



Standard Practice for Estimation of Heat Gain or Loss Through Ceilings Under Attics Containing Radiant Barriers by Use of a Computer Program¹

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1. Scope

1.1 This practice covers the estimation of heat gain or loss through ceilings under attics containing radiant barriers by use of a computer program. The computer program included as an adjunct to this practice provides a calculational procedure for estimating the heat loss or gain through the ceiling under an attic containing a truss or rafter mounted radiant barrier. The program also is applicable to the estimation of heat loss or gain through ceilings under an attic without a radiant barrier. This procedure utilizes hour-by-hour weather data to estimate the hour-by-hour ceiling heat flows. The interior of the house below the ceiling is assumed to be maintained at a constant temperature. At present, the procedure is applicable to sloped-roof attics with rectangular floor plans having an unshaded gabled roof and a horizontal ceiling. It is not applicable to structures with flat roofs, vaulted ceilings, or cathedral ceilings. The calculational accuracy also is limited by the quality of physical property data for the construction materials, principally the insulation and the radiant barrier, and by the quality of the weather data.

1.2 Under some circumstances, interactions between radiant barriers and HVAC ducts in attics can have a significant effect on the thermal performance of a building. Ducts are included in an extension of the computer model given in the appendix.

1.3 The values stated in either SI units or inch-pound units are to be regarded separately as standard. The values stated in each system may not be exact equivalents; therefore, each system shall be used independently of the other. Combining values from the two systems may result in non-conformance with the standard.

¹ This practice is under the jurisdiction of ASTM Committee C16 on Thermal Insulation and is the direct responsibility of Subcommittee C16.21 on Reflective Insulation.

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2. Referenced Documents

2.1 *ASTM Standards*:²

C168 Terminology Relating to Thermal Insulation

2.2 *ANSI Standards*:

X3.5 Flow Chart Symbols and Their Usage in Information Processing³

X3.9 Standard for Fortran Programming Language³

2.3 *ASTM Adjuncts*:

Computer Program for Estimation of Heat Gain or Loss through Ceilings Under Attics Containing Radiant Barriers⁴

3. Terminology

3.1 *Definitions*—For definitions of terms used in this practice, refer to Terminology C168.

3.2 *Symbols*—Symbols will be introduced and defined in the detailed description of the development.

4. Summary of Practice

4.1 The procedures used in this practice are based on the thermal response factor method for calculating dynamic heat conduction through multilayer slabs (1, 2),⁵ along with a model for convective and radiative heat exchanges inside and outside the attic.

4.2 The operation of the computer program involves the following steps:

² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ Available from American National Standards Institute (ANSI), 25 W. 43rd St., 4th Floor, New York, NY 10036, <http://www.ansi.org>.

⁴ Available from ASTM International Headquarters. Order Adjunct No. ADJC1340.

⁵ The boldface numbers in parentheses refer to the list of references at the end of this standard.

4.2.1 Response Factors—A separate computer program must be used to calculate the thermal response factors of the solid materials surrounding the attic. Input to this program would consist of the thermal conductivity, specific heat, density, and thickness of each layer, or the thermal resistance of the layer if it has negligible density, and the fraction of the cross-sectional area occupied by the framing. Output of such a program would be a set of response factors for use as input to the main program. The adjunct to this practice contains data files with response factors for several typical attic constructions.

4.2.2 Data Input to the Main Program—This input includes the response factors, total hemispherical emittances of the inside and outside surfaces of the attic envelope, solar absorptances of the outside surfaces of the attic envelope, length and width of the attic, slopes of the two roof sections, distance between attic floor and roof at edge of attic, orientation of house, vent areas and type of vents, water vapor permeances of attic surfaces, area of exposed wood inside attic, mass of wood in attic, initial moisture content of wood in attic, rate of exfiltration of air from house into attic space, latitude and longitude, time zone indicator, solar reflectance of the ground, indoor temperature, and indoor humidity.

4.2.3 Analysis—Using hourly weather data consisting of outdoor temperature and humidity ratio, atmospheric pressure, total horizontal and direct solar radiation, wind speed and direction, cloud amount, cloud type, and atmospheric clearness number, the computer program calculates the inside and outside temperatures of the attic envelope and the temperature of the air inside the attic. Using these temperatures, the program calculates the heat flux through the ceiling.

4.2.4 Output—The hourly heat flux through the ceiling is written to a file which can be used for further processing, such as seasonal or annual heat gains or losses.

5. Significance and Use

5.1 Manufacturers of radiant barriers express the performance of their products in terms of the total hemispherical emittance. The purpose of a radiant barrier is to decrease the radiation heat transfer across the attic air space, and hence, to decrease the heat loss or gain through the ceiling below the attic. The amount of decrease in heat flow will depend upon a number of factors, such as weather conditions, amount of mass or reflective insulation in the attic, solar absorptance of the roof, geometry of the attic and roof, and amount and type of attic ventilation. Because of the infinite combinations of these factors, it is not practical to publish data for each possible case.

5.2 The calculation of heat loss or gain of a system containing radiant barriers is mathematically complex, and because of the iterative nature of the method, it is best handled by computers.

5.3 Computers are now widely available to most producers and consumers of radiant barriers to permit the use of this practice.

5.4 The user of this practice may wish to modify the data input to represent accurately the structure. The computer program also may be modified to meet individual needs. Also,

additional calculations may be desired, for example, to sum the hourly heat flows in some fashion to obtain estimates of seasonal or annual energy usages. This might be done using the hourly data as inputs to a whole-house model, and by choosing house balance points to use as cutoff points in the summations.

6. Method of Calculation

6.1 Approach:

6.1.1 This calculation of heat loss or gain requires that the following be known:

6.1.1.1 The thermal conductivity, specific heat, and density of the construction materials (that is, insulation, plywood, roofing materials, sheathing, gypsum board);

6.1.1.2 The total hemispherical emittance of all materials facing the attic air space;

6.1.1.3 The solar absorptance of the exterior surfaces of the attic (that is, the roof and gables);

6.1.1.4 The geometry of the attic;

6.1.1.5 The moisture permeance and storage properties of the materials facing the attic space; and

6.1.1.6 The weather conditions.

6.1.2 The solution is a computer procedure that estimates temperatures of both sides of the components of the attic envelope and the temperature of the air in the attic space, uses these estimates of temperatures to refine estimates of convection and radiation heat transfer coefficients, reestimates the temperatures using the new heat transfer coefficients, continues iterating on the temperatures and heat transfer coefficients until convergence is reached, and uses the last estimates of temperatures to calculate the heat gain or loss through the ceiling. This procedure is repeated for each hour of the simulation period (typically a full year).

6.2 Development of Equations—The model that is the basis for this practice is based on the model developed by B. Peavy (3), which was later extended by Wilkes (4-6). The sketch of an attic given in Fig. 1 shows the various heat transfer mechanisms that occur within an attic. Although the sketch shows

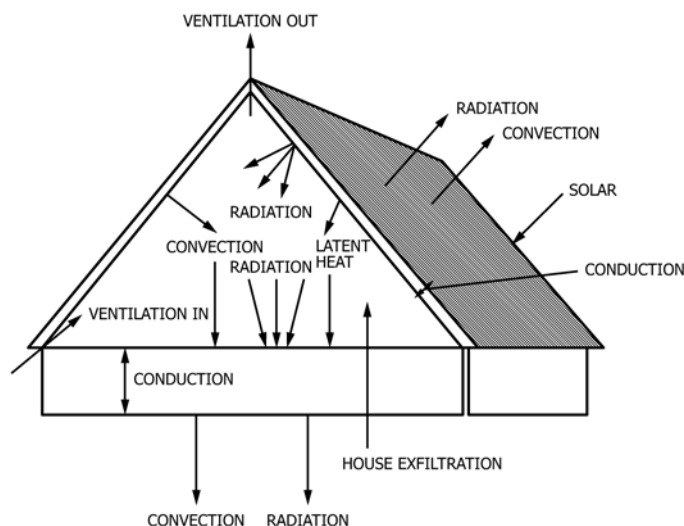


FIG. 1 Schematic of Residential Attic Showing Heat Transfer Phenomena

ventilation occurring at soffit and ridge vents, the location of the vents may be at other locations, such as at the gables. The model treats all of these phenomena through a system of heat balance equations at the interior and exterior surfaces of the ceiling, roof sections, and gables, as well as a heat balance on the air mass within the attic. To handle the case of raised trusses, short vertical walls at the eaves also are included. Each of the surfaces is assumed to be isothermal; thus, for an attic consisting of a ceiling, two roof sections, two gables, two vertical eave sections, and one air space, a total of 15 heat balance equations is used.

6.3 Equations—Conduction:

6.3.1 The model developed here utilizes the thermal response factor method to analyze conduction through building envelope sections. The thermal response factor method was developed by Mitalas and Arseneault (1) and was extended by Kusuda (2). The method is based on an exact analytical solution of the heat conduction equation for one-dimensional heat flow through a multilayer slab having temperature-independent thermal properties. The only approximation is that the surface temperatures are taken to vary linearly with time between time steps. For analysis of buildings, the time step is normally taken to be 1 h. The response factor equations relate the heat fluxes at the surfaces of the slab to the present and previous temperatures at the two surfaces. The equations are:

$$QI = \sum_{j=0}^{\infty} Z'(j) (TIS(j) - TR) - \sum_{j=0}^{\infty} Y'(j) (TOS(j) - TR) \quad (1)$$

$$QO = \sum_{j=0}^{\infty} Y'(j) (TIS(j) - TR) - \sum_{j=0}^{\infty} X'(j) (TOS(j) - TR) \quad (2)$$

where:

QI	= heat flux at inside surface at present time (note that the positive heat flow direction is from the inside to the outside), W/m^2 [Btu/h-ft ²],
QO	= heat flux at outside surface at present time, W/m^2 [Btu/h-ft ²],
$TIS(j)$	= temperature at inside surface j hours previous to present time, °C [°F],
$TOS(j)$	= temperature at outside surface j hours previous to present time, °C [°F]
$X'(j), Y'(j), Z'(j)$	= response factors, $W/m^2 \cdot K$ [Btu/h-ft ² ·°F], and
TR	= reference temperature, °C [°F].

6.3.2 The response factors are determined from a sequence of calculations that involve the thermal diffusivity, thermal conductivity, specific heat, density, and thickness of each of the layers in the multilayer slab. An efficient computer program for calculating the response factors has been developed by George Walton of the National Institute of Standards and Technology (NIST) (7).

6.3.2.1 The efficiency of the response factor calculations can be increased by making use of the fact that after a sufficient number of terms, the ratio of two consecutive response factors becomes constant. This is expressed by:

$$\frac{X'(j+1)}{X'(j)} = \frac{Y'(j+1)}{Y'(j)} = \frac{Z'(j+1)}{Z'(j)} = CR \text{ for } j \geq N \quad (3)$$

CR = the common ratio, and
 N = a sufficiently large number.

6.3.2.2 The common ratio is used to define a new set of functions, called the first order conduction transfer functions or simply the conduction transfer functions, $X(j)$, $Y(j)$, and $Z(j)$, which are given by:

$$X(0) = X'(0) \quad (4)$$

$$Y(0) = Y'(0) \quad (5)$$

$$Z(0) = Z'(0) \quad (6)$$

$$X(j) = X'(j) - CR X'(j-1) \text{ for } j \leq N \quad (7)$$

$$Y(j) = Y'(j) - CR Y'(j-1) \text{ for } j \leq N \quad (8)$$

$$Z(j) = Z'(j) - CR Z'(j-1) \text{ for } j \leq N \quad (9)$$

$$X(j) = 0 \text{ for } j > N \quad (10)$$

$$Y(j) = 0 \text{ for } j > N \quad (11)$$

$$Z(j) = 0 \text{ for } j > N \quad (12)$$

6.3.2.3 With the conduction transfer functions, the heat fluxes and surface temperatures are related by:

$$QI = \sum_{j=0}^N Z(j) (TIS(j) - TR) \quad (13)$$

$$- \sum_{j=0}^N Y(j) (TOS(j) - TR) + CR QI'$$

$$QO = \sum_{j=0}^N Y(j) (TIS(j) - TR) \quad (14)$$

$$- \sum_{j=0}^N X(j) (TOS(j) - TR) + CR QO'$$

where:

QI' = heat flux at inside surface at previous time step,
 QO' = heat flux at outside surface at previous time step, and
 N = number of significant conduction transfer functions.

6.3.2.4 When parallel heat flow paths occur in an envelope component, separate response factors for each path may be needed. If the boundary temperatures of the two paths may be assumed to be equal, however, then the response factors may be added together as:

$$X' = A_1 X'_1 + A_2 X'_2 \quad (15)$$

where:

A_1, A_2 = area fractions for paths 1 and 2, and
 $(X'_1, X'_2), (Y'_1, Y'_2) \text{ and } (Z'_1, Z'_2)$ = the response factors for paths 1 and 2.

Parallel conduction transfer functions may be calculated from these parallel response factors, provided that the common ratio for the path with the largest number of significant terms is used.

