

Chapter 12

Transient Structural Simulations

12.1 Basics of Structural Dynamics

12.2 Lifting Fork

12.3 Two-Story Building

12.4 Ball and Rod

12.5 Guitar String

12.6 Review

Section 12.1

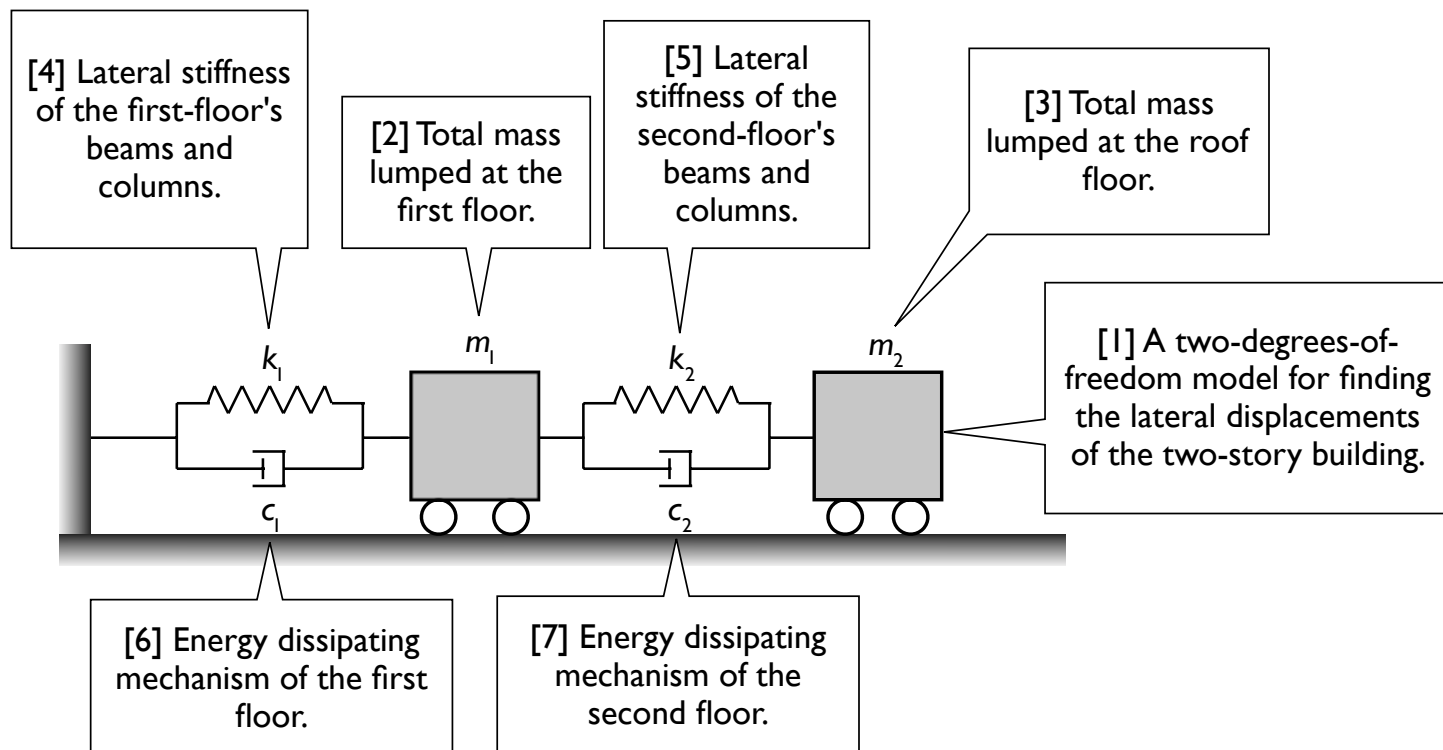
Basics of Structural Dynamics

Key Concepts

- Lumped Mass Model
- Single Degree of Freedom Model
- Undamped Free Vibration
- Damped Free Vibration
- Damping Coefficient
- Damping Mechanisms

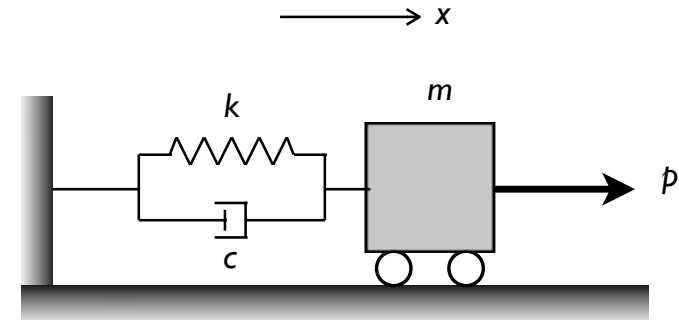
- Viscous Damping
- Material Damping
- Coulomb Friction
- Modal Analysis
- Harmonic Response Analysis
- Transient Structural Analysis
- Explicit Dynamics
- Response Spectrum Analysis
- Random Vibration Analysis

Lumped Mass Model: The Two-Story Building



Single Degree of Freedom Model

$$\begin{aligned}\sum F &= ma \\ p - kx - c\dot{x} &= m\ddot{x} \\ m\ddot{x} + c\dot{x} + kx &= p\end{aligned}$$



- We will use this single-degree-of-freedom lumped mass model to explain some basic behavior of dynamic response.
- The results can be conceptually extended to general multiple-degrees-of-freedom cases.

