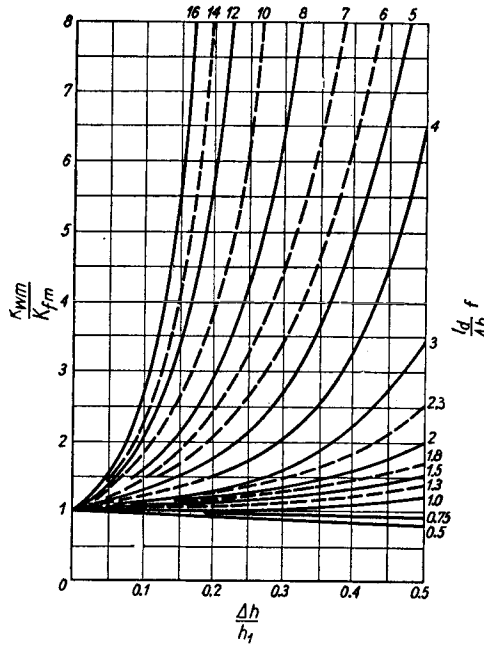


Fig. 4.133. Trinks' curves for determination of ratio of  $K_{wm}/K_{fm}$  [22]

(f) Therefore, the mean resistance to deformation  $K_{wm}$  is

$$K_{wm} = \left( \frac{K_{wm}}{K_{fm}} \right) K_{fx} = 1.17 \times 65.02 = 76.07 \text{ kg/mm}^2$$

The diagram shown in Fig. 4.134 is used for determination of roll flattening. As the modulus of elasticity is known, hence

$$100 \frac{K_{wm}}{E} = 100 \frac{76.07}{2.12 \times 10^{-4}} = 0.359$$

and

$$\frac{100 \Delta h}{R} = \frac{100 \times 1.14}{127} = 0.9$$

From these values and using Fig. 4.134

$$100 \frac{l'_d}{R} = 10.40$$

(g) Now the deformed length of contact can be found, and compared with the assumed value. For the case considered

$$l'_d = \left( \frac{100 l'_d}{R} \right) \frac{R}{100} = 13.208 \text{ mm}$$

(h) Comparison with the assumed value of  $l'_d$  shows a sufficient agreement. If there is too great a difference between assumed and calculated values, then a closer value of  $l'_d$  should be chosen, and the whole calculation repeated.

(i) Now  $l'_d$  is known, the projected area of contact can be found

$$F'_d = b_m l'_d = 3354.83 \text{ mm}^2$$

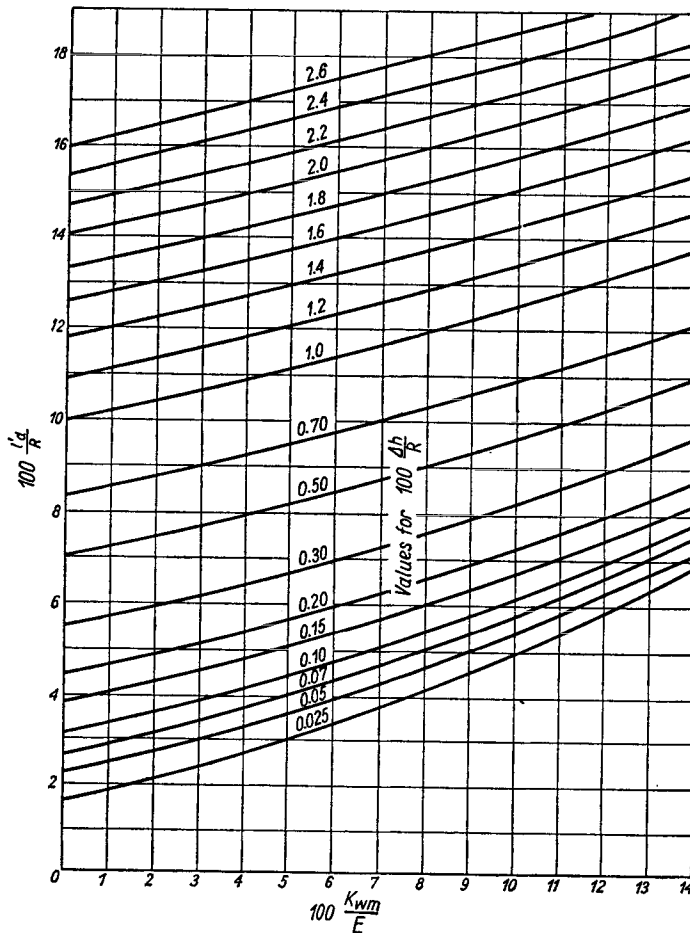


Fig. 4.134. Hitchcock's curves for determination of roll flattening [22]

The roll force taking into account roll flattening can now be calculated

$$P = K_{wm} F'_d = 76.07 \times 3354.83 = 255.2 \text{ t}$$

The measured roll force was 224.54 t, i.e. about 30 t less than the calculated value.

Comparison of calculated values given in Table 4.23 shows that this method gives satisfactory results if a suitable value of  $K_f$  is chosen. The error in all passes except the first is positive and varies from +10%–30%. Greater variations between calculation and experiment are found with high reductions [73].

#### 4.4.8. KOROLEV–NIKOLAYEVSKI'S METHOD OF CALCULATION OF ROLL FORCE IN COLD ROLLING

Korolev–Nikolayevski's method takes into account both roll flattening and work hardening of the rolled stock. Calculation is based on following formulae:

(a) Formula for calculation of resistance to deformation

$$K_{wm} = \frac{9500}{D} \frac{l_d'^2 - l_d^2}{l_d'} \quad (4.170)$$

where

$D$  = roll diameter, mm,

$l_d'$  = deformed length of contact, mm,

$l_d$  = projected length of contact for undeformed roll, mm,

$l_d \approx \sqrt{R \Delta h}$ .

(b) The deformed length of contact of the flattened roll is obtained from formula

$$l_d' = l_d \sqrt{a + b A l_d} \quad (4.171)$$

where  $a = 1 - b$ , and the coefficient  $b$  is equal to

$$b = \frac{2 - \varepsilon_h}{9500 f \varepsilon_h} \left( \eta K_{fm} - \frac{\sigma_1 + \sigma_2}{2} \right)$$

and for  $A$  the expression

$$A = \left( \frac{1}{1 - \varepsilon_h} \right)^{f/\Delta h}$$

should be substituted.

$K_m = (K_1 + K_2)/2$  = mean yield stress in plane strain,

$K_1 = \eta K_{f1}$  and  $K_2 = \eta K_{f2}$ ,

$f$  = coefficient of friction between roll and stock,

$\varepsilon_h = \Delta h/h_1$  = relative draught.

The best way of explaining this method of calculation is to take a numerical example, and once more the roll force arising during the first pass given in Table 4.22 is calculated.

First the coefficients  $a$ ,  $b$  and  $A$  should be determined. The values of yield stresses  $K_{f1}$  and  $K_{f2}$  are read off once more from Fig. 4.116 (curve 3).

$$K_{f1} = 23.78 \text{ kg/mm}^2 \quad \text{and} \quad K_{f2} = 74.5 \text{ kg/mm}^2$$

$$\begin{aligned} b &= \frac{2 - \varepsilon_h}{9500 f \varepsilon_h} - \left( \eta K_{fm} - \frac{\sigma_1 + \sigma_2}{2} \right) \\ &= \frac{2 - 0.45}{9500 \times 0.09 \times 0.45} \left( 1.155 \frac{23.78 + 74.5}{2} - \frac{0 + 15.75}{2} \right) = 0.196 \\ a &= 1 - 0.196 = 0.804 \\ A &= \left( \frac{1}{1 - \varepsilon_h} \right)^{f/\Delta h} = \left( \frac{1}{1 - 0.45} \right)^{0.09/1.14} = 1.0484 \end{aligned}$$



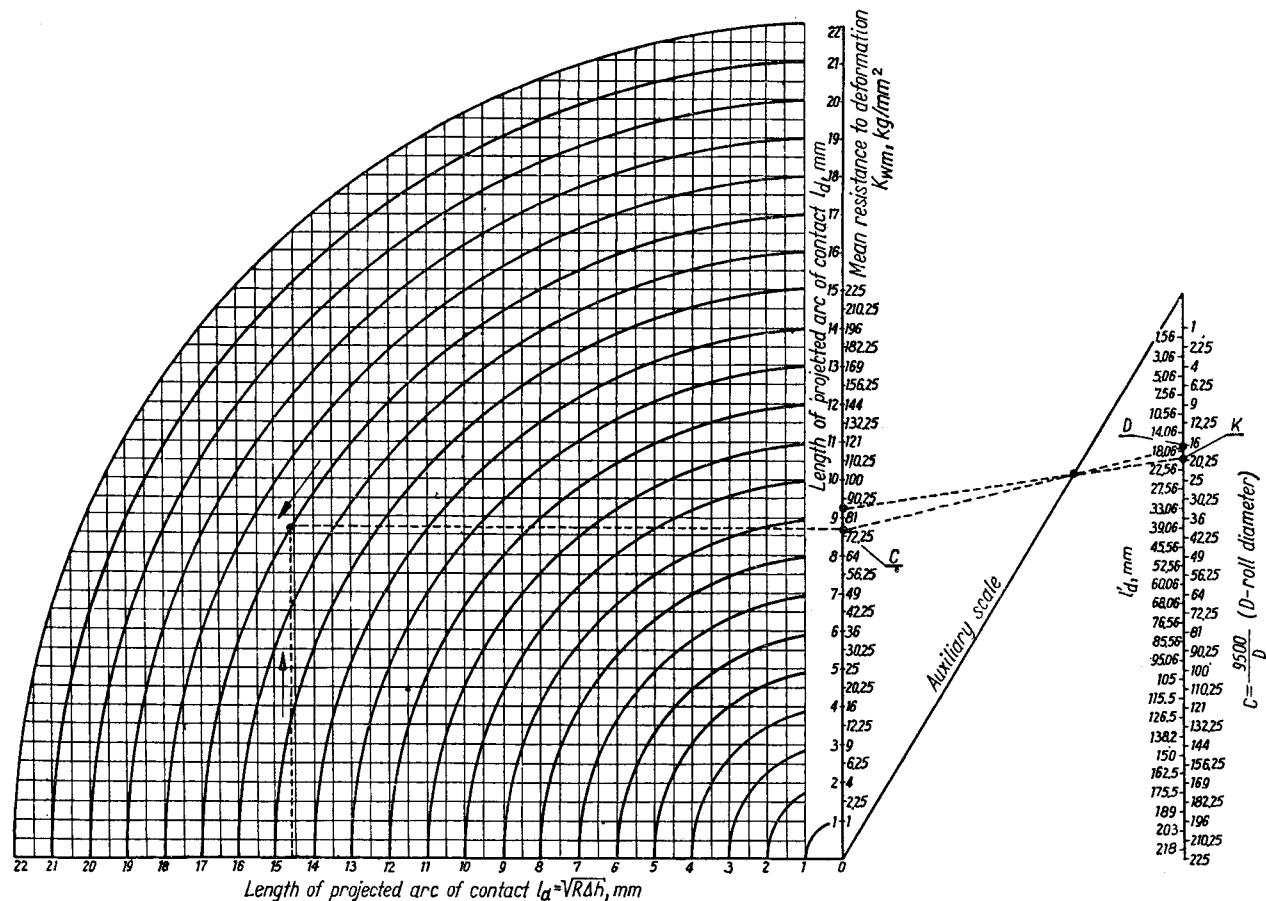


Fig. 4.135. Nomogram for determination of  $K_{wm}$  and  $l_d'$  according to Korolev-Nikolayevski [79]

Next the projected length of contact should be calculated, for the undeformed roll this is

$$l_d \simeq \sqrt{R \Delta h} = \sqrt{127 \times 1.14} = 12.032 \text{ mm}$$

A deformed length of contact equal to  $l'_d = 12.98 \text{ mm}$  is assumed, then from (4.171)

$$l'_d = l_d \sqrt{a + b A l'_d} = 12.03 \sqrt{0.804 + 0.196 \times 1.0484^{12.98}} = 12.98 \text{ mm}$$

As the assumed and calculated values of  $l'_d$  are equal, the mean resistance to deformation  $K_{wm}$  can be found from equation (4.170). If the calculated value of deformed length of contact  $l'_d$  differs from the assumed one, then this value should be corrected and the calculation repeated. The deformed length of contact may also be found graphically. A coordinate system is chosen where the abscissa shows  $l'_d$  and  $l_d$ , and the ordinate shows the ratio  $l'_d/l_d$ . In this system the straight line  $x_1 = l'_d/l_d$  and the curve of

$$x_2 = \sqrt{a + b A l'_d}$$

is drawn for known values of  $l_d$ ,  $a$ ,  $b$ , and  $A$ . The point of intersection of the straight line  $x_1$  and curve represents the required value of  $l'_d$ .

Knowing  $l'_d$  the mean resistance to deformation  $K_{wm}$  can be found from (4.170)

$$K_{wm} = \frac{9500}{D} \frac{l_d'^2 - l_d^2}{l'_d} = \frac{9500}{254} \frac{12.98^2 - 12.03^2}{12.98} = 68.29 \text{ kg/mm}^2$$

Hence, the total roll force

$$P = b_m l'_d K_{wm} = 254 \times 12.98 \times 68.29 = 225.16 \text{ t}$$

In the work of Korolev and Nikolayevski a nomogram was given [79] for quick calculation of resistance to deformation  $K_{wm}$  when the deformed length of contact  $l'_d$  is known. It is shown in Fig. 4.135.

Results obtained by using this method are also compared in Table 4.23. In some passes there is a very good agreement between calculation and measurement. For other passes the errors obtained are somewhat greater, and reach even to +24.2%. Only in the second pass greater errors of up to +41% occur (Table 4.23). This table shows that results depend to a certain extent on strip tension  $\sigma_1$  and  $\sigma_2$ , and on reduction, but the relation is difficult to define.

#### 4.5. Calculation of Roll Torque and Power in Cold Rolling

For practical purposes the simple formula

$$M_w = 2 P a \quad (4.172)$$

is most often used,

where

$M_w$  = roll torque for two rolls,

$P$  = roll force,

$a$  = torque arm.

Several methods of calculation of roll force  $P$  have been discussed in the previous chapter, and are now used for calculation of roll torque.

Assuming that during rolling the stock does not spread, and smooth rolls of equal diameters are used, then the shaded area in Fig. 4.60 represents the roll force per mm width of stock. This area is the sum of pressure elements along the roll gap. The roll

force  $P$  is the resultant of these pressures, and its line of action passes through the centre of gravity of the pressure diagram.

According to Tselikov [59] such results are valid only with the following assumptions:

- (a) both rolls are driven and have equal diameters and peripheral speeds,
- (b) the rolled stock has uniform strength properties,
- (c) the metal flow is uniform, and no other forces, apart from roll force, are acting.

#### 4.5.1. FORD'S METHOD OF CALCULATION OF ROLL TORQUE

There are other methods besides these previously discussed for determination of roll torque during cold rolling, e.g. Ford's method [147], where roll flattening is taken into account.

From theoretical considerations Ford proposes the following formula for torque for unit width for one roll

$$M_w = RR' \int_0^{\alpha'} K \varphi d\varphi + \left( \frac{\sigma_1 h_1 - \sigma_2 h_2}{2} \right) R \quad (4.173)$$

where

$K = 1.155 K_f =$  constrained yield stress,

$\alpha' =$  bite angle for deformed roll, in radians,

$\sigma_1 =$  stress due to back tension, at point of entry,

$\sigma_2 =$  stress due to front tension, at point of exit.

When rolling without strip tension equation (4.173) becomes

$$M_w = RR' \int_0^{\alpha'} K \varphi dp \quad (4.174)$$

According to Ford this equation can be solved graphically.

#### 4.5.2. TSELIKOV'S METHOD OF CALCULATION OF ROLL TORQUE

Tselikov's formula for roll torque takes into account the deformation of rolls. For two rolls the torque is

$$M_w = K_{mw} \frac{\delta}{\alpha} (b_1 + b_2) R \Delta h \quad (4.175)$$

where  $\delta$  is the angle corresponding to the point of action of the force (Fig. 4.60). The effect of elastic deformation and flattening of rolls is greater at the point of exit of the rolled strip, and with increasing elastic deformation of the roll the point of action of the force  $P$  shifts towards the rolling direction (Fig. 4.114).

For a simple case of rolling as shown in Fig. 4.60, when assuming  $\delta/\alpha = 0.5$  the formula for roll torque becomes

$$M_w = P (\sqrt{R \Delta h + x_1^2} - x_1) \quad (4.176)$$

where  $x_1$  — increase in length of projected arc of contact  $l_d$  in the rolling direction on the exit side, due to roll flattening (Fig. 4.114).

$x_1$  is calculated as follows:

$$x_1 = \frac{8(1-\mu)^2 2R}{E} K_{wm} \quad (4.177)$$

where

$\mu = 1/m =$  Poisson's ratio, for the roll material,

$E =$  Young's modulus for the roll material,

$K_{wm} =$  mean resistance to deformation.

For steel rolls the approximate formula

$$x_1 \approx \frac{K_{wm} R}{9500} \quad (4.178)$$

may be used.

#### 4.5.3. BLAND, FORD AND ELLIS'S GRAPHICAL METHOD OF CALCULATION OF ROLL TORQUE

Calculation of roll torques in cold rolling by Bland, Ford and Ellis's graphical method is simpler and quicker than that for determination of roll force. The rolling torque is determined as follows:

$$M_w = 2b \frac{R h_1^2}{h_2} K f_2 10^{-3} \quad (4.179)$$

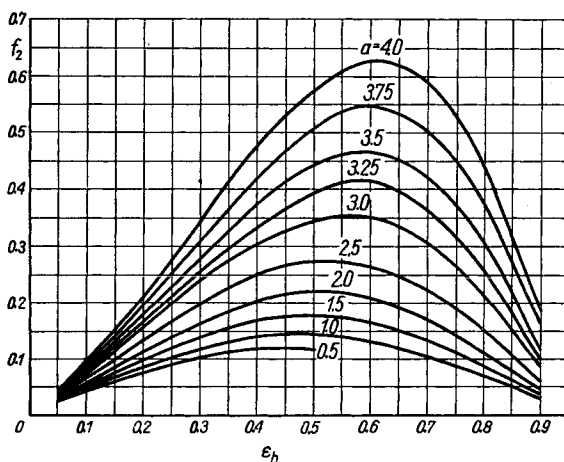


Fig. 4.136. Function of  $f_2$  ( $a$ ,  $\epsilon_h$ ) for determination of rolling torque in cold rolling

where

$K =$  yield stress in plane strain of the rolled stock, kg/mm<sup>2</sup>,

$f_2 =$  dimensionless coefficient determined from Fig. 4.136, depending on relative draught per pass and the value  $a = f\sqrt{R'/h_2}$ ,

$R' =$  deformed roll radius, determined by equation (4.150), mm.

Formula (4.179) is valid for cold rolling without applied strip tension. If tension is

to be taken into account, it must be remembered that the torque decreases due to front tension, and increases due to back tension.

These calculations are based on the following assumptions:

- (a) both rolls are driven, and are of equal diameter and peripheral speed,
- (b) strength properties of the rolled stock are uniform,
- (c) rolls are elastically deformed and flattened.

#### 4.5.4. FORD'S METHOD OF CALCULATION OF ROLL TORQUE WITH APPLIED TENSION

For calculation of roll torque, the following relation has been published by Ford [147]

$$M' = RR' \int_0^{\alpha'} K \varphi d\varphi + \left( \frac{\sigma_1 h_1 - \sigma_2 h_2}{2} \right) R = M'_w + \left( \frac{\sigma_1 h_1 - \sigma_2 h_2}{2} \right) R \quad (4.180)$$

where

$M$  = roll torque for rolling with applied front and back tension for both rolls per mm stock width, kgm/mm,

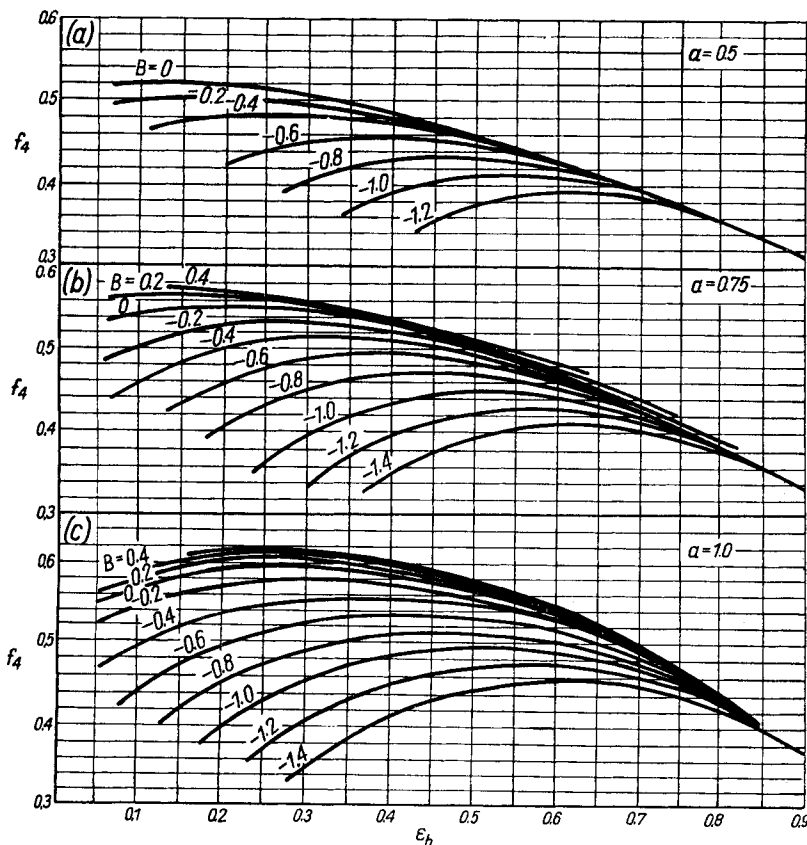


Fig. 4.137. Function of  $f_4(\alpha, \epsilon_h, B)$  for determination of rolling torque in cold rolling with applied tension: (a) for  $\alpha = 0.5$ , (b) for  $\alpha = 0.75$ , (c) for  $\alpha = 1.0$

$M'_w$  = roll torque for rolling without tension for both rolls per mm stock width,  
kgm/mm,  
 $R$  = radius of undeformed roll, mm.

#### 4.5.5. BLAND, FORD AND ELLIS'S METHOD OF CALCULATION OF ROLL TORQUE WITH APPLIED TENSION

By this method the torque per mm stock width acting upon one roll is expressed as

$$M' = RR' \left( \int_0^{\alpha'} p_x \varphi d\varphi + \frac{\sigma_1 h_1 - \sigma_2 h_2}{2R'} \right) \quad (4.181)$$

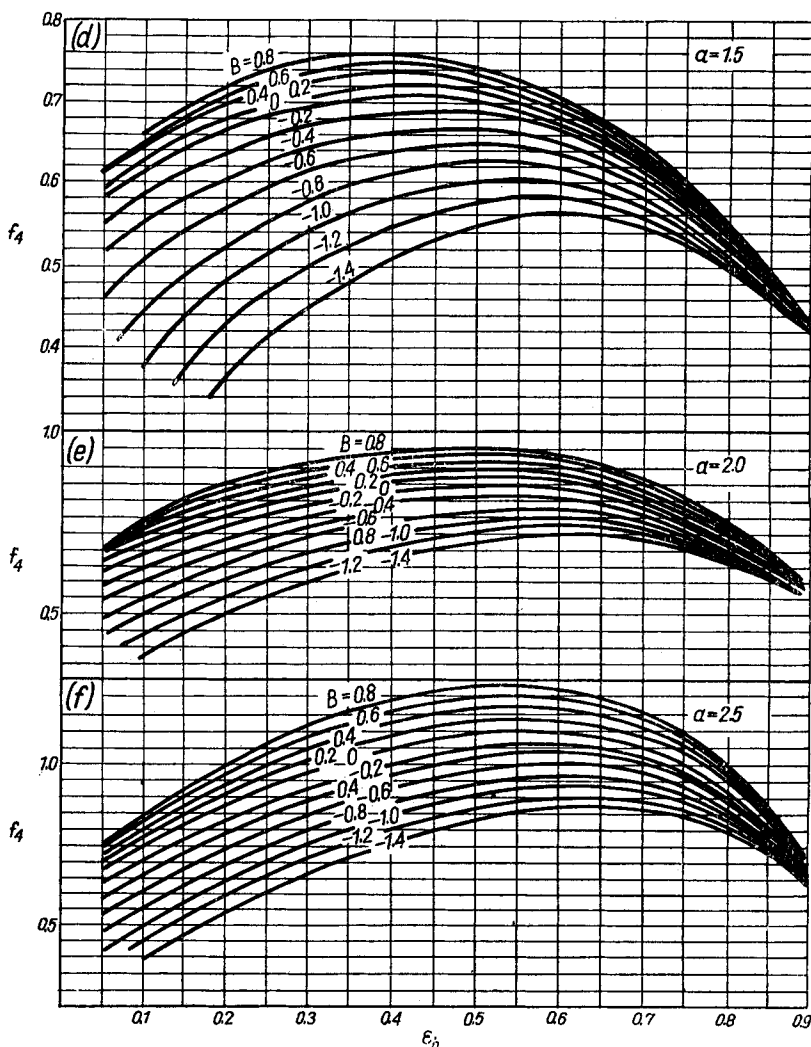


Fig. 4.137 (cont.). Function of  $f_4(a, \epsilon_h, B)$  for determination of rolling torque in cold rolling with applied tension: (d) for  $a = 1.5$ , (e) for  $a = 2.0$ , (f) for  $a = 2.5$

Since the integral in this equation is very difficult to calculate, a graphical method has been worked out assuming that the mean yield stress in plane strain is constant along the roll gap and equal to

$$K_m = \frac{\int_0^a 1.15 K_f}{\alpha}$$

The method is based on solution of the equation

$$M' = RK_m(h_1 - h_2) \left( 1 - \frac{\sigma_1}{K_m} \right) f_4(a, \varepsilon_h, B) \quad (4.182)$$

where

$M'$  = torque per mm width of the stock, for one roll,

$K_m$  = mean yield stress in plane strain of the rolled stock,

$f_4(a, \varepsilon_h, B)$  = dimensionless torque function, read off from Figs. 4.137a-f.

To determine the total roll torque the result obtained from equation (4.182) must be multiplied by 2 and by the width of the rolled stock.

#### 4.5.6. OTHER METHODS OF CALCULATION OF ROLL TORQUE

The accurate determination of lever arm  $a$  is of great importance for calculation of roll torque. Lever arm  $a$  as a function of length of arc of contact  $l_d$  is given by equation (4.75), where

$$c = \frac{a}{l_d} = \frac{a}{\sqrt{R\Delta h}}$$

Underwood [34] states that for practical calculations of cold rolling with applied tension the value of ratio of lever arm to length of arc of contact  $c$  lies in the range 0.38–0.42.

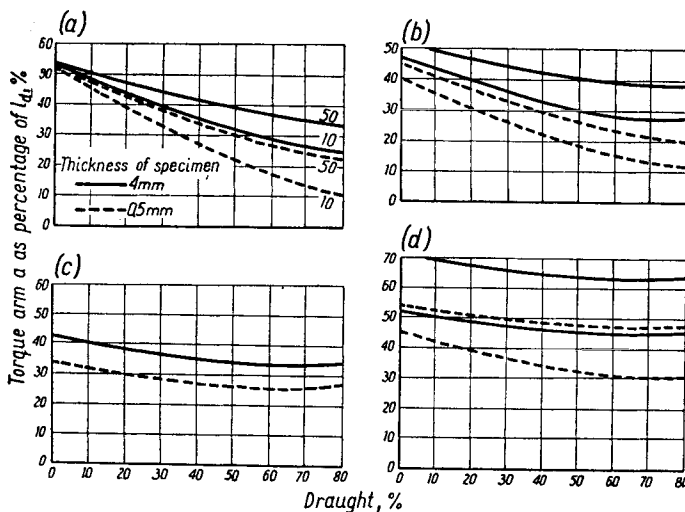


Fig. 4.138. Torque lever arm  $a$  as percentage of  $l_d$  in cold rolling of: (a) deep-drawing steel and brass, (b) copper, (c) duraluminium, (d) aluminium [82]

Figure 4.138 shows the results of measurements carried out by Emicke and Lucas [90]. It shows the ratio of lever arm to length of arc of contact as a function of percentage draught for two thicknesses.

The value of ratio of lever arm to length of arc of contact can be also determined from the roll work formula as follows: (4.82)

$$a = \frac{l_d}{2} \frac{\gamma}{1-\gamma} \log_e \frac{1}{\gamma}$$

According to Ford [72] when calculating the lever arm  $a$  first the ratio  $c_1 = a/l_d$  which takes into account roll flattening should be determined, from which multiplying the value of  $c_1$  by  $l_d$  the required lever arm  $a$  is obtained.

$c_1$  is calculated from the equation

$$c_1 = 0.5 \sqrt{\frac{R'}{R} - (0.5 - c_2)} \sqrt{\frac{R'}{R}} \quad (4.183)$$

where

$R'$  = radius of deformed roll, calculated from formula (4.150), (Fig. 4.115),

$c_2$  = coefficient taken from Table 4.24,

$R$  = radius of undeformed roll.

TABLE 4.24  
VALUES OF COEFFICIENT  $c_2$  GIVEN BY FORD

| Rolled material | Strip thickness mm | Rolling condition | Percentage draught % | Roll surface | Coefficient $c_2$ |
|-----------------|--------------------|-------------------|----------------------|--------------|-------------------|
| 0.2% C steel    | below 2.5          | lubricated        | 10-70                | polished     | 0.47              |
| copper          | below 2.5          | lubricated        | 10-70                | polished     | 0.49              |
| copper          | below 1.25         | lubricated        | 10-70                | matt         | 0.43              |
| copper          | below 2.0          | lubricated        | 10-70                | matt         | 0.42              |
| copper          | below 2.5          | lubricated        | 10-70                | matt         | 0.41              |
| 0.2% C steel    | below 2.5          | lubricated        | 10-70                | matt         | 0.43              |
| 0.2% C steel    | below 2.5          | dry               | 10-70                | matt         | 0.44              |

Tselikov's curves of energy consumption [87] already given, can be used for calculation of roll torque.

Both Figs. 4.139 and 4.140 show the energy consumption during cold rolling of strip or sheet in kWh per ton of the rolled stock as a function of thickness. These curves can be used similarly as for hot rolling, i.e. values of roll torque plus torque due to additional friction can be found. Rolling conditions for plotting these curves are compared in Table 4.25.

Nomograms were published by Emicke and Lucas for determination of roll forces and torques in cold rolling of steel (Figs. 4.141 and 4.142).

An example of calculation by this method is given in Table 4.26.

For determining roll force and torque by means of this nomogram the following procedure is used. From Table 4.26 the initial thickness  $h_i = 1.5$  mm is read off, which corresponds to point 1 in Fig. 4.141(a). From this point a vertical line is drawn upwards up to a curve corresponding to a percentage draught of 12% (giving point 2). A horizontal line drawn through point 2 cuts the left-hand scale at a point corresponding to roll force of 25.5 t in the first pass (point 3).



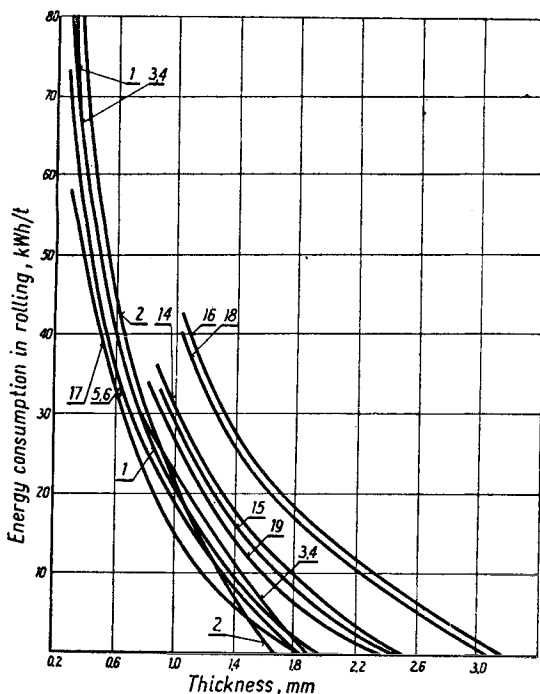


Fig. 4.139. Energy consumption curves for cold rolling of low-carbon steels, as a function of strip thickness. For rolling conditions see Table 4.25

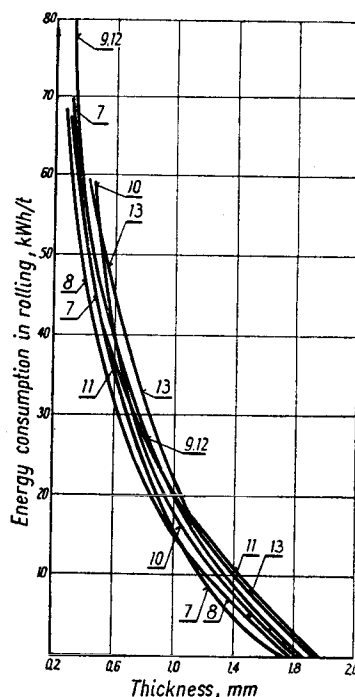


Fig. 4.140. Energy consumption curves for cold rolling of low-carbon steels as a function of strip thickness. For rolling conditions see Table 4.25

This procedure can be repeated as long as the rolled stock is not excessively work-hardened and then it must be annealed. For determining the roll force in the second pass a different procedure should be used due to work-hardening of the rolled stock during the first pass.

From point 1 in Fig. 4.141(a) a vertical line is drawn up to intersect with the curve corresponding to a percentage draught of 6% (point 4) and this line is produced upwards to the curve corresponding to a percentage draught of 6% in part (b) (point 5). A horizontal line is drawn through point 5 up to point 6, and through it a line parallel to lines connecting parts (b) and (c), giving point 7. From 7 a vertical line is drawn downwards to intersect the horizontal line drawn through point 4 at point 8 on part (c). Through this point a line is drawn parallel to the curve plotted in part (c) to meet the right-hand ordinate at point 11.

To take into account work-hardening of the initial stock, a vertical line through point 1 is produced downwards to part (d) to a value corresponding to percentage draught in the first pass. This is 12% in the first pass, thus point 9 is found. A horizontal line drawn through 9 to the right-hand ordinate gives point 10. Through this point a line is drawn parallel to lines joining corresponding points on parts (d) and (e) meeting the horizontal axis at point 12. A vertical line drawn through 12 intersects the horizontal line through point 11, giving point 13. A line through 13 parallel to the lines drawn in part (e), gives point 14 on the vertical axis, corresponding to a roll force of 24 t.

The rolling torque is determined similarly by means of the nomogram shown in Fig. 4.142. The same example as before is considered. Starting at point 1 of part (a) (Fig. 4.142) corresponding to the initial thickness of 1.5 mm a vertical line is drawn, intersecting the curve of 12% draught at point 2. As the rolled stock is not initially work-hardened point 3 is determined on the curve of 0% deformation. Projecting this point onto the right-hand ordinate of part (b) gives point 4. Then a parallel line is drawn to lines connecting part (b) and (c), determining point 5, and a vertical line drawn through 5 gives point 6 on part (c). Joining point 6 to the centre of this coordinate system, then this line intersects the parallel line drawn through point 2 at point 7. A vertical line through 7 intersects the guiding

TABLE 4.25  
DATA FOR ENERGY CONSUMPTION CURVES SHOWN IN FIGS. 4.139 AND 4.140

| Curve               | Dimensions of rolled sheet and strip |                          |           | Rolling speed<br>m/sec | Work roll diameter<br>mm | Number of stands |
|---------------------|--------------------------------------|--------------------------|-----------|------------------------|--------------------------|------------------|
|                     | Initial thickness<br>$h_1$           | Final thickness<br>$h_2$ | Width     |                        |                          |                  |
|                     | mm                                   | mm                       | mm        |                        |                          |                  |
| A. Continuous mills |                                      |                          |           |                        |                          |                  |
| 1                   | 1.9/2.0                              | 0.24                     | 685       | —                      | —                        | 5                |
| 2                   | 1.63                                 | 0.22/0.30                | 700/800   | 4.7/7.2                | —                        | 5                |
| 3                   | 1.9                                  | 0.26/0.30                | 650/825   | 7.1                    | —                        | 5                |
| 4                   | 1.9                                  | 0.21/0.24                | 650/825   | 7.1                    | —                        | 5                |
| 5                   | 1.83                                 | 0.22/0.30                | 750/850   | 6.6/7.0                | —                        | 4                |
| 6                   | 1.83                                 | 0.22/0.30                | 570/810   | 4.7/5.0                | —                        | 4                |
| 7                   | 1.65                                 | 0.22/0.33                | 660/825   | 4.8/6.1                | 380/500                  | 4                |
| 8                   | 1.8                                  | 0.20/0.35                | 585/685   | 4.8/6.1                | 380/500                  | 4                |
| 9                   | 1.83/2.0                             | 0.25/0.30                | 735/840   | 7.4/7.9                | 380/500                  | 4                |
| 10                  | 1.65/2.0                             | 0.40/0.70                | 635/830   | 3.1/3.8                | 500                      | 3                |
| 11                  | 1.7/2.1                              | 0.30/0.50                | 560/875   | 4.2/7.1                | 380/500                  | 4                |
| 12                  | 1.8                                  | 0.4                      | 750       | 5.8                    | —                        | 4                |
| 13                  | 1.75/2.0                             | 0.4/0.6                  | 935       | 2.5/3.0                | 380/500                  | 3                |
| 14                  | 2.4/2.7                              | 0.9/1.2                  | 900/2000  | —                      | 500                      | 3                |
| 15                  | 2.3/2.7                              | 0.9/1.1                  | 900/1900  | —                      | 500                      | 3                |
| 16                  | 3.2                                  | 0.9/1.2                  | 1000/1700 | —                      | 480                      | 3                |
| B. Reversing mills  |                                      |                          |           |                        |                          |                  |
| 17                  | 1.8                                  | 0.37                     | 750       | —                      | 400/450                  | 1                |
| 18                  | 3.0                                  | 0.90                     | 1750      | —                      | 520                      | 1                |
| 19                  | 2.4                                  | 0.80                     | 1750      | —                      | 520                      | 1                |

TABLE 4.26  
ROLL FORCES ARISING DURING ROLLING OF DEEP-DRAWING SOFT STEEL

| Roll diameter<br>mm | Width<br>mm | Pass No. | Initial thickness<br>$h_1$<br>mm | Percentage draught<br>% | Work hardening<br>% | Roll force<br>t | Roll torque<br>kgm |
|---------------------|-------------|----------|----------------------------------|-------------------------|---------------------|-----------------|--------------------|
| 200                 | 100         | 1        | 1.5                              | 12                      | 6                   | 25.5            | 116                |
|                     |             | 2        | 1.32                             | 6                       | 12                  | 24.1            | 62                 |
|                     |             | 3        | 1.24                             | 4                       | 17                  | 22.1            | 41                 |
|                     |             |          | final thickness<br>1.19          | —                       | —                   | —               | —                  |

line at point 8, and a horizontal line drawn through 8 gives point 9 on the right-hand ordinate of part (c). Joining this point to the centre of coordinate system of part (d) the intersecting of this line with the vertical line corresponding to roll force, point 10 is found. A horizontal line drawn through 10 to the right-hand ordinate gives the required roll torque at point 11 (125 tm).

Part (d) of the nomogram shows three different scales for roll force and roll torque. The scale to be used depends on the magnitude of roll force and torque. For the case considered the roll force is 25.5 t, and therefore the first scale should be used.



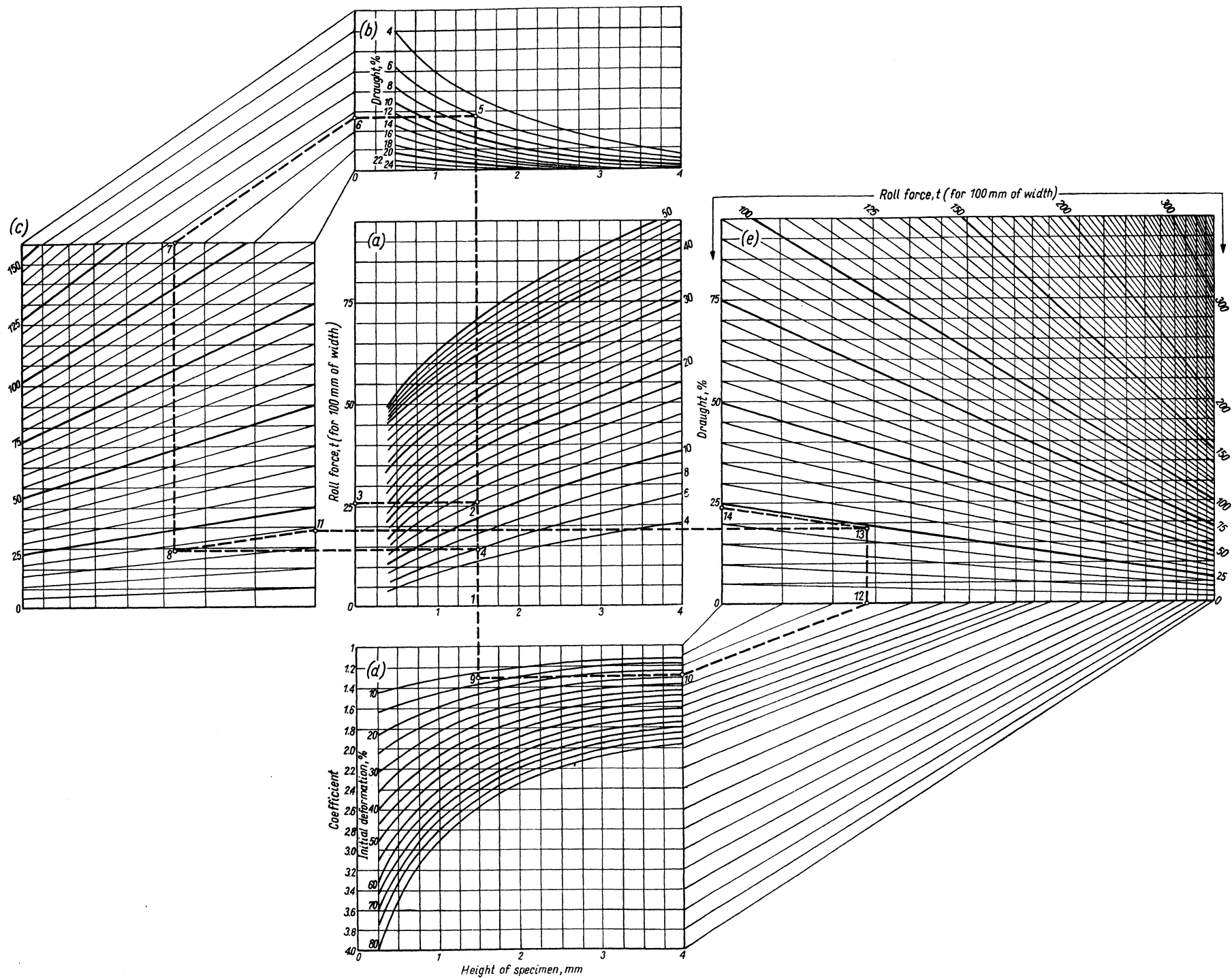


Fig. 4.141. Nomogram for determination of roll forces in cold rolling [83]: (a) roll diameter 200 mm, (b) initial thickness from 0.5–4 mm, (c) draught per pass from 10–50% (d) initial deformation 0–80%, (e) smooth and ground rolls, (f) lubricated with paraffin

For the second pass, where initial work-hardening with a percentage draught of 12% occurs and the draught in the pass equals 6%, point 22 is obtained similarly. This point corresponds to a roll torque of 62.5 tm.

Formulae published by SKF [77], for determination of drive power in cold rolling, especially on 4-high and 6-high mills, represent a further development of the methods already given for calculation of pure roll torque and additional friction torque on 2-high and 3-high mills. The second formula is also applicable to 4-high and 6-high mills.

For 2-high and 3-high mills the formula for power calculation is as follows:

$$N_w = (1.05-1.10) \frac{P w_2}{102} \left( \frac{\sqrt{R_1 \Delta h}}{R_1} + \frac{f_1 d_1}{R_1} \right) + N_i \quad (4.184)$$

and for a 4-high mill

$$N_w = (1.05-1.10) \frac{P w_2}{102} \left( \frac{\sqrt{R_1 \Delta h}}{R_1} + \frac{f_2 d_2}{R_2} \right) + N_i \quad (4.185)$$

where

$N_w$  = driving power of the main motor,

$P$  = roll force, measured experimentally, or calculated by means of nomograms,

$w_2$  = speed of exit of the strip from roll gap,

$R_1$  = radius of work rolls,

$R_2$  = radius of back-up rolls,

$d_1$  = neck diameter of work rolls,

$d_2$  = neck diameter of back-up rolls,

$\Delta h$  = absolute draught,

$f_1$  = coefficient of friction of work roll bearings,

$f_2$  = coefficient of friction of back-up roll bearings,

$N_i$  = idling power of the mill.

The coefficient 1.05–1.10 takes into account the efficiency of gears mounted into the drive, the smaller value of 1.05 corresponds to single-reduction and the greater one of 1.10 to double-reduction gears.

When work roll radii differ from each other and are equal to  $R_{1a}$  and  $R_{1b}$ , then into equations (4.184) and (4.185) for the roll radius  $R_1$  the mean value of

$$R_m = \frac{2 R_{1a} R_{1b}}{R_{1a} + R_{1b}}$$

should be substituted.

For cluster mills (6-high mill) formula (4.143) takes the form

$$N_w = (1.05-1.10) \left( \frac{\sqrt{R_1 \Delta h}}{R_1} + \frac{f_2 d_2 \sqrt{2}}{R_2} \right) + N_i \quad (4.186)$$

#### 4.5.7. EFFECT OF STRIP TENSION

Reducing metal stock between two flat and smooth dies, i.e. a cubic element from initial height of  $h_1$  to a final one of  $h_2$ , neglecting frictional forces between surface of the stock and dies, and assuming that  $\sigma$  is the horizontal stress produced in the metal along its direction of flow, then for a constant constrained yield stress in plane strain the work of deformation is equal to that for simple compression.

Denoting  $K_w$  as the stress acting between die surface and cubic element, then based on the condition of plasticity

$$K_w = \eta K_f - \sigma$$

The work done by resultant of vertical pressure is

$$A_v = V(\eta K_f - \sigma) \int_{h_2}^{h_1} \frac{dh}{h}$$

The work done by the resultant of horizontal pressure is

$$A_h = \sigma b \int_{l_1}^{l_2} h dl = V \sigma \int_{l_1}^{l_2} \frac{dl}{l}$$

In these equations  $V$  denotes the volume of the deformed stock, i.e.  $h_1 b_1 l_1 = h_2 b_2 l_2$ .

The total theoretical work of deformation is the sum of both values, i.e.

$$A_o = A_v + A_h = V(\eta K_f - \sigma) \int_{h_2}^{h_1} \frac{dh}{h} + V \sigma \int_{l_1}^{l_2} \frac{dl}{l}$$

hence

$$A_o = V(\eta K_f - \sigma) \log_e \frac{h_1}{h_2} + V \sigma \log_e \frac{l_2}{l_1} \quad (4.187)$$

As for constant width  $h_1 l_1 = h_2 l_2$ , then this equation becomes the known relation for theoretical work of deformation

$$A_o = \eta V K_f \log_e \frac{h_1}{h_2}$$

These considerations are purely theoretical ones. Formula (4.187) virtually never represents true conditions, due to the considerably varying frictional conditions between roll surface and stock.

Strip tension in cold rolling can be produced by the coiler, as also by differences in peripheral speeds between adjacent stands when rolling in continuous mills. In actual rolling conditions tension acting in the rolling direction is called front tension, and tension acting opposite to the rolling direction is called back tension (Fig. 4.130).

Therefore, strip tension is caused by front or back tension, or both of them acting simultaneously.

If the coiler acts on the strip with a tensile force  $Q_2$  kg, and the decoiler exerts a braking force  $Q_1$  kg (Fig. 4.130), then denoting strip tensions as stresses  $\sigma_2$  and  $\sigma_1$  [kg/mm<sup>2</sup>], respectively, hence from equations (4.159) and (4.160)

$$Q_1 = \sigma_1 b h_1 \quad \text{and} \quad Q_2 = \sigma_2 b h_2$$

For a constant tension  $Q_2$  and speed of exit of the strip of  $w_2$  the power required for the coiler is determined as

$$N_{c2} = Q_2 w_2 = \sigma_2 b h_2 w_2 \quad (4.188)$$

The recovered power of the decoiler is equal to

$$N_{c1} = Q_1 v_1 = \sigma_1 b h_1 v_1 \quad (4.189)$$

From the condition of continuity  $v_1 h_1 = w_2 h_2$ , hence

$$N_{c1} = \sigma_1 b h_2 w_2 \quad (4.190)$$

For the layout shown in Fig. 4.130 for a reversible mill with one coiler and decoiler the following distribution of power among the separate machines is obtained

$$N_{roll} + N_{c1(br)} = N_m + N_{c2} \quad (4.191)$$

assuming that power of the braking decoiler is not recovered. Due to the tension produced, the front coiler assists the main motor to some extent, whereas the uncoiler increases the motor load by its braking effect; where

$N_{roll}$  = power required for pure rolling,

$N_m$  = power of the main motor, driving the work rolls,

$N_{c2}$  = power of the coiler (tensional reel),

$N_{c1}$  = power of the decoiler (braking reel), delivered or lost.

Substituting relations previously derived for power calculation into equation (4.191), then for a speed ratio of  $w_2/v_r = r$

$$2 \frac{c}{r} b K_{wm} w_2 \Delta h + \sigma_1 v_1 b h_1 = N_m + \sigma_2 w_2 b h_2 \quad (4.192)$$

This equation expresses a general relation, where the mill considered can be driven with or without strip tension, and with work rolls driven or not.

In modern rolling mills electrical braking of the decoiler is used, which acts as a generator and supplies energy to the grid. For such cases, after subtraction of all losses, about 60% of braking power can be recovered, as confirmed by practical experience. Hence

$$N_m = N_r + N_{c1} - 0.6 N_{x1} = N_r + 0.4 N_{c1} \quad (4.193)$$

If  $s$  is denoted as

$$s = \frac{\text{peripheral speed of the coiler}}{\text{peripheral speed of the roll}} = \frac{v_c}{v_r}$$

and  $x$  as

$$x = \frac{\text{mean coil diameter}}{\text{diameter of the coiler drum}} = \frac{d_c}{d_d}$$

hence, thermal losses arising during coiling are equal to

$$Q \left[ 1 + \frac{s}{100} (x+1) \right] \quad (4.194)$$

Hence, the power of coiler with thermal losses taken into account is

$$N_c = Q v_r \left( 1 + \frac{s}{100} \right) x \quad (4.195)$$

or

$$N_c = \sigma b h v_r \left[ \left( 1 + \frac{s}{100} \right) x \right] \quad (4.196)$$

Knowing dimensions of the coils (equations (4.159) and (4.160)) then for given rolling conditions values of  $Q$ ,  $\sigma$ ,  $b$  and  $h$  can be substituted.

For a rolling mill, with anti-friction or force-lubricated bearings frictional losses in the bearings can be neglected, and the formula becomes

$$N_m = b \Delta h K_{wm} w_2 + Q_2 w_2 \left( 1 + \frac{s}{100} \right) x + Q_1 v_1 \quad (4.197)$$

This power is relative to the shaft of the main motor, and consists of roll power and power of strip tensions for respective couplings. Assuming the efficiency of the pinion stand of a cold rolling mill to be equal to  $\eta_1 = 0.95$ , and that of the gears  $\eta_2 = 0.95$ , i.e.  $\eta_1 = \eta_2 = 0.95$ , then when working with coupling, the following formula for the power of the main motor of the mill is found

$$N_m = \frac{0.8 b \Delta h K_{wm} w_2 + (2\pi n/60) f_1 P d}{0.9025} + Q_2 w_2 \left( 1 + \frac{s}{100} \right) x + Q_1 v_1 \text{ kgm/sec} \quad (4.198)$$

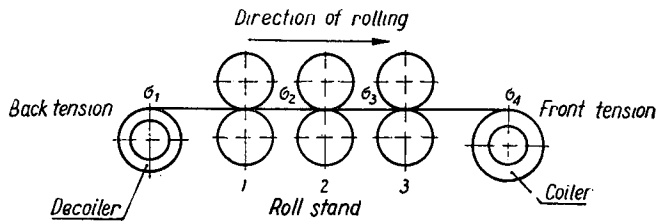


Fig. 4.143. Schematic layout of a 3-stand tandem mill for cold rolling of strip with coiler and decoiler

When rolling in tandem then a different power distribution is found depending on rolling conditions. For example, if a three-stand continuous mill is considered with individually driven stands and front and back coilers also individually driven (Fig. 4.143).

Denoting:

- $\sigma_1$  = strip tension before the first stand,
- $\sigma_2$  = strip tension between first and second stand,
- $\sigma_3$  = strip tension between second and third stand,
- $\sigma_4$  = strip tension between third stand and coiler,
- $b$  = strip width,

$h_1$  and  $v_1$  = initial thickness and entry speed of the strip before the first stand,  
 $h_2$  and  $v_2$  = initial thickness and entry speed of the strip before the second stand,  
 $h_3$  and  $v_3$  = initial thickness and entry speed of the strip before the third stand,  
 $h_4$  and  $v_4$  = final thickness and exit speed of the strip after the third stand,  
 then

$$bh_1 v_1 = bh_2 v_2 = bh_3 v_3 = bh_4 v_4 = V_s \quad (4.199)$$

$V_s$  = volume of metal passing through the train per unit of time.

Values of power due to strip tension equal in particular stands of the train are given by:

- front tension after the third stand =  $\sigma_4 bh_4 v_4 = \sigma_4 V_s$ ,
- back tension at the third stand =  $\sigma_3 V_s$  = front tension at the second stand,
- back tension at the second stand =  $\sigma_2 V_s$  = front tension at the first stand,
- back tension before the first stand =  $\sigma_1 V_s$ .



Denoting power of the main motors for driving work rolls at stands 1 to 3 as  $N_{m1}$ ,  $N_{m2}$ , and  $N_{m3}$ , and the power required for pure rolling without friction losses in the driving system as  $N_{w1}$ ,  $N_{w2}$ , and  $N_{w3}$ , then for the total mill the following equation is found:

$$V_s \sigma_4 + N_{m3} + N_{m2} + N_{m1} = N_{w1} + N_{w2} + N_{w3} + V_s \sigma_1 \quad (4.200)$$

This equation is a development of equation (4.191) for continuous three-stand mills, and gives the relation between driving power of individual stands, coiler and uncoiler, and pure roll power in these three stands. It does not include strip tension between stands or power recovery.

The values of  $N_{w1}$ ,  $N_{w2}$  and  $N_{w3}$  decrease due to strip tension acting between stands, hence the effect of front and back tension is similar to the sum of the effects for one stand only. For 100% power recovery

$$V_s \sigma_4 + N_{m1} + N_{m2} + N_{m3} = N_{w1} + N_{w2} + N_{w3} \quad (4.201)$$

For a continuous mill of  $n$  stands, the power distribution is given by the formula

$$V_s \sigma_{n+1} + N_{m1} + N_{m2} + \dots + N_{mn} = N_{w1} + N_{w2} + \dots + N_{wn} + V_s \sigma_1 \quad (4.202)$$

For practical purposes strip tension in cold rolling can be determined as follows: It is known, that for a coefficient of spread of  $\beta = 1$  the constrained yield stress equals  $K = 1.155 K_f$ . For ideal rolling conditions without friction between roll surface and stock there is a direct relation between strip tension and roll force, e.g. if a stress of  $0.5 K$  is produced in the strip due to tension, then a roll pressure of  $0.5 K$  is sufficient for rolling. For such cases the following table can be evolved.

|                                                   |     |          |         |         |     |
|---------------------------------------------------|-----|----------|---------|---------|-----|
| Tensile stress in the strip                       | 0   | 0.25 $K$ | 0.5 $K$ | 0.8 $K$ | $K$ |
| Pressure required for deformation in the roll gap | $K$ | 0.75 $K$ | 0.5 $K$ | 0.2 $K$ | 0   |

Therefore, for frictionless rolling a general relation can be stated

$$K = K_{\text{tension}} + K_{\text{roll}}$$

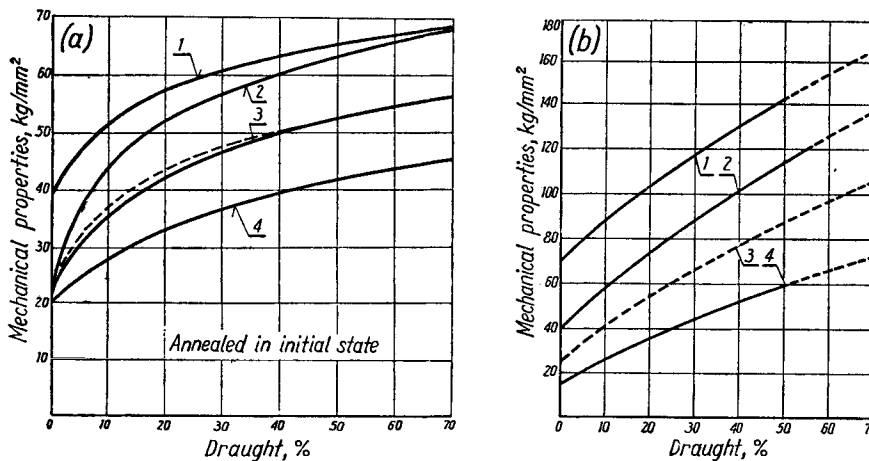


Fig. 4.144. Mechanical properties of various steels [91]: (a) mild steel of 0.08% C, 0.22% Mn, 0.08% Si, 0.008% P, 0.033% S and 0.2% Cu, (b) 18Cr18Ni stainless steel (see Appendix 1). Curves of: 1 — tensile strength  $U_T$ , 2 — yield point  $Y_{P0.2}$ , 3 — mean stress between yield and elastic limit, 4 — elastic limit  $\sigma_{0.01}$

Frictional forces occurring during cold rolling are small, due to coefficient of friction varying from 0.04 to 0.12. Friction causes considerable differences between tension stresses arising in the strip due to front tension, and those from back tension. These differences depend on the coefficient of friction, and increase together with it.

In practice, strip tension of from  $0.3 K$ – $0.5 K$  is used in cold rolling of steel. Strip tension as large as  $K$  cannot be used, due to danger of strip breaks, i.e.

$$\sigma_1 \leq 0.5K \quad \text{and} \quad \sigma_2 \leq 0.5K \quad (4.203)$$

From their own experiments Emicke and Lucas [83] state that the practical upper limit of strip tension (Figs. 4.144a and b) is

$$\sigma_{\max} \leq \frac{\sigma_{0.01} + \sigma_{0.2}}{2} \quad (4.204)$$

where

$\sigma_{0.01}$  = limit of elasticity of the rolled stock,

$\sigma_{0.2}$  = yield point of the rolled stock.

Obviously  $\sigma_{\max}$  increases with increasing work-hardening of the rolled stock.

#### 4.6. Power of the Main Driving Motors

Both DC and AC motors are used to drive rolling mills, depending on the type of mill and working conditions. For rolling mills, where speed control is not required, asynchronous AC motors can be used, where roll work does not require a flywheel, or the necessity of joint running with a motor with flywheel. For mills requiring speed control a DC motor must be used, but in this case the main motor will be AC.

To determine the required power of the main driving motor, the following data must be known:

- (a) rolling mill type, number and layout of stands, roll diameter and barrel length, number of roll revolutions per minute,
- (b) mill output,
- (c) rolling programme, steel quality, weight and dimensions of initial ingot or billets, dimensions and weight of semiproducts or products,
- (d) number of passes, roll pass design with known intermediate and total elongation coefficients in each pass,
- (e) time in each pass with corresponding idle times,
- (f) range of speed control with indication whether control takes place with  $P = \text{const}$  or  $M = \text{const}$ .

From the diagram of static loading of the rolling train or stand (Figs. 4.145a–d) and the following notation:

$$\left. \begin{aligned} M_1 &= \text{roll torque in the first pass} \\ M_2 &= \text{roll torque in the second pass} \\ M_n &= \text{roll torque in the } n\text{-th pass} \end{aligned} \right\} \text{relative to shaft of the main motor}$$

$$\begin{aligned} t_1 &= \text{pass time of the first pass,} \\ t_2 &= \text{pass time of the second pass,} \\ t_n &= \text{pass time of the } n\text{-th pass,} \\ M_i &= \text{idling torque,} \\ t'_1 &= \text{idle time after the first pass,} \end{aligned}$$

- $t'_2$  = idle time after the second pass,  
 $t'_n$  = idle time after the  $n$ -th pass,  
 $t_t$  = total time of the rolling cycle,

then for the torque distribution shown in Fig. 4.145 the mean square of effective torque equals

$$M_k^2 = \frac{M_1^2 t_1 + M_i^2 t'_1 + M_2^2 t_2 + M_i^2 t'_2 + \dots + M_n^2 t_n + M_i^2 t'_n}{t_t} \quad (4.205)$$

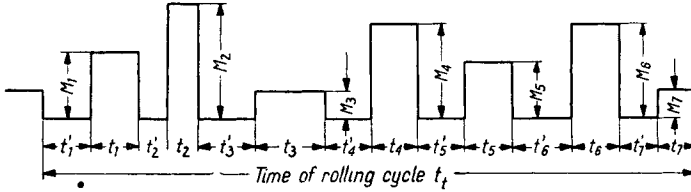


Fig. 4.145. Diagrammatic representation of the root mean square torque [92]

To determine the effective roll torque for reversing stands for passes where the motor rpm exceeds the nominal value, i.e.  $n_{\text{pass}} > n_{\text{nom}}$ , the calculated torque must be multiplied by the ratio  $n_{\text{pass}}/n_{\text{nom}}$ , e.g. if the motor rpm in the fifth pass for the stock being in the gap equals  $n_5$ , then the effective torque  $M'_5$  becomes

$$M'_5 = M_5 \frac{n_5}{n_{\text{nom}}}$$

The nominal torque of the motor can be determined from the mean square torque by using the relation

$$M_{\text{nom}} \leq M_k \quad (4.206)$$

This relation is also used to check overheating of the motor. The second condition to determine the nominal torque is

$$M_{\text{nom}} = 1.2-1.3 M_m \quad (4.207)$$

where  $M_m$  = mean static torque calculated from Fig. 4.145 as follows:

$$M_m = \frac{M_1 t_1 + M_2 t_2 + \dots + M_n t_n}{t_t} \quad (4.208)$$

In this equation  $M_1, M_2 \dots$  etc. denote static torques in particular passes,  $t_1, t_2 \dots$  etc. the corresponding pass times, and  $t_t$  total time of rolling cycle. The coefficient of 1.2–1.3 in (4.207) is to take into account motor heating.

Two conditions should be fulfilled when calculating the power of the main driving motor [10]:

(a) The temperature of any part of the motor should not exceed permissible values. This condition is expressed by the relation (4.206)

$$M_{\text{nom}} \leq M_k$$

$M_{\text{nom}}$  = nominal torque of the motor,

$M_k$  = mean square torque according to relation (4.205).

(b) Momentary peak loading should not exceed the permissible maximum torque or power for the given motor.

The starting-up motor torque should be checked, as sufficient to give the drive the necessary acceleration.

When choosing a motor from a factory catalogue or specification the overload capacity should be checked. The maximum permissible momentary overloading of the motor  $\alpha_m$ , i.e. ratio of maximum torque to nominal torque, is governed by the relation [87]

$$\alpha_m M_{\text{nom}} \geq M_{\text{max}} \quad (4.209)$$

or

$$\alpha_m = \frac{M_{\text{max}}}{M_{\text{nom}}} \quad (4.210)$$

Values for overloading coefficient  $\alpha_m$  for various types of motor are given in Table 4.27 [87].

Overloading of the main driving motor occurs due to unequal loading in particular passes, or also due to operational mistakes. Among these may be too great draughts or insufficient and unequal heating of the rolled stock, causing considerably greater roll forces and torques than calculated ones.

TABLE 4.27  
PERMISSIBLE OVERLOADING OF ELECTRIC MOTORS

| Design of motor                                | Permissible overloading<br>$\alpha_m = M_{\text{max}}/M_{\text{nom}}$ |
|------------------------------------------------|-----------------------------------------------------------------------|
| Normal DC motor                                | 2                                                                     |
| DC motor of special design                     | 3-4                                                                   |
| Asynchronous squirrel cage motor               | 1.8-2                                                                 |
| Asynchronous slip-ring motor                   | 2-2.5                                                                 |
| Asynchronous slip-ring motor for hoists        | 2-2.5                                                                 |
| Asynchronous high-slip and squirrel cage motor | 2-3                                                                   |
| Asynchronous double-squirrel-cage motor        | 1.8-2.7                                                               |
| Synchronous motor of normal design             | 2                                                                     |
| Synchronous motor of special design            | 3.5-4                                                                 |
| AC collector motor                             | 2-2.5                                                                 |

The calculated nominal power of the motor is relative to the motor shaft. After determining nominal torque from equations (4.206) or (4.207) it should be verified that

$$M_{\text{nom}} = \sqrt[3]{M_k^2 M_m} \quad (4.211)$$

where the mean square torque is expressed as

$$M_k = \sqrt{\int \frac{M^2 dt}{t_i}} \quad (4.212)$$

and the mean torque  $M_m$  is found from equation (4.208).

For practical purposes  $M_k$  is determined from equation (4.205).

When the nominal (basic) motor rpm is known, then nominal power is found from nominal torque

$$N_{\text{nom}} = \frac{2\pi}{60} M_{\text{nom}} n_{\text{nom}} \quad \text{kgm/sec} \quad (4.213)$$

Transforming this relation into kW or HP the following equations are found

$$N_{\text{nom}} = \frac{1}{975} M_n n_n = 0.00102 M_n n_n \quad (4.214)$$

or

$$N_{\text{nom}} = \frac{1}{726} M_n n_n = 0.0014 M_n n_n \quad (4.215)$$

As the motor is working often with slip expressed as

$$s_n = \frac{n_0 - n_n}{n_0} \quad (4.216)$$

and depending on motor power varies within 3 and 10%, and should be taken into account. For a drive with a flywheel and slip regulator, greater slip is obtained than for a drive without such equipment.

When considering slip then equations for motor power are

$$N_{\text{nom}} = \frac{1}{975} M_n n_0 (1 - s_n) \quad \text{kW} \quad (4.217)$$

and

$$N_{\text{nom}} = \frac{1}{726} M_n n_0 (1 - s_n) \quad \text{HP} \quad (4.218)$$

where

$n_0$  = idling speed of the motor,

$n_n$  = rated motor speed.

For increasing slip the mean speed of the rolled stock and output of the mill decrease causing greater losses. Therefore, the drop in rpm of the main driving motor should not exceed 8–10% at nominal torque.

The operational time of the main driving motor of a single train, single stand mill (Figs. 4.100b and 4.145) consists of:

- (a) pass times during which the motor is working,
- (b) idle times during which the motor is idling.

During pass time the motor is subjected to static torque, which is sometimes greater than the rotational torque supplied by the motor. When working with a flywheel this difference is covered by the flywheel giving back a part of the accumulated kinetic energy, with decrease in rpm.

During idle times the main driving motor is subjected to idling torque loading only. During this time motor torque exceeds the static idling torque, therefore this excess goes to increase the kinetic energy of the flywheel, and serves then to overcome the load peak occurring during the next pass.

When working without a flywheel the motor is overloaded in relation to the nominal torque. The pass schedule should be worked out so that peak loads do not exceed values given in Table 4.27. In many cases a flywheel is used to even out the load on the driving motor between pass times and idle times. Taking the effect of flywheel into account as in equation (4.132) then the torque acting upon the motor shaft is [59]

$$M_m = M_{st} \mp \frac{GD^2}{375} \frac{dn}{dt}$$

where

$M_{st}$  = static torque acting on motor shaft,

$GD^2$  = dynamic torque of the flywheel and other rotating elements of the stand, relative to the motor shaft,

$dn/dt$  = acceleration or deceleration of the motor shaft, rpm per unit of time.

The motor load decreases only when its speed decreases. Then the acceleration  $dn/dt$  becomes negative and therefore, the second expression in equation (4.132) becomes negative, giving a negative dynamic torque. To solve equation (4.132) it is necessary to know the relation between speed and torque of the motor, or time. When calculating drives with flywheel, in which case asynchronous motors are mostly used it is usually assumed that the speed drop is proportional to the increase in loading, i.e.

$$n = n_0 - \frac{n_0}{M_n} s_n M_m \quad (4.219)$$

The expression in this equation  $s_n = (n_0 - n)/n_0$  is called nominal slip, where  $n_0$  and  $n_n$  are idling and nominal rpm of the main driving motor.

The speed drop per unit of time  $dn/dt$  is found from equation (4.219) by differentiating. Substituting this into equation (4.132), hence

$$M_m - M_{st} = -\frac{GD^2}{375} \frac{s_n}{M_n} n_0 \frac{dM_m}{dt} \quad (4.220)$$

If the constant

$$\frac{GD^2 n_0 s_n}{375 M_n} = T \quad (4.221)$$

then the basic differential equation for calculation of drives with flywheel becomes

$$\frac{dM_m}{M_m - M_{st}} = -\frac{1}{T} dt \quad (4.222)$$

The constant  $T$  occurring in this relation depends on the dimensions of the flywheel and motor characteristics and is called the electromechanical constant of the drive. It has the dimensions of time, and is the product of nominal slip and time of running up of the flywheel from stop up to the idling speed  $n_0$ , assuming that during the period of running up the motor gives nominal torque  $M_n$  and no loading occurs by means of static torques.

Integrating equation (4.222), then

$$\log_e(M_m - M_{st}) = -\frac{t}{T} + C$$

The constant  $C$  is found from boundary conditions, e.g. for  $t = 0$

$$M_m = M_0 \quad \text{and} \quad M_{st} = 0$$

Substituting these values, then Haag's formula [59] is found

$$M_m = M_{st} - (M_{st} - M_0) e^{-t/T} \quad (4.223)$$

This equation shows that the torque on the motor shaft varies according to an exponential curve, whose asymptote is a straight line parallel to the  $t$ -axis, at a distance equal to  $M_{st}$  (Fig. 4.146). The greater the electromechanical constant  $T$ , i.e. the higher the torque of running up of the flywheel, the steeper is the curve (see dotted line in Fig. 4.147).

For idling operation, as  $M_{st} = M_i$  and  $M_0 > M_i$ , equation (4.223) becomes

$$M_m = M_i + (M_0 - M_i) e^{-t/T} \quad (4.224)$$

By using equations (4.223) and (4.224) loading diagrams can be drawn for any pass or break. When many passes are involved the analytical solution of this problem is very laborious, and a graphical method by means of appropriate master curves is recommended. The curve

$$M_m = M_{st \max} (1 - e^{-t/T}) \quad (4.225)$$

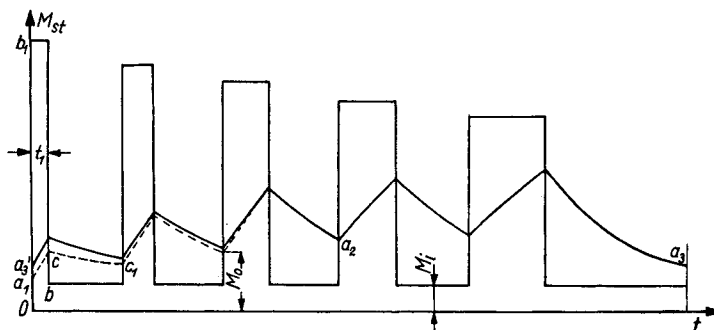


Fig. 4.146. Drive loading with flywheel [92]

should be drawn on stiff cardboard (Fig. 4.148) on the same scale as the loading diagram is intended to be drawn.

$M_{st \max}$  = maximum static torque assumed approximately equal to the maximum static torque from the torque distribution diagram.

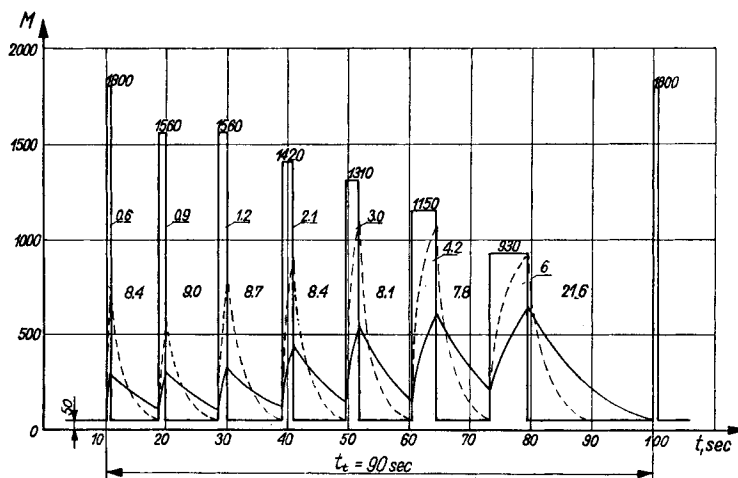


Fig. 4.147. Loading of the drive for different time constants for a motor working with flywheel

It is sufficient to draw the part of the curve from  $t = 0$  up to  $t \approx 4T$  since the curve and its asymptote are indistinguishable at this point. This template is then cut

out and can be used by laying it on respective loading diagrams and drawing round it. It should be laid on the static torque diagram so that  $M = M_0$  and the asymptote of the curve is parallel to the axis of time for the appropriate value of  $M_{st}$ . Drawing round the contour of the template for the given pass or idle time, gives the required section of loading diagram very simply (Figs. 4.149a and b).

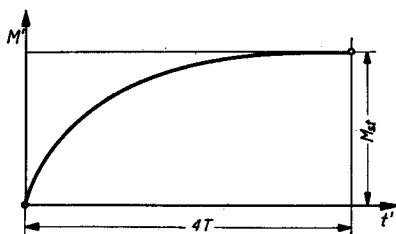


Fig. 4.148. Template for torque distribution curve [87]

As an example; the torque distribution shown in Fig. 4.146 is examined. This diagram was drawn from the acceleration and deceleration curves shown in Fig. 4.150. Both curves serve for determination of torque distribution during the pass. During the pass the static roll torque is greater than the motor torque.

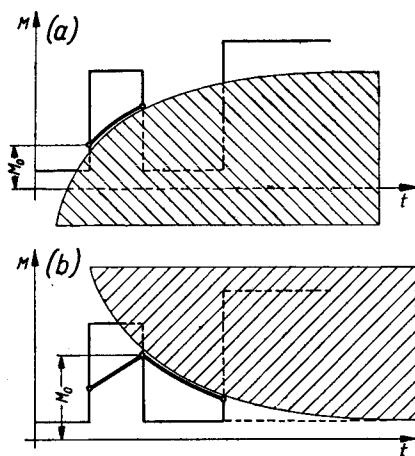


Fig. 4.149. Determination of torque distribution for motor with flywheel using the template shown in Fig. 4.148 [87]: (a) for  $M_{st} > M_m$ , (b) for  $M_{st} < M_m$

It is assumed that at the beginning of the rolling cycle the motor torque  $M_1$  is equal to the idling torque of the mill, then the deceleration curve in Fig. 4.146 is drawn starting from point  $a_1$ . At this point (at the beginning of first pass), the static torque  $M_3$  is considerably greater than the motor torque  $M_1$ , and the difference  $(M_3 - M_1)$  is plotted in Fig. 4.150 as an ordinate downwards from the horizontal line  $AA$ , giving point  $a$ . From this point the torque curve follows the deceleration curve, and after a pass time of  $t_1$  point  $b$  is found. Its distance from the horizontal line  $AA$  corresponds to a difference in torques of  $M_3 - M_1$ . Sector  $a-b$  of the curve should be drawn from



Fig. 4.150, or more easily from the master curve of the loading diagram. It corresponds to the sector  $a_1c$ .

Similarly the torque distribution for breaks between passes can be drawn, using the acceleration curve in Fig. 4.150, where the horizontal asymptote is now formed by the idling torque of the mill.

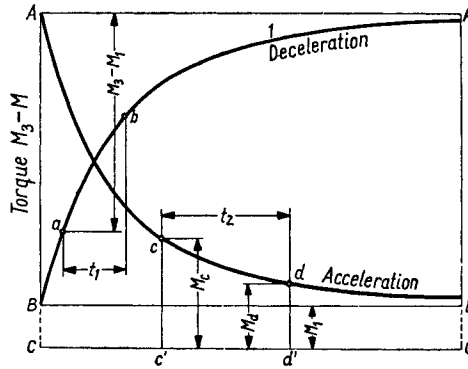


Fig. 4.150. Diagram of torque for determination of mean square torque

Beginning with the first pass, the distribution of torque during pass and idle times is drawn on loading diagrams as described until the last point  $a_3$ , is found as shown in Fig. 4.146. It corresponds to the driving motor loading at the beginning of the second rolling cycle, when the succeeding ingot enters the roll gap. This load may be greater or less than that at the beginning of the first rolling cycle, therefore starting from point  $a_3$ , a second torque curve is drawn, representing the torque distribution at the end of the cycle. These operations should be repeated until equal values at the beginning and end of the rolling cycle are found, e.g. point  $a_2$ . The final curve for torque distribution for this example is shown in Fig. 4.146, the full line. If in Fig. 4.150 a line  $CC$  is drawn parallel to  $BB$  and at a distance corresponding to the idling torque of the mill  $M_1$ , then intercepts  $cc'$  and  $dd'$  are proportional to torque  $M_c$  and  $M_d$  at the beginning and end of the rolling cycle considered.

#### Example

It is required to find the motor power and flywheel torque for driving of a roughing 3-high mill. The loading diagram is shown in Fig. 4.147. Rated motor speed  $n = 735$  rpm. Total time of the cycle  $t_t = 90$  sec.

From the diagram in Fig. 4.147 the following mean square torque is found (4.205)

$$M_k = \sqrt{\frac{1800^2 \times 0.6 + 1560^2 \times 0.9 + 1560^2 \times 1.2 + 1420^2 \times 2.1 + 1310^2 \times 3 + 1150^2 \times 4.2 + 930^2 \times 6 + 50^2 (8.4 + 9 + 8.7 + 8.4 + 8.1 + 7.8 + 21.6)}{90}} = 551 \text{ kgm}$$

From equation (4.208) the mean static torque is:

$$M_m = \frac{1800 \times 0.6 + 1560 \times 0.9 + 1560 \times 1.2 + 1420 \times 2.1 + 1310 \times 3 + 1150 \times 4.2 + 930 \times 6 + 50 (8.4 + 9 + 8.7 + 8.4 + 8.1 + 7.8 + 21.6)}{90} = 281 \text{ kgm}$$

Substituting this into equation (4.211), the nominal torque is

$$M_n = \sqrt[3]{M_m M_k^2} = \sqrt[3]{281 \times 551^2} = 352 \text{ kgm}$$

If it is also assumed that the coefficient of maximum overloading of the main driving motor equals 1.8, and at the maximum torque a slip of  $s_k = 12\%$  occurs, then a nominal slip of

$$s_n = \frac{s_k}{\alpha_m} = \frac{0.12}{1.8} = 0.06667$$

is found.

If it is further assumed that maximum torque occurring during the first pass of the rolled billet, i.e. the maximum static torque, is  $M_{st \max} = 1800$  kgm, then from equations (4.137) and (4.138) the flywheel torque will be

$$GD^2 = \frac{375 M_n}{s_n n_0} \frac{t}{\log_e \frac{M_{st} - M_i}{M_{st} - k M_n}} = \frac{375 \times 352}{750 \times 0.0667} \frac{0.6}{\log_e \frac{1800 - 50}{1800 - 1.8 \times 352}} = 3906 \text{ kgm}^2$$

From equation (4.221) the electromechanical constant is

$$T = \frac{GD^2 n_0}{375 M_n} s_n = \frac{3906 \times 750 \times 0.0667}{375 \times 352} = 1.48 \text{ sec}$$

A master curve can be drawn based on equation (4.225)

$$M_m = M_{st \max} (1 - e^{-t/T}) = 1800(1 - e^{-t/1.48})$$

Using this template the curve  $M = f(t)$ , i.e. load of the main driving motor, is drawn onto the load diagram. After drawing these curves it can be seen that the motor should reach its maximum torque during the sixth, and not first pass, as had been previously assumed. This same result is also found when determining the maximum value of the expression

$$\frac{t}{\log_e \frac{M_{st} - M_i}{M_{st} - k M_n}}$$

Therefore for flywheel torque the design pass is the sixth pass with a pass time of  $t_6 = 4.2$  sec and torque of  $M_6 = 1150$  kgm, hence from equations (4.137) and (4.138)

$$GD^2 = \frac{375 \times 352}{750 \times 0.0667} \frac{4.2}{\log_e \frac{1150 - 50}{1150 - 1.8 \times 352}} = 14,637 \text{ kgm}^2$$

For this torque, the electromechanical constant  $T$  from equation (4.221) will be

$$T = \frac{14,637 \times 750 \times 0.0667}{375 \times 352} = 5.547 \text{ sec}$$

Due to the considerable change in the value of  $T$ , a new template must be made from the equation

$$M_m = 1800(1 - e^{-t/5.547})$$

from which a new curve should be drawn on the load diagram.

Knowing the distribution of  $M = f(t)$ , the substituted torque can be determined.

To determine the magnitude of  $M_k$ , particular sections of the curve should be replaced by straight lines, thus the area of the curve can be calculated as a series of trapezia

$$M_k = M_n = \sqrt{\frac{9,470,000}{90}} = 324.2 \text{ kgm}$$

The overloading of the motor should also be checked. It follows from the load diagram that maximum torque occurs during the sixth pass, and is equal to 660 kgm, and this torque should be delivered by the motor. This value corresponds to a nominal torque of

$$M_n = \frac{M_{\max}}{\alpha_m} = \frac{660}{1.8} = 366 \text{ kgm}$$

Knowing the nominal torque, then from equation (4.217) the nominal power is equal to

$$N_n = \frac{M_n n_0 (1 - s_n)}{975} = \frac{366 \times 750 (1 - 0.02)}{975} = 276 \text{ kW}$$

at a nominal slip of

$$s_n = \frac{750 - 735}{750} = 0.02$$

Hence the required dynamic torque of the drive is

$$GD^2 = \frac{375 \times 366 \times 5.547}{0.0667 \times 750} = 15,250 \text{ kgm}^2$$

This total dynamic torque must be delivered by flywheel and rotor of the driving motor, i.e. to calculate the magnitude of the flywheel the dynamic torque of the rotor should be subtracted from the total torque of  $GD^2$ . Dynamic torque of remaining elements are generally neglected, due to their small values.

Next the motor power can be determined from energy consumption curves shown in Figs. 4.101–4.112, by the following method. Based on values of coefficient of elongation  $\lambda_1, \lambda_2, \lambda_3$  etc. for the first, second, third pass etc. values of  $A_1, A_2, A_3$  etc. can be read off from the appropriate diagrams. The value of  $A_1$  is obtained as a function of  $\lambda_1$ ,  $A_1 + A_2$  as a function of  $\lambda_2$ ,  $A_1 + A_2 + A_3$  as a function of  $\lambda_3$ , etc. For a pass with coefficient of elongation increasing from  $\lambda_1$  to  $\lambda_2$  there is an energy consumption per ton of the rolled stock of  $A = A_2 - A_1$ . From equation (4.139) the motor power required for this pass is equal to

$$N_m = N_w + N_{fr} = \frac{A \times 3600 G}{t}$$

where

$N_{fr}$  = power required due to additional friction,

$A$  = energy consumption during the pass, determined from corresponding diagram, kWh/t,

$G$  = tonnage of the rolled stock, t,

$t$  = pass time of the pass considered, sec.

On reversible mills there is a drop in the roll rpm when the stock enters the roll gap, after which it increases up to the normal rolling speed and once more drops shortly before the end of the pass (Fig. 4.99a). Therefore, the pass time can be divided into three periods, acceleration, constant speed and finally deceleration. Expressing angular acceleration and deceleration in rpm per unit of time and denoting them as  $\varepsilon_a$  or  $\varepsilon_d$ , respectively, then for the three periods of operation the following driving torques are obtained:

(1) period of acceleration

$$M_a = M_{st} + \frac{GD^2}{375} \varepsilon_a \quad (4.226)$$

(2) period of constant speed

$$M_w = M_{st} \quad (4.227)$$

(3) period of deceleration

$$M_d = M_{st} - \frac{GD^2}{375} \varepsilon_d \quad (4.228)$$

where  $GD^2$  = dynamic torque of all rotating elements of the drive including rotor of the motor, relative to motor shaft.

For DC shunt motors generally used for reversible mills,  $\varepsilon_a$  and  $\varepsilon_d$  are usually assumed to be constant. Then the three operation periods are shown on the torque diagram in Fig. 4.99b as three rectangles. Between two succeeding passes, i.e. during the idle time, the following load occurs:

for the acceleration period

$$M_{ba} = M_i + \frac{GD^2}{375} \varepsilon_a \quad (4.229)$$

for the deceleration period (braking)

$$M_{bd} = M_i - \frac{GD^2}{375} \varepsilon_d \quad (4.230)$$

The acceleration and deceleration  $\varepsilon_a$  and  $\varepsilon_d$  depend on the motor characteristics and electrical layout of the mill. For heavy product mills these values are usually taken as

$$\varepsilon_a \approx 30\text{--}80 \text{ rpm per sec} \quad \text{and} \quad \varepsilon_d \approx 60\text{--}160 \text{ rpm per sec}$$

A practical example will be discussed for DC shunt motors, since many problems arise during their use in reversible mills.

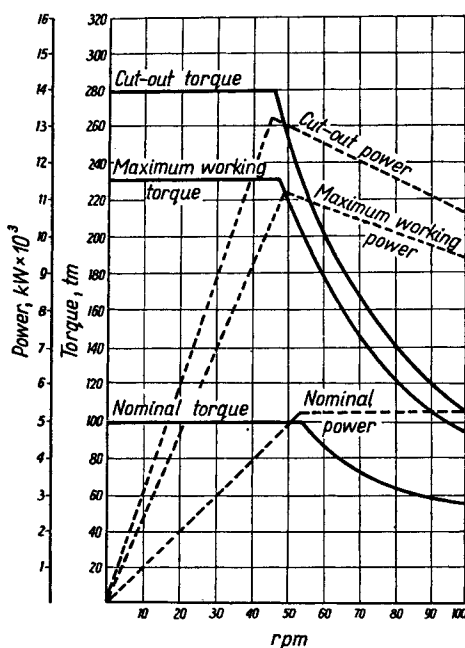


Fig. 4.151. Characteristics of a DC motor for a 2-high reversible blooming mill. Motor power 5000 kW [10]

The characteristics of a DC shunt motor for driving reversible 2-high blooming mills are shown in Fig. 4.151 of nominal power 5000 kW = 6800 HP with a range of rpm from 0–50–100, i.e. nominal speed  $n_n = 50$  rpm [10].

It can be seen from the diagram that the maximum motor torque is 2.3 times the nominal value and at this level the nominal speed  $n_n = 50$  rpm is held. The maximum

cut-out torque is approximately 20% higher than the maximum working torque. At higher speeds within 50 and 100 rpm there is a drop in torque, only the nominal power stays constant, whereas the maximum working power and cut-out power of the motor decrease. These variations are not so apparent in blooming mills, due to the fact that large torques are only required in the first passes. After a certain considerable elongation there is a drop in torque, due to the large reduction in cross-section of the rolled stock. At this stage an increased speed is required to enable rolling long blooms at higher speeds.

When the roll pass design is worked out and for each pass, and the roll force and torque calculated, the diagram shown in Fig. 4.151 can be used to determine the appropriate working torques which are drawn onto the motor diagram. All these torques should be below the curve for maximum permissible working torque of the motor. If the torque at any point exceeds this value, the pass design should be corrected or speed of the motor decreased. From this diagram it can also be checked, if the motor is insufficiently utilized, i.e. if torque values arising in particular passes are considerably smaller than the maximum permissible torque. If this is the case, then again the pass design must be corrected or again the speed must be increased. This method is designed to approach ideal conditions for rolling and pass design, and by maximum utilization of the given motor to achieve highest possible draughts and rolling speeds.

For practical conditions when calculating roll power, the SKF equations (4.184), (4.185), and (4.186), are often used. They present a further development and application of known formulae for determining roll torque and additional friction torque by means of roll force for 2-high, 4-high, and 6-high mills.

#### Example No. 1

To calculate motor torque and power. On a reversible mill a bloom of low carbon steel of initial dimensions of  $h_1 = 200$  mm,  $b_1 = 250$  mm,  $l_1 = 2000$  mm is rolled down to final dimensions of  $h_2 = 150$  mm,  $b_2 = 262$  mm,  $l_2 = 2545$  mm. There is a rolling temperature of  $1000^\circ\text{C}$ , and steel rolls of  $D = 700$  mm of rough surface are used. The rolls rotate at 95 rpm, corresponding to a rolling speed of  $v_r = 3.1$  m/sec

From equation (2.108) the roll force

$$P = F_d K_{wm}$$

From Ekelund's formula (4.36) the mean resistance to deformation  $K_{wm} = 9.11$  kg/mm<sup>2</sup>. The projected area of contact is 34,133 mm<sup>2</sup>.

Therefore roll force  $P$  can be found

$$P = F_d K_{wm} = 34,133 \times 9.11 = 310,955 \text{ kg}$$

From equation (4.66) the roll torque for two rolls is

$$M_w = 2Pa$$

The torque lever arm for hot rolling can be determined empirically from equation (4.77)

$$a \approx \frac{l_d}{2} \approx \frac{\sqrt{R \Delta h}}{2} = \frac{132.3}{2} = 66.15 \text{ mm}$$

Or using equation (4.81)

$$a = \frac{l_d}{2} \frac{\gamma}{1-\gamma} \frac{b_2}{b_m} \log_e \frac{1}{\gamma}$$

The reduction coefficient  $\gamma$  is given by

$$\gamma = \frac{h_2}{h_1} = \frac{150}{200} = 0.75$$

Substituting this value

$$a = \frac{132.3}{2} \frac{0.75}{0.25} \frac{262}{258} \log_e \frac{1}{0.75} = 58.0 \text{ mm}$$

Substituting corresponding values the roll torque is found

$$M_w = 2Pa = 2 \times 310,955 \times 0.058 = 36,070 \text{ kgm}$$

If it is assumed that the lever arm  $a \simeq l_d/2$ , then from equation (4.78) a roll torque of

$$M_w = K_{wm} b_m R \Delta h = 9.11 \times 258 \times 350 \times 50 = 41,136 \text{ kgm}$$

is found, i.e. a greater value than from equation (4.81).

Then from equation (4.105) the total torque relative to the axis of rolls is found

$$M_t = M_w + M_f + M_i \pm M_d$$

As the rolling torque is already known, the additional friction torque should be determined

$$M_f = M_{f1} + M_{f2}$$

From equation (4.107)

$$M_{f1} = P f_1 d$$

Assuming the roll neck diameter as equal to  $d = 385 \text{ mm}$  and the coefficient of bearing friction as  $f_1 = 0.07$ , hence

$$M_{f1} = 310,955 \times 0.07 \times 0.385 = 8380 \text{ kgm}$$

$M_{f2}$  can be found from formula (4.110)

$$M_{f2} = (M_w + M_{f1}) \left( \frac{1}{\eta_1 \eta_2} - 1 \right)$$

Efficiencies of the pinion stand and gear are estimated to be equal to  $\eta_1 = \eta_2 = 0.95$ , hence the following value for  $M_{f2}$  is obtained

$$M_{f2} = (36,070 + 8380) \left( \frac{1}{0.95 \times 0.95} - 1 \right) = 4800 \text{ kgm}$$

For comparison, these values will be calculated by the simplified method as in equation (4.113) with the assumption  $a \simeq l_d/2$  and  $1/\eta = 1.05$  for a single reduction, or  $1/\eta = 1.10$  for a double reduction gear

$$M_w + M_f = (1.05 - 1.10) P (\sqrt{R \Delta h} + d f_1)$$

$$M_w + M_f = 1.10 \times 310,955 (\sqrt{0.350 \times 0.050} + 0.385 \times 0.07) = 54,454 \text{ kgm}$$

These torques are calculated relative to the axis of roll. To express this relative to the motor shaft,  $M_w$  and  $M_f$  must be divided through by the gear ratio  $i$ , i.e. the ratio of roll and motor shaft revolutions.

From equation (4.115) the idling torque is

$$M_i = \sum_{i=1}^{i=n} \frac{G_i f_i d_i}{2 i_i}$$

Due to lack of detailed data on mill design, the idling torque will be calculated from an estimated value of idling power of approximately 240 kW based upon practical experience. Hence, the idling torque is

$$M_i = \frac{60 \times 102}{2 \pi n} N_i = \frac{1,468,800}{533.8} = 275 \text{ kgm}$$

The dynamic torque is calculated from equation (4.120). Assuming the moment of inertia of all rotating elements of the drive  $GD^2 = 65,000 \text{ kgm}^2$

$$M_d = \frac{65,000}{375} \times 20 = 3461 \text{ kgm}$$

Tabulating the individual expression of the equation

$$M_t = M_w + M_f + M_i + M_d$$

| Lever arm $a$ from<br>equation (4.81) | Lever arm $a$ , assumed<br>equal to $l_d/2$ |
|---------------------------------------|---------------------------------------------|
| $M_w$ 36,070 kgm                      | $M_w$ 41,136 kgm                            |
| $M_f$ 13,180 kgm                      | $M_f$ 13,323 kgm                            |
| $M_i$ 275 kgm                         | $M_i$ 275 kgm                               |
| $M_d$ 3,461 kgm                       | $M_d$ 3,461 kgm                             |
| $M_t$ 52,986 kgm                      | $M_t$ 58,195 kgm                            |

Similar calculations for remaining passes should be done. The values of total torque  $M_t$  should be then divided through by the gear ratio

$$i = \frac{\text{motor number of revolutions}}{\text{roll number of revolutions}}$$

to express torque relative to motor shaft.

Knowing the values of torque  $M_1, M_2, M_3 \dots M_n$ , total rolling time and pass respective times, the torque diagram can be drawn, and the mean square torque relative to the motor shaft determined from equation (4.205). Hence, the power at the motor shaft is determined from formulae (4.213)–(4.218). It has to be taken into account that torque and power relative to motor shaft have been determined allowing for all drive losses, which are usually included in the efficiency coefficient. The mean square torque value is used for checking the main driving motor for thermal loading during operation.

#### Example No. 2

To calculate the roll force arising during rolling of symmetrical sections. A square of initial dimensions of  $h_1 = 17.7$  mm and  $b_1 = 17.7$  mm and initial cross-sectional area  $F_1 = 314$  mm<sup>2</sup>, is rolled to an oval of final dimensions of  $h_2 = 10$  mm,  $b_2 = 30$  mm and cross-sectional area  $F_2 = 200$  mm<sup>2</sup>. Mild steel is rolled at a temperature 1000°C by chilled rolls of mean nominal diameter of 300 mm.

From equation (3.5) the mean height is given by

$$\Delta h_m = h_{1m} - h_{2m} = \frac{F_1}{b_1} - \frac{F_2}{b_2} = \frac{314}{17.7} - \frac{200}{30} = 17.7 - 6.7 = 11 \text{ mm}$$

From equation (4.15) the mean projected area of contact is

$$l_{dm} \simeq \sqrt{R_m \Delta h_m}$$

and the mean work roll radius

$$R_m = \frac{D_n - h_m}{2}$$

where

$D_n$  = mean nominal roll diameter,

$h_m$  = mean height of the delivered stock.

For the case considered  $D_n = 300$  mm and  $h_{2m} = 200/30 = 6.7$  mm. Hence, the mean working diameter of rolls is

$$D_m = 300 - 6.7 = 293.3 \text{ mm}$$

and

$$R_m = \frac{293.3}{2} = 146.7 \text{ mm}$$

In the case of unequal mean working diameters, the mean working diameter of the pass is calculated from equation (3.31) and (3.32)

$$D_{ma} = \frac{D_{w1} + D_{w2}}{2} - h_m + s$$

When assuming for the case considered  $D_{w1} = 295$  mm and  $D_{w2} = 285$  mm, then for  $h_m = 6.7$  mm and  $s = 3.0$  mm

$$D_{ma} = \frac{285 + 295}{2} - 6.7 + 3.0 = 286.3 \text{ mm}$$

and

$$R_{ma} = 143.1 \text{ mm}$$

and also

$$l_{dm} = \sqrt{143.1 \times 11.0} = 39.67 \text{ mm}$$

$$b_m = \frac{b_1 + 2b_2}{3} = \frac{17.7 + 2 \times 30}{3} = 25.9 \text{ mm}$$

Then the projected area of contact is given by

$$F_{dm} = l_{dm} b_m = 39.67 \times 25.9 = 1027.4 \text{ mm}^2$$

Further calculation should be done as for example No. 1. Mean resistance to deformation is determined by means of known formulae and methods, but the additional friction at the side walls of the pass should also be taken into account, calculated from equation (4.60) and Table 4.6.

To calculate the design load on drive components between main motor and rolls, the maximum operational torque should be known. Accurate calculation presents difficulties, due to technological variations in rolling processes, and in practice rolling torque values greater than calculated ones can occur.

The following assumptions for roll pass design are recommended:

(1) When calculating from permissible overloading of the electric motor, the maximum torque equals:

for reversible driving motors

$$M_{\max} = 2.5\text{--}3.0 M_{\text{nom}}$$

for non-reversible driving motors

$$M_{\max} = 1.5\text{--}2.0 M_{\text{nom}}$$

(2) For drive without flywheel the maximum possible torque is limited by the motor, which has been chosen based upon given rolling conditions, when taking into account conditions given under 1.

(3) For drive with flywheel the maximum torque can reach very great values, during rapid deceleration of the flywheel. The maximum torque may reach

$$M_{\max} = (4\text{--}6) M_{\text{nom}}$$

When calculating permissible overloading of separate driving elements of a rolling mill, such as gears, pinion stand, spindles and rolls, from maximum torque, the gear ratio should always be taken into account.



## CHAPTER 5

# ROLLS AND THEIR PERMISSIBLE LOADING

### 5.1. Rolls and Roll Strength Calculations

Rolls used in steelworks can be divided into four groups *A*, *B*, *C* and *D*, as shown in Table 5.1 [93–98].

*A. Common cast iron rolls* are cast in clay moulds. The structure is of perlitic grey cast iron. Covering the mould inside with a refractory clay lining gives semi-chilled smooth rolls. Refractory lined cooling segments are used to evoke more intensive local cooling for obtaining semi-chilled grooved rolls with local graphite dispersion in the basic structure, of greater hardness.

*B. Chilled cast iron rolls* are cast in cast iron moulds. The surface of the casting in contact with the mould wall is of white iron. Its thickness is chosen depending on roll requirements. The core is of grey iron with a dense, fine-grained structure able to withstand high bending stresses. There is an intermediate zone of mottled iron composed of perlite graphite and cementite between core and chilled layer.

Non-alloy cast iron rolls are sufficient for a hardness requirement of the chilled layer not exceeding 500 Brinell units. For a hardness of more than 500 Brinell alloyed rolls are used. Hardness up to 650–700 Brinell can be achieved by adding 1–2% Cr and 4–5% Ni to the roll material.

When increasing the silicon content of the roll material, semi-chilled rolls are obtained of a 5–10 mm thick chilled layer which is machined off.

*C. Duplex rolls.* Duplex rolls consisting of two different alloys are frequently made to obtain a roll with a very hard surface layer and a core relatively resistant to impact and bending stresses. Three different methods are used:

- (a) the roll sleeve consists of highly alloyed cast iron, the core is of common low-alloyed grey cast iron of high bending resistance,
- (b) the roll sleeve is of alloyed grey cast iron, the core is of common steel,
- (c) the roll sleeve is of alloy steel, the core is of non-alloy carbon steel.

In this way rolls of high surface hardness up to 100 Shore are produced, which are more resistant to abrasion than rolls made from only one material.

*D. Steel rolls* are made from plain or alloyed cast steel, or forged from plain or alloy steels. There are considerable differences in methods of determination of roll hardness, and it should be taken into account that when using the Shore method very great apparent discrepancies may occur if it is not noted which type of scleroscope has been used for the case considered. Figure 5.1 gives a comparison of measurements of Brinell hardness measured by scleroscopes of different types. Curve 1 has been plotted for a German scleroscope, curve 2 for a scleroscope of American design, curve 3 is from the Werkstoff-Handbuch (Stahl und Eisen 1940), curve 4 is for the Reindl and Niberding scleroscope, and curve 5 for the English Shore scleroscope. Roll hardness

TABLE 5.1  
COMPARISON OF DIFFERENT TYPES OF ROLLS

| Roll type                                                                              | Hardness of<br>the barrel<br>(Shore) | Hardness of<br>the barrel<br>(HB) | Chemical composition, % |                     |                   |                     |                     |     |     |     |
|----------------------------------------------------------------------------------------|--------------------------------------|-----------------------------------|-------------------------|---------------------|-------------------|---------------------|---------------------|-----|-----|-----|
|                                                                                        |                                      |                                   | C                       | Mn                  | Si                | P                   | S                   | Cr  | Mo  | Ni  |
| A. Cast iron rolls                                                                     |                                      |                                   |                         |                     |                   |                     |                     |     |     |     |
| (1) Ordinary soft cast iron rolls                                                      | 30                                   | 180-260                           | —                       | —                   | —                 | —                   | —                   | —   | —   | —   |
| (2) Semi-chilled rolls                                                                 | —                                    | 220-280                           | 2.2                     | 0.5                 | 0.5               | 0.2                 | 0.06                | —   | —   | —   |
| (a) plain cylindrical rolls                                                            | —                                    | 200-260                           | 3.0                     | 1.2                 | 1.3               | 0.3                 | 0.12                | —   | —   | —   |
| (b) section rolls                                                                      | —                                    | 200-260                           | 3.0                     | 1.2                 | 1.3               | 0.3                 | 0.12                | —   | —   | —   |
| B. Chilled rolls                                                                       |                                      |                                   |                         |                     |                   |                     |                     |     |     |     |
| (1) Plate mill rolls (thickness of the chilled layer<br>12-25 mm)                      | 55-65                                | 340-420                           | $\frac{2.6}{3.0}$       | $\frac{0.5}{0.8}$   | $\frac{0.5}{0.7}$ | $\frac{0.2}{0.4}$   | $\frac{0.07}{0.12}$ | —   | —   | —   |
| (2) Sheet mill rolls                                                                   |                                      |                                   |                         |                     |                   |                     |                     |     |     |     |
| (a) roughing trains                                                                    | 50-65 (German)<br>60-65 (English)    | 340-400                           | $\frac{2.6}{3.3}$       | $\frac{0.23}{0.65}$ | $\frac{0.6}{0.8}$ | $\frac{0.35}{0.55}$ | $\frac{0.07}{0.12}$ | —   | †   | —   |
| (b) finishing trains (thickness of the chilled layer<br>12-25 mm)                      | 60-70                                | 380-460                           | —                       | —                   | —                 | —                   | —                   | —   | —   | —   |
| (c) finishing trains for very thin sheets (thickness of<br>the chilled layer 15-20 mm) | 70-75 (German)<br>65-70 (English)    | 430-500                           | $\frac{2.7}{3.5}$       | $\frac{0.23}{0.65}$ | $\frac{0.6}{0.8}$ | $\frac{0.35}{0.55}$ | $\frac{0.07}{0.12}$ | —   | †   | —   |
| (d) skin passing mills (for special purposes)                                          |                                      |                                   |                         |                     |                   |                     |                     |     |     |     |
| type I                                                                                 | 70-75                                | 450-520                           | 2.75                    | 0.25                | 0.35              | 0.2                 | 0.05                | 0.5 | 0.5 | 1.5 |
| type II                                                                                | 85-95                                | 570-670                           | 3.5                     | 0.45                | 0.75              | 0.5                 | 0.15                | 1.5 | 1.0 | 6.0 |
| type III                                                                               | 95-100                               | 650-700                           | —                       | —                   | —                 | —                   | —                   | —   | —   | —   |
| (e) cooled rolls                                                                       | 65-70                                | 400-460                           | —                       | —                   | —                 | —                   | —                   | —   | —   | —   |
| (3) Section rolls                                                                      |                                      |                                   |                         |                     |                   |                     |                     |     |     |     |
| (a) cast with smooth barrel                                                            |                                      |                                   |                         |                     |                   |                     |                     |     |     |     |
| (b) with cast chilled passes                                                           |                                      |                                   |                         |                     |                   |                     |                     |     |     |     |
| roll diameter 250 to 300 mm                                                            | 65-70                                | 420-500                           | —                       | —                   | —                 | —                   | —                   | —   | —   | —   |
| roll diameter 300 to 600 mm                                                            | 55-60                                | 400-480                           | —                       | —                   | —                 | —                   | —                   | —   | —   | —   |

| Roll type                                                               | Hardness of the barrel (Shore) | Hardness of the barrel (HB) | Chemical composition, % |                   |                   |         |         |                   |    |                   |
|-------------------------------------------------------------------------|--------------------------------|-----------------------------|-------------------------|-------------------|-------------------|---------|---------|-------------------|----|-------------------|
|                                                                         |                                |                             | C                       | Mn                | Si                | P       | S       | Cr                | Mo | Ni                |
| (4) Polishing rolls for strip (thickness of the chilled layer 15–25 mm) |                                |                             |                         |                   |                   |         |         |                   |    |                   |
| (a) hot rolling                                                         | 70–75                          | 450–520                     | see B2c                 |                   |                   |         |         |                   |    |                   |
| (b) cold rolling special rolls (chilled layer 15–25 mm)                 |                                |                             |                         |                   |                   |         |         |                   |    |                   |
| quality I                                                               | 70–75                          | 450–520                     | see B2d                 |                   |                   |         |         |                   |    |                   |
| quality II                                                              | 85–95                          | 570–670                     |                         |                   |                   |         |         |                   |    |                   |
| quality III                                                             | 95–100                         | 650–700                     |                         |                   |                   |         |         |                   |    |                   |
| (5) Alloy chilled rolls (chilled layer up to 18 mm)                     | 60–68                          | —                           | 3.6                     | $\frac{0.2}{0.8}$ | $\frac{0.4}{0.7}$ | to 0.65 | to 0.12 | $\frac{0.4}{0.8}$ | —  | $\frac{0.8}{1.4}$ |
| (6) Semi-chilled rolls                                                  | —                              | 220–280                     | —                       | —                 | —                 | —       | —       | —                 | —  | —                 |
| (7) Chilled rolls with spheroidal graphite (chilled layer 10–18 mm)     | 38–35                          | —                           | 3.6                     | 0.5               | 1.2               | —       | —       | $\frac{0.4}{0.8}$ | —  | $\frac{0.8}{1.4}$ |

*C. Duplex Rolls*

|                               |        |        |                   |                   |                   |        |                   |                    |                     |      |
|-------------------------------|--------|--------|-------------------|-------------------|-------------------|--------|-------------------|--------------------|---------------------|------|
| (a) Cast iron rolls           |        |        |                   |                   |                   |        |                   |                    |                     |      |
| quality I                     | 86–92  | —      | —                 | —                 | —                 | —      | —                 | —                  | —                   | —    |
| quality II                    | 95–100 | —      | —                 | —                 | —                 | —      | —                 | —                  | —                   | —    |
| of alloyed chill iron sleeve  | —      | sleeve | 3.8               | 2.8               | 0.8               | —      | —                 | 1.5                | —                   | 2.5  |
| soft cast iron core           | —      | core   | $\frac{2.6}{3.1}$ | 0.8               | $\frac{1.0}{1.3}$ | —      | —                 | —                  | —                   | —    |
| (b) cast iron sleeve          | —      | sleeve | $\frac{3.8}{3.9}$ | 0.3               | $\frac{0.6}{0.7}$ | < 0.05 | $\frac{0.2}{0.3}$ | $\frac{1.2}{1.5}$  | $\frac{1.0}{4.0}$   | —    |
| steel core (0.6–0.7% C steel) | —      | core   | —                 | —                 | —                 | —      | —                 | —                  | —                   | —    |
| (c) steel sleeve              | 67–75  | sleeve | $\frac{0.5}{0.6}$ | $\frac{0.5}{0.6}$ | $\frac{0.2}{0.3}$ | < 0.04 | < 0.04            | $\frac{1.5}{1.75}$ | $\frac{0.35}{0.45}$ | —    |
| steel core                    | 50     | core   | $\frac{0.5}{0.6}$ | $\frac{0.5}{0.6}$ | $\frac{0.3}{0.4}$ | < 0.03 | < 0.03            | 0.75               | 0.75                | 0.75 |

TABLE 5.1 (cont.)

| Roll type                                     | Hardness of<br>the barrel<br>(Shore) | Hardness of<br>the barrel<br>(HB)               | Chemical composition, %                              |                     |                    |             |             |                    |                     |    |
|-----------------------------------------------|--------------------------------------|-------------------------------------------------|------------------------------------------------------|---------------------|--------------------|-------------|-------------|--------------------|---------------------|----|
|                                               |                                      |                                                 | C                                                    | Mn                  | Si                 | P           | S           | Cr                 | Mo                  | Ni |
| D. Steel rolls                                |                                      |                                                 |                                                      |                     |                    |             |             |                    |                     |    |
| (1) Cast steel rolls                          |                                      | Tensile<br>strength<br>$U_T$ kg/mm <sup>2</sup> |                                                      |                     |                    |             |             |                    |                     |    |
| (a) for blooming mills                        | —                                    | —                                               | 0.25                                                 | $\frac{0.60}{0.70}$ | 0.40               | max<br>0.05 | max<br>0.05 | —                  | —                   | —  |
| (b) roughing stands for heavy products        | —                                    | —                                               | 0.40                                                 | $\frac{0.60}{0.70}$ | 0.40               | max<br>0.05 | max<br>0.05 | —                  | —                   | —  |
| (c) finishing stands for heavy products       | —                                    | 60–80                                           | $\frac{0.5}{0.8}$                                    | $\frac{0.6}{0.8}$   | $\frac{0.2}{0.35}$ | max<br>0.06 | max<br>0.06 | —                  | —                   | —  |
| (d) roughing stands for small products        | 20–30                                | 50–60                                           | normal cast steel                                    |                     |                    |             |             |                    |                     |    |
| (e) alloyed cast steel rolls for cold rolling | 65–80                                | —                                               | chromium steel, Cr–Mo steel, Cr–Ni steel             |                     |                    |             |             |                    |                     |    |
| (f) cast carbon steel rolls with steel facing | —                                    | —                                               | Hard- and wear-resisting-steel electrodes for facing |                     |                    |             |             |                    |                     |    |
| (2) Forged steel rolls                        |                                      |                                                 |                                                      |                     |                    |             |             |                    |                     |    |
| (a) carbon steel rolls                        | —                                    | 60–80                                           | see D1abc                                            |                     |                    |             |             |                    |                     |    |
| (b) roughing stands of light-loaded stands    | 20–30                                | 50–60                                           | ordinary carbon steel                                |                     |                    |             |             |                    |                     |    |
| (c) alloy steels for work rolls               |                                      |                                                 |                                                      |                     |                    |             |             |                    |                     |    |
| roll barrel                                   | 90–100                               | —                                               | $\frac{0.75}{0.90}$                                  | $\frac{0.2}{0.4}$   | $\frac{0.25}{0.4}$ | max<br>0.03 | max<br>0.03 | $\frac{1.6}{2.1}$  | 0.2                 | —  |
| roll neck                                     | 45–55                                | —                                               | —                                                    | —                   | —                  | —           | —           | —                  | —                   | —  |
| (d) alloy steels for back-up rolls            | 65–75                                | —                                               | $\frac{0.75}{0.9}$                                   | $\frac{0.5}{0.8}$   | $\frac{0.2}{0.3}$  | max<br>0.03 | max<br>0.03 | $\frac{1.5}{1.75}$ | $\frac{0.35}{0.45}$ | —  |

† Often with addition of 0.25–0.40% Mo

NOTE: Shore and Brinell hardness values are only for orientation. An exact relation between Shore and Brinell hardness is impossible due to entirely different methods of measurement

may be measured on any of these hardness scales, but the chosen scale should be carefully noted and used exclusively [10].

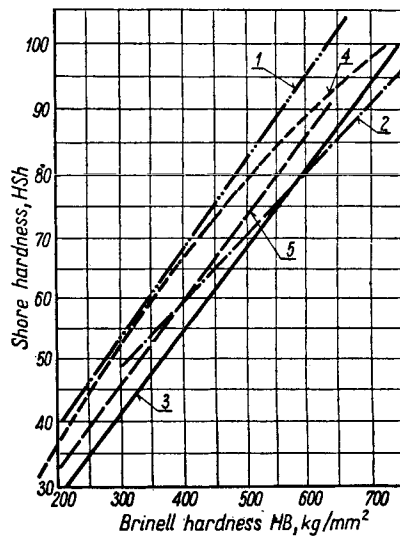


Fig. 5.1. Relation between roll hardness measured by means of different Shore scleroscopes and Brinell hardness [10]: 1 — German scleroscope, 2 — American scleroscope (Foundry Trade Journal 25.8.1938), 3 — "Baildon" steelworks tables, from Werkstoffhandbuch, Stahl und Eisen 1940, 4 — Reindle and Niberling scleroscope, 5 — English Shore scleroscope B10S No. 737

Hardness results measured by the German Schuchard-Schütte scleroscope differ considerably from those measured by the English one, due to the fact that 100 on the German Shore hardness scale corresponds approximately to 85 or 86 on the English scale.

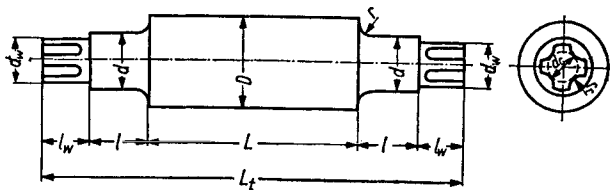


Fig. 5.2. Main dimensions of a roll [10]

The term hardness includes several properties, such as resistance to scratching, resistance to abrasion, resistance to deformation, high modulus of elasticity etc. A single hardness testing method does not give an estimate of all of these properties, but usually only one of them is checked. For example the Brinell test is a measure of the resistance to deformation of the material tested, elasticity and damping of elastic deformation capability are checked when using a scleroscope.

The range of application of different types of roll is given in Table 5.2.

Figure 5.2 shows the basic dimensions of a roll. The total length of the roll

$$L_t = L + 2l + 2l_w$$

where

$L$  = barrel length,

$l$  = length of neck,

$l_w$  = length of wobbler.

$D$  denotes the roll barrel diameter and  $d$  the diameter of the roll neck. The outside diameter of the wobbler is  $d_w$  and the diameter of the core is  $d_c$ .

TABLE 5.2  
TYPES OF ROLLS FOR VARIOUS ROLLING MILLS

| No. | Rolling mill                                                | Type of rolls                                                                                                                                                                                                                                                                                                                                                            |
|-----|-------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1   | Blooming mills                                              | Forged carbon steel or cast steel rolls, for smaller stands also cast iron rolls                                                                                                                                                                                                                                                                                         |
| 2   | Heavy product mills (rails, beams)                          | Forged steel rolls, semi-chilled cast iron rolls                                                                                                                                                                                                                                                                                                                         |
| 3   | Small section mills (rounds, squares, wire rod, flats etc.) | For roughing stands cast steel or forged rolls of 50–60 kg/mm <sup>2</sup> tensile strength, or cast steel rolls faced with 30 HGSA or EJ 707* electrodes. For intermediate stands spheroidal cast iron rolls, cast with plain cylindrical barrels. For finishing stands chilled rolls only. If alloyed rolls are used they should be cast with plain cylindrical barrel |
| 4   | Sheet bar mills                                             | For roughing stands, steel or semi-chilled rolls. For finishing stands chilled rolls of 55–60 Shore                                                                                                                                                                                                                                                                      |
| 5   | Plate mills                                                 | Chilled rolls (of lower hardness) or semi-chilled rolls                                                                                                                                                                                                                                                                                                                  |
| 6   | Sheet mills (uncooled rolls)                                | For roughing stands, chilled rolls (of lower hardness) and for finishing stands chilled rolls                                                                                                                                                                                                                                                                            |
| 7   | Skin passing mills for sheet                                | Hardened steel or chilled rolls of martensitic–ledeburitic structure                                                                                                                                                                                                                                                                                                     |
| 8   | Cold rolling mills                                          | As for 7                                                                                                                                                                                                                                                                                                                                                                 |

\* See Appendix 1.

TABLE 5.3  
RATIO OF BARREL LENGTH TO DIAMETER FOR VARIOUS ROLLING MILLS, FROM TSELIKOV [87]

| No. | Rolling mill type                                         | Ratio $L/D$ |
|-----|-----------------------------------------------------------|-------------|
| 1   | Blooming mills                                            | 2.2–2.7     |
| 2   | Roughing stands                                           | 2.2–3.0     |
| 3   | Section mills with shallow-grooved passes, lightly loaded | 2.9–3.2     |
| 4   | Section mills with deep-grooved passes, or heavily loaded | 1.5–2.5     |
| 5   | 2- and 3-high plate mills                                 | 2.2–2.8     |
| 6   | 2-high sheet mills                                        | 1.5–2.2     |
| 7   | 2-high mills for very thin sheets                         | 1.3–1.5     |
| 8   | Work rolls of 4-high mills                                | 2.5–4.0     |
| 9   | Back-up rolls of 4-high mills                             | 1.0–2.5     |

TABLE 5.4  
RELATIVE DIMENSIONS OF ROLL NECKS AND WOBBLERS FROM TSELIKOV [87]

| No. | Rolling mill type                                                            | Roll neck diameter<br>$d$ | Roll neck length<br>$l$ | Wobbler core diameter<br>$d_c$ | Wobbler length<br>$l_w$ |
|-----|------------------------------------------------------------------------------|---------------------------|-------------------------|--------------------------------|-------------------------|
| 1   | Blooming mills, also heavy, medium and fine product mills, rod and bar mills | $0.55 D$                  | $(0.92-1.2)d$           | $d-10-15 \text{ mm}$           | $(0.7-0.8)d_c$          |
| 2   | Plate mills                                                                  | $0.67 D$                  | $(0.83-1.0)d$           | $(0.9-0.94)d$                  | $(0.7-0.75)d_c$         |
| 3   | 2-high sheet mills                                                           | $0.75 D$                  | $0.8 d$                 | $0.85 d$                       | $0.7 d_c$               |
| 4   | Stands of continuous mills                                                   | $(0.57-0.65)D$            | $1.2 d$                 | $d-10 \text{ mm}$              | $0.75 d_c$              |

Generally the wobbler has four grooves, occasionally wobblers with more or less grooves are used.

Tables 5.3 and 5.4 show the basic roll proportions as in Fig. 5.2, i.e. ratios of barrel length to roll diameter and dimensions of roll necks and wobblers for rolls of different rolling mills.

Calculation of stresses occurring in a roll during rolling is essential for checking the rolling sequence, for roll pass design and pass scheduling and also provides a basis for rolling mill design.

These calculations are concerned firstly with determination of roll forces and torques occurring during rolling due to the action of the rolled stock on the rolls, and secondly with the phenomena of roll deformation, due to stress, which causes the rolls to deflect and flatten.

The methods generally used for strength calculations of rolls do not solve this problem correctly since they are based only on considerations of mechanical statics, and treat the rolls as elements under the action of forces which are constant with respect to time. Such an assumption seems to be approximately in agreement with practical conditions for many cases, specially for long passes, where there is only slight variation of forces and stresses during the pass. Generally the roll forces and stresses increase at the end of the pass for very long rolled bars due to cooling of the rolled stock, with consequent increase of resistance to deformation. This is usually associated with increase in thickness at the end of the rolled stock, resulting in additional increase of roll force and stresses. However force and stress vary only slightly with time, and therefore approximately static loading of the roll can be assumed.

Quite different conditions occur at the moment of bite of the rolled stock by the unloaded rolls at the beginning of the pass, and at the moment of delivery from rolls. These phenomena can often be compared with blows delivered against the great inertial mass of the roll.

The inertial mass or weight of rolls can be as much as some tens of tons. Forces acting upon a roll may be distributed along the greater part of the barrel or limited to a relatively small range. The forces which occur depend firstly on the rolling speed, and thus on the rotational speed of the rolls. The higher the value of rotational speed of the rolls the more violent is the impact, and the closer is the approach to dynamic loading. The second factor is the magnitude of the force which is acting; the greater

its magnitude, the greater is its influence. It often happens that due to dynamic loading, stresses arising during a short period of time cannot be fully transferred to the roll barrel and bearings. If these stresses are high, it is possible that the roll may break at the point of action of the force, i.e. at the barrel, sometimes at the point of maximum diameter, while other considerably weaker parts, e.g. roll necks, and particularly breaker blocks positioned under the adjusting screw and intended as safety devices, remain undamaged. Similar violent change from loaded to unloaded condition is experienced at the end of the pass. The dynamic theory of roll loading has not yet been investigated scientifically.

It may be questioned, why rolls are not calculated for alternating loading, since a roll is essentially an element which during operation is subjected to periodic alternating loading. This is usually in the form of unidirectional torsion stresses of varying magnitude, and alternating bending stresses also of varying magnitude. In reversible mills torsion stresses also become alternating. However, it should be taken into account that this stress arises during the pass only.

Although roll loading is a classic example of alternating loading, strength calculations are based only on static loading, since fatigue strength is not taken as a design factor.

Rolls must be periodically turned or dressed, due to gradual wear of the roll surface during rolling. This results in the reduction of the roll diameter, and the replacement of the roll due to reduction of working diameter below the allowable minimum becomes necessary earlier than would be the case if fatigue failure due to alternating forces were the criterion. This is one reason why fatigue strength is not considered as a design factor. Replacement of the roll due to reduction of diameter is usually the case in practice, though typical fatigue cracks can be met with before rolls are worn to their minimum diameter due to normal wear. Another reason why fatigue strength is not taken into account in strength calculations of rolls is the type of loading of their surface. There is some work-hardening of the roll material at the surface due to pressure acting between roll and rolled stock during cold or hot rolling. It is known that due to work-hardening during rolling, a fatigue strength of the metal two or even three times greater is obtained. For this reason the detrimental influence of craze cracking which always occurs on the roll surface is considerably reduced, and does not cause fatigue failure.

Forces arising during rolling are transferred by the roll neck bearings, to the adjusting screw and roll housing. As is known, the roll force  $P$  (Fig. 5.3) depends on the following factors:

- (a) the projected area of contact between roll and stock  $F_d$ ,
- (b) the resistance to deformation of the rolled stock  $K_{wm}$ , which is generally assumed to be a constant mean value along the projected arc of contact (Fig. 4.3). Hence, as in formula (2.108)

$$P = F_d K_{wm}$$

This relation takes into account only vertical components of roll pressure, horizontal components being neglected. These horizontal components are taken into account only when rolling sheet and strip where tension due to external forces causes an additional increase in horizontal stresses.

Roll deflection should also be considered (Fig. 5.4). Due to the deflection of rolls under the action of roll force  $P$ , in the extreme edge layers of the roll, denoted by circles (1 and 4), tensile stresses arise, while the internal layers, denoted as (3 and 6), are under



compression. The circle (2-5) represents the neutral line of the cross-section. The neutral line of the roll is neither under the action of tension, nor of compression.

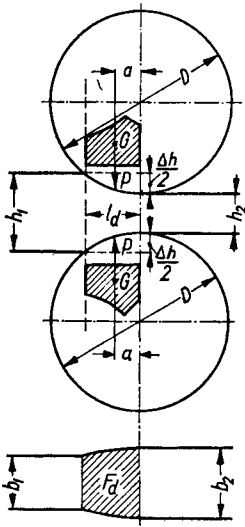


**Fig. 5.3. Vertical pressure distribution on a roll**

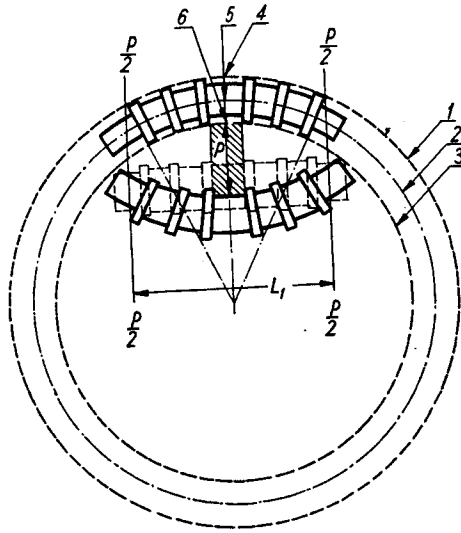


**Fig. 5.4. Roll deflection (exaggerated) [22]**

Assuming the simplest case when rolling a narrow bar at the barrel centre, then each roll neck or bearing is under the action of a force  $P/2$  (Fig. 5.5). To determine bending stresses and deflection of the roll, a half of it is considered to be a cantilever

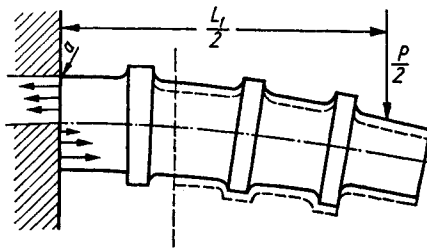


**Fig. 5.5. Deflection of a half roll [22]**

fixed at the barrel centre and loaded with a force of  $P/2$  acting at its end [22]. The bending moment at the fixed cross-section is equal to

$$M_b = \frac{P}{2} \frac{L_1}{2} = \frac{PL_1}{4} \quad (5.1)$$

where

$P$  = roll force acting between roll and stock,

$L_1$  = length of loaded roll measured from half neck length.

The maximum bending stress in the roll barrel occurs at the extreme layer and is equal to

$$\sigma_b = \frac{M_b}{W_b} \quad (5.2)$$

where  $W_b$  = sectional bending modulus.

For a round cross-section the moment of inertia is

$$I = \frac{\pi D^4}{64} \quad (5.3)$$

where  $D$  = roll diameter at a given point.

Therefore, the stress arising in the extreme fibres at a distance of  $D/2$  from the neutral line is

$$\sigma_b = \frac{M_b \frac{D}{2}}{\frac{\pi D^4}{64}} = \frac{M_b}{\frac{\pi D^3}{32}} \approx \frac{M_b}{0.1 D^3} = \frac{10 M_b}{D^3} \quad (5.4)$$

where  $\pi D^3/32$  = bending modulus of a round cross-section.

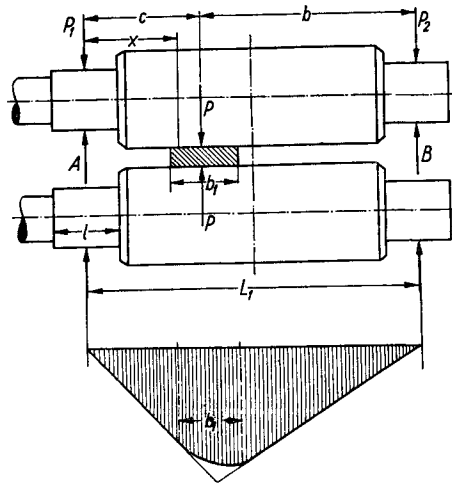


Fig. 5.6. Forces and bending moment acting on a roll when rolling flats of width  $b_1$

If rolling does not take place at the barrel centre, but to one side (Fig. 5.6), then the roll necks are unequally loaded [22]. It is generally assumed that the bearing force acts at the centre of the roll neck of length  $l$ . Trinks recommends assuming the bearing force acting at a distance of  $l/3$  from the end of the roll barrel [22]. The roll force  $P$  is resolved into two bearing forces on the necks,  $P_1$  and  $P_2$ . These forces  $P_1$  and  $P_2$

can be calculated since their moments about the point of action of  $P$  are equal, therefore if the distance between lines of action of  $P$  and  $P_1$  is  $c$  and between  $P$  and  $P_2$  is  $b$ , then

$$P_1 = A = \frac{Pb}{L_1} \quad (5.5)$$

$$P_2 = B = \frac{Pc}{L_1} \quad (5.6)$$

where  $A$  and  $B$  denote the vertical reactions at the bearings.

If the roll load is not concentrated at a point, but uniformly distributed along a width of  $b_1$ , as shown in Fig. 5.6, then the triangular B.M. diagram as for concentrated loading becomes rounded off at its peak. The distance between the apex of the triangle and the top of the parabolic diagram for distributed loading over width  $b_1$ , represents the decrease in maximum B.M.

