



Essential structure of S - N curve: Prediction of fatigue life and fatigue limit of defective materials and nature of scatter

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ABSTRACT

Historically, S - N curve has been expressed in the form of an exponential relationship between applied stress σ and number of cycles to failure N_f . This paper elucidates the essential structure of S - N curve based on the prediction method for fatigue life and limit of materials containing defects from the viewpoint of mechanics of small crack. The extended application of the proposed method to step loading and variable amplitude loading is discussed. The question about the applicability of Miner's rule is also discussed from the viewpoint of continuous decrease in fatigue limit with number of stress cycles and crack growth.

1. Introduction

It seems that the phenomenon of metal fatigue was recognized in the beginning of the Industrial Revolution in Europe [1]. The terminology of "fatigue" was first used by Jean-Victor Poncelet, French mathematician and engineer, in his book in 1841 [2]. Later, F. Brathwaite, British engineer, first used the English term "fatigue" in his paper 1850s. Circa 1850, the fatigue behavior of steels was systematically investigated for the first time by Wöhler [3,4]. Since the concept of metal fatigue itself was not clear at that time, the mechanics and microscopic aspects of S - N data were not well understood. It was natural that the exact understanding of metal fatigue was very difficult in the days of Wöhler. However, regarding the interpretation and prediction of S - N data, what substantial progress have we made since the days of Wöhler? The objective of Wöhler's study was the acquisition of the basic fatigue data for railroad axles in which the study on the effect of defects contained in materials on fatigue was not explicitly considered. In the days of Wöhler, the confirmation and determination of endurance cycles was the first priority for the safe design of railroad axles. This situation continues to the present day. Thus, currently we have to still continue fatigue tests and acquire S - N data for safe fatigue design.

In 1910s, Basquin [5] expressed S - N data obtained by Wöhler et al. in the form of the following exponential equation as in Fig. 1. Equation (1) is called the Basquin model.

$$\sigma = C_1 N^{C_2} \quad (1)$$

where, σ : stress, N : number of cycles to failure, C_1 , C_2 : material constants.

The concept of the fatigue limit was not recognized in the Basquin model. Later in 1914, Stromeyer [6] analyzed Wöhler's data and carried out bending and torsional fatigue tests on various materials to verify the existence of definite fatigue limit and determined the relationship between the applied stress and the number of cycles N as Eq. (2).

$$\sigma = \sigma_w + C(10^6/N)^{1/4} \quad (2)$$

where, σ : stress, σ_w : fatigue limit, N : number of cycles to failure, C : material constant.

Namely, the Stromeyer law expresses S - N data by truncating the curve of Basquin model at fatigue limit σ_w (Fig. 1). Stromeyer plotted the applied stress σ on the ordinate and $C(10^6/N)^{1/4}$ on the abscissa and extrapolated the data point to $C(10^6/N)^{1/4} = 0$, i.e. the abscissa = 0, and the ordinate of applied stress σ was defined as fatigue limit σ_w . However, the *knee* point [7], N_{knee} , was not defined explicitly. It is interesting that Stromeyer noted the relationship between fatigue limit and heat evolution from a specimen during fatigue testing [6].

Palmgren [8] proposed a model similar to the Basquin model and the Stromeyer law as Eq. (3) based on the endurance test of ball bearings.

$$\sigma = C(aN + e)^x + \sigma_w \quad (3)$$

where, C , a , and e are material constants and x is a negative number.

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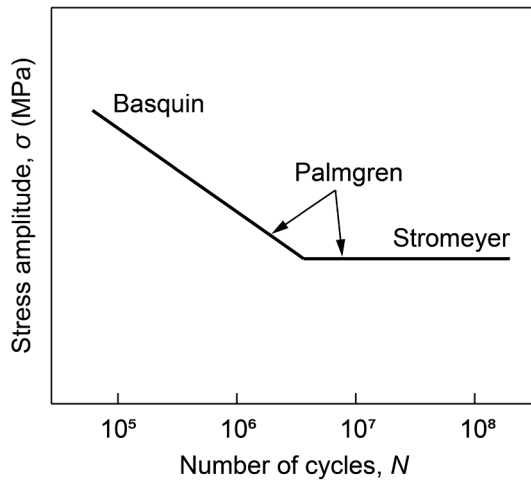


Fig. 1. Curve fitting models of S - N data.

Weibull [9] proposed a model in which the statistical scatter near fatigue limit is considered. Weibull had the principal objection against the usually accepted model of the S - N curve as two straight lines which may give the impression of a *knee* without any physical significance. However, physical or mechanistic factors that cause the scatter have not been discussed in these models. According to the discussion with Socie [10] in terms of his experience that he was requested by an industry engineer who had never conducted a fatigue test or analysis to propose a method of fatigue life prediction for a component within $\pm 10\%$ accuracy, he explained to the industry engineer that it is impossible. Industry engineers who are not familiar with fatigue phenomena are likely to misunderstand that fatigue life prediction should be easier than other technical problems.

Later, even nowadays other modified versions similar to Eqs. (1)–(3) have been repeatedly proposed [11]. However, these models, though having the name of a *model*, are basically made by *curve fitting* procedures for fatigue test data and there has been no discussion based on mechanics of cracks or defects which should critically influence the form of an S - N curve. The equations obtained by curve fitting procedure are naturally dependent on specimen shapes, specimen sizes and materials, though it is well known that the qualities of materials having even the same name are different depending on suppliers of every material, for example commercial carbon steels, tool steels, bearing steels (such as SAE52100), spring steels and aluminum alloys.

Therefore, an equation of S - N curve fitted to a material supplied by one manufacturer does not necessarily represent the fatigue properties of other materials with the same composition and heat treatment produced by other suppliers and accordingly the data fitting procedure has to be repeated on every material and supplier. Such a method cannot be used especially for the prediction of S - N curve of materials containing defects. Moreover, as explained later, it is difficult to use such types of models for the prediction of *fatigue life under variable amplitude loading*. It must be noted that Miner's rule, Palmgren-Miner rule [8,12] or modified Miner's rule cannot be used in combination with this kind of curve fitting approach, because it is difficult to correctly assess the fatigue damage by stress cycles with an amplitude below the fatigue limit. It must be also noted that Miner's rule and Modified Miner's rule (or its variations [13–18]) do not consider the sequence effect of load application on fatigue damage counting. Thus, the models made without the basis of mechanics fail in the fatigue life prediction problems of defective materials under variable amplitude loading. This paper analyzes the essential structure of S - N curve and presents the mechanistic view of the fatigue phenomena which is useful for the prediction of fatigue life and fatigue limit and can contribute to reliable fatigue design in presence of defects.

S - N curve of high cycle fatigue is composed of the following three parts.

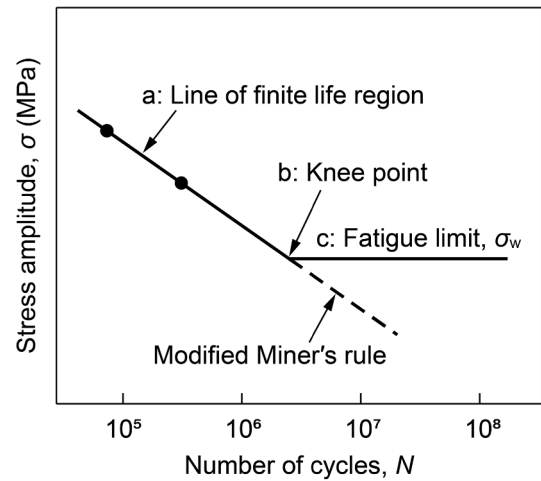


Fig. 2. Composition of S - N curve: Line of finite life regime, knee point N_{knee} and fatigue limit σ_w .

- The inclined line of finite life region (N_f in logarithmic scale) which shows longer fatigue life N_f with decreasing stress amplitude σ (Fig. 2).
- The minimum number of fatigue cycles N_{knee} (knee point) at which fatigue failure does not occur with additional stress cycles (Fig. 2).
- The horizontal line of fatigue limit σ_w below which no fatigue failure occurs.

The Basquin model [5] as Eq. (1), Stromeyer model [6] as Eq. (2) and the Weibull model with adding failure probability analysis [9], are made to express S - N data such as Fig. 2 by a *curve fitting technique*. Although these models have a name of *model* and gave some contributions to the fatigue research and fatigue design in the early stage of fatigue study, they were not a *model* made on the basis of the physical meaning of the data points. Even if we manipulate the equations of these models, we cannot derive a mechanistic understanding of fatigue phenomena. What is necessary for reliable fatigue design is to elucidate the essential structure of S - N curve and the related influential factors based on the fatigue process from the beginning of stress cycle to the final fracture.

2. S - N data of materials containing defects and the essential meaning of S - N curve

2.1. Correlation among S - N curves of series of specimens containing defects with different sizes

If we test specimens containing defects, the S - N data will be an S - N curve lower than defect free specimens. This feature is schematically expressed as Fig. 3. Curves A, B and C show respectively the S - N data of the specimens containing a defect with size d_A , d_B and d_C ($d_A < d_B < d_C$). Naturally the fatigue lives N_f are short in the order of Series C, Series B and Series A ($N_{fC} < N_{fB} < N_{fA}$) and the fatigue limits decrease in the order σ_{wA} , σ_{wB} and σ_{wC} ($\sigma_{wC} < \sigma_{wB} < \sigma_{wA}$). The slope of the S - N curve, C_2 in Eq. (1), is nearly constant although a slight decrease is observed for larger defect sizes. Historically the determination of S - N curve as Fig. 3 has been an important objective of fatigue testing. However, S - N curves such as Curves A, B and C have been treated only as separate experimental results and the correlations among these curves have not been studied in depth.