Undamped Free Vibration

If no external forces exist, the equation for the one-degree-of-freedom system becomes

$$m\ddot{x} + c\dot{x} + kx = 0$$

If the damping is negligible, then the equation becomes

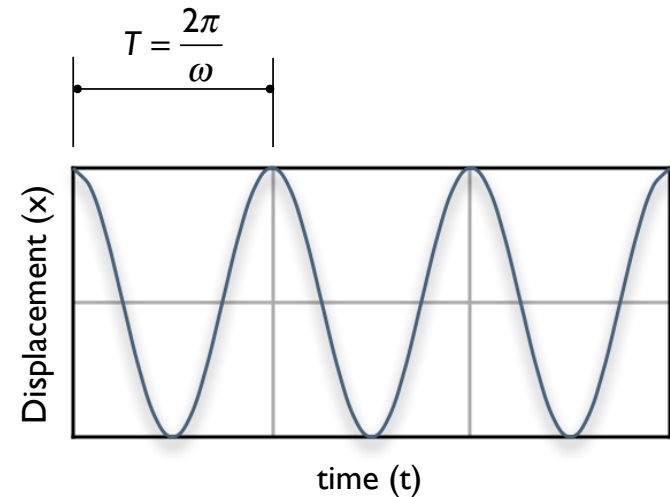
$$m\ddot{x} + kx = 0$$

The solution

$$x = A \sin(\omega t + B)$$

Natural frequency: $\omega = \sqrt{\frac{k}{m}}$ (rad/s) or $f = \frac{\omega}{2\pi}$ (Hz)

Natural period: $T = \frac{1}{f}$



Damped Free Vibration

$$m\ddot{x} + c\dot{x} + kx = 0$$

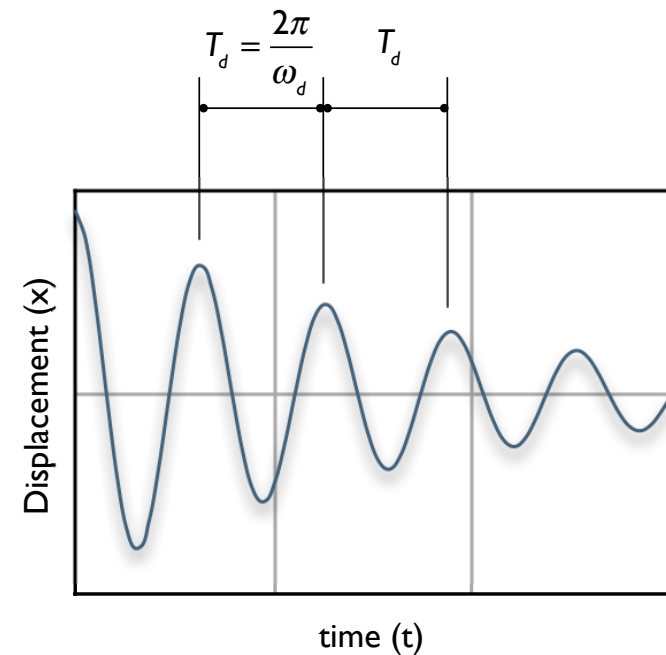
If the damping c is small (smaller than c_c), then the general solution is

$$x = Ae^{-\xi\omega t} \sin(\omega_d t + B)$$

where

$$\omega_d = \omega\sqrt{1-\xi^2}, \quad \xi = \frac{c}{c_c}, \quad c_c = 2m\omega$$

The quantity c_c is called the *critical damping coefficient* and the quantity ξ is called the *damping ratio*.



Damping Mechanisms

- Damping is the collection of all energy dissipating mechanisms.
- In a structural system, all energy dissipating mechanisms come down to one word: friction. Three categories of frictions can be identified:
 - friction between the structure and its surrounding fluid, called *viscous damping*;
 - internal friction in the material, called *material damping*, *solid damping*, or *elastic hysteresis*;
 - friction in the connection between structural members, called *dry friction* or *Coulomb friction*.

Analysis Systems

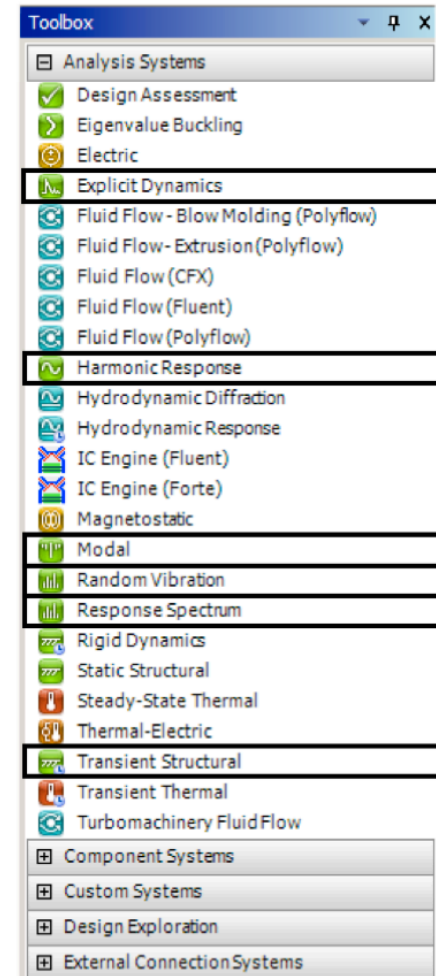
The foregoing concepts may be generalized to multiple-degrees-of-freedom cases,