In the case of a single roll force  $P$  (Fig. 5.6), the bending moment due to bearing reaction  $A$  about point of action of  $P$  is

$$M_b = Ac = P \frac{bc}{L_1} \quad (5.7)$$

In the case of a roll force uniformly distributed along a width  $b_1$ , from Fig. 5.6, the bending moment is

$$M_b = P \left( \frac{bc}{L_1} - \frac{b_1}{8} \right) \quad (5.8)$$

The bending moment at any point of the barrel at a distance  $x$  from the line of action of  $P_1$  can be determined as follows (Fig. 5.6):

(a) for  $x \leq c - b_1/2$

$$M_b = Ax = \frac{Pb}{L_1} x \quad (5.9)$$

(b) for  $x = c$

$$M_b = Ac = P \frac{b_1}{8} \quad (5.10)$$

(c) for  $x \geq c + b_1/2$  from Fig. 5.6

$$M_b = B(L_1 - x) = \frac{Pc}{L_1} (L_1 - x) \quad (5.11)$$

In the case where the barrel is not plain cylindrical but grooved, there is an increase in bending moment due to concentration of stresses depending on the fairing radii, and the bending moment equation becomes

$$M_b = 0.1 D^3 \sigma_b \alpha_k \quad (5.12)$$

where  $\alpha_k$  = coefficient taking into account stress concentrations and depending on  $D/d$  (the ratio of diameters of adjacent sections) and  $r/d$  (the rounding radius divided by the smaller diameter). Values of coefficient  $\alpha_k$  for different cases calculated by Geleji [99] can be read off from Figs. 5.7–5.10.

However, results obtained from calculations of bending stress give values that are too high, and sometimes exceed the strength of the roll material. The reason for this

is often the incorrect choice of the form coefficient  $\alpha_k$ . The simpler calculation method from equation (5.7) is often used for grooved rolls.

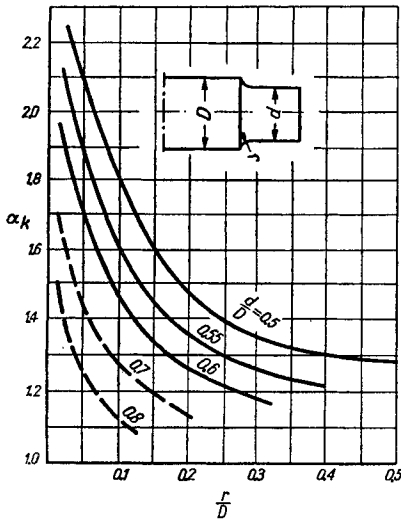


Fig. 5.7. Form factor  $\alpha_k$  for bending stresses in a roll with varying diameter [99]

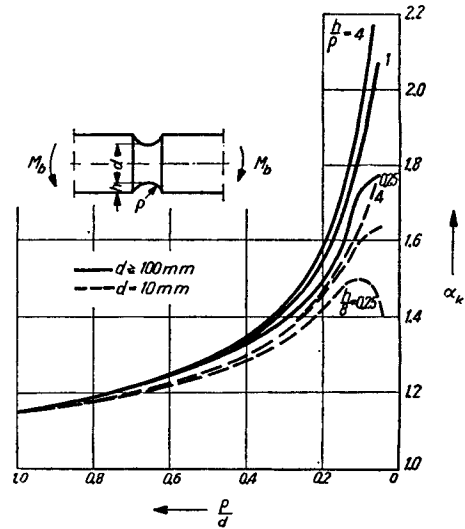


Fig. 5.8. Form factor  $\alpha_k$  for bending stresses in a grooved roll [99]

Roll force and bending moment in each groove and pass must be determined when using grooved rolls. The maximum bending moment  $M_{\max}$  gives the design loading for the calculation of roll diameter. Roll neck loading must also be considered, taking into account bending and torsional moments.

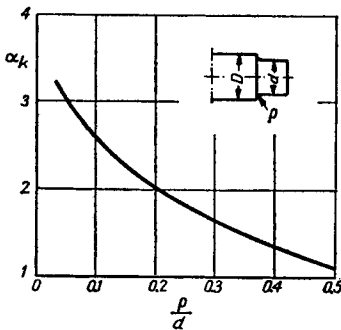


Fig. 5.9. Form factor  $\alpha_k$  for bending stresses in a roll neck [99]

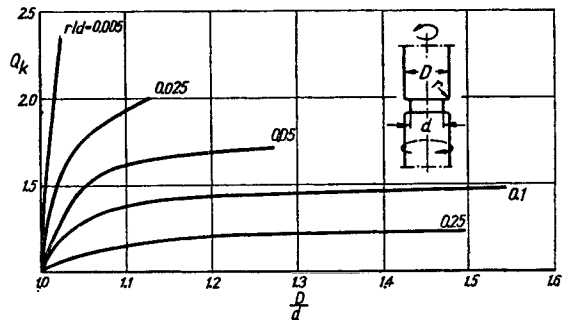


Fig. 5.10. Form factor  $\alpha_k$  for torsion stresses in a grooved roll [99]

The roll neck loading is calculated similarly as for the barrel, but for the calculation of bending stress a concentrated loading on the neck is assumed. When the roll force  $P$  is acting at the barrel centre, the roll neck is acted on by a force  $P/2$ . This force is acting as the neck centre (Fig. 5.6). Hence

$$M_{bn} = \frac{P}{2} \frac{l}{2} = \frac{Pl}{4} \quad (5.13)$$

where  $l$  = length of the roll neck.

If the force  $P$  is acting at an arbitrary point on the barrel then there are unequal bending moments at the necks. Reactions  $A$  and  $B$  can be calculated (Fig. 5.6) and hence bending moments at the necks, i.e.

$$M'_{bn} = A \frac{l}{2} \quad (5.14)$$

and

$$M''_{bn} = B \frac{l}{2} \quad (5.15)$$

Therefore bending stresses produced at the necks are given by

$$\sigma_{bn} = \frac{M_{bn}}{0.1 d^3} \quad (5.16)$$

where  $d$  = diameter of the roll neck.

If the stress concentrations due to variations of section are to be taken into account the equation (5.16) becomes

$$\sigma_{bn} = \frac{M_{bn}}{0.1 d^3} \alpha_k \quad (5.17)$$

where  $\alpha_k$  = form coefficient for the stress concentration at the point at which roll barrel and neck join, equal to 1.3 for  $r/d = 0.1-0.15$  according to Tselikov [87] (or from Fig. 5.9).

It should be remembered that the form coefficient does not give the maximum bending stress which will occur at some point of roll or neck depending on variations of section, it indicates by how much the bending stresses at any point are increased above the directly calculated value, i.e.

$$\sigma_{\max} = \alpha_k \sigma$$

The roll is not only subject to bending stresses due to roll force, but also to torsional stresses due to roll torque. On the roll barrel, the torsion moment is equal to the roll torque (Fig. 5.3) i.e.

$$M_t = M_w = Pa$$

where

$P$  = roll force,

$a$  = roll torque lever arm, determined from equations (4.79) to (4.81).

Frequently  $a \approx l_d/2$  is assumed, and in this case roll torque equals

$$M_t = M_w \approx P \frac{l_d}{2} \approx 0.5 P \sqrt{R \Delta h}$$

This torque produces torsional stresses in the roll barrel usually considerably smaller than bending stresses, and therefore often neglected in roll strength calculations. They can be taken into account using the relation

$$\tau = \frac{M_t R}{\text{moment of inertia of cross-section}} \approx \frac{M_t \frac{D}{2}}{0.1 D^4} \approx \frac{M_t}{0.2 D^3} \quad (5.18)$$

For grooved rolls, the stress concentration, due to variation of cross-section should be taken into account (Fig. 5.10)

$$\tau \simeq \frac{M_t}{0.2D} \alpha_k \quad (5.19)$$

The torque acting on the roll neck can be calculated similarly, the frictional torque at the neck being added to roll torque. The frictional torque at one roll neck is

$$M_{fn} = \frac{Rf_1 d}{2 \times 2} \quad (5.20)$$

Hence the total torque at the roll neck is

$$M_{tn} = P \left( a + \frac{f_1 d}{2} \right) \quad (5.21)$$

since the roll neck must transmit both bearing frictional torques, i.e.  $2(P/2)f_1(d/2)$ .

Assuming the torque lever arm of the roll force to be  $a \simeq l_d/2$ , then

$$M_{tn} = \frac{P}{2} (\sqrt{R \Delta h} + f_1 d) \quad (5.22)$$

For the considered case the direction of roll force  $P$  is not taken into account, i.e. not only the vertical component but total roll force should be substituted.

When rolling several passes in one groove the maximum force  $P_{\max}$  is used for calculations. The torsional stress in the roll neck is calculated as for the roll barrel

$$\tau_n = \frac{M_{tn}}{0.2d^3} \quad (5.23)$$

If the roll barrel or the neck are subjected to simultaneous bending and torsional moments, then the equivalent moment used for strength calculations is for steel rolls

$$M'_b = \sqrt{M_b^2 + 0.75 M_t^2} \quad (5.24)$$

and for cast iron rolls

$$M''_b = 0.35 M_b + 0.65 \sqrt{M_b^2 + M_t^2} \quad (5.25)$$

Corresponding formulae for calculation of compound stresses under simultaneous action of bending and torsion are:

for steel rolls

$$\sigma'_b = \sqrt{\sigma_b^2 + 3\tau^2} \quad (5.26)$$

and for cast iron rolls

$$\sigma''_b = 0.375 \sigma_b + 0.625 \sqrt{\sigma_b^2 + 4\tau^2} \quad (5.27)$$

The roll is considered as a tool, therefore considerable wear occurs during both hot and cold rolling. Rolls are usually replaced on account of excessive wear before achieving the number of stress cycles that would cause fatigue failure. The diameters of all section and sheet mill rolls and work rolls of cold rolling stands vary from a value  $D_p$  (for a new roll i.e. for primary diameter) to a value  $D_d$  (for a worn out roll i.e. for dead diameter). Permissible values of ratio  $D_d/D_p$  for turning or dressing of rolls are

given at the beginning of Chapter 7. All strength calculations should be based on minimum roll diameters, i.e.  $D_d$  at which the roll is finally removed and scrapped.

The difference between the roll diameters  $D_p$  and  $D_d$  assumed for calculations represents a certain reserve which allows the rolling technologist to estimate appropriate draughts in particular passes. For all rolls, combined bending and torsional stresses arising during the pass are considered as design cases. Fatigue cracks must be expected in back-up rolls of cold mills and hot mills or in soft steel rolls faced with hard steel electrodes. For these rolls values given in Table 5.5 are valid. Fatigue cracks will not be dealt with here.

TABLE 5.5  
PERMISSIBLE STRESSES

| Material of rolls (acc. to SES)* | $U_T$<br>kg/mm <sup>2</sup> | $Y_P$<br>kg/mm <sup>2</sup> | Permissible stresses, kg/cm <sup>2</sup> |                                               |                                             |
|----------------------------------|-----------------------------|-----------------------------|------------------------------------------|-----------------------------------------------|---------------------------------------------|
|                                  |                             |                             | $\sigma_{per}$<br>static                 | $\sigma'_{per}$<br>variable<br>unidirectional | $\sigma''_{per}$<br>variable<br>alternating |
| St70<br>Cast steel               | 70–85                       | 35–44                       | 2100–2500                                | 1400–2000                                     | 1150–1700                                   |
| St45                             | 45–52                       | 22–28                       | 1100–1600                                | 750–1000                                      | 600–850                                     |
| St52                             | 52–60                       | 25–32                       | 1300–1800                                | 800–1100                                      | 700–950                                     |
| St60<br>Cast iron                | 60–70                       | 28–80                       | 1400–2000                                | 900–1300                                      | 800–1100                                    |
| 14                               | 11–16                       | —                           | 400–550                                  | 300–450                                       | 250–300                                     |
| 18                               | 15–20                       | —                           | 500–700                                  | 400–550                                       | 250–350                                     |

\* See Appendix 1.

Roll wobblers are calculated for torsion only, and for strength calculations the same torque as for the roll neck is used. The torsional stress is equal to

$$\tau = \frac{M_t}{0.07 d_w^3} \quad (5.28)$$

where the core diameter of the wobbler  $d_c = 0.66 d_w$  (Fig. 5.2).

The torsional stress from (5.28) should not exceed [100]:

(a) for reversing mills with cast iron rolls 160 kg/cm<sup>2</sup>, and with steel rolls 720 kg/cm<sup>2</sup>,

(b) for non-reversing mills with cast iron rolls 320 kg/cm<sup>2</sup>, and with steel rolls 860 kg/cm<sup>2</sup>.

Strength calculation of rolls should be based on permissible stresses given in Table 5.6. These values correspond to a safety factor of  $s = 5$ , i.e. permissible stress  $\sigma_{per} = U_T/5$ , where  $U_T$  is the ultimate tensile strength of the material. This assumption of permissible stress is used for steel rolls due to lack of direct data for the value of the yield point  $Y_P$ . Permissible shear stress is assumed as  $0.75 \sigma$ , i.e. permissible torsional stress is equal to  $\tau_{per} = 0.75 \sigma_{per}$ .

For more recent strength calculations, a value of permissible stress relative to yield point is quoted with a safety factor of  $s = 1.4$  for static loading.

The fatigue limit of almost all engineering materials differs little from the yield point which is assumed related to  $U_T$  the ultimate tensile stress.

The permissible static tensile stress is assumed proportional to the yield point.

In the case of periodic alternating forces, strength calculations should be based on fatigue strength, to which the permissible stress will be related. Denoting the permissible stress as  $\sigma_{\text{per}}$  (Table 5.5), then

for static loading

$$\sigma_{\text{per}} = \frac{Y_P}{s} = \frac{Y_P}{(1.3-2.0)} \quad (5.29)$$

for variable unidirectional loading

$$\sigma'_{\text{per}} = \frac{\sigma_{wI}}{s} = \frac{\sigma_{wI}}{(2.0-3.0)} \quad (5.30)$$

for variable alternating loading

$$\sigma''_{\text{per}} = \frac{\sigma_{wII}}{s} = \frac{\sigma_{wII}}{(2.0-3.0)} \quad (5.31)$$

where  $s$  is the safety factor, chosen according to loading conditions.

TABLE 5.6

PERMISSIBLE STRESSES FOR ROLLS FROM TRINKS, TSELIKOV AND K. OCHĘDUSZKO [10]

| Material of rolls               | $U_T$<br>kg/mm <sup>2</sup> | Permissible<br>bending stress<br>$\sigma_{b\text{per}}$<br>kg/mm <sup>2</sup> | Young's<br>modulus<br>$E$<br>kg/mm <sup>2</sup> | Modulus of<br>rigidity<br>$G$ †<br>kg/mm <sup>2</sup> |
|---------------------------------|-----------------------------|-------------------------------------------------------------------------------|-------------------------------------------------|-------------------------------------------------------|
| Soft cast iron                  | 11-16                       | 250-300                                                                       | 8500                                            | 3360                                                  |
| Chilled iron                    | 17-21                       | 600-750                                                                       | 8500-10,000                                     | 3360-4000                                             |
| Semi-chilled iron               | 18-25                       | 600-750                                                                       | 7500-9000                                       | 3000-3600                                             |
| Molybdenum alloyed chilled iron | 35-40                       | 600-750                                                                       | 10,000-14,000                                   | 4000-5600                                             |
| Alloyed cast iron               | 21-28                       | 600-750                                                                       | 9000-14,000                                     | 3600-5600                                             |
| Cast steel                      | 50-60                       | 1000-1200                                                                     | 20,000                                          | 7700                                                  |
| Soft carbon steel               | 32-45                       | 600-900                                                                       | 20,000                                          | 7700                                                  |
| Carbon steel of medium hardness | 50-60                       | 1000-1200                                                                     | 21,000                                          | 8100                                                  |
| Hard carbon steel               | 60-70                       | 1200-1400                                                                     | 21,500                                          | 8500                                                  |
| Alloyed steel                   | 80-90                       | 1600-1800                                                                     | 22,000                                          | 8600                                                  |
| High-alloyed steel              | > 100                       | > 2000                                                                        | 22,500                                          | 8700                                                  |

†  $G$  is calculated from the formula  $G = \frac{E}{2(1+\mu)}$ , where  $\mu$  = Poisson's number: for steel  $\mu = 0.25-0.3$ , for cast iron  $\mu = 0.23-0.27$ .

For brittle materials, e.g. grey cast iron, it is assumed that  $Y_P \approx U_T$ . Values of permissible stresses are given in Table 5.5, according to Tauscher [101]. Higher values for cast steel should be used for metal in a fully annealed condition. For cast iron the higher values correspond to smaller cross-sections, and the lower ones for larger sections approximating to roll cross-sections. The temperature should also be taken into account. Values given in Table 5.5. correspond to 20°C. The roll strength can suddenly decrease when heating rolls to a higher temperature. Values of permissible stresses given in Table 5.5 do not correspond to true loading conditions of rolls, which are subject to compound stresses consisting of variable alternating bending stresses and variable unidirectional torsional stresses, except for rolls of reversible mills, for which smaller values of  $\sigma_{wII}$  are valid.



Permissible shear stresses can be assumed as in Table 5.5:  
for steel

$$\tau_{\text{per}} = 0.6-0.65\sigma'_{\text{per}}$$

for cast iron

$$\tau_{\text{per}} = 1.0\sigma'_{\text{per}}$$

According to Tselikov [59], the permissible loading of roll necks in most cases is determined by bending strength, where the allowable load  $P$  is such that

$$P = \frac{W\sigma_{b\text{per}}}{c-r} \quad (5.32)$$

where

- $W$  = bending section modulus of the roll neck,
- $\sigma_{b\text{per}}$  = permissible bending stress of material of roll neck,
- $c$  = distance between centre of screw-down and edge planes of roll barrel,
- $r$  = fillet radius between roll neck and barrel.

From this formula,  $P$  can be expressed as a function of neck diameter. Expressing values of  $W$ ,  $c$  and  $r$  relative to neck diameter  $d$ , i.e.  $c = 0.55d$  and  $r = 0.1d$ , as usually assumed for practical purposes, then

$$P \approx 0.22d^2\sigma_{b\text{per}} \quad (5.33)$$

Therefore, for cast iron rolls where permissible bending stress is  $\sigma_{b\text{per}} = 700 \text{ kg/cm}^2$ , the maximum neck force  $P$  is

$$P \approx 0.154d^2 \quad \text{tons} \quad (5.34)$$

For cast steel rolls where permissible bending stress  $\sigma_{b\text{per}} = 1200 \text{ kg/cm}^2$ , the neck force  $P$  is

$$P \approx 0.264d^2 \quad \text{tons} \quad (5.35)$$

where  $d$  is expressed in cm.

According to Emicke [102] the mean bearing pressure on the bearing surface of neck should not exceed 200–230  $\text{kg/cm}^2$ . This value is for all bearings materials, i.e. for metal and plastic bearings.

W. Henky [102] gives a permissible bearing pressure of 90–100  $\text{kg/cm}^2$  for plastic bearings at a peripheral speed of neck of 3.5 m/sec. This value increases to 120–250  $\text{kg/cm}^2$  for an increase in peripheral speed of the neck up to 5.5 m/sec.

Considerably higher values of permissible bearing pressure are given by Tselikov [87], e.g.  $p = 150-500 \text{ kg/cm}^2$ , and values of  $p v_n = 400-2000 \text{ kg/cm}^2$ , where  $v_n$  is the peripheral speed of neck in m/sec, assuming efficient cooling of the roll necks.

If the total allowable roll force is calculated from the maximum allowable bending stress, Emicke has shown that the bearing load may be a design case limiting the rolling programme. It may be seen from Fig. 5.11 that a permissible stress of  $\sigma_{b\text{per}} = 6 \text{ kg/mm}^2$  gives a total roll force of 900 t, which corresponds to a specific bearing pressure of 225  $\text{kg/cm}^2$ , i.e. the limiting value.

#### Example [103]

The strength of rolls shown in Figs. 5.12 and 5.13 is to be calculated numerically and graphically. Ingots of  $575 \times 575 \times 1500 \text{ mm}$  are rolled down to blooms of  $210 \times 188 \text{ mm}$  cross-section in 15 passes. The chemical composition of the rolled steel is as follows: 0.14% C, 0.45% Mn, 0.045% P and 0.045% S.

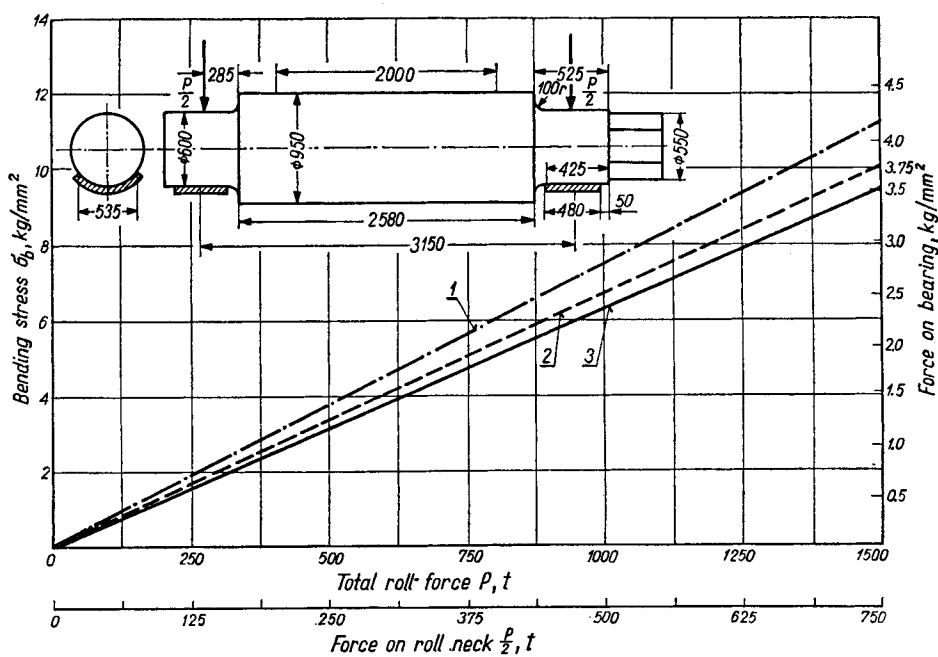


Fig. 5.11. Bending stresses along roll barrel and neck, and bearing pressure of a cast iron roll of 950 mm diameter, when rolling plate [102]: 1 — effective stress  $\sigma_2$  at the roll neck for a torsional stress of  $\tau = 500 \text{ kg/mm}^2$  at the wobbler, 2 — bending stress at the roll neck  $\sigma_b$ , 3 — bending stress on the roll barrel  $\sigma'_b$

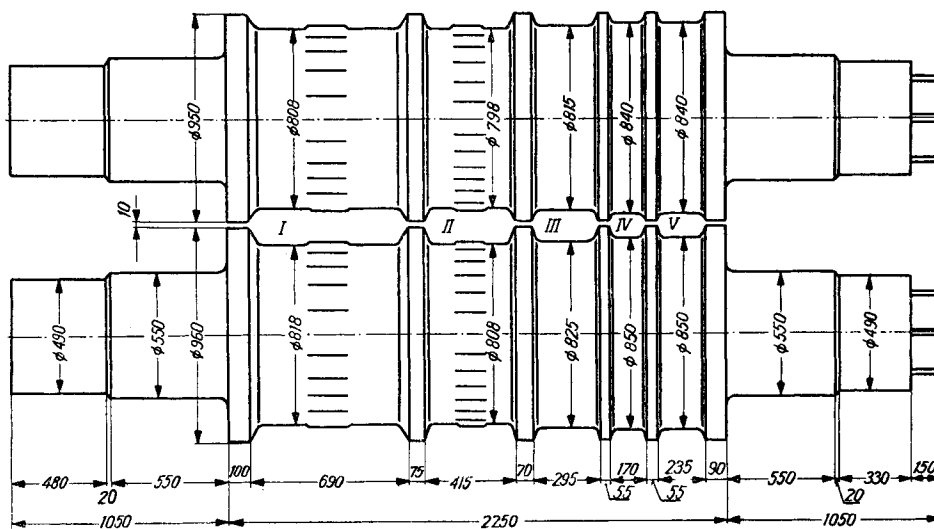


Fig. 5.12. Roll set of a blooming mill for rolling square and flat blooms

The pass schedule and corresponding dimensions are given in Table 5.7. The rolls are cut with five box passes for the complete rolling cycle. For the rolling of blooms of  $210 \times 188$  mm three passes are sufficient with the working diameter equal to 808, 798 and 815 mm. Underdraught is applied, i.e. the bottom roll has a diameter 10 mm greater than the top one.

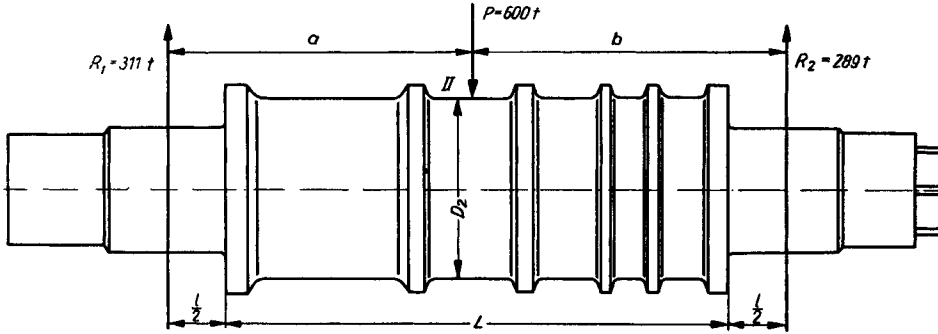


Fig. 5.13. Roll loading for stress calculation

Bite angles for individual passes are found from formula (3.9)

$$\cos \alpha = 1 - \frac{\Delta h}{D}$$

The values of the length of projected arc of contact  $l_d$  are found from equation

$$l_d \approx \sqrt{R \Delta h}$$

Values of projected area of contact given in the next column in Table 5.7, were determined as follows:

$$F_d = \frac{2b_2 + b_1}{3} l_d$$

Mean resistance to deformation  $K_{\text{wm}} = P/F_d$ . The maximum calculated roll force is 600 t and occurs during the eighth pass.

The bearing force is calculated from

$$A = R_l = \frac{Pb}{a+b} \quad (5.36)$$

and the maximum value occurs for the 4th pass.

The maximum bending moment

$$M_b = Ac = 392 \text{ tm}$$

occurs during pass No. 8, and is calculated from equation (5.7).

The section modulus of the roll for a mean diameter of 800 mm is equal to

$$W_b = \frac{\pi D^3}{32} \approx 0.1 D^3 = 51,200 \text{ cm}^3$$

According to equation (5.4), the bending stress in the roll barrel at pass No. 8 is therefore

$$\sigma_b = \frac{M_b}{0.1 D^3} = \frac{392 \times 10^5}{0.1 \times 80^3} = 765 \text{ kg/cm}^2$$

St60 forged steel rolls are used. The permissible stress for this material under variable alternating loading is 1050 kg/cm<sup>2</sup> according to Table 5.5. The smaller of the two values was chosen because of the relatively large cross-section.

The form coefficient  $\alpha_k$  allowing for stress concentration due to variation in cross-section, is not taken into account, as this problem is not yet sufficiently clarified for rolls.

The maximum roll force of 600 t occurs during pass No. 8 at groove II, as shown in Table 5.7.

TABLE  
PASS SCHEDULE FOR ROLLING OF 575 × 575 mm

| Pass groove No. | Pass No. | Width<br>mm | Thick-ness<br>mm | Absolute draught<br>mm | Draught coefficient<br>$\gamma$ | Cross-section<br>mm <sup>2</sup> | Length<br>m | Coeffi-<br>cient<br>of<br>elonga-<br>tion<br>$\lambda$ | Roll<br>diameter<br>mm |
|-----------------|----------|-------------|------------------|------------------------|---------------------------------|----------------------------------|-------------|--------------------------------------------------------|------------------------|
| I               | —        | 575         | 575              | —                      | —                               | 330,500                          | 1.55        | —                                                      | —                      |
|                 | 1        | 600         | 520              | 55                     | 0.9043                          | 312,000                          | 1.64        | 1.0593                                                 | 810                    |
|                 | 2        | 610         | 470              | 50                     | 0.9038                          | 286,700                          | 1.79        | 1.0882                                                 | 810                    |
|                 | 3        | 620         | 410              | 60                     | 0.8728                          | 254,200                          | 2.00        | 1.1279                                                 | 810                    |
|                 | 4        | 630         | 360              | 50                     | 0.8780                          | 226,800                          | 2.25        | 1.1208                                                 | 810                    |
| II              | 5        | 380         | 520              | 110                    | 0.8254                          | 197,600                          | 2.60        | 1.1478                                                 | 800                    |
|                 | 6        | 400         | 460              | 60                     | 0.8846                          | 184,000                          | 2.80        | 1.0739                                                 | 800                    |
|                 | 7        | 413         | 370              | 90                     | 0.8043                          | 152,810                          | 3.35        | 1.2041                                                 | 800                    |
|                 | 8        | 390         | 320              | 93                     | 0.8649                          | 124,800                          | 4.11        | 1.2244                                                 | 800                    |
| III             | 9        | 405         | 270              | 50                     | 0.8438                          | 109,350                          | 4.70        | 1.1413                                                 | 800                    |
|                 | 10       | 290         | 300              | 105                    | 0.7407                          | 87,000                           | 5.90        | 1.2568                                                 | 820                    |
|                 | 11       | 310         | 230              | 70                     | 0.7667                          | 71,300                           | 7.22        | 1.2202                                                 | 820                    |
|                 | 12       | 240         | 230              | 80                     | 0.7419                          | 55,200                           | 9.32        | 1.2917                                                 | 820                    |
|                 | 13       | 250         | 205              | 25                     | 0.8913                          | 51,200                           | 10.00       | 1.0771                                                 | 820                    |
|                 | 14       | 215         | 205              | 45                     | 0.8200                          | 44,000                           | 11.65       | 1.1648                                                 | 820                    |
|                 | 15       | 210         | 188              | 27                     | 0.9171                          | 39,500                           | 12.82       | 1.1139                                                 | 820                    |

From the figures the following roll dimensions are found:

$$L = 2250 \text{ mm} \quad a = 1350 \text{ mm}$$

$$l = 550 \text{ mm} \quad b = 1450 \text{ mm}$$

$$L_1 = L + 2 \frac{l}{2} = a + b = 2800 \text{ mm}$$

The reactions  $R_1$  and  $R_2$  at both bearings  $A$  and  $B$  can be found

$$R_1(L + l) = Pb$$

and

$$R_1 = \frac{600 \times 1450}{2800} = 311 \text{ t}$$

$$R_2 = \frac{600 \times 1350}{2800} = 289 \text{ t}$$

The bending moment at the critical cross-section is (5.7)

$$M_b = \frac{600,000 \times 145 \times 135}{280} = 41,985,000 \text{ kgcm}$$

and the roll torque

$$M_d = M_w = Pa = 600,000 \times \frac{18.8}{2} = 563,000 \text{ kgcm}$$

Then from equation (5.24) the compound moment is

$$M'_c = \sqrt{M_b^2 + 0.75 M_d^2} = \sqrt{41,985,000^2 + 0.75 \times 563,000^2} = 42,012,000 \text{ kgcm}$$

5.7

INGOTS TO 210×188 mm BLOOMS [103]

| Bite angle<br>$\alpha$ | Projected length of arc of contact<br>mm | Projected area of contact<br>mm <sup>2</sup> | $K_{\psi m}$<br>kg/mm <sup>2</sup> | Roll force<br>$P$<br>t | Reaction at the roll neck<br>kg | Bending moment<br>tm | Bending modulus<br>cm <sup>3</sup> | Bending stress in the roll barrel<br>kg/cm <sup>2</sup> |
|------------------------|------------------------------------------|----------------------------------------------|------------------------------------|------------------------|---------------------------------|----------------------|------------------------------------|---------------------------------------------------------|
| —                      | —                                        | —                                            | —                                  | —                      | —                               | —                    | —                                  | —                                                       |
| 21°26'                 | 148                                      | 86,876                                       | 5.0                                | 434.4                  | 330,000                         | 208                  | 53,140                             | 391                                                     |
| 20°15'                 | 140                                      | 84,700                                       | 5.4                                | 457.4                  | 347,000                         | 219                  | 53,140                             | 412                                                     |
| 22°13'                 | 153                                      | 94,095                                       | 6.0                                | 564.6                  | 419,000                         | 270                  | 53,140                             | 508                                                     |
| 20°15'                 | 140                                      | 87,500                                       | 6.4                                | 560.0                  | 415,000                         | 268                  | 53,140                             | 504                                                     |
| 30°24'                 | 202                                      | 74,740                                       | 6.9                                | 515.7                  | 268,000                         | 338                  | 51,200                             | 660                                                     |
| 22°11'                 | 151                                      | 58,890                                       | 7.4                                | 435.8                  | 226,000                         | 285                  | 51,200                             | 556                                                     |
| 27°26'                 | 184                                      | 74,880                                       | 7.9                                | 591.5                  | 307,000                         | 387                  | 51,200                             | 750                                                     |
| 28°3'                  | 188                                      | 71,440                                       | 8.4                                | 600.0                  | 311,000                         | 392                  | 51,200                             | 755                                                     |
| 20°22'                 | 139                                      | 55,180                                       | 8.9                                | 491.1                  | 255,000                         | 321                  | 51,200                             | 625                                                     |
| 29°28'                 | 201                                      | 56,280                                       | 9.4                                | 529.0                  | 189,000                         | 317                  | 55,136                             | 570                                                     |
| 23°51'                 | 165                                      | 49,500                                       | 10.0                               | 495.0                  | 177,000                         | 297                  | 55,136                             | 538                                                     |
| 25°31'                 | 176                                      | 41,360                                       | 10.4                               | 430.1                  | 154,000                         | 258                  | 55,136                             | 467                                                     |
| 14°27'                 | 104                                      | 25,480                                       | 11.0                               | 280.3                  | 101,000                         | 170                  | 55,136                             | 308                                                     |
| 19°17'                 | 135                                      | 28,350                                       | 11.4                               | 323.2                  | 116,000                         | 195                  | 55,136                             | 350                                                     |
| 14°30'                 | 105                                      | 21,850                                       | 12.0                               | 262.2                  | 94,000                          | 158                  | 55,136                             | 286                                                     |

This value corresponds to a compound bending stress of

$$\sigma_c = \frac{M_c}{W_b}$$

where the section modulus is

$$W_b = \frac{\pi D^3}{32} = \frac{3.14 \times 79.8^3}{32} = 49,892 \text{ cm}^3$$

and hence

$$\sigma_c = \frac{42,012,000}{49,892} = 842 \text{ kg/cm}^2$$

The permissible stress for forged rolls is equal to 1050 kg/cm<sup>2</sup>.

For the roll neck the following dimensions are shown in Fig. 5.14:

$$\begin{aligned} d &= 550 \text{ mm} & l &= 550 \text{ mm} & a &= 720 \text{ mm} & L_1 &= 2800 \text{ mm} \\ d_1 &= 490 \text{ mm} & l_1 &= 480 \text{ mm} & b &= 2080 \text{ mm} & D &= 808 \text{ mm} \end{aligned}$$

The roll neck undergoes maximum loading when rolling in the immediately adjacent groove. For the case considered this is groove No. 1 where during the third pass a maximum roll force of 564.6 t occurs.

From equation (5.5) the reaction can be calculated

$$A = R_1 = \frac{Pb}{L_1} = 564.6 \times \frac{2080}{2800} = 419 \text{ t}$$

This reaction gives a bending moment at the roll neck

$$M_b = A \frac{l}{2} = 419,000 \times 27.5 = 11,522,000 \text{ kgcm}$$

Torque at the neck, from equation (5.22) is equal to

$$M_d \approx \frac{P}{2} (\sqrt{R \Delta h} + f_1 d)$$

$$M_d = \frac{564,000}{2} (15.3 + 0.1 \times 55) = 5,865,000 \text{ kgcm}$$

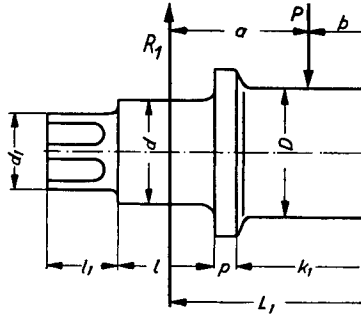


Fig. 5.14. Loading at the roll neck for stress calculation

The compound moment for steel rolls is (5.24)

$$M'_c = \sqrt{M_b^2 + 0.75 M_d^2} = \sqrt{11,522,000^2 + 0.75 \times 5,865,000^2} = 12,580,000 \text{ kgcm}$$

The section modulus is

$$W_b = \frac{\pi d^3}{32} = \frac{3.14 \times 55^3}{32} = 16,334 \text{ cm}^3$$

and hence a compound stress of

$$\sigma_b = \frac{M_b}{W_b} = \frac{12,580,000}{16,334} = 770 \text{ kg/cm}^2$$

is obtained.

Bearing pressure is calculated from

$$p = \frac{R_1}{\varphi d l} = \frac{419,000}{0.65 \times 55 \times 55} = 213 \text{ kg/cm}^2$$

where  $d = 550 \text{ mm}$ ,  $l = 550 \text{ mm}$  and  $\varphi = 0.65$  (coefficient allowing for gird angle of bearing bush).

The permissible loading is  $250 \text{ kg/cm}^2$  for highest roll forces, and  $200\text{--}250 \text{ kg/cm}^2$  for lower ones.

**Graphical method of roll strength calculation [22].** The roll is assumed to be a beam of variable cross-section supported at two points (Fig. 5.15). A bloom of  $315 \times 250 \text{ mm}$  is rolled down to a final cross-section of  $322 \times 190 \text{ mm}$  with an initial rolling temperature of  $1170^\circ\text{C}$  and final temperature  $1080^\circ\text{C}$ . The measured roll force of  $P = 672 \text{ t}$  is considered as uniformly distributed across the width of the bloom and is reacted by the two bearing forces, acting at the neck centres, at a distance of half bearing length from the edge of the roll barrel. The roll force of  $672,000 \text{ kg}$  is marked off on a scale of  $1 \text{ mm} = 10,000 \text{ kg}$  on a vertical line which is the reaction  $R_1$  produced. Then any pole  $O$  is chosen at a distance of e.g.  $h_0 = 60 \text{ mm}$ . After drawing in the force diagram, the triangle  $cde$  is drawn, representing the distribution of bending moment. Then this triangle is repositioned so that its base line  $ce$  is horizontal, i.e.  $ce$  become  $ce_1$ , and triangle  $cd_1e_1$  is substituted for triangle  $cde$ .

Assuming that the scale of roll drawing is 1:10, on the force scale 1 mm = 10,000 kg and  $h_0 = 60$  mm, then the bending moment will be

$M_b = \text{length of the ordinate (in mm)} \times h_0 (\text{in mm}) \times \text{scale of roll diagram} \times \text{scale of force diagram in kg,}$

i.e.  $M_b = \text{ordinate in mm} \times 60 \times 10 \times 10,000 \text{ kg/mm.}$

The apex of the B.M. diagram must be rounded off to a parabolic form, since the roll force is uniformly distributed over the bloom width. The line  $O-l$  parallel to  $ce$  in

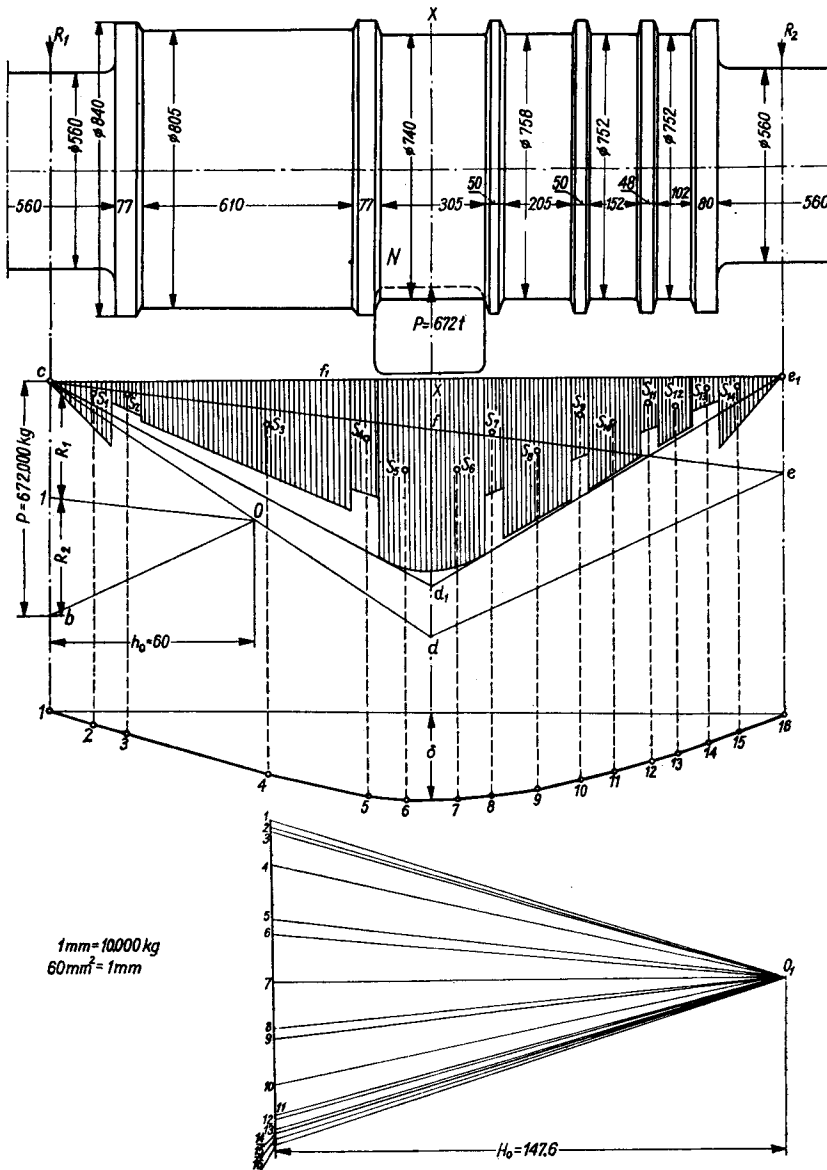


Fig. 5.15. Graphical determination of bending stresses and deflection for a blooming mill roll [22]

the triangle of forces, divides the total roll force  $P$  in the proportion of the bearing forces  $R_1$  and  $R_2$ .

The triangular bending moment diagram  $cd_1e_1$  shown in Fig. 5.15 corresponds to a constant roll diameter of  $D = 740$  mm and roll force acting in the plane of  $x-x$ , i.e. for simplification the rigidity  $EI$  has been assumed to be constant along the whole roll. The bending moment must be reduced relatively to a constant value of  $EI_0$ , taking into account variation of diameter of roll. This is done by multiplying each ordinate by the ratio  $EI_0/EI$ , where  $EI$  is the product of Young's modulus and the moment of inertia at any point. For this case  $E = \text{const.}$

For the centre cross-section

$$I_0 = 0.05 \times 74^4 = 1,499,000 \text{ cm}^4$$

and for the bullhead where  $D = 805$  mm

$$I = 0.05 \times 8.05^4 = 2,099,000 \text{ cm}^4$$

giving a ratio of

$$\frac{I_0}{I} = \frac{1,499,000}{2,099,000} = 0.714$$

Multiplying the ordinate of the bending moment diagram  $cd_1e_1$  by this ratio a corrected value is found taking into account variation of cross-section.

The ordinates on the corrected B.M. diagram are equal to uncorrected values multiplied by  $74^4/D^4$ , where  $D$  is the roll diameter at the point considered. Correcting the B.M. diagram over the roll length between reactions, gives a new diagram allowing for variations in roll diameter (shaded area).

The deflection curve for the roll is also drawn into Fig. 5.15. Roll deflection in any sector is found by multiplying the area of the corresponding part of the B.M. diagram by the distance of its centre of gravity from the line of action of the reaction. and hence the line of deflection of the roll can be found graphically. The B.M. diagram is divided into sections as shown in the figure, and straight lines perpendicular to the base  $c-c_1$  are drawn through the centres of gravity of these partial areas. Then a suitable scale of area is chosen e.g.  $60 \text{ mm}^2 = 1 \text{ mm}$  and the polygon of forces for these partial areas is drawn to this scale, denoting the forces as  $1-2$ ,  $2-3$  etc. up to  $15-16$ . The pole  $O_1$  is chosen at a distance of  $H_0 = 147.6$  mm from the base line and lines  $1-O_1$ ,  $2-O_1$  etc up to  $16-O_1$  are drawn through  $O_1$  and points  $1, 2, 3, 4$  representing the magnitude of forces  $1-2$ ,  $2-3$  etc. Starting from point  $1$ , positioned on the line of action of  $R$  line  $1-2$  is drawn parallel to  $O_1 1$  to intersect with the line through the centre of gravity  $S_1$  of the partial area. Through this point of intersection  $2$ , a line parallel to line  $2-O_1$  is drawn, and similarly point  $3$  is found. This process is continued until by joining points  $1$  and  $16$  the closed polygon  $1-16$  is finally obtained, giving the deflection curve of the roll. Then the maximum value of roll deflection, at the cross-section  $x-x$  is equal to

$$f = \delta \frac{L^3 h_0 K F H_0}{I_0 E} \quad (5.37)$$

where

$\delta$  = maximum deflection of the deflection curve determined graphically, mm,

$L$  = scale length (for this case 1:10),

$h_0$  = distance of pole from the base line in the first polygon of forces  $Obc$ , mm,

$H_0$  = distance of pole from the base line in the second polygon of forces  $O_1 1-16$ , mm,



$K$  = scale of force (for this case 1 mm = 10,000 kg),  
 $F$  = scale of area (for this case 1 mm = 60 mm<sup>2</sup>),  
 $E$  = Young's modulus (for cast iron = 10,500 kg/mm<sup>2</sup>),  
 $I_0$  = moment of inertia ( $0.05 \times 740^4$  mm<sup>2</sup>).

Hence, for cross-section  $x-x$

$$f = 23.3 \frac{10^3 \times 60 \times 10,000 \times 60 \times 147.6}{10,500 \times 14,990,000,000} = 0.787 \text{ mm}$$

The following values are also found for cross-section  $x-x$

$$M_b = 60,000 \times 5.61 = 33,660,000 \text{ kgcm},$$

$$M_d = 672,000 \times 8.7 = 5,846,000 \text{ kgcm},$$

$$W_b = 0.1 \times 74^3 = 40,522 \text{ cm}^3,$$

$$W_d = 0.2 \times 74^3 = 81,045 \text{ cm}^3,$$

$$\sigma_b = 830 \text{ kg/cm}^2,$$

$$\tau = 73.5 \text{ kg/cm}^2.$$

## 5.2. Roll Deflection and Other Factors Affecting Variation of Thickness During Rolling of Sheet and Strip

Under the action of the roll force the rolls deflect and the roll gap increases in comparison with the unloaded condition. The roll force can be determined for any particular loading case by special measuring devices or by calculation from formulae or experimental data.

To determine the roll deflection it is also necessary to know the roll force distribution along the roll barrel. For large roll diameters, not only deflection due to bending moments, but also deflection due to shear must be considered [104]. The total deflection is

$$f = f_1 + f_2 \quad (5.38)$$

where

$f$  = maximum roll deflection,

$f_1$  = roll deflection due to bending stresses,

$f_2$  = roll deflection due to shear stresses.

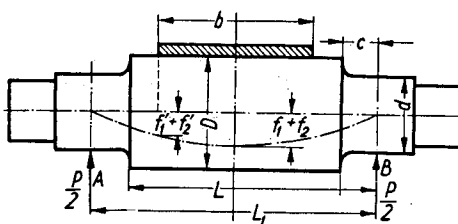


Fig. 5.16. Determination of roll deflection

For calculation of total deflection for a roll shown in Fig. 5.16:

$E$  = Young's modulus of roll material, kg/mm<sup>2</sup>,

$G$  = modulus of rigidity of roll material, kg/mm<sup>2</sup>,

$D$  = diameter of roll barrel, mm,

$d$  = diameter of roll neck, mm,

$L$  = barrel length of the roll, mm,

$L_1$  = distance between reactions, mm,

$b$  = width of the rolled stock, mm,

$c$  = distance between reaction and edge plane of the roll barrel, mm,

$P = K_{wm}F_d$  = roll force, kg.

When rolling wide sheet and strip the roll force is assumed to be uniformly distributed along the total width of the rolled stock [105].

The following is the general form of the formula for calculation of roll deflection  $f_1$  at barrel centre for symmetrical loading of the roll barrel

$$f_1 = \frac{P}{18.8ED^4} \left\{ 8L_1^3 - 4L_1b^2 + b^3 + 64c^3 \left[ \left( \frac{D}{d} \right)^4 - 1 \right] \right\} \quad (5.39)$$

At the same point there is a deflection  $f_2$  due to shear stress which is equal to

$$f_2 = \frac{P}{G\pi D^2} \left\{ L_1 - \frac{b}{2} + 2c \left[ \left( \frac{D}{d} \right)^2 - 1 \right] \right\} \quad (5.40)$$

The modulus of elasticity  $E$  for different roll materials can be taken from Table 5.6. The corresponding modulus of rigidity is determined from the well-known relation

$$G = \frac{E}{2(1+\mu)} \quad (5.41)$$

Poisson's ratio  $\mu$  is approximately equal to 0.33 for steel, and therefore, for steel rolls  $G \approx (3/8)E$  can be substituted. As the value of  $E$  for carbon steels varies from about 19,000 to 22,000 kg/mm<sup>2</sup>, the modulus of rigidity  $G$  is approximately equal to 7000–8300 kg/mm<sup>2</sup>. For cast iron rolls the value of  $G$  varies from about 2900 to 4000 kg/mm<sup>2</sup>, depending on material quality (Table 5.6).

The deflections at the sheet edge, at a distance of  $b/2$  from the barrel centre, due to bending and shear are

$$f'_1 = \frac{P}{2.36ED^4} \left\{ L_1^3 - 2L_1b^2 + b^3 + 8c^3 \left[ \left( \frac{D}{d} \right)^4 - 1 \right] \right\} \quad (5.42)$$

and

$$f'_2 = \frac{P}{G\pi D^2} \left\{ L_1 - b + 2c \left[ \left( \frac{D}{d} \right)^2 - 1 \right] \right\} \quad (5.43)$$

respectively.

The difference in roll deflection between the centre and edge of the sheet must be known to specify the crown form of the roll. For deflection due to bending

$$\Delta f_1 = f_1 - f'_1 \quad (5.44)$$

and due to shear

$$\Delta f_2 = f_2 - f'_2 \quad (5.45)$$

Therefore the total deflection, i.e. the total difference in thickness of the sheet is

$$f = \Delta f_1 + \Delta f_2 = (f_1 - f'_1) + (f_2 - f'_2) \quad (5.46)$$

#### Example

Pass schedule for a 1100 blooming-slabbing mill ( $D = 1100$  mm) [159] is given in Table 5.8.

To calculate roll forces by the Golovin-Tiagunov method, the mean resistance to deformation is given by equation (4.34)

$$K_{wm} = \left[ 1 + f \left( \frac{l_d}{h_m} - 1 \right) \right] k_t Y_P$$

and from (4.31) for rolling temperature above  $t_m = 575^\circ\text{C}$  the value of  $k_t$  is given by

$$k_t = \frac{t_m - 75 - t}{1500}$$

where

$k_t$  = coefficient of constrained yield stress, from Fig. 4.33,

$t$  = rolling temperature,  $^\circ\text{C}$ ,

$t_m$  = temperature of fusion of the rolled stock,  $^\circ\text{C}$ ,

$Y_P$  = constrained yield stress of the rolled stock at  $20^\circ\text{C}$  from Fig. 4.34.

TABLE 5.8  
PASS SCHEDULE

| Pass No. | Pass groove | Slab dimensions |           | Absolute draught $\Delta h$<br>mm | Absolute spread $\Delta b$<br>mm | Reduction<br>% | Coefficient of elongation<br>$\lambda$ | Bite angle<br>$\alpha$ |
|----------|-------------|-----------------|-----------|-----------------------------------|----------------------------------|----------------|----------------------------------------|------------------------|
|          |             | $h$<br>mm       | $b$<br>mm |                                   |                                  |                |                                        |                        |
| 0        | —           | 1100            | 650       | —                                 | —                                | —              | —                                      | —                      |
| 1        | B           | 1070            | 650       | 30                                | 0                                | 2.72           | 1.030                                  | $13^\circ 30'$         |
| 2        | B           | 1050            | 650       | 20                                | 0                                | 1.87           | 1.020                                  | $11^\circ$             |
| 3        | B           | 610             | 1052      | 40                                | 2                                | 6.15           | 1.060                                  | $16^\circ 35'$         |
| 4        | B           | 565             | 1056      | 45                                | 4                                | 7.39           | 1.070                                  | $16^\circ 35'$         |
| 5        | B           | 520             | 1062      | 45                                | 6                                | 7.95           | 1.080                                  | $16^\circ 35'$         |
| 6        | B           | 475             | 1068      | 45                                | 8                                | 8.65           | 1.080                                  | $16^\circ 35'$         |
| 7        | B           | 430             | 1075      | 45                                | 7                                | 9.49           | 1.090                                  | $16^\circ 35'$         |
| 8        | B           | 385             | 1083      | 45                                | 8                                | 10.49          | 1.10                                   | $16^\circ 35'$         |
| 9        | B           | 340             | 1093      | 45                                | 10                               | 11.70          | 1.120                                  | $16^\circ 35'$         |
| 10       | B           | 300             | 1102      | 30                                | 9                                | 11.75          | 1.120                                  | $15^\circ 40'$         |
| 11       | I           | 1040            | 300       | 62                                | 0                                | 5.62           | 1.060                                  | $19^\circ 50'$         |
| 12       | I           | 981             | 300       | 59                                | 0                                | 5.67           | 1.060                                  | $19^\circ 20'$         |
| 13       | B           | 260             | 993       | 40                                | 12                               | 13.33          | 1.155                                  | $15^\circ 40'$         |
| 14       | B           | 220             | 1006      | 40                                | 13                               | 15.40          | 1.180                                  | $15^\circ 40'$         |
| 15       | B           | 180             | 1019      | 40                                | 113                              | 18.18          | 1.220                                  | $15^\circ 40'$         |
| 16       | B           | 150             | 1030      | 30                                | 11                               | 16.70          | 1.280                                  | $13^\circ 30'$         |
| 17       | B           | 130             | 1040      | 20                                | 10                               | 13.30          | 1.155                                  | $11^\circ$             |

The length of contact and mean width of the rolled stock are determined from

$$l_d \approx \sqrt{R \Delta h} \quad \text{and} \quad b_m = \frac{2b_2 + b_1}{3}$$

Hence, e.g. for pass No. 8 and 15 the following roll forces are obtained:

Pass No. 8:  $t = 1140^\circ\text{C}$ ,  $k_t = 0.157$ ,  $b_m = 1078$  mm,

$$l_d \approx \sqrt{540 \times 45} \approx 155,$$

$$K_{wm} = \left[ 1 + \frac{1}{3} \left( \frac{155}{407} - 1 \right) \right] 0.157 \times 50 = 6.3 \text{ kg/mm}^2,$$

$$P = l_d b_m K_{wm} = 155 \times 1078 \times 6.3 = 1,050,000 \text{ kg} = 1050 \text{ t.}$$

Pass No. 15:  $t = 1090^\circ\text{C}$ ,  $k_t = 0.196$ ,  $b_m = 1012$  mm,

$$l_d = \sqrt{540 \times 40} = 147 \text{ mm},$$

$$K_{wm} = \left[ 1 + \frac{1}{3} \left( \frac{147}{200} - 1 \right) \right] 0.196 \times 50 = 9.6 \text{ kg/mm}^2,$$

$$P = l_d b_m K_{wm} = 147 \times 1012 \times 9.6 = 1,420,000 \text{ kg} = 1420 \text{ t.}$$

Calculating roll torque by the approximate method

$$M_w = 2Pa \approx 2P \frac{l_d}{2} = Pl_d$$

Therefore

$$M_{w8} \approx P \sqrt{R \Delta h} = 1050 \sqrt{540 \times 45} \approx 164 \text{ tm}$$

and

$$M_{w15} \approx P \sqrt{R \Delta h} = 1420 \sqrt{540 \times 40} \approx 208.6 \text{ tm}$$

These results are tabulated in Table 5.9, and also roll forces and torques calculated from Cook and McCrum's formulae (4.54) and (4.55).

Roll forces and torques compared in Table 5.9 have been plotted on a histogram in Figs. 5.17 and 5.18. As permissible values of roll force and torque, maximum permissible values reduced by 10% for corresponding passes have been assumed.

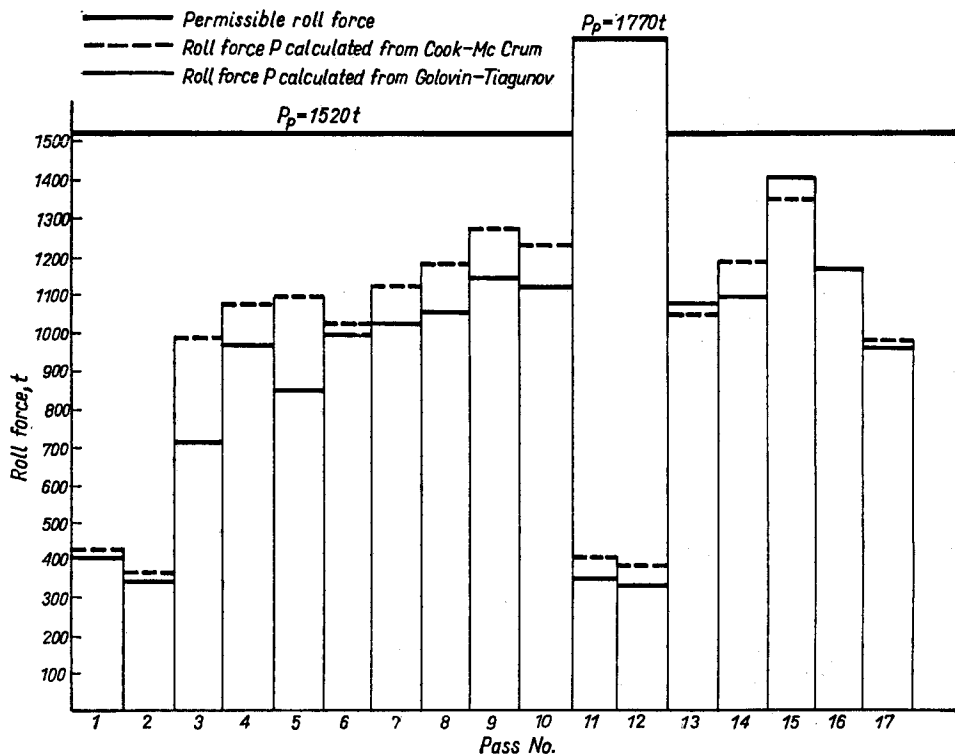


Fig. 5.17. Permissible and calculated roll force in separate passes

The roll drawing is shown in Fig. 5.19. Permissible maximum values of roll force and torque have been calculated by means of formulae (5.5) to (5.25). Similarly as in Fig. 5.15, the distribution of bending moment and deflection line of the roll have been determined. The maximum roll force arising during pass No. 15 has been assumed to be equal to  $P = 1420 \text{ t}$ . It should be noticed that for graphical determination of the line of deflection, bending moments only have been taken into account, and not shear.

TABLE 5.9  
CALCULATION OF ROLL FORCES

| Pass<br>No. | Roll forces calculated from Cook and McCrum |                          |                          |                            |               |                 |          |             | Roll forces calculated from<br>Golovin-Tiagunov |          |             |
|-------------|---------------------------------------------|--------------------------|--------------------------|----------------------------|---------------|-----------------|----------|-------------|-------------------------------------------------|----------|-------------|
|             | $R/h_2$                                     | $q$<br>sec <sup>-1</sup> | $n$<br>min <sup>-1</sup> | $q_m$<br>sec <sup>-1</sup> | $P'$<br>kg/mm | $M_w$<br>kgm/mm | $P$<br>t | $M_w$<br>tm | $K_{wm}$<br>kg/mm                               | $P$<br>t | $M_w$<br>tm |
| 0           | —                                           | —                        | —                        | —                          | —             | —               | —        | —           | —                                               | —        | —           |
| 1           | 0.505                                       | 0.0114                   | 25                       | 0.285                      | 645           | 65,400          | 425      | 42.5        | 5.2                                             | 410      | 52          |
| 2           | 0.515                                       | 0.0104                   | 25                       | 0.26                       | 580           | 57,000          | 378      | 34.1        | 5.1                                             | 346      | 36          |
| 3           | 0.885                                       | 0.0228                   | 25                       | 0.685                      | 945           | 134,000         | 990      | 130         | 4.55                                            | 715      | 105         |
| 4           | 0.955                                       | 0.0266                   | 25                       | 0.798                      | 1030          | 158,000         | 1080     | 166         | 5.95                                            | 970      | 151         |
| 5           | 1.04                                        | 0.0276                   | 25                       | 0.83                       | 1038          | 159,000         | 1100     | 169         | 5.2                                             | 855      | 133         |
| 6           | 1.139                                       | 0.0315                   | 25                       | 0.94                       | 980           | 151,000         | 1030     | 162         | 6.05                                            | 1000     | 156         |
| 7           | 1.258                                       | 0.0344                   | 25                       | 1.03                       | 1049          | 165,000         | 1130     | 172.5       | 6.1                                             | 1025     | 160         |
| 8           | 1.4                                         | 0.0378                   | 37.5                     | 1.13                       | 1090          | 157,000         | 1190     | 170         | 6.3                                             | 1050     | 164         |
| 9           | 1.59                                        | 0.042                    | 37.5                     | 1.57                       | 1169          | 156,000         | 1280     | 170.25      | 6.8                                             | 1150     | 179         |
| 10          | 1.8                                         | 0.045                    | 37.5                     | 1.68                       | 1119          | 154,000         | 1230     | 170         | 6.95                                            | 1125     | 165         |
| 11          | 0.465                                       | 0.0164                   | 37.5                     | 0.615                      | 1370          | 172,000         | 410      | 53          | 6.45                                            | 350      | 60.2        |
| 12          | 0.495                                       | 0.0168                   | 37.5                     | 0.63                       | 1330          | 170,000         | 399      | 51          | 6.4                                             | 331      | 56          |
| 13          | 2.4                                         | 0.0515                   | 37.5                     | 1.93                       | 1060          | 122,000         | 1050     | 121.8       | 7.45                                            | 1080     | 160         |
| 14          | 2.46                                        | 0.0594                   | 50                       | 2.98                       | 1190          | 148,000         | 1190     | 148         | 7.45                                            | 1095     | 160         |
| 15          | 3.0                                         | 0.069                    | 50                       | 3.46                       | 1350          | 171,000         | 1360     | 174         | 9.6                                             | 1420     | 208.6       |
| 16          | 3.6                                         | 0.0734                   | 60                       | 4.4                        | 1180          | 124,500         | 1180     | 128         | 9.1                                             | 1180     | 150         |
| 17          | 4.15                                        | 0.0715                   | 60                       | 4.28                       | 985           | 84,000          | 985      | 87.5        | 9.0                                             | 965      | 100         |

When calculating roll deflection it should be remembered that the roll has different diameters at the four grooves. From equation (5.38) the total deflection is

$$f = f_1 + f_2$$

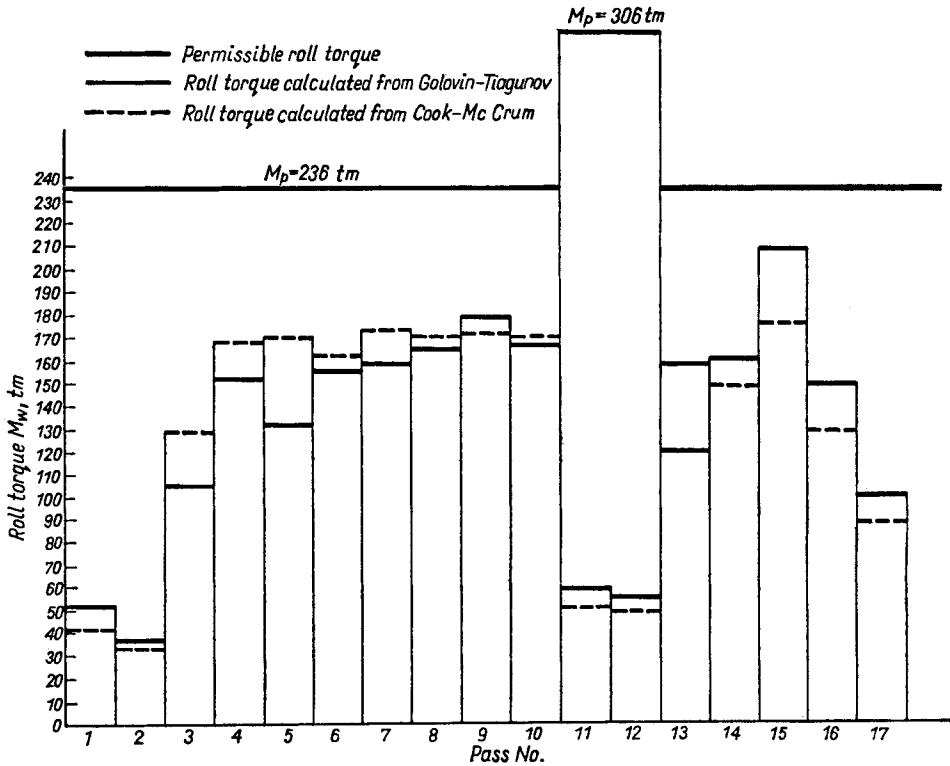


Fig. 5.18. Permissible and calculated rolling torque in separate passes

Both components can be determined as follows (Fig. 5.19):

$$f_1 = \frac{\partial U_1}{\partial R} \quad \text{for } U_1 = \int \frac{M^2 dx}{2EI} \quad \text{due to bending moment} \quad f_2 = \frac{\partial U_2}{\partial R} \quad \text{for } U_2 = \int \frac{Q^2 dx}{2GF} \quad \text{due to shear}$$

For  $x = 0$  to  $x = l_5$

$$M = R_B x$$

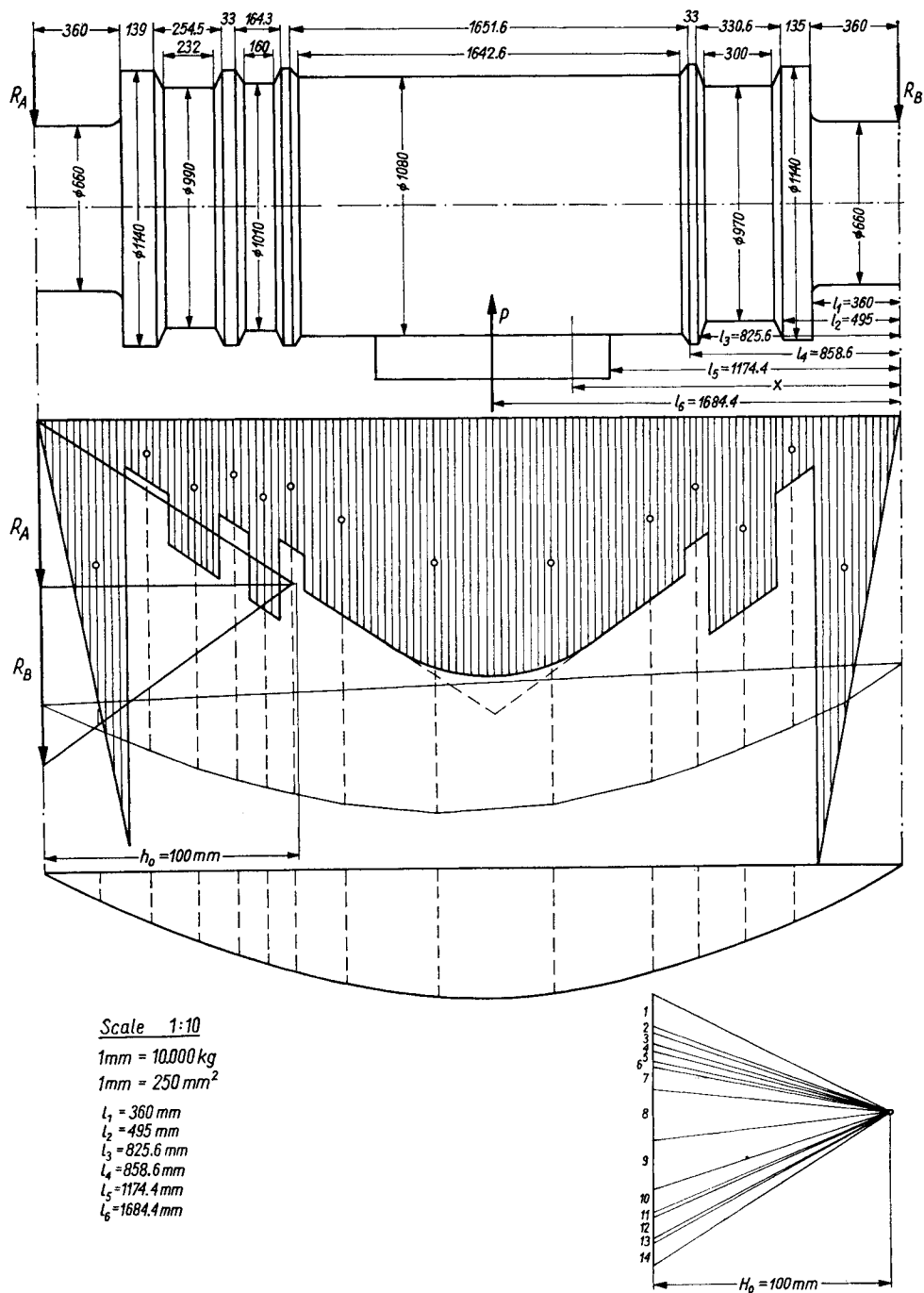
and for  $x = l_5$  to  $x = l_6$

$$M = R_B x - \frac{q}{2} (x - l_5)^2$$

$$R_B = \frac{L - l_6}{L} \times P = \frac{1804}{3488.4} \times 1420 = 734 \text{ tons}$$

$$R_A = P - R_B = 686 \text{ tons}$$

$$q = \frac{P}{b_m} = \frac{1,420,000}{102} = 14,000 \text{ kg/cm}$$



For both cases  $\partial M/\partial R = x$ , hence the component  $f_1$  can be determined from the following equation:

$$f_1 = \frac{1}{EI_n} \int_0^{l_1} R_B x^2 dx + \frac{1}{EI_f} \int_{l_1}^{l_2} R_B x^2 dx + \frac{1}{EI_1} \int_{l_2}^{l_3} R_B x^2 dx + \frac{1}{EI_f} \int_{l_3}^{l_4} R_B x^2 dx \\ + \frac{1}{EI_b} \int_{l_4}^{l_5} R_B x^2 dx + \frac{1}{EI_b} \int_{l_5}^{l_6} \left[ R_B x - \frac{q}{2} (x - l_5)^2 \right] x dx$$

The shearing force is determined from

$$Q = \frac{\partial M}{\partial x} = R_B \quad \text{or} \quad Q = R_B - q(x - l_5)$$

and therefore its share in roll deflection can be determined from the following relation:

$$f_2 = \frac{1}{GF_n} \int_0^{l_1} R_B dx + \frac{1}{GF_f} \int_{l_1}^{l_2} R_B dx + \frac{1}{GF_1} \int_{l_2}^{l_3} R_B dx + \frac{1}{GF_f} \int_{l_3}^{l_4} R_B dx \\ + \frac{1}{GF_b} \int_{l_4}^{l_5} R_B dx + \frac{1}{GF_b} \int_{l_5}^{l_6} [R_B - q(x - l_5)] dx$$

After integration

$$f_1 = \frac{1}{EI_n} R_B \frac{l_1^3}{3} + \frac{1}{EI_f} R_B \frac{l_2^3 - l_1^3}{3} + \frac{1}{EI_1} R_B \frac{l_3^3 - l_2^3}{3} + \frac{1}{EI_f} R_B \frac{l_4^3 - l_3^3}{3} \\ + \frac{1}{EI_b} R_B \frac{l_5^3 - l_4^3}{3} + \frac{1}{EI_b} \left[ R_B \frac{l_6^3 - l_5^3}{3} - \frac{q}{2} \left( \frac{l_6^4 - l_5^4}{4} - 2l_5 \frac{l_6^3 - l_5^3}{3} + l_5^2 \frac{l_6^2 - l_5^2}{2} \right) \right]$$

Moment of inertia for any cross-sections (Fig. 5.19) is equal to  $I_x = \pi d_x^4/64$ :

$$I_n = \frac{\pi \times 66^4}{64} = 930,000 \text{ cm}^4, \quad I_f = \frac{\pi \times 114^4}{64} = 8,290,000 \text{ cm}^4 \\ I_1 = \frac{\pi \times 97^4}{64} = 4,340,000 \text{ cm}^4, \quad I_b = \frac{\pi \times 108^4}{64} = 6,670,000 \text{ cm}^4.$$

Substituting  $E$  = modulus of elasticity for steel =  $2.1 \times 10^6$  kg/cm<sup>2</sup>

$$f_1 = \frac{734,000}{2.1 \times 10^6 \times 3} \left[ \frac{36^3}{0.93 \times 10^6} + \frac{49.5^3 - 36^3}{8.29 \times 10^6} + \frac{82.6^3 - 49.5^3}{4.34 \times 10^6} \right. \\ + \frac{85.9^3 - 82.6^3}{8.29 \times 10^6} + \frac{117.4^3 - 85.9^3}{6.67 \times 10^6} + \frac{168.4^3 - 117.4^3}{6.67 \times 10^6} \\ \left. - \frac{14,000}{8 \times 734,000 \times 6.67 \times 10^6} (3 \times 168.4^4 - 8 \times 168.4^3 \times 117.4 \right. \\ \left. + 6 \times 168.4^2 \times 117.4^2 - 117.4^4) \right]$$

$$f_1 = 0.1165(0.0502 + 0.0090 + 0.1015 + 0.0085 + 0.1503 + 0.4715 - 0.0296)$$

$$f_1 = 0.0889 \text{ cm} = 0.889 \text{ mm}$$



When integrating for shear force

$$f_2 = \frac{1}{GF_n} R_B l_1 + \frac{1}{GF_f} R_B (l_2 - l_1) + \frac{1}{GF_1} R_B (l_3 - l_2) + \frac{1}{GF_f} R_B (l_4 - l_3) \\ + \frac{1}{GF_b} R_B (l_5 - l_4) + \frac{1}{GF_b} \left[ R_B (l_6 - l_5) - q \frac{l_6^2 - l_5^2}{2} + q l_5 (l_6 - l_5) \right] \\ f_2 = \frac{R_B}{G} \left[ \frac{l_1}{F_n} + \frac{l_2 - l_1}{F_f} + \frac{l_3 - l_2}{F_1} + \frac{l_4 - l_3}{F_f} + \frac{l_5 - l_4}{F_b} + \frac{l_6 - l_5}{F_b} - \frac{q(l_6 - l_5)^2}{2R_B F_b} \right]$$

Areas for corresponding cross-sections are equal to  $F_x = \pi d_x^2/4$ :

$$F_n = \frac{\pi \times 66^2}{4} = 3430 \text{ cm}^2, \quad F_f = \frac{\pi \times 114^2}{4} = 10,230 \text{ cm}^2 \\ F_1 = \frac{\pi \times 97^2}{4} = 7410 \text{ cm}^2, \quad F_b = \frac{\pi \times 108^2}{4} = 9190 \text{ cm}^2$$

Therefore, for the modulus of rigidity of  $G \approx 3/8E \approx 0.7875 \times 10^6 \text{ kg/cm}^2$ :

$$f_2 = \frac{734,000}{0.7875 \times 10^6} \left[ \frac{36}{3.43 \times 10^3} + \frac{49.5 - 36}{10.15 \times 10^3} + \frac{82.6 - 49.5}{7.41 \times 10^3} + \frac{85.9 - 82.6}{10.23 \times 10^3} \right. \\ \left. + \frac{117.4 - 85.9}{9.19 \times 10^3} + \frac{168.4 - 117.4}{9.19 \times 10^3} - \frac{14,000(168.4 - 117.4)^2}{2 \times 734,000 \times 9.19 \times 10^3} \right] \\ f_2 = 0.933(10.49 + 1.32 + 4.47 + 0.33 + 3.50 + 5.51 - 2.66) \times 10^{-3} \\ f_2 = 0.0213 \text{ cm} = 0.213 \text{ mm}$$

Hence, the total roll deflection is equal to

$$f = f_1 + f_2 = 0.889 + 0.213 = 1.102 \text{ mm}$$

Values of modulus of rigidity for other metals are shown in Table 5.6.

The maximum roll deflection determined graphically (Fig. 5.15) is equal to

$$f = \frac{L^3 h_0 K F H_0}{EI} \delta$$

where

- $\delta = 55.8 \text{ mm}$  from the drawing,
- $L = 10$  (scale for length 1:10),
- $h_0 = 100 \text{ mm}$  (distance of pole from base line in the force diagram),
- $H_0 = 100 \text{ mm}$  (distance of pole from base line in the second force diagram),
- $K = 10,000 \text{ kg}$  (scale of forces),
- $F = 250 \text{ mm}^2$  (second scale of forces),
- $E = 21,000 \text{ kg/mm}^2$ ,
- $I = 6.67 \times 10^{10} \text{ mm}^4$  (reduced moment of inertia).

$$f = 55.8 \frac{1000 \times 100 \times 10,000 \times 250 \times 100}{21,000 \times 6.67 \times 10^{10}} = 0.996 \text{ mm}$$

During hot rolling of sheet and strip, and also during cold rolling of wide and narrow strip it can be observed that cross-sections of the product are not exactly rectangular, but are most often convex, as shown in Fig. 5.20a. A concave shape shown in Fig. 5.20c is less often found, while correctly rectangular shapes, as shown in Fig. 5.20b are rarely obtained.

It is left to the skill of the roll designer to choose a roll design which ensures that these variations from the true rectangular shape do not exceed permissible tolerances, and also that the surface of the final product is flat and free from twisting and rippling at the centre and edges [104].

If a dead flat sheet is rolled on a roll pair of convex crown, then a product with a cross-section similar to that shown in Fig. 5.20c is obtained, where the surplus metal

flows away from the centre. Since there is almost no spread during rolling the metal tends to flow longitudinally, and so-called “tongues” are formed at ends of the sheet. For a concave roll gap (Fig. 5.20a), the surplus metal flows away from the edges of

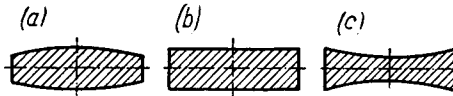


Fig. 5.20. Cross-sections of rolled sheet and strip: (a) convex, (b) rectangular, (c) concave

the sheet, and so-called “ears” arise. This occurs as long as the metal is highly ductile and its particles can easily change their relative positions, i.e. “flow”. If however, a decrease of temperature or other factors decrease the ductility of the metal, the relative motion of the particles i.e. slip, becomes limited. This occurs also in cold rolling due to considerable decrease in ductility of the deformed material. The displaced surplus metal must find a new place and ruffling occurs at the sheet centre (Fig. 5.20c) or sheet edge (Fig. 5.20a).

This is easily understandable, since with a concave roll section, the less compressed and more rigid centre of the strip shows much less tendency to elongate than the edges. The heavily compressed edges cannot however elongate freely since they are constrained by the centre, hence ruffles arise in this place.

The choice of correct cross-sectional form is of special importance in the rolling of flat products, when passing from one set of rolls to another, or to a further hot or cold rolling process. In such a case, a wrong or insufficiently accurate choice of crown for the roll barrel before or after a particular pass, leads to a cross-sectional form of the rolled stock unsuitable for succeeding passes. This in turn, leads to production difficulties and to the rolling of defective or reject products.

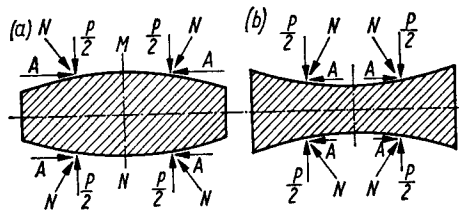


Fig. 5.21. Forces acting upon the rolled stock during rolling of sheet and strip: (a) for a concave camber, (b) for a convex camber

The roll camber form shown in Fig. 5.20a is the most useful one for practical rolling. Figure 5.21a shows the cross-section of strip rolled in this way. For a sheet thicker at the centre than at the edge, as the roll force  $N$  is normal to the surface of the rolled stock, the horizontal component  $A$  acts towards the plane of symmetry  $M-N$  of the sheet. If for any reason the centre plane of a sheet or strip deviates from the plane  $M-N$ , these forces tend to return it to its former position [10].

However, for the cross-section shown in Fig. 5.21b any deviation of the sheet from the centre plane of the roll stand gives rise to lateral forces, increasing with increase in deviation, and tending to throw the sheet towards the edge of roll. For this reason sheet is usually rolled with a slight convexity. When rolling sheet or strip on

a multi-stand train, the camber of the rolled stock should increase continuously from the last towards the first stand. Simultaneously, in any pass, all parts of the cross-section of the rolled stock should be subjected to the same elongation.

This condition can only be neglected for cold rolling on stands equipped with coiling reels, where strip tension ensures correct guiding. For all other cases the sheet being rolled should be thicker at the centre than at the edge.

Since in this case there is a tendency to riffling, draughts in any pass should be uniformly distributed across the whole width of the cross-section. Therefore, thickness variations (camber) across the sheet width should increase from the last to the first pass [104], according to equation

$$e_i = e_n \frac{h_i}{h_n} \quad (5.47)$$

where

$e_i$  = difference in thickness between centre and edge of the sheet in the  $i$ -th pass,

$e_n$  = difference in thickness between centre and edge in the last,  $n$ -th, pass,

$h_i$  = sheet thickness in the  $i$ -th pass,

$h_n$  = sheet thickness in the last,  $n$ -th, pass.

Sheet thickness measurements may be carried out at the centre or the edge of the sheet so long as they are both carried out in the same place.

Differences in thickness between centre and edge of sheets are from 2 to 10%. The minimum value of 2% applies to modern efficient mills, and 10% to rolling of sheet on old type sheet mills using old-fashioned rolling methods.

The cross-sectional form of the finished sheet or strip depends mainly on the shape of the roll gap during the pass and the shape of the initial cross-section. For continuous hot strip rolling mills this cross-sectional form also depends on the distance between stands, and on other additional factors.

The roll camber, and therefore the shape of the roll gap for cold unloaded rolls differs considerably from the true rolling conditions for heated and loaded rolls. An unsuitable finished shape of product can be corrected by turning the roll to another camber so long as other factors, e.g. roll force, rolling temperature etc., remain unchanged. The influence of each factor will be considered in turn, assuming each time that all other factors remain constant. This same principle is applied in rolling practice, where it is endeavoured to keep all factors constant, including roll force.

The roll camber, and therefore the shape of the roll gap during the pass, depends on the following factors:

- (a) mechanical factors,
- (b) thermal factors,
- (c) roll wear during operation,
- (d) design features of rolls, roll stands and lay-outs.

Mechanical factors include roll deflection which has already been considered, and flattening.

Roll flattening causes change in length of the arc of contact, which can be calculated from (4.149) and (4.150). Therefore to find the total roll deflection, not only bending moment and shear must be considered, but also deflection due to roll flattening, taken as the difference in radii of the unflattened and flattened roll.

For roll pressure uniformly distributed along the roll barrel, the influence of roll flattening on the variation of sheet thickness is insignificant, since this condition

corresponds to an approximately constant value of roll flattening across the width of the rolled sheet. However, for non-uniform roll pressure distribution across the width of the rolled stock, roll flattening becomes an important factor. It should be remembered that under operational conditions a variable roll pressure distribution almost always occurs.

Since the smaller the roll diameter, the less the flattening, therefore the influence of roll flattening on a 4-high mill (of small diameter of work rolls) will be less than for a 2-high mill of roll diameter corresponding approximately to that of back-up rolls of the 4-high mill.

Roll deflection due to flattening accounts for approximately 20–30% of total roll deflection.

Roll camber is also influenced by thermal conditions during rolling. The mechanical work of deformation required for deformation of the rolled stock during the pass is transformed into heat, which results in increase of temperature of the rolled stock. Simultaneously, during hot rolling the rolled stock coming into contact with the colder roll renders heat to it by conduction and radiation.

During cold rolling the rolls are heated because of the transformation of mechanical work into heat only, but this exchange is greater than during hot rolling.

Increase in temperature of the roll barrel results in increase of diameter, due to thermal expansion, but this causes variations of thickness of the rolled stock only if the distribution of temperature is non-uniform.

Temperature distribution of a roll is influenced by:

- (a) conduction to the roll barrel from the rolled stock and transformation of work of deformation into heat,
- (b) heat taken off by the cooling water from the roll,
- (c) thermal losses through roll necks to bearings, housings and surrounding air,
- (d) thermal losses due to radiation.

The calculation of the effect of these factors on temperature distribution can be only approximate, due to considerable difficulties in expressing them by mathematical formulae. This problem is further complicated by the fact that heat conduction to the roll barrel takes place only periodically during the pass.

If the length of the rolled stock is not great, e.g. for cold rolling on continuous mills, these periodic variations of temperature only affect the roll surface to a few millimetres depth. This causes only insignificant variations in roll diameter, and therefore can be neglected. It can be assumed that there is a continuous flow of heat from the roll to the roll necks, and thence to bearing bushes, and from bearings to screw down and roll housing. Heat is also lost to the surrounding atmosphere and taken off by the cooling water from the barrel and necks. Thus the temperature of roll barrel depends on draught, time of contact between roll and stock, temperature of rolled stock and quantity of cooling water.

The resulting roll camber due to thermal expansion, i.e. the increase in roll diameter at any point on the barrel relative to that at the edge of barrel, is given by

$$W = \alpha R(t - t_1) \quad (5.48)$$

where

$\alpha$  = coefficient of linear thermal expansion of the roll, which depends on roll diameter,

$R$  = roll radius, mm,

$t$  = temperature at any point of the roll barrel, °C,

$t_1$  = temperature at the edge of the roll barrel, °C.

With over-intensive cooling of roll barrel the surface temperature decreases, resulting in decrease of roll camber, especially at the barrel centre. Insufficient cooling however gives an increase of temperature and hence of camber.

If the temperature of the rolls is constant and the roll pressure is approximately uniformly distributed along the roll barrel, then the geometrical form of the roll gap can be determined approximately from equation (5.48). For all other conditions roll camber must be determined experimentally [104].

There are two means of varying the shape of the roll gap, by changing the roll deflection or roll diameter. Roll deflection can only be changed by change of draught. Since the rolling process demands a minimum number of passes with fixed draughts, this method is almost never used. It is especially unsuitable for continuous trains where draughts and reductions are directly related to rolling speed.

Change of temperature or of temperature differences along the roll barrel can be used to obtain variation of roll camber by overall or local heating of roll, or by overall or local cooling. Hot rolls can be cooled by air cooling along the whole roll barrel or locally, using cold or preheated air. Other methods of roll cooling have been found to be detrimental and dangerous for these rolls. Additional heating of rolls can be provided by gas burners. Using any of these methods the temperature distribution along the roll can be controlled very exactly, and the desired roll camber obtained.

Variation of surface temperature of the roll should be as small as possible in cold rolling, as it causes variations of thickness of the rolled stock. The ideal rolling condition is when the heat lost by cooling is equal to that gained during rolling.

In practice two methods are used for the cooling of rolls. The first method consists of cooling the roll barrel by water spray jets in connected groups, so that increasing or decreasing the supply of water to a given group of jets, the temperature distribution along the roll barrel can be controlled. The second method, often applied, is to use a cooling box, fitted over a sector with a wrapping angle of 60 to 90° along the whole barrel length, divided into several longitudinal compartments, and fitted closely to the rolls. The coolant, usually 5% oil and 95% water emulsion, is conducted by a pipe to the top of the cooling box, and taken away by two pipes. With good sealing the coolant losses are insignificant. This method gives very accurate temperature control along the roll barrel, and has the advantage that the coolant is not in contact with the rolled stock, so that any required lubricant can be used for rolling. If this method gives insufficient cooling, then the quantity of circulating coolant can be increased by increasing the pressure in the feed pipe. This method of roll cooling is insufficient for high speed rolling, and spray-jet cooling must be used. Oil emulsions are used for rolling of steel, but are unsuitable for rolling of aluminium.

The older method of cooling rolls by drilling holes and passing water through them is unsuitable, due to inefficiency. Rolls heat at the surface, and this heat is conducted through the roll material towards its centre, where it is transferred to the cooling water, flowing through the drilled hole. This heat abstraction from roll surface to the core takes a long time, and if also the quantity of heat conducted is not constant, the roll camber varies after a certain time, influencing the rolling conditions.

The required shape of roll gap is obtained by accurate grinding when the rolls are changed. Roll camber can be used to control variations in thickness of the rolled sheet only if the following conditions are satisfied [104]:

- (a) uniform temperature distribution along the roll barrel is maintained,

- (b) constant temperature of roll barrel is maintained by means of uniform rolling sequence,
- (c) draught is uniformly distributed up to the last pass when rolling in several grooves on the same stand,
- (d) rolls are changed as soon as they are so worn that the permissible tolerance of the rolled stock is exceeded.

When rolling in a continuous system, where only one pass is done in each stand, the whole train must be designed to provide uniform draught distribution over the whole cross-section, and guiding of the sheet or strip to keep the convexity at the centre. This convexity is gradually reduced as in equation (5.47).

Emicke and Lucas [105] recommend the following data for concave roll camber:

|                       |              |
|-----------------------|--------------|
| first roughing stand  | 0.43 mm      |
| second roughing stand | 0.24 mm      |
| third roughing stand  | 0.16 mm      |
| finishing stand       | 0.10–0.12 mm |

When rolling sheet in 2-high stands of hot rolls (i.e. uncooled), rolls are dressed to a concave camber of 0.2–0.5 mm, depending on roll diameter, length and working temperature. The optimum value should be chosen by experiment.

Cold rolls for 3-high sheet mills (i.e. cooled rolls) are dressed to a convex camber of 0.10–0.15 mm. In both cases chilled cast iron rolls are used.

Hot rolls for sheet are cooled during rolling by cold or preheated compressed air, supplied from groups of jets. For preheating of rolls groups of gas burners are used. Optimum operational conditions for hot rolls are reached in the range 300–400°C, a temperature of about 400°C at barrel surface being the best. Temperature drop between roll centre and barrel edge depends on barrel length, and has an average value of 30 to 50°C. A roll temperature above 400°C is not recommended for the following reasons:

- (a) mechanical strength of the roll material decreases by about 30%,
- (b) at 450°C graphitization of the chilled layer begins,
- (c) at 450–500°C the scale that forms on the sheet surface begins to stick to the roll and to be pressed into it.

For cold rolling, rolls are usually dressed to a convex camber of from 0 to 0.10 mm, depending on length of roll barrel and type of metal rolled, as the longer the roll barrel, and the harder the metal rolled, the greater the convexity of camber that should be used. For 4-high mills, back-up rolls are often heated at the centre of the barrel to give a slightly increased diameter. Roll camber of a 2-high stand should be twice as great as for a 4-high stand, as may be deduced from the preceding considerations. Since the shape of the rolling gap depends closely on the roll force, which in turn is dependent on strip tension and lubrication, therefore control of these factors gives a method of obtaining a required shape of roll gap, and thus of maintaining dimensions of rolled stock within the range of allowable tolerances. For operational conditions roll camber, strip tension and lubricant, should be determined jointly.

Barrels of cold rolls are usually ground to a convex camber on an arc. However when rolls are strongly pressed together as in rolling of very thin strip, it may happen that due to roll flattening the barrel edges are in contact even after the strip is between the rolls. To avoid this, the barrel is ground near the edges to a considerably greater radius  $R_1$  than at the centre ( $R$ ), as shown in Fig. 5.22 [10].

This is of special importance for cold rolling of very narrow strip approximately 2.5 mm wide, where in order to avoid contact the whole barrel is ground with a large convexity, except for a narrow part at the centre (Fig. 5.23).

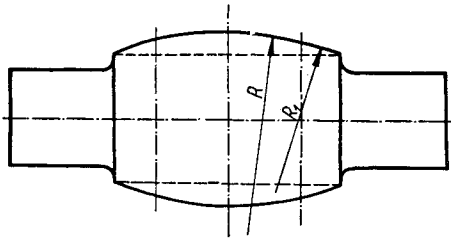


Fig. 5.22. Roll camber for cold rolling of narrow and thin strip

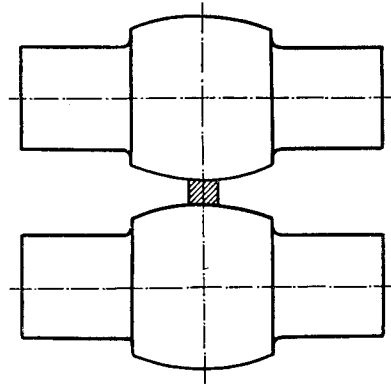


Fig. 5.23. Roll camber for cold rolling of very narrow strip

So far this chapter has been concerned with variations of thickness and shape of the cross-section of rolled sheet or strip. Differences in thickness between the beginning and the end of rolled sheet or strip should also be considered. The basic reason for this difference is the increase in resistance to deformation chiefly due to drop in temperature of strip, between the time of rolling the beginning and the time of rolling the end.

The end of the strip after leaving the furnace must wait to enter the rolls, and is therefore always cooler than the beginning. This is especially important in continuous systems. This effect can be minimized by non-uniform heating of the stock (i.e. heating the end more than the beginning) or by the use of reheating furnaces. This increase in resistance to deformation results in greater roll deflection, and increase of elastic deformation of bearings and housings, which has the effect of increasing variations in dimensions of the rolled stock.

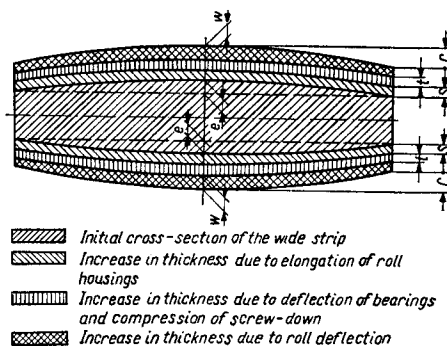


Fig. 5.24. Increase in thickness of a wide strip just before leaving the roll

Figure 5.24 shows schematically the increase in thickness of wide strip during the last pass [104]. Value  $e$  denotes half camber before the pass, required for sufficient guiding of the rolled strip, and  $s$ —the half increase in thickness due to housing

elongation, which is constant across the total width of the rolled stock. The half value of increase in thickness  $t$  due to elastic deformation of chocks, bearings and screw downs, is also constant across the width of stock. However, the increase in thickness of the sheet due to roll deflection  $w$  varies across the width and is greater than that due to previous factors. These differences depend on the increase in resistance to deformation.

Since the smaller the roll force, the smaller the differences in thickness between beginning and end of the strip, therefore draughts of 4–15% are used in the last stand of continuous trains, thus considerably reducing the differences in thickness between beginning and end of the rolled strip.

So far newly ground rolls only have been considered. However, during operation they are gradually worn down, causing still greater differences in thickness of the rolled stock. Roll wear is caused mainly by friction between roll surface and stock. The peripheral speed of rolls and speed of the rolled stock are equal to each other at the neutral plane only. Before the neutral plane is the zone of backward slip, where the stock moves more slowly than roll surface, and beyond the neutral plane is the zone of forward slip where the stock moves faster than the rolls. Also due to roll camber the roll diameter varies along roll barrel, giving variations in peripheral speeds. When two rolls are working together, they are almost never of equal diameter, and when both are driven then variations in the rotational speed of the motors cause further differences in the peripheral speeds of rolls [104].

Since the resistance to deformation of the rolled stock does not allow an additional metal flow to any extent, and in the considered cases there is no spread, therefore these differences in speed can only be equalized by additional slip, i.e. additional friction between roll surface and stock.

For 4-high and cluster mills additional friction between work and back-up rolls occurs, (every roll working directly with the work roll is assumed as a back-up roll) since roll diameters and barrel shapes are never equal to each other. This also causes differences in peripheral speed and unequal wear of rolls.

Roll wear is kept to a minimum by the use of hard material resistant to abrasion, e.g. chilled cast iron or hardened alloy steel with a hardness of 570–670 Brinell.

Under operational conditions, a correctly chosen rolling schedule should keep down roll wear, and minimize dimensional variations of the finished product.

When rolling wide and thin products, the edges are always cooler than the centre, due to more rapid cooling during rolling. This is the case when rolling sheet and strip. When rolling such products roll ends get more wear than the centre, due to greater resistance to deformation. When starting the rolling programme with rolling of narrow sheet, the rolls would get more wear at their centres, and changing later to rolling of wider sheet would be impossible without regrinding of the barrel. However, when starting with rolling of wide sheet to the allowable limit of roll wear, then the change to rolling of narrower sheet can be made, without regrinding of rolls, since the wear is evenly distributed instead of being concentrated at the centre.

The determination of roll camber when rolling sheet and strip, is of great importance.

The derived equations (5.38) to (5.46) can be used to determine roll deflection at the centre and edge of sheet or strip (Fig. 5.16), but they are unsuitable for calculations to determine roll camber for hot and cold rolling of wide flat sections.

For this purpose Emicke and Lucas [105, 149] and also Larke [62] evolved supplementary equations for the calculation of roll camber at every point of the roll, taking into account all the factors directly affecting roll shape.



Symbols used denote [62] (Fig. 5.25):

$P_c$  = maximum roll load, assumed uniformly distributed along the roll barrel, used to calculate the camber needed to offset the effect of deflection, t,  
 $P_x$  = roll load at point  $x$ , t,

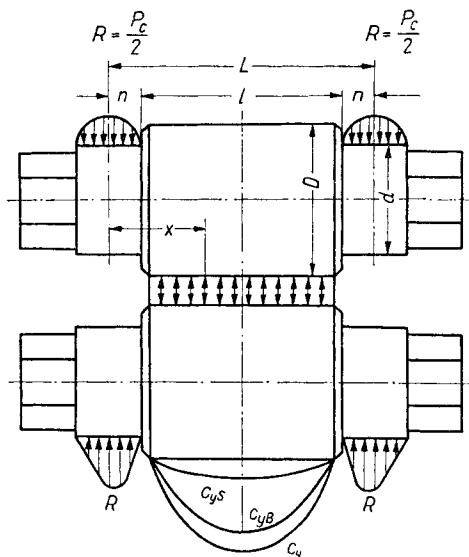


Fig. 5.25. Factors involved in the design of roll cambers

- $R$  = reaction to roll load, t,  
 $l$  = length of roll barrel, mm,  
 $n$  = distance from line of action of reaction  $R$  at the roll neck to the beginning of the roll barrel, mm,  
 $L = l + 2n$  = distance between reactions, mm,  
 $c_{yB}$  = deflection (or crown) at any point on the roll barrel due to bending stresses, mm,  
 $c_{yS}$  = deflection (or crown) at any point on the roll barrel due to shear stresses, mm,  
 $c_y = c_{yB} + c_{yS}$  = total deflection or crown of roll barrel, mm,  
 $x$  = ordinate along the roll barrel, measured from reaction  $R$ , mm,  
 $D$  = diameter of roll barrel, mm,  
 $d$  = diameter of roll neck, mm,  
 $I_D = \pi D^4/64$  and  $I_d = \pi d^4/64$  = moments of inertia of roll barrel and neck,  
 $M_b$  = bending moment at any point of roll barrel or neck, t cm,  
 $\sigma''_{per}$  = maximum permissible bending stress for the roll, from Table 5.5 or 5.6, kg/cm<sup>2</sup>,  
 $\Delta$  = dimensionless stress concentration factor, depending on  $H = D - d$ ,  
 $r$  = fairing radius between neck and roll barrel, mm,  
 $b$  = width of rolled stock, mm,  
 $m_{yB}$  = change of roll camber due to bending stresses, mm,  
 $m_{yS}$  = change of roll camber due to shear stresses, mm,  
 $m_y = m_{yB} + m_{yS}$  = total change of roll camber, mm,

$E$  = Young's modulus of elasticity for the roll material (Table 5.6), kg/cm<sup>2</sup>,  
 $G$  = modulus of rigidity of the roll material (Table 5.6), kg/cm<sup>2</sup>,  $G \approx 3E/8$ ,  
 $t_m$  = temperature midway along roll barrel, °C,  
 $t_1$  = temperature at ends of roll barrel, °C,  
 $t$  = temperature at any point  $x$  along roll barrel, °C,  
 $\alpha$  = coefficient of linear expansion per °C, of the roll material,  
 $x_1 = (x-n)$  = distance along the roll barrel, measured from end of barrel, mm,  
 $K$  = a factor based on the assumption of approximately parabolic deflection of barrel,  
 $y_t$  = negative camber at any point  $x_1$  along the roll barrel, to offset the effects of thermal expansion, mm,  
 $y_f = c_y - y_t$  = true camber to which hot rolls should be ground, mm.

Roll deflection for a cold roll due to bending and shear stresses (Fig. 5.25) is given by

$$c_{yB} = P_c \frac{x-n}{24EI} \left[ \frac{(x-n)^3}{l} + L^2 + 2ln - 2x(x+n) \right] \quad (5.49)$$

and

$$c_{yS} = \frac{2P_c}{GD^2} \left[ \frac{x(L-x) - n(L-n)}{l} \right] \quad (5.50)$$

Hence the total deflection at any point between  $x = n$  and  $x = L/2$  is given by

$$c_y = c_{yB} + c_{yS} \quad (5.51)$$

From these equations it can be seen that at the ends of the roll barrel, where  $x = n$  or  $x = l-n$ , the deflection is equal to zero ( $c_y = 0$ ), therefore this point is chosen as the origin of the deflection curve (Fig. 5.25).

The maximum deflection occurs at  $x = L/2$ , i.e.

$$c_{yB\max} = \frac{P_c l^2}{6ED^2} (5l + 24n) \quad (5.52)$$

$$c_{yS\max} = \frac{P_c l}{2\pi GD^2} \quad (5.53)$$

$$c_{y\max} = c_{yB\max} + c_{yS\max} \quad (5.54)$$

To illustrate the use of these equations a numerical example for cold rolling will be considered [62].

#### Example

For the roll shown in Fig. 5.26 the required roll camber is to be determined from the roll force  $P_c$ , shown uniformly distributed over the full length  $l$  of the roll barrel. The bending moment  $M_b$  where the neck merges into the barrel of the roll, i.e. where  $x = n$  is

$$M_b = \frac{1}{2} P_c n$$

Hence the maximum permissible roll load  $P_c$  is

$$P_c = \frac{\pi d^3 \sigma'_{per}}{16n\Delta} \quad (5.55)$$

To find  $P_c$  first  $\Delta$  must be found, where  $\Delta$  is a coefficient allowing for stress concentration at the place where the fillet radius  $r$  blends the neck of the roll into the barrel.

From Fig. 5.27 for a ratio of  $r/d = 0.1$  and  $H/2r = (D-d)/2r = 1.68$  a value of  $\Delta = 1.73$  is read off [62, 160].

For a permissible strength of  $14 \text{ kg/mm}^2$  the stress at the position where the neck merges into the barrel of the roll reaches a value of  $\sigma'_{\text{per}} = 14 \times 1.73 = 24 \text{ kg/mm}^2$ .

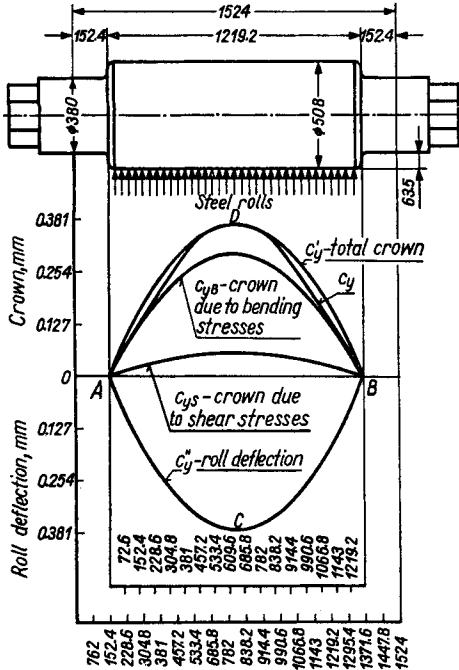


Fig. 5.26. Deflections due to bending and shearing forces and resultant camber [160]

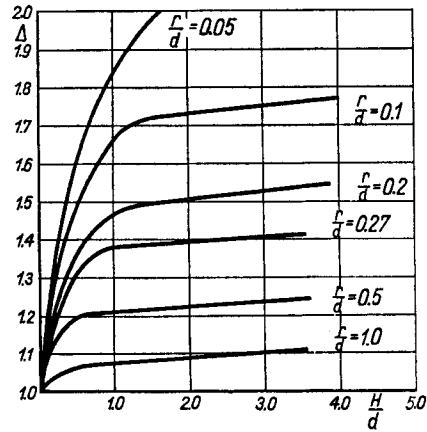


Fig. 5.27. Curves for obtaining stress concentration factor  $\Delta$  [160]

Hence, substituting corresponding values in equation (5.55), gives a permissible roll load of

$$P_c = 574 \text{ t}$$

From the known maximum permissible roll load, the condition for minimum permissible width of the rolled sheet can be derived, and expressed as

$$b = 2L - \frac{4\pi D^3 \Delta}{d^3} \quad (5.56)$$

For the example shown in Fig. 5.26 this minimum width is 560 mm. Equation (5.56) has been obtained by transposing equation (5.55) to

$$\sigma'_{\text{per}} = \frac{4P_c(2L-1)}{\pi D^3}$$

Knowing the value of the maximum permissible load which is assumed uniformly distributed over the full length of the roll barrel, the next step is to compute the deflection curve for this loading condition. Substituting in equations (5.49) and (5.50) the numerical values given in Fig. 5.26, and taking  $P_c = 574 \text{ t}$

$$c_{yB} = 0.3(x-15.24)10^{-9}[(x-15.24)^3 - 244 \times (x+15.24) + 328.3 \times 10^4] \quad (5.57)$$

and

$$c_{yS} = 1.44 \times 10^{-6}[x(15.24-x) - 2080] \quad (5.58)$$

Deflections at points between  $x = 152.4 \text{ mm}$ , i.e. at the beginning of the roll barrel, and  $x = 762 \text{ mm}$ , i.e. midway between the reactions, have been computed, and the results are shown in Table 5.10.

The total deflection, shown in column 5, is given by

$$c_y = c_{yB} + c_{yS}$$

and is plotted in Fig. 5.26 as curve  $ADB$ . It should be noted from Table 5.10 that for this roll, where the ratio of barrel diameter to barrel length is 0.42, the deflection due to shear is about 15% of the total deflection.

TABLE 5.10  
DETAILS OF CONVEX CAMBER FOR 508 × 1220 mm STEEL ROLLS  
 $P_c$  = design load (for camber) = 574 t

| Distance along roll, mm                               |                                                 | Amount of camber, mm × 10 <sup>-4</sup>  |                                        |                                     |                             | Per cent camber<br>due to shear<br>forces<br>$c_{yS}/c_y$<br>% |
|-------------------------------------------------------|-------------------------------------------------|------------------------------------------|----------------------------------------|-------------------------------------|-----------------------------|----------------------------------------------------------------|
| measured from<br>line of action<br>of reaction<br>$x$ | measured from<br>end of roll<br>barrel<br>$x_1$ | due to<br>bending<br>from equ.<br>(5.57) | due to<br>shear<br>from equ.<br>(5.58) | total<br>$c_{yB} + c_{yS}$<br>$c_y$ | approx.<br>values<br>$c'_y$ |                                                                |
|                                                       |                                                 | $c_{yB}$                                 | $c_{yS}$                               |                                     |                             |                                                                |
| 152 or 1370                                           | 0 or 1220                                       | 0                                        | 0                                      | 0                                   | 0                           | —                                                              |
| 230 or 1300                                           | 76 or 1150                                      | 711                                      | 127                                    | 838                                 | 888                         | 16                                                             |
| 305 or 1220                                           | 152 or 1070                                     | 1346                                     | 228                                    | 1574                                | 1675                        | 15                                                             |
| 380 or 1150                                           | 234 or 1000                                     | 1905                                     | 330                                    | 2235                                | 2335                        | 15                                                             |
| 457 or 1070                                           | 305 or 915                                      | 2385                                     | 407                                    | 2792                                | 2870                        | 14                                                             |
| 530 or 1000                                           | 380 or 840                                      | 2770                                     | 457                                    | 3227                                | 3275                        | 14                                                             |
| 610 or 915                                            | 457 or 762                                      | 3063                                     | 508                                    | 3581                                | 3580                        | 14                                                             |
| 685 or 840                                            | 530 or 685                                      | 3226                                     | 533                                    | 3759                                | 3750                        | 14                                                             |
| 762                                                   | 610                                             | 3302                                     | 533                                    | 3835                                | 3840                        | 14                                                             |

In Fig. 5.26 the curve  $ACB$ , which is the mirror image of curve  $ADB$  represents, greatly magnified, the shape of the deflected roll contour. If both the top and bottom rolls were ground to this camber and a force of 574 t was uniformly distributed across the full length of the roll barrel, then the curve  $ACB$  would deflect to the horizontal line  $AB$ , i.e. the upper and lower rolls in the mill would be parallel across the full length of their barrels.

Slightly different conditions occur when rolls are loaded with uniformly distributed forces, acting not over the full length of the roll barrel, but over a given length of the barrel only. In this case, the component deflections of that portion of the roll face in contact with rolled stock of width  $b$  are given by

$$m_{yB} = \frac{P_x}{384EI} \left[ 8(3L^2 - 2b^2)(x - n) - 32(x^3 - n^3) + \frac{1}{b}(2x - L - b)^4 \right] \quad (5.59)$$

and

$$m_{yS} = \frac{P_x}{2\pi GD^2} \left[ \frac{4x(L - x) - (L - b)^2 - 4bn}{b} \right] \quad (5.60)$$

where  $x$  lies between  $1/2(L - b)$  and  $L/2$ . Hence, the total deflection is

$$m_y = m_{yB} + m_{yS}$$

The index  $m$  indicates that this deflection is for a parallel roll and not for a cambered roll. To maintain the thickness over the cross-section of the rolled stock it is only necessary to find the roll deflection at the centre and edges of rolled stock.

$$m_{yS_{\max}} = 1.4 \times 10^{-5} P_x$$
$$m_{Y \max} = 8.654 \times 10^{-5} P_x = Y_B \quad (5.61)$$

Technical drawing of a shaft with a crown, showing dimensions and deflection curves. The shaft has a total length of 1524 mm, with a central section of 1219.2 mm. The diameter is  $\phi 508$ . The crown height is 63.5 mm. The shaft is supported by bearings with reaction forces  $R_1$  and  $R_2$ . The crown is defined by a radius  $R_x = 412t$ . The deflection curves are shown as  $m_y$  (solid line) and  $Deflection$  (dashed line). The vertical distances from the crown to the deflection curves are  $Y_A - Y_B = 0.0275$  and  $Y_D - Y_E = 0.279$ . The vertical distances from the crown to the deflection curves are  $Y_E = 457.2$  and  $Y_D = 0.383$ . The vertical distances from the crown to the deflection curves are  $Y_B = 0.356$  and  $Y_A = 0.383$ . The horizontal distances from the crown to the deflection curves are  $x_1$  and  $x$ . The horizontal distances from the crown to the deflection curves are 762 and 609.6. The horizontal distances from the crown to the deflection curves are 152.4 and 152.4. The horizontal distances from the crown to the deflection curves are 1219.2 and 1219.2. The horizontal distances from the crown to the deflection curves are 1524 and 1524.

$$m_{yBedge} = 5.194 \times 10^{-5} P_x, \quad m_{ySedge} = 0.932 \times 10^{-5} P_x$$
$$m_{Y_{\text{edge}}} = 6.126 \times 10^{-5} P_x = Y_E \quad (5.62)$$

$Y_E$  = deflection at the edges of the stock using parallel rolls.

From this diagram it can be seen that the condition necessary to produce a parallel cross-section is

$$Y_D - Y_E = Y_A - Y_B \quad (5.63)$$

From Table 5.10, when  $x = 457$  mm, i.e. the distance from reaction  $R$  to edge of stock in Fig. 5.28

$$Y_D = 0.279 \text{ mm} = 0.0279 \text{ cm}$$

and when  $x = 762$  mm, i.e. halfway along the roll barrel

$$Y_A = 0.383 \text{ mm} = 0.0383 \text{ cm}$$

Substituting these values for  $Y_A$  and  $Y_D$  into equations (5.61) to (5.63), then

$$0.0279 - 6.126 \times 10^{-5} P_x = 0.0383 - 8.654 \times 10^{-5} P_x$$

and hence

$$P_x = 412 \text{ t}$$

This load uniformly distributed over the 610 mm wide stock will give parallel roll surfaces over the full length of the 1220 mm roll barrel. From equation (5.62) the deflection at edge of stock will be

$$Y_E = 6.126 \times 10^{-5} \times 412 = 0.02519 \text{ cm} = 0.2519 \text{ mm}$$

and at centre of the roll barrel from equation (5.61)

$$Y_B = 8.654 \times 10^{-5} \times 412 = 0.0356 \text{ cm} = 0.356 \text{ mm}$$

At point  $E$  in Fig. 5.28 where the edges of the stock contact the roll face,  $Y_D = 0.2794$  mm (Table 5.10). Hence, the surface of the roll at this point lies below the horizontal line, where

$$Y_D - Y_E = 0.2794 - 0.2519 = 0.0275 \text{ mm}$$

At centre of roll barrel (point  $C$ ) the deflection is

$$Y_A = 0.3835 \text{ mm}$$

hence displacement below line  $AB$  is

$$Y_A - Y_B = 0.3835 - 0.356 = 0.0275 \text{ mm}$$

Thus it can be seen that the top and bottom rolls are parallel over a distance of 610 mm, as shown in Fig. 5.28, which enables a strip with parallel surfaces to be rolled.

From Fig. 5.29 [160], in which

$$Y_B - Y_A = Y_1 \quad \text{and} \quad Y_E - Y_D = Y_2$$

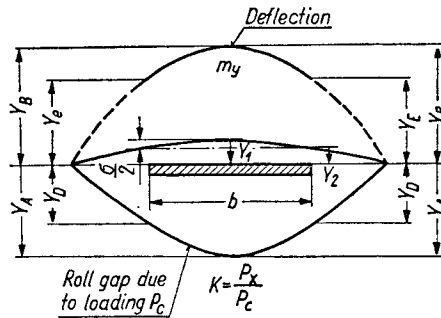


Fig. 5.29. Cross-sectional thickness variation when  $K$  is too large

the total change of thickness  $\delta$ , across the width of the rolled stock  $b$ , is

$$\delta = 2(Y_1 - Y_2) \quad (5.64)$$

For rolled stock 610 mm wide,  $Y_A$  and  $Y_D$  are read from Table 5.10

$$Y_A = 0.3835 \text{ mm} \quad \text{and} \quad Y_D = 0.2794 \text{ mm}$$

It will be seen from equations (5.61) and (5.62) that  $Y_B$  and  $Y_E$  are linear functions of the roll force  $P_x$ . Hence, if two values of roll force, together with the corresponding values of thickness variation  $\delta$  are plotted as a straight line, other values of roll force and associated thickness variation can be found for a strip of width 610 mm.

If the ratio  $P_x/P_c$  is defined as

$$K = \frac{P_x}{P_c}$$

(5.65)

then for the known values of  $P_x = 412$  t and  $P_c = 574$ , the ratio

$$K = \frac{412}{574} = 0.718$$

This value of  $K$  corresponds to the condition shown in Fig. 5.28, where the variation in thickness  $\delta$  equals zero over the full width of the rolled stock.

When no roll force is developed, i.e.  $P_x$  and  $K$  each equal zero, and there is no movement in the roll gap, then  $Y_B = Y_E = 0$ .

Then from equation (5.64) the change of thickness

$$\delta = -2(Y_A - Y_D) = -0.208 \text{ mm}$$

In plotting the curve in Fig. 5.30, the values of

|             |     |        |
|-------------|-----|--------|
| $P_x$ t     | 412 | 0      |
| $\delta$ mm | 0   | -0.208 |

have been used. From this curve the variation in thickness across 610 mm wide stock can be read for values of  $K$  between 0.2 and 1.0. The resulting cross-sectional shapes, much exaggerated, are drawn in Fig. 5.31. From this figure it can be seen that change of  $K$  or  $P_x$  gives a change of camber. The only value of  $K$ , for which 610 mm wide stock can be rolled dead flat, is  $K = 0.718$ .

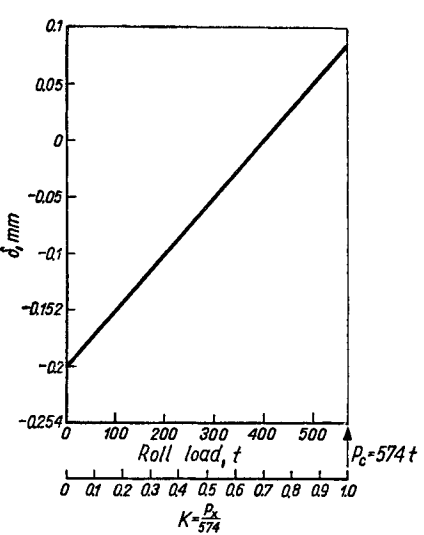


Fig. 5.30. Linear relation between rolling load and variation of cross-sectional thickness

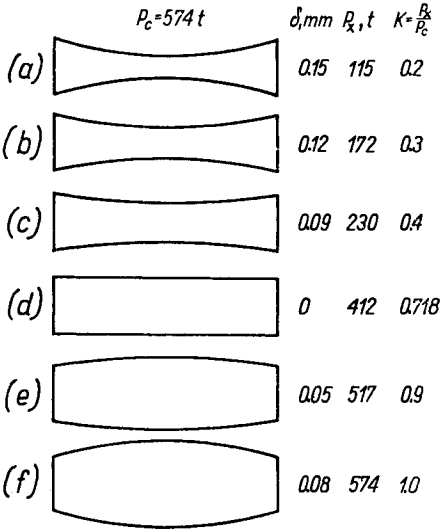


Fig. 5.31. Variation of cross-sectional thickness  $\delta$ , due to change of roll force  $P_x$ , width 610 mm  
 $P_x = 574$  tons [160]

It has been shown, cf. Figs. 5.30 and 5.31, that if  $K$  is greater or less than the optimum value, e.g. 0.718 for the given case, the cross-section of the rolled material will not be parallel sided.

From Fig. 5.29 it can also be seen that the extent of the variation from the parallel depends on the displacement of  $Y_1$  relative to  $Y_2$ . When  $Y_1$  is greater than  $Y_2$ , the value of  $K$  is too high and the cross-section of the rolled stock will be thicker at the centre, as shown in the two last sections in Fig. 5.31. In order to obtain a parallel-sided section, i.e.  $\delta = 0$ ,  $Y_2$  and  $Y_1$  must lie in the same horizontal plane, i.e. as from Fig. 5.29, the following condition must be fulfilled:

$$(Y_B - Y_A) - (Y_E - Y_D) = 0 \quad (5.66)$$

Since  $Y_B - Y_A = Y_1$  and  $Y_E - Y_D = Y_2$  this condition becomes

$$Y_1 - Y_2 = 0$$

In equation (5.66)

$$\begin{aligned} Y_A &= c_{yB\max} + c_{yS\max} && \text{from equation (5.54)} \\ Y_B &= c_{yB} + c_{yS} && \text{for } x = L - b \quad \text{from equation (5.51)} \\ Y_D &= m_{yB\max} + m_{yS\max} && \text{for } x = L/2 \quad \text{from equations (5.59) and (5.60)} \\ Y_E &= m_{yB\max} + m_{yS\max} && \text{for } x = L - b \quad \text{from equations (5.59) and (5.60)} \end{aligned}$$

Thus  $Y_B$  and  $Y_E$  are each functions of  $P_x$ , while  $Y_A$  and  $Y_D$  are functions of  $P_c$ , so if  $K = P_x/P_c$ , equation (5.66) can be solved to find  $K$ , the solution being

$$K = \frac{(6 - 24\alpha^2 - \beta^2) + z}{(1 - 2\alpha)\left(12 - 7\beta + \frac{z}{\beta}\right)} \quad (5.67)$$

where

$$\alpha = \frac{n}{L}, \quad \beta = \frac{b}{L} \quad \text{and} \quad z = 7.5 \frac{D^2}{L^2}$$

In this formula, the modulus of rigidity  $G$  of the roll, is taken as  $G \approx 3/8 E$ , and other symbols as previously. Equation (5.67) gives the condition required to obtain parallel-sided cross-sections of the rolled stock.

Assuming that the camber curve is parabolic, then according to Larke [62, 160], equations (5.49) and (5.50) can be written in the form

$$c_y = \frac{P_c}{3\pi ED} \left[ 10l + 48n + \frac{18D^2}{l} \right] (l - x_1) x_1 \quad (5.68)$$

where  $x_1 = 0$  at the beginning of the roll barrel and  $x_1 = l/2$  midway along the barrel length. When  $x_1 = 0$ , then also  $c_y = 0$ . Applying equation (5.68) to the previously solved example, where  $P_c = 574$  t, the following simple expression is obtained

$$c_y = 1.03 \times 10^{-5} (122 - x_1) x_1 \quad (5.69)$$

Similarly the general equations (5.59) and (5.60), can be replaced by

$$m_y = \frac{P_x}{3\pi ED^2} [16l^2(l + 3n) - 2b^2(4l - b) + 15D^2(2l - b)] (l - x_1) x_1 \quad (5.70)$$

There are different conditions for hot rolling, as roll deflection and thermal expansion occur simultaneously. The factors influencing the shape of the roll gap have already been considered at the beginning of this chapter.



In the following numerical example, the shape of the roll gap is determined for a 2-high mill for hot rolling of sheet with various widths.

Using uncooled rolls, temperatures of 360–400°C occur at the middle of the roll barrel and approximately 275°C for the ends [62, 160], i.e. there is an operating temperature gradient of 85°C.

Maximum increase of diameter will occur at the middle of the roll barrel, the radial increase being given by

$$y_{t \max} = \frac{1}{2} D \alpha (t_m - t_e)$$

where

$\alpha$  = linear coefficient of thermal expansion of roll material,

$t_m$  = temperature at middle of roll barrel, °C,

$t_e$  = temperature at ends of the roll barrel, °C.

Assuming that these temperature conditions are stable, then to roll flat sheets the rolls should be ground concave, as shown in Fig. 5.20. After reaching the operating temperature the barrel becomes slightly convex due to differences in thermal expansion. When the roll deflects under the action of the roll force arising during rolling, a constant roll gap is formed, giving parallel-sided sheets with only insignificant variations in thickness between centre and edge.

The form of the required concave curve can be written as

$$y_t = k(l - x_1)x_1$$

When  $x_1 = l/2$ , i.e. at the middle of the roll barrel, the maximum concavity is

$$y_{t \max} = -R\alpha(t_m - t_e) = -\alpha R \Delta t$$

where  $R$  = roll radius at the end of the roll barrel.

Comparing these equations

$$k = -\frac{2D\alpha(t_m - t_e)}{l^2}$$

The equation for the concave camber needed to offset the effect of the temperature gradient only is, therefore

$$y_t = \frac{-2D\alpha(t_m - t_e)(l - x_1)x_1}{l^2} \quad (5.71)$$

A negative value signifies that the required camber is concave.

As an example of the use of this equation a cast-iron roll with dimensions shown in Fig. 5.32 will be considered, the temperatures of the middle and end of the roll barrel being 360°C and 275°C, respectively, with the coefficient of expansion

$$\alpha = 12.8 \times 10^{-6} \text{ per } ^\circ\text{C}$$

Substituting these values, and the dimensions of  $D$  and  $l$  given in Fig. 5.32, then equation (5.71) reduces to

$$y_t = 1.1 \times 10^{-5}(x_1 - 127)x_1 \quad (5.72)$$

The calculated values are shown in column 3 of Table 5.11. The curve to which the rolls must be ground to offset the rise in temperature is obtained by plotting  $y_t$  against  $x_1$  as shown in Fig. 5.32.

Both temperature and deflection effects follow a parabolic law, therefore the total deflection can be expressed as

$$c_y = 40.5 \frac{b}{lD^4} (l-x_1)x_1 \quad (5.73)$$

Assuming the width of the sheet to be  $b = 1000$  mm, equation (5.73) becomes

$$c_y = 0.75 \times 10^{-6} (127 - x_1)x_1$$

Using this equation the convex camber needed to offset the deflection of the roll has been calculated (see column 4 in Table 5.11).

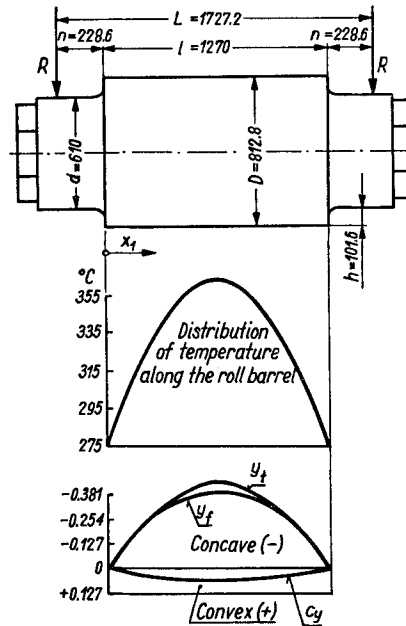


Fig. 5.32. Concave roll camber to offset high temperature gradient for a sheet mill

Both curves are included in Fig. 5.32. The thick-lined concave curve  $y_f$  found by subtracting the curve of deflection  $c_y$  from that due to thermal expansion, represents the final radial camber, to which both rolls should be ground. When the temperature gradient shown in Fig. 5.32 has been attained, the stock rolled will be flat.

From the maximum deflection of rolls of 0.03 mm, the maximum load when rolling 1000 mm wide material is calculated as 140 t.

Assuming that the distribution of temperature along the roll barrel is of parabolic form, then knowing the values of  $t_m$  and  $t_e$  the temperature  $t$  at any point  $x_1$  along the barrel can be expressed as [62, 160]

$$t = t_e + \frac{4(t_m - t_e)}{l^2} (l - x_1)x_1 \quad (5.74)$$

For this example equation (5.74) becomes

$$t = 275 + 0.021 (127 - x_1)x_1 \quad (5.75)$$

TABLE 5.11  
TEMPERATURE GRADIENT AND DEFLECTION OFFSET BY CONCAVE  
CAMBER OF AN 815 × 1270 mm ALLOY IRON ROLL AS IN FIG. 5.32

| Distance<br>along roll barrel<br>$x_1$<br><br>mm | Temperature<br>along roll<br>barrel*<br><br>$t$<br><br>°C | Amount of camber to<br>offset, mm                      |                                                      | Resultant<br>camber, mm<br><br>concave‡‡<br><br>$y_f \times 10^4$ |
|--------------------------------------------------|-----------------------------------------------------------|--------------------------------------------------------|------------------------------------------------------|-------------------------------------------------------------------|
|                                                  |                                                           | due to<br>temperature<br>concave†<br>$y_t \times 10^4$ | due to<br>deflection<br>convex‡<br>$c_y \times 10^4$ |                                                                   |
| 0 or 1270                                        | 275                                                       | 0                                                      | 0                                                    | 0                                                                 |
| 50.8 or 1220                                     | 288                                                       | 685                                                    | 51                                                   | 634                                                               |
| 101.6 or 1170                                    | 300                                                       | 1295                                                   | 102                                                  | 1193                                                              |
| 203.2 or 1070                                    | 321                                                       | 2387                                                   | 152                                                  | 2235                                                              |
| 305 or 965                                       | 337                                                       | 3225                                                   | 229                                                  | 2996                                                              |
| 407 or 863                                       | 349                                                       | 3880                                                   | 254                                                  | 3606                                                              |
| 508 or 762                                       | 357                                                       | 4280                                                   | 280                                                  | 4000                                                              |
| 558 or 712                                       | 359                                                       | 4370                                                   | 305                                                  | 4065                                                              |
| 635                                              | 360                                                       | 4430                                                   | 305                                                  | 4125                                                              |

\*  $t = 275 + 0.021(127 - x_1)x_1$ , †  $y_t = 1.1 \times 10^{-5}(x_1 - 127)x_1$

‡ From equation  $c_y = 0.75 - 10^{-6}(127 - x_1)x_1$

‡‡ From equation  $y_f = y_t - c_y$

Temperature values calculated from this equation are shown in Table 5.11, column 1, and plotted against  $x_1$  in Fig. 5.32. Larke [62, 160] does not give numerical examples for hot and cold rolling in modern 4-high mills. Therefore the method of Emicke and Lukas [149] based on A. Winkler is used [149]. Elastic and thermal deformations of work and back-up rolls for 4-high mills are considered [149]. The rolls shown in Fig. 5.33 are for rolling of plates up to 2000 mm wide. Work rolls are of diameter 700 mm and back-up rolls of diameter 1100 mm, and barrel length of 2300 mm.

