

**K. Mukhanov Design
of metal structures**

624.014
Muk

К. К. МУХАНОВ

Металлические конструкции

Основы проектирования

СТРОИИЗДАТ • МОСКВА

Design of metal structures

K. K. MUKHANOV

TRANSLATED FROM THE RUSSIAN

BY G. LEIB

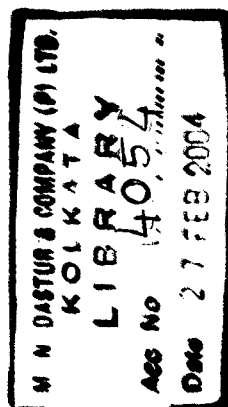
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MIR PUBLISHERS • MOSCOW • 1968

REVISED FROM THE RUSSIAN 1963 EDITION

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На английском языке

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Notation

- h = depth of beam, girder or truss, size (leg) of weld
 I = moment of inertia
 i = inclination, linear stiffness, pitch, ratio
 J = polar moment of inertia
 k = factor, length coefficient, stress concentration factor
 k_{as} = asymmetry coefficient
 k_d = section area distribution factor
 k_{hom} = homogeneity factor
 k_n = normal stress distribution factor
 k_{rs} = range ratio of stress cycle
 k_s = service condition factor
 k_w = fillet weld area reduction factor
 L = length, span
 M = bending moment
 m = number, ratio, relative or unit eccentricity
 m_1 = equivalent eccentricity
 N = number of rivets, number
 n = load factor, number, ratio
 P = load, axial force
 P = unit dead load
 Q = shear force, lifting capacity of crane
 Q' = load-carrying capacity
 q = unit live load, unit shearing force
 R = radius of sphere, resistance or strength of material
 r = radius, radius of gyration, reaction, ratio
 r_a = rise of arch
 r_0 = radius of curvature of a deflected beam or cable
 S = a dimension, length of arch, statical moment of area
 s = dimensionless quantity, ratio
 T_1 = meridional force in shell
 T_2 = annular force in shell
 t = dimensionless quantity, temperature difference, thickness
 U = energy
 u = dimensionless quantity, unit energy
 V = potential energy
 W = section modulus, work
 X = section modulus (redundant) force
 x = unknown distance
 z = distance from centre of gravity, distance from neutral axis, length of zone of local load action
- A = area
 a = coefficient, dimension, spacing of stiffeners, opening at weld, pitch of rivets
 a_k = notch toughness
 B = spacing of arches
 b = coefficient, dimension, root length of weld, spacing of floor beams, spacing of frames, width
 C = coefficient, factor, parameter
 C_a = aerodynamical coefficient
 c = coefficient, dimension, distance from centre of gravity of flange to neutral axis, load combination factor
 c_{sn} = snow distribution factor
 c_{st} = linear stiffness ratio
 c_1, c_2, c_3, c_4 = parameters used in determining conditions of column stability
 D = diameter, dimension, pair of forces, quantity characterizing cable load
 d = diameter, dimension
 E = modulus of elasticity in tension or compression (Young's modulus)
 e = arm of moment, dimension, eccentricity
 F = force
 F_{br} = braking force
 F_h = horizontal force, thrust
 F_l = longitudinal force
 F_r = reaction
 F_{res} = resultant force
 F_{sh} = shearing force
 F_t = tensile force
 F' = allowable force
 f = coefficient of friction, factor, safety factor
 f_{sh} = shearing force per unit length
 G = shear modulus of elasticity, weight
 g = unit weight
 H = height

GREEK LETTER SYMBOLS

α (alpha)	= angle, coefficient, factor, included angle of weld, parameter
β (beta)	= angle, ratio, self-stressing factor
γ (gamma)	= factor, unit weight
Δ (delta)	= displacement, increment, permanent set, temperature strain, yield or pliancy
δ (delta)	= deflection, unit displacement
ϵ (epsilon)	= longitudinal or normal strain
ζ (zeta)	= coefficient
η (eta)	= coefficient, yield of supports
θ (theta)	= angle, parameter
λ (lambda)	= slenderness ratio
μ (mu)	= Poisson's ratio, ratio, relative or unit eccentricity
μ_a	= unit strain factor
ν (nu)	= ratio of forces
ξ (xi)	= relative distance of neutral axis from edge of section
π (pi)	= ratio of length of circle to diameter (3.14...)
ρ (rho)	= core radius of cross section
σ (sigma)	= normal stress
σ'	= allowable stress
τ (tau)	= shearing stress
ϕ (phi)	= an angle, buckling factor
χ (chi)	= curvature of section
ψ (psi)	= factor, strength factor

SUBSCRIPTS

a	= actual, angle, arch, asymmetry, axial, axis
act	= active
al	= aluminium
$anch$	= anchor
ang	= angle
b	= back of angle, beam, bearer, bell, bending, bolt, bottom
bot	= bottom
bp	= base plate
br	= bearing rope, bracing beam, braking, branch
c	= combined, compression, concentrated, contour, crane
cb	= collar beam
ch	= chord
col	= column
con	= concrete, construction, controlled
cr	= critical, crushing
$cr-e$	= end crushing
$cr-l$	= local crushing

$c-r$	= compression of rollers
cr	= crane wheel
cyl	= cylindrical
d	= design, diagonal, distribution, dome
E	= Euler
e	= effective, elastic, end (edge), external, external load
ec	= eccentric
emb	= embedded
en	= endurance
eq	= equivalent
f	= fillet weld, flooring, footing, form
fb	= flooring beam
fd	= foundation
fl	= flange
g	= gusset
gd	= guide carrier
gr	= gross
h	= hinge, horizontal
hor	= horizontal
hom	= homogeneity
i	= internal
j	= joint
l	= limiting, local, longitudinal
loc	= local
m	= main (beam), mean, minor
max	= maximum
min	= minimum
n	= net, normal
o	= opening, open-web, original, outer
opt	= optimal
p	= panel, plane, plastic
pl	= batten plate, plate
pr	= proportional
ps	= prestressed
r	= rated, reaction, reactive, ring, rivet, roof
red	= reduced
ref	= reference
rel	= relative
req	= required
res	= resultant
s	= secondary (beam), service, service conditions, span, strap, support
ser	= service
sh	= shank, shearing, shell
sn	= snow

NOTATION

sph = sphere
st = steel, stiffness, strength
str = stress
suc = suction
t = tension, thread, tie, top, torsion, truss
tb = tie bar
tel = telescope
tk = tank
tr = traverse
u = ultimate, unit, unsupported
v = vertical, vibration
w = web, weld, wind
y = yield, yield point

SYMBOLS USED ON DRAWINGS

$\frac{0.00}{\text{---}}$ = elevation; the number adjacent to the symbol shows the elevation in metres
 $\varnothing$ = diameter
 $\#$ = number
 $L\sigma^{\frac{1}{2}}$ = angle or angles
 $-$ = plate
 I = I beam
 $\square$ = channel
 H = H beam

Note. All dimensions on drawings are in millimetres, unless otherwise specified.

Transliteration of the Russian Alphabet

Russian	Transliteration	Russian	Transliteration
Аа	a	Рр	r
Бб	b	Сс	s
Вв	v	Тт	t
Гг	g	Уу	u
Дд	d	Фф	f
Ее	e	Хх	kh
Жж	zh	Цц	ts
Зз	z	Чч	ch
Ии	i	Шш	sh
Йй	k	Щщ	shch
Кк	l	Ъъ	"
Лл	m	Ыы	y
Мм	n	Ьь	,
Нн	o	Ээ	é
Оо	p	Юю	yu
Пп		Яя	ya

Fundamentals of design

This text sets forth methods of designing and analyzing metal engineering structures (of steel and aluminium). The first two chapters are devoted to the fundamentals of designing and the theory of analyzing metal structures and structural members with account of the material working not only in the elastic, but also in the elastoplastic stage. Chapters 3-5 describe various structural shapes and methods of joining together structural elements, the actual behaviour of the joints and their investigation, as well as certain industrial requirements which the design of structures must meet. In Chapters 6-8 the reader will find a detailed consideration of the principal elements of metal structures such as beams, girders, trusses, and columns, as well as information on crane girders and eccentrically loaded columns.

The design of metal structures consisting of separate structural elements is the subject matter of Chapters 9 and 10. The exposition of this material is based on examples of industrial buildings and some special large-span and high structures. The last chapter sets forth the fundamentals of designing continuous sheet-metal structures (steel shells).

All the material contained in the book conforms to the standards for designing steel structures and structures of aluminium alloys, as well as to the general building standards and regulations followed in the USSR.

Metal structural members are mainly designed for taking the loads acting on a structure. The constructional shape of a structure is determined by the combination of the main constituent parts and elements such as beams, girders, trusses, columns or shells that form this structure. The best constructional shape of a structure and its elements is selected in the process of designing, which is a creative process wherein many solutions may be obtained. The duty of the designer is to find in each case the most appropriate and rational solution for his designing problems that will conform to the modern level of development of science and engineering.

1-1. FIELD OF APPLICATION AND NOMENCLATURE OF METAL STRUCTURAL MEMBERS

Steel structural members have found the widest use in the following types of structures, which can be divided into two groups.

A. *Framework or skeleton systems* having as their main elements beams, girders, trusses, and columns, such as:

(1) the frameworks of industrial buildings and structures (mainly the shops of iron-and-steel and machine-building works) with their internal members such as crane girders, platforms, and so forth (Fig. 1-1a-c);

(2) railway, highway and urban large-span bridges;

(3) civic multistorey buildings (Fig. 1-2), exhibition pavilions, various vaults, roofs, floors, domes, etc.;

(4) special-purpose buildings such as hangars, shipbuilding slips, and so on;

(5) special structures, for example towers and masts, headworks of mines, oil derricks, hydraulic engineering structures, trestles, cranes, etc.

B. *Shell systems* which are largely made up of plates or sheets and include, among others:

(1) gas holders and tanks for the storage and distribution of gases;

(2) tanks and reservoirs for the storage of liquids;

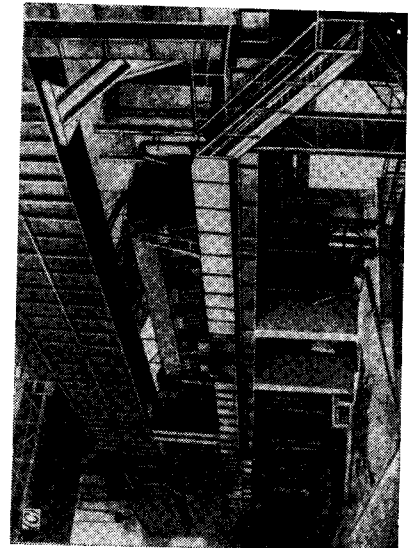
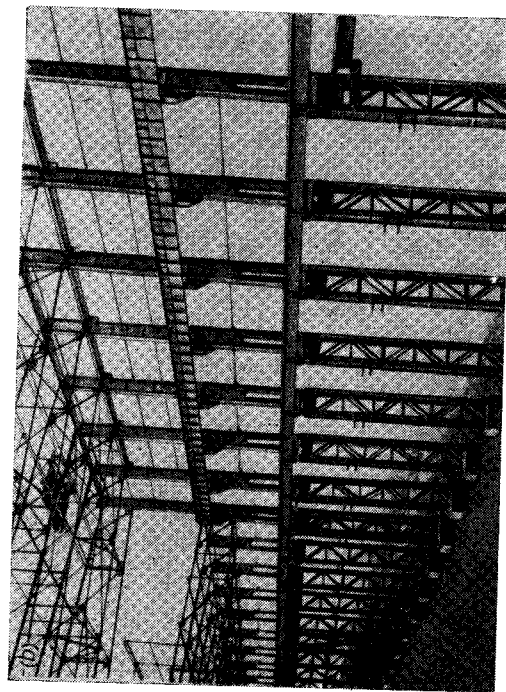


Fig. 1-1. Erection of shops of iron and steel works:
a—open-hearth shop; b—pipe electric welding shop;
c—platform and crane girder of open-hearth shop

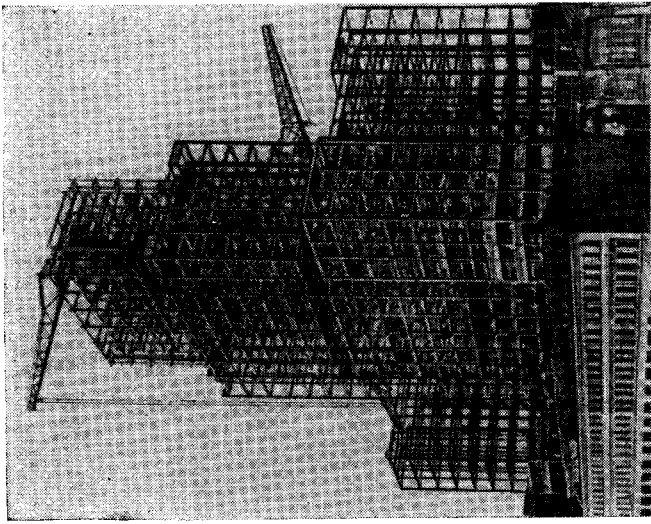


Fig. 1-2. Erection of multistorey building in Moscow

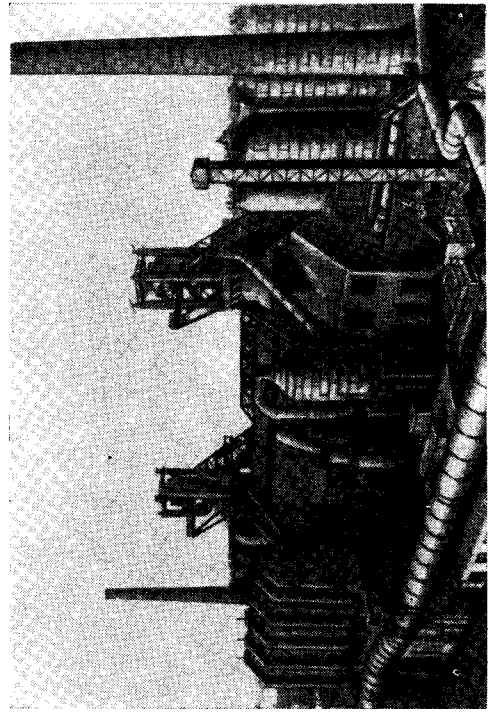


Fig. 1-3. Blast-furnace plant

- (3) bins and bunkers for the storage and handling of loose materials;
- (4) special structures such as blast furnaces, air heaters, gas scrubbers (Fig. 1-3);
- (5) large-diameter piping employed at iron-and-steel, coke and by-product works, hydroelectric power plants, for oil and gas pipelines.

Owing to their low specific weight and high resistance to corrosion, structural members made from aluminium alloys have found use first of all in enclosing members such as roof cladding, wall panels, window sashes, etc., and also in the chemical and petroleum industries.

Aluminium members are employed in structures where the own weight of the structure is of considerable importance (transfer cranes, drawbridges, the large-span roofs of pavilions, structures erected in seismic areas and in regions difficult of access).

1-2. MERITS AND DRAWBACKS OF METAL MEMBERS

The principal merits of steel members are:

- (1) the ability to resist high loads with a comparatively light weight and small size of the members, due to the high strength of steel; notwithstanding the high specific weight of steel (7.85 t/m^3) members made from this material have a lower weight in comparison with members of other construction materials. Owing to the high strength of the material, steel members are small in size, which makes them convenient for transportation;
- (2) their gas- and watertightness, which is due to the high density of steel;
- (3) a long service life, determined by the high and homogeneous strength and density properties of steel;
- (4) the possibility of industrializing construction work, attained by the use of prefabricated members with mechanized erection thereof at the construction site;
- (5) the possibility of readily disassembling or replacing steel members, which makes it easier to reinforce or replace parts of a structure.

The principal drawback of steel members and elements is their susceptibility to corrosion, which necessitates their painting or the use of other methods for their protection.

Among the merits of aluminium alloys are their low specific weight (2.7 t/m^3) with a comparatively high strength and their high resistance to corrosion.

The shortcomings of aluminium members include the relatively low value of the modulus of elasticity of alloys ($710,000 \text{ kg/cm}^2$),

which determines the increased deformability of these members, as well as their low heat resistance (the mechanical properties of the alloy begin to deteriorate at a temperature above 100°C , while at a temperature over 200°C creep begins).

1-3. BRIEF HISTORICAL REVIEW

A long time has passed since metal was first used in construction. The first metal members to be used in Russia were apparently the iron jack-rafters in the Kremlin palace (seventeenth century). The year 1725 saw the appearance of the first load-bearing cast iron members in a roof over a 12-metre span in a tower at the Nevsky Works, after which cast iron, which has a good compressive strength, found application in arch-type members and columns.

The buildings of factories and plants at the beginning of the nineteenth century were quite primitive and consisted of brick walls and combined wood and iron roof members.

In the thirties of last century, after the appearance of the steam engine, the erection of plants with large shop spans was commenced. The trusses for them were made from wrought iron with riveted joints (1835). In the sixties, with the appearance of converter and open-hearth steel, the erection was commenced of multiple-span shops with trusses resting on steel members.

In the middle of the nineteenth century there was undertaken the erection of some large and very complicated bridges. The erection of bridges required the working out of the theory of structure design, to the development of which Russian scientists and engineers made a substantial contribution. In 1856 engineer D. Zhuravsky (1822-1891) worked out the theory of designing some types of open-web trusses and was the first to point out the existence of shear stresses in bending. Prof. F. Yasnitsky (1856-1899) worked out a method of designing the compression elements of steel structures that is still partly in use at present.