$$[M]\{\ddot{D}\} + [C]\{\dot{D}\} + [K]\{D\} = \{F\}$$

Where $\{D\}$ is the nodal displacements vector, $\{F\}$ is the nodal external forces vector, $[M]$ is called the *mass matrix*, $[C]$ is called the *damping matrix*, and $[K]$ is the *stiffness matrix*.

Note that when the dynamic effects (inertia effect and damping effect) are neglected, it reduces to a static structural analysis system,

$$[K]\{D\} = \{F\}$$



Modal Analysis

$$[M]\{\ddot{D}\} + [C]\{\dot{D}\} + [K]\{D\} = 0$$

For a problem of n degrees of freedom, it has at most n solutions, denoted by $\{D_i\}, i = 1, 2, \dots, n$. These solutions are called *mode shapes* of the structure. Each mode shape $\{D_i\}$ can be excited by an external excitation of frequency ω_i , called the natural frequency of the mode.

In a modal analysis, since we are usually interested only in the natural frequencies and the shapes of the vibration modes, the damping effect is usually neglected to simplify the calculation,

$$[M]\{\ddot{D}\} + [K]\{D\} = 0$$

Harmonic Response Analysis

$$[M]\{\ddot{D}\} + [C]\{\dot{D}\} + [K]\{D\} = \{F\}$$

Harmonic Response analysis solves a special form of the equation, in which the external force on i th degree of freedom is of the form

$$F_i = A_i \sin(\Omega t + \phi_i)$$

where A_i is the amplitude of the force, ϕ_i is the phase angle of the force, and Ω is the angular frequency of the external force. The steady-state solution of the equation will be of the form

$$D_i = B_i \sin(\Omega t + \phi_i)$$

The goal of the harmonic response analysis is to find the magnitude B_i and the phase angle ϕ_i , under a range of frequencies of the external force.

Transient Structural Analysis

$$[M]\{\ddot{D}\} + [C]\{\dot{D}\} + [K]\{D\} = \{F\}$$

Transient Structural analysis solves the general form of the equation. External force $\{F\}$ can be time-dependent forces. All nonlinearities can be included. It uses a ***direct integration*** method to calculate the dynamic response.

The direct integration method used in **Transient Structural** analysis is called an ***implicit integration*** method.

Explicit Dynamics

$$[M]\{\ddot{D}\} + [C]\{\dot{D}\} + [K]\{D\} = \{F\}$$

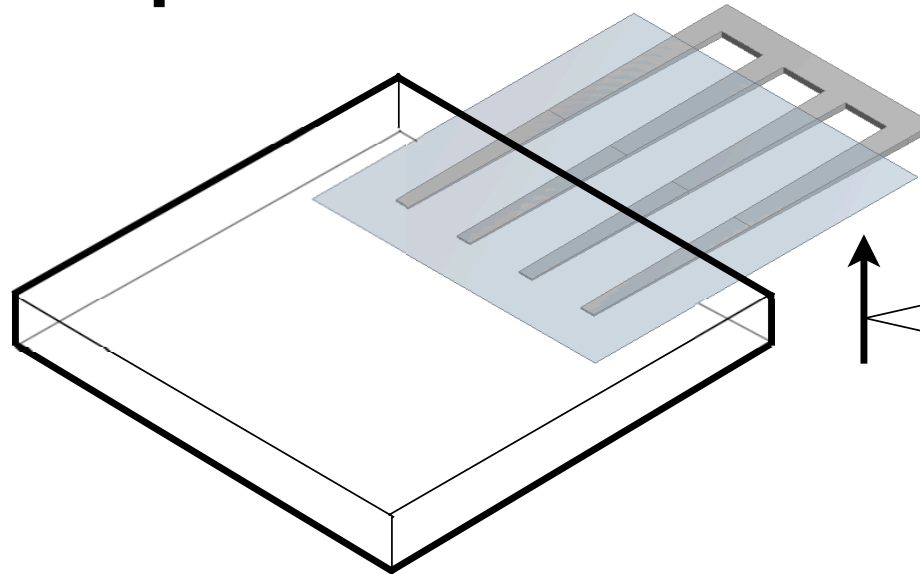
Similar to **Transient Structural**, **Explicit Dynamics** also solves the general form of equation. External force $\{F\}$ can be time-dependent forces. All nonlinearities can be included. It also uses a direct integration method to calculate the dynamic response.

The direct integration method used in **Explicit Dynamic** analysis is called an **explicit integration** method.

