



Standard Practice for Use of Control Charts in Statistical Process Control¹

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1. Scope

1.1 This practice provides guidance for the use of control charts in statistical process control programs, which improve process quality through reducing variation by identifying and eliminating the effect of special causes of variation.

1.2 Control charts are used to continually monitor product or process characteristics to determine whether or not a process is in a state of statistical control. When this state is attained, the process characteristic will, at least approximately, vary within certain limits at a given probability.

1.3 This practice applies to variables data (characteristics measured on a continuous numerical scale) and to attributes data (characteristics measured as percentages, fractions, or counts of occurrences in a defined interval of time or space).

1.4 The system of units for this practice is not specified. Dimensional quantities in the practice are presented only as illustrations of calculation methods. The examples are not binding on products or test methods treated.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*²

E456 Terminology Relating to Quality and Statistics

E1994 Practice for Use of Process Oriented AOQL and LTPD Sampling Plans

E2234 Practice for Sampling a Stream of Product by Attributes Indexed by AQL

E2281 Practice for Process and Measurement Capability Indices

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² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

3. Terminology

3.1 *Definitions*—See Terminology E456 for a more extensive listing of statistical terms.

3.1.1 *assignable cause, n*—factor that contributes to variation in a process or product output that is feasible to detect and identify (see *special cause*).

3.1.1.1 *Discussion*—Many factors will contribute to variation, but it may not be feasible (economically or otherwise) to identify some of them.

3.1.2 *attributes data, n*—observed values or test results that indicate the presence or absence of specific characteristics or counts of occurrences of events in time or space.

3.1.3 *average run length (ARL), n*—the average number of times that a process will have been sampled and evaluated before a shift in process level is signaled.

3.1.3.1 *Discussion*—A long ARL is desirable for a process located at its specified level (so as to minimize calling for unneeded investigation or corrective action) and a short ARL is desirable for a process shifted to some undesirable level (so that corrective action will be called for promptly). ARL curves are used to describe the relative quickness in detecting level shifts of various control chart systems (see section 5.4). The average number of units that will have been produced before a shift in level is signaled may also be of interest from an economic standpoint.

3.1.4 *c chart, n*—control chart that monitors the count of occurrences of an event in a defined increment of time or space

3.1.5 *center line, n*—line on a control chart depicting the average level of the statistic being monitored.

3.1.6 *chance cause, n*—source of inherent random variation in a process which is predictable within statistical limits (see *common cause*).

3.1.6.1 *Discussion*—Chance causes may be unidentifiable, or may have known origins that are not easily controllable or cost effective to eliminate.

3.1.7 *common cause, n*—(see *chance cause*).

3.1.8 *control chart, n*—chart on which are plotted a statistical measure of a subgroup versus time of sampling along with limits based on the statistical distribution of that measure so as to indicate how much common, or chance, cause variation is inherent in the process or product.

3.1.9 *control chart factor, n* —a tabulated constant, depending on sample size, used to convert specified statistics or parameters into a central line value or control limit appropriate to the control chart.

3.1.10 *control limits, n* —limits on a control chart that are used as criteria for signaling the need for action or judging whether a set of data does or does not indicate a state of statistical control based on a prescribed degree of risk.

3.1.10.1 *Discussion*—For example, typical three-sigma limits carry a risk of 0.135 % of being out of control (on one side of the center line) when the process is actually in control and the statistic has a normal distribution.

3.1.11 *EWMA chart, n* —control chart that monitors the exponentially weighted moving averages of consecutive subgroups.

3.1.12 *exponentially weighted moving average (EWMA), n* —weighted average of time ordered data where the weights of past observations decrease geometrically with age.

3.1.12.1 *Discussion*—Data used for the EWMA may consist of individual observations, averages, fractions, numbers defective, or counts.

3.1.13 *I chart, n* —control chart that monitors the individual subgroup observations.

3.1.14 *lower control limit (LCL), n* —minimum value of the control chart statistic that indicates statistical control.

3.1.15 *MR chart, n* —control chart that monitors the moving range of consecutive individual subgroup observations.

3.1.16 *p chart, n* —control chart that monitors the fraction of occurrences of an event.

3.1.17 *R chart, n* —control chart that monitors the range of observations within a subgroup.

3.1.18 *rational subgroup, n* —subgroup chosen to minimize the variability within subgroups and maximize the variability between subgroups (see *subgroup*).

3.1.18.1 *Discussion*—Variation within the subgroup is assumed to be due only to common, or chance, cause variation, that is, the variation is believed to be homogeneous. If using a range or standard deviation chart, this chart should be in statistical control. This implies that any assignable, or special, cause variation will show up as differences between the subgroups on a corresponding X chart.

3.1.19 *s chart, n* —control chart that monitors the standard deviations of subgroup observations.

3.1.20 *special cause, n* —(see *assignable cause*).

3.1.21 *state of statistical control, n* —process condition when only common causes are operating on the process.

3.1.21.1 *Discussion*—In the strict sense, a process being in a state of statistical control implies that successive values of the characteristic have the statistical character of a sequence of observations drawn independently from a common distribution.

3.1.22 *statistical process control (SPC), n* —set of techniques for improving the quality of process output by reducing variability through the use of one or more control charts and a corrective action strategy used to bring the process back into a state of statistical control.

3.1.23 *subgroup, n* —set of observations on outputs sampled from a process at a particular time.

3.1.24 *upper control limit (UCL), n* —maximum value of the control chart statistic that indicates statistical control.

3.1.25 *variables data, n* —observations or test results defined on a continuous scale.

3.1.26 *warning limits, n* —limits on a control chart that are two standard errors below and above the centerline.

3.1.27 *X-bar chart, n* —control chart that monitors the average of observations within a subgroup.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *average count ($\bar{c}$), n* —arithmetic average of subgroup counts.

3.2.2 *average moving range ($\overline{MR}$), n* —arithmetic average of subgroup moving ranges.

3.2.3 *average proportion ($\bar{p}$), n* —arithmetic average of subgroup proportions.

3.2.4 *average range ($\bar{R}$), n* —arithmetic average of subgroup ranges.

3.2.5 *average standard deviation ($\bar{s}$), n* —arithmetic average of subgroup sample standard deviations.

3.2.6 *grand average ($\bar{\bar{X}}$), n* —average of subgroup averages.

3.2.7 *moving range (MR), n* —absolute difference between two adjacent subgroup observations in an I chart.

3.2.8 *observation, n* —a single value of a process output for charting purposes.

3.2.8.1 *Discussion*—This term has a different meaning than the term defined in Terminology E456, which refers there to a component of a test result.

3.2.9 *process, n* —set of interrelated or interacting activities that convert input into outputs.

3.2.10 *subgroup average ($\bar{X}_i$), n* —average for the i^{th} subgroup in an X -bar chart.

3.2.11 *subgroup count (c_i), n* —count for the i^{th} subgroup in a c chart.

3.2.12 *subgroup individual observation (X_i), n* —value of the single observation for the i^{th} subgroup in an I chart.

3.2.13 *subgroup moving range (MR_i), n* —moving range for the i^{th} subgroup in an MR chart.

3.2.13.1 *Discussion*—If there are k subgroups, there will be $k-1$ moving ranges.

3.2.14 *subgroup proportion (p_i), n* —proportion for the i^{th} subgroup in a p chart.

3.2.15 *subgroup range (R_i), n* —range of the observations for the i^{th} subgroup in an R chart.

3.2.16 *subgroup standard deviation (s_i), n* —sample standard deviation of the observations for the i^{th} subgroup in an s chart.

3.3 Symbols:

A_2	= Factor for converting the average range to three standard errors for the $\bar{X}$ -bar chart (Table 1)
A_3	= Factor for converting the average standard deviation to three standard errors of the average for the $\bar{X}$ -bar chart (Table 1)
B_3, B_4	= Factors for converting the average standard deviation to three-sigma limits for the s chart (Table 1)
c_i	= Counts of the observed occurrences of events in the i^{th} subgroup (10.2.1)
$\bar{c}$	= Average of the k subgroup counts (10.2.1)
c_4	= Factor for converting the average standard deviation to an unbiased estimate of sigma (see σ) (Table 1)
$\bar{d}_2$	= Factor for converting the average range to an estimate of sigma (see σ) (Table 1)
D_3, D_4	= Factors for converting the average range to three-sigma limits for the R chart (Table 1)
k	= Number of subgroups used in calculation of control limits (6.2.1)
MR_i	= Absolute value of the difference of the observations in the $(i-1)^{\text{th}}$ and the i^{th} subgroups in a MR chart (8.2.1)
$\overline{MR}$	= Average of the subgroup moving ranges (8.2.2.1)
n	= Subgroup size, number of observations in a subgroup (5.1.3)
p_i	= Proportion of the observed occurrences of events in the i^{th} subgroup (9.2.1)
$\bar{p}$	= Average of the k subgroup proportions (9.2.1)
R_i	= Range of the observations in the i^{th} subgroup for the R chart (6.2.1.2)
$\bar{R}$	= Average of the k subgroup ranges (6.2.2)
s_i	= Sample standard deviation of the observations in the i^{th} subgroup for the s chart (7.2.1)
$\frac{s_z}{s}$	= Standard error of the EWMA statistic (11.2.1.2)
s	= Average of the k subgroup standard deviations (7.2.2)
X_i	= Single observation in the i^{th} subgroup for the $\bar{X}$ -bar chart (8.2.1)
X_{ij}	= The j^{th} observation in the i^{th} subgroup for the $\bar{X}$ -bar chart (6.2.1)

$\bar{X}_i$	= Average of the i^{th} subgroup observations for the $\bar{X}$ -bar chart (6.2.1)
$\bar{\bar{X}}$	= Average of the individual observations over k subgroups for the $\bar{X}$ chart (8.2.2)
$\bar{\bar{X}}$	= Average of the k subgroup averages for the $\bar{X}$ -bar chart (6.2.2)
Y_i	= Value of the statistic being monitored by an EWMA chart at time i (11.2.1)
Z_i	= Exponentially-weighted average (EWMA) statistic at time i (11.2.1)
λ	= Factor ($0 < \lambda < 1$) which determines the weighing of data in the EWMA statistic (11.2.1)
$\hat{\sigma}$	= Estimated common cause standard deviation of the process (6.2.4)
$\hat{\sigma}_c$	= Standard error of c , the number of observed counts (10.2.1.2)
$\hat{\sigma}_p$	= Standard error of p , the proportion of observed occurrences (9.2.2.4)

4. Significance and Use

4.1 This practice describes the use of control charts as a tool for use in statistical process control (SPC). Control charts were developed by Shewhart (1) in the 1920s and are still in wide use today. SPC is a branch of statistical quality control (2, 3), which also encompasses process capability analysis and acceptance sampling inspection. Process capability analysis, as described in Practice E2281, requires the use of SPC in some of its procedures. Acceptance sampling inspection, described in Practices E1994 and E2234, requires the use of SPC so as to minimize rejection.

