

Standard Practice for Calculating Sample Size to Estimate, With Specified Precision, the Average for a Characteristic of a Lot or Process¹

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This standard has been approved for use by agencies of the U.S. Department of Defense.

^{ε1} NOTE—Editorial corrections were made to 8.4.1.2 in November 2011.

1. Scope

1.1 This practice covers simple methods for calculating how many units to include in a random sample in order to estimate with a specified precision, a measure of quality for all the units of a lot of material, or produced by a process. This practice will clearly indicate the sample size required to estimate the average value of some property or the fraction of nonconforming items produced by a production process during the time interval covered by the random sample. If the process is not in a state of statistical control, the result will not have predictive value for immediate (future) production. The practice treats the common situation where the sampling units can be considered to exhibit a single (overall) source of variability; it does not treat multi-level sources of variability.

2. Referenced Documents

2.1 *ASTM Standards*:²

E456 Terminology Relating to Quality and Statistics

3. Terminology

3.1 *Definitions*—Unless otherwise noted, all statistical terms are defined in Terminology E456.

3.2 *Symbols*—Symbols used in all equations are defined as follows:

E = the maximum acceptable difference between the true average and the sample average.

e = E/μ , maximum acceptable difference expressed as a fraction of μ .
 f = degrees of freedom for a standard deviation estimate (7.5).
 k = the total number of samples available from the same or similar lots.
 μ = lot or process mean or expected value of X , the result of measuring all the units in the lot or process.
 μ_0 = an advance estimate of μ .
 N = size of the lot.
 n = size of the sample taken from a lot or process.
 n_j = size of sample j .
 n_L = size of the sample from a finite lot (7.4).
 p' = fraction of a lot or process whose units have the nonconforming characteristic under investigation.
 p_0 = an advance estimate of p' .
 p = fraction nonconforming in the sample.
 R = range of a set of sampling values. The largest minus the smallest observation.
 R_j = range of sample j .
 $\bar{R}$ = $\sum_{j=1}^k R_j/k$, average of the range of k samples, all of the same size (8.2.2).
 σ = lot or process standard deviation of X , the result of measuring all of the units of a finite lot or process.
 σ_0 = an advance estimate of σ .
 s = $\left[\sum_{i=1}^n (X_i - \bar{X})^2 / (n-1)^{1/2} \right]$, an estimate of the standard deviation σ from n observation, X_i , $i = 1$ to n .
 $\bar{s}$ = $\sum_{j=1}^k s_j/k$, average s from k samples all of the same size (8.2.1).
 s_p = pooled (weighted average) s from k samples, not all of the same size (8.2).
 s_j = standard deviation of sample j .
 V_o = an advance estimate of V , equal to δ_o/μ_o .
 v = $s/\bar{X}$, the coefficient of variation estimated from the sample.
 v_p = pooled (weighted average) of v from k samples (8.3).

¹ This practice is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.10 on Sampling / Statistics.

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² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

- v_j = coefficient of variation from sample j .
 $\bar{X}$ = numerical value of the characteristic of an individual unit being measured.
 $\bar{\bar{X}}$ = $\sum_{i=1}^n X_i/n_i$ average of n observations, X_i , $i = 1$ to n .

4. Significance and Use

4.1 This practice is intended for use in determining the sample size required to estimate, with specified precision, a measure of quality of a lot or process. The practice applies when quality is expressed as either the lot average for a given property, or as the lot fraction not conforming to prescribed standards. The level of a characteristic may often be taken as an indication of the quality of a material. If so, an estimate of the average value of that characteristic or of the fraction of the observed values that do not conform to a specification for that characteristic becomes a measure of quality with respect to that characteristic. This practice is intended for use in determining the sample size required to estimate, with specified precision, such a measure of the quality of a lot or process either as an average value or as a fraction not conforming to a specified value.

5. Empirical Knowledge Needed

5.1 Some empirical knowledge of the problem is desirable in advance.

5.1.1 We may have some idea about the standard deviation of the characteristic.

5.1.2 If we have not had enough experience to give a precise estimate for the standard deviation, we may be able to state our belief about the range or spread of the characteristic from its lowest to its highest value and possibly about the shape of the distribution of the characteristic; for instance, we might be able to say whether most of the values lie at one end of the range, or are mostly in the middle, or run rather uniformly from one end to the other (Section 9).

