

SUMMARY OF COURSE FORMULAS



PHYSICAL PROPERTIES OF FLUIDS

KINEMATIC VISCOSITY

$$\nu \text{ (centistokes)} = \frac{\mu \text{ (centipoise)}}{\rho \text{ (grams per cm}^3\text{)}}$$

WEIGHT DENSITY OF LIQUIDS

WEIGHT DENSITY

- ▶ The weight density or specific weight of a substance is its weight per unit volume

$$\rho \text{ (weight density)} = \frac{m \text{ (weight)}}{V \text{ (volume)}}$$

POUNDS / CUBIC FOOT

SPECIFIC VOLUME

- The specific volume is the reciprocal of the weight density

$$V \text{ (specific volume)} = \frac{1}{\rho \text{ (weight density)}}$$

WEIGHT DENSITY OF GASES AND VAPORS

IDEAL GASES

$$\rho \text{ (weight density)} = \frac{144 \times P}{R \times T}$$

Diagram illustrating the equation for weight density (ρ) of an ideal gas:

- P : ABSOLUTE PRESSURE (PSIA)
- R : INDIVIDUAL GAS CONSTANT
- T : ABSOLUTE TEMPERATURE (RANKINE °R)

$$R = \frac{1545.35}{MW}$$

Diagram illustrating the equation for the individual gas constant (R):

- MW : GAS MOLECULAR WEIGHT (GRAM/MOL)

SPECIFIC GRAVITY

SPECIFIC GRAVITY

- ▶ The specific gravity is a **relative measure** of the weight density

$$S = \frac{\rho(\text{liquid at specified temperature})}{\rho(\text{water at } 60^{\circ}\text{F})}$$

SPECIFIC GRAVITY

▶ API scale

Oils

▶ BAUME scale

Heavier than water

▶ BAUME scale

Lighter than water

$$S (60^{\circ}\text{F}) = \frac{141.5}{131.5 + \text{Deg API}}$$

$$S (60^{\circ}\text{F}) = \frac{140}{130 + \text{Deg BAUME}}$$

$$S (60^{\circ}\text{F}) = \frac{145}{145 - \text{Deg BAUME}}$$

SPECIFIC GRAVITY

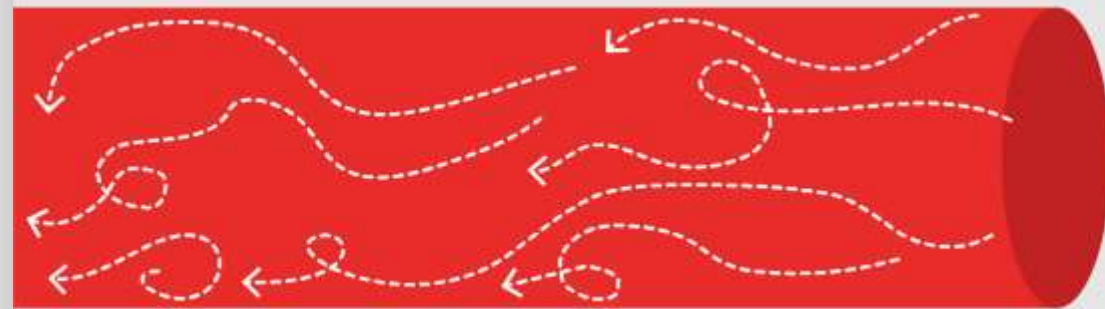
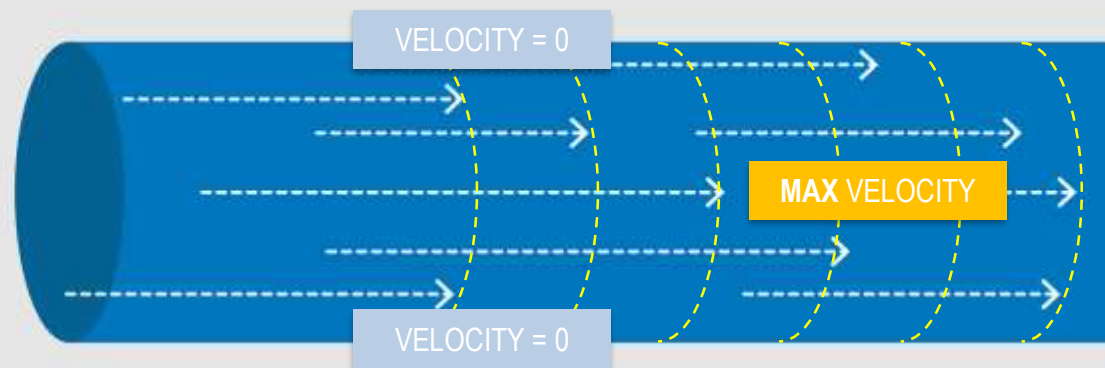
GASES

$$S = \frac{\text{MW (gas)}}{\text{MW (air)}}$$



NATURE OF FLOW IN PIPE : LAMINAR AND **TURBULENT** FLOW

LAMINAR
FLOW



TURBULENT
FLOW

MEAN VELOCITY OF FLOW

VELOCITY

- ▶ MEAN
- ▶ AVERAGE VELOCITY
- ▶ AT A GIVEN CROSS SECTION

$$\text{Velocity (ft/s)} = \frac{\text{Rate of flow (ft}^3/\text{s)}}{\text{Cross sectional area (ft}^2\text{)}}$$

VELOCITY

- ▶ MEAN
- ▶ AVERAGE VELOCITY
- ▶ AT A GIVEN CROSS SECTION

$$\text{Velocity (ft/s)} = \frac{\text{Rate of flow (lb/s)}}{\text{Cross sectional area (ft}^2\text{) x weight density (lb/ft}^3\text{)}}$$

► REASONABLE VELOCITY FOR THE FLOW OF WATER THROUGH PIPE

SERVICE CONDITION	REASONABLE VELOCITY (ft/s)
Boiler feed	8 to 15
Pump suction	4 to 7
General services	4 to 10
City	up to 7

► REASONABLE VELOCITY FOR THE FLOW OF STEAM THROUGH PIPE

CONDITION OF STEAM	PRESSURE (psig)	SERVICE	REASONABLE VELOCITY (ft/min)
Saturated	0 to 25	Heating	4000 to 6000
	25 and up	Process piping	6000 to 10000
Superheated	200 and up	Boiler and turbine leads	7000 to 20000

REYNOLDS NUMBER

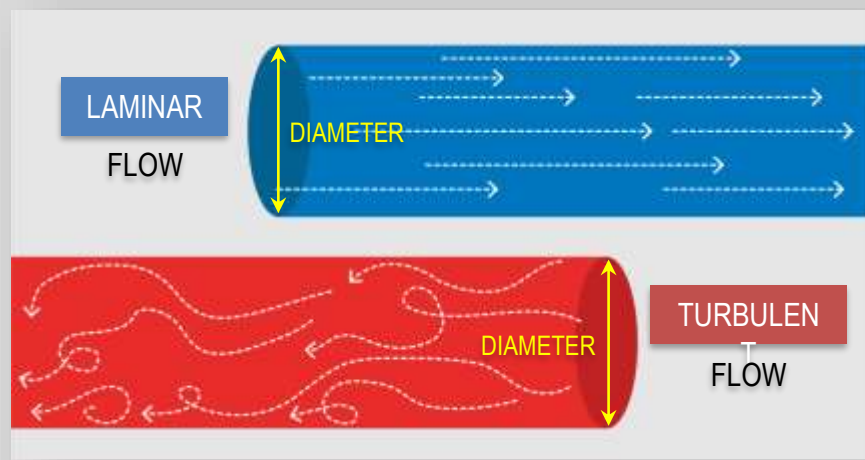


OSBORNE REYNOLDS

DENSITY

VISCOSITY

VELOCITY





OSBORNE REYNOLDS

$$Re = \frac{Dv\rho}{\mu}$$

D : Pipe internal diameter (ft)

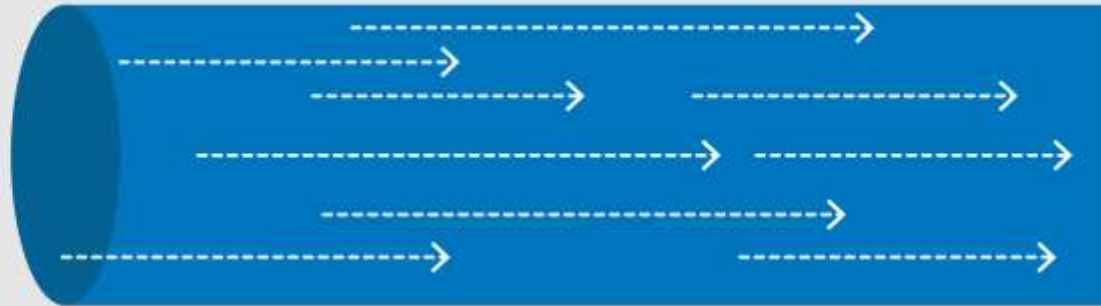
v : Mean velocity of flow (ft/s)

ρ : Weight density of fluid (lb/ft³)

μ : Absolute viscosity (lb/ft.s)

LAMINAR

$Re < 2000$



TURBULENT

$Re > 4000$



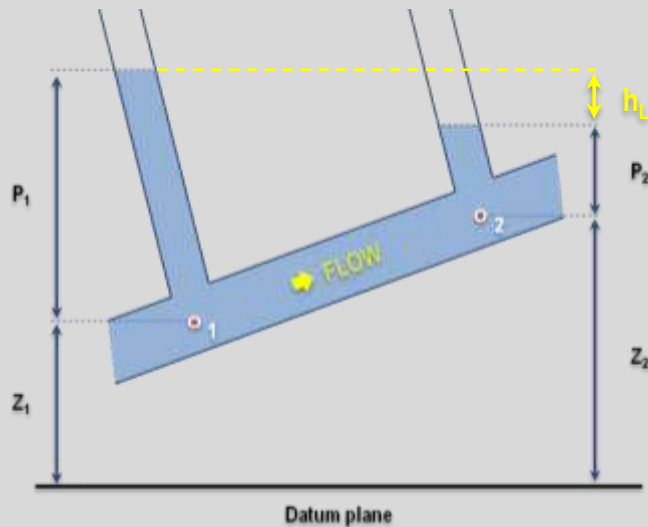
BERNOULLI'S THEOREM



DANIEL BERNOULLI

- ▶ The Bernoulli theorem is a means of expressing the application of the law of conservation of energy to the flow of fluids in a conduit

$$Z_1 + \frac{144 P_1}{\rho} + \frac{v_1^2}{2g} = Z_2 + \frac{144 P_2}{\rho} + \frac{v_2^2}{2g} + \boxed{h_L}$$



H : Total head or fluid energy (ft)

Z : Potential head or Elevation above reference level (ft)

P : Gauge pressure (psig)

ρ : Weight density of fluid (lb/ft³)

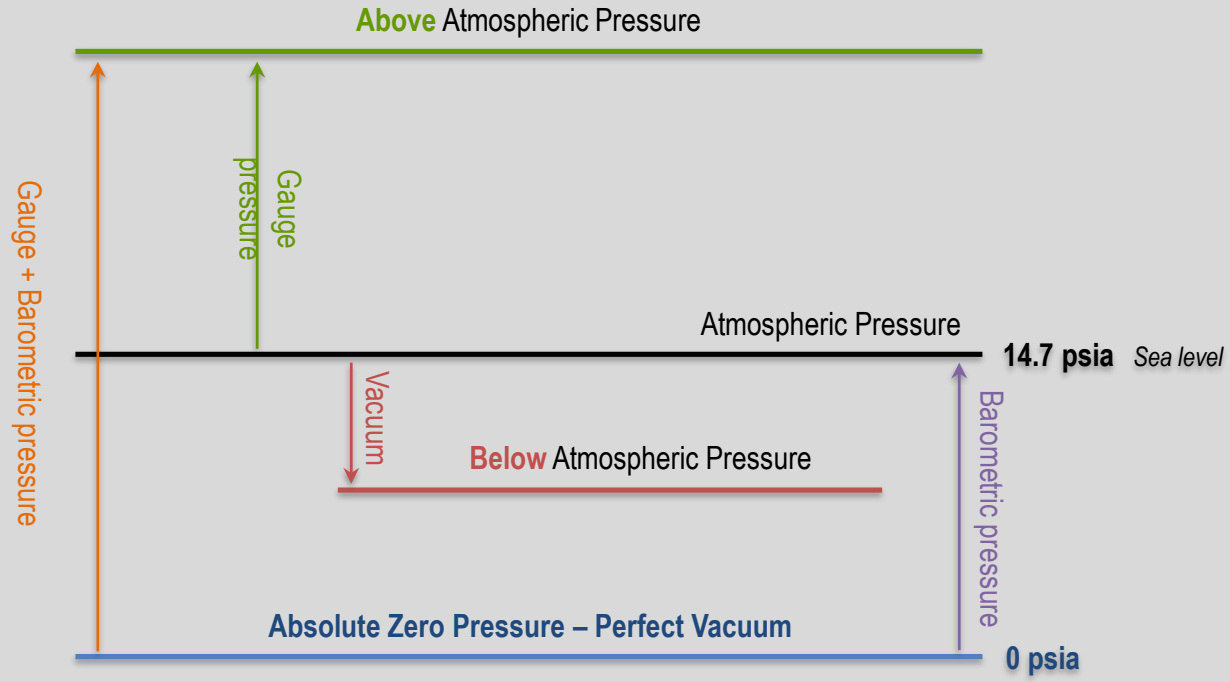
v : Mean velocity of flow (ft/s)

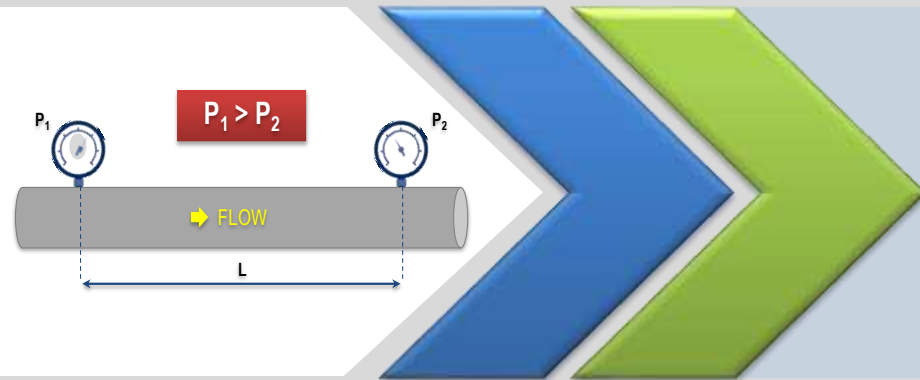
g : Gravitational acceleration = 32.174 ft/s²

h_L : Loss of static pressure head due to fluid flow (ft)



MEASUREMENT OF PRESSURE

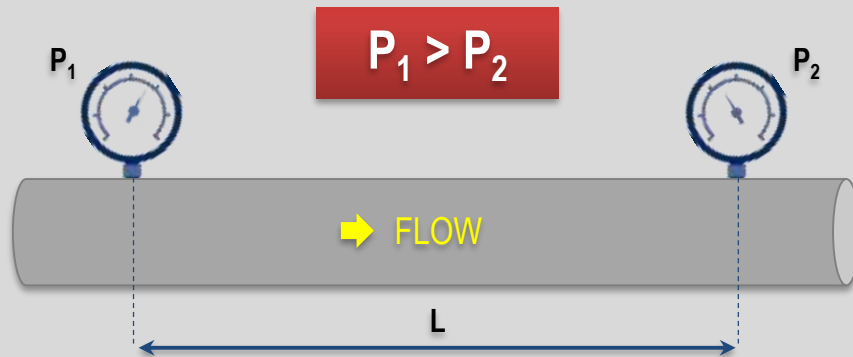




HEAD LOSS AND PRESSURE DROP THROUGH PIPE



HENRY DARCY



$$h_L = f \frac{L}{D} \frac{v^2}{2g}$$

h_L : Loss of static pressure head due to fluid flow (ft)

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

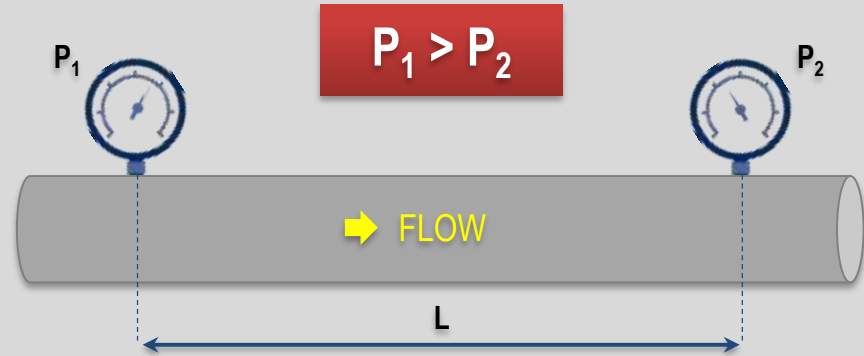
D : Pipe internal diameter (ft)

v : Mean velocity of flow (ft/s)

g : Gravitational acceleration = 32.174 ft/s²



HENRY DARCY



$$\Delta P = \frac{\rho}{144} \left(f \frac{L}{D} \frac{v^2}{2g} \right)$$

h_L : Loss of static pressure head due to fluid flow (ft)

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

D : Pipe internal diameter (ft)

v : Mean velocity of flow (ft/s)

g : Gravitational acceleration = 32.174 ft/s²

$$\Delta P = \frac{\rho}{144} \left(f \frac{L}{D} \frac{v^2}{2g} \right)$$

ΔP : Pressure differential (psi)

ρ : Weight density of fluid (lb/ft³)

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

D : Pipe internal diameter (ft)

v : Mean velocity of flow (ft/s)

g : Gravitational acceleration = 32.174 ft/s²

- ▶ Pipe of constant diameter
- ▶ Carrying fluids of constant weight density
- ▶ In straight pipe

Horizontal

Vertical

Sloping

FRICTION FACTOR



HENRY DARCY

$$\Delta P = \frac{\rho}{144} \left(f \frac{L}{D} \frac{v^2}{2g} \right)$$

ΔP : Pressure differential (psi)

ρ : Weight density of fluid (lb/ft³)

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

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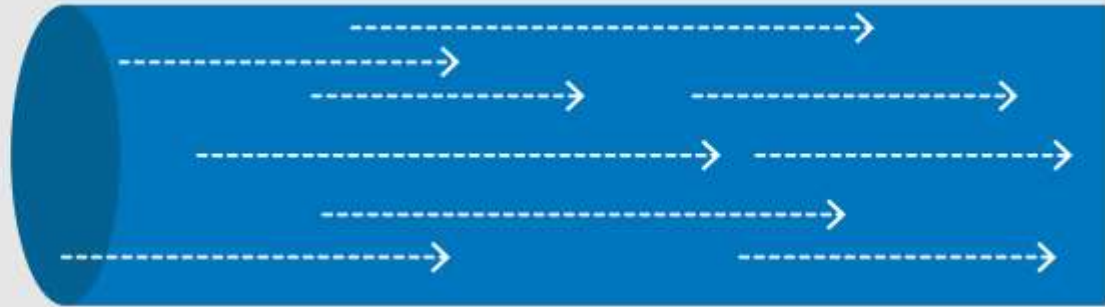
v : Mean velocity of flow (ft/s)

g : Gravitational acceleration = 32.174 ft/s²

LAMINAR

$Re < 2000$

$f \sim Re$



TURBULENT

$Re > 4000$

$f \sim Re + \text{pipe wall}$

$$\Delta P = \frac{\rho}{144} \left(f \frac{L}{D} \frac{v^2}{2g} \right)$$

ΔP : Pressure differential (psi)

ρ : Weight density of fluid (lb/ft³)

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

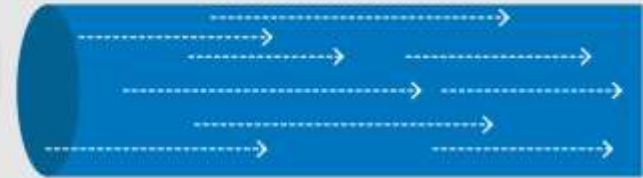
D : Pipe internal diameter (ft)

v : Mean velocity of flow (ft/s)

g : Gravitational acceleration = 32.174 ft/s²

LAMINAR

$Re < 2000$



$$f = \frac{64}{Re} = \frac{64\mu}{124dv\rho}$$

μ : Absolute viscosity (cP)

d : Internal diameter (in)

$$\Delta P = 6.68 \times 10^{-4} \frac{\mu L v}{d^2}$$

ΔP : Pressure differential (psi)

μ : Absolute viscosity (cP)

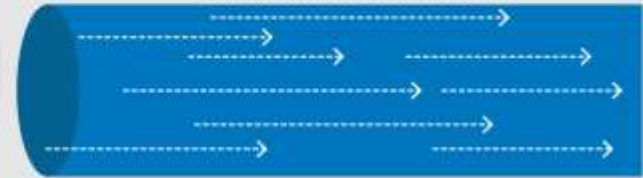
L : Length of pipe (ft)

v : Mean velocity of flow (ft/s)

d : Internal diameter (in)