4.2 *Principles of SPC*—A process may be defined as a set of interrelated activities that convert inputs into outputs. SPC uses various statistical methodologies to improve the quality of a process by reducing the variability of one or more of its outputs, for example, a quality characteristic of a product or service.

4.2.1 A certain amount of variability will exist in all process outputs regardless of how well the process is designed or maintained. A process operating with only this inherent variability is said to be in a state of statistical control, with its output variability subject only to chance, or common, causes.

TABLE 1 Control Chart Factors

n	for $\bar{X}$ -Bar and R Charts				for $\bar{X}$ -Bar and S Charts			
	A_2	D_3	D_4	d_2	A_3	B_3	B_4	c_4
2	1.880	0	3.267	1.128	2.659	0	3.267	0.7979
3	1.023	0	2.575	1.693	1.954	0	2.568	0.8862
4	0.729	0	2.282	2.059	1.628	0	2.266	0.9213
5	0.577	0	2.114	2.326	1.427	0	2.089	0.9400
6	0.483	0	2.004	2.534	1.287	0.030	1.970	0.9515
7	0.419	0.076	1.924	2.704	1.182	0.118	1.882	0.9594
8	0.373	0.136	1.864	2.847	1.099	0.185	1.815	0.9650
9	0.337	0.184	1.816	2.970	1.032	0.239	1.761	0.9693
10	0.308	0.223	1.777	3.078	0.975	0.284	1.716	0.9727

Note: for larger numbers of n , see Ref. (11).

4.2.2 Process upsets, said to be due to assignable, or special causes, are manifested by changes in the output level, such as a spike, shift, trend, or by changes in the variability of an output. The control chart is the basic analytical tool in SPC and is used to detect the occurrence of special causes operating on the process.

4.2.3 When the control chart signals the presence of a special cause, other SPC tools, such as flow charts, brainstorming, cause-and-effect diagrams, or Pareto analysis, described in various references (3-7), are used to identify the special cause. Special causes, when identified, are either eliminated or controlled. When special cause variation is eliminated, process variability is reduced to its inherent variability, and control charts then function as a process monitor. Further reduction in variation would require modification of the process itself.

4.3 The use of control charts to adjust one or more process inputs is not recommended, although a control chart may signal the need to do so. Process adjustment schemes are outside the scope of this practice and are discussed by Box and Luceño (8).

5. Control Chart Principles and Usage

5.1 One or more observations of an output characteristic are periodically sampled from a process at a defined frequency. A control chart is basically a time plot summarizing these observations using a sample statistic, which is a function of the observations. The observations sampled at a particular time point constitute a subgroup. Control limits are plotted on the chart based on the sampling distribution of the sample statistic being evaluated (see 5.2 for further discussion).

NOTE 1—Subgroup statistics commonly used are the average, range, standard deviation, percentage or fraction of an occurrence of an event among multiple opportunities, or the number of occurrences during a defined time period or in a defined space.

5.1.1 The subgroup sampling frequency is determined by practical considerations, such as time and cost of an observation, the process dynamics (how quickly the output responds to upsets), and consequences of not reacting promptly to a process upset.

NOTE 2—Sampling at too high of a frequency may introduce correlation between successive subgroups. This is referred to as autocorrelation. Control charts that can handle this type of correlation are outside the scope of this practice.

NOTE 3—Rules for nonrandomness (see 5.2.2) assume that the plotted points on the chart are independent of one another. This shall be kept in mind when determining the sampling frequency for the control charts discussed in this practice.

5.1.2 The sampling plan for collecting subgroup observations should be designed to minimize the variation of observations within a subgroup and to maximize variation between subgroups. This is termed *rational subgrouping*. This gives the best chance for the within-subgroup variation to estimate only the inherent, or common-cause, process variation.

NOTE 4—For example, to obtain hourly rational subgroups of size four in a product-filling operation, four bottles should be sampled within a short time span, rather than sampling one bottle every 15 min. Sampling over 1 h allows the admission of special cause variation as a component of within-subgroup variation.

5.1.3 The subgroup size, n , is the number of observations per subgroup. For ease of interpretation of the control chart, the subgroup size should be fixed. Control chart techniques are available for handling variable subgroup sizes, but these are outside the scope of this practice.

5.2 The control chart is a plot of the subgroup statistic in time order. The chart also features a center line, representing the time-averaged value of the statistic, and the lower and upper control limits, that are located at $\pm$ three standard errors of the statistic around the center line. The center line and control limits are calculated from the process data and are not based in any way on specification limits. The presence of a special cause is indicated by a subgroup statistic falling outside the control limits.

5.2.1 The use of three standard errors for control limits (so-called “three-sigma limits”) was chosen by Shewhart (1), and therefore are also known as Shewhart Limits. Shewhart chose these limits to balance the two risks of: (1) failing to signal the presence of a special cause when one occurs, and (2) occurrence of an out-of-control signal when the process is actually in a state of statistical control (a false alarm).

5.2.2 Special cause variation may also be indicated by certain nonrandom patterns of the plotted subgroup statistic, as detected by using the so-called Western Electric Rules (2). To implement these rules, additional limits are shown on the chart at $\pm$ two standard errors (warning limits) and at $\pm$ one standard error (see 7.3 for example).

5.2.2.1 *Western Electric Rules*—A shift in the process level is indicated if:

- (1) One value falls outside either control limit,
- (2) Two out of three consecutive values fall outside the warning limits on the same side,
- (3) Four out of five consecutive values fall outside the $\pm$ one-sigma limits on the same side, and
- (4) Eight consecutive values either fall above or fall below the center line.

5.2.2.2 Other Western Electric rules indicate less common situations of nonrandom behavior:

- (1) Six consecutive values in a row are steadily increasing or decreasing (trend),
- (2) Fifteen consecutive values are all within the $\pm$ one-sigma limits on either side of the center line,
- (3) Fourteen consecutive values are alternating up and down, and
- (4) Eight consecutive values are outside the $\pm$ one-sigma limits.

5.2.2.3 These rules should be used judiciously since they will increase the risk of a false alarm, in which the control chart indicates lack of statistical control when only common causes are operating. The effect of using each of the rules, and groups of these rules, on false alarm incidence is discussed by Champ and Woodall (9).

5.3 This practice describes the use of control charts for variables and attributes data.

5.3.1 Variables data represent observations obtained by observing and recording the magnitude of an output characteristic measured on a continuous numerical scale. Control charts

are described for monitoring process variability and process level, and these two types of charts are used as a unit for process monitoring.

5.3.1.1 For multiple observations per subgroup, the subgroup average is the statistic for monitoring process level ($\bar{X}$ -bar chart) and either the subgroup range (R chart), or the subgroup standard deviation (s chart) is used for monitoring process variability. The range is easier to calculate and is nearly as efficient as the standard deviation for small subgroup sizes. The $\bar{X}$ -bar, R chart combination is discussed in Section 6. The $\bar{X}$ -bar, s chart combination is discussed in Section 7.

NOTE 5—For processes producing discrete items, a subgroup usually consists of multiple observations. The subgroup size is often five or less, but larger subgroup sizes may be used if measurement ease and cost are low. The larger the subgroup size, the more sensitive the control chart is to smaller shifts in the process level (see 5.4).

5.3.1.2 For single observations per subgroup, the subgroup individual observation is the statistic for monitoring process level (I chart) and the subgroup moving range is used for monitoring process variability (MR chart). The I , MR chart combination is discussed in Section 8.

NOTE 6—For batch or continuous processes producing bulk material, often only a single observation is taken per subgroup, as multiple observations would only reflect measurement variation.

5.3.2 Attributes data consist of two types: (1) observations representing the frequency of occurrence of an event in the subgroup, for example, the number or percentage of defective units in a subgroup of inspected units, or (2) observations representing the count of occurrences of an event in a defined interval of time or unit of space, for example, numbers of auto accidents per month in a given region. For such data, the variability is a function of the process average, so that only a single control chart is needed.

NOTE 7—The subgroup size for attributes data, because of their lower cost and quicker measurement, is usually much greater than for numeric observations.

5.3.2.1 For monitoring the frequency of occurrences, the statistic is the proportion or fraction of events per unit (p chart). An alternate statistic is the number of occurrences for a given number of units (np chart) and these charts are described in Section 9.

5.3.2.2 For monitoring the count of occurrences over a time or space interval, the statistic is the count (c chart) described in Section 10.

5.3.3 The EWMA chart plots the exponentially weighted moving average statistic which is described by Hunter (14). The EWMA may be calculated for individual observations and averages of multiple observations of variables data, and for percent defective, or counts of occurrences over time or space for attributes data. The calculations for the EWMA chart are defined and discussed in Section 11, and examples of the EWMA chart as a supplementary chart are given in Appendix X1.

5.3.3.1 The EWMA chart is also a useful supplementary control chart to the previously discussed charts in SPC, and is a particularly good companion chart to the I chart for individual observations. The EWMA reacts more quickly to

smaller shifts in the process characteristic, on the order of 1.5 standard errors or less, whereas the Shewhart-based charts are more sensitive to larger shifts.

5.3.3.2 The EMWA chart is also used in process adjustment schemes where the EWMA statistic is used to locate the local mean of a non-stationary process and as a forecast of the next observation from the process. This usage is beyond the scope of this practice but is discussed by Box and Paniagua-Quiñones (15) and by Lucas and Saccucci (16).

5.4 *Subgroup Size and Average Run Length*—The average run length (ARL) is a measure of how quickly the control chart signals a sustained process shift of a given magnitude in the output characteristic being monitored. It is defined as the average number of subgroups needed to respond to a process shift of h sigma units, where sigma is the intrinsic standard deviation estimated by σ (see 6.2.4). The theoretical background for this relationship is developed in Montgomery (3), and Fig. 1 gives the curves relating ARL to the process shift for selected subgroup sizes in an $\bar{X}$ -bar chart. An ARL = 1 means that the next subgroup will have a very high probability of detecting the shift.

5.5 The role of a control chart changes as the SPC program evolves. An SPC program can be organized into three stages (10).

5.5.1 *Stage A, Process Evaluation*—Historical data from the process are plotted on control charts to assess the current state of the process, and control limits from this data are calculated for further use. See Ref 11 for a more complete discussion on the use of control charts for data analysis. Ideally, it is recommended that 100 or more numeric data points be collected for this stage. For single observations per subgroup at least 30 data points should be collected (5, 6). For attributes, a total of 20 to 25 subgroups of data are recommended. At this stage, it will be difficult to find special causes, but it would be useful to compile a list of possible sources for these for use in the next stage.

5.5.2 *Stage B, Process Improvement*—Process data are collected in real time and control charts, using limits calculated in Stage A, are used to detect special causes for identification and resolution. A team approach is vital for finding the sources of special cause variation, and process understanding will be increased. This stage is completed when further use of the control chart indicates that a state of statistical control exists.