S - N data for materials manufactured by the additive manufacturing (AM) process usually show a large scatter in fatigue lives and fatigue limit. The scatter is caused by the different size of defects at fatigue fracture origins, i.e. mostly the largest defect contained in the surface layer of an individual AM specimen. As an assumption, if several AM

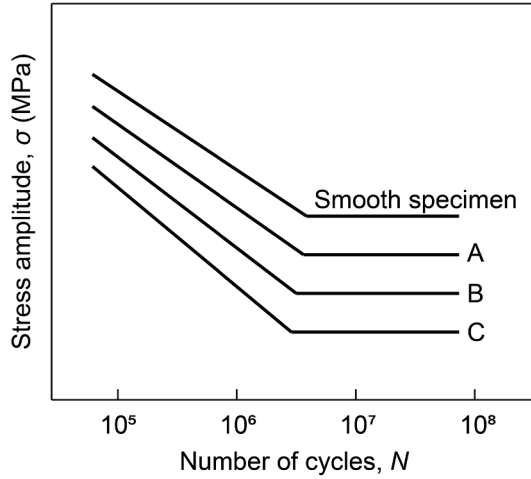


Fig. 3. *S-N* data for specimen Series A, B and C containing defects with different sizes. The sizes of defects are d_A , d_B , and d_C ($d_A < d_B < d_C$) for specimen Series A, B and C.

specimen series having a defect with the same size in each series are prepared, separate *S-N* curves for each defect size will be obtained as in Fig. 3. However, since it is impossible to prepare such specimen series intentionally, apparently one set of *S-N* data which are composed with a mixture of separate *S-N* data such as Curves A, B and C are obtained in actual fatigue test and a large scatter will be observed. So far, this kind of *S-N* data or *S-N* curves has been reported without further pursuit as the typical fatigue properties of various materials. Although some papers treat the scatter of fatigue lives in *S-N* data as a normal distribution, there is no theoretical justification that the scatter of *S-N* data obeys the normal distribution as indicated by Murakami et al. [19,20]. If we assume that the gauge part of one cylindrical fatigue specimen is composed of 10 sub-smaller circular plate specimens, the fatigue failure of the specimen occurs at the weakest sub-specimen. Although the fatigue strengths of the 10 sub-specimens in one specimen are thought to obey the normal distribution, the statistical distribution of the fatigue strength of the failed specimens should obey the statistics of extremes distribution [19,21–28].

This paper elucidates the essential structure of *S-N* curves by decomposing the mutually important correlations of *S-N* curves such as Curves A, B and C in Fig. 3 from the viewpoint of the effects of small defects and small cracks. As explained later, this viewpoint is a basic condition for the fatigue life prediction under variable amplitude loading.

2.2. Fatigue limit of specimens containing defects and the knee point of *S-N* curve

In order to elucidate the essential structure of *S-N* curve, the fatigue limit σ_w , the knee point, N_{knee} , and the slope of the *S-N* curve will be investigated on specimens containing defects for various materials. First of all, the horizontal line of *S-N* curve, the fatigue limit σ_w and the size of defect, $\sqrt{area}$, at fatigue fracture origin will be investigated. The fatigue limit σ_w of specimens containing a small defect with the size $\sqrt{area}$ has been studied in detail and can be predicted for many metallic materials by Eq. (4) of the $\sqrt{area}$ parameter model [20,29,30].

$$\sigma_w = C(HV + 120)/(\sqrt{area})^{1/6} \quad (4)$$

where, the units of the quantities in Eq. (4) are σ_w : MPa, HV : kgf/mm²,

$\sqrt{area}$: μm . $\sqrt{area}$ is the representative size of a defect and is defined by the square root of the area obtained by projecting the defect onto a plane perpendicular to applied stress. The constant C is $C = 1.43$ for a surface defect and $C = 1.56$ for an internal defect [20,29,30]. The $\sqrt{area}$ parameter model can be used as the powerful tool for the prediction of fatigue limit, the horizontal line of *S-N* curve. If a specimen does not contain a definite defect, the fatigue limit can be approximately estimated by $\sigma_w = 1.6HV$ [31,32]. The details of the model are explained in Ref. [20,29,30]. Using this model, the fatigue limits (for example σ_{wA} , σ_{wB} and σ_{wC} in Fig. 3), namely the horizontal lines of Fig. 2 can be predicted.

Therefore, if we have two fatigue data points in the finite life regime (Fig. 2) as the minimum condition, we can approximately determine the knee point N_{knee} as the intersection point of the finite life line and the horizontal line of the fatigue limit σ_w . The existence of the knee point is mechanically related to the fact that fatigue limit is determined when an initiated crack after some initial growth [20,29,30] becomes non-propagating. It follows that the fatigue limit is not a critical stress for crack initiation but the threshold stress at which the initiated crack ceases growth. Cracks can become nonpropagating when they reach a microstructural barrier such as a grain boundary. In many materials the crack closure phenomenon [33–35] is the dominant mechanism. Three types of crack closure mechanisms were reported, plasticity induced crack closure [33–35], oxide induced crack closure [36,37] and surface roughness induced crack closure [38,39]. Among these three mechanisms plasticity induced crack closure [33–35] is the most important mechanism. Without understanding the phenomenon of nonpropagating cracks, it is impossible to understand the essential structure of *S-N* curve. In some exceptional problems such as very high cycle fatigue (VHCF), fatigue cracks initiate at nonmetallic inclusions which trap hydrogen and the hydrogen enhances the elimination of distinct fatigue limit (knee point) by the mechanism of hydrogen induced fatigue crack growth [20,40]. Another case of the elimination of distinct fatigue limit is VHCF of Ti alloys in which crack initiates at a large internal α phase or at a group of α phases adjacent to each other having preferential orientation to applied stress [41,42]. In these cases, the clear knee point due to the mechanism of nonpropagating crack does not necessarily appear and the phenomena must be treated as exceptional cases.

2.3. Structure of *S-N* curve: Unification of the data by the normalized *S-N* curve

Fig. 3 schematically illustrates a group of *S-N* curves of specimens containing defects with different sizes. If the stress σ higher than fatigue limit σ_w for each defect, d_A , d_B and d_C ($d_A < d_B < d_C$), is applied, fatigue failure occurs in a finite life. Therefore, the driving force is not the absolute value of the applied stress σ but should be measured as the value of the stress σ relative to fatigue limit σ_w .

Namely, σ/σ_w represents the driving force and accordingly determines *S-N* curve. In other words, the relative value of σ/σ_w for $\sigma/\sigma_w > 1$ effectively controls the fatigue life N_f . Thus, plotting *S-N* data of Fig. 3 in terms of σ/σ_w in the ordinate, we should have another type of *S-N* curves as in Fig. 4. Although the knee points, N_{knee} , for Curves A, B and C may not be exactly the same value, the values of N_{knee} for the cases of small defects are within a narrow range for many materials.

Test data for a medium carbon steel is shown in Fig. 5 (a). Fig. 5 (b) shows the same data normalized by the fatigue limit σ/σ_w . This *S-N* curve will be called the *normalized S-N curve* in the following discussion and is expressed as

$$N = C(\sigma/\sigma_w)^m \text{ for } \sigma > \sigma_w \quad (5)$$

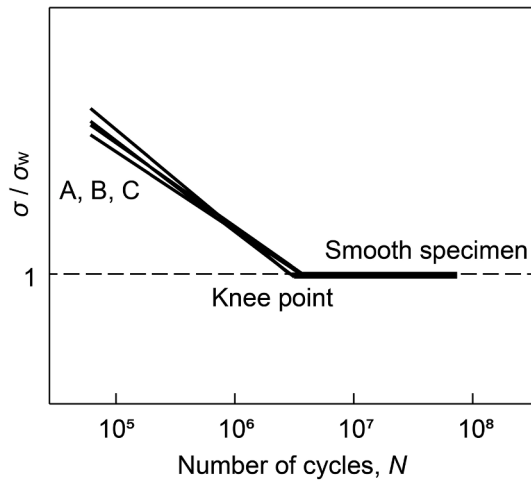


Fig. 4. $\sigma/\sigma_w - N$ curves (Normalized $S-N$ curves) made from Fig. 3. Fatigue limit σ_w , $\sigma_w = C(HV + 120)/(\sqrt{area})^{1/6}$.

The $S-N$ curves normalized by the fatigue limit σ_w calculated by Eq. (4) play an extremely important role to understand the structure of $S-N$ curves. For example, as explained later, the analysis of $S-N$ data based on the method of Fig. 4 is useful for the interpretation of $S-N$ data of AM materials in which the size of defects has a large scatter. In the following, the normalized $S-N$ data such as Fig. 4 for various materials will be investigated.

3. Interpretation of $S-N$ data of additively manufactured (AM) materials

It is well known that $S-N$ data of AM materials have a large scatter. AM materials are manufactured by special processing techniques different from those for wrought materials and it is therefore difficult in principle to avoid various types of defects such as lack of fusion and gas

pore. Fig. 6 (a) shows the original $S-N$ data of Ti-6Al-4V, and Fig. 6 (b) is the normalized $S-N$ data made by the method of Fig. 4 [44,45]. The values of σ_w of the specimens which ran out 10^7 cycles at the previous stress σ were tested again at a higher stress until the specimen fractured and the size of the defect at fracture origin, $\sqrt{area}$, was identified to calculate the value of σ/σ_w .

Fig. 7 (a) and (b) show the $S-N$ data and the normalized $S-N$ data for AM Ni-based superalloy 718 manufactured by two different processes, Batches A and B [46]. It is interesting to see that although the original $S-N$ data for Batch A are plotted separately from Batch B, the normalized $S-N$ data are almost overlapped. This is because the size of the defect at fracture origin is taken into consideration for calculation of the fatigue limit σ_w of individual specimens.