6.3.2.5 The original derivation of the response factor technique relied upon the assumption of temperature-independent

thermal properties. An approximate method has been developed to account for the temperature dependence of the thermal properties (5). The thermal transmission coefficient of the component is taken to vary linearly with temperature as:

$$U = U_{TR} [1 + b(T - TR)] \quad (16)$$

The conduction transfer function equations then become:

$$QI = \sum_{j=0}^N Z(j) (TIS(j) - TR) \quad (17)$$

$$\begin{aligned} & - \sum_{j=0}^N Y(j) (TOS(j) - TR) + CR QI \\ & + b/2 \sum_{j=0}^N Z(j) (TIS(j) - TR)^2 - b/2 \sum_{j=0}^N Y(j) (TOS(j) - TR)^2 \\ QO = & \sum_{j=0}^N Y(j) (TIS(j) - TR) \quad (18) \\ & - \sum_{j=0}^N X(j) (TOS(j) - TR) + CR QO \\ & + b/2 \sum_{j=0}^N Y(j) (TIS(j) - TR)^2 - b/2 \sum_{j=0}^N X(j) (TOS(j) - TR)^2 \end{aligned}$$

These equations are used in the system of heat balance equations.

6.4 Equations—Convection:

6.4.1 Convection heat transfer from the interior and exterior surfaces of the envelope components is calculated using correlations from the literature (8). The coefficients are based on correlations that have been developed for isolated isothermal flat plates. The correlations are in the form of a Nusselt number, Nu , as a function of a Rayleigh number, Ra , Grashof number, Gr , or a Reynolds number, Re , where:

$$Nu = hL/k \quad (19)$$

$$Ra = \frac{g\beta p C_p \Delta T L^3}{\nu k} \quad (20)$$

$$Gr = Ra/Pr \quad (21)$$

$$Pr = \nu/\alpha \quad (22)$$

$$Re = VL/\nu \quad (23)$$

and:

- h = convection heat transfer coefficient, $W/m^2 \cdot K$ [$Btu/h \cdot ft^2 \cdot ^\circ F$],
- L = characteristic length of plate, m [ft],
- k = thermal conductivity of air, $W/m \cdot K$ [$Btu/h \cdot ft \cdot ^\circ F$],
- g = acceleration of gravity, m/s^2 [ft/h^2],
- β = volume coefficient of expansion of air, K^{-1} [$^{\circ}R^{-1}$],
- ρ = density of air, kg/m^3 [lb/ft^3],
- C_p = specific heat of air, $J/kg \cdot K$ [$Btu/lb \cdot ^\circ F$],
- ΔT = temperature difference between surface and air, K [$^{\circ}F$],
- ν = kinematic viscosity of air, m^2/s [ft^2/h],
- Pr = Prandtl number for air, dimensionless,
- α = thermal diffusivity of air, m^2/s [ft^2/h], and
- V = velocity of air stream, m/s (ft/h).

6.4.2 The model accounts for the temperature dependence of the properties of air by evaluating them at the film

temperature, which is defined as the average of the temperatures of the surface and the air. Relationships for the temperature dependent properties were obtained from NBS Circular 564 (9).

6.4.2.1 The model utilizes correlations that have been developed for various orientations of the plate with respect to gravity and for the direction of heat flow (up versus down). Correlations for both laminar and turbulent flow are used, with the choice depending upon the magnitude of the Rayleigh number for natural convection and of the Reynolds number for forced convection. Separate coefficients are calculated for natural and forced flow, and a mixed coefficient is calculated by taking the third root of the sum of the cubes of the two separate coefficients (10).

6.4.2.2 The correlations used in the model are given in Table 1. They account for the effects of surface-to-air temperature difference, heat flow direction, film temperature, surface size, and surface orientation. The correlations are contained in a subroutine called HCON. Values of surface temperature, air temperature, plate tilt angle, plate characteristic length, a flag to denote whether the plate faces up or down, and air speed past the plate are passed to the subroutine. The subroutine returns the mixed convection coefficient. For exterior surfaces, the air speed is taken to be the wind speed obtained from meteorological data. For interior surfaces, a crude estimate of air speed is obtained by dividing the attic ventilation volume flow rate by an average cross-sectional area for the attic. For the interior surfaces, a crude estimate for air speed should suffice, since natural convection should dominate over forced convection.

6.5 Equations—Radiation:

6.5.1 Radiation interchanges within the attic space are handled using the enclosure method described by Sparrow and Cess (11). With this method, each of the surfaces is assumed to be plane, opaque, gray, and isothermal, to be diffusely emitting and reflecting, and to have a uniform radiant flux over the surface. The assumption of diffuse emission and reflection for radiant barrier surfaces may be questioned. Very flat radiant

TABLE 1 Correlations for Convection Coefficients

I. Natural Convection:

A. Horizontal surface, heat flow up

$$Nu = 0.54 Ra^{1/4} \text{ for } Ra < 8 \times 10^6$$

$$Nu = 0.15 Ra^{1/3} \text{ for } Ra > 8 \times 10^6$$

B. Horizontal surface, heat flow down

$$Nu = 0.58 Ra^{0.2}$$

C. Vertical surface

$$Nu = 0.59 Ra^{1/4} \text{ for } Ra < 1 \times 10^9$$

$$Nu = 0.10 Ra^{1/3} \text{ for } Ra > 1 \times 10^9$$

D. Nearly horizontal surface (tilt angle less than 2°), heat flow down

$$Nu = 0.58 Ra^{0.2}$$

E. Tilted surfaces (greater than 2° tilt), heat flow down

$$Nu = 0.56 (Ra \sin(\Phi))^{1/4} \quad \Phi = \text{tilt angle}$$

F. Tilted surface, heat flow up

$$Nu = 0.56 (Ra \sin(\Phi))^{1/4} \text{ for } Ra/Pr < Gr_c$$

$$Nu = 0.14 (Ra^{1/3} - (Gr_c Pr)^{1/3})$$

$$+ 0.56 (Gr_c Pr \sin(\Phi))^{1/4} \text{ for } Ra/Pr > Gr_c$$

$$Gr_c = 1 \times 10^6 \text{ for } \Phi < 15^\circ$$

$$Gr_c = 10^{(0.1/(1.1870 + 0.0870 \times \Phi))} \text{ for } 15^\circ < \Phi < 75^\circ$$

$$Gr_c = 5 \times 10^9 \text{ for } \Phi > 75^\circ$$

II. Forced Convection:

$$Nu = 0.664 Pr^{1/3} Re^{1/2} \text{ for } Re < 5 \times 10^5$$

$$Nu = Pr^{1/3} (0.037 Re^{0.8} - 850) \text{ for } Re > 5 \times 10^5$$

barrier surfaces would be expected to exhibit specular properties. Real radiant barrier surfaces usually are not flat, and hence the diffuse assumption is probably appropriate.

6.5.1.1 The net radiant heat flux away from surface i is given by:

$$Q_{ri} = \sum_{j=1}^N G_{ij} \sigma (T_i^4 - T_j^4) \quad (24)$$

with the following relations:

$$G_{ij} = \frac{\varepsilon_i}{1 - \varepsilon_i} \Psi_{ij} \quad (25)$$

Ψ_{ij} = inverse of the matrix χ_{ij}

$$\chi_{ij} = \frac{\delta_{ij} - (1 - \varepsilon_i) F_{ij}}{\varepsilon_i}$$

$$\delta_{ij} = 0 \text{ for } i \neq j; \delta_{ij} = 1 \text{ for } i = j$$

where:

F_{ij} = radiation view factor from surface i to surface j ,
 ε_i = emittance of surface i ,
 σ = Stefan-Boltzmann constant, $5.6696 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ [$1.714 \times 10^{-9} \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4$], and
 N = number of surfaces in enclosure.

6.5.1.2 The heat flux equation may be cast into a form that is linear in the temperatures by factoring as:

$$Q_{ri} = \sum_{j=1}^N G_{ij} \sigma (T_i^2 + T_j^2)(T_i + T_j)(T_i - T_j) \quad (26)$$

$$= \sum_{j=1}^N HR_{ij} (T_i - T_j)$$

6.5.1.3 The quantities appearing in these equations are calculated in several subprograms: subroutine VIEW2, and functions FMN, FP, and HRAD. Function FMN calculates the view factor between two rectangular surfaces that meet at an angle and share a common edge, while function FP calculates the view factor between two parallel surfaces that have the same dimensions. Expressions for the two view factors are given by Sparrow and Cess (11). The functions themselves are based on those given by Peavy, with some necessary modifications (3). The functions have been verified by comparison with the calculations of Feingold (12). Subroutine VIEW2 calculates the view factors among the surfaces that face the attic space, using view factor algebra and calls to functions FMN and FP. The G_{ij} matrix is also calculated within VIEW2. The HR_{ij} matrix is calculated by function HRAD using G_{ij} , T_i , and T_j as inputs.

6.5.1.4 The algorithm for radiation interchanges allows each of the surfaces to have different emittances. The algorithm accounts for all interreflections within the enclosure and properly accounts for the T^4 (Stefan-Boltzmann) law for radiation exchange.

6.6 Equations—Ventilation:

6.6.1 Heat transfer to the ventilation air is treated by an extension of the method used by Peavy (3). With this method, the temperature of the air is assumed to increase (or decrease) as it moves along a flow path and picks up heat by convection

from the surrounding surfaces. This is expressed by the following first order ordinary differential equation:

$$\dot{m} C_p \frac{dT}{dx} = \sum_{j=1}^M A(j) HC(j) (TIS(j) - T) \quad (27)$$

$\dot{m}$ = mass flow rate of ventilation air, kg/s [lb/h],
 C_p = specific heat of air, J/kg · K [Btu/lb · °F],
 T = air temperature at position, °C [°F],
 x = normalized position (0–1) along flow path,
 $A(j)$ = area of surface j in contact with ventilation air, m² [ft²],
 $HC(j)$ = convection heat transfer coefficient at surface j , W/m² · K [Btu/h · ft² · °F],
 $TIS(j)$ = temperature of surface j , °C [°F], and,
 M = number of surfaces in contact with ventilation air.

6.6.2 In the absence of a detailed picture of the ventilation air flow pattern within the attic, the simplifying assumption has been made that equal proportions of area of each surface are contacted as the air flows along a differential length of its flow path. Integrating this equation yields the following expression for the temperature as a function of flow path position:

$$T = T_o \exp(-ax) + b[1 - \exp(-ax)] \quad (28)$$

where:

T_o = temperature at $x = 0$,
 a = $\sum_{j=1}^M A(j) HC(j) / \dot{m} C_p$, and

$$b = \frac{\sum_{j=1}^M A(j) HC(j) TIS(j)}{\sum_{j=1}^M A(j) HC(j)}$$

6.6.3 An average air temperature is defined by:

$$TA = \int_0^1 T dx = b[1 + (\exp(-a) - 1)/a] - T_o/a[\exp(-a) - 1] \quad (29)$$

and the temperature of the air exiting from the attic at $x = 1$ is:

$$TE = b[1 - \exp(-a)] + T_o \exp(-a) \quad (30)$$

6.6.4 The flow rate of ventilation air is determined from a combination of stack and wind pressure effects. For the stack effect calculation, the air inside the attic space is assumed to be well-mixed, and relations from Chapter 22 of the 1985 ASHRAE Handbook of Fundamentals are used (13). The volume flow rate due to the stack effect is given by:

$$\dot{V}_s = C_D A [2 gh (TA - T_o) / TA]^{1/2} \text{ for } TA > T_o \quad (31)$$

$$\dot{V}_s = C_D A [2 gh (T_o - TA) / T_o]^{1/2} \text{ for } T_o > TA \quad (32)$$

where:

$\dot{V}_s$ = volume flow rate due to stack effect, m³/s [ft³/h],
 C_D = discharge coefficient taken to be 0.65,
 A = the lesser of the net free areas of the vent inlet and outlet, m² [ft²],
 h = height of the neutral pressure level from the lower opening, m [ft],

T_o = absolute temperature of inlet air, K [°R], and
 TA = average absolute temperature of air in attic, K [°R].

6.6.5 The height of the neutral pressure level is given by:

$$h = \frac{H}{1 + \left(\frac{AI}{AO}\right)^2 \frac{TA}{T_o}} \text{ for } TA > T_o \quad (33)$$

$$h = \frac{H}{1 + \left(\frac{AI}{AO}\right)^2 \frac{T_o}{TA}} \text{ for } T_o > TA \quad (34)$$

where:

H = difference in elevation between inlet and outlet vents, m [ft],
 AI = net free area of inlet vents, m^2 [ft²], and
 AO = net free area of outlet vents, m^2 [ft²].

6.6.6 The volume flow rate due to wind pressure is given by:

$$\dot{V}_w = C_F A WS \quad (35)$$

where:

C_F = discharge coefficient,
 A = the lesser of the net free areas of the vent inlet and outlet, m^2 [ft²], and
 WS = wind speed, m/s [ft/h].

6.6.7 From the data given by Burch and Treado (14), the discharge coefficients for the wind pressure effect are estimated to be 0.38 for soffit and ridge vents and 0.54 for soffit and gable vents. For soffit vents only, C_F is estimated from

$$C_F = 0.089 + 0.132 \sin^2 D \quad (36)$$

where:

D = wind direction, measured from a direction parallel to the ridge, radians.