The development of bridge construction is connected with the work of eminent Russian scientists and engineers, namely, Prof. N. Beluhsky (1845-1922), Prof. L. Proskuryakov (1858-1926), Academician E. Paton (1870-1953), Prof. I. Prokofyev (1877-1958), Prof. N. Streletsky, and others. They created wonderful bridges of original designs, developed the theory of bridge designing and founded a Russian school of bridge construction.

At the end of the nineteenth century industrial buildings underwent further development in connection with the appearance of electrical overhead travelling (bridge) cranes, which required special cranesways—crane girders. At this period the building of all-steel shop frames with rigid cranesways was commenced.

A variety of original steel structures was designed at the end of the nineteenth and the beginning of the twentieth century by Academician V. Shukhov (1853-1939). He was the author of scores of various structures. Of special interest is his work involving the creation of dimensional structures in the form of hyperboloids made from rectangular elements (Fig. 1-4).

The years 1930-1931 saw the wide introduction of a new method of connecting steel members, viz., electric welding, invented by the engineers N. Benardos and N. Slavyanov in the eighties of last century.

The technical equipment of industry, especially in the fields of ferrous metallurgy and machine building, improved considerably. The increase in the capacity of cranes raised the requirements which the structural members of a building had to meet in service, and this led to the necessity of erecting new buildings for shops with frame-type skeletons.

During these years there appeared large bodies of design and research organizations, whose work resulted in the creation of a Soviet school of designing steel structures.

The further development of steel structures was accompanied by the introduction of low-alloy steels, the increase in the employment of new rational rolled and bent shapes, simplification of design, unification and standardization of the elements of structures and further mechanization and automation of their fabrication.

1-4. REQUIREMENTS WHICH METAL MEMBERS MUST MEET

Metal members designed to take external loads must meet a number of requirements without which their normal service will be impossible, namely, the member must correspond to its application in service, it must have the required load-carrying capacity, i.e., it must be strong and stable, and its stiffness should be adequate.

The basic principles followed in designing structures are the development of structural members having a minimum weight, the lowest possible labour requirements for fabrication, and which will permit the structures to be erected in the shortest possible time.

For this reason designs of metal structural members must comply with the following requirements:

(1) General (Design) Requirements

The shape of a member and its parts should be adapted to service conditions and be convenient for protection against corrosion, the accumulation of dust, dirt, etc.

The member must have the most advantageous dimensions making possible complete utilization of its material, the tendency being to make the greatest possible use of standardized structural elements. The structure must be well-proportioned and have a good appearance.

(2) Fabrication Requirements

It is desirable that the greatest possible number of elements used in a structure be identical. An order placed for the serial production of members with a minimum number of various types will cut the cost and labour required for the fabrication of the elements or parts of the members several times.

The design must take into account the possibilities of the fabricating plants (the use of jigs and various contrivances, the characteristics of the machine tools, the capacity of the cranes, the kind of welding equipment available, and so on).

The elements should have a simple shape and consist of the minimum possible number of parts, with a minimum of welds or rivets. The member should be easy to assemble.

(3) Erection Requirements

A member or a part thereof shipped from a plant (a so-called *shipping element*) must have dimensions within the clearance gauge of the railway or other kind of transport that will deliver it to the erection site.

The tendency should be to use the minimum number of erection elements in a structure with the greatest possible interchangeability thereof. The connections of elements should be simple and convenient for assembly. Attention should be given to the possibility of using large prefabricated members in the erection of structures.

Members that meet all the above requirements can be called high-quality and rational ones. The shapes of structural members are not fixed, they keep pace with the general development of science and engineering.

1-5. ORGANIZATION OF DESIGNING

Depending on their complexity, structures are designed in two or three stages, viz., the preliminary project report, the detailed project report and the working project.

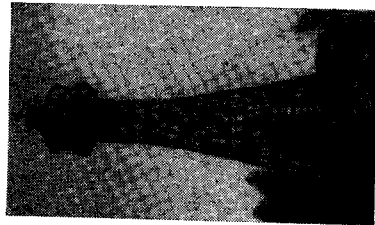


Fig. 1-4. Water tower designed by V. Shukhov

The *preliminary project report* has the aim of determining whether complicated structures are feasible from the technical and economical standpoints.

The *detailed project report* is a document wherein the main technical problems are solved and the cost of construction is estimated. It contains general architectural and erection drawings (top views and cross sections) indicating the material of the members, their approximate weight, a list of the building materials and quantities of work, as well as estimates. The approved detailed project report serves for drawing up the working project.

The *working project* of metal structural members consists of two stages. In the first stage the final design of the structure is determined, all the members are analyzed and the sections of all their elements are selected, the general drawings and drawings of complicated assemblies are prepared, and all the members are co-ordinated with the technological, transportation, power supply, sanitary engineering, architectural, building and other parts of the project. A specification of the metal required is also drawn up in this stage. The working drawings in the second stage, which show all the details of the members, are worked out, as a rule, by the design offices of the fabricating plants on the basis of the first stage of the working project. They are drawings of separate member elements that will be shipped ready for erection from the plants (shipping elements, standard elements), as well as erection drawings with the corresponding marking.

1-6. METHODS OF ANALYZING METAL MEMBERS

The designing of metal structures begins with the drawing up and comparison of several alternatives of structure solutions meeting the stipulated service requirements. Next the design is selected that best complies with all the requirements relating to metal members.

The analysis of members is aimed at checking the strength, stability and rigidity of the *preliminarily selected* solution of the structure, which will make it possible to determine more precisely the dimensions and ensure both reliability and the lowest possible cost.

Structures and their members and elements are analyzed on the basis of the theory of the strength of materials and structural mechanics. The main purpose of this theory is to determine the internal stresses that are induced in members under the action of applied loads.

With this aim in view there should be first drawn up what is called the *general layout of a structure* consisting of separate elements (beams, rods), temporarily disregarding the actual shape of their cross

sections. The supporting connections of the elements are endowed here with certain theoretical properties (hinged, elastically restrained or fixed connections, etc.). After having determined the stresses in the various elements according to the general layout adopted as the basis of designing, the element sections are selected, the load-carrying capacity is checked and the connection is designed to meet the stipulated conditions.

Sometimes more precise methods of determining the stresses are necessary to take into account the development of plastic deformations. The mathematical complicity of these methods, however, often makes it necessary to use in formulas a number of coefficients and factors whose values are given in various tables.

Previously steel structures were investigated by means of the method of elastic design, based on allowable stresses. At present a more perfect method of analysis has been introduced, viz., the method of limit design, worked out by Soviet scientists headed by Prof. N. Streletsky. These principles under the "Building Standards and Regulations" published in the USSR, Part II of which is devoted to the designing of structural members and foundations.

For a number of structures (bridges, hydraulic engineering structures, blast furnaces, etc.) special recommendations and instructions are worked out for designing that reflect the specific conditions of their service. At present certain kinds of steel members are still investigated by means of the old method of elastic design.

(1) Limit Design

In this method of design each member is considered in its limiting state. The *limiting state* of a member denotes such a state wherein it does not any longer meet the service requirements, i. e., either it loses its ability to withstand external loads, or it acquires an inadmissible deformation or local damage.

Two limiting states have been established for metal members, namely,

(a) the first limiting state, also known as the condition of non-destruction, determined by the load-carrying capacity (strength, stability or endurance). The requirements of this state must be met by all metal members;

(b) the second limiting state, determined by the development of excessive deformations (deflections and displacements). This state is used for checking members in which the magnitude of the deformations may limit the possibility of their service.

The *first limiting state* is expressed by the inequality

$$F \leq Q' \quad (1-1)$$

where F = design force in a member caused by the combined action of all the design loads P in their most unfavourable combination

Q' = load-carrying capacity of the member, which is a function of the geometrical dimensions of the member and the design strength of the material R .

The maximum values of the loads tolerated in the normal service of members, which are established by the building standards and regulations, are called the *working or service loads* P_s (see Appendix I, Table 1). The design loads P used in the analysis of members (for the condition of non-destruction) are taken somewhat greater than the service ones. The *design load* is determined as the product of the service load and a load factor n (greater than unity), which takes into account the danger of an increase in load over its service value due to possible load changes, viz.,

$$P = P_s n \quad (1-2)$$

The values of the factor n are given in Table 1 of Appendix I. Thus the behaviour of members is considered under the action of the design, and not the service, loads. The values of the design loads are used to determine for the members the design forces (axial force F , bending moment M , etc.), which are found in accordance with the general rules of the strength of materials and structural mechanics.

The right-hand term of the basic inequality (1-1), the load-carrying capacity of a member Q' , depends upon the ultimate strength of the material, i.e., its resistance to the action of the various forces, which is characterized by the mechanical properties of the material and is called the service resistance or strength R_s , as well as upon the geometrical characteristics of the cross section (the section area A , the section modulus W , etc.).

For structural steels the service tensile, compressive and bending strength is taken equal to the minimum value of the yield point guaranteed by the corresponding USSR State Standards (GOST), i.e.,

$$R_s = \sigma_y$$

(for the most widespread grade of structural steel, BCr. 3, the value of $R_s = 2,300$ kg/cm²).

In some instances, when the service of members working under tension is also possible after the metal has reached the yield point (for example, cylindrical tanks, etc.), and also for materials that do not have yielding properties (for example, high-strength wire, iron castings, etc.) the service tensile strength of the steel is taken equal to the minimum guaranteed value of the ultimate tensile strength, i.e.,

$$R_s = \sigma_u$$

For example, for steel of grade BCr. 3 (the designations of the various grades of steel used in USSR State Standards and followed in this book are described in the next Chapter) the service tensile strength $R_s = 3,800$ kg/cm².

The service tensile, compressive and bending strength of aluminium alloys is taken equal to the smaller of the following two quantities:

1. Seven-tenths of the minimum guaranteed value of the ultimate tensile strength, i.e., $0.7 \sigma_u$.
2. The arbitrary yield point, also known as the *offset stress* $\sigma_{y0.2}$, corresponding to the stress producing a unit permanent elongation of 0.2%.

The design strength R is obtained by multiplying the service strength by a *homogeneity factor* k_{hom} (less than unity) that takes into account the danger of a reduction in the strength of the material in comparison with its service strength due to possible variations in the mechanical properties of the material:

$$R = k_{hom} R_s^1 \quad (1-3)$$

For ordinary carbon steels (when $R_s = \sigma_y$) the factor $k_{hom} = 0.9$, while for high-quality (low-alloy) steels and for aluminium alloys $k_{hom} = 0.85$.

Thus the *design strength* R is a stress equal to the lowest possible value of the yield point (or the ultimate tensile strength) of a material, which is taken as the limiting strength of the member.

To ensure safety of a structure it is necessary to take account of possible deviations from the normal service conditions, as well as special features of work of the member (for example, the particularly strenuous working conditions of girders supporting certain types of overhead cranes; a corroding medium causing an increased rate of metal corrosion; constant work of the members at the limit loads with only slight variation thereof; and where there is a high probability that the stresses produced by these loads coincide with the lowest value of the strength of the material).

To take account of all these circumstances, there is introduced a safety factor known as the *factor of service conditions* of structural members and their elements k_s , which reduces the value of the design strength when necessary, and thus we have

$$R = k_{hom} k_s R_s \quad (1-4)$$

Thus, for example, when designing crane girders for overhead cranes with strenuous service conditions, such as are encountered in

¹⁾ The values of the service strengths and the homogeneity factors are contained in Chapter II-A.10-62 of the Building Standards and Regulations, headed "Structural Members and Foundations. Basic Prescriptions for Designing". This manual is in general use in the USSR.

the shops of iron and steel works, a service condition factor of $k_s = 0.9$ is introduced, while when designing a tank that works under conditions of a practically constant load produced by the material filling it (with a small load factor) the value assigned to the service condition factor is usually 0.72 to 0.80.

The values of the service condition factor for structural members and their elements are given in Appendix I, Table 2.

In all instances when the design strength is determined in accordance with the ultimate tensile strength (and not the yield point) the design strength is additionally corrected by a factor taking into account the service conditions of the material in the member (less than unity), that allows for uneven distribution of the stresses when the material is working beyond the yield point.

Finally, the values of the design strengths for riveted and bolted connections are corrected by a factor of connection service conditions that takes into account the kind and nature of behaviour of the connection, as well as the quality of the holes.

The values of the last two factors mentioned above are contained in the Building Standards and Regulations.

Thus the first limiting condition (1-4) for checking the strength can now be expressed as

$$F \leq AR \quad \text{or} \quad M \leq WR \quad (1-5)$$

where F and M are the design axial forces and bending moments caused by the design loads (with regard to the load factors n).

When investigating a contemplated member first the section of an element is selected and then the stress caused by the design forces is checked. This must never exceed the design strength (corrected by any or all of the safety factors indicated above that are essential for ensuring reliable service of the structure).

The basic formulas for checking the strength of members being designed will thereupon take the following form

$$\sigma = \frac{F}{A_n} \leq R \quad \text{or} \quad \sigma = \frac{M}{W_n} \leq R \quad (1-6)$$

where σ = design stresses in element (caused by design loads)

A_n = net area of section (holes subtracted)

W_n = net section modulus

R = design strength of material, taken from the Building Standards and Regulations.

Several categories of stresses are distinguished, viz., principal, local, additional and internal stresses.

The *principal stresses* are stresses that develop inside a body as the result of balancing the action of external loads. They are taken into account in analysis.

Uneven distribution of the force field over a cross section caused by a sharp change in the section or by the presence of a hole therein leads to a *local concentration of stresses*. In plastic materials, which include structural steel, these local excessive stresses are equalized upon increasing of the load and are therefore not considered in investigations.

The *additional stresses* are those appearing in the elements of a member upon the application of external loads by additional restraints that are not required for maintaining the general equilibrium of the system (for example, rigid connection of the elements at the joints

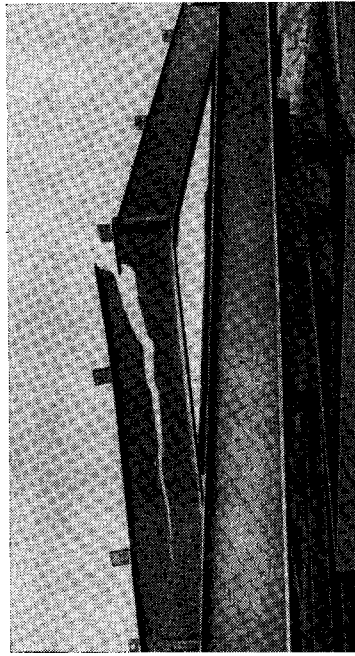


Fig. 1-5. Failure of I beam caused by improperly applied weld

of a truss). These stresses do not affect the equilibrium of the system as a whole, and in members of plastic materials are not generally taken into account in investigations.

The *internal stresses* arise not as the result of the application of external loads, but owing to the action of heat (thermal effect) on the metal under restricted conditions of deformation. Depending on the cause originating them, they are often also called thermal (temperature or heat), shrinkage, welding, residual, inherent, initial stresses, etc.

An example of internal stresses are the internal initial stresses in rolled I beams appearing as the result of their uneven cooling after being rolled. Internal stresses do not reduce the margin of safety of a member, since they are mutually balanced therein. They must be taken into consideration, however, when as the result of some external cause internal equilibrium is violated; thus, for example, Fig. 1-5 depicts the failure of an I beam after the improper application of a weld at a fillet of the beam, which disturbed the equilibrium of the internal stresses.

2. *Temporary or live loads* acting for a long period, known as *movable loads*—for example the weight of stationary equipment, the pressure of gases, liquids and warehouses, libraries, theatres; the continuous thermal action of equipment.

3. *Live loads* acting for a short time, called *moving loads*—for example cranes and other mechanical handling equipment, the load of the occupants of buildings, wind loads, temperature (climatic) action, erection and other loads.

4. *Special loads*—for example loads caused by earthquakes, accidents, the settlement of foundations, etc.

As a rule not one, but various combinations of loads act on a structure. The probability of the simultaneous action of the maximum loads of all kinds on a structure is very small, and a structure designed for such a combination of loads would have an excessive margin of safety. The values of the most frequently encountered service loads and load factors, as well as the combinations of loads that should be taken into account when designing structures, are established by the Building Standards and Regulations, which provide for the following three categories of load combinations:

1. *main combinations*, consisting of the dead loads, movable loads and one, most important, moving load;
2. *additional combinations*, consisting of the dead loads, movable loads and all the moving loads;
3. *special combinations*, consisting of the dead loads, movable loads, possible moving loads and one of the special loads.

When these combinations are considered the vertical and horizontal loads induced by overhead cranes are treated as one moving load.

In exceptional cases the main combination also takes into consideration the combined action of a snow load and one or two overhead cranes (except cranes with light and medium duties).

When designing members with regard to the additional load combinations, the values of the design live loads (or of the stresses in the members corresponding to them) should be multiplied by a load combination factor $c=0.9$, while when taking account of special combinations a factor $c=0.8$ should be employed.

For special structures (bridges, hydraulic engineering structures, special cranes, etc.) the loads and their combinations are established by special documents.

The own weight of the steel structural members is usually only a small fraction of the total load and can be preliminarily established on the basis of an approximate estimate, or from similar existing structures (see, for example, Table 9-3).

The service live loads are given in various building standards and codes. The wind load is considered not only as the result of active pressure on the windward wall, but also as being produced by the suction acting on the roof and the inner side of the opposite wall.