Section 12.2

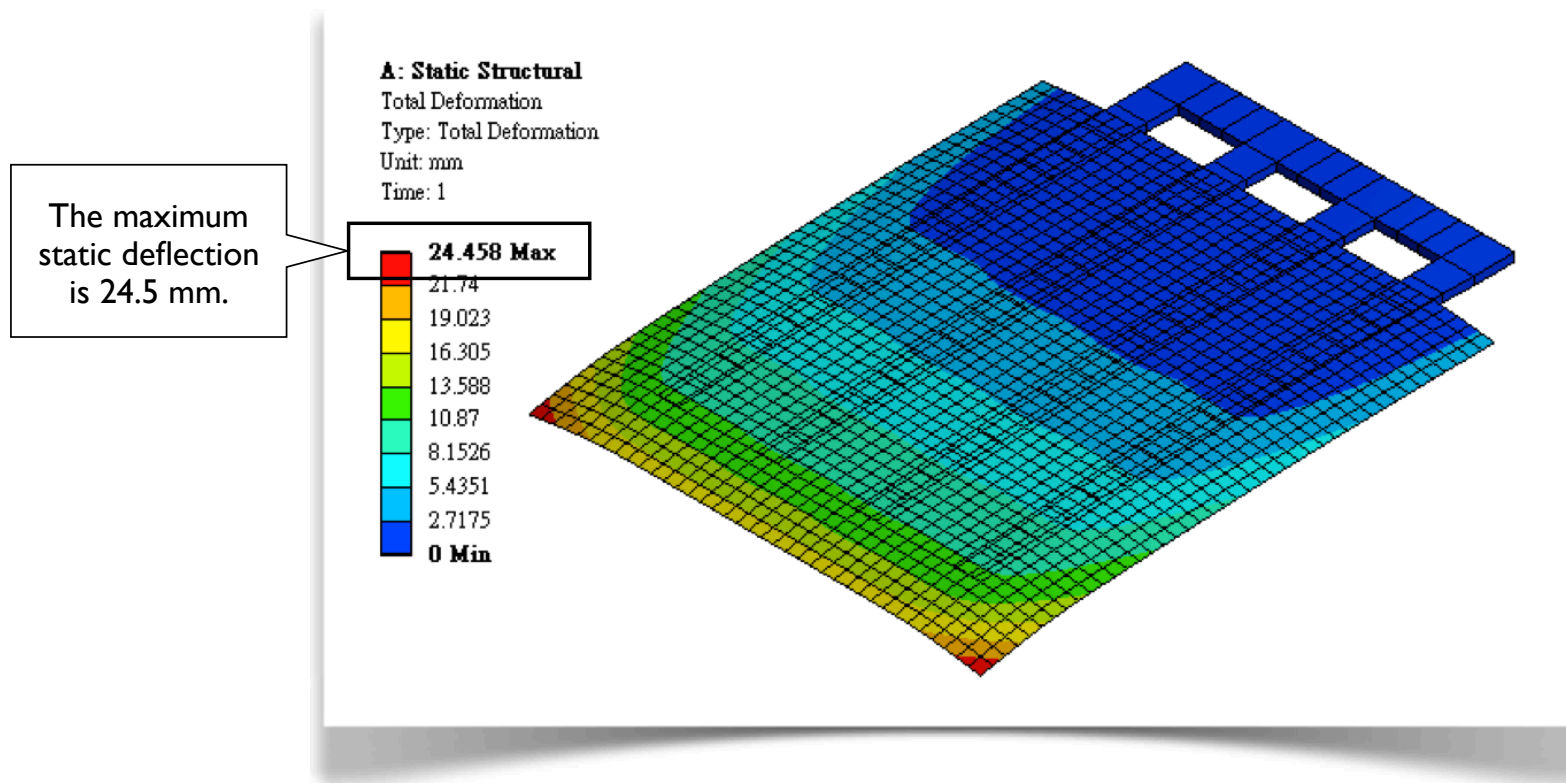
Lifting Fork

Problem Description

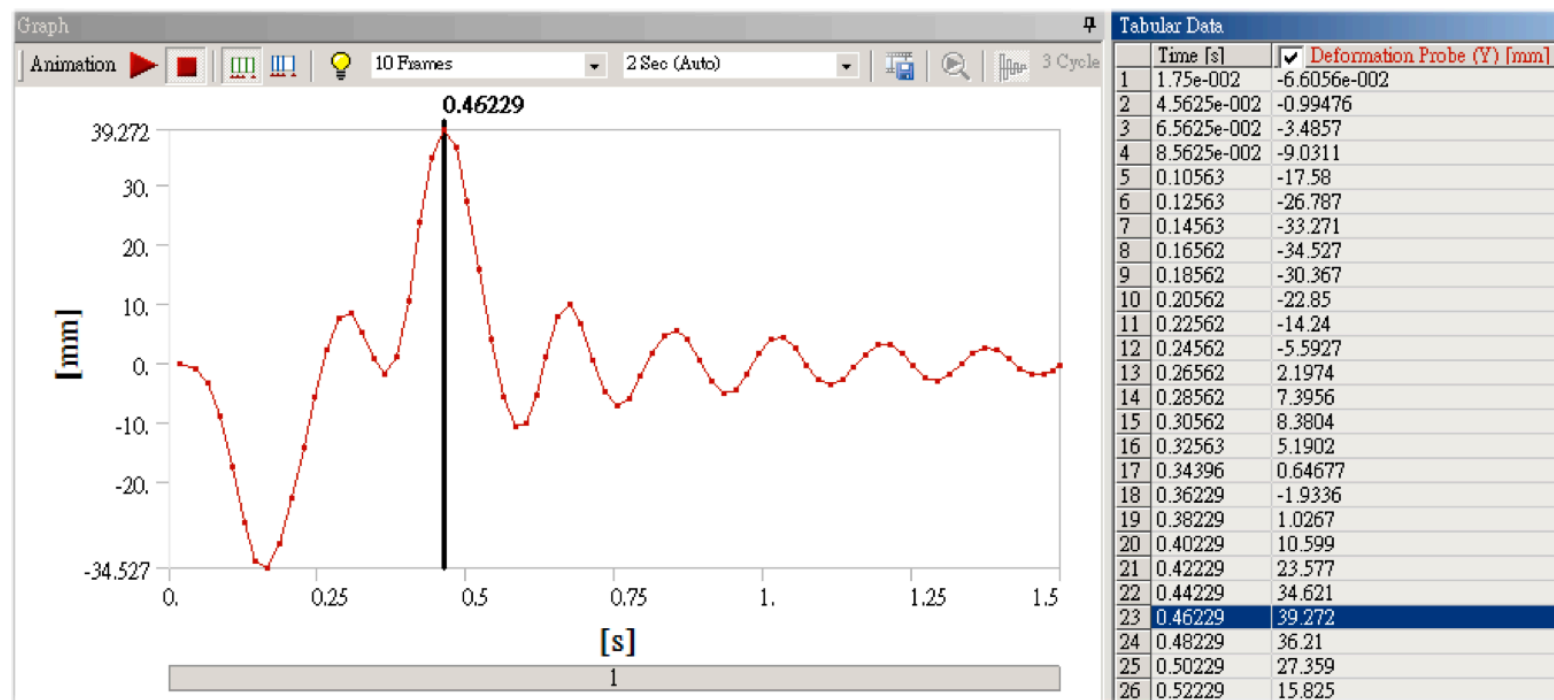


During the handling, the fork accelerates upward to a velocity of 1.5 m/s in 0.3 second, and then decelerates to a full stop in another 0.3 second, causing the glass panel to vibrate.

Static Structural Simulation