6.6.8 Mass flow rates for the stack and wind pressure effects are obtained by multiplying the volume flow rates by the density of the air. A total mass flow rate is given by:

$$\dot{m} = (\dot{m}_s^2 + \dot{m}_w^2)^{1/2} \quad (37)$$

6.6.9 Finally, the adjustment recommended by ASHRAE is made to account for unequal inlet and outlet vent areas:

$$\dot{m} = [1 + 0.4077 (1 - (A_{\min}/A_{\max})^{1.5})] (\dot{m}_s^2 + \dot{m}_w^2)^{1/2} \quad (38)$$

where the quantity in parentheses is a fit to the graph in the 1985 ASHRAE Handbook of Fundamentals (13). The air flow due to exfiltration of air from the house into the attic space is added to the flow due to the stack and wind pressure effects. The algorithm for estimating ventilation rates is embodied in a subroutine called VENT.

6.6.10 Although some other results in the literature are in qualitative agreement (15,16), there appears to be a good deal of uncertainty in this ventilation algorithm. D. M. Burch, in a private conversation with K. E. Wilkes in 1987, has suggested that the ventilation rates he and Treado measured may be in error because of problems of mixing tracer gases in an attic. D. Ober, in a private conversation with K. E. Wilkes in 1988, has suggested that a better stack ventilation rate might be obtained by using the roof temperature rather than the average air temperature as the driving force during the daytime hours.

Additional experimental data appear to be needed to develop better algorithms for ventilation.

6.7 Equations—Moisture:

6.7.1 Approximations have been built into the models to account for the latent heat effects due to sorption and desorption of moisture at the wood surfaces that face the attic space. These generally follow the suggestions given by Burch et al (17) and Cleary (18). In this model, the wood surface is assumed to be in moisture equilibrium with a thin layer of air adjacent to the surface. The humidity ratio of this layer of air is given by (18).

$$\omega_s = (b + c \cdot u + d \cdot u^2 + e \cdot u^3) \exp(T/a) \quad (39)$$

where:

ω_s = humidity ratio of air near wood surface, kg of water/kg of dry air [lb of water/lb of dry air],
 T = temperature of wood surface, °C [°F],
 u = moisture content of wood (dimensionless fraction),
 a, b, c, d = constants from fit to equilibrium moisture content data.

6.7.2 The rate of transfer of moisture from the air in the attic space to the surface is given by:

$$\dot{m}_w = h_w (\omega_a - \omega_s) \quad (40)$$

where:

$\dot{m}$ = mass flow rate of moisture per unit area of exposed wood surface, kg/s · m² [lb/h·ft²],
 ω_a = humidity ratio of air in attic space, kg of water/kg of dry air [lb of water/lb of dry air], and
 h_w = mass transfer coefficient, kg/s·m² [lb/h·ft²].

6.7.3 The mass transfer coefficient is obtained from the analogy between heat and mass transfer as:

$$\frac{h_c}{h_w C_P} = \left(\frac{\alpha}{D}\right)^{2/3} \approx 1 \quad (41)$$

where:

h_c = convection heat transfer coefficient, W/m² · K [Btu/h·ft²·°F],
 C_P = specific heat of air, J/kg·K [Btu/lb·°F],
 α = thermal diffusivity of air, m²/s [ft²/h], and
 D = coefficient for diffusion of water vapor through air, m²/s [ft²/h].

6.7.4 The humidity ratio of the attic air is obtained by performing a steady-state moisture balance on the attic space, including diffusion of moisture through the boundary surfaces, convection of moisture into the attic space from the outside air and from exfiltration from the house, convection of moisture out of the attic space by the ventilation air, and moisture transfer to or from the wood surfaces. The attic moisture balance is given by:

$$\sum_{j=1}^M A(j) \text{Perm}(j) (P(j) - P_a) + \dot{m}_v (\omega_o - \omega_a) + \dot{m}_E (\omega_i - \omega_a) + \sum_{j=1}^M h_w(j) A'(j) (\omega_s(j) - \omega_a) = 0 \quad (42)$$

where:

- $Perm(j)$ = water vapor permeance of surface j ,
- $\dot{m}_v$ = mass flow rate of ventilation air,
- $\dot{m}_E$ = mass flow rate of exfiltration air,
- $\omega_s(j)$ = humidity ratio at surface j ,
- ω_a = humidity ratio of air in attic space,
- ω_o = humidity ratio of outside air,
- ω_i = humidity ratio of indoor air,
- $P(j)$ = partial pressure of water vapor in air on outside of surface j ,
- P_a = partial pressure of water vapor in attic air,
- $h_w(j)$ = water vapor mass transfer coefficient at surface j ,
- $A(j)$ = area of surface j , and
- $A'(j)$ = exposed wood area at surface j .

6.7.4.1 When the humidity ratio of the attic air and the mass transfer rates have been calculated, the heat transferred to the surface by latent heat effects, Q_{lat} , is given by

$$Q_{lat} = \dot{m}_v h_v \quad (43)$$

where:

h_v = latent heat of vaporization of water. For this model, h_v has been taken to have a constant value of 2 466 kJ/kg [1060 Btu/lb].

6.7.4.2 The mass transfer to a surface is used to estimate a new surface moisture content, assuming that only a thin layer of wood participates in the moisture exchanges. It should be noted that the intent of this treatment of moisture is only to estimate the effect of latent heats on the heat flow rates and not to determine the accumulation of moisture itself. An estimation of moisture accumulation rates over long periods of time would require a more detailed treatment than is used here.

6.7.5 The algorithms for calculating moisture effects are embodied in several subprograms. The humidity ratio at the wood surface is calculated by function WDHUM. Subroutine PSY is used to calculate humidity ratios and partial pressures of water vapor from known values of air temperature and relative humidity. The mass transfer coefficients are calculated in subroutine HMASS, and the moisture balance is performed in subroutine MOIST.

6.8 Equations—Heat Balances:

6.8.1 The heat balance equations combine the heat flows by various mechanisms at the inside and outside surfaces of the components of the attic envelope and on the attic air mass. The heat balance at an interior surface, that is, one facing the attic space, is obtained by summing the contributions due to conduction through the component, radiation interchanges with each of the other surfaces that it sees, convection exchanges with the attic air mass, and latent heat loads due to moisture sorption/desorption. The interior heat balances are:

$$\begin{aligned} & \sum_{j=0}^N Z(i,j) (TIS(i,j) - TR) - \sum_{j=0}^N Y(i,j) (TOS(i,j) - TR) \quad (44) \\ & + b(i)/2 \sum_{j=0}^N Z(i,j) (TIS(i,j) - TR)^2 - b(i)/2 \sum_{j=0}^N Y(i,j) (TOS(i,j) \\ & - TR)^2 + CR(i) QO'(i) + HC(i) (TIS(i,0) - TA) \end{aligned}$$

$$+ \sum_{k=1}^M \sum_{k \neq i} HR(i,k) (TIS(i,0) - TIS(k,0)) - \dot{m}_v(i) h_v = 0$$

6.8.1.1 In Eq 44, the index i refers to the surface for which the heat balance is being written. Index k refers to the other surfaces that face the attic space, and j is the index for the conduction transfer function time sequence, with $j = 0$ representing the current time. An equation of this form is obtained for each of the surfaces facing the attic space.

6.8.2 The heat balance at the exterior surface relates the heat conducted through the envelope surface to the heat convected to the outdoor air, the heat radiated by the surface to the surroundings, and the solar radiation absorbed by the surface. Since the exterior surface of the bottom of the attic is the ceiling of the house, the balance is modified to allow convection to the indoor air, radiation to the surfaces of the room below the attic, and no direct absorption of solar radiation. The exterior heat balances have the form:

$$\begin{aligned} & \sum_{j=0}^N Y(i,j) (TIS(i,j) - TR) - \sum_{j=0}^N X(i,j) (TOS(i,j) - TR) \quad (45) \\ & + b(i)/2 \sum_{j=0}^N Y(i,j) (TIS(i,j) - TR)^2 \\ & - b(i)/2 \sum_{j=0}^N X(i,j) (TOS(i,j) - TR)^2 \\ & + CR(i) QO'(i) + HC(i) (T_o - TOS(i,0)) \\ & + HR(i) (T_s - TOS(i,0)) + \alpha(i) Q_s(i) = 0 \end{aligned}$$

where:

- T_o = temperature of outside air,
- T_s = temperature of surroundings,
- $\alpha(i)$ = solar absorptance of surface i , and
- $Q_s(i)$ = solar radiation incident on surface i .

6.8.2.1 The heat balance on the attic air mass accounts for the heat convected to the air from each of the surfaces facing the attic space, convection of outdoor air into the space, and convection of attic air out of the space. With the model given above for the temperature rise of the ventilation air, the attic air heat balance is given by:

$$\begin{aligned} & \sum_{i=1}^M A(i) HC(i) TIS(i,0) - \frac{C_1}{1 + C_2 C_3} TA \quad (46) \\ & = \dot{m}_v C_p T_o \frac{C_3}{1 + C_2 C_3} \end{aligned}$$

where:

- $C_1 = \sum_{i=1}^M A(i) HC(i)$,
- $C_2 = \frac{\dot{m}_v C_p}{C}$, and
- $C_3 = \exp(-1/C_2) - 1$.

6.8.2.2 The system of heat balance equations is arranged in a matrix form with the interior and exterior surface temperatures and the air temperatures at the current time step being the unknown quantities. Values for these temperatures at previous time steps are known. The matrix equation may be written as:

$$(AA)(T) = (BB) \quad (47)$$

where:

AA = a square matrix of coefficients,
 T = a vector of unknown temperatures, and
 BB = a known factor.

Detailed expressions for the elements of the matrices are lengthy. They may be found by inspecting the Fortran code for the computer program and will not be repeated here.

6.8.3 This system of equations is solved by Gauss-Jordan elimination, using a subroutine named SOLVP, which was developed by Peavy (given in Kusuda (19)). Since many of the coefficients in the square matrix and the constant vector depend upon the unknown temperatures, the system of equations is solved iteratively. When the temperatures have been determined, the heat flows are calculated using the conduction transfer function equations.

6.8.4 To avoid a numerical instability associated with the latent heat term in the heat balance on interior surfaces, the surface humidity ratio is expanded in a Taylor series such that:

$$\begin{aligned} \dot{m}_w(i) &= h_w(i) A'(i) (\omega_a - \omega_s(i)) \\ &= h_w(i) A'(i) \left[\left(\omega_a - \omega'_s(i) - \frac{d\omega_s(i)}{dT} (TIS(i) - TIS'(i)) \right) \right] \end{aligned} \quad (48)$$

where:

$\omega'_s(i)$ = is the surface humidity ratio evaluated at a previously estimated surface temperature $TIS'(i)$.

The coefficients of $TIS(i)$ are included in the (AA) matrix while the other terms are included in the (BB) vector. With this scheme, the solutions have been found to converge.

6.8.5 The system of equations is set up to allow the model to be driven by weather conditions with all the surface and air temperatures being unknown. For comparison of the model with experiments, often it is helpful to use measured values of exterior roof and ceiling temperatures as boundary conditions. The model is set up to allow any of the surface temperatures to be forced to its measured value. This is accomplished using a method suggested by D. Ober, in a private conversation with K. E. Wilkes in 1988, wherein the diagonal element of (AA) that corresponds to the forced temperature is multiplied by a very large number and the corresponding element of (BB) is set to the product of the new element of (AA) and the known temperature.

7. Computer Program

7.1 General:

7.1.1 A computer program that embodies the model described in this practice is available from ASTM as an adjunct. This program is only an example. Users of the practice may develop their own program if they wish. The computer program in the adjunct is written in Fortran 77.

NOTE 1—Identical versions of this computer program have been compiled successfully and run on mainframe and personal computers. Only minor modifications necessary for conformance to the resident operating system were required for operation.

7.1.2 The program consists of a main program and several subroutines. Other subroutines may be added to make the program more applicable to the specific problems of individual users.

7.1.3 A flow chart for the program is given in Fig. 2. The program starts by initializing all temperatures to 75°F and all heat fluxes to zero through the use of DATA statements. DATA statements also are used to set various other quantities to zero such as the elements of the (AA) matrix. Many of these elements will remain at zero while others are recalculated later in the program.

7.1.3.1 Next, material property and geometrical input data are read in. This consists of conduction transfer functions, solar absorptances, total hemispherical emittances, the length and width of the attic, roof pitches, height of eave walls, orientation of the house, vent inlet and outlet areas and type of vents, water vapor permeances, wood surface areas and participating masses, and wood moisture contents. The house exfiltration rate and the latent heat of vaporization also are read as inputs. By setting the latent heat to zero, moisture effects may be ignored. The input read in also includes a series of flags that specify whether surface temperatures are to be forced to known values or are to be calculated from weather conditions.

7.1.3.2 The program then calls subroutine VIEW2. The attic length, width, roof pitches, height of eave walls and emittances of the surfaces facing the attic space are passed into VIEW2. The subroutine utilizes function FMN to calculate view factors between two rectangular surfaces that share an edge and function FP to calculate view factors between two equal parallel rectangles. The subroutine then uses view factor algebra to calculate the view factors among the surfaces that face the attic space and the overall view factor matrix G_{ij} , which is passed back out of the subroutine.

7.1.3.3 Next, the program calculates characteristic lengths and areas of surfaces. For the ceiling, the characteristic length is taken to be the average of the length and width of the attic. For a roof surface, it is taken to be the distance from the eave to the ridge. For gables, it is taken to be the average height, and for the eave walls, it is taken to be the height. These characteristic lengths are the ones normally chosen for use in correlations for convection heat transfer coefficients.