LAMINAR

$Re < 2000$



POISEUILLE

$$\Delta P = \frac{\rho}{144} \left(f \frac{L}{D} \frac{v^2}{2g} \right)$$

ΔP : Pressure differential (psi)

ρ : Weight density of fluid (lb/ft³)

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

D : Pipe internal diameter (ft)

v : Mean velocity of flow (ft/s)

g : Gravitational acceleration = 32.174 ft/s²



$$\text{Relative roughness} = \frac{\varepsilon}{D}$$

ε : Roughness of the pipe walls (ft)

D : Pipe internal diameter (ft)

FRICTION FACTOR :

COLEBROOK EQUATION

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

f : Darcy friction factor (unitless)

ε : Roughness of the pipe walls (ft)

D : Pipe internal diameter (ft)

Re : Reynolds number (unitless)

APPROXIMATIONS OF COLEBROOK

COLEBROOK EQUATION

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}} \right)$$

f : Darcy friction factor (unitless)

ε : Roughness of the pipe walls (ft)

D : Pipe internal diameter (ft)

Re : Reynolds number (unitless)

SERGHIDE EQUATION

$$f = \left(A - \frac{(B - A)^2}{C - 2B + A} \right)^{-2}$$

Where :

$$A = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{12}{Re} \right)$$

$$B = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{2.51A}{Re} \right)$$

$$C = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{2.51B}{Re} \right)$$

SWAMEE-JAIN EQUATION

$$f = \frac{0.25}{\left(\log \left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right)^2}$$

ε : Roughness of the pipe walls (ft)

D : Pipe internal diameter (ft)

HAZEN-WILLIAMS FORMULA FOR FLOW OF WATER

HAZEN-WILLIAMS FORMULA

$$\Delta P_{per\ foot} = 4.52 \frac{Q^{1.85}}{C^{1.85} d^{4.87}}$$

ΔP : Pressure differential (psi)

Q : Rate of flow (gpm)

C : Hazen-Williams factor

d : Internal diameter (in)

- ▶ Fully **turbulent** flow
- ▶ Fluids **similar** to 60°F water

HAZEN-WILLIAMS FORMULA

$$\Delta P_{per\ foot} = 4.52 \frac{Q^{1.85}}{C^{1.85} d^{4.87}}$$

ΔP : Pressure differential (psi)

Q : Rate of flow (gpm)

C : Hazen-Williams factor

d : Internal diameter (in)

PIPE OF TUBE	HAZEN-WILLIAMS C VALUE
Unlined cast or ductile iron	100
Galvanized steel	120
Plastic	150
Cement lined cast	140
Copper tube or stainless steel	150



COMPRESSIBLE FLOW IN PIPE

- ▶ PRESSURE
- ▶ SPECIFIC VOLUME

POLYTROPIC

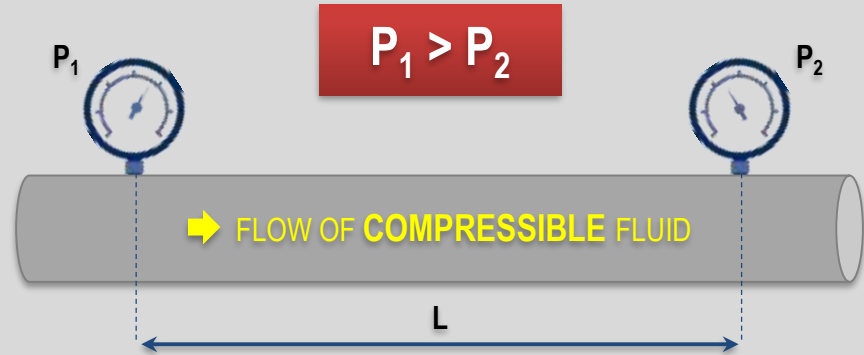
$$PV^n = C$$

P : Absolute pressure (psia)

V : Specific volume of fluid (ft³/lb)

n : Constant

C : Constant



- ▶ PRESSURE
- ▶ SPECIFIC VOLUME

ADIABATIC

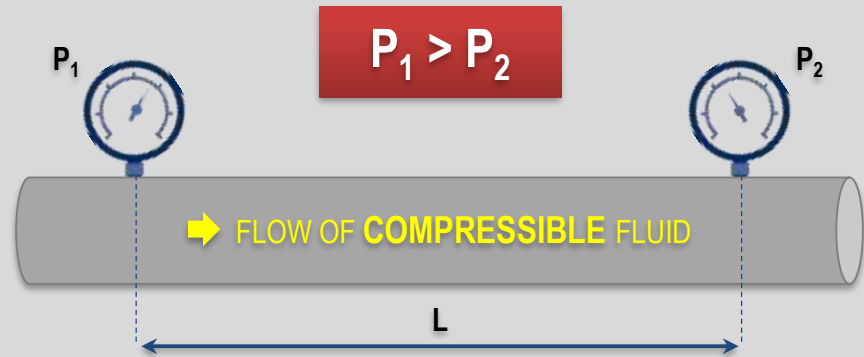
$$PV^n = C$$

P : Absolute pressure (psia)

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n : Constant

C : Constant



- ▶ PRESSURE
- ▶ SPECIFIC VOLUME

ADIABATIC

$$PV^k = C$$

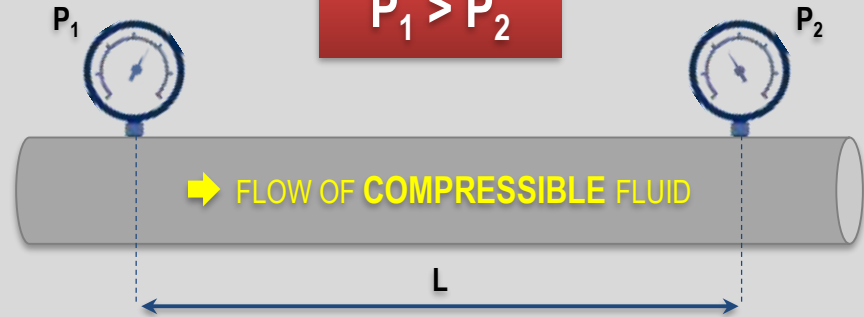
P : Absolute pressure (psia)

V : Specific volume of fluid (ft³/lb)

k : ratio C_p/C_v

C : Constant

$$P_1 > P_2$$



C_p : Specific heat at constant pressure

C_v : Specific heat at constant volume

- ▶ PRESSURE
- ▶ SPECIFIC VOLUME

ISOTHERMAL

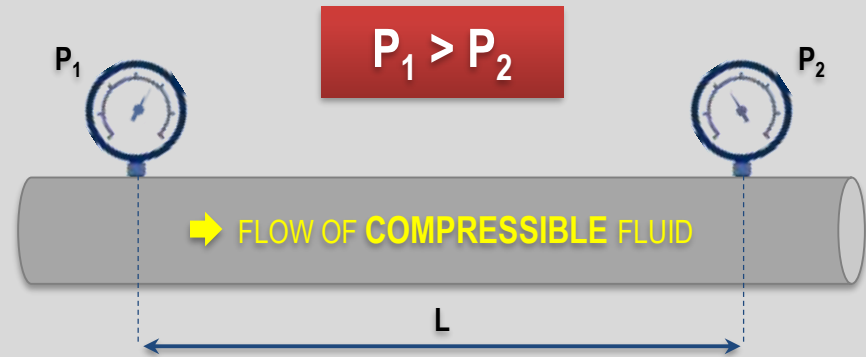
$$PV^n = C$$

P : Absolute pressure (psia)

V : Specific volume of fluid (ft³/lb)

n : Constant

C : Constant



- ▶ PRESSURE
- ▶ SPECIFIC VOLUME

ISOTHERMAL

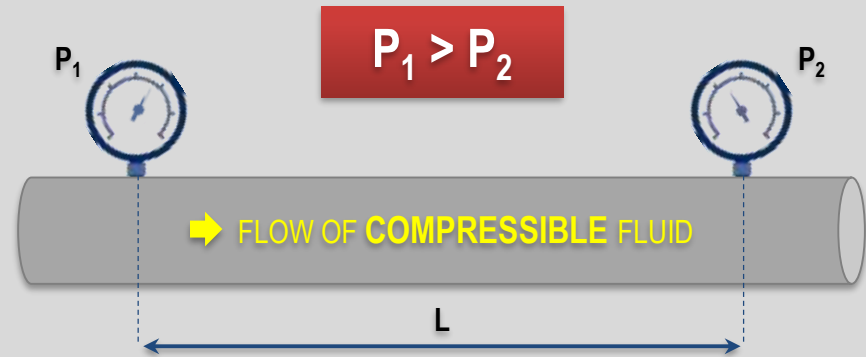
$$PV^n = C$$

P : Absolute pressure (psia)

V : Specific volume of fluid (ft³/lb)

n : Constant = 1

C : Constant



DEFINITION OF A PERFECT GAS

PERFECT GAS

$$PV = nRT$$

- ▶ Constant specific heat ratio $k = C_p/C_v$
- ▶ $k = C_p/C_v$ varies **only** with temperature

P : Absolute pressure (atm)

V : Volume of fluid (L)

n : Number of moles (moles)

R : Constant = 0.08206

T : Temperature (°K)

H₂

O₂

N₂

CO

k = 1.4

SPEED OF SOUND AND MACH NUMBER

SPEED OF SOUND

PERFECT
GAS

- It is the speed at which the pressure wave is transmitted

$$c = \sqrt{kgRT}$$

c : Speed of sound in a fluid (ft/s)

k : Specific heat ratio = C_p/C_v

g : Gravitational acceleration = 32.174 ft/s²

R : Individual gas constant = 1545.35/MW

MW : Gas molecular weight

T : Absolute temperature in degrees Rankine (°R)

MACH NUMBER

- ▶ A dimensionless ratio of the velocity of the fluid to the speed of sound in the fluid at local conditions

$$M = \frac{v}{c}$$

M : Mach number (unitless)

v : Mean velocity of flow (ft/s)

c : Speed of sound in a fluid (ft/s)

COMPLETE ISOTHERMAL EQUATION

$$w^2 = \frac{144gA^2}{V_1 \left(f \frac{L}{D} + 2 \ln \frac{P_1}{P_2} \right)} \frac{P_1^2 - P_2^2}{P_1}$$

w : Rate of flow (lb/s)

g : Gravitational acceleration = 32.174 ft/s²

A : Cross section area (ft²)

V_1 : Specific volume of fluid **upstream** (ft³/lb)

P_1 : Absolute pressure **upstream** (psia)

P_2 : Absolute pressure **downstream** (psia)

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

D : Pipe internal diameter (ft)

ISOTHERMAL



ASSUMPTIONS

- ▶ Isothermal flow
- ▶ Average velocity
- ▶ No mechanical work
- ▶ Constant friction factor
- ▶ Steady flow
- ▶ Straight horizontal pipeline
- ▶ Perfect gas law

SIMPLIFIED ISOTHERMAL GAS PIPELINE EQUATION



ASSUMPTIONS

- ▶ Isothermal flow
- ▶ No mechanical work
- ▶ Steady flow
- ▶ Perfect gas law
- ▶ Average velocity
- ▶ Constant friction factor
- ▶ Straight horizontal pipeline
- ▶ **Long pipeline to neglect acceleration**

$$w^2 = \frac{144gDA^2}{V_1 f L} \frac{P_1^2 - P_2^2}{P_1}$$

w : Rate of flow (lb/s)

g : Gravitational acceleration = 32.174 ft/s²

A : Cross section area (ft²)

V_1 : Specific volume of fluid **upstream** (ft³/lb)

P_1 : Absolute pressure **upstream** (psia)

P_2 : Absolute pressure **downstream** (psia)

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

D : Pipe internal diameter (ft)



ASSUMPTIONS

- ▶ Isothermal flow
- ▶ No mechanical work
- ▶ Steady flow
- ▶ Perfect gas law
- ▶ Average velocity
- ▶ Constant friction factor
- ▶ Straight horizontal pipeline
- ▶ **Long pipeline to neglect acceleration**

$$q_s = 3.2308 \left(\frac{T_b}{P_b} \right) \left(\frac{P_1^2 - P_2^2}{f L_m T_{avg} S_g} \right)^{0.5} d^{2.5}$$

q_s : Rate of flow at **standard conditions** (ft³/h) (scfh)

T_b : Absolute temperature at **standard conditions** = 520 °R

P_b : Absolute pressure at **standard conditions** = 14.7 psia

P_1 : Absolute pressure **upstream** (psia)

P_2 : Absolute pressure **downstream** (psia)

f : Darcy friction factor (unitless)

L_m : Length of pipe in miles (mi)

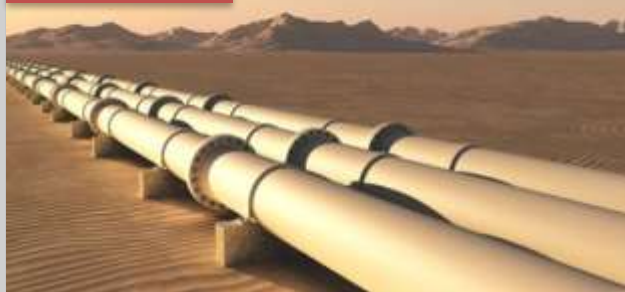
T_{avg} : Average temperature in degrees Rankine (°R)

S_g : Specific gravity of gas relative to air = MW(gas)/29

d : Pipe internal diameter (in)

OTHER COMMONLY USED EQUATIONS FOR COMPRESSIBLE FLOW IN LONG PIPELINES

GAS PIPELINE



ASSUMPTIONS

- ▶ Isothermal flow
- ▶ No mechanical work
- ▶ Steady flow
- ▶ Perfect gas law
- ▶ Average velocity
- ▶ Constant friction factor
- ▶ Straight horizontal pipeline
- ▶ **Long pipeline to neglect acceleration**

WEYMOUTH

FULLY TURBULENT FLOW

$$q_s = 18.062 \left(\frac{T_b}{P_b} \right) \left(\frac{P_1^2 - P_2^2}{L_m T_{avg} S_g} \right)^{0.5} d^{2.667}$$

q_s : Rate of flow at **standard conditions** (ft³/h) (scfh)

T_b : Absolute temperature at **standard conditions** = 520 °R

P_b : Absolute pressure at **standard conditions** = 14.7 psia

P_1 : Absolute pressure **upstream** (psia)

P_2 : Absolute pressure **downstream** (psia)

L_m : Length of pipe in miles (mi)

T_{avg} : Average temperature in degrees Rankine (°R)

S_g : Specific gravity of gas relative to air = MW(gas)/29

d : Pipe internal diameter (in)

GAS PIPELINE



ASSUMPTIONS

- ▶ Isothermal flow
- ▶ No mechanical work
- ▶ Steady flow
- ▶ Perfect gas law
- ▶ Average velocity
- ▶ Constant friction factor
- ▶ Straight horizontal pipeline
- ▶ **Long pipeline to neglect acceleration**

PANHANDLE A

PARTIALLY TURBULENT FLOW

$$q_s = 18.161 E \left(\frac{T_b}{P_b} \right)^{1.0788} \left(\frac{P_1^2 - P_2^2}{L_m T_{avg} S_g^{0.8539}} \right)^{0.5394} d^{2.6182}$$

q_s : Rate of flow at **standard conditions** (ft³/h) (scfh)

T_b : Absolute temperature at **standard conditions** = 520 °R

P_b : Absolute pressure at **standard conditions** = 14.7 psia

P_1 : Absolute pressure **upstream** (psia)

P_2 : Absolute pressure **downstream** (psia)

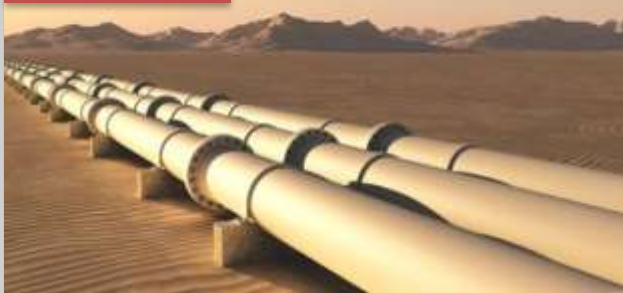
L_m : Length of pipe in miles (mi)

T_{avg} : Average temperature in degrees Rankine (°R)

S_g : Specific gravity of gas relative to air = MW(gas)/29

d : Pipe internal diameter (in)

GAS PIPELINE



ASSUMPTIONS

- ▶ Isothermal flow
- ▶ No mechanical work
- ▶ Steady flow
- ▶ Perfect gas law
- ▶ Average velocity
- ▶ Constant friction factor
- ▶ Straight horizontal pipeline
- ▶ **Long pipeline to neglect acceleration**

PANHANDLE B

FULLY TURBULENT FLOW

$$q_s = 30.708 E \left(\frac{T_b}{P_b} \right)^{1.02} \left(\frac{P_1^2 - P_2^2}{L_m T_{avg} S_g^{0.961}} \right)^{0.510} d^{2.53}$$

q_s : Rate of flow at **standard conditions** (ft³/h) (scfh)

T_b : Absolute temperature at **standard conditions** = 520 °R

P_b : Absolute pressure at **standard conditions** = 14.7 psia

P_1 : Absolute pressure **upstream** (psia)

P_2 : Absolute pressure **downstream** (psia)

L_m : Length of pipe in miles (mi)

T_{avg} : Average temperature in degrees Rankine (°R)

S_g : Specific gravity of gas relative to air = MW(gas)/29

d : Pipe internal diameter (in)



► Partially

► Fully turbulent flow

$$\frac{1}{\sqrt{f}} = 2F_f \log \left(\frac{Re\sqrt{f}}{2.825} \right)$$

$F_f = 0.9 \sim 0.99$

$$q_s = 3.2308 \left(\frac{T_b}{P_b} \right) \left(\frac{P_1^2 - P_2^2}{f L_m T_{avg} S_g} \right)^{0.5} d^{2.5}$$

q_s : Rate of flow at **standard conditions** (ft³/h) (scfh)

T_b : Absolute temperature at **standard conditions** = 520 °R

P_b : Absolute pressure at **standard conditions** = 14.7 psia

P_1 : Absolute pressure **upstream** (psia)

P_2 : Absolute pressure **downstream** (psia)

f : **Darcy friction factor** (unitless)

L_m : Length of pipe in miles (mi)

T_{avg} : Average temperature in degrees Rankine (°R)

S_g : Specific gravity of gas relative to air = MW(gas)/29

d : Pipe internal diameter (in)



► Partially

► Fully turbulent flow

$$\frac{1}{\sqrt{f}} = 2 \log \left(\frac{3.7D}{\varepsilon} \right)$$

$$q_s = 3.2308 \left(\frac{T_b}{P_b} \right) \left(\frac{P_1^2 - P_2^2}{f L_m T_{avg} S_g} \right)^{0.5} d^{2.5}$$

q_s : Rate of flow at **standard conditions** (ft³/h) (scfh)

T_b : Absolute temperature at **standard conditions** = 520 °R

P_b : Absolute pressure at **standard conditions** = 14.7 psia

P_1 : Absolute pressure **upstream** (psia)

P_2 : Absolute pressure **downstream** (psia)

f : **Darcy friction factor** (unitless)

L_m : Length of pipe in miles (mi)

T_{avg} : Average temperature in degrees Rankine (°R)

S_g : Specific gravity of gas relative to air = MW(gas)/29

d : Pipe internal diameter (in)

MODIFICATIONS TO THE ISOTHERMAL FLOW EQUATION

COMPRESSIBILITY FACTOR

- ▶ Accounts for real gas behavior that deviates from the ideal gas equation

POTENTIAL ENERGY

- ▶ Accounts for elevation changes

WEYMOUTH

$$q_s = 18.062 \left(\frac{T_b}{P_b} \right) \left(\frac{P_1^2 - P_2^2}{L_m T_{avg} S_g} \right)^{0.5} d^{2.667}$$

q_s : Rate of flow at **standard conditions** (ft³/h) (scfh)

T_b : Absolute temperature at **standard conditions** = 520 °R

P_b : Absolute pressure at **standard conditions** = 14.7 psia

P_1 : Absolute pressure **upstream** (psia)

P_2 : Absolute pressure **downstream** (psia)

L_m : Length of pipe in miles (mi)

T_{avg} : Average temperature in degrees Rankine (°R)

S_g : Specific gravity of gas relative to air = MW(gas)/29

d : Pipe internal diameter (in)

GAS PIPELINE



**~35 - 100
psig**

COMPRESSIBILITY FACTOR

**~1000 - 3200
psig**

COMPRESSIBILITY FACTOR

$$Z = \frac{P\bar{V}}{RT}$$

Z : Compressibility factor (unitless)

P : Absolute pressure in lb/ft² (psfa)

V : Specific volume of fluid (ft³/lb)

R : Individual gas constant = 1545.35/MW

T : Absolute temperature (°R)

COMPRESSIBILITY FACTOR

- ▶ Evaluated at **average conditions** of the flowing gas
- ▶ T_{avg} : **average** temperature of the flowing gas
- ▶ P_{avg} : **arithmetic mean** of the upstream and downstream pressures

$$\Delta P < 0.2\%$$

$$\Delta P > 0.2\%$$

$$P_{\text{avg}} = \frac{2(P_1)^3 - (P_2)^3}{3(P_1)^2 - (P_2)^2}$$

P_1 : Absolute upstream pressure in lb/in² (psia)

