



Standard Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio by Sonic Resonance¹

This standard is issued under the fixed designation E1875; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This test method covers the determination of the dynamic elastic properties of elastic materials. Specimens of these materials possess specific mechanical resonant frequencies that are determined by the modulus of elasticity, mass, and geometry of the test specimen. Therefore, the dynamic elastic properties of a material can be computed if the geometry, mass, and mechanical resonant frequencies of a suitable test specimen of that material can be measured. Dynamic Young's modulus is determined using the resonant frequency in the flexural mode of vibration. The dynamic shear modulus, or modulus of rigidity, is found using torsional resonant vibrations. Dynamic Young's modulus and dynamic shear modulus are used to compute Poisson's ratio.

1.2 This test method is specifically appropriate for materials that are elastic, homogeneous, and isotropic (1).² Materials of a composite character (particulate, whisker, or fiber reinforced) may be tested by this test method with the understanding that the character (volume fraction, size, morphology, distribution, orientation, elastic properties, and interfacial bonding) of the reinforcement in the test specimen will have a direct effect on the elastic properties. These reinforcement effects must be considered in interpreting the test results for composites. This test method is not satisfactory for specimens that have cracks or voids that are major discontinuities in the specimen. Neither is the test method satisfactory when these materials cannot be fabricated in a uniform rectangular or circular cross section.

1.3 A high-temperature furnace and cryogenic cabinet are described for measuring the dynamic elastic moduli as a function of temperature from –195 to 1200°C.

1.4 Modification of this test method for use in quality control is possible. A range of acceptable resonant frequencies is determined for a specimen with a particular geometry and

mass. Any specimen with a frequency response falling outside this frequency range is rejected. The actual modulus of each specimen need not be determined as long as the limits of the selected frequency range are known to include the resonant frequency that the specimen must possess if its geometry and mass are within specified tolerances.

1.5 There are material-specific ASTM standards that cover the determination of resonance frequencies and elastic properties of specific materials by sonic resonance or by impulse excitation of vibration. Test Methods C215, C623, C747, C848, C1198, and C1259 may differ from this test method in several areas (for example; sample size, dimensional tolerances, sample preparation). The testing of these materials shall be done in compliance with these material specific standards. Where possible, the procedures, sample specifications, and calculations are consistent with these test methods.

1.6 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 ASTM Standards:³

C215 Test Method for Fundamental Transverse, Longitudinal, and Torsional Resonant Frequencies of Concrete Specimens

C623 Test Method for Young's Modulus, Shear Modulus, and Poisson's Ratio for Glass and Glass-Ceramics by Resonance

C747 Test Method for Moduli of Elasticity and Fundamental Frequencies of Carbon and Graphite Materials by Sonic Resonance

¹ This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.04 on Uniaxial Testing.

Current edition approved Nov. 1, 2013. Published May 2014. Originally approved in 1997. Last previous edition approved in 2008 as E1875-08. DOI: 10.1520/E1875-13.

² The boldface numbers in parentheses refer to a list of references at the end of this standard.

³ For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

*A Summary of Changes section appears at the end of this standard

- C848** Test Method for Young's Modulus, Shear Modulus, and Poisson's Ratio For Ceramic Whitewares by Resonance
- C1198** Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio for Advanced Ceramics by Sonic Resonance
- C1259** Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio for Advanced Ceramics by Impulse Excitation of Vibration
- E6** Terminology Relating to Methods of Mechanical Testing
- E177** Practice for Use of the Terms Precision and Bias in ASTM Test Methods
- E691** Practice for Conducting an Interlaboratory Study to Determine the Precision of a Test Method

3. Terminology

3.1 *Definitions*: Terms common to mechanical testing.

3.1.1 *dynamic mechanical measurement, n*—a technique in which either the modulus or damping, or both, of a substance under oscillatory applied force or displacement is measured as a function of temperature, frequency, or time, or a combination thereof.

3.1.2 *elastic limit* [FL^{-2}], *n*—the greatest stress that a material is capable of sustaining without permanent strain remaining upon complete release of the stress.

3.1.2.1 *Discussion*—Due to practical considerations in determining the elastic limit, measurements of strain using a small force, rather than zero force, are usually taken as the initial and final reference. **(E6)**

3.1.3 *modulus of elasticity* [FL^{-2}], *n*—the ratio of stress to corresponding strain below the proportional limit.

3.1.3.1 *Discussion*—The stress-strain relationships of many materials do not conform to Hooke's law throughout the elastic range, but deviate therefrom even at stresses well below the elastic limit. For such materials, the slope of either the tangent to the stress-strain curve at the origin or at a low stress, the secant drawn from the origin to any specified point on the stress-strain curve, or the chord connecting any two specified points on the stress-strain curve is usually taken to be the "modulus of elasticity." In these cases, the modulus should be designated as the "tangent modulus," the "secant modulus," or the "chord modulus," and the point or points on the stress-strain curve described. Thus, for materials where the stress-strain relationship is curvilinear rather than linear, one of the four following terms may be used:

(a) *initial tangent modulus* [FL^{-2}], *n*—the slope of the stress-strain curve at the origin.

(b) *tangent modulus* [FL^{-2}], *n*—the slope of the stress-strain curve at any specified stress or strain.