5.5.3 *Stage C, Process Monitoring*—The control chart is used to monitor the process to confirm continually the state of statistical control and to react to new special causes entering the system or the reoccurrence of previous special causes. In the latter case, an out-of-control action plan (OCAP) can be developed to deal with this situation (6, 12). Update the control limits periodically or if process changes have occurred.

NOTE 8—Some practitioners combine Stages A and B into a Phase I and denote Stage C as Phase II (10).

6. Control Charts for Multiple Numerical Measurements ($\bar{X}$ -Bar, R Charts)

6.1 *Control Chart Usage*—These control charts are used for subgroups consisting of multiple numerical measurements. The $\bar{X}$ -bar chart is used for monitoring the process level, and the R

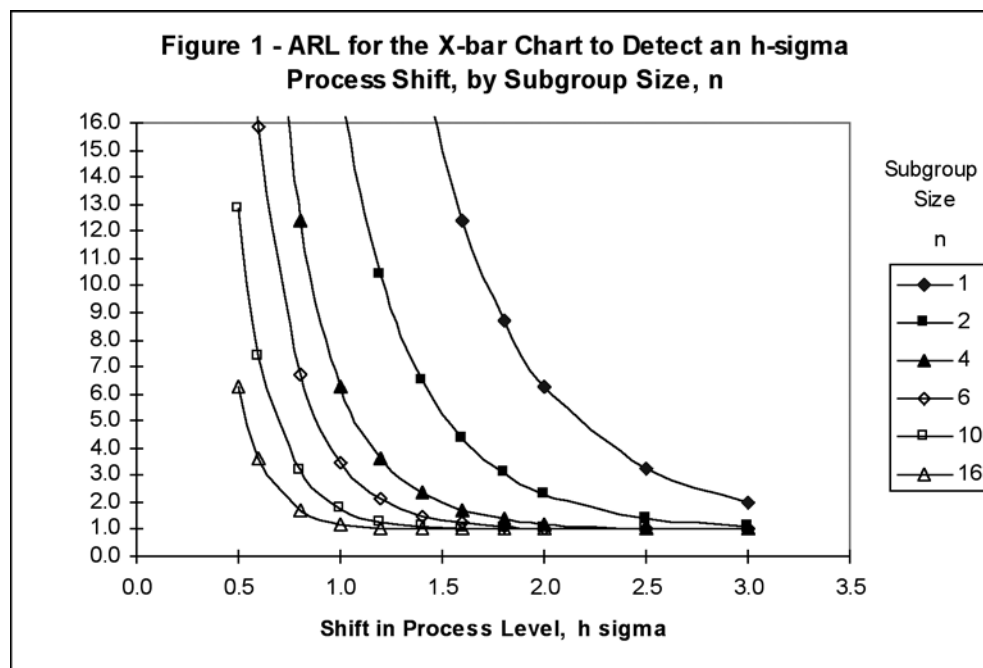


FIG. 1 ARL for the X-Bar Chart to Detect an h -Sigma Process Shift by Subgroup Size, n

chart is used for monitoring the short-term variability. The two charts use the same subgroup data and are used as a unit for SPC purposes.

6.2 Control Chart Setup and Calculations

6.2.1 Denote an observation X_{ij} , as the j^{th} observation, $j = 1, \dots, n$, in the i^{th} subgroup $i = 1, \dots, k$. For each of the k subgroups, calculate the i^{th} subgroup average,

$$\bar{X}_i = \sum_{j=1}^n X_{ij} / n = (X_{i1} + X_{i2} \dots + X_{in}) / n \quad (1)$$

6.2.1.1 Averages may be rounded to one more significant figure than the data.

6.2.1.2 For each of the k subgroups, calculate the i^{th} subgroup range, the difference between the largest and the smallest observation in the subgroup.

$$R_i = \text{Max}(X_{i1}, \dots, X_{in}) - \text{Min}(X_{i1}, \dots, X_{in}) \quad (2)$$

6.2.1.3 The averages and ranges are plotted as dots on the X-bar chart and the R chart, respectively. The dots may be connected by lines, if desired.

6.2.2 Calculate the grand average and the average range over all k subgroups:

$$\bar{\bar{X}} = \sum_{i=1}^k \bar{X}_i / k = (\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_k) / k \quad (3)$$

$$\bar{R} = \sum_{i=1}^k R_i / k = (R_1 + R_2 + \dots + R_k) / k \quad (4)$$

6.2.2.1 These values are used for the center lines on the control chart, which are usually depicted as solid lines on the control chart, and may be rounded to one more significant figure than the data.

6.2.3 Using the control chart factors in Table 1, calculate the lower control limits (LCL) and upper control limits (UCL) for the two charts.

6.2.3.1 For the X-bar chart:

$$LCL = \bar{\bar{X}} - A_2 \bar{R} \quad (5)$$

$$UCL = \bar{\bar{X}} + A_2 \bar{R} \quad (6)$$

6.2.3.2 For the R chart:

$$LCL = D_3 \bar{R} \quad (7)$$

$$UCL = D_4 \bar{R} \quad (8)$$

6.2.3.3 The control limits are usually depicted as dotted lines on the control chart.

6.2.4 An estimate of the inherent (common cause) standard deviation may be calculated as follows:

$$\hat{\sigma} = \bar{R} / d_2 \quad (9)$$

6.2.4.1 This estimate is useful in process capability studies (see Practice E2281).

6.2.5 Subgroup statistics falling outside the control limits on the X-bar chart or the R chart indicate the presence of a special cause. The Western Electric Rules may also be applied to the X-bar and R chart (see 5.2.2).

6.3 Example—Liquid Product Filling into Bottles—At a frequency of 30 min, four consecutive bottles are pulled from the filling line and weighed. The observations, subgroup averages, and subgroup ranges are listed in Table 2, and the grand average and average range are calculated at the bottom of the table.

6.3.1 The control limits are calculated as follows:

6.3.1.1 X-Bar chart:

$$LCL = 246.44 - (0.729)(5.92) = 242.12 \quad (10)$$

$$UCL = 246.44 + (0.729)(5.92) = 250.76 \quad (11)$$

TABLE 2 Example of X-Bar, R Chart for Bottle-Filling Operation

Subgroup	Bottle 1	Bottle 2	Bottle 3	Bottle 4	Average	Range
1	246.5	250.7	246.1	250.2	248.38	4.6
2	246.5	243.7	241.7	248.0	244.98	6.3
3	246.5	243.3	250.1	243.5	245.85	6.8
4	246.5	248.5	250.5	242.0	246.88	8.5
5	246.5	242.9	248.0	249.4	246.70	6.5
6	246.7	250.6	246.0	246.1	247.35	4.6
7	246.6	247.3	251.6	248.8	248.58	5.0
8	246.5	249.6	246.6	243.6	246.58	6.0
9	246.4	251.1	247.7	245.5	247.68	5.6
10	246.4	245.7	245.8	247.0	246.23	1.3
11	246.5	242.6	241.5	248.3	244.73	6.8
12	246.4	247.3	244.1	243.3	245.28	4.0
13	246.4	250.1	249.0	245.3	247.70	4.8
14	246.3	247.8	239.4	245.7	244.80	8.4
15	246.6	242.7	244.1	249.7	245.78	7.0
16	246.6	248.4	246.8	251.0	248.20	4.4
17	246.4	246.0	250.3	246.2	247.23	4.3
18	246.5	250.2	243.2	246.9	246.70	7.0
19	246.4	247.5	246.6	244.8	246.33	2.7
20	246.3	248.4	244.6	244.9	246.05	3.8
21	246.5	244.7	243.0	248.0	245.55	5.0
22	246.6	249.2	250.5	242.6	247.23	7.9
23	246.5	249.7	240.7	246.7	245.90	9.0
24	246.6	244.0	238.5	243.0	243.03	8.1
25	246.5	251.5	248.9	242.0	247.23	9.5
Grand average				246.44		
Average range						5.92

6.3.1.2 R chart:

$$LCL = (0)(5.92) = 0 \quad (12)$$

$$UCL = (2.282)(5.92) = 13.51 \quad (13)$$

6.3.1.3 Estimate of inherent standard deviation:

$$\sigma = 5.92 / 2.326 = 2.55 \quad (14)$$

6.3.1.4 The control charts are shown in Fig. 2 and Fig. 3. Both charts indicate that the filling weights are in statistical control.

7. Control Charts for Multiple Numerical Measurements (X-Bar, s Charts)

7.1 Control Chart Usage—These control charts are used for subgroups consisting of multiple numerical measurements, the X-bar chart for monitoring the process level, and the s chart for monitoring the short-term variability. The two charts use the same subgroup data and are used as a unit for SPC purposes.

7.2 Control Chart Setup and Calculations

7.2.1 Denote an observation X_{ij} , as the j^{th} observation, $j = 1, \dots, n$, in the i^{th} subgroup, $i = 1, \dots, k$. For each of the k

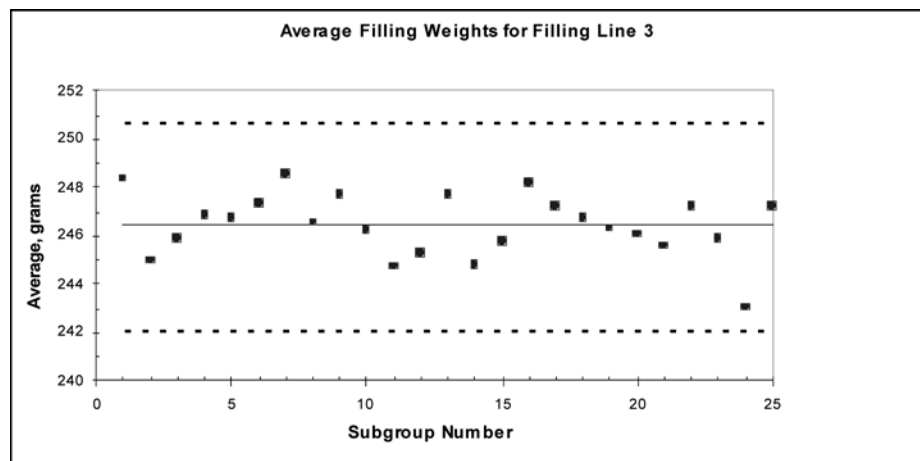


FIG. 2 X-Bar Chart for Filling Line 3

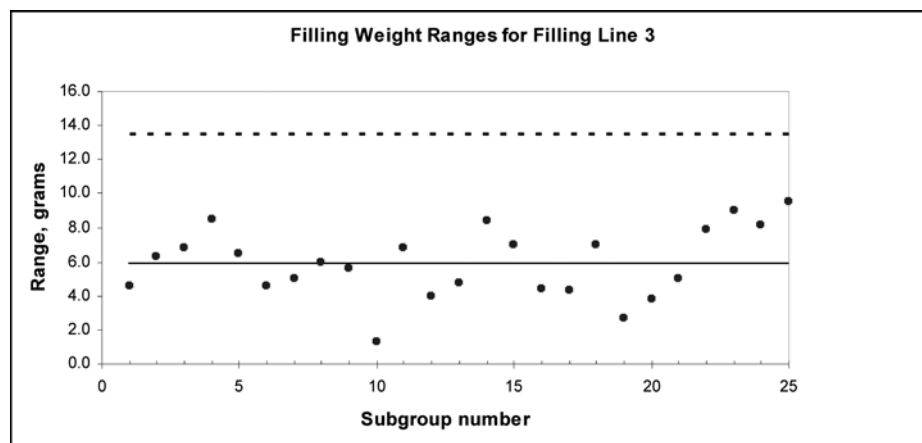


FIG. 3 Range Chart for Filling Line 3

subgroups, calculate the i^{th} subgroup average and the i^{th} subgroup standard deviation:

$$\bar{X}_i = \sum_{j=1}^n X_{ij} / n = (X_{i1} + X_{i2} + \dots + X_{in}) / n \quad (15)$$

$$s_i = \sqrt{\sum_{j=1}^n (X_{ij} - \bar{X}_i)^2 / (n - 1)} \quad (16)$$

7.2.1.1 Averages may be rounded to one more significant figure than the data.

7.2.1.2 Sample standard deviations may be rounded to two or three significant figures.

7.2.1.3 The averages and standard deviations are plotted as dots on the $\bar{X}$ -bar chart and the s chart, respectively. The dots may be connected by lines, if desired.

7.2.2 Calculate the grand average and the average standard deviation over all k subgroups:

$$\bar{\bar{X}} = \sum_{i=1}^k \bar{X}_i / k = (\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_k) / k \quad (17)$$

$$\bar{s} = \sum_{i=1}^k s_i / k = (s_1 + s_2 + \dots + s_k) / k \quad (18)$$

7.2.2.1 These values are used for the center lines on the control chart, usually depicted as solid lines, and may be rounded to the same number of significant figures as the subgroup statistics.

7.2.3 Using the control chart factors in Table 1, calculate the LCL and UCL for the two charts.

7.2.3.1 For the $\bar{X}$ -bar chart:

$$LCL = \bar{\bar{X}} - A_3 \bar{s} \quad (19)$$

$$UCL = \bar{\bar{X}} + A_3 \bar{s} \quad (20)$$

7.2.3.2 For the s chart:

$$LCL = B_3 \bar{s} \quad (21)$$

$$UCL = B_4 \bar{s} \quad (22)$$

7.2.3.3 The control limits are usually depicted by dotted lines on the control charts.

7.2.4 An estimate of the inherent (common cause) standard deviation may be calculated as follows:

$$\hat{\sigma} = \bar{s} / c_4 \quad (23)$$

7.2.5 Subgroup statistics falling outside the control limits on the $\bar{X}$ -bar chart or the s chart indicate the presence of a special cause.

7.3 Example—Vitamin tablets are compressed from blended granulated powder and tablet hardness is measured on ten tablets each hour. The observations, subgroup averages, and subgroup standard deviations are listed in Table 3, and the grand average and average range are calculated at the bottom of the table.

7.3.1 The control limits are calculated as follows:

7.3.1.1 $\bar{X}$ -bar chart:

$$LCL = 24.141 - (0.975)(1.352) = 22.823 \quad (24)$$

$$UCL = 24.141 + (0.975)(1.352) = 25.459 \quad (25)$$

7.3.1.2 The s chart:

$$LCL = (0.284)(1.352) = 0.384 \quad (26)$$

$$UCL = (1.716)(1.352) = 2.320 \quad (27)$$

7.3.2 The two-sigma warning limits and the one-sigma limits are also calculated for the $\bar{X}$ -bar chart to illustrate the use of the Western Electric Rules.

7.3.3 The warning limits and one-sigma limits for the $\bar{X}$ -bar chart were calculated as follows.

7.3.3.1 Warning limits:

$$LCL = 24.141 - 2(0.975)(1.352)/3 = 23.262 \quad (28)$$

$$UCL = 24.141 + 2(0.975)(1.352)/3 = 25.020 \quad (29)$$

7.3.3.2 One-sigma limits:

$$LCL = 24.141 - (0.975)(1.352)/3 = 23.702 \quad (30)$$

TABLE 3 Example of X-Bar, S Chart for Tablet Hardness

Subgroup	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	Avg	Std
1	21.3	19.5	21.3	23.1	22.4	24.6	23.4	22.4	21.4	22.9	22.23	1.419
2	21.4	22.2	22.1	23.3	23.9	22.9	21.6	24.6	25.7	24.1	23.18	1.399
3	23.9	24.2	22.8	22.9	25.9	21.4	23.1	20.5	23.6	23.8	23.21	1.494
4	23.4	26.3	24.4	25.3	22.0	25.8	22.7	26.5	21.6	25.0	24.30	1.781
5	25.6	22.9	24.6	23.8	23.6	22.7	27.1	25.3	25.1	25.5	24.62	1.372
6	26.8	25.7	25.6	24.8	25.7	26.0	23.6	22.8	22.1	24.7	24.78	1.507
7	26.6	24.0	25.0	26.3	25.3	25.2	25.0	24.1	22.6	26.0	25.01	1.199
8	26.7	25.9	22.6	23.8	27.3	26.7	25.7	24.1	25.2	25.2	25.32	1.470
9	25.0	24.8	22.4	22.9	24.9	24.4	22.5	22.7	23.8	24.0	23.74	1.037
10	26.1	25.3	25.7	25.0	24.7	26.2	23.6	24.2	25.1	24.3	25.02	0.844
Center line											24.141	1.352
LCL											22.823	0.384
UCL											25.459	2.320
Lower warning limit											23.262	
Upper warning limit											25.020	
Lower one-sigma limit											23.702	
Upper one-sigma limit											24.580	

$$UCL = 24.141 + (0.975)(1.352)/3 = 24.580 \quad (31)$$

7.3.3.3 Estimate of inherent standard deviation:

$$\hat{\sigma} = 1.352 / 0.9727 = 1.39 \quad (32)$$

7.3.3.4 The control charts are shown in Fig. 4 and Fig. 5. The *s* chart indicates statistical control in the process variation.

7.3.4 The X-bar chart gives several out-of-control signals:

7.3.4.1 Subgroup 1—Below the LCL.

7.3.4.2 Subgroups 2 and 3—Two points outside the warning limit on the same side.

7.3.4.3 Subgroups 6, 7, and 8—End points of six points in a row steadily increasing.

7.3.4.4 Subgroup 10—Four out of five points on the same side of the upper one-sigma limits.

7.3.5 It appears that the process level has been steadily increasing during the run. Some possible special causes are particle segregation in the feed hopper or a drift in the press settings.

8. Control Charts for Single Numerical Measurements (*I*, *MR* Charts)

8.1 Control Chart Usage—These control charts are used for subgroups consisting of a single numerical measurement. The *I* chart is used for monitoring the process level and the *MR*

chart is used for monitoring the short-term variability. The two charts are used as a unit for SPC purposes, although some practitioners state that the *MR* chart does not add value and recommend against its use for other than calculating the control limits for the *I* chart (13).

8.2 Control Chart Setup and Calculations

8.2.1 Denote the observation, X_i , as the individual observation in the i^{th} subgroup, $i = 1, 2, \dots, k$.

8.2.1.1 Note that the first subgroup will not have a moving range. For the $k-1$ subgroups, $i = 2, \dots, k$ calculate the moving range, the absolute value of the difference between two successive values:

$$MR_i = |X_i - X_{i-1}| \quad (33)$$

8.2.1.2 The individual values and moving ranges are plotted as dots on the *I* chart and the *MR* chart, respectively. The dots may be connected by lines, if desired.

8.2.2 Calculate the average of the observations over all k subgroups:

$$\bar{X} = \sum_{i=1}^k X_i / k = (X_1 + X_2 + \dots + X_k) / k \quad (34)$$

Also calculate the average moving range for the $k-1$ subgroups:

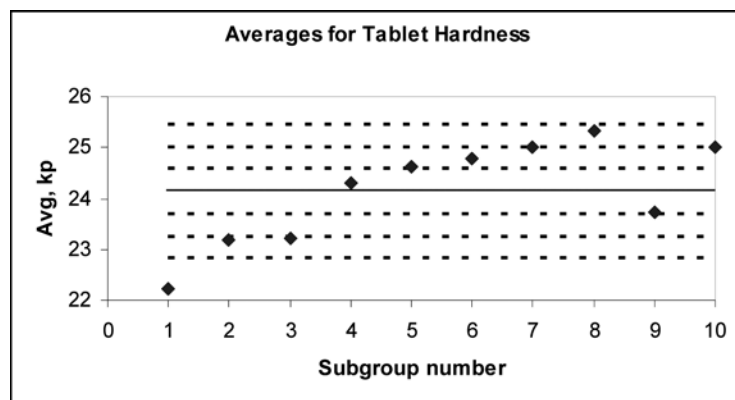


FIG. 4 X-Bar Chart for Tablet Hardness

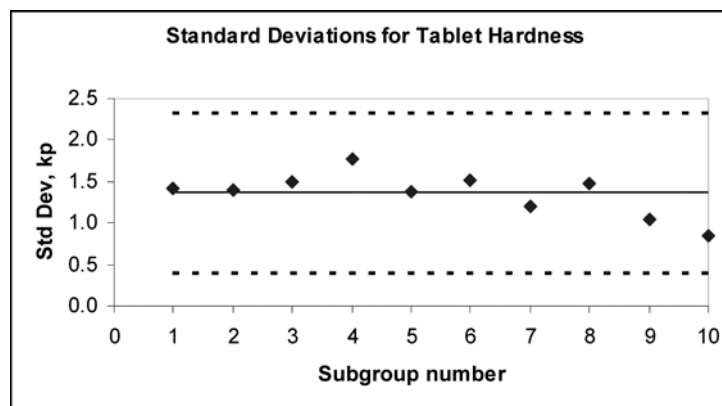


FIG. 5 s Chart for Tablet Hardness

$$\overline{MR} = \sum_{i=2}^k MR_i / (k - 1) = (MR_2 + MR_3 + \dots + MR_k) / (k - 1) \quad (35)$$

8.2.2.1 These values are used for the center lines on the control charts, usually depicted as a solid line, and may be rounded to one more significant figure than the data.

