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GATING AND RISER OF ALUMINUM

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Sequential Filling of Mold Cavities

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ABSTRACT

A technique has been proposed whereby the sequential filling of mold cavities can be accomplished making use of a runner dam, or weir. Through this procedure, individual mold cavities can be poured at a rate more suitable to that necessary to obtain optimum casting results. This technique makes it possible to fill the mold cavities of multiple mold cavity molds one at a time, or in a given predicted and calculated sequence, in full accordance with the filling rates calculated using most of the well-known calculation methods. The technique also permits the orderly filling of pattern impressions where multiple inserts are used, and where it is desired to isolate the filling of each impression. In those cases where the riser ingate is positioned above the normal parting line to effect sequential filling, riser efficiency is substantially improved.

INTRODUCTION

Proper, effective and efficient design of gating systems has long been the goal of the foundryman, and a number of systems have been proposed to accomplish this task. These systems may vary significantly, but they are usually based upon sound principles derived from the application of hydraulic flow to molten metals. It has been suggested¹ that the gating system be designed to accomplish the following objectives:

- 1) fill the mold rapidly, without laps or requiring excessively high pouring temperatures;
- 2) reduce or prevent agitation or turbulence and the formation of dross in the mold;
- 3) prevent slag, scum, dross and eroded sand from entering the casting by way of the gating system;
- 4) prevent aspiration of air or mold gases into the metal stream;
- 5) avoid erosion of molds and cores;
- 6) aid in obtaining suitable thermal gradients to attain directional solidification and minimize distortion in the casting;
- 7) obtain a maximum casting yield and minimum grinding costs; and,
- 8) provide for ease of pouring, using available ladle and crane equipment.

Gating systems purporting to meet these objectives may be pressurized or unpressurized, the differences being related primarily to the location of the flow control orifice (the choke) within the system. Choke location at, or near, the base of the sprue renders the gating system unpressurized and full control of the metal flow is achieved when the sprue is filled.

This type of gating system permits a low velocity of metal flow to be realized in the runners and gates, and separation of slag and/or dross in the runners before the metal enters the mold cavity. In this type of system a variety of runner and gate locations have been advocated. With the runner located in the drag and the gates in the cope, the runner is filled prior to the entry of any metal into the mold cavity. Furthermore, this geometry insures that the runner will always remain full regardless of the details of pouring, e. g., interrupted pour, inadequate pouring rate to keep the system full, etc. On the other

hand, pressurized gating systems have the choke at, or near, the casting gate. In these systems full control of metal flow is not achieved until the entire gating system is filled. Entry of metal into the mold cavity is controlled by the respective gates.

THE GATING SYSTEM

The selection of a suitable gating system, whether it be pressurized or unpressurized, is the result of a number of considerations which may be unique to a given casting, e. g. casting geometry, yield requirements, space available in the mold, need for risers, etc. The principles behind both pressurized and unpressurized gating systems consider similar factors in the calculation of component geometry:

- 1) casting weight;
- 2) critical casting section thickness;
- 3) pouring rate;
- 4) metal fluidity;
- 5) type of sprue;
- 6) type of runners;
- 7) type and location of ingates; and,
- 8) sprue height or cope height.

Examples presented for the application of most gating systems first consider the production of a single casting with single or multiple ingates. Where multiple ingates are involved, procedures have been developed so that the flow rate of metal through each ingate is the same. This entails the introduction of procedures such as tapering or stepping down the runner, adjustment of the gate area, etc. When multiple castings are to be produced from a single gate system, the usual procedure is to design the gating system so that the flow rate of metal through each ingate is the same or, in other words, all of the mold cavities fill at the same time and rate. However, calculation of the gating system where multiple mold cavities are involved usually results in mold cavity filling times which are substantially longer than those determined for a single mold cavity. In fact, the foundryman often finds it impossible to deliver metal through the gating system rapidly enough to fill a multiple mold cavity system at a rate adequate to match that based on the mold-filling rate of a single mold cavity. The result often experienced is that the multiple mold cavity arrangement yields defects related to slow pouring rates, e. g. cold shuts, misruns, entrapping slag and dross, pinholes, chilled edges in gray and ductile irons, etc.

To illustrate this, one procedure^{2,3} to calculate a gating system bases the pouring time for gray cast iron on the following formula (the results of which have frequently been presented graphically):

$$t = K(0.95 + T/0.853) \cdot W^{1/2}$$

where

- t = pouring time in sec
- W = casting weight in lb
- K = fluidity factor, and
- T = critical section thickness in inches.

Consider a 7.5 lb gray iron casting having a critical section thickness of 0.5 in. (and a fluidity factor of 1.0). This results in a pouring time of 4.2 sec (1.8 lb per sec). As a first approximation this may be considered the time to fill the mold cavity.

Now, consider the production of four of these same castings from a common gating system (the usual procedure to accomplish this is to multiply the casting weight by four). The resulting pouring time would then be 8.4 sec (3.6 lb per sec), a mold cavity filling time which has doubled. If eight castings were to be made from a common gating system, this procedure results in a pouring time of 11.9 sec (5.0 lb per sec). To maintain the same mold cavity filling time as that of a single mold cavity

Table 1. Calculated Pouring Time for 7.5 lb Gray Iron Casting

No. of Mold Cavities	Pouring Time, Sec	Pouring Rate, lb/sec	Pouring Rate, lb/sec, to Yield Mold Filling Rate Equal to Single Mold Cavity
1	4.2	1.8	1.8
2	5.9	2.5	3.6
4	8.4	3.6	7.1
8	11.9	5.0	14.3
12	14.6	6.2	21.4
16	16.8	7.1	28.6

gating system requires a significant increase in the pouring rate. This is illustrated by the calculated data in Table 1.

ALTERNATIVE GATING DESIGNS

While these numbers relate to a hypothetical casting, the change in pouring rates required to fill the mold cavity at the same rate for each case is dramatic and characteristic of a problem often experienced in the foundry. Since there are practical limitations to manipulating the pouring rate to obtain a constant mold cavity filling rate, the foundryman may have to consider alternative procedures in gating design.

For example, the foundryman often finds that the limiting factor controlling the maximum pouring rate is the rate at which metal can be poured from the pouring ladles used in a particular plant, and the ability to provide a pouring basin capable of receiving metal at that maximum pouring rate. Many foundrymen have found that it is desirable to pour at a rate which is substantially faster than that determined from most calculation methods. In these cases, the pouring rate may be determined by first determining the maximum rate at which metal can be delivered from a given pouring ladle, and then using a value somewhat less than that to insure that the pourer can reproducibly fill mold cavities at that pouring rate. Even these rapid pouring rates may be inadequate to fill the individual mold cavities at a satisfactory rate. It is proposed that this can be accomplished by sequentially filling the mold cavities.

As an example of the change that the sequential filling of mold cavities will make, consider the same 7.5 lb gray iron casting. The gating system design for pouring two castings from a common gating system calls for a pouring time of 5.9 sec (pouring rate of 2.5 lb per sec). If eight cavities were to be poured in a single mold, it would be desirable to fill them in pairs with a filling time of 5.9 sec for each pair of mold cavities. The total filling time would then be 23.6 sec at a pouring rate of 2.5 lb per sec. To accomplish this requires that a runner dam, or weir divert the flow of iron into two mold cavities at a time.

THE SEQUENTIAL FILLING SYSTEM

An example of a system that will result in this sequential filling of mold cavities is presented in Fig. 1. In this case a small (7.5 lb gray iron) shaft is being poured with eight castings on the plate. An unpressurized gating system is shown here using a drag runner and cope gates. Flow control is established by the sprue. It is evident that mold cavities 1 and 2 will fill completely before the level of metal in the runner rises to pass over runner dam A. When the metal runs over runner dam A, mold cavities 3 and 4 will be filled. Similarly, mold cavities 5 and 6 will fill only after the metal runs over runner dam B, and mold cavities 7 and 8 will fill only after the metal runs over runner dam C. By this technique the mold cavities are filled sequentially in pairs so that each set of mold cavities can be filled rapidly enough to minimize the occurrence of slow mold filling problems.

The runner dams must be molded well to prevent the washing

away of molding sand. It is, however, possible that a core could be used to form the runner dam. In that case it may be possible to reduce the spacing between mold cavities considerably.

Note that while the individual mold cavities fill in 5.9 sec (based upon the first approximation calculations shown in the previous table), the total mold filling time for this system is about 23.6 sec. This is considerably longer than the 11.9 sec pouring time calculated on the basis of a pouring time equivalent to that required to fill a single mold cavity. But, the pouring rate is reduced to one-half that by the use of this sequential filling procedure applied to dual mold cavities. While the total pouring time for a given mold may be extended as a result of the sequential filling procedure, a pouring rate which is reproducibly obtainable can be realized, and an individual mold cavity filling rate can be obtained which results in optimum casting results.

It must be stressed at this point that the numbers cited in the aforementioned calculations are only referenced as an example. There is no inference that this calculation method yields optimum casting results, or that it is a preferred and recommended procedure.

Another example of this technique is presented in Fig. 2. In this case a ductile iron cam shaft is produced with a single riser feeding two cam shafts at the stub end. An indefinite number of these pairs of cam shafts are connected by a common gating system. To obtain sequential filling of each riser and pair of cam shafts, the runner (only a segment of the series of castings is shown in Fig. 2) has been moved to a position higher than the top of the cam. To accomplish this means that the parting line must be offset in the region of the runner. This results in the flow of iron across the top portion of each riser, keeping that portion of the riser hot. The last portion of the entire assemblage of cam shafts in the mold to be filled will be the top portion of the risers. This greatly increases the efficiency of the riser, particularly since the differential between the temperature of the iron in the riser and that in the mold cavity will improve feed metal transfer from the riser.

Displacement of the runner and gates to the upper portion of the riser also enables more efficient use of space on the pattern plate, since there is no need to provide for a runner dam, or weir, in the system (it is built into the riser system).

USE OF MULTIPLE INSERT PATTERNS

The procedure of sequential filling of mold cavities is particularly attractive when multiple insert patterns are used in the production of large molds. In this case the technique described can be applied to control the filling of the individual impressions, thus eliminating the problems often experienced with this pattern arrangement where a given impression fills in a manner dependent upon the other insert patterns mounted with it.

For example, consider the hypothetical arrangement presented in Fig. 3. In this case the pattern plate is equipped with

Fig. 1. Example of sequential filling of a small gray cast iron shaft using runner dams (A, B and C) to control the metal flow into pairs of mold cavities (1 and 2, 3 and 4, 5 and 6, 7 and 8).

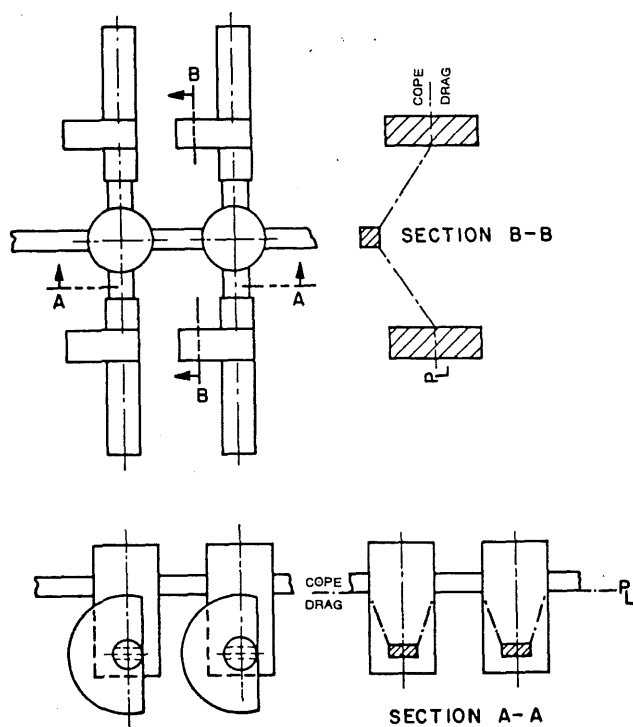
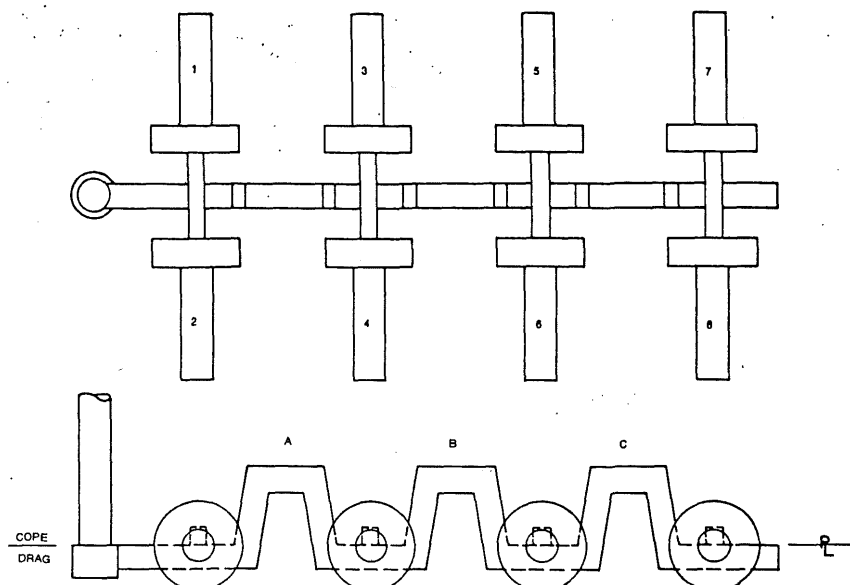


Fig. 2. Sequential filling of mold cavities using an elevated runner to distribute metal into the riser of adjacent pairs of mold cavities.

a fixed sprue location and a fixed "master runner." Various inserts can be mounted in this pattern plate and a variety of runner systems may result depending upon the number of inserts, and the number of individual mold cavities.

Figure 3 presents a simplified combination of insert patterns, one of which is located primarily in the drag and the other split between the cope and the drag. It is preferable to pour these individual castings in a manner such that the filling of one mold does not adversely affect the filling of the other mold cavity. The common master runner involved in this case would normally result in filling the two mold cavities somewhat individually until one mold cavity was filled to the level of the gates. The

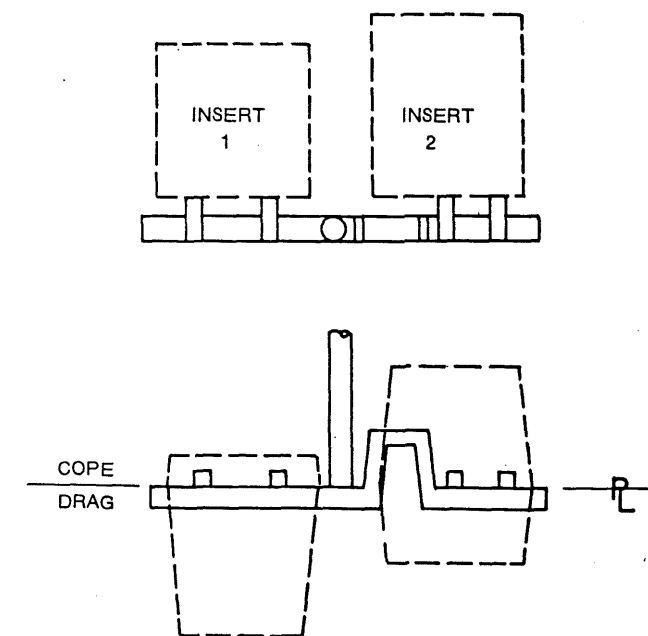


Fig. 3. Example illustrating the application of the sequential filling procedure to a situation where pattern inserts are employed.

filling of that mold cavity would then cease until the other mold cavity reached that level. Further filling of the mold cavities (the cope portion) will occur at a rate dependent upon the mass of those casting sections and the ability of the gating system to provide metal to the castings. However, inclusion of a runner dam, or weir, in the master runner causes one mold cavity to be filled completely before filling of the other mold cavity is initiated.

In a similar manner, a variety of other mold cavity combinations can be filled in an improved manner. Multiple insert mold cavities often involve castings which would best be treated individually, e. g. one casting which should be poured at a rapid

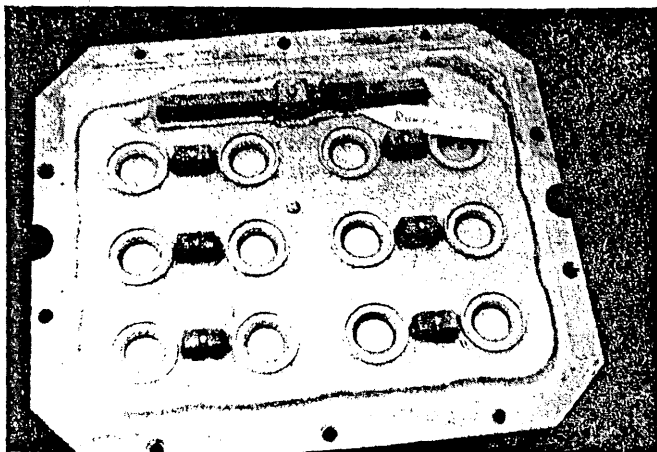


Fig. 4. Cope view of a production pattern showing the print for the core that forms a runner dam to divide the gating system into two sections.

pouring rate and another which should be poured more slowly; or, one casting which is chunky and the other which is heavily cored. Modification of the gating system using the principle of sequential filling of mold cavities enables solution of these problems.

CONCLUSIONS

Application of the sequential filling procedure requires that an appropriate gating system be used in conjunction with this technique. In the examples set forth, the system was unpressurized. A pressurized gating system (where flow control is established at the gates) results in a back-up of metal in the runner, thereby causing metal to flow over the runner dam, or weir, prior to the complete filling of the mold cavity (or cavities) designated.

In summary, a technique has been proposed whereby the sequential filling of mold cavities can be accomplished making use of a runner dam, or weir. Through this procedure, individual mold cavities can be poured at a rate more suitable to that necessary to obtain optimum casting results.

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3. J. M. Svoboda, *Basic Principles of Gating and Riser* (Des Plaines, IL: American Foundrymen's Society, 1973).

APPENDIX

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In the Belleville Foundry of Sperry New Holland, the sequential method of filling mold cavities has been applied with con-

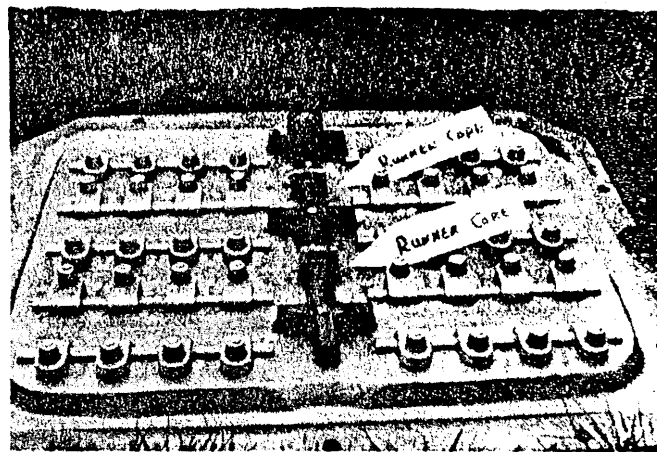


Fig. 5. Drag view of a lift pivot pattern showing the coreprints for two runner dam cores.

siderable success to a number of jobs. An oil sand core has been designed for the application and is now produced in a three impression dump box. The core is two inches in overall height, with one inch in the drag coreprint leaving one inch of the core to serve as a runner dam in the cope.

An example of the application of this technique to a ductile iron casting is shown in Fig. 4. Originally the gating system was similar to that shown but without the runner dam. Severe problems were experienced with cold shuts and misruns to the extent that scrap levels of 41, 55 and 54% were realized. When first experimenting with the system, each section of the mold was not treated as a complete gating system. As a result, the casting ingates were acting as the choke and misruns were still being experienced. Adjustments to the ingate size resulted in successful casting production. Since applying the sequential filling techniques, the scrap rates for misrun and cold shuts are negligible.

A second example is shown in Fig. 5. In this case, this gray iron lift pivot had experienced a substantial amount of cold shuts in the 3/64 inch wall between cored holes. Use of the sequential filling technique resulted in being able to fill these castings in series: first eight castings, then sixteen castings, and then the final sixteen castings. Scrap due to misrun or cold shut has virtually been eliminated.

Presently the sequential filling method of gating is being used on five different jobs, and there is sufficient confidence in its success that a number of new jobs are being rigged using this technique. More impressions can often be placed on a plate since each section can be treated as a separate mold, thus increasing the yield.

The system is not a cure-all, but may be applied to those jobs where problems are being experienced due to the filling process of the mold. However, when applied properly, the results have been encouraging.

Side Pour — A Unique Approach for Producing Thin Wall Premium Quality Castings in Aluminum

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ABSTRACT

Teledyne Cast Products used their unique side pour method to produce ten Air Launched Cruise Missile (ALCM) tank number 4 test castings for Boeing's "Manufacturing Methodology Improvement for Casting Ductility" program. These parts were cast of AA-357-T6 alloy. They exceeded grade B x-ray requirements, showed no porosity or dross in penetrant inspection, and met tensile property requirements of 40 KSI yield, 48 KSI tensile and 5% elongation in critical stress areas and 34 KSI yield, 45 KSI tensile and 4% elongation in noncritical areas. The castings contained sections up to 2 inches thick, large thin wall areas of 0.130 to 0.190 in., and isolated reinforcement ribs.

A casting machine was used to pour sand molds from two sides through two continuous ingate slots. Metal flowed directly into the slots from a trough as the mold was lowered past the trough.

The side pour method provided nonturbulent metal flow at a uniform rate and at a uniform temperature at all levels in the mold. Therefore, solidification could be closely controlled. Side pouring gave a higher degree of quality with reliability, reproducibility and higher mechanical properties than conventional top pouring, which uses down sprues.

INTRODUCTION

A reproducible manufacturing method was needed for making ten simulated Air Launch Cruise Missile (ALCM) tank number 4 test castings. A high degree of reliability was required with respect to metallurgical quality and optimum mechanical properties. These castings were ordered by the Advanced Structural Concepts Group of the Boeing Military Airplane Company to be used for evaluation in their "Manufacturing Methodology Improvement for Casting Ductility" program. Boeing is performing this work under the sponsorship of the Materials Laboratory, Air Force Wright Aeronautical Laboratories.

The ten ALCM number 4 tank test castings, AA-357-T6, were required to meet grade B x-ray standards, 40 KSI yield, 48 KSI tensile and 5% elongation in critical stress areas, and 34 KSI yield, 45 KSI tensile and 4% elongation in noncritical areas. Castings contained heavy critical sections up to 2 inches thick, large thin wall sections of 0.160 in. $\pm$ 30 in. and internal reinforcement ribs (see Fig. 1).

The castings were made to confirm previously completed manufacturing control parameters and nondestructive testing phases^{1,2} of the Boeing ductility program. Therefore, foundry variables had to be closely controlled to produce castings with a high degree of metallurgical reliability and reproducibility. On the basis of past production experience, Teledyne Cast Products selected their side pour method³ as the foundry practice that would be most likely to satisfy these requirements.

Results from 3 years of production experience demonstrated that side pouring offered distinct advantages over the conventional method of top pouring down a sprue. Among the advantages are the ability to pour slowly with nonturbulent metal flow, precise mechanically controlled pouring rate and uniform flow rate and metal temperature at all elevations in the gating systems. Equipment and practices used to side pour conventional dry sand molds with two-up type gating is described.



PROCEDURE

Equipment

Molding and metal pouring equipment was designed and constructed to derive maximum benefit from the known advantages of side pouring. Pattern equipment was designed so that all runners, gates and risers could be insulated. Multiple tiers of runners were provided with closely spaced gates so that metal did not flow far after entering the mold. Feeding distances were minimized to obtain rapid directional solidification.

Since the castings were for testing and not intended to be operational, pattern equipment was designed with several minor modifications to simplify tooling construction and thus reduce cost. The pattern equipment consisted of five core boxes for making a drag, a cope, three cheeks and two internal cores. By using loose pieces to vary the configuration, three cheek mold sections could be made in the same core box. The drag and each of the three cheeks had conventional runners and slot type gates that also served as side risers. For side pouring, the runners were connected into two ingate slots that opened to the outside of the mold. These slot ingates, which extended from the drag through the cope, were used instead of conventional down sprues.

A casting machine was used for filling the mold from two sides. The machine consisted of an electrical motor driven elevator platform and a molten metal distribution system. The elevator platform was encompassed on three sides by a stationary trough that had one centrally located pouring basin. At the two locations where metal entered the mold, the trough

Fig. 1. ALCM tank number 4 casting. Note the isolated reinforcement rib and other internal details. The casting is approximately 23 x 12 x 14 in. and weighs 49 pounds.

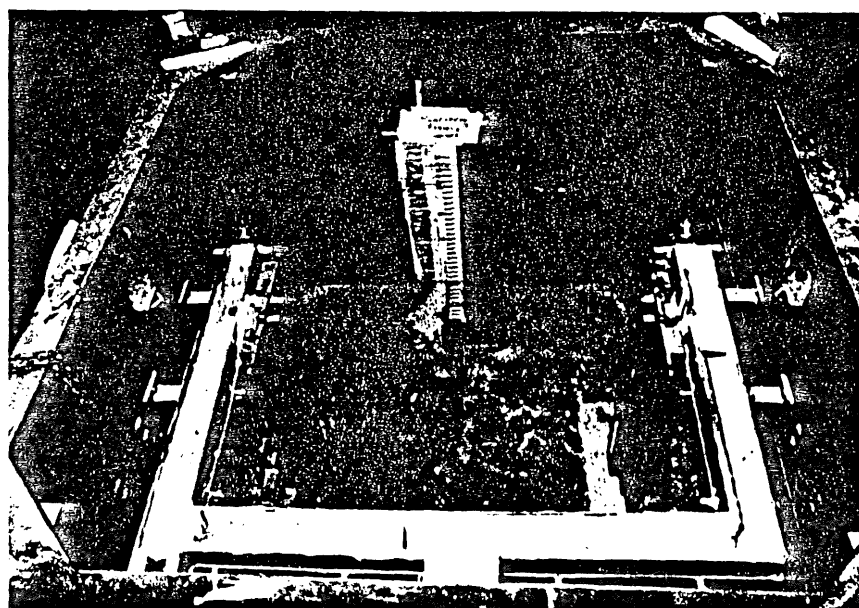


Fig. 2. The casting machine used for side pouring ALCM tank number 4 castings. The elevator platform appears at the center and is encompassed on three sides by the pouring trough. Slotted guide plate assemblies appear on the right and left sides of the platform, extending up past the trough.

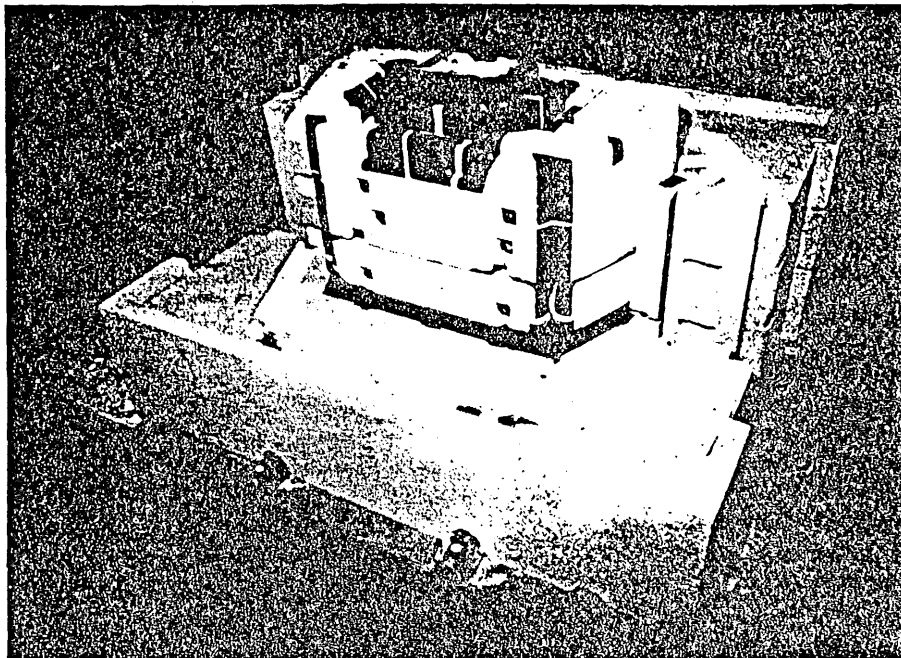


Fig. 3a. Partial assembly of mold, showing the drag, two internal cores, and two half cheek sections.

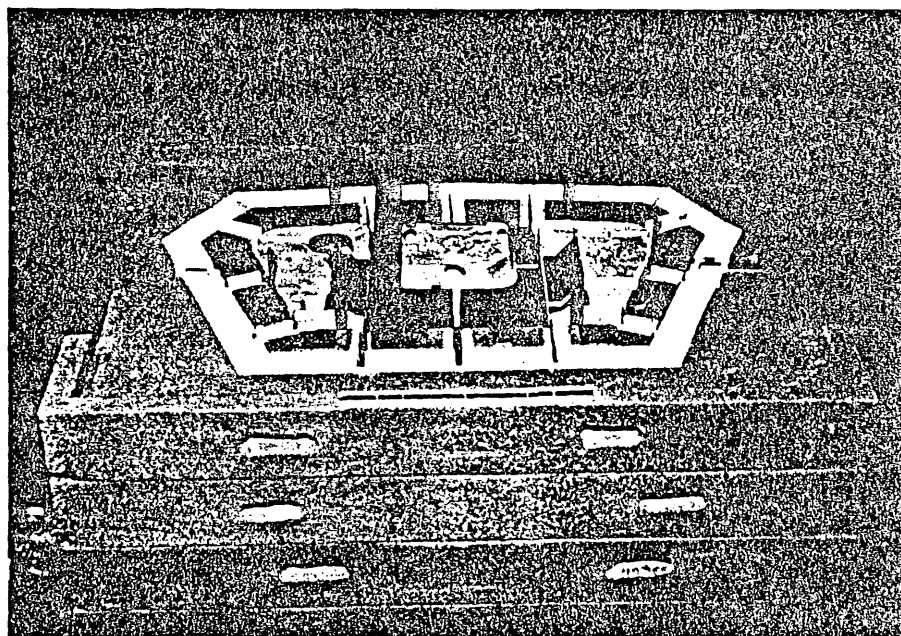


Fig. 3b. Mold assembly prior to setting the cope. The assembly as shown consisted of a drag, three cheeks and two internal cores. Notice the completely insulated gate and runner system that is typical of the gate and runner systems located in each of the three cheeks and the drag.

was attached to two slotted guide plate assemblies. These guides positioned the mold, retained the ingate slot insulators and sealed the side of the mold to prevent run-outs. Figure 2 shows a bird's-eye view of the casting machine.

During the side pouring operation, composite molds were lowered past the stationary trough. Metal flowed from the insulated trough through the slotted guide plate assembly and then was distributed into two sides of the mold through

insulated ingate slot recesses. Metal entered the pouring basin with the mold all the way up so that the drag was positioned opposite the pouring trough and bottom of the slotted ingate. Metal entry started at the drag level and progressed up the side of the mold to the cope.

Core Making and Mold Assembly

Internal cores were made with self-set sodium silicate bonded

round grain sand. Chemically bonded sand was used to make the drag, three cheeks and the cope. Gates and runners were insulated with ceramic fiber board. Aluminum and iron form chills were used throughout the mold. Molds were assembled on a specially designed bottom platen that also served as an integral part of a clamping and lifting fixture. The mold assembling operation consisted of setting components in the following sequence: the drag, the first internal core, the first cheek, the second and third cheeks, the second internal core and finally the cope.

Figure 3a shows the drag, two internal cores, and two half cheek sections. Note the insulated runner system and slot gates. Figure 3b shows the mold assembly just prior to setting the cope. After the cope was set, mold assemblies were clamped and then hoisted onto the casting machine platform. Ingate slot insulators were then inserted into the sides of the mold, bridging from the trough to the mold. Figure 4 shows a completed and poured mold assembly.

Melting and Pouring

Castings were poured of AA-357 sodium modified ingot. Metal chemistry was selected by Boeing on the basis of the results of preceding work.^{1,2,4,5} Melts were scheduled to be of the following chemistry range: 6.5% to 7.0% silicon, 0.58% to 0.62% magnesium, 0.15% to 0.20% titanium, 0.10% maximum iron and 0.06% to 0.07% beryllium.

Metal was melted and the chemistry was adjusted while in a gas-fired silicon carbide crucible furnace. Then the metal was transferred to a coated steel crucible heated in a gas-fired furnace setting. After transferring, temperature was adjusted and the metal was degassed with a nitrogen -15% chlorine gas

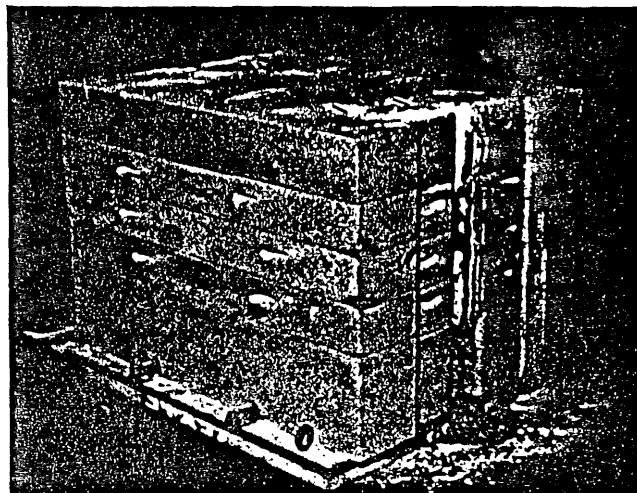


Fig. 4. A mold shown after side pouring. Notice the insulator that protrudes from the side of the mold. Before pouring, insulators were inserted into the two ingate slots recessed in the side of the mold. The slot ingates distribute metal from the pouring trough to the gate and runner system within the mold.

mixture. During the last few minutes of degassing, Ti-B grain refiner was added. Metal was heated rapidly to pouring temperature and then poured directly from the steel crucible into the trough system of the casting machine.

Metal entered into the side of the mold at about 1375F. The mold was lowered to give a metal pouring rate of about 1½ pounds per second. Figures 5a and 5b show a mold being poured in the casting machine.

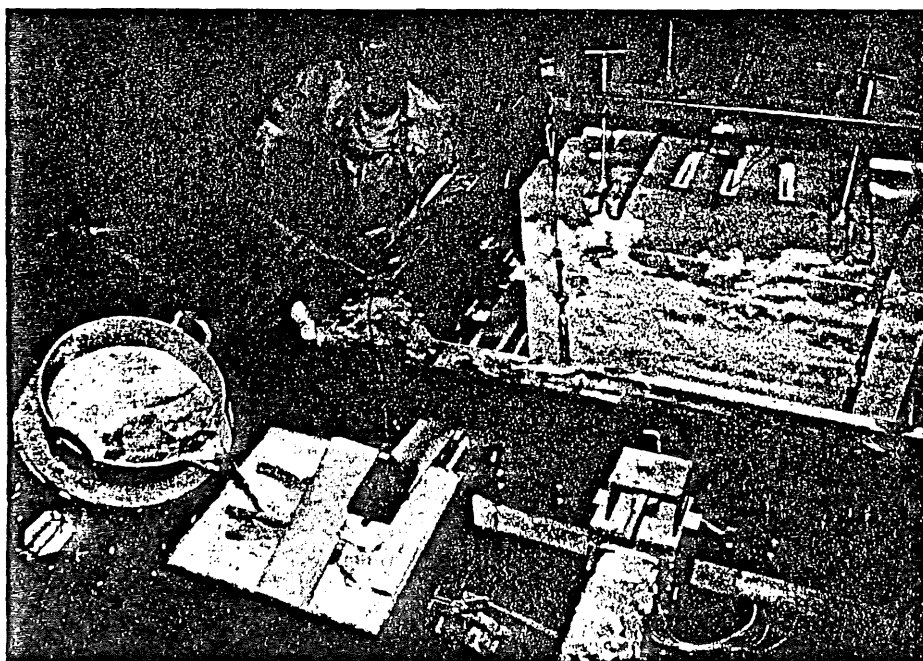


Fig. 5a. The start of a mold being poured in the casting machine. Metal was poured directly from a crucible. The outside of the crucible was insulated to minimize heat loss. Metal flowed from the pouring basin into an insulated u-shaped trough. While keeping the trough full, the mold was lowered so that metal flowed from the trough into two insulated slotted ingates. Metal was distributed from the slot ingates into four tiers of runners in the sand mold, then through slot gates attached to the casting cavity.

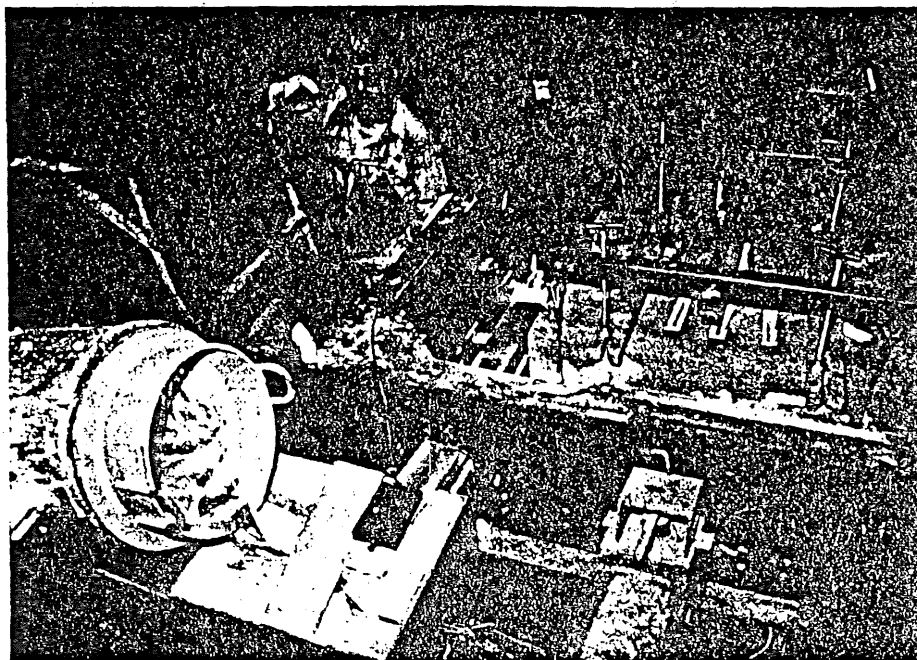


Fig. 5b. The end of a pour in the casting machine. Note that the top of the mold is level with the metal trough.

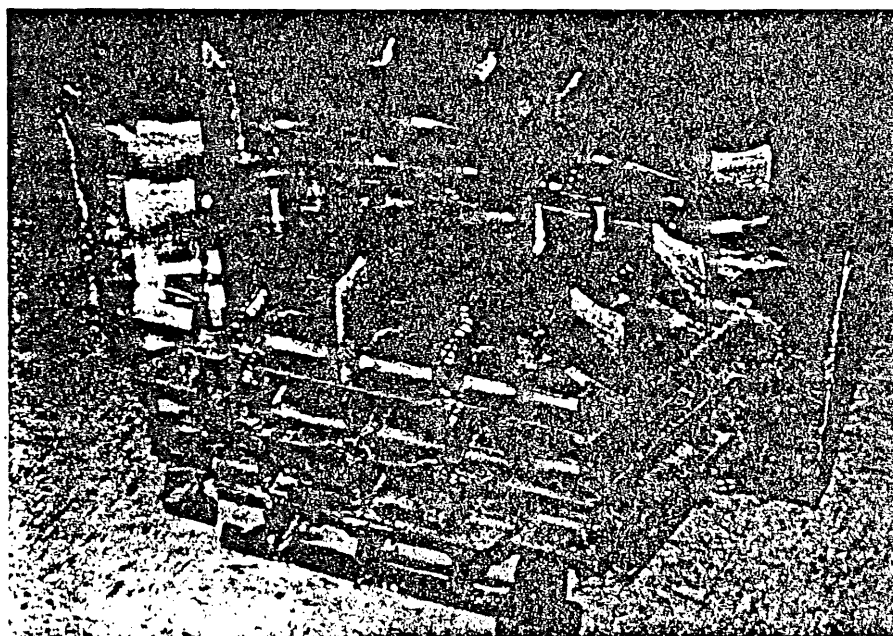


Fig. 6. ALCM tank number 4 test casting with gating system. Notice four tiers of runners that distributed metal from the two external slotted ingates to the continuous internal slot gates that extend the full height of the casting.

RESULTS AND DISCUSSION

The side pour method was used to cast eleven ALCM number 4 tanks (Fig. 6). Of these, one was destructively tested for foundry control to qualify the casting. Mechanical properties were determined on tensile specimens excised from 36 locations selected by Boeing. Requirements for mechanical properties were met, and in most areas, exceeded by a considerable amount. Table 1 shows results of mechanical property tests.

Mechanical properties tests performed by Boeing on the other ten castings are reported in the subsequent Quarterly Report of references.^{1,2,4,5}

Penetrant evaluations indicated that the eleven castings were free of dross and porosity. Meeting grade B radiographic requirements all over the casting consistently was not a problem.

Side pouring has many advantages over the conventional practice of pouring down a sprue. Pouring rate can be precisely

Table 1. Results of Tensile Tests for Foundry Control

High Stress Areas 40-48-5%								Low Stress Areas 34-45-4%							
SPEC NO.	Fty KSI	Ftu KSI	%e	SPEC NO.	Fty KSI	Ftu KSI	%e	SPEC NO.	Fty KSI	Ftu KSI	%e	SPEC NO.	Fty KSI	Ftu KSI	%e
cc-1	46.3	53.3	11.0	24	43.0	51.7	5.0	6	45.2	53.6	8.0	17	48.2	53.6	6.0
cc-2	44.9	53.0	10.0	25	45.3	53.2	5.0	7	43.0	51.7	5.0	18	42.6	51.3	6.0
cc-3	43.6	53.3	9.0	26	46.2	50.9	5.0	8	41.7	52.7	8.0	19	43.3	53.3	9.0 _R
cc-4	46.2	52.9	9.0	28	42.5	53.3	8.0	9	43.8	53.6	7.0	20	44.0	53.2	9.0 _{EE}
1	46.8	52.4	5.0	29	44.2	52.2	6.0	10	43.2	53.2	7.0	21	42.9	52.0	8.0 _R
2	45.8	52.7	6.0					11	42.6	53.6	10.0	27	41.5	52.0	8.0
3	45.7	52.7	5.0					12	43.5	53.6	8.0	30	45.8	51.8	5.0
4	45.8	53.1	8.0					13	42.4	51.1	6.0	32	43.9	51.2	5.0
5	45.9	53.6	7.0					14	42.7	53.4	12.0	33	43.0	50.9	4.0
22	46.4	53.3	5.0 _{EGg}					15	43.2	51.9	6.0	34	43.0	52.0	6.0
23	45.0	53.3	7.0					16	47.6	53.8	7.0	36	43.0	53.8	11.0

controlled mechanically. Metal enters the mold under negligible hydrostatic head and very low velocity, resulting in nonturbulent flow. Since inertia effects are eliminated, metal does not establish preferential flow patterns through the casting. The entering metal remains at a constant temperature at all elevations in the mold.

As a result of the low velocity and nonturbulent metal flow characteristics, generation of dross and porosity is prevented by side pouring. Because of the more favorable thermal conditions, thermal gradients and directional solidification can be closely controlled to produce high strength castings. Side pouring produces castings with a higher degree of metallurgical reliability and reproducibility than does conventional top pouring down sprues.

CONCLUSIONS

1. A unique side pour method that is an established foundry production practice at Teledyne Cast Products was used to produce ten Air Launched Cruise Missile (ALCM) tank number 4 test castings for Boeing's "Manufacturing Methodology Improvement For Casting Ductility" program.
2. A casting machine was used for introducing metal from the sides into sand molds through insulated slot ingates instead of the conventional practice of top pouring down sprues.
3. Castings in AA-357-T6 alloys met grade B radiographic requirements, showed no porosity or dross in penetrant inspection, and met tensile property requirements of 40 KSI yield, 48 KSI tensile and 5% elongation in critical stress areas, and 34 KSI yield, 45 KSI tensile and 4% elongation in noncritical areas. The castings had sections up to 2 inches thick together with large thin wall areas of 0.130 to 0.190 in. thick and also has isolated internal reinforcement ribs.

4. Side pouring provided nonturbulent metal flow at a uniform rate, with negligible hydrostatic head and a constant temperature at all levels in the mold, thereby minimizing generations of dross and porosity. Preferential metal flow patterns in the casting are prevented and solidification is closely controlled.

5. Side pour gave a higher degree of metallurgical reliability and reproducibility, and consistently higher mechanical properties than the conventional practice of top pouring down sprues.

ACKNOWLEDGMENT

The author gratefully acknowledges the guidance and contributions of Erik Laid who developed the side pour method and the casting machine used by Teledyne Cast Products for production castings.

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Solidification Simulation Based on the Edge Function Approach

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ABSTRACT

Cost effective evaluation and optimization of sand casting system designs often requires numerical simulation on digital computers. Current simulation techniques that compute the temperature field within the mold and casting enclosure must operate on large amounts of memory cells that are available only in large-scale computers. This in turn hampers a widespread application of available simulation schemes in the foundry industry.

A new approach, the q-method, has evolved from recent studies of the heat transfer within sand mold enclosures. It eliminates the need to compute the temperature field within the sand mold, where the majority of the discretized volume elements are located, and thus virtually extends the simulation capacity of the more cost-justifiable mini- or macro-computers. The advantage of the q-method over conventional schemes is demonstrated through a simple example.

INTRODUCTION

Cost effective evaluation and optimization of casting system designs often requires numerical simulation on digital computers. Recent advances in this field have been reviewed by Erickson.¹ In order to simulate numerically the solidification of sand casting systems, one is required to discretize the space involved into small volume elements that can be associated with finite difference or finite element routines. The fact that sand molds are much larger in size than castings often results in 70 to 80% of the volume elements being located on the mold side of the interface.

Since the temperature history of the mold is, in practice, of little interest to the foundryman, a computationally efficient simulation scheme can be achieved if the heat dissipation into the mold can be represented by a convective heat transfer coefficient, instead of a continuity in temperature, acting at the mold-metal interface, and thus only the volume elements in the casting need to be manipulated.

This concept has been proposed independently by Niyama² and Wei, Hansen and Berry.³ The term q-method was used by the latter group to describe the approach. Niyama² applied a uniform, but time-dependent, heat flux function, obtainable from Schwarz's⁴ closed form solution of one-dimensional mold and casting systems, at the surface of two- and three-dimensional castings, in the form of a convective heat transfer coefficient, and recorded a significant saving in computation time as compared with a conventional scheme that takes into account the mold elements.