Roll deflection due to bending stresses:

- (1) at the point  $x = n$  (end of the roll barrel)

$$c_{yB1} = \frac{16lbn}{(E_1d^4 + E_2D^4)3\pi} \left[ \frac{3L^2}{4} - \frac{b^2}{4} - n^2 \left( 3 - \frac{2E_1E_2d^4D^4}{E_1d_1^4} \right) \right] \quad (5.76)$$

- (2) at the point  $x = c$  (edge of the rolled stock)

$$c_{yB2} = \frac{4lb}{(E_1d^4 + E_2D^4)3\pi} \left[ L^3 + b^3 + 2Lb^2 + 8n^3 \left( \frac{E_1d^4 + E_2D^4}{E_1d_1^4} - 1 \right) \right] \quad (5.77)$$

- (3) at the point  $x = L/2$  (middle of roll barrel)

$$c_{yB3} = \frac{lb}{(E_1d^4 + E_2D^4)6\pi} \left[ 8L^3 - 4Lb^2 + b^3 + 64n^3 \left( \frac{E_1d^4 + E_2D^4}{E_1d_1^4} - 1 \right) \right] \quad (5.78)$$

Roll deflection due to shear stresses:

- (1) at the point  $x = n$

$$c_{yS1} = \frac{2lbn}{G_1d_1^2\pi} \quad (5.79)$$

- (2) at the point  $x = c$

$$c_{yS2} = \frac{lb}{(G_1d^2 + G_2D^2)\pi} \left[ L - b - 2n \left( \frac{G_1d^2 + G_2D^2}{G_1d_1^2} - 1 \right) \right] \quad (5.80)$$

(3) at the point  $x = L/2$

$$c_{yS3} = \frac{lb}{(G_1 d^2 + G_2 D^2) \pi} \left[ L - \frac{b}{2} - 2n \left( \frac{G_1 d^2 + G_2 D^2}{G_1 d_1^2} - 1 \right) \right] \quad (5.81)$$

where (see Table 5.6)

$E_1$  and  $G_1$  = moduli of elasticity and rigidity of the material of back-up rolls,

$E_2$  and  $G_2$  = moduli of elasticity and rigidity of the material of work rolls,

$d$  = back-up roll diameter,

$d_1$  = roll neck diameter of the back-up roll,

$c$  = distance between sheet or strip edge and reaction,

$D$  = work roll diameter.

The back-up rolls are of Cr-Ni steel with

$$E_1 = 2,200,000 \text{ kg/cm}^2 \quad \text{and} \quad G_1 = 848,000 \text{ kg/cm}^2$$

The work rolls are of chilled iron with

$$E_2 = 1,340,000 \text{ kg/cm}^2 \quad \text{and} \quad G_2 = 552,000 \text{ kg/cm}^2$$

The roll force is 1000 t and all values in Fig. 5.33 are related to this force.

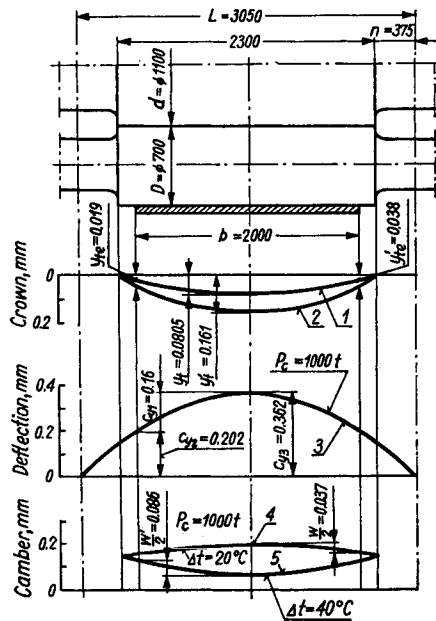


Fig. 5.33. Determination of roll camber for a 4-high mill [149]: 1 — deflection along the barrel at 20°C, 2 — deflection along the barrel at 40°C, 3 — total deflection of work and back-up rolls due to bending and torque, 4 — difference of camber between the roll end and the given point, measured along the barrel, for a temperature gradient of 20°C between centre and end of rolls, 5 — as 4, with temperature gradient 40°C

Plain cylindrical rolls, i.e. ground without camber, have been assumed, and a temperature gradient of 20 or 40°C between middle and ends of the work roll barrel.

The effect of temperature variation along the roll barrel on the geometrical shape of the roll is calculated from equation (5.71), and shown on curve  $y_t$  in Fig. 5.33.

A temperature difference of  $\Delta t = 20^\circ\text{C}$  along the roll barrel gives a thickness variation between centre and edge of the rolled material of

$$y_t - y_{te} = 0.0805 - 0.019 = 0.0615 \text{ mm (curve 1)}$$

as shown in Fig. 5.33.

A temperature difference of  $\Delta t = 40^\circ\text{C}$  along the roll barrel, gives a thickness variation of

$$y'_t - y'_{te} = 0.161 - 0.038 = 0.123 \text{ mm (curve 2)}$$

The temperature of the back-up rolls has been assumed to be constant over the full length of the barrel.

Curve 3 in Fig. 5.33 shows the total deflection of work and back-up rolls due to bending and shearing stresses. For a roll force of 1000 t the thickness variation between centre and edge of the sheet is

$$c_{y1} = c_{y3} - c_{y2} = 0.362 - 0.202 = 0.16 \text{ mm (curve 3)}$$

Since it is assumed that only the work rolls are subject to thermal expansion, therefore twice the value of their convex camber must be taken [149].

Hence

$$\frac{w}{2} = c_{y1} - 2y_t \quad (5.82)$$

For a roll force of 1000 t, temperature gradient of  $20^\circ\text{C}$ , and a 2000 mm wide plate, the thickness variation between edge and centre of the plate due to deflection and thermal expansion is

$$\frac{w}{2} = c_{y1} - 2(y_t - y_{te}) = 0.16 - 2 \times 0.0615 = 0.037 \text{ mm (curve 4)}$$

For a temperature gradient of  $\Delta t = 40^\circ\text{C}$

$$\frac{w'}{2} = 0.086 \text{ mm (curve 5)}$$

The thickness variation of the plate has been plotted in Fig. 5.34 as a function of plate width. Curve 3 gives the difference in deflections of the work roll determined by curve 1 and 2, between centre and

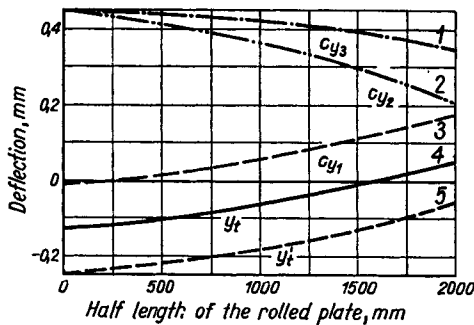


Fig. 5.34. Distribution of roll deflection and thermal expansion over the half length of the roll barrel: 1 — deflection midway along the barrel length, 2 — deflection at the edge of the rolled sheet, 3 — variation in deflection over the sheet width, 4 — roll camber due to thermal expansion at  $\Delta t = 20^\circ\text{C}$ , 5 — roll camber due to thermal expansion at  $\Delta t = 40^\circ\text{C}$

edge of the plate. The influence of thermal expansion based on temperature gradients of 20 or  $40^\circ\text{C}$  along the roll barrel, is shown by both curves 3 and 4. It can be seen from Fig. 5.34 that for a total roll force of 1000 t the influence of deflection (curve 3) and thermal expansion (curve 4) is offset for a width of approximately 1650 mm ( $b/2 = 825 \text{ mm}$ ). i.e. under these conditions a parallel roll gap is maintained, giving a parallel rolled plate.



whereas torque values (5.84) show a tendency to decrease in the passes following annealing.

In hot rolling the stock cools gradually during rolling with a consequent increase of resistance to deformation. A similar effect occurs during cold rolling, due to work-hardening with increasing deformation. Also for both hot and cold rolling, as the thickness of the strip is reduced the frictional forces between roll surface and stock increase, with a corresponding increase in resistance to deformation. Due to this dual increase in resistance to deformation with increasing deformation during both hot and cold rolling, the draught must be reduced with increasing number of passes.

If the characteristics of the main driving motor are known (Fig. 4.151), then the rolling programme should be planned to give optimum motor loading in every pass.

The draught should be gradually reduced to compensate for the effect of the increasing resistance to deformation on the roll force  $P$  and on the roll work  $A$

$$A = aK_{wm}V \frac{h_1 - h_2}{h_1} \quad (5.86)$$

where

$a$  = coefficient,

$V = b_1 h_1 v_1 = b_2 h_2 v_2$  = volume of material transferred in unit time.

The relation between resistance to deformation  $K_{wm}$  and total roll force  $P$  is given by (2.108) and (4.12)

$$P = K_{wm} b_m \sqrt{(h_1 - h_2) R}$$

To maintain a constant roll force  $P$  the absolute draught must be reduced in proportion to the square of the mean resistance to deformation  $K_{wm}$ . This can be expressed mathematically as follows:

$$\frac{\Delta h_I}{\Delta h_{II}} = \frac{(h_1 - h_2)_I}{(h_2 - h_3)_{II}} = \left( \frac{K_{wII}}{K_{wI}} \right)^2 \quad (5.87)$$

To solve this equation the variation of resistance to deformation  $K_{wm}$  in the given rolling conditions must be known. Instead of this complicated calculation, the total roll force  $P$  in each pass can be measured directly, and a correction factor for practical conditions applied.

For cold rolling, although the roll pressure and energy consumption should be kept constant, the draught should be reduced in the last passes to prevent damage to the metal. This may also be necessary during hot rolling, if a drop in rolling temperature causes a significant decrease in the ductility of the metal. Zaroshchinski [111] gives the following method for blooming mills. The roll force is given by

$$P = K_{wm} b_m \sqrt{R(h_1 - h_2)}$$

$K_{wm}$  is the mean resistance to deformation determined from some known method and  $b_m$  is the mean width of the bloom. Substituting this expression for  $P$  into equation (4.113) gives an equation for the roll torque

$$M_w = 1.05 K_{wm} b_m \sqrt{R(h_1 - h_2)} [\sqrt{R(h_1 - h_2)} + kR] \quad (5.88)$$

where  $df_1$  from equation (4.114) is denoted by  $kR$ .

From this equation the absolute draught can be found as a function of  $K_{wm}$ ,  $b_m$ ,  $R$  and  $M_w$ . Substituting

$$1.05K_{wm}b_m = k_1 \quad \text{and} \quad \sqrt{R(h_1 - h_2)} \approx l_d$$

into equation (5.88) then a quadratic equation for the length of contact  $l_d$  is found

$$M_w = k_1 l_d^2 + k_1 k R l_d$$

Solving this equation

$$(h_1 - h_2) = \left[ \frac{(kR)^2}{4} - kR \sqrt{\frac{(kR)^2}{4} + \frac{M_w}{k_1}} + \frac{(kR)^2}{4} + \frac{M_w}{k_1} \right] \frac{1}{R}$$

Terms in  $(kR)^2/4$  are very small and can be neglected. Assuming  $m \approx 0.75$  as most often used for blooming mills then  $k \approx 0.110$ . For example, with a roll radius of  $R = 500$  mm,  $(kR)^2/4 = 755$ , and dividing by  $R$  gives a value  $\approx 1.50$ . Thus the sum of both terms in  $(kR)^2/2$  is 3.0, and the term  $(kR)^2/4$  under the root sign will be even smaller. Neglecting these values, the previous equation becomes

$$(h_1 - h_2) = \frac{1}{R} \left( \frac{M_w}{1.05K_{wm}b_m} - kR \sqrt{\frac{M_w}{1.05K_{wm}b_m}} \right) \quad (5.89)$$

The value of  $kR$  varies for different rolling mills and for different grooves of the same stand. Therefore for accurate calculation of absolute draughts, values for  $k$  and  $R$  corresponding to given rolling conditions should be chosen. For practical purposes a mean value of  $kR$  corresponding to mean rolling conditions can be assumed, since the error relative to exact values of absolute draught of  $h_1 - h_2$  is negligible (2–5%). For example, a blooming mill with nominal diameter of rolls of 1000–1100 mm, will have a working roll diameter at the bull-head and box passes of 750–1000 mm, allowing for normal roll wear of about 10%. Hence the average working roll diameter will be 900 mm, and for  $k = 0.110$  the term  $kR \approx 50$ , and the equation (5.89) can be written

$$(h_1 - h_2) = \frac{1}{R} \left( \frac{M_w}{1.05K_{wm}b_m} - 50 \sqrt{\frac{M_w}{1.05K_{wm}b_m}} \right) \quad (5.90)$$

Using this equation the maximum permissible absolute draught for the given rolling conditions can be determined. It can be seen from equation (5.90) that the absolute draught depends on the magnitude of the term  $K_{wm}b_m$ . The greater this value the smaller the draught required to maintain a constant motor torque. The absolute draught is also inversely proportional to the roll diameter.

The maximum roll torque, corresponding to the maximum loading of the driving motor, is given by the equation (4.105)

$$M_w = M_{\text{mot}} - M_d - M_i$$

For calculation, the value of  $M_{\text{mot}}$  is assumed to be about 20% smaller than the cut-out torque, leaving a necessary reserve for unforeseen cases of overloading of the motor, e.g. exceeding the calculated draught when rolling ingots at too low a rolling temperature etc.

The method described here can be used to determine analytically the maximum permissible values of draught for a given motor power. However, it is often more convenient to use a graphical method. Motors with a power of about 5200 kW are most often used for driving of blooming mills, with maximum torque equal to 300 tm



at 50 rpm. At greater rpm the torque decreases proportionally. The maximum allowable working torque of this motor can be assumed at 250 tm ( $300 \times 0.83$ ).

From operational experience, for acceleration of rotating elements of the roll stand and rotor of the driving motor, a torque of 1.9 tm per 1 rpm/sec is required. For modern mills values for acceleration of 40 rpm/sec, and deceleration of 80 rpm/sec can be assumed. For the given motor an acceleration of 20 rpm/sec and a deceleration of 40 rpm/sec can be taken. Hence

$$M_d = 1.9 \times 20 = 38 \text{ tm}$$

The idling torque of the stand and motor is assumed as equal to

$$M_i = 5 \text{ tm}$$

Therefore the allowable roll torque is

$$M_w = 250 - 38 - 5 = 207 \text{ tm}$$

corresponding to a speed of 50 rpm. For a greater speed e.g. 60, 75 or 90 rpm, the following roll torques are obtained

| $n =$   | 60  | 75  | 90 rpm |
|---------|-----|-----|--------|
| $M_w =$ | 165 | 124 | 96 tm  |

Based on these values of  $M_w$  diagrams for determination of absolute draught  $h_1 - h_2$  as a function of  $K_{wm} b_m$  for a roll diameter of  $D = 1000$  mm, can be plotted. It can be seen from equation (5.90) that these curves can be used for other roll diameters when multiplied by an appropriate factor.

Similarly, these curves can be used for other speeds, taking mean values of  $h_1 - h_2$ . Values of absolute draughts are given in Table 5.13, and Fig. 5.35 shows  $h_1 - h_2$  plotted against  $K_{wm} b_m$ .

TABLE 5.13  
ABSOLUTE DRAUGHTS AS A FUNCTION OF TORQUE OF THE DRIVING MOTOR

| $K_{wm} b_m$<br>kg/mm | $h_1 - h_2$ , mm                   |                                    |                                    |                                   |
|-----------------------|------------------------------------|------------------------------------|------------------------------------|-----------------------------------|
|                       | $M_w = 207 \text{ tm}$<br>$n = 50$ | $M_w = 165 \text{ tm}$<br>$n = 60$ | $M_w = 124 \text{ tm}$<br>$n = 75$ | $M_w = 96 \text{ tm}$<br>$n = 90$ |
| 1000                  | 350 *                              | 274 *                              | 200 *                              | 150 *                             |
| 1500                  | 226 *                              | 177 *                              | 128 *                              | 97 *                              |
| 2000                  | 166 *                              | 129 *                              | 94 *                               | 70 *                              |
| 2500                  | 129 *                              | 100                                | 73                                 | 54                                |
| 3000                  | 89                                 | 69                                 | 49                                 | 36                                |
| 4000                  | 77                                 | 59                                 | 42                                 | 31                                |
| 4500                  | 67                                 | 51                                 | 36                                 | 27                                |
| 5000                  | 59                                 | 45                                 | 32                                 | 23                                |
| 5500                  | 53                                 | 40                                 | 28                                 | 20                                |
| 6000                  | 48                                 | 36                                 | 25                                 | 18                                |
| 6500                  | 43                                 | 33                                 | 22                                 | 16                                |
| 7000                  | 39                                 | 30                                 | 20                                 | 15                                |
| 7500                  | 36                                 | 27                                 | 18                                 | 14                                |
| 8000                  | 34                                 | 25                                 | 17                                 | 13                                |

\* Conditions not occurring in practice.

If the value of  $K_{wm}b_m$  is known, the permissible absolute draught for a given pass can be read off directly from this diagram for the required speed. Similar curves can be drawn also for other rolling mills driven by motors of different characteristics.

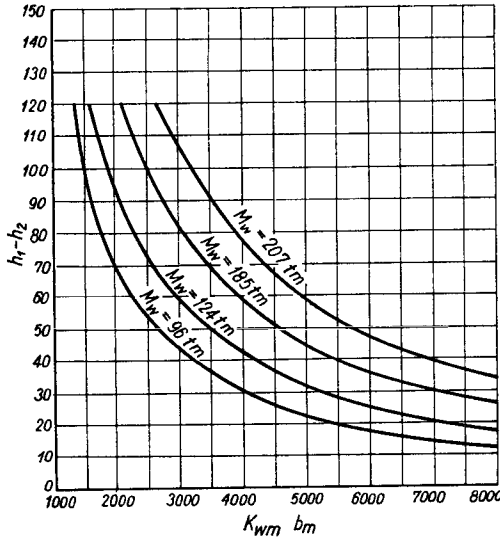


Fig. 5.35. Absolute draught  $h_1-h_2$  as a function of  $K_{wm}b_m$

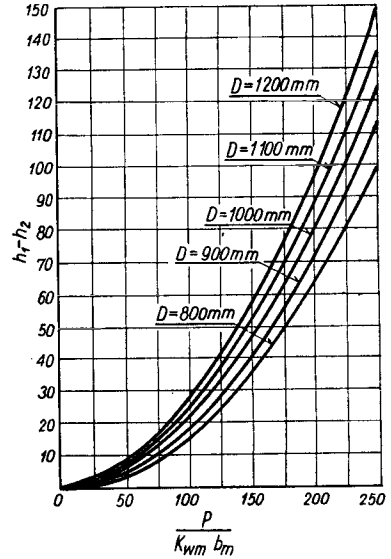


Fig. 5.36. Permissible absolute draught  $h_1-h_2$  as a function of roll diameter

The absolute draught can also be determined as a function of roll strength.

At the point of action of the roll force at a distance  $x$  from the centre of roll neck the B.M. in the roll is (5.11)

$$M_b = P \frac{x(L_1 - x)}{L_1}$$

where  $L_1$  = distance between reactions acting at the centres of roll necks.  
Substituting this value of bending moment into equation (5.11)

$$P = \sigma_b W_b \frac{L_1}{x(L_1 - x)}$$

or

$$P = 0.1 D^3 \sigma_b \frac{L_1}{x(L_1 - x)} \quad (5.91)$$

where

$\sigma_p$  = allowable bending stress for the roll material,  
 $W_b$  = bending modulus of the roll.

The roll force  $P$  found from equation (5.91) is the maximum to which the roll can be loaded for a given roll diameter  $D$ , and known permissible bending stress depending on roll material, and known position of the roll force. The following relation between absolute draught and roll force can be calculated from Zaroshchinski's equation [111]

$$h_1 - h_2 = \frac{1}{R} \left( \frac{P}{K_{wm}b_m} \right)^2 \quad (5.92)$$

All values in this equation are known, and therefore using equations (5.91) and (5.92) the permissible absolute draught relative to roll strength can be determined.

If in equation (5.92) the expression  $(P/K_{wm}b_m)$  is assumed as an independent variable, then the curve of absolute draught has a parabolic form (Fig. 5.36). These curves were drawn for roll diameters of 800–1200 mm and data for a roll diameter of 1000 mm are given in Table 5.14 [111].

TABLE 5.14  
VALUES FOR DRAUGHT DIAGRAMS (Fig. 5.36)

| $P/K_{wm}b_m$ | $(P/K_{wm}b_m)^2$ | $h_1-h_2$ , mm<br>for $D = 1000$ mm |
|---------------|-------------------|-------------------------------------|
| 50            | 2500              | 5                                   |
| 100           | 10,000            | 20                                  |
| 150           | 22,500            | 45                                  |
| 200           | 40,000            | 80                                  |
| 250           | 62,500            | 125                                 |
| 300           | 90,000            | 180                                 |

When rolling plates, blooms and slabs, the choice of draught depends on the composition of the metal of the rolled stock, the motor power and permissible loading of the components of the roll stands, particularly the rolls. Only in exceptional cases is the power of the main driving motor insufficient for draught values based on mechanical strength of the rolls. In such cases it is necessary to check the chosen draught scheme against motor loading.

The maximum absolute draught for 3-high Lauth mills can be found from equation

$$P = K_{wm}b_m \sqrt{\frac{D_1D_2}{D_1+D_2}}(h_1-h_2)$$

i.e.

$$h_1-h_2 = \frac{D_1+D_2}{D_1D_2} \left( \frac{P}{K_{wm}b_m} \right)^2$$

where

$D_1$  = diameter of the top or bottom roll,

$D_2$  = diameter of the middle roll.

Or substituting  $D_2/D_1 = i$

$$h_1-h_2 = \frac{i+1}{iD_1} \left( \frac{P}{K_{wm}b_m} \right)^2$$

The ratio of diameters  $i = 0.65-0.75$ , with an average 0.7. Assuming  $i \approx 0.7$  gives results with errors of up to  $\pm 4\%$ , which in practice are negligible. With this assumption the approximate equation becomes

$$h_1-h_2 = \frac{2.4}{D_1} \left( \frac{P}{K_{wm}b_m} \right)^2 \quad (5.93)$$

In this equation  $P$  denotes the maximum permissible roll force, for given  $K_{wm}b_m$  and  $D_1$ , and corresponds to the maximum absolute draught  $h_1-h_2$ . The roll force  $P$  can be expressed as a function of bending moment since

$$M_b = P \left( \frac{a}{4} - \frac{b_m}{8} \right) = \frac{P}{8} (2a - b_m)$$

therefore

$$P = \frac{8M_b}{2a - b_m}$$

In this equation  $a$  is the distance between the reactions which act at the middle of the roll necks (barrel length + length of one roll neck), and  $b_m$  is the mean width of the rolled sheet.

If the known relation

$$M_b = 0.1 \sigma_{b_{per}} D_1^3$$

is substituted into the preceding equation

$$P = \frac{0.8 \sigma_{b_{per}} D_1^3}{2a - b_m} \quad (5.94)$$

Putting  $b_m/a = m$ ,  $a/D_1 = n$ , and  $K = 0.8 \sigma_{b_{per}}$  into equation (5.94)

$$P = \frac{K a^2}{n^3 (2 - m)}$$

From Table 5.6, for chilled iron rolls  $\sigma_{b_{per}} = 600 \text{ kg/cm}^2$ , and for cast steel rolls  $\sigma_b = 1000 \text{ kg/cm}^2$  can be assumed. Substituting values of  $P$ ,  $D_1$ ,  $b_m$  and  $\sigma_{b_{per}} = 6 \text{ kg/mm}^2$  into equation (5.93), then for chilled iron rolls

$$\Delta h = h_1 - h_2 = \frac{38.4}{n^5} \frac{a}{[K_{wm} m (2 - m)]^2} \quad (5.95)$$

From equation (5.94) for rolling of sheet the permissible absolute draught can be expressed as a function of  $m$ ,  $n$ ,  $a$ ,  $i$  and  $K_{wm}$  as follows:

$$\Delta h = h_1 - h_2 = \frac{(1+i) a K^2}{n^5 i} \left[ \frac{1}{m(2-m) K_{wm}} \right]^2 \quad (5.96)$$

Using equation (5.96) a nomogram for the determination of permissible absolute draughts for the given rolling mill can be worked out as a function of loading per unit length of the roll barrel. This nomogram consists of a family of curves, corresponding to particular values of the product  $m(2-m)$ , which for practical purposes lie between 0.4 and 0.9.

Nomograms for determining roll force and torque during rolling of plates published by Emicke [106] are shown in Figs. 5.37 and 5.38.

#### Example 1

The nomograms shown in Figs. 5.37 and 5.38 for determination of roll force and torque during rolling of plates are applied. Data: initial thickness  $h_1 = 60 \text{ mm}$ , draught in the first pass 10%, rolling temperature  $1000^\circ\text{C}$ , roll diameter 1000 mm and width of the plate 1250 mm.

In part (a) of the nomogram in Fig. 5.37 the point corresponding to the initial thickness is marked on the vertical scale (point 1) from which a horizontal line is drawn to the curve of 10% draught (point 2). From this point a vertical line is drawn to point 3 at the intersection with the curve for temperature 1000°C in part (b). The horizontal line drawn through this point intersects the straight line for roll diameter of 1000 mm at point 4 in part (c). A vertical line is drawn through this point to give point 5 at the intersection with the straight line for plate width of 1250 mm. The horizontal line drawn through point 5 to the right-hand ordinate (point 6) gives the required roll force of 680 t.

Plotting this value in part (d) (point 7) and drawing a vertical line to the point of intersection with the straight line for diameter of 1000 mm gives point 8, from which a horizontal line to the left-hand ordinate gives point 9 where the bearing pressure of 1.32 kg/mm<sup>2</sup> can be read off. The corresponding point on the right-hand ordinate gives the bending stress in the roll barrel of 3.35 kg/mm<sup>2</sup> (point 10).

Similarly, the rolling torque can be found from Fig. 5.38. Knowing the initial thickness  $h_1 = 60$  mm and percentage draught 10%, points 1 and 2 are plotted in part (a), and a vertical line is drawn to intersect with the straight line for temperature of 1000° (point 3). A horizontal line through this point gives point 4 in part (c) on the straight line for roll diameter 1000 mm, and a vertical line drawn through point 4 to part (d) gives point 5 for a plate width of 1250 mm. The horizontal line from point 5 to the right-hand ordinate gives the required roll torque of 58 tm (point 6).

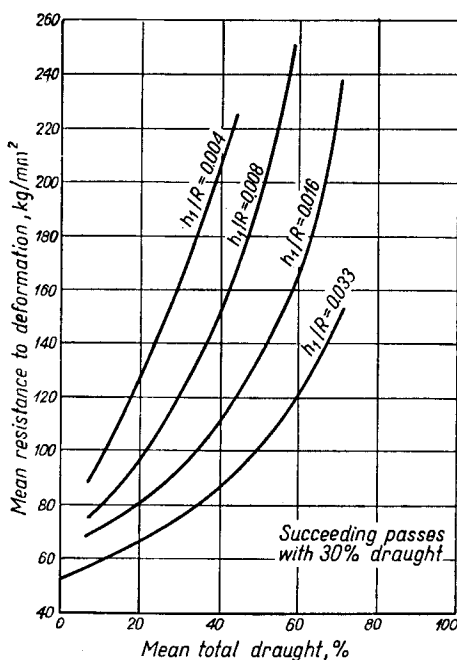


Fig. 5.39. The mean resistance to deformation during cold rolling of 0.15% C steel, as a function of the ratio of  $h_1/R$  and mean total percentage draught (from SKF) [77]

To find the additional frictional torque, point 7 is plotted on the right-hand ordinate, corresponding to roll force 680 t, and through it a horizontal line is drawn to intersect with the straight line for roll diameter of 1000 mm (point 8). A vertical line from point 8 intersects the straight line for the coefficient of bearing friction which in this case is equal to 0.06 at point 9. From point 9 a horizontal line is drawn to point 10 on the left-hand ordinate, giving the additional frictional torque of 28 tm.

For practical purposes during cold rolling measured values given by Wallquist from SKF tables [77] are applicable (Fig 5.39). In this figure the mean resistance to deformation is plotted against mean total percentage draught for a rolling mill of diameter 215 mm. These four curves correspond to different ratios of  $h_1/R$ . If the mean total draught is known the value of resistance to deformation can be read off from this diagram, and the total roll force calculated. The mean total draught

should always be related to the undeformed condition after annealing, e.g. if a strip is pre-rolled after annealing using a draught of 20%, then for the next pass with 30% draught, the mean total draught will be

$$\frac{20+30}{2} = 25\%$$

For cold rolling, similarly as for hot rolling, draught calculations are based on the assumption of constant roll force during successive passes. Maintaining a constant roll force during cold rolling is also a necessary condition for accurate rolling of the product.

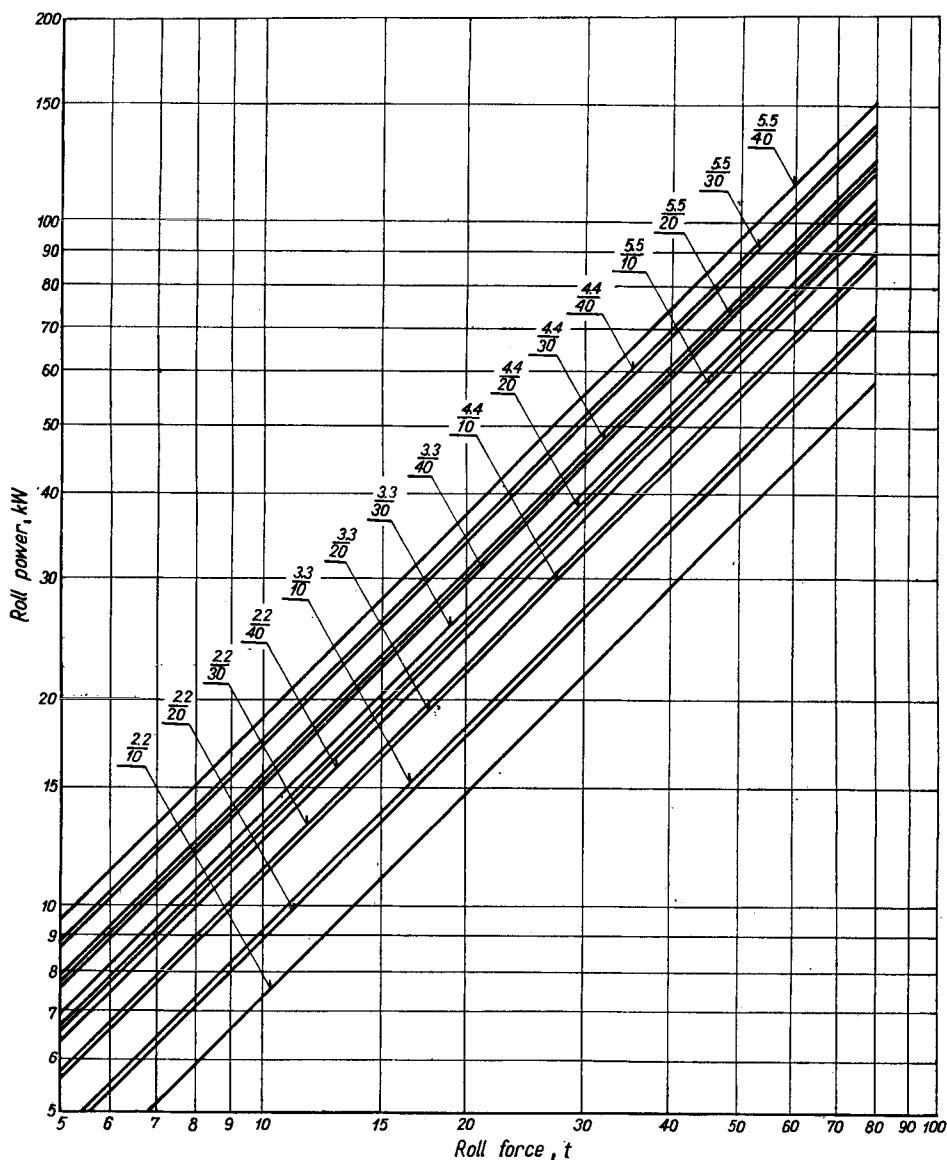


Fig. 5.40. Roll force and power during hot rolling of 0.1% C steel strip at  $n = 150$  rpm,  $b_m = 85.5$  mm. Values in the numerator denote initial thickness, values in the denominator percentage draught per pass [85], rolling temperature 750–1150°C

Constant roll force ensures constant roll deflection and flattening. Variations in roll deflection cause either thickness variation between centre and edge, or rippling [10].

#### Example 2

Figure 5.39 is used to determine the permissible draught for a given roll force during cold rolling.

It is required to calculate a draughting schedule assuming a constant roll force during cold rolling of a 150 mm wide strip of 0.15% C steel. Initial thickness of the strip is 2 mm and final thickness 0.75 mm. The mill is equipped with steel rolls of the following dimensions: roll diameter  $D = 200$  mm, barrel length  $L = 200$  mm, roll neck diameter  $d = 125$  mm, neck length  $l = 150$  mm. The roll is drilled to an inside diameter of 30 mm.

From Table 5.6 the maximum permissible bending stress for the steel of the roll is  $1000 \text{ kg/cm}^2$  hence the roll can be loaded with  $P_{\max} = 128 \text{ t}$ . A maximum working load of  $P = 120 \text{ t}$  is assumed. The mean resistance to deformation  $K_{wm}$  for 0.15% C steel can be determined from Fig. 5.39 [77]. Since there are two unknowns i.e. draught and roll force, and only one equation

$$P = K_{wm} b_m \sqrt{R \Delta h} \quad (5.97)$$

therefore first one value must be estimated and corrected by the method of successive approximations until equation (5.97) is satisfied. For  $h_1/R = 0.02$  and estimated draught of 37.5%, from Fig. 5.39  $K_{wm} = 100 \text{ kg/mm}^2$ , and  $\Delta h = 0.75 \text{ mm}$  and  $h_2 = 1.25 \text{ mm}$ , therefore

$$P = K_{wm} b_m \sqrt{R \Delta h} = 100 \times 150 \sqrt{100 \times 0.75} = 130 \text{ t} \quad (5.98)$$

As this value is too high, the estimated draught is decreased to 35%, i.e.  $h_1 = 2 \text{ mm}$ ,  $h_2 = 1.3 \text{ mm}$  and  $\Delta h = 0.7 \text{ mm}$ . Using this draught, from Fig. 5.39 a value of  $K_{wm} = 96 \text{ kg/mm}^2$  is found and substituting these new values into equation (5.97) gives

$$P = K_{wm} b_m \sqrt{R \Delta h} = 96 \times 150 \sqrt{100 \times 0.7} = 120.5 \text{ t}$$

which corresponds to the initial assumed value.

For rapid determination of roll power consumption and force in hot rolling the diagram shown in Fig. 5.40 (from SKF) [85] can be used. These values are for 0.1% C low-carbon steel, with a roll diameter of 265 mm, initial thicknesses from 2.2 to 5.5 mm, and width 85.5 mm. Numbers quoted by each straight line denote as follows: numerator = initial thickness  $h_1$  of the strip, denominator = current percentage draught in the pass. In using this diagram difficulties arise in the selection of rolling conditions which correspond with sufficient accuracy to experimental conditions.

### 5.4. Measurements in Rolling

Essential measurements required in modern rolling techniques are:

- (a) measurement of roll force,
- (b) measurement of rolling torque or rolling power,
- (c) measurement of front and back-tension during cold rolling of strip,
- (d) measurement of strip gauge.

Measurements can be made by electrical or hydraulic methods but in modern mills only electrical methods are used. The electrical method of measurement consists in translating mechanical values, or changes of value, into electrical impulses or changes of impulse, which can then be measured by some standard electrical technique.

Variations of mechanical quantities are translated into electrical impulses by means of a sensitive measuring device. These electrical impulses are generally measured as changes in potential, resistance, induction or capacity. These values are measured by electronic measuring devices of great sensitivity and the measured impulses can

be amplified as required by valve amplifiers. This amplification can be of linear type or follow a curve, to give optimum conditions of measurement. The accuracy of measurement depends chiefly on the sensitivity of the sensing device, i.e. its ability to react to very small changes in the value being measured [112].

Among numerous methods of electrical measurement, many are now of only historical interest.

Among modern methods the induction method is still frequently used, while the electrical resistance strain gauge method is of increasing importance, and only these two methods will be discussed here.

#### 5.4.1. MEASUREMENT OF ROLL FORCE AND TORQUE BY MEANS OF STRAIN GAUGES [112]

The resistance of any element of an electrical circuit depends on the geometrical dimensions of this element and the resistivity of the material. If such an element in the form of wire or rod, elongates due to tensile forces, its cross-section will decrease, and if there is a parallel increase in resistivity of the material, this can result in a considerable increase in the resistance of the element. Based on this fact the relation between the resistance of an element made from material of known electrical and mechanical properties, and the force acting upon it, and the deformation  $\Delta L$  caused by this force can be expressed exactly, in the form

$$\frac{\Delta R}{R} = S \frac{\Delta L}{L} \quad (5.99)$$

where  $S$  is the so called resistance factor determining the sensitivity of the resistance element, and  $\Delta R$  = increase in resistance of the element with an elongation of  $\Delta L$ .

The value of  $S$  is constant for a given material and usually varies between 2 and 4 for materials used in practice. To maintain small dimensions of strain gauges, the resistance wire is bent in a hairpin coil (Fig. 5.41) and fixed to thin paper by elastic moisture-resistant glue. Strain gauges are distinguished as low-resistance ones i.e.

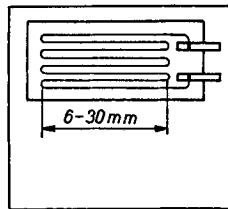


Fig. 5.41. Single strain gauge

having a resistance of 100–200 ohms, and high-resistance ones, having a resistance of 500–1000 ohms. The latter are used for measuring variable stresses and vibrations.

Strain gauges should fulfil the following requirements:

(a) high sensitivity, i.e. great change of resistance with deformation. The sensitivity of nickel–chromium alloys is much greater ( $S = 3\text{--}3.5$ ) than that of cupro-nickel alloys ( $S = 2\text{--}2.2$ ),



(b) high elasticity and tensile strength of resistance wire, allowing considerable elongation during measurement, without plastic deformation,

(c) small variation of resistance with temperature. The temperature resistance coefficient of nickel-chromium alloys is considerably greater than for cupro-nickel alloys.

Strain gauges are mostly used for the measurement of stresses caused by external forces in component parts of mechanical constructions, and also for the measurement of vibrations in structural members.

For unidirectional stresses or forces, a simple strain gauge as shown in Fig. 5.41 is used. However, if at a given point of the structural member multidirectional stresses occur, several single strain gauges can be used, or preferably a rosette type strain gauge [112], as shown in Fig. 5.42. The magnitude and direction of the maximum stress can be calculated from formulae depending on the lay-out.

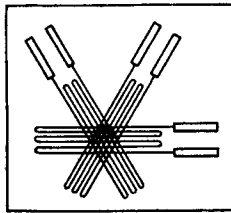


Fig. 5.42. Strain gauges in rosette form

For measuring stresses and vibrations occurring in structural members, strain gauges are fixed to the tested member near the critical cross-sections, and connected to measuring devices. These are normally electronic devices equipped with recording pens or oscilloscopes, which enable continuous recordings and observations of stress time variations.

Bridge networks (Fig. 5.43) are used to measure the change of resistance of strain gauges. Two similar strain gauges are connected into separate arms of an AC bridge

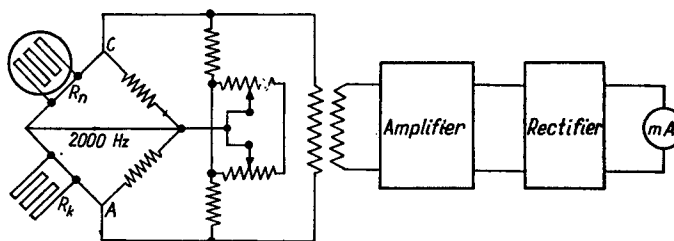


Fig. 5.43. Lay-out for measuring stress

(2000 Hz); the strain gauge  $R_n$  serves for direct measurement (the active strain gauge), while the strain gauge  $R_k$  serves for compensation of thermal influence, and is therefore positioned near  $R_n$  in such a way that no force acts on it. When tensile forces act on the gauge  $R_n$ , the balance of the bridge is upset causing a change of voltage at the bridge output terminals, which is proportional to the tensile force deforming the resistance

element of the strain gauge. After rectification, this voltage can drive a galvanometer, or it can be used to trace the curve of variation of the force as a function of time on the screen of the oscilloscope.

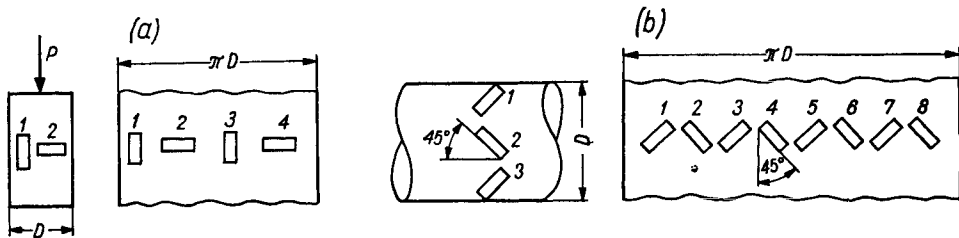


Fig. 5.44. Various strain gauge lay-outs (1 to 4 or 1 to 8) [107]

A strain gauge apparatus for measurement of roll load and torque was built in the Rolling and Forging Division of the Institute of Ferrous Metallurgy. Fig. 5.44 shows the positioning of strain gauges [107]:

- (a) on the surface of the measuring column of the 100 t-loadmeter,
- (b) on the surface of the roll spindle.

Strain gauge connections, and the method of connecting strain gauges systems of each loadmeter with the recording apparatus, are shown in Fig. 5.45 [107].

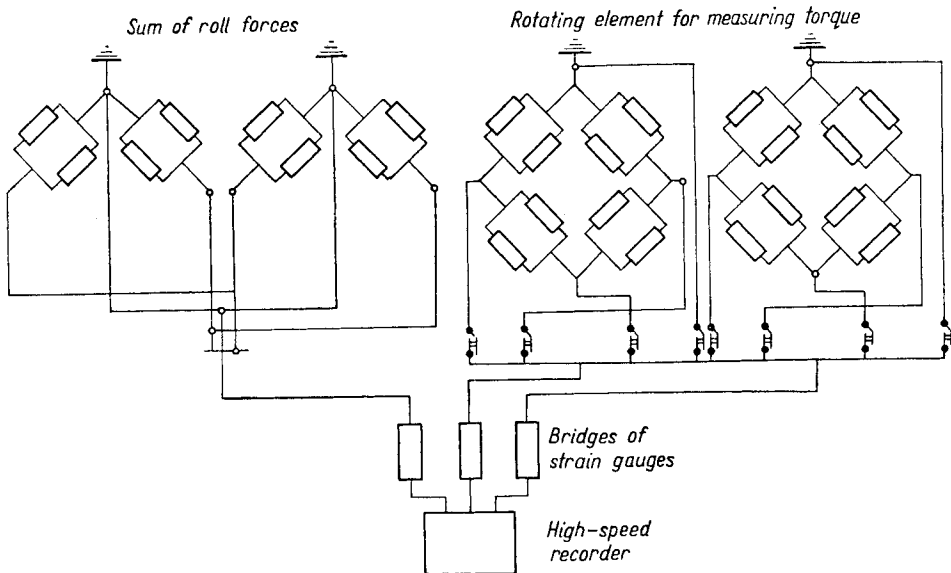


Fig. 5.45. Circuit between strain gauges and indicating apparatus, when measuring roll force and torque [107]

Strain gauges are connected in partial bridge networks for loadmeters, and in complete bridge networks for torque-meters.

The loadmeters were calibrated in an "Amsler" hydraulic press, and the torque-meters by means of a spring dynamometer with direct read-off on indicator and recording tape.

In both cases calibration was carried out using the complete amplifying and recording equipment, and the results were recorded with a Hottinger high-speed recorder. Examples of calibration curves for roll load and torque are given in Figs. 5.46 and 5.47. It can be seen from these figures that the calibration curve is a straight line, where the tangent of the angle of slope is the appropriate conversion constant.

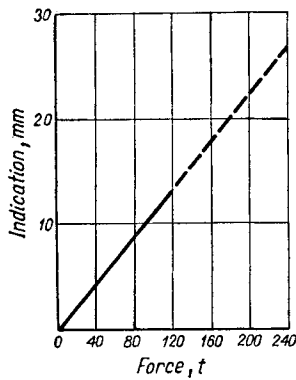


Fig. 5.46. Diagram of calibration of a 100 t loadmeter [107]

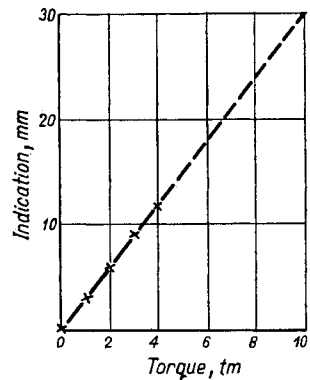


Fig. 5.47. Diagram of calibration of a 12 tm torquemeter [107]

For operational measurements of rolling loads up to 500 t the loadmeter shown in Fig. 5.48 was found to work satisfactorily.

The measuring column 1, made from heat-treatable alloy steel, transfers the total measured roll load. Strain gauges 2 are fixed to the surface of the column, and protected

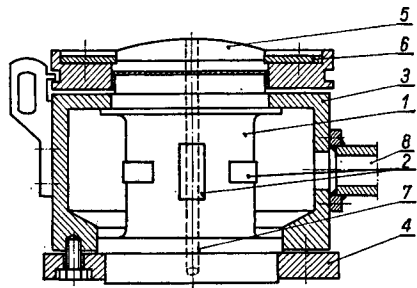


Fig. 5.48. Strain gauge loadmeter for rolling measurements

from atmospheric influence. A steel housing 3 provides additional protection against atmospheric and mechanical damage. The steel column 1 is screwed to a square base plate 4. Plate 4 fixes the loadmeter to the upper roll housing, protecting it against motion of the adjusting screw during measuring. The loadmeter is fitted to the shape of the head of the adjusting screw by means of a cap 5 with a curvature corresponding exactly to that of the screw head. Centring rings 6 maintain the position of cap 5. Rotating parts are force-lubricated through a lubricating duct 7 with dia. 5 mm, drilled through the centre of the column. The connections between strain gauges and the indicating apparatus are led through the vent pipe 8 which is fixed to the steel housing 3. A general view of a load cell is shown in Fig. 5.49.

A torquemeter for experimental roll stands is shown in Fig. 5.50, indicating the positioning of strain gauges 1 on the spindle surface 2 at an angle of  $45^\circ$  to the spindle axis, i.e. in the direction of principal stresses.

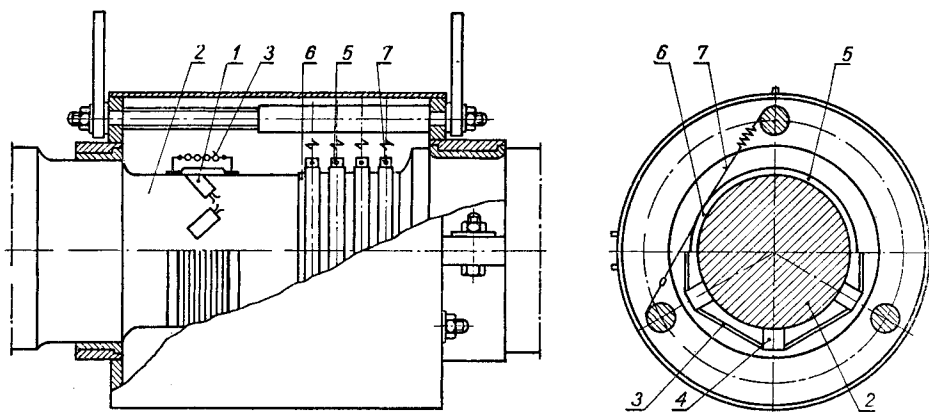


Fig. 5.50. Basic design of a torquemeter

As in the loadmeter, the strain gauge leads are connected to copper busbars 3, fixed to bakelite blocks 4. The connection scheme for strain gauges in the complete bridge lay-out is identical with that for the loadmeter, shown in Fig. 5.45.

A system of slip rings 5 fixed to the insulating flange 6 at the spindle surface 2, and collector brushes 7 mounted in the torquemeter housing transfer the signal coming from the rotating spindle to the fixed torquemeter housing. The torquemeter and part of the spindle are shown in Fig. 5.51.

#### 5.4.2. MEASUREMENT OF ROLL FORCE AND TORQUE BY INDUCTION SENSORS

Induction load and torquemeters are usually in the form of coils with or without iron core, in single coils or coupled groups of coils. The sensing element can be a choke or transformer with an air gap in the core, where the magnitude of this gap depends on the position of a movable armature or membrane. Variations in magnitude of the gap result in variation of inductance, or directly in variation of voltage [107].

The sensing devices are used for determining the rolling load and the torque acting on the spindle during rolling. An apparatus for measuring the elastic deformation of the roll housing, from which the rolling load can be deduced, is shown in Fig. 5.52. It consists of a measuring head 2 and pin 6, made from metal with the same thermal expansion characteristics as the roll housing. The measuring head and pin are screwed separately to the roll housing, to prevent mechanical movement. At the top (i.e. the head side) of the measuring pin is a regulating screw 7 with a special thread, secured by a lock nut. The steel pin from the measuring head rests on the top of the measuring pin, and is hardened and ground. The measuring pin and head are boxed in a thick-walled cover directly fixed to the roll housing, for better heat conduction and protection against atmospheric influences. The induction sensing device proper is positioned inside the measuring head (Fig. 5.53). Fig. 5.54 shows that this device consists of coils 1 and 2 connected to resistances 3 and 4, and the electrical indicating apparatus. The

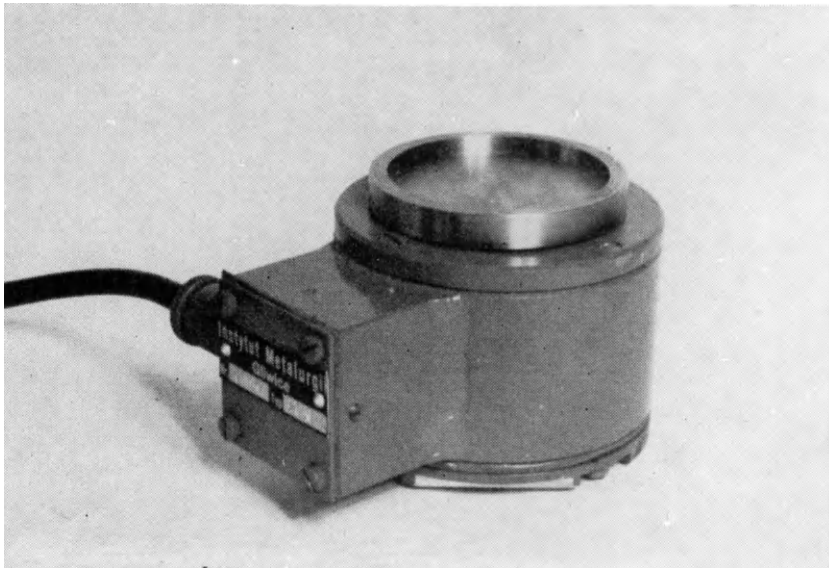


Fig. 5.49. General view of a strain gauge load cell

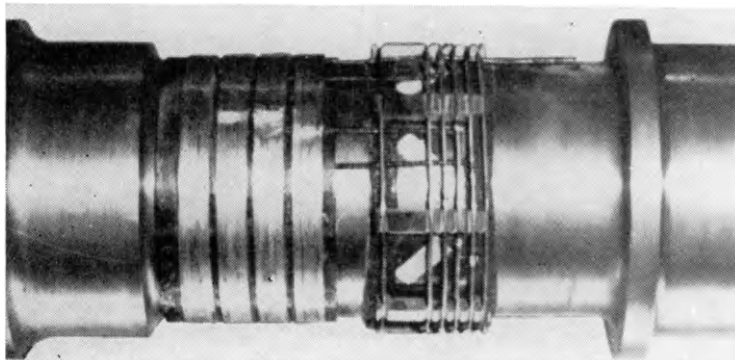


Fig. 5.51. View of the torquemeter shown in Fig. 5.50

resistance bridge system is connected to the *AC* grid *RS* through the iron resistance 5, which takes the form of electronic valves and a transformer, connected into this circuit. Due to this connection any displacement of the armature 4 (Fig. 5.53) from its central

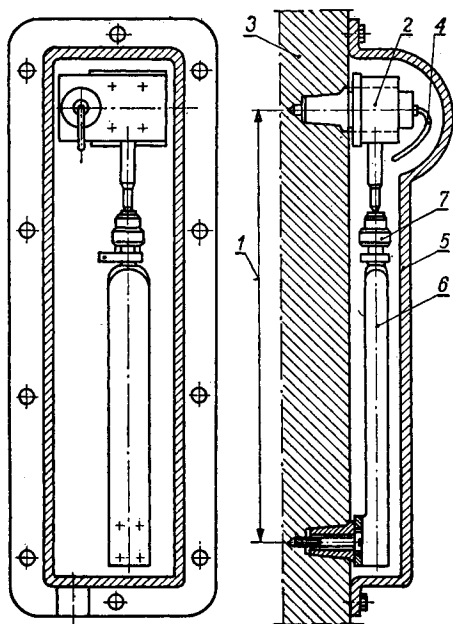


Fig. 5.52. Induction loadmeter [112]: 1 — measuring length, 2 — measuring head, 3 — housing, 4 — power supply, 5 — protective cover, 6 — measuring pin, 7 — adjusting screw

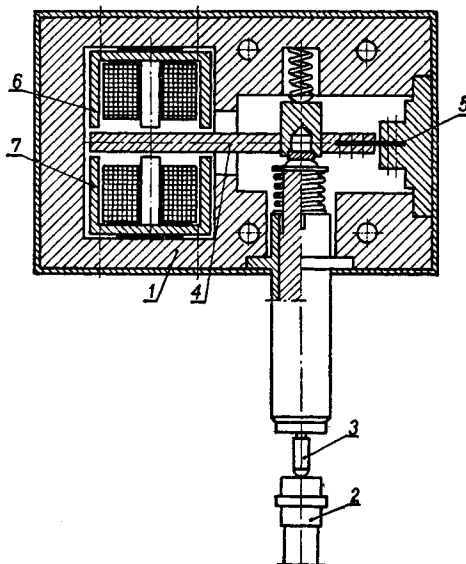


Fig. 5.53. Measuring head of induction loadmeter [112]: 1 — housing, 2 — measuring pin, 3 — sensory pin, 4 — armature, 5 — spring, 6 and 7 — coils

position induces a voltage in the coils, the magnitude of which will be shown on the electrical indicator. The iron resistance ensures that measured values will not be affected by variations of  $\pm 10\%$  in the grid voltage. When the roll stand is not in operation the zero point of this lay-out is fixed by means of the potentiometer 6 (Fig. 5.54). Elastic stresses arise in the roll housing during rolling, which cause the armature to

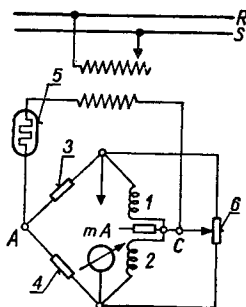


Fig. 5.54. Resistance measurement bridge for an induction loadmeter [112]: 1 and 2 — coils, 3 and 4 — resistances, 5 — iron resistance, 6 — potentiometer

be displaced from its central position, which induces a voltage at the indicator leads. This voltage is led to the rectifier so that sensitive measuring devices can be used. These devices indicate the value of the rolling load directly in tons after being calibrated experimentally, or by calculations based on measured values of elongation. These indicating devices are sometimes equipped with automatic recording apparatus.

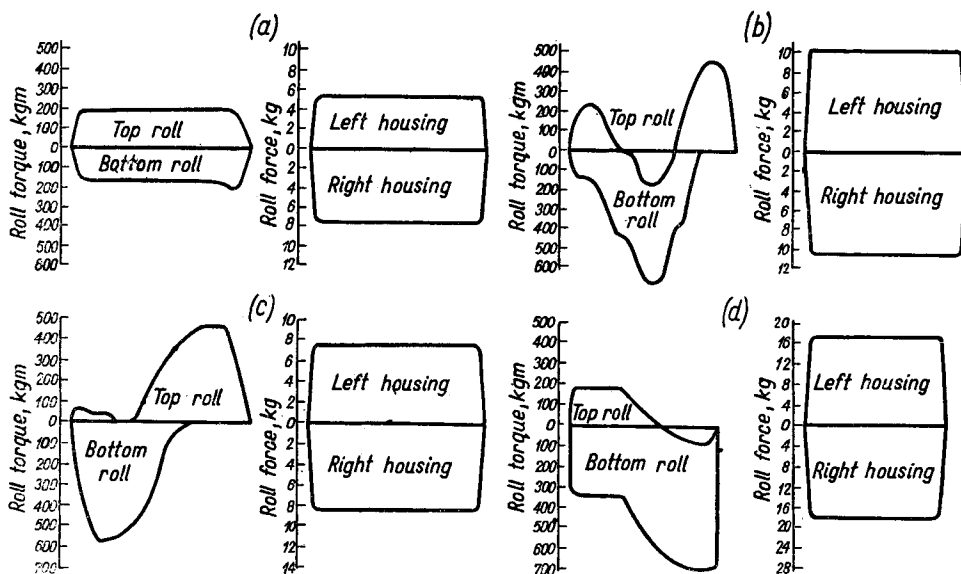


Fig. 5.55. Examples of different distribution of load and roll torque between the top and bottom rolls [86]

In operational conditions the electrical indicators are positioned so that one of them (or two in the case of a reversing mill) is on the stand, and the second one on the control desk.

Emicke investigated the distribution of total rolling torque between the two rolls. Fig. 5.55 gives four oscillograms taken from this work, showing non-uniform distribution of total rolling torque between the top and bottom rolls with a constant rolling force. This problem is also discussed by Zouhar [158].

#### 5.4.3. DEVICES FOR CONTINUOUS MEASUREMENT OF STRIP THICKNESS

Continuous gauge measurement in cold rolling provides continuous control during the rolling process, which gives improved operation and yield of the mills, and reduces the quantity of reject products in which the strip thickness tolerances are exceeded [112].

An instrument for continuous gauge measurement of strip will be described, the so-called flying micrometer. It consists of two hardened steel rollers, which can approach or recede from each other when the rolled strip passes between them. The bottom roller is connected with an armature positioned between two electromagnets, by means of a double-arm lever. Variation in thickness of the rolled strip causes a displacement of the electromagnet armature, which in turn excites a current in the windings of

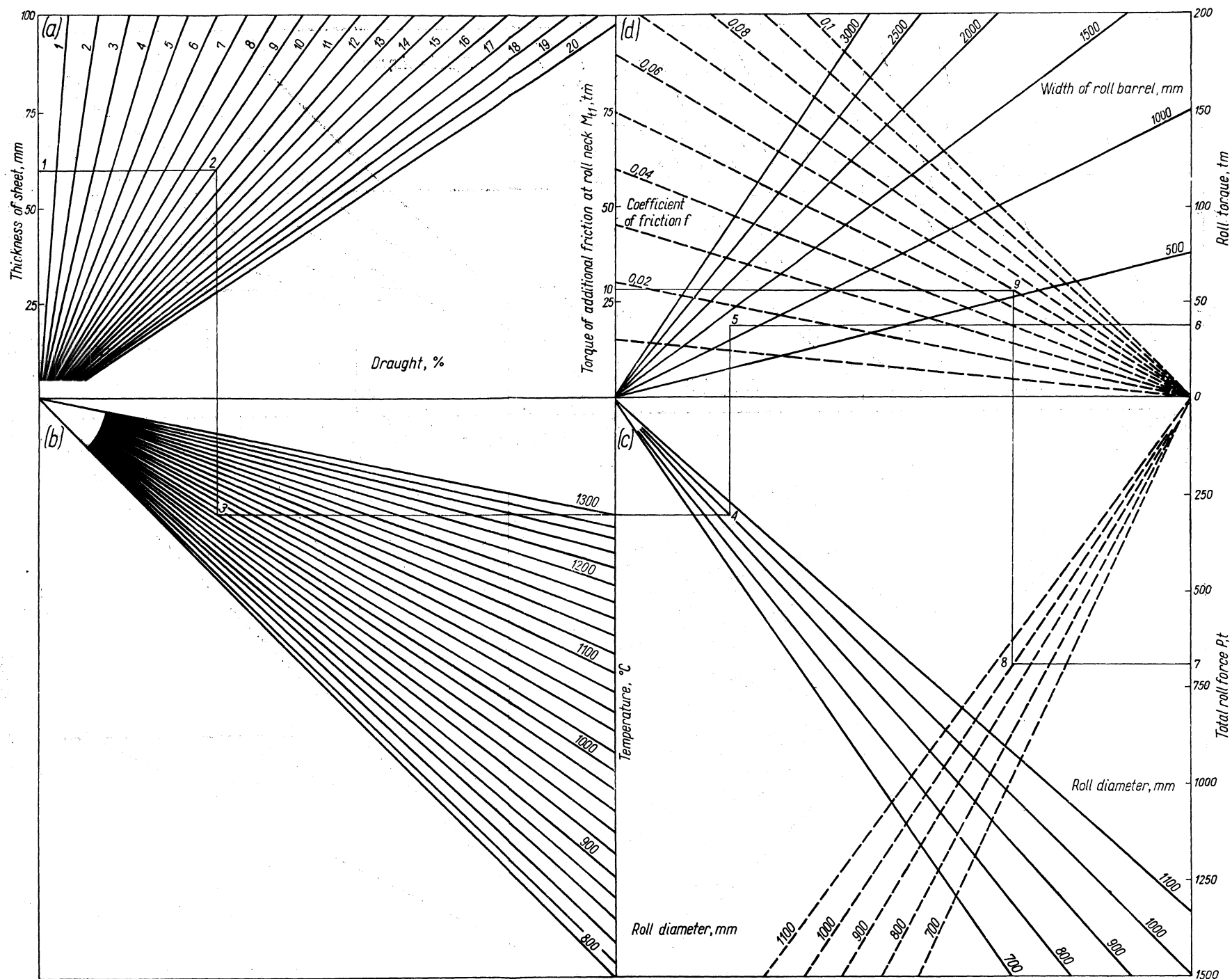


Fig. 5.38. Nomogram for determination of rolling torque during rolling of plate 5–100 mm thick, 500–3000 mm wide, from St37 steel (see Appendix 1). Roll diameters from 700 to 1110 mm, draught 1–20%, rolling temperature 800–1300°C, coefficient of friction in roll neck bearing 0.01–0.1 [106]



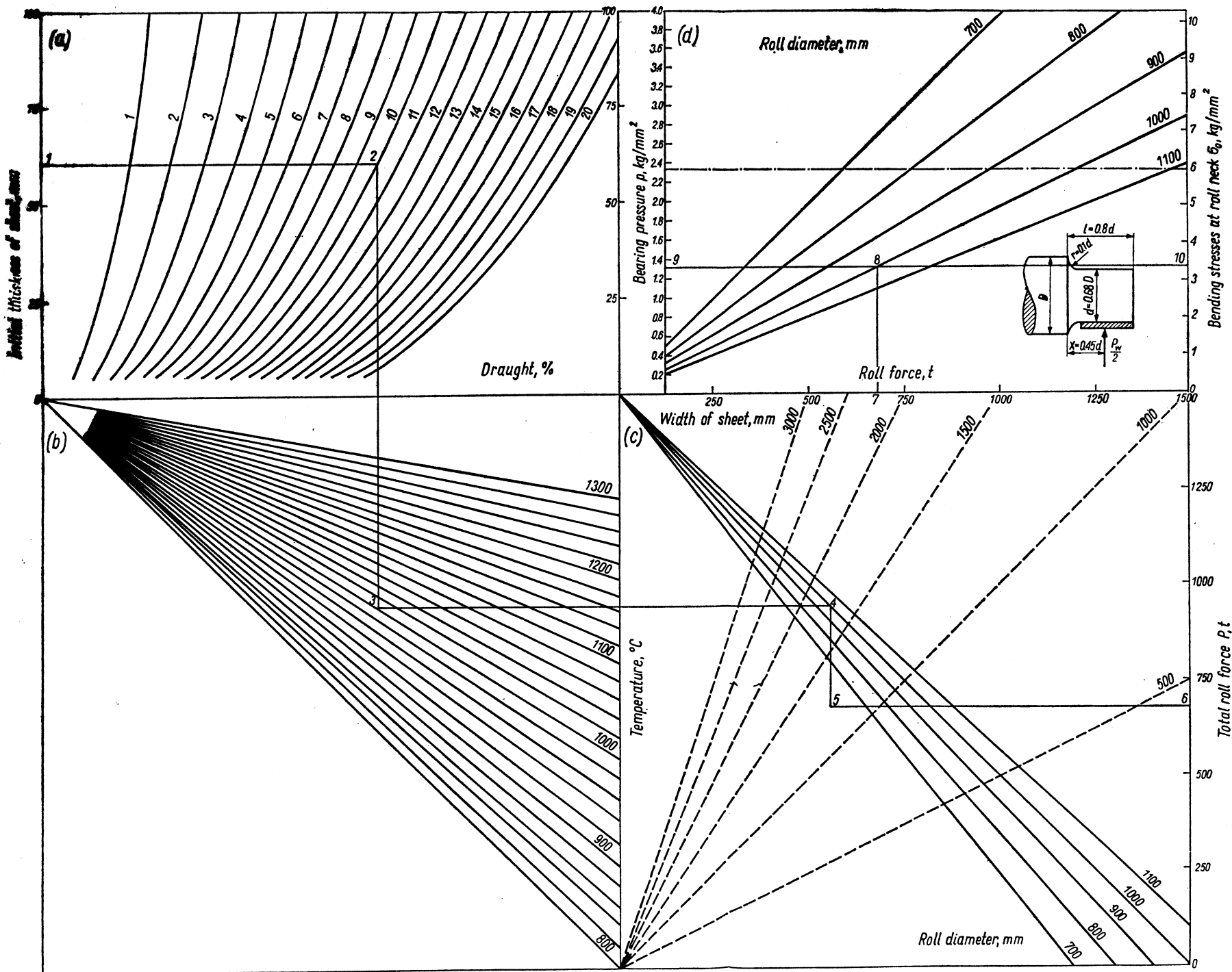


Fig. 5.37. Nomogram for determination of roll force during rolling of plate 5–100 mm thick, 500–3000 mm wide, from St37 steel (see Appendix 1). Roll diameters from 700 to 1110 mm, draught 1–20%, rolling temperature 800–1300°C [106]

the electromagnet coils. The magnetic fields of these coils act on both rollers. Any variation of current due to variation of the distance between the rollers is shown on a suitably enlarged scale by a sensitive electrical indicator.

Contactless gauges have recently been successfully applied, working with X- or  $\beta$ -rays. The principle of gauge measurement with X-rays is as follows. If a bundle of X-rays meets a strip of absorbent metal (e.g. steel), some of the rays pass through the strip unchanged, and the rest are absorbed by the metal itself (neglecting the insignificant number of rays reflected as diffused X-rays). If a given bundle of X-rays originating from a common radiating source passes through a layer of metal of uniform thickness, then at any point the same quantity of rays pass through.

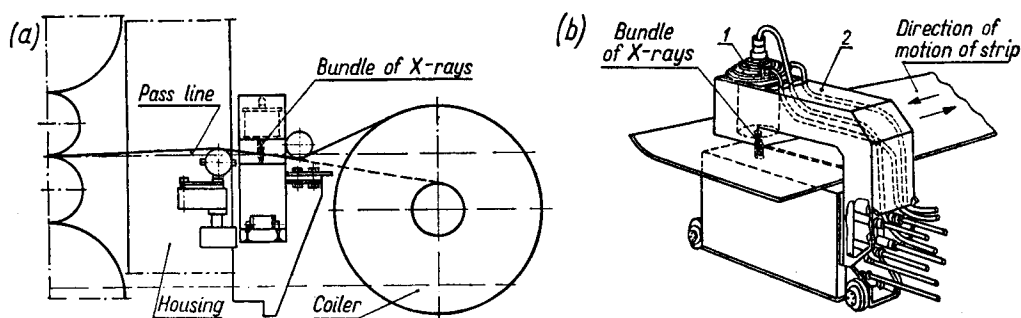


Fig. 5.56. Measuring device located between roll stand and coiler [112]: (a) scheme, (b) view, 1 — upper measuring head, 2 — measuring lever

A typical arrangement with a radiation gauge placed between the finishing stand and coiler is shown in Fig. 5.56. Pinch rollers maintain a constant position of the strip passing through the gauge. A horizontal position of the strip passing the gauge is not essential, but the angle between the strip and the vertical bundle of rays must be constant [112].

A radiation gauge for cold rolling is shown in Fig. 5.56. A lever mounted in the housing of the gauge maintains the upper detector in a constant position in relation to the bundle of rays [112]. The gap of the gauge is about 100 mm in length and its depth is about 760 mm, to measure the thickness at strip centre. There is a continuous circulation of cooling distilled water or anti-freeze solution between the housing, upper detector and cooler. Intensive radiation and prolonged working periods with X-rays are dangerous for the operating crew.

It is important to equip the network feeding the X-ray tube with a system of safety devices, which switch off automatically immediately an operator enters the danger region of X-radiation. The X-ray tube housing is constructed so that it can be mounted below the guide plate. The measuring bundle of X-rays is directed through a hole in the plate and vertically upwards. The upper detector is positioned in the path of the measuring bundle, i.e. above the measured strip.

The gauge measures the thickness of the strip passing over the hole in the plate. The upper detector and housing are connected electrically with the control panel. One dial showing the nominal value of strip thickness, and a second dial showing deviations from the nominal value are positioned so as to be clearly visible by the controller. From the reading of this second dial, the operator can control the strip

thickness from the control desk. If required the strip gauge can be recorded by a recorder, controlled from the control desk. Regulation of cooling is of great importance.

A schematic lay-out of another type of radiation gauge is shown in Fig. 5.57. The wedges shown in this figure are elongated tapered metal plates with flat ground surfaces. Each wedge can move forwards or backwards perpendicularly to the bundle of rays

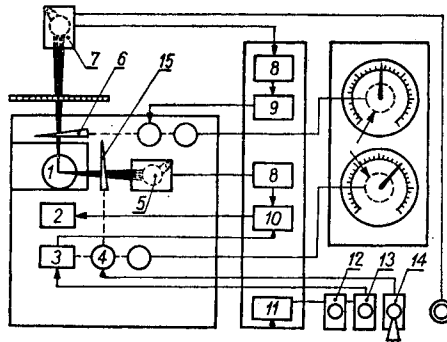


Fig. 5.57. Schematic lay-out of a thickness measuring device: 1 — X-ray tubes, 2 — transformer, 3 — self-adjusting transformer, 4 — motor, 5 — lower detector, 6 — deviation wedge, 7 — upper detector, 8 — demodulator, 9 — wedge regulator, 10 — X-ray voltage regulator, 11 — grid voltage regulator, 12 — off switch, 13 — on switch, 14 — selector, 15 — measuring wedge

so that the thickness of the specimen placed in the path of the X-rays [112] can be measured. During rolling an auxiliary motor automatically adjusts the deviation wedge to maintain the relation: thickness of the deviation wedge + strip thickness = thickness of the measuring wedge (set separately for each required strip thickness). This measurement is continuously recorded, and based on this the operator can correct the roll adjustment.

Two bundles of X-rays are sent by the X-ray tube. The horizontal bundle passes through the measuring wedge to the photocell and amplifier set, located in the lower detector. The vertical beam passes through the deviation wedge and the strip, to a similar photocell and amplifier set located above the strip. The useful part of the measuring bundle is cylindrical in shape and about 25 mm in dia.

Each amplifier compares the light falling on it from the fluorescent screen activated by the X-rays with that coming from a standard light source. Gas discharge lamps, one mounted near the amplifier in the main measuring head, and a second similar one near the amplifier in the lower detector are used as light sources. Both lamps are fed from a common source of current.

X-rays are emitted at each positive half cycle of the 60-cycle feed current and the standard light is emitted at each negative half cycle.

Every variation of light intensity at the lower detector acts on the sensitive control system, which automatically equalizes the voltage of the X-ray tube 1. When the strip thickness changes the measuring wedge is displaced, and the tube voltage is regulated by the regulator. The penetrating power of X-rays is thus regulated so that the energy at the fluorescent screen is constant.

Any variation in light intensity at the upper measuring head automatically regulates the position of the deviation wedge so that the change of thickness becomes equal

to zero. Therefore the position of this wedge is always a function of strip thickness, and the difference between actual and nominal value of thickness of the rolled strip is indicated on one or several dials by means of electrical relays.

The gauge is equipped with an adjusting system and auxiliary voltage control system, both continuously controlled by a special device. Therefore, the only possible deviations are those between the two measuring systems. The accuracy can be controlled occasionally by putting a strip of known thickness in the field of the measuring beam. If the hand of the deviation dial does not show zero, it can be zeroed by means of a potentiometer. This operation, lasting only 1 minute, is all that is necessary for scale correction.

To adjust the gauge for a new thickness, the operator turns the selecting switch and observes the scale until the servomotor brings the measuring wedge into the correct position. The servomotor also drives the automatic regulator which adjusts the tube voltage to a value corresponding to the new conditions. Values can be read off directly from the scale, as due to the symmetrical taper of wedges there are linear relations between measured and indicated values.

#### 5.4.4. MEASUREMENT OF STRIP TENSION

Modern rolling mills working with front and back tension should be equipped with devices for measuring these forces acting on the strip. One such device based on the principle of induction sensors is shown in Fig. 5.58. The tensiometer is located between

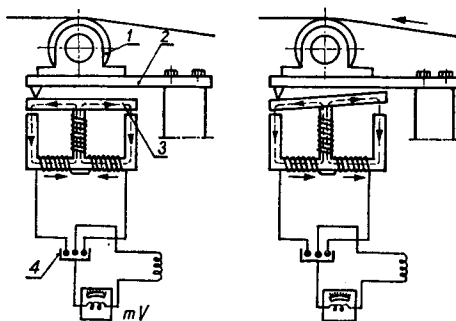


Fig. 5.58. Device for measurement of strip tension (schematic) [112]: 1 — pinch roll, 2 — spring, 3 — armature, 4 — rectifier

the roll stand and coiler or decoiler, depending on whether front or back tension is being measured. When measuring both forces simultaneously two tensiometers are used, located one at each side of the stand. The rolled strip passing from the stand to the coiler exerts a pressure on the roller 1 which is mounted on an elastic lever, so that the upper part of its perimeter is slightly above a straight line tangential to the roll perimeter and the coiler. If the rolled strip is not in tension, i.e. no force acts on the measuring roller, then the air gap between the armature and the shackle is everywhere constant, resulting in an equal current flow in both branches. When tension is applied, the strip presses the measuring roller down causing displacement of the movable armature about its fulcrum. This causes a simultaneous decrease of the left-hand gap and an increase of the right-hand gap, inducing a higher voltage at the left

side, and lower at the right side. The difference of voltage is proportional to the displacement of the armature and therefore to the tensile force. An electrical indicator showing voltage variations is included in the circuit, and calibrated directly in kg.

### 5.5. Evaluation of Measurements in Rolling Mills

The evaluation of measurements in rolling mills is based on the analysis of measured results by means of various formulae and diagrams already given. Such an analysis gives a correct estimate of the degree of utilization of rolling mills, and also indicates if the range of draughts applied is causing overloading, which can lead to roll fracture and stoppages.

As an example, results of measurements and the rolling analysis of a blooming mill with rolls of  $D = 750$  mm [110] will be discussed. Frequently occurring roll fractures made it necessary to determine the magnitude of forces during rolling.

Besides seeking the reason for roll fractures, it was required to determine experimentally the degree of loading and utilization of the mill. For this purpose the following values were measured:

- (1) roll force,
- (2) roll power,
- (3) decrease of rpm,
- (4) armature current,
- (5) magnitude of draught being applied,
- (6) temperature of the rolled stock,
- (7) pass and idle time.

The blooming mill was a 3-high adjustable type with rolls 750 mm in dia. and barrel length of 2150 mm (Fig. 5.59). As stock, ingots marked L25 and L20 of following dimensions were used:

|      |                             |
|------|-----------------------------|
| L 25 | 480/455 × 410/385 × 1500 mm |
| L 20 | 480/420 × 410/350 × 1500 mm |

The rolling programme included blooms 140 mm square and slabs of (160–360) × (100–160) mm cross-section.

The ingot heated in the pusher furnace was supplied to the mill by the roller table. The first six passes were carried out without turning in pass grooves No. *I–Ia* (Fig. 5.59) positioned in the central part of the barrel. In these passes the ingot was reduced from the mean initial thickness of 450 mm down to 225 mm, i.e. the total absolute draught was equal to 225 mm. The stock was then turned 90° and led into pass groove No. *II*.

In pass groove No. *II* and *IIa* four passes were carried out to reduce the stock from a thickness of 410 to 170 mm, i.e. a total absolute draught  $\Sigma \Delta h = 240$  mm.

After a second turn the stock was rolled in pass grooves No. *III–IIIa* or *III'–III'a* in two passes with an absolute draught of  $255 - 135 = 120$  mm.

After turning, the last pass was carried out in pass groove No. *IV* to the final dimensions of 140 × 140 mm.

Measurements were carried out during normal mill operation. Derived parameters, such as draught, spread, rate of deformation and resistance to deformation were calculated. Three series of measurements were carried out, covering the whole rolling programme. The stock was often insufficiently heated owing to the small capacity of the

Technical drawing of a multi-ported pipe. The drawing shows a cross-section of the pipe with five ports labeled I, II, III, III', and IV. The ports are arranged in a row. The dimensions are as follows:

- Port I: Diameter  $\phi 660$ , length  $l_a$ .
- Port II: Diameter  $\phi 580$ , length  $l_a$ .
- Port III: Diameter  $\phi 619$ , length  $l_a$ .
- Port III': Diameter  $\phi 619$ , length  $l_a$ .
- Port IV: Diameter  $\phi 640$ , length  $l_a$ .

The overall dimensions of the pipe are:

- Total length: 2670
- Distance from left end to Port I: 1245
- Distance between Port I and Port III: 1691
- Distance between Port III and Port IV: 979
- Distance from Port IV to right end: 451
- Distance from left end to Port II: 767.5
- Distance from Port II to Port I: 260
- Distance from Port III to Port III': 707

To facilitate the analysis, the results of measurements are presented in the form of diagrams (Figs. 5.60, 5.61 and 5.62) showing absolute draught, relative draught, roll force, rolling power and mean resistance to deformation as functions of the elongation coefficient  $\lambda$  which is shown on a logarithmic scale.

The same values for St5 steel are shown in Fig. 5.61. In comparison with Fig. 5.60 greater roll forces occur during the first passes for approximately similar values of draught, due to greater value of resistance to deformation for this steel.

In the remaining pass grooves *II*, *IIa*, *III*, *IIIa* and *IV* considerably lower roll forces occur than in pass grooves *I-Ia* previously discussed.

\* See Appendix 1.

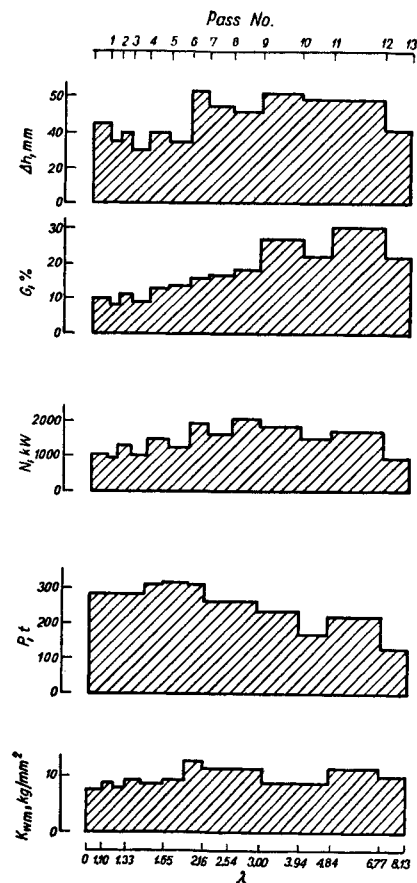


Fig. 5.60. Results of measurements when rolling L20 ingots of St37S steel (see Appendix 1) to blooms of  $140 \times 140$  mm [110]

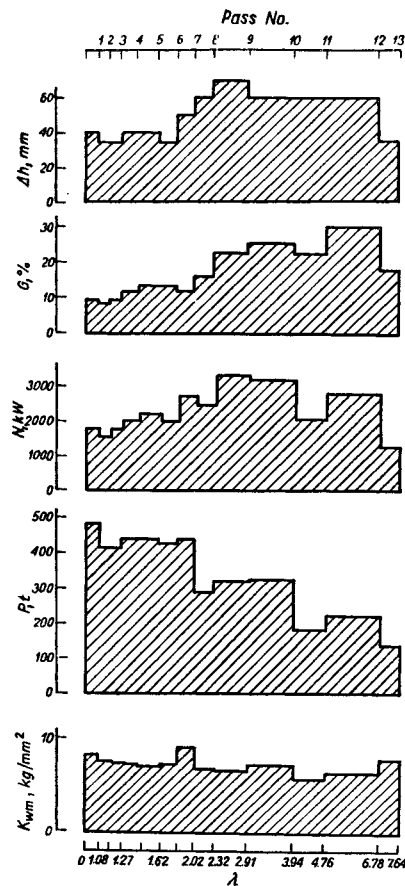


Fig. 5.61. Results of measurements when rolling L20 ingots of St5 steel (see Appendix 1) to blooms of  $160 \times 140$  mm [110]

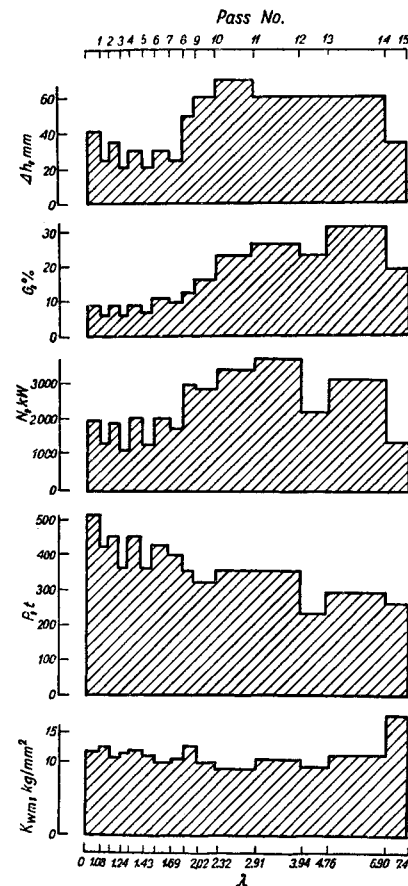


Fig. 5.62. Results of measurements when rolling insufficiently heated L20 ingots of St5 steel (see Appendix 1) to blooms of  $160 \times 140$  mm [710]

The analysis of power consumption shows a characteristic increase in the last passes (No. 11 and 12) of roll pass grooves *II* and *IIa* with simultaneous increase of roll force. Spread calculations indicate that this increase is due to additional friction on the side walls of the pass.

To check roll loading and determine the cause of roll fractures in passes *II* and *IIa*, the bending stresses at this place were calculated.

Bending stresses are calculated from the relation  $\sigma_b = M_b/W_b$ . Permissible bending stresses of  $\sigma_{b\text{ per}} = 1300 \text{ kg/cm}^2$  for forged steel rolls and  $2000 \text{ kg/cm}^2$  for alloyed steel rolls are assumed. Results are given in Table 5.15 and compared with measured values. Values are calculated for both new and dressed rolls.

TABLE 5.15  
MAXIMUM ROLL FORCES AS A FUNCTION OF PERMISSIBLE BENDING STRESSES

| Pass groove No.   | New rolls           |                                                                                                            |      | Dressed rolls               |                                                                                                            |      | Maximum measured roll force |     |
|-------------------|---------------------|------------------------------------------------------------------------------------------------------------|------|-----------------------------|------------------------------------------------------------------------------------------------------------|------|-----------------------------|-----|
|                   | working diameter mm | maximum roll force for $\sigma_{b\text{ per}} =$<br>1300 kg/cm <sup>2</sup> 2000 kg/cm <sup>2</sup><br>t t |      | working diameter of roll mm | maximum roll force for $\sigma_{b\text{ per}} =$<br>1300 kg/cm <sup>2</sup> 2000 kg/cm <sup>2</sup><br>t t |      | pass groove                 | t   |
| <i>I-Ia</i>       | 660                 | 563                                                                                                        | 865  | 600                         | 425                                                                                                        | 652  | top                         | 511 |
| <i>II-IIa</i>     | 580                 | 464                                                                                                        | 715  | 520                         | 335                                                                                                        | 518  | bottom                      | 426 |
| <i>III-IIIa</i>   | 619                 | 495                                                                                                        | 763  | 559                         | 366                                                                                                        | 564  | top                         | 356 |
| <i>III'-III'a</i> | 619                 | 592                                                                                                        | 910  | 559                         | 437                                                                                                        | 672  | bottom                      | 356 |
| <i>IV</i>         | 640                 | 905                                                                                                        | 1400 | 580                         | 676                                                                                                        | 1040 | top                         | —   |
|                   |                     |                                                                                                            |      |                             |                                                                                                            |      | bottom                      | 229 |
|                   |                     |                                                                                                            |      |                             |                                                                                                            |      |                             | 287 |
|                   |                     |                                                                                                            |      |                             |                                                                                                            |      |                             | 262 |
|                   |                     |                                                                                                            |      |                             |                                                                                                            |      |                             | —   |

It can be seen from this table that for a permissible bending stress of  $\sigma_{b\text{ per}} = 1300 \text{ kg/cm}^2$  and new rolls, the maximum measured roll force at pass groove *I* almost reaches the permissible value determined from strength calculation. After dressing the permissible load values are exceeded. For alloyed steel rolls with a permissible stress of  $2000 \text{ kg/cm}^2$  measured loads do not exceed permissible limits for either new or dressed rolls. This is also true for pass groove *II*. In succeeding passes measured values of roll force are considerably less than permissible values.

The driving power is not fully utilized, so that the mill could be loaded considerably more, but rolling capacity is limited by the roll strength. The variations shown in values of the measured roll force during the last passes indicate unequal heating of the ingot.

Roll force and torque for rolling of plate, measured by Emicke [102], are shown in Figs. 5.63 and 5.64. Slabs of a weight of 1.07 t of St37 steel\*, were rolled in 40 passes down to plates of  $7760 \times 2060 \times 8.85 \text{ mm}$ . The advantage of choosing values of roll force and torque most suited to the properties of the rolled metal and the characteristics of the driving motor, is clearly shown by this diagram.

\* See Appendix 1.



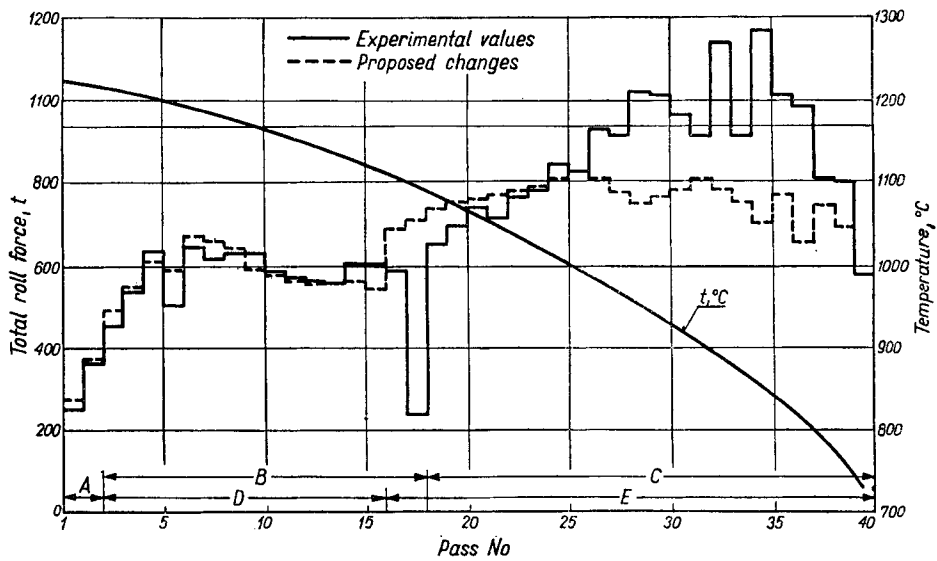


Fig. 5.63. Distribution of roll force and rolling temperature during rolling of St37 steel (see Appendix 1) plates in a 950 mm diameter reversible mill: A — 2 longitudinal passes, B — 16 transverse passes, C — 21 longitudinal passes, D — 14 transverse passes, E — 23 longitudinal passes,  $\sigma_{b\text{ per}} = 7 \text{ kg/mm}^2$ ,  $p = 2.33 \text{ kg/mm}^2$  [102]

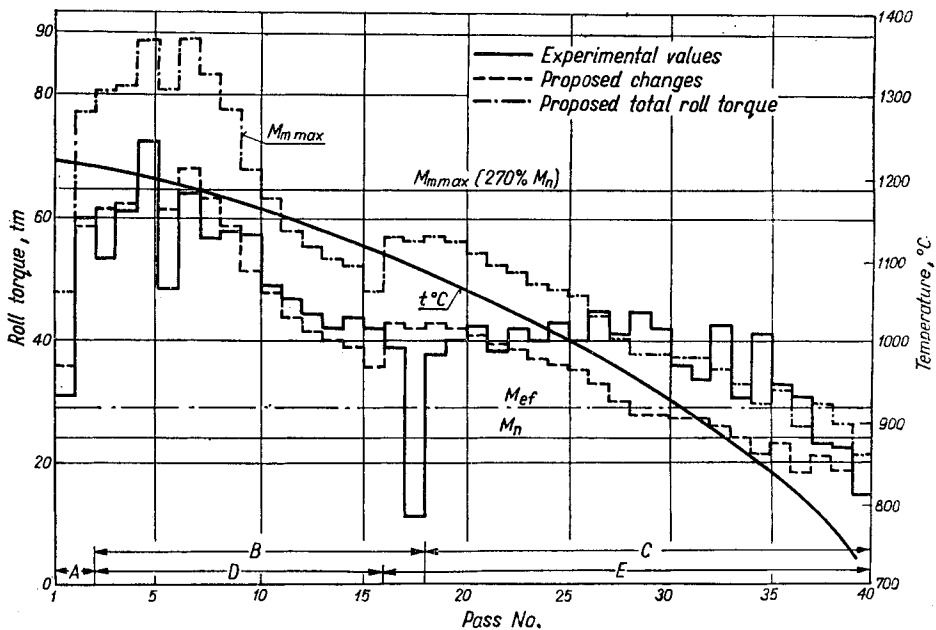


Fig. 5.64. Distribution of rolling torque, frictional torque at roll neck and rolling temperature during rolling of St37 steel (see Appendix 1) plates in a reversible mill 950 mm in dia.

The values of permissible bending stresses at barrel and neck and the permissible bearing load for a cast iron roll of diameter 950 mm are shown in Fig. 5.11 [102]. The values of measured roll forces and torques are shown in Figs. 5.63 and 5.64 as full lines. The proposed changes of rolling schedule are shown as broken lines.

In Fig. 5.63 a maximum permissible rolling load of 950 t is drawn in, corresponding to a permissible bending stress of 700 kg/cm<sup>2</sup>. It can be shown that during the last passes from No. 27 upwards, this limiting force is exceeded, and bending stresses of up to 9 kg/mm<sup>2</sup> occur.

The total rolling torque about the rolling axis (Fig. 5.64) shows a very varied distribution. A maximum rolling torque of 65.7 tm occurs during the first diagonal passes, i.e. more than 270 per cent of the nominal torque of 24 tm. If the torque due to bearing friction is added to these values then considerable overloading of the driving motor occurs during the first passes.

A new draughting schedule was proposed to reduce the overloading of the driving motor in the first passes, and that of rolls during the last ones; the new values are drawn in Figs. 5.63 and 5.64 as broken lines. Only small changes were proposed for the first ten passes, as the motor was already overloaded, and full utilization of this rolling mill was only possible by changing the driving motor for one of twice the power. On this assumption a new draughting schedule was designed.

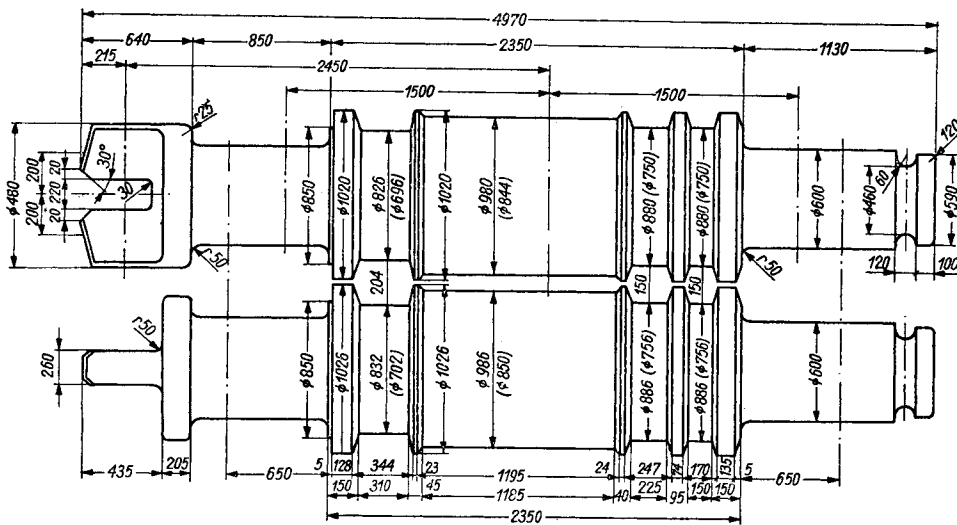


Fig. 5.65. Roll set of a 1000 mm blooming mill

As a further example of the method of evaluation of rolling measurements, the results of measurements for a blooming mill will be analysed. A 2-high reversible blooming mill of 1000 mm dia. has a barrel length of 2350 mm and is equipped with manipulators on both sides and a turning device on the side of the feeding roller table. The roll lay-out is shown in Fig. 5.65. This mill is driven through a pinion stand by a DC motor type MP7000-50 with the following characteristics:

|                  |                              |                  |                         |
|------------------|------------------------------|------------------|-------------------------|
| nominal voltage  | $U_n = 750 \text{ V}$        | motor power      | $N_n = 4416 \text{ kW}$ |
| nominal current  | $I_n = 6400 \text{ A}$       | motor efficiency | $\eta = 0.921$          |
| rotational speed | $n_n = 0-50-120 \text{ rpm}$ | nominal torque   | $M_n = 86 \text{ tm}$   |

The moment of inertia of the rotational elements of the main driving motor and rolling stand is

$$GD^2 = 461 \text{ tm}^2$$

The main motor is fed by two Ilgner sets, working in parallel. The idling torque  $M_i = 5 \text{ tm}$ , and the maximum torque  $M_{\max} = 3M_n = 258 \text{ tm}$ .

The screw-down is equipped with two DC-motors with a power of 100 kW each, and a maximum upward speed of 124 mm/sec and downward speed of 86 mm/sec.

The maximum roll opening is 1300 mm.

For this mill ingots 520/610 mm square are used, weighing  $A = 4.7 \text{ t}$ ,  $B = 5 \text{ t}$  and  $C = 6 \text{ t}$ , and also 6 t heavy slabs  $D$  with and  $E$  without mould top.

Roll forces have been measured by means of a special sensing device designed at the Institute of Ferrous Metallurgy, Gliwice, fixed to the roll housing since the mounting of conventional loadmeters under the screw-down and chock was difficult. This device consists of a bar with a narrowed section to which electric resistance strain gauges are fixed and connected up in a complete resistance bridge circuit. Two of these units were fixed to the outside of each roll housing, and connected so that the sum of the readings gives the roll force.

An instrument of this type is shown in Fig. 5.66 fixed to the outside of a roll housing. The device with its lead is clearly visible at the roll housing.

For calibration and testing of this instrument, ring-shaped standard load cells shown in Fig. 5.67 were used, with an outside diameter of 250 mm, inside diameter of 120 mm, and a height of 70 mm. Eight strain gauges are fixed to the outside of the steel column, and connected up into a bridge circuit. After calibration on a 1000 t Amsler testing machine, these loadmeters were mounted between the screw-down and the upper chock. During rolling the readings of both loadmeters were recorded, thus calibrating the instrument fixed to the roll housing.

Besides roll forces, the following quantities were measured:

- (a) power at the AC current side,
- (b) voltage of the main driving motor,
- (c) armature current of the main driving motor,
- (d) rpm of the main driving motor,
- (e) temperature of the bearings (to check cooling conditions),
- (f) rolling and idle times,
- (g) time and speed of adjusting of the top roll,
- (h) motor power variation, as a function of time,
- (i) time and direction of manipulation of manipulators,
- (j) time of operation and direction of rotation of roller tables,
- (k) time of operation of the tilting device.

Temperatures were measured by means of radiation pyrometers, after the second and after the last pass.

The value of the temperature measured after the last pass was corrected.

Roll dimensions used for measurements are shown in brackets in Fig. 5.65. It can be seen that the rolls had been dressed several times and were close to scrapping. Figure 5.68 shows the bending moment diagram along the roll length, under the action of measured roll forces. These values can be compared with maximum allowable values of bending moment calculated from the mechanical strength of the roll, which are also drawn in Fig. 5.68.

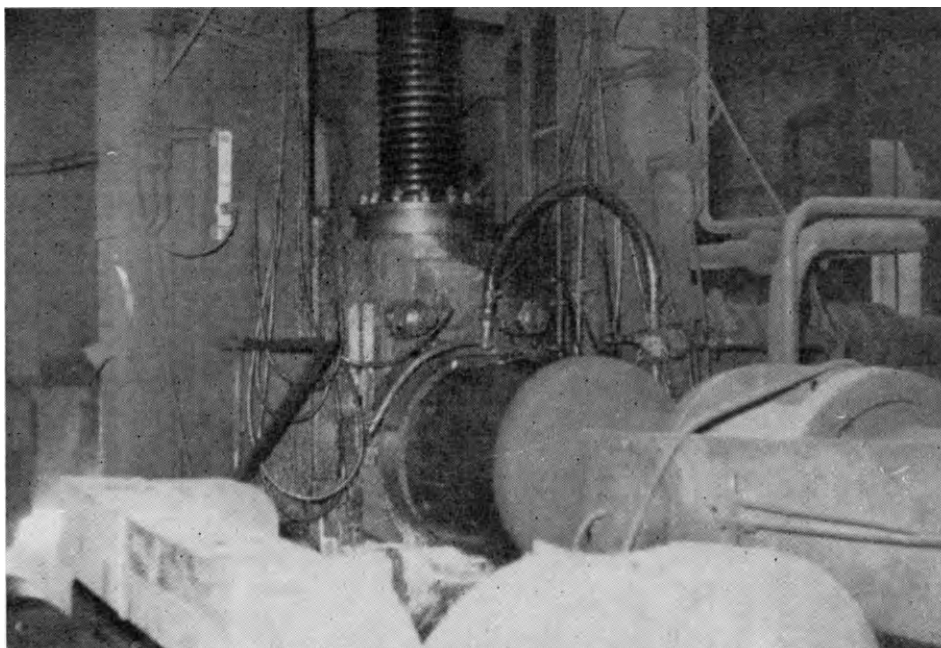


Fig. 5.66. Roll stand with strain gauge device

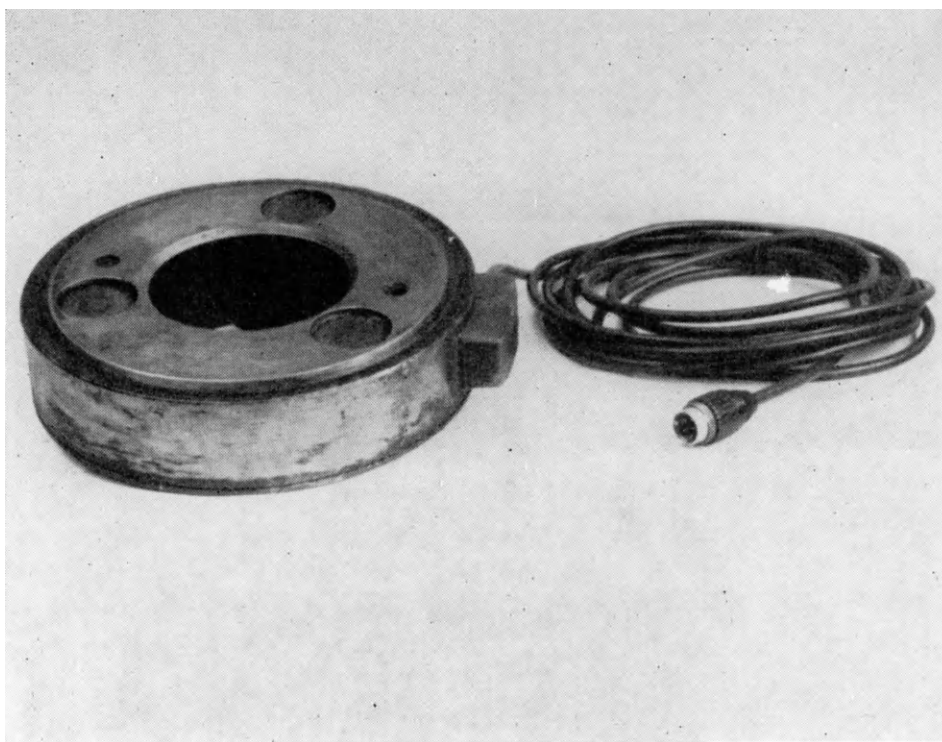


Fig. 5.67. 800 t strain gauge load cell

Knowing the calculated permissible roll forces based on the mechanical strength of roll and the maximum permissible bite angle for this blooming mill, then the following values of absolute draught  $\Delta h$  are found:

|                             |                             |
|-----------------------------|-----------------------------|
| <i>For plain ingots</i>     |                             |
| Slabbing pass               | $\Delta h = 80 \text{ mm}$  |
| Pass groove No. I           | $\Delta h = 120 \text{ mm}$ |
| Pass grooves No. III and IV | $\Delta h = 110 \text{ mm}$ |
| <i>For slabs</i>            |                             |
| Slabbing pass               | $\Delta h = 50 \text{ mm}$  |
| Pass grooves                | $\Delta h = 80 \text{ mm}$  |

Calculated results and rolling conditions for an A-type ingot are given in Table 5.16. From strength calculations of the roll and results of measurements a diagram of the degree of draught exploitation is shown in Fig. 5.69, taking into account the value of maximum bite angle. Comparison of values given in Fig. 5.69a, b and c, with those in Table 5.16, indicates insufficient exploitation.

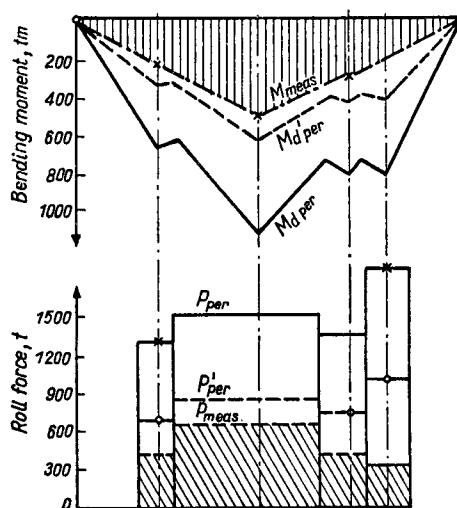


Fig. 5.68. Diagram of actual and permissible values of roll force  $P$  and bending moment  $M_d$  for the new blooming mill and  $P'$  and  $M_d'$  for the roll after the last dressing before scrap-ping shown in Fig. 5.65

Measured results show that a 19-pass schedule with a mean absolute draught of approximately 45 mm and a mean roll force of approximately 350 t, could be considerably improved. It would be possible to reduce it to a 13-pass schedule with a mean draught  $h_m$  of 65 mm, which would allow the rolling cycle to be reduced to 80 sec, where

$$h_m = \frac{1.15}{n} [(h_1 - h_n)(b_1 - b_n)] = \frac{1.15}{19} [(610 - 240)(520 - 240)] \approx 65 \text{ mm}$$

During the first passes the total roll forces are low while the values of resistance to deformation are relatively high compared with what would be expected for a sufficiently heated ingot.

TABLE  
RESULTS OF MEASUREMENTS CARRIED OUT

| Pass No. | Pass groove | Working diameter of roll | Dimensions of ingot after pass |           |                   | Absolute draught $\Delta h$ | Absolute spread $\Delta b$ | Percentage reduction | Pass time $t_p$ |
|----------|-------------|--------------------------|--------------------------------|-----------|-------------------|-----------------------------|----------------------------|----------------------|-----------------|
|          |             |                          | thickness $h$                  | width $b$ | cross-section $F$ |                             |                            |                      |                 |
|          |             | mm                       | mm                             | mm        | cm <sup>2</sup>   | mm                          | mm                         | %                    | sec             |
| 1        | 2           | 3                        | 4                              | 5         | 6                 | 7                           | 8                          | 9                    | 10              |
| 0        | —           | —                        | 610                            | 610       | —                 | —                           | —                          | —                    | —               |
| 1        | B           | 847                      | 568                            | 610       | 3460              | 42                          | —                          | 6.5                  | 0.9             |
| 2        | B           | 847                      | 518                            | 620       | 3210              | 50                          | 10                         | 8.8                  | 1.3             |
| 3        | B           | 847                      | 580                            | 525       | 3040              | 40                          | 7                          | 6.5                  | 0.9             |
| 4        | B           | 847                      | 537                            | 530       | 2840              | 43                          | 5                          | 6.9                  | 1.4             |
| 5        | B           | 847                      | 499                            | 535       | 2665              | 38                          | 5                          | 7.3                  | 1.6             |
| 6        | B           | 847                      | 457                            | 540       | 2465              | 42                          | 5                          | 8.0                  | 1.5             |
| 7        | B           | 847                      | 418                            | 545       | 2280              | 39                          | 5                          | 8.7                  | 1.9             |
| 8        | B           | 847                      | 378                            | 550       | 2080              | 40                          | 5                          | 9.5                  | 1.8             |
| 9        | B           | 847                      | 340                            | 555       | 1885              | 38                          | 5                          | 10.5                 | 2.0             |
| 10       | B           | 847                      | 310                            | 560       | 1735              | 30                          | 5                          | 8.8                  | 1.5             |
| 11       | I           | 699                      | 510                            | 315       | 1605              | 50                          | 5                          | 8.9                  | 3.0             |
| 12       | I           | 699                      | 446                            | 320       | 1425              | 64                          | 5                          | 9.8                  | 2.5             |
| 13       | I           | 699                      | 391                            | 325       | 1270              | 55                          | 5                          | 13.0                 | 2.1             |
| 14       | B           | 699                      | 341                            | 330       | 1125              | 50                          | 5                          | 15.0                 | 2.9             |
| 15       | B           | 847                      | 280                            | 350       | 980               | 50                          | 9                          | 15.2                 | 2.3             |
| 16       | III         | 847                      | 231                            | 360       | 823               | 49                          | 10                         | 17.9                 | 3.0             |
| 17       | III         | 753                      | 260                            | 245       | 634               | 100                         | 14                         | 27.8                 | 5.9             |
| 18       | III         | 753                      | 236                            | 250       | 590               | 24                          | 5                          | 0.78                 | 4.4             |
| 19       | B           | 847                      | 240                            | 240       | 576               | 10                          | 4                          | 0.40                 | 4.8             |

#### Technical data

Dimensions of ingot  $\square$  610/ $\square$  520 mm

Final dimensions of bloom 240×240 mm

Weight 4.7 t

Steel 175 P

Initial rolling temperature 1170°C

Final rolling temperature 1060°C

Rolling was carried out at a rather low temperature, i.e. in the range 1060–1170° C.

Undoubtedly lower values of roll forces would have been obtained if the material had been sufficiently heated, which is all the more reason for reducing the number of passes from 19 to 13.

Bearing in mind that measurements were carried out with rolls that had been dressed several times, it is reasonable to assume that with a new roll set the main driving motor would be more heavily loaded, due to increase in roll forces. Therefore the initial rolling temperature should be raised to at least 1200° C, which would have the effect of decreasing resistance to deformation.

The total rolling torque  $M_k$  about the motor shaft, plus the thermal loading of the motor expressed by the effective torque  $M_e$  gives the loading of the main driving motor.

## 5.16

ON A 1000 mm BLOOMING MILL

| Idling<br>time $t_i$ | Rolling<br>time $t_w$ | Main motor               |                  |             |                                           |                                                          | Roll stand             |                        |                                                       |
|----------------------|-----------------------|--------------------------|------------------|-------------|-------------------------------------------|----------------------------------------------------------|------------------------|------------------------|-------------------------------------------------------|
|                      |                       | number of revolutions at |                  |             | max.<br>armature<br>current<br>$I_{\max}$ | torque<br>$M_{\max}$<br>corres-<br>ponding<br>$I_{\max}$ | total roll force       |                        | mean<br>resistance<br>to de-<br>formation<br>$K_{wm}$ |
|                      |                       | entry<br>$n_e$           | exit<br>$n_{ex}$ | $n_{I\max}$ |                                           |                                                          | peak<br>value<br>$P_p$ | mean<br>value<br>$P_m$ |                                                       |
| sec                  | sec                   | rpm                      | rpm              | rpm         | kA                                        | tm                                                       | t                      | t                      | kg/mm <sup>2</sup>                                    |
| 11                   | 12                    | 13                       | 14               | 15          | 16                                        | 17                                                       | 18                     | 19                     | 20                                                    |
| —                    | —                     | —                        | —                | —           | —                                         | —                                                        | —                      | —                      | —                                                     |
| 1.7                  | 2.6                   | 26.2                     | 45.6             | 30.8        | 7.1                                       | 90.2                                                     | 360                    | 290                    | 3.65                                                  |
| 11.6                 | 12.9                  | 28.5                     | 53.6             | 47.6        | 8.4                                       | 107.5                                                    | 500                    | 470                    | 5.23                                                  |
| 1.9                  | 2.8                   | 29.6                     | 45.6             | 33.6        | 7.5                                       | 95.5                                                     | 340                    | 300                    | 4.38                                                  |
| 1.7                  | 3.1                   | 23.9                     | 45.6             | 47.6        | 8.1                                       | 103.6                                                    | 510                    | 450                    | 6.48                                                  |
| 1.9                  | 3.5                   | 8.0                      | 45.6             | 25.2        | 9.4                                       | 121.1                                                    | 460                    | 450                    | 6.48                                                  |
| 2.0                  | 3.5                   | 14.8                     | 59.3             | 30.8        | 9.5                                       | 122.3                                                    | 510                    | 500                    | 7.11                                                  |
| 2.2                  | 4.1                   | 11.4                     | 55.9             | 28.0        | 9.0                                       | 115.5                                                    | 500                    | 450                    | 6.33                                                  |
| 2.2                  | 4.0                   | 17.1                     | 64.0             | 28.0        | 9.5                                       | 122.3                                                    | 500                    | 490                    | 6.83                                                  |
| 2.7                  | 4.7                   | 12.5                     | 59.3             | 25.2        | 9.0                                       | 115.5                                                    | 480                    | 460                    | 6.38                                                  |
| 6.0                  | 7.5                   | 28.5                     | 68.4             | 61.6        | 7.5                                       | 95.5                                                     | 450                    | 420                    | 6.63                                                  |
| 1.9                  | 4.9                   | 26.2                     | 61.6             | 33.6        | 6.1                                       | 76.7                                                     | 230                    | 210                    | 5.03                                                  |
| 3.0                  | 5.5                   | 0.0                      | 55.9             | 30.8        | 6.8                                       | 86.2                                                     | 290                    | 260                    | 6.15                                                  |
| 2.3                  | 4.4                   | 36.5                     | 68.4             | 67.2        | 6.5                                       | 82.0                                                     | 270                    | 260                    | 5.51                                                  |
| 4.6                  | 7.5                   | 11.4                     | 69.5             | 16.8        | 6.9                                       | 87.4                                                     | 270                    | 250                    | 5.23                                                  |
| 3.7                  | 6.0                   | 36.5                     | 61.6             | 61.6        | 8.5                                       | 108.9                                                    | 420                    | 410                    | 8.03                                                  |
| 12.5                 | 15.5                  | 10.3                     | 68.4             | 21.0        | 8.0                                       | 102.1                                                    | 440                    | 400                    | 7.63                                                  |
| 6.1                  | 12.0                  | 5.7                      | 61.6             | 58.8        | 12.4                                      | 161.1                                                    | 490                    | 460                    | 9.63                                                  |
| 6.1                  | 10.5                  | 10.3                     | 61.6             | 22.4        | 4.4                                       | 53.9                                                     | 190                    | 160                    | 7.38                                                  |
| 12.7                 | 17.5                  | 18.2                     | 69.5             | 63.0        | 4.0                                       | 48.6                                                     | 170                    | 140                    | 8.93                                                  |

 $N_n = 4416 \text{ kW}$ ,  $n = 50/120 \text{ rpm}$ Total rolling time  $t_{\text{tot}} = 123.3 \text{ sec}$ Mean power consumption on the AC side  $131 \times 10^3 \text{ kW}$ Specific energy consumption  $9.4 \text{ kWh/t}$ Effective current of the main motor  $4.68 \text{ kA}$ 

The total torque acting on the motor shaft in its steady-state condition (during the last four to six passes) can be determined from the relation

$$M_k = 0.975 \frac{N_k}{n} \quad (5.100)$$

where

$N_k$  = power transmitted by the motor at the coupling, kW,

$n$  = rotational speed of the armature, rpm.

The effective power at the coupling is equal to the power delivered, being the product of voltage  $V$  and current  $I$ , minus losses for copper, brushes and idling operation.

For intermediate conditions of work of the motor with voltage control of the rpm, and the excitation flux  $\Phi = \text{const}$ , the torque  $M_k$  is proportional to the current, i.e.

$$M_k = cI \quad (5.101)$$

where the factor  $c = M_n/I_n(\text{tm/A})$ . Motor torque as a function of speed is shown in Fig. 5.70, i.e. curves for nominal torque  $M_n$ , maximum torque  $M_{\max}$  and cut-out torque  $M_c$ . The cut-out torque shown in Fig. 5.70 was determined by measuring the cut-out current at the main switch.

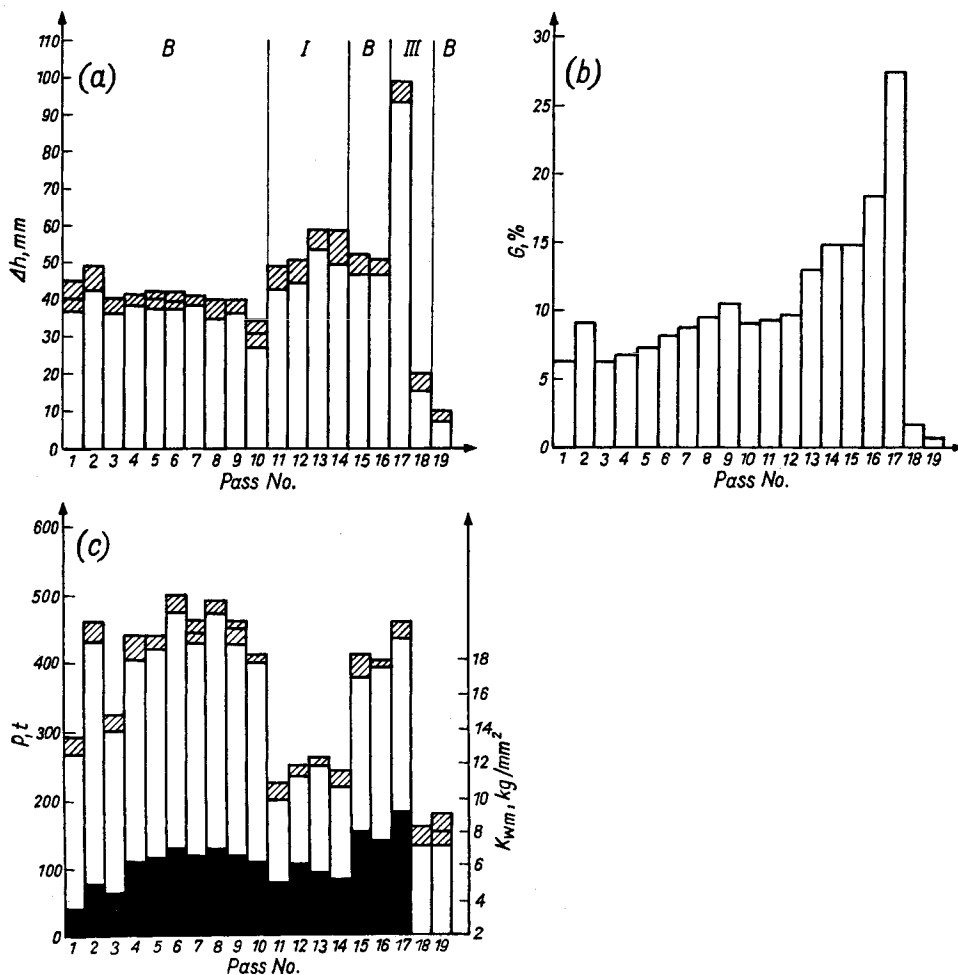


Fig. 5.69. Distribution of absolute and percentage draught and roll force when rolling a 4.7 t ingot *A* to a 240×240 mm bloom. The shaded area indicates scatter of results.  
B, I, III — passes from Fig. 5.65

To analyse the torque distribution, the maximum armature current  $I_{\max}$  and speed  $n_{I_{\max}}$  were measured, and from these values the torque  $M_{I_{\max}}$  was determined, allowing for the idling torque  $M_i$ . These values are given in columns 15, 16 and 17 of Table 5.16, and shown graphically in Fig. 5.71. The maximum rolling torque occurred during the



17-th pass and was equal to  $M_{17}=161$  tm. The corresponding speed was  $n_{17} = 58.8$  rpm, which gives for this case the relation  $M_{17}/M_{\max}$  equal to 80.6%. By adjusting the main switch, the maximum torque could be increased to  $M'_{\max} = 235$  tm, giving a value of  $M_n/M'_{\max} = 67.7\%$ . To avoid frequent switching out, it would also be necessary to

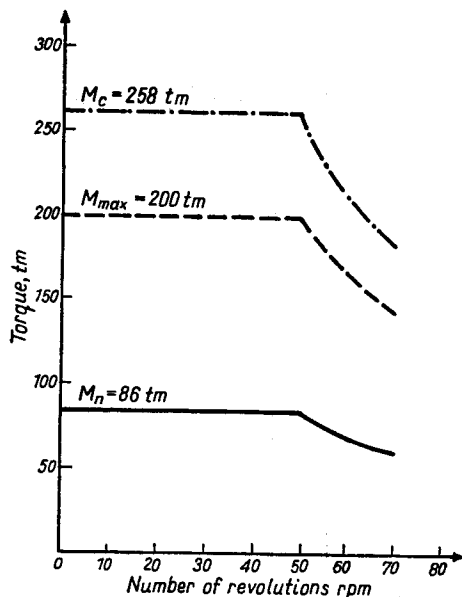


Fig. 5.70. Torque characteristics of the main driving motor of a blooming mill

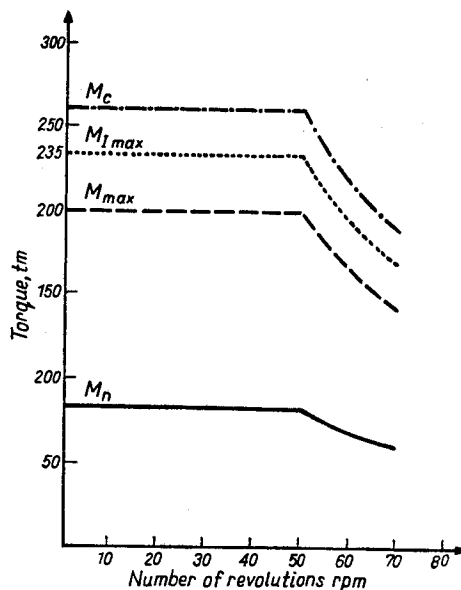


Fig. 5.71. Torque characteristics when rolling an A-type ingot

install an automatic current limiting device for the motor armature. The cut-out switches would act as an auxiliary safety device guarding against overloading of the main driving motor.

The thermal loading of the motor is determined by the effective torque  $M_e$  or effective current, which can be calculated from formula

$$I_e = \sqrt{\frac{\sum I_x^2 t_x}{\sum t_x}} \quad (5.102)$$

or relative to the nominal current  $I_n$  of the motor

$$I'_e = \frac{I_e}{I_n} 100\% \quad (5.103)$$

For the case considered a ratio of

$$I'_e = 73.2\%$$

was found for one total rolling cycle. In all these passes the main driving motor is not fully loaded. The thermal loading ratio becomes still smaller if idle times are included in the denominator of equation (5.102), as during this time the motor is unloaded and can cool.

To analyse the energy consumption per ton of the rolled stock (kWh/t) an elongation factor  $\lambda_t$  is introduced, where

$$\lambda_t = \frac{F_0}{F_n} \tag{5.104}$$

and

- $F_0$  = cross-section of the ingot,
- $F_n$  = cross-section of the bloom.

On this basis the ingots were divided into several groups. The *A*-ingots were those with a factor  $\lambda_t = 6.7$ .

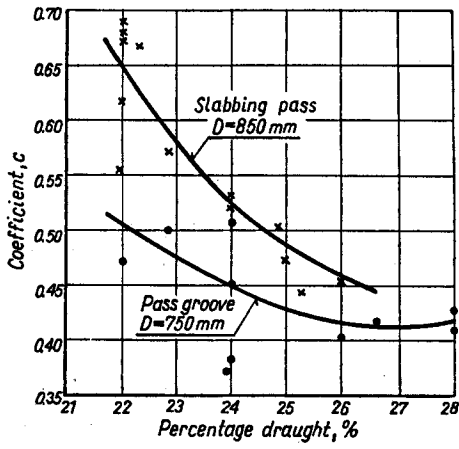


Fig. 5.72. Coefficient *c* of the lever arm as a function of percentage draught and roll diameter

TABLE 5.17  
ROLLING CYCLE FOR INGOTS WITH PASS AND AUXILIARY TIMES

| No | Rolled section                                  | Pass time  |                          | Auxiliary time |                          | Rolling time | Waiting time for the next ingot | Rolling cycle for ingots |
|----|-------------------------------------------------|------------|--------------------------|----------------|--------------------------|--------------|---------------------------------|--------------------------|
|    |                                                 | mean value | fraction of rolling time | mean value     | fraction of rolling time |              |                                 |                          |
|    |                                                 | sec        | %                        | sec            | %                        |              |                                 |                          |
| 1  | 2                                               | 3          | 4                        | 5              | 6                        | 7            | 8                               | 9                        |
| 1  | Bloom 130×230 mm<br>Ingot 5T—"B"<br>19 passes   | 64.35      | 49.5                     | 59.2           | 45.6                     | 123.55       | 6.4                             | 129.95                   |
| 2  | Bloom 130×230 mm<br>Ingot 6T—"C"<br>21 passes   | 79.4       | 52.9                     | 62.1           | 41.3                     | 141.5        | 8.7                             | 150.2                    |
| 3  | Bloom 240×240 mm<br>Ingot 4.7T—"A"<br>19 passes | 45.7       | 35.3                     | 77.1           | 57.3                     | 122.8        | 9.6                             | 132.4                    |
| 4  | Slab 220×800 mm<br>Ingot 6T—"D"<br>23 passes    | 51.8       | 34.7                     | 83.5           | 56.1                     | 135.5        | 13.4                            | 148.9                    |

TABLE 5.18

COMPARISON OF VALUES OF ROLLING SPEED, TIME, AND TORQUE WITH AN OPTIMUM PASS SCHEDULE WHEN ROLLING A 5 t INGOT TO  
130×230 mm BLOOM (SLAB)

| Pass No. | Pass groove No. | Roll speed         |                     |                      |                       |                                 |                               | Time                     |                            |                               |                                |                                         |                            |                          |                        |                             | Maximum torque $M_{\max}$<br>tm | Overloading factor<br>$M_{\max}/M_a$ | Effective mean torque $M_{ef}$<br>tm |
|----------|-----------------|--------------------|---------------------|----------------------|-----------------------|---------------------------------|-------------------------------|--------------------------|----------------------------|-------------------------------|--------------------------------|-----------------------------------------|----------------------------|--------------------------|------------------------|-----------------------------|---------------------------------|--------------------------------------|--------------------------------------|
|          |                 | Entry $n_e$<br>rpm | I motor step<br>rpm | II motor step<br>rpm | III motor step<br>rpm | Maximum speed $n_{\max}$<br>rpm | Speed at exit $n_{ex}$<br>rpm | Acceleration time<br>sec | Pass time for $n_I$<br>sec | Pass time for $n_{II}$<br>sec | Pass time for $n_{III}$<br>sec | Pass time for $n_{\text{const}}$<br>sec | Pass time for $n_I$<br>sec | Deceleration time<br>sec | Pass time $t_p$<br>sec | Auxiliary time $t_a$<br>sec |                                 |                                      |                                      |
| 1        | 2               | 3                  | 4                   | 5                    | 6                     | 7                               | 8                             | 9                        | 10                         | 11                            | 12                             | 13                                      | 14                         | 15                       | 16                     | 17                          | 18                              | 19                                   | 20                                   |
| 1        | B               | 16                 | 33                  | —                    | —                     | 33                              | 20                            | 0.59                     | 0.63                       | —                             | —                              | —                                       | 0.28                       | 0.43                     | 0.91                   | 2.07                        | 123                             | 0.476                                | 69.5                                 |
| 2        | B               | 20                 | 50                  | —                    | —                     | 50                              | 50                            | 0.74                     | 1.11                       | —                             | —                              | 0.06                                    | —                          | 1.07                     | 1.17                   | 4.10                        | 125                             | 0.484                                |                                      |
| 3        | B               | 16                 | 50                  | —                    | —                     | 50                              | 43.5                          | 0.59                     | 1.26                       | —                             | —                              | —                                       | 0.14                       | 0.94                     | 1.40                   | 2.07                        | 104                             | 0.404                                |                                      |
| 4        | B               | 20                 | 50                  | —                    | —                     | 50                              | 32                            | 0.74                     | 1.11                       | —                             | —                              | —                                       | 0.38                       | 0.69                     | 1.49                   | 2.07                        | 115                             | 0.446                                |                                      |
| 5        | B               | 22                 | 50                  | —                    | —                     | 50                              | 37                            | 0.81                     | 1.04                       | —                             | —                              | 0.20                                    | 0.27                       | 0.80                     | 1.51                   | 2.07                        | 124                             | 0.483                                |                                      |
| 6        | B               | 22                 | 50                  | —                    | —                     | 50                              | 36                            | 0.81                     | 1.04                       | —                             | —                              | 0.31                                    | 0.30                       | 0.77                     | 1.65                   | 1.88                        | 134                             | 0.518                                |                                      |
| 7        | B               | 25                 | 50                  | —                    | —                     | 50                              | 32                            | 0.93                     | 0.93                       | —                             | —                              | 0.48                                    | 0.39                       | 0.67                     | 1.80                   | 1.68                        | 125                             | 0.484                                |                                      |
| 8        | B               | 25                 | 50                  | 60                   | —                     | 60                              | 50                            | 0.93                     | 0.93                       | 0.42                          | —                              | 0.12                                    | 0.21                       | 1.07                     | 1.68                   | 6.00                        | 113                             | 0.437                                |                                      |
| 9        | I               | 16                 | 50                  | 60                   | —                     | 60                              | 38                            | 0.59                     | 1.26                       | 0.42                          | —                              | 0.23                                    | 0.47                       | 0.81                     | 2.38                   | 2.67                        | 112.5                           | 0.435                                |                                      |
| 10       | I               | 18                 | 50                  | 60                   | —                     | 60                              | 38                            | 0.67                     | 1.19                       | 0.42                          | —                              | 0.69                                    | 0.47                       | 0.81                     | 2.77                   | 2.67                        | 125                             | 0.484                                |                                      |
| 11       | I               | 20                 | 50                  | 60                   | —                     | 60                              | 38                            | 0.74                     | 1.11                       | 0.42                          | —                              | 1.53                                    | 0.47                       | 0.81                     | 3.53                   | 2.00                        | 141                             | 0.545                                |                                      |
| 12       | B               | 25                 | 50                  | 60                   | —                     | 60                              | 60                            | 0.93                     | 0.93                       | 0.42                          | —                              | 3.00                                    | —                          | 1.29                     | 4.35                   | 6.00                        | 157                             | 0.608                                |                                      |
| 13       | III             | 20                 | 50                  | 60                   | 70                    | 70                              | 40                            | 0.74                     | 1.11                       | 0.42                          | 0.71                           | 2.56                                    | 0.64                       | 0.86                     | 5.44                   | 2.15                        | 84                              | 0.325                                |                                      |
| 14       | III             | 25                 | 50                  | 60                   | 70                    | 70                              | 60                            | 0.93                     | 0.93                       | 0.42                          | 0.71                           | 3.98                                    | 0.21                       | 1.29                     | 6.25                   | 6.00                        | 85.5                            | 0.330                                |                                      |
| 15       | B               | 25                 | 50                  | 60                   | 70                    | 70                              | 70                            | 0.93                     | 0.93                       | 0.42                          | 0.71                           | 4.63                                    | —                          | 1.50                     | 6.69                   | 5.69                        | 73.6                            | 0.285                                |                                      |





of 4.1 sec and with manipulation between passes of 6 sec (measured values). These values agree with calculated ones.