With dynamic loads that create impacts and cause vibrations of a structure (cranes, trains) the same service loads are used in analysis, but after being multiplied, besides the usual load factor, by a special dynamic load factor.

Materials and their behaviour in members

2-1. MATERIALS USED IN METAL MEMBERS

(1) Carbon and Low-Alloy Steels

Depending upon its carbon content, carbon steel is classified as:

- (a) low-carbon soft or mild steel containing from 0.09 to 0.22% carbon (used mainly in construction);
- (b) medium-carbon or medium steel containing 0.25-0.5% carbon (employed chiefly in machine-building);
- (c) high-carbon or hard steel, with a carbon content varying from 0.6 to 1.2%, used for making tools.

Steel always contains admixtures of manganese, silicon, phosphorus and sulphur whose total content does not generally exceed one per cent. Phosphorus and sulphur are harmful impurities, but they cannot be completely eliminated in the process of manufacturing steel. A phosphorus content of over 0.045% renders the steel brittle at low temperatures (cold short). A sulphur content exceeding 0.055% causes the steel to be red short or brittle when heated, i.e., facilitates the formation of cracks in a hot state.

The greater part of structural steel is made by the open-hearth process. Until recently converter (Bessemer and Thomas) steels ordinarily contained a somewhat larger amount of harmful impurities than their open-hearth counterparts. They sometimes contained dissolved gases (nitrogen, oxygen, etc.) which are also harmful admixtures.

At present, owing to the improvements in the process of producing converter steel (the top blowing of oxygen), its widespread use in construction has become possible.

By adding alloying elements such as manganese, nickel, chromium and others, it is possible to produce a high-strength alloy steel. In ordinary construction such steels are seldom used owing to their high cost. In many instances low-alloy steel is employed in the construction of large and critical structures.

The main structural steel is low-carbon steel, obtained by the open-hearth or the new converter process, rimming, killed or semi-killed. Its distinguishing properties are poor hardening, high plasticity and good weldability.

Killed steel advantageously differs from rimming steel in that the solidification of the steel in the ingot moulds takes place quietly, without violent evolution of the gases contained in the steel in large quantities. This is attained by introducing deoxidizing agents such as silicon, aluminium and others that bind the gases ("kill" the steel), forming slag. There is less possibility of gas bubbles forming in the steel, around which there may concentrate nonmetallic inclusions, for example various sulphur compounds, that have a harmful effect on steel in rolling (exfoliation) and welding, especially in thick plates. Killed steel costs more than rimming steel, and for this reason it should be used only for certain kinds of critical structures, especially welded ones.

There also exists semi-killed steel that has not been completely deoxidized, obtainable in special ingot moulds.

These three kinds of steel are used in welded members as follows:

1. Killed steel—in members designed for a service temperature below -30°C , and also (regardless of the service temperature) in members working in especially strenuous conditions—under dynamic and vibrating loads.
2. Semi-killed steel (designated "nc")—in the main load-carrying members of roofings and ceilings (trusses, frame collar-beams, beams).
3. Rimming steel (designated "kr")—in the remaining instances. Somewhat more relaxed conditions for selecting the kind of steel to be used are possible for structural members having no welded connections.

Iron and steel works give information on the chemical composition of each melt of steel, which is entered in a special certificate. The maximum contents of chemical elements in steel are established in the USSR by the State Standard covering the delivery of ordinary quality carbon steel (GOST 380-60). There are various grades of steel in use, the grade depending upon the contents of various elements. The grades of structural steel are listed in Table 2-1.

The main structural steel is grade Cr. 3. This steel, owing to the relatively low content of carbon (as a rule less than 0.22%) and silicon (less than 0.3%) has good welding properties.

Steel of grade Cr. 0 is not manufactured specially, but is obtained as the result of rejection of other grades of steel both for chemical and mechanical reasons, and therefore it may be used only in the elements of members that are not analyzed.

Open-hearth steel is delivered according to GOST 380-60, while converter steel—to GOST 9543-60, with division into 3 groups, group A having guaranteed mechanical properties, group B—guaranteed chemical composition, and group B with both mechanical properties and chemical composition guaranteed.

Seeing that the same guaranteed mechanical properties and design strength are established, as a rule, for definite groups of carbon steel grades, regardless of the method of manufacture and the conditions of delivery thereof, these groups for purposes of simplification in the following exposition will be named "steel 3", "steel 4" and "steel 5", covering all the grades of the respective steels.

TABLE 2-1 Chemical Composition of Structural Steels (Open-Hearth)
(According to GOST 380-60)

Grade of steel	Content of elements, %			
	Carbon	Silicon	Manganese	Phosphorus Sulfur not over
Cr. 0 Cr. 2 кп	Not over 0.23 0.09-0.15	— Not over 0.07	— 0.25-0.50	0.07 0.045
Cr. 3 кп	0.14-0.22	Not over 0.07	0.30-0.60	0.045
Cr. 3 пс	0.14-0.22	Not over 0.17	0.40-0.65	0.045
Cr. 3	0.14-0.22	0.12-0.30	0.40-0.65	0.045
Cr. 4 кп	0.18-0.27	Not over 0.07	0.40-0.70	0.045
Cr. 4	0.18-0.27	0.12-0.30	0.40-0.70	0.045
Cr. 5	0.28-0.37	0.15-0.35	0.50-0.80	0.045

Since structural members require both definite guarantees in respect to mechanical properties, and compliance with limitations in respect to chemical composition, then steel of subgroup B will be the main kind used for them.

Low-alloy steels are delivered according to GOST 5058-57¹⁾ and to special specifications and are used in the main in heavy members, as well as in members that see service at low temperatures.

The designation of low-alloy steel grades, consisting of letters and numbers, mainly characterizes the chemical composition of the steel. The left-hand figures indicate the average carbon content in hundredths of a per cent, the letters after these figures denote the constituents, viz., Г for manganese, С for silicon, Х for chromium, Н for nickel, Д for copper, Т for titanium, М for molybdenum, etc. The numbers following these letters indicate the approximate content of the element in per cent (in whole numbers) exceeding one per cent. If the quantity of the constituents is less than 0.3% they are not indicated in the grade designation.

¹⁾ An asterisk following the designation of a USSR State Standard (GOST) indicates a revised edition of the original standard.

Low-alloy steels of grades 15XCHД and 10XCHД (naturally alloyed, from the Orsk-Khalilov deposits) are used in construction.

Of the cheaper steels, containing no nickel, grades 14Г2, 15ГС, 10Г2С, 10Г2СД, 09Г2Т, 09Г2Т(М) and others are employed. The use of low-alloy steels leads to a reduction in the weight of the members of about 15%. The cost of the members, however, remains practically the same. A considerably greater economy of steel in structural members can be obtained by using heat-treated low-alloy or low-carbon steels, in which the yield point and the ultimate strength are considerably higher with a very slight reduction in plasticity. Heat treatment consists in heating the steel to a temperature of 900-930 deg C (above the upper critical point) and hardening (quenching) in water without or with following tempering.

There will be soon available heat-treated steels of grades Т30, Т45, Т60 and Т75 (here the figure indicates the lowest value of the yield point of the steel). GOST 9458-60 provides meanwhile for the delivery of only grade Т30 steel obtained on the basis of low-carbon rimming steel 3. The introduction in the nearest future of the remaining types of heat-treated steels, which will lead to a great reduction in the weight of members and a general saving in costs of about 25-30%, will be of very great significance.

(2) Aluminium Alloys

Aluminium alloys containing various elements such as magnesium, manganese, silicon, copper and others are used in structural members. For brevity we shall call members made from aluminium alloys aluminium ones.

Aluminium alloys are divided into *casting* ones, used in machine building, and *malleable* or *forging* ones (worked under pressure—in presses, by extrusion, rolling, stamping, etc.) used in construction.

The alloys are given the required strength not only by adding the appropriate components, but also by mechanical action consisting in cold deformation of the billets—cold working (cold hardening, drawing), while for some of the alloys heat treatment (hardening, ageing, etc.) is employed.

The alloys are designated in accordance with the alloying elements as follows:

AMr denotes aluminium-magnesium alloys (in the grade designation AMr6 the number 6 indicates that the alloy contains about six per cent of magnesium);

AMn stands for an aluminium-manganese alloy;

AB (Avial) and АД are alloys of aluminium with magnesium and silicon;

Д4, Д16, etc. designate duralumins (the figure indicates the number of the alloy), the basic components of these alloys being aluminium, magnesium and copper;

В means high-strength alloys (В65, В92 and others) consisting mainly of aluminium, magnesium, copper and zinc. These alloys are more costly.

Alloys designated by the letters АД correspond to aluminium (А) malleable (Д) alloys of an international standard. The figures following the letters show the number of the alloy (АД31, АД33).

The state in which the material is delivered is also designated by letters: М denotes annealed (soft) material, Т—hardened and naturally aged (Т1—artificially aged), Н—cold worked, П—semicold worked.

The strength of the alloys can be increased by heat treatment from 1.3 to 1.5 times, but this will be accompanied by a reduction in the unit elongation. Duralumins, Avials and high-strength alloys are strengthened by heat treatment. Grade Д16-Т duralumin is a strong alloy recommended for riveted structural members, but it is difficult to weld, since annealing of the weld zone takes place; this sometimes leads to the formation of cracks. Duralumin has a lower resistance to corrosion.

The heat-treated alloys АВ and АД can be used for welded members provided that welding is followed by heat treatment; this is necessary for increasing the strength of the weld, since after welding this strength will be about 60% of that of the basic metal.

The alloys АМр and АМн are not strengthened by heat treatment. The alloys АМр6 and АД33, owing to their good weldability and relatively good mechanical properties, have found the widest use in welded members; they have a comparatively high resistance to corrosion. The alloy АМн has a low strength, and a high resistance to corrosion, it can be welded and is comparatively cheap. It can be employed for enclosures and guards.

2-2. PRINCIPAL MECHANICAL PROPERTIES OF STEEL. BEHAVIOUR OF STEEL UNDER TENSION. BRITTLENESS

The most important mechanical properties of steel are its strength, elasticity and plasticity, characterized by stresses and elongations, as well as its tendency to brittle destruction, which in an indirect way is determined by the toughness or impact strength of the material. *Strength* is determined by the resistance of the material to external loads and forces. *Elasticity* is the property of a material to restore its initial shape after removal of the external loads. *Plasticity* is the reverse of elasticity, i.e., the property of a material not to return to its initial dimensions after removal of the external loads or, in

other words, the property of obtaining permanent sets. *Brittleness* is characterized by destruction of the material upon slight deformation.

The relation between stresses and strains for various materials is established experimentally. The most simple and reliable test is the tension test of specimens, which is used to determine the strength of steel and also its elastic and plastic properties.

Other characteristics of steel are its *notch toughness* (see page 50), as well as the angle when testing for bending in a cold state, which characterizes the state of the surface and is a simple field test of the plasticity and ductility of the material.

(1) Behaviour of Steel Under Tension

Should a specimen be subjected to tension by gradually increasing the load P , and should the resulting elongations ΔL be measured, the results can be used to plot an experimental tension diagram of elongation versus load. For convenience in comparison with similar charts such a diagram is usually plotted in stress against unit elongation (also called normal or longitudinal strain)

$$\sigma = \frac{P}{A} \text{ kg/cm}^2; \quad \epsilon = \frac{\Delta L}{L_0} 100\% \quad (2-1)$$

where σ = normal stress

A = original cross-sectional area of the specimen

ϵ = longitudinal strain in per cent

L_0 = original length of specimen.

The magnitude of the longitudinal strain depends upon the length and the cross-sectional area of the specimen.

Two types of specimens, namely, long and short ones, are in use (GOST 1497-61).

An experimentally obtained tension diagram for carbon steel 3 is illustrated in Fig. 2-1a.

First the relation between the stresses and the strains follows the law of a straight line, i.e., they are proportional to each other. This is expressed by a linear equation (Hooke's law)

$$\sigma = E\epsilon \text{ kg/cm}^2 \quad (2-2)$$

Here E is a constant factor of proportionality, known as the *modulus of elasticity in tension*, the *modulus of elasticity for axial loading* or *Young's modulus*. For steel $E=2,100,000 \text{ kg/cm}^2$.

The highest stress in a material, after which the relation between stress and strain no longer remains linear, is called the *proportional limit* σ_{pr} (Fig. 2-1a). Somewhat higher than this point is the *elastic*

limit σ_e corresponding to the maximum stress which can be developed in a material without causing a permanent set.

When low-carbon steels are loaded above the proportional limit, the curve in the stress-strain diagram for tension moves away from the straight line and, after rising smoothly, makes a jump (forming at high loading speeds a characteristic "tooth") and then with insignificant fluctuations runs parallel to the x -axis (Fig. 2-1a). The specimen grows in length without any increase in load, the material flows or yields. That normal, practically constant stress which causes an increase of strain without an increase in load, is called the *yield point* σ_y .

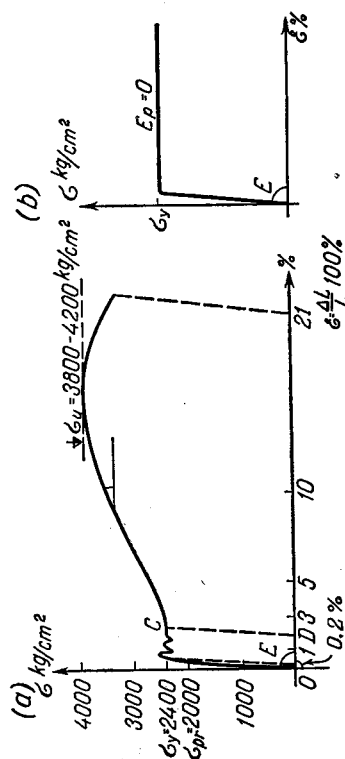


Fig. 2-1. Stress-strain diagram for steel 3

The horizontal section of the diagram, called the *yield area*, is obtained in low-carbon steels for strains varying from 0.2 to 2.5%. The existence of a yield area in the diagram of a material is a positive factor in the behaviour of steel structural members.

In other steels, not low-carbon ones, the transition to the plastic stage takes place gradually, and thus the elastic limit and the yield point in these steels do not in essence differ from each other. In such steels the yield point, which in this instance is called the *offset stress* or *yield strength*, is taken by convention equal to the stress corresponding to a permanent set of 0.2%.

Upon removal of the load from the specimen after it has received plastic deformation, the unloading diagram follows the straight line $C-D$ parallel to the straight line of elastic loading (Fig. 2-1a).

When the strain reaches a certain value (about 2.5% for steel 3) the material stops yielding and again becomes capable of resisting loads. It behaves as if it has acquired new strength. Here the stress-strain relation will follow a curvilinear law, with a rapid increase of the strains. A neck forms in the specimen and, finally, it completely ruptures.

The maximum stress a material can withstand, which characterizes its strength, is determined by the maximum stress in the process of destruction (referred to the initial cross-sectional area of the specimen). This stress is known as the *ultimate stress* σ_u (*ultimate tensile strength*).

The total strain, measured after destruction of the specimen, is a measure of the plasticity of the steel.

Since the modulus of plastic deformations E_p is small (Fig. 2-1a), it is often taken equal to zero in theoretical investigations when considering the elastoplastic behaviour of steel, an idealized tension diagram of an ideal elastoplastic material being adopted for all kinds of steel (Fig. 2-1b, Prandtl's Diagram).

Thus the most important mechanical properties characterizing the behaviour of steel are the yield point, the ultimate tensile strength and the longitudinal strain. These properties together with the chemical composition are indicated in the certificates that are supplied with each shipment of metal.

State Standards of the USSR and special specifications for the delivery of structural steels guarantee their mechanical properties as set forth in Table 2-2.

(2) Structure of Steel and Phenomenon of Flow

Low-carbon steel is a homogeneous crystalline material consisting of fine crystals of ferrite, forming grains, and pearlite (a mixture of cementite Fe_3C and ferrite) mainly located along the joints of the ferrite grains and forming a sort of a "grid" or disseminations between the grains (Fig. 2-2, the dark lines). Pearlite is considerably harder than ferrite and is more brittle. In the course of elastic strain under the action of externally applied loads, the forces of interaction between the atoms of the crystals change, and as a result the shape of the crystals is distorted. After removal of the loads, the initial shape is restored. In the plastic strain of low-carbon steel tension specimens there can be noticed the appearance of characteristic lines known as *flow lines* (Chernov and Lüders lines) that are directed at an angle of 45 deg to the line of action of the tensile forces (Fig. 2-3). These lines, which are noticeable to the naked eye, are traces of plastic displacements of the layers of metal that remain after removal of the load. Their direction corresponds in the main to that of the maximum shear stresses.

There exists a hypothesis assuming that since in low-carbon steels the pearlite, which is located at the boundaries of the grains in a comparatively thin layer, is considerably harder than the ferrite, it will first hold back the plastic strain of the latter. However, at the spots subjected to the greatest pressure of the ferrite crystals when

they move and turn under loads exceeding the elastic limit, local destruction of the brittle pearlite is possible. When this happens, the energy accumulated by the plastic ferrite will lead to its increased displacement. Upon displacement of a large number of the grains noticeable plastic displacements will be formed. This explains the appearance of a "tooth" and the presence of the yield area in the tension diagram.

Thus the yield area is the consequence of the energy being released that had been accumulated in the grains of ferrite and restrained by the pearlite grid.