The error introduced by neglecting the corner effect on the distribution of heat flux over the mold-metal interface was also noted. Wei, Hansen and Berry³ proposed a corner-adjusted heat flux function at two-dimensional, rectangular corners and reported a good agreement between results of a conventional scheme and the q-method.

Recent studies^{5,6} of the heat transfer within sand mold enclosures have revealed an important similarity characteristic in Jaeger's⁷ classic solution for the transient temperature field in a wedge enclosure subjected to a constant surface temperature. Namely, the conduction heat flux at the surface of such a wedge enclosure can be decomposed into the product of the corner-free, one-dimensional surface heat flux and an edge function (E-function) that is dependent only upon a Fourier number. This similarity characteristic allows one to construct a convenient form for the surface heat flux function in two-dimensional, non-rectangular corners.

This paper proposes a general form for the effective heat transfer coefficient at mold-casting interfaces which, in effect, eliminates the need to compute the temperature field within sand molds. Conversion of the chilling effect of sand molds into effective heat transfer coefficients is illustrated through a simple example. The advantage of the q-method over conventional schemes is viewed via comparisons of numerical results.

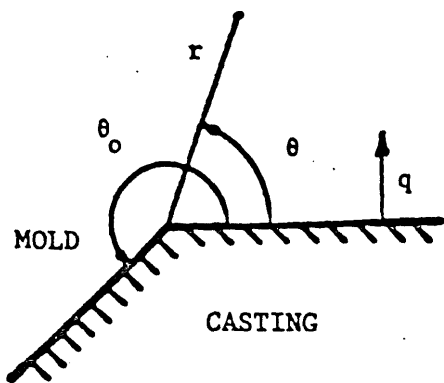


Fig. 1. An idealized 2D sand casting system.

HEAT FLUXES OVER A WEDGE AND A COMPOSITE OF WEDGES

In the sand casting of short freezing range alloys, the release of latent heat from the molten metal results in an approximately uniform temperature at the mold-casting interface near the solidus. A practical idealization for analysis of the heat transfer within the sand mold is to treat the mold as an isolated enclosure subjected, at the cavity wall, to an isothermal heat source. In addition, the outer edge of the mold enclosure can be regarded as semi-infinite since, during the solidification period, the temperature variation there is usually insignificant.⁸

Figure 1 shows an idealized two-dimensional sand casting system. With the mathematically convenient assumptions that the mold enclosure is initially at zero temperature and the isothermal heat source at the cavity wall (mold-casting interface) is at a unit temperature, it has been shown⁶ that the

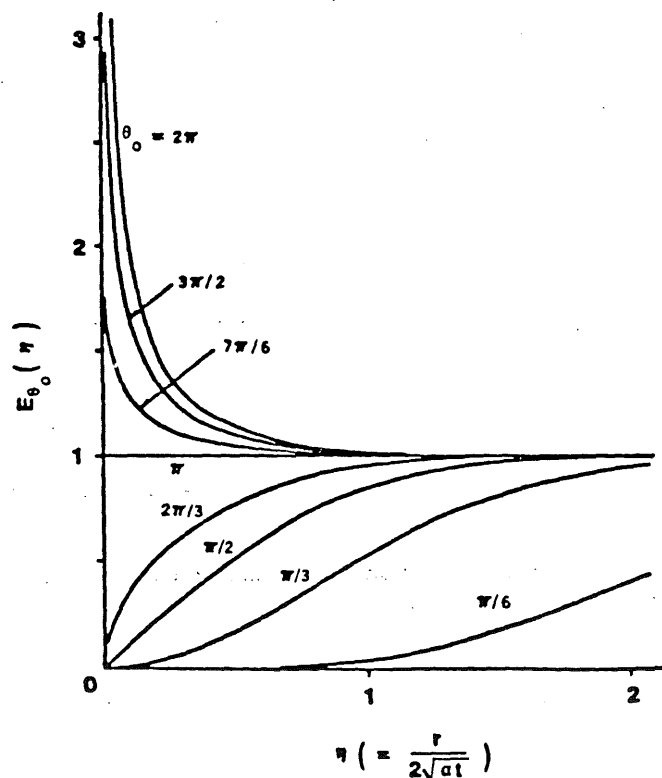


Fig. 2. Graphical representation of the E-function vs. $r/2\sqrt{\alpha t}$.

heat flux, per unit area and time, at the surface of the mold enclosure, $q(r,t)$, is the product of the one-dimensional, corner-free surface heat flux and an edge function (E-function) that is dependent only upon a Fourier number. Symbolically,

$$q(r,t) = q_{1d}(t) \cdot E_{\theta_0}(r_0) \quad (1)$$

$$\text{where } q_{1d}(t) = \frac{K}{\sqrt{\pi \alpha t}}$$

$$\text{and } E_{\theta_0}(r_0) = \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(2n+1)\pi}{\theta_0^2} \int_0^{\infty} u^2 r_0 \frac{J_{\nu}(u) du}{u}$$

In the above equations, Fo is the Fourier number $\alpha t/r^2$, J_{ν} is the Bessel function of the first kind of order ν , and $\nu = (2n+1)\pi/\theta_0$ for $n=0,1,2,\dots$. A graphical representation of the E-function plotted against $r/2\sqrt{\alpha t}$, with θ_0 as a parameter, has also been reported in reference 6, and is reproduced here as Fig. 2.

The similarity exhibited by the heat flux over a wedge enclosure under conduction is especially useful in the design of sand casting systems. Investigating the solidification process of a two-dimensional cross-section of a practical shaped sand casting system is very often an effective approach to optimizing the rigging designs.⁹⁻¹⁴ In sand casting systems, the release of latent heat into the mold through the solidified layer helps maintain the mold-metal interface at a stable temperature.⁸ An infinite, ideal system comprised of a wedge shaped molten metal and a geometrically complementary wedge shaped, dried sand mold would thus promote, in the mold enclosure, a thermal field in close proximity to that in the same enclosure subjected to a constant temperature boundary condition. Therefore, the similarity characteristic introduced earlier can serve as a convenient tool for a compact description of the corner induced divergence or convergence of heat flow across the mold-metal interface.

A laboratory-scale sand casting system was designed, in a separate project,¹⁵ to reveal the divergence of heat flow due to two-dimensional corners ($\theta_0 = 3\pi/2$). The experiment has been successful in providing physical evidence for the aforementioned similarity. The above arguments regarding

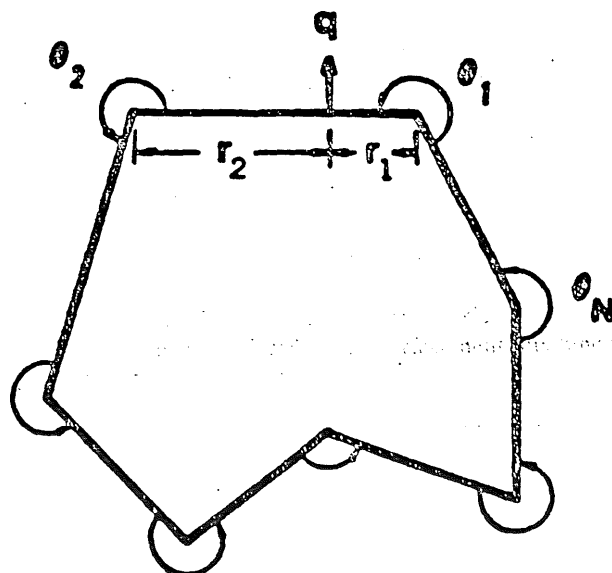


Fig. 3. Cross-section of an infinite solid having a cavity of the shape of a polygonal cylinder.

the infinite, ideal sand casting system can be generalized for practical systems with finite geometries.

An infinite solid having a cavity of the shape of a polygonal cylinder can be viewed as a composite of wedges. Figure 3 shows a perpendicular cross-section of such a solid. A closed form analysis of the two-dimensional heat conduction within the enclosure bounded internally by the cavity wall and subjected to an isothermal heat source imposed at the cavity wall is far beyond current reach. A quantitative understanding of such heat transfer systems has to rely on discretized numerical schemes.

However, the existence of the similarity characteristic of the heat flux over a single wedge prompts one to formulate an approximate form for the heat flux over the cavity that is valid for moderately small time intervals. The term moderately small time intervals is essential here, since the sole purpose of the approximate formulation is to describe the heat flow into a shaped mold while the casting solidifies.

Two basic approximate forms for the heat flux over the cavity wall can be deduced from the similarity characteristic. The approximations can be regarded, according to the mode of handling the interaction of neighboring corners, as additive and multiplicative. The first proposed form for the heat flow rate q , as illustrated in Fig. 3, is

$$q_{add} = q_{1d} \left[\varepsilon_{\theta_1} \left(\frac{x_1}{2\sqrt{\alpha t}} \right) + \varepsilon_{\theta_2} \left(\frac{x_2}{2\sqrt{\alpha t}} \right) - 1 \right] \quad (2)$$

for which the resultant corner effect on the total heat flow rate entering the enclosure is the algebraic sum of the individual corner effects. Expression (2) implies the assumption that neighboring corners do not interact with one another. The second approximate form presumes a certain corner interaction that

$$q_{mul} = q_{1d} \varepsilon_{\theta_1} \left(\frac{x_1}{2\sqrt{\alpha t}} \right) \varepsilon_{\theta_2} \left(\frac{x_2}{2\sqrt{\alpha t}} \right) \quad (3)$$

The practicality of these proposed forms can be examined through numerical simulation of the heat transport in such enclosures, or through comparisons with existing closed form solutions. The localized nature of the corner influence, as pointed out in reference 6, warrants numerical proximity between expressions (2) and (3) in moderately small time

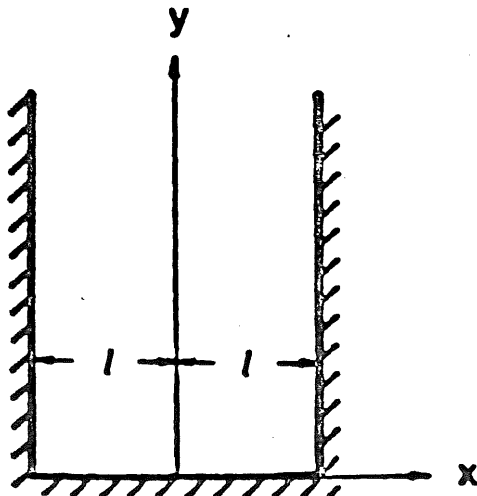


Fig. 4. Semi-infinite rectangular enclosure.

intervals. The following compares the proposed q -functions (2) and (3) with the exact solution for semi-infinite slabs.

Very little information is available on the heat conduction within a composite wedge. Carslaw and Jaeger¹⁶ obtained, via a product approach, several theoretical solutions for externally bounded enclosures, e.g., infinite slabs, semi-infinite slabs and finite rectangles. Although in foundry practice such heat convergent mold enclosures would correspond to cores or mold inserts, for which the q -technique is not applicable due to thermal saturation and strong corner interaction in the enclosure, a comparison with the proposed general forms may help clarify the superimposing nature of corner interaction. The analytical solution for the temperature field in the semi-infinite rectangular enclosure, as shown in Fig. 4, is subjected to zero initial temperature and unit surface temperature is¹⁶

$$\begin{aligned} T(x, y, t) &= 1 - \operatorname{erf} \left(\frac{y}{2\sqrt{\alpha t}} \right) \left\{ 1 - \sum_{n=0}^{\infty} (-1)^n \left[\operatorname{erfc} \frac{(2n+1)l-x}{2\sqrt{\alpha t}} \right. \right. \\ &\quad \left. \left. + \operatorname{erfc} \frac{(2n+1)l+x}{2\sqrt{\alpha t}} \right] \right\} \\ q &= -K \frac{\partial T}{\partial y} \Big|_{y=0} \end{aligned} \quad (4)$$

The heat flow rate per unit area at $y = 0$ is thus

$$\begin{aligned} &= \frac{K}{\sqrt{\pi \alpha t}} \left\{ 1 - \sum_{n=0}^{\infty} (-1)^n \left[\operatorname{erfc} \frac{(2n+1)l-x/l}{2\sqrt{\alpha t}/l^2} \right. \right. \\ &\quad \left. \left. + \operatorname{erfc} \frac{(2n+1)l+x/l}{2\sqrt{\alpha t}/l^2} \right] \right\} \end{aligned} \quad (5)$$

The dimensionless parameter $\alpha t/l^2$ will be used to indicate the accuracy of the proposed formulations, as follows, in describing the heat flux over $y = 0$.

$$q_{add} = \frac{K}{\sqrt{\pi \alpha t}} \left[\operatorname{erf} \frac{1-x/l}{2\sqrt{\alpha t}/l^2} + \operatorname{erf} \frac{1+x/l}{2\sqrt{\alpha t}/l^2} - 1 \right] \quad (6)$$

and

$$q_{mul} = \frac{K}{\sqrt{\pi \alpha t}} \left[\operatorname{erf} \frac{1-x/l}{2\sqrt{\alpha t}/l^2} \operatorname{erf} \frac{1+x/l}{2\sqrt{\alpha t}/l^2} \right] \quad (7)$$

The accuracy of the proposed q -functions is presented in the form of ratio of the proposed formulations to the exact solution. The accuracy is found to deteriorate, gradually, with increased $\alpha t/l^2$. Figure 5 shows the accuracy at $\alpha t/l^2 = 0.18$, plotted against dimensionless x/l . Consider a sand mold ($\alpha = 0.45 \text{ mm}^2/\text{sec}$) having the geometry shown in Fig. 4 and that $l = 300 \text{ mm}$, Fig. 5 would represent the accuracy at $t = 10 \text{ hr}$, a condition which obviously refers to an exceedingly heavy section casting. The existence of interaction between the two corners at $t = 10 \text{ hr}$ is certain since in that time interval the corner influence has diffused about 460 mm (18 in.) away from the edge. [The limit of the region under edge influence is defined as the point where the edge function (Fig. 2) has a value of $0.99(\theta_0 < \pi)$ or $1.01(\theta_0 > \pi)$.] The results prompt the statement that, in relatively small time intervals, the resultant corner effect on the heat flux over a composite wedge is, to a good approximation, the algebraic sum of the respective corner effects.

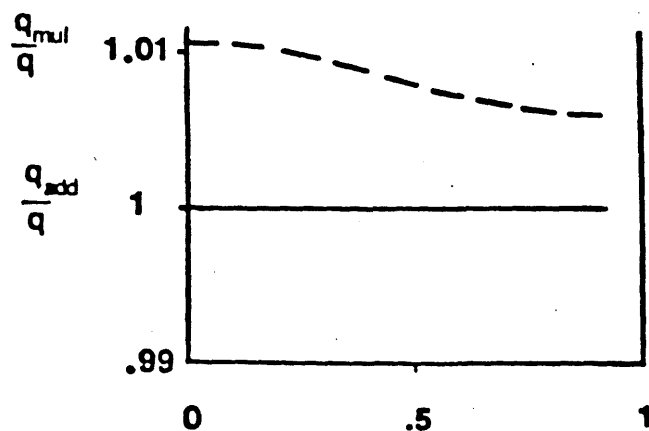


Fig. 5. Comparison of approximate q -functions, (6) and (7), with the analytical solution, (5); $\alpha/l^2 = 0.18$.

THE q -METHOD

One of the principal problems in numerical simulation of physical systems, such as the cooling and solidification of sand castings, is the great number of volume elements needed to ensure accuracy in the result of the computations. Since, typically, the majority of these elements will be in the mold, it is obvious that great savings would be obtained if the cooling effect of the mold is described in a compact analytical fashion. In this study, only silica sand molds are examined where it is reasonable to assume that, during solidification, the temperature in both the sand and the metal at the interface is identical.

The q -method is concerned only with the metal part of a sand casting system and replaces the discretized volume elements in the sand mold with hypothetical heat transfer coefficients. The analytic-numerical technique would eliminate a major portion of computing costs and, more significantly, would extend the solution capacity of small-scale computing machines that have rather limited memory space but are more economically adoptable than large-scale computers, to independent foundries.

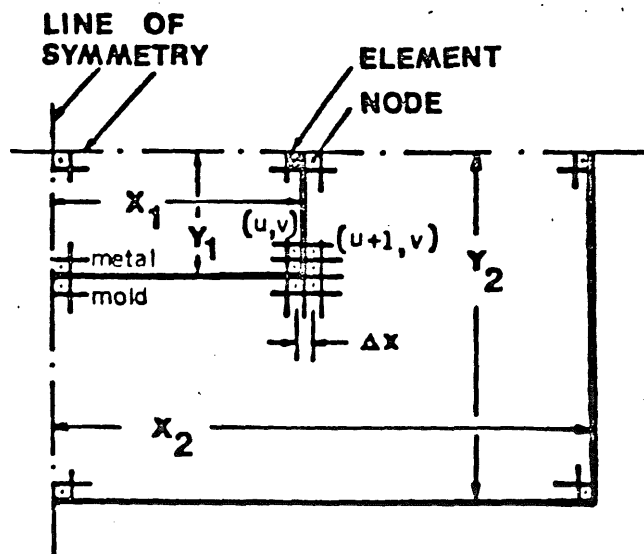


Fig. 6. Quartered cross-section of a two-dimensional sand casting system.

The fact that dissipation of the latent heat of a pure molten metal or eutectic tends to maintain the mold-metal interface temperature near the solidification temperature while the casting solidifies,⁸ along with the above proposed heat flux function over a composite of wedges, enables one to conceive of a hypothetical heat transfer coefficient acting between the casting surface and an isothermal heat sink at the initial mold temperature. The q -method differs from applying a constant temperature boundary condition to the casting surface in that the boundary temperature in the discretized space occupied by the casting is allowed to vary with time and position. Such relaxation is necessary for regions near corner edges, particularly so for poorly heat-conductive cast metals.

It is convenient to consider an infinite compacted sand mold having a cavity as shown in Fig. 3. In practice, a finite-size mold can be considered as infinite as long as the temperature at the outer edge of the mold does not show significant variations. The complication of turbulent flows upon pouring the melt into the mold is usually simplified by assuming instantaneous filling of a quiescent liquid metal.⁹⁻¹¹

It is also known that superheated liquid metals, after being poured into cavities, dissipate their superheat by convection in a short time before a significant amount of the frozen skin has formed at the cold boundary.¹⁷ The understanding and treatment of convection flows, induced by superheat, in solidification processes has formed a separate subject which has just begun to receive specific attentions in the literature.¹⁷⁻²⁴ A full description of convection flows due to superheat is considered beyond the scope of the paper. The following discussions, therefore, assume a zero superheat.

The q -method treats the sand casting system as a spatially constrained molten metal subjected to a flux boundary condition

$$-K_c \frac{\partial T}{\partial n} = h(T_i - T_\infty) \quad (8)$$

where K_c is the thermal conductivity of the cast metal, $\partial T/\partial n$ represents the outward temperature gradient at the boundary, h is the hypothetical heat transfer coefficient used to replace the chilling effect of the mold, T_i is temperature at the casting boundary and T_∞ is the initial mold temperature. It follows, from (2), that h has the approximate form

$$h = \frac{K_m}{\sqrt{\pi \alpha_m t}} \left[E_{\theta_1} \left(\frac{r_1}{2\sqrt{\alpha_m t}} \right) + E_{\theta_2} \left(\frac{r_2}{2\sqrt{\alpha_m t}} \right) - 1 \right] \quad (9)$$

with subscripts 1 and 2 representing the two neighboring corners, as shown in Fig. 3.

The utility of such hypothetical heat transfer coefficients in simulation of the solidification process of shaped sand castings can be demonstrated by comparing computational results obtained with and without the hypothetical heat transfer coefficient. A comparison will be performed on a two-dimensional, high purity aluminum/silica sand system. The properties of high purity aluminum and compacted, dried, clay bonded silica sand as functions of temperature are available in the literature.²⁵

Figure 6 shows the quartered cross-section of the system. The space is discretized into square volume elements. The cross-sectional dimensions of the system are: $x_1 = 30$ mm (1.2 in.), $y_1 = 20$ mm (0.8 in.), $x_2 = 90$ mm (3.5 in.) and $y_2 = 60$

mm (2.4 in.). Dimension of all volume elements is 2 x 2 mm (0.8 in.). A reliable finite difference enthalpy scheme^{9,11} is adopted in solving the heat conduction within the mold and casting.

The advantage of the enthalpy model in handling solidification heat transfer problems is well known.^{9,26} The heat conduction equation, expressed in terms of an enthalpy function and a modified temperature function,^{9,11,27} can be transformed into

$$\frac{\partial H}{\partial t} = v^2 \theta \quad (10)$$

where

$$H(T) = \int_{T_0}^T \left(\frac{\rho C(T)}{K_0} + \frac{\rho}{K_0} \frac{\partial L}{\partial T} \right) dT \quad (11)$$

and

$$\theta(T) = \frac{1}{K_0} \int_{T_0}^T K(T) dT \quad (12)$$

with K_0 representing the thermal conductivity at a reference temperature T_0 . The above formulation incorporates the temperature dependence of specific heat and thermal conductivity, but assumes a constant density medium.

Introducing the enthalpy function H eliminates the difficulty of handling the release of latent heat at the moving phase change boundary in terms of a temperature variable, while the modified temperature function θ facilitates consideration of the temperature dependence of thermal conductivity. To enable a one-to-one functional relationship between H and T , the release of latent heat (L) of pure aluminum is assumed to take place in a temperature range of 1°C (33.8°F).

A conventional simulation scheme involves discretization of the space occupied by the casting and mold, tabulation of H and θ from given property data, transformation of equation (10) into an explicit finite difference form, and computation of values of H and T at each element node at increasing time steps.¹¹ At the mold-metal interface, continuity of temperature and heat flow has to be satisfied at every time step.

The q -method, by replacing the chilling effect of the mold with a hypothetical heat transfer coefficient, eliminates the laborious computation for the temperature field in the mold.

However, in order to apply the previously derived hypothetical heat transfer coefficient, (9), to the casting boundary, one is required to select a set of representative properties for the mold. In the present example, the following properties are used to describe the heat transport in the mold.

$$K_m = 0.06 \text{ W/cm}\cdot\text{C}; \rho_m = 1.52 \text{ g/cm}^3, \text{ and}$$

$$C_m = 0.24 \text{ cal/g}\cdot\text{C} (= 1.005 \text{ J/g}\cdot\text{C}).$$

To demonstrate the practicality of the q -method, the solidification process of the two-dimensional aluminum casting is simulated via three different schemes: a) the conventional approach described above, that incorporates the temperature dependence of mold properties; b) the conventional approach using constant mold properties; c) the q -method.

An instantaneous filling of quiescent molten aluminum in the mold initially at room temperature is assumed in the computations. The pouring temperature is taken as the melting point to avoid complications due to convection flows.¹⁷⁻²⁴ The time step Δt is chosen such that the stability criterion of explicit finite difference schemes^{11,28} is satisfied.

Figure 7 illustrates the resultant solidification front movements. The solidification front contours are determined by connecting nodal points having an H value that is correspondent with the end of solidification. In this particular example, applying the q -technique reduces the number of nodes needed to be treated from 1350 to 150. The saving in computing cost is thus obvious. Comparison of Figs. 7a and 7b leads to the conclusion that the heat transport in the mold can be reasonably described by a set of constant properties. On the other hand, good agreement between Figs. 7b and 7c shows that the cost effectiveness of the q -method does not result in any loss of accuracy. The above analysis has thus demonstrated the practicality and advantage of the q -method in simulation of the solidification process of sand casting systems.

CONCLUSION

This paper has presented a new approach to simulation of the solidification process of sand casting systems. The similarity characteristic exhibited by the heat flow across mold-casting interfaces forms the theoretical basis for the q -method, which, in simulation of the solidification process of two-dimensional

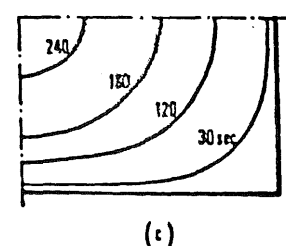
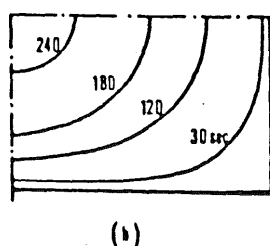
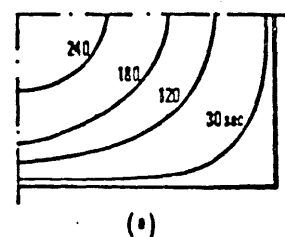


Fig. 7. Computed solidification front movement via a) conventional approach incorporating the temperature dependence of mold properties; b) conventional approach using constant mold properties; and c) the q -method.

shaped sand castings, replaces a multitude of volume elements in the mold with a hypothetical heat transfer coefficient acting at the mold-metal interface. Comparison of the results of a conventional scheme that solves for the transient temperature field in the whole mold and casting system and of the q-method has suggested that the q-method is capable of producing accurate results with a considerable saving in computing cost. Furthermore, the q-method will, in effect, broaden the solution capacity of storage limited computer systems by focusing the numerical effort on the metal part of a sand casting system.

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NOMENCLATURE

C	Specific heat
erf	Error function
erfc	Complementary error function
E	Edge function
Fo	Fourier number
h	Heat transfer coefficient
H	Enthalpy function
J	Bessel function of the first kind
K	Thermal conductivity
L	Latent heat
q	Heat flow per unit time and area
t	Time
T	Temperature
α	Thermal diffusivity
θ_0	Wedge angle
Θ	Modified temperature function
ρ	Density

Subscripts

c	casting
m	mold
ld	one-dimensional

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Geometric Modeling of Mold Aggregate, Superheat, Edge Effects, Feeding Distance, Chills, and Solidification Macrostructure

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ABSTRACT

Geometric modeling in the sense implied here is the use of geometric parameters to describe the effects of the flow of heat resulting from solidification and cooling of the molten metal poured into molds. Obviously, the volume of a casting limits the amount of heat which can enter the system. This heat transfers out of the casting through its surface area into the mold. Recognition of the role of these two geometric parameters in casting solidification was first described by Chvorinov,¹ through the rule:

$$\text{Log } t \approx \text{Log } (V/A)^2$$

where t is the solidification time in minutes, V is the volume of the casting, and A is the surface area of the casting. Chvorinov's rule was initially shown to be a straight line log-log relationship for steel castings solidified in silica molds (Fig. 1). The calculated value of V/A for a casting or section of a casting is called its section modulus, SM ; or $SM = V/A$. The slope of the Chvorinov rule lines in Fig. 1 can be described in the same units as thermal diffusivity— mm^2/sec . When considered superficially, V/A does not include the effects of casting process variables such as superheat, change in mold or core aggregate, casting design or shape effect, risering, and many others. However, Wlodawer² has made a most extensive interpretation of SM as applied to a number of these variables in steel castings. For example, Fig. 1 indicates how different mold aggregates change the solidification time, t , versus V/A relationship as compared with silica molds. Wlodawer's work dealt to a major extent with use of SM in the analysis of steel casting designs and their necessary risering for soundness.² However, he did not apply the section modulus idea to the interaction of metal and mold thermal properties with the casting design, solidification processes, and foundry process variables. This paper presents the relationship of geometric parameters including section modulus to thermal properties as concepts that can be used to develop computer programs. The specific computer programs are not presented but will be reported separately. Rather, presentation of concepts behind the programming are intended herein.

SOLIDIFICATION

When a skin of solidified metal develops at the surface of a casting, a volume, V_s , of solid metal has formed and given up its heat through the casting surface area, SA_c . The section modulus of the solidified skin is V_s / SA_c . As solidification proceeds, the solidified skin grows to complete solidification, then V_s increases to the volume of the casting, V_c , and the section modulus increases from V_s / SA_c to V_c / SA_c —the latter being Chvorinov's rule

for the entire casting. The formation and growth of the skin and associated heat transfer representing complete solidification occurs as V_s increases from 0 to V_c , while SA_c remains constant. The use of SM to simulate solidification wave-fronts was suggested by Kotschi.³ Equations for the simulation in a cylinder cross-section and the more general case are noted in references 4 and 5 respectively. An example of such section modulus wave-fronts are shown in Fig. 2, left, for a cross-section of a ring casting. In the case shown in Fig. 2, left, it is assumed that there is no superheat, all surfaces conduct heat equally, and no risers are attached. For these conditions, Fig. 2 shows two late freezing regions separated by a thinner early freezing section as would be expected from the design. In order to introduce the effects of potential process variables, the thermal properties of the cast metal will be expressed as geometric parameters that can be applied through the section modulus wave-front concept.

VOLUME AND HEAT CONTENT

The heat energy entering the system depends on the temperature of the metal after the mold is filled as well as the volume of metal poured.

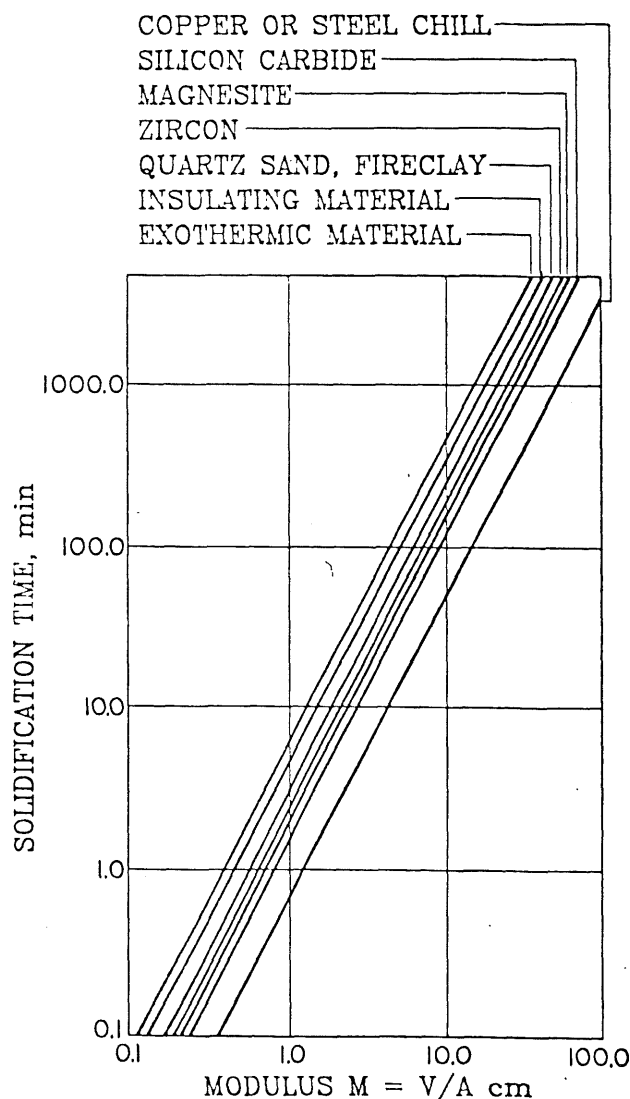


Fig. 1. Graph showing Chvorinov's rule for steel cast in silica and other mold aggregates.

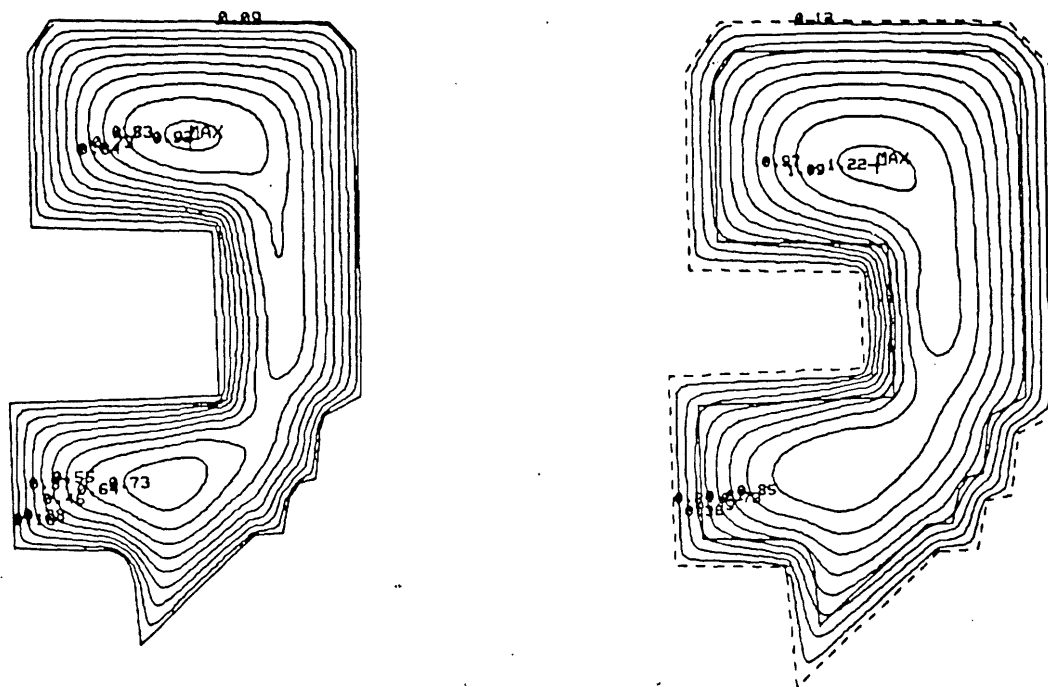


Fig. 2. Left: Solidification wave fronts in cross-section of ring casting for conditions of no superheat or attached riser. Note the two late-freezing heavier regions separated by narrow section. Right: Superheat energy of 5% shown as dotted line added around cross-section at left. Note that solidification within the actual cross-section does not begin until the wave fronts cross the solid line section outline, and occurs at the corners first.

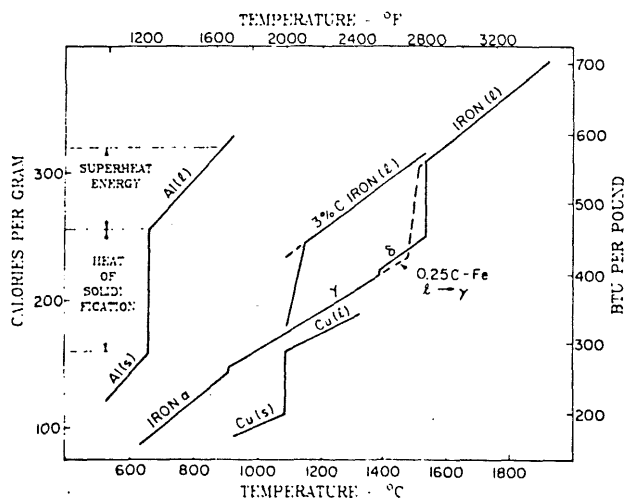


Fig. 3. Relationship of temperature and cal/gm heat content of Al, Cu, steel, and cast iron. Superheat energy above the solidus divided by the heat of fusion provides the ratio used to calculate the superheat volume correction.

Heat content of Al, Cu, and Fe as related to temperature is given in Table 1. Table 2 provides the same data for a 3% C pig iron which is used to represent cast irons. A graph of calories per gm versus temperature for Al, Cu, and Fe is shown in Figs. 3 and 4. Figure 3 also includes estimated values for a 0.25% C-Fe alloy when solidification is completed as gamma iron. A more detailed comparison of Fe and cast iron data is provided in Fig. 4. Figure 5 shows data for the comparison on an equivalent volume basis, calories per cubic centimeter.

SUPERHEAT ENERGY

The geometric model of superheat is based on Chvorinov's rule. Solidification time obviously depends on the heat content of the casting volume. Without superheat, only heat of solidification is transferred (Table 1). With superheat, the heat to cool the molten metal from the superheated temperature to the liquidus temperature must also be transferred. This is illustrated in Fig. 3 for Al superheated to 880C (1616F). The numbers can be obtained from Table 1. While superheat puts more heat energy into the same casting volume as melt temperature is increased, the same increase in heat energy could occur at the liquidus temperature by enlarging the casting volume proportionally. For illustration, consider one gram of molten Al at 933.5K (660.5C, 1221F)—the liquidus temperature. For solidification, $Al(l) \rightarrow Al(s)$, heat given off is $6890 - 4310 = 2580$ cal/gm atom. If the liquid metal temperature is raised to 1000K (727C, 1341F), the heat content then is 7191 cal/gm atom, or, the total heat to be transferred $7191 - 4310 = 2881$ cal/gm atom to the solidus or an additional 301 cal/gm atom. Energy superheat then may be expressed as a ratio: $(2580 + 301) / 2580 = 1.1167$. But if the volume of the casting were increased by this ratio without superheat, its heat content would be as much as the original volume with superheat at 727C (1340F). The geometric equivalent of superheat in this case then requires a 11.67% increase in casting volume so that the new volume will release as much heat when solidification occurs at the liquidus-solidus temperature of 660.5C (1221F) as would be released by the original volume from the higher temperature of 727C (1340F). Linearly, the dimensions are increased by the cube root of the volume ratio. Thus, the geometric modeling of superheat can be accomplished by increasing an original casting volume by the ratio of heat evolved over the superheating-solidus temperature range, divided by the heat evolved over the liquidus-solidus temperature range. The effect of superheat energy increase is shown in Fig

Table 1. Heat Content of Al, Cu and Fe*

K	°C	°F	Cal/gm atm.	Cal/gm.	Cal/cm ³	Btu/lb solid	Btu/lb liquid	Btu/in ³ solid	Btu/in ³ liquid
Al									
800	527.0	980.6	3286	121.80	316.8	219.6	—	20.60	—
900	627.0	—	4047	150.10	383.7	270.1	—	25.00	—
933(s)	660.5	1221.0	4310	159.80	407.5	288.0	—	26.50	—
933(γ)	660.5	1221.0	6890	255.50	604.9	—	460.0	—	39.30
973	700.0	1292.0	7191	266.60	628.4	—	481.0	—	40.90
1000	727.0	1341.0	7396	274.20	644.7	—	494.0	—	41.90
1100	827.0	1520.6	8155	302.40	707.0	—	545.0	—	46.00
1200	927.0	1701.0	8914	330.50	768.0	—	—	—	50.00
Cu									
1200	927.0	1701.0	5886	92.60	785.0	167.0	—	51.10	—
1300	1027.0	1881.0	6634	104.40	876.0	188.0	—	57.00	—
1356.6(s)	1083.6	1982.5	7077	111.30	931.0	200.6	—	60.50	—
1356.6(γ)	1083.6	1982.5	10197	160.40	1272.0	—	289.0	—	82.70
1400	1127.0	2061.0	10536	165.70	1309.0	—	299.0	—	85.10
1500	1227.0	2241.0	11316	178.00	1394.0	—	321.0	—	90.60
1600	1327.0	2421.0	12096	190.00	1475.0	—	343.0	—	95.90
Fe									
800	527.0	981.0	3717	66.60	525.8	120.0	—	34.20	—
1000	727.0	1341.0	5820	104.20	823.2	192.2	—	53.53	—
1100	827.0	1521.0	7154	128.10	1012.0	230.9	—	65.80	—
1184(α)	911.0	1672.0	8030	143.80	1136.0	265.2	—	73.90	—
1184(γ)	911.0	1672.0	8245	147.60	1166.3	266.1	—	75.80	—
1200	927.0	1701.0	8375	150.00	1184.6	270.2	—	77.00	—
1400	1127.0	2061.0	10041	179.80	1420.0	324.1	—	92.40	—
1500	1227.0	2241.0	10904	195.20	1542.0	351.9	—	100.30	—
1600	1327.0	2421.0	11787	211.00	1667.3	380.4	—	108.40	—
1665(γ)	1392.0	2506.0	12372	221.52	1750.0	399.3	—	113.80	—
1665(δ)	1392.0	2506.0	12572	225.10	1778.0	405.8	—	115.60	—
1700	1427.0	2601.0	12917	231.30	1827.1	416.9	—	118.80	—
1800	1527.0	2781.0	13920	249.00	1970.0	449.3	—	128.00	—
1809(γ)	1536.0	2797.0	14012	251.00	1982.0	452.3	—	128.90	—
1809(γ)	1536.0	2797.0	17312	310.00	2448.8	558.8	558.8	—	152.20
1900	1627.0	2961.0	18313	327.90	2590.0	626.6	591.1	—	168.45
2000	1727.0	3141.0	19413	347.60	2746.0	—	626.7	—	178.60
2200	1927.0	3501.0	21613	387.00	3057.0	—	697.6	—	198.80
0.25C-Fe**									
400	127.0	261.0	637	11.40	90.1	20.6	—	5.86	—
500	227.0	441.0	1319	23.60	186.6	42.6	—	12.10	—
600	327.0	621.0	2057	36.80	291.0	66.4	—	18.90	—
800	527.0	981.0	3717	66.60	525.8	120.0	—	34.20	—
1000	727.0	1341.0	5820	104.20	823.2	192.2	—	53.50	—
1100	827.0	1521.0	7154	128.10	1012.0	230.9	—	65.80	—
1200	927.0	1701.0	8375	150.00	1184.6	270.2	—	77.00	—
1400	1127.0	2061.0	10041	179.80	1420.0	324.1	—	92.40	—
1500	1227.0	2241.0	10904	195.20	1420.0	351.9	—	100.30	—
1600	1327.0	2421.0	11787	211.00	1667.3	380.4	—	108.40	—
1665(γ)	1392.0	2506.0	12372	—	221.5	1750.0	399.3	113.80	—
1665(γ)	1392.0	2506.0 No γ-δ; γ metastable							
1700	1427.0	2601.0	12690	227.20	1795.0	409.6	—	116.70	—
1773(s)	1480.0	2696.0	13150	235.50	1860.0	424.0	—	121.00	—
Metastable γ⁻									
1787(γ)	1514.0	2757.0	17060	305.50	2413.0	—	550.6	—	156.00
1800	1527.0	2781.0	17213	308.20	2435.0	—	555.6	—	158.00
1809	1536.0	2797.0	17312	310.00	2449.0	—	558.8	—	159.00
1900	1627.0	2961.0	18313	327.90	2590.0	—	591.1	—	168.00
2000	1727.0	3141.0	19413	347.60	2746.0	—	626.7	—	178.00
2200	1927.0	3501.0	21613	387.00	3057.0	—	697.6	—	198.00

* Hultgren et al. "Select Values of the Thermodynamic Properties of the Elements." American Society for Metals, 1973.

** Fe density of 7.9 assumed.

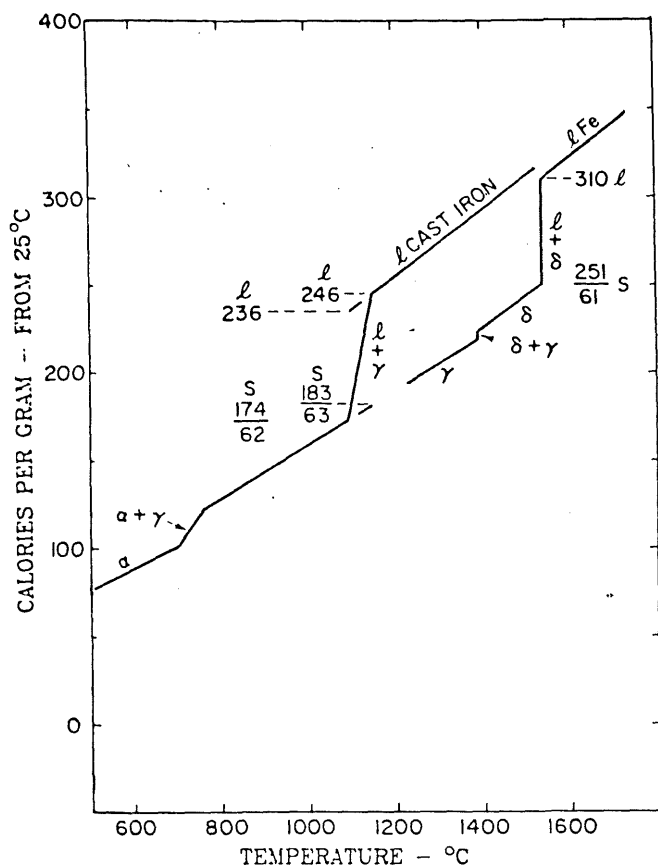


Fig. 4. Relationship of temperature and callgm for 0.25%C steel and 3%C cast iron.

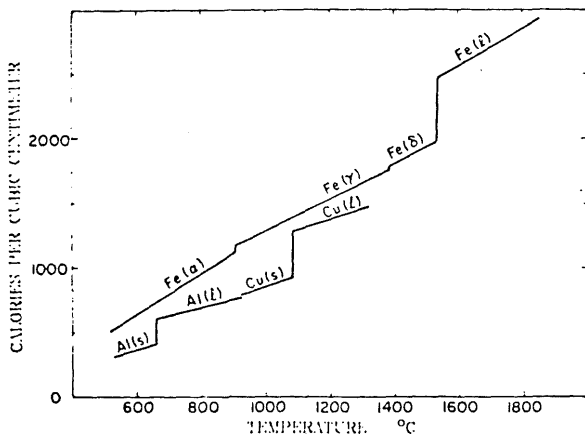
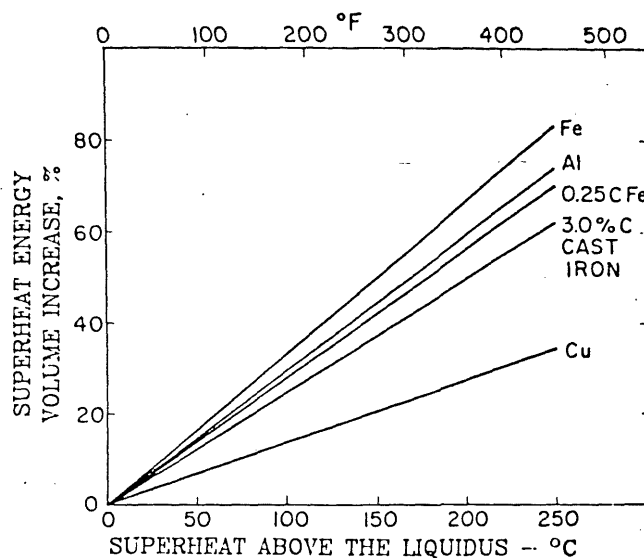


Fig. 5. Relationship of temperature and callcm³ for Al, Cu, and Fe.

2, right. The enlarged casting volume is shown as the dotted line enveloping the original solid casting section (Fig. 2, right). Superheat delays solidification so that it occurs first at the corners and later throughout the section. Two late freezing regions are still present, as in Fig. 2, left, with no superheat.