Fig. 8 shows the $S-N$ data and the normalized $S-N$ data made by replotting the original data reported by Meneghetti et al. [47]. It must be noted that, in Fig. 8(b), there are many specimens which failed below the predicted fatigue limit σ_w , i.e. $\sigma/\sigma_w < 1$. The reason will be discussed in detail in Section 5 (Causes of scatter of $S-N$ data).

The sizes of defects in AM materials are in general not constant and have a large scatter. The size and number of defects are larger in low quality AM materials. Since the individual AM specimens contain different defects in size, location and number, the fatigue limit of one series of specimens is not constant and then the fatigue life also varies depending on individual specimens. This fact has not been recognized in some existing papers and the scatter of fatigue limits and fatigue lives have been interpreted in the manner of Weibull model. Now, if we assume that we can prepare a series of specimens which contain a defect with the same size as shown Fig. 9, we can apply the same method explained in Section 2 for the artificial defect.

However, since in reality the size of defects necessarily has some scatter, $S-N$ curves cannot be expressed with one representative $S-N$ curve and the individual data points are interpreted as one point on the several groups of $S-N$ curves explained in Section 2. The several groups of $S-N$ curves such as Curves A, B and C in Fig. 3 and Fig. 4, which are not recognized explicitly in AM materials, have been simply interpreted as the scatter of fatigue life. In such a case, the width of the scatter band of

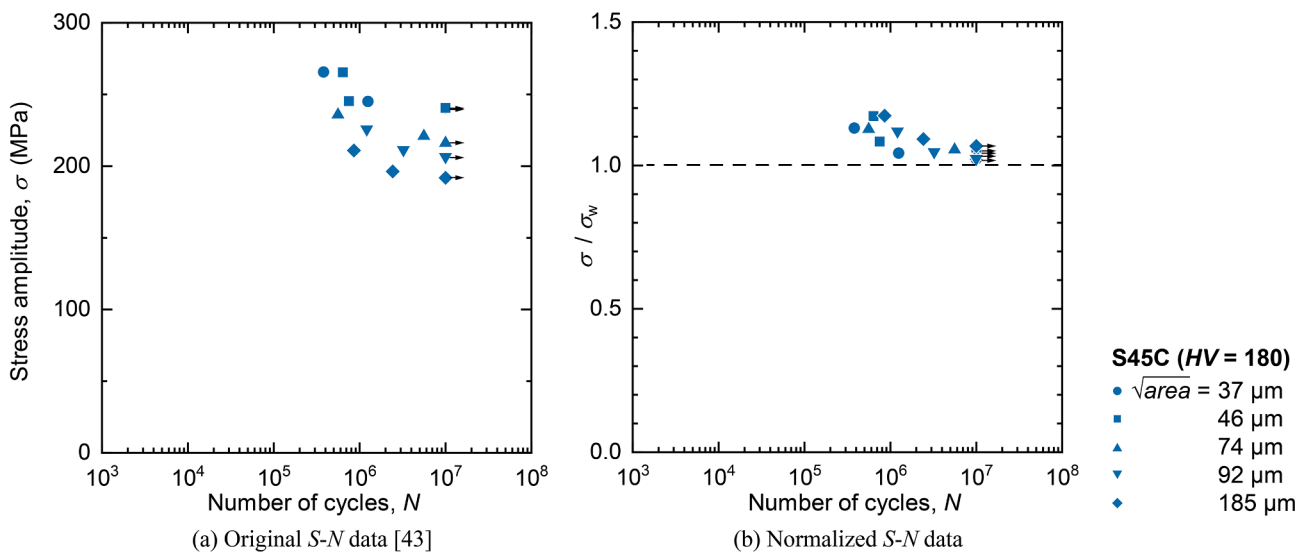


Fig. 5. Normalized $S-N$ data. Material: Annealed 0.46% carbon steel.

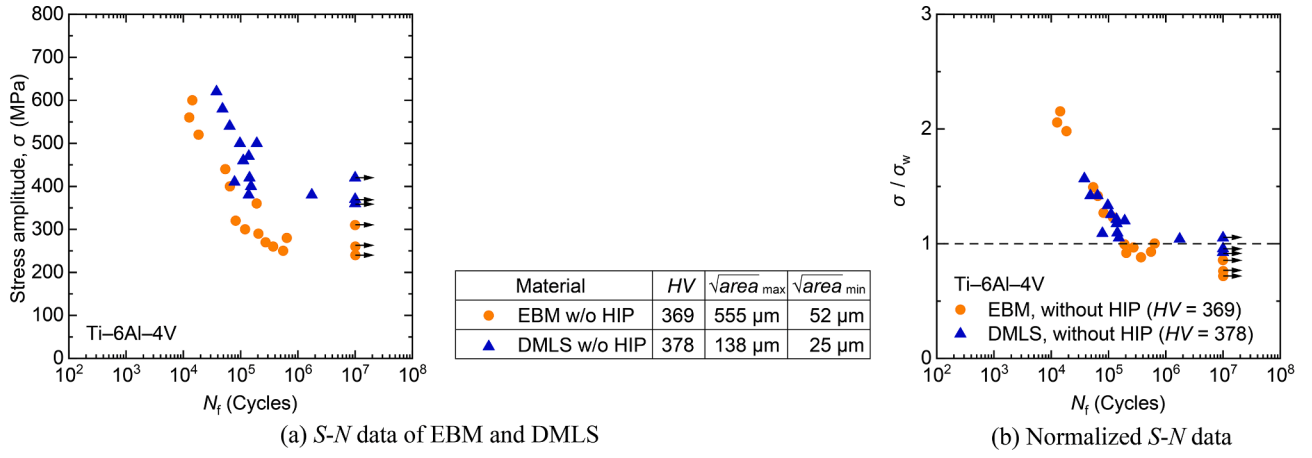


Fig. 6. S - N data of AM Ti-6Al-4V and the normalized S - N data of Ti-6Al-4V [44,45]. EBM: Electron Beam Melting, DMLS: Direct Metal Laser Sintering. Rotating bending fatigue.

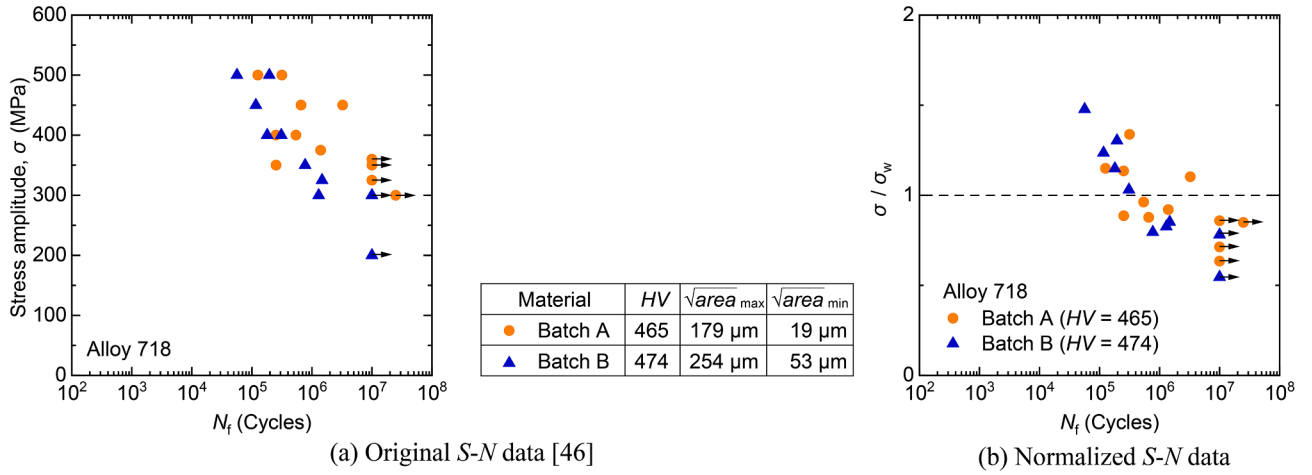


Fig. 7. S - N data of AM Ni-based superalloy 718 and the normalized S - N data. Tension-compression fatigue.

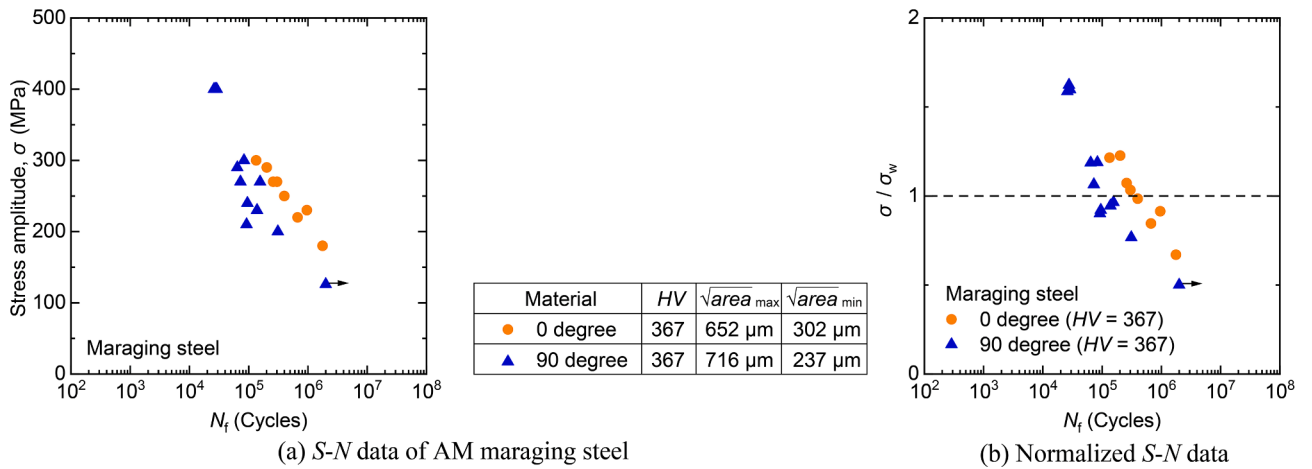


Fig. 8. S - N data of AM maraging steel [47] and the normalized S - N data. Tension-compression fatigue.