5.2 If the aim is to estimate the fraction nonconforming, then each unit can be assigned a value of 0 or 1 (conforming or nonconforming), and the standard deviation as well as the shape of the distribution depends only on p' , the fraction nonconforming in the lot or process. Some rough idea concerning the size of p' is therefore needed, which may be derived from preliminary sampling or from previous experience.

5.3 More knowledge permits a smaller sample size. Seldom will there be difficulty in acquiring enough information to compute the required size of sample. A sample that is larger than the equations indicate is used in actual practice when the empirical knowledge is sketchy to start with and when the desired precision is critical.

5.4 The precision of the estimate made from a random sample may itself be estimated from the sample. This estimation of the precision from one sample makes it possible to fix more economically the sample size for the next sample of a similar material. In other words, information concerning the process, and the material produced thereby, accumulates and should be used.

6. Precision Desired

6.1 The approximate precision desired for the estimate must be prescribed. That is, it must be decided what maximum deviation, E , can be tolerated between the estimate to be made from the sample and the result that would be obtained by measuring every unit in the lot or process.

6.2 In some cases, the maximum allowable sampling error is expressed as a proportion, e , or a percentage, $100 e$. For example, one may wish to make an estimate of the sulfur content of coal within 1 %, or $e = 0.01$.

7. Equations for Calculating Sample Size

7.1 Based on a normal distribution for the characteristic, the equation for the size, n , of the sample is as follows:

$$n = (3\sigma_e/E)^2 \quad (1)$$

The multiplier 3 is a factor corresponding to a low probability that the difference between the sample estimate and the result of measuring (by the same methods) all the units in the lot or process is greater than E . The value 3 is recommended for general use. With the multiplier 3, and with a lot or process standard deviation equal to the advance estimate, it is practically certain that the sampling error will not exceed E . Where a lesser degree of certainty is desired a smaller multiplier may be used (Note 1).

NOTE 1—For example, multiplying by 2 in place of 3 gives a probability of about 45 parts in 1000 that the sampling error will exceed E . Although distributions met in practice may not be normal, the following text table (based on the normal distribution) indicates approximate probabilities:

Factor	Approximate Probability of Exceeding E
3	0.003 or 3 in 1000 (practical certainty)
2.56	0.010 or 10 in 1000
2	0.045 or 45 in 1000
1.96	0.050 or 50 in 1000 (1 in 20)
1.64	0.100 or 100 in 1000 (1 in 10)

7.1.1 If a lot of material has a highly asymmetric distribution in the characteristic measured, the sample size as calculated in Eq 1 may not be adequate. There are two things to do when asymmetry is suspected.

7.1.1.1 Probe the material with a view to discovering, for example, extra-high values, or possibly spotty runs of abnormal character, in order to approximate roughly the amount of the asymmetry for use with statistical theory and adjustment of the sample size if necessary.

7.1.1.2 Search the lot for abnormal material and segregate it for separate treatment.

7.2 There are some materials for which σ varies approximately with μ , in which case $V (= \sigma/\mu)$ remains approximately constant from large to small values of μ .

7.2.1 For the situation of 7.2, the equation for the sample size, n , is as follows:

$$n = (3 V_e/e)^2 \quad (2)$$

If the relative error, e , is to be the same for all values of μ , then everything on the right-hand side of Eq 2 is a constant; hence n is also a constant, which means that the same sample size n would be required for all values of μ .

7.3 If the problem is to estimate the lot fraction nonconforming, then σ_o^2 is replaced by $p_o(1 - p_o)$ so that Eq 1 becomes:

$$n = (3/E)^2 p_o(1 - p_o) \quad (3)$$

7.4 When the average for the production process is not needed, but rather the average of a particular lot is needed, then the required sample size is less than Eq 1, Eq 2, and Eq 3 indicate. The sample size for estimating the average of the finite lot will be:

$$n_L = n/[1 + (n/N)] \quad (4)$$

where n is the value computed from Eq 1, Eq 2, or Eq 3. This reduction in sample size is usually of little importance unless n is 10 % or more of N .