Owing to the presence of a large number of variously oriented crys-



Fig. 2-2. Structure of steel 3

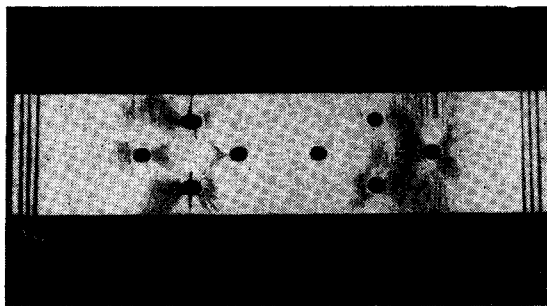


Fig. 2-3. Flow lines in stretched strip

tals, steel may be considered as a homogeneous material, notwithstanding the heterogeneity of its microstructure.

Upon loading a specimen above its yield point, when the entire yield area is used up (i.e., when the energy accumulated in the ferrite grains is exhausted) the material acquires the ability of further load resistance, and the tension diagram becomes curved, reflecting the uniform development of plastic strains in the whole mass of the metal up to the moment of destruction.

Failure occurs as a result of the accumulation of large plastic shear strains and the development of local shearing stresses that disrupt the atomic bonds. A fine-grain crystal structure can be observed in the fracture.

In case a macrocrystalline structure is formed (for instance when the metal is heated to a certain temperature) the yield area is reduced and the yield point lowers, which is explained by the reduction in the

TABLE 2-2 Mechanical Properties of Structural Steels.
GOST 380-60 and GOST 5058-57*

Kind of steel	Grade of steel	Yield point by rolled stock thickness groups, kg/mm ² , at least			Ultimate tensile strength, kg/mm ²	Longitudinal strain, per cent, at least		Test for bending through 180° in cold state (t = thickness of specimen, d = diameter of mandrel)
		1	2	3		long specimen	short specimen	
Carbon	Cr. 0	—	—	—	At least 32	18	22	d = 2t
	BCr. 2kn	22	20	19	34-42	26	31	d = 0
	BCr. 3	24	23	22	38-40	23	27	d = 0.5t
	BCr. 3kn	24	23	21	41-43	22	26	
					44-47	21	25	
	BCr. 4				42-44	21	25	d = 2t
Low-alloy	BCr. 4kn	26	25	24	45-48	20	24	
					49-52	19	23	
	BCr. 5	28	27	26	50-53	17	21	d = 3t
					54-57	16	20	
					58-62	15	19	
Low-alloy		Thickness of rolled stock	Yield point					
	14Г2	4-10 11-20	34 33	48 47	18 18	—	—	d = 2t
	15ГC	4-10 11-20	35 34	50 48	18 18	—	—	d = 2t
	10Г2C							
	10Г2CД (МК)	4-32	35	50	18	—	—	d = 2t

NOTE. The rolled stock thickness groups are established as follows:

1st group—section steel (angles, rods, bars)—up to 40 mm inc usive; steel shapes (I beams, channels)—up to 15 mm inclusive; in web thickness; sheets and wide strips—(from 4 to 20 mm inclusive;
2nd group—section steel—over 40 to 100 mm; shapes over 15 to 20 mm, sheets and wide strips—over 20 to 40 mm;
3rd group—section steel—over 100 to 250 mm; shapes—over 20 mm, sheets and wide strips—over 40 to 60 mm.

The unit strain energy or work in uniaxial tension can be represented as the ratio between the elastic strain energy in tension (equal to half the product of the final value of the force and the final elongation) and the volume of the body

$$u = \frac{U}{AL} = \frac{0.5P \Delta L}{AL} = \frac{0.5P}{AL} \frac{EA}{EA} \frac{P^2}{2A^2E} = \frac{P^2}{2AE} \quad (2-3)$$

The unit energy in a combined stressed state of an elementary parallelepiped with dimensions equal to unity in a body, having no shear stresses in its principal planes, and with the principal normal stresses σ_1 , σ_2 and σ_3 applied to its sides, will be expressed as follows:

$$u_c = \frac{1}{2} \sigma_1 \epsilon_1 + \frac{1}{2} \sigma_2 \epsilon_2 + \frac{1}{2} \sigma_3 \epsilon_3$$

in which ϵ_1 , ϵ_2 and ϵ_3 are the respective strains.

Since $\epsilon_1 = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} - \frac{\mu \sigma_3}{E}$, where μ is the factor of lateral strain (Poisson's ratio) taken equal to 0.3 for steel (ϵ_2 and ϵ_3 are expressed in a similar manner), then substitution yields

$$u_c = \frac{1}{2E} [(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1 \sigma_2 + \sigma_1 \sigma_3 + \sigma_2 \sigma_3)] \quad (2-4)$$

Let us separate the part of the energy that corresponds only to a change in the form of the body characteristic of plastic strain (plastic flow). As is known, for this end a value of $\mu = 0.5$ should be taken. The unit energy in a combined stressed state spent for changing only the form of the body ($u_{c.f}$) will be found from expression (2-4) with 0.5 substituted for μ .

The condition of plasticity in a combined stressed state, according to the maximum energy theory, will be obtained by equating the specific energies, viz., $u = u_{c.f}$. This condition can be expressed in the form of a reduced stress if a square root is extracted from both parts of expressions (2-3) and (2-4) with 0.5 substituted for μ :

$$\sigma_{red} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1 \sigma_2 + \sigma_1 \sigma_3 + \sigma_2 \sigma_3)} = \sigma_y \quad (2-5)$$

Thus the reduced stress is the stress in such a uniaxial stressed state, which in respect to the condition of transition to a plastic state corresponds to the given combined stressed state; in other words, the reduced stress is a quantity equivalent to the yield point in a uniaxial stressed state.

For a plane or biaxial stressed state, for example in a cut out element of a shell, where the normal stresses σ_1 and σ_2 develop in two mutually perpendicular directions, the limiting state will be expressed

TABLE 2-2 (Concluded)

Kind of steel	Grade of steel	Thickness of rolled stock, mm	Yield point, kg/mm ²	Ultimate tensile strength, kg/mm ²	Longitudinal strain, per cent, at least		Test for bending through 180° in cold state (t=thickness of specimen, d=diameter of mandrel)
					long specimen	short specimen	
Low-alloy	15XCHД (HJ12)	4-32	35	52	18	—	d=2t
	10XCHД	4-32 33-40	40 37	54 51	16 15	— —	d=2t d=2t

total length of contact along the boundaries of the grains with respect to that observed in a fine-grain structure.

In alloy steels and in steels containing over 0.3% of carbon the pearlite inclusions have larger dimensions and do not form a grid or network surrounding the ferrite grains. For this reason deformation takes place more smoothly, there is no "tooth" in the tension diagram, and the yield area is reduced or completely disappears; the steel becomes harder. Thus a large yield area is characteristic of low-carbon steels with a carbon content varying from 0.1 to 0.3%.

(3) Behaviour of Steel Subjected to Combined Stresses

Upon performance of a tensile test, the yield point σ_y is determined for the specimen when it is stressed only in one direction. If a combination of stresses is involved (for example, the combined action of normal and shear stresses in bending) the transition to the plastic state, in accordance with the maximum energy theory of strength, is characterized by the ultimate value of the unit work of deformation of a body (when only the shape, and not the volume, of the body changes under the influence of crystal displacement).

This transition to a plastic state is usually expressed through a reduced or equivalent stress by taking it equal to the yield point found in simple tension.

The deduction of the condition of plasticity in the maximum energy theory is based on the assumption that under a combined stress the potential strain energy per unit of volume should not exceed the potential strain energy in uniaxial tension.

through the following reduced stress ($\sigma_3 = 0$)

$$\sigma_{red} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} \leq R \quad (2-6)$$

In simple bending, when there are two components of the stressed state, namely, normal and shear stresses, for example in a simple beam having the stresses $\sigma_x = \frac{M_x}{W_x}$ and $\tau_{xy} = \frac{QS_x}{I_x l}$, the main stresses σ_1 and σ_2 , as is known, will be

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \quad \text{and} \quad \sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \quad (2-7)$$

Substitution of these expressions in formula (2-6) gives the reduced stress for bending

$$\sigma_{red} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \leq R \quad (2-8)$$

From this expression it is possible to obtain the condition of yielding for the maximum possible values of the shear stresses in pure shear, i.e., when $\sigma_x = 0$ (assuming that $R = \sigma_y$):

$$\tau_y = \frac{\sigma_y}{\sqrt{3}} = 0.577\sigma_y \cong 0.6\sigma_y \quad (2-9)$$

The shear modulus of elasticity (or simply the shear modulus) G is taken equal to 840,000 kg/cm² for steel.

For a biaxial stressed state when three components of this stressed state are present (σ_x , σ_y and τ_{xy}), the reduced stress will be

$$\sigma_{red} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} \leq R \quad (2-10)$$

If in a biaxial stressed state the stress in a plate in one direction reaches the tensile yield point (for example, $\sigma_1 = +\sigma_y$), then in the other direction the material will practically not be capable of withstanding either compressive or tensile stresses ($\sigma_2 = 0$). To permit the material to work ($\sigma_2 \neq 0$) a reduction of the stress σ_1 is necessary, i.e., σ_1 should be less than σ_y .

Should the stress in a plate in one direction reach the compressive yield point ($\sigma_1 = -\sigma_y$), then the plate can still take considerable compressive stresses in the other direction ($-\sigma_2$), but is practically unable to withstand tensile forces ($+\sigma_2 = 0$).

(4) Brittleness in Steels

Brittleness is characterized by failure of the material upon small deformations within the limits of elastic behaviour. In this case failure takes place by breaking off of the material, while the tension diagram has the form illustrated in Fig. 2-4a, without a yield area and a plastic stage.

Brittleness may appear in steels not only as a result of different qualities thereof, depending upon their chemical composition and heat or mechanical treatment, but also as a result of certain conditions

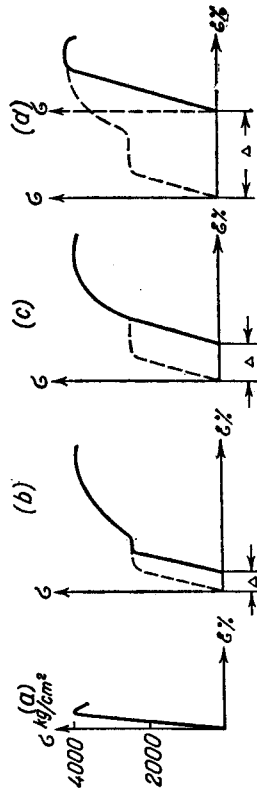


Fig. 2-4. Tension diagrams (Δ = permanent set)

of service characterized by the distribution of the force field. The latter induces a combined stressed state which depends on the shape of the body (Fig. 2-5d). The conditions facilitating the appearance of brittleness include phenomena leading to strain hardening, ageing,

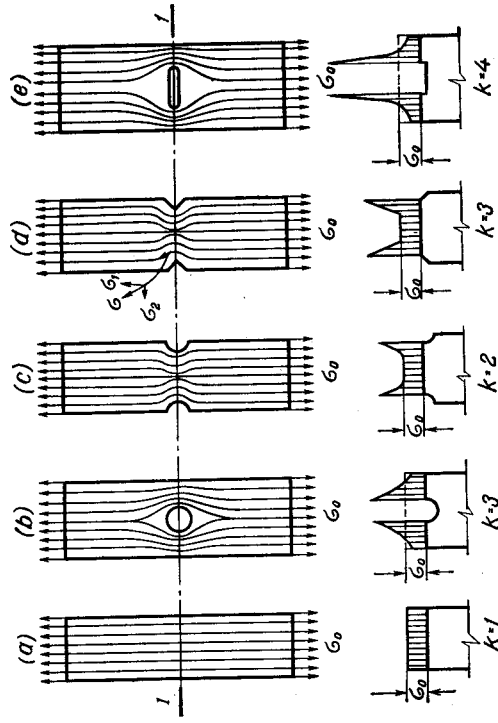


Fig. 2-5. Lines of constant stresses (k = stress concentration factor)

uneven distribution of the stresses, and also various temperature changes. The brittle state of a material often facilitates the appearance of cracks and leads to collapse of the member. For this reason designers should avoid creating such conditions that may lead to brittle failure of structural members.

Strain Hardening. If steel is subjected to tension until a plastic state is reached and is then unloaded, a permanent set will appear (Fig. 2-4b). Upon repeated loading of the specimen after a certain "rest" of the material the steel will again behave elastically, following the unloading curve, and then it will follow the usual path of a tension diagram obtained in single loading. The same will occur if unloading is commenced after the whole yield area has been used up (Fig. 2-4c). It is obvious that upon repeated loading the material will not have any yield area. Figure 2-4d shows a diagram for an instance when a preliminarily tensioned specimen obtained a large permanent set of the same sign, and its rupture took place almost within the limits of elastic work with small strains. Such an increase of the elastic range as a result of preceding plastic deformation is called *work-hardening*, or *strain hardening*. The effect of strain hardening is to increase brittleness; this effect cannot be considered as a positive property of steel structural members.

In some cases, when the reduction in the elongation is not of great significance, strain hardening is used to increase the yield point and strength (for example in the thin reinforcing wire employed for prestressed members). Strain hardening takes place when elements are bent, along the edges of holes being driven in members, when cutting in shears, etc.

Ageing. Ageing denotes a change in the properties of a material in time without any noticeable alteration of its microstructure.

In the process of ageing, hardening of the material takes place, accompanied by a reduction in plasticity and ductility, and an increase of brittleness. Two kinds of ageing are distinguished, namely, *thermal* (*dispersion* or *precipitation hardening*) and *strain ageing*. Thermal ageing takes place after the material has been heated to comparatively low temperatures (*artificial ageing*, also called *temper hardening*) or (in alloys) goes on at the ambient temperature (*natural ageing*). Strain ageing occurs after plastic deformation at temperatures below the temperature of recrystallization.

The phenomenon taking place in ageing consists in the gradual tendency to pass from a fixed, less stable structure of a supersaturated solid solution, to a more stable state.

With time the excessive component will be liberated from the supersaturated solid solution in the form of the most minute particles (in a dispersed state). For example, in low-carbon steels there will be

¹⁾ With a rapidly changing variable load beyond elastic limits, without "resting" of the material, unloading and loading will form small loops in the diagram instead of following one straight line.

liberated inclusions of cementite and nitrides, i. e., compounds of iron with nitrogen that prevent displacement of the ferrite crystals.

As a result the yield area decreases and, although the elastic properties of the steel improve, its elongation is considerably reduced, and thus the brittleness grows.

The process of ageing depends upon the composition of the steel and the technology followed in its production.

Thermal ageing is employed for hardening aluminium alloys.

Uneven Distribution of Stresses. Attention was devoted above to the behaviour of smooth specimens of a regular shape, wherein the stresses in all sections removed from the place of load application are distributed uniformly.

Upon drawing lines of constant stresses or forces, we shall obtain rectilinear forcelines inside the specimen (Fig. 2-5a) that determine a linear uniaxial stressed state.

If a hole is made in a flat specimen or its sides are notched (Fig. 2-5b-e) the forcelines will change their direction and flow around the new boundaries. Concentration of the forcelines characterizes a growth of the stresses, while deviation of these lines from a straight direction is an indication of the existence of stresses acting in two directions and bending the path of the lines (Fig. 2-5d). In this instance there will be observed a plane (biaxial) stressed state that appears exclusively as a result of changes in the design of the specimen boundaries. As is known from the course in the strength of materials, in a plane stressed state the maximum shearing stress is equal to half the difference between the principal stresses

$$\tau = \frac{\sigma_1 - \sigma_2}{2}$$

When the difference between the two principal stresses is small or equal to zero (when $\sigma_1 = \sigma_2$), the shearing stress will also be small or equal to zero.

Yielding of a material is to a considerable extent the result of displacements under the influence of shearing stresses. This is why with small values of τ and high values of σ_1 and σ_2 the yield point of the material sharply increases, while for $\tau=0$ it disappears completely, and failure takes place by rupture of the material within the limits of elastic strains, i. e., in the form of brittle fracture. Thus, with an increase in the sharpness of the notch (slot), the plasticity will be sharply reduced (as well as the elongation) and the ultimate strength will grow (Fig. 2-6).

A similar result is obtained upon extending a flat specimen by means of identical forces acting in two directions (an element of a boiler,

a gas holder or an element inside the closed contour of a weld tensioned as the result of shrinkage thereof upon cooling).

In analyzing the elements of structural members the local stresses are not usually determined, since the point of primary interest is the ultimate strength of the element as a whole, determined by the mean, uniformly distributed stress. When designing parts, however, especially in welded members, the local stresses must be taken into account. Local stresses relating to the category of principal ones may be omitted in analysis only when plastic steels are used, wherein equalization of the stresses is ensured.