(c) *secant modulus* [FL^{-2}], *n*—the slope of the secant drawn from the origin to any specified point on the stress-strain curve.

(d) *chord modulus* [FL^{-2}], *n*—the slope of the chord drawn between any two specified points on the stress-strain curve below the elastic limit of the material.

3.1.3.2 *Discussion*—Modulus of elasticity, like stress, is expressed in force per unit of area (pounds per square inch, etc.).

3.1.4 *Poisson's ratio, μ , n* —the negative of the ratio of transverse strain to the corresponding axial strain resulting from an axial stress below the proportional limit of the material.

3.1.4.1 *Discussion*—Poisson's ratio may be negative for some materials, for example, a tensile transverse strain will result from a tensile axial strain.

3.1.4.2 *Discussion*—Poisson's ratio will have more than one value if the material is not isotropic. **(E6)**

3.1.5 *proportional limit* [FL^{-2}], *n*—the greatest stress that a material is capable of sustaining without deviation from proportionality of stress to strain (Hooke's law).

3.1.5.1 *Discussion*—Many experiments have shown that values observed for the proportional limit vary greatly with the sensitivity and accuracy of the testing equipment, eccentricity of loading, the scale to which the stress-strain diagram is plotted, and other factors. When determination of proportional limit is required, the procedure and the sensitivity of the test equipment should be specified. **(E6)**

3.1.6 *shear modulus (G)* [FL^{-2}], *n*—the ratio of shear stress to corresponding shear strain below the proportional limit, also called *torsional modulus* and *modulus of rigidity*.

3.1.6.1 *Discussion*—The value of the shear modulus may depend on the direction in which it is measured if the material is not isotropic. Wood, many plastics and certain metals are markedly anisotropic. Deviations from isotropy should be suspected if the shear modulus differs from that determined by substituting independently measured values of Young's modulus, *E*, and Poisson's ratio, μ , in the relation:

$$G = E/[2(1 + \mu)]$$

3.1.6.2 *Discussion*—In general, it is advisable in reporting values of shear modulus to state the range of stress over which it is measured.

3.1.7 *Young's modulus (E)* [FL^{-2}], *n*—the ratio of tensile or compressive stress to corresponding strain below the proportional limit of the material. **(E6)**

3.2 *Definitions of Terms Specific to This Standard*:

3.2.1 *anti-nodes, n*—two or more locations in an unconstrained slender rod or bar in resonance that have local maximum displacements.

3.2.1.1 *Discussion*—For the fundamental flexure resonance, the anti-nodes are located at the two ends and the center of the specimen.

3.2.2 *elastic, adj*—the property of a material such that an application of stress within the elastic limit of that material making up the body being stressed will cause an instantaneous and uniform deformation that will be eliminated upon removal of the stress, with the body returning instantly to its original size and shape without energy loss.

3.2.2.1 *Discussion*—Most elastic materials conform to this definition well enough to make this resonance test valid.

3.2.3 *flexural vibrations, n*—oscillations that occur in a slender rod or bar in a vertical plane normal to the length dimension.

3.2.4 *homogeneous, adj*—the condition of a specimen such that the composition and density are uniform, such that any smaller specimen taken from the original is representative of the whole.

3.2.4.1 *Discussion*—Practically, as long as the geometrical dimensions of the test specimen are large with respect to the size of individual grains, crystals, or components, the body can be considered homogeneous.

3.2.5 *isotropic, adj*—the condition of a specimen such that the values of the elastic properties are the same in all directions in the material.

3.2.5.1 *Discussion*—Materials are considered isotropic on a macroscopic scale, if they are homogeneous and there is a random distribution and orientation of phases, crystallites, and components.

3.2.6 *nodes, n*—one or more locations of a slender rod or bar in resonance that have a constant zero displacement.

3.2.6.1 *Discussion*—For the fundamental flexural resonance, the nodes are located at $0.224 L$ from each end, where L is the length of the specimen.

3.2.7 *resonance, n*—state of slender rod or bar driven into one of the modes of vibration described in 3.2.3 or 3.2.9 when the imposed frequency is such that the resultant displacements for a given amount of driving force are at a maximum.

3.2.7.1 *Discussion*—The resonant frequencies are natural vibration frequencies that are determined by the modulus of elasticity, mass, and dimensions of the test specimen.

3.2.8 *slender rod or bar, n*—in dynamic elastic property testing, a specimen whose ratio of length to minimum cross-sectional dimension is at least five and preferably in the range from 20 to 25.

3.2.9 *torsional vibrations, n*—oscillations that occur in each cross-sectional plane of a slender rod or bar, such that the plane twists around the length dimension axis.

4. Summary of Test Method

4.1 This test method measures the resonant frequencies of test specimens of suitable geometry by exciting them at continuously variable frequencies. Mechanical excitation of the bars is provided through the use of a transducer that transforms a cyclic electrical signal into a cyclic mechanical force on the specimen. A second transducer senses the resulting mechanical vibrations of the specimen and transforms them into an electrical signal. The amplitude and frequency of the signal are measured by an oscilloscope or other means to detect resonance. The resonant frequencies, dimensions, and mass of the specimen are used to calculate dynamic Young's modulus and dynamic shear modulus.