7.5 When the information on the standard deviation is limited, a sample size larger than indicated in Eq 1, Eq 2, and Eq 3 may be appropriate. When the advance estimate σ_0 is based on f degrees of freedom, the sample size in Eq 1 may be replaced by:

$$n = (3\sigma_0/E)^2 (1 + \sqrt{2/f}) \quad (5)$$

NOTE 2—The standard error of a sample variance with f degrees of freedom, based on the normal distribution, is $\sqrt{2\sigma^4/f}$. The factor $(1 + \sqrt{2/f})$ has the effect of increasing the preliminary estimate σ_0^2 by one times its standard error.

8. Reduction of Empirical Knowledge to a Numerical Value of σ_o (Data for Previous Samples Available)

8.1 This section illustrates the use of the equations in Section 7 when there are data for previous samples.

8.2 For Eq 1—An estimate of σ_o can be obtained from previous sets of data. The standard deviation, s , from any given sample is computed as:

$$s = \left[\sum_{i=1}^n (X_i - \bar{X})^2 / (n - 1) \right]^{1/2} \quad (6)$$

The s value is a sample estimate of σ_o . A better, more stable value for σ_o may be computed by pooling the s values obtained from several samples from similar lots. The pooled s value s_p for k samples is obtained by a weighted averaging of the k results from use of Eq 6.

$$s_p = \left[\sum_{j=1}^k (n_j - 1) s_j^2 / \sum_{j=1}^k (n_j - 1) \right]^{1/2} \quad (7)$$

8.2.1 If each of the previous data sets contains the same number of measurements, n_j , then a simpler, but slightly less efficient estimate for σ_o may be made by using an average ($\bar{s}$) of the s values obtained from the several previous samples. The calculated $\bar{s}$ value will in general be a slightly biased estimate of σ_o . An unbiased estimate of σ_o is computed as follows:

$$\sigma_o = \frac{\bar{s}}{c_4} \quad (8)$$

where the value of the correction factor, c_4 , depends on the size of the individual data sets (n_j) (Table 1³).

TABLE 1 Values of the Correction Factor C_4 and d_2 for Selected Sample Sizes n_j^A

Sample Size ³ , (n_j)	C_4	d_2
2	.798	1.13
4	.921	2.06
5	.940	2.33
8	.965	2.85
10	.973	3.08

^A As n_j becomes large, C_4 approaches 1.000.

8.2.2 An even simpler, and slightly less efficient estimate for σ_o may be computed by using the average range ($\bar{R}$) taken from the several previous data sets that have the same group size.

$$\sigma_o = \frac{\bar{R}}{d_2} \quad (9)$$

The factor, d_2 , from Table 1 is needed to convert the average range into an unbiased estimate of σ_o .

8.2.3 Example 1—Use of $\bar{s}$.

8.2.3.1 Problem—To compute the sample size needed to estimate the average transverse strength of a lot of bricks when the value of E is 50 psi, and practical certainty is desired.

8.2.3.2 Solution—From the data of three previous lots, the values of the estimated standard deviation were found to be 215, 192, and 202 psi based on samples of 100 bricks. The average of these three standard deviations is 203 psi. The c_4 value is essentially unity when Eq 1 gives the following equation for the required size of sample to give a maximum sampling error of 50 psi:

$$n = [(3 \times 203)/50]^2 = 12.2^2 = 149 \text{ bricks} \quad (10)$$

8.3 For Eq 2—If σ varies approximately proportionately with μ for the characteristic of the material to be measured, compute the average, $\bar{X}$, the standard deviation, s , and the coefficient of variation v for each sample. The pooled V value v_p for k samples, not necessarily of the same size, is obtained by a weighted average of the k results. Then use Eq 2.

$$v_p = \left[\sum_{j=1}^k (n_j - 1) v_j^2 / \sum_{j=1}^k (n_j - 1) \right]^{1/2} \quad (11)$$

8.3.1 Example 2—Use of V , the estimated coefficient of variation:

8.3.1.1 Problem—To compute the sample size needed to estimate the average abrasion resistance (that is, average number of cycles) of a material when the value of e is 0.10 or 10 %, and practical certainty is desired.