Superheat Volume Increase

The geometric model of superheat just described is expressed as a volume ratio or percentage volume increase. This volume increase can only be applied if the casting is presented as a three-dimensional model. The 3D percentage volume increase must be corrected to an areal-percentage increase when a two-dimensional cross-section of the casting is being studied. The cube root of percentage volume



	660	700	800	900	°C
Al	1220	1300	1400	1500	°F
	1083	1100	1200	1300	°C
Cu	1983	2000	2100	2200	°F
	1536	1600	1700	1750	°C
Fe	2797	2900	3000	3100	°F
	1514	1600	1700	1700	°C
Fe 0.25%C	2757	2800	2900	3000	°F
	1149	1200	1300	1400	°C
Fe 3.0%C	2100	2200	2300	2400	°F

Fig. 6. Percentage volume increase for superheat energy in relation to superheat temperature.

increase is the increase in linear dimensions and is squared to obtain the areal percentage. Both volume and areal-increase percentages as related to superheating temperature for Al, Cu, Fe, and 0.25%C steel are listed in Table 3, and for cast-iron (3%C pig iron) in Table 4. A graph of percentage volume increase vs. superheating temperature is given in Fig. 6. Equations approximating the percentage volume increase corresponding to superheating temperature can be easily written. Typical pouring superheat temperatures in the foundry are about 100°C (212°F) for Al, Cu, Fe, and steel and about 200°C (392°F) for cast irons. Hence the volume increase for the first four metals is in the range of 25-33% or 8-10% areal. For cast iron at 200°C (392°F) superheat the volume increase is about 50% or 13.6% areal.

Programming Superheat

Correction for energy superheat has been accomplished in 2D section modulus wave-front program by increasing the casting cross-sectional area of interest by an amount corresponding to the areal increase for the selected superheat from Tables 3 and 4, but corrected for scale factors. In subsequent 3D programs, the volume increase will be used. The areal increase is outlined around the actual cross-section as shown in Fig. 2, right, and Fig. 7. Micro-computer programs can be areal- or volume-corrected for superheat in a similar way. Figure 7 will be discussed after the casting heat transfer surface area is considered.

Table 2. Heat Content of 3% C Pig Iron*

°K	°C	°F	Cal/gm(s)	Cal/gm(1)	Btu/lb(s)	Btu/lb(1)
3%C Pig Iron						
589	316	600	49.8		89.6	
		900	65.5		117.8	
755	482	1000	74.3		133.7	
922	649	1200	93.0		167.4	
977	704	1300	104.2 (α)		187.5 (α)	
1033	760	1400	122.8 (γ)		221.0 (γ)	
1089	816	1500	131.1		236.0	
1144	871	1600	139.5		251.0	
1200	927	1700	148.0		266.1	
1255	982	1800	156.5		281.7	
1311	1038	1900	165.0		297.0	
1366	1093	2000	174.0	236	313.0	425
1422	1149	2100	183.0	246	329.0(s)	443
1477	1204	2200	191.7	256	345.0	461
1533	1260	2300		266		479
1589	1316	2400		276		497
1644	1371	2500		286		515
1700	1427	2600		296		533
1755	1482	2700		—		—
1811	1538	2800		316		569

* The Cupola and Its Operation, American Foundrymen's Society

Table 3. Volume Increase for Energy Superheat

T	Cal.gm	H	H(1-s)	% Volume	% Area	% Linear	Superheat	
°C	atm.	H	+ Superheat	Ratio%	Increase	Increase	Increase	°C °F
AL								
660.5	1221	4310(s)	—					
660.5	1221	6890(l)	—					
	ΔH =	— S =	2580	2580	1.0	0	0	0
700.0	1291	7191	301	2881	1.1167	11.67	7.64	39.5 71
727.0	1341	7396	506	3086	1.1961	19.61	12.68	66.5 120
827.0	1527	8155	1265	3845	1.4903	49.03	30.47	14.22 166.5 300
Cu								
1083.6	1982.5	7077(s)						
1083.6	1982.5	10197(l)						
	ΔH =	— S =	3120	3120	1.0	0	0	0
1127.0	2061.0	10536	339	3459	1.1087	10.87	7.12	3.50 43.4 78.1
1227.0	2241.0	11316	1119	4239	1.3586	35.87	22.67	10.76 143.4 258.1
1327.0	2421.0					60.90	37.30	17.18 243.4 438.1
Fe								
1536	2797	14012(s)						
1536	2797	17312(l)						
	ΔH =	— S =	3300	3300	1.0	0	0	0
1627	2961	18313	1001	4301	1.3033	30.33	19.31	9.23 91 164
1727	3141	19413	2101	5401	1.6367	63.67	38.90	17.84 191 344
Fe - 0.25%C, T_M = 1514°C - 1480°C								
1480(s)		13161						
1514(l)		17060	3900		1.0	0	0	0
1600	2912	18000	940	4840	1.2410	24.10	15.48	7.46 86 155
1700	3092	19100	1100	5940	1.5231	52.31	32.38	15.06 186 335

Table 4. Volume Increase for Energy Superheat of 3% C Pig Iron (Cast Iron)

°C	T °F	Cal/gm	H	H(1-s)-s + Superheat	Ratio	% Volume Increase	% Area Increase	% Linear Increase	Superheat	
									°C	°F
1093	2000	174(s)	—	—						
1149	2100	246(l)	—							
	L - S		72	72	1.0	—	—	0	0	0
1204	2200	256	10	82	1.1389	13.89	9.06	4.43	55.6	100
1316	2400	276	30	102	1.4167	41.67	26.14	12.31	166.7	300
1427	2600	296	50	122	1.6944	69.44	42.13	19.21	277.8	500
1538	1800	316	70	142	1.9722	97.22	52.27	25.41	388.9	700

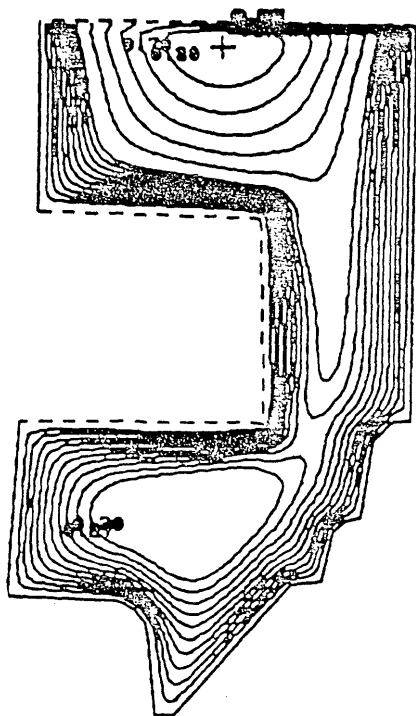
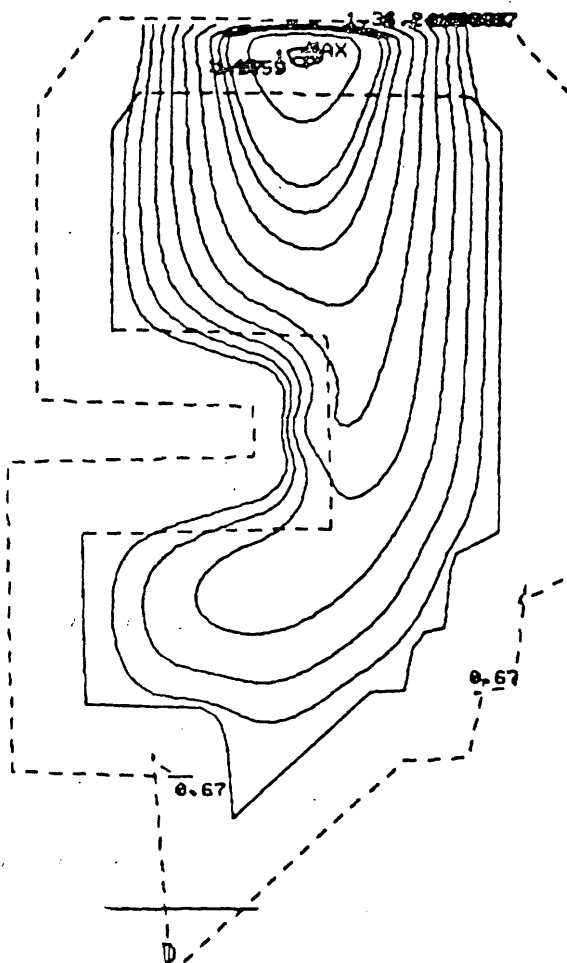


Fig. 7. Left: Solidification wavefronts in ring casting cross-section with SA factor of 0.02 at top riser connection and 0.08 at core recess. (Compare with Fig. 2, left.)

GEOMETRICALLY MODELING THE CASTING HEAT-TRANSFER SURFACES

All surfaces of the casting can transfer some of the heat energy out of the casting, except for those surfaces that are removed by riser and gate connections. The geometric model for all casting or mold surfaces does not involve heat transfer calculations but rather utilizes a ratio value which is similar to a change in thermal diffusivity of the affected surface. Since most foundry molds are silica aggregates, silica is assigned a ratio value of 1.0 to describe its heat transfer capabilities. Another aggregate of higher thermal



Right: Same figure with 10% superheat energy input. (Compare with Fig. 7, left.) Note feeding path is open from riser through the narrow section to the bottom as a result of superheat.

diffusivity such as zircon sand will transfer heat more quickly and cause the casting to solidify earlier. This is revealed in Fig. 1 which shows Chvorinov's rule for silica, zircon, and many other mold materials. Figure 1 shows that zircon sand reduces the freezing time for steel an amount dependent on the section modulus for silica. Hence, the section modulus can be corrected by increasing the actual casting surface area by an amount that will lower the freezing time in silica to that in zircon. A value higher than 1.0 applied to SA in the SM calculation, then, implies that the material will shorten solidification time while one of lower ratio

Table 5. Chilling Ability and Ratios to Silica of Various Mold Materials

Mold material	Solidification times t_0 for 6 in step 1 sphere, min.	Ratio t_0 silica t_0 material	Ratio $\sqrt{t_0}$ silica $\sqrt{t_0}$ material	Typical mold density lbs/ft ³	Solidification time, t_0 cast iron bar, 2.5 in. D				Suggested surface area ratios for programs
					Ratio	Min.	t_0 silica t_0 material	$\sqrt{t_0}$ silica $\sqrt{t_0}$ material	
Copper chill	4.2	4.05	2.02						
Steel chill	4.3	3.95	2.00	490	4.9				1.5-2.0
Graphite	5.1	3.33	1.84						
Copper shot	6.3	2.70	1.68						
Steel shot	9.0	1.89	1.38						
Silicon carbide	10.4	1.63	1.28						
Magnesite	11.2	1.52	1.24						
Alumina	12.1	1.40	1.18						
Chrome ore	13.4	1.27	1.12	166	1.6	11.1	1.33	1.15	1.2-1.6
Zircon	13.8	1.12	1.12	180	1.8	7.6	1.95	1.40	1.2-1.8
Rutile	14.5	1.17	1.08						
Chamotte	15.2	1.12	1.06						
Forsterite (olivine)	15.8	1.08	1.04	120	1.2	13.7	1.08	1.04	1.1-1.3
Silica	17.0	1.00	1.00	100	1.0	14.8	1.00	1.00	0.9-1.10
Alumina bubbles	20.0	0.85	0.92						
40% Diatomaceous earth									
Mulled	23.2	0.73	0.86						
Not mulled	27.0	0.63	0.80						
Thermit-silica compound	31.0	0.55	0.74						

Columns 1-3 abstracted from ref. 9.

Columns 4-5 experimental data.

Column 6 estimated values from a variety of conflicting thermal diffusivity data.

will lengthen low ratios, and hot surfaces such as riser connections will have ratios near zero. These ratios are akin to the effect of thermal diffusivity and are defined as the surface area multiplying effect of changing a surface from silica to some other material. Table 5 lists examples of mold aggregate ratios to enter into computer programs. Because the literature values of mold thermal diffusivity are uncertain, only recommended ratios are given.

For programming purposes, the geometric surface area of a casting is entered into the data base from a drawing or other program. The geometric area is arbitrarily assumed to have a value of 1.0 per unit area. When part of the silica is replaced by another aggregate, chill, or riser connection, the appropriate factor correcting that location is entered. Figure 7, left, provides an example of the solidification wave front changes caused when a riser connection, top, assigned a value of 0.02 and a core assigned a value of 0.80 are entered into the program for the casting shown in Fig. 2, where all surfaces were given the normal value of 1.0. Surfaces where the ratio is different from 1.0 are indicated by dotted lines and the ratios are recorded, Fig. 7, left.

The profound changes in solidification shown in Fig. 7, left, illustrate the important effect of riser attachments. The riser connection must provide an open channel or feeding path to the sections to be fed. Both Fig. 2 and 7, left, show feeding path pinch-off in the narrow section with a later-freezing region below it where shrinkage would occur.

COMBINATION OF VOLUME AND SURFACE AREA EFFECTS

Since risering, riser connections, mold surface, and superheat are always prevalent in the casting process their interaction must be

considered. The corrections for surface effects and superheat were described earlier. The effect of superheat energy on a specific job is shown in Fig. 7, right. To produce this wave front analysis, 10% superheat increase was entered into the program. The early freezing fronts were not drawn so that a clear picture of the superheat energy effect could be observed. Note that the bottom section freezes first, then wave fronts progress through the narrow section, then the upper heavier section, and finally through the riser connection. It is possible that 10% superheat is excessive in this case since surface shrinkage might occur at inside corners. However, the important point shown here is that the casting does not freeze favorably until the process variables of casting shape, riser connection, and superheat are all accounted. The core material in this case was not important while the other variables were overriding in the particular casting. Without considering the superheat and riser variables in Figs. 2 and 7, the use of a chill might have been thought necessary. The wave front program deals with these variables correctly because of the dependence of the section modulus parameters on fundamental thermal properties.

SOLIDIFICATION AND TEMPERATURE GRADIENTS

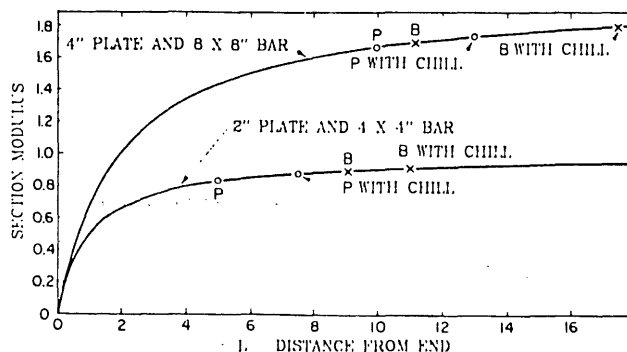
Solidification gradients and related temperature gradients may be modeled through section modulus gradients. This is most easily illustrated by comparing the gradients at the ends of plates and bars risered from one end. Tables 6 and 7 present data for steel plates and bars, respectively, abstracted from the published work of Pellini et al.^{6,7,10} The section modulus gradient from the end of a plate is $V/A = T \times L / (T + 2L)$ and end of a bar is $V/A = E \times L / (E + 4L)$ —where L is the distance from the end, T is the infinite plate thickness, and $E \times E$ is the bar cross section. A plot of SM versus distance from the end of 2 and 4 in. (5.08 and 10.16 cm) T plates and bars with $E \times E$ of 4 x 4 in. (10.16 x 10.16 cm)

Table 6. Infinite Plate and Chill Width, $T \times L$: $V/A = T \times L \div T + 2L$

T, plate Thickness plate	L, Sound end		Chill TxL	L, Sound end		Chill, V/A	
	No chill	V/A		With chill	V/A	$\frac{T \times L}{2T + 2L}$	$\frac{V}{2A}$
0.5	1.25	0.210	—	—	—		
1.0	2.50	0.417	L = 1T = 1 L = 1T = 2	3.5 3.5	0.438 0.438		
1.5	3.75	0.625	—	—	—		
2.0	5.00	0.833	L = 1/4T = 0.5 L = 1/2T = 1.0 L = 1T = 2.0 L = 2T = 4.0	6.0 6.5 7.5 7.5	0.857 0.867 0.882 0.882		
3.0	7.50	1.250	L = 1/2T = 1.5	9.5	1.295		
4.0	10.00	1.670	L = 1/2T = 2.0 L = T = 4.0 L = 2T = 8.0	10.0 12.5 13.0	1.670 1.724 1.733	0.670 1.000 1.333	0.335 0.500 0.667

Table 7. Square Bars $T \times T \times L$: $V/A = EL \div E + 4L$

Bare edge E x E inches	L, Sound end with		Chill E x E x L inches	L, Sound end with	
	No chill; inches	V/A		Chill; inches	V/A
2x2	4.10	0.450	—	—	—
4x4	7.10	0.880	4x4 l = 1/4T = 1 l = 1/2T = 2 l = 1/2T = 2 l = 1T = 4 l = 1.5T = 6	8.5 10.5 11.0 10.5 11.0	0.895 0.910 0.920 0.910 0.920
6x6	9.50	1.295	6x6 l = 1/4T = 1.5 l = 1/2T = 3.0 l = 1/2T = 3.0 l = 1T = 6.0	11.0 13.5 13.0 13.0	1.320 1.350 1.345 1.345
8x8	11.25	1.700	8x8 l = 1/4T = 2 l = 1/2T = 4	13.5 17.5	1.740 1.795

Fig. 8. Section modulus of 2.0 and 4.0 in. (5.08 and 10.16 cm) plate and 4 x 4 in. (10.16 x 10.16 cm) and 8 x 8 in. (20.32 x 20.32 cm) bar versus distance from the end. Length of the sound end is indicated on the SM curve.^{6,7}

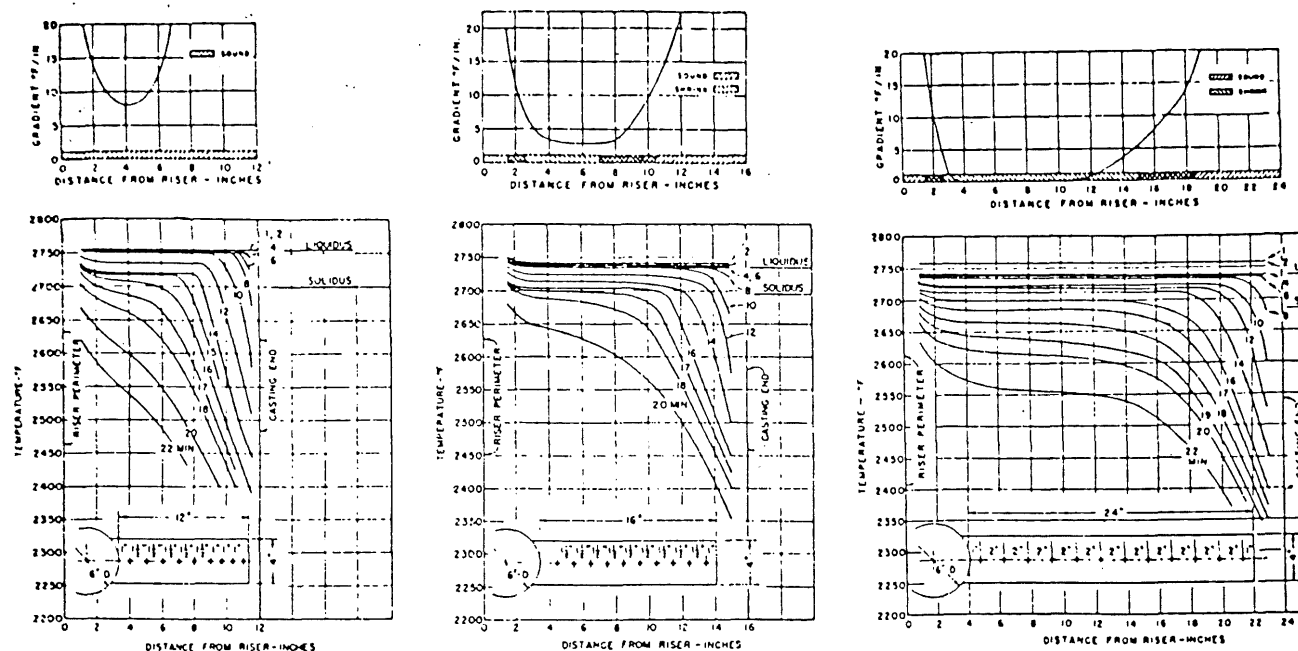


Fig. 9. Temperature gradients and sound end and riser lengths in a 4 x 4 in. (10.16 x 10.16 cm) bar.¹⁰

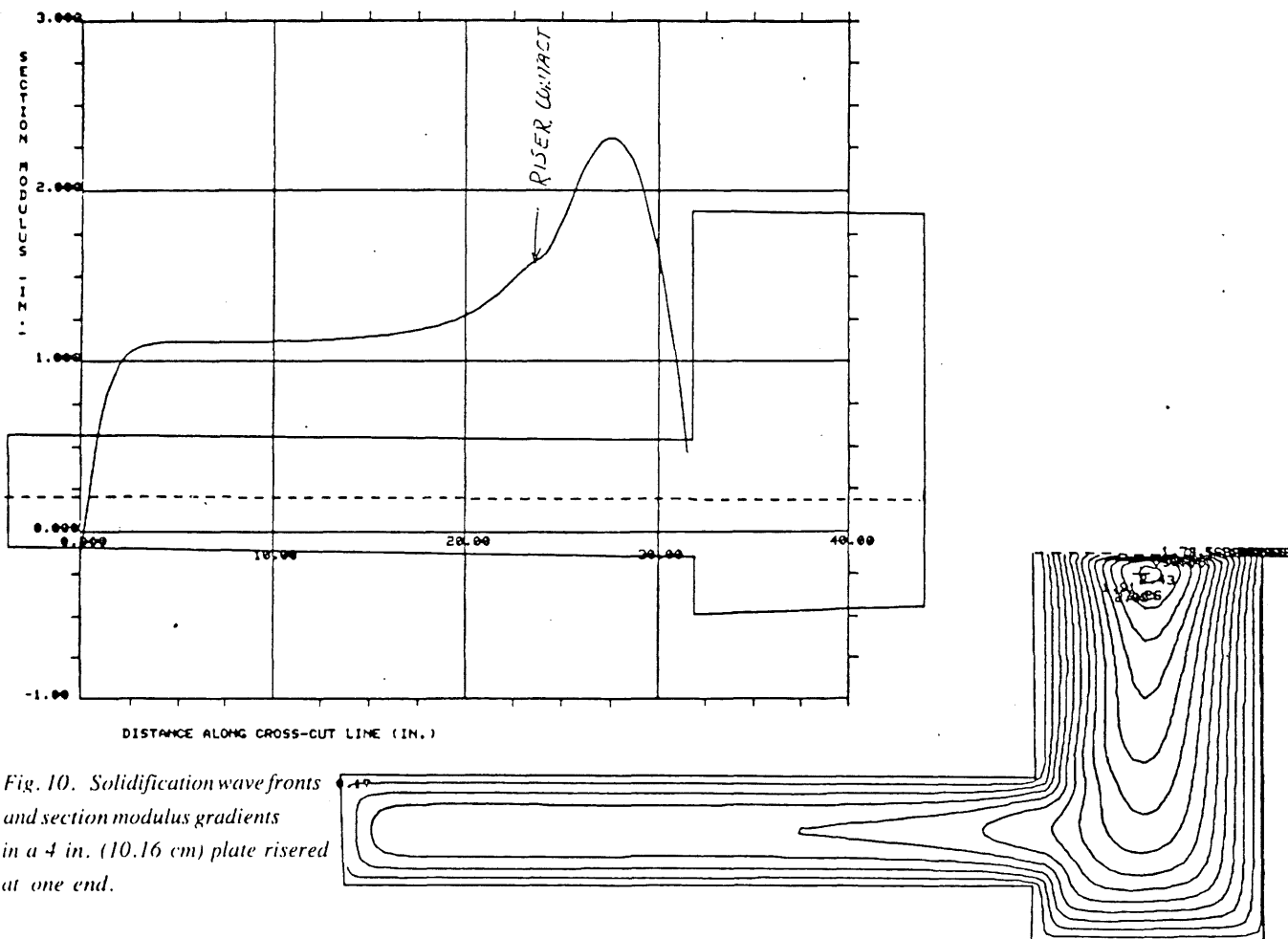


Fig. 10. Solidification wave fronts and section modulus gradients in a 4 in. (10.16 cm) plate risered at one end.

and 8 x 8 in (20.32 x 20.32 cm) is shown in Fig. 8. The length of sound end in each case is shown on the curve. The SM gradient at the end parallels temperature gradients actually measured in the bars. Figure 9,¹⁰ shows the temperature gradients existing in 4 x 4 in. (10.16 x 10.16 cm) bars of various lengths both at the bar

end and at the riser end. A steep gradient is accompanied by casting soundness at each end with shrinkage between them. The SM gradient can be modeled into a cross-section using the wave front model as shown in Fig. 10. The steep section modulus curves are shown at both the bar and the riser end in Fig. 10. The slope of

Table 8. Ratio of V/A Sound End Length to V/A Infinite Plate or Bar
With and Without Chills and Associated Taper

T, in.	Plate	Sound end	Ratio	Chilled plates W/1T chill ratio	
	Plate V/A	V/A	$e(V/A) \div P(V/A)$	$e_c(V/A)$	$e_c[(V/A) \div p(V/A)]$
0.50	0.25	0.210	0.840	—	—
1.0	0.50	0.420	0.820	0.438	0.876
1.5	0.75	0.625	0.833	—	—
2.0	1.00	0.833	0.833	0.882	0.882
3.0	1.50	1.250	0.833	1.295	0.863
4.0	2.00	1.670	0.835	1.724	0.862
			Ave. 0.832		Ave. 0.871

Bars			Chilled Bars		
2x2	0.5	0.445	0.890	—	—
4x4	1.0	0.880	0.880	0.915	0.915
6x6	1.5	1.295	0.860	1.345	0.897
8x8	2.0	1.700	0.850	1.795	0.898
			Ave. 0.873		Ave. 0.903

the SM curves of Fig. 10 would more directly mirror the temperature gradient curve of Fig. 9. The location of the cross-section having the SM gradients is indicated by the dotted line in Fig. 10. The casting cross-section and SM curves in Fig. 10 are not of the same scale.

The SM and gradient where the bar and riser ends become unsound suggests that there may be a parameter that limits feeding. Table 8 reveals that in plates where $T = 0.5$ to 4.0 in. (12.7 to 101.6 mm), the ratio of SM of the sound end to the SM of the infinite plate averages 0.832 with a range of only 0.82 - 0.84 . (The points for 2.0 and 4.0 in. (5.08 and 10.16 cm) T plates were indicated on the SM gradient curve, Fig. 8.) If a chill is added, the ratio average increases to 0.871 , indicating that where the SM of the plate end reaches 83.2% of the infinite plate SM, $T/2$, feeding will no longer occur toward that point from the farther reaches of the plate. When a chill is used at the end of the plate, the ratio increases to 87.1% of the infinite plate SM. With respect to bars, with and without chills at the end, the values are 90.3% and 87.3% , respectively, of the infinite bar SM, $E/4$.

The limiting feeding ratio appears to provide a very useful generalization since the point on the SM versus L curve where the end SM = $0.83T/2$ is the location where taper should begin on the plate in order for feeding to continue. Further, the amount of taper may be estimated as $(1.0 - .83) 100 = 17\%$. This is an increase in plate thickness from the sound end location, T_{SE} of 17% per inch beyond the sound end. For bars without chills the taper will be 13% per inch beyond the sound end. This may appear to be very large. However, as can be seen in Fig. 10, this effect of taper is produced thermally at the riser end of the plate or bar. Hence, the taper need only be used where the end effect and riser effect do not overlap. The numbers developed here were for a 0.20 - 0.35% C steel.^{6,7} Greater taper might be required for higher percent carbon.

Experimentally, taper was found to increase the distance fed to radiographic soundness from 5.0 in. (12.7 cm) with no taper to

8.0 in. (20.32 cm) with 10% taper in 0.25% C steel plates.⁸ Greater lengths were not investigated. However, no provision was made to eliminate taper near the riser. This could have been done with a resultant decrease in the amount of taper metal added. Evidently, feeding distance from risers to sound ends should be reinvestigated to determine how end effect, riser thermal effect, and taper can be most efficiently used to maximize feeding distance. Composition will also effect feeding distance.

CHILLS

Chills provide another tool for increasing temperature and solidification gradients. They may be used to increase solidification rate or solidified skin thickness in order to avoid two main defects: internal shrinkage and hot tears. Improvement of mechanical properties and microstructure through finer grain and phase size can also be obtained with chills. Also, the distance fed between risers to create end effects may be increased by using chills.

The effect of the chill is dependent on its thermal properties. The important thermal properties of the chill material appear to be heat capacity, density, thermal conductivity, and diffusivity. The most commonly used chill materials are steel and cast iron. Heat transfer factors such as metal temperature, chill coating, air gap formation, interface temperature, and temperature gradients all influence the effect of the chill on solidification. However, these factors all function geometrically through the heat transfer surface area of the chill at the casting and the volume of the chill capable of receiving the heat.

Chill Surface Area

The chill is expected to have contact with the molten metal when the mold cavity is filled. The contact area is selected by practical considerations of the need for chilling and surface accessibility. Usually, though not always, the entire chill is coated with a thin refractory layer to prevent welding of the chill to the casting. In

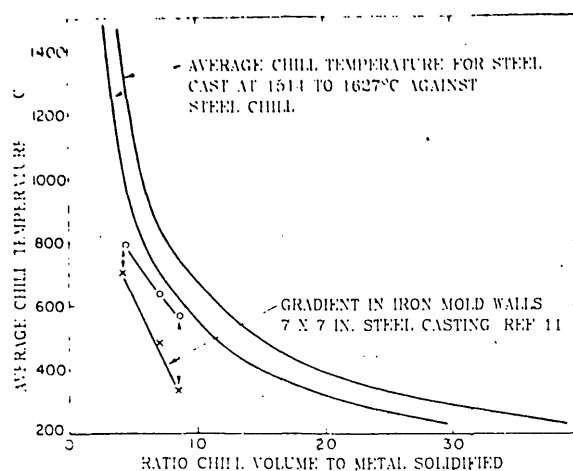


Fig. 11. Chill volume ratio versus average chill temperature for steel cast against steel or cast-iron chills.¹¹

addition, shortly after solidification begins an air (or mold atmosphere) gap may form, especially if the chill is on an upper surface. The heat transfer temperature across the interface may differ considerably depending on these conditions. Nevertheless, the heat must be transferred through the chill surface area exposed to the heat from the casting.

One method suggested for estimating the heat transfer capability of the chill surface is to assign the chill a ratio comparing it with the surface area of the sand aggregate adjacent to the chill. The ratio implies that the chill contributes additional surface area for heat transfer, and in effect, is like a multiplier of the same area as sand. Investigations of chill ratios suggests a ratio of about 3.0 without an air gap and 2.0 with an air gap. With air gap, variation of the factor may be from about 1.2–2.0, as the gap and refractory layer increase.¹ Details establishing the actual ratio for several aggregates are given in Table 5.⁹ Aggregates such as zircon, olivine, and chromite are also used for their chilling effect compared to silica.

Chill Volume

The chill volume must be sufficient to receive the heat from the metal volume that must be solidified to provide the desired chilling effects. The required casting volume can be estimated from a drawing or determined from either solidification wave front or microcomputer programs. Chilling molten steel will be used as an example. Table 1 provides the heat content data for 0.25 C-Fe steel. Since the temperature of the molten metal must be known, let's say that it is 1627°C (2961°F). The liquidus temperature is

1514°C (2797°F) and the solidus is 1480°C (2696°F) when austenite is formed. The heat transferred out when the molten metal finally solidifies, then, is $327.9 - 235.5 = 92.4$ cal/gm (166.3 Btu/lb), using Table 1. The volume of the chill needed to receive this heat requires knowledge of the maximum temperature reached at the interface and gradient in the chill. The latter depends in part on the thickness of the chill. Since these values are not known in advance, assumptions may be made for simplicity. Assuming no temperature gradient in the chill, the chill size can be calculated as a function of the average temperature reached by the chill. For example, if the chill reaches an average temperature of 627°C (1161°F), then above 25°C (77°F), the chill can receive 83.8 cal/gm (Table 1). The ratio of chill weight to the metal solidified then is $92.4/83.8 = 1.10$; or chill volume is estimated as about 1.10 times the volume solidified. The ratio of chill volume to metal volume can be calculated in this way for different average chill temperatures and molten metal pouring temperatures from the liquidus upward and plotted as shown for steel in Fig. 11. Correction for chill density versus molten metal density is unnecessary in this case since both are steel.

At an average chill temperature of 627°C (1161°F) and superheat temperature of 1577°C (2871°F) the chill volume ratio is $318 - 235.5/83.8 = 0.984$. Table 9 lists the chill volume ratios related to molten metal temperature and average chill temperature plotted in Fig. 11. The chill volume ratio is defined as the volume (or weight) of solidified metal, or the heat energy given up when the metal volume solidifies divided by the heat energy absorbed when the temperature of the chill rises to its average. When the two volumes are equal, the ratio is 1.0 and the average chill temperature is about 600–650°C (1112–1202°F), as shown in Fig. 11.^{6,7} From Fig. 11 superheat is observed to have a minor effect unless the chill ratio is below 0.50. Figure 11 applies to steel cast against a steel or cast iron chill.

The relationship of chill ratio to average chill temperature shown in Fig. 11 plots as a straight line on log-log coordinates in Fig. 12. Also shown in Fig. 12 are chill ratio versus average chill temperature for Al cast against iron, Al cast against Cu chills, and cast iron cast against cast iron chills. The latter pair shows a greater effect of superheat because molten cast iron is more superheated than the other metals when cast at 2600°F (1427°C).

Chill Effects and Dimensions

The volume of a chill needed to solidify a given steel casting volume may be predicted from Fig. 11, and an estimate of the average chill temperature expected. The effect of the chill on solidification and feeding depends upon where the chill is located on the casting as well as its volume. On plate or bar castings, the chill may be located at an end or edge distant from risers or at a

Table 9. Ratio of Chill Volume to Metal Solidified for 0.25 C-Fe

Molten metal Temperature		Superheat		Ratio chill volume to metal solidified at average chill temperature											
°C	°C	°C	°F	127C	227	327	527	627	727	827	927	1127	1327	1480	
SHT	°F			261F	441	621	981	1161	1341	1521	1701	2061	2421	2696	
1514	2757	0	0	6.14	2.97	1.90	1.05	0.835	0.671	0.546	0.467	0.389	0.3320	0.297	
1527	2781	13	24	6.38	3.08	1.98	1.09	0.868	0.698	0.568	0.485	0.404	0.3450	0.309	
1536	2797	22	40	6.54	3.16	2.02	1.12	0.889	0.715	0.582	0.497	0.414	0.3530	0.316	
1577	2871	63	114	7.24	3.50	2.24	1.24	0.984	0.792	0.644	0.550	0.459	0.3591	0.350	
1627	2961	113	204	8.11	3.91	2.51	1.39	1.100	0.886	0.721	0.616	0.514	0.4380	0.392	

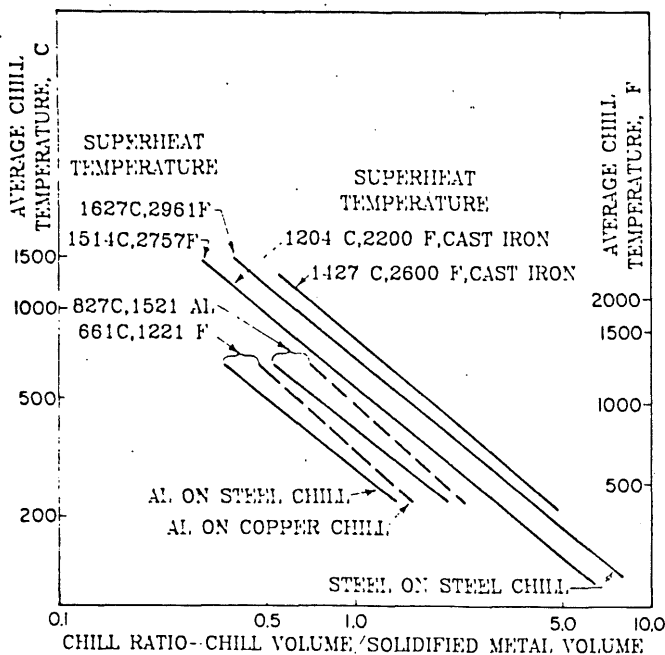


Fig. 12. Log-log plot of chill volume ratio to average chill temperature for Al cast on iron or copper chills, and steel and cast iron cast on steel or cast-iron chills.

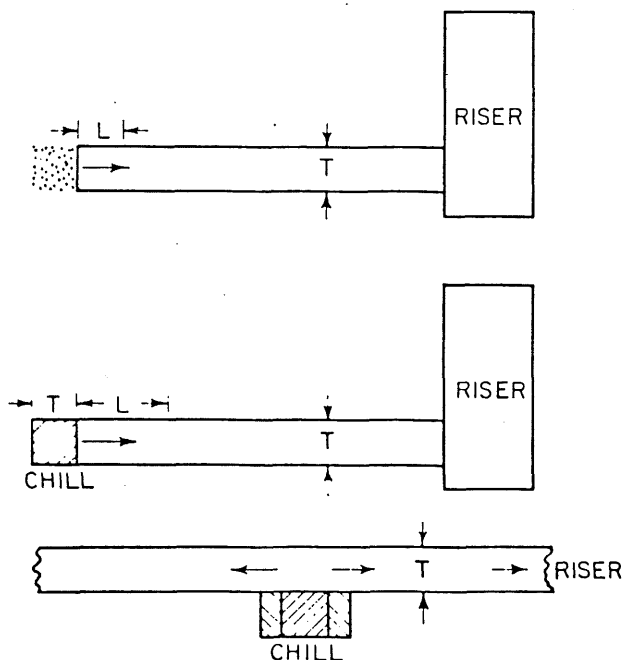


Fig. 13. Chills located at plate or bar ends, and at a point between risers.

central location where there is no end or edge effect (Fig. 12). The effect of the chill is determined by the sound end length produced by the chill compared with the sound end length produced when no chill is used and the casting length exceeds the distance which can be fed by the riser.^{6,7,10-12} This comparison is summarized in Tables 6-8. The section modulus for sound end lengths with and without chills is compared graphically in Fig. 14 for plates, and in Fig. 15 for bars. The experiments included chills of thickness $\frac{1}{4}T$, $\frac{1}{2}T$, $1T$ and $2T$ in this plate direction as indicated in Table 6. Increasing chill thickness resulted in increasing sound

end length until a $1T$ thickness is used, as shown in Figs. 14 and 15. The section modulus of the sound end length increases only slightly with increasing chill thickness beyond $1T$ (Fig. 14 and Tables 6-8). In Figs. 14 and 15, the increase in sound end length caused by the chill is shown by the distance of separation between the SM versus L curves with and without end chills for plates and bars. The separation amounts to T in the case of plates, and E in the case of bars. Since this was caused by a chill of T or E thickness, the volume ratio of chill to metal solidified is 1.0. The predicted average temperature from Fig. 11 would then be about 550-670°C (1022-1238°F). Measurement of chill temperature gradients was reported by Myskowski, Bishop, and Pellini¹⁰ for a 2E chill on a 4 x 4 in. bar. The ratio is 2.0 since only 1E of additional metal is solidified at the sound end. For a ratio of 2.0, Fig. 22 shows average chill temperature to be about 350°C (660°F), which is within the gradient range of 500-650°F (260-343°C) shown at 1E in the 2E chill.¹⁰

The following is an example of chill temperature behavior.¹¹ A 7 x 7 in. square bar, 20 in. high, with 12 in. high risers was cast into a chill mold comprised of gray iron walls. The chill walls were 1.5, 2.5, and 4.125 in. thick and extremely surrounded by sand. Thermal analysis of the chills and casting showed temperature gradients at the time of complete solidification with the 7 x 7 in. square bar steel casting as follows:

Chill wall, inches	Ratio chill to casting volume, or SM ratio	Chill temperature, °C at solidification		Gradient, °C
		max.	min.	
1.5	0.43	801	704	97
2.5	0.71	638	482	156
4.125	0.85	566	316	250

The maximum and minimum temperatures are plotted against chill ratio in Fig. 11. The gradient is below the calculated average temperature. However, this may be due to the development of an air gap which would delay heat transfer into the chill and increase heat loss of the iron slab mold into the sand surrounding it.

Chill dimensions for chilling steel and iron castings can be surmised from the observations cited. Firstly, the thickness of a chill need not be greater than T for a plate, or E for a bar. Only if some special effect is desired is a different thickness useful. For example, less than maximum chilling effect can be obtained with a thinner chill if desired. Secondly, the area covered by the chill must be sufficient to solidify the casting volume of importance. The chill volume depends on the area covered which in turn depends on the casting volume to be solidified. A chill ratio of 1.0 is most common. Casting volume solidified can only be increased by increasing the chill area in contact with the casting and not by increasing its thickness.¹¹ The chill dimensions, then, are $T \times L_c \times W_c$, where T is plate thickness or its SM equivalent, and L_c and W_c are the casting surface area dimensions that describe the volume of metal to be solidified.

FEEDING DISTANCE

Feeding distance information is available for cast steel,² white cast iron,¹³ and ductile iron ferrous alloys,⁸ whereas very little feeding distance information is available for non-ferrous alloys. A comparison of feeding distance information for the three ferrous alloys i

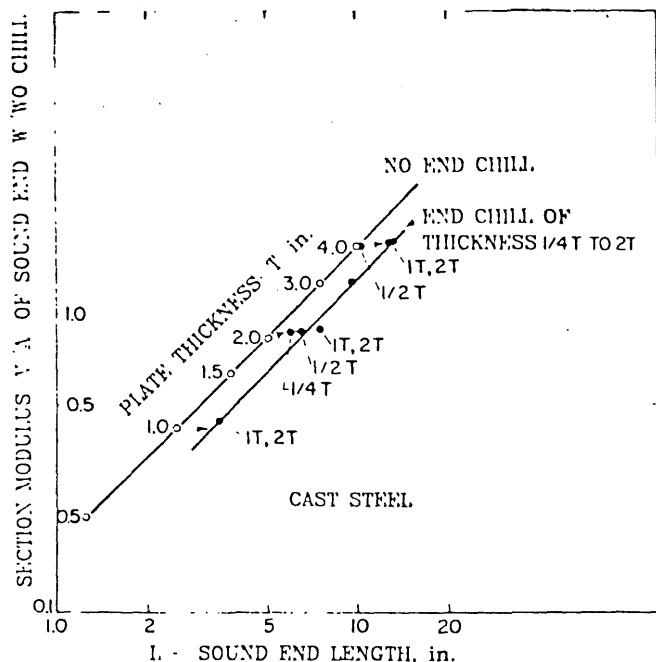


Fig. 14. Sound end length versus plate and section modulus for chills of $1/4T$ to $2T$ in thickness.

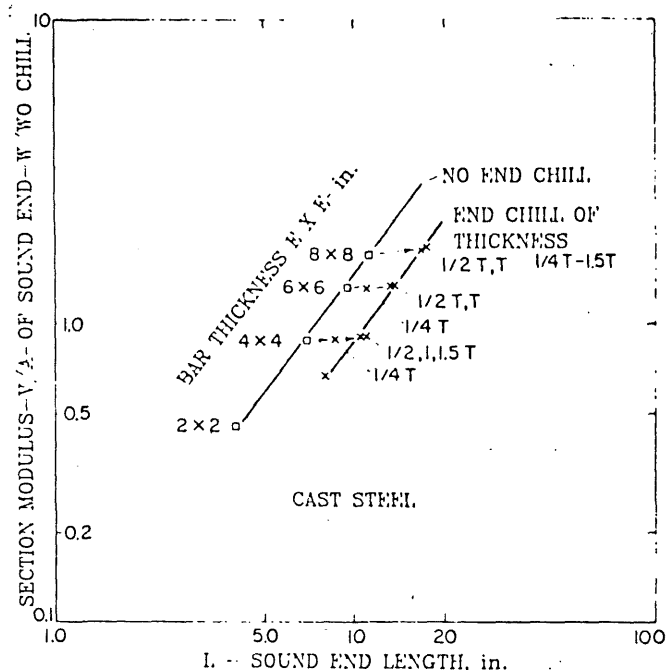


Fig. 15. Sound end length versus bar end section modulus for chills of $1/4E$ to $2E$ in thickness.

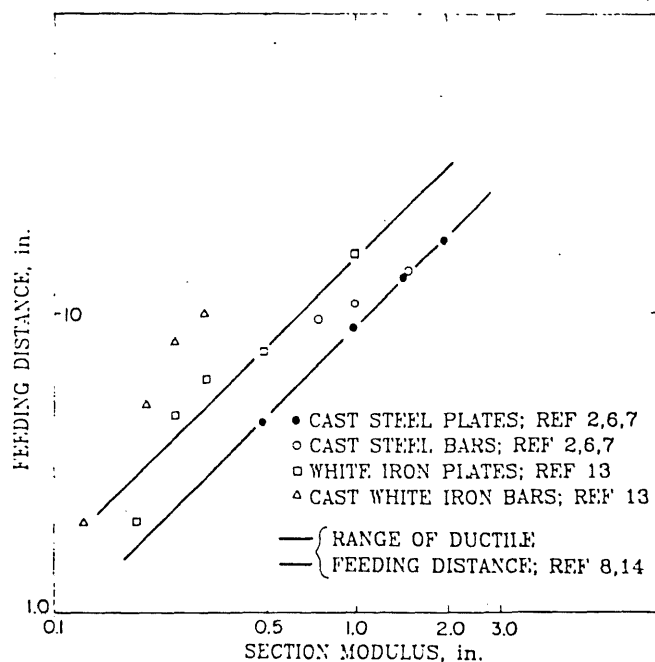


Fig. 16. Comparison of feeding distance for cast steel, white cast iron, and ductile iron.

shown in Fig. 16. The comparison in Fig. 16 of the distance fed from a riser toward the end of plates of the same SM value reveals remarkably similar feeding behavior by three ferrous alloys of greatly differing solidification behavior. If this comparison is sustained in future research it suggests that consideration of geometric features and related solidification gradients is at least as important as the influence of the alloy involved. Obviously, there are differences related to the solidification mode. However, the starting point is comparison under equal solidification gradients, a geometric thermal property feature, rather than ferrous alloy. Furthermore, risers must be properly sized and feeding paths must be open before feeding distance is actually measured. Probably the taper necessary

to extend feeding between the plate end and the riser end will be found to be more related to the alloy solidification mode than is the feeding distance, which only joins the two ends.

SOLIDIFICATION MACROSTRUCTURE

Columnar solidification macrostructure may be modeled geometrically from the section modulus wave front program. Figure 17 illustrates the wave fronts in a 4.0 in. (10.16 cm) square bar cross-section. Heat transfer from the bar will be normal to the wave fronts. Figure 18 shows the columnar structure that is predicted. The number of columnar grains can be input so that a nucleation and growth effect can be simulated. Bending of columnar grains toward convergence planes emanating from the corners is evident (Fig. 18). An example of this structure for a stainless steel bar cross-section is shown in Fig. 19, left. The macrostructure in Fig. 18 does not show regions of fine grain chill near the surface nor equiaxed grains in the center as is frequently observed. However, this can be built into the computer program and will then be revealed in the macrostructure if that feature becomes desirable. Chvorinov's rule can be used to generate the model geometrically. Figure 19, right, illustrates the macrostructure which may be expected in the casting cross-section shown in Fig. 2. The convergence planes at the section thickness change and internally are potential trouble spots with respect to hot tears, shrinkage, or stress cracks.