8.2.3 Calculate the control limits, *LCL* and *UCL*, for the two charts.

8.2.3.1 For the *I* chart:

$$LCL = \bar{X} - 2.66\overline{MR} \quad (36)$$

$$UCL = \bar{X} + 2.66\overline{MR} \quad (37)$$

8.2.3.2 For the *MR* chart:

$$LCL = 0 \quad (38)$$

$$UCL = 3.27\overline{MR} \quad (39)$$

8.2.3.3 The control limits are usually depicted as dotted lines on the charts.

8.2.4 An estimate of the inherent (common cause) standard deviation may be calculated as follows:

$$\hat{\sigma} = \overline{MR} / 1.128 \quad (40)$$

8.2.5 Subgroup statistics falling outside the control limits on the *I* chart or the *MR* chart indicate the presence of a special cause. The Western Electric Rules may also be applied to the *I* chart (see 5.2.1).

8.3 Example—Batches of polymer are sampled and analyzed for an impurity reported as weight percent. The values for 30 batches are listed in Table 4 along with the calculated moving ranges. The average impurity and average moving range are also listed along with the *LCL* and *UCL* for the charts.

8.3.1 The control limits are calculated as follows:

8.3.1.1 *I* chart

$$LCL = 1.437 - (2.66)(0.165) = 0.998 \quad (41)$$

$$UCL = 1.437 + (2.66)(0.165) = 1.877 \quad (42)$$

8.3.1.2 *MR* chart

TABLE 4 Example of *I*, *MR* Chart for % Impurity in Polymer Batches

Subgroup	Impurity	<i>MR</i>
1	1.39	
2	1.42	0.03
3	1.42	0.00
4	1.39	0.03
5	1.40	0.01
6	1.46	0.06
7	1.70	0.24
8	1.33	0.37
9	1.36	0.03
10	1.50	0.14
11	1.38	0.12
12	1.28	0.10
13	1.68	0.40
14	1.35	0.33
15	1.38	0.03
16	1.30	0.08
17	1.44	0.14
18	1.47	0.03
19	1.38	0.09
20	1.54	0.16
21	1.38	0.16
22	1.34	0.04
23	1.91	0.57
24	1.24	0.67
25	1.45	0.21
26	1.55	0.10
27	1.35	0.20
28	1.45	0.10
29	1.56	0.11
30	1.32	0.24
Center	1.437	0.165
<i>LCL</i>	0.998	0
<i>UCL</i>	1.877	0.540

$$LCL = (0)(0.165) = 0 \quad (43)$$

$$UCL = (3.27)(0.165) = 0.540 \quad (44)$$

8.3.2 The *I* chart indicates an out-of-control point at Subgroup 23 (Fig. 6). The *MR* chart indicates out-of-control points at Subgroups 23 and 24 (Fig. 7) and this was a result of the high impurity value for Subgroup 24. This illustrates that the *MR* chart is affected by the successive differences between individual observations, and thus, it is more difficult to exclude special cause variation from entering the picture in the *MR* chart.

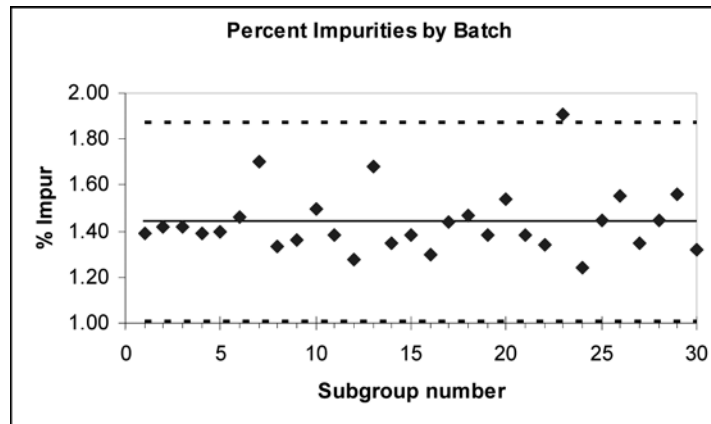


FIG. 6 / Chart for Batch Impurities

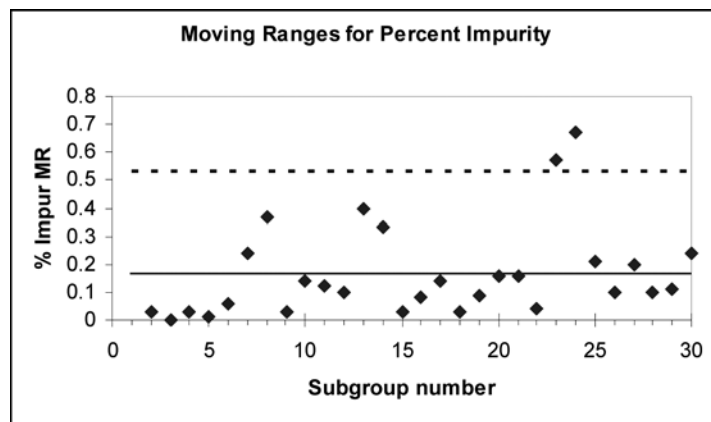


FIG. 7 MR Chart for % Impurities

9. Control Charts for Fraction and Number of Occurrences (p , np Charts)

9.1 *Control Chart Usage*—These control charts are used for subgroups consisting of the fraction occurrence of an event, the p chart, or number of occurrences, the np chart. For example, the occurrence could be nonconformance of a manufactured unit with respect to a specification limit.

9.2 Control Chart Setup and Calculations

9.2.1 Denote an observation X_{ij} , as the j^{th} observation in the i^{th} subgroup, where $X_{ij} = 1$ if there was an occurrence of an event, for example, defect, and $X_{ij} = 0$ if there is no occurrence. Let X_i denote the number of occurrences for the i^{th} subgroup

$$X_i = \sum_{j=1}^n X_{ij} \quad (45)$$

9.2.1.1 Calculate the fraction occurrence p_i for the i^{th} subgroup.

$$p_i = \sum_{j=1}^n X_{ij} / n = X_i / n \quad (46)$$

9.2.1.2 The fractions are plotted as dots on the p chart. The numbers of occurrences are plotted as dots on the np chart. The dots may be connected by lines, if desired.

9.2.2 Calculate the average fraction occurrence over all k subgroups:

$$\bar{p} = \sum_{i=1}^k p_i / k = (p_1 + p_2 + \dots + p_k) / k \quad (47)$$

9.2.2.1 This value is used for the center line on the p chart.

9.2.2.2 The center line for the np chart is np

9.2.2.3 Center lines are usually depicted as solid lines on the control chart.

9.2.2.4 Calculate the standard error for p :

$$\sigma_p = \sqrt{\bar{p}(1 - \bar{p}) / n} \quad (48)$$

9.2.3 Calculate the control limits, LCL and UCL , for the two charts.

9.2.3.1 For the p chart:

$$LCL = \bar{p} - 3\sigma_p = \bar{p} - 3\sqrt{\bar{p}(1 - \bar{p}) / n} \quad (49)$$

$$UCL = \bar{p} + 3\sigma_p = \bar{p} + 3\sqrt{\bar{p}(1 - \bar{p}) / n} \quad (50)$$

If the calculated LCL is negative, this limit is set to zero.

9.2.3.2 Control limits are usually depicted as dotted lines on the control charts.

9.2.3.3 The p chart can be used for variable n , but the limits shall be calculated and plotted for each value of n , which will result in an uneven appearance.

9.2.4 The np chart center line is np with control limits as follows:

9.2.4.1 For the np chart:

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} \quad (51)$$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \quad (52)$$

If the calculated LCL is negative then this limit is set to zero.

NOTE 9—The np chart can only be used when the sample size for each subgroup is constant.

9.2.5 Subgroup statistics falling outside the control limits on the p chart or the np chart indicate the presence of a special cause.

9.3 Example—Cartons are inspected each shift in samples of 200 for minor (cosmetic) defects (such as tearing, dents, or scoring). Table 5 lists the number of nonconforming cartons (X) and the fraction defective (p) for 30 inspections. The p chart is shown in Fig. 8 and the np chart is shown in Fig. 9. The np chart is identical to the p chart but with the vertical scale multiplied by n . Special cause variation is indicated for Subgroups 15 and 23.

9.3.1 The control limits are calculated as follows:

9.3.1.1 p chart:

$$LCL = 0.058 - 3\sqrt{(0.058)(1-0.058)/200} = 0.008 \quad (53)$$

$$UCL = 0.058 + 3\sqrt{(0.058)(1-0.058)/200} = 0.107 \quad (54)$$

9.3.1.2 np chart:

$$LCL = 11.6 - 3\sqrt{(200)(0.058)(1-0.058)} = 1.7 \quad (55)$$

$$UCL = 11.6 + 3\sqrt{(200)(0.058)(1-0.058)} = 21.5 \quad (56)$$

9.3.2 Except for a change in the vertical scale, the two control charts are identical.

Out-of-control signals are indicated at Subgroups 15 and 23.

TABLE 5 Example of P Chart for Nonconforming Cartons in Samples of 200

Sub	X	p	Sub	X	p
1	12	0.060	21	20	0.100
2	15	0.075	22	18	0.090
3	8	0.040	23	24	0.120
4	10	0.050	24	15	0.075
5	4	0.020	25	9	0.045
6	7	0.035	26	12	0.060
7	16	0.080	27	7	0.035
8	9	0.045	28	13	0.065
9	14	0.070	29	9	0.045
10	10	0.050	30	6	0.030
11	5	0.025			
12	6	0.030	Average	11.6	0.058
13	17	0.085	LCL	1.7	0.008
14	12	0.060	UCL	21.5	0.107
15	22	0.110	NP		P
16	8	0.040			
17	10	0.050			
18	5	0.025			
19	13	0.065			
20	11	0.055			

10. Control Charts for Counts of Occurrences in a Defined Time or Space Increment (c Chart)

10.1 Control Chart Usage—These control charts are used for subgroups consisting of the counts of occurrences of an event over a defined time or space within which there are multiple opportunities for occurrence. For example, the occurrence could be number of knots of a specified diameter on a wood panel or the number of minor injuries per 10 000 h worked in a manufacturing plant.

10.2 Control Chart Setup and Calculations

10.2.1 Denote the number of occurrences in the subgroup as c_i for the i^{th} subgroup. Calculate the average count over all k subgroups:

$$\bar{c} = \sum_{i=1}^k c_i / k = (c_1 + c_2 + \dots + c_k) / k \quad (57)$$

10.2.1.1 This value is used for the center line on the c chart. Center lines are usually depicted as solid lines on the control chart.

10.2.1.2 Calculate the standard error of c :

$$\sigma_c = \sqrt{\bar{c}} \quad (58)$$

10.2.2 Calculate the LCL and UCL for the chart.

10.2.2.1 For the c chart:

$$LCL = \bar{c} - 3\sigma_c = \bar{c} - 3\sqrt{\bar{c}} \quad (59)$$

$$UCL = \bar{c} + 3\sigma_c = \bar{c} + 3\sqrt{\bar{c}} \quad (60)$$

If the calculated LCL is negative then this limit is set to zero.