The maximum operating speed of the manipulators was equal to 1.58 m/sec. From these values the pass and auxiliary time were calculated. Knowing the adjusting time of the top roll, reversing time for the bloom and motor and the assumed rpm for biting and exiting, it is possible to determine a minimum auxiliary time between two successive passes. Assuming that tilting begins at the moment the bloom end leaves the stand, then the tilting and reversing time is equal to

$$t_k = t_x + t'_k \quad (5.105)$$

where

$t_k$  = minimum auxiliary time between two succeeding passes, including tilting,

$t_x$  = reversing time of the bloom at the roll table,

$t'_k$  = time pause for bloom tilting.

The time  $t_x$  can be calculated from the formula

$$t_x = \frac{2v_r}{fg} \quad (5.106)$$

where

$v_r$  = rolling speed,

$f = 0.3$  = coefficient of friction between the rolled stock and roll table,

$g$  = acceleration due to gravity.

The time pause for bloom tilting can be calculated from the following relation:

$$t'_k = \frac{2fg(2s-l) + 2v_s^2 + (v_s - v_r)^2}{2fgv_s} \quad (5.107)$$

where

$s$  = distance from roll axis to the first ingot tilting hook,

$l$  = bloom length in the given pass,

$v_s$  = operating speed of the roll table,

$v_r$  = rolling speed.

When calculating the minimum auxiliary time between two succeeding passes it was found that this is often decided by the adjusting times of the top roll, which are relatively long, due to insufficient acceleration of the screw-down motors.

The results of calculation are given in Tables 5.18 and 5.19 and are shown graphically in Fig. 5.73.

It can be seen from the calculated values of the effective torque that although the number of passes is reduced to 15, and the rolling time of one ingot to 82 sec, the main driving motor is still loaded to 82.5% only. This indicates that the rolling time could be further reduced, by decreasing the auxiliary time (increasing the acceleration of the screw-down motors), and by increasing the rpm of the main driving motor up to 70 rpm during the first passes.

Analysis of measured results makes it possible to work out a rolling schedule mathematically, ensuring a considerable increase in mill output from 1 million to as much as 1.5 million tons per year.

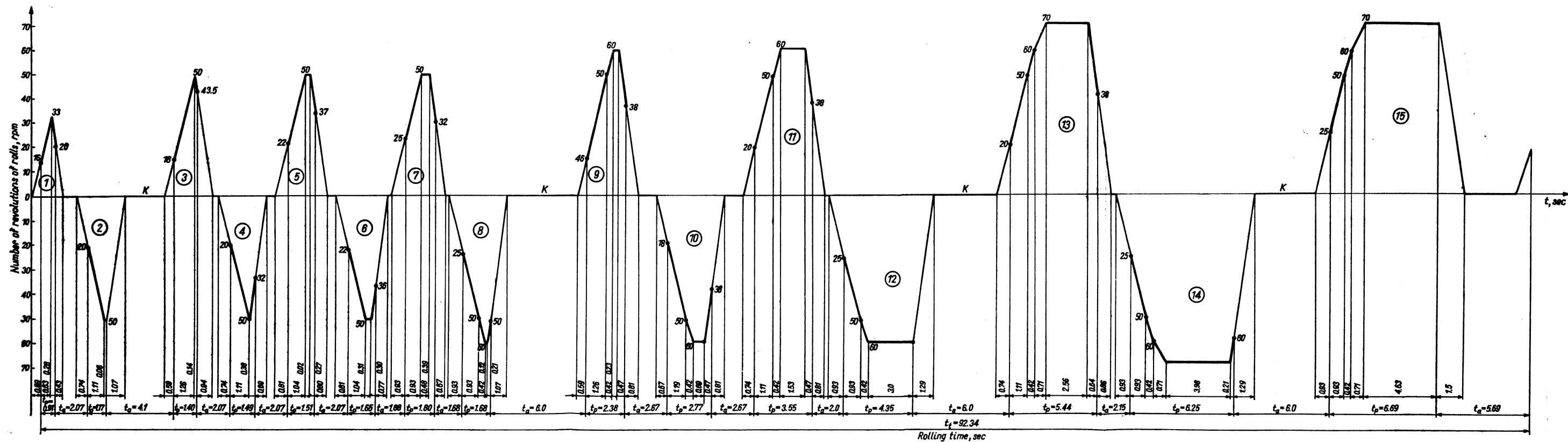


Fig. 5.73. Speed distribution during rolling of a  $\varnothing 630/\varnothing 530$  mm ingot down to a  $130 \times 230$  mm final cross-section in 15 passes. Scale 1 mm = 0.1 sec

## CHAPTER 6

# HOT ROLLING WITH HEAVY DRAUGHTS

### 6.1. Elongation and Spread During Hot Rolling of Mild Steel with Heavy Draughts

The problem of hot rolling is a very large one, consequently it is considered in several parts.

The purpose of the first part is the examination and analysis of behaviour of mild steel when rolled with heavy draughts, and hence to find the correlation between the coefficients  $\beta$  and  $\gamma$  for small values of  $\gamma$  in the range from 0.5 to 0.1.

So far very few experiments have been carried out in this field, previous experiments having been in the range from 0.9 to 0.5. It was decided to carry out experiments on a very large scale and to present the results, after analysis, by appropriate diagrams and formulae.

The results obtained were analysed on the basis of Z. Wusatowski's formula, thus a comparison was made between the actual spread and values calculated by means of the author's formulae (3.45)–(3.48).

*Description of experiments.* Experiments were carried out using Schloemann's 2-high rolling mill stand in the Institute of Ferrous Metallurgy [166–170]. For these investigations rolls with diameters 308 and 305 mm were used. The initial series of tests were performed on rolls with diameter 308 mm. Due to roll wear, the rolls were later sent to be dressed to 305 mm dia.

For rolling investigations mild steel was used with the composition: 0.19% C, 0.31% Mn, 0.10% Si, 0.022% P, 0.031% S.

Initial dimensions of specimens were: 20×20 mm, 20×40 mm, 15×100 mm and 10×110 mm. The stock was cut cold with a shearing machine into lengths of 600 mm. The dimensions of every specimen were checked with a slide gauge to an accuracy of  $\pm 0.1$  mm. Four kinds of specimens with various values of width to height ratio  $\delta_w = b_1/h_1$  (1, 2, 6.6, 11) were rolled. As a result, various values of  $\varepsilon_w = h_1/D$  were obtained varying from 0.03246 to 0.06493.

In the initial calculations absolute draughts were calculated for draughts from 5 to 90%, also draught coefficients and bite angles for the lower draught values.

These calculations for specimen dimensions used are shown in Table 6.1.

For every draught, three specimens for each of the four chosen width to height ratios were rolled. Rolling was performed at four temperatures: about 900, 1000, 1100 and 1200° C. Lower temperatures were not used since due to heavy draughts there was danger of roll fracture.

During rolling it became apparent that due to low engine power,  $N = 51.5$  kW, it was not possible to roll specimens with dimensions 15×100 mm and 10×110 mm with draughts of 70% and 80% at a temperature of 900°C, also specimens



with dimensions  $15 \times 100$  mm and  $10 \times 110$  mm with draughts of 90% at the whole temperature range investigated. The rolling speed (0.19 m/sec) was read from an oscillograph tape, which recorded simultaneously the speed of the outgoing metal, the peripheral speed of the rolls and pressure.

TABLE 6.1  
DRAUGHT COEFFICIENT, ABSOLUTE DRAUGHT AND ANGLES OF BITE

| $h_1$<br>mm               | $b_1$<br>mm | $h_2$<br>mm | $\Delta h$<br>mm | $\gamma$ | $D$<br>mm | Angle<br>of bite |                   |                   |
|---------------------------|-------------|-------------|------------------|----------|-----------|------------------|-------------------|-------------------|
| (a) For a square 20×20 mm |             |             |                  |          |           |                  |                   |                   |
| 20                        | 20          | 10          | 10               | 0.5      | 308       | 18°30'           | 19°30'            |                   |
| 20                        | 20          | 8           | 12               | 0.4      | 308       |                  |                   |                   |
| 20                        | 20          | 6           | 14               | 0.3      | 308       |                  |                   |                   |
| 20                        | 20          | 4           | 16               | 0.2      | 308       |                  |                   |                   |
| 20                        | 20          | 2           | 18               | 0.1      | 308       |                  |                   |                   |
| (b) For a flat 20×40 mm   |             |             |                  |          |           |                  |                   |                   |
| 20                        | 40          | 10          | 10               | 0.5      | 305       | 18°40'           | 19°10'            |                   |
| 20                        | 40          | 8           | 12               | 0.4      | 305       |                  |                   |                   |
| 20                        | 40          | 6           | 14               | 0.3      | 305       |                  |                   |                   |
| 20                        | 40          | 4           | 16               | 0.2      | 305       |                  |                   |                   |
| 20                        | 40          | 2           | 18               | 0.1      | 305       |                  |                   |                   |
| $h_1$<br>mm               | $b_1$<br>mm | $h_2$<br>mm | $\Delta h$<br>mm | $\gamma$ | $D$<br>mm | Angle<br>of bite | $D^\dagger$<br>mm | Angle<br>of bite† |
| (c) For a flat 15×100 mm  |             |             |                  |          |           |                  |                   |                   |
| 15                        | 100         | 7.5         | 7.5              | 0.5      | 308       | 16°0'            | 305               | 16°10'            |
| 15                        | 100         | 6.0         | 9.0              | 0.4      | 308       |                  |                   |                   |
| 15                        | 100         | 4.5         | 10.5             | 0.3      | 308       |                  |                   |                   |
| 15                        | 100         | 3.0         | 12.0             | 0.2      | 308       |                  |                   |                   |
| 15                        | 100         | 1.5         | 13.5             | 0.1      | 308       |                  |                   |                   |
| (d) For a flat 10×110 mm  |             |             |                  |          |           |                  |                   |                   |
| 10                        | 110         | 5           | 5                | 0.5      | 308       | 13°10'           | 305               | 13°10'            |
| 10                        | 110         | 4           | 6                | 0.4      | 308       |                  |                   |                   |
| 10                        | 110         | 3           | 7                | 0.3      | 308       |                  |                   |                   |
| 10                        | 110         | 2           | 8                | 0.2      | 308       |                  |                   |                   |
| 10                        | 110         | 1           | 9                | 0.1      | 308       |                  |                   |                   |

† Values after dressing.

† Values after dressing.

The general procedure for the investigations was as follows:

- determination of roll speed,
- rolling of test specimen in order to verify the correct setting of rolls,
- rolling out of three specimens for the given draught,
- setting of roll for a new draught, followed by checking the rolling out of the next three specimens, and so on until the chosen draught ranges have been covered.

The stock for all tests was heated together in a gas fired box-type heating furnace and in this way the temperature was maintained within more or less stable limits.

The temperature was measured with an optical pyrometer "Pyro" without corrections. The pyrometer was previously checked in the Physics Laboratory of the Institute of Metallurgy.

The results obtained were used to determine the relationship between  $\beta$ ,  $\gamma$ ,  $\lambda$  under heavy draughts. The calculations showed that Z. Wusatowski's formula (3.45)

$$\beta = \gamma^{-W}$$

gives considerable errors when heavy draughts are used.

Expressing this mathematically

$$\beta = f(\gamma, \delta_w, \varepsilon_w)$$

$$\lambda = g(\gamma, \delta_w, \varepsilon_w)$$

with the constant volume (2.8)

$$\gamma\beta\lambda = 1$$

In the limiting case, e.g. when  $\beta = 1$  or when  $\lambda = 1$  the equation takes the form

$$\lambda = \frac{1}{\gamma} \quad \text{or} \quad \beta = \frac{1}{\gamma}$$

These are hyperbolic equations.

The relationship for heavy draughts is found to be

$$\beta = f(\gamma, \delta_w, \varepsilon_w)$$

The equation which satisfies this relationship in a general form would be an equation of type

$$\beta = \gamma^{-f(\delta_w, \varepsilon_w)} = \gamma^{-W_1} \quad (6.1)$$

When the variation of  $-W_1$  relative to  $\delta_w$  and  $\varepsilon_w$  was examined it was found that the equation which satisfies the necessary conditions is of the form

$$\beta = \gamma^{-10^{-3.457\varepsilon_w^{0.968}}\delta_w} \quad (6.2)$$

where

$$-W_1 = -10^{-3.457\varepsilon_w^{0.968}}\delta_w \quad (6.3)$$

Substituting the value  $W_1$  for  $W$  in the formula (3.45) the required values are obtained. Since the coefficients of spread and elongation are connected by the relationship (2.8) for the same rolling processes [7], when  $-W_1$  has been calculated for the spread the elongation coefficient can be determined from

$$\lambda = \gamma^{(W_1-1)} \quad (6.4)$$

Comparing the previous formula (3.45) with the new one (6.2) it seems that they differ only in the exponential coefficients. In order to compare the variation of values of

$W$  and  $W_1$  they were calculated for constant  $\varepsilon_w = 0.06$ , and  $\delta_w$  varying from 0.5 to 11. The values were plotted on the diagram in Fig. 6.1 in the system

$$W_1 = f(\delta_w, \varepsilon_w) = \text{constant}$$

It can be seen in this diagram that  $W_1$  takes values greater than  $W$ .

To avoid the arduous calculation of the coefficient  $W_1$  from equation (6.3), a diagram was worked out (Fig. 6.2) which enables  $W_1$  to be read directly, given the known indices  $\delta_w$  and  $\varepsilon_w$ .

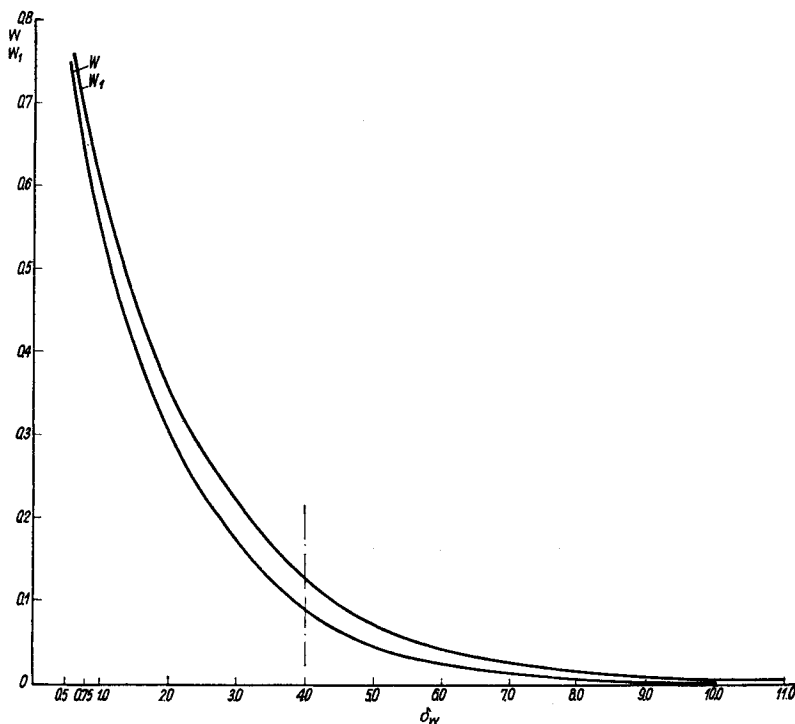


Fig. 6.1. The graph of exponential coefficient  $W$  and  $W_1$  of the functions  $\gamma^{-W}$  and  $\gamma^{-W_1}$  plotted against  $\delta_w = b_1/h_1$  for  $\varepsilon_w = h_1/D = 0.06 = \text{constant}$

When checking the error it was found that the exponential coefficient  $W_1$  gives good results for flats with a width to height ratio greater than four.

This error varies between the limits  $-0.1$  to  $-5.9\%$  for flats with the ratio  $b_1/h_1 = 11.0$  and from  $-0.38$  to  $-6.8\%$  for flats with the ratio  $b_1/h_1 = 6.6$ .

The mean error varies within the limits  $\pm 3\%$ , but the mean error calculated by means of exponent  $W$  varies within the limits of  $\pm 7\%$ . For the spread to height ratio equal to two, this error would be from  $+0.25$  to  $-7.5\%$ .

Much worse is the case of squares, particularly of those which were rolled with draughts 80% and 90%. The derived relationship gives a great scatter of errors from  $-0.45\%$  to 39%.

It appeared that the change of friction conditions affects the spread to a high degree causing a great increase of spread when rough rolls are used and conversely

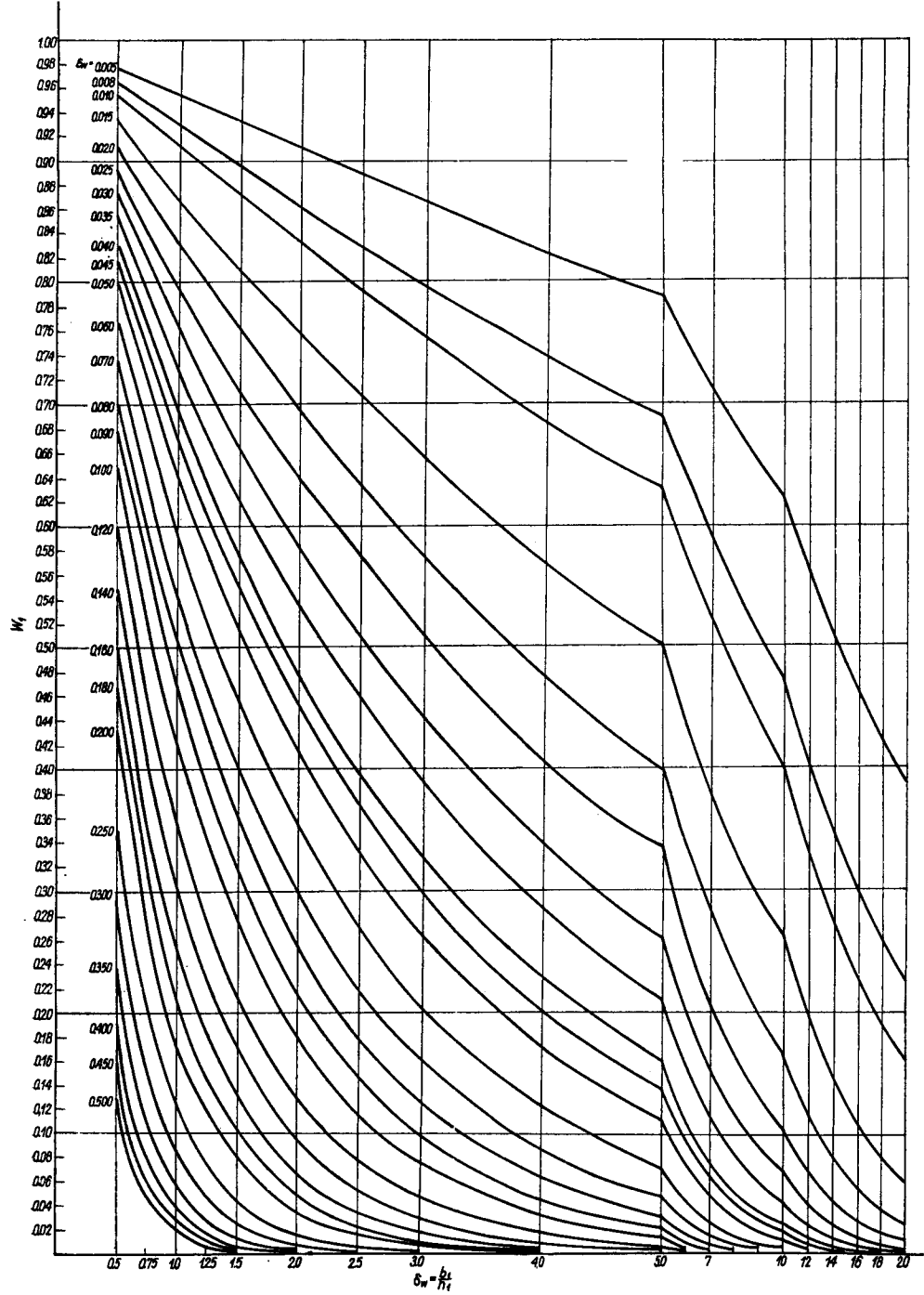


Fig. 6.2. Diagram of exponential coefficient  $W_1$  plotted against factors  $\delta_w$  and  $\epsilon_w$

with smooth rolls. Since in both cases  $\delta_w$  and  $\varepsilon_w$  for the squares stayed more or less the same, the same spread would be expected with the draught more or less stable.

## 6.2. Forward Slip and No-slip Angle in Rolling Mild Steel with Heavy Draughts

From analyses and experiments [54], [57] it is known that nearly all previous formulae for no-slip angle, rolling speed and forward slip are valid only for cases with constant width.

For rolling with heavy draughts only equations that take into account rolling spread and constant volume principles can be used.

From equation (3.116) the following expression (3.145) is derived for the speed at the neutral plane

$$\frac{v_N}{\lambda_N} = \frac{w_2}{\lambda} = v_1$$

In this case  $\lambda_N$  will be calculated rather differently. A publication on cases of rolling mild steel with heavy draughts [166], defines an exponent  $W_1$  where

$$\lambda_N = \gamma_N^{W_1-1} \quad (6.5)$$

TABLE 6.2  
THE RESULTS OF FORWARD SLIP TESTS

| No. | $t$<br>°C | $a_0$<br>mm | $l_p$<br>mm | $\gamma = h_1/h_0$ | $\beta = b_1/b_0$ | $\delta_w = b_0/h_0$ | $s = l_p/a_0$ † | $\alpha^\circ$ | $\delta^\circ$ | $\alpha/\delta$ |
|-----|-----------|-------------|-------------|--------------------|-------------------|----------------------|-----------------|----------------|----------------|-----------------|
| 1   | 940       | 490         | 500         | 0.5113             | 1.5000            | 1.0075               | 1.0204          | 14°25'         | 4°40'          | 0.3237          |
| 2   | 920       | 490         | 500         | 0.5067             | 1.5375            | 1.0101               | 1.0204          | 14°28'         | 4°28'          | 0.2996          |
| 3   | 910       | 490         | 501         | 0.5126             | 1.5300            | 1.0100               | 1.0224          | 14°22'         | 4°40'          | 0.3248          |
| 4   | 960       | 365         | 380         | 0.5050             | 1.2356            | 1.9950               | 1.0411          | 14°40'         | 3°55'          | 0.2670          |
| 5   | 930       | 365         | 382         | 0.5099             | 1.2630            | 1.9950               | 1.0466          | 14°40'         | 4°0'           | 0.2727          |
| 6   | 920       | 365         | 381         | 0.5075             | 1.2612            | 2.0000               | 1.0438          | 14°40'         | 3°57'          | 0.2693          |
| 7   | 890       | 490         | 530         | 0.5340             | 1.0360            | 6.9047               | 1.0816          | 12°8'          | 3°50'          | 0.3021          |
| 8   | 890       | 490         | 532         | 0.4795             | 1.0561            | 6.8972               | 1.0857          | 12°45'         | 3°53'          | 0.2941          |
| 9   | 900       | 485         | 525         | 0.5302             | 1.6502            | 6.8221               | 1.0825          | 12°14'         | 4°0'           | 0.3269          |
| 10  | 920       | 365         | 397         | 0.5049             | 1.0337            | 11.771               | 1.0874          | 10°24'         | 3°10'          | 0.3044          |
| 11  | 920       | 365         | 398         | 0.4900             | 1.0447            | 10.940               | 1.0904          | 10°30'         | 3°10'          | 0.3015          |
| 12  | 890       | 365         | 397         | 0.5098             | 1.0291            | 10.784               | 1.0874          | 10°24'         | 3°10'          | 0.3044          |
| 13  | 1000      | 490         | 498         | 0.4840             | 1.4569            | 1.0025               | 1.0164          | 14°48'         | 3°40'          | 0.2477          |
| 14  | 1040      | 490         | 497         | 0.4836             | 1.6725            | 1.0000               | 1.0143          | 14°50'         | 4°0'           | 0.2697          |
| 15  | 980       | 490         | 497         | 0.4874             | 1.5544            | 0.9975               | 1.0143          | 14°45'         | 4°0'           | 0.2712          |
| 16  | 1040      | 365         | 383         | 0.4350             | 1.2300            | 2.0000               | 1.0493          | 15°38'         | 4°0'           | 0.2558          |
| 17  | 1040      | 365         | 377         | 0.5149             | 1.2214            | 1.9900               | 1.0329          | 14°33'         | 3°40'          | 0.2520          |
| 18  | 1020      | 365         | 377         | 0.5099             | 1.2239            | 1.9900               | 1.0329          | 14°40'         | 3°34'          | 0.2556          |
| 19  | 1000      | 490         | 518         | 0.5236             | 1.0501            | 6.8108               | 1.0571          | 12°16'         | 3°25'          | 0.2785          |
| 20  | 1000      | 490         | 525         | 0.5171             | 1.0524            | 6.9246               | 1.0653          | 12°16'         | 3°25'          | 0.2785          |
| 21  | 1010      | 490         | 525         | 0.5151             | 1.0614            | 6.7474               | 1.0714          | 12°26'         | 3°30'          | 0.2815          |
| 22  | 1040      | 490         | 515         | 0.5100             | 1.0375            | 10.0940              | 1.0510          | 10°15'         | 2°30'          | 0.2439          |
| 23  | 1020      | 490         | 514         | 0.5098             | 1.0310            | 10.7941              | 1.0490          | 10°20'         | 2°30'          | 0.2419          |
| 24  | 1030      | 490         | 516         | 0.5250             | 1.0390            | 11.0250              | 1.0531          | 10°5'          | 2°35'          | 0.2561          |

† See formula (3.153).

Similarly for neutral angle a relation different from the previous ones is found by taking into account equation (6.5)

$$\frac{\cos \delta}{\left[ \frac{D - D \cos \delta - h_2}{h_1} \right]^{w_1 - 1}} = \frac{w_2}{\lambda v_r} \quad (6.6)$$

For a practical solution of equation (6.6) the nomogram Fig. 3.69 is used, and the exponential coefficient will be calculated differently, taking into account heavy draughts.

To minimize the influence of accidental factors on the measured values, three specimens for every  $b_1/h_1$  relation were rolled at the same temperature with the same roll setting. Forward slip values were measured in two ways; by measuring the distance between rags on rolls and on rolled specimen and after recalculating them for  $w_2/v_r$ .

*Elaboration of results.* The results of these experiments were collected in the form of tables, as for example Table 6.2. On the basis of these tables groups of diagrams were evolved representing:

- (1) *The relation between forward slips and spread coefficient  $\beta$  to draught coefficient  $\gamma$*

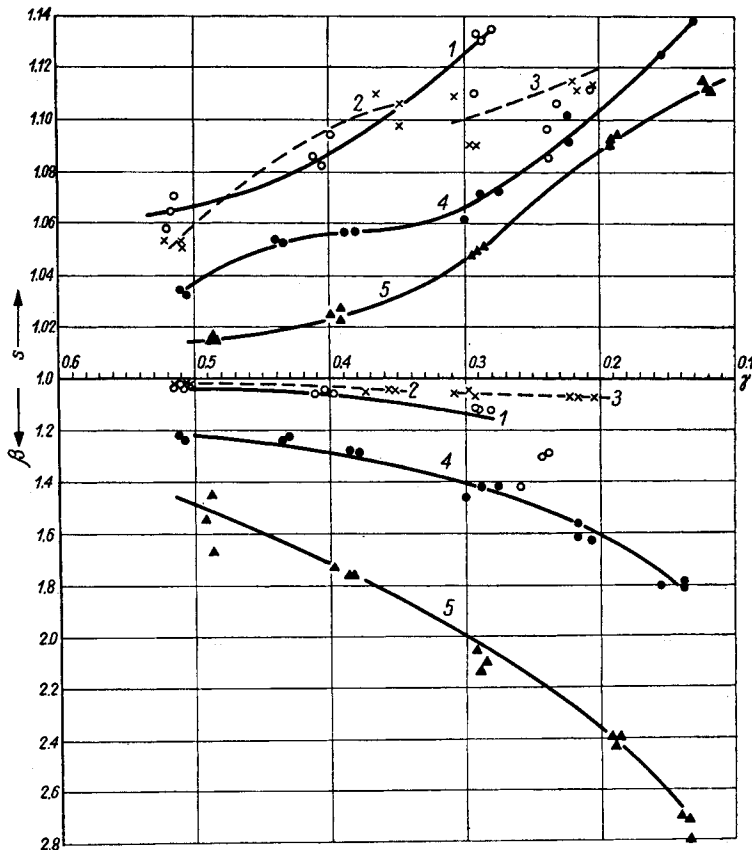


Fig. 6.3. Forward slip and  $\beta$  relative to  $\gamma$  at temperature 1000°C: 1 —  $b_1/h_1 = 6.6$ ,  $D = 308$  mm; 2 —  $b_1/h_1 = 11$ ,  $D = 308$  mm; 3 —  $b_1/h_1 = 11$ ,  $D = 305$  mm; 4 —  $b_1/h_1 = 1$ ,  $D = 305$  mm; 5 —  $b_1/h_1 = 1$ ,  $D = 308$  mm

Forward slip is defined as the relation

$$s = \frac{w_2}{v_r}$$

Figures 6.3 and 6.4 show that the forward slip and spread coefficient  $\beta$  increase with decrease of draught coefficient, independently of temperature value and the  $b_1/h_1$  relation. The spread coefficient  $\beta$  reaches the maximum value in the rolling of squares, while the forward slip reaches the maximum in the rolling of flats. The value of the coefficient  $\beta$  with  $\gamma$  approaching 0.1 does not exceed 3.0. Forward slip on the other hand does not exceed the value of 1.16, i.e. 16%.

The greater the spread coefficient at constant temperature and constant draught coefficient, the smaller is the forward slip.

For the relationship  $b_1/h_1 = 1$  the forward slip at first increases slowly, then after a certain  $\gamma$  which depends on the temperature reached, a more rapid increase of forward slip is observed while the spread coefficient increases more uniformly.

At a temperature of 1200°C (Fig. 6.4) the rapid forward slip increase begins at draught coefficients  $\gamma = 0.2$ . With increase of temperature the threshold of the more

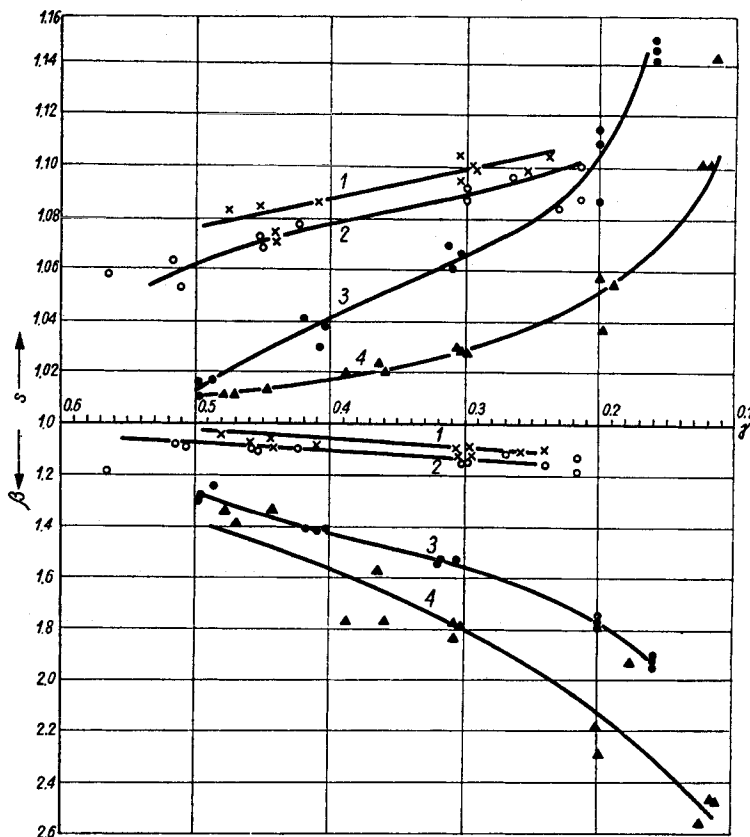


Fig. 6.4. Forward slip and  $\beta$  relative to  $\gamma$  at temperature 1200°C: 1 —  $b_1/h_1 = 11$ ,  $D = 308$  mm; 2 —  $b_1/h_1 = 6.6$ ,  $D = 308$  mm; 3 —  $b_1/h_1 = 2$ ,  $D = 305$  mm; 4 —  $b_1/h_1 = 1$ ,  $D = 305$  mm

rapid forward slip increase tends towards higher values, and at a temperature of  $1000^{\circ}\text{C}$  it reaches the value of  $\gamma = 0.4-0.33$ .

A similar trend is shown by the curves for  $b_1/h_1 = 2$ . The curves giving the change of the coefficient of spread  $\beta$  plotted against the coefficient of draught  $\gamma$  show greater increase of  $\beta$  with decrease of  $\gamma$  for  $b_1/h_1 = 1$  than for  $b_1/h_1 = 2$ .

(2) *Relation between forward slip and temperature*

The relation between forward slip and temperature with constant coefficient of draught and particular values of  $b_1/h_1$  is shown in Figs. 6.5 and 6.6.

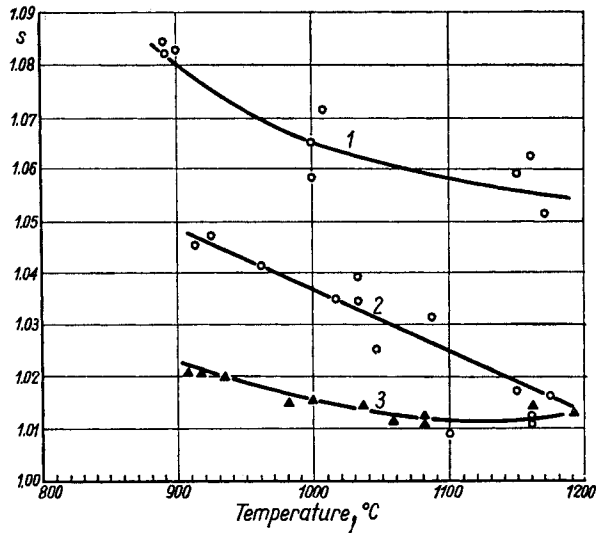


Fig. 6.5. Forward slip plotted against temperature for  $\gamma = 0.5$ : 1 —  $b_1/h_1 = 6.6$ ,  $D = 308$  mm; 2 —  $b_1/h_1 = 2$ ,  $D = 305$  mm; 3 —  $b_1/h_1 = 1$ ,  $D = 308$  mm

Rolling conditions are the same as given for forward slip and spread for increasing draught. The curves plotted in Figs. 6.5 and 6.6 show that with increase of rolling temperature, other factors being constant, forward slip drops off. Figs. 6.5 and 6.6

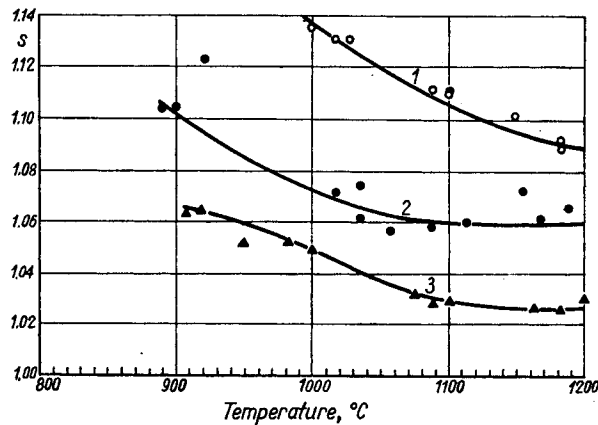


Fig. 6.6. Forward slip plotted against temperature for  $\gamma = 0.3$ . Symbols as in Fig. 6.5



show also that for  $b_1/h_1 = 1$  at a temperature of  $900^\circ\text{C}$  to about  $1100^\circ\text{C}$  the forward slip drop with temperature increase is greater than for temperatures  $>1100^\circ\text{C}$ , where forward slip values remain almost at a constant level.

A similar trend is shown by the curves where forward slip is plotted against temperature for the remaining  $b_1/h_1$  values. On the other hand the curves for  $b_1/h_1 = 1$  or 2 have a rather different form since the forward slip decrease is more uniform. The shape of these curves agrees with rolling theory since the friction coefficient, beginning with a temperature of about  $700^\circ\text{C}$  where it reaches its maximum value, falls off with temperature increase, consequently the forward slip value should also decrease.

(3) *The relation between  $\delta/\alpha$  ratio and spread coefficient  $\beta$  and draught coefficient  $\gamma$*

The relation between ratio  $\delta/\alpha$  and spread coefficient  $\beta$  and draught coefficient  $\gamma$  with constant  $b_1/h_1$  for several temperatures is shown in Figs. 6.7 and 6.8. Rolling conditions were the same as before. The curves showing  $\delta/\alpha$  plotted against  $\gamma$  for  $b_1/h_1 = 1$  (Fig. 6.7) and for  $b_1/h_1 = 6.6$  at temperatures of  $1000$  and  $1200^\circ\text{C}$  (Fig. 6.8)

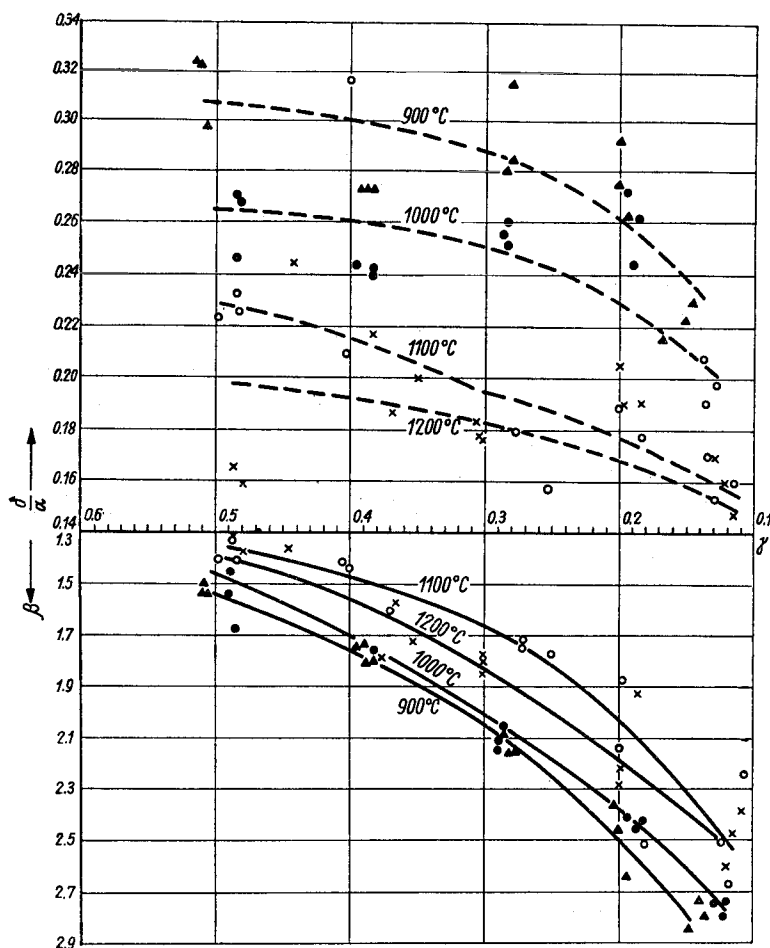


Fig. 6.7.  $\delta/\alpha$  ratio and  $\beta$  plotted against  $\gamma$  for  $b_1/h_1 = 1$

have an assumed form due to the points obtained being very scattered, thus a true form could not be plotted. From Figs. 6.7 and 6.8 it can be seen that with the increase of draught coefficient  $\gamma$  the ratio  $\delta/\alpha$  decreases while the spread coefficient  $\beta$  increases.

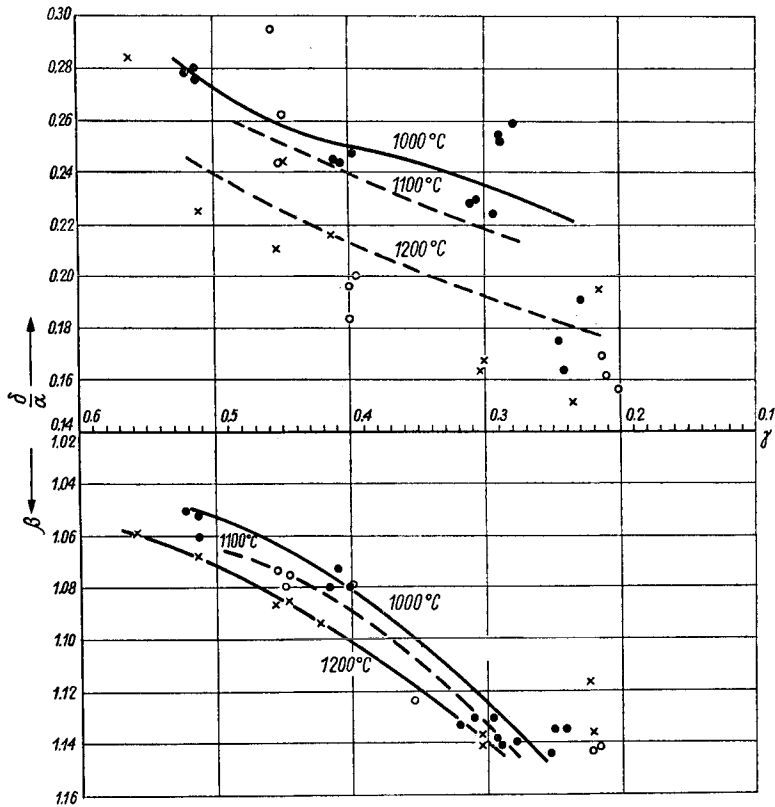


Fig. 6.8.  $\delta/\alpha$  ratio and  $\beta$  plotted against  $\gamma$  for  $b_1/h_1 = 6.6$

Considering the diagram for  $b_1/h_1 = 6.6$  (Fig. 6.8) it can be seen that the higher the rolling temperature the greater is the spread coefficient and the smaller the  $\delta/\alpha$  ratio, i.e. greater spread corresponds to smaller  $\delta/\alpha$  values.

### 6.3. Roll Force and Torque in Rolling of Mild Steel

Experiments were carried out using a 2-high rolling-mill (Fig. 6.9). Two 100 t strain gauge load cells were built in under the roll adjusting screws, while the rolls were coupled to the drive by measuring links, each capable of transmitting a 12 tm torque. Forces and torques which occur during rolling were measured by a tensiometric method.

Diagrams evolved from the data obtained (Figs. 6.10 and 6.11) give the relation between roll force and draught coefficient  $\gamma$  for a constant width to height ratio of specimens and constant rolling temperature. The diagrams were obtained by plotting two or three tape recorded values for each measurement in the system: force-draught coefficient. Ranges of scatter were obtained, through which mean curves were drawn.

Analysis of forces acting at the lower draughts, within limits of 10–50% ( $\gamma = 0.9$ –0.5), shows that the curves obtained for force as a function of draught with constant  $b_1/h_1$  ratio and temperature, are almost rectilinear; also the beginnings of curves obtained for draughts above 50% ( $\gamma < 0.5$ ) are the continuations of the previous ones.

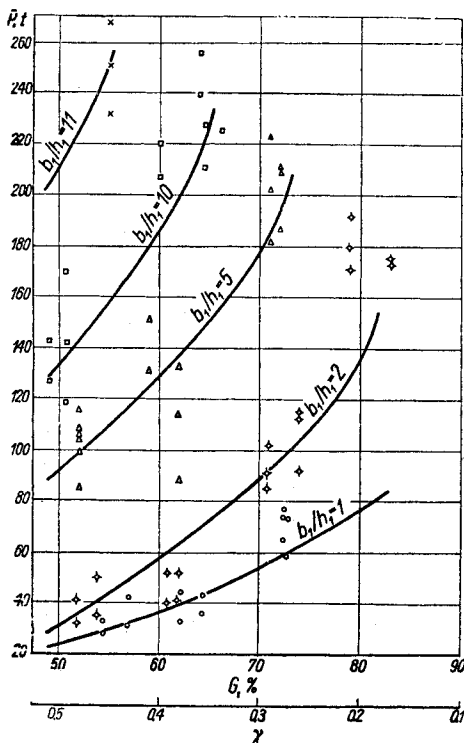


Fig. 6.10. Force measured as a function of draught at 900°C and fixed form factors

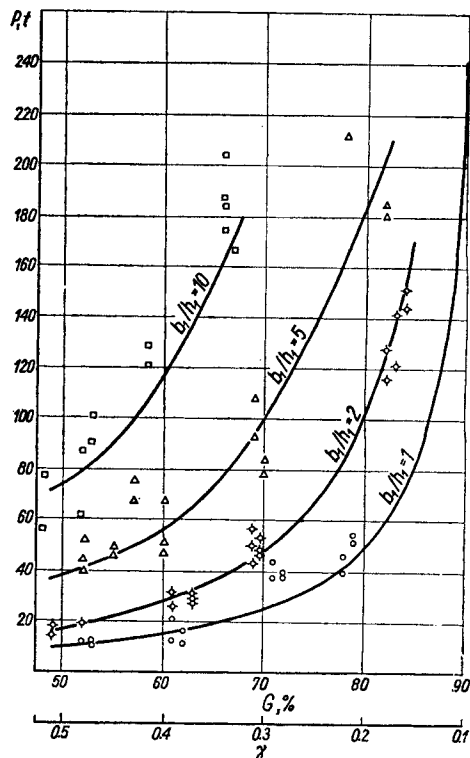


Fig. 6.11. Force measured as a function of draught at 1100°C and fixed form factors

For a given draught coefficient, increase of temperature gives a decrease of roll force value (Figs. 6.10 and 6.11). Independently of decrease of force values with increasing temperature these curves show a steeper gradient with decreasing draught coefficient, and for a temperature of 1100°C the curve showing the force–draught coefficient relation for  $b_1/h_1 = 1$  is close to an equiaxial hyperbola.

The large spread and the relatively small size of specimens causes great heat loss to the rolls and as a result the specimen is partially cooled. This is particularly troublesome with draught coefficients of the order of 0.1 (draught 90%, Fig. 6.11).

With the change of draught coefficient from 0.5 to 0.2 at a temperature of 1100°C and for form factor  $b_1/h_1 = 1$  the roll force becomes five times greater, and for  $\gamma = 0.1$  it is more than 20 times greater.

The analysis of results shows that one of the essential factors also having a great effect on the force magnitude is the ratio of specimen height to roll diameter.

With heavy draughts and an increase of  $b_1/h_1$  ratio, decrease of  $h_1/D$  ratio and constant roll diameter, considerable forces arise, which can limit the magnitude of applied draught, due to considerations of roll strength.

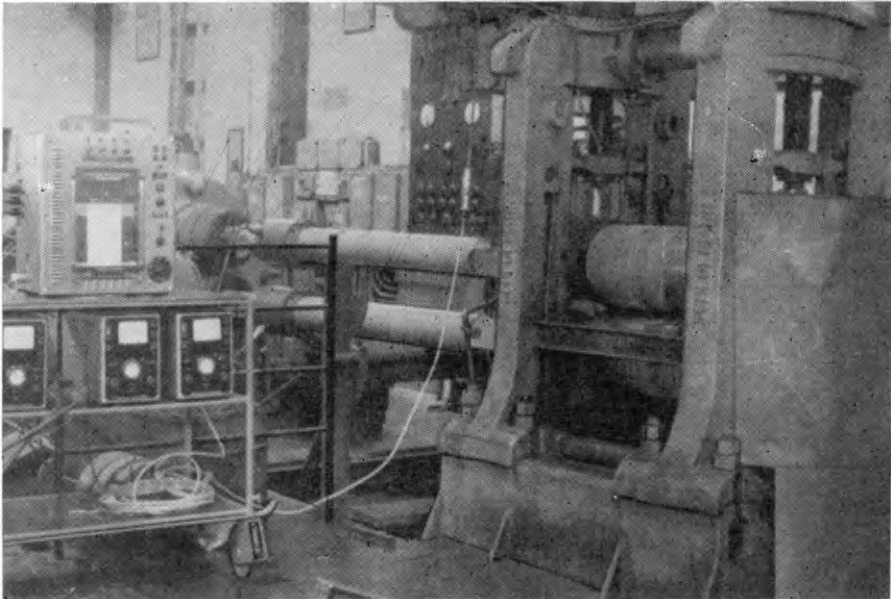


Fig. 6.9. Experimental rolling mill and its instrumentation

Torque measurements were made by recording separately upper and lower roll torques.

After being read from recording tape these torques were added and a total torque was obtained.

Diagrams in Figs. 6.12 and 6.13 show the relation between total torque and draught coefficient at a given temperature and with constant height to width ratios, for every specimen two or three values were read at the same points as for pressure.

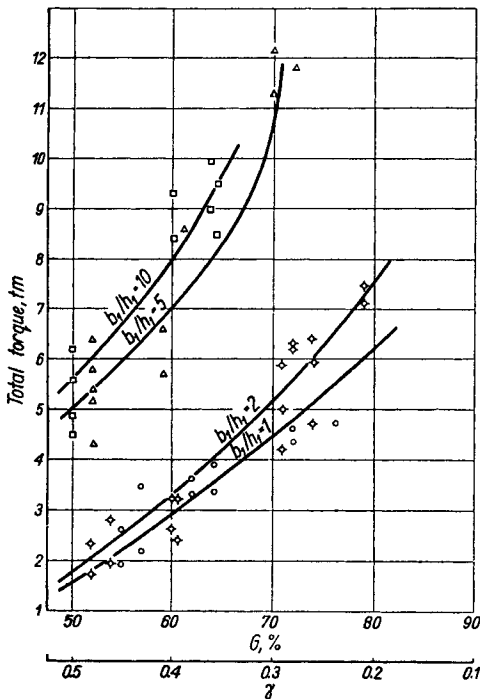


Fig. 6.12. Total torque as a function of draught at 900°C and fixed form factors

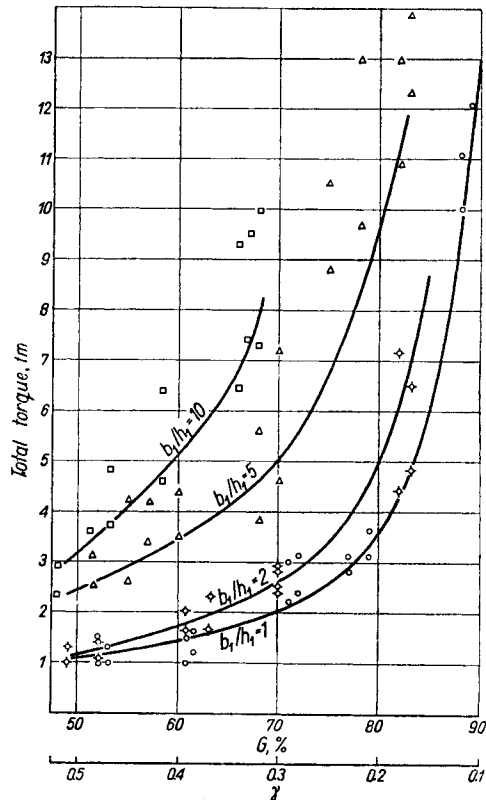


Fig. 6.13. Total torque as a function of draught at 1100°C and fixed form factors

These values for the total torque to draught coefficient relation show a certain scatter, through which mean curves were plotted.

Comparing the behaviour of curves of rolling torque and of force relative to draught coefficient, it is found that they show essentially the same form. This is reasonable when it is remembered that torque, being the product of vector force and moment arm, must vary directly as its factors.

Since actually the torque arm varies only within small limits, the most important factor in torque magnitude is the force. As force increases the rolling torque increases and conversely.

The almost rectilinear run of the total torque curve with increasing draught, tends to an increase of slope above a certain draught coefficient value, at the given

temperature. This increase of slope becomes more noticeable with rise of temperatures; e.g. at temperature 900°C for square specimens 40×40 mm (Fig. 6.12) the torque curve is almost rectilinear, while for temperature 1100°C (Fig. 6.13) the corresponding curve is an equiaxial hyperbola.

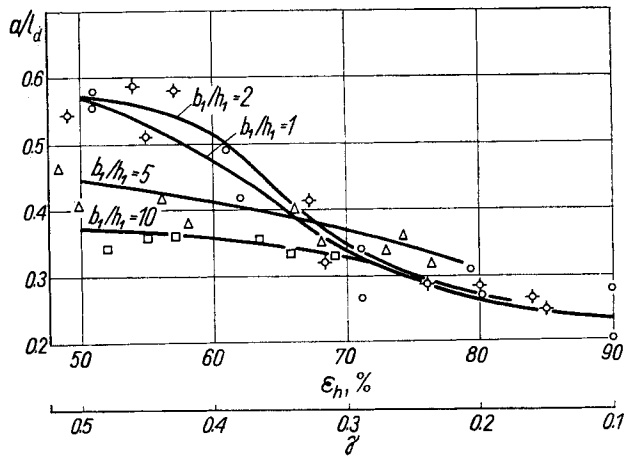


Fig. 6.14. Torque arm  $a$  (expressed as ratio  $a/l_d$ ) as a function of draught for various form factor values at temperature 900°C

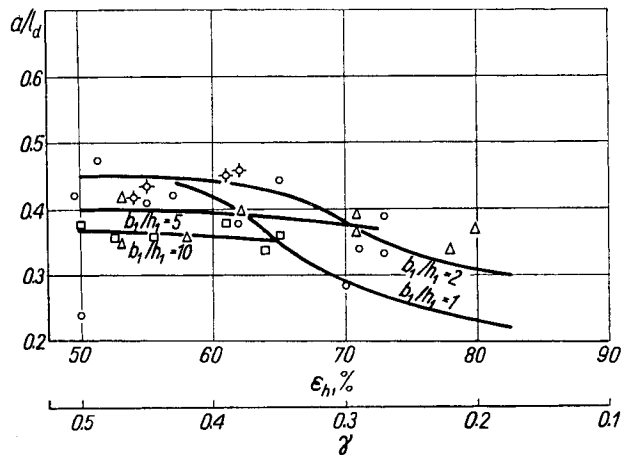


Fig. 6.15. Torque arm  $a$  (expressed as ratio  $a/l_d$ ) as a function of draught at temperature 1100°C, for various form factor values

At constant temperature the torque magnitude is noticeably dependent on width to height ratio: e.g. at temperature 900°C for small  $b_1/h_1$  ratios in the draught coefficient range from 0.5 to 0.2 the torque becomes four times greater (Fig. 6.12), whereas for large  $b_1/h_1$  ratios in the draught coefficient range from 0.5 to 0.3 it is about twice as great.

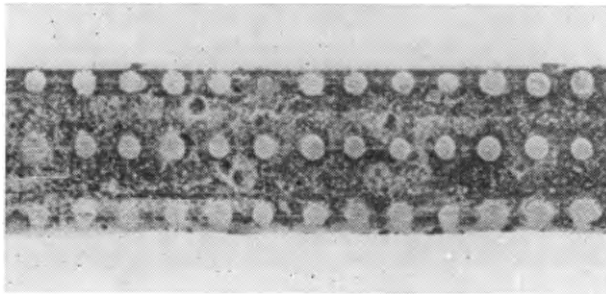


Fig. 6.16. Specimens with three rows of pins, before rolling

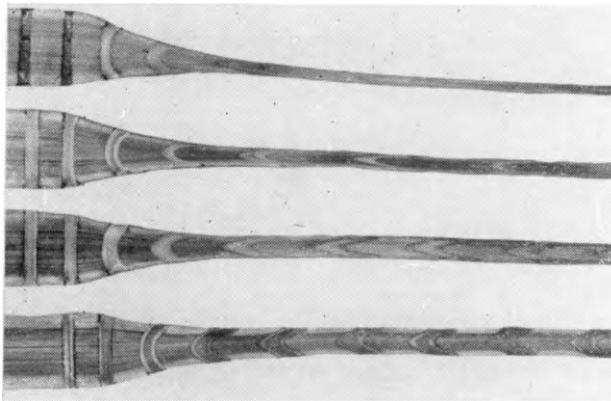


Fig. 6.17. Part-rolled specimens, sectioned lengthwise, and etched, rolled with draughts of 60, 70, 80 and 90%

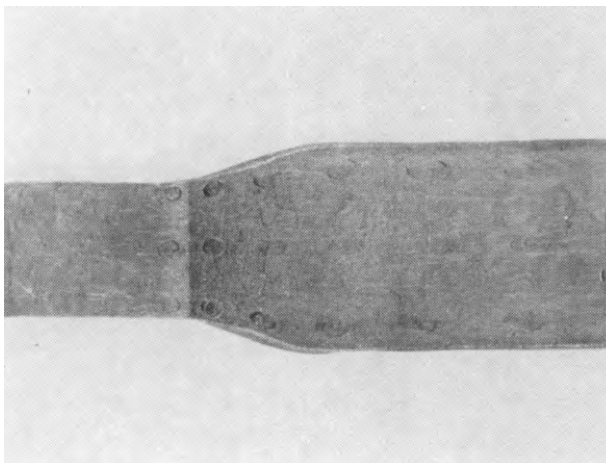


Fig. 6.18. Specimen with pins, rolled with draught of 80%

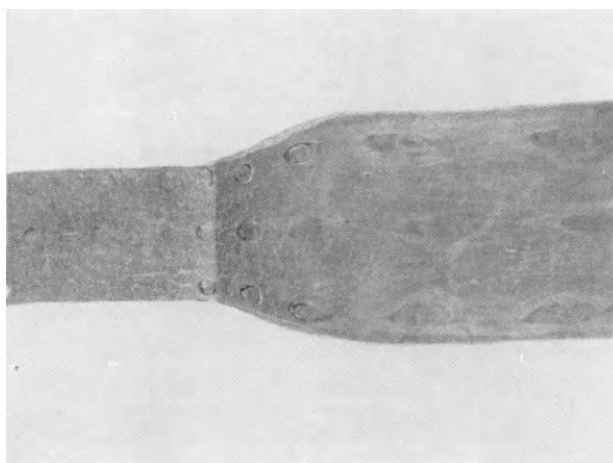


Fig. 6.19. Specimen with pins, rolled with draught of 90%

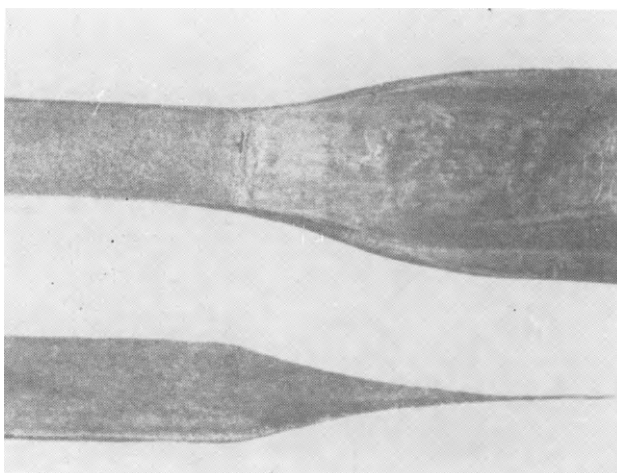


Fig. 6.20. Specimen 20×20 mm rolled with draught of 96%.  
Rough rolls



Independently of analysis of forces and torque, experiments carried out allowed of partial analysis of torque lever arm behaviour. During investigations the total torque was measured, from which the additional friction torque was subtracted, and the rolling torque thus found divided by twice the force to give the rolling torque arm.

These calculations were carried out separately for each specimen. The values obtained refer to the half length of the arc of contact, as normally used in calculations, in order to compare the values.

The calculated values were plotted in Figs. 6.14 and 6.15 showing the relation of torque lever arm  $a$  and draught coefficient, for constant  $b_1/h_1$  ratio and given temperatures.

Comparing the diagrams for these temperatures, it can be noted that with the temperature increase these values become higher while at constant temperature these values decrease with decreasing draught coefficient.

This change is very marked for ratios  $b_1/h_1 = 1$  and 2, but shows less with increase of the width to height ratio.

The curves for the small width to height ratios are not of rectilinear form, but curves for the ratio of  $b_1/h_1 \geq 4$  have an almost rectilinear form.

#### 6.4. Flow of Metal and Structural Phenomena in Rolling

Investigations of metal flow under heavy draughts were carried out on specimens with cross-section of  $20 \times 40$  mm and length of 300 mm, by drilling them in the centre section over a length of 120 mm, with three rows of holes, with intervals of 10 mm between the holes. Pins of 3.5 mm diameter, and of chemical composition close to that of steel investigated were forced into these drilled holes. Specimens (Fig. 6.16) were heated to  $1150^\circ\text{C}$  in a resistance box furnace.

The specimens were rolled on an experimental 2-high rolling mill in such a manner that rolling was broken off when the part of the specimen with pins was rolled out.

Two such experiments were carried out for each draught of 60, 70, 80 and 90% on smooth rolls with a diameter of 305 mm.

The rolling speed was the same as in previous investigations, and did not exceed 0.2 m/sec.

Figure 6.17 shows specimens rolled with draughts of 60, 70, 80 and 90% which were next cut lengthwise and etched.

On specimens cut along the rolling axis (Fig. 6.17) three layers with different relative flow speed can be distinguished. The first layer arises at the surface of contact between metal being rolled and the rolls. Considerable sticking of metal to the rolls due to frictional forces causes part of the metal in the immediate area of the rolls to slip parallel to the rolls and in the direction of rolling. Because of slip, the pins in direct contact with the rolls did not move up to a certain depth, as far as could be observed.

The second layer lies immediately next to the first one and comprises only a small part of the specimen rolled. In the third and middle layer, also the greatest one, a great metal retardation relative to the outer layers occurs. The second medial layer is formed as a result of the action of these two layers. Comparing the behaviour of pins with increasing draught (Fig. 6.17) it can be seen that metal flow speed also increases. This causes varying speeds of metal flow simultaneously at different sectional heights of the specimen investigated.

The formation of these three layers with different relative speeds gives rise—as the difference increases—to defects in the form of edge delamination of specimens being rolled.

Figures 6.18 and 6.19 show the top surfaces of the specimens in Fig. 6.17. Further characteristics which appear with heavy draughts can be distinguished.

With draught increase (Fig. 6.18) a greater elongation and spread of metal appear due to the displacement of its particles.

As the deformation of pins on these figures shows, the metal in the centre of a specimen flows more rapidly forward while with increasing draught the side pins move further away from the middle pins.

The metal spread however does not occur over the whole cross-section in the described manner. With a rough roll surface the top and bottom layers of the rolled specimen tend to flow rather in the direction of rolling with smaller side displacement.

The spread is here in the main due to the flow out of metal from the middle parts (Fig. 6.20).

Independently of the described phenomena the investigations carried out provide new data in regard to the retardation zone, sticking and forward slip, as well as speed distribution and unit pressure.

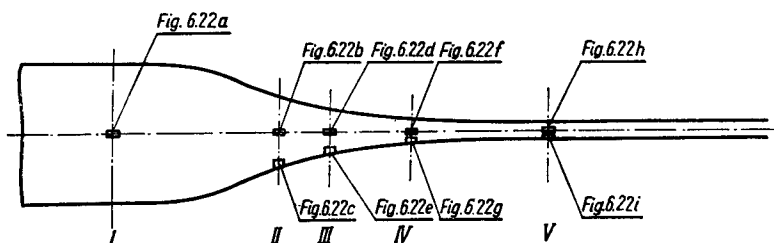


Fig. 6.21. Method of taking a sample for metallographic examination and hardness measurement on a specimen rolled from a square of 40 mm

With heavy draughts the range of plastic working is different in three directions. Hence the metal shows a varying structure and shows varying resistance properties. Therefore experiments were carried out to examine the changes of structure and hardness on a bar of cross-section  $40 \times 40$  mm, which was subjected to a draught of 50–90% along the rolling axis. Figure 6.21 shows the method of preparing specimens for metallographical tests, and the points selected for hardness tests.

Mild steel rolled with heavy draughts shows varying structure across the sections, depending on the degree of draught and on the heat abstraction by rolls. Analysing the microstructures in Fig. 6.22 certain characteristic changes may be found, different in the inner and extreme layers.

The structure seen in Fig. 6.22 is for the steel after heating in an oven. The metal after having been rolled and then heated shows large ferrite grains and non-uniformly distributed pearlite. These grains under a draught of the order of 50% are partly broken up and with draught increase they disintegrate more and more and the structure consists of uniformly distributed and ever finer ferrite and pearlite grains (Figs. 6.22b, d, f, h).

The structure of the extreme layers can be considered as corresponding to cold plastic deformation, since the heat abstraction by the rolls is here so great that recrystallization hardly occurs at all. As a result of draught, work hardening occurs. Above 50%

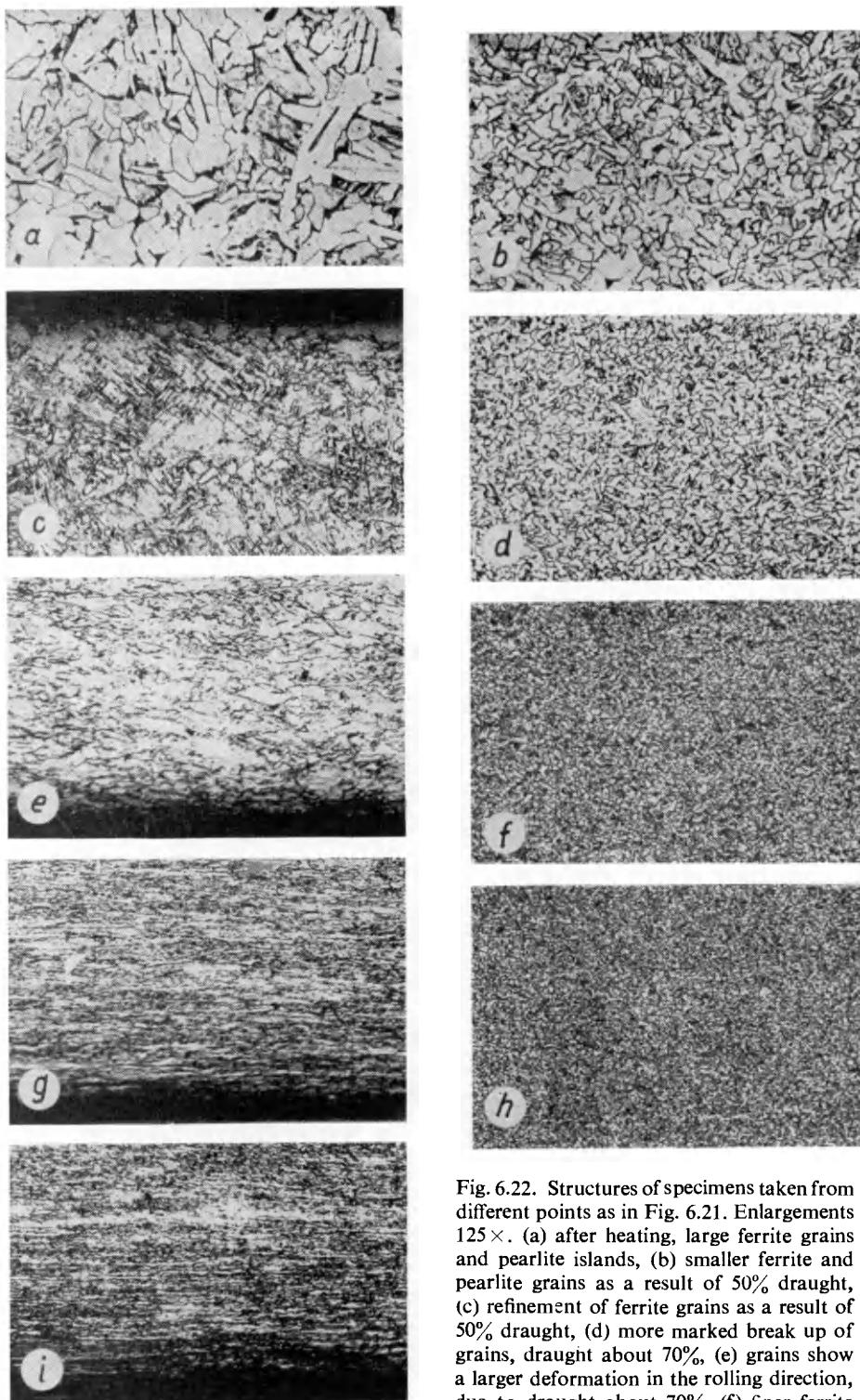


Fig. 6.22. Structures of specimens taken from different points as in Fig. 6.21. Enlargements  $125\times$ . (a) after heating, large ferrite grains and pearlite islands, (b) smaller ferrite and pearlite grains as a result of 50% draught, (c) refinement of ferrite grains as a result of 50% draught, (d) more marked break up of grains, draught about 70%, (e) grains show a larger deformation in the rolling direction, due to draught about 70%, (f) finer ferrite and pearlite grains, uniformly arranged, draught about 80%, (g) broken fine ferrite and pearlite grains, draught about 80%, (h) very marked breaking up of grains after rolling, draught about 90%, (i) retained very marked fibrous structure, which does not recrystallize

reduction, a considerable refinement of grain begins (Fig. 6.22c), and a marked elongation of grain appears with draught of about 70% after which the grain elongates more and more (Fig. 6.22e). For higher draughts the structure appearance becomes uniform and shows texture features—with unidirectionality of fibres (Figs. 6.22g, i). Work hardening is indicated not only by the change of structure, but also by the hardness increase, shown in Table 6.3.

TABLE 6.3  
RESULTS OF HARDNESS MEASUREMENTS OF SPECIMEN IN FIG. 6.21

| Tests point                                 | <i>I</i> | <i>II</i> | <i>III</i> | <i>IV</i> | <i>V</i> |
|---------------------------------------------|----------|-----------|------------|-----------|----------|
| Middle                                      | 112      | 135       | 148        | 187       | 203      |
| Edge                                        | 123      | 171       | 183        | 199       | 210      |
| Hardness data are Vicker's values HV = 5 kg |          |           |            |           |          |

From these microstructures a considerable refinement of grains in the centre with increasing deformation can be observed. The heat abstraction from the interior of the specimen was less intensive than for extreme layers. This allowed a greater degree of recrystallization of the central part, unlike the edges where the fibrous structure remained.

## CHAPTER 7

# PRINCIPLES OF ROLL PASS DESIGN

### 7.1. Basic Definitions

Roll pass designing comprises the preparation of designs for [113]:

- (a) all roll passes that are necessary to obtain a chosen profile or group of profiles, forming a complete unit for the particular set of rolls which are working together,
- (b) distribution of passes along the roll barrels,
- (c) appropriate pass patterns.

A rolling pass is a single rolling action, i.e. passing the metal being rolled between the rolls.

The profile determines a finished product, its cross-sectional shape, the limiting outline of pass or the limiting outline of metal being rolled in a given roll pass.

A hot section is a section corrected for heat expansion. The dimensions of a hot section are calculated as the product of the dimensions of the cold section chosen and the heat expansion at the rolling temperature. The heat expansion linear coefficient equals 1.013–1.015.

A pass represents the counterpart of the hot section shape, cut on the surface of the two rolls working together jointly.

Passes can be active or dead, as well as open or closed.

An active pass is intended to be rolled, and is formed by two active grooves (Fig. 7.1, 1 and 2).

A dead pass is not intended to be rolled, it forms part of 3-high systems, with a staggered arrangement of passes. It is made by one active groove cut in the middle roll, and a second dead one in the top or bottom roll (Fig. 7.1, 3 and 4).

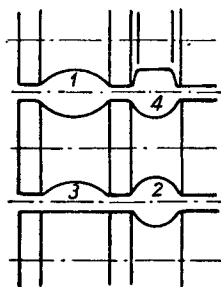


Fig. 7.1. Dead and active passes in a roll: 1, 2 — active passes; 3, 4 — dead passes

An open pass (Figs. 7.2a and b) is composed of two concave grooves or of one concave groove and one convex, in which case the line of clearance between the rolls makes an angle of less than  $60^\circ$  with the rolling line.

A closed pass (Fig. 7.2c, d) is formed by a concave groove and a convex one. The line of clearance makes an angle of  $60\text{--}90^\circ$  with the rolling line. Open and closed parts of passes may be found together in the same pass.

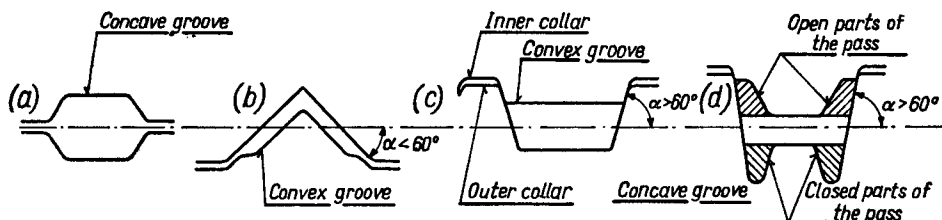


Fig. 7.2. Examples of open and closed passes

An open part of the pass is the name given to a part cut off by a line parallel to the line of rolling, the outline of which is formed by two rolls. The outline of a closed part is formed by a single roll (down-shaded parts in Fig. 7.2d).

A groove (Fig. 7.2) is a part of a pass cut on a single roll. Two grooves working jointly in the rolling plane form a pass.

Grooves may be of concave type lying between two external collars, or of convex type between two internal collars, or of diagonal type lying between one external collar and one internal collar (Fig. 7.3). There are also active grooves and dead ones, corresponding to active and dead passes.

The groove depth should not exceed  $1/6$  of the theoretical roll diameter.

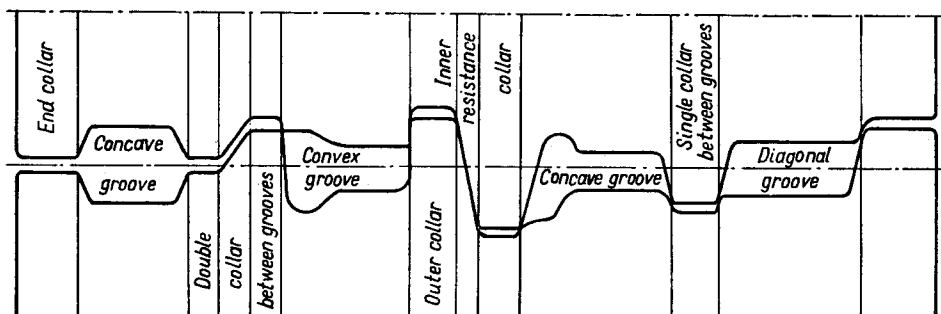


Fig. 7.3. Types of grooves and collars on a roll [113]

The groove taper is the inclination to the vertical of the side walls of the groove, cut into the roll. Taper is used to facilitate the introduction of metal into rolls and metal exit from rolls, to counteract the metal pick-up on rolls and also to facilitate redressing. In finished roll passes a taper of from  $0.5$  to  $3\%$  is used, in forming passes of  $3\text{--}10\%$ , in roughing passes  $6\text{--}15\%$ , in blooming mills  $6\text{--}20\%$ , where the lower values are for thinner sections, the greater values for thicker ones (Fig. 7.4).

Rounding or dulling of grooves is used to prevent the outflow of metal. For metal being rolled at a temperature of about  $1100^\circ\text{C}$  the minimum dulling is determined by formula (Fig. 7.5b)

$$c = \left( 3.61 - 0.485 \frac{b_1}{h_1} \right) (h_1 - h_2) \left( \frac{h_1 - h_2}{h_1} \right)^2 \quad (7.1)$$

The rounding or dulling is usually taken somewhat greater than calculated since with temperature decrease the spread increases, hence the danger of overflow increases. Over-large rounding can be reduced even during rolling. Increase of roundings radius gives increased roll strength.

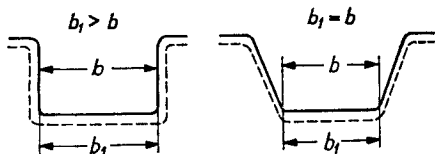


Fig. 7.4. Tapers of pass grooves [88]

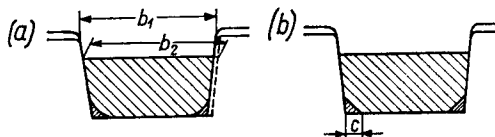


Fig. 7.5. Form of groove [88]: (a) rounded, (b) chamfered

The collar is that part of the roll barrel which separates the adjacent grooves (passes), forms the side walls of passes, facilitates roll setting and resists the axial forces that arise during rolling.

There are several kinds of collar (Fig. 7.3):

- (1) An end collar separates the roll and face from the groove. The width of these collars is normally greater than the width of inter-groove collars (inner collars).
- (2) An inner collar separates two adjacent grooves.
- (3) A single collar separates similar grooves i.e. convex from convex, concave from concave.
- (4) A double collar is composed of a collar with stepped cylindrical surfaces joined by a conical surface.
- (5) A resistance collar or lock is similar to the double one, but the conical surface is steeper. It resists the axial forces, and therefore prevents movement of the rolls along their axes during rolling.
- (6) An outer collar has a diameter larger than that of the adjacent groove.
- (7) An inner collar has a diameter smaller than that of the adjacent groove.

The width of a collar is at least equal to the groove depth for cast iron rolls, and is somewhat greater than half groove depth for cast steel rolls. The number of collars equals the number of passes  $+1$ .

The passive part of the roll is the sum of all collar widths. The active part is the sum of all groove widths.

Draught is a term used to express the reduction of linear dimension of a section or of its part.

Draught can be direct, that is a dimensional reduction of a section or of its element in a direction vertical to the roll axis. Lateral draught is a reduction of shape dimension in a direction parallel to the roll axis. Oblique draught is a reduction of shape dimension in the direction inclined to the roll axis.

Overdraught or underdraught (Fig. 7.6) is the difference between the working diameters in a given pass. Overdraught occurs when the working diameter of the upper roll is larger, underdraught when the working diameter of the lower roll is larger.

In the case of equal roll diameters the metal being rolled would leave the rolls either vertically to the rolling plane, or would bend up or down due to other influences e.g. difference in temperature between upper and lower parts of metal, different degree of smoothness of the upper and lower grooves. To guard against such contingencies upper and lower strippers must be used at the rolls exit, which hinders rolling.





The neutral line  $L_n$  is a straight line in the plane of the roll axes, parallel to and midway between them; the peripheral speeds of theoretical rolls which meet along this line are equal.

The neutral axis of pass  $A_{np}$  is a straight line in the pass plane, determined theoretically in order to position the pass in the rolls.

The rolling line  $L_r$  is an auxiliary straight line parallel to the roll axes and positioned relative to the neutral axis of rolls either lower by the amount of the overdraught, or, more rarely, higher by the amount of the underdraught. The neutral axes of the passes are positioned relative to this line which is shown on the roll design drawings.

The nominal roll diameter  $D_n$  is a diameter that designates the roll stand. This diameter is equal to the pitch diameter of pinions in the finishing stand rounded off upwards to the nearest multiple of 5 mm for diameters up to 500 mm, and to a multiple of 10 mm for diameters above 500 mm.

For example, if the pitch diameter of the pinions is equal to 278 mm, the nominal diameter is then equal to 280 mm and the stand is called a 280 mm roll mill. For adjustable rolls and those which have no pinion stands, the nominal diameter in 2-high mills is equal to the distance between the axes of new rolls set with the gap.

The theoretical diameter  $D_t$  in a 3-high mill is equal to half the distance between the outer roll axes, and in 2-high mills it is equal to the distance between the roll axes when the rolls are set with the gap.

Rolling diameters  $D_{ru}$ ,  $D_{rl}$  separately determined for each roll (upper and lower), are equal to double the distance between the rolling line and the axis of the given roll.

True diameters  $D_u$ ,  $D_l$  are diameters at the particular points on the roll barrel surface.

The following diameters are also distinguished:

- (a) working diameters referring to passes,
- (b) barrel diameter, measured for grooved rolls on the outer collar of the drive side, and for smooth rolls on the edge of the working part of the barrel,
- (c) maximum diameter, i.e. the largest true roll diameter,
- (d) minimum diameter, i.e. the smallest true roll diameter.

These primary diameters  $D_p$  theoretical, rolling, and true ones, determine the dimensions of the new roll. Due to roll wear and redressing these diameters decrease.

Dead diameters  $D_d$  theoretical, rolling, and true ones, determine the dimensions of the rolls which have been redressed for the last time.

The useful thickness of a roll is limited by roll strength, roll quality, in the case of the chilled rolls by the depth of the chilled layer, and finally by roll stand design. It is calculated as half the dead diameter.

The degree of roll wear is expressed as a % of useful roll thickness worn away.

Ratio of dead diameter to primary diameter (generally found in publications):

| <i>Rolls</i>                   | $D_d/D_p$ |
|--------------------------------|-----------|
| Blooming mills                 | 0.86      |
| Heavy mills                    | 0.84-0.9  |
| Medium mills                   | 0.86-0.9  |
| Small mills                    | 0.88-0.9  |
| Reversing plate mills          | 0.90-0.91 |
| 3-high mills with Lauth rolls: |           |
| chilled                        | 0.95      |
| semi-chilled                   | 0.90      |
| Sheet mills                    | 0.95-0.97 |

*Roll pass names and their sequences*

Roll passes named according to their action:

- (1) blooming (roughing) pass
- (2) breaking-down pass
- (3) edging pass
- (4) knifing pass
- (5) forming (sizing) pass

Roll passes named according to their shape (Fig. 7.8):

- (1) open box pass (a)
- (2) gothic square pass (b)
- (3) diamond pass: sharp (c) rounded (d)
- (4) oval passes: common oval (e), hexagonal (f), flattened oval (g)
- (5) bar passes: open square pass (h), rounded open square (i), round, etc.
- (6) section passes: angle pass, channel bar pass, rail pass, etc.

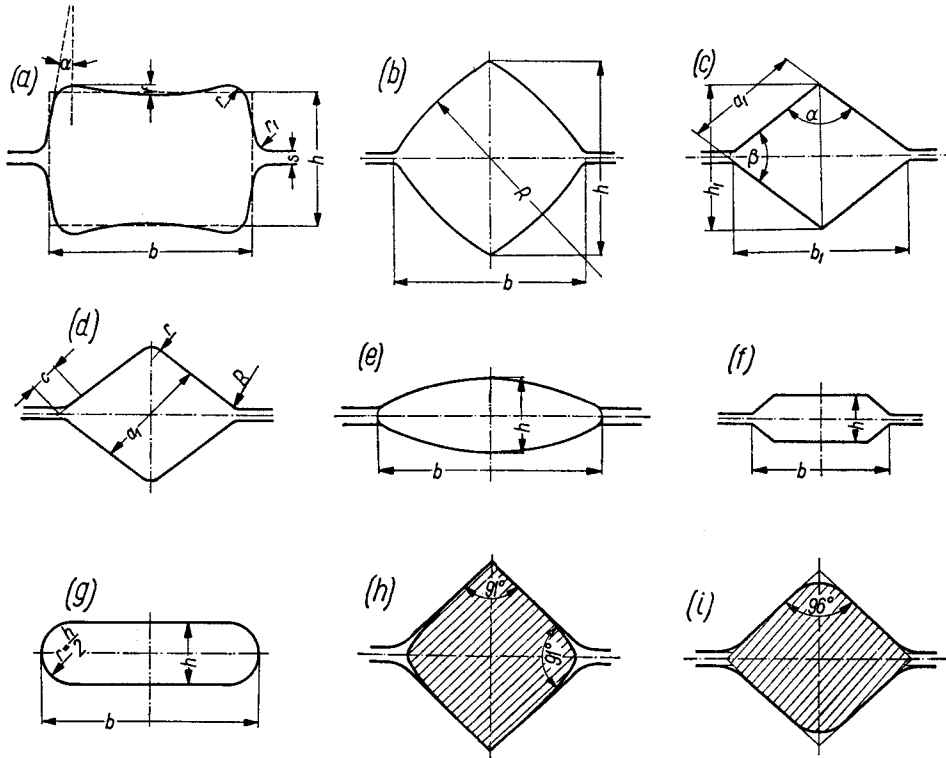


Fig. 7.8. Types of pass, according to shape

The names of bar and section passes correspond to the names of finished passes. Roll passes named in order of rolling sequence:

- (1) introductory pass
- (2) preparatory pass
- (3) leader pass
- (4) finishing pass

The names of basic breaking-down pass sequences (Fig. 7.9):

- (1) open box (a)
- (2) gothic square–gothic square (b)
- (3) square–square (c)
- (4) oval–oval (d)
- (5) diamond–diamond (e)
- (6) square–diamond (f)
- (7) square–oval (g)
- (8) round–oval (h)

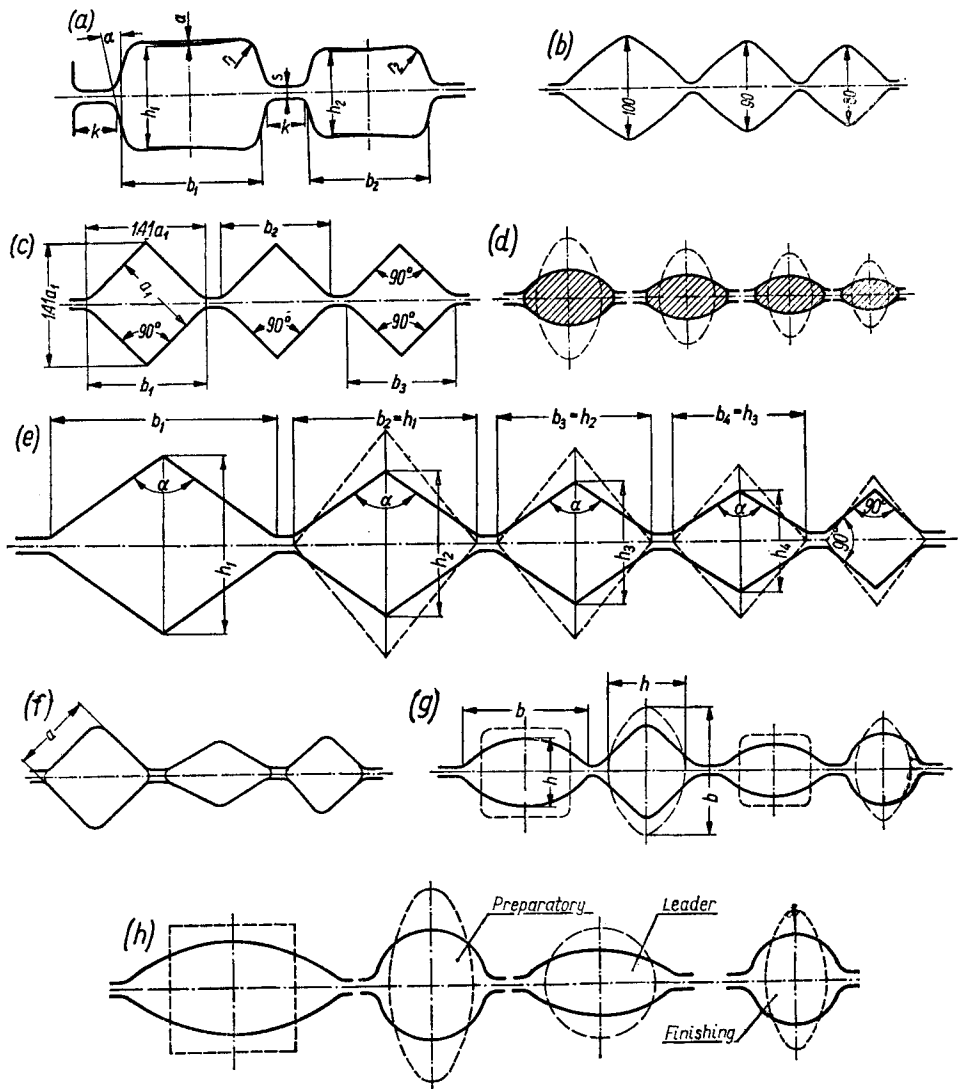


Fig. 7.9. Principal breaking-down systems

The names of pass sequences according to pass positioning in rolls:

- (1) horizontal sequence (Fig. 7.10a) corresponds to roll passes in 2-high rolls,
- (2) vertical sequence (Fig. 7.10b) occurs in 3-high roll systems, where the roll passes are cut off along the single vertical axis, hence the middle roll groove is common for the lower and upper passes,

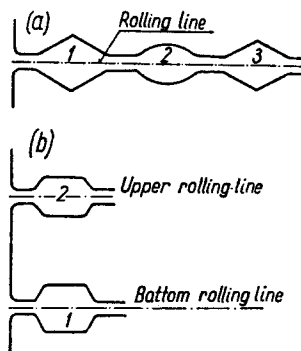


Fig. 7.10. Examples of pass arrangement in rolls: 1, 2, 3 — successive passes

- (3) staggered roll pass sequence, where the successive passes in 3-high rolls lie on axes shifted with regard to each other (Fig. 7.1).

## 7.2. Roll Pass Design and Basic Formulae

The purpose of roll pass design is:

- (a) to ensure the production of correct profile within permissible dimensional limits with a good surface finish free of surface defects, at the same time keeping the internal stress in the section being rolled at a minimum,
- (b) to ensure the maximum output at minimum cost,
- (c) to ease the working conditions of the rolling crew,
- (d) to reduce the roll wear to a minimum.

Basic requirements for correct roll pass designing are as follows:

- (a) the shapes of sections in particular roll passes must ensure a free metal flow in the roll gap,
- (b) the draughts in particular parts of profile in finishing passes, particularly in section rolling, must be selected so that the metal flow from one part to another is reduced to a minimum,
- (c) if necessary, different draughts in particular parts of profile can be used in the early forming passes where the section is a large one and the metal is hot, but in the last three passes a uniform draught must be attained in all parts of the pass; in the last passes the effect of roll adjustment on draught change should be remembered,
- (d) the true expansion must be taken into account in the calculation of the pass width; too narrow a pass with great expansion causes rapid wear,
- (e) draughts should be distributed so as to ensure as far as possible uniform wear, if greater wear of particular passes occurs interchangeable passes must be used,
- (f) draughts should be uniformly distributed in particular passes to avoid the overloading of drive installations and rolls,

(g) when determining the final shape of the section, provision should be made for design correction by local re-turning, and not by redressing of rolls as a whole; in particular, greater rounding or dulling should be used than that resulting from calculation,

(h) an optimum number of passes should be used; too great a number lowers the output of the roll stand, too small a number causes excessive roll wear, and the danger of the incidence of roll fracture or of rolling defects,

(i) the number and arrangement of passes in particular rolls should assure the most uniform possible exploitation of all the passes in each stand,

(j) rolls should automatically and easily grip the material being rolled and lead it out in the rolling direction without danger of twisting and wrapping on the rolls.

Before roll design is undertaken, an analysis of available installations should be made which will decide the dimensions (weight, cross-section, shape, length etc.) that the ingot or bloom and also the finished product should have.

For the given type of pass designing suitable shapes of passes must be chosen from the following types:

- (1) box pass
- (2) gothic square pass
- (3) diamond pass
- (4) oval pass
- (5) square pass
- (6) round pass
- (7) edging pass
- (8) flat pass
- (9) shaped pass

The most suitable breaking-down pass sequence should be chosen from the following:

- (1) box passes, (2) gothic square–gothic square (outdated system), (3) diamond–diamond
- (4) square–diamond–square, (5) square–oval–square, (6) round–oval–round.

Before choosing the system of roll pass design, similar existing ones in the works should be studied. In the rolling of sections the order and position of pass closing should be decided beginning with the finishing pass. The pass closing should be so chosen that a small under- or overfilling would not disqualify the product. Finally for the finished pass itself the surfaces which must be faultlessly rolled should be determined.

*Calculation of number of passes.* With known cross-sectional area of an ingot  $F_0$  and of the required product  $F_n$  and assuming a mean elongation coefficient of  $\lambda_m$ , the number of passes may be calculated from equation (2.17)

$$n = \frac{\log F_0 - \log F_n}{\log \lambda_m} = \frac{\log \lambda_t}{\log \lambda_m}$$

The mean elongation coefficient may be calculated if the number of passes is given and the initial and finished cross-sections are known (2.16)

$$\lambda_m = \sqrt[n]{\frac{F_0}{F_n}} = \sqrt[n]{\lambda_t}$$

In rolling only square or rectangular sections, e.g. blooming mills, let the ingot cross-section dimensions be  $A$  and  $B$  and the dimensions of the outgoing bloom cross-section  $a$  and  $a$ .

Then taking an average value for  $\Delta b/b_1$  e.g. 10%, and the mean draught value  $\Delta h_m$  the total draught will be

$$\Sigma \Delta h = [(A-a) + (B-a)] \times 1.10 \quad (7.2)$$

and the number of passes

$$n = \frac{\Sigma \Delta h}{\Delta h_m} = \frac{(A+B-2a) \times 1.10}{\Delta h_m} \quad (7.3)$$

In some publications formulae of another form in terms of mean relative reduction are given

$$U_m = 1 - \frac{1}{\lambda_m} \quad (7.4)$$

hence

$$\lambda_m = \frac{1}{1 - U_m} \quad (7.5)$$

In rolling with unrestricted spread the pass design may be calculated by means of the draught coefficient  $\gamma$  or of the heights ratio  $h_0/h_1 = Z_h$ . For pass design where the spread is partly or almost entirely limited the individual passes should be calculated

$$\lambda = \frac{l_2}{l_1} = \frac{F_1}{F_2}$$

The successive cross-sections are calculated from the elongation coefficient formula, relative to their area. (Table 7.1). For correct calculation the finished cross-section according to formula (2.13) should be used

$$\begin{aligned} F_{n-1} &= F_n \lambda_n \\ F_{n-2} &= F_{n-1} \lambda_{n-1} \\ F_1 &= F_2 \lambda_2 \\ F_0 &= F_1 \lambda_1 \end{aligned}$$

TABLE 7.1  
RELATION BETWEEN ELONGATION COEFFICIENT  $\lambda$ , RELATIVE  
REDUCTION IN CROSS-SECTION  $U$  AND REDUCTION FACTOR  $A$

| In terms of: |                   |                   |             |         |                 |
|--------------|-------------------|-------------------|-------------|---------|-----------------|
| value        | $F_1$ and $F_2$   | $l_1$ and $l_2$   | $U$         | $A$     | $\lambda$       |
| $U$          | $(F_1 - F_2)/F_1$ | $(l_2 - l_1)/l_2$ | $U$         | $1 - A$ | $1 - 1/\lambda$ |
| $A$          | $F_2/F_1$         | $l_1/l_2$         | $1 - U$     | $A$     | $1/\lambda$     |
| $\lambda$    | $F_1/F_2$         | $l_2/l_1$         | $1/(1 - U)$ | $1/A$   | $\lambda$       |

When the end cross-section is known and the successive elongation coefficients are assumed, the initial cross-section required may be calculated without calculation of intermediate passes (2.14)

$$F_0 = F_n \lambda_n \lambda_{n-1} \dots \lambda_2 \lambda_1$$

In preliminary calculations it is sometimes sufficient to assume  $\lambda_m$  and the number of passes. Then

$$F_0 = F_n \lambda_m^n$$

In calculations where the initial cross-section is known, the end cross-section required can be calculated from

$$F_n = \frac{F_0}{\lambda_m^n}$$

In pass calculation for a semi-product with a simple section, calculation begins from ingot or bloom cross-section to obtain the required semi-product cross-section i.e. from  $F_0$  to  $F_n$ . For rolling of finished products it should be reversed, i.e. from  $F_n$  to  $F_0$ . In this case if the end cross-section is determined as  $F_I$  the following formula is obtained [115]

$$\begin{aligned} F_{II} &= F_I \lambda_I, \\ F_{III} &= F_{II} \lambda_{II} = F_I \lambda_I \lambda_{II} \\ F_{IV} &= F_{III} \lambda_{III} = F_I \lambda_I \lambda_{II} \lambda_{III} \\ F_x &= F_{x-1} \lambda_{x-1} = F_I \lambda_I \lambda_{II} \lambda_{III} \dots \lambda_{x-1} \end{aligned} \quad (7.6)$$

In this way the preliminary cross-section is obtained as:

$$F_{x+1} = F_x \lambda_x = F_I \lambda_I \lambda_{II} \lambda_{III} \dots \lambda_{x+1} \lambda_x$$

where

$$\lambda_I = \frac{F_{II}}{F_I}, \quad \lambda_{II} = \frac{F_{III}}{F_{II}}, \quad \lambda_x = \frac{F_{x+1}}{F_x} \quad (7.7)$$

If all the coefficients  $\lambda$  are equal

$$\lambda_I = \lambda_{II} \dots \lambda_x = C \quad (7.8)$$

where

$$\begin{aligned} F_{II} &= F_I \lambda \\ F_{III} &= F_{II} \lambda = F_I \lambda^2 \\ F_{IV} &= F_{III} \lambda = F_I \lambda^3 \\ F_x &= F_{x-1} \lambda = F_I \lambda^{x-1} \\ F_{x+1} &= F_I \lambda^x \end{aligned} \quad (7.9)$$

hence

$$\lambda = \lambda_m = \sqrt[x]{\frac{F_{x+1}}{F_I}}$$

and from this

$$x = \frac{\log F_{x+1} - \log F_I}{\log \lambda} \quad (7.10)$$

Table 7.2 gives values of elongation coefficients according to Trinks [22] in particular passes where reduction in successive passes varies from 5 to 40%.

TABLE 7.2  
TOTAL ELONGATION COEFFICIENT  $\lambda_t$  AT DIFFERENT REDUCTIONS  
RELATIVE TO THE NUMBER OF PASSES

| No.<br>of<br>passes | $\lambda_t$ for percentage reductions of: |       |       |       |       |       |       |       |
|---------------------|-------------------------------------------|-------|-------|-------|-------|-------|-------|-------|
|                     | 5                                         | 10    | 15    | 20    | 25    | 30    | 35    | 40    |
| 1                   | 1.053                                     | 1.111 | 1.177 | 1.250 | 1.333 | 1.429 | 1.538 | 1.667 |
| 2                   | 1.108                                     | 1.235 | 1.384 | 1.563 | 1.777 | 2.041 | 2.367 | 2.779 |
| 3                   | 1.167                                     | 1.372 | 1.628 | 1.953 | 2.369 | 2.916 | 3.642 | 4.63  |
| 4                   | 1.228                                     | 1.524 | 1.915 | 2.441 | 3.160 | 4.165 | 5.610 | 7.72  |
| 5                   | 1.293                                     | 1.694 | 2.254 | 3.052 | 4.215 | 5.95  | 8.625 | 12.87 |
| 6                   | 1.361                                     | 1.882 | 2.65  | 3.815 | 5.620 | 8.50  | 13.27 | 21.4  |
| 7                   | 1.432                                     | 2.090 | 3.12  | 4.77  | 7.49  | 12.15 | 20.40 | 35.7  |
| 8                   | 1.508                                     | 2.323 | 3.67  | 5.96  | 9.98  | 17.35 | 31.0  | 59.5  |
| 9                   | 1.587                                     | 2.581 | 4.32  | 7.45  | 13.32 | 24.8  | 48.3  | 99.2  |
| 10                  | 1.670                                     | 2.868 | 5.08  | 9.31  | 17.75 | 35.4  | 74.2  | 166   |
| 11                  | 1.758                                     | 3.188 | 5.98  | 11.64 | 23.65 | 50.5  | 114   | 276   |
| 12                  | 1.851                                     | 3.540 | 7.03  | 14.55 | 31.5  | 72.2  | 176   | 460   |
| 13                  | 1.948                                     | 3.93  | 8.27  | 18.20 | 42.0  | 103   | 271   | 765   |
| 14                  | 2.051                                     | 4.37  | 9.72  | 22.75 | 56.1  | 147   | 407   | 1280  |
| 15                  | 2.159                                     | 4.86  | 11.43 | 28.40 | 74.8  | 211   | 640   | 2120  |
| 16                  | 2.273                                     | 5.40  | 13.47 | 35.50 | 99.6  | 301   | 985   | 3530  |
| 17                  | 2.392                                     | 6.00  | 15.82 | 44.40 | 133   | 430   | 1515  | 5900  |
| 18                  | 2.518                                     | 6.66  | 18.62 | 55.50 | 177   | 615   | 2320  | 9800  |
| 19                  | 2.651                                     | 7.40  | 21.9  | 69.30 | 236   | 880   | 3580  | —     |
| 20                  | 2.791                                     | 8.23  | 25.8  | 86.70 | 315   | 1260  | 5500  | —     |
| 21                  | 2.938                                     | 9.14  | 30.4  | 108.0 | 420   | 1790  | 8500  | —     |
| 22                  | 3.092                                     | 10.15 | 35.7  | 135.5 | 560   | 2560  | —     | —     |
| 23                  | 3.255                                     | 11.29 | 42.0  | 169.0 | 745   | 3650  | —     | —     |
| 24                  | 3.427                                     | 12.53 | 49.4  | 211.0 | 990   | 5210  | —     | —     |
| 25                  | 3.607                                     | 13.92 | 58.1  | 264.0 | 1325  | 7420  | —     | —     |

### Example

If from a bloom of  $200 \times 200$  mm a billet of  $50 \times 50$  mm is required, then

$$\lambda_t = \frac{200 \times 200}{50 \times 50} = 16$$

From Table 7.2 it is shown that with a reduction of 25% in every pass, after 9 passes  $\lambda_t = 13.32$ , and after 10 passes  $\lambda_t = 17.15$ . Therefore 10 passes will be sufficient with the reduction somewhat less than 25%.

A similar section is to be rolled with reduction of finishing cross-section of 5%, of primary cross-section of 10%, and of the intermediate passes of 30%,  $\lambda_t = 16$ . The elongation coefficients of finishing and preliminary passes are  $1.053 \times 1.111 = 1.168$ . For intermediate passes  $\lambda_t = 16/1.168 = 13.7$ .

With a reduction of 30%, after 7 passes  $\lambda_t = 12.15$  and after 8 passes  $\lambda_t = 17.35$ .

If rolling with  $\lambda_t = 13.7$  is required, then reductions should be a little lower than 30% rolling with 10 passes: the preliminary pass, 8 breaking-down passes and the finishing one.

In all these calculations the effect of temperature change on the product being rolled is neglected.



For this reason rather large elongation coefficients should be taken in the early passes, since the metal is more ductile at this stage, afterwards they should be decreased gradually as the bar cools until the finishing pass is reached.

Then progressive elongation coefficients should be used [115]

$$\begin{aligned}\lambda_{II} &= \lambda_I + Z \\ \lambda_{III} &= \lambda_{II} + Z = \lambda_I + 2Z \\ \lambda_{IV} &= \lambda_{III} + Z = \lambda_I + 3Z \\ \lambda_x &= \lambda_{x-1} + Z = \lambda_I + (x-1)Z\end{aligned}\quad (7.11)$$

In general

$$F_{II} = F_I \lambda_I \text{ etc.}$$

In this case  $F_I$  means the finished section.

If exact calculation of spread coefficients with the assumed draught coefficient is possible then the correctness of calculations can be checked by the constant volume condition (2.8)

$$\gamma \beta \lambda = 1.0$$

Relations given apply only to processes where the metal passes the successive stages of deformation according to the constant volume principle (2.1).

An additional relationship for roll pass design is derived by K. Rytel [116] from the author's formulae (3.45) to (3.47) in the form (3.48)

$$\beta = \lambda^{W/(1-W)}$$

Substituting this into (2.21)

$$\beta_t = \lambda_1^{W_1/(1-W_1)} \lambda_2^{W_2/(1-W_2)} \lambda_3^{W_3/(1-W_3)} \dots \lambda_i^{W_i/(1-W_i)} \dots \lambda_n^{W_n/(1-W_n)} \quad (7.12)$$

where the exponential coefficient  $W$  is a function of form factor and roll factors

$$\delta_{wt} = \frac{b_i}{h_i}, \quad \varepsilon_{wt} = \frac{h_i}{D_i}$$

Roll pass designing may be commenced on the basis of equations (2.20) to (2.22) and (3.45) to (3.48), from which the following equations are obtained:

$$\begin{aligned}\gamma_t &= \gamma_1 \gamma_2 \gamma_3 \dots \gamma_i \dots \gamma_n \\ \beta_t &= \gamma_1^{-W_1} \gamma_2^{-W_2} \gamma_3^{-W_3} \dots \gamma_i^{-W_i} \dots \gamma_n^{-W_n}\end{aligned}\quad (7.13)$$

$$\lambda_t = \gamma_1^{W_1-1} \gamma_2^{W_2-1} \gamma_3^{W_3-1} \dots \gamma_i^{W_i-1} \dots \gamma_n^{W_n-1} \quad (7.14)$$

or formulae according to (2.14)

$$\begin{aligned}\gamma_t &= \gamma_1 \gamma_2 \gamma_3 \dots \gamma_i \dots \gamma_n \\ \beta_t &= \lambda_1^{W_1/(1-W_1)} \lambda_2^{W_2/(1-W_2)} \lambda_3^{W_3/(1-W_3)} \dots \lambda_i^{W_i/(1-W_i)} \dots \lambda_n^{W_n/(1-W_n)}\end{aligned}\quad (7.15)$$

$$\lambda_t = \lambda_1 \lambda_2 \lambda_3 \dots \lambda_i \dots \lambda_n$$

Checking  $\gamma_t \beta_t \lambda_t = 1$

Similarly substituting the mean values  $\gamma_m$

$$\beta = \gamma_m^{-W_m} \quad (3.56) \quad \text{and} \quad \lambda = \gamma_m^{W_m-1} \quad (3.58)$$

the following equations are obtained

$$\gamma_t = \gamma_{1m} \gamma_{2m} \gamma_{3m} \dots \gamma_{im} \dots \gamma_{nm} \quad (7.16)$$

$$\beta_t = \gamma_{1m}^{-W_{1m}} \gamma_{2m}^{-W_{2m}} \gamma_{3m}^{-W_{3m}} \dots \gamma_{im}^{-W_{im}} \dots \gamma_{nm}^{-W_{nm}} \quad (7.17)$$

$$\lambda_t = \gamma_{1m}^{W_{1m}-1} \gamma_{2m}^{W_{2m}-1} \gamma_{3m}^{W_{3m}-1} \dots \gamma_{im}^{W_{im}-1} \dots \gamma_{nm}^{W_{nm}-1} \quad (7.18)$$

and the elongation coefficient  $\lambda$  can be found using

$$\beta = \lambda^{W_m/(1-W_m)}$$

$$\gamma_t = \gamma_{1m} \gamma_{2m} \gamma_{3m} \dots \gamma_{im} \dots \gamma_{nm} \quad (7.19)$$

$$\beta_t = \lambda_1^{W_{1m}/(1-W_{1m})} \lambda_2^{W_{2m}/(1-W_{2m})} \lambda_3^{W_{3m}/(1-W_{3m})} \dots \lambda_i^{W_{im}/(1-W_{im})} \dots \lambda_n^{W_{nm}/(1-W_{nm})} \quad (7.20)$$

$$\lambda_t = \lambda_1 \lambda_2 \lambda_3 \dots \lambda_i \dots \lambda_n$$

The products of corresponding coefficients must always satisfy:

$$\gamma \beta \lambda = 1$$

General principles governing the exact calculations of areas of regular shapes and sections for hot rolling will be given.

Another quantity which must be calculated in roll pass designing is the angle of bite. Since the problem has been considered in detail in part 3 it will be neglected here, but it should be noted that the permissible values of this angle must be calculated and checked every time.

Calculations already given for successive sectional areas, angle of bite and pass dimensions, are by no means all the calculations required for roll pass design. In more difficult cases it is recommended that:

- (1) the minimum admissible diameter in the deepest pass should be calculated, and if necessary the bending and torsional stress in barrel and neck, to avoid excessive weakening of roll,
- (2) deflection and flattening of the rolls should be calculated,
- (3) the principle of suitable reduction distribution between particular passes should be applied consistently,
- (4) the correct pass filling should be remembered,
- (5) appropriate taper in passes must be selected in order to avoid metal jamming in rolls,
- (6) the different cooling rates of thicker and thinner parts of profile must be taken in to account,
- (7) the correct guiding of bar being rolled must be ensured. The determination of elongation coefficients is a necessary condition for calculation of the various profile sections. These coefficients are dependent on the type of the roll stand, its construction, shape of pass, the angle of bite, and the roll strength and drive power.

### 7.2.1. BREAKING-DOWN SEQUENCES [145]

A breaking-down sequence is the system of successive passes used to reduce the sectional area of metal being rolled as quickly as possible. Usually every alternate pass has a similar shape, but with ever decreasing dimensions. The most frequently used breaking-down

sequences are: box pass, diamond pass, square–diamond–square and square–oval–square (Fig. 7.9).

*Box pass sequence.* This sequence is used for rolling squares greater than 75×75 mm.

The design principles and shapes of these passes are similar to those for box passes of blooming mills, the taper of pass being assumed at 10–15% and the elongation coefficient  $\lambda = 1.25\text{--}1.33$  for squares of side greater than 100 mm, and  $\lambda = 1.33\text{--}1.54$  for

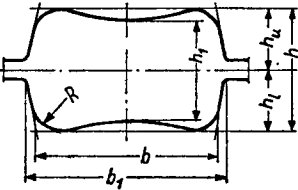


Fig. 7.11. Box pass construction [145]

squares of side up to 100 mm. Pitch lines of such passes do not pass through their centres of symmetry (in this case), since these passes are used in 3-high blooming mills and roughing mills (also most frequently 3-high). In these cases the groove depth of middle roll (Fig. 7.11) is assumed

$$\frac{h_1 + h_2}{4}$$

Then

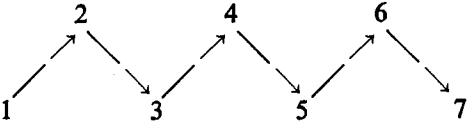
$$h_{1l} = h_1 - h_{1u} \quad \text{and} \quad h_{2u} = h_2 - h_{2l}$$
$$h_{1u} = h_{2l}$$

On this assumption the clearances between grooves in all passes agree. Where there is a danger of outflow of metal, the assumption

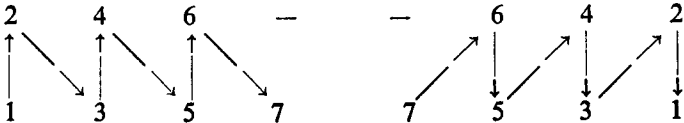
$$0.65h_2 \geq h_{2u} \geq 0.5h_2$$

is sometimes applied.

Passes in roughers are cut either in a skew sequence e.g.



or in a vertical one e.g.



In the first case it is possible to roll three blooms simultaneously, depending on drive power, with the following combination of passes:

1-3-5, 2-4-6, 3-5-7, 4-6, 1-5-7, 2-6, 1-3-7, 2-4 and from the beginning 1-3-5 etc.  
In the second case two blooms may be rolled simultaneously:  
1-5, 2-6, 3-7, 4 and from the beginning 1-5 etc.

*Diamond pass sequence.* This sequence takes the place of the obsolete gothic square pass sequence. The advantages of the diamond pass sequence are:

- (1) sufficiently exact square profiles can be obtained if metal is passed twice in the same pass groove, turning through  $90^\circ$  between first and second passes,
- (2) there is little danger of metal twisting in the pass (more certain guiding),
- (3) greater elongations may be used,
- (4) roll gaps can be controlled, and hence profile dimensions can be regulated.

Disadvantages of diamond pass sequence are:

- (1) difficult bite of metal by rolls (at moment of bite the area of metal in contact with rolls is very small),
- (2) elongation is limited by angle of bite.

Diamond pass sequences are used in roughing rolls to obtain a large number of blooms with square section (small variations of section). The elongation coefficient in these cases is small and does not exceed 1.3.

The diamond pass sequence is also used as a breaking-down pass sequence to obtain rapid reduction of sectional areas. The elongation coefficient then increases to 1.5.

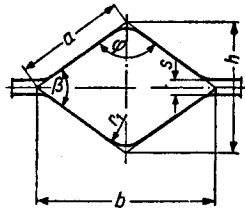


Fig. 7.12. Diamond pass construction [145]

The method of construction of diamond passes (Fig. 7.12) is often based on the principle that the width of the next pass is equal to the height of the previous pass, hence  $b_2 = h_1$ .

If diamond passes are used as breaking-down ones, the width of the next pass is greater than the height of the previous pass by the amount of the spread, i.e.

$$b_2 = h_1 + \Delta b \quad (7.21)$$

The diamond section area is

$$F = \frac{bh}{2} \quad (7.22)$$

The length of diagonals is

$$\left. \begin{aligned} h &= b \tan \frac{\beta}{2} & a &= \frac{b}{2 \sin \frac{\varphi}{2}} \\ b &= h \tan \frac{\varphi}{2} & r &= 0.10-0.2 h \end{aligned} \right\} \quad (7.23)$$

The elongation coefficient depends on the diamond apex angle, which depends on the angle of bite.

Hence for larger sectional areas smaller apex angles  $\varphi$  are chosen, giving a smaller elongation coefficient  $\lambda$ ; and for smaller section areas the larger apex angle  $\varphi$  and the higher coefficient  $\lambda$ :

for sections 3600–2400 mm<sup>2</sup>: the angle  $\varphi = 100^\circ$ ,  $\lambda = 1.23$   
 for sections 2400–1200 mm<sup>2</sup>: the angle  $\varphi = 110^\circ$ ,  $\lambda = 1.27$   
 for sections 1200–400 mm<sup>2</sup>: the angle  $\varphi = 117$ – $120^\circ$ ,  $\lambda = 1.31$   
 for sections up to 400 mm<sup>2</sup>: the angle  $\varphi = 128^\circ$ ,  $\lambda = 1.41$

These values are only to be taken as a guide, the apex angle  $\varphi$  should be adjusted to the permissible angle of bite.

*Square–diamond–square sequence.* This sequence is used in roughing stands instead of the diamond–diamond sequence in order to avoid rolling metal twice through the same pass. The elongation coefficients of successive passes of this sequence will be given in Table 7.5.

The elongation coefficient is taken as equal from one square to the next, with a value of 1.5–2.0.

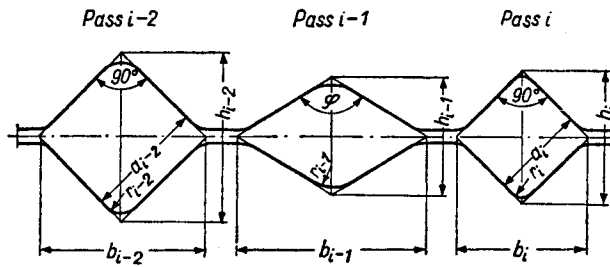


Fig. 7.13. Square–diamond–square sequence. Construction and dimensions [145]

The calculation of successive passes by Holzweiler's method (Fig. 7.13) for a given square of side  $a_i$  [145] gives

pass  $i$

$$\left. \begin{aligned} b_i &= h_i = a_i \sqrt{2} = 1.414 a_i \\ r_i &\approx 0.2 a_i \end{aligned} \right\} \quad (7.24)$$

pass  $i-1$

$$\left. \begin{aligned} h_{i-1} &= 1.372 a_i - 0.758 \sqrt{a_i} \\ b_{i-1} &= 1.758 a_i + 2.024 \sqrt{a_i} \\ r_{i-1} &\approx 0.2 h_{i-1} \end{aligned} \right\} \quad (7.25)$$

pass  $i-2$

$$\left. \begin{aligned} a_{i-2} &= 0.505 h_{i-1} + 0.346 b_{i-1} \\ h_{i-2} &= b_{i-2} = 1.414 a_{i-2} \\ r_{i-2} &\approx 0.2 a_{i-2} \end{aligned} \right\} \quad (7.26)$$

Further passes are calculated in the same manner as  $i-1$  and  $i-2$ .

**Square-oval-square sequence.** This sequence is in common use for the designing of roughing and leader rolls, for quick elongation especially of medium and small sections (not greater than 22,500 mm<sup>2</sup>).

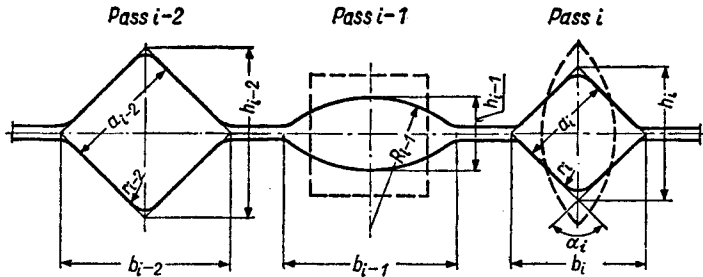


Fig. 7.14. Square-oval-square sequence. Construction and dimensions [145]

Square passes are rolled so that the one set of diagonals is vertical and the other parallel to the rolls axes, oval passes are rolled so that the major axes are horizontal (Fig. 7.14).

A square is put flat into an oval, but an oval is put into a square with its major axis in the vertical diagonal of the square.

If the oval section is assumed to be composed of two equal arcs of circle, its area is approximately

$$F = \frac{2bh}{2.85} \quad (7.27)$$

or more exactly,

$$F = \frac{4\pi}{360} \left( \frac{b^2 + h^2}{4h} \right) \arctan \frac{2}{\frac{b}{h} - \frac{h}{b}} - \frac{b}{4h} (b^2 - h^2) \quad (7.28)$$

*Calculation of oval dimensions by Mercader's method*

$$b = a + c(2a - h) \quad (7.29)$$

where

$b$  = width of oval pass following after a square pass of side  $a$ ,

$h$  = oval height (assumed value),

$c = 0.43-0.50$  (constant value), for smaller ovals the greater value is used, for larger ovals the smaller value,

if  $c = 0.465$  then

$$b = 1.93a - 0.465h$$

*Example*

The side of the previous square  $a = 15.6$  mm, the oval height is assumed  $h = 9.86$  mm, then  $b = 15.6 + 0.50(2 \times 15.6 - 9.86) = 26.27$  mm.

*Calculation of pass dimensions by Holzweiler method*

The basic pass for calculations is pass  $i$  (the smaller square) of dimensions:  $a_i$  (given),  $\alpha_i = 90^\circ$  and  $r_i = 0.1 a_i$ .

Dimensions of pass  $i-1$  (oval):

$$h_{i-1} = 0.885a_i - 0.215\sqrt{a_i}$$

$$b_{i-1} = 2.715h_{i-1} + 0.635\sqrt{h_{i-1}}$$

$$R_{i-1} = \frac{h_{i-1}^2 + b_{i-1}^2}{4h_{i-1}},$$

Dimensions of pass  $i-2$  (larger square):

$$a_{i-2} = 1.688h_{i-1}$$

$$h_{i-2} = b_{i-2} = 1.414a_{i-2}$$

$$r_{i-2} = 0.2a_{i-2}.$$

*Calculation of pass dimensions by Holzweiler method (Fig. 7.15) for square-flat oval-square sequence*

The dimensions of pass  $i$  (smaller square):  $a_i$  (given),  $\alpha_i = 90^\circ$ ,  $r_i = 0.1 a_i$ .

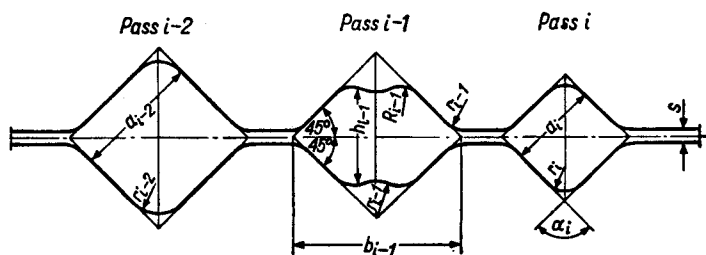


Fig. 7.15. Square-flat oval-square sequence. Construction and dimensions [145]

The dimensions of pass  $i-1$  (flat oval):  $b_{i-1} = 2a_i$ ,  $h_{i-1} = a_i$ ,  $R_{i-1} = 1/3 - 1/2h_{i-1}$ ,  $r_{i-1} = 2/3R_{i-1}$ .

The dimensions of pass  $i-2$  (larger square):  $a_{i-2} = 1.414 a_i$ ,  $r_{i-2} = 0.2 a_{i-2}$ .

## 7.2.2. FINISHING ROLL PASSES FOR BAR ROLLING [145]

*Square bars.* Square bars are rolled either by hand or with a guide. Rolling with a guide allows of larger draughts also greater length of bars and higher rolling speeds.

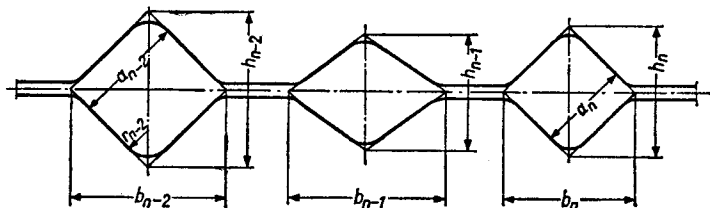


Fig. 7.16. Preparatory square-sizing diamond-finishing square sequence. Construction and dimensions [145]

For a square section with sharp corners the following pass sequence is used, working back from the finishing pass:

- (1) a finishing square-sizing diamond-preparatory square (Fig. 7.16),
- (2) finishing square-sizing diamond-preparatory diamond-preparatory square,
- (3) a series of successive squares.

To facilitate the more accurate setting of rolls, finishing square and sizing diamonds should be cut in separate rolls.

**The finishing square.** The finishing square pass is cut out in rolls with the diagonal on the rolling line. The apex angle equals  $90^\circ$ .  $b = h = 1.414 a$ . For a square with side greater than 15 mm,  $b = 1.42 a$  and  $h = 1.40 a$  are taken.

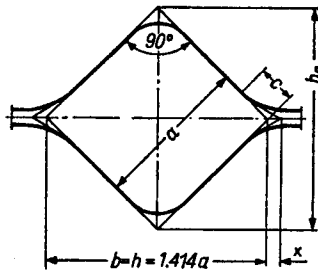


Fig. 7.17. Finishing square construction after Sedlaczek [145]

For squares with the side  $a \leq 20$  mm Sedlaczek assumes the diagonals equal, i.e.  $b = h$ , and for larger squares while keeping an apex angle of  $90^\circ$  he uses a side opening  $x \approx 0.02 a$  over the length  $c = 0.2 a$  and a corner radius  $r \approx 0.05 a$  (Fig. 7.17). The roll gap equals 1–5 mm, minimum width of collars 5–10 mm, and edge collars are suitably wider to allow the building of a cage.

**The sizing diamond.** The area of the sizing diamond is calculated as the product of elongation coefficient  $\lambda$  and the finished square area, taking elongation coefficient  $\lambda_n = 1.07$ –1.15. The lengths of the diagonals of the sizing diamond are then calculated for the given section area relative to the assumed apex angle.

For varying values of side of square  $a$  these angles are assumed as follows:

|                        |                                   |
|------------------------|-----------------------------------|
| $a = 5\text{--}13$ mm  | $\varphi = 116\text{--}112^\circ$ |
| $a = 14\text{--}25$ mm | $\varphi = 112\text{--}108^\circ$ |
| $a = 26\text{--}35$ mm | $\varphi = 108\text{--}105^\circ$ |
| $a = 36\text{--}45$ mm | $\varphi = 102^\circ$             |
| $a = 46\text{--}35$ mm | $\varphi = 98^\circ$              |

The calculated sizing diamonds should be checked for side spread. According to Vinogradov for a square with side

|             |                               |
|-------------|-------------------------------|
| $a < 20$ mm | $\Delta b = 1.5\text{--}2$ mm |
| $a > 20$ mm | $\Delta b = 2\text{--}3$ mm   |



According to Brovot, for finishing squares dimensions of sides are 5–25 mm.

If a preparatory diamond is used,  $\lambda_{n-1} = 1.26$ – $1.315$  is assumed. The length of diagonals however should be checked for spreading.

*Holzweiler's method of calculation of successive pass dimensions*

Pass  $n$ , a finishing square with side  $a$ :

$$a_n = 1.015a, \quad h_n = 1.414a_n, \quad r_n = 0 \quad (7.30)$$

Pass  $n-1$ , a sizing diamond:

$$h_{n-1} = 1.495a - 0.895\sqrt{a}, \quad b_{n-1} = 1.395a + 1.895\sqrt{a}, \quad r_{n-1} = 0. \quad (7.31)$$

Pass  $n-2$ , a preparatory square:

$$a_{n-2} = 0.423h_{n-1} + 0.354b_{n-1}, \quad h_{n-2} = b_{n-2} = 1.414a_{n-2}, \quad r_{n-2} = 0.1a_{n-2}. \quad (7.32)$$

**Round bars.** In rolling round bars the following sequences are used:

- (1) preparatory square–sizing oval–finished round,
- (2) preparatory square–preparatory oval–sizing oval–finished round.

For alloy steel the following sequences are also used: preparatory square–flat–preparatory oval–sizing oval–finished round or round–oval–round. The square is put flat into the oval, whereas the oval is put with the longer axis vertical i.e. after turning through  $90^\circ$ . An oval into an oval is put in flat, or for alloy steels with the longer axis vertical, i.e. after turning through  $90^\circ$ .

The successive profiles are calculated starting from the pass area of the finished profile.

*Holzweiler method.* (Fig. 7.18) For  $d_c = 6$ – $30$  mm.

The diameter of finished round pass  $d_f = 1.015 d_c$ , where  $d_c$  = sectional diameter of cold product.

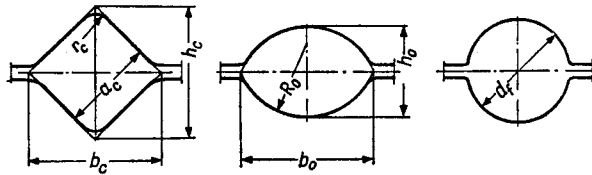


Fig. 7.18. Sizing oval–finishing round sequence. Construction and dimensions [145]

*Sizing oval pass.*

$$h_o = d_c - 0.67\sqrt{d_c}, \quad b_o = 1.33d_c + \sqrt{d_c}, \quad R_o = \frac{h_o^2 + b_o^2}{4h_o} \quad (7.33)$$

*Preparatory square pass.*

$$a_s = 1.164h_o + 0.564\sqrt{h_o}, \quad h_s = b_s = 1.414a_s, \quad r_s = 0.1a_s \quad (7.34)$$

where  $a_s$  = side of square,  $b_s$  = diagonal of square.

Further passes consist of stretching sequence square–diamond–square or square–oval–square.

With diameters  $d_c$  greater than 30 mm the following formulae should be used:  
round pass diameter

$$d_f = 1.013 d_c \quad (7.35)$$

Sizing oval dimensions

$$h_0 = d_c - 4 \text{ mm}, \quad b_0 = 1.18 d_c + 10 \text{ mm}, \quad R_0 = \frac{h_0^2 + b_0^2}{4 h_0} \quad (7.36)$$

Dimensions of preparatory square pass:

$$a_s = 1.03 d_c + 4 \text{ mm}, \quad h_s = 1.414 a_s, \quad b_s = 1.425 a_s + 3 \text{ mm}, \quad r_s = 0.2 d_c + 3 \text{ mm} \quad (7.37)$$

TABLE 7.3

$h/d$  AND  $b/h$  FOR OVAL PASSES USED IN ROLLING OF WIRE ROD  
AND ROUND BARS RELATIVE TO DIAMETER OF FINISHED PRODUCT [145]

| Diameter<br>of finished<br>product<br>mm | Sizing oval |           | Preparatory<br>square of side<br>$a_k$<br>mm |
|------------------------------------------|-------------|-----------|----------------------------------------------|
|                                          | $h/d_f$     | $b/h$     |                                              |
| 5-7                                      | 0.69-0.78   | 2.35-2.50 | $d_f$                                        |
| 7-9                                      | 0.75-0.85   | 1.75-2.10 | $d_f$                                        |
| 9-11                                     | 0.80-0.85   | 1.67-1.80 | $d_f + (0.25-0.5)$                           |
| 12-15                                    | 0.80-0.87   | 1.60-1.70 | $d_f + 1$                                    |
| 16-20                                    | 0.87-0.90   | 1.50-1.60 | $d_f + 2$                                    |
| 21-25                                    | 0.90-0.92   | 1.45-1.50 | $d_f + 3$                                    |
| 26-35                                    | 0.90-0.94   | 1.47-1.50 | $d_f + 3$                                    |
| 36-40                                    | 0.90        | 1.40-1.50 | $d_f + (4-7)$                                |
| 42-50                                    | 0.91        | 1.4       | $d_f + (8-12)$                               |
| 52-60                                    | 0.92        | 1.4       | $d_f + (12-15)$                              |
| 65-80                                    | 0.92        | 1.4       | $d_f + (12-15)$                              |

From practical data, Vinogradov suggests the calculation of successive passes for the preparatory square-sizing oval-finished round sequence as given in Table 7.3.