SUMMARY

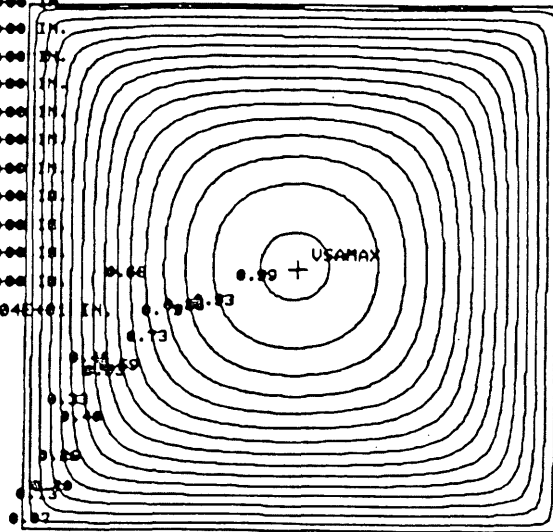
The relationship between geometric parameters and thermal parameters of materials for the purpose of geometric alloy modeling casting process variables and casting design interaction has been reviewed. Casting volume correlates directly with heat energy of the molten metal entering the mold and may be expressed in units of cal/gm (Btu/lb) or cal/cm³ (Btu/in.³). Solidification of the metal results in heat transferred over time. Chvorinov's rule relates casting volume, heat energy and solidification time to heat transfer through thermal diffusivity with units of mm²/min, $[(V/A)^2/t]$. The

INPUT NUMBER OF
WAVEFRONT LINES
DESIRED?
-->

IS
WANT SET RANGE
OF VOLUME TO
SURFACE AREA
RATIOS?
-->

N

U/SA = 0.0000E+01 IN.
U/SA = 0.1388E+00 IN.
U/SA = 0.1068E+00 IN.
U/SA = 0.2843E+00 IN.
U/SA = 0.3304E+00 IN.
U/SA = 0.3065E+00 IN.
U/SA = 0.4625E+00 IN.
U/SA = 0.5288E+00 IN.
U/SA = 0.5947E+00 IN.
U/SA = 0.6608E+00 IN.
U/SA = 0.7269E+00 IN.
U/SA = 0.7929E+00 IN.
U/SA = 0.8590E+00 IN.
U/SA = 0.9251E+00 IN.
U/SA = 0.9912E+00 IN.
USAMAX = 0.1004E+01 IN.



CAST

SAND

NORM

S-HEAT

DRAW

GRAPH

CLEAR

EXIT

STATUS: CASTING NOT MOLD NORMALIZED NO SUPERHEAT

Fig. 17. Wavefront solidification mode in 4 x 4 in. (10.16 x 10.16 cm) square cross-section.

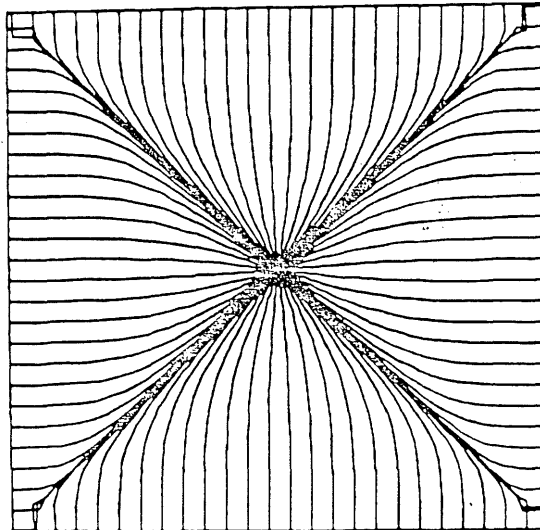
NUMBER GRADIENTS?
1-100 -->

50

INDICATE LOCATION.

NUMBER GRADIENTS?
1-100 -->

100



DRAW

PICK

S-HEAT

FEED

SHAPE

WHOLE

COMBINE

CLEAR

EXIT

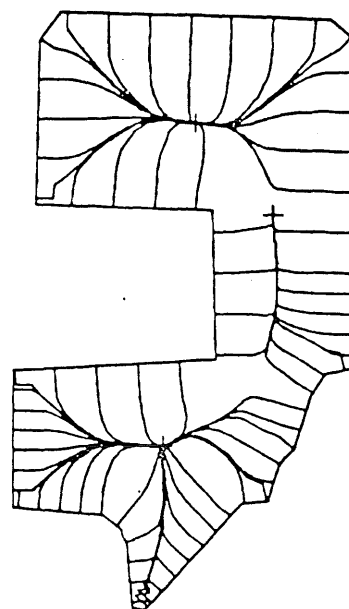
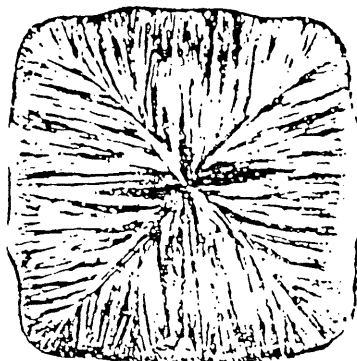
STATUS: CASTING NOT MOLD RAW NO SUPERHEAT

Fig. 18. Columnar macrostructure from Fig. 17.

NUMBER GRADIENTS? 50
1-100 ->
NUMBER GRADIENTS? 100
1-100 ->

GRADIENT VALUES

LIBR Principles of Metal Casting



CAST
MOLD
WAVES
RANGE
PICK
NORMAL
INPUT
TRANS
SCALE
S-HEAT
CLEAR
EXIT

STATUS: CASTING NOT MOLD RAW NO SUPERHEAT

Fig. 19. Left: Example of columnar macrostructure in stainless steel cross-section.¹⁵

Right: Columnar macrostructure in casting cross-section shown in Fig. 2.

mold affects the diffusivity value since it acts at the casting surface. Mold aggregate or chill ratios are essentially diffusivity factors in which silica is assigned a value of 1.0 and other materials increase or decrease the heat transfer capability according to the casting surface area. Mold aggregates and riser contacts can both be modeled by the use of the surface area ratios. The units are unchanged.

Superheat is modeled as an increase in energy input for a given casting volume. The volume is increased by an amount above the actual volume which represents the additional energy of superheat above the liquidus. The units are unchanged.

Section modulus gradients can be used to model temperature gradients. Both represent the consequences of thermal diffusivity acting over distance. Such gradients establish the sound ends on plates and bars and at risers. Failure to overlap soundness at risers and ends leads to unsoundness between them. Hence, there is a limited feeding distance. This can be offset by introducing taper between the two sound regions to extend the region of favorable SM gradient. The taper may be as much as 17% for unchilled steel plates and is applied between the sound end and the riser end.

Chills are modeled as heat sinks capable of receiving heat from the metal volume solidified. A chill volume/metal solidified ratio of 1.0 has been established as the maximum effective ratio. Chill dimensions will then be 1T (or 1E) in thickness with a length and

width of the area necessary to chill the desired volume. This may require chilling on both sides of a section.

Feeding distance in three ferrous alloys has been shown to be more dependent on plate geometry than on alloy type. This may be a consequence of SM gradients being related to geometry rather than to alloy type. Taper necessary to join end effects may be more related to alloy type. Gradients, whether SM or temperature, have the most influence on feeding—provided that risers are properly designed.

Material thermal properties, casting process variables, and design interactions can be successfully modeled through correlation with geometric parameters. These geometric parameters can also be used to model columnar solidification macrostructure by considering the path of heat transfer normal to solidification wave fronts.

ACKNOWLEDGEMENTS

Financial sponsorship of a consortium of foundries is gratefully acknowledged by the authors. The programming of J. Garcia, S. Neises, D. Gantenbein, P. Thermanson, D. Whealon, D. Schmidt, and M. Ganter has made these analyses possible. Liaison with the foundry industry committee chaired by L. S. Krueger is recognized as an important factor contributing to continued research in the line of applications.

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Experimental Analysis of Wave Fronts in Standard Casting Sections And in Finned Castings

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INTRODUCTION

Most castings are composed of a combination of simple shapes such as plates, bars, ribs, bosses, and L-, T-, V- and X-sections. It has, therefore, been common to evaluate the manufacturability of castings by examining the solidification of the various sections of the casting. A variety of techniques have been used in this respect, including numerical solutions (finite difference, finite element) to heat transfer problems¹⁻⁸ and geometric analysis of the casting modulus (volume/surface area ratio).⁹⁻¹⁵ Many of these techniques have been of great value to the operating foundryman, but others are of lesser interest since they are overly complex or economically unattractive.

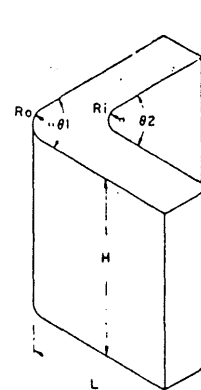
The casting designer and the foundryman need to develop a "feel" for the way a given casting solidifies and the potential a casting exhibits for shrinkage formation. The purpose of this experimental program was to contribute to this knowledge base through the analysis of a series of castings produced by decanting the liquid at various stages of solidification to determine the progress of the solidification wave fronts.¹⁶⁻²⁰ The results obtained were subsequently evaluated to enable the prediction of wave front development in other castings.²¹⁻²⁷

EXPERIMENTAL PROCEDURE

A series of castings was produced in silica sand air set molds by casting pure aluminum (99.9+%) into the mold cavity, allowing the casting to solidify for a given period of time, and then decanting the remaining liquid from the mold cavity. These experiments were conducted to study two aspects of casting geometry: corner configurations (Fig. 1) and fin configurations (Fig. 2). The results reported herein were confirmed using a number of other molten metals exhibiting different thermal properties.²⁸

Two types of corner configurations were studied: an L-corner and a V-corner. One set of L-corner geometries was composed of two intersecting (90-degree) plates 2.0-in. (5.08-cm) thick having inside and outside radii of 0.5 in. (1.27 cm) and 2.0 in. (5.08 cm). A smaller L-corner geometry 1 1/16-in. (2.70-cm) thick with sharp corners was also included. Three V-corner geometries having various angles of intersection of their 1 1/16-in. (5.00-cm) thick plates, i.e., 60, 90, and 120 degrees, were also considered. The arm lengths and heights were selected to avoid end effects at the plate intersection.

The fin configurations studied were attached to a heavy plate section 2 1/16-in. (7.5-cm) thick and were of a constant thickness (0.138 in. (0.35 cm)). The length of the fin was varied from 0.45 in. (1.15 cm) to 2.64 in. (6.7 cm). This geometry was selected since it not only offered a semi-infinite plate, but permitted solidifi-



	T	L	H	R ₀	R ₁	θ ₁	θ ₂
Large L Section	2.0	8.0	8.0	-	-	90°	90°
	2.0	8.0	8.0	0.5	0.5	90°	90°
	2.0	8.0	8.0	2.0	2.0	90°	90°
Small L Section	1.06	4.45	8.0	-	-	90°	90°
V Section	2.0	6.10	7.87	-	-	60°	60°
	2.0	8.0	7.87	-	-	90°	90°
	2.0	8.46	7.87	-	-	120°	120°

(Linear dimensions presented in inches, angles in degrees)

Fig. 1. Dimensions of the L- and V-sections used for the decantation experiments.

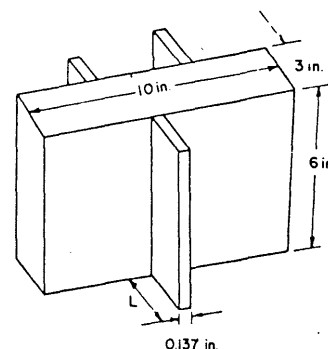


Fig. 2. Dimensions of the castings used to study the fin effect. The length (L) of the fin varied: 0.45 in. (1.15 cm), 0.67 in. (1.7 cm), 0.91 in. (2.3 cm), 1.07 in. (2.7 cm), 1.46 in. (3.7 cm), 1.85 in. (4.7 cm), and 2.24 in. (5.7 cm). The thickness of the fin was maintained at 0.137 in. (0.35 cm). Average thickness of cast fins was 0.132 in. (0.336 cm).

cation of the fins without interference from the plate. Two sets of experiments were conducted: the first used fin lengths of 0.45, 0.91, and 1.50 in. (1.15, 2.3, and 3.8 cm); and the second used fin lengths of 0.67, 1.06, 1.46, 1.85, 2.24, and 2.64 in. (1.7, 2.7, 3.7, 4.7, 5.7, and 6.7 cm).

The decantation tests were conducted by melting the pure aluminum in a medium frequency induction furnace, degassing at 1400F (760C), and pouring from the melting crucible into the molds at about 1300F (704C). A digital readout thermocouple was used to monitor temperature of the molten aluminum.

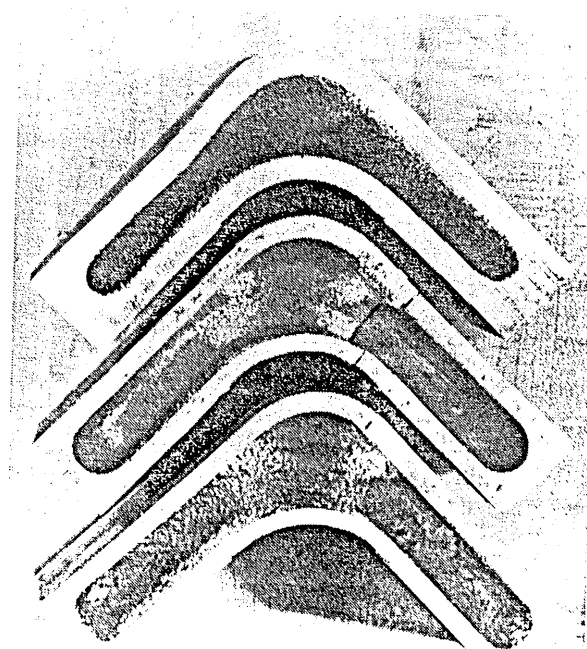


Fig. 3. Cross-section of the L-section castings produced by decantation after 160, 230, and 340 sec. (from bottom to top, respectively): a) corners at 90 degrees; b) radii (outside and inside) of 0.5 in. (1.27 cm); c) radii (outside and inside) of 2.0 in. (5.08 cm).

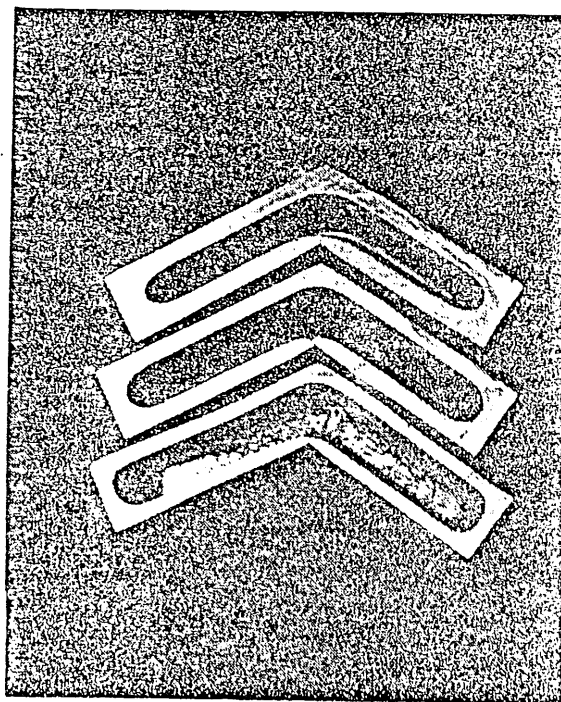
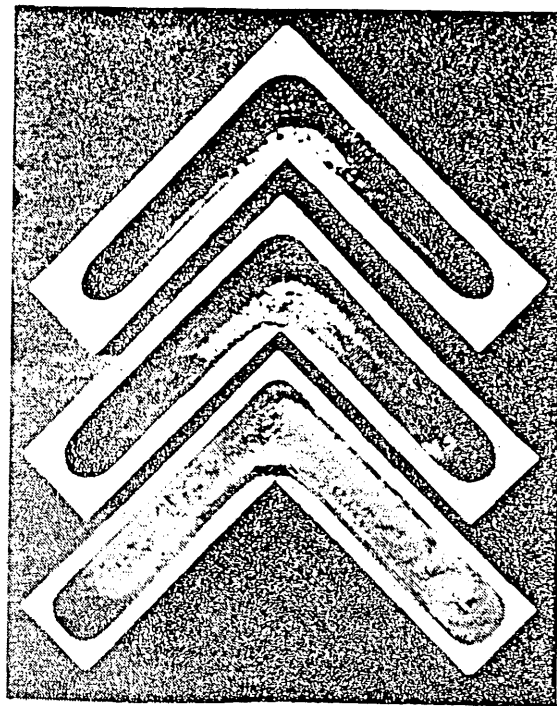
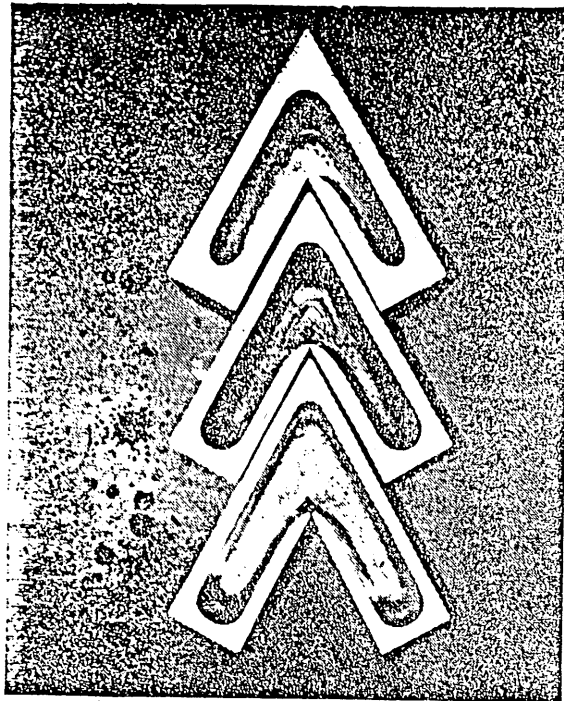


Fig. 4. Cross-sections of the V-section castings produced by decantation after 160, 230, and 340 sec. (from bottom to top, respectively): a) corners at 60 degrees; b) corners at 90 degrees; c) corners at 120 degrees.

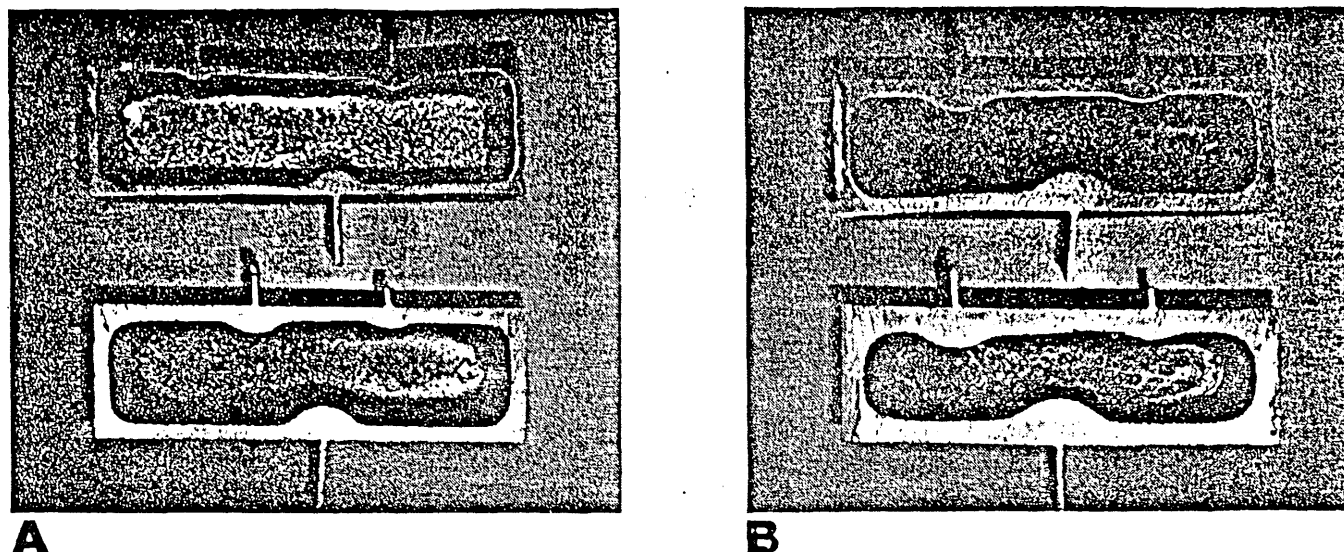


Fig. 5. Cross-sections of the fin effect castings illustrating the wave fronts established at various decantation times. The fin lengths were 0.45 in. (1.15 cm), 0.91 in. (2.3 cm), and 1.47 in. (3.7 cm) with a fin thickness of 0.138 in. (0.35 cm): a) decantation times of 90 sec. (top) and 160 sec. (bottom); b) decantation times of 230 sec. (top) and 340 sec. (bottom).

The corner configurations were decanted after solidification times of 160, 230, and 340 sec., and were sectioned for evaluation at about 4 in. (10 cm) from the bottom. The fin configurations were decanted after solidification times of 90, 160, 230, and 340 sec. and were sectioned for evaluation at about midlength.

A few of these castings were reproduced using other metals having different thermal properties in order to make comparisons of the wave front movements.

EXPERIMENTAL RESULTS

Sections of decanted L-corner geometries [2.0-in. (5.08-cm) plates] are presented in Figure 3. Three corner radii have been evaluated: a sharp radius (Fig. 3a), a 0.5-in. (1.27 cm) radius (Fig. 3b), and a 2.0-in. (5.08-cm) radius (Fig. 3c). For each of these geometries decanted sections were included after 160, 230, and 340 sec. solidification times. Similar results were obtained using other metals, e.g., leads, copper, etc., although the solidification times required to achieve a given planar casting thickness differed.

In the same manner, sections of decanted V-corner geometries are presented in Figure 4. Intersection angles of 60 degrees (Fig. 4a), 90 degrees (Fig. 4b), and 120 degrees (Fig. 4c) are shown after having solidified for 160, 230, and 340 sec.

Sections from decanted fin castings are presented in Figures 5 and 6. In the first series, Figure 5, three 0.138-in. (0.35-cm) thick fins were considered with fin lengths of 0.45 in. (1.15 cm), 0.91 in. (2.3 cm), and 1.5 in. (3.8 cm) on one casting. (The wave fronts depicted confirm that the fins exhibited independent effects over the time period considered.) Decantation of these castings occurred after solidification times of 230 and 340 sec. The significant chilling effect of the fins, and the effect of fin length, is readily apparent. The second series, Figure 6, illustrates the effect of fin lengths of 0.67 in. (1.7 cm) to 2.64 in. (6.7 cm) over solidification times of 90–340 sec.

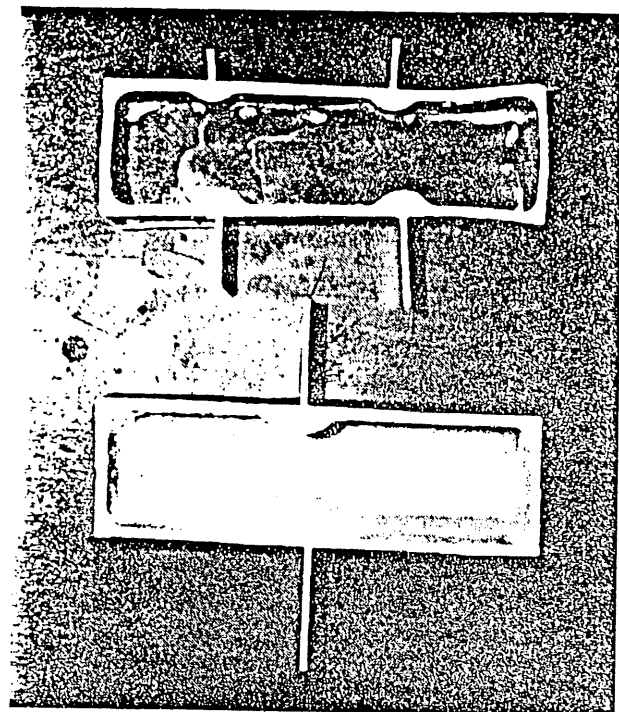
One additional series of experiments was conducted to assess the effect of a plate appendage on the solidification of a plate section. Decantation from plate castings $2\frac{1}{16}$ -in. (7.5-cm) thick to which $\frac{3}{16}$ -in. (2.46-cm) wide by $5\frac{1}{2}$ -in. (13.9-cm) long plate appendages were attached resulted in wave fronts depicted in Figure 7. This series illustrates the change from "T-section" solidification to the "fin effect" occurring as solidification progresses. It should be apparent that once the fin is solid, the heat and thermal diffusivity of the solid serves to increase the rate of cooling at this intersection.

DISCUSSION AND ANALYSIS OF RESULTS

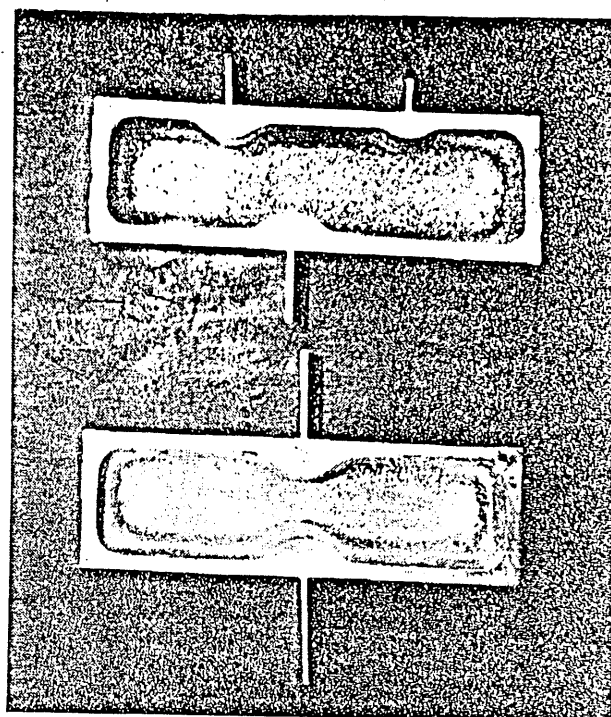
Decantation Experiments

"These decantation studies, conducted using a relatively pure aluminum (99.9+%), provide an opportunity to more clearly evaluate the effect of geometry on the solidification process. The use of pure aluminum enables a reasonably planar liquid-solid interface to be realized, clearly demarking the solidification isotherm [1220F (660C)] at the time of decantation. The relative solidification rates at corners, fins, etc., can be directly related to the wall thickness solidified at a flat surface. It should be realized that while these experiments were conducted using a rather unique metal, the results are applicable to alloys which solidify by other modes (long or short range solid solution alloys, eutectic-containing alloys, etc.) and will differ as the thermal properties of the materials differ.

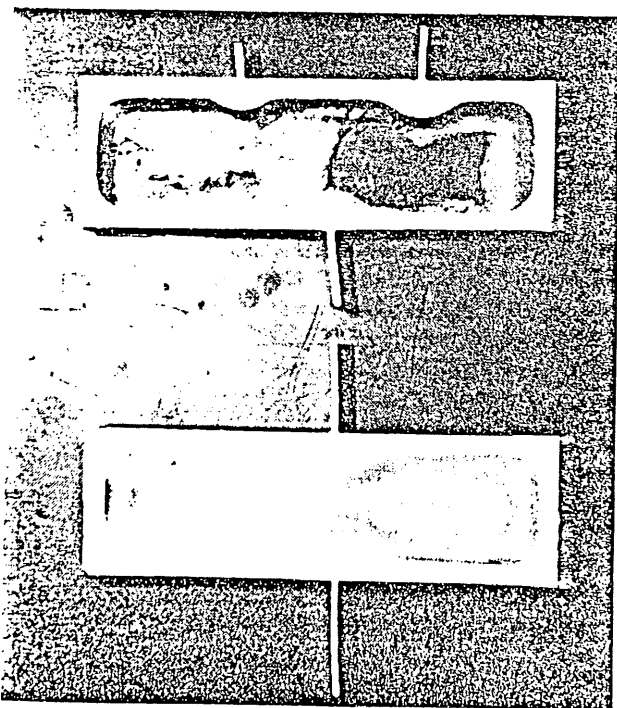
Solidification of the 90-degree L-corner with sharp corners was shown in Figure 3a. It is apparent that over the range of solidification times studied the divergent heat flow from the outside corners resulted in a thicker solid wall, and that solidification was retarded by the convergent heat flow at the inside corner. In fact, it may be noted that the sharp inside corner never completely solidified, and that a separation persists in that region.



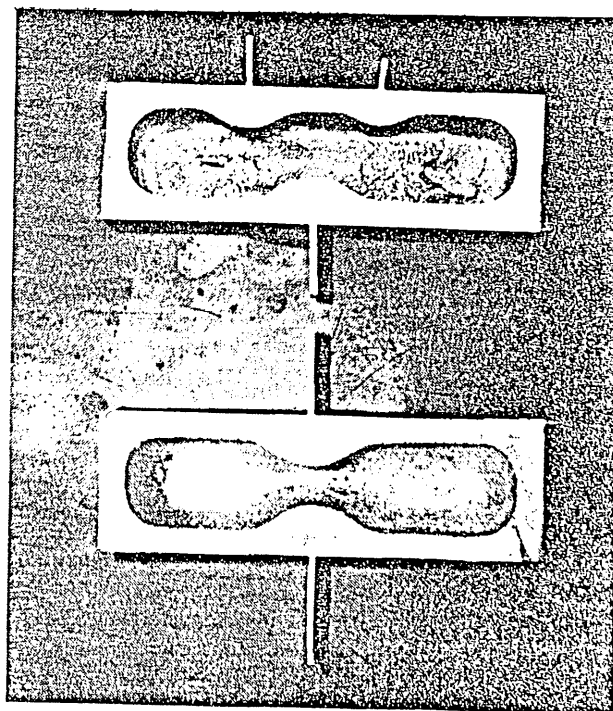
A



B



C



D

Fig. 6. Cross-sections of castings used to study the fin effect. All fins were 0.138 in. (0.35 cm) thick. Fin lengths were 0.67, 1.06, 1.46, 1.85, 2.24, and 2.64 in. (1.7, 2.7, 3.7, 4.7, 5.7, and 6.7 cm); a) casting decanted after 90 sec; b) casting decanted after 160 sec; c) casting decanted after 340 sec.

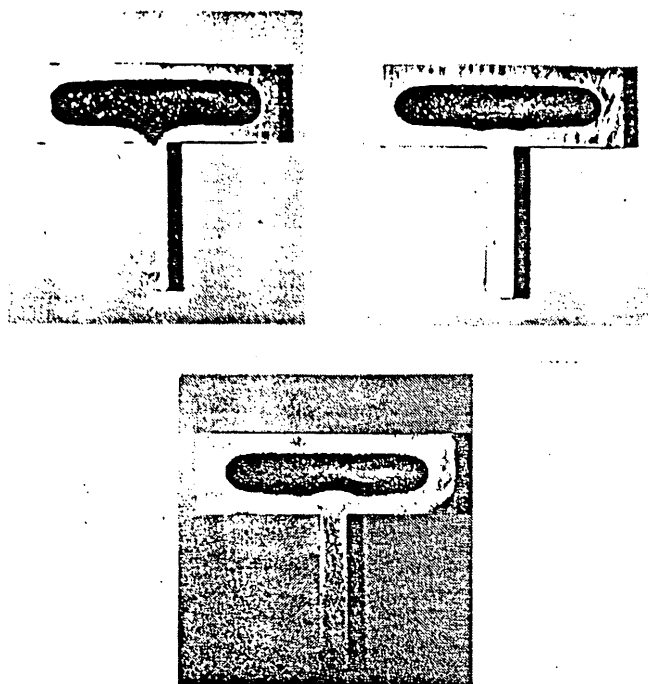


Fig. 7. Cross-sections of casting produced with thicker fin to illustrate the transition from the hot spot effect to the fin effect as the appendage solidifies. The fin appendage was 0.98-in. (2.48 cm) wide by 5.47-in. (13.9-cm) long. Dimensionless solidification times (S/T) of 0.61, 0.69, and 0.71 are illustrated in Figures a, b, and c, respectively.

This inside corner effect was lessened as the corner radius was enlarged (Figs. 3b and 3c). Note that where the corner radius was 2.0 in. (5.08 cm) the wall thickness around the corner appears essentially the same as along the flat wall. It may also be apparent that the corner effect extends to a further distance from the corner as the wall thickness becomes greater.

Sharp corners were also used in evaluating V-corner configurations (Fig. 4), with the same phenomenon occurring at the inside corner for the 60, 90, and 120 degree geometries. As would be expected, the thickness solidified at the outside corner increases as the angle of intersection decreases (accompanied by a reduced thickness solidified at the inside corner).

The decantation experiments with finned castings demonstrate the "chilling effect" of fins, noted to relate to the length of the fin (Figs. 5 and 6). The thickness solidified at a fin may be increased by over two times at the base of the fin. This effect, of course, is attributed to the rate of heat extraction afforded by the solidified fin.

Solidification of L- and V-Sections

A schematic diagram of the solidification of an L-section is presented in Figure 8. This diagram represents a typical section of an L-section with sharp corners, and may be used to further discuss the solidification isotherms (wave fronts) established in L and V-sections and typified by the cross-sections shown in Figures 3 and 4.

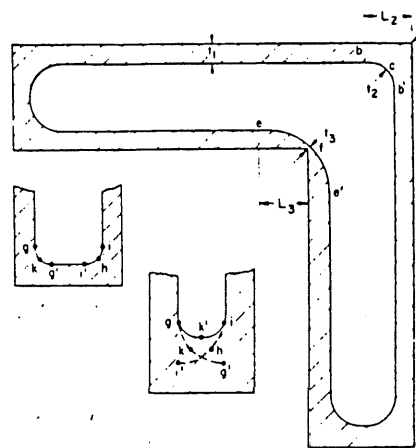


Fig. 8. Schematic representation of the solidification of an L-section casting used to describe the corner and end effects observed: t_1 = thickness of planar wall section; t_2 = thickness solidified at outside corner; t_3 = thickness solidified at inside corner; L_2 = distance over which the outside corner effect was observed; L_3 = distance over which the inside corner effect was observed. $b-c-b'$ = solidification front influenced by the outside corner. $e-f-e'$ = solidification front influenced by the inside corner. $g-k-g'$ and $i-k-i'$ = solidification fronts at the end corners.

It may be noted that the planar wall thickness, t_1 , is uniform along the flat wall section of the casting and develops a thickness proportional to the square root of the decantation time. Solidification at the external corner develops a greater thickness, t_2 , of solid at any given time than is developed at t_1 . Likewise, the thickness of solid developed at the inside corner is designated t_3 . As solidification progresses, and a greater thickness of solid occurs at the outside corner, t_2 , that effect influences an ever increasing distance of solidified metal away from the corner, L_2 being a measure of that corner effect. In a similar manner, the influence of the inside corner, t_3 , is observed over the distance L_3 .

This identical effect of the outside corner was observed at the two ends of the L-section, and a sketch of this effect is also included in Figure 8. The same shape of the solid wave front developed at each corner of the ends as developed at the outside corner of the L, and this continued to develop until the respective L_2 distance from one corner met the L_2 distance from the other corner. (For purpose of discussion, the wave front included within the L_2 distance of the L-section has been denoted as $b-c-b'$, and the equivalent effect has been labeled $g-k-g'$ and $i-k-i'$ at the end corners.) When point g' meets point i' the effect of the outside corners becomes the well-known "end effect," and the thickness solidified is subsequently greater than that expected from the effect of each corner by itself. (In a similar manner, the wave front at the inside corners, $e-f-e'$, develops with the L_3 distance.)

Relative thicknesses solidified at three solidification times for L-sections produced with three different radii (outside and inside corners) are presented in Table 1. The ratios presented here compare the outside corner to the planar casting wall (t_2/t_1), the inside corner to the planar casting wall (t_3/t_1), and the corner effect distances to the thickness solidified at the planar casting wall (L_2/t_1).

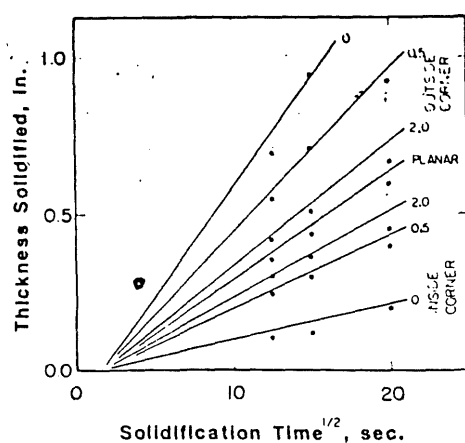


Fig. 9. Development of the solidification wave front in the L-section casting noting the effect of the corner radii.

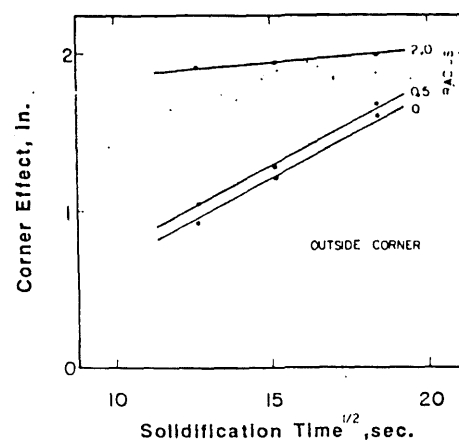


Fig. 10. Effect of the outside corner radius on the distance over which the corner effect was observed in L-section castings.

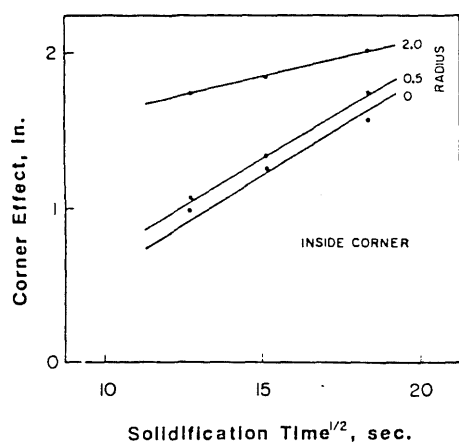


Fig. 11. Effect of the inside corner radius on the distance over which the corner effect was observed in L-section castings.

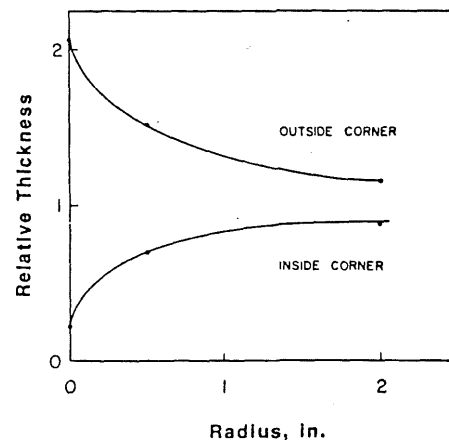


Fig. 12. Effect of increasing radius on the relative thickness (t_2/t_1 or t_3/t_1) at the outside and inside corners in L-section castings.

for the outside corner, and L_3/t_1 for the inside corner). The actual measured thickness solidified is plotted as a function of solidification time in Figure 9. Thickness solidified is approximately proportional to the square root of the solidification time:

$$x = a + b \cdot t^{(1/2)}$$

where x is the thickness solidified, t is the solidification time, and the constants a and b vary with the geometry as follows:

	a	b
Radius, outside corner: 0.0 in.	-0.740	0.200
0.5	-0.526	0.148
2.0	-0.397	0.112
Planar wall:	-0.345	0.098
Radius, inside corner: 0.0 in.	-0.253	0.078
0.5	-0.230	0.066
2.0	-0.114	0.033

The distance over which this effect was observed is plotted in Figures 10 and 11. It is apparent that the corner effect increases with solidification time for the smaller radii, and that distance over which the corner effect is experienced becomes similar for all radii at longer solidification times.

It might also be expected that the relative thickness of the casting solidified at either the outside (t_2/t_1) or inside (t_3/t_1) corner would decrease with increasing fillet radius. This effect is shown in Figure 12 where the curves sketched could be expected to be asymptotic to a ratio of 1.0. Over the range of data studied these curves may be approximated by the following equations:

$$\text{Outside radius: } t_2/t_1 = -0.49 \cdot r^{0.36} + 2.06$$

$$\text{Outside radius: } t_2/t_1 = 0.48 \cdot r^{0.22} + 0.20$$

where r is the outside or inside corner radius in cm.

The distance over which the corner effect is observed appears to be almost the same for inside and outside corners of the L-section,

Table 1. Effect of Fillet Radius on the Relative Thickness Solidified and the distance from the Corner Over Which the Effect was Observed in L-Section Castings

Fillet Radius	Solidification Times, sec.	Relative Thickness Solidified, or Distance Over Which the Corner Effect was Observed			
		Outside Corner t_2/t_1	Inside Corner t_2/t_1	Outside Corner L_2/t_1	Inside Corner L_3/t_1
0	160	1.92	2.85	0.29	2.92
	230	2.10	2.91	0.31	2.78
	340	2.16	2.74	0.33	2.87
	Av/SD	2.06/0.12	2.83/0.09	0.31/0.02	2.86/0.07
0.5 in. (1.27 cm.)	160	1.47	3.09	0.71	3.10
	230	1.60	2.97	0.67	3.04
	340	1.52	2.95	0.73	3.12
	Av/SD	1.53/0.07	3.00/0.08	0.70/0.03	3.09/0.04
2.0 in. (5.08 cm.)	160	1.20	5.70	0.90	5.19
	230	1.19	4.50	0.83	4.19
	340	1.17	3.56	0.87	4.33
	Av/SD	1.19/0.02	4.59/1.07	0.87/0.04	4.33/0.80

The average (Av) and standard deviation (SD) are noted for each fillet radius.

but increases with increasing fillet radius. These relationships may be approximated by:

$$\text{Outside radius: } \log (L_2/t_1) = 0.043*r + 0.439$$

$$\text{Inside radius: } \log (L_3/t_1) = 0.036*r + 0.452$$

Relative thicknesses solidified to sharp cornered V-sections of three different inside corner angles are presented in Table 2, while the actual measured thickness solidified is plotted as a function of solidification time in Figure 13. It is readily observed that solidification at the sharp inside corner is severely limited. The distance over which this effect occurs is plotted in Figures 14a and 14b for the inside and outside corners, respectively. The corner effect is realized over a greater distance when the corners (inside and outside) are sharper.

As with the L-sections, the thickness solidified is proportional to the square root of the solidification time at the corners and along the planar wall (Fig. 13). The constants a and b in this relationship vary with casting geometry as follows:

		a	b
Outside corner angle:	300	-0.404	0.121
	270	-0.277	0.078
	240	-0.185	0.058
Planar wall:	180	-0.122	0.036
Inside corner angle:	120	-0.048	0.012
	90	-0.020	0.005
	60	-0.015	0.004

The corner effect distance for the inside and outside corners, Figures 14a and 14b, is also a function of the square root of the solidification time and may be expressed using the following constants:

	a	b
Outside corner angle	300-0.471	0.128
	270	-0.359
	240	-0.325
Inside corner angle:	120	-0.271
	90	-0.329
	60	-0.510

Over the range of casting geometries studied, the relative thickness solidified at the corners of the V-sections (t_2/t_1 or t_3/t_1 , for

Table 2. Effect of Corner Angle on the Relative Thickness Solidified and the Distance from the Corner Over Which the Effect was Observed in V-Section Castings

Inside Corner Angle	Outside Corner Angle	Solidification Times, sec.	Relative Thickness Solidified, or Distance Over Which the Corner Effect was Observed			
			Outside Corner t_2/t_1	Inside Corner t_3/t_1	Outside Corner L_2/t_1	Inside Corner L_3/t_1
60	300	160	3.29	3.99	-	3.39
		230	3.18	4.08	0.08	3.42
		340	3.25	4.19	0.08	3.74
		Av/SD	3.24/0.06	4.09/0.10	0.00/0.00	3.52/0.19
90	270	160	1.92	2.93	0.16	3.01
		230	2.10	2.78	0.15	3.13
		340	2.10	2.83	0.15	3.03
		Av/SD	2.04/0.10	2.85/0.08	0.15/0.01	3.06/0.06
120	240	160	1.64	2.64	0.40	2.86
		230	1.56	2.25	0.34	2.67
		340	1.65	2.25	0.37	2.81
		Av/SD	1.62/0.05	2.38/0.23	0.37/0.03	2.78/0.10

The average (Av) and standard deviation (SD) are noted for each corner angle.

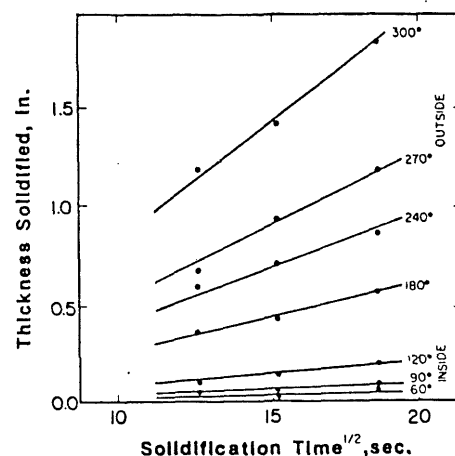


Fig. 13. Development of the solidification in V-section castings as a function of the corner angles.

the inside or outside corners) can be related to the corner angle as depicted in Figure 15. With the corner angle expressed as x and the relative thickness solidified as y , the following exponential relationship applies:

$$y = 0.0434 * \exp (0.0143x)$$

Analysis of the Fin Effect

The fin effect was illustrated in Figures 5-7 for a number of combinations of fin geometries and solidification times. Figure 16 is presented as a schematic illustration of the fin effect as illustrated in these photographs. The fin is considered to have a length L and a thickness T . The casting has developed a solid surface $a-b-f-d-e$ when the planar wall of the casting has solidified to a thickness S .

If one were to ignore the fin effect, the solidification wave from $a-b-c-d-e$ would be predicted on the basis of the application of the analysis derived from the solidification at the corners of castings (L-sections). Actually this effect would be the result of a wave front $a-b-c-b'-c'$ from one corner and $e-d-e-d'-e'$ from the other corner. The actual wave front realized is $a-b-f-d-e$ which exhibits an increased cross-sectional area denoted by Δf , an effect which can be ascribed to the presence of the fin.

