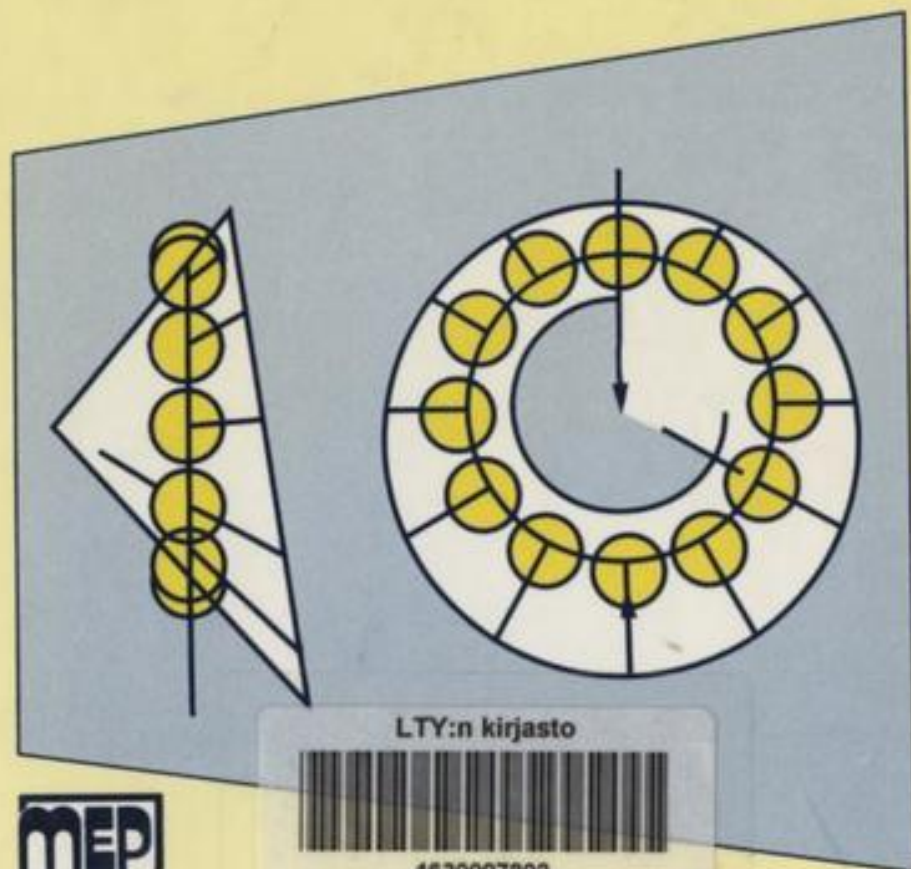


Analysis of Rolling Element Bearings



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Wan Changsen

Translated by Wan Changsen and Zhang Zhaoying

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ANALYSIS OF ROLLING ELEMENT BEARINGS

W. C. Chong

Translated by

W. C. Chong

and

Zhang Zhong

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TEKNILLISEN YLIOPISTON
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Preface

Rolling bearings are widely used as important components to support the relative motion between two machine parts. To be sure, if there were no bearings all commercial transport vehicles, such as cars, buses and airplanes could not work at all. On the other hand, the development of automotive, aeronautic and astronautic industries in this century has given significant motivations to the advancement in design and manufacturing technology of rolling bearings. One problem of special interest in these application areas is how to design and select rolling bearings based on different operating conditions to reach their excellent performance, long life and high reliability.

The book is organized to give a clear and logical sequence of topics regarding the analytical methods of rolling bearings. It progresses systematically from the description of geometry, kinematics, load distribution, stress and deformation in rolling bearings to the calculation of static load rating, dynamic load rating and life of rolling bearings. These are addressed in Chapter Two to Chapter Fourteen except for Chapter Six.

I wrote this book because I also believe that engineers need to be exposed to some new topics that have come to take an important role in rolling bearing analysis over the past two decades, such as elastohydrodynamic lubrication, optimization of design and computer aided design in rolling bearings. To respond to these needs my book provides more detailed description in Chapters Six, Fifteen and Seventeen.

Some of my experiences during my long period of working at the Luoyang Research Institute of Bearings in China are included in this book for discussion and exchange. Since the development of calculation and analysis of rolling bearings is the result of the work of many people from a wide variety of disciplines, this book would not have been possible without the many contributions that have appeared in the open literature. I am therefore very grateful to the scientists and engineers who have shared their results through publication and technical journals.

The original book is written primarily for Chinese readers with the aim to give them more systematic methods for analyzing rolling bearings. Upon recent research results some supplements and revisions

have been completed in this English edition. Now, it gives me great pleasure to publish it in English, particularly to translate it by myself. I truly believe that this gives me a good chance to discuss with more people. Of course whether or how far I have succeeded in this book in living up to my intention above awaits judgement from my readers.

I wish to express my gratitude here to all my friends who have given me their valuable suggestions during the time of writing this book. I am especially grateful to Mr. Charles A. Stanton of the American Telephone and Telegraph Company who was kind enough to provide beneficial comments for this book. Finally I want to thank the people at the press who have greatly helped in the publication of this book.

Wan Changsen, March 1990

Notation

Symbols which only appear locally, and are defined on the spot, are not listed.

A	Distance between curvature centres of inner and outer raceway grooves
a	Semi-major axis of contact ellipse
a_1	Life adjustment factor for reliability
a_2	Life adjustment factor for material
a_3	Life adjustment factor for operating conditions
b	Semi-minor axis of contact ellipse
C	Dynamic load rating
C_0	Static load rating
C_E	Modified coefficient for materials
C_h	Modified coefficient for thermal effects
C_T	Dynamic load rating considering temperature effects
D	Outside diameter of bearing
D_b	Diameter of rolling element
D_e	Raceway diameter of outer ring
D_i	Raceway diameter of inner ring
D_m	Pitch diameter of bearing
d	Bore diameter of bearing
E	Modulus of elasticity
E_0	Equivalent modulus of elasticity
e	Eccentricity of contact ellipse; slope parameter of Weibull distribution function
F	Applied load; probability of failure
F_a	Applied axial load
F_r	Applied radial load
F_{ax}	Axial preload
f	Curvature radius coefficient of raceway groove
f_e	Curvature radius coefficient of outer raceway groove
f_h	Life factor
f_i	Curvature radius coefficient of inner raceway groove
f_n	Speed factor
f_T	Temperature factor
f_V	Permissible wear factor

G	Shear modulus of elasticity
H	Film thickness parameter
H_0	Central film thickness parameter
H_{min}	Minimum film thickness parameter
h	Film thickness
h_0	Central film thickness
h_{min}	Minimum film thickness
i	Number of rows
J	Mass moment of inertia
J_a	Axial load integral
J_m	Moment load integral
J_r	Radial load integral
K	Load-deflection factor
K_l	Load-deflection factor of line contact
K_p	Load-deflection factor of point contact
k	Ratio b/a
L	Rating life of bearing
L_h	Rating life, hours
L_m	Average life
l_e	Effective roller length
M	Total friction moment of bearing
M_R	Rolling friction torque due to elastic hysteresis
M_s	Friction torque due to spinning slip
m_a	Semi-major axis coefficient of contact ellipse
m_b	Semi-minor axis coefficient of contact ellipse
N	Number of stress cycles, revolutions $\times 10^6$
n	Rotational speed; total number of testing bearings
n_e	Rotational speed of outer ring
n_i	Rotation speed of inner ring
n_m	Rotational speed of cage or orbital rotational speed of rolling elements
P	Equivalent dynamic load
P_0	Equivalent static load
P_m	Average equivalent dynamic load
p	Life exponent
Q	Rolling element load
Q_0	Load of maximum loaded rolling element
Q_ψ	Rolling element load at angular location ψ
Q_c	Basic dynamic load of rolling element

q	Line density of load distribution
R	Curvature radius
R_e	Equivalent curvature radius in rolling direction
R_y	Equivalent curvature radius in the direction transverse to rolling
R_o	Raceway radius of outer ring
R_b	Curvature radius of spherical roller at longitudinal section
R_i	Raceway radius of inner ring
r	Curvature radius of raceway groove
r_e	Curvature radius of outer raceway groove
r_i	Curvature radius of inner raceway groove
S	Probability of survival
T	Temperature
t	Grease life
U	Velocity parameter
u	Average velocity
u_a	Free endplay
u_r	Diameter clearance
V	Permissible clearance enlargement
v	Velocity at contact point
v_e	Velocity at contact point of outer raceway
v_i	Velocity at contact point of inner raceway
v_m	Velocity at pitch diameter
W	Load parameter in line contact
W'	Load parameter in point contact
X	Radial load factor
X_0	Static radial factor
Y	Axial load factor
Y_0	Static axial factor
Z	Number of rolling elements per row
Z_0	Depth of maximum orthogonal shear stress
α	Contact angle
α_e	Contact angle of outer raceway
α_i	Contact angle of inner raceway
α_1	Pressure coefficient of viscosity
β	Half taper of roller; characteristic parameter of Weibull distribution function
γ	$D_0 \cos \alpha / D_m$

CHAPTER ONE

δ	Relative approach of contact bodies
δ_e	Relative contact approach at outer raceway
δ_i	Relative contact approach at inner raceway
δ_a	Relative axial displacement between inner and outer rings
δ_r	Relative radial displacement of inner and outer rings
δ_p	Permanent deformation
ε	Load distribution factor
η	Reduction factor; dynamic viscosity
η_0	Dynamic viscosity at atmospheric pressure
θ	Misalignment angle (or limiting tilting angle)
λ	Reduction factor
Λ	Film thickness parameter
ν	Poisson's ratio; reduction factor
τ_0	Maximum orthogonal shear stress
ρ	Curvature
$\Sigma \rho$	Curvature sum
$F(\rho)$	Curvature function
ϕ	Osculation
ψ	Angular position of rolling element
ψ_L	Angular extent of load zone
ω	Angular speed
ω_e	Angular speed of outer ring
ω_i	Angular speed of inner ring
ω_b	Rotational angular speed of a rolling element around its own axis
ω_m	Orbital angular speed of a rolling element

Subscripts

e	Outer ring
i	Inner ring
b	Rolling element
m	Cage
a	Axial
r	Radial
n	Vertical

CHAPTER ONE

Types of rolling bearings

Rolling bearings are widely used to support motion including rotation, oscillation and linear motion of a body relative to another. In comparison with sliding bearings, rolling bearings are primarily based upon rolling motion of balls or rollers. Generally, rolling bearings are composed of two rings, a group of rolling elements and a cage. In some special cases, they may be without one ring or both rings.

Bearing rings and rolling elements are mostly produced from the through-hardened chromium steel. To meet different requirements of bearing life and reliability, several metallurgical methods, such as electric steel, vacuum degassed steel, electric-slag remelted steel and vacuum remelted steel etc, can be chosen. The hardness of bearing rings and rolling elements is usually in the range HR_C61 to 65. Rolling bearings running in extreme ambient conditions, such as corrosion or high temperature, are produced from stainless steels or high alloy tool steels. Anti-magnetic bearings are produced from the beryllium bronze. Materials for cages should possess properties of high strength, low gravity, excellent self-lubrication and wear-resistance and so on. Low carbon steels, austenitic stainless steels, brass, aluminium alloy, phenolic resin, nylon, PTFE, etc, are commonly used.

By comparison with sliding bearings, the advantages of rolling bearings are:

- (1) much lower friction moment, therefore less friction power loss;
- (2) valid over a wide range of load, speed and operating temperature and less sensitive to variations of their operating conditions;
- (3) easy to be lubricated and maintained;
- (4) capable to support combined radial, axial and moment loads;
- (5) less axial length;
- (6) internationally standardized for their principal dimensions, therefore excellently exchangeable and convenient to be replaced.

On the other hand, rolling bearings also have some disadvantages;

- (1) larger radial sizes for most types of bearings;
- (2) short service life if at high speed or under heavy load;
- (3) greater vibration and noises.

In the last twenty years, much great progress has been made in bear-

ing design and manufacturing technology. The above disadvantages are relatively improved. For example, in many cases, needle roller bearings with cage can be applied instead of sliding bearings due to their less radial sizes. To meet the requirements for electric motors, low noise and vibration rolling bearings have been developed.

As early as two thousand years ago, human beings found that less effort was required to move massive stones on rollers than to slide them over the same ground. This was an early rough sense of rolling motion. In the early part of the 20th century, the first motivation for advancement in the manufacturing technology of rolling bearings came from requirements of the automobile industry. Due to the continuous development of aircraft, aerospace, nuclear energy and computer industries after the Second World War, further advanced requirements for performances and service life of rolling bearings have been put forward. Quite a number of institutes or labs in which the technology of rolling bearings is arranged as a primary research project has been established in many countries. Structural design, calculating theory, testing methods and manufacturing technology regarding rolling bearings have been remarkably progressed. Contemporary design of rolling bearings is based on the computing analysis of their system performances. The computer aided design, particularly persona-computer dialogue type, is successfully applied to predict performances of bearings and their system. Relevant analyzing methods will be described in Chapters Fifteen to Seventeen of this book. Although the outlines of rolling bearings seem to be constant, significant improvements in performance, fatigue life and reliability of rolling bearings have been achieved during the last three decades.

A number of types of rolling bearings may be classified in several ways. According to the direction of load which is primarily capable of being carried by the bearings, there are the following four types.

- (1) Radial bearings which are primarily used to carry radial loads, that is, the loads in the direction perpendicular to corresponding bearing axes. Many radial bearings are able to carry axial loads simultaneously upon their internal structures. Radial bearings are most widely used in applications of bearings.
- (2) Radial-thrust bearings which are capable of carrying combined radial and axial loads. The contact angle of this type of bearings is less than 45° .
- (3) Thrust-radial bearings which are primarily used to carry axial

loads, and are also capable of carrying certain radial loads at the same time. The contact angle of this type of bearing is greater than 45° .

- (4) Thrust bearings which are solely used to carry axial loads. On the other hand, according to the rolling elements used in bearings there are the following two types.
 - (1) Ball bearings in which rolling elements are balls mostly made of steel. In order to reduce centrifugal forces of balls at high speed, hollow balls welded from half balls are developed in some labs. But the practical application will not be seen in the near future.
 - (2) Roller bearings in which rolling elements are cylindrical, tapered or needle rollers. In order to improve stress distribution and lubrication in the bearings, hollow cylindrical and hollow tapered rollers have been used in machine tools. But these are only limited to a few types of bearings.

Comparisons between ball and roller bearings are listed in Table 1.1. Generally, roller bearings may carry heavier loads than those of ball bearings with the same external dimensions. Therefore, the roller bearings will generally achieve longer lives in the same operating conditions.

Several common types of rolling bearings and their characteristics are described and more attention is paid to the introduction of various new prospective structures.

Table 1.1 Comparisons between ball and roller bearings

	Ball bearings	Roller bearings
Load capacity	Light	Heavy
Speed limit	Higher	High
Friction coefficient	Lower	Low
Shockproof property	Not good	Good
Life	Short	Long

BALL BEARINGS

Deep groove ball bearings

Fig.1.1 shows a typical deep groove ball bearing. This type of bearing is used in a considerable variety of applications and in mostly mass-produced. Their major characteristics are

- (1) The bearings are primarily to carry radial loads, and also capable of carrying certain axial loads. As the bearings are subjected to pure radial loads their contact angles equal zero. Axial load capacities of the bearings are affected by their diametral clearances. Increased diametral clearances enable heavier axial loads to be carried.
- (2) The bearings have a low friction coefficient and valid for running at high speed, particularly under axial loads.
- (3) Internal structures of the bearings are very simple. By comparison with other types of rolling bearings, it is easy for them to be mass-produced at high grades of accuracy.
- (4) Deep groove ball bearings of mass-production are commonly fitted with pressed steel cages. For particular applications, such as bore diameter $d > 200$ mm or at high speed, machined brass cages or reinforced polyamide cages are used.

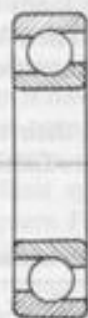


Fig. 1.1 Deep groove ball bearing

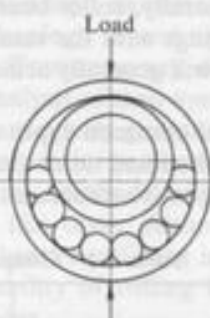


Fig. 1.2 Method of assembly

- (5) Assembling method of this type of bearings is shown by Fig. 1.2. Firstly, the inner ring is moved to an eccentric position relative to the outer ring. A certain amount of balls are filled and the size and number of balls are limited by the assembling angle. Then the inner ring can be slightly pressed and shifted to the concentric position with the outer ring. The balls are allocated uniformly and a cage is inserted.
- (6) International Standard ISO 15 specifies various diameter series of radial rolling bearings. The relative dimensional relation of differ-

ent diameter series of the bearings with the same bore size is shown in Fig. 1.3.

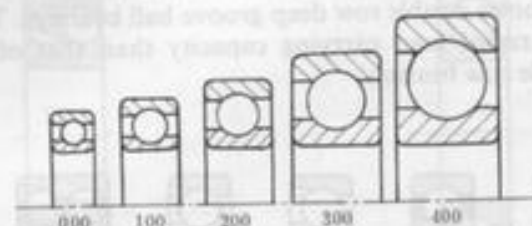


Fig. 1.3 Relative dimensions of different diameter series

To meet different requirements of applications, deep groove ball bearings can be produced in a wide variety of structures, as shown in Fig. 1.4. They are

Deep groove ball bearings with shields. Fig. 1.4(a) shows this variety of bearings. The shields form sealing gaps with the lands of the inner ring. Their function is to prevent grease from leaking out and to protect the bearings from ingress of dirt or particles. A certain amount of grease is filled into each of the bearings. Therefore, it is convenient to maintain and commonly unnecessary to relubricate during their entire service life. Depending on requirements, the bearings with one or two shields can be supplied.

Deep groove ball bearings with seals. Fig. 1.4(b) shows this variety of bearings. The seals are made of oil-resistant synthetic rubber with a sheet steel reinforcement. Their lips are in touch with the lands of the inner ring. This variety of bearing possesses excellent sealing performance. Because of the high frictional loss their limiting speeds are usually lower than those of other varieties of bearings. Similar to the bearings with shields, these bearings are filled with grease in advance. No relubrication is required during their entire service life. Depending on requirements, the bearings with one or two seals can be supplied.

Fig. 1.4(c) shows deep groove ball bearings with filling slots. A filling slot through which more balls can be inserted into the bearing is designed on the shoulders of both outer and inner rings. Then the radial load carrying capacity is higher than that of bearings without a filling slot. However, its axial load carrying capacity is reduced. This variety of bearings can be provided without a cage. But due to the sliding fric-

tion between adjacent balls in this case, they are not valid for applications at high speed. To cover this shortage, one variety of the bearings with filling slots and cage is developed, as shown in Fig. 1.4(d).

Fig. 1.4(e) shows double row deep groove ball bearings. They have a much higher radial load carrying capacity than that of the corresponding single row bearing.

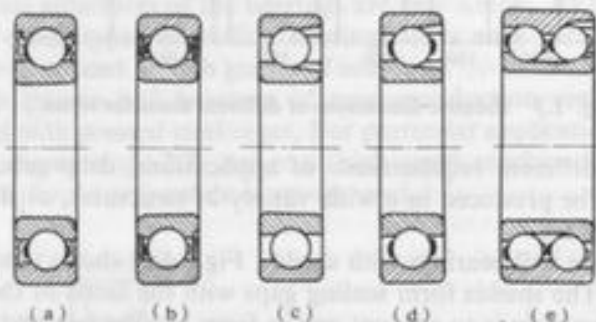


Fig. 1.4 Different varieties of deep groove ball bearings. (a) with shields, (b) with seals, (c) with filling slots, (d) with filling slots and cage, (e) double row bearing

Deep groove ball bearings find their typical applications, such as gearboxes, electric motors, instruments and meters, household electric appliances, internal combustion engines, agricultural machinery, etc.

Self-aligning ball bearings

Fig. 1.5 shows a typical self-aligning ball bearing. The characteristics of this type of bearings are

- (1) The outer raceway of a self-aligning ball bearing is a part of a sphere, whose curvature centre is on the axis of the bearing. Therefore, this type of bearing possesses a self-aligning characteristic, allowing the misalignment of shaft and housing from manufacturing and mounting deviation, of shaft bending deflection etc. The maximum permissible relative misalignment should not exceed 3° .
- (2) Less axial load carrying capacity is due to their smaller contact angles.
- (3) A self-aligning ball bearing with a tapered sleeve and locknut adapter may be simply mounted on a shaft of uniform diameter

without the requirement of a locating shoulder.



Fig. 1.5 Self-aligning ball bearing



Fig. 1.6 Angular contact ball bearing

This type of bearing is primarily used in place of either a power shaft which is easy to bend or greater mounting deviation, such as textile machinery, steel-framed conveyors, agricultural machinery, etc.

Angular contact ball bearings

Fig. 1.6 shows a typical angular contact ball bearing. This type of bearing is able to carry combined radial and axial loads or pure axial loads. Their major characteristics are

- (1) The nominal contact angles of the bearings are 15° , 25° and 35° . In the cases of high accuracy or at high speed, it is necessary to select the bearings with contact angle 15° . The greater contact angles are used for applications of heavy axial loads.
- (2) The assembling method of the bearings is different from that of the deep groove ball bearings. An outer ring expanded by heating is axially assembled with a package of an inner ring, cage and balls. Generally, more balls can be inserted than those of a deep groove ball bearing with the same external dimensions. Therefore, the dynamic load rating is greater.
- (3) In this type of bearing, the loads are transmitted from one raceway to another at a certain contact angle. They are particularly suitable for carrying combined radial and axial loads. Two angular contact bearings must be mounted opposed to each other with the axial clearance eliminated when a pure radial load acts.

- (4) To increase the stiffness of the entire bearing system, the bearings may be mounted in pairs, such as face-to-face and back-to-back arrangements shown in Fig.1.7.

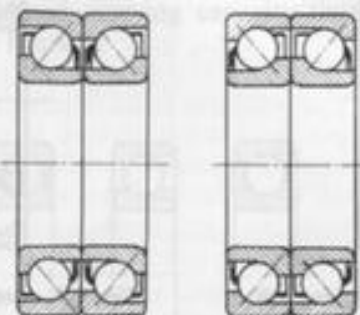


Fig. 1.7 Duplex pairs of bearings

This type of bearing is primarily used in applications of high speed, or high accuracy conditions, as well as the cases of combined radial and axial loads such as grinding spindles, headstock spindles, electric motors at high frequency, centrifuges, gyroscopes, etc.

Angular contact ball bearings—magneto type

This type of bearing is one of the variety of angular contact ball bearings, as shown in Fig.1.8. Their major characteristics are

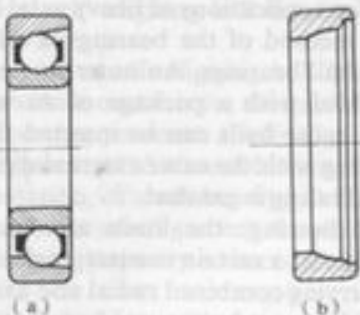


Fig. 1.8 Angular contact ball bearings—magneto type

- (1) The outer ring of the bearing has only one shoulder and therefore it can be separated from its corresponding inner ring. This type of bearing is suitable for the application which requires the installation of bearings at limiting places.
- (2) The axial load carrying capacity of the bearing is less than that of other angular contact ball bearings due to the lower height of bearing shoulders.
- (3) Only the bearings having less than 50 mm outside diameter are usually produced.

This type of bearing is mainly used in the applications, such as small electric generators, instruments and meters, household electrical appliances, etc.

Angular contact ball bearings—split inner ring type

The inner ring of this type of bearing consists of two separable inner rings, as shown in Fig.1.9. Their primary characteristics are

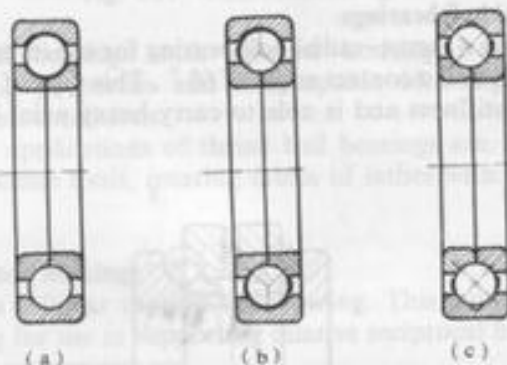


Fig. 1.9 Angular contact ball bearings—split inner ring type. (a) split inner ring type, (b) three-point contact type, (c) four-point contact type

- (1) Each ball in the bearing contacts with inner raceways at two points under pure radial load. Upon the internal structure of the bearings, a ball may contact with the outer raceway either at one point, or at two points. Correspondingly, they are called angular contact ball bearings with three-point contact type or four-point contact type. The axial clearance of the bearing with four-point

contact type is much less than that of the bearing with three-point contact type for the same external dimensions and diametral clearance.

- (2) These bearings are able to carry axial loads in both directions and to carry a certain amount of radial loads.
- (3) This type of bearing usually has a machined solid cage from brass or alloy steel. Sometimes, the surface of a cage is silver-plated in order to improve its self-lubricating performance. The bearings are separable, i.e. the inner rings and the outer ring with ball set can be mounted separately.
- (4) To simplify the mounting and locating structure, a new type of these bearings with flanged outer ring has been successfully developed.

This type of bearing is primarily for use as on aircraft gas turbine bearing. Sufficient lubricating and cooling are of extreme importance due to their strict operating conditions including heavy loads, high speeds and high temperature environment.

Thrust-radial ball bearings

Fig.1.10 shows a thrust-radial ball bearing for use in machine tools. It has more balls and a contact angle of 60° . This type of bearing has excellent axial stiffness and is able to carry heavy axial loads and light radial loads.

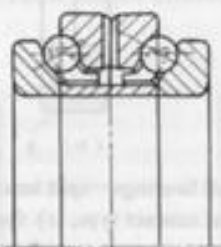


Fig. 1.10 Thrust-radial ball bearing

Thrust-radial ball bearings are primarily used, for lathe, boring and drilling spindles and are often combined with double-row cylindrical roller bearings for use.

Thrust ball bearings

Thrust ball bearings are separable, as shown in Fig.1.10. Their major characteristics are

- (1) The contact angle of the bearings is 90° , and therefore they are solely able to carry axial loads. Single direction thrust ball bearings are suitable for carrying axial loads in one direction. Double direction thrust ball bearings are able to carry axial loads in both directions.
- (2) Balls of the bearing press to the outward side of raceways. Therefore thrust ball bearings are not suitable for running at high speed.



Fig. 1.11 Thrust ball bearing

- (3) Thrust ball bearings with sphered housing washers possess self-aligning properties and are capable of compensating for misalignment conditions.

The typical applications of thrust ball bearings are, for example, spindles in machine tools, rotating tables of lathes with vertical spindles, etc.

Linear motion ball bearings

Fig.1.12 shows a linear motion ball bearing. This belongs to a new type of bearing for use in supporting relative reciprocal linear motion. Their major characteristics are

- (1) A linear motion ball bearing consists of an outer ring, cage and several rows of balls between the outer ring and shaft. The outer ring is in linear motion relative to the shaft. Balls roll over the raceways reciprocally and recirculate in passageways of the cage.
- (2) The frictional coefficients of the bearings generally are in the range 0.001 to 0.003, which is much less than those of sliding bushes.
- (3) The bearings with seals at both ends are filled with grease in advance, so that they are lubricated for their entire service life and are maintenance-free. No special lubricating system is required.

- (4) Low diametral clearance leads to high accuracy of linear motion. The diametral clearance of a variety of the bearings can be adjusted to reach zero or even negative clearance in order to satisfy the requirement of extremely high accuracy.

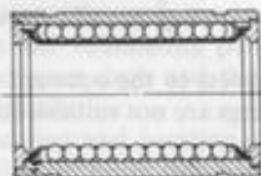


Fig. 1.12 Linear motion ball bearing

Linear motion ball bearings find their typical applications, such as reciprocal mechanisms in machine tools with numerical control, automatically recording devices, follow-up devices, as well as guiding columns in pressing dies.

Miniature ball bearings

Generally, ball bearings having less than 10 mm bore diameter or 28 mm outside diameter are termed miniature ball bearings. To meet the different requirements for instruments and meters, various structural varieties shown in Fig. 1.13 are developed. Their major characteristics are

- (1) Miniature ball bearings with shields for use in synchro motors and servo motors are required to meet supplementary requirements of friction moments, such as easy to start, their running stability, and so on.
- (2) To satisfy the requirement of high accuracy and speed for gyroscopic motors, angular contact ball bearings with or without flanged outer ring at the high grade are mostly selected. The bearings with a flanged outer ring simplify mounting structures and reduce errors. The outer ring of a miniature ball bearing can be designed as an end-cover of a gyroscopic motor and then it is called an end-cover bearing. Various typical varieties of miniature ball bearings for use in gyroscopes are shown in Fig. 1.13(a) to (c).
- (3) Bearings for use in gyroscopic frames are required to possess

extra-low friction moment in order to reduce the drift ratio of the navigation. Besides general radial ball bearings, several special structures of bearings are developed. Fig. 1.13(d) shows a radial ball bearing with straight outer raceway. It has a much lower friction moment but cannot be axially located itself. Fig. 1.13(e) shows a three-ring bearing. Its middle-ring is continuously rotated by a gear-driven device. Therefore the navigating accuracy is remarkably improved due to the lower running friction moment instead of the starting friction moment.

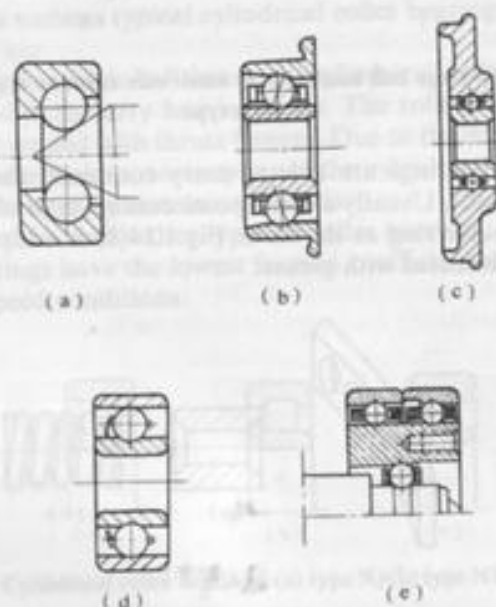


Fig. 1.13 Miniature ball bearings for gyroscopes. (a) angular contact ball bearing, (b) with flanged outer ring, (c) end-cover bearing, (d) bearing with straight outer raceway, (e) three-ring bearing

Extra-large ball bearings

Bearings having more than 440 mm outside diameter are called extra-large bearings. Usually, the rings of the bearings whose outside diameters exceed two meters are made in piece-combined structures. Typical extra-large ball bearings are shown in Fig. 1.14.

- (1) Bearings with steel-wire raceways are shown in Fig. 1.14(a). The inner and outer rings are made of common carbon steel with steel wire inserts. These bearings possess advantages, such as low cost and good agility, and therefore they are commonly used.

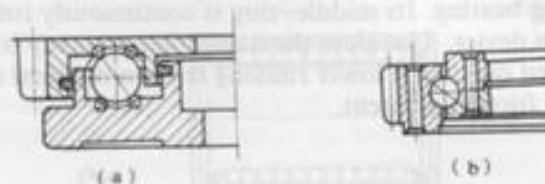


Fig. 1.14 Extra-large ball bearings. (a) steel-wire raceway type, (b) four-point contact type

- (2) Turntable bearings are able to carry combined radial, axial and moment loads. Usually a four-point contact type of bearings with a geared outer ring as shown in Fig. 1.14(b) is adopted. The bearings are lubricated with grease.

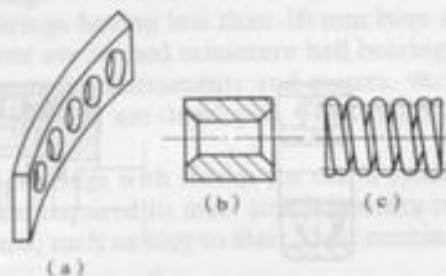


Fig. 1.15 Cages for extra-large bearing. (a) piecewise cage, (b) separating washer, (c) spring

- (3) Various special cages for use in extra-large ball bearings are developed besides conventional cages. Fig. 1.15(a) shows a piecewise cage which is usually made of brass. Fig. 1.15(b) shows a separating washer made of brass or nylon. And a spring which can also be used as a separating element is shown in Fig. 1.15(c). However, these types of cages are only valid for the applications at low speed

or oscillating motion.

- (4) The design and selection of extra-large ball bearings are primarily based on the calculation of static load rating. Fatigue damage is not the major failure mode for these bearings.

Extra-large ball bearings find their applications in mine machinery, constructional machines, heavy machine tools, moveable launching, etc.

ROLLER BEARINGS

Cylindrical roller bearings

Fig. 1.16 shows various typical cylindrical roller bearings. Their major characteristics are

- (1) By comparison with ball bearings, cylindrical roller bearings are generally able to carry heavier loads. The rollers of the bearings are usually guided with thrust flanges. Due to the different configurations of the flanges, some types of the cylindrical roller bearings may be able to carry certain axial loads.
- (2) By comparison with other types of roller bearings, the cylindrical roller bearings have the lowest friction coefficient and are suitable for high speed conditions.

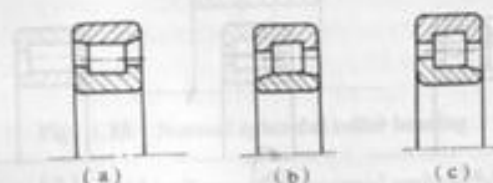


Fig. 1.16 Cylindrical roller bearings. (a) type N, (b) type NU, (c) type NJ

- (3) Upon the configurations of the flanges, various basic types of cylindrical roller bearings are shown in Fig. 1.16(a) to (c). Type NU has two flanges on the outer ring and no flange on the inner ring. Type N has two flanges on the inner ring and no flange on the outer ring. Both are applied as floating bearings, that is, variation in the axial direction due to thermal expansion can be accommodated. Type NJ has two flanges on the outer ring and one on the inner ring, and therefore it can carry a certain axial load in one direction.

- (4) The permissible relative angular misalignments of cylindrical roller bearings are strictly limited to a few minutes of arc. Therefore high manufacturing accuracy of shaft and housing is required.
- (5) In order to increase the radial load carrying capacity of the bearings, several new types of cylindrical roller bearings are developed. Fig.1.17(a) shows an enhanced type of cylindrical roller bearing. They have more rollers, larger diameters and lengths of rollers with the same external dimensions as normal bearings. The edge-stress concentration can be much improved if a modified profile of rollers is adopted. The crowning values of rollers may be decided upon the loads acting on bearings. Standard crowning values of rollers for cylindrical roller bearings are introduced. For the convenience of comparison, the original type of cylindrical roller bearing is shown in Fig.1.17(b). The enhanced type is designated by a suffix "E". Fig.1.17(c) shows a full complement type of cylindrical roller bearings without cage. They are assembled with the greatest possible number of rollers, so that they are able to carry heavier radial loads. The major disadvantage of the full complement type is their lower limiting speeds because of the sliding friction between rollers. Therefore, they are usually valid for the applications under heavy loads and at low speeds.

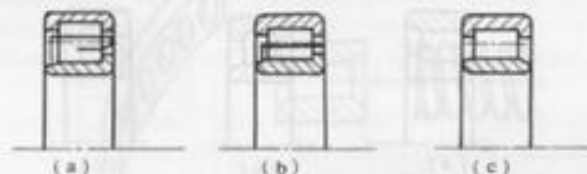


Fig. 1.17 Improvement of cylindrical roller bearings. (a) enhanced type, (b) normal type, (c) full complement type

- (6) Double row cylindrical roller bearings possess higher radial stiffness and are primarily applied to spindles in lathes and boring machines. For the adjustment of the diametral clearance, the bearings with tapered bore 1 : 12 are used. The operating clearance of a bearing may be changed through the axial displacement of its inner ring on the shaft.

Cylindrical roller bearings find their primary applications in medium or large electric motors, axleboxes of rail vehicles, spindles of ma-

chine tools, rolling mills, vibrating screens, etc.

Spherical roller bearings

Fig.1.18 shows a normal spherical roller bearing. The major characteristics are

- (1) The outer raceway of a spherical roller bearing is a part of a sphere, whose curvature centre is on the axis of the bearing. Similar to self-aligning ball bearings, therefore, this type of bearing possesses self-aligning performance, allowing the misalignment of shaft and housing.
- (2) These bearings are able to carry heavy radial loads and shock loads, as well as certain axial loads due to the line contact between rollers and raceways.
- (3) These bearings are not suitable for applications at high speeds, because the high osculation between rollers and raceways, as well as curve generatrices leads to large sliding coefficients.

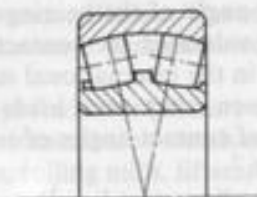


Fig. 1.18 Normal spherical roller bearing

- (4) The non-symmetrical rollers of a normal spherical roller bearing are pushed toward the integral middle guide during its operation. Recently, a new variety of spherical roller bearings, which have a large number of long, symmetrical rollers of large diameter, as well as a non-integral guide positioned between the two rows of rollers have been developed. The non-integral guide can adjust to a certain position automatically upon the ratio between radial and axial loads. Obviously, the new variety of bearings has a very high radial load carrying capacity.
- (5) A spherical roller bearing with an adapter sleeve may be simply mounted on a shaft of uniform diameter without the need for a locating shoulder.

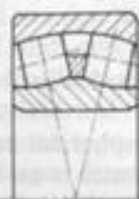


Fig. 1.19 Enhanced spherical roller bearing

Spherical roller bearings are primarily used in the applications, such as rolling mills, paper-making machines, construction machinery, crushers, printing machines, vibrating screens, etc.

Tapered roller bearings

Tapered roller bearings are shown in Fig.1.20. They are able to carry either combined radial and axial loads or pure axial loads. Their major characteristics are

- (1) The nominal contact angle of the bearings is in the range 10° to 30° . Generally, the values of the contact angle are divided into six series as specified in the international standard. Larger contact angles should be chosen when axial loads acting on bearings are heavy. The majority of contact angles of tapered roller bearings is in the range 10° to 16° .
- (2) The extensions of the all contact lines between the rollers and the inner or outer raceway should theoretically intersect at one common point on the bearing axis.
- (3) The rollers in a loaded bearing are pressed onto the big guide of the inner ring by the components of internal contact loads due to different tapered cone angles of the inner and outer raceways. The sliding friction between roller ends and the big guide sets a limit to limiting speeds of the bearings. Recently, high speed tapered roller bearings have been successfully developed through improvements of the guide lubrication.
- (4) Similar to angular contact ball bearings, two tapered roller bearings are generally mounted opposed to each other. The preload of tapered roller bearings is only obtained by the adjustment of axial location when the bearings are mounted. Double or four-row tapered roller bearings are used for extremely heavy radial loads.



Fig. 1.20 Tapered roller bearing

- (5) An enhanced type of tapered roller bearings having more rollers, larger diameters and lengths of rollers with the same external dimensions as normal bearings has been developed to provide high load carry capacity and stiffness. The modified line contact between rollers and raceways eliminates edge stresses and extends the bearing life.

Tapered roller bearings find their primary applications in front and rear wheels of trucks, transmission mechanisms in machine tools, axleboxes of rail vehicles, rolling mills, lifters, etc.

Needle roller bearings

Generally, the bearings with rolling elements being in the following ranges, roller diameter $D_b < 5$ mm and l_b / D_b ratio > 3 , are called needle roller bearings. Fig.1.21 shows various types of needle roller bearings. Their major characteristics are

- (1) By comparison with other types of bearings needle roller bearings have the smallest cross-sections. In many cases, they can be used instead of sliding bearings.
- (2) Needle roller bearings can be provided without inner ring or outer ring, as well as even without either ring, that is, needle roller and cage assemblies. Then, the corresponding mounting shafts and housings should reach the same manufacturing accuracy and hardness as normal bearing rings.
- (3) Full complement needle roller bearings are able to carry heavier

radial loads, but they are not valid for applications at high speeds due to the roller skewing. On the other hand, needle roller bearings with cage can be used at high speeds, but their radial load carrying capacity is reduced slightly.

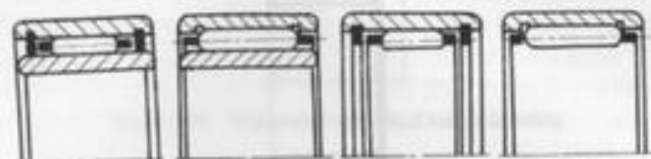


Fig. 1.21 Various types of needle roller bearings

- (4) The manufacturing accuracy of needle rollers is lower than that of cylindrical rollers and the sliding of needle rollers will often occur in the unloading region of a bearing. Therefore, the friction coefficient of needle roller bearings is greater than that of cylindrical roller bearings.

Needle roller bearings find their applications in gearboxes, universal joints, crankshafts and connecting rods, hydraulic equipment, textile machines, etc.

Thrust roller bearings

On the basis of shapes of the rollers used, there are various types of thrust roller bearings shown in Fig.1.22. Their major characteristics are

- (1) Thrust roller bearings are primarily used for carrying axial loads. The axial load carrying capacity and stiffness are much higher than those of thrust ball bearings due to line contact.
- (2) More internal sliding in thrust roller bearings sets a limit to the speed of bearings. Cylindrical roller thrust bearings shown in Fig.1.22(a) have serious sliding between rollers and raceways. Therefore, they are limited to running at low speeds. Needle roller thrust bearings shown in Fig.1.22(b) are suitable for the application where the small mounting height is required. But they have the same disadvantage as thrust cylindrical roller bearings. Fig.1.22(c) shows a tapered roller thrust bearing. The extensions of contact lines between rollers and raceways in the bearing intersect on the bearing axis. Thus the above sliding is significantly reduced.

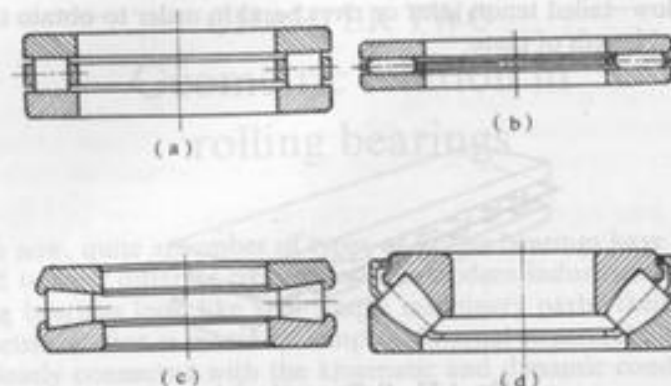


Fig. 1.22 Various types of thrust roller bearings. (a) cylindrical roller type, (b) needle roller type, (c) tapered roller type, (d) spherical roller type

- (3) By comparison with other types of thrust roller bearings, spherical roller thrust bearings shown in Fig.1.22(d) are able to carry heavy axial loads and certain radial loads simultaneously. They also possess the self-aligning capacity, which makes them insensitive to misalignment of the shaft relative to the housing. Because asymmetrical spherical rollers and specially shaped raceways are designed, they are therefore suitable for relative high speed operation.

Spherical roller thrust bearings find their primary applications in vertical electric motors, propeller shafts, arm cranes, turn tables in machine tools, etc.

Rolling guide plates

Fig.1.23 shows an operating example of rolling guide plates which are commonly used to support reciprocating linear motion. Their major characteristics are

- (1) A rolling guide plate consists of a group of needle rollers and a cage. Mounting surfaces of the slide carriage and the sliding guide should reach or approach the technical requirements of raceways. Cages are usually made of plastic or steel plate. The former is suitable for low speed operation and operating at temperatures of no more than 100°C and the latter has the wider range of applications. Several guide plates can be connected through the

swallow-tailed tenon joint or rivet bond in order to obtain the required length of plate.

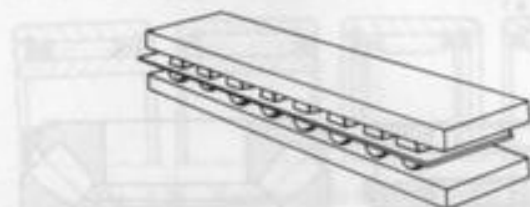


Fig. 1.23 Rolling guide plate

- (2) The guide plates have two types of the plain guide plates and the "V" shaped guide plates. In the latter, two rows of needle rollers are positioned with an angle of 90° . These two types of plates are often used together and they are preloaded to guarantee the high motion accuracy.

Rolling guide plates find their primary applications in sliding guides of machine tools.

CHAPTER TWO

Geometric relation in rolling bearings

Up to now, quite a number of types of rolling bearings have been designed to meet different requirements in modern industries. Although rolling bearings look like very simple machinery parts, their internal geometric relation is relatively complex. Internal structural parameters are closely connected with the kinematic and dynamic conditions in rolling bearings. Therefore they have important influences on bearing characteristics, such as load distribution, stress and deformation, stiffness, friction, lubrication, vibration, noise, bearing life, etc. For example, the different combinations of groove curvature radii of inner and outer raceways in a bearing effect its internal relative sliding significantly, or a slight change of the roller generatrix may remarkably improve the life of a cylindrical roller bearing. A primary purpose of the rolling bearing design is to determine the most suitable geometric structure and corresponding parameters for the satisfaction of different functions and life requirements.

These parameters primarily include rolling element diameter D , and in the case of roller bearings the roller effective length l , number of rolling elements Z , pitch circle diameter D_m , raceway groove curvature radii R_i and R_o , diameter clearance u , contact angle α , etc. The geometric relation will be further complicated due to the effects of the stress and deformation under loading conditions. Usually the analysis for this is required to solve a statically indeterminate system. In this chapter, only the geometric relation in non-loaded bearings is considered.

BALL BEARINGS

Osculation

Osculation is the ratio of the curvature radius of a rolling element to that of its matched raceway in a direction transverse to the rolling direction. An osculation shows the osculating degree at the contact point between the rolling element and the raceway. It has an important influence on the load distribution, load capacity, life and friction

moment of the bearing.

As shown in Fig.2.1, for ball bearings, the osculation is expressed as

$$\varphi = \frac{D_b}{2r} \quad (2.1)$$

In the bearing design, the following equation is often used to determine the groove curvature radius of a raceway

$$r = fD_b \quad (2.2)$$

where

f = curvature radius coefficient of a raceway groove.

The range of value f is between 0.515 and 0.525.

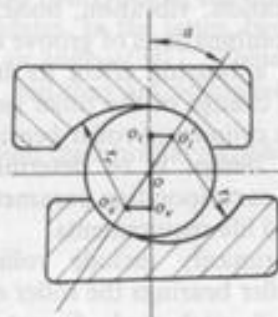


Fig. 2.1 Geometric parameters of a ball bearing

Hence

$$\varphi = \frac{1}{2f} \quad (2.3)$$

The groove curvature radius coefficients of inner and outer raceways may be different, that is, f_i and f_o correspondingly. Fig.2.1 shows the positions of the groove curvature centres before and after the relative axial movement between inner and outer rings. In this figure, it is convenient to analyze the internal geometric relation in a bearing by an assumption of the fixed ball centre.

Contact angle

A contact angle is the angle formed by the connecting line of the two contact points between the rolling element and both raceways and a radial plane of the bearing. The contact angle for a stationary and unloading bearing in which the rolling elements are in stress-free contact with the raceways is termed the initial contact angle, or nominal contact angle.

For ball bearings, the contact angle is the angle formed by the connecting line of the two contact points between the ball and both raceways and a radial plane of the bearing. Fig.2.2 shows the contact angles for two common types of ball bearings.

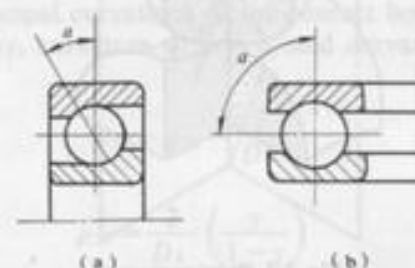


Fig. 2.2 Contact angles of ball bearings. (a) an angular contact ball bearing, (b) a thrust ball bearing

The contact angle is an important structural parameter for rolling bearings. It has a great influence on load distribution, kinematic relation, life, etc.

The contact angle formed in the operating condition of a bearing is termed the operating contact angle. It usually becomes larger than the initial contact angle if under an axial load. For an angular contact ball bearing running at high speed, different contact angles with the inner raceway and the outer raceway occur. This will be described in Chapters Five and Sixteen.

Curvature

If two bodies contact each other in an unloaded state at a point then this state is called point contact, as shown in Fig. 2.3. The curvatures of body 1 at the contact point in a pair of principal planes, perpendicular to each other, are expressed as ρ_{11} and ρ_{12} , similar principal curvatures ρ_{21} and

ρ_{22} refer to body 2. The principal curvatures are marked by two subscripts, the first refers to the body and the second refers to the principal plane. For a convex surface i.e. the centre of curvature is inside the body, the sign of ρ is positive. In reverse, the negative is for a concave surface i.e. the centre of curvature is outside the body.

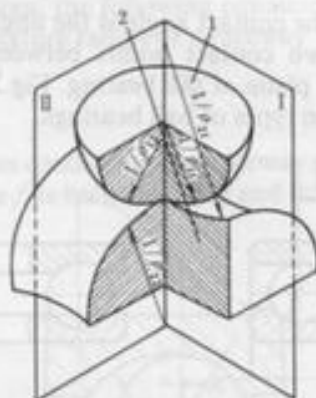


Fig. 2.3 Point contact

The following definitions are introduced. The curvature sum is the sum total of curvatures of the two bodies at the contact point.

$$\sum \rho = \rho_{11} + \rho_{12} + \rho_{21} + \rho_{22} \quad (2.4)$$

The curvature function is

$$F(\rho) = \frac{\sqrt{(\rho_{11} - \rho_{12})^2 + 2(\rho_{11} - \rho_{12})(\rho_{21} - \rho_{22})\cos 2\omega + (\rho_{21} - \rho_{22})^2}}{\sum \rho} \quad (2.5)$$

where

ω = angle between the corresponding principal planes of two contact bodies.

The corresponding principal planes of the two contact bodies in rolling bearings coincide with each other, therefore $\omega = 0$, $\cos 2\omega = 1$. Equation (2.5) changes to

$$F(\rho) = \frac{(\rho_{11} - \rho_{12}) + (\rho_{21} - \rho_{22})}{\sum \rho} \quad (2.6)$$

In this case, the curvature function is also termed curvature difference. It should be noted that $F(\rho)$ is always positive.

Taking an angular contact ball bearing for example, as shown in Fig. 2.1.

Introducing

$$\gamma = \frac{D_b \cos \alpha}{D_m} \quad (2.7)$$

Then, the principal curvatures of the contact between the ball and the inner raceway, curvature sum $\sum \rho_i$ and curvature function $F(\rho_i)$ respectively are

$$\rho_{11} = \rho_{12} = \frac{2}{D_b} \quad (2.8)$$

$$\rho_{21} = \frac{2}{D_b} \left(\frac{\gamma}{1 - \gamma} \right) \quad (2.9)$$

$$\rho_{22} = -\frac{1}{f_i D_b} \quad (2.10)$$

$$\sum \rho_i = \frac{1}{D_b} \left(4 - \frac{1}{f_i} + \frac{2\gamma}{1 - \gamma} \right) \quad (2.11)$$

$$F(\rho_i) = \frac{\frac{1}{f_i} + \frac{2\gamma}{1 - \gamma}}{4 - \frac{1}{f_i} + \frac{2\gamma}{1 - \gamma}} \quad (2.12)$$

Similarly, for the contact between the ball and the outer raceway

$$\rho_{11} = \rho_{12} = \frac{2}{D_b} \quad (2.13)$$

$$\rho_{21} = -\frac{2}{D_b} \left(\frac{\gamma}{1 + \gamma} \right) \quad (2.14)$$

$$\rho_{22} = -\frac{1}{f_e D_b} \quad (2.15)$$

$$\Sigma \rho_e = \frac{1}{D_b} \left(4 - \frac{1}{f_e} - \frac{2\gamma}{1+\gamma} \right) \quad (2.16)$$

$$F(\rho_e) = \frac{\frac{1}{f_e} - \frac{2\gamma}{1+\gamma}}{4 - \frac{1}{f_e} - \frac{2\gamma}{1+\gamma}} \quad (2.17)$$

Calculating formulae of curvatures, curvature sum and function $F(\rho)$ for various types of ball bearings are listed in Table 2.1. It should be noted that if groove curvature radius coefficients of the inner and outer raceways are the same, i.e. $f_i = f_e$, the function $F(\rho)$ at the inner raceway contact would be greater than that at the outer raceway contact. As values of f_i and f_e increase the value of $F(\rho)$ correspondingly decreases.

Clearance

The definition of the clearance is the amount of movement of one ring relative to another ring in the radial or axial direction from one limiting position to another under no load. Respectively it is termed diametral or axial clearance according to its moving direction.

The clearance is an important parameter of a bearing. It effects the load distribution, the vibration, noises and the bearing life. Therefore, it should be selected carefully, taking into consideration of the operating conditions of the bearing.

Table 2.2 gives the diametral clearance standard of deep groove ball bearings. The basic group of diametral clearances is valid for general operating conditions. The values in the third and fourth groups of diametral clearances are greater than those in the basic group. They are valid for the conditions of the tight fitting of the bearing on the shaft and housing, or large temperature difference between the inner and outer rings, as well as lower friction moment. The values in the second group of diametral clearances are less than those in the basic group. They are valid for the conditions of high running accuracy, or strictly limited relative movement in the axial direction.

Table 2.1 Calculating formulae of curvatures for ball bearings

Bearing types	Radial and angular contact ball bearings	Self-aligning ball bearings	Thrust ball bearings	
Fig.				
Ball and inner ring or shaft washer	ρ_{11}	$2 / D_b$	$2 / D_b$	
	ρ_{12}	$2 / D_b$	$2 / D_b$	
	ρ_{21}	$\frac{2}{D_b} \left(\frac{\gamma}{1-\gamma} \right)$	$2 \cos \alpha / D_i$	0
	ρ_{22}	$-1 / (f_i D_b)$	$-1 / (f_i D_b)$	$-1 / (f_i D_b)$
	$\Sigma \rho_i$	$\frac{1}{D_b} \left(4 - \frac{1}{f_i} + \frac{2\gamma}{1-\gamma} \right)$	$\frac{4}{D_b} - \frac{1}{f_i D_b} + \frac{2 \cos \alpha}{D_i}$	$\frac{1}{D_b} \left(4 - \frac{1}{f_i} \right)$
	$F(\rho_i)$	$\frac{\frac{1}{f_i} + \frac{2\gamma}{1-\gamma}}{4 - \frac{1}{f_i} + \frac{2\gamma}{1-\gamma}}$	$\frac{\frac{1}{f_i D_b} + \frac{2 \cos \alpha}{D_i}}{\frac{4}{D_b} - \frac{1}{f_i D_b} + \frac{2 \cos \alpha}{D_i}}$	$\frac{1}{4f_i - 1}$
Ball and outer ring or housing washer	ρ_{11}	$2 / D_b$	$2 / D_b$	
	ρ_{12}	$2 / D_b$	$2 / D_b$	
	ρ_{21}	$-\frac{2}{D_b} \left(\frac{\gamma}{1+\gamma} \right)$	$-1 / R_e$	0
	ρ_{22}	$-1 / (f_e D_b)$	$-1 / R_e$	$-1 / (f_e D_b)$
	$\Sigma \rho_e$	$\frac{1}{D_b} \left(4 - \frac{1}{f_e} - \frac{2\gamma}{1+\gamma} \right)$	$\frac{4}{D_b} - \frac{2}{R_e}$	$\frac{1}{D_b} \left(4 - \frac{1}{f_e} \right)$
	$F(\rho_e)$	$\frac{\frac{1}{f_e} - \frac{2\gamma}{1+\gamma}}{4 - \frac{1}{f_e} - \frac{2\gamma}{1+\gamma}}$	0	$\frac{1}{4f_e - 1}$

Table 2.2 Diametral clearances of radial ball bearings μm

Bore diameter mm		Group 2		Group 0		Group 3		Group 4		Group 5	
Over	Inclu.	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
2.5	6	0	7	2	13	8	23	—	—	—	—
6	10	0	7	2	13	8	23	14	29	20	37
10	18	0	9	3	18	11	25	18	33	25	45
18	24	0	10	5	20	13	28	20	36	28	48
24	30	1	11	5	20	13	28	23	41	30	53
30	40	1	11	6	20	15	33	28	46	40	64
40	50	1	11	6	23	18	36	30	51	45	73
50	65	1	15	8	28	23	43	38	61	55	90
65	80	1	15	10	30	25	51	46	71	65	105
80	100	1	18	12	36	30	58	53	84	75	120
100	120	2	20	15	41	36	66	61	97	90	140
120	140	2	23	18	48	41	81	71	114	105	160
140	160	2	23	18	53	46	91	81	130	120	180
160	180	2	25	20	61	53	102	91	147	135	200
180	200	2	30	25	71	63	117	107	163	150	230

Table 2.3 Diametral clearances of self-aligning ball bearings with cylindrical bore μm

Bore diameter mm		Group 2		Group 0		Group 3		Group 4		Group 5	
Over	Inclu.	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
2.5	6	1	8	5	15	10	20	15	25	21	33
6	10	2	9	6	17	12	25	19	33	27	42
10	14	2	10	6	19	13	26	21	35	30	48
14	18	3	12	8	21	15	28	23	37	32	50
18	24	4	14	10	23	17	30	25	39	34	52
24	30	5	16	11	24	19	35	29	46	40	58
30	40	6	18	13	29	23	40	34	53	46	66
40	50	6	19	14	31	25	44	37	57	50	71
50	65	7	21	16	36	30	50	45	69	62	88
65	80	8	24	18	40	35	60	54	83	76	108
80	100	9	27	22	48	42	70	64	96	89	124
100	120	10	31	25	56	50	83	75	114	105	145
120	140	10	38	30	68	60	100	90	135	125	175
140	160	15	44	35	80	70	120	110	161	150	210

Table 2.4 Diametral clearances of self-aligning ball bearings with tapered bore μm

Bore diameter mm		Group 2		Group 0		Group 3		Group 4		Group 5	
Over	Inclu.	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
18	24	7	17	13	26	20	33	28	42	37	55
24	30	9	20	15	28	23	39	33	50	44	62
30	40	12	24	19	35	29	46	40	59	52	72
40	50	14	27	22	39	33	52	45	65	58	79
50	65	18	32	27	47	41	61	56	80	73	99
65	80	23	39	35	57	50	75	69	98	91	123
80	100	29	47	42	68	62	90	84	116	109	144
100	120	35	56	50	81	75	108	100	139	130	170
120	140	40	68	60	98	90	130	120	165	155	205
140	160	45	74	65	110	100	150	140	191	180	240

Tables 2.3 and 2.4 list the diametral clearance standards of self-aligning ball bearings with cylindrical bore and with tapered bore respectively.

Relation between clearance and contact angle

A certain geometric relation between the diametral clearance and the initial contact angle of various types of ball bearings exists.

Deep groove ball bearings

The inner and the outer rings of a deep groove ball bearing move relatively in the axial direction until stress-free contact between the ball and the raceways. Fig.2.4(a) shows a deep groove ball bearing with diametral clearance u_r and Fig.2.4(b) shows the position after this relative axial movement.

It can be seen that the distance $O'O'_e$ between groove curvature centres of inner and outer raceways is

$$A = r_i + r_o - D_b \quad (2.18)$$

Using equation (2.2) gives

$$A = (f_e + f_i - 1)D_b \quad (2.19)$$

The relationship of the diametral clearance and the contact angle can be derived from the figure

$$\mu_r = 2(1 - \cos \alpha)(f_e + f_i - 1)D_b \quad (2.20)$$

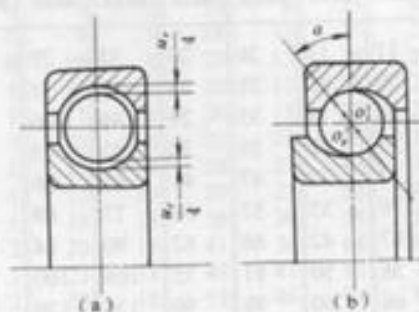


Fig. 2.4 Clearance and contact angle. (a) diametral clearance, (b) initial contact angle

And the relationship of the axial clearance and the contact angle is

$$u_a = 2 \sin \alpha (f_e + f_i - 1)D_b \quad (2.21)$$

Combining equation (2.20) with equation (2.21) yields the relationship of the diametral clearance and the axial clearance

$$u_a = \sqrt{4(f_e + f_i - 1)D_b u_r - u_r^2} \quad (2.22)$$

Equation (2.22) can be approximately written by

$$u_a \approx 2\sqrt{(f_e + f_i - 1)D_b u_r} \quad (2.23)$$

Fig. 2.5 shows the relation among the axial clearance, the diametral clearance and the initial contact angle. In the figure, symbol B is

$$B = f_e + f_i - 1 \quad (2.24)$$

For standard deep groove ball bearings, $B = 0.04$. Then, from equation (2.23)

$$u_a \approx 0.4\sqrt{D_b u_r} \quad (2.25)$$

Angular contact ball bearings

The initial contact angle of an angular contact ball bearing is connected with the diametral clearance, the raceway groove curvature radius and the ball diameter in the bearing. It can be deduced from equation (2.20), that

$$\cos \alpha = 1 - \frac{u_r}{2(f_e + f_i - 1)D_b} \quad (2.26)$$

Angular contact ball bearings with split inner rings

Fig. 2.6 shows an angular contact ball bearing with split inner rings. It can be imagined that the split inner rings are formed by eliminating a shim from an entire inner ring of a corresponding deep groove ball bearing.

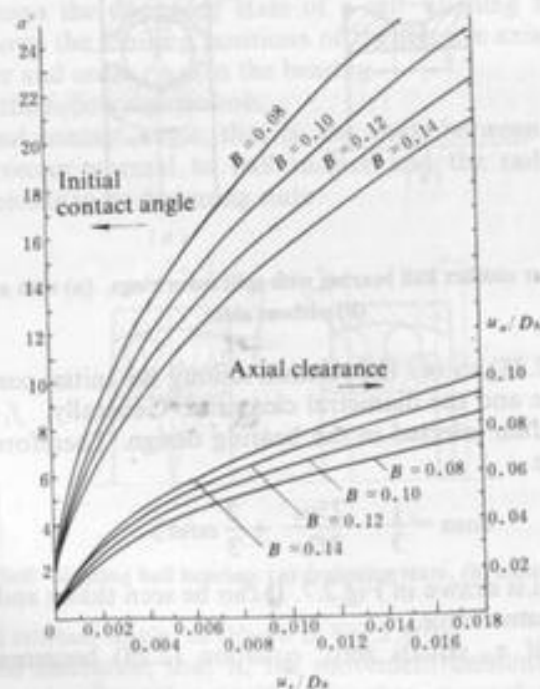


Fig. 2.5 Axial clearance, diametral clearance and initial contact angle

In the bearing design, the shim angle α_D and the initial contact angle α are important parameters. It can be seen from Fig.2.6 that

$$\sin \alpha_D = \frac{h}{(2f_i - 1)D_b} \quad (2.27)$$

where

h = width of the shim.

The following equation may be derived

$$\cos \alpha = 1 - \frac{u_r}{2(f_e + f_i - 1)D_b} + \frac{(2f_i - 1)(1 - \cos \alpha_D)}{2(f_e + f_i - 1)} \quad (2.28)$$

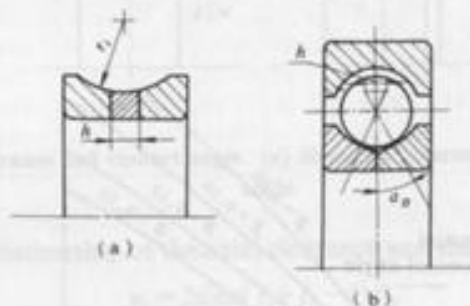


Fig. 2.6 Angular contact ball bearing with split inner rings. (a) with a shim, (b) without shim

Equation (2.28) shows the relation among the initial contact angle, the shim angle and the diametral clearance. Generally, $f_i = 0.54$ and $f_e = 0.52$ are often selected in the bearing design. Therefore, equation (2.28) becomes

$$\cos \alpha = \frac{1}{3} - \frac{25u_r}{3D_b} + \frac{2}{3} \cos \alpha_D \quad (2.29)$$

This relation is drawn in Fig.2.7. It can be seen that α and α_D usually are not the same value.

Obviously, if α_D equals zero, equation (2.28) becomes equation (2.26).

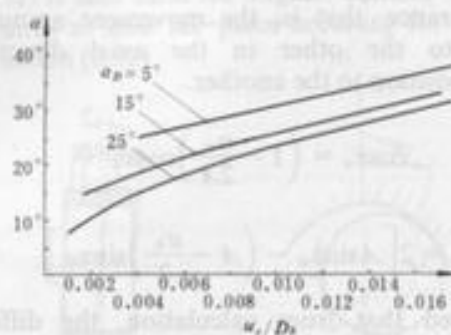


Fig. 2.7 Relation among α , u_r and α_D

Self-aligning ball bearings

Fig.2.8(a) shows the designing state of a self-aligning ball bearing. Fig.2.8(b) shows the limiting positions of the relative axial movement between inner and outer rings in the bearing.

Introduce the following symbols:

α_0 = nominal contact angle, that is, the angle between the contact load vector normal to ball surface and the radial plane of the bearing in its designing state

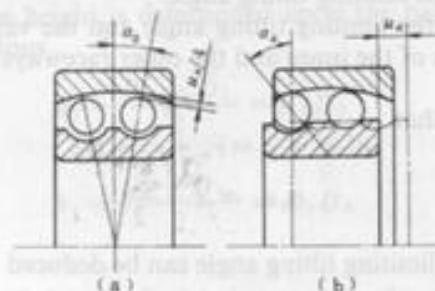


Fig. 2.8 Self-aligning ball bearing. (a) designing state, (b) axial movement

α_0 = initial contact angle, as shown in Fig.2.8(b)

u_a = normal clearance, that is, the movement amount of one ring relative to the other in the direction normal to the outer raceway at contact point from one limiting position to the

other

u_a = axial clearance, that is, the movement amount of one ring relative to the other in the axial direction from one limiting position to the another.

then

$$\cos \alpha_a = \left(1 - \frac{u_a}{2A}\right) \cos \alpha_0 \quad (2.30)$$

$$u_a = 2 \left[A \sin \alpha_a - \left(A - \frac{u_a}{2}\right) \sin \alpha_0 \right] \quad (2.31)$$

It is concluded that from calculation, the different kinds of clearances and contact angles have the following order according to their values

$$u_a > u_r > u_n$$

$$\alpha_a > \alpha_0$$

Limiting tilting angle

For a deep groove ball bearing with a certain amount of diametral clearance, its inner ring may be tilted from the central position under no load. Generally, the maximum relative angle between the inner and the outer ring axes without any additional stress to bearing components is termed the limiting tilting angle.

Fig. 2.9 shows the limiting tilting angle and the variation of groove curvature centres of the inner and the outer raceways in a deep groove ball bearing.

It can be seen that

$$o'_1 o_e = A = r_e + r_i - D_b$$

$$o_1 o_e = A - \frac{u_r}{2}$$

Therefore, the limiting tilting angle can be deduced

$$\cos \theta = \frac{\left(\frac{D_e}{2} + r_i - D_b - \frac{u_r}{2}\right)^2 + \left(\frac{D_e}{2} - r_e\right)^2 - (r_e + r_i - D_b)^2}{2 \left(\frac{D_e}{2} + r_i - D_b - \frac{u_r}{2}\right) \left(\frac{D_e}{2} - r_e\right)} \quad (2.32)$$

Equation (2.32) is also valid for angular contact ball bearings. The calculation requires at least six-place accuracy for the terms on the right side of equation (2.32).

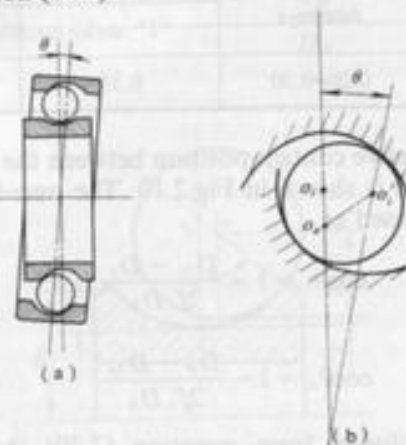


Fig. 2.9 Limiting tilting angle in a deep groove ball bearing. (a) limiting tilting angle, (b) groove curvature centres

Land of the bearing ring

Height of the land of a bearing ring affects the axial load capacity of the bearing. The height is determined upon the ball diameter by the following equations

$$h_i = \frac{D_{ii} - D_i}{2} = K_i D_b \quad (2.33)$$

$$h_e = \frac{D_{ie} - D_{ie}}{2} = K_i D_b \quad (2.34)$$

where

h_i and h_e = heights of the inner and the outer ring lands respectively

D_{ii} and D_{ie} = diameters of the inner and the outer ring lands respectively

K_i = height coefficient of the land, connected with diametral series of bearings, values of K_i are listed in Table 2.5.

Table 2.5 Land height coefficient

Diametral series	Miniature bearings	Deep groove ball bearings	
		"1" series	"2", "3", "4" series
K_1	0.20–0.30	0.35	0.40

The limiting relative contact position between the inner ring and the ball under no load is shown in Fig.2.10. The one-half of the central angle can be expressed as

$$\cos \theta_i = 1 - \frac{D_{ii} - D_i}{2f_i D_b} \quad (2.35)$$

$$\cos \theta_e = 1 - \frac{D_e - D_{ie}}{2f_e D_b} \quad (2.36)$$

It can be concluded from equation (2.20) that increasing the diametral clearance brings about a corresponding increase of the initial contact angle. The diametral clearance which makes the initial contact angle equal to the one-half of the central angle is termed the maximum theoretic diametral clearance. The actually selected diametral clearance is less than this value. The maximum theoretic diametral clearance is calculated by

$$u_{rmax} = \frac{2(f_e + f_i - 1)h_i}{f_i} \quad (2.37)$$

The relation of the height of land and its axial load capacity will be described in Chapter Thirteen.

Cage

Fig.2.11 shows a pressed steel cage. The functions of the cage are to keep rolling elements equally-spaced and to guide the movement of rolling elements, as well as improve lubrication conditions. Cages with different materials and structures can be chosen in the same type of bearings in order to meet different requirements for bearing performances.

Generally, a cage may deviate from its designed position in the radial direction due to the gap between each cage pocket and ball. The

amount of the radial deviation is calculated by the following equation

$$\epsilon_c = \frac{2 \left(R_c \cos \theta_i^{-1} \frac{0.96 B_c}{2 R_c} - R_c + K_c \right)}{0.96 B_c - D_b \sin \theta_i^{-1}} \quad (2.38)$$

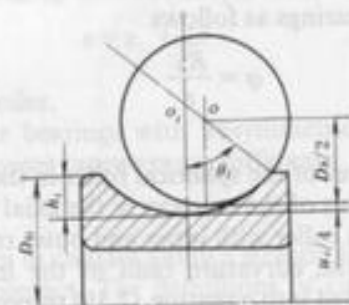


Fig. 2.10 Land of a bearing

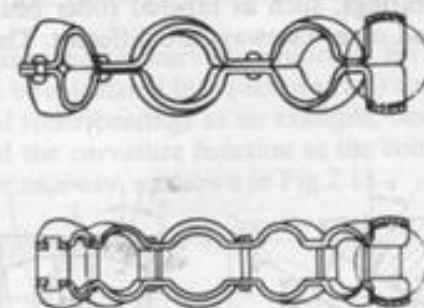


Fig. 2.11 Pressed steel cage

where

ϵ_c = radial deviation of the cage

B_c = cage width

R_c = curvature radius of pocket

K_c = pocket depth.

Obviously, the cage should avoid contact with any ring of the bearing during running as it takes the maximum radial deviation.

ROLLER BEARINGS

Osculation

If generatrices of the roller and the raceway in a roller bearing are curves the term of osculation can also be used. Similarly, it is defined for spherical roller bearings as follows

$$\varphi = \frac{R_b}{r} \quad (2.39)$$

where

R_b = curvature radius of the spherical roller in the axial plane

r = curvature radius of the raceway in the axial plane.

The osculation of a roller with inner and outer raceways may be different. In this case the curvature radii of the inner and the outer raceways are substituted into equation (2.39) respectively.

Contact angle

The definition of the contact angle is also valid for roller bearings. For some types of bearings, such as tapered roller bearings, the contact angles at inner and outer raceways are different. The contact angle of

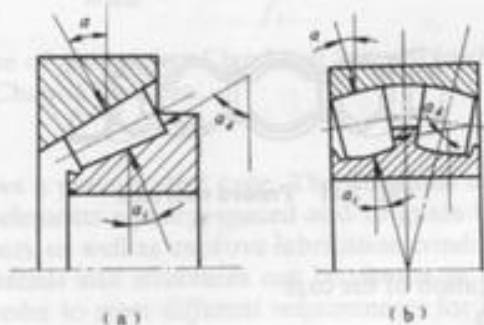


Fig. 2.12 Contact angles of roller bearings. (a) tapered roller bearings, (b) spherical roller bearings

cylindrical roller bearings is zero. Fig.2.12 shows the contact angles of tapered roller bearings and spherical roller bearings with unsymmetric rollers.

Usually, the contact angle α of a tapered roller bearing means the contact angle between the tapered roller and the outer raceway. Assuming that the contact angle between the tapered roller and the inner raceway is expressed as α_i , then one may obtain

$$\alpha = \alpha_i + 2\beta \quad (2.40)$$

where

β = half taper of roller.

In spherical roller bearings with unsymmetric rollers, the contact angles at inner and outer raceways are different.

Principal curvature

A state of two bodies in contact along a straight line or curve under no load is called line contact. The definitions of curvature sum $\Sigma\rho$ and curvature function $F(\rho)$ are valid for line contact conditions.

For a roller bearing under no load, the contact between the roller and the raceway is either a line contact, such as cylindrical roller bearings and tapered roller bearings, or a point contact, such as spherical roller bearings.

The curvature sum and the curvature function for various types of roller bearings can be calculated by equations (2.4) and (2.6).

Taking spherical roller bearings as an example, one may obtain the curvature sum and the curvature function at the contact between the roller and the inner raceway, as shown in Fig.2.13.

$$\Sigma\rho_i = \frac{2}{D_b} + \frac{1}{R_b} + \frac{2\cos\alpha_i}{D_i} - \frac{1}{r_i} \quad (2.41)$$

$$F(\rho_i) = \frac{\frac{2}{D_b} - \frac{1}{R_b} + \frac{2\cos\alpha_i}{D_i} + \frac{1}{r_i}}{\frac{2}{D_b} + \frac{1}{R_b} + \frac{2\cos\alpha_i}{D_i} - \frac{1}{r_i}} \quad (2.42)$$

Similarly, for the contact between the roller and the outer raceway

$$\Sigma\rho_e = \frac{2}{D_b} + \frac{1}{R_b} - \frac{2}{R_e} \quad (2.43)$$

$$F(\rho_r) = \frac{\frac{2}{D_b} - \frac{1}{R_b}}{\frac{2}{D_b} + \frac{1}{R_b} - \frac{2}{R_r}} \quad (2.44)$$

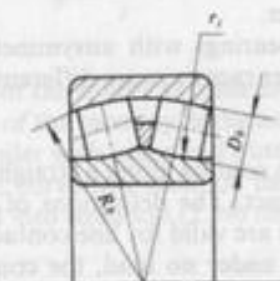


Fig. 2.13 Spherical roller bearing

It can be seen that if in the axial plane the curvature radius of the roller R_b equals that of the raceway r_i (or R_r), then the curvature function is equal to one. Obviously, this is valid for cylindrical and tapered roller bearings.

Calculating formulae of principal curvatures, curvature sum and function for various types of roller bearings are listed in Table 2.6.

Clearance

Table 2.7 lists the diametral clearance standard of cylindrical roller bearings with cylindrical bore. Tables 2.8 and 2.9 list the diametral clearance standard of spherical roller bearings with cylindrical bore and with tapered bore. The diametral clearances in basic group are valid for general operating conditions and are selected preferentially.

The certain geometric relation among the diametral clearance, the axial clearance and the initial contact angle for various types of roller bearings exists.

Table 2.6 Calculating formulae of curvatures for roller bearings

Bearing types	Cylindrical and needle roller bearings		Tapered roller bearings	Spherical roller bearings
Figure				
Roller and inner ring	ρ_{11}	$2 / D_b$	$2 / \cos \beta / D_b$	$2 / D_b$
	ρ_{12}	0	0	$1 / R_b$
	ρ_{21}	$2 / D_i$	$2 \cos \alpha_i / D_i$	$2 \cos \alpha_i / D_i$
	ρ_{22}	0	0	$-1 / r_i$
	$\Sigma \rho_i$	$\frac{2}{D_b} + \frac{2}{D_i}$	$\frac{2 \cos \beta}{D_b} + \frac{2 \cos \alpha_i}{D_i}$	$\frac{2}{D_b} + \frac{1}{R_b} + \frac{2 \cos \alpha_i}{D_i} - \frac{1}{r_i}$
	$F(\rho_i)$	1	1	$\frac{\frac{2}{D_b} - \frac{1}{R_b} + \frac{2 \cos \alpha_i}{D_i} + \frac{1}{r_i}}{\frac{2}{D_b} + \frac{1}{R_b} + \frac{2 \cos \alpha_i}{D_i} - \frac{1}{r_i}}$
Roller and outer ring	ρ_{11}	$2 / D_b$	$2 \cos \beta / D_b$	$2 / D_b$
	ρ_{12}	0	0	$1 / R_b$
	ρ_{21}	$-2 / D_e$	$-2 \cos \alpha / D_e$	$-1 / r_e$
	ρ_{22}	0	0	$-1 / r_e$
	$\Sigma \rho_e$	$\frac{2}{D_b} - \frac{2}{D_e}$	$\frac{2 \cos \beta}{D_b} - \frac{2 \cos \alpha}{D_e}$	$\frac{2}{D_b} + \frac{1}{R_b} - \frac{2}{D_e}$
	$F(\rho_e)$	1	1	$\frac{\frac{2}{D_b} - \frac{1}{R_b} - \frac{2}{D_e}}{\frac{2}{D_b} + \frac{1}{R_b} - \frac{2}{D_e}}$

Table 2.7 Diametral clearances of cylindrical roller bearings

 μm

Bore diameter mm		Group 2				Group 0			
		Exchangeable				Exchangeable			
		Unexchangeable				Unexchangeable			
Over	Inclu.	Min	Min	Max	Max	Min	Min	Max	Max
	10	0	10	20	30	10	20	30	40
10	18	0	10	20	30	10	20	30	40
18	24	0	10	20	30	10	20	30	40
24	30	0	10	25	30	10	25	35	45
30	40	0	12	25	35	15	25	40	50
40	50	5	15	30	40	20	30	45	55
50	65	5	15	35	45	20	35	50	65
65	80	5	20	40	55	25	40	60	75
80	100	10	25	45	60	30	45	70	80
100	120	10	25	50	65	35	50	80	90
120	140	10	30	60	75	40	60	90	105
140	160	15	35	65	80	50	65	100	115
160	180	20	35	75	85	60	75	110	125
180	200	25	40	80	95	65	80	120	135
200	225	30	45	90	105	75	90	135	150
225	250	40	50	100	115	90	100	150	165
250	280	45	55	110	125	100	110	165	180
280	315	50	60	120	135	110	120	180	195
315	355	55	65	135	145	125	135	200	215
355	400	65	75	150	160	140	150	225	235
400	450	70	85	170	190	155	170	255	275
450	500	85	95	190	205	180	190	285	300

(continued)

Group 3				Group 4				Group 5			
Exchangeable				Exchangeable				Exchangeable			
Unexchangeable				Unexchangeable				Unexchangeable			
Min	Min	Max	Max	Min	Min	Max	Max	Min	Min	Max	Max
25	35	45	55	35	45	55	65	—	—	—	—
25	35	45	55	35	45	55	65	55	65	75	85
25	35	45	55	35	45	55	65	55	65	75	85
30	40	50	65	40	50	60	70	60	70	80	90
35	45	55	70	45	55	70	80	70	80	95	105
40	50	65	75	55	65	80	90	85	95	110	120
45	55	75	90	65	75	90	105	100	110	130	140
55	70	90	105	75	90	110	125	115	130	150	165
65	80	105	115	90	105	125	140	145	155	180	195
80	95	120	135	105	120	145	160	165	180	205	220
90	105	135	155	115	135	160	180	185	200	230	250
100	115	150	165	130	150	180	195	210	225	260	275
110	125	165	175	150	165	200	215	235	250	285	300
125	140	180	195	165	180	220	235	260	275	315	330
140	155	200	215	180	200	240	255	290	305	350	365
155	170	215	230	205	215	265	280	320	330	380	395
175	185	240	255	230	240	295	310	355	370	420	435
195	205	265	280	255	265	325	340	400	410	470	485
215	225	295	305	280	295	360	370	440	455	520	530
245	255	330	340	320	330	405	415	500	510	585	595
270	285	370	390	355	370	455	465	555	565	650	675
300	315	410	420	395	410	505	515	620	625	720	740

Table 2.8 Diametral clearances of spherical roller bearings with cylindrical bore μm

Bore diameter mm		Group 2		Group 0		Group 3		Group 4		Group 5	
Over	Inclu.	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
14	18	10	20	20	35	35	45	45	60	60	75
18	24	10	20	20	35	35	45	45	60	60	75
24	30	15	25	25	40	40	55	55	75	75	95
30	40	15	30	30	45	45	60	60	80	80	100
40	50	20	35	35	55	55	75	75	100	100	125
50	65	20	40	40	65	65	90	90	120	120	150
65	80	30	50	50	80	80	110	110	145	145	180
80	100	35	60	60	100	100	135	135	180	180	225
100	120	40	75	75	120	120	160	160	210	210	260
120	140	50	95	95	145	145	190	190	240	240	300
140	160	60	110	110	170	170	220	220	280	280	350
160	180	65	120	120	180	180	240	240	310	310	390
180	200	70	130	130	200	200	260	260	340	340	430
200	225	80	140	140	220	220	290	290	380	380	470
225	250	90	150	150	240	240	320	320	420	420	520
250	280	100	170	170	260	260	350	350	460	460	570
280	315	110	190	190	280	280	370	370	500	500	630
315	355	120	200	200	310	310	410	410	550	550	690
355	400	130	220	220	340	340	450	450	600	600	750
400	450	140	240	240	370	370	500	500	660	660	820
450	500	140	260	260	410	410	550	550	720	720	900
500	560	150	280	280	440	440	600	600	780	780	1000
560	630	170	310	310	480	480	650	650	850	850	1100
630	710	190	350	350	530	530	700	700	920	920	1190
710	800	210	390	390	580	580	770	770	1010	1010	1300
800	900	230	430	430	650	650	860	860	1120	1120	1440
900	1000	260	480	480	710	710	930	930	1220	1220	1570

Table 2.9 Diametral clearances of spherical roller bearings with tapered bore μm

Bore diameter mm		Group 2		Group 0		Group 3		Group 4		Group 5	
Over	Inclu.	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
18	24	15	25	25	35	35	45	45	60	60	75
24	30	20	30	30	40	40	55	55	75	75	95
30	40	25	35	35	50	50	65	65	85	85	105
40	50	30	45	45	60	60	80	80	100	100	130
50	65	40	55	55	75	75	95	95	120	120	160
65	80	50	70	70	95	95	120	120	150	150	200
80	100	55	80	80	110	110	140	140	180	180	230
100	120	65	100	100	135	135	170	170	220	220	280
120	140	80	120	120	160	160	200	200	260	260	330
140	160	90	130	130	180	180	230	230	300	300	380
160	180	100	140	140	200	200	260	260	340	340	430
180	200	110	160	160	220	220	290	290	370	370	470
200	225	120	180	180	250	250	320	320	410	410	520
225	250	140	200	200	270	270	350	350	450	450	570
250	280	150	220	220	300	300	390	390	490	490	620
280	315	170	240	240	330	330	430	430	540	540	680
315	355	190	270	270	360	360	470	470	590	590	740
355	400	210	300	300	400	400	520	520	650	650	820
400	450	230	330	330	440	440	570	570	720	720	910
450	500	260	370	370	490	490	630	630	790	790	1000
500	560	290	410	410	540	540	680	680	870	870	1100
560	630	320	460	460	600	600	760	760	980	980	1230
630	710	350	510	510	670	670	850	850	1090	1090	1360
710	800	390	570	570	750	750	960	960	1220	1220	1500
800	900	440	640	640	840	840	1070	1070	1370	1370	1690
900	1000	490	710	710	930	930	1190	1190	1520	1520	1860

Tapered roller bearings

A single tapered roller bearing has no diametral clearance because its rings can be separated. After the bearing is mounted on a shaft and housing, adjusting the relative axial position of its inner and outer rings leads to produce a diametral and axial clearances, as shown in Fig. 2.14. Therefore

$$u_r = 2u_a \tan \alpha \quad (2.45)$$

Spherical roller bearings

Fig. 2.15 shows a spherical roller bearing and its relation between a roller and its outer raceway.