5. Significance and Use

5.1 This test method has advantages in certain respects over the use of static loading systems for measuring moduli.

5.1.1 This test method is nondestructive in nature. Only minute stresses are applied to the specimen, thus minimizing the possibility of fracture.

5.1.2 The period of time during which measurement stress is applied and removed is of the order of hundreds of

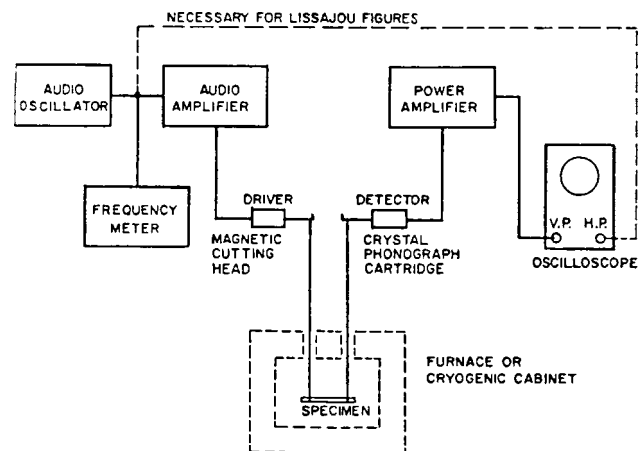


FIG. 1 Block Diagram of a Typical Test Apparatus

microseconds. With this test method it is feasible to perform measurements at high temperatures, where delayed elastic and creep effects would invalidate modulus of elasticity measurements calculated from static loading.

5.2 This test method is suitable for detecting whether a material meets the specifications, if cognizance is given to one important fact in materials are often sensitive to thermal history. Therefore, the thermal history of a test specimen must be considered in comparing experimental values of moduli to reference or standard values. Specimen descriptions should include any specific thermal treatments that the specimens have received.

6. Apparatus

6.1 The test apparatus is shown in Fig. 1. It consists of a variable-frequency audio oscillator, used to generate a sinusoidal voltage, and a power amplifier and suitable transducer to convert the electrical signal to a mechanical driving vibration. A frequency meter (preferably digital) monitors the audio oscillator output to provide an accurate frequency determination. A suitable suspension-coupling system supports the test specimen. Another transducer acts to detect mechanical vibration in the specimen and to convert it into an electrical signal that is passed through an amplifier and displayed on an indicating meter. The meter may be a voltmeter, microammeter, or oscilloscope. An oscilloscope is recommended because it enables the operator to positively identify resonances, including higher order harmonics, by Lissajous figure analysis. If a Lissajous figure is desired, the output of the oscillator is also coupled to the horizontal plates of the oscilloscope. If temperature-dependent data are desired, a suitable furnace or cryogenic chamber is used. Details of the equipment are as follows:

6.2 *Audio Oscillator*, having a continuously variable frequency output from about 100 Hz to at least 30 kHz. Frequency drift shall not exceed 1 Hz/min for any given setting.

6.3 *Audio Amplifier*, having a power output sufficient to ensure that the type of transducer used can excite any specimen the mass of which falls within a specified range.

6.4 Transducers—Two are required; one used as a driver may be a speaker of the tweeter type or a magnetic cutting head or other similar device depending on the type of coupling chosen for use between the transducer and the specimen. The other transducer, used as a detector, may be a crystal or magnetic reluctance type of phonograph cartridge. A capacitive pickup may be used if desired. An electromagnetic coupling system with an attached metal foil may also be used, with due consideration for effects of the foil on the natural vibration of the test bar. The frequency response of the transducer across the frequency range of interest shall have at least a 6.5 kHz bandwidth before -3 dB power loss occurs.

6.5 Power Amplifier, in the detector circuit shall be impedance matched with the type of detector transducer selected and shall serve as a prescope amplifier.

6.6 Oscilloscope, any model suitable for general laboratory work.

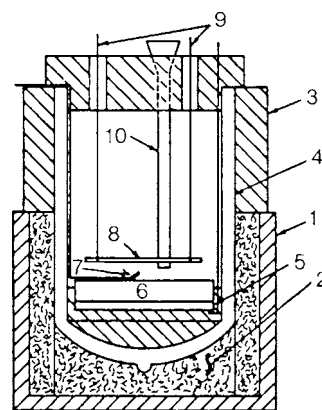
6.7 Frequency Counter, preferably digital, shall be able to measure frequencies to within ± 1 Hz.

6.8 Furnace—If data at an elevated temperature are desired, a furnace shall be used that is capable of controlled heating and cooling. It shall have a specimen zone large enough for the specimen to be uniform in temperature within $\pm 5^\circ\text{C}$ along its length through the range of temperatures encountered in testing. It is recommended that an independent thermocouple be placed in close proximity to (within 5 mm), but not touching, the center of the specimen to accurately measure temperature during heating and cooling.

6.9 Cryogenic Chamber—For data at cryogenic temperatures, any chamber shall suffice that shall be capable of controlled heating/cooling, frost-free and uniform in temperature within $\pm 5^\circ\text{C}$ over the length of the specimen at any selected temperature. A suitable cryogenic chamber is shown in Fig. 2 (2). It is recommended that an independent thermocouple be placed in close proximity to (within 5 mm), but not touching, the center of the specimen to accurately measure temperature during heating and cooling.