P_2 : Absolute downstream pressure in lb/in² (psia)

POTENTIAL ENERGY

- Accounts for elevation changes

$$\varphi = 0.0375 \frac{S_g \Delta z (P_{avg})^2}{T_{avg} Z_{avg}}$$

S_g : Specific gravity of gas relative to air = $MW(\text{gas})/29$

Δz : Elevation above reference level (ft)

P_{avg} : Average absolute pressure in lb/in² (psia)

Z_{avg} : Average Compressibility factor (unitless)

T_{avg} : Average absolute temperature (°R)

$$q_s = 3.2308 \left(\frac{T_b}{P_b} \right) \left(\frac{(P_1)^2 - (P_2)^2 - \phi}{f L_m T_{avg} Z_{avg} S_g} \right)^{0.5} d^{2.5}$$

q_s : Rate of flow at **standard conditions** (ft³/h) (scfh)

T_b : Absolute temperature at **standard conditions** = 520 °R

P_b : Absolute pressure at **standard conditions** = 14.7 psia

P_1 : Absolute pressure **upstream** (psia)

P_2 : Absolute pressure **downstream** (psia)

ϕ : **Potential energy to account for elevation changes**

L_m : Length of pipe in miles (ml)

T_{avg} : Average temperature in degrees Rankine (°R)

Z_{avg} : Average Compressibility factor (unitless)

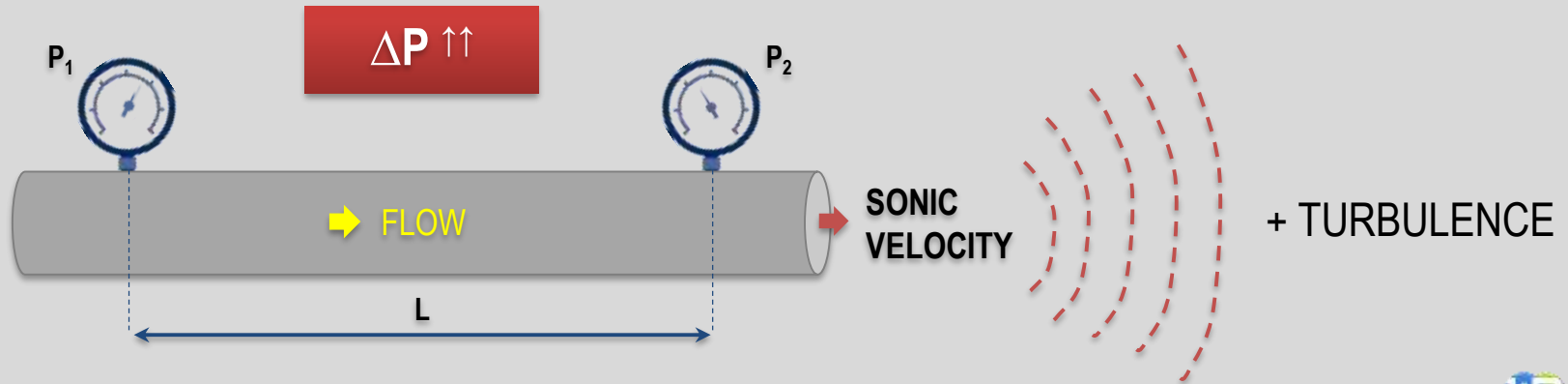
S_g : Specific gravity of gas relative to air = MW(gas)/29

d : Pipe internal diameter (in)

LIMITING FLOW OF GASES AND VAPORS

CHOKED FLOW

- ▶ The mass flow rate of a compressible fluid in a pipe, with a given upstream pressure will approach a **certain maximum** which it cannot exceed no matter how much the downstream pressure is reduced
- ▶ The maximum velocity of a compressible fluid in a pipe is limited by the **speed of sound** in the fluid



SONIC VELOCITY

$$v_s = c = \sqrt{kgRT}$$

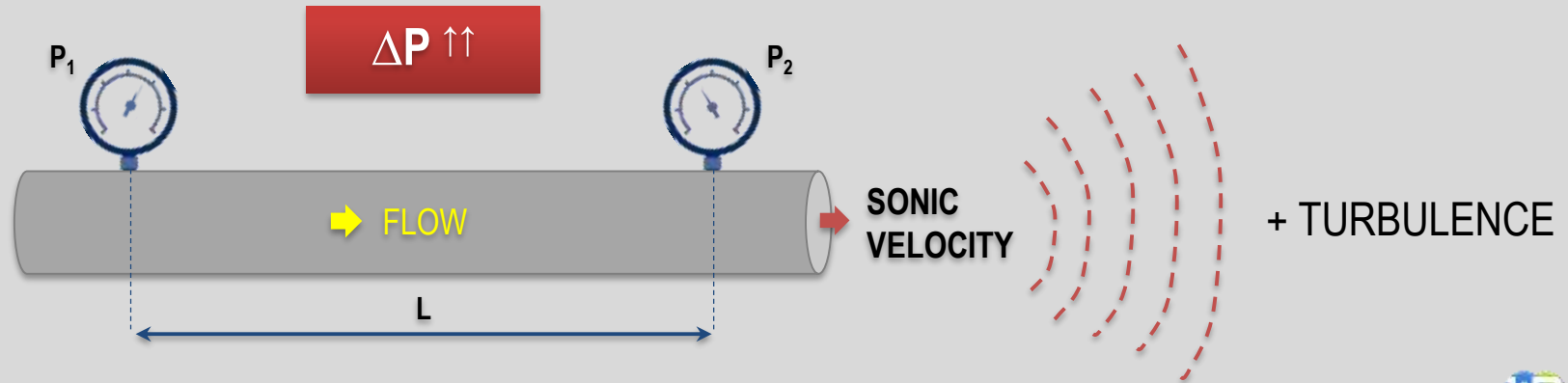
c : Speed of sound in a fluid (ft/s)

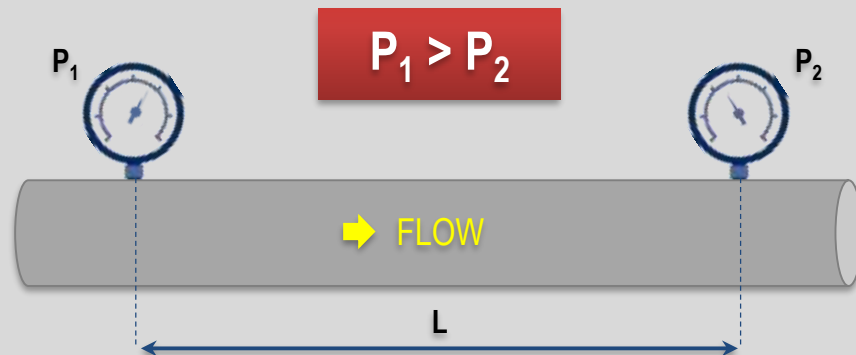
k : Specific heat ratio = C_p/C_v

g : Gravitational acceleration = 32.174 ft/s²

R : Individual gas constant = 1545.35/MW

T : Absolute temperature in degrees Rankine (°R)





$$w = 0.525Yd^2 \sqrt{\frac{\Delta P}{K\bar{V}}}$$

w : Rate of flow (lb/s)

Y : Net expansion factor for compressible flow (unitless)

d : Pipe internal diameter (in)

ΔP : **Gauge** pressure (psig)

K : Resistance coefficient (unitless)

$\bar{V}$: Specific volume of fluid (ft³/lb)

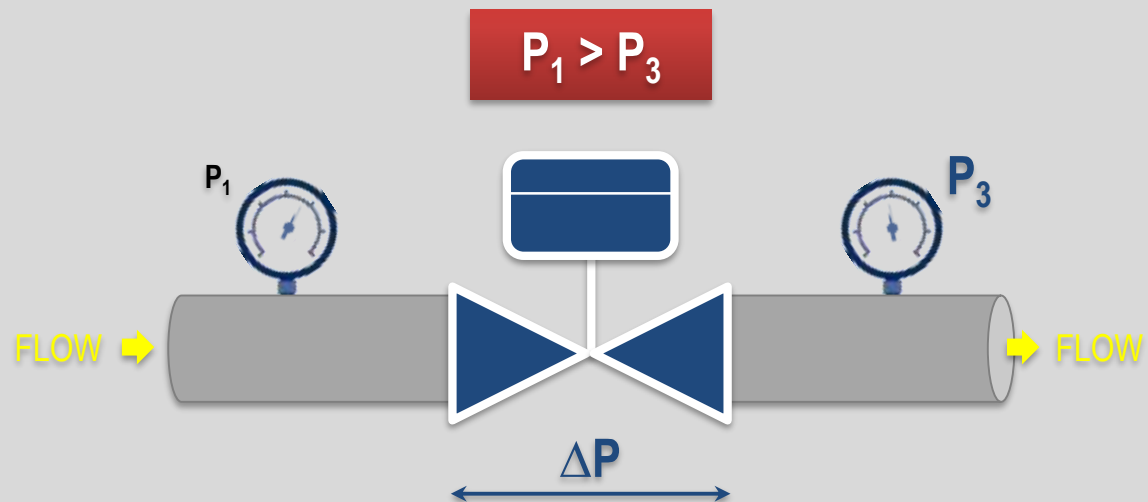


FLOW OF FLUIDS THROUGH VALVES AND FITTINGS

PRESSURE DROP ATTRIBUTED TO VALVES AND FITTINGS



- ▶ Any impediment in the pipe which changes the direction of the whole stream or even part of it, will :
 - ▶ alter the characteristic flow pattern
 - ▶ create turbulence
 - ▶ cause an energy loss greater than normally accompanying flow in straight pipe



- ▶ The loss of pressure produced by a valve or fitting consists of :

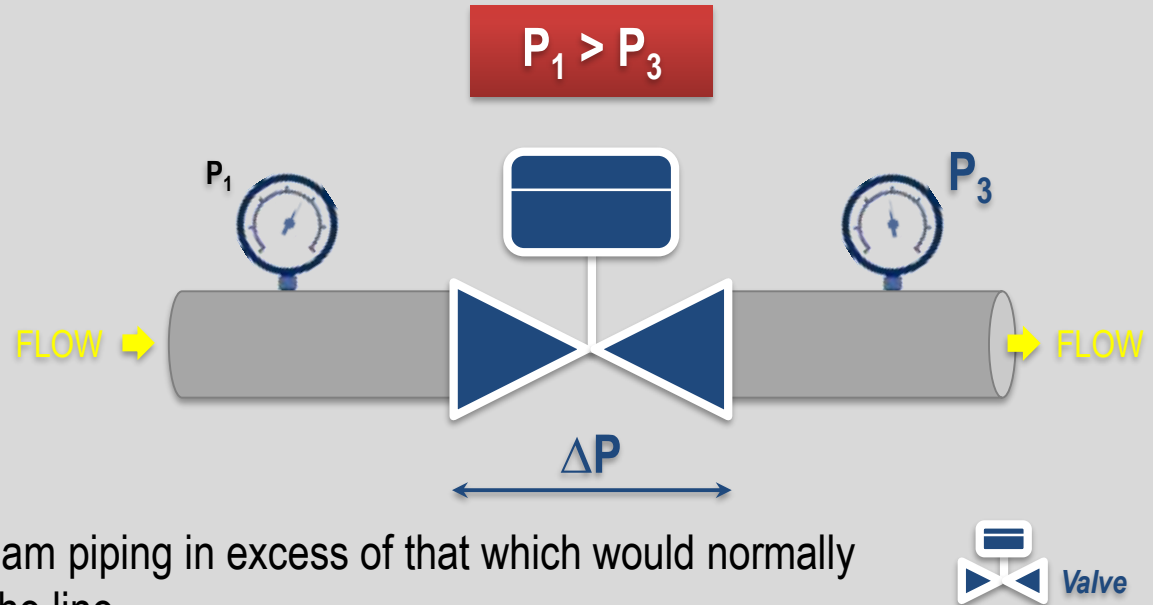
- ▶ The pressure drop within the valve itself

SMALL

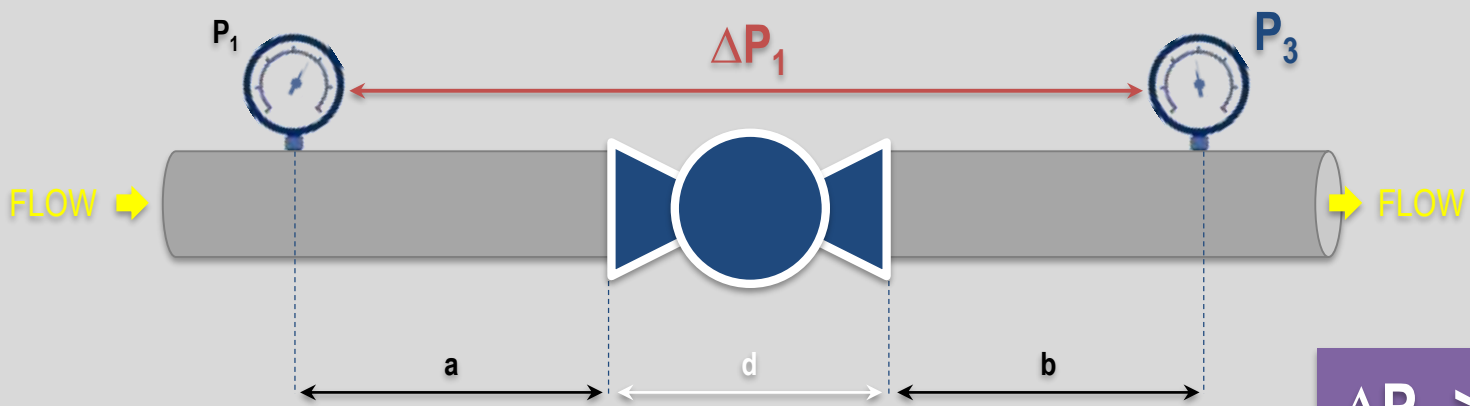
- ▶ The pressure drop in the upstream piping in excess of that which would normally occur if there were no valve in the line

LARGE

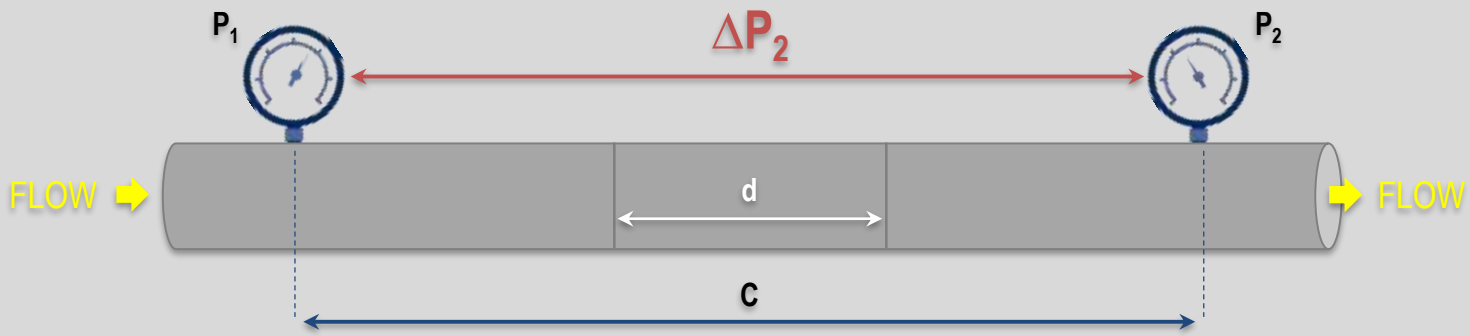
- ▶ The pressure drop in the downstream piping in excess of that which would normally occur if there were no valve in the line

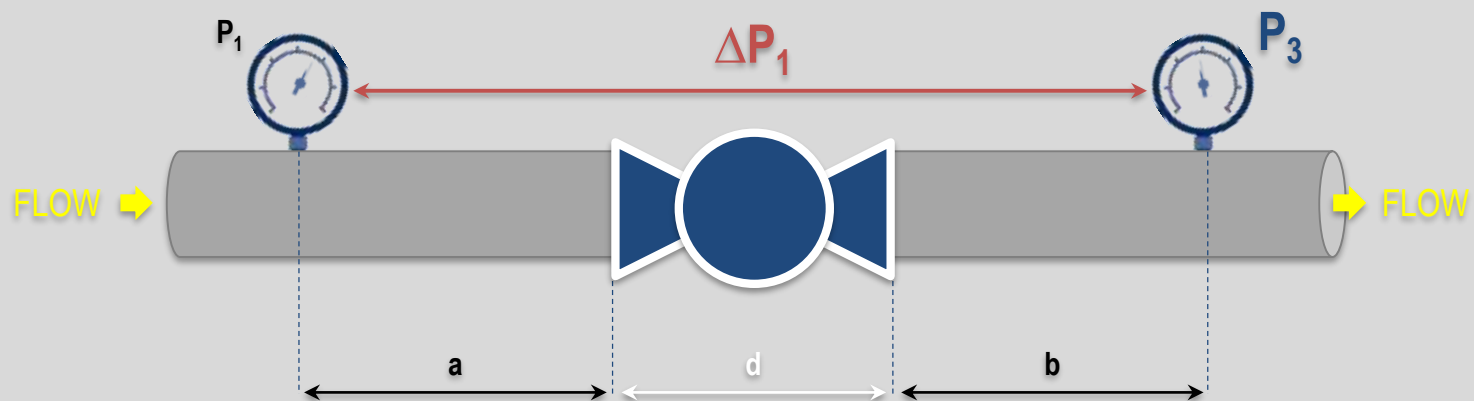


FLOW OF FLUIDS THROUGH VALVES AND FITTINGS



$\Delta P_1 > \Delta P_2$





$$\boxed{\text{Valve loss}} = \Delta P_1 - \text{Pipe section "a + b" loss}$$

Resistance coefficient "K"

RELATIONSHIP OF PRESSURE DROP TO VELOCITY OF FLOW

$$\text{Head loss } \left(\frac{\text{valves}}{\text{fittings}} \right) \approx \text{Velocity}^{\text{cste power}}$$

TURBULENT

$Re > 4000$

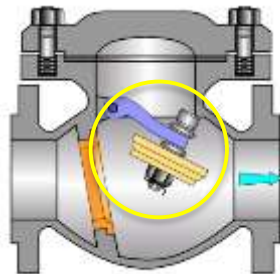
$\sim 1.8 - 2.1$

$$\text{Head loss } \left(\frac{\text{valves}}{\text{fittings}} \right) \approx \text{Velocity}^{\text{cste power}}$$

TURBULENT

$$Re > 4000$$

$$= 2$$



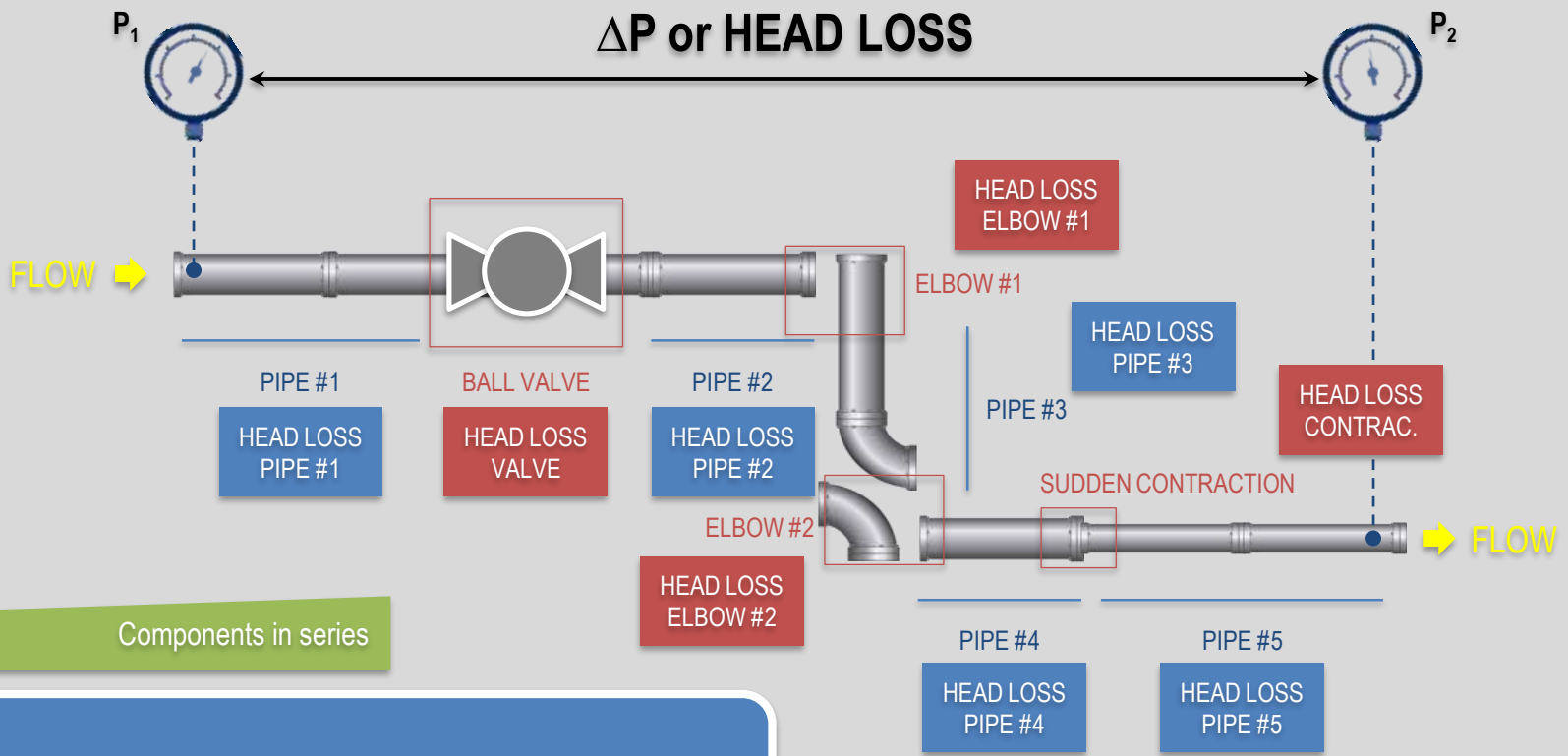
Wide open position

$$\text{Head loss} \left(\frac{\text{valves}}{\text{fittings}} \right) \approx \text{Velocity}^2$$