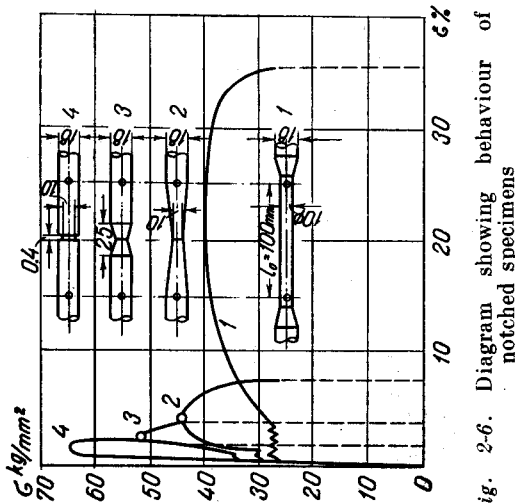


Fig. 2-6. Diagram showing behaviour of notched specimens

the steel in a fracture obtains a coarse-grain structure and becomes more brittle (blue brittleness). At this temperature it is bad practice to deform the steel or subject it to blows. Upon a further increase in temperature this property disappears, but rapid decreasing of the yield point and ultimate strength values will begin. Thus, for example, for steel 3 at a temperature of about 500 deg C we have $\sigma_y = 1,400 \text{ kg/cm}^2$ and $\sigma_u = 2,500 \text{ kg/cm}^2$, while at a temperature of about 600 deg the corresponding values will be 400 and 1,500 kg/cm^2 , i.e., the yield point rapidly approaches zero. At a temperature of about 600-650 deg C thermal plasticity sets in. Heating for a long time at temperatures above 700 deg (a cherry-red colour) facilitates the growth of crystals and the formation of a coarse-grain structure. This phenomenon is called *overheating* and is the cause of poorer mechanical properties of steel.

Upon heating for a long time in air to a temperature approaching the melting point (a bright yellow-white colour) overburning of the metal is possible. Overburned metal is a cause for the rejection of members. Temperatures below freezing point improve somewhat the strength of steel, but this is accompanied by a growth of brittleness. At tem-

peratures below -10 deg C the plasticity of steel begins to decline at a noticeable rate; at temperatures below -45 deg (for steel 3) and -60 deg (for low-alloy steel) the steel becomes brittle.

Fatigue of Metal. By the fatigue of metal is meant the phenomenon of its failure under the action of repeated (several million times) reversed or simply alternating loading at values of the stresses below the ultimate strength (for example, the rupture of wire when it is bent back and forth several times). The ability of metal to resist such destruction is called its *endurance*, while the stress at which the metal

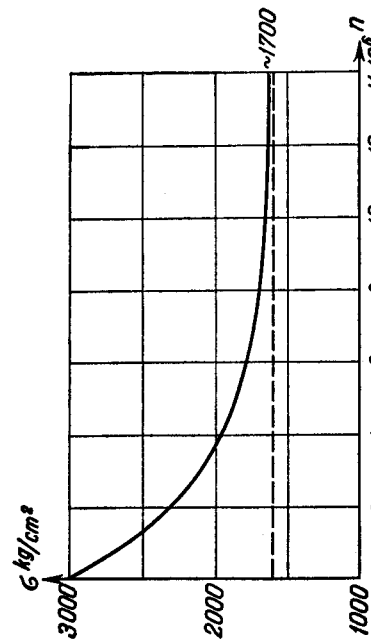


Fig. 2-7. Vibration strength (fatigue) curve

fails is known as its *fatigue* or *vibration strength* σ_v . Figure 2-7 shows a curve of the vibration strength for grade Cr. 3 steel with completely reversed loading cycles against the number of cycles n . With a great number of cycles the vibration strength curve asymptotically approaches a certain limit known as the *endurance (fatigue) limit*. Its value equals approximately four tenths of the ultimate tensile strength (2-11)

$$\sigma_v \approx 0.4\sigma_u$$

The vibration strength of a material is reduced by the presence of notches, sharp changes in section and other stress concentrators that create a combined stress state. Upon the failure of steel as a result of fatigue a brittle fracture is generally obtained, and it may be presumed that destruction commences with the appearance of some sort of microcracks.

Building standards and regulations prescribe a reduction of the design strengths for members and connections that are subjected to alternating or vibrating loads by introducing a factor (γ) less than unity [see expression (2-41)], and also draw special attention to the

necessity of employing designs that will not produce considerable concentrations of stresses.

Testing Steel for Impact Strength. In view of the harmful influence of brittleness in steels, which facilitates the appearance of cracks, it

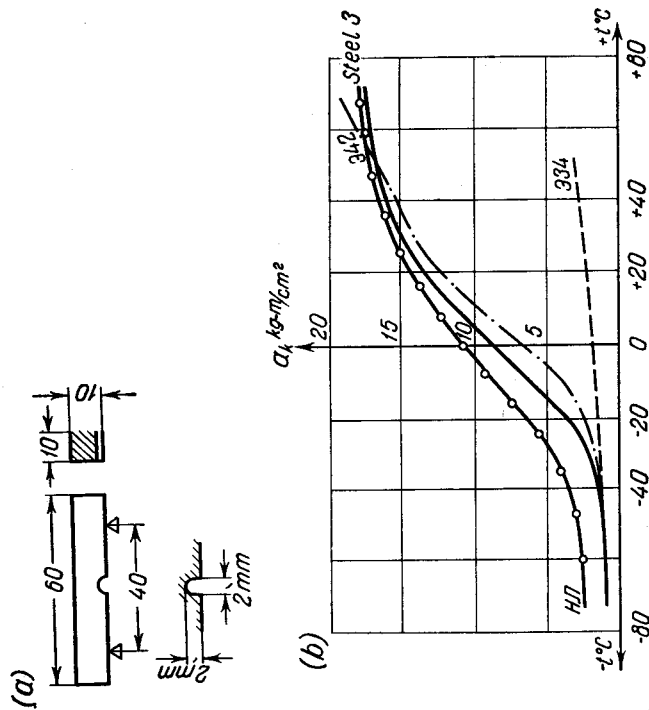


Fig. 2-8. Testing of steel for notch toughness: — steel 3; - - - metal deposited using electrodes 3421; . . . ditto, electrodes 334; —o—o— steel HJI (low-alloy)

is necessary to have the possibility of making a comparative appraisal to permit designers to judge how the steel withstands the transition to a brittle state.

For this purpose use can be made of the resistance of steel to impacts. The measure of the strength of steel under impact loads is the magnitude of the work required to destroy the specimen, and not the destructing force or stress. The unit work, i.e., the work spent for the impact destruction of a standard notched specimen (Fig. 2-8a) per unit of cross-sectional area (in kilogram-metres per sq cm) is known as the *notch toughness*. In plastic steels *ductility* characterizes the energy absorbed by a unit of volume of the material in plastic deformation. Notch toughness is a comparative characteristic.

Low-carbon steels at a normal temperature have a notch toughness of 7 to 10 kg-m/cm². The notch toughness is shown versus temperature in Fig. 2-8b.

2-3. PRINCIPAL MECHANICAL PROPERTIES OF ALUMINIUM ALLOYS

The mechanical properties of aluminium alloys depend not only on their chemical composition, but also on the conditions of their machining and working. The principal difference between the behaviour of aluminium alloys and that of steel consists in their being more liable to deformation, i.e., less rigid. In aluminium alloys the modulus of elasticity in tension $E=710,000$ kg/cm², while the shear modulus of elasticity $G=270,000$ kg/cm², which is about one-third of the corresponding values for steel. For this reason, under equal stresses the deflections of aluminium members are three times greater. Poisson's ratio $\mu=0.3$ for aluminium. In tension diagrams of aluminium alloys there are no yield areas (Fig. 2-9). The yield strength is by convention taken equal to the stress producing a strain of $\epsilon=0.2\%$. At temperatures above 100 deg C a certain reduction of the strength characteristics is observed, while beginning from about 200 deg C creep appears. The coefficient of thermal expansion of aluminium is 0.00023, which is twice that of steel. At reduced temperatures all the mechanical properties of aluminium alloys improve. The notch toughness of the alloys at normal temperature is lower than that of steel (about 3 kg-m/cm²) and decreases very noticeably at temperatures below freezing point. Table 2-3 contains the mechanical properties of certain aluminium alloys.

The merits of aluminium alloys include:

- (1) a comparatively high strength with a low weight of the material itself, which has a great importance for large-span, mobile and enclosing members;
- (2) a high workability when subjected to pressing, rolling or forging, which makes it possible to manufacture articles of a complicated configuration;
- (3) a high resistance to corrosion;

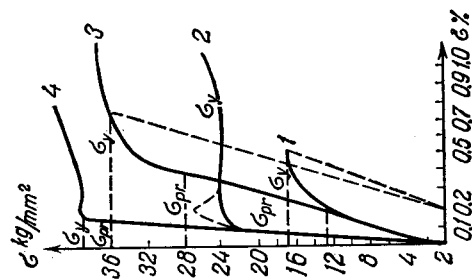


Fig. 2-9. Tension diagram of aluminium alloys in comparison with steels: 1—alloy AMr 6; 2—steel 3; 3—alloy Al6-T; 4—steel 15XCHD

TABLE 2-3 Mechanical Properties of Aluminium Alloys

Grade of alloy	State of delivery	Ultimate strength, kg/mm ²	Yield point, kg/mm ²	Longitudinal strain, %	Ultimate shearing strength, kg/mm ²	Endurance limit, kg/mm ²	Brinell hardness number Bhn, kg/mm ²	USSR Standard or specification
Д1	Д1-Т	36	22	12	—	—	—	GOST 4977-52
Д16	Д16-Т Д16-М	40-49 21	30 11	10 18	28 13	—	105 42	GOST 4977-52
AB	AB-Т1 AB-М	33 15	28 —	10 20	20 —	9.8 4.5	95 30	AMTY 258-55 AMTY 252-57
В92	В92-Т	36	20	20	—	—	—	AMTY 258-55 AMTY 252-57
АМг6	АМг 6-М	32	16	15	—	—	70	AMTY 443-59
АМц	АМц-М АМц-П	13 16	5 13	20 16	—	5.5 6.5	30 40	AMTY 252-57

(4) the retaining of its high mechanical characteristics at temperatures below freezing point;

(5) the absence of sparks when struck with various articles.

The drawbacks of aluminium alloys are:

- (1) a relatively low modulus of elasticity;
- (2) a higher coefficient of thermal expansion;
- (3) the relative complicity of making connections;
- (4) the short supply and, as yet, high cost;
- (5) the comparatively low refractoriness.

2-4. BEHAVIOUR OF MATERIAL IN COMPRESSION. PROBLEM OF STABILITY

(1) Behaviour of Steel in Compression

Steel when subjected to compression in short elements behaves the same as in tension. The magnitudes of the yield point, the yield area and the modulus of elasticity are equal to those obtained in tension.

It is not possible, however, to destruct by compression short specimens made from plastic steel and thus determine the ultimate strength of the material, since the specimen flattens out.

An absolutely different picture will be observed in long compressed elements whose length is several times greater than the width of their cross section (slender elements). In this case the element may lose its load-carrying capacity, i.e., the ability to withstand external forces, not as a result of failure of the material, but owing to the loss of stability (buckling).

(2) Problem of Stability

In steel structural members the problem of stability is of a very great significance. Underestimation of this factor may lead to disastrous results.

If a straight rod is compressed by an axially applied force P , then the rod will initially remain straight, and this state of equilibrium will be stable. The stable state of equilibrium of an elastic rod is characterized by the rod returning to its original position after performing insignificant damped oscillations if it had been loaded and received an insignificant possible deflection due to any cause, after which the action of this cause stopped. This takes place because the external compressive force cannot overcome the resistance of the rod when it has been subjected to insignificant bending (deflection of its axis due to a small disturbing transverse force); the internal elastic work of deformation involved in the bending of the rod, resulting from deflection of its axis (the potential energy of bending ΔV), is greater than the external work (ΔW) done by the compressive force when the ends of the rod approach each other in bending: $\Delta V > \Delta W$ (Fig. 2-10a). Upon a further increase in the compressive force it may reach such a value that its work will be equal to the work of deformation in bending induced by any small disturbing factor. In this instance we have $\Delta V = \Delta W$ and the compressive force reaches its critical value P_{cr} . Thus a straight rod when loaded with a force up to the critical state has a linear shape in the stable state of equilibrium. When the force reaches its critical value, the linear shape of equilibrium stops being stable, the rod may buckle in the plane of least stiffness and it will now have a new curvilinear shape of stable equilibrium.

That magnitude of the force which causes the original stable form of equilibrium of the rod to become unstable is known as the *critical force* (or *load*).

If there is a slight initial curvature of the rod (or a slight eccentricity of the point of compressive force application) the rod upon an increase in the load will immediately deflect from a straight position.

This deflection is first small, however, and only when the compressive force approaches its critical value (differing from it by not more than one per cent) do the deflections become considerable. This indicates a transition to an unstable state. Thus the unstable state of equilibrium is characterized by great displacements taking place even with a

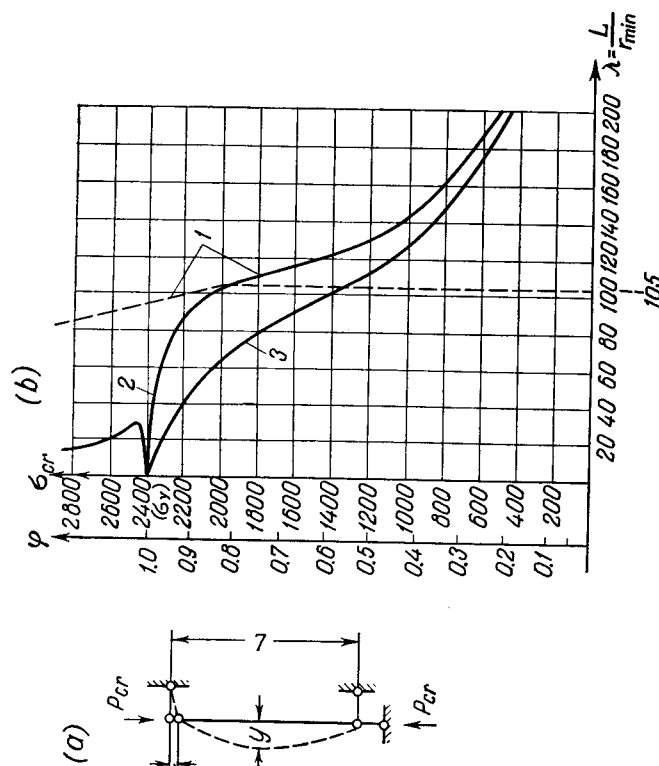


Fig. 2-10. Buckling of axially compressed rods:
a—basic case; b—curves of critical stresses and buckling factor for steel 3; 1—Euler's curve; 2—curve of critical stresses with account taken of plastic behaviour of material; 3—curve of factor ϕ

small increase of the load. A further increase of the compressive force above the critical value calls forth ever growing deflections, and the rod will lose its load-carrying capacity. It should be noted that different values of the critical load correspond to different end conditions of the rod. For the axially compressed rod shown in Fig. 2-10a, having hinged end constraints (the basic case), the critical load was determined by the renowned mathematician L. Euler in 1744 as follows

$$P_{cr} = \frac{\pi^2 EI_{min}}{L^2} \quad (2-12)$$

The stress that is induced in the rod by the critical force is known as the *critical stress*

$$\sigma_{cr} = \frac{P_{cr}}{A_{gr}} = \frac{\pi^2 EI_{min}}{L^2 A_{gr}} = \frac{\pi^2 E r_{min}^2}{L^2} = \frac{\pi^2 E}{\lambda^2} \quad (2-13)$$

where $r_{min} = \sqrt{\frac{I_{min}}{A_{gr}}}$ = minimum radius of gyration

A_{gr} = gross cross-sectional area of the rod

$\lambda = \frac{L}{r_{min}}$ = slenderness ratio of the rod, equal to the ratio between the length of the rod and the minimum radius of gyration of its cross section.

A glance at expression (2-13) will show that the critical stress depends upon the slenderness ratio of the rod (since the numerator $\pi^2 E$ is a constant quantity), while the slenderness ratio is a quantity that depends only on the geometrical dimensions of the rod. Hence the possibility of increasing the value of the critical stress by changing the slenderness ratio of the rod (mainly by increasing the radius of gyration of the cross section) is in the hands of the designer and he must use this possibility rationally.

Thus the critical stress may also be considered as a parameter that shows whether the section of an element designed for service under compression has been advantageously selected.

The graph of Euler's formula (2-13) is a hyperbola (Fig. 2-10b, Curve 1).

The critical stresses determined from Euler's formula are true only for a constant modulus of elasticity E , i.e., within elastic limits (more exactly, within proportional limits), and this can take place only with high slenderness ratios (exceeding 105), which follows from expression (2-13)

$$\lambda = \sqrt{\frac{\pi^2 E}{\sigma_{pr}}} = 3.14 \sqrt{\frac{2,100,000}{2,000}} = 105$$

Here $\sigma_{pr} = 2,000$ kg/cm² is the proportional limit for steel 3.

The critical stresses for rods (columns, struts) with a small (below 30) and medium (from 30 to 100) slenderness ratios are obtained above the proportional limit but, of course, below the yield point. The theoretical determination of the critical stresses for such rods is rather complicated owing to buckling taking place upon the partial development of plastic deformations and with a variable modulus of elasticity.

As a result of numerous experiments that have confirmed the correctness of the theoretical conclusions, critical stresses have been established for rods with small and medium slenderness ratios, their values being shown in the form of a curve in Fig. 2-10b (Section 2).

The load-carrying capacity of a compressed member is also noticeably affected by the local stability of its elements, which depends on the slenderness ratio of the flanges, webs or other elements forming the cross section of the member. The slenderness ratios of these elements are determined by the ratio between their characteristic dimension (width of flanges or depth of section web) and their thickness, viz., $\frac{b}{t}$ or $\frac{h}{t}$. Figure 2-11 shows the loss of load-carrying capacity of an axially loaded column as the result of a loss of local stability of its flanges and web.¹⁾

Thus, the load-carrying capacity of a compressed element may be exhausted owing to two causes:

- (1) as the result of the stress in the member reaching the yield point (loss of strength);
 - (2) owing to the stress in the member reaching its critical value (loss of stability).
- These two causes, which are absolutely different in their nature, should never be confused.