7.1.3.4 In the next step, the program reads a line of hourly weather data. This reading includes the outdoor temperature, incident solar radiation, wind speed, wind direction, and outdoor humidity ratio. At this step, the program also is set up to read in measured values for surface temperatures and ventilation rate. If the appropriate flags are set, the program will use the indicated measured surface temperatures or ventilation rate in the calculations. If this option is elected, a separate input file is required.

7.1.3.5 If the flag is set so that the measured ventilation rate is not to be used, then the program makes a call to subroutine VENT. Quantities passed into the subroutine are outdoor air temperature, average air temperature within the air space, wind speed and direction, difference in elevation between inlet and outlet vents, inlet and outlet vent areas, and a flag to indicate the type of vent. The subroutine uses the algorithms described above for the stack and wind pressure effects to calculate

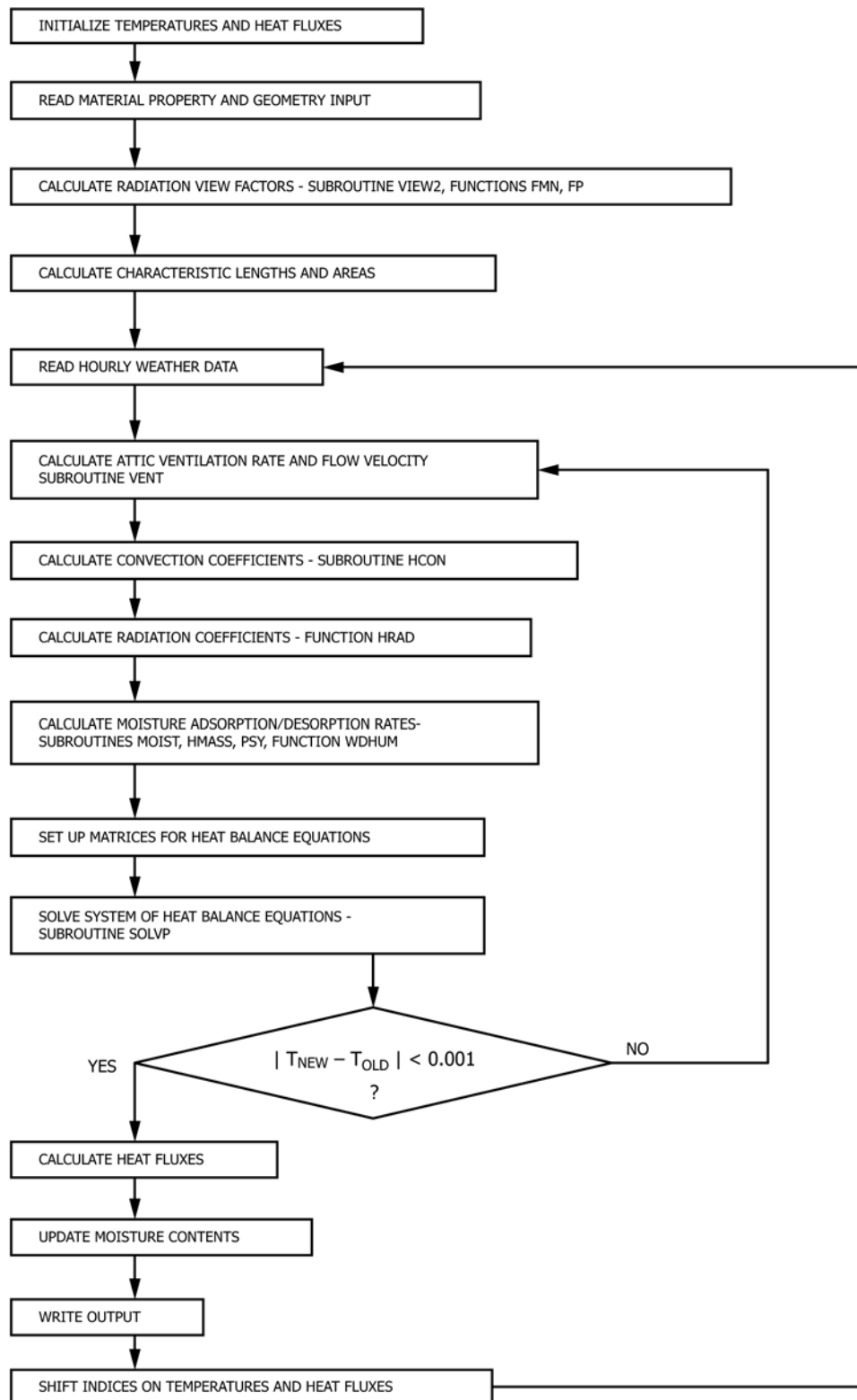


FIG. 2 Flow Chart for Attic Model

ventilation volume and mass flow rates, and the product of the mass flow rate and the specific heat of air. These last three quantities are passed back out of the subroutine. If the flag is

set to use the measured ventilation volume flow rate, VENT is bypassed and the measured volume flow rate is used to calculate the mass flow rate and the product of the mass flow

rate and the specific heat. Next, a crude estimate for the flow velocity is calculated by dividing the volume flow rate by an average cross sectional area for the attic.

7.1.3.6 The program then makes several calls to subroutine HCON to calculate convection heat transfer coefficients for the interior and exterior surfaces. Quantities passed into HCON are the previous estimates of the surface temperature and the adjacent average air temperature, surface tilt angle, characteristic length of surface, a flag to indicate whether the surface faces up or down, and the air speed past the surface. For interior surfaces, the crude estimate of ventilation flow velocity is used. For exterior surfaces, the wind speed is used (except for the exterior of the ceiling, where the air speed is set to zero). HCON utilizes the correlations described above to calculate natural, forced, and mixed convection heat transfer coefficients. The mixed coefficient is passed back out of HCON.

7.1.3.7 Radiation heat transfer coefficients are calculated using the HRAD function. The function uses the previous estimates of the two bounding temperatures and either the appropriate element of the G_{ij} matrix (for an interior surface) or the emittance (for an exterior surface).

7.1.3.8 If the latent heat is not set close to zero, the program calls subroutine MOIST, which calculates moisture sorption/desorption rates for the surfaces facing each of the air space. The quantities passed into MOIST are the previous estimates of the surface temperatures, the outdoor and indoor air temperatures, the previous estimate of the average air temperature within the space, the convection heat transfer coefficients, surface areas, wood surface areas and masses, wood moisture contents, indoor and outdoor relative humidities, water vapor permeances, attic ventilation mass flow rate, and house exfiltration mass flow rate. The subroutine calculates wood surface humidity ratios from the surface moisture content and temperature using function WDHUM. Subroutine PSY is used to calculate humidity ratios and water vapor partial pressures for the indoor and outdoor air using the given temperatures and relative humidities. Mass transfer coefficients are calculated using subroutine HMASS, which uses the analogy between heat and mass transfer.

7.1.3.9 The humidity ratio of the air in the air space then is calculated from a moisture balance on the space. This calculation is done iteratively since the moisture balance involves both the humidity ratio and the partial pressure. With the humidity ratio of the air established, the moisture flow rates from Eq 48 are calculated and pass back out of the subroutine. If the latent heat is set close to zero, all of these calculations are bypassed.

7.1.3.10 At this point, all of the quantities for the matrices (AA) and (BB) are available. The matrices are set up in the main program and are solved using subroutine SOLVP. The result of this solution is a new set of estimates for the surface and air space temperatures. The new estimates are compared with the old estimates. If any differences are greater than 0.0005°C [0.001°F], then the program goes back to subroutine VENT and new elements for the matrices of the heat balance equations are calculated using the newly estimated temperatures.

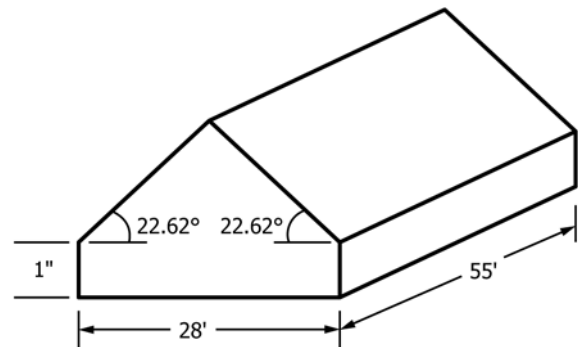


FIG. 3 Geometry of Attic for Example Problem

7.1.3.11 When all of the new estimates for surface and air temperatures agree with old estimates to within 0.0005°C [0.001°F], or when the limit of 15 iterations has been reached, heat fluxes are calculated using the conduction transfer function equations, moisture contents are updated, and results are written out for the current hour. The WRITE and FORMAT statements may need to be changed to suit the user's needs.

7.1.3.12 The time indices on the temperatures and heat fluxes are shifted, the program goes back to read another line of weather data and the calculations proceed as before for the next hour.

8. Example Calculation

8.1 The gabled attic shown in Fig. 3 was modeled using the computer program. The construction and properties of the various surfaces of the attic are given in Tables 2 and 3. The ridge of the attic is oriented in the east-west direction, the attic has soffit vents with an area of 0.64m^2 [6.84ft^2] and a ridge vent with an area of 0.32m^2 [3.42ft^2]. The interior of the house is maintained at 23°C [74°F] and 50 % relative humidity. No air from the house exfiltrates into the attic space. The house is located in Phoenix, AZ, and the reflectance of the ground is 0.2.

8.2 An input file for the computer program is given in Fig. 4, and a key to this file is given in Table 4. A file containing weather data for input to the program is given in Fig. 5, and a

TABLE 2 Construction and Properties of Attic Surfaces for Example Problem

Material	Thickness, in.	Thermal Conductivity, Btu/(h·ft·°F)	Specific Heat, Btu/lb·°F	Density, lb/ft ³
Ceiling:				
Gypsum board	0.5	0.0926	0.26	50
R-19 insulation	6.25	0.02741	0.19	0.6
Joists ^A	0.2917	0.06833	0.39	28
Roof (both sides):				
Shingles	0.25	0.04734	0.30	70
Felt	0.085	0.04734	0.36	70
Plywood	0.5	0.0667	0.29	34
Rafters ^A	3.5	0.06833	0.39	28
Gables (both ends):				
Hardboard siding	0.4375	0.1242	0.28	40
Studs	1.5	0.06833	0.39	28
Eave walls (both sides):				
Hardboard siding	0.4375	0.1242	0.28	40

^ATrusses are 24 in. on center.

**TABLE 3 Properties of Surfaces for Example Problem**

Property	Ceiling	Roof (both sides)	Gables (both ends)	Eave Walls (both sides)
Solar absorptance	...	0.9	0.7	0.7
Outside emittance	0.9	0.9	0.9	0.9
Interior emittance	0.9	0.05	0.05	0.9
Water vapor permeance	0.08	0.322	10	10
Exposed wood area/ projected area	0	1.29	1.19	1.0
Wood mass/projected area	0.0365	0.871	0.888	0.833
Initial moisture content	0.09	0.09	0.09	0.09

key to the file is given in [Table 5](#). The weather data consists of typical meteorological year weather data for one day in the summer. The data for this one day are repeated three times in the file for startup purposes.

8.3 [Fig. 6](#) contains the output file, which is an hourly listing of the ceiling heat flux, in Btu/h·ft², and [Fig. 7](#) contains a plot of the heat flux.

9. Report

9.1 The results of calculations performed in accordance with this practice may be used as design data for specific job conditions or may be used in general form to represent the performance of a particular product or system. When the results will be used for comparison of performance of similar products, it is recommended that reference be made to the specific constants used in the calculation. These references should include:

9.1.1 Name and other identification of products or components.

9.1.2 Source of weather conditions used in calculations and location being simulated.

9.1.3 Surface total hemispherical emittances and solar absorptances used in calculations.

9.1.4 All other input parameters, including other material properties and geometry of attic.

9.1.5 The resulting heat gain or loss.

9.1.6 Either tabular or graphical representation of the results of the calculations may be used. No attempt is made to recommend the format of this presentation of results.

10. Precision and Bias

10.1 The precision of this practice is a function of the computer equipment used to generate the calculational results. In many typical computers normally used, seven significant digits are resident in the computer for calculations. Adjustments to this level can be made through the use of *double precision*, however, for the intended purpose of this practice, standard levels of precision are adequate. The formatting of the output results however has been structured to provide a resolution of better than 0.1 % for the typical expected levels of heat flux.

10.2 Many factors influence the accuracy of a calculational procedure used for predicting heat flux results. These factors include computer resolution, accuracy of input data, and the applicability of the assumptions used in the method for the system under study. By comparison of model predictions with measured heat flux data, Wilkes (5) concluded that calculated cumulative heat flows under summer conditions were accurate to 5–10 % for attics without radiant barriers and were accurate to about 15 % for attics with radiant barriers. A significant source of uncertainty in the calculations for a radiant barrier attached near the roof was in the effective emittance of the combination of radiant barrier and exposed wood surfaces.