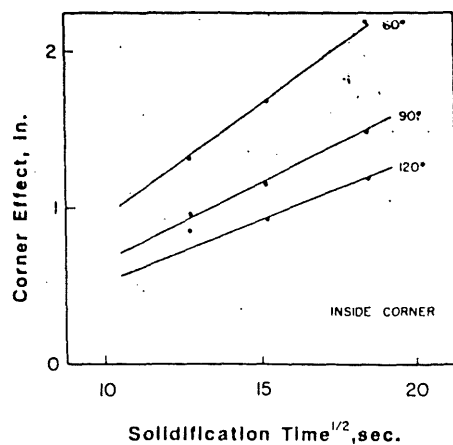


Fig. 14a. Effect of the angle of the inside corner on the distance over which the corner effect was observed in V-section castings.

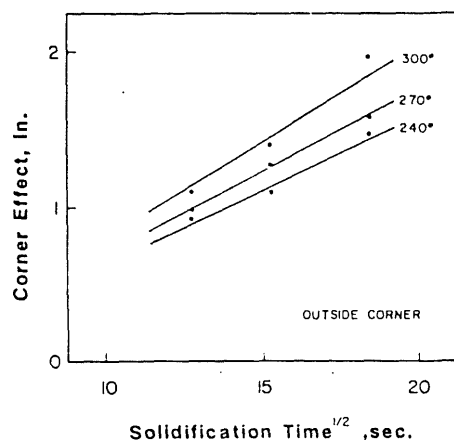


Fig. 14b. Effect of the angle of the outside corner on the distance over which the corner effect was observed in V-section castings.

This analysis also applies to the corner effect referred to in Figure 8. Solid develops at the corners and generates the surfaces g-k-g' and i-k-i' which converge when points g' and i' meet. Further solidification causes these points to cross over each other, and the thickness of solid formed at the end is greater than predicted from the corner solidification. This end effect is shown schematically in Figure 8 and has been evaluated quantitatively elsewhere.²⁸

Because solidification time is proportional to the thickness solidified, and since the fin increases the solidification rate at this junction, a dimensionless solidification time has been defined as the ratio of the planar solidification thickness S to the thickness of the fin T (actually the average thickness of the fin which was determined to be 0.132 in. (0.336 cm)). The increase in the fin effect area, Δf , as a function of this dimensionless solidification time (S/T) is illustrated in Figure 17 for a range of fin lengths. As would be expected, the fin effect area is increased with increasing fin length, and the effect is more significant as the solidification process proceeds (i.e., as the planar wall becomes thicker).

It could be anticipated that a fin effect would not be realized until the fin appendage solidified, or until the thickness solidified

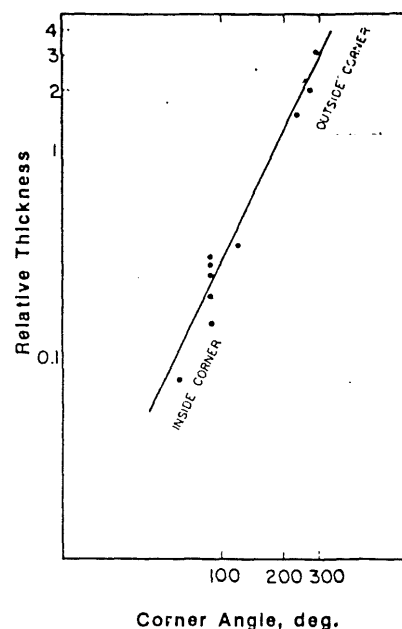


Fig. 15. Influence of the corner angle on the relative thickness (t_2/t_1 or t_3/t_1) solidified in V-section castings.

S equalled $T/2$. With a solid fin, the fin is much more effective in extracting heat from this section of the casting. If the data used to generate Figure 17 were used to obtain a "best fit" curve for each fin length, and the intercept determined for a zero fin effect area, that intercept would be a dimensionless solidification time of 0.62 not far removed from the expected value of 0.50 (half the fin thickness). This seems to support the analysis presented.

The fin effect can be generated generally to yield the following relationship which can predict the fin effect area based on the fin geometry, L and T , and the solidification time or thickness solidified, S :

$$\Delta f = 4.10 (L/T)^{0.63} (S - 0.62T)$$

While this relationship was generated on the basis of thin fins [0.132 in. (0.336 cm)], it has been successfully employed for predicting the fin effect with thicker fins.²⁸

Simulation of these wave fronts has been accomplished using the following steps:

- 1) The casting, or casting section, is sketched on a graphics screen.
- 2) Points are generated on the graphics screen along the solidification front both in the planar region of the shape (controlling points) and the region to be evaluated. (The shape of the solidification front in the region to be evaluated is determined by the geometry of that region and the extent of solidification occurring to that point, determined by the thickness solidified along a planar wall.)
- 3) The points so determined are used to generate an equation characterizing that geometry (cubic spline functions and computer graphics are used for this purpose).^{27,28}

Using this procedure, the solidification wave fronts developed during the solidification of a variety of T-section geometries can be predicted.²⁸ Typical results of this type of analysis are presented in Figure 18. In this series of computer sketches, the length of

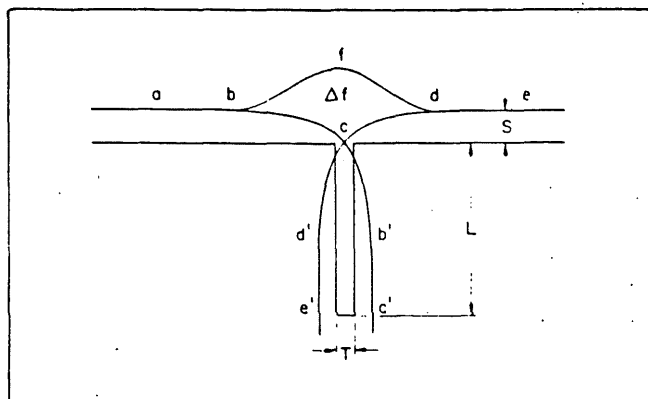


Fig. 16. Schematic representation of the development of the fin effect during solidification. S denotes the thickness solidified along the planar wall, while the surface $a-b-f-d-e$ depicts the solidification front formed as influenced by the fin. The surfaces $a-b-c-b'-c'$ and $e-d-c-d'-e'$ denote the wave front effect attributed to the corner geometry on each side of the fin. The difference between these two wave front surfaces, Δf , is attributed to the fin effect, and has been termed the fin effect area.

fins of 0.38-in. (1.0-cm) thickness has been varied to depict the change in the solidification wave front pattern. The relative shape of the wave fronts may be compared to the casting sections illustrated in Figures 5-7.

This analysis of the fin effect is restricted somewhat by the fact that that experimental data was obtained using pure aluminum, and the thermal properties of aluminum may differ appreciably from those of a given alloy in question. As a result, the magnitude of a given fin effect geometry in steel or cast iron may be significantly less than that obtained from aluminum, although the general effect will be the same and the principles by which the fin effect is realized will be similar.

Wave Front Analysis Applied to Casting Production

The most apparent application of solidification wave front analysis is to the determination of the shrinkage potential in a given casting design. Most castings are composed of planar surfaces, inside and outside corners, and fins or appendages; the features which have been considered in this work. The wave front analysis will predict one aspect of the potential development of shrinkage in a given casting. If the casting contains isolated regions generated by the wave front, these regions will not be capable of being fed by a source of feed metal and will generate shrinkage. (A design that does not contain isolated unfed regions is not necessarily assured of being sound, since the casting must also solidify with a temperature gradient capable of insuring the supply of feed metal to the solidifying front.)

A simple geometry is illustrated in Figure 19. In this case the casting section is a square with a square cored hole at the center. (It is assumed that the core does not become saturated with heat, i.e., that the core is able to extract heat from the solidifying metal in the same manner as the mold.) The model based on the solidification of L- and V-sections, and the analysis of the end effect, was used to generate the computer sketches shown in this figure.

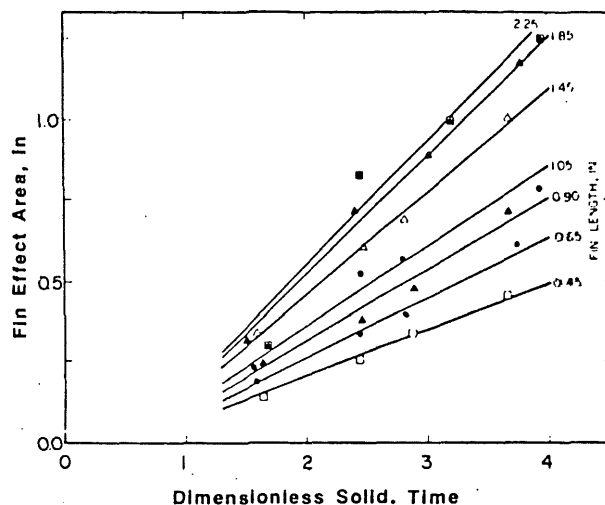


Fig. 17. Development of the fin effect as a function of the fin length. The dimensionless solidification time is defined as the ratio of the planar wall thickness solidified to the thickness of the fin.

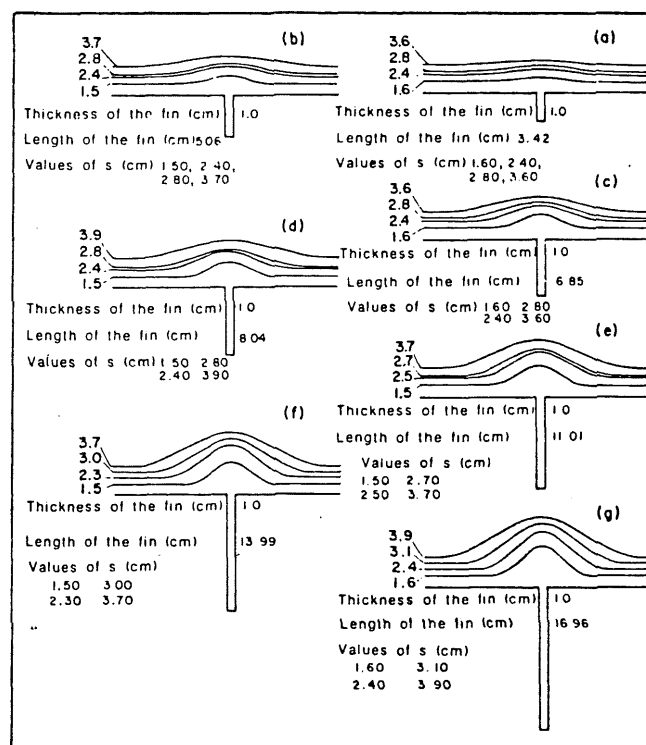


Fig. 18. Calculation of solidification wave fronts as affected by various lengths of fin appendages.

As the wave front progresses it is observed that isolated liquid regions are formed in the "thermal center" of the section, and that this geometry represents potential shrinkage problems.

A more characteristic casting geometry is illustrated in Figure 20. This gear blank casting is composed essentially of the H-shaped cross-section rotated around a central axis. For purposes of this analysis it may be assumed that the casting is to be produced with risers attached to the outer rim, and that it is desired to feed the casting hub through the rim and web sections. A model of this geometry (using blind riser, with no provision shown for insulating

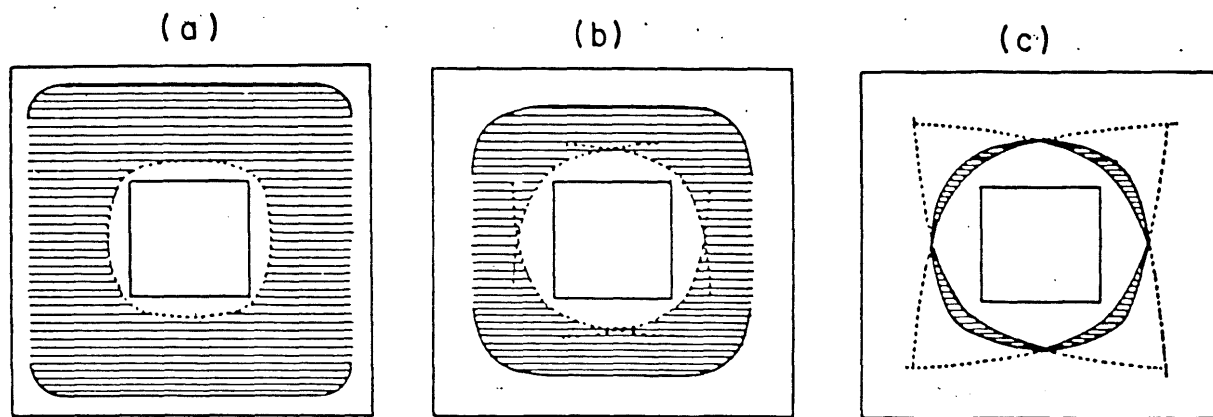


Fig. 19. Solidification profiles in square cross-section of casting with square cross-section core. The regions denoted with "+" represent locations where unfed liquid regions solidify (potential shrinkage areas).

the top or inducing pipe formation, etc.) is presented in Figure 21. The series of sketches illustrates the development of the wave fronts, and the isolation of a liquid region at the hub (Fig. 21c) while the riser effectively supplies feed metal to the web and rim. An additional, though smaller, isolated liquid region develops at the web-rim junction just prior to solidification of the casting and attached riser (Fig. 21d). This model, then, predicts that significant shrinkage will form at the hub-web junction and that limited shrinkage may develop at the web-rim junction. (The effect of other variables such as temperature gradients, flow through heating from the gating system employed, etc. has not been included in this model, but would have a direct impact on the production of this casting.)

While casting geometries may be quite complex, an evaluation of the effect of corner geometry, end effects and the effect of appendages may be used to evaluate the manufacturability of given casting sections.

SUMMARY

Using the decantation technique to document the solidification profiles developed for a number of casting section geometries, the solidification wave fronts generated at corners, as a result of the end effect, and due to the presence of fins of castings have been discussed. Using relatively pure metals to conduct these decantation experiments has enabled the wave fronts to be clearly discerned, since the solidification temperature was essentially a fixed temperature.

A review of this data illustrates the relative effect of the corner angle, the fillet radius, and the inside and outside corners on the development of solid at these locations compared to that developed along a planar casting wall. These effects apply throughout the casting until the developing corner wave fronts "overlap," thereby generating the well-known end or fin effect. A technique has been considered to quantify this end or fin effect, and used to determine the effect of fin geometry on the solidification which results.

Of greatest significance is for the foundryman to gain a "feel" for the geometric factors affecting the solidification of castings. The geometry of corners influences the development of shrinkage at these locations, as does the end effect. The fin effect is known to induce more rapid solidification in these regions but is not usually employed with a detailed knowledge of its effectiveness.

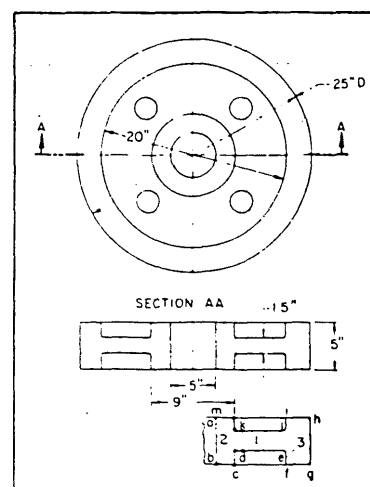


Fig. 20. Gear blank casting used to evaluate solidification and shrinkage potential.

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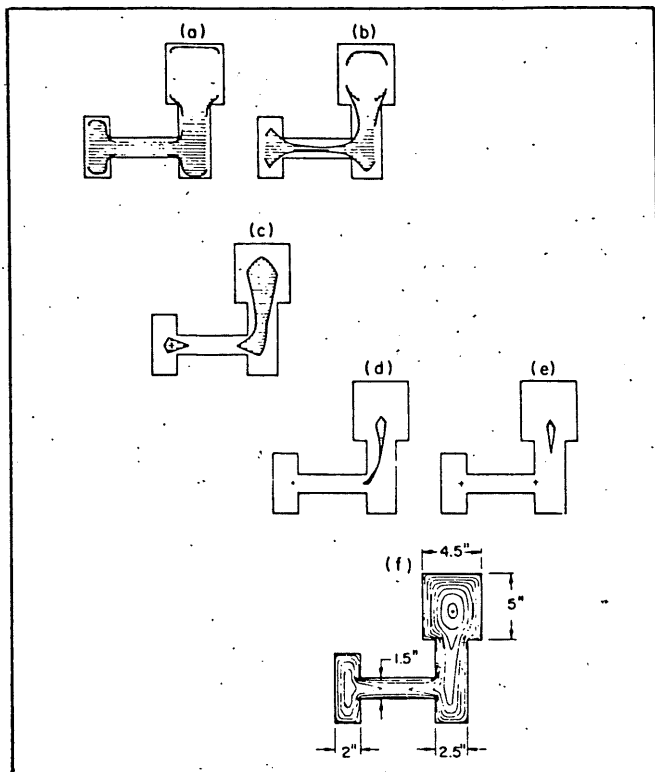


Fig. 21. Solidification profiles determined on top risered cross-section of the gear blank casting shown in Figure 20. Regions of liquid and solid are shown at various solidification stages, with the areas of potential shrinkage denoted with an "+".

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DISCUSSIONS

Question: How did turbulence in pouring affect dimensions of the test specimen? (G. Gooden, Pres., Bentoven, Caracas, Venezuela)

Answer: The authors are somewhat uncertain as to the intent of this question and therefore apologize for rephrasing it to state: "How did turbulence in pouring affect the measured thickness solidified?" Insofar as turbulence tends to eliminate any nonuniformity in temperature within the molten metal in the mold cavity, it is advantageous in obtaining reproducible wave fronts. However, where the pouring process deposits molten metal of varying superheat in different segments of the mold cavity, the onset of solidification may be affected and the plots of thickness solidified vs. square root of time will be moved to the left as the superheat is diminished.

Question: How does the metal flow wave front react in the lost foam process? (W. Brighton, Tech. Adv., Cummins Engine Co., Inc., Columbus, IN)

Answer: Substantial mold cavity turbulence occurs during the filling of the mold cavity where evaporated polystyrene patterns are used. However, solidification after mold filling is complete should be similar to conventional processes. The higher density of the unbonded sand mold will affect heat and thermal diffusivity, but the general nature of the wave fronts will be similar.

Question: 1) The plots of solidification thickness vs. square root of time would be linear (as shown) and also pass through the origin, if the metal were cast at the solidification temperature. Was the displacement of the intersection of these lines to the right of the origin caused by superheat of the cast metal? If so, what was the superheat of the metals studied? (If not, why?)

2) Would you comment on application of this experimental technique to long range freezing of alloys? (R. D. Pehlke, Prof., University of Michigan, Dept. of Met. & Mat. Eng., Ann Arbor, MI)

Answer: 1) Professor Pehlke is correct in his observation that the plots of solidification thickness vs. square root of solidification time are displaced due to the need to dissipate superheat. Temperatures at which pouring was conducted are noted in the written procedure.

2) The technique described should be applicable to all alloys, even though data was obtained using pure aluminum. Pure aluminum enabled the accurate determination of wave fronts (isotherms) as a function of time. Even relatively small amounts of contamination resulted in an iron planar casting wall (i.e., a dendritic interface) which could not be dependably decanted nor evaluated to determine the thickness where fully solidified. However, the same information (isothermal wave fronts) is of value when assessing the solidification of long freezing range alloys or alloys containing eutectics. The evaluator is concerned with establishing the isotherm for 100% solid and relating that to the geometry of the casting. Feed metal unit is supplied to that 100% solid interface to avoid the potential of shrinkage. It is easier to provide this feed metal if the solid plus liquid zone is narrow (short dendritic), so that the ability to provide feed metal may also be assessed through the wave front analysis.

Question: How do you calculate the thickness or movement of the wave fronts? (R. C. Creese, Assoc. Prof., West Virginia University, Morgantown, WV)

Answer: Initially, the wave fronts were determined in test castings where, after a given solidification time, the thickness of solid formed at various points in the casting was measured. Similar data was obtained at other solidification times so that the thickness solidified vs. solidification time could be determined for a number of geometric conditions. This information was then used to predict the thickness solidified in a nonplanar geometry as a formation of the thickness solidified at a planar wall (wave front).

An Easy and Inexpensive Technique to Study the Solidification of Castings in Three Dimensions

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INTRODUCTION

Rigging and gating of castings has a direct bearing on casting manufacturing costs. Both the overall quality of the casting produced and the casting yield (i. e., the ratio of pounds of metal poured to pounds of salable casting(s)) depends on the success of these systems. Furthermore, much of the engineering of these systems is still based largely on experience and trial and error. Thus, the costs to the foundry industry resulting from low yield castings and scrap related to rigging and gating are extremely high. This is particularly true of job shop operations in which once a successful casting is produced low production does not allow for yield improvement experimentation. Because of the trial and error nature of much of this work, the time required to produce a successful rigging and gating system can often lead to customer shipment delays adding even further to the costs.

If the tremendous costs associated with rigging and gating are ever to be reduced the answer must come from the application of engineering principles to the sizing and placement of risers and gating system components. Great strides have been made toward this end in the last few decades but one key ingredient has always been missing. Techniques are available to calculate the diameter and height of risers to ensure their proper function¹⁻³ and methods which calculate the dimensions of runners, gates and sprues are available.⁴ Although these techniques can be laborious, the application of them can be made quite simple through the use of computers, as shown by Kotschi and Kleist.⁵ However, although the dimensions of risers and gating system components can be calculated, the placement of risers, gates, runners and sprue(s) on pattern equipment remains a process of trial and error experimentation and/or hard-won experience. The reason for this unfortunate circumstance is the lack of methods to determine the manner in which three-dimensional shapes solidify. Were this key ingredient available, rigging and gating could be done to a great extent on a computer. A successful system should evaluate the effects of chills, padding, insulating and exothermic feeding aids as well as the rigging and gating system. Since such a system would allow all this to be done prior to the existence of pattern equipment, many manufacturing methods could be evaluated without the need for experimental castings. Provided the computer costs for the solidification studies do not become excessive, the cost savings potential of this approach to foundry engineering is enormous.

LITERATURE REVIEW

The recognized importance of understanding the manner in which three-dimensional shapes solidify has led to a great deal of work directed toward this problem. The early work of

Chvorinov⁶ showed a relationship between the casting modulus (V/SA or the volume of the casting/surface area) and the solidification time. This was an important beginning to understanding this complex problem.

Pehlke, in a series of articles,⁷⁻⁹ applied classical heat transfer to the problem. This approach provided for more sophistication than the simple relation developed by Chvorinov. However, this approach requires knowledge of the thermal diffusivity, density, and other physical and mechanical properties of all the components comprising each case, i. e. the metal, the mold, chills, feeding aids, etc. Since many of these parameters are often unknown or difficult to determine, the application of classical heat transfer to the widely-varied combinations encountered throughout the foundry industry will require a great deal of further research. Furthermore, the mathematics encountered in the use of classical heat transfer applied to three-dimensional solidification are rigorous. Thus, the use of this technique, even when computers are used, has to date only been applied to very simple shapes. Although at some point in the distant future classical heat transfer may provide the best final solution to the three-dimensional solidification problem, for the present it is impractical in all but those cases involving fairly simple geometries. Furthermore, without considerable improvements in computers or a significant reduction in the rigor of the mathematics, the application of classical heat transfer to the problem facing the foundryman may not be economically justifiable. It must be remembered that to tap the economic benefits of three-dimensional solidification analysis the computer costs must be kept low. Thus, simple models and non-rigorous mathematical techniques must be sought.

Because of the complex problems of solving three-dimensional heat transfer by the classical heat transfer approach, the emphasis of work has shifted to the use of some relatively new approaches known as the FEM (Finite Element Methods) and FDM (Finite Difference Methods) by such authors as Myers.¹⁰ These techniques can be used to provide approximated solutions to the three-dimensional solidification problem through the use of a computer. The true importance of these techniques stems from the fact that by using a finite number of points and by making a number of simplifying assumptions, the techniques can produce solutions of practical engineering importance. Unfortunately, these techniques are not a panacea. The problem once again is cost. The cost for a single three-dimensional solution will usually be in the range of a few hundred dollars for computer usage. Initially such computer costs do not seem overly high. But the most useful way for foundrymen and design engineers to use three-dimensional solidification analysis will involve the use of such computer software a number of times before the final decisions on gating, rigging, or part geometry are made. This is because various designs or manufacturing methods will need to be evaluated based on the knowledge gained from previous runs of the computer software, i. e. if a problem is spotted on one run by the foundry or design engineer, he will want to evaluate the possibility of perhaps using a chill or improved placement of a riser on the next run. Furthermore, yield studies require a number of manufacturing methods to be evaluated from which the lowest cost technique can be selected. When viewed in this way, it is to be expected that ten runs of the software for each casting or casting design might be common. Thus, the few hundreds of dollars for a single analysis by the FEM or FDM techniques when viewed in this way represent thousands of dollars for each casting studied. To those foundries involved in high production such costs will provide economic payback. However, for shorter run jobbing-type foundries such costs will in general outweigh the potential savings. Thus, there still exists a need for a three-dimensional solidification analysis which is less costly than either the FEM, FDM or classical heat transfer techniques. This is particularly true if such analysis is to be of

real value to the shorter run and jobbing portions of the foundry industry. Furthermore, if a less expensive technique were available it could be used as a roughing procedure in conjunction with the more sophisticated FEM, FDM or classical systems to reduce the costs of using these systems.

Because of its inherent simplicity it seems reasonable to reconsider the relationship developed by Chvorinov.⁶ Based on this general approach, Heine¹¹ and Kotschi and Loper¹²⁻¹⁴ in a later series of articles, developed a relationship between the dimensions of casting sections and the solidification sequence. This information is quite useful but it requires that complex castings be mentally separated into a set of simple casting components such as "T", "X", "L" sections, etc. Since this work was based on casting modulus (V/SA), the results are dependent only on the geometry of the sections involved. This approach thus does not require that all the physical and heat transfer properties of the mold components and metal be known in advance. Because the heat transfer and physical properties are neglected in this work, a certain amount of lost precision must be expected.

Although the casting modulus approach is somewhat primitive, many of the benefits formerly only associated with either classical heat transfer, or FEM or FDM techniques, could be realized by using the approach discussed by Kotschi and Loper.¹⁴ As discussed by these authors the casting modulus is composed of two terms: the volume and the surface area. By considering these two terms in the broadest sense, the volume term can be equated to heat content and the surface area can be related to the rate of heat extraction. When viewed in this way, differences in thermal diffusivity between a chill and the molding sand could be accommodated by a multiplying factor to the surface area term of the casting modulus. Similarly, if the

effects of an exothermic material were to be studied, an appropriate multiplying factor could be applied to the volume term of the casting modulus. In this way many of the complex relationships such as the effects of exothermic and/or insulating material, the effects of convergent and divergent heat flow, the effects of chills, and other heat transfer effects can be accommodated within the framework of the casting modulus approach. Although the casting modulus approach lacks much of the rigor of the classical heat transfer methods it could well provide a basis for an adequate engineering approach to the solution of the three-dimensional heat transfer problem.

DISCUSSION

In order to obtain a low cost method of evaluating three-dimensional solidification, a simpler mathematical approach must be sought. If computer processing, which translates into computer costs, is to be reduced, a method must be employed which contains fewer and less complex mathematical relationships. This can be accomplished by the application of casting modulus principles in a special form. This analog can also be made to account for variations in thermal diffusivity between mold media; to consider the effects of exothermic and/or insulating materials; to take into account the effects of convergent and divergent heat flow; and many other benefits formally only attributed to the use of classical heat transfer, FEM, or FDM techniques.

With the loss of mathematical rigor, obviously there is some information available with the other techniques which the simplified analog cannot provide. Since heat transfer rates are not used in the method, the actual solidification time for a portion or the entire casting cannot be determined using this

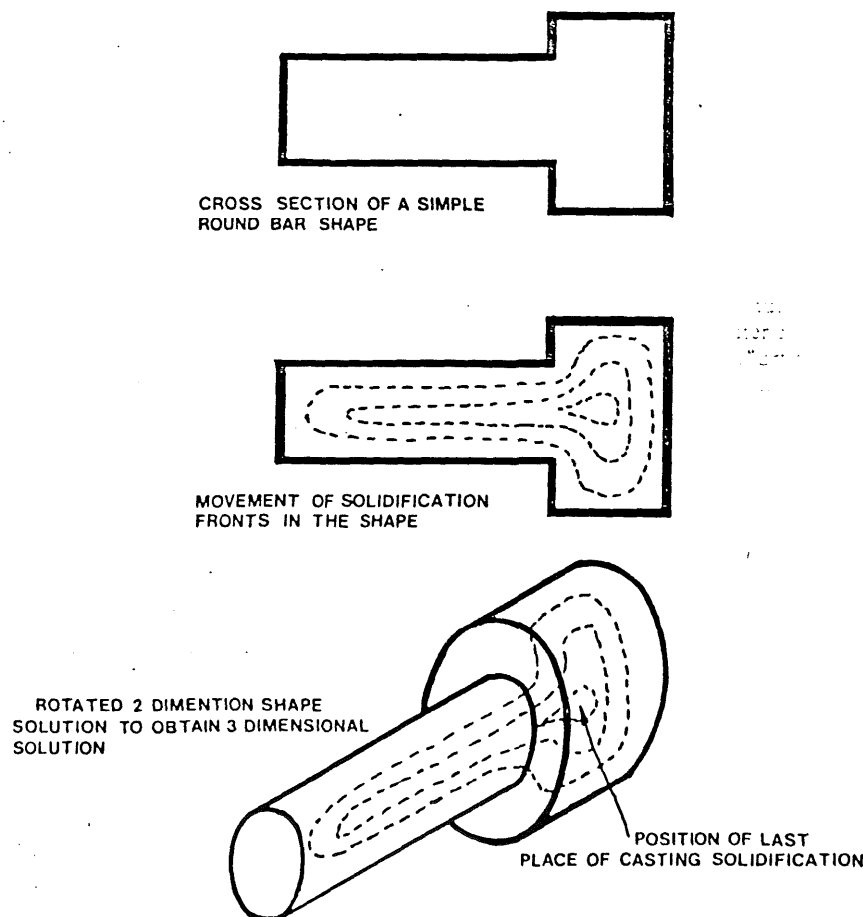


Fig. 1. The application of a 2-dimensional solution to a 3-dimensional solution by rotation of the 2-dimensional results.

simplified method. This may at first seem a great loss jeopardizing the use of the technique. However, in most applications required by either foundry or design engineers the time required for solidification is relatively unimportant provided the solidification sequence of the various casting sections can be obtained. If the solidification sequence can be determined, the actual time required for each event to take place is of little value, its only purpose then being its ability to predict the time required between pouring and shakeout.

The problem of solidification sequencing of a three-dimensional shape can be somewhat simplified by recognizing that once a two-dimensional slice of a three-dimensional shape can be properly sequenced, the three-dimensional sequence can also be determined. Consider the bar-shaped casting shown in Fig. 1. By determining the solidification sequence of the two-dimensional slice passing through the center line, the three-dimensional solution can then be obtained simply by rotation. Geometric symmetry, however, is not essential. Consider the casting shown in Fig. 2. Although this casting is symmetric about its vertical center, the solidification sequence of the three-

dimensional shape can be determined through the use of two-dimensional slices independent of the symmetry, provided slicing of the shape is done properly.

Consider the infinite plate section shown in Fig. 3. It can easily be shown that the overall casting modulus for this section is $T/2$. Casting modulus = casting volume/surface area, or $V/A = LWT/2LW = T/2$.

According to Chvorinov the value of the casting modulus is proportional to the solidification time. Thus, if the proportionality factors were equal within a given casting, the casting modulus could be used to determine the solidification sequence of various portions of the casting. This is basically the manner in which the casting modulus has been used in the past. By using it in a slightly different context it can provide a far more useful method of determining the solidification sequence. Again consider the infinite plate section shown in Fig. 3. From this model a generalized equation can be derived which gives the casting modulus at various depths from the casting surface to the center of the plate as follows:

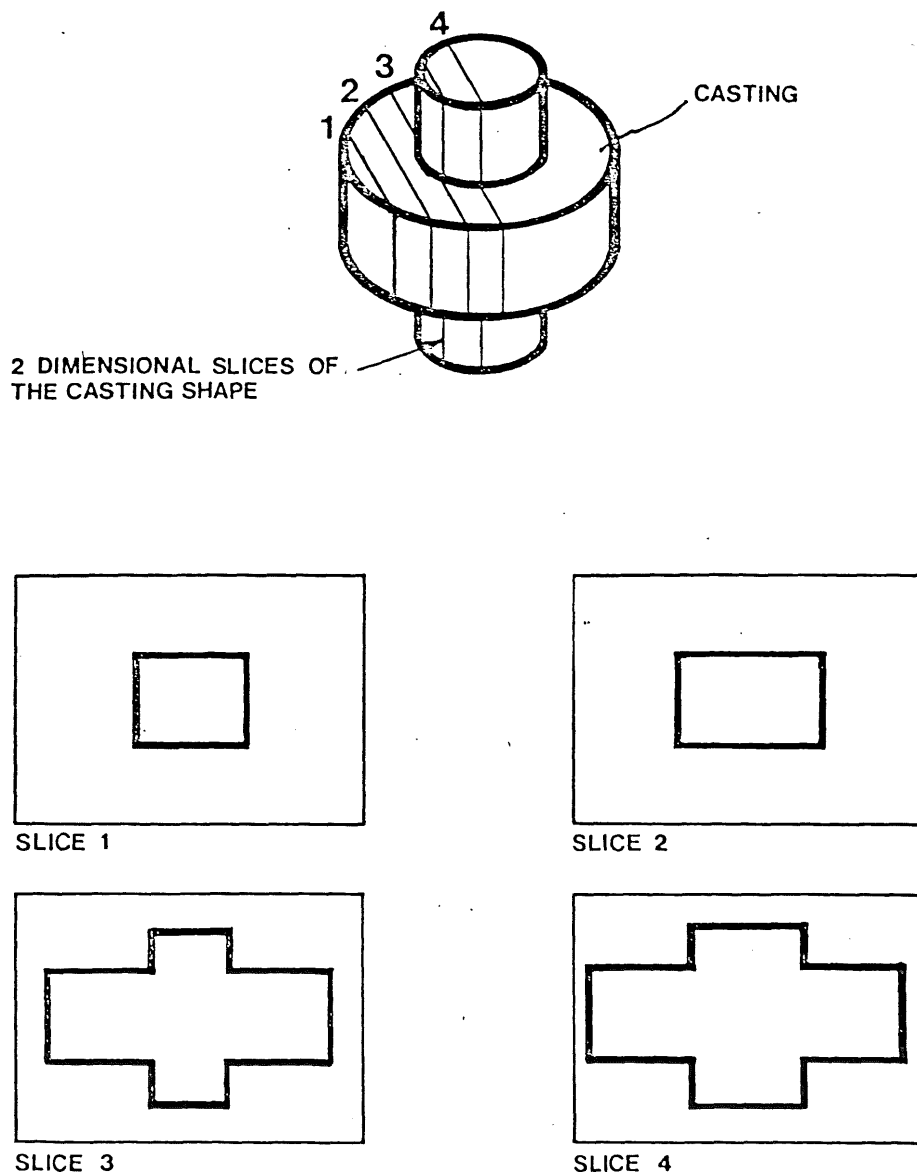


Fig. 2. The slicing technique used to simulate 3-dimensional shapes as 2-dimensional slices.

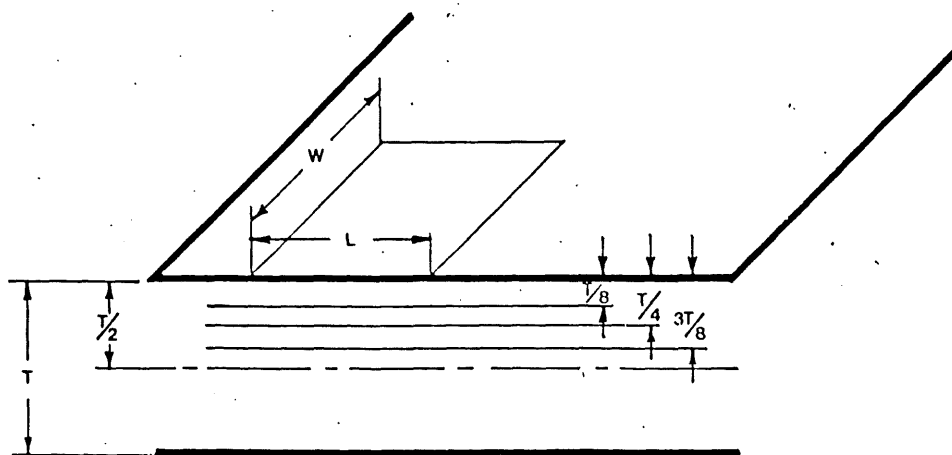


Fig. 3. Infinite plate section.

$$V/A = LW\Delta T, LW = \Delta T$$

(ΔT = distance from the plate surface to the depth of interest)

Note that this expression predicts that there exist isocasting modulus lines or contours of equivalent V/A which extend through the plate in three dimensions. Further, at both surfaces of the plate the iso V/A contour lines or K-contour lines have a value of 0 and at the plate center line the contour takes on a value of $T/2$. Since the K-contour lines are proportional to the solidification time based on Chvorinov's work, they must also predict the relative movement of the solidification fronts through the shape. For the example shown in Fig. 3 this approach predicts that solidification would begin at the plates' surfaces where $V/A = 0$ and proceed toward the center line of the plate where $V/A = T/2$. Thus, the known solidification behavior for infinite plates is predicted by the K-contour technique. Note that although the time at which the solidification front passes any given plate thickness is unknown, the progression of the solid front is predicted by this method.

As a result of the section of three-dimensional shapes, a series of dimension slices are produced. Since the method proposed requires the graphical representation of resulting two-dimensional slices, before proceeding with the development of the algorithm for corners, the various types of two-dimensional geometry potentially encountered must be considered. Based on the current practices of drafting, it seems likely that regardless of the shape of the two-dimensional slices they could be represented by a series of straight lines and portions of circles. This simple system should prove to be adequate with the major loss of rigor being in those portions of shapes usually drawn with French curves and like devices. Since these areas could be approximated with various chords of circles of varying radii this should cause little difficulty. However, for the purpose of application of the K-contour technique, the types of corners encountered in two-dimensional sections are important. The corners which develop can be categorized into either internal or external corners, as shown in Fig. 4. External corners are those in which the metal occupies the space whose angle of juncture is less than 180° and internal corners are those in which the metal occupies the space whose angle of juncture is greater than 180° . Each type of corner, i. e. internal and external, can then either be acute, obtuse, or square depending on the form of the juncture angle. Finally, the corner may also contain a radius of variable sizes. Various types of corners with various radii are shown in Fig. 4. By using the proposed system, the geometry of the two-dimensional slices can be developed merely by using a series of straight lines and radii and the types of corners encountered will either be internal or external.

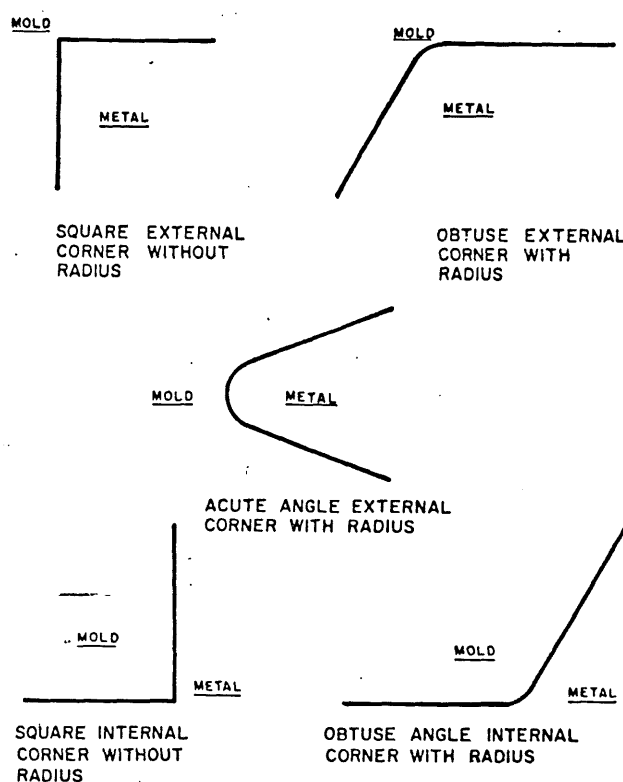
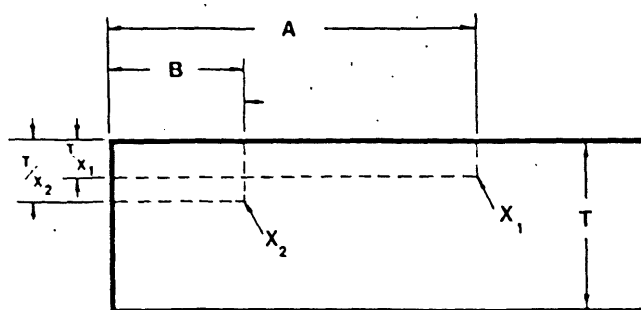


Fig. 4. Types of corners encountered in 2-dimensional sectioning.

The application of the K-contour technique to flat plates was discussed earlier. To accommodate the potential two-dimensional shapes, two other cases, i. e. internal and external corners, must be considered. It is important to distinguish between internal and external corners from a heat transfer standpoint, since they represent two different conditions. External corners represent those cases in which the heat absorptive volume is less than the heat content volume and thus involves convergent heat transfer. If the K-contour technique is to be of engineering value, it must show a similar behavior to the actual solidification behavior associated with either of these two cases.



Determination of X_1 on the $T/10$ Contour

$$\frac{V}{SA} = \frac{T}{10} = \frac{A(T/X_1)}{A + (T/X_1)} \quad \text{Thus} \quad X_1 = 10 - \frac{T}{A}$$

$$\text{For } X_2 \quad X_2 = 10 - \frac{T}{B}$$

Fig. 5. A semi-infinite plate showing the model for K-contour development for external corners.

EXTERNAL CORNERS

The semi-infinite plate shown in Fig. 5 is composed of two square external corners without radii. Since the K-contours will vary from 0 at the surface to $T/2$ at the center line of the plate, a contour equal to $T/10$ must lie somewhere between the surface and center line of the plate. At some distance A from the end of the plate there exists a point X which lies on the $T/10$ contour. The distance from the surface of the plate to X_1 equals T/X_1 .

Thus, the surface area for the two-dimensional slice at X_1 equals:

$$SA = A + T X_1$$

And the volume equals:

$$V = A T X_1$$

Since the X_1 point must lie on the $T/10$ contour:

$$T/10 = V/SA = A T/X_1 / (A + T/X_1)$$

Thus:

$$TA + T^2 X_1 = 10A T X_1 \text{ and } X_1 = (10 - T/A).$$

Thus, the coordinates of the point X_1 are $A, T/X_1$. Similarly, as can be seen in Fig. 5 for the point X_2 :

$$X_2 = (10 - T/B)$$

By continuing this process and plotting the results for the square external corner, Fig. 6 is obtained. Note that the general expected shape of a solidification front is obtained in this manner.

INTERNAL CORNERS

Consider the square internal corner without radius shown in Fig. 7. To determine the K-contour for this type of corner, the method described in Fig. 7 can be used as follows.

For the $T/10$ K-contour:

X_1 must lie on the $T/10$ K-contour and thus

$$V/SA = T/10 = [A T/X_1 + (T/X_1)^2/2]/A$$

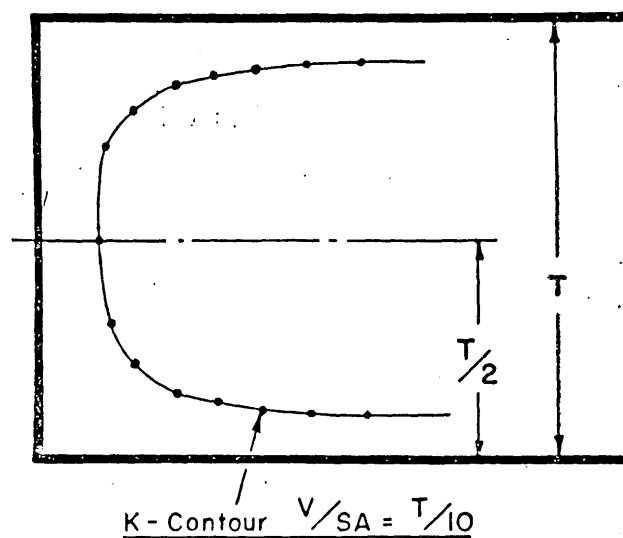


Fig. 6. The $T/10$ K-contour for a 5-in. thick semi-infinite plate.

Thus,

$$X_1 = (+ 20/T + [(20/T)^2 + 80/TA]^{1/2})/(4/T)$$

and for the point X_2 on the $T/10$ K-contour

$$X_2 = (+ 20/T + [(20/T)^2 + 80/TA]^{1/2})/(4/T)$$

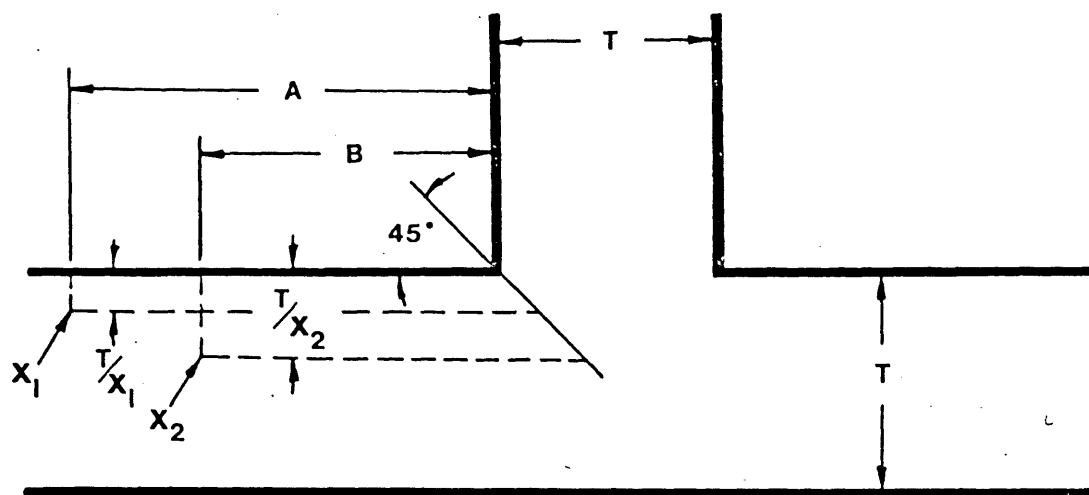
By continuing this process, the curves for K-contours of $T/10, T/8, T/6, T/4$ and $T/2$ can be obtained, as shown in Fig. 8.