10.2.2.2 Control limits are usually depicted as dotted lines on the control charts.

10.2.3 Subgroup statistics falling outside the control limits on the c chart indicate the presence of a special cause.

10.3 Example—The number of minor injuries per month are tracked for a manufacturing plant with a stable workforce over a two-year period (Table 6). A c chart (Fig. 10) indicated that the injuries evolved from a common cause system. Although the eight minor injuries during Month 10 seemed unusually high, it is within the normal range of variation, and it might not be fruitful to investigate for a special cause.

10.3.1 The control limits are calculated as follows:

10.3.1.1 c chart:

$$LCL = 3.3 - 3\sqrt{3.3} = -2.2, \text{ so set limit to } 0 \quad (61)$$

$$UCL = 3.3 + 3\sqrt{3.3} = 8.7 \quad (62)$$

11. Control Chart Using the Exponentially-Weighted Moving Average (EWMA Chart)

11.1 Control Chart Usage—The EWMA control chart uses the exponentially-weighted moving average (EWMA) statistic, a type of weighted average of numerical subgroup statistics, where the weights decrease geometrically with the age of the subgroup. A weighting factor λ is used to control the rate of decrease of the weights with time.

11.1.1 Denote as Y_i the subgroup statistic at time i . This statistic may be an individual observation or an average, a

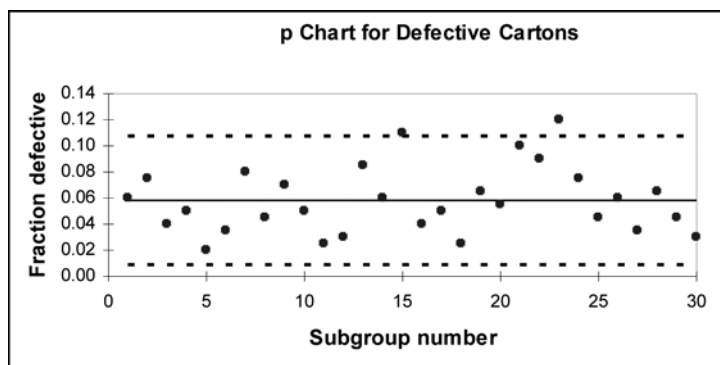


FIG. 8 *p* Chart for Nonconforming Cartons

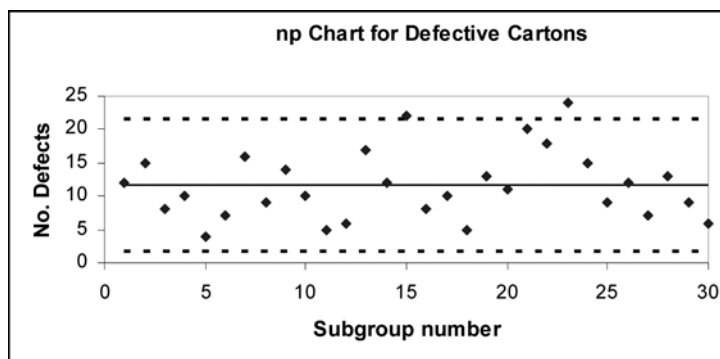


FIG. 9 *np* Chart for Nonconforming Cartons

TABLE 6 *c* Chart Example

Month	<i>c</i>	Month	<i>c</i>
1	5	13	0
2	3	14	4
3	2	15	6
4	4	16	2
5	0	17	4
6	3	18	5
7	4	19	3
8	1	20	6
9	4	21	2
10	8	22	3
11	3	23	1
12	1	24	5
Average		3.3	
LCL		-2.2	
UCL		8.7	

percentage, or a count based on multiple observations. The EWMA statistic at time i is denoted Z_i and is calculated as:

$$Z_i = \lambda Y_i + (1 - \lambda) Z_{i-1} \quad (63)$$

where Z_{i-1} is the EWMA at the immediately preceding time $i-1$ and λ is the weighting factor $0 < \lambda < 1$ for combining the current subgroup statistic Y_i and the preceding EWMA to obtain the current EWMA.

11.1.1.1 Typical values of λ used for EWMA charts range between 0.1 and 0.5, but the performance of the EWMA chart is often robust to the value chosen. A value for λ can be determined empirically (8, 15) in two ways: (1) minimization of the forecasting errors ($Y_i - Z_{i-1}$) in the data set, or (2)

choosing a value that maximizes the chance of detecting certain sizes of shifts in the average.

11.2 Control Chart Setup and Calculations:

11.2.1 The control limits for the EWMA chart depend on the stage of the SPC program (see 5.5). In Stage A there are little or no historical process estimates so the EWMA control limits must compensate for limited initial data, but in Stages B and C the EWMA can utilize historical estimates from previous stages and no such compensation is necessary.

11.2.1.1 The calculation of the EWMA for the first time point, Z_1 , needs an initial EWMA value Z_0 to combine with the subgroup statistic Y_1 . In Stage A, Z_0 can either be (1) set equal to Y_1 , (2) to the average of the subgroup statistics in the initial set of data to be plotted, or (3) can be back-forecasted by conducting the EWMA in reverse and Z_0 will be the last value in the reversed EWMA series (15). In subsequent stages, Z_0 can be the last EWMA from the preceding stage. The effect of the starting value Z_0 on the EWMA dissipates rapidly with time. The Z_0 value is plotted as the center line on the EWMA chart and is usually depicted as a solid line.

11.2.1.2 The successive EWMA values are calculated in time order using the current subgroup statistic and previous EWMA in (Eq 63). Plot the EWMA values as dots on the chart or as a connected line if the EWMA is plotted together with the companion statistic.

11.2.1.3 When historical information is available calculate the standard error of Z_i :

$$s_z = \hat{\sigma} \sqrt{\frac{\lambda}{(2 - \lambda)}} \quad (64)$$

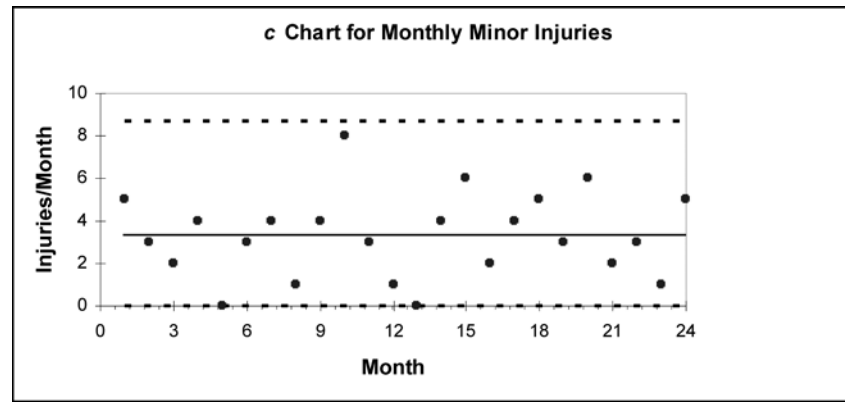


FIG. 10 c Chart for Monthly Minor Injuries

where $\hat{\sigma}$ depends on the subgroup statistic of the companion chart as follows:

- (1) averages from the X-bar, R chart, use equation (Eq 9) in 6.2.4,
- (2) averages from the X-bar, s chart, use equation (Eq 23) in 7.2.4,
- (3) observations from the I, MR chart, use equation (Eq 40) in 8.2.4,
- (4) fraction occurrences from the p chart, use equation (Eq 48) in 9.2.2.4,
- (5) numbers of occurrences from the np chart, use n times equation (Eq 48),
- (6) counts of occurrences from the c chart, use equation (Eq 58) in 10.2.1.2.

11.2.1.4 When no historical information is available, the initial EWMA values are based on very little data and the exact equation for the standard error of the EWMA must be used, which is:

$$s_{z_i} = \hat{\sigma} \sqrt{\frac{\lambda}{(2-\lambda)} [1-(1-\lambda)^{2i}]} \quad (65)$$

for the i^{th} subgroup. The term inside the brackets diminishes rapidly over time and so that the equation eventually converges to (Eq 64).

11.2.2 Calculate the lower control limit (LCL) and the upper control limit (UCL).

11.2.2.1 For the EWMA chart:

$$LCL = Z_0 - 3s_z \quad (66)$$

$$UCL = Z_0 + 3s_z \quad (67)$$

11.2.2.2 Control limits are usually depicted as dotted lines on the control charts, but if combined with a chart for the companion statistic, the control limits for the EWMA can be shown as dashed lines.

11.2.3 EWMA statistics falling outside the control limits on the control chart may indicate the presence of a special cause requiring further investigation.

11.3 Example—The EWMA chart is used as a supplementary chart to the I chart (see 8.3) used for initial evaluation of the impurity level of a batch polymer process. The I chart in Fig. 6 indicated a single out-of-control value at Subgroup (Batch) 23. Assuming that the data listed in Table 4 was the initial data set in Stage A of this SPC project, there was no

historical data available. Hence the starting EWMA value Z_0 used the average of the 30 batches which was equal to 1.44%, and the EWMA values are calculated from (Eq 63) and listed in Table 7.

11.3.1 The control limits were calculated by equation (Eq 65) and also appear in Table 7. The control limits appear closer to the center line for the initial subgroups, and converge to the values that would be calculated by equation (Eq 64). The EWMA Chart shown in Fig. 11 gave no indication of an out-of-control situation at Subgroup 23 since the out-of-control signal was a transitory shift in level (a spike). The EWMA does not react as quickly as the I chart to this type of special cause variation.