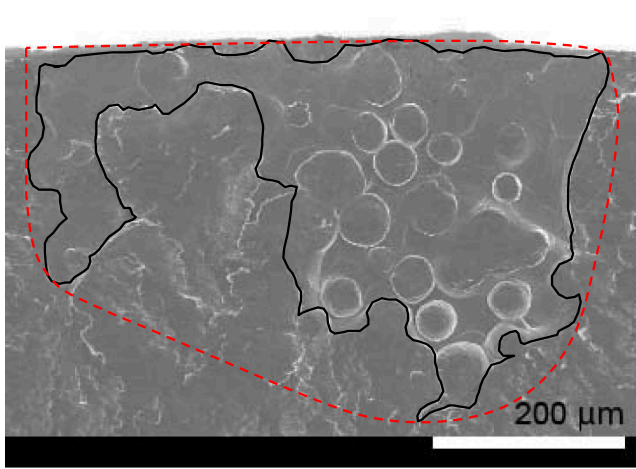


Fig. 9. Defect at fatigue fracture origin observed at fatigue fracture origin of AM Ti-6Al-4V by EBM without HIP [44,45]. The size of the defect, $\sqrt{\text{area}} = 454 \mu\text{m}$. This defect is plotted with the mark $\circ^*$ in the statistics of extremes data of Fig. 10.

fatigue lives can be predicted by using the statistics of extremes data (Fig. 10) [44,45] of the size of defects observed at the fatigue fracture origin (e.g. Fig. 9) in combination with the method of the normalized $S-N$ curves (Fig. 4). If such predictions are necessary before fatigue test, similar defect data of statistics of extremes on the as-processed AM material can be used for this purpose [44–46].

4. $S-N$ data of other materials and the normalized $S-N$ data

Fig. 11 shows $S-N$ data and the normalized $S-N$ data for various steels [43,48–52] including several data points for specimens containing

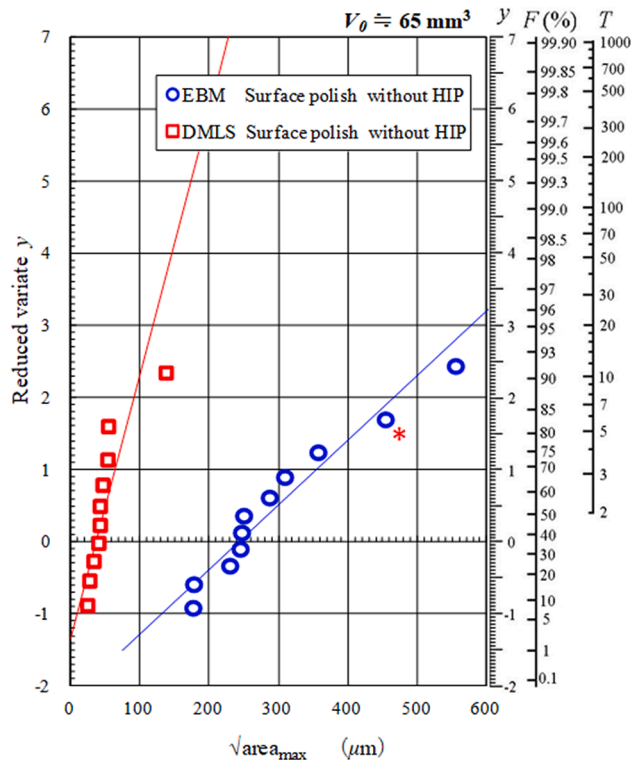


Fig. 10. Statistics of extremes of defects at fracture origin observed at fatigue fracture origin of AM Ti-6Al-4V alloy [44,45]. $V_0 \approx 65 \text{ mm}^3$: Control volume of surface annual zone of one specimen under stress higher than 90% of nominal stress. The second largest defect of EBM marked by $\circ^*$ is shown in Fig. 9.

defects. Although the difference in fatigue lives at a constant stress level looks like scatter, this scatter almost disappears in the normalized $S-N$ data of Fig. 11 (a-2). This trend is the same in Fig. 11 (b-2), (c-2) and (d-2). Note that the data points for 300M steel [52] in Fig. 11 (d-1) shows the largest scatter in fatigue life at the same stress level, the data are uniquely rearranged in the normalized $S-N$ data in Fig. 11 (d-2). The data points for 300 M steel fall above σ/σ_w of one in Fig. 11 (d-2). This is because the specimens have been shot peened [52] and have compressive residual stress at the fatigue fracture origins, but the fatigue limits were calculated by Eq. (4) for zero mean stress.

The knee point N_{knee} of steels having Vickers hardness lower than $HV = 400$ is distinct and ranges around $N_{\text{knee}} \approx 10^5 - 10^6$. On the other hand, N_{knee} for heat-treated steels such as bearing steels and low alloy steels with $HV > 400$ is not distinct as Fig. 11 (b-1) and (b-2). The fatigue fracture origins of these hard steels are mostly at nonmetallic inclusions. The reason for nondistinct knee point is presumed to be the influence of hydrogen trapped at nonmetallic inclusions [20,40]. The effect of hydrogen on VHCF is attributed to hydrogen induced fatigue crack growth which weakens the plasticity induced crack closure mechanism and eventually the distinct knee point disappears [20,40].

5. Causes of scatter of $S-N$ data

There are three main factors as the cause of scatter of $S-N$ data.

- Factor 1: Scatter of defect sizes contained in materials.
- Factor 2: Variability of the microstructure.
- Factor 3: Poor alignment of testing machine and specimens.

Factor 1: Scatter depends on presence of initial defects. If the scatter of defects is small, the scatter of fatigue life and fatigue limit is small. Typical examples have been shown in Fig. 11.

Factor 2: In smooth specimens containing no definite defect, a crack which initiates in the microstructure corresponds to an initial defect. In this case, fatigue crack initiates in a grain preferentially oriented to the direction of applied stress or along grain boundary. Such a fatigue crack is mechanically equivalent to an initial defect but the size is dependent on the grain size and the orientation. For such reasons, the scatter of fatigue data of smooth specimens is larger than that of specimens containing an artificial defect with constant size larger than grain size. Factor 2 influences Factor 1 to some extent if there is variation of microstructure around the initial defect. It must be noted that many grains yield and plastic deformation occurs at high applied stresses, so that mechanical behavior of microstructure is averaged over many grains resulting in less scatter of fatigue lives N_f . However, by decreasing the applied stress close to fatigue limit, the scatter of crystallographic slip behavior (plastic deformation) among grains increases inside one specimen and the growth of initiated cracks is influenced by the unyielded grains resulting in large scatter in crack growth rate and fatigue lives N_f . Presence of many nonpropagating cracks at the fatigue limit in low and medium carbon steels is the evidence of this phenomenon [20,29,30]. It seems that this phenomenon occurs in combination with the crack closure mechanism which is sensitive to crack tip deformation.

Factor 3: It must be noted that poor alignment of the testing machine and specimens can frequently be a cause for the scatter in fatigue life and strength even if specimens contain no apparent defect. For example, it is well known that machine vibration in rotating bending fatigue test significantly influences the test results. The typical causes of machine vibration in rotating bending test are as follows, (1) the poor alignment of specimen attachment to machine, (2) the resonant vibration of testing machine due to the loss of damping capability by the temperature rise of bearings sometime after start of operation when the number of machine rotation is close to the resonance vibration, and (3) building vibrations during fatigue testing [53].