11. Keywords

11.1 attic radiant barrier system; computer program; heat gain; heat loss

C1340/C1340M – 10 (2015)

										Row										Row
0	0	0	0	0	0	0	0	0	0	1	1.378291871167D-04	1.273475199657D-04	1.234330829909D-04							35
0	0	0	0	0	0	0	0	0	0	2	2.326726793748D-05	2.198298620464D-05	2.088207445969D-05							36
0										3	3.968702189298D-06	3.759072498190D-06	3.562723535596D-06							37
13	5.297757243125D-02	8.120386020702D-01	0							4	6.777330258734D-07	6.421197823789D-07	6.084212372946D-07							38
7.466638753913D-02	3.296111635260D-02	6.329589156026D-01								5	1.157516029329D-07	1.096727646893D-07	1.039140135711D-07							39
-8.081856552558D-02	-1.131177163705D-02	-1.072584603927D+00								6	1.976979039206D-08	1.873162668348D-08	1.774799628159D-08							40
1.607573153106D-02	-1.191553986429D-02	4.471300601487D-01								7	3.376586523790D-09	3.199274487260D-09	3.031273803920D-09							41
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5.537768644386D-06	4.727784241798D-05	4.153680331032D-04								9	3.016973590501D+00	2.654359059909D+00	2.981945246195D+00							43
1.380214832286D-06	1.190345771433D-05	1.029478608764D-04								10	-4.165555544567D-01	-4.201537925157D-02	-3.801915149702D-01							44
3.452580027148D-07	2.980496677172D-06	2.573693243892D-05								11	9.800719684440D-03	-2.124919702908D-03	8.465033821940D-03							45
8.639422165610D-08	7.458846284631D-07	6.439790581926D-06								12	2.408209180406D-08	1.885644335746D-08	1.476481616605D-08							46
2.161922353515D-08	1.866514164267D-07	1.611478536661D-06								13		2.763702754562D+00	5.553561006528D-02	0						47
5.409996000263D-09	4.670770927139D-08	4.032562565449D-07								14	3.016973590501D+00	2.654359059909D+00	2.981945246195D+00							48
1.353798256872D-09	1.168814551653D-08	1.009109015561D-07								15	-4.165555544567D-01	-4.201537925157D-02	-3.801915149702D-01							49
3.387746877255D-10	2.924843488629D-09	2.525196027162D-08								16	9.800719684440D-03	-2.124919702908D-03	8.465033821940D-03							50
8.477503184473D-11	7.319132818230D-10	6.319054571052D-09								17	2.408209180406D-08	1.885644335746D-08	1.476481616605D-08							51
	7.831770447500D-01	6.433285642988D-01	0							18		3.405649900000D+00	1.787297931241D-36	0						52
1.208496084373D+00	6.097210027624D-01	1.101969373618D+00								19	3.541767161767D+00	3.337591148759D+00	3.541767161767D+00							53
-1.188415531935D+00	-2.290645733438D-01	-1.013811108372D+00								20	-1.361173420051D-01	6.805867100255D-02	-1.361173420051D-01							54
2.581284413707D-01	-1.020951969292D-01	1.901995116329D-01								21	2.432822437717D-37	-1.216411218859D-37	3.152602707844D-19							55
9.620012442075D-04	6.217810497892D-04	8.304889670224D-04								22		3.405649900000D+00	1.787297931241D-36	0						56
1.378291871167D-04	1.273475199657D-04	1.234330829909D-04								23	3.541767161767D+00	3.337591148759D+00	3.541767161767D+00							57
2.326726793748D-05	2.198298620464D-05	2.088207445969D-05								24	-1.361173420051D-01	6.805867100255D-02	-1.361173420051D-01							58
3.968702189298D-06	3.759072498190D-06	3.562723535596D-06								25	2.432822437717D-37	-1.216411218859D-37	3.152602707844D-19							59
6.777330258734D-07	6.421197823789D-07	6.084212372946D-07								26	0.9	0.9	0.9	0.7	0.7	0.7	0.7	0.7	0.7	60
1.157516029329D-07	1.096727646893D-07	1.039140135711D-07								27	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	61
1.976979039206D-08	1.873162668348D-08	1.774799628159D-08								28	0.9	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	62
3.376586523790D-09	3.199274487260D-09	3.031273803920D-09								29	55	28	22.62	22.62	90	0.083				63
	7.831770447500D-01	6.433285642988D-01	0							30	6.84	3.42	1							64
1.208496084373D+00	6.097210027624D-01	1.101969373618D+00								31	0.08	0.322	0.322	10	10	10	10	10	10	65
-1.188415531935D+00	-2.290645733438D-01	-1.013811108372D+00								32	0	1.29	1.29	1.19	1.19	1	1	1	1	66
2.581284413707D-01	-1.020951969292D-01	1.901995116329D-01								33	0.0365	0.871	0.871	0.888	0.888	0.833	0.833	0.833	0.833	67
9.620012442075D-04	6.217810497892D-04	8.304889670224D-04								34	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	68
											1060									69
											0									70
											33.4	112	7	0.95	0.2	1				71
											74	50	0	0						72

FIG. 4 Input Data for Example Problem (See Table 4 for Key to Input Data)

TABLE 4 Key to Input Data in Fig. 4

Rows 1-3	Flags for specified surface temperature and ventilation rate: 0 = unspecified 1 = specified
Rows 4-17	Conduction transfer functions for ceiling
Rows 18-29	Conduction transfer functions for east side of roof
Rows 30-41	Conduction transfer functions for west side of roof
Rows 42-46	Conduction transfer functions for south gable
Rows 47-51	Conduction transfer functions for north gable
Rows 52-55	Conduction transfer functions for east eave wall
Rows 56-59	Conduction transfer functions for west eave wall ^A
Row 60	Solar absorptances of attic surfaces ^B
Row 61	Total hemispherical emittances of surfaces facing away from attic space
Row 62	Total hemispherical emittances of surfaces facing attic space
Row 63	Attic length (ft), attic width (ft), pitch of east side of roof (degrees), pitch of west side of roof (degrees), angle between ridge line and north-south direction (degrees, 90 = east-west ridge), height of eave walls (ft)
Row 64	Vent inlet (ft ²), vent outlet area (ft ²), type of vents (1 = soffit and ridge, 2 = soffit and ridge, 3 = soffit vents only)
Row 65	Water vapor permeances of attic surfaces (perms)
Row 66	Ratio of total area of exposed wood projected area of surface
Row 67	Mass of wood per unit projected area of attic surface (lb/ft ²)
Row 68	Initial moisture content of wood (weight fraction)
Row 69	Heat of vaporization of water (Btu/lb); set = 0 to bypass all moisture algorithms
Row 70	Rate of exfiltration of air from house into attic space (lb/ft)
Row 71	Latitude (degrees), longitude (degrees), time zone (5 = Eastern, 6 = Central, 7 = Mountain, 8 = Pacific), atmospheric clearness, solar reflectance of ground, flag for solar radiation input (1 = measured total horizontal and direct, 2 = measured total horizontal only, 3 = no solar measured, but measured cloud cover; use 1 with weather data as in example)
Row 72	Air temperature inside house (°F), relative humidity inside house (% RH), flags for ventilation (0 = both sides of attic vented, 1 = attic vented on only one side), measured ventilation rate (0.0 if not measured)

^ANomenclature for directions assumes the ridge runs in the north-south direction.

^BColumns for this and succeeding lines are ceiling, east side of roof, west side of roof, south gable, north gable, east eave wall, and west eave wall.

C1340/C1340M – 10 (2015)

Day of Year	Hour of Day	Outdoor Air Temp	Atmospheric Pressure	Cloud Amount	Wind Direction	Humidity Ratio	Horizontal Solar	Direct Solar	Cloud Type	Wind Speed	Atmospheric Clearness
185	1	91	14.096	0	270.0	0.0085	0	0	2	4.61	1.05
185	2	90	14.096	0	247.5	0.0087	0	0	2	4.61	1.05
185	3	87	14.096	0	180.0	0.0087	0	0	2	4.61	1.05
185	4	85	14.145	0	135.0	0.0091	0	0	2	4.61	1.05
185	5	82	14.145	0	90.0	0.0098	0	0	2	4.61	1.05
185	6	84	14.145	0	90.0	0.0101	1	0	2	4.61	1.05
185	7	86	14.145	0	90.0	0.0096	21	4	2	5.76	1.05
185	8	88	14.145	0	90.0	0.0099	91	78	2	5.76	1.05
185	9	92	14.145	0	45.0	0.0090	190	234	2	5.76	1.05
185	10	95	14.145	0	22.5	0.0083	254	272	2	6.91	1.05
185	11	99	14.145	0	0.0	0.0081	299	281	2	6.91	1.05
185	12	102	14.096	0	337.5	0.0074	325	284	2	8.06	1.05
185	13	104	14.096	0	315.0	0.0070	334	284	2	9.21	1.05
185	14	107	14.096	0	292.5	0.0071	327	284	2	10.36	1.05
185	15	108	14.047	0	270.0	0.0061	302	282	2	10.36	1.05
185	16	109	14.047	0	247.5	0.0059	258	274	2	10.36	1.05
185	17	110	14.047	0	247.5	0.0050	195	238	2	10.36	1.05
185	18	108	14.047	0	247.5	0.0062	98	90	2	10.36	1.05
185	19	106	14.047	0	247.5	0.0066	24	5	2	11.52	1.05
185	20	104	13.998	0	270.0	0.0071	2	0	2	11.52	1.05
185	21	101	14.047	0	202.5	0.0070	0	0	2	9.21	1.05
185	22	98	14.047	1	157.5	0.0077	0	0	1	6.91	1.05
185	23	95	14.047	2	90.0	0.0076	0	0	2	4.61	1.05
185	24	94	14.047	3	112.5	0.0071	0	0	2	6.91	1.05
185	1	91	14.096	0	270.0	0.0085	0	0	2	4.61	1.05
185	2	90	14.096	0	247.5	0.0087	0	0	2	4.61	1.05
185	3	87	14.096	0	180.0	0.0087	0	0	2	4.61	1.05
185	4	85	14.145	0	135.0	0.0091	0	0	2	4.61	1.05
185	5	82	14.145	0	90.0	0.0098	0	0	2	4.61	1.05
185	6	84	14.145	0	90.0	0.0101	1	0	2	4.61	1.05
185	7	86	14.145	0	90.0	0.0096	21	4	2	5.76	1.05
185	8	88	14.145	0	90.0	0.0099	91	78	2	5.76	1.05
185	9	92	14.145	0	45.0	0.0090	190	234	2	5.76	1.05
185	10	95	14.145	0	22.5	0.0083	254	272	2	6.91	1.05
185	11	99	14.145	0	0.0	0.0081	299	281	2	6.91	1.05
185	12	102	14.096	0	337.5	0.0074	325	284	2	8.06	1.05
185	13	104	14.096	0	315.0	0.0070	334	284	2	9.21	1.05
185	14	107	14.096	0	292.5	0.0071	327	284	2	10.36	1.05
185	15	108	14.047	0	270.0	0.0061	302	282	2	10.36	1.05
185	16	109	14.047	0	247.5	0.0059	258	274	2	10.36	1.05
185	17	110	14.047	0	247.5	0.0050	195	238	2	10.36	1.05
185	18	108	14.047	0	247.5	0.0062	98	90	2	10.36	1.05
185	19	106	14.047	0	247.5	0.0066	24	5	2	11.52	1.05
185	20	104	13.998	0	270.0	0.0071	2	0	2	11.52	1.05
185	21	101	14.047	0	202.5	0.0070	0	0	2	9.21	1.05
185	22	98	14.047	1	157.5	0.0077	0	0	1	6.91	1.05
185	23	95	14.047	2	90.0	0.0076	0	0	2	4.61	1.05
185	24	94	14.047	3	112.5	0.0071	0	0	2	6.91	1.05
185	1	91	14.096	0	270.0	0.0085	0	0	2	4.61	1.05
185	2	90	14.096	0	247.5	0.0087	0	0	2	4.61	1.05
185	3	87	14.096	0	180.0	0.0087	0	0	2	4.61	1.05
185	4	85	14.145	0	135.0	0.0091	0	0	2	4.61	1.05
185	5	82	14.145	0	90.0	0.0098	0	0	2	4.61	1.05
185	6	84	14.145	0	90.0	0.0101	1	0	2	4.61	1.05
185	7	86	14.145	0	90.0	0.0096	21	4	2	5.76	1.05
185	8	88	14.145	0	90.0	0.0099	91	78	2	5.76	1.05
185	9	92	14.145	0	45.0	0.0090	190	234	2	5.76	1.05
185	10	95	14.145	0	22.5	0.0083	254	272	2	6.91	1.05

FIG. 5 Weather Data for Example Problem (see Table 5 for Units and Further Definition)

C1340/C1340M – 10 (2015)