VALID FOR CHECK VALVES

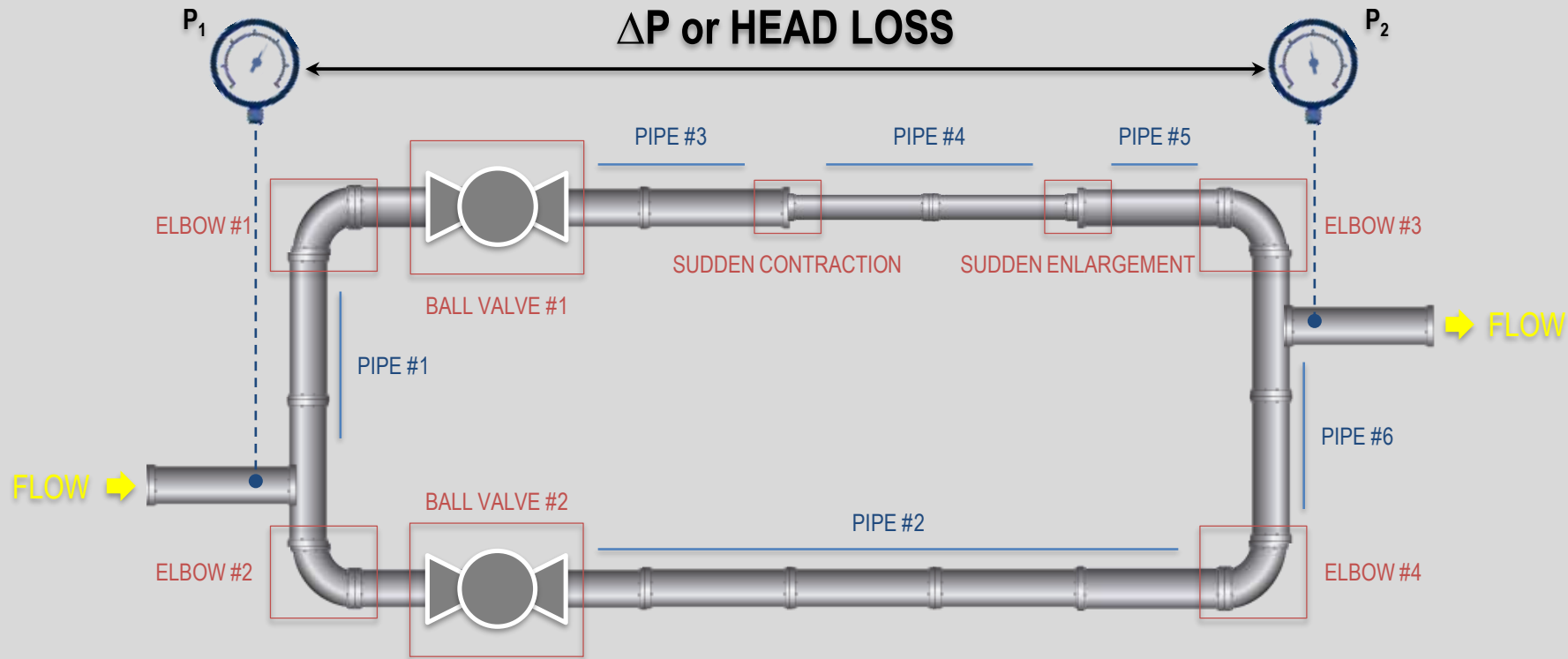
- ▶ Only if there is sufficient flow to hold the disc in a **wide open position**

RESISTANCE COEFFICIENT “K” FOR PIPELINES, VALVES AND FITTINGS IN SERIES AND PARALLEL



Components in series

$$K_{\text{total}} = K_1 + K_2 + K_3 + \dots + K_n$$



Components in parallel

$$\frac{1}{\sqrt{K_{\text{total}}}} = \frac{1}{\sqrt{K_1}} + \frac{1}{\sqrt{K_2}} + \frac{1}{\sqrt{K_3}} + \dots + \frac{1}{\sqrt{K_n}}$$

FLOW COEFFICIENT “ C_v ”



- ▶ Express the valve **capacity** and the valve **flow characteristics** in terms of the flow coefficient C_v
- ▶ K_v is the **metric equivalent** of C_v

$$K_v = 0.865C_v$$

- ▶ C_v is also used to characterize the hydraulic performance of other components

SIMPLE DEFINITION OF “C_v”

- ▶ Amount of water flow
- ▶ At 60°F
- ▶ In gallons per minute (gpm)
- ▶ At a pressure drop of 1 pound per square inch (1 psi)

SIMPLE DEFINITION OF “C_v”

- For fluids other than water :

$$C_v = Q \sqrt{\frac{S}{\Delta P}}$$

C_v : Flow coefficient for valves or piping components

Q : Rate of flow (gpm)

S : Specific gravity of liquids at specified temperature relative to water at 60°F and 14.7 psia

ΔP : Pressure differential in lb/in² (psi)

RELATION BETWEEN “C_v” AND “K”

$$C_v = 29.84 \frac{d^2}{\sqrt{K}}$$

C_v : Flow coefficient for valves or piping components

d : Pipe internal diameter (in)

K : Resistance coefficient (unitless)

$$h_L = K \frac{v^2}{2g}$$

$$C_v = Q \sqrt{\frac{S}{\Delta P}}$$

h_L : Loss of static pressure head due to fluid flow (ft)

K : Resistance coefficient (unitless)

v : Mean velocity of flow (ft/s)

g : Gravitational acceleration = 32.174 ft/s²

C_v : Flow coefficient for valves or piping components

Q : Rate of flow (gpm)

S : Specific gravity of liquids at specified temperature relative to water at 60°F and 14.7 psia

ΔP : Pressure differential in lb/in² (psi)

FLOW COEFFICIENT “ C_v ” FOR PIPELINES, VALVES AND FITTINGS IN SERIES AND PARALLEL

SERIES

$$\frac{1}{C_{v_{\text{total}}}^2} = \frac{1}{C_{v_1}^2} + \frac{1}{C_{v_2}^2} + \frac{1}{C_{v_3}^2} + \dots + \frac{1}{C_{v_n}^2}$$

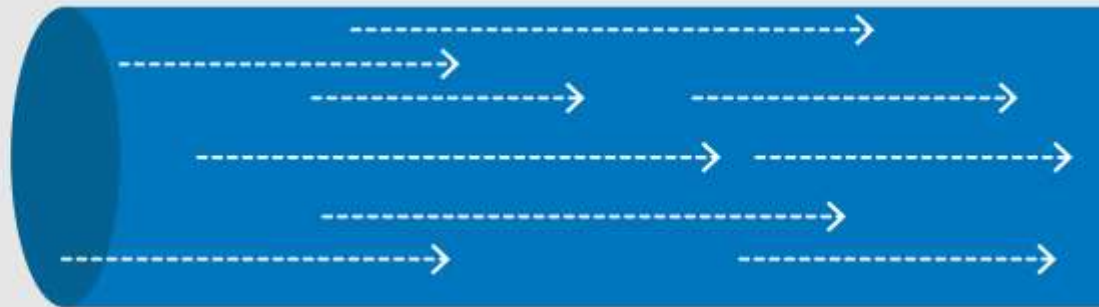
PARALLEL

$$C_{v_{\text{total}}} = C_{v_1} + C_{v_2} + C_{v_3} + \dots + C_{v_n}$$

LAMINAR FLOW CONDITIONS

LAMINAR

$Re < 2000$



TURBULENT

$Re > 4000$

LAMINAR

$$f = \frac{64}{Re}$$

POISEUILLE

$$h_L = 0.0962 \frac{\mu L v}{d^2 \rho}$$

HEAD LOSS EQUATION

ALL FLOWS

$$h_L = f \frac{L}{D} \frac{v^2}{2g}$$

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

D : Pipe internal diameter (ft)

v : Mean velocity of flow (ft/s)

Re : Reynolds number (unitless)

 μ : Absolute viscosity (cP)

d : Internal diameter (in)

 ρ : Weight density of fluid (lb/ft³)g : Gravitational acceleration = 32.174 ft/s²

HEAD LOSS EQUATION

$$h_L = f \frac{L}{D} \frac{v^2}{2g}$$

$$Re = \frac{Dv\rho}{\mu}$$

$$K = f \frac{L}{D}$$

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

D : Pipe internal diameter (ft)

v : Mean velocity of flow (ft/s)

Re : Reynolds number (unitless)

μ : Absolute viscosity (cP)

d : Internal diameter (in)

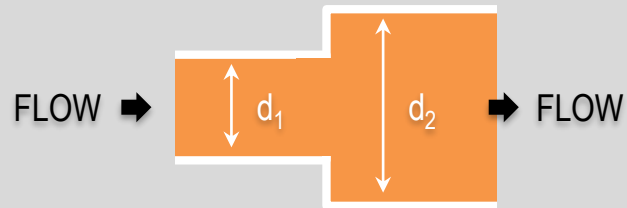
ρ : Weight density of fluid (lb/ft³)

g : Gravitational acceleration = 32.174 ft/s²

CONTRACTION AND ENLARGEMENT

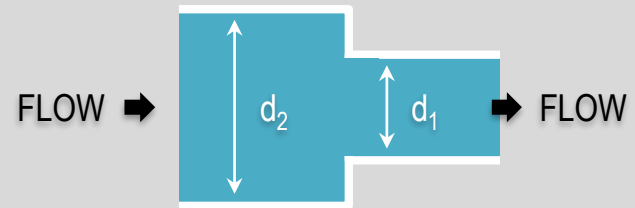
ENLARGEMENT

$$K_1 = \left(1 - \frac{d_1^2}{d_2^2}\right)^2$$



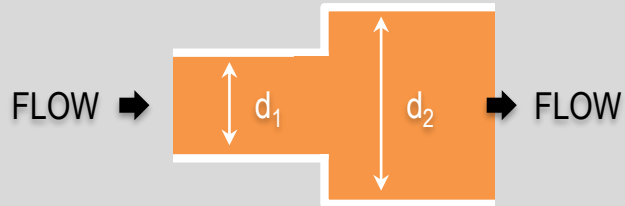
CONTRACTION

$$K_1 = 0.5 \left(1 - \frac{d_1^2}{d_2^2}\right)$$



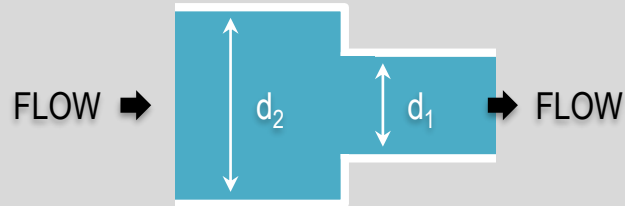
ENLARGEMENT

$$K_1 = (1 - \beta^2)^2$$

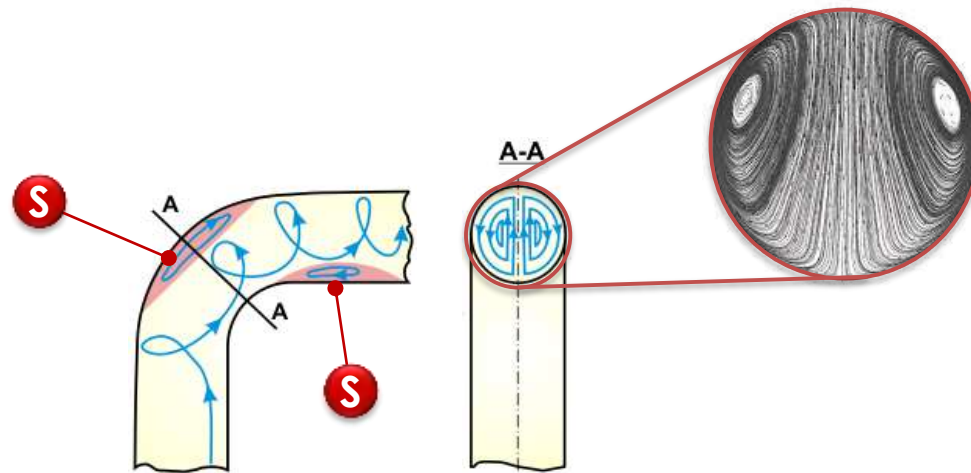


CONTRACTION

$$K_1 = 0.5(1 - \beta^2)$$



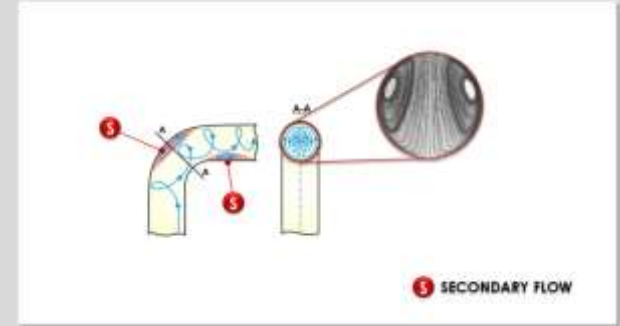
RESISTANCE OF BENDS



S SECONDARY FLOW

RESISTANCE OF BENDS TO FLOW

- ▶ Loss due to curvature “ h_c ”
- ▶ Excess loss in the downstream tangent “ h_p ”
- ▶ Loss due to pipe length “ h_L ”



$$h_{\text{total}} = \underbrace{h_c + h_p}_{h_b} + h_L$$

VELOCITY HEAD

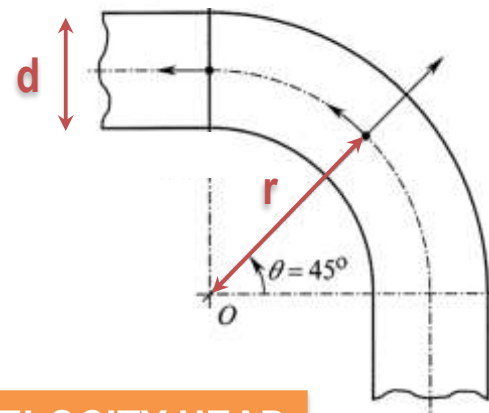
$$h_b = K_b \frac{v^2}{2g}$$

Pipe Friction Data for Schedule 40 Clean Commercial Steel Pipe with Flow in Zone of Complete Turbulence

Nominal Size	1/2"	3/4"	1"	1 1/4"	1 1/2"	2"	2 1/2"	3"	4"	5, 6"	8"	10-14"	16-22"	24-36"
Friction Factor (f_T)	.026	.024	.022	.021	.020	.019	.018	.017	.016	.015	.014	.013	.012	.011

r/d	K	r/d	K
1	$20 f_T$	8	$24 f_T$
1.5	$14 f_T$	10	$30 f_T$
2	$12 f_T$	12	$34 f_T$
3	$12 f_T$	14	$38 f_T$
4	$14 f_T$	16	$42 f_T$
6	$17 f_T$	20	$50 f_T$

**90° PIPE BENDS AND FLANGED
OR BUTT-WELDING 90° ELBOWS**



VELOCITY HEAD

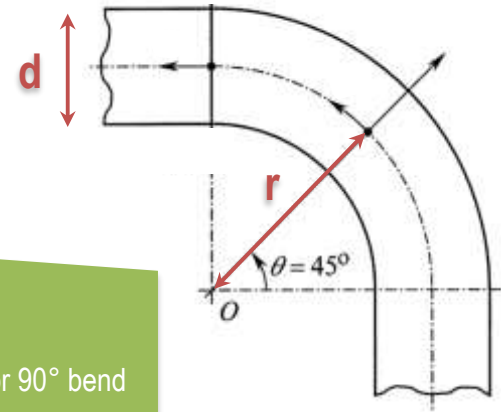
$$h_b = K_b \frac{v^2}{2g}$$

Pipe Friction Data for Schedule 40 Clean Commercial Steel Pipe with Flow in Zone of Complete Turbulence

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Friction Factor (f_T)	.026	.024	.022	.021	.020	.019	.018	.017	.016	.015	.014	.013	.012	.011

r/d	K	r/d	K
1	$20 f_T$	8	$24 f_T$
1.5	$14 f_T$	10	$30 f_T$
2	$12 f_T$	12	$34 f_T$
3	$12 f_T$	14	$38 f_T$
4	$14 f_T$	16	$42 f_T$
6	$17 f_T$	20	$50 f_T$

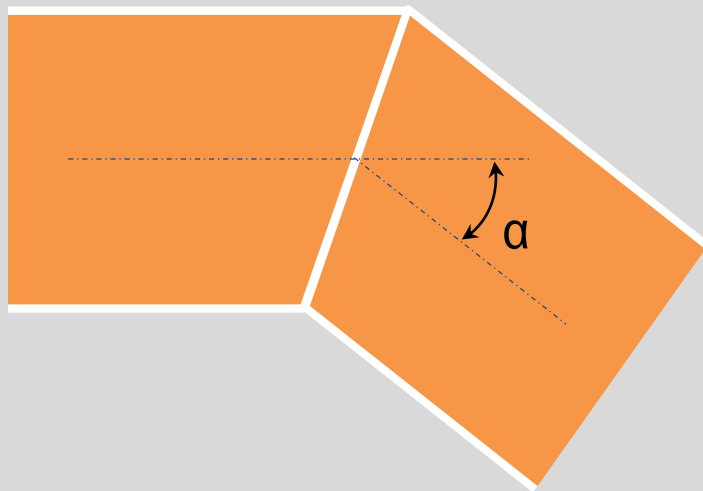
**90° PIPE BENDS AND FLANGED
OR BUTT-WELDING 90° ELBOWS**



n : Number of 90° bends

K : Resistance coefficient for 90° bend

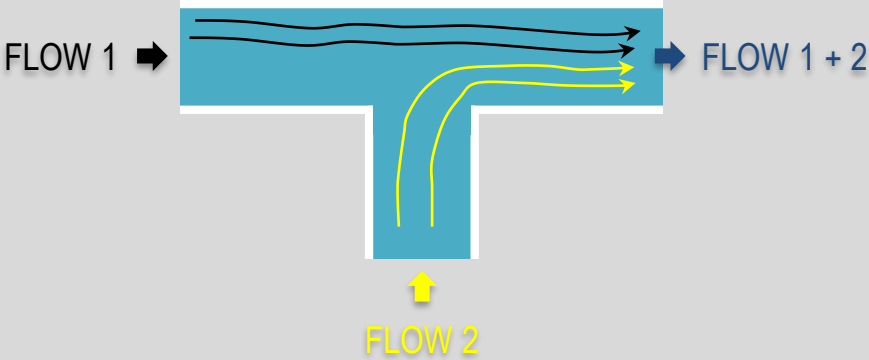
$$K_b = (n - 1) \left(0.25\pi f_T \frac{r}{d} + 0.5K \right) + K$$



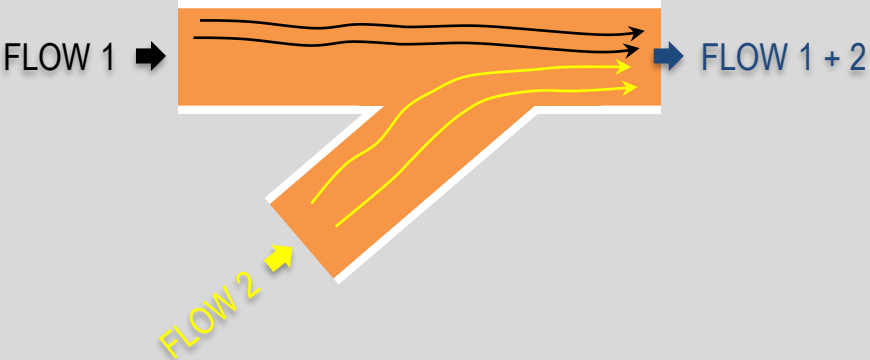
α	K
0°	$2 f_T$
15°	$4 f_T$
30°	$8 f_T$
45°	$15 f_T$
60°	$25 f_T$
75°	$40 f_T$
90°	$60 f_T$