As an additional factor of Factor 3, we need to pay attention to the

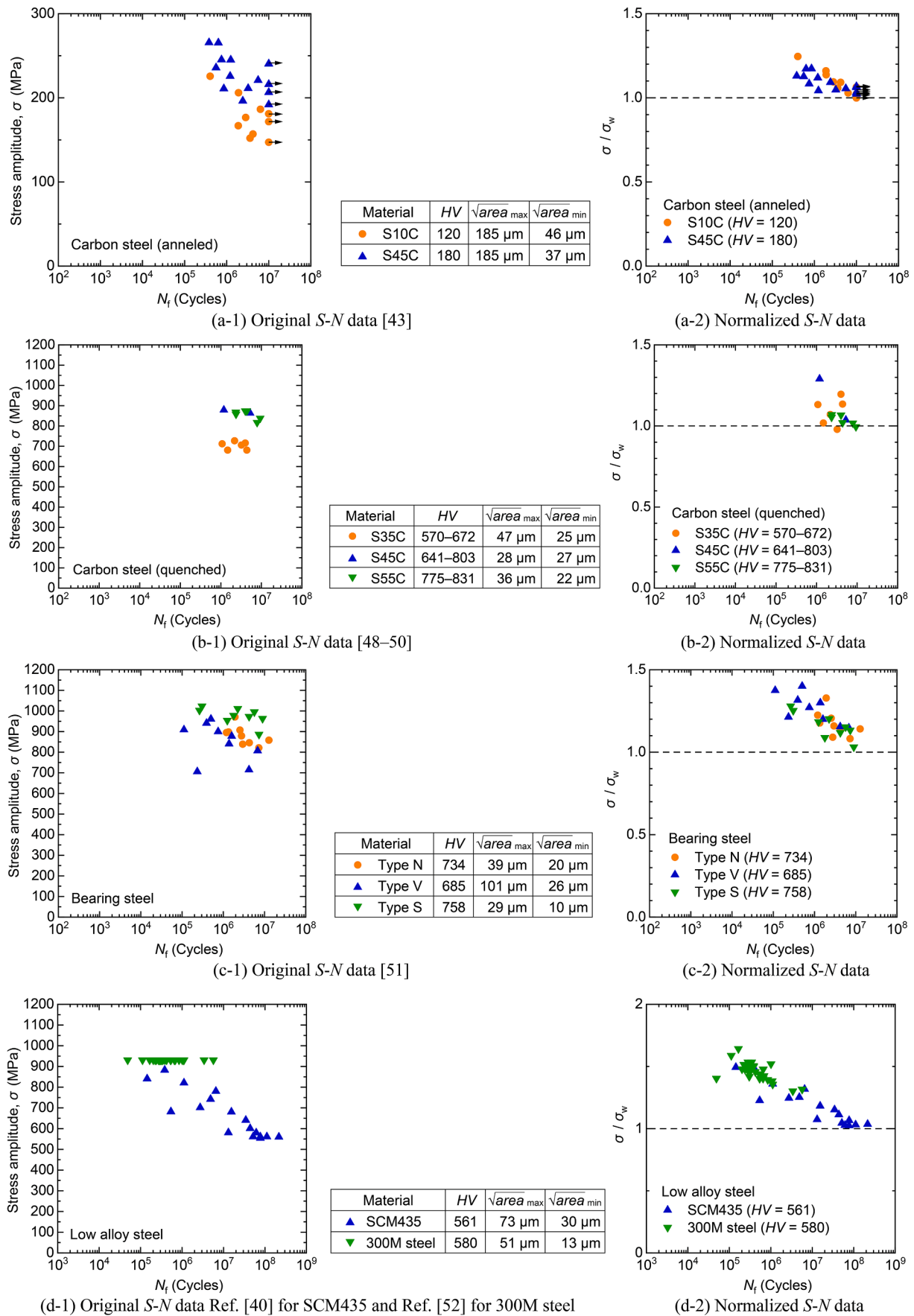


Fig. 11. $S-N$ data of various materials and the normalized $S-N$ data. Source data (a-1), (b-1), (c-1) and (d-1). Normalized $S-N$ data (a-2), (b-2), (c-2) and (d-2).

effect of residual stress. Regularly distributed residual stress introduced on purpose by several industrial techniques to increase fatigue strength does not become the main cause of scatter of fatigue strength and life. However, we have to note that irregularly distributed residual stress created by such as the additive manufacturing process [54,55] causes distortion of specimens and eventual scatter of fatigue strength and life.

If fatigue tests are carried out using specimens with distortion or a fatigue testing machine with poor alignment, the resultant S - N data have no consistency and will have a large scatter. Discussion on fatigue mechanisms and models based on such inconsistent data does not guarantee the correct solution. Usually it has not been explicitly mentioned in published papers that the alignment of testing machine and specimens is precisely checked and this problem has not been discussed in public. The detailed explanation by Meneghetti et al. [47] on this problem gives a useful and constructive information to be considered. Fig. 8 is the replot of the data reported in Ref. [47]. Fig. 8 (b) (AM maraging steel) has clearly revealed many data points of fatigue failure under $\sigma/\sigma_w < 1$ in the normalized S - N curve. The main cause of the unusually low fatigue strength is presumed to be the distortion of specimens as pointed out by the authors of Ref. [47]. According to the discussion by the authors of Ref. [47], there is a possibility that the transition zone of the threshold stress intensity factor ΔK_{th} from small crack to long crack is influenced by the specimen distortion and residual stress. The authors of the present paper have recognized the problem of the transition zone from small crack to long crack [20,56–59]. However, it is very difficult to discuss the scatter problem of Fig. 8 (Ref. [47]) from the viewpoint of the transition zone, because the data are presumed to include the effects of mixture of several factors such as residual stress, specimen distortion and the crack size transition zone.

According to the survey by the authors of the present paper, the problem of poor alignment of testing machines and specimens is becoming more serious. If fatigue fracture origins of hard steels having $HV > 400$ are mostly at specimen surface, bent specimens should be suspected. The brief conclusion on this problem is that the quality of fatigue test data is different depending on testing laboratories in universities and industries. If unusually low fatigue strength data are reported [60–62], this factor should be suspected.

6. The minimum data necessary for the prediction of S - N curve

Large numbers of fatigue tests have been carried out all over the world. How many fatigue tests are necessary for quick determination of

approximate S - N curve? Knowing the minimum number of data for reliable prediction of an S - N curve is useful for efficient fatigue tests and minimizing number of fatigue specimens. However, needless to say this plan can be achieved based on the condition of correct alignment of testing machines and specimens.

According to the method explained in this paper, the necessary and minimum data for determining an S - N curve are the Vickers hardness HV of the material in question, the size of the defect, $\sqrt{area}$, and two data points. As shown in Fig. 2, connecting two data points, S - N curve above fatigue limit σ_w can be approximately determined. This line is denoted by Line (a). The horizontal line of fatigue limit σ_w is denoted by Line (c) which can be determined by Eq. (4). The intersection of Line (a) and Line (c) is the knee point N_{knee} .

If additional data are prepared, the value of slope of S - N curve, m , and the value of C in Eq. (5) can be revised for determination of more reliable S - N curve. If we can establish this method, it is not necessary to carry out so many fatigue tests.

The advantage of the formulation of S - N curve of this method is not limited to one series of specimens containing a defect with the same size. Extending the method in combination with the crack growth rate analysis as explained in Section 7, S - N curves for other series of specimens containing defects with different size can be predicted.

Another advantage of the present method is the application to the prediction of the scatter band in S - N curve for the materials such as AM materials containing defects with various sizes. Although the size of the defect at fatigue fracture origin for AM materials, cast irons, powder metals and hard steels is unknown before fatigue testing, the scatter band of S - N data can be predicted with minimum number of fatigue tests in combination with the statistics of extremes data of defects and nonmetallic inclusions.

Fig. 12 shows an example of the method applied to the prediction of the lower band (the shortest fatigue life) of the scatter of S - N curve for AM material of Ti-6Al-4V. In the prediction, the statistics of extremes data of Fig. 10 was used to predict the largest defect by assuming the number of specimens 200. Numbers in the figure represent the values of $\sqrt{area}$ for each specimen. Thus, the collections of the statistics of extremes data of defects for AM materials [44–46,63,64] will be useful for the determination of the lower bound of fatigue limit and fatigue life.

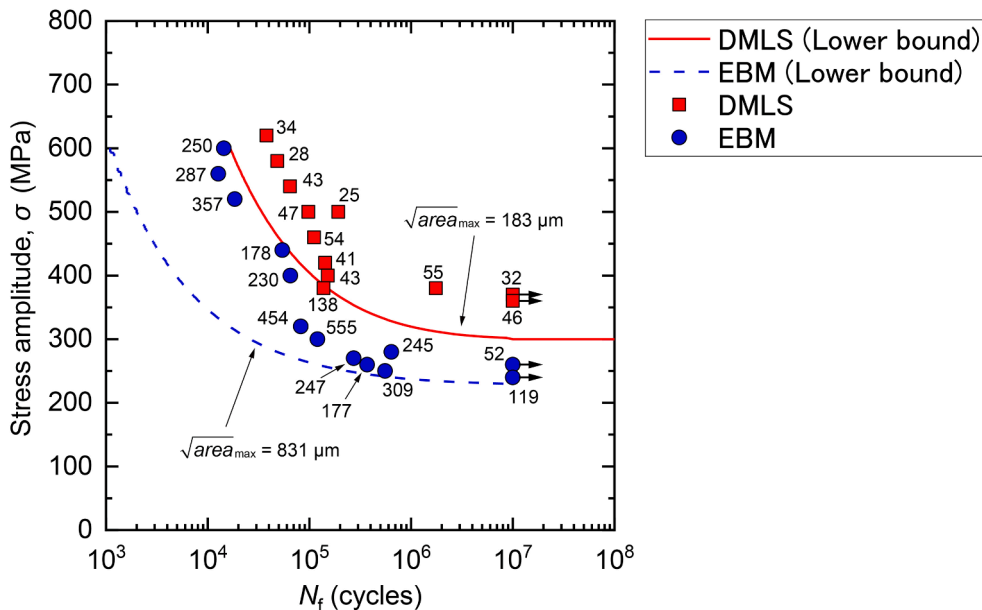


Fig. 12. Lower bound (the shortest fatigue life) for the fatigue life of the additively manufactured Ti-6Al-4V calculated based the statistics of extreme values of defects for 200 specimens (Fig. 10) [44,45]. Details of the computations are given in the Appendix as well as determining the constants C^* and m^* ($C^* = 10^{-4.2}$, $m^* = 2.0$) in Eq. (7). The predicted largest values of defects, $\sqrt{area}_{max}$, for 200 specimens are $\sqrt{area}_{max} = 183 \mu m$ for DMLS and $\sqrt{area}_{max} = 831 \mu m$ for EBM from Fig. 10. Numbers in the figure associated with the data points indicate the value of $\sqrt{area}$ at fracture origin in each specimen. DMLS: Direct Metal Laser Sintering, EBM: Electron Beam Melting.