TABLE 7 Example of EWMA Chart for Polymer Impurities

Subgroup	Impurity	MR	EWMA	LCL	CL	UCL
1	1.39	...	1.44	1.35	1.44	1.53
2	1.42	0.03	1.43	1.32	1.44	1.55
3	1.42	0.00	1.43	1.31	1.44	1.56
4	1.39	0.03	1.42	1.30	1.44	1.57
5	1.40	0.01	1.42	1.30	1.44	1.58
6	1.46	0.06	1.43	1.30	1.44	1.58
7	1.70	0.24	1.48	1.29	1.44	1.58
8	1.33	0.37	1.45	1.29	1.44	1.58
9	1.36	0.03	1.43	1.29	1.44	1.58
10	1.50	0.14	1.45	1.29	1.44	1.58
11	1.38	0.12	1.43	1.29	1.44	1.58
12	1.28	0.10	1.40	1.29	1.44	1.58
13	1.68	0.40	1.46	1.29	1.44	1.58
14	1.35	0.33	1.44	1.29	1.44	1.58
15	1.38	0.03	1.43	1.29	1.44	1.58
16	1.30	0.08	1.40	1.29	1.44	1.58
17	1.44	0.14	1.41	1.29	1.44	1.58
18	1.47	0.03	1.42	1.29	1.44	1.58
19	1.38	0.09	1.41	1.29	1.44	1.58
20	1.54	0.16	1.44	1.29	1.44	1.58
21	1.38	0.16	1.43	1.29	1.44	1.58
22	1.34	0.04	1.41	1.29	1.44	1.58
23	1.91	0.57	1.51	1.29	1.44	1.58
24	1.24	0.67	1.46	1.29	1.44	1.58
25	1.45	0.21	1.45	1.29	1.44	1.58
26	1.55	0.10	1.47	1.29	1.44	1.58
27	1.35	0.20	1.45	1.29	1.44	1.58
28	1.45	0.10	1.45	1.29	1.44	1.58
29	1.56	0.11	1.47	1.29	1.44	1.58
30	1.32	0.24	1.44	1.29	1.44	1.58
Average		1.44	0.17			
Standard deviation			0.146			

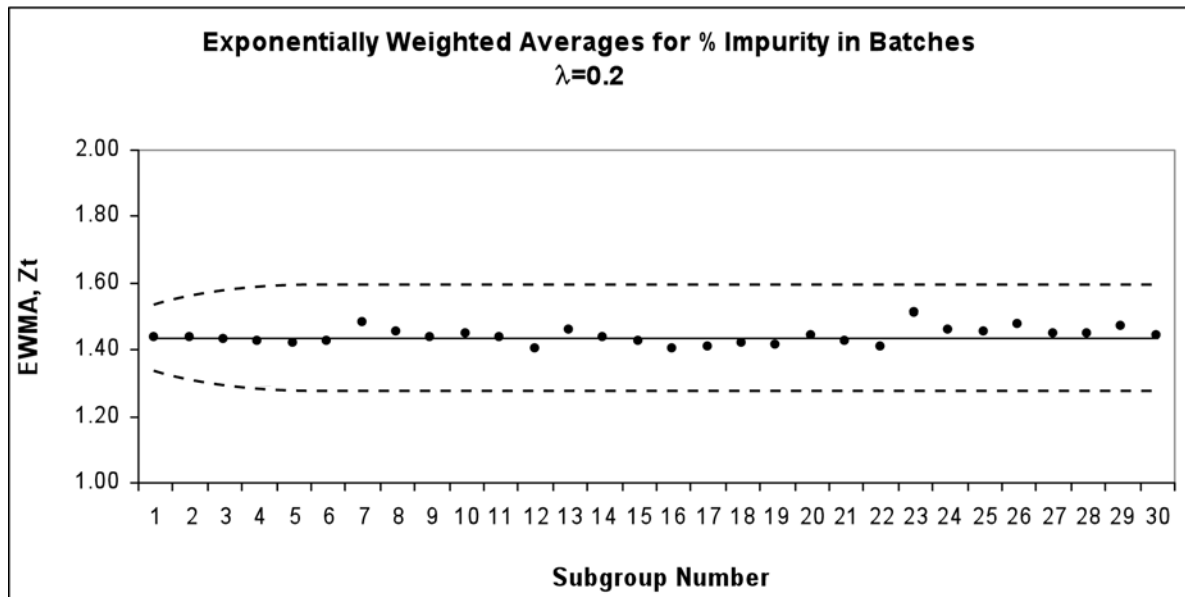


FIG. 11 EWMA Chart for Process Impurity

11.4 Example—Process yields are tracked daily on a five day per week operation. A successful SPC program had been conducted on this process, bringing the average yield up to 95%, and the program was now in Stage C, or monitoring mode, using a combined I chart and a EWMA chart with $\lambda = 0.2$. Table 8 lists the yields (Y_i) for the current month of production, the calculated EWMA values (Z_i) and the moving ranges (MR_i). For this chart the historical average yield of 95.4% was used for the starting EWMA (Z_0) and the center line of the control chart. The historical moving range of 1.24% was used to estimate $\hat{\sigma}$ for calculating the EWMA control limits.

11.4.1 The control limits are calculated as follows:

I Chart: (see 8.3.1.1)

$$LCL = 95.4 - (2.66) (1.24) = 92.1 \quad (68)$$

$$UCL = 95.4 + (2.66) (1.24) = 98.7 \quad (69)$$

MR Chart (not plotted):

$$UCL = (3.27) (1.24) = 4.1 \quad (70)$$

EWMA Chart:

$$\hat{\sigma} = 1.24 / 1.128 + 1.099 \quad (71)$$

$$s_z = 1.099 \sqrt{0.2/(2-0.2)} + (1.099) \sqrt{1/3} = 0.366 \quad (72)$$

$$LCL = 95.4 - (3) (0.366) = 94.3 \quad (73)$$

$$UCL = 95.4 + (3) (0.366) = 96.5 \quad (74)$$

It is noteworthy that EWMA control limits are equivalent to the one-sigma limits for the I chart when $\lambda = 0.2$.

11.4.2 The I chart and EWMA control charts are plotted together (Fig. 12) with the I chart control limits depicted as dotted lines and the EWMA chart control limits depicted as dashed lines.

11.4.2.1 The I chart did not indicate a shift, even though there appeared to be a decrease in yield starting around Day 12 or 13. If the Western Electric rules were applied for detections of a shift (5.2.2.1), a shift would also not have been detected:

(1) the lower warning limit, midway between the I Chart LCL and EWMA Chart LCL, is 93.2, and no two consecutive points fall below this limit,

(2) there are no cases where four out of five consecutive points fall below the lower one-sigma limit of the I Chart (equal to the EWMA lower control limit), and

(3) there are no cases of eight consecutive points below the center line. There were also no moving ranges above the UCL for the MR Chart (not plotted).

11.4.2.2 The EWMA chart indicated out-of-control points at Days 15, 17, and 20. The downward shift appeared to be on the order of 1% yield or about one standard deviation. An investigation was conducted to find the special cause of the lower yields.

TABLE 8 EWMA and Chart 1 Example

Day	Yield	EWMA	MR	
1	96.1	95.5	...	
2	98.3	96.1	2.2	
3	94.7	95.8	3.6	Historical Data
4	93.6	95.4	1.1	Average 95.4
5	95.6	95.4	2.0	MRBAR 1.24
6	96.7	95.7	1.1	
7	95.7	95.7	1.0	I Chart Limits
8	96.9	95.9	1.2	UCL 98.7
9	93.9	95.5	3.0	LCL 92.1
10	95.4	95.5	1.5	
11	97.0	95.8	1.6	EWMA Chart Limits
12	94.5	95.5	2.5	UCL 96.5
13	93.6	95.1	0.9	LCL 92.1
14	93.2	94.8	0.4	
15	92.2	94.2	1.0	Center Line
16	94.5	94.3	2.3	CL 95.4
17	93.3	94.1	1.2	
18	95.5	94.4	2.2	MR Chart Limits
19	94.5	94.4	1.0	UCL 4.1
20	93.3	94.2	1.2	LCL 0.0

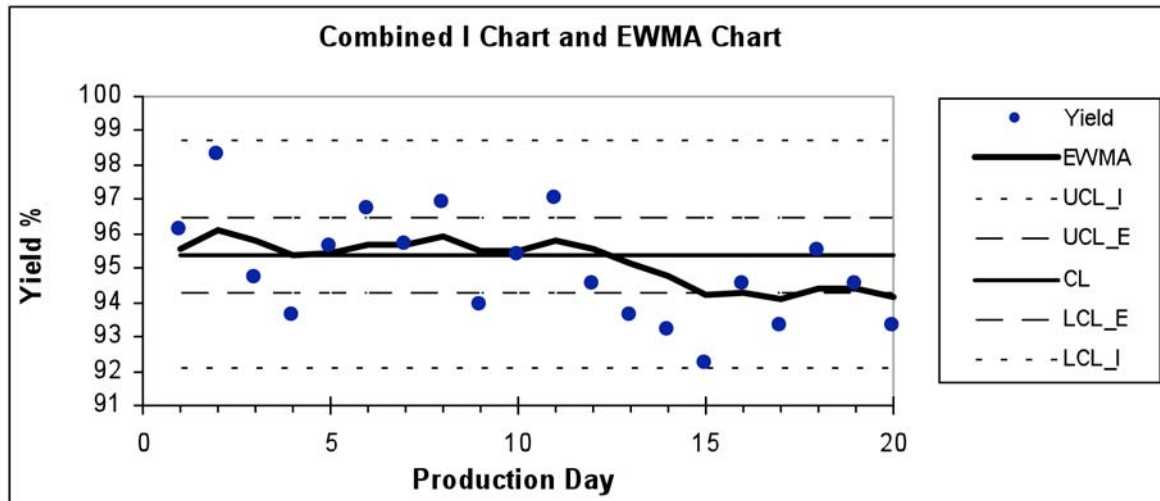


FIG. 12 Combined I Chart and EWMA Chart for Process Yield

12. Keywords

12.1 common cause; control chart; control limits; rational subgroup; special cause; state of statistical control; statistical process control

APPENDIX

X1. EXAMPLES OF EWMA CHARTS AS SUPPLEMENTARY CHARTS TO SHEWHART CHARTS

X1.1 Companion chart to X-bar, R-Chart (see 6.3 Example—Liquid Product Filling into Bottles)

X1.1.1 At a frequency of 30 min, four consecutive bottles were pulled from the filling line and weighed. The observations, subgroup averages, and subgroup ranges are listed in Table 2, and the grand average and average range are calculated at the bottom of the table. The average weight, Y_i , the EWMA, Z_i , and the EWMA control limits are shown in Table X1.1.

X1.1.2 The control limits are calculated as follows:

EWMA chart using $Z_0 = 246.44$, $s_z = 1.355$ and $\lambda = 0.2$:

$$LCL = 246.44 - 3(1.355) \sqrt{\frac{0.2}{2-0.2}} \sqrt{1-(1-0.2)^{2i}} \quad (X1.1)$$

$$UCL = 246.44 + 3(1.355) \sqrt{\frac{0.2}{2-0.2}} \sqrt{1-(1-0.2)^{2i}} \quad (X1.2)$$

X1.1.3 The control chart is shown in Fig. X1.1. The EWMA chart indicates that the filling weights are in statistical control.