MITRE BENDS

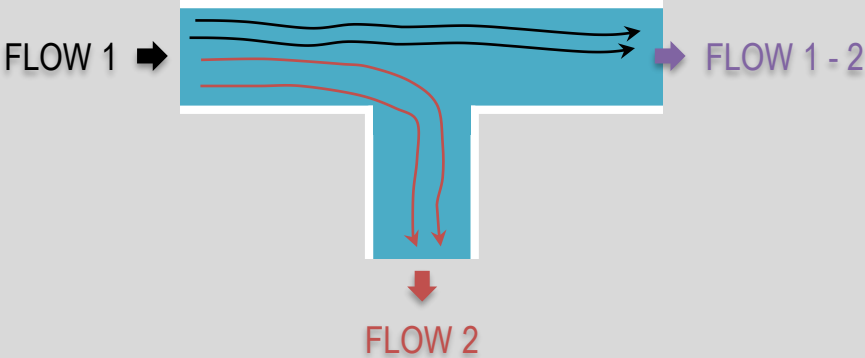
HYDRAULIC RESISTANCE OF TEES AND WYES



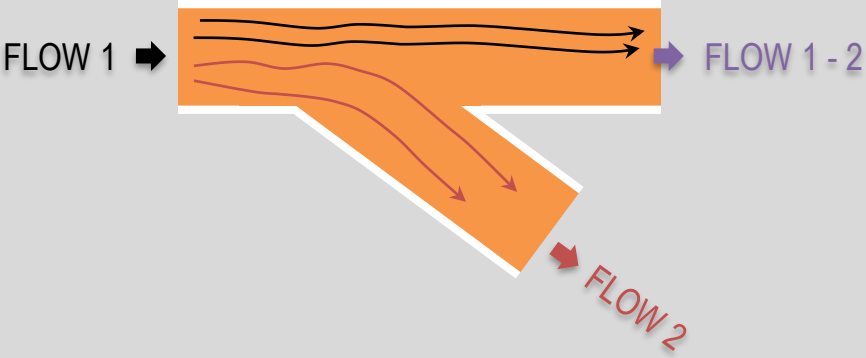
TEES



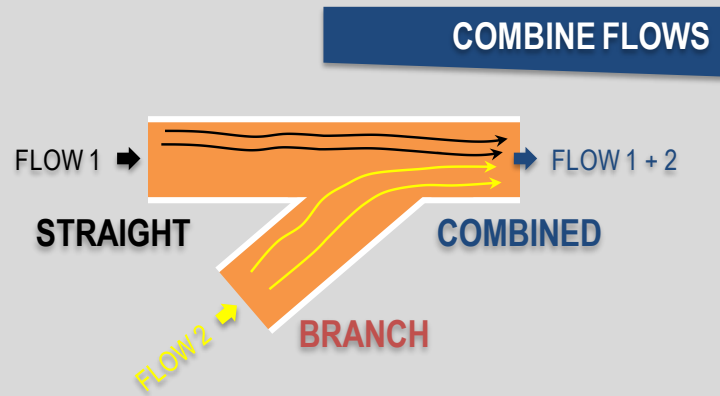
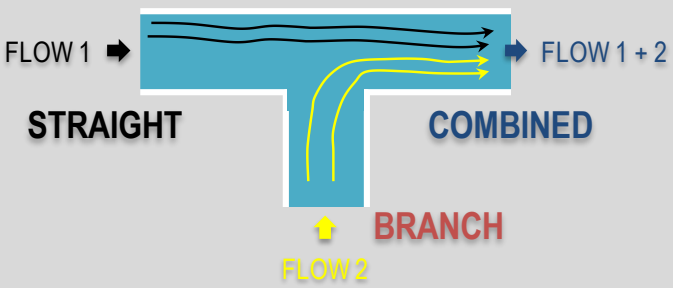
WYES



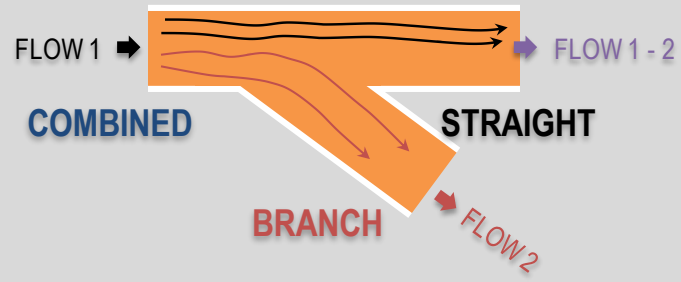
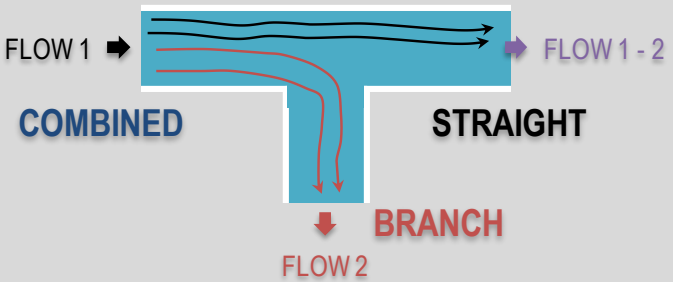
TEES



WYES

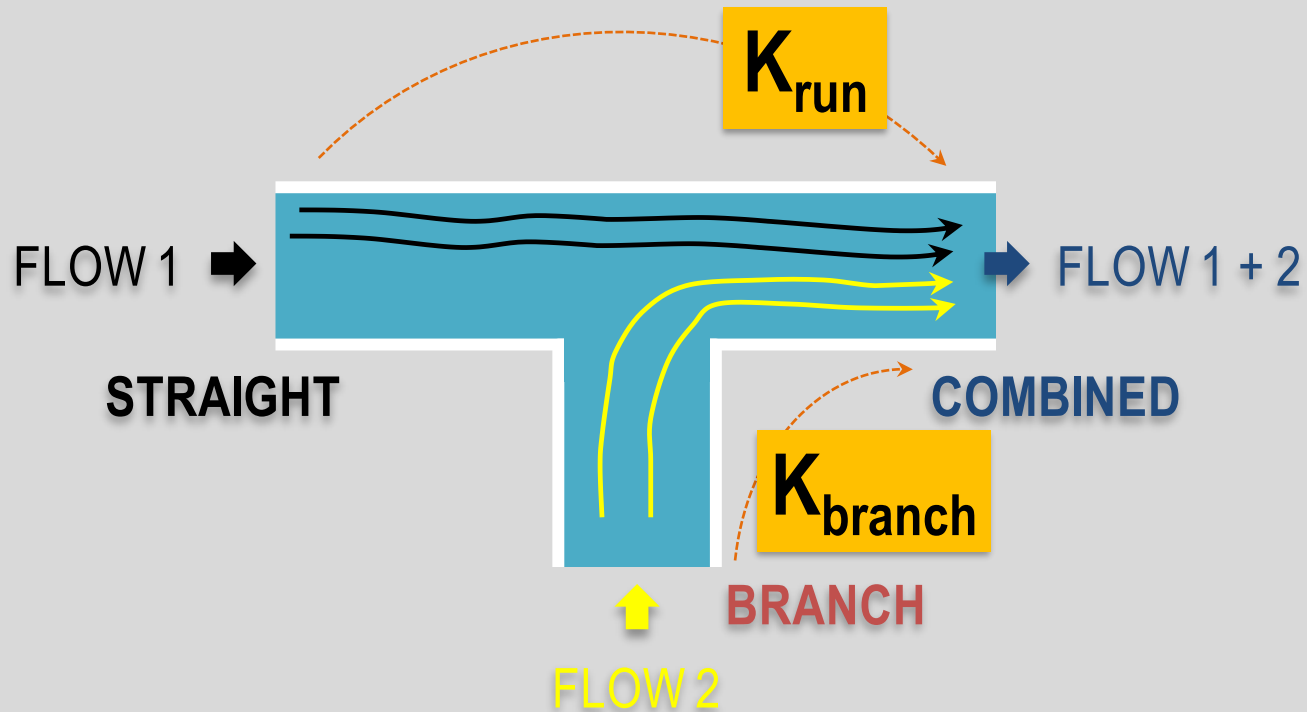


DIVIDE FLOW



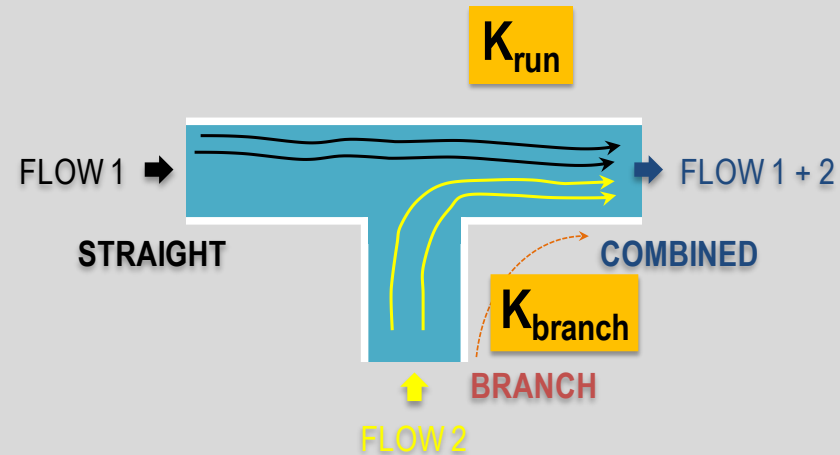
TEES

WYES



K_{run} AND K_{branch} DEPEND ON :

- ▶ Cross sectional area ratios of the legs
- ▶ The angle between the legs
- ▶ The ratio of the flow rates
- ▶ Converging or diverging ?



$$\frac{A_{branch}}{A_{combined}} = \frac{d_{branch}^2}{d_{combined}^2} = \left(\frac{d_{branch}}{d_{combined}} \right)^2 = \beta^2$$

HYDRAULIC RESISTANCE OF TEES AND WYES

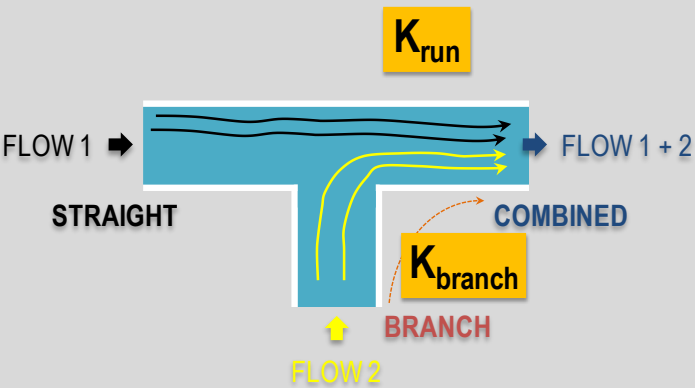
1. CONVERGING FLOW

FLOW OF FLUIDS THROUGH VALVES AND FITTINGS

		K _{branch}				K _{run}			
Angle	C	D	E	F	C	D	E	F	
30	See green table →	1	2	1.74	1	0	1	1.74	
45		1	2	1.74	1	0	1	1.74	
60		1	2	1	1	0	1	1	
90		1	2	0	Use equation in green box				

$$K_{run} \approx 1.55 \frac{Q_{branch}}{Q_{comb}} - \left(\frac{Q_{branch}}{Q_{comb}} \right)^2$$

$$K_{branch} \text{ or } K_{run} = C \left[1 + D \left(\frac{Q_{branch}}{Q_{comb}} \frac{1}{\beta_{branch}^2} \right)^2 - E \left(1 - \frac{Q_{branch}}{Q_{comb}} \right)^2 - F \frac{1}{\beta_{branch}^2} \left(\frac{Q_{branch}}{Q_{comb}} \right)^2 \right]$$



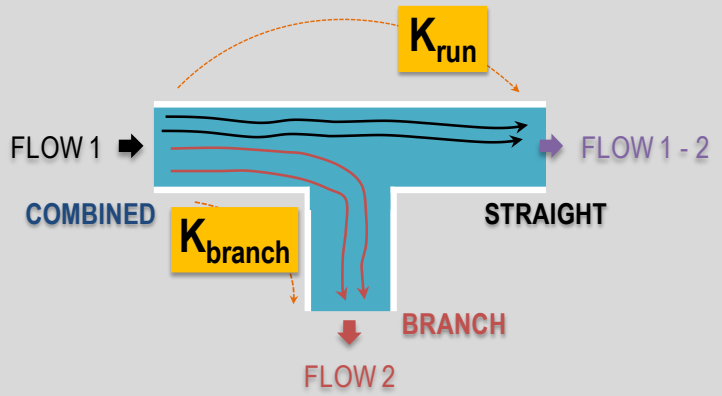
		Q _{branch} / Q _{comb}	
		≤ 0.4	> 0.4
β^2_{branch}	≤ 0.35	C = 1	
	> 0.35	$C = 0.9 \left(1 - \frac{Q_{branch}}{Q_{comb}} \right)$	C = 0.55

HYDRAULIC RESISTANCE OF TEES AND WYES

2. DIVERGING FLOW

BRANCH

α	G	H	J
0 – 60°	See green table →	1	2
$\alpha = 90^\circ$ at $\beta_{branch} \leq 2/3$	1	1	2
$\alpha = 90^\circ$ at $\beta_{branch} = 1$	$G = 1 + 0.3 \left(\frac{Q_{branch}}{Q_{comb}} \right)^2$	0.3	0

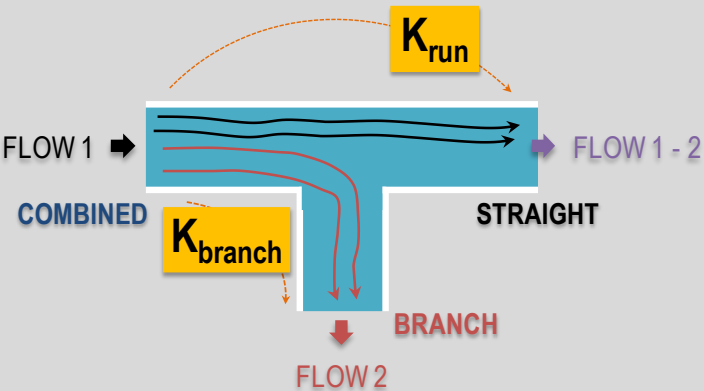


Q_{branch} / Q_{comb}			
		≤ 0.4	> 0.4
β_{branch}^2	≤ 0.35	$G = 1.1 - 0.7 \frac{Q_{branch}}{Q_{comb}}$	$G = 0.85$
	> 0.35	$G = 1 - 0.6 \frac{Q_{branch}}{Q_{comb}}$	$G = 0.6$

$$K_{branch} = G \left[1 + H \left(\frac{Q_{branch}}{Q_{comb}} \frac{1}{\beta_{branch}^2} \right)^2 - J \left(\frac{Q_{branch}}{Q_{comb}} \frac{1}{\beta_{branch}^2} \right) \cos \alpha \right]$$

RUN

$$K_{run} = M \left(\frac{Q_{branch}}{Q_{comb}} \right)^2$$



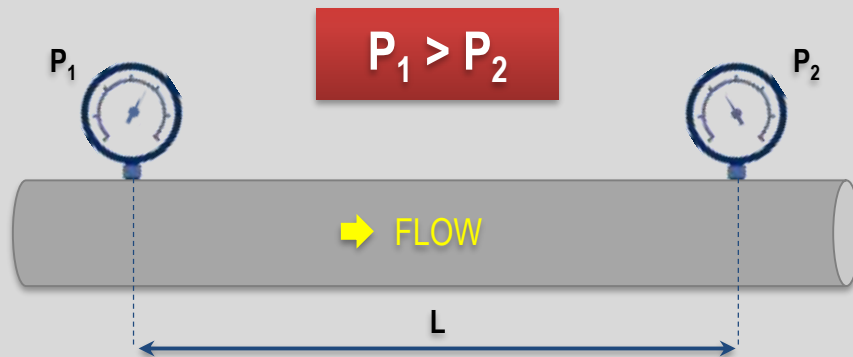
		Q_{branch} / Q_{comb}	
		≤ 0.5	> 0.5
β^2_{branch}	≤ 0.4	$M = 0.4$	
	> 0.4	$M = 2 \left(2 \frac{Q_{branch}}{Q_{comb}} - 1 \right)$	$M = 0.3 \left(2 \frac{Q_{branch}}{Q_{comb}} - 1 \right)$

DISCHARGE OF FLUIDS THROUGH VALVES, FITTINGS AND PIPE



HENRY DARCY

$$\Delta P = \frac{\rho}{144} \left(f \frac{L}{D} \frac{v^2}{2g} \right)$$



h_L : Loss of static pressure head due to fluid flow (ft)

f : Darcy friction factor (unitless)

L : Length of pipe (ft)

D : Pipe internal diameter (ft)

v : Mean velocity of flow (ft/s)

g : Gravitational acceleration = 32.174 ft/s²

$$h_L = 0.002593 \frac{KQ^2}{d^5}$$

$$Q = 19.64 d^2 \sqrt{\frac{h_L}{K}}$$

h_L : Loss of static pressure head due to fluid flow (ft)

K : Resistance coefficient (unitless)

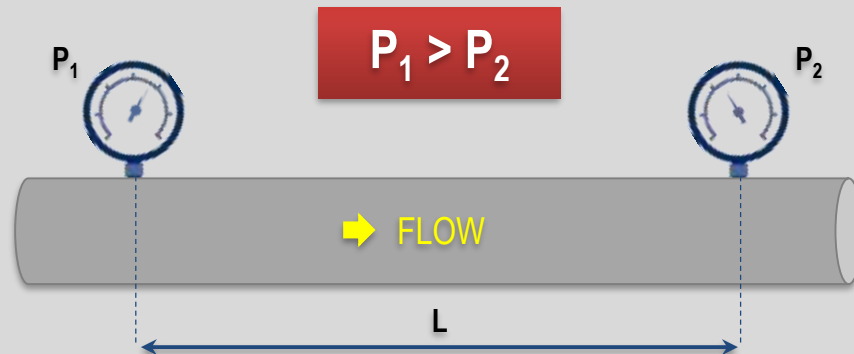
Q : Rate of fluid flow (gpm)

d : Internal diameter (in)

K : Sum of all the resistances
in the piping system



$$w = 0.525 Y d^2 \sqrt{\frac{\Delta P}{K \bar{V}_1}}$$



w : Rate of flow (lb/s)

Y : Net expansion factor for compressible flow (unitless)

d : Pipe internal diameter (in)

ΔP : Gauge pressure (psig)

K : Resistance coefficient (unitless)

V : Specific volume of fluid (ft³/lb)



REGULATING FLOW WITH CONTROL VALVES

INHERENT CHARACTERISTIC CURVE

INHERENT CHARACTERISTIC CURVE

- ▶ Plot of the valve position vs **flow coefficient C_v**
- ▶ Flow rate in gpm of 60°F
- ▶ At various position of valve travel
- ▶ With fixed $\Delta P = 1$ psi

$$C_v = \frac{Q}{F_p \sqrt{\frac{\Delta P}{S}}}$$

C_v : Valve flow coefficient

Q : Rate of flow (gpm)

F_p : Piping geometry factor (unitless)

ΔP : Pressure differential (psi)