7. Perspective for fatigue life prediction under variable amplitude loadings: Conditions for applicability of Miner's rule

Service loadings in machine components are usually not constant amplitude. Therefore, fatigue life prediction of actual components and structures under variable amplitude loadings is of practical importance. Miner's rule [12] is often used for fatigue life prediction under variable amplitude loading because of simplicity. In Miner's rule, fatigue damage D_i is linearly accumulated such that one cycle of the stress σ_i produces a fatigue damage of $1/N_i$ where N_i is the constant amplitude fatigue life at a stress of σ_i . In a variable amplitude loading history the total accumulation of fatigue damage D is calculated as follows.

$$D = \sum (n_i/N_i) \quad (6)$$

where, the individual cycles, n_i : each with their own mean and amplitude is identified with a suitable cycle counting scheme.

Here, how should we count the contribution of a stress σ_i below σ_w to D ? In Miner's rule, a cycle of stress below the initial fatigue limit $\sigma_{w,initial}$ will not cause any fatigue damage. In this discussion, we must note that the value of σ_w continuously decreases with crack growth as a function of crack size throughout the lifetime, and the fatigue life N_f at the stress below σ_w becomes ∞ . This situation is schematically explained by Fig. 13. Therefore, as a result, stress cycles of $\sigma_i < \sigma_{w,initial}$ that were initially nondamaging become damaging once σ_i exceeds the fatigue

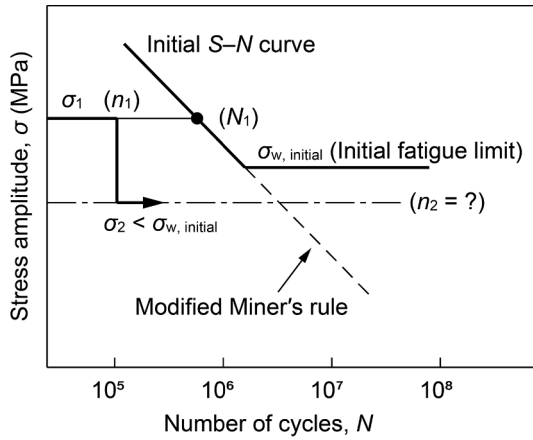
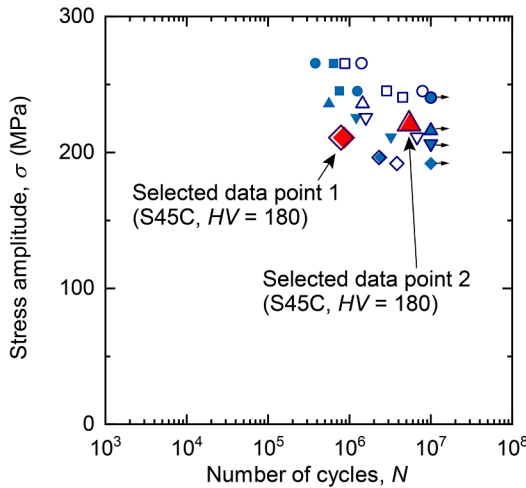
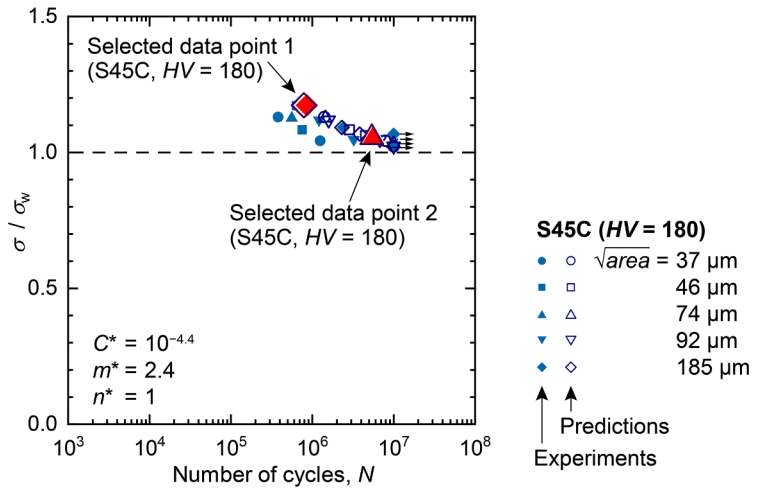


Fig. 13. Concept of Modified Miner's rule.



(a) Original S-N data and selected two data points



(b) Normalized S-N data of the original data and predicted points

Fig. 14. Comparison between the original data and predictions by using two selected data points. (a) the original S-N data [43] and (b) normalized S-N data.

limit for a critical crack size.

Historically, modified Miner's rule was proposed as a convenient solution. In modified Miner's rule, the straight line of finite life regime of S-N curve is extended downward below the initial fatigue limit (Fig. 13) and a fictitious fatigue life (the dashed line in Fig. 13) is assumed for convenience. However, the mechanistic basis of Miner's rule has not been fully discussed in case of high cycle fatigue [20,65]. If we simply apply this rule to actual components, an unusually large safety factor may be necessary in some service loading conditions.

However, if we understand correctly the essential structure of the S-N curve of specimens containing defects, we can consider the continuous decrease in fatigue limit σ_w with number of cycles for fatigue design. If subsequent applied stress σ_{i+1} is lower than the fatigue limit σ_w at the end of stress σ_i , the damage by σ_{i+1} can be ignored. Thus, for the correct estimation of the fatigue damage, D , it is rational to consider the correlation between the applied stress σ_i and the fatigue limit σ_w which continuously decreases with number of cycles, i.e. growth of crack. In the stage of the present discussion, to avoid the complicated phenomenon in small crack problems, we ignore the effect of retardation or acceleration of crack growth after overloading.

One example of this method will be introduced in the following. First of all, it is necessary to elucidate the correlation between the S-N curve with the form $N_f = C(\sigma/\sigma_w)^m$ and the growth process of small crack. The basic approach adopted here is similar to the method explained in the reference [20,65]. Then,

$$\frac{da}{dN} = C^* (\sigma/\sigma_w - 1)^{m^*} a^{n^*} \quad (7)$$

where, $a = \sqrt{area}$ is defect size, and C^* , m^* and n^* are material constants.

It must be noted here that σ_w is the fatigue limit which decreases with every cycle continuously as a function of a ($=\sqrt{area}$) and that σ_w can be expressed by Eq. (4). The term $(\sigma/\sigma_w - 1)$ in Eq. (7) is a function of a ($=\sqrt{area}$) and expresses the effective driving force, which increases cycle-by-cycle since σ_w decreases with an increase in a . By integrating this equation, N_f and growth process of small crack can be connected.

Since Eq. (7) cannot be integrated in a closed form, the numerical integral is needed to calculate predicted N_f . Presuming $n^* = 1$ is reasonable by considering the data reported in existing papers [66–69], at least two data points are necessary to determine C^* and m^* . This tentative assumption of $n^* = 1$ is to be reasonable if we consider the analogy of the present model to the Dugdale model [70], in which the plastic zone size ahead of crack tip, r_p , is linearly proportional to crack

length. If the prediction results contradict with the experimental results, the value of n^* must be modified. Once C^* , m^* and n^* are determined based on two or three fatigue data points, Eq. (7) can be used extendedly to other arbitrary loading history. Details of the numerical integral applied in the present study are explained in the Appendix.

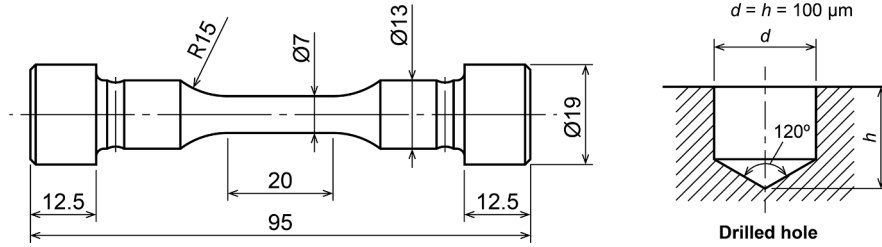
First, the method was applied to an annealed medium carbon (0.46 %C, JIS S45C) steel under constant amplitude loading. Fig. 14 shows (a) the original $S-N$ data [43] and (b) the normalized $S-N$ data, which were obtained at a stress ratio, R , of -1 using specimens having various sizes of drilled hole. Selecting the two experimental data points obtained for $\sqrt{area} = 185 \mu\text{m}$ (cf., red diamond in Fig. 14) and $\sqrt{area} = 74 \mu\text{m}$ (cf., red triangle in Fig. 14), we determined the constants as $C^* = 10^{-4.4}$, $m^* = 2.4$ and $n^* = 1.0$ based on Eq. (7). The predictions based on Eq. (7) for other defect sizes are in good agreement with the experiments.