6.10 Specimen Suspension—Any method of specimen suspension shall be used that is adequate for the temperatures encountered in testing and that allows the specimen to vibrate without significant restriction. Thread suspension is the system of choice for cryogenic and high-temperature testing. (See Fig. 1 and Fig. 3.) Common cotton thread, silica-glass fiber thread, oxidation-resistant nickel (or platinum) alloy wire, or platinum wire may be used. If metal wire suspension is used in the furnace, coupling characteristics will be improved if, outside the temperature zone, the wire is coupled to cotton thread, and the thread is coupled to the transducer. The specimen should be initially suspended at distances of approximately $0.1 L$ from each end. The specimen should not be suspended at its fundamental flexural node locations ($0.224 L$ from each end). The suspension point distances can be adjusted experimentally to maximize the vibrational deflection and resulting signal. For torsional vibration, the axes of suspension shall be off-center from the longitudinal axis of the specimen (shown in Fig. 3).

6.11 Specimen Supports—If the specimen is supported on direct contact supports, the supports shall permit the specimen



- NOTE 1—Legend:
 1 = Cylindrical glass jar
 2 = Glass wool
 3 = Plastic foam
 4 = Vacuum jar
 5 = Heater disk
 6 = Copper plate
 7 = Thermocouple
 8 = Sample
 9 = Suspension wires
 10 = Fill port for liquid

FIG. 2 Detail Drawing of a Typical Cryogenic Chamber

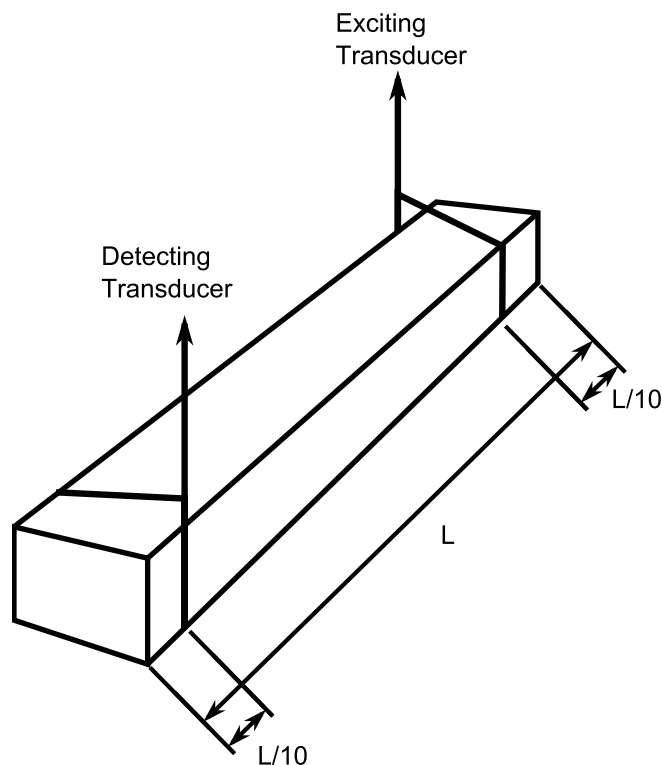


FIG. 3 Specimen Positioned for Measurement of Flexural and Torsional Resonant Frequencies Using Thread or Wire Suspension

to oscillate without significant restriction in the desired mode. This is accomplished for flexural modes by supporting the specimen at its transverse fundamental node locations ($0.224 L$ from each end). In torsional modes the specimen should be supported at its center point. The supports should have minimal area in contact with the specimen and shall be cork, rubber, or

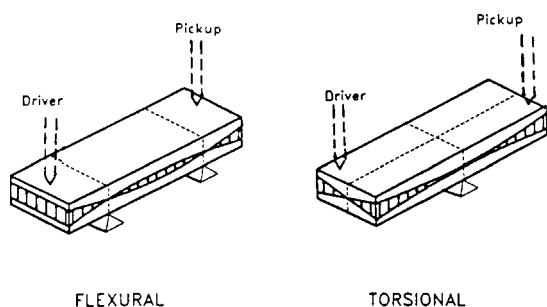


FIG. 4 Specimen Positioned for Measurement of Flexural and Torsional Resonant Frequencies Using Direct Support and Direct Contact Transducers

similar material. In order to properly identify resonant frequencies, the transducers should be movable along the total specimen length and width. See Fig. 4. The transducer contact pressure should be consistent with good response and minimal interference with the free vibration of the specimen.

7. Test Specimen

7.1 Prepare the specimens so that they are either rectangular or circular in cross section. Either geometry may be used to measure both dynamic Young's modulus and dynamic shear modulus. However, experimental difficulties in obtaining torsional resonant frequencies for a cylindrical specimen usually preclude its use in determining shear modulus, although the equations for computing shear modulus with a cylindrical specimen are both simpler and more accurate than those used with a rectangular bar.