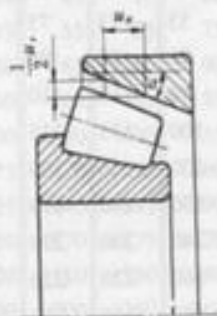


Fig. 2.14 Tapered roller bearing

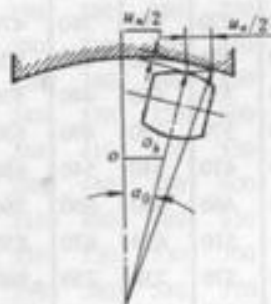


Fig. 2.15 Spherical roller bearing

Introducing the following symbols

α_0 = nominal contact angle, that is, the angle between the contact load vector normal to the roller and the radial plane of the bearing in its designing state

α_s = initial contact angle at the limiting axial position

u_n = normal clearance, similar to self-aligning ball bearings.

It can be obtained from the figure

$$(r_e - R_b) \cos \alpha_s = \left(r_e - R_b - \frac{u_n}{2} \right) \cos \alpha_0 \quad (2.46)$$

therefore

$$\alpha_s = \arccos \left[\left(1 - \frac{u_n}{2(r_e - R_b)} \right) \cos \alpha_0 \right] \quad (2.47)$$

and then

$$u_s = 2 \left[(r_e - R_b) \sin \alpha_s - \left(r_e - R_b - \frac{u_n}{2} \right) \sin \alpha_0 \right] \quad (2.48)$$

Generatrix shape of cylindrical roller

To avoid edge stresses at the ends of the cylindrical roller, the rollers are generally manufactured into a slightly crowned shape, as shown in Fig. 2.16.

There are two types of crowned cylindrical rollers. One of them is the rollers with convex generatrix, called the entire convex type. The other is designed with a slightly crowned transition zone between the central straight line and the corner, called the modified line type.

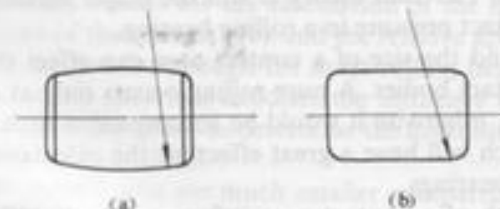


Fig. 2.16 Crowned cylindrical rollers. (a) entire convex type, (b) modified line type

CHAPTER THREE

Contact pressure and deformation

Contact pressure and deformation refer to those produced in the area nearing the contact point between two bodies with curved surfaces which are pressed against each other. With a rolling bearing for example, point or line contacts between rolling elements and raceways occur initially in a loaded rolling bearing. Then, small contact areas are formed to transmit the load to rolling elements. At the same time, the contacting bodies would approach each other due to the deformation. The purpose of this chapter is to give calculation methods for the contact pressure distribution, the shapes and sizes of contact areas, as well as the relative approaches of two bodies. These are necessary for consideration of the load distribution, rigidity, lubrication, friction and vibration in rolling bearings.

The contact pressure has an important influence on the contact fatigue and the wear of rolling bearings. Although a load acting on contact bodies may not be heavy, its contact pressure induced on the contact area is usually large. In general, the contact pressure in a rolling bearing is between 2000 MPa and 4000 MPa. The contact fatigue life chiefly depends on the magnitude of the contact pressure. In 1881, Hertz established the classical solution for the contact pressure and deformation of two bodies contacting at a point initially. A number of experiments show satisfactory agreement with the calculated results. Up to now, the Hertzian theory is still the major method for calculation of the contact pressure in a rolling bearing.

The shape and the size of a contact area can effect the friction between two contact bodies. A pure rolling occurs only at an ideal point or line contact, otherwise it would be accompanied with various types of sliding, which will have a great effect on the calculation of the friction in rolling bearings.

The approach of a contact area influences the rigidity of a rolling bearing. Due to the nonlinear relation between the contact load and its corresponding approach the rigidity of a rolling bearing can be increased significantly by a preload method.

In the first paragraph of this chapter, the basic formulae of the Hertzian theory, including the contact pressure, sizes of the contact

area and the relative approach are introduced. The application of them to rolling bearings is described in the second paragraph. For convenience, the computational programs for the contact pressure and the deformation in a rolling bearing are introduced in FORTRAN and PASCAL languages.

The expressions of the shear stress and the maximum orthogonal shear stress below the contact surface are given in the third paragraph. Some influential factors, such as surface defects, the shear stress at the contact surface, are also explained.

HERTZIAN THEORY OF THE ELASTIC CONTACT

Basic assumptions

Hertz found the solution of the contact theory upon the following assumptions.

(1) All contact deformation occurs in the elastic range and it follows the Hooke Law.

Generally, rolling elements and rings of a rolling bearing are made of the hardened bearing steel. In the conventional condition, the total maximum plastic deformation at a contact area among the bearing components does not exceed 0.01% of the rolling element diameter. Therefore, this assumption is acceptable for rolling bearings.

(2) The contact load is perpendicular to the surface, that is, the contact surfaces are perfectly smooth and the friction force at the surfaces is neglected.

This assumption makes it easier to find the solution of the contact theory and to simplify its mathematical expression. Experiments indicate that satisfactory results for the calculation of the maximum contact pressure, sizes of the contact area and the relative approach can be obtained. These are precise enough for most rolling bearings. On the other hand, one must take into account the influence of the friction force at the surface if one needs to determine the maximum shear stress below the contact surface [12].

(3) Sizes of the contact area are much smaller compared with the curvature radii of the contact bodies.

This assumption may allow for the simplification of the geometric relation in the contact theory using the basic solution of a semi-infinite solid (or plane) subjected to a concentrated force on its boundary. Precisely speaking, this is not available to most types of rolling bearings. For example, the contact areas in a deep groove ball bearing are

clearly curved and their major axes are not small compared with the curvature radii. But the interesting thing is that experiments show satisfactory agreement with the computed results based on this assumption. Therefore, it is still used on most rolling bearings.

Hertzian theory of three dimensional contact problems

For the convenience of application, some basic expressions are stated briefly. More details can be found in the elasticity theory (13), (14).

Geometric conditions of a contact

Assume that two bodies contact at the point O initially, as shown in Fig.3.1. The common tangential plane that passes through the point O is taken as the x_1y_1 plane. z_1 and z_2 axes are the coordinates of the contact bodies 1 and 2 respectively.

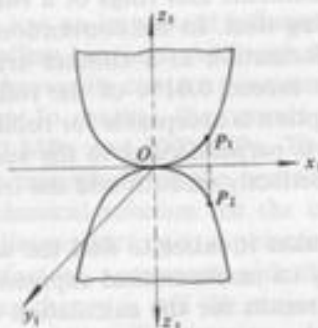


Fig. 3.1 Elastic contact

Considering that the ratio of the sizes of the contact area to its curvature radii is very small, the contact surfaces close to the point O are expressed as

$$\left. \begin{aligned} z_1 &= A_1x_1^2 + B_1y_1^2 + 2H_1x_1y_1 \\ z_2 &= A_2x_1^2 + B_2y_1^2 + 2H_2x_1y_1 \end{aligned} \right\} \quad (3.1)$$

Taking two points P_1 and P_2 on the contact surfaces with the coordinates (x_1, y_1) , the distance between them before loading is

$$z_1 + z_2 = (A_1 + A_2)x_1^2 + 2(H_1 + H_2)x_1y_1 + (B_1 + B_2)y_1^2$$

Choosing such a new set of coordinates (x, y) makes the coefficient of the xy product term zero. Hence,

$$z_1 + z_2 = A_0x^2 + B_0y^2 \quad (3.2)$$

In which, $A_1, B_1, H_1, A_2, B_2, H_2$, and A_0, B_0 are the coefficients of the equation of second order.

After loading, the points P_1 and P_2 gain their displacements w_1 and w_2 along the z_1 and z_2 axes respectively and then coincide with each other. Therefore

$$w_1 + w_2 = \delta - A_0x^2 - B_0y^2 \quad (3.3)$$

where

δ = relative elastic approach of the two bodies.

Relation between contact pressure and displacement

Boussinesq gave an elasticity solution of a concentrate force acting at the boundary of the semi-infinite solid. Based on this, the problem having a pressure distribution at the solid boundary can be derived. Fig.3.2 shows the pressure $p(x, y)$ within a contact area at the boundary of the semi-infinite solid.

It can be proved that the displacement of a point (x, y) at the boundary along the direction of the z axis is

$$w = \frac{1-\nu^2}{\pi E} \iint_a \frac{p(\xi, \eta) d\xi d\eta}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} \quad (3.4)$$

If the contact area is an ellipse, that is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (3.5)$$

Assuming the contact pressure follows a semi-ellipsoidal distribution shown in Fig.3.3, one may express

$$p(x, y) = p_0 \sqrt{1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2} \quad (3.6)$$

where

a, b = semi-major and semi-minor axes respectively

p_0 = pressure at the centre of the contact ellipse.

Substituting equations (3.5) and (3.6) into equation (3.4) and integrating it over the contact area, then

$$w = \frac{1-\nu^2}{E} b p_0 \left[K(e) - D(e) \frac{x^2}{a^2} - B(e) \frac{y^2}{b^2} \right] \quad (3.7)$$

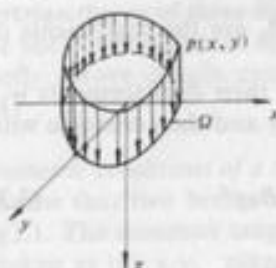


Fig. 3.2 Pressure acting at the semi-infinite solid

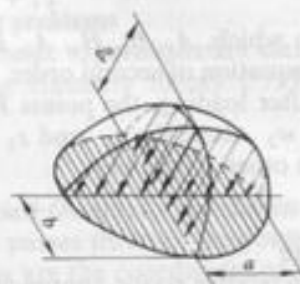


Fig. 3.3 Semi-ellipsoidal distribution of the pressure

where

$K(e)$ = the complete elliptic integral of the first kind

$$K(e) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1-e^2 \sin^2 \varphi}} \quad (3.8)$$

$D(e)$ and $B(e)$ = combination of the complete elliptic integrals

$$D(e) = \frac{1}{e^2} [K(e) - L(e)] \quad (3.9)$$

$$B(e) = K(e) - D(e) \quad (3.10)$$

$L(e)$ = the complete elliptic integral of the second kind

$$L(e) = \int_0^{\pi/2} \sqrt{1-e^2 \sin^2 \varphi} d\varphi \quad (3.11)$$

e = eccentricity of the contact ellipse, it is

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2} \quad (3.12)$$

Given the value e , $K(e)$ and $L(e)$ can be obtained from mathematical tables. Then $D(e)$ and $B(e)$ are computed.

Static equilibrium condition

The integration of the pressure distribution over the contact area should be equal to the total load, that is

$$Q = \iint_{\Omega} p(x, y) d\Omega \quad (3.13)$$

where

Q = total load.

Substituting equation (3.6) into equation (3.13) and integrating it over the contact area, then

$$p_0 = \frac{3}{2} \frac{Q}{\pi ab} \quad (3.14)$$

Obviously, $Q / (\pi ab)$ is the average pressure over the elliptic area. It can be seen that the maximum pressure is one and a half times the average pressure, while the contact pressure follows the semi-ellipsoidal distribution.

Solving process of contact problems

Assume that material constants of the two contact bodies are ν_1, E_1 and ν_2, E_2 respectively. An elastic contact under the action of a load Q is shown in Fig. 3.1. Substitution of equation (3.4) into equation (3.3) leads to a integral equation of the pressure $p(x, y)$

$$\left(\frac{1-\nu_1^2}{\pi E_1} + \frac{1-\nu_2^2}{\pi E_2} \right) \iint_{\Omega} \frac{p(\xi, \eta) d\xi d\eta}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} = \delta - A_0 x^2 - B_0 y^2 \quad (3.15)$$

This shows the relation among $p(\xi, \eta)$, Ω and δ . It is hard to solve because the contact area Ω is unknown. Hertz presumed that Ω is an elliptic area and $p(\xi, \eta)$ follows a semi-ellipsoidal distribution. He found the solution of the equation.

On the basis of this consideration, substitution of equation (3.7) into equation (3.3) deduces that the following equation should be valid for all the points at the contact area.

$$\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right) b p_0 \left[K(e) - D(e) \frac{x^2}{a^2} - B(e) \frac{y^2}{b^2} \right]$$

$$\equiv \delta - A_0 x^2 - B_0 y^2 \quad (3.16)$$

Comparing the coefficients of the same order term and using equation (3.14), one may obtain

$$\frac{A_0}{B_0} = \frac{(1-e^2)D(e)}{B(e)} \quad (3.17)$$

$$a = m_a \sqrt{\frac{3Q}{2\Sigma\rho} \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)} \quad (3.18)$$

$$b = m_b \sqrt{\frac{3Q}{2\Sigma\rho} \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)} \quad (3.19)$$

$$\delta = \frac{2K(e)}{\pi m_a} \sqrt{\frac{1}{8} \left[\frac{3}{2} \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right) \right]^2 Q^2 \Sigma\rho} \quad (3.20)$$

where

m_a, m_b = semi-major and semi-minor axis coefficients of the contact ellipse. They are expressed as

$$m_a = \sqrt{\frac{2L(e)}{\pi k^2}} \quad (3.21)$$

$$m_b = \sqrt{\frac{2L(e)k}{\pi}} \quad (3.22)$$

$$k = \frac{b}{a} \quad (3.23)$$

Equations (3.14), (3.18) to (3.23) are the solution for the three dimensional contact problems. These are difficult to apply since the eccentricity of the contact ellipse is unknown. Upon the coordinate transformation corresponding to equation (3.2), it can be proved

$$\frac{B_0 - A_0}{B_0 + A_0} = F(\rho) \quad (3.24)$$

Using equation (3.17), one may derive

$$F(\rho) = \frac{(1+k^2)L(e) - 2k^2K(e)}{(1-k^2)L(e)} \quad (3.25)$$

Consequently, if given the curvatures of the two elastic bodies, the curvature sum $\Sigma\rho$ and the curvature function $F(\rho)$ are calculated using equations (2.4) to (2.6) in Chapter Two. Then the eccentricity of the contact ellipse is determined from equation (3.25). After that, m_a and m_b are given by equations (3.21) and (3.22). As a result, the solution of the contact problem is obtained using equations (3.14), (3.18) to (3.20).

Basic formulae of two dimensional contact problems

Similar to the solving process used for the three dimensional contact problems, one may derive the solution of the two dimensional case. The basic formulae for calculation of the contact pressure and the deformation are introduced without their derivation.

Two dimensional case

Fig.3.4 shows that two cylinders with parallel axes are pressed by a

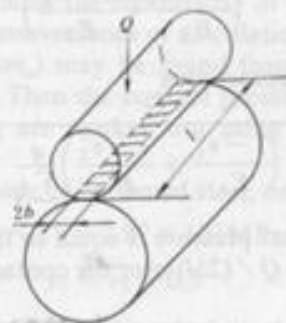


Fig. 3.4 Contact between two elastic cylinders

uniform pressure with its linear density q . The previous assumptions are still valid for this case. Strictly speaking, the third assumption should be, the width of the contact area is much smaller compared with the curvature radii of the contact cylinders. And another assumption, that is that the length of the cylinders is much longer than their diameters, is added. Obviously this assumption is approximately valid for roller bearings.

Symbols used are shown in Fig.3.4, such as the cylinder length l , the

total load Q and the linear density of the load q . Under the loading, the rectangular contact area with the width $2b$ and the length l , as well as the relative approach is formed.

The linear density of the load is

$$q = \frac{Q}{l} \quad (3.26)$$

Contact pressure and the size of contact area

The contact pressure follows an elliptic distribution, that is

$$p(y) = p_0 \sqrt{1 - \left(\frac{y}{b}\right)^2} \quad (3.27)$$

Maximum contact pressure

$$p_0 = \sqrt{\frac{2}{\pi} \frac{q}{\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right) \sum \rho}} \quad (3.28)$$

Semi-width of the contact area

$$b = \sqrt{\frac{4}{\pi} \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right) \frac{q}{\sum \rho}} \quad (3.29)$$

The maximum contact pressure is equal to the $4/\pi$ times of the average pressure which is $Q/(2bl)$ over the contact area.

Relative elastic approach

No precise solution exists for the relative elastic approach in the two dimensional case. Palmgren established the following approximate formula for the estimation of the relative approach

$$\delta = 3.81 \left[\frac{2(1-\nu^2)}{\pi E} \right]^{0.9} \frac{Q^{0.9}}{l^{0.8}} \quad (3.30)$$

Experiments indicate that the calculated results show satisfactory agreement with tests. Therefore, equation (3.30) can be applied for determination of the contact deformation in various type of roller bearings. It also shows that the diameter of a cylinder has no influence on

the relative approach.

CONTACT PRESSURE AND DEFORMATION IN ROLLING BEARINGS

For the convenience of application in rolling bearings, adjustments to the formulae of the Hertzian theory are made. Two cases of the point and line contact are considered separately. In general, various types of ball bearings belong to the point contact. And roller bearings belong to the line contact except lightly loading spherical roller bearings, which are considered as the point contact conditions if the length of their major-axes of the contact ellipses does not exceed their roller length.

Contact pressure and deformation in rolling bearings

Point contact

As mentioned before, the solution of point contact problems can be obtained using equations (3.14) and (3.18) to (3.23). As can be seen, it is more difficult to determine the eccentricity of contact ellipse e from equation (3.25). For the convenience of calculation, the coefficients m_a , m_b , $m_a m_b$ and $2K(e)/(\pi m_a)$ may be found from Table 3.1 upon the curvature function $F(\rho)$. Then the contact pressure and the deformation in a rolling bearing are worked out using equations (3.14) and (3.18) to (3.20).

For rolling bearings made of hardened steel, one may take

$$E_1 = E_2 = 2.07 \times 10^5 \text{ MPa}$$

$$\nu_1 = \nu_2 = 0.3$$

The calculating equations for the contact pressure and the deformation are simplified as

$$a = 0.0236 m_a \left(\frac{Q}{\sum \rho} \right)^{1/3} \quad (3.31)$$

$$b = 0.0236 m_b \left(\frac{Q}{\sum \rho} \right)^{1/3} \quad (3.32)$$

$$p_0 = \frac{858}{m_a m_b} [Q(\sum \rho)^2]^{1/3} \quad (3.33)$$

Table 3.1 Values of m_a , m_b , $m_a m_b$ and $2K/(\pi m_a)$

$F(\rho)$	m_a	m_b	$m_a m_b$	$\frac{2K}{\pi m_a}$	$F(\rho)$	m_a	m_b	$m_a m_b$	$\frac{2K}{\pi m_a}$
0.9995	23.95	0.163	3.91	0.171	0.9845	6.55	0.312	2.04	0.430
0.9990	18.53	0.185	3.43	0.207	0.9840	6.47	0.314	2.03	0.433
0.9985	15.77	0.201	3.17	0.230	0.9835	6.40	0.316	2.02	0.437
0.9980	14.25	0.212	3.02	0.249	0.9830	6.33	0.317	2.01	0.440
0.9975	13.15	0.220	2.89	0.266	0.9825	6.26	0.319	2.00	0.444
0.9970	12.26	0.228	2.80	0.279	0.9820	6.19	0.321	1.99	0.447
0.9965	11.58	0.235	2.72	0.291	0.9815	6.12	0.323	1.98	0.450
0.9960	11.02	0.241	2.65	0.302	0.9810	6.06	0.325	1.97	0.453
0.9955	10.53	0.246	2.59	0.311	0.9805	6.00	0.327	1.96	0.456
0.9950	10.15	0.251	2.54	0.320	0.9800	5.95	0.328	1.95	0.459
0.9945	9.77	0.256	2.50	0.328	0.9795	5.89	0.330	1.94	0.462
0.9940	9.46	0.260	2.46	0.336	0.9790	5.83	0.332	1.93	0.465
0.9935	9.17	0.264	2.42	0.343	0.9785	5.78	0.333	1.92	0.468
0.9930	8.92	0.268	2.39	0.350	0.9780	5.72	0.335	1.92	0.470
0.9925	8.68	0.271	2.36	0.356	0.9775	5.67	0.336	1.91	0.473
0.9920	8.47	0.275	2.33	0.362	0.9770	5.63	0.338	1.90	0.476
0.9915	8.27	0.278	2.30	0.368	0.9765	5.58	0.339	1.89	0.478
0.9910	8.10	0.281	2.28	0.373	0.9760	5.53	0.340	1.88	0.481
0.9905	7.93	0.284	2.25	0.379	0.9755	5.49	0.342	1.88	0.483
0.9900	7.76	0.287	2.23	0.384	0.9750	5.44	0.343	1.87	0.486
0.9895	7.62	0.289	2.21	0.388	0.9745	5.39	0.345	1.86	0.489
0.9890	7.49	0.292	2.19	0.393	0.9740	5.35	0.346	1.85	0.491
0.9885	7.37	0.294	2.17	0.398	0.9735	5.32	0.347	1.85	0.493
0.9880	7.25	0.297	2.15	0.402	0.9730	5.28	0.349	1.84	0.495
0.9875	7.13	0.299	2.13	0.407	0.9725	5.24	0.350	1.83	0.498
0.9870	7.02	0.301	2.11	0.410	0.9720	5.20	0.351	1.83	0.500
0.9865	6.93	0.303	2.10	0.416	0.9715	5.16	0.353	1.82	0.502
0.9860	6.84	0.305	2.09	0.420	0.9710	5.13	0.354	1.81	0.505
0.9855	6.74	0.307	2.07	0.423	0.9705	5.09	0.355	1.81	0.507
0.9850	6.64	0.310	2.06	0.427	0.9700	5.05	0.357	1.80	0.509

(continued)

$F(\rho)$	m_a	m_b	$m_a m_b$	$\frac{2K}{\pi m_a}$	$F(\rho)$	m_a	m_b	$m_a m_b$	$\frac{2K}{\pi m_a}$
0.969	4.98	0.359	1.79	0.513	0.928	3.55	0.428	1.52	0.630
0.968	4.92	0.361	1.78	0.518	0.926	3.51	0.431	1.51	0.634
0.967	4.86	0.363	1.77	0.522	0.924	3.47	0.433	1.50	0.638
0.966	4.81	0.365	1.76	0.526	0.922	3.43	0.436	1.50	0.642
0.965	4.76	0.367	1.75	0.530	0.920	3.40	0.438	1.49	0.646
0.964	4.70	0.369	1.74	0.533	0.918	3.36	0.441	1.48	0.650
0.963	4.65	0.371	1.73	0.536	0.916	3.33	0.443	1.47	0.653
0.962	4.61	0.374	1.72	0.540	0.914	3.30	0.445	1.47	0.657
0.961	4.56	0.376	1.71	0.543	0.912	3.27	0.448	1.46	0.660
0.960	4.51	0.378	1.70	0.546	0.910	3.23	0.450	1.45	0.664
0.959	4.47	0.380	1.70	0.550	0.908	3.20	0.452	1.45	0.667
0.958	4.42	0.382	1.69	0.553	0.906	3.17	0.454	1.44	0.671
0.957	4.38	0.384	1.68	0.556	0.904	3.15	0.456	1.44	0.674
0.956	4.34	0.386	1.67	0.559	0.902	3.12	0.459	1.43	0.677
0.955	4.30	0.388	1.67	0.562	0.900	3.09	0.461	1.42	0.680
0.954	4.26	0.390	1.66	0.565	0.895	3.03	0.466	1.41	0.688
0.953	4.22	0.391	1.65	0.568	0.890	2.97	0.471	1.40	0.695
0.952	4.19	0.393	1.65	0.571	0.885	2.92	0.476	1.39	0.702
0.951	4.15	0.394	1.64	0.574	0.880	2.86	0.481	1.38	0.709
0.950	4.12	0.396	1.63	0.577	0.875	2.82	0.485	1.37	0.715
0.943	4.05	0.399	1.62	0.583	0.870	2.77	0.490	1.36	0.721
0.946	3.99	0.403	1.61	0.588	0.865	2.72	0.494	1.35	0.727
0.944	3.94	0.406	1.60	0.593	0.860	2.68	0.498	1.34	0.733
0.942	3.88	0.409	1.59	0.598	0.855	2.64	0.502	1.33	0.739
0.940	3.83	0.412	1.58	0.603	0.850	2.60	0.507	1.32	0.745
0.938	3.78	0.415	1.57	0.608	0.840	2.53	0.515	1.30	0.755
0.936	3.73	0.418	1.56	0.613	0.830	2.46	0.523	1.29	0.765
0.934	3.68	0.420	1.55	0.618	0.820	2.40	0.530	1.27	0.774
0.932	3.63	0.423	1.54	0.622	0.81	2.35	0.537	1.26	0.783
0.930	3.59	0.426	1.53	0.626	0.80	2.30	0.544	1.25	0.792

(continued)

$F(\rho)$	m_a	m_b	$m_a m_b$	$\frac{2K}{\pi m_a}$	$F(\rho)$	m_a	m_b	$m_a m_b$	$\frac{2K}{\pi m_a}$
0.75	2.07	0.577	1.20	0.829	0.350	1.29	0.796	1.03	0.971
0.70	1.91	0.607	1.16	0.859	0.300	1.24	0.824	1.02	0.979
0.65	1.77	0.637	1.13	0.884	0.250	1.19	0.850	1.01	0.986
0.60	1.66	0.664	1.10	0.904	0.200	1.15	0.879	1.01	0.991
0.55	1.57	0.690	1.08	0.922	0.150	1.11	0.908	1.01	0.994
0.50	1.48	0.718	1.06	0.938	0.100	1.07	0.938	1.00	0.997
0.45	1.41	0.745	1.05	0.951	0.050	1.03	0.969	1.00	0.999
0.40	1.35	0.771	1.04	0.962	0	1	1	1	1

$$\delta = 2.79 \times 10^{-4} \frac{2K}{\pi m_a} (Q^2 \sum \rho)^{1/3} \quad (3.34)$$

In equations (3.31) to (3.34), units used are as follows: a , b , δ in mm, ρ in $1/\text{mm}$, Q in Newtons and p_0 in MPa.

Example, an angular contact ball bearing 7208, pitch diameter $D_m = 60$ mm, ball diameter $D_b = 12.7$ mm, curvature radius of the inner groove $r_i = 6.54$ mm, raceway diameter at the contact point with the inner ring $D_i = 48.6$ mm, contact angle $\alpha = 25^\circ$, contact load between a ball and the inner raceway $Q = 1765$ N. Calculate the maximum contact pressure and the sizes of the contact ellipse.

Calculating the curvature sum and the curvature function gives

$$\sum \rho = 0.199 \quad 1/\text{mm}$$

$$F(\rho) = 0.955$$

From Table 3.1

$$m_a = 4.30, \quad m_b = 0.388, \quad m_a m_b = 1.67$$

The maximum contact pressure

$$\begin{aligned} p_0 &= \frac{858}{m_a m_b} [Q(\sum \rho)^2]^{1/3} \\ &= \frac{858}{1.67} (1765 \times 0.199^2)^{1/3} \end{aligned}$$

$$= 2116 \text{ MPa}$$

The semi-major and the semi-minor axes of the contact ellipse

$$\begin{aligned} a &= 0.0236 m_a \left(\frac{Q}{\sum \rho} \right)^{1/3} \\ &= 0.0236 \times 4.30 \times \left(\frac{1765}{0.199} \right)^{1/3} \\ &= 2.10 \text{ mm} \end{aligned}$$

$$\begin{aligned} b &= 0.0236 m_b \left(\frac{Q}{\sum \rho} \right)^{1/3} \\ &= 0.0236 \times 0.388 \times \left(\frac{1765}{0.199} \right)^{1/3} \\ &= 0.19 \text{ mm} \end{aligned}$$

Line contact

Similarly, the calculation equations of the contact pressure and the deformation for roller bearings made of hardened steel are

$$b = 3.34 \times 10^{-3} \left(\frac{Q}{\sum \rho} \right)^{1/2} \quad (3.35)$$

$$p_0 = 190.6 \left(\frac{Q \sum \rho}{l} \right)^{1/2} \quad (3.36)$$

$$\delta = 3.83 \times 10^{-5} \frac{Q^{0.9}}{l^{0.8}} \quad (3.37)$$

In equation (3.35) to (3.37), units used are as follows: b and l , i.e. the effective length of rollers l_e in mm, ρ in $1/\text{mm}$, Q in Newtons and p_0 in MPa.

Modification for different materials

Introduce the following modified coefficient

$$C_E = \frac{E}{2(1-\nu^2)} \left(\frac{1-\nu_1^2}{E_1^2} + \frac{1-\nu_2^2}{E_2^2} \right) \quad (3.38)$$

in which, E , E_1 , E_2 , and ν , ν_1 , ν_2 express the moduli of elasticity and Poisson ratios for bearing steel and materials 1 and 2 respectively. Values of the modified coefficients C_E for different materials are listed in Table 3.2. Equations (3.31) to (3.37) can still be applied to the calculation of roller bearings made of different materials, but it is necessary to add modified factors given in Table 3.3.

Table 3.2 Modified coefficients C_E

Materials		C_E
1	2	
Steel	Steel	1
	Cast iron	1.59
	Bronze	1.46
	Brass	1.68
Cast iron	Cast iron	2.18
	Bronze	2.06
	Brass	2.27
Bronze	Bronze	1.93
	Brass	2.16
Brass	Brass	2.36

Table 3.3 Modified factors for different materials

Contact types	Required quantities	Equations	Modified factors
Point contact	a	(3.31)	$C_E^{1/3}$
	b	(3.32)	$C_E^{1/3}$
	p_0	(3.33)	$C_E^{2/3}$
	δ	(3.34)	$C_E^{0.9}$
Line contact	b	(3.35)	$C_E^{1/2}$
	p_0	(3.36)	$C_E^{1/2}$
	δ	(3.37)	$C_E^{0.9}$

For example, equations (3.31) and (3.32) respectively are

$$a = 0.0236m_a C_E^{1/3} \left(\frac{Q}{\sum \rho} \right)^{1/3}$$

$$b = 0.0236m_b C_E^{1/3} \left(\frac{Q}{\sum \rho} \right)^{1/3}$$

Computer programs COSDE

Design the computer programs upon equations (3.31) to (3.38). Symbols and their explanations are listed in Table 3.4.

Table 3.4 Symbols in the program

Symbols	Variables	Explanation
DB	D_b	Rolling element diameter or mean diameter for tapered roller
DBL	D_{bl}	Two times of the curvature radius of a rolling element in the bearing longitudinal section
	l	Effective length of a roller
DEI	D_e or D_i	Raceway diameter at the contact point
REI	R_e or R_i	Groove curvature radius of the raceway
AFA	α	Contact angle, radian
BETA	β	Half taper of a roller, radian. For other rolling element, $\beta = 0$
J		J = 2 for the contact with the inner ring J = 1 for the contact with the outer ring
AFAI	α_i	Contact angle between a roller and inner raceway
E1, E2	E_1, E_2	Modulus of elasticity
V1, V2	ν_1, ν_2	Poisson ratio
QMA	Q or q	Contact load
LINC		Contact types, value 0.0 for point contact, 1.0 for line contact
THU	$\sum \rho$	
AA	a	
BB	b	
PMAX	p_0	
DELTA	δ	

Two groups of input data

- (1) Geometric data, including the roller diameter, effective length of a

roller, raceway diameter, groove curvature radius, contact angle, half taper of a roller, the constants marked the inner or outer ring, etc.

- (2) Mechanical data, including modulus of elasticity, Poisson ratio, the contact load, the contact conditions, etc.

Programs

A program named COSDEA in the FORTRAN language is presented in Table 3.5. It is composed of a main program and an interpolation subroutine, which is used for determining coefficients. The output data are a , b , p_0 and δ .

Determining the coefficients by interpolation may cause some deviations due to the capacity limitation of the input data group. To improve on this, a program named COSDEB in the PASCAL language is designed, shown in Table 3.6. Symbols used in the program are the same as those in the program COSDEA. This program is written upon the Reference [18], which can determine the eccentricity of contact ellipse e directly through the calculation of the first and second complete elliptic integrals.

Table 3.5 Program COSDEA for the contact pressure

```

PROGRAM COSDEA (INPUT, OUTPUT)
DIMENSION CSTA(40), CMUA(40), CMUB(40), CMDA(40)
DATA(CSTA(I), I=1, 40) / 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.55,
1 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.88, 0.90, 0.91, 0.92, 0.93,
2 0.94, 0.95, 0.955, 0.96, 0.965, 0.97, 0.972, 0.974, 0.976, 0.978, 0.98,
3 0.982, 0.984, 0.986, 0.988, 0.99, 0.992, 0.994, 0.996, 0.997, 0.998,
4 0.999 /
DATA(CMUA(I), I=1, 40) / 1.0, 1.07, 1.15, 1.24, 1.35, 1.48, 1.57,
1 1.66, 1.77, 1.91, 2.07, 2.30, 2.60, 2.86, 3.09, 3.23, 3.40, 3.59, 3.83,
2 4.12, 4.30, 4.51, 4.76, 5.05, 5.20, 5.35, 5.53, 5.72, 5.94, 6.19, 6.47,
3 6.84, 7.25, 7.76, 8.47, 9.46, 11.02, 12.26, 14.25, 18.53 /
DATA(CMUB(I), I=1, 40) / 1.0, 0.938, 0.879, 0.824, 0.771, 0.718,
1 0.690, 0.664, 0.637, 0.607, 0.577, 0.544, 0.507, 0.481, 0.461, 0.450,
2 0.438, 0.426, 0.412, 0.396, 0.388, 0.378, 0.367, 0.357, 0.351, 0.346,
3 0.340, 0.335, 0.328, 0.321, 0.314, 0.305, 0.297, 0.287, 0.275, 0.260,
4 0.241, 0.228, 0.212, 0.185 /
DATA(CMDA(I), I=1, 40) / 1.0, 0.997, 0.991, 0.979, 0.962,
1 0.938, 0.922, 0.904, 0.884, 0.859, 0.829, 0.792, 0.745, 0.709,

```

(continued)

```

2 0.680, 0.664, 0.646, 0.626, 0.603, 0.577, 0.562, 0.546, 0.530,
3 0.509, 0.500, 0.491, 0.481, 0.470, 0.459, 0.447, 0.433, 0.420,
4 0.402, 0.384, 0.362, 0.336, 0.302, 0.279, 0.249, 0.207 /
READ(9, 501) DB, DBL, DEI, REI, AFA, BETA, J
READ(9, 502) E1, E2, V1, V2, QMA, LINC
IF (LINC.EQ.0.0) GO TO 90
READ(9, 503) AFEI
90 WRITE(12, 500)
ROX1=2 * COS(BETA) / DB
ROX2=2 * (-1) * J * COS(AFA) / DEI
IF (LINC.EQ.1) GO TO 100
ROY1=2 / DBL
ROY2=-1 / REI
WRITE(12, 504) QMA
A=ROX1+ROX2
B=ROY1+ROY2
THU=A+B
CST=(A-B) / THU
CALL CLI(CST, CMA, CSTA, CMUA, 40)
CALL CLI(CST, CMB, CSTA, CMUB, 40)
CALL CLI(CST, CMD, CSTA, CMDA, 40)
PI=3.14159
EP=(1.0-V1 * * 2) / E1+(1.0-V2 * * 2) / E2
AA=CMA * (1.5 * QMA * EP / THU) * * (1 / 3)
BB=CMB * (1.5 * QMA * EP / THU) * * (1 / 3)
DELTA=CMD * ((1.5 * EP * QMA) * * 2 * THU / 8) * * (1 / 3)
PMAX=1.5 * QMA / (A * B * PI)
WRITE(12, 505) AA, BB
GO TO 110
100 QMA=QMA / DBL
WRITE(12, 506) QMA
BB=SQRT(4 * EP * QMA / (PI * THU))
WRITE(12, 507) BB
DELTA=3.81 * (EP / PI) * * 0.9 * (QMA) * * 0.9 * DBL * * 0.1
PMAX=SQRT(2 * QMA / EP * THU / PI)
110 WRITE(12, 508) DELTA
WRITE(12, 509) PMAX
500 FORMAT(40X, 25HRESULT OF CONTACT STRESS)

```

(continued)

```

501 FORMAT(7E10.3)
502 FORMAT(6E10.3)
503 FORMAT(F10.6)
504 FORMAT(3HQ, 4IX, E12.5)
505 FORMAT(35H HALF HERTZ WIDTH(ROLLING DIR.),
1 MM, 9X, E12.5, 8X, 38H HALF HERTZ WIDTH (PERP
2 TO ROLLING), MM, 2X, E12.5)
506 FORMAT (33H EQUIV. LINE CONTACT LOAD (MPA),
1 11X, E12.5)
507 FORMAT (35H HALF HERTZ WIDTH (ROLLING DIR.),
1 MM, 9X, E12.5)
508 FORMAT (20H CONTACT APPROACH, MM, 24X, E12.5)
509 FORMAT (24H HERTZ PRESSURE (MPA), 20X,
1 E12.5)
STOP
END
SUBROUTINE CLI(X, Y, XO, YO, N)
DIMENSION XO(N), YO(N)
- IF(N-1)1, 2, 3
1 Y=0
GO TO 50
2 Y=YO(1)
GO TO 50
3 ML=1
MU=N
8 IF(MU-ML-1)15, 15, 9
9 M=(MU+ML)/2
IF(XO(1)-XO(2))11, 2, 10
10 IF(XO(M)-X)13, 12, 14
11 IF(X-XO(M))13, 12, 14
12 Y=YO(M)
GO TO 50
13 MU=M
GO TO 8
14 ML=M
GO TO 8
15 Y=YO(ML)+(YO(MU)-(YO(ML))*((X-XO(ML))/
1 (XO(HU)-XO(ML)))

```

(continued)

```

50 RETURN
END

```

Table 3.6 Program COSDEB for the contact pressure

```

PROGRAM COSDEB
CONST PI=3.14159; EPS=1E-6; N=16; NN=14; NN1=15;
TYPE SERIES=ARRAY [1..N] OF REAL;
VAR I: INTEGER;
DB, DM, REI, AFA, R1, R2, R, QMA, SR, C, AM, A, T, TI, U, W, E,
EI, D, V, ANA, ANB, AKC, AMC, ANC, ACC, J, AH, PMA, AA,
BB, DELTA: REAL; AK, BL, BK: SERIES;
FUNCTION AKNE(KK: INTEGER; P: REAL; AMAS: SERIES): REAL;
VAR AKNEI: REAL;
BEGIN
AKNEI:=1;
FOR I:=1 TO KK DO
AKNKI:=AKNEI+AMAS[I]*EXP((2*D*LN(P)));
AKNE:=PI*AKNEI/2
END;
FUNCTION AKNT(KK: INTEGER; P: REAL; AMAS, BMAS: SERIES):
REAL;
VAR AKNTI: REAL;
BEGIN
AKNTI:=LN(4/P);
FOR I:=1 TO KK DO
AKNTI:=AKNTI+AMAS[I]*(LN(4/P)-BMAS[I])*EXP
((2*I)*LN(P));
AKNT:=AKNTI
END;
FUNCTION ALNE(KK: INTEGER; P: REAL; AMAS, BMAS: SERIES):
REAL;
VAR ALNEI: REAL;
BEGIN
ALNEI:=1;
FOR I:=1 TO KK DO
ALNEI:=ALNEI+AMAS[I]/(2*I-1)*EXP((2*I)*LN(P));
ALNE:=PI*ALNEI/2
END;

```

(continued)

```

FUNCTION ALNT(KK: INTEGER; P: REAL; AMAS, BMSA: SERIES):
  REAL;
VAR ALNTI: REAL;
BEGIN
  ALNTI := 1;
  FOR I := 1 TO KK DO
    ALNTI := ALNTI + 2 * AMAS[I] * I * (LN(4 / P) - BMSA[I]) * EXP
      ((2 * I) * LN(P)) / (2 * I - 1);
    ALNT := ALNTI
  END;
FUNCTION AN(X, C: REAL): REAL;
BEGIN
  AN := EXP(1 / 3) * LN(2 * X * (C + 1) / C / PI)
END;
FUNCTION BN(X, Y, Z, C: REAL): REAL;
BEGIN
  BN := EXP((1 / 3) * LN(2 * (C + 1) * (X - Y) * Z / PI))
END;
PROCEDURE BASIC;
BEGIN
  IF J = 1 THEN R := 1 ELSE R := -1;
  R1 := DB / REI / 2;
  R2 := 1 + R * DM / DB / COS(AFA);
  SR := EXP((1 / 3) * LN(4 - 2 * (R1 + 1 / R2)));
  C := (1 - R1) * (1 + 1 / (R2 - 1))
END;
PROCEDURE TAD;
BEGIN
  T1 := SQRT(1 - E1 * E1);
  A := AKNE(NN, E1, AK);
  D := (A - ALNE(NN, E1, AK)) / (E1 * E1)
END
PROCEDURE UWT;
BEGIN
  U := 8 * (2 * (C + 2) * LN(4 / T) + C - 4) / (4 * (C - 4)
    * LN(4 / T) - 3 * C);
  W := 64 * (((C + 1) * (AKNT(NN, T, AK, BK) - AKNT(1,
    T, AK, BK)) - ALNT(NN, T, AK, BL) + ALNT(1, T, AK, BL))

```

(continued)

```

  * (T * T) - C * (ALNT(NN + 1, T, AK, BL) - ALNT(2, T, AK,
    BL) + 1)) / (4 * (C - 4) * LN(4 / T) - 3 * C);
  T1 := SQRT(-U - SQRT((U * U) - W))
END;
PROCEDURE TAD;
BEGIN
  E1 := SQRT(1 - T1 * T1);
  A := AKNT(NN, T1, AK, BK);
  D := (A - ALNT(NN, T1, AK, BL)) / (E1 * E1)
END;
PROCEDURE VEI;
BEGIN
  V := 64 * (C - 1) / (C + 3) / 3 - 256 / (C + 3) / PI / 3 *
    (ALNE(NN, E, AK) - ALNE(2, E, AK) - (C + 1) * (AKNE
      (NN, E, AK) - AKNE(2, E, AK)) + (C + 1) * (AKNE(NN + 1,
      E, AK) - AKNE(3, E, AK) - ALNE(NN + 1, E, AK) + ALNE
      (3, E, AK)) / (E * E));
  E1 := SQRT(SQRT(16 * ((C + 5) / (C + 3)) * ((C + 5) / (C
    + 3)) / (9 - V) - 4 * (C + 5) / (C + 3) / 3)
END;
BEGIN
  READ(DB, DM, REI, AFA, J, QMA);
  BASIC;
  A := 0; AM := 1;
  IF C = 1 THEN BEGIN
    E1 := 0; TAD;
  END
  ELSE BEGIN
    FOR I := 1 TO 15 DO
      BEGIN
        AM := AM * (2 * I - 1) / I;
        A := A + 1 / (2 * I - 1);
        AK[I] := (AM / (EXP(1 * LN
          (2)))) * (AM / (EXP(1 * LN
          (2))));
        BK[I] := A;
        BL[I] := A - 5 / I / (2 * I - 1)
      END
    END
  END

```

(continued)

```

END;
IF C < 0.4 THEN BEGIN
    T = SQRT(-2 * C / LN(C / 16));
    UWT;
    WHILE ABS(T1 - T) > EPS DO
        BEGIN
            T = T1; UWT;
        END;
    TAD;
END;
ELSE BEGIN
    E = SQRT(4 * (C+5) * (SQRT
    (1+12 * (1-C) * (C+3) / ((C+5)
    * (C+5))) - 1) / (C+3) / 3);
    VE1;
    WHILE ABS(E1 - E) > EPS DO
        BEGIN
            E = E1; VE1;
        END;
    TAD;
END;
ANA = AN(D, C); ANB = BN(A, D, T1, C);
AKC = 857.668 / ANA / ANB * (SR * SR);
AMC = 0.02359 * ANA / SR;
ANC = 0.02359 * ANB / SR;
ACC = 2.7838E-3 * A / ANA / PI * SR;
AH = EXP((1 / 3) * LN(QMA / (DB * DB)));
PMA = AKC * AH;
AA = AMC * AH * DB;
BB = ANC * AH * DB;
DELTA = ACC * (AH * AH) * DB;
WRITELN('HERTZ STRESS IS', PMA);
WRITELN('CONTACT AREA OF', AA, 'AND', BB);
WRITELN('CONTACT APPROACH IS', DELTA);
END.

```

It should be explained that the program COSDEA in Table 3.5 is valid for all types of ball and roller bearings without any limitation on

bearing materials, but the program COSDEB is only valid for ball bearings made of steel.

Following units are used in the programs

$DB = D_b$, mm;
 $DM = D_m$, mm;
 $AA = a$, mm;
 $BB = b$, mm;
 $REI = R_e$ or R_p , mm;
 $DELTA = \delta$, mm;
 $AFA = \alpha$, radian;
 $J = 1$ for outer ring, 2 for inner ring;
 $QMA = Q$, N;
 $PMA = p_0$, MPa.

STRESS DISTRIBUTION BELOW CONTACT SURFACE

Investigations indicate that the contact fatigue of a rolling bearing is a process of the initiation and the propagation of fatigue cracks. They initiate either from a point below contact surfaces or from a point at the surfaces. Besides defects at or below the surfaces, the initiation and the propagation are mainly effected by the stress distribution below the

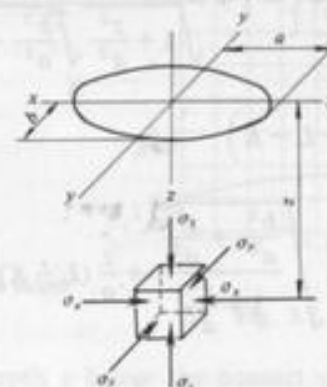


Fig. 3.5 Principal stresses

surfaces, especially the shear stress distribution. Therefore, it is important to analyse the stress condition below a contact surface in order to improve the reliability and the life of rolling bearings.

Maximum static shear stress

Fig.3.5 shows an elastic half-space subjected to the action of the contact pressure. Analyses indicate that the maximum static shear stress occurs at a certain depth along the z axis below the contact surface.

Easy to find, shear stresses τ_{xy} , τ_{yz} , τ_{zx} along the z axis are equal to zero owing to the symmetry. The stress expressions along the z axis are given from Reference [18].

$$\sigma_x = -p_0 \frac{b}{a} \left\{ 1 - \sqrt{\frac{\frac{b^2}{a^2} + \frac{z^2}{a^2}}{1 + \frac{z^2}{a^2}}} + 2\frac{z}{a}(L - K) \right. \\ \left. - 2\nu \left[1 - \frac{a^2}{b^2} \sqrt{\frac{\frac{b^2}{a^2} + \frac{z^2}{a^2}}{1 + \frac{z^2}{a^2}}} + \frac{z}{a} \left(\frac{a^2}{b^2} L - K \right) \right] \right\} \quad (3.39)$$

$$\sigma_y = -p_0 \frac{b}{a} \left\{ -1 + \frac{1 + \frac{z^2}{a^2} \left(2\frac{a^2}{b^2} - 1 \right)}{\sqrt{1 + \frac{z^2}{a^2}} \sqrt{\frac{b^2}{a^2} + \frac{z^2}{a^2}}} \right. \\ \left. - 2\frac{z}{a} \left(\frac{a^2}{b^2} L - K \right) \right. \\ \left. + 2\nu \left[1 - \sqrt{\frac{\frac{b^2}{a^2} + \frac{z^2}{a^2}}{1 + \frac{z^2}{a^2}}} + \frac{z}{a}(L - K) \right] \right\} \quad (3.40)$$

$$\sigma_z = -p_0 \frac{1}{\sqrt{1 + \frac{z^2}{a^2}} \sqrt{1 + \frac{z^2}{b^2}}} \quad (3.41)$$

where

K, L = first and second elliptic integrals respectively

$$K(e, \psi) = \int_0^\psi \frac{d\phi}{\sqrt{1 - e^2 \sin^2 \phi}} \quad (3.42)$$

$$L(e, \psi) = \int_0^\psi \sqrt{1 - e^2 \sin^2 \phi} d\phi \quad (3.43)$$

ψ = parameter, that is

$$\psi = \arctg \frac{z}{a} \quad (3.44)$$

e = eccentricity of contact ellipse, expressed by equation (3.12).

The calculation gives that the maximum shear stress τ_{xy} is

$$\tau_{xy} = \frac{1}{2}(\sigma_z - \sigma_y)$$

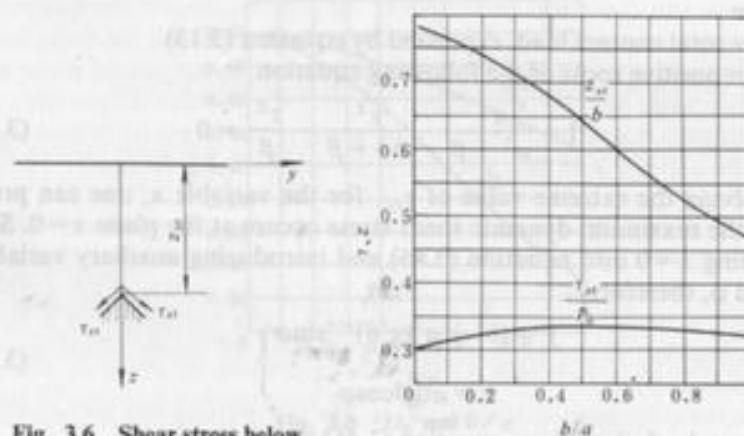


Fig. 3.6 Shear stress below the surface

Fig. 3.7 τ_{xy} and b/a

It acts at a certain depth z below the contact surface, as shown in Fig.3.6. τ_{xy} is called a maximum static shear stress.

The magnitude of the maximum static shear stress τ_{xy} and its existing depth are connected with the ratio b/a of the contact ellipse shown in Fig.3.7. τ_{xy} reaches its maximum value of $0.325p_0$ at a depth z of $0.62b$ when b/a is approximately equal to 0.5.

The shear stress of a point positioning at a depth z varies between zero and τ_{xy} while a roller rolls over the contact surface.

Maximum dynamic shear stress

Lundberg and Palmgren indicated that if a rolling element rolls along the direction of the y axis shown in Fig.3.5, the shear stress parallel to the rolling direction below the surface will vary according to an alternate stress pattern. At a certain depth, its amplitude reaches its maximum value, called a maximum dynamic shear stress or a maximum orthogonal shear stress. This depth is expressed as z_0 .

Analyses deduce that the shear stress at a point (x, y, z) below the contact surface is

$$\tau_{xy} = -\frac{3Qyz^2}{2\pi} \frac{[(b^2 + \beta_0)\beta_0]^{-3/2}(a^2 + \beta_0)^{-1/2}}{\left[\left(\frac{x}{a^2 + \beta_0}\right)^2 + \left(\frac{y}{b^2 + \beta_0}\right)^2 + \left(\frac{z}{\beta_0}\right)^2\right]} \quad (3.45)$$

where

Q = total contact load, expressed by equation (3.13)

β_0 = positive roots of the following equation

$$1 - \frac{x^2}{a^2 + \beta} - \frac{y^2}{b^2 + \beta} - \frac{z^2}{\beta} = 0 \quad (3.46)$$

Solving the extreme value of τ_{xy} for the variable x , one can prove that the maximum dynamic shear stress occurs at the plane $x=0$. Substituting $x=0$ into equation (3.45) and introducing auxiliary variables θ and φ , therefore

$$\begin{cases} y = (b^2 + a^2 \operatorname{tg}^2 \theta)^{1/2} \sin \varphi \\ z = a \operatorname{tg} \theta \cos \varphi \end{cases} \quad (3.47)$$

It can be derived from equation (3.46) that

$$\beta_0 = a^2 \operatorname{tg}^2 \theta$$

Then, equation (3.45) becomes

$$\tau_{xy} = \frac{3Q}{2\pi} \frac{\cos^2 \varphi \sin \varphi \sin \theta}{a^2 \operatorname{tg}^2 \theta + b^2 \cos^2 \varphi} \quad (3.48)$$

Calculating the extreme value of τ_{xy} for the auxiliary variables θ and φ , the maximum shear stress τ_0 at the depth z_0 below the contact sur-

face and its distance $\pm y_0$ in the rolling direction are obtained

$$\left. \begin{aligned} \frac{z_0}{b} &= \frac{1}{(t+1)\sqrt{2t-1}} \\ \frac{y_0}{b} &= \frac{t}{t+1} \sqrt{\frac{2t+1}{2t-1}} \\ \frac{\tau_0}{p_0} &= \frac{\sqrt{2t-1}}{2t(t+1)} \end{aligned} \right\} \quad (3.49)$$

where

t = auxiliary parameter, determined by the follow equation upon the sizes of the contact surface

$$\frac{b}{a} = \sqrt{(t^2 - 1)(2t - 1)} \quad (3.50)$$

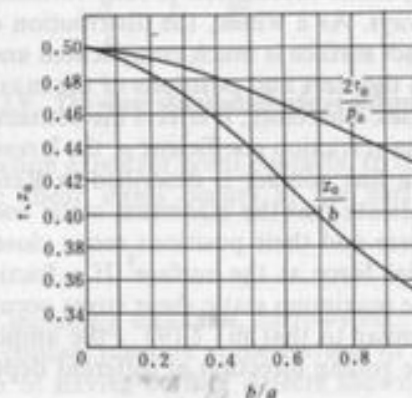


Fig. 3.8 $2\tau_0$ and b/a

θ and φ are given as follows

$$\left. \begin{aligned} \operatorname{tg}^2 \varphi &= t \\ \operatorname{tg}^2 \theta &= t - 1 \end{aligned} \right\} \quad (3.51)$$

The amplitude of maximum dynamic shear stress τ_0 or the depth z_0 as a function of b/a is shown in Fig.3.8. For any value of b/a , $2\tau_0$ is larger than τ_{xy} . On the other hand, the absolute value of τ_{xy} is greater

than τ_0 .

Investigations indicate that the maximum dynamic shear stress τ_0 is the most significant factor in the initiation and the propagation of contact fatigue cracks. The depth z_0 is also important to bearing designs. If the bearing is made of carbonized steel, the thickness of its carbonized layer should be deeper than z_0 .

INFLUENTIAL FACTORS ON CONTACT STRESS

Surface tangential force

In the foregoing, only the load applied perpendicular to the surface was considered. Actually, various types of sliding, including the orbital and spinning slips, as well as the microslip at the surface, occur between rolling elements and raceways in rolling bearings. Obviously, any kind of sliding is accompanied by a tangential frictional force at the surface. On the other hand, a tangential frictional force also exists if the elastohydrodynamic lubrication is fully formed between rolling elements and raceways. As a whole, the distribution of the tangential force over the contact surface is much complicated and the force has a significant effect on the sizes and positions of the maximum static and dynamic shear stresses. Therefore, it effects the contact fatigue life.

Assuming a constant friction coefficient μ , the stress distribution below the surface in a line contact is described in References (19), (20). Analyses indicate that the maximum static and dynamic shear stress tend to increase and their positions move closer to the surface due to the tangential force at the surface. If a friction coefficient is larger than $1/9$, the maximum static shear stress occurs at the surface. Using a method similar to that in (19), the amplitude of a shear stress parallel to the rolling direction at different depths is computed, as shown in Fig.3.9.

For the convenience of calculation, the shear stress can be expressed as follows (21)

$$\frac{\tau_{xy}}{p_0} = C_1 \left(\frac{z}{b} \right)^{C_2} \exp \left[-C_3 D \left(\frac{z}{b} \right) \right] \quad (3.52)$$

where

C_1 , C_2 , and D = coefficients, listed in Table 7.3.

It can be seen that the maximum dynamic shear stress increases and its position moves closer to the surface as the friction coefficient increases. The shear stress reaches $0.285p_0$ at the friction coefficient

0.11.



Fig. 3.9 The shear stress amplitude at different depths

A similar variation exists for point contacts. In the point contact, the maximum static shear stress occurs at the surface at the friction coefficient 0.25.

Surface defects

Surface defects, including scratches, furrows etc., have a remarkable influence on the contact pressure at the surface. In Reference (20), a contact problem of having surface defects shown in Fig.3.10 is analysed. Following assumptions are proposed.

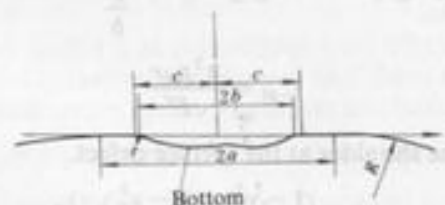


Fig. 3.10 A surface defect

- (1) A plane strain condition is approximated.
 - (2) The width and the depth of the defect are very small. It can be thought of as an elastic contact of two half-planes.
 - (3) At a point far away from the defect, i.e. $x \rightarrow \infty$, the contact pressure tends to equal the value p_0 which is the one without defect.
- The width of a surface defect is expressed by $2c$. The contact region is

$$a > |x| > b$$

The shape of a defect bottom has no influence on the contact pressure when it has not entered into contact. Using the solving process of the complex variable function in the theory of elasticity, the pressure at the contact surface is obtained

$$p(x) = \left(\frac{E'}{2\pi^2 r} \right) [I(x) - I(-x)] \quad (3.53)$$

where

$$I(x) = (x - c) \ln \left| \frac{(x + b)\eta - \sqrt{x^2 - b^2}}{(x + b)\eta + \sqrt{x^2 - b^2}} \right|$$

$$\eta = \operatorname{tg} \left(0.5 \cos^{-1} \frac{b}{c} \right)$$

b = lower shoulder of the contact region, it can be decided using the following equation

$$\ln \left[\sqrt{\left(\frac{c}{b} \right)^2 - 1} + \frac{c}{b} \right] - \frac{\sqrt{\left(\frac{c}{b} \right)^2 - 1}}{\frac{c}{b}} = C_{01}$$

$$C_{01} = \frac{\pi^2 p_0 r}{c E'}$$

r = radius of the shoulder at the surface defect.

$$E' = \left(\frac{1 - \nu_1^2}{\pi E_1} + \frac{1 - \nu_2^2}{\pi E_2} \right)^{-1}$$

Fig.3.11 shows typical curves. In which, C_{01} is a dimensionless parameter. $P_{\max}$ approximately is equal to $4p_0$ when $C_{01} = 0.01$. The calculation of the shear stress below the surface reveals that the stress reaches its maximum value $P_{\max}$ at a point x_0 accompanied by severe plastic deformation.

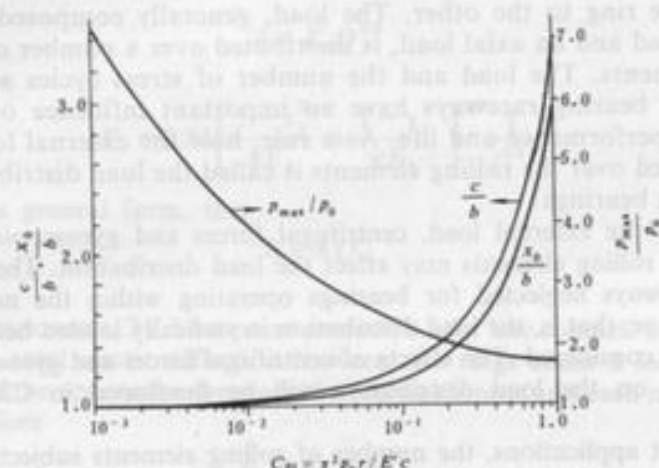


Fig. 3.11 Typical curves

CHAPTER FOUR

Load distribution in rolling bearings

The rolling elements in a rolling bearing transmit the external load from one ring to the other. The load, generally composed of a radial load and an axial load, is distributed over a number of rolling elements. The load and the number of stress cycles at one point of bearing raceways have an important influence on the bearing performance and life. As a rule, how the external load is distributed over the rolling elements is called the load distribution in rolling bearings.

Besides the external load, centrifugal forces and gyroscopic moments of rolling elements may effect the load distribution. They are nearly always neglected for bearings operating within the normal speed range; that is, the load distribution in statically loaded bearings would be considered. The effects of centrifugal forces and gyroscopic moments on the load distribution will be developed in Chapter Sixteen.

In most applications, the number of rolling elements subjected to the load in a rolling bearing is over two, so that the support is a statically indeterminate system. To analyse this, it is first necessary to introduce load-deflection relationships for a contact between rolling elements and raceways. The deflections considered in the determination of a load distribution are contact deformations only. It is assumed that the rolling bearings are supported in stiff housings and rigid shafts. For a rolling bearing with a flexible ring or in a flexible housing, the deformation of the ring bending or the housing has to be taken into account. Generally, finite element methods may lead to satisfactory solutions.

In this chapter, the load distribution in a rolling bearing subjected to a radial or an axial load is explained at first. Then that under the action of radial, axial and moment loading is considered.

LOAD-DEFLECTION RELATION

It can be seen from equation (3.20), for point contact

$$\delta = K_p Q^{2/3} \quad (4.1)$$

where

$$K_p = \frac{2K(e)}{\pi m_e} \sqrt{\frac{1}{8} \left[\frac{3}{2} \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right) \right]^2 \Sigma \rho} \quad (4.2)$$

Similarly, for line contact

$$\delta = K_l Q^{0.9} \quad (4.3)$$

where

$$K_l = 3.81 \left[\frac{1-\nu_1^2}{\pi E_1} + \frac{1-\nu_2^2}{\pi E_2} \right]^{0.9} \frac{1}{l^{0.8}} \quad (4.4)$$

In a general form, then

$$\delta = K Q^t \quad (4.5)$$

where

t = exponents, $2/3$ for point contact, 0.9 for line contact.

The total elastic deflection between two rings under a load is the sum of the approaches between the rolling element and each raceway. Therefore

$$\delta_n = \delta_i + \delta_e \quad (4.6)$$

where

δ_n = total elastic deflection between two rings

δ_i = elastic approach between a rolling element and an inner raceway

δ_e = elastic approach between a rolling element and an outer raceway.

Obviously, it can be expressed as

$$\delta_n = K_n Q^t \quad (4.7)$$

where

K_n = coefficient.

For point contact

$$K_n = 2.79 \times 10^{-4} \left[C_k^{2/3} \frac{2K(e)}{\pi m_e} (\Sigma \rho_i)^{1/3} + C_k^{2/3} \frac{2K(e)}{\pi m_e} (\Sigma \rho_o)^{1/3} \right] \quad (4.8)$$

For line contact

$$K_s = 7.66 \times 10^{-5} \frac{C_E^{0.9} Q^{0.9}}{l^{0.8}} \quad (4.9)$$

LOAD DISTRIBUTION IN RADIAL BEARINGS

The load distribution in radial ball or roller bearings under a radial load is analysed. Two conditions with and without clearance are considered separately.

Radial bearings with zero clearance

This was first investigated by Stribeck (1). Assume that the centre of a rolling element is located on the radial loading line, as shown in Fig.4.1. In this case, the rolling elements in the lower half-cycle are loaded.

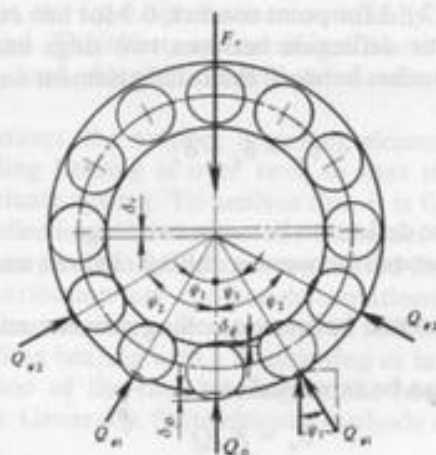


Fig. 4.1 Load distribution in radial bearings

The following symbols are introduced:

- ψ = position angle between the radial loading line and the centre line of a rolling element marked by i shown in Fig.4.1, then, $\psi_0 = 0^\circ$, $\psi_i = 360^\circ i / Z$, $i = 1, 2, \dots, n$
- Q_ψ = rolling element load at the position angle ψ
- δ_r = relative displacement along the radial loading line between two

rings

δ_ψ = total elastic deflection at the position angle ψ

$\delta_{\max}$ = total elastic deflection along the radial loading line due to the contact deformation.

Establishing the static equilibrium condition of the inner ring gives

$$F_r = Q_0 + 2 \sum Q_\psi \cos \psi \quad (4.10)$$

where

F_r = external radial load.

Upon the deformation harmonization, the elastic deflection at the position angle ψ is

$$\delta_\psi = \delta_{\max} \cos \psi \quad (4.11)$$

From equation (4.7), the relation between a rolling element load and its corresponding deflection may be expressed as

$$\frac{Q_\psi}{Q_0} = \left(\frac{\delta_\psi}{\delta_{\max}} \right)^{1/1} \quad (4.12)$$

Substituting equation (4.11) into the previous equation yields

$$Q_\psi = Q_0 \cos^{1/1} \psi \quad (4.13)$$

Combining equations (4.13) and (4.10) gives

$$Q_0 = \frac{F_r}{Z J_r} \quad (4.14)$$

where

$$J_r = \frac{1 + 2 \sum \cos^{1/1} \psi}{Z} \quad (4.15)$$

For different number Z of rolling elements, values of $1/J_r$ may be determined, as listed in Table 4.1. It can be seen that the value of $1/J_r$ tends to be a constant as the symbol Z increases, 4.37 for ball bearings and 4.08 for roller bearings. Therefore, the maximum rolling element load in a radial bearing subjected to a radial load is as follows:

For ball bearings

$$Q_0 = \frac{4.37 F_r}{Z} \quad (4.16)$$

or, for roller bearings

$$Q_0 = \frac{4.08F_r}{Z} \quad (4.17)$$

Table 4.1 Values of $1/J_r$

Number of rolling element	6	7	8	10	12	15	20
Ball bearings	4.43	4.34	4.35	4.38	4.36	4.37	4.36
Roller bearings	4.10	4.03	4.07	4.10	4.08	4.07	4.08

The effect of radial clearance of a radial bearing on the load zone and the rolling element load will be described later. The following equations can be applied for the calculation of the maximum rolling element load in a radial bearing approximately.

For ball bearings

$$Q_0 = \frac{5F_r}{Z} \quad (4.18)$$

For roller bearings

$$Q_0 = \frac{4.6F_r}{Z} \quad (4.19)$$

Radial bearings having radial clearance

A radial bearing having a certain amount of radial clearance u_r is subjected to a radial load F_r , as shown in Fig.4.2. The total deflection at the position angle ψ is

$$\delta_\psi = \left(\delta_{\max} + \frac{u_r}{2} \right) \cos\psi - \frac{u_r}{2}$$

It can be rewritten in the following

$$\delta_\psi = \delta_{\max} \left[1 - \frac{1}{2\varepsilon} (1 - \cos\psi) \right] \quad (4.20)$$

where

$$\varepsilon = \frac{1}{2} \left(1 - \frac{u_r}{2\delta_{\max} + u_r} \right) \quad (4.21)$$

ε = load distribution factor.

Substituting $\delta_\psi = 0$ into equation (4.20), the angular extent of load zone in the bearing can be obtained

$$\psi_L = \arccos \frac{u_r}{2\delta_{\max} + u_r} \quad (4.22)$$

As seen, if $u_r = 0$, then $\varepsilon = 0.5$ and $\psi_L = \pi/2$; or $u_r > 0$, then $\varepsilon < 0.5$ and $\psi_L < \pi/2$; or $u_r < 0$, then $\varepsilon > 0.5$ and $\psi_L > \pi/2$, as shown in Fig.4.3. Obviously, if $\psi_L < 360^\circ/Z$, only one rolling element is loaded.

Substitution of equation (4.20) into equation (4.12) gives the rolling element load at the position angle ψ by

$$Q_\psi = Q_0 \left[1 - \frac{1}{2\varepsilon} (1 - \cos\psi) \right]^{1/\varepsilon} \quad (4.23)$$

Using the equilibrium equation (4.10) leads to

$$F_r = ZQ_0 \frac{\sum \left[1 - \frac{1}{2\varepsilon} (1 - \cos\psi) \right]^{1/\varepsilon} \cos\psi}{Z}$$

The factor on the right side is written as a integral form, then

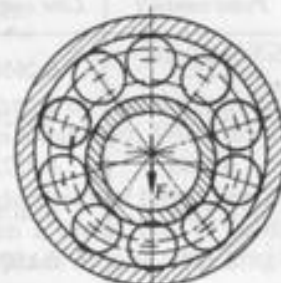


Fig. 4.2 Effect of clearance

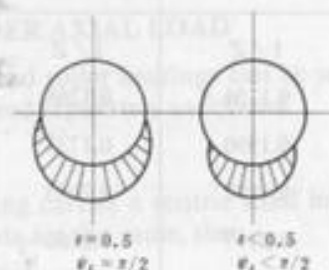


Fig. 4.3 Load distribution factor

$$F_r = ZQ_0 J_r(\varepsilon) \quad (4.24)$$

where

$$J_r(\epsilon) = \frac{1}{2\pi} \int_{-\psi_L}^{\psi_L} \left[1 - \frac{1}{2\epsilon} (1 - \cos\psi) \right]^{1/t} \cos\psi d\psi \quad (4.25)$$

Values of $1/t$ are $3/2$ for ball bearings or 1.1 for roller bearings. The relation between $J_r(\epsilon)$ and ϵ is presented in Table 4.2. It can also be found from Fig. 4.4.

The maximum rolling element load

$$Q_0 = \frac{F_r}{ZJ_r(\epsilon)} \quad (4.26)$$

Given internal geometric parameters, the radial clearance and the radial load of a bearing, the contact loads among its rolling elements will be determined. Considering that the integral value $J_r(\epsilon)$ is connected with $\delta_{\max}$ and $\delta_{\max}$ is required to be calculated from $J_r(\epsilon)$, one may obtain the rolling element load by an iterative method. Its computing process is as follows.

- (1) To compute the coefficient K_n by equations (4.8) and (4.9).
- (2) To select an initial approximate value of the maximum rolling element load. For example, the calculating results from equation (4.16) or (4.17) can be used for the initial value.

Table 4.2 Values of $J_r(\epsilon)$

ϵ	Point contact	Line contact	ϵ	Point contact	Line contact
0	$1/Z$	$1/Z$	0.80	0.2559	0.2658
0.1	0.1156	0.1268	0.90	0.2576	0.2628
0.2	0.1590	0.1737	1.00	0.2546	0.2523
0.3	0.1892	0.2055	1.25	0.2289	0.2078
0.4	0.2117	0.2286	1.67	0.1871	0.1589
0.5	0.2288	0.2453	2.50	0.1339	0.1075
0.6	0.2416	0.2568	5.00	0.0711	0.0544
0.7	0.2505	0.2636	∞	0	0

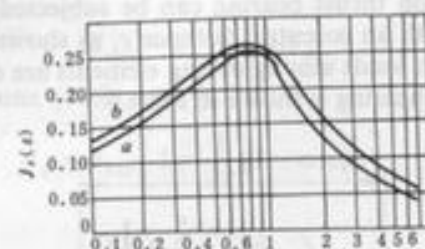


Fig. 4.4 Values of $J_r(\epsilon)$

- (3) To determine the total elastic deflection from equation (4.7) using the approximate value of the maximum rolling element load.
- (4) To decide the load distribution factor ϵ from equation (4.21) upon the given radial clearance u_r and the deflection $\delta_{\max}$.
- (5) To find the corresponding value of $J_r(\epsilon)$ from Table 4.2.
- (6) To establish the first approximate value of the maximum rolling element load Q_0 using equation (4.14) and to compare with its initial value. If their difference exceeds a permissible error, the computation should be returned to the third step and computed again until a satisfactory solution is obtained.
- (7) To compute the rolling element loads.

A program in BASIC language for this computing is listed in Chapter Six.

BEARINGS UNDER AXIAL LOAD

Thrust or angular contact ball and roller bearings can be subjected to an axial load. These are considered separately as follows.

Single direction thrust bearings

When a thrust ball or roller bearing carries a centric axial load, contact loads among its rolling elements are the same, then

$$Q = \frac{F_a}{Z \sin \alpha} \quad (4.27)$$

where

α = actual contact angle of the bearing subjected to the axial load. If

its initial contact angle is less than 20° , the actual contact angle of the bearing is larger than that; it will be described later.

A single direction thrust bearing can be subjected to an eccentric thrust load F_a with an eccentric distance e , as shown in Fig.4.5. Obviously, the contact loads among rolling elements are different. A load distribution in the bearing is shown in Fig.4.6.

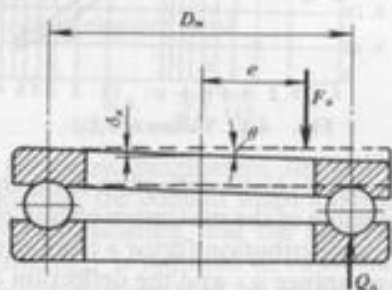


Fig. 4.5 Bearing subjected to an eccentric load

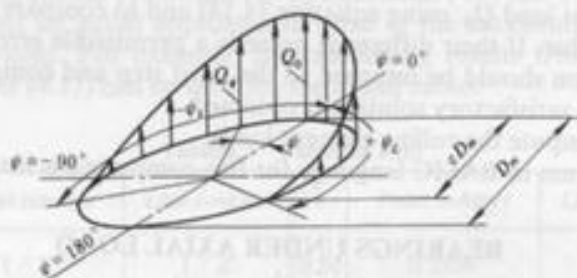


Fig. 4.6 Load distribution in a thrust bearing

Taking a position corresponding to the maximally loaded rolling element as $\psi = 0$, the total axial deflection at a position angle ψ is

$$\delta_\psi = \delta_a + \frac{1}{2} \theta D_m \cos \psi \quad (4.28)$$

where

δ_a = relative approach between two rings along their centric line
 θ = relative tilting angle between two rings shown in Fig.4.5

D_m = pitch diameter.

The total axial deflection of the maximally loaded rolling element is

$$\delta_{\max} = \delta_a + \frac{1}{2} \theta D_m \quad (4.29)$$

From equations (4.28) and (4.29)

$$\delta_\psi = \delta_{\max} \left[1 - \frac{1}{2\varepsilon} (1 - \cos \psi) \right] \quad (4.30)$$

where

$$\varepsilon = \frac{1}{2} \left(1 + \frac{2\delta_a}{\theta D_m} \right) \quad (4.31)$$

The angular extent of load zone

$$\psi_L = \cos^{-1} \left(-\frac{2\delta_a}{\theta D_m} \right) \quad (4.32)$$

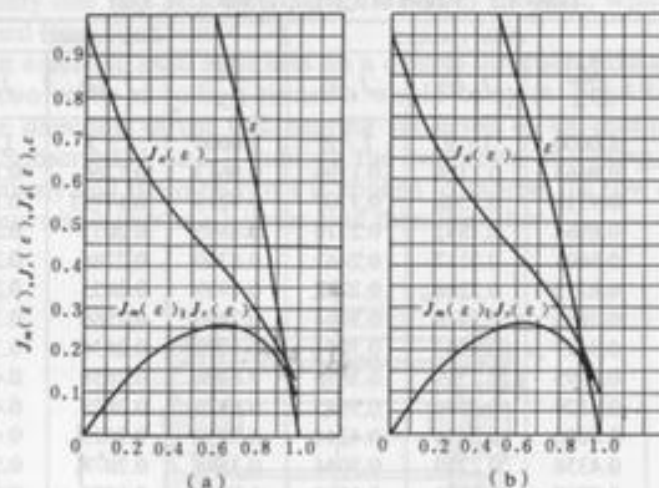


Fig. 4.7 Load distribution integral of single direction thrust bearings. (a) point contact, (b) line contact

Correspondingly, the rolling element load at the position angle ψ is

$$Q_\psi = Q_0 \left[1 - \frac{1}{2\varepsilon} (1 - \cos \psi) \right]^{1/2} \quad (4.33)$$

From equilibrium conditions

$$F_a = 2Q_0 \sum_{\psi=0}^{\psi_L} \left[1 - \frac{1}{2\epsilon} (1 - \cos\psi) \right]^{1/2} \sin\psi$$

$$M = eF_a = \frac{1}{2} D_m \sum_{\psi=0}^{\psi_L} Q_0 \cos\psi \sin\psi$$

Similarly, an axial load and a moment integrals are introduced

$$J_a(\epsilon) = \frac{1}{2\pi} \int_{-\psi_L}^{+\psi_L} \left[1 - \frac{1}{2\epsilon} (1 - \cos\psi) \right]^{1/2} d\psi \quad (4.34)$$

$$J_m(\epsilon) = \frac{1}{2\pi} \int_{-\psi_L}^{+\psi_L} \left[1 - \frac{1}{2\epsilon} (1 - \cos\psi) \right]^{1/2} \cos\psi d\psi \quad (4.35)$$

Table 4.3 Values of $J_m(\epsilon)$, $J_a(\epsilon)$ and $J_s(\epsilon)$, $J_d(\epsilon)$

ϵ	Point contact			Line contact		
	$\frac{2e}{D_m}$	$J_m(\epsilon)$	$J_a(\epsilon)$	$\frac{2e}{D_m}$	$J_m(\epsilon)$	$J_a(\epsilon)$
0.0	1.0000	1/Z	1/Z	1.0000	1/Z	1/Z
0.1	0.9663	0.1156	0.1196	0.9613	0.1268	0.1319
0.2	0.9318	0.1590	0.1707	0.9215	0.17737	0.1885
0.3	0.8964	0.1892	0.2110	0.8805	0.2055	0.2334
0.4	0.8601	0.2117	0.2462	0.8380	0.2286	0.2728
0.5	0.8225	0.2288	0.2782	0.7939	0.2453	0.3090
0.6	0.7835	0.2416	0.3084	0.7480	0.2568	0.3433
0.7	0.7427	0.2505	0.3374	0.6999	0.2636	0.3766
0.8	0.6995	0.2559	0.3658	0.6486	0.2658	0.4098
0.9	0.6529	0.2576	0.3945	0.5920	0.2628	0.4439
1.0	0.6000	0.2546	0.4244	0.5238	0.2523	0.4817
1.25	0.4338	0.2289	0.5044	0.3598	0.2078	0.5775
1.67	0.3088	0.1871	0.6060	0.2340	0.1589	0.6790
2.5	0.1850	0.1339	0.7240	0.1372	0.1075	0.7837
5.0	0.0831	0.0711	0.8558	0.0611	0.0544	0.8909
∞	0	0	1.0000	0	0	1.0000
ϵ	$\frac{F_s \tan \alpha}{F_a}$	$J_s(\epsilon)$	$J_d(\epsilon)$	$\frac{F_s \tan \alpha}{F_a}$	$J_s(\epsilon)$	$J_d(\epsilon)$
	Point contact			Line contact		

Comparing with equation (4.25) yields

$$J_m(\epsilon) = J'_m(\epsilon)$$

Values of $J_a(\epsilon)$ and $J_m(\epsilon)$ may be obtained from Table 4.3 or Fig.4.7. Therefore,

$$F_a = 2Q_0 J_a(\epsilon) \sin\alpha \quad (4.36)$$

$$M = eF_a = \frac{1}{2} Q_0 D_m J_m(\epsilon) \sin\alpha \quad (4.37)$$

Equations (4.36) and (4.37) can be used for the calculation of a maximum rolling element load Q_0 .

Double direction thrust bearings

When a double direction thrust bearing is subjected to a centric axial load only one row of rolling elements carries the load, which can be obtained from equation (4.27).

If an eccentric axial load acts on a double direction thrust bearing, their two rows of rolling elements would carry it. Fig.4.8 shows a double direction thrust ball bearing subjected to an eccentric axial load. Subscripts 1 and 2 indicate the two rows of rolling elements respectively and the subscript 1 is applied to express the row of rolling elements which carries a heavier load than the other.

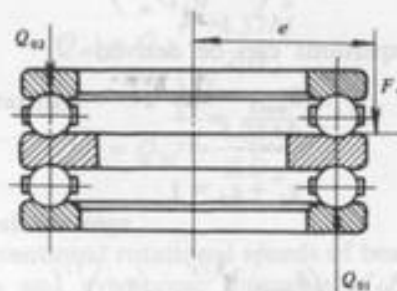


Fig. 4.8 Double direction thrust bearings

Based on deformation harmonization, then

$$\delta_{a1} = -\delta_{a2} \quad (4.38)$$

$$\theta_1 = \theta_2 \quad (4.39)$$

where

δ_{a1} and δ_{a2} = relative axial approaches between the rings contacted with the first and second rows of rolling elements respectively

θ_1 and θ_2 = relative tilting angles between the rings contacted with the first and second rows of rolling elements respectively.

It can be seen from Fig. 4.8

$$\delta_{\max 1} = \delta_{a1} + \frac{1}{2} \theta_1 D_m \quad (4.40)$$

$$\delta_{\max 2} = -\delta_{a1} + \frac{1}{2} \theta_1 D_m \quad (4.41)$$

Similarly, if introducing

$$\varepsilon_1 = \frac{1}{2} \left(1 + \frac{2\delta_{a1}}{\theta_1 D_m} \right) \quad (4.42)$$

$$\varepsilon_2 = \frac{1}{2} \left(1 - \frac{2\delta_{a1}}{\theta_1 D_m} \right) \quad (4.43)$$

The following equations can be derived

$$\frac{\delta_{\max 2}}{\delta_{\max 1}} = \frac{\varepsilon_2}{\varepsilon_1} \quad (4.44)$$

$$\varepsilon_1 + \varepsilon_2 = 1 \quad (4.45)$$

and

$$\frac{Q_{02}}{Q_{01}} = \left(\frac{\delta_{\max 2}}{\delta_{\max 1}} \right)^{1/1} = \left(\frac{\varepsilon_2}{\varepsilon_1} \right)^{1/1} \quad (4.46)$$

From the equilibrium condition of the central ring, hence

$$F_a = F_{a1} - F_{a2} = ZQ_{01}J_a(\varepsilon_1, \varepsilon_2)\sin\alpha \quad (4.47)$$

$$M = M_1 + M_2 = \frac{1}{2} ZQ_{01}D_mJ_m(\varepsilon_1, \varepsilon_2)\sin\alpha \quad (4.48)$$

where

$$J_a(\varepsilon_1, \varepsilon_2) = J_a(\varepsilon_1) - \frac{Q_{02}}{Q_{01}}J_a(\varepsilon_2) \quad (4.49)$$

$$J_m(\varepsilon_1, \varepsilon_2) = J_m(\varepsilon_1) + \frac{Q_{02}}{Q_{01}}J_m(\varepsilon_2) \quad (4.50)$$

Q_{01} and Q_{02} indicate the maximum rolling element loads in the first and second rows of rolling elements respectively. Using equations (4.47) and (4.48) gives

$$\frac{J_m(\varepsilon_1, \varepsilon_2)}{J_a(\varepsilon_1, \varepsilon_2)} = \frac{2e}{D_m} \quad (4.51)$$

Values of $J_a(\varepsilon_1, \varepsilon_2)$, $J_m(\varepsilon_1, \varepsilon_2)$, ε_1 and ε_2 are obtained from Table 4.4 upon $2e/D_m$. If the eccentric distance $e < 0.3 D_m$ for ball bearings or $e < 0.2619 D_m$ for roller bearings, only one row of rolling elements carries the load and then the calculation can be conducted according to that of a single-direction thrust bearing. If the eccentric distance e is more than the above values, the other row of rolling elements will be loaded.

$e \rightarrow \infty$ reflects that a pure moment load acts. In this case, for ball bearings

$$Q_{01} = Q_{02} = \frac{4.37M}{ZD_m} \quad (4.52)$$

and for roller bearings

$$Q_{01} = Q_{02} = \frac{4.08M}{ZD_m} \quad (4.53)$$

Angular contact ball bearings

Generally, at conventional rotational speeds of bearings, the effects of centrifugal forces and gyroscopic moments of balls are neglected. Contact angles of the inner and outer raceways are the same and values of the angles increase as the axial load enhances. As seen from Fig. 4.9, due to the effects of contact deformations under a centric axial load, a relative axial approach δ_a and an elastic deflection δ_n along

the contact line occur. Then the initial contact angle α_0 transmits to an actual contact angle α .