*Finished passes of round profile* [145]. For profiles of small diameters the construction as shown in Fig. 7.19 is used (Vinogradov).

Profile width  $b = AC = 1.02 d_c$ . The profile height  $h = BD = 1.01 d_c$ . Lines  $AC$  and  $BD$  are perpendicular to and bisect each other. The profile arcs are drawn from points  $O_1$  and  $O_2$  on the vertical axis.

Length  $AM = BM$ ,  $CN = DN$ . Diameter  $d_c$  denotes the profile dimension in a cold state. For  $d_c$  less than 25 mm (Fig. 7.20) the pass is machined accurately round. However it is advisable after machining to widen the pass up to 1/4 of groove depth with a cutter of diameter greater than the true diameter by 0.5-1 mm i.e. of a radius  $d_1/2 = d/2 + (0.25-0.5)$  mm.

For bars of diameters greater than 25 mm the construction shown in Fig. 7.21 is used.

Two straight lines inclined to both axes at an angle of  $45^\circ$  determine points  $A$ ,  $B$ ,  $C$ ,  $D$  on the circumference. With a radius equal to  $AB = BC = CD = DA$  from points  $A$ ,  $B$ ,  $C$ ,  $D$  the centres  $O_1$ ,  $O_2$ ,  $O_3$ ,  $O_4$  are successively determined.

From these centres with radius  $R = AB$  the pass sides are widened. The cornering radius  $r = 0.1 d_c$  and roll gap is 2–5 mm.

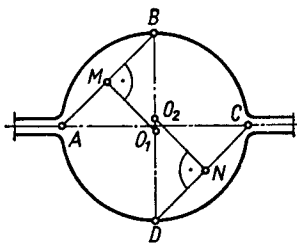


Fig. 7.19. Finishing round pass construction [145]

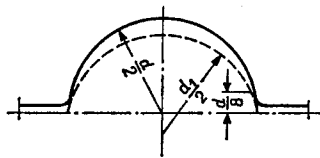


Fig. 7.20. Finishing round pass construction with diameter less than 25 mm [145]

For profiles of diameters  $d_c$  greater than 60 mm Sedlaczek uses a finished pass construction as in Fig. 7.22.

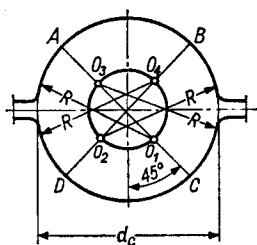


Fig. 7.21. Finishing round pass construction with diameter greater than 25 mm [145]

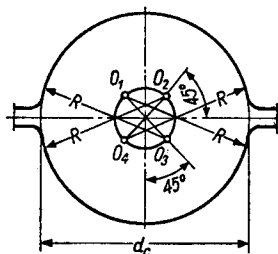


Fig. 7.22. Finishing round pass construction with diameter greater than 60 mm after Sedlaczek [145]

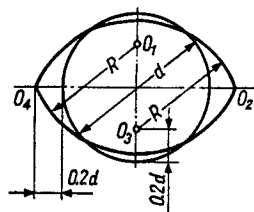


Fig. 7.23. Finishing oval pass construction [145]

Two straight lines intersecting with the pass axes at an angle of  $45^\circ$  determine on the inner circle of diameter  $d_c/8$  the circle centres  $O_1, O_2, O_3, O_4$ . From these centres and with radius  $R = 1/2 d_c + 1/16 d_c$  the pass sides are widened.

*Sizing ovals.* From among several constructions the simplest one is a single radius sizing oval construction with calculated lengths of diagonals (comp. Table 7.3). The geometrical determination of the oval without calculation of dimensions is also used (Fig. 7.23).

On the axes of the finished profile of diameter  $d$  intercepts of length  $0.2 d$  are cut off, determining points  $O_1, O_2, O_3$  and  $O_4$ . With a radius equal to  $O_1 O_2$  arcs are drawn; one with centre  $O_1$  defines the lower arc and with centre  $O_3$  defines the upper one.

For round profiles of diameter greater than 80 mm, a triple radius oval is sometimes used (Fig. 7.24). In this case  $b = 1.288 d$ ,  $h = 0.93 d$ ,  $R_1 = d - 1$  mm,  $R_2 = 0.74 d$  and  $R_3 = 0.38 d$ .

According to Sedlaczek for bars of diameter  $d = 60$ –90 mm single radius sizing ovals should be used with dimensions

$$\left. \begin{aligned} b &= 1.32 - 1.37 d \\ h &= 0.92 - 0.95 d \end{aligned} \right\} R = \frac{b^2 + h^2}{4h} \quad (7.38)$$

and for bars of diameter  $d = 90\text{--}160$  mm (Fig. 7.25) double radius sizing ovals with dimensions

$$\left. \begin{aligned} b &= 1.31\text{--}1.37d & R_1 &\approx 0.78d \\ h &= 0.94\text{--}0.97d & R_2 &\approx \frac{b^2 + h^2}{4h} \end{aligned} \right\} \quad (7.39)$$

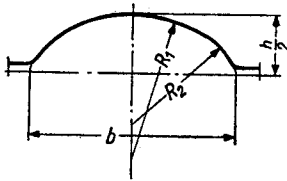


Fig. 7.24. Double radius oval [145]

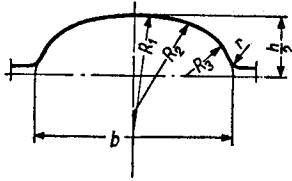


Fig. 7.25. Triple radius oval [145]

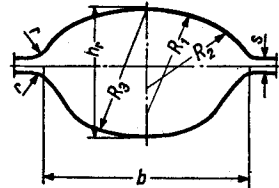


Fig. 7.26. Preliminary oval in 3-high rolling mill [145]

**Preparatory ovals.** In the case of the vertical pass sequence (the groove cut in the middle roll of 3-high stands is common for sizing oval and preparatory oval) (Fig. 7.26) [145]  $b = h$ ,  $h = 1.08d$ ,  $R_1 = R_2$  similarly as for sizing oval,  $R_3 = h$ ,  $R_4 = h/3$ , pass settling = 30%.

According to Sedlaczek for bars of diameter  $d = 90\text{--}160$  mm the following dimensions are recommended (Fig. 7.25)

$$\left. \begin{aligned} b &= 1.28\text{--}1.4d & R_1 &= h \\ h &= 1.05\text{--}1.15d & R_2 &\approx 0.51R_1 \\ & & r &= 0.1\text{--}0.12h \end{aligned} \right\} \quad (7.40)$$

Table 7.4 shows an example of designing preparatory and sizing ovals [145].

TABLE 7.4

PASS DESIGN WITH PREPARATORY OVAL AND SIZING OVAL  
calculated from Sedlaczek's formulae for round bars with diameter  $d = 90\text{--}125$  mm for  
3-high rolling mill  $D = 700$  mm [145]

| Pass dimensions | Pass No.                  |            |            |             |             |             |            |            |
|-----------------|---------------------------|------------|------------|-------------|-------------|-------------|------------|------------|
|                 | 1                         | 2          | 3          | 4           | 5           | 6           | 7          | 8          |
|                 | section diameter $d$ , mm |            |            |             |             |             |            |            |
|                 | 125 to 115                | 110 to 100 | 97.5 to 90 | 125 to 120  | 115 and 110 | 105 and 100 | 97.5 to 95 | 92.5 to 90 |
|                 | Pass type                 |            |            |             |             |             |            |            |
|                 | preparatory oval          |            |            | sizing oval |             |             |            |            |
| $b$ , mm        | 160.39                    | 141.38     | 125.53     | 164.5       | 151.5       | 138.5       | 128.75     | 122.25     |
| $h$ , mm        | 131.25                    | 115.5      | 100.5      | 117         | 107         | 97          | 92         | 87         |
| $R_1$ , mm      | 131.25                    | 115.5      | 100.5      | 94          | 86          | 78.5        | 74         | 70         |
| $R_2$ , mm      | 67.5                      | 59.5       | 51.5       | 86.33       | 79.66       | 73          | 68         | 64.66      |
| $r$ , mm        | 15.25                     | 13.4       | 11.8       | 15.25       | 14.0        | 12.8        | 11.8       | 11.2       |

The preparatory and sizing ovals are calculated with Sedlacek formulae for round bars of diameter  $d = 90\text{--}125$  mm for 3-high roll stand where  $D = 700$  mm.

*Wire rod.* Wire rod is characterized by a small cross-sectional area, a very great length and also the manner of delivery (wire is delivered in coils). To avoid rapid cooling of metal during rolling, a high rolling speed is used. In looper mills this speed may be up to 9.2 m/sec, and in continuous mills up to 60 m/sec. Standard pass designing is used, the finishing pass for a round wire rod being circular, and for other profiles appropriate to required dimensions and shapes.

In the finishing pass  $\lambda = 1.07\text{--}1.14$ .

According to Felland, the sizing oval dimensions for particular diameters  $d$  of wire rod are calculated from ratios  $b/h$  and  $h/d$  which are given in the following table [145]:

| $d$<br>mm | $b/h$    | $h/b$    |
|-----------|----------|----------|
| 5-7       | 2.41-2.5 | 0.7-0.69 |
| 8-9       | 2.08     | 0.75     |
| 10-14     | 1.8      | 0.8      |

For constructional pass calculations the equivalent equations for round bars may also be used

$$h = 0.99d_f - 1.6 \text{ mm}, \quad b = 1.3d + 2 \text{ mm} \quad (7.41)$$

the oval will then be filled by metal to a width of

$$b_p = 1.18d + 2 \text{ mm} \quad (7.42)$$

The side of the preparatory square is assumed to be

$$a = d_f$$

The width of intergroove collars is 4-5 mm and of edge collars 35-50 mm. The roll gap is 0.5-1.0 mm and the loop length between two stages is

$$L_n = 1.05 \left[ \left( v_1 - v_2 \frac{F_2}{F_1} \right) (T_1 - t) + v_1 t \right] \quad (7.43)$$

where

1.05 = factor allowing for forward slip in stands I and II,

$v_1, v_2$  = exit speed from passes of stands I and II,

$F_1, F_2$  = area of exit section from stands I and II,

$t$  = time necessary to pass the leading end of bar from stand I to stand II,

$T_1$  = time of passing through stand I.

*Flat bars (flats).* Flat bars are rolled either in box passes (width of flat greater than 70 mm), or in plain rolls (or also in stepped rolls). In order to obtain sharp corners the side spread must be accurately calculated, and to avoid overfill rounding or chamfer is used. Taper in the finishing pass is 1-3%, in the leader pass 4% and in further passes

5%. Draughts are determined from the calculated parabola for the given roll set. Geuze recommends the calculation of the number of roll passes and the magnitude of successive draughts relative to the degree of draught.

Degree of absolute draught

$$C = \frac{h_0}{h_n} = \frac{1}{\gamma_t} \quad (7.44)$$

and the draught degree in a single roll pass

$$C = \frac{h_{i-1}}{h_i} = \frac{1}{\gamma_i} \quad (7.45)$$

In rolling flats in plain or stepped rolls the draughts and number of roll passes are calculated similarly as for rolling in passes.

To obtain the required widths and sharp corners for flat bars 1–3 edging passes are used (Fig. 7.27), of which one serves as a pre-finishing pass.

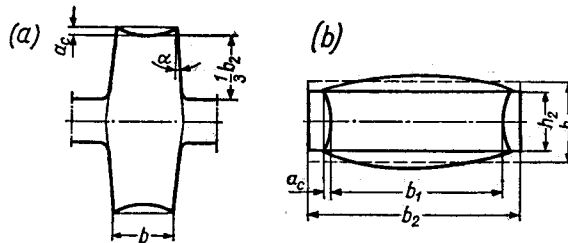


Fig. 7.27. Edging pass: (a) construction, (b) dimensioning of edging pass and flat

In the edging pass the percentage draught is from 5–15%, the camber of groove bottom  $a_c = 0.4\text{--}0.75$  mm, the taper of side walls is about 10% and the width of the bottom of the pass is equal at least to the thickness of the ingoing flat.

According to Hammond (Fig. 7.27) the groove depth for an edging pass is equal to 1/3 of the width of flat, and the groove width for flats of various thicknesses for [145]:

- thickness 3.2–12.7 mm is greater by 0.8 mm,
- thickness 12.7–25.4 mm is greater by 1.6 mm,
- thickness 25.4–38.0 mm is greater by 3.2 mm,
- thickness above 38.0 mm is greater by 6.4 mm.

To obtain sharp corners Lendl recommends calculation of bottom camber  $a_c$  in an edging pass (Fig. 7.27) according to formula

$$a_c = c \frac{h_1 - h_2}{h_1} \quad (7.46)$$

where  $c = 1.4$ , if  $h_1/h_2 \leq 1.3$ .

To calculate the dimensions of an exit square with side  $a$

$$a = \frac{b + ch}{1 + c}$$

TABLE 7.5  
REDUCTIONS AND ELONGATION COEFFICIENTS USED IN ROLL PASS DESIGNING [10]

| Rolling conditions                                                                                                                    | Reduction<br>$U$<br>% | Elongation<br>coefficient<br>$\lambda$ |
|---------------------------------------------------------------------------------------------------------------------------------------|-----------------------|----------------------------------------|
| (1) In blooming mills (from Emicke) for squares with side of 600–400 mm [117, 118]                                                    | 7–13                  | 1.07–1.15                              |
| For squares with side of:                                                                                                             |                       |                                        |
| 400–300 mm                                                                                                                            | 7–17                  | 1.07–1.20                              |
| 300–250 mm                                                                                                                            | 7–21                  | 1.07–1.27                              |
| 250–150 mm                                                                                                                            | 13–25                 | 1.15–1.33                              |
| 150–100 mm                                                                                                                            | 17–30                 | 1.20–1.43                              |
| 100 mm                                                                                                                                | 21–35                 | 1.27–1.54                              |
| (2) For box passes of preliminary roll stands (from Emicke) [118]                                                                     |                       |                                        |
| Square sections with side:                                                                                                            |                       |                                        |
| greater than 100 mm                                                                                                                   | 20–25                 | 1.25–1.33                              |
| less than 100 mm                                                                                                                      | 25–35                 | 1.33–1.54                              |
| If the value $\lambda > 1.5$ in box passes causes difficulties, a breaking-down sequence square–diamond of square–oval should be used |                       |                                        |
| (3) For squares, gothic square and diamond passes (old type breaking-down sequences) [118]                                            |                       |                                        |
| (a) Small squares series with side up to 32 mm, stepped every 1 mm, or squares series rolled twice across the same pass               | 7                     | 1.07                                   |
| Medium large squares with side from 32 to 50 mm, stepped every 2 mm                                                                   | 13                    | 1.15                                   |
| Large squares with side from 52–80 mm, stepped every 3 mm or with side $> 80$ mm, stepped every 5 mm                                  | 15                    | 1.18                                   |
| (b) Large gothic square passes*                                                                                                       |                       |                                        |
| $h/b = 8/9$ , width 100–200 mm                                                                                                        | 11                    | 1.12                                   |
| Medium gothic square passes                                                                                                           |                       |                                        |
| $h/b = 7/8$ , width 40–100 mm                                                                                                         | 12.5                  | 1.14                                   |
| Small gothic square passes $h/b = 6/7$ , width less than 40 mm                                                                        | 14.3                  | 1.17                                   |
| (c) Breaking-down sequences square–diamond–square and diamond–diamond [118]                                                           |                       |                                        |
| Diamond with an apex angle of $100^\circ$ in square                                                                                   |                       |                                        |
| For sections 3200–2400 mm <sup>2</sup>                                                                                                | 14                    | 1.16                                   |
| Square in a diamond with an apex angle of $100^\circ$                                                                                 |                       |                                        |
| For sections 3200–2400 mm <sup>2</sup>                                                                                                | 24                    | 1.32                                   |
| Diamond with an apex angle of $100^\circ$ in a diamond                                                                                |                       |                                        |
| with an apex angle of $100^\circ$                                                                                                     |                       |                                        |
| For sections 3200–2400 mm <sup>2</sup>                                                                                                | 19                    | 1.23                                   |
| Diamond with an apex angle of $110^\circ$ in a square                                                                                 |                       |                                        |
| For sections 2400–1200 mm <sup>2</sup>                                                                                                | 19                    | 1.23                                   |
| Square in a diamond with an apex angle of $110^\circ$                                                                                 |                       |                                        |
| For section 2400–1200 mm <sup>2</sup>                                                                                                 | 24                    | 1.32                                   |
| Diamond with an apex angle of $110^\circ$ in a diamond                                                                                |                       |                                        |
| with an apex angle of $110^\circ$                                                                                                     |                       |                                        |
| For sections 2400–1200 mm <sup>2</sup>                                                                                                | 21.5                  | 1.27                                   |
| Diamond with an apex angle of $117$ – $120^\circ$ in a square                                                                         |                       |                                        |
| For sections 1200–400 mm <sup>2</sup>                                                                                                 | 19                    | 1.23                                   |

TABLE 7.5. (cont.)

| Rolling conditions                                                                                                             | Reduction<br>$U$<br>% | Elongation<br>coefficient<br>$\lambda$ |
|--------------------------------------------------------------------------------------------------------------------------------|-----------------------|----------------------------------------|
| Square in a diamond with an apex angle of 117–120°<br>For sections 1200–400 mm <sup>2</sup>                                    | 28                    | 1.39                                   |
| Diamond with an apex angle of 117–120° in<br>a diamond with an apex angle of 117–120°<br>For sections 1200–400 mm <sup>2</sup> | 23.5                  | 1.31                                   |
| Diamond with an apex angle of 128° in a square<br>For sections less than 400 mm <sup>2</sup>                                   | 26                    | 1.35                                   |
| Square in a diamond with an apex angle of 128°<br>For sections less than 400 mm <sup>2</sup>                                   | 32                    | 1.47                                   |
| Diamond with an apex angle of 128° in a diamond<br>with apex angle of 128°<br>For sections less than 400 mm <sup>2</sup>       | 29                    | 1.41                                   |
| (4) Breaking-down sequence square-oval-square for common<br>ovals $F = 2.2bh$                                                  |                       |                                        |
| (a) From Hütte [119]                                                                                                           |                       |                                        |
| Square with side of 20–25 mm in ovals $b/h = 3.5$                                                                              | 50–55†                | 2.0–2.22                               |
| Ovals $b/h = 3.5$ in squares                                                                                                   | 35–38                 | 1.54–1.61                              |
| Squares with side of 15–20–30 mm in ovals $b/h = 3.0$                                                                          | 40–45                 | 1.67–1.82                              |
| Ovals $b/h = 3$ in squares                                                                                                     | 30                    | 1.43                                   |
| Squares with side of 10–15 and 30–40 mm in ovals<br>$b/h = 2.5$                                                                | 35–40                 | 1.54–1.68                              |
| Ovals $b/h = 2.5$ in squares                                                                                                   | 26                    | 1.35                                   |
| Square with side of 5–10 mm and greater than 40 mm in<br>ovals $b/h = 2$                                                       | 30–35                 | 1.43–1.54                              |
| Ovals $b/h = 2$ in squares                                                                                                     | 21                    | 1.37                                   |
| (b) From Mercader [118, 119]                                                                                                   |                       |                                        |
| Squares with side 20–25 mm in ovals $b/h = 4.34$                                                                               | 53.5–46               | 2.14–1.85                              |
| Ovals $b/h = 4.34$ in squares                                                                                                  | 37.5–32.2             | 1.6–1.47                               |
| Squares with side of 15–20 mm and 25–30 mm in ovals<br>$b/h = 3.56$                                                            | 46.0–40               | 1.85–1.67                              |
| Ovals $b/h = 3.56$ in squares                                                                                                  | 32.2–28               | 1.47–1.39                              |
| Squares with side of 10–15 mm and 30–40 mm in ovals<br>$b/h = 3.14$                                                            | 40–37.5               | 1.67–1.60                              |
| Ovals $b/h = 3.14$ in squares                                                                                                  | 28                    | 1.39                                   |
| Squares with side of 5–10 mm and greater than 40 mm<br>in ovals $b/h = 2.75$                                                   | 33.9–32.2             | 1.51–1.47                              |
| Ovals $b/h = 2.75$ in squares                                                                                                  | 23                    | 1.32                                   |
| In Mercader's method:<br>$h/c = 0.4–0.6$ ( $h$ = oval height, $c$ = square side)                                               |                       |                                        |
| (c) From Tafel for wire [37]                                                                                                   |                       |                                        |
| Squares in breaking-down oval $b/h = 2.9–3.2$                                                                                  | 43–26                 | 1.75–1.35                              |
| Ovals $b/h = 2.9–3.2$ in squares                                                                                               | 23–27                 | 1.3–1.2                                |
| (d) For finishing passes [119]                                                                                                 |                       |                                        |
| Leader square in a leader oval and in a finished round                                                                         | 35                    | 1.54                                   |
| Leader oval in a finished round                                                                                                | 12                    | 1.14                                   |
| Leader square in a leader oval                                                                                                 | 21                    | 1.37                                   |
| For large round passes greater than 60 mm diameter                                                                             |                       |                                        |
| Leader square in leader oval I                                                                                                 | 14                    | 1.16                                   |
| Leader oval I in a leader oval II                                                                                              | 13                    | 1.15                                   |
| Leader oval II in a finished round                                                                                             | 12                    | 1.14                                   |



TABLE 7.5 (cont.)

| Rolling conditions                                                          | Reduction<br>$U$<br>% | Elongation<br>coefficient<br>$\lambda$ |
|-----------------------------------------------------------------------------|-----------------------|----------------------------------------|
| (5) Breaking-down sequence oval-round-oval from Struk and Ludyga:           |                       |                                        |
| Round of diameter 15–40 mm in an oval $b/h = 3.5$                           | 41                    | 1.7                                    |
| Oval $b/h = 3.5$ in a round                                                 | 23                    | 1.3                                    |
| Round of diameter 10–40 mm in an oval $b/h = 3$                             | 28.5–38               | 1.4–1.6                                |
| Oval $b/h = 3$ in a round                                                   | 22                    | 1.28                                   |
| Round of diameter 6–40 mm in an oval $b/h = 2.5$                            | 23–28.5               | 1.3–1.4                                |
| Oval $b/h = 2.5$ in a round                                                 | 19                    | 1.23                                   |
| Round of diameter 6–40 mm in an oval $b/h = 2$                              | 19.5                  | 1.24                                   |
| Oval $b/h = 2$ in a round                                                   | 16                    | 1.19                                   |
| The end roll pass:                                                          |                       |                                        |
| Round in an oval $b/h = 1.6$                                                | up to 15              | up to 1.18                             |
| Oval $b/h = 1.6$ in a round                                                 | up to 11              | up to 1.12                             |
| (6) Flats and band steel rolling [119]                                      |                       |                                        |
| For preliminary roll passes and in the middle range from 100 to 6 mm height | 50–30                 | 2.0–1.43                               |
| For the roll passes less than 6 mm height                                   | 45–30                 | 1.82–1.43                              |
| For the finishing and edging roll passes                                    | 10–5                  | 1.11–1.05                              |
| (7) Rolling of flat blooms and sheet slabs [119]                            |                       |                                        |
| Greater than 50 mm height and for edging passes                             | 6–20                  | 1.06–1.25                              |
| Less than 50 mm of height                                                   | 18–40                 | 1.22–1.67                              |
| (8) Rolling of sections [145]                                               |                       |                                        |
| for knifing passes                                                          | 17–48                 | 1.2–1.9                                |
| for forming passes                                                          | 23–38                 | 1.3–1.6                                |
| for leader passes                                                           | 17–23                 | 1.2–1.3                                |
| for finishing passes                                                        | 5–17                  | 1.05–1.2                               |

N. B. The degree of draught is defined as the sum of draughts applied to the steel in one or several passes, between particular heat treatments to remove strain hardening of metal.

\*  $h$  = diagonal height of gothic square pass,  $b$  = diagonal width of gothic square pass.

† Obtaining this value seems doubtful, flatter ovals with  $b/h = 3.8$  should be taken.

where

$c \approx 0.2$  for thick flats,

$c \approx 0.25$  for thin flats,

and

$$\left. \begin{aligned} a &= \frac{2h+b}{3} & \text{for } b < 20 \\ \text{or} & & \\ a &= \frac{2(b+h)}{3} & \text{for } b \geq 20 \end{aligned} \right\} \quad (7.47)$$

where

$b$  = width of finished flat,

$h$  = height of finished flat.

Hammond recommends

using 3 edging passes,  $a = 0.8(b+h)$ ,

using 2 edging passes,  $a = 0.7(b+h)$ .



The values of mean elongation or draught coefficients are decided experimentally on a basis of existing design systems. Soviet data give the mean elongation coefficients as [145]

$$\lambda_m = 1.12-1.20 \text{ for mild steel}$$

or

$$\lambda_m = 1.10-1.14 \text{ for carbon steel.}$$

TABLE 7.6  
MEAN ELONGATION COEFFICIENT  $\lambda_m$  RELATIVE TO TOTAL ELONGATION  
COEFFICIENT  $\lambda_t$  AND NUMBER OF PASSES  $n$  [145]

| $\lambda_t = F_0/F_n$ | $\lambda_m$ |          |          |          |          |          |
|-----------------------|-------------|----------|----------|----------|----------|----------|
|                       | $n = 23$    | $n = 21$ | $n = 19$ | $n = 17$ | $n = 15$ | $n = 13$ |
| 10                    | 1.105       | 1.116    | 1.129    | 1.145    | 1.166    | 1.195    |
| 9                     | 1.100       | 1.110    | 1.123    | 1.138    | 1.158    | 1.184    |
| 8                     | 1.095       | 1.104    | 1.116    | 1.130    | 1.149    | 1.173    |
| 7                     | 1.088       | 1.097    | 1.108    | 1.121    | 1.139    | 1.161    |
| 6                     | 1.081       | 1.089    | 1.099    | 1.111    | 1.127    | 1.148    |

In general the following elongation coefficients are assumed depending on sectional dimensions of metal being rolled:

for sections with a side greater than 600 mm  $\lambda = 1.07-1.15$

for sections with a side of 600-400 mm  $\lambda = 1.07-1.20$

for sections with a side of 400-250 mm  $\lambda = 1.15-1.33$

for sections with a side of 250-100 mm  $\lambda = 1.27-1.54$

In rolling square or rectangular sections exclusively e.g. in blooming mills, the ingot section dimensions are denoted as  $A$  and  $B$ , and the exit bloom side dimensions as  $a$ .

If the mean spread value of  $\Delta b/b_1$  is assumed as 10% and the mean draught value  $\Delta h_m$ , then the total draught can be calculated

$$\Sigma \Delta h = [(A-a) + (B-a)] \times 1.10 \quad (7.48)$$

and the number of roll passes

$$n = \frac{\Sigma \Delta h}{\Delta h_m} = \frac{(A+B-2a) \times 1.10}{\Delta h_m} \quad (7.49)$$

and the absolute mean draughts

$$\Delta h_m = 64-78 \text{ mm for mild steels}$$

or

$$\Delta h_m = 61-70 \text{ mm for carbon steels.}$$

The greatest absolute draughts used at present in the initial passes are from 80-120 mm, depending on roll diameters [162].

Draughts, reductions and elongation coefficients, at present used for modern blooming mills in the Soviet Union are given in Table 7.7 [162].

TABLE 7.7  
CHARACTERISTICS OF DRAUGHT CONDITIONS OF INGOTS ROLLED IN BLOOMING MILLS IN SOVIET METALLURGICAL PLANTS

| Type of steel†                          | Cross-sectional dimensions of ingot<br><br>mm | Weight of ingot<br><br>t | Bloom section<br><br>mm | Number of roll passes<br><br>$n$ | Total elongation coefficient<br><br>$\lambda_t$ | Mean elongation coefficient<br><br>$\lambda_m$ | Draught in roll pass, mm |                   | Maximum angle of bite<br><br>$\alpha_{\max}$ | Maximum width to height ratio of bloom for edging |
|-----------------------------------------|-----------------------------------------------|--------------------------|-------------------------|----------------------------------|-------------------------------------------------|------------------------------------------------|--------------------------|-------------------|----------------------------------------------|---------------------------------------------------|
|                                         |                                               |                          |                         |                                  |                                                 |                                                | max $\Delta h$           | mean $\Delta h_m$ |                                              |                                                   |
| Blooming mill 1150                      |                                               |                          |                         |                                  |                                                 |                                                |                          |                   |                                              |                                                   |
| Alloy and high quality carbon steel     | $\frac{740 \times 740}{590 \times 590}$       | 6.3                      | 255 × 255               | 19                               | 6.95                                            | 1.10                                           | 80                       | 60                | 23°30′                                       | 1.46                                              |
| Low carbon killed and rimming steel     | $\frac{700 \times 620}{760 \times 680}$       | 6.7                      | 255 × 250               | 17                               | 7.44                                            | 1.13                                           | 80                       | 63                | 23°30′                                       | 1.46                                              |
| Blooming mill 1100                      |                                               |                          |                         |                                  |                                                 |                                                |                          |                   |                                              |                                                   |
| High quality carbon and low alloy steel | $\frac{690 \times 690}{535 \times 535}$       | 5.8                      | 330 × 320               | 11                               | 3.55                                            | 1.12                                           | 120                      | 77                | 29°30′                                       | 1.42                                              |
| Rimming steel                           | $\frac{720 \times 650}{760 \times 690}$       | 6.8                      | 330 × 320               | 13                               | 4.7                                             | 1.13                                           | 120                      | 71                | 29°30′                                       | 1.60                                              |
| Rail open hearth steel                  | $\frac{740 \times 740}{585 \times 585}$       | 6.8                      | 330 × 320               | 15                               | 4.15                                            | 1.10                                           | 120                      | 65                | 19°30′                                       | 1.63                                              |
| Rimming steel                           | $\frac{720 \times 650}{760 \times 690}$       | 6.8                      | Shapes for I-bars       | 15                               | 4.65                                            | 1.11                                           | 100                      | 75                | 25°55′                                       | 1.53                                              |
| Blooming mill 1150                      |                                               |                          |                         |                                  |                                                 |                                                |                          |                   |                                              |                                                   |
| Rimming                                 | $\frac{525 \times 625}{630 \times 730}$       | 5.3                      | 240 × 240               | 13                               | 6.8                                             | 1.16                                           | 100                      | 76                | 25°55′                                       | 1.78                                              |
| Rimming                                 | $\frac{525 \times 625}{630 \times 730}$       | 5.3                      | 225 × 225               | 15                               | 7.7                                             | 1.15                                           | 100                      | 69                | 25°55′                                       | 1.78                                              |
| High quality killed carbon steel        | $\frac{670 \times 670}{490 \times 490}$       | 4.5                      | 250 × 250               | 15                               | 5.38                                            | 1.11                                           | 100                      | 64                | 25°55′                                       | 1.48                                              |
| Ditto                                   | $\frac{670 \times 670}{490 \times 490}$       | 4.5                      | 240 × 240               | 13                               | 5.84                                            | 1.14                                           | 100                      | 73                | 25°55′                                       | 1.48                                              |

† See Appendix 1.

The calculated lateral spread  $\Delta b$  approximately equals

$$\begin{aligned}\Delta b &= 6\% & \text{for } h &= 600 \text{ mm} \\ \Delta b &= 10\% & \text{for } h &= 250 \text{ mm} \\ \Delta b &= 21\% & \text{for } h &= 100 \text{ mm}\end{aligned}$$

where  $h$  = the height of metal being rolled. On the average  $\Delta b = 10\%$  is taken, or for smaller passes  $\Delta b = 15\%$ .

After preliminary calculations of sectional dimension of metal being rolled in the particular roll passes, the number and sequence of edging passes should be determined. To obtain better and more uniform working of metal and also a clean surface, a large number of edging passes is desirable. However, increasing the number of edging passes prolongs the idle rolling time, hence the output of roll stand falls.

For this reason alloy steels are edged after every roll pass. In rolling mild steels however, the first edging pass is made after two roll passes (to remove ingot taper), the next 2–3 edging passes after every 4 roll passes, further ones after every 2 roll passes, and before the end roll pass the metal is always edged.

*Checking of draught magnitudes.* When pass dimensions of successive passes are calculated and an edging pass scheme worked out, draughts can be calculated, which must be checked relative to the angle of bite, drive motor power and roll strength.

The maximum permissible draught relative to the angle of bite [162] can be checked by means of the formula

$$\Delta h_{\max} = D_w \left( 1 - \frac{1}{\sqrt{1+f^2}} \right) \quad (7.50)$$

where

$$D_w = 0.9 D_0 = 0.9 \times 1100 = 990 \text{ mm},$$

$f$  = friction coefficient,

and if  $f = 0.44$

$$\Delta h_{\max} = 990 \left( 1 - \frac{1}{\sqrt{1+0.44^2}} \right) = 82 \text{ mm}$$

$$\Delta h_m = 0.95 \Delta h_{\max} = 78 \text{ mm}$$

According to Tafel and Schneider (Fig. 3.53) when rolling mild steel at a temperature of 1200°C on rough rolls of a blooming mill, the permissible angle of bite decreases from 32–30° when the peripheral speed of rolls  $v$ , increases from 0–2 m/sec, with smooth rolls this angle decreases from 24–21°. When  $v$ , increases to 3 m/sec the permissible angle of bite decreases rapidly to 12° but further speed increase causes only a very small decrease of this angle. For checking the absolute draught relative to drive motor power, the formula (5.89) is used

$$\Delta h = \frac{1}{R_{mw}} \left( \frac{M_w}{1.05 K_{wm} b_m} - k R_{mw} \sqrt{\frac{M_w}{1.05 K_{wm} b_m}} \right)$$

where

$R_{mw}$  = mean effective radius,  $R_{mw} = 2(R_1 R_2)/(R_1 + R_2)$ ,

$M_w$  = true rolling torque corresponding to maximum motor load,

$M_w = M_m - (M_d + M_i)$ , where

$M_m$  = total driving torque,  $M_m \approx 0.8$  cut-out torque,

$M_d$  = dynamic torque [formula (4.120)],  $M_d \approx 1.9$  tm for 1 rpm/sec,

$M_i$  = idling torque,  $M_i \approx 5$  tm,

$K_{wm}$  = mean deformation resistance of metal,

$b_m$  = mean sectional width of metal being rolled,

$k$  = factor allowing for bearing friction,  $k = 2fd/D$  where  $f$  = friction coefficient,  $d$  = neck diameter,  $D$  = effective roll diameter,

$1.05 = 1/\eta_1$  = factor allowing for friction in teeth of pinions.

The draught can be checked relative to the roll strength from the roll force formula (2.108) (4.11)

$$P = K_{wm} b \sqrt{R_m \Delta h}$$

transforming this equation gives (5.92)

$$\Delta h = \frac{1}{R_m} \left( \frac{p}{K_{wm} b} \right)^2$$

where

$$P = 0.1 D^3 \sigma_p \frac{a+b}{ab}$$

and

$a, b$  = distances of line of action of force  $P$  from half length of necks,

$\sigma_p$  = permissible roll stress.

*Shape and dimensions of passes.* A blooming mill is an adjustable mill. With successive adjustments of the setting height of the upper roll the metal can be rolled several times through one pass. Construction and dimensions of a box pass is shown in Fig. 7.11.

The groove depth should not exceed  $1/6$  roll diameter, and the pass height when rolls are set on a normal gap should be less than or equal to the width of the next pass, hence  $h_m \leq b_{i+1}$ .

The pass width should be greater than or equal to the height of the pass before turning over (the last roll pass) and also limited spread of all the roll passes in the given pass groove should be ensured.

The camber of the groove bottom should be about 5 mm, but its use is not essential; in the first and final passes camber is not used.

A groove taper of 10–20% is used. The groove rounding radius to the side wall is assumed from  $0.08b$ – $0.15b$  (usually 10% of the groove width). For alloy steels, rounding with the same radius in all the passes is advisable.

The rounding radius  $r$  of collars is used at about 5% of groove width. A roll gap of 10–20 mm and an underdraught of 5–10 mm is used.

The height of raised or cut notches on the groove bottom should be  $h = 4$ –6 mm and the distance between them is taken about 1.5 notch width.

In 2-high rolls the shape and dimensions of the two collaborating grooves are identical.

#### Example

Given: barrel diameter  $D_t = 1090$  mm, length  $L = 2800$  mm, square ingot tapered upwards with dimensions  $630 \times 630$  mm at the bottom,  $570 \times 570$  mm at the top, of weight 8.0 t, underdraught  $g = 10$  mm.

Design of a blooming mill rolls to get square bloom  $300 \times 300$  mm is required.

(1) Calculation of total elongation coefficient  $\lambda_t$ : mean ingot section is

$$\left( \frac{630+570}{2} \right)^2 = 600^2, \quad \text{hence} \quad \lambda_t = \frac{600^2}{300^2} = 4$$

(2) Calculation of number of roll passes

$$n = \frac{\log \lambda_t}{\log \lambda_m} \quad \lambda_m = 1.165 \quad n = 9$$

(3) Determination of chamfer scheme.

Successive edging passes after 2, 4, 2 passes are assumed, i.e. 3 edging passes. One pair of opposite sides of ingot will be constrained in  $2+2=4$  roll passes, the other in  $4+1=5$  roll passes.

(4) Calculation of mean draughts for each pair of opposite sides of ingot.

The mean spread value is assumed about 15%. Hence the total draught for one pair of opposite sides of ingot will be

$$(630-300) \times 1.15 = 380 \text{ mm}$$

and the mean draught

$$\text{for I pair of sides } 380 : 4 = 95 \text{ mm}$$

$$\text{for II pair of sides } 380 : 5 = 76 \text{ mm}$$

(5) Determination of preliminary draught scheme

|                                     |                                    |
|-------------------------------------|------------------------------------|
| for the I pair of sides:            | for the II pair of sides:          |
| 2 roll passes $\times 80 = 160$ mm  | 4 roll passes $\times 80 = 320$ mm |
| 2 roll passes $\times 110 = 220$ mm | 1 roll pass $\times 20 = 20$ mm    |
| 4 roll passes $\times 95 = 380$ mm  | 5 roll passes $\times 68 = 340$ mm |

(6) Roll pass arrangement.

In calculating the section dimensions of an ingot rolled in particular passes the assumed draughts are such that the mean draught between edging passes corresponds to the agreed draught scheme. Absolute spread is taken from practical data. The sectional areas of the ingot and the first three roll passes (tapered ingot) are calculated relative to the mean sections.

Table 7.8 shows the roll pass arrangement, the letters *ch* denote edging passes.

TABLE 7.8  
ROLL PASSES FOR BLOOMING MILLS (FIG 7.28)

| Roll pass No. | Pass groove | Dimensions of ingot after roll pass, mm |          | Draught      |      | Spread       |      | Pass area <i>F</i><br>cm <sup>2</sup> | $\lambda$ | Length after roll pass<br>m | Angle of bite<br>$\alpha$ |
|---------------|-------------|-----------------------------------------|----------|--------------|------|--------------|------|---------------------------------------|-----------|-----------------------------|---------------------------|
|               |             | <i>h</i>                                | <i>b</i> | absol.<br>mm | %    | absol.<br>mm | %    |                                       |           |                             |                           |
| 0             | —           | 630                                     | 630      | —            | —    | —            | —    | 3969                                  | —         | 2                           |                           |
| 1             | B           | 550                                     | 630      | 80           | 12.7 | —            | —    | 3460                                  | 1.06      | 2.12                        | 22°7'                     |
| 2             | B           | 470                                     | 630      | 80           | 14.5 | —            | —    | 2960                                  | 1.17      | 2.48                        | 22°7'                     |
| <i>ch*</i>    |             |                                         |          |              |      |              |      |                                       |           |                             |                           |
| 3             | B           | 550                                     | 480      | 80           | 12.7 | 10           | 12.5 | 2640                                  | 1.12      | 2.78                        | 22°7'                     |
| 4             | B           | 460                                     | 490      | 90           | 16.3 | 10           | 11.1 | 2250                                  | 1.175     | 3.26                        | 23°18'                    |
| 5             | B           | 380                                     | 500      | 80           | 17.4 | 10           | 12.5 | 1900                                  | 1.18      | 3.86                        | 22°7'                     |
| 6             | B           | 300                                     | 510      | 80           | 21.0 | 10           | 12.5 | 1530                                  | 1.24      | 4.80                        | 22°7'                     |
| <i>ch*</i>    |             |                                         |          |              |      |              |      |                                       |           |                             |                           |
| 7             | I           | 400                                     | 310      | 110          | 21.5 | 10           | 9.1  | 1240                                  | 1.23      | 5.90                        | 27°17'                    |
| 8             | I           | 295                                     | 320      | 105          | 26.3 | 10           | 9.5  | 945                                   | 1.31      | 7.73                        | 26°45'                    |
| <i>ch*</i>    |             |                                         |          |              |      |              |      |                                       |           |                             |                           |
| 9             | B           | 300                                     | 300      | 20           | 6.8  | 5            | 25   | 900                                   | 1.05      | 8.1                         | 10°50'                    |

\* *ch* — edging pass

Checking from formulae for highest and mean draughts

$$\Delta h_{\max} = D_w \left( 1 - \frac{1}{\sqrt{1+f^2}} \right)$$
$$D_w \approx 0.9 D_0 \text{ (for passes)}$$

or

$$D_w = D_{\text{true}} \text{ (for barrel)}$$
$$f = \text{friction coefficient} = 0.45$$

for barrel

$$\Delta h_{\max} = 1090 \left( 1 - \frac{1}{\sqrt{1+0.45^2}} \right) = 96 \text{ mm}$$

for pass

$$\Delta h_{\max} = 0.9 \times 1150 \left( 1 - \frac{1}{\sqrt{1+0.45^2}} \right) = 91 \text{ mm}$$

$$\text{Check: } 0.95 \times 96 = 91 \text{ mm}$$

$$\Delta h_m = 0.95 \Delta h_{\max} = 0.95 \times 91 = 87 \text{ mm}$$

The values given in Table 7.8 are permissible.

TABLE 7.9  
DIMENSIONS OF PASSES, DIAMETERS OF ROLLS, AND ANGLES OF BITE (FIG. 7.28)

| Pass groove | Roll pass No. | Pass dimensions, mm |        |     |             |       | Diameters, mm |       | $\alpha_{\max}$ |
|-------------|---------------|---------------------|--------|-----|-------------|-------|---------------|-------|-----------------|
|             |               | $b$                 | $b_1$  | $h$ | $h_l = h_u$ | $h_1$ | $D_l$         | $D_u$ |                 |
| Barrel I    | 1 to 6 and 9  | 1865.9              | 1834.9 | 106 | 103         | 90    | 1100          | 1090  | 23°18'          |
|             |               | 300                 | 330.6  | 190 | 95          | 190   | 990           | 980   | 27°17'          |

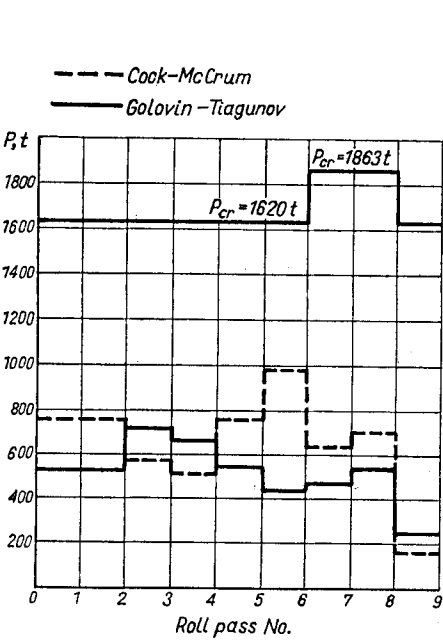


Fig. 7.29. Roll forces calculated from roll pass programme (Table 7.8)

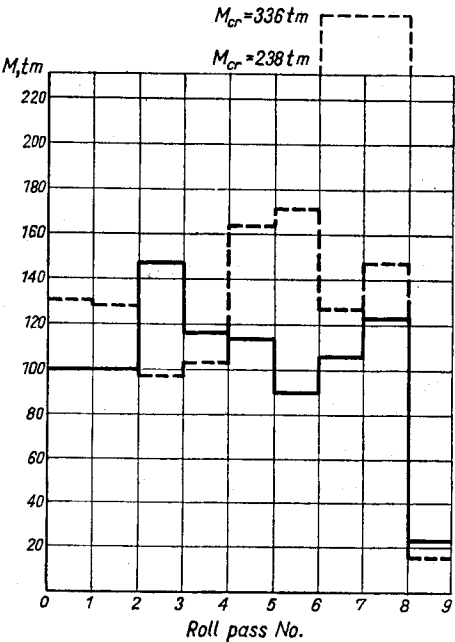


Fig. 7.30. Rolling torques calculated from roll pass programme (Table 7.8)



Since the high absolute draughts at points 7 and 8 are in edging passes where areas are small, it has been shown in practice that they are permissible for these ranges of bite, pressures and torques.

(7) Table of passes, effective diameters and bite angles.

Active diameters are calculated from the formulae given in Sec. 7.8. The angle of bite  $\alpha$  is calculated for the maximum draught used in a given pass, Table 7.9.

(8) The roll drawing is given in Fig. 7.29.

The pass design for a blooming mill of  $D = 1100$  mm given in Table 7.8 and in Fig. 7.29 was checked by the calculation of roll forces by Cook and McCrum's method (formulae (4.50) and (4.59) and diagrams in Figs. 4.39 to 4.42) also Golovin-Tiagunov's method (Figs. 4.32 to 4.34 and formulae (4.30) to (4.34)). When the roll force was calculated, the torque was also calculated by the same methods.

Figures 7.29 and 7.30 show these calculated values of roll force and torque. The values from Cook and McCrum's method are always greater than those from Golovin-Tiagunov's method.

The values of torques and forces given in Figs. 7.29 and 7.30 show that critical forces  $P_{cr}$  and torque  $M_{cr}$  are not reached in any pass, i.e. the rolls show a considerable reserve of mechanical strength.

### 7.3. Graphical Method of Roll Pass Design

Computation methods of roll pass design are often too long and tedious to be convenient for practical everyday conditions, especially for control cases, where accurate calculations and profile design by specialists is not essential. For these cases a simple quick and uniform method is needed, which can be used for as many as possible of the regular roll pass designs.

The best choice is a graphical method, which although less exact than computation, has the advantage of simplicity, clearness and speed [122].

This method is especially suitable for the determination of initial profiles, number of passes required, choice of pass design during rolling, and most important, to provide a quick and simple method of checking calculations. It is based of course on the same theoretical and practical principles as the computation method, but in a simplified form [122].

For drawing these diagrams the following relationships are used:

$$U = \frac{F_1 - F_2}{F_1} = 1 - \frac{F_2}{F_1}$$

where

$U$  = relative reduction of rolled stock,

$F_1 = b_1 h_1$  = sectional area before roll pass,

$F_2 = b_2 h_2$  = sectional area after roll pass, and from (3.6)

$$\lambda = \frac{l_2}{l_1} = \frac{F_1}{F_2}$$

where

$\lambda$  = elongation coefficient,

$l_1, l_2$  = length before and after roll pass.

In the case where there is no spread i.e.  $b_2/b_1 \approx 1$ , the above formulae take a simplified form

$$U \approx 1 - \frac{h_2}{h_1} = 1 - \frac{1}{\gamma}$$

$$\lambda \approx \frac{h_1}{h_2} = \frac{1}{1 - U} = \frac{1}{\gamma}$$

where  $\gamma = h_2/h_1$  is the draught coefficient.

The diagram in Fig. 7.31 is based on simplified formulae assuming that the width of metal being rolled does not change, and consists of two parts: the upper part above the zero line and the lower one below this line.

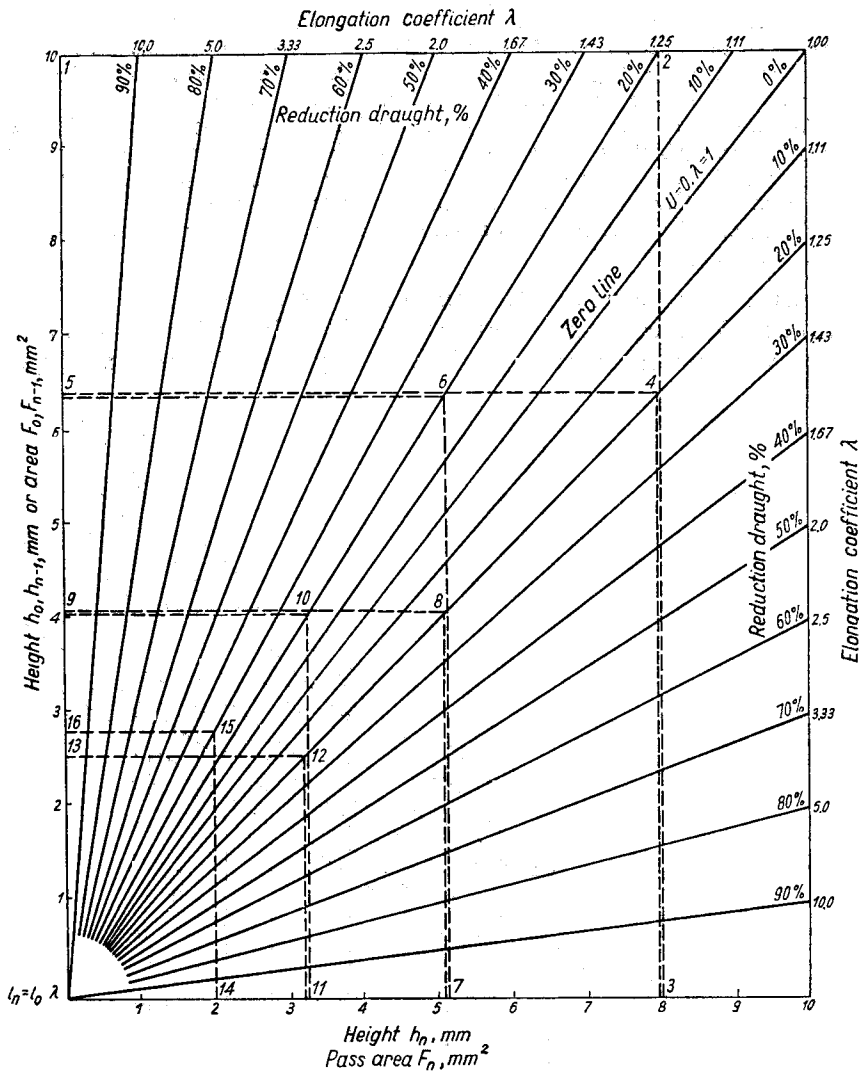


Fig. 7.31. Nomogram for determination of draughts and reductions

The upper and lower areas are divided for every 10% value of draught from 0–100%. The diagonal lines show the values of % draught and the corresponding values of elongation coefficient  $\lambda$ .

On the perpendicular scale are values of stock thicknesses in mm, on the horizontal scale the corresponding thicknesses of sheet or strip in mm.

This diagram applies for all ranges of thickness. For example, if a value for small thickness is to be found, the value for 10 times greater thickness is read off and the result divided by 10.

*Example*

Sheet is to be rolled from sheet slab 10 mm thick with draught of 20%. Moving from the point corresponding to  $h_0 = 10$  mm on the ordinate (point 1) horizontally to the straight line corresponding to upper draught 20% (point 2) and dropping a perpendicular to the abscissa gives the values  $h_n$  or  $h_1 = 8$  mm (point 3). From point 3 a vertical to the lower line of 20% draughts gives point 4, corresponding on the ordinate to a thickness of 6.4 mm (point 5). From point 5 a horizontal line cuts the line for the upper 20% draught at point 6, giving on the abscissa (scale  $h_n$ ) the value of  $h_n = 5.12$  mm (point 7). Proceeding in this way further thicknesses 4.1 mm (point 9), 3.3 mm (point 11) and 2.64 (point 16) are found. Thus a series of successive thicknesses is obtained: 10; 8; 6.4; 5.12; 4.1; 3.3; 2.64 mm.

Similarly by working backwards the slab thickness can be found starting from the finished section  $h_n$  or  $h_1 = 2$  mm (point 14), then the foregoing height e.g. for 30% draught is required (point 15). Passing to the next 30% draught line this gives a thickness of 2.86 mm on the ordinate (point 16) which corresponds to  $h_{n-1}$  or  $h_0$ . Proceeding in this way the preliminary thickness of slab can be found for cases where spread is not important.

This diagram can be used to determine the changes of pass area using instead of thickness the sectional area, and instead of draughts using reductions and elongation coefficients. In the same manner a series of successively decreasing or increasing sectional

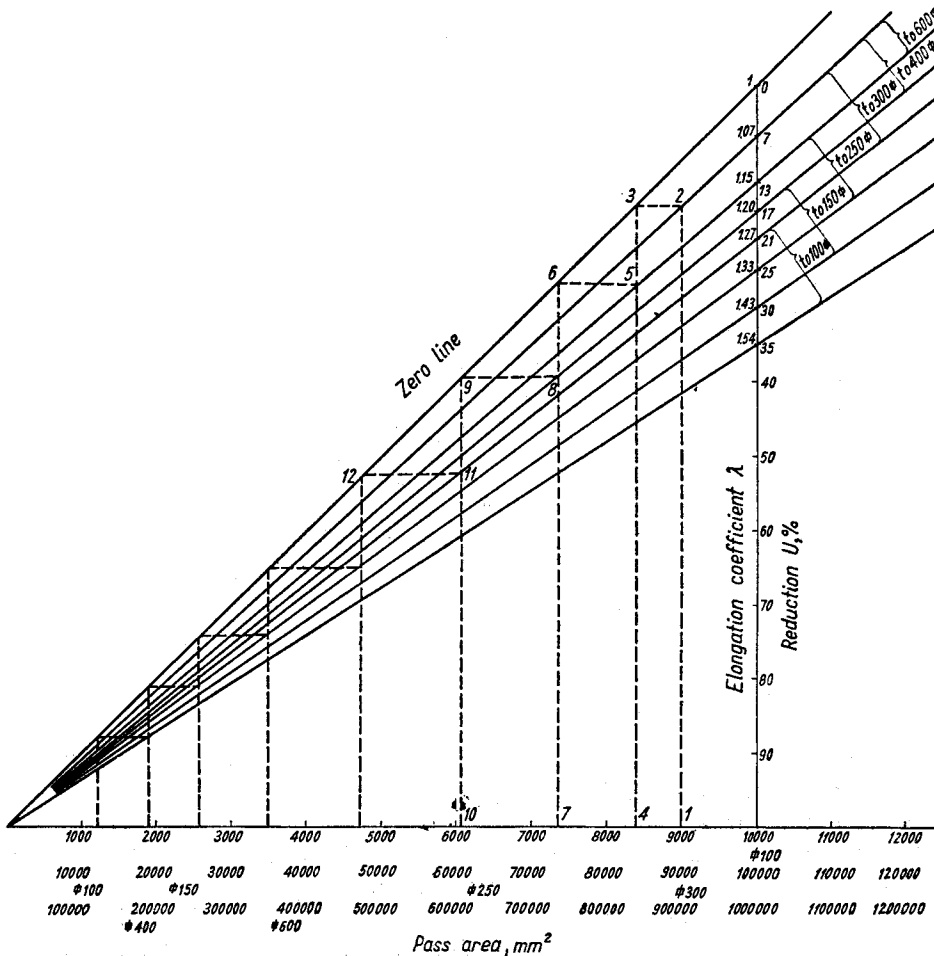


Fig. 7.32. Graphical roll pass designing for blooming mills

areas can be found. Intermediate values of reductions or draughts applied can be found by interpolation.

Similarly successive values of sectional area can be obtained from equation

$$\lambda = \frac{F_0}{F_1}$$

Simply multiplying or dividing by the coefficient  $\lambda$  gives the next smaller or larger size. These examples are given to facilitate the understanding of Figs. 7.32 and 7.33 showing the standard type of pass designing.

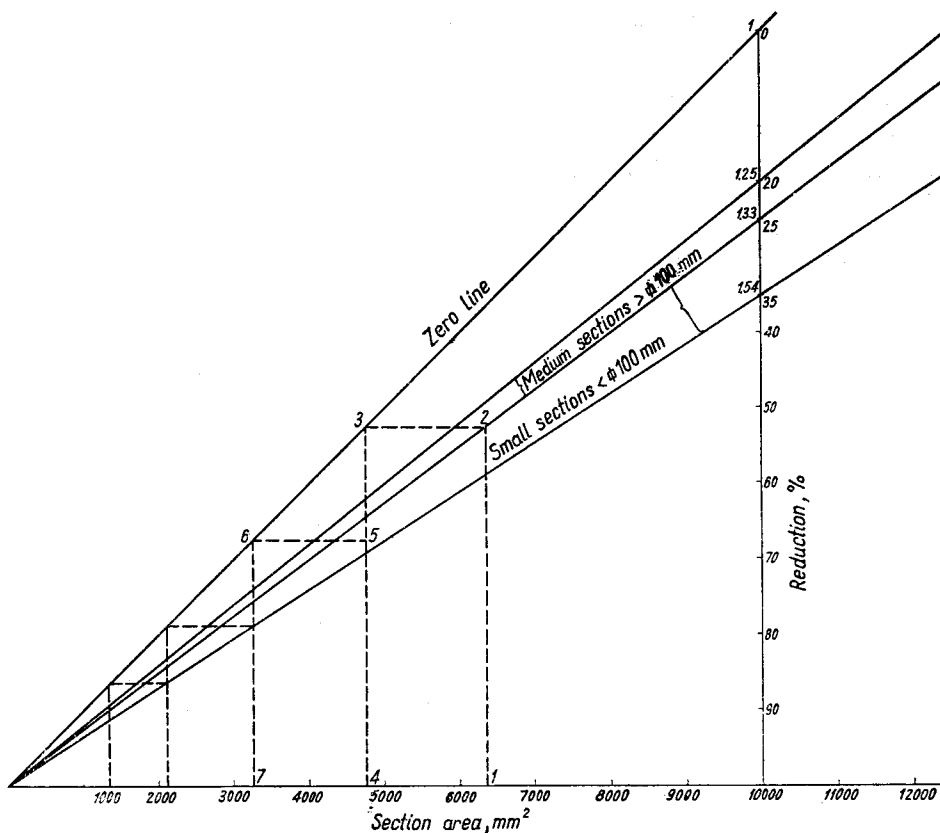


Fig. 7.33. Graphical roll pass designing of box-passes in roughing mills

In Fig. 7.32 the nomogram for a blooming mill pass design is given. On the abscissa the sectional area is given in mm², on the ordinate the reduction  $U$  in % and the corresponding elongation coefficient. Straight lines corresponding to various permissible reductions relative to the section being rolled and the zero line i.e. the line corresponding to  $U = 0$  and  $\lambda = 1$ , are given. To obtain the successive passes, the point where the ordinate corresponding to the section area of stock, in this case the ingot, cuts the line for a given reduction and the zero line i.e. the line corresponding to  $U = 0$  and  $\lambda = 1$ , are given. To obtain the successive passes, the point where the ordinate corresponding to the section area of stock, in this case the ingot, cuts the line for a given reduction and defines a point from which a horizontal line is drawn to the zero line, intersecting it at a point from which a vertical line is drawn to the next required fixed reduction. This intersection gives a point which determines the next successive section, and the vertical drawn to the abscissa gives the area of required section.

*Example*

An ingot of  $300 \times 300 = 90,000 \text{ mm}^2$  is given. The range of permissible reductions for  $300 \times 300 \text{ mm}$  is 7–21%. The first reduction is chosen small e.g. 7%, hence from the value 90,000 on abscissa i.e. point 1 a line is drawn to point 2 on the line corresponding to reduction of 7%. From this point a horizontal line is drawn to intersect the neutral line (point 3), from which a vertical line to the abscissa determines the sectional area  $84,000 \text{ mm}^2$  at point 4, and intersects the line for 13% reduction at point 5, determining the next section. Similarly point 7 is found giving an area of  $73,500 \text{ mm}^2$ . Proceeding in this way and choosing suitable reductions the last and smallest section for the blooming mill can be found and also intermediate sections can be found as required. If intermediate reductions are to be used the appropriate line can be drawn into the diagram, and proceeding as before, the successive sectional areas can be found.

To obtain the true area the value read off should be multiplied by the appropriate power of 10.

The nomogram in Fig. 7.33 is for box pass designing in roughing mills.

This nomogram is similar to the previous one, but the lines are drawn for different ranges of reductions as used in roughing stands of mills rolling thinner sections. Proceeding as before points 1, 2, 3, 4, 5, 6, 7, are found.

In roll pass designing for sheet slabs (Fig. 7.34) the method is similar, i.e. either the sectional area or thickness can be found. It should be noted that the range of draughts used depends on thickness. Hence for thickness greater than 100 mm draughts

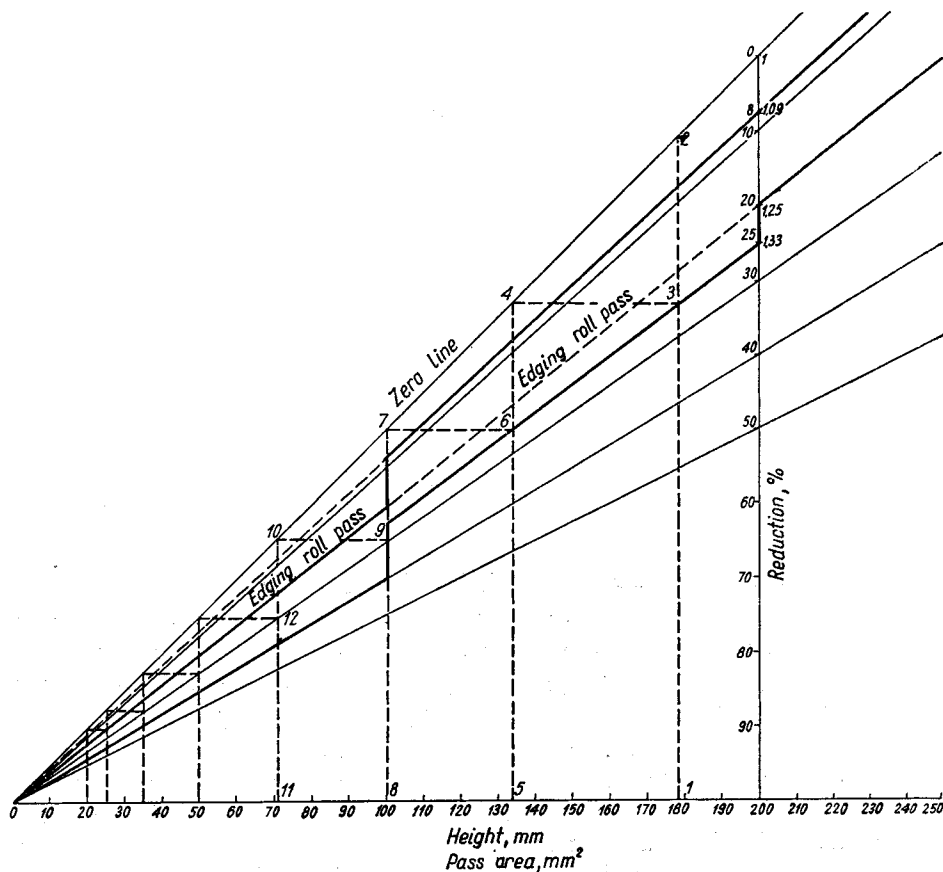


Fig. 7.34. Graphical roll pass designing for sheet bars

of 8–25% are to be used, and for lower than 100 mm, of 20–40%. The edging draughts, e.g. for flats, always vary from 6–20%. Proceeding thus within the limits of permissible draughts (the area enclosed by thicker lines), a series of intermediate passes is found.

### Example

The roll pass design for sheet bars can be found in a similar manner. Sheet of 20 mm height is to be rolled from sheet bars 178 mm thick. From point 1 a vertical line is drawn to the zero line (point 2).

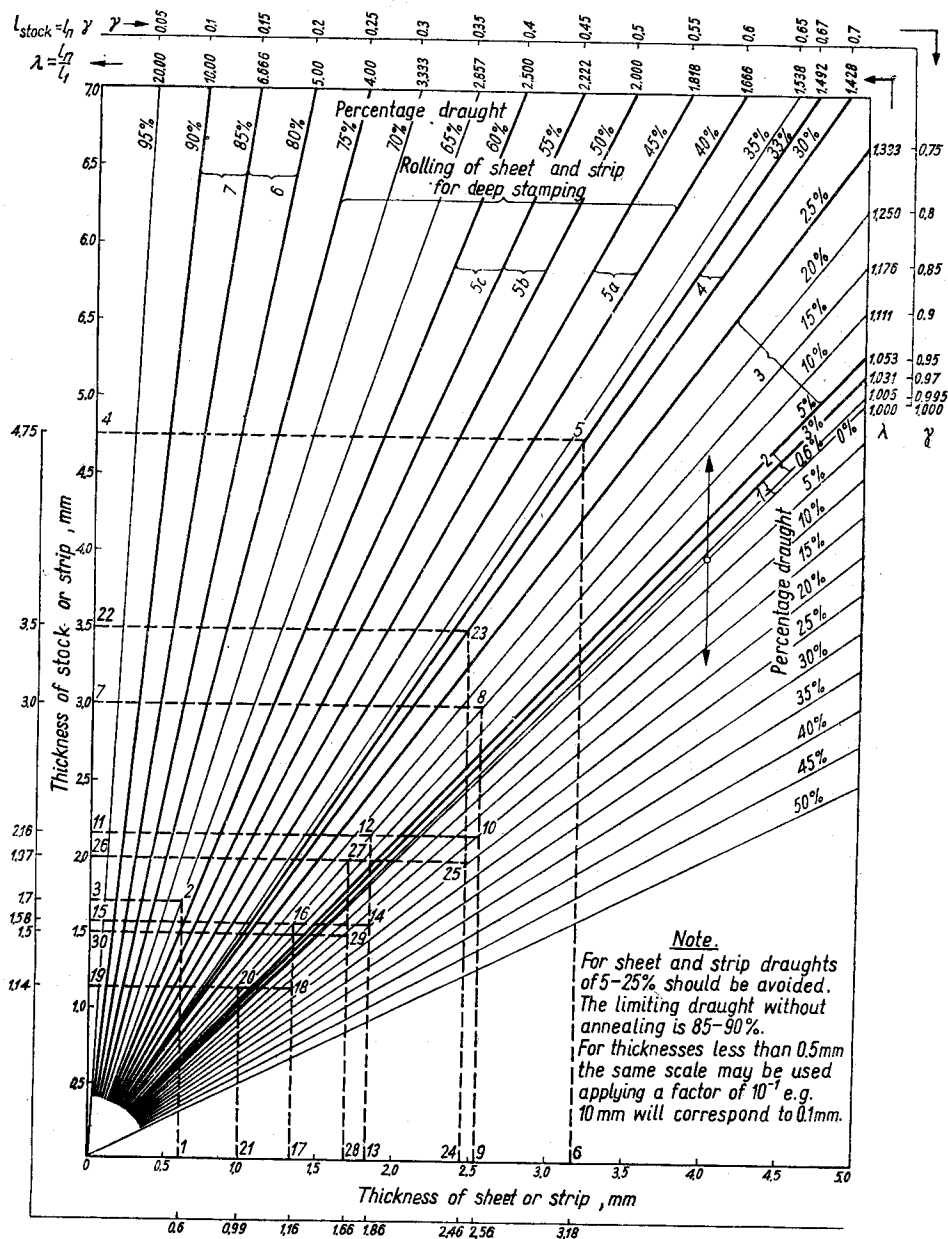


**Fig. 7.35. Graphical roll pass designing for cold rolling**

For roughing passes draughts of 25% are used (point 3). From this point a horizontal is drawn to the zero line (point 4) corresponding on the horizontal axis to a height of 133 mm (point 5) and cutting the next draught of 25% (point 6). The horizontal to the zero line gives point 7 corresponding to a height of 100 mm (point 8). Increasing the draught to 30% (point 9), the horizontal to the zero line gives point 10 corresponding to a height of 70 mm (point 11). The next intersection with 30% draught gives point 12 etc. Further steps are shown in the figure.

In roll pass designing for cold rolling the nomogram (Fig. 7.35) can be used. In this diagram suggested work areas are shown, based on rolling practice [120].

| Cold rolling of strips and sheets                                                                        | Draught<br>% | Elongation<br>coefficient<br>$\lambda$ |
|----------------------------------------------------------------------------------------------------------|--------------|----------------------------------------|
| (1) End sizing range for sheet or strip                                                                  | 0.5–3.0      | 1.005–1.031                            |
| (2) Interpass sizing range for strip or sheet                                                            | 0.5–5.0      | 1.005–1.053                            |
| (3) Range of harmful large recrystallization                                                             | 5–25         | 1.053–1.250                            |
| (4) Normal range of cold rolling                                                                         | 30–33        | 1.482–1.492                            |
| (5) Rolling of sheet and strip for deep drawing. This range is sub-<br>divided into three parts          | 40–75        | 1.666–4.0                              |
| (a) for steels with high strength properties                                                             | 40–45        | 1.666–1.818                            |
| (b) for aluminium killed steels                                                                          | 50–55        | 2.0–2.222                              |
| (c) for strip and sheet from rimming steel                                                               | 55–60        | 2.222–2.5                              |
| Higher draughts than these are used for extra deep drawing<br>sheet and strip, and for car bodywork etc. |              |                                        |
| (6) Rolling of tinned sheet and strip                                                                    | 80–85        | 5.0–6.666                              |
| (7) Limiting draught without interpass annealing                                                         | 85–90        | 6.666–10.0                             |

The limiting draught is variable and may often be exceeded or not attained, depending on the rolling method. For example when unequal draughts are used, decreasing from pass to pass, a smaller value of total draught is obtained than when rolling with fixed draughts.

Rolling with front or back tension always gives higher values of draught since cold rolling with tension partially eliminates the effects of strain hardening [121].

If the thickness of the initial stock for a given end product is required, starting from the point on the horizontal scale corresponding to end height, passing over the zero line, the intersection with the line of total draught gives on the vertical scale the thickness of required stock. The stock length is obtained by multiplying the length of the end product by the draught coefficient appropriate to the total draught, read off from the other scale.

#### Example 1

Initial stock is required for deep drawing sheet 0.6 mm thick and 2000 mm long with total draught of 65%.

The ingoing slab must be 1.7 mm thick (point 1, 2, 3) and  $2000 \gamma = 2000 \times 0.35 = 700$  mm long.

When the slab is given and the thickness of the finished product is required with a given total draught, the process is reversed: starting from the left vertical scale at the point corresponding to slab thickness, and fixing a point on the line of given draught, then on the horizontal scale the end thickness is read off. The end product length is found by multiplying the slab length by the elongation coefficient  $\lambda$ , read from the diagram.

#### Example 2

To find the length and thickness rolled from band iron 4.75 mm thick and 200 m long with the total draught of 33%.

Thickness will be 3.18 mm (points 4, 5, 6) and length  $200\lambda = 200 \times 1.492 = 298.40$  m, since  $U = 33\%$  corresponds to  $\lambda = 1.492$  on the upper vertical scale.

For the calculation of particular passes the auxiliary diagram below the neutral line can be used.

Starting from the thickness of a given stock in the vertical scale, and moving horizontally to the line corresponding to the first draught value, in the horizontal scale below the thickness after rolling pass can be read off, then from the same value the required draught value for the next pass can be found in the lower auxiliary diagram, and the next thickness can be read off on the left vertical scale.

This operation is repeated until the final thickness is reached. Alternatively, starting from final thickness the initial thickness can be found. The various lengths can be calculated multiplying by the appropriate  $\lambda$  or  $\gamma$  values for the given draught.

The zones outlined in the diagram indicate what total and intermediate draughts can be used in a given case. This is explained by examples 3 and 4.

#### Example 3

The successive thicknesses are required in rolling of strip with fixed draughts of 15% from stock 3.0 mm thick.

The successive thicknesses will be: 3.0 mm, 2.56 mm, 2.16 mm, 1.86 mm, 1.58 mm, 1.36 mm, 1.14 mm, 0.99 mm marked in diagram by points 7 to 21.

#### Example 4

The successive thicknesses are to be determined in rolling strips with the varying draughts of 30, 20, 15, 10% from stock 3.5 mm thick.

The required thicknesses are: 3.5 mm, 2.46 mm, 1.97 mm, 1.66 mm, 1.5 mm in successive roll passes, indicated in diagram by points 22–30.

### 7.4. Transformation of Formulae for Roll Pass Design

Formulae (3.45) to (3.47) cannot be used for actual roll pass design in their present form. It should be noticed that previous calculations begin from a larger pass and work to a smaller one, whereas in roll designing this is reversed: calculations start from the smallest pass and work successively to the largest, i.e. initial pass.

To use these formulae for roll pass designing, and in particular to obtain the required coefficients of spread and elongation, formulae (3.46) and (3.47) are transformed as follows:

$$\varepsilon'_w = \frac{h_2}{D} \quad \delta' = \frac{F_1}{h_2^2}$$

then

$$\lambda = \gamma^{-[1-10^{-1.269(\varepsilon'_w \times 1/\gamma)^{0.556}\delta'^{\gamma^2}}]} \quad (7.51)$$

If it is assumed that

$$E = 1.269(\varepsilon'_w)^{0.556}\delta' \quad (7.52)$$

the equation (7.51) becomes

$$\lambda = \gamma^{-(1-10^{-E\gamma^{1.444}})} \quad (7.53)$$

from which the draught coefficient  $\gamma$  can be calculated.

To find a quick solution to this equation, a diagram was evolved (Fig. 7.36) from which the required draught coefficient may be read off by means of a ruler.



As an example a section is considered having end dimensions:  $h_2 = 20$  mm,  $b_2 = 55$  mm, the working diameter of rolls  $D = 370$  mm. The elongation coefficient in the roll pass is assumed to be  $\lambda = 1.300$  with free spread. The end and initial sectional areas are:

$$F_2 = b_2 h_2 = 1100 \text{ mm}^2$$

$$F_1 = F_2 \lambda = 1100 \times 1.300 = 1430 \text{ mm}^2$$

and the required coefficients

$$\varepsilon'_w = \frac{h_2}{D} = \frac{20}{370} = 0.054$$

$$\delta' = \frac{F_2}{h_2^2} = \frac{1430}{400} = 3.57$$

In Fig. 7.36 the line joining points  $\varepsilon_w$  and  $\delta'$  is produced to intersect with scale  $E$  (point  $A$ ). From point  $A$  a parallel is drawn to the  $\gamma$  axis to intersect with the curve corresponding to the elongation coefficient in this roll pass. (In this case  $\lambda = 1.300$ , point  $B$ ). From  $B$  a vertical is drawn to the  $\gamma$  axis to find the draught coefficient for the given section. In this case  $\gamma$  can be read off as

$$\gamma = 0.687$$

hence the initial dimensions of the section should be

$$h_1 = \frac{h_2}{\gamma} = \frac{20}{0.687} = 29.1 \text{ mm}$$

$$b_1 = \frac{F_1}{h_1} = \frac{1430}{29.1} = 49.1 \text{ mm}$$

Checking

$$\varepsilon_w = \frac{29.1}{370} = 0.0786$$

$$\delta = \frac{49.1}{29.1} = 1.69$$

$$\gamma = \frac{20}{29.1} = 0.687$$

The spread coefficient calculated from equation (3.45) is

$$\beta = 1.121$$

then

$$b_2 = b_1 \beta = 49.1 \times 1.121 = 55 \text{ mm}$$

The elongation coefficient is

$$\lambda = \frac{1}{\beta \gamma} = \frac{1}{1.121 \times 0.687} = 1.300$$

To calculate the spread and elongation coefficients from equations (3.45) to (3.47) and (7.51) to (7.53), A. Wojtylak constructed a slide rule (Fig. 7.37) which enables a quick and accurate solution of these equations. The method of using this slide rule is illustrated by a numerical example.

Given values:  $h_1 = 32$  mm,  $h_2 = 24$  mm,  $b_1 = 46$  mm,  $D = 485$  mm. The coefficients  $\varepsilon_w$ ,  $\delta_w$ , and  $\gamma$  are calculated from these values using the bottom scale  $E$  and scale  $\delta$  on the slide rule. These are normal logarithmic scales and are used as on a standard logarithmic slide rule. Then

$$\varepsilon_w = \frac{h_1}{D} = \frac{32}{485} = 0.066$$

$$\delta_w = \frac{b_1}{h_1} = \frac{46}{32} = 1.44$$

$$\gamma = \frac{h_2}{h_1} = \frac{24}{32} = 0.75$$

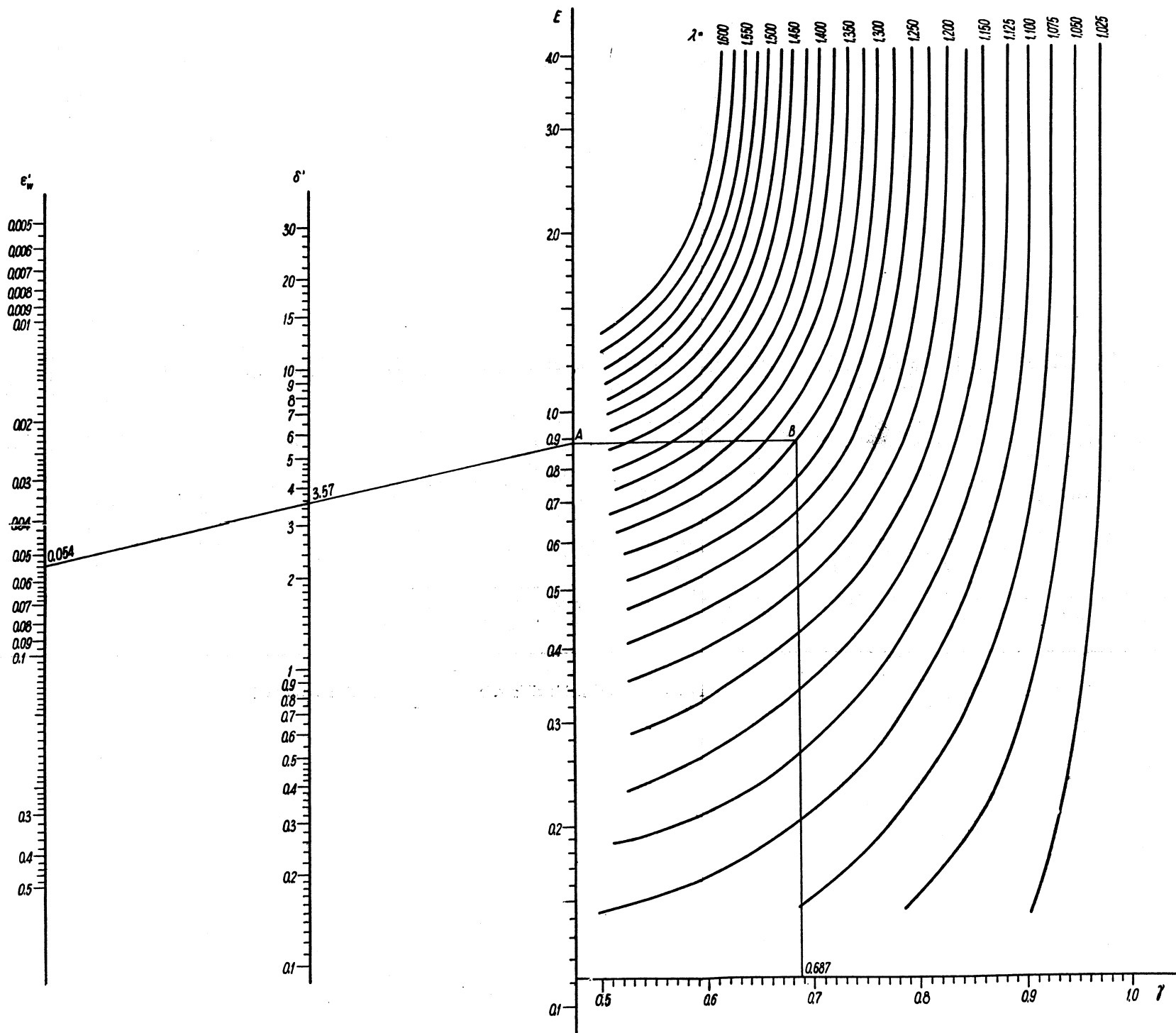


Fig. 7.36. Nomogram for determination of draught coefficient in pass design

Then the slide rule is set with the hairline of the cursor on the calculated value  $\varepsilon_w = 0.066$  on the  $\varepsilon_w$  scale of slide rule (Fig. 7.38a), and on value 1 on the  $\delta$  scale (Fig. 7.38b) on the middle part of slide rule, then the cursor is moved to align with the value  $\delta = 1.44$  on the same  $\delta$  scale (Fig. 7.38c) and on the  $E$  scale the value of  $E$  can be read off. In this example  $E = 0.4025$ . From equation (7.52)

$$E = 1.269(\varepsilon'_w)^{0.556} \delta'$$

Then the cursor is set on the previously read value  $E = 0.4025$  on the upper  $E$  scale (Fig. 7.38d), the middle part of slide rule is moved to the right until the calculated draught coefficient  $\gamma$  (in this example  $\gamma = 0.75$ ) in the  $\gamma$  scale coincides with the beginning of the upper  $E$  scale, i.e. with 0. Then on the  $\beta$ ,  $\lambda$  scale the value of spread coefficient  $\beta$  can be read off. In this case  $\beta = 1.1205$ .

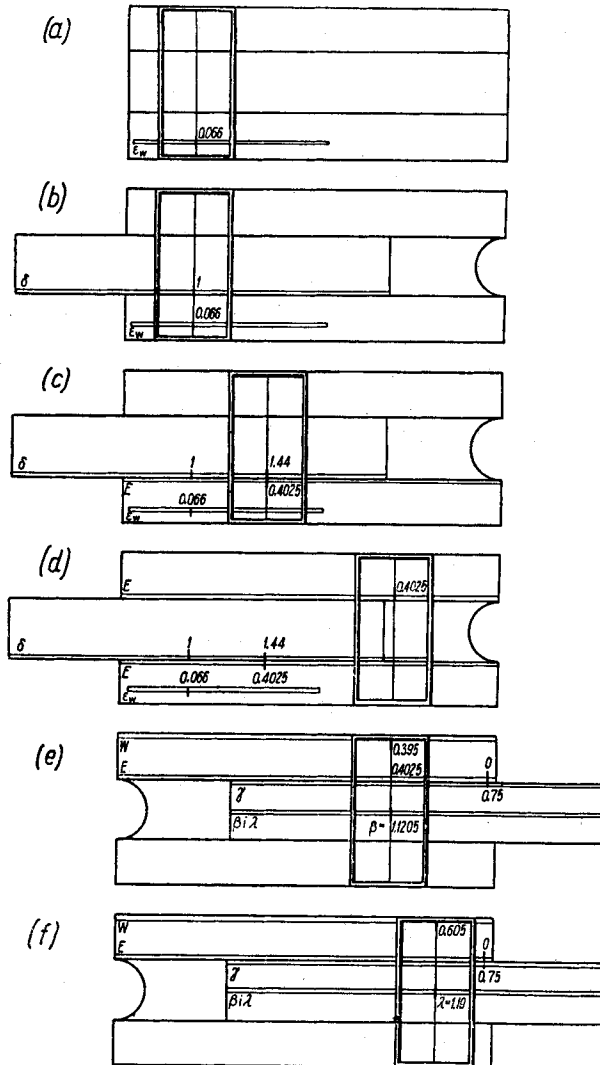


Fig. 7.38. Method of using Wojtylak's slide rule

To calculate the elongation coefficient, at the same time  $W$  can be read off from the upper  $W$  scale, where  $W$  is the exponential coefficient in equation (3.46)

$$\beta = \gamma^{-W}$$

In this case  $W = 0.395$  (Fig. 7.38e). Next the value  $1 - W$  is calculated since from (3.47)

$$\lambda = \gamma^{-(1-W)}$$

In this example

$$1 - W = 1 - 0.395 = 0.605$$

The cursor is moved to the value  $1 - W = 0.605$  in the  $W$  scale (Fig. 7.38f) and on the  $\beta, \lambda$  scale the value of  $\lambda$  can be read off. For this example  $\lambda = 1.19$ .

Checking

$$\gamma \beta \lambda = 0.75 \times 1.1205 \times 1.19 = 1.00004625$$

It can be seen that this slide rule gives very accurate results.

Instead of using the slide rule from Fig. 7.38, a slide rule constructed by E. Otto (Fig. 7.39) may be used to facilitate calculations. It is composed of two horizontal scales for the values  $\delta'$  and  $\epsilon'_w$ . The  $\epsilon'_w$  scale is a mobile one, drawn on the tongue of the slide rule. Above the tongue is a family of curves covering the whole range of  $\lambda$  coefficients in practical use. The  $\gamma$  scale is placed on the cursor so that  $l$  on this scale moves along the contact line of the mobile scale  $\epsilon'_w$  with the  $\lambda$  curves.

The method of using this slide rule is as follows: The number 1.0 on the  $\epsilon'_w$  scale is aligned with the previously calculated value  $\delta'$  on the mobile scale. Then the cursor is moved to the previously calculated value  $\epsilon'_w$  so that the mobile scale intersects the  $\lambda$  curves, i.e. the value  $\gamma$  required for roll pass designing is read off at the point of intersection of the  $\gamma$  scale with the appropriate  $\lambda$  curve assumed for the given pass, or alternatively if  $\gamma$  is known,  $\lambda$  can be read off.

#### Example

The size of pre-finished flat is required with an assumed elongation coefficient  $\lambda = 1.200$  for a finished profile  $12 \times 5$  mm with rounded sides. Mild steel is rolled on rolls of effective diameter  $D = 310$  mm, with a smooth barrel.

Area of cold section  $12 \times 5 = 60 \text{ mm}^2$

Area of hot section  $F_n = 60 \times 1.013 \times 1.013 = 61.6 \text{ mm}^2$

$$\lambda = \frac{F_{n-1}}{F_n} = 1.200$$

From this

$$F_{n-1} = \lambda F_n = 61.6 \times 1.200 = 73.9 \text{ mm}^2$$

Calculating the auxiliary indices

$$\epsilon'_w = \frac{h_n}{D} = \frac{5}{310} = 0.016$$

$$\delta' = \frac{F_{n-1}}{h_n^2} = \frac{73.9}{25} = 2.96$$

The mobile  $\epsilon'_w$  scale of slide rule is set so that value 1.0 aligns with the calculated value  $\delta' = 2.96$  (Fig. 7.39).

Then the cursor is moved to the calculated value  $\epsilon'_w = 0.016$  and at the intersection of mobile scale  $\gamma$  with  $\lambda$  curves for the assumed value  $\lambda = 1.200$  the value of  $\gamma = 0.560$  is read off. Since

$$\gamma = \frac{h_n}{h_{n-1}} = 0.560$$

hence

$$h_{n-1} = \frac{h_n}{\gamma} = \frac{5}{0.560} = 8.9 \text{ mm}$$

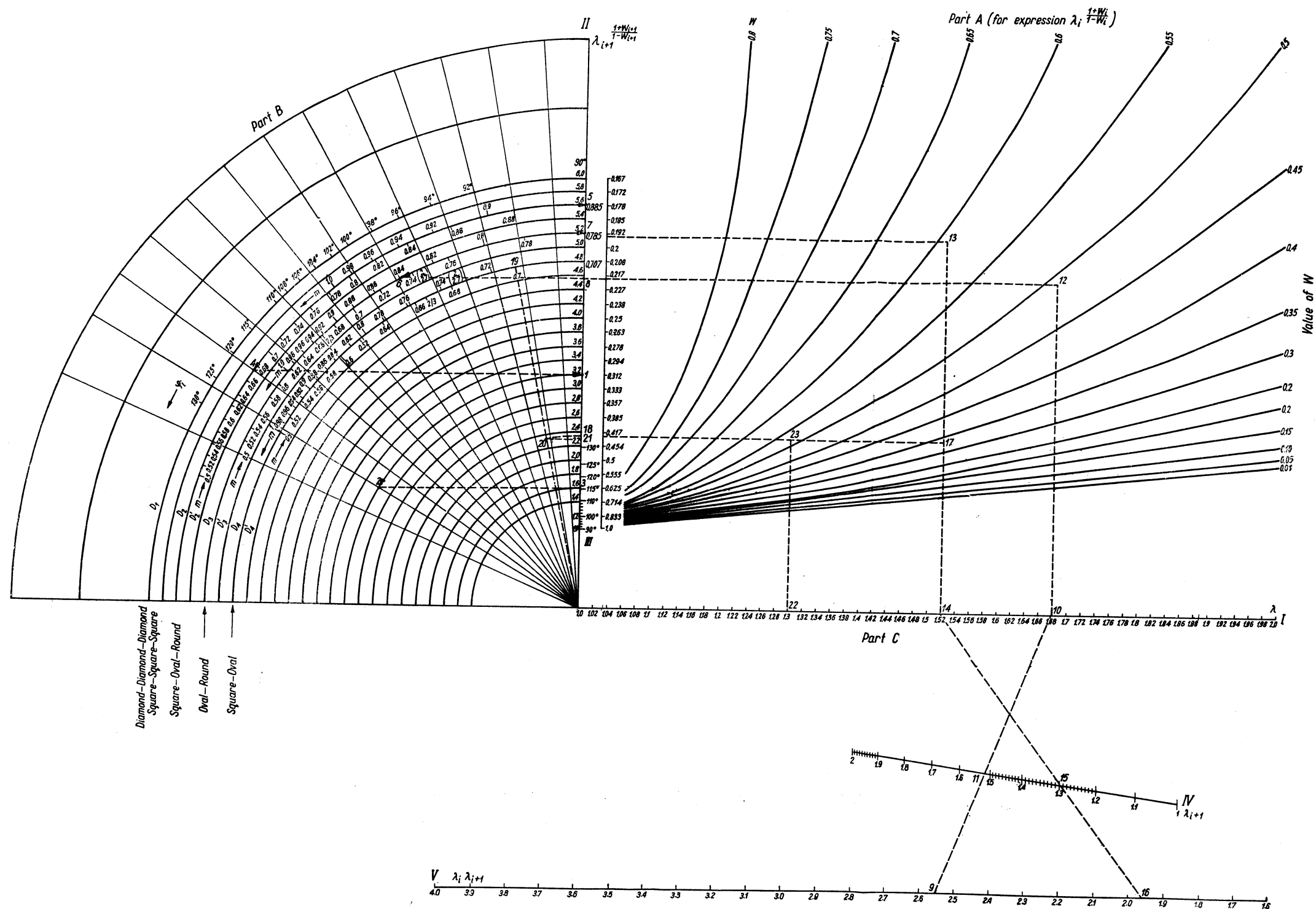


Fig. 7.41. Nomogram for determination of breaking-down systems: square-diamond, oval-round, diamond-diamond and square-square



The breadth  $b_{n-1}$  can be calculated in two ways: from relation

$$b_{n-1} = \frac{F_{n-1}}{h_{n-1}} = \frac{73.9}{8.9} = 8.3 \text{ mm}$$

or from the condition

$$\gamma \beta \lambda = 1$$

### 7.5. Design of Regular Sections and Breaking-down Systems by the Mean Heights Method

The method of simultaneous calculation of all passes for the given pass design should ensure a sequence of passes giving the chosen finished section from the given stock. Other variations are then also possible, e.g. when from a single initial section several different sections are to be rolled with different numbers of passes. For this purpose the mathematical method of simultaneous calculation of all the passes [116] can be applied.

This method does not require the repetition of calculations to check results with assumed values. Also a change of assumptions does not require the repetition of too much calculation. In the way a great economy of computing time can be attained as well as the greatest accuracy.

In this work the method of mean heights is used, i.e. instead of the given section a corresponding rectangle with the same width and area is assumed (Fig. 3.3). Therefore for the height  $h$  of the substituted rectangle the mean height of the profile,  $h_m$ , is taken. Hence three values are given:  $b$ ,  $h_m$  and  $F$ , where  $b h_m = F$ . Equation (2.13), called the elongation condition, ensures the required change of one of them only, namely  $F$ . Hence another equation is needed, which can be found from spread conditions (3.48). The pass design system should be first established, i.e. the sectional shape should be chosen for every pass, thus it must be decided whether it should be a square, a diamond, or an oval of flattened or convex shape. The choice of one shape or another has already been sufficiently discussed [42], and further consideration is not now necessary. It is sufficient to establish for each pass the ratio of mean height to maximum height  $m_i$

$$m_i = \frac{h_m}{h_{i, \max}} \quad (7.54)$$

and to take into account whether the bar between the last and present pass is rotated about its longitudinal axis and if so whether through 45 or 90°. Bar rotation here means rotation of the axis of symmetry relative to the rolling axis.

The method of taking into account these rotations is considered further on when the formulae which govern the required shapes are derived. These formulae were derived first for the simple cases and then generalized [116]. The various known rolling methods are divided into three groups:

(1) The bar being rolled is not rotated about its axis and the rolling plane does not vary. Rolling of flats belongs to this group, if perpendicular edging passes are not used.

(2) The bar being rolled is rotated through 90° after each pass (square–diamond sequence).

(3) The bar being rolled is rotated alternately through 45 or 90° (square–oval sequence).

Considering the rotation or non-rotation of the bar being rolled in successive passes, i.e. the changing of height into width at every pass, the section outgoing from pass  $i$  and the section ingoing to pass  $i+1$  must be carefully differentiated. Therefore the following symbols are used:

$h_i, b_i, \delta_i$  — for the bar outgoing from pass  $i$ ,

$h'_i, b'_i, \delta'_i$  — for the ingoing bar for the next pass  $i+1$ .

When rotation does not take place the symbols do not vary. From the pass design of rapid elongation systems the form factor  $\delta$  from its definition is expressed as

$$\delta_{i-1} = \frac{b_{i-1}}{h_{i-1,m}} = \frac{\text{maximum width after the } (i-1)\text{th pass}}{\text{mean height after the } (i-1)\text{th pass}}$$

for the  $i$ -th pass the width of bar is  $b'_{i-1}$  and height  $h'_{i-1}$ . If there is no  $90^\circ$  rotation before the pass then

$$b'_{i-1} = b_{i-1} \quad \text{and} \quad h'_{i-1,m} = h_{i-1,m}$$

hence

$$\delta'_{i-1} = \frac{b_{i-1}}{h_{i-1,m}} = \delta_{i-1} \quad (7.55)$$

With  $90^\circ$  rotation height becomes a width (Fig. 7.40) and width becomes a height, then

$b'_{i-1} = h_{i-1,\max}$  (the width for the  $i$ -th pass is the maximum height of the  $(i-1)$ th pass)

$h'_{i-1} = b_{i-1}$  (the maximum height before the  $i$ -th pass is the width after the  $(i-1)$ th pass)

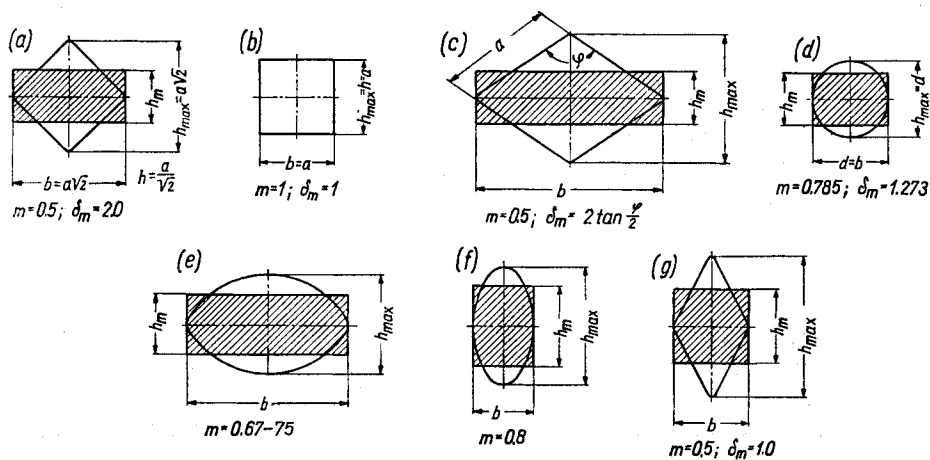


Fig. 7.40. Examples of determining  $m$  and  $\delta_m$  for different types of section [116]

When the mean width before the rotation is known the maximum height can be calculated by means of value  $m$ , established for the given shape of pass and the ratio of mean height to maximum height. Figure 7.40 gives a method of determination of  $m$  for several typical breaking-down passes.

In this case

$$h_{i-1,\max} = \frac{h_{i-1,m}}{m_{i-1}} \quad \text{or} \quad h_{i-1,m} = m_{i-1} h_{i-1,\max}$$



and

$$\delta'_{i-1} = \frac{b'_{i-1}}{h'_{i-1,m}}$$

Substituting for  $h'_{i-1,m}$  and  $b'_{i-1,m}$  the values obtained in  $(i-1)$ th pass

$$\delta'_{i-1} = \frac{h_{i-1,m}}{m_{i-1}^2 b_{i-1}} = \frac{1}{\delta_{i-1} m_{i-1}^2} \quad (7.56)$$

Examples of dimensioning typical shapes of bars and passes are given in Fig. 7.40. For a square which exits on a diagonal, is rotated through  $45^\circ$  and enters the oval on the flat (Fig. 7.40b),  $\delta_i = 1$ .

For a square placed obliquely in the rolls (Fig. 7.40a) with the diagonal parallel to the rolls axis, the width equals the diagonal, and the mean height equals half of the diagonal, hence

$$\delta_i = 2 \quad \text{and} \quad m_i = 0.5$$

A diamond with an apex angle  $\varphi$  and a side  $a$  (Fig. 7.40c) has a width  $b = 2a \sin(\varphi/2)$  and height  $h_m = 2am \cos(\varphi/2)$ .

For diamonds and squares obliquely placed  $m = 0.5$ , hence

$$h = a \cos \frac{\varphi}{2}$$

and

$$\delta_m = 2 \tan \frac{\varphi}{2}$$

A diamond can be divided into four triangles, each of which is replaced by a rectangle with width equal to half the height of the triangle (Fig. 7.40c) hence  $m = 0.5$ . Maximum width and height are calculated for sharp corners, without considering the roundings. The actual  $h_m$  and  $b$  are about 5% smaller. All flats, and squares, placed with side parallel to the roll axis, have  $m = 1$  (Fig. 7.40b).

A round section with diameter  $d$  has  $\delta$  and  $m$  constant (Fig. 7.40d) irrespective of the rotation, since

$$m = \frac{h_m}{d} \quad \text{as} \quad h_{\max} = d$$

The mean height can be obtained by dividing the area by the width, equal in this case to the diameter

$$h_m = \frac{\pi d^2}{4d} = \frac{\pi d}{4}$$

and substituting, for a round section

$$m = \frac{\pi}{4} = 0.785$$

Substituting the appropriate values in place of  $b$  and  $h_m$

$$\delta = \frac{1}{m} = 1.273$$

For ovals, pyramids and gothic squares,  $\delta_i$  and  $m$  depend on their shapes and do not take a constant value [42].

For an oval the actual dimensions are used (Fig. 7.40e) since calculated  $\delta_i$  and  $m$  would deviate too much from the actual ones, and expressions for  $\delta_i$  and  $m$  would take too complicated a form for geometric figures.

In practice the approximate equation for the calculation of flat oval is used

$$F = fbh$$

where  $f$  takes different values [116]. Some authors assume

$$f = \frac{2}{3} \quad \text{and others} \quad \frac{2}{2.7}$$

In this formula the maximum height and width are used, i.e. the assumed  $f$  equals  $m$ . In practice  $m = 2/3$  or  $m = 2/2.7$  is used. Both values are plotted on a convenient scale in the nomogram Fig. 7.41, part (B). The value  $m$  can be also determined from the nomograms in Figs. 3.32, 3.36, 3.37 and 3.38.

In Fig. 7.42 the detailed construction of a gothic square is given. A gothic square is made from a common square by making its sides convex with the radius  $R$  and chamfering the corners. The dimension  $x$  determines the convexity of square sides and is

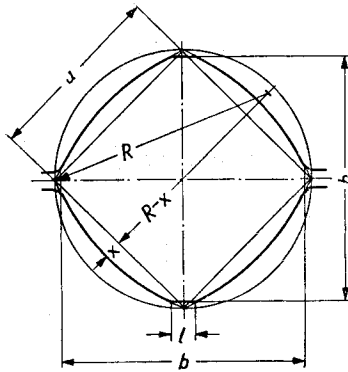


Fig. 7.42. Gothic square construction

chosen by taking into account the spread, so that the outline of the gothic square lies between the outlines of a common square and of a circle.

Hence it is assumed that

$$\left. \begin{aligned} x &= 0.0625a - 0.1a - 0.134a \\ R &= 2.0a - 1.3a - 1.0a \\ R &= 2.83R_1 - 1.84R_1 - 1.42R_1 \end{aligned} \right\} \quad (7.57)$$

where

$a$  = side of square,

$R_1$  = radius of the circle circumscribing the square.

The value of  $R$  is found from equation

$$R = \frac{a^2}{8x} - \frac{x}{2} \quad (7.58)$$

derived from simple geometrical relationships.

The length of the corner edge is fixed at  $l = 0.1a$ . This construction of the gothic square is the simplest, intended to form a system, having the advantages of the oval-round system and simultaneously eliminating its defects or at least reducing them.

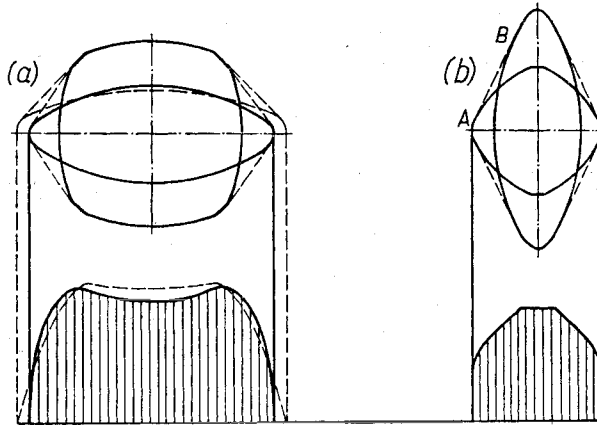


Fig. 7.43. Distribution of local elongations across the pass width of:  
(a) oval passes, (b) gothic square passes

The basic principles of rolling a gothic square in an oval pass and an oval in a gothic square are illustrated in Fig. 7.43, in which the distribution of elongations in these passes is shown. The draught (or elongation) distribution in the oval pass is good. A relatively small deflection appears in the middle of diagram, and almost disappears with the use of a flatter oval, thus the draught distribution is more uniform than in the oval-square sequence, where non-uniformity of draughts in the oval pass is very marked.

The gothic square is caught by the large surface of the oval pass and the middle of the square and its edges are deformed almost simultaneously, thus eliminating the forward slip of edges and hitching on the guide. This type of deformation of gothic square ensures more uniform wear of the oval pass than would be the case with a square pass. Distribution of elongations in the gothic square pass (Fig. 7.43b) is also more convenient than in the common square pass. There is no deflection in the middle of diagram which is so characteristic for a common square. In a gothic square there is room for the displacement of a larger volume of metal into the convex parts. This serves to prevent overfilling of the pass, but at the same time it reduces elongations relative to those obtained in a common square pass. Nevertheless these elongations are greater than those in the round pass which is seen by comparing diagrams for  $\lambda_{i+1} = f(\lambda_i)$ .

The gothic pass shape ensures a smoother metal flow than the square pass thus avoiding the setting up of internal stresses which may appear in the common square, specially at lower temperatures, i.e. in the finishing passes.

The bite of the oval stock by the gothic square avoids the formation of wrinkle marks in the square corners, which can take place in the common square.

Hence the following advantages of the oval-gothic square sequence can be summarized:

(1) rolling with great elongation is possible, both in the oval pass  $\lambda = 1.8-2.0$ , and in the gothic square pass  $\lambda = 1.53$  (greater than in the oval-round sequence),

(2) distribution of draughts and elongations is better than in the oval-square sequence,

(3) elimination of wrinkle marks (a very small chance of their arising, and only when rolling gothic square in an oval),

(4) fluent transition from one section to the next,

(5) uniform temperature distribution throughout the section due to the lack of sharp corners,

(6) continual re-working of the corners of the square.

Disadvantages:

(1) poor oval leading in the gothic square and a possibility of overturning the gothic square in the oval pass, due to its rounded form,

(2) considerable differences between the elongations in the oval pass and gothic square pass.

To determine  $\delta$  for gothic square several calculations are required. The gothic square was assumed with  $R = 1.3a$  hence  $x = 0.1a$  (given in Fig. 7.44).

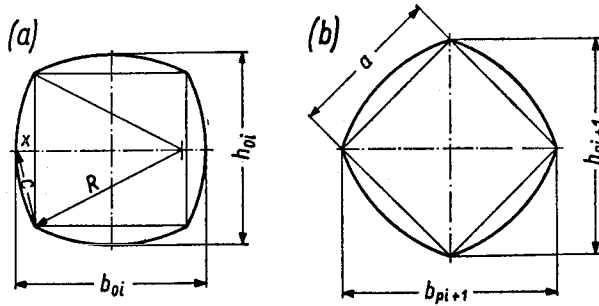


Fig. 7.44. Calculation of form factors: (a) when presenting square to oval, (b) when presenting oval to square

The sectional area was calculated from equation

$$F = a^2 + \frac{4}{15} x(6a + 8c) \quad (7.59)$$

where

$$c = \sqrt{x^2 + \frac{a^2}{4}} = 0.51a$$

$$F = a^2 + \frac{4}{15} 0.1a(6a + 8 \times 0.51a) = 1.2688a^2 \quad (7.60)$$

Thus the mean height equals

$$h_{im} = \frac{F}{b_i} = \frac{1.2688a^2}{1.2a} = 1.057a \quad (7.61)$$

and

$$\delta_{im} = \frac{b_i}{h_{im}} = \frac{1.2a}{1.057a} = 1.138 \quad (7.62)$$

After turning the oval in a square

$$h_{i+1,m} = \frac{F}{b_{i+1}} = \frac{1.2688a^2}{a\sqrt{2}} = 0.898a \quad (7.63)$$

hence

$$\delta'_{i+1} = \frac{b_{i+1}}{h_{i+1}} = \frac{1.4142a}{0.898a} = 1.575 \quad (7.64)$$

The edging pass is here disregarded, its effect is reduced by applying a simplified formula for gothic square area. The joint effect of these two factors is negligible and their omission facilitates the calculations.

For every pass design there are as many unknown quantities as there are passes, i.e. knowing one value in a pass of given shape and size it is possible to calculate all the others. The elongation condition from formula (2.13) allows the calculation of one of the unknowns, the others should be obtained from spread formula (3.48).

$$b_i = b_{i-1}\beta_{i-1} = b_{i-1}\lambda_i^{W_i/(1-W_i)}$$

$$h_{im} = h_{i-1,m}\gamma_i = h_{i-1,m}\lambda_i^{-1/(1-W_i)}$$

and

$$\frac{b'_{i-1}}{h'_{i-1,m}} = \delta'_{i-1}$$

After substituting

$$\delta_i = \delta'_{i-1}\lambda_i^{(1+W_i)/(1-W_i)} \quad (7.65)$$

Formula (7.56) gives the relation between the form factors before and after turning through 90°. If between the  $(i-1)$  th and  $i$ -th passes a turn is made, then instead of  $\delta'_{i-1}$  the right hand part of equation (7.56) is inserted in (7.65) with appropriate indices, and from the dimensions of the  $(i+1)$  th pass  $\delta'_{i+1}$  can be calculated, and from the dimensions of the  $(i-1)$  th pass  $\delta_i$  can be calculated

$$\delta_i = \frac{\lambda_i^{(1+W_i)/(1-W_i)}}{\delta'_{i-1}m_1^2} \quad (7.66)$$

If there is no rotation, from (7.55)

$$\delta_i = \delta'_{i-1}\lambda_i^{(1+W_i)/(1-W_i)}$$

Similarly  $\delta_{i-1}$  is calculated, substituting into equation (7.66)

$$\delta_n = \delta_0 \lambda_1^{\pm(1+W_1)/(1-W_1)} \lambda_2^{\pm(1+W_2)/(1-W_2)} \dots \lambda_n^{\pm(1+W_n)/(1-W_n)} m_1^{\pm 2} m_2^{\pm 2} \dots m_n^{\pm 2} \quad (7.67)$$

The sign of the exponential coefficient depends on the number of 90° rotations made after the considered roll pass. If this is an odd number the negative sign (−) is taken, if even then the positive sign (+) is taken, due to the fact that every rotation switches these values from numerator to denominator and conversely, as in equation (7.66).

The relationship (7.67) is called the shape condition, which must be satisfied to obtain the required size and shape of the finished profile.

To calculate the exponential coefficient  $W$  from equation (3.46), the form factor  $\delta_w$  and roll factor  $\varepsilon_w$  in every roll pass must be known. These values can at first only be assumed and then checked by trial and error. This method would not facilitate the design calculation, therefore it must be correspondingly modified.

In quick breaking-down sequences there are profile shapes in every second roll pass where  $\delta_w$  and  $m$  are known. Therefore the long formula (7.67) can be divided into several formulae, each including only two successive roll passes.

An equation of this type will be considered for different systems. In these considerations subscripts  $i-1$  and  $i+1$  relate to the roll pass with known  $\delta_i$  and  $m$ .

In the square-oval sequence the square enters into the oval in a parallel position, Fig. 7.40b, hence

$$\delta_i = \delta'_{i-1} \lambda_i^{(1-w_d)/(1-w_i)} \quad (7.68)$$

The oval after rotation through  $90^\circ$  (Fig. 7.40f) enters the square along the diagonal (Fig. 7.40a), which is expressed as

$$\delta_{i+1} = \frac{\lambda_{i+1}^{(1+w_{i+1})/(1-w_{i+1})}}{\delta'_{i-1} \lambda_i^{(1+w_d)/(1-w_d)} m_i^2} \quad (7.69)$$

The second equation is given by the elongation condition (2.13) expressed for these two passes.

It is known that  $\delta'_{i-1} = 1$  for a square entering parallel into the pass (Fig. 7.40b) while  $\delta_{i+2} = 2$  for a square entering obliquely into the rolls (Fig. 7.40a). After substituting corresponding values equation (7.69) becomes

$$2m_i^2 \lambda_i^{(1+w_d)/(1-w_d)} = \lambda_{i+1}^{(1+w_{i+1})/(1-w_{i+1})} \quad (7.70)$$

In the oval-round sequence, before the  $i$ -th pass rotation has no significance (Fig. 7.40d) since always  $\delta = 1.273$ . Before the second pass the oval is rotated through  $90^\circ$  (Fig. 7.40f).

The end formula will then take the form

$$(1.273m_i)^2 \lambda_i^{(1+w_d)/(1-w_d)} = \lambda_{i+1}^{(1+w_{i+1})/(1-w_{i+1})} \quad (7.71)$$

Equations (7.71) and (7.70) differ in the numerical factor only. The method of solution can be the same for both equations.

Somewhat different values must be inserted in equation (7.67) for the square-oval-round sequence. For the square entering flat  $\delta = 1$ , for the round  $\delta$  has a constant value 1.273, hence the equation takes the form:

$$1.273^2 m_i^2 \lambda_i^{(1+w_d)/(1-w_d)} = \lambda_{i+1}^{(1+w_{i+1})/(1-w_{i+1})} \quad (7.72)$$

The square-square, square-diamond and diamond-diamond sequences are simpler to design since every pass is placed along the diagonal as in Fig. 7.40a, c, g. After each pass there is a  $90^\circ$  rotation so that the stock is in the diagonal position again. The relation between the form factors is determined by equation (7.66). It is already known that for a diamond

$$\delta = 2 \tan \frac{\varphi}{2}$$

This relation also applies for squares entering the rolls obliquely. Substituting it into equation (7.68) and taking  $m = 0.5$  gives

$$\tan \frac{\varphi_i}{2} \tan \frac{\varphi_{i+1}}{2} = \lambda_i^{(1+w_d)/(1-w_d)} \quad (7.73)$$

To solve this equation (as well as the previous ones)  $\varepsilon_w$  must be assumed. The value  $\lambda_{i-1}$  determines the elongation condition. From the relation (7.73) it can be checked whether  $\varphi$ , the obtuse angle of the diamond, was correctly assumed.

Pass design may now be commenced. The assumption of values of elongation coefficients  $\lambda$  which satisfy the elongation and spread conditions, ensures obtaining from the given stock with the given number of passes a finished profile of required shape and area. First the initial sectional size must be decided. It is a square with a side of  $h_0$ . Next the size of a bar to be obtained from it is given, i.e. area of  $F_n$  and the shape coefficient  $\delta_{n+1}$ . The number of passes is determined by the equation

$$n = \frac{2 \log h_0 - \log F_n}{\log \lambda_m} \quad (7.74)$$

obtained by transformation from formula (2.13)

Next the type of pass for particular roll passes must be decided in order to select for each the appropriate formula for calculation. If the breaking-down sequence square-diamond, square-square or diamond-diamond is chosen, each  $\lambda_i$  must be assumed so that the elongation condition (2.13) is satisfied, from which the apex angles of each successive pass can be calculated. If angles smaller than calculated ones are taken there is a danger of overfilling, or if angles greater than the calculated ones are taken, underfilling occurs. The magnitude of  $\tan \varphi/2$  is thus closely linked to the form factor  $\delta$ . The calculation of the second coefficient  $\varepsilon_w$  when calculating from the preliminary section to the finished section is not difficult.

The design calculations for the breaking-down sequences oval-round or oval-square are more difficult. For these, instead of elongation coefficients in particular roll passes, the products of these coefficients in the roll pass forming the oval section of bar and in the next one,  $\lambda'_i$ , are assumed. These products must satisfy the equation

$$\lambda' = \lambda_i \lambda_{i+1} \quad (7.75)$$

which is a modified elongation condition for products

$$\lambda_i = \frac{F_0}{F_n} = \lambda'_1 \lambda'_2 \dots \lambda'_i \dots \lambda'_n \quad (7.76)$$

Hence two passes are being solved at the same time by means of equations (7.75) and (7.70), (7.71) or (7.72), depending on which pass design method is used.

To solve this system of equations  $\delta_{i-1}$ ,  $\varepsilon_{w,i-1}$ ,  $\lambda_i$  and  $\varepsilon_{wi}$  must be known. The form factor  $\delta_i$  and roll factor  $\varepsilon_w$  for the  $(i-1)$ th roll pass are known, but for the  $i$ -th roll pass they can be calculated when  $\lambda_i$  is assumed. It is known that an oval can fill correctly only one round or square section, therefore in the roll pass from an oval into a square or into a round, for the given oval  $\lambda_{i+1}$  is fixed, once  $\lambda_i$  is assumed. Then  $\lambda_{i+1}$  can be calculated from equation (7.70) or (7.71).

The equation (7.76) is not necessarily also satisfied, and if not then either the assumed  $\lambda'_i$  may be changed so as to satisfy equation (7.75), or leaving  $\lambda'_i$  unchanged  $\lambda'$  must be changed. This second eventuality may lead to unacceptable results in the final stage. Table 7.2 gives a guide for choice of suitable  $\lambda$  values with the assumed number of passes.

For easier solution of the system of equations (7.70) (7.73) and (7.76) a diagram was evolved as shown in Fig. 7.41. This diagram is composed of three parts *A*, *B*, *C*. From part *A*  $\lambda_i^{(1+W_D)/(1-W_D)}$  can be determined with known  $\lambda$ , the values of which are given on scale *I*, and with calculated  $W$ .

$\lambda_i^{(1+W_D)/(1-W_D)}$  is read off from scale *II* for these values. Part *B* is an auxiliary one, to facilitate the solution of equations (7.70)–(7.73). A series of arcuate scales correspond to different breaking-down systems.

The  $D_1$  scale is adapted to solve the equation (7.73) for diamond–diamond, diamond–square and square–square sequences. From a point corresponding to the  $\lambda_i^{(1+W_i)/(1-W_i)}$  value on the scale *II* e.g. 3.2 (point 1) an arc is drawn to the intersection with a radius giving on the  $D_1$  scale the apex angle of the diamond, e.g.  $\angle 127^\circ$  (point 2) or of a square from the  $i$ -th roll pass. A straight line drawn from this point perpendicular to scale *II* determines on it an apex angle of diamond or square in the  $(i+1)$ th roll pass. This value is read off on scale *II* as  $\angle 116^\circ$  (point 3).

Scales  $D_2$  and  $D'_2$  are for square–oval–round sequences,  $D_3$  and  $D'_3$  for oval–round,  $D_4$  and  $D'_4$  for square–oval sequences. They have scales calculated for products of  $m_i^2$  and corresponding  $\delta$ . Since these values are fixed for every sequence on these scales only the corresponding values of  $m$  are drawn.

If the assumed  $m_i$  is in the lower part of scale ( $D'_2$ ,  $D'_3$  or  $D'_4$ ) e.g.  $D_2 \approx 0.68$  (point 4) then from point 1 on scale *II* corresponding to the calculated  $\lambda_i^{(1+W_i)/(1-W_i)}$  a straight line is drawn to intersect with a radius for given  $m$ , returning along the arc to scale *II*, at point 5. If however  $m_i$  is on the upper part of scale ( $D_2$ ,  $D_3$ , or  $D_4$ ) proceeding along the arc (e.g.  $D_3 = 0.84$ , point 6) to the intersection with the radius determining  $m$  from point 7, and back again with a straight line to scale *II*, gives point 8. Point 8 found in this way on scale *II* determines the value of

$$\lambda_{i+1}^{(1+W_{i+1})/(1-W_{i+1})}$$

With known  $W_{i+1}$  and  $\lambda_i$  and  $\delta_i$ , it is not difficult to determine  $\lambda_{i+1}$ . It is found on scale *IV*. In this case, however, two roll passes are considered simultaneously, and they must in addition satisfy the constant volume principle. For this purpose part *C* of diagram 7.41 will be used.

On scale *V* of nomogram point 9 corresponds to assumed  $\lambda'_i = 2.55$ . Joining it with a straight line from  $\lambda_i$  in scale *I*, point 10, the intersection with the middle scale *IV* gives a certain  $\lambda_{i+1}$  at point 11. Its value 1.52 as a rule will be different from that obtained on the  $\lambda = 1.68$  axis. Thus there are two possibilities: the value obtained from the diagram is greater than that obtained from the nomogram or it is lower, here  $1.52 < 1.68$ .

The behaviour of  $\lambda_{i+1}$  with changing  $\lambda_i$  is now analysed. If  $\lambda_i$  is increased  $W_i$  does not change, being independent of  $\lambda_i$ . The  $\delta_{i+1}$  value increases, and the steeper is the run of the  $W_i$  curve close to the intersection point with abscissa  $\lambda_i$ , the more will be the increase. The  $m$  value does not change, but starting from the higher  $\delta_i$  the increase of  $m$  value will be obtained after a repeated passing on the scale *II*.

The increase of  $\lambda_i$  gives a decrease of area in the  $i$ -th roll pass, and hence a smaller value of  $h_i$ . The height also decreases as the breadth increases with growing spread, i.e.  $\varepsilon_{w,i+1}$  decreases as  $\lambda_i$  increases. The exponential coefficient  $W_{i+1}$  decreases with increase of  $\delta_i$  though this decrease is lessened by the  $\varepsilon_{wi}$  increase, which has the opposite effect to  $\delta_{i+1}$ , but not to any great extent especially for ovals with  $\delta > 2.5$ . An explanation of this is given in Fig. 3.18, where  $W = f(\delta, \varepsilon_w)$ .

Increase of  $\delta_i$  and decrease of  $W_{i+1}$  moves  $\lambda_{i+1}$  towards greater values. From the relative positions of possible areas of intersection (scale *I*) with corresponding  $W$ , it can be seen that the  $W_{i+1}$  curve rises more gently than  $W_i$  curve, hence the increase of  $\lambda_{i+1}$  will be greater than that of  $\lambda_i$ . This should be taken into account when correcting  $\lambda_i$ . With decreasing  $\lambda_i$  the changes will be reversed, so that a certain decrease of  $\lambda_{i+1}$  will be found. However, the conclusion that  $\lambda_i$  changes more slowly than  $\lambda_{i+1}$  still holds true.



In this work, for the oval the shape factor  $m$  is taken without any additional definition. This means that the  $(i+1)$ th pass need not be a regular oval, it can be a small pyramid (Fig. 7.8f) or some other similar section. The closer the value of  $m_{i+1}$  to the value which on the scale coincides with the value  $\delta$  the smaller will be differences between  $\lambda_i$  and  $\lambda_{i+1}$ , hence the more completely design possibilities can be exploited. This gives indications for the design of the shape of the  $(i+1)$ th pass.

Roll pass designing proceeds as follows: the larger pass is given with sectional area  $F_{i-1}$ , the parameters of the smaller pass with sectional area  $F_i$  are required. These having been found, the next pass with area  $F_{i+1}$  can be calculated.

For a known  $\lambda_i = 1.68$  (point 10) a vertical is drawn to the intersection with the  $W$  scale for a known  $W_i = 0.49$  (point 12). From here the parallel to scale  $I$  is drawn to the intersection with the scale  $II$  (point 8).

Since  $m = 0.84$  is on the upper scale  $D_3$ , point 8 on scale  $II$  is joined to point 6 on scale  $D_3$ , by a straight line, and returning along the arc to scale  $II$  point 7 is obtained, corresponding to a value of 5.2.

Proceeding horizontally to the  $W$  curves the value  $W_{i+1} = 0.559$  (point 13) is found and the intersection with a vertical drawn for  $\lambda_{i+1} = 1.52$  gives point 14.

The value of  $\lambda_{i+1} = 1.52$  corresponds to the 1.52 value in scale  $IV$ . Hence the problem has been solved correctly.

If the  $\lambda_{i+1}$  values found on scales  $I$  and  $IV$  are different, the previously mentioned correction methods should be used until agreement is reached. If the  $m$  values occur on the lower part of arc, the method is to proceed as for determining  $D_2$  (points 1, 4 and 5).

The characteristics of square-oval, gothic square-oval and round-oval breaking-down sequences may be defined by diagrams for  $\lambda_{i+1} = f(\lambda_i)$ , showing the elongations obtained in adjacent passes of the given design system. These diagrams are worked out from equations (7.65)–(7.69), from which

$$\lambda_{i+1} = (\delta_i \delta'_{i+1} m_i^2 \lambda_i^{(1+W_i)/(1-W_i)})^{(1-W_{i+1})/(1+W_{i+1})} \quad (7.77)$$

These equations were derived from two basic formulae:

- (1) principle of constant volume (2.8)  $\gamma \beta \lambda = 1$
- (2) Wusatowski's spread formula (3.45)  $\beta = \gamma^{-W_i}$

where

$$W_i = 10^{-1.269 \delta_i \varepsilon_w^{0.556}} \quad (7.78)$$

$$W_{i+1} = W_i \delta_i^{-1.444} m_i^{-1.444} \lambda_i^{-(1+0.444 W_i)/(1-W_i)}$$

$\delta_i = b_i/h_m$ ,  
 $m = h_m/h_{\max}$  = profile form factor,  
 $\delta_i$  = form factor before edging pass,  
 $\delta'_{i+1}$  = form factor after edging pass,  
 $\varepsilon_w = h_i/D_i$  index taking into account roll diameter.

This formula gives a relationship between the elongation coefficients of two adjacent passes of the given design system. Its mathematical form is very difficult to analyse, therefore for oval-square, oval-gothic square and oval-round sequences suitable diagrams were evolved, Figs. 7.45, 7.46, 7.47.

The elongation curves were plotted for the following values:

- |                          |                      |                          |              |
|--------------------------|----------------------|--------------------------|--------------|
| (1) Oval-square sequence | $\delta_i = 1$ ,     | $\delta'_{i+1} = 1$ ,    | $m_i = 0.79$ |
| (2) Oval-gothic square   | $\delta_i = 1.138$ , | $\delta'_{i+1} = 1.575$  | $m_i = 0.79$ |
| (3) Oval-round           | $\delta_i = 1.27$ ,  | $\delta'_{i+1} = 1.27$ , | $m_i = 0.79$ |

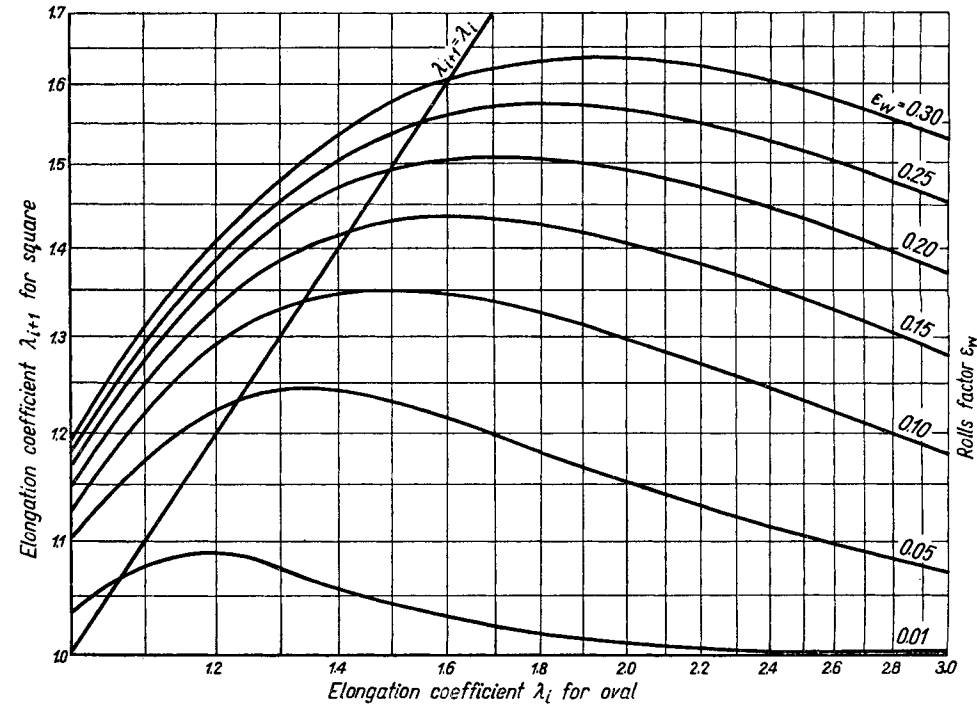


Fig. 7.45. Diagram of elongations in two successive passes of oval-square sequence for various values of  $\epsilon_w$

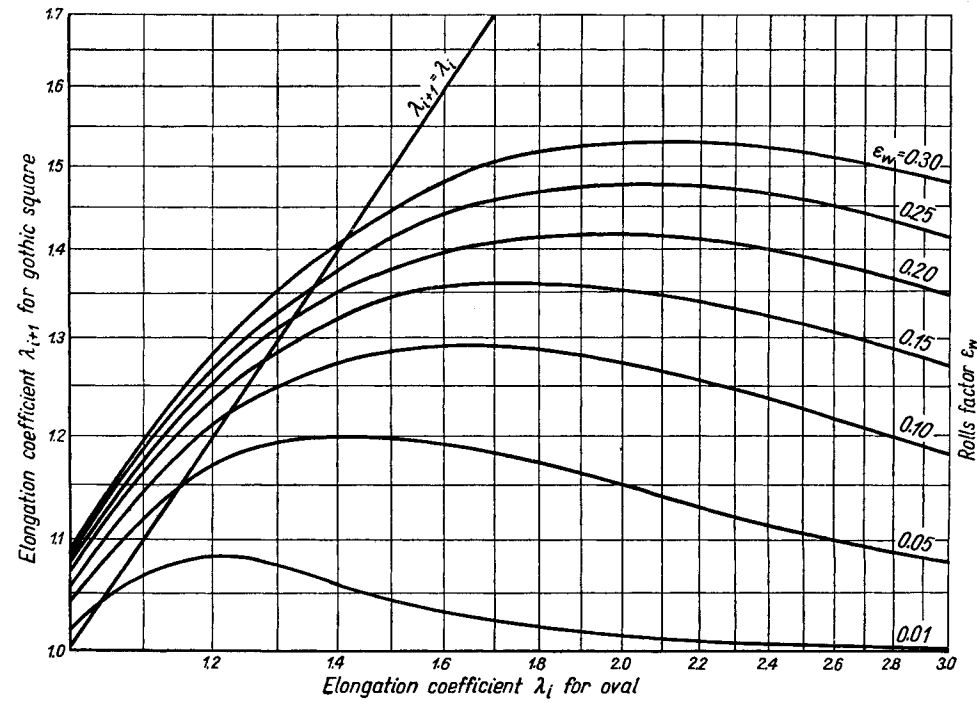


Fig. 7.46. Diagram of elongations in two successive passes of oval-gothic square sequence for various values of  $\epsilon_w$

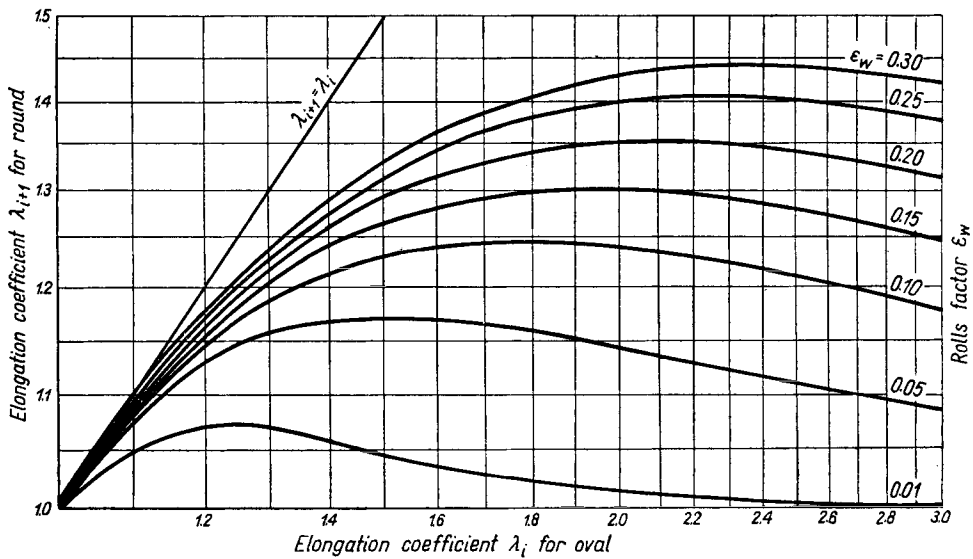


Fig. 7.47. Diagram of elongations in two successive passes of oval-round sequence for various values of  $\varepsilon_w$

For all sequences values of  $\varepsilon_w$  to 0.3 were taken rising by intervals of 0.05. Value  $m_i$  for an oval was determined from Fig. 3.32, taking a mean value.