7.2 Resonant frequencies for a given specimen are functions of the bar dimensions as well as its mass and moduli; therefore, dimensions should be selected with this relationship in mind. Make selection of size so that, for an estimated modulus of elasticity, the resonant frequencies measured will fall within the range of frequency response of the transducers used. A slender rod with a ratio of length to minimum cross-sectional dimension greater than 5 and approximately 25 is preferred for ease in calculation. For shear modulus measurements of rectangular bars, a ratio of width to thickness of five is recommended for minimizing experimental difficulties.

7.2.1 These specimen sizes should produce a fundamental flexural resonant frequency in the range from 1000 to 10 000 Hz and a fundamental torsional resonant frequency in the range from 10 000 to 30 000 Hz. Specimens shall have a minimum mass of 5 g to avoid coupling effects; any size of specimen that has a suitable length-to-cross section ratio in terms of frequency response and meets the mass minimum may be used. Maximum specimen size and mass are determined primarily by the power of the test system and physical space capabilities.

7.3 All surfaces on the rectangular specimen shall be flat. Opposite surfaces across the length, thickness, and width shall be parallel to within 0.1 %. The cylindrical specimen shall be round and constant in diameter to within 0.1 %.

7.4 Specimen mass shall be determined to within 0.1 %

7.5 Specimen length shall be measured to within 0.1 %. The thickness and width of the rectangular specimen shall be

TABLE 1 Effects of Variable Error on Modulus Calculation

Variable	Measurement Error	Variable Exponent in Modulus Equation	Calculation Error
Frequency (f)	0.1 %	f^2	0.2 %
Length (L)	0.1 %	L^3	0.3 %
Mass (m)	0.1 %	m	0.1 %
Width (b)	0.1 %	b^{-1}	0.1 %
Thickness (t)	0.1 %	t^{-3}	0.3 %
Diameter (D)	0.1 %	D^{-4}	0.4 %

measured to within 0.1 % at three locations and an average determined. The diameter of the cylindrical specimen shall be measured to within 0.1 % at three locations and an average determined.

7.6 Table 1 illustrates how uncertainties in the measured parameters influence the calculated modulus of elasticity. It shows that calculations are most sensitive to error in the measurement of the thickness. Special care must be taken when measuring the thickness of samples with a thickness of less than 3 mm.

8. Procedure

8.1 Procedure A—Room-Temperature Testing:

8.1.1 Switch on all electrical equipment and allow to stabilize in accordance with the manufacturer's recommendations. Suspend or support the specimen properly (see Fig. 3 and Fig. 4). Activate the equipment so that power adequate to excite the specimen is delivered to the driving transducer. Set the gain of the detector circuit high enough to detect vibration in the specimen and to display it on the oscilloscope screen with sufficient amplitude to measure accurately the frequency at which the signal amplitude is maximized. Adjust the oscilloscope so that a sharply defined horizontal baseline exists when the specimen is not excited. Scan frequencies with the audio oscillator until specimen resonance is indicated by a sinusoidal pattern of maximum amplitude on the oscilloscope or by a single closed loop Lissajous pattern. (It is recommended that the frequency scan start at a low frequency and then increase.) To verify that the frequency is fundamental and not an overtone, either the node/anti-node locations or one or more overtones should be identified (see Note 1). If a determination of the shear modulus is made, offset the coupling to the transducers so that the torsional mode of vibration may be induced and detected. (See Fig. 3 and Fig. 4.)

NOTE 1—The proper identification of the fundamental flexural mode is important, because spurious frequencies inherent in the system may interfere, especially when greater excitation power and detection sensitivity are required for work with a specimen that has a poor response. The location of the nodes for the fundamental and the first four overtones are indicated in Fig. 5. One method to locate the nodes on the specimen is to move the detector along the length of the specimen; a node is indicated when the output amplitude goes to zero. An anti-node is indicated when the output amplitude reaches a local maximum. Another node location method (used often with string suspensions) is to lay a thin rod across the specimen at a presumed node or anti-node location. If the output amplitude is not affected, then the rod is on a node; if the output amplitude goes to zero, then the location is an anti-node. When several resonant flexural frequencies have been identified, the lowest frequency can be verified as the fundamental, if the numerical ratios of the first three overtone frequencies to the lowest frequency are 2.7, 5.4, and 8.9. Note that these ratios are for a Bernoulli-Euler (simple) beam under ideal

HARMONIC	FLEXURAL	FRACTIONAL DISTANCE FROM END FOR EACH NODAL LOCATION
1F		0.224
2F		0.132, 0.500
3F		0.0944, 0.3558
4F		0.0733, 0.277, 0.500
5F		0.060, 0.224, 0.409
TORSIONAL		
1T		0.500
2T		0.250
3T		0.167, 0.500
4T		0.125, 0.375

FIG. 5 Dynamic Modulus Resonant Modes and Nodal Locations Tracking Guide Template

conditions. Typically the ratios will be slightly lower.

8.1.2 Find and verify (see **Note 2**) the fundamental resonant frequency in the torsion mode. The dimensions and mass of the specimen may be measured before or after the test.

NOTE 2—Identification of the fundamental torsional mode is based on the same approaches used in identifying the flexural modes; node identification or frequency ratios, or both. **Fig. 5** locates the node positions for torsional vibrations. The ratios of the first three torsional overtones to the fundamental torsional frequency are 2, 3, and 4.