Table 4.4 Values of J_r , J_a and J_m , J_s

e_1	e_2	Point contact				Line contact			
		$\frac{F_r \tan \alpha}{F_a}$	J_r	J_a	$\frac{Q_{a2}}{Q_{a1}}$	$\frac{F_r \tan \alpha}{F_a}$	J_r	J_a	$\frac{Q_{a2}}{Q_{a1}}$
0.5	0.5	∞	0.4577	0	1	∞	0.4906	0	1
0.6	0.4	2.046	0.3568	0.1744	0.544	2.389	0.4031	0.1687	0.640
0.7	0.3	1.092	0.3036	0.2782	0.281	1.210	0.3445	0.2847	0.394
0.8	0.2	0.8005	0.2758	0.3445	0.125	0.8232	0.3036	0.3688	0.218
0.9	0.1	0.6713	0.2618	0.3900	0.037	0.6343	0.2741	0.4321	0.089
1.0	0	0.6000	0.2546	0.4244	0	0.5238	0.2523	0.4817	0
1.25	0	0.4338	0.2289	0.5044	0	0.3598	0.2078	0.5775	0
1.67	0	0.3088	0.1871	0.6060	0	0.2340	0.1589	0.6790	0
2.5	0	0.1850	0.1339	0.7240	0	0.1372	0.1075	0.7837	0
5.0	0	0.0831	0.0711	0.8558	0	0.0611	0.0544	0.8909	0
∞	0	0	0	1.0000	0	0	0	1.0000	0
e_1	e_2	$\frac{2e}{D_m}$	J_m	J_s	$\frac{Q_{s2}}{Q_{s1}}$	$\frac{2e}{D_m}$	J_m	J_s	$\frac{Q_{s2}}{Q_{s1}}$
		Point contact				Line contact			

It can be seen from Fig.4.9

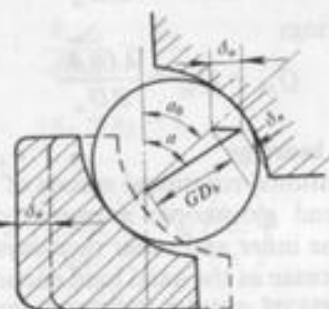


Fig. 4.9 Contact angle in an angular contact ball bearing

$$(\delta_n + GD_b) \cos \alpha = GD_b \cos \alpha_0$$

where

$$G = f_r + f_t - 1$$

therefore,

$$\delta_n = GD_b \left(\frac{\cos \alpha_0}{\cos \alpha} - 1 \right) \quad (4.54)$$

Substituting equation (4.27) into equation (4.7) and using equation (4.54) yield

$$\frac{F_a K_n^{3/2}}{Z(GD_b)^{3/2}} = \sin \alpha \left(\frac{\cos \alpha_0}{\cos \alpha} - 1 \right)^{3/2} \quad (4.55)$$

K_n depends on actual contact angles and internal geometric parameters in bearings. If given an axial load, the actual contact angle can be determined from equation (4.55) by iterative techniques. Jones assumes that

$$K_n = \frac{kG}{D_b^{1/3}} \quad (4.56)$$

The constant k is solely connected with the curvature sum of inner and outer raceways. Its values are listed in Table 4.5.

Table 4.5 Constant k

G	0.03	0.035	0.04	0.05
k	0.01411	0.01253	0.01129	0.00952

Substituting equation (4.56) into equation (4.55) gives

$$\frac{F_a k^{3/2}}{ZD_b^2} = \sin \alpha \left(\frac{\cos \alpha_0}{\cos \alpha} - 1 \right)^{3/2} \quad (4.57)$$

It may be solved by the Newton-Raphson method and its iterative equation is

$$\alpha^{(i+1)} = \alpha^{(i)} +$$

$$\frac{F_a k^{3/2}}{Z D_b^2} - \sin \alpha^{(j)} \left(\frac{\cos \alpha_0}{\cos \alpha^{(j)}} - 1 \right)^{3/2} \\ \cos \alpha^{(j)} \left(\frac{\cos \alpha_0}{\cos \alpha^{(j)}} - 1 \right)^{3/2} + \frac{3}{2} \cos \alpha_0 \tan^2 \alpha^{(j)} \left(\frac{\cos \alpha_0}{\cos \alpha^{(j)}} - 1 \right)^{1/2} \quad (4.58)$$

The $(j+1)$ -th iterative value $\alpha^{(j+1)}$ is computed from the j -th value $\alpha^{(j)}$. A satisfactory solution will be obtained until the difference between $\alpha^{(j+1)}$ and $\alpha^{(j)}$ is less than a permissible error or equal to it.

If the initial contact angle α_0 is not equal to zero, equation (4.57) may be changed to

$$\frac{F_a^{2/3} k}{(Z D_b^2)^{2/3}} - \sin^{2/3} (\alpha_0 + x) \left[\frac{\cos \alpha_0}{\cos (\alpha_0 + x)} - 1 \right] = 0$$

In which, α is replaced by $\alpha_0 + x$. The left side is represented by its Taylor series and the square of x is retained. Then the actual contact angle α can be approximately calculated using the following equation

$$\alpha = \alpha_0 + \frac{3}{2} \frac{\sin 2\alpha}{6 + \cos^2 \alpha} \\ \times \left[\sqrt{1 + \frac{2}{3} \frac{6 + \cos^2 \alpha}{\sin^2 \alpha} \left(\frac{F_a k^{3/2}}{Z D_b^2 \sin \alpha_0} \right)^{2/3}} - 1 \right] \quad (4.59)$$

The more the initial contact angle α_0 and the less the axial load F_a , the higher accuracy the computation of equation (4.59) has.

BEARINGS UNDER COMBINED RADIAL AND AXIAL LOADS

An angular contact ball bearing or a tapered roller bearing can be simultaneously subjected to combined radial and axial loads.

Single row bearings

A approximate calculation is proposed upon the following assumptions.

- (1) The relative tilting of rings is neglected, i.e. the inner and outer rings of the bearing remain parallel.
- (2) The contact angles of all rolling elements loaded are the same, and the actual contact angle is given.

The inner and outer rings of the bearing are relatively performed deflection δ_a in the axial direction and δ_r in the radial direction, as shown in Fig. 4.10.

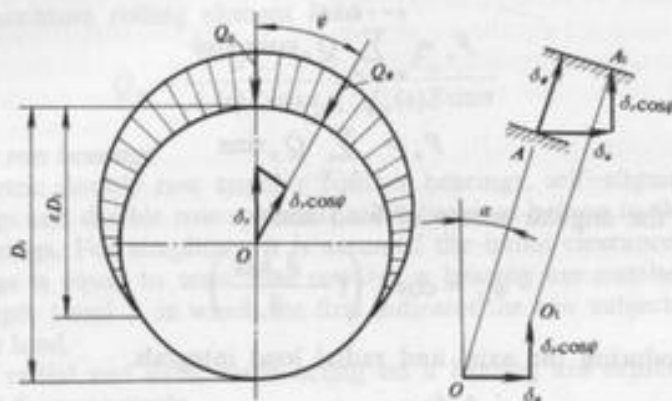


Fig. 4.10 Deflection caused by combined loads

The total relative deflection of the rings at the position angle ψ from the maximally loaded rolling element is

$$\delta_\psi = \delta_a \sin \alpha + \delta_r \cos \alpha \cos \psi \quad (4.60)$$

The maximum elastic deflection at the $\psi = 0$

$$\delta_{\max} = \delta_a \sin \alpha + \delta_r \cos \alpha \quad (4.61)$$

Combining equations (4.60) and (4.61) and introducing

$$\varepsilon = \frac{1}{2} \left(1 + \frac{\delta_a \tan \alpha}{\delta_r} \right) \quad (4.62)$$

therefore

$$\delta_\psi = \delta_{\max} \left[1 - \frac{1}{2\varepsilon} (1 - \cos \psi) \right] \quad (4.63)$$

In form, it is the same to equation (4.20). Using equation (4.21) gives

$$Q_\psi = Q_0 \left[1 - \frac{1}{2\varepsilon} (1 - \cos\psi) \right]^{1/2} \quad (4.64)$$

From the equilibrium condition

$$F_r = \sum_{\psi=-\psi_L}^{+\psi_L} Q_\psi \cos\alpha \cos\psi$$

$$F_a = \sum_{\psi=-\psi_L}^{+\psi_L} Q_\psi \sin\alpha$$

where the angular extent of load zone

$$\psi_L = \cos^{-1} \left(1 - \frac{\delta_a \tan\alpha}{\delta_r} \right) \quad (4.65)$$

Introducing the axial and radial load integrals

$$J_a(\varepsilon) = \frac{1}{2\pi} \int_{-\psi_L}^{+\psi_L} \left[1 - \frac{1}{2\varepsilon} (1 - \cos\psi) \right]^{1/2} d\psi \quad (4.66)$$

$$J_r(\varepsilon) = \frac{1}{2\pi} \int_{-\psi_L}^{+\psi_L} \left[1 - \frac{1}{2\varepsilon} (1 - \cos\psi) \right]^{1/2} \cos\psi d\psi \quad (4.67)$$

then

$$F_r = ZQ_0 J_r(\varepsilon) \cos\alpha \quad (4.68)$$

$$F_a = ZQ_0 J_a(\varepsilon) \sin\alpha \quad (4.69)$$

Dividing equation (4.68) by equation (4.69) yields

$$\frac{F_r \tan\alpha}{F_a} = \frac{J_r(\varepsilon)}{J_a(\varepsilon)} \quad (4.70)$$

Comparing equation (4.66) and (4.67) with equations (4.34) and (4.35), it can be seen that they are of the same form. Given values of F_r , F_a and α , then ε , $J_r(\varepsilon)$ and $J_a(\varepsilon)$ as functions of $F_r \tan\alpha / F_a$ are listed in Table 4.3 and Fig. 4.7.

Setting $\theta_\psi = 0$ in equation (4.64), the angular extent of load zone is expressed

$$\psi_L = \cos^{-1} (1 - 2\varepsilon)$$

If $\varepsilon > 1$, then

$$\psi_L = \pi$$

The maximum rolling element load

$$Q_0 = \frac{F_r}{J_r(\varepsilon) Z \cos\alpha} = \frac{F_a}{J_a(\varepsilon) Z \sin\alpha} \quad (4.71)$$

Double row bearings

Symmetric double row angular contact bearings, self-aligning ball bearings and double row spherical roller bearings belong to this type of bearings. For simplicity, it is assumed the initial clearance in the bearings is equal to zero. The rows of a bearing are numbered by subscripts 1 and 2, in which the first indicates the row subjected to a heavier load.

The radial and axial loads acting on a bearing are expressed by F_r and F_a respectively.

From deformation harmonization

$$\begin{cases} \delta_{r1} = \delta_{r2} \\ \delta_{a1} = -\delta_{a2} \end{cases} \quad (4.72)$$

where

δ_{r1} and δ_{r2} = relative radial approaches between the rings contacted with the first and second rows of rolling elements respectively

δ_{a1} and δ_{a2} = relative axial approaches between the rings contacted with the first and second rows of rolling elements respectively.

Similarly, from equation (4.61) and (4.63)

$$\frac{\delta_{\max 2}}{\delta_{\max 1}} = \frac{\varepsilon_2}{\varepsilon_1} \quad (4.73)$$

$$\varepsilon_1 + \varepsilon_2 = 1 \quad (4.74)$$

where

$$\varepsilon_1 = \frac{1}{2} \left(1 + \frac{\delta_{a1} \tan\alpha}{\delta_{r1}} \right) \quad (4.75)$$

$$\varepsilon_2 = \frac{1}{2} \left(1 + \frac{\delta_{a2} \operatorname{tg} \alpha}{\delta_{r2}} \right) \quad (4.76)$$

Equation (4.74) is valid only when the two rows of rolling elements in a bearing are simultaneously loaded. If only one row of rolling elements is subjected to a load, then

$$\varepsilon_1 \geq 1, \varepsilon_2 = 0$$

From equation (4.5)

$$\frac{Q_{02}}{Q_{01}} = \left(\frac{\varepsilon_2}{\varepsilon_1} \right)^{1/1} \quad (4.77)$$

Using the equilibrium condition, therefore

$$F_r = F_{r1} + F_{r2}$$

$$F_a = F_{a1} - F_{a2}$$

Similar to the foregoing derivation, the following equations are obtained

$$F_r = Z Q_{01} J_r(\varepsilon_1, \varepsilon_2) \cos \alpha \quad (4.78)$$

$$F_a = Z Q_{01} J_a(\varepsilon_1, \varepsilon_2) \sin \alpha \quad (4.79)$$

where

$$J_a(\varepsilon_1, \varepsilon_2) = J_a(\varepsilon_1) - \frac{Q_{02}}{Q_{01}} J_a(\varepsilon_2) \quad (4.80)$$

$$J_r(\varepsilon_1, \varepsilon_2) = J_r(\varepsilon_1) + \frac{Q_{02}}{Q_{01}} J_r(\varepsilon_2) \quad (4.81)$$

$$\frac{J_r(\varepsilon_1, \varepsilon_2)}{J_a(\varepsilon_1, \varepsilon_2)} = \frac{F_r \operatorname{tg} \alpha}{F_a} \quad (4.82)$$

Comparing these with equations (4.49) to (4.51), it can be seen that $J_a(\varepsilon_1, \varepsilon_2)$, $J_r(\varepsilon_1, \varepsilon_2)$, ε_1 and ε_2 are obtained from Table 4.4 upon $F_r \operatorname{tg} \alpha / F_a$. If $F_a > 1.667 F_r \operatorname{tg} \alpha$ for ball bearings or $F_a > 1.909 F_r \operatorname{tg} \alpha$ for roller bearings, only one row of rolling elements carries the load and then the calculation can be conducted according to that of a single row bearing.

The maximum rolling element load Q_{01} is determined by equation (4.78) or (4.79). It should be noted that the previous calculating method is approximate. The assumptions for single row bearings also remain.

Variation of contact angles

In contrast to the previous statements the assumption regarding the same contact angles is given up, but the parallel rings of the bearing are still assumed (22).

Variation of the contact angle in a single row angular contact ball bearing under combined radial and axial loads is considered. The relative displacement of raceway curvature centres at the section of an angular position from the centre of the maximally loaded rolling element is shown in Fig.4.11. The following relationship may be established from this figure

$$(\delta_a + G D_b)^2 = (G D_b \cos \alpha_0 + \delta_r \cos \psi)^2 + (G D_b \sin \alpha_0 + \delta_a)^2$$

The meanings of symbols shown in this equation are the same of those in the previous paragraph. It is rewritten as

$$\delta_a = G D_b \left\{ \left[\left(\cos \alpha_0 + \frac{\delta_r}{G D_b} \cos \psi \right)^2 + \left(\sin \alpha_0 + \frac{\delta_a}{G D_b} \right)^2 \right]^{1/2} - 1 \right\} \quad (4.83)$$

From equation (4.1)

$$Q_a = \left(\frac{G D_b}{K_r} \right)^{3/2} \left\{ \left[\left(\cos \alpha_0 + \frac{\delta_r}{G D_b} \cos \psi \right)^2 + \left(\sin \alpha_0 + \frac{\delta_a}{G D_b} \right)^2 \right]^{1/2} - 1 \right\}^{3/2} \quad (4.84)$$

It can be seen from Fig.4.11

$$\sin \alpha_a = \frac{\sin \alpha_0 + \frac{\delta_a}{G D_b}}{\left[\left(\cos \alpha_0 + \frac{\delta_r}{G D_b} \cos \psi \right)^2 + \left(\sin \alpha_0 + \frac{\delta_a}{G D_b} \right)^2 \right]^{1/2}} \quad (4.85)$$

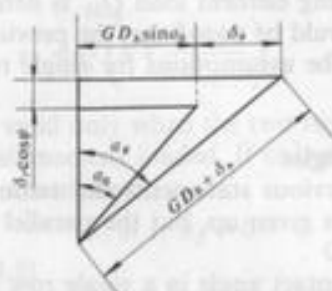


Fig. 4.11 Relative position of curvature centres

where

α_ψ = actual contact angle at an angular position ψ from the centre of the maximally loaded rolling element.

If given δ_r , δ_a and structural parameters of a bearing, the actual contact angle at any angular position ψ can be determined from equation (4.85). For $\psi=0$, i.e. the maximally loaded rolling element, the actual contact angle reaches its minimum value. The angle increases as the absolute value of ψ becomes larger. It reaches its maximum value at $\psi=\pi$.

Setting $Q=0$ in equation (4.84), the angular extent of the load zone is obtained

$$\psi_L = \cos^{-1} \left\{ \frac{\left[1 - \left(\sin \alpha_0 + \frac{\delta_a}{GD_b} \right)^2 \right]^{1/2} - \cos \alpha_0}{\frac{\delta_r}{GD_b}} \right\} \quad (4.86)$$

Using the equilibrium condition and introducing the radial and axial load integrals yield

$$F_r = \frac{Z(GD_b)^{3/2}}{\pi K_p} J_r \quad (4.87)$$

$$F_a = \frac{Z(GD_b)^{3/2}}{\pi K_p} J_a \quad (4.88)$$

where

$$J_a = \int_0^{\psi_L} \frac{\left(\sin \alpha_0 + \frac{\delta_a}{GD_b} \right) \left\{ \left[\left(\cos \alpha_0 + \frac{\delta_r}{GD_b} \cos \psi \right)^2 + \left(\cos \alpha_0 + \frac{\delta_r}{GD_b} \cos \psi \right)^2 + \left(\sin \alpha_0 + \frac{\delta_a}{GD_b} \right)^2 \right]^{1/2} - 1 \right\}^{3/2}}{\left[\left(\cos \alpha_0 + \frac{\delta_r}{GD_b} \cos \psi \right)^2 + \left(\sin \alpha_0 + \frac{\delta_a}{GD_b} \right)^2 \right]^{1/2}} d\psi \quad (4.89)$$

$$J_r = \int_0^{\psi_L} \left\{ \frac{\left(\cos \alpha_0 + \frac{\delta_r}{GD_b} \cos \psi \right)}{\left[\left(\cos \alpha_0 + \frac{\delta_r}{GD_b} \cos \psi \right)^2 + \left(\sin \alpha_0 + \frac{\delta_a}{GD_b} \right)^2 \right]^{1/2}} \times \cos \psi \left\{ \left[\left(\cos \alpha_0 + \frac{\delta_r}{GD_b} \cos \psi \right)^2 + \left(\sin \alpha_0 + \frac{\delta_a}{GD_b} \right)^2 \right]^{1/2} - 1 \right\}^{3/2} \right\} d\psi \quad (4.90)$$

The above integrals may be numerically solved by computers. Given α_0 , $\delta_a/(GD_b)$ and $\delta_r/(GD_b)$, the integrals are computed. Then, values of F_r and F_a are determined by equations (4.87) and (4.88) correspondingly. In reverse, an iterative process is required. Setting $\psi=0$ in equation (4.84), one may give the maximum rolling element load.

Since the table for values of J_a and J_r has not been completed, the above calculation will be more complicated. For the convenience of application, the assumption regarding the same contact angles is still adopted.

BEARINGS UNDER COMBINED RADIAL, AXIAL AND MOMENT LOADS

If a bearing is subjected to combined radial, axial and moment loads

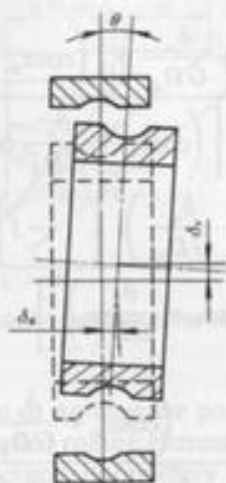


Fig. 4.12 Relative position of rings

in which the moment vector is vertical to the axis of the bearing, its two rings will generate relative displacements, including relative axial approach δ_a , relative radial approach δ_r , relative tilting angle θ .

As shown from Fig. 4.12, assuming that the outer ring is fixed, the inner ring will be relatively displaced to the fixed ring under a combined load. Fig. 4.13 shows the positions of the groove curvature centres of the inner and outer raceways before and after the action of the combined load. The locus radius of the groove curvature centres of the inner raceway is expressed by

$$R_i = 0.5D_m + (r_i - 0.5D_b)\cos\alpha_0 \quad (4.91)$$

Similarly, the circle radius of the groove curvature centres of the outer raceway is

$$R_e = 0.5D_m - (r_e - 0.5D_b)\cos\alpha_0 \quad (4.92)$$

then

$$R_i - R_e = GD_b \cos\alpha_0$$

ψ indicates an angle between any rolling element and the maximally

loaded rolling element. It can be seen that the distance between the groove curvature centres of the inner and outer raceways at an angular position ψ after loading is

$$r = [(GD_b \sin\alpha_0 + \delta_a + R_i \theta \cos\psi)^2 + (GD_b \cos\alpha_0 + \delta_r \cos\psi)^2]^{1/2}$$

Introducing the following dimensionless expression

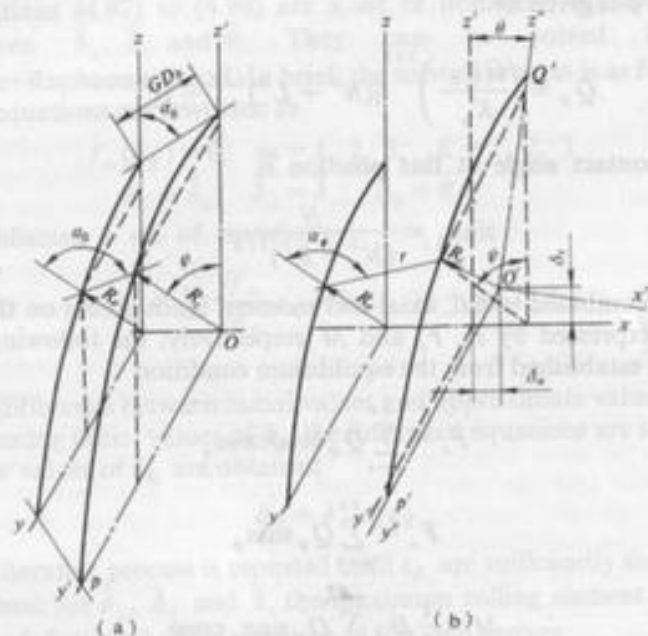


Fig. 4.13 Relative position of groove curvature centres. (a) before loading, (b) after loading

$$\bar{\delta}_a = \frac{\delta_a}{GD_b}; \quad \bar{\delta}_r = \frac{\delta_r}{GD_b}; \quad \bar{\theta} = \frac{\theta R_i}{GD_b}$$

and defining

$$N = \sin\alpha_0 + \bar{\delta}_a + \bar{\theta} \cos\psi$$

$$L = \cos\alpha_0 + \bar{\delta}_r \cos\psi$$

one obtains

$$r = GD_b(N^2 + L^2)^{1/2} \quad (4.93)$$

The total deflection at the angular position ψ

$$\delta_\psi = GD_b[(N^2 + L^2)^{1/2} - 1] \quad (4.94)$$

Considering equation (4.1), the rolling element load at the angular position ψ is given by

$$Q_\psi = \left(\frac{GD_b}{K_r}\right)^{3/2} [(N^2 + L^2)^{1/2} - 1]^{3/2} \quad (4.95)$$

The contact angle at this position is

$$\sin \alpha_\psi = \frac{N}{(N^2 + L^2)^{1/2}} \quad (4.96)$$

If the combined radial, axial and moment loads acting on the bearing are expressed by F_r , F_a and M respectively, the following equations are established from the equilibrium condition

$$F_r = \sum_{\psi=0}^{2\pi} Q_\psi \cos \psi \cos \alpha_\psi$$

$$F_a = \sum_{\psi=0}^{2\pi} Q_\psi \sin \alpha_\psi$$

$$M = \frac{1}{2} D_m \sum_{\psi=0}^{2\pi} Q_\psi \sin \alpha_\psi \cos \psi$$

Rearranging them, one may deduce

$$F_r = \left(\frac{GD_b}{K_r}\right)^{3/2} \times \sum_{\psi=0}^{2\pi} [(N^2 + L^2)^{1/2} - 1]^{3/2} \frac{L}{(N^2 + L^2)^{1/2}} \cos \psi \quad (4.97)$$

$$F_a = \left(\frac{GD_b}{K_r}\right)^{3/2} \sum_{\psi=0}^{2\pi} [(N^2 + L^2)^{1/2} - 1]^{3/2} \frac{N}{(N^2 + L^2)^{1/2}} \quad (4.98)$$

$$M = \frac{1}{2} D_m \left(\frac{GD_b}{K_r}\right)^{3/2} \sum_{\psi=0}^{2\pi} [(N^2 + L^2)^{1/2} - 1]^{3/2} \frac{N}{(N^2 + L^2)^{1/2}} \cos \psi \quad (4.99)$$

Equations (4.97) to (4.99) are a set of nonlinear equations with unknowns δ_a , δ_r and θ . They can be solved by the Newton-Raphson method. In brief, the solving process is as follows:

The equations are rewritten as

$$f_m(\delta_K) = 0; \quad m = 1, 2, 3; \quad K = 1, 2, 3$$

where

$$\delta_1 = \delta_a; \quad \delta_2 = \delta_r; \quad \delta_3 = \theta$$

Establishing a set of equations

$$f_m + \sum_{K=1}^3 \frac{\partial f_m}{\partial \delta_K} \varepsilon_K = 0; \quad m = 1, 2, 3$$

where

ε_K = difference between initial values and approximate values of δ_K .

Assuming initial values of δ_K , the linearized equations are solved for ε_K . New values of δ_K are obtained

$$\delta'_K = \delta_K + \varepsilon_K$$

The iterative process is repeated until ε_K are sufficiently small. Having solved for δ_a , δ_r and θ , the maximum rolling element load can be solved. Setting $\psi = 0$ in equation (4.95), one derives

$$Q_0 = \left(\frac{GD_b}{K_r}\right)^{3/2} \{[(\sin \alpha_0 + \delta_a + \bar{\theta})^2 + (\cos \alpha_0 + \delta_r)^2]^{1/2} - 1\}^{3/2} \quad (4.100)$$

The foregoing computing process should be conducted by computers. For some special conditions, the equations may also be solved by a perturbation method.

CHAPTER FIVE

Kinematics of rolling bearings

It is somewhat more complicated to describe kinematic relations of rolling bearings, particularly those of rolling elements. Generally rolling elements rotate around their own axes, and simultaneously they orbit round the bearing axis. In addition, their rolling motions are accompanied by a certain degree of sliding occurring on the raceways.

In ball bearings, ball motions are three-dimensional problems with six degrees of freedom. A ball may have a rotational sliding normal to the contact surface, called spinning, if its contact angle is not zero. At the same time, the ball can also have another kind of motion caused by gyroscopic moments. Besides orbit, rotation and skidding of a roller, a slight skew of the roller in roller bearings may exist, that is, the roller axis does not coincide with the rolling axis.

Structural parameters, operating environment, lubrication condition and manufacturing accuracy of bearings can affect their internal kinematic relations. Generally speaking, the higher the clearances and the lighter the bearing loads, the more serious is the internal sliding of bearings which takes place. Roller bearings used in aircraft engines are sometimes assembled with "out of round" outer raceways to yield a certain amount of preload in the radial direction in order to reduce skidding.

In this chapter, simple kinematic relations of rolling bearings are described. General kinematic relations including sliding and spinning of ball bearings are explained in detail.

SIMPLE KINEMATIC RELATIONS

The following assumptions are made to analyse the simple kinematic relations of rolling bearings.

- (1) Bearing elements are rigid; that is, the effect of contact deformations is neglected.
- (2) Rolling element motions on raceways are pure rolling. The linear velocity of the surface of a rolling element is identical to that of inner and outer raceways at the contact points.
- (3) The influence of radial clearances is not considered.
- (4) The effect of lubrication oil films is ignored.

Fig.5.1 illustrates an angular contact ball bearing. Symbols n_i , n_e represent the rotational speeds of the inner, outer rings of the rolling bearing respectively. A clockwise rotation is regarded as positive while anticlockwise rotation is then negative. The linear velocity of the inner or outer raceway at the contact points correspondingly is

$$v_i = \frac{\pi}{60} n_i D_m \left(1 - \frac{D_b}{D_m} \cos \alpha \right) \quad (5.1)$$

$$v_e = \frac{\pi}{60} n_e D_m \left(1 + \frac{D_b}{D_m} \cos \alpha \right) \quad (5.2)$$

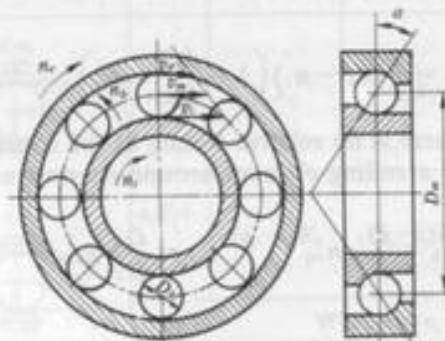


Fig. 5.1 Simple kinematic relations of bearings

As can be seen from Fig.5.1, the linear velocity at the centre of rolling elements is equal to the mean of the inner and outer raceway velocities at contact points. Considering equations (5.1) and (5.2), therefore

$$v_m = \frac{\pi}{120} n_i D_m \left(1 - \frac{D_b}{D_m} \cos \alpha \right) + \frac{\pi}{120} n_e D_m \left(1 + \frac{D_b}{D_m} \cos \alpha \right) \quad (5.3)$$

The linear velocity of the cage pitch circle is

$$v_m = \frac{\pi}{60} n_m D_m$$

then the rotational speed of the cage is derived as

$$n_m = \frac{1}{2} \left[n_i \left(1 - \frac{D_b}{D_m} \cos \alpha \right) + n_e \left(1 + \frac{D_b}{D_m} \cos \alpha \right) \right] \quad (5.4)$$

The rotational speed of the cage relative to the inner ring, i.e. the difference between absolute rotational speed of the cage and that of the inner ring can be written as follows

$$n_{mi} = \frac{1}{2} (n_e - n_i) \left(1 + \frac{D_b}{D_m} \cos \alpha \right) \quad (5.5)$$

Similarly, the rotational speed of the outer ring relative to the cage is

$$n_{em} = \frac{1}{2} (n_e - n_i) \left(1 - \frac{D_b}{D_m} \cos \alpha \right) \quad (5.6)$$

Assuming that there is no relative sliding at the contact points, the rotational speed of a rolling element around its own axis can be expressed by

$$n_b = \frac{D_i}{D_b} n_{mi} \quad n_b = \frac{D_e}{D_b} n_{em}$$

It may be deduced

$$n_b = \frac{1}{2} \frac{D_m}{D_b} (n_e - n_i) \left(1 - \frac{D_b}{D_m} \cos \alpha \right) \left(1 + \frac{D_b}{D_m} \cos \alpha \right) \quad (5.7)$$

Table 5.1 gives simple kinematic relations of radial and angular contact bearings. Kinematic relations of thrust bearings are listed in Table 5.2. These can be obtained by setting $\alpha = 90^\circ$ from corresponding equations in Table 5.1.

In order to calculate bearing life, it is necessary to know how many rolling elements pass a given point on the inner or outer raceway per unit time or one complete revolution of a ring. While the whole set of rolling elements revolves for a cycle relative to a ring, all the elements, with the number designated by Z , will pass each contact point on the raceway of the ring. The number of rolling elements passing a given point on the inner ring per unit time is Zn_{mi} .

Thus

$$Zn_{mi} = \frac{Z}{2} (n_e - n_i) \left(1 + \frac{D_b}{D_m} \cos \alpha \right) \quad (5.8)$$

Table 5.1 Kinematic relations of radial and angular contact bearings

Rotating ring	Fig.	Cage speed n_m	Cage speed relative to inner ring n_{mi}	Outer ring speed relative to cage n_{em}	Ball rotational speed around its own axis n_b
Inner ring		$n_i K_1$	$-n_i K_2$	$-n_i K_1$	$-n_i \times \frac{2D_m K_1 K_2}{D_b}$
Outer ring		$n_e K_2$	$n_e K_2$	$n_e K_1$	$n_e \times \frac{2D_m K_1 K_2}{D_b}$
Two rings rotate in same direction		$n_i K_1 + n_e K_2$	$(n_e - n_i) K_2$	$(n_e - n_i) K_1$	$(n_e - n_i) \times \frac{2D_m K_1 K_2}{D_b}$
		When $n_i = n_e = n$			
		0	0	0	0
Two rings rotate in opposite direction		$n_i K_1 - n_e K_2$	$-(n_e + n_i) K_2$	$-(n_e + n_i) K_1$	$-(n_e + n_i) \times \frac{2D_m K_1 K_2}{D_b}$
		When $v_i = v_e$			
		0	n_i	$-n_e$	0
Notes	For radial and angular contact bearings except tapered roller bearings $K_1 = \frac{1}{2} \left(1 - \frac{D_b}{D_m} \cos \alpha \right); \quad K_2 = \frac{1}{2} \left(1 + \frac{D_b}{D_m} \cos \alpha \right)$ For tapered roller bearings $K_1 = 0.5 [\operatorname{tg}(\alpha - \beta) - \operatorname{tg} \beta] \operatorname{ctg}(\alpha - \beta);$ $K_2 = 0.5 [\operatorname{tg}(\alpha - \beta) + \operatorname{tg} \beta] \operatorname{ctg}(\alpha - \beta)$				

Table 5.2 Kinematic relations of thrust bearings

Rotating ring	Bearing element speeds			Rolling contact number at a point on the raceway per minute
	Cage n_m	Ball (roller) n_b	Cage relative to ring n'_m	
One ring rotates n	$\frac{n}{2}$	$\frac{n}{2} \frac{D_m}{D_b}$	$\frac{n}{2}$	$\frac{nZ}{2}$
Two rings rotate in same direction n_1, n_2	$\frac{n_1 + n_2}{2}$	$\left(\frac{n_1 - n_2}{2}\right) \frac{D_m}{D_b}$	$\frac{n_1 - n_2}{2}$	$\left(\frac{n_1 - n_2}{2}\right) Z$
Two rings rotate in opposite direction n_1, n_2	$\frac{n_1 - n_2}{2}$	$\left(\frac{n_1 + n_2}{2}\right) \frac{D_m}{D_b}$	$\frac{n_1 + n_2}{2}$	$\left(\frac{n_1 + n_2}{2}\right) Z$

By a similar process to the outer ring, it is Zn_{em} , and hence

$$Zn_{em} = \frac{Z}{2} (n_e - n_i) \left(1 - \frac{D_b}{D_m} \cos \alpha\right) \quad (5.9)$$

Relative rotational speeds of a whole set of rolling elements should be regarded as positive value. The Table 5.2 shows the rolling contact number per minute passing a given point on raceways in thrust bearings.

The rolling element number passing a point is normally not identical to the stress cycle number at this point. A point is loaded only when it is located in the load zone of rolling bearings. If the spacing angle of a load distribution zone is expressed by $2\psi_L$, stress cycle number per minute at a point on the raceways of radial and angular contact bearings is given by Table 5.3.

Table 5.3 Stress cycle number of bearings

Rotating ring	Stress cycle number per minute	
	Inner ring	Outer ring
Inner ring	$n_i Z K_2 \frac{2\psi_L}{360}$	$n_i Z K_1$
Outer ring	$n_e Z K_2$	$n_e Z K_1 \frac{2\psi_L}{360}$
Note	K_1, K_2 are shown in Table 5.1	

GENERAL KINEMATIC RELATIONS IN BALL BEARINGS

Kinematic relations are much more complicated if sliding between balls and rings is considered. For example, in the case of an angular contact ball bearing running at high speed, more assumptions are required to find a reasonable solution to their general kinematic relations.

Coordinate systems

To analyse an instantaneous position of an angular contact ball bearing running at high speed, coordinate axes systems as shown in Fig. 5.2 are defined.

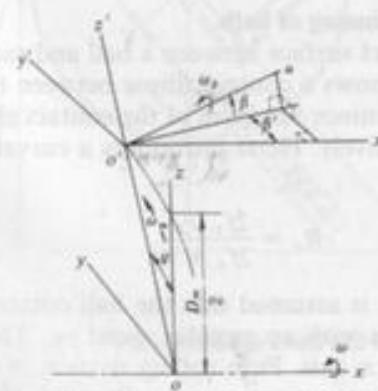


Fig. 5.2 Coordinate system

The following are symbols used in the analysis

x, y, z = a fixed set of coordinates with the x axis coincident with the bearing axis

x', y', z' = a set of coordinates with the x' axis parallel to x axis.

The set of coordinates rotates at the revolution speed of balls round the x axis

$o'u$ = the rotational axis of the ball about its own axis

β = the angle between the rotational axis of the ball $o'u$ and the $x'o'y'$ plane

β' = the angle between the projection of the $o'u$ axis on the $x'o'y'$ plane and the x' axis

ψ = the angle between the z axis and z' axis, that is, the angular position of the ball on pitch circle.

Projections of the rotational speed vector of the ball at coordinate axes of the $x'y'z'$ coordinate system are represented as $\omega_{x'}$, $\omega_{y'}$ and $\omega_{z'}$ respectively.

hence

$$\omega_b = (\omega_{x'}^2 + \omega_{y'}^2 + \omega_{z'}^2)^{1/2} \quad (5.10)$$

and

$$\omega_{x'} = \omega_b \cos \beta \cos \beta' \quad (5.11)$$

$$\omega_{y'} = \omega_b \cos \beta \sin \beta' \quad (5.12)$$

$$\omega_{z'} = \omega_b \sin \beta \quad (5.13)$$

Orbit, rotation and spinning of balls

A shape of the contact surface between a ball and raceway under load is elliptical. Fig. 5.3 shows a contact ellipse between the ball and outer raceway. Major and minor semi-axes of the contact ellipse are signified by a_e and b_e respectively. Hertz introduces a curvature radius of the contact surface as

$$R_e = \frac{2f_e D_b}{2f_e + 1} \quad (5.14)$$

For convenience it is assumed that the ball centre is fixed, and the outer raceway rotates with an angular speed ω_e . The direction of ω_e is collinear with the x axis. Pure rolling motion of the ball over the raceway occurs, at most, at two points. Because of symmetry about the minor axis of the contact ellipse, their unknown radius is defined as r'_e and it can be obtained later.

As shown in Fig. 5.3, a linear velocity of a point (x_e, y_e) on the con-

tact surface of the outer raceway in the rolling direction is given by

$$v_{1e} = -\frac{\omega_e}{2} D_m - \left\{ (R_e^2 - x_e^2)^{1/2} - (R_e^2 - a_e^2)^{1/2} \right. \\ \left. + \left[\left(\frac{D_b}{2} \right)^2 - a_e^2 \right]^{1/2} \right\} \omega_e \cos \alpha_e \quad (5.15)$$

Correspondingly, a point (x_e, y_e) on the contact surface of the ball has a linear velocity in the rolling direction as follows

$$v_{2e} = -(\omega_{x'} \cos \alpha_e + \omega_{y'} \sin \alpha_e) \left\{ (R_e^2 - x_e^2)^{1/2} - (R_e^2 - a_e^2)^{1/2} \right. \\ \left. + \left[\left(\frac{D_b}{2} \right)^2 - a_e^2 \right]^{1/2} \right\} \quad (5.16)$$

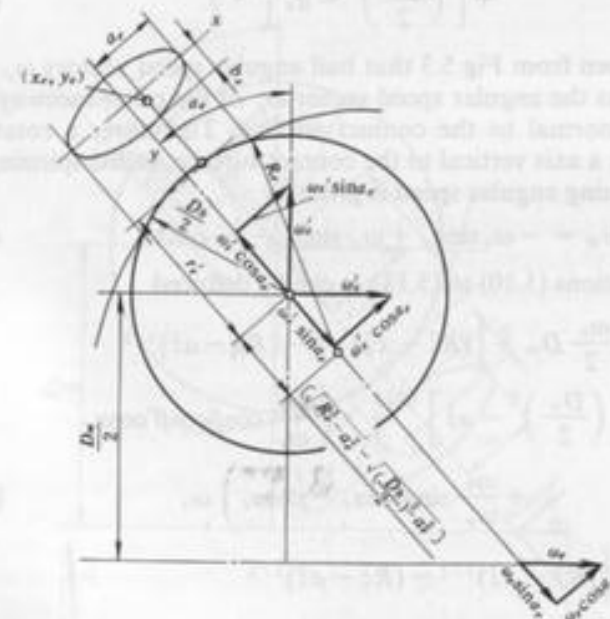


Fig. 5.3 Contact between a ball and outer raceway

Sliding of the outer raceway over the ball in the rolling direction is equal to the difference between the linear velocities v_{1e} and v_{2e} .

Hence, the sliding velocity is

$$v_{ye} = -\frac{\omega_e}{2} D_m + (\omega_x \cos \alpha_e + \omega_z \sin \alpha_e - \omega_e \cos \alpha_e) \\ \times \left\{ (R_i^2 - x_i^2)^{1/2} - (R_i^2 - a_i^2)^{1/2} + \left[\left(\frac{D_h}{2} \right)^2 - a_i^2 \right]^{1/2} \right\} \quad (5.17)$$

A vector component ω_y of the rotational angular speed ω_b of the ball causes a sliding in the direction of the major axis of the contact ellipse. The sliding velocity can be obtained by

$$v_{xe} = -\omega_y \left\{ (R_i^2 - x_i^2)^{1/2} - (R_i^2 - a_i^2)^{1/2} \right. \\ \left. + \left[\left(\frac{D_h}{2} \right)^2 - a_i^2 \right]^{1/2} \right\} \quad (5.18)$$

It can be seen from Fig. 5.3 that ball angular speed vectors ω_x and ω_z , as well as the angular speed vector ω_e of the outer raceway have components normal to the contact surface. Therefore, a rotational motion about a axis vertical to the contact surface, called spinning, exists. The spinning angular speed is given by

$$\omega_{se} = -\omega_x \sin \alpha_e + \omega_z \sin \alpha_e - \omega_e \cos \alpha_e \quad (5.19)$$

From equations (5.10) to (5.13), it can be deduced

$$v_{ye} = -\frac{\omega_e}{2} D_m + \left\{ (R_i^2 - x_i^2)^{1/2} - (R_i^2 - a_i^2)^{1/2} \right. \\ \left. + \left[\left(\frac{D_h}{2} \right)^2 - a_i^2 \right]^{1/2} \right\} \times \left(\frac{\omega_b}{\omega_e} \cos \beta \cos \beta' \cos \alpha_e \right. \\ \left. + \frac{\omega_b}{\omega_e} \sin \beta \sin \alpha_e - \cos \alpha_e \right) \omega_e \quad (5.20)$$

$$v_{xe} = - \left\{ (R_i^2 - x_i^2)^{1/2} - (R_i^2 - a_i^2)^{1/2} \right. \\ \left. + \left[\left(\frac{D_h}{2} \right)^2 - a_i^2 \right]^{1/2} \right\} \omega_e \left(\frac{\omega_b}{\omega_e} \right) \cos \beta \sin \beta' \quad (5.21)$$

$$\omega_{se} = \left(\frac{\omega_b}{\omega_e} \cos \beta \cos \beta' \sin \alpha_e - \frac{\omega_b}{\omega_e} \sin \beta \cos \alpha_e - \sin \alpha_e \right) \omega_e \quad (5.22)$$

The linear velocity of the ball at the rolling radius r'_e equals that of the outer raceway, and hence

$$\left(\frac{D_m}{2 \cos \alpha_e} + r'_e \right) \omega_e \cos \alpha_e = r'_e (\omega_x \cos \alpha_e + \omega_z \sin \alpha_e)$$

By using equations (5.11) and (5.13), it becomes

$$\frac{\omega_b}{\omega_e} = \frac{(D_m/2) + r'_e \cos \alpha_e}{r'_e (\cos \beta \cos \beta' \cos \alpha_e + \sin \beta \sin \alpha_e)} \quad (5.23)$$

Similarly, for the contact between the ball and inner raceway shown in Fig. 5.4, the following equations can be derived

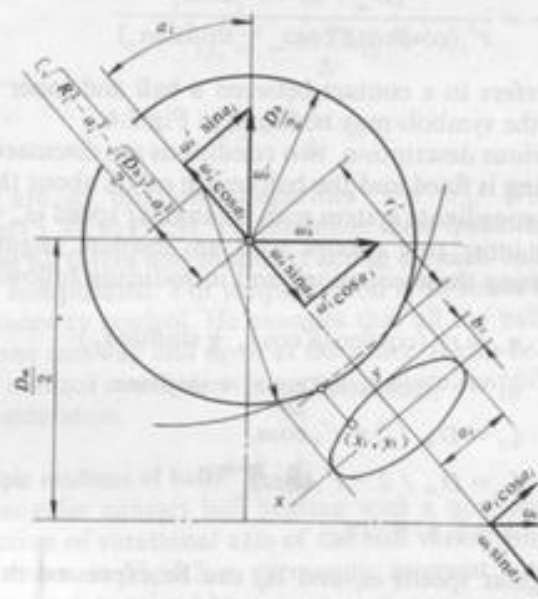


Fig. 5.4 Contact between a ball and inner raceway

$$v_{ye} = -\frac{\omega_i}{2} D_m - \left\{ (R_i^2 - x_i^2)^{1/2} - (R_i^2 - a_i^2)^{1/2} \right. \\ \left. + \left[\left(\frac{D_h}{2} \right)^2 - a_i^2 \right]^{1/2} \right\} \times \left(\frac{\omega_b}{\omega_i} \cos \beta \sin \beta' \cos \alpha_i \right.$$

$$+ \frac{\omega_b}{\omega_i} \sin \beta \sin \alpha_i - \cos \alpha_i \Big) \omega_i \quad (5.24)$$

$$v_{xi} = - \left\{ (R_i^2 - x_i^2)^{1/2} - (R_i^2 - a_i^2)^{1/2} + \left[\left(\frac{D_b}{2} \right)^2 - a_i^2 \right]^{1/2} \right\} \omega_i \left(\frac{\omega_b}{\omega_i} \right) \cos \beta \sin \beta' \quad (5.25)$$

$$\omega_{xi} = \left(- \frac{\omega_b}{\omega_i} \cos \beta \cos \beta' \sin \alpha_i + \frac{\omega_b}{\omega_i} \sin \beta \cos \alpha_i + \sin \alpha_i \right) \omega_i \quad (5.26)$$

$$\frac{\omega_b}{\omega_i} = \frac{-(D_m/2) + r'_i \cos \alpha_i}{r'_i (\cos \beta \cos \beta' \cos \alpha_i + \sin \beta \sin \alpha_i)} \quad (5.27)$$

The suffix i refers to a contact between a ball and inner raceway. The meaning of the symbols may be found in Fig. 5.4.

Using the previous description, two conditions are discussed.

- (1) The outer ring is fixed and the ball centre orbits about the x axis of the fixed coordinate system with an angular speed $\omega_m = -\omega_e$.

Therefore the inner ring rotates with an absolute angular speed $\omega = \omega_i + \omega_m$. By using these equations and introducing following equations

$$\eta_e = r'_e (\cos \beta \cos \beta' \cos \alpha_e + \sin \beta \sin \alpha_e)$$

$$\eta_i = r'_i (\cos \beta \cos \beta' \cos \alpha_i + \sin \beta \sin \alpha_i)$$

$$\zeta_e = D_m / 2 + r'_e \cos \alpha_e$$

$$\zeta_i = D_m / 2 - r'_i \cos \alpha_i$$

$$\Delta = \zeta_e \eta_i + \zeta_i \eta_e$$

The relative angular speeds ω_i and ω_e can be expressed in terms of the absolute angular speed of the inner ring

$$\omega_i = \frac{\zeta_e \eta_i}{\Delta} \omega \quad (5.28)$$

$$\omega_e = - \frac{\zeta_i \eta_e}{\Delta} \omega \quad (5.29)$$

and the rotational angular speed of the ball around its own axis is

$$\omega_b = - \frac{\zeta_e \zeta_i}{\Delta} \omega \quad (5.30)$$

- (2) The inner ring is fixed and the centre of the ball orbits about the x axis of the fixed coordinate system with an angular speed $\omega_m = \omega_i$.

Therefore the outer ring rotates with an absolute angular speed $\omega = \omega_m + \omega_e$.

Similarly, the relative angular speeds ω_e and ω_i , as well as the rotational angular speed of the ball can be represented in terms of the absolute angular speed of the outer ring.

$$\omega_e = \frac{\zeta_i \eta_e}{\Delta} \omega \quad (5.31)$$

$$\omega_i = - \frac{\zeta_e \eta_i}{\Delta} \omega \quad (5.32)$$

$$\omega_b = \frac{\zeta_e \zeta_i}{\Delta} \omega \quad (5.33)$$

There are six unknown quantities r'_e , r'_i , β' , β , α_e and α_i in equations (5.20) to (5.23). To determine these quantities, it is necessary to analyse all forces and moments acting on each ball. Obviously, this is rather complicated. For simplification A.B. Jones introduces a concept of raceway control. He assumes that all the ball spinning occurs only at one raceway and none at the other. It can be thought that the raceway control assumption is a approximate simplification and practical consideration.

Gyroscopic motions of balls

For an angular contact ball bearing with a non-zero contact angle, the direction of rotational axis of the ball varies continuously. Therefore as shown in Fig. 5.2, a gyroscopic moment acts on the ball. Its value may be determined by

$$M_{gy'} = J \omega_m \omega_b \sin \beta \quad (5.34)$$

And it leads to the rotation about y' axis of the ball. A gyroscopic moment about z' axis also exists if the angular speed ω_y is not zero. It can be written as

$$M_{gz'} = J \omega_m \omega_b \cos \beta \sin \beta' \quad (5.35)$$

where

J = the mass moment of inertia of the ball.

The motion caused by a gyroscopic moment should be avoided because it brings about ball sliding on raceways. Testing indicates that the sliding friction coefficient between balls and raceways is 0.01 to 0.06. If the sliding friction coefficient is sufficient, the sliding may be prevented. Jones states that the following inequality should be satisfied

$$0.02QD_b > M_{gy} \quad (5.36)$$

where

Q = contact load of the ball.

All the calculation may be outstandingly simplified in the absence of gyroscopic motion. Thus the angle β' equals zero. From the equations (5.11) and (5.12), the following equations pertain

$$\begin{aligned}\omega_{\beta'} &= \omega_b \cos \beta \\ \omega_{\beta'} &= 0\end{aligned}$$

Using equations (5.23) and (5.27), we can obtain

$$\frac{\omega_b}{\omega_e} = \frac{\left(\frac{D_m}{2}\right) + r'_e \cos \alpha_e}{r'_e (\cos \beta \cos \alpha_e + \sin \beta \sin \alpha_e)} \quad (5.37)$$

$$\frac{\omega_b}{\omega_i} = \frac{-\left(\frac{D_m}{2}\right) + r'_i \cos \alpha_i}{r'_i (\cos \beta \cos \alpha_i + \sin \beta \sin \alpha_i)} \quad (5.38)$$

Allowing for the very small difference between both r'_e , r'_i and the ball radius, it is assumed that

$$r'_e \approx r'_i \approx \frac{D_b}{2}$$

and

$$\gamma' = \frac{D_b}{D_m} \quad (5.39)$$

From equations (5.30) and (5.33), the ratio of the rotational angular

speed of the ball to the absolute angular speed of a ring can be determined as follows

$$\frac{\omega_b}{\omega} = \frac{\pm 1}{\left(\frac{\cos \alpha_e + \text{tg} \beta \sin \alpha_e}{1 + \gamma' \cos \alpha_e} + \frac{\cos \alpha_i + \text{tg} \beta \sin \alpha_i}{1 - \gamma' \cos \alpha_i}\right) \gamma' \cos \beta} \quad (5.40)$$

In which positive is used for a rotating outer ring, and negative is for a rotating inner ring.

By using equation (5.29), the ratio of the orbital angular speed of a ball to the absolute angular speed of a ring in rotating inner ring is expressed by

$$\frac{\omega_m}{\omega} = \frac{1}{1 + \frac{(1 + \gamma' \cos \alpha_e)(\cos \alpha_i + \text{tg} \beta \sin \alpha_i)}{(1 - \gamma' \cos \alpha_i)(\cos \alpha_e + \text{tg} \beta \sin \alpha_e)}} \quad (5.41)$$

Similarly, the ratio of orbital angular speed of a ball to absolute angular speed of a ring in the rotating outer ring becomes

$$\frac{\omega_m}{\omega} = \frac{1}{1 + \frac{(1 - \gamma' \cos \alpha_i)(\cos \alpha_e + \text{tg} \beta \sin \alpha_e)}{(1 + \gamma' \cos \alpha_e)(\cos \alpha_i + \text{tg} \beta \sin \alpha_i)}} \quad (5.42)$$

Raceway control assumption

Having the raceway control assumption, we may simplify the calculation remarkably. Assuming that there is no spinning on the outer raceway, e.g. outer raceway control, and substituting the equation (5.37) into (5.22) and setting ω_m equal to zero, we may obtain

$$\text{tg} \beta = \frac{\sin \alpha_e}{\cos \alpha_e + \gamma'} \quad (5.43)$$

Similarly, if there is no spinning on the inner raceway, e.g., inner raceway control, we have

$$\text{tg} \beta = \frac{\sin \alpha_i}{\cos \alpha_i - \gamma'} \quad (5.44)$$

It can be seen, given the values of α_e , α_i and γ' , the value of $\text{tg} \beta$ may be determined based upon the raceway control assumption. Then ω_b/ω can be decided. In a similar process, ω_m/ω may be calculated

by using equations (5.41) and (5.42). The equations of ω_m / ω are listed in Table 5.4.

Table 5.4 Equations of ω_m / ω

Rotating ring	Outer raceway control	Inner raceway control
Inner ring	$\frac{1 - \gamma' \cos \alpha_i}{1 + \cos(\alpha_i - \alpha_e)}$	$\frac{\cos(\alpha_i - \alpha_e) - \gamma' \cos \alpha_e}{1 + \cos(\alpha_i - \alpha_e)}$
Outer ring	$\frac{\cos(\alpha_i - \alpha_e) + \gamma' \cos \alpha_i}{1 + \cos(\alpha_i - \alpha_e)}$	$\frac{1 + \gamma' \cos \alpha_e}{1 + \cos(\alpha_i - \alpha_e)}$

Jones gives a condition to determine what kind of raceway control takes place. From (23), the sliding friction moment of a spinning motion is obtained

$$M_s = \frac{3}{8} \mu_s Q a L(e) \quad (5.45)$$

where

M_s = the sliding friction moment

μ_s = the coefficient of sliding friction

$L(e)$ = the complete elliptic integrals of the second kind. It can be found from equation (3.11).

An assumption is introduced that pure rolling motion, e.g., no spinning occurs at the contact point having larger sliding friction moment. Upon the equilibrium equations of moments, it is an outer raceway control if the following inequality is satisfied

$$Q_e a_e L_e(e) \cos(\alpha_i - \alpha_e) > Q_i a_i L_i(e) \quad (5.46)$$

or it is an inner raceway control if the inequality is satisfied

$$Q_i a_i L_i(e) \cos(\alpha_i - \alpha_e) > Q_e a_e L_e(e) \quad (5.47)$$

In which, suffixes e and i refer to the value at the contact point on the outer and inner raceway respectively. Outer raceway control is often used in high speed ball bearings due to the significant influence of the ball centrifugal forces.

Measuring the orbital and rotational angular speed and comparing

with the calculations from equation (5.40) and the equations listed in Table 5.4, we may examine the efficiency of the raceway control assumption.

Fig. 5.5 shows the testing results introduced by (4). Solid lines are calculated curves based upon the raceway control assumption. It can be concluded that the assumption is acceptable, particularly in the low and high speed condition.

7204, $\alpha_e = 20^\circ$, $F_e = 98.1 \text{ N}$

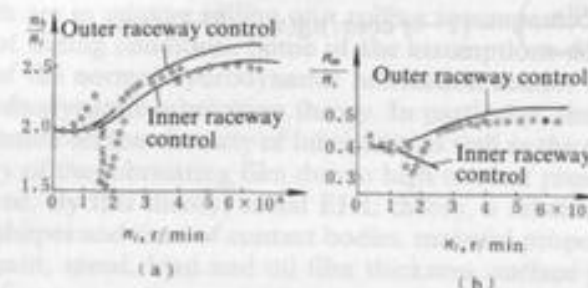


Fig. 5.5 Orbital and rotational angular speed. (a) rotation, (b) orbit

Spin to roll ratio

Ball spinning motions bring about a frictional power dissipation and it is manifested in the form of heat generation. It is hoped to be reduced as much as possible. The ratio of the angular speed of the spinning motion of a ball at the raceway contact to its rolling angular speed is called spin to roll ratio. Obviously, a small value of a spin to roll ratio indicates that the sliding is relatively small, and that the design of the bearing is reasonable. Assuming

$$r'_e \approx r'_i \approx \frac{D_b}{2}$$

the ball rolling angular speed relative to the outer raceway is

$$\omega_{roll} = -\omega_e \frac{D_m}{D_b} = -\frac{\omega_e}{\gamma'}$$

From equation (5.22) and $\beta' = 0$, therefore

$$\omega_w = \omega_o \sin(\alpha_o - \beta) - \omega_i \sin \alpha_i \quad (5.48)$$

Considering the two equations and using equation (5.37), the spin to roll ratio of the ball to the outer raceway is determined by

$$\left(\frac{\omega_w}{\omega_{roll}} \right)_o = -(1 + \gamma' \cos \alpha_o) \operatorname{tg}(\alpha_o - \beta) + \gamma' \sin \alpha_o \quad (5.49)$$

Similarly, the spin to roll ratio of the ball to the inner raceway is

$$\left(\frac{\omega_w}{\omega_{roll}} \right)_i = (1 - \gamma' \cos \alpha_i) \operatorname{tg}(\alpha_i - \beta) + \gamma' \sin \alpha_i \quad (5.50)$$

CHAPTER SIX

Elastohydrodynamic lubrication in rolling bearings

The theory of elastohydrodynamic lubrication deals with the lubricating behaviour of a hydrodynamic oil film between two elastic bodies which are in relative rolling or a rolling accompanied by a certain degree of sliding condition. Some of the assumptions adopted in the theory of the normal hydrodynamic lubrication cannot be applied to elastohydrodynamic lubrication theory. In particular, the influence of high pressure on the viscosity of lubricants as well as the change of the geometry of the lubricating film due to high contact pressure must be considered. By this theory, called EHL theory, a relationship among surface shapes and sizes of contact bodies, material property, viscosity of lubricant, speed, load and oil film thickness, surface pressure and friction force, as well as operating temperature has been established. In practice, the main problem is how to determine the minimum oil film thickness and the friction force.

Generally, the modes of failure for relatively rolling or sliding machine components are mainly contact fatigue and wear. In the early stage, people found that some bearings and gears did not show any remarkable wear after their long terms of service. This raised the question as to whether the rolling surfaces were completely separated by oil film during their operating period. This could not be explained by the normal hydrodynamic lubrication theory because values of calculated oil film thicknesses were much smaller than those of surface roughnesses. In 1949, Grubin assumed that the surfaces of the bounding solids would adopt the shape produced by dry contact and derived an approximate equation for oil film thicknesses. And then Dowson presented numerically computed results by an iterative process in 1959. After that, the theory of the EHL has been developing rapidly. Various measuring methods for oil film thicknesses and pressure have been discovered. Up to now, it has virtually been concluded that the theory of the EHL can successfully predict the minimum thickness of a lubrication oil film, but in some cases it still cannot give satisfactory results for tangential friction forces.

The theory of the EHL plays an important part in the design and the application of rolling bearings, such as bearings for gyroscopic motors, main-shaft bearings for aircraft engines, as well as precise main-shaft bearings for machine tools. Experiments indicate that if a bearing works with a sufficient elastohydrodynamic lubrication, its fatigue life will increase significantly. In the International Standard ISO281, a modified factor is defined to reflect the effect of oil film lubrication on the bearing life.

In the first paragraph of this chapter, the main points of the EHL theory, including equations for the oil film thickness, calculating methods for the tangential friction force, experimental and measuring methods, etc. are described.

In the following paragraphs, the calculation of the oil film thicknesses in roller and ball bearing is introduced. For the convenience of application, an estimating method for rolling bearings is also proposed.

In the appendix of this chapter, a program for the load distribution and the oil film thickness in a rolling bearing is listed in the BASIC language.

THE THEORY OF ELASTOHYDRODYNAMIC LUBRICATION

Development of the theory of elastohydrodynamic lubrication

The basic equation of hydrodynamic lubrication theory is the Reynolds equation, as shown in Fig.6.1.

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\eta} \frac{\partial p}{\partial z} \right) = 12 \frac{\partial}{\partial x} (uh) \quad (6.1)$$

in which

h = oil film thickness

η = dynamic viscosity of lubricants

p = oil film pressure

u = average velocity, $u = (u_1 + u_2) / 2$

x, z = coordinate variables.

Martin analysed the lubrication problem of rolling bodies in 1916. He made the following assumptions.

- (1) The rolling bodies are rigid, that is, the effect of their contact deformation is neglected.

- (2) Lubricants are isoviscous.
- (3) Isothermal lubrication is considered.

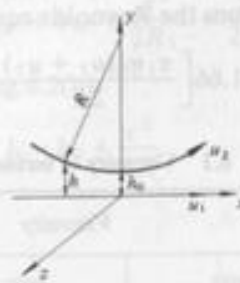


Fig. 6.1 Coordinate system

Then, the minimum oil film thickness of two relatively rolling cylinders was derived

$$\frac{h_0}{R} = 2.45 \eta_0 \frac{u_1 + u_2}{q} \quad (6.2)$$

where

R = equivalent curvature radius, $R = R_1 R_2 / (R_1 + R_2)$, R_1 and R_2 are the curvature radii of two cylinders correspondingly

η_0 = dynamic viscosity at atmospheric pressure

q = line density of load distribution.

The oil film thickness given by equation (6.2) is too small to compare with practice. For example, in a cylinder roller bearing with 45mm bore diameter, the rotational speed is 1000 r.p.m., the radial load is 19.6 kN, the minimum oil film thickness determined by equation (6.2) is only 0.008 μm .

In general, the viscosity of lubricants increases as the operating pressure rises. The viscosity variation of turbine oil with operating pressure is listed in Table 6.1. It can be seen that the viscosity variation is significant and it can be expressed by an exponent function

$$\eta = \eta_0 \exp(\alpha_1 p) \quad (6.3)$$

where

η = viscosity at operating pressure p

α_1 = pressure coefficient of viscosity.

Values of the pressure coefficient of viscosity for the normal mineral oil are listed in Table 6.2. Considering the influence of the viscosity-pressure characteristics on oil film, the minimum oil film thickness can be solved from the Reynolds equation

$$\frac{h_0}{R} = 1.66 \left[\frac{\alpha_1 \eta_0 (u_1 + u_2)}{2R} \right]^{2/3} \quad (6.4)$$

Table 6.1 Viscosity of turbine oil

Pressure MPa	Viscosity (Pa.s)		
	30°C	60°C	90°C
Ambient	0.084	0.021	0.0085
34.32	0.185	0.042	0.0145
68.65	0.375	0.074	0.025
102.97	0.725	0.130	0.039

Table 6.2 Pressure coefficient of viscosity $10^{-8} \times 1/\text{Pa}$

Temperature °C	Cyclo-alkanes base			Paraffin base		
	Spindle oil	Engine oil	Heavy oil	Spindle oil	Engine oil	Heavy oil
30	2.1	2.6	2.8	2.2	2.4	3.4
60	1.6	2.0	2.3	1.9	2.1	2.8
90	1.3	1.6	1.8	1.4	1.6	2.2

The film thickness values given from this equation are still smaller than those measured in tests. This indicates that the oil film thickness in the lubrications of rolling cylinders is not satisfactorily determined by the hydrodynamic lubrication theory of rigid bodies. It is necessary to consider the elastic deformation of contact bodies.

The problem and basic equations

For convenience a contact problem of two elastic cylinders is equivalent to that of an elastic cylinder near a rigid half-plane, as shown in

Fig.6.2. The radius of the equivalent cylinder is determined as follows. The oil film thickness as shown in Fig.6.2(a)

$$h = h_{01} + h_{02} + \frac{x^2}{2R_1} + \frac{x^2}{2R_2} \quad (6.5)$$

Correspondingly in Fig.6.2(b)

$$h = h_0 + \frac{x^2}{2R} \quad (6.6)$$

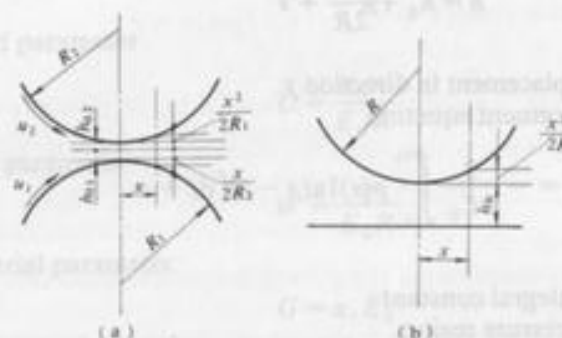


Fig. 6.2 Equivalent cylinder. (a) two cylinders, (b) equivalent cylinder

in which, the radius of the equivalent cylinder is

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad (6.7)$$

Similarly, in order to maintain the same relationship between a load and its displacement, the equivalent modulus of elasticity is defined as

$$\frac{1}{E_0} = \frac{1}{2} \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right) \quad (6.8)$$

where

E_1 and E_2 = moduli of elasticity of the cylinders 1 and 2 respectively

ν_1 and ν_2 = Poisson ratios of the cylinders 1 and 2 respectively.

Based on these, the original problem is transited to that of hydrodynamic lubrication between the equivalent elastic cylinder and

a rigid half-plane.

For the isothermal elastohydrodynamic lubrication of a line contact, its basic equations are as follows

Reynolds equation

$$\frac{d}{dx} \left(\frac{h^3}{12\eta} \frac{dp}{dx} \right) = \frac{1}{2} (u_1 + u_2) \frac{dh}{dx} \quad (6.9)$$

Oil film thickness equation

$$h = h_0 + \frac{x^2}{2R} + v \quad (6.10)$$

where

v = elastic displacement in direction y .

Pressure-displacement equation

$$v = -\frac{2}{\pi E_0} \int_{x_1}^{x_2} p(s) \ln(x-s)^2 ds + c \quad (6.11)$$

where

c = integral constant

x_1 and x_2 = pressure region.

Considering equation (6.3) and boundary conditions, then

$$p = 0, \text{ when } x \rightarrow -\infty$$

$$p = \frac{dp}{dx} = 0, \text{ when } x = x_2$$

and the equilibrium condition

$$q = \int_{x_1}^{x_2} p(x) dx \quad (6.12)$$

One may deduce the solution and determine the constant. It will be described later.

Dimensionless parameters

If thermal effects are neglected, the oil film thickness is expressed by the following function

$$h_0 = f(x_1, \eta_0, u, R, q, E_0)$$

For the convenience of comparison, the function can be simplified by an expression among several dimensionless parameters. Dowson's expression is commonly used, that is

$$H_0 = \varphi(U, W, G) \quad (6.13)$$

where

dimensionless oil film thickness, i.e. film thickness parameter

$$H_0 = \frac{h_0}{R}$$

speed parameter

$$U = \frac{\eta_0 u}{E_0 R}$$

load parameter

$$W = \frac{q}{E_0 R}$$

material parameter

$$G = \alpha_1 E_0$$

Using these parameters, Martin equation (6.2) is expressed by

$$H = 4.9 \frac{U}{W} \quad (6.14)$$

then, equation (6.4) is rewritten as

$$H = 1.66(GU)^{2/3} \quad (6.15)$$

An approximate solution

Grubin assumed that the deformed shape of the cylinders in the EHL lubrication will be the same as that in a dry contact, and a high pressure is formed in the entry region to the Hertzian zone. Then, he gave an equation for calculation of the oil film thickness.

Introducing the replacement variables

$$Q = \frac{1}{\alpha_1 E_0} [1 - \exp(-\alpha_1 p)] \quad (6.16)$$

the pressure formed at the entry edge of the Hertzian zone is solved from the Reynolds equation (6.4)

$$Q_{z=-b} = 48 \left(\frac{W}{2\pi} \right)^{1/2} U \left(\frac{\pi}{2W} \right)^2 \times 0.0986 \left(\frac{\pi H_0}{2W} \right)^{-11/8} \quad (6.17)$$

Considering that high pressure exists at the entry, $\exp(-\alpha_1 p) \ll 1$. So from equation (6.16)

$$Q_{z=-b} \approx \frac{1}{\alpha_1 E_0}$$

substituting it into equation (6.17), the dimensionless film thickness parameter can be expressed as

$$H_0 = \frac{1.95(GU)^{3/11}}{W^{1/11}} \quad (6.18)$$

An interesting thing is that the approximate equation is very close to the numerical solution which is widely used nowadays.

Numerical methods

In the numerical calculation, three basic numerical processes are included.

- (1) Given geometry of contact bodies or oil film shape obtained from its previous iterative value, and the pressure distribution is established by the integration of the Reynolds equation.
- (2) Given or iterated the pressure distribution, the corresponding oil film shape is determined using the inverse solution of the Reynolds equation.
- (3) The elastic displacements are computed from equation (6.11).

In the computation, process (1) is applied to the low pressure region, while process (2) is used in the high pressure region. Combining iterative methods with the technique of modification often leads to satisfactory results. The convergent speed varies with the different regions in a contact area. Generally speaking, in the entry region, a fairly accurate oil film shape can be found in two or three cycles, but more than ten cycles are needed to find a solution in the high pressure region.

A program in FORTRAN language is presented in Reference (26) by the Newton-Raphson method. Numerical examples indicate that the convergent character is satisfactory using this program if the Hertzian contact pressure and the Grubin solution for a film

thickness are taken as initial values.

Thermal effects

The temperature in highly loaded contacts may remarkably increase at high rolling speeds due to the frictional heat. This will reduce the effective viscosity in the contact. Therefore an increase in operating temperature of the lubricant will lessen the film thickness. Murch and Wilson (27) solved the Reynolds equation and the energy equation by using the following viscosity-temperature expression

$$\eta = \eta_0 \exp[\alpha_1 p - \delta_1 (T - T_0)] \quad (6.19)$$

where

η = viscosity of the lubricant at temperature T and pressure p

δ_1 = viscosity-temperature index

T_0 = initial temperature.

Their results show that the film thickness decreases as the rolling speed rises and it can be expressed by

$$H_k = C_k H_0 \quad (6.20)$$

where

H_k = modified film thickness

C_k = modified coefficient for thermal effects

C_k may be calculated by

$$C_k = \frac{3.94}{3.94 + L^{0.62}} \quad (6.21)$$

$$L = \frac{\eta_0 u^2 \delta_1}{k_l} \quad (6.22)$$

where

k_l = heat conductivity of lubricants

L = factor of thermal effects.

The computed results are compared with experiments, as shown in Fig. 6.3.

Hence, the actual film thickness can be determined by equation (6.20). In Reference (29), an oil film thickness between rollers and the inner raceway in a cylinder roller bearing is given by this method, as seen in Fig. 6.4. It is concluded that the thermal effects may be neg-

lected at low or medium rotational speeds, but they must be considered at high rotational speeds or near the limit speeds of bearings.

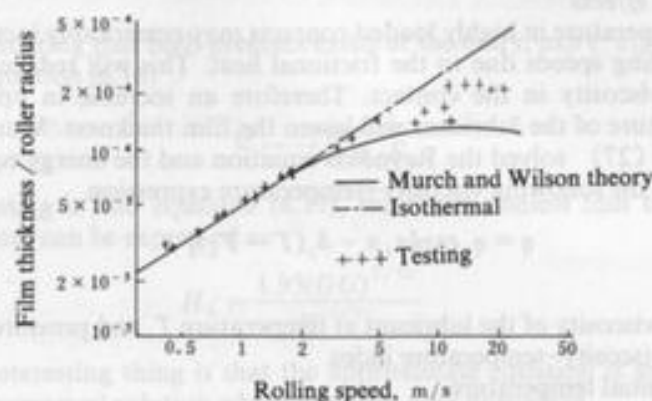


Fig. 6.3 Thermal effects

The effects of sliding in the contact area on the oil film thickness is examined by Cheng [30]. His results show that the influence of the sliding is scarcely observed, and it can be neglected in practice.

Effects of surface roughness

Any actual surface always has a certain degree of roughness. For a lubricating situation, peaks of a rough surface probably penetrate through the oil film and directly contact with the opposite surface. This situation is called partial elastohydrodynamic lubrication.

For the convenience of application, a film thickness parameter is introduced, that is

$$\Lambda = \frac{h}{\sqrt{\sigma_1^2 + \sigma_2^2}} \quad (6.23)$$

where

σ_1 and σ_2 = root mean square values of the roughnesses of the surfaces of the two contact bodies respectively.

Experiments indicate that when Λ is less than three, the effects of the roughness should be considered. In Reference [32], the influence of the surface roughness orientation and values, as well as the

film thickness on the lubricating load capacity is analysed. The conclusion is that transversely oriented surfaces tend to increase the film thickness and its load capacity, but longitudinally oriented surfaces have a reverse effect, as shown in Fig.6.5. It should be noted that the abscissa in Fig.6.5 expresses the ratio of the maximum value of a roughness to the oil film thickness.

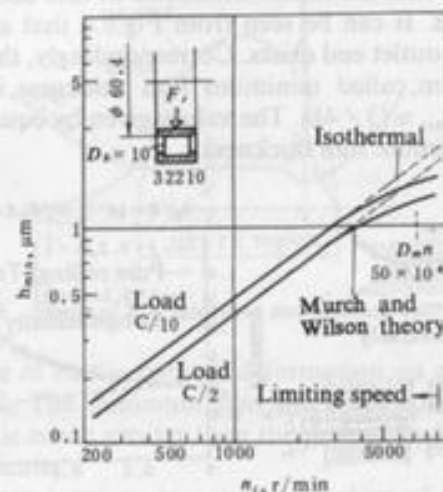


Fig. 6.4 Oil film thickness in a roller bearing

Film thickness in line contact

Dowson presented the numerical solution for an isothermal elastohydrodynamic lubrication in a line contact. The calculating expression for a minimum film thickness is

$$H_{\min} = \frac{h_{\min}}{R} = 1.6 \frac{G^{0.6} U^{0.7}}{W^{0.13}} \quad (6.24)$$

A modified expression was proposed in 1967

$$H_{\min} = \frac{h_{\min}}{R} = 2.65 \frac{G^{0.54} U^{0.7}}{W^{0.13}} \quad (6.25)$$

where

$H_{\min}$ = minimum film thickness parameter

$h_{\min}$ = minimum film thickness.

The shape of a typical oil film and the pressure distribution are shown in Fig. 6.6. In the most part of the oil film region, the film thickness, called central film thickness or nominated film thickness, and expressed by h_0 , almost keeps constant. The value given by Grubin's equation (6.18) is the central oil film thickness. Unless specially noted, a film thickness mentioned in this book means a central film thickness. It can be seen from Fig. 6.6 that a constriction in the oil film at the outlet end exists. Correspondingly, the film thickness at the constriction, called minimum film thickness, is expressed by $h_{\min}$. Generally, $h_{\min} \approx (3/4)h_0$. The values given by equation (6.24) and (6.25) are the minimum film thickness.

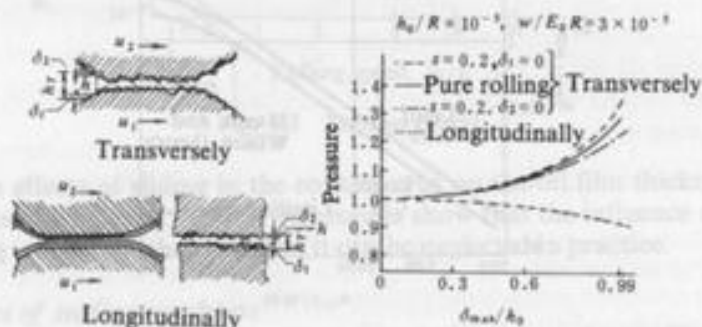


Fig. 6.5 Effects of surface roughness on pressure

As seen from equations (6.24) and (6.25), their coefficients are quite different. However, considering that $G \approx 5000$ for components made of steel and lubricants of the mineral oil, we may calculate $1.6G^{0.6} \approx 2.65G^{0.54}$. Actually, the same values will be obtained by the two equations.

The following characteristics are concluded from the elastohydrodynamic lubrication

- (1) Film thicknesses are only slightly dependent on load. This is one of the most important results of the elastohydrodynamic lubrication theory. As seen from the Martin's equation (6.2), the film thickness is inversely proportional to the load, while from equation (6.24) it is inversely proportional to only 0.13 power of the load. Fig. 6.7 shows differences among these equations. In this

figure, curve *a* indicates the method including deformation and variable viscosity and curve *b* represents the method including variable viscosity, but curve *c* expresses the isoviscosity method only. The elastic deformation should be considered for contact surfaces under a heavy load.

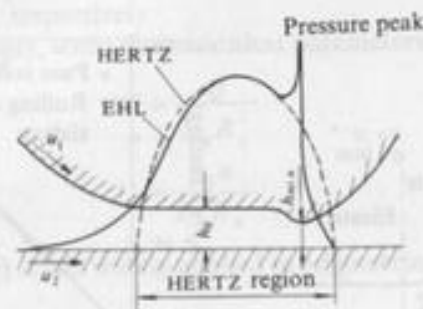


Fig. 6.6 Shape of film thickness and pressure distribution

- (2) The influence of elastic contact deformation on an film thickness is substantial. The minimum film thickness computed from the EHL theory is much greater than the general hydrodynamic theory would indicate.
- (3) The speed parameter, so to say, the product of the speed u and the viscosity η_0 of a lubricant at the entry, has a significant influence on the film thickness and the pressure distribution. As seen from equation (6.24), the film thickness is proportional to 0.7 power of the speed parameter. On the other hand, even though a similar relation between the film thickness and the material parameter exists, the influence of material parameters is not remarkable due to the very narrow range of their variation.
- (4) The film thickness remains constant throughout most of the contact area. A peak of the pressure distribution at the outlet end corresponds to a constriction in the oil film. The pressure peak is higher than the Hertzian contact pressure when its speed parameter $U > 10^{-12}$. And the peak will move toward the entry in the contact region as the speed parameter increases. Recently, it has been concluded that the pressure peak has no remarkable influence on the fatigue life of contact bodies.
- (5) The effects of lubricant compressibility are to reduce values of the

peak and to move it toward the outlet, particularly drastic at high speeds. The minimum film thickness of a compressible lubricant is almost the same as that of an incompressible lubricant. But the central film thickness slightly decreases.

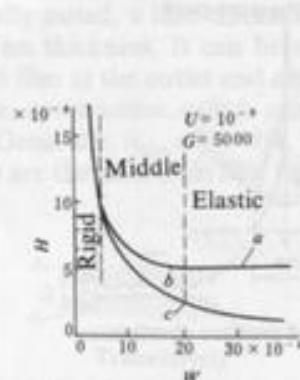


Fig. 6.7 Comparison of calculating methods

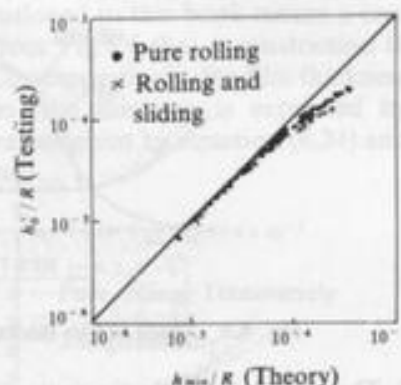


Fig. 6.8 Comparison of calculation with tests

The Dowson's equation has been proved by various experiments. Fig. 6.8 shows the experimental results compared with calculated values. It can be seen that the satisfactory agreement between them exists except when $H_{min} > 10^{-4}$.

Film thickness in point contact

The elastohydrodynamic lubrication in a point contact is more complicated. Various researchers have given some approximate equations and numerically computed solutions.

Archard and Cowking first proposed an approximate calculating equation for the film thickness in a point contact. This is

$$H_0 = 2.04 \frac{(\phi G U)^{0.74}}{W^{0.074}} \quad (6.26)$$

where

ϕ = modified factor for the side-leakage

$$\phi = \left(1 + \frac{2}{3} \frac{R_x}{R_y}\right)^{-1} \quad (6.27)$$

R_x and R_y = equivalent curvature radii in the directions x and y respectively
and correspondingly, some dimensionless parameters

$$W' = \frac{Q}{E_0 R_x^2} \quad (6.28)$$

$$U = \frac{\eta_0 u}{E_0 R_x} \quad (6.29)$$

Equation (6.26) is still widely applied in engineering, particularly for rolling bearings.

Cheng gave another approximate formula having more valuable application for a general elliptic contact, but its calculation is more complicated. The equation and its constants are the following

$$H_0 = C(GU)^{n_1} \left(\frac{p_0}{E_0}\right)^{n_2} \quad (6.30)$$

where

C , n_1 and n_2 = constants, listed in Table 6.3

a , b and p_0 = major and minor semi-axes in a point contact, as well as Hertzian contact pressure respectively.

Hamrock and Dowson suggested an expression of numerical solutions for the elastohydrodynamic lubrication in a point contact.

Table 6.3 Values of constants C , n_1 and n_2

a/b	C	n_1	n_2
5	1.625	0.74	-0.22
2	1.560	0.736	-0.209
1	1.415	0.725	-0.174
0.5	1.145	0.688	-0.066

Central film thickness parameter

$$H_0 = 2.69 \frac{U^{0.67} G^{0.53}}{W^{0.067}} (1 - 0.61 e^{-0.734}) \quad (6.31)$$

Minimum film thickness parameter

$$H_{\min} = 3.63 \frac{U^{0.68} G^{0.49}}{W^{0.073}} (1 - e^{-0.68k}) \quad (6.32)$$

where

$k = \text{ratio } a/b$.

For convenience, the following approximate equation can be used because a and b are unknown

$$k = 1.0339 \left(\frac{R_y}{R_x} \right)^{0.636} \quad (6.33)$$

It can be seen that the influential relationships among the parameters in a point contact are similar to those in a line contact.

All the foregoing analyses are based on an assumption of the adequate supply of lubricants. Sometimes, the volume of lubricant entering the contact is insufficient to form a full oil film. This is called starvation. In rolling bearings, it is not easy for lubricants to be replenished into the contact surfaces between rollers and raceways on time, particularly in high speed bearings.

The starvation can be analysed by setting different lubricating boundaries. The lower integral limit in equation (6.11) may be taken as -5 to reflect the adequate lubrication. As shown in Fig. 6.9, an entry boundary is expressed by m . The numerical computation indicates that the film thickness will start to decline when the m is reduced to a certain value m^* . The value m^* is taken from Reference (40)

$$m^* = 1 + 3.06 \left[\left(\frac{R_y}{b} \right)^2 H_0 \right]^{0.38} \quad (6.34)$$

or

$$m^* = 1 + 3.34 \left[\left(\frac{R_y}{b} \right)^2 H_{\min} \right]^{0.56} \quad (6.35)$$

in which, b is determined by the following equation

$$b = \left[\frac{6\epsilon Q}{\pi k^2 E_0 \left(\frac{1}{R_x} + \frac{1}{R_y} \right)} \right]^{1/3} \quad (6.36)$$

$$\epsilon = 1 + \frac{3}{5} \frac{R_y}{R_x} \quad (6.37)$$

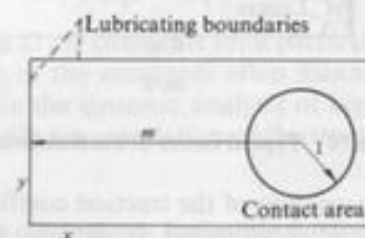


Fig. 6.9 Entry boundary of lubricants

If $m < m^*$, i.e. the lubricant starvation, the film thickness should be modified by a reduction factor

$$H_{0s} = H_0 \left(\frac{m-1}{m^*-1} \right)^{0.29} \quad (6.38)$$

$$H_{\min s} = H_{\min} \left(\frac{m-1}{m^*-1} \right)^{0.25} \quad (6.39)$$

where

H_{0s} = central film thickness parameter in starvation

$H_{\min s}$ = minimum film thickness parameter in starvation.

Traction force in elastohydrodynamic lubrication

Besides the calculation of a film thickness in elastohydrodynamic lubrication, the prediction of a tangential traction force is another important topic, which is still very attractive to many researchers. Typical experimental curves of traction forces are shown in Fig. 6.10. The experiments were conducted on the testing machine of two disc type. The following symbols are introduced: relative sliding velocity $\Delta u = u_2 - u_1$, sliding-rolling ratio $\Delta u / u$, normal pressed load between the two discs Q as well as traction force F .

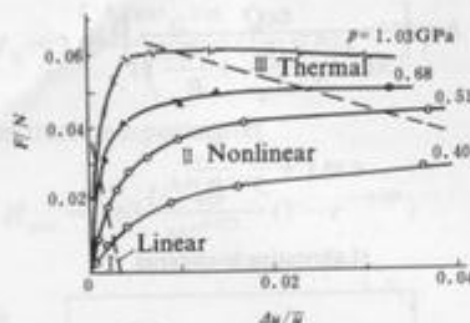


Fig. 6.10 Typical curves of traction forces

It can be seen that variation of the traction coefficient with the sliding-rolling ratio is more complicated. In general, the variation is divided into three regions: I is a linear region, in which traction coefficient initially increases linearly with sliding. And then, II is a nonlinear region, in which the rate of increase of the traction coefficient declines as the sliding velocity increases. The coefficient usually reaches its maximum value. Finally, III indicates a thermal region, in which the rate of increase of the traction coefficient continues to decline, or becomes even slightly negative. Thermal effects must be taken into account in this region.