185	11	99	14.145	0	0.0	0.0081	299	281	2	6.91	1.05
185	12	102	14.096	0	337.5	0.0074	325	284	2	8.06	1.05
185	13	104	14.096	0	315.0	0.0070	334	284	2	9.21	1.05
185	14	107	14.096	0	292.5	0.0071	327	284	2	10.36	1.05
185	15	108	14.047	0	270.0	0.0061	302	282	2	10.36	1.05
185	16	109	14.047	0	247.5	0.0059	258	274	2	10.36	1.05
185	17	110	14.047	0	247.5	0.0050	195	238	2	10.36	1.05
185	18	108	14.047	0	247.5	0.0062	98	90	2	10.36	1.05
185	19	106	14.047	0	247.5	0.0066	24	5	2	11.52	1.05
185	20	104	13.998	0	270.0	0.0071	2	0	2	11.52	1.05
185	21	101	14.047	0	202.5	0.0070	0	0	2	9.21	1.05
185	22	98	14.047	1	157.5	0.0077	0	0	1	6.91	1.05
185	23	95	14.047	2	90.0	0.0076	0	0	2	4.61	1.05
185	24	94	14.047	3	112.5	0.0071	0	0	2	6.91	1.05
185	1	91	14.096	0	270.0	0.0085	0	0	2	4.61	1.05
185	2	90	14.096	0	247.5	0.0087	0	0	2	4.61	1.05
185	3	87	14.096	0	180.0	0.0087	0	0	2	4.61	1.05
185	4	85	14.145	0	135.0	0.0091	0	0	2	4.61	1.05
185	5	82	14.145	0	90.0	0.0098	0	0	2	4.61	1.05
185	6	84	14.145	0	90.0	0.0101	1	0	2	4.61	1.05
185	7	86	14.145	0	90.0	0.0096	21	4	2	5.76	1.05
185	8	88	14.145	0	90.0	0.0099	91	78	2	5.76	1.05
185	9	92	14.145	0	45.0	0.0090	190	234	2	5.76	1.05
185	10	95	14.145	0	22.5	0.0083	254	272	2	6.91	1.05
185	11	99	14.145	0	0.0	0.0081	299	281	2	6.91	1.05
185	12	102	14.096	0	337.5	0.0074	325	284	2	8.06	1.05
185	13	104	14.096	0	315.0	0.0070	334	284	2	9.21	1.05
185	14	107	14.096	0	292.5	0.0071	327	284	2	10.36	1.05
185	15	108	14.047	0	270.0	0.0061	302	282	2	10.36	1.05
185	16	109	14.047	0	247.5	0.0059	258	274	2	10.36	1.05
185	17	110	14.047	0	247.5	0.0050	195	238	2	10.36	1.05
185	18	108	14.047	0	247.5	0.0062	98	90	2	10.36	1.05
185	19	106	14.047	0	247.5	0.0066	24	5	2	11.52	1.05
185	20	104	13.998	0	270.0	0.0071	2	0	2	11.52	1.05
185	21	101	14.047	0	202.5	0.0070	0	0	2	9.21	1.05
185	22	98	14.047	1	157.5	0.0077	0	0	1	6.91	1.05
185	23	95	14.047	2	90.0	0.0076	0	0	2	4.61	1.05
185	24	94	14.047	3	112.5	0.0071	0	0	2	6.91	1.05

FIG. 5 (continued)

TABLE 5 Key to Weather Data in Fig. 5

Col. 1	Day of year (1 = January 1)
Col. 2	Hour of day (1 = 1:00 am)
Col. 3	Outdoor air temperature, °F
Col. 4	Atmospheric pressure, psi
Col. 5	Cloud amount (0 to 10)
Col. 6	Wind direction (0 = from north)
Col. 7	Outside humidity ratio
Col. 8	Horizontal solar radiation (Btu/h-ft ²)
Col. 9	Direct solar radiation (Btu/h-ft ²)
Col. 10	Cloud type
Col. 11	Wind speed, mph
Col. 12	Atmospheric clearness

Ceiling Heat Flux,
Btu/h • ft²

0.4571	0.9038
0.3212	0.7908
0.2581	0.6883
0.2098	0.5942
0.1626	0.5015
0.1434	0.4146
0.1774	0.3321
0.3089	0.2850
0.5480	0.2984
0.8240	0.4101
1.1196	0.6413
1.4532	0.9228
1.7644	1.2325
1.8945	1.5749
2.0143	1.8029
2.0964	1.9449
2.0822	2.0400
1.9506	2.0970
1.7188	2.1161
1.4630	1.9773
1.2387	1.7293
1.0581	1.4679
0.9004	1.2435
0.7875	1.0620
0.6855	0.9039
0.5919	0.7908
0.4996	0.6884
0.4130	0.5942
0.3308	0.5016
0.2839	0.4146
0.2975	0.3321
0.4094	0.2850
0.6407	0.2984
0.9223	0.4101
1.2321	0.6413
1.5746	0.9228
1.8026	1.2325
1.9448	1.5750
2.0398	1.8029
2.0968	1.9449
2.1160	2.0400
1.9772	2.0970
1.7292	2.1161
1.4678	1.9773
1.2434	1.7293
1.0620	1.4679
	1.2435
	1.0620
	0.9039
	0.7908

FIG. 6 Output File for Example Problem

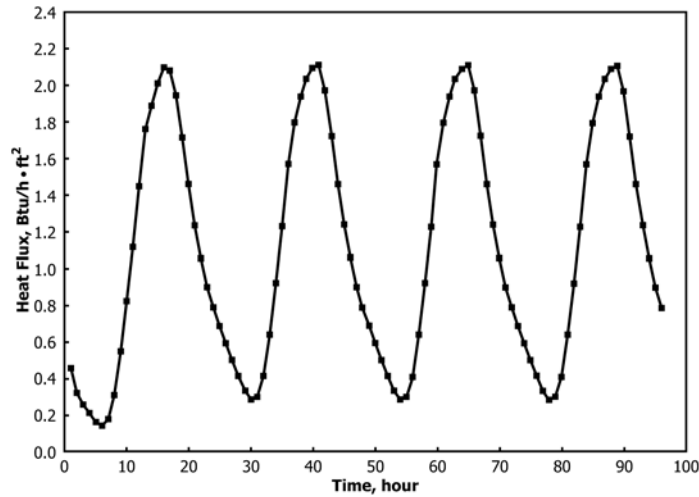


FIG. 7 Hourly Ceiling Heat Fluxes Calculated for Example Problem. The Summation of the Hourly Heat Fluxes is 107 Btu/ft²

APPENDIX

(Nonmandatory Information)

X1. COMPUTER PROGRAM WITH QUASI-STEADY MODEL FOR HVAC DUCTS IN ATTICS

INTRODUCTION

This annex describes additional features of the computer program that allow the modeling to include HVAC ducts that are run in attic spaces. The duct part of the model is quasi-steady and is based on the model developed by M. Modera, et al. (20). It is assumed that the heat capacity of the duct walls is negligible, that axial conduction along the duct may be ignored in comparison with the energy carried by air flowing through the duct, and that no heat is stored in the air inside the duct. For supply ducts, air may leak from the duct at a constant rate per unit length for a particular run of duct. For return ducts, air may leak into the duct at a constant rate per unit length. The computer program and description were developed as part of ASHRAE Research Project 717 (21).

X1.1 Supply Ducts

X1.1.1 The model is developed by considering heat balances on a differential length of supply duct. Nomenclature for the model is as follows:

dx	= length of differential section of duct
$\dot{m}$	= mass flow rate of air into differential length
a	= mass flow rate of air leakage from duct per unit length
$\dot{m} - a dx$	= mass flow rate of air out of differential length
T	= temperature of air entering differential length of duct
$T + dT$	= temperature of air exiting differential length of duct
T_{wi}	= temperature of inside wall of duct
T_{wo}	= temperature of outside wall of duct
P_i	= perimeter of inside wall of duct
P_o	= perimeter of outside wall of duct

h_i	= convection heat transfer coefficient at inside wall of duct
h_o	= convection heat transfer coefficient at outside wall of duct
h_{rj}	= radiation heat transfer coefficient between outside wall of duct and surrounding surface, j
T_{attic}	= temperature of air in attic space
T_j	= temperature of surrounding surface, j
k	= thermal conductivity of duct insulation

X1.1.2 A heat balance on the air flowing through the duct may be written as:

$$\dot{m} C_p T = (\dot{m} - a dx) C_p (T + dT) + a dx C_p T + h_i P_i (T - T_{wi}) dx \quad (X1.1)$$

X1.1.3 Letting “ $\dot{m}_{in} - ax$ ” be equal to the mass flow at position “ x ,” the heat balance on the flowing air may be rewritten as:

$$(\dot{m}_{in} - ax)C_p dT + h_i P_i (T - T_{wi}) dx = 0 \quad (X1.2)$$

X1.1.4 Assuming the duct to be round, the heat transferred by convection from the air stream to the inside wall of the duct may be equated to the heat transferred by conduction across the duct wall as:

$$h_i P_i (T - T_{wi}) = \frac{2\pi k (T_{wi} - T_{wo})}{\ln(P_o/P_i)} \quad (X1.3)$$

and the heat transferred through the duct wall by conduction may be equated to the sum of the heat transferred by convection from the outside duct surface to the attic air and the heat transferred by radiation from the outside duct surface to the N surfaces that make up the attic enclosure:

$$\frac{2\pi k (T_{wi} - T_{wo})}{\ln(P_o/P_i)} = h_o P_o (T_{wo} - T_{attic}) + \sum_{j=1}^N h_{rj} (T_{wo} - T_j) \quad (X1.4)$$

X1.1.5 It should be noted that the above two equations for round ducts are given as an example; the computer model also has an option for rectangular ducts, with appropriate formulas for heat conduction through the duct walls.

X1.1.6 We now define an effective temperature of the duct surroundings as:

$$T_s = \frac{h_o T_{attic} + \sum_{j=1}^N h_{rj} T_j}{h_o + \sum_{j=1}^N h_{rj}} \quad (X1.5)$$

X1.1.7 We next define an overall heat transfer coefficient, U' , as:

$$U' = \frac{1}{\frac{1}{h_i P_i} + \frac{\ln(P_o/P_i)}{2\pi k} + \frac{1}{P_o \left[h_o + \sum_{j=1}^N h_{rj} \right]}} \quad (X1.6)$$

X1.1.8 With these definitions, **Eq X1.2** may be rewritten as:

$$(\dot{m}_{in} - ax)C_p dT + U' (T - T_s) dx = 0 \quad (X1.7)$$

X1.1.9 If $a \neq 0$, **Eq X1.7** may be integrated to give the air stream temperature, T , at position, x , as:

$$T = T_s + (T_{in} - T_s) \left(\frac{\dot{m}_{in} - ax}{\dot{m}_{in}} \right)^{U'/aC_p} \quad (X1.8)$$

where T_{in} is the air stream temperature at $x = 0$. If $a = 0$, **Eq X1.7** may be integrated to give:

$$T = T_s + (T_{in} - T_s) \exp\left(-\frac{U'x}{\dot{m}_{in}C_p}\right) \quad (X1.9)$$

X1.1.10 The average air temperature inside the duct section of length L is obtained from:

$$T = \frac{1}{L} \int_0^L T dx \quad (X1.10)$$

X1.1.11 For “ a ” not equal to zero, this integral gives:

$$T = T_s + (T_{in} - T_s) \left(\frac{\dot{m}_{in}}{aL} \right) \left[\frac{1 - \left(1 - \frac{aL}{\dot{m}_{in}} \right)^{U'/aC_p + 1}}{U'/aC_p + 1} \right] \quad (X1.11)$$

X1.1.12 For $a = 0$, this integral gives:

$$T = T_s + \frac{(T_{in} - T_s)}{L} \left(\frac{\dot{m}_{in}C_p}{U'} \right) \left[1 - \exp\left(-\frac{U'L}{\dot{m}_{in}C_p}\right) \right] \quad (X1.12)$$

X1.1.13 The average temperature of the outside of the duct wall, T_{wo} , is related to T by:

$$T_{wo} = T - U' \left(\frac{1}{h_i P_i} + \frac{\ln(P_o/P_i)}{2\pi k} \right) (T - T_s) \quad (X1.13)$$

X1.1.14 The heat lost by convection from the duct of length L to the attic air is then given by:

$$Q_{conv} = h_o P_o L (T_{wo} - T_{attic}) \quad (X1.14)$$

and the heat lost by radiation from the duct of length L to the N surfaces comprising the attic enclosure is given by:

$$Q_{rad} = \sum_{j=1}^N h_{rj} P_o L (T_{wo} - T_j) \quad (X1.15)$$

X1.1.15 A heat balance on the duct of length L gives:

$$\dot{m}_{in} C_p T_{in} = \quad (X1.16)$$

$$(\dot{m}_{in} - aL)C_p T_L + h_o P_o L (T_{wo} - T_{attic}) + \sum_{j=1}^N h_{rj} P_o L (T_{wo} - T_j) + Q_{leaks}$$

where T_L is the air stream temperature at the exit from the duct and Q_{leaks} is the heat lost from the duct to the attic air space due to leakage of air from the duct.

X1.1.16 For a system of supply ducts, the volume flow rates (CFM) entering each of the duct sections would be specified, along with the temperature of the air entering the first duct section. The volume air flow rates are converted to mass flows by assuming the air to have the standard density of 0.075 lb/ft³. The above equations are then used to calculate the temperature of the air exiting the first section, which then becomes the temperature of the air entering the next section. By keeping track of inlet and exit temperatures, the temperature profile along a branching system of supply ducts may be calculated, finally giving the temperature of air exiting from supply registers into the interior space of a building.