S : Specific gravity of liquid

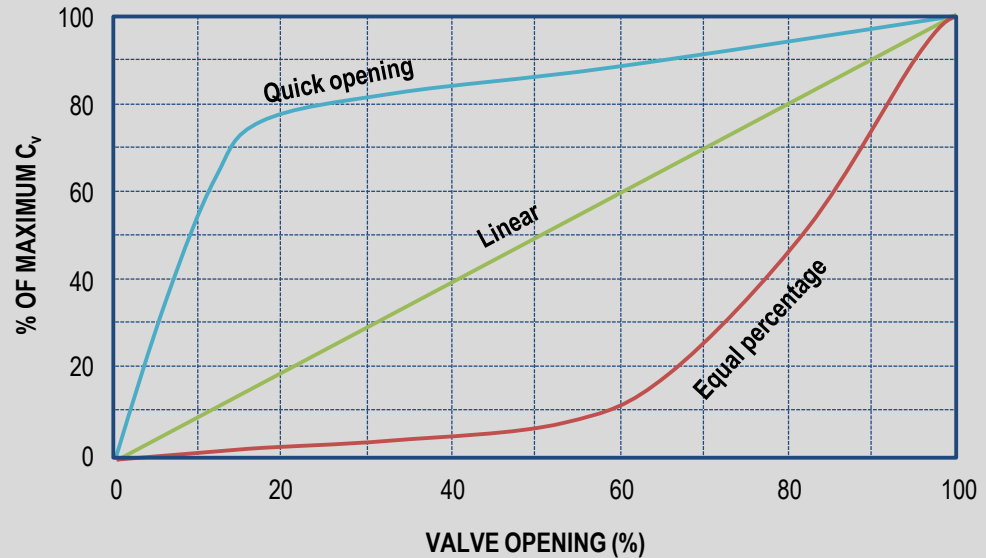


INHERENT CHARACTERISTIC CURVE

Quick opening

Linear

Equal percentage



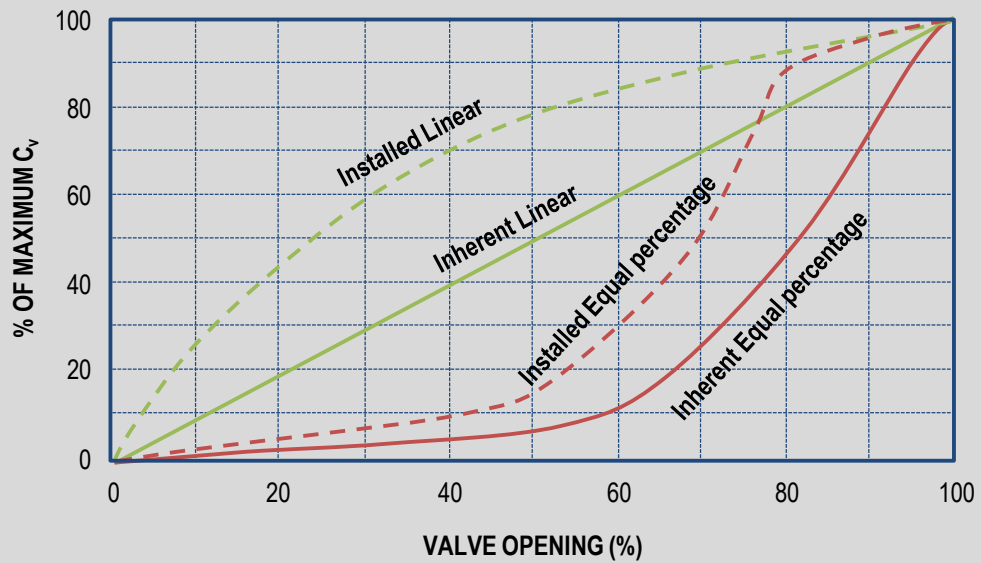
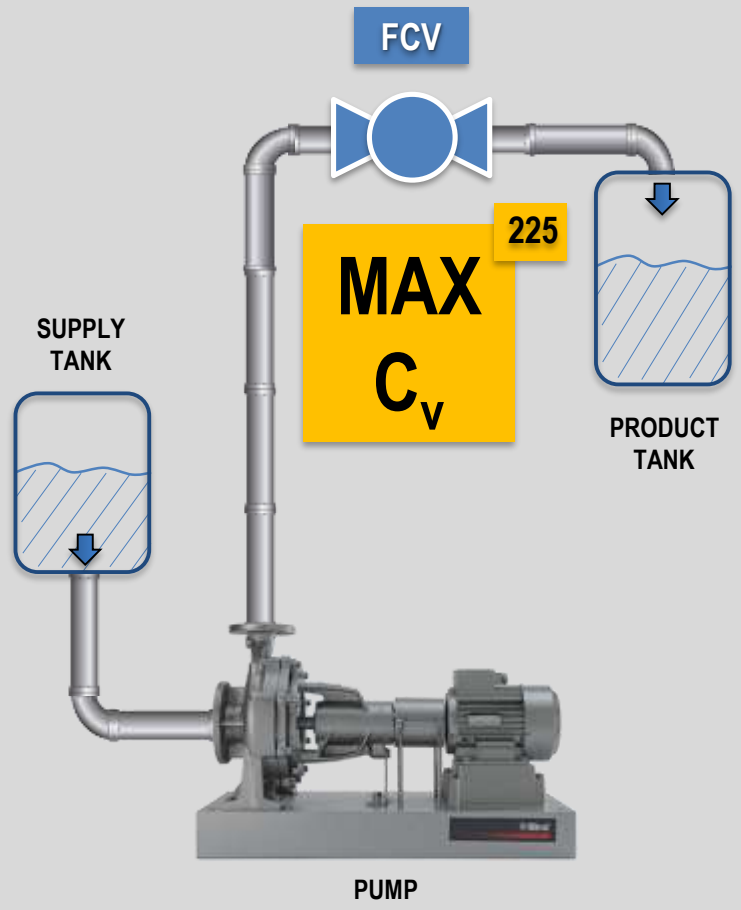
INSTALLED CHARACTERISTIC CURVE

INSTALLED CHARACTERISTIC CURVE

- ▶ Plot of the valve position vs % **maximum flow rate**
- ▶ Installed curve # Inherent curve
- ▶ Installed curve shifted **UP** and to the **LEFT**



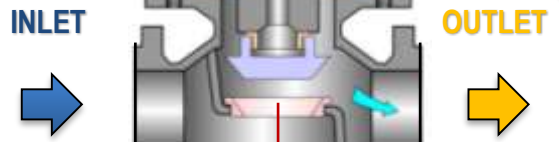
REGULATING FLOW WITH CONTROL VALVES



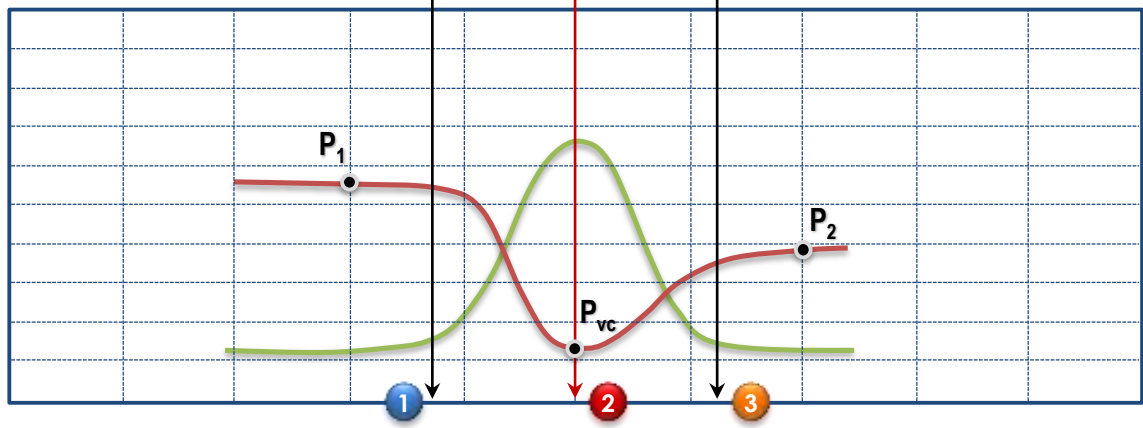
PRESSURE, VELOCITY AND ENERGY PROFILES

- 1 INLET
- 2 VENA CONTRACTA
- 3 OUTLET

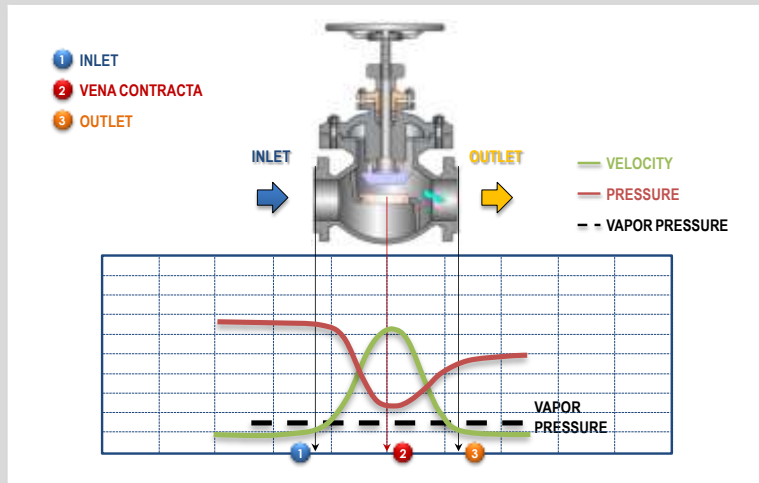
$$F_L = \sqrt{\frac{P_1 - P_2}{P_1 - P_{vc}}}$$



— VELOCITY
— PRESSURE



CAVITATION, CHOCKED FLOW AND FLASHING



DOWNSTREAM PRESSURE



FLOW RATE

$$C_v = \frac{Q}{F_p \sqrt{\frac{\Delta P}{S}}}$$

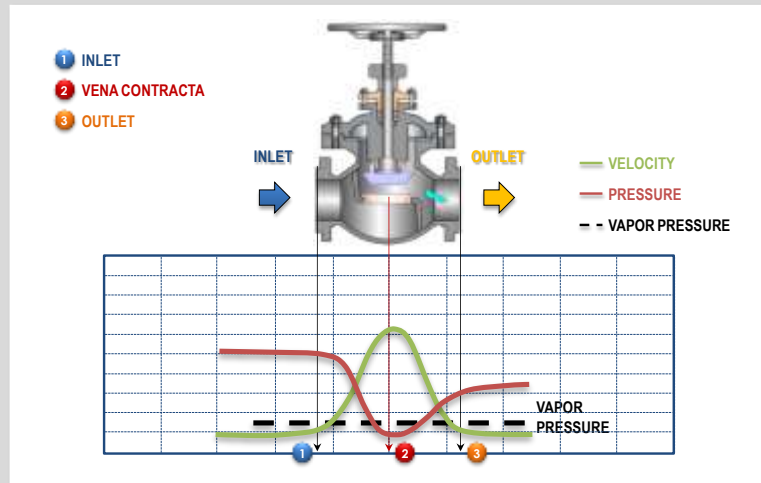
C_v : Valve flow coefficient

Q : Rate of flow (gpm)

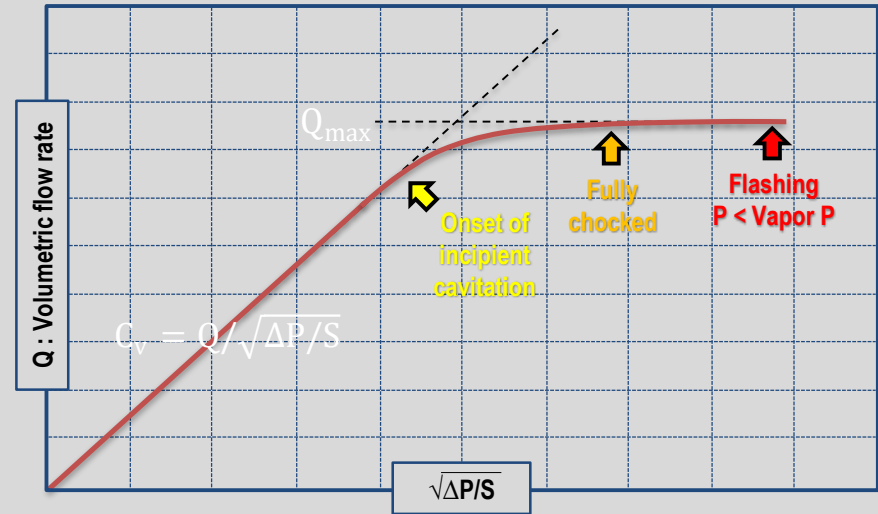
F_p : Piping geometry factor (unitless)

ΔP : Pressure differential (psi)

S : Specific gravity of liquid



CAVITATION



- ▶ Severe erosion and damage to the valve
- ▶ Increased noise levels & vibration
- ▶ Flow rate deviates from that predicted by C_v

SIZING FOR **IN**COMPRESSIBLE FLOW

- ✓ NON CHOCKED
- ✓ TURBULENT FLOW
- ✓ WITH OR WITHOUT FITTINGS

$$C_v = \frac{Q}{F_p \sqrt{\frac{\Delta P}{S}}}$$

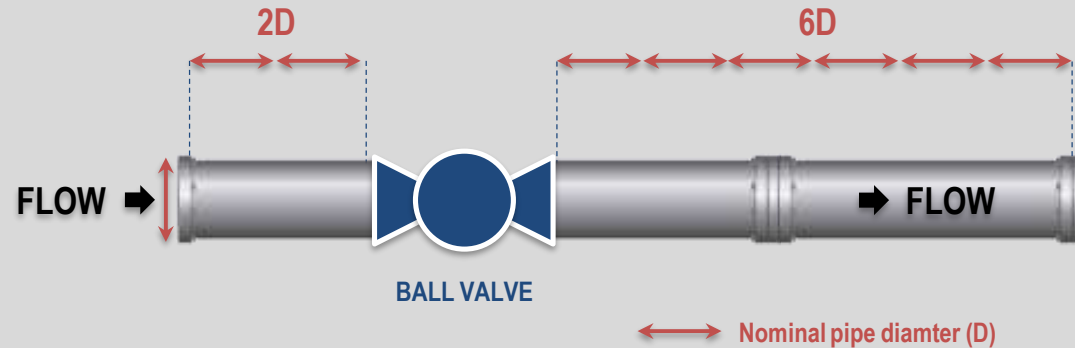
C_v : Valve flow coefficient

Q : Rate of flow (gpm)

F_p : Piping geometry factor (unitless)

ΔP : Pressure differential (psi)

S : Specific gravity of liquid



$$F_p = \frac{1}{\sqrt{1 + \frac{\sum K}{890} \left(\frac{C_v}{d_{nom,v}^2} \right)^2}}$$

$$C_v = \frac{Q}{F_p \sqrt{\frac{\Delta P}{S}}}$$

C_v : Valve flow coefficient

Q : Rate of flow (gpm)

F_p : Piping geometry factor (unitless)

ΔP : Pressure differential (psi)

S : Specific gravity of liquid

$$\sum K = K_1 + K_2 + K_{B1} + K_{B2}$$

K_1 : Upstream fittings resistance coefficient

K_2 : Downstream fittings resistance coefficient

K_{B1} : Inlet Bernoulli coefficient = $1 - (d_{nom,v} / d_{nom,1})^4$

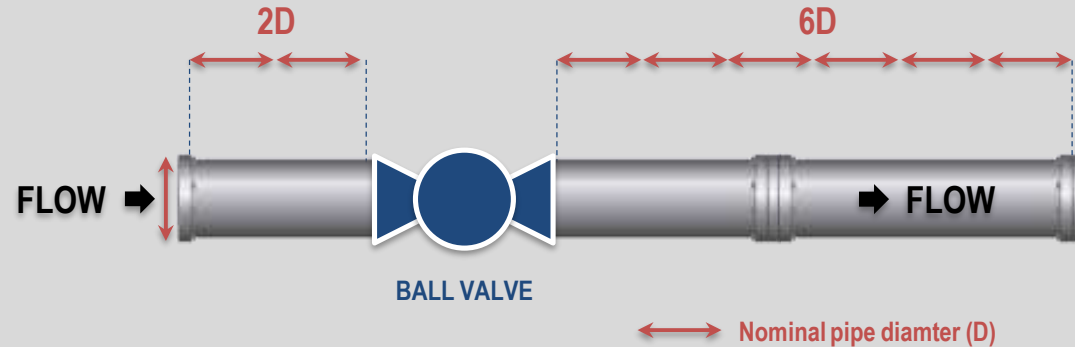
K_{B2} : Outlet Bernoulli coefficient = $1 - (d_{nom,v} / d_{nom,2})^4$

$d_{nom,v}$: Nominal valve size (in)

$d_{nom,1}$: Nominal pipe size **upstream** (in)

$d_{nom,2}$: Nominal pipe size **downstream** (in)

C_v : Flow coefficient of assumed valve size at 100% open



$$F_p = \frac{1}{\sqrt{1 + \frac{\sum K}{890} \left(\frac{C_v}{d_{nom,v}^2} \right)^2}}$$

$$C_v = \frac{Q}{F_p \sqrt{\frac{\Delta P}{S}}}$$

C_v : Valve flow coefficient

Q : Rate of flow (gpm)

F_p : Piping geometry factor (unitless)

ΔP : Pressure differential (psi)

S : Specific gravity of liquid

$$\sum K = K_1 + K_2 + K_{B1} - K_{B2}$$

K_1 : Upstream fittings resistance coefficient

K_2 : Downstream fittings resistance coefficient

K_{B1} : Inlet Bernoulli coefficient = $1 - (d_{nom,v} / d_{nom,1})^4$

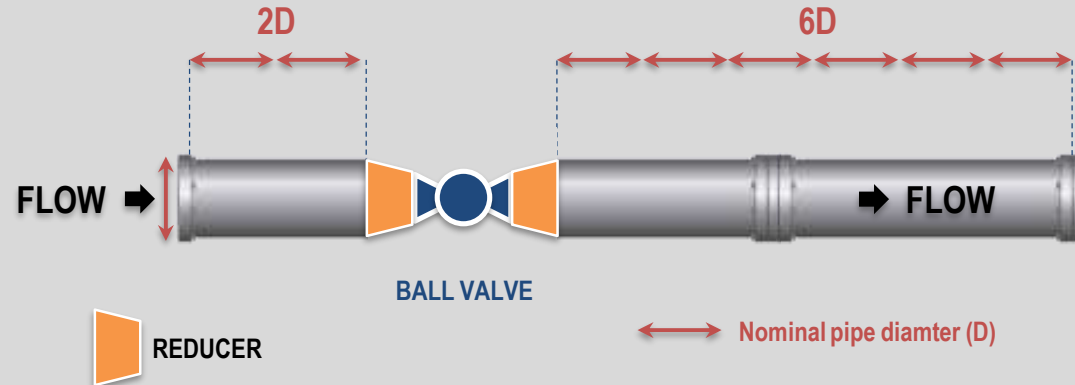
K_{B2} : Outlet Bernoulli coefficient = $1 - (d_{nom,v} / d_{nom,2})^4$

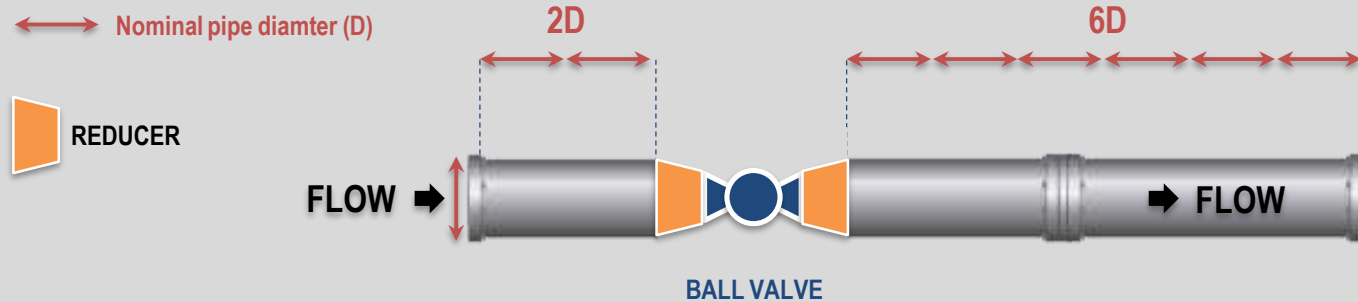
$d_{nom,v}$: Nominal valve size (in)

$d_{nom,1}$: Nominal pipe size **upstream** (in)

$d_{nom,2}$: Nominal pipe size **downstream** (in)

C_v : Flow coefficient of assumed valve size at 100% open





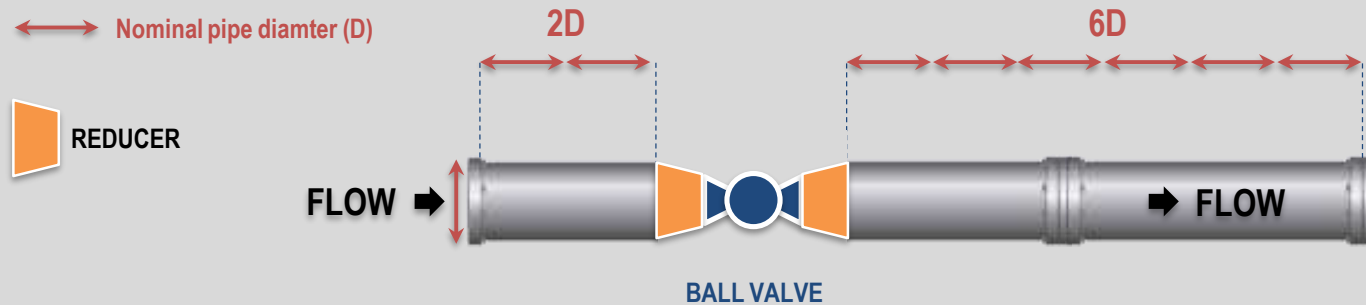
$$K_{\text{reducer}}^{\text{inlet}} = 0.5 \left[1 - \left(\frac{d_{\text{nom},v}}{d_{\text{nom},1}} \right)^2 \right]^2$$

$$K_{\text{reducer}}^{\text{outlet}} = 1 \left[1 - \left(\frac{d_{\text{nom},v}}{d_{\text{nom},2}} \right)^2 \right]^2$$

$d_{\text{nom},v}$: Nominal valve size (in)

$d_{\text{nom},1}$: Nominal pipe size **upstream** (in)

$d_{\text{nom},2}$: Nominal pipe size **downstream** (in)