The form factors  $\delta$  for square and round are easy to determine.

Substituting appropriate values in the formula for  $\lambda_{i+1} = f(\lambda_i)$  curves were obtained for  $\varepsilon_w$  for each of the considered designing systems.

The curves in Figs. 7.45, 7.46, 7.47 have a characteristic feature: they increase to certain maximum values, different for different  $\varepsilon_w$ , and then drop, but less steeply. For each sequence, oval-square, oval-gothic square or oval-round, the maxima of the curves increase and shift to the right as  $\varepsilon_w$  increases.

The maximum elongation values that are practically attainable are:

- |                                          |                      |
|------------------------------------------|----------------------|
| (1) Oval-square sequence, in oval        | $\lambda = 2.0-2.2$  |
| in square                                | $\lambda = 1.6-1.64$ |
| (2) Oval-gothic square sequence, in oval | $\lambda = 1.8-2.0$  |
| in gothic square                         | $\lambda = 1.5-1.53$ |
| (3) Oval-round sequence, in oval         | $\lambda = 1.8$      |
| in round                                 | $\lambda = 1.4-1.43$ |

From the diagrams it can be seen very clearly that the elongations in the oval passes are considerably greater than for the alternate passes of the given sequence.

In the oval-square sequence it is possible to obtain equal or nearly equal elongations in two adjacent passes; the oval-gothic square sequence gives less possibility, and in the oval-round sequence there is no possibility of obtaining equal elongations in adjacent passes.

Hence, the oval-gothic square system gives intermediate elongations, between those of oval-square and oval-round systems.

#### Example

A square  $52 \times 52$  mm is to be rolled in 6 stands of a continuous system to a square  $19 \times 19$  mm. The roll diameters of the middle group are  $D = 350$  mm.

Calculation of total elongation coefficient

$$\lambda_t = \frac{52^2}{19^2} = 7.49$$

As breaking-down passes the square-oval sequence is assumed with equal elongations for the squares entering the ovals with  $m_0 = 0.7$ .

Hence

$$\lambda' = \lambda_{\text{oval}} \lambda_{sq} = \sqrt[3]{\lambda_t} = \sqrt[3]{7.49} = 1.957$$

The cube root is taken since the 6 passes are divided into 3 groups of 2.

Now the sizes and dimensions of the particular square profiles can be calculated. For squares with rounded corners:

ingoing square  $52 \times 52$  mm,  $F_0 \approx 2650$  mm<sup>2</sup>,  $m_{sq} = 0.98$ ,  
 first intermediate square  $37.2 \times 37.2$  mm,  $F_I = 1356.2$  mm<sup>2</sup>,  $m_{sq} = 0.98$ ,  
 second intermediate square  $26.6 \times 26.6$  mm,  $F_{II} = 693$  mm<sup>2</sup>,  $m_{sq} = 0.98$ ,  
 finished square  $19.0 \times 19.0$  mm,  $F_n = 353.8$  mm<sup>2</sup>,  $m_{sq} = 0.98$ .

The area of each square is given by

$$F_I = \frac{F_0}{\lambda'} = \frac{2650}{1.957} = 1356.2 \text{ mm}^2$$

$$F_{II} = \frac{F_I}{\lambda'} = \frac{1356.2}{1.957} = 693 \text{ mm}^2 \text{ etc.}$$

Considering the first pair of passes, the square  $52 \times 52$  mm is rolled in the  $i$ -th roll pass, into an oval and in the  $(i+1)$ th pass into a square of  $37.2 \times 37.2$  mm.

Substituting in formula (7.70) for the square-oval-square sequence

$$2m_i^2 \lambda_i^{(1+W_i)/(1-W_i)} = \lambda_{i+1}^{(1+W_{i+1})/(1-W_{i+1})}$$

On Wojtylak's rule,  $W_i$  is calculated as a function of  $\delta_i$  and  $\varepsilon_{wi}$  as follows:  
 for a square  $52 \times 52$  mm

$$\delta_{i-1} = 1, \quad \varepsilon_{wi} = \frac{h_{i-1}}{D_i} = \frac{52}{326} \approx 0.160$$

for these values

$$D_i = D_t - h_{im} = 350 - 24 = 326 \text{ mm}$$

Calculating from Wojtylak's slide rule (Fig. 7.37) or from diagrams (Fig. 3.18)  $W_i = 0.344$  and hence the exponential coefficient  $(1+W_i)/(1-W_i) = 2.049$ .

Here  $\lambda_i = 1.52$  was taken on scale  $I$ , Fig. 7.41, (point 14) and joined to  $\lambda_i \lambda_{i+1} = 1.957$  on scale  $V$  (point 16), crossing scale  $IV$  to give  $\lambda_{i+1} = 1.3$  (point 15). From  $\lambda_i = 1.52$  in the diagram Fig. 7.41 (scale  $I$ , point 14) a vertical is drawn to the  $W$  curves and  $W_i = 0.344$  (point 17) is found. From point 17 a horizontal is drawn to intersect with scale  $II$ , where the value of 2.36 (point 18) is read off.

This value corresponds to

$$\lambda_i^{(1+W_i)/(1-W_i)} = 1.52^{2.049} = 2.358$$

From point 18 an arc is drawn to intersect with the straight line drawn from  $D_4$  scale (for  $m = 0.7$  and square-oval sequence). This intersection gives point 20. From point 20 a horizontal is drawn to scale  $II$  and the value 2.29 (point 21) is read off. This value satisfies both sides of equation (7.70)

$$2m_i^2 \lambda_i^{(1+W_i)/(1-W_i)} = 2.29 = \lambda_{i+1}^{(1+W_{i+1})/(1-W_{i+1})}$$

Again from the diagram in Fig. 7.41 on scale  $I$  the value  $\lambda_{i+1} = 1.3$  (point 22) is plotted. From 22 a vertical is drawn upwards to intersect with the horizontal 17-18 at point 23. Point 23 gives the required value  $W_{i+1} = 0.52$ . Next

$$\varepsilon_{w, i+1} = \frac{58}{313} \approx 0.185 \text{ is found}$$

It should be taken into account that the oval obtained will be turned and enter horizontally into the square, as in Fig. 7.40f.

For the oval  $h_{im} = 58$  mm is assumed. With  $W_{i+1} = 0.52$  and  $\varepsilon_{m, i+1} = 0.185$  known, from Wojtylak's slide rule (Fig. 7.37) or from nomogram Fig. 3.18  $\delta_i = 0.58$  is determined. This is the form factor for an oval introduced vertically into a square.

Taking into account rotation, from (7.56)

$$\delta'_i = \frac{1}{\delta_i m_i^2} = \frac{1}{0.58 \times 0.7^2} = 3.52$$

$$\delta_i = \frac{b}{h_{im}}$$

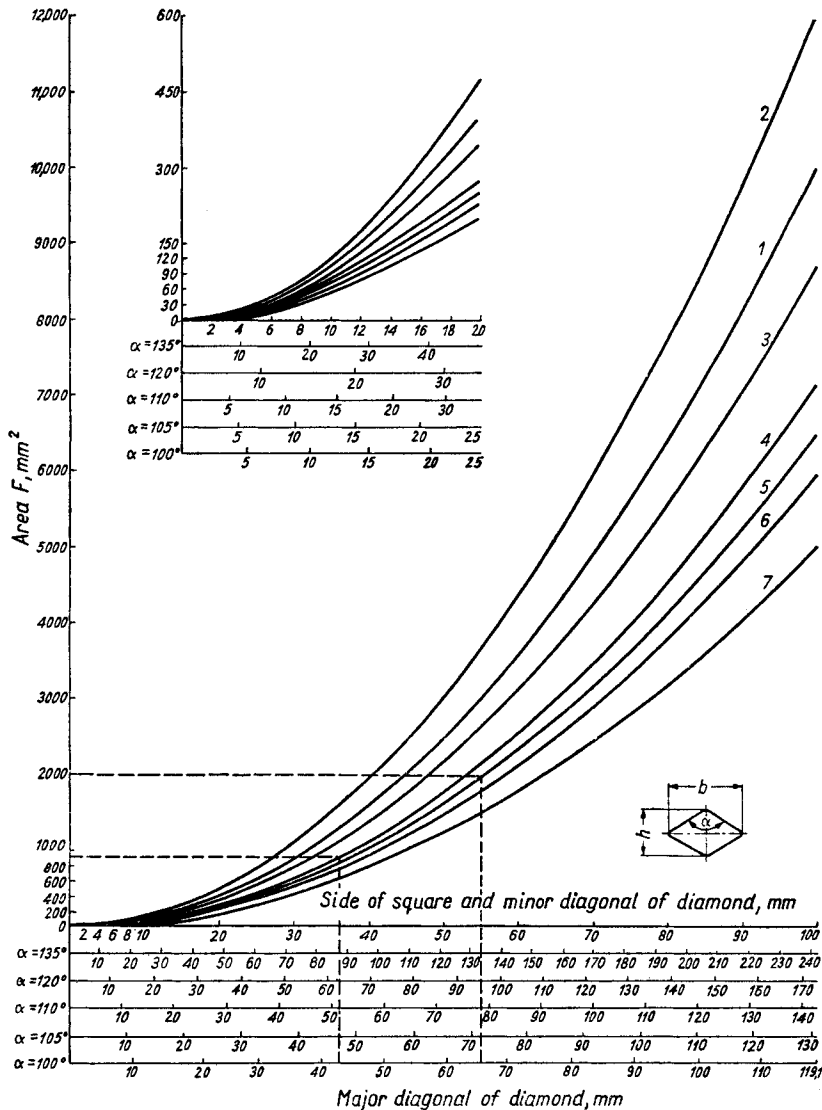


Fig. 7.48. Determination of areas and dimensions of squares and diamonds: 1 — curve for square, 2 — curve for diamond with  $\alpha = 135^\circ$ , 3 — curve for diamond with  $\alpha = 120^\circ$ , 4 — curve for diamond with  $\alpha = 110^\circ$ , 5 — curve for diamond with  $\alpha = 105^\circ$ , 6 — curve for diamond with  $\alpha = 100^\circ$ , 7 — curve for diagonal of square

Assuming

$$F_i = \frac{F_0}{\lambda_i} = \frac{2650}{1.52} = 1743.4 \text{ mm}^2$$

then

$$h_{im} = \sqrt{\frac{F_i}{\delta_i}} = \sqrt{\frac{1743.4}{3.52}} = 22.3 \text{ mm}$$

$$b_i = h_{im} \delta_i = 22.3 \times 3.52 = 78.5 \text{ mm}$$

and

$$h_{i, \max} = \frac{h_{im}}{m_i} = \frac{22.3}{0.7} = 31.9 \text{ mm}$$

The calculated values of the oval  $h_{i, \max} = 31.9$  and  $b_i = 78.5$  agree well with those met in practice.

The calculated  $\lambda_i$  and  $\lambda_{i+1}$  and oval dimensions obtained from them are correct.

In the further roll passes the roll diameter will remain constant but the heights will decrease, with a corresponding decrease of  $\varepsilon_w$ . This causes a spread increase hence the further ovals must be slimmer, i.e.  $\delta_i$  must be greater than calculated in this example.

In order to avoid the recalculation of sides of the square and diamond Fig. 7.48 is used. On the ordinate are the sectional areas in  $\text{mm}^2$ , on the abscissa are several scales. The first scale refers to the side of the square and the shorter to the diagonal of the diamond, the others give the longer diagonal of the diamond relative to its apex angle. With known area and apex angle of the diamond a horizontal is drawn to intersect with the corresponding curve, and the vertical to the abscissa gives on the first scale the shorter diagonal of the diamond, and on the corresponding further scale the longer diagonal of the diamond, e.g.  $F = 935 \text{ mm}^2$ ,  $\sphericalangle = 110^\circ$ ,  $h = 36 \text{ mm}$ ,  $b = 52 \text{ mm}$ .

## 7.6. Theoretical Principles of Section Pass Design

Sections are defined as rolled products of considerable length and constant cross-section in the form of a closed geometric figure where the tangent to, or the elongation of at least one side cuts the section. Regular sections are those which have at least one axis of symmetry. Amongst these are: equal leg angle sections, T-sections, channels and I-sections. The mean height of section or part of section is determined by replacing with an equivalent rectangle [123, 126 and 129].

The method of spread determination has been described when considering the principles of pass design for particular shapes.

The mean draught coefficient (3.4) is (Figs. 3.3b and 7.49)

$$\gamma_m = \frac{h_{2m}}{h_{1m}} = \frac{F_2}{F_1} \frac{b_1}{b_2}$$

where

$F_1$  = sectional area before roll pass,

$b_1$  = greatest sectional width before roll pass,

$F_2$  = sectional area after roll pass,

$b_2$  = greatest sectional width after roll pass.

In Fig. 7.49

$$h_{1m} = \frac{F_1}{b_1}$$

and in general

$$F = h_m b$$

where  $b$  is the largest width up to which a pass will be filled (Fig. 7.49).

In such cases, as well as the elongation conditions, further relations linking the height or width ratios are required, if the pass is to be correctly filled.

As an example the behaviour of a section made up of two rectangular bars *A* and *B* joined as in Fig. 7.50 is considered.

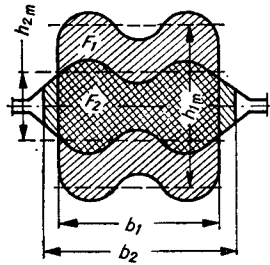


Fig. 7.49. Determination of mean height of section

Assuming that the two parts are exposed to different draughts where

$$\frac{h_{A2}}{h_{A1}} < \frac{h_{B2}}{h_{B1}}$$

If both parts are rolled separately (Fig. 7.50) then the perpendicular part *A* will show less elongation than part *B*. Rolling both parts together, *A* will retard the elongation of *B* depending on the relative masses, while part *B* will drag along the less elongated part *A*. As a result the bar receives an intermediate elongation (Fig. 7.50c). The

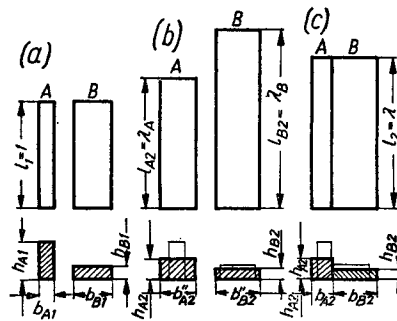


Fig. 7.50. Rolling of two-part section: (a) both bars before roll pass, (b) two bars rolled separately, (c) both bars rolled together

amount of metal, corresponding to the length by which the elongation of retarded part has been curtailed, causes its spread to increase. If the spread is limited by roll pass contour, the metal will flow to the adjacent part of the pass or will flow out as a fin. Depending on the masses, the part with less elongation *A* will undergo less spread than if it were rolled independently, since the amount of metal which should give a higher spread is pulled along by the elongated part to even up the length.

This kind of metal flow is nearly always observed when rolling regular sections. Cases in which, for example, web and flanges of T or I-sections have the same elongations are very rare. Usually the various parts of the section have different elongations,

then evening up of elongations will take place or associated with it metal flow from flanges to web or vice versa.

It is important in roll pass designing to determine the mean elongation coefficient in order to ensure correct filling of the next pass. This coefficient is defined as the average of coefficients calculated for free elongation of the several parts of the section. In calculating these coefficients the whole section is divided into parts with approximately the same draught across their breadth. The free elongation coefficients can be calculated from equation  $\lambda = 1/\beta\gamma$  where the draught and spread coefficients are known. The free elongation coefficient can be calculated directly from formula (3.47) without knowing the spread coefficient

$$\lambda = \gamma^{-(1-W)}$$

In section rolling the problem of metal flow is still more complicated. The correct definition of mean height and spread is not sufficient, but other additional factors must be taken into account:

- (1) calculation of elongation and free spread of open parts of the section,
- (2) correct determination of mean elongation coefficient of the whole section,
- (3) determination of the amount of metal flow from one part of section to another during rolling.

When rolling certain simpler sections it is sometimes sufficient to proceed as for rolling of regular sections, i.e. to determine the mean draught coefficient. This method is also used to calculate the metal flow in the parts of section where free elongation or spread occurs.

Assuming a shape of sectional area

$$F_1 = F_{A1} + F_{B1} + F_{C1} \quad (7.79)$$

which after deformation takes the form shown schematically in Fig. 7.51

$$F_2 = F_{A2} + F_{B2} + F_{C2} \quad (7.80)$$

As component parts of the section are assumed those parts that have more or less even draught over the whole width (Fig. 7.51).

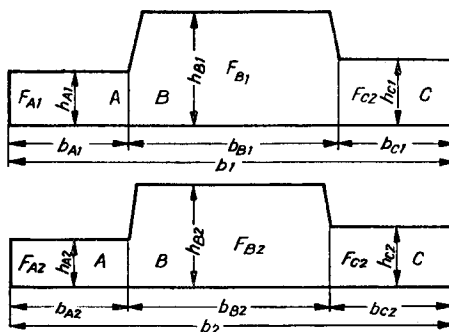


Fig. 7.51. Section of a simple shape composed of parts *A*, *B*, *C* [129]

It is assumed that the section in Fig. 7.51 is rolled with entirely free spread.

Then for the particular parts of section  $F_{A1}$ ,  $F_{B1}$ ,  $F_{C1}$  the following equations may be stated (see equation 3.45):

$$\beta_A = \gamma_A^{-W_A}, \quad \beta_B = \gamma_B^{-W_B}, \quad \beta_C = \gamma_C^{-W_C} \quad (7.81)$$



Expressing this for the whole section

$$\beta_t = \frac{b_{2t}}{b_{1t}} = \beta_A \beta_B \beta_C$$

For the whole section the constant volume principle must be satisfied, hence

$$\gamma_t \beta_t \lambda_t = 1$$

A section will now be considered similar to that in Fig. 7.51 but composed of  $N$  parts.

The volume of the section before and after deformation should remain fixed; then the product of the total coefficients of deformation must be also 1, according to equation (2.8).

The particular total deformation coefficients of a section may be expressed as the product of deformation coefficients of several components of a given section, then  $\gamma_t \beta_t \lambda_t = 1$ , similarly as in equations (2.20)–(2.22) [129]

$$\left. \begin{aligned} \gamma_{tw} &= \gamma_A \gamma_B \gamma_C \dots \gamma_I \dots \gamma_N \\ \beta_{tw} &= \beta_A \beta_B \beta_C \dots \beta_I \dots \beta_N \\ \lambda_{tw} &= \lambda_A \lambda_B \lambda_C \dots \lambda_I \dots \lambda_N \\ \downarrow &\quad \downarrow \downarrow \downarrow \quad \downarrow \quad \downarrow \\ 1 &\quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \end{aligned} \right\} \quad (7.82)$$

Products:

From equation (7.82) it may be seen that the product of the respective deformation coefficients for each part of the section is 1.

The subscript  $w$  indicates that the relation refers exclusively to one pass. Formulae in this form would represent the changes in the section components considered independently of each other. This however is not true, since all the components are closely integrated and after rolling a mean deformation of the whole section is obtained. The equation (7.82) will be valid in this form with some entirely free manner of deformation of the shape (Fig. 7.51).

It is known that if when rolling a certain amount of metal flows out from one part, changing the deformation coefficients relative to (7.82), this amount of metal flows to an adjoining part or parts and changes the deformation coefficients in such a way that the constant volume principle is true for the whole section. Hence if one part of the section undergoes a change, another part must change in an exactly opposite sense. Hence instead of equation (7.82) for metal flow in component parts of the section the equation should be written [129]

$$\left. \begin{aligned} \gamma_{tp} &= \gamma_{Ap} \gamma_{Bp} \gamma_{Cp} \dots \gamma_{Np} \\ \beta_{tp} &= \beta_{Ap} \beta_{Bp} \beta_{Cp} \dots \beta_{Np} \\ \lambda_{tp} &= \lambda_{Ap} \lambda_{Bp} \lambda_{Cp} \dots \lambda_{Np} \\ \downarrow &\quad \downarrow \downarrow \downarrow \quad \downarrow \quad \downarrow \\ 1 &\quad 1 \quad 1 \quad 1 \quad 1 \end{aligned} \right\} \quad (7.83)$$

where subscript  $p$  indicates that the given coefficient is for metal flow out in a section from part to part, hence it is a real deformation coefficient.

Figure 7.52 shows schematically the rolling of a shape composed of parts  $A$  and  $B$ . After deformation an amount of metal  $F_x$  has flowed out from part  $A$  to part  $B$ .

From equations (2.8) and (7.82) for a shape composed of two parts  $A$  and  $B$  the following relations are obtained as in Fig. 7.52 [129]

$$\gamma_{ip}\beta_{ip}\lambda_{ip} = \gamma_A\gamma_B\beta_{Ap}\beta_{Bp}\lambda_{Ap}\lambda_{Bp} = 1 \quad (7.84)$$

and for a shape composed of three parts  $A, B, C$

$$\gamma_A\gamma_B\gamma_C\beta_{Ap}\beta_{Bp}\beta_{Cp}\lambda_{Ap}\lambda_{Bp}\lambda_{Cp}\gamma_{ip}\beta_{ip}\lambda_{ip} = 1 \quad (7.85)$$

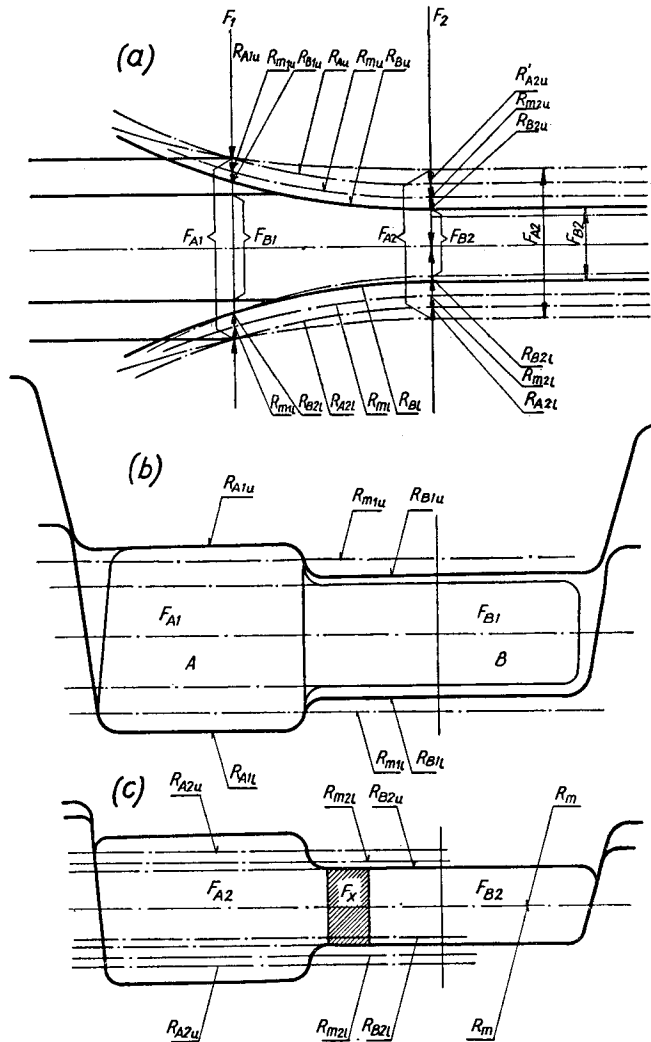


Fig. 7.52. Rolling of section composed of parts  $A$  and  $B$ . From part  $A$  an amount of metal  $F_x$  flows to part  $B$ : (a) longitudinal section, (b) cross-section before pass, (c) cross-section after pass

The subscript  $p$  may be omitted for the  $\gamma$  coefficients, since the value of  $\gamma$  does not vary with or without metal flow.

For sections composed of two components, similarly the equations (7.83) can be written

$$\left. \begin{aligned} \gamma_i &= \gamma_A \gamma_B = \gamma_{mw}^2 \\ \beta_{ip} &= \beta_{Ap} \beta_{Bp} = \beta_{mw}^2 \\ \lambda_{ip} &= \lambda_{Ap} \lambda_{Bp} = \lambda_{mw}^2 \end{aligned} \right\} \quad (7.86)$$

If the section is composed of two symmetrical components (e.g. an equal leg angle section, Fig. 7.53), the pairs of coefficients should be equal

$$\gamma_A = \gamma_B$$

$$\beta_A = \beta_B$$

$$\lambda_A = \lambda_B$$

In such case equation (7.86) is satisfied immediately. If these coefficients are different then it is known that after deformation the profile as a whole must obey the constant volume principle, hence if the value of one coefficient increases the value of

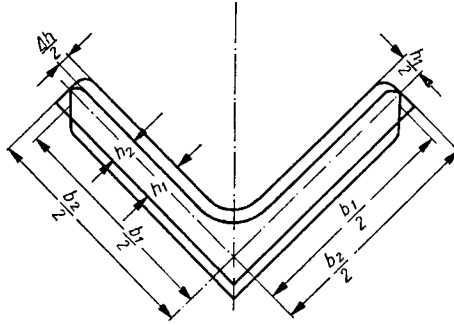


Fig. 7.53. Equal-leg angle as example of section with  $F_A = F_B$

the other must decrease appropriately; the mean value of the coefficient, however, does not change.

Then the equations can be written

$$\left. \begin{aligned} \gamma_m &= \sqrt{\gamma_A \gamma_B} \\ \beta_m &= \sqrt{\beta_{Ap} \beta_{Bp}} \\ \lambda_m &= \sqrt{\lambda_{Ap} \lambda_{Bp}} \end{aligned} \right\} \quad (7.87)$$

From equations (7.86) and (7.87) it can be seen also that

$$\gamma_{mw} \beta_{mw} \lambda_{mw} = \sqrt{\gamma_A \gamma_B} \sqrt{\beta_{Ap} \beta_{Bp}} \sqrt{\lambda_{Ap} \lambda_{Bp}} = 1$$

Similarly, for profiles composed of  $N$  parts the formulae (7.87) take a more general form

$$\left. \begin{aligned} \gamma_m &= \sqrt[N]{\gamma_A \gamma_B \gamma_C \dots \gamma_I \dots \gamma_N} \\ \beta_m &= \sqrt[N]{\beta_{Ap} \beta_{Bp} \beta_{Cp} \dots \beta_{Ip} \dots \beta_{Np}} \\ \lambda_m &= \sqrt[N]{\lambda_{Ap} \lambda_{Bp} \lambda_{Cp} \dots \lambda_{Ip} \dots \lambda_{Np}} \end{aligned} \right\} \quad (7.88)$$

These relations (7.86) and (7.87) facilitate the calculation of particular coefficients e.g.

$$\lambda_{Ap} = \frac{\lambda_m^2}{\lambda_{Bp}} \quad (7.89)$$

and for a profile composed of three elements

$$\lambda_{Ap}\lambda_{Bp}\lambda_{Cp} = \lambda_m^3$$

hence

$$\lambda_{Ap} = \frac{\lambda_m^3}{\lambda_{Bp}\lambda_{Cp}}$$

Similarly coefficients  $\lambda_{Cp}$  and  $\lambda_{Bp}$  and coefficients  $\gamma$  and  $\beta$  can be expressed in this form.

In a previous work the author derived formulae to calculate mean elongation coefficients of a section composed of  $N$  parts

$$\lambda_{mw} = \frac{\sum_{I=1}^{I=N} F_{I2} \lambda_I}{\sum_{I=1}^{I=N} F_{I2}} \quad (7.90)$$

For two elements this formula takes the form

$$\lambda_{mw} = \frac{F_{A2} \lambda_A + F_{B2} \lambda_B}{F_{A2} + F_{B2}} \quad (7.91)$$

The subscript  $w$  (pass groove) is introduced in order to distinguish  $\lambda_{mw}$  from  $\lambda_m$  calculated from equation (2.13). In these formulae:

$F_{A2}$  = the area of part  $A$  after roll pass (Fig. 7.52),

$F_{B2}$  = the area of part  $B$  after roll pass (Fig. 7.52),

$\lambda_{mw}$  = the mean pass elongation coefficient,

$\lambda_A$  = the elongation coefficient of part  $A = F_{A1}/F_{A2}$ ,

$\lambda_B$  = the elongation coefficient of part  $B = F_{B1}/F_{B2}$ ,

$R_A$  = the mean effective radius of part  $A$ ,

$R_B$  = the mean effective radius of part  $B$ ,

$R_m$  = the mean effective radius of a pass.

$F_{I2}$  in the formula (7.90) indicates the area of some component part of the section after deformation, i.e. its true dimensions.

Formulae in this form were derived independently by the author [54], and J. Górecki [127].

A detailed analysis [128] showed that formulae (7.90) and (7.91) have the basically correct form, therefore they will be used from now on.

Formulae (7.88) and (7.89) give large errors and they are not recommended for practical calculations.

Besides the formulae for mean elongation coefficient (7.90) and (7.91) already given, another basic relationship is also known in the form

$$\lambda_{mw} = \frac{F_2}{F_1} = \frac{F_{A1} + F_{B1} + F_{C1} + \dots + F_{I1} + \dots + F_{N1}}{F_{A2} + F_{B2} + F_{C2} + \dots + F_{I2} + \dots + F_{N2}} \quad (7.92)$$

Since to calculate  $\lambda_{mw}$  two formulae of different form are known, (7.90) and (7.92), the author found the relation between them. The unknown quantities are:  $\lambda_{mw}$ ,  $\lambda_A$

and  $\lambda_B$ . As equation (7.92) does not express the relation between  $\lambda_A$ ,  $\lambda_B$  and  $\lambda_N$  it is suitably transformed. First a section with two components  $A$  and  $B$  is considered, then equation (7.90) takes the form

$$\lambda_{mw} = \frac{F_{A2}\lambda_A + F_{B2}\lambda_B}{F_{A2} + F_{B2}}$$

and formula (7.92) for two components becomes

$$\lambda_{mw} = \frac{F_1}{F_2} = \frac{F_{A1} + F_{B1}}{F_{A2} + F_{B2}} \quad (7.93)$$

Since

$$F_{A1} = \lambda_A F_{A2} \quad (7.94)$$

$$F_{B1} = \lambda_B F_{B2} \quad (7.95)$$

substituting these values in equation (7.93)

$$\lambda_{mw} = \frac{F_{A2}\lambda_A + F_{B2}\lambda_B}{F_{A2} + F_{B2}}$$

It is seen that equations (7.91) and (7.93) are identical.

Besides formulae to calculate  $\lambda_{mw}$ , formulae to calculate  $F_x$ , the amount of metal flowing from one part of pass to another, will be used (Figs. 7.52 and 7.54).

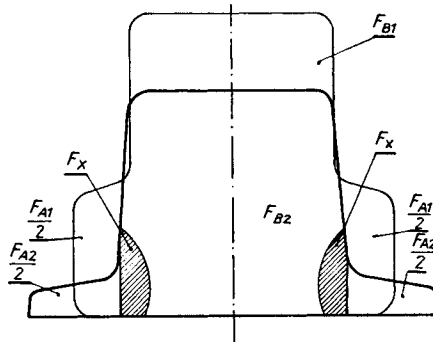


Fig. 7.54. Metal flow from one part of section to another. Thin line—ingoing profile, thick line—pass shape,  $F_x$ —amount of metal flowing out from part  $A$  [127]

J. Górecki's formulae [127] will be applied, as being most convenient [52 and 130].

For  $\lambda_A > \lambda_B$

$$F_{xA} = F_{A2} \left( \frac{\lambda_A}{\lambda_{mw}} - 1 \right) \quad (7.96)$$

or  $\lambda_A > \lambda_B$

$$F_{xB} = F_{B2} \left( 1 - \frac{\lambda_B}{\lambda_{mw}} \right) \quad (7.97)$$

For  $\lambda_B > \lambda_A$

$$F_{xA} = F_{A2} \left( 1 - \frac{\lambda_A}{\lambda_{mw}} \right) \quad (7.98)$$

or  $\lambda_B > \lambda_A$

$$F_{xB} = F_{B2} \left( \frac{\lambda_B}{\lambda_{mw}} - 1 \right) \quad (7.99)$$

and a further series of relations.

For  $\lambda_A > \lambda_B$

$$F_{xA} = F_{A1} \left( \frac{\lambda_A - \lambda_{mw}}{\lambda_A \lambda_{mw}} \right) \quad (7.100)$$

or  $\lambda_A > \lambda_B$

$$F_{xB} = F_{B1} \left( \frac{\lambda_{mw} - \lambda_B}{\lambda_{mw} \lambda_B} \right) \quad (7.101)$$

For  $\lambda_B > \lambda_A$

$$F_{xA} = F_{A1} \left( \frac{\lambda_{mw} - \lambda_A}{\lambda_{mw} \lambda_A} \right) \quad (7.102)$$

or  $\lambda_B > \lambda_A$

$$F_{xB} = F_{B1} \left( \frac{\lambda_B - \lambda_{mw}}{\lambda_B \lambda_{mw}} \right) \quad (7.103)$$

In general, formulae (7.96) (7.103) can be written:

for  $\lambda_N > \lambda_{mw}$

$$F_{xN} = F_{N2} \left( \frac{\lambda_N}{\lambda_{mw}} - 1 \right) \quad (7.104)$$

or for  $\lambda_{mw} > \lambda_N$

$$F_{xN} = F_{N2} \left( 1 - \frac{\lambda_N}{\lambda_{mw}} \right) \quad (7.105)$$

To determine the amount and direction of metal flow transfer simple indices  $k$  and  $k'$  should be taken, [129] such that:

|                                              |                                      |                                                  |
|----------------------------------------------|--------------------------------------|--------------------------------------------------|
| $k = \frac{\lambda_A}{\lambda_{mw}}$         | $k$ may take values                  |                                                  |
|                                              | $k > 1$<br>metal flows from part $A$ | or $k < 1$<br>metal flows to part $A$ (7.106)    |
| and<br>$k' = \frac{\lambda_B}{\lambda_{mw}}$ | $k'$ may take values                 |                                                  |
|                                              | $k' < 1$<br>metal flows to part $B$  | or $k' > 1$<br>metal flows from part $B$ (7.107) |

Then

$$k = 1/k' \quad (7.108)$$

If  $k = k' = 1$  there is no metal flow over.

These relations can be used to determine flow of metal from part  $A$  to part  $B$  in the following manner:

$$k = \frac{\lambda_A}{\lambda_{mw}} = \frac{F_{A1} l_1}{F_{A2} l_2} = \frac{V_{A1}}{V_{A2}} \quad (7.109)$$

From this

$$V_{xA} = V_{A1} - V_{A2} \quad (7.110)$$

and similarly

$$k' = \frac{\lambda_B}{\lambda_{mw}} = \frac{F_{B1}}{F_{B2}} \frac{l_1}{l_2} = \frac{V_{B1}}{V_{B2}} \quad (7.111)$$

and

$$V_{xB} = V_{B1} - V_{B2} \quad (7.112)$$

### 7.6.1. PASS DESIGN METHODS FOR SECTIONS COMPOSED OF TWO ELEMENTS

Figure 7.55 shows a profile of cross-section  $F_2 = F_{A2} + F_{B2}$  with dimensions  $h_{A2}$ ,  $b_{A2}$  and  $h_{B2}$ ,  $b_{B2}$ .

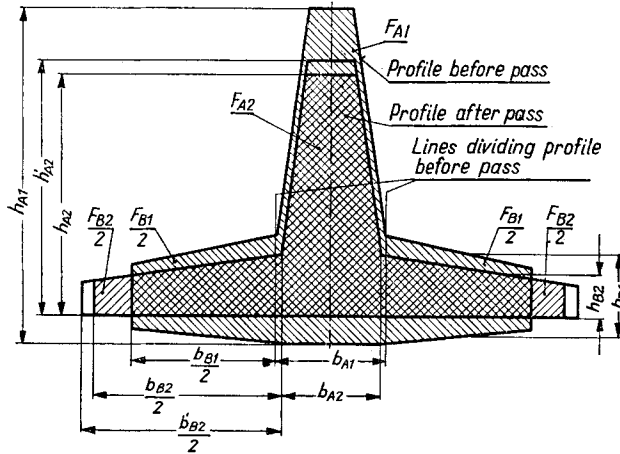


Fig. 7.55. Rolling of simple shape formed of two elements

The required dimensions of ingoing section and its total cross-section are:  $F_1 = F_{A1} + F_{B1}$  and  $F_{A1}$ ,  $h_{A1}$ ,  $b_{A1}$ ,  $F_{B1}$ ,  $h_{B1}$ ,  $b_{B1}$ , i.e. 7 unknowns altogether.

These can be reduced to 4 since there are additional relations between area and its dimensions

$$F_{A1} = h_{A1} b_{A1} \quad (7.113)$$

$$F_{B1} = h_{B1} b_{B1} \quad (7.114)$$

$$F_1 = F_{A1} + F_{B1} \quad (7.115)$$

$h_{A1}$  and  $h_{B1}$  are mean heights usually calculated as the sectional area divided by the filled breadth, thus  $h_{A1m} = F_{A1}/b_{A1}$ . For simplicity, symbol  $h_{A1}$  is used to indicate  $h_{A1m}$ . Knowing two dimensions of a part of profile, the third one can be calculated.

Symbols  $A$  and  $B$  have so far been used quite arbitrarily. The area  $F_1 = F_{A1} + F_{B1}$  is found assuming a value of  $\lambda_{mw1}$  which satisfies the general equation (2.13) for the chosen pass design system of  $n$  roll passes, then

$$\lambda_t = \lambda_{m1}^n = \lambda_{mw1} \lambda_{mw2} \lambda_{mw3} \dots \lambda_{mwi} \dots \lambda_{mwn}$$

Assuming  $\lambda_{mw2}$  for a chosen pass designing system, the following relation can be expressed:

$$F_1 = F_{A1} + F_{B1} = F_2 \lambda_{mw2} \quad (7.116)$$

Since the section is composed of two parts with unequal elongations, the additional equation (7.91) must be satisfied

$$\lambda_{mw2} = \frac{F_{A2} \lambda_A + F_{B2} \lambda_B}{F_{A2} + F_{B2}}$$

Here  $\lambda_{mw}$  indicates the mean elongation coefficient of the pass, to distinguish it from that obtained from equation (2.16) where

$$\lambda_m = \sqrt[n]{\lambda_t}$$

Thus two further unknowns  $\lambda_A$  and  $\lambda_B$  are introduced, so that there are 6 real unknowns

$$F_{A1}, \lambda_A, b_{A1}, F_{B1}, \lambda_B, b_{B1}$$

They must satisfy equations (7.90)–(7.95)

$$\lambda_{mw} = \frac{F_1}{F_2} = \frac{F_{A1} + F_{B1}}{F_{A2} + F_{B2}}$$

which corresponds to equation (7.91)

There are additional relations between profile elements

$$\lambda_A F_{A2} = F_{A1}$$

$$\lambda_B F_{B2} = F_{B1}$$

therefore the real number of unknowns is reduced to 4

$$\begin{array}{l} F_{N1} \quad \text{or} \quad \lambda_A \quad \text{and} \quad b_{A1} \\ F_{B1} \quad \text{or} \quad \lambda_B \quad \text{and} \quad b_{B1} \end{array}$$

If one of coefficients  $\lambda_A$  or  $\lambda_B$  is assumed, the number of unknowns does not change, since  $\lambda_{mw}$  will be then required as well as the other coefficient e.g.  $\lambda_B$ .

The following three cases are considered:

$$\lambda_A > \lambda_{mw} > \lambda_B \quad (7.117)$$

$$\lambda_A \approx \lambda_{mw} \approx \lambda_B \quad (7.118)$$

$$\lambda_A < \lambda_{mw} < \lambda_B \quad (7.119)$$

With the condition (7.118) the elongation coefficients of both parts are equal, hence there is no metal flow from one part to another.

The case where relation (7.117) holds will now be considered, i.e.

$$\lambda_A < \lambda_{mw} < \lambda_B$$

In a pass design for a simple section, there are theoretically three different cases (Fig. 7.56) [129]:

- (1) Part  $A'$  has completely restricted spread, part  $B$  has free spread.
- (2) Parts  $A$  and  $B$  have partly restricted spread.
- (3) Parts  $A$  and  $B$  have free spread.



Other cases are not considered, e.g. where both parts have completely or partly restricted spread, since these conditions can only refer to closed part C.

Partial or completely restricted spread of both parts makes impossible the free passing of metal from one part to another. This surplus cannot be accommodated in

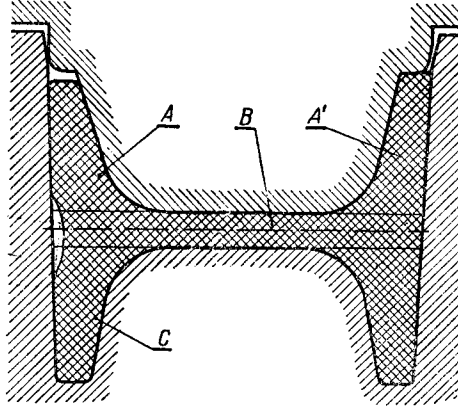


Fig. 7.56. Parts of shape section: *A* — with free spread, *B* — with partly restricted spread, *A'* and *C* — with totally restricted spread

the pass, and causes overfill of metal from one or both sides of the pass. Therefore these are not suitable designing methods.

In practice cases 2 and 3 are met with most often.

Considering the first case, since the spread of part *A'* is completely restricted then  $b_{A1} \approx b_{A2}$ , and simplified equations are found

$$\lambda_A = \frac{h_{A1}}{h_{A2}} = \frac{F_{A1}}{F_{A2}} = \frac{1}{\gamma_A} \quad (7.120)$$

As  $h_{A2}$  is known, formula (7.120) gives  $\lambda_A$  and basic dimensions of part *A'*.

In a pass where (7.117) holds good, a certain amount of metal  $F_x$  flows from part *A'* to *B*, given by formula (7.96)

$$F_{xA2} = F_{A2} \left( \frac{\lambda_A}{\lambda_{mw}} - 1 \right)$$

Since  $\lambda_A > \lambda_{mw}$  then  $F_{xA2}$  has (+) sign, which indicates that part *A'* is yielding metal, that is part *B* will increase by this amount relative to its size before roll pass, i.e. the area will increase or decrease by  $F_x$  after roll pass, therefore

$$|F_{xA2}| = |F_{xB2}| \quad (7.121)$$

where  $|F_{xB2}|$  is calculated from (7.97)

$$F_{xB2} = F_{B2} \left( 1 - \frac{\lambda_B}{\lambda_{mw}} \right)$$

Since  $\lambda_B < \lambda_{mw}$ ,  $F_{xB2}$  has a (−) sign, indicating that this amount of metal flows over from part *A'*.

In general, the equation taking into account metal flow transference as in (7.104) (7.96) and (7.97), may be written

$$F_{A2}\lambda_A - F_A\lambda_{mw} = F_{xA2}\lambda_{mw} = F_{xA1} \quad (7.122)$$

and from this

$$F_{A1} - F_{A2}\lambda_{mw} = F_{xA2}\lambda_{mw} = F_{xA1} \quad (7.123)$$

where  $\lambda_A > \lambda_B$

$$F_{A1} = (F_{A2} + F_{xA2})\lambda_{mw} \quad (7.124)$$

and similarly

$$F_{B1} = (F_{B2} - F_{xB1})\lambda_{mw} \quad (7.125)$$

Substituting from (7.96) and (7.97)

$$F_{A1} = \left[ F_{A2} + F_{A2} \left( \frac{\lambda_A}{\lambda_{mw}} - 1 \right) \right] \lambda_{mw} \quad (7.126)$$

$$F_{B1} = \left[ F_{B2} + F_{B2} \left( 1 - \frac{\lambda_B}{\lambda_{mw}} \right) \right] \lambda_{mw} \quad (7.127)$$

Equations (7.126) and (7.127) define the size of  $F_{A1}$  and  $F_{B2}$ , taking into account metal flow.

Cross-sectional areas can be expressed as follows:

$$F_1 = F_{A1} \pm F_{xA1} + F_{B1} \mp F_{xB1}$$

and

$$F_2 = F_{A2} \pm F_{xA2} + F_{B2} \mp F_{xB2}$$

The true mean elongation coefficient  $\lambda_{mw}$  can then be written as

$$\lambda_{mw} = \frac{F_1}{F_2} = \frac{F_{A1} \pm F_{xA1} + F_{B1} \mp F_{xB1}}{F_{A2} \pm F_{xA2} + F_{B2} \mp F_{xB2}} \quad (7.128)$$

As an amount  $F_{xA}$  flows from part  $A$  to part  $B$ , hence area  $F_A$  will be decreased by this amount, hence for  $\lambda_A > \lambda_B$  reduced area  $F_{Ax}$  becomes

$$F_{Ax} = F_{A1} - F_{xA1} = F_{A2}\lambda_{mw} \quad (7.129)$$

and correspondingly

$$F_{Bx} = F_{B1} + F_{xB1} = F_{B2}\lambda_{mw} \quad (7.130)$$

where from (7.122) and (7.123)

$$F_{xA1} = F_{xA2}\lambda_{mw}, \quad F_{xB1} = F_{xB2}\lambda_{mw}$$

$F_{Bx}$  and  $F_{Ax}$  indicate areas of parts of the section when metal transference from one part to another has been taken into account.

$\lambda_B$  can now be calculated by comparing equations (7.96) and (7.97), where metal flows from part  $A'$  to part  $B$ .

$$\lambda_B = \left[ F_{B2} - F_{A2} \left( \frac{\lambda_A}{\lambda_{mw}} - 1 \right) \right] \frac{\lambda_{mw}}{F_{B2}} \quad (7.131)$$

$\lambda_B$  can be also calculated from equation (7.109) where

$$\lambda_B = k'\lambda_{mw}$$

From known  $\lambda_B$  and formula (7.130)  $F_{Bx}$  can be calculated.

The free spread of part  $B$  can be calculated from the equation

$$\beta_{B\text{act}} = \frac{b_{B2\text{act}}}{b_{B1\text{act}}} = \lambda_B^{W_B/(1-W_B)} \quad (7.132)$$

i.e.

$$b_{B1\text{act}} = \frac{b_{B2\text{act}}}{\lambda_B^{W_B/(1-W_B)}} \quad (7.133)$$

where  $W_B$  is the exponential coefficient of the author's formulae for the elongation and spread coefficients of part  $B$  (7.81) [32]. Then from (7.114) the last unknown can be found

$$h_{B1} = \frac{F_{B1}}{b_{B1}}$$

With metal passing from part  $A'$  to  $B$ , the constant volume condition  $\gamma_B \beta_B \lambda_B = 1$  may be used, since this applies to component  $B$  only.

The second case is now considered where there is partly restricted spread in part  $A$  and part  $B$ .

If the spread of part  $A'$  is partly restricted, then three quantities must be assumed:  $b_{A2} < b_{A2\text{free}}$ ,  $\lambda_{mw}$  and  $\lambda_A$ , where the  $b_{A2\text{free}}$  value can be calculated from (7.132). Knowing these values

$$\begin{aligned} F_{A1} &= F_{A2} \lambda_A \\ F_{A1z} &= F_{A2z} \lambda_{mw} \end{aligned}$$

can be calculated, and

$$h_{A1} = \frac{F_{A1}}{b_{A1}}$$

It is assumed that

$$\lambda_A > \lambda_{mw} > \lambda_B$$

If there is a restricted spread of part  $A'$ , then the amount of metal passing from part  $A'$  must be equal to the amount of metal passing to part  $B$ . From equations (7.121), (7.96) and (7.97) equation (7.131) can be found

$$\lambda_B = \left[ F_{B2} - F_{A2} \left( 1 - \frac{\lambda_A}{\lambda_{mw}} \right) \right] \frac{\lambda_{mw}}{F_{B2}}$$

$\lambda_B$  can also be found from equation (7.107). Knowing  $\lambda_B$

$$F_{B1} = F_{B2} \lambda_B$$

and

$$F_{B1z} = F_{B2} \lambda_{mw}$$

To find the correct dimensions of part  $B$ , they must be calculated from (7.132)

$$b_{B1\text{act}} = \frac{b_{B2\text{act}}}{\lambda_B^{W_B/(1-W_B)}} \quad \text{and} \quad h_{B1\text{act}} = \frac{F_{B1\text{act}}}{b_{B1\text{act}}}$$

It should be noted that a change of condition (7.117)  $\lambda_A > \lambda_{mw} > \lambda_B$  to  $\lambda_A < \lambda_{mw} < \lambda_B$  (7.119) implies a change of direction of metal flow. For  $\lambda_A > \lambda_{mw} > \lambda_B$ , metal flows from  $F_A$  to  $F_B$ , i.e. from a less intensively elongated area to a more elongated area. Whereas for  $\lambda_A < \lambda_{mw} < \lambda_B$  it is reversed, i.e. metal flows from  $F_B$  to  $F_A$ , again from

a less elongated to a more elongated area. In this case the metal flow formulae must be appropriately changed.

Case 3 is now considered, with free spread of both parts  $A$  and  $B$ , and  $\lambda_A \simeq \lambda_B$ .

If  $\lambda_A$  is assumed, then from equation (7.132), which gives the most correct solution

$$b_{A1} = \frac{b_{A2}}{\lambda_A^{W_A/(1-W_A)}}$$

and from (7.94)

$$F_{A1} = F_{A2} \lambda_A$$

and from (7.113)

$$h_{A1} = \frac{F_{A1}}{b_{A1}}$$

Similarly  $\lambda_B$  for part  $B$  is calculated from equation (7.131)

$$\lambda_B = \frac{\lambda_{mw} F_2 - F_{A2} \lambda_A}{F_{B2}}$$

From this, the dimensions of part  $B$  are found at once by means of equation (7.132)

$$b_{B1} = \frac{b_{B2}}{\lambda_B^{W_B/(1-W_B)}}$$

then

$$F_{B1} = F_{B2} \lambda_B$$

and

$$h_{B1} = \frac{F_{B1}}{b_{B1}}$$

If with free spread of parts  $A$  and  $B$ , condition (7.117)  $\lambda_A > \lambda_{mw} > \lambda_B$  or (7.119) is true, then the method must be modified.

A value of  $\lambda_A$  is assumed from the metal flow equations (7.121), (7.104), (7.105), and then  $\lambda_B$  can be found. Next  $F_{B2}$  is calculated from  $F_{B1} = F_{B2} \lambda_B$ , and then  $b_{B1}$  is calculated from equation (7.132).

Since in calculating  $F_{B1}$  the metal flow is considered,  $F_{A1}$  can be calculated directly:

$$F_{A1} = F_1 - F_{B1}$$

The dimensions of part  $F_{A1}$  can be calculated as for part  $B$ , i.e. from equation (7.132).

### 7.6.2. PASS DESIGN OF SECTIONS COMPOSED OF THREE ELEMENTS

In Figure 7.57 a shape composed of three elements is shown schematically. The dimensions of the finished section are known

$$F_2 = F_{A2} + F_{B2} + F_{C2}$$

also the dimensions of its components

$$h_{A2}, \quad b_{A2}, \quad h_{B2}, \quad b_{B2}, \quad \text{and} \quad h_{C2}, \quad b_{C2}$$

The cross-sectional dimensions of the preceding pass are required, i.e. its area

$$F_1 = F_{A1} + F_{B1} + F_{C1}$$

and the dimensions of its various components

$$F_{A1}, h_{A1}, b_{A1}, F_{B1}, h_{B1}, b_{B1}, F_{C1}, h_{C1}, b_{C1}$$

9 unknowns altogether.

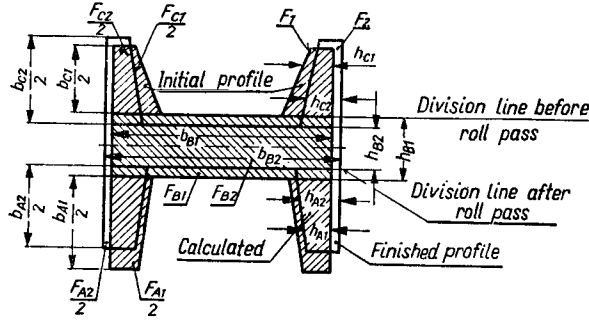


Fig. 7.57. Rolling of simple shape formed of three elements

From known relations between an area and its dimensions (7.113) and (7.114)

$$F_{A1} = h_{A1} b_{A1}$$

$$F_{B1} = h_{B1} b_{B1}$$

$$F_{C1} = h_{C1} b_{C1}$$

thus the number of unknowns is reduced to six.

As the section is composed of three parts *A*, *B*, *C* and they are non-uniformly elongated, the additional equation (7.90) must be satisfied

$$\lambda_{mw} = \frac{F_{A2} \lambda_A + F_{B2} \lambda_B + F_{C2} \lambda_C}{F_{A2} + F_{B2} + F_{C2}}$$

The section components are related as in (7.94) and (7.95), where metal flow from one part to another is expressed as

$$F_{A1} = \lambda_A F_{A2}$$

$$F_{B1} = \lambda_B F_{B2}$$

$$F_{C1} = \lambda_C F_{C2}$$

Hence three new unknowns  $\lambda_A$ ,  $\lambda_B$ ,  $\lambda_C$  are introduced.

Assuming that  $\lambda_{mw}$  is defined for the given pass design system, then for a given section there are 6 actual unknowns

$$\lambda_A, b_{A1}, \lambda_B, b_{B1}, \lambda_C, b_{C1}$$

or

$$F_{A1}, b_{A1}, F_{B1}, b_{B1}, F_{C1}, b_{C1}$$

In this case the equation

$$\lambda_{mw} = \sqrt[3]{\lambda_A \lambda_B \lambda_C}$$

is also used, in order to solve a problem with so many unknowns.

If equation (7.86) is used to solve the  $\lambda$  equations, relations similar to those obtained for a shape composed of two parts are found, i.e.

$$\lambda_A \lambda_B \lambda_C = \lambda_{mw}^3$$

therefore

$$\lambda_A = \frac{\lambda_{mw}^3}{\lambda_B \lambda_C}$$

and similarly for  $\lambda_B$  and  $\lambda_C$  formulae (7.90), (7.91), and also

$$\lambda_{mw} = \frac{F_{A2}\lambda_A + F_{B2}\lambda_B + F_{C2}\lambda_C}{F_{A2} + F_{B2} + F_{C2}}$$

Thus it is more convenient to assume a value of  $\lambda_{mw}$  and reduce the number of unknowns to two,  $\lambda_B$  and  $\lambda_C$ . Then the problem reduces to solving a set of equations similar to those in the case of a simple two-part shape.

From the analysis of pass design of simple sections the following cases of metal flow from one part of cross-section to another are distinguished:

|   | Completely closed part | Partly closed part    | Free spreading part              |
|---|------------------------|-----------------------|----------------------------------|
| 1 | <i>A</i>               | <i>B</i>              | <i>C</i>                         |
| 2 | <i>A</i> and <i>B</i>  | —                     | <i>C</i>                         |
| 3 | —                      | <i>A</i> and <i>B</i> | <i>C</i>                         |
| 4 | <i>A</i>               | —                     | <i>B</i> and <i>C</i>            |
| 5 | —                      | <i>B</i>              | <i>A</i> and <i>C</i>            |
| 6 | —                      | —                     | <i>A</i> , <i>B</i> and <i>C</i> |

There are only 6 practical cases out of 9 theoretically possible ones, for the three remaining cases correct pass designing is impossible, hence only these 6 cases will be analysed.

The successive partial elongation coefficients are assumed. Since there are no restrictions as to definition of section parts, the following equations can be written:

$$\lambda_A = \lambda_B = \lambda_C = \lambda_{mw} \quad (7.134)$$

$$\lambda_A > \lambda_B > \lambda_C \quad (7.135)$$

or

$$\lambda_A = \lambda_B > \lambda_C \quad (7.136)$$

$$\lambda_A > \lambda_B = \lambda_C \quad (7.137)$$

Metal flow from one part to another will depend on which of these conditions applies. Only in equation (7.134), where all the coefficients are equal, will there be no metal flow.

In this case the  $\lambda_{mw}$  value and partial coefficients can be determined as before, which gives the optimum solution.

Rolling cases where equations (7.136) and (7.137) apply can be transformed to conform with the case already analysed: i.e. a shape composed of two components having different elongation coefficients. These cases may then be solved as at the beginning of this chapter, hence they will not be considered here, but the elongation

conditions (7.134) and (7.135) for the 6 cases of shapes composed of three elements  $A$ ,  $B$ ,  $C$  will be analysed.

For conditions as in (7.134), there will be no metal flow from one part to another, therefore the flow equations are not required.

For conditions as in (7.135), metal flows from one part of the section to another, and the constant volume principle leads to the assumption that the amounts of metal being transferred must be equivalent to each other, i.e.

$$\pm F_{xA} = \pm F_{xB} = \pm F_{xC} \quad (7.138)$$

and

$$F = (F_{A1} \pm F_{xA}) + (F_{B1} \pm F_{xB}) + (F_{C1} \pm F_{xC}) \quad (7.139)$$

If metal flow is restricted to two parts, the calculation of the third part may be neglected and equations (7.138) and (7.139) can be calculated as for a two-part profile since this covers with conditions in (7.136) and (7.137).

If the metal passing takes place with conditions in (7.135) it occurs in all the three components.

Since  $\lambda_A$  was assumed as the largest coefficient, then from equation (7.106) it may be seen that from part  $A$  a certain amount of metal will flow out to parts  $B$  and  $C$ .

In this case the method of calculation of area  $F_{xA}$  will be unchanged, i.e. from equation (7.96)

$$F_{xA} = F_{A2} \left( \frac{\lambda_A}{\lambda_{mw}} - 1 \right)$$

and in this case

$$|F_{xA}| = |F_{xB} + F_{xC}| \quad (7.140)$$

Then for parts  $B$  and  $C$  equation (7.97) may be written

$$\left. \begin{aligned} F_{xB} &= F_{B2} \left( 1 - \frac{\lambda_B}{\lambda_{mw}} \right) \\ F_{xC} &= F_{C2} \left( 1 - \frac{\lambda_C}{\lambda_{mw}} \right) \end{aligned} \right\} \quad (7.141)$$

or

If values with a negative sign are found, it means that an amount of metal instead of flowing to that part, flows out from it, for instance the whole amount of metal from part  $B$  equal to  $F_{xB}$  flows to part  $C$ .

Substituting equations (7.96), (7.97) and (7.105) into (7.139) and taking into consideration metal flow

$$F_{A2} \left( \frac{\lambda_A}{\lambda_{mw}} - 1 \right) = F_{B2} \left( 1 - \frac{\lambda_B}{\lambda_{mw}} \right) + F_{C2} \left( 1 - \frac{\lambda_C}{\lambda_{mw}} \right) \quad (7.142)$$

The 6 cases for relative conditions of parts  $A$ ,  $B$ ,  $C$ , which are capable of solution will now be analysed.

In case 1 part  $A$  is completely closed, part  $B$  has spread partly restricted, part  $C$  is open. To solve part  $A$ ,  $b_{A1} \approx b_{A2}$  is assumed.

From equation (7.120)

$$\lambda_A = \frac{F_{A1}}{F_{A2}} = \frac{h_{A1}}{h_{A2}} = \frac{1}{\gamma_A}$$

$h_{A2}$  being known,  $h_{A1}$  assumed, hence draught coefficient of this part will be  $\gamma_A$ . Then for part  $A$  the amount of metal transferred  $F_{xA}$  is calculated from equation (7.96).

As part  $B$  is partly restricted in spread it may be assumed that  $b_{B1} < b_{B1\text{free}}$ .

The breadth  $b_{B1\text{free}}$  is calculated from the equation  $\beta_B = \gamma_B^{-W}$ . Knowing  $\lambda_A$  from equation (7.90),  $\lambda_B$  can be found, and hence  $F_{B1}$ .

The dimensions of part  $B$  are calculated from equation

$$F_{B1} = F_{B2}\lambda_{mw}$$

therefore

$$h_{B1\text{act}} = \frac{F_{B2}\lambda_{mw}}{b_{B1\text{act}}} = \frac{F_{B1\text{act}}}{b_{B1\text{act}}}$$

The amount of metal passing to or from part  $B$  is

$$F_{xB} = F_{B2} \left( \frac{\lambda_B}{\lambda_{mw}} - 1 \right)$$

Then from equation

$$|F_{xA}| = |F_{xB} + F_{xC}|$$

$F_{xC}$  is found.

From relation (7.97)  $\lambda_C$  can be found

$$\lambda_C = \lambda_{mw} \left( 1 + \frac{F_{xC}}{F_{C2}} \right)$$

Considering part  $C$

$$F_{C1} = \lambda_C F_{C2} \quad \text{and} \quad F_{C1z} = F_{C2}\lambda_{mw}$$

The solution of part  $C$  is not difficult, since from equation (7.132)

$$\beta_C = \frac{b_{C2}}{b_{C1}} = \lambda_C^{W_C/(1-W_C)}$$

and from this  $b_{C1\text{act}}$  can be found.

From equation (7.114)

$$h_{C1\text{act}} = \frac{F_{C1\text{act}}}{b_{C1\text{act}}}$$

In case 2 parts  $A$  and  $B$  are completely closed, and part  $C$  open, it is assumed that  $b_{A1} \approx b_{A2}$ , then from equation (7.120)

$$\lambda_A = \frac{F_{A1}}{F_{A2}} = \frac{h_{A1}}{h_{A2}} = \frac{1}{\gamma_A}$$

Similarly for part  $B$ , if  $b_{B1} \approx b_{B2}$  is assumed, then  $\lambda_B$  can also be found (7.120)

$$\lambda_B = \frac{h_{B1}}{h_{B2}} = \frac{F_{B1}}{F_{B2}} = \frac{1}{\gamma_B}$$



As in part *B* the spread is completely restricted, and since  $b_{B1} \cong b_{B2}$  is assumed, from part *B* a certain amount of metal flows out to part *C*, i.e.

$$F_{xB} = F_{B2} \left( \frac{\lambda_B}{\lambda_{mw}} - 1 \right)$$

or also

$$|F_{xA} + F_{xB}| = |F_{xC}|$$

Then  $\lambda_A$  is calculated,  $F_{xA}$  is determined,  $\lambda_B$  can be found from equation (7.142), and from equation (7.97)

$$\lambda_C = \lambda_{mw} \left( 1 + \frac{F_{xC}}{F_{C2}} \right)$$

It remains to solve part *C* only, with an area

$$F_{C1} = \lambda_C F_{C2}$$

and

$$F_{C1z} = \lambda_{mw} F_{C2}$$

This part can be calculated from (7.132)

$$\beta_C = \frac{b_{C2 \text{ act}}}{b_{C1 \text{ act}}} = \lambda_C^{w_C/(1-w_C)}$$

and from this  $b_{C1}$  can be found.

From equation (7.114)

$$h_{C1 \text{ act}} = \frac{F_{C1 \text{ act}}}{b_{C1 \text{ act}}}$$

Case 3 where parts *A* and *B* have restricted spread can be solved similarly, assuming  $b_{A1}$ ,  $\lambda_A$ ,  $b_{B1}$ , and calculating  $\lambda_B$  and  $\lambda_C$ , and then in a similar manner the whole profile can be solved.

Case 4 can also be solved similarly. It is assumed that part *A* has completely restricted spread,  $b_{A1} \cong b_{A2}$ , and parts *B* and *C* have free spread.

Then  $\lambda_A$  for the closed part can be found from equation (7.120) and  $\lambda_B$  can be found from equation (7.142) with the condition (7.140)

$$|F_{xA}| = |F_{xB} + F_{xC}|$$

With known  $\lambda_B$  and  $F_{B2}$ , part *B* can be solved by means of free spread formula (7.132).

Next part *C* can be solved, after first calculating  $F_{C2}$  and  $\lambda_C$ .

In case 5 part *B* is partly closed, while parts *A* and *C* are open.

Since the spread of part *B* is partly restricted, two values must be assumed:  $b_{B1} < b_{B1 \text{ free}}$  and  $\lambda_B$ .  
Then

$$F_{B1} = F_{B2} \lambda_B$$

$$F_{B1z} = F_{B2} \lambda_{mw}$$

and

$$h_{B1 \text{ act}} = \frac{F_{B1 \text{ act}}}{b_{B1 \text{ act}}}$$

A certain amount of metal will flow out from part  $B$

$$F_{xB} = F_{B2} \left( \frac{\lambda_B}{\lambda_{mw}} - 1 \right)$$

This metal flows to parts  $A$  and  $C$  in such a way that

$$|F_{xB}| = |F_{xA} + F_{xC}|$$

Equation systems (7.142), (7.104) and (7.105) are transformed so that  $\lambda_A$  and  $\lambda_B$  can be calculated from them, assuming that with  $\lambda_B$  known  $\lambda_A$  is required.

$\lambda_A$  and  $\lambda_B$  being known,  $F_{A1}$  and  $F_{C1}$  are calculated and then these sections can be solved by means of equation (7.131).

The last case remains to be analysed, where  $A$ ,  $B$ ,  $C$  are open and have free spread.

If  $\lambda_A \approx \lambda_B \approx \lambda_C \approx \lambda_{mw}$ , then assuming  $\lambda_{mw} \approx \lambda_A$  from (7.133).

$$b_{A1 \text{ act}} = \frac{b_{A2 \text{ act}}}{\lambda_A^{W_A/(1-W_A)}}$$

and from (7.94)

$$F_{A1 \text{ act}} = F_{A2 \text{ act}} \lambda_A$$

and from (7.113)

$$h_{A1 \text{ act}} = \frac{F_{A1 \text{ act}}}{b_{A1}}$$

With  $\lambda_A$  and  $\lambda_C$  known, parts  $B$  and  $C$  can be solved similarly.

If the differences between  $\lambda_A$ ,  $\lambda_B$ ,  $\lambda_C$  are small, they can be calculated from equations similar to (7.90) and (7.92) as follows:

$$\lambda_{mw} = \frac{F_{A2} \lambda_A + F_{B2} \lambda_B + F_{C2} \lambda_C}{F_{A2} + F_{B2} + F_{C2}}$$

$$\lambda_{mw} = \frac{F_{A1} + F_{B1} + F_{C1}}{F_{A2} + F_{B2} + F_{C2}}$$

where  $\lambda_A$ ,  $\lambda_B$  and  $\lambda_C$  are known from equations (7.94) and (7.95), and substituting them in the preceding equation for  $\lambda_{mw}$  a second degree equation for  $\lambda_{mw}$  is found.

However if

$$\lambda_A > \lambda_B > \lambda_C$$

metal will flow from part  $A$  to parts  $B$  and  $C$ . Then  $\lambda_A$  is assumed and

$$|F_{xA}| = |F_{xB} + F_{xC}|$$

must be calculated from (7.138).

Knowing these values  $\lambda_B$  and  $\lambda_C$  can be found from (7.142) and (7.90) and also  $F_{B1}$  and  $F_{C1}$ .

The areas can be found without any great difficulty by means of equation (7.132), and next  $F_{B1 \text{ act}}$  and  $F_{C1 \text{ act}}$  can be calculated, and then

$$\beta_B = \lambda_B^{W_B/(1-W_B)} \quad \text{and} \quad \beta_C = \lambda_C^{W_C/(1-W_C)}$$

The results of the author's experiments to investigate the accuracy of these formulae in real cases, where only one part of profile is directly compressed by the collars of settling and box-passes, are shown in Fig. 7.58.

As stock, square bars  $42 \times 42$  mm were used heated to  $1200^\circ\text{C}$ . Firstly these tests were used to compare calculated results of the effective elongation coefficient, with those found from equation (7.92)

$$\lambda_{\text{act}} = \frac{F_1}{F_2}$$

Both areas were found by accurate planimetric measurement from the templates. First  $F_A$  was measured and  $F_B$  was calculated as the difference  $F - F_A$ .

The preceding theoretical conclusions were checked as follows:

- (1) Prints were made from rolling specimens of sections Figs. 7.58 and 7.59.
- (2) The whole section and its parts were accurately planimetered to find the areas, and dimensioned as in Figs. 7.58 and 7.59.
- (3) From ratio  $F_1/F_2$ ,  $\lambda_{\text{act}}$  was calculated.
- (4)  $\lambda_{mw}$  was calculated from equations (7.90), (7.92).

Next all the rolled specimens were analysed in detail, the successive cross-sectional areas and heights and breadths of parts of sections were measured for each roll pass. All these dimensions and the total mean and partial coefficients  $\gamma$ ,  $\beta$ ,  $\lambda$  calculated from measured dimensions, are given in Table 7.10.

The mean deformation coefficients of parts of sections were calculated from equation (7.87). These results are also given in Table 7.10.

It may be seen from Table 7.10, that calculated values do not differ from true ones, and the products of the coefficients are always about 1.0. This proves that the constant volume principle is obeyed, thus these formulae can be used to calculate the particular shapes.

At the same time the dimensions of every rolling specimen have been calculated theoretically.

It was assumed that each profile is composed of two parts  $A$  and  $B$ , and that part  $A$  has almost completely restricted spread while part  $B$  can spread freely, i.e. it corresponds to the first case of those previously described as possible of solution.

Such passes can be solved by means of formulae (7.93), (7.120), (7.122), (7.123), (7.126), (7.127), (7.129), (7.130), (7.131).

Mean pass dimensions are found from formulae (7.132), (7.133), (7.113) and (7.114).

All the areas, partial and mean deformation coefficients and the calculated dimensions are given in Table 7.11. This table also gives the error of calculated free spread widths of passes relative to these measured from the rolling samples.

The error in this case is small, and indicates that these results are satisfactory.

#### Example

Calculation of another case is shown in Fig. 7.59. Given:

|                              |                              |
|------------------------------|------------------------------|
| $F_{0t} = 1764 \text{ mm}^2$ | $F_{1t} = 1434 \text{ mm}^2$ |
| $F_{A0} = 903 \text{ mm}^2$  | $F_{A1} = 812 \text{ mm}^2$  |
| $F_{B0} = 861 \text{ mm}^2$  | $F_{B1} = 622 \text{ mm}^2$  |
| $b_{A0} = 21.5 \text{ mm}$   | $b_{A1} = 21.5 \text{ mm}$   |
| $b_{B0} = 20.5 \text{ mm}$   | $b_{B1} = 27.45 \text{ mm}$  |

$h_{A0}$ ,  $h_{B0}$  and  $h_{A1}$ ,  $h_{B1}$  are to be calculated.

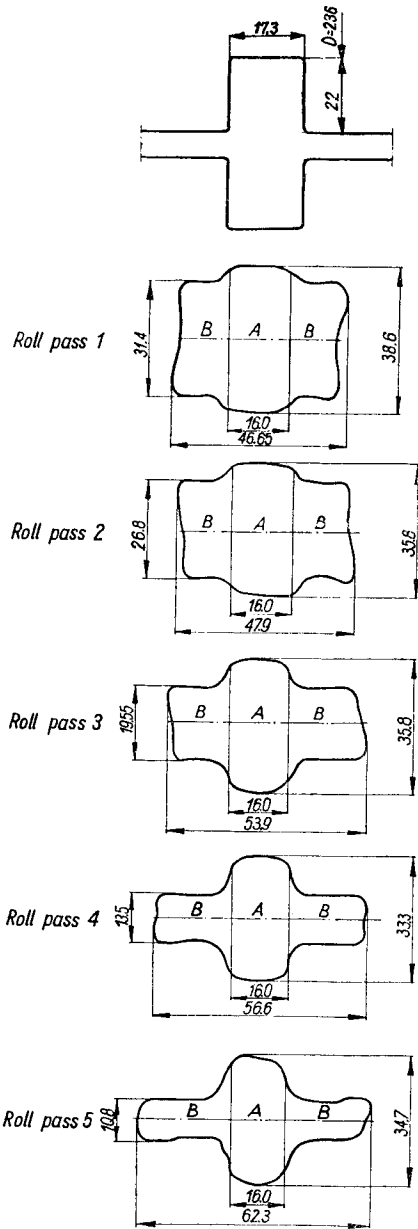


Fig. 7.58. Templates from the author's rolling experiments in 5 roll passes with different ratios of centre part to lateral part

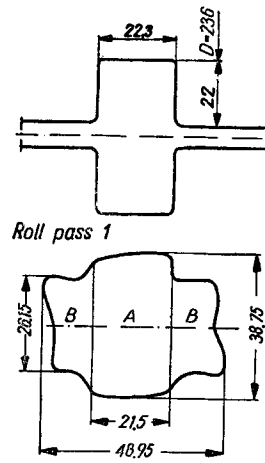


Fig. 7.59. Template from the author's rolling experiments with different ratio of centre part to lateral part, after one roll pass

TABLE 7.10  
SECTIONAL DIMENSIONS AND CALCULATED COEFFICIENTS OF ROLLED SAMPLES (Fig. 7.58)

| Roll pass No. | $F_t$<br>mm <sup>2</sup> | $F_A$<br>mm <sup>2</sup> | $F_B$<br>mm <sup>2</sup> | $h_{Am}$<br>mm | $b_A$<br>mm | $h_{Bm}$<br>mm | $b_B$<br>mm | $\lambda_{A\text{act}}$ | $\lambda_{B\text{act}}$ | $\beta_{A\text{act}}$ | $\beta_{B\text{act}}$ | $\gamma_{A\text{act}}$ | $\gamma_{B\text{act}}$ | $\gamma_{t\text{act}}$ | $\beta_{t\text{act}}$ | $\lambda_{t\text{act}}$ |
|---------------|--------------------------|--------------------------|--------------------------|----------------|-------------|----------------|-------------|-------------------------|-------------------------|-----------------------|-----------------------|------------------------|------------------------|------------------------|-----------------------|-------------------------|
| 0             | 1764                     | 672                      | 1092                     | 42             | 16.0        | 42             | 26          |                         |                         |                       |                       |                        |                        |                        |                       |                         |
| 1             | 1494                     | 616                      | 878                      | 38.5           | 16.0        | 28.65          | 30.65       | 1.0909                  | 1.2437                  | 1.0000                | 1.1788                | 0.9167                 | 0.6821                 | 0.6253                 | 1.1788                | 1.3568                  |
| 2             | 1336                     | 552                      | 784                      | 34.5           | 16.0        | 24.58          | 31.9        | 1.1159                  | 1.1198                  | 1.0000                | 1.0407                | 0.8961                 | 0.8579                 | 0.7688                 | 1.0407                | 1.2497                  |
| 3             | 1240                     | 548                      | 692                      | 34.25          | 16.0        | 18.26          | 37.9        | 1.0073                  | 1.1329                  | 1.0000                | 1.1881                | 0.9928                 | 0.7429                 | 0.7376                 | 1.1881                | 1.1412                  |
| 4             | 1028                     | 512                      | 516                      | 32.0           | 16.0        | 12.7           | 40.6        | 1.0703                  | 1.3411                  | 1.0000                | 1.0712                | 0.9343                 | 0.6955                 | 0.6498                 | 1.0712                | 1.4354                  |
| 5             | 1000                     | 508                      | 492                      | 31.75          | 16.0        | 10.63          | 46.3        | 1.0079                  | 1.0487                  | 1.0000                | 1.1404                | 0.9922                 | 0.8370                 | 0.8305                 | 1.1404                | 1.0571                  |

Roll pass 1

$$\gamma_m = \sqrt{0.6253} = 0.79077$$

$$\beta_m = \sqrt{1.1788} = 1.08573$$

$$\lambda_m = \sqrt{1.3568} = 1.16482 \quad \gamma_m \beta_m \lambda_m = 1.00007$$

Roll pass 2

$$\gamma_m = \sqrt{0.7688} = 0.8768$$

$$\beta_m = \sqrt{1.0407} = 1.02015$$

$$\lambda_m = \sqrt{1.2497} = 1.1179 \quad \gamma_m \beta_m \lambda_m = 0.99993$$

Roll pass 3

$$\gamma_m = \sqrt{0.7376} = 0.85884$$

$$\beta_m = \sqrt{1.1881} = 1.0990$$

$$\lambda_m = \sqrt{1.1412} = 1.06827 \quad \gamma_m \beta_m \lambda_m = 1.00004$$

Roll pass 4

$$\gamma_m = \sqrt{0.6498} = 0.8061$$

$$\beta_m = \sqrt{1.0712} = 1.035$$

$$\lambda_m = \sqrt{1.4354} = 1.1981 \quad \gamma_m \beta_m \lambda_m = 0.9996$$

Roll pass 5

$$\gamma_m = \sqrt{0.8305} = 0.9113$$

$$\beta_m = \sqrt{1.1404} = 1.0679$$

$$\lambda_m = \sqrt{1.0571} = 1.02816 \quad \gamma_m \beta_m \lambda_m = 1.0006$$

TABLE  
AREAS, DIMENSIONS AND DEFORMATION

| Pass No. | $\lambda_{A \text{ act}} = h_{A1}/h_{A2}$ | $\lambda_{\text{act}} = F_{1t}/F_{2t}$ | $\lambda_{B \text{ cal}}$ | $\beta_{B \text{ cal}}$ | $b_{B1 \text{ cal}}$ mm |
|----------|-------------------------------------------|----------------------------------------|---------------------------|-------------------------|-------------------------|
| 0        | 1.0909                                    | 1.1807                                 | 1.2438                    | 1.181                   | 25.95                   |
| 1        | 1.1159                                    | 1.11826                                | 1.1199                    | 1.040                   | 30.67                   |
| 2        | 1.0073                                    | 1.0774                                 | 1.1329                    | 1.186                   | 31.95                   |
| 3        | 1.0703                                    | 1.2062                                 | 1.3415                    | 1.069                   | 37.98                   |
| 4        | 1.0079                                    | 1.0280                                 | 1.0487                    | 1.140                   | 40.59                   |

$$h_{A0} = \frac{F_{A0}}{b_{A0}} = \frac{903}{21.5} = 42 \text{ mm}$$

$$h_{B0} = \frac{F_{B0}}{b_{B0}} = \frac{861}{20.5} = 42 \text{ mm}$$

$$h_{A1} = \frac{F_{A1}}{b_{A1}} = \frac{812}{21.5} = 37.76 \text{ mm}$$

$$h_{B1} = \frac{F_{B1}}{b_{B1}} = \frac{622}{27.45} = 22.66 \text{ mm}$$

The coefficients  $\gamma$ ,  $\beta$ ,  $\lambda$  are to be calculated

$$\lambda_A = \frac{F_{A0}}{F_{A1}} = \frac{903}{812} = 1.1121$$

$$\lambda_B = \frac{F_{B0}}{F_{B1}} = \frac{861}{622} = 1.3842$$

$$\beta_A = \frac{b_{A1}}{b_{A0}} = \frac{21.5}{21.5} = 1.0000$$

$$\beta_B = \frac{b_{B1}}{b_{B0}} = \frac{27.45}{20.5} = 1.3390$$

$$\gamma_A = \frac{h_{A1}}{h_{A0}} = \frac{37.76}{42} = 0.8990$$

$$\gamma_B = \frac{h_{B1}}{h_{B0}} = \frac{22.66}{42} = 0.5395$$

$$\gamma_t = \gamma_A \gamma_B = 0.4850$$

$$\gamma_m = \sqrt{\gamma_A \gamma_B} = \sqrt{0.4850} = 0.6966$$

$$\beta_t = \beta_A \beta_B = 1.3390$$

$$\beta_m = \sqrt{\beta_A \beta_B} = \sqrt{1.3390} = 1.1572$$

$$\lambda_t = \lambda_A \lambda_B = 1.5394$$

$$\lambda_m = \sqrt{\lambda_A \lambda_B} = \sqrt{1.5394} = 1.2407$$

$$\gamma_m \beta_m \lambda_m = 1.0001$$

Then  $\lambda_{\text{act}}$  is calculated

$$\lambda_{\text{act}} = \frac{F_0}{F_1} = 1.2301$$

The metal flow is determined from equations (7.96), (7.97), (7.98), and (7.99)

$$F_{xA} = F_{A1} \left( 1 - \frac{\lambda_A}{\lambda_{\text{act}}} \right) \quad \text{for } \lambda_A < \lambda_B$$

$$F_{xA} = F_{A1} \left( \frac{\lambda_A}{\lambda_{\text{act}}} - 1 \right) \quad \text{for } \lambda_A > \lambda_B$$

$$F_{xA} = 812 \left( \frac{1.1123}{1.2301} - 1 \right) = 77.89 \text{ mm}^2$$

and

$$F_{xB} = F_{B1} \left( \frac{\lambda_B}{\lambda_{\text{act}}} - 1 \right) \quad \text{for } \lambda_A < \lambda_B$$

$$F_{xB} = F_{B1} \left( 1 - \frac{\lambda_B}{\lambda_{\text{act}}} \right) \quad \text{for } \lambda_A > \lambda_B$$

$$F_{xB} = 622 \left( \frac{1.3842}{1.2301} - 1 \right) = 77.91 \text{ mm}^2$$

7.11

## COEFFICIENTS OF SECTIONS

| Error $\Delta b_B$<br>% | $h_{B1 \text{ cal}}$<br>mm | $F_{xA \text{ cal}}$<br>mm <sup>2</sup> | $F_{xB \text{ cal}}$<br>mm <sup>2</sup> | $F_{A1 \text{ cal}}$<br>mm <sup>2</sup> | $F_{b1 \text{ cal}}$<br>mm <sup>2</sup> |
|-------------------------|----------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|-----------------------------------------|
| 0.19                    | 42.08                      | 46.91                                   | 46.84                                   | 671.94                                  | 1092.0                                  |
| -0.06                   | 28.63                      | 1.47                                    | 1.13                                    | 616.01                                  | 878.0                                   |
| -0.16                   | 24.54                      | 35.66                                   | 34.64                                   | 552.06                                  | 783.91                                  |
| -0.21                   | 18.22                      | 57.88                                   | 57.67                                   | 547.73                                  | 692.2                                   |
| 0.02                    | 12.71                      | 9.94                                    | 9.94                                    | 512.0                                   | 515.99                                  |

From (7.131)

$$\lambda_{B \text{ cal}} = \left[ F_{B1} + F_{A1} \left( 1 - \frac{\lambda_A}{\lambda_{\text{act}}} \right) \frac{\lambda_{\text{act}}}{F_{B2}} \right]$$

$$\lambda_{B \text{ cal}} = \left( 1 + \frac{F_{xB}}{F_{B2}} \right) \lambda_{\text{act}} \quad \text{for } \lambda_A < \lambda_B$$

$$\lambda_{B \text{ cal}} = \left( 1 - \frac{F_{xB}}{F_{B2}} \right) \lambda_{\text{act}} \quad \text{for } \lambda_A > \lambda_B$$

$$\lambda_{B \text{ cal}} = 1.3842$$

From (7.120)

$$\lambda_{A \text{ cal}} = \frac{h_{A1}}{h_{A2}} = \frac{1}{\gamma_A} = \frac{F_{A0}}{F_{A1}} = 1.1123$$

$$\beta_{B \text{ cal}} = \frac{b_{B1}}{b_B} = \lambda_{B \text{ cal}}^{W_{B1}/(1-W_{B1})}$$

 $W_B$  and  $1 - W_B$  can be calculated on the special slide rule or from formulae (3.46) and (3.48)

$$W_B = 10^{-1.269 \delta \varepsilon_w^{0.556}}$$

$$\beta_{B \text{ cal}} = 1.3370$$

$$b_{B \text{ cal}} = \frac{b_{B1}}{\beta_{B \text{ cal}}} = \frac{27.45}{1.3370} = 20.53 \text{ mm}$$

Then  $F_{A0 \text{ cal}}$  and  $F_{B0 \text{ cal}}$  can be calculated from (7.126) and (7.127)

$$F_{A0 \text{ cal}} = (F_{A1} + F_{xA}) \lambda_{\text{act}} \quad \text{for } \lambda_B > \lambda_A$$

$$F_{A0 \text{ cal}} = (F_{A1} - F_{xA}) \lambda_{\text{act}} \quad \text{for } \lambda_B < \lambda_A$$

$$F_{A0} = (812 + 77.89) \times 1.2301 = 903.2 \text{ mm}^2$$

$$F_{B0} = (F_{B1} - F_{xB}) \lambda_{\text{act}} \quad \text{for } \lambda_B > \lambda_A$$

$$F_{B0} = (F_{B1} + F_{xB}) \lambda_{\text{act}} \quad \text{for } \lambda_B < \lambda_A$$

$$F_{B0} = (622 - 77.91) \times 1.2301 = 861.0 \text{ mm}^2$$

Next it is calculated

$$h_{B \text{ cal}} = \frac{F_{B0}}{b_{B \text{ cal}}} = \frac{861.0}{20.53} = 41.9 \text{ mm}$$

## 7.6.3. PASS DESIGN FOR ANGLES, T-BEAMS, CHANNELS AND I-BEAMS [174]

The pass design method for sections is similar to that previously described, but the calculations start from the finished section, and work back to the initial one.

Otto's slide rule (Fig. 7.39) which has already been described in detail, is useful for the calculation of the free elongation coefficients of the various parts of the section from Wusatowski's formula.

By means of this slide rule  $\lambda_A$  and  $\lambda_B$  can be calculated, or  $\gamma_A$  and  $\gamma_B$  of profile parts with free elongation, while from equation (3.3) the elongation coefficients of the closed part are found, assuming that it will be correctly filled, i.e. with the assumed  $\lambda$  the required  $\beta$  will be obtained.

Since in the closed part  $b_1 = b_2$  and  $\beta \approx 1.0$ , the elongation coefficient can be calculated from

$$\lambda = \frac{1}{\gamma}$$

i.e. in this case

$$\lambda \approx \frac{h_1}{h_2}$$

The consideration of pass design for the whole range of sections would take too much space, therefore the solution of pass design problems for several simple sections will be described, based on the actual flow of metal in the section being rolled.

The elongation coefficient is the most important design factor. In the case of an irregular section this value can define the whole pass system, if elongation coefficients of the component parts of section can be defined. The commonly used division of I-beams into a web and open and closed flanges, is not sufficiently accurate for the definition of partial elongation coefficients. For this purpose the limits of particular section components where the process of deformation is approximately similar must also be precisely defined.

In practice the following divisions of irregular profiles are used:

- (a) Brovot's division (Fig. 7.60a),
- (b) Tafel's division (Fig. 7.60b),
- (c) Sokolov's division (Fig. 7.60d).

According to Brovot a section composed of web and flanges is divided as in Fig. 7.60a, whereas Tafel divided a similar section as in Fig. 7.60b. He considers Brovot's

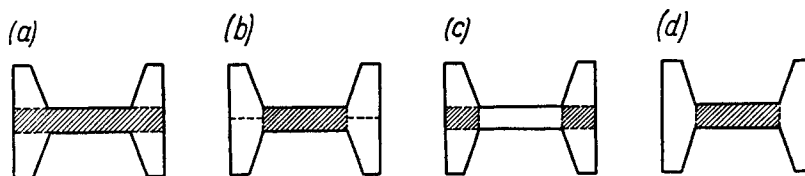


Fig. 7.60. Different methods of dividing I-beam after: (a) Brovot, (b) Tafel, (c) Cramer (d) Sokolov

division as incorrect. According to Tafel, in a section divided as proposed by Brovot, the shaded part of web in Fig. 7.60c would be exposed to the same direct draughts as the remaining parts of web, whereas actually parts of the web are exposed to the same draughts as the flanges.

Cramer analysed the partial elongation coefficient for profile components according to both Brovot and Tafel divisions, and came to the following conclusions:



In the case of Tafel's division, material from part of web limited by points  $ABCD$  (Fig. 7.61) in every roll pass must not only fill a part  $A'B'C'D'$  but first of all widen the web by an amount shown by oblique shading and then additionally fill the part

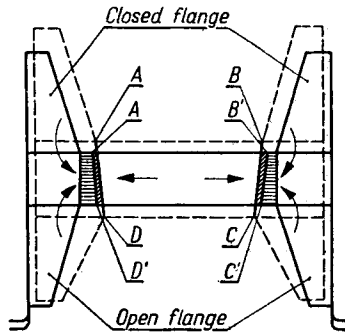


Fig. 7.61. Directions of metal flow during deformation in a pass

horizontally shaded. In fact material passes to the shaded part not only from the web, but also from the flanges. It is seen that with Tafel's division, too small elongation coefficients are obtained for the web, whereas with Brovot's division the elongation coefficients of flanges are too small.

Cramer proposes the assumption of Tafel's division for flanges, but Brovot's division for the web. However, this gives partial elongation coefficients always greater than the mean elongation coefficient of the whole section.

Sokolov worked out a method for I-beams using the division given in Fig. 7.60d. It can be seen that this division does not consider open and closed flange parts, which gives a different distribution of elongation coefficients in a web and flanges.

Cramer's experiments show that for analysis of deformation coefficients the proper interpretation of obtained results is more important than the assumed division of section.

To facilitate the selection from the existing systems of the most suitable division of section for purposes of analysis, and to ensure correct interpretation of results obtained, the run of elongation coefficients for Brovot's, Tafel's and Sokolov's divisions for pass design of sample I-beams 120\* is described.

Figure 7.62 shows a system of curves for partial elongation coefficients with Brovot's division. It can be seen from the run of curves, that in the first roll passes the web elongation coefficients  $\lambda_w$  are significantly greater than the corresponding flange coefficients  $\lambda_f$ . Beginning from the roll pass where the web sectional area exceeds the flange sectional area, the coefficients  $\lambda_{fo}$  are greater than the corresponding web coefficients  $\lambda_w$ .

A comparison of curves for  $\lambda_{fo}$  and  $\lambda_{fel}$  shows that in all roll passes values of  $\lambda_{fo}$  for the open flanges are greater than corresponding ones for closed flanges.

All coefficients follow smooth curves, except in the finishing pass, where a sudden break is observed.

The drop of the  $\lambda_{fo}$  curve in Fig. 7.62 in the first roll passes can be explained as follows: if the flange dimensions  $h$  and  $b$  (7.63) do not change after a roll pass, and at the same time the web is deformed by  $\Delta d/2$ , then according to Brovot's division the

\* See Appendix 2.

open flange area increases by  $\Delta F$  and the open flange elongation coefficient in an extreme case can take a negative value.

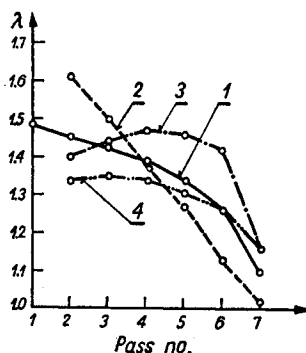


Fig. 7.62. Part elongation coefficients for I-beam 120 (see Appendix 2) for Brovot's division: 1 —  $\lambda_t$  total of pass, 2 —  $\lambda_w$  of web, 3 —  $\lambda_{fo}$  of open flange, 4 —  $\lambda_{fcl}$  of closed flange

In general the conclusion can be drawn that Brovot's division gives lower elongation coefficients for the flanges than other methods, especially in the first roll passes.

The run of  $\lambda$  curves for Tafel's division is given in Fig. 7.64. Here the values of  $\lambda_w$  in all the roll passes are lower than  $\lambda_f$  and a comparison of  $\lambda_{fo}$  with  $\lambda_{fcl}$  shows that in all the passes the elongation coefficients for open flanges are greater than for closed ones.

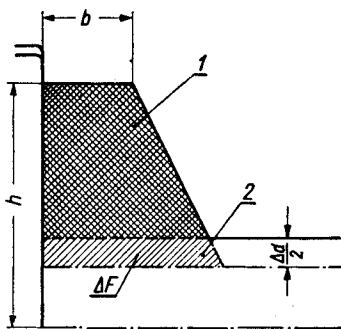


Fig. 7.63. Open flange deformation with Brovot's division: 1 — before roll pass, 2 — after roll pass

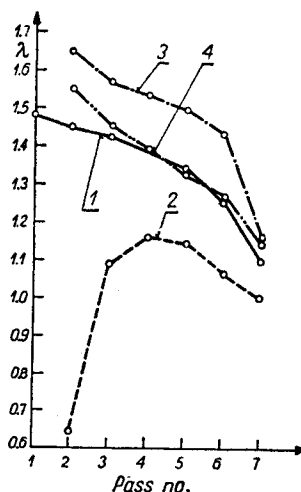


Fig. 7.64. Part elongation coefficients for I-beam 120 (see Appendix 2) with Tafel's division: 1 —  $\lambda_t$  total of pass, 2 —  $\lambda_w$  of web, 3 —  $\lambda_{fo}$  of open flange, 4 —  $\lambda_{fcl}$  of closed flange

It may be concluded in general that with Tafel's division  $\lambda$  coefficients found for the web are too small.

For this reason Cramer proposes a division (Fig. 7.60c) where the shaded profile parts are taken into account twice over in calculation of elongation coefficients.

The curves for total and partial elongation coefficients for this division are shown in Fig. 7.65. In this way a more uniform curve run is obtained, but the partial elongation coefficients always take greater values than  $\lambda_t$ , which is considered as a defect of Cramer's division.

The curves of elongation coefficients for Sokolov's division are shown in Fig. 7.66. From the form of these curves it can be seen that in the first roll passes the web is more intensively deformed than the flanges, and from the moment when web and flanges show equal values of  $\lambda$  the  $\lambda_w$  values are greater than  $\lambda_{fcl}$ .

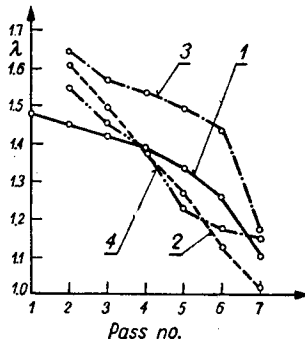


Fig. 7.65. Part elongation coefficients for I-beam 120 (see Appendix 2) with Cramer's division: 1 —  $\lambda_t$  total of pass, 2 —  $\lambda_w$  of web, 3 —  $\lambda_{fo}$  of open flange, 4 —  $\lambda_{fcl}$  of closed flange

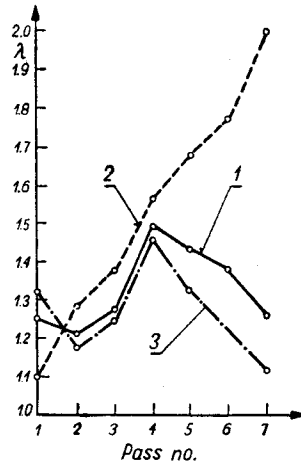


Fig. 7.66. Part elongation coefficients for I-beam 120 (see Appendix 2) with Sokolov's division: 1 —  $\lambda_t$  total of pass, 2 —  $\lambda_w$  of web, 3 —  $\lambda_f$  of flange

This division, in comparison with previous ones, shows a closer approach to the true deformation pattern, but nevertheless it cannot be considered as the best.

From this analysis it may be concluded that the method of division has a basic influence on the values obtained.

#### 7.6.4. ROLL PASS DESIGN OF ANGLES

To determine dimensions of the particular passes it is sufficient to know the end dimensions of profile, diameter of rolls and elongation coefficient. From equation (7.53) the draught coefficient which the initial section must have to give the required elongation coefficient, can be calculated. With known draught coefficient, the initial dimensions of section can be calculated.

The final dimensions of an angle are assumed to be  $b_2$  and  $h_2$  (Fig. 7.53) and the elongation coefficient equal to  $\lambda_1$ . From equation (3.47)

$$\lambda_1 = \gamma^{-(1-W)}$$

All the components of this formula are known already except the effective roll diameter which can be calculated from formula

$$D_{\text{effective}} = D_{\text{neutr}} - h_{2m}$$

where  $D_{\text{neutr}}$  = neutral diameter of roll.

Then the following values can be calculated:

$$h_1 = \frac{h_2}{\gamma_1}$$

$$b_1 = \frac{F_1}{h_1} = \frac{F_1}{h_2/\gamma_1} = \frac{F_1}{h_2} \gamma_1$$

The area  $F_1$  can be calculated knowing the elongation coefficient

$$F_1 = F_2 \lambda_1 = b_2 h_2 \lambda_1$$

In the finishing passes the breadth of the ingoing profile (hot dimensions) should be somewhat larger than the calculated ones by about 0.5–1.0%, to obtain sharp edges of the finished product.

With this method the most important matter is the correct choice of elongation coefficients in the particular roll passes.

If the ingoing bloom area is  $F_0$  and the finished section area  $F_n$ , relation (2.13) must be satisfied i.e.

$$\frac{F_0}{F_n} = \lambda_1 \lambda_2 \lambda_3 \dots \lambda_n$$

where  $\lambda_1, \lambda_2, \lambda_3 \dots \lambda_n$  are the elongation coefficients in consecutive roll passes.

In the final roll pass it is usual to take lower elongation coefficients than in the previous passes to avoid the setting up of stresses in the finished section due to lowered rolling temperature.

The spread and elongation can be calculated from equations (3.45) and (3.46), then with known spread coefficient the breadth of angle legs after a roll pass can be checked from

$$b_2 = \beta b_1$$

#### Example

The last, fifth roll pass is calculated, in the rolling of the angle  $75 \times 75 \times 10$  mm\*. This roll pass design taken from Dehez's work (Fig. 7.67) is obsolescent, but serves to illustrate the calculation method [10].

The profile leg dimensions before the fifth roll pass are as follows: (from the fourth roll pass)

$$\frac{1}{2} b_1 = 74.9 \text{ mm}$$

$$h_1 = 12.7 \text{ mm}$$

The leg thickness after the roll pass  $h_2 = 10.0$  mm.

The auxiliary coefficients for calculation of  $\beta$  and  $\lambda$  are

$$\epsilon_w = \frac{h_1}{D} = \frac{12.7}{450} = 0.0282$$

$$\delta_w = \frac{b_1}{h_1} = \frac{2 \times 74.9}{12.7} = 11.79$$

$$\gamma = \frac{h_2}{h_1} = \frac{10.0}{12.7} = 0.787$$

\* See Appendix 2.

The effective roll diameter  $D = 450$  mm. It is calculated as the difference between the neutral diameter and end height ( $h_2$ ). Hence the spread and elongation coefficients can be found

$$\beta = 1.00223 \quad \lambda = 1.267$$

Calculated leg breadth after roll pass is

$$\frac{1}{2}b_2 = \beta \frac{1}{2}b_1 = 1.00223 \times 74.9 = 75.1 \text{ mm}$$

The actual measured leg breadth is

$$\frac{1}{2}b'_2 = 76.2 \text{ mm}$$

The difference is

$$\frac{1}{2}(b_2 - b'_2) = 75.1 - 76.2 = -1.1 \text{ mm, i.e. } -1.5\%$$

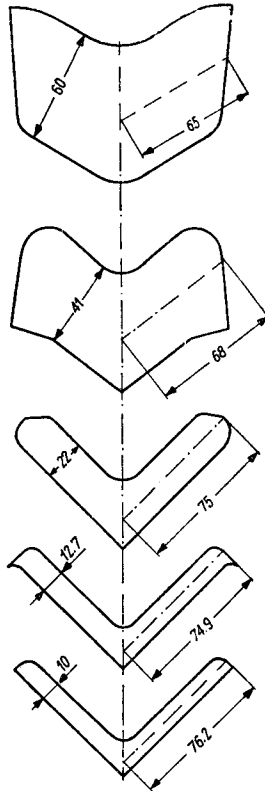


Fig. 7.67 Pass design of angle section  $75 \times 75$  from Dehez [123]

The breadth is well chosen, the difference occurs since the leg when spreading tends to retain its original outline of end shape (Fig. 7.68), with  $bb$  parallel to  $aa$ . In the final end shape of leg given by the end pass, the pass roundness on the inner edge forces the metal to fill the sharp edge of the section. In Fig. 7.68 the displaced areas are shaded, and the pass leg will be accurately filled by metal.

Checking the spread coefficient in particular roll passes shows that the pass design was well chosen and gives good results in practice.

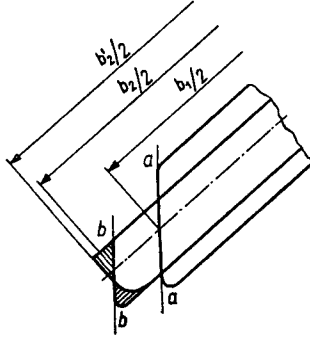


Fig. 7.68. Filling of angle legs [123]

#### 7.6.5. ROLL PASS DESIGN OF T-BEAMS

In T-beam roll pass design great difficulty is encountered in determining the elongation coefficients in particular roll passes for the various design methods. The T-shape section is composed of two parts exposed to different draughts and elongations (Fig. 7.55). Besides the mean elongation coefficient of the whole section, the free elongation coefficient of one part of the section must be assumed. The part with a smaller cross-sectional area should have a smaller elongation coefficient than the mean elongation coefficient of the whole profile. If the reverse is true, this part having its elongation tendency restricted by the greater area of the adjacent part, could very easily buckle.

The part of the section in the open part of pass should also have an elongation coefficient smaller than  $\lambda_m$ . In the finishing passes, metal flow from one part of the section to the other should be restricted. This condition will be fulfilled if

$$F_{A1} = (F_{A2} \pm F_x) \lambda_{mw} \quad \text{and} \quad F_{B1} = (F_{B2} \pm F_x) \lambda_{mw}$$

where  $F_x \rightarrow 0$ .

If the dimensions of finished profile, the rolling temperature and two elongation coefficients are known,  $\lambda_B$  can be calculated from equation

$$\lambda_B = \frac{\lambda_{mw}(F_{A2} + F_{B2}) - F_{A2}\lambda_A}{F_{B2}} \quad (7.143)$$

Knowing  $\lambda_A$ ,  $\lambda_B$  and  $\lambda_{mw}$  the web height before the roll pass (Fig. 7.57) can be calculated from equation (7.120)

$$h_{A1} = h_{A2} \lambda_A$$

From  $F_{A1}$  and  $h_{A1}$  the web breadth can be calculated (7.113)

$$b_{A1} = \frac{F_{A1}}{h_{A1}}$$

It is a mean width, since the pass sides must be slightly tapered to avoid sticking in the pass.

Knowing elongation coefficient  $\lambda_B$ , area  $F_{B2}$  and flange dimension  $h_{B2}$ , the draught coefficient which should act on the flange can be read off from the slide rule (Figs. 7.37 and 7.39).

Then the initial dimensions can be calculated as in angle section designing (7.120)

$$h_{B1} = \frac{h_{B2}}{\gamma_B}$$

and from (7.114)

$$b_{B1} = \frac{F_{B1}}{h_{B1}}$$

This method of calculation can be used only for the last three or four, or with large profiles for the last five passes. In the initial passes, and especially in shearing passes, great allowances must be made for metal flow between parts of the section. Metal should flow from parts with greater elongation coefficients to parts with lower coefficients. Since in the initial passes  $\lambda_A < \lambda_B$  is assumed, substituting in equations (7.124) and (7.125) gives

$$F_{A1} = (F_{A2} - F_x)\lambda_{mw} = \left[ F_{A2} - F_{B2} \left( 1 - \frac{\lambda_B}{\lambda_{mw}} \right) \right] \lambda_{mw} \quad (7.144)$$

$$F_{B1} = (F_{B2} + F_x)\lambda_{mw} = \left[ F_{B2} + F_{A2} \left( \frac{\lambda_A}{\lambda_{mw}} - 1 \right) \right] \lambda_{mw} \quad (7.145)$$

The value  $F_x$  should be chosen so that the last result gives an initial bloom section that is square or rectangular. Further calculations are exactly similar to the previous ones.

It is also possible to calculate the pass design with other initial data. If end profile dimensions, roll diameter and the flange draught coefficient  $\gamma_A$  are known, i.e. data for the closed part of the pass, then from equation (7.120)

$$\lambda_A = \frac{1}{\gamma_A}$$

If also  $\gamma_B$  for the flange is known,  $\lambda_B$  can be calculated by means of Otto's slide rule, and next from equation (7.91)  $\lambda_{mw}$  can be found. Further steps are similar to those previously described.

#### 7.6.6. ROLL PASS DESIGN OF CHANNELS AND I-BEAMS

As an example the channel in Fig. 7.69 is considered. If dimensions of the particular passes are required, it is sufficient to know the finished profile dimensions, the neutral roll diameter and mean elongation coefficient in particular roll passes and either the assumed number of roll passes or the approximate initial bloom dimensions.

The channel section is composed of two parts which are subjected to different draughts and elongations. Hence besides the mean elongation coefficient of the whole section the free elongation coefficient of one of the parts must be assumed.

The part of the section in the closed part of pass should have a smaller elongation coefficient than the mean elongation coefficient of the whole section.

If the finished profile dimensions at the rolling temperature and two elongation coefficients are known, then the dimensions of the preceding pass can be calculated. It is assumed initially that the metal does not flow from one part to the other in the finishing roll passes. This condition is satisfied if

$$F_{A1} = F_{A2} \lambda_{mw}$$

$$F_{B1} = F_{B2} \lambda_{mw}$$

With  $F_{A1}$ ,  $F_{B2}$ ,  $\lambda_{mw}$ , and  $\lambda_A$  known,  $\lambda_B$  can be calculated. With known  $\lambda_A$  and  $\lambda_B$  the draughts which must be applied to give flanges and web the required elongation coefficients can be determined.

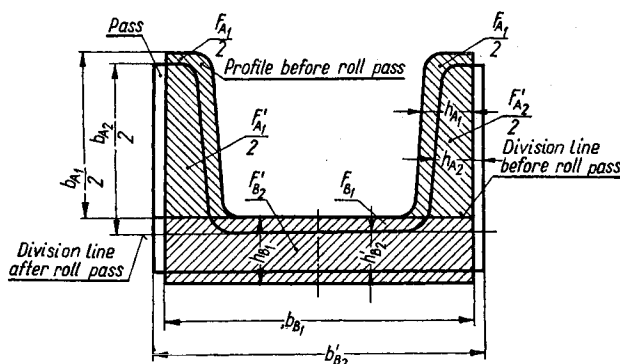


Fig. 7.69. Rolling of channel

With  $\gamma_A$  and  $\gamma_B$  known the principal dimensions of web and flanges before the pass can be calculated (7.113) and (7.120)

$$h_{A1} = \frac{h_{A2}}{\gamma_A} \quad h_{B1} = \frac{h_{B2}}{\gamma_B}$$

$$b_{A1} = \frac{F_{A1}}{h_{A1}} \quad b_{B1} = \frac{F_{B1}}{h_{B1}}$$

In the initial passes metal flow cannot be avoided due to the differing partial elongation coefficients. Therefore from (7.124) and (7.125)

$$F_{A1} = (F_{A2} \pm F_x) \lambda_{mw}$$

$$F_{B1} = (F_{B2} \pm F_x) \lambda_{mw}$$

where  $F_x$  is the amount of metal flowing from web to the flanges calculated from relative areas or from equations (7.104) and (7.105). The  $F_x$  value should be so chosen that the resulting initial bloom will have a square or rectangular cross-section. Further calculations are similar to the preceding ones.

This calculation method gives mean pass dimensions without taking into account rounding of corners, taper in a closed pass, and the angle of divergence of legs in an angle section etc. Calculation of these quantities should be based on the practical experience of a skilled roll designer.

The correct choice of elongation coefficients in the particular passes according to type of pass used is of paramount importance.



The problem of speeding up the rolling of channels and I-beams can be limited to establishing the least number of roughing roll passes required, from the consideration of theoretical shape design and lateral flow of metal.

Theoretically, in order to obtain small and medium-sized shapes with double symmetry, the number of passes should be 4 to 5, on condition that rolling begins from a suitable preliminary section. These passes should be: 1 shearing pass, 1 to 2 roughing, 1 leader and 1 finishing pass.

All transverse metal flow should take place in the shearing and forming passes, so that virtually no flow occurs in the leader and finishing passes, in order to reduce the internal stresses in the finished product, and also reduce roll wear.

As an example, the pass design to give a 200 mm channel\* in 7 roll passes is shown (Fig. 7.70) with elongation coefficient [115]

$$\lambda_t = 1.55 \times 1.48 \times 1.44 \times 1.38 \times 1.39 \times 1.24 \times 1.10 = 8.65$$

$$\lambda_m = 1.36$$

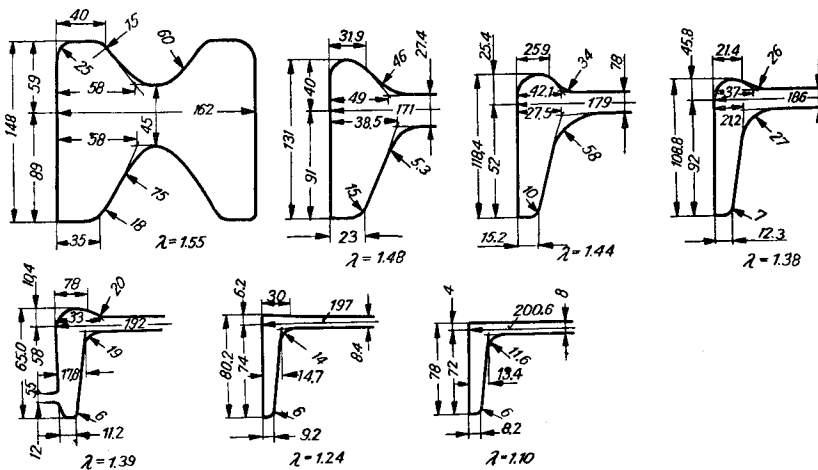


Fig. 7.70. Rolling of 200 mm channel (see Appendix 2) in 7 roll passes [115]

Another example (Fig. 7.71) shows the pass design of the same 200 mm channel in 5 roll passes where

$$\lambda_t = 1.55 \times 1.82 \times 1.60 \times 1.40 \times 1.14 = 7.2$$

$$\lambda_m = 1.5$$

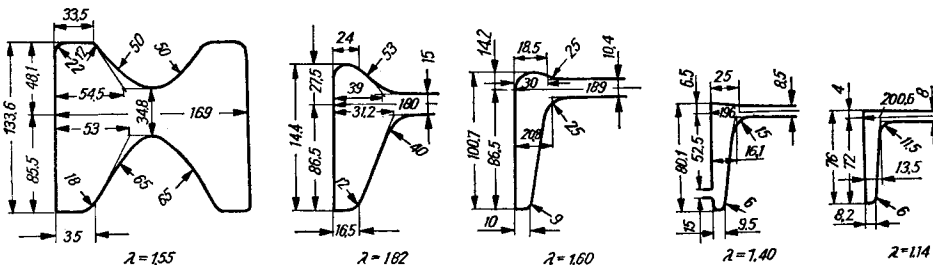


Fig. 7.71. Rolling of 200 mm channel (see Appendix 2) in 5 roll passes [115]

\* See Appendix 2.

Both these examples have many defects, such as non-equalizing of metal flow in last and finishing roll pass, which in particular should be eliminated before practical application.

When planning the speeding up of shape rolling two factors neglected so far must be taken into account, roll strength and drive power.

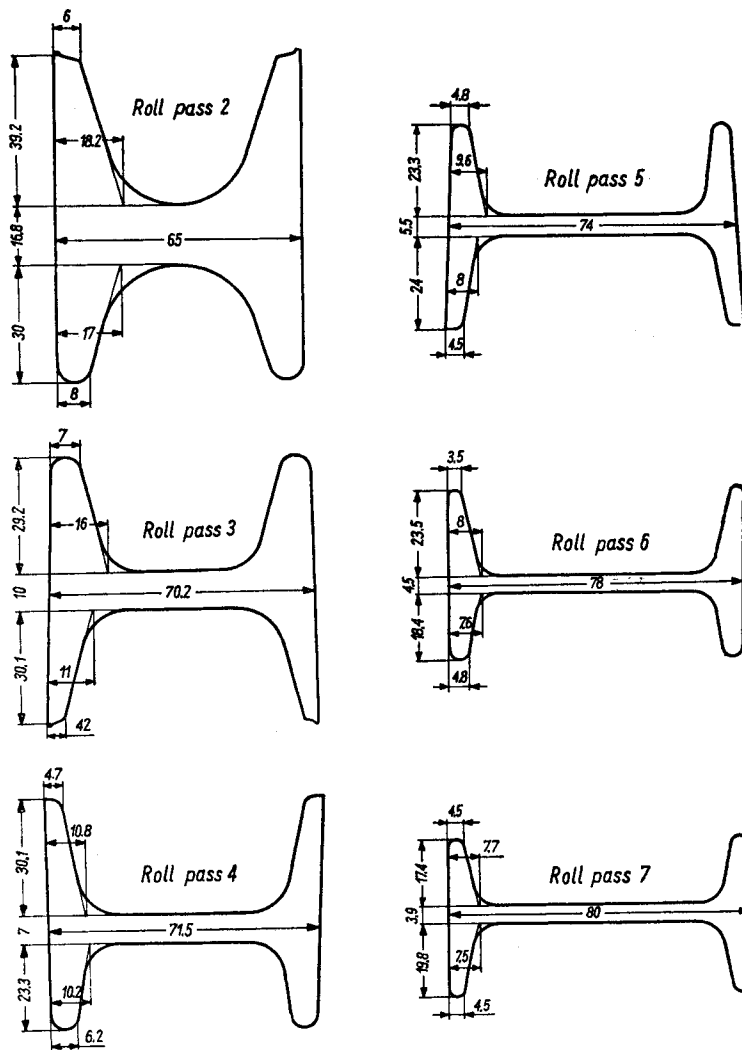


Fig. 7.72. Sections of successive roll passes of I-beams 80 (see Appendix 2) with new rolls [10]

In modern high-efficiency roll pass design roll strength should be checked, i.e. roll forces must satisfy the relationship (5.83)

$$P_1 \cong P_2 \cong P_3 \cong \dots \cong P_n \leq P_{\text{per}} \quad \text{for checking of rolls,}$$

and torques must satisfy the following relationship (5.84):

$$M_{w1} \cong M_{w2} \cong M_{w3} \cong \dots \cong M_{wn} \leq M_{\text{per}} \quad \text{for checking of drive.}$$

This method was formerly used for blooming mills, but is recommended for all modern high-efficiency pass design to reduce the number of roll passes.

Samples were taken from particular roll passes during rolling of the I-beam 80\* (Fig. 7.72).

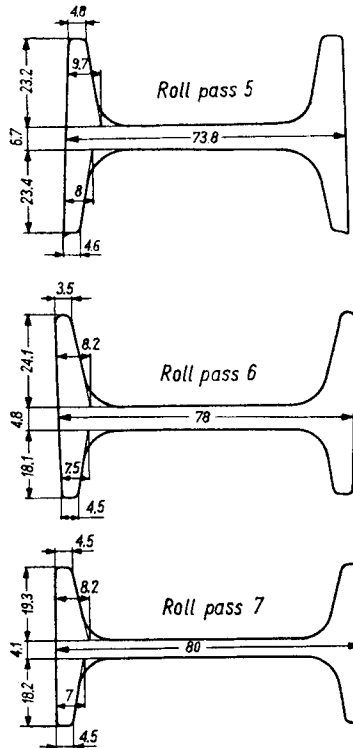


Fig. 7.73. Sections of three last roll passes of I-beam 80 (see Appendix 2) (Fig. 7.72) with worn rolls [10]

Comparison of results obtained from calculation and measurement are given in Table 7.12.

In the second and third roll passes spread is over-restricted, as is shown by over-filled sections from these passes. The finished section is for the most part underfilled and without sharp cut edges. In the open part of pass the flange should have the same width as in the closed part, i.e. 19.8 mm, hence 2.4 mm is lacking to complete filling.

For the same section, samples from the three last roll passes were taken, with the rolls already worn (Fig. 7.73).

In order to check theoretical calculations and compare with the actual metal flow and consequent roll wear, an I-beam 80\* was checked during rolling. Calculations were made for new rolls, and then samples were taken from the three last roll passes of the same section near the end of rolling, when the rolls were worn. It is found that metal flow affects roll wear. The pass where most metal displacement occurred showed the most wear. Another cause of wear is restricted spread, when metal pressure on the sides is very great. A change of free elongation of section parts is desirable.

\* See Appendix 2.

TABLE  
CALCULATION OF I-BEAM

| Roll<br>pass<br>No. | Web            |                |                |                          | Flange in the closed part of pass |                |                            |                          |
|---------------------|----------------|----------------|----------------|--------------------------|-----------------------------------|----------------|----------------------------|--------------------------|
|                     | $b_{B1}$<br>mm | $h_{B1}$<br>mm | $h_{B2}$<br>mm | $F_B$<br>mm <sup>2</sup> | $\frac{1}{2} b_{A1}$<br>mm        | $h_{A1}$<br>mm | $\frac{1}{2} b_{A2}$<br>mm | $F_A$<br>mm <sup>2</sup> |
| 2                   | 64.0           | 36.8           | 16.8           | 2355                     | 38.5                              | 13.45          | 30.0                       | 1035                     |
| 3                   | 65.0           | 16.8           | 10.0           | 1091                     | 39.2                              | 12.1           | 29.2                       | 948                      |
| 4                   | 70.2           | 10.0           | 7.0            | 702                      | 30.1                              | 7.6            | 23.3                       | 457                      |
| 5                   | 71.5           | 7.0            | 5.5            | 501                      | 30.1                              | 7.75           | 23.3                       | 467                      |
| 6                   | 74.0           | 5.5            | 4.5            | 407                      | 24.0                              | 6.25           | 18.4                       | 300                      |
| 7                   | 78.0           | 4.5            | 3.9            | 351                      | 23.3                              | 5.75           | 19.8                       | 270                      |

\* See Appendix 2.

In the fifth roll pass the spread of the flange in the open profile part was restricted. In the section marked fins were seen. In the finishing pass the flanges are underfilled and the edges of section are not sharp. This is caused by over-large flange elongation in the closed profile part.

Also the flange width entering an open profile part should be somewhat greater.

All the results of calculation of new and worn passes are given in Table 7.13.

Free and mean spreads and widths of flanges in particular roll passes for the I-80\* section with new rolls and worn rolls are given in Fig. 7.74.

#### 7.6.7. K. RYTEL'S METHOD OF PASS DESIGN, PARTICULARLY FOR I-BEAMS [131]

K. Rytel has published an ingenious and original method of shape designing such that all parts have the same elongation coefficient, i.e. from equation (7.134)

$$\lambda_{mw} \simeq \lambda_A \simeq \lambda_B \simeq \lambda_C \simeq \lambda_I \simeq \dots \simeq \lambda_N$$

Using the equations relating coefficients (7.92) the area ratios can be found

$$\lambda_{mw} = \frac{F_1}{F_2} \simeq \frac{F_{A1} + F_{B1}}{F_{A2} + F_{B2}} \simeq \frac{F_{A1}}{F_{A2}} \simeq \frac{F_{B1}}{F_{B2}} \quad (7.146)$$

In this case there is no metal flow from one part to the other since they are uniformly elongated.

There is also no problem to determine elongation coefficients as they are all equal to  $\lambda_{mw}$ , and roll pass design is very simple.

The areas of particular components were calculated by means of auxiliary formulae. For the flange

$$\lambda_{mw} \gamma_B \beta_B = 1 \quad (7.147)$$

\* See Appendix 2.

7.12  
80 SECTION\*

| Flange in the open part of pass |                |                |                          | Elongation coefficients |             |             |                |                          |
|---------------------------------|----------------|----------------|--------------------------|-------------------------|-------------|-------------|----------------|--------------------------|
| $\frac{1}{2} b_{C1}$<br>mm      | $h_{C1}$<br>mm | $h_{C2}$<br>mm | $F_C$<br>mm <sup>2</sup> | $\lambda_A$             | $\lambda_B$ | $\lambda_C$ | $\lambda_{mw}$ | $F_x$<br>mm <sup>2</sup> |
| 27.2                            | 18.7           | 12.1           | 1017                     | 1.283                   | 1.695       | 1.373       | 1.551          | 84.0                     |
| 30.0                            | 12.5           | 7.6            | 750                      | 1.342                   | 1.500       | 1.483       | 1.469          | 343.0                    |
| 29.2                            | 11.0           | 7.75           | 642                      | 1.292                   | 1.368       | 1.328       | 1.346          | 40.0                     |
| 23.3                            | 8.2            | 6.25           | 382                      | 1.292                   | 1.240       | 1.235       | 1.260          | 2.0                      |
| 23.3                            | 7.7            | 5.75           | 359                      | 1.304                   | 1.207       | 1.256       | 1.253          | 25.0                     |
| 18.4                            | 6.2            | 6.1            | 228                      | 1.187                   | 1.147       | 1.015       | 1.122          | 13.0                     |
|                                 |                |                |                          |                         |             |             |                | 9.5                      |
|                                 |                |                |                          |                         |             |             |                | 2.0                      |
|                                 |                |                |                          |                         |             |             |                | 12.0                     |
|                                 |                |                |                          |                         |             |             |                | 14.0                     |
|                                 |                |                |                          |                         |             |             |                | 4.0                      |

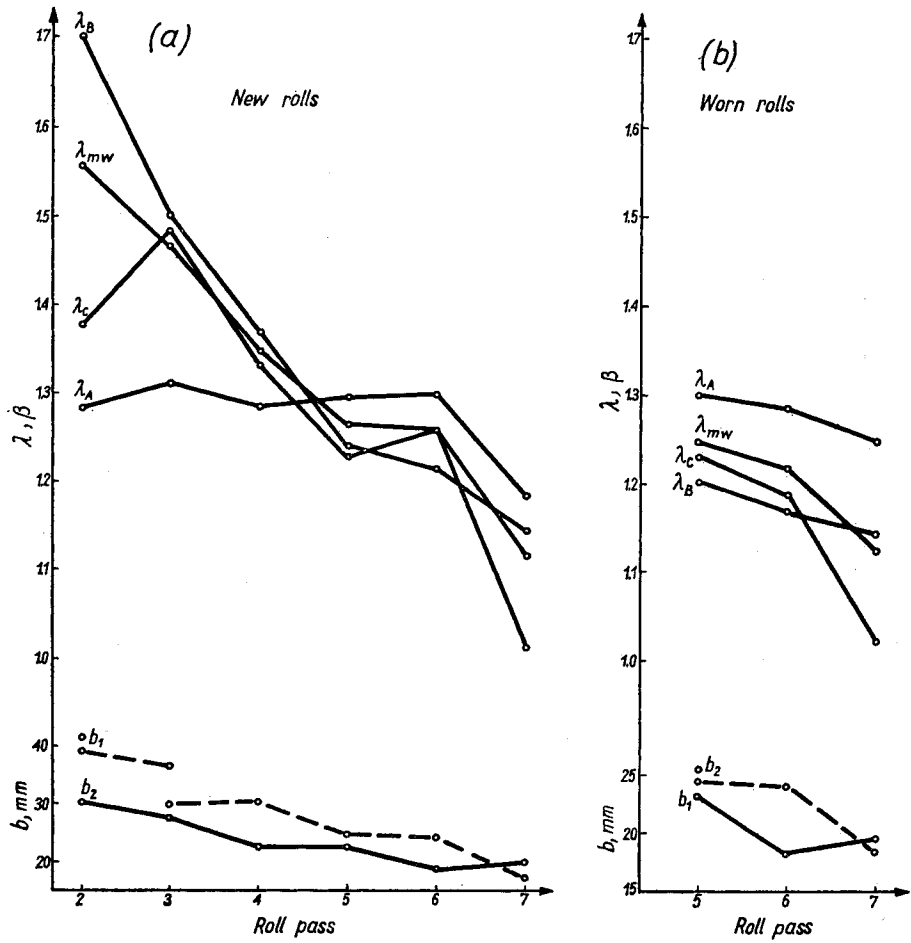
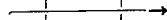
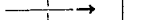
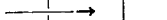


Fig. 7.74. Elongation coefficients and widths of flanges of I-beams 80 (see Appendix 2) in successive roll passes with: (a) new rolls and (b) worn rolls [123]

TABLE 7.13  
RESULTS OF CALCULATION OF THREE LAST PASSES FOR I-BEAM 80 SECTION\* (FIGS. 7.72 and 7.73)

| Pass<br>No.   | Web<br>mm |          |          |          | Flange in the closed part of pass<br>mm |             |          |          | Flange in the open part of pass<br>mm |             |          |          | Direction of metal<br>flow                                                            |       |       | Amount of<br>metal<br>transferred |
|---------------|-----------|----------|----------|----------|-----------------------------------------|-------------|----------|----------|---------------------------------------|-------------|----------|----------|---------------------------------------------------------------------------------------|-------|-------|-----------------------------------|
|               | $b_{B1}$  | $b_{B2}$ | $h_{B1}$ | $h_{B2}$ | $1/2b_{A1}$                             | $1/2b_{A2}$ | $h_{A1}$ | $h_{A2}$ | $1/2b_{C1}$                           | $1/2b_{C2}$ | $h_{C1}$ | $h_{C2}$ | $F_A$                                                                                 | $F_B$ | $F_C$ | $F_x$ , mm <sup>2</sup>           |
|               | 1         | 2        | 3        | 4        | 5                                       | 6           | 7        | 8        | 9                                     | 10          | 11       | 12       | 13                                                                                    | 14    | 15    | 16                                |
| In new rolls  |           |          |          |          |                                         |             |          |          |                                       |             |          |          |                                                                                       |       |       |                                   |
| 5             | 71.5      | 74.0     | 7.0      | 5.5      | 30.1                                    | 23.3        | 7.75     | 7.7      | 23.3                                  | 24.4        | 8.2      | 6.25     |    |       |       | 9.5                               |
|               |           |          |          |          |                                         |             |          |          |                                       |             |          |          |    |       |       | 2.0                               |
| 6             | 74.0      | 78.0     | 5.5      | 4.5      | 24.0                                    | 18.4        | 6.25     | 6.2      | 23.3                                  | 23.5        | 7.7      | 5.75     |    |       |       | 12.0                              |
|               |           |          |          |          |                                         |             |          |          |                                       |             |          |          |    |       |       | 14.0                              |
| 7             | 78.0      | 80.0     | 4.5      | 3.9      | 23.5                                    | 19.8        | 5.75     | 6.0      | 18.4                                  | 17.4        | 6.2      | 6.1      |    |       |       | 4.0                               |
|               |           |          |          |          |                                         |             |          |          |                                       |             |          |          |    |       |       |                                   |
| In worn rolls |           |          |          |          |                                         |             |          |          |                                       |             |          |          |                                                                                       |       |       |                                   |
| 5             | 71.5      | 73.8     | 7.0      | 5.7      | 30.1                                    | 23.2        | 7.75     | 7.25     | 23.3                                  | 23.4        | 8.2      | 6.3      |    |       |       | 21.0                              |
|               |           |          |          |          |                                         |             |          |          |                                       |             |          |          |    |       |       | 15.5                              |
| 6             | 73.8      | 78.0     | 5.7      | 4.8      | 23.4                                    | 18.1        | 6.3      | 6.0      | 23.2                                  | 24.0        | 7.25     | 5.85     |    |       |       | 25.0                              |
|               |           |          |          |          |                                         |             |          |          |                                       |             |          |          |  |       |       | 1.5                               |
| 7             | 78.0      | 80.0     | 4.8      | 4.1      | 24.1                                    | 19.3        | 5.85     | 6.35     | 18.1                                  | 18.2        | 6.0      | 5.75     |  |       |       | 9.0                               |
|               |           |          |          |          |                                         |             |          |          |                                       |             |          |          |  |       |       | 4.0                               |

\* See Appendix 2.

and hence

$$h_{B1} = \frac{h_{B2}}{\gamma_B} \quad (7.148)$$

Also

$$h_{B1} = h_{B0} + h_{Bcl} \quad (7.149)$$

where

$h_{Bcl}$  = height of closed part of flange,

$h_{B0}$  = height of open part of flange.