Transient Structural Simulation



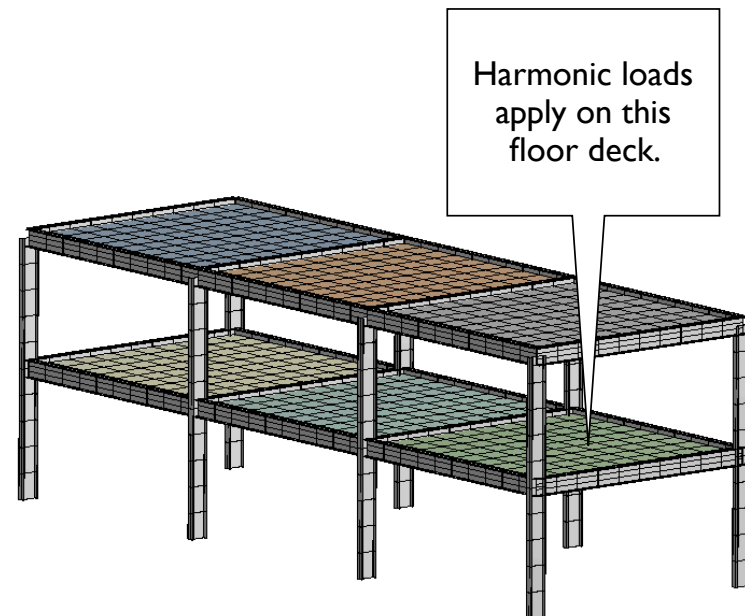
Section 12.3

Two-Story Building

Problem Description

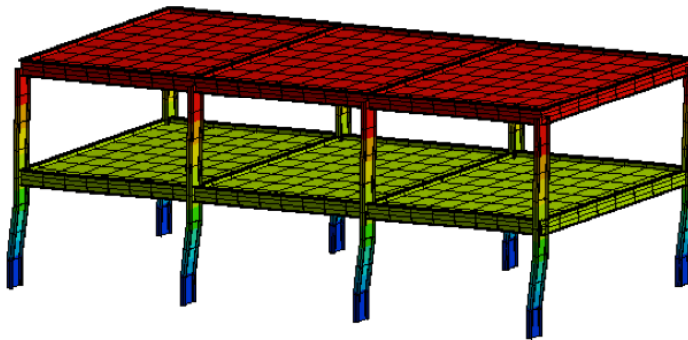
Two scenarios are investigated:

- Harmonic load of magnitude of 10 psf due to the dancing on the floor.
- Harmonic load of magnitude of 0.1 psf due to rotations of a machine.