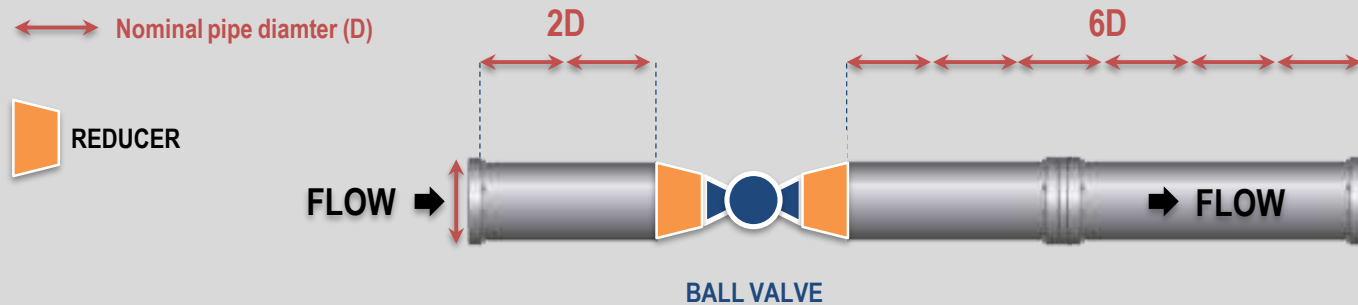


$$K_{\text{reducer}}^{\text{inlet}} = 0.5 \left[1 - \left(\frac{d_{\text{nom},v}}{d_{\text{nom},1}} \right)^2 \right]^2$$

$$K_{\text{reducer}}^{\text{outlet}} = 1 \left[1 - \left(\frac{d_{\text{nom},v}}{d_{\text{nom},2}} \right)^2 \right]^2$$

~~$$K_{B1} = 1 - \left(\frac{d_{\text{nom},v}}{d_{\text{nom},1}} \right)^4$$~~

~~$$K_{B2} = 1 - \left(\frac{d_{\text{nom},v}}{d_{\text{nom},2}} \right)^4$$~~



$$\sum K = 1.5 \left[1 - \left(\frac{d_{\text{nom},v}}{d_{\text{nom}}} \right)^2 \right]^2$$

$d_{\text{nom},v}$: Nominal valve size (in)

d_{nom} : Nominal pipe size (in)

$$F_p = \frac{1}{\sqrt{1 + \frac{\sum K}{890} \left(\frac{C_v}{d_{nom,v}^2} \right)^2}}$$

$$C_v = \frac{Q}{F_p \sqrt{\frac{\Delta P}{S}}}$$

C_v : Valve flow coefficient

Q : Rate of flow (gpm)

F_p : Piping geometry factor (unitless)

ΔP : Pressure differential (psi)

S : Specific gravity of liquids

$$\sum K = K_1 + K_2 + K_{B1} - K_{B2}$$

K_1 : Upstream fittings resistance coefficient

K_2 : Downstream fittings resistance coefficient

K_{B1} : Inlet Bernoulli coefficient = $1 - (d_{nom,v} / d_{nom,1})^4$

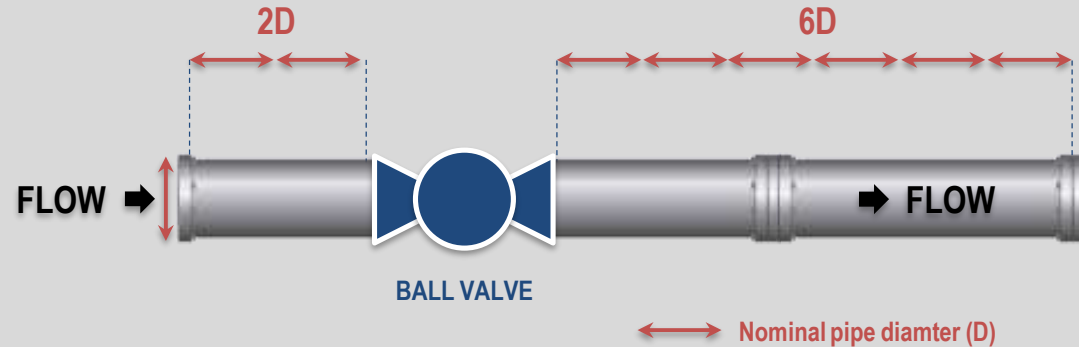
K_{B2} : Outlet Bernoulli coefficient = $1 - (d_{nom,v} / d_{nom,2})^4$

$d_{nom,v}$: Nominal valve size (in)

$d_{nom,1}$: Nominal pipe size **upstream** (in)

$d_{nom,2}$: Nominal pipe size **downstream** (in)

C_v : Flow coefficient of assumed valve size at 100% open



**MAXIMUM FLOW
RATE****at which choking occurs**

$$Q_{\max} = F_L C_v \sqrt{\frac{P_1 - F_F P_v}{S}}$$

**MAXIMUM
 ΔP** **at which choking occurs**

$$\Delta P_{\max} = F_L^2 (P_1 - F_F P_v) \qquad \Delta P_{\max} = \left(\frac{F_{LP}}{F_P} \right)^2 (P_1 - F_F P_v)$$

 F_L : Liquid pressure recovery factor (unitless) C_v : Flow coefficient for valves or piping components P_1 : Absolute **up**stream pressure (psia) F_F : Liquid critical pressure ratio factor (unitless) P_v : Absolute fluid vapor pressure (psia) S : Specific gravity of liquids F_{LP} : Combined piping geometry and liquid pressure recovery factor (unitless) F_P : Piping geometry factor (unitless)

F_F : Liquid critical pressure ratio factor

$$F_F = 0.96 - 0.28 \sqrt{\frac{P_v}{P_c}}$$

P_v : Absolute fluid vapor pressure (psia)

P_c : Fluid critical pressure (psia)

F_{LP} : Combined piping geometry and liquid pressure recovery factor

$$F_{LP} = \frac{F_L}{\sqrt{1 + F_L^2 \frac{\sum K_i}{890} \left(\frac{C_v}{d_{nom,v}} \right)^2}}$$

F_L : Liquid pressure recovery factor (unitless)

F_p : Piping geometry factor (unitless)

C_v : Flow coefficient for valves or piping components

$d_{nom,v}$: Nominal valve size (in)

$\sum K_i = K_1 + K_{B1}$

K_1 : Upstream fittings resistance coefficient

K_{B1} : Inlet Bernoulli coefficient = $1 - (d_{nom,v} / d_{nom,1})^4$

SIZING FOR COMPRESSIBLE FLOW

SIZING AND SELECTION

$$0.667 \leq Y = 1 - \frac{x}{3F_k x_T} \leq 1$$

$$x = \frac{\Delta P}{P_1}$$

$$F_k = \frac{C_p}{C_v} / 1.4$$

x_T : Critical pressure drop factor
for a valve **without** fittings

COMPRESSIBILITY

- Expansion factor “Y”

APPLICABLE ONLY TO

- Single phase gases or vapors

SIZING AND SELECTION

WITH
FITTINGS

$$Y = 1 - \frac{x}{3F_k x_{TP}}$$

$$x_{TP} = \frac{x_T / F_p^2}{1 + x_T \frac{\sum K_i}{1000} \left(\frac{C_v}{d_{nom,v}^2} \right)^2}$$

$$x = \frac{\Delta P}{P_1} \quad F_k = \frac{C_p}{C_v / 1.4}$$

COMPRESSIBILITY

- Expansion factor “Y”

APPLICABLE ONLY TO

- Single phase gases or vapors

MASS FLOW
RATE

$$C_v = \frac{W}{63.3 F_P Y \sqrt{x P_1 \rho_1}}$$

VOLUMETRIC
FLOW RATE

$$C_v = \frac{q_h}{1360 F_P P_1 Y \sqrt{\frac{x}{S_g T_1 Z}}}$$

C_v : Flow coefficient for valves or piping components

W : Rate of flow (lb/h)

Y : Expansion factor (unitless)

x : Pressure drop ratio (unitless)

P_1 : Absolute **upstream** pressure (psia)

ρ_1 : Weight density of fluid **upstream** (lb/ft³)

q_h : Rate of flow at **standard conditions** (scfh)

S_g : Specific gravity of a gas relative to air

T_1 : Absolute **upstream** temperature (°Rankine)

Z_f : Compressibility factor (unitless)

CONVERSION OF C_v TO K_v

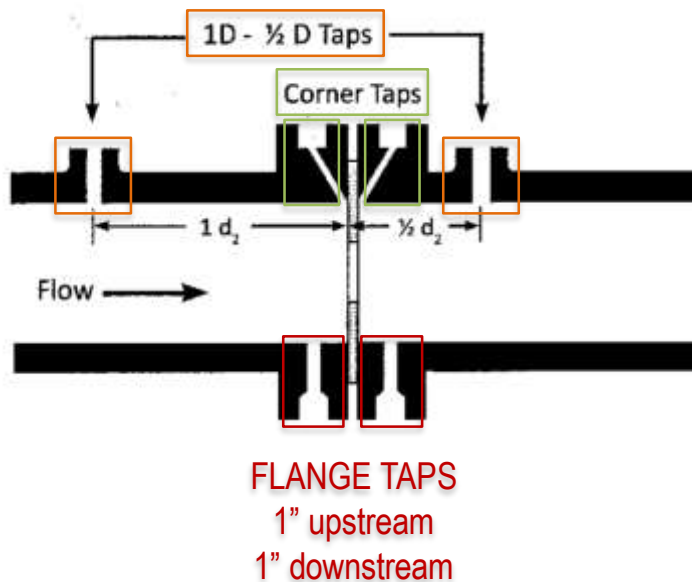
CONVERSION OF C_v TO K_v

$$K_v = 0.865C_v$$



MEASURING FLOW WITH DIFFERENTIAL PRESSURE METERS

LIMITS OF USE OF ORIFICE PLATES



LIMITS OF USE

CORNER OR 1D - 1/2 D TAPS

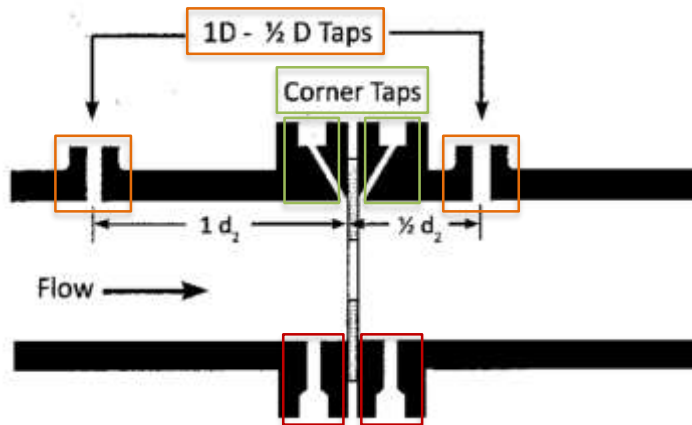
$$d_1 \geq 0.5 \text{ inch}$$

$$2 \text{ inch} \leq d_2 \leq 40 \text{ inch}$$

$$0.10 \leq \beta \leq 0.75$$

$$Re \geq 5000 \text{ for } 0.10 \leq \beta \leq 0.56$$

$$Re \geq 16000\beta^2 \text{ for } \beta > 0.56$$



FLANGE TAPS
1" upstream
1" downstream

LIMITS OF USE

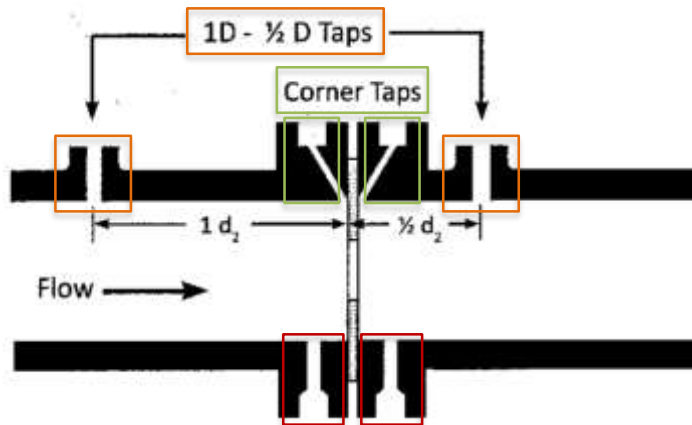
FLANGE TAPS

$$d_1 \geq 0.5 \text{ inch}$$

$$2 \text{ inch} \leq d_2 \leq 40 \text{ inch}$$

$$0.10 \leq \beta \leq 0.75$$

$$Re \geq 5000 \text{ and } Re > 4.318\beta^2 d_2$$



FLANGE TAPS
1" upstream
1" downstream

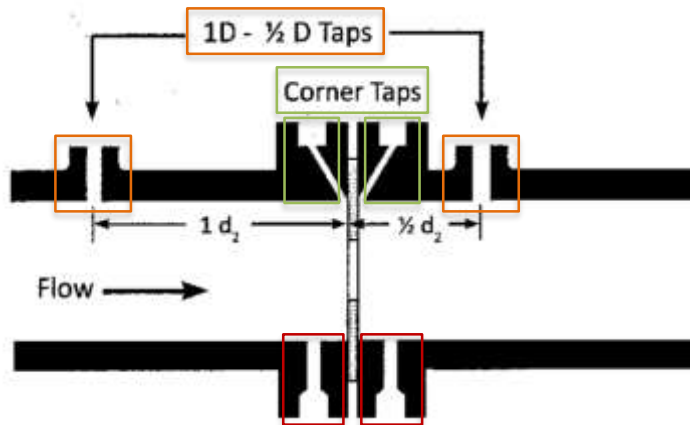
LIMITS OF USE

GASES

$$0.8 < \frac{P_2}{P_1} < 1$$

PRESSURE DIFFERENTIAL

$$\Delta P \leq 36.31 \text{ psi}$$



FLANGE TAPS
1" upstream
1" downstream

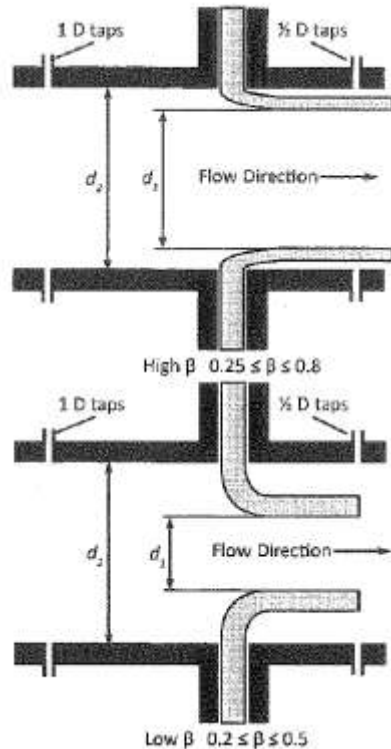
LIMITS OF USE

STRAIGHT LENGTHS OF PIPE

ASME MFC-3M
STANDARD

LIMITS OF USE OF FLOW NOZZLES

LONG RADIUS NOZZLE



LIMITS OF USE

LONG RADIUS NOZZLES

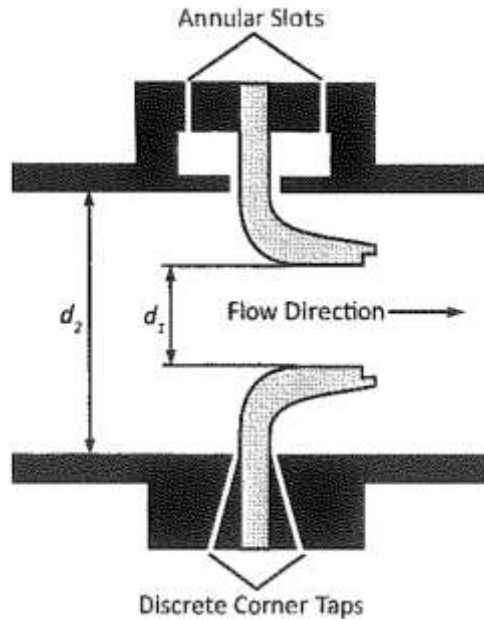
$$2 \text{ inch} \leq d_2 \leq 25 \text{ inch}$$

$$0.20 \leq \beta \leq 0.80$$

$$10^4 \leq Re \leq 10^7$$

$$\varepsilon/d_2 \leq 3.2 \times 10^{-4}$$

ISA 1932 NOZZLE

LIMITS OF USE

ISA 1932 NOZZLE

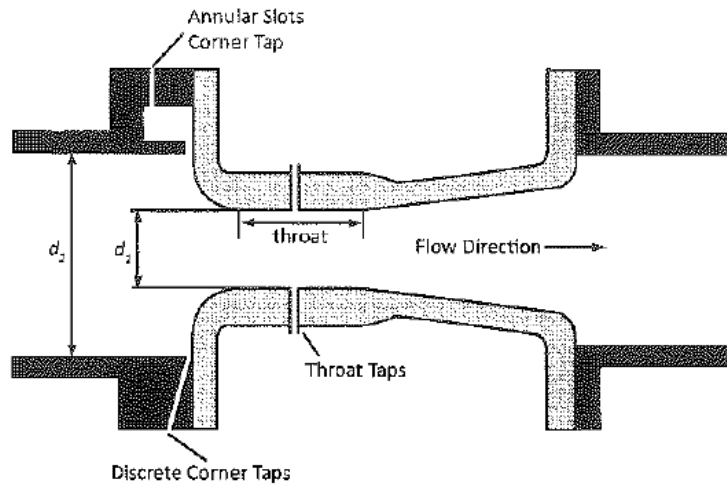
$$2 \text{ inch} \leq d_2 \leq 20 \text{ inch}$$

$$0.30 \leq \beta \leq 0.80$$

$$7 \times 10^4 \leq Re \leq 10^7 \text{ for } 0.30 \leq \beta \leq 0.44$$

$$2 \times 10^4 \leq Re \leq 10^7 \text{ for } 0.44 \leq \beta \leq 0.80$$

VENTURI NOZZLE



LIMITS OF USE

VENTURI NOZZLE

$$2.5 \text{ inch} \leq d_2 \leq 20 \text{ inch}$$

$$d_1 \geq 2 \text{ inch}$$

$$0.316 \leq \beta \leq 0.775$$

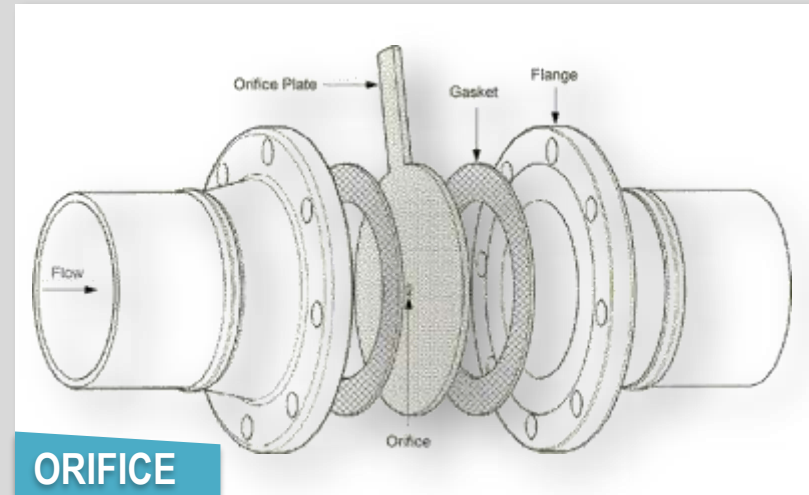
$$1.5 \times 10^5 \leq Re \leq 2 \times 10^6$$



NOZZLE

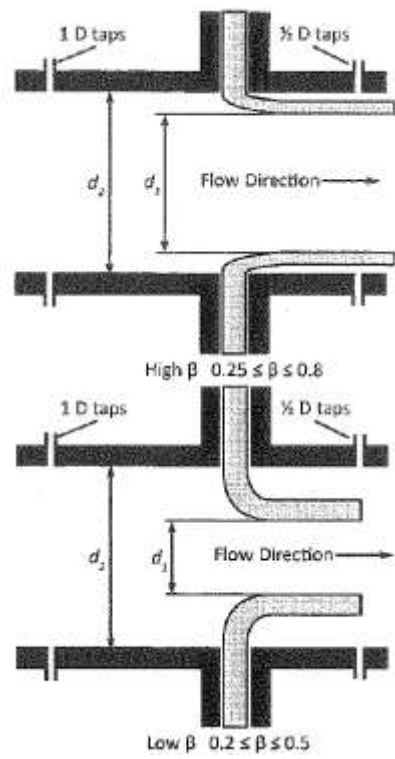
- ▶ High temperature
- ▶ High velocity
- ▶ Sensitive to flow conditions