The limiting states of compressed rods in respect to strength and stability are expressed by the condition that the stresses in the member should be

$$(1) \sigma \leq \sigma_y; \quad (2) \sigma \leq \sigma_{cr}$$

in which σ = stresses in member caused by design loads.

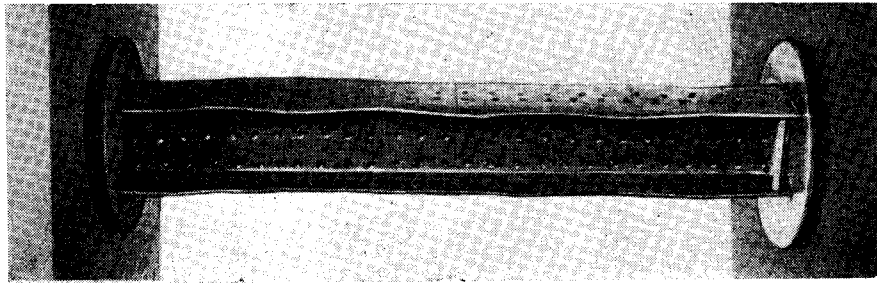
If the ratio between the two limiting stresses is denoted

$$\varphi = \frac{\sigma_{cr}}{\sigma_y} \quad (\text{whence } \sigma_{cr} = \varphi \sigma_y)$$

then the second check for stability may be written as follows (the design strength R being taken as the lowest yield point)

$$\sigma \leq \varphi R$$

Fig. 2-11. Testing model of column in compression. Loss of local stability of flanges and web



¹⁾ The loss of local stability in the elements of sections due to the action of normal or shearing stresses will be considered in Sections 6-4 and 6-7

or, in order to simplify the calculations and the comparison of their results, in the form of the working formula

$$\sigma = \frac{F}{\varphi A_{gr}} \leq R \quad (2-14)$$

The factor φ , which reduces the design strength to values ensuring stable equilibrium, is known as the *buckling factor*. The building standards and regulations establish the values of the buckling factor with regard to the influence of chance eccentricities

$$\varphi = \frac{\sigma_{cr}}{\sigma_y} \times \frac{\sigma_{cr \cdot e}}{\sigma_{cr}} = \frac{\sigma_{cr \cdot e}}{\sigma_y} \quad (2-15)$$

In the above expression $\sigma_{cr \cdot e}$ is the critical stress of a rod compressed by a force that is applied with a possible chance eccentricity of e .

The buckling factor is of interest to designers only when it is less than unity, seeing that otherwise $\sigma_{cr} > \sigma_y$, and the case of the loss of load-carrying capacity due to the lack of strength now becomes dangerous. Since it characterizes the critical stresses, the buckling factor φ is a function of the slenderness ratio of a rod. Curve 3 in Fig. 2-10b illustrates the buckling factor. The values of this factor for steel 3 and low-alloy steel are contained in Table 1 of Appendix II.

Analysis of aluminium alloy members in compression is similar to that of their steel counterparts. The values of the buckling factor φ for aluminium alloys are set forth in Table 2 of Appendix II.

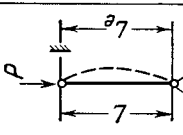
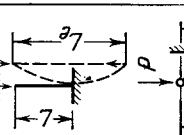
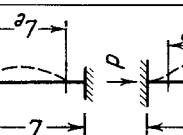
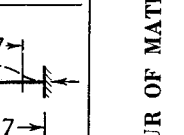
The values of the critical stresses and, therefore, of the buckling factor, depend to a very great extent upon the end conditions of the compressed member. The values of the buckling factor that are tabulated are determined for the basic end conditions of rods, viz., for rods with both ends pinned (such ends are also called pivoted, hinged or round ends). For other end conditions, the shape of the buckling curve changes, but it can be reduced to the basic case by substituting for the actual length L the *effective*, or *equivalent length* L_e , for which purpose the length of the rod should be multiplied by a length coefficient k . Hence the slenderness ratio of a rod with any end conditions will be determined from the expression

$$\lambda = \frac{L_e}{r} = \frac{kL}{r} \quad (2-16)$$

This method of analyzing buckling by using the effective lengths was proposed by prof. F. Yasinisky (1894) who devoted much time to the problems of buckling.

The values of the length coefficient k for some end conditions are given in Table 2-4.

TABLE 2-4 Effective Lengths of Compression Rods

Type of end conditions	Sketches showing actual and effective lengths	Length coefficient k	Effective length of rod $L_e = kL$	Remarks
1. Both ends pinned		1	$L_e = L$	Rotation is possible at both ends, but horizontal displacements of the rod ends are impossible
2. Lower end fixed, upper end free		2	$L_e = 2L$	Horizontal displacements and rotation of upper end are possible
3. Lower end fixed, upper end pinned		0.7	$L_e = 0.7L$	Rotation is possible at upper end, horizontal displacements of both ends are impossible
4. Both ends fixed		0.5	$L_e = 0.5L$	No rotation or horizontal displacement is possible at either end

2-5. BEHAVIOUR OF MATERIAL IN BENDING AND TORSION

(1) Limiting State of Steel Beams in Bending for Strength Analysis

When a beam is subjected to bending (Fig. 2-42a) within elastic limits, a triangular diagram of normal stresses (Fig. 2-42b) will be obtained in the cross sections of the beam. The maximum value of these stresses in the extreme fibres will be determined from the expression

$$\sigma_x = \pm \frac{M}{W_x} \quad (2-17)$$

Upon an increase in the load the extreme fibres will be the first ones to reach the yield point, after which the growth of the stresses in them will cease (with continuing increase in strains). Yielding of

the material spreads along the depth of the cross section, an elastic core remaining in the middle part of the section (Fig. 2-42c) and the latter will be in an elastoplastic state. An increase in the load brings up to the yield point all the fibres in the heaviest loaded cross section, characterized by a rectangular stress diagram (Fig. 2-42d), and at the neutral axis, at the spot where the greatest moment is acting, a so-called *plastic hinge* will be formed. The spreading of yielding along the length of the beam is shown by the shaded area in Fig. 2-42e. Under the influence of such action a large increase in the strains will take place at the location of the plastic hinge, the beam will sag, but

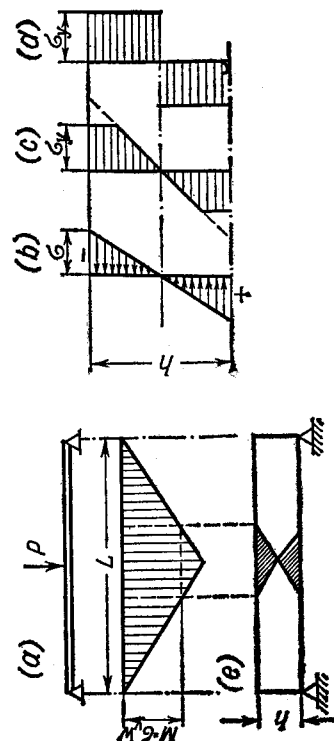


Fig. 2-12. Sequence of stress development upon plastic behaviour of beam in bending

will not fail. In this instance the beam usually loses either its total stability, or local stability of certain of its parts¹⁾. The appearance of a plastic hinge transforms a simple beam into a changeable system. The maximum moment corresponding to the plastic hinge can be determined from the expression

$$M_p = \sigma_y \int y dA = \sigma_y 2S = \sigma_y W_p \quad (2-18)$$

where W_p is the plastic section modulus, equal to the double statical moment of half the cross-sectional area relative to the axis passing through the centre of gravity.

The plastic section modulus W_p is somewhat greater than the ordinary (elastic) section modulus W_e . Thus, for a rectangular section $W_p = \frac{bh^2}{4}$ and for rolled sections $W_p = cW_e$ (the values of c are given in Table 2-5).

¹⁾ The loss of total and local stability in beams subjected to bending will be discussed in Section 6-4.

TABLE 2-5 Values of Coefficient c for Rolled Sections

With bending in plane of	I beams (GOST 8239-56*)	Channels (GOST 8240-56*)
Web	1.12	1.13
Flange	1.5	1.8

The Building Standards and Regulations allow account to be taken of the development of plastic strains for simple rolled beams (I beams and channels) constrained to prevent buckling and carrying a statical load. The values of the plastic section moduli in this instance are taken equal to $W_p = 1.12 W_e$ for bending in the plane of the web and $W_p = 1.2W_e$ for bending parallel to the flanges (and not $1.5W_e$ or $1.8W_e$, as indicated in Table 2-5) in order to reduce the edge strains.

The Building Standards and Regulations also permit account to be taken of the development of plastic strains for constant-section welded beams with a ratio between the width of the compressed flange overhang and the flange thickness of $\frac{b}{t} \leq 10$, and a ratio between the depth of the web and its thickness of $\frac{h}{t} \leq 80$ (for steel 3).

At spots with the greatest bending moments large shearing stresses are not tolerated. They should comply with the inequality $\tau \leq 0.3R$.

When a long zone of pure bending is present, the corresponding section modulus, in order to avoid excessive strains, is taken equal to half the sum of the elastic and plastic moduli, viz., $0.5 (W_e + W_p)$.

In continuous beams the formation of plastic hinges is taken as the limiting state, but on condition that the system remains unchanged.

In the analysis of continuous beams (rolled and welded) that comply with all the limitations indicated above for simple beams the Building Standards and Regulations permit the design bending moments to be determined on the basis of equalization of the support and the span moments (provided that adjacent spans differ by not more than 20%).

In this instance the values of the design moment are taken equal to:

$$M = \frac{M_{01}}{1 + D/L} \quad \text{or} \quad M = \frac{1}{2} M_{02}$$

the two values

in which M_{01} and M_{02} = maximum bending moments in extreme and intermediate spans respectively, computed as for a freely supported simple beam

D = distance to the extreme support from the section corresponding to the moment M_{01}

L = extreme span.

2. In simple and continuous beams with fixed ends

$$M = 0.5M_0$$

where M_0 is the greatest value of the moments calculated as for a beam with pinned supports.

In all cases when the design moments are determined on the assumption of plastic strains developing (equalization of the moments) the beam strength should be checked in accordance with formula (2-17) using the elastic section modulus. In analyzing beams made from aluminium alloys the development of plastic strains is not taken into consideration.

(2) Limiting State Strength Analysis of Steel Beams in Bending Under Simultaneous Action of Moments and Shear Forces

With simultaneous action in a beam subjected to bending of normal and shearing stresses σ and τ , the transition of the beam to a plastic state is expressed, as indicated in Sec. 2-2, (3), through the *equivalent* or *reduced stress*, which establishes the relation between σ and τ at the moment when yielding appears at an extreme point, or any other point of the cross section.

Until a plastic hinge is obtained, however, the beam being bent can still be additionally loaded. In this instance if large shearing stresses ($\tau > 0.4R$) are present yielding begins to spread over the cross section more rapidly than if only normal stresses were acting. Obviously, if the shearing stress τ grows, the normal stress σ should decrease, inasmuch as the equivalent stress must not be greater than the design strength R . The relation between σ and τ , or between the corresponding values of M and Q acting together, at which a plastic hinge is obtained, has not been precisely determined for various cross sections. The boundary line between the areas of the elastoplastic and the plastic states in the coordinate plane σ - τ may be taken as a convex curve (or surface if there are several components of the stressed state). Every point of this line or surface is a point where a plastic hinge is formed and a maximum point of the elastoplastic state, and for this reason a tangent to the line will lie outside it. For convenience of notation, let us express the coordinates of the

boundary line (surface) of transition to a plastic state by dimensionless quantities

$$s = \frac{MQ}{M_p} < 1 \text{ and } t = \frac{Q^M}{Q_p} < 1$$

where MQ = maximum moment observed upon the formation of a plastic hinge with the simultaneous action of a shear force Q

M_p = maximum moment observed upon the formation of a plastic hinge with action only of the moment

Q^M = maximum shear force observed upon the formation of a plastic hinge with the simultaneous action of a moment M

Q_p = maximum shear force observed upon the formation of a plastic hinge with action only of the shear force.

It is obvious that with common action of M and Q a plastic hinge will be formed with lower values of the forces than when they act separately.

Investigations conducted by B. Broude have resulted in the following equation for the boundary curve

$$s^2 + t^2 + as^2t^2 = 1 \quad (2-19)$$

in which a is a factor varying from -0.8 to -0.9 for I sections.

Assuming that $MQ = \sigma W$ and that the shear force is mainly taken by the web of the beam, we shall obtain for I beams

$$s = \frac{MQ}{M_p} = \frac{\sigma M}{\sigma_y W_p} = 0.89 \frac{\sigma}{\sigma_y}$$

$$t = \frac{Q^M}{Q_p} = \frac{\tau_{m^A w}}{\frac{\sigma_y}{V^3} A_w} = \frac{\tau_m V^3}{\sigma_y}$$

Substitution of these values for s and t in equation (2-19) yields

$$\frac{\sigma^2}{\sigma_y^2} (0.89)^2 + \frac{3\tau_m^2}{\sigma_y^2} - a \frac{3\tau_m^2 \sigma^2 (0.89)^2}{\sigma_y^4} = 1$$

whence, assuming that $\sigma_y = R$, we shall determine, with certain approximations to increase the margin of safety, the ultimate equivalent or reduced stress in a beam corresponding to the moment when a plastic hinge is formed

$$\sigma_{h.red} = \sqrt{\frac{3}{4}\sigma^2 + 3\tau^2 \left(1 - \frac{1}{2} \frac{\sigma^2}{R^2}\right)} \leq R \quad (2-20)$$

Here σ is the maximum absolute value of the boundary stress in the web of the beam in elastic behaviour of the material, and

$\tau = \frac{Q}{h_w t}$ is the mean shearing stress in the web of the beam.

Checking of the reduced stress in the webs of beams with the aid of expression (2-20) is required when τ is greater than $0.4 R$.

(3) Behaviour of Material in Torsion

Torsion may appear in metal structural members when a load is applied that bends an element outside the plane passing through the axis of symmetry of the section. The torsional strength of the separate elements of metal members is very small, and it is therefore essential to avoid designs permitting torsion.

Assume that we have a beam which is not symmetrical with respect to its vertical axis, and that has a channel section which a load is applied to, directed at right angles to the centre line of the beam with an eccentricity to the left or the right from the web, as shown in Fig. 2-13a. The beam will be twisted to the left or the right. Hence, upon shifting the plane in which the load is acting to the left we shall find such a position thereof in which only bending (without torsion) will be observed. This plane of action of a force perpendicular to the centre line of a beam intersects the axis of symmetry in sections that are symmetrical with respect to their horizontal centre line at a point known as the *centre of bending* (Fig. 2-13b).

The stresses and strains when an element is in torsion, as distinct from tension, depend upon the shape of its cross section. It is well known that in a round solid body, which a twisting moment or torque is applied to, only shearing stresses develop that are distributed over the section in a triangular diagram (Fig. 2-13c). Here two cross sections, while turning with respect to each other, remain plane, perpendicular to the axis. The maximum shearing stresses in this instance will be

$$\tau_{max} = \frac{M_t}{W_p} = \frac{M_t r}{J}$$

where M_t = twisting moment

$W_p \approx 0.2d^3$ = polar section modulus

$J \approx 0.1d^4$ = polar moment of inertia

r and d = radius and diameter of round bar, respectively.

In a tube or pipe the shearing stresses can be approximately taken as uniformly distributed ones (Fig. 2-13d). For thin-walled pipes in which the diameter to wall thickness ratio exceeds five, i.e., $\frac{d}{t} > 5$, it

In this expression $I_t = \frac{bt^3}{3}$ is the moment of inertia of the section in torsion. In this instance after twisting the cross sections do not remain in the main plane.

In a rectangular box section made up of plates, the shearing stresses, the same as in a round pipe or tube, will be distributed along the periphery of the section uniformly over the thickness of the plate, having their centre at the middle of the contour, and not at the middle of each plate (Fig. 2-13e). Such a section with a closed contour has a high twisting strength.

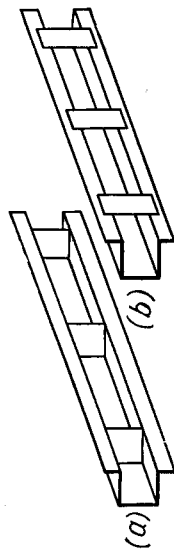


Fig. 2-14. Bracing of open sections with transverse membranes or batten plates

Immediately this contour is cut open, however, its twisting strength will be equal to that of the unfolded plate. I sections can be considered as such open sections.

The stresses in such sections are determined from expression (2-21), the moment of inertia in torsion being taken equal to

$$I_t = \alpha \sum \frac{bt^3}{3} \quad (2-22)$$

in which for an I section $\alpha = 1.3$, for channels $\alpha = 1.42$, for angles $\alpha = 1$, for welded beams with stiffening ribs welded to the flanges $\alpha = 1.5$, for riveted beams $\alpha = 0.5$.

Thus, for example, for an I section (Fig. 2-13g) we obtain

$$I_t = \alpha \sum \frac{bt^3}{3} = \frac{1.3}{3} (b_1 t_1^3 + b_2 t_2^3 + b_3 t_3^3)$$

The installation of transverse membranes (Fig. 2-14a) and especially of transverse batten plates (Fig. 2-14b) which close the open section at some spots increases the torsional strength of the element many times.