Note the K-contours shown in Fig. 8 become asymptotic to zero at the corner of the section. This contour shape results because the dimension A or B which represents the surface area goes to zero as the end of the corner is approached for internal sections without fillet radii. The K-contours which result for internal corners without fillet radii point out the difficulties often associated with corners of this type. Such corners are prone to hot tearing because the convergent heat flow associated with the sharp corner tends to keep the corner liquid, as predicted by the K-contours, and the sharp corner tends to concentrate casting stresses at this point. When castings having such sharp internal corners are shaken out prematurely evidence of the lack of solidification at the section are often found. A casting exhibiting such behavior is shown in Fig. 9.

Since internal corners without fillet radii are usually avoided by the foundry industry due to the potential for either hot tears or lack of solidification associated with the corners, a method of applying the K-contour technique to sections both internal and external having radii must be developed. Currently there are several methods being considered to incorporate radii into internal and external corners, but further work will be required before a method which predicts the solidification behavior can be selected.

COMPARISONS OF K-CONTOURS WITH ACTUAL CASTING BEHAVIOR

To simulate both internal and external corners in a single casting the "L"-shaped pattern shown in Fig. 10 was selected. From this pattern self-set molds were produced in a square core mold box maintaining at least $2\frac{1}{2}$ in. of sand surrounding all portions of the pattern. The completed molds are open faced and contain no gating system or risers. Three molds of this



x_1 On The $T/10$ Contour

$$\frac{V}{SA} = \frac{T}{10} = \frac{A(T/x_1) + 0.5(T/x_1)^2}{A}$$

Thus if

$$\begin{aligned} a &= 1/0.5T \\ b &= 10/0.5T \\ c &= 10/A \end{aligned}$$

Then

$$x_1 = \frac{\pm b + \sqrt{b^2 + 4ac}}{2a}$$

x_2 On The $T/10$ Contour

$$x_2 = \frac{10/0.5T + [(10/0.5T)^2 + 40(1/0.5TB)]^{1/2}}{(2/0.5T)}$$

Fig. 7. Square internal corner without fillet radii and model to develop K-contours.

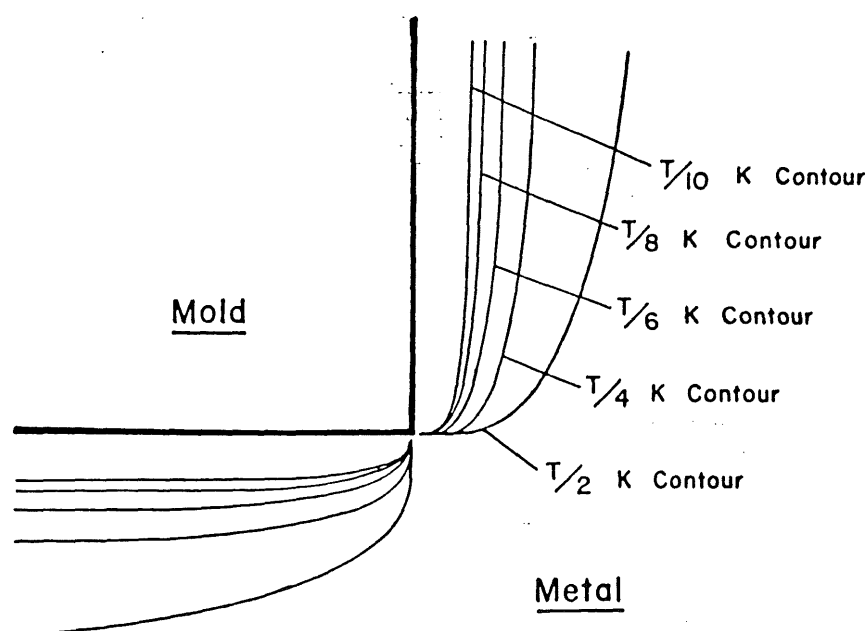


Fig. 8. The K-contour obtained from the model shown in Fig. 7.

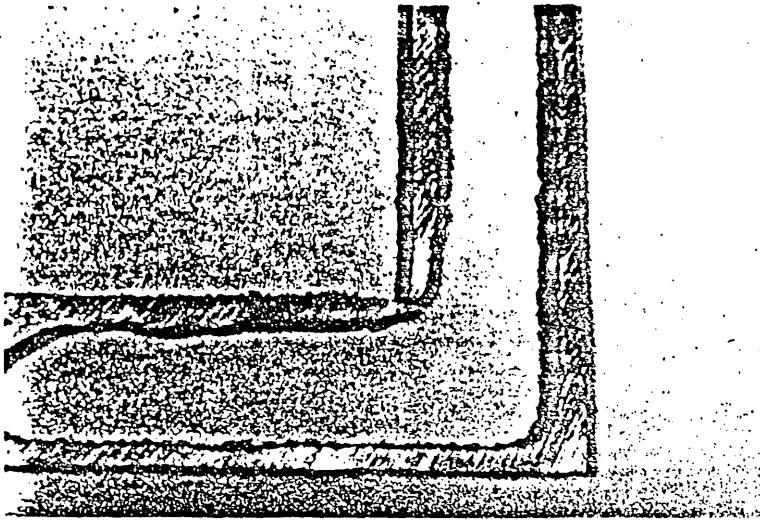


Fig. 9. Photograph of a decanted "L" casting with a 90° method of connecting the K-contours with internal corners having fillet radii.

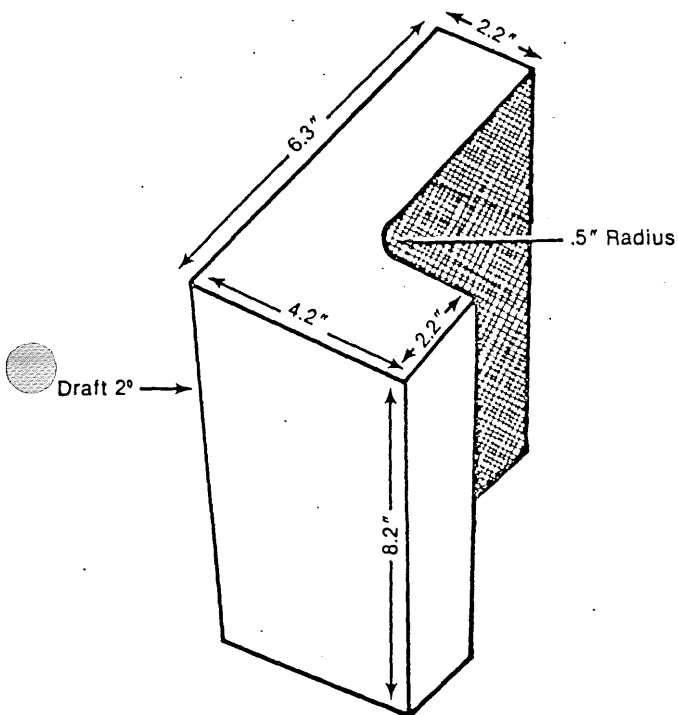


Fig. 10. Pattern used for pour-out tests.

design were poured from the same heat of WCB grade steel. After pouring each casting it was covered with a rather thick layer of hot topping compound to prevent solidification at the mold's open face. At 55 seconds, 2 minutes and 3 minutes the unsolidified metal was removed from the casting by tipping the mold on its side. After shakeout each casting was sectioned, as shown in Fig. 11, to produce the 1/2-in. thick sections shown in Fig. 11. The completed sections from this test are shown in Fig. 12.

Because the castings produced in this test contain an internal corner with a 0.5 inch radius the K-contours shown in Fig. 8 are inappropriate. To overcome this problem barring the existence of a single appropriate model applicable to corners with radii, it

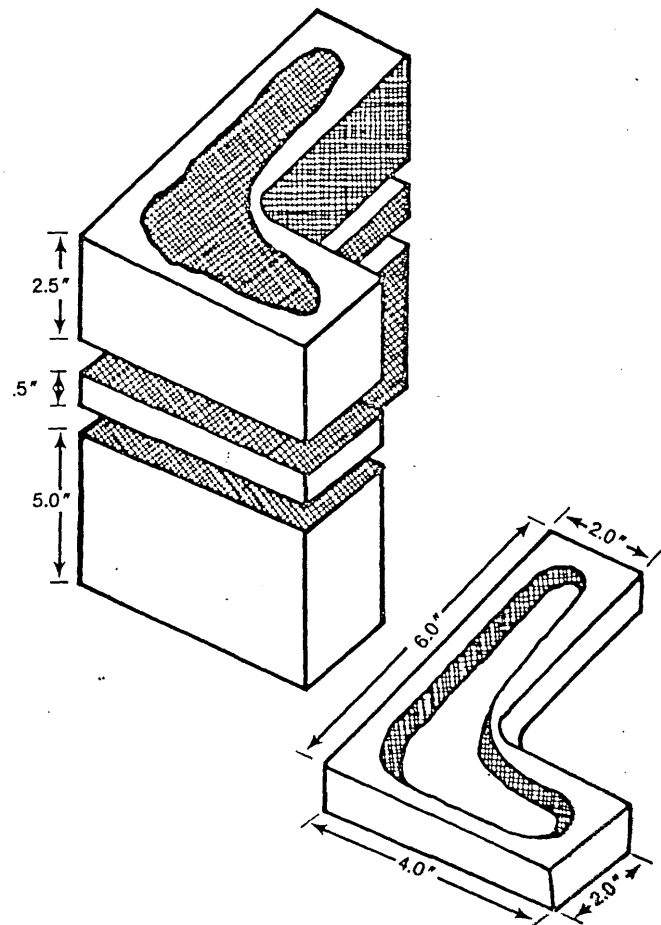


Fig. 11. Pour-out casting showing position of section used to check computer model.

was decided to join the appropriate K-contours by means of a 45° line, as shown in Fig. 13. This method of connecting the K-contours provides a convenient method of joining them. This method is not presented as the definitive method of representing

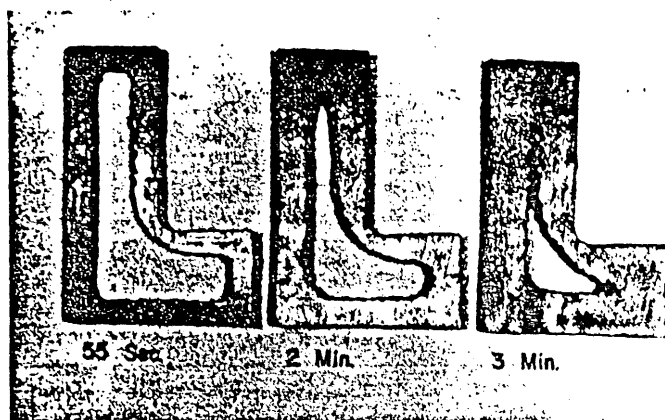


Fig. 12. Photograph of decanted "L" sections showing the progression of the solid front through the sections.

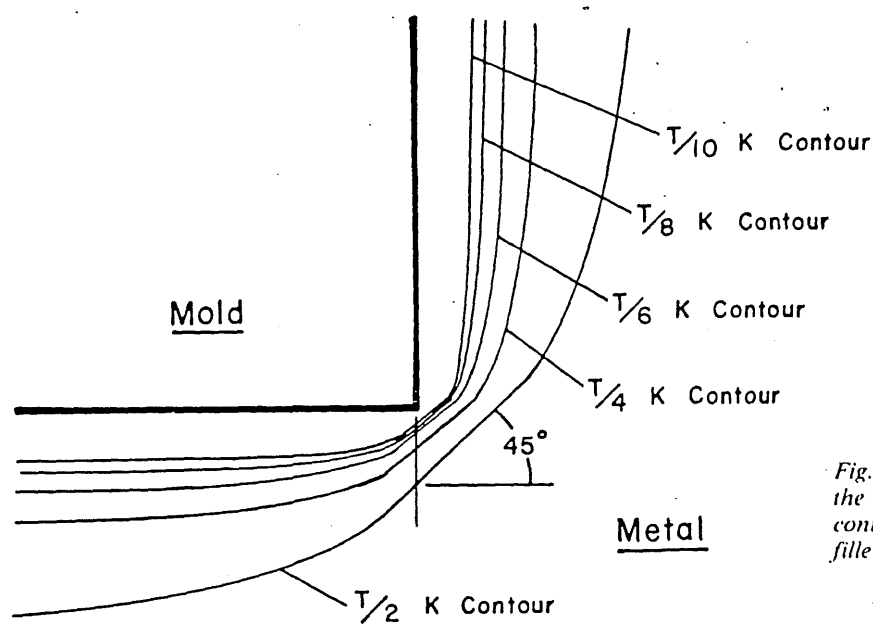
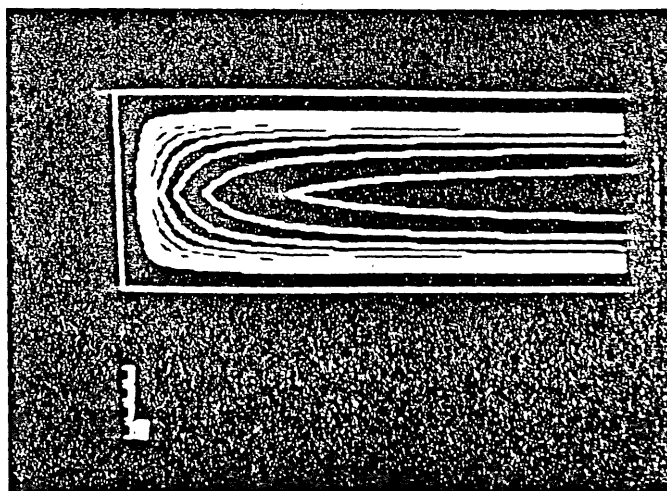


Fig. 13. The 45° method of connecting the K-contours to allow comparison contours with internal corners having fillet radii.

Fig. 14. Photograph of the computer graphics output for a square external corner.



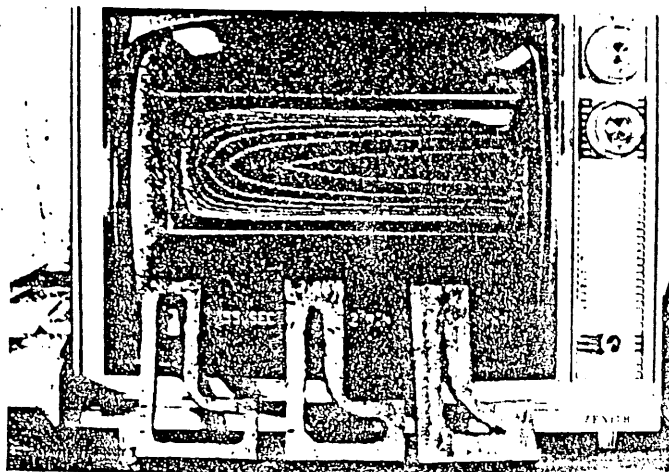


Fig. 15. Photograph of the computer graphics output for a square external corner and the "L" casting sections obtained experimentally.

Fig. 16. The experimental "L" section castings when viewed in a manner similar to the computer graphics outputs.

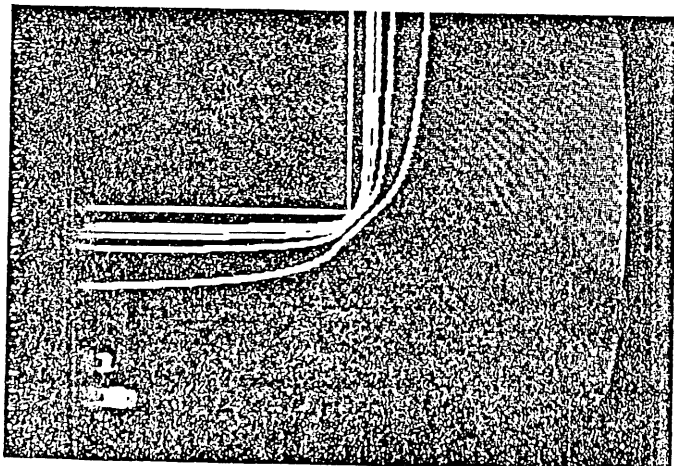
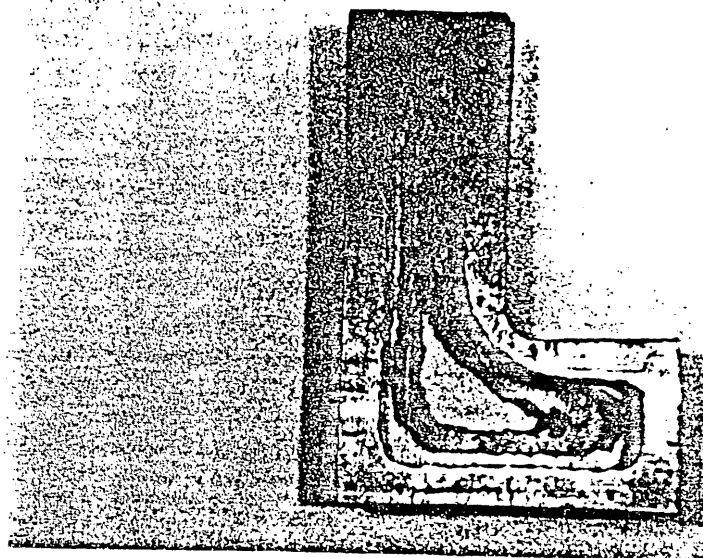


Fig. 17. Photograph of the computer graphics output for the 45° connection technique of K-contour derived for internal corner comparisons.

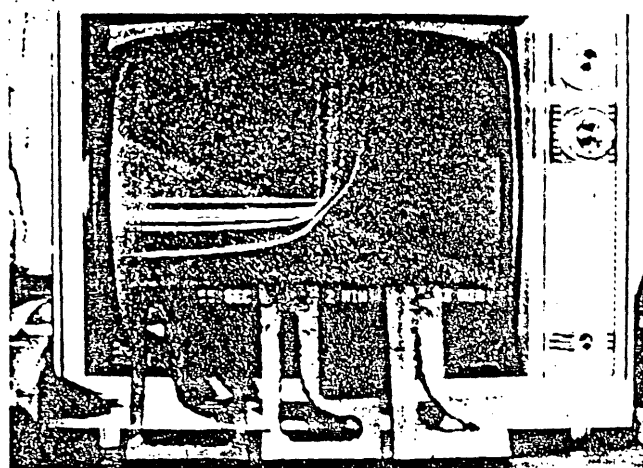


Fig. 18. Photograph of the 45° connection technique of K-contour generation and the "L" casting sections obtained experimentally.

K-contours associated with corners composed of radii, rather than this method is presented for comparison purposes only to the "L"-section casting produced experimentally.

To facilitate the production of K-contours of the various types a series of computer programs which use computer graphics were written for square external corners, square internal corners and the 45° interconnected contours used to represent internal corners with radii. Figure 14 is a photograph of the computer graphics output for a square external corner. Note that the shape of the K-contours begins as fairly rectangular in shape with rounded corners, then progresses to a "U"-shape, and finally to "V"-shaped contours. When this progression of shapes is compared to the actual progression of the solid front of the experimental "L"-shape castings a fairly close agreement is achieved, as seen in Fig. 15. Note the progression of the solid fronts in the experimental castings when viewed in a similar manner to computer graphics output as shown in Fig. 16.

Figure 17 is a photograph of the computer graphics output of the square internal corner using the 45° connector. Again note the relatively good correlation between the shape of the K-contours and the actual casting solid front positions, as seen in Fig. 18.

It should also be remembered that the K-contour positions and shape can be altered by applying various factors to the localized surface area and volume terms. This means that some

adjustments could be made to produce even better correlation between the K-contours and the actual positions of the solid front. It also means that the effects of variations of thermal properties within a mold can be accommodated by the technique such as the effects of chills, cores, exothermic or insulating materials, etc.

Although the K-contour technique has not been fully perfected, it does show great promise as an inexpensive technique to study the solidification of castings in three dimensions. The loss of mathematical rigor in its use requires that the relation of solid front position with time be lost. However, its ability to predict the manner in which three-dimensional objects solidify may provide both the small jobbing and large production foundryman with a useful tool for rigging and gating castings. A tool of this type in the hands of the foundryman's customers, the design engineers, could also provide great benefits to both the foundry industry and its potential customers.

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Determination of Thermal Diffusivity of Al-13% Si Alloy by Monitoring Casting Solidification and Cooling

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ABSTRACT

Experimental measurements of the temperature distribution along the central line perpendicular to the wide face of a 12 x 12 x 3-in. plate casting were used to determine the thermal diffusivity of Al-13% Si alloy as a function of temperature.

The experimental measurements were numerically analyzed using two techniques. First, the experimental data were analyzed in terms of ratio of the rate of change of temperature with time compared to the rate of change of thermal gradient. These values of alpha were then fitted by regression analysis as a polynomial function of temperature. Secondly, the thermal measurements were fitted by polynomial regression analysis to provide analytical expressions for the temperature as a function of distance at various times. These functions were then used to compute values of alpha which were then fitted to a polynomial expression as a function of temperature. The two results were found to be in good agreement.

The thermal properties determined in this fashion have been used to simulate solidification of cylindrical-shaped castings of this alloy. Measured thermal profiles in several castings compare closely with the simulated results.

INTRODUCTION

In order to successfully simulate solidification of castings, thermal properties of the materials involved must be known as functions of temperature. Unfortunately, information about the thermal properties of many alloys at elevated temperatures is very limited.

The data presently available for Al-Si alloys basically are those collected in the *Metals Reference* book and presented in a recent research paper.^{1,2} Some thermal parameters used to compute heat transfer in Al-Si castings at elevated temperature have been estimated from properties reported for room temperature.

One of the most important thermal parameters is the thermal conductivity, which is in part the subject of this study. However, because the specific heat and density were not defined as a function of temperature, it was more appropriate to utilize the thermal diffusivity, defined as $k/\rho C_p$.

This present research involves determination of the thermal diffusivity of an aluminum-silicon alloy from experimental measurements of the temperature distribution along the central line perpendicular to the wide face of a plate casting, 12 x 12 x 3-in. thick, wherein the heat flow can be assumed to be one dimensional. The experimental measurements were analyzed using two numerical

techniques. The reliability of these thermal properties were then tested by simulation of solidification of cylindrical-shaped castings of this alloy which were thermally monitored to provide a critical comparison with the calculations.

EXPERIMENTAL PROCEDURE

Casting Geometry and Materials

For this study, two casting geometries were selected, each for a different purpose. A plate-shaped casting, 12 x 12 x 3 in., with a bottom gate and top riser, was used to determine the thermal diffusivity of an aluminum-silicon alloy. A vertical, cylindrical-shaped casting, previously described, was used to test the determined thermal properties used in a computer simulation by comparison with thermal monitoring measurements.³

A near eutectic composition aluminum 13 wt. % silicon alloy was used in both casting experiments. Each casting was poured three times at 1292F (702C). The eutectic solidification temperature of this alloy has been reported to be 1070F (577C).⁴ The alloy was superheated to 1328F (720C) in a 3000 cycle induction heated lift coil unit. A clay-graphite crucible was used for both melting and pouring.

A dried silica sand for which the thermal properties are well-defined was chosen as a molding material for both castings. The molds were hand rammed from the composition: 4 wt. % Western bentonite, 1 wt. % dextrin, 2.5 wt. % water, and the balance AFS 85 mesh Wedron silica sand.

The hardness of the molds as rammed was controlled within the AFS hardness range 85-90. The molds were dried in a gas oven at 350F (177C) for 24 hr. and cooled to room temperature. The densities of the dried molds ranged from 100-105 lb/ft³ (1.60-1.65 g/cm³). The thermal properties of this molding material have been fully documented.⁵

Thermocouple Locations and Temperature Measurement

Solidification of the two castings was carefully monitored by placing the junctions of chromel-alumel thermocouples at specific locations in the castings and in the sand. The thermocouples were enclosed in 1-mm (0.04-in.) bare thin-walled quartz glass tubes. Figure 1 shows the five thermocouple locations in the plate castings used for determination of thermal diffusivity of the aluminum-silicon alloy. Thermocouple positions were varied among the given locations to assure unidirectional heat flow in these experiments. The positions of the thermocouples in the cylindrical castings, identical to those reported earlier, are shown in Figure 2.³

The temperature measurement system consisted of a scanning device, a digital thermometer and a microcomputer for transmitting the outputs of the digital thermometer to a recording device. The monitored temperatures were recorded continuously until the temperatures fell below 800F (427C) in the two types of castings. The interval between readings was 2.5 sec. The total cycle time for all thermocouples in the array was 12.5 sec. for the plate castings and 20 sec. for the cylinder castings.

Computer Implementation

Two programs were written to compute the thermal diffusivity and a finite difference approximation procedure was developed to simulate heat transfer during solidification of the cylindrical test casting. These programs were implemented on a large mainframe computer (Amdahl 5860) using the Michigan Terminal system (MTS), and an Apollo Supermicro computer (DN 300).

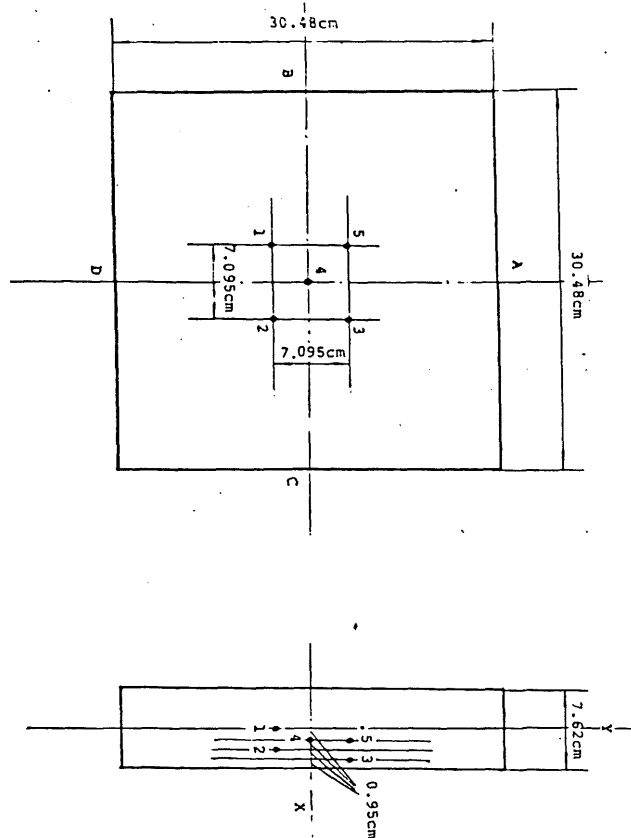


Fig. 1. Thermocouple locations in the plate casting.

Calculation of Thermal Diffusivity

In the present study, two numerical techniques were utilized to calculate the thermal diffusivity of aluminum-13% silicon alloy based on thermal monitoring of plate castings. Both techniques were based on the following premises:

- 1) The heat flux from the central plane of the plate was symmetrical about that plane.
- 2) The casting was considered to be an infinite plate with respect to heat flow. The heat flux in the central area of the plate was assumed to be perpendicular to the wide face of the plate.
- 3) The temperature near the central area of the wide face of the plate was uniform on planes parallel to the wide face, and these temperatures were accurately monitored by thermocouples placed near the central area of the wide face at various distances below the surface.

The latter two assumptions are supported by experimental documentation as presented in Table 1, where data from the initial plate casting, A, show negligibly small temperature differences between thermocouples on the same vertical plane, 2.85 cm from the surface; point 4 in Figure 1 on the centerline perpendicular to the wide face, and point 5, about 5 cm away. In contrast, the differences between temperatures measured at point 4 in Figure 1 and point 2, which is displaced 0.95 cm closer to the surface, are substantially larger.

Table 1 also presents a summary of the temperature gradients between these locations determined as the casting cooled below

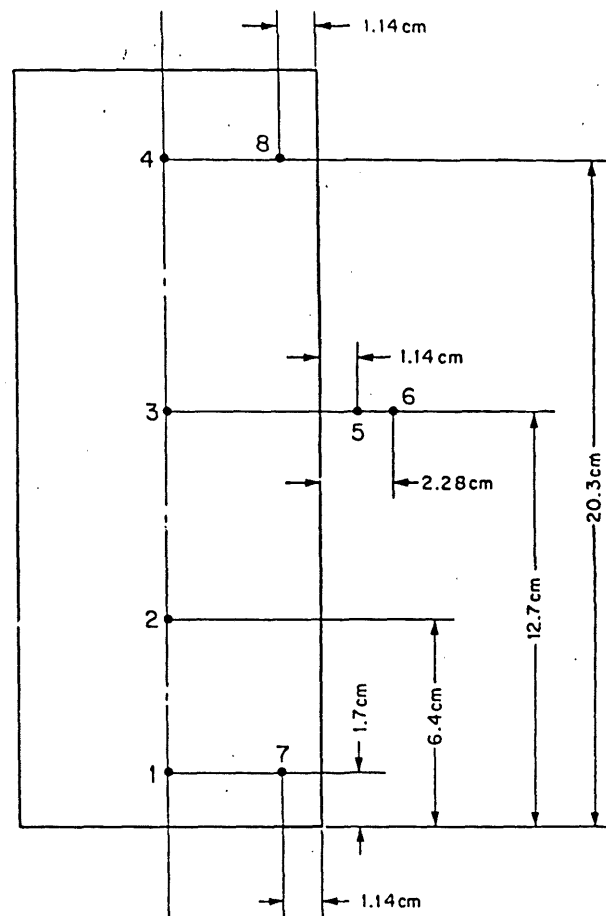


Fig. 2. Thermocouple locations in the cylindrical casting and mold.

the eutectic temperature. As the casting cooled, the measured temperature gradient between points 4 and 5 in Figure 1 decreased from 0.174 to 0.045F/cm, whereas the gradient perpendicular to the surface between points 4 and 2 decreased from 2.196 to 0.18F/cm.

In two other identical castings, plate castings B and C, the location of the comparison temperatures corresponding to locations 4 and 5 of Figure 1 were moved to positions 0.95 cm and 2.85 cm from the surface, respectively. Similar results were found with respect to the measured thermal gradients. Thus, it was concluded that unidirectional heat flow could be assumed perpendicular to the wide face of the plate castings.

Method I for Calculation of Thermal Diffusivity

The Fourier equation for unsteady state heat transfer can be expressed as:

$$(\partial^2 T / \partial x^2) = (1/\alpha) (\partial T / \partial t) \quad (1)$$

and can be rearranged to:

$$\alpha = (\partial T / \partial t) / (\partial^2 T / \partial x^2) \quad (2)$$

where α is a function of temperature, and hence of location and time. Equation 2 can be expressed in finite difference form as:

$$\alpha = [(T_{i+1}^{j+1} - T_i^j) / (T_{i+1}^j - 2T_i^j + T_{i-1}^j)] \cdot ((\Delta x)^2 / \Delta t) \quad (3)$$

where α represents the values of thermal diffusivity at location

Table 1. Temperature Gradient in Different Directions from Point 4 Below the Eutectic Temperature in Plate Casting A

Time (s)	Temperature Difference (°F)		Temperature Gradient (°F/CM)	
	No 4 and 2	No 4 and 5	No 4 and 2	No 4 and 5
0	2.0860	0.6630	2.196	0.174
200	1.9091	1.6701	2.009	0.258
400	2.3438	1.1732	2.467	0.308
600	3.0793	0.8533	3.241	0.224
800	3.4431	0.8504	3.624	0.223
1000	1.7319	0.7102	1.823	0.186
1200	1.7581	0.5232	1.851	0.137
1400	0.7957	0.3531	0.838	0.093
1600	0.6433	0.1824	0.677	0.047
1800	0.7627	0.1832	0.803	0.048
2000	0.1714	0.1736	0.280	0.045

Table 2. Selected Values of Thermal Diffusivity

a(t)=a T ² +b T +c Unit: cm ² /s			
		Coefficient	Standard Error
First Method	1070°F to 1200°F	a=0.8558E-5	a=0.8732E-5
		b=-0.1743E-1	b=0.1982E-1
		c=9.3717	c=11.241
	800°F to 1070°F	a=-0.4113E-5	a=0.6937E-6
Second Method	1070°F to 1200°F	b=0.6163E-2	b=0.1298E-2
		c=-1.6405	c=0.6039
		a=0.1477E-5	a=0.6949E-5
		b=-0.1061E-2	b=0.1578E-1
Method	800°F to 1070°F	c=-0.1256	c=8.9457
		a=-0.5301E-5	a=0.6201E-6
		b=0.8508E-2	b=0.1160E-2
		c=-2.8116	c=0.5399

which can be either of the two interior planes where temperature T_i is measured at time, t , and a next time interval, $t + \Delta t$.

In the present experiments, the distance interval between each pair of thermocouple locations, Δx , was 0.95 cm. The digital thermometer monitoring system devoted 2.5 sec. to each reading so that the cycle to complete the measurement of temperature output of all five thermocouples (including the thermocouple for comparison) required 12.5 sec. Each plate casting cooled continuously for about 150 min., until the temperature of the castings reached 800°F. Thus, the experiments provided sufficient data to determine the thermal diffusivity in the liquid and in the solid below the eutectic temperature.

First, the experimental temperature data were interpolated, using a spline interpolation technique, to determine the value of temperature at each location as a function of time. The calculated temperature at location i for $t + \Delta t$ could be determined from the extrapolation, as could the temperatures at the adjoining nodal points $i-1$ and $i+1$. The values of $\alpha(T_i)$ were then calculated using Equation 3 for the three interior nodal locations, including the central plane where $(2T_{i+1}^t - 2T_i^t)$ was substituted for $(2T_{i+1}^t - 2T_i^t + T_{i+1}^t)$ in Equation 3 by reflection.

Nine sets of $\alpha(T_i)$ values resulted from this procedure based on the three nodal point calculations from each of three castings based on the cooling curves in the liquid and cooling curves below the eutectic in the solid. These sets of $\alpha(T_i)$ data were fitted by a second order polynomial with good agreement. The functions for α developed by combining one data set from each plate casting are presented in Table 2.

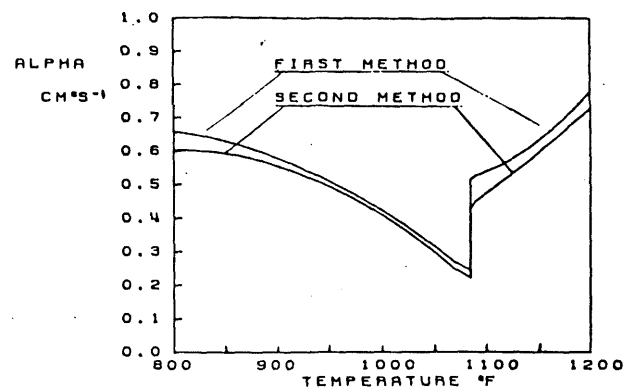


Fig. 3. Thermal diffusivity of Al-13% Si alloy.

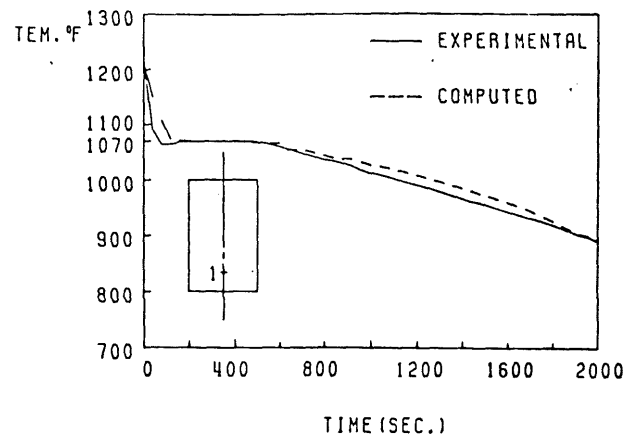


Fig. 4. The experimental and the computed temperature-time profiles at location 1 in casting A.

Method II for Calculation of Thermal Diffusivity

Essentially, this method is similar to the first method, also using one-dimensional heat transfer (Equation 1), and considering each given x position at each specified time, i.e., at each specified temperature. Since $\alpha = \alpha(T_i)$ is essentially constant in the region of T_i , Equation 1 can be written as:

$$\alpha(T_i) = \frac{(\partial T / \partial t) |_{t, x_i}}{(\partial^2 T / \partial X^2) |_{t, x_i}} \quad (4)$$

In order to calculate the thermal diffusivity at a particular temperature, it is necessary to know how the temperature at x_i varied with time in the region of t_i and how the temperature at t_i varied with distance in the region of x_i at the given time. Using a least squares regression technique, functional relationships between temperature, T , and distance, x , $T(x_1)$, $T(x_2)$, ..., $T(x_m)$, at each time t_i , were fitted successfully by a second order polynomial. The technique utilized is detailed in Appendix A. From these relations, values of $\partial^2 T / \partial X^2 |_{t_i, x_i}$ were calculated. Then, assuming the temperature variation with time to be linear during the short period of time for the determination of temperature by the spline fitting technique described above, where $\Delta t = 2.5$ sec., $(\partial T / \partial t) |_{t_i, x_i}$ can be expressed as:

$$(\partial T / \partial t) |_{t_i, x_i} = (T(x_i)^{t_i + \Delta t} - T(x_i)^{t_i}) / \Delta t \quad (5)$$

Thus, the right hand side of Equation 4 was known and $\alpha(T_i)$ was thus defined. Then, using a least squares regression technique

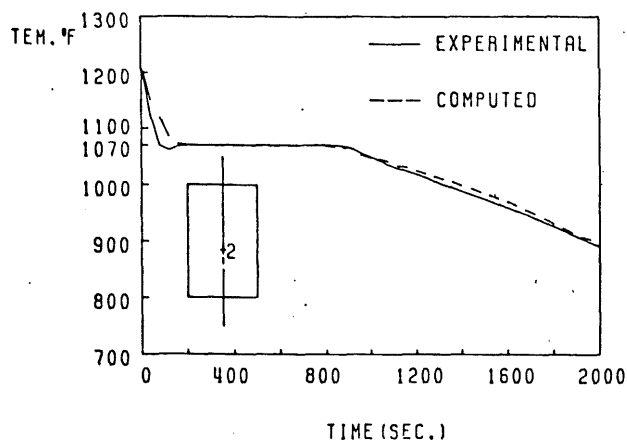


Fig. 5. The experimental and the computed temperature-time profiles at location 2 in casting A.

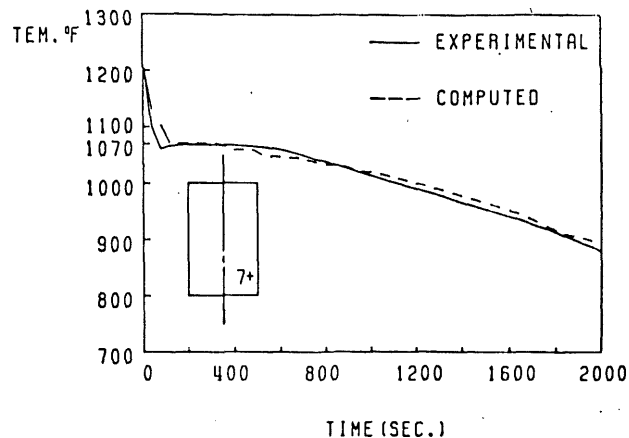


Fig. 8. The experimental and the computed temperature-time profiles at location 7 in casting A.

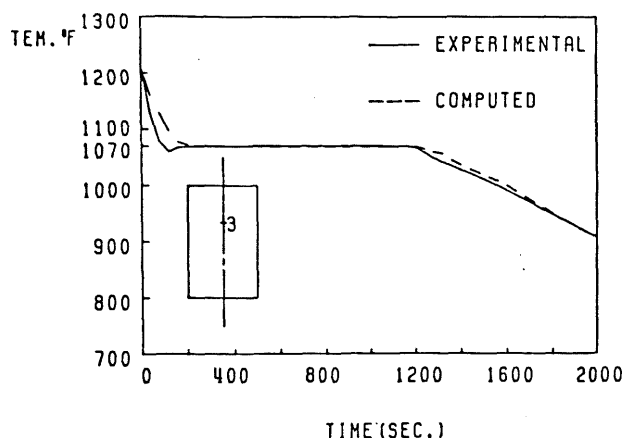


Fig. 6. The experimental and the computed temperature-time profiles at location 3 in casting A.

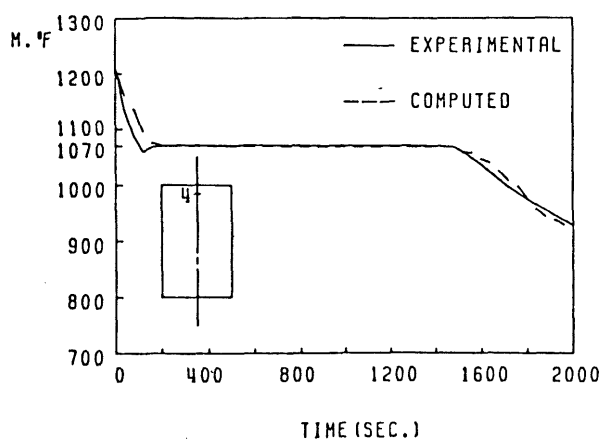


Fig. 7. The experimental and the computed temperature-time profiles at location 4 in casting A.

Table 3. The Experimental and the Computed Eutectic Solidification Time at Six Locations in Casting A

		Unit: Second					
		No1	No2	No3	No4	No7	No8
Experimental	Start	160	180	200	180	120	200
	Finish	380	780	1160	1460	280	1100
Computed	Start	161	161	171	176	121	191
	Finish	385	781	1151	1461	281	1066
Eutectic-zone Time	Exp.	220	600	960	1274	160	900
	Comp.	225	620	980	1285	161	875
Errors		+5	-20	+20	+11	+1	-25
Relative Errors		+2.27%	+3.33%	+2.08%	+0.86%	+0.63%	-2.78%

again to fit all $\alpha(T_i)$ values, functional relationships between thermal diffusivity and temperature were established.

The functions developed using this method based on the combination of one data set from each plate casting are presented in Table 2. The relationships from the two methods are in close agreement.

Figure 3 shows the thermal diffusivity of the aluminum-13% silicon alloy as a function of temperature as determined from the thermal monitoring of plate castings based on method I and II.

The results of both calculation methods are nearly identical. The values of thermal diffusivity determined by the second method are slightly smaller. The estimated standard deviation of the $\alpha(T)$ function fitting the combined thermal diffusivity data by method I are slightly larger than those of method II. The reason can be attributed to the function $T(x)$ used to fit experimental temperature data in method II. The precision of thermal diffusivity mainly depends on the precision of the thermocouple positions.

Simulation of Solidification of Cylindrical Castings

As a part of the present investigation, solidification of a cylindrical casting of Al-13% Si was simulated. The thermal explicit diffusivity results from method I were utilized in the simulation. A finite-different method similar to that described previously was used.³

For axisymmetric heat flow, the heat transfer expression for a cylindrical-shaped casting can be expressed as:

$$(\partial^2 T / \partial r^2) + (1/r) (\partial T / \partial r) + (\partial^2 T / \partial z^2) = (1/\alpha) (\partial T / \partial t) \quad (6)$$

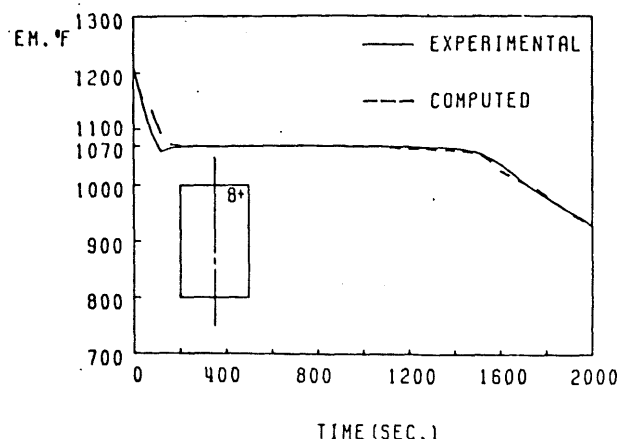


Fig. 9. The experimental and the computed temperature-time profiles at location 8 in casting A.

Table 4. The Experimental and the Computed Eutectic Solidification Time at Six Locations in Casting B

		Unit: Second					
		No1	No2	No3	No4	No7	No8
Experimental	Start	167	185	185	185	140	140
	Finish	387	705	1105	1345	300	1000
Computed	Start	161	171	181	181	121	181
	Finish	386	686	1081	1301	271	1021
Eutectic-zone Time	Exp.	220	520	920	1160	160	860
	Comp.	225	515	900	1120	150	840
Errors		+5	-5	-20	-40	-10	-20
Relative Errors		+2.27%	-0.96%	-2.17%	-3.45%	-6.25%	-2.33%

In order to solve this equation, some initial and boundary conditions were assumed as follows:

- 1) After liquid metal filled the mold, the initial temperature distribution of the liquid metal in the mold is homogeneous at a specified temperature.
- 2) The heat losses from the surface of the mold are small enough to assume that the surface is at room temperature.
- 3) The bottom of the casting and corresponding part of the sand mold are assumed to be in contact with a specific thermal resistance during solidification. The vertical surface of the casting and corresponding mold wall are assumed to be in contact with a thermal resistance only until a gap forms.
- 4) When a gap forms, the heat flux across the metal-sand interface will be considered in one dimension, and will be equal to the product of a heat transfer coefficient, H_{ci} , and the temperature difference across the interface, ΔT .
- 5) The central line of the vertical cylinder is considered to be insulating in the heat transfer analysis since the shape of the cylinder provides radial symmetry.
- 6) The top surface of the casting is assumed to be insulated after the furnace located above the mold is turned off.