- ▶ Sensitive to flow conditions

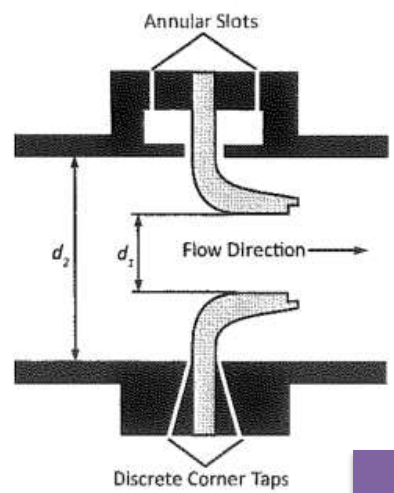


ORIFICE

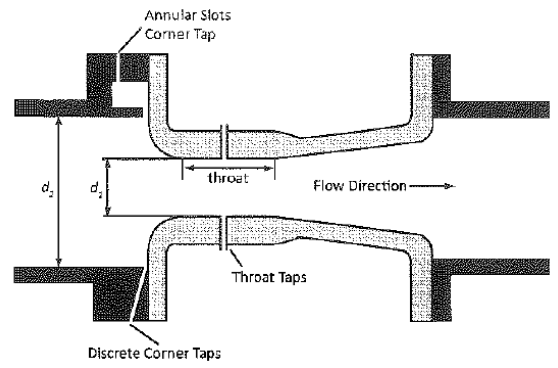
LONG RADIUS NOZZLE



ISA 1932 NOZZLE



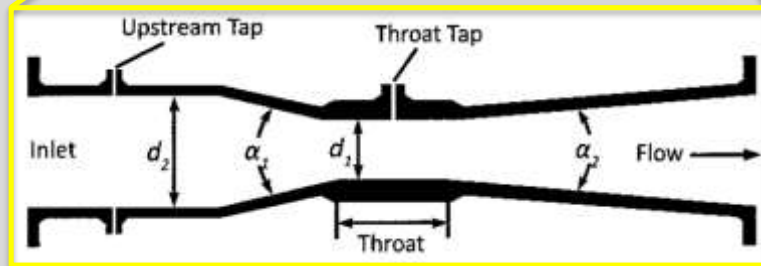
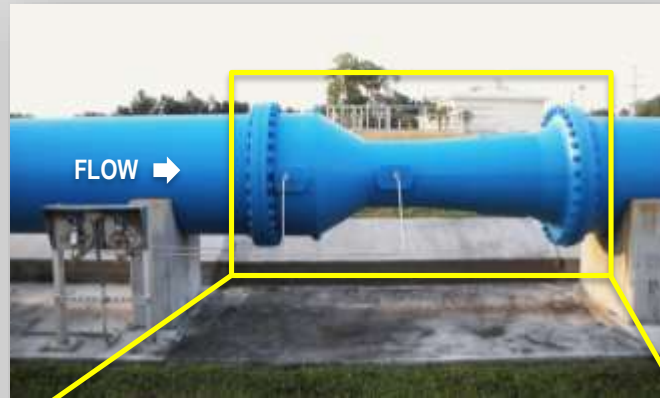
VENTURI NOZZLE



STRAIGHT LENGTHS OF PIPE
ASME MFC-3M
STANDARD

LIMITS OF USE OF VENTURI METERS

“AS CAST”



LIMITS OF USE

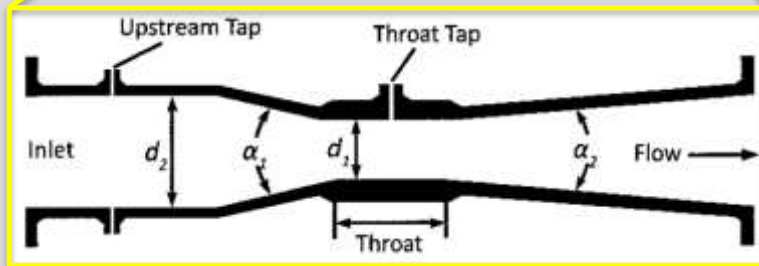
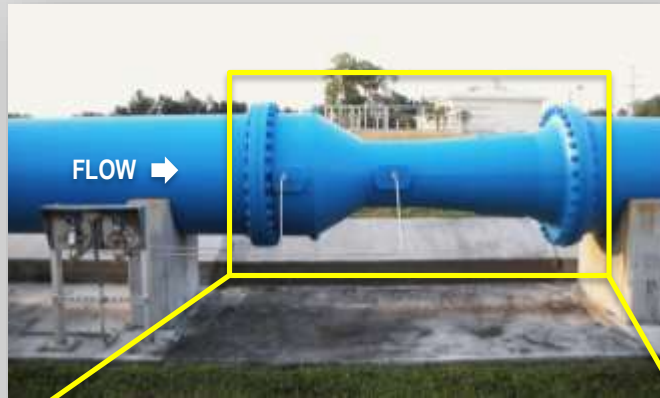
“AS CAST” CONVERGENT SECTION

$$4 \text{ inch} \leq d_2 \leq 48 \text{ inch}$$

$$0.30 \leq \beta \leq 0.75$$

$$2 \times 10^5 \leq Re \leq 6 \times 10^6$$

MACHINED



LIMITS OF USE

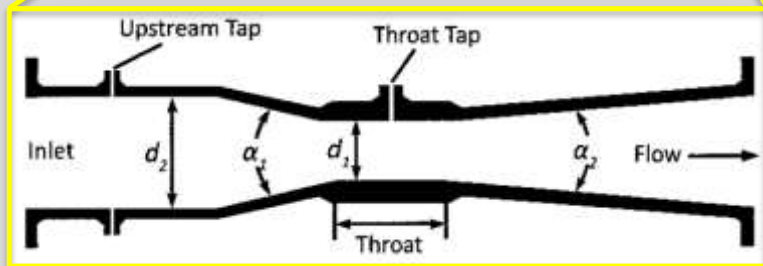
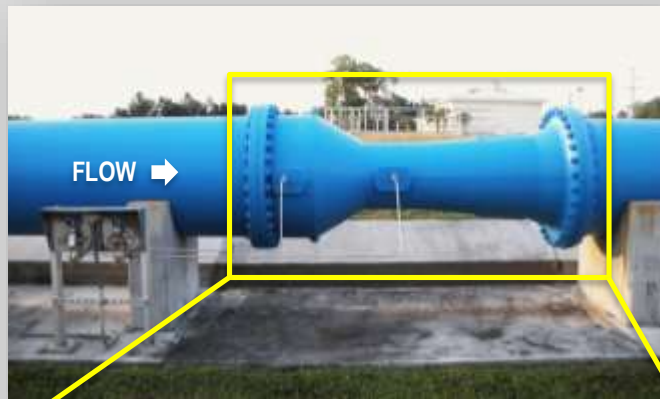
MACHINED CONVERGENT SECTION

$$2 \text{ inch} \leq d_2 \leq 10 \text{ inch}$$

$$0.30 \leq \beta \leq 0.75$$

$$2 \times 10^5 \leq Re \leq 6 \times 10^6$$

ROUGH-WELDED



LIMITS OF USE

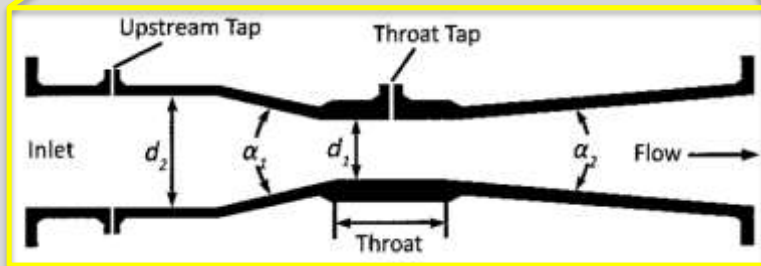
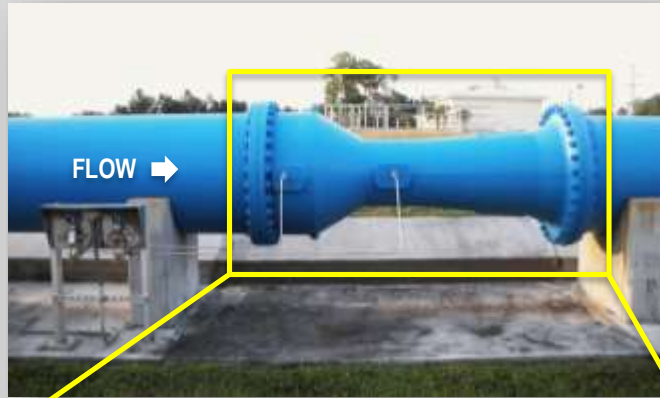
ROUGH-WELDED CONVERGENT SECTION

$$4 \text{ inch} \leq d_2 \leq 48 \text{ inch}$$

$$0.30 \leq \beta \leq 0.75$$

$$2 \times 10^5 \leq Re \leq 6 \times 10^6$$

ALL VENTURI



LIMITS OF USE

- ▶ High flow rates
- ▶ High pressure recovery
- ▶ High energy recovery
- ▶ Unrecovered pressure $< 10\% \Delta P$



LIQUID FLOW THROUGH ORIFICES, NOZZLES AND VENTURI

DIFFERENTIAL PRESSURE AND PRESSURE LOSS

1

DIFFERENTIAL PRESSURE (ΔP or D_p)

The difference between the **absolute** pressures at the **upstream** and **downstream** taps, referred to as :

$$dP \text{ or } \Delta P$$

2

PRESSURE LOSS (NRPD)

The **permanent** pressure loss or **non-recoverable** pressure drop (NRPD) which is the difference in static pressure between the pressure measured on **upstream** side of the primary device and that measured on the **downstream** side

2

PRESSURE LOSS (NRPD)

The **permanent** pressure loss or **non-recoverable** pressure drop (NRPD) which is the difference in static pressure between the pressure measured on **up**stream side of the primary device and that measured on the **down**stream side

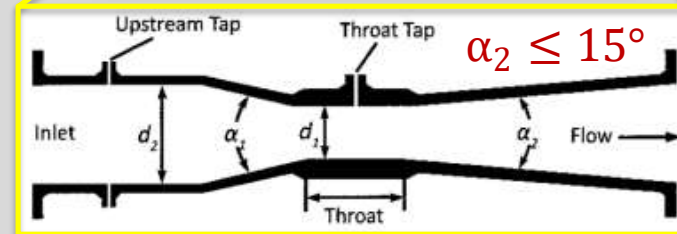
$$\text{NRPD} = \Delta P \left[\frac{\sqrt{1 - \beta^4 (1 - C_d^2)} - C_d \beta^2}{\sqrt{1 - \beta^4 (1 - C_d^2)} + C_d \beta^2} \right]$$

C_d : Discharge coefficient for orifices and nozzles

2

PRESSURE LOSS (NRPD)

The **permanent** pressure loss or **non-recoverable** pressure drop (NRPD) which is the difference in static pressure between the pressure measured on **upstream** side of the primary device and that measured on the **downstream** side



$$\text{NRPD} \approx 5\% - 20\% \Delta P + \Delta P_{\text{straight pipe}}$$

PRESSURE LOSS COEFFICIENT

PRESSURE LOSS COEFFICIENT

$$K = \left[\frac{\sqrt{1 - \beta^4(1 - C_d^2)}}{C_d \beta^2} - 1 \right]^2$$

C_d : Discharge coefficient for orifices and nozzles

RATE OF FLOW

RATE OF FLOW

$$q = C_d A \sqrt{2gh_L}$$

$$q = \frac{C_d A}{\sqrt{1 - \beta^4}} \sqrt{2gh_L}$$

q : Rate of flow at flowing conditions (ft^3/s)

C_d : Discharge coefficient for orifices and nozzles

A : Cross sectional area (ft^2)

g : Gravitational acceleration = 32.174 ft/s^2

h_L : Loss of static pressure head due to fluid flow (ft)

β : Ratio of small to large diameter (Unitless)

FLOW COEFFICIENT “C”

$$C = \frac{C_d}{\sqrt{1 - \beta^4}}$$

C_d : Discharge coefficient for orifices and nozzles

β : Ratio of small to large diameter (Unitless)

RATE OF FLOW

$$q = \frac{C_d A}{\sqrt{1 - \beta^4}} \sqrt{2gh_L}$$

q : Rate of flow at flowing conditions (ft³/s)

C_d : Discharge coefficient for orifices and nozzles

A : Cross sectional area (ft²)

g : Gravitational acceleration = 32.174 ft/s²

h_L : Loss of static pressure head due to fluid flow (ft)

β : Ratio of small to large diameter (Unitless)

RATE OF FLOW

$$q = CA\sqrt{2gh_L}$$

q : Rate of flow at flowing conditions (ft³/s)

C : Flow coefficient

A : Cross sectional area (ft²)

g : Gravitational acceleration = 32.174 ft/s²

h_L : Loss of static pressure head due to fluid flow (ft)

RATE OF FLOW

$$q = CA \sqrt{\frac{2 \times 144 g \Delta P}{\rho}}$$

q : Rate of flow at flowing conditions (ft³/s)

C : Flow coefficient

A : Cross sectional area (ft²)

g : Gravitational acceleration = 32.174 ft/s²

ΔP : Pressure differential (psi)

ρ : Weight density of fluid (lb/ft³)

RATE OF FLOW

- ▶ Incompressible fluids
- ▶ Discharging to atmosphere

$$q = CA \sqrt{\frac{2 \times 144 g \Delta P}{\rho}}$$

q : Rate of flow at flowing conditions (ft³/s)

C : Flow coefficient

A : Cross sectional area (ft²)

g : Gravitational acceleration = 32.174 ft/s²

ΔP : Pressure differential (psi)

ρ : Weight density of fluid (lb/ft³)

DISCHARGE COEFFICIENT “ C_d ”

ORIFICE PLATES

Reader-Harris / Gallagher equation

$$C_d = 0.5961 + 0.0261\beta^2 - 0.216\beta^8 + 0.000521 \left(\frac{10^6\beta}{Re} \right)^{0.7} + \left(0.0188 + 0.0063 \left(\frac{19000\beta}{Re} \right)^{0.8} \right) \beta^{3.5} \left(\frac{10^6}{Re} \right)^{0.3} \\ + (0.043 + 0.08e^{-10L_1} - 0.123e^{-7L_1}) \left(1 - 0.11 \left(\frac{19000\beta}{Re} \right)^{0.8} \right) \frac{\beta^4}{1 - \beta^4} - 0.031(M_2 - 0.8M_2^{1.1})\beta^{1.3}$$

β : diameter ratio = d_1/d_2

Re : Pipe Reynolds number

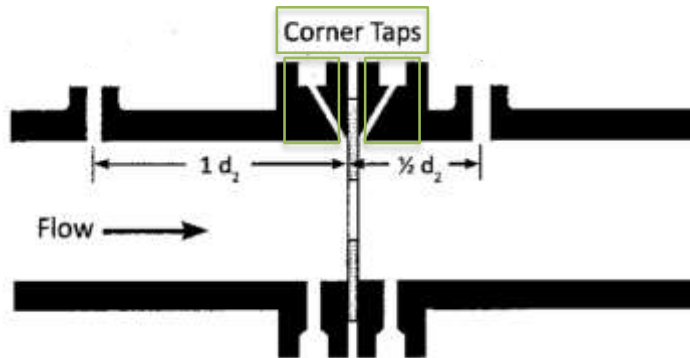
L_1 : Ratio of the distance of the upstream tap from the upstream face of the plate and the pipe diameter

L_2 : Ratio of the distance of the downstream tap from the downstream face of the plate and the pipe diameter

$$M_2 = \frac{2L_2}{1 - \beta}$$

If $d_2 < 2.8$ inch

+ $0.011(0.75 - \beta)(2.8 - d_2)$



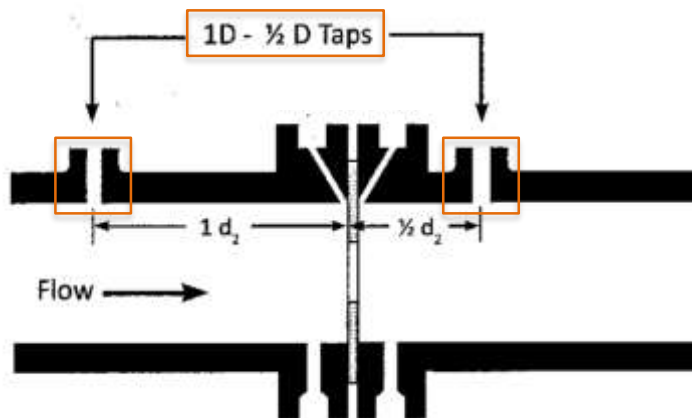
**ASME MFC-3M
STANDARD**

CORNER TAPS

$$L_1 = L_2 = 0$$

L_1 : Ratio of the distance of the upstream tap from the upstream face of the plate and the pipe diameter

L_2 : Ratio of the distance of the downstream tap from the downstream face of the plate and the pipe diameter



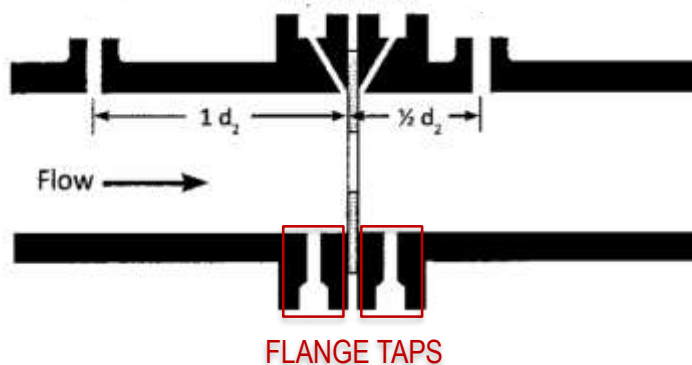
**ASME MFC-3M
STANDARD**

1D – ½ D TAPS

$$L_1 = 1 \quad L_2 = 0.47$$

L_1 : Ratio of the distance of the upstream tap from the upstream face of the plate and the pipe diameter

L_2 : Ratio of the distance of the downstream tap from the downstream face of the plate and the pipe diameter



**ASME MFC-3M
STANDARD**

FLANGE TAPS

$$L_1 = L_2 = \frac{1}{d_2}$$

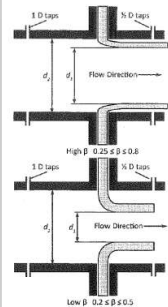
L_1 : Ratio of the distance of the upstream tap from the upstream face of the plate and the pipe diameter

L_2 : Ratio of the distance of the downstream tap from the downstream face of the plate and the pipe diameter

DISCHARGE COEFFICIENT “ C_d ”

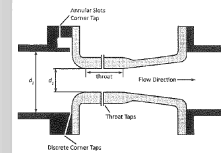
FLOW NOZZLES

LONG RADIUS NOZZLE



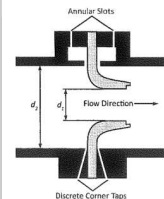
$$C_d = 0.9965 - 0.00653\beta^{0.5} \left(\frac{10^6}{Re} \right)^{0.5}$$

VENTURI NOZZLE



$$C_d = 0.9858 - 0.196\beta^{4.5}$$

ISA 1932 NOZZLE



$$C_d = 0.99 - 0.2262\beta^{4.1} - (0.00175\beta^2 - 0.0033\beta^{4.15}) \left(\frac{10^6}{Re} \right)^{1.15}$$

β : diameter ratio = d_1/d_2

DISCHARGE COEFFICIENT “ C_d ”

VENTURI METERS

“AS CAST”

$$C_d = 0.984$$

MACHINED

$$C_d = 0.995$$

ROUGH-WELDED

$$C_d = 0.985$$



VENTURI

ASME MFC-3M
STANDARD



COMPRESSIBLE FLOW THROUGH ORIFICES, NOZZLES AND VENTURI

FLOW OF GASES AND NET EXPANSIBILITY FACTOR “Y”

FLOW OF GASES AND VAPORS

$$q = YCA \sqrt{\frac{2 \times 144 g \Delta P}{\rho}}$$

q : Rate of flow at flowing conditions (ft³/s)

Y : Net expansibility factor (Unitless)

C : Flow coefficient

A : Cross sectional area (ft²)

g : Gravitational acceleration = 32.174 ft/s²

ΔP : Pressure differential (psi)

ρ : Weight density of fluid (lb/ft³)

NET EXPANSIBILITY FACTOR “Y”

- ▶ The specific heat ratio “k”
- ▶ The ratio β of orifice diameter to inlet diameter
- ▶ Ratio of downstream to upstream **absolute** pressures

$$q = YCA \sqrt{\frac{2 \times 144 g \Delta P}{\rho}}$$

q : Rate of flow at flowing conditions (ft³/s)

Y : Net expansibility factor (Unitless)

C : Flow coefficient

A : Cross sectional area (ft²)

g : Gravitational acceleration = 32.174 ft/s²

ΔP : Pressure differential (psi)

ρ : Weight density of fluid (lb/ft³)

NET EXPANSIBILITY FACTOR “Y”

$$Y = 1 - (0.351 + 0.256\beta^4 + 0.93\beta^8) \left(1 - \left(\frac{P_2}{P_1} \right)^{\frac{1}{k}} \right)$$

β : Ratio of orifice diameter to inlet diameter

P_1 : Upstream absolute pressure (psia)

P_2 : Downstream absolute pressure (psia)

k : Specific heat ratio

NET EXPANSIBILITY FACTOR “Y”

$$Y = \left[\frac{\left(k \left(\frac{P_2}{P_1} \right)^{\frac{2}{k}} \right)}{(k-1)} \frac{(1-\beta^4)}{\left(1 - \beta^4 \left(\frac{P_2}{P_1} \right)^{\frac{2}{k}} \right)} \frac{\left(1 - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right)}{\left(1 - \left(\frac{P_2}{P_1} \right) \right)} \right]^{0.5}$$

β : Ratio of orifice diameter to inlet diameter

P_1 : Upstream absolute pressure (psia)

k : Specific heat ratio

P_2 : Downstream absolute pressure (psia)