8.2 *Procedure B—Elevated-Temperature Testing*—Determine the mass, dimensions, and resonant frequencies at room temperature in air as outlined in **8.1**. Place the specimen in the furnace and adjust the driver-detector system so that all the frequencies to be measured can be detected without further adjustment. Determine the resonant frequencies at room temperature in the furnace cavity with the furnace doors closed, etc., as will be the case at elevated temperatures. Heat the furnace at a controlled rate that does not exceed 150°C/h. Take data at 25°C intervals or at 15 min intervals as dictated by heating rate and specimen composition. Follow the change in resonant frequencies with time and temperature closely to avoid losing the identity of each frequency. (The overtones in flexure and the fundamental in torsion may be difficult to differentiate if not followed closely; spurious frequencies inherent in the system may also appear at temperatures above 600°C using certain types of suspensions, particularly wire.) If desired, data may also be taken on cooling. It must be remembered, however, that high temperatures may alter the specimen either reversibly or permanently (for example, phase change, devitrification, or microcracking). Such potential changes should be considered in planning the range of test temperatures and in interpreting test results as a function of temperature. Dimensions and mass of the specimen should be measured both before and after the test to check for permanent thermal effects. Measurements should be made to the precision described in **8.1**.

8.3 *Procedure C—Cryogenic Testing*—Determine the mass, dimensions, and resonant frequencies in air at room temperature, as outlined in **8.1**. Measure the resonant frequencies at room temperature in the cryogenic chamber. Take the chamber to the minimum temperature desired (see warning statement below), monitoring frequencies as the chamber is

cooled. Allow the specimen to stabilize at minimum temperature for at least 15 min. Heating rate should not exceed 50°C/h, and data may be taken at intervals of 10 min or 15°C or as desired. Dimensions and mass of the specimen should be measured both before and after the test to check for permanent thermal effects. Measurements should be made to the precision described in **8.1**. Take care to remove water vapor from the chamber by flushing with dry nitrogen gas prior to chilling so that frost deposits on the specimen do not cause anomalous results.

9. Calculation

9.1 *Dynamic Young's Modulus (1, 3)*—For the fundamental in flexure of a rectangular bar calculate as follows **(3)**:

$$E = 0.9465 (mf_f^2/b)(L^3/t^3)T_1 \quad (1)$$

where:

- E = Young's modulus, Pa,
- m = mass of the bar, g, (see **Note 3**),
- b = width of the bar, mm, (see **Note 3**),
- L = length of the bar, mm, (see **Note 3**),
- t = thickness of the bar, mm, (see **Note 3**),
- f_f = fundamental resonant frequency of bar in flexure, Hz, and
- T_1 = correction factor for fundamental flexural mode to account for finite thickness of bar, Poisson's ratio, and so forth.

and:

$$T_1 = 1 + 6.585 (1 + 0.0752 \mu + 0.8109 \mu^2)(t/L)^2 - 0.868 (t/L)^4 \quad (2)$$

$$- \left[\frac{8.340 (1 + 0.2023 \mu + 2.173 \mu^2)(t/L)^4}{1.000 + 6.338 (1 + 0.1408 \mu + 1.536 \mu^2)(t/L)^2} \right]$$

where:

μ = Poisson's ratio.

NOTE 3—In the modulus of elasticity equations the mass and length terms are given in units of grams and millimetres. However, the defined equations can also be used with mass and length terms in units of kilograms and metres with no changes in terms or exponents.

9.1.1 If $L/t \geq 20$, T_1 can be simplified to:

$$T_1 = [1.000 + 6.585 (t/L)^2] \quad (3)$$

and E can be calculated directly.

9.1.2 If $L/t < 20$ and Poisson's ratio is known, then T_1 can be calculated directly from **Eq 2** and then used to calculate E .

9.1.3 If $L/t < 20$ and Poisson's ratio is not known, then an initial Poisson's ratio must be assumed to start the computations. An iterative process is then used to determine a value of Poisson's ratio, based on experimental Young's modulus and shear modulus.

9.1.3.1 Determine the fundamental flexural and torsional resonant frequency of the rectangular test specimen, as described in **8.1**. Using **Eq 7** and **Eq 8**, calculate the dynamic shear modulus of the test specimen for the fundamental torsional resonant frequency and the dimensions and mass of the specimen.

9.1.3.2 Using **Eq 1** and **Eq 2**, calculate the dynamic Young's modulus of the rectangular test specimen from the fundamental

flexural resonant frequency, the dimensions, and mass of the specimen and the initial/iterative Poisson's ratio. Care must be exercised in using consistent units for all the parameters throughout the computations.

9.1.3.3 The dynamic shear modulus and Young's modulus values calculated in 9.1.3.1 and 9.1.3.2 are substituted into Eq 11 for Poisson's ratio satisfying conditions. A new value for Poisson's ratio is calculated for another iteration starting at 9.1.3.2.

9.1.3.4 The steps in 9.1.3.2 through 9.1.3.3 are repeated until no significant difference (2 % or less) is observed between the last iterative value and the final computed value of the Poisson's ratio.