The second application of the method was planned and performed as Fig. 15. The fatigue behaviors were predicted for medium carbon steel specimens containing a small artificial defect under a two-level stress

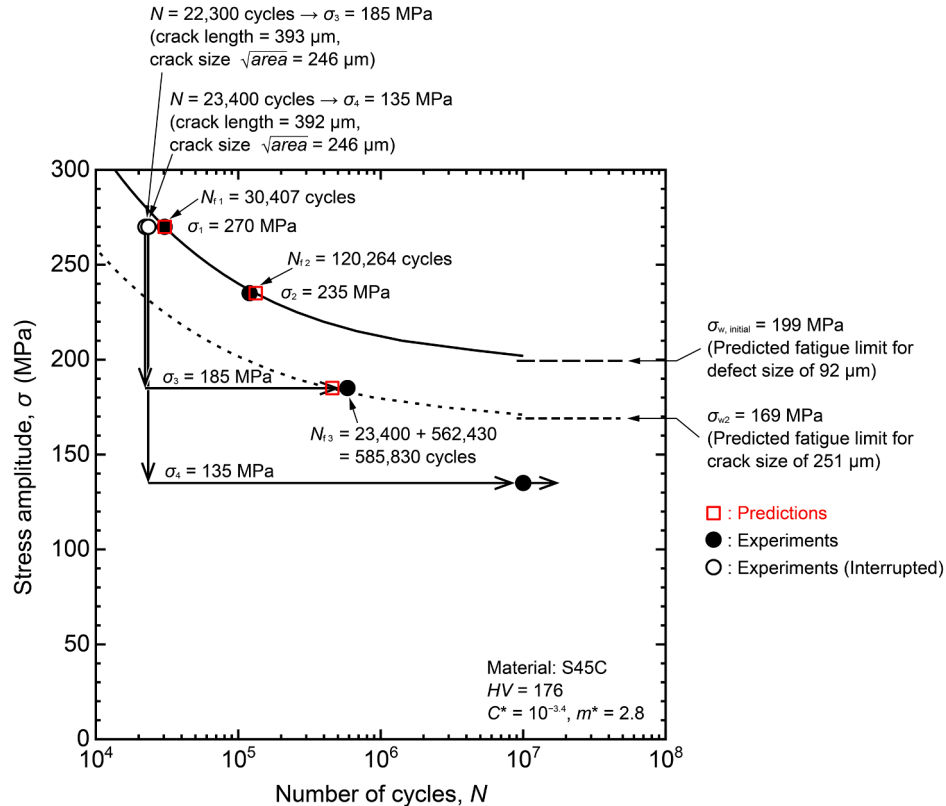
test. The completely reversed tension-compression fatigue test with $R = -1$ was performed by MTS servo-hydraulic fatigue testing machine under frequency of 10 Hz for short fatigue life test and 20 Hz for relatively long fatigue life test close to fatigue limit.

Specimens were machined after annealing the original 0.47% C (JIS S45C) steel bars. The distortion of all the specimens after machining was confirmed to be $< 10 \mu\text{m}$ at the mid point of the specimens. Afterward, the specimens were polished by emery papers followed by electro-polishing for the observation of crack growth behavior. In order to check the alignment of fatigue testing machine, four strain gauges were attached to every specimen before test at the equal quarter points of the circumferential direction at the part of 13 mm diameter (Fig. 15 (a)). The bending strain for setting specimens to fatigue testing machine was strictly controlled within $\pm 2.5 \mu\epsilon$.

A drilled hole with $100 \mu\text{m}$ diameter (d) and depth (h), $d = h = 100 \mu\text{m}$ ($\sqrt{area} = 92 \mu\text{m}$), (Fig. 15 (a)) was introduced onto specimen surface.



(a) Specimen containing an initial artificial drilled hole with $100 \mu\text{m}$ diameter (d) and depth (h), $d = h = 100 \mu\text{m}$. ($\sqrt{area} = 92 \mu\text{m}$), $HV = 176$. The predicted initial fatigue limit $\sigma_{w,initial} \sim 200 \text{ MPa}$.



(b) Step loading fatigue test results and analysis.

Fig. 15. Basic experiments for investigating step loading.

The initial fatigue limit $\sigma_{w,initial}$ of the specimen containing the 100 μm hole can be predicted by Eq. (4) to be $\sigma_{w,initial} = \sim 199$ MPa for $HV = 176$ (Average of 12 measurements with standard deviation 4.7).

Four fatigue tests were conducted as follows to evaluate the validity of the method.

(1) Fatigue tests:

Two constant-amplitude tests were conducted, at $\sigma_1 = 270$ MPa with a resulting fatigue life of 30,407 cycles and at $\sigma_2 = 235$ MPa with a fatigue life of 120,264 cycles.

The third test of $\sigma_1 = 270$ MPa for 22,300 cycles followed by $\sigma_3 = 185$ MPa (10% higher than the predicted fatigue limit σ_{w2}) until failure of an additional 562,430 cycles.

The fourth test of $\sigma_1 = 270$ MPa for 23,400 cycles followed by $\sigma_4 = 135$ MPa (20% lower than the predicted fatigue limit σ_{w2}) was stopped after an additional 10^7 cycles.

The crack length measured on specimen surface at the moment of stress-switching in the third and fourth tests was ~ 400 μm ($\sqrt{area} = \sim 250$ μm).

(2) Determination of C^* , m^* and n^* :

Based on two fatigue data obtained by tests under constant amplitude

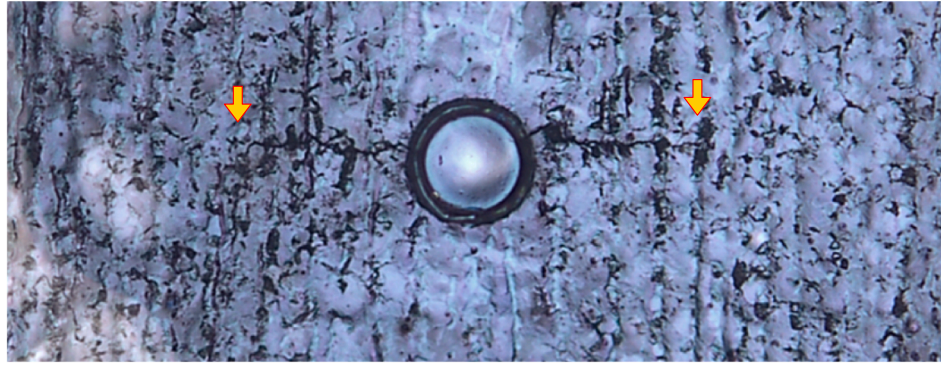
stresses, $\sigma_1 = 270$ MPa ($N_{f1} = 30,407$ cycles) and $\sigma_2 = 235$ MPa ($N_{f2} = 120,264$ cycles), C^* , m^* and n^* in Eq. (7) were determined as $C^* = 10^{-3.4}$, $m^* = 2.8$ (and $n^* = 1.0$) by the method explained in Appendix. The resulting S - N curve from the model is shown as the solid and dashed line in Fig. 15 (b). These values, C^* , m^* and n^* , were used to predict the total fatigue life for the step loading test by integrating Eq. (7).

(3) Fatigue life prediction for step loading tests:

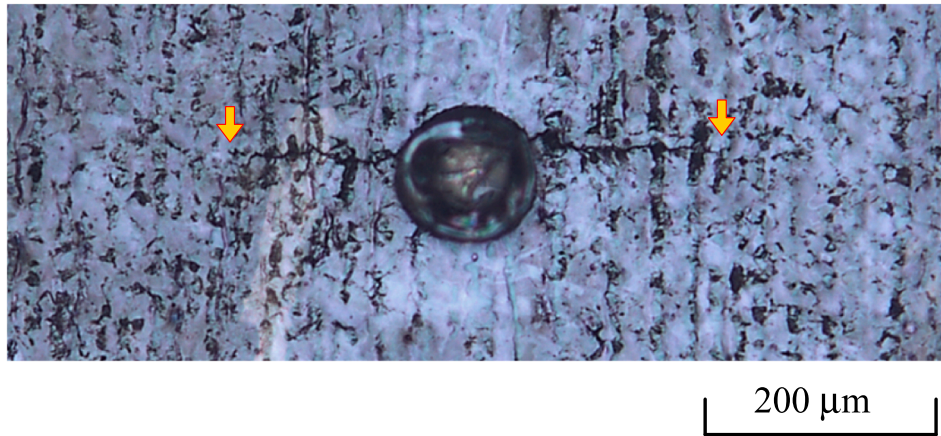
The third test: The calculated number of cycles N_{step1} for crack growth to $\sqrt{area} = 251$ μm under $\sigma_1 = 270$ MPa was $N_{step1} = 23,000$ cycles. At the second step, the number of cycles N_{step2} for crack growth to $\sqrt{area} = 1000$ μm under $\sigma_3 = 185$ MPa was calculated as $N_{step2} = 434,000$ cycles. In this calculation, the cycles after crack size 1000 μm was neglected, because it is well known that remaining life after crack size 1000 μm is very short. Then, the calculated total number of cycles to failure $N_{f, step1} + step2 = N_{step1} + N_{step2} = 23,000 + 434,000 = 457,000$ cycles (the red square mark $\square$ in Fig. 15 (b)). Since the experimental value is $N_{f, step1} + step2 = 585,830$, the prediction is considered acceptable.

The fourth test: At the first step, $\sigma_1 = 270$ MPa was applied for 23,400 cycles. Fig. 16 (a) shows the crack before switching to the second step. The total length of crack emanating from the hole of 100 μm diameter was 392 μm .

The second step test of $\sigma_4 = 135$ MPa (20% lower than the predicted



(a) Crack emanating from the hole of 100 μm diameter and depth at the stress $\sigma_1 = 270$ MPa and $N = 23,400$ cycles. Crack length = 392 μm .



(b) Nonpropagating crack emanated from the crack of (a) after switching the stress from $\sigma_1 = 270$ MPa to $\sigma_4 = 135$ MPa. $N = 10^7$ cycles at σ_4 (Runout). Total crack length = 432 μm .
The length l of nonpropagating crack: $l_{left} = 12$ μm and $l_{right} = 28$ μm .

Fig. 16. Crack growth and nonpropagation at the step loading test (the fourth test) by (a) the first step, $\sigma_1 = 270$ MPa and $N = 23,400$ cycles and (b) the second step test, $\sigma_4 = 135$ MPa (20% lower than the predicted fatigue limit σ_{w2}) after an additional $N = 10^7$ cycles.