Except for the linear region, traction forces calculated by a Newton fluid normally are much higher than their experimental values. Therefore, it is not suitable to compute traction forces in high speed bearings by using a Newton fluid. For the improvement, various nonlinear viscous-plastic lubricant models are introduced. A satisfactory agreement between the theory and experiment may be obtained in two disc testing by these models. However, due to complex relationships between the shear stress and the shear strain rate, much sophisticated computation is needed for various machine components like rolling bearings.

Besides using different nonlinear lubricant models, some other treatments can also be applied for the calculation of rolling bearings. In a simple way, it is permissible to assume a constant value of the traction coefficient, such as 0.007, in order to obtain some approximate or qualitative results. Alternatively some semi-empiric formulae

are used to simulate the relation of traction and sliding. As an example, assuming the traction coefficient is a function of the Hertzian contact pressure and the sliding-rolling ratio, Reference [44] gives the following expression

$$f = A\Delta u \exp\left(-\frac{BC\Delta u}{p_0}\right) \left[1 - \exp\left(-\frac{B-p_0}{B}\right)\right] + D[1 - \exp(-C\Delta u)] \quad (6.40)$$

where, A , B , C and D are constants for a particular lubricant.

The adjustment of the constants often means that they fit in with tests sufficiently. In the dynamic analysis of high speed ball bearings, using equation (6.40) for calculation of the traction force can give satisfactory results.

Experiments and measurements in elastohydrodynamic lubrication

Owing to very small contact regions and extremely thin film thicknesses, it is not convenient to measure lubricating parameters in the EHL lubrication. During the previous decades, a number of measuring methods, which are used for measuring the thickness and the shape of an oil film, the pressure distribution, the traction force, the temperature distribution, etc., were developed. Most of them were based on two disc machines.

Measurement of film thickness

Generally, electric-resistance, electric-capacitance, X-ray and optical interference methods are applied.

Electric-resistance method.

A certain value of voltage such as 100 mV, is applied between two relatively rolling bodies. An oil film thickness can be determined by measuring the electric current across the oil film. The main weakness of the resistance method is that an oil film resistance is connected with the voltage applied, temperature, pressure, additives in the oil, as well as the surface roughness of the contact bodies. Therefore, the resistance method usually cannot lead to a satisfactory measurement of film thickness. Secondly, this method only gives a signal to show what magnitude a film thickness is, while the shape of the oil film cannot be detected. The advantage of the method is to judge whether an oil film is formed or not. Up to now, the electric-resistance method is still

used, particularly in partial elastohydrodynamic lubrication.

Electric-capacitance method.

The basic principle of an electric-capacitance method is simple. An oil film thickness can be deduced from measurements of the capacitance among the disc, the oil film and the opposite disc using a low-voltage, high-frequency signal. Various electric-capacitance methods have been developed. One of the methods is shown in Fig. 6.11. The measurement is conducted on a machine of two discs composed of a glass and a steel disc. A thin platinum electrode, having a width of 0.025 to 0.05 mm and a thickness of 0.1 to 0.15 μm , is evaporated onto the glass disc. The film thickness may be determined from measurements of the electric-capacitance. The film shape is portrayed on an oscilloscope when the platinum electrode sweeps over the whole oil film region during the running of the lower glass disc.

X-ray method.

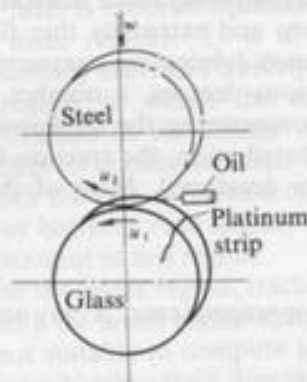


Fig. 6.11 Electric-capacitance method

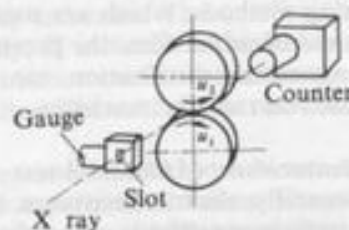


Fig. 6.12 X-ray method

Another effective method is based on a direct X-ray transmission technique. As shown in Fig. 6.12, a beam of X-ray having a width of 0.076 μm passes through the oil film and the amount of the X-ray beam is measured by a radiation counter in order to deduce the film thickness. A major advantage of this method is that there is no limit on disc material. Therefore the effects of different elasticity moduli of materials are avoided.

Optical interference method.

An optical interference method can be applied to measurements of

the film thickness in point contact. As shown in Fig. 6.13, an optical glass presses onto a steel ball and a beam of single-colour or laser light passes through the glass. Based on the Newton-ring effect, optical interference rings occur. Dark colour rings are displayed when the film thickness $h = n\lambda / 2\cos\theta$. In which, n is the strip order of the interference, λ expresses the optical wave length and θ angle of the optical refraction. On the other hand, bright colour rings are displayed when $h = (n+1/2)\lambda / 2\cos\theta$. The thickness and the shape of an oil film can be deduced. Optical interference methods are widely used in engineering application. For example, the film thickness in a thrust ball bearing can be measured by this method if a piece of transparent material is embedded into the upper raceway of the bearing.

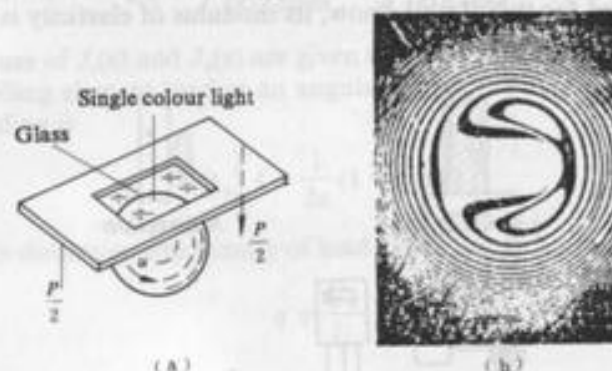


Fig. 6.13 Optical interference method. (a) key diagram, (b) optical interference diagram

Measurement of pressure distribution

Various methods, including small-hole measuring, electric-resistance and optical elasticity, are used. In general, the purpose of measuring pressure distribution is to examine the elastohydrodynamic lubrication theory.

Small-hole measuring method.

Using very small hole connected with a indicator, we may measure the pressure in an oil film. The small hole can sweep over the oil film region to determine variation of the pressure. The less the hole diameter, the higher the measuring accuracy will be. Fig. 6.14 shows the measuring principle of this method.

Electric-resistance method.

This is similar to the electric-capacitance method for a film thickness measurement. A thin constant strip, having a width of 0.05 mm and a thickness of 0.025 to 0.05 μm , is plated onto the glass disc. The electric resistance of a strip is approximately 4400 ohms. Variation of the transverse section of a constant strip under pressure will effect its electric resistance. Therefore, measurements of the resistance may give the pressure in an oil film. A pressure distribution can be displayed on an oscilloscope when the strip sweeps over the oil film region.

Optical elasticity method.

A pressure distribution can be measured using interference strips of the polarized light. The weakness of this method is that epoxy resin should be used for this. As we know, its modulus of elasticity is much lower than that of steel.

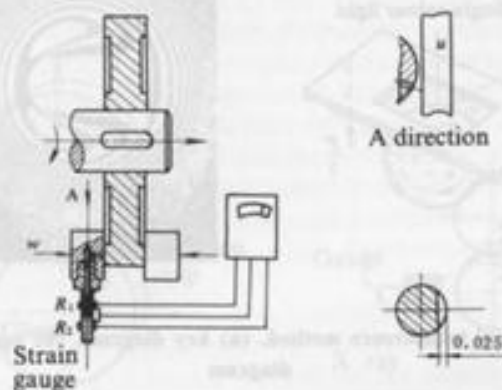


Fig. 6.14 Small-hole measuring method

CALCULATION OF FILM THICKNESS IN ROLLER BEARINGS

In a roller bearing, relative rolling and sliding at contact surfaces between rollers and raceways usually exist under high contact pressure. The Hertzian contact pressure normally ranges from 2000 MPa to 4000 MPa. These conditions should be analysed according to the

elastohydrodynamic lubrication theory.

In general, contact between rollers and raceways in a roller bearing belongs to line contact. Calculating equations of the minimum film thickness, i.e. equations (6.24) and (6.25), may be applied for these conditions.

Main steps for the calculation of film thickness are as follows

- (1) To calculate maximum rolling element load.

For a roller bearing subjected to radial and axial loads simultaneously, its maximum rolling element load can be calculated by equation (4.71), that is

$$Q_0 = \frac{F_r}{J_r(\epsilon)Z\cos\alpha} = \frac{F_a}{J_a(\epsilon)Z\sin\alpha} \quad (4.71)$$

Values of $J_r(\epsilon)$ and $J_a(\epsilon)$ are given from Table 4.3. Correspondingly, the rolling element load at an angular position ψ from the acting line of loading is

$$Q_\psi = Q_0 \left[1 - \frac{1}{2\epsilon} (1 - \cos\psi) \right]^{1/2} \quad (4.64)$$

- (2) To determine line density of load distribution

$$q = \frac{Q_0}{l_r} \quad (6.41)$$

- (3) To enumerate equivalent curvature radius.

From Table 2.6, the equivalent curvature radius

$$R = \left(\frac{2}{D_b} \pm \frac{2}{\frac{D_m}{\cos\alpha} \mp D_b} \right)^{-1} = \frac{D_b}{2} (1 \mp \gamma) \quad (6.42)$$

where

$$\gamma = \frac{D_b}{D_m} \cos\alpha$$

In equation (6.42) the upper sign refers to the inner ring and the lower sign refers to the outer ring.

- (4) To evaluate average velocity.

Assuming that the centre of a roller is fixed, we may consider the velocity relation shown in Fig.6.15. For the condition of the rotating in-

ner ring, the average velocity at the contact between rollers and inner or outer raceway upon equations (5.5) to (5.7) can be expressed as

$$u = \frac{\pi}{120} n_i D_m \left[1 - \left(\frac{D_b}{D_m} \cos \alpha \right)^2 \right] = \frac{\pi}{120} n_i D_m (1 - \gamma^2) \quad (6.43)$$

(5) To introduce equivalent modulus of elasticity

$$E_0 = \frac{E}{1 - \nu^2} \quad (10.13)$$

$E = 2.07 \times 10^5$ MPa, $\nu = 0.3$ for the contact of steel to steel, therefore $E_0 = 2.25 \times 10^5$ MPa.

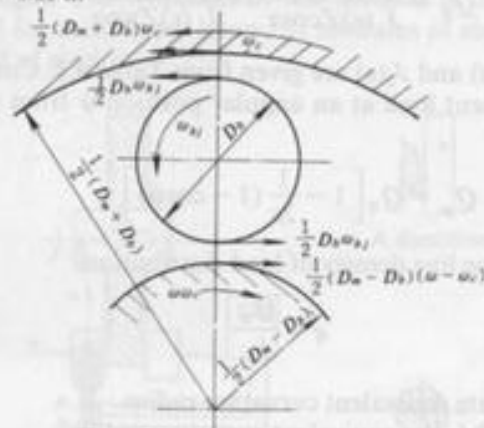


Fig. 6.15 Velocity at a contact

(6) To select lubricant parameters.

η_0 is the viscosity of lubricants at the inlet, in the unit of Pa · s.

α_1 is the pressure coefficient of viscosity. $\alpha_1 = 0.5 \times 10^{-8}$ to 3.6×10^{-8} 1/Pa can be used for estimation.

(7) To work out dimensionless parameters.

From equation (6.13)

$$H_{\min} = \frac{h_{\min}}{R}; \quad H_0 = \frac{h_0}{R}$$

$$U = \frac{\eta_0 u}{E_0 R}; \quad W = \frac{q}{E_0 R}$$

$$G = \alpha_1 E_0$$

In the calculation, it is necessary to consider the roller contact with inner or outer raceway respectively.

(8) To obtain film thickness.

Substituting the above data into equation (6.18) yields the central film thickness parameter. And equation (6.24) or (6.25) gives the minimum film thickness parameter.

As an example, the minimum film thickness can be calculated from equation (6.25).

For a roller and inner raceway

$$h_{\min} = 0.154 \alpha_1^{0.54} (\eta_0 n_i)^{0.7} D_b^{0.43} D_m^{0.7} (1 - \gamma)^{1.13} (1 + \gamma)^{0.7} \frac{E_0^{-0.03}}{q^{0.13}} \quad (6.44)$$

For a roller and outer raceway

$$h_{\min} = 0.154 \alpha_1^{0.54} (\eta_0 n_i)^{0.7} D_b^{0.43} D_m^{0.7} (1 + \gamma)^{1.13} (1 - \gamma)^{0.7} \frac{E_0^{-0.03}}{q^{0.13}} \quad (6.45)$$

It can be seen that a ratio of a film thickness at inner raceway to that at outer raceway is

$$\frac{(h_{\min})_i}{(h_{\min})_e} = \left(\frac{1 - \gamma}{1 + \gamma} \right)^{0.43} \quad (6.46)$$

The value of the ratio is less than 1. Therefore, normally only the film thickness at a contact between roller and inner raceway is examined.

Units in the above equations: $h_{\min}$, D_b and D_m in m, α_1 in 1/Pa, η_0 in Pa · s, n_i in r/min, E_0 in Pa, q in N/m.

Example. A cylindrical roller bearing, roller diameter 16 mm, effective length of a roller 15.4 mm, number of rollers 16, pitch diameter 82 mm, radial load 1000 N, rotational speed 2500 rpm, viscosity of lubricant 0.02 Pa · s, pressure coefficient of viscosity 2.3×10^{-8} 1/Pa, calculate the minimum film thickness at a contact between a roller and inner raceway.

From equation (4.71), the maximum rolling element load

$$Q_0 = \frac{F_r}{J_r(e)Z \cos \alpha} = \frac{10000}{0.2453 \times 10} = 4076.6 \text{ N}$$

From equation (6.41), the load at each effective length of a roller

$$q = \frac{Q_0}{l_e} = \frac{4076.6}{0.0154} = 264717 \text{ N/m}$$

Substituting into equation (6.44) gives the minimum film thickness at a contact between a roller and inner raceway

$$\begin{aligned} h_{\min} &= 0.154 \alpha_1^{0.54} (\eta_0 n_1)^{0.7} D_b^{0.43} D_m^{0.7} (1-\gamma)^{1.13} (1+\gamma)^{0.7} \frac{E_0^{-0.03}}{q^{0.13}} \\ &= 0.154 (2.3 \times 10^{-8})^{0.54} (0.02 \times 2500)^{0.7} (0.016)^{0.43} (0.082)^{0.7} \\ &\quad \times (1-0.195)^{1.13} (1+0.195)^{0.7} \times \frac{(2.25 \times 10^{11})^{-0.03}}{264717^{0.13}} \\ &= 4.19 \times 10^{-7} \text{ m} \end{aligned}$$

that is, the minimum film thickness $h_{\min} \approx 0.42 \mu\text{m}$.

CALCULATION OF FILM THICKNESS IN BALL BEARINGS

In a ball bearing, contact between balls and raceways belongs to point contact. Calculating equations of the film thickness at point contact introduced in the first paragraph of this chapter may be applied for this condition. For convenience, equations (6.31) and (6.32) can be used.

Main steps for the calculation of film thickness are as follows

- (1) To calculate maximum rolling element load by equation (4.71).
- (2) To enumerate equivalent curvature radius.

The equivalent curvature radius in the rolling direction of a ball is

$$R_x = \frac{D_b}{2} (1 \mp \gamma)$$

It is the same as equation (6.42) that the upper sign refers to the inner ring while the lower sign refers to the outer ring.

For the contact between balls and inner raceway, the equivalent curvature radius in the direction normal to the rolling of the ball is

$$R_y = \frac{f_i D_b}{2f_i - 1} \quad (6.47)$$

For the contact between balls and outer raceway

$$R_y = \frac{f_o D_b}{2f_o - 1} \quad (6.48)$$

The elliptic ratio at the contact is estimated by equation (6.33). That is,

$$k = 1.0339 \left(\frac{R_y}{R_x} \right)^{0.636} \quad (6.33)$$

- (3) To evaluate average velocity.
- (4) To introduce equivalent modulus of elasticity.
- (5) To select lubricant parameters.
- (6) To work out dimensionless parameters.

Expressions are

$$\begin{aligned} H_{\min} &= \frac{h_{\min}}{R_x}; & H_0 &= \frac{h_0}{R_x} \\ U &= \frac{\eta_0 U}{E_0 R_x}; & W &= \frac{Q}{E_0 R_x^2} \\ G &= \alpha_1 E_0 \end{aligned}$$

It is necessary to consider the ball contact with inner or outer raceway respectively.

- (7) To obtain film thickness.

Substituting the above data into equation (6.31) yields the central film thickness parameter. And equation (6.32) gives the minimum film thickness parameter.

- (8) To consider a reduction factor for starvation.

Estimating the inlet position m of lubricant at the contact region upon the actual operating condition, we may calculate the value of m using equations (6.35) to (6.37). Then, the minimum film thickness parameter for starvation is

$$H_{\min s} = H_{\min} \left(\frac{m-1}{m^* - 1} \right)^{0.25} \quad (6.39)$$

ESTIMATION OF FILM THICKNESS IN ROLLING BEARINGS

In the previous statement, the film thickness parameter Λ expressed by equation (6.23) indicates the formation degree of an oil film lubrication. To increase the reliability and the life of bearings, it is hoped to thicken the film thickness in bearings. But the calculation of a film thickness is more complicated and it is necessary to know the relevant values of internal geometric parameters in a bearing. For simplicity, an estimation method is proposed. Even though only an approximate value is given by the method, it is a relatively reliable way of telling what kind of lubricating condition will occur in a bearing.

Film thickness parameter Λ may be estimated by the following equation

$$\Lambda = K_A D_m (\alpha_1 \eta_0 n_i)^{0.73} P_0^{-0.09} \quad (6.49)$$

where

K_A = constants connected with bearing types, as listed in Table 6.4

$(\alpha_1 \eta_0)^{0.73}$ = values shown in Fig. 6.16 depending on kinematic viscosity of lubricants at operating temperature

$n_i^{0.73}$ = values given in Fig. 6.17

P_0 = equivalent static load of a bearing, in N.

Table 6.4 Values of K_A

Bearing type	K_A	Bearing type	K_A
Deep groove ball bearings	2210	Cylindrical roller bearings	2330
Self-aligning ball bearings	2210	Tapered roller bearings	2210
Angular contact ball bearings	2460	Thrust ball bearings	1970
Spherical roller bearings	2700	Spherical roller thrust bearings	2820

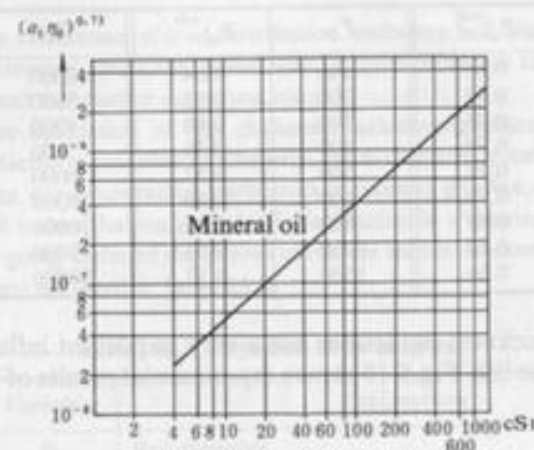


Fig. 6.16 Values of $(\alpha_1 \eta_0)^{0.73}$

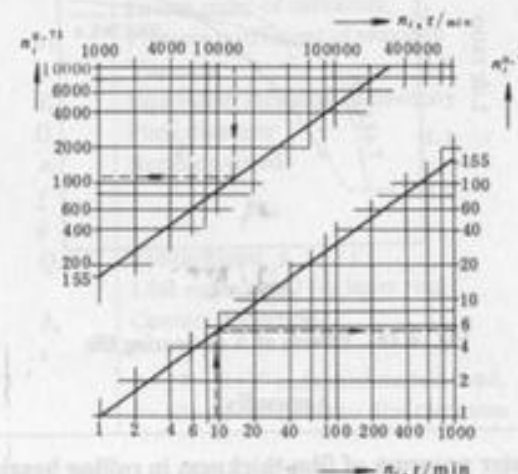
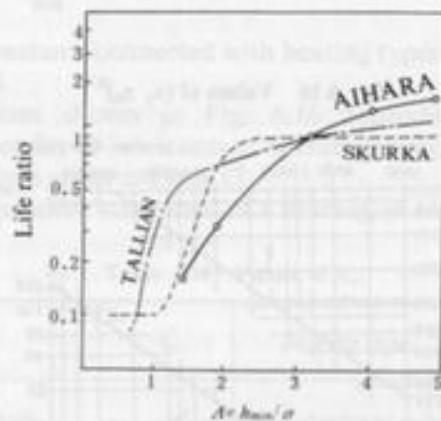


Fig. 6.17 Values of $n_i^{0.73}$

Table 6.5 Values of $P_0^{-0.09}$

P_0	$P_0^{-0.09}$	P_0	$P_0^{-0.09}$	P_0	$P_0^{-0.09}$
10	0.81	150	0.64	2000	0.50
15	0.78	200	0.62	3000	0.49
20	0.76	300	0.60	4000	0.47
30	0.74	400	0.58	5000	0.46
40	0.72	500	0.57	6000	0.46
50	0.70	600	0.56	8000	0.45
60	0.69	800	0.55	10000	0.44
80	0.67	1000	0.54	15000	0.42
100	0.66	1500	0.52	20000	0.41

The film thickness parameter has a very important influence on the bearing fatigue life. Fig. 6.18 shows experimental results of these effects (29).

Fig. 6.18 Effects of Λ on bearing life

Appendix

Computer program of film thickness in rolling bearings

As an example, a program in BASIC language is written for calculation of the load distribution and the film thickness in a deep groove ball bearing. It can be run using a PC1211 pocketable computer.

1. Symbols and input data

Symbols and their explanations are listed in Table 6A.1. Two groups of input data are

- (1) data for the calculation of load distribution, including ball diameter, number of balls, diametral clearance, radial load, groove curvature radius, the constants marked the inner or outer ring, etc.;
- (2) data for the calculation of film thickness, including pressure coefficient of viscosity, viscosity, modulus of elasticity, pitch diameter, rotational speed of an inner ring, curvature radius coefficients of raceway grooves, etc.

Setting $H=0$ means that only the load distribution in a bearing is computed. I indicates the group order of the groove curvature radius. Its corresponding values of coefficients are listed in Table 6A.2.

Table 6A.1 Symbols in the program

Symbols	Variables	Explanations
D	D_b	Ball diameter
Z	Z	Number of balls
U	u_r	Diametral clearance
F	F	Radial load
I		Group order of curvature
C	α_1	Pressure coefficient of viscosity
K	η	Viscosity
E	E_0	Equivalent modulus of elasticity
O	D_m	Pitch diameter
V	n_i	Rotation speed
R	f	
P	ψ	
Q	Q_i	Contact load
H		1 for outer ring 2 for inner ring
G	δ_0	Contact deflection
T	ϵ	
$\Lambda(I)$		$I=27-N+27$ indicates contact load, $I=60-N+60$ refers to film thickness

Table 6A.2 Group order of groove curvature radius

I	1	2	3	4	5	6	7	8	9
f	0.515	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59

2. Program

A program is presented in Table 6A.3. It is composed of a main program and two subroutines. Statements 10 to 18 are input data. Statements 20 to 65 are for the calculation of contact deflection coefficients. Statements 70 to 170 are for the iteration of load distribution in a bearing. Statements 175 to 230 are for the calculation of film thickness. Subroutines include the Simpson integral and the function.

Table 6A.3 Program

```

10 INPUT "D="; D, "Z="; Z, "U="; U, "F="; F, "I="; I,
   "H="; H
15 IF H = 0 GOTO 20
18 INPUT "C="; C, "K="; K, "E="; E, "O="; O, "V="; V,
   "R="; R
20 IF I < 3 LET S = 0.194207228 : GOTO 35
25 IF I < 6 LET S = 0.241252263 : I = I - 3 : GOTO 35
30 IF I < 9 LET S = 0.273277285 : I = I - 6
35 FOR W = 1 TO I
40 S = S * 10 ^ (3 * W)
45 Y = INT (S)
50 S = (S - Y) / 10 ^ (3 * W)
55 NEXT W
60 S = Y / 1.0E5
65 S = S / D ^ (1 / 3)
70 Q = 4.5 * F / Z
75 G = S * Q ^ (2 / 3)
80 T = (1 - U / (2 * G + U)) / 2
90 GO SUB 400
100 W = F / Z / J
110 IF ABS (Q - W) >= F * 3.0E-4 LET Q = W : GOTO 75
120 P = 2 * pi / Z
130 N = INT (L / P)
135 Y = T
140 FOR I = 27 TO N + 27
150 A (I) = W * (1 - 1 / 2 * (1 - cos (P * (I - 27)))) / Y ^ 1.5
155 T = 180 * P * (I - 27) / pi

```

(continued)

```

160 PRINT "P="; T, "Q="; A (I)
170 NEXT I
175 IF H = 0 GOTO 230
180 A = R * D / (2 * R - 1)
190 IF H = 2 LET X = D * (1 - D / O) / 2
195 IF H = 1 LET X = D * (1 + D / O) / 2
200 B = 3.63 * C ^ 0.49 / E ^ 0.117 * (K * pi / 120 * V * O * (1 - (D / O) ^
   2)) ^ 0.68
205 FOR I = 60 TO N + 60
210 A (I) = B * X ^ 0.466 * (1 - 1 / EXP (0.703 * (A / X) ^ 0.636)) / A (I - 33)
   ^ 0.073
215 T = 180 * P * (I - 60) / pi
220 PRINT "P="; T, "H="; A (I)
225 NEXT I
230 END
400 IF T > 1 LET L = pi : GOTO 420
410 L = ACS (1 - 2 * T) : M = 8
420 B = L / M / 2
430 A = 0 : X = 0 : GOSUB 500
440 A = Y + A : X = X + B : GOSUB 500
450 A = Y * 4 + A : X = X + B
460 IF (1 - 1 / 2 * T * (1 - cos X)) < 0 GOTO 475
470 GOSUB 500
475 A = Y + A : M = M - 1
480 IF M < 0 GOTO 440
485 J = A * B / 3 / pi : BEEP 2
490 RETURN
500 Y = cos X * (1 - 1 / T / 2 * (1 - cos X)) ^ 1.5
510 RETURN

```

3. Explanation

Similarly, the program can be applied for the calculation of the load distribution and the film thickness in a cylindrical roller bearing using replacements of equations (4.1) and (4.23) as well as the film thickness equation by the corresponding equations of roller bearings.

Units used in the program: D_z , u_r , D_m and δ in mm, F in kgf, η in cp, α_1 in mm^2/kgf , E_0 in kgf/mm^2 and n_i in rpm.

CHAPTER SEVEN

Contact fatigue

Generally, rolling bearings have various models of failure, such as contact fatigue, wear, corrosion, over-heat, accuracy loss, friction moment increase and severe vibration or noise and so on. For example, wear often occurs due to dirt or foreign particles entering the bearing; and severe rust takes place when water or acids enter the bearing. Some of the failure models can be avoided through proper design and correct maintenance. But others, like the contact fatigue, cannot be avoided completely. Generally speaking, if a rolling bearing is working in proper loading and lubricating conditions without abrasive and corrosive reagents entering it, then its major model of failure is the material fatigue of its rolling surface layer after a certain running time, i.e. contact fatigue. Therefore, the life of a rolling bearing means its fatigue life, that is, the total number of revolutions, or the number of hours at a given constant speed which the bearing is capable of enduring before the first sign of fatigue appears at any rolling surface of the bearing.

The contact fatigue behaves like the common material fatigue, such as life dispersion, stress dependence on life etc. On the other hand, the contact fatigue has its own characteristics, which mainly are uneven stress distribution concentrated on to the surface layer, close relation to the geometric pattern and the physical property of contact surfaces. Hence, contact fatigue is more complicated than normal material fatigue. Up to now, the mechanism of contact fatigue is still a research project.

In the early stage, it was supposed that a fatigue crack is initiated at a place where shear stress is maximum. And then, it propagates to the surface and spalling or pitting will occur. This is called the maximum static shear stress theory.

Later, Lundberg and Palmgren proposed the maximum dynamic shear stress theory, which explains that the occurrence of a fatigue crack depends on the maximum orthogonal shear stress parallel to the rolling direction at a certain depth below the contact surface. As it propagates to the surface, spalling or pitting will occur (16). On the

basis of this theory, a more complete calculation theory regarding the bearing fatigue life is established using probability and statistics. Up to now, it has widely been applied to the life calculation of rolling bearings and is specified in ISO International Standards (49). In the first paragraph of this chapter, the maximum dynamic shear stress theory is introduced.

Due to the development of the advanced metallurgical technology and detective methods of cracks, it is found that fatigue cracks initiate not only below the contact surface but also at the contact surface. Upon this assumption, Chiu and Tallian proposed an engineering model of contact fatigue and established its basic calculating method (51), (52), (53). This model will be described in the second paragraph.

To apply the concept of the fracture mechanics to the contact fatigue is a very important progress. A summary of the statistical fracture theory of the contact fatigue is explained in the third paragraph.

In the last paragraph of this chapter, the effects of various factors on the contact fatigue life will be mentioned.

THE MAXIMUM DYNAMIC SHEAR STRESS THEORY OF CONTACT FATIGUE

A typical contact fatigue spalling is shown in Fig.7.1. The sizes of a spalling or pitting and its depth are connected either with loading and lubricating conditions or with the initial defect distribution of the bearing material. As a fatigue spalling occurs, uneven running and uncommon vibration or noise take place. Therefore, the occurrence of the bearing fatigue is generally detected or monitored by vibration and temperature measuring instruments.

Life distribution of rolling bearings

A great number of experiments indicate that the fatigue life is highly dispersive. It is evident from both tests and practical applications that seemingly identical bearings operating under the same conditions have quite different lives, such as the longest life may be ten or even hundreds times more than the shortest one. Hence, it is necessary to treat the bearing life as a random variable by statistics. Generally, the bearing life means a life at a certain probability of survival, such as 0.90.

Some symbols relating to the bearing life and their definitions are described as follows:

L = bearing life, as a random variable. This means the total number of revolutions, or the number of hours at a given constant speed, which the bearing is capable of enduring before the first sign of fatigue appears at any rolling surface of the bearing.

L_{10} = rating life. This means the total number of revolutions, or the number of hours at a given constant speed, which the 90% of a group of identical bearings may endure or exceed before any fatigue damage occurs in the bearing.

L_{50} = median life. This means the total number of revolutions, or the number of hours at a given constant speed, which the 50% of a group of identical bearings may endure or exceed before any fatigue damage occurs in the bearing.

L_m = mean life. Which means the average life value of a group of identical bearings. Due to the bearing life distribution is asymmetric, L_m is not equal to L_{50} .

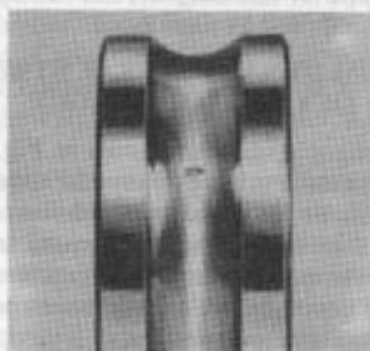


Fig. 7.1 Contact fatigue spalling

The probability of survival may be calculated according to the following

$$S = \frac{N_s}{N} \quad (7.1)$$

where

N_s = number of bearings enduring or exceeding the life L_s

N = total number of bearings in a group.

L_s expresses the life corresponding to the probability of survival.

Customarily, the number 100 (1- S) is used as a subscript, such as $S = 0.90$ corresponding to the bearing life L_{10} . For a single bearing, the probability of survival is that the bearing may reach or exceed L_s . This is also called life reliability.

The probability of failure is

$$F = 1 - S \quad (7.2)$$

Experiments prove that, the life of a group of identical bearings under the same loading follows the Weibull distribution function with two parameters, that is, the probability that the bearing life is less and equal to L is

$$F(L) = 1 - \exp \left[- \left(\frac{L}{\beta} \right)^e \right] \quad (7.3)$$

where

e = slope parameter, which expresses the life dispersion of a group of bearings

β = characteristic parameter, which corresponds to the bearing life at failure probability 0.632.

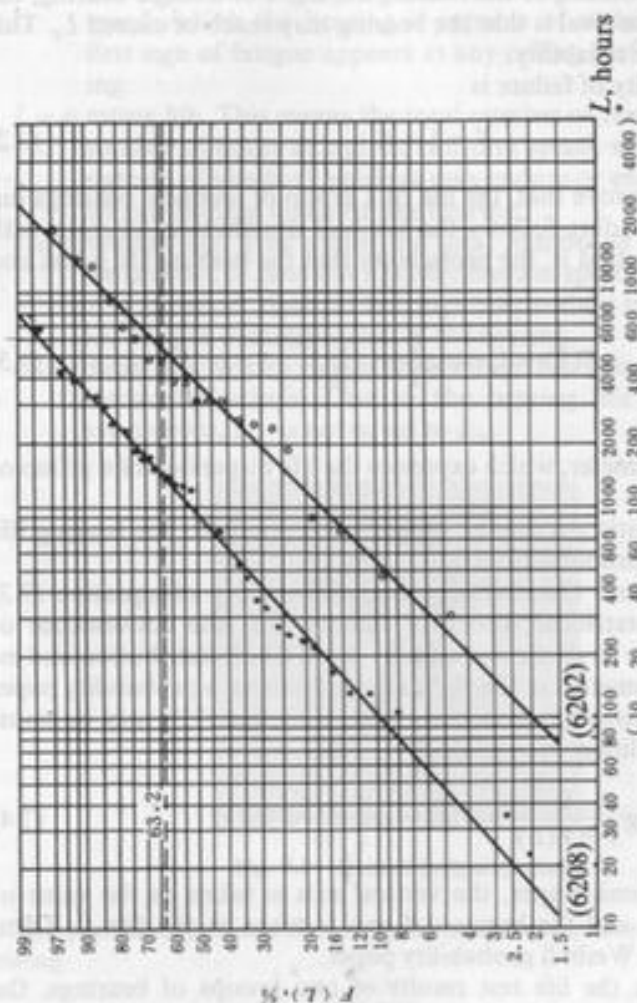
Freudenthal and Gumbel approved the validity of equation (7.3) based on the statistical theory of fatigue. For the convenience of evaluation of the fit of the test data to the Weibull distribution and estimation of parameters in the Weibull distribution, a probability paper of the Weibull distribution is often used. Equation (7.3) may be transformed into the linear relationship

$$\lg \lg \frac{1}{1 - F(L)} = e \lg L - e \lg \beta - 0.36222 \quad (7.4)$$

On a logarithmic paper, the vertical axis is taken as the value of $\lg(1 / (1 - F(L)))$ and the horizontal axis is taken as the life L . Then, this is called the Weibull probability paper.

Fig. 7.2 shows the life test results of two groups of bearings, the bearing designation 6208, 37 sets and the designation 6202, 21 sets. It can be seen that the test data are randomly very close to straight lines and this indicates the test data follow the Weibull distribution.

If $L = L_{10}$, then $S = 0.9$. Substituting it into equation (7.3) and using equation (7.2) gives



• These refer to bearings 6202
Fig. 7.2 Life test results of deep groove ball bearings

$$\lg \frac{1}{S} = 0.0458 \left(\frac{L}{L_{10}} \right)^e \quad (7.5)$$

Given the rating life of a bearing and the slope parameter, the bearing life L corresponding to any probability of survival i.e. reliability, can be deduced by equation (7.5).

Tallian analysed the life test data of 2520 sets of bearings with different types. Each group of the test data is normalized by the following variable replacement, i.e.

$$y = \left(\frac{L}{L_{50}} \right)^e \ln 2$$

there, the normalized test data are compared with the exponent-type distribution

$$F = 1 - \exp(-y)$$

The results express that the normalized test data fit the exponent-type distribution quite well within the range $S = 0.40 - 0.90$. At the high probability of survival, the test data deviate from the straight line towards the longer life side, as shown in Fig. 7.3.

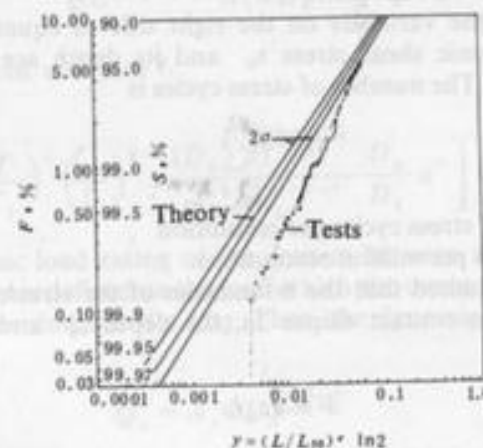


Fig. 7.3 Evaluation of life distribution

It was assumed that y has a fixed value 0.004 when $F < 0.1\%$, i.e. $S > 99.9\%$. Respectively, the minimum life is:

for ball bearings $L_{\min} = 0.053L_{90}$

for roller bearings $L_{\min} = 0.055L_{90}$

Dynamic load rating of a contact

On the basis of the Weibull statistical theory for material strength, Lundberg and Palmgren proposed that the occurrence of a fatigue crack depends on the maximum dynamic shear stress parallel to the rolling direction below the contact surface and the depth at which this shear stress exists effects the probability of failure. Empirically, the following relationship is assumed

$$\lg \frac{1}{S} \propto \frac{\tau_0^c N^e}{z_0^h} V \quad (7.6)$$

where

τ_0 = maximum dynamic shear stress

z_0 = depth of the maximum dynamic shear stress

N = number of stress cycles $\times 10^6$

V = volume under stress

c, e, h = unknown exponents.

Considering the variables on the right side of equation (7.6), the maximum dynamic shear stress τ_0 and its depth are expressed by equation (3.49). The number of stress cycles is

$$N \propto uL \quad (7.7)$$

where

u = number of stress cycles per revolution

L = fatigue life per million revolutions.

Then, it is assumed that the boundaries of the stressed volume are the width of the contact ellipse $2a$, the depth z_0 and the raceway length l , that is,

$$V \propto az_0 l \quad (7.8)$$

Substituting this into equation (7.6) and using equations (3.14), (3.18) and (3.19) yield

$$\lg \frac{1}{S} \propto \frac{T^e}{\zeta^{h-1}} \left(\frac{E_0 D_b \sum \rho}{3m_a m_b^2} \right)^{\frac{c+h-1}{2}} \left(\frac{D_b}{a} \right)^{\frac{e-h-1}{2}} \times \left(\frac{Q}{D_b^2} \right)^{\frac{e-h+1}{2}} D_a D_b^{2-h} u^e L^e \quad (7.9)$$

where

$T = \tau_0 / p_0$

$\zeta = z_0 / b$

$$\frac{1}{E_0} = \frac{1}{2} \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)$$

Q = rolling element load

D_a = raceway diameter at the contact point

m_a and m_b = constants in the Hertzian contact formula.

For the application of a given bearing, the probability of survival S is a constant. Considering the point and line contact conditions separately, one may express T and ζ corresponding to $b/a=0$ and $b/a=1$ as T_0, ζ_0 and T_1, ζ_1 respectively. Then, for point contact

$$QL^{\frac{3e}{c-h+2}} = A_1 \Phi D_b^{\frac{2c+h-5}{c-h+2}} \quad (7.10)$$

where

A_1 = material constant

and

$$\Phi = \left[\left(\frac{T}{T_1} \right)^e \left(\frac{\zeta_1}{\zeta} \right)^{h-1} \left(\frac{D_b \sum \rho}{m_a^{c-1} m_b^{c+h-1}} \right)^{\frac{2c+h-2}{2}} \frac{D_a}{D_b} u^e \right]^{-\frac{3}{c-h+2}} \quad (7.11)$$

The dynamic load rating of the contact between a rolling element and a raceway is defined as the load which the contact will endure for one million revolutions of a ring. Then, the dynamic load rating of a contact is

$$Q_c = A_1 \Phi D_b^{\frac{2c+h-5}{c-h+2}} \quad (7.12)$$

Combining with equation (7.10) gives the following relationship between the rolling element load and the bearing life

$$L = \left(\frac{Q_c}{Q} \right)^{\frac{c-h+2}{2e}} \quad (7.13)$$

Similarly, for line contact

$$QL^{\frac{2e}{c-h+1}} = B_1 \Psi D_b^{\frac{c+h-3}{c-h+1}} l_e^{\frac{c-h-1}{c-h+1}} \quad (7.14)$$

$$\Psi = \left[(D_b \sum \rho)^{\frac{c+h-1}{2}} \frac{D_s}{D_b} u^e \right]^{-\frac{2}{c-h+1}} \quad (7.15)$$

$$Q_c = B_1 \Psi D_b^{\frac{c+h-3}{c-h+1}} l_e^{\frac{c-h-1}{c-h+1}} \quad (7.16)$$

$$L = \left(\frac{Q_c}{Q} \right)^{\frac{c-h+1}{2e}} \quad (7.17)$$

where

l_e = effective length of a roller

B_1 = material constant connected with A_1 .

Equations (7.13) and (7.17) may be written as an unified form

$$L = \left(\frac{Q_c}{Q} \right)^f \quad (7.18)$$

where

for point contact

$$p = \frac{c-h+2}{3e} \quad (7.19)$$

for line contact

$$p = \frac{c-h+1}{2e} \quad (7.20)$$

Determination of exponents

It is necessary to decide exponents c , h , e and p based on test results.

Slope parameter e

A number of tests indicates that e is valued at 1.1 to 1.5. For the excellent quality of material, such as the vacuum degassed steel, the higher

value of e is taken. Palmgren suggested that $e = 10/9$ for ball bearings and $e = 9/8$ for roller bearings. Lieblein verified the validity of the value of e by a statistical method [58].

Life exponent p

Life tests under different loading levels indicate that $p = 3$ refers to point contact and $p = 4$ refers to line contact.

Exponents c , h

Substitution of values of e and p into equations (7.19) and (7.20) yields

$$c-h=8$$

Material constants c and h are not dependent on contact conditions. On the basis of life test results for ball bearings with different values of D_b , it should have

$$\frac{2c+h-5}{c-h+2} = 1.8$$

Then, values of c and h can be deduced, as listed in Table 7.1.

Table 7.1 Values of exponents

Contact condition	p	e	c	h
Point contact	3	10/9	31/3	7/3
Line contact	4	9/8	31/3	7/3

Substituting the above values of exponents into equations (7.12), (7.11), (7.16) and (7.15), regarding the dynamic load rating of a contact yields:

for point contact

$$Q_c = A_1 \Phi D_b^{1.8} \quad (7.21)$$

$$\Phi = \left(\frac{T_1}{T} \right)^{3.1} \left(\frac{\xi}{\xi_1} \right)^{0.4} \frac{m_s^{2.8} m_b^{3.5}}{(D_b \sum \rho)^{2.1}} \left(\frac{D_b}{D_s} \right)^{0.3} u^{-1/3} \quad (7.22)$$

for line contact

$$Q_c = B_1 \Psi D_b^{28/27} l_e^{7/9} \quad (7.23)$$

$$\Psi = (D_b \sum \rho)^{-15/27} \left(\frac{D_b}{D_n} \right)^{2/9} u^{-1/4} \quad (7.24)$$

$$B_1 = 3.517 A_1^{20/27}$$

ENGINEERING MODEL OF CONTACT FATIGUE FAILURE

On the basis of the Manson-Coffin Law, reflecting the relation between the plastic strain caused by a defect and the fatigue life, Chiu and Tallian analysed the stress concentration due to the defects below and at the surface and proposed an engineering model of contact fatigue failure. Some assumptions are introduced during its derivation. Even though the empiric constants are hardly to be determined, it can offer the satisfactory explanation for several influential factors.

Various propagation modes of contact fatigue

Chiu and Tallian divided an entire contact fatigue process into three stages, i.e. the initiation of fatigue cracks, the propagation of fatigue cracks and the formation of fatigue spalling. At the first stage, the fatigue crack is initiated around a defect existing below or at the contact surface. And at the next stage, the crack is gradually propagated. At the last stage, the crack is propagated rapidly and a spalling is formed.

As mentioned above, Palmgren proposed that fatigue cracks are initiated at the depth where the maximum dynamic shear stress occurs below the surface and are propagated to the surface, eventually forming a spalling. The assumption was verified by some metallurgical observations of fatigue damages, such as described by Martin (59). Since the late sixties, the spalling formed from the initiation of a crack below the surface has gradually reduced due to the improvement of material quality. On the other hand, the spalling formed from the initiation of a crack at the surface is often found. These enlighten us on the consideration of different crack propagation modes. The fatigue failure is the combined result due to the two crack propagation modes, i.e. crack initiations below and at the surface.

The propagation of the crack initiated at the surface can be divided into two conditions.

(1) Cracks caused by surface pre-defects, such as surface pre-crack,

furrows formed in the manufacturing process, indents, inclusions near the surface, etc. These defects lead to cracks and spallings due to effects of the stress concentration under the reversal action of the contact pressure.

- (2) Cracks caused by the contact between asperities at the opposite rough surfaces. Due to the contact of asperities at the rough surfaces during the operating process, surface defects, such as pittings, dents etc. are formed as a result of the effects of the stress concentration. And then the defects are propagated and a spalling occurs.

Chiu and Tallian called the defect below a surface by name defect *A*. And respectively the surface pre-defect by name defect *B*, the defect due to the asperity contact by name defect *C*.

As a result of the combination of various crack propagation modes, the contact fatigue life, in the unit of number of stress cycles with a certain probability of survival can be derived

$$N^{-\epsilon} = N_{ss}^{-\epsilon} + N_s^{-\epsilon} \quad (7.25)$$

where

N = contact fatigue life

ϵ = slope parameter in the Weibull distribution

N_s = contact fatigue life initiated from the defect at the surface

N_{ss} = contact fatigue life initiated from the defect below the surface.

Equation (7.25) is derived by the product law of probability for independent events. It can be thought that N_s is connected with surface conditions of contact components and the lubrication, while N_{ss} is not connected with these and depends on the material quality.

Crack propagation below the surface

Normally, the defects, such as non-metallic inclusions or voids etc, exist in the component material. Every defect will cause initiation and propagation of a crack, which means that there is a corresponding fatigue life. For any component, the defects follow a random distribution, that is, sizes, shapes and positions of the defects are random. Then, the contact fatigue life of the entire component depends on the combining action of the random defects.

Manson-Coffin Law

Manson-Coffin indicated that the propagation of a fatigue crack depends on the plastic strain around a defect and the local toughness. It can be written as

$$N_{ss} = C_p \left(\frac{D}{\gamma_p} \right)^{\zeta} \quad (7.26)$$

where

γ_p = plastic strain around a defect

D = local toughness, which expresses the material capacity enduring the plastic strain

ζ = material constant

C_p = quantity connected with the shear stress and the material properties. It is given by

$$C_p = c A_p$$

where

A_p = critical size of the crack

c = material constant.

Contact fatigue life determined by a defect

The plastic strain caused by a defect is connected with the shear stress at the defect, the defect sizes, its stress concentration and the material properties etc. Chiu and Tallian derived the following expression upon some assumptions

$$\gamma_p = 2.9 f_2(\sigma_u) \tau^{4.6} \rho^{0.2} x^{1.2} \left[(K_t)_{\infty}^{-1} \right]^{1.2}$$

where

σ_u = yielding strength of the material

$f_2(\cdot)$ = unknown function

τ = shear stress at the defect

ρ = defect size, such as the minor axis of a elliptic inclusion

x = modified coefficient for the stress concentration factor

$(K_t)_{\infty}$ = theoretical stress concentration factor of the defect in the stress field of an infinite space.

The local toughness D is

$$D = f_1(\sigma_u)(\sigma_h + 30000)$$

where

$f_1(\cdot)$ = unknown function

σ_h = three-dimensional compressive stress. It should be noted that the inch system is applied in the engineering model.

Substituting the above into equation (7.26) gives the contact fatigue life determined by a defect

$$N_{ss} = C_p [f_2(\sigma_u)]^{-\zeta} p_0^{-3.6\zeta} t_R^{-\zeta} \rho^{-0.2\zeta} [(K_t)_{\infty} - 1]^{-1.2\zeta} x^{-1.2\zeta} \quad (7.27)$$

where

$f_2(\cdot)$ = unknown function

t_R = dimensionless parameter of a shear stress, it can be written as

$$t_R = \frac{(0.5\tau / p_0)^{4.6}}{\sigma_h / p_0 + 0.1}$$

Contact fatigue life of an entire component

Considering the combining action of random defects, the following assumptions are made.

- (1) The distribution of defects is sparse and the fatigue life determined by each defect is independent of each other.
- (2) The distribution of the defect number follows the Poisson distribution function.
- (3) The distribution of the defect size in a unit volume follows the power function

$$m[1 - F(\rho)] = \frac{\Phi_2}{\rho^{\mu}} \quad (7.28)$$

where

$F(\rho)$ = distribution function of the defect size

m = average number in a unit of volume

Φ_2, μ = unknown constants.

Strictly speaking, equation (7.28) is an approximate expression of the distribution. On the basis of the above assumptions, the probability that the entire component will endure the number of stress cycles N_{ss} can be deduced

$$-\ln S(N_{ss}) = \Phi_0 \Phi_1 \Phi_2 \Phi_3 \Phi_4 N_{ss}^{5\mu/\zeta} \quad (7.29)$$

In which, Φ_0 , Φ_1 , Φ_3 and Φ_4 are expressed by the following equations respectively

$$\begin{aligned}\Phi_0 &= [C_p^{-1/2} f_3(\sigma_u)]^{5\mu} \\ \Phi_1 &= \int_0^{K'_i} [(K_i)_\infty]^{-1} g[(K_i)_\infty] d(K_i)_\infty \\ \Phi_3 &= \int_{\bar{s}_A} t_R^{5\mu} d\bar{s}_A; \quad \bar{s}_A = \frac{s_A}{az_0} \\ \Phi_4 &= \int_l x^{6\mu} az_0 p_0^{18\mu} dl\end{aligned}$$

where

K'_i = actual maximum value of $(K_i)_\infty$

$g(\cdot)$ = distribution density function of $(K_i)_\infty$

s_A = cross-section normal to the motion direction

l = length of a raceway.

It can be seen that equation (7.29) is similar to equation (7.6), but the influential factors considered here are more precise than in equation (7.6). Of course the number of unknown constants and functions in equation (7.29) increases.

Crack propagation at the surface

The crack propagation at a surface is probably initiated either from the surface pre-defect or from the asperity contact of rough surfaces.

Crack propagation initiated from the surface pre-defect

Chiu and Tallian analysed the defect of a furrow type, as shown in Fig. 3.10. Using the Manson-Coffin Law (7.26), it is assumed that the ratio of the plastic strain at the furrow defect to the local toughness is

$$\frac{\gamma_F}{D} = f_3'(\sigma_u) c^{a_1} p_0^{b_1}$$

where

c = half-width of the furrow

$f_3'(\cdot)$ = unknown function

a_1 and b_1 = unknown exponents.

Substitution of the ratio into equation (7.26) gives the contact fatigue life determined by a furrow

$$N_s = C_p [f_3'(\sigma_u)]^{-1} c^{-a_1} p_0^{-b_1}$$

Similar to the assumption of equation (7.28), it is assumed that the furrow width distribution follows a power function, that is,

$$m'[1 - G(c)] = \frac{\Phi_2'}{c^{\mu'}} \quad (7.30)$$

where

m' = average number of furrows per a unit of surface

$G(c)$ = distribution function of the furrow width

Φ_2' and μ' = unknown constants.

Considering the combining action of random furrows, one may derive the probability of survival that the entire component will endure the number of stress cycles N_s

$$-\ln S(N_s) = \Phi_0' \Phi_2' \Phi_3' \Phi_4' N_s^{m'/a_1} \quad (7.31)$$

where

Φ_0' , Φ_3' and Φ_4' = factors connected with the stress distribution and the material properties.

Crack propagation initiated from the asperity contact of rough surfaces

The asperity contact of rough surfaces leads the gradual formation of a group of micro-pits. Chiu and Tallian analysed the asperity contact of rough surfaces, as shown in Fig. 7.4. Fig. 7.4(a) shows a real

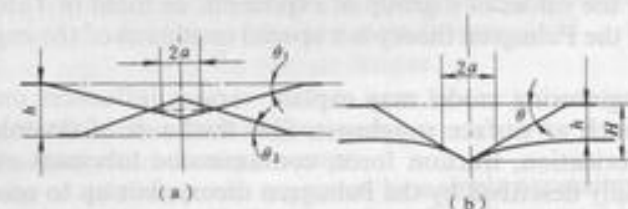


Fig. 7.4 Asperity contact of rough surfaces. (a) a contact of two asperities, (b) a contact between an asperity and a plane

asperity contact and Fig. 7.4(b) shows a simplified model, which is the contact between a rigid asperity and a smooth elastic half-plane.

By comparison with the contact fatigue life of the entire component, the time required to form a group of micro-pits may be neglected.

It is assumed that the ratio of the plastic strain as the group of micro-pits is formed at the asperity contact to the local toughness is

$$\frac{\gamma_p}{D} = f_3'(\sigma_s) \theta^{b_1} \left[c \left(\frac{h}{H} \right) \right]^{a_1}$$

where

θ = slope angle of the asperity

h = film thickness of elastohydrodynamic lubrication

$c(\cdot)$ = unknown function of the micro-pit diameter c and the ratio h/H

H = height of the asperity.

Similar to the derivation of equation (7.31), assuming an approximate distribution function of the micro-pit diameter like equation (7.30) and considering the combining action of random micro-pits, the probability of survival that the entire component will endure the number of stress cycles N_f can be deduced.

Determination of exponents

Obviously, the engineering model contains many unknown exponents, which are very difficult to determine by experiments. Chiu and Tallian compared the engineering model with the Palmgren theory and gave the values of a group of exponents, as listed in Table 7.2. In this way, the Palmgren theory is a special condition of the engineering model.

The engineering model may explain various influences on contact fatigue, such as surface roughness, film thickness of elastohydrodynamic lubrication, traction force, contaminated lubricant etc. These are not fully described by the Palmgren theory, but up to now the engineering model still cannot replace the Palmgren theory for the application in machine design because the exponents in the engineering model need to be determined through further research.

Table 7.2 Exponents in the engineering model

Exponents	Crack propagation below the surface	Crack propagation at the surface	Palmgren theory
Exponents of N_m or N_s	$5\mu/\zeta$	$\mu' = a_1\zeta$ $\mu'/a_1 = 2.9$	$e = 1.1$
Exponents of P_0 or τ	$\zeta = 2.6$ $18\mu = 10.44$	$\zeta = 2.6$ $b_1\mu'/a_1 = 10.44$ $b_1 = 3.6$ $a_1 = 0.2$	$c = 10.3$
Exponent in defect distribution	$\mu = 0.58$	$\mu' = 0.58$	
Exponent of shear stress depth	$0.42\mu = 0.24$	Non	$h = 2.3$
Exponent of stressed volume	1	1	1

STATISTICAL FRACTURE THEORY OF CONTACT FATIGUE

To analyse the contact fatigue by fracture mechanics is a recent significant development. Muro explained the contact fatigue phenomena about a surface crack propagated to form a spalling by the concept of the stress intensity factor (63). Hill proposed that the second type of shear crack is a possible fracture mechanism (64). A statistical fracture theory developed by Hunt and McCartney is effective on analysis of the static strength and the common fatigue (65), (66). It is possible to apply this to the contact fatigue.

Failure probability of contact fatigue

The statistical fracture theory assumes

- (1) The size of a defect can be expressed by a single parameter.
- (2) The defects are sparsely distributed so that the mutual influences of the defects are neglected.
- (3) The entire component can be treated as a set of small units, in

which the stress condition in each unit is uniform. The component fails as any one of the units fractures.

Similar to the above engineering model, it is assumed that the contact fatigue is the result of propagations of various types of defects and cracks. Three types are considered, that is, the crack propagation initiated from a defect below the surface or from a surface pre-defect, as well as from an asperity contact of rough surfaces. Any of the three type of cracks is propagated to form a spalling, then the contact fatigue occurs. Using these, the probability of failure of the contact fatigue is expressed by

$$F(N) = 1 - \exp[-Q_b(N) - Q_s(N) - Q_a(N)] \quad (7.32)$$

$$Q_b(N) = \int_{V_b} \int_{x_{bc}}^{\infty} q_b(x, N) dx dV \quad (7.33)$$

$$Q_s(N) = \int_{V_s} \int_{x_{sc}}^{\infty} q_s(x, N) dx dV \quad (7.34)$$

$$Q_a(N) = \int_{V_a} \int_{x_{ac}}^{\infty} q_a(x, N) dx dV \quad (7.35)$$

where

$q(x, N)$ = probability density function of the crack length in a unit of volume at the time of number of stress cycles N

N = number of stress cycles

x = length of a crack

V = volume

Subscripts used in equations (7.32) to (7.35) are as follows

b = below a surface

s = surface pre-defect

a = asperity contact of rough surfaces

c = critical value.

Paris equation and random differential equation

Paris gives the propagation equation of a fatigue crack. It is widely used in the calculation of the fatigue life.

Assuming that the dynamic shear stress parallel to the rolling direc-

tion effects the contact fatigue life, one may rewrite the Paris equation as

$$\frac{dC_i}{dN} = \beta(\Delta K_{II})^m \quad (7.36)$$

Considering that

$$\Delta K_{II} = Y\Delta\tau_{xy}\sqrt{\pi C_i} \quad (7.37)$$

equation (7.36) is changed to

$$\frac{dC_i}{dN} = \xi C_i^{m/2} \quad (7.38)$$

where

C_i = length of the crack

K_{II} = stress intensity factor

β and m = constants

Y = modified geometric factor. Since the stress condition in a small unit is uniform, Y is approximately constant

τ_{xy} = shear stress parallel to the rolling direction

$$\xi = \pi^{m/2} \beta (Y\Delta\tau_{xy})^m$$

Assume that the random pre-cracks which follow a probability density function $f_0(x)$ exist in the component at the time $N=0$. For any given time of N , the crack length $C_i(N)$ is a random variable. $C_i(N)$ may be treated as a random process. Therefore, the prediction of a fatigue life becomes the determination of the statistical characteristics of the random process, that is, the probability that the crack size reaches or exceeds a critical value at the time of N . The Paris equation becomes a differential equation with random initial conditions.

The initial crack length is expressed by C_0 . Then, the determinate solution is

$$C_i(N) = \left(\frac{2-m}{2} \xi N + C_0^{\frac{2-m}{2}} \right)^{\frac{2}{2-m}}$$

The probability density function $f(x, N)$ of the crack size $C_i(N)$ is

$$f(x, N) = f_0 \left[x_0 = \left(x^{\frac{2-m}{2}} - \frac{2-m}{2} \xi N \right)^{\frac{2}{2-m}} \right] \cdot |J| \quad (7.39)$$

in which, J is the following equation

$$J = \left(x^{\frac{2-m}{2}} - \frac{2-m}{2} \xi N \right)^{\frac{m}{2-m}} \cdot x^{-\frac{m}{2}}$$

Shear stress below a surface and its nonlinear fit

It is necessary to know the amplitude of the shear stress in the integral of equations (7.33) to (7.35). On the basis of the solutions by Smith (19) and Lundberg (16), the following expression is made through a nonlinear fit

$$\frac{\tau_{xy\max}}{P_0} = C_1 \left(\frac{z}{b} \right)^{C_2} \exp \left[-C_2 D \left(\frac{z}{b} \right) \right] \quad (7.40)$$

Values of the parameters C_1 , C_2 and D are listed in Table 7.3. z is the depth starting from the surface.

Table 7.3 Values of C_1 , C_2 and D

Contact conditions	Friction coefficients	C_1	C_2	D
Two dimensions	0.00	0.4708	0.3741	2.0000
	0.03	0.4626	0.3411	2.0833
	0.07	0.5709	0.3652	2.1277
	0.11	0.5346	0.3555	2.2222
Axisymmetric	0.00	0.6354	0.5311	2.8571

Calculation of contact fatigue failure probability

Obviously, the key of calculation of the failure probability of the contact fatigue is the integral of equations (7.33) to (7.35). A numerical computation is commonly required due to its complexity. As an example of a disc-type sample of contact fatigue, the propagation of a fatigue crack initiated from the subsurface is analysed.

To calculate the probability density function of the pre-crack sizes Various distributions can be applied, such as the normal distribution, the Poisson distribution, etc. Since the fatigue life mainly depends on cracks with large sizes, the following power distribution may be approximately adopted, i.e.

$$f_0(x) = \frac{\mu}{v x^{\mu+1}} \quad (7.41)$$

where

μ and v = unknown constants of the distribution density function.

To express the stress intensity factor

Martin found that fatigue cracks often occur along the direction having a certain angle, called the Martin angle, with the surface. The angle between 22° to 24° is for point contact and 30° angle for line contact, as shown in Fig. 7.5.

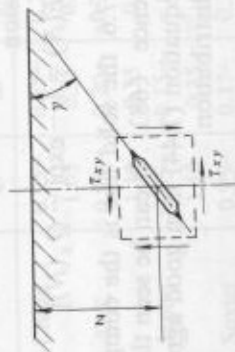


Fig. 7.5 Angular position of fatigue crack

Considering the propagation of a crack along the direction of the Martin angle, one may find its stress intensity factor as follows (67)

$$K_{II} = \tau_{xy} \sin^2 \gamma \sqrt{\pi C_I} \quad (7.42)$$

To determine the critical crack size

Using the maximum shear stress principle, a crack will reach its critical value if

$$K_{II} = K_{IIc} \quad (7.43)$$

To compute the probability of failure

Substituting equation (7.39) into equation (7.33), one can compute the integral Q_b . The integral volume V_b is the volume bounded by the side surface of the ring and the depth $z = 1.5b$. By comparison with the similar integrals, $z = 1.0b$ is taken in the Palmgren theory and $z = 0.5b$ in the engineering model. Relevant constants are

$$m = 2.5$$

$$\mu = 0.65$$

$$K_{uc} \approx 0.9K_{1c} = 15.3 \text{ MPa}\sqrt{\text{m}}$$

$$\beta = 2.368 \times 10^{-8}$$

$$p_0 = 2.14 \text{ GPa}$$

$$\text{Disc radius } R = 12.7 \text{ mm}$$

For a given number of stress cycles N , the failure probability of the contact fatigue initiated only from the subsurface crack propagation is the following expression

$$F(N) = 1 - \exp[-Q_b(N)] \quad (7.44)$$

As shown in Fig. 7.6, the solid line is the computed result and test data are from Reference [68]. It can be seen that the failure probability expressed by equation (7.44) is in good agreement with the two parameter Weibull distribution.

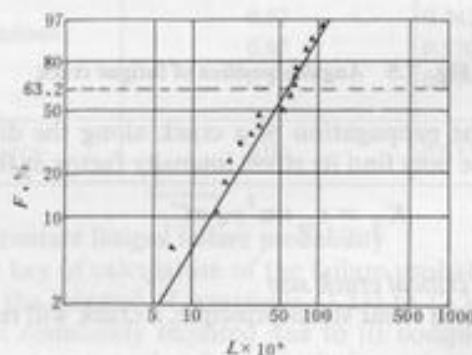


Fig. 7.6 Contact fatigue life distribution

INFLUENTIAL FACTORS ON CONTACT FATIGUE

Many factors, such as design, manufacturing technique, material, lubrication, operating conditions, etc. have important influences on the contact fatigue. Due to the dispersion of fatigue test data and the mutual influence of these factors, it is more difficult to describe the influences in numerical quantities accurately. In the following, the influences of material and manufacturing technique are explained.

Material and metallurgical methods

Rolling bearing steel must possess high strength, good hardenability, excellent fracture toughness and wear properties. In the great majority of cases, a high-carbon chromium bearing steel is applied worldwide. Its chemical composition is listed in Table 7.4.

Table 7.4 Main chemical composition of the bearing steel

Designations	Countries	Chemical composition				
		C	Si	Mn	Cr	Mo
GCr 15	China	0.95–1.05	0.15–0.35	0.20–0.40	1.30–1.65	—
SAE 52100	U.S.	0.98–1.10	0.20–0.35	0.25–0.45	1.30–1.60	<0.06
SUJ 2	Japan	0.95–1.10	0.15–0.35	<0.50	1.30–1.60	<0.08
III X 15	Soviet Union	0.95–1.05	0.17–0.37	0.20–0.40	1.30–1.65	—

Influential factors on the bearing fatigue life

- (1) As silicon content increases, bearing fatigue life increases remarkably. Fig. 7.7 shows the test results of contact fatigue tests. As silicon content increases to 1.65%, the median life doubles approximately. Silicon can improve the capacity of the bearing steel for tempering-resistant. Hence, the residual compressive stress of a surface layer can be maintained. The occurrence of a fatigue spalling at early stage decreases.
- (2) Oxide inclusions have a harmful influence on the contact fatigue. Oxide inclusions are one kind of hard particles, which bring about the stress concentration in the material and lead to form fatigue

cracks. Fig. 7.8 shows the test results, which were obtained from a life test of self-aligning ball bearings. It can be seen that the more the number of oxide inclusions whose size are over 0.03 mm, the lower the bearing life will be.

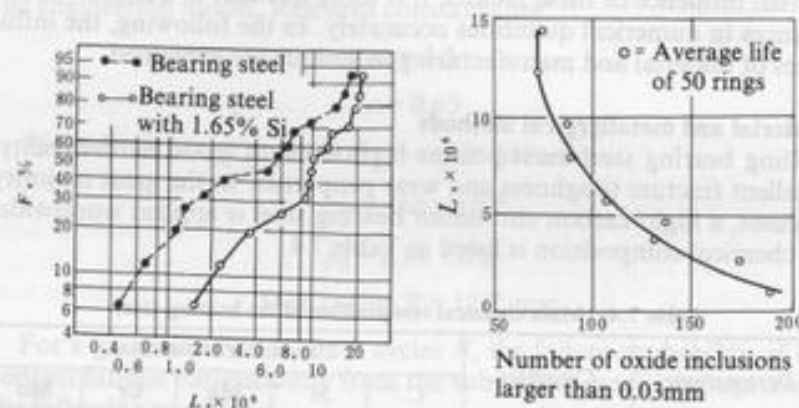


Fig. 7.7 Effect of silicon content on fatigue life

Fig. 7.8 Effect of oxide inclusions on fatigue life

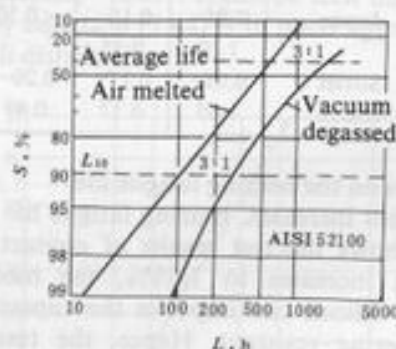


Fig. 7.9 Life of vacuum degassed steel

- (3) Metallurgical methods have an important effect on the contact fatigue. Generally speaking, the vacuum arc remelting process espe-

ially in combination with the vacuum induction melting process (VIM-VAR) produces an outstandingly clean material. In consideration of the economy, the vacuum degassed bearing steel is commonly used. Experiments indicate that the fatigue life of the vacuum degassed bearing steel is 2.5 to 3 times that of the conventional bearing steel. Fig. 7.9 shows a test result by Reference (69).

It should be noted that test results presented by different references are distinct. Table 7.5 shows the test results of deep groove ball bearings 6206 given by the Japan Bearing Industry Association.

The electric-slag remelting process may reduce oxygen content in the bearing steel, but it also reduces sulphurets.

Table 7.5 Results of life tests

Symbols	Bearing material	Number of test bearings	Life ratio	
			L_{10}	L_{50}
	Rating life		1.0	1.0
A	SUJ2 air melted	255	2.5	2.1
B	SUJ2 air melted	57	3.0	3.0
C	SUJ2 vacuum degassed	125	4.7	4.7
D	SUJ2 vacuum degassed	121	5.3	8.0

Forging methods

The forging process changes the fibrous direction in the bearing steel. Experiments conclude that the metallic fibres parallel to the rolling surface contribute most to the fatigue life and the metallic fibres vertical to the rolling surface contribute the least. Fig. 7.10 shows the relation between the ratio of the ball fatigue and the fibre direction. The polar regions are the weakest areas.

In a consideration of the fibrous distribution, steel tube with rolled raceway may lead to a longer fatigue life.

Heat treatment methods

The structure of hardened and tempered bearing steel should be of fine grains and martensite.

- (1) As carbon content in martensite is in the range of 0.5—0.45%, the fatigue life of the bearing steel is the optimum. Fig. 7.11 shows the

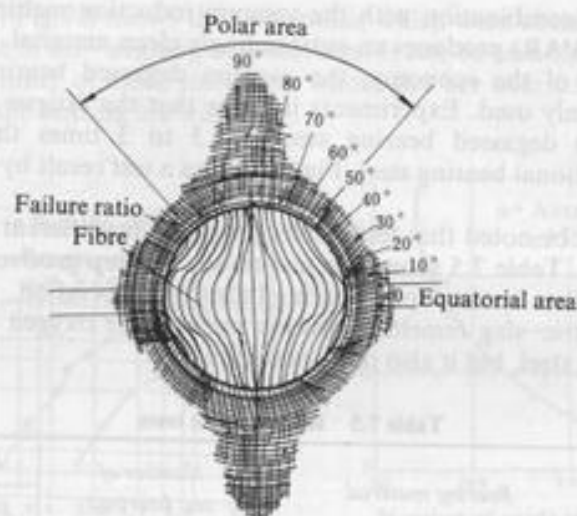


Fig. 7.10 Effect of fibrous distribution on fatigue

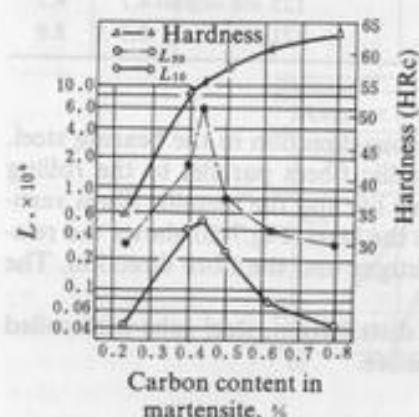


Fig. 7.11 Effect of carbon content in martensite on fatigue life

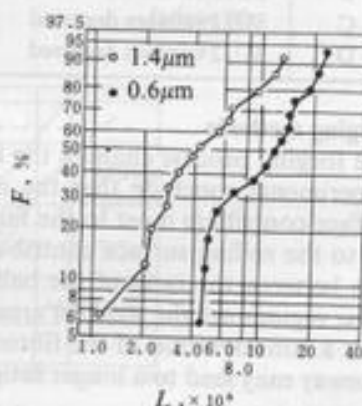


Fig. 7.12 Effect of carbonide size on fatigue life

life test results of thrust bearings by Reference (69). If carbon

content in martensite is less than 0.45%, the strength of the martensite decreases. In reverse, if carbon content in martensite is over 0.5%, its brittleness increases.

- (2) Large sizes of carbonides have a harmful influence on the fatigue life. Fig. 7.12 shows a typical test result (69). Carbon content in martensite was controlled around 0.5% in this test. A comparison between average carbonide sizes $0.6 \mu\text{m}$ and $1.4 \mu\text{m}$ was made. It can be seen that the samples with fine carbonides have the longer fatigue life.
- (3) If the hardness of a bearing component is lower than HR_{c60} , its contact fatigue life will decrease remarkably. Various empiric equations show the effect of the hardness on the fatigue life. Reference (70) suggests the following expression

$$\frac{L_1}{L_2} = \exp[0.01(R_{c2} - R_{c1})]$$

where

L_1 and $L_2 = L_{10}$ lives corresponding to values of the hardness HR_{c1} and HR_{c2}

R_{c1} and R_{c2} = values of the hardness HR_{c1} and HR_{c2} .

If hardness is less than HR_{c60} , the following relation between the fatigue life and the hardness HV is used

$$L \propto \text{HV}^{6.7}$$

Manufacturing methods

Precise manufacturing technique, such as grinding, superfinishing etc. may have important effects on the structure of surface layer, the residual stress and the surface roughness etc.

- (1) surface residual compressive stress can slow down the initiation and propagation of fatigue cracks and improve the bearing fatigue life. The following equation expresses the effect of the residual stress on the maximum dynamic shear stress

$$(\tau_0)_r = -\tau_0 - \frac{S_r}{2} \quad (7.45)$$

where

$(\tau_0)_r$ = maximum dynamic shear stress modified due to a residual

stress

S_r = residual stress, negative value for a compressive stress.

As a residual compressive stress exists, (τ_0) , decreases and the fatigue life increases.

- (2) Grinding affected layer causes a decrease in the contact fatigue life of bearing components. The temperature distribution on a component surface during grinding may change its metallurgical structure of the surface layer.

CHAPTER EIGHT

Dynamic load rating and equivalent dynamic load

Generally, the life of a rolling bearing means its fatigue life, that is, the total number of revolutions, or the number of hours at a given constant speed, which the bearing is capable of enduring before the first sign of fatigue appears at any rolling surface of the bearing. Besides contact fatigue, there are some other failure modes of bearings, such as wear, plastic indentation, scuffing, efficacy loss of grease, etc. The fatigue and other failures may cause the bearing to lose its running agility, to increase the friction moment and temperature level of the bearing, as well as to enlarge its vibration, noise and so on.

Contact fatigue

Fatigue phenomena occur on the operating surfaces of rolling bearings. At the beginning, fatigue cracks initiate from microcracks at or below the surfaces of a bearing. With further operation the cracks propagate. Finally pitting and spalling appear.

Research indicates that if a rolling bearing in operation is properly lubricated and aligned, kept free of abrasives and moisture, and properly loaded, the major failure mode is the contact fatigue appeared at the rolling surfaces of the bearing.

The theory of the dynamic load rating created by Lundberg and Palmgren is a basis of the calculation of the fatigue life of a rolling bearing. The calculation methods of the dynamic load rating and equivalent dynamic load of the rolling bearing specified by the International Standard ISO 281 are introduced in this chapter. Life adjustment factors and their tendency of modifications are also described.

Wear

The relative sliding among rolling elements and bearing raceways as well as the cage occurs in a rolling bearing. Characteristics of the sliding will be described in Chapter Fourteen. The relative sliding may bring about wear of the bearing components. Particularly when some

metallic powders, abrasives or other hard particles enter into the bearing, wear develops rapidly. Severe wear will cause damage of the raceway surface quality, reduce the rotating accuracy of the bearing, and increase its clearance and noise. Finally the bearing ceases to fulfil its proper performance. The total number of revolutions or number of operating hours at certain constant speeds before the bearing loses its normal operating performance due to wear is called the wear life of a bearing.

Rolling bearings used in machine tools, construction and agricultural machinery often show that considerable wear eventually does occur owing to dirt and foreign particles entering into the bearings. For other conditions, such as precise miniature bearings used in instruments and meters, the calculated fatigue life is much longer than the wear life due to very light load. Then, the wear is a main failure mode of this type of bearings. It should be noticed that the calculation method of the bearing wear life is only an approximate estimation. In Chapter Eleven, an empirical formula proposed by FAG Bearing Company will be introduced.

Plastic indentation

Plastic indentations corresponding to the positions of rolling elements appear at raceway surfaces when an excessive load or impact load acts on a bearing. As a result, vibrations and noises increase severely. The bearing would not be fit for further service.

It is easy to understand that plastic indentations occur under the instantaneous acting of the load. To avoid these, it is necessary to estimate the possible maximum load and select the correct type and size of the bearing upon the static load rating.

Scuffing

Rolling bearing damage may take place for a sudden rise in operating temperature without a change of the operating conditions due to high friction occurring among relative rolling or sliding surfaces in a bearing, especially a high speed bearing. The extreme temperature may lead to efficacy loss of lubricant, changes of material structures and metal adhesion. This kind of damage is called scuffing. It often happens in the operation of high speed bearings owing to improper designs and insufficient lubrication. To avoid these, it is necessary to improve the bearing design for the reduction of the sliding friction and

provide sufficient lubrication.

Efficacy loss of grease

The service life of the lubrication grease is limited. Mechanical stresses, oxidation, as well as separation of the grease into base oil and thickener may accelerate the grease distress. Eventually, the increase of the frictional temperature causes scuffing of operating surfaces of the bearing. Therefore it is necessary to replenish or renew the grease filling periodically. The recommended relubrication intervals including a safety margin are determined by practical experiences. These will be described in Chapter Twelve.

DYNAMIC LOAD RATING

Using the basics of the Lundberg and Palmgren assumptions, the equations of dynamic load rating of a contact (7.21) and (7.23) can be deduced. Furthermore, one can derive the dynamic load rating of a rolling bearing.

Simplification of dynamic load rating of a contact

The equations of curvature sum $\Sigma\rho$ and curvature function $F(\rho)$ are listed in Tables 2.1 and 2.6.

For a point contact

$$[1+F(\rho)]\frac{D_b}{2}\Sigma\rho = \frac{2}{1+\gamma} \quad (8.1)$$

$$[1-F(\rho)]\frac{D_b}{2}\Sigma\rho = D_b\left(\frac{1}{R} - \frac{1}{r}\right) \quad (8.2)$$

where

R = radius of curvature of a rolling element in a plane normal to the rolling direction, for balls $R = D_b/2$.

From equations (5.8) and (5.9), the number of contact cycles of the inner and outer raceways can be given by

$$u = \frac{Z}{2}(1 \pm \gamma) \quad (8.3)$$

Substituting equations (8.1) and (8.3) into equation (7.22) yields

$$\Phi = 0.06855\Omega_1 \frac{(1 \mp \gamma)^{1.8}}{(1 \pm \gamma)^{1/3}} \left(\frac{D_b}{D_m}\right)^{0.3} Z^{-1/3} \quad (8.4)$$

where

$$\Omega_1 = [1 + F(\rho)]^{2.1} \left(\frac{T_1}{T}\right)^{3.1} \left(\frac{\xi}{\xi_1}\right)^{0.4} m_a^{2.8} m_b^{3.5} \quad (8.5)$$

For inner and outer raceways, the respective diameters D_i and D_e have taken the place of D_a in equation (7.22)

Defining a function

$$\Omega = \frac{1 - F(\rho)}{1 + F(\rho)} = \frac{D_b}{2R} \frac{r - R}{R} (1 \mp \gamma) \quad (8.6)$$

Lundberg et al. indicate that Ω_1 may be expressed as a function of Ω approximately, that is

$$\Omega_1 = 1.3\Omega^{0.41} \quad (8.7)$$

Substituting equations (8.4), (8.6) and (8.7) into equation (7.21) gives the following expression for the dynamic load rating of a point contact

$$Q_c = A \left(\frac{2R}{D_b} \frac{r}{r - R}\right)^{0.41} \frac{(1 \mp \gamma)^{1.39}}{(1 \pm \gamma)^{1/3}} \left(\frac{D_b}{D_m}\right)^{0.3} D_b^{1.8} Z^{-1/3} \quad (8.8)$$

Similarly, the expression for the dynamic load rating of a line contact is derived from equations (7.23) and (7.24)

$$Q_c = B \frac{(1 \mp \gamma)^{29/27}}{(1 \pm \gamma)^{1/4}} \left(\frac{D_b}{D_m}\right)^{2/9} D_b^{29/27} l_e^{7/9} Z^{-1/4} \quad (8.9)$$

In the above equations, the upper signs refer to the inner ring and the lower signs refer to the outer ring.

Constants A and B are determined upon materials. For the standard through hardened bearing steel and in the Newton unit, experiments give the values of $A = 98.07$ and $B = 551.15$.

Dynamic load ratings of inner and outer rings

The loads carried by rolling elements are varied from one element to another in a bearing. Supposing the situation of a rotating inner ring

and a stationary outer ring, one may analyse the dynamic load ratings of an inner and outer rings separately.

Dynamic load rating of an inner ring

While the inner ring rotates, a point on its raceway undertakes load cycles of a same amplitude. On the basis of the accumulated damage principle of the fatigue, Lundberg et al. assume that the equivalent load of rolling elements in an inner ring is as follows

$$Q_{iq} = \left[\frac{1}{2\pi} \int_{-\psi_L}^{+\psi_L} Q'(\psi) d\psi \right]^{1/s} \quad (8.10)$$

where

ψ_L = angular extent of the load zone of a bearing

s = constant, $s = 3$ for a point contact and 4 for line contact.

Substituting equation (4.64) of the load distribution into equation (8.10) yields

$$Q_{iq} = Q_0 J_1 \quad (8.11)$$

where

Q_0 = the maximum rolling element load

J_1 = the load distribution integral, which can also express a ratio of the equivalent load of rolling elements in the rotating ring to the maximum rolling element load.

$$J_1 = \left\{ \frac{1}{2\pi} \int_{-\psi_L}^{+\psi_L} \left[1 - \frac{1}{2\epsilon} (1 - \cos\psi) \right]^{s/4} d\psi \right\}^{1/s} \quad (8.12)$$

For point contact

$$\frac{s}{t} = 3 \times \frac{3}{2} = \frac{9}{2}$$

For line contact

$$\frac{s}{t} = 4 \times 1.1 = 4.4$$

Substituting equation (4.68) into equation (8.11) yields

$$Q_{iq} = \frac{F_r J_1}{Z \cos z J_r} \quad (8.13)$$

If only half of a raceway of the bearing sustains the load, i.e. the load distribution factor ϵ is 0.5, and the equivalent rolling element load of the inner ring Q_{ie} reaches the dynamic load rating of rolling elements, expressed as $Q_{ie} F_r$ in equation (8.13) becomes the corresponding dynamic load rating of an inner ring. Therefore

$$C_i = Q_{ie} Z \cos \alpha \frac{J_2(0.5)}{J_1(0.5)} \quad (8.14)$$

For point contact

$$C_i = 0.407 Q_{ie} Z \cos \alpha \quad (8.15)$$

For line contact

$$C_i = 0.3767 Q_{ie} Z \cos \alpha \quad (8.16)$$

Dynamic load rating of an outer ring

While the outer ring is stationary, every point on its raceway undertakes various different load cycles. Upon the product law of probability, the survival probability of a whole outer ring is equal to the product of the survival probability of the individual parts of the outer ring. The equivalent load of rolling elements in an outer ring

$$Q_{eq} = \left[\frac{1}{2\pi} \int_{-\phi_L}^{+\phi_L} Q^w(\psi) d\psi \right]^{1/w} \quad (8.17)$$

where

w = constant, 10/3 for point contact and 9/2 for line contact.

Substituting equation (4.64) of the load distribution into equation (8.17) gives

$$Q_{eq} = Q_0 J_2 \quad (8.18)$$

where

J_2 = the load distribution integral, which can also express a ratio of the equivalent load of rolling elements in the stationary ring to the maximum rolling element load.

$$J_2 = \left\{ \frac{1}{2\pi} \int_{-\phi_L}^{+\phi_L} \left[1 - \frac{1}{2\epsilon} (1 - \cos \psi) \right]^{w/\epsilon} d\psi \right\}^{1/w} \quad (8.19)$$

For point contact

$$\frac{\omega}{i} = \frac{10}{3} \times \frac{3}{2} = 5$$

For line contact

$$\frac{\omega}{i} = \frac{9}{2} \times 1.1 = 4.95$$

Substituting of equation (4.68) into equation (8.11) yields

$$Q_{eq} = \frac{F_r J_2}{Z \cos \alpha J_1} \quad (8.20)$$

Similar to the inner ring condition, if the equivalent rolling element load of the outer ring reaches the dynamic load rating of rolling elements, expressed by $Q_{oe} F_r$ in equation (8.20) becomes the corresponding dynamic load rating of an outer ring. Then

$$C_e = Q_{oe} Z \cos \alpha \frac{J_2(0.5)}{J_1(0.5)} \quad (8.21)$$

For point contact

$$C_e = 0.389 Q_{oe} Z \cos \alpha \quad (8.22)$$

For line contact

$$C_e = 0.3634 Q_{oe} Z \cos \alpha \quad (8.23)$$

Calculation of dynamic load rating of a bearing

The survival probability of a whole bearing is the product of the survival probability of the inner and outer rings. Therefore, the dynamic load rating of a bearing may be expressed

$$C = \left[1 + \left(\frac{C_i}{C_e} \right)^w \right]^{-1/w} C_i \quad (8.24)$$

If there are i -row rolling elements, the survival probability of the entire bearing is the product of the survival probability of the individual rows. Assuming that every row has the same dynamic load rating expressed by C' as others, the dynamic load rating of the entire bearing is

$$C = i^{1-\frac{1}{\alpha}} C' \quad (8.25)$$

Particularly, double row bearings, that is $i=2$

For point contact

$$C = 1.62 C'$$

For line contact

$$C = 1.71 C'$$

On the basis of experiments, some modifications are made to the calculation of the dynamic load rating.