Since in the closed part  $\beta_B = 1.0$  to 1.05 then

$$\lambda_B \approx \frac{1}{\gamma_B}$$

hence

$$h_{B1} = h_{B2} \lambda_B \beta_B \quad (7.150)$$

The mean widths can be calculated from relation

$$\frac{F_{B2}}{F_2} = \frac{F_{B1}}{F_1} \quad (7.151)$$

which also results from the relation of coefficients. For the web the following relation should also be satisfied:

$$\lambda_{mw} \gamma_A \beta_A = 1 \quad (7.152)$$

Here  $\beta_A$  is close to 1.0 and part  $A$  can easily be calculated similarly as for part  $B$ .

To calculate I-beams K. Rytel uses a general equation [131]

$$\lambda_{mw, i} \gamma_{fl, i} = \sqrt{(\lambda_i \gamma_{A1} + 0.6)^2 + 0.64 \left( \frac{i-1}{n-1} \right)^2} - 0.6 \quad (7.153)$$

where

$\gamma_{fl, i}$  = flange draught coefficient,

$i$  = successive roll pass,

$n$  = number of roll passes.

In practice this roll pass design is divided into the following operations:

(1) The assumption of the number of roughing roll passes  $n$ , and elongation coefficients  $\lambda$  in every roll pass, and flange coefficient  $\gamma_{fl, 1}$  after the first roll pass (shearing pass).

(2) Calculation of bloom.

(3) Calculation of section area  $F$  in all roll passes from  $F_{i-1} = F_i \lambda_i$  and the calculation of areas of particular profile parts, i.e. web and flanges from equation (7.151). The flange area is considered to be 1/4 of the difference between the area of the whole profile and the area of the web. This division is necessary since a flange shape is differently designed in an open pass and in a closed one.

(4) Calculation of  $\lambda_i \gamma_{fl, i}$  from (7.153) and since  $\lambda_i$  is already known, the calculation of  $\gamma_{fl, i}$ .

(5) Calculation of total flange height in every roll pass (7.148), beginning from the known height of finished profile flanges.

- (6) Calculation of flange heights rolled in the open and closed passes from formulae (7.149) and (7.150), on a basis of assumed  $\beta$  values in the closed profile parts.
- (7) Calculation of flange mean widths (from profile area and height).
- (8) Calculation of web dimensions. The calculated value of web dimensions should be taken as mean ones and the web width should ensure appropriate pass taper. For this reason, pass drawings should be prepared at this stage.
- (9) Finalizing of drawings, assumption of fillet radii, neutral axis, and design positioning in rolls.

As an example the calculation procedure is given for the design of a barrel I-beam in four forming passes ( $n = 4$ ) [131]. The finished profile area  $F_n = 270 \text{ mm}^2$ , flange height  $h_{fl,n} = 24 \text{ mm}$ , section width  $b_n = 32 \text{ mm}$ .

In the finished profile the web area  $F_{w4} = 90 \text{ mm}^2$ , and the flange area  $F_{fl,4} = 45 \text{ mm}^2$ . The flange area is 1/6 of the whole profile area, the web 1/3 of it.

This proportion is kept throughout the whole design as equal partial elongation coefficients are assumed.

The initial section is a rectangle  $21 \times 46.5 \text{ mm}$  of area  $F_0 = 970 \text{ mm}^2$ .

It is assumed that  $\lambda_1 = 1.51$ ,  $\lambda_2 = 1.48$ ,  $\lambda_3 = 1.34$ ,  $\lambda_4 = 1.20$  and  $\gamma_{fl,i} = 0.82$ .

From condition (2.13) the bloom area can be found

$$F_0 = F_n \lambda_1 \lambda_2 \lambda_3 \lambda_4 = 270 \times 1.51 \times 1.48 \times 1.34 \times 1.20 = 970 \text{ mm}^2$$

$$\lambda_1 \gamma_{fl,i} = 1.51 \times 0.82 = 1.24$$

After the first roll pass the total area:  $F_1 = 642 \text{ mm}^2$ , the web area  $F_{w1} = 214 \text{ mm}^2$ , the flanges area  $F_{fl,1} = 107 \text{ mm}^2$ .

After the second roll pass

$$F_2 = \frac{F_1}{\lambda_2} = 434 \text{ mm}^2 \quad F_{w2} = 144.8 \text{ mm}^2, \quad F_{fl,2} = 72.4 \text{ mm}^2$$

After the third roll pass

$$F_3 = 324 \text{ mm}^2, \quad F_{w3} = 108 \text{ mm}^2, \quad F_{fl,3} = 54.0 \text{ mm}^2$$

After the fourth roll pass, the finished profile is obtained with dimensions as given

Next the  $\lambda_i \gamma_{fl,i}$  product in every roll pass must be calculated. In the first roll pass  $\lambda_1 \gamma_{fl,i} = 1.24$ . Further roll passes can be calculated from equation (7.153).

$$\lambda_2 \gamma_{fl,2} = \sqrt{(1.24 + 0.6)^2 - 0.64 \left( \frac{2-1}{4-1} \right)^2} - 0.6 = 1.22$$

$$\lambda_3 \gamma_{fl,3} = \sqrt{(1.24 + 0.6)^2 - 0.64 \left( \frac{3-1}{4-1} \right)^2} - 0.6 = 1.16$$

$$\lambda_4 \gamma_{fl,4} = \sqrt{(1.24 + 0.6)^2 - 0.64 \left( \frac{4-1}{4-1} \right)^2} - 0.6 = 1.06$$

As  $\lambda_i$  values were already assumed, the draught coefficient of the whole flanges (open and closed parts together) can be calculated in all the roll passes.

In the second roll pass

$$\gamma_{fl,2} = \frac{1.22}{1.48} = 0.824$$

In the third roll pass

$$\gamma_{fl,3} = \frac{1.16}{1.34} = 0.866$$

In the fourth roll pass

$$\gamma_{fl,4} = \frac{1.06}{1.20} = 0.883$$



The height calculation of the whole flange in the roll passes is now simple. Beginning from the end, since  $h_{fl,4} = 24$  mm is known,

$$h_{fl,3} = \frac{h_{fl,4}}{\gamma_{fl,4}} = \frac{24}{0.883} = 27.2 \text{ mm}$$

$$h_{fl,2} = \frac{27.2}{0.866} = 31.4 \text{ mm}$$

$$h_{fl,1} = \frac{31.4}{0.824} = 38.1 \text{ mm}$$

$$h_{fl,0} = \frac{h_{fl,1}}{\gamma_{fl,1}} = \frac{38.1}{0.820} = 46.5 \text{ mm}$$

Next  $\beta_{cl} = 1.05$  is assumed.

The height of flange rolled in the closed part in the last roll pass  $h_{cl,4} = 12$  mm. From (7.150) for  $i = 4$

$$h_{o,3} = 12 \times 1.2 \times 1.05 = 15.1 \text{ mm}$$

From equation (7.149) the height of the closed part in the leader pass can be calculated

$$h_{cl,3} = h_{fl,3} - h_{o,3} = 27.2 - 15.1 = 12.1 \text{ mm}$$

The height of flange part in the intermediate pass is calculated as in the leader pass

$$h_{o,2} = 12.1 \times 1.34 \times 1.05 = 17.0 \text{ mm}$$

$$h_{cl,2} = 31.4 - 17.0 = 14.4 \text{ mm}$$

and in the shearing pass

$$h_{o,1} = 14.5 \times 1.48 \times 1.05 = 22.4 \text{ mm}$$

$$h_{cl,1} = 38.1 - 22.4 = 15.7 \text{ mm}$$

The mean flange width is calculated dividing its area by the height.  
For the first pass

$$b_{o,1} = \frac{107}{22.4} = 4.78 \text{ mm}$$

$$b_{cl,1} = \frac{107}{15.7} = 6.82 \text{ mm}$$

for the second pass

$$b_{o,2} = \frac{72.4}{17.0} = 4.26 \text{ mm}$$

$$b_{cl,2} = \frac{72.4}{14.4} = 5.03 \text{ mm}$$

for the third

$$b_{o,3} = \frac{54.0}{15.1} = 3.58 \text{ mm}$$

$$b_{cl,3} = \frac{54.0}{12.1} = 4.46 \text{ mm}$$

The dimensions in the fourth and last roll pass define the shape of the finished product and need not be calculated.

The mean widths are considered as auxiliary dimensions to facilitate pass drawing. Figure 7.75 shows the method of drawing of flange shapes, after the side wall taper is determined, and how these shapes may be corrected as required.

The analysis of the roll pass design of the "Pokal" section used for mine walling [130] is given.

A steelworks in the German Democratic Republic asked for advice on pass design for rolling the "Pokal" section, as shown in Fig. 7.76.

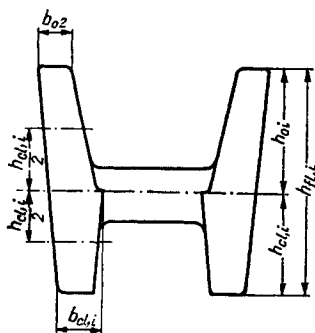


Fig. 7.75. Division of I-beam section after Rytel

For this analysis equations for the mean elongation coefficient were used (7.90)–(7.93), and also Górecki's equations for metal flow from one part of section to the other (7.96)–(7.105).

In Fig. 7.77 a schematic representation of metal flow is shown in the individual roll passes, the corresponding data are tabulated in Table 7.14.

Even after corrections the profile was not correctly filled (Fig. 7.78).

A second pass design project based primarily on Holzweiler's formulae was carried out in the German Democratic Republic, and also proved unsatisfactory. Figure 7.79 gives the schematic representation of metal flow for this project, for which data are given in Table 7.15. This design gave an unsatisfactory filling of passes, as shown in Fig. 7.80.

Figure 7.81 shows the roll pass design worked out by K. Rytel [131] in which equal values of elongation coefficients of all parts of section are used, to minimize metal flow from one part to another. In this design the number of roll passes was reduced to 7, as against 9 used previously.

First the areas of the particular parts of section were divided into rectangles, which in turn were replaced by equivalent trapezoids. Then suitable tapers and fillets were introduced, without reducing the sectional part areas. Only in the shearing and next pass (passes 1 and 2) it was necessary to introduce considerable changes of section by means of suitable fillets shown in Fig. 7.81. All the non-uniform metal flow was concentrated in these two roll passes. Checking by planimeter measurements, it was aimed to maintain constant areas of the various parts of the section.

The following elongation coefficients were chosen:

$$\lambda_1 = 1.3, \quad \lambda_2 = 1.3, \quad \lambda_3 = 1.3, \quad \lambda_4 = 1.24,$$

$$\lambda_5 = 1.19, \quad \lambda_6 = 1.16, \quad \lambda_7 = 1.1$$

Table 7.16 gives individual pass dimensions for roll passes 1–7. Practical tests showed the complete correctness of this design.

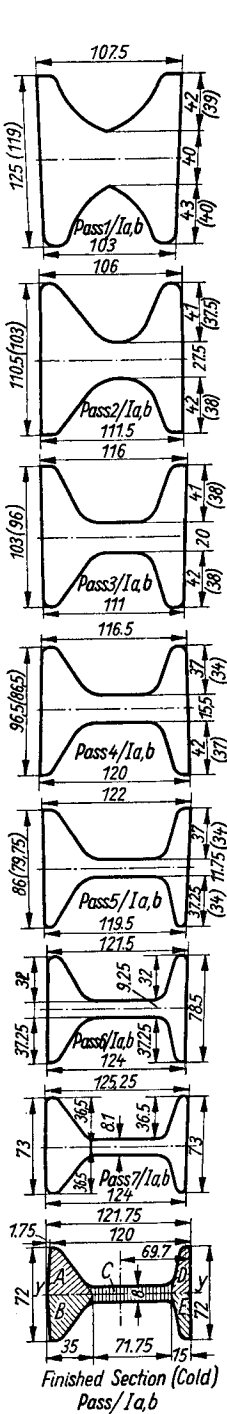


Fig. 7.76. First roll pass design of "Pokal" section [130] (dimensions in parenthesis concern worn rolls)

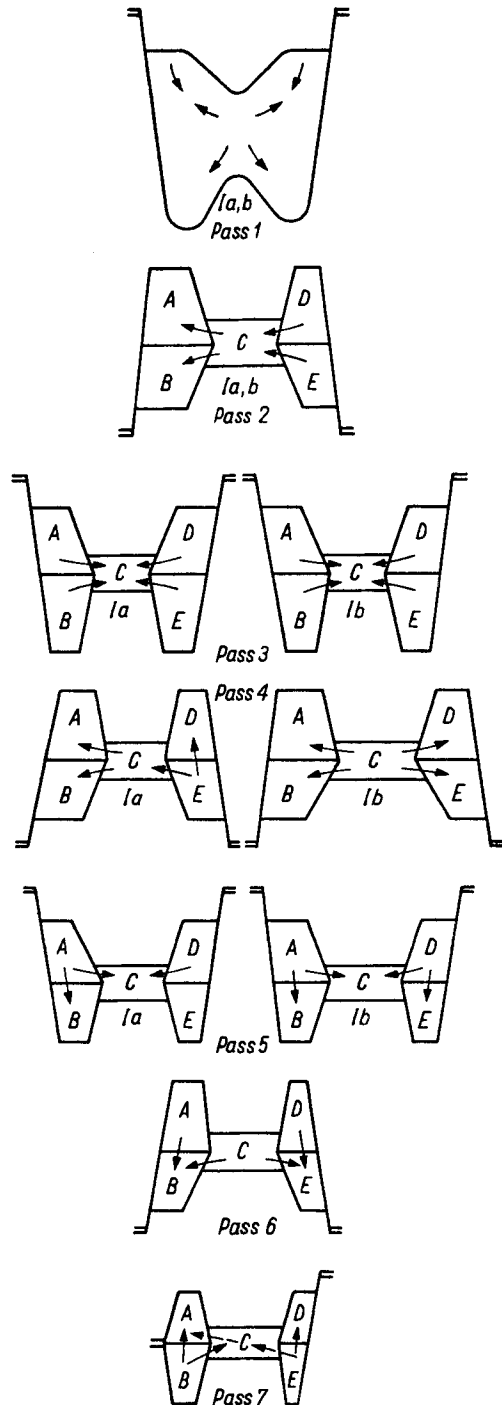


Fig. 7.77. Division of section and metal flow scheme for first roll pass design of "Pokal" section [130]

TABLE 7.14  
DATA FOR FIG. 7.77 [130]

| Pass groove     | Roll pass No. | Section                |             | Part A                   |             |                             | Part B                   |             |                             | Part C                   |             |                             | Part D                   |             |                             | Part E                   |             |                             |
|-----------------|---------------|------------------------|-------------|--------------------------|-------------|-----------------------------|--------------------------|-------------|-----------------------------|--------------------------|-------------|-----------------------------|--------------------------|-------------|-----------------------------|--------------------------|-------------|-----------------------------|
|                 |               | $F$<br>mm <sup>2</sup> | $\lambda_m$ | $F_A$<br>mm <sup>2</sup> | $\lambda_A$ | $F_{x1}$<br>mm <sup>2</sup> | $F_B$<br>mm <sup>2</sup> | $\lambda_B$ | $F_{x1}$<br>mm <sup>2</sup> | $F_C$<br>mm <sup>2</sup> | $\lambda_C$ | $F_{x1}$<br>mm <sup>2</sup> | $F_D$<br>mm <sup>2</sup> | $\lambda_D$ | $F_{x1}$<br>mm <sup>2</sup> | $F_E$<br>mm <sup>2</sup> | $\lambda_E$ | $F_{x1}$<br>mm <sup>2</sup> |
| 0               | 0             | 1955 × 92              | —           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           |
| I Knifing pass  | 1             | —                      | —           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           |
| II Knifing pass | 2             | —                      | —           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           |
| 1               | 3             | —                      | —           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           |
| 2               | 4             | 7360                   | 1.258       | 1830                     | —           | —                           | 1630                     | —           | —                           | 1130                     | —           | —                           | 1570                     | —           | —                           | 1200                     | —           | —                           |
| 3               | 5             | 6618                   | 1.112       | 1540                     | 1.189       | +100                        | 1410                     | 1.154       | + 54                        | 1660                     | 0.681       | −644                        | 995                      | 1.58        | +418                        | 1013                     | 1.185       | +66                         |
| 4               | 6             | 4801                   | 1.379       | 1220                     | 1.263       | −103                        | 1200                     | 1.175       | −178                        | 930                      | 1.785       | +273                        | 731                      | 1.36        | − 10                        | 720                      | 1.41        | +77                         |
| 5               | 7             | 3946                   | 1.218       | 966                      | 1.263       | + 37                        | 1020                     | 1.175       | − 34                        | 823                      | 1.130       | − 60                        | 543                      | 1.348       | + 58                        | 592                      | 1.217       | 0                           |
| 6               | 8             | 3361                   | 1.173       | 798                      | 1.213       | + 28                        | 910                      | 1.12        | − 41                        | 685                      | 1.202       | + 16                        | 457                      | 1.19        | + 5                         | 511                      | 1.16        | − 6                         |
| 7               | 9             | 3049                   | 1.101       | 784                      | 1.007       | − 69                        | 794                      | 1.148       | + 32                        | 615                      | 1.112       | + 6                         | 423                      | 1.08        | − 9                         | 423                      | 1.21        | +42                         |

TABLE 7.15  
DATA FOR FIGS. 7.79 AND 7.80 [130]

| Pass groove     | Roll pass No. | Section                |             | Part A                   |             |                             | Part B                   |             |                             | Part C                   |             |                             | Part D                   |             |                             | Part E                   |             |                             |
|-----------------|---------------|------------------------|-------------|--------------------------|-------------|-----------------------------|--------------------------|-------------|-----------------------------|--------------------------|-------------|-----------------------------|--------------------------|-------------|-----------------------------|--------------------------|-------------|-----------------------------|
|                 |               | $F$<br>mm <sup>2</sup> | $\lambda_m$ | $F_A$<br>mm <sup>2</sup> | $\lambda_A$ | $F_{x1}$<br>mm <sup>2</sup> | $F_B$<br>mm <sup>2</sup> | $\lambda_B$ | $F_{x1}$<br>mm <sup>2</sup> | $F_C$<br>mm <sup>2</sup> | $\lambda_C$ | $F_{x1}$<br>mm <sup>2</sup> | $F_D$<br>mm <sup>2</sup> | $\lambda_D$ | $F_{x1}$<br>mm <sup>2</sup> | $F_E$<br>mm <sup>2</sup> | $\lambda_E$ | $F_{x1}$<br>mm <sup>2</sup> |
| 0               | 0             | 1955 × 92              | —           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           |
| I Knifing pass  | 1             | —                      | —           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           |
| II Knifing pass | 2             | —                      | —           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           | —                        | —           | —                           |
| 1               | 3             | 8575                   | —           | 1370                     | —           | —                           | 1370                     | —           | —                           | 3795                     | —           | —                           | 1020                     | —           | —                           | 1020                     | —           | —                           |
| 2               | 4             | 6975                   | 1.229       | 1300                     | 1.053       | −185                        | 1300                     | 1.053       | −185                        | 2655                     | 1.429       | +430                        | 808                      | 1.139       | −51                         | 840                      | 1.215       | −10                         |
| 3               | 5             | 5720                   | 1.22        | 1130                     | 1.15        | − 63                        | 1130                     | 1.15        | − 63                        | 2020                     | 1.313       | +153                        | 720                      | 1.222       | + 1                         | 720                      | 1.168       | −31                         |
| 4               | 6             | 4750                   | 1.203       | 980                      | 1.152       | − 42                        | 980                      | 1.152       | − 42                        | 1600                     | 1.263       | + 80                        | 595                      | 1.21        | + 3                         | 595                      | 1.21        | + 3                         |
| 5               | 7             | 3960                   | 1.2         | 840                      | 1.116       | − 25                        | 840                      | 1.166       | − 25                        | 1310                     | 1.221       | + 26                        | 485                      | 1.226       | +11                         | 485                      | 1.226       | +11                         |
| 6               | 8             | 3398                   | 1.167       | 735                      | 1.141       | − 15                        | 735                      | 1.141       | − 15                        | 1110                     | 1.18        | + 13                        | 409                      | 1.185       | + 6                         | 409                      | 1.185       | + 6                         |
| 7               | 9             | 3059                   | 1.11        | 675                      | 1.09        | − 12                        | 765                      | 1.09        | − 12                        | 985                      | 1.128       | + 16                        | 362                      | 1.13        | + 6                         | 362                      | 1.13        | + 6                         |

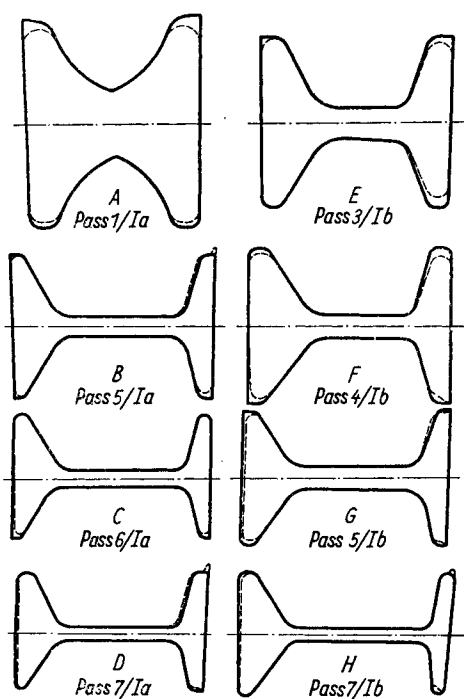


Fig. 7.78. First roll pass design prints of "Pokal" section: A-D before correction, E-H after correction. Insufficient section filling [130]

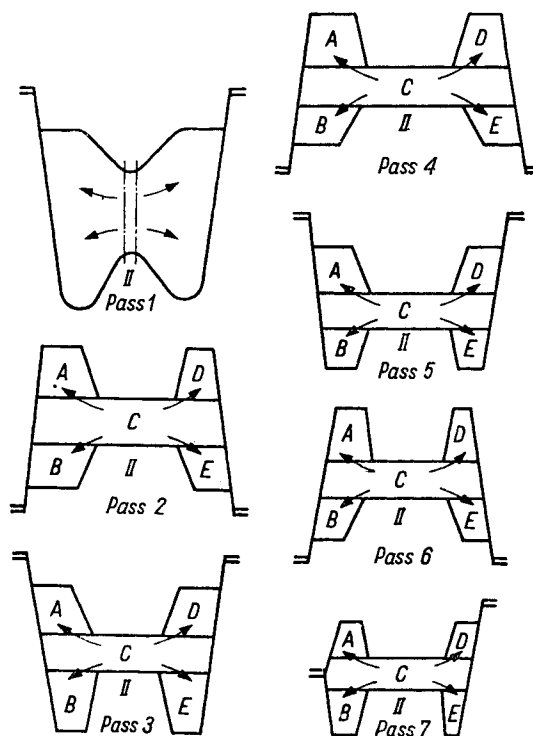


Fig. 7.79. The second roll pass design of "Pokal" profile (from Holzweiler). Division of section and metal flow scheme

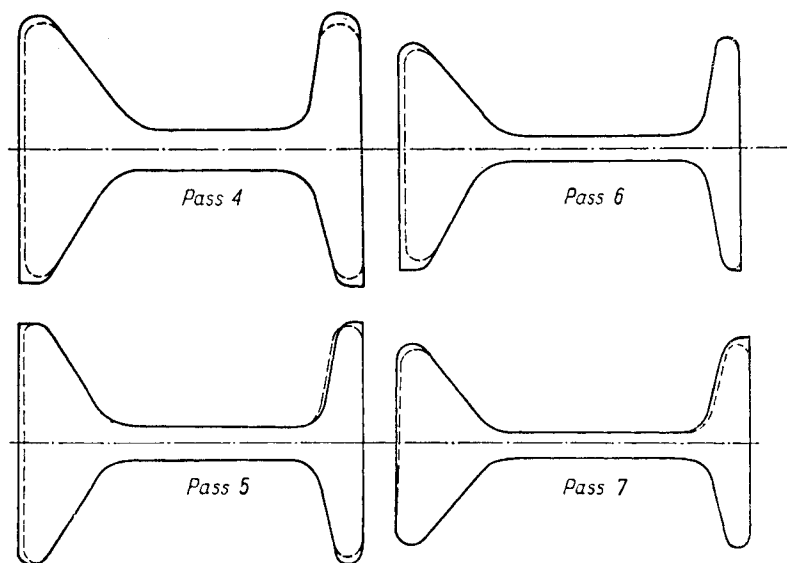


Fig. 7.80. The last four passes of "Pokal" section (Holzweiler method) showing insufficient section filling [130]

TABLE  
DATA FOR

| Roll pass No. | $\lambda$ | $F_A$<br>mm <sup>2</sup> | $F_{A/B}$<br>mm <sup>2</sup> | $F_C$<br>mm <sup>2</sup> | $F_{D/E}$<br>mm <sup>2</sup> | $\gamma_{fl} \lambda$ | $\gamma_{fl}$ |
|---------------|-----------|--------------------------|------------------------------|--------------------------|------------------------------|-----------------------|---------------|
| 0             | —         | 127.3                    | 32.0                         | 29.2                     | 17.5                         | —                     | —             |
| 1             | 1.30      | 98.0                     | 24.6                         | 22.4                     | 13.2                         | 1.18                  | 0.908         |
| 2             | 1.30      | 75.2                     | 18.9                         | 17.26                    | 10.07                        | 1.17                  | 0.900         |
| 3             | 1.30      | 57.9                     | 14.6                         | 13.27                    | 7.81                         | 1.16                  | 0.893         |
| 4             | 1.24      | 46.7                     | 11.75                        | 10.70                    | 6.25                         | 1.15                  | 0.927         |
| 5             | 1.19      | 39.2                     | 9.86                         | 9.00                     | 5.23                         | 1.10                  | 0.925         |
| 6             | 1.15      | 34.1                     | 8.58                         | 7.82                     | 4.56                         | 1.05                  | 0.914         |
| 7             | 1.10      | 31.0                     | 7.80                         | 7.10                     | 4.15                         | 1.00                  | 0.908         |

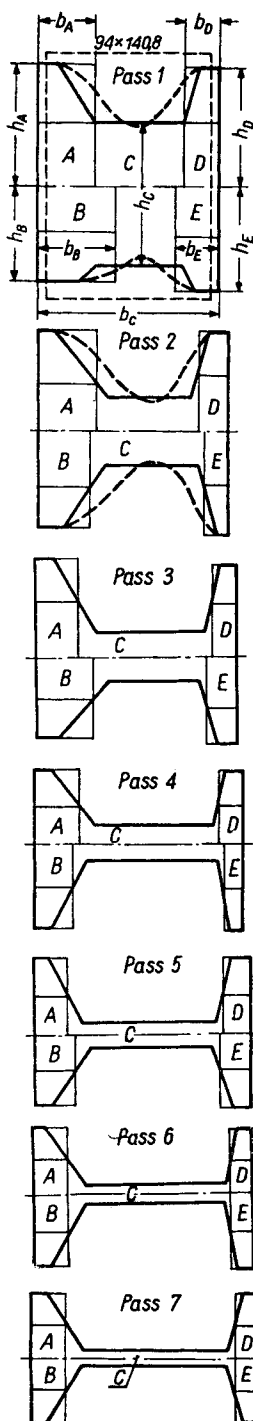


Fig. 7.81. Roll pass design for "Pokal" profile after Rytel [131]

The greatest elongations were chosen in the first passes, in order to avoid the additional metal flow from part to part in the finishing roll passes. In the next passes lower elongation coefficients were chosen, assuming elongation coefficients as nearly as possible equal in the various parts of section (7.134).

The total sectional area and the areas of the parts of the section can be calculated assuming equal elongation coefficients.

The successive areas of parts of section can be found by multiplying these partial areas in the finished section by corresponding elongation coefficients ((7.94) and (7.95)).

$$F_{A,i-1} = \lambda_i F_{A,i}$$

The product  $\lambda_i \gamma_{fl,i} \approx 1.18$  was taken as constant.  $\gamma_{fl,i}$  is here the draught coefficient of the whole flange in the first roll pass, assuming some uncertainty in the calculation of the shearing pass.

For further roll passes the products  $\lambda_i \gamma_{fl,i}$  are calculated from equation (7.153).

Knowing  $\lambda_i \gamma_{fl,i}$  values,  $\gamma_{fl,i}$  can be calculated dividing by  $\lambda_i$ .

The corresponding values are shown in Table 7.16.

The flange may be considered as parts *A* and *B*, or as parts *D* and *E*. During rolling they are differently formed, when parts *A* and *D* are rolled in the open part of the pass, parts *B* and *E* will be rolled in the closed part of the pass and vice versa.

The height of the part entering the closed pass part can be found by multiplying the height after the pass by the corresponding elongation coefficient and taking into account the spread coefficient in the closed pass part,  $\beta_{cl}$ . It was assumed that  $\beta_{cl} = 1.05$  for the 3rd and 6th roll passes and for the others  $\beta_{cl} = 1.00$ . At the end in part *E* for

7.16

FIG. 7.81

| $h_{AB}$ | $h_A$ | $h_B$ | $b_A$ | $b_B$ | $h_{AB/DE}$ | $h_D$ | $h_E$ | $b_D$ | $b_E$ | $h_C$ | $b_C$ |
|----------|-------|-------|-------|-------|-------------|-------|-------|-------|-------|-------|-------|
| mm       | mm    | mm    | mm    | mm    |             | mm    | mm    | mm    | mm    | mm    | mm    |
| 140.8    | —     |       |       |       |             |       |       |       |       |       | 90.0  |
| 127.8    | 73.2  | 54.6  | 33.6  | 45.0  | 127.8       | 69.6  | 58.2  | 19.0  | 22.7  | 78.0  | 104.5 |
| 115.0    | 56.2  | 58.8  | 33.6  | 32.1  | 115.0       | 53.6  | 61.4  | 19.0  | 16.60 | 41.6  | 108.0 |
| 102.8    | 59.7  | 43.1  | 24.5  | 33.8  | 102.8       | 55.7  | 47.1  | 14.0  | 16.60 | 26.9  | 113.0 |
| 95.3     | 45.9  | 49.4  | 25.6  | 23.8  | 95.3        | 42.8  | 52.5  | 14.60 | 11.90 | 16.8  | 117.5 |
| 88.0     | 48.5  | 39.5  | 20.6  | 25.0  | 88.0        | 46.0  | 42.0  | 11.36 | 12.45 | 12.6  | 120.0 |
| 80.4     | 40.2  | 40.2  | 21.7  | 21.7  | 80.4        | 38.1  | 42.3  | 11.96 | 10.80 | 9.6   | 122.5 |
| 73.0     | 36.5  | 36.5  | 21.7  | 21.7  | 73.0        | 36.5  | 36.5  | 11.36 | 11.36 | 8.10  | 123.0 |

the 7th roll pass  $\beta_7 = 1.05$ . Thus the individual pass part dimensions can be calculated from the finishing pass.

Part *E* in the 7th roll pass is rolled in the closed part, hence its height in the previous roll pass can be found as follows:

$$h_{E6} = h_{E7} 1.05 = 42.5 \text{ mm}$$

The height changes of parts *D* and *E* in the 6th roll pass were calculated by means of  $\gamma_{fi}$ .

The height of the open part

$$h_{D6} = h_{DE6} - h_{E6} = 38.1 \text{ mm}$$

For rolling in the open part of the pass most often  $h_{o,i-1} > h_{o,i}$ , where  $h_o$  = the height of the open part.

This inequality is valid for all passes.

The heights of these parts should be calculated as for the 6th pass already given.

The values of width  $b_0$  given in Table 7.16 are mean ones, calculated by dividing the area by height, but they are of great help in pass construction. The total widths are taken from the previous design given in Figs. 7.76 and 7.79, since they were correctly filled. The calculated profile part was assumed as an equivalent rectangle, shown with a faint line in Fig. 7.81.

In fact the flanges take the form of a trapezoid with rounded corners. To calculate such trapezoids three equations are needed: the elongation coefficient equation, equation (7.153), and the  $\beta_{fi}$  coefficient equation.

All the dimensions vertical to the rolling axis are assumed as the heights, the parallel ones as the widths.

Thus it can occur that  $\gamma > 1$  and  $\beta < 1$ .

The parts of section rolled in the  $(i-1)$ th roll pass in the closed part of pass are considered. In this pass a flange end thickness  $a_i$  is found, which must satisfy equation  $a_i = a_{i-1}\beta$ .

In the closed parts it is assumed that  $\beta_{ci} \approx 1$ , or  $\beta_{ci} \approx 1.05$ . Since  $a_n$  is known, then from relation  $a_i = a_{i-1}\beta$ ,  $a_i$  for all the closed parts can be calculated.

In the open pass part

$$\beta_{oi} = \frac{b_i}{b_{i-1}}$$

If the width in every pass is assumed the calculation of  $\beta_o$  is not a difficult matter. The flange thickness  $g$  is given by relation

$$\frac{g_i + a_i}{2} = b_i$$

Table 7.17 gives values of  $\beta$ ,  $a$  and  $g$  in all the pass parts in successive passes.

TABLE 7.17  
SPREAD COEFFICIENTS AND THICKNESSES AT END OF FLANGE FOOT FOR ROLL PASS DESIGN  
(FIG. 7.81)

| No. | A       |      |      | B       |      |      | D       |      |      | E       |      |      |
|-----|---------|------|------|---------|------|------|---------|------|------|---------|------|------|
|     | $\beta$ | $a$  | $g$  | $\beta$ | $a$  | $g$  | $\beta$ | $a$  | $g$  | $\beta$ | $a$  | $g$  |
| 1   | —       | 12.4 | 54.8 |         | 18.0 | 72.0 |         | 13.0 | 25.0 | —       | 15.0 | 30.4 |
| 2   | 1.00    | 12.4 | 54.8 | 0.715   | 12.8 | 51.4 | 1.00    | 13.0 | 25.0 | 0.731   | 11.6 | 21.6 |
| 3   | 0.73    | 9.0  | 40.0 | 1.00    | 12.8 | 54.8 | 0.736   | 9.9  | 18.1 | 1.00    | 11.6 | 21.6 |
| 4   | 1.05    | 9.5  | 41.7 | 0.705   | 9.0  | 37.8 | 1.05    | 10.4 | 18.8 | 0.716   | 8.3  | 15.5 |
| 5   | 0.797   | 7.5  | 33.6 | 1.05    | 9.4  | 40.6 | 0.778   | 8.1  | 14.6 | 1.05    | 8.7  | 16.2 |
| 6   | 1.05    | 8.0  | 35.5 | 0.856   | 8.0  | 33.5 | 1.05    | 8.5  | 15.4 | 0.886   | 7.6  | 14.0 |
| 7   | 1.00    | 8.0  | 35.5 | 1.00    | 8.0  | 35.5 | 0.950   | 8.0  | 15.2 | 1.05    | 8.0  | 15.2 |

The parts  $A$  and  $D$  were designed to be rolled in the closed part of the shearing pass.

#### 7.6.8. KNIFING PASSES

Of the passes commonly used in pass design methods one of the most characteristic is the primary forming pass, called the knifing pass.

In this pass the forming of the flange perpendicular to the web begins. In further passes the thickness of flanges and web is gradually reduced.

The proper construction of the knifing pass and the correct determination of the ingoing bloom height so as to fill the whole section are of great importance in designing, since in this pass the non-uniform draughts and metal flow occur.

The bloom height when entering the knifing pass should be somewhat greater than the pass height. Metz recommends a bloom height 7–20% greater than the pass height; while Holzweiler and Dahl recommend a pass height 5–10 mm less than the bloom height [115].

Besides such general indications there are also mathematical methods of bloom size calculation based on a more or less correct interpretation of metal flow during rolling.

Lendl [123, 126] calculates the filling of flanges in the knifing pass from equations (7.90) and (7.91), while calculating the mean elongation coefficient. The section is divided as in Fig. 7.82.

The elongation coefficient of the web is

$$\lambda_B = \frac{b_{B1}h_1}{F_{B2} + (b_2 - b_1)h_{A2}} \quad (7.154)$$



where in Fig. 7.82 and formula (7.154)

$F_1 = b_1 h_1 = F_{A1} + F_{B1}$  = area of initial bloom section,

$F_{A1} = 2h_1 b_{A1}$  = area of side parts of bloom section,

$F_{B1} = b_{B1} h_1$  = area of middle part of bloom,

$F_{B2}$  = area of middle part of pass,

$(b_2 - b_1) h_{A2}$  = approximate area of edge parts of pass which is filled with metal from the compressed middle parts of bloom.

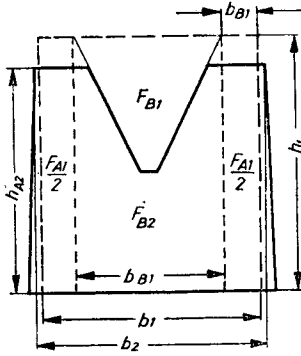


Fig. 7.82. Knifing pass division according to Lendl [123]

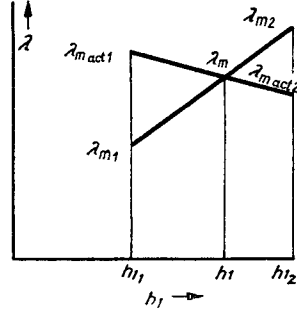


Fig. 7.83. Bloom height determination according to Lendl [123]

The assumed elongation coefficient of flanges  $\lambda_A \approx 1.0$  i.e.  $F_{A1} \approx F_{A2}$  since the flanges are not compressed by the rolls but only drawn along by the web. With known shape and dimensions of the knifing pass the bloom height  $h_1$  required to fill the pass can be calculated from equations (7.90) and (7.92) and from

$$\frac{F_{A1} + F_{B1}}{F_{A2} + F_{B2}} = \lambda_{mw}$$

$$b_1 h_1 = F_{A1} + F_{B1}$$

This height can be obtained by drawing (Fig. 7.83).

On the abscissa the bloom height is measured and on the ordinate the mean elongation coefficient. An arbitrary bloom height  $h_{11}$  is assumed and  $\lambda_{m1}$  calculated. This value of  $\lambda_m$  is plotted in the diagram for a corresponding assumed  $h_{12}$ . Then the actual mean elongation coefficient is calculated

$$\lambda_{mact1} = \frac{F_{A1} + F_{B1}}{F_{A2} + F_{B2}}$$

which is also plotted on the diagram corresponding to assumed  $h_{11}$ . If these points do not coincide, another value of bloom height  $h_{12}$  is assumed and  $\lambda_{m2}$  and  $\lambda_{mact12}$  are calculated again.

Joining  $\lambda_{m1}$  with  $\lambda_{m2}$  and  $\lambda_{mact1}$  with  $\lambda_{mact2}$  at the intersection of these lines the bloom height  $h_1$  is found, at which  $\lambda_m = \lambda_{mact}$ . This means that with bloom height  $= h_1$  the pass will be completely filled.

To compare the described methods of calculation of knifing passes, a knifing pass for channel 80 was calculated.

Table 7.18 gives the bloom and pass dimensions and flange heights, calculated by Lendl's methods.

TABLE 7.18  
ACTUAL DIMENSIONS OF BLOOM AND KNIFING PASS OF CHANNEL 80

| Bloom dimensions, mm |       | Pass area, mm <sup>2</sup> |          |          |
|----------------------|-------|----------------------------|----------|----------|
| $b_1$                | $h_1$ | $F_2$                      | $F_{A2}$ | $F_{B2}$ |
| 66.0                 | 130.0 | 5964.0                     | 3740.0   | 2224.0   |

#### 7.6.9. GENERAL PRINCIPLES OF SECTIONS PASS DESIGN [113, 130, 150]

(1) Large elongation coefficients and great variations of elongations between the various components of section are permitted in the initial roll passes where the metal is hot, the section is large and well rounded making an easy transition from one part to another. In such conditions metal flow is facilitated.

(2) In the finishing roll passes, especially in the final pass, where the metal is already cooler, the temperature varies between particular components, the section is small and there are relatively small fillets in passing from one component to the other, the lateral flow of metal is considerably restricted. For this reason elongation coefficients as nearly uniform as possible should be used in calculating the various components of the pass.

The finishing roll passes, above all the finished section, should be rolled as nearly as possible without metal flow, i.e. with  $F_x \approx 0$ . This is possible if  $\lambda_A \approx \lambda_B \approx \lambda_{mw}$ . If non-uniform elongations are used the following snags should be anticipated:

- (a) overfilling in the open flanges where  $\lambda_A > \lambda_{mw}$ ,
- (b) web buckling where  $\lambda_A > \lambda_{mw}$ ,
- (c) buckling or sticking in the closed flanges where  $\lambda_A > \lambda_{mw}$ ,
- (d) underfilling of open or closed flanges where  $\lambda_B < \lambda_{mw}$ .

Nevertheless if the flanges are thin, it is wise to design for underfilling rather than overfilling, since it is easier to add metal than to take it off.

(3) With assumed non-uniform elongations metal flow between adjacent components should be allowed for. The case where  $\lambda_A > \lambda_B = \lambda_C = \lambda_D > \lambda_E$  must not be allowed, since metal cannot flow from part *A* to part *E* through parts *D* and *C*.

(4) When fillets are too small the metal flow to an adjacent part is hindered, which can cause buckling of the overfilled part. It should be borne in mind that excessive lateral flow of metal, especially occurring at lower temperatures, tends to cause rapid roll wear.

(5) In the open parts side draughts should be used, smaller in thinner parts, greater in the thicker parts.

In closed profile parts direct draughts should be used, and to avoid sticking draughts should be lower than in the open parts.

(6) Besides temperature, section height and fillets, the relative areas of various parts of section have an important influence on metal flow. If  $\lambda_A > \lambda_B$  the influence

of the greater area of part  $A$  on part  $B$  will be such that not the whole calculated amount  $F_x$  will flow to  $B$ , hence part  $B$  will be underfilled. If  $\lambda_A > \lambda_B$  the part  $B$  will be overfilled, despite the fact that part  $A$  holds back part  $B$ .

This can be observed in knifing passes where a wide but shallow cut causes metal to flow more easily than a narrower and deeper one.

(7) For shapes with a thin web and high flanges (e.g. I-beams, channels) the finished pass should be designed with a somewhat lower draught on the web than on the flange. It will be noticed that when regulating the rolls, a slight lifting of the upper roll causes a much greater change of percentage draught on the web than on the flange.

(8) In the case of an asymmetric section the knifing pass (frequently the first roll pass) should divide the bloom approximately in proportion to the elements of the finished section.

(9) In places where under- or overfilling may occur, larger rounding radii should be used in the preceding pass than appears from calculation. If underfilling occurs the rounding radii can easily be reduced; in the case of overfilling the roll as a whole must be dressed.

(10) When choosing pass width and taper in successive roll passes the principle should be applied that the flange enters into the closed part of pass to a depth of at least  $2/3$  of the height, while in the open part a depth of a few millimetres is sufficient.

(11) The pass closing should be changed alternately, and successive roll passes with the pass closing in the same place should be avoided.

(12) In practice rolls very rarely stay exactly as they were designed during rolling. With a small opening or closing of roll the elongation coefficients and metal flow differ from the assumed ones. This is another reason why finishing and pre-finishing profiles should be designed as nearly as possible without metal flow.

(13) The hot section width should be calculated with a negative tolerance. With roll dressing the pass will be widened.

#### 7.6.10. ROLL WEAR

Inefficient roll design, in which true spread has not been taken into account, causes increased roll wear and power consumption.

A section compressed by a force  $P$  in a pass as in Fig. 3.101 is considered. A force  $mP$  acts on the side walls, where  $m$  is a factor depending on the manner of metal flow and on its restriction in the pass. For passes with free spread  $m = 0$ , with spread completely restricted according to Trinks [22]  $m = 0.3$ . In knifing passes  $m$  is greater than in rectangular ones and is dependent on the aperture angle of the shearing part of pass and on the plastic resistance of metal. On the basis of experiments with steel bars Trinks showed that at an aperture angle of  $60^\circ$  and temperature  $980^\circ$  the factor  $m = 0.4-0.45$ , while with metal heated to  $1200^\circ\text{C}$  in similar conditions  $m = 0.45-0.60$  [22].

With lateral pressures, a pass becomes worn over its whole perimeter proportionally to the product of pressure and slip speed. If possible the spread should not be restricted. The dimensions of the finished section should be kept within certain standard defined limits, and rolling can be continued as long as the sectional dimensions lie within the upper limits of tolerance. Hence the more rapidly a pass is worn the less steel can be rolled because of the necessity of more frequent roll dressing.

Roll wear is also affected by metal flow inside the section at right angles to the rolling axis. Metal being forced to change its direction of flow when passing from one part of section to another exerts additional pressure and also rubs on the pass, causing greater wear in these places. If metal flow from one part of the section to another cannot be avoided it should be directed from a thinner part to a thicker part but not conversely, and should be applied only in the initial passes when the metal is more ductile. This can be attained by appropriate pass construction. In practice this is often impossible because of the relative positioning of these parts.

### 7.7. Principles of Roll Pass Design for Continuous Mills

A mill arrangement, where at least two stands are standing one after another and working together is called a continuous mill. (Fig. 7.84). A continuous rolling process is defined as one where rolling takes place simultaneously in at least two mill stands. This is not always the case in a continuous mill system. In continuous groups of roughing mills, for instance as in roughing mills for strip, the space between mill stands is greater than the length of metal being rolled at any one moment, thus after every pass metal leaves the rolls freely and is never simultaneously in two mill stands. In a completely continuous train of mill stands (e.g. in rod mill) it can occur that one end of the billet after having been rolled is coiled on the coilers and at the same time the other end is still in the furnace. In this case rolling takes place in all mill stands simultaneously.

Continuous trains of mill stands can therefore be divided into two groups:

- (1) Where the rolling process does not take place continuously and the metal leaves the respective rolls freely.
- (2) Where the rolling process takes place continuously.

In the first rolling method the length of metal being rolled is usually short, e.g. ingots, billets, slabs etc. The spacings between mill stands are large, which necessitates very long shops. Simultaneous rolling in mill stands set as closely together as possible is advantageous. The short distance between rolls means only a small temperature drop during the roll pass, which reduces rolling loads, power consumption and roll wear. In rolling large sections heavy draughts can be used, since the metal is forced towards the next pass, which allows greater angles of bite to be used.

In cold rolling, a continuous train enables the simultaneous use of large tensions between the mill stands which allows one to increase the draughts in the passes. When rolling in the various stands separately the rolling processes take place independently and each stand should be considered independently.

When rolling in all the stands simultaneously the rolling processes should be closely connected. In this case the amount of metal passing through the particular mill stands in unit time should be constant, i.e. their output should be constant.

This value is obtained by multiplying the outgoing cross-sectional area  $F_x$  by speed  $w_x$  of the given mill stand. Since metal spread and forward slip which occur in rolling are dependent on many factors, these factors should be taken into account as accurately as possible, and introduced into calculations for continuous rolling. If this is neglected the roll pass calculations will not be exact, the amounts of metal passing in unit time will be different, which could cause pulling or pushing of metal between the roll passes.

Pulling can sometimes be so great as to cause rupture during rolling or a large reduction of section. Since pulling occurs only when metal is simultaneously in at least

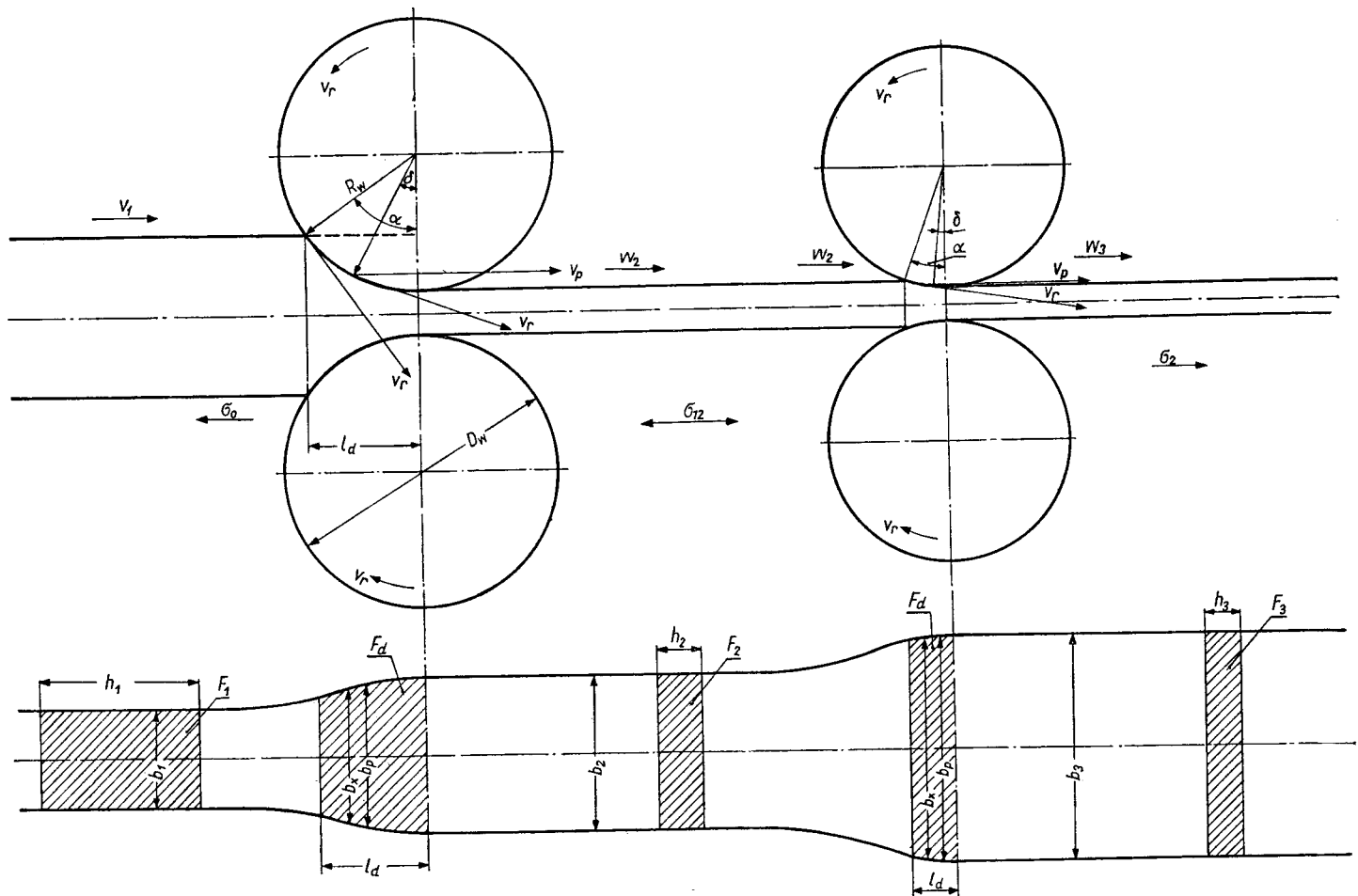


Fig. 7.84. Two stands in a continuous system

two stands it is obvious that the cross-sections at the beginning, the end and the middle of rolled stock will be different. If this occurs in many stands simultaneously these differences increase and the finished product is defective. The sections obtained in particular roll passes will be incorrect, in certain places they will be under- or overfilled. This may cause interruptions in rolling, since the deformed section will not twist correctly in the guides between the stands, or may cause the wipers to be knocked out.

If pushing takes place between the passes, interruption of the rolling process may or may not occur, depending on the size of the section of metal being rolled. Thin sections such as band, wire rod, sheet, that loop and bend easily, can be rolled with small loops forming between mills. Then pushing will not occur and the finished section will be correct. The loops facilitate the rolling process which takes place independently in the individual roll passes. This method of rolling is similar to that where a bar emerges freely from the rolls between passes. Thus the rolling method with loops being formed is the most suitable one for thin sections.

In rolling large rigid sections this rolling method can cause great damage to the mills by overloading, roll breaking, excessive pass wear etc. For such sections pushing between rolls is more detrimental than a small pull. Since calculation methods can never fully represent rolling conditions (variations of temperature due to friction, continual wear at sides of pass etc.), a somewhat greater inter-stand tension should be assumed to avoid pushing in particular roll passes. However, this tension should not be too great since it may cause section reduction, excessive noise, deflection of stands and excessive power consumption. From the relation (Fig. 7.84)

$$F_i w_i = F_i \frac{\pi D_{wi} n_i}{60} s_i = \text{constant} \quad (7.155)$$

where

$F_i$  = cross-sectional area after  $i$ -th roll pass,

$w_i$  = exit speed after  $i$ -th roll pass,

$D_{wi}$  = mean effective diameter of roll in the  $i$ -th roll pass,

$n_i$  = roll rpm in  $i$ -th pass,

$s_i$  = forward slip in the  $i$ -th roll pass and from equation (3.104) or (3.150)

$$s_i = \frac{w_i}{v_{ri}}$$

It can be seen that the rolling process depends only on the variations of three quantities:

- (1) sectional area  $F_i$ ,
- (2) mean effective diameter  $D_{wi}$ ,
- (3) roll rpm  $n_i$ .

These points will be considered in order.

### 7.7.1. EFFECT OF INITIAL CROSS-SECTION

The cross-section  $F_i$  of an arbitrary roll pass can be changed most easily by change of roll setting. If too large a loop is formed between two mill stands as a result of too small an output from the next stand, the loops can be reduced in two ways: by lowering the roll setting of the first mill stand which will reduce the amount of metal passing in unit time to the second mill stand, or by increasing the roll setting of the second mill stand which will increase the amount of metal being rolled, i.e. the output of that stand

will be increased. If the loop between particular stands is too large, the procedure must be reversed.

The simplest method of operation is when rolling with loop formation. A similar method is used for rolling of large sections with tension, but in this case the roller cannot verify the exact rolling condition.

There is less possibility of controlling the rolling process by roll setting, since to obtain a required end cross-section the sizes of intermediate passes are assumed in designing as well as the tension between particular mills. Large deviations from these values cannot be used because of the danger of disrupting rolling conditions of the preceding or next roll pass. Also the correct setting of rolls in this case is tedious and requires much work and experience. For roughing mills in a continuous system, difficulties are encountered both in roll pass design calculations and in rolling, due to the fact that an effective diameter is assumed and often rpm as well. Similarly the projection of successive passes is made difficult. It is obvious that if the whole mill is driven by a single motor, rolling conditions worsen since all the factors directly affect each other.

If a set of mill stands driven by a single motor is divided into two separate groups, with such a distance between them that the metal being rolled momentarily lies freely on the roller tables, the rolling conditions of the first group do not affect the second group. The roller then has greater freedom to control the rolling process by roll setting, variation of diameter or number of revolutions. This arrangement of mill stands is more advantageous than the previous one.

The greater the number of small groups of separately driven rolls in a layout, the easier the servicing and working. The best solution is an individual, controlled drive of each stand.

For a mill train with common drive rolling large rigid sections, free exit of metal after each roll pass should be aimed for. This is possible with small lengths but with large ones it becomes impossible due to temperature decrease, increased rolling time and great length of shops. In this case a division into two trains with individual controlled drive is more convenient. Then the output of the last mill stand in one train can be matched with that of the first mill stand in the other train, and the distances between the trains can also be reduced.

In a well set out and well working continuous train it is necessary to change the roll setting from time to time in order to compensate for variations caused by wear of roll and bearings wear. With fixed rolling conditions the rolling load is constant. However, if the roll setting is changed the rolling load also changes. Hence variations caused by wear of rolls and bearings can be compensated for by maintaining a fixed rolling load. When the measuring equipment indicates that the rolling load is reduced, then by adjusting the setting screw the roll pressure can be brought to the previous value, and rolling conditions can be maintained.

It must of course be remembered, that change of other factors may cause a change of rolling load, e.g. change of temperature, friction, chemical composition of metal, pulling or pushing between mills etc.

#### 7.7.2. EFFECT OF ROLL DIAMETER

A continuous rolling process can be influenced by the change of roll diameter. If too big a loop is formed between mill stands, this difference can be balanced by substituting a roll of larger diameter in the next mill stand or a roll of smaller diameter in

the preceding mill stand. If the loop formed is too small, the procedure should be reversed, thus reducing the peripheral speed of rolls.

In rolling with inter-stand tension, the procedure is not so simple. If the tension is too great, putting rolls of smaller diameter in the next stand or of larger diameter in the previous stand, will improve rolling conditions. However, if the rolls are too thin, pushing can be caused between the two stands, which may be dangerous with large cross-sections.

When rolling in shaped passes, if too great a tension between mills is encountered, this can be corrected by reducing the effective diameter. This is done by cutting the passes of the next stand more deeply, which reduces its rolling speed. It is obvious that to do this the rolls must be taken out.

Such roll dressing can only be used to a limited degree, so as not to affect the rolling processes in the preceding or following passes.

If a roll breaks and must be replaced, rolling conditions will be changed. If new rolls are fitted their diameter may be too great relative to existing rolling conditions, or very worn rolls may have too small a diameter. The rebuilding of the whole layout during work is impracticable, but if the rolls have a constant speed a change of diameter can only be balanced by change of roll setting in particular stands. This is a difficult problem which requires considerable experience, and the conditions depend on the number of stands driven by a single motor. Hence it can again be concluded that the best solution is to install stands with individual drive.

### 7.7.3. EFFECT OF ROLL SPEED

The stand output can be controlled by varying the roll speeds. When two stands are working together their output is the same if the exit speed of one equals the entry speed of the next, i.e. if  $w_2 = v_2$ , with  $F$  constant, where:  $w_2$  = exit speed of the first stand,  $v_2$  = entry speed of the second stand (Fig. 7.84).

If  $w_2 < v_2$ , this means that the rotational speed of the second stand is too high, which for a fixed elongation coefficient  $\lambda = F_1/F_2$  causes inter-stand tension.

Conversely, if  $w_2 > v_2$ , the output of the first stand is higher than that of the following stand and looping or pushing occurs. This can be eliminated by adjusting the rotational speed of roll, but this is only possible when each stand has an individual, controlled drive. If the layout is divided into several trains driven by separate motors, this speed equalizing is only possible between trains, but not within one train.

Hence it is seen that the optimum method is to have an individual drive of each stand, which provides a means of controlling and correcting the rolling process. In rolling with a loop an experienced roller can regulate roll rpm so that only small loops are formed, just sufficient to ensure independent working of the stands.

It must be remembered that with D.C. drive the roll rpm do not remain fixed but vary during working, which must be balanced by the roll operator. These variations are especially large at the moment of biting the metal by the rolls. In large rolling mills with a fixed rolling programme automatic regulation is used. This problem will not be discussed here since it is concerned with the drive control of electric motors. Further information can be found in [136, 139].

Much worse conditions are found when rolling large sections, as in the moment of biting of the metal by the rolls the roll rpm are reduced, which upsets the rolling



equilibrium. Since the roll speed returns to normal only after a certain time, momentary pushing takes place which gradually changes to pulling. This is very dangerous for the rolling process, and for this reason motors with only a small speed drop are recommended for mill drives.

In rolling without looping an operator can only detect changes in the rolling process when great deviations from the normal occur. The following faults can be detected: bending and buckling of metal, large cross-sectional changes, undue noise during rolling, hammering of bearings, and large power consumption. It is desirable that measuring equipment should be installed to indicate the magnitude of the tension and motor power consumption. Then according to the indicated values of roll pressure, torque or power consumption, the operator can regulate the drive motor revolutions. Measurement of tensions in this way can be inaccurate, since changes of roll pressure, torque and power are influenced by many different factors. Not even the best operator, however, can continuously regulate roll speeds so as to balance all the changes occurring during the rolling process, such as changes of rolling temperature, friction conditions, bearing wear, roll wear etc.

Hence complete automation of the process is recommended through the installation of control equipment for regulation of roll speed [138, 139].

#### 7.7.4. PRINCIPLES OF CONTINUOUS TRAIN DESIGN AND ROLLING SPEED CALCULATION

Until now the design of continuous trains has been based on the calculation of the amount of metal passing through the roll gap, as follows (Fig. 7.84).

Every roll stand passes a constant amount of metal, then

$$F_1 v_{r1} = F_2 v_{r2} = F_3 v_{r3} = \dots F_n v_{rn} = \text{constant} \quad (7.156)$$

In this formula  $F_1, F_2 \dots F_n$  are the successive cross-sectional areas of the bar being rolled:  $v_{r1}, v_{r2}, v_{r3} \dots v_{rn}$  are the peripheral speeds of rolls in corresponding stands.

Since from formula (3.101)

$$v_r = \frac{\pi D_w n}{60} \quad \text{and} \quad R_w = \frac{D_w}{2}$$

where  $R_w$  and  $D_w$  are effective working radius and diameter of roll, putting these values in equation (7.56) gives

$$F_1 \frac{2\pi}{60} R_{w1} n_1 = F_2 \frac{2\pi}{60} R_{w2} n_2 = \dots F_n \frac{2\pi}{60} R_{wn} n_n = \text{constant}$$

Dividing the whole equation by  $2\pi/60$

$$F_1 R_{w1} n_1 = F_2 R_{w2} n_2 = F_3 R_{w3} n_3 = \dots F_n R_{wn} n_n = \text{constant} \quad (7.157)$$

Formula (7.157) gives a method commonly used until now for design calculations of continuous systems. To avoid looping or pushing between stands a small pulling of 1% is designed for, very often by means of suitable overdraught. The value of 1.01 is assumed to correspond approximately to the forward slip of metal. In such cases it can be calculated that

$$\frac{F_2 R_{w2} n_2}{F_1 R_{w1} n_1} = 1.01 \quad (7.158)$$

Pulling is applied when rolling larger sections such as blooms, billets, thick bars, sheet slabs, plate etc. since even very slight pushing of metal being rolled could damage the stand, rolls or fittings.

Sections which easily bend or twist such as thin bars, wire rods, thin sheet slabs, band steel and sheet strips can be rolled with small loops between roll passes.

The derived formulae (7.156) to (7.158) refer to rolling of flat sections on the smooth straight part of the roll barrel. If in the passes instead of  $R_w$ ,  $\rho$  is substituted then formula (7.157) takes the form

$$F_1 \rho_1 n_1 = F_2 \rho_2 n_2 = \dots = F_n \rho_n n_n = \text{constant} \quad (7.159)$$

where  $\rho_1, \rho_2, \dots, \rho_n$  are radii of centres of gravity of sections.

This assumption of metal speed through the rolls is incorrect, for three reasons:

- (1) the metal exit and entry speeds in the rolls are not taken into account,
- (2) the correct position of the line of mean rolling speed for the pass is not taken into account,
- (3) the mean section height in the pass is not taken into account.

In the first case, when rolling flat sections on the plain barrel from the preceding formulae:

$$F_0 v_1 = F_1 w_1 = F_1 v_2 = F_2 w_2 = \dots = F_{n-1} v_n = F_n w_n = \text{constant} \quad (7.160)$$

where

$v_1, v_2, \dots, v_n$  = entry speeds to the corresponding pass,

$w_1, w_2, \dots, w_n$  = exit speeds from the corresponding pass.

Considering the preliminary cross-section  $F_0$  and knowing

$$F_0, \lambda_1 \text{ and } v_r$$

then the entry speed  $v_1$  for a known peripheral speed of rolls can be calculated from (3.126) and (3.149) and Fig. 3.69 if there is spread, or without spread from (3.179) or (3.187) and (Figs. 3.91–3.95).

The exit speed and the succeeding cross-sections and speeds can be found from the known elongation coefficient equations

$$\lambda_1 = \frac{w_1}{v_1} = \frac{F_0}{F_1}, \quad \lambda_2 = \frac{w_2}{v_2} = \frac{F_1}{F_2} \text{ etc.}$$

If  $F_n, \lambda_n$  and  $v_r$  are known,  $w_n$  can be calculated similarly and then

$$\lambda_n = \frac{w_n}{v_n} = \frac{F_{n-1}}{F_n} \text{ etc. as in (7.160)}$$

For the calculation of metal flow speed the following formulae are used:

Since  $w_1 = (1 + s_1)v_r$ , where  $s_1$  is forward slip, rolling with spread can be found from (3.102) or (3.104) and (3.149).

$$F_1(1 + s_1)v_{r1} = F_2(1 + s_2)v_{r2} = F_3(1 + s_3)v_{r3} = \dots = F_n(1 + s_n)v_{rn} = \text{constant} \quad (7.161)$$

Neglecting  $2\pi/60$  then

$$\begin{aligned} F_1 R_{w1} n_1 (1 + s_1) &= F_2 R_{w2} n_2 (1 + s_2) \\ &= F_3 R_{w3} n_3 (1 + s_3) = \dots = F_n R_{wn} n_n (1 + s_n) = \text{constant} \end{aligned} \quad (7.162)$$

or  $s$  can be found more simply from (3.137)

$$F_1 R_{w1} n_1 s_1 = F_2 R_{w2} n_2 s_2 = F_3 R_{w3} n_3 s_3 = \dots = F_n R_{wn} n_n s_n = \text{constant} \quad (7.163)$$

If the exit speed is lower than the entry speed to the next stand, then pulling takes place, increasing the entry speed to the next stand by 1–3%. Then

$$\begin{aligned} v_2 &= w_1 + w_1(0.01-0.03) \\ v_n &= w_{n-1} + w_{n-1}(0.01-0.03) \end{aligned} \quad (7.164)$$

In continuous trains where the stands are geared together the following relation between the elongation coefficients and roll speeds is useful:

$$\lambda_t = \lambda_1 \lambda_2 \dots \lambda_i \dots \lambda_n = \lambda_1 \frac{n_2}{n_1} \frac{n_3}{n_2} \dots \frac{n_{i+1}}{n_i} \dots \frac{n_n}{n_{n-1}} = \lambda_1 \frac{n_n}{n_1} \quad (7.165)$$

If a loop is formed between stands

$$w_1 > v_2$$

$$w_{n-1} > v_n$$

Since the time of rolling of the bar in the first stand

$$t_1 = \frac{l_1}{v_1} = \frac{l_2}{w_1} \quad (7.166)$$

where

$l_1$  = length before roll pass,

$l_2$  = length after roll pass,

then

$$L_t = (w_1 - v_2) \frac{l_2}{w_1} \quad (7.167)$$

where  $L_t$  = theoretical length of loop.

If the stands are set out with a certain distance  $A$  between them, the time taken by the bar to cover the distance  $A$  should be included. Then the actual loop length will be

$$L_{act} = (w_1 - v_2)(t_1 - t')$$

where

$$t' = \frac{A}{w_1}$$

From this

$$L_{act} = (w_1 - v_2) \left( \frac{l_2}{w_1} - \frac{A}{w_1} \right) \quad (7.168)$$

$$L_{act} = (L_t - A) \left( 1 - \frac{v_2}{w_1} \right) \quad (7.169)$$

$$L_{act} = L_t - A \frac{L_t}{l_2} \quad (7.170)$$

#### 7.7.5. CALCULATION OF ROLL SPEED IN ROLLING OF BARS IN SYMMETRICAL PASSES

Calculations for rolling of symmetrical sections are similar to those for rectangular sections already considered but the following equation is used:

$$\begin{aligned} F_0 v_{1m} &= F_1 w_{1m} = F_1 v_{2m} = F_2 w_{2m} \\ &= F_2 v_{3m} = F_3 w_{3m} = \dots = F_{n-1} v_{nm} = F_n w_{nm} = \text{constant} \end{aligned} \quad (7.171)$$

where  $v_m$  and  $w_m$  are the mean entry and exit speed, respectively.

With  $F_0$ ,  $\lambda_1$  and  $v_r$  given,  $v_{rm}$  is calculated by means of equation (3.197) and the forward slip from (3.149). From this  $w_{1m}$  can be found, and then further calculations are made using given elongation coefficients

$$\lambda_1 = \frac{w_{1m}}{v_{1m}} = \frac{F_0}{F_1} \text{ etc.}$$

$v_{1m}$  can be also calculated from formulae (3.126), (3.149). If  $F_n$ ,  $\lambda_n$  and  $v_n$  are given the method is similar;  $v_{rm}$  is calculated from (3.197), forward slip from (3.149) and from known elongation coefficient

$$\lambda_n = \frac{w_{nm}}{v_{nm}} = \frac{F_{n-1}}{F_n}$$

From this the intermediate values are found.

The loop length is calculated from equations (7.166) to (7.170) substituting the mean speeds.

Roll design with symmetrical passes is similar as before, the forward slip  $s_f$  is calculated taking into account spread, mean heights and mean effective radii, which gives

$$\begin{aligned} F_1(1+s_1)v_{rm1} &= F_2(1+s_2)v_{rm2} \\ &= F_3(1+s_3)v_{rm3} = \dots = F_n(1+s_n)v_{rmn} = \text{constant} \end{aligned}$$

where the peripheral rolling speed  $v_{rm}$  is calculated from formula (3.197). Neglecting  $2\pi/60$

$$\begin{aligned} F_1 R_{m1} n_1 (1+s_1) &= F_2 R_{m2} n_2 (1+s_2) \\ &= F_3 R_{m3} n_3 (1+s_3) = \dots = F_n R_{mn} n_n (1+s_n) = \text{constant} \end{aligned} \quad (7.172)$$

or for  $s$  calculated from (3.149) a simpler formula can be found

$$F_1 R_{m1} n_1 s_1 = F_2 R_{m2} n_2 s_2 = F_3 R_{m3} n_3 s_3 = \dots = F_n R_{mn} n_n s_n = \text{constant} \quad (7.173)$$

$R_m$  is the mean effective radius in the pass (3.30)

$$R_m = \frac{D_m}{2} = \frac{D_t - h_{im}}{2}$$

where  $h_{im}$  is the mean height of bar leaving the pass i.e.  $h_{im} = F_i/b_i$ .

Considering practical methods to determine the speed and pulling or pushing between stands, from the relation between elongation coefficients calculated from considerations of area and speed where

$$\begin{aligned} \lambda_F &\doteq \lambda_v \\ \lambda_F &= \frac{F_1}{F_2} \end{aligned} \quad (7.174)$$

and

$$\lambda_v = \frac{R_2 n_2 s_2}{R_1 n_1 s_1} \quad \text{or} \quad \frac{D_2 n_2 s_2}{D_1 n_1 s_1} = \frac{v_2}{v_1} \quad (7.175)$$

if forward slip is determined from (3.137).

#### Example 1

Two roll passes are given

$$\begin{array}{lll} h_1 = 22 \text{ mm}, & b_1 = 196 \text{ mm}, & F_1 = 4312 \text{ mm}^2 \\ h_2 = 15 \text{ mm}, & b_2 = 198 \text{ mm}, & F_2 = 2970 \text{ mm}^2 \end{array}$$

then

$$\lambda_F = \frac{4312}{2970} = 1.451$$

$$D_1 = 450 \text{ mm}, \quad n_1 = 50 \text{ rpm}, \quad s_1 = 1.02$$

$$D_2 = 466 \text{ mm}, \quad n_2 = 70 \text{ rpm}, \quad s_2 = 1.05$$

$$\lambda_v = \frac{466 \times 70 \times 1.05}{450 \times 50 \times 1.02} = 1.492$$

$$\Delta\lambda = \lambda_F - \lambda_v = 1.451 - 1.492 = -0.041$$

A pull of  $-0.041$  is found, which would be easy to balance. A (+) sign would indicate a push of the same magnitude. As a second example the effect of change of thickness on pull or push is given.

Taking data from above example, but  $h_1 = 21$  mm instead of 22 mm

$$F_1 = 21 \times 196 = 4116 \text{ mm}^2$$

Then

$$\lambda_F = \frac{F_1}{F_2} = \frac{4116}{2906} = 1.385$$

$$\Delta\lambda = \lambda_F - \lambda_v = 1.385 - 1.492 = -0.107$$

a large back tension is obtained:  $-0.107$ .

With the same data but  $h_2 = 16$  mm

$$F_2 = 198 \times 16 = 3168 \text{ mm}^2$$

$$\lambda_F = \frac{F_1}{F_2} = \frac{4312}{3168} = 1.361$$

$$\Delta\lambda = 1.361 - 1.492 = -0.131$$

a very large pull is found.

Changing  $h_2$  to 14 mm

$$F_2 = 198 \times 14 = 2772 \text{ mm}^2$$

$$\lambda_F = \frac{F_1}{F_2} = \frac{4312}{2772} = 1.555$$

$$\Delta\lambda = \lambda_F - \lambda_v = 1.555 - 1.492 = +0.063$$

hence pushing is found.

These examples show the importance of correct roll setting.

Differences in pushing or pulling can be balanced by suitable choice of roll diameters or rolling speed.

#### Example 2

A roll pass design was required for a round bar of 14.4 mm dia. in a 6-stand continuous train. Preliminary square of  $41 \times 41$  mm. Due to wrinkle marks and local decarburization an oval-round pass design system was chosen. Data is tabulated in Table 7.19.

Calculation of dimensions and areas of ovals and round.

Square  $h_0 = b_0 = 41$  mm,  $F_0 = 1630 \text{ mm}^2$  (planimeter measurement of actual section).

TABLE 7.19  
VALUES FOR EXAMPLE ILLUSTRATED IN FIG. 7.85

| Pass<br>No. | Roll<br>stand<br>No. | Dimensions<br>mm | Cross-<br>section<br>$F$<br>mm <sup>2</sup> | Coefficient |         | Entry<br>speed<br>$v$<br>m/sec | Exit<br>speed<br>$w$<br>m/sec | $Fw$<br>mm <sup>3</sup> /sec | Rotational<br>speed<br>$n$<br>rpm | Theoreti-<br>cal dia.<br>$D_t$<br>mm | Effective<br>dia.<br>$D_w$<br>mm | Bite<br>angle<br>$\alpha^\circ$ | $F_d$<br>mm <sup>2</sup> | Roll<br>load<br>$P$<br>t |
|-------------|----------------------|------------------|---------------------------------------------|-------------|---------|--------------------------------|-------------------------------|------------------------------|-----------------------------------|--------------------------------------|----------------------------------|---------------------------------|--------------------------|--------------------------|
|             |                      |                  |                                             | $\lambda$   | $\beta$ |                                |                               |                              |                                   |                                      |                                  |                                 |                          |                          |
| 1           | 2                    | 3                | 4                                           | 5           | 6       | 7                              | 8                             | 9                            | 10                                | 11                                   | 12                               | 13                              | 14                       | 15                       |
| 0           |                      | square 41 × 41   | $F_0 = 1630$                                |             |         |                                |                               |                              |                                   |                                      |                                  |                                 |                          |                          |
| 1           | I                    | oval 21.1 × 67.5 | $F_1 = 950$                                 | 1.72        | 1.43    | 0.282                          | 0.486                         | $4.62 \times 10^5$           | 29.5                              | 330                                  | 313.8                            | 20°30'                          | 3300                     | 28                       |
| 2           | II                   | round Ø29.5      | $F_2 = 683.5$                               | 1.39        | 1.33    | 0.486                          | 0.676                         | $4.62 \times 10^5$           | 42                                | 330                                  | 305.7                            | 20°20'                          | 1950                     | 17.9                     |
| 3           | III                  | oval 14.8 × 43   | $F_3 = 424.5$                               | 1.61        | 1.25    | 0.676                          | 1.088                         | $4.62 \times 10^5$           | 78                                | 280                                  | 267.9                            | 19°10'                          | 1650                     | 16.8                     |
| 4           | IV                   | round Ø20.1      | $F_4 = 316.8$                               | 1.34        | 1.20    | 1.088                          | 1.458                         | $4.62 \times 10^5$           | 105                               | 280                                  | 262.2                            | 24°30'                          | 852                      | 9.7                      |
| 5           | V                    | oval 10.4 × 30.1 | $F_5 = 208.4$                               | 1.52        | 1.33    | 1.458                          | 2.216                         | $4.62 \times 10^5$           | 154                               | 280                                  | 271.2                            | 15°30'                          | 925                      | 11.3                     |
| 6           | VI                   | round Ø14.2      | $F_6 = 159.1$                               | 1.31        | 1.33    | 2.216                          | 2.903                         | $4.62 \times 10^5$           | 198.5                             | 285                                  | 273.5                            | 20°10'                          | 536                      | 7.3                      |

Calculation of oval in Stand I (Figs. 3.32, 3.34, 3.35).

Assuming  $b/h = 3.2$

$$F_1 = \frac{F_0}{\lambda_1} = \frac{1630}{1.72} = 950 \text{ mm}^2$$

$$b_{1w} = \sqrt{4.8 F_1} = \sqrt{4.8 \times 950} = 67.5 \text{ mm (theoretical width of pass, Fig. 7.85, Stand I)}$$

$$h_1 = \frac{b_{1w}}{3.2} = \frac{67.5}{3.2} = 21.1 \text{ mm}$$

$$R_1 = \frac{b_{1w}^2 + h_1^2}{4 h_1} = 59.3 \text{ mm}$$

$$b_1 = b_0 \beta_1 = 41 \times 1.43 = 58.6 \text{ mm}$$

$$h_{1m} = \frac{F_1}{b_1} = \frac{950}{58.6} = 16.2 \text{ mm}$$

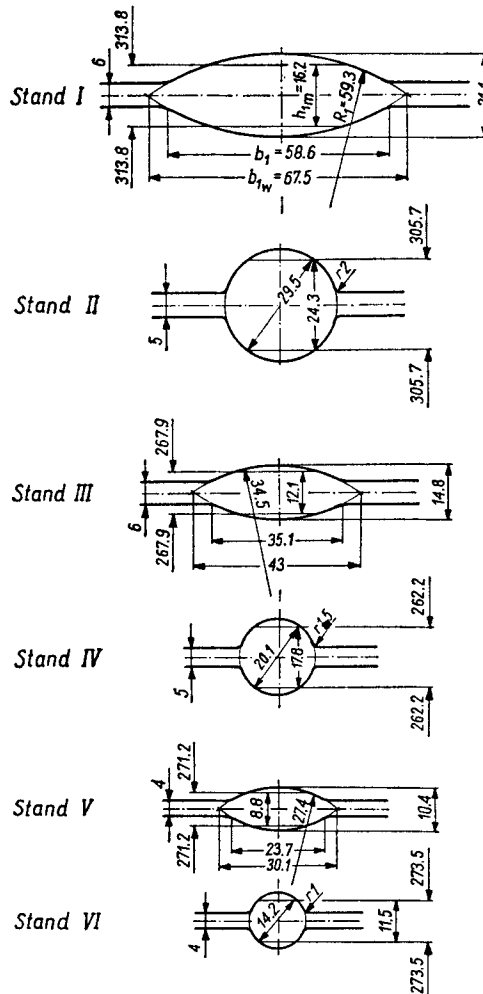


Fig. 7.85. Pass design for a six-stand continuous train

Theoretical roll diameter

$$D_t = 330.0 \text{ mm}$$

Effective diameter

$$D_{w1} = D_t - h_{1m} = 330.0 - 16.2 = 313.8 \text{ mm}$$

Output rate

$$F_1 w_1 = 4.62 \times 10^5 \text{ mm}^3/\text{sec}$$

$$w_1 = \frac{4.62 \times 10^5}{950} = 0.486 \text{ m/sec}$$

$$v_1 = \frac{w_1}{\lambda_1} = \frac{0.486}{1.72} = 0.282 \text{ m/sec}$$

*Calculation of round in Stand II*

$$F_2 = \frac{F_1}{\lambda_2} = \frac{950.0}{1.39} = 683.5 \text{ mm}^2$$

$$d_{2w} = \sqrt{\frac{4F_2}{\pi}} = \sqrt{\frac{4 \times 683.5}{3.14}} = 29.5 \text{ mm}$$

$$b_2 = h_1 \beta_2 = 21.1 \times 1.33 = 28.1 \text{ mm}$$

$$h_{2m} = \frac{F_2}{b_2} = \frac{683.5}{28.1} = 24.3 \text{ mm}$$

Theoretical diameter

$$D_t = 330.0 \text{ mm}$$

Effective diameter

$$D_{w2} = D_t - h_{2m} = 330.0 - 24.3 = 305.7 \text{ mm}$$

$$F_2 w_2 = 4.62 \times 10^5 \text{ mm}^3/\text{sec}$$

$$w_2 = \frac{4.62 \times 10^5}{683.5} = 0.676 \text{ m/sec}$$

$$w_1 = v_2 = \frac{w_2}{\lambda_2} = \frac{0.676}{1.39} = 0.486 \text{ m/sec}$$

*Calculation of oval in Stand III*Assuming  $b/h = 2.9$ 

$$F_3 = \frac{F_2}{\lambda_3} = \frac{683.5}{1.61} = 424.5 \text{ mm}^2$$

$$b_{3w} = \sqrt{4.35 F_3} = \sqrt{4.35 \times 424.5} = 43.0 \text{ mm}$$

$$h_3 = \frac{b_{3w}}{2.9} = \frac{43.0}{2.9} = 14.8 \text{ mm}$$

$$R_3 = \frac{b_{3w}^2 + h_3^2}{4h_3} = 34.9 \text{ mm}$$

$$b_3 = b_2 \beta_3 = 28.1 \times 1.25 = 35.1 \text{ mm}$$

$$h_{3m} = \frac{F_3}{b_3} = \frac{424.5}{35.1} = 12.1 \text{ mm}$$

$$D_t = 280.0 \text{ mm}$$

$$D_{w3} = D_t - h_{3m} = 280.0 - 12.1 = 267.9 \text{ mm}$$

$$F_3 w_3 = 4.62 \times 10^5 \text{ mm}^3/\text{sec}$$

$$w_3 = \frac{4.62 \times 10^5}{F_3} = \frac{4.62 \times 10^5}{424.5} = 1.088 \text{ m/sec}$$

$$w_2 = v_3 = \frac{w_3}{\lambda_3} = \frac{1.088}{1.61} = 0.676 \text{ m/sec}$$



*Calculation of round in Stand IV*

$$F_4 = \frac{F_3}{\lambda_4} = \frac{424.5}{1.34} = 316.8 \text{ mm}^2$$

$$h_4 = d_4 = \sqrt{\frac{4F_4}{\pi}} = \sqrt{\frac{4 \times 316.8}{3.14}} = 20.1 \text{ mm}$$

$$\beta_4 = 1.20$$

$$b_4 = h_3 \beta_4 = 14.8 \times 1.20 = 17.8 \text{ mm}$$

$$h_{4m} = \frac{F_4}{b_4} = \frac{316.8}{17.8} = 17.8 \text{ mm}$$

$$D_t = 280.0 \text{ mm}$$

$$D_{w4} = D_t - h_{4m} = 280.0 - 17.8 = 262.2 \text{ mm}$$

$$v_4 = w_3 = 1.088 \text{ m/sec}$$

$$w_4 = v_4 \lambda_4 = 1.088 \times 1.34 = 1.458 \text{ m/sec}$$

$$F_4 w_4 = 316.8 \times 1.458 = 4.62 \times 10^5 \text{ mm}^3/\text{sec}$$

*Calculation of oval in Stand V*

Assuming  $b/h = 2.9$

$$F_5 = \frac{F_4}{\lambda_5} = \frac{316.8}{1.52} = 208.4 \text{ mm}^2$$

$$b_{5w} = \sqrt{4.35 F_5} = 30.1 \text{ mm}$$

$$h_5 = \frac{b_{5w}}{2.9} = \frac{30.1}{2.9} = 10.4 \text{ mm}$$

$$R = \frac{b_{5w}^2 + h_5^2}{4h_5} = 24.4 \text{ mm}$$

$$\beta_5 = 1.33$$

$$b_5 = \beta_5 b_4 = 1.33 \times 17.8 = 23.7 \text{ mm}$$

$$h_{5m} = \frac{F_5}{b_5} = \frac{208.4}{23.7} = 8.8 \text{ mm}$$

$$D_t = 280.0 \text{ mm}$$

$$D_{w5} = D_t - h_{5m} = 280.0 - 8.8 = 271.2 \text{ mm}$$

$$v_5 = w_4 = 1.458 \text{ m/sec}$$

$$w_5 = v_5 \lambda_5 = 1.458 \times 1.52 = 2.216 \text{ m/sec}$$

$$w_5 = \frac{4.62 \times 10^5}{F_5} = \frac{4.62 \times 10^5}{208.4} = 2.216 \text{ m/sec}$$

*Calculation of round in Stand VI*

$$F_6 = \frac{F_5}{\lambda_6} = \frac{208.4}{1.31} = 159.1 \text{ mm}^2$$

$$h_6 = d_6 = \sqrt{\frac{4F_6}{\pi}} = \sqrt{\frac{4 \times 159.1}{3.14}} = 14.2 \text{ mm}$$

$$\beta_6 = 1.33$$

$$b_6 = \beta_6 h_5 = 1.33 \times 10.4 = 13.8 \text{ mm}$$

$$h_{6m} = \frac{F_6}{b_6} = \frac{159.1}{13.8} = 11.5 \text{ mm}$$

$$D_t = 285.0 \text{ mm}$$

$$D_{w6} = D_t - h_{6m} = 285.0 - 11.5 = 273.5 \text{ mm}$$

$$v_6 = w_5 = 2.216 \text{ m/sec}$$

$$w_6 = v_6 \lambda_6 = 2.216 \times 1.31 = 2.903 \text{ m/sec}$$

$$F_6 w_6 = 4.62 \times 10^5 \text{ mm}^3/\text{sec}$$

$$w_6 = \frac{4.62 \times 10^5}{159.1} = 2.904 \text{ m/sec}$$

Table 7.19 gives the results of these pass design calculations, the pass dimensions are shown Fig. 7.85. It will be recalculated once more by using  $\beta$ .

The values of roll pressure  $P$  were measured during rolling. The pass programme was well designed since it may be seen from Table 7.19 that  $\lambda$  is always  $> \beta$ , which is difficult to attain for a small section and large roll diameter. In the final pass the coefficients equalize, i.e.  $\lambda \approx \beta$ .

## 7.8. Effective Radii and Neutral Lines

### 7.8.1. CALCULATION OF ROLL DIAMETERS IN SECTION MILLS BY C.MURSKI'S METHOD [132, 145]

Roll pass design is not only concerned with the determination of the shape and dimensions of successive passes, but also with their proper positioning in the rolls. For this reason, the neutral axis of every pass must be determined, i.e. the horizontal line which defines the pass position in the rolls, and also serves as an auxiliary line for the calculation of effective diameters [132, 145].

In principle, in order to obtain equal speeds of metal flow for the metal in both rolls, and thus theoretically a free and horizontal exit of stock from rolls, or assuming over- or underdraughts and the optimum speed of flow, the neutral axis of the pass is assumed to be the rolling line.

This principle obtains for passes in horizontal or staggered systems, but in a vertical system, where the choice of draught values in passes is especially important, the neutral line of pass lies above or below the rolling line, according to the kind of draught used.

In all the formulae evolved here, overdraught is assumed, since this type of draught is used in most cases. Underdraughts are commonly used in blooming and roughing mills for ingots or blooms, and in certain cases where deep and sharp cut grooves are used in the closed lower parts of pass, and in vertical systems.

If underdraught is used, the opposite sign should be used for corresponding values in these formulae.

It should be noted that although figures and calculation examples are given for box-passes in order to facilitate the presentation of the problem, the formulae given are valid for all shapes.

The problem will be considered for three principal pass systems [132, 145]:

(a) The horizontal system where the successive roll passes take place in pass grooves cut in the rolls of 2-high or double 2-high mills (Fig. 7.86) [132, 145].

(b) The staggered system in 3-high mills, where the successive roll passes take place in pass grooves which do not lie one above another (Fig. 7.87) [132, 145].

(c) The vertical system in 3-high mills where the successive roll passes take place in pass grooves lying one above another, i.e. where the middle roll provides a common groove for upper and lower rolls (Fig. 7.88) [132, 145].

Roll diameters for a horizontal pass system (Fig. 7.86) are calculated after the pass design is fixed. Thus pass heights  $h$ , the upper  $h_u$  and lower  $h_l$  resulting from the neutral axis position, diameter  $D_t$  and the value of overdraught  $g$  assumed by the designer are known.

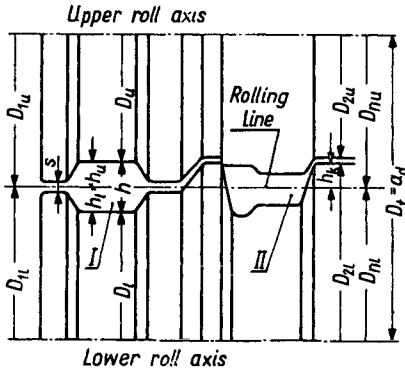


Fig. 7.86. Determination of mean and effective roll diameters for passes in horizontal system of 2-high rolling mills [145]

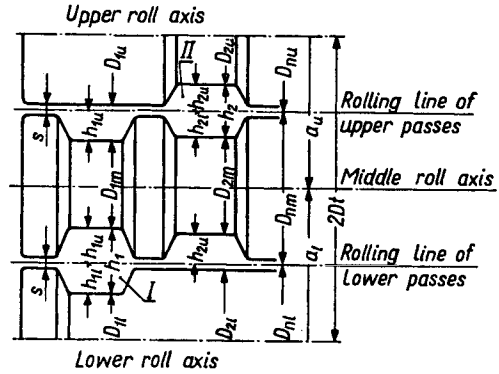


Fig. 7.87. Determination of mean and effective diameters for passes in staggered system of 3-high rolling mills [145]

The neutral and effective diameters in a horizontal pass system are calculated so that by placing the neutral line of pass on the rolling line, the assumed draughts are obtained.

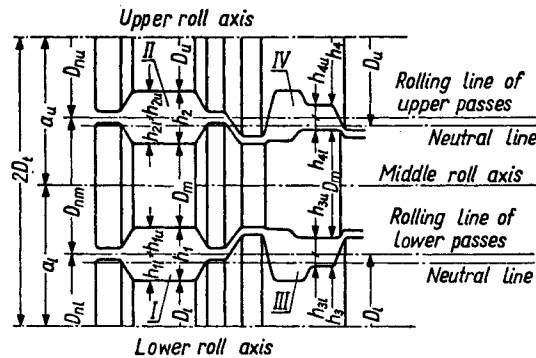


Fig. 7.88. Determination of mean and effective diameters for passes in vertical system of 3-high rolling mills [145]

The following formulae apply:

$$D_t = a_d \quad (7.176)$$

Rolling diameters

$$D_{nt} = D_t - \frac{g}{2} \quad (7.177)$$

$$D_{nu} = D_t + \frac{g}{2} \quad (7.178)$$

Effective pass diameters

$$D_l = D_{nl} - 2h_l \quad (7.179)$$

$$D_u = D_{nu} - 2h_u \quad (7.180)$$

In the case of regular open passes where the rolling line passes through the centre of the roll gap  $s$ , for the calculation of effective diameters at the collars it is assumed that

$$h_l = h_u = \frac{s}{2}$$

For the closed passes the roll gap lies below or above the line of pass closing at a distance of  $s = 5\text{--}30$  mm, the larger values of  $s$  corresponding to larger roll diameters or to large axial forces.

For the case shown in Fig. 7.86 pass groove  $II$ , the collar diameters

$$D_{2l} = D_{nl} + 2h_k \quad (7.181)$$

$$D_{2u} = D_{nu} - 2(h_k + s) \quad (7.182)$$

where  $h_k$  is the height of the extreme collar above the rolling line.

When checking calculations for neutral and effective diameters, they are always taken relative to a constant value for a given design system, that is the distance between roll axes.

Then

$$D_{nl} + D_{nu} = 2a_d = \text{constant} \quad (7.183)$$

For passes

$$D_l + 2h + D_u = 2a_d = \text{constant} \quad (7.184)$$

For collars

$$D_l + 2s + D_u = 2a_d = \text{constant} \quad (7.185)$$

*Example*

Given:

$$D_t = 800 \text{ mm}$$

Regular pass height

$$h_l = 200 \text{ mm}$$

$$h_{1l} = h_{1u} = 100 \text{ mm}$$

Irregular pass height

$$h_k = 125 \text{ mm}$$

It is assumed:

Overdraught  $g = 10$  mm

The gap between the extreme collars = 10 mm

The gap between the inter-pass collars = 12 mm

Hence

$$a_d = 800 \text{ mm}$$

$$D_{nl} = 800 - \frac{10}{2} = 795 \text{ mm}$$

$$D_{nu} = 800 + \frac{10}{2} = 805 \text{ mm}$$

For regular pass

$$D_l = 795 - 2 \times 100 = 595 \text{ mm}$$

$$D_u = 805 - 2 \times 100 = 605 \text{ mm}$$

Then the assumed overdraught is found

$$D_u - D_l = 605 - 595 = 10 \text{ mm} = g$$

For extreme collar

$$D_l = 795 - 2 \times 5 = 785 \text{ mm}$$

$$D_u = 805 - 2 \times 5 = 795 \text{ mm}$$

For inter-pass collar

$$D_l = 795 - 2 \times 6 = 783 \text{ mm}$$

$$D_u = 805 - 2 \times 6 = 793 \text{ mm}$$

For collar of the irregular pass

$$D_l = 795 + 2 \times 125 = 1045 \text{ mm}$$

$$D_u = 805 - 2 \times (125 + 12) = 531 \text{ mm}$$

Checking

$$2a_d = 2 \times 800 = 1600 \text{ mm} = \text{constant}$$

Neutral diameters

$$795 + 805 = 1600 \text{ mm}$$

For regular pass

$$595 + 2 \times 200 + 605 = 1600 \text{ mm}$$

For extreme collar

$$785 + 2 \times 10 + 795 = 1600 \text{ mm}$$

For inter-pass collar

$$783 + 2 \times 12 + 793 = 1600 \text{ mm}$$

For collar of irregular pass

$$1045 + 2 \times 12 + 531 = 1600 \text{ mm}$$

The calculation of neutral diameters for a staggered pass system (Fig. 7.87) is similar to that for the horizontal one. The neutral axis of passes when placed in the rolling line must give effective diameters with the assumed over- or underdraughts.

Roll design determines  $D_l$  and the heights of passes and of pass parts divided by the neutral axis, hence  $h$ ,  $h_l$  and  $h_u$  are known. Certain values are assumed by the designer: the overdraught in the lower pass grooves  $g_l$ , the overdraught in the upper pass grooves  $g_u$  and roll gap  $s$ . These values being known, the roll diameters can be calculated from formulae

$$a_l = D_l - \frac{g_l + g_u}{4} \quad (7.186)$$

$$a_u = D_l + \frac{g_l + g_u}{4} \quad (7.187)$$

For rolling diameters

$$D_{nl} = D_l - \frac{3g_l + g_u}{4} \quad (7.188)$$

$$D_{nm} = D_l + \frac{g_l - g_u}{4} \quad (7.189)$$

$$D_{nu} = D_l + \frac{g_l + 3g_u}{4} \quad (7.190)$$

The neutral axis being placed on the rolling line, the effective diameters are calculated similarly as for vertical pass system.

Hence for the pass groove I (lower) (Fig. 7.87)

$$D_{1l} = D_{nl} - 2h_{1l} \quad (7.191)$$

$$D_{1m} = D_{nm} - 2h_{1u} \quad (7.192)$$

For the pass groove II (upper)

$$D_{2m} = D_{nm} - 2h_{2m} \quad (7.193)$$

$$D_{2u} = D_{nu} - 2h_{2u} \quad (7.194)$$

For checking, calculations are taken relative to constant values; then

$$a_l + a_u = 2D_t \quad (7.195)$$

$$D_{nl} + D_{nm} = 2a_l = \text{constant} \quad (7.196)$$

$$D_{nm} + D_{nu} = 2a_u = \text{constant} \quad (7.197)$$

$$D_l + 2h_1 + D_m = 2a_l = \text{constant} \quad (7.198)$$

$$D_m + 2h_2 + D_u = 2a_u = \text{constant} \quad (7.199)$$

#### Example

Given:  $D_t = 800$  mm,  $h_1 = 200$  mm,  $h_{1l} = h_{1u} = 100$  mm,  $h_2 = 160$  mm,  $h_{2l} = h_{2u} = 80$  mm,  $g_l = 10$  mm,  $g_u = 12$  mm,  $s_1 = 10$  mm (gap between the extreme collars),  $s_2 = 11$  mm (gap between the inter-pass collars). Calculations are from formulae (7.186) to (7.199):

$$a_l = 800 - \frac{10+12}{4} = 794.5 \text{ mm}$$

$$a_u = 800 + \frac{10+12}{4} = 805.5 \text{ mm}$$

$$D_{nl} = 800 - \frac{30+12}{4} = 789.5 \text{ mm}$$

$$D_{nm} = 800 + \frac{10-12}{4} = 799.5 \text{ mm}$$

$$D_{nu} = 800 + \frac{10+36}{4} = 811.5 \text{ mm}$$

The effective diameters for the first pass will be calculated assuming that the upper pass is "dead" and the upper groove diameter in the upper pass is equal to the extreme collar diameter

$$D_{1l} = 789.5 - 2 \times 100 = 589.5 \text{ mm}$$

$$D_{1m} = 799.5 - 2 \times 100 = 599.5 \text{ mm}$$

$$D_{1u} = 811.5 - 2 \times 5 = 801.5 \text{ mm}$$

The effective diameters for the second pass are calculated as for the first, remembering that the diameter of the lower groove of the lower pass is equal to the diameter of the inter-pass collar

$$D_{2l} = 789.5 - 2 \times 5.5 = 778.5 \text{ mm}$$

$$D_{2m} = 799.5 - 2 \times 80 = 639.5 \text{ mm}$$

$$D_{2u} = 811.5 - 2 \times 80 = 651.5 \text{ mm}$$

Checking:

(a) Distance between axes

$$794.5 + 805.5 = 2 \times 800$$

$$1600 = 1600 \text{ mm}$$

(b) Neutral diameters

$$789.5 + 799.5 = 2 \times 794.5$$

$$1589 = 1589 \text{ mm}$$

$$799.5 + 811.5 = 2 \times 805.5$$

$$1611 = 1611 \text{ mm}$$

(c) Effective diameters: *Pass I*

$$589.5 + 2 \times 200 + 599.5 = 1589 \text{ mm}$$

$$599.5 + 2 \times 105 + 801.5 = 1611 \text{ mm}$$

N.B. The upper pass height is the sum of  $h_{1u} + s_1/2$ , that is

$$100 + 5 = 105 \text{ mm}$$

(d) Effective diameter: *Pass II*

$$778.5 + 2 \times 85.5 + 639.5 = 1589 \text{ mm}$$

$$639.5 + 2 \times 160 + 651.5 = 1611 \text{ mm}$$

Thus the calculations are correct since the equalities are fulfilled. The calculated effective diameters satisfy the assumption relative to draughts where

$$D_{1m} - D_{1l} = 599.5 - 589.5 = 10 = g_l$$

$$D_{2u} - D_{2m} = 651.5 - 639.5 = 12 = g_u$$

The calculation of diameters in the vertical system (Fig. 7.88) is more complex than for the systems already considered. Due to the different heights of passes forming a pair, that is of passes cut one above another, the rolling line cannot divide the passes along their neutral axis. To obtain the optimum division of pass height by the rolling line, the principle is commonly applied that

$$h_{1u} = h_{2l} = \frac{h_1 + h_2}{4} \quad (7.200)$$

then

$$\Delta h_{1l} = h_1 - h_{1u} \quad (7.201)$$

$$\Delta h_{2u} = h_2 - h_{2l} \quad (7.202)$$

If this principle is observed, in the case of overdraughts the neutral axis of passes will lie below the rolling line, at a distance  $c$ , where

$$c = \frac{h_1 - h_2}{4}$$

In the case of underdraughts the neutral axis of pass will lie a distance  $c$  above the rolling line. Since there are different values of  $h_1$  and  $h_2$  for particular pairs of passes a constant value  $c$  will be obtained, different for each pair, and hence different values of overdraughts or underdraughts. For this reason the choice of optimum over- or underdraughts is extremely important for proper rolling and for the maximum exploitation of rolls. This is of even greater importance when in many cases a diameter of the middle roll larger than that of the extreme ones is used in order to increase roll exploitation. Thus for new rolls, overdraught in the lower passes and underdraught in the upper ones is used intentionally. This division of diameters is justified when the middle roll, the groove of which works in both lower and upper pass, is worn more than the outer rolls. With successive dressing, roll diameters equalize and eventually

the "dead 3-high" should be composed of the thinner middle roll and two thicker outer rolls. Taking this into account, a designer should fix draughts for new rolls such that when these rolls reach the "dead" stage i.e. after the last re-turning, these draughts can be used without danger of roll or gear parts breaking. The following formulae, valid for all passes in the given rolls, assist a choice of such draughts

$$\Delta h - g_l = k_1 = \text{constant} \quad (7.203)$$

$$\Delta h - g_u = k_2 = \text{constant} \quad (7.204)$$

where

$g_l$  = lower draught,

$g_u$  = upper draught.

These relations being known, a designer should establish optimum draughts for particular passes before calculating diameters. Examples given later will explain these formulae in detail.

Values  $g_l$  and  $g_u$  for all the passes are related by

$$g_l - g_u = \text{constant} \quad (7.205)$$

To simplify the calculation it is sufficient to calculate the value of  $g_l$  for the further pairs of passes from equation (7.203), and the value of  $g_u$  can be readily obtained from equation (7.205).

With the pass design fixed and  $D_t$  known and with assumed optimum  $g_l$  and  $g_u$ , the roll diameters can be calculated from the following formulae:

$$a_l = D_t + \frac{\Delta h}{2} - \frac{g_l + g_u}{4} \quad (7.206)$$

$$a_u = D_t - \frac{\Delta h}{2} + \frac{g_l + g_u}{4} \quad (7.207)$$

Rolling diameters

$$D_{nl} = D_t + \Delta h + \frac{3g_l + g_u}{4} \quad (7.208)$$

$$D_{nm} = D_t + \frac{g_l - g_u}{4} \quad (7.209)$$

$$D_{nu} = D_t - \Delta h + \frac{g_l + 3g_u}{4} \quad (7.210)$$

The effective diameters are calculated from the principles used in previous methods, then

$$D_l = D_{nl} - 2h_{1l} \quad (7.211)$$

$$D_m = D_{nm} - 2h_{1u} \quad (7.212)$$

$$D_u = D_{nu} - 2h_{2u} \quad (7.213)$$

where  $h_{1l}$ ,  $h_{1u}$  and  $h_2$  are obtained not from the division of pass height by the neutral axis of pass, but from the division by the rolling line as in formulae (7.200) to (7.202).

The effective diameters of the collars are calculated as for effective diameters of the grooves but

$$\Delta h_{1l} = h_{1u} - h_{2u} = \frac{s}{2}$$



Checking is related to constant values.

Then

$$a_l + a_u = 2D_t \quad (7.214)$$

$$D_{nl} + D_{nm} = 2a_l = \text{constant} \quad (7.215)$$

$$D_{nm} + D_{nu} = 2a_u = \text{constant} \quad (7.216)$$

$$D_l + 2h_l + D_m = 2a_l = \text{constant} \quad (7.217)$$

$$D_m + 2h_2 + D_u = 2a_u = \text{constant} \quad (7.218)$$

### Example

The roll diameters are to be calculated for rolling sheet bar in a mill of  $D_t = 800$  mm, where passes are to be cut in a vertical system. Overdraughts are assumed for passes I and II:  $g_l = 22$  mm. Roll gap  $s = 10$  mm. Roll pass design is to give the following heights:

| Pass                            | I  | II | III  | IV   | V    | VI  |
|---------------------------------|----|----|------|------|------|-----|
| $h$                             | 80 | 56 | 39.2 | 27.4 | 19.2 | 16  |
| $\Delta h$ for a pair of passes |    | 24 |      | 11.8 |      | 3.2 |

First of all one investigates overdraughts obtained in all passes as a result of the assumed overdraught for passes I and II. Since it is assumed that overdraughts are taken in pairs, and thus are equal for the lower and upper passes, they will also be equal in pairs for the following passes, and it will be sufficient to calculate the one value only.

From equation (7.203)

$$\Delta h - g_l = 24 - 22 = 2 \text{ mm} = k = \text{constant}$$

and from this for passes III and IV

$$\Delta h - k_l = g_l = g_u$$

$$11.8 - 2 = 9.8 \text{ mm}$$

$$g_l = g_u = 9.8 \text{ mm}$$

and for passes V and VI

$$\Delta h - k = 3.2 - 2 = 1.2 \text{ mm}$$

hence

$$g_l = g_u = 1.2 \text{ mm}$$

Assuming these draughts as permissible, the corresponding diameters will be calculated

$$a_l = 800 + \frac{24}{2} - \frac{22+22}{4} = 801 \text{ mm}$$

$$a_u = 800 - \frac{24}{2} + \frac{22+22}{4} = 799 \text{ mm}$$

$$D_{nl} = 800 + 24 - \frac{3 \times 22 + 22}{4} = 802 \text{ mm}$$

$$D_{nm} = 800 + \frac{22-22}{4} = 800 \text{ mm}$$

$$D_{nu} = 800 - 24 + \frac{22+3 \times 22}{4} = 798 \text{ mm}$$

For passes I and II

$$h_{1u} = \frac{80+56}{4} = 34 \text{ mm}$$

$$h_{1l} = 80 - 34 = 46 \text{ mm}$$

$$h_{2u} = 56 - 34 = 22 \text{ mm}$$

$$D_1 = 802 - 2 \times 46 = 710 \text{ mm}$$

$$D_m = 800 - 2 \times 34 = 732 \text{ mm}$$

$$D_u = 798 - 2 \times 22 = 754 \text{ mm}$$

and from this

$$g_l = 754 - 732 = 22 \text{ mm}$$

$$g_u = 754 - 732 = 22 \text{ mm}$$

The calculated draughts agree with the assumed ones.

For passes III and IV

$$h_{3u} = \frac{39.2+27.4}{4} = 16.65 \text{ mm}$$

$$h_{3l} = 39.2 - 16.65 = 22.55 \text{ mm}$$

$$h_{4u} = 27.4 - 16.65 = 10.75 \text{ mm}$$

$$D_l = 800 - 2 \times 16.65 = 766.7 \text{ mm}$$

$$D_u = 798 - 2 \times 10.75 = 776.5 \text{ mm}$$

From this

$$g_l = 766.7 - 756.9 = 9.8 \text{ mm}$$

$$g_u = 776.5 - 766.7 = 9.8 \text{ mm}$$

For passes V and VI

$$h_{5u} = \frac{19.2+16}{4} = 8.8 \text{ mm}$$

$$h_{5l} = 19.2 - 8.8 = 10.4 \text{ mm}$$

$$h_{6u} = 16 - 8.8 = 7.2 \text{ mm}$$

$$D_l = 802 - 2 \times 10.4 = 781.2 \text{ mm}$$

$$D_m = 800 - 2 \times 8.8 = 782.4 \text{ mm}$$

$$D_u = 798 - 2 \times 7.2 = 783.6 \text{ mm}$$

Then

$$g_l = 782.4 - 781.2 = 1.2 \text{ mm}$$

$$g_u = 783.6 - 782.4 = 1.2 \text{ mm}$$

Checking:

$$801 + 799 = 2 \times 800 \text{ mm}$$

$$1600 = 1600 \text{ mm}$$

Neutral diameters

$$802 + 800 = 2 \times 801 \text{ mm}$$

$$1602 = 1602 \text{ mm}$$

$$800 + 798 = 2 \times 799 \text{ mm}$$

$$1598 = 1598 \text{ mm}$$

Pass I

$$710 + 160 + 732 = 2 \times 801 \text{ mm}$$

$$1602 = 1602 \text{ mm}$$

Pass II

$$732 + 112 + 754 = 2 \times 799 \text{ mm}$$

$$1598 = 1598 \text{ mm}$$

Pass III

$$756.9 + 78.4 + 766.7 = 1602 \text{ mm}$$

$$1602 = 1602 \text{ mm}$$

Pass IV

$$766.7 + 54.8 + 776.5 = 1598 \text{ mm}$$

$$1598 = 1598 \text{ mm}$$

Pass V

$$781.2 + 38.4 + 782.4 = 1602 \text{ mm}$$

$$1602 = 1602 \text{ mm}$$

Pass VI

$$782.4 + 32 + 783.6 = 1598 \text{ mm}$$

$$1598 = 1598 \text{ mm}$$

Extreme collars

$$890 + 20 + 692 = 1602 \text{ mm}$$

$$1602 = 1602 \text{ mm}$$

$$692 + 20 + 886 = 1598 \text{ mm}$$

$$1598 = 1598 \text{ mm}$$

It is seen that the calculations are correct, and the assumed draughts are obtained.

### 7.8.2. EFFECTIVE RADII AND NEUTRAL AXES OF PASSES IN THE ROLLING OF SHAPES

The correct determination of the neutral axis of a pass is a necessary condition of efficient work, especially in rolling of shapes. If two collaborating grooves have different mean effective diameters, the roll with the larger diameter will drag along the section being rolled and cause additional torque and pressure on the pinions. As the metal leaves the pass the rolls come back to their original position, causing an impact on the whole line of drive. The greater the play in the connecting links, the greater are these impacts, which are dangerous and can cause a roll or teeth of a pinion to break. The roll with the smaller effective diameter retards the metal passing through the rolls, increasing the forward slip. This causes additional surface defects of metal. In such conditions the rolls wear out more quickly, the motor power consumption is also greater in order to overcome frictional resistance [54, 133, 161].

Rolling in passes with different effective diameters has a detrimental effect on the metal, giving rise to tensile stresses which often exceed its cohesion. Technical literature gives many methods of calculation of the neutral axis and effective diameter of the pass, but there is a lack of information on the correct choice of the formula. For a better understanding of the problem a roll pass scheme will be analysed.

It is assumed that a section as in Fig. 7.89 is rolled, where the exit speed of section from the upper roll is determined by  $R_{um}$  the mean upper effective radius, and for the lower roll by  $R_{lm}$  the mean lower effective radius. The neutral axis of pass 0-0 must be so placed on the neutral line of rolls that these radii will be equal, hence

$$R_{um} = R_{lm} = R_m \quad (7.219)$$

Only in this way will equal peripheral speeds of upper and lower rolls be obtained, i.e.

$$v_{rum} = v_{rlm} = v_{rm} \quad (7.220)$$

hence

$$\frac{\pi D_{um} n}{60} = \frac{\pi D_{lm} n}{60} = \frac{\pi D_m n}{60} \quad (7.221)$$

if rpm of both rolls are equal.

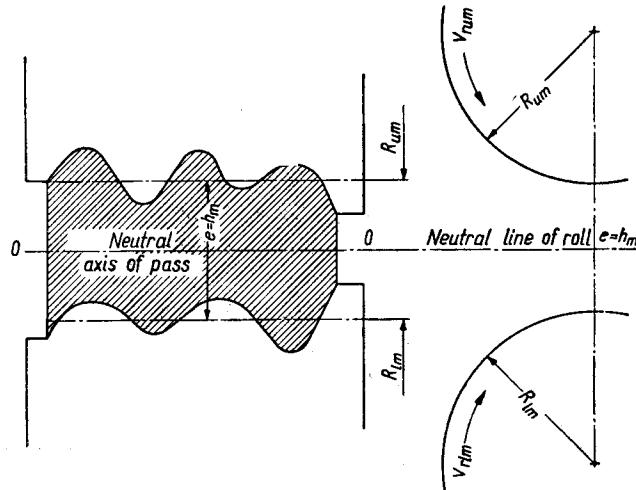


Fig. 7.89. Rolling scheme of a section for determination of neutral axis and of mean effective radii [133], [161]

Theoretically the position of neutral axis of pass 0-0 fulfilling the relations (7.219) and (7.220) can be found by dividing the distance  $e$  by half, i.e.

$$\frac{e}{2} = \frac{h_m}{2}$$

Actually the effect of the upper roll is somewhat different from that of the lower, but a correct estimation of its effect and correct division of the length  $e = h_m$  is a very difficult matter.

Relatively easier is the analysis of rolling conditions of each roll separately with a suitable correction of assumed  $R_{um}$  and  $R_{lm}$ . These values having been determined, the neutral axis position can be established by dividing  $e = h_m$  in half [133, 144].

As an example the pass in Fig. 7.90 is considered. The rolls are of equal diameter, hence  $R_u = R_l$ , and their speeds are equal, and in the whole pass the pressures on the walls are approximately uniform, then as may be seen from the figure, placing the neutral line of pass low gives different rolling speeds for the two grooves, since  $R_a$  is considerably less than  $R_b$ . Therefore the mean effective radius of the upper groove will be less than that of the lower groove, hence

$$R_{lm} > R_{um}$$

and hence the mean speed of the lower groove is greater than that of the upper one

$$v_{rlm} > v_{rum}$$

It is clear that turning Fig. 7.90 through  $180^\circ$  will only cause a change of conditions in the sense that

$$v_{ru} > v_{rl}$$

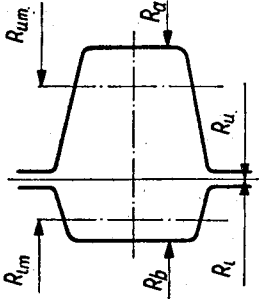


Fig. 7.90. Pass neutral axis positioned too low

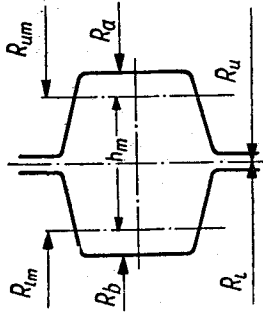


Fig. 7.91. Correct position of pass neutral axis  $R_u = R_l$ ,  
 $v_u = v_l$

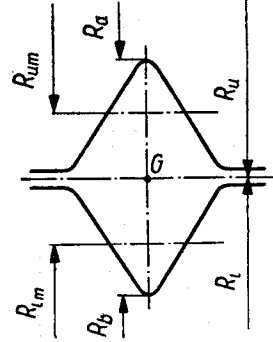


Fig. 7.92. Neutral axis position for pass having double symmetry

Placing the neutral axis of pass as in Fig. 7.91 where not only  $R_u = R_l$  but also  $R_a = R_b$  gives the condition

$$R_{lm} = R_{um}$$

hence the mean effective radius of upper roll groove is equal to the mean effective radius of the lower groove, so that

$$v_{rum} = v_{rlm}$$

i.e. the mean rolling speeds of the two rolls are equal.

Double symmetrical passes will be considered where pressures on the walls are slightly different from each other (Fig. 7.92). In this case

$$R_{um} = \frac{R_u + R_a}{2} \quad (7.222)$$

$$R_{lm} = \frac{R_l + R_b}{2} \quad (7.223)$$

The radius corresponding to the mean speed is given by the condition

$$R_m = R_{lm} = R_{um} \quad (7.224)$$

Calculation of the position of the neutral axis of the pass as the line of equal rolling speeds for the two rolls is given for a schematic pass outline assumed to be composed of horizontal lengths  $a$  and vertical or diagonal lengths  $p$  (Fig. 3.100). The mean effective radius of the upper roll groove will be (3.199)

$$R_{um} = \frac{a_1 R_1 + a_2 R_2 + a_3 R_3 + a_4 R_4 + \left( \frac{R_1 + R_2}{2} \right) p_1 + \left( \frac{R_2 + R_3}{2} \right) p_2 + \left( \frac{R_3 + R_4}{2} \right) p_3}{a_1 + a_2 + a_3 + a_4 + p_1 + p_2 + p_3}$$

For the lower roll groove there is a similar formula (3.200).

$$R_{lm} = \frac{a_I R_I + a_{II} R_{II} + a_{III} R_{III} + \left( \frac{R_I + R_{II}}{2} \right) p_I + \left( \frac{R_{II} + R_{III}}{2} \right) p_{II}}{a_I + a_{II} + a_{III} + p_I + p_{II}}$$

From these data the mean effective radius of the pass can be calculated from (7.224)

$$R_{m \text{ pass}} = \frac{R_{um} + R_{lm}}{2}$$

Formulae (3.199) and (3.200) are only valid with the assumption of uniform pressures in every part of roll groove [133 and 144].

To obtain relation (7.224) the neutral axis of the pass must be positioned so that the mean exit speeds of the upper and lower grooves are equal. This is only possible if the neutral axis of the pass bisects the distance between the axes defined by  $R_{lm}$  and  $R_{um}$  (Fig. 7.89), and it must be placed in this position [133 and 144].

A more correct method of determining the position of the neutral axis of pass would take into account the effect of different lengths of sectors of the pass outline and also of the different pressures caused by draughts on these lengths. This can be expressed as a general equation for any given roll

$$R_{mw} = \frac{\sum a_i R_i + \sum \left( \frac{R_i + R_{i+1}}{2} \right) m p_i}{\sum (a_i + m p_i)} \quad (7.225)$$

In equation (7.225)  $m$  is found from the section dealing with the effect of additional friction in the pass, Table 3.18.

In the case of compound passes the sections are divided into simple elements such as flange, neck, web, head, leg etc.

The mean effective radii of each constituent part of pass are calculated separately. Then the neutral axis of pass can be found (Fig. 7.93) [133 and 144].

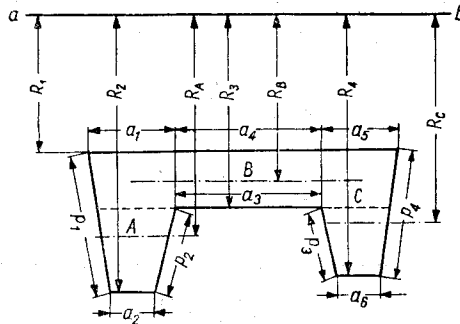


Fig. 7.93. Illustration of author's III method of determining the position of the neutral axis

The pass shape is drawn out as it will be cut in the rolls and an arbitrary straight line  $a-b$  is drawn, relative to which all the diameters will be calculated.

The pass is divided into three separate areas  $A$ ,  $B$  and  $C$ . The division is shown by the thin line. The pass can be also divided as shown by the broken line, depending on the chosen rolling methods.

Neglecting the pressure difference on the different walls, from formula (3.199) [133 and 144]

$$R_A = \frac{a_1 R_1 + a_2 R_2 + p_1 \left( \frac{R_1 + R_2}{2} \right) + p_2 \left( \frac{R_2 + R_3}{2} \right)}{a_1 + a_2 + p_1 + p_2} \quad (7.226)$$

$$R_B = \frac{a_3 R_3 + a_4 R_1}{a_3 + a_4} \quad (7.227)$$

$$R_C = \frac{a_6 R_4 + a_5 R_1 + p_3 \left( \frac{R_3 + R_4}{2} \right) + p_4 \left( \frac{R_4 + R_1}{2} \right)}{a_6 + a_5 + p_3 + p_4} \quad (7.228)$$

where  $R_A$ ,  $R_B$ ,  $R_C$  are the mean effective radii of the corresponding areas into which the pass was divided. From these values the mean effective radius can be calculated

$$R_m = \frac{F_{A2} \lambda_A R_A + F_{B2} \lambda_B R_B + F_{C2} \lambda_C R_C}{(F_{A2} + F_{B2} + F_{C2}) \lambda_m} \quad (7.229)$$

and hence the position of the neutral axis, where

$F_{A2}$ ,  $F_{B2}$  and  $F_{C2}$  = areas of the pass parts  $A$ ,  $B$ ,  $C$ ,

$\lambda_A$ ,  $\lambda_B$  and  $\lambda_C$  = elongation coefficients of pass parts  $A$ ,  $B$ ,  $C$ ,  $\lambda_A = F_{A1}/F_{A2}$ ,

$\lambda_B = F_{B1}/F_{B2}$ ,  $\lambda_C = F_{C1}/F_{C2}$

$\lambda_m$  = mean elongation coefficient of the whole pass =  $F_1/F_2$ .

The methods considered neglect non-uniform metal flow during rolling. Therefore they can be used in finishing roll passes where these phenomena are intentionally limited but if used in roughing roll passes very great differences may be found due to metal flow.

By transforming, formulae can be found taking into account the effect of metal flow on the mean effective diameters of a given part of the pass in rolling of sections.

$$\left. \begin{aligned} R_{A2} &= \frac{R_{A1}}{1 + \frac{\lambda_A - \lambda_m}{\lambda_m \lambda_A}} \\ R_{B2} &= \frac{R_{B1}}{1 + \frac{\lambda_m - \lambda_B}{\lambda_m \lambda_B}} \end{aligned} \right\} \quad (7.230)$$

With known  $R_{A2}$  and  $R_{B2}$  calculated from formula (7.230),  $R_m$  can be calculated from formula (7.229)

$$R_m = \frac{F_{A2} R_A \lambda_A + F_{B2} R_B \lambda_B}{(F_{A2} + F_{B2}) \lambda_m}$$

It was assumed that if the effective radii were correctly defined and the neutral axis correctly positioned (Fig. 7.89), then if the torques of the upper and lower rolls are equal the following relations should be found:

$$R_{um} = R_{lm} = R_m$$

$$v_{rum} = v_{rlm} = v_{rm}$$

$$M_{ru} = M_{rl} = 0.5 M_r$$

$$e_u = e_l = 0.5 h_m$$

$$e_u + e_l = e = h_m$$

and hence

$$\frac{M_{ru}}{M_{rl}} = \frac{e_l}{e_u} = 1$$

Hence the position of the neutral axis of the pass can be calculated from the upper and lower roll torques.

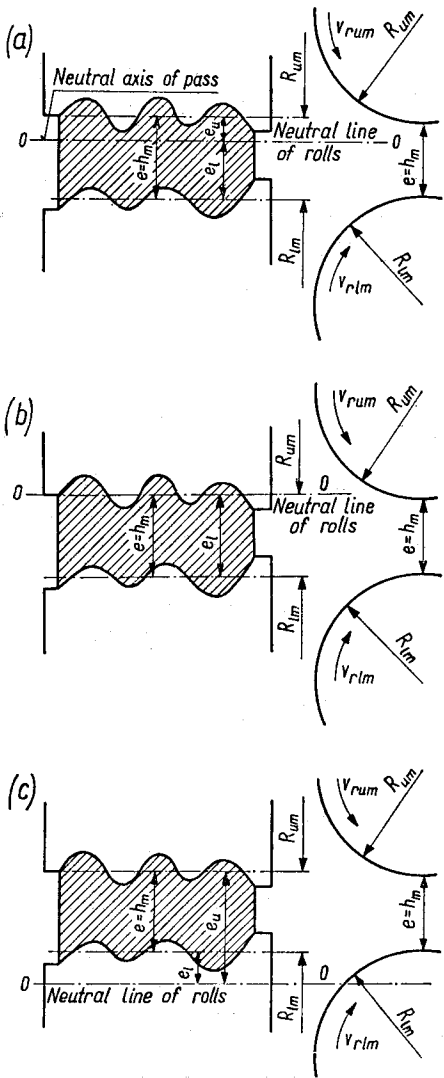


Fig. 7.94. Different positions of neutral axis according to the partition of the rolling torque between upper and lower rolls

But if  $M_{ru} > M_{rl}$  then from condition of torque equilibrium (Fig. 7.94a)

$$M_{ru}e_u = M_{rl}e_l \quad (7.231)$$

hence

$$e_l > e_u$$



Calculating  $e_u$  and  $e_l$  the distance by which the neutral axis of the pass must be shifted to equalize the torques again, can be found (Figs. 7.89 and 7.94) from

$$M_{ru}e_u = M_{rl}e_l$$

and

$$e_u + e_l = h_m$$

From these equations  $e_l$  and  $e_u$  can be found

$$\left. \begin{aligned} e_l &= h_m \frac{M_{ru}}{M_r} \\ e_u &= h_m \frac{M_{rl}}{M_r} \end{aligned} \right\} \quad (7.232)$$

Similar relations can be found if  $M_l > M_u$ , then  $e_u > e_l$ . From these formulae further conclusions can be drawn (Fig. 7.94b):

Where the position of the neutral axis corresponds to the effective radius then

$$\left. \begin{aligned} e_u &= 0 & M_{ru} &= 0 \\ e_l &= e = h_m \end{aligned} \right\} \quad (7.233)$$

and

$$M_r = M_{rl} \text{ (double torque value)}$$

Similar results will be obtained if (Fig. 7.94c)

$$\left. \begin{aligned} e_l &= 0 & \text{then } M_{rl} &= 0 \\ e_u &= e = h_m \end{aligned} \right\} \quad (7.234)$$

and

$$M_r = M_{ru} \text{ (double torque value)}$$

In Fig. 7.94c the neutral axis lies below the lower effective radius  $R_{lm}$ .

In such a case  $e_l$  takes values  $-e_l$ , and the torque becomes negative  $-M_{rl}e_l$ .

The upper roll must transmit a positive torque equal to the total torque plus the lower roll torque, i.e.

$$2M_{ru}e_u = -M_{rl}e_l$$

This is a very unfavourable case since the rolls are non-uniformly loaded, and the upper roll must transmit double torque plus the torque from the lower roll, giving excessive loading, and causes excessive wear, especially of the upper roll.

#### Example 1

As a calculation example the 10th roll pass of a 300 mm channel\* is taken. The roll barrel dimensions are 2000×835 mm. This section was chosen since it is large and a comparison of methods can very readily be made [133].

\* See Appendix 2.

The general data are as follows:

(a) the ingoing section (Fig. 7.95)

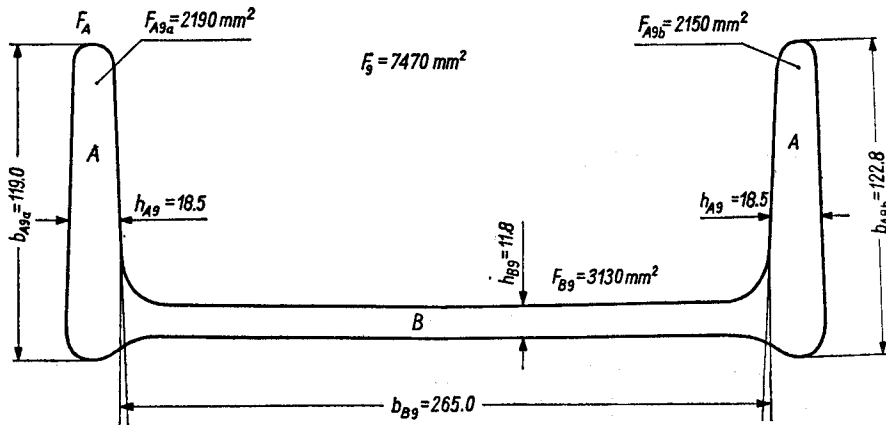


Fig. 7.95. Section of channel 300 (see Appendix 2) before 10th roll pass

(b) the exit section (Fig. 7.96)

Flange:

$$\begin{aligned} F_{A9a} &= 2190 \text{ mm}^2 & F_{A10a} &= 1800 \text{ mm}^2 \\ F_{A9b} &= 2150 \text{ mm}^2 & F_{A10b} &= 1840 \text{ mm}^2 \\ F_{A9} &= 4340 \text{ mm}^2 & F_{A10} &= 3640 \text{ mm}^2 \\ h_{A9} &= 18.5 \text{ mm} & h_{A10} &= 17.5 \text{ mm} \\ b_{A9a} &= 119.0 \text{ mm} & b_{A10b} &= 101 \text{ mm} \\ b_{A9b} &= 122.8 \text{ mm} \end{aligned}$$

$$\lambda_A = 1.1923$$

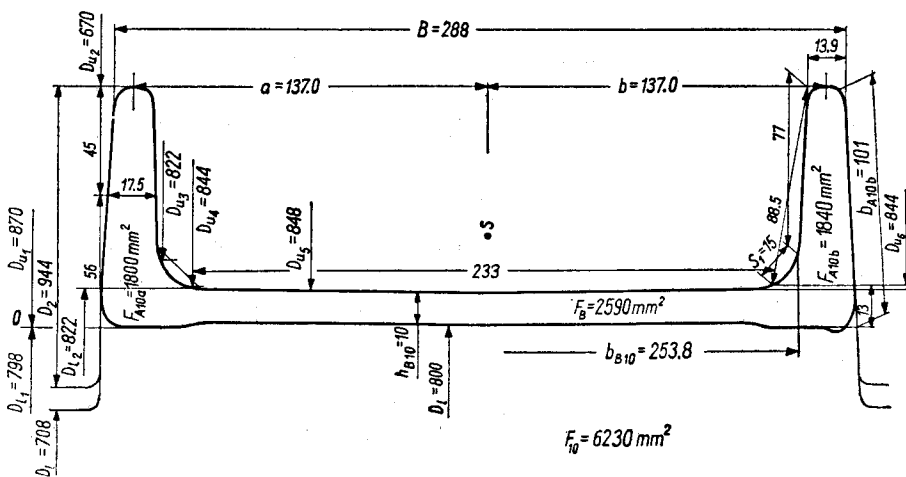


Fig. 7.96. Section of channel 300 (see Appendix 2) after 10th roll pass

Web:

$$F_{B9} = 3130 \text{ mm}^2 \quad F_{B10} = 2590 \text{ mm}^2$$

$$b_{B9} = 265.0 \text{ mm} \quad b_{B10} = 253.8 \text{ mm}$$

$$h_{B9} = 11.8 \text{ mm} \quad h_{B10} = 10.0 \text{ mm}$$

$$\lambda_B = 1.2085$$

$$F_9 = 7470 \text{ mm}^2 \quad F_{10} = 6230 \text{ mm}^2$$

$$\lambda_m = \frac{F_9}{F_{10}} = \frac{7470}{6230} = 1.1991$$

Different methods will be considered, and the diameters for this section will be calculated and compared. Such a comparison enables one to evaluate the practical value of the various methods.