8.3.1.2 Solution—There are no data from previous samples of this same material, but data for six samples of similar materials show a wide range of resistance. However, the values of estimated standard deviation are approximately proportional to the observed averages, as shown in the following text table:

Lot No.	Sample Size	Avg Cycles	Standard Deviation	Coefficient of Variation, %
1	10	90	13	14
2	10	190	32	17
3	10	350	45	13
4	10	450	71	16
5	10	1000	120	12
6	10	3550	680	19
Pooled				15.4

³ ASTM Manual on Presentation of Data and Control Chart Analysis, ASTM MNL 7A, 2002, Part 3.

The use of the pooled coefficient of variation for V_o in Eq 2 gives the following for the required size of sample to give a maximum sampling error not more than 10 % of the expected value:

$$n = [(3 \times 15.4)/10]^2 = 21.3 \rightarrow 22 \text{ test specimens} \quad (12)$$

8.3.1.3 If a maximum allowable error of 5 % were needed, the required sample size would be 86 specimens. The data supplied by the prescribed sample will be useful for the study in hand and also for the next investigation of similar material.

8.4 For Eq 3—Compute the estimated fraction nonconforming, p , for each sample. Then for the weighted average use the following equation:

$$p = \frac{\text{total number nonconforming in all samples}}{\text{total number of units in all samples}} \quad (13)$$

8.4.1 Example 3—Use of p :

8.4.1.1 Problem—To compute the size of sample needed to estimate the fraction nonconforming in a lot of alloy steel track bolts and nuts when the value of E is 0.04, and practical certainty is desired.

8.4.1.2 Solution—The data in the following table from four previous lots were used for an advance estimate of p :

Lot No.	Sample Size	Number Nonconforming	Fraction Nonconforming
1	75	3	0.040
2	100	10	0.100
3	90	4	0.044
4	125	4	0.032
Total	390	21	

$$\begin{aligned} p &= 21/390 = 0.054 \\ n &= (3/0.04)^2 (0.054) (0.946) \\ &= [(9 \times 0.0511)/0.0016] = 287.4 = 288 \end{aligned}$$

If the value of E were 0.01 the required sample size would be 4600. With a lot size of 2000, Eq 4 gives $n_L = 1394$ items. Although this value of n_L represents about 70 % of the lot, the example illustrates the sample size required to achieve the value of E with practical certainty.

9. Reduction of Empirical Knowledge to a Numerical Value for σ_o (No Data from Previous Samples of the Same or Like Material Available)

9.1 This section illustrates the use of the equations in Section 7 when there are no actual observed values for the computation of σ_o .

9.2 For Eq 1—From past experience, try to discover what the smallest (a) and largest (b) values of the characteristic are likely to be. If this is not known, obtain this information from

some other source. Try to picture how the other observed values may be distributed. A few simple observations and questions concerning the past behavior of the process, the usual procedure of blending, mixing, stacking, storing, etc., and knowledge concerning the aging of material and the usual practice of withdrawing the material (last in, first out; or last in, last out) will usually elicit sufficient information to distinguish between one form of distribution and another (Fig. 1). In case of doubt, or in case the specified precision E is a critical matter, the rectangular distribution may be used. The price of the extra protection afforded by the rectangular distribution is a larger sample, owing to the larger standard deviation thereof.

9.2.1 The standard deviation estimated from one of the formulas of Fig. 1 as based on the largest and smallest values, may be used as an advance estimate of σ_o in Eq 1. This method of advance estimation is acceptable and is often preferable to doubtful observed values of s , $\bar{s}$, or $\bar{r}$.

9.2.2 Example 4—Use of σ_o from Fig. 1.

9.2.2.1 Problem (same as Example 1)—To compute the sample size needed to estimate the average transverse strength of a lot of bricks when the value of E is 50 psi.

9.2.2.2 Solution—From past experience the spread of values of transverse strength for a lot of bricks has been about 1200 psi. The values were heaped up in the middle of this band, but not necessarily normally distributed.