Influence of the rolling element diameter

The dynamic load rating would not be proportional to $D_b^{1.8}$ if the diameter of rolling elements $D_b > 25.4$ mm. A function is defined as follows

$$F(D_b) = \begin{cases} D_b^{1.8}, & D_b \leq 25.4 \text{ mm} \\ 3.647 D_b^{1.4}, & D_b > 25.4 \text{ mm} \end{cases}$$

Influence of the manufacturing and mounting deviation

The manufacturing and mounting deviation would make the actual value of the dynamic load rating of a bearing lower than its calculated value. Introducing reduction factors λ and η , their values and the upper limits of the curvature radii of raceways in ball bearings are listed in Table 8.1.

Table 8.1 Upper limits of raceway curvature radius and values of λ and η

Bearing type	Radius of raceway curvature		Reduction factor	
	r_i	r_e	λ	η
Deep groove ball bearings, single and double row angular contact ball bearings	$0.52 D_b$		0.95	—
Double row radial ball bearings	$0.52 D_b$		0.90	—
Self-aligning ball bearings	$0.53 D_b$	$0.5 \left(\frac{1}{\gamma} + 1 \right) D_b$	1	—
Single row angular contact ball bearings (magneto bearings)	$0.52 D_b$	∞	0.95	—
Thrust ball bearings	$0.54 D_b$		0.90	$1 - \frac{\sin \alpha}{3}$

Table 8.2 Values λ and ν for roller bearings

Contact type	λ	ν	$\lambda \nu$
Line contact at both raceways	0.45	1.36	0.61
Line contact at one raceway and point contact at another raceway	0.54	1.26	0.68
Modified line contact, roller length $l_r < 2.5 D_b$	0.61	1.36	0.83
Tapered roller bearings and spherical roller bearings both with modified line contact	0.65	1.2	0.78

Table 8.3 Calculation of dynamic load ratings

	Ball bearings		Roller bearings			
C	$f_c f_d^{0.3} Z^{2/3} F(D_b)$		$f_c f_d^{1/9} Z^{3/4} F(D_b)$			
$F(D_b)$	$D_b^{1.8}, D_b < 25.4 \text{ mm}$ $3.647 D_b^{1.4}, D_b > 25.4 \text{ mm}$		$D_b^{29/27}$			
f_c	$\lambda g \cdot f \cdot f_2 \left(\frac{2r_i}{2r_i - D_b} \right)^{0.41}$		$\lambda g \cdot f \cdot f_2$			
g_c	$\left[1 + \left(\frac{C_i}{C_e} \right)^{10/3} \right]^{-0.3}$		$\left[1 + \left(\frac{C_i}{C_e} \right)^{9/2} \right]^{-2/9}$			
$\frac{C_i}{C_e}$	$f_3 \left[\frac{r_i (2r_e - D_b)}{r_e (2r_i - D_b)} \right]^{0.41}$		f_3			
	Radial bearing	Thrust bearing		Radial bearing	Thrust bearing	
		$\alpha \neq 90^\circ$	$\alpha = 90^\circ$		$\alpha \neq 90^\circ$	$\alpha = 90^\circ$
γ	$\frac{D_b \cos \alpha}{D_m}$		$\frac{D_b}{D_m}$	$\frac{D_b \cos \alpha}{D_m}$		$\frac{D_b}{D_m}$
f_e	$(\cos \alpha)^{0.7}$	$(\cos \alpha)^{0.7} \tan \alpha$	1	$(\cos \alpha)^{7/9}$	$(\cos \alpha)^{7/9} \tan \alpha$	1
f_1	39.91	98.07 η		207.9	551.15 η	
f_2	$\frac{\gamma^{0.3} (1 - \gamma)^{1.39}}{(1 + \gamma)^{1/3}}$		$\gamma^{0.3}$	$\frac{\gamma^{2/9} (1 - \gamma)^{29/27}}{(1 + \gamma)^{1/4}}$		$\gamma^{2/9}$
f_3	1.04 f_4	f_4	1	1.04 f_4	f_4	1
f_4	$\left[\frac{1 - \gamma}{1 + \gamma} \right]^{1.72}$			$\left[\frac{1 - \gamma}{1 + \gamma} \right]^{143/108}$		

Reduction factor v of roller bearings

Depending on the conditions of the loading and contacts, point or line contact may exist in a roller bearing. A reduction factor v accounting for edge loading is introduced. Values of λ and v for different contact types are presented in Table 8.2.

The reduction factor η for roller bearings is connected with their contact angles and expressed by the following equation

$$\eta = 1 - 0.15 \sin \alpha \quad (8.26)$$

Substitution of equations (8.8), (8.9), (8.15), (8.16), (8.22) and (8.23) into equation (8.24) for the different bearing types gives the expression for the dynamic load ratings of rolling bearings. They are listed in Table 8.3.

Factor f_c

The factor f_c for the dynamic load rating is connected with the bearing structure and sizes. It also partly reflects the influence of materials and manufacturing deviation.

In Standard ISO 281, the values of factor f_c for different bearing types are given.

Given values of the size and number of rolling elements, the pitch diameter, as well as the contact angle, the dynamic load rating of a bearing can be calculated.

Deep groove ball bearings and angular contact ball bearings

It can be seen from Table 8.3 that the dynamic load rating of deep groove ball bearings or angular contact ball bearings is determined using the follow equation

$$C = f_c (i \cos \alpha)^{0.7} Z^{2/3} F(D_b) \quad (8.27)$$

The factor f_c depends on the ratio $D_b \cos \alpha / D_m$ and its values are listed in Table 8.4.

The values f_c in Table 8.4 are valid for following curvature radii of raceways.

For deep groove ball bearings and angular contact ball bearings

$$r_i < 0.52 D_b; \quad r_e < 0.53 D_b$$

For self-aligning ball bearings

$$r_i < 0.53 D_b$$

Thrust ball bearings

The dynamic load rating of thrust ball bearings with the contact angle $\alpha = 90^\circ$ is determined

Table 8.4 Factor f_c for radial ball bearings

$\gamma = \frac{D_b \cos \alpha}{D_m}$	Single row radial ball bearings, single and double row angular contact ball bearings	Double row radial ball bearings	Self-aligning ball bearings	Single row angular contact ball bearings (magneto type)
0.05	46.7	44.2	17.3	16.2
0.06	49.1	46.5	18.6	17.4
0.07	51.1	48.4	19.9	18.5
0.08	52.8	50.0	21.1	19.5
0.09	54.3	51.4	22.3	20.6
0.10	55.5	52.6	23.4	21.5
0.12	57.5	54.5	25.6	23.4
0.14	58.8	55.7	27.7	25.3
0.16	59.6	56.5	29.7	27.1
0.18	59.9	56.8	31.7	28.8
0.20	59.9	56.8	33.5	30.5
0.22	59.6	56.5	35.2	32.1
0.24	59.0	55.9	36.8	33.7
0.26	58.2	55.1	38.2	35.2
0.28	57.1	54.1	39.4	36.6
0.30	56.0	53.0	40.3	37.8
0.32	54.6	51.8	40.9	38.9
0.34	53.2	50.4	41.2	39.8
0.36	51.7	48.9	41.3	40.4
0.38	50.0	47.4	41.0	40.8
0.40	48.4	45.8	40.4	40.9

$$C = f_c Z^{2/3} F(D_b) \quad (8.28)$$

For thrust ball bearings with the contact angle $\alpha \neq 90^\circ$, the dynamic load rating is

$$C = f_c (\cos \alpha)^{0.7} \tan \alpha Z^{2/3} F(D_b) \quad (8.29)$$

Table 8.5 Factor f_c for thrust ball bearings

$\frac{D_b}{D_m}$	f_c	$\frac{D_b \cos \alpha}{D_m}$	f_c		
	$\alpha = 90^\circ$		$\alpha = 45^\circ$	$\alpha = 60^\circ$	$\alpha = 75^\circ$
0.01	36.7	0.01	42.1	39.2	37.3
0.02	45.2	0.02	51.7	48.1	45.9
0.03	51.1	0.03	58.2	54.2	51.7
0.04	55.7	0.04	63.3	58.9	56.1
0.05	59.5	0.05	67.3	62.6	59.7
0.06	62.9	0.06	70.7	65.8	62.7
0.07	65.8	0.07	73.5	68.4	65.2
0.08	68.5	0.08	75.9	70.7	67.3
0.09	71.0	0.09	78.0	72.6	69.2
0.10	73.3	0.10	79.7	74.2	70.7
0.12	77.4	0.12	82.3	76.6	
0.14	81.1	0.14	84.1	78.3	
0.16	84.4	0.16	85.1	79.2	
0.18	87.4	0.18	85.5	79.6	
0.20	90.2	0.20	85.4	79.5	
0.22	92.8	0.22	84.9		
0.24	95.3	0.24	84.0		
0.26	97.6	0.26	82.8		
0.28	99.8	0.28	81.3		
0.30	101.9	0.30	79.6		
0.32	103.9				
0.34	105.8				

* For $\alpha > 45^\circ$, values f_c are given by the linear interpolation.

The values f_c are listed in Table 8.5. They are valid for following curvature radii of raceways

$$r_i = r_e < 0.54 D_b$$

For multi-row single direction thrust ball bearings, the dynamic load rating is

$$C = (Z_1 + Z_2 + \dots + Z_n) \left[\left(\frac{Z_1}{C_1} \right)^{10/3} + \left(\frac{Z_2}{C_2} \right)^{10/3} + \dots + \left(\frac{Z_n}{C_n} \right)^{10/3} \right]^{-0.3} \quad (8.30)$$

Table 8.6 Factor f_c for radial roller bearings

$\gamma = \frac{D_b \cos \alpha}{D_m}$	$\lambda_v = 0.61$	$\lambda_v = 0.68$	$\lambda_v = 0.73$	$\lambda_v = 0.83$
0.01	38.3	42.7	45.8	52.1
0.02	44.6	49.7	53.5	60.8
0.03	48.9	54.5	58.5	66.5
0.04	52.0	57.9	62.2	70.7
0.05	54.5	60.7	65.1	74.1
0.06	56.5	63.0	67.6	76.9
0.07	58.2	64.9	69.7	79.2
0.08	59.6	66.5	71.4	81.2
0.09	60.9	67.9	72.9	82.8
0.10	61.9	69.1	74.1	84.2
0.12	63.5	70.7	75.9	86.4
0.14	64.5	71.9	77.2	87.7
0.16	65.0	72.5	77.9	88.5
0.18	65.2	72.8	78.1	88.8
0.20	65.1	72.7	78.0	88.7
0.22	64.8	72.3	77.6	88.2
0.24	64.3	71.6	76.9	87.5
0.26	63.5	70.8	76.0	86.4
0.28	62.6	69.8	75.0	85.2
0.30	61.5	68.6	73.7	83.8

where

$Z_1, Z_2, \dots$ = number of balls in each row

$C_1, C_2, \dots$ = dynamic load rating of each row

n = number of rows.

Radial roller bearings

The dynamic load rating of radial roller bearings is

$$C = f_c (il_e \cos \alpha)^{7/9} Z^{3/4} D^{29/27} \quad (8.31)$$

The values f_c are listed in Table 8.6.

Thrust roller bearings

The dynamic load rating of thrust roller bearings with the contact angle $\alpha = 90^\circ$ is determined

$$C = f_c l_e^{7/9} Z^{3/4} D^{29/27} \quad (8.32)$$

For thrust ball bearings with the contact angle $\alpha \neq 90^\circ$, the dynamic load rating is

$$C = f_c (l_e \cos \alpha)^{7/9} \tan \alpha Z^{3/4} D^{29/27} \quad (8.33)$$

The values f_c corresponding to $\lambda_v = 0.73$ are listed in Table 8.7.

For multi-row single direction thrust roller bearings, the dynamic load rating is

$$C = (Z_1 l_{e1} + Z_2 l_{e2} + \dots + Z_n l_{en}) \left[\left(\frac{Z_1 l_{e1}}{C_1} \right)^{9/2} + \left(\frac{Z_2 l_{e2}}{C_2} \right)^{9/2} + \dots + \left(\frac{Z_n l_{en}}{C_n} \right)^{9/2} \right]^{-2/9}$$

where

$Z_1, Z_2, \dots$ = number of rollers in each row

$l_{e1}, l_{e2}, \dots$ = effective length of each row

$C_1, C_2, \dots$ = dynamic load rating of each row

n = number of rows.

The selection of λ_v is a matter of great importance to the calculation of values f_c for radial and thrust roller bearings. In general, if the contact stress of the roller which carries the maximum rolling element load is evenly distributed over its length, larger values of λ_v can be chosen, such as 0.73 or 0.83. And then, if this roller has a very large stress concentration over its length, or rollers lack precise guiding, or the ratio of the roller length to its diameter is more than 2.5, smaller values of λ_v should be selected. Larger values of λ_v are not valid for thrust roller bearings owing to the fact that the significant sliding exists at the contact areas between the rollers and the raceways.

For the preceding calculation, both D_b and l_e are given in millimeters and C is obtained in Newtons.

Table 8.7 Factor f_c for thrust roller bearings

$\frac{D_b}{D_m}$	f_c	$\frac{D_b \cos \alpha}{D_m}$	f_c		
	$\alpha = 90^\circ$		$\alpha = 50^\circ$	$\alpha = 65^\circ$	$\alpha = 80^\circ$
0.01	105.4	0.01	109.7	107.1	105.6
0.02	122.9	0.02	127.8	124.7	123.0
0.03	134.5	0.03	139.5	136.2	134.3
0.04	143.4	0.04	148.3	144.7	142.8
0.05	150.7	0.05	155.2	151.5	149.4
0.06	156.9	0.06	160.9	157.0	154.9
0.07	162.4	0.07	165.6	161.6	159.4
0.08	167.2	0.08	169.5	165.5	163.2
0.09	171.7	0.09	172.8	168.7	166.4
0.10	175.7	0.10	175.5	171.4	169.0
0.12	183.0	0.12	179.7	175.4	173.0
0.14	189.4	0.14	182.3	177.9	175.5
0.16	195.1	0.16	183.7	179.3	
0.18	200.3	0.18	184.1	179.7	
0.20	205.0	0.20	183.7	179.3	
0.22	209.4	0.22	182.6		
0.24	213.5	0.24	180.9		
0.26	217.3	0.26	178.7		
0.28	220.9				
0.30	224.3				

* valid for $45^\circ < \alpha < 60^\circ$

** valid for $60^\circ < \alpha < 75^\circ$

*** valid for $75^\circ < \alpha < 90^\circ$

EQUIVALENT DYNAMIC LOAD

In the previous section, the dynamic load rating of rolling bearings is deduced under several assumed conditions. In many applications the actual load acting on a bearing does not satisfy these conditions. A constant radial or axial force must be presumed for the rating life calculation. The bearing under this force, called equivalent dynamic load, will endure for the same life as it does under the actual load.

Calculating equation of bearing life

Using equation (7.9) for the given bearings yields

$$\lg \frac{1}{S} \propto F_r^p L^e$$

where

S = survival probability of a bearing

L = bearing life in 10^6 revolutions.

Therefore,

$$\lg \frac{1}{S} = k F_r^p L^e$$

Considering that when $S = 0.90$ and $L = 1$, the dynamic load rating of rolling bearings is equal to a radial load component F_r required to stress half the circumference of the raceway, one may derive an expression for k .

Rearranging the foregoing equation

$$\lg \frac{1}{S} = \left(\frac{F_r}{C} \right)^p L^e \lg \frac{1}{0.9}$$

If L is replaced with the rating life L_{10} and F_r which follows the assumed conditions is the equivalent dynamic load, expressed as P , the calculating equation of the bearing rating life is

$$L_{10} = \left(\frac{C}{P} \right)^p \quad (8.34)$$

where

p = constant, 3 for ball bearings and 10/3 for roller bearings.

For simplification, equation (8.34) can be written

$$L = \left(\frac{C}{P} \right)^p \quad (8.35)$$

It should be noticed that L in equation (8.35) still expresses the bearing rating life, i.e. the life with the probability of failure 0.10. The correctness of equation (8.34) is tested by a great number of experiments, particularly the life tests of a same type of bearings under the action of different loads.

Equation (8.34) or (8.35) is the basic equation for the life calculation of rolling bearings.

Equivalent dynamic load of radial and angular contact bearings

Equation (7.18) indicates the relation between the rating life, the dynamic load rating of a contact and the actual rolling element load. It can be applied to the contact at the inner or outer raceways respectively, that is

$$L_i = \left(\frac{Q_{ec}}{Q_{iq}} \right)^p \quad (8.36)$$

and

$$L_e = \left(\frac{Q_{ec}}{Q_{eq}} \right)^p \quad (8.37)$$

where

L_i, L_e = rating life respective to the contact at the inner and outer raceways

Q_{ec}, Q_{eq} = dynamic load rating of a contact at the inner and outer raceways

Q_{iq}, Q_{eq} = equivalent dynamic load respective to the inner and outer raceways.

Substituting equations (8.13) and (8.14) into equation (8.36) yields

$$L_i = \left[\frac{C_i}{F_r} \frac{J_1(0.5)J_r(\epsilon)}{J_r(0.5)J_1(\epsilon)} \right]^p = \left(\frac{C_i}{P_i} \right)^p \quad (8.38)$$

where

$$P_i = F_r \frac{J_r(0.5)J_1(\epsilon)}{J_1(0.5)J_r(\epsilon)} \quad (8.39)$$

Similarly, substitution of equations (8.20) and (8.21) into equation (8.37) gives

$$L_e = \left[\frac{C_e}{F_r} \frac{J_2(0.5)J_r(\epsilon)}{J_r(0.5)J_2(\epsilon)} \right]^p = \left(\frac{C_e}{P_e} \right)^p \quad (8.40)$$

where

$$P_e = F_r \frac{J_r(0.5)J_2(\epsilon)}{J_2(0.5)J_r(\epsilon)} \quad (8.41)$$

P_i and P_e express the equivalent dynamic load of the inner and outer rings respectively. According to the product law of probability,

the survival probability of a bearing is equal to the product of the survival probability of the inner and outer rings. The following expression can be obtained

$$\left(\frac{P}{C}\right)^w = \left(\frac{P_i}{C_i}\right)^w + \left(\frac{P_e}{C_e}\right)^w \quad (8.42)$$

Substituting equations (8.39) and (8.41) into equation (8.42) yields

$$\frac{F_r}{P} = \frac{J_r(\epsilon)}{J_r(0.5)} \left[\left(\frac{C}{C_i} \frac{J_1(\epsilon)}{J_1(0.5)} \right)^w + \left(\frac{C}{C_e} \frac{J_2(\epsilon)}{J_2(0.5)} \right)^w \right]^{-1/w} \quad (8.43)$$

Similarly, one may derive the following equation

$$\frac{F_a \operatorname{ctg} \alpha}{P} = \frac{J_a(\epsilon)}{J_r(0.5)} \left[\left(\frac{C}{C_i} \frac{J_1(\epsilon)}{J_1(0.5)} \right)^w + \left(\frac{C}{C_e} \frac{J_2(\epsilon)}{J_2(0.5)} \right)^w \right]^{-1/w} \quad (8.44)$$

It can be seen from equations (8.43) and (8.44) that the equations of F_r/P and $(F_a \operatorname{ctg} \alpha)/P$ only depend on the load distribution factor ϵ . Fig 8.1 shows the equivalent dynamic load of deep groove ball bearings. The solid line indicates the calculating expression of F_r/P and $(F_a \operatorname{ctg} \alpha)/P$. A good correspondence is found between the calculated results and the test data.

To simplify for applications, the solid line is replaced with two sections of the straight lines. Then, the following expression can be established

$$\left. \begin{aligned} P &= F_r, & \text{when } F_a/F_r \leq \zeta \operatorname{tg} \alpha \equiv e \\ P &= XF_r + YF_a, & \text{when } F_a/F_r > \zeta \operatorname{tg} \alpha \equiv e \end{aligned} \right\} \quad (8.45)$$

where

X = radial load factor

Y = axial load factor

ζ = constants connected with bearing types, $\zeta = 1.1$ for radial ball bearings, 1.25 for angular contact ball bearings and 1.5 for radial roller bearings

e = parameter connected with ζ and the contact angle α . It should be noticed that this parameter e is in distinction from the slope parameter of the Weibull distribution.

Factors X and Y for various types of radial and angular contact bearings are presented in Tables 8.8 and 8.9. The contact angles of radial and angular contact ball bearings vary with the axial load, especially bearings with small initial contact angles. It is necessary to consider the variation of the contact angle for the determination of the equivalent dynamic load.

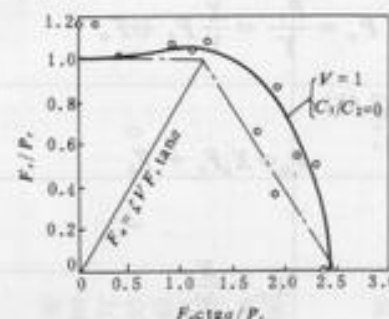


Fig. 8.1 Equivalent dynamic load

Equivalent dynamic load of thrust bearings

The dynamic load rating of a thrust bearing refers to a central axial load. The axial and radial load applied to the bearing should be converted to an equivalent dynamic load of a pure axial load.

In a similar way to the foregoing derivation of the equivalent dynamic load of radial bearings, one may deduce that for thrust bearings.

For distinction, equation (8.45) can be written as follows

$$P_r = XF_r + YF_a \quad (8.46)$$

Considering that both the radial equivalent dynamic load and the axial equivalent dynamic load give equal results of the life calculation for the same bearing, one may establish

$$\frac{C_r}{P_r} = \frac{C_a}{P_a} \quad (8.47)$$

where

C_r, C_a = dynamic load rating expressed in terms of the radial and axial load respectively

(continued)

Bearing type	Relative axial load	Single row bearing				Double row bearing			
		$\frac{F_a}{F_r} \leq e$		$\frac{F_a}{F_r} > e$		$\frac{F_a}{F_r} \leq e$		$\frac{F_a}{F_r} > e$	
		X	Y	X	Y	X	Y	X	Y
Angular contact ball bearing	α	$\frac{F_a}{C_0}$	$\frac{F_a}{ZD_0^2}$						
		0.014	0.172						
		0.029	0.345						
		0.057	0.689						
	10°	0.086	1.03	0	0.46	1	0	0.75	0.29
		0.11	1.38						0.32
		0.17	2.07						0.36
		0.29	3.45						0.38
		0.43	5.17						0.40
		0.015	0.172						0.43
Self-aligning ball bearing	15°	0.029	0.345	0	0.44	1	0	0.72	0.46
		0.058	0.689						0.47
		0.087	1.03						0.50
		0.12	1.38						0.55
		0.17	2.07						0.56
		0.29	3.45						0.57
		0.44	5.17						0.68
		0.015	0.172						0.80
		0.029	0.345						0.95
		0.058	0.689						1.14
Angular contact ball bearing—magnet type	20°	0.015	0.172	0.43	0.40	0.43	0.40	0.65	1.51gx
	25°	0.029	0.345	0.41	0.39	0.41	0.39	0.63	1.51gx
	30°	0.058	0.689	0.39	0.37	0.39	0.37	0.60	1.51gx
	35°	0.087	1.03	0.37	0.35	0.37	0.35	0.57	1.51gx
	40°	0.12	1.38	0.35	0.33	0.35	0.33	0.54	1.51gx
	45°	0.17	2.07	0.33	0.33	0.33	0.33	0.54	1.51gx
		0.29	3.45	0.33	0.33	0.33	0.33	0.54	1.51gx
		0.44	5.17	0.33	0.33	0.33	0.33	0.54	1.51gx
		0.015	0.172	0.33	0.33	0.33	0.33	0.54	1.51gx
		0.029	0.345	0.33	0.33	0.33	0.33	0.54	1.51gx

In many operating conditions, the load and the rotational speed vary either randomly or according to a cycle. It is necessary to determine a mean equivalent dynamic load under which at the mean speed the bearing would have the same life as the one in the actual condition of the load and the speed.

Table 8.9 Factors X and Y for radial and angular contact roller bearings

Bearing type	$\frac{F_a}{F_r} \leq e$		$\frac{F_a}{F_r} > e$		e
	X	Y	X	Y	
Single row bearing $\alpha \neq 0$	1	0	0.4	0.4ctgx	1.5tgx
Double row bearing $\alpha \neq 0$	1	0.45ctgx	0.67	0.67ctgx	1.5tgx

Table 8.10 Factors X and Y for thrust ball bearings

α	Single* direction		Double direction				e
	$\frac{F_x}{F_r} > e$		$\frac{F_x}{F_r} \leq e$		$\frac{F_x}{F_r} > e$		
	X	Y	X	Y	X	Y	
45 °	0.66	1	1.18	0.59	0.66	1	1.25
50 °	0.73		1.37	0.57	0.73		1.49
55 °	0.81		1.60	0.56	0.81		1.79
60 °	0.92		1.90	0.55	0.92		2.17
65 °	1.06		2.30	0.54	1.06		2.68
70 °	1.28		2.90	0.53	1.28		3.43
75 °	1.66		3.89	0.52	1.66		4.67
80 °	2.43		5.86	0.52	2.43		7.09
85 °	4.80		11.75	0.51	4.80		14.29
$\alpha \neq 90^\circ$	$1.25 \operatorname{tg} \alpha$ $\left(1 - \frac{2}{3} \sin \alpha\right)$	1	$\frac{20}{13} \operatorname{tg} \alpha$ $\left(1 - \frac{1}{3} \sin \alpha\right)$	$\frac{10}{13}$ $\left(1 - \frac{1}{3} \sin \alpha\right)$	$1.25 \operatorname{tg} \alpha$ $\left(1 - \frac{2}{3} \sin \alpha\right)$	1	$1.25 \operatorname{tg} \alpha$

* Single direction thrust ball bearings are not valid for the condition of $F_a/F_r < e$.

On the basis of the linear accumulated damage principle of fatigue, the following equation is established when a bearing under the equivalent dynamic load $P_1, P_2, \dots, P_n$ has worked correspondingly on 10^6 revolutions of $N_1, N_2, \dots, N_n$

$$\sum_{i=1}^n \frac{N_i}{L_i} = 1 \quad (8.52)$$

in which, $L_1, L_2, \dots, L_n$ are the rating lives of the bearing respectively under the action of $P_1, P_2, \dots, P_n$.

Table 8.11 Factors X and Y for thrust roller bearings

Bearing type	$\frac{F_a}{F_r} \leq e$		$\frac{F_a}{F_r} > e$		e
	X	Y	X	Y	
Single direction $\alpha \neq 90^\circ$	—*	—*	$\tan \alpha$	1	$1.5 \tan \alpha$
Double direction $\alpha \neq 90^\circ$	$1.5 \tan \alpha$	0.67	$\tan \alpha$	1	$1.5 \tan \alpha$

* Single direction roller bearings are not valid for the condition of $F_a / F_r < e$.

In integral format, the mean equivalent dynamic load can be obtained

$$P_m = \left[\frac{1}{L} \int_0^L (XF_r + YF_a)^p dN \right]^{1/p} \quad (8.53)$$

Equation (8.53) is more difficult to solve because L is unknown. For a cycle load of period N_0 , equation (8.53) becomes

$$P_m = \left[\frac{1}{N_0} \int_0^{N_0} (XF_r + YF_a)^p dN \right]^{1/p} \quad (8.54)$$

If the equivalent dynamic load is made up of a number of values $P_1, P_2, \dots, P_n$, corresponding to the speeds $n_1, n_2, \dots, n_n$, and the operating time $t_1, t_2, \dots, t_n$, the mean equivalent dynamic load is

$$P_m = \left[\frac{P_1^p n_1 t_1 + P_2^p n_2 t_2 + \dots + P_n^p n_n t_n}{n_1 t_1 + n_2 t_2 + \dots + n_n t_n} \right]^{1/p} \quad (8.55)$$

If the equivalent dynamic load is made up of a number of values $P_1, P_2, \dots, P_n$, at the constant speed, corresponding to the number of 10^6 revolutions $N_1, N_2, \dots, N_n$, the mean equivalent dynamic load is

$$P_m = \left[\frac{P_1^p N_1 + P_2^p N_2 + \dots + P_n^p N_n}{N_0} \right]^{1/p} \quad (8.56)$$

where

N_0 = a cycle load of period and $N_0 = N_1 + N_2 + \dots + N_n$.

In some special cases, the mean equivalent dynamic load can be defined. Fig 8.2 shows a case where only two loads act with different duration t . The mean equivalent dynamic load can be determined from this figure.

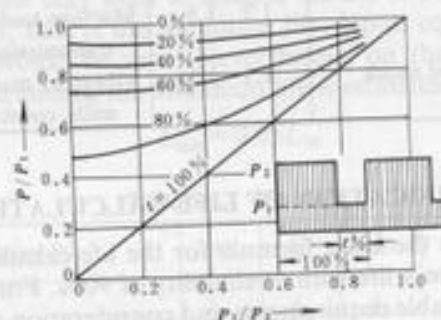


Fig. 8.2 Mean equivalent dynamic load

If a bearing load varies approximately linearly between $P_{\min}$ and $P_{\max}$ as shown in Fig. 8.3, the following approximate estimate can be used

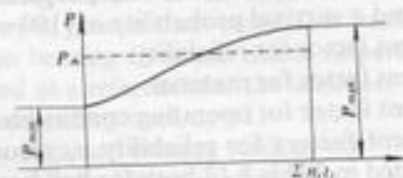


Fig. 8.3 Approximately linear variation of load

$$P_n = \frac{P_{\min} + 2P_{\max}}{3} \quad (8.57)$$

Vibration, shock or other additional dynamic loads may occur in various machines and equipment. Their influence on the rating life of bearings is included in the calculation by an operation factor f_p listed in Table 8.12. The equivalent dynamic load is

$$P = f_p (XF_r + YF_a) \quad (8.58)$$

Table 8.12 Factor f_p

Loading properties	f_p	Examples
Stationary or slight shock	1.0–1.2	Motors, air-conditioners
General shock	1.2–1.3	Machine tools, blowers transmission equipment
Severe vibration and shock machinery	1.5–3.0	Vibratory machines, rolling mills, construction

MODIFICATION OF LIFE CALCULATION

Equation (8.35) is the basic formula for the life calculation of rolling bearings, leading to a life with a reliability of 90%. For satisfaction to various highly reliable requirements and consideration of the influence of material and lubrication improvements, the Standard ISO 281 proposes the following revised life equation

$$L_n = a_1 a_2 a_3 \left(\frac{C}{P} \right)^p \quad (8.59)$$

where

L_n = modified life for different material properties and operating conditions and a survival probability of (100– n)%

a_1 = life adjustment factor for reliability

a_2 = life adjustment factor for material

a_3 = life adjustment factor for operating conditions.

The life adjustment factors for reliability a_1 recommended by the standard are presented in Table 8.13 both for ball bearings and for roller bearings.

It can be deduced from the Weibull distribution

$$\frac{L_n}{L_{10}} = \left(\frac{\ln S}{\ln 0.9} \right)^{1/\alpha} \quad (8.60)$$

where

L_n = bearing life corresponding to a survival probability S

$\alpha = 1.5$

S = survival probability, $S = (100 - n)\%$.

Hence, factor a_1 is

$$a_1 = \frac{L_n}{L_{10}} = \left(\frac{\ln S}{\ln 0.9} \right)^{1/1.5} \quad (8.61)$$

Sometimes, one may want to analyse the life of bearings with a reliability of 100%, that is the minimum life. Upon equation (8.11), the minimum life would be zero. Nevertheless, on the basis of a great number of experiments, the minimum life is estimated at

$$L_{\min} = 0.05L_{10}$$

Table 8.13 Factor a_1

Reliability %	90	95	96	97	98	99
L_n	L_{10}	L_5	L_4	L_3	L_2	L_1
a_1	1.0	0.62	0.53	0.44	0.33	0.21

Factor a_2 takes into account the metallurgical methods and properties of the material, as well as its heat treatment. The conventional rating life calculation in which the value of a_2 is equal to 1 is based on the standard bearing steel. For an improved steel or special heat treatment, factor a_2 is greater than unity. It is hard to give the precise values because the bearing lives are rather dispersive. The following values are proposed as a reference for the selection.

For the standard bearing steel, $a_2 = 1$.

For the vacuum degassed bearing steel, $a_2 = 3$.

For the vacuum remelted bearing steel, $a_2 = 5$.

Factor a_3 takes into account various lubrication conditions, including oil film thickness, viscosity of lubricant, cleanliness, etc.,

which have a positive or negative influence on the rating life of bearings. The conventional rating life calculation in which the value of a_3 is equal to 1 is based on the normal lubrication. If the viscosity of a lubricant at the operating temperature is lower than $13 \times 10^{-6} \text{ m}^2/\text{s}$ (13cSt) for ball bearings, or lower than $2 \times 10^{-5} \text{ m}^2/\text{s}$ (20cSt) for roller bearings, or if the value of $D_m n$ is less than 10000 mm · rpm, factor a_3 is less than unity. On the other hand, if a bearing works with a hydrodynamic state of lubrication, that is, where no metal-to-metal contact exists between rolling elements and raceways, factor a_3 is greater than unity.

It is convenient to judge a lubrication condition using a film thickness parameter Λ described in Chapter Six. The following values of a_3 are proposed: If Λ is less than 0.8, i.e. the bearing will probably have a certain degree of surface distress which can lead to rapid fatigue failure of the rolling surfaces, factor a_3 is less than unity. If the region of Λ is between 0.8 and 4, factor a_3 is equal to 1. On the other hand, if Λ is equal to or greater than 4, i.e. the bearing works with a



Fig. 8.4 Required viscosity

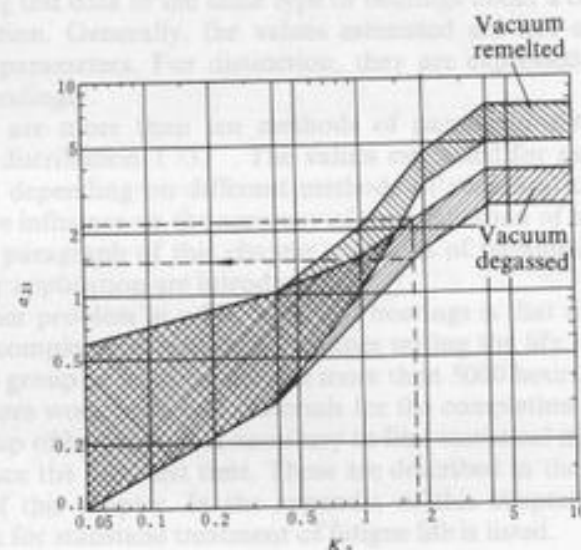
sufficient elastohydrodynamic lubrication, factor a_3 can be chosen as 2.

Experiments indicate that factors a_2 and a_3 are not independent of each other. It is not possible to choose their values separately.

The SKF Company suggests combining factors a_2 and a_3 into one factor called a_{23} . Then, equation (8.59) is rearranged as follows

$$L_n = a_1 a_{23} \left(\frac{C}{P} \right)^r \quad (8.62)$$

Factor a_{23} may be determined from Fig. 8.4 and 8.5. The selected viscosity v of a lubricant is divided by the viscosity v_1 found from Fig. 8.4 using the rotational speed and the average diameter of the bearing, that is

Fig. 8.5 Modified factor a_{23}

Some estimation methods which are described in this paragraph. They are graphic method, least-squares, optimum, contact, as well as maximum likelihood estimation. For convenience of understanding, some mathematical characteristics are introduced briefly.

$$K_v = \frac{v}{v_1} \quad (8.63)$$

Then, factor a_{23} can be chosen from Fig.8.5. It should be noted that ν is the kinematic viscosity at the operating temperature of the bearing.

When the ratio of ν / ν_1 is less than 0.4, the values for two kinds of materials are the same. It means that in this case the bearing life can not be increased even though a high quality material is used.



Fig. 8.5 Required viscosity

CHAPTER NINE

Statistical treatments of life experiments

Having mentioned it in Chapter Seven, it is necessary to analyse test data by statistical methods because fatigue lives of bearings are extremely dispersive. Bearing fatigue life is a random variable mostly accepted to follow a Weibull probability distribution shown by equation (7.3). The purpose of statistical treatments is to estimate the values of shape parameter e and scale parameter β of Weibull distribution using test data of the same type of bearings under a certain testing specification. Generally, the values estimated are not equal to true ones of parameters. For distinction, they are expressed by $\hat{e}$ and $\hat{\beta}$ correspondingly.

There are more than ten methods of parameter estimations for Weibull distribution (73). The values estimated for parameters are different depending on different methods of statistical treatments. It may have influence on the accuracy of the evaluation of test results. In the first paragraph of this chapter, methods of parameter estimation and their application are introduced.

Another problem in a life testing of bearings is that it takes a long time to complete life testing. Sometimes testing the life of a few bearings in a group of bearings may be more than 5000 hours. It consumes much more work force and materials for the completion of life testing of a group of bearings. It is necessary to find statistical methods which can reduce the total test time. These are described in the second paragraph of this chapter. In the appendix of this chapter, a computer program for statistical treatment of fatigue life is listed.

PARAMETER ESTIMATIONS OF WEIBULL DISTRIBUTION

Some estimation methods widely used are described in this paragraph. They are graphic method, improved least-squares, optimum linear constant, as well as maximum likelihood estimation. For convenience of understanding, some mathematical characteristics are introduced briefly.

Mathematical characteristics of Weibull distribution

Probability density

Having a differentiation on equation (7.3), the probability density of Weibull distribution can be obtained

$$f(L) = \frac{e}{\beta} \left(\frac{L}{\beta}\right)^{e-1} \exp\left[-\left(\frac{L}{\beta}\right)^e\right] \quad (9.1)$$

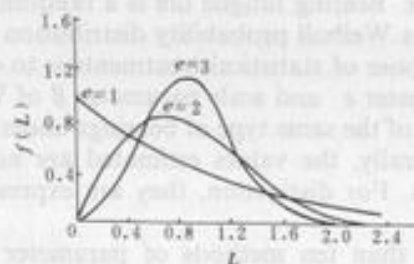


Fig. 9.1 Probability density function

The curves of probability density functions are shown in Fig. 9.1. It can be seen that the shape of the distribution is skew. $e=1$ for exponential distribution, $e=2$ for Rayleigh distribution, and curves approximate to the normal distribution while $3.2 < e < 3.7$.

Mathematical expectation

According to the average value in statistics, we have the following mathematical expectation

$$E(L) = \beta \Gamma\left(1 + \frac{1}{e}\right) \quad (9.2)$$

where

$\Gamma(x)$ = function of x variable, it can be found out from its table.

Square deviation

$$\sigma_0^2(L) = \beta^2 \left[\Gamma\left(1 + \frac{2}{e}\right) - \Gamma^2\left(1 + \frac{1}{e}\right) \right] \quad (9.3)$$

More detail can be found in (57).

Graphic method of parameter estimation

By using the graphic method it is possible: (1) to estimate parameters of Weibull distribution quickly; (2) to evaluate the fitness of Weibull distribution and test results; (3) to show bearing life corresponding to any probability of failure. Even though its accuracy is not good, it still is an effective method for parameter estimation.

The main contents of the graphic method are as follows.

Probability of failure corresponding to variables of order statistics

Data obtained from a life testing of bearings are arranged according to their actual value, therefore

$$L_1 \leq L_2 \leq \dots \leq L_i \leq \dots \leq L_n$$

L_i is called a variable of order statistics.

Obviously, L_i is a random variable, the probability of failure corresponding to the variable L_i is

$$F(L_i) = 1 - \exp\left[-\left(\frac{L_i}{\beta}\right)^e\right] \quad (9.4)$$

The probability of failure also is a random variable. It may be shown that the expected value of $F(L_i)$ is

$$\overline{F(L_i)} = \frac{i}{n+1} \quad (9.4)$$

Normally, the vertical axis indicates the probability of failure of variable L_i and the horizontal axis is L_i . Test data are expressed in the probability paper. All these points are located round a straight line, as shown in Fig. 7.2.

Graphic method (a) of parameter estimation

Substituting $L = \beta$ into equation (7.3), it can be shown that $F(\beta) = 0.632$. By drawing a straight line by estimation close to all the test points, the bearing life corresponding to an intersection of the straight line and the horizontal line $F(L) = 63.2\%$ is β . Assuming that an angle between the straight line and the horizontal axis is θ , the estimated value of e is

$$\hat{e} = \lg \theta \quad (9.5)$$

If two points L_1 and L_2 are taken at a certain distance and their probability axes $F(L_1)$ and $F(L_2)$ respectively are found, the estimated value of e can be calculated according to the following equation

$$\hat{e} = \frac{\lg \lg \frac{1}{1-F(L_1)} - \lg \lg \frac{1}{1-F(L_2)}}{\lg L_1 - \lg L_2} \quad (9.6)$$

Graphic method (b) of parameter estimation

The probability of Weibull distribution may be slightly modified, as shown in Fig. 9.2. The right axis and the upper axis $\ln L$ are on a normal scale, but the lower axis L is on a natural logarithmic scale.

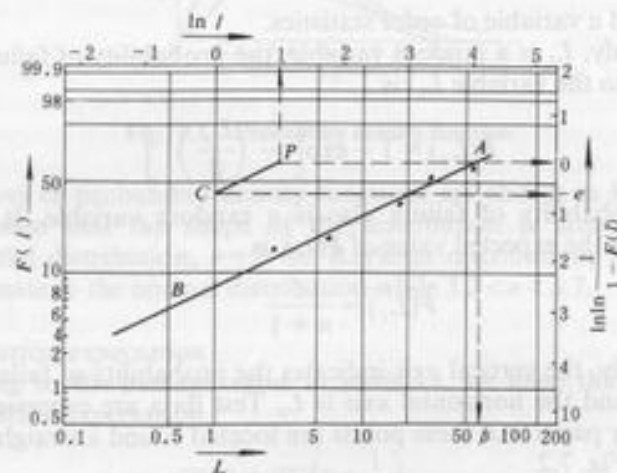


Fig. 9.2 Graphic method (b)

A straight line can be drawn as described in the graphic method (a). Drawing another straight line parallel through point P , whose axes are $\ln L = 1$ and $\ln \ln \{1/[1-F(L)]\} = 0$. It intersects the vertical line $\ln L = 0$ at point C . Then the correct coordinate value of point C is the value of $-\hat{e}$. A horizontal line from point P intersects the first line at

point A . Therefore lower coordinate value of point A is the value of β .

Improved least-squares method

It is necessary to introduce analytical methods of parameter estimations when high precision is needed. One of the commonly used analytical methods is the improved least-squares method. Estimated values of parameters e and β can be calculated according to the following equations

$$\frac{1}{2.30258\hat{e}} = \frac{S_{\lg L}}{\sigma_n} \quad (9.7)$$

$$\lg \hat{\beta} = \overline{\lg L} + \frac{y_n}{2.30258\hat{e}} \quad (9.8)$$

where

$$\overline{\lg L} = \frac{1}{n} \sum_{i=1}^n \lg L_i \quad (9.9)$$

$$S_{\lg L} = \sqrt{\frac{n}{n-1} (\lg^2 L - \overline{\lg L}^2)} \quad (9.10)$$

$$\lg^2 L = \frac{1}{n} \sum_{i=1}^n \lg^2 L_i \quad (9.11)$$

Values of σ_n, y_n for various numbers of testing bearings are presented in Table 9.1.

Example. Analyse data of a life test for tapered roller bearings 30208. A total of nineteen bearings were tested under a radial load of 10780 N and a thrust load of 3430 N at a rotational speed of 4500 rpm. Life data are listed in Table 9.2.

$\lg L_i$ and $\lg^2 L_i$ are presented in Table 9.2. From equations (9.9) to (9.11)

$$\overline{\lg L} = 3.3738; \quad S_{\lg L} = 0.3676$$

From Table 9.1

$$\sigma_n = 1.0566; \quad y_n = 0.5220$$

Substituting these values into equations (9.7) and (9.8) yields

$$\hat{e} = 1.2483; \quad \hat{\beta} = 3593$$

Optimum linear constant method

In (73), an adoption of an optimum linear constant method is suggested, which is both precise and easy to calculate. It will be explained later that this method may also be applied to test data of a truncated sample for reduction of testing time.

An estimate of parameters e and β can be obtained according to the following equations

$$\hat{e} = \frac{1}{2.3026 \sum_{i=1}^n a_{in} \lg L_i} \quad (9.12)$$

Table 9.1 Values of σ_n, y_n

n	y_n	σ_n	n	y_n	σ_n
8	0.4843	0.9043	30	0.53622	1.11238
9	0.4902	0.9288	31	0.5371	1.1159
10	0.4952	0.9497	32	0.5380	1.1193
11	0.4996	0.9676	33	0.5388	1.1226
12	0.5035	0.9833	34	0.5396	1.1255
13	0.5070	0.9972	35	0.54034	1.12847
14	0.5100	1.0095	36	0.5410	1.1313
15	0.5128	1.02057	37	0.5418	1.1339
16	0.5157	1.0316	38	0.5424	1.1363
17	0.5181	1.0411	39	0.5430	1.1388
18	0.5202	1.0493	40	0.54362	1.14132
19	0.5220	1.0566	41	0.5442	1.1436
20	0.52355	1.06283	42	0.5448	1.1458
21	0.5252	1.0696	43	0.5453	1.1480
22	0.5268	1.0754	44	0.5458	1.1499
23	0.5283	1.0811	45	0.54630	1.15185
24	0.5296	1.0864	46	0.5468	1.1538
25	0.53086	1.09145	47	0.5473	1.1557
26	0.5320	1.0961	48	0.5477	1.1574
27	0.5332	1.1004	49	0.5481	1.1590
28	0.5343	1.1047	50	0.54854	1.16066
29	0.5353	1.1086	60	0.55208	1.17467

$$\lg \hat{\beta} = \sum_{i=1}^n b_{in} \lg L_i \quad (9.13)$$

where

a_{in}, b_{in} = coefficients of the optimum linear constant estimation.

These can be found out from (75). Table 9.3 gives the values for $n = 18, 19$ and 20.

The estimated parameters e and β for the above example given in Table 9.2 can be shown as

$$\hat{e} = 2.2397; \quad \hat{\beta} = 3245.5$$

Table 9.2 Improved least-squares estimation

Order i	Failure probability $\frac{i}{n+1} \%$	Life t_i	$\lg t_i$	$\lg^2 t_i$
1	5	120	2.0792	4.3231
2	10	1189	3.0752	9.4569
3	15	1242	3.0941	9.5735
4	20	1335	3.1255	9.7688
5	25	1940	3.2878	10.8096
6	30	2007	3.3025	10.9065
7	35	2254.5	3.3531	11.2433
8	40	2656	3.4242	11.7251
9	45	3211.5	3.5067	12.2969
10	50	3246	3.5113	12.3292
11	55	3364	3.5269	12.4390
12	60	3378	3.5287	12.4517
13	65	3647.5	3.5620	12.6878
14	70	3672	3.5649	12.7085
15	75	3686.5	3.5666	12.7206
16	80	3794.5	3.5792	12.8107
17	85	4108.5	3.6137	13.0588
18	90	5016	3.7004	13.6930
19	95	5022.5	3.7009	13.6967
$\sum_{i=1}^n$			64.1029	218.6997

Table 9.3 Values of a_m and b_m

i	$n=18$		$n=19$		$n=20$	
	a_m	b_m	a_m	b_m	a_m	b_m
1	-0.03817	0.01509	-0.03600	0.01428	-0.03406	0.01355
2	-0.04097	0.01933	-0.03860	0.01812	-0.03648	0.01704
3	-0.04222	0.02326	-0.03983	0.02166	-0.03769	0.02026
4	-0.04250	0.02709	-0.04020	0.02511	-0.03812	0.02338
5	-0.04196	0.03091	-0.03987	0.02853	-0.03795	0.02646
6	-0.04068	0.03477	-0.03890	0.03198	-0.03721	0.02957
7	-0.03863	0.03873	-0.03728	0.03549	-0.03593	0.03271
8	-0.03577	0.04282	-0.03500	0.03911	-0.03409	0.03593
9	-0.03199	0.04710	-0.03198	0.04286	-0.03165	0.03926
10	-0.02716	0.05161	-0.02812	0.04679	-0.02853	0.04272
11	-0.02104	0.05644	-0.02328	0.05096	-0.02463	0.04636
12	-0.01330	0.06169	-0.01723	0.05542	-0.01981	0.05022
13	-0.00341	0.06748	-0.00966	0.06027	-0.01386	0.05435
14	0.00949	0.07403	-0.00006	0.06563	-0.00646	0.05886
15	0.02694	0.08171	0.01240	0.07170	0.00287	0.06384
16	0.05213	0.09122	0.02918	0.07883	0.01490	0.06950
17	0.09353	0.10431	0.05331	0.08765	0.03105	0.07613
18	0.23569	0.13241	0.09283	0.09980	0.05420	0.08435
19			0.22829	0.12582	0.09203	0.09567
20					0.22142	0.11986

Maximum likelihood estimation

Considering a life test of n bearings which leads to the failure of bearings at lives $L_1, L_2, \dots, L_n$ a likelihood function may be adopted

$$\tilde{L}(L_1, \dots, L_n, e, \beta) = \prod_{i=1}^n \left(\frac{e}{\beta^e} \right) L_i^{e-1} \exp \left[- \left(\frac{L_i}{\beta} \right)^e \right]$$

Taking the natural logarithm of the likelihood function, and solving the maximum of the function, estimate equations of parameters e and β are as follows

$$\frac{1}{\hat{e}} + \frac{1}{n} \sum_{i=1}^n \ln L_i - \frac{\sum_{i=1}^n L_i^e \ln L_i}{\sum_{i=1}^n L_i^e} = 0 \quad (9.14)$$

$$\hat{\beta} = \left(\frac{\sum_{i=1}^n L_i^e}{n} \right)^{1/e} \quad (9.15)$$

The estimate of parameter $\hat{e}$ can be obtained from equation (9.14), which is a nonlinear equation. The Newton-Raphson method works well over the parameter range met in life tests of bearings. Given the value of $\hat{e}$, it is easy to determine $\hat{\beta}$ from equation (9.15).

Rating life, i.e. life whose probability of failure equals 0.10, is

$$\hat{L}_{10} = \hat{\beta}(0.10536)^{1/\hat{e}} \quad (9.16)$$

It will be mentioned later that estimates of parameters obtained from equations (9.14) and (9.15) have to be modified to find out unbiased estimates of the parameters.

Consider the example listed in Table 9.2. The estimates may be solved by using the program listed in the appendix of this chapter.

$$\hat{e} = 2.2174; \quad \hat{\beta} = 3212.0$$

It can be seen that for a same group of test data various methods of parameter estimations will lead to different estimates of parameters. For comparison, a concept of effectiveness is introduced. Supposing both $\hat{\theta}_n$ and $\hat{\theta}'_n$ are unbiased estimates and the following inequality is available for any size n of a sample,

$$\sigma^2(\hat{\theta}_n) < \sigma^2(\hat{\theta}'_n)$$

then we state that $\hat{\theta}_n$ is more effective than $\hat{\theta}'_n$.

To compare effectiveness of least-squares, moment, extreme value and maximum likelihood methods, Monte carlo methods are used in (78). It is concluded that the maximum likelihood method is more effective than others.

TREATMENT METHODS OF ACCELERATED TESTS

For reduction of total test time, various testing methods are utilized, such as increasing test load or speed, reducing numbers of test bearings, as well as a special programed load. On the other hand, it is possible to reduce the dispersion of test data upon statistical principle. Therefore estimate methods which have equivalent precision can be

reached by using less test data. These methods can be simple and can reduce total test time.

Sudden death test

The sudden death test is a method commonly used in dividing a life test group into a number of equally sized subgroups and testing until the occurrence of the first failure in each subgroup. This method is widely applied by most bearing manufacturing companies in the world.

Theoretical consideration

Probability of surviving L hours can be expressed as follows

$$S(L) = 1 - F(L) = \exp \left[- \left(\frac{L}{\beta} \right)^e \right] \quad (9.17)$$

A group of m bearings is tested under fixed operating conditions until the first one of them fails. Assuming that failures of the bearings are independent from each other, the probability of surviving L hours for the one which has the shortest life among the m bearings is

$$S(L)_m = [S(L)]^m = \exp \left[- \left(\frac{m^{1/e} L}{\beta} \right)^e \right] \quad (9.18)$$

Introducing

$$\beta^* = \beta m^{-1/e} \quad (9.19)$$

It can be seen that $S(L)_m$ also follows a Weibull distribution, which has a different scale parameter β^* from the original one.

It has been deduced that the square deviation $\sigma^2(L)_m$ of the distribution expressed by equation (9.18) has the following relationship with the square deviation $\sigma^2(L)$ of the original distribution

$$\frac{\sigma^2(L)_m}{\sigma^2(L)} = m^{-1/e} \quad (9.20)$$

$m > 1$, $e > 0$, therefore $m^{1/e} < 1$, $\sigma^2(L)_m < \sigma^2(L)$. It indicates that the distribution of equation (9.18) has less dispersion than the original one. The relationship between $\sigma^2(L)_m / \sigma^2(L)$ and e, m is shown in Fig. 9.3. In general, e is between 0.8 and 1.5, and m is taken as 2 to 4. Therefore the dispersion is reduced dramatically.

These offer a possible way of obtaining results equivalent to common methods by using less failure data.

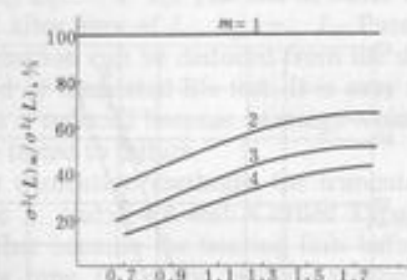


Fig. 9.3 Values of $\sigma^2(L)_m / \sigma^2(L)$

Testing methods

Consider that a bearing test is conducted on l sub-groups of size m each for a total sample size of $n = ml$. The test will yield life data of l failures, each being the shortest one in its sub-group. These failure lives are treated according to a sample of size l and the corresponding parameters $\hat{e}$ and $\hat{\beta}^*$ are estimated. Then the parameter estimates of the original distribution will be $\hat{e}$ and $\hat{\beta}$. The $\hat{\beta}$ should be

$$\hat{\beta} = \hat{\beta}^* m^{1/e} \quad (9.21)$$

Obviously, various estimate methods described in the previous paragraph are all available for the sudden death tests. However, the graphic method may be adopted, as shown in Fig. 9.4. l failure lives are marked on a probability paper of Weibull distribution as a conventional method. A straight line is drawn close to all the life data. A vertical line through a point corresponding to probability of failure 50% on the experienced line intersects the horizontal line of probability 15.9%, for condition $m=4$ at point C. Then the straight line parallel to the experienced line is the original distribution.

Experimental comparison

Consider a group of deep groove ball bearings 6302. 37 bearings were tested under a radial load of 1577.8 N and at rotational speed 4000 rpm. Test data and their life distributions are shown in Fig. 9.5. It can

be seen that there are only small differences, no more than 15%, between the two methods.

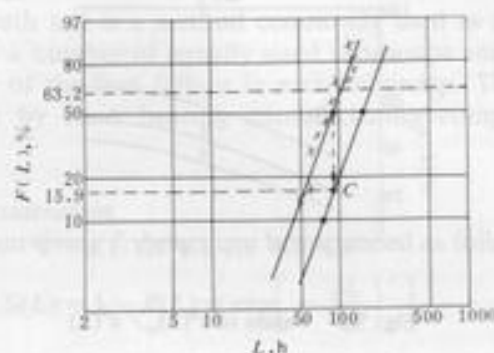


Fig. 9.4 A graphic method for sudden death test

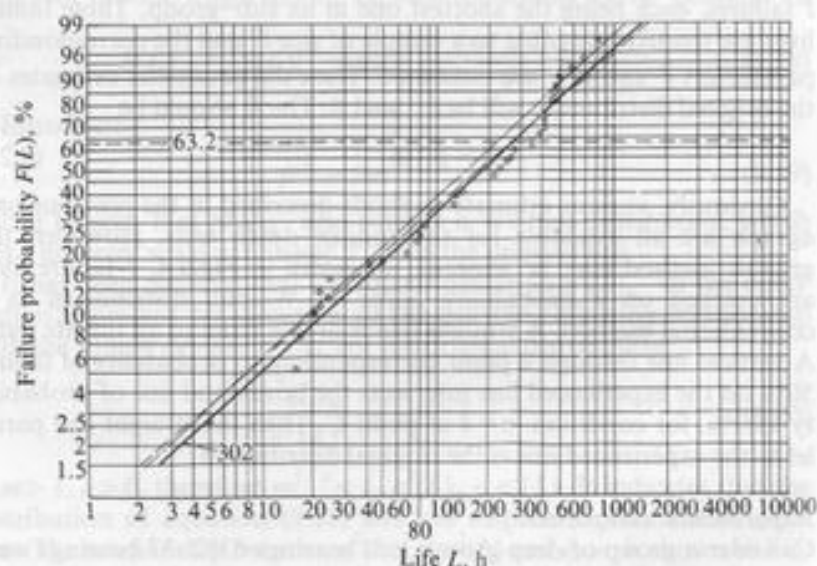


Fig. 9.5 Experimental comparison of sudden death tests

Truncated life test

Consider a life test of n bearings which leads to r failure data of the bearing at lives $L_1, L_2, \dots, L_r$. The test of other $n-r$ unfailed bearings is suspended after lives of $L_1, L_2, \dots, L_r$. Parameter estimates of original life distribution can be deduced from life data obtained. This is called a method of truncated life test. It is easy to understand that the total test time is reduced because bearings which have longer lives do not have to be tested to failure.

Upon different truncation methods, the truncated life test may be divided into Type I and II. A test is called Type I of truncation if the test ceases either because the bearing fails before a prescribed accumulated testing time L_0 or the bearing reaches the accumulated time L_0 without failure. The number of bearings failed is r . On the other hand, a test is called Type II of truncation if the test ceases after a prescribed number r of failures occurs.

Several methods of statistical treatments can be applied to truncated life tests. Here, only the optimum linear constant estimate and maximum likelihood estimate are described in brief.

Optimum linear constant estimate

Consider that n bearings are tested with a truncated number r . Life data of bearings failed are listed as follows according to their values

$$L_1 < L_2 < \dots < L_r$$

Parameters e and β may be estimated by using the following equations

$$\hat{e} = \frac{1}{2.3026 \sum_{i=1}^r a_{irn} \lg L_i} \quad (9.22)$$

$$\lg \hat{\beta} = \sum_{i=1}^r b_{irn} \lg L_i \quad (9.23)$$

where

a_{irn}, b_{irn} = coefficients of the optimum linear constant estimates with sample size n and truncated number r , which may be found out from (75).

Example. n tapered roller bearings 30206 were tested with a trun-

cated number 12 under a radial load 5880 N and an axial load 3332 N at rotational speed 2500 rpm.

Their test results are shown in Fig. 9.6. It can be seen that there are very small differences between the conventional life distribution and the truncated one. A calculation indicates that the relative deviation of the parameter $\hat{e}$ is 6.1% and that of $\hat{\beta}$ is 7%. A very interesting figure is that the relative deviation of the ration life is only 1.7%.

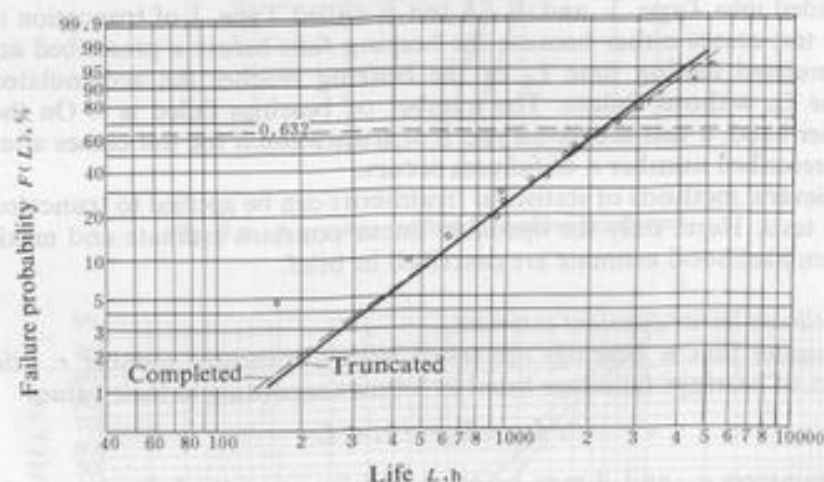


Fig. 9.6 A comparison of conventional and truncated tests

Maximum likelihood estimate

Consider that n bearings are tested with a truncated number r . The same procedure used in the deduction of equations (9.14) and (9.15) is adopted. The estimate equations of parameters $\hat{e}$ and $\hat{\beta}$ can be derived as follows

$$\frac{1}{\hat{e}} + \frac{1}{r} \sum_{i=1}^r \ln L_i - \frac{\sum_{i=1}^r L_i^{\hat{\beta}} \ln L_i}{\sum_{i=1}^r L_i^{\hat{\beta}}} = 0 \quad (9.24)$$

$$\hat{\beta} = \left(\frac{\sum_{i=1}^r L_i^{\hat{\beta}}}{n} \right)^{1/\hat{\beta}} \quad (9.25)$$

It should be pointed out that the failure lives are unnecessarily less than the lives at suspension. In other words, the test can be conducted more flexibly to permit selection of different accumulated time $L_{p+1}, \dots, L_r$.

For further inquiry, an unbiased estimate is introduced. An unbiased estimate is a value for which the average of a number of independent repeated estimates will probably converge to it. Considering repeated samples from a Weibull distribution with size n and truncated number r , it may be approved that the following two quantities follow distributions which only depend upon n and r but not upon the parameters of the Weibull distribution.

$$u(r, n) = e \ln \frac{\hat{L}_{10}}{L_{10}} \quad (9.26)$$

$$v(r, n) = \frac{\hat{e}}{e} \quad (9.27)$$

Introducing a p -th percentage point of u and v , that is

$$\text{Prob}\{u(r, n) < u_p\} = P$$

It can be deduced that a mean unbiased estimate of the shape parameter e is

$$\hat{e}' = \frac{\hat{e}}{\bar{v}} \quad (9.28)$$

And a median unbiased estimate of life L is calculated as

$$\hat{L}'_{10} = \left[\exp\left(\frac{-u_{0.50}}{\hat{e}'}\right) \right] \hat{L}_{10} \quad (9.29)$$

where, values of $\bar{v}$ and $u_{0.50}$ are given in Table 9.4.

In conclusion, main steps of the maximum likelihood estimate in truncated tests are as follows

- (1) Estimates of parameters $\hat{e}$ and $\hat{\beta}$ are solved from equations (9.24) and (9.25).

- (2) An estimate of rating life $\hat{L}_{10}$ is calculated from equation (9.16).
 (3) Unbiased estimates $\hat{e}'$ and $\hat{L}_{10}'$ are given by equations (9.28) and (9.29).

A program listed in the appendix of this chapter may be used to solve equation (9.24).

Application of maximum likelihood estimate

Confidence limits

On the basis of maximum likelihood estimates, confidence limits of the rating life L_{10} and the shape parameter e may be determined. It can be seen from equation (9.26) that the following inequalities are true with 90% probability

$$u_{0.05}(r, n) < u(r, n) = \hat{e} \ln \frac{\hat{L}_{10}}{L_{10}} < u_{0.95}(r, n)$$

Solving the inequalities for L_{10} yields

$$\left[\exp \left(-\frac{u_{0.95}}{\hat{e}} \right) \right] \hat{L}_{10} < L_{10} < \left[\exp \left(-\frac{u_{0.05}}{\hat{e}} \right) \right] \hat{L}_{10}$$

In general, confidence limits with a probability $100(1-\alpha)\%$ for L_{10} is

$$\left[\exp \left(-\frac{u_{1-\alpha/2}}{\hat{e}} \right) \right] \hat{L}_{10} < L_{10} < \left[\exp \left(-\frac{u_{\alpha/2}}{\hat{e}} \right) \right] \hat{L}_{10} \quad (9.30)$$

It means that the true value of L_{10} will be covered with the confidence interval in probability $100(1-\alpha)\%$.

Similarly, confidence limits with a probability $100(1-\alpha)\%$ for the parameter e is

$$\frac{\hat{e}}{v_{1-\alpha/2}} < e < \frac{\hat{e}}{v_{\alpha/2}} \quad (9.31)$$

Values of $u_{1-\alpha/2}$, $u_{\alpha/2}$, $v_{1-\alpha/2}$, $v_{\alpha/2}$ are given in Table 9.4 (82).

Hypothesis tests

It is generally necessary to decide whether a hypothesis is accepted or not, e.g. whether bearing lives will increase or not after adopting an improvement on the design or manufacturing technology. In statistics,

it is called the hypothesis test.

Table 9.4 Percentage points of $u(r, n)$ and $v(r, n)$

n	r	$u_{0.05}$	$u_{0.10}$	$u_{0.50}$	$u_{0.90}$	$u_{0.95}$	$v_{0.05}$	$v_{0.10}$	$v_{0.50}$	$\bar{v}$	$v_{0.90}$	$v_{0.95}$
10	2	-1.11	-1.61	0.180	5.91	9.97	0.654	0.856	2.72	7.45	14.7	31.7
	3	-1.16	-1.03	0.300	3.87	6.27	0.619	0.729	1.65	2.72	4.98	7.41
	1	-0.969	-0.769	0.340	3.31	1.88	0.646	0.743	1.43	1.85	3.27	4.42
	5	-0.918	-0.716	0.349	2.64	3.78	0.650	0.750	1.30	1.56	2.59	3.36
	7	-0.932	-0.696	0.304	2.00	2.71	0.686	0.779	1.20	1.32	1.98	2.30
	10	-0.869	-0.682	0.208	1.56	1.97	0.739	0.800	1.11	1.16	1.60	1.78
20	3	-2.85	-1.60	0.0198	1.85	3.15	0.639	0.768	1.70	2.82	4.98	7.360
	4	-1.18	-0.858	0.104	1.92	2.91	0.622	0.740	1.47	1.95	3.42	4.84
	5	-0.883	-0.691	0.125	1.67	2.66	0.637	0.741	1.34	1.60	2.70	3.40
	6	-0.804	-0.601	0.156	1.56	2.24	0.651	0.751	1.26	1.46	2.41	2.89
	7	-0.778	-0.585	0.148	1.46	2.03	0.663	0.747	1.22	1.36	2.12	2.50
	10	-0.702	-0.539	0.178	1.29	1.74	0.716	0.781	1.15	1.22	1.73	2.03
	15	-0.708	-0.539	0.116	1.01	1.39	0.740	0.804	1.07	1.11	1.45	1.61
	20	-0.679	-0.518	0.106	0.920	1.25	0.788	0.841	1.05	1.07	1.34	1.43
30	3	-4.31	-2.59	-0.187	1.27	2.07	0.623	0.763	1.74	2.95	5.51	8.30
	4	-1.89	-1.27	0.0630	1.14	1.86	0.654	0.762	1.51	1.99	3.64	4.78
	5	-1.10	-0.797	0.0216	1.27	1.88	0.650	0.749	1.35	1.65	2.85	3.58
	6	-0.865	-0.645	0.0698	1.12	1.57	0.663	0.762	1.28	1.48	2.37	2.88
	7	-0.783	-0.574	0.0761	1.26	1.76	0.681	0.753	1.22	1.38	2.16	2.62
	8	-0.713	-0.532	0.0850	1.08	1.46	0.686	0.768	1.18	1.30	1.97	2.27
	9	-0.641	-0.512	0.105	1.08	1.51	0.689	0.766	1.16	1.26	1.85	2.15
	10	-0.625	-0.493	0.0787	1.08	1.40	0.705	0.784	1.14	1.23	1.79	2.03
	15	-0.595	-0.460	0.109	0.965	1.27	0.750	0.809	1.09	1.14	1.55	1.69
	20	-0.570	-0.431	0.0680	0.731	0.924	0.826	0.868	1.04	1.05	1.25	1.33

Question 1. Assume that a type of bearing has original rating life $L_{10(0)}$. A sample of bearings will be taken for life tests after some improvements of bearings are made. Using the maximum likelihood method, the shape and scale parameters and rating life are estimated as $\hat{e}$, $\hat{\beta}$ and $\hat{L}_{10}$ respectively. Compare the life L_{10} of improved bear-

ings with the original bearing life.

Some basic hypotheses are introduced as follows

Basic hypothesis

$$H_0: L_{10} = L_{10(0)}$$

indicates that the life L_{10} of improved bearings is the same as the original.

The second hypothesis expresses a deviation from the original life $L_{10(0)}$. There are three possibilities, that is

$$H_1: L_{10} < L_{10(0)}$$

In this hypothesis, one is concerned about whether the bearing life L_{10} is shorter than the original.

$$H_1: L_{10} > L_{10(0)}$$

Similarly, this means that one is concerned about whether the bearing life L_{10} is longer than the original.

$$H_1: L_{10} \neq L_{10(0)}$$

And this means that one has no predetermined idea.

To test, a quantity

$$\hat{e} \ln \frac{\hat{L}_{10}}{L_{10(0)}}$$

is calculated and compared with $u(r, n)$ given in Table 9.4. In which $\hat{e}$ and $\hat{L}_{10}$ are unbiased estimates. The acceptance criteria are summarized in Table 9.5.

Table 9.5 Acceptance criteria

Second hypothesis H_1	Acceptance criteria for basic hypothesis H_0
$L_{10} < L_{10(0)}$	$u_a(r, n) < \hat{e} \ln \frac{\hat{L}_{10}}{L_{10(0)}}$
$L_{10} > L_{10(0)}$	$\hat{e} \ln \frac{\hat{L}_{10}}{L_{10(0)}} < u_{1-a}(r, n)$
$L_{10} \neq L_{10(0)}$	$u_{a/2}(r, n) < \hat{e} \ln \frac{\hat{L}_{10}}{L_{10(0)}} < u_{1-a/2}(r, n)$

Example (82). An equipment manufacturer offers a new finishing process to improve the life of ball bearings. To test this improvement, 10 bearings finished by the new process are tested until all failure under conditions for which the original life $L_{10(0)}$ is 98 hours. Upon the test results, the maximum likelihood estimate gives $\hat{L}_{10} = 126$ h, $\hat{e} = 1.57$.

Assuming

$$H_0: L_{10} = L_{10(0)} = 98$$

$$H_1: L_{10} > L_{10(0)}$$

and $a = 0.1$, therefore, from Table 9.4

$$u_{0.90}(10, 10) = 1.56$$

Test the hypothesis

$$\hat{e} \ln \frac{\hat{L}_{10}}{L_{10(0)}} = 1.57 \times \ln \frac{126}{98} = 0.395 < 1.56$$

This indicates that the basic hypothesis is accepted. There is no essential improvement on the bearing life by the new finishing process.

Similarly, hypothesis tests can be conducted for the shape parameter e . Its acceptance criteria are listed in Table 9.6. In which, e_0 is the original shape parameter.

Table 9.6 Acceptance criteria

Second hypothesis	Acceptance criteria for second hypothesis
$e < e_0$	$v_a(r, n) < \frac{\hat{e}}{e_0}$
$e > e_0$	$\frac{\hat{e}}{e_0} < v_{1-a}(r, n)$
$e \neq e_0$	$v_{a/2}(r, n) < \frac{\hat{e}}{e_0} < v_{1-a/2}(r, n)$

Question 2. It is necessary to test whether there is a significant difference between two populations of bearings.

Two bearing samples from different populations are tested under the same conditions. Their estimates are $\hat{L}_{10(1)}$, $\hat{L}_{10(2)}$, $\hat{e}_1$, and $\hat{e}_2$ by

using subscripts to indicate population number.

Acceptance criteria for tests of Question 2 are listed in Table 9.7. In which, values of $w_a(n_1, r_1, n_2, r_2)$ and $t_a(n_1, r_1, n_2, r_2)$ are given in Table 9.8. n_1, n_2, r_1, r_2 express the size of testing bearings and truncated numbers respectively.

Calculation of sudden death tests

It can be seen that estimates $\hat{e}^*$, $\hat{\beta}^*$ and $\hat{L}_{10}^*$ are obtained if l failure data of testing bearings are treated according to conventional estimate methods. Then parameter estimates of n bearings are

$$\hat{e} = \hat{e}^*$$

$$\hat{\beta} = \hat{\beta}^* m^{1/l}$$

$$\hat{L}_{10} = \hat{L}_{10}^* m^{1/l}$$

Table 9.7 Acceptance criteria

Basic hypothesis H_0	$L_{10(1)} = L_{10(2)}$
Second hypothesis H_1	Acceptance criteria for H_0
$L_{10(1)} < L_{10(2)}$	$t_a(n_1, r_1, n_2, r_2) < (\hat{e}_1 + \hat{e}_2) \ln \frac{\hat{L}_{10(1)}}{\hat{L}_{10(2)}}$
$L_{10(1)} > L_{10(2)}$	$(\hat{e}_1 + \hat{e}_2) \ln \frac{\hat{L}_{10(1)}}{\hat{L}_{10(2)}} < t_{1-a}(n_1, r_1, n_2, r_2)$
$L_{10(1)} \neq L_{10(2)}$	$t_{a/2}(n_1, r_1, n_2, r_2) < (\hat{e}_1 + \hat{e}_2) \ln \frac{\hat{L}_{10(1)}}{\hat{L}_{10(2)}} < t_{1-a/2}(n_1, r_1, n_2, r_2)$
Basic hypothesis H_0	$e_1 = e_2$
Second hypothesis H_1	Acceptance criteria for H_0
$e_1 < e_2$	$w_a(n_1, r_1, n_2, r_2) < \frac{\hat{e}_1}{\hat{e}_2}$
$e_1 > e_2$	$\frac{\hat{e}_1}{\hat{e}_2} < w_{1-a}(n_1, r_1, n_2, r_2)$
$e_1 \neq e_2$	$w_{a/2}(n_1, r_1, n_2, r_2) < \frac{\hat{e}_1}{\hat{e}_2} < w_{1-a/2}(n_1, r_1, n_2, r_2)$

Table 9.8 Values of $w_a(n, r, n, r)$, $t_a(n, r, n, r)$

n	r	$w_{0.05}$	$w_{0.10}$	$\bar{w}$	$w_{0.90}$	$w_{0.95}$	$t_{0.90}$	$t_{0.95}$
10	2	—	—	—	—	—	—	—
	3	0.149	0.245	2.04	4.08	6.71	7.42	11.4
	4	0.234	0.351	1.42	2.85	4.27	5.06	7.40
	5	0.312	0.416	1.27	2.40	3.20	4.06	5.86
	7	0.413	0.504	1.17	1.99	2.42	3.56	4.77
	10	0.527	0.616	1.08	1.62	1.90	2.98	3.99
20	3	0.171	0.267	1.89	3.75	5.83	5.63	8.82
	4	0.240	0.336	1.45	2.98	4.16	3.80	5.48
	5	0.317	0.402	1.32	2.49	3.15	3.24	4.40
	6	0.337	0.437	1.30	2.29	2.97	3.00	4.21
	7	0.386	0.478	1.22	2.09	2.59	2.82	3.99
	10	0.482	0.562	1.09	1.78	2.08	2.45	3.40
	15	0.590	0.662	1.03	1.51	1.70	2.26	2.87
	20	0.649	0.718	1.04	1.39	1.54	2.04	2.67
30	3	0.147	0.241	2.16	4.16	6.81	6.08	8.90
	4	0.237	0.350	1.50	2.86	4.21	3.54	5.09
	5	0.294	0.395	1.34	2.53	3.41	2.76	4.13
	6	0.363	0.453	1.23	2.21	2.75	2.44	3.29
	7	0.378	0.476	1.20	2.10	2.65	2.47	3.36
	8	0.426	0.521	1.15	1.92	2.35	2.30	3.02
	9	0.450	0.536	1.11	1.87	2.22	2.25	2.93
	10	0.466	0.566	1.09	1.76	2.14	2.09	2.68
	15	0.554	0.631	1.06	1.58	1.81	1.92	2.48
	20	0.606	0.689	1.03	1.42	1.63	1.63	2.10
	30	0.706	0.769	1.03	1.30	1.42	1.63	2.10

The median unbiased estimate of rating life L_{10} is

$$\hat{L}'_{10} = \hat{L}_{10} \exp \left[- \frac{q_{0.50}(l, m)}{\hat{e}} \right] \quad (9.32)$$

The median unbiased estimate of the shape parameter e

$$\hat{e}' = \frac{\hat{e}}{v_{0.50}(l, l)} \quad (9.33)$$

Confidence limits of rating life L_{10} given by a sudden death test with $(1-\alpha)\%$ probability are as follows

$$\hat{L}_{10} \exp\left[-\frac{q_{1-\alpha/2}(l, m)}{\hat{e}}\right] < L_{10} < \hat{L}_{10} \exp\left[-\frac{q_{\alpha/2}(l, m)}{\hat{e}}\right] \quad (9.34)$$

Similarly, confidence limits of the shape parameter e

$$\frac{\hat{e}}{v_{1-\alpha/2}(l, l)} < e < \frac{\hat{e}}{v_{\alpha/2}(l, l)} \quad (9.35)$$

where

Values of $q(l, m)$ and $v(l, l)$ are given in Tables 9.9 and 9.10 respectively.

In [80], a comparison between sudden death and truncated life tests has been made. The median ratio of upper to lower confidence limits is used as a criterion. It is easy to know that the less the ratio, i.e., a narrow confidence interval, the higher the accuracy of parameter estimates. Computed results indicate that the estimate accuracy of sudden death tests is better than those of truncated life tests if

$$g(l, m, a) < h(r, n, a)$$

Values of $g(l, m, a)$ and $h(r, n, a)$ are given in Table 9.11. In which, $(1-\alpha)\%$ is normally called a confidence probability.

Table 9.9 Values of $q(l, m)$

l	m	$q_{0.05}$	$q_{0.10}$	$q_{0.15}$	$q_{0.50}$	$q_{0.85}$	$q_{0.90}$
2	5	-16.8	-3.49	-2.02	+0.128	3.04	4.42
2	10	-29.7	-7.18	-4.01	-0.334	1.20	1.88
3	5	-1.79	-1.19	-0.853	+0.163	1.82	2.44
3	10	-2.91	-1.89	-1.34	-0.133	0.866	1.19
4	5	-1.19	-0.878	-0.681	0.099	1.21	1.63
5	2	-1.02	-0.788	-0.632	0.302	1.80	2.27
5	3	-0.980	-0.760	-0.597	0.208	1.43	1.82
5	4	-0.954	-0.739	-0.593	0.140	1.21	1.53
5	6	-0.997	-0.748	-0.679	0.045	0.889	1.14

Table 9.10 Values of $v(l, l)$

l	$v_{0.05}$	$v_{0.10}$	$v_{0.15}$	$v_{0.50}$	$v_{0.85}$	$v_{0.90}$	$v_{0.95}$
2	0.654	0.791	0.934	2.15	7.57	11.2	22.8
3	0.650	0.767	0.864	1.53	3.34	4.11	5.71
4	0.653	0.751	0.837	1.32	2.38	2.80	3.73
5	0.668	0.765	0.832	1.23	2.01	2.28	2.83
6	0.714	0.782	0.843	1.20	1.82	2.06	2.41

Table 9.11 Values of $g(l, m, a)$ and $h(r, n, a)$

No. of bearings n	No. of failures r	No. of groups l	subgroup size m	90% confidence probability		70% confidence probability	
				$h(r, n, 0.10)$	$g(l, m, 0.10)$	$h(r, n, 0.30)$	$g(l, m, 0.30)$
10	2	2	5	10.72	11.06	2.53	2.35
20	2	2	10	12.39	15.26	2.66	2.42
15	3	3	5	3.98	3.63	1.68	1.75
30	3	3	10	3.64	3.09	1.56	1.44
20	4	4	5	2.77	2.78	1.40	1.43
10	5	5	2	3.88	3.48	2.02	1.98
15	5	5	3	2.97	2.94	1.59	1.65
20	5	5	4	2.62	2.58	1.37	1.46
30	5	5	6	2.19	2.13	1.15	1.28
30	6	6	5	1.95	2.00	1.07	1.14

It can be seen that values of $g(l, m, a)$ and $h(r, n, a)$ are connected with the size of the bearing sample, number of failures and groups, as well as confidence probabilities. Both methods above have recently been adopted by different companies.

Appendix Computer program

As described above, the maximum likelihood estimates of parameters e and β are the solutions of equations (9.24) and (9.25). A program is written in BASIC language.

Table 9 A.1 Symbols

Symbols	Variables	Explanations
A	a	
B	$f(e)$	Left value of equation 4
D	e	
F	L_i	Variable input
G	i	$i = 1 - n$
H	h	Taking 0.0001
I	$26 + i$	
J	$26 + i$	
M	L_T	
N	n	
P	$\sum_{i=1}^r \ln L_i$	
Q	e	
R	r	
S	$\sum_{i=1}^r \ln L_i^*$	
T	$\sum_{i=1}^r L_i^* \ln L_i$	
U	$\sum_{i=1}^r L_i^*$	
X_0		Initial value of e
Y	$f(e)$	Left value of equation 4
A(I)	L_i	$i = 26 + i$
NO		Total number of bearings
FNO		Number of bearings failed
LT	L_T	
SLOPE	e	

Table 9 A.2 Program for the estimation of parameters e and β

```

10 INPUT "NO,"N,"FNO"; R,"LT"; M
20 I=27:FOR G=1 TO N
30 PAUSE USING "#####,####"; G:INPUT F
40 A(I)=F:I=I+1
50 NEXT G
60 INPUT "X0="; Q,"H="; H
70 GOSUB 400
80 GOSUB 500
90 Y=B:Q=H+Q
100 GOSUB 400
110 GOSUB 500
120 D=Q:Q=D-H*Y/(B-Y)
130 IF ABS(D-Q)>=E-4 GOTO 70
140 BEEP 2:PAUSE "SLOPE", Q
150 U=0
160 I=27:FOR G=1 TO N
170 U=U+A(I)/Q:I=I+1
180 NEXT G
190 A=(U/R)/(1/Q)
200 PRINT "A="; A;"Q="; Q
210 END
400 P=0:S=0:T=0
420 I=27:FOR J=1 TO R
430 P=P+LN A(I)
440 S=S+A(I)/Q
450 T=T+A(I)/Q*LN A(I)
460 I=I+1
470 NEXT J
480 RETURN
500 B=(T+(N-R)*M/Q*LN M)/(S+(N-R)*M/Q-P/R-1/Q
510 RETURN

```

1. Symbols and input

Symbols and their explanations are summarized in Table 9A.1. There are three groups of input data.

(1) Test conditions, including sample size, number of bearings failed and trun-

late permanent deformations.

For point contact

$$\delta_p = 1.30 \times 10^{-7} \frac{Q^2}{D_b} (\rho_{11} + \rho_{21})(\rho_{12} + \rho_{22}) \quad (10.1)$$

For line contact

$$\delta_p = \frac{2.12 \times 10^{-11}}{\sqrt{D_b}} \left(\frac{Q}{l_e} \sqrt{\rho_1 + \rho_2} \right)^3 \quad (10.2)$$

where

δ_p = total permanent deformation of a rolling element-raceway contact, mm

Q = contact load, N

D_b = rolling element diameter, mm

ρ_{11} and ρ_{12} = curvatures of body I in planes 1 and 2, 1/mm

ρ_{21} and ρ_{22} = curvatures of body II in planes 1 and 2, 1/mm

ρ_1 and ρ_2 = curvatures of bodies I and II at the contact centre in a plane vertical to the contact line, 1/mm

l_e = effective length of the roller.

In the case of a line contact, the permanent deformation at the end of a roller is larger than that in the central part of the roller. Equation (10.2) represents the permanent deformation at the end of a roller. Palmgren indicated that in the total permanent deformation, approximately two thirds occur in the raceway and one third in the rolling element. But Kurota's test shows that the concrete proportions are connected with the shapes of contact bodies and the metallurgical structure, and also it is possible to have the same permanent deformation between the rolling element and the raceway.

Q in equation (10.1) is replaced by the maximum rolling element load, expressed as Q_0 . Using equations (2.8) to (2.15) and assuming the total permanent deformation of 0.0001 of the rolling element diameter under load Q_0 , one may deduce that for point contact

$$Q_0 = 13.87 D_b^2 \left[\frac{2f(1+\gamma)}{2f-1} \right]^{1/2} \quad (10.3)$$

Similarly, for line contact

$$Q_0 = 118.7 l_e D_b (1+\gamma)^{1/2} \quad (10.4)$$

where

f = ratio of the raceway groove curvature radius to the ball diameter

$\gamma = D_b \cos \alpha / D_m$

α = nominal contact angle of a bearing.

In equations (10.3) and (10.4), the upper sign refers to the rolling element-inner raceway contact and the lower sign refers to the rolling element-outer raceway contact.