X1.2 Return Ducts

X1.2.1 A heat balance on the air flowing through a differential length of return duct may be written as:

$$\dot{m} C_p T + a dx C_p T_{attic} = (\dot{m} + a dx) C_p (T + dT) + h_i P_i (T - T_{wi}) dx \quad (X1.17)$$

X1.2.2 Using the definitions of T_s and U' given in **Eq X1.5 and X1.6**, neglecting second order terms, and letting $\dot{m} = (\dot{m}_{in} + ax)$, this heat balance may be simplified to:

$$[aC_p (T - T_{attic}) + U' (T - T_s)] dx + (\dot{m} + ax) C_p dT = 0 \quad (X1.18)$$

X1.2.3 We now define a new effective temperature by:

$$T_e = \frac{aC_p T_{attic} + U' T_s}{aC_p + U'} \quad (X1.19)$$

X1.2.4 Then **Eq X1.18** becomes:

$$(aC_p + U') (T - T_e) dx + (\dot{m}_{in} + ax) C_p dT = 0 \quad (X1.20)$$

X1.2.5 If $a \neq 0$, **Eq X1.20** may be integrated to give:

$$T = T_e + (T_{in} - T_e) \left(\frac{\dot{m}_{in} + ax}{\dot{m}_{in}} \right)^{\left(\frac{aC_p + U'}{aC_p} \right)} \quad (X1.21)$$

X1.2.6 If $a = 0$, integration of Eq X1.20 yields Eq X1.9 again. For $a \neq 0$, the air temperature inside the duct averaged over the length of the duct is given by:

$$T = T_e + (T_{in} - T_e) \left(\frac{\dot{m}_{in}}{aL} \right) \left[\frac{1 - \left(1 + \frac{aL}{\dot{m}_{in}} \right)^{U'/aC_p + 2}}{U'/aC_p + 2} \right] \quad (X1.22)$$

X1.3 Radiation View Factors

X1.3.1 The above model utilizes radiation heat transfer coefficients between outside wall of duct and surrounding surface (h_{rj}). As with the main attic model, these coefficients are calculated from radiation view factors. Approximate view factors for radiation interactions between ducts and the attic surfaces were developed under the following simplifying assumptions:

X1.3.1.1 For radiation heat transfer calculations, each duct may be treated as being isothermal, gray, and diffusely emitting and reflecting.

X1.3.1.2 The ducts are small enough that view factors between any two duct runs may be considered negligible compared with view factors between the ducts and the attic surfaces.

X1.3.1.3 The view factor between any duct run and the floor of the attic enclosure is 0.5.

X1.3.1.4 The view factor from a duct to a particular attic surface is the same for all duct segments.

X1.3.1.5 The presence of ducts modifies only those view factors that involve the attic floor. For example, it is assumed that the ducts do not change the view factor between the two roof surfaces nor the view factors between the two gables nor the view factors between either of the roof surfaces and either of the gables.

X1.3.1.6 The presence of ducts reduces the view factors from the attic floor to the other attic surfaces by a factor that is the same for all of the other attic surfaces.

X1.4 Convection Heat Transfer Coefficients

X1.4.1 The convection heat transfer coefficient on the interior of the duct is calculated from:

$$h_i = [0.00368 + 1.5 \times 10^{-6} (T_a - 80)] \dot{m}_{avg} D_h^{-0.2} \quad (X1.23)$$

where T_a is the average air temperature inside the duct, $\dot{m}_{avg}$ is the average mass flow rate per unit area in the duct and D_h is the hydraulic diameter of the duct. Eq X1.23 is the same as that used in ASHRAE SP 43 project (22).

X1.4.2 Convection heat transfer coefficients on the duct exterior surfaces are calculated from Nusselt-Rayleigh number correlations. The Rayleigh number is based on the difference in temperature between the duct outside surface and the attic air space, and the characteristic height of the duct. The characteristic height is the outside diameter of a round duct, or the outside height of a rectangular duct. If the Rayleigh number is less than 10^9 , the following correlation for laminar natural convection is used:

$$Nu = 0.53 Ra^{1/4} \quad (X1.24)$$

TABLE X1.1 Parameters for Forced Convection from a Round Duct

Re	C	n
Re ≤ 4	0.989	0.33
4 ≤ Re ≤ 40	0.911	0.385
40 ≤ Re ≤ 4000	0.683	0.466
4000 ≤ Re ≤ 40 000	0.193	0.618
Re ≥ 40 000	0.0266	0.805

X1.4.3 If the Rayleigh number is greater than 10^9 , the correlation for turbulent natural convection is used:

$$Nu = 0.13 Ra^{1/3} \quad (X1.25)$$

X1.4.4 A forced convection coefficient is calculated, using the Reynolds number based on the average speed of air flow through the attic and the characteristic height of the duct. The forced convection coefficient is calculated from:

$$Nu = C Re^n Pr^{1/3} \quad (X1.26)$$

X1.4.5 For a rectangular duct, C is 0.102 and n is 0.675. For a round duct, C and n depend upon the Reynolds number as shown in Table X1.1 (23). A mixed convection coefficient is then calculated by taking the third root of the sum of the cubes of the natural and forced coefficients.

X1.5 Computer Program with Ducts

X1.5.1 A new computer program that includes ducts is called RB-DUCT.FOR. It was developed from the program without ducts by adding new subroutines specifically for ducts and by modifying the main part of the program to incorporate the effects of ducts. The subroutines added are: DUCTREAD to read input data for the duct system, DUCTSS for the quasi-steady duct model algorithms, VIEW2D to calculate the radiation viewfactors, and HCOND to calculate the convection heat transfer coefficients at the exterior surfaces of the ducts. The new subroutines and modifications to the main program were developed as part of ASHRAE Research Project 717 (21).

X1.5.2 *Input Parameters for Duct Model*—This section describes the inputs required for the duct model. Fig. 4 and Table 4 of the main body of the standard describe the input data for an example problem. The example input contains 72 lines. When using the duct model, additional input parameters are entered after Line 72 of the example problem. The new input parameters are as follows:

X1.5.2.1 *Line 73*—Flag to indicate whether the duct model is being used (0 = ducts not included; 1 = ducts included).

X1.5.2.2 *Line 74*—Number of supply duct runs in the attic; number of return duct runs (both integers).

X1.5.2.3 *Line 75*—Temperature of the air as it leaves the conditioning equipment (°F).

X1.5.2.4 *Lines 76-XXX*—These lines are grouped into sets of three lines, with one set for each of the supply and return ducts. Inputs for the supply ducts are listed first, followed by input for the return ducts. The number of lines in this total group should equal three times the sum of the two entries on Line 74. Lines 76, 77, and 78 will now be described. Lines 79-XXX follow the same pattern.

X1.5.2.5 *Line 76*—There are eight entries on this line:

(1) Integer identification of the duct run. Ducts should be numbered sequentially, starting with the run nearest the air-handling equipment.

(2) Integer node numbers for inlet and exit of duct run. The node numbers identify how the duct runs are connected to each other. The exit for one duct run will be the inlet for one or more duct runs next downstream, and the junction will occur at a particular node.

(3) Flag for shape of the duct (0 = a round duct; 1 = rectangular duct). All ducts do not have to have the same shape.

(4) Thermal conductivities (Btu/h-ft-°F) for an inner and an outer layer of duct insulation. Even if only one layer of insulation is to be modeled, non-zero values for both conductivities must be entered as input, in order to avoid a divide-check.

(5) Heat capacity per unit length of the duct materials (including the duct wall and insulation), in Btu/ft-°F.

(6) Length of the duct run (ft).

X1.5.2.6 Line 77—There are four entries on this line:

(1) Volume flow rate of air that enters the duct length (CFM).

NOTE X1.1—CFMs are assumed to be based on standard indoor conditions of 68°F and 14.696 psia, for which the density of air is 0.075 lb/ft³. The computer program does not check for flow balances at the nodes. The user must account for flow balances, including duct leakages.

(2) Rate of leakage to or from the duct per unit of length (CFM/ft).

(3) Emittance of the external surface of the duct, dimensionless.

(4) Integer to identify the zone of the house that the duct supplies air to. At present, this parameter is not used in the program.

X1.5.2.7 Line 78—If the duct run is round, Line 78 will contain three entries:

(1) Inside diameter of the inner layer of the duct insulation system (ft).

(2) Outside diameter of the inner layer (and inside diameter of the outer layer) of the duct insulation system (ft).

(3) Outside diameter of the outer layer of the duct insulation system (ft).

X1.5.2.8 If the duct run is rectangular, Line 78 will contain six entries:

(1) Inside width of the inner layer of the duct insulation system (ft).

(2) Inside height of the inner layer of the duct insulation system (ft).

(3) Outside width of the inner layer of the duct insulation system (ft).

(4) Outside height of the inner layer of the duct insulation system (ft).

(5) Outside width of the outer layer of the duct insulation system (ft).

(6) Outside height of the outer layer of the duct insulation system (ft).

X1.5.2.9 Lines 76 to 78 are repeated for each of the duct runs in the duct system. The program is dimensioned to allow up to 25 duct runs, in any combination of supply and return ducts.

TABLE X1.2 Duct Input Parameters for Example Problem

Input Parameter	Duct Section			
	1	2	3	4
Inlet node number	1	2	2	5
Outlet node number	2	3	4	6
Shape	Round	Round	Round	Round
Insulation Thermal Conductivity (Btu/h-ft-°F):				
Inner layer	0.021	0.021	0.021	0.021
Outer layer	0.021	0.021	0.021	0.021
Heat capacity (Btu/ft-°F)	0.075	0.075	0.075	0.075
Length, ft	15	15	25	20
Inlet Volumetric Flow Rate, CFM	1000	400	585	1000
Leakage, CFM/ft	1	1	1	0
Emittance	0.9	0.9	0.9	0.9
Diameters, ft:				
Inner	1.0	0.5	1.0	1.0
Middle	1.083	0.583	1.083	1.083
Outer	1.167	0.667	1.167	1.167

X1.5.3 Example Problem—The house described in Section 8 is used, with a simple HVAC duct system that consists of a supply system with three sections and one return section. The supply system consists of a main trunk 40 ft long with a branch 15 ft downstream. The branch is 15 ft long. The main trunk is a round duct with an inside diameter of 1 ft covered with R-4 insulation (1 in. thick with a thermal conductivity of 0.021 Btu/h-ft-°F), and an exterior emittance of 0.9. The branch is a round duct with an inside diameter of 0.5 ft also covered with R-4 insulation and an exterior emittance of 0.9. Conditioned air at 55°F and 1000 CFM enters the main trunk, and 400 CFM enters the branch. All supply ducts have a leakage of 1 CFM/ft. The return duct is similar to the main trunk, but is 20 ft long, has a flow of 1000 CFM and is leak-free. Parameters for the system are summarized in **Table X1.2**. **Table X1.3** shows the required input data file. The output file is given in **Table X1.4**, and a plot of calculated duct temperatures is given in **Fig. X1.1**.

X1.6 Precision and Bias

X1.6.1 Limited validation of the algorithms for ducts in an attic has been obtained by Petrie, et al. (24). Laboratory tests were conducted on a 14 by 16 ft (4.3 by 4.9 m) attic having a 6 in. (152 mm) diameter galvanized duct insulated with

TABLE X1.3 Example Additional Input Data for Model with Ducts

										Line
1										73
3	1									74
55.0										75
1	1	2	0	0.021	0.021	0.075	15.0			76
1000.0	1.00	0.90	0							77
1.000	1.083	1.167								78
2	2	3	0	0.021	0.021	0.075	15.0			79
400.0	1.00	0.90	0							80
0.500	0.583	0.667								81
3	2	4	0	0.021	0.021	0.075	25.0			82
585.0	1.00	0.90	0							83
1.000	1.083	1.167								84
4	5	6	0	0.021	0.021	0.075	20.0			85
1000.0	0.00	0.90	0							86
1.000	1.083	1.167								87