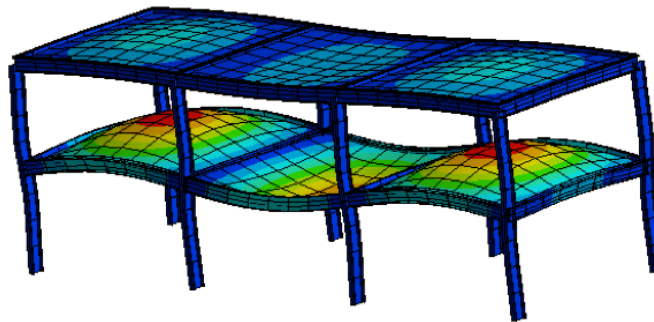


Modal Analysis

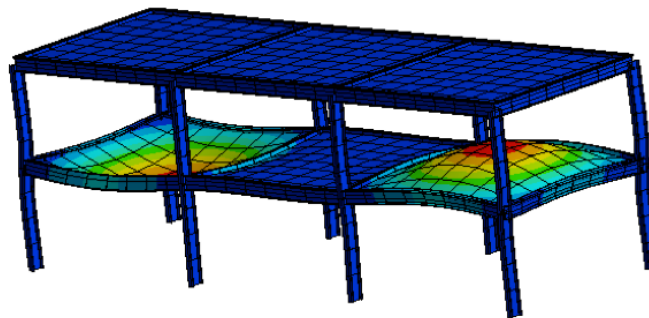
Tabular Data		
	Mode	☒ Frequency [Hz]
1	1.	1.5188
2	2.	4.0914
3	3.	4.1122
4	4.	4.8765
5	5.	9.3552
6	6.	9.5907
7	7.	9.9077
8	8.	10.333
9	9.	10.721
10	10.	11.499
11	11.	13.632
12	12.	16.308
13	13.	17.538
14	14.	18.227
15	15.	19.299
16	16.	20.091
17	17.	20.828
18	18.	21.181
19	19.	21.654
20	20.	21.934



[1] The first mode (1.52 Hz).



[2] The sixth mode (9.59 Hz).

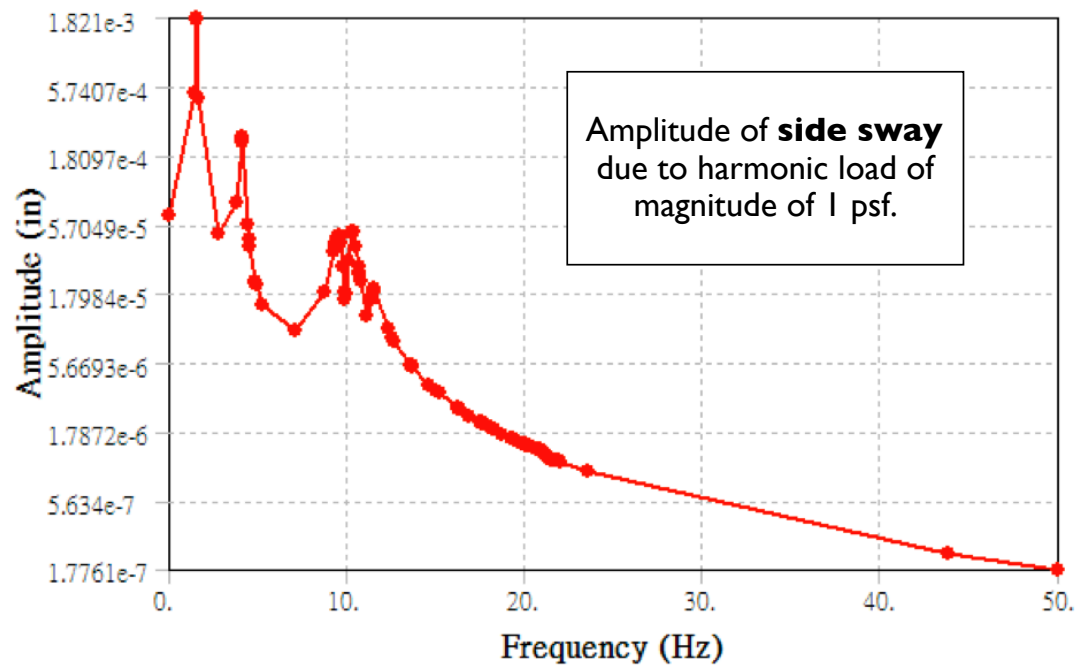


[3] The eighth mode (10.3 Hz)

- The dancing frequency is close to the fundamental mode (1.55 Hz), that's why we pay attention to this mode, which is a side sway mode (in X-direction).
- For the rotatory machine, we are concerned about the floor vibrations in vertical direction. That's why we pay attention at the sixth and eighth modes.