9.2.2.3 The isosceles triangle distribution in Fig. 1 appears to be most appropriate, the advance estimate σ_o is 1200/4.9 = 245 psi. Then

$$n = [(3 \times 245)/50]^2 = 14.7^2 = 216.1 = 217 \text{ bricks} \quad (14)$$

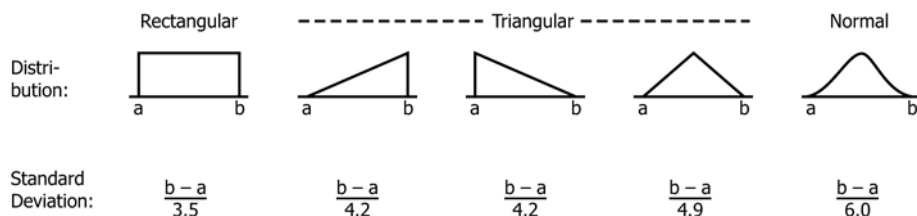
9.2.2.4 The difference in sample size between 217 and 149 bricks (found in Example 1) is the cost of sketchy knowledge.

9.3 For Eq 2—In general, the knowledge that the use of V_o instead of σ_o is preferable would be obtained from the analysis of actual data in which case the methods of Section 8 apply.

9.4 For Eq 3—From past experience, estimate approximately the band within which the fraction nonconforming is likely to lie. Turn to Fig. 2 and read off the value of $\sigma_o^2 = p'(1 - p')$ for the middle of the possible range of p' and use it in Eq 8. In case the desired precision is a critical matter, use the largest value of σ_o^2 within the possible range of p' .

10. Consideration of Cost for Sampling and Testing

10.1 After the required size of sample to meet a prescribed precision is computed from Eq 1, Eq 2, or Eq 3, the next step is to compute the cost of testing this size of sample. If the cost is too great, it may be possible to relax the required precision



NOTE 1—What is shown here for the normal distribution is somewhat arbitrary, because the normal distribution has no finite endpoints.

FIG. 1 Some Types of Distributions and Their Standard Deviations

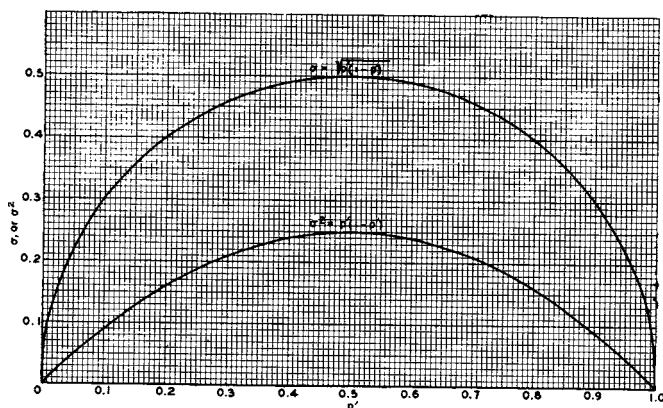


FIG. 2 Values of σ , or $(\sigma)^2$, Corresponding to Values of p'

(or the equivalent, which is to accept an increase in the probability (Section 7) that the sampling error may exceed the maximum error E) and to reduce the size of the sample.

10.2 Eq 1 gives n in terms of a prescribed precision, but we may solve it for E in terms of a given n and thus discover the precision possible for a given cost that is, $E = 3\sigma_o/\sqrt{n}$. The same may be done for Eq 2 and Eq 3.

10.3 It is necessary to specify either E or the allowable cost; otherwise there is no proper size of sample.

11. Selection of the Sample

11.1 In order to make any estimate for a lot or for a process, on the basis of a sample, it is necessary to select the units in the sample at random. An acceptable procedure to ensure a random selection is the use random numbers. Lack of predictability, such as a mechanical arm sweeping over a conveyor belt, does not yield a random sample.

11.2 In the use of random numbers, the material must first be broken up in some manner into *sampling units*. Moreover, each sampling unit must be identifiable by a serial number, actual, or by some rule. For packaged articles, a rule is easy; the package contains a certain number of articles in definite layers, arranged in a particular way, and it is easy to devise some system for numbering the articles. In the case of bulk material like ore, or coal, or a barrel of bolts or nuts, the problem of defining usable sampling units must take place at an earlier stage of manufacture or in the process of moving the materials.

11.3 It is not the purpose of this practice cover the handling of materials, nor to find ways by which one can with surety discover the way to a satisfactory type of sampling unit. Instead, it is assumed that a suitable sampling unit has been defined and then the aim is to answer the question of how many to draw.

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