9.1.3.5 Self-consistent values for the moduli are thus obtained.

9.1.4 For the fundamental in flexure of a rod of circular cross section calculate as follows (3)

$$E = 1.6067 (L^3/D^4) (m f_f^2) T_1' \quad (4)$$

where:

D = diameter of rod, mm, (see Note 3), and

T_1' = correction factor for fundamental flexural mode to account for finite diameter of rod, Poisson's ratio, and so forth.

and

$$T_1' = 1 + 4.939(1 + 0.0752 \mu + 0.8109 \mu^2) (D/L)^2 - 0.4883 (D/L)^4 - \left[\frac{4.691(1 + 0.2023 \mu + 2.173 \mu^2)(D/L)^4}{1.000 + 4.754(1 + 0.1408 \mu + 1.536 \mu^2)(D/L)^2} \right] \quad (5)$$

9.1.4.1 If $L/D \geq 20$, the T_1 can be simplified to the following:

$$T_1' = [1.000 + 4.939 (D/L)^2] \quad (6)$$

9.1.4.2 If $L/D < 20$ and Poisson's ratio is known, then T_1' can be calculated directly from Eq 5 and then used to calculate E .

9.1.4.3 If $L/D < 20$ and Poisson's ratio is not known, then an initial Poisson's ratio must be assumed to start the computations. Final values for Poisson's ratio, the dynamic Young's modulus, and dynamic shear modulus are determined, using the same method described in 9.1.3.1 – 9.1.3.5 and the modulus of elasticity equations for circular bars (See Eq 4, and Eq 5, and Eq 10).

9.2 Dynamic Shear Modulus (4):

9.2.1 For the fundamental torsion of a rectangular bar calculate as follows (4):

$$G = \frac{4 L m f_t^2}{b t} [B/(1+A)] \quad (7)$$

where:

G = dynamic shear modulus, Pa,

f_t = fundamental resonant frequency of bar in torsion, Hz,

$$B = \left[\frac{b/t + t/b}{4(t/b) - 2.52(t/b)^2 + 0.21(t/b)^6} \right] \quad (8)$$

and

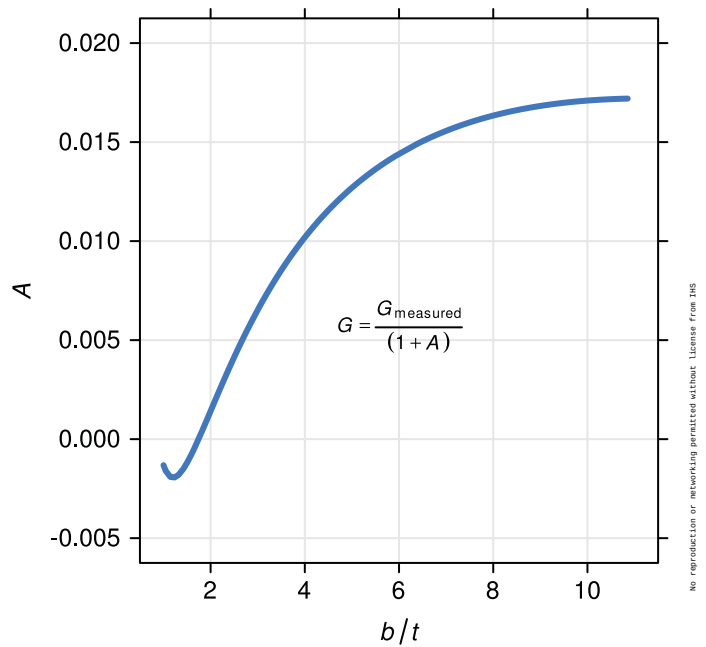


FIG. 6 Plot of the Shear Modulus Correction Term A

A = an empirical correction factor dependent on the width-to-thickness ratio of the test specimen (Ref 5). This correction factor has an effect of less than 2 % and can be omitted, unless accuracies of better than 2 % are desired. (See Fig. 6 for a plot of A as a function of the width-to-thickness ratio.) A rational equation fitted to the points from (5) is given in Eq 9.

$$A = \frac{0.5062 - 0.8776 (b/t) + 0.3504 (b/t)^2 - 0.0078 (b/t)^3}{12.03 (b/t) + 9.892 (b/t)^2} \quad (9)$$

9.2.2 For the fundamental torsion of a cylindrical rod calculate as follows:

$$G = 16 m f_t^2 \left(\frac{L}{\pi D^2} \right) \quad (10)$$

9.3 Calculate Poisson's ratio as follows:

$$\mu = \left(\frac{E}{2G} \right) - 1 \quad (11)$$

where:

μ = Poisson's ratio

E = Young's modulus, and

G = shear modulus.

9.4 Calculate moduli at elevated and cryogenic temperatures as follows:

$$M_T = M_o \left[\frac{f_T}{f_o} \right]^2 \left[\frac{1}{(1 + \alpha \Delta T)} \right] \quad (12)$$

where:

M_T = modulus at temperature T (either Young's modulus, E , or shear modulus, G),

M_o = modulus at room temperature (either Young's modulus, E , or shear modulus, G),

f_T = resonant frequency in furnace or cryogenic chamber at temperature T ,

TABLE 2 Precision Obtained from the Intralaboratory Study(6)

Parameter	Average	Repeatability Standard Deviation	Repeatability Limit
Young's Modulus, GPa	382.95	7.682	21.510
Shear Modulus, GPa	155.51	2.927	8.196
Poisson's ratio	0.231	0.0014	0.0040

- f_o = resonant frequency at room temperature in furnace or cryogenic chamber,
 α = average linear thermal expansion (mm/mm·°C) from room temperature to test temperature, and
 ΔT = temperature differential in °C between the test temperature T and room temperature.