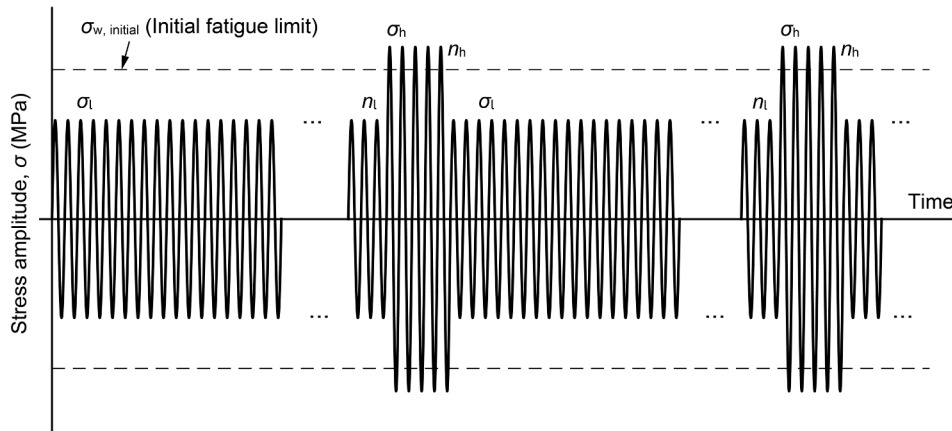


Fig. 17. Step loading having the alternation of low and high stress cycle blocks.

fatigue limit σ_{w2}) was stopped after an additional 10^7 cycles. Fig. 16 (b) shows the nonpropagating cracks emanated from the crack of Fig. 16 (a) after switching the stress from $\sigma_1 = 270$ MPa to $\sigma_4 = 135$ MPa for $N = 10^7$ cycles (Runout). The total crack length was $432 \mu\text{m}$ including the length l of nonpropagating cracks, $l_{\text{left}} = 12 \mu\text{m}$ and $l_{\text{right}} = 28 \mu\text{m}$. Thus, the result of the fourth test verifies that the application of stress below the fatigue limit calculated by Eq. (4) for the crack size at moments is nondamaging and it is not necessary to count the cycles as a damage.

The results of these fatigue tests reveal the simple mechanism of the effect of step loading and sequence on fatigue damage.

By considering the decrease in fatigue limit with crack growth under variable amplitude or step loading, we can quantitatively count the damage caused by varying stress amplitudes based on the model of Eq. (7). For example, if a stress amplitude σ is lower than σ_w at the end of previous stress cycles, the effect of the additional cycles of σ should not be counted as fatigue damage.

If we assume a step loading like Fig. 17 in which the blocks of low σ_l and high stress σ_h cycles are alternately repeated, we need to note the variation of fatigue damage given by these blocks from the early stage of loading to the later stage of fatigue loading. Even if the damage given by the low stress block at the early stage of loading can be ignored, the damage given by the same block of stress cycles at the later stage is gradually induced, because the crack grows at the beginning by the repetition of the block of high stress σ_h and eventually after the size of the crack exceeds the critical value $\sqrt{\text{area}_{\text{crit}}}$ for low stress σ_l , the block of low stress σ_l contributes to the damage. The critical value $\sqrt{\text{area}_{\text{crit}}}$ is calculated by equating the low stress σ_l to the fatigue limit σ_w in Eq. (4). Thus, even if the block of low stress σ_l is non-damaging at the early stage of stress cycle, the effect must be taken into consideration after some repetitions of the block of high stress σ_h . At the same time, the contribution of one block of high stress σ_h on fatigue damage is not constant from the beginning to end but continuously varies as well. This means that the sequence of low and high stress applications definitely influences the damage and eventually the fatigue life under variable amplitude loading.

The results of the more systematic experiments and detailed analyses of fatigue under several step loadings and variable amplitude loadings will be reported in a separate paper based on the method developed in this paper, and the validity of existing models [12–18,71,72] will be examined.

8. Conclusions

In order to elucidate the essential structure of S - N curve such as $\sigma = C_1 N^{C_2}$, this paper discussed a prediction method for fatigue life and fatigue limit of materials containing various defects and the nature of scatter of fatigue strength from the mechanistic viewpoint of small crack.

The conclusions are summarized as follows.

- (1) Existing models for S - N curves, though having the name of a *model*, are basically made by *curve fitting* procedures and cannot be applied to the prediction of fatigue life and fatigue limit of materials containing various defects.
- (2) Replotting the S - N data to the normalized S - N data, σ/σ_w vs N , is useful for analyzing the essential structure of S - N curves and for checking the validity of the original S - N data. If many data are plotted below the predicted fatigue limit σ_w , based on the initial size of defect $\sqrt{\text{area}}$ i.e. at $\sigma/\sigma_w < 1$, the bad alignment of testing machines and specimens must be suspected.
- (3) The effective driving force for fatigue crack growth is not the absolute value of the applied stress σ . The net amount of the stress above the fatigue limit $(\sigma/\sigma_w - 1)$ determines the fatigue crack growth rate. In this analysis, it must be noted that the value of σ_w continuously varies at throughout the lifetime until the final fracture occurs.
- (4) S - N curves of specimens having defects with different initial sizes have mutual correlations. If S - N curve of one series of specimens is determined, S - N curves of other series can be predicted based on the crack growth analysis developed by the S - N curves of the first series of specimens. The scatter of S - N data of AM materials can be predicted by this analysis in combination with the statistics of extremes of defects such as lack of fusion.
- (5) The analytical method for S - N curves developed in this paper can be extensively applied to fatigue under step loadings and variable amplitude loadings. The reason for unsuccessful results of application of Miner's rule is pointed out in terms of continuously decreasing fatigue limit σ_w under fatigue cycling.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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problems. The discussion with Dr. Yamashita was successfully reflected in the present paper. The authors would like to thank Dr. Yoshihiro Fukushima for technical assistance with the experiments.

Appendix A. Determination of C^* , m^* and n^*

A1. Definition of the objective function O

The error E of the prediction $N_{f \text{ pred}}$ compared to the experimental value N_f was evaluated in a logarithmic scale.

$$E = \log(N_{f \text{ pred}}) - \log(N_f) \quad (\text{A1})$$

where, $N_{f \text{ pred}}$ was determined by integrating Eq. (7) in the main text by assuming an appropriate incremental cycle ΔN , for example $\Delta N = 100$.

The objective function O (C^* , m^* , n^*) is defined as the average value of square of E_i for one experimental data point as Eq. (A2).

$$O = \frac{\sum_{i=1}^M E_i^2}{M} \quad (\text{A2})$$

where, M is the number of data points.

A2. Minimization of the objective function O by variation of C^* and m^*

In the present paper, n^* was assumed to be 1 based on the tendencies of existing data [66–69]. The optimum C^* and m^* values were determined to minimize O . To seek an optimum set of C^* and m^* values, O was calculated for each C^* and m^* . For this analysis, at least two data-points of (σ, N_f) are necessary to determine C^* and m^* .

Once the optimum values of C^* and m^* are determined, Eq. (7) can be extendedly applied not only to the prediction of fatigue lives of specimens containing defects with various sizes under constant amplitude loading but also to the fatigue damage counting for step loadings.

A3. Results of analysis

The optimum values of C^* and m^* for Ti-6Al-4V were determined to be $10^{-4.2}$ and 2.0, respectively. The comparison between the prediction and the experimental data are given in Fig. A1. In Fig. A1, two data points marked by large red symbols were selected for the calculation. The predictions including fatigue limits are in good agreement with the experimental data.

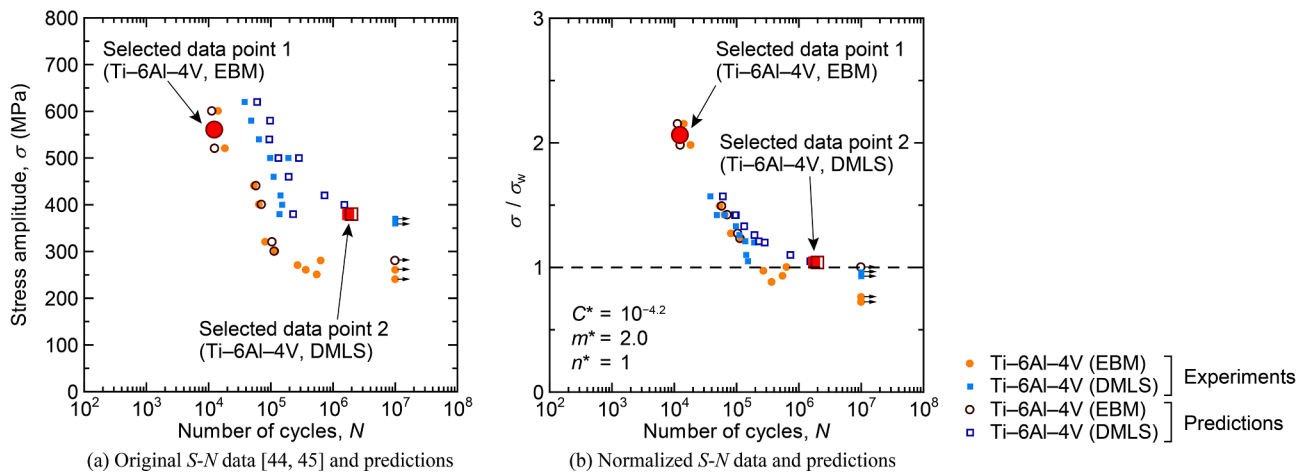


Fig. A1. Comparison between original data and predictions. Two large red symbols indicate the datasets selected for the calculation of C^* and m^* . $HV = 369$ for EBM, $HV = 378$ for DMLS (see also Fig. 6).

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