In a closed thin-walled section the unit shearing force induced by the flow of shearing stresses appearing in torsion will be constant over the whole section

$$q_{sh} = \tau t \text{ kg/cm} \quad (2-23)$$

The external twisting moment is balanced by an internal one equal to the sum of the products of the forces per unit of length $L\Delta$ and the

3-2136

can be assumed, with an error of less than five per cent, that $W = 2\pi r^2 t$ and $I = 2\pi r^3 t$.

In a rectangular strip with a solid section (Fig. 2-13f), when $\frac{b}{t} > 8-10$, the shearing stresses in torsion will be distributed unevenly,

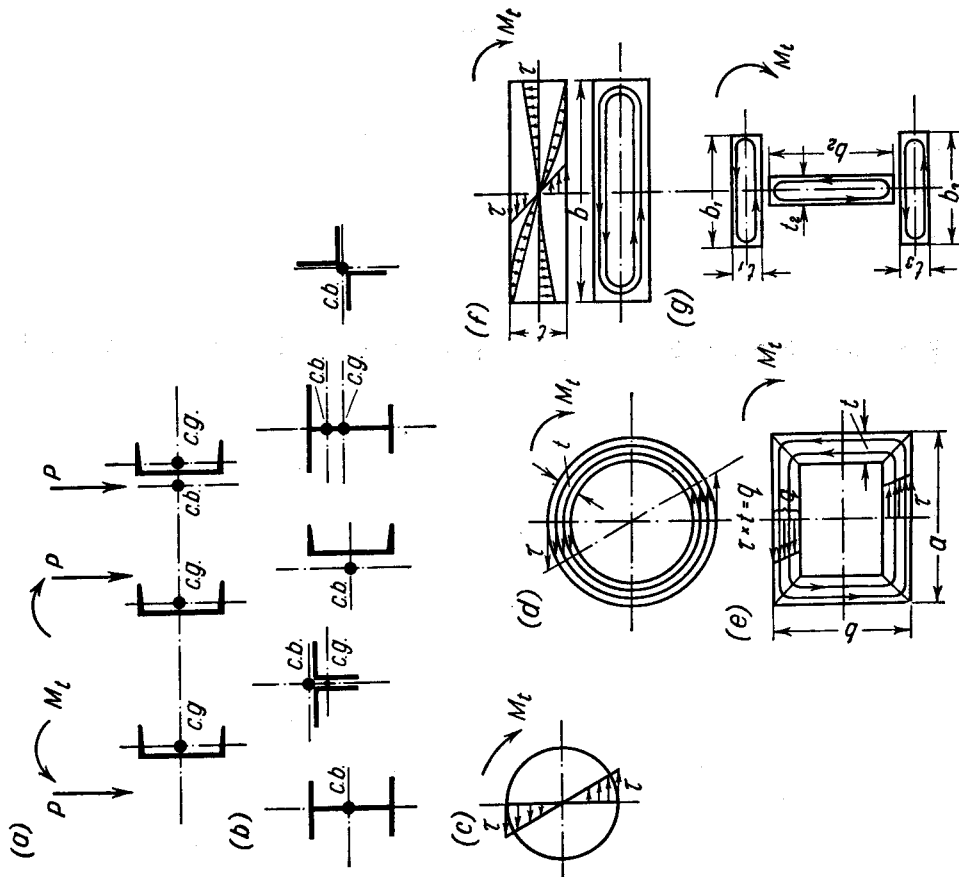


Fig. 2-13. Torsional behaviour of sections

their maximum value being found on the edges at the middle of the long side of the section. This value will be

$$\tau_{max} = \frac{M_t t}{I_t} \quad (2-21)$$

arm r over the whole contour

$$M_t = q_{sh} \sum r \Delta L = q_{sh} A_c \quad (2-24)$$

in which $A_c = \sum r \Delta L = \text{double area of section contour}$.
Hence

$$q_{sh} = \frac{M_t}{A_c} \quad \text{and} \quad \tau = \frac{q_{sh}}{t} = \frac{M_t}{A_c t} \quad (2-25)$$

For rectangular and round closed sections (Fig. 2-13e and d) we shall have, respectively

$$\tau = \frac{M_t}{2abt} \text{ kg/cm}^2 \quad \text{and} \quad \tau = \frac{M_t}{2\pi r^3 t} \text{ kg/cm}^2$$

2-6. BEHAVIOUR OF MATERIAL IN ECCENTRIC TENSION AND COMPRESSION

(1) Eccentric Tension

If an axial tensile force F and a bending moment M act simultaneously on an element, or if it is placed in tension by a force applied eccentrically with respect to its axis, the strength in the elastic stage is checked by means of the expression

$$\sigma = \frac{F}{A_n} + \frac{M_x}{W_{n,x}} = \frac{F}{A_n} \left(1 + \frac{e_x}{\rho_x} \right) = \sigma_a (1 + m_x) \leq R \quad (2-26)$$

where A_n = net cross section, cm^2

$W_{n,x}$ = net section modulus, cm^3

$e_x = \frac{M_x}{F}$ = eccentricity of force application, cm

$\rho_x = \frac{W_x}{A}$ = core (kern) radius of cross section, cm

$\sigma_a = \frac{F}{A_n}$ = axial stress induced by longitudinal force

$m_x = \frac{e_x}{\rho_x} = \frac{M_x}{F} \frac{A}{W_x}$ = relative or unit eccentricity.

When account is taken of the behaviour of a material in the plastic stage (which is tolerated by the Building Standards and Regulations for welded and rolled elements of steel members that are not directly subjected to dynamic loads), the load-carrying capacity of the element is higher than when only its elastic behaviour is taken into consideration. The limiting state for an eccentrically tensioned element which a moment M and a force F are applied to can be determined in the same way as that for a beam in bending when the moment M and the force Q are applied to it [see Sec. 2-5, (2)]. For this purpose it is necessary to determine the relation between M and F at which a plastic hinge will be obtained, and thus find the boundary lines separating the elastic, elastoplastic and plastic areas of behaviour of

the material in a member. If we designate by $F_y = A\sigma_y$ the force at which a yield stress is reached in a section (in the absence of a moment, Fig. 2-15a) and by $M_y = W_y \sigma_y$ the moment corresponding to the formation of a plastic hinge (in the absence of a longitudinal force, Fig. 2-15b), then with the simultaneous action of F and M their limiting values, obviously, will be

$$\begin{aligned} F_1^M &< F_y \quad \text{and} \quad M_1^F < M_y, \text{ i. e.,} \\ \frac{F_1^M}{F_y} &= \frac{F_1^M}{F_y} < 1 \quad \text{and} \quad \mu = \frac{M_1^F}{M_y} = \frac{M_1^F}{W_y \sigma_y} < 1 \end{aligned} \quad (2-27)$$

With a gradual increase of the eccentrically applied force P (Fig. 2-15c) the stress diagrams in the section, after having passed the stages shown in Fig. 2-15d and e, will reach the limiting diagram with a plastic hinge that is offset from the axis of the element (Fig. 2-15f). It may be considered here that the moment is taken by an internal pair of forces resulting from stresses in equal areas of the compressed and part of the tensioned zones of the stress diagrams, whereas the

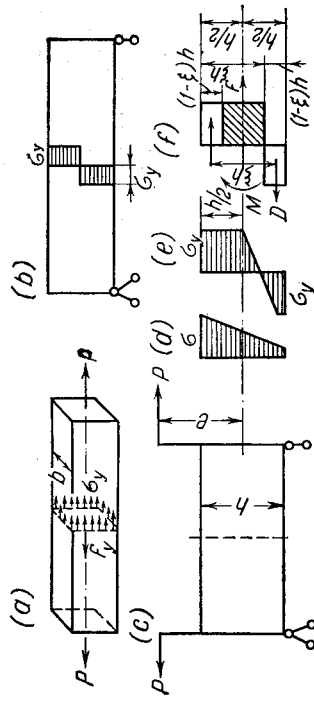


Fig. 2-15. Limiting stress diagrams of eccentrically tensioned element

longitudinal force is taken by the middle symmetrical part of the stress diagram shaded in Fig. 2-15f.

Thus, for a rectangular section (with a thickness b and a depth h) we obtain

$$\begin{aligned} M_1^F &= D\xi h = \sigma_y b (1 - \xi) h \xi h = \sigma_y b h^2 \xi (1 - \xi) \\ F_1^M &= \sigma_y b h (2\xi - 1) \end{aligned}$$

and

$$\begin{aligned} \frac{F_1^M}{A\sigma_y} &= 2\xi - 1; \quad \mu = \frac{M_1^F}{W_y \sigma_y} = \frac{4M_1^F}{b h^2 \sigma_y} = 4\xi (1 - \xi) \end{aligned} \quad (2-28)$$

in which the quantity $\xi < 1$ designates the relative distance of the neutral line from the edge of the section. Upon excluding the quantity ξ from equations (2-28) we shall obtain the sought for equation of the boundary curve in dimensionless quantities, in the form of a parabola (Fig. 2-16)

$$\nu^2 + \mu = 1 \quad \text{or} \quad \left(\frac{F^M}{F_y} \right) + \frac{M^F}{M_y} = 1 \quad (2-29)$$

For I sections the boundary lines (surfaces) are nearer to a straight line (plane); for this reason the Building Standards and Regulations, to ensure a margin of safety, propose to compute the strength of solid-web eccentrically tensioned elements that are not subjected to the direct action of dynamic loads from the formula

$$\left(\frac{F}{A_y R} \right)^{1/2} + \frac{M_x}{W_{x,y} R} \leq 1 \quad (2-30)$$

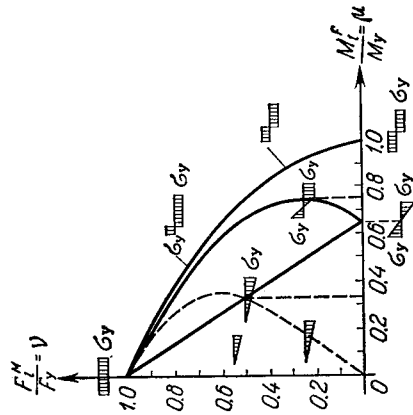


Fig. 2-16. Boundary curves of stressed state areas with simultaneous action of bending moments M and longitudinal forces F

(2) Eccentrically Compressed and Compressed and Bent Elements

Upon the application of a compressive force with a certain eccentricity an element is subjected to eccentric compression. Bending also appears together with this compression and, correspondingly, a deflection (Fig. 2-17a). When a longitudinal axial force and a lateral load that induces bending are applied together, the element will be both compressed and bent (Fig. 1-17b). The behaviour of elements of these two types differ somewhat from each other, mainly in the limiting state at low slenderness ratios. Nevertheless (with an increase of the margin of safety) compressed and bent elements, when considering their critical states, are equalled to eccentrically compressed ones with an eccentricity of $e = \frac{M}{F}$. Eccentric compression, notwithstanding the existence of a deflection from the very beginning of load application, also relates to the problem of stability, and not only to that of strength; moreover, axial compression is in essence a particular case of eccentric compression with a small chance eccentricity. Indeed, the load-carrying capacity of each element (it should not be

forgotten that a loss of stability is always connected with a loss of load-carrying capacity) is characterized by only one value of the strain corresponding to each value of the load applied to the element. In the given instance with a growth of the force P (Fig. 2-17a) the external moment will also increase owing to a greater deflection, viz., $M_e = P(e + \delta)$. As soon as the stress in the extreme fibre of the bent element at one side of the section passes the yield point, then in accordance with the condition of equilibrium in elastoplastic bodies, upon an increase of the moment (and the deflection) the normal force

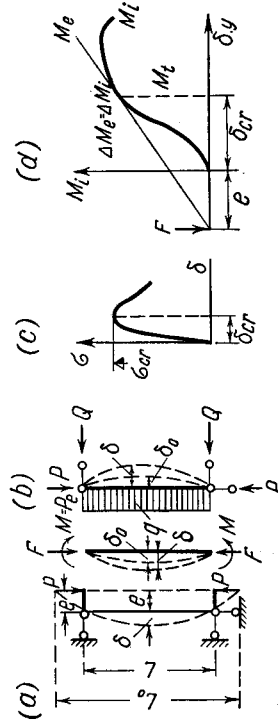


Fig. 2-17. Behaviour of eccentrically compressed element

[see equation (2-29)] should decrease, i.e., the value of the force as a function of the deflection will pass through its maximum and begin to diminish.

The stress-strain curve shown in Fig. 2-17c has a rising line of stable equilibrium until the critical value (critical stress) is reached, after which the strains begin to grow without any increase of the load. Further retaining of equilibrium is possible only upon reduction of the load, and it is this that characterizes the loss of stable equilibrium

$$\frac{d\sigma}{d\delta} = 0$$

Upon the loss of stability, equilibrium is disturbed between the external force factors and the internal forces which develop inside the section of the element, the latter having a certain possible curvature.

Generally with small and medium slenderness ratios of an element, the limiting state of equilibrium is reached in elastoplastic behaviour of the material. The main condition for the critical state, in which the load-carrying capacity of an eccentrically compressed element is exhausted, i.e., limiting equilibrium is reached, will be equality of the increments of the external moment ΔM_e and the internal moment ΔM_i

$$\Delta M_e = \Delta M_i$$

This condition for a critical state makes it possible to determine the critical forces with the aid of the graphoanalytical method, by plotting a curve of M_i against the deflection δ and drawing a straight line $M_e = F(e + \delta)$. The eccentricity e is laid off to the left of the origin of coordinates (Fig. 2-17d). Here is shown a curve of M_i plotted against the strain δ for a certain definite value of σ equal to σ_0 . This curve can be considered as a characteristic of the section. It does not depend on the external load, but is determined only by the shape of the section, its moment of inertia, slenderness ratio and other geometrical characteristics connected with possible curvature of the element or its deflection. The curve of M_i in the region of elastic behaviour is concave (a slow growth of the deflection) and in the plastic region convex, and it has a maximum corresponding to the formation of a plastic hinge.

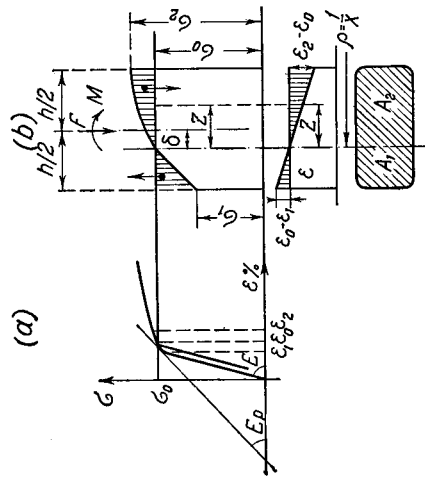


Fig. 2-18. Distribution of stresses upon loss of stability in eccentrically compressed element

during its elastoplastic behaviour. This position depends on the tension diagram of the material and, accordingly, on the value of the modulus of elasticity in compression or tension. It also depends on the shape of the bent centre line and on that of the element cross section.

For each given value of the mean stress $\sigma_0 = \frac{F}{A}$ beyond the proportional limit, the tension diagram can be used to find the modulus of plastic strains E_p by drawing a tangent to point (σ_0, ϵ_0) (Fig. 2-18a). On the basis of the hypothesis of plane sections, the longitudinal strains ϵ_1 and ϵ_2 are directly proportional to the distance z from the neutral line, the curvature χ related by a differential expression to the deflection y being the proportionality coefficient

$$\epsilon = \chi z; \chi = \frac{1}{\rho} = y'' = \frac{P_y}{EI}$$

With definite curvature of the element and shape of its section the stresses σ_1 and σ_2 (Fig. 2-18b) can be found, while from the equality

of the stress diagrams in bending $\int_{A_1} \sigma_1 dA = \int_{A_2} \sigma_2 dA$ the distance to the neutral line z will be determined. Hence the internal moment due to the bending which the curvature of the element corresponds to can be found from the expression

$$M_i = \int_{A_1} \sigma_1 z dA + \int_{A_2} \sigma_2 z dA = E \int_{A_1} \epsilon_1 z dA + \int_{A_2} E_p \epsilon_2 z dA$$

This internal moment characterizes the section. Let us lay off the eccentricity of the force F along the x -axis to the left of the origin of coordinates (Fig. 2-17d), and draw a line tangent to the curve M_i . This straight line will represent the external moment $M_e = F(e + \delta)$. At the meeting point the condition $\Delta M_e = \Delta M_i$ will be complied with and thus the values of M_{cr} and δ_{cr} will be determined that correspond to the respective M_i . Hence, the critical force will be found equal to

$$F_{cr} = \frac{M_{i,cr}}{e + \delta_{cr}} \quad (2-33)$$

or, correspondingly, the critical stress

$$\sigma_{cr} = \frac{F_{cr}}{A} \quad (2-34)$$

It is obvious that the critical stresses of an eccentrically compressed element are less than those of an axially compressed one.

Indeed, by continuing the curve of the elastic line of the element (Fig. 2-17a) up to the line connecting the longitudinal forces, we shall obtain an imaginary axially compressed element with a length and a slenderness ratio greater than those of the eccentrically compressed element and, therefore, $\sigma_{ec,cr} < \sigma_{0cr}$.

A glance at Fig. 2-17d will show that the meeting point of the straight line M_e always lies to the left of the maximum point of the curve M_i which characterizes the plastic hinge. Thus it has been established that in the critical state of an eccentrically compressed pinned-end (statically determinate) element there will always be present an elastic core, which is mainly the part that resists bending. In statically indeterminate systems having elastic constraints the formation of a plastic hinge is possible. Redistribution of the forces or a change in the adopted design scheme of the structure will then take place.