TABLE X1.4 Output File for Example Problem with Ducts

Ceiling Heat Flux Btu/h-ft ²	Temperatures, F Nodes 1–6					
0.4393	55.000	55.237	55.569	55.889	74.000	74.072
0.2831	55.000	55.237	55.568	55.887	74.000	74.071
0.2057	55.000	55.222	55.532	55.831	74.000	74.052
0.1479	55.000	55.210	55.504	55.788	74.000	74.037
0.0929	55.000	55.195	55.467	55.730	74.000	74.018
0.0713	55.000	55.203	55.486	55.760	74.000	74.027
0.1082	55.000	55.227	55.544	55.850	74.000	74.059
0.2304	55.000	55.273	55.653	56.021	74.000	74.118
0.4449	55.000	55.337	55.806	56.261	74.000	74.202
0.6891	55.000	55.388	55.928	56.454	74.000	74.270
0.9480	55.000	55.447	56.068	56.674	74.000	74.347
1.2500	55.000	55.516	56.232	56.933	74.000	74.438
1.5696	55.000	55.578	56.378	57.163	74.000	74.519
1.6891	55.000	55.550	56.312	57.058	74.000	74.482
1.8027	55.000	55.591	56.410	57.213	74.000	74.536
1.8946	55.000	55.592	56.412	57.216	74.000	74.538
1.8637	55.000	55.562	56.342	57.105	74.000	74.498
1.7205	55.000	55.512	56.224	56.918	74.000	74.432
1.5150	55.000	55.461	56.103	56.726	74.000	74.364
1.3032	55.000	55.420	56.006	56.573	74.000	74.311
1.1150	55.000	55.381	55.913	56.427	74.000	74.259
0.9497	55.000	55.351	55.842	56.315	74.000	74.220
0.7938	55.000	55.316	55.758	56.184	74.000	74.175
0.6881	55.000	55.310	55.743	56.161	74.000	74.167
0.5874	55.000	55.281	55.673	56.052	74.000	74.129
0.4941	55.000	55.266	55.638	55.998	74.000	74.110
0.4024	55.000	55.244	55.584	55.914	74.000	74.081
0.3160	55.000	55.228	55.545	55.853	74.000	74.060
0.2323	55.000	55.208	55.498	55.780	74.000	74.035
0.1882	55.000	55.215	55.514	55.804	74.000	74.043
0.2087	55.000	55.237	55.569	55.889	74.000	74.072
0.3145	55.000	55.279	55.669	56.047	74.000	74.127
0.5225	55.000	55.346	55.828	56.297	74.000	74.215
0.7704	55.000	55.401	55.958	56.501	74.000	74.286
1.0412	55.000	55.464	56.108	56.736	74.000	74.369
1.3664	55.000	55.540	56.288	57.021	74.000	74.469
1.6087	55.000	55.559	56.334	57.094	74.000	74.495
1.7569	55.000	55.577	56.376	57.160	74.000	74.518
1.8386	55.000	55.580	56.383	57.170	74.000	74.521
1.8750	55.000	55.580	56.384	57.171	74.000	74.521
1.8834	55.000	55.577	56.377	57.160	74.000	74.517
1.7547	55.000	55.515	56.231	56.928	74.000	74.436
1.5322	55.000	55.460	56.101	56.723	74.000	74.363
1.3111	55.000	55.420	56.005	56.572	74.000	74.310
1.1195	55.000	55.381	55.912	56.425	74.000	74.259
0.9538	55.000	55.351	55.842	56.316	74.000	74.221
0.7975	55.000	55.316	55.758	56.184	74.000	74.175
0.6911	55.000	55.310	55.743	56.161	74.000	74.167
0.5898	55.000	55.281	55.673	56.052	74.000	74.129
0.4961	55.000	55.266	55.638	55.998	74.000	74.110
0.4040	55.000	55.244	55.584	55.914	74.000	74.081
0.3174	55.000	55.228	55.545	55.853	74.000	74.060
0.2334	55.000	55.208	55.498	55.780	74.000	74.035
0.1892	55.000	55.215	55.514	55.804	74.000	74.043
0.2094	55.000	55.237	55.569	55.889	74.000	74.072
0.3152	55.000	55.279	55.669	56.047	74.000	74.127
0.5230	55.000	55.346	55.828	56.297	74.000	74.215
0.7708	55.000	55.401	55.958	56.501	74.000	74.286
1.0415	55.000	55.464	56.108	56.736	74.000	74.369
1.3666	55.000	55.540	56.288	57.021	74.000	74.469
1.6089	55.000	55.559	56.334	57.094	74.000	74.495
1.7571	55.000	55.577	56.376	57.160	74.000	74.518
1.8388	55.000	55.580	56.383	57.170	74.000	74.521
1.8751	55.000	55.580	56.384	57.171	74.000	74.521
1.8835	55.000	55.577	56.377	57.160	74.000	74.517
1.7548	55.000	55.515	56.231	56.928	74.000	74.436
1.5323	55.000	55.460	56.101	56.723	74.000	74.363
1.3112	55.000	55.420	56.005	56.572	74.000	74.310
1.1195	55.000	55.381	55.912	56.425	74.000	74.259
0.9538	55.000	55.351	55.842	56.316	74.000	74.221
0.7975	55.000	55.316	55.758	56.184	74.000	74.175
0.6912	55.000	55.310	55.743	56.161	74.000	74.167
0.5898	55.000	55.281	55.673	56.052	74.000	74.129
0.4961	55.000	55.266	55.638	55.998	74.000	74.110

TABLE X1.4 *Continued*

Ceiling Heat Flux Btu/h-ft ²	Temperatures, F Nodes 1–6					
0.4040	55.000	55.244	55.584	55.914	74.000	74.081
0.3174	55.000	55.228	55.545	55.853	74.000	74.060
0.2334	55.000	55.208	55.498	55.780	74.000	74.035
0.1892	55.000	55.215	55.514	55.804	74.000	74.043
0.2094	55.000	55.237	55.569	55.889	74.000	74.072
0.3152	55.000	55.279	55.669	56.047	74.000	74.127
0.5230	55.000	55.346	55.828	56.297	74.000	74.215
0.7708	55.000	55.401	55.958	56.501	74.000	74.286
1.0415	55.000	55.464	56.108	56.736	74.000	74.369
1.3666	55.000	55.540	56.288	57.021	74.000	74.469
1.6089	55.000	55.559	56.334	57.094	74.000	74.495
1.7571	55.000	55.577	56.376	57.160	74.000	74.518
1.8388	55.000	55.580	56.383	57.170	74.000	74.521
1.8751	55.000	55.580	56.384	57.171	74.000	74.521
1.8835	55.000	55.577	56.377	57.160	74.000	74.517
1.7548	55.000	55.515	56.231	56.928	74.000	74.436
1.5323	55.000	55.460	56.101	56.723	74.000	74.363
1.3112	55.000	55.420	56.005	56.572	74.000	74.310
1.1195	55.000	55.381	55.912	56.425	74.000	74.259
0.9538	55.000	55.351	55.842	56.316	74.000	74.221
0.7975	55.000	55.316	55.758	56.184	74.000	74.175
0.6912	55.000	55.310	55.743	56.161	74.000	74.167

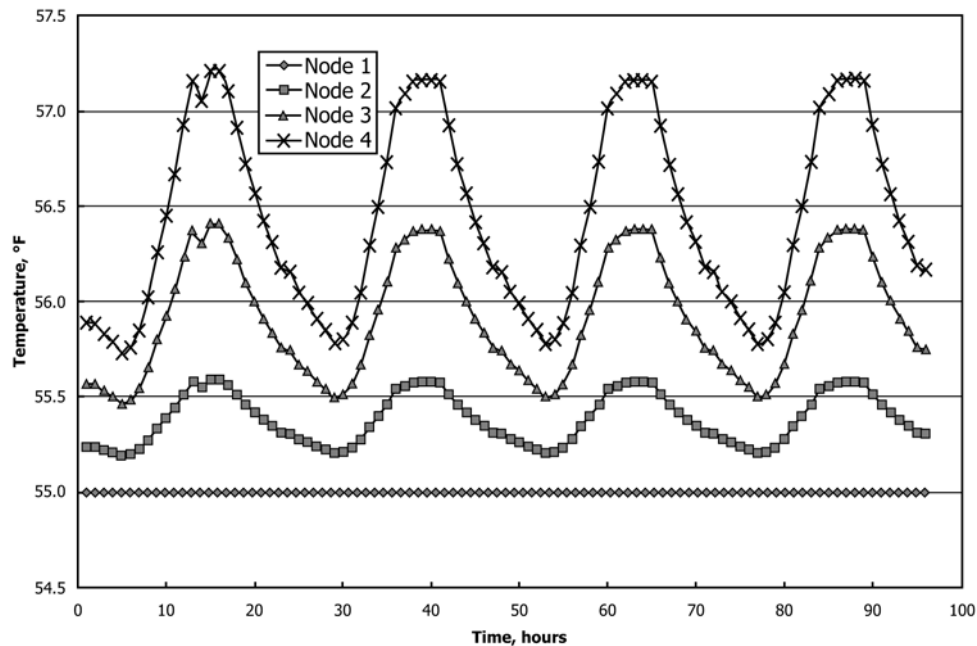


FIG. X1.1 Hourly Duct Temperatures Calculated for the Example Problem

foil-covered fiberglass batt duct insulation. The duct was installed in a U-shape with long runs near the eaves and a shorter run along one gable. The duct system was 31 ft (9.4 m) long, including two 90 degree elbows. Tests were conducted under both summer and winter conditions, with and without a radiant barrier mounted under the roof, and with varying levels

of ventilation. Duct air temperature changes from duct inlet to outlet ranged from 0.9°F to 1.5°F (0.5°C to 0.8°C). Differences between model predictions and measured duct air temperature changes averaged +0.16°F (0.09°C) at the mild winter conditions and +0.02°F (+0.01°C) at severe summer conditions.

REFERENCES

- (1) Mitalas, G. P., and Arseneault, J. G., "Fortran IV Program to Calculate Heat Flux Response Factors for Multi-Layer Slabs," DBR Computer Program No. 23, Division of Building Research, National Research Council of Canada, 1967.
- (2) Kusuda, T., "Thermal Response Factors for Multi-Layer Structures of Various Heat Conduction Systems," *ASHRAE Trans.*, Vol 75, Part 1, pp. 246–271 (1969).
- (3) Peavy, B. A., "A Model for Predicting the Thermal Performance of Ventilated Attics," in *Summer Attic and Whole-House Ventilation*, NBS Special Publication 548, pp. 119–149 (1979).
- (4) Wilkes, K. E., "Modeling of Residential Attics," *Thermal Performance of the Exterior Envelopes of Buildings*, ASHRAE SP 28, American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., pp. 436–455 (1979).
- (5) Wilkes, K. E., "Dynamic Thermal Performance of Walls and Ceilings/Attics," *Proceedings, ASHRAE/DOE Conference on Thermal Performance of the Exterior Envelopes of Buildings II*, ASHRAE SP 38, pp. 131–159 (1983).
- (6) Wilkes, K. E., "Thermal Model of Attic Systems with Radiant Barriers," ORNL/CON-262, Oak Ridge National Laboratory, Oak Ridge, Tennessee, 1991.
- (7) Walton, G., NIST, 1987.
- (8) Holman, J. P., *Heat Transfer*, 5th ed., pp. 191, 202, 272–286 (1981).
- (9) Hilsenrath, J., et al., "Tables of Thermodynamic and Transport Properties of Air, Argon, Carbon Dioxide, Carbon Monoxide, Hydrogen, Nitrogen, Oxygen, and Steam," NBS Circular 564, U.S. Dept. of Commerce (1960).
- (10) Chen, T. S., Armaly, B. F., and Ramachandran, N., "Correlations for Laminar Mixed convection Flows on Vertical, Inclined, and Horizontal Flat Plates," *ASME Journal of Heat Transfer*. Vol 108, pp. 835–840 (1986).
- (11) Sparrow, E. M., and Cess, R. D., *Radiation Heat Transfer*, pp. 86–90, Brooks/Cole Publishing Company, Belmont, CA (1966).
- (12) Feingold, A., "Radiant-Interchange Configuration Factors Between Various Selected Plane Surfaces," *Proc. Roy. Soc., A*, Vol 292, pp. 51–60 (1965).
- (13) Chapter 22, *ASHRAE Handbook of Fundamentals*, American Society of Heating, Refrigerating, and Air Conditioning Engineers, Atlanta, GA (1985).
- (14) Burch, D. M., and Treado, S. J., "Ventilating Residences and Their Attics for Energy Conservation—An Experimental Study," in *Summer Attic and Whole-House Ventilation*, NBS Special Publication 548, pp. 73–104 (1979).
- (15) Hinrichs, H. S., "Comparative Study of the Effectiveness of Fixed Ventilating Louvers," *ASHRAE Trans.*, Vol 68, pp. 297–309 (1962).
- (16) Chandra, S., Fairey III, P. W., Kerestecioglu, A. A., and Kamel, A. A., "Wind Tunnel and Full-Scale Data on Airflow from Natural Ventilation and Ceiling Fans," *ASHRAE Trans.*, Vol 92, Part 2B, pp. 804–814 (1986).
- (17) Burch, D. M., Lemay, M. R., Rian, B. J., and Parker, E. J., "Experimental Validation of an Attic Condensation Model," *ASHRAE Trans.*, Vol 20, Part 2A, pp. 59–77, 1984.
- (18) Cleary, P. G., "Moisture Control by Attic Ventilation—An In-Situ Study," *ASHRAE Trans.*, Vol 91, Part 1, pp. 227–239 (1985).
- (19) Kusuda, T., *NBSLD, the Computer Program for Heating and Cooling Loads in Buildings*, NBS Building Science Series 69, pp. 52d, U.S. Department of Commerce (July 1976).
- (20) Modera, M. P., Jansky, R., "Residential Air-Distribution Systems: Interactions with the Building Envelope," ASHRAE/DOE/BTECC Conference on Thermal Performance of the Exterior Envelopes of Buildings V, pp. 623–631, 1992.
- (21) Ober, D. G., and Wilkes, K. E., "Final Report on ASHRAE RP 717, An Energy Calculation Model for Attics, Including Radiant Barriers" 1997.
- (22) Jakob, F. E., Fischer, R. D., and Flanigan, R. D., "Experimental Validation of the Duct Submodel for the SP43 Simulation Model," *ASHRAE Trans.*, pp. 1499–1514, 1987.
- (23) Holman, J. P., *Heat Transfer*, 5th Ed., pp. 243–247, 1981.
- (24) Petrie, T. W., Wilkes, K. E., Childs, P. W., and Christian, J. E., "Effects of Radiant Barriers and Attic Ventilation on Residential Attics and Attic Duct Systems: New Tools for Measuring and Modeling," *ASHRAE Trans.*, Vol 104, Part 2, pp. 1175–1192, 1998.

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