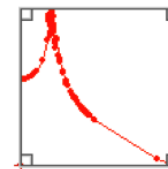
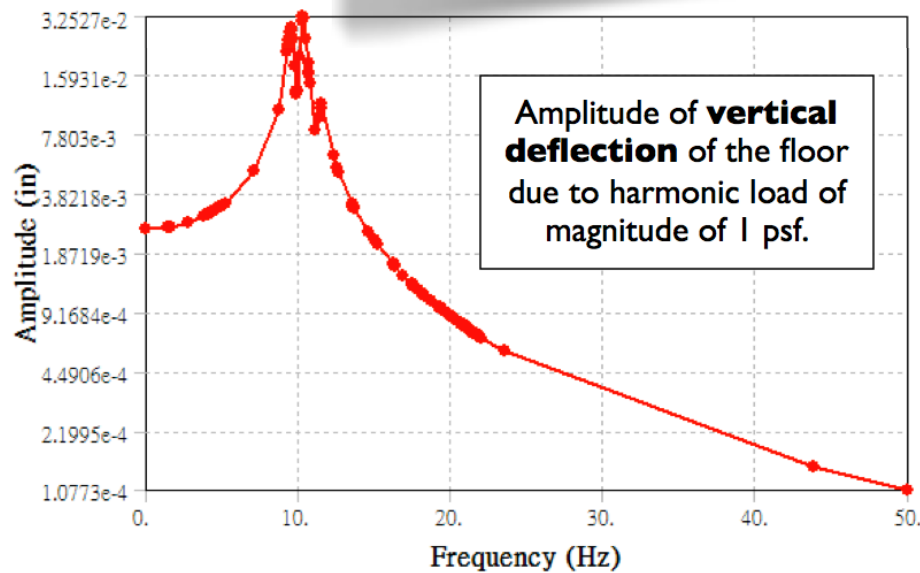
Side Sway Due to Dancing

At dancing frequency of 1.52 Hz, the structure is excited such that the maximum X-displacement is 0.0182 in (0.46 mm). This value is too small to be worried about.



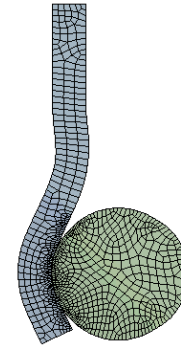
Vertical Deflection of the Floor Due to Rotatory Machine

Although high frequencies do excite the floor, but the values are very small. At frequency of 10.3 Hz, the excitation reaches a maximum of 0.0033 in (0.1 times of 0.033 in), or 0.084 mm. The value is too small to cause an issue.