The permanent deformation is connected with body hardness, surfacial residual stress, residual austenite content, metallurgical structure, etc. Even though in the same testing condition, measured permanent deformations are probably different. Experiments prove that the Hertzian theory of the contact pressure is valid for the condition having a small amount of the permanent deformation. The contact permanent deformation has a connection with the maximum static shear stress below the surface. As mentioned in Chapter Three, this shear stress is proportional to the maximum contact pressure. Therefore, it is reasonable to determine the static load rating of a bearing based on the maximum contact pressure.

STATIC LOAD RATING OF ROLLING BEARINGS

As mentioned above, the static load rating means a load which causes a total permanent deformation of 0.0001 of the rolling element diameter to form at the centre of the most heavily stressed rolling element-raceway contact. In this paragraph, the former international standard of the static load rating is described. And then, the new international standard about that is explained.

Former calculating method of static load rating

When half the circumference of the rolling element complement is stressed, then from equation (4.68)

$$F_r = Q_0 J_r (0.5) Z \cos \alpha \quad (10.5)$$

Substituting equation (10.3) into this equation and considering that this radial load is equal to the static load rating C_0 , one may derive that the static load ratings of the deep groove and the angular contact ball bearings are as follows

$$C_0 = 13.87 J_r (0.5) \left[\frac{2f(1+\gamma)}{2f-1} \right]^{1/2} Z D_b^2 \cos \alpha \quad (10.6)$$

Using a symbol f_0

$$f_0 = 13.87 J_r (0.5) \left[\frac{2f(1-\gamma)}{2f-1} \right]^{1/2}$$

And for i -row bearings

$$C_0 = f_0 i Z D_b^2 \cos \alpha \quad (10.7)$$

For thrust ball bearings, the static load rating is the centric axial load. From equations (4.27) and (10.3), it may be expressed as

$$C_0 = f_0 Z D_b^2 \sin \alpha \quad (10.8)$$

where

$$f_0 = 13.87 \left[\frac{2f(1-\gamma)}{2f-1} \right]^{1/2}$$

Similarly, for radial and angular contact roller bearings, the static load rating is given from equations (4.68) and (10.4)

$$C_0 = f_0 i Z D_b l_e \cos \alpha \quad (10.9)$$

where

$$f_0 = 118.7 J_r (0.5) (1-\gamma)^{1/2}$$

For thrust roller bearings, the static load rating is written from equation (4.27) and (10.4)

$$C_0 = f_0 Z D_b l_e \sin \alpha \quad (10.10)$$

where

$$f_0 = 118.7 (1-\gamma)^{1/2}$$

It can be seen that the values of f_0 should be connected with bearing types and their internal geometric parameters. But in the former International Standard ISO76, f_0 is specified to depend only on bearing types. For a certain type of standard bearings, it is approximately assumed that f_0 is not dependent on their internal geometric parameters. The values of f_0 are listed in Table 10.1. Obviously, this assumption may bring about some deviations, as shown in Fig. 10.1. The figure indicates the relation between f_0 and γ in radial ball and roller bearings.

The dotted line expresses values listed in Table 10.1 and the solid line is the calculated values. The conventional range of values γ for rolling bearings is marked in the figure.

In equations (10.6) to (10.10), the unit of the static load rating is Newton.

Table 10.1 Values of f_0

Bearing types	f_0	Bearing types	f_0
Deep groove ball bearings	12.26	Thrust ball bearings	49.00
Angular contact ball bearings	12.26	Cylindrical roller bearings	21.58
Angular contact ball bearings (magneto bearings)	2.94	Spherical roller bearings	21.58
Self-aligning ball bearings	3.33	Tapered roller bearings	21.58
		Thrust ball bearings	98.07

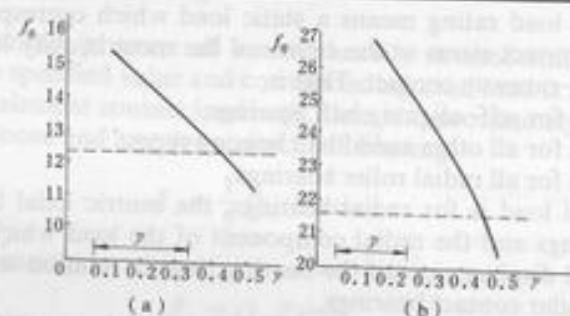


Fig. 10.1 Relation between f_0 and γ . (a) deep groove ball bearings, (b) cylindrical roller bearings

The static load rating is not the kind of load which causes bearing components to be fractured. Experiments show that the loads which may fracture bearing components generally is greater than $8 C_0$. For rotating bearings without strict requirements regarding quiet running, the permissible static load may exceed the corresponding static load rating of the bearing. On the other hand, for precision bearings or the bearings subjected to pronounced shock loads, the permissible static load should be chosen according to the corresponding static load rating divided by a factor of safety which is more than one.

New calculating method of static load rating

The new calculating method of the static load rating was first proposed by AFBMA and was accepted in 1986.

Definition

The new static load rating is maintained at approximately the same level as that at which a total permanent deformation of 0.0001 of the rolling element diameter at the centre of the most heavily stressed rolling element-raceway contact exists. Experiments show that the maximum contact pressure corresponding to the permanent deformation is connected with these types of rolling bearings. Generally, the maximum contact pressure at the centre of the most heavily stressed rolling element-raceway contact is in the range from 4000 to 4600 MPa.

Upon these, a definition of the static load rating is specified as follows:

The static load rating means a static load which corresponds to a calculated contact stress at the centre of the most heavily loaded rolling element-raceway contact. That is,

- 4600 MPa for self-aligning ball bearings;
- 4200 MPa for all other radial ball bearing types;
- 4000 MPa for all radial roller bearings.

The radial load is for radial bearings, the centric axial load is for thrust bearings and the radial component of the load which causes a purely radial displacement of the bearing rings in relation to each other is for angular contact bearings.

In this way, the calculated value of the contact pressure depends on the external load, internal geometric parameters of the bearing, elastic constants of the bearing material, etc. and is independent of the metallurgical structure and surfacial residual stress and so on. Therefore, it can be calculated accurately. Of course, the actual total permanent deformation at the centre of the most heavily stressed rolling element-raceway contact is probably larger or less than 0.0001 of the rolling element diameter.

The unit used to the static load rating is the IU system, i.e. in Newton.

Static load rating of ball bearings

From equations (3.14), (3.18) and (3.19), one may derive the following

relationship between the contact load of a rolling element and the contact pressure

$$Q = p_0 \frac{32\pi}{3E_0} \frac{1}{k} \left[\frac{L(e)}{\sum \rho} \right]^2 \quad (10.11)$$

where

e is the solution of the equation

$$1 - 2 \left(\frac{k}{e} \right)^2 \left[\frac{K(e)}{L(e)} - 1 \right] - F(\rho) = 0 \quad (10.12)$$

$$E_0 = \frac{E}{1 - \nu^2} \quad (10.13)$$

$$k = \frac{b}{a} = (1 - e^2)^{1/2} \quad (10.14)$$

If the contact pressure of a most heavily stressed rolling element reaches the specified value and corresponding axial load is F , as well as the rolling element contact load is Q_0 , then it approximately is: for deep groove and angular contact ball bearings

$$F_r = \frac{1}{5} Q_0 i Z \cos \alpha \quad (10.15)$$

for thrust bearings

$$F_a = Q_0 Z \sin \alpha \quad (10.16)$$

Substituting equation (10.11) into this equation and introducing a symbol f_0 , one may obtain the basically same equation as that in the former method.

for deep groove and angular contact ball bearings

$$C_0 = f_0 i Z D_s^2 \cos \alpha \quad (10.17)$$

for thrust bearings

$$C_0 = f_0 Z D_s^2 \sin \alpha \quad (10.18)$$

Introducing the following constants

$$E = 2.07 \times 10^5 \text{ MPa}$$

$$\nu = 0.3$$

and for deep groove and angular contact ball bearings

$$\frac{R_i}{D_b} = 0.52; \quad \frac{R_e}{D_b} = 0.53$$

for thrust ball bearings

$$\frac{R}{D_b} = 0.54$$

It is deduced that the expressions of f_0 are as follows:
for deep groove and angular contact ball bearings

$$f_0 = 2.072 \left(\frac{p_0}{4000} \right)^3 \frac{1}{k} \left[\frac{L(e)}{2 + \frac{\gamma}{1-\gamma} - \frac{1}{1.04}} \right]^2 \quad (10.19)$$

for self-aligning ball bearings

$$f_0 = 2.072 \left(\frac{p_0}{4000} \right)^3 \left[\frac{\pi}{4} (1 + \gamma) \right]^2 \quad (10.20)$$

for thrust ball bearings

$$f_0 = 10.361 \left(\frac{p_0}{4000} \right)^3 \frac{1}{k} \left[\frac{L(e)}{2 + \frac{\gamma}{1-\gamma} - \frac{1}{1.08}} \right]^2 \quad (10.21)$$

Static load rating of roller bearings

Similarly, the fundamentally same equation as that of the former static load rating can be derived.

For deep groove and angular contact roller bearings

$$C_0 = f_0 i Z l_s D_b \cos \alpha \quad (10.22)$$

where

$$f_0 = 44(1 - \gamma) \quad (10.23)$$

For thrust roller bearings

$$C_0 = f_0 Z l_s D_b \sin \alpha \quad (10.24)$$

where

$$f_0 = 220(1 - \gamma) \quad (10.25)$$

Comparison with the former calculating method

In comparison with the former static load rating, Fig. 10.2 shows the relation between the relative ratio of the static load rating and the value γ . The vertical coordinate is the ratio of the new static load rating to the corresponding former one.

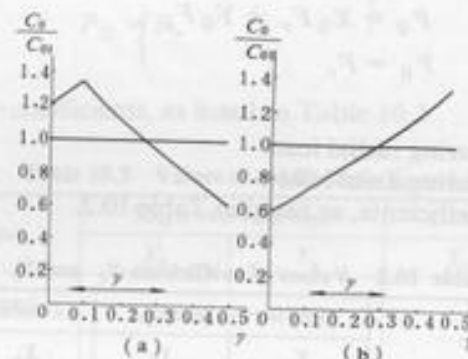


Fig. 10.2 Relation between C_0/C_{01} and γ . (a) radial ball bearings, (b) self-aligning ball bearings

It should be noted that for most types of bearings the new static load rating generally increases, particularly for cylindrical roller bearings. In reverse, the new static load rating decreases for self-aligning bearings. The agreement between theory and experiments is improved for the static load rating using the new method.

EQUIVALENT STATIC LOAD OF ROLLING BEARINGS

If the actual load acting on a bearing is different from the assumed conditions in determining the static load rating of the bearing, such as combined radial and axial loads, then it is necessary to convert the actual load into an equivalent static load. It is an assumed load which would cause the same contact stress at the centre of the most heavily loaded rolling element-raceway contact as that which occurs under

the actual load conditions. The radial load is for radial bearings, the centric axial load is for thrust bearings and the radial component of the load which causes a purely radial displacement of the bearing rings in relation to each other is for angular contact bearings.

Equivalent static load of deep groove and angular contact ball bearings

The equivalent static load of deep groove or angular contact ball bearings is the greater one between the two values given by the following equations

$$\left. \begin{aligned} P_0 &= X_0 F_r + Y_0 F_a \\ P_0 &= F_r \end{aligned} \right\} \quad (10.26)$$

where

F_r = bearing radial load

F_a = bearing axial load

X_0 and Y_0 = coefficients, as listed in Table 10.2.

Table 10.2 Values of coefficients X_0 and Y_0

Bearing type	Single row bearings		Double row bearings	
	X_0	Y_0	X_0	Y_0
Deep groove ball bearings*	0.6	0.5	0.6	0.5
Angular contact ball bearings				
$\alpha = 15^\circ$	0.5	0.46	1	0.92
$\alpha = 20^\circ$	0.5	0.42	1	0.84
$\alpha = 25^\circ$	0.5	0.38	1	0.76
$\alpha = 30^\circ$	0.5	0.33	1	0.66
$\alpha = 35^\circ$	0.5	0.29	1	0.58
$\alpha = 40^\circ$	0.5	0.26	1	0.52
$\alpha = 45^\circ$	0.5	0.22	1	0.44
Self-aligning ball bearings	0.5	0.22ctgz	1	0.44ctgz

* Permissible maximum value of F_a / C_0 depends on bearing design (internal clearance and raceway groove depth).

For two identical single row deep groove or angular contact ball bearings mounted side by side on the same shaft such that they oper-

ate as a unit in "back-to-back" or "face-to-face" arrangement, use the X_0 and Y_0 values for a double row bearing. On the other hand, for two identical bearings in "tandem" arrangement, use the X_0 and Y_0 values for a single row bearing.

Equivalent static load of radial and tapered roller bearings

The equivalent static load of spherical or tapered roller bearings is the greater one between the values given by the following equations:

$$\left. \begin{aligned} P_0 &= X_0 F_r + Y_0 F_a \\ P_0 &= F_r \end{aligned} \right\} \quad (10.27)$$

where

X_0 and Y_0 = coefficients, as listed in Table 10.3.

Table 10.3 Values of coefficients X_0 and Y_0

Bearing type	Single row bearings		Double row bearings	
	X_0	Y_0	X_0	Y_0
Spherical roller and tapered roller bearings	0.5	0.22ctgz	1	0.44ctgz

Equivalent static load of thrust bearings

The static load rating of thrust ball or roller bearings refers to the centric axial load. Because a thrust bearing with $\alpha < 90^\circ$ can also be subjected to a radial load, it is necessary to calculate the equivalent static load by the following equation

$$P_0 = 2.3 \text{tg} \alpha F_r + F_a \quad (10.28)$$

This equation is valid for all ratios of radial to axial load in the case of double direction bearings. For single direction bearings, it is valid where $F_r / F_a < 0.44 \text{ctg} \alpha$ and gives satisfactory but less conservative values of P_0 for F_r / F_a up to $0.67 \text{ctg} \alpha$.

When a thrust bearing with $\alpha = 90^\circ$ is subjected to an eccentric axial load, it is necessary to calculate the equivalent static load. In this case, the ratio of the equivalent static load to the actual eccentric load is equal to the ratio of the maximum rolling element load in the action of

where

L_h = bearing rating life, hours

n = rotational speed of bearing, rpm.

For simplification of the calculation, the speed factor and life factor are introduced

$$f_s = \left(\frac{33\frac{1}{3}}{n} \right)^{1/3} \quad (11.2)$$

$$f_h = \left(\frac{L_h}{500} \right)^{1/3} \quad (11.3)$$

Then, equation (11.1) converts to

$$C = \frac{f_h}{f_s} P \quad (11.4)$$

The relationships of speed factor f_s , life factor f_h and rating life L_h are presented in Table 11.1.

Table 11.1 Life factor and speed factor

Bearing type	Ball bearings	Roller bearings
Rating life L_h	$L_h = 500 f_s^3$	$L_h = 500 f_s^{10/3}$
Life factor f_h	$f_h = f_s \frac{C}{P}$	$f_h = f_s \frac{C}{P}$
Speed factor f_s	$f_s = \left(\frac{33\frac{1}{3}}{n} \right)^{1/3}$	$f_s = \left(\frac{33\frac{1}{3}}{n} \right)^{3/10}$

Upon the bearing rotational speed, the speed factor f_s can be obtained from Tables 11.2 and 11.3. Correspondingly, the life factor f_h is picked upon the basis of the bearing rating life from Tables 11.4 and 11.5. The rating life can be calculated according to $n = 10$ rpm if the bearing rotational speed is within 1 to 10 rpm. Besides, it is reasonable to select a bearing upon its static load rating if its rotational speed is less than 1 rpm.

Table 11.2 f_s values of ball bearings

n	f_s	n	f_s	n	f_s	n	f_s	n	f_s	n	f_s
10	1.494	56	0.841	210	0.541	780	0.350	3200	0.218	10000	0.149
11	1.447	58	0.831	220	0.533	800	0.347	3300	0.216	10500	0.147
12	1.405	60	0.822	230	0.525	820	0.344	3400	0.214	11000	0.145
13	1.369	62	0.813	240	0.518	840	0.341	3500	0.212	11500	0.143
14	1.335	64	0.805	250	0.511	860	0.339	3600	0.210	12000	0.141
15	1.305	66	0.797	260	0.504	880	0.336	3700	0.208	12500	0.139
16	1.277	68	0.788	270	0.498	900	0.333	3800	0.206	13000	0.137
17	1.252	70	0.781	280	0.492	920	0.331	3900	0.205	13500	0.135
18	1.228	72	0.774	290	0.487	940	0.329	4000	0.203	14000	0.134
19	1.206	74	0.767	300	0.481	960	0.326	4100	0.201	14500	0.132
20	1.186	76	0.760	310	0.476	980	0.324	4200	0.199	15000	0.131
21	1.166	78	0.753	320	0.471	1000	0.322	4300	0.198	16000	0.128
22	1.148	80	0.747	330	0.466	1050	0.317	4400	0.196	17000	0.125
23	1.132	82	0.741	340	0.461	1100	0.312	4500	0.195	18000	0.123
24	1.116	84	0.735	350	0.457	1150	0.307	4600	0.193	19000	0.121
25	1.100	86	0.729	360	0.453	1200	0.303	4700	0.192	20000	0.119
26	1.086	88	0.724	370	0.449	1250	0.299	4800	0.191	21000	0.117
27	1.073	90	0.718	380	0.444	1300	0.295	4900	0.190	22000	0.115
28	1.060	92	0.713	390	0.441	1350	0.291	5000	0.188	23000	0.113
29	1.048	94	0.708	400	0.437	1400	0.288	5200	0.186	24000	0.112
30	1.036	96	0.703	410	0.433	1450	0.284	5400	0.183	25000	0.110
31	1.025	98	0.698	420	0.430	1500	0.281	5600	0.181	26000	0.109
32	1.014	100	0.693	430	0.426	1550	0.278	5800	0.179	27000	0.107
33	1.003	105	0.682	440	0.423	1600	0.275	6000	0.177	28000	0.106
34	0.994	110	0.672	450	0.420	1650	0.272	6200	0.175	29000	0.105
35	0.984	115	0.662	460	0.417	1700	0.270	6400	0.173		
36	0.975	120	0.652	470	0.414	1750	0.267	6600	0.172		
37	0.966	125	0.644	480	0.411	1800	0.265	6800	0.170		
38	0.958	130	0.635	490	0.408	1850	0.262	7000	0.168		
39	0.949	135	0.627	500	0.406	1900	0.260	7200	0.167		
40	0.941	140	0.620	520	0.400	1950	0.258	7400	0.165		
41	0.933	145	0.613	540	0.395	2000	0.255	7600	0.164		
42	0.926	150	0.606	560	0.390	2100	0.251	7800	0.162		
43	0.919	155	0.599	580	0.386	2200	0.247	8000	0.161		
44	0.912	160	0.593	600	0.382	2300	0.244	8200	0.160		
45	0.905	165	0.586	620	0.378	2400	0.240	8400	0.158		
46	0.898	170	0.581	640	0.374	2500	0.237	8600	0.157		
47	0.892	175	0.575	660	0.370	2600	0.234	8800	0.156		
48	0.885	180	0.570	680	0.366	2700	0.231	9000	0.155		
49	0.880	185	0.565	700	0.363	2800	0.228	9200	0.154		
50	0.874	190	0.560	720	0.359	2900	0.226	9400	0.153		
52	0.863	195	0.555	740	0.356	3000	0.223	9600	0.152		
54	0.851	200	0.550	760	0.353	3100	0.221	9800	0.150		

Table 11.3 f_n values of roller bearings

n	f_n	n	f_n	n	f_n	n	f_n	n	f_n	n	f_n
10	1.435	54	0.865	195	0.589	720	0.398	2800	0.265	9000	0.187
11	1.395	56	0.856	200	0.584	740	0.395	2900	0.262	9200	0.185
12	1.359	58	0.847	210	0.576	760	0.391	3000	0.259	9400	0.184
13	1.326	60	0.838	220	0.568	780	0.388	3100	0.257	9600	0.183
14	1.297	62	0.830	230	0.560	800	0.385	3200	0.254	9800	0.182
15	1.271	64	0.822	240	0.553	820	0.383	3300	0.252	10000	0.181
16	1.246	66	0.815	250	0.546	840	0.380	3400	0.250	10500	0.178
17	1.224	68	0.807	260	0.540	860	0.377	3500	0.248	11000	0.176
18	1.203	70	0.800	270	0.534	880	0.375	3600	0.246	11500	0.173
19	1.184	72	0.794	280	0.528	900	0.372	3700	0.243	12000	0.171
20	1.166	74	0.787	290	0.523	920	0.370	3800	0.242	12500	0.169
21	1.149	76	0.781	300	0.517	940	0.367	3900	0.240	13000	0.167
22	1.133	78	0.775	310	0.512	960	0.365	4000	0.238	13500	0.165
23	1.118	80	0.769	320	0.507	980	0.363	4100	0.236	14000	0.163
24	1.104	82	0.763	330	0.503	1000	0.361	4200	0.234	14500	0.162
25	1.090	84	0.758	340	0.498	1050	0.355	4300	0.233	15000	0.160
26	1.077	86	0.753	350	0.494	1100	0.350	4400	0.231	16000	0.157
27	1.065	88	0.747	360	0.490	1150	0.346	4500	0.230	17000	0.154
28	1.054	90	0.742	370	0.486	1200	0.341	4600	0.228	18000	0.152
29	1.043	92	0.737	380	0.482	1250	0.337	4700	0.227	19000	0.149
30	1.032	94	0.733	390	0.478	1300	0.333	4800	0.225	20000	0.147
31	1.022	96	0.728	400	0.475	1350	0.329	4900	0.224	21000	0.145
32	1.012	98	0.724	410	0.471	1400	0.326	5000	0.222	22000	0.143
33	1.003	100	0.719	420	0.467	1450	0.322	5200	0.220	23000	0.141
34	0.994	105	0.709	430	0.464	1500	0.319	5400	0.217	24000	0.139
35	0.986	110	0.699	440	0.461	1550	0.316	5600	0.215	25000	0.137
36	0.977	115	0.690	450	0.458	1600	0.313	5800	0.213	26000	0.136
37	0.969	120	0.681	460	0.455	1650	0.310	6000	0.211	27000	0.134
38	0.962	125	0.673	470	0.452	1700	0.307	6200	0.209	28000	0.133
39	0.954	130	0.665	480	0.449	1750	0.305	6400	0.207	29000	0.131
40	0.947	135	0.657	490	0.447	1800	0.302	6600	0.205	30000	0.130
41	0.940	140	0.650	500	0.444	1850	0.300	6800	0.203		
42	0.933	145	0.643	520	0.439	1900	0.297	7000	0.201		
43	0.927	150	0.637	540	0.434	1950	0.295	7200	0.199		
44	0.920	155	0.631	560	0.429	2000	0.293	7400	0.198		
45	0.914	160	0.625	580	0.425	2100	0.289	7600	0.196		
46	0.908	165	0.619	600	0.420	2200	0.285	7800	0.195		
47	0.902	170	0.613	620	0.416	2300	0.281	8000	0.193		
48	0.896	175	0.608	640	0.412	2400	0.277	8200	0.192		
49	0.891	180	0.603	660	0.408	2500	0.274	8400	0.190		
50	0.886	185	0.598	680	0.405	2600	0.271	8600	0.189		
52	0.875	190	0.593	700	0.401	2700	0.268	8800	0.188		

Table 11.4 f_h values of ball bearings

L_h	f_h	L_h	f_h	L_h	f_h	L_h	f_h	L_h	f_h
100	0.585	400	0.928	1250	1.360	4500	2.08	15000	3.11
105	0.595	410	0.936	1300	1.375	4600	2.10	15500	3.14
110	0.604	420	0.944	1350	1.395	4700	2.11	16000	3.18
115	0.613	430	0.951	1400	1.410	4800	2.13	16500	3.21
120	0.622	440	0.959	1450	1.425	4900	2.14	17000	3.24
125	0.631	450	0.966	1500	1.445	5000	2.15	17500	3.27
130	0.639	460	0.973	1550	1.460	5200	2.18	18000	3.30
135	0.647	470	0.980	1600	1.475	5400	2.21	18500	3.33
140	0.654	480	0.987	1650	1.490	5600	2.24	19000	3.36
145	0.662	490	0.994	1700	1.505	5800	2.27	19500	3.39
150	0.670	500	1.000	1750	1.520	6000	2.29	20000	3.42
155	0.677	520	1.015	1800	1.535	6200	2.32	21000	3.48
160	0.684	540	1.025	1850	1.545	6400	2.34	22000	3.53
165	0.691	560	1.040	1900	1.560	6600	2.37	23000	3.58
170	0.698	580	1.050	1950	1.575	6800	2.39	24000	3.63
175	0.705	600	1.065	2000	1.590	7000	2.41	25000	3.68
180	0.712	620	1.075	2100	1.615	7200	2.43	26000	3.73
185	0.718	640	1.085	2200	1.640	7400	2.46	27000	3.78
190	0.724	660	1.100	2300	1.665	7600	2.48	28000	3.82
195	0.731	680	1.110	2400	1.690	7800	2.50	29000	3.87
200	0.737	700	1.120	2500	1.710	8000	2.52	30000	3.91
210	0.749	720	1.130	2600	1.730	8200	2.54	31000	3.96
220	0.761	740	1.140	2700	1.755	8400	2.56	32000	4.00
230	0.772	760	1.150	2800	1.775	8600	2.58	33000	4.04
240	0.783	780	1.160	2900	1.795	8800	2.60	34000	4.08
250	0.794	800	1.170	3000	1.815	9000	2.62	35000	4.12
260	0.804	820	1.180	3100	1.835	9200	2.64	36000	4.16
270	0.814	840	1.190	3200	1.855	9400	2.66	38000	4.24
280	0.824	860	1.200	3300	1.875	9600	2.68	40000	4.31
290	0.834	880	1.205	3400	1.895	9800	2.70	45000	4.48
300	0.843	900	1.215	3500	1.910	10000	2.71	50000	4.64
310	0.852	920	1.225	3600	1.930	10500	2.76	55000	4.80
320	0.861	940	1.235	3700	1.950	11000	2.80	60000	4.94
330	0.870	960	1.245	3800	1.965	11500	2.85	65000	5.07
340	0.879	980	1.250	3900	1.985	12000	2.89	70000	5.19
350	0.888	1000	1.260	4000	2.00	12500	2.93	75000	5.30
360	0.896	1050	1.280	4100	2.02	13000	2.96	80000	5.43
370	0.905	1100	1.300	4200	2.03	13500	3.00	85000	5.55
380	0.913	1150	1.320	4300	2.05	14000	3.04	90000	5.65
390	0.921	1200	1.340	4400	2.07	14500	3.07	100000	5.85

Table 11.5 f_h values of roller bearings

L_h	f_h	L_h	f_h	L_h	f_h	L_h	f_h	L_h	f_h
100	0.617	400	0.935	1250	1.315	4500	1.935	15000	2.77
105	0.626	410	0.942	1300	1.330	4600	1.945	15500	2.80
110	0.635	420	0.949	1350	1.345	4700	1.960	16000	2.83
115	0.643	430	0.956	1400	1.360	4800	1.970	16500	2.85
120	0.652	440	0.962	1450	1.375	4900	1.985	17000	2.88
125	0.660	450	0.969	1500	1.390	5000	2.00	17500	2.91
130	0.668	460	0.975	1550	1.405	5200	2.02	18000	2.93
135	0.675	470	0.982	1600	1.420	5400	2.04	18500	2.95
140	0.683	480	0.988	1650	1.430	5600	2.06	19000	2.98
145	0.690	490	0.994	1700	1.445	5800	2.09	19500	3.00
150	0.697	500	1.000	1750	1.455	6000	2.11	20000	3.02
155	0.704	520	1.010	1800	1.470	6200	2.13	21000	3.07
160	0.710	540	1.025	1850	1.480	6400	2.15	22000	3.11
165	0.717	560	1.035	1900	1.490	6600	2.17	23000	3.15
170	0.723	580	1.045	1950	1.505	6800	2.19	24000	3.19
175	0.730	600	1.055	2000	1.515	7000	2.21	25000	3.23
180	0.736	620	1.065	2100	1.540	7200	2.23	26000	3.27
185	0.742	640	1.075	2200	1.560	7400	2.24	27000	3.31
190	0.748	660	1.085	2300	1.580	7600	2.26	28000	3.35
195	0.754	680	1.095	2400	1.600	7800	2.28	29000	3.38
200	0.760	700	1.105	2500	1.620	8000	2.30	30000	3.42
210	0.771	720	1.115	2600	1.640	8200	2.31	31000	3.45
220	0.782	740	1.125	2700	1.660	8400	2.33	32000	3.48
230	0.792	760	1.135	2800	1.675	8600	2.35	33000	3.51
240	0.802	780	1.145	2900	1.695	8800	2.36	34000	3.55
250	0.812	800	1.150	3000	1.710	9000	2.38	35000	3.58
260	0.822	820	1.160	3100	1.730	9200	2.40	36000	3.61
270	0.831	840	1.170	3200	1.745	9400	2.41	38000	3.67
280	0.840	860	1.180	3300	1.760	9600	2.43	40000	3.72
290	0.849	880	1.185	3400	1.775	9800	2.44	45000	3.86
300	0.858	900	1.190	3500	1.795	10000	2.46	50000	3.98
310	0.866	920	1.200	3600	1.810	10500	2.49	55000	4.10
320	0.875	940	1.210	3700	1.825	11000	2.53	60000	4.20
330	0.883	960	1.215	3800	1.840	11500	2.56	65000	4.30
340	0.891	980	1.225	3900	1.850	12000	2.59	70000	4.40
350	0.898	1000	1.230	4000	1.865	12500	2.63	75000	4.50
360	0.906	1050	1.250	4100	1.880	13000	2.66	80000	4.58
370	0.914	1100	1.270	4200	1.895	13500	2.69	85000	4.66
380	0.921	1150	1.285	4300	1.905	14000	2.72	90000	4.75
390	0.928	1200	1.300	4400	1.910	14500	2.75	100000	4.90

Table 11.6 Values of factor f_T

T	120	125	150	175	200	225	250	300
f_T	1	0.95	0.90	0.85	0.80	0.75	0.70	0.60

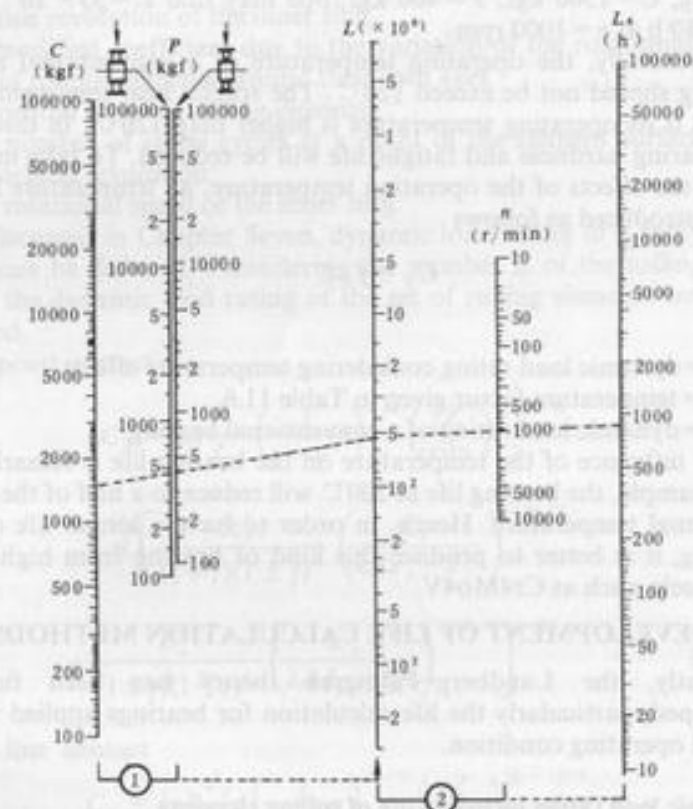


Fig. 11.1 Nomograph for rating life

In case of estimation of the bearing life, a nomograph shown in Fig. 11.1 can also be used. Drawing a straight line through two points

corresponding to values of the dynamic load rating, in kgf, and the equivalent dynamic load, in kgf, its extension intersects the axis L at a point which shows the rating life in million revolutions. Then, another straight line is drawn from the point through a point which is the bearing rotational speed, and extended to intersect the axis L . Its intersection indicates the rating life of the bearing in operating hours. An example is shown by dotted lines in Fig. 11.1. A deep groove ball bearing, $C=1500$ kgf, $P=400$ kgf, one may find $L=53 \times 10^6$, and $L_h=880$ h at $n=1000$ rpm.

Generally, the operating temperature of a conventional rolling bearing should not be exceed 120°C . The special heat treatment is required if its operating temperature is higher than 120°C . In this case, the bearing hardness and fatigue life will be reduced. To take into account the effects of the operating temperature, a temperature factor f_T is introduced as follows

$$C_T = f_T C \quad (11.5)$$

where

C_T = dynamic load rating considering temperature effects

f_T = temperature factor given in Table 11.6

C = dynamic load rating of a conventional bearing.

The influence of the temperature on the bearing life is remarkable. For example, the bearing life at 200°C will reduce to a half of the value at normal temperature. Hence, in order to have a longer life of the bearing, it is better to produce this kind of bearing from high alloy tool steels, such as Cr4Mo4V.

DEVELOPMENT OF LIFE CALCULATION METHODS

Currently, the Lundberg-Palmgren theory has been further developed, particularly the life calculation for bearings applied under special operating condition.

Dynamic load rating including life of rolling elements

It can be seen from equation (8.24) that only the fatigue failure of inner and outer rings are considered when the equation of the dynamic load rating is derived. Nevertheless, the fatigue failure of rolling elements has a large proportion in reality. In (4), an improved calculation method of the bearing life is proposed.

The number of stress cycles at a extreme small part of the rolling element surface per revolution of the inner ring is

$$u_{bi} = \frac{k_b n_m u_{bm}}{n_i} \quad (11.6)$$

where

u_{bi} = number of stress cycles at one point of a rolling element per one revolution of the inner ring

k_b = modified coefficient due to the variation of the rotational axes of rolling elements around their own axes

n_m = orbital speed of rolling elements

u_{bm} = number of stress cycles at a point of the element surface per orbital revolution

n_i = rotational speed of the inner ring.

As discussed in Chapter Seven, dynamic load rating of a rolling element can be deduced. Considering the number Z of the rolling elements, the dynamic load rating of the set of rolling elements may be obtained.

For point contact

$$C_b = \lambda A \frac{J_r}{J_1} \left(\frac{1}{1-\gamma^2} \right)^{0.3\epsilon} \left(\frac{\gamma Z}{\cos \alpha} \right)^{0.3(\epsilon-1)} \\ \times \left[\frac{k_i^e}{(1 \mp \gamma)(1 \pm \gamma)^e} \left(\frac{C_i}{\lambda A J_r / J_1} \right)^{-10/3} \right. \\ \left. + \frac{k_e^e}{(1 \pm \gamma)(1 \mp \gamma)^e} \left(\frac{C_e}{\lambda A J_r / J_2} \right)^{-10/3} \right]^{-0.3} \quad (11.7)$$

For line contact

$$C_b = \lambda v B \frac{J_r}{J_1} \left(\frac{1}{1-\gamma^2} \right)^{2\epsilon/9} \left(\frac{\gamma Z}{\cos \alpha} \right)^{2(\epsilon-1)/9} \\ \times \left[\frac{k_i^e}{(1 \mp \gamma)(1 \pm \gamma)^e} \left(\frac{C_i}{\lambda v B J_r / J_1} \right)^{-9/2} \right. \\ \left. + \frac{k_e^e}{(1 \pm \gamma)(1 \mp \gamma)^e} \left(\frac{C_e}{\lambda v B J_r / J_2} \right)^{-9/2} \right]^{-0.3}$$

$$+ \frac{k_e}{(1 \pm \gamma)(1 \mp \gamma)^e} \left(\frac{C_e}{\lambda v B J_1 / J_2} \right)^{-9/2} \Big]^{-2/9} \quad (11.8)$$

where, A and B are constants relevant to materials. K_i and K_e are constants relevant to values of K_b . K_b equals 1 for roller bearings, as well as for ball bearings under axial loads. For other conditions, then $0 < K_b < 1$. J_1 and J_2 are explained in Chapter Eight.

Obviously, the dynamic load rating of the inner and outer rings can be calculated using equations described in Chapter Eight. Similar to equation (8.24), the dynamic load rating of the bearing is

$$C = \left[1 + \left(\frac{C_i}{C_e} \right)^m + \left(\frac{C_e}{C_i} \right)^m \right]^{-1/m} C_i \quad (11.9)$$

It can be seen that if taking the same constants A and B as are used in Chapter Eight, the value C determined by equation (11.9) is smaller than that determined by equation (8.24).

From a theoretical point of view, it is necessary to consider the influence of the rolling element lives on the dynamic load ratings. But it is too complicated to calculate. In reality, the constants A and B are adjusted to fit testing results and equation (11.9) is not applied.

Life calculation for high speed bearings

Generally, bearings whose values $D_m n$ exceed 1×10^6 mm · rpm are called high speed bearings. In practice, the values $D_m n$ of some bearings may be more than 2×10^6 mm · rpm. For miniature bearings, if their rotational speeds are higher than 1.6 times of limiting speeds, they are also considered to be high speed bearings even though their values $D_m n$ are not high enough.

Due to an increase of centrifugal forces of rolling elements in high speed bearings, the distribution of applied loads among rolling elements is changed. These will be explained in detail in Chapter Sixteen. In 1952, A.B. Jones [4] took into account the influence of the centrifugal forces and derived the life calculation equation of a high speed ball bearing under a radial load or only pure axial load. His analysis indicated that the centrifugal forces of balls might reduce bearing life significantly. Ball centrifugal forces cause contact angles of inner raceways to increase and contact angles of outer raceway to decrease.

Jones gave the life equations for high speed ball bearings

$$L_e = L \left(\frac{\sin \alpha_i}{\sin \alpha_e} \right)^3 \left(\frac{1 - \gamma_i}{1 - \gamma_e} \right)^{4.17} \left(\frac{2 + 2\gamma_i}{2 + \gamma_i + \gamma_e} \right) \times \left[\frac{1 + (C_i / C_e)^{10/3}}{1 + (C_e / C_i)^{10/3}} \right]^{0.9} \quad (11.10)$$

$$\left(\frac{C_i}{C_e} \right)_e = \frac{\sin \alpha_i}{\sin \alpha_e} \times \left[\frac{f_i(2f_e - 1)}{f_e(2f_i - 1)} \right]^{0.41} \left(\frac{1 - \gamma_i}{1 + \gamma_e} \right)^{1.39} \left(\frac{2 - \gamma_i - \gamma_e}{2 + \gamma_i + \gamma_e} \right)^{1/3} \quad (11.11)$$

where

α_i, α_e = contact angles of inner and outer raceways

$\gamma_i = D_b \cos \alpha_i / D_m$

$\gamma_e = D_b \cos \alpha_e / D_m$

L = bearing life not including the influence of centrifugal forces

C_i / C_e = ratio of the dynamic load rating of the inner ring to that of the outer ring.

New methods will be described in Chapters Sixteen and Seventeen. Considering the internal geometry, kinematics, contact deformation, gyroscopic moments and centrifugal forces, etc. in bearings, a set of simultaneous nonlinear equations of differential equations can be solved to obtain the load distribution among rolling elements.

As seen from equations (8.36) and (8.37), the dynamic load ratings of the inner and outer rings respectively are

$$L_i = \left(\frac{Q_{ic}}{Q_{iq}} \right)^p \quad (8.36)$$

and

$$L_e = \left(\frac{Q_{ec}}{Q_{eq}} \right)^p \quad (8.37)$$

where, the equivalent rolling element loads Q_{iq}, Q_{eq} of the inner and outer rings can be determined using equations (8.10) and (8.17) upon

obtained $Q(\psi)$.

Then the rating life of a high speed bearing may be derived by the following equation

$$L = (L_i^{-e} + L_e^{-e})^{-1/e} \quad (11.12)$$

where, e is the shape parameter of the Weibull distribution. For ball bearing, $e = 10/9$ and for roller bearing $e = 9/8$.

Due to their complexity, the above equations are solved by computers.

Influence of misalignment on bearing life

Various types of misalignment of a bearing ring relative to another ring may happen during mounting due to influence of manufacturing deviation, shaft deflection and out of line, etc. A bearing without self-aligning performance may cause an increase in the rolling element loads and effect the bearing life.

Influence of misalignment on ball bearings

As described in Chapter Four, if a rolling bearing carries combined radial, axial and moment loads, the basic analysis is to determine the relative radial displacement δ_r , relative axial displacement δ_a and the relative angular misalignment θ . Given three variables among them, other variables can be solved. Of course, the computational process is more complicated.

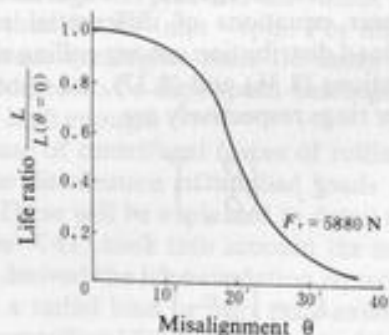


Fig. 11.2 Influence of misalignment on ball bearing life

One of the cases is commonly analysed. Given the radial load F_r , axial load F_a and relative angular misalignment θ , therefore δ_r , δ_a and M can be computed using equations (4.97) and (4.99). Then using equation (4.95) derives $Q(\psi)$.

The dynamic load ratings of the inner and outer rings are computed from equations (8.36) and (8.37). And the bearing rating life will be obtained using equation (11.12).

Fig.11.2 shows the computed results of the influence of misalignment over the bearing life in a deep groove ball bearing, which has zero radial clearance and carries a radial load of 5880 N.

Influence of misalignment on roller bearings

While having misalignment of the inner and outer rings in a roller bearing, the load distribution among rollers and the load density along the roller length are changed. Computational methods for these will be introduced in Chapter Seventeen. Since the load density along the roller length is not constant, the roller is divided over its effective length into several pieces of disc with thickness w . The probability of survival of a whole roller is equal to the product of the probabilities of survival of every individual parts. Then, the equivalent roller load per roller length is

$$q_e = \left(\frac{1}{l_e} \int_0^{l_e} q^{9/2} dx \right)^{2/9} \quad (11.13)$$

Hence, the equivalent roller load at angular position ψ_j is

$$Q_j = l_e^{7/9} \left(\int_0^{l_e} q_j^{9/2} dx \right)^{2/9} \quad (11.14)$$

where

q_j = load density along the roller length at angular position ψ_j .

Equation (11.14) may also be changed to

$$Q_j = l_e^{7/9} \omega^{2/9} \left(\sum_{i=1}^n q_{ji}^{9/2} \right)^{2/9} \quad (11.15)$$

where

q_i = load density of the i -th piece of disc at angular position ψ_i .

Easy to see that another equivalent is made due to the variation of the load density along the roller length.

Using equation (8.36) yields the following rating life of the inner ring

$$L_i = \left[\frac{Q_{ic}}{\left(\frac{1}{Z} \sum_{j=1}^i Q_j^4 \right)^{1/4}} \right]^4 \quad (11.16)$$

where

Q_{ic} = dynamic load rating of the inner raceway contact which is determined by equation (8.9).

Similarly, the rating life of the outer raceway contact can be deduced from equation (8.37)

$$L_e = \left[\frac{Q_{ec}}{\left(\frac{1}{Z} \sum_{j=1}^i Q_j^{9/2} \right)^{2/9}} \right]^4 \quad (11.17)$$

where

Q_{ec} = dynamic load rating of the outer raceway contact.

For simplification, all the descriptions above are based on the fixed outer ring and rotating inner ring.

The rating life of the bearing can be derived using equation (11.12). Fig. 11.3 shows the computed results of the influence of the relative misalignment over the bearing life in a radial cylindrical roller bearing. It can be seen that the life reduces significantly while the relative misalignment increases.

Life calculation of oscillating bearings

A bearing which does not turn through a complete revolution and oscillates reciprocally is called an oscillating bearing. Express its frequency of oscillation as n cycles per minute and its angular amplitude as φ . To determine the life of an oscillating bearing, its applied load can be equivalent to a load of a rotating bearing.

It can be concluded from equations (7.10) and (7.14) that

$$Q'u = \text{const.}$$

Consider an equivalent rotating bearing which has the same life as

the oscillating bearing. Therefore

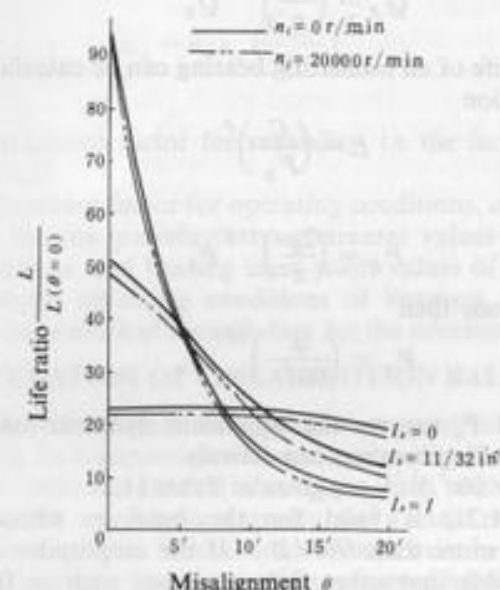


Fig. 11.3 Influence of misalignment on roller bearing life

$$Q_R = \left(\frac{u_0}{u_R} \right)^{1/p} Q_0 \quad (11.18)$$

In which, subscript R expresses the corresponding quantity of the equivalent rotating bearing, and subscript 0 expresses that of the oscillating bearing.

The stressed arc length during one complete revolution of a rotating bearing is $2\pi r$, and that during one complete oscillation of an oscillating bearing is $4\varphi r$. In which, r is the raceway radius and φ is the angular amplitude of the oscillation. Then

$$\frac{u_0}{u_R} = \frac{2\varphi}{\pi} \quad (11.19)$$

Substitution of equation (11.18) yields

$$Q_R = \left(\frac{2\varphi}{\pi} \right)^{1/p} Q_0 \quad (11.20)$$

Therefore, the life of an oscillating bearing can be calculated using the following equation

$$L = \left(\frac{C}{P_R} \right)^p \quad (11.21)$$

where

$$P_R = \left(\frac{2\varphi}{\pi} \right)^{1/p} P_0 \quad (11.22)$$

For φ in degrees, then

$$P_R = \left(\frac{\varphi}{90^\circ} \right)^{1/p} P_0 \quad (11.23)$$

Where, P_R and P_0 express the equivalent dynamic load of the rotating and oscillating bearings respectively.

Values of $(\varphi/90^\circ)^{1/p}$ are given in Table 11.7.

Equation (11.21) is valid for the bearings whose oscillating amplitudes are more than $90^\circ/Z$. If the amplitudes are less than that, it is possible that other failure modes, such as fretting wear, indentation at raceways will occur. Equation (11.21) is not valid for this situation.

Table 11.7 Values of $(\varphi/90^\circ)^{1/p}$

φ°	5	10	15	20	30	45	60	75	90
Ball bearings	0.38	0.48	0.55	0.61	0.69	0.79	0.87	0.94	1.0
Roller bearings	0.42	0.52	0.58	0.64	0.72	0.81	0.89	0.95	1.0

A modified proposal for life calculation

ISO/TC4 discussed a modified proposal for the life calculation of bearing. An effective dynamic load rating is introduced, that is

$$C_{EF} = b_m C \quad (11.24)$$

where

b_m = modified coefficients of materials and manufacturing quality

C = dynamic load rating given in Table 8.3.

Equation (8.29) may be written as follows

$$L_n = a_n a_c \left(\frac{C_{EF}}{P} \right)^p \quad (11.25)$$

where

a_n = life adjustment factor for reliability, i.e the factor a_1 given in Table 8.13

a_c = life adjustment factor for operating conditions, $a_c = a_3$.

Therefore, bearing manufacturers guarantee values of effective dynamic load ratings, and bearing users select values of factors a_n and a_c upon different operating conditions of bearings. Unfortunately, until now we have not had enough data for the selection of b_m and a_c .

LIFE CALCULATION OF LINEAR MOTION BALL BEARINGS

Linear motion ball bearings are applied to support relative reciprocal linear motions. In comparison with sliding bushes, linear motion ball bearings have some remarkable advantages, such as low friction, high accuracy, long life, etc. A typical structure of a linear motion ball bearing is shown in Fig. 11.4. Several rows of balls exist between the outer raceway and the shaft. The outer raceway is parallel to the axis of the shaft and balls roll over the raceway reciprocally under an applied load.

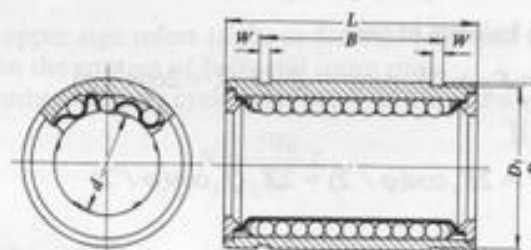


Fig 11.4 Linear motion ball bearing

Main failure modes of this type of bearings are contact fatigue and wear. Up to now, the contact fatigue still is one of the bases of the life

calculation owing to shortage of an ideal method for the wear life calculation.

Dynamic load rating

The dynamic load rating of a linear motion ball bearing refers to a central radial load under which the bearing may operate to a rating life of accumulated moving length of 50 kilometers. In other words, under this load 90 per cent of bearings will reach or exceed their accumulated moving length of 50 kilometers. On the basis of the Lundberg and Palmgren theory, the dynamic load rating of linear motion ball bearings can be deduced. Two types of relative ball positions, called case I and II respectively, are shown in Fig. 11.5.

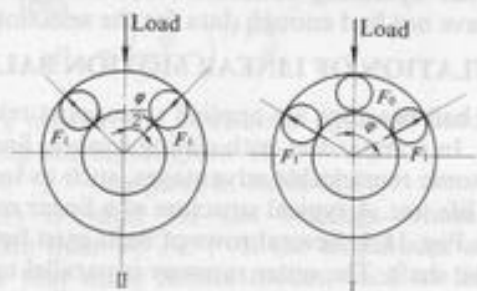


Fig. 11.5 Typical ball positions

For the force balance in case I

$$P = F_0 + 2F_1 \cos \varphi = Z_f Q_0 (1 + 2\cos^{2.5} \varphi) \quad (11.26)$$

And for case II

$$P = 2F_1 \cos(\varphi / 2) = 2Z_f Q_1 \cos(\varphi / 2) \quad (11.27)$$

where

- P = central radial load of the bearing
- F_0, F_1 = load carried by one row of balls
- Q_0, Q_1 = load carried by a ball
- Z_f = average number of balls simultaneously carried the load in one row of balls
- φ = central angle shown in the Fig., $\varphi = 360^\circ / i$

i = number of rows.

Fig. 11.6 shows the internal contact condition and contact ellipse of a linear motion ball bearing. It can be seen that the directions of elliptical major-axes at the outer and inner raceway contacts are different.

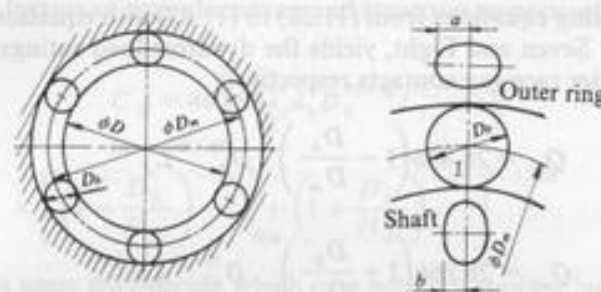


Fig. 11.6 Internal contact condition of a bearing

The curvature sum at the contact

$$\sum \rho = \frac{4}{D_b} \pm \frac{2}{D_m \mp D_b} \quad (11.28)$$

And the curvature function

$$F(\rho) = \frac{D_b}{2D_m \mp D_b} \quad (11.29)$$

where, the upper sign refers to the contact of balls and shaft, the lower sign refers to the contact of balls and outer ring.

Total number of stress cycles at a point of the raceway is

$$N = \frac{50u}{l_s} L \quad (11.30)$$

where

u = number of stress cycles at a point of the raceway during a stroke

l_s = stroke length at one direction in mm

L = accumulated stroke length in 50 km.

Stressed volumes of an inner and outer raceway contacts respectively are

$$V_i \propto (bZ_0 D_n)_i \quad (11.31)$$

$$V_e \propto (aZ_0 D_n)_e \quad (11.32)$$

in which, D_n is the length of the stressed raceway.

Substituting equations from (11.28) to (11.32) into equation (7.6), as in Chapter Seven and Eight, yields the dynamic load ratings of an inner and outer raceway contacts respectively

$$Q_{ie} = 28.866 \left(1 - \frac{D_b}{D_m}\right)^{1.4} D_b^{73/30} L_f^{-0.3} \quad (11.33)$$

$$Q_{ee} = 28.866 \left(1 + \frac{D_b}{D_m}\right)^{0.51} D_b^{73/30} L_f^{-0.3} \quad (11.34)$$

where

L_f = effective length of the raceway.

The dynamic load ratings of contacts between one row of balls and the shaft or the outer raceway are

$$C_i = Z_f Q_{ie} \quad (11.35)$$

$$C_e = Z_f Q_{ee} \quad (11.36)$$

The dynamic load rating of one row of balls is

$$C_1 = (C_i^{-10/3} + C_e^{-10/3})^{-3/10} \quad (11.37)$$

By the product law of probability, the dynamic load rating of a bearing in which several rows of balls carry the load will be

$$C = C_1 \left[\sum \left(\frac{F_i}{P} \right)^{10/3} \right]^{-3/10} \quad (11.38)$$

Therefore, the dynamic load rating of a bearing in case I is given by

$$C_1 = 28.866 \lambda_i \lambda_e D_b^{43/30} L_f^{0.7} \frac{1 + 2\cos^{2.5} \varphi}{(1 + 2\cos^5 \varphi)^{3/10}}$$

$$\times \left[\left(1 - \frac{D_b}{D_m}\right)^{-14/3} + \left(1 + \frac{D_b}{D_m}\right)^{-1.7} \right]^{-3/10} \quad (11.39)$$

If $\varphi > 90^\circ$ in equation (11.39), $\varphi = 90^\circ$ is substituted. λ_i and λ_e are modified factors of manufacturing and structure respectively. For case II

$$C_{II} = 46.893 \lambda_i \lambda_e D_b^{43/30} L_f^{0.7} \cos \frac{\varphi}{2}$$

$$\times \left[\left(1 - \frac{D_b}{D_m}\right)^{-14/3} + \left(1 + \frac{D_b}{D_m}\right)^{-1.7} \right]^{-3/10} \quad (11.40)$$

Bearing users may decide which case actually applies, but smaller a value between C_I and C_{II} is commonly selected for safety. The ratios of C_{II} to C_I are listed in Table 11.8. It can be seen that C_{II} is more than C_I except $i = 3$.

Table 11.8 Values of C_{II} / C_I

i	3	4	5	6
C_{II} / C_I	0.8122	1.1487	1.1901	1.0584

Calculation of rating life

The rating life of a linear motion ball bearing refers to accumulated stroke length, or operating hours at a certain reciprocal frequency, for which 90 per cent of a sufficiently large group of apparently identical bearings can be expected to reach or exceed.

The basic equation of the rating life is

$$L = \left(\frac{C}{P} \right)^3 \quad (11.41)$$

in which, L is in 50 km.

For a life of operating hours, this is

$$L = 2.4 \times 10^{-6} n L_s l_s \quad (11.42)$$

where

n = number of reciprocal cycles

L_h = life in operating hours.

For simplification, a life factor is introduced

$$f_L = L^{-1/3} \quad (11.43)$$

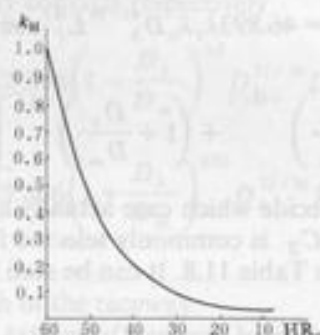


Fig. 11.7 Modified factor for hardness

Table 11.9 Life factor f_L

$L (\times 50\text{km})$	f_L	$L (\times 50\text{km})$	f_L	$L (\times 50\text{km})$	f_L
1	1	10	0.464	100	0.215
2	0.794	20	0.368	200	0.171
3	0.693	30	0.322	300	0.149
4	0.630	40	0.292	400	0.136
5	0.585	50	0.271	500	0.126
6	0.550	60	0.255		
7	0.523	70	0.243		
8	0.500	80	0.232		
9	0.481	90	0.223		

Values of f_L corresponding to L are presented in Table 11.9. Equation (11.41) becomes

$$f_L = \frac{P}{C} \quad (11.44)$$

Using equation (11.44), one can select a bearing on the basis of its given load and required rating life. On the other hand, one can also calculate the rating life on the basis of the given type of bearings and the bearing load.

If the hardness of the operating surface of a shaft is lower than HR_{C60} , it should be modified by the following equation

$$C = \frac{P}{f_L k_H} \quad (11.45)$$

where

k_H = modified factor for hardness, given in Fig. 11.7.

LIFE CALCULATION OF ROLLING SCREWS

A rolling screw is one type of drive in which both bolt and nut have spiral raceways filled with rows of recirculating balls. It increases the driving efficiency and accuracy significantly due to rolling friction instead of sliding friction.

The structure of the rolling screw is shown in Fig. 11.8. In view of the similarity of the manufacturing technology many bearing industry companies produce rolling screws. Their calculation methods of the static load rating and rating life are similar to rolling bearings.

In a screw, an angle between a normal to the raceway at the ball contact point and the transverse section of the bolt is called the contact angle. Generally, it is 45° .

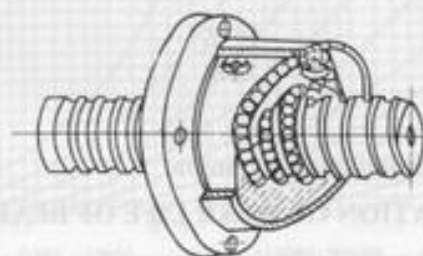


Fig. 11.8 Rolling screw

Dynamic load rating

The dynamic load rating of a rolling screw refers to an axial load un-

der which 90 per cent of apparently identical screws can be expected to reach its rating life of 10^6 revolutions without any sign of fatigue.

Similar to equation (8.29), the dynamic load rating of rolling screws is

$$C_a = f_c (i_r \cos \alpha)^{0.7} \tan \alpha Z^{2/3} D_b^{1.8} \quad (11.46)$$

where

i_r = number of spiral circles times number of rows

Z = number of balls per spiral circle

f_c = being concerned with geometrical parameters, manufacturing accuracy and materials. For the groove curvature radius $R < 0.52D_b$, values of f_c may be found up from Table 8.5.

The unit of dynamic load ratings in equation (11.46) is Newton.

Calculation of rating life

The basic equation of the rating life of rolling screw is

$$L = \left(\frac{C_a}{P_a} \right)^3 \quad (11.47)$$

where

P_a = axial load.

In mathematics, equation (11.47) is the same with equation (8.35). The unit of the rating life L is 10^6 revolutions. If the rating life is expressed by operating hours, and it is marked L_h , then

$$L_h = \frac{10^6 L}{60n} \quad (11.48)$$

where

n is number of revolutions per minute.

CALCULATION OF WEAR LIFE OF BEARINGS

As mentioned in Chapter Eight, bearings used in various kinds of machinery, such as machine tool spindles, gyroscopes may fail in a manner of wear instead of fatigue. The wear occurs at the contact surfaces of rings and rolling elements, or at sliding surfaces of the cage. The severe wear can cause a decrease in the operating performances of bearings. For distinction from the fatigue life, the operating life of a bear-

ing is called wear life when it loses its normal performance due to wear.

Permissible wear factor

Wear causes a reduction in bearing accuracy and an increase in bearing clearances, which effect the running performances of the bearing. A simple way is to use the increase of clearances as a measure of wear [91]. The permissible radial clearance enlargement V in μm can be determined by tests on the basis of different bearing types, sizes and requirements.

A permissible wear factor is introduced by the following equation

$$f_v = \frac{V}{e_0} \quad (11.49)$$

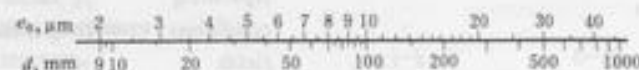


Fig. 11.9 Wear constants of bearings

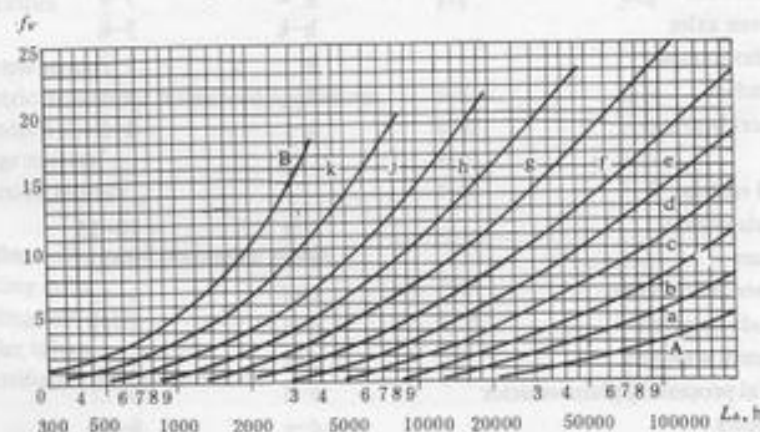


Fig. 11.10 Permissible wear factor and wear life

where

e_0 = wear constant which is dependent on the bore diameter of the

bearing and indicates the wear velocity, in μm . It is shown in Fig. 11.9.

Obviously, operating conditions of bearings have an important influence over the wear life. The relations between the permissible wear factor and the wear life are shown in Fig. 11.10. The area given by curves *A* and *B* is divided into ten fields marked *a-k*. A transition from *a* to *k* shows that environmental and operating conditions become more unfavourable and wear increases more severely.

Bearing applications covering a wide range of machinery and equipment and their fields of operating conditions, as well as factor f_v are listed in Table 11.10. It is suggested that the lower values of f_v be selected for more strict applications.

Table 11.10 Operating conditions and permissible wear factor (91)

Application	Operating conditions fields	f_v factor for permissible wear
Motor vehicles		
Gearboxes	g-k	5-8
Driven axles	h-k	3-6
Water pumps	k	5-7
Clutches	k	5-7
Wheel bearings	h-i	4-6
Rail vehicles		
Haulage cars	f-h	12-15
Trams	e-f	8-12
Passenger coaches	c-d	8-12
Goods waggons	c-d	8-12
Mineral waggons	c-d	8-12
Metal processing plant vehicles	c-d	8-12
Railcars	d-e	6-10
Locomotives / outer bearing	d-e	6-10
Locomotives / inner bearing	d-e	6-10
Transmission gears for rail vehicles	c-d	3-6

(continued)

Application	Operating conditions fields	f_v factor for permissible wear
Shipbuilding		
Ship's propeller thrust blocks	e-f	6-10
Ship's propeller shaft bearings	e-f	15-20
Large marine gears	c-d	5-10
Agricultural machinery		
Tractors	i-k	6-10
Self-propelled agricultural machines	k	8-15
Machines for seasonal use	k	10-30
Construction machinery		
Crawler tractors	i-k	6-10
Loaders	i-k	6-10
Excavators	i-k	8-15
Vibrators	f-i	3-4
Electric motors		
Electric motors for household appliances	i-k	3-5
Standard motors	c-d	3-5
Large motors	b-d	3-5
Traction motors	c-d	3-5
Rolling mills, metal processing plants		
Rolling mills	f-g	6-10
Rolling mill gears	e-d	6-12
Roller tables	e-f	8-12
Centrifugal casting machines	e-f	8-12
Machine tools		
Lathe spindles	a-b	0.5-1.5
Milling spindles	a-b	0.5-1.5
Boring spindles	b-c	1-2
Grinding spindles	c-d	0.5-1

(continued)

Application	Operating conditions fields	f_v factor for permissible wear
Headstock spindles	a-b	0.5-1
Machine tool gears	c-d	3-8
Presses / flywheel	d-f	3-8
Presses / eccentric shaft	d-e	3-8
Electric tools	g-h	3-8
Pneumatic tools	g-h	3-8
Woodworking machines		
Milling cutters	e-f	1.5-3
Cutter shafts	e-f	1.5-3
Frame saws / main bearings	e-g	3-4
Frame saws / con rod bearings	c-d	2-3
Gears / general engineering		
Universal gears	d-e	3-8
Gear motors	d-e	3-8
Large-sized gearboxes, fixed	c-d	6-10
Materials handling equipment		
Belt drives / opencast mining	c-d	5-10
Belt conveyor idlers	g-k	10-20
Belt pulleys	e-f	10-15
Bucket wheel excavators / bucket wheel	e-g	12-15
Winding cable sheaves	c-d	8-12
Pump, blowers, compressors		
Small fans and blowers	f-h	5-8
Medium-sized fans	d-f	3-5
Large fans	c-d	3-5
Centrifugal pumps	d-f	3-5
Compressors	d-f	3-5
Centrifuges, stirrers		
Centrifuges	d-e	2-4

(continued)

Application	Operating conditions fields	f_v factor for permissible wear
Large stirrers	g-h	8-15
Crushers, mills, screens, etc.		
Jaw crushers	f-g	8-12
Cone crushers, roll crushers	f-g	8-10
Beater mills	c-d	4-6
Hammer mills, impact mills	d-e	5-8
Tube mills	f-g	12-18
Vibrating mills	f-g	3-5
Vibrating screens	e-f	4-6
Briquette presses	e-g	8-12
Rotary kiln support rollers	f-g	12-18
Paper-making machines and printing machines		
Paper-making machines / wet section	b-c	7-10
Paper-making machines / dryer section	a-b	10-15
Paper-making machines / refiners	b-c	5-8
Paper-making machines / calenders	a-b	4-8
Printing machines	a-b	3-4
Textile machinery		
Spinning machines / spindles	a-c	1-2
Power looms, knitting machines	a-e	2-8
Plastics processing machinery		
Screw extruders for plastic materials	e-f	8-12
Rubber and plastic sheeting calenders	e-f	8-12

Calculation of wear life

Given the permissible clearance enlargement of a bearing, it is possible to determine its wear life. The main steps are the following

- (1) to obtain the value of the wear constant e_0 upon the bearing bore diameter from Fig.11.9;

- (2) to calculate the permissible wear factor f_v using equation (11.49);
- (3) to select the corresponding operating condition field from Table 11.10;
- (4) to draw a horizontal line corresponding to the wear constant in Fig. 11.10 and to intersect the boundaries of the field selected. Then, the abscissas of the intersections are the maximum and minimum values of its wear life.

Example. Consider the wear life of a deep groove ball bearing 6220 for an application of standard motors. Its permissible clearance enlargement is $50 \mu\text{m}$.

Its wear constant is $10 \mu\text{m}$ from Fig. 11.9 upon the bearing bore diameter of 100 mm. And using equation (11.49) yields that the permissible wear factor is five. Furthermore from Table 11.10, the condition field d is selected. Therefore, the wear life of the bearing is 16000 to 26000 hours.

If the permissible wear factor f_v of a bearing and its bore diameter as given, then the clearance enlargement of the bearing till the failure can be deduced using equation (11.49).

It can be seen that the wear life calculation of a bearing only refers to an approximate value.

Special tests are needed in the case of strict requirements.

CHAPTER TWELVE

Grease life in rolling bearings

In the majority of applications, rolling bearings are lubricated by grease because the bearing arrangement design can be relatively simple. Grease is more easily retained in the bearing and it provides sufficient protection against foreign particles and water.

Grease-lubricated bearings are widely applied to various types of machines, such as electric motors, measuring instruments, machine tools, automobiles, computers, robots, video recorders, electric fans, etc. In most of the applications, since the bearing speed and the external load are not high then calculated fatigue lives of bearings would probably be several tens of thousands of hours or even more. Bearings will possibly be damaged in other mode, that is, efficacy loss of grease. The sudden increasing of frictional temperature causes scuffing of operating surfaces of bearings. The total operating hours of a grease-lubricated bearing from starting running until efficacy loss of grease is called the grease life.

Sometimes, rolling bearings must be relubricated if the life of grease used is shorter than the expected service life of the bearing. Relubrication should be undertaken at a time when the lubrication of the bearing is still valid. A bearing may be relubricated several times. Therefore, it is necessary to predict the grease life of a bearing and to specify the appropriate relubrication interval. These will be introduced in the first paragraph of this chapter.

Grease life also is dispersive. By comparison with the dispersion of fatigue life, the range of its dispersion is narrow.

CALCULATING METHOD FOR GREASE LIFE

The calculating method for the grease life of bearings is primarily based on the analysis of test data. Firstly, the types of grease commonly used and their testing methods are described.

Types of grease

Conventional grease is composed of base oil, grease thickener and additives. Generally, base oil 70–95%, grease thickener 30–5% and

with small amount of additives.

Base oils may be mineral or synthetic. The lubricating properties of a grease are mainly determined by its base oil. For use in low temperature of high speed condition, the grease with a low viscosity base oil is selected and for high temperature or heavy loading condition, the grease with a high viscosity base oil is selected.

Grease may be classified according to its thickeners, which are soaps, such as calcium, lithium, sodium, barium, etc. and agents, such as silicagel, alumina, colourings, etc. Usually, the operating temperature of a bearing should be 10°C to 20°C lower than the drop point of the grease used. The water resistance of grease depends on its thickener. Sodium base grease is unstable in water or high humidity operating conditions, because it absorbs water to a certain extent to form an emulsion.

Additives are primarily used to increase functions of antioxygen, antirust, EP, etc.

For applications with heavy load or shock load, greases containing different EP additives are required. On the other hand, for applications where a long grease life is required, greases containing antioxidant additive are used.

Functions of conventional greases are listed in Table 12.1. Penetration is an important index. The specification of the penetration means a depth that a cone which weighs 150 grams penetrates into the grease at temperature 25°C within five seconds, in 1/10 mm unit. The penetration indicates "softness" of a grease. Various penetrations and their applications are listed in Table 12.2.

Usually, mixing different types of greases together in application should be avoided. Also for similar types of greases, a mixture of different additives may have a harmful effect. The anticorrosive properties of greases are improved by the addition of rust inhibitors.

Testing methods for grease life

To determine grease life by tests, some standard testing methods of the grease properties and life are specified, as shown in Table 12.3.

On the whole, the testing methods may be classified as the following two kinds: one is to test until efficacy loss of the grease and to count the total testing time and the other is to specify the standard testing time and to measure the variations of the grease properties. The primary structures of testing equipment are similar to bearing life testing

Table 12.1 Functions of greases

Name Base oil Properties	Lithium base grease		Sodium base grease	Calcium base grease	Mixed base grease
	Mineral oil	Cylinder oil	Mineral oil	Mineral oil	Mineral oil
Drop point °C	170-190	200-250	170-200	70-90	160-190
Range of operating temperature °C	-20-+100	-50-+160	-20-+120	-20-+60	-20-+80
Permissible rotational speed* %	70	70	70	40	70
Mechanical stability	Good	Good	Good	Bad	Good
Load capacity	Medium	Weak	Strong to medium	Weak	Strong to medium
Water resistance	Good	Good	Bad	Good	Bad if containing sodium base
Antirust	Good	Bad	Good to bad	Good	Good to medium
Instruction	Widely used in rolling bearings	Mainly used in high temperature, not valid for high speed and heavy load conditions	Not valid for water or high humidity	Load capacity can be improved by EP additive	For large ball or roller bearings

* modified percentages to limiting speeds in a bearing catalogue.

machines. The differences from the latter are that only a fixed type of bearings is used in grease testing and the testing load mostly is much less than that in life testing.

Table 12.2 Penetrations and applications

Penetration index	0	1	2	3	4
Values of penetration	385–355	340–310	295–265	250–220	205–175
Application	For use in fretting wear conditions	For use in low temperature conditions	General shielded or sealed bearings	General sealed bearings for use in high temperature	For use in high temperature conditions

Besides the testing of grease life, many testing methods for physical and chemical properties of greases are also specified. These include viscosity, flash point, water content, corrosion, separation, heat-resistance, etc. The description for these would be out of the sphere of this book.

Experiments show that the life of identical grease under the same testing conditions is dispersive. The longest life of the grease may probably be five to ten times of its shortest life. The statistical treatment methods described in Chapter Nine can be applied. If without special explanation, a grease life means the average value of several test data.

Some characteristics found by tests are helpful to evaluate grease life and to reduce testing time. Those are

- (1) Noises of a bearing are closely connected with the grease life. If noises of a running bearing with grease lubrication increase slowly, then the grease life will be longer. The time required for these noises of a bearing add 6 dB to their initial value approximately is $1/2$ to $1/3$ of the grease life;
- (2) Grease quantity filled in a bearing influences the grease life of the bearing. If grease quantity filled in a bearing respectively is 5%

Table 12.3 Standard testing methods of greases

	ASTM-D1974 grease life	Federal test standard 791B		IP-168 Grease function test	DIN-51806
		Method 331.2	Method 331.1		
Test bearing	Deep groove ball bearing 30BC02 J 06	Deep groove ball bearing 6204K	Bearing made of high temperature steel 6204ABEC3	Deep groove ball bearings RMB040	Spherical roller bearings 22312
Grease quantity	6 ± 0.1 g for two	3.0 ± 0.1 g	3.2 ± 0.1 g	30 ml	50 g
Rotational speed	3500 r/min	10000 r/min	10000 r/min	10000 r/min	1500 or 2500 r/min
Testing temperature	125°C	~176°C	~232°C	~200°C	~200°C
Load	Radial 11.2 kgf Axial 18.0 kgf	Radial 1.4 kgf Axial 2.3 kgf	Radial 6.8 kgf Axial 2.3 kgf	Radial 22.7–136.1 kgf	Radial 850 kgf
Testing time	20 hours for loading, 4 hours for unloading, cycle testing	21.5 hours for loading, 2.5 hours for unloading, cycle testing	20 hours for loading, 4 hours for unloading, total 72 hours	If normal to 50 hours then testing until 500 hours	6 hours ~20 days
Damage index	Motor stops Temperature is 10°C above a controlled value Continuous noises last more than 10 seconds	Motor stops Grease leaks out too much Bearing stops	Shaft torque exceeds 300% of conventional value Temperature is 10°C above a controlled value		

and 35% of free-space of the bearing, the ratio of their grease lives approximately is 1 : 3.

Calculating methods of grease life

On the basis of test data, the average total operating time until efficacy loss of grease, i.e. grease life, while the grease quantity filled is around 30% of free-space of a shielded or sealed bearing may be estimated by the following empiric equations.

Wilcock formula

Considering the effects of operating temperature, rotational speed, external load, bearing sizes, etc., Wilcock proposed the equation

$$\log t = 4.73 - (T - 17.2) \times (0.0104 + 8.46n \times 10^{-7}) - 0.03 \frac{n F_r^{1.5}}{C^{1.9}} \quad (12.1)$$

where

t = average life of grease, h

T = operating temperature of the bearing, °C

n = rotational speed of the bearing, rmp

F_r = radial load acting on the bearing, kgf

C = dynamic load rating of the bearing, kgf.

Equation (12.1) was presented in the fifties. Due to the improvement of grease properties, the actual life possibly exceeds the corresponding calculated value.

NSK formula

NSK Bearing Corporation in Japan suggests the following formula

$$\log t = 6.54 - 2.6 \frac{n}{n_{\max}} - \left(0.025 - 0.012 \frac{n}{n_{\max}} \right) T \quad (12.2)$$

where

$n_{\max}$ = limiting speed for grease lubrication specified in a catalogue.

Equation (12.2) is valid under the following conditions

$$0.25 \leq \frac{n}{n_{\max}} \leq 1, \quad 40^\circ\text{C} \leq T < 120^\circ\text{C}$$

If $n/n_{\max}$ is less than 0.25, then $n/n_{\max}$ is replaced by 0.25. And if

operating temperature T is lower than 40°C , then T is replaced by 40.

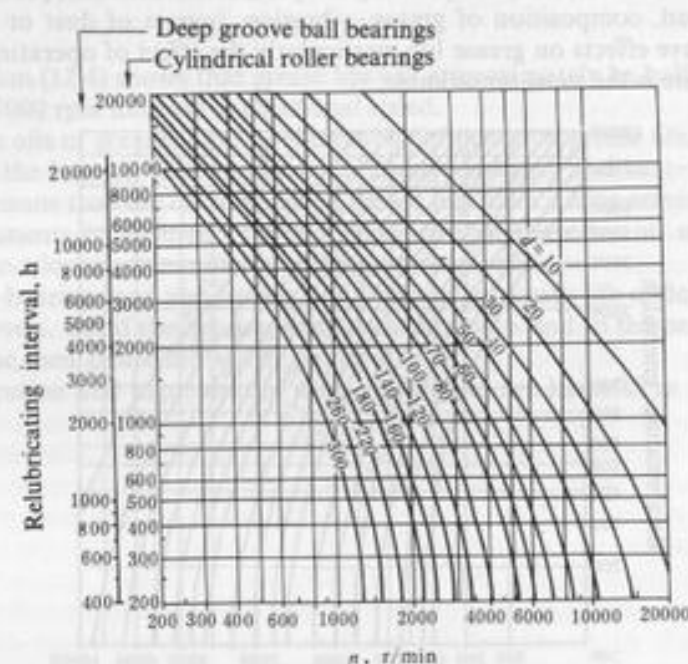


Fig. 12.1 Relubrication intervals for radial ball and roller bearings

It can be seen that some factors, such as grease quantity filled, types and compositions of grease, are not taken into account in equations (12.1) and (12.2). Therefore, calculated results according to the equations only give approximate average values.

The above equations are not valid for opened-types of bearings with grease lubrication. In these cases, relubrication intervals of the bearings can be found by curve figures or tables. Fig.12.1 and Fig.12.2 respectively show relubrication intervals for radial ball or roller bearings and for tapered or spherical roller bearings. These figures are suitable for bearing temperatures not exceeding 70°C . The intervals obtained from the figures are approximately halved for every 15°C increase in bearing temperature above 70°C .

INFLUENTIAL FACTORS ON GREASE LIFE

Various factors, such as operating temperature, rotational speed, bearing load, composition of grease, vibration, ingress of dust or water, etc. have effects on grease life, particularly the effect of operating temperature is the most remarkable.

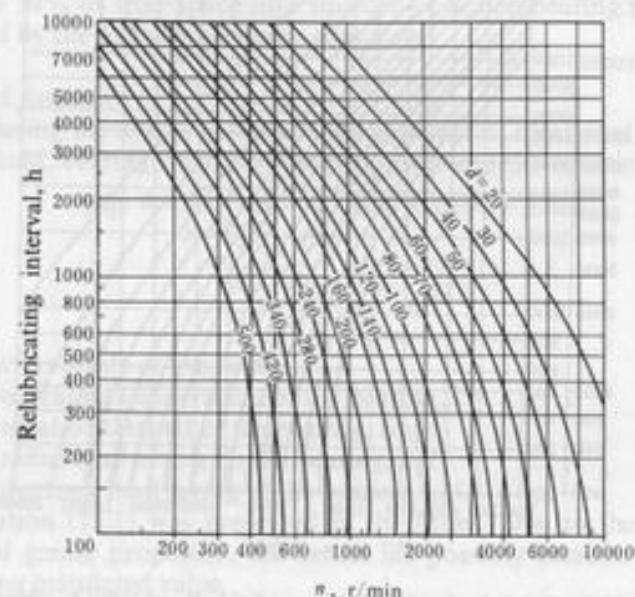


Fig. 12.2 Relubrication intervals for tapered and spherical roller bearings

The effect of temperature. Reference (92) gives a relationship between the grease life and the operating temperature of a bearing. The testing conditions are deep groove ball bearings with shields 6305ZZ and with seals 6305 2RS, grease quantity filled 3.4 gram, testing rotational speed 3000 rpm. Test results are treated by a least square method and the following equation is given.

$$\lg t = 5.85 - 0.022T \quad (12.3)$$

Equation (12.3) shows that grease life will approximately be halved for every 14°C increase in bearing temperature.

The effect of rotational speed. Similarly, five different types of

greases are tested in a fixed temperature 120°C at different rotational speeds. It gives a relationship between the grease life and the rotational speed of a bearing as follows:

$$\lg t = 3.73 - 0.00016n \quad (12.4)$$

Equation (12.4) shows that grease life will approximately be halved for every 2000 rpm increase in rotational speed.

Base oils of greases have a remarkable influence on grease life. Generally, the base oil with excellent oxidation resistance and heat-resistance means that the corresponding grease will have a long service life. Experiments in Reference (92) indicate that the ester oil usually leads to a longer grease life than that of mineral oils.

The bearing load almost has no influence on grease life under light load levels. But if the bearing load is too heavy to lead to temperature increase then grease life will be reduced.

Vibration and ingress of dust or particles are harmful to grease life.

CHAPTER THIRTEEN

Operating performance of rolling bearings
—stiffness

In Chapter Four, the load distribution in rolling bearings and the relative displacement between two rings under different loading conditions were described. Generally speaking, a bearing subjected to combined radial, axial and moment loads will bring about relative radial displacement, relative axial displacement and relative tilting angle between two rings. These relative elastic displacements will have an important effect on the bearing applications and reflect one of the operating performances of the bearing. Normally, the definition of the stiffness of a bearing is an external load required to produce one unit of the relative elastic displacement between two rings of the bearing. According to the directions of relative displacements, there are radial stiffness, axial stiffness and angular stiffness. In the first paragraph of this chapter, some calculating methods commonly used for the relative elastic displacement and the stiffness are explained. Similarly, the bending deformation of bearing rings is neglected in the calculation and only elastic contact deformation at the rolling element-raceway contact is considered.

Different types of machinery have different requirements for the bearing stiffness. For example, it is extremely important in machine tool applications, in order to ensure the high productivity and manufacturing accuracy of the machine tool. The proper stiffness of bearings for high speed spindles is required to avoid the resonance of the spindles. To reduce the acceleration square deviation, it is necessary for gyroscopic motor bearings to have the same radial and axial stiffnesses.

It is possible to increase bearing stiffness by preload. Angular contact ball bearings and tapered roller bearings are generally adjusted against each other by axial displacement of the inner and the outer rings until a certain preload is obtained in the bearing arrangement. In the second paragraph of this chapter, the principle of preload for rolling bearings and its calculating method are described.

In the last paragraph, for the convenience of the design and application of bearings, the calculating method of the limiting thrust load for deep groove and angular contact ball bearings is given.

DEFLECTION AND STIFFNESS OF ROLLING BEARINGS

The basis of elastic deflection calculation is the Hertzian contact theory. As described in Chapter Four, it is necessary to solve a set of nonlinear algebraic equations for calculation of the relative deflection between two rings in a bearing under its loading condition. Generally, this is conducted by a computer. In practice for simple loading conditions, the calculating expressions for the relative deflection between two rings can be given.

As can be seen from Chapter Four, the maximum relative elastic deflection at $\psi_r = 0$ in a bearing subjected to combined radial and axial loads may be determined by equation (4.61)

$$\delta_{\max} = \delta_a \sin \alpha + \delta_r \cos \alpha \quad (4.61)$$

If only relative axial deflection between two rings exists, i.e. $\delta_r = 0$, then

$$\delta_a = \frac{\delta_{\max}}{\sin \alpha} \quad (13.1)$$

If only relative radial deflection exists, i.e. $\delta_a = 0$, then

$$\delta_r = \frac{\delta_{\max}}{\cos \alpha} \quad (13.2)$$

Considering equations (4.1) to (4.5), one may deduce the elastic deformation formulae for standard bearings made of the bearing steel, that is, relative deflection expressions between two rings.

In condition of the pure axial deflection, i.e. $\delta_r = 0$, for angular contact ball bearings

$$\delta_a = \frac{0.00044}{\sin \alpha} \left(\frac{Q_0^2}{D_b} \right)^{1/3} \quad (13.3)$$

for self-aligning ball bearings

$$\delta_a = \frac{0.0007}{\sin \alpha} \left(\frac{Q_0^2}{D_b} \right)^{1/3} \quad (13.4)$$

for roller bearings with the line contact both on inner and outer raceways

$$\delta_a = \frac{0.000077}{\sin \alpha} \frac{Q_0^{0.9}}{l_e^{0.8}} \quad (13.5)$$

for thrust ball bearings

$$\delta_a = \frac{0.00052}{\sin \alpha} \left(\frac{Q_0^2}{D_b} \right)^{1/3} \quad (13.6)$$

In condition of the pure radial deflection, i.e. $\delta_a = 0$ for deep groove and angular contact ball bearings

$$\delta_r = \frac{0.00044}{\cos \alpha} \left(\frac{Q_0^2}{D_b} \right)^{1/3} \quad (13.7)$$

for self-aligning ball bearings

$$\delta_r = \frac{0.0007}{\cos \alpha} \left(\frac{Q_0^2}{D_b} \right)^{1/3} \quad (13.8)$$

for roller bearings with the line contact both on inner and outer rings

$$\delta_r = \frac{0.000077}{\cos \alpha} \frac{Q_0^{0.9}}{l_e^{0.8}} \quad (13.9)$$

For roller bearings with the point contact on one raceway and the line contact on another raceway, Palmgren gave an approximate equation to calculate the elastic deflection for rolling bearings (1).