Thus the critical stress of an eccentrically compressed element, while depending on δ_{cr} , does not depend on the external load, as shown by equation (2-33). It can be expressed as a function of the slenderness ratio of the element and the relative eccentricity con-

loss of stability. The values of the coefficient η are given in Table 9 of Appendix II.

$$m = \frac{e}{\rho}$$

in which $e = \frac{M}{F}$ = eccentricity, cm

$\rho = \frac{N}{A}$ = core radius, cm.

The equation for the stress caused by eccentric compression, the same as in eccentric tension [see expression (2-26)] contains the

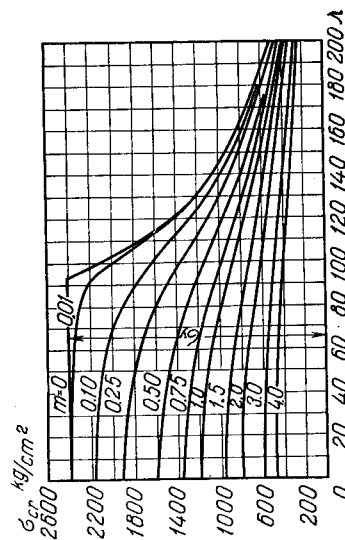


Fig. 2-19. Curves of critical stresses for eccentrically compressed element versus slenderness ratio (for ideal elastoplastic material)

relative eccentricity m (for compression the plus sign has been taken, as usual)

$$\sigma = \frac{F}{A} + \frac{M}{W} = \sigma_0 (1 + m)$$

Figure 2-19 shows theoretical curves of the critical stresses versus the slenderness ratio λ for eccentrically compressed elements having a rectangular shape, which makes possible a general solution in a finite form (Yezhek's solution). The curves are given for various values of the relative eccentricity m . The upper curve in Fig. 2-19 for $m = 0$ corresponds to axial compression and repeats the relevant curve given in Fig. 2-10. With sections other than a rectangular one the influence of the section shape is accounted for in the Building Standards and Regulations by the introduction of an equivalent eccentricity m_1 . The latter is equal to the product of a coefficient η allowing for the shape of the section, and the relative eccentricity m , viz., $m_1 = \eta m$. The coefficient η takes into account the development of plastic strains in the element accompanying the

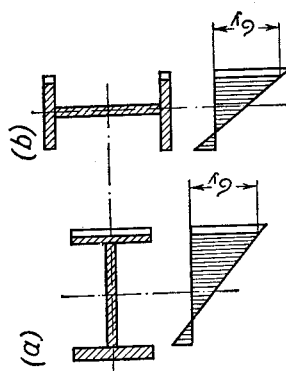


Fig. 2-20. Regions of propagation of plastic strains and weakening of sections in an I element: a—eccentricity in plane of web; b—eccentricity perpendicular to web

$$\varphi_{ec} = \frac{\sigma_{ec,cr}}{\sigma_y} \quad (2-35)$$

whence $\sigma_{ec,cr} = \varphi_{ec} \sigma_y$. Thus the equation for checking the stability of an eccentrically compressed element can be written as follows

$$\sigma = \frac{F}{A} \leq \sigma_{ec,cr} = \varphi_{ec} \sigma_y$$

or in the working form (taking $\sigma = R$)

$$\sigma = \frac{F}{\varphi_{ec} A} \leq R \quad (2-36)$$

where φ_{ec} is a factor that reduces the design strength for the given element to the value of the critical stress for an eccentrically compressed element.

The values of the factor φ_{ec} are given in Tables 7 and 8 of Appendix II against the slenderness ratio of the element λ and the equivalent eccentricity

$$m_1 = \eta m = \eta \frac{e}{\rho} = \eta \frac{A}{W} \quad (2-37)$$

Here $e = \frac{M}{F}$ = eccentricity of force in plane of bending, cm
 M = design moment, which is equal to the maximum moment within the limits of the column length for columns of frame systems with a constant section, to the maximum moment over the length of the part with a constant section for stepped columns, to the moment in the constraint for cantilevers, to the values given in Table 2-6 for elements with pinned ends

F = longitudinal force in section under consideration
 $\rho = \frac{W_{gr}}{A_{gr}}$ = core radius of section, cm

W_{gr} = section modulus for the most compressed fibre.
 Ordinarily when selecting a section the value of ρ is first found from the equation

$$\rho = \frac{W}{A} = \frac{I}{zA} = \frac{r^2}{z} \quad (2-38)$$

where r = radius of gyration of the section (approximate values of r are given in Tables 7-2 and 8-3)

z = distance from neutral axis to farthest compressed fibre.
 In Table 2-6 the moment M_1 is the maximum one within the limits of the middle third of the element length; in all cases M is taken not less than $0.5M_{max}$.

TABLE 2-6 Design Moments M for Elements with Pinned Ends

m	$0 \leq \lambda < 120$	$\lambda \geq 120$
$m \leq 3$	$M_2 = M_{max} - \frac{\lambda}{120} \times (M_{max} - M_1)$	M_1
$3 < m \leq 20$	$M_2 + \frac{m-3}{17} \times (M_{max} - M_2)$	$M_1 + \frac{m-3}{17} \times (M_{max} - M_1)$

With values of m_1 exceeding 4, in addition to the standard formula (2-36), eccentrically compressed elements may be analyzed by means of Yasinsky's binominal expression

$$\sigma = \frac{F}{\varphi A_{gr}} + \frac{M}{W} = \frac{F}{A_{gr}} \left(\frac{1}{\varphi} + \frac{e}{\rho} \right) \leq R \quad (2-39)$$

which does not take complete account of the development of plastic strains and entirely omits the influence of the section shape. The inaccuracy of this expression is insignificant (from five to eight per cent).

With a large equivalent eccentricity ($m_1 \geq 20$) the influence of the longitudinal force is small and no checking of stability is required. The element is checked for strength in bending by equation (2-17).

Besides checking for stability in the plane of action of the moment, it is always required to check an eccentrically compressed element for stability at right angles to the plane of action of the moment (see Chapter 8).

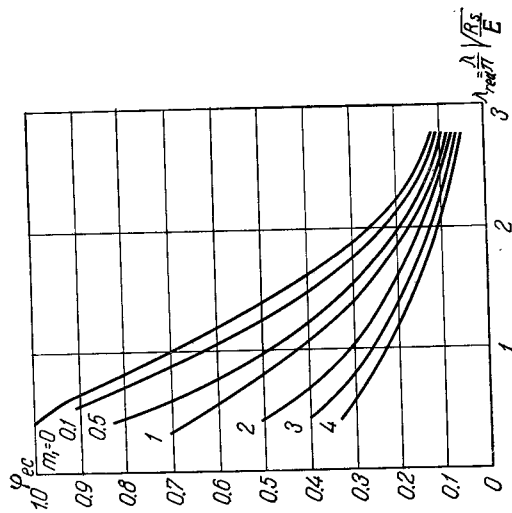


Fig. 2-21. Curves of buckling factors φ_{ec} for aluminium alloy elements

For eccentrically compressed elements made from aluminium alloys the buckling factor φ_{ec} is determined depending upon the equivalent eccentricity $m_1 = \eta e \frac{A}{W}$ and the relative slenderness ratio λ_{rel} , the latter being computed from the equation

$$\lambda_{rel} = \frac{\lambda}{\pi} \sqrt{\frac{R_s}{E}} = f \lambda \quad (2-40)$$

where $\lambda = \frac{L}{r}$ = slenderness ratio in the plane of action of the moment
 $r = \sqrt{\frac{I}{A}}$ = radius of gyration in plane of action of moment
 f = factor taken (with mean value of $E = 710,000$ kg/cm²) from Table 2-7.

The values of φ_{ec} depending upon m_1 and λ_{rel} are given graphically in Fig. 2-21 (for elements with a rectangular section). The coefficient

η is taken according to Table 9 of Appendix II. With relative eccentricities of $m > 10$ the factors φ_{ec} , regardless of the type of element section, are taken from Table 2-8.

TABLE 2-7 Values of Factor f

Grade of alloy	f	Grade of alloy	f
AMn-M	0.0093	AB-T1 and AMr61	0.0174
AMr-M and AЛ-34-T	0.0108	Л1-T	0.0182
AB-T, AЛ31-T1 and AMn-II	0.0137	Л16-T	0.0242
AMr-6 and AMr-II	0.0152	B92 (sheet)	0.017
AЛ33-T1	0.0166	B92 (section)	0.0207

TABLE 2-8 Factors φ_{ec} for All Types of Aluminium Alloy Sections with High Eccentricities

m	λ_{red}						
	0.4	0.8	1.2	1.6	2.0	2.4	3.0
10	0.094	0.087	0.081	0.078	0.068	0.061	0.050
15	0.063	0.060	0.058	0.056	0.051	0.042	0.039

2-7. DESIGN STRENGTHS AND ALLOWABLE STRESSES

(1) Design Strengths for Steel Members

Steel members are analyzed in accordance with the Building Standards and Regulations, as indicated in Chapter 1, first of all for the load-carrying capacity of the member elements, i.e., for strength, stability and in some cases for endurance.

The strength and stability are analyzed for the action of the design loads (with the introduction of load factors). Here the design stresses must not exceed the design strengths of the material.

The values of the design strengths of rolled steel, as well as of carbon steel and grey iron castings (for cases when it is not necessary to allow for the factors of service conditions of the elements) established by the standards are set forth in Tables 2-9, 2-10 and 2-11.

TABLE 2-9 Design Strengths R in kg/cm² for Rolled Steel Shapes

Stress state	Sym- bol	Grade of rolled steel				
		Carbon		Low-alloy		
		Steel 3, Steel 4	Steel 5	14Г2, 15Г1С	10Г2С, 10Г2СН, 15ХСН	10ХСН
				$t \leq 20$ mm	$t = 21$ - 32 mm	
Tension, compression and bending	R	2,100	2,300	2,900	2,800	2,900
Shear	R_{sh}	1,300	1,400	1,700	1,700	1,700
Crushing of end face (with matched surfaces)	R_{cr-e}	3,200	3,400	4,300	4,200	4,300
Local crushing with tight contact	R_{cr-l}	4,600	1,700	2,200	2,100	2,200
Diametrical compression of rollers with free contact (in members with restricted movement)	R_{c-r}	80	90	110	110	130

NOTE. The values of the design strengths indicated in Table 2-9 have been established:
(a) for rolled steel shapes of ordinary quality;

strips, angles, round and square sections with a thickness t up to 100 mm inclusive; I beams and channels with a thickness t up to 20 mm inclusive (in these sections the thickness is measured over the web); steel sheets, plates and wide strips with a thickness t up to 40 mm inclusive;

(b) for rolled low-alloy steel shapes, (strips, angles, round and square sections, I beams, channels, sheets, plates and wide strips) with a thickness from 4 to 32 mm inclusive.

TABLE 2-10 Design Strengths R in kg/cm² for Carbon Steel Castings

Stress state	Sym- bol	Grades of carbon steel castings	
		15Г1	35Г1
Tension, compression and bending	R	1,500	2,100
Shear	R_{sh}	900	1,300
Crushing of end face	R_{cr-e}	2,300	3,200
Local crushing with tight contact	R_{c-r-l}	1,100	1,600
Diametrical compression of rollers with free contact (in members with restricted movement)	R_{c-r}	60	80

TABLE 2-41 Design Strengths R in kg/cm² for Grey Iron Castings

Stress state	Sym- bol	Grades of grey iron castings			
		Cч 12-28, Cч 15-32	Cч 18-36, Cч 21-40	Cч 24-44, Cч 28-48	
Axial compression, with bending, if any	R_c	1,600	1,800		2,100
Tension with bending	R_b	450	600		800
Shear	R_{sh}	350	450		600
Crushing of end face	R_{cr-e}	2,400	2,700		3,200

(2) Taking Account of Variable Load Influence When Analyzing Endurance of Steel Members

Steel members that are directly subjected to repeated moving loads in buildings and structures with heavy conditions of work ¹⁾ (for example, the crane girders of steel foundries) as well as members that are directly subjected to a regular vibrating load, must be analyzed for endurance.

The endurance of members subjected to variable loads (Sec. 2-2, para. 4, c, e, f) depends to a considerable degree upon the shape of the member and the related uneven distribution of the force lines causing a concentration of stresses which facilitates brittle failure.

When investigating the endurance of steel members the design strengths are reduced by introducing a factor γ determined from the expression

$$\gamma = \frac{1}{(ak_e \pm b) - (ak_e \mp b)k_{rs}} \quad (2-41)$$

Here a and b = coefficients taken from Table 2-42

k_e = effective stress concentration factor, which is the ratio of the fatigue limit for a smooth flat specimen to that of a specimen with the given stress concentration (the values of k_e are given in Table 2-43)

¹⁾ Buildings and structures with heavy conditions of work include shops and plants with continuous round-the-clock operation of cranes with very heavy and very heavy continuous duties, making it necessary to repair the cranes and the crane tracks without interrupting production. For a more detailed exposition see Chapter 9.

$k_{rs} = \frac{\sigma_{min}}{\sigma_{max}}$ = range ratio of stress cycle, where σ_{min} and σ_{max} are the minimum and maximum absolute values of the stresses in the element being investigated, measured (each with its own sign) from the design load level, without using the dynamic load factor, nor the buckling factors characterizing the stable state of the elements (φ , φ_{ec} and φ_b) and the factors k_b , contained in Table 6-11.

TABLE 2-42 Values of Coefficients a and b

Material of member	Kind of structural member			
	Crane girders and trusses, as well as beams of working platforms and elements of bin and unloading trestle members		Members subjected to action of total continuously applied alternating design load (for example beams supporting an electric motor)	
	a	b	a	b
Carbon steel	0.75	0.3	0.90	0.3
Low-alloy steel	0.80	0.3	0.95	0.3

The upper signs in the denominator of expression (2-41) are taken for cases when the stress with the highest absolute value is tension, and the lower signs—when this stress is compression.

In all instances the value of the factor γ is not taken greater than unity.

The elements of members are checked for stability and endurance separately. The endurance is investigated for the action of the design loads (without regard to the load factors). Thus the working expressions for checking the endurance will have the following form

$$\sigma = \frac{F_s}{A} \leq \gamma R \quad (2-42)$$

or

$$\sigma = \frac{M_s}{W} \leq \gamma R \quad (2-43)$$

in which F_s and M_s are the force and moment, respectively, determined from the service loads without applying any load factors.

For elements subjected only to alternating compressive loads the factor γ is taken equal to unity.

TABLE 2-13 Values of Effective Stress Concentration Factors k_e for Base Metal

Cross sections being analyzed	Values of factor k_e for	
	Carbon steel	Low-alloy steel
Base metal with rolled, cut off or machined edges	1.0	1.0
Ditto with edges processed by gas cutting:		
(a) machine	1.1	1.2
(b) hand	1.4	1.8
Base metal at unmachined butt weld	1.4	1.8
Ditto, but with machining of weld:		
(a) when joining sheets of identical thickness and width	1.0	1.0
(b) when joining sheets of various width	1.2	1.4
(c) when joining sheets of various thickness	1.3	1.6
Base metal at place of transition to transverse lap fillet weld with a leg ratio of 1:1.5	2.7	3.7
Base metal in lap joint at ends of longitudinal fillet weld	3.4	4.4
Base metal near membranes and webs connected by means of fillet welds to tension flanges of beam and elements of trusses:		
(a) with hand welding	1.6	2.2
(b) with semi-automatic welding	1.3	1.5
Base metal at connections to constructive elements following them:		
(a) rectangular gusset plates welded to elements by means of butt or T joints, without machining	2.5	3.5
(b) gusset plates with smooth curvilinear form and machining of butt or T welded joints to element	1.2	1.4
(c) gusset plates connected to element by means of lap joints with welding along contour, without machining of joints	2.5	3.5
Base metal at cross section:		
(a) of connection with double-shear rivets or bolts	1.6	1.9
(b) ditto, with use of high-strength bolts	1.2	1.4
(c) of connection with single-shear rivets or bolts	2.0	2.4

(3) Allowable Stresses for Steel Members

When employing the method of elastic design (allowable stresses) for analysis, as indicated in Chapter 1, the stresses in a member are determined according to the forces computed for the service loads (without applying the load factors). These stresses must not exceed the allowable ones established for this method by the relevant standards and regulations for the design of steel structural members, and indicated in Table 2-14.

TABLE 2-14 Allowable Stresses for Rolled Steel

Stress state	Rolled steel of grades			
	Cr.0 and Cr.2		Cr.3	Low-alloy (HET)
	Allowable stresses in kg/cm ² for			
	Main loads	Main and additional loads	Main loads	Main and additional loads
Tension, compression and bending	1,400	1,600	1,800	2,250
Shear (in bending)	900	1,000	1,100	1,350
Crushing of end face	2,100	2,400	2,700	3,400
Local crushing with tight contact	1,100	1,300	1,450	1,800
Diametrical compression of rollers with free contact	50	60	70	85
				100

(4) Design Strengths for Aluminium Alloy Members

The design strengths for structural members of aluminium alloys established by USSR building standards are given in Tables 2-15 and 2-16.

TABLE 2-15 Principal Design Strengths R (Tensile, Compressive and Bending) for Alloys not Hardened by Heat Treatment, kg/cm²

Values of R for grades and state of alloys				
AMr-M	AMn-U	AMr-M	AMr-II	AMr6-M
500	1,000	700	1,400	1,800

NOTE. For the designation of the alloys see page 35.