In this calculation, the latent heat of fusion was transformed into an equivalent number of degrees by dividing the latent heat of fusion by the specific heat. When the temperature of a node is below the melting point of the metal (1070°F), the temperature difference was calculated and the node temperature was compensated relative to the melting point at each time step until the sum of added degrees over some time period was equal to the equivalent

Table 5. The Experimental and the Computed Eutectic Solidification Time at Six Locations in Casting C

		Unit: Second					
		No1	No2	No3	No4	No7	No8
Experimental	Start	136	150	170	189	None	180
	Finish	376	670	1070	1389	None	1060
Computed	Start	161	171	181	181	121	181
	Finish	386	686	1081	1301	271	1021
Eutectic-zone Time	Exp.	240	520	900	1200	None	880
	Comp.	225	515	900	1120	150	840
Errors		-15	-5	0	-80	None	-40
Relative Errors		-6.25%	-0.96%	-0.00%	-6.67%	None	-4.55%

Table 6. The Experimental and the Computed Solidification Temperature at 1500 Seconds in Casting A

		Temperature (°F)							
		No1	No2	No3	No4	No5	No6	No7	No8
Exp.	941	966	1003	1061	884	596	940	1051	
	973	985	1011	1051	841	566	969	1048	
Errors	+32	+19	+8	-10	-43	-30	+29	-3	
	Relative Errors	+3.40%	+1.97%	+0.80%	-0.94%	-4.86%	-5.03%	+3.09%	-0.30%

Table 7. The Experimental and the Computed Solidification Temperature at 1500 Seconds in Casting B

		Temperature (°F)							
		No1	No2	No3	No4	No5	No6	No7	No8
Exp.	958	987	1027	1064	882	489	958	1058	
	966	1001	1013	1046	817	516	944	1036	
Errors	+8	+14	-14	-18	-65	-27	-14	-22	
	Relative Errors	+0.84%	+1.42%	-1.36%	-1.69%	-7.40%	-5.57%	-1.46%	-2.08%

Table 8. The Experimental and the Computed Solidification Temperature at 1500 Seconds in Casting C

		Temperature (°F)							
		No1	No2	No3	No4	No5	No6	No7	No8
Exp.	941	969	1009	1056	862	476	913	1051	
	966	1001	1013	1046	817	516	944	1036	
Errors	+15	+32	+4	-10	-45	+40	+31	-15	
	Relative Errors	+0.85%	+3.30%	+0.39%	-0.59%	-5.22%	+8.40%	+3.39%	-1.43%

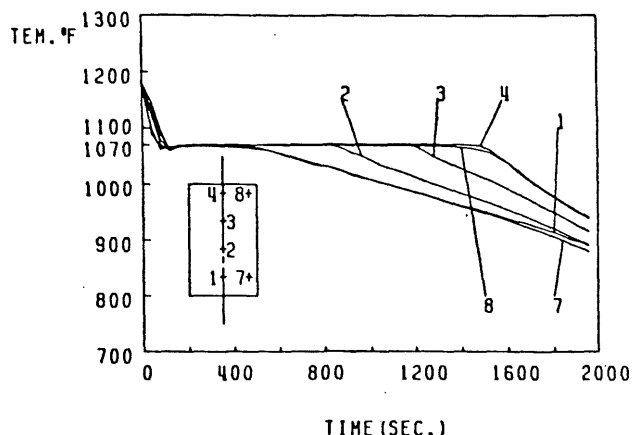


Fig. 10. The experimental temperature-time profiles for Al-13% Si casting A.

number of degrees, this signified that all the latent heat of fusion had been released, and the temperature of the nodes was allowed to fall again.

The heat transfer coefficient involved in the calculation was estimated from previous results.^{6,7} The various explicit nodal equations are detailed in Appendix B.

For purposes of comparison, three cylindrical castings—A, B, and C—were poured in the aluminum-13% silicon alloy with thermocouples placed at several locations in the castings. The experimental and computed results are in very good agreement. In order to compare the experimental results with the computed results directly, six pairs of curves from casting A are shown in Figures 4–9. The solid lines in Figures 4–9 represent the experimental results, and the dashed lines represent the computed results. The experimental and the computed eutectic solidification times for castings A, B, and C are shown in Tables 3, 4, and 5. Tables 6, 7, and 8 display a comparison of results of measured temperatures after 1500 sec. in castings A, B, and C with the computed results. As described before, the agreement between experimental and computed results is considered to be excellent.

The correlation between experimental and computed results for locations in the mold were not as satisfactory. The reason might be the estimate of the heat transfer coefficient, but more likely is the accuracy with which the thermocouples can be placed in close proximity to a curved surface.

SUMMARY

The thermal diffusivity of aluminum-13% silicon alloy has been determined above and below the eutectic solidification temperature from temperature measurements during cooling and solidification of plate-shaped castings. Two numerical methods were utilized to develop temperature dependent expressions for the thermal diffusivity as a function of temperature.

The experimentally derived thermal property was used in the simulation of solidification of a cylindrical casting of the same alloy. Excellent agreement was obtained between computed temperature-time curves at various locations in the casting and the results of thermal monitoring of three castings.

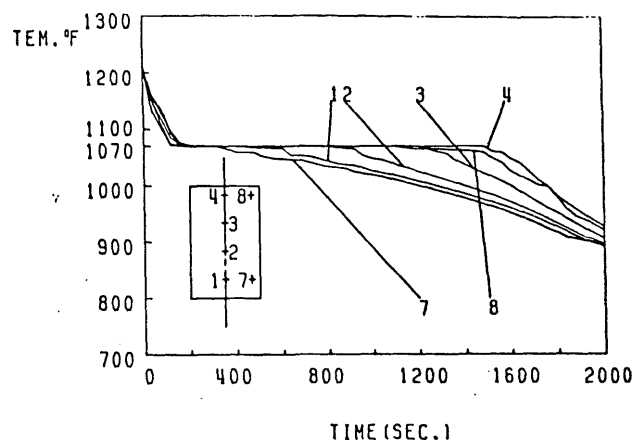


Fig. 11. The computed temperature-time profiles for Al-13% Si casting A.

ACKNOWLEDGMENT

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APPENDIX A

Thermal Diffusivity from a Power Series Fit of Temperature Profiles

For approximating a one-dimensional temperature distribution in a casting, a power series (second order) can be suggested as:

$$T(x) = A_0 + A_1x + A_2x^2 \quad (A-1)$$

Equation 1 can be used to fit the experimental data. The coefficients (A_0, A_1, A_2) of this best fit are:

$$\begin{bmatrix} m & \sum_{i=1}^m x_i & \sum_{i=1}^m x_i^2 \\ \sum_{i=1}^m x_i & \sum_{i=1}^m x_i^2 & \sum_{i=1}^m x_i^3 \\ \sum_{i=1}^m x_i^2 & \sum_{i=1}^m x_i^3 & \sum_{i=1}^m x_i^4 \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^m T_i \\ \sum_{i=1}^m T_i x_i \\ \sum_{i=1}^m T_i x_i^2 \end{bmatrix} \quad (A-2)$$

Here, m is the number of x locations where thermocouples are placed. Using the measured experimental data in Equation A-2 yields the coefficients (A_0^i, A_1^i, A_2^i). Thus, at time t_i :

$$T(x)^{t_i} = A_0^{t_i} + A_1^{t_i}x + A_2^{t_i}x^2 \quad (A-3)$$

At time $t_i + \Delta t$:

$$T(x)^{t_i + \Delta t} = A_0^{t_i + \Delta t} + A_1^{t_i + \Delta t}x + A_2^{t_i + \Delta t}x^2 \quad (A-4)$$

These are combined with Equations 4 and 5 from the test to provide:

$$a(\tau_i) = \frac{(\lambda_0^t i + \Delta t - \lambda_0^t i) + (\lambda_1^t i + \Delta t - \lambda_1^t i)x_i + (\lambda_2^t i + \Delta t - \lambda_2^t i)x_i^2}{2 \lambda_2^t i \Delta t} \quad (A-5)$$

APPENDIX B

Finite Difference Approximations for Various Nodes

For the nodes at the central lines of the cylindrical casting, the finite difference approximations (FDA) were expressed as:

$$\tau_{i,j}^{t+\Delta t} = \tau_{i,j}^t + 4M(\tau_{i+1,j}^t - \tau_{i,j}^t) + M(\tau_{i,j-1}^t - \tau_{i,j}^t - \tau_{i,j}^t) + M(\tau_{i,j+1}^t - \tau_{i,j}^t) \quad (B-1)$$

For the nodes that are at the top of the casting or the mold, if the top surface is assumed to be insulating, the FDA's will be:

$$\tau_{i,j}^{t+\Delta t} = \tau_{i,j}^t + M(1 - \frac{1}{2n})(\tau_{i-1,j}^t - \tau_{i,j}^t) + M(1 + \frac{1}{2n})(\tau_{i+1,j}^t - \tau_{i,j}^t) + 2M(\tau_{i,j-1}^t - \tau_{i,j}^t) \quad (B-2)$$

For the top surfaces in contact with a heat generator, the FDA's will be:

$$\tau_{i,j}^{t+\Delta t} = \tau_{i,j}^t + M(1 - \frac{1}{2n})(\tau_{i-1,j}^t - \tau_{i,j}^t) + M(1 + \frac{1}{2n})(\tau_{i+1,j}^t - \tau_{i,j}^t) + 2M(\tau_{i,j-1}^t - \tau_{i,j}^t) + 2M(\frac{\Delta r}{K})(q'') \quad (B-3)$$

For the nodes which are at the metal side of the interface after a gap forms, the FDA's are:

$$\tau_{i,j}^{t+\Delta t} = M[2\tau_{i,j}^t + \tau_{i,j}^t + \frac{8n\Delta r H_G}{K(4n-1)} \tau_{i,j}^t] + [1 - M(\frac{8n \Delta r H_G}{K(4n-1)} + 4)] \tau_{i,j}^t \quad (B-4)$$

For the normal nodes in the bulk, the FDA's are:

$$\tau_{i,j}^{t+\Delta t} = \tau_{i,j}^t + M(1 - \frac{1}{2n})(\tau_{i-1,j}^t - \tau_{i,j}^t) + M(1 + \frac{1}{2n})(\tau_{i+1,j}^t - \tau_{i,j}^t) + 2M(\tau_{i,j-1}^t - \tau_{i,j}^t) \quad (B-5)$$

NOMENCLATURE

T	Temperature
t	Time
X	Distance in Cartesian coordinates
r, z	Distance in cylindrical coordinate system
m	Numbers of data points
Φ	Angle in cylindrical coordinate system
α	Thermal conductivity
C_p	Specific heat
ρ	Density
H	Heat transfer coefficient
M	Fourier number
n	Numbers of nodes in r direction
Δt	Time step
$\Delta r, \Delta z$	Space increments in r, z direction, respectively

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DISCUSSIONS

Question: Do you attach any significance to the apparent step-wise effect recorded in your cooling curves (experimental)? (C. R. Loper, Jr., University of Wisconsin, Madison, WI)

Answer: The experimental cooling curves (Fig. 10) are quite smooth. The calculated cooling curves (Fig. 11) show some "steps" that are related to the numerical method and the grid spacings in time and space which were selected for the calculation.

Question: This work was done on the eutectic silicon. What effects on the procedure used would be necessary if an off-eutectic alloy were used? (A. Kearney, Trialco Inc., Chicago Heights, IL)

Answer: This simulation used an accounting of the latent heat of fusion released at the eutectic temperature based on an equivalent number of degrees determined by dividing the latent heat by the specific heat. For an "off-eutectic" composition the release of latent heat could be distributed over the freezing range, and incorporated in the calculation as a peak in the specific heat curve.

Question: In the calculation of thermal diffusivity did you assume the thermal conductivity to be independent of temperature? (Prof. G. P. Reddy, IIT Bombay, Bombay, India)

Answer: The thermal conductivity was not used independently. The thermal diffusivity was determined numerically from experimental measurements. Two polynomials were fitted to the results giving the thermal diffusivity as a second order function of temperature above and below the eutectic temperature.

Question: What type of reference junction was used on the thermocouples? (George Gooden, Bentover, Caracas, Venezuela, S.A.)

Answer: A Newport digital thermometer was used to monitor the output of the chromel-alumel thermocouples. The instrument contains a chromel-alumel junction which is utilized for reference temperature compensation.

Question: Were effects of convection prior to solidification included in the simulation work or was the maximum temperature of the incoming metal adjusted in any way? (J. T. Berry, Prof., School of Mech. Eng., Georgia Inst. of Technology)

Answer: Convection in the melt, in both the plate casting and in the cylindrical casting was not considered in this work. The pouring temperature for each of the three castings was 1292F. In the computations, the measured local mold cavity temperatures (1200–1220F) on mold filling were used as starting temperatures for simulation of the cylindrical castings.

Calculation of Feeding Range Data for Hypoeutectic A-357 Alloy Using FEM Solidification Model Results

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ABSTRACT

The presence or absence of unsoundness due to solidification shrinkage is dependent upon the range of action of risers in sand castings. As established by Pellini and co-workers,¹⁻³ the feeding range of a riser for a given alloy is an empirical estimate of this range of action. This measure depends upon many factors including casting geometry, the thermal properties of the mold and casting materials, and the mode of solidification of the cast alloy. In recent years a great deal of work has focused upon modeling the solidification of sand castings using numerical solutions to the non-linear heat conduction problem.⁴⁻⁸ These models are capable of providing information about temperature fields. Davies⁹ has used information derived from finite difference models (FDM) in determining feeding ranges for various cast alloys using a simple fluid flow model which includes an empirical calibration based upon the data of Pellini, et al.¹⁻³

In this study a finite element (FEM) formulation of the heat conduction problem is incorporated to solve for the temperature field in a simple plate casting of A-357 alloy in silica sand. Information derived from the numerical solution is then used to estimate feeding range for the alloy in the manner outlined by Davies.⁹ The results are compared with those reported by Davies⁹ for Al 7% Si alloy and are discussed in terms of their implication of a distribution of dispersed shrinkage along the length of the plate casting.

INTRODUCTION

It is a well-established principle, perhaps due to the pioneering work of Pellini and his co-workers,³ that the presence or absence of unsoundness due to solidification shrinkage is not only dependent upon the freezing order of joined sections, but also upon the range of action of the risers in sand castings. Although dealing primarily with short freezing range alloys (SFRA), Pellini et al. established the concept of a feeding range attributable to a given alloy as the distance over which a riser could supply feed metal, producing a sound cross-section, in plate and bar castings. This distance was given as a multiple of the plate or bar thickness. Unsoundness in the SFRA is, primarily, the presence of centerline shrinkage. In long freezing range alloys (LFRA), unsoundness is primarily dispersed interdendritic porosity. In LFRA feeding of solidification shrinkage occurs via "mass feeding" and interdendritic fluid flow, with the former term distinguishing the bulk flow of liquid plus solid crystals. This is because solidification occurs in a "pasty" manner, with the entire casting or major parts of it consisting of a mixture of solid and liquid phases. In the SFRA, on the other hand, solidification proceeds via growth of the solid from the mold walls with a more or less distinguishable boundary between liquid and solid phases. The end result is that the feeding of solidification proceeds by virtue of capillary flow through the resulting channel.

The differences between the modes of feeding and resulting forms of unsoundness in SFRA and LFRA are illustrated schematically in Figure 1. Only a limited amount of work has been done to characterize feeding ranges for LFRA. No empirical data similar to that of Pellini et al. are available for the many alloys of commercial importance falling into this category.

Piwonka and Flemings¹⁰ have shown that flow through "mushy" zones obeys relations similar to those employed in flow through other types of porous beds. However, data applicable to feeding ranges of risers was not reported. Davies⁹ has calculated feeding ranges for several alloys in plate castings by employing the velocity of the 80% solidified front isotherm calculated from a finite difference formulation of the differential enthalpy equation in a simple model of fluid flow. The aspects of Davies' analysis important to the problem studied here are as follows: for laminar flow through a tube of radius, r , and length, ℓ , the pressure drop is

$$\Delta P = \frac{8\mu\ell\dot{Q}}{\pi r^4} \quad (1)$$

where μ is the viscosity of the fluid, and $\dot{Q}$ is the volumetric flow rate. By conservation of mass, the flow rate required to feed solidification shrinkage is

$$\dot{Q} = V_s \pi r^2 m \quad (2)$$

where m is the solidification contraction and V_s is the solidus velocity. Equating 1 and 2 and solving for ℓ yields the capillary feeding distance, since the flow rate $\dot{Q}$ is just the amount necessary to feed shrinkage. Thus:

$$\ell = \ell_f = \frac{\Delta P r^2}{8\mu m V_s} \quad (3)$$

where ℓ_f is the feeding distance. The result is highly approximate at best for metals solidifying with a plane front. Davies seeks to correct Equation 3 to incorporate the effects of a not-so-clearly defined solidification front by means of an empirical correction factor, B . Since we can write $\Delta P = \gamma P_o$ where P_o is the pressure due to metallostatic head and γ a constant, Equation 3 can be

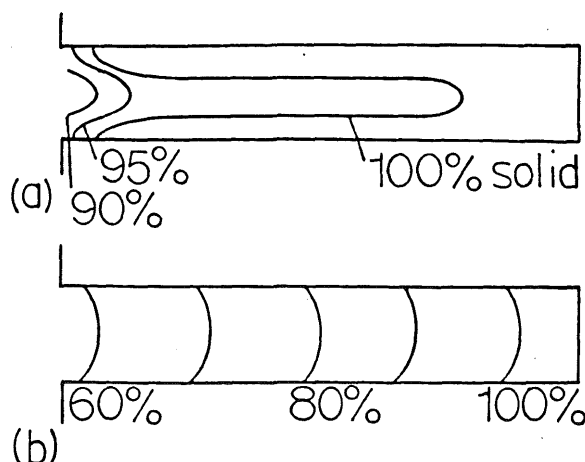


Fig. 1. Solidification pattern at a given time, schematically, a) pure metals; b) alloy with a wide solidification range.⁹

rewritten as

$$\ell_f = \frac{B P_o r^2}{\mu m V_s} = \frac{B' h \rho_e r^2}{\mu m V_s} \quad (4)$$

where h is the height of the metalostatic head and ρ_e is the density of the liquid metal. Davies then chooses B' such that a maximum feeding distance of 4.5 S results, where S is the plate thickness. In doing so, he employs data involving 0.6% C steel cast in silica sand. Since this empirical value (4.5 S) is generally accepted for this combination of alloy and mold material, this step is probably permissible. The resulting model is then used to calculate feeding ranges for some pure metals, eutectics and SFRA by employing solidus velocities obtained from FDM solutions to the differential enthalpy equation.

To calculate feeding ranges for LFRA, Davies uses the same basic model with a modified approach which attempts to incorporate observed differences in the nature of feeding in these types of alloys. During the first part of solidification mass feeding (movement of liquid and solidified crystals) takes place until a solid dendrite network prevents this from occurring. This is generally believed to occur at approximately 70% solidified. When mass feeding stops, feeding will continue via interdendritic fluid flow. To obtain some estimate of the length of interdendritic feeding distance, Davies uses a simple idealization of the dendrite network geometry to calculate an effective channel radius to be used in Equation 4. He then finds the mean velocity of the 80% solid front from the FDM solution and substitutes this for V_s to obtain ℓ_f as the interdendritic feeding distance. The 80% solid front velocity is chosen as it is observed that feeding terminates near that point in the solidification (80%), perhaps due to surface tension and other effects which are difficult to analyze.

Davies has calculated feeding range data for Al 7% Si alloy which is close in composition to alloy A-357 (Al 7% Si 0.6% Mg), having the same hypoeutectic character which gives a mixed dendritic/eutectic solidification mode. He found the end contribution to be 5.5 S and the feeder contribution to be 4.5 S yielding a maximum feeding distance of 10 S , for the 7% silicon alloy. It must be noted here that Davies treated the Al 7% Si alloy as an SFRA. Thus, the value of feeding range was that estimated to be required to prevent centerline shrinkage. He notes in his discussion that plates cast in Al 7% Si which were ten times the thickness in length (equal to maximum feeding range) showed no centerline shrinkage but *did* show some microporosity. Davies also calculated interdendritic feeding distance for Al 2% Si alloy (an LFRA) and estimated percent microporosity from his data. He found that in simulations without an end chill, percent microshrinkage varied somewhat along the plate but was relatively constant at 1.5%. When a simulation was of a casting with an end chill, however, he found a lower average percent porosity which increased from 0.0% at the chilled end to about 1.5% at the feeder end. His results, for the unchilled plate, however, agreed qualitatively with experimental results.

Numerical Analysis

A numerical analysis of the heat transfer problem with the dimensions of a plate casting as shown in Figure 2 using liquidus and solidus temperatures for A-357 alloy has been performed.

A finite element model has been developed which can simulate the solidification of sand castings assuming uniform initial temperatures in the casting and mold. The model can take into account

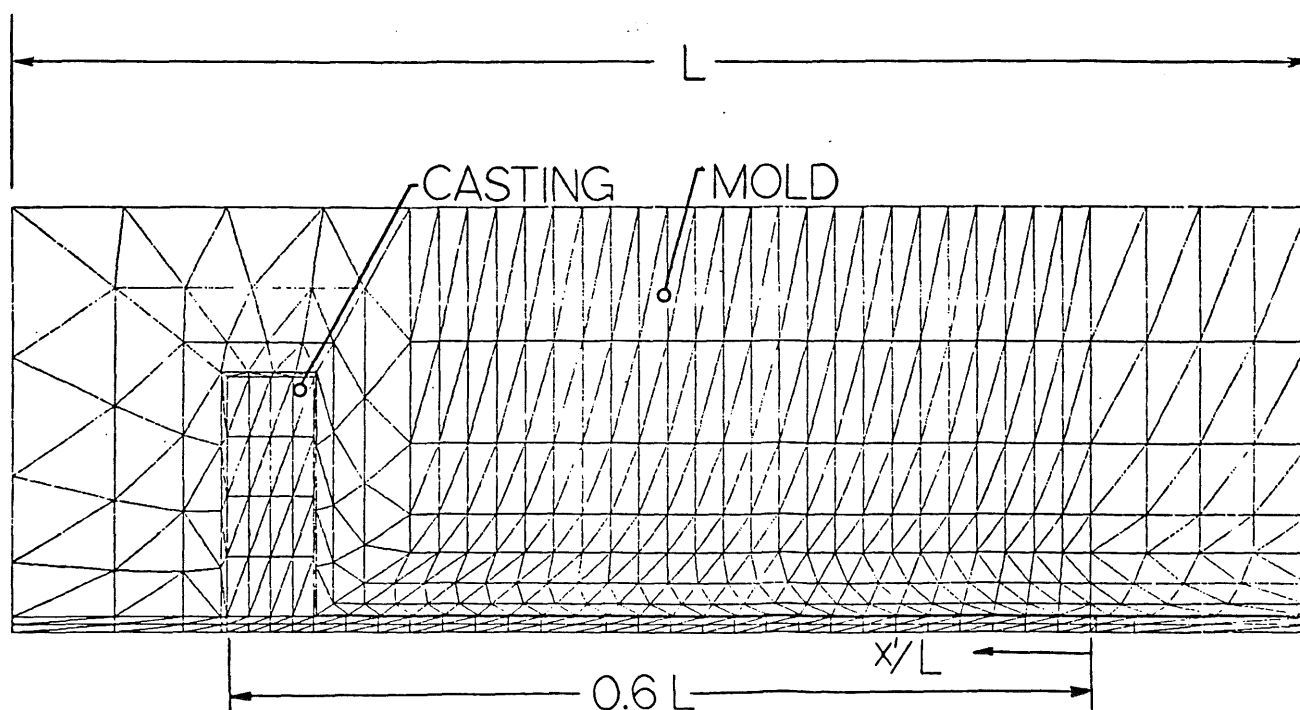


Fig. 2. Two-dimensional finite element mesh used in the numerical analysis.

temperature dependent thermal properties of the mold and cast material and will allow latent heat generation at constant temperature or over a temperature range. These characteristics make the model suited for studying the solidification of the plate casting considered in this analysis. Specifically, a model for the solidification of the hypoeutectic A-357 alloy has been formulated for this study.

The model involves a finite element formulation of the differential enthalpy equation first solved using finite differences by Sarjant and Slack¹¹ and used extensively by Hansen^{12,13} in solidification studies. The finite difference formulation has been shown by Meyer,¹⁴ via a rigorous mathematical proof to converge to the weak solution of the Stefan problem as the time step and node spacing approach zero. A summary of the formulation as used in this study follows. A more complete description has been given.¹⁵

For a metal at rest with density ρ specific heat C , and thermal conductivity k , conservation of energy can be written

$$\frac{\partial E}{\partial t} = \nabla \cdot k \nabla T \quad (5)$$

where T is temperature, t is time and

$$E = \int_0^{T-T_s} (\rho C + \rho \lambda \delta) dT \quad (6)$$

In Equation 6 T_s is the liquidus temperature and λ is the latent heat of fusion with δ producing a discontinuity of strength λ in the function E at $T = T_s$. A modified temperature scale is then defined as

$$\phi = \frac{C_s}{\lambda} \int_0^{T-T_s} \frac{k}{k_0} dT \quad (7)$$

where C_s is the specific heat at $T = T_s$ and k_0 is the thermal conductivity at a reference temperature. Equation 5 can then be written as

$$\frac{\partial H}{\partial Fo} = \nabla^2 \phi \quad (8)$$

$$H = \frac{1}{\lambda} \int_0^{T-T_s} (C + \lambda \delta) dT, \quad (9)$$

∇^2 is the dimensionless Laplacian operator and $Fo = k_0 t / \rho C_s L^2$. For the mold material

$$\frac{\rho_m}{\rho} \frac{\partial H_m}{\partial Fo} = \frac{k_{om}}{k_0} \nabla^2 \phi_m \quad (10)$$

where the subscript m denotes mold material and

$$H_m = \int_0^{T-T_s} \frac{C_m}{\lambda} dT \quad (11)$$

$$\phi_m = \frac{C_s}{\lambda} \int_0^{T-T_s} \frac{k_m}{k_{om}} dT \quad (12)$$

Equations 8 and 10 are the governing field equations for the mold and solidifying metal subject to the initial conditions

$$H = H_0 \text{ in } \Omega \quad (13)$$

and boundary conditions

$$H = H_{S_1} \text{ on } S_1 \quad (14)$$

$$\nabla \phi \cdot \hat{n} + \frac{C_s}{Bi} (\phi - \phi_\infty) = 0 \text{ on } S_2 \quad (15)$$

where

$$\overline{Bi} = Bi (T - T_\infty) / (\phi - \phi_\infty) \quad (16)$$

$$Bi = \frac{hL}{k_0} \quad (17)$$

h is the heat transfer coefficient acting at S_2 . In Equations 13–17 the appropriate properties are chosen depending on whether the boundary lies on the mold or metal.

In order to model the hypoeutectic character of A-357 alloy (latent heat generation over a temperature range followed by generation at a constant temperature), Equation 9 can be modified as

$$H = \int_0^{T-T_s} \left(\frac{C}{\lambda} + \delta' + \frac{\partial f}{\partial T} \right) dT \quad (18)$$

where T_e is the eutectic temperature. The fraction solidified, f , during crystallization of the primary phase is determined by assuming equilibrium conditions and using the lever rule

$$f = \frac{T_s - T}{(T_m - T)(1 - \gamma)} \quad (19)$$

where γ is the equilibrium partition coefficient, T_s is the liquidus temperature and T_m is the melting temperature of the pure metal. It follows from Equation 18 that $H = 0$ at $T = T_e$. Letting $H = 1$ at $T = T_s$, it follows that $H = 1 - f$ during primary solidification. Latent heat generation at the eutectic temperature is accounted for by choosing δ' such that $\delta' = 1 - f_e$ at $T = T_e$ where f_e is the fraction solidified from Equation 19 at $T = T_e$. Thus, a one-to-one correspondence exists between H and ϕ for the alloy except at the eutectic point where $0 < H < 1 - f_e$.

The finite element equations are derived using the standard Galerkin method procedure¹⁶ with the interpolation functions being the same for H as for ϕ . This procedure yields the following matrix equations:

$$-[[K_{vm}] + [K_{s_{2m}}]] \{\phi_m\} = [C_m] \{\dot{H}_m\} - \{R_{s_{2m}}\} \quad (20)$$

$$-[[K_v] + [K_{s_2}]] \{\phi\} = [C] \{\dot{H}\} - \{R_{s_2}\} \quad (21)$$

The element matrices are

$$[K_v]^{(e)} = \int_{V(e)} \nabla^* [N]^T \cdot \nabla^* [N] dV^{(e)} \quad (22)$$

$$[K_{vm}]^{(e)} = \frac{k_{om}}{k_o} \int_{V(e)} \nabla^* [N]^T \cdot \nabla^* [N] dV^{(e)} \quad (23)$$

$$[K_{s_2}]^{(e)} = \frac{C_s}{\lambda} \frac{1}{B\Gamma} \int_{S_2(e)} [N]^T [N] dS_2^{(e)} \quad (24)$$

$$[K_{s_{2m}}]^{(e)} = \frac{C_s}{\lambda} \frac{1}{B\Gamma_m} \int_{S_2(e)} [N]^T [N] dS_2^{(e)} \quad (25)$$

$$[C]^{(e)} = \int_{V(e)} [N]^T [N] dV^{(e)} \quad (26)$$

$$[C_m]^{(e)} = \frac{\rho_m}{\rho} \int_{V(e)} [N]^T [N] dV^{(e)} \quad (27)$$

$$\{R_{s_2}\}^{(e)} = \frac{C_s}{\lambda} \frac{1}{B\Gamma} \int_{S_2(e)} [N]^T \phi_\infty dS_2^{(e)} \quad (28)$$

$$\{R_{s_{2m}}\}^{(e)} = \frac{C_s}{\lambda} \frac{1}{B\Gamma_m} \int_{S_2(e)} [N]^T \phi_\infty dS_2^{(e)} \quad (29)$$

In Equations 20–29, $\{\phi\}$ and $\{H\}$ are the column vectors of nodal modified temperatures and time derivative of nodal dimensionless enthalpies respectively. The matrices $[N]$ are the interpolation functions and $V^{(e)}$ denote the volume and S_2 surface of the element respectively. The time derivative may be replaced by

$$\{\dot{H}\}_\beta = \frac{\{H\}^{n+1} - \{H\}^n}{\Delta Fo} \quad (30)$$

and the nodal modified temperatures evaluated at time level $n\Delta Fo + \beta\Delta Fo$ by

$$\{\phi\}_\beta = \beta\{\phi\}^{n+1} + (1 - \beta)\{\phi\}^n \quad (31)$$

The system of equations to be solved at each time step is then

$$\frac{[C]}{\Delta Fo} \{H\}^{n+1} = \frac{[C]}{\Delta Fo} \{H\}^n - \beta [K] \{\phi\}^{n+1} - (1 - \beta) [K] \{\phi\}^n + \{R\} \quad (32)$$

For $\beta = 0$ the algorithm is fully explicit and the $\{\phi\}^n$ can be determined from the $\{H\}^n$. For $0 < \beta < 1$ iteration is required within a time step since the $\{\phi\}^{n+1}$ are not known until the $\{H\}^{n+1}$ are calculated. In this study $\beta = 1/2$ and a “lumped” capacitance procedure were used. Mass lumping is a common practice and has been discussed in the literature.^{17–19} The properties and parameters used in this study to model A-357 alloy solidification are listed in Table 1. The model phase diagram derived from these parameters (nominal 7% Si) is shown in Figure 3. The applicable dimensionless enthalpy modified temperature relationship (Equations C-18 and C-19) is shown in Figure 4. Symmetry about the centerline of the

Table 1. Properties and Parameters Used in the Numerical Analysis

$\rho = 2.67 \text{ g/cm}^3$	ref [9]
$\rho_m = 1.50 \text{ g/cm}^3$	ref [9]
$C = C_s = 1.05 \text{ J/g}^\circ\text{C}$	ref [9]
$C_m = 1.12 \text{ J/g}^\circ\text{C}$ (Silica Sand)	ref [9]
$k = k_o = 1.87 \text{ W/cm}^\circ\text{C}$	ref [9]
$k_m = k_{om} = 0.0084 \text{ W/cm}^\circ\text{C}$ (Silica Sand)	ref [9]
$\lambda = 432.6 \text{ J/g}$	ref [9]
$\gamma = 0.13$	ref [20]
$T_s = 613^\circ\text{C}$	ref [21]
$T_e = 557^\circ\text{C}$	ref [21]
$T_m = 660^\circ\text{C}$	ref [21]
$L = 76.2 \text{ cm}$	

plate was assumed in order to reduce computational effort. This is not expected to introduce significant error except perhaps in regions near the riser. Insulated boundary conditions were applied at the outer mold boundaries. This was found to be acceptable since semi-infinite conditions were maintained for the solidification time involved. The metal was assumed to be initially at the liquidus temperature and the mold was initially at room temperature. Constant properties as listed in Table 1 were assumed to be distributed evenly over the entire solidification period so that the fraction of latent heat liberated during primary solidification was equal to the fraction solidified during that period and the remainder of the latent heat was released at the eutectic temperature.

ANALYSIS AND RESULTS

The location along the center line of the plate of various fraction solidified fronts versus time is shown in Figure 5. From this diagram it can be seen that at approximately $Fo = 0.027$ ($t = 4$ min) after the start of solidification, a large region along the centerline of the plate is nearly solidified. The capillary length between a given fraction solid curve chosen and the 100% solid curve is the vertical distance between these curves. Capillary length is plotted in Figure 6 versus position along the centerline of the plate for various fraction solid curves. The capillary feeding distance, ℓ_f , can be calculated using Equation 4 and the properties or parameters listed in Table 2 along with the velocity of the 100% solid front, V_s . The latter velocity is calculated from the numerical solution. The feeding distance ℓ_f versus distance along the centerline of the plate is also shown in Figure 6. The limits of centerline shrinkage are then the points of intersection of the curves for capillary length and the capillary feeding distance, ℓ_f . The feeding ranges, assuming feeding problems starting at various fractions solidified, are given in Table 3 as multiples of the plate thickness, S/L . These results are in close agreement with those obtained by Davies⁹ for Al 7% Si alloy.

Because of the hypoeutectic character of A-357 alloy, it is felt that it is not meaningful to treat the alloy in a manner similar to the way in which Davies⁹ treated Al 2% Si alloy (having the character of an LFRA). This is because eutectic solidification be-

gins at 62.5% solid in the model of A-357 alloy, whereas dendritic growth continues up to some much larger fraction solid in Al 2% Si. However, some qualitative assessment of the variation in microporosity along the plate length can be made if we simply assert that feeding through the solid dendrite network becomes more difficult as the solid fraction increases. Thus, the amount of microporosity will depend upon the imbalance between the ability to feed (volumetric flow rate of liquid metal through the network) and the tendency to consume (velocities of the various fraction solid fronts). Because of the gradual increase in the rate of movement of the 70% through 100% solid fronts followed by very rapid movement in the center of the plate followed by a gradual decrease, the interdendritic feeding distance will gradually decrease, then rapidly decrease, then gradually increase. This should lead to a microporosity profile that gradually increases, rapidly reaches a plateau at the middle of the plate, and then falls off rapidly near the riser end. Thus, a uniform distribution of porosity should be obtained in the center of the plate.

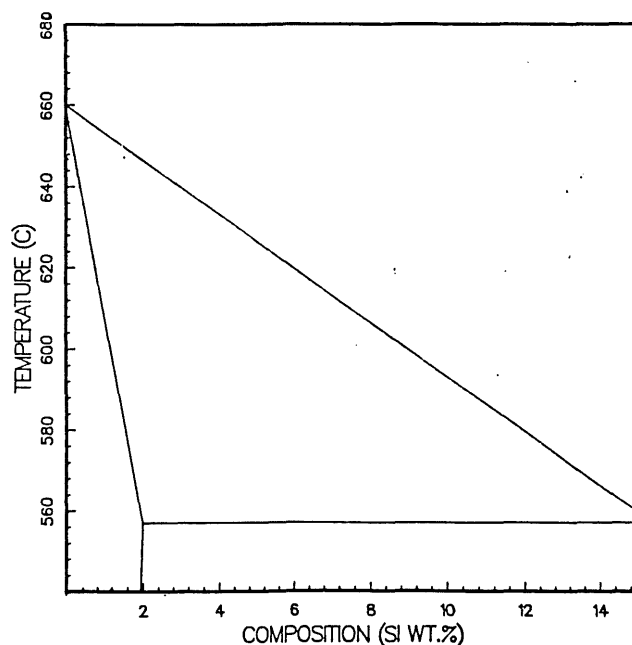


Fig. 3. Model phase diagram derived from the parameters listed in Table C-1 and used in the numerical analysis.

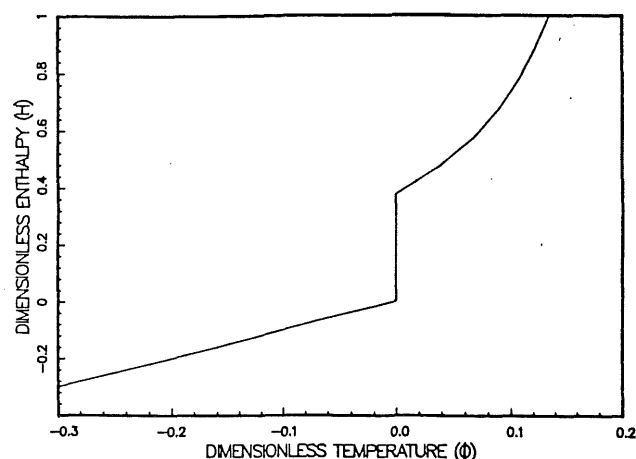


Fig. 4. Dimensionless enthalpy vs. modified temperature for A-357 alloy.

Table 2. Properties and Parameters Used to Calculate ℓ_f from Equation (C-4)

$B' = 0.22$	ref [9]
$\rho = 2.3 \text{ g/cm}^3$	ref [9]
$r = 0.75 \times 10^{-2} \text{ cm}$	ref [9]
$\mu = 0.030 \text{ poise}$	ref [9]
$m = 0.05$	ref [9]
$h = 14.1875 \text{ cm}$	

Table 3. Feeding Ranges for Feeding Problems Starting at Various Fractions Solid for A-357 Alloy

Fraction Solid	Feeding Range		
	end	feeder	max
90%	6.75 (S/L)	3.75 (S/L)	10.5 (S/L)
80%	5.2 (S/L)	3.3 (S/L)	8.5 (S/L)
70%	4.4 (S/L)	3.0 (S/L)	7.4 (S/L)
Eutectic	0 (S/L)	2.2 (S/L)	2.2 (S/L)

CONCLUSIONS AND OBSERVATIONS

1. The results of the analysis carried out in this study are in good agreement with the results of Davies.⁹
2. Approximate quantitative assessment of design parameters to prevent gross (centerline) shrinkage porosity for the alloy modeled in this study (A-357) alloy and other alloys of commercial importance can be derived from temperature field data obtained from a numerical solution to the solidification problem.
3. Qualitative assessment of the distribution of microporosity (interdendritic shrinkage) can be obtained for a particular casting geometry from the temperature field data.

ACKNOWLEDGMENTS

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NOMENCLATURE

- Bi Biot number, hL/k_0
 $\bar{Bi}$ modified Biot number, $Bi (T - T_\infty)/(\phi - \phi_\infty)$
 B^1 empirical correction factor
 C heat capacity of cast material
 C_s heat capacity of cast material at liquidus temperature
 E enthalpy
 f fraction solidified
 Fo Fourier number of cast material $k_0 t / \rho C_s L^2$
 ΔFo dimensionless time step size
 h metallostatic heat height in Equation C-4, heat transfer coefficient in Equation C-17

- H dimensionless enthalpy of cast material
 H_0 initial dimensionless enthalpy of cast material
 k thermal conductivity of cast material
 k_0 reference thermal conductivity of cast material
 L characteristic length (length of mold)
 m solidification contraction
 $\hat{n}$ unit normal vector to surface
 ℓ length of capillary
 ΔP pressure difference
 P_0 metallostatic pressure
 Q volumetric flow rate
 r capillary radius
 S plate thickness
 S_1 specified temperature boundary surface
 S_2 specified convection boundary surface
 t time
 T temperature
 T_e eutectic temperature
 T_m melting point of pure metal
 T_s liquidus temperature
 T_∞ ambient temperature
 V_s velocity of 100% solid front
 x' distance along centerline measured from end of plate
 β fraction of a time step at which FEM equations are solved
 γ equilibrium partition coefficient for Si in Al
 δ Dirac delta function evaluated at liquidus temperature
 δ' delta function of strength $1 - f_e$ evaluated at the eutectic temperature
 λ latent heat of fusion of cast material
 μ liquid metal viscosity
 ρ density of cast material
 $\rho \ell$ density of liquid metal

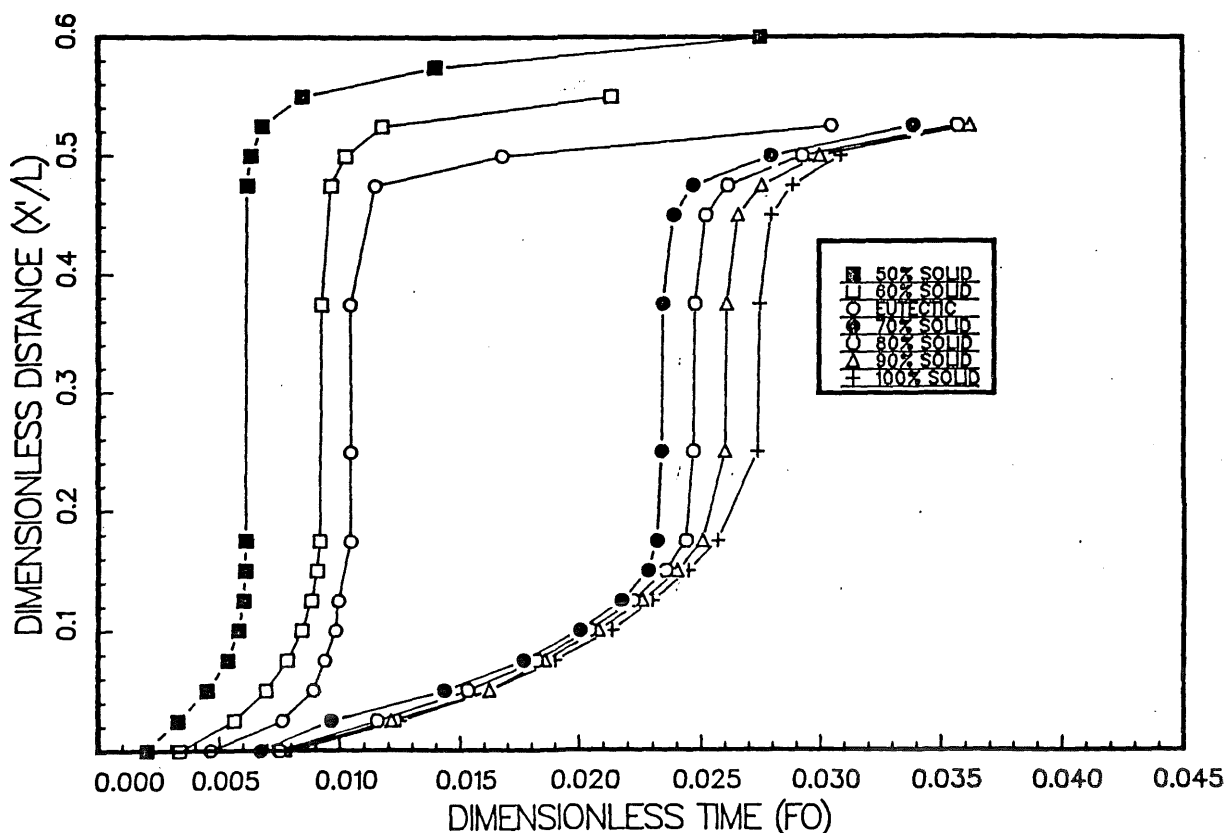


Fig. 5. Location along the centerline of the plate of various fraction solid fronts vs. time.

ϕ modified dimensionless temperature of cast material
 Ω solution domain

SUBSCRIPTS

m denotes mold material
 e denotes eutectic

SUPERSCRIPTS

n time step
 (e) denotes element
 T denotes transpose of matrix

MATRICES

$[K_v]$ volume integral of stiffness matrix
 $[K_{s_2}]$ surface integral of stiffness matrix
 $[C]$ capacitance matrix
 $\{H\}$ dimensionless nodal enthalpy vector
 $\{H\}$ time derivative of nodal dimensionless enthalpy vector
 $\{\phi\}$ dimensionless nodal modified temperature vector
 $[N]$ interpolation functions
 $\{R_{s_2}\}$ vector due to surface integral

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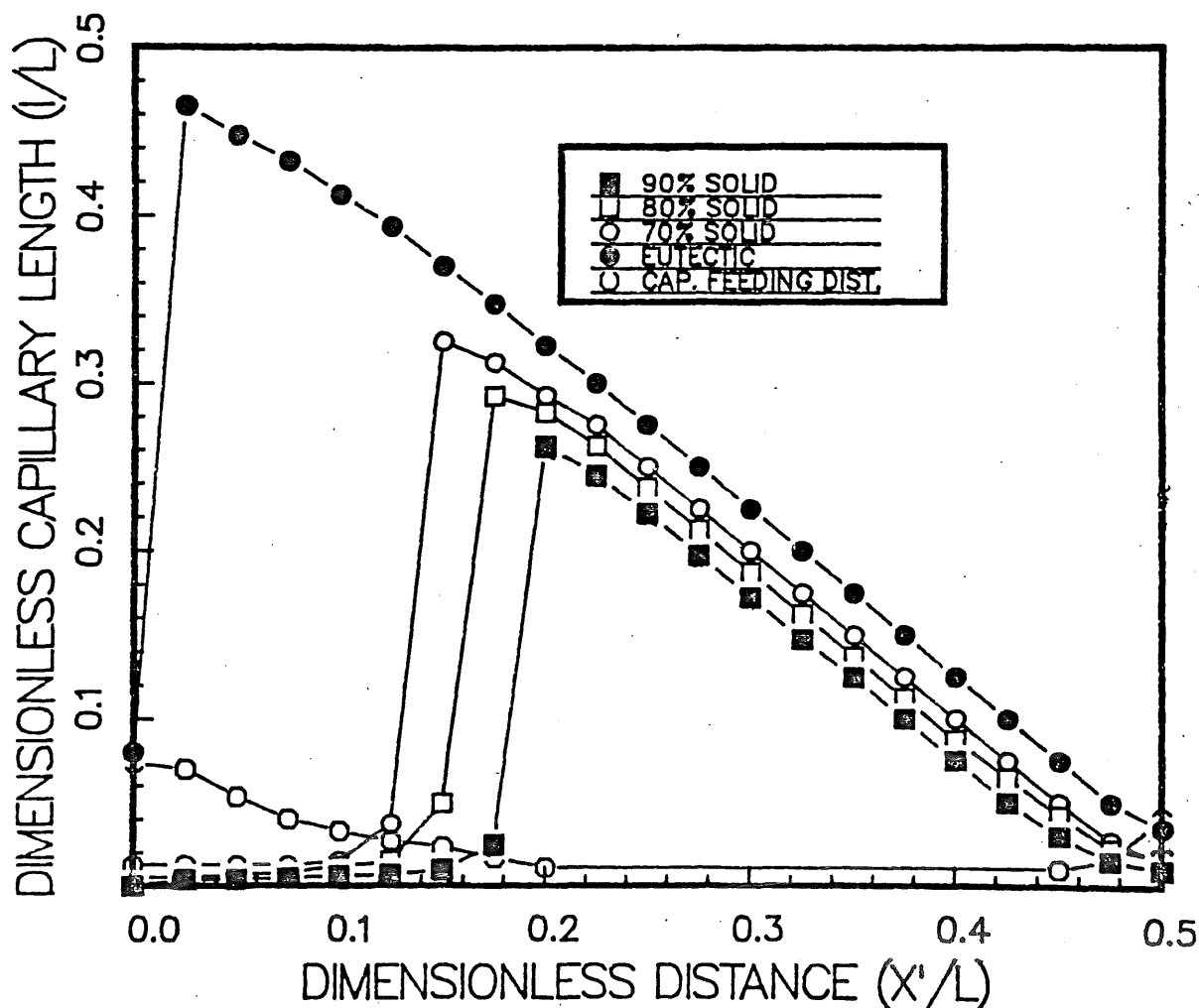


Fig. 6. Capillary length and Capillary feeding distance vs. distance along the plate centerline.