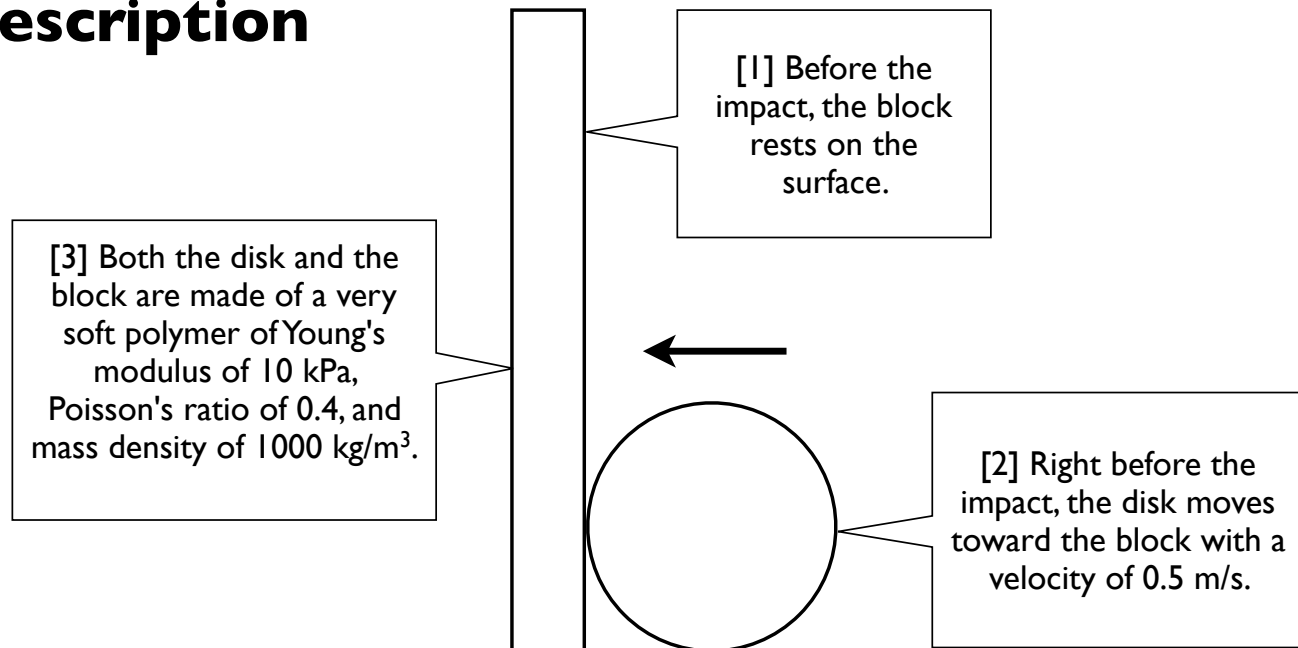


Section 12.4

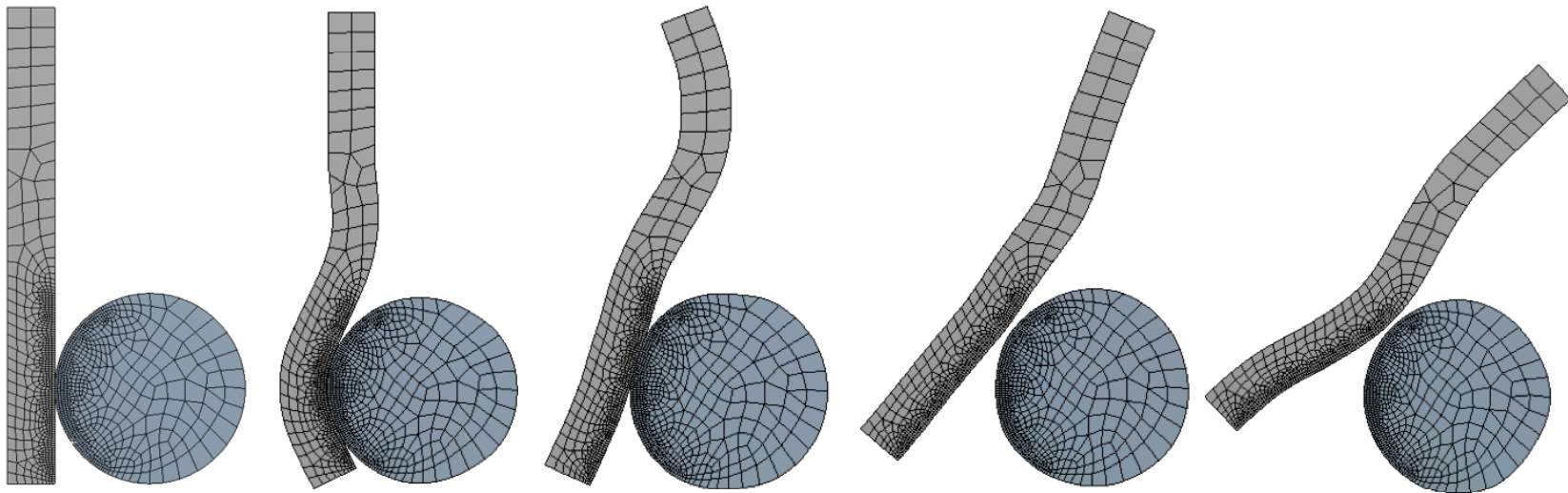
Disk and Block



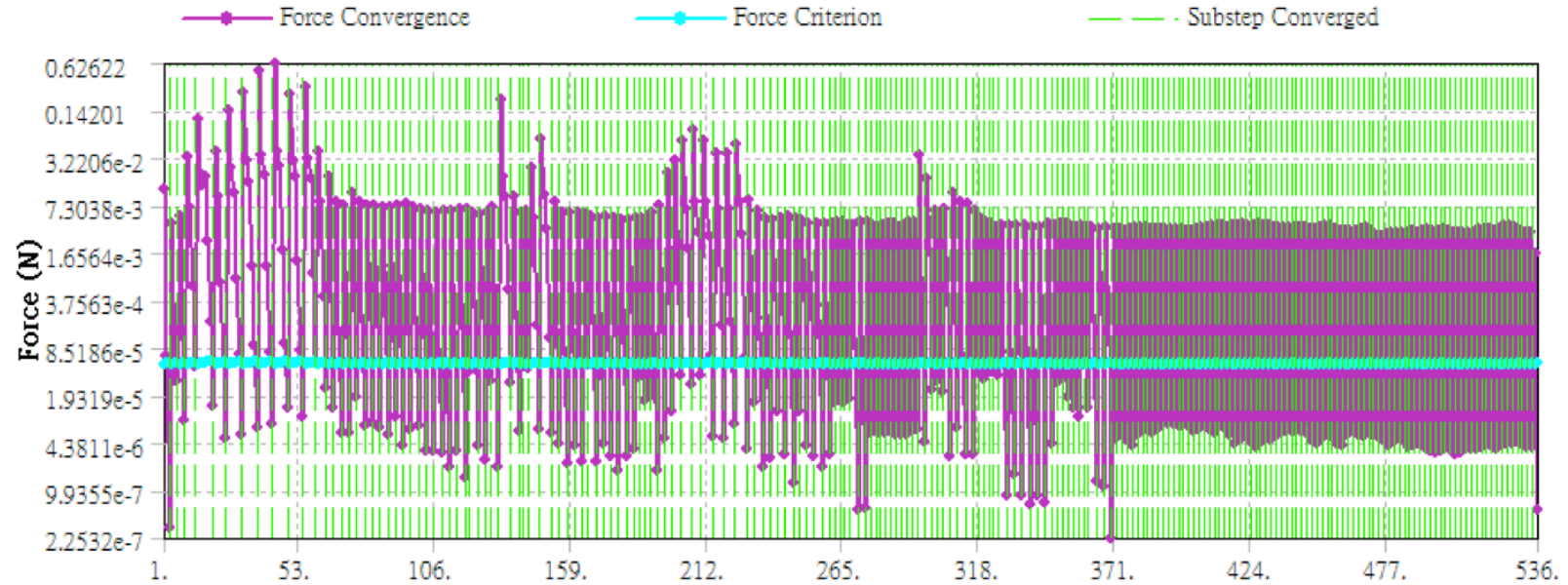
Problem Description



Results



Solution Behavior



Section 12.5

Guitar String

The main purpose of this exercise is to demonstrate how to use the results of a static simulation as the initial condition of a transient dynamic simulation