10. Report

10.1 Report the following information:

10.1.1 Identification of specific tests performed and apparatus used, with a description of any deviations from the described test method.

10.1.2 Complete description of material(s) tested stating composition, number of specimens, specimen geometry and mass, specimen history, and any treatments to which the specimens have been subjected. Comments on surface finish, edge conditions, observed changes after cryogenic or high-temperature testing, and so forth shall be included where pertinent.

10.1.3 Name of the person performing the test and date of the test.

10.1.4 Laboratory notebook number and page on which test data are recorded or the computer data file name, if used, or both.

10.1.5 Numerical values obtained for measured resonant frequencies, the dynamic Young's modulus, dynamic shear modulus, Poisson's ratio, and temperature of measurement for each specimen tested.

11. Precision and Bias

11.1 An intralaboratory study by Dickson and Wachtman (6) on 40 sintered polycrystalline alumina bar-type specimens demonstrated an uncertainty of 0.2 % for the dynamic shear modulus and 0.4 % for dynamic Young's modulus. The uncertainty included both random and systematic errors. This estimate was based upon uncertainties of 0.1 % on the thickness, width, and mass measurements; an estimate of 0.1 % on the equations; and measurements of torsional frequency to 0.0007 % and of flexural frequency to 0.0015 %. The latter uncertainties were based upon frequency uncertainties of 0.08 Hz in torsion and 0.03 Hz in flexure. If frequencies are measured to an accuracy of 1 Hz, the uncertainty for frequency would be larger, but only 0.05 % in flexural and 0.01 % in torsion. The values reported in Table 2 have been calculated for the results of the interlaboratory study in accordance with Practice E691. See ASTM Research Report No. E28-1032.⁴

11.2 An additional interlaboratory study (7) for the measurement of the dynamic Young's modulus of two metallic materials (UNS N06600 and UNS N19907) was conducted in accordance with Practice E691. See Table 3 and ASTM Research Report No. E28-1031.⁵

11.3 The terms of repeatability limit and reproducibility limit are used as specified in Practice E177.

11.4 Spinner and Tefft (1) report that the measured frequencies of bulkier specimens are least affected by the method of coupling or the position of the supports with respect to the nodes. In view of these considerations a conservative generic estimate of the bias for resonant frequencies is about 1 part in 4000 for flexural resonance. For torsional resonant frequencies Spinner and Valore (5) estimate the bias as one part in 2000 for flat specimens and one part in 10 000 for square specimens.

11.5 A propagation of errors analysis of the equations for E and G using the stated tolerances for dimensions, mass and frequency measurements in this test method has shown that a 1 % error in the measurement of the key variables produces a range of errors in the calculation of the modulus of elasticity based on the variable exponent of in the equations. Table 1 gives the calculation error effects of errors in the different experiment variables.

11.6 It is expected that the two major sources of experimental variation in the modulus of elasticity values for this test method will be in two measurements – the fundamental frequency and the smallest dimension (thickness/diameter) of the test bars. If a fundamental resonance frequency of 6000 Hz is measurable to an accuracy of 18 Hz (0.3 %) and a 3 mm thick bar is measured to an accuracy of 0.01 mm (0.3 %) the error in the thickness measurement will have the greater effect on the modulus of elasticity calculation (0.9 % for thickness error versus 0.6 % for frequency error).

11.7 *Bias*—Since there is no accepted reference material, method, or laboratory suitable for determining the bias for the procedure in this test method, no meaningful statement on bias can be formulated.

12. Keywords

12.1 dynamic; elastic properties; modulus of elasticity; Poisson's ratio; resonance; resonant beam; shear modulus; Young's modulus

⁴ Supporting data have been filed at ASTM International Headquarters and may be obtained by requesting Research Report RR:E28-1032.

⁵ Supporting data have been filed at ASTM International Headquarters and may be obtained by requesting Research Report RR:E28-1031.

TABLE 3 Precision Obtained from the Interlaboratory Study (7)

Parameter	Material	Average	Repeatability Standard Deviation	Reproducibility Standard Deviation	Repeatability Limit	Reproducibility Limit
E (GPa)	UNS N06600	213.28	2.69	5.09	7.54	14.25
	UNS N19907	159.28	2.78	3.25	7.79	9.09

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SUMMARY OF CHANGES

Committee E28 has identified the location of selected changes to this standard since the last issue (E1875–08) that may impact the use of this standard.

- (1) Section 3 was revised.
- (2) Elastic modulus was revised to modulus of elasticity in 1.1, 5.1.2, 7.2, 7.6, 9.1.4.3, 11.5, 11.6, 12, and Note 3.
- (3) Sections 1.5, 5.2, 6.6, 6.8, 6.10, 7.1, 8.2, 9.1.1, and 10.1.1 were revised.
- (4) Note 1 was revised.
- (5) Fig. 3 and Fig. 6 were revised.
- (6) Reference section was updated.

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