X1.2 Companion chart to p Chart (see 9.3 Example—Nonconforming cartons)

X1.2.1 Cartons are inspected each shift in samples of 200 for minor (cosmetic) defects (such as tearing, dents, or scoring). From Table 5, Table X1.2 lists the number of nonconforming cartons (X) and the fraction defective (p) for 30 inspections. The average fraction defective was 0.058 and was used as the EWMA starting value Z_0 , and the standard deviation $\hat{\sigma}$ was 0.0165 by (Eq 48). The EWMA was calcu-

TABLE X1.1 Example of Bottle-Filling Operation with EWMA ($\lambda=0.2$)

Subgroup	Average	EWMA	LCL	CL	UCL
1	248.38	246.83	245.63	246.44	247.25
2	244.98	246.46	245.40	246.44	247.48
3	245.85	246.32	245.28	246.44	247.60
4	246.88	246.43	245.20	246.44	247.68
5	246.70	246.49	245.16	246.44	247.72
6	247.35	246.66	245.13	246.44	247.75
7	248.58	246.87	245.11	246.44	247.77
8	246.58	246.81	245.10	246.44	247.78
9	247.58	246.69	245.10	246.44	247.78
10	246.58	246.60	245.09	246.44	247.79
11	244.73	246.10	245.09	246.44	247.79
12	245.28	245.93	245.09	246.44	247.79
13	247.70	246.69	245.09	246.44	247.79
14	244.80	246.31	245.09	246.44	247.79
15	245.78	246.31	245.09	246.44	247.79
16	248.20	246.69	245.09	246.44	247.79
17	247.23	246.60	245.09	246.44	247.79
18	246.70	246.62	245.08	246.44	247.80
19	246.33	246.42	245.08	246.44	247.80
20	246.05	246.34	245.08	246.44	247.80
21	245.55	246.26	245.08	246.44	247.80
22	247.23	246.46	245.08	246.44	247.80
23	245.90	246.33	245.08	246.44	247.80
24	243.03	245.67	245.08	246.44	247.80
25	247.23	246.60	245.08	246.44	247.80

lated by equation (Eq 63) and the EWMA control limits were calculated by equation (Eq 65) as listed in Table X1.2.

X1.2.2 The EMWA chart is shown in Fig. X1.2 and gave evidence of a lack of statistical control at subgroups 23 and 24.

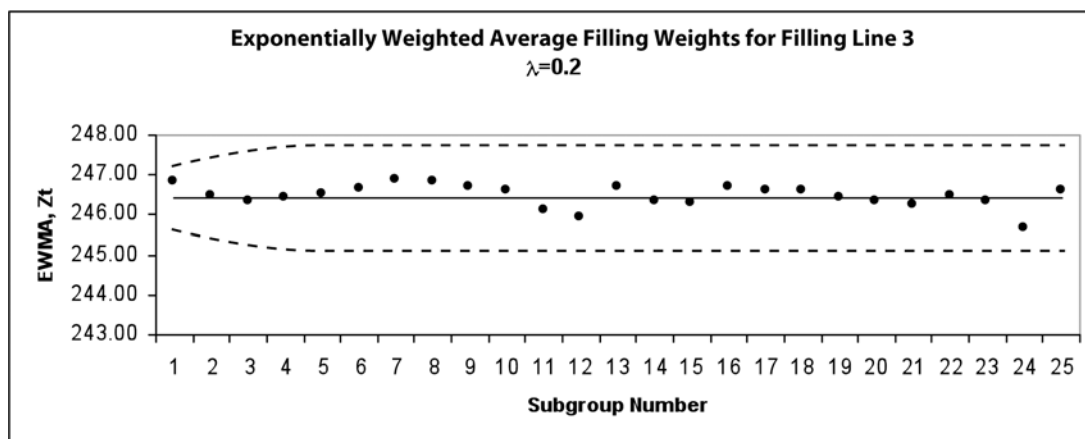


FIG. X1.1 EWMA Chart for Filling Line 3

TABLE X1.2 Example—Nonconforming Cartons with EWMA ($\lambda=0.4$)

Subgroup	X	p	EWMA	LCL	UCL
1	12	0.060	0.058	0.038	0.078
2	15	0.075	0.065	0.035	0.081
3	8	0.040	0.055	0.034	0.082
4	10	0.050	0.053	0.033	0.083
5	4	0.020	0.040	0.033	0.083
6	7	0.035	0.038	0.033	0.083
7	16	0.080	0.055	0.033	0.083
8	9	0.045	0.051	0.033	0.083
9	14	0.070	0.058	0.033	0.083
10	10	0.050	0.055	0.033	0.083
11	5	0.025	0.043	0.033	0.083
12	6	0.030	0.038	0.033	0.083
13	17	0.085	0.057	0.033	0.083
14	12	0.060	0.058	0.033	0.083
15	22	0.110	0.079	0.033	0.083
16	8	0.040	0.063	0.033	0.083
17	10	0.050	0.058	0.033	0.083
18	5	0.025	0.045	0.033	0.083
19	13	0.065	0.053	0.033	0.083
20	11	0.055	0.054	0.033	0.083
21	20	0.100	0.072	0.033	0.083
22	18	0.090	0.079	0.033	0.083
23	24	0.120	0.096	0.033	0.083
24	15	0.075	0.087	0.033	0.083
25	9	0.045	0.070	0.033	0.083
26	12	0.060	0.066	0.033	0.083
27	7	0.035	0.054	0.033	0.083
28	13	0.065	0.058	0.033	0.083
29	9	0.045	0.053	0.033	0.083
30	6	0.030	0.044	0.033	0.083
Average		0.058	λ	0.4	
Standard deviation		0.0615			

X1.3 Companion chart to c Chart (see 10.3 Example—Minor injuries)

X1.3.1 The number of minor injuries per month are tracked for a manufacturing plant with a stable workforce over a two-year period in Table 6. The average count was 3.3 and was used as the EWMA starting value Z_0 , and the standard

deviation $\hat{\sigma}$ was 1.82 by (Eq 58). The EWMA was calculated by (Eq 63) and the control limits were calculated by equation (Eq 65) as listed in Table X1.3.

X1.3.2 The EWMA chart is shown in Fig. X1.3 and indicated no lack of statistical control as was also indicated by the c Chart.

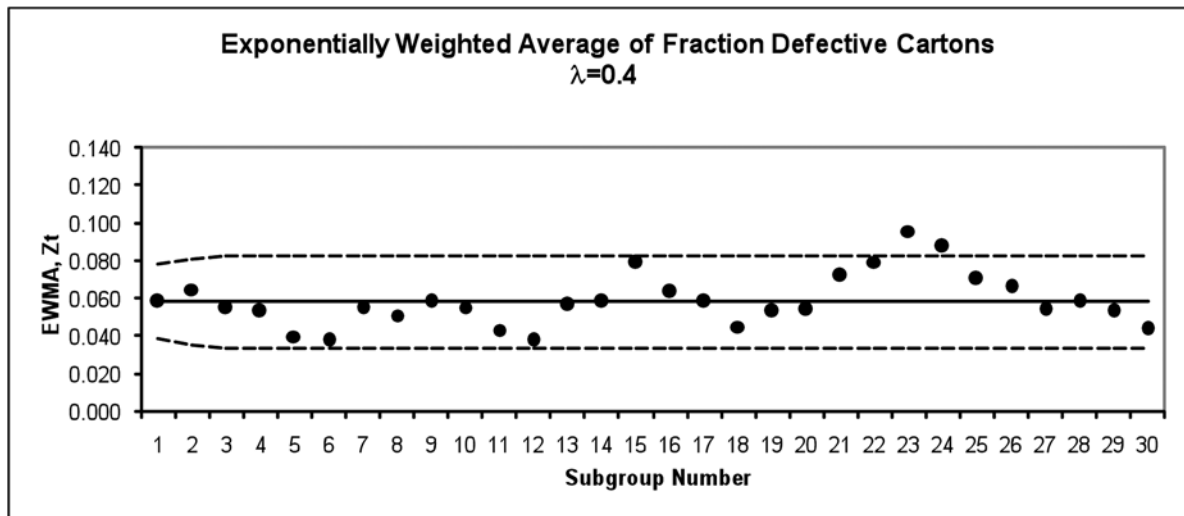


FIG. X1.2 EWMA Chart for Nonconforming Cartons

TABLE X1.3 EWMA Chart Calculations for Monthly Minor Injuries

Month	<i>c</i>	EWMA	LCL	UCL
1	5	3.3	1.1	5.5
2	3	3.2	0.8	5.8
3	2	2.7	0.6	6.0
4	4	3.2	0.6	6.0
5	0	1.9	0.6	6.0
6	3	2.4	0.6	6.0
7	4	3.0	0.6	6.0
8	1	2.2	0.6	6.0
9	4	2.9	0.6	6.0
10	8	5.0	0.6	6.0
11	3	4.2	0.6	6.0
12	1	2.9	0.6	6.0
13	0	1.7	0.6	6.0
14	4	2.6	0.6	6.0
15	6	4.0	0.6	6.0
16	2	3.2	0.6	6.0
17	4	3.5	0.6	6.0
18	5	4.1	0.6	6.0
19	3	3.7	0.6	6.0
20	6	4.6	0.6	6.0
21	2	3.6	0.6	6.0
22	3	3.3	0.6	6.0
23	1	2.4	0.6	6.0
24	5	3.4	0.6	6.0
Average			3.3	
Standard deviation			1.82	

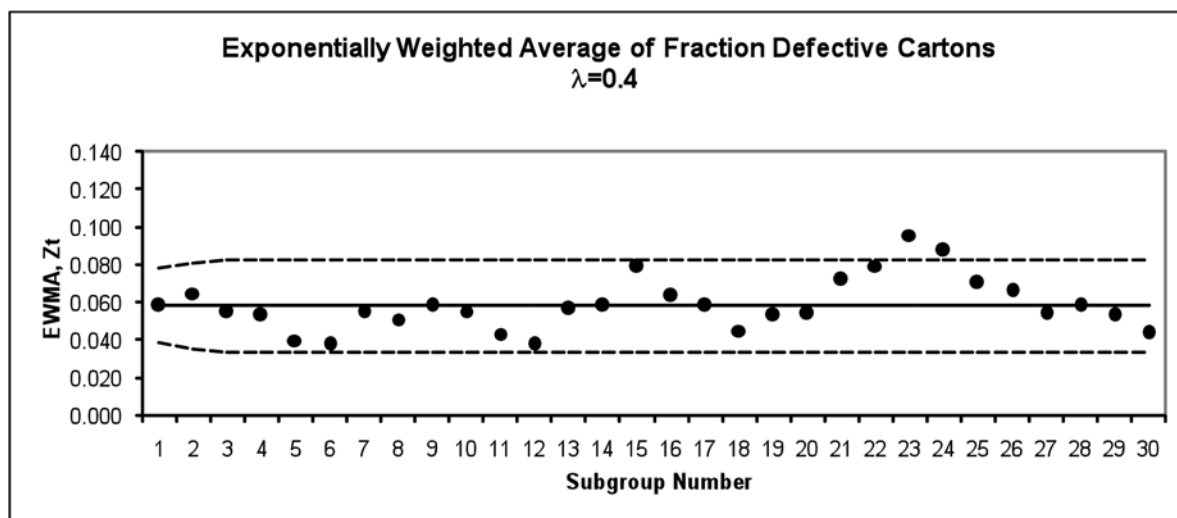


FIG. X1.3 EWMA Chart for Monthly Minor Injuries

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