In condition of the pure axial deflection

$$\delta_a = \frac{0.00018}{\sin \alpha} \frac{Q_0^{3/4}}{l_e^{1/2}} \quad (13.10)$$

In condition of the pure radial deflection

$$\delta_r = \frac{0.00018}{\cos \alpha} \frac{Q_0^{3/4}}{l_e^{1/2}} \quad (13.11)$$

where

Q_0 = maximum rolling element load, N

D_b = rolling element diameter, mm

l_e = effective length of a roller, mm

δ_a = elastic axial deflection, mm

δ_r = elastic radial deflection, mm.

It should be noted that for a deep groove ball bearing with zero initial contact angle, a pure axial load will cause a larger axial deflection. Its value is connected with the axial load, ball diameter, clearance, groove curvature radius, etc. It is hard to express the value by a simple formula.

As mentioned above, the bearing stiffness is an external load required to produce one unit of the relative elastic displacement between two rings of the bearing. It can be expressed as

$$\Psi = \frac{\partial F}{\partial \delta} \quad (13.12)$$

where

F = load acting on the bearing

δ = relative elastic displacement between two rings in the F direction, such as radial, axial or tilting directions.

On the basis of the load distribution in rolling bearings described in Chapter Four, substitution of the equation for the maximum rolling element load into the corresponding equations (13.3) to (13.11) yields the calculating methods for the bearing stiffness. Such as, for deep groove ball bearings, from equation (4.68)

$$Q_0 = \frac{F_r}{Z J_1(\epsilon) \cos \alpha} \quad (13.13)$$

Considering the effect of a diametral clearance, the following equation is commonly used

$$Q_0 = \frac{5F_r}{Z \cos \alpha} \quad (13.14)$$

Substituting into equation (13.7), then

$$\delta_r = \frac{0.00129 F_r^{2/3}}{Z^{2/3} D_b^{1/3} \cos^{5/3} \alpha} \quad (13.15)$$

Combining with equation (13.12) gives the radial stiffness formula for deep groove ball bearings

$$\Psi_r = 32375ZD_b^{1/2}\delta_r^{1/2}\cos^{5/2}\alpha \quad (13.16)$$

It can be seen that this radial stiffness is nonlinear to the deflection. It is possible to increase the bearing stiffness by an elastic radial pre-deflection.

In practice, axial preload is usually used to increase the bearing stiffness. A similar method can be applied to deduce an approximate formula for the bearing stiffness. Substitution of equation (4.27) into equation (13.3) gives the relation between an axial load and its corresponding relative deflection

$$\delta_a = \frac{0.00044F_a^{2/3}}{Z^{2/3}D_b^{1/3}\sin^{5/3}\alpha} \quad (13.17)$$

Therefore, the axial bearing stiffness is

$$\Psi_a = 162520ZD_b^{1/2}\delta_a^{1/2}\sin^{5/2}\alpha \quad (13.18)$$

In equations (13.17) and (13.18), the variation of a contact angle due to the action of an axial load is not considered, so these are approximate formulae. If the initial contact angle is greater than 20° , the formulae are still certainly accurate.

Similarly, for thrust ball bearings

$$\delta_a = \frac{0.00052F_a^{2/3}}{Z^{2/3}D_b^{1/3}\sin^{5/3}\alpha} \quad (13.19)$$

The axial stiffness of the bearing

$$\Psi_a = 126500ZD_b^{1/2}\delta_a^{1/2}\sin^{5/2}\alpha \quad (13.20)$$

Substituting equation (4.27) into equation (13.5) yields the similar expression for tapered roller bearings with the line contact on inner and outer raceways

$$\delta_a = \frac{0.000077F_a^{0.9}}{Z^{0.9}I_r^{0.8}\sin^{1.9}\alpha} \quad (13.21)$$

Correspondingly, the axial stiffness is

$$\Psi_a = 41335ZI_r^{8/9}\delta_a^{1/9}\sin^{19/9}\alpha \quad (13.22)$$

It can be seen that for tapered roller bearings with the line contact on inner and outer raceways, the axial deflection varies with load to the 0.9 power and the axial stiffness varies with deflection to the 0.1 power. Therefore, it can be approximately assumed that the axial deflection is proportional to load and then the axial stiffness keeps being constant. The relevant calculation will be significantly simplified.

For any given bearing, the curve exhibited the relation between a axial load and its deflection can be drawn upon equations (13.17), (13.19) or (13.21).

PRELOADING FOR ROLLING BEARINGS

As mentioned above, the purpose of preloading is to keep bearing raceways and rolling elements in a loading condition by pre-deflection between them. The preloading is one of the operating characteristics of rolling bearings. Usually, sliding bearings do not work in preloading condition. Various effects can be obtained by preloading. Those are (1) to improve the running accuracy of a bearing arrangement; (2) to increase bearing stiffness; (3) to reduce vibration and noises of the bearing; (4) to decrease the spinning sliding and orbital sliding of rolling elements.

The preloading can be divided into the axial preloading and the radial one according to the direction of the preload. Commonly, the axial preloading is widely used.

Based on the preloading methods, it may also be divided into the fixed-position preloading and fixed-pressure preloading.

Fixed-position preloading

Fixed-position preloading is one type of axial preloading which keeps the relative position of bearings constant. As shown in Fig.13.1, a certain preloading can be obtained by matched spacers.

The diagram of axial load vs bearing deflection of two identical angular contact ball bearings are mounted in a pair is shown in Fig.13.2.

The horizontal coordinate is axial deflection δ_a and the vertical coordinate is axial load F_a . Curves I and II respectively express the relations of load vs deflection of the bearings I and II. The intersec-

tion δ of two curves indicates the same axial deflection δ_{a0} to be produced for bearings I and II under the axial preload F_{a0} . An external axial load F_a produces a heavier load on bearing I and an additional axial deflection δ_a of bearing I; correspondingly, the load and deflection of bearing II are reduced.

It can be seen from Fig. 13.2, the axial deflections of bearings I and II respectively are

$$\delta_{aI} = \delta_{a0} + \delta_a$$

$$\delta_{aII} = \delta_{a0} - \delta_a$$

Correspondingly, the axial loads are

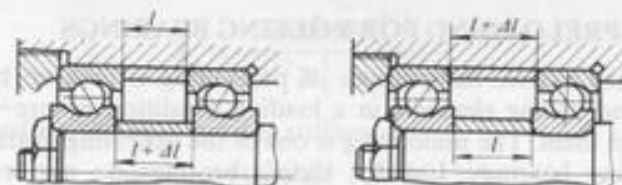


Fig. 13.1 Fixed-position preloading

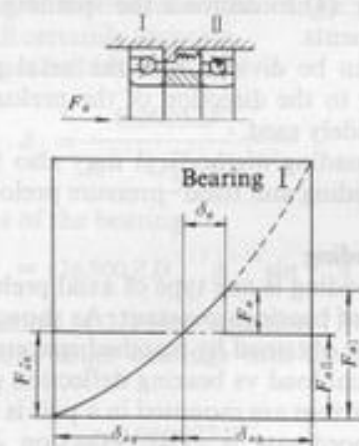


Fig. 13.2 Load and deflection in fixed-position preloading

$$F_{aI} = F_{a0} + \Delta F_{aI}$$

$$F_{aII} = F_{a0} - \Delta F_{aII}$$

From the equilibrium of forces

$$F_a = F_{aI} - F_{aII}$$

The axial deflection of the entire bearing system under the axial load F_a is only δ_a . Therefore, angular contact ball bearings mounted in a pair may increase the stiffness of an entire bearing system by preloading.

If the external load continues to increase until $\Delta F_{aII} = F_{a0}$, the axial deflection of the bearing system δ_a will equal δ_{a0} , that is, bearing II will be completely unloaded.

An external load under which one of the bearings in the system is completely unstressed is called the unload. For a pair of identical angular contact ball bearings, the unload is

$$F_{au} = 2^{3/2} F_{a0} = 2.83 F_{a0} \quad (13.23)$$

If an external load is greater than the unload, then only one of the bearings is completely subjected to the load. And thus the stiffness corresponds to that of the single bearing.

It can be seen from equation (13.22), that the axial stiffness of a tapered roller bearing varies with deflection to the 0.1 power only. If two identical tapered roller bearings are mounted in a pair, the diagram of axial load vs bearing deflection is shown in Fig. 13.3. Similar to ball

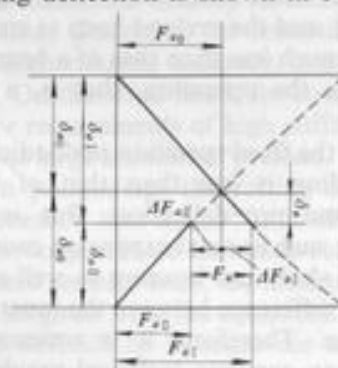


Fig. 13.3 Fixed-position preloading of tapered roller bearings

bearings, the pre-deflection under preload F_{a0} is δ_{a0} . An external axial load causes an additional deflection δ_a . Because the curves are approximately straight lines, by comparison with single tapered roller bearing, the stiffness of tapered roller bearings mounted in a pair may be increased by two times.

For a pair of identical tapered roller bearings, the unload is

$$F_{ax} = 2F_{a0} \quad (13.24)$$

Fixed-pressure preloading

Fixed-pressure preloading is one type of axial preloading which keeps the axial preload at constant in the operation. As shown in Fig.13.4, a certain preloading can be obtained by adjustment of the compressive amount of a spring.

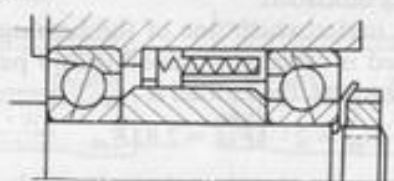


Fig. 13.4 Fixed-pressure preloading

If two identical angular contact ball bearings are mounted in a pair, the diagram of axial load vs bearing deflection is shown in Fig.13.5. The pre-deflection under preload F_{a0} is δ_{a0} . After an external axial load F_a is applied, the axial deflection of bearing I increases by δ_{a1} , while that of bearing II and the preload keep at constant. Because the stiffness of a spring is much less than that of a bearing, the preload of a spring is constant in the operation, that is, a horizontal line in Fig.13.5.

By comparison with the fixed-position preloading, the stiffness of a fixed-pressure preloading is less than that of the fixed-position preloading at the same pre-deflection. But in a fixed-position preloading, the factors, such as axial expansion caused by temperature difference between the shaft and housing as well as radial expansion caused by temperature difference between the inner and outer rings effect the pre-deflection. Therefore, it is necessary to choose the preloading method upon concrete technical requirements. Generally speaking, fixed-position preloading is suitable for high-stiffness re-

quirements and fixed-pressure preloading for high-speed applications.

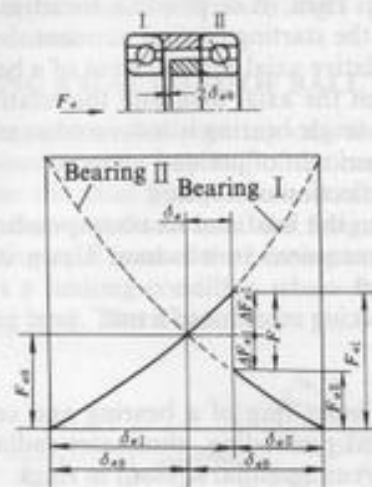


Fig. 13.5 Load and deflection in fixed-pressure preloading

Selection of preload amount

The amount of preload to a rolling bearing is limited because friction moment and operating temperature increase when the preload increases and there is a reduction in bearing life. An optimum preload is that which gives the maximum bearing stiffness for the smallest possible increase in friction. Generally, for conditions of high speed and light load or for a high running accuracy, it is better to select a light amount of preload. On the other hand, for conditions of middle-class or heavy load or for requirements of high stiffness, it is suitable to select a middle-class or large amount of preload.

In fixed-position preloading, the rolling elements and raceways should be kept in contact, that is, the "unloading" condition should be avoided. In order to guarantee that each rolling element in a bearing carries a non-zero contact load, the minimum amount of preload may be determined as given from the equations in Table 13.1.

In practice, it is difficult to measure the actual preload applied. Usually, the following methods are used for adjusting the amount of preload.

(1) To measure the starting friction moment of a bearing.

The relation between the starting friction moment and axial load is measured in advance. Then, it is possible to adjust the amount of preload by measuring the starting friction moment.

(2) To measure the relative axial displacement of a bearing.

The relation between the axial load and the relative axial displacement of two rings in a single bearing is measured in advance. Then it is possible to adjust the amount of preload upon it.

(3) To measure the deflection of a spring.

The relation between the load and its corresponding deflection of a preloading spring is measured in advance. Upon it, the amount of preload can be adjusted.

(4) To measure the locking moment of a nut.

Radial preloading

Tight fits expand the inner ring of a bearing and contract the outer ring. This, called radial preloading, eliminates radial clearances and produces radial and circumferential stresses in rings.

Table 13.1 Minimum preload in fixed-position preloading

Bearing type	Load condition	Minimum preload F_{dmin}
Angular contact ball bearings	pure axial load	$F_{dmin} = 0.35F_a$
	Combined radial and axial loads	$\begin{cases} F_{dmin} \geq 1.7F_{r1} \tan \alpha_1 - 0.5F_a \\ F_{dmin} \geq 1.7F_{r2} \tan \alpha_2 + 0.5F_a \end{cases}$ The greater of the two values
Tapered roller bearings	Pure axial load	$F_{dmin} = 0.5F_a$
	Combined radial and axial loads	$\begin{cases} F_{dmin} \geq 1.9F_{r1} \tan \alpha_1 - 0.5F_a \\ F_{dmin} \geq 1.9F_{r2} \tan \alpha_2 + 0.5F_a \end{cases}$ The greater of the two values
Symbols	F_{r1} = radial load acting on bearing I F_{r2} = radial load acting on bearing II α_1 = contact angle of bearing I α_2 = contact angle of bearing II	

The major purpose of a radial preloading is to increase the number of rolling elements in the loading region of a bearing. Therefore, the orbital sliding in a bearing, particularly high speed cylindrical roller bearing, can be reduced.

LIMITING AXIAL LOAD OF BALL BEARINGS

Commonly, a deep groove ball bearing can also be subjected to an axial load. It is necessary that any ball of the bearing should not override the land under the axial load. Otherwise, stress concentration occurs and it will cause a reduction in the fatigue life of the bearing. Upon this, a limiting axial load of the bearing can be determined.

Fig. 13.6 shows a limiting condition where the balls just touch the edge of the bearing land. Therefore

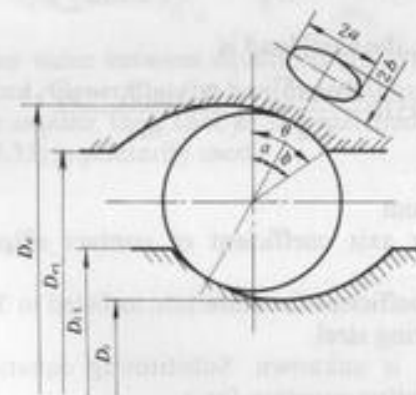


Fig. 13.6 Limiting contact condition

$$\varphi = \theta_c - a \quad (13.25)$$

$$\theta_c = \cos^{-1} \left(1 - \frac{D_e - D_{e1}}{D_b} \right) \quad (13.26)$$

where

θ_c = angle between the land edge of outer ring and the central line of outer raceway

φ = angular extent of the semi-major axis of the contact ellipse

D_e = outer raceway diameter

D_{el} = outer land diameter.

Because the contact deformation is small, an approximate equation is established

$$\sin \phi = \frac{2a}{D_b} \quad (13.27)$$

where

a = semi-major axis of the contact ellipse.

For pure axial loading conditions, combining equations (4.27), (3.31), (13.25) and (13.27) yields

$$\sin(\theta_e - \alpha) = \frac{0.0472 m_a C_E^{1/3}}{D_b} \left(\frac{F_a}{Z \sin \alpha \sum \rho} \right)^{1/3} \quad (13.28)$$

Correspondingly, the axial load is

$$F_a = 9510 \frac{Z \sin \alpha \sum \rho}{C_E} \left(\frac{D_b \sin(\theta_e - \alpha)}{m_a} \right)^3 \quad (13.29)$$

where

$\sum \rho$ = curvature sum

m_a = semi-major axis coefficient of contact ellipse, as listed in Table 3.1

C_E = modified coefficient for materials, as listed in Table 3.2, $C_E = 1$ for the bearing steel.

The value of α is unknown. Substituting equation (13.29) into (4.57) gives a calculating equation for α .

$$\sin(\theta_e - \alpha) = 0.0472 m_a C_E^{1/3} \frac{\left(\frac{\cos \alpha_0}{\cos \alpha} - 1 \right)^{1/2}}{k^{1/2} (D_b \sum \rho)^{1/3}} \quad (13.30)$$

It is necessary to iterate it numerically, using such as the Newton-Raphson method. After α is given, the limiting axial load can be determined by equation (13.29).

Similarly for the inner raceway, the limiting axial load can be determined by the following steps.

First, determining the contact angle

$$\sin(\theta_i - \alpha) = 0.0472 m_a C_E^{1/3} \frac{\left(\frac{\cos \alpha_0}{\cos \alpha} - 1 \right)^{1/2}}{k^{1/2} (D_b \sum \rho)^{1/3}} \quad (13.31)$$

where

θ_i = angle between the land edge of inner ring and the central line of inner raceway. It may be calculated

$$\theta_i = \cos^{-1} \left(1 - \frac{D_{il} - D_i}{D_b} \right) \quad (13.32)$$

And then, deciding the limiting axial load

$$F_a = 9510 \frac{Z \sin \alpha \sum \rho}{C_E} \left(\frac{D_b \sin(\theta_i - \alpha)}{m_a} \right)^3 \quad (13.33)$$

The smaller value between equations (13.29) and (13.33) is the limiting axial load. Generally, the limiting axial load corresponding to an inner ring is smaller than that of the matched outer ring. Therefore, equation (13.33) is primarily used.

CHAPTER FOURTEEN

Operating performance of rolling bearings—friction moment

Friction moment is one of the important characteristics of rolling bearings. It has a great influence on energy loss. In order to reduce the friction loss, one type of bearings, named "energy-saving bearings" is proposed. The friction moment may cause temperature increase in the operating process of bearings, or even efficacy loss of the grease, severe wear, etc. In some special conditions, the friction moment effects the accuracy of precise machinery operations and informational transportations. For example, values and variations of friction moments of miniature bearings in gyroscopes have a significant influence on the navigation deviation. Some special bearings need to be inspected for their friction moments in order to guarantee their quality.

Generally, rolling bearings have lower values of friction moments than other bearings. Friction moments of rolling bearings are connected with many influential factors including rolling friction, sliding friction and viscous fluid friction and so on. Formation mechanisms of the friction moment in bearings are explained in the first paragraph of this chapter.

From the point of view of engineering applications, it is necessary to be able to calculate the value of a bearing's friction moment if given the type and size of a bearing and operating conditions including its load and speed. Up to now, some predicting methods for friction moments of rolling bearings have been developed. These will be introduced in the second paragraph of this chapter.

Experiments indicate that the friction moments for given bearings with the same type and outline are very dispersive. Even for one bearing, the friction moment varies with its running time. Therefore, a calculated friction moment of a bearing is only an estimated value in its conventional operating conditions. It is better to measure the friction moment of a bearing using special instruments if an accurate value is required.

FORMATION MECHANISMS OF FRICTION MOMENTS IN ROLLING BEARINGS

Friction moments of rolling bearings are composed of the following aspects

- (1) pure friction caused by the elastic hysteresis of bearing material;
- (2) friction caused by micro-sliding and micro-slip at rolling contact surfaces;
- (3) friction brought about due to spinning motions of rolling elements;
- (4) pure sliding friction at some contact surfaces;
- (5) viscous friction of lubrication.

Pure rolling friction caused by elastic hysteresis of material

Elastic deformation of material below a contact surface occurs as a rolling element under loading passes on a raceway. When the loading is eliminated, a major part of the deformation is recovered. Experiments indicate that as the load increases again the deformation corresponding to a given stress is always less than the deformation at the same stress value but under a decreasing load, that is, a hysteresis loop exists. This phenomena is called an elastic hysteresis, which reflects a certain degree of energy loss and exhibits a rolling frictional resistance.

Using a simple model, one may explain a pure rolling friction caused by the elastic hysteresis. Fig.14.1 shows that a rigid cylinder rolls on an elastic half-space. As mentioned in Chapter Three, the pressure per length of the cylinder at a distance x from its contact centre is

$$p(x) = \frac{2Q}{\pi b} \left[1 - \left(\frac{x}{b} \right)^2 \right]^{1/2} \quad (14.1)$$

where

Q = contact load of the cylinder

b = half width of the contact area.

Vertical displacement of the elastic body surface at a distance x from the contact centre approximately is

$$\delta(x) = \delta_{\max} - \frac{x^2}{2R} \quad (14.2)$$

A work done by the pressure at width dx and passed forward one unit of distance is expressed as

$$p(x)dx \frac{\partial \delta(x)}{\partial x}$$

Integrating it from zero to b , a work done by the pressure on a front part of a contact area and passed forward one unit of distance is derived. Then

$$N = \int_0^b p(x) \frac{\partial \delta(x)}{\partial x} dx = \frac{2Qb}{3\pi R} \quad (14.3)$$

Substituting equation (3.29) into the above equation

$$N = \frac{Q^{3/2}}{R^{1/2}} \left[\frac{16}{9\pi^3} \left(1 - \frac{\nu^2}{E} \right) \right]^{1/2} \quad (14.4)$$

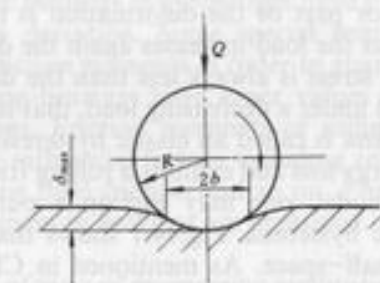


Fig. 14.1 Rolling of a rigid cylinder

Suppose that the energy loss caused by the elastic hysteresis during a rolling process is a certain ratio of the work N , that is, $\alpha_n N$. Because the work done by the friction force is equal to the energy loss, a rolling friction coefficient can be determined as follows:

$$f_R = \frac{F_R}{Q} = \frac{\alpha_n N}{Q}$$

where

F_R = friction force.

Substituting equation (14.4) into this equation yields

$$f_R = \frac{4\alpha_n}{3\pi} \left[\frac{Q}{R} \left(\frac{1-\nu^2}{\pi E} \right) \right]^{1/2}$$

where

R = radius of the cylinder.

Therefore, the rolling friction moment caused by an elastic hysteresis can be calculated by the following equation

$$M_R = f_R Q = \frac{4\alpha_n}{3\pi} Q^{3/2} \left(\frac{1-\nu^2}{\pi E R} \right)^{1/2} \quad (14.5)$$

Similarly, a rolling friction coefficient for a rigid ball rolling on an elastic body is

$$f_R = \frac{3\alpha_n}{16} \left[\frac{3Q}{4R^2} \left(\frac{1-\nu^2}{E} \right) \right]^{1/3} \quad (14.6)$$

And a corresponding rolling friction moment is

$$M_R = \frac{3\alpha_n}{16} Q^{4/3} \left[\frac{3}{4R^2} \left(\frac{1-\nu^2}{E} \right) \right]^{1/3} \quad (14.7)$$

It can be seen that this type of rolling friction moment is connected with the contact load, the material constants, the radius of a rolling element, the energy loss percent, etc. A rolling friction moment is proportional to the $3/2$ power of a contact load in line contact, while it is proportional to the $4/3$ power of a contact load in point contact.

Furthermore, considering that a rigid ball rolls along a raceway, one may deduce an expression regarding the pure rolling friction moment due to elastic hysteresis as follows:

$$\begin{aligned} M_R &= \int_{-a}^a \int_0^{\sqrt{1-\frac{y^2}{a^2}}} 2\alpha_n p_0 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^{1/2} y dy dx \\ &= \frac{3}{8} \alpha_n b Q \end{aligned} \quad (14.8)$$

An interesting characteristic is that the friction moment is still proportional to the $4/3$ power of a contact load.

Friction caused by micro-sliding

A micro-sliding at the contact area between a ball and inner raceway in a deep groove ball bearing under radial load is considered, as shown in Fig. 14.2. The linear velocity of an inner raceway surface is proportional to the distance from the axis. The velocity v_i varies in the axial direction. Similarly, the linear velocity v_b of a ball is also proportional to the distance from its own rotational axis. Then, it can be seen that pure rolling only occurs at two points and sliding at central and side regions of the contact area will take place in different directions. This is called micro-sliding.

Supposing that the tangential force at any unit area of contact surface varies directly as the contact pressure, Reference (2) gives an expression of the friction moment caused by the micro-sliding between a ball and raceway.

Experiments point out that friction caused by the micro-sliding is larger than that by the elastic hysteresis if a relative difference between the radii of the ball and the raceway groove is less than 20%.

Friction caused by spinning sliding

As mentioned in the second paragraph of Chapter Five, a ball may have a rotational motion normal to the contact surface. Sliding caused by this motion is called spinning sliding.

Its corresponding friction moment is given by equation (5.45)

$$M_s = \frac{3}{8} \mu_s Q a L(e) \quad (5.45)$$

Because the vector of the friction moment is in the direction normal to the contact area, its axial component exhibits as a friction moment of the bearing, i.e.

$$M_s \sin \alpha = \frac{3}{8} \mu_s Q a L(e) \sin \alpha \quad (14.9)$$

Considering $a \propto Q^{1/3}$, the friction moment caused by spinning sliding is also proportional to the $4/3$ power of contact load.

Spinning sliding may take place in thrust roller or thrust needle bearings, as shown in Fig. 14.3. Supposing that two rings rotate in opposite directions, one may express linear velocities of a roller and raceway as in the figure. It can be seen that the roller will rotate in the direction normal to the contact area, that is, spinning sliding occurs.

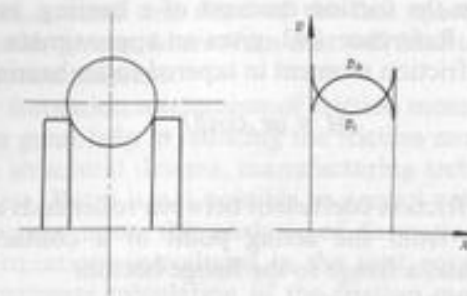


Fig. 14.2 Micro-sliding



Fig. 14.3 Spinning sliding in roller bearing

Pure sliding friction at some contact surfaces

In rolling bearings, pure sliding friction occurs at some contact surfaces, such as sliding between rolling elements and cage pockets, or between a cage and its guiding surface of a ring, as well as between roller ends and inner and outer ring flanges.

Sliding friction between rolling elements and cage pockets is hardly calculated. And sliding friction between a cage and its guiding surface of a ring may be estimated according to the hydrodynamic lubrication theory of a short journal bearing. For a properly balanced cage with a small running eccentricity, Petroff's equation for two concentric hollow cylinders filled with viscous fluid can be applied.

In addition, sliding friction between roller ends and ring flanges is not negligent in the friction moment of a bearing, especially tapered roller bearings. Reference (4) gives an approximate formula for calculation of the friction moment in tapered roller bearings.

$$M = \mu e_f \cos \beta F_a \quad (14.10)$$

where

- μ = sliding friction coefficient between roller ends and ring flanges
- e_f = distance from the acting point of a contact load between a roller and a flange to the flange bottom
- β = half taper of a roller
- F_a = axial load of a bearing.

Viscous friction of lubricants

Lubricants and lubricating methods have an important influence on friction moments of bearings. Due to the action of lubricants, an elastohydrodynamic lubrication film may be formed between rolling elements and raceways, and friction coefficients between sliding contact surfaces may vary. During the rotation of a bearing, each rolling element is subjected to a viscous drag force caused by a gaseous mixture. Reference (100) proposes to estimate the viscous drag force on each rolling element by the following equations.

For ball bearings

$$F = \frac{\pi}{32} C_a \rho (D_s D_m \omega_m)^2 \quad (14.11)$$

For roller bearings

$$F = \frac{1}{8} C_a \rho D_s l (D_m \omega_m)^2 \quad (14.12)$$

where

- ρ = density of gaseous mixture
- C_a = drag coefficient, connected with lubricant properties and rolling element sizes.

Grease is widely used to lubricate rolling bearings under conventional operating conditions. In a grease-lubricated bearing, ratios of various influential factors contributing to the bearing friction moment are approximately listed in Table 14.1. Friction moments of

grease-lubricated bearings are connected with the type and amount of grease. In general, around one third of the free space in a bearing should be filled with grease. More than that will lead to a significant increase in the friction moment.

Analysing the formation mechanism of friction moments in rolling bearings can be a great help in reducing the friction moment through improvement on structural designs, manufacturing technology, lubricating methods, etc. But it is not possible to predict values of friction moments of bearings upon the analysis of formation mechanism. Some empirical equations introduced in the next paragraph can be applied for approximate calculation of the friction moments for the bearing selection.

Table 14.1 Composition of friction moment factors

Item	Percent %
Rolling and sliding friction in bearings	20–30
Grease drag resistance	50–60
Seal friction	30–10

CALCULATION OF FRICTION MOMENT IN ROLLING BEARINGS

Approximate calculation

For different types of rolling bearings, their friction moment may be approximately calculated by the following equation

$$M = \frac{1}{2} \mu d F \quad (14.13)$$

where

- μ = friction coefficient of bearings, as listed in Table 14.2
- d = bore diameter of a bearing
- F = load acting on a bearing, radial load refers to radial bearings and axial load refers to thrust bearings.

Friction coefficients given in Table 14.2 are valued in certain ranges. Higher values of the coefficient are chosen for $F/C < 0.05$ and lower

values for $F/C > 0.1$. It can be seen that it is better to select deep groove ball bearings, self-aligning ball bearings or cylindrical roller bearings if lower friction moments of bearings are required.

Table 14.2 Friction coefficients of rolling bearings

Types of bearings	μ
Deep groove ball bearings	0.0015–0.0022
Self-aligning ball bearings	0.0010–0.0018
Cylindrical roller bearings	0.0011–0.0022
Spherical roller bearings	0.0018–0.0025
Needle roller bearings	0.0025–0.0040
Angular contact ball bearings	0.0018–0.0025
Tapered roller bearings	0.0018–0.0028
Thrust ball bearings	0.0013–0.0020
Spherical roller thrust bearings	0.0018–0.0030

When a bearing starts to rotate, its friction moment at the initial stage may be higher than the value calculated from equation (14.13).

More accurate calculation

Based on measurement results of bearing friction moments, Palmgren proposed the following formula for more accurate calculation (94)

$$M = M_0 + M_1 \quad (14.14)$$

where

M_0 = friction moment connected with bearing types, speeds and properties of lubricants, named load-independent friction moment, N · mm

M_1 = friction moment connected with loads acting on bearings, named load-dependent friction moment, N · mm.

Calculation of M_0

M_0 results from hydrodynamic losses of a lubricant and depends on the viscosity and quantity of the lubricant, as well as the rolling velocity. It can be calculated

$$\begin{aligned} \text{if } vn > 2000 \quad M_0 &= 10^{-7} f_0 (vn)^{2/3} D_m^3 \\ \text{or} \\ \text{if } vn < 2000 \quad M_0 &= 160 \times 10^{-7} f_0 D_m^3 \end{aligned} \quad (14.15)$$

where

D_m = pitch diameter of a bearing, mm

f_0 = factor dependent on bearing design and lubricating methods as listed in Table 14.3

n = bearing speed, rpm

v = kinematic viscosity of lubricant at the operating temperature (for grease, viscosity of the base oil should be adopted), mm^2/s .

Lower values are for light series bearings, and higher values for heavy series bearings.

Calculation of M_1

M_1 results from the elastic hysteresis of the material and the micro-sliding in a bearing, and depends on the load acting on the bearing. It can be calculated by

$$M_1 = f_1 P_1 D_m \quad (14.16)$$

where

f_1 = factor dependent on the bearing type and the load, as listed in Table 14.4

P_1 = calculating load which determines the friction moment, N.

In Table 14.4, P_0 is the equivalent static load, in Newton. C_0 is the static load rating of the bearing, in Newton. Y is the axial load factor when $F_a/F_r > e$.

Calculation of M_2

If cylindrical roller bearings are subjected to an axial load and a radial load, an additional moment should be included. Therefore, the total friction moment of the bearings is

$$M = M_0 + M_1 + M_2 \quad (14.17)$$

in which

$$M_2 = f_2 F_a D_m \quad (14.18)$$

where

Table 14.3 Values of f_0

Types of bearings	Oil mist lubrication	Oil bath or grease	Oil bath (vertical shaft), oil jet
Deep groove ball bearings	0.7-1	1.5-2	3-4
Self-aligning ball bearings	0.7-1	1.5-2	3-4
Angular contact ball bearings			
single row	1	2	4
double row	2	4	8
Cylindrical roller bearings			
with cage	1-1.5	2-3	4-6
full complement	-	2.5-4	-
Spherical roller bearings	2-3	4-6	8-12
Tapered roller bearings	1.5-2	3-4	6-8
Thrust ball bearings	0.7-1	1.5-2	3-4
Cylinder roller thrust bearings	-	2.5	5
Spherical roller thrust bearings	-	3-4	6-8

Table 14.4 Calculation of f_1 and P_1

Types of bearings	F_1	P_1^*
Deep groove ball bearings	$0.0009(P_0 / C_0)^{0.55}$	$3F_a - 0.1F_r$
Self-aligning ball bearings	$0.0003(P_0 / C_0)^{0.4}$	$1.4YF_a - 0.1F_r$
Angular contact ball bearings		
single row	$0.0013(P_0 / C_0)^{0.33}$	$F_a - 0.1F_r$
double row	$0.001(P_0 / C_0)^{0.33}$	$1.4F_a - 0.1F_r$
Cylindrical roller bearings		
with cage	0.00025-0.0003**	F_r^{***}
full complement	0.00045	F_r^{***}
Spherical roller bearings	0.0004-0.0005**	$1.2YF_a$
Tapered roller bearings	0.0004-0.0005**	$2YF_a$
Thrust ball bearings	$0.0012(P_0 / C_0)^{0.33}$	F_a
Cylinder roller thrust bearings	0.0018	F_a
Spherical roller thrust bearings	0.0005-0.0006**	$F_a (F_{rmax} \leq 0.55F_a)$

- notes: * if $P_1 < F_r$, use $P_1 = F_r$
 ** the smaller values are for light series bearings and the larger for heavy series bearings
 *** for cylindrical roller bearings subjected to an axial load, it is needed to consider an additional moment M_2 , as shown by equation (14.18)

f_2 = factor dependent on bearing design and lubricating methods, as shown in Table 14.5.

Table 14.5 Values of f_2 for cylindrical roller bearings

Types of bearings	Oil lubrication	Grease lubrication
With cage	0.006	0.009
Full complement	0.003	0.006

Values of f_2 are valid for a viscosity ratio $K_v = 1.5$ seen in equation (8.63) and F_a / F_r must not exceed 0.4.

CHAPTER FIFTEEN

Optimization of design for rolling bearings

The design of rolling bearings has a fundamental influence on the performance, life and reliability of the bearings. Consequently, this also affects the operating quality and the economization of machines on which the bearings are used. The responsibility of bearing designers is to select an optimal scheme from all the possible alternative designs, or simply put to make the best choice.

A general procedure of a bearing design is shown in Fig.5.1. At first, a proposal of a selection of bearing types is put forward upon technical requirements for the bearing and relevant design standards, as well as testing data. Secondly, a numerical computing is made according to a certain mathematical model. Then, the computed result is examined to see whether all the requirements can be met by it. If they can, the design is carried onto the next step; if not, modifications and computations are made again until the requirements are fulfilled. Thirdly, a structural design is carried out to decide all sizes and parameters in the bearing. Finally, a detailed drawing of the bearing is laid out and necessary technical documents are worked out. Of course, in a broad sense, the bearing design has to be checked through manufacturing and testing. By way of explanation for design methods, the step shown in Fig.15.1 is a complete designing procedure.

Obviously, the best design scheme should have optimal technical and economic indexes. For this purpose, various methods are adopted. A method of intuitional optimization means selecting the best from several designing schemes. And a method of testing optimization denotes modifying a design by sample tests gradually in order to seek the best one. Also a method of evolving optimization represents the renewal of a design for improvement upon comments of customers or users. By comparison with the above, a method of mathematical optimization has advantages of strict timesaving and economic characteristics. Developments of computer technology have been of great help to the mathematical optimization. Computer-aided design (CAD) is a dynamic, new field of engineering science. In

fact, the mathematical optimization is the base and the core of the CAD.

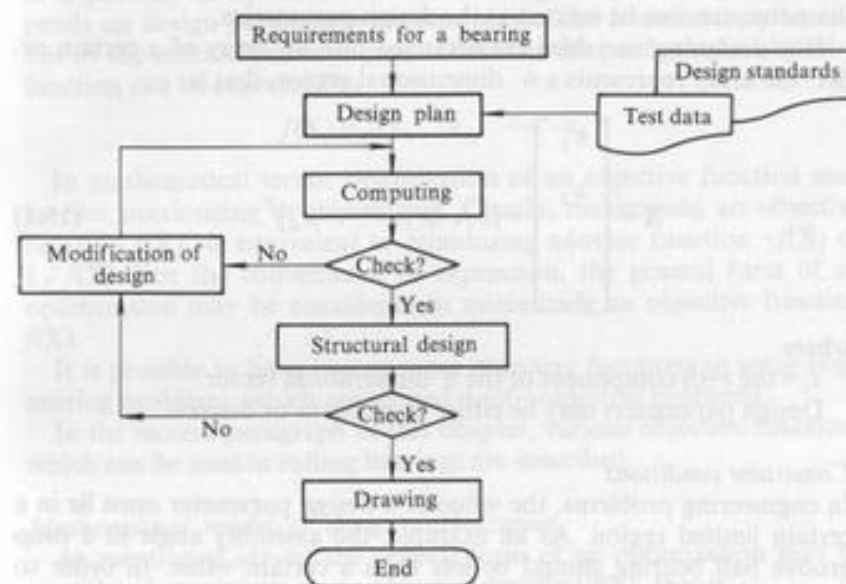


Fig. 15.1 A flow chart for a bearing design

In this chapter, design methods of the mathematical optimization for rolling bearings are described. Basic concepts and solving methods of the optimization are introduced in the first paragraph. Then, various objective functions in the bearing design are proposed and their practical applications in rolling bearings are given in the following paragraphs.

BASIC CONCEPTS OF MATHEMATICAL OPTIMIZATION OF DESIGN

General mathematical form of design optimization

Design parameters

It is often necessary to decide some variables during a designing process. The mutually independent variables which must be determined in

a design are called design parameters. The design of a product is completely decided as the parameters are given. For example, in the design of a rolling bearing, diameter and number of rolling elements, pitch diameter, etc. can be treated as the design parameters.

If n designing variables are arranged into an array of a certain order, the array represents a n dimensional vector, that is

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = [x_1, x_2, \dots, x_n]^T \quad (15.1)$$

where

x_i = the i -th component of the n dimensional vector.

Design parameters may be either continuous or discrete.

Constraint conditions

In engineering problems, the value of a design parameter must lie in a certain limited region. As an example, the assembly angle in a deep groove ball bearing should be less than a certain value. In order to complete the formulation of the optimization problem, some restrictions must be imposed on the values of the design parameters. These restrictions are called constraint conditions, which can be divided into equality constraints and inequality constraints. The mathematical expressions of the two groups of constraint conditions respectively are

$$h_j(\mathbf{X}) = h_j(x_1, x_2, \dots, x_n) = 0 \quad j = 1, 2, \dots, n_h \quad (15.2)$$

$$g_l(\mathbf{X}) = g_l(x_1, x_2, \dots, x_n) \geq 0 \quad l = 1, 2, \dots, n_g \quad (15.3)$$

A surface given by $g_l(\mathbf{X}) = 0$ divides the space into two parts. In one of them, the constraint condition $g_l(\mathbf{X}) > 0$ is satisfied and it is not satisfied in the other. Any point $\mathbf{X}$ that satisfies all the inequality constraints (15.3) is said to be feasible. The set of all feasible points is termed the feasible region. In reverse, the set of all points other than feasible points is said to be the nonfeasible region. Obviously, a design scheme whose parameters are given in a nonfeasible region cannot be accepted.

Objective function

A function which is applied to evaluate "goodness" of design schemes of a product in optimization is called objective function which depends on design parameters in some ways. The function may be any one of the technical or economic indexes of the product. An objective function can be expressed by

$$f(\mathbf{X}) = f(x_1, x_2, \dots, x_n)$$

In mathematical terms, optimization of an objective function may involve maximizing or minimizing. Clearly, maximizing an objective function $f(\mathbf{X})$ is equivalent to minimizing another function $-f(\mathbf{X})$ or $1/f(\mathbf{X})$. For the convenience of expression, the general form of an optimization may be considered as minimizing an objective function $f(\mathbf{X})$.

It is possible to have two or more objective functions in some engineering problems, which are termed multi-objective problems.

In the second paragraph of this chapter, various objective functions which can be used in rolling bearings are described.

Mathematical model of design optimization

As mentioned above, the general form of an optimization may be considered as minimizing an objective function $f(\mathbf{X})$, that is

$$\min f(\mathbf{X}) \quad \mathbf{X} \in R^n \quad (15.4)$$

subject to

$$h_j(\mathbf{X}) = 0 \quad j = 1, 2, \dots, n_h$$

$$g_l(\mathbf{X}) \geq 0 \quad l = 1, 2, \dots, n_g$$

where, R^n expresses n dimensional Euclidean space.

Any point that satisfies equation (15.4) and its constraint conditions is called an optimum point. Of course, it may be either a local optimum point or a global optimum point. From the engineering point of view, the global optimum point is usually needed. Therefore, one may solve an optimization problem by starting from different initial values of design parameters and examining whether the same global optimum point is converged to.

In the design of rolling bearings, the number of design parameters generally does not exceed ten.

Unconstrained optimization methods

An unconstrained optimization problem is to determine an n dimensional design parameter $\mathbf{X}$ without any constraint to it in order for the value of objective function $f(\mathbf{X})$ to be minimum.

Of course, constraint conditions often exist in engineering problems. But unconstrained optimization problems are easy to solve and any constrained optimization problem can be transformed into an unconstrained one. Various standard computing programs are proposed for unconstrained optimization problems (95), (96).

Mathematically, solving a set of nonlinear equations

$$\nabla f(\mathbf{X}) = 0$$

one may obtain a solution of the minimum problem. Then, the solution should be judged by sufficient conditions. However, it usually is more difficult to solve a set of nonlinear equations like this. Different search methods are often applied. Considering the characteristics of an objective function $f(\mathbf{X})$, a special iterative process is adopted. It seems reasonable to decrease $f(\mathbf{X})$ at every iteration until an optimum point is obtained. The main steps are

- (1) to select an initial point $\mathbf{X}^{(0)}$ nearer to the minimum point;
- (2) to choose a search direction at iterated point $\mathbf{X}^{(k)}$ ($k=0, 1, 2, \dots$) for reduction of the function values;
- (3) to search in the direction $\mathbf{S}^{(k)}$ and to obtain a new design point $\mathbf{X}^{(k+1)}$ for satisfaction of the inequality $f(\mathbf{X}^{(k+1)}) < f(\mathbf{X}^{(k)})$;
- (4) to check whether the following inequality

$$\left| \frac{f(\mathbf{X}^{(k+1)}) - f(\mathbf{X}^{(k)})}{f(\mathbf{X}^{(k)})} \right| \leq \varepsilon$$

is valid at the new design point $\mathbf{X}^{(k+1)}$, where ε is a given small value determining accuracy. If it is, the point $\mathbf{X}^{(k+1)}$ would be the approximate minimum point, otherwise a continuous search is required.

Unconstrained optimization methods include Newton's method, variable-metric method, conjugate-gradient method, conjugate-direction method, etc. The Powell's method applied for the design optimization of rolling bearings belongs to the conjugate-direction method. Standard programs in FORTRAN language for this are described in Reference (96).

Constrained optimization methods

Design optimization of rolling bearings is constrained. It can be solved by direct search methods or by transforming it into an unconstrained problem. The direct search method is based on function comparison techniques. It is suitable for problems in which design parameters are discrete variables. But its disadvantage in general cases is that few guarantees can be made concerning convergence. Primal methods applied for the design optimization of rolling bearings are indirect methods, in which the optimization problem is transformed into an unconstrained problem and then an appropriate standard program is used for the unconstrained problem.

Penalty function methods can be effectively applied for a variety of engineering problems. The basic concept of the methods is to combine an objective function and its corresponding constrained conditions into a new function, termed penalty function, and the original problem is transformed into an unconstrained optimization of the following objective function

$$\min \Phi(\mathbf{X}, r_1^{(k)}, r_2^{(k)}) = \min \left\{ f(\mathbf{X}) + r_1^{(k)} \sum_{j=1}^{n_g} G[g_j(\mathbf{X})] + r_2^{(k)} \sum_{j=1}^{n_h} H[h_j(\mathbf{X})] \right\}$$

When a point $\mathbf{X}$ does not satisfy constraints, values of terms $r_1^{(k)} G[g_j(\mathbf{X})]$ and $r_2^{(k)} H[h_j(\mathbf{X})]$ would be larger, therefore a "penalty" is imposed on the function $\Phi(\mathbf{X}, r_1^{(k)}, r_2^{(k)})$. Only when at the optimum point constraints are satisfied, $\Phi(\mathbf{X}, r_1^{(k)}, r_2^{(k)})$ may reach its optimum value. $r_1^{(k)}$ and $r_2^{(k)}$ are called penalty parameters. $r_1^{(k)}$ and $r_2^{(k)}$ may be progressively increased (or progressively decreased) sequences as the search number k increases. On the basis of search points inside or outside the feasible region, the methods are termed internal or external point methods.

Functions $G(g)$ and $H(h)$ are a certain form of composite functions which are composed of inequality constraint functions $g_j(\mathbf{X})$ and equality constraint functions $h_j(\mathbf{X})$. For example, a penalty function for internal point methods may be

$$\Phi(\mathbf{X}, r^{(k)}) = f(\mathbf{X}) + \frac{1}{\sqrt{r^{(k)}}} \sum_{j=1}^{n_g} [h_j(\mathbf{X})]^2 + r^{(k)} \sum_{i=1}^{n_g} \frac{1}{g_i(\mathbf{X})} \quad (15.5)$$

where

$r^{(k)}$ = penalty parameters, progressively decrease positive sequence, i.e.

$$r^{(1)} > r^{(2)} > \dots > 0$$

The second and third terms on the right side of equation (15.5) are named penalty terms. The third term also has another name, barrier term. Values of the third term approach infinity as search points reach constraint boundaries. In this way, the search points cannot jump over the boundaries.

A penalty function for external point methods is

$$\Phi(\mathbf{X}, r^{(k)}) = f(\mathbf{X}) + r^{(k)} \sum_{j=1}^{n_g} [h_j(\mathbf{X})]^2 + r^{(k)} \sum_{i=1}^{n_g} [\min(g_i(\mathbf{X}), 0)]^2 \quad (15.6)$$

where

$r^{(k)}$ = penalty parameters, progressively increase positive sequence, i.e.

$$0 < r^{(1)} < r^{(2)} < \dots < r^{(k)}$$

$$\min(g_i(\mathbf{X}), 0) = \begin{cases} g_i(\mathbf{X}) & \text{(when outside} \\ & \text{feasible region)} \\ 0 & \text{(when in feasible} \\ & \text{region)} \end{cases} \quad (15.7)$$

The second and third terms on the right side of equation (15.6) are named penalty terms. Values of both the two terms are more than zero when search points are not in the feasible region.

The computing steps of internal or external point methods are as follows

- (1) to select an initial penalty parameter $r^{(0)}$, such as $r^{(0)} = 1$;
- (2) to choose an initial point $\mathbf{X}^{(0)}$ inside or outside the feasible region respectively and to set $k = 1$;
- (3) to start from point $\mathbf{X}^{(k-1)}$ and to obtain an optimum solution

$\mathbf{X}^{(k)}$ by the optimization of function $\Phi(\mathbf{X}, r^{(k)})$;

- (4) to check its accuracy, that is, to examine whether inequalities

$$|f(\mathbf{X}^{(k)}) - f(\mathbf{X}^{(k-1)})| < \varepsilon_1$$

and

$$|\mathbf{X}^{(k)} - \mathbf{X}^{(k-1)}| < \varepsilon_2$$

are valid for given small positive values ε_1 and ε_2 . If they are, $\mathbf{X}^{(k)}$ would be the optimum solution, otherwise a continuous search is required;

- (5) to set $r^{(k+1)} = \beta_1 r^{(k)}$, $\beta_1 = 0.1-0.5$ for internal point method and $\beta_1 = 5-10$ for external point method, and to let $k = k+1$ turn to step three.

Because the optimization of function $\Phi(\mathbf{X}, r^{(k)})$ belongs to unconstrained problems, standard programs which are presented in References (95), (96) can be used.

OBJECTIVE FUNCTIONS OF DESIGN OPTIMIZATION FOR ROLLING BEARINGS

On the basis of operating requirements, different objective functions for rolling bearings may be proposed.

Requirement for the longest fatigue life

As mentioned in Chapter Seven, in normal operating conditions of rolling bearings, the main mode of failure is contact fatigue. If without a special note, the bearing life means contact fatigue life. Therefore, a basic requirement for conventional rolling bearings is the longest fatigue life. It can be derived from equation (8.35) that for given sizes of the bearing outline, the dynamic load rating C should be the maximum.

For convenience of application of existing programs, a negative value or reciprocal of dynamic load rating C may be taken as an objective function. Therefore, the above requirement leads to a minimization problem, that is

$$\min f(\mathbf{X}) = \min[-C]$$

For example, for deep groove ball bearings

$$\min f(\mathbf{X}) = \begin{cases} \min[-f_e Z^{2/3} D_b^{1/3}] & D_b \leq 25.4 \text{ mm} \\ \min[-3.647 f_e Z^{2/3} D_b^{1/4}] & D_b > 25.4 \text{ mm} \end{cases} \quad (15.8)$$

where

$$f_e = 37.91 \left\{ 1 + \left[1.04 \left(\frac{1-\gamma}{1+\gamma} \right)^{1.72} \left(\frac{f_i(2f_e-1)}{f_e(2f_i-1)} \right)^{0.41} \right]^{10/3} \right\}^{-0.3} \\ \times \left[\frac{\gamma^{0.3}(1-\gamma)^{1.39}}{(1+\gamma)^{1/3}} \right] \left[\frac{2f_i}{2f_e-1} \right]^{0.41} \quad (15.9)$$

For cylindrical roller bearings

$$\min f(\mathbf{X}) = \min[-f_e I_e^{7/2} Z^{3/4} D_b^{29/27}] \quad (15.10)$$

where

$$f_e = 207.9 \lambda v \gamma^{2/9} \frac{(1-\gamma)^{29/27}}{(1+\gamma)^{1/4}} \\ \times \left\{ 1 + \left[1.04 \left(\frac{1-\gamma}{1+\gamma} \right)^{143/108} \right]^{9/2} \right\}^{-2/9} \quad (15.11)$$

Requirement for the longest wear life

Experiments indicate that for miniature bearings and spindle bearings of precision machine tools, etc., their accuracy and performance decrease gradually due to wear at rolling surfaces of the bearings. A requirement for these types of bearings is the least amount of wear. It can be thought that if a film thickness parameter Λ in a bearing increases, the wear in the bearing will markedly be reduced. Therefore the optimization problem for these types of bearings is that for given sizes of the bearing outline and operating conditions, the film thickness should be the maximum.

Similarly, taking a negative value or reciprocal of a film thickness as an objective function leads to a minimization problem, that is

$$\min f(\mathbf{X}) = \min[-h] \quad (15.12)$$

For example, for deep groove or angular contact ball bearings, using equation (6.32) yields

$$-h_{\min} = -3.63 \alpha_1^{0.49} R_x^{0.466} E_0^{-0.117} Q^{-0.073} [\pi n_i D_m \eta_0 \\ \times (1-\gamma^2)/120]^{0.68} \times [1 - \exp(-0.703(R_y/R_x)^{0.636})] \quad (15.12a)$$

For cylindrical roller bearings, using equation (6.44) yields

$$-h_{\min} = -0.154 \alpha_1^{0.54} (\eta_0 n_i)^{0.7} D_b^{0.43} D_m^{0.7} \\ \times (1-\gamma)^{1.13} (1+\gamma)^{0.7} \frac{E_0^{-0.03}}{q^{0.13}} \quad (15.12b)$$

Requirement for the lowest friction moment

Friction moment is an important dynamic performance of rolling bearings. In some special conditions, such as frame bearings for gyroscopes, energy-saving bearings in artificial satellites, etc., the friction moment becomes the first important requirement of the bearings. It can be seen that the optimization problem for these types of bearings is the minimum of the friction moment, that is

$$\min f(\mathbf{X}) = \min(M)$$

But from Chapter Fourteen, the friction moment of rolling bearings is formed upon spinning, micro-slip, elastic hysteresis, etc. It is more difficult to give an accuracy expression for these. Equation (14.14) may be used for calculation of the friction moment of a bearing, but it does not involve internal geometric parameters of the bearing. In Reference [97], the friction moment is taken as an objective function whose expression is

$$\min f(\mathbf{X}) = \min(M_E + M_D + M_S) \quad (15.13)$$

where

M_E = friction moment caused by elastic hysteresis at the contact between rolling elements and raceways

M_D = friction moment caused by micro-slip at the contact between rolling elements and raceways

M_S = friction moment caused by spinning at the contact between

rolling elements and raceways.

Requirement for the smallest value of spin to roll ratio

As mentioned in Chapter Five, the spin to roll ratio indicates the relative rolling and sliding condition of balls on raceways. The higher the value of a spin to roll ratio, the more severe the wear and frictional heat will be. Obviously, the best design of high speed ball bearings should have the ratio as small as possible. From the raceway control theory, balls are controlled by the outer raceway at high speed, and spinning of balls occurs on the inner raceway. Therefore, a spin to roll ratio at contact between balls and inner raceway may be taken as an objective function. Its optimization is

$$\min f(X) = \min \left[\left(\frac{\omega_{sl}}{\omega_{roll}} \right)_i \right] \quad (15.14)$$

From equation (5.51)

$$\min f(X) = \min [(1 - \gamma' \cos \alpha_i) \operatorname{tg}(\alpha_i - \beta) + \gamma' \sin \alpha_i] \quad (15.15)$$

It is more complicated to find the solution. Because contact angles α_i and α_o at inner and outer raceway are connected with internal structural parameters and operating conditions of a bearing, equation (15.15) should be simultaneously solved with a set of equations in the raceway control theory.

In engineering applications, several groups of internal structural parameters of a bearing are chosen in advance. Then, the value of spin to roll ratio for each group of parameters is computed and the smallest among them is selected. Obviously, the parameters selected are a better design, but not the best one.

It is different from the optimization of fatigue life. The minimization of spin to roll ratio is closely connected with operating conditions. An optimum solution in a certain condition possibly is not an optimum solution in another condition.

Requirement for the maximum of static load rating

Some bearings, such as extra-large bearings, control bearings in airplanes, etc., work at low speed. Their basic technical requirement is a support capacity for static loads. For given sizes of the bearing outline, the static load rating C_0 should be the maximum.

A negative value or reciprocal of a static load rating may be taken as an objective function, that is

$$\min f(X) = \min(-C_0) \quad (15.16)$$

For example, for deep groove and angular contact ball bearings

$$\min f(X) = \min(-f_0 i Z D_b^2 \cos \alpha) \quad (15.17)$$

where, f_0 is substituted by equation (10.19).

For cylindrical and tapered roller bearings

$$\min f(X) = \min[-44(1 - \gamma) i Z l_r D_b \cos \alpha] \quad (15.18)$$

Multi-objective functions

In some cases, more than one technical requirement is demanded. For example, at the same time, the static load rating and the dynamic load rating of a bearing are required to be maximum, or the maximum dynamic load rating and the minimum friction moment are needed, etc. The optimization problems which require several technical indexes to be optimum simultaneously are called optimization of multi-objective functions. In general, that several objectives synchronistically reach their optimum values is impossible. A certain compromise of the several objectives is arranged in order to obtain a better acceptable design for every objective. Of course, this kind of optimization solution possibly is not the only one. It is effected by optimization methods and subjective judgements of a designer.

Two treatment methods for design optimization of rolling bearings are introduced.

Primary objective method

Several technical and economic requirements, that is, objective functions for a bearing are set one by one in order of their importance. Then an optimization of each of the objective functions is solved successively.

For the first objective function, the optimum function value is

$$f_1^* = \min f_1(X)$$

Then, the second objective function is optimized with an additional constraint

$$f_1(\mathbf{X}) - (f_1^* + \Delta_1) \leq 0 \quad (15.19)$$

where

Δ_1 = permissible relaxed value of the first objective function.

The same procedures are made until the last optimization is completed.

Weighted combination method

Suppose that there are m objective functions $f_1(\mathbf{X}), f_2(\mathbf{X}), \dots, f_m(\mathbf{X})$ in a design optimization of a rolling bearing and estimates of upper limits and lower limits for variations of each objective function are expressed as

$$a_i \leq f_i(\mathbf{X}) \leq b_i \quad i = 1, 2, \dots, m \quad (15.20)$$

then a set of weighted factors w_i ($i = 1, 2, \dots, m$) may be chosen and the following equation is valid

$$\sum_{i=1}^m w_i = 1 \quad (15.21)$$

Obviously, weight factor w_i indicates a relative importance of the i -th objective function in the design optimization. If $w_i = 1/m$, they are uniform weight factors.

Structuring a unified objective function

$$f(\mathbf{X}) = \sum_{i=1}^m w_i \frac{f_i(\mathbf{X}) - a_i}{b_i - a_i} \quad (15.22)$$

one may transform an original multi-objective problem into the optimization of the single-objective function.

APPLICATIONS OF OPTIMIZATION METHODS IN BEARING DESIGN

Usually, bearings designed for applications in special operating conditions including rotational speeds and loads are called special bearings, which will be explained in next two chapters. Bearings designed for general applications are called general bearings or standard bearings. In this paragraph, the design optimization for general bearings is described. Its objective function is the dynamic load rating of bearings.

Design optimization for deep groove ball bearings

Objective function

The objective function for deep groove ball bearings is expressed as equations (15.8) and (15.9).

Design parameters

Design parameters are internal structural sizes and other variables, called main parameters, which need to be determined in the bearing design.

Consider five design parameters, that is

$$\mathbf{X} = [D_b, Z, D_m, f_e, f_i]^T \quad (15.23)$$

Because $\gamma = D_b / D_m$ is not independent, it is not listed in the design parameters.

In some cases, given $f_e = 0.53$ and $f_i = 0.52$, the number of the design parameters reduces to three.

Constraint conditions

For the convenience of the bearing assembly, number and diameter of balls should satisfy the following requirement, i.e.

$$2(Z-1)\arcsin \frac{D_b}{D_m} \leq \varphi_0 \quad (15.24)$$

Therefore, a constraint condition

$$g_1(\mathbf{X}) = \frac{\varphi_0}{2\sin^{-1}(D_b/D_m)} - Z + 1 \geq 0 \quad (15.25)$$

where

φ_0 = maximum tolerable assemble angle.

Diameter of balls should be within an empirical arrangement, that is

$$K_{Dmin} \frac{D-d}{2} \leq D_b \leq K_{Dmax} \frac{D-d}{2} \quad (15.26)$$

where

K_{Dmin} and K_{Dmax} = minimum and maximum values of ball diameter, empirical constants which are connected with diametral series of bearings

D and d = outside and bore diameters of bearings.

Then, corresponding constraint conditions

$$g_2(\mathbf{X}) = 2D_b - K_{D_{\min}}(D - d) \geq 0 \quad (15.27)$$

$$g_3(\mathbf{X}) = K_{D_{\max}}(D - d) - 2D_b \geq 0 \quad (15.28)$$

In order to guarantee the running mobility of bearings, the difference between the pitch diameter and the average diameter in a bearing should be less than a certain given value.

Therefore,

$$g_4(\mathbf{X}) = D_m - (0.5 - e)(D + d) \geq 0 \quad (15.29)$$

$$g_5(\mathbf{X}) = (0.5 + e)(D + d) - D_m \geq 0 \quad (15.30)$$

where

e = given constant.

In design optimization of bearings, a pitch diameter usually is larger than the corresponding average diameter in a bearing. Hence, a thickness of a bearing ring at outer raceway bottom should not be less than εD_b . ε is a given constant. Therefore a constraint condition is

$$g_6(\mathbf{X}) = 0.5(D - D_m - D_b) - \varepsilon D_b \geq 0 \quad (15.31)$$

Groove curvature radii of inner and outer raceways in a bearing should not be less than $0.515 D_b$. No upper limit is specified for the groove curvature radius. If it is larger than $0.52 D_b$, the dynamic load rating of the bearing will decline.

Constraint conditions

$$g_7(\mathbf{X}) = f_i \geq 0.515 \quad (15.32)$$

$$g_8(\mathbf{X}) = f_e \geq 0.515 \quad (15.33)$$

In summary, a design optimization problem is summed up as an optimization of objective function (15.8) and eight inequality constraints.

Penalty function and explanation of solving methods

Using external point method of penalty function, a penalty function upon equation (15.6) may be established

$$\Phi(\mathbf{X}, r^{(k)}) = f(\mathbf{X}) + r^{(k)} \sum_{i=1}^8 [\min(g_i(\mathbf{X}), 0)]^2 \quad (15.34)$$

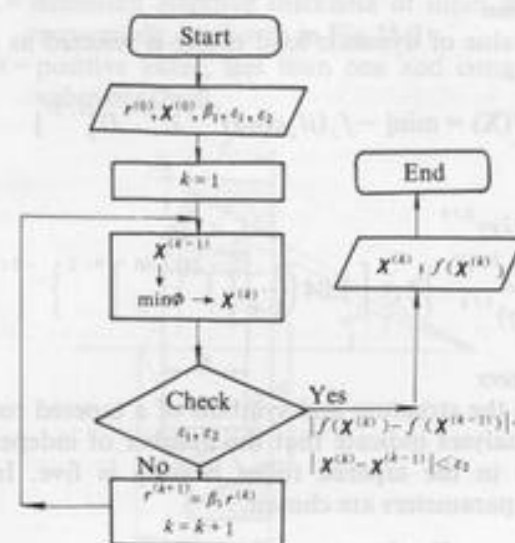


Fig. 15.2 A flow chart of program for external point method of penalty function

The design problem is transformed into an unconstrained optimization. Then its solution can be determined by Powell's method.

Fig.15.2 shows a flow chart of a program for the external point method of penalty function. An initial penalty parameter $r^{(0)}$ is decided by testing. On the whole, the objective function and penalty terms should be adjusted to a same grade in quantity. Coefficient β_1 of penalty parameters is valued between 5 and 10.

It should be noted that the diameter of balls is a discrete variable chosen from a set of standard values and the number of balls is a variable of the whole number type. In previous optimization, both are treated as continuous variables. Therefore, some modifications are needed for this difference. It will be explained later.

The solution obtained is possibly only a local optimum. In general, it is better to select several initial points and to see whether the same solution is arrived at. Finally a satisfactory solution is chosen as a global optimum of the original problem.

Design optimization for tapered roller bearings

Objective function

The negative value of dynamic load rating is selected as an objective function, that is

$$\min f(\mathbf{X}) = \min[-f_e (il_r \cos \alpha)^{7/9} Z^{3/4} D_b^{29/27}] \quad (15.35)$$

where

$$f_e = 207.9 \lambda \nu \gamma^{2/9} \times \frac{(1-\gamma)^{29/27}}{(1+\gamma)^{1/4}} \left\{ 1 + \left[1.04 \left(\frac{1-\gamma}{1+\gamma} \right)^{143/108} \right]^{9/2} \right\}^{-2/9} \quad (15.36)$$

Design parameters

Fig.15.3 shows the structure and symbols of a tapered roller bearing. Geometrical analyses indicate that the number of independent structural variables in the tapered roller bearing is five. In Reference (98), design parameters are chosen.

$$\mathbf{X} = [a_0, a_1, \beta, D_1, \alpha]^T \quad (15.37)$$

where

a_0 and a_1 = thickness of the big and small flanges of an inner ring respectively

β = half taper of a roller

D_1 = minimum diameter of outer raceway

α = contact angle.

Clearly, another set of design parameters is also possibly taken.

Constraint conditions

Reference (98) proposes following inequality constraints upon internal geometrical relation and strength check for the bearing.

From empirical relationships of minimum effective thicknesses of inner and outer rings

$$g_1(\mathbf{X}) = S_i - 0.08(D-d) \geq 0 \quad (15.38)$$

$$g_2(\mathbf{X}) = S_i - S_e \geq 0 \quad (15.39)$$

$$g_3(\mathbf{X}) = S_e - S_i + \delta \geq 0 \quad (15.40)$$

where

S_i and S_e = minimum effective thickness of inner and outer rings respectively, as shown in Fig.15.3

δ = positive value, less than one and connected with the value of $(D-d)$.

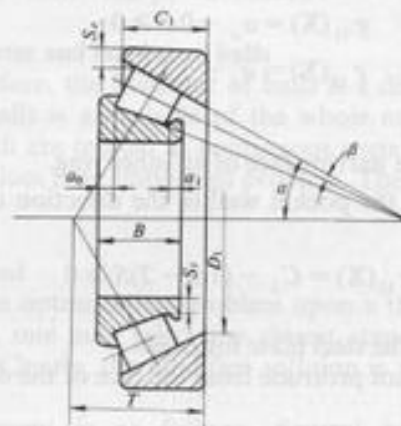


Fig. 15.3 Design parameters of tapered roller bearings

From empirical relationships of flange thicknesses of inner and outer rings

$$g_4(\mathbf{X}) = a_0 - K_{a0min} B \geq 0 \quad (15.41)$$

$$g_5(\mathbf{X}) = K_{a0max} B - a_0 \geq 0 \quad (15.42)$$

$$g_6(\mathbf{X}) = a_1 - K_{a1min} B \geq 0 \quad (15.43)$$

$$g_7(\mathbf{X}) = K_{a1max} B - a_1 \geq 0 \quad (15.44)$$

Diameter at contact point between the outer raceway and the big end of a roller must be less than the maximum diameter of the outer raceway, therefore

$$g_8(\mathbf{X}) = D_1 + 2C_1 \tan \alpha - 2\rho \sin \alpha \geq 0 \quad (15.45)$$

where

C_1 = width of an outer ring

ρ = spherical surface radius of the big flange of an inner ring.

And a series of requirements for lower limits

$$g_9(\mathbf{X}) = \beta - 1^\circ \geq 0 \quad (15.46)$$

$$g_{10}(\mathbf{X}) = \alpha - 10^\circ \geq 0 \quad (15.47)$$

minimum width a_i of the narrow face of an outer ring should be

$$g_{11}(\mathbf{X}) = a_i - 0.3 \geq 0 \quad (15.48)$$

$$g_{12}(\mathbf{X}) = a_i - 0.3 \geq 0 \quad (15.49)$$

where

a_i = width of the narrow face of an inner ring.

Thickness C_s of the pocket wall in the direction of circle in a cage should satisfy

$$g_{13}(\mathbf{X}) = C_s - (1.8 - 2)S \geq 0 \quad (15.50)$$

where

S = thickness of the steel plate for cages.

All rollers must not protrude from the face of the outer rings.

Then

$$g_{14}(\mathbf{X}) = C_{2min} \geq 0 \quad (15.51)$$

$$g_{15}(\mathbf{X}) = C_{3min} \geq 0 \quad (15.52)$$

where

C_{2min} and C_{3min} = minimum distances of the contact points between two ends of a roller and the outer raceway from the faces of an outer ring respectively.

Deviation of the total width (abutment width) T should be less than 0.002 mm, i.e.

$$g_{16}(\mathbf{X}) = 0.002 - \Delta T \geq 0 \quad (15.53)$$

In summary, a design optimization problem is summed up as an optimization of objective function (15.35) and sixteen inequality constraints.

Penalty function

Using external point methods of penalty function, a penalty function may be established

$$\Phi(\mathbf{X}, r^{(k)}) = f(\mathbf{X}) + r^{(k)} \sum_{i=1}^{16} [\min(g_i(\mathbf{X}), 0)]^2 \quad (15.54)$$

Its solving method is similar to that in deep groove ball bearings. The initial penalty parameter $r^{(0)}$ and the coefficient β_1 are decided by testing. Reference [98] sets $\beta_1 = 2$, that is, $r^{(k+1)} = 2r^{(k)}$.

Selection of diameter and number of balls

As mentioned before, the diameter of balls is a discrete variable and the number of balls is a variable of the whole number type. In the optimization, both are treated as continuous variables. How to select their optimum values is an important problem. There are two methods for this.

Comparison method

Having solved the optimization problem upon a treatment of continuous parameters, one may select the closest standard diameter and number of balls. Clearly, the modified solution is a better design, but not the best one.

Another treatment is as follows. Several ball diameters and numbers, which are near to the optimum values, are selected. Then, one may examine whether different combinations from them satisfy the constraints. Finally, a group of parameters which leads to a maximum dynamic load rating, is an approximate optimum solution. This method is simple and reliable.

Penalty function method for discrete variables

Design variables $\mathbf{X}$ can be expressed as

$$\mathbf{X}_d = [D_b, Z]^T \in R_d \quad (15.55)$$

where

R_d = subset of discrete design variables.

In the program, all possible values of a discrete variable may be arranged as an array in order to be convenient for comparison.

Taking deep groove ball bearings for example, one may change the penalty function (15.34) into

$$\Phi(\mathbf{X}, r_1^{(k)}, r_2^{(k)}) = f(\mathbf{X}) + r_1^{(k)} \sum_{i=1}^{16} [\min(g_i(\mathbf{X}), 0)]^2 + r_2^{(k)} Q(\mathbf{X}_d) \quad (15.56)$$

where

$r_1^{(k)}$ and $r_2^{(k)}$ = weighted penalty parameter, progressively increasing positive sequence

$Q(X_d)$ = penalty term of a discrete variable, which may be expressed by

$$Q(X_d) = \begin{cases} = 0, & (X_d \in R_d, \text{ when a design point satisfies the discrete values}) \\ > 0, & (X_d \notin R_d, \text{ when a design point does not satisfy the discrete values}) \end{cases} \quad (15.57)$$

When a design point does not satisfy the discrete values, $Q(X_d)$ takes an action of penalty. $Q(X_d)$ may be defined as

$$Q(X_d) = \sum_{i=1}^2 [4q_i(1-q_i)]^{\beta(k)} \quad (15.58)$$

where

$\beta(k)$ = exponents, progressively decrease positive sequence.

Generally, $\beta(0) > 1$ is set and

$$\beta(k+1) = \frac{\beta(k)}{1.2}$$

when i equals to one and two, q_i is respectively expressed as

$$q_1 = \frac{D_b - D_{bl}}{D_{bu} - D_{bl}}$$

$$q_2 = \frac{Z - Z_l}{Z_u - Z_l}$$

where

D_{bu} and D_{bl} = two discrete points next to the value D_b , i.e.

$$D_{bl} < D_b < D_{bu}$$

Z_u and Z_l = two discrete points next to the value Z , i.e.

$$Z_l < Z < Z_u$$

It should be noted that programs for this are more complicated and sometimes, one may need to try computing in order to select a set of proper parameters.

CHAPTER SIXTEEN

Computer aided design of high speed ball bearings

Even though the function of rolling bearings is very simple, motions of rolling elements and forces acting on the elements are much more complicated. For high speed ball bearings, some dynamic factors, such as ball centrifugal forces and gyroscopic moments, have a great important influence on the bearing design. To consider these factors, a quasi-static analytical design method for high speed ball bearings is developed. On the basis of a set of equilibrium equations, internal geometrical relationships, rotational speeds of balls, forces acting on balls and deflections in the bearing are analysed. By comparison with the general static design, contact angles, load distribution, orbit and rotational speeds of balls, as well as internal sliding in the bearings are remarkably different. Therefore, internal structural parameters of the bearings should be changed correspondingly.

Jones first proposed the theoretical basis of the quasi-static analytical design method in 1959 and 1960. He adopted an assumption of the raceway control, that is, spinning occurs only on one raceway and no spinning on the other. Then the equilibrium equations of balls and rings are transformed into a set of nonlinear algebraic equations, which may be solved by a Newton-Raphson method. In the first paragraph of this chapter, the quasi-static analytical design method, termed raceway control theory, is explained. From the early sixties to late seventies, high speed ball bearings were mostly designed based on this theory.

Considering the elastohydrodynamic lubrication, Harris developed the quasi-static analytical design method for high speed ball bearings in 1971 (99) and improved it in 1973 (100). Boness computed tractive forces by an empirical equation and obtained a good correspondence with experiments (101). These methods will be described in the second paragraph of this chapter.

To analyse the whole dynamic process of a bearing, integrate the differential equation of motion for each bearing component. Positions, speeds of rolling elements and the cage, and other internal

sliding in the bearing at any time can be numerically deduced. This is called a dynamic analytical design method, which actually gives a real simulation of a bearing performance. This method was initially submitted by Walter in 1971 (102) and systematically developed by Gupta (103), (104). Due to the complexity of the equations and their influential factors, the method requires a substantially longer computing time and it is hardly applied to engineering problems.

Neither the quasi-static method nor the dynamic method is able to derive its accurate analytical solution. Either of both the two methods can only be numerically solved by computers.

RACEWAY CONTROL THEORY

An angular contact ball bearing rotates at high speed under combined radial, thrust and moment loads. Due to the influence of ball centrifugal forces the contact angle of the bearing can be changed, so that inner raceway contact angles have a tendency to increase and outer raceway contact angles to decrease. The higher the speed of a bearing, the larger the difference between inner and outer raceway contact angles will be.

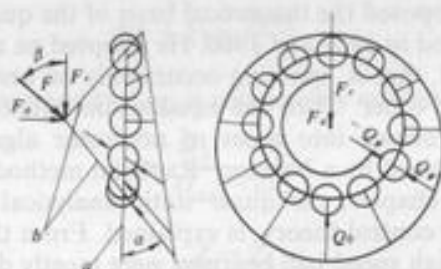


Fig. 16.1 Angular positions of balls

Fig. 16.1 shows angular position of balls in the bearing. Obviously, the force acting on each ball varies with different positions of balls.

In Fig. 16.1, the angular distance between two neighbouring balls is

$$\Delta\psi = \frac{2\pi}{Z}$$

and the angular position of the j -th ball

$$\psi_j = \frac{2\pi(j-1)}{Z}$$

Geometrical relation of deformations

Fig. 16.2 shows the relative position of the ball centres at angular position ψ_j . Assume that the outer raceway is fixed in space. The groove curvature centre of the inner raceway may have a displacement relative to that of the outer raceway. Furthermore, the ball centre moves due to the variations of inner and outer raceway contact angles.

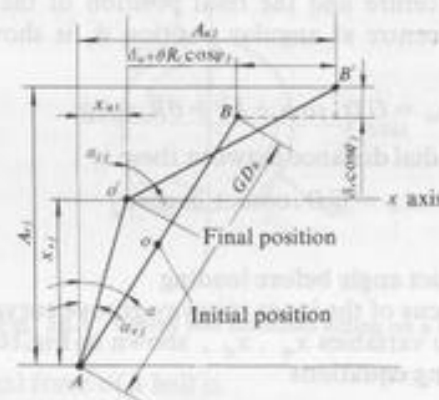


Fig. 16.2 Relative position of curvature centres

The distance between inner and outer raceway groove curvature centres before loading is

$$GD_b = (f_o + f_i - 1)D_b \quad (16.1)$$

After loading, the ball centre o moves to point o' and the inner raceway groove curvature centre B changes to B' . Therefore the distance between the final position of the ball centre and the fixed outer raceway groove curvature centre at any angular position ψ_j is

$$\Delta_{oj} = (f_o - 0.5)D_b + \delta_{oj} \quad (16.2)$$

Similarly, the distance between the final position of the ball centre and the final inner raceway groove curvature centre at any angular

position ψ_j is

$$\Delta_{ij} = (f_i - 0.5)D_b + \delta_{ij} \quad (16.3)$$

where

δ_{ej} and δ_{ij} = contact deflections normal to the contact areas at angular position ψ_j on outer and inner raceways respectively.

If the relative axial displacement, the radial displacement and the tilting angle of inner and outer rings are expressed as δ_a , δ_r and θ respectively, the axial distance between the fixed outer raceway groove curvature centre and the final position of the inner raceway groove curvature centre at angular position ψ_j is shown in Fig.16.2, that is,

$$A_{aj} = GD_b \sin \alpha + \delta_a + \theta R_i \cos \psi_j \quad (16.4)$$

Similarly, the radial distance between these is

$$A_{rj} = GD_b \cos \alpha + \delta_r \cos \psi_j \quad (16.5)$$

where

α = initial contact angle before loading

R_i = radius of locus of the inner raceway groove curvature centre.

Introducing two variables x_{aj} , x_{rj} , shown in Fig.16.2, one may establish the following equations

$$\cos \alpha_{ej} = \frac{x_{rj}}{(f_e - 0.5)D_b + \delta_{ej}} \quad (16.6)$$

$$\sin \alpha_{ej} = \frac{x_{aj}}{(f_e - 0.5)D_b + \delta_{ej}} \quad (16.7)$$

$$\cos \alpha_{ij} = \frac{A_{rj} - x_{rj}}{(f_i - 0.5)D_b + \delta_{ij}} \quad (16.8)$$

$$\sin \alpha_{ij} = \frac{A_{aj} - x_{aj}}{(f_i - 0.5)D_b + \delta_{ij}} \quad (16.9)$$

and geometric relationships regarding the change of ball centre positions

$$(A_{aj} - x_{aj})^2 + (A_{rj} - x_{rj})^2 - [(f_i - 0.5)D_b + \delta_{ij}]^2 = 0 \quad (16.10)$$

$$x_{aj}^2 + x_{rj}^2 - [(f_e - 0.5)D_b + \delta_{ej}]^2 = 0 \quad (16.11)$$

Analysis of forces

In a high speed ball bearing, centrifugal forces of balls lead to changes of the contact deflections and the contact angles. The variation of a ball axis about which the ball rotates causes a gyroscopic moment and corresponding frictional resistance. Fig.16.3 shows the forces and moments acting on a ball.

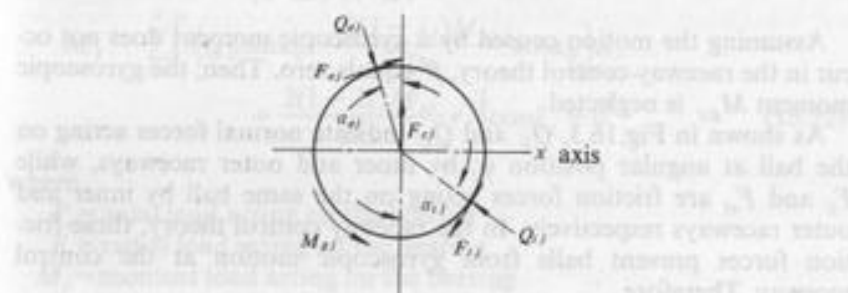


Fig. 16.3 Forces and moments acting on a ball

The centrifugal force of a ball is

$$F_e = \frac{\pi}{12} \rho D_b^3 D_m \omega_m^2 \quad (16.12)$$

Considering equations in Table 5.4, equation (16.12) applied to the ball at angular position ψ_j can be rewritten as

$$F_{ej} = \frac{\pi}{12} \rho D_b^3 D_m \omega^2 \left(\frac{\omega_m}{\omega} \right)_j \quad (16.13)$$

where

ρ = density of ball material

F_{ej} = centrifugal force of the ball at angular position ψ_j .

Similarly from equation (5.34), gyroscopic moment of the ball at angular position ψ_j is

$$M_{ej} = J \omega^2 \sin \beta \left(\frac{\omega_b}{\omega} \right)_j \left(\frac{\omega_m}{\omega} \right)_j \quad (16.14)$$

where

J = mass moment of inertia

$$J = \frac{1}{60} \rho \pi D_b^5 \quad (16.15)$$

If ω_y is not equal to zero, a gyroscopic moment about axis z exists. As seen from equation (5.35), then

$$M_{gz} = J \omega^2 \cos \beta \sin \beta' \left(\frac{\omega_b}{\omega} \right)_j \left(\frac{\omega_m}{\omega} \right)_j \quad (16.16)$$

Assuming the motion caused by a gyroscopic moment does not occur in the raceway control theory, β' equals zero. Then, the gyroscopic moment M_{gz} is neglected.

As shown in Fig. 16.3, Q_{ij} and Q_{ej} indicate normal forces acting on the ball at angular position ψ_j by inner and outer raceways, while F_{ij} and F_{ej} are friction forces acting on the same ball by inner and outer raceways respectively. In the raceway control theory, these friction forces prevent balls from gyroscopic motion at the control raceway. Therefore

$$F_{ej} = \frac{2\lambda_j M_{ej}}{D_b} \quad (16.17a)$$

$$F_{ij} = \frac{2(1-\lambda_j)M_{ej}}{D_b} \quad (16.17b)$$

in which, values of λ_j are based on control conditions. $\lambda_j = 1$ refers to the outer raceway control and $\lambda_j = 0$ refers to the inner raceway control.

In the quasi-static method, considering the equilibrium of forces acting on the ball in the horizontal and the vertical directions, i.e.

$$Q_{ij} \sin \alpha_{ij} - Q_{ej} \sin \alpha_{ej} - \frac{2M_{ej}}{D_b} [(1-\lambda_j) \cos \alpha_{ij} - \lambda_j \cos \alpha_{ej}] = 0 \quad (16.18)$$

$$Q_{ij} \cos \alpha_{ij} - Q_{ej} \cos \alpha_{ej} + \frac{2M_{ej}}{D_b} [(1-\lambda_j) \sin \alpha_{ij} - \lambda_j \sin \alpha_{ej}] + F_{ej} = 0 \quad (16.19)$$

and the equilibrium of forces acting on the inner ring in the radial and the axial directions

$$F_a - \sum_{j=1}^z \left[Q_{ij} \sin \alpha_{ij} - \frac{2(1-\lambda_j)M_{ej}}{D_b} \cos \alpha_{ij} \right] = 0 \quad (16.20)$$

$$F_r - \sum_{j=1}^z \left[Q_{ij} \cos \alpha_{ij} + \frac{2(1-\lambda_j)M_{ej}}{D_b} \sin \alpha_{ij} \right] = 0 \quad (16.21)$$

as well as the equilibrium condition of moment about axis y , that is

$$M_y - \sum_{j=1}^z \left[\left(Q_{ij} \sin \alpha_{ij} - \frac{2(1-\lambda_j)M_{ej}}{D_b} \cos \alpha_{ij} \right) R_i + \frac{2(1-\lambda_j)M_{ej}}{D_b} r_i \right] \cos \psi_j = 0 \quad (16.22)$$

where

F_a = axial load acting on the bearing

F_r = radial load acting on the bearing

M_y = moment load acting on the bearing

R_i = radius of locus of the inner raceway groove curvature centre.