I Author's method: (Fig. 3.100 and formulae (3.199) and (3.200))

$$R_{um} = \frac{a_1 R_1 + a_2 R_2 + a_3 R_3 + \left( \frac{R_1 + R_2}{2} \right) p_1 + \left( \frac{R_3 + R_4}{2} \right) p_2}{a_1 + a_2 + a_3 + p_1 + p_2}$$

$$R_{lm} = \frac{a_1 R_I + a_{II} R_{II} + \left( \frac{R_I + R_{II}}{2} \right) p_I}{a_1 + a_{II} + p_I}$$

Substituting it is found

$$R_{um} = \frac{233 \times 424 + 2 \times 13.9 \times 335 + 15(422 + 411) + 77(411 + 335) + 101(437 + 335)}{233 + 27.8 + 30 + 154 + 202}$$

$$= 395.5 \text{ mm}$$

$$D_{um} = 2R_{um} = 2 \times 395.5 = 791 \text{ mm}$$

$$R_{lm} = \frac{253.8 \times 400 + 2 \times 22.6 \times 399}{253.8 + 45.2} = 399.84 \text{ mm}$$

$$D_{lm} = 2R_{lm} = 799.68 \text{ mm}$$

$$D_{um} = 791 \text{ mm}$$

According to Fig. 7.97

$$h_m = e = m = 39.66, \quad \frac{m}{2} = 19.83$$

and from this

$$D_0 = 830 \text{ mm}$$

II Author's method [formula (7.225)] is based on the introduction of coefficients  $m$  over the diagonal lengths according to Table 3.18 [141]. With  $m = 0.2$  the following value is found:

$$R_{um} = \frac{233 \times 424 + 2 \times 13.9 \times 335 + 0.2[(422 + 411)15 + 77(411 + 335) + 101(437 + 335)]}{233 + 27.8 + 0.2 \times 30 + 154 + 202}$$

$$= 407.23 \text{ mm}$$

$$D_{um} = 2R_{um} = 814.46 \text{ mm}$$

$$R_{lm} = \frac{253.8 \times 400 + 45.2 \times 399}{253.8 + 45.2} = 399.84 \text{ mm}$$

$$D_{lm} = 2R_{lm} = 799.68 \text{ mm}$$

Hence in Fig. 7.97

$$h_m = e = m = 27.30 \text{ mm}, \quad \frac{m}{2} = 13.65 \text{ mm}$$

and for the neutral line

$$D_0 = 844 \text{ mm}$$

III Author's method (7.226) to (7.229) and Fig. 7.93

$$R_B = \frac{253.8 \times 400 + 233 \times 424 + 2 \times 15 \left( \frac{424 + 411}{2} \right)}{253.8 + 233 + 2 \times 15} = 411.84 \text{ mm}$$

$$D_B = 2R_B = 823.68 \text{ mm}$$

$$R_A = \frac{2(22.6 \times 400 + 13.9 \times 335) + 15(424 + 411) + 77(411 + 335) + 101(335 + 435)}{45.2 + 27.8 + 154 + 202} = 379.03 \text{ mm}$$

$$D_A = 2R_A = 758.06 \text{ mm}$$

$$R_m = \frac{3640 \times 1.1923 \times 379.03 + 2590 \times 1.2085 \times 411.84}{6230 \times 1.199} = 391.39 \text{ mm}$$

Hence in Fig. 7.97

$$D_m = 2R_m = 782.78 \text{ mm}$$

Dividing  $m$  in half

$$m = 51.22, \quad \frac{m}{2} = 25.69 \text{ mm}$$

Hence

$$D_0 = 834.00 \text{ mm}$$

IV Author's method based on equations (7.230) and Fig. 7.97. Substituting

$$R_{A2} = \frac{379.03}{1 + \left( \frac{1.1923 - 1.199}{1.199 \times 1.1923} \right)} = 380.81 \text{ mm}$$

$$D_{A2} = 2R_{A2} = 761.62 \text{ mm}$$

$$R_{B2} = \frac{411.84}{1 + \left( \frac{1.199 - 1.2085}{1.199 \times 1.2085} \right)} = 414.56 \text{ mm}$$

$$D_{B2} = 2R_{B2} = 829.12 \text{ mm}$$

$$R_m = \frac{3640 \times 1.1923 \times 380.81 + 2590 \times 1.2085 \times 414.56}{6230 \times 1.199} = 394.96 \text{ mm}$$

Hence

$$D_m = 789.92 \text{ mm}$$

For

$$m = 44.56 \text{ mm}, \quad \frac{m}{2} = \frac{44.56}{2} = 22.3 \text{ mm}$$

and from this, reading from (Fig. 7.97)

$$D_0 = 834.5 \text{ mm}$$

Calculations of this practical example by these methods of positioning of neutral axis in the pass give markedly divergent results [133] (Fig. 7.97).

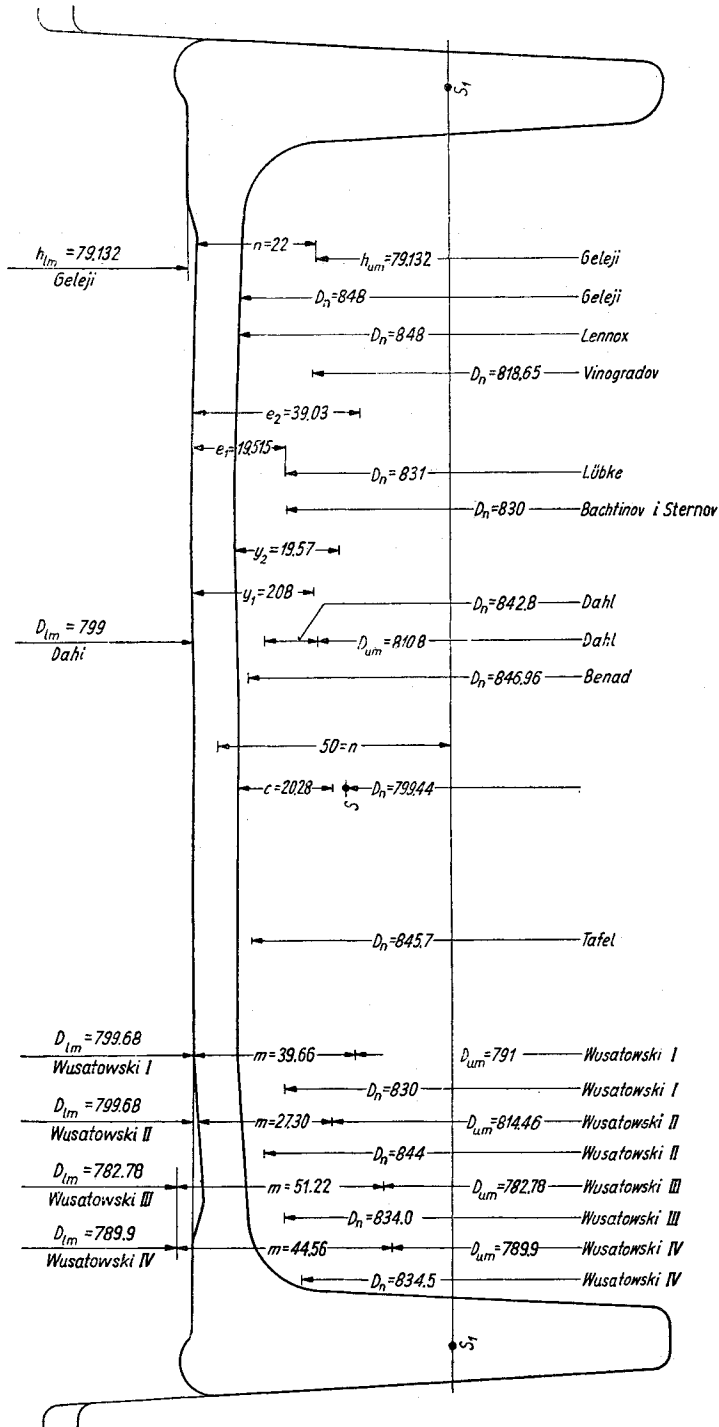


Fig. 7.97. Profile section of channel 300 (see Appendix 2) after 10th roll pass. From Fig. 7.96 with plotted diameters and neutral axes calculated by different methods. The section before roll pass as in Fig. 7.95

Methods by which the section is positioned high in the pass: Geleji and Lennox—848 mm, Benad—846.96 mm, Tafel—845.7 mm, Wusatowski II—844.0 mm and Dahl—842.8 mm [133].

The section is positioned somewhat lower in the pass in the methods of: Lübke—831.0 mm, Wusatowski I—830.0 mm, Wusatowski III—834.0 mm and Wusatowski IV—834.5 mm. The shape is positioned still lower in the pass in the methods: Dahl-Vinogradov—818.65 mm and the centre of gravity method—799.4 mm.

From this comparison it is seen that there are three different groups of methods in which similar results are obtained:

Group I: Geleji, Lennox, Benad, Tafel and Wusatowski II.

Group II: Lübke, Bachtinov-Sternov and Wusatowski I, III and IV.

Group III: Dahl-Vinogradov, centre of gravity method [133].

Including the effect of metal flow as in Wusatowski IV raises the position of the neutral axis by about 0.5 mm. It would be of interest to analyse this effect in the preliminary roll passes where the metal flow is considerably greater.

In the pass under consideration the elongation coefficients  $\lambda_A$  and  $\lambda_B$  are nearly equal to  $\lambda_m$ , i.e. there is almost no metal flow in the pass and its effect on the pass positioning is almost undetectable [133].

### Example 2

Turbine blades were designed and rolled in the experimental roll stand of the Institute of Ferrous Metallurgy in Gliwice with the following sections:

- (1) blade section P157,
- (2) blade section P167p,
- (3) blade section P170p.

The general pass design methods have already been considered. Figures 7.98, 7.99 and 7.100 show the passes, Fig. 7.98 the finished and pre-finished pass for section P157, Fig. 7.99 the same for section P167p and Fig. 7.100 for section P170p.

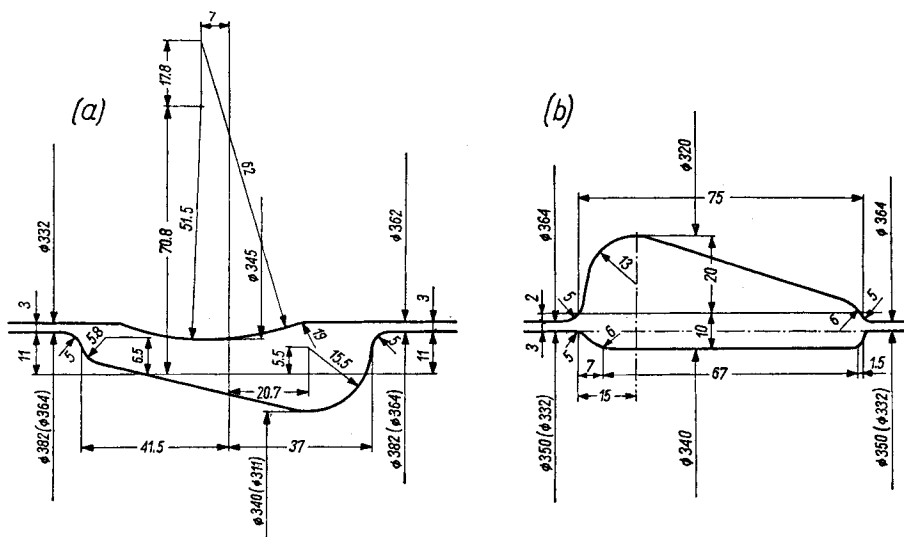
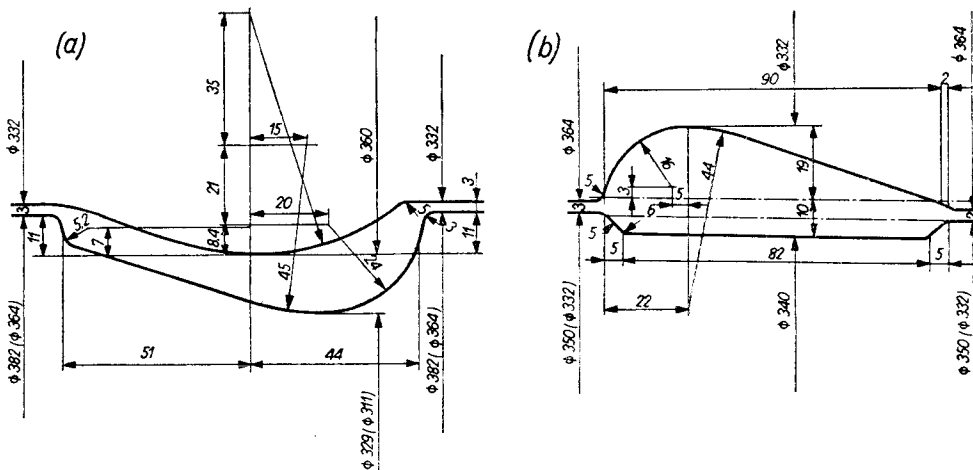
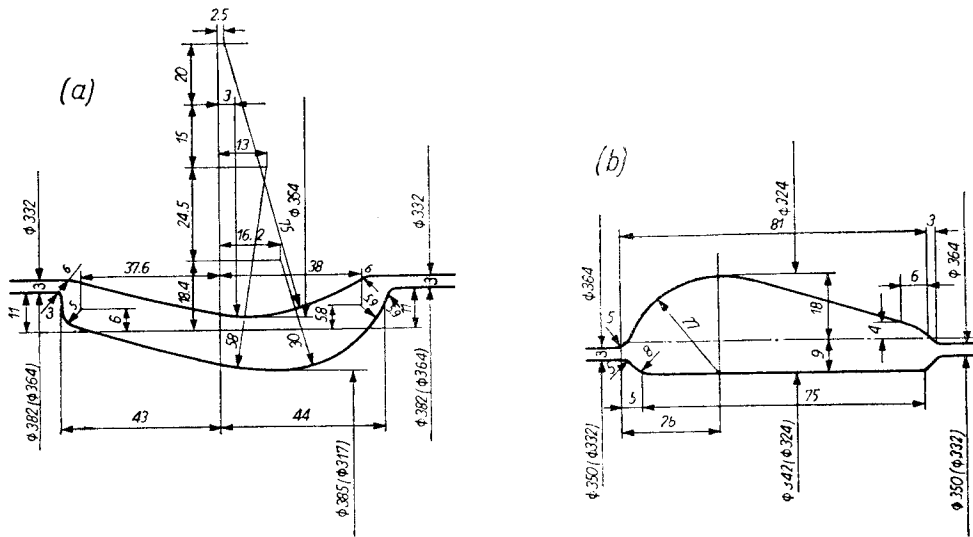


Fig. 7.98. Finishing and leader pass of turbine blades section P157 (the values in brackets are from tests given in Table 7.20): (a) finishing pass, (b) leader pass [161]



Effective diameters of these sections are calculated by means of formulae (3.199) and (3.200), i.e. using Wusatowski's I and II methods, and equation (7.225).

Wusatowski's III and IV methods are not suitable for this pass shape and they were not used.

**Effective diameters:**

For the leader section P157

$$R_{um} = 170.0 \text{ mm}$$

$$R_{lm} = 170.5 \text{ mm}$$

$$R_m = 170.25 \text{ mm}$$

For the finishing section P157

$$R_{um} = 169.0 \text{ mm}$$

$$R_{lm} = 177.6 \text{ mm}$$

$$R_m = 173.3 \text{ mm}$$

For the leader section P167p

$$R_{um} = 170.2 \text{ mm}$$

$$R_{lm} = 171.3 \text{ mm}$$

$$R_m = 170.75 \text{ mm}$$

For the finishing section P167p

$$R_{um} = 173.0 \text{ mm}$$

$$R_{lm} = 173.0 \text{ mm}$$

$$R_m = 173.0 \text{ mm}$$

For the leader section P170p

$$R_{um} = 169.3 \text{ mm}$$

$$R_{lm} = 170.4 \text{ mm}$$

$$R_m = 169.8 \text{ mm}$$

For the finishing section P170p

$$R_{um} = 174.3 \text{ mm}$$

$$R_{lm} = 174.3 \text{ mm}$$

$$R_m = 174.3 \text{ mm}$$

As a result of a preliminary sample blade rolling for steam turbine TK1 20, it was established that omitting the roughing pass did not adversely affect the finished blade section.

Therefore it was decided to roll the blades in the leader and finishing passes (Figs. 7.98; 7.99; 7.100) using the following ingoing stock dimensions:

for section P157      a flat  $67 \times 35 \text{ mm}$

for section P167p      a flat  $72 \times 35 \text{ mm}$

for section P170p      a flat  $82 \times 35 \text{ mm}$

In experiments the following steels were rolled: 1H13 with 0.15% C, 13.2% Cr, 0.6% Mn, 0.6% Ni and for purposes of comparison: Mild steel St 37 with 0.15% C and Steel 35 with 0.36% C, 0.6% Mn, 0.35% Si.

During rolling of these sections other measurements were also made which will be discussed later in more detail. Of these measurements, only results for rolling torques are given here, both for individual rolls and summed, since they are useful for solving torque and rolling speed problems.

Table 7.20 shows the results of measurements of rolling torque for turbine blades from 1H13 steel.

It is seen from this table that large errors were made in the positioning of sections P157II, P167I and P167II.

The values of measured torques do not differ widely. However for sections P157II, P167I, P167 II the neutral axis of pass was repositioned according to data given in Table 7.20 and Fig. 7.101. These displacements were introduced as the simplest way to equalize the upper and lower roll torques.

Check measurements confirmed the correctness of these changes.

(1) It was found that the position of the neutral axis of the pass depends on the method of calculation of the effective diameters of the pass and due to wide variations of these methods the calculation of diameters is the weak point of the whole problem.



(2) Experiments proved that the simplest method is to assume calculated effective diameters, and then equalize the differences of speed and torques by suitable adjustment of the position of the pass neutral axis.

TABLE 7.20  
RESULTS OF MEASUREMENTS OF ROLLING TORQUE FOR TURBINE BLADES

| Section | $M_{ru}$<br>tm | $M_{rl}$<br>tm | $M_{total}$<br>tm | $h_m = e$<br>mm | $e_u$<br>mm | $e_l$<br>mm | $R_0$<br>mm | $R'_0$<br>mm |
|---------|----------------|----------------|-------------------|-----------------|-------------|-------------|-------------|--------------|
| PI57I   | 0.703          | 0.669          | 1.37              | 21.01           | 10.23       | 10.76       | 181.0       | 181.26       |
| PI57II  | 0.376          | 1.64           | 2.016             | 14.4            | 11.72       | 2.68        | 184.8       | 180.2        |
| PI67I   | 0.469          | 0.946          | 1.414             | 20.88           | 13.96       | 6.92        | 181.7       | 177.22       |
| PI67II  | 1.383          | 1.693          | 3.076             | 13.72           | 7.6         | 6.12        | 179.85      | 179.12       |
| PI70I   | 0.822          | 0.943          | 1.765             | 20.42           | 10.45       | 10.00       | 180.6       | 180.4        |
| PI70II  | 1.85           | 1.994          | 3.844             | 13.24           | 6.89        | 6.35        | 180.9       | 180.65       |

$$R_0 = R_l + h_m/2$$

$$R'_0 = R_l + e_l$$

(3) Both preliminary and final measurement showed that the method evolved for torque correction by the change of neutral axis position provides the most correct solution.

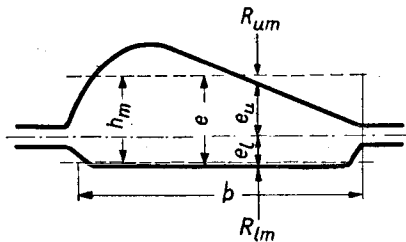


Fig. 7.101. Positioning of neutral line of pass for equalization of rolling torques [161]

(4) The conclusions refer to the rolling of simple sections where Wusatowski's I and II methods are applicable. For complex shapes or those in which large metal flow takes place, i.e. where Wusatowski's III and IV methods must be used, a further preliminary investigation should be made in order to check the conformity of derived formulae.

(5) Every dressing of the rolls gives a change of effective diameters, but the condition  $R_{um} = R_{lm} = R_m$  and  $e_u = e_l = 0.5 h_m$  which determines the neutral axis position (Fig. 7.89 and 7.94) must be maintained. If torque differences are found, the position of the neutral axis must be corrected at the next roll dressing to maintain these conditions which determine further correct roll dressing.

## 7.9. Analysis of the Rolling of Small Sections

The analysis of flat rolling processes [150, 142] has been treated by many authors, and the problem is sufficiently known and documented.

Less work has been done concerning the exact analysis of rolling in breaking-down

passes. Three recent works must be noted: Zouhar [157], Wallquist [163] and Pobedin and Drozd [164].

When rolling sections for TK1 20 turbine blades on an experimental 2-high rolling mill, at the same time a complete analysis of rolling conditions was made using the available measuring installations [150].

A complete analysis was carried out for three turbine blade sections, for which the finished shapes are shown in Fig. 7.102. Each section was rolled in two roll passes,

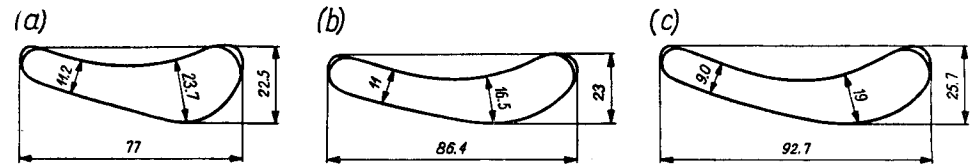


Fig. 7.102. Finished sections of turbine blades Pl67p: (a) finished rolled section Pl57, (b) finished rolled section Pl67p, (c) finished rolled section Pl70p

since in a single roll only six roll grooves could be cut due to the rolling mill type: 2-high reversing with  $D = 360$  mm,  $L = 800$  mm, driven by DC motor  $N = 50$  kW, fed from a Leonard set [55, 56, 142].

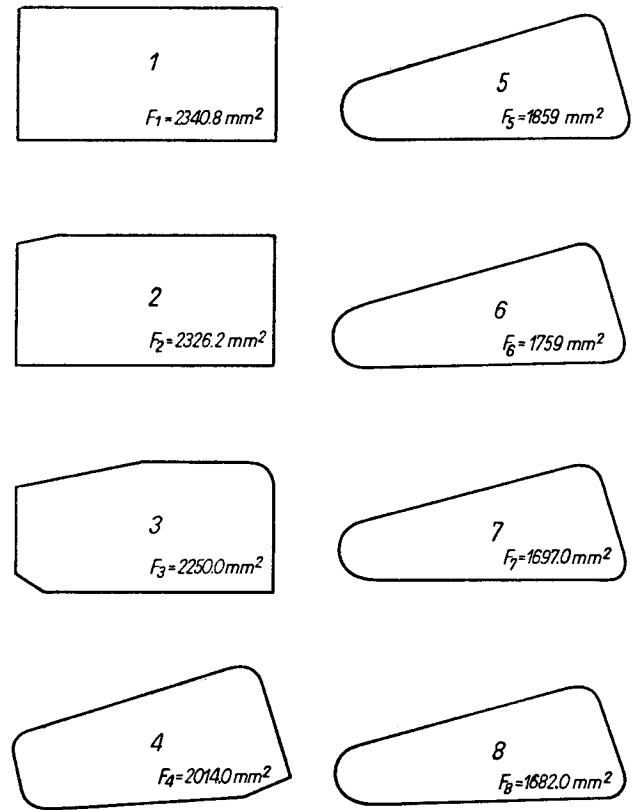


Fig. 7.103. Leader pass profiles of sectioned turbine blades Pl57

Pass and roll design were chiefly based on Z. Wusatowski's formulae and methods [125, 150].

In Figure 7.98 the leader and finishing shapes of P157 section are shown. The other sections were designed similarly.

Figures 7.99 and 7.100 show the corresponding passes: Fig. 7.99 the finishing and leader passes for P167p, Fig. 7.100 for P170p.

The required gradation of roll diameters could not be carried out because of the short distances between the pass grooves, resulting from a narrow roll barrel (length 800 mm).

As stock, flats were taken 35 mm thick and of width depending on the sections:

|            |             |
|------------|-------------|
| for P157,  | width 67 mm |
| for P167p, | width 72 mm |
| for P170p, | width 82 mm |

For tests the following steels suitable for turbine blades were rolled: Steel 1H13 with 0.15% C, 13.2% Cr, 0.6% Mn, 0.6% Ni, and for comparison St37 mild steel with 0.15% C and steel 35 with 0.36% C, 0.6% Mn, 0.35% Si.

When rolling the flats, the basic relations [55, 56] introduced by the author were used without simplifications, based on the constant volume conditions. Then two basic equations (3.110) and (3.116) are found, defining the metal deformation in the roll gap (Figs. 7.103 and 7.104)

$$F_1 l_1 = h_1 b_1 l_1 = h_x b_x l_x = h_p b_p l_p = h_2 b_2 l_2 = F_2 l_2$$

$$F_1 v_1 = h_1 b_1 v_1 = h_x b_x v_x = h_2 b_2 v_2 = F_2 v_2$$

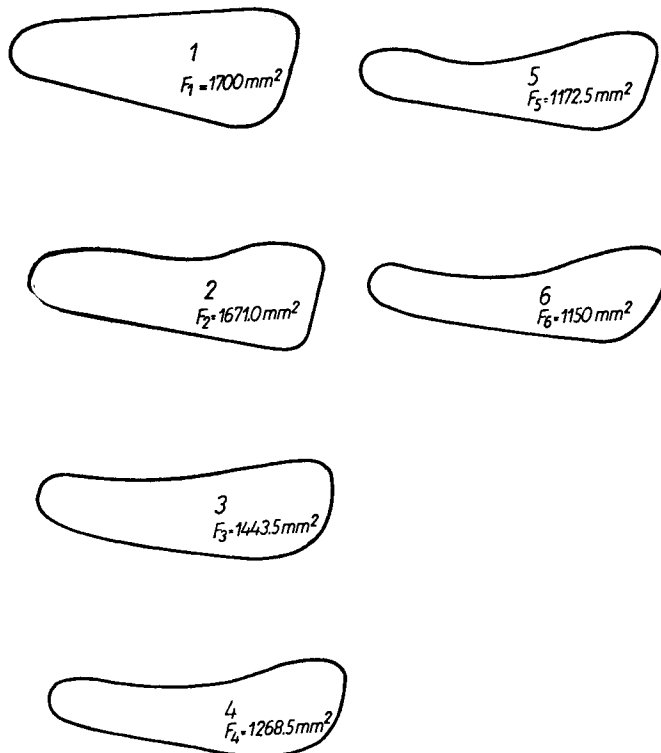


Fig. 7.104. Leader pass profiles of sectioned blades P157

where

- $F_1, h_1, b_1, l_1$  = sectional area, height, width and length before roll pass,  
 $F_2, h_2, b_2, l_2$  = sectional area, height, width and length after roll pass,  
 $h_x, b_x, l_x$  = height, width, length in a plane at a distance  $x$  from the exit plane,  
 $h_p, b_p, l_p$  = height, width, length in the pitch plane,  
 $v_1$  = mean entry speed relative to area  $F_1$ ,  
 $w_2$  = mean exit speed after the roll pass relative to area  $F_2$ ,  
 $v_x$  = mean speed at an arbitrary distance  $x$  from the exit plane,  
 $v_p$  = mean speed in the pitch plane.

Dividing equation (3.110) by initial values  $h_1 b_1 l_1$  the following relation is found for the whole roll gap

$$\gamma \beta \lambda = \gamma_x \beta_x \lambda_x = \gamma_p \beta_p \lambda_p \quad (7.235)$$

Similarly dividing equation (3.116) by  $h_1 b_1$  a second relation is found, valid for the whole roll gap

$$v_1 = \gamma_x \beta_x v_x = \gamma_p \beta_p v_p = \gamma \beta w_2 \quad (7.236)$$

This equation can also be written more simply

$$v_1 = \frac{v_x}{\lambda_x} = \frac{v_p}{\lambda_p} = \frac{w_2}{\lambda_2} \quad (7.237)$$

For practical solution of equations (7.235)–(7.237) in the roll gap an additional relation is needed, defining the pitch plane angle, in the following form [54, 55, 56]:

$$\frac{\cos \delta}{\left[ \frac{D - D \cos \delta + h_2}{h_1} \right]^{W-1}} = \frac{w_2}{\lambda v_r}$$

where  $v_r$  = known or measured peripheral rolling speed.

In practice the equation is solved by means of nomograms (Fig. 3.69).

To calculate changes of width in the roll gap the additional known relation may be used [156]

$$\beta_x = d \gamma_x^{-W}$$

where

- $d$  = correction factor to Wusatowski's formula for alloy steel,  
 $W$  = exponential coefficient relative to initial conditions.

The ratio of metal flow speed in any cross-section  $v_x$  to the peripheral speed of rolls in this cross-section  $v_{xr}$  defines the forward slip of metal relative to the rolls at any point of arc of contact [55 and 56]

$$s_x = \frac{v_x}{v_{xr}}$$

where

$$v_{xr} = v_r \cos \varphi_x$$

In the case of metal sticking to the roll, this forward slip gives the internal displacement of metal particles in the given pass.

The ratio  $v_x$  to  $v_q$  gives the coefficient of speed flow change in the metal itself independent of the peripheral speed of rolls. This coefficient is expressed as [55]

$$i_x = \frac{v_x}{v_p}$$

To adapt equations (7.235) to (7.237) to the rolling of sections it is necessary to introduce certain modifications. The mean height of section must be introduced (Fig. 7.101)

$$h_m = \frac{F}{b}$$

where

$F$  = sectional area,

$b$  = maximum width of section.

The most convenient method of defining conditions in the roll gap in the rolling of sections is to use equation (7.237) relating speeds and corresponding local elongation coefficients. The given speed is the mean speed of metal flow at a given point and the elongation coefficient is the mean elongation coefficient in the given pass, i.e.

$$\lambda_{xm} = \frac{F_1}{F_x} \quad (7.238)$$

To obtain data on phenomena in the roll gap taking place during the rolling of turbine blades, incompletely rolled samples were taken. This was not difficult since the rolling speed was about 30 mm/sec, so that when a roll was lifted and the sample taken out, conditions in the roll gap could be examined. On the sample obtained, first the exit plane was marked and the arc of contact was divided into 10 mm lengths up to the entry plane. For each of these cross-sections the maximum width  $b_x$  was measured as accurately as possible and areas  $F_x$  were integrated by planimeter. In this way data were obtained for calculation of  $h_{xm}$ . Templates of several sample cross-sections for the leader and finishing section P157 are shown in Figs. 7.103 and 7.104. Results for other sections are not given due to lack of space.

As a comparison, calculations of spread coefficients were made from equation (3.128), taking into account the correction  $d$  for alloy steel.

A calculation example was made for the leader pass P157, results are tabulated in Table 7.21.

During rolling the exit speed of metal from rolls was also measured [55] and [56]. Given known exit speed, then  $v_x$ ,  $v_{xr}$ ,  $s_x$ ,  $i_x$  were calculated from formulae (7.235) to (7.237), and plotted together with corresponding coefficients  $\gamma_x$ ,  $\beta_x$ ,  $\lambda_x$  in the diagrams in Figs. 7.105 to 7.110. In these diagrams the values of pitch plane angle are given, determined from the nomogram for equation (3.149) and calculated from the relation

$$\sin \delta_{calc} = \frac{l_{dp}}{R_w} \quad (7.239)$$

where

$l_{dp}$  = length of projected arc of contact measured from the exit plane to the point where  $s = 1$ ,

$R_w$  = effective radius of pass.

TABLE 7.21  
LEADER PASS OF PROFILE P157

| Pass No. | $l_{dx}$<br>mm | $F_x$<br>mm <sup>2</sup> | $b_x$<br>mm | $h_{mx}$<br>mm | $\lambda_{mx}$ | $\gamma_{mx}$ | $\beta_{mx}$ | $\beta_{calc}$ | $d$   | $d\beta_{calc}$ | Error<br>% | $\varphi_x$ | $v_{xr}$ | $v_x$ | $s_x$ | $i_x$ |
|----------|----------------|--------------------------|-------------|----------------|----------------|---------------|--------------|----------------|-------|-----------------|------------|-------------|----------|-------|-------|-------|
| 1        | 66             | 2340.8                   | 66.5        | 35.2           | 1.000          | 1.000         | 1.000        | 1.000          | —     | —               | —          | 22°48'      | 28.6     | 23.0  | 0.79  | 0.742 |
| 2        | 60             | 2326.2                   | 67.0        | 34.7           | 1.0062         | 0.9863        | 1.0075       | 1.0034         | 1.01  | 1.013           | +0.5       | 20°18'      | 29.0     | 23.1  | 0.797 | 0.745 |
| 3        | 50             | 2250.0                   | 69.5        | 32.6           | 1.0400         | 0.929         | 1.030        | 1.016          | 1.01  | 1.026           | -0.4       | 17°40'      | 29.7     | 23.9  | 0.805 | 0.772 |
| 4        | 40             | 2014.0                   | 71.7        | 28.1           | 1.161          | 0.797         | 1.08         | 1.053          | 1.003 | 1.055           | -1.9       | 13°35'      | 30.2     | 26.7  | 0.884 | 0.86  |
| 5        | 30             | 1859.0                   | 74.5        | 25.0           | 1.257          | 0.718         | 1.105        | 1.078          | 1.027 | 1.107           | +0.18      | 10°10'      | 30.6     | 28.9  | 0.945 | 0.932 |
| 6        | 20             | 1759.0                   | 75.6        | 23.1           | 1.335          | 0.656         | 1.138        | 1.098          | 1.045 | 1.148           | +0.87      | 6°45'       | 30.9     | 30.7  | 0.994 | 0.99  |
| 7        | 10             | 1697.0                   | 77.5        | 21.9           | 1.376          | 0.622         | 1.165        | 1.105          | 1.050 | 1.16            | -0.4       | 3°22'       | 31.0     | 31.6  | 1.019 | 1.02  |
| 8        | 0              | 1682.0                   | 77.7        | 21.1           | 1.388          | 0.614         | 1.17         | 1.11           | 1.055 | 1.17            | 0.0        | 0°00'       | 31.1     | 31.9  | 1.026 | 1.03  |

Pass No. as in Figs. 7.103 and 7.104

$l_{dx}$  = length of projected arc of contact from the pass to the exit plane

$F_x$  = pass area

$b_x$  = maximum width of pass

$h_{mx}$  = mean height of pass

$\lambda_{mx}$  = mean elongation coefficient

$\beta_{mx}$  = mean spread coefficient

$\beta_{calc}$  = spread coefficient calculated from equation (3.128)

$d$  = correction to Wusatowski's formula for alloy steel

$\varphi_x$  = angle of pass

$v_{xr}$  = peripheral speed of rolls in relation to  $\varphi_x$

$v_x$  = speed of flow in pass

$s_x$  = from formula (3.137)

$i_x$  = from formula (3.138)

Analysis of samples shows good agreement between measured spread values and those calculated from equation (3.128). The maximum difference is 3.7%, the mean less than 1%.

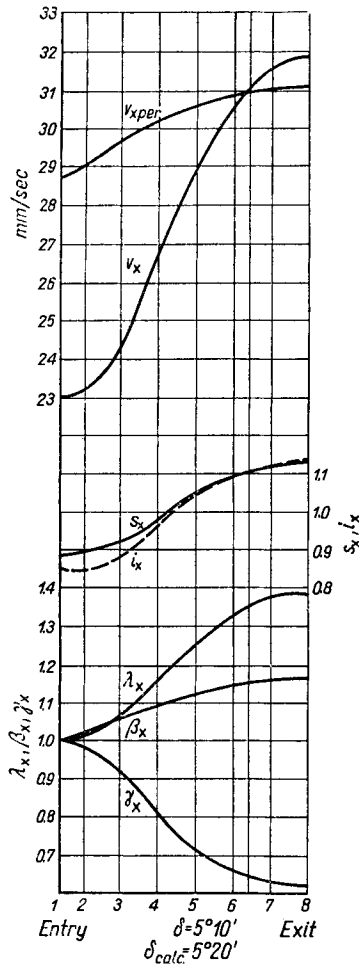


Fig. 7.105. Changes of values along arc of contact of leader turbine blade section P157

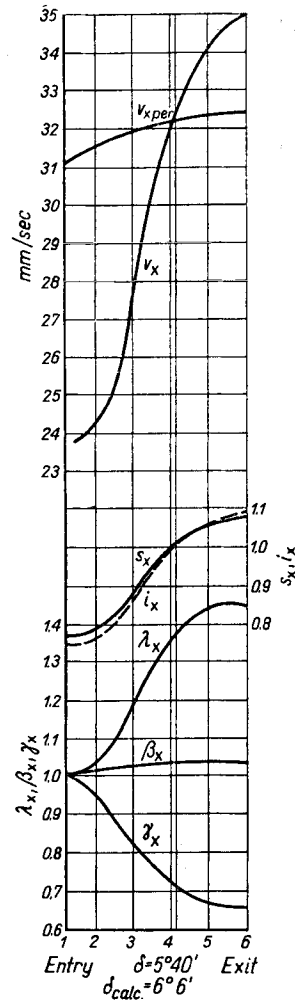


Fig. 7.106. Changes of values along arc of contact of leader turbine blade section P157

During rolling the following processes were recorded on the thermosensitive tape:

- (1) rotational speed of roll,
- (2) exit stock speed,
- (3) total roll force  $P$ ,
- (4) upper rolling torque  $M_u$ ,
- (5) lower rolling torque  $M_l$ .

For these measurements, apparatus from the firm Hohinger Messtechnik was used, consisting of three measuring amplifiers Kk1 SII/5 and a four-channel recorder He4 from the same firm.

The torque magnitude was measured only as a sum

$$M_m = M_w + M_f$$

where

$M_w$  = true rolling torque relative to motor shaft. This is a torque required to overcome resistance to deformation and frictional forces arising between the roll surface and the metal. In these experiments the upper and lower roll torque were measured separately, as described later.

$M_f$  = torque arising in the roll bearings calculated as in (4.107),  $M_f = Pdf$  where:  $P$  = load on the bearings,  $d$  = diameter of roll neck,  $f$  = coefficient of friction between roll neck and bearing.

Load cells and torque meters with a permissible load of 4.5 tm were used.

Figure 7.107 shows pressure changes in the leader and ultimate passes for the three tested sections. The results of pressure measurements for steel 1H13, 35 and St37\* are given.

Figure 7.108 shows the sum of the measured rolling torques. With known torque and roll forces for the particular sections the torque arm was calculated from the equation

$$M_w = 2Pa$$

where

$M_w$  = rolling torque,

$P$  = pressure,

$a$  = torque arm.

The value of the ratio of torque arm to projected arc of contact was found for various steels by rolling similar sections and gave average values:

$$1H13 \quad (0.436-0.67)l_d$$

$$35 \quad (0.404-0.62)l_d$$

$$St\ 37 \quad (0.36-0.59)l_d$$

i.e. different values for different steels.

To determine the mean resistance to deformation the following relation was used:

$$K_{wm} = \frac{P}{F_{dm}}$$

where

$K_{wm}$  = mean deformation resistance,

$P$  = roll pressure equal to the metal pressure on rolls,

$F_{dm}$  = horizontal projection of the area of contact of metal with the rolls surface =  $l_{dm}b_m$ ,

$l_{dm}$  = mean length of projected arc of contact projection, calculated from (4.15),

$$l_{dm} = \sqrt{R_{mw}\Delta h_m - \Delta h_m^2/4} \quad \text{where: } R_{mw} = \text{mean effective roll radius,}$$

$$\Delta h_m = h_{1m} - h_{2m}.$$

The second term of formula (4.11) can be calculated with sufficient accuracy as a spread increase

$$b_x = b_1\beta_x$$

where  $b_1$  = initial breadth.

\* See Appendix 1.



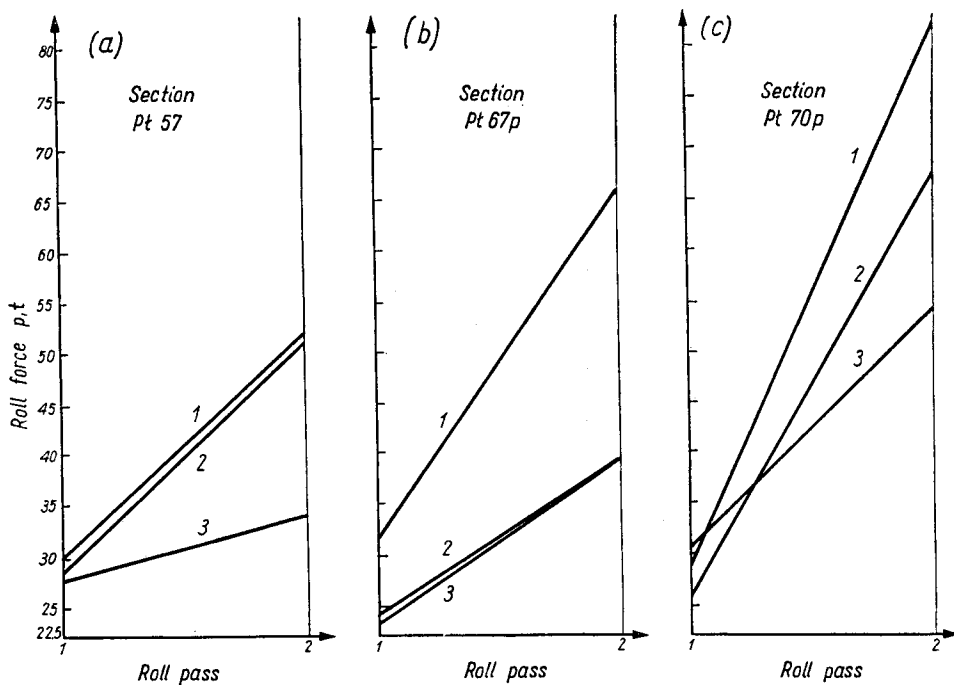


Fig. 7.107. Diagrams of measured roll force during rolling of turbine blades: (a) Pt57, (b) Pt67p, (c) Pt70p, from steel (see Appendix 1), 1 — 1H13, 2 — 35, 3 — St37

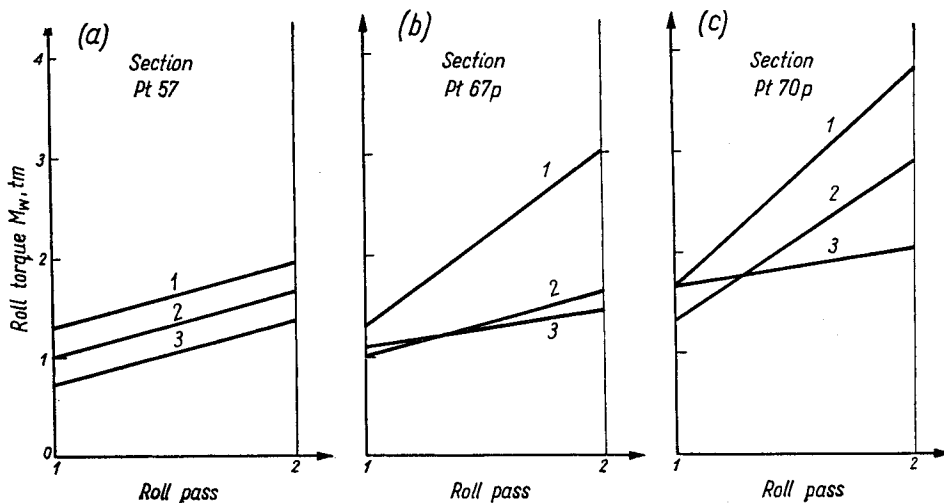


Fig. 7.108. Diagrams of measured total torques in rolling of blades: (a) Pt57, (b) Pt67p, (c) Pt70p, from steel (see Appendix 1), 1 — 1H13, 2 — 35, 3 — St37

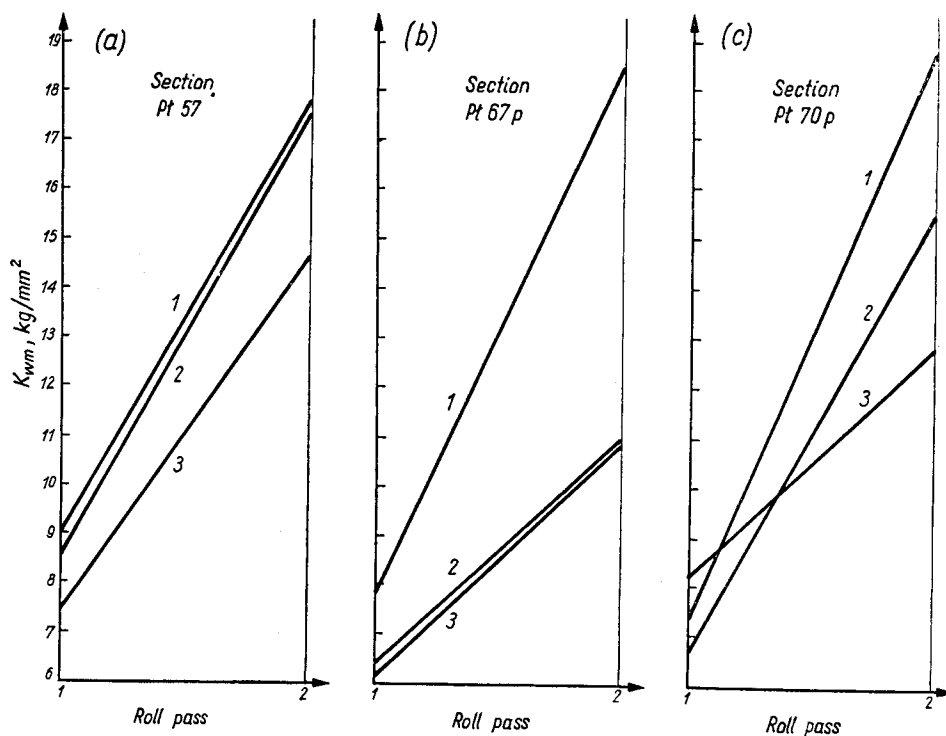


Fig. 7.109. Diagrams of mean deformation resistance in rolling of turbine blades: (a) Pt57, (b) Pt67p, (c) Pt70p, from steel (see Appendix 1), 1 — 1H13, 2 — 35, 3 — St37

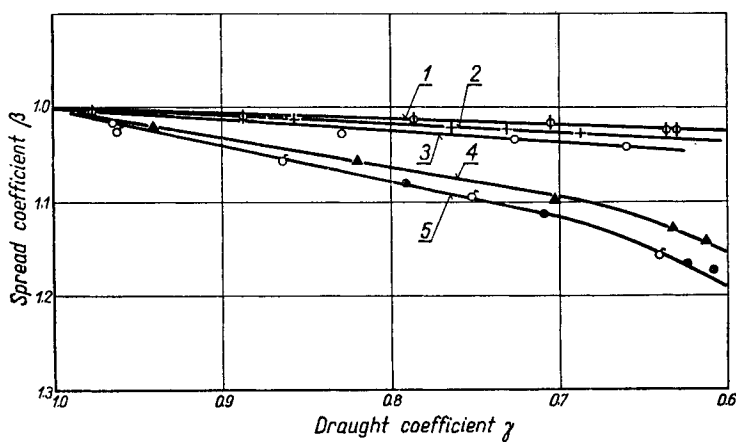


Fig. 7.110. Coefficient  $\beta$  plotted against  $\gamma$  for steel 1H13 at temperature 1000°C in rolling of turbine blade: 1 — Pt70p,  $\delta_w = 4.349$  — finishing pass, 2 — Pt67p,  $\delta_w = 4.299$  — finishing pass, 3 — Pt57,  $\delta_w = 3.580$  — finishing pass, 4 — Pt70p,  $\delta_w = 2.429$  — leader pass, 5 — Pt57,  $\delta_w = 1.889$  and Pt67p,  $\delta_w = 1.993$  — leader pass

Variations of mean resistance to deformation in the passes for three steels are illustrated in Fig. 7.109. This figure shows that values of  $K_{wm}$  vary relative to the type of steel, and for 1H13 the slope of the curve is considerably steeper than for the other steels. The P157 section shows a curve of a different shape from the others, which can be explained by the influence of greater sectional thickness.

This analysis of conditions arising during the rolling of turbine blades for TH120 turbine was carried out on a laboratory rolling mill. However, as this was actually a production job, the initial and end conditions were imposed and could not be changed, since the technological conditions had to be maintained. This explains the fractional values of form-factor, roll-factor, and the applied draughts.

Figure 7.110 shows the measured values of spread coefficient taken from rolling of samples from steel 1H13 in the leader and finishing passes (Figs. 7.98, 7.99, 7.100) at a temperature of 1000°C.

The diagram in Fig. 7.110 shows the relation between spread coefficients and draught coefficient  $\gamma$  for different values of factor  $\delta_w = b_1/h_1$  measured for finishing and leader passes.

Analysing the changes of spread coefficient in relation to form factor and draught coefficient (Fig. 7.110) it can be seen that irregular and asymmetric shapes behave similarly to those rolled on the plain barrel. Diagrams showing similar relations for the sections rolled on a plain barrel are shown in [150].

The results of measurement of upper and lower roll torques are used to correct positioning of neutral axis of pass by equalizing the torques of the two rolls.

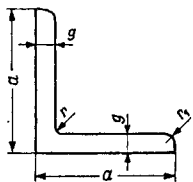
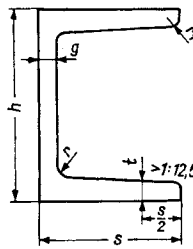
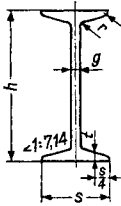
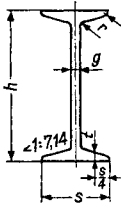
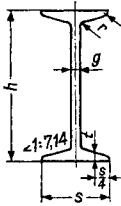
## APPENDIX 1

### TABLE OF STEELS USED IN EXAMPLES

| Grade of steel<br>(mark)              | Chemical composition % |           |           |                   |          |          |
|---------------------------------------|------------------------|-----------|-----------|-------------------|----------|----------|
|                                       | C                      | Mn        | Si        | other<br>elements | P<br>max | S<br>max |
| Mild steel                            | max 0.15               | —         | —         | —                 | 0.040    | 0.040    |
| Carbon steel of<br>medium hardness    | 0.15–0.35              | 0.5       | 0.2       | —                 | 0.040    | 0.040    |
| Hard carbon steel                     | min 0.35               | 0.5       | 0.2       | —                 | 0.040    | 0.040    |
| Armco                                 | max 0.03               | max 0.03  | max 0.002 | —                 | —        | —        |
| 10                                    | 0.07–0.14              | 0.35–0.65 | 0.17–0.37 | —                 | 0.040    | 0.040    |
| 1010                                  | 0.10                   | 0.35–0.65 | 0.17–0.37 | —                 | 0.040    | 0.040    |
| St37                                  | 0.10–0.20              | 0.40–0.65 | 0.12–0.30 | —                 | 0.040    | 0.040    |
| St37S                                 | 0.10–0.20              | 0.40–0.65 | 0.12–0.30 | —                 | 0.040    | 0.040    |
| St3                                   | 0.14–0.22              | 0.40–0.65 | 0.12–0.30 | —                 | 0.040    | 0.040    |
| 1020                                  | 0.15                   | 0.35–0.65 | 0.17–0.37 | —                 | 0.040    | 0.040    |
| St45                                  | 0.21–0.28              | 0.50–0.80 | 0.17–0.37 | —                 | 0.050    | 0.055    |
| St52                                  | max 0.22               | max 1.2   | max 0.5   | —                 | 0.050    | 0.055    |
| St5                                   | 0.28–0.38              | 0.50–0.80 | 0.15–0.35 | —                 | 0.050    | 0.055    |
| 35                                    | 0.32–0.40              | 0.50–0.80 | 0.17–0.37 | —                 | 0.040    | 0.045    |
| St60                                  | 0.38–0.49              | 0.50–0.80 | 0.15–0.35 | —                 | 0.050    | 0.055    |
| St70                                  | 0.46–0.66              | 0.57–0.95 | 0.08–0.35 | —                 | 0.050    | 0.055    |
| Open-hearth rail<br>steel (Table 7.7) | 0.68–0.80              | 0.70–1.05 | 0.10–0.30 | —                 | 0.040    | 0.040    |
| Chromium steel                        | 0.75–0.90              | —         | —         | Cr 1.6–2.1        | 0.030    | 0.030    |
| 1H13                                  | max 0.15               | max 0.6   | max 0.6   | Cr 12–14          | 0.035    | 0.030    |
| Acid-resisting steel                  | max 0.07               | max 2.0   | max 1.2   | Cr18, Ni8         | 0.035    | 0.030    |

APPENDIX 2

TABLE OF SECTIONS USED IN EXAMPLES

| Shape type      | Symbol   | Drawing                                                                             | Dimensions, mm |     |     |                |      |                | Area<br>cm <sup>2</sup> | Weight<br>1m, kg |
|-----------------|----------|-------------------------------------------------------------------------------------|----------------|-----|-----|----------------|------|----------------|-------------------------|------------------|
| Equal leg angle | 75×75×10 |    | a              | g   | r   | r <sub>1</sub> |      |                |                         |                  |
|                 |          |                                                                                     | 75             | 10  | 90  | 30             |      |                |                         |                  |
| Channel         | 200      |   | h              | s   | g   | t              | r    | r <sub>1</sub> |                         |                  |
|                 | 300      |                                                                                     | 200            | 75  | 8.5 | 11.5           | 11.5 | 6              |                         |                  |
| I-beam          | 80       |  | h              | s   | g   | t              | r    | r <sub>1</sub> |                         |                  |
|                 | 120      |                                                                                     | 300            | 100 | 10  | 16             | 16   | 8              |                         |                  |
| I-beam          | 80       |  | h              | s   | g   | t              | r    | r <sub>1</sub> |                         |                  |
|                 | 120      |                                                                                     | 80             | 42  | 3.9 | 5.9            | 3.9  | 2.3            |                         |                  |
| I-beam          | 80       |  | h              | s   | g   | t              | r    | r <sub>1</sub> |                         |                  |
|                 | 120      |                                                                                     | 120            | 58  | 5.1 | 7.7            | 5.1  | 3.1            |                         |                  |

## REFERENCES

1. Mechanik. Poradnik Techniczny (*Mechanics. Technical Handbook*), vol. III, No. 1-2, Warszawa 1957.
2. BERUGERS, W. G., Rekristallisation. Verformter Zustand und Erholung (*Recrystallization, Cold-working and Relaxation*), Leipzig, 1941.
3. WUSATOWSKI, Z., Wpływ przemian substancji międzykrystalicznej na własności miękkich stali węglowych (The influence of changes of the intercrystalline substance on the properties of low carbon steel), *Prace Inst. Mechan.* 1953, p. 293-333.
4. WUSATOWSKI, Z., Zjawiska wydzielania się cementytu z roztworu stałego i utwardzenie dyspersyjne stali miękkiej (Precipitation of cementite from the solid solution and dispersion hardening of low carbon steel). *Hutnik*, 20, 1953, p. 299-333.
5. PUPPE, J., and STAUBER, G., Walzwerkwesen (*Rolling Mill Practice*), vol. 1, 1929, vol. 2, 1934 vol. 3, 1939.
6. SAMOKHOTSKI, A., and KUNIAVSKI, M., Metalovedenie (*Physical Metallurgy*), Moscow, 1952.
7. COOK, M., and RICHARDS, T., Fundamental aspects of the cold working of metals, *J. Inst. Metal.* 78, 1951, p. 463-482.
8. KRUPKOWSKI, A., and BALICKI, M., Nowa teoria rekrytalizacji metali zgniecionych (A new theory of recrystallization of work-hardened metals), *Hutnik*, 1938, p. 624-643.
9. MALKIEWICZ, T., Obróbka cieplna stali (*Heat Treatment of Steel*), Kraków, 1954.
10. WUSATOWSKI, Z., Podstawy procesu walcowania (*Fundamentals of Rolling*), Katowice, 1952.
11. WUSATOWSKI, Z., Próba krytyki dotychczasowych pojęć z przeróbki plastycznej (Critical estimation of contemporary terminology in metal working), *Zeszyty Naukowe Politechniki Śląskiej, Mechanika*, 1955, No. 2, p. 3-17.
12. WUSATOWSKI, Z., Właściwe sposoby określania odkształceń plastycznych (Proper methods of determining plastic deformation), *Zeszyty Naukowe Politechniki Śląskiej, Mechanika*, 1955, No. 4, p. 3.
13. GUBKIN, S. I., Deformiruemost metallov (*Formability of Metals*), Moscow, 1953.
14. OBERHOFFER, P., Das technische Eisen (*Technical Iron*), Berlin, 1936.
15. WUSATOWSKI, Z., Obliczanie nacisku walców w procesie walcowania (Calculation of roll forces in rolling), *Prace badawcze GIMO*, 1949, p. 233-290.
16. NADAI, A. L., The flow of metals under various stress conditions, *Proc. Inst. Mech. Engrs. Appl. Mech.* 157, 1947, p. 128-160.
17. HUBER, M. T., Czasopismo Techniczne (*Technical Journal*), Lwów, 1904.
18. LODE, W., Versuche über den Einfluss der mittleren Hauptspannung auf die Fließgrenze (Investigations on the influence of the mean principal stress on the yield stress), *Z. Physik*, 36, 1926, p. 913-939.
19. WUSATOWSKI, Z., Próba określenia właściwej wielkości naprężeń granicznych przy trójosiowym odkształceniu plastycznym (Determination of the correct magnitude of constrained yield stress during triaxial plastic deformation), *Arch. Górn. i Hutn. PAN*, 1, 1956, No. 4, p. 283-317.
20. GELEJI, A., Die Berechnung der Kräfte und des Arbeitsbedarfes bei der Formgebung im bildsamen Zustande der Metalle (*Calculation of Forces and Power Consumption During Metal Working*), Budapest, 1955, II. ed.
21. UNKSOV, E., Inzhenernye metody raschota usilii pri obrabotke metallov davleniem (*Engineering Methods for Calculation of Forces in Metal Working*), Mashgiz, Moscow, 1955.
22. TRINKS, W., Kalibrowanie walców (*Roll Pass Design*), Katowice, 1948, 1949.
23. SACHS, G., *Fundamentals of the Working of Metals*, London, 1954.

24. SIEBEL, E., Technologische Mechanik der bildsamen Verformung (Technological mechanics of metal working), *Arch. Eisenhüttenw.* **18**, 1944, p. 13-22.
25. COOK, P. M., *Proceedings of Conference on Properties of Materials at High Rate of Strain*, Institution of Mechanical Engineers, 1957.
26. POLAKOWSKI, H., The compression test in relation to cold rolling, *J. Iron and Steel Inst.* **163**, 1949, p. 205-256.
27. UNKSOV, E., *Novye issledovaniya v oblasti kusnechnoi tekhnologii (New Investigations in the Technology of Forging)*, Moscow, 1950.
28. UNKSOV, E., *Elementy teorii i novye protsessy obrabotki metallov davleniem (Basic Theory and New Processes of Metal Working)*, Moscow, 1954.
29. WUSATOWSKI, Z., Krytyczne omówienie nowych teorii walcowania (Critical appraisal of new theories of rolling), *Prace GIMet.* **3**, 1951, p. 389-416.
30. SIEBEL, E., *Die Formgebung im bildsamen Zustande (Formability in Metal Working)*, Düsseldorf, 1932.
31. WUSATOWSKI, Z., Praca i moc w procesie walcowania (Work and power in rolling), *Prace Badawcze GIMO*, **2**, 1950, p. 215-267.
32. WUSATOWSKI, Z., Gniot, wydłużenie i roztłoczenie w procesie walcowania na gorąco (Draught, elongation and spread in hot rolling), *Prace Badawcze GIMO*, 1949, p. 27-58.
33. ALEKSANDROV, P. A., PISANKO, W. A., and BOGATYREV, W. A., *Ratsionalizacia kalibrovok valkov sortovykh prokatnykh stanov (Improvement of Roll Pass Design of Section Rolling Mills)*, Kharkov, 1938.
34. UNDERWOOD, L. R., *The Rolling of Metals*, vol. 1, London, 1952.
35. OROWAN, E., The calculation of roll pressure in hot and cold flat rolling, *Proc. Inst. Mech. Engrs.* **150**, 1943, p. 140-162.
36. BLAND, D. R., and FORD, H., The calculation of roll force and torque in cold strip rolling with tension, *Proc. Inst. Mech. Engrs.* **159**, 1948, p. 144-163.
37. TAFEL, W., *Walzen und Walzenkalibrieren (Rolling and Roll Pass Design)*, Dortmund, 1923.
38. KONCEWICZ, S., Analiza i porównanie wzorów do obliczania roztłoczenia przy walcowaniu na gorąco (Analysis and comparison of formulae for the calculation of spread in hot rolling), *Zeszyty Naukowe Politechniki Śląskiej. Mechanika*, 1956, No. 3, p. 47-63.
39. WUSATOWSKI, Z., and WUSATOWSKI, R., Wpływ szybkości, temperatury i rodzaju walców na roztłoczenie i wydłużenie w procesie walcowania (Influence of speed, temperature and type of rolls on spread and elongation in rolling), *Prace Badawcze GIMO*, 1950, No. 2, p. 111-122.
40. WUSATOWSKI, Z., and SZOSTAK, R., Ustalenie poprawek dla obliczenia wydłużenia i roztłoczenia przy walcowaniu na gorąco stali stopowych (Determination of correction factors for the calculation of elongation and spread in the hot rolling of alloy steels), *Prace IHM*, **6**, 1954, p. 217/220.
41. WUSATOWSKI, Z., and KRYWULT, J., Ustalenie poprawek do wzorów Z. Wusatowskiego dla kilku stali stopowych (Determination of correction factors to Z. Wusatowski's formulae for some alloy steels), *Zeszyty Naukowe Politechniki Śląskiej. Mechanika*, 1958, No. 5, p. 55-87.
42. WUSATOWSKI, Z., and WUSATOWSKI, R., Możliwości matematycznego określenia płynięcia metalu w profilach regularnych (The possibility of mathematical determination of metal flow in regular sections), *Prace IMet.* **4**, 1952, p. 273-292.
43. FILASIEWICZ, K., Skrypt "Walcownictwo" (Script "Rolling Mill Practice"), Gliwice, 1947.
44. Obrabotka metallov davleniem (Plastic deformation of metals), *Sbornik statiej*, Moscow, **3**, 1953, **4**, 1954.
45. TIAGUNOV, W. A., *Rezhimy prokatki na reversivnykh stanakh (Rolling Schedules for Reversing Mills)*, Sverdlovsk, 1954.
46. KOROLEV, A. A., *Novye issledovaniya deformacii metalla pri prokatke (New Investigations in the Deformation of Metals During Rolling)*, Moscow, 1953.
47. *Sbornik XXXIII. Tekhnologicheskie protsessy obrabotki stali i splovov (Technological Processes for Mechanical Working of Steel and Alloys)*, Moscow, 1955.
48. HOFF, H., and DAHL, T., *Grundlagen des Walzverfahrens (Fundamentals of Rolling)*, Düsseldorf, 1950.
49. UNDERWOOD, L. R., Cold rolling, *Metal Ind.* 1949, p. 1-12.
50. WUSATOWSKI, Z., Zmiany współczynnika tarcia w zależności od szybkości walcowania (Variation of coefficient of friction as a function of rolling speed), *Hutnik*, **22**, 1956, p. 98-99.
51. WUSATOWSKI, Z., *Piece walcownicze i kuźnicze (Heating Furnaces for Rolling and Forging Practice)*, vols. 1 and 2, Katowice, 1954.

52. NEUMANN, H., *Wissenschaftliche Ergebnisse auf dem Gebiet der Kalibrierung (Scientific Results in the Field of Roll Pass Design)*, Riesa, 1956.
53. JAGLARZ, Z., Określenie temperatur blach w czasie walcowania według Tiagunova (Determination of temperature of steel sheets during rolling according to Tiagunov). *Wiadomości Hutnicze*, 1951, No. 11, p. 8–11.
54. WUSATOWSKI, Z., Obliczanie szybkości w procesie walcowania (Calculation of speed in rolling). *Prace GIMet.* 4, 1952, p. 1–47.
55. WUSATOWSKI, Z., Analiza zjawisk w szczelinie walców przy zachodzącym roztłoczeniu (Analysis of phenomena in the roll gap taking into account spread of the rolled stock), *Arch. Hutn. PAN*, 4, 1956, p. 141–169.
56. WUSATOWSKI, Z., and SZALAJDA, K., Kąt płaszczyzny podziałowej i wyprzedzanie przy walcowaniu z roztłoczeniem (Neutral angle and forward slip during rolling with spread), *Arch. Hutn. PAN*, 2, 1957, p. 127–166.
57. WUSATOWSKI, Z., Kąt płaszczyzny podziałowej w procesie walcowania na gorąco i na zimno (Neutral angle in hot and cold rolling), *Arch. Górn. i Hutn. PAN*, 3, 1955, p. 11–42.
58. GALAJ, A. G., *Materiały po teorii prokatki (Source Material on the Theory of Rolling)*, vol. 1, Moscow, 1934, vol. 2, Moscow, 1934, vol. 3, Moscow, 1940, vol. 4, Moscow, 1948.
59. TSELIKOV, A., *Stress and Strain in Metal Rolling*, Moscow, 1967.
60. Special Report No. 34, First report of the rolling mill research sub-committee, London, 1948.
61. SIEBEL, E., *Die Berechnung des Walzdruckes. Wälzlager für Walzwerke (Calculation of Roll Force Anti-friction Bearings for Rolling Mills)*, Schweinfurt, 1941.
62. LARKE, E. C., *The Rolling of Sheet, Strip and Plate*, London, 1963.
63. SMITH, C. L., SCOTT, F. H., and SYLWESTROWICZ, W., Pressure distribution between stock and rolls in hot and cold flat rolling, *J. Iron Steel Inst.* 1952, p. 347–359.
64. WUSATOWSKI, Z., Pomiar nacisku wzdłuż łuku styku przy walcowaniu na gorąco i na zimno (The measurement of roll pressure along the arc of contact in hot and cold rolling), *Hutnik*, 1954, p. 48–51.
65. LUEG, W., Spannungsverteilung und Werkstofffluss im Walzspalt (The distribution of stress and metal flow in the roll gap), *Stahl u. Eisen*, 1933, p. 346–352.
66. WUSATOWSKI, Z., Zagadnienie prędkości walcowania (Investigation of rolling speed), *Hutnik*, 1960, No. 7–8.
67. COOK, P. M., and MCCRUM, A. W., *The Calculation of Load and Torque in Hot Flat Rolling*, B.I.S.R.A., Sheffield, 1958.
68. WUSATOWSKI, Z., and BALA, S., Porównanie metod obliczania nacisku walców w procesie walcowania na gorąco (Comparison of methods of calculation of roll force in hot rolling), *Prace IMH* 6, 1954, p. 120–132.
69. WUSATOWSKI, Z., Porównanie najlepszych metod obliczania nacisków przy walcowaniu na gorąco (Comparison of the best methods of calculating the roll force in hot rolling), *Hutnik*, 1956, *Biuletyn IMH*, 5–8.
70. WUSATOWSKI, Z., Porównanie metod radzieckich obliczania nacisku walców (Comparison of Soviet methods for the calculation of roll force), *Zeszyty Naukowe Politechniki Śląskiej, Mechanika*, 1956, No. 3, p. 19–46.
71. FILASIEWICZ, K., GALANTY, A., and WUSATOWSKI, Z., Porównanie metod obliczania nacisków walców w procesie walcowania taśm na zimno bez naciągu i przeciwności (Comparison of methods for the calculation of roll force in cold rolling of strip without tension), *Prace IMH*, 5, 1953, p. 57–81.
72. FILASIEWICZ, K., GALANTY, A., and WUSATOWSKI, Z., Porównanie metod obliczania momentów i mocy właściwej w procesie walcowania na zimno bez naciągu i przeciwności (Comparison of methods for the calculation of rolling torque and specific power in cold rolling without tension), *Prace IMH*, 3, 1953, p. 199–200.
73. GALANTY, A., Porównanie metod obliczania nacisku walców oraz momentu walcowania przy walcowaniu taśm na zimno z naciągiem i przeciwności (Comparison of methods for the calculation of roll force and torque in cold rolling of strip with tension), *Prace IMH*, 8, 1956, p. 225–230.
74. WINTON, D. A., Nomogram for the evaluation of the Hitchcock formula for roll flattening, *Sheet Metals Ind.* 1951, p. 17–18.
75. POLAKOWSKI, H., An examination of modern theories of rolling in the light of rolling mill practice, *Sheet Metals Ind.* 27, 1950, 28, 1951, 29, 1952, 30, 1953, 31, 1954.



76. SKF. Walzdruck und Leistungsbedarf beim Warmwalzen von Bandstahl (SKF. Roll pressure and power consumption in hot strip rolling), *Kugellager-Z.* 1946, No. 1, p. 15–24.
77. SKF. Kaltwalzwerke mit SKF-Rollenlagern (SKF. Cold Rolling Mills Working with SKF-Roller Bearings), Berlin.
78. Akademia Nauk USSR. Problemy metallurgii (Academy of Science of the USSR. Problems of Metallurgy), Moscow, 1953.
79. KOROLEV, A. A., and NIKOLAYEVSKI, G. M., Mekhanicheskoe oborudovanie prokatnykh cekhov (Mechanical Equipment of Rolling Mills), Moscow, 1955.
80. PAVLOV, J. M., Teoria prokatki (The Theory of Rolling), Moscow, 1950.
81. EMICKE, O., Kalt- und Warmwalzen von Leichtmetallblechen (Cold and hot rolling of light metal sheets), *Z. VDI*, 1943, 87, p. 435–438.
82. EMICKE, O., and LUCAS, A., Das Walzen von Leichtmetallen zu Blechen und Bändern (Rolling of Light Metals into Sheet and Strip), Freiberg, 1944.
83. EMICKE, O., and LUCAS, A., Grundlegende Untersuchungen über das Kalt- und Warmwalzen von Metallen zu Blechen und Bändern, insbesondere von Aluminium und Aluminium-Knetlegierungen sowie vergleichsweise von Elektrolytblei und Stählen (Investigation of cold and hot rolling of metals into sheet and strip, particularly for aluminium and its alloys, and electrolytic lead and steel for comparison). *Z. Metallkunde*, 34, 1942, No. 2, p. 25–38; No. 3, p. 49–58; *Z. VDI* 87, 1953, p. 435–438; *Kalt-Walz-Welt*, 1943, No. 3/6.
84. JURETZEK, G., Walzdrücke und Drehmomente beim Walzen auf Flachbahnen mit Ober- und Unterdruck (Roll force and torque when rolling at the bullhead with top and bottom draught), *Freiberger Forschungshefte*, E. 16, *Metallformung*, 1957, p. 58–81.
85. WALLQUIST, G., Neue Untersuchungen über Walzdrücke, Leistungsbedarf usw. beim Warmwalzen von Bandstahl (New investigations of roll force, power consumption etc. in hot flat rolling), *Kugellager-Z.* 1950, p. 1–17.
86. ZOUHAR, G., Beitrag zur Frage der Verteilung des Gesamtdrehmoments auf Ober- und Unterwalze beim Walzen (A contribution to the problem of distribution of total rolling torque between top and bottom rolls during rolling), *Freiberger Forschungshefte*, 1956, 9, p. 83–109.
87. Mashinostroenie. Entsiklopedicheskij Spravochnik (Mechanical Engineering. Encyclopaedic Handbook), vol. 8, Moscow, 1948.
88. FILASIEWICZ, K., Technologia metali (The Technology of Metals), Katowice, 1954.
89. WUSATOWSKI, Z., Pomiar pracy i mocy przy walcowaniu (Measurement of work and power during rolling), *Zeszyty Naukowe Politechniki Śląskiej. Mechanika*, 1955, No. 2, p. 43–54.
90. EMICKE, O., and LUCAS, K. H., Die vereinfachte Ermittlung des Gesamtwalzdruckes und Umformungsdrehmoments beim Walzen von Metallen und Metallegierungen (A simplified method for the determination of total roll force and torque during the rolling of metals and alloys), *Metallurgie u. Giessereitechnik*, No. 2, 1952, p. 2–14.
91. EMICKE, O., Einige grundlegende Fragen bei der Kaltwalzung von Breitband aus Flusstahl, Tiefziehqualität (Some basic problems in the cold rolling of wide strip of mild steel of deep-drawing quality), *Technik*, 4, 1949, p. 435–440.
92. MONKIEWICZ, T., Napęd elektryczny (Electrical Drives), Warsaw, 1951.
93. WUSATOWSKI, Z., and MAJ, S., Walce półtwarde i utwardzone (Semi-chilled and chilled rolls), *Hutnik*, 14, 1947, p. 529–534.
94. WUSATOWSKI, Z., and KOWALSKI, W., Obróbka cieplna walców żeliwnych utwardzonych dla walcowni blachy cienkiej (Heat treatment of chilled cast iron rolls for sheet mills), *Hutnik*, 15, 1948, p. 491–492.
95. WUSATOWSKI, Z., and HACZEWSKI, W., Walce dla walcowania szerokich taśm (Rolls for wide strip mills), *Hutnik*, 16, 1949, p. 57–59.
96. WUSATOWSKI, Z., Niemieckie walce żeliwne (German cast iron rolls), *Hutnik*, 17, 1950, p. 250–256.
97. WUSATOWSKI, Z., Niemieckie walce stalowe (German steel rolls), *Hutnik*, 17, 1950, p. 340–342.
98. WUSATOWSKI, Z., Niemieckie walce zespolone (German duplex rolls), *Hutnik*, 17, 1950, p. 342–345.
99. GELEJI, A., Walzwerks- und Schmiedemaschinen (Rolling and Forging Machines), Berlin, 1954.
100. SEROVATIN, A. J., Metodika raschota oborudavania prokatnykh tsekhov (A Method for Calculation of Rolling Mill Facilities), Kharkov, 1955.
101. TAUSCHER, H., Zulässige Spannungen für standardisierte und genormte Werkstoffe des Maschinenbaues (Permissible Stresses for Materials Covered by Standardized Norms of the Engineering Industry), Leipzig, 1955.

102. EMICKE, O., Zur Frage der statischen und dynamischen Belastung von Duo-Umkehr-Grobblech-Walzwerken und ihres Antriebs (Investigations in the static and dynamic loading of two-high reversible plate mills and their drive), *Metallurgie*, **5**, 1955, p. 19-34.
103. T.V.V. Vybrane kapitoly z mechaniki valcovani (*Chosen Chapters from the Mechanics of Rolling*), Praha, 1952.
104. WUSATOWSKI, Z., Czynniki wpływające na odchyłki grubości przy walcowaniu blach i taśm (Factors influencing gauge variation during rolling of sheet and strip), *Hutnik*, **17**, 1950, p. 415-423.
105. EMICKE, O., and LUCAS, H., Einflüsse auf die Walzgenauigkeit beim Warmwalzen von Blechen und Bändern und ihre Nachprüfung durch Versuch und Rechnung (Factors influencing the stock accuracy during the hot rolling of sheet and strip, and their examination by experiment and calculation), *Neue Hütte*, **1**, 257-274.
106. EMICKE, O., Netztafel zur vereinfachten zeichnerischen Ermittlung von Walzdruck, Umformung und Lagerreibungs-drehmoment (Nomograms for simplified graphical determination of roll force, rolling torque and torque due to additional bearing friction), *Metallurgie*, **5**, 1955, p. 44-50.
107. RĄBALSKI, J., and JOŃCA, J., Pomiar nacisku i momentu obrotowego przy walcowaniu na gorąco (Measurement of roll force and torque in hot rolling), *Publikacje wybrane*, Katowice, 1958, No. 6, p. 57-72.
108. EMICKE, O., Beanspruchung, Abnutzung und Schleifen von Walzen für die Herstellung von Grob-, Mittel- und Feinblechen (Stressing, wear and grinding of rolls for the rolling of plate and sheet) *Stahl u. Eisen*, **58**, 1938, p. 73-82.
109. WUSATOWSKI, Z., and LUDYGA, J., Wpływ dodatkowego tarcia w wykrojach regularnych (Influence of additional friction in regular passes), *Arch. Hutn. PAN*, **5**, 1960, No. 2, p. 195-243.
110. BALA, S., JOŃCA, J., and RĄBALSKI, J., Pomiar i analiza procesów walcowania na zginiaczu  $D = 750$  mm (Measurements and analysis of the rolling process on a  $D = 750$  mm blooming mill), *Publikacje wybrane*, Katowice, 1958, No. 6, p. 85-104.
111. ZAROSHCHINSKI, M., Walcowanie stali (*The Rolling of Steel*), Katowice, 1952.
112. GALANTY, A., Urządzenia pomiarowe przy walcowaniu na zimno (Measuring devices in cold rolling), *Hutnik*, **22**, 1955, p. 210-216.
113. MURSKI, C., and GÓRECKI, J., Podstawy kalibrowania walców (The fundamentals of roll pass design), *BIPROHUT*, Gliwice, 1957.
114. CHEKMAREV, A. P., and NEFEDOV, A. A., Prokatka na valkakh neravnogo diametra (Rolling on rolls of unequal diameters), *Obrabotka Metallov Davleniem*, IV, Moscow, 1956, p. 3-21.
115. HOFF, H., and DAHL, T., Walzen und Kalibrieren (*Rolling and Roll Pass Design*), Düsseldorf, 1954.
116. RYTEL, K., Kalibrowanie profili regularnych i systemów wydłużających (*Roll Pass Design of Regular Sections and Stretching Systems*), Maszynopis pracy doktorskiej na Politechnice Śląskiej w Gliwicach (Typescript of doctoral thesis at the Technical University of Gliwice).
117. EMICKE, O., and ALLHAUSEN, H., Zeichnerische Bestimmung der Stichfolge beim Walzen frei breiter Querschnitte (Graphical determination of pass sequence during rolling of free-spreading cross-sections), *Stahl u. Eisen*, **54**, 1934, p. 492-500 and 512-517.
118. EMICKE, O., Stand und Fortentwicklung der Walzernkalibrierung (Present state and development of roll pass designing) *Paper presented at the Haus der Technik*, Essen, May 1937.
119. Hütte. Taschenbuch für Eisenhüttenleute (*Handbook for Ferrous Metallurgists*), Berlin 1961.
120. WUSATOWSKI, Z., Graficzne wyznaczanie wymiarów wsadu i kolejnych przepustów dla walcowni zimnych (Graphical determination of stock dimensions and pass sequence in cold rolling), *Hutnik*, **21**, 1954, *Biuletyn IMH*, No. 6, p. 21-23.
121. LESKIEWICZ, W., WUSATOWSKI, Z., PORCZYŃSKI, A., and GALANTY, A., Graniczny zginiot stali węglowej walcowanej bez wyżarzania pośredniego (Limiting draught of cold rolled carbon steel without intermediate annealing), *Hutnik*, **20**, 1953, *Biuletyn IMH*, No. 1, p. 1-3.
122. WUSATOWSKI, Z., Kalibrowanie walców sposobem graficznym (Graphical method of roll pass design), *Hutnik*, **14**, 1947, p. 206-219.
123. WUSATOWSKI, Z., and WOJTYŁAK, A., Płynięcie metali, wydłużenie i rozłoczenie przy walcowaniu kształtowników regularnych (Metal flow, elongation and spread during rolling of regular sections), *Prace GIMet*, **3**, 1951, p. 23-46.
124. WUSATOWSKI, Z., and WUSATOWSKI, R., Próba przystosowania wzorów płynięcia metali do kalibrowania walców (An attempt to apply formulae of metal flow to roll pass design), *Prace badawcze IMH*, **7**, 1955, p. 115-123.
125. WUSATOWSKI, Z., Intensywność odkształcania w procesie walcowania (The rate of deformation in rolling), *Prace IH*, **10**, 1958, p. 120-124.

126. WUSATOWSKI, Z., and WOJTYŁAK, A., Analiza płynięcia metalu w kształtownikach nieregularnych i niesymetrycznych (Analysis of metal flow in irregular and non-symmetrical sections), *Prace IMet*, 4, 1952, p. 99–109.
127. GÓRECKI, J., Średni współczynnik wydłużenia liniowego przy walcowaniu (The mean linear elongation coefficient in rolling), *Hutnik*, 19, 1952, p. 197–201.
128. WUSATOWSKI, Z., and RYTEL, K., Analiza wzorów na współczynnik średniego wydłużenia kształtowników (Analysis of formulae for the determination of the mean elongation factor for sections), *Prace IMH*, 7, 1955, p. 252–259.
129. WUSATOWSKI, Z., Podstawy kalibrowania kształtowników (Fundamentals of pass design of sections) *Arch. Hutn. PAN*, 2, 1957, p. 325–376.
130. NEUMANN, H., and LEHMANN, F., Ermittlung des Querflusses als Hilfsmittel beim Kalibrieren von Formstahl (Determination of transverse flow as a help in the pass design of sections), *Neue Hütte*, 2, 1957, p. 307–317.
131. RYTEL, K., Kalibrowanie dwuteowników (Pass design of I-beams), *Prace IMH*, 8, 1956, p. 79–84.
132. MURSKI, C., Obliczanie średnic walców walcowni bruzdowych (The calculation of roll diameter for section mills), *Hutnik*, 17, 1950, p. 207–216.
133. WUSATOWSKI, Z., and ORZEŁ, E., Wyznaczanie średnic czynnych i osi obojętnej wykroju przy walcowaniu kształtowników (Determination of effective diameter and neutral axis of the pass, during the rolling of sections), *Zeszyty Naukowe Politechniki Śląskiej. Mechanika*, 1958, No. 5 p. 89–119.
134. WUSATOWSKI, Z., Walcowanie sposobem ciągłym (The continuous rolling process), *Hutnik*, 12, 1945, p. 34–39.
135. WUSATOWSKI, Z., Zasady walcowania ciągłego. Referat plenarny na Naukowy Zjazd Hutników (Principles of continuous rolling. Plenary paper presented at the Scientific Meeting of Metallurgists), PAN, Kraków, 1954, VII, p. 1–38.
136. WUSATOWSKI, Z., Zasady walcowania ciągłego (Principles of continuous rolling), *Wiadomości Hutnicze*, 10, 1954, p. 194–198.
137. SEBARD, W., Walcowanie na zimno taśmy stalowej (Cold rolling of steel strip), *Jernkont. Ann.*, 1951, p. 287–384.
138. TSELUSTKIN, A. B., and ROSEMAN, A. R., Avtomaticheskoe upravlenie prokatnymi stanami (*Automatic Control of Rolling Mills*), Moscow, 1955.
139. MOROZOV, D. P., Issledovanie avtomatizirovannogo elektropivoda prokatnykh stanov (*Investigation of Automated Electrical Drives of Rolling Mills*), Moscow, 1952.
140. WUSATOWSKI, Z., Stal walcowana. Nazwy i określenia półwyrobów i wyrobów, (*Rolled steel. Names and definitions of semi-products and products*), *Hutnik*, 17, 1950, p. 361–363.
141. WUSATOWSKI, Z., and LUDYGA, J., Wpływ dodatkowego tarcia w wykrojach regularnych (Influence of additional friction in regular pass grooves), *Arch. Hutn. PAN*, 5, 1960, p. 195–243.
142. WUSATOWSKI, Z., Zagadnienia walcownicze (Some rolling problems), *Hutnik*, 26, 1959, p. 289–296.
143. TYCHOWSKI, F., Studia nad niektórymi funkcjami teorii plastyczności (Investigations of some functions of the theory of plasticity), *Arch. Hutn. PAN*, 4, 1959, p. 3–43.
144. WUSATOWSKI, Z., Laboratorium przeróbki plastycznej z atlasem. Cz. 1. Walcownictwo (Laboratory exercises of mechanical working with atlas. Part I. Rolling), Script. Politechnika Śląska, Gliwice, 1960.
145. WUSATOWSKI, Z., and MURSKI, C., Przeróbka plastyczna. Walcownictwo. Mały Poradnik Hutnika (*Mechanical Working. Rolling. The Small Handbook for Metallurgists*), v. 2, WGH, Katowice, 1959.
146. WUSATOWSKI, Z., and Collaborators, Przeróbka plastyczna. Walcownictwo. Słownictwo i określenia (Mechanical working. Rolling. Terminology and definitions), P.K.N., Warszawa, 1960 Projekt.
147. Research on the rolling strip. A symposium of selected papers 1948–1958, B.I.S.R.A. London, 1960.
148. MANITIUS, J., and Collaborators, Hutnicze napędy elektryczne (*Electrical Drives in Steel Works*), WGH, Katowice, 1959.
149. ZOUHAR, G., Umformungskräfte beim Walzen in Streckkaliberreihen (Forces during rolling in roughing pass sequences), *Neue Hütte*, 6, 1961, p. 291–300.
150. WUSATOWSKI, Z., Podstawy walcowania (*Fundamentals of Rolling*), WGH, Katowice, 1960.
151. GALLAJ, J. S., Materiały po teorii prokatki (*Source Materials on the Theory of Rolling*), vol. V, Moscow, 1960.

152. GALLAJ, J. S., Materiały po teorii prokatki (*Materials on the Theory of Rolling*), vol. VI, Moscow, 1960.
153. WEBER, K. H., Berechnung zur Ermittlung von Walzkraft und Drehmoment beim Warmwalzen (Calculation of roll force and torque during hot rolling), *Freiberger Forschungshefte*, Berlin, 1959, B. 46.
154. WEBER, K. H., Berechnung von Walzkraft und Drehmoment beim Kaltwalzen mit und ohne Bandzug (Calculation of roll force and torque during cold rolling with or without applied strip tensions), *Freiberger Forschungshefte*, B. 49, Berlin, 1960.
155. WUSATOWSKI, Z., and Collaborators, Wykaz publikacji polskich z zakresu walcownictwa do roku 1957 (*List of Polish Publications on Rolling up to 1957*), Gliwice, 1958.
156. WUSATOWSKI, Z., Średnice czynne i osie obojętne przy walcowaniu kształtowników (Effective diameters and neutral axis during rolling of sections), *Problemy Projektowe*, 9, 1961, No. 7, p. 201–205.
157. ZOUHAR, G., Umformungskräfte beim Walzen in Streckkaliberreihen (Forces during rolling in roughing pass sequences), *Freiberger Forschungshefte*, B. 52, Berlin, Akademie Verlag, 1960.
158. Zouhar, G., Siłowe vozdeistva pri prokatke v vytiazhnykh kalibrakh. (Forces during rolling in roughing pass sequences), Moscow, 1963.
159. JANAS, K., Zależność ciężaru właściwego drutu od zwiększającego się zgniotu (The relation between specific weight of wire and increasing reduction), *Hutnik*, 26, 1959, p. 113–120.
160. LARKE, E. C., Prokatka listowego i polosowego metalla (*The Rolling of Sheet, Plate and Strip*), Moscow, 1959.
161. WUSATOWSKI, Z., Arbeitende Walzen-Halbmesser und neutrale Linie des Kalibers beim Walzen von Formstahl (Effective radii and neutral axis during rolling of sections), *Freiberger Forschungshefte*, 1962, B. 69, p. 49–62.
162. POLUKHIN, P. J., FEDOROV, M. N., KOROLEV, A. A., and MATEJEV, I. M., Prokatnoje proizvodstvo (*Rolling Practice*), Moscow, 1960.
163. WALLQUIST, G., Investigation of the influence of different factors on roll pressure, energy consumption, spread and forward slip in hot rolling, *Jernkont. Ann.*, 144, 1960, No. 3, p. 193/255.
164. POBEDIN, S. J. S., and DROZD, W. G., Opredelenie usilij pri prokatke v vytiazhnykh kalibrakh (Determination of forces during rolling in stretching passes), *Trudy WNIMETMASH, Sbornik* 2, 1961, p. 179–192.
165. ORZECZOWSKI, Z., Szybka metoda obliczania zespołu ciągłego (A method for quick determination of pass design for continuous trains), *Hutnik*, 27, 1960, p. 224–229.
166. WUSATOWSKI, Z., and HODERNY, B., Wydłużenie i rozszerzenie przy walcowaniu na gorąco dużymi gniotami miękkiej stali (Elongation and spread during hot rolling of mild steel with high reductions), *Arch. Hutn. PAN*, 3, 1958, p. 161–179.
167. WUSATOWSKI, Z., and KUNERT, F., Wyprowadzenie i kąt płaszczyzny podziałowej przy walcowaniu dużymi gniotami miękkiej stali (Forward slip and neutral angle during rolling of mild steel with high reductions), *Arch. Hutn. PAN*, 3, 1958, p. 187–199.
168. WUSATOWSKI, Z., and HODERNY, B., Płynięcie metalu i zjawiska strukturalne przy walcowaniu na gorąco dużymi gniotami (Metal flow and structural changes during hot rolling with high reductions), *Arch. Hutn. PAN*, 4, 1959, p. 63–80.
169. WUSATOWSKI, Z., and HODERNY, B., Siły i momenty przy walcowaniu na gorąco dużymi gniotami miękkiej stali (Roll force and torque during hot rolling of mild steel with high reductions), *Arch. Hutn. PAN*, 4, 1959, p. 60–88.
170. WUSATOWSKI, Z., and HODERNY, B., Das Walzen von weichem Stahl mit grossen Abnahmen (Rolling of mild steel with high reductions), *Berg- und Hüttenmännische Monatshefte, Leoben*, 103, 1958, p. 136–142.
171. WUSATOWSKI, Z., and SZALAJDA, Z., Warmwalzen von weichem Stahl mit grossen Abnahmen. Voreilung und Walzgeschwindigkeit (Hot rolling of mild steel with high reductions. Forward slip and rolling speed), *Berg- und Hüttenmännische Monatshefte, Leoben*, 103, 1958, p. 153–160.
172. WUSATOWSKI, Z., and HODERNY, B., Das Walzen von weichem Stahl mit grossen Abnahmen. Teil III. Das Fliessen des Metals und strukturelle Erscheinungen beim Warmwalzen (The rolling of mild steel with high reductions. Part III. Metal flow and structural phenomena during hot rolling), *Berg- und Hüttenmännische Monatshefte, Leoben*, 103, 1958, p. 136–142, 175–180.
173. WUSATOWSKI, Z., and HODERNY, B., Analyse der Kräfte und Momente beim Warmwalzen von weichem Stahl mit grossen Höhenabnahmen (Analysis of roll force and torque during hot rolling of mild steel with high reductions), *Rev. Techn. Luxembourgoise*, 52, 1960, p. 111–122.

174. LUDYGA, J., STRUK, S., and WUSATOWSKI, Z., Usprawnienie walcowania dwuteowników i ceowników (Improvements in the rolling of beams and channels), *Sprawozdanie IMŻ*, No. 804, Gliwice, 1960.
175. WUSATOWSKI, Z., and LENARD, K., Określenie rozszerzenia i wydłużenia miękkiej stali przy walcowaniu na zimno (Determination of spread and elongation during cold rolling of mild steel), *Zeszyty Naukowe Politechniki Śląskiej. Mechanika*, Gliwice, 1958, p. 41–53.
176. KRUPKOWSKI, A., Annales de l'Académie Polonaise des Sciences (Proceedings of the Polish Academy of Sciences), *Techniques*, 7, 1946, p. 113.
177. TRUSZKOWSKI, W., *Archiwum Górnictwa i Hutnictwa*, 3, 1955, p. 345.
178. TRUSZKOWSKI, W., Analiza procesu deformacji w procesie rozciągania względnego niejednorodnego metalu (Analysis of deformation of non-homogeneous metal in relative tensile test), *Archiwum Hutnictwa*, 4, 1959, p. 283–322.
179. KONCEWICZ, St., Prędkość odkształcania względnego w procesie walcowania (Rate of relative deformation in rolling), *Zeszyty Naukowe Politechniki Śląskiej. Mechanika*, No. 12, 1962, p. 3–39.
180. DOBRUCKI, W., Hipoteza tarcia szepno-poślizgowego na powierzchniach styku ciał sztywnych (narzędzia) z ciałem sprężysto-plastycznym (Hypothesis of stick-slip friction on surfaces of contact of rigid bodies (tool) with an elastic-plastic body), *Obróbka Plastyczna*, Poznań, 2, 1962, p. 449–488.
181. KONCEWICZ, S., Wyprzedzanie i kąt płaszczyzny podziałowej przy walcowaniu na gorąco z uwzględnieniem rozszerzenia (Forward slip and neutral angle in hot rolling with occurring spread), Doctoral thesis, manuscript, 1961.
182. STONE, M. D., *Iron and Steel Engineer*, No. 6, 1961.
183. WUSATOWSKI, Z., and KUSCHKA, W., Ocena teorii walcowania w świetle pomiarów (An appraisal of rolling theories in the light of measurements), *Arch. Hutn. PAN*, 8, 1963, p. 259–280.
184. CHERTAVSKIKH, A. K., *Trenie i smazka pri obrabotke metallov (Friction and Lubrication in Working of Metals)*, Metallurgizdat, Moscow, 1955.

## INDEX

- Absolute draught 20, 23, 74–77, 80, 124, 217, 441–5, 478, 496
  - maximum 71, 74, 445
  - mean 71, 74, 524
- Absolute elongation 20, 30, 74
- Absolute error 21
- Absolute reduction 20, 74
- Absolute spread 20, 74
- Absolute temperature 21
- Accumulation of energy in crystal lattices 12
- Active pass 494
- Actual work of plastic deformation 68, 266, 309
- Affinity of metal for oxygen, chemical 19
- Angle
  - apex 110
  - of bite 46, 75–79, 81–82, 120–36, 147, 183, 275, 476, 525, 529
  - maximum 79, 81, 122, 127, 525
  - of contact 79–80, 123, 125–7
  - maximum 126
  - of distortion 33
  - of friction 21, 120–36
  - neutral 145, 158–202, 483
  - rolling 75, 78, 150
- Angle pass 499
- Anisotropic structure 11
- Annealing 11
  - subcritical 14
- Apex angle 110
- Arc of contact 77–78, 83, 153, 220
- Area of contact 85, 219–20
- Average cold work, degree of 51
- Axis
  - neutral 498, 624–47
  - roll 70, 497
- Back tension 206, 459
  - measurement of 212
- Backward slip 147, 152–3
- Bakhtinov's formula *see* Formula for the calculation of free spread
- Bar pass 499
- Bar rolling 512
- Barrel 392–3, 424, 498
- Bearing pressure 403–8
- Behaviour of metals, plastic 21, 31
- Bending modulus 396
- Bending moment 395, 397–9
- Bending stress 396, 399, 404
- Bite 120–36
  - angle of 46, 75–79, 81–82, 120–36, 147, 183, 275, 476, 525, 529
- Blooming mill 316, 392, 417, 441, 460, 502
- Blooming mill pass design 523–30
- Blooming (roughing) pass 499
- Blowholes, subcutaneous 22
- Boczwar relation *see* Relation between recrystallization temperature and melting temperature
- Body-centred cubic crystal lattice 6
- Boundary conditions
  - for criterion of yielding 36
  - for plastic strain 40
- Box pass 275–92
  - open 499, 502
- Box pass sequence 500, 508
- Breadth 20, 70
- Breaking-down pass 292–309, 499
- Breaking-down pass sequence 500, 507–12
  - diamond 509
  - gothic square–gothic square 500
  - open box 500, 508
  - oval–oval 500
  - square–diamond–square 510
  - square–oval–square 511
  - square–square 500
- Breaking-down systems design 541–56
- Brittleness 46
- Burning of steel 19
- Calculated roll force 414
- Calculated roll torque 416
- Calculated value 21
- Calculation
  - of elliptical ovals 112
  - of flat ovals 112–14
  - of form factors 546
  - of forward slip 160
  - of hexagonal ovals 114–18
  - of mean yield stress 335
  - of number of passes 502–3, 519, 523–4, 549
  - of oval dimensions 511
  - of pass dimensions 511–12, 514
  - of resultant roll force 273

Calculation (*cont.*)

- of roll camber 426-9
  - of roll force 229-66, 273-5, 278-82, 289, 294-9, 331-58, 415
    - Bland, Ford and Ellis method 349-53
    - Cook-McCrum's method 238, 258-66
    - Ekelund's method 234, 254-66, 341
    - Geleji's method 237, 258-66
    - Golovin-Tiagunov's method 231, 254-66
    - Korolev-Nikolayevski's method 356-8
    - Orovan-Pascoe's method 235, 254-66
    - Samarin's method 250, 254-66
    - Siebel's method 250, 254-66, 275, 342
    - Trink's method 353-6
    - Tselikov's method 229, 254-66, 341
  - of roll strength 387-411
  - of roll torque 267, 278, 285-6, 291, 294-9, 309-19, 358-64
    - Bland, Ford and Ellis method 360, 362
    - Ford's method 361
    - Tselikov's method 359
  - of rolling speed 144-202, 615-24
  - of rolling temperature 136-44
  - of simple ovals 110-12
- Camber 426-9
- Capacity for deformation *see* Plasticity
- Change in shape 20
- Channel bar pass 499
- Channels, pass design for 556, 589-94
- Chemical affinity of metal for oxygen 19
- Chilled cast iron rolls 387
- Closed pass 494-5
- Close-packed hexagonal crystal lattice 6
- Coarsening of grain 17-18
- Coefficient
- of deformation 24, 39-40, 71, 559
  - of draught 20, 24, 71-72, 84-120, 149, 162, 478, 484-90, 537
    - maximum 71, 74
    - mean 71, 74, 155, 556
  - of dry friction 59
  - of elongation 20, 24, 26, 48-49, 71-73, 84-120, 149, 151, 321-7, 479, 503-7, 520-4, 549, 606
    - mean 502, 524-5, 558, 562
    - total 321-7, 505, 524-5
  - of forward slip 146, 177-82, 195
    - as a function of draught 177-82
  - of friction 59, 61-64, 120-36, 311
    - mean 128
  - of kinematic friction 21, 59
  - of loss 199
  - of rolling efficiency 311, 342
  - of spread 20, 24, 72, 84-120, 151, 153, 276, 483-6
    - of static friction 21, 59
- Coercive force 11
- Cogging mill 316
- Cohesive force 3
- Cold deformation 8
- Cold plastic working of metals 2, 8-11, 15, 42
  - critical 14
- Cold rolling 203, 211, 275, 331-72, 440-1, 536
- Cold working range 46
- Collar 496
  - double 496
  - end 496
  - inner 496
  - outer 496
  - resistance 496
  - single 496
- Collective recrystallization 13
- Common cast iron rolls 387
- Compression 39
- Compression patterns 38
- Compressive forces 3
- Compressive strength 19
- Compressive stresses 59
- Concave groove 495
- Condition
  - of constant volume 22-24, 70, 88, 159, 506
  - of plasticity 30-42
  - of yielding *see* Criterion of yielding
- Constancy of volume 21-30, 37, 88
- Constant volume condition 22-24, 70, 88, 159, 506
- Constant volume relationship 24
- Constrained yield stress 21, 42, 51, 211, 220, 222, 226, 231, 239
  - mean 220, 229
- Contact
  - angle of 79-80, 123, 125-7
  - arc of 77-78, 83, 153, 220
  - area of 85, 219-20
- Continuous mills, roll pass design for 610-24
- Continuous rolling process 610
- Continuous train design 615-24
- Contraction 20
- Convex groove 495
- Cooling box 423
- Cooling of rolls 423
- Correction factor 93-107
  - for condition of rolling surface 93-95
  - for grade of steel 93-107
  - for rolling speed 93-95
  - for temperature 93-95
- Criterion of yielding 35-36, 42, 63
  - boundary conditions for 36
- Critical cold work 14
- Cross-sectional area 20, 70
- Crown 427
- Crystal lattices 6, 12
  - accumulation of energy in 12
  - body-centred cubic 6
  - close-packed hexagonal 6
  - face-centred cubic 6
- Crystal recovery 12

- Crystallization processes 18
  - internal 14
- Crystallographic orientation 5
- Crystallographic planes 5
- Cubic lattice structure 7
- Curve of spread 217
  
- Dead diameter 498
- Dead pass 494
- Deformation 1–2
  - coefficient of 24, 39–40, 71, 559
  - elastic 2
  - in rolling 70
  - intensity of 45
  - intercrystalline 4–5
  - of a cubic element 30
  - of grains 5
  - of polycrystalline aggregates 4–8
  - of single crystals 4–8
  - parallelepipedal 21
  - permanent 2–3
  - plastic 2, 38
  - relative 23, 43
  - strain energy of 34
  - total 2–3
  - velocity of 43
- Deformation-energy theory 34, 41
- Deformation work 21
  - specific 21
- Degree
  - of cold work 8–13, 51
    - average 51
    - formula for 9–10
  - of deformation 9, 12–14
  - of plasticity 3
  - of strain-hardening 15
- Dependence
  - of ductility on temperature 18–19
  - of grain-growth on temperature 16
- Design
  - of breaking-down systems 541–56
  - of continuous train 615–24
  - of regular sections 541–56
  - of roll cambers 427
  - of roll pass 494–660
- Determination
  - of areas of squares and diamonds 555
  - of average yield stress 47
  - of mean and effective roll diameters 625
  - of mean height of section 557
  - of roll camber 438
  - of roll deflection 411
  - of roll forces in cold rolling 331–58
  - of roll forces in hot rolling 216–29
  - of roll speed 478
  - of spread 87
  - of torque distribution 378
- Diagonal groove 495
- Diagram
  - of elongations in two successive passes 552 3
  - of recrystallization, space 13–14
  - of strain-hardening 9
- Diameter 20
  - barrel 498
- Diamond pass 219, 499, 502
- Diamond pass sequence 509
- Direct draught 496
- Direction of rolling 70
- Directions of slip in crystal lattices 6
- Displacement 5, 33
  - absolute 23
  - relative 3
- Distortion, angle of 33
- Distribution of compressive stresses 59
- Disturbance of equilibrium
  - on grain boundaries 4–5
  - within the grains 4
- Double collar 496
- Draught 20, 23, 84–120
  - absolute 20, 23, 74–77, 80, 124, 441–5, 478, 496
    - maximum 71, 74, 445
    - mean 71, 74, 524
  - coefficient of 20, 24, 71–72, 84–120, 149, 162, 478, 484–90, 537
    - maximum 71, 74
    - mean 71, 556
  - direct 496
  - lateral 496
  - maximum 71
  - natural 20
  - oblique 496
  - percentage 71
    - maximum 71, 74
    - mean 71, 74
  - relative 20, 74
    - maximum 71
    - mean 71
  - total 524
  - velocity of 44
- Drawing 1
- Drive 1
- Drive loading with flywheel 377
- Driving motors, power of 372–86
- Driving torque, total 310
- Dry friction 60, 128
  - coefficient of 59
- Ductility 11, 18–19, 46
- Dulling of grooves 495–6
- Duplex rolls 387
- Dynamic torque 310, 314
  
- Economic rolling, theoretical limit of 90–92
- Edging pass 499, 502, 519, 526
- Effect
  - of initial cross-section 612



- Effect (*cont.*)  
 of roll diameter 613  
 of roll speed 614  
 of strip tension 367–72  
 Effective radius 624–47  
 Effective roll diameter, mean 107  
 Effective work of plastic deformation 68  
 Efficiency of process 21  
 Effort of material relative to yield point 21  
 Ekelund's formula  
 for the calculation of roll force 234  
 for the calculation of yield stress 57  
 Elastic deformation 2  
 Elastic disturbance zone 207–8  
 Elastic limit 2  
 Elastic strain 2, 21, 23  
 Elasticity  
 modulus of 3, 21, 30  
 theory of 30  
 Elliptical ovals, calculation of 112  
 Elongation 2, 20, 67, 84–120, 477–82  
 absolute 20, 30, 74  
 coefficient of 20, 24, 26, 48–49, 71–73, 84–120, 149, 151, 321–7, 479, 503–7, 520–4, 549 606  
 mean 502, 524–5, 558, 562  
 total 321–7, 505, 524–5  
 during hot rolling 477–82  
 natural 20  
 relative 20  
 End collar 496  
 Energy consumption in rolling 321–9, 331 364–6  
 Entry speed of stock 72, 145  
 Error  
 absolute 21  
 per cent 21  
 relative 21  
 Error ratio 21  
 Evaluation of rolling measurements 460–76  
 Exit speed of stock 72, 145, 175  
 Experimental rolling mill 488–9  
 Exponential coefficient 480–1  
 Extension 2  
 Extrusion 1
- Face-centred cubic crystal lattice 6  
 Fatigue cracks 401  
 Fatigue limit 401  
 Finishing pass 499, 512  
 Finishing square 513  
 Fink's formula *see* Formula for the calculation of the coefficient of forward slip  
 Flat bars 518  
 Flat ovals, calculation of 112–14  
 Flat pass 502  
 Flat rolling processes 647–60
- Flow lines 3–4, 214  
 Flow of metal 82–84, 147, 491–3, 557–9, 563–4, 573, 583  
 Flying micrometer 456  
 Flywheel 314, 319–22  
 kinetic energy of 319  
 Force 21  
 coercive 11  
 cohesive 3  
 compressive 3  
 frictional 59–62, 122  
 Ford's formula *see* Formula for the calculation of degree of cold work  
 Forging 1  
 Form factor 88, 99, 107, 264–5, 398, 546  
 mean 107  
 Formation  
 of recrystallization nuclei 12  
 of twin crystals *see* Twinning  
 Forming pass 499  
 Formula for the calculation  
 of angle of bite 76–78  
 of average stress 35  
 of coefficient of elongation 93  
 of coefficient of forward slip 147  
 of coefficient of spread 93  
 of correction factor 93  
 of degree of cold work 9–10, 51  
 of effective work of deformation 68  
 of form factor 88  
 of free spread 86  
 of heat gain 139  
 of heat loss due to convection 139  
 of internal friction of viscous liquids 33  
 of length of sticking zone 208  
 of lever arm 270  
 of mean resistance to deformation 275  
 of neutral angle 183, 192  
 of plastic slip 33  
 of pressure of metal on the tool 65  
 of projected length of contact of a roll 332  
 of resistance to deformation 203, 356  
 of roll deflection 412  
 of roll factor 88  
 of roll force 229, 231, 233–9, 347, 349  
 of roll power 309  
 of rolling angle 78  
 of spread of alloy steels 95  
 of temperature drop 141–4  
 of theoretical work of deformation 266  
 of work of deformation 266  
 of yield criterion 35  
 of yield stress 50, 57  
 Formulae  
 for roll pass design 537–41  
 for the length of sticking zone 208  
 Forward slip 146, 152–3, 158–82, 482–7  
 coefficient of 146, 177–82, 195

- Fragmentation of grains 4, 7
- Free rolling 123
- Free spread 86
- Friction 59, 120–36
  - angle of 21, 120–36
  - coefficient of 59, 61–64, 120–36, 311
  - mean 128
- Frictional forces 59–62, 122
- Frictional resistance 63–64
- Frictional torque 311–14
- Front tension 206, 459
  
- Golovin-Tiagunov's formula for roll force 231
- Gothic square construction 544
- Gothic square–gothic square pass sequence 500
- Gothic square pass 499, 502
- Grade of steel, correction factor for 93–107
- Grain boundaries 4, 7
- Grain fragmentation 4, 7
- Grain growth 13, 16
- Grain slip 4–5
- Graphical roll pass design 530–7
- Groove 85, 495
  - concave 495
  - convex 495
  - diagonal 495
- Groove depth 85, 495
- Groove taper 495–6
- Groove width 85
- Group displacements of metal particles 4
  
- Haag's formula 376
- Heavy draughts 477–93
- Height
  - of pass 218
  - of stock 20, 70
- Hexagonal ovals, calculation of 114–18
- Higher temperature range 46
- Hitchcock's formula *see* Formula for the calculation of projected length of contact of a roll
- Holzweiler method 514
- Hooke's law 30
- Hot plastic working 2, 15–19, 42, 51
  - incomplete 16
- Hot rolling 203, 211, 216–66, 275, 441
  - of mild steel 477–91
  - with heavy draughts 477–93
- Hot section 494
- Hot working range 46
- Huber criterion of yielding 35
- Hypothesis of parallelepipedal deformation 21, 66
  
- Idling torque 310, 314, 443
- Incomplete hot plastic working 16
- Induction loadmeter 454–6
- Induction sensor 454–6
  
- Inertia energy of flywheel 319–22
- Influence
  - of absolute draught and roll diameter on the coefficient of friction 124
  - of coefficient of friction on the resistance to deformation 206
  - of cold work and temperature on grain size, 14
  - of cold work on grain size 13
  - of cold work on steel density 10
  - of deformation and temperature on recrystallization 15
  - of deformation rate on ductility and yield stress 46, 51
  - of front and back tension on the resistance to deformation 206
  - of hot working on grain size 16–17
  - of roll diameter on the resistance to deformation 206
  - of temperature on ductility of steel 18–19
  - of temperature on strain rate 46
  - of work hardening on roll pressure distribution 205
- Initial cross-section 612
- Inner collar 496
- Intensity of deformation 45
- Intercrystalline deformation 4–5
- Intercrystalline matter 4, 6, 14, 19
- Intermediate mill 316
- Internal crystallization process 14
- Introductory pass 499
  
- Kinematic friction, coefficient of 21, 59
- Knifing pass 499, 606–9
- Korolev formulae *see* Formulae for the length of sticking zone
  
- Lateral draught 496
- Laws of plastic deformation 21–30
- Leader pass 499, 648–9, 652
- Length 20, 70
- Lever arm 270
- Limit
  - of elastic deformation *see* Elastic limit
  - of permanent (plastic) deformation *see* Yield point
- Limited plasticity 83
- Line
  - of deflection 414
  - neutral 145, 395, 498, 624–47
- Load 21
- Loading of rolls, permissible 387–486
- Loadmeter 452–4
  - induction 454–6
- Longitudinal rolling 1, 69, 145
- Loss, coefficient of 199
- Lowest temperature range 46
- Lüders lines *see* Flow lines

- Magnetic permeability 11
- Mass 21
- Maximum absolute draught 71, 74, 445
- Maximum angle of bite 79, 81, 122, 127, 525
- Maximum angle of contact 126
- Maximum coefficient of draught 71, 74
- Maximum diameter 498
- Maximum draught 71
- Maximum percentage draught 71, 74
- Maximum permissible draught 526
- Maximum relative draught 71
- Maximum roll forces as a function of permissible bending stresses 463
- Maximum shear theory 33
- Maximum stress theory 33
- Mean absolute draught 71, 74, 524
- Mean coefficient of draught 71, 74, 155, 556
- Mean coefficient of elongation 502, 524–5, 558, 562
- Mean constrained yield stress 220, 229
- Mean effective roll diameter 107
- Mean form factor 107
- Mean height 71
- Mean height of the pass 218
- Mean heights method of pass design 541–56
- Mean percentage draught 71, 74
- Mean peripheral speed of rolls 195
- Mean relative draught 71
- Mean relative reduction 503
- Mean resistance to deformation 21, 68, 220, 250–3, 260–4, 283–4, 300–2, 305–6, 343–4, 654
- Mean resistance to flow 220
- Mean roll factor 107
- Mean speed of metal flow 152
- Mean strain rate 44, 221–2
- Mean working diameter 195
- Mean yield stress 51, 226–8, 335–6, 341, 352
- Measurement
  - of back tension 212
  - of exit speed 175
  - of roll force 449–56
    - by induction sensors 454–6
    - by means of strain gauges 450–4
  - of roll pressure distribution 211–16
  - of roll torque 449–56
    - by induction sensors 454–6
    - by means of strain gauges 450–4
  - of strip tension 449, 459
  - of strip thickness 449, 456–60
- Measurements
  - in rolling 449–76
  - in rolling mills 460–76
- Mechanism of bite 120–36
- Metal flow 82–84, 147, 491–3, 557–9, 563–4, 573, 583
  - velocity of 84, 147–8, 151, 153, 491–3, 557–9, 563–4
  - mean 152
- Metal flow lines 2–3, 214
- Methods of rolling 1
- Mild steel, hot rolling of 477–91
- Mill, blooming 316, 392, 417, 441, 460, 502
- Minimum diameter 498
- Modulus
  - of elasticity 3, 21, 30
  - of plasticity 21, 32
  - of rigidity 21, 412
  - of strain hardening 21
- Moment of inertia 418
- Motor power 440
- Natural draught 20
- Natural elongation 20
- Natural spread 20
- Navier and Stokes formulae *see* Formula for the calculation of internal friction of viscous liquids
- Neck 392–3, 398
- Neutral angle 145, 158–202, 483
- Neutral axis 498, 624–47
- Neutral line 145, 395, 498, 624–47
- Neutral plane 124, 145
- Neutral zone 203
- Nominal roll diameter 498
- Nomogram for determination
  - of bite angle 81
  - of breaking-down systems 544–5
  - of coefficient of deformation 25
  - of draught coefficient in pass design 538–9
  - of draughts and reductions 531
  - of mean strain rate 223–5
  - of neutral angle 161, 190–1
  - of number of passes 27
  - of peripheral speed of rolls 146
  - of permissible absolute draughts 446
  - of radiation loss 138
  - of roll forces 366–7, 446–7
  - of roll torque 366–7, 466–7
  - of the areas of various ovals 113–17
- Non-uniform deformation of metal 83
- No-slip angle 482–7
- Number of passes 27, 502–3, 519, 523–4, 549
- Oblique draught 496
- Open box pass 499, 502
- Open pass 494
- Optimum rolling programme 440
- Orowan-Pascoe formulae for the calculation of roll force 235
- Otto's slide rule 540, 582
- Outer collar 496
- Oval-oval pass sequence 500
- Oval pass 499, 502
- Ovals
  - types of 110
  - elliptical 112

Ovals (*cont.*)

- flat 112-14
- hexagonal 114-18
- simple 110-12

Overdraught 496-7

Overheating of steel 19

Overloading of motors, permissible 374

Parallelepipedal deformation, hypothesis of 21, 66

Pass 494

- active 494
- angle 499
- blooming (roughing) 499
- box 275-92
- breaking-down 292-309, 499
- closed 494-5
- dead 494
- edging 499, 502, 519, 526
- finishing 499, 512
- flat 502
- forming 499
- introductory 499
- knifing 499, 606-9
- leader 499, 648-9
- preparatory 499
- section 499

Pass design 494-660

- blooming mill 523-30
- mean heights method of 541-56
- sections 556-610

Pass dimensions 511-12, 514, 529

Pass neutral axis 635

Pass schedule for rolling 406, 413

Pass sequences 500-2

- breaking-down 500, 507-12

Pavlov-Ekelund formula for the calculation of neutral angle 183, 193

Per cent error 21

Percentage draught 74, 162  
maximum 71, 74

Percentage reduction 20, 72, 74, 229

Peripheral speed of rolls 145  
mean 195Permanent plastic deformation 2-3  
limit of *see* Yield point

Permeability, magnetic 11

Permissible draught, maximum 526

Permissible loading  
of rolling mills 440-9  
of rolls 387-486

Permissible overloading of motors 374

Permissible roll force 414, 467

Permissible roll torque 416

Permissible stresses for rolls 401-2

Phenomena in plastic working 2-4

Phenomenon  
of cold work 18

of friction 59-60

Pinion stand 1

Plane, neutral 124, 145

Planes of slip in crystal lattices 6

Plastic behaviour of metals 21, 31

Plastic deformation 1-2, 16, 38  
laws of 21-30

of polycrystalline aggregates 4-8

of single crystals 4-8

work of 65-68, 266, 309, 368

Plastic disturbance zone 207-8

Plastic state of solid matter 3

Plastic strain 2, 21, 23, 26, 28, 30, 32

boundary conditions for 40

Plastic working of metals 1-19

principles of 20-68

Plasticity 6, 9

condition of 30-42

modulus of 21, 32

Plate mill 316, 392

Poisson's constant 21, 31-32

Poisson's ratio 21, 31

Polycrystalline aggregates 4-8

interaction of stresses in 7

plastic deformation of 4-8

Power 21

of driving motors 372-86

Power consumption in rolling 322-31

Practical condition for rolling 124

Preparatory oval 517

Preparatory pass 499

Preparatory square pass 502

Pressing 1

Principal strain 21

Principal stresses 21

Principle of constant volume 159

Principles

of plastic working 20-68

of roll pass design 494-660

Profile 494

Profile section of channel 643

Properties of steel, influence of cold working  
10-12

Radiation gauge 457-8

Radiation loss in rolling bars 138

Rail pass 499

Range

cold working 46

higher temperature 46

hot working 46

lowest temperature 46

Rate

of deformation 16, 42-46

of strain 44-47, 52

of strain-hardening 9

Reconstruction of space lattice 14

- Recrystallization 8, 11–18
  - collective 13
  - incomplete 16
  - space diagrams of 13–14
- Recrystallization nuclei 12, 16
- Recrystallization range 15–16
- Recrystallization temperature 9, 12
- Reduction 520–2
  - absolute 20, 74
  - percentage 72
  - relative 20, 48–49, 72, 74
  - mean 503
  - total 20
- Regions
  - of limited plasticity 83
  - of plastic deformation 63
- Regular sections 541–56
  - design of 541–56
- Reheating 11–12
- Relation
  - between coefficient of elongation and percentage reduction 72
  - between constrained yield stress and strain 67
  - between forward slip and spread coefficient to draught coefficient 483
  - between forward slip and temperature 485
  - between friction coefficient and surface condition 61
  - between recrystallization temperature and melting temperature 12
  - between yield stress and deformation 49
  - between yield stress and the strain rate and temperature 52
- Relative deformation 23, 43
- Relative displacement 3
- Relative draught 20, 74
  - maximum 71
  - mean 71
- Relative elongation 20
- Relative error 21
- Relative reduction 20, 48–49, 72, 74
  - mean 503
- Relative spread 20
- Relative strain, velocity of 44
- Resistance collar 496
- Resistance to deformation 6, 58–65, 68, 204, 220, 233–4, 250, 275, 278, 281–4, 290, 294, 305, 341–6, 356, 441
  - mean 21, 68, 220, 250–3, 260–4, 283–4, 300–2, 305–6, 343–4, 654
  - specific 21
- Resistance to flow 63, 203
- Resistance to friction 63–64, 204
- Resultant roll force 273–5
- Rigidity, modulus of 21, 412
- Rod mill 316
- Roll axis 70, 497
- Roll barrel 392–3, 424, 498
- Roll camber 420–40
  - calculation of 426–9
  - design of 427
- Roll clearance 108–10
- Roll cooling 423
- Roll deflection 394–5, 411–40, 507
- Roll diameter 80, 613, 624–5
  - nominal 498
- Roll factor 88, 107
  - effective, mean 107
- Roll flattening 421, 507
- Roll force 394, 398–9, 411–12, 415, 440–1, 444–6, 487–8
  - calculated 414
  - calculation of 229–66, 273–5, 278–82, 289, 294–9, 331–58, 415
  - determination of 216–29, 331–58
  - due to bending stresses 411–12, 416, 437
  - due to shear stresses 411–12, 416, 437
  - in cold rolling 331–58
  - in hot rolling 216–66
  - measurement of 449–56
  - permissible 414, 467
  - resultant 273–5
  - total 411–12, 416, 419, 428, 430, 436
- Roll gap 80, 82, 125, 204, 411, 423–5, 435, 497
- Roll hardness 387, 391
- Roll heating 423
- Roll jump 497
- Roll neck 392–3, 398
- Roll pass design 494–660
  - basic definitions of 494–501
  - formulae for 501–30, 537–41
  - graphical method of 530–7
  - theoretical principles of 556–647
- Roll power, calculation of 309–31, 367
- Roll pressure 203–29
- Roll pressure distribution 204–7, 213–16
  - measurement of 211–16
- Roll setting 497
- Roll speed 478, 615–24
- Roll strength calculation 387–411
- Roll torque 310, 399, 416, 440–2, 446, 487–91
  - calculated 416
  - calculation of 267, 278, 285–6, 291, 294–9, 309–19, 358–64
  - characteristics of 471
  - dynamic 310
  - measurement of 449–56
  - permissible 416
  - static 311, 318
- Roll wear 394, 426, 498, 609
- Roll wobblers 392–3, 401
- Rolling
  - basic concepts in 69–75
  - bite in 120–36
  - deformation in 70
  - efficiency coefficient of 311, 342

**Rolling (*cont.*)**

flow of metal in 82–84  
 friction in 120–36  
 geometrical relations in 75–82  
 in box passes 275–92  
 in breaking-down passes 292–309  
 longitudinal 1, 69, 145  
 measurements in 449–76  
 methods of 1  
 of bars 107–20  
 of sheet 411–40  
 of small sections 647–60  
 of strip 411–40  
 skew 1–2, 69  
 symbols in 69–75  
 transverse 1–2, 69  
 trigonometrical relations in 75–82  
 with slip 185  
 with spread 158–82  
 Rolling analysis 460–76  
 Rolling angle 75, 78, 150  
 Rolling equipment 1  
 Rolling line 498  
 Rolling load 487–91  
 Rolling mills 1  
   experimental 488–9  
   laboratory 211  
   measurements in 460–76  
   permissible loading of 440–9  
 Rolling process  
   continuous 610  
   flat 647–60  
 Rolling programme, optimum 440  
 Rolling schedule for sheets 330  
 Rolling speed 93–94, 305  
   calculation of 144–202, 615–24  
 Rolling temperature 136–44  
   range of 99  
 Rolling work 322–31  
 Rolls 387–476  
   calculation of 387–411  
   chilled cast iron 387  
   common cast iron 387  
   duplex 387  
   steel 387  
 Roughing mill 316  
 Round bars 514  
 Round pass 502  
 Round profile 515  
 Rounding of grooves 495–6  
 Samarín's formula for the calculation of roll  
   force 233  
 Seamless tubes 2  
 Section mill 316, 392, 624  
 Section pass 499

**Sections**

equal leg angle 556, 561, 581–8  
 I- 556, 581–5, 589–606  
   pass design for 556–610  
   regular 556  
   T- 556, 581–5, 588–9  
 Sedlaczek's formula 86  
 Shape sections 567  
 Shaped pass 502  
 Shear components 20  
 Shear stress 6  
   components 21  
 Shearing force 418–19  
 Sheet mill 316, 392  
 Sheets 536  
 Siebel's correction factor 275  
 Siebel's equation for the mean resistance to  
   deformation 275  
 Siebel's formula *see* Formula for the calculation  
   of free spread  
 Simple ovals, calculation of 110–12  
 Sim's formula for the calculation of roll force 239  
 Single collar 496  
 Single crystals, plastic deformation of 4–8  
 Sizing diamond 513  
 Sizing oval pass 514  
 Sizing ovals 516–17  
 Skew rolling 1–2, 69  
 Slide rule  
   Otto's 540, 582  
   Wojtylak's 538  
 Slip, backward 147, 152–3  
 Slip along definite crystallographic planes *see*  
   Displacement  
 Slip friction 63–64, 82  
 Slip of grains *see* Grain slip  
 Slip theories in rolling 210  
 Slip velocity 33  
 Slipping 220  
 Slipping zone 207–8  
 Small sections rolling 647–57  
 Solidus line 19  
 Space diagram of recrystallization 13–14  
 Specific deformation work 21  
 Specific roll work 294–9  
 Specific weight 21–22  
 Speed distribution during rolling 476–7  
 Speed of rolls, peripheral 145, 195  
 Speed of stock  
   entry 72, 145  
   exit 72, 145, 175  
 Splitting of grains 7  
 Spray-jet cooling of rolls 423  
 Spread 20, 84–120, 158–82, 477–82, 488  
   absolute 20, 74  
   coefficient of 20, 24, 72, 84–120, 151, 153, 276,  
     483–6  
   curve of 217

- Spread (*cont.*)
  - free 86
  - natural 20
  - relative 20
- Square bars 512
- Square-diamond-square pass sequence 510
- Square-oval-square pass sequence 511
- Square pass 502
- Square-square pass sequence 500
- Stamping 1
- Stands of rolls 1
- State
  - of strain 20
  - of stress 21, 31
  - of stress-strain in triaxial deformation 32
  - of stress-strain without lateral spread 35
  - plastic 3
- Static friction 63
  - coefficient of 21, 59
- Static load diagram 318
- Static torque 311, 318
- Static yield stress 21
- Steel rolls 387
- Stefan-Boltzmann relation 136
- Sticking 220
- Sticking zone 207-8
- Strain 2, 20
  - elastic 2, 21, 23
  - plastic 2, 21, 23, 26, 28, 30, 32
  - principal 21
  - total 2, 21, 31
- Strain energy of deformation 34
- Strain gauge 450-4
- Strain rate 44, 46, 52, 220-5
  - mean 44, 221-2
- Strain-hardening 8-11, 15
  - modulus of 21
- Strain-hardening curves 48-50
- Strain-hardening formulae 49
- Strain-hardening rate 9
- Strength
  - calculation of rolls 387-411
  - compressive 19
- Stress 2, 21
  - normal 21
  - shear 21
  - state of 21, 31
- Stress components 21
- Stress concentration factor 429
- Stress function 36
- Stressless zone 207-8
- Stress-strain curve 30, 42, 67
- Stress-strain diagram 52
- Stress-strain relationships 30-42
- Stress-strain state 32, 35
- Strip tension
  - effect 367-72
  - measurement of 449, 459
- Strip thickness, measurement of 449, 456-60
- Strippers 496
- Strips 536
- Structure, anisotropic 11
- Subcritical annealing 14
- Subcutaneous blowholes 22
- Summary temperature drop 142
- Tafel and Sedlaczek's formulae *see* Formula for the calculation of free spread
- Taper 495-6
- Temperature 21
  - absolute 21
  - of fusion 4
  - of recrystallization 51
- Temperature distribution of a roll 422
- Temperature drop, summary 142
- Temperature range, rolling 99
- Tensile force 2
- Tensile strength 2, 8
- Tensiometer 459
- Tension 39
  - back 206, 459
  - front 206, 459
- Theoretical limit of economic rolling 90-92
- Theoretical work of plastic deformation 66-68, 266
- Theory
  - of elasticity 30
  - of maximum shear 33
  - of maximum stress 33
- Thickness of plates 20
- Thickness ratio *see* Roll factor
- Thickness variation during rolling 411-40
- Tiagunov's formula *see* Formula for the calculation of temperature drop
- Time 21
- Torque
  - dynamic 310, 314
  - idling 310, 314, 443
  - on the motor shaft 310, 316
  - static 311, 318
- Torquemeter 452-4
  - induction 454-6
- Total coefficient of elongation 321-7, 505, 524-5
- Total deformation 2-3
- Total draught 524
- Total reduction 20
- Total roll force 411-12, 416, 419, 428, 430, 436
- Total strain 2, 21, 31
- Transverse rolling 1, 69
- Triaxial deformation of a cubic element 22
- True diameter 498
- Tselikov's formula
  - for the calculation of free spread 86
  - for the calculation of neutral angle 183, 192

- Tselikov's formula (*cont.*)  
     for the calculation of roll force 229
- Tubes 2
- Twin crystals 55
- Twinning 5
- Underdraught 496–7
- Value  
     calculated 21  
     measured 21
- Variation  
     of angle of bite with rolling speed 130  
     of coefficient of friction with rolling speed 131, 135  
     of coefficient of friction with temperature 130, 132  
     of coefficient of friction with type of lubricant 134, 135  
     of coefficient of friction with various testing conditions 61  
     of correction factor  $c$  with rolling speed 94  
     of correction factor  $d$  with draught coefficient 97, 100, 102, 104, 106  
     of correction factor  $d$  with temperature 98, 101, 103, 105  
     of yield stress with temperature 58
- Velocity  
     of deformation 43  
     of draught 44  
     of metal flow 84, 147–8, 151–3, 491–3, 557–9, 563–4  
     of relative strain 44
- Volume 20  
     constancy of 21–30, 37, 70, 88
- Wear of the roll surface 394, 426, 498, 609
- Weight, specific 21–22
- Wire rod 518
- Wobbler 392–3, 401
- Wojtylak's slide rule 538
- Work hardened zone 207–8
- Work of plastic deformation 65–68, 266–70, 309, 368  
     actual 68, 266, 309  
     effective 68  
     theoretical 66–68, 266
- Working diameter 498  
     mean 195
- Working radius, mean 197, 201
- Wusatowski's formula 89, 477, 479, 551
- Yield criterion 35, 42, 63
- Yield point 2, 33  
     effort of material relative to 21
- Yield stress 18–21, 42–58, 337–40, 342  
     constrained 21, 42, 51, 211, 220, 222, 226, 231, 239  
     mean 51, 226–8, 335–6, 341, 352  
     static 21
- Yield stress curves 47, 52–57, 226–7
- Young's modulus *see* Modulus of elasticity
- Zolotnikov's formula *see* Formula for the calculation of free spread
- Zone  
     elastic disturbance 207–8  
     neutral 203  
     plastic disturbance 207–8  
     slipping 207–8  
     sticking 207–8  
     stressless 207–8  
     work hardened 207–8