Load-deflection relation

The relation of a load to its contact deflection between the ball and the raceway is expressed by equation (4.1). It may be rewritten as

$$Q = \frac{1}{K_p^{3/2}} \delta^{3/2} \quad (16.23)$$

K_p is connected with the contact angle. Therefore, it varies with angular position ψ_j . Then equation (16.23) is applied to the inner and the outer raceways at angular position ψ_j . Respectively, these are

$$Q_{ij} = K_{ij} \delta_{ij}^{3/2} \quad (16.24)$$

$$Q_{ej} = K_{ej} \delta_{ej}^{3/2} \quad (16.25)$$

The calculation of K_{ij} and K_{ej} is conducted by equations (16.23) and (4.2). Substituting equations (16.24) and (16.25) as well as equations (16.6) to (16.9) into equations (16.18) to (16.22) gives the following equations of the equilibrium.

$$\frac{2\lambda_j M_{gj} x_{rj}}{D_b} - K_{ej} \delta_{ej}^{3/2} x_{aj} + \frac{K_{ij} \delta_{ij}^{3/2} (A_{aj} - x_{aj})}{(f_i - 0.5)D_b + \delta_{ij}} - \frac{2(1 - \lambda_j) M_{gj} (A_{rj} - x_{rj})}{D_b} = 0 \quad (16.26)$$

$$\frac{K_{ej} \delta_{ej}^{3/2} x_{rj} + \frac{2\lambda_j M_{gj} x_{rj}}{D_b}}{(f_e - 0.5)D_b + \delta_{ej}} - \frac{K_{ij} \delta_{ij}^{3/2} (A_{rj} - x_{rj})}{(f_i - 0.5)D_b + \delta_{ij}} - \frac{2(1 - \lambda_j) M_{gj} (A_{rj} - x_{rj})}{D_b} - F_{ej} = 0 \quad (16.27)$$

$$F_a - \sum_{j=1}^Z \left[\frac{K_{ij} (A_{aj} - x_{aj}) \delta_{ij}^{3/2} - \frac{2(1 - \lambda_j) M_{gj} (A_{rj} - x_{rj})}{D_b}}{(f_i - 0.5)D_b + \delta_{ij}} \right] = 0 \quad (16.28)$$

$$F_r - \sum_{j=1}^Z \left[\frac{K_{ij} (A_{rj} - x_{rj}) \delta_{ij}^{3/2} + \frac{2(1 - \lambda_j) M_{gj} (A_{aj} - x_{aj})}{D_b}}{(f_i - 0.5)D_b + \delta_{ij}} \right] = 0 \quad (16.29)$$

$$M_y - \sum_{j=1}^Z \left\{ \frac{K_{ij} (A_{aj} - x_{aj}) \delta_{ij}^{3/2} - \frac{2(1 - \lambda_j) M_{gj} (A_{rj} - x_{rj})}{D_b}}{(f_i - 0.5)D_b + \delta_{ij}} R_i + 2(1 - \lambda_j) f_i M_{gj} \right\} \cos \psi_j = 0 \quad (16.30)$$

A set of basic equations and solving method

Geometric equations (16.10) and (16.11), and equilibrium equations (16.26) to (16.30) are a set of basic equations for calculation of high

speed ball bearings. j is valued from 1 to Z . The number of the equations is $(4Z+3)$. And $(4Z+3)$ unknown variables are x_{aj} , x_{rj} , δ_{ij} , δ_{ej} , δ_a , δ_r , and θ . If taking the symmetry of ball positions shown in Fig.16.1, the number of equations or unknown variables may be reduced to $(2Z+7)$ or $(2Z+5)$.

Besides the set of basic equations, some supplementary equations are used during the solving process. Such as equations (16.4) and (16.5) which are applied for A_{aj} and A_{rj} , equations (16.23) and (4.2) for K_{ij} and K_{ej} , equation (16.14) for M_{gj} and equation (5.40) for $(\omega_b / \omega)_j$. An equation for $(\omega_m / \omega)_j$ may be selected from Table 5.4. R_i is determined by

$$R_i = \frac{D_m}{2} + (f_i - 0.5)D_b \cos \alpha \quad (16.31)$$

Obviously, it is impossible to solve these equations by a strict analytical method. The Newton-Raphson method is usually applied for the simultaneous nonlinear equations.

Some high speed ball bearings are mainly subjected to an axial load, such as high speed ball bearings for internal grinding spindles, motor bearings for gyroscopes, etc. In these conditions, effects of the radial load and moment load are neglected. Therefore, it may be considered that the geometric relation, forces, loads and deflections are not precisely dependent on angular positions of balls. The basic equations for calculation of high speed ball bearings are significantly simplified, that is, they are composed of five equations: geometric relationships (16.10) and (16.11), equilibrium equations (16.26) to (16.28). Unknown variables are x_a , x_r , δ_i , δ_e and δ_a . For the convenience of the description, the unknown variables are expressed as δ_k , $k=1, 2, \dots, 5$, i.e. $\delta_1 = x_a$, $\delta_2 = x_r$, $\delta_3 = \delta_i$, $\delta_4 = \delta_e$, $\delta_5 = \delta_a$.

A flow chart for calculation of a high speed ball bearing subjected to an axial load is shown in Fig.16.4. Input data include geometric parameters of the bearing, material constants, load and speed, as well as initial values and permissible errors. The computation may be conducted until the iterative errors

$$e_k = \delta_k^{(i+1)} - \delta_k^{(i)}, k=1, 2, \dots, 5$$

are small enough. Explanations for the subroutines are listed in Table 16.1. Besides $\delta_k (k=1, 2, \dots, 5)$, contact angles at inner and outer

raceways, contact load, contact pressure, cage speed, ball rotation speed and spin to roll ratio can also be computed.

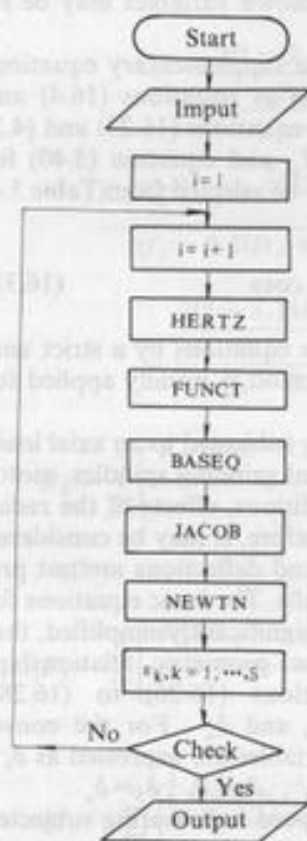


Fig. 16.4 A flow chart for high speed ball bearings

The basic equations for a high speed ball bearing subjected to an axial load may be solved for variables α_i , α_e , δ_i , δ_e and δ_a . The meanings of the five unknown variables are obvious and it is easy to specify their initial values.

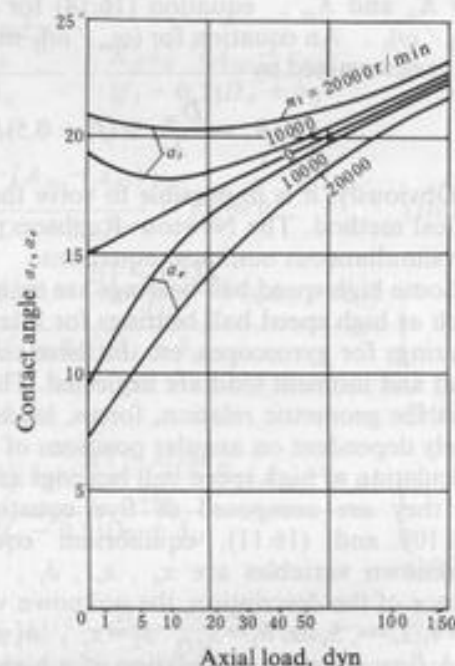


Fig. 16.5 Variations of contact angles

Reference [105] gives a numerical example. An angular contact ball bearing is subjected to a pure axial load at high speeds. Its bore diameter $d = 5$ mm, initial contact angle $\alpha = 15^\circ$. Fig. 16.5 shows the dependence of the inner and outer raceway contact angles of the bearing on axial load and speed. It can be seen that the contact angles

Table 16.1 Explanations

Subroutines	Brief explanations
HERTZ	Given geometric parameters, curvatures, sizes of contact area and K_i , K_e are computed
FUNCT	Given initial values, operating conditions, gyroscopic moments, A_i and A_e are determined
BASEQ	To establish basic equations
JACOB	To compute partial derivation of the function on the left side of basic equations
NEWTN	To linearize nonlinear equations and to solve them by the iterative method

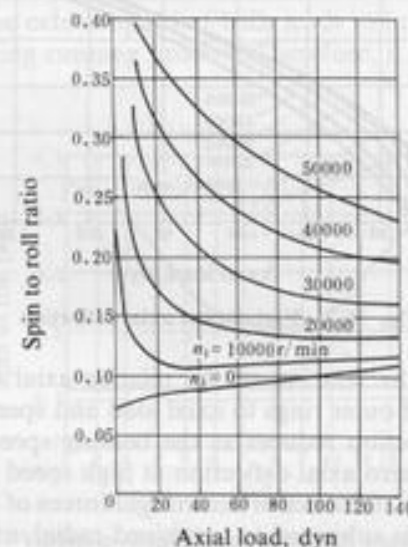


Fig. 16.6 Variation of spin to roll ratio

increase as the axial load rises. The outer raceway contact angle decreases and the inner raceway contact angle increases at high speeds. For example, for the axial load 5 dyn and bearing speed 2000 rpm, the outer raceway contact angle is 10.6° and the inner raceway contact angle 20.5° . The difference between the inner and outer raceway contact angles declines as the axial load increases.

Fig. 16.6 shows the dependence of the spin to roll ratio along the inner raceway on axial load and speed. It can be seen that the spin to roll ratio rises as the bearing speed increases and increasing the axial load at high speed may reduce the spin to roll ratio remarkably. The spin to roll ratio reflects spinning conditions of a ball on the raceway. Its larger values indicate that more serious spinning sliding and wear will occur. As seen from this figure, the spin to roll ratio may reach a relatively high value under a low axial load and at high speed. To eliminate the harmful influence, a certain value of axial load must act on high speed ball bearings.

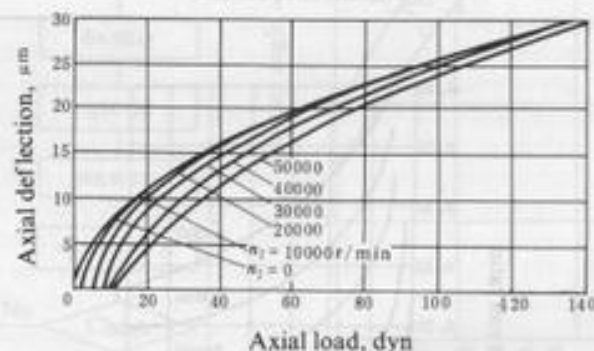


Fig. 16.7 Variation of axial deflection

Fig. 16.7 shows the relation of the relative axial deflection δ_a between the inner and outer rings to axial load and speed. It can be seen that the axial deflection reduces as the bearing speed increases. It is possible to have a zero axial deflection at high speed and under a low axial load due to the influence of centrifugal forces of balls.

When a bearing is subjected to combined radial, axial and moment loads, the contact load and the contact angle vary with angular positions in the bearing, then in turn the orbital and rotational speeds of

balls, as well as the spin to roll ratio vary with angular positions in the bearing. An example of the variations of contact angles and spin to roll ratios with angular positions is shown in Fig. 16.8 and Fig. 16.9.

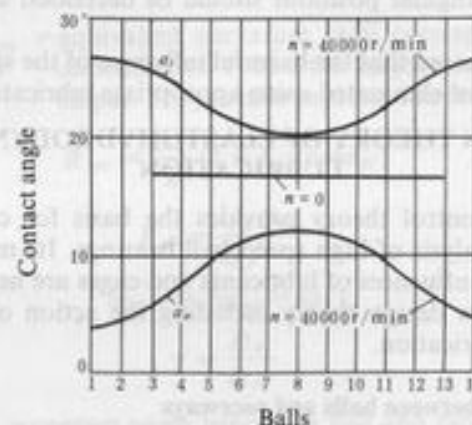


Fig. 16.8 Variation of contact angles with angular positions

Variation of the orbital speed of balls leads to impact between balls and the cage during running process. Therefore, it may enhance wear and heating.

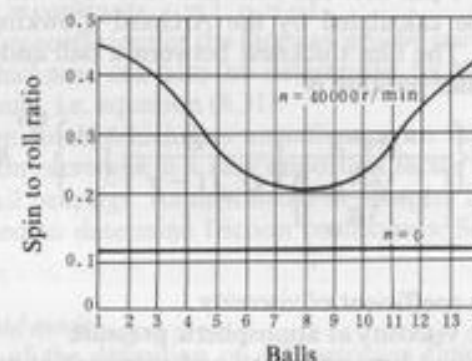


Fig. 16.9 Variation of spin to roll ratio with angular positions

It may be concluded that the design of a high speed ball bearing

must be combined with its operating conditions. The spin to roll ratio may be used as an important index to show if the design of a high speed ball bearing is correct. The value of the spin to roll ratio and its variation with angular positions should be decreased as low as possible.

It should be noted that the harmful influence of the spin to roll ratio can be reduced or eliminated using appropriate lubricating methods.

DESIGN THEORY OF ELASTOHYDRODYNAMIC LUBRICATION

The raceway control theory provides the basis for considering the quasi-static analysis of high speed ball bearings. Its major disadvantage is that the influences of lubricants and cages are neglected. Harris developed a new design theory including the action of the elastohydrodynamic lubrication.

Traction forces between balls and raceways

For the convenience of description, the subscripts adopted by Harris in Reference [100] are used in this paragraph, that is, subscript $p=1$ refers to a quantity related to the outer ring and subscript $p=2$ refers to that related to the inner ring.

If a rolling bearing in operation is properly lubricated and at high enough speed, then an elastohydrodynamic lubrication oil film of a thickness 0.3 to 1 μm between balls and raceways will exist. The film thickness can be calculated by the Archard-Cowling formula, i.e. equation (6.26). The film thickness between a ball and inner or outer raceway at angular position is

$$h_{pj} = 2.04 \left\{ \frac{\alpha_1 \eta_0 u_{pj}}{\left[1 + \frac{2R_{ypj}}{3R_{xpj}} \right]} \left[\frac{E}{Q_{pj}(1-\nu^2)} \right]^{0.1} \right\}^{0.74} R_{ypj}^{0.408} \quad (16.32)$$

where

α_1 = pressure coefficient of viscosity

η_0 = dynamic viscosity at atmospheric pressure

Q_{pj} = contact load between a ball and raceways at angular position ψ_j

u_{pj} = average velocity of a ball and raceways at angular position ψ_j , then

$$u_{pj} = \frac{D_b}{4} \left[\omega_{pj} \left(\frac{1}{\gamma} + c_p \cos \alpha_{pj} \right) + \omega_{xj} \cos \alpha_{pj} + \omega_{zj} \sin \alpha_{pj} \right] \quad (16.33)$$

R_{ypj} and R_{xpj} = equivalent curvature radii correspondingly in the directions of Y and the major-axis of the contact ellipse. They can be expressed as follows

$$R_{ypj} = \frac{D_b}{2} (1 + c_p \gamma' \cos \alpha_{pj}) \quad (16.34)$$

$$R_{xpj} = \frac{f_p D_b}{2f_p - 1} \quad (16.35)$$

$$\omega_{pj} = c_p (\omega_{mj} - \omega_p) \quad (16.36)$$

$$\gamma' = \frac{D_b}{D_m} \quad (16.37)$$

α_{pj} = contact angle between a ball and raceways at angular position ψ_j , as expressed by equations (16.6) to (16.9)

ω_{xj} and ω_{zj} = components of the rotational angular speed of a ball around its own axis in the direction x and z correspondingly, as seen from equation (5.10)

ω_{mj} = orbital angular speed of balls at angular position ψ_j

ω_p = bearing angular speed

c_p = constants, $c_1 = 1$, $c_2 = -1$

f_p = coefficient of the raceway groove curvature radius.

The film thickness can also be determined by the Hamrock and Dowson formula, i.e. equation (6.31).

The traction force, which has an influence on the sliding and the contact load on raceways, is a very important factor in the analysis of high speed ball bearings. As mentioned in Chapter Six, several methods are applied to determine friction coefficients. Some of them are the following.

Newtonian fluid model

On the basis of the definition of a Newtonian fluid, the fluid shear stress τ is proportional to the local fluid velocity gradient in the z direction, i.e.

$$\tau = \eta \frac{\partial v}{\partial z}$$

Considering that a film thickness in a bearing is very thin, such as 0.1 to 1 μm , one may simplify the calculation of the shear stress as follows

$$\tau = \eta \frac{v}{h} \quad (16.38)$$

where

v = sliding velocity

h = film thickness

η = viscosity connected with the pressure and the operating temperature, it can be expressed at constant operating temperature

$$\eta = d_1 e^{(d_2 + d_3 p)^{1/2}} \quad (16.39)$$

where

d_1, d_2 and d_3 = constants dependent on lubricant properties

p = pressure, for a point contact

$$p = \frac{3Q_{pj}}{2\pi a_{pj} b_{pj}} (1 - s^2 - t^2)^{1/2} \quad (16.40)$$

a_{pj} and b_{pj} = semi-major and semi-minor axes of the contact area between a ball and raceways at angular position ψ_j

s and t = dimensionless coordinates, that is

$$s = \frac{x_p}{a_{pj}}; \quad t = \frac{y_p}{b_{pj}}$$

x_p = coordinate from the contact ellipse centre in the direction of the major axis

y_p = coordinate from the contact ellipse centre in the direction of the minor axis.

Relative sliding velocities between a ball and raceways in the y_p direction are expressed as equations (5.17) and (5.24). For the convenience of calculation of the traction force, the relative sliding velocity can be rewritten

$$v_{ypj} = \omega_{pj} \left[\frac{D_m}{2} + c_p r_{xpj} \cos(\alpha_{pj} + \theta_{xpj}) \right] - \omega_{sj} r_{xpj} \cos(\alpha_{pj} + \theta_{xpj}) - \omega_{sj} r_{xpj} \sin(\alpha_{pj} + \theta_{xpj}) \quad (16.41)$$

where

r_{xpj} = distance from the ball centre to a point on the contact area at angular position ψ_j

$$r_{xpj} = \left\{ \left[(R_p^2 - x_p^2)^{1/2} - (R_p^2 - a_{pj}^2)^{1/2} + \left(\frac{D_b^2}{4} - a_{pj}^2 \right)^{1/2} \right]^2 + x_p^2 \right\}^{1/2} \quad (16.42)$$

$$R_p = \frac{2f_p D_b}{2f_p + 1} \quad (16.43)$$

θ_{xpj} = angular coordinate of a point on the contact area of the ball at angular position ψ_j

As shown from Fig. 5.3, θ_{xpj} is expressed as

$$\theta_{xpj} = \arctg \left(\frac{x_p}{r_{xpj}} \right) \quad (16.44)$$

Relative sliding velocities in the direction of the major-axis of the contact ellipse are determined by equations (5.18) and (5.25), which can be approximately calculated by

$$v_{xpj} = \frac{1}{2} D_b \omega_{pj} \quad (16.45)$$

In summary from above, components of the traction force on the contact area in the direction of y_p and x_p are

$$F_{ypj} = a_{pj} b_{pj} \int_{-1}^{+1} \int_{-(1-s^2)^{1/2}}^{+(1-s^2)^{1/2}} \tau_{ypj} dt ds \quad (16.46)$$

$$F_{xpj} = a_{pj} b_{pj} \int_{-1}^{+1} \int_{-(1-t^2)^{1/2}}^{+(1-t^2)^{1/2}} \tau_{xpj} dr ds \quad (16.47)$$

The traction force on the contact area

$$F_{pj} = (F_{ypj}^2 + F_{xpj}^2)^{1/2} \quad (16.48)$$

Semi-empiric model

On the basis of the elastohydrodynamic lubrication theory and experimental data, Gu proposed a semi-empiric model. It is assumed that the friction coefficients in the rolling direction are decided by the following three parameters, that is,

$$G_1 = \frac{\eta_0 v}{\rho_0 h} ; G_2 = \frac{\beta_1 \eta_0 v^2}{8K_f} ; G_3 = \gamma p_0$$

where

v = relative sliding velocity

p_0 = Hertzian contact pressure

h = film thickness

K_f = thermal conductivity coefficient of a lubricant

γ and β_1 = coefficients of the viscosity-pressure-temperature equation of a lubricant.

$$\eta = \eta_0 \exp[\gamma p - \beta_1 (T - T_0)]$$

The relation of friction coefficients and G_1 , G_2 , G_3 is usually designated by curves or listed data.

Empiric model

Gentle and Cameron proposed that the friction coefficient can be expressed by equation (6.40) upon experimental data. Values of A , B , C and D are decided by experiments. For example, Reference (106) gives a group of the following values

$$A = 0.27; B = 0.43 \text{ GPa};$$

$$C = 6.93; D = 0.022$$

As seen from equation (6.40), although it is nonlinear, it avoids the calculation of integrals in traction forces.

Constant friction coefficient

For the convenience of comparison, a constant value of the friction coefficient is assumed. Reference (103) takes the coefficient as 0.007. Obviously, the calculation is significantly simplified. It will be described later that constant friction coefficient sometimes gives satisfactory appropriate results.

Forces acting on balls by lubricant and cage

The jet oil and oil mist lubrication are often used in high speed ball bearings. Besides traction forces in the elastohydrodynamic lubrication film, the oil drag forces of lubricant and the friction forces between balls and cage pockets takes place.

Oil drag force

A ball orbits about the bearing axis within a space of oil-air mixture and is subjected to the oil drag force of the lubricant. It can be estimated by the following

$$F_{dj} = \frac{\pi \rho C_d D_b^2 (D_m \omega_m)^{1.95}}{32g} \quad (16.49)$$

where

ρ = density of oil-air mixture

C_d = resistance coefficient

g = acceleration of gravity.

The direction of F_{dj} is opposite to the orbital direction of the ball.

Normal load between balls and cage

If the rotational speed of a ball is not equal to that of the cage, the ball will give impetus to the cage or in reverse. They are in the tangential direction of the bearing pitch circle and vertical to the wall of the cage pockets, correspondingly expressed by Q_{cj} and $-Q_{cj}$. But their values are hardly estimated.

Friction forces between balls and cage

Friction forces exist between balls and cage pockets. The values of the forces, expressed as F_{cfj} , can be calculated using the hydrodynamic lubrication theory.

Basic equations and solution summary

Similar to the raceway control theory, geometric relation, analysis of forces and load-deflection relation are separately considered.

The geometric relationships of variations of the ball centre position are expressed by equations (16.10) and (16.11), which are merged into the following

$$\sum_{k=1}^2 (v_p A_{kj} + c_p x_{kj})^2 - [(f_p - 0.5)D_b + \delta_{pj}]^2 = 0 \quad (16.50)$$

where

$v_p = \text{constant } p=1, v_1=0; p=2, v_2=1$

$A_{kj} = \text{corresponding to } A_{1j} = A_{aj}, A_{2j} = A_{ej}$

$x_{kj} = \text{corresponding to } x_{1j} = x_{aj}, x_{2j} = x_{ej}$

and the relationship

$$\alpha_{pj} = \arctg \frac{v_p A_{1j} + c_p x_{1j}}{v_p A_{2j} + c_p x_{2j}} \quad (16.51)$$

Considering the equilibrium equations of a ball at angular position ψ_j , as shown in Fig. 16.10

$$\sum_{p=1}^2 c_p (Q_{pj} \cos \alpha_{pj} - F_{xpj} \sin \alpha_{pj}) - F_{cj} - F_{ej} = 0 \quad (16.52)$$

$$\sum_{p=1}^2 c_p (Q_{pj} \sin \alpha_{pj} + F_{xpj} \cos \alpha_{pj}) = 0 \quad (16.53)$$

and dynamic equations of the ball

$$\sum_{p=1}^2 c_p F_{ypj} + F_{dj} - Q_{cj} = \frac{m D_m \omega_{mj}}{2} \frac{d\omega_{mj}}{d\psi} \quad (16.54)$$

$$\frac{D_b}{2} \sum_{p=1}^2 F_{xpj} - M_{ej} = J \omega_{mj} \frac{d\omega_{mj}}{d\psi} \quad (16.55)$$

$$\sum_{p=1}^2 M_{xpj} - \frac{D_b}{2} F_{zcj} = J \omega_{mj} \frac{d\omega_{mj}}{d\psi} \quad (16.56)$$

$$\sum_{p=1}^2 M_{zpj} - \frac{D_b}{2} F_{xcj} - M_{ezj} = J \omega_{mj} \frac{d\omega_{mj}}{d\psi} \quad (16.57)$$

where

$m = \text{mass of the ball}$

F_{zcj} and F_{xcj} = components of F_{cj} in the directions of z and x axes respectively

M_{xpj} and M_{zpj} = moments of traction forces about x and z axes

respectively

$$M_{xpj} = a_{pj} b_{pj} \int_{-1}^1 \int_{-(1-x^2)^{1/2}}^{(1-x^2)^{1/2}} \tau_{ypj} r_{xpj} \cos(\alpha_{pj} + \theta_{xpj}) d\tau ds \quad (16.58)$$

$$M_{zpj} = a_{pj} b_{pj} \int_{-1}^1 \int_{-(1-x^2)^{1/2}}^{(1-x^2)^{1/2}} \tau_{ypj} r_{xpj} \sin(\alpha_{pj} + \theta_{xpj}) d\tau ds \quad (16.59)$$

Load-deflection relationships are the same as equations (16.24) and (16.25).

If the normal load between balls and the cage is neglected, i.e. $Q_{cj} = 0$, then ω_{mj} is a constant, expressed as ω_m .

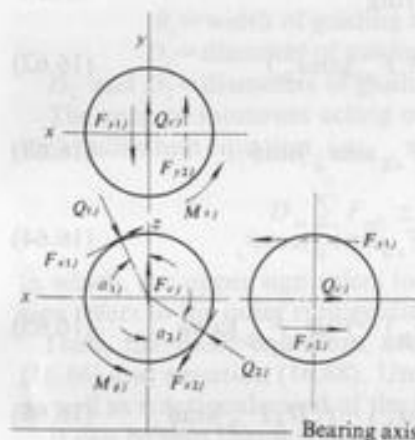


Fig. 16.10 Ball equilibrium

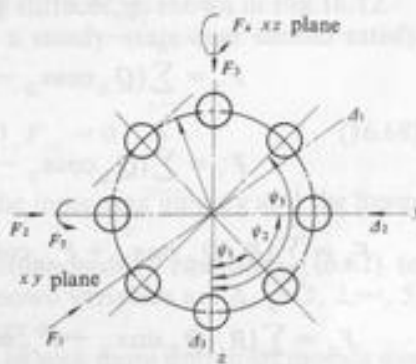


Fig. 16.11 Loading condition of a bearing

For a ball at fixed angular position ψ_j , there are eight equations, that is, equations (16.50), (16.52) to (16.57). If given relative deflections including displacement and tilting angle of bearing rings and the cage rotational speed, then unknown variables Q_{pj} , α_{pj} , ω_{mj} , ω_{vj} , ω_{zj} and F_{cj} can be solved. Due to nonlinearity the equations are solved numerically by the Newton-Raphson method.

Usually, relative deflections of bearing rings and the cage rotational

speed are unknown and only loads acting on the bearing and the rotational speed of the bearing are given in actual engineering problems.

Considering a loading condition shown in Fig. 16.11 external loads generally are axial load F_1 , radial loads F_2 and F_3 in the directions of y and z axes and moment loads F_4 and F_5 . Correspondingly five relative deflections of rings are expressed as Δ_i , $i = 1, 2, \dots, 5$.

Similar to the derivation of equations (16.4) and (16.5), the variations of the raceway groove curvature centre

$$A_{1j} = GD_0 \sin \alpha_j + \Delta_1 + R_1 (\Delta_4 \cos \psi_j + \Delta_5 \sin \psi_j) \quad (16.60)$$

$$A_{2j} = GD_0 \cos \alpha_j + \Delta_2 \sin \psi_j + \Delta_3 \cos \psi_j \quad (16.61)$$

From the equilibrium of the inner ring

$$F_1 = \sum_{j=1}^z (Q_{2j} \sin \alpha_{2j} + F_{x2j} \cos \alpha_{2j}) \quad (16.62)$$

$$F_2 = \sum_{j=1}^z (Q_{2j} \cos \alpha_{2j} - F_{x2j} \sin \alpha_{2j}) \sin \psi_j \quad (16.63)$$

$$F_3 = \sum_{j=1}^z (Q_{2j} \cos \alpha_{2j} - F_{x2j} \sin \alpha_{2j}) \cos \psi_j \quad (16.64)$$

$$F_4 = \sum_{j=1}^z [R_1 (Q_{2j} \sin \alpha_{2j} + F_{x2j} \cos \alpha_{2j}) - f_2 D_0 F_{x2j}] \cos \psi_j \quad (16.65)$$

$$F_5 = \sum_{j=1}^z [R_1 (Q_{2j} \sin \alpha_{2j} + F_{x2j} \cos \alpha_{2j}) - f_2 D_0 F_{x2j}] \sin \psi_j \quad (16.66)$$

Furthermore, consider the equilibrium of the cage. Cages are often guided by the inner or outer ring in high speed ball bearings, as shown in Fig. 16.12.

The cage is subjected to the friction forces of balls on pocket surfaces and the traction force of oil film on the guiding surface. The latter may be estimated as follows

$$F_{cL} = \eta_0 \frac{\pi B_c c_p D_c (\omega_m - \omega_p)}{1 - \frac{D_1}{D_2}} \quad (16.67)$$

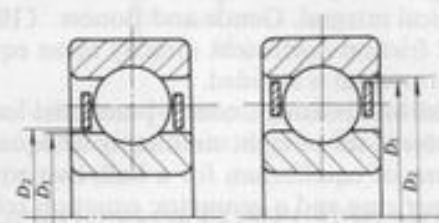


Fig. 16.12 Guiding of the cage

where

B_c = width of guiding surface of a cage

D_c = diameter of guiding surface of a cage

D_1 and D_2 = diameters of guiding surfaces, as shown in Fig. 16.12.

The sum of moments acting on a steady-state cage should satisfy the equilibrium equation, i.e.

$$D_m \sum_{j=1}^z F_{cj} \pm D_c F_{cL} = 0 \quad (16.68)$$

in which, the upper sign refers to the inner ring guiding and the lower sign refers to the outer ring guiding.

Then, the basic equations are composed of equations (16.62) to (16.66) and equation (16.68). Unknown variables are Δ_i , $i = 1, 2, \dots, 5$, as well as rotational speed of the cage ω_m .

It can be seen that it is necessary to seek more simplified models due to the complexity of the above equations.

Some simplified models

As mentioned before, a high speed ball bearing subjected to pure axial load is a typical application. It is valuable to introduce two simple models.

Gentle-Boness model

This model is similar to the Harris's method described above except calculation of traction forces in the lubrication film. The calculation of traction forces based on the Newtonian fluid is required to deal with

the relative sliding velocity at each point of contact area, and then to compute a numerical integral. Gentle and Boness (107) proposed to consider the total friction coefficient directly upon equation (6.40) so that the numerical integral is avoided.

For a high speed ball bearing under a pure axial load, the problem is simplified to solve a set of eight simultaneous equations, which include five equations of equilibrium for a ball, two equations of equilibrium for the inner ring and a geometric equation reflecting the variation of the curvature centre position of raceway grooves. Eight unknown variables are contact angles α_i and α_o , contact loads Q_i and Q_o , relative sliding velocities V_i and V_o , as well as altitude angles β and β' of a ball.

Computations indicate that the selection of initial values has a great important influence on the convergence of the equations.

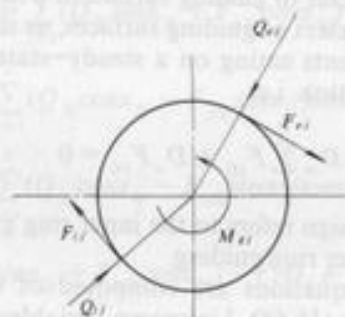


Fig. 16.13 Forces acting on a ball

Assumption of the constant friction coefficient

Considering the loading condition of a ball shown in Fig.16.13, its gyroscopic moment is balanced by the traction force of the lubrication film. Therefore

$$M_{ei} = 0.5D_b(F_i + F_o)$$

Values of friction coefficients are based on data given by two-disc testing machines.

Assuming that traction coefficients at contact areas of the inner and outer raceways are about the same, i.e.

$$\frac{F_i}{Q_i} = \frac{F_o}{Q_o} \quad (16.69)$$

then, the traction forces can be simply decided. Based on this assumption, Reference (108) gives an example of high speed ball bearings applied to machine tools.

APPLICATIONS OF COMPUTING MODELS

The computer aided design of rolling bearings has been developing successfully since the middle of the sixties. In order to obtain the high reliability of rolling bearings for special applications, it is necessary to predict the performance and the life of a rolling bearing more accurately. On the other hand, the development of the tribological theory and the successful application of computers offer a possibility for these. Recently, some special programs of the computer aided design for rolling bearings have been submitted for applications, or even commodities.

Typical input and output

In References (109), (110), the program functions of the computer aided design for rolling bearings are evaluated. Most of them are developed by the Mechanical Technology Incorporation and the SKF Company.

Computer program upon raceway control theory

Program TABACY developed by the SKF Company may be applied to analyse a high speed ball bearing subjected to combined axial and radial loads, as well as a relative tilting of inner and outer rings. The outer raceway control is assumed, that is, all the spinning occurs at contact areas between balls and the inner raceway.

Input data include groove curvature radii r_i and r_o of inner and outer raceways, inner and outer raceway diameters D_i and D_o , ball diameter D_b , radial clearance u_r , number of balls Z , tilting angle of shaft θ , rotational speed n , radial load F_r , modulus of elasticity E , Poisson ratio ν , material density ρ , viscosity of lubricant η_0 , pressure coefficient of viscosity α_1 , etc.

Output data include contact loads Q_i and Q_o between ball and inner or outer raceway at angular position ψ_j , contact angles α_j , and

Table 16.2 Print format of program TABACY

INPUT	GROOVE		RING		BALL
	RADIUS(IN)		DIAMETER(IN)		DIAMETER
	INNER	OUTER	INNER	OUTER	(IN)
	.3575	.3575	3.6904	4.3804	.6875
DIAMETER	NUMBER	SHAFT	SHAFT	RADIAL	THRUST
CLEARANCE	OF	DEFLECTION	SPEED	LOAD	LOAD
(IN)	BALL	SLOPE(DEC)	(RPM)	(LBS)	(LBS)
.0052	16.	.1000	15000.	317.	800.
YOUNGS	POISSONS	MATERIAL	LUBRICANT	PRESSURE	
MODULUS		DENSITY	VISCOSITY	EXPONENT	
(PSI)	RATIO	(LB/IN ³ * 3)	(LB-SEC/IN ² * 2)	(IN * 2/LB)	
.2900+08	.25	.283	.1218-04	.1100-03	
OUTPUT	CONTACT FORCES		CONTACT ANGLE		ORBITAL
BALL	(LBS)		(DEG)		VELOCITY
No.	OUTER	INNER	OUTER	INNER	(RPM)
1	.2856+03	.1789+03	.1027+02	.1916+02	.6332+04
2	.2566+03	.1490+03	.1006+02	.2062+02	.6359+04
3	.2069+03	.9671+02	.9311+01	.2500+02	.6463+04
4	.1838+03	.6800+02	.8703+01	.3116+02	.6660+04
5	.1841+03	.6191+02	.9155+01	.3741+02	.6896+04
6	.1945+03	.6779+02	.1063+02	.4275+02	.7106+04
7	.2066+03	.7833+02	.1250+02	.4671+02	.7252+04
8	.2157+03	.8779+02	.1404+02	.4909+02	.7329+04
9	.2190+03	.9142+02	.1463+02	.4988+02	.7351+04
BALL	RPM ABOUT	VECTOR	CENTRIFUGAL	GYROSCOPIC	
	OWN CENTER	ATTITUDE	FORCE	MOMENT	
No.		(DEG.)	(LBS)	(LB-IN)	
1	-.4344+05	.8780+01	.1107+03	-.2714+01	
2	-.4363+05	.8601+01	.1117+03	-.2682+01	
3	-.4435+05	.7960+01	.1154+03	-.2567+01	
4	-.4571+05	.7440+01	.1225+03	-.2549+01	
5	-.4733+05	.7827+01	.1314+03	-.2874+01	
6	-.4874+05	.9085+01	.1395+03	-.3537+01	
7	-.4970+05	.1069+02	.1452+03	-.4324+01	
8	-.5018+05	.1201+02	.1483+03	-.4947+01	
9	-.5032+05	.1251+02	.1493+03	-.5182+01	
LIO FATIGUE LIFE (MIL.REV.)					
	OUTER	INNER	TOTAL		
	.6877+04	.7557+04	.3839+04		
MINIMUM FILM THICKNESS (IN)			RESISTING		
OUTER	INNER		MOMENT(LB-IN)		
.2461-03	.2260-03		-.1380-03		

Table 16.3 Question-answer procedure of program STARBRG

THE PROGRAM WILL RETURN TO WHICH QUESTION(S)
 IF 99999 IS ENTERED AS THE LAST VALUE OF QUESTIONS 4-6
 1. ENTER TITLE? BALL BEARING FOR MAIN SHAFT
 2. IS IT UNDER PURE THRUST LOAD ONLY? YES
 3. DO YOU WANT AN EXPLANATION OF TERMINOLOGY? YES
 Z = NUMBER OF BALLS
 TR = THRUST LOAD, NEWTONS
 RPM = REVOLUTIONS PER MIN.
 D = BALL DIAMETER, MM
 FO = OUTER RACE CONFORMITY
 FI = INNER RACE CONFORMITY
 BET = NO LOAD CONTACT ANGLE, DEG.
 SRR = SPIN TO ROLL RATIO
 CAP = LOAD CAPACITY OF A CONTACT, NEWTONS
 DM = PITCH DIAMETER, MM
 CP = DYNAMIC VISCOSITY AT OPERATING TEMP.,
 CENTIPOISE
 ALFA = PRESSURE VISCOSITY INDEX, MM² / N
 SUT = OIL-AIR SURFACE TENSION, N / M
 CAP = LOAD CAPACITY PER RING, NEWTON
 4. DO YOU WANT TO COMPUTE FILM THICKNESS? YES
 ENTER IN ORDER SHOWN: CP, ALFA, SUT? 2.5, 0.0145,
 0.028
 5. ENTER IN ORDER SHOWN: Z, D, DM, FO, FI, BET?
 21, 20.6375, 159.4885, 0.521, 0.516, 26
 6. ENTER TR, RPM? 9996, 20000

α_{ej} , orbital speed of ball n_{mj} , rotational speed of ball around its own axis n_{bj} , ball altitude angle β_j , ball centrifugal force F_{cj} , gyroscopic moment M_{gj} , bearing life L_{10} , minimum film thickness h_l and h_e , external moment M , etc.

The print format of program TABACY is listed in Table 16.2. It should be noticed that the film thicknesses are computed separately upon the bearing parameters and its operating conditions.

They are not the solution of basic equations including the elastohydrodynamic lubrication theory.

Table 16.4 Print format of program STARBRG

BALL BEARING FOR MAIN SHAFT		
INPUT		
NO. OF BALLS	21.	
BALL DIAMETER	20.64 MM	
OUTER RING CONFORMITY	.5210	
INNER RING SPEED	20000. RPM	
THRUST LOAD	9996.N	
DYNAMIC VISCOSITY	2.5 C.P.	
PRESSURE VISCOSITY INDEX	.0145MM2 / N	
PITCH DIAMETER	159.49 MM	
INNER RING CONFORMITY	.5160	
FREE CONTACT ANGLE	26.00 DGR.	
SURFACE TENSION OF OIL	.0280 N / M	
OUTPUT		
	OUTER RING	INNER RING
CONTACT FORCE(N)	3742.4	670.8
CONTACT ANGLE (DGR.)	7.3	45.2
ROLLING VELOCITY(M / S)	95.7	74.7
MAX. PRESSURE(N / MM2)	1799.3	1101.8
SEMI-MAJOR AXIS(MM)	2.718	1.691
AXIS RATIO OF ELLIPSE	7.4	9.8
LOAD CAPACITY(N)	9360.3	6461.5
SPIN ROLL RATIO	0.002	0.699
CAGE SPEED	10156.RPM	
END FILM THICKNESS(MM)	0.00042	0.00037
MENISCUS DISTANCE(MM)	0.1223	0.1112
STARV. REDUCTION FACTOR	0.5479	0.6167
WHICH QUESTION(S)?(O = STOP)?O		

Computer program for elastohydrodynamic lubrication

Program STARBRG developed by the SKF Company belongs to the person-computer conversation type. On the basis of the elastohydrodynamic lubrication theory, it can compute internal parameters of a high speed ball bearing subjected to a pure axial load, contact load of balls, contact angle, film thickness in starvation, etc.

The question-answer procedure shown in program STARBRG is presented in Table 16.3. Its print format of input and output data is listed in Table 16.4.

Input data include number of ball Z , ball diameter D_b , pitch diameter D_m , groove curvature radius coefficients f_i and f_e , rotational speed n_i of inner ring, axial load F_a , initial contact angle α , lubricant viscosity η_0 , pressure coefficient of viscosity α_1 , surface tension of lubricant T_l , etc.

Output data include contact loads Q_i and Q_e between balls and inner or outer raceway contact angles α_i and α_e , rolling velocities u_{roll} and u_{eroll} , sizes of contact ellipse a_i , a_e , b_i and b_e , Hertzian contact pressure p_{oi} and p_{oe} , dynamic load ratings C_i and C_e , spin to roll ratios $(\omega_s / \omega_{roll})_i$ and $(\omega_s / \omega_{roll})_e$, cage speed n_m , film thicknesses h_i and h_e , reduction factor of starvation, etc.

Brief of dynamic design theory

Considering the entire dynamic process of a rolling bearing from the beginning of its rotation, one may establish a set of the differential equations of motion of the bearing elements, that is, the motion equations of the ball mass centre in an inertial coordinate system

$$m\ddot{x} = F_x \quad (16.70)$$

$$m(\ddot{r} - r\dot{\phi}^2) = F_r \quad (16.71)$$

$$m(r\ddot{\phi} + 2\dot{r}\dot{\phi}) = F_\phi \quad (16.72)$$

and ball rotation motion around its mass centre

$$I\ddot{\omega}_x = G_x \quad (16.73)$$

$$I\ddot{\omega}_y - I\omega_z\dot{\phi} = G_y \quad (16.74)$$

$$I\ddot{\omega}_z + I\omega_y\dot{\phi} = G_z \quad (16.75)$$

The coordinate system is shown in Fig.16.14. Forces and moments acting on a ball are expressed as F_x , F_r , F_ϕ and G_x , G_y , G_z respectively. The components of the ball angular speed are ω_x , ω_y and ω_z .

It is necessary to deal with several supplementary expressions, which are interactions between balls and rings, interactions between balls and cage, drag force of lubricant, traction force, etc. No kinematic constraint about the ball motion is needed. Given the initial conditions, the differential equations can be solved numerically to give a

real simulation of bearing performance. It is easy to see, that the dynamic design theory can be used to describe the positions of balls and the cage, internal sliding at any time and to analyse the transition process of motion, such as the sudden variations of load or speed, unstable motion of the cage, etc. All these could not be solved by the quasi-static design theory.

The comparison between the quasi-static and the dynamic design theories is made, as listed in Table 16.5. Obviously, the more the considered factors, the more complex the computation will be.

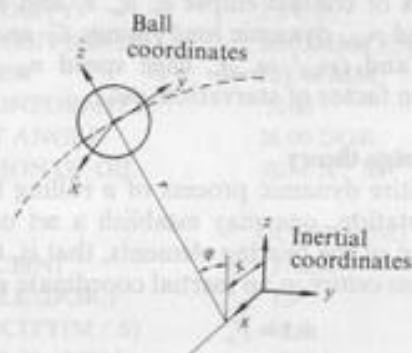


Fig. 16.14 Coordinate system

In so far as the load distribution and the bearing life are concerned, the quasi-static design theory may give satisfactory results. On the other hand, although the dynamic design theory is superior theoretically, up to now it is still hard to obtain practical applications of the dynamic design theory.

Selection instruction

The following aspects have to be considered during the selection of design theory models.

- (1) The raceway control theory can provide reliable prediction regarding the load distribution and the fatigue life of rolling bearings. If the traction force of the elastohydrodynamic lubrication in the bearing is higher, then the calculation of internal motion relationships is also approximately valid.

Table 16.5 Comparison between two theories

	Quasi-static theory	Dynamic theory
Equations	Nonlinear algebraic equations of static equilibrium	Differential equations of motion
Constraints	Kinematic constraints are required	No constraint is required
Computing convergence	Convergence is connected with the traction force model	Convergence is not connected with the traction force model
Time dependence	No time dependence	Solution is a function of the time
Stability analysis	Impossible to consider motion instability of bearing components	Valid for motion instability process of bearing components
Computing time	Less computing time	Much longer computing time is needed
Effectiveness	Effective for computing load distribution, life and stiffness	Effective for real-time simulation of bearing motion performance

Table 16.6 Comparison of computed results

Parameters	Raceway control theory	Dynamic design theory	Dynamic design theory
Angular speed of a ball (r/min)			
ω_x	57670	57410	57950
ω_y	0	51	121
ω_z	3591	3990	580
Orbital speed of balls (r/min)	7255	7214	7247
Spin to roll ratio			
outer ring	0	0.00833	0.0823
inner ring	0.689	0.679	0.754
Contact angle (deg)			
outer ring	4.074	4.068	5.358
inner ring	37.70	37.68	37.51
Contact load (N)			
outer ring	488.97	480.07	484.52
inner ring	76.57	76.61	76.93
Notes		Semi-empirical model of traction force	Constant traction coefficient 0.007

In Reference (103), the computed results on the raceway control theory are compared with those of the dynamic design theory, as shown in Table 15.6.

The bearing parameters considered are as follows:

$Z=19$, $f_i=f_e=0.52$, $D_b=15.081$ mm, $D_m=105$ mm, $\alpha=25^\circ$, $F_a=889.84$ N, $n_i=15000$ rpm.

It can be seen that the computed results of contact loads, contact angles and orbital rotational speed of balls upon the two design theories have no major difference. But the computed angular speeds of balls are dependent on the design theories, especially at low traction coefficient.

- (2) The elastohydrodynamic lubrication theory can predict the internal sliding velocity and the rotational speeds of rolling elements. But the convergence is very much connected with the traction force model selected and initial iterative values.

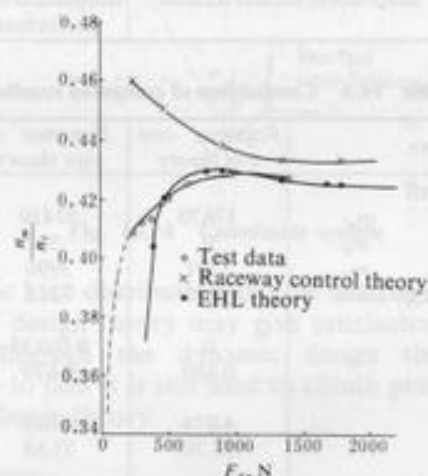


Fig. 16.15 Rotational speed of cage

One of the common models of failure in high speed ball bearings is that the orbital sliding of balls damages the inner raceway. The elastohydrodynamic lubrication design theory can give a more effective analysis than the raceway control theory. Reference (107) presents the influence of the different design methods on the cage speed

and compares this with test data, as shown in Fig. 16.15. It can be seen that the elastohydrodynamic lubrication theory gives more satisfactory prediction. A considerable distinction is found between the test data and the raceway control theory.

- (3) The dynamic design theory can simulate the transition process of the bearing motion. Then, it may evaluate the influences of different factors and decide which to choose in order to reduce internal sliding and wear of a bearing in special operating conditions.

CHAPTER SEVENTEEN

Computer aided design of high speed roller bearings

One type of the typical high speed roller bearings is the cylindrical roller bearing applied to the main-shaft of aircraft gas turbine engine. Their major operating characteristics are

- (1) extremely high rotational speed at about $DN = 1.5 \times 10^6$ to 2.5×10^6 , DN is a measure of bearing speed, the bore diameter D in mm, multiplied by the rotational speed in rpm;
- (2) light radial load.

Due to the influence of centrifugal forces at high speed, rollers are pressed to the outer raceway of the bearing. Then, the traction force between rollers and the inner raceway is not enough to prevent the relative sliding of rollers. Sometimes, severe wear takes place.

The mechanism of the relative sliding can be successfully analysed by the elastohydrodynamic lubrication theory. Dowson first proposed an elastohydrodynamic lubrication model for cylindrical roller bearings [10]. Harris established a basic mathematic model of high speed roller bearings [2]. This model is further developed in References [111] and [112]. Similar to the quasi-static design theory of ball bearings, the geometric and mechanical relationships are considered and a set of nonlinear algebraic equations is derived. Therefore, it is called the quasi-static design theory of high speed roller bearings, which will be described in the first paragraph of this chapter.

To analyse high speed roller bearings, Kleckner and Pirvics developed a complicated program, which includes a main program and 131 subroutines. Using this program, the effects of load, speed and misalignment of the bearing, as well as the outer ring temperature and the amount of heat transferred to the lubricant can be predicted quite well [113]. Gupta presented the dynamic design theory of cylindrical roller bearings in 1979. In the second paragraph, the effect of the bearing misalignment on the load distribution will be explained.

DESIGN THEORY OF ELASTOHYDRODYNAMIC LUBRICATION

Assume a high speed roller bearing subjected to a radial load. Due to the action of centrifugal forces, rollers are pressed to the outer raceway and are separated from the inner raceway at the unloading region. Similar to ball bearings, the contact load varies with the angular positions of rollers in cylindrical roller bearings.

Motion conditions of rollers

Fig.17.1 shows a steady-state running condition of a high speed roller bearing. For consideration of the elastohydrodynamic lubrication, the major influential factors concerned are the relative sliding velocities between the roller and raceways, as well as the average velocities at contact areas.

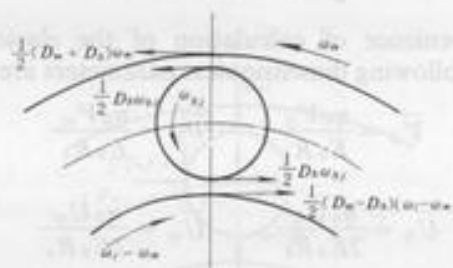


Fig. 17.1 Motion condition of a cylindrical roller bearing

The angular speed of inner ring is expressed as ω_i . An equivalent relative motion condition is proposed, in which the roller centre is fixed and the outer ring rotates in the opposite direction from the inner ring.

The relative sliding velocity components at contact points between the roller and inner or outer raceway are

$$V_{ij} = \frac{1}{2} (D_o - D_r)(\omega_i - \omega_o) - \frac{1}{2} D_r \omega_r \quad (17.1)$$

$$V_{oj} = \frac{1}{2} (D_o + D_r)\omega_o - \frac{1}{2} D_r \omega_r \quad (17.2)$$

Substituting $\gamma = D_r / D_o$ into the above equations yields

$$V_{ij} = \frac{D_m}{2} [(1 - \gamma)(\omega_i - \omega_m) - \gamma\omega_{hj}] \quad (17.3)$$

$$V_{ej} = \frac{D_m}{2} [(1 + \gamma)\omega_m - \gamma\omega_{hj}] \quad (17.4)$$

where

ω_m = angular speed of the cage

ω_{hj} = rotational angular speed of the roller around its own axis at angular position ψ_j .

Similarly, the average velocities at the same contact points between the roller and raceways at angular position ψ_j

$$U_{ij} = \frac{D_m}{2} [(1 - \gamma)(\omega_i - \omega_m) + \gamma\omega_{hj}] \quad (17.5)$$

$$U_{ej} = \frac{D_m}{2} [(1 + \gamma)\omega_m - \gamma\omega_{hj}] \quad (17.6)$$

For the convenience of calculation of the elastohydrodynamic lubrication, the following dimensionless parameters are introduced

$$\bar{V}_{ij} = \frac{\eta_0 V_{ij}}{E_0 R_i}; \quad \bar{V}_{ej} = \frac{\eta_0 V_{ej}}{E_0 R_e}$$

$$\bar{U}_{ij} = \frac{\eta_0 U_{ij}}{2E_0 R_i}; \quad \bar{U}_{ej} = \frac{\eta_0 U_{ej}}{2E_0 R_e}$$

where

R_i and R_e = equivalent curvature radii at contact points with the inner and the outer raceways respectively

E_0 = equivalent modulus of elasticity, as shown by equation (6.8).

Pure rolling occurs at the unloading region on the outer raceway, that is, $V_{ej} = 0$. From equation (17.4), then

$$\omega_{hs} = \left(\frac{1 + \gamma}{\gamma} \right) \omega_m \quad (17.7)$$

ω_{hs} indicates the rotational angular speed of the roller around its own axis at the unloading region and is not dependent on angular position ψ_j . Correspondingly, the average velocity between the roller and the outer raceway at unloading region is

$$U_{es} = \frac{D_m}{2} [(1 + \gamma)\omega_m - \gamma\omega_{hs}] \quad (17.8)$$

and its dimensionless expression

$$\bar{U}_{es} = \frac{\eta_0 U_{es}}{2E_0 R_e}$$

Analysis of forces

For simplicity, a plane problem is considered, as shown in Fig. 17.2. The following assumptions are proposed

- (1) the influence of a gap between roller and pocket of a cage is neglected and a roller entering into the unload region from loading region is completed at an instant;
- (2) the contact load between a roller and the cage is vertical to the side wall of the cage pocket.

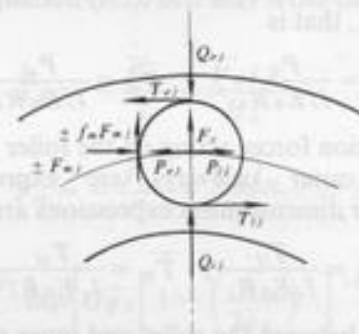


Fig. 17.2 Force analysis of a roller

The orbital angular speed is expressed as ω_m . Then, the centrifugal force of a roller

$$F_c = \frac{1}{2} m_b D_m \omega_m^2 \quad (17.9)$$

where

m_b = the roller mass.

The forces P_{ij} and P_{ej} acting on a roller at angular position

ψ_j by inner and outer raceways. Dowson suggested that while the material factor is 5000 they can be calculated by the following equations (10).

$$\bar{P}_i = 1.43(1 - \gamma)\bar{U}_i^{0.71}$$

$$\bar{P}_o = 1.43(1 + \gamma)\bar{U}_o^{0.71}$$

Harris considered that

$$1.43(1 \pm \gamma)\bar{U}^{0.71}(5000/G)^{0.3} = 18.4(1 \pm \gamma)\bar{U}^{0.71}G^{-0.3}$$

and proposed approximate equations

$$\bar{P}_i = 18.4(1 - \gamma)G^{-0.3}\bar{U}_i^{0.7} \quad (17.10)$$

$$\bar{P}_o = 18.4(1 + \gamma)G^{-0.3}\bar{U}_o^{0.7} \quad (17.11)$$

where

$\bar{P}_i$ and $\bar{P}_o$ = dimensionless pressures corresponding to P_i and P_o , that is

$$\bar{P}_i = \frac{P_i}{l_e E_0 R_i}; \quad \bar{P}_o = \frac{P_o}{l_e E_0 R_o}$$

The tangential friction forces acting on the roller at angular position ψ_j by inner and outer raceways are expressed as T_i and T_o respectively. Their dimensionless expressions are

$$\bar{T}_i = \frac{T_i}{l_e E_0 R_i}; \quad \bar{T}_o = \frac{T_o}{l_e E_0 R_o}$$

The normal forces between the roller and inner or outer raceway at angular position ψ_j are represented by Q_i and Q_o . Assuming that the bearing is in a steady-state running condition, the forces acting on the roller should satisfy the following equations of equilibrium

$$P_i + T_i - P_o - T_o \pm F_{mj} = 0 \quad (17.12)$$

and

$$Q_i + F_c - Q_o \pm f_m F_{mj} = 0 \quad (17.13)$$

where

F_{mj} = force acting on the roller by a pocket of the cage. It should be noted that directions of the force are different in loading and unloading regions

f_m = friction coefficient between the roller and the cage pocket.

From the equation of moment equilibrium

$$T_i - T_o \pm f_m F_{mj} = 0 \quad (17.14)$$

Similar to ball bearings, the tangential traction forces acting on rollers may be calculated by different models.

Newtonian fluid model

Considering an elastohydrodynamic lubrication model regarding a roller rolling with sliding on a plane, Dowson and Harris separately assumed that the tangential traction force is composed of a pure rolling traction force and a sliding traction force. They are represented in the following dimensionless expression

$$\bar{T} = \frac{T}{E_0 R} = -\frac{\bar{P}}{2} + \frac{V}{E_0 R} \int \frac{\eta}{h} dx$$

Using viscosity equation (6.3), one may write the tangential traction force as follows

$$\bar{T}_i = -\frac{\bar{P}_i}{2} + \frac{\bar{V}_i I_i}{H_i} \quad (17.15)$$

$$\bar{T}_o = -\frac{\bar{P}_o}{2} + \frac{\bar{V}_o I_o}{H_o} \quad (17.16)$$

where

$$I_i = 2 \int_0^{\bar{q}_i} \exp \left\{ G \bar{q}_i \left[1 - \left(\frac{\bar{x}}{2 \bar{q}_i} \right)^2 \right]^{1/2} \right\} d\bar{x} \quad (17.17)$$

$$I_o = 2 \int_0^{\bar{q}_o} \exp \left\{ G \bar{q}_o \left[1 - \left(\frac{\bar{x}}{2 \bar{q}_o} \right)^2 \right]^{1/2} \right\} d\bar{x} \quad (17.18)$$

$$\bar{q}_i = \sqrt{\frac{Q_i}{2\pi}}; \quad \bar{q}_o = \sqrt{\frac{Q_o}{2\pi}}$$

H_i and H_o = film thicknesses calculated by equation (6.24).

The integrals in equations (17.17) and (17.18) must be numerically computed.

Semi-empiric model

As mentioned in Chapter Sixteen, it is assumed that the traction coefficient depends on three parameters, that is, G_1 , G_2 and G_3 . Then, the tangential traction forces acting on the roller at angular position ψ_j by inner and outer raceways are

$$T_{ij} = -9.2G^{-0.3}\bar{U}_{ij}^{0.7}l_e E_0 R_i + \frac{|V_{ij}|}{V_{ij}} f(T_e) P_{ij} \quad (17.19)$$

$$T_{ej} = -9.2G^{-0.3}\bar{U}_{ej}^{0.7}l_e E_0 R_e + \frac{|V_{ej}|}{V_{ej}} f(T_e) P_{ej} \quad (17.20)$$

where

$f(T_e)$ = friction coefficient, a function of G_1 , G_2 , G_3 , and temperature T . It can be written as

$$f(T_e) = F(G_1, G_2, G_3, 30^\circ\text{C}) \left(\frac{30}{T_e} \right)^{0.149}$$

T_e = temperature at entry of the elastohydrodynamic lubrication, $^\circ\text{C}$.

Constant friction coefficient

The calculation is significantly simplified. It is only used for a direct comparison.

Geometric relation of deflections

Total elastic contact deflection between a roller and raceways at angular position ψ_j is

$$\delta_j = \delta_{ij} + \delta_{ej} \quad (17.21)$$

where

δ_{ij} and δ_{ej} = elastic contact deflection between the roller and inner or outer raceway respectively.

From equations (4.3) and (6.24), the elastic deflections may be expressed as

$$\delta_{ij} = K_i Q_{ij}^{0.9} + Q_{ij} \left(\frac{\partial h_{ij}}{\partial Q_{ij}} \right)_{Q_{ij}} \quad (17.22)$$

$$\delta_{ej} = K_e Q_{ej}^{0.9} + Q_{ej} \left(\frac{\partial h_{ej}}{\partial Q_{ej}} \right)_{Q_{ej}} \quad (17.23)$$

The film thicknesses between the roller and inner or outer raceway at angular position ψ_j are determined by equation (6.24). It can be seen that the deflections should satisfy the following

$$\delta_j = \Delta x \sin \psi_j + \Delta y \cos \psi_j - \frac{u_r}{2} + h_{ij} + h_{ej} \quad (17.24)$$

where

Δx = deflection of the inner ring in the x direction

Δy = deflection of the inner ring in the y direction

u_r = radial clearance of the bearing.

Basic equations

The calculation theory for high speed roller bearings is summed up to solve a set of basic equations, which include equilibrium equations (17.12), (17.13), (17.14) and geometric relationship of deflections (17.24), as well as an equilibrium equation of ring and cage. Therefore,

$$F_x = \sum_{j=1}^z Q_{ij} \sin \psi_j \quad (17.25)$$

$$F_y = \sum_{j=1}^z Q_{ij} \cos \psi_j \quad (17.26)$$

and

$$\sum_{j=1}^z F_{mj} = 0 \quad (17.27)$$

where

F_x and F_y = external load components acting on a bearing.

Number of the basic equations are $4Z+3$. Unknown variables are rotational angular speed of a roller ω_{ij} , normal loads acting on a roller Q_{ij} and Q_{ej} , force acting on a roller by a cage F_{mj} , angular speed of cage ω_m , as well as relative deflection between two rings Δx and Δy . Some supplementary expressions introduced above are required in the calculation. Obviously, the nonlinear simultaneous equations must be numerically solved, such as the Newton-Raphson method.

Using the symmetrical condition, one may reduce the number of the basic equations. To choose initial values properly will decrease required times of the iteratives. For example, initial values of ω_m and ω_{ij} may be established by the simple kinematic relations (5.4) and (5.7).

If some assumptions are added for simplification, it will be more convenient to solve the basic equations.

Simplified equations

To simplify these equations, Harris proposed the following assumptions

- (1) in the unloading region, the tangential traction force acting on a roller through oil film by an inner raceway is neglected;
- (2) in unloading region, the values of forces acting on rollers by a cage are the same, expressed as F_{mu} ;
- (3) in the loading region, the values of forces acting on rollers by a cage are also the same, expressed as F_m .

From assumption (1), equation (17.12) in the unloading region is simplified to the following form, which is not dependent on the angular position

$$\bar{P}_e + \bar{T}_e - \bar{F}_{mu} = 0 \quad (17.28)$$

where, the symbol $\bar{F}_{mu}$ is

$$\bar{F}_{mu} = \frac{F_{mu}}{l_e E_0 R_e}$$

Substituting equations (17.11) and (17.16) into equation (17.28) yields

$$-9.2(1+2\gamma)G^{-0.3}\bar{U}_{e0}^{0.7} + \bar{F}_{mu} = 0 \quad (17.29)$$

Using the equilibrium equation of the cage and from assumptions (2) and (3), the force F_m acting on a roller by the cage in loading region can be deduced. Its dimensionless expression is

$$\bar{F}_m = \frac{Z_a \bar{F}_{mu}}{Z - Z_a} \quad (17.30)$$

Considering the maximally loading roller position in loading region and indicating its corresponding quantities by a subscript "0", the equilibrium equation (17.12) may be rewritten as

$$\bar{P}_0 + \bar{T}_0 - \frac{R_e}{R_i} \left(\bar{P}_{e0} + \bar{T}_{e0} + \frac{Z_a \bar{F}_{mu}}{Z - Z_a} \right) = 0 \quad (17.31)$$

Substitution of equations (17.10), (17.11), (17.15), (17.16) and film thickness equation (6.24) into equation (17.31) gives

$$\begin{aligned} & -9.2G^{-0.3}\bar{U}_{00}^{0.7} + \frac{\bar{V}_{e0}I_{e0}\bar{Q}_{00}^{0.13}}{1.6G^{0.6}\bar{U}_{00}^{0.7}} + 18.4(1-\gamma)G^{-0.3}\bar{U}_{00}^{0.7} \\ & - \frac{R_e}{R_i} \left[-9.2G^{-0.3}\bar{U}_{00}^{0.7} + \frac{\bar{V}_{e0}I_{e0}\bar{Q}_{00}^{0.13}}{1.6G^{0.6}\bar{U}_{00}^{0.7}} \right. \\ & \left. + 18.4(1+\gamma)G^{-0.3}\bar{U}_{00}^{0.7} + \frac{Z_a \bar{F}_{mu}}{Z - Z_a} \right] = 0 \end{aligned} \quad (17.32)$$

Neglecting the friction forces between rollers and pockets of the cage and using the equilibrium equation (17.14) of the roller in loading region yield the following dimensionless equation

$$\bar{T}_0 + \frac{R_e}{R_i} \bar{T}_{e0} = 0 \quad (17.33)$$

For the maximally loading roller position, substitution of equations (17.15) and (17.16) into (17.33) gives

$$\begin{aligned} & -9.2G^{-0.3}\bar{U}_{00}^{0.7} + \frac{\bar{V}_{e0}I_{e0}\bar{Q}_{00}^{0.13}}{1.6G^{0.6}\bar{U}_{00}^{0.7}} \\ & + \frac{R_e}{R_i} \left(-9.2G^{-0.3}\bar{U}_{00}^{0.7} + \frac{\bar{V}_{e0}I_{e0}\bar{Q}_{00}^{0.13}}{1.6G^{0.6}\bar{U}_{00}^{0.7}} \right) = 0 \end{aligned} \quad (17.34)$$

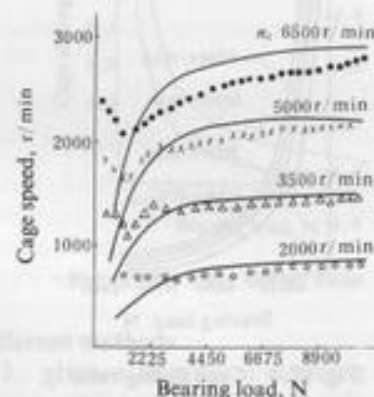


Fig. 17.3 Rotational speed of the cage

Then equations (17.7), (17.29), (17.32) and (17.34) are a set of nonlinear equations. Unknown variables include rotational angular speed of the roller ω_{b0} , angular speed of the cage ω_m , rotational angular speed of the roller in unloading region ω_{bu} , the force acting on the roller by the cage in unloading region F_{mu} , etc.

Harris gave a numerical example by these simplified equations. A cylindrical roller bearing, roller diameter $D_b = 14$ mm, number of rollers $Z = 36$, effective length of a roller $l_e = 20$ mm, pitch diameter $D_m = 182.88$ mm, radial clearance $u_r = 0.064$ mm, lubricant MIL-L-7808. The computed relation between the rotational speed of the cage and the bearing load is shown in Fig. 17.3.

For the convenience of comparison, a cage sliding ratio is defined

$$S_f = 1 - \frac{\omega_m}{\omega_{mt}}$$

where

ω_{mt} = computed angular speed of the cage upon the simple kinematic relationship.

The relation between the cage sliding ratio and the bearing load is shown in Fig. 17.4.

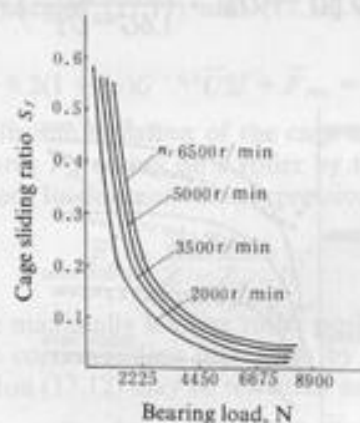


Fig. 17.4 Cage sliding ratio

From the computed results shown in the above figures, the following conclusions can be obtained.

- (1) An approximate correspondence is found between the prediction of rotational speed of the cage and test data. The higher the bearing load, the more accurate the prediction will be. On the other hand, the higher the bearing speed, the larger the deviation between the prediction and the test data will be. These indicate that the simplified equations have a certain limit to application.
- (2) The cage sliding ratio is relatively larger at the lower bearing load. Therefore, a light load or out of loading is harmful or even dangerous to cylindrical roller bearings.
- (3) Reducing the number of rollers properly may decrease the cage sliding. It can be seen from Fig. 17.5 that proper reduction of the number of rollers, for example, from $Z = 36$ to $Z = 18$, leads to increase the load on each roller and to decrease the sliding significantly.

For the selection of a bearing, it is necessary to consider all of its operating conditions. Making a choice solely upon the dynamic load rating is not always suitable.

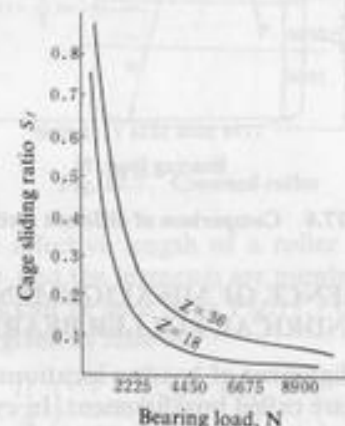


Fig. 17.5 Cage sliding ratio

Comparison of different methods

Reference [11] gives a numerical example of a cylindrical roller bearing for the comparison of different calculating methods. The geometric parameters of the bearing are: bore diameter 65 mm, roller diameter 21 mm, effective length of a roller 17.68 mm, number of rollers

lers 12, pitch diameter 102.49 mm, experimental rotational speed 15100 rpm, lubricant MIL-L-7808. The comparison between the calculation and experimental data is shown in Fig.17.6.

Curve 1 represents the computed results based on the simplified equations and Curve 2 represents those based on the above basic equations. Finally the influences of friction at the cage guiding surface and the lubricant drag resistance are supplemented in addition to the above basic equations. Curve 3 indicates the results of this improvement. It can be seen that Curve 3 is very close to the experimental data. Of course, it requires correspondingly more complicated computation.

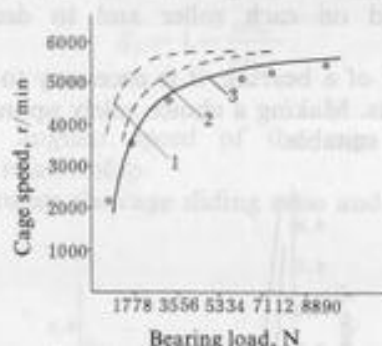


Fig. 17.6 Comparison of different methods

INFLUENCE OF MISALIGNMENT ON CYLINDRICAL ROLLER BEARINGS

Inaccuracies in the alignment of bearing locations due to mounting or shaft deflection, etc. are called misalignment. In cylindrical roller bearings, the misalignment leads to nonuniform loading over the roller length. Particularly at high speed, it brings about irregular rotations of the roller and the increase of stress at the roller end. Therefore, the misalignment has a great influence on the bearing life. Obviously, how to calculate the load variation along the length of a roller in contact is the basic problem for influential analysis of the misalignment. Harris proposed a method for analysing the roller axial load distribution without consideration of the centrifugal forces of rollers (2).

Program CYBEAN developed by Kleckner (113) may be used to compute the effect of the bearing misalignment. In this paragraph, an improved method in Reference (112) is introduced. It can consider the influence of the centrifugal forces of rollers for high speed roller bearings. On the other hand, it does not require solving a large number of nonlinear equations and the convergence of its solving method will be more rapid.

Geometric relation of deformations

To eliminate the stress concentration at the roller ends, commonly crowned rollers shown in Fig.17.7 are used. The central part contour of the rollers are straight lines.

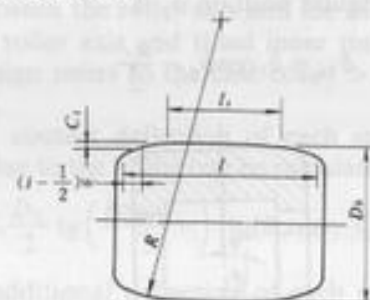


Fig. 17.7 Crowned roller

Assume that the effective length of a roller is divided into n segments with width w and the segments are numbered from the left end, i.e. $i = 1, 2, \dots, n$. It can be seen from Fig.17.7 that the crown drop of the i -th segment is given as follows

$$C_i = \left(R^2 - \frac{l_e^2}{4} \right)^{1/2} - \left\{ R^2 - \left[\frac{l_e}{2} - \left(i - \frac{1}{2} \right) w \right]^2 \right\}^{1/2}$$

$$0 \leq \left(i - \frac{1}{2} \right) w \leq \frac{l_e - l_s}{2}$$

$$C_i = 0; \quad \frac{l_e - l_s}{2} \leq \left(i - \frac{1}{2} \right) w \leq \frac{l_e + l_s}{2} \quad (17.35)$$

$$C_i = \left(R^2 - \frac{l_i^2}{4} \right)^{1/2} - \left\{ R^2 - \left[\left(i - \frac{1}{2} \right) w - \frac{l_e}{2} \right]^2 \right\}^{1/2}$$

$$\frac{l_e + l_i}{2} \leq \left(i - \frac{1}{2} \right) w \leq l_e$$

where

l_i = straight line length of the roller generatrix.

The geometric relation of a relative misalignment between inner and outer raceways is shown in Fig. 17.8. The angular position of a roller, counted from the top of the bearing, is expressed as ψ_j . Due to a radial displacement of the misaligned inner ring δ_r , the radial deflection of the roller at angular position ψ_j is

$$\delta_{\psi j} = \delta_r \cos \psi_j - \frac{u_r}{2} \quad (17.36)$$

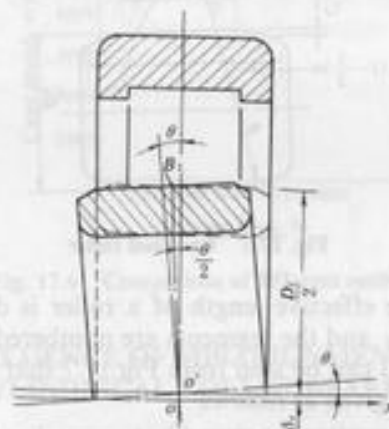


Fig. 17.8 Misalignment of bearing

Symbol θ indicates a tilting angle of the inner ring relative to the outer ring. It can be seen from the figure, only the point B_i on the inner raceway keeps the same position as that before the misalignment. The x coordinate of the point B_i is

$$x_{B_i} = -\frac{D_i}{2} \operatorname{tg} \frac{\theta}{2}$$

For angular position ψ_j , θ is replaced by $\theta \cos \psi_j$ in the above equation. Then

$$x_{B_i} = -\frac{D_i}{2} \operatorname{tg} \frac{\theta \cos \psi_j}{2} \quad (17.37)$$

Symbol $\delta_{\psi j e}$ expresses a relative approach between the roller axis and the outer ring at point B_i in the radial direction. Similarly, a relative approach between the roller axis and the inner ring is

$$\delta_{\psi j i} = \delta_r \cos \psi_j - \frac{u_r}{2} - \delta_{\psi j e} \quad (17.38)$$

The tilting angle of a roller at angular position ψ_j is expressed as β_j . The angle between the roller axis and the outer ring is $\pm \beta_j$, and that between the roller axis and tilted inner ring is $\theta \cos \psi_j \pm \beta_j$, in which the upper sign refers to the case $\cos \psi_j > 0$ and the lower sign refers to $\cos \psi_j < 0$.

The additional contact deflection of each segment for the inner raceway contact due to the tilting can be calculated

$$C_{xit} = \left[x_i \frac{D_i}{2} \operatorname{tg} \left(\frac{\theta \cos \psi_j}{2} \right) \right] \operatorname{tg} (\theta \cos \psi_j \mp \beta_j) - C_i \quad (17.39)$$

Similarly, the additional deflection of each segment for the outer raceway contact

$$C_{xio} = \left[x_i + \frac{D_i}{2} \operatorname{tg} \left(\frac{\theta \cos \psi_j}{2} \right) \right] \operatorname{tg} (\pm \beta_j) - C_i \quad (17.40)$$

where

$$x_i = -\frac{l_e}{2} + \left(i - \frac{1}{2} \right) w$$

The total contact deflection of the i -th segment at angular position ψ_j consists of the inner raceway contact deflection $\delta_{\psi j i}$ and the outer raceway contact deflection $\delta_{\psi j e}$, that is,

$$\delta_{\psi j} = \delta_r \cos \psi_j - \frac{u_r}{2} - \delta_{\psi j e}$$

$$+ \left[x_i + \frac{D_i}{2} \operatorname{tg} \left(\frac{\theta \cos \psi_j}{2} \right) \right] \operatorname{tg} (\theta \cos \psi_j \mp \beta_j) - C_i \quad (17.41)$$

and

$$\delta_{\psi j} = \delta_{\psi j} + \left[x_i + \frac{D_i}{2} \operatorname{tg} \left(\frac{\theta \cos \psi_j}{2} \right) \right] \operatorname{tg}(\pm \beta_j) - C_i \quad (17.42)$$

Relation between load and deflection

It can be derived from equation (3.30) or (3.37) that the elastic approach of a line contact may be determined by the following

$$\delta = K_a \frac{Q^{0.9}}{l_e^{0.8}} \quad (17.43)$$

where

$K_a = \text{constant}$.

$$K_a = 3.81 \left[\frac{2(1-\nu^2)}{\pi E} \right]^{0.9}$$

Symbol q indicates the contact load per unit effective length, i.e. the load density

$$q = \frac{Q}{l_e} \quad (17.44)$$

Combining with equation (17.43) gives

$$q = \frac{\delta^{1.11}}{K_a^{1.11} l_e^{0.11}} \quad (17.45)$$

It is easy to see that the influence of the effective length of a roller on the load density is quite small. Using equations (17.41) and (17.42) for the load densities, one may derive the load density of the i -th segment at angular position ψ_j for the inner raceway contact

$$q_{\psi i} = \frac{\delta_{\psi i}^{1.11}}{K_a^{1.11} l_e^{0.11}} \quad (17.46)$$

and for the outer raceway contact

$$q_{\psi e} = \frac{\delta_{\psi e}^{1.11}}{K_a^{1.11} l_e^{0.11}} \quad (17.47)$$

If $\delta_{\psi i}$ or $\delta_{\psi e} < 0$, the i -th segment of roller j separates from its corresponding contact and its contact load is zero.

Analysis of forces

As shown in Fig.17.9, considering the equilibrium of the roller at

angular position ψ_j gives

$$w \sum_{i=1}^n q_{\psi i} - w \sum_{i=1}^n q_{\psi e} + F_c = 0 \quad (17.48)$$

and

$$\sum_{i=1}^n q_{\psi i} \left[\frac{l_e}{2} - \left(i - \frac{1}{2} \right) w \right] - \sum_{i=1}^n q_{\psi e} \left[\frac{l_e}{2} - \left(i - \frac{1}{2} \right) w \right] = 0 \quad (17.49)$$

where

F_c = centrifugal force of a roller calculated by equation (17.9). In which, the orbital angular speed of a roller may be determined by the simple kinematic relation.

If given δ_r and θ , one may solve the simultaneous equations (17.48), (17.49) and obtain the values of unknown variables $\delta_{\psi i}$ and β_j at angular position ψ_j . Then, the variation of the contact load along the roller length can be deduced.

If the centrifugal forces of rollers are neglected at low speed, then from equation (17.48), $q_{\psi i} = q_{\psi e}$.

From equations (17.41) and (17.42), we obtain

$$\delta_{\psi i} = \frac{1}{2} \left(\delta_r \cos \psi_j - \frac{u_r}{2} \right)$$

$$\beta_j = \frac{\theta \cos \psi_j}{2}$$

The calculation will be much simpler.

The total contact load between the roller and the inner raceway at angular position ψ_j is

$$Q_j = w \sum_{i=1}^n q_{\psi i} \quad (17.50)$$

If the bearing is subjected to a radial load F_r and a moment load M which makes the misalignment, one may consider the equilibrium of the bearing ring. Then

$$F_r - \sum_{j=1}^n Q_j \cos \psi_j = 0 \quad (17.51)$$

and

$$M - \sum_{j=1}^z Q_j e_j \cos \psi_j = 0 \quad (17.52)$$

where

e_j = eccentricity of the contact load for the inner raceway at angular position ψ_j , i.e.

$$e_j = \frac{\sum_{i=1}^n q_{ji} \left[\frac{l_z}{2} - \left(i - \frac{1}{2} \right) w \right]}{\sum_{i=1}^n q_{ji}} \quad (17.53)$$

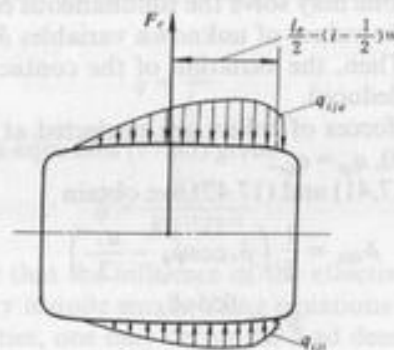


Fig. 17.9 Force equilibrium of a roller

If given external loads F and M , unknown variables δ_r and θ may be solved from equations (17.51) and (17.52). On the other hand, if given F and θ , then δ_r and M can be solved.

In the Newton-Raphson iterative process, it is necessary to find the derivative of $\delta_{\psi j}$ with respect to δ_r . By comparison with the neglect of the centrifugal forces of rollers, it is assumed that

$$\delta_{\psi j} = \alpha \left(\delta_r \cos \psi_j - \frac{u_r}{2} \right) \quad (17.54)$$

From equation (17.41), the derivative

$$\frac{\partial \delta_{\psi j}}{\partial \delta_r} = \cos \psi_j - \alpha \cos \psi_j$$

Combining with equation (17.54), then

$$\frac{\partial \delta_{\psi j}}{\partial \delta_r} = \left(1 - \frac{\delta_{\psi j}}{\delta_r \cos \psi_j - 0.5 u_r} \right) \cos \psi_j \quad (17.55)$$

It should be noted that, the iterative process will be conducted in this way. Firstly, δ_r and θ are solved upon given F and M . And then, substitution of the values of δ_r and θ into the equations to find $\delta_{\psi j}$ and β_j . Repeat again until the convergence.

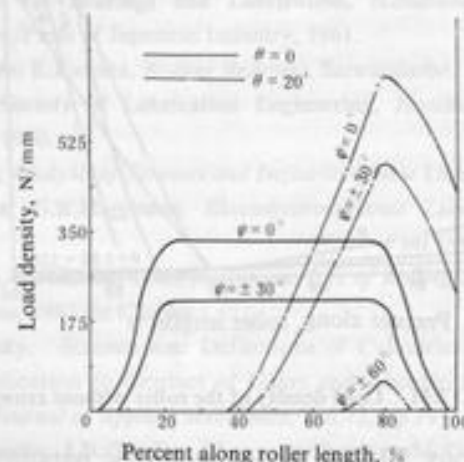


Fig. 17.10 Load density of the roller with crown

After solving the load distribution in the bearing and the load variation along the roller length, one may calculate the bearing life upon the dynamic load rating theory. Similar to the raceway control theory of ball bearings, the results regarding the load distribution and the load variation along the roller length are reliable.

A numerical example

Reference [112] gives an example by the above method: a cylin-

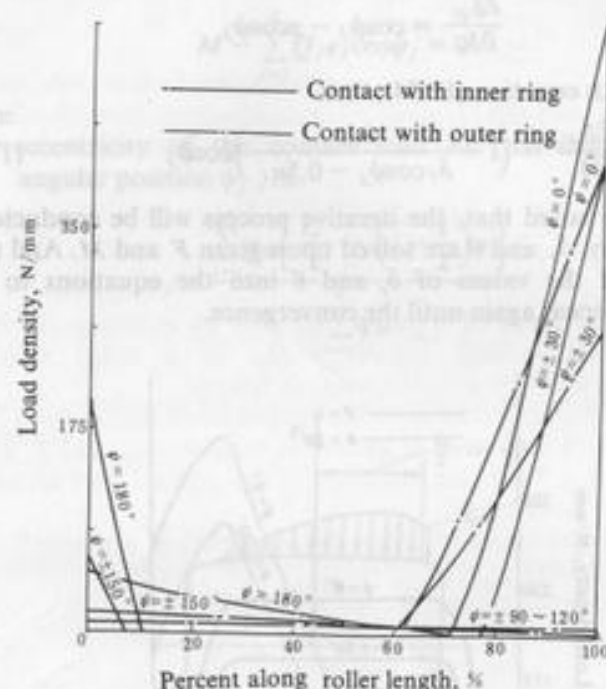


Fig. 17.11 Load density of the roller without crown

drical roller bearing NU308. Major geometric parameters are: roller diameter 14 mm, effective length of the roller 14.68 mm, length of the straight generatrix of the roller 8.73 mm and 14.68 mm, roller crown radius 1270 mm, inner raceway diameter 51.918 mm, outer raceway diameter 79.959 mm. The variations of the load density along the roller length are shown in Fig. 17.10 and 17.11.

It can be seen that, the misalignment of cylindrical roller bearings has a remarkable influence on the load non-uniform distribution along the roller length, particularly for rollers with an entirely straight generatrix.

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Analysis of Rolling Element Bearings provides a comprehensive introduction to the theory and design of rolling element bearings. Broadly falling into two parts, it first of all deals with the fundamental design principles involved - geometric relationships, static and dynamic loads, load distribution, stresses, deformations, and lifetimes. It then addresses current developments in bearing theory, including the mechanism of contact fatigue, elasto-hydrodynamic lubrication theory, optimal design, and CAD.

Available now for the first time in English, *Analysis of Rolling Element Bearings* can be strongly recommended to all mechanical engineers, designers, and technologists concerned with the specification, design, installation, and operation of rolling element bearings.

Wan Changsen is Chairman of the Friction Institution of the China Mechanical Engineering Society. He has been awarded two prizes by the National Science Conference, and has received five awards from the State Commission for Defence Science and Technology and the Ministry of Industrial Machinery - for achievements in developing special bearings for space vehicles, for the nuclear industry, for nuclear submarines, for the vertical spindles of precision machine tools, and for ultra-high speed dental drills.

Four times the head of Chinese delegations to meetings of the ISO Bearing Technology Committee, his papers have been widely published, both in China and elsewhere. His work in the field of bearing technology is widely acclaimed.