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Risering by Computer Assisted Geometric Modeling

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ABSTRACT

An overall information flow plan for computer aided engineering of casting processing and design is revealed. Application of the plan to geometric modeling of risering systems for castings is presented. The freezing order of casting sections is determined by obtaining the section modulus of the geometric subshapes comprising the design. The section moduli are obtained using a minicomputer program and graphics tablet input from the casting drawing. Volume and weight of the casting are obtained in the same way. An alternative method based on section modulus solidification wave fronts revealing directional freezing from casting, through riser contacts, to the riser obtained with a main frame computer program is also presented. Riserer by the geometric technique with cylindrical or tapered risers for specific examples with casting results illustrated.

INTRODUCTION

Castings and their necessary risering and gating systems are geometric objects of greater or lesser complexity of shape. The complete system may be modeled by building an assembly of primitive geometric sub-shapes such as plates, cubes, cylinders, bars, cones, tubes, etc. to establish a three dimensional geometric data base for engineering calculations. Building a complex configuration from its geometric sub-shapes is known as solid geometric modeling. Solid geometric modeling holds

promise as a major tool of computer aided engineering for the analysis of casting designs and related foundry process engineering. This paper will present some aspects of the use of solid geometric modeling for computer aided engineering of the risering of castings. Actually, risering is presently done by a mental process very parallel to solid geometric modeling whether this is consciously recognized or not. Computer aided engineering causes this situation to be recognized.

SOLID GEOMETRIC MODELING

For risering purposes, the design of the casting is analyzed for directional solidification, location of the last regions to solidify, hot spots, feeding paths and distances, casting volume, riser locations and section modulus. Risers and the feeding system may then be designed. The casting design may be modeled and analyzed either by two dimensional or three dimensional methods. Presently the principles of two dimensional analysis for risering are fairly well developed. But, computer software to model the casting geometry and permit rapid analysis for risering is just beginning to become available. Also, 2D analysis is limited to certain shapes. For complex shapes, 3D geometric modeling may be necessary. Much greater difficulty with the modeling system, considerably longer computing time and engineering man-hours are involved when 3D modeling of the geometry is required.

In either case, modeling the system provides the capability for casting design analysis and foundry process engineering for maximizing yield through the most efficient risering system. New programs for the currently available computer equipment are necessary to accomplish this. Developing these programs and related processing principles is the objective of the CAST METALS PROGRAM of research established by a consortium of foundries mainly from Wisconsin, in the Computer Aided Engineering Center of the College of Engineering of the University of Wisconsin-Madison.

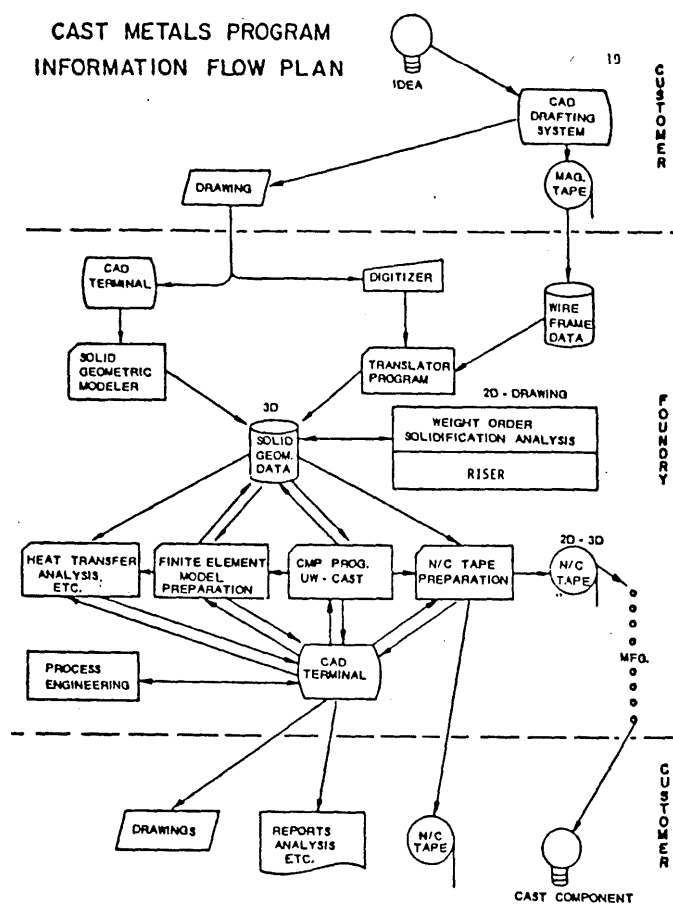


Fig. 1. Cast metals program information flow plan.

Solid geometric modeling by both 3D and 2D techniques is important research area in the Cast Metals Program. Its key role is shown on the Information Flow Plan for the project, Fig. 1. The top area, Fig. 1, represents the component design of a foundry customer presented either as presently, on a conventional 2D drawing or blueprint, or in the case of certain companies on magnetic tape. The drawing or tape, of course,

represents the finished component design without processing allowances.

When the blueprint or tape is received by the foundry, process engineering may begin after the solid geometric data base is developed, middle area Fig. 1. Wire frame data from computerized drafting systems may be converted to a solid geometric data base via a Translator program developed at UW.¹ Or, the blueprint may be converted to the solid geometric data base either by digitizing or at a computer aided design terminal. After digitizing the blueprint geometry, the translator program produces the solid geometry data base. Or at the CAD workstation, the solid geometric modeling software program permits the operator to directly model the casting from the blueprint. This is done by inputting primitive solid geometric subshapes such as plates, cylinders, cubes, cones, etc. and joining them as required by the print.

Figure 2 illustrates how an air-cooled cylinder casting can be modeled. An assembly of discs (1) is joined with a cylinder (3), to produce the finned shape (2). Next the bore is removed, Bosses (4) are added (5) and boss holes removed (6) and a sector of fins is removed to produce the air-cooled cylinder model (7). The model can be rotated and blend lines removed (7). Cross-sectioning can also be done. This geometric assembly process is familiar to foundry engineers because it mimics the current technique for determining total casting volume (and weight), i.e. to estimate the sum of all the primitive sub-shape volumes making up the total casting based on component blueprint dimensions. This tedious process is now built into computer software. Having built the solid geometric model of the component, the geometric data base is established and further engineering may proceed.

From the casting viewpoint, many important questions can be answered from the solid geometric data base. The CAST program, middle area, Fig. 1, provides videoscreen output including casting isometric views, perspective views from any angle, choice of section views, perspective views of sections, casting volume-weight-surface area calculation, casting section modulus calculation, cross section slicing and section modulus of late freezing sections and riser calculations. The most favorable coring of internal cavities can be pictured from the viewing and sectioning capabilities of CAST. Increasing dimensions for allowances and resultant changes in volume and solidification order can be evaluated through interactive computing by the program.

Without geometric modeling the computer input for CAST comprises digitized 3D points and lines from a drawing and identification of surfaces, order by bounding edges, material vs. non-material sides of surfaces to form the geometric model. This is similar to the modeling path via the Translator to the geometric data base, Fig. 1. The modeling path via the CAD terminal and geometric modeler does not require the digitizing step and is believed to be more amenable to casting process needs and also less costly. In either case, 3D finite element analysis, preparation for heat transfer studies of solidification for complete casting soundness, stress analysis and casting redesign for improved efficiency in utilization of metal properties may also be carried out from the data base.

All aspects of foundry processing including gating and material handling, tooling, lastings alloy properties, control of dimensions and inspection and other production question can be optimally engineered from the solid geometric data base, Fig. 1. NC tape for machining the paper and finally the casting can be

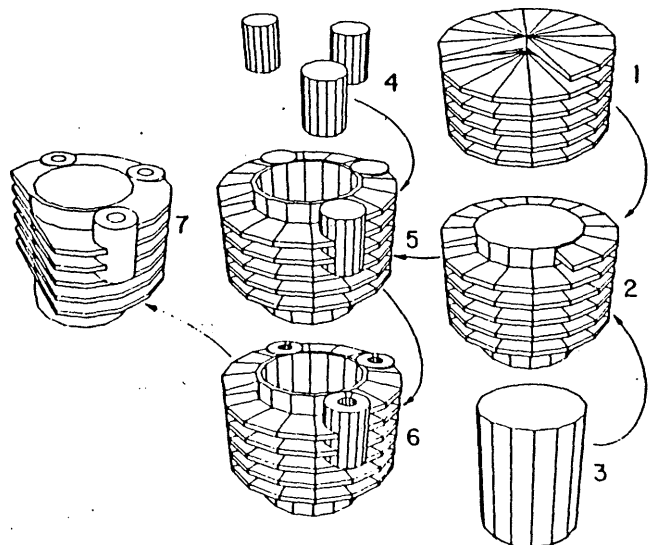


Fig. 2. Schematic representation of geometric modeling by assembling sub-shapes.

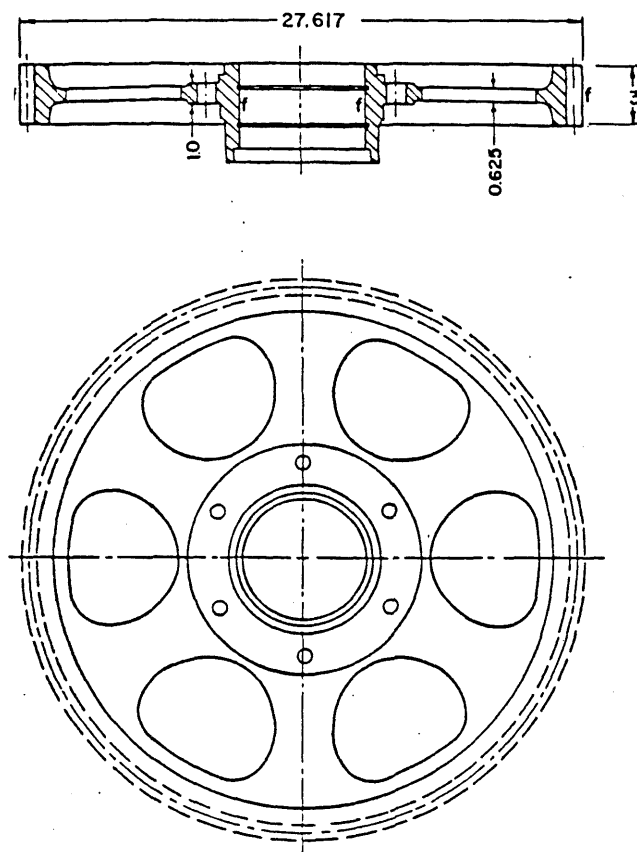


Fig. 3. Drawing of gear.

prepared from the solid geometric data base, Fig. 1. These capabilities are expected to eventually result from the UW CAST METALS PROGRAM. The final consequence of this approach to computer aided casting design and process engineering is expected to be the most functional and cost efficient cast component or product.

TWO DIMENSIONAL ANALYSIS

While casting process engineering and computer aided design becomes feasible from a 3D geometric data base, the expenditure of time and money for a 3D analysis may not be necessary for all aspects of casting processing. For castings of appropriate shape, a 2D analysis can accomplish some of the same objectives. Actually the conventional machine drawing serves as the 2D geometric data base from which engineering analyses are presently made. This is suggested in the middle of the CMP Information Flow Plan, Fig. 1. Casting shapes exhibiting symmetry with respect to an axis can most easily benefit from 2D analysis. Castings of revolution such as gears, wheels, pulleys or their sectors and extrusions such as plates, bars, rods, levers and arms can have the needed axial symmetry.

For 2D analysis, computer programs employing the Apple II or II Plus computer with disk drive, graphics digitizing tablet and pen, printer and video TV monitor have been specifically developed for calculating casting volume, weight and freezing order; the former called WEIGHT and the latter called ORDER (2).

The functioning of the program is best illustrated by means of

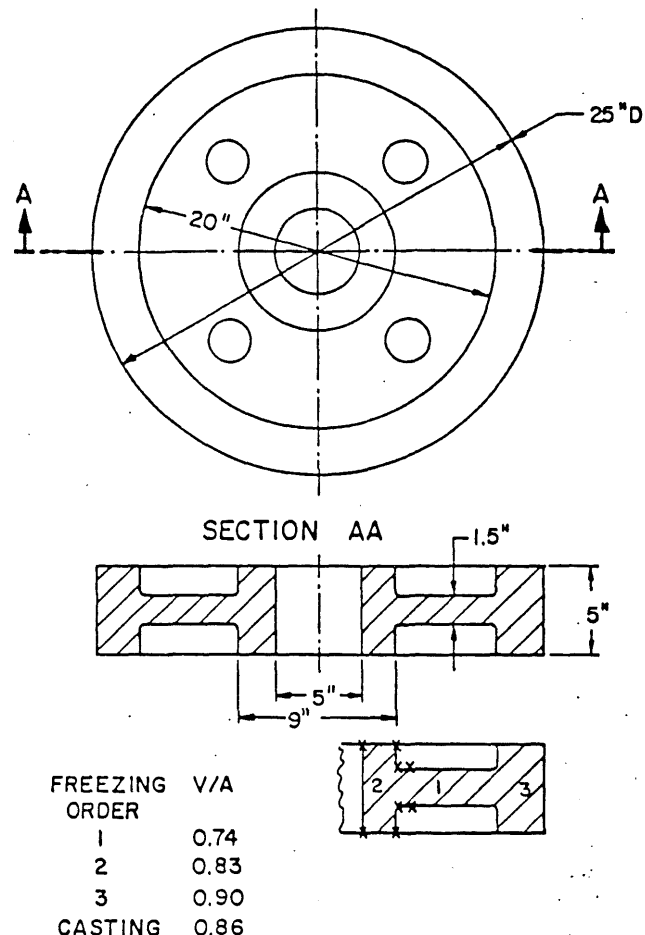


Fig. 4. Gear blank drawing.

a videotape. However, for the present reader's convenience a brief description of the tape presentation will be offered using the drawing of a gear shown in Fig. 3. The blueprint of the casting is placed on the digitizing tablet. Two points on the axis of rotation are entered first from the blueprint using the digitizing tablet and pen. The dimensional scale is then established by entering two points of known distance on the blueprint so that all subsequent dimensions are scaled accordingly.

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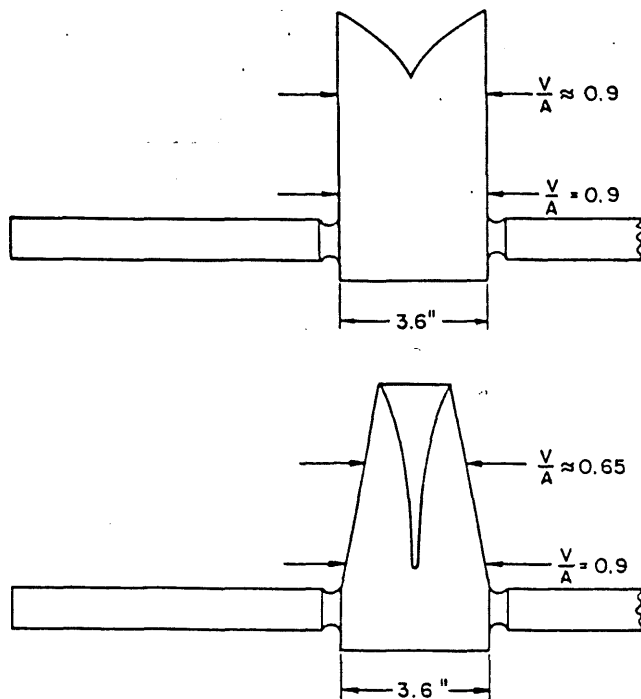


Fig. 5. Piping cavity in conventional cylindrical risers versus hollow tapered risers.

PIPING IN CYLINDRICAL vs. TAPERED RISER

modulus. Section modulus is defined as the casting volume divided by its surface area transferring heat out of the casting; V/A . In a 2D cross section, section modulus is estimated by cross-sectional area divided by perimeter.

This is the assumption on which the 2D solidification order analysis presented on the videotape is based. The order of solidification for the casting in Fig. 3 is web first, hub T-junction second and flange T-junction last as shown on the videotape.

Figure 4 provides another gear blank casting example of section modulus — freezing order determination by this technique. The modulus of the entire casting was found to be 0.86 in. The hub T-junction modulus was determined to be 0.83 in., the web 0.74 in. and the flange T-junction 0.90 in. Freezing order and modulus is shown as the video screen as in Fig. 4 and may be recorded by a printer. Both T-junctions freeze later than the web and therefore require risering. Web section may be thick and so that only the flange T-junction needs to be fed. With section moduli known the Riser system can then be designed.

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However, tapered risers can accomplish this just as well as cylindrical risers. The tapered riser shown in Fig. 5, lower, has V/A riser $> 1.2 V/A$ casting at its largest section near the base but narrows in diameter toward its top. Several examples of open and blind tapered side risers which fed a one inch steel plate are shown in Fig. 6. The tapered riser is intended to leave a hollow shell that results from the most efficient use of feed metal.

The thickness frozen at different levels in a tapered riser may be 2D modeled. The thermal wall or thickness frozen that keeps the pipe open must become thicker as time increases during the feeding period as can be seen in the risers of Fig. 6. Thickness frozen follows the general equation $TF = c\sqrt{t} + b$ where TF = thickness frozen, t = time and c and b are constants.

Thickness frozen may be calculated in several ways or empirically determined from sectioned tapered type risers such as those shown in Fig. 6. Geometric analysis can be approached via the section modulus, V/A and Chvorinov's rule, $\log t = K \log (V/A)$. The equation for steel adapted from Fig. 19 of C. W. Briggs, AFS Transactions 1962, is $\log t = 1.8739 \log V/A + 1.13487$, where t = minutes and V/A = inches. However, this equation is not necessary for modeling the risers.

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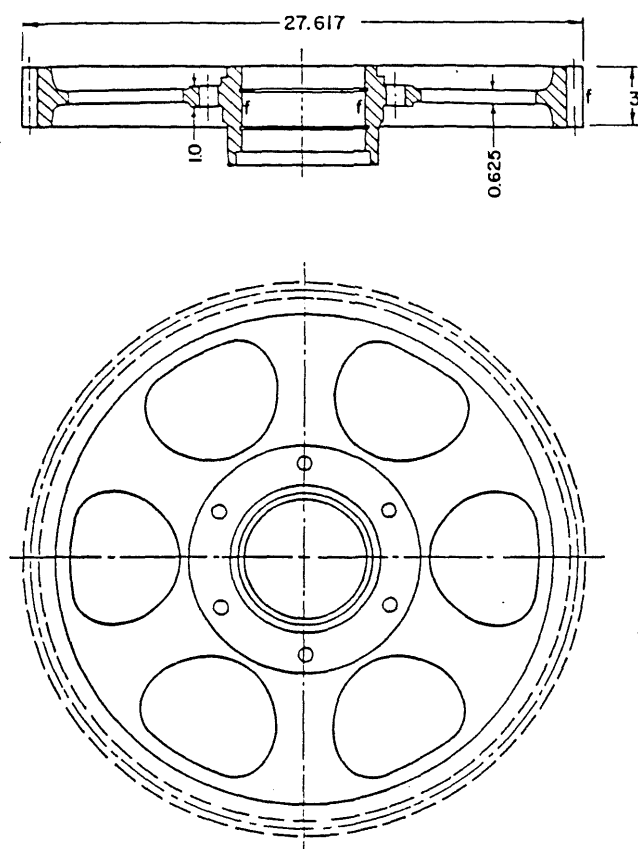


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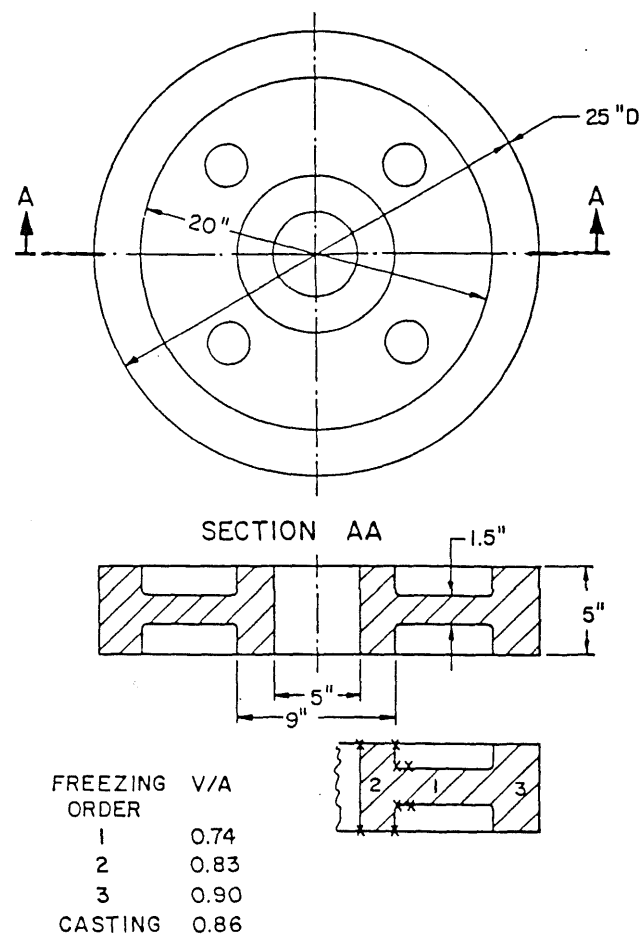


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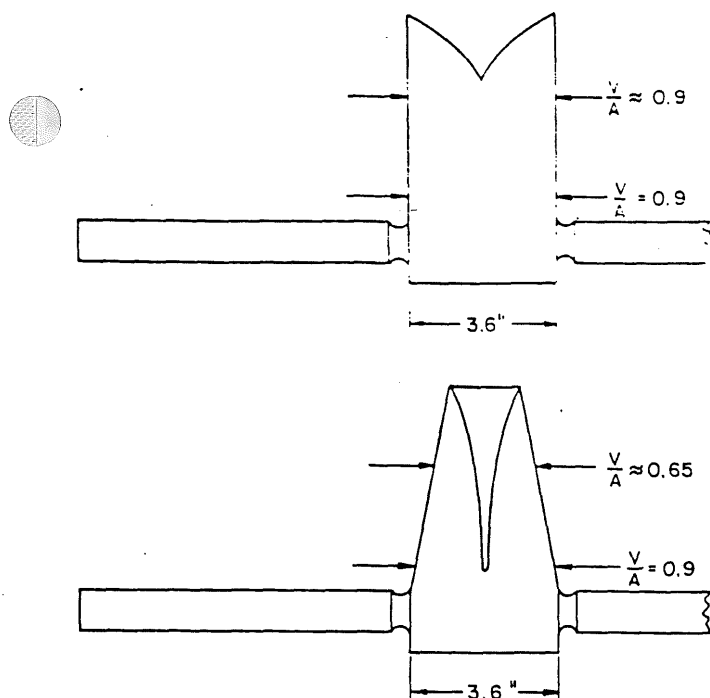


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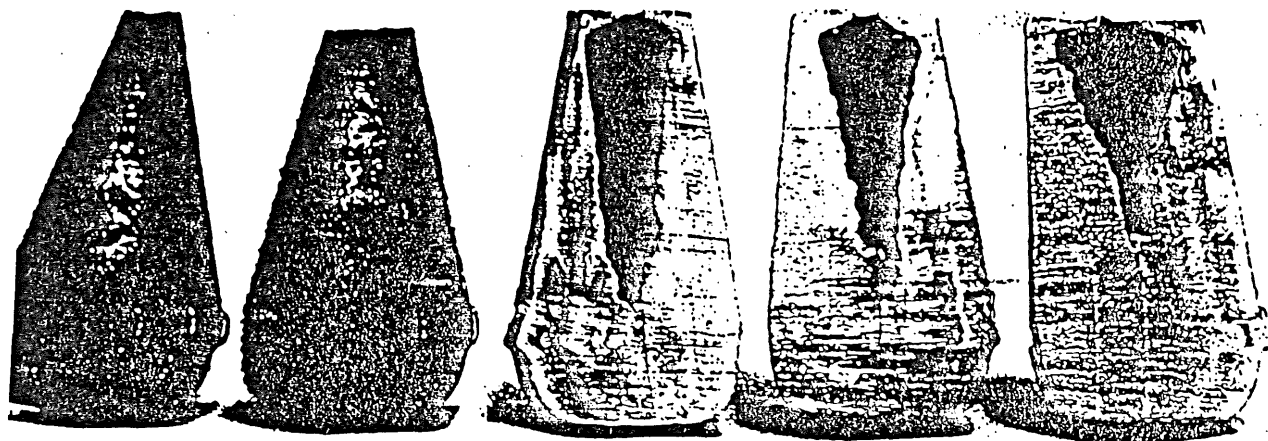


Fig. 6. Tapered blind and open side risers used to feed 1 in. carbon steel plates.

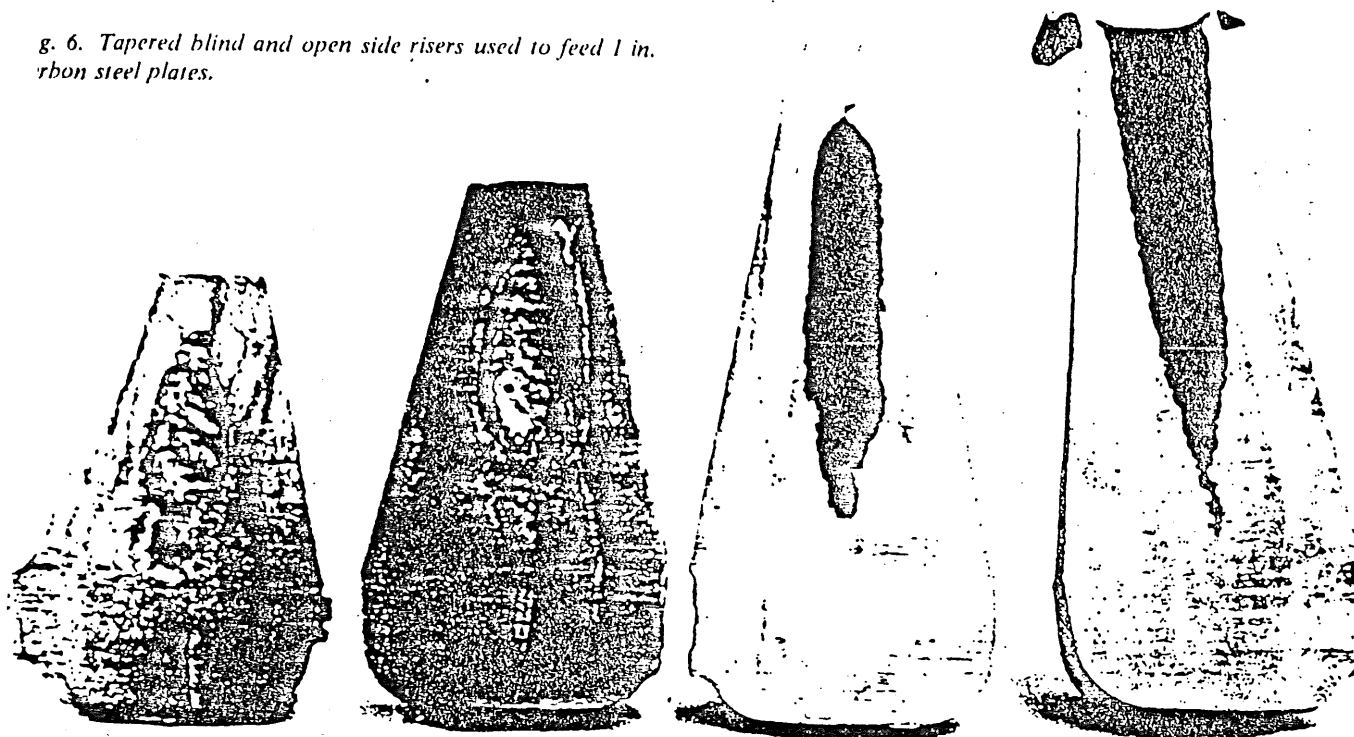
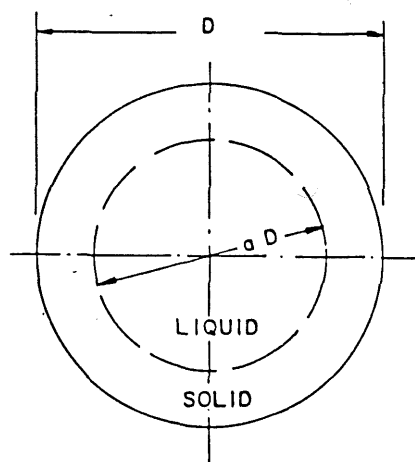


Fig. 7. Equations for calculating the section modulus, V/A , of the thickness frozen, TF , in a riser.



$$\text{EQN. 1 THICKNESS FROZEN, } TF = \frac{D - aD}{2}$$

$$\text{SECTION MODULUS OF TF} = \frac{V}{A} = \left(\frac{\pi D^2}{4} - \frac{\pi a^2 D^2}{4} \right) \div \pi D$$

$$\text{EQN. 2 } \frac{V}{A} = \frac{D(1 - a^2)}{4}$$

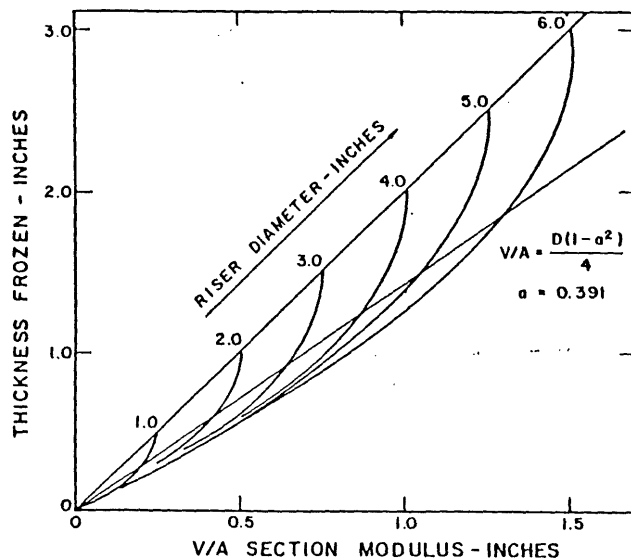


Fig. 8. Graphical relationship of thickness frozen, TF, to section modulus of the thickness frozen for riser diameters of 1.0 to 6.0 in.

Tapered Riser Design

The extent to which tapered risers actually behave, as suggested by Fig. 8, can be measured from the thickness frozen at a given diameter of a number of sectioned risers such as the ones shown in Fig. 6. The line on Fig. 8 labeled $a = 0.391$ is an example of this behavior in many ductile iron risers. The taper design can be varied as suggested in AFS preprint 82-20, Ref. 5 and as is evident in Fig. 6 and Fig. 9. The top diameter, maximum bottom diameter and height are obvious design features of importance as discussed in Ref. 5. The bottom diameter, D_R is designed from equation 3, $D_R = 2W + D_p$, where W is the casting's effective section thickness at the riser's location and D_p is the diameter of the pipe calculated by the geometric risering technique for cylindrical risers, Ref. 5 and 6. The effective section thickness $W = 2 \times V/A$ where V/A is the section modulus at the casting

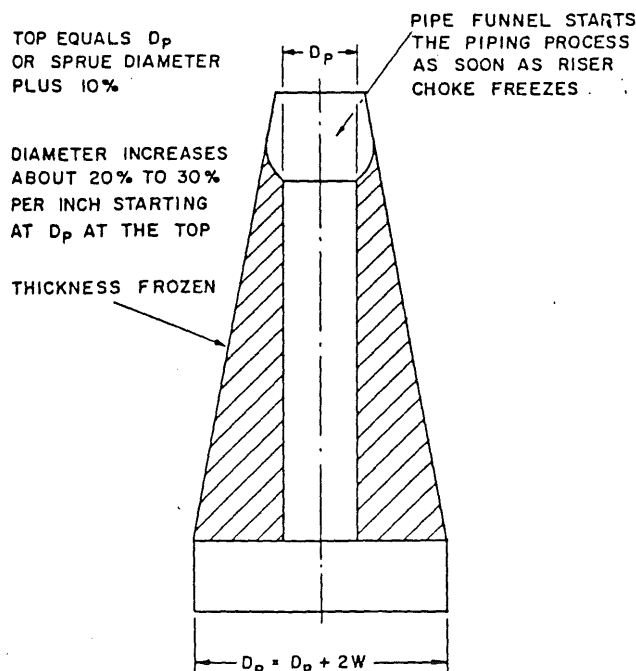


Fig. 9. Example of design system for tall tapered risers.

where the riser is to be attached. Equation 3 then becomes $D_R = f \times V/A + D_p$.

The geometric technique of risering has been amply publicized in the literature, Ref. 5-7, and will therefore not be discussed again. However, calculation of D_p requires feed metal constants for steel. Feed metal volume required is expressed by equation 4 as F_m , cu. in. = $K(V_c + V_R)$ where V_c = casting volume, cu. in.; V_R = riser volume, cu. in. and K is a constant dependent on section thickness. From experiments on 1.0 in. carbon steel plate castings, $K = 2.5$ to 3.0%. If the conservative assumption is made that yield is 50% or greater, true when tapered risers are used, then equation 4 becomes $FM = K \times 2 V_c$.

The value of K is being studied for geometric and metal factors. The value of D_p is then calculated from $FM = \pi(D_p^3 \lambda H_p / 4)$ where H_p is the riser height chosen on the basis of mold rigging or cope height. The feeding section of the riser can then be completely dimensioned. A typical design is shown in Fig. 9 and an example of a tapered open top riser from a commercial steel casting is shown in Fig. 10. Gating design is an important feature of this risering system. The gating is designed with a choke to cause it to freeze off quickly so that piping begins early after pouring.⁵

Optimizing riser shape and efficiency is an objective of CMP research. The same feed metal volume might be supplied by tapered risers of varying height and constant D_R at the base, Fig. 11. The shortest hollow riser will provide the highest yield. A stepped riser cylinder may be more efficient than a tapered riser. Fig. 12 shows stepped risers of cast carbon steel feeding 1.0 in. plates tapered to increase feeding distance. Efficiency calls for feed metal volume sufficient to feed both casting and riser leaving a hollow shell of minimum weight. Hollow shell tapered or stepped risers provide riser yield in the 55-70% range when yield is defined as casting weight divided by casting weight plus riser weight. The development of improved yield risering is another of the objectives of the CMP.

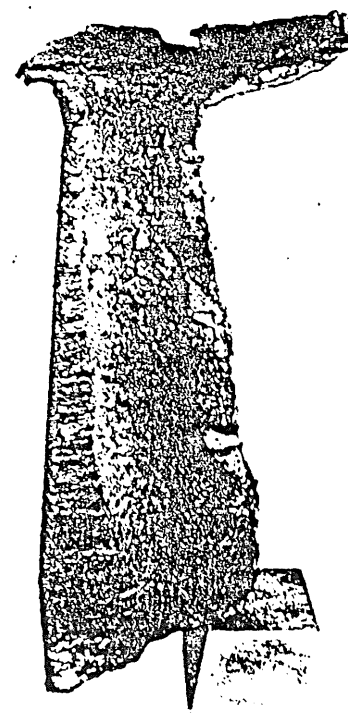


Fig. 10. Tapered open top riser from a commercial steel casting

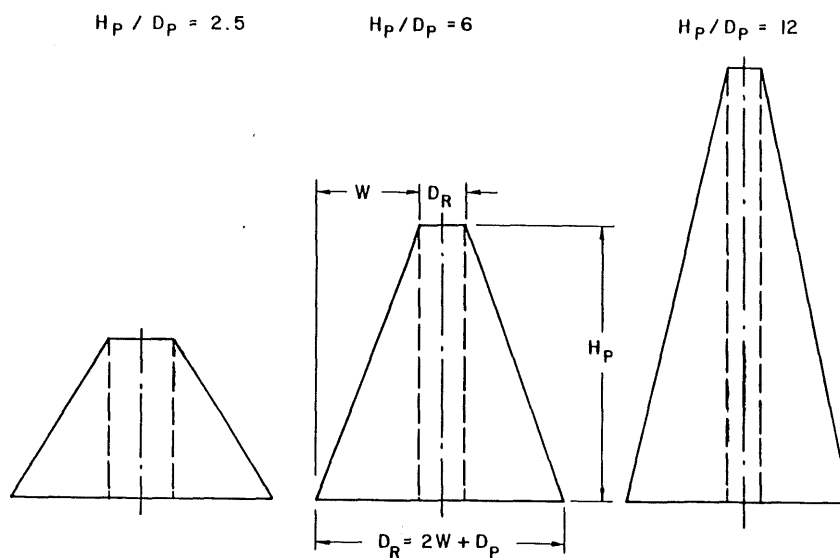


Fig. 11. Equal feed metal volumes are designed into each of the three shapes shown.

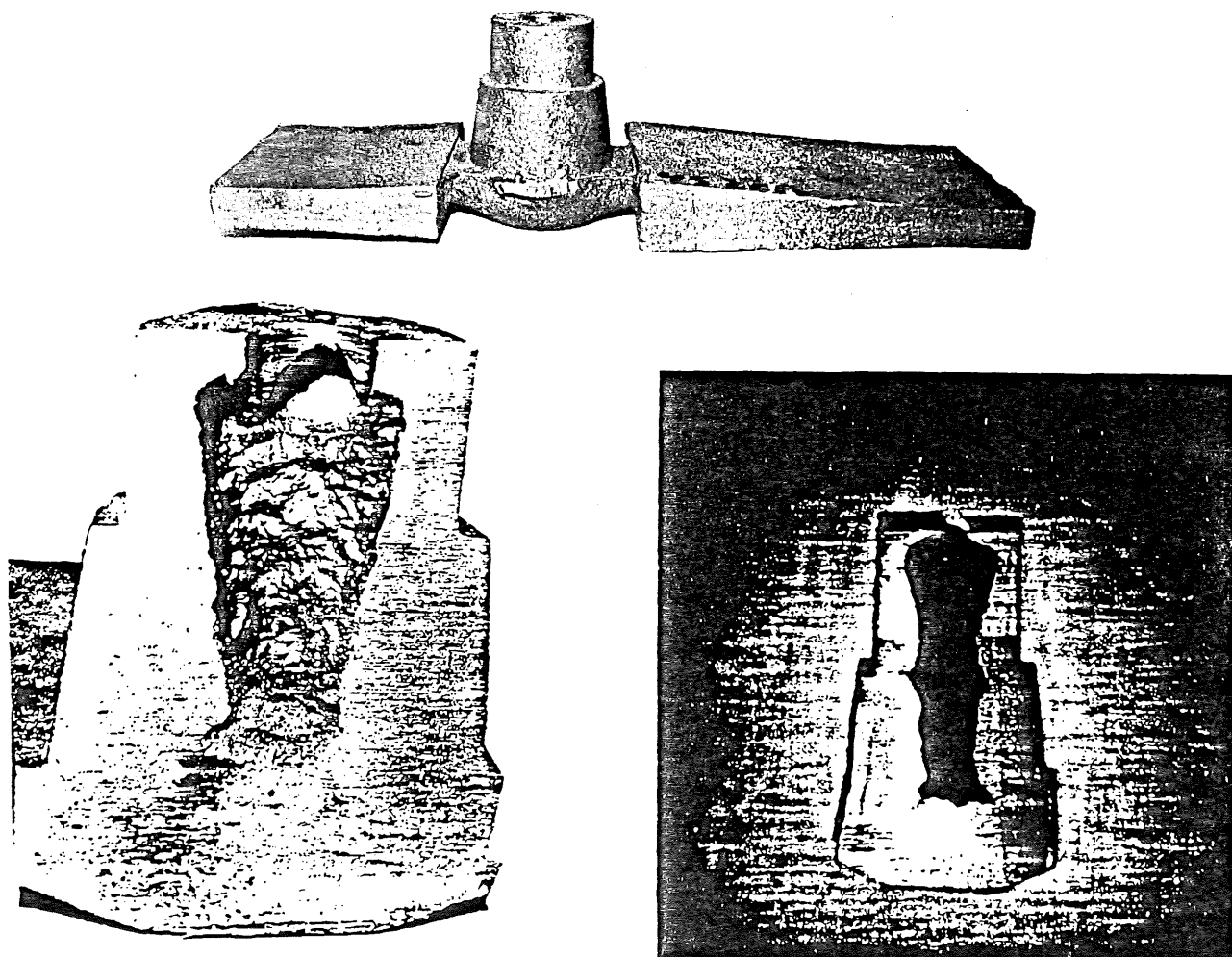


Fig. 12. Photograph of castings and stepped cylinders riser feeding 1.0 in. thick plates tapered 10% in thickness and 5 x 9 in. and 5 x 5 in. in width and length.

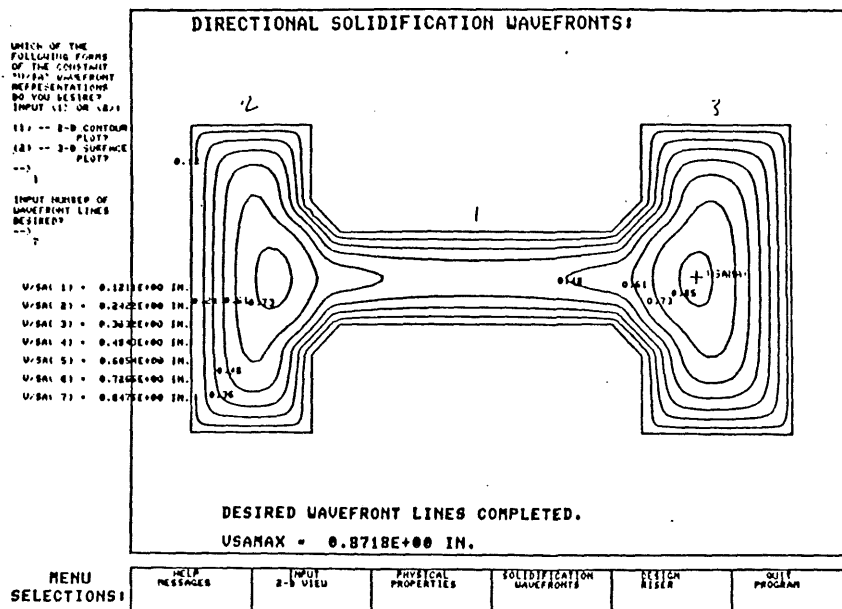


Fig. 13. Solidification wave fronts and their section modulus for gear blank in Fig. 4.

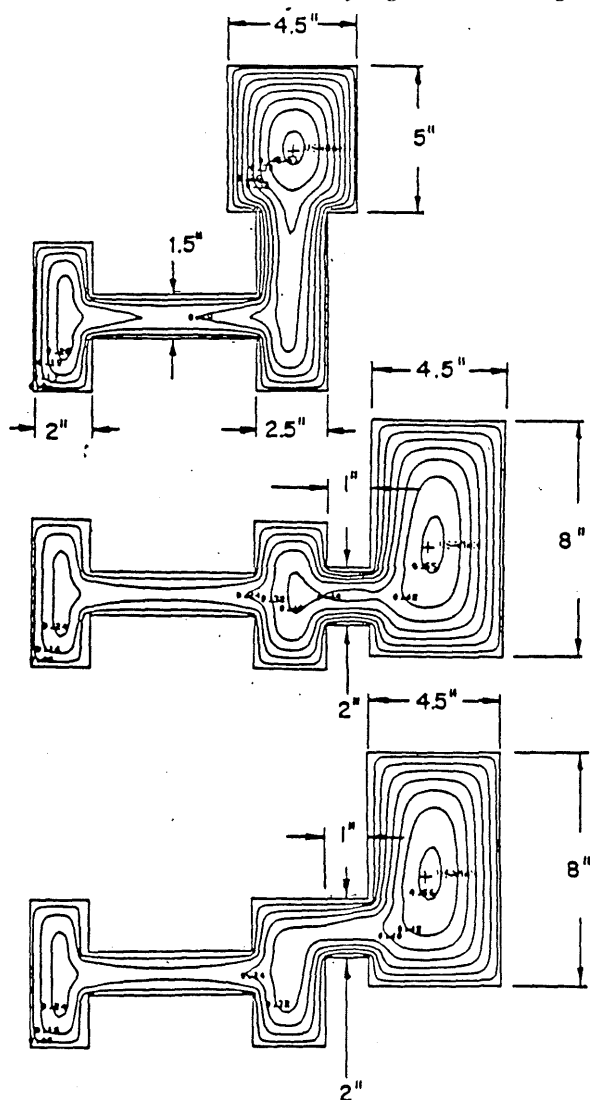


Fig. 14. Effect of risers and riser contacts on solidification wave fronts in gear blank casting of Fig. 12.

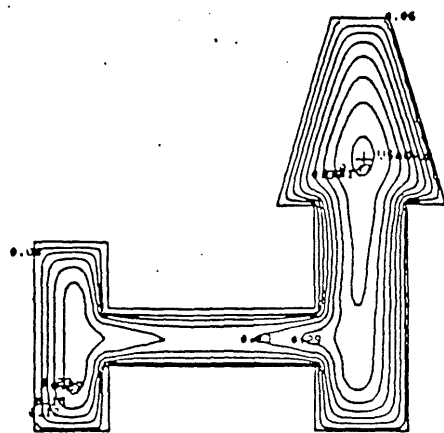
Geometric modeling and analysis can also be applied to riser contacts, bases of side risers and taper to promote feeding in various sections as has been described in articles in the bibliography of Ref. 6. Obviously, the entire system requires directional solidification from faster freezing casting regions toward slower regions, through riser contacts and finally into the riser. Meanwhile feed metal is transferred from the riser leaving behind a time dependent solidified thickness that reflects the solidification process in both the casting and the riser. The thickness solidified in the casting can also be modeled geometrically by 2D and 3D analysis of solidification fronts similar to the analysis suggested for risers.

MODELING OF SOLIDIFICATION FRONTS

Classical heat transfer calculations by finite element or difference techniques have been used to simulate the progress of solidification wave fronts with time from the casting surface to the end of solidification. Even for simple 2D shapes such as infinite cylinders, plates, bars, etc. large amounts of programming time and expense are required. Even larger amounts are necessary for 3D analysis and of course the solid geometric data base must be available to prepare the finite element grid. In addition the geometry must be described by the 3D data base in a way that permits the calculation methods to proceed, Fig. 1. To the extent that they are practical, 2D analyses should be developed as an alternative.

Solidification wave fronts may be geometrically modeled using the Chvorinov principle. The riser analyses of Fig. 6 and 7 provide an example. Computer software for calculating solidification fronts in casting cross sections has been developed through CMP research. The program results will be described in the case of the simple gear blank shown in Fig. 4.

The WEIGHT ORDER program gives the overall section modulus as 0.86; and freezing order as (1) the web as 0.74; (2) the hub junction as 0.83; and (3) the flange junction as 0.9. The



DESIRED WAVEFRONT LINES COMPLETED.
USAMAX = 0.4805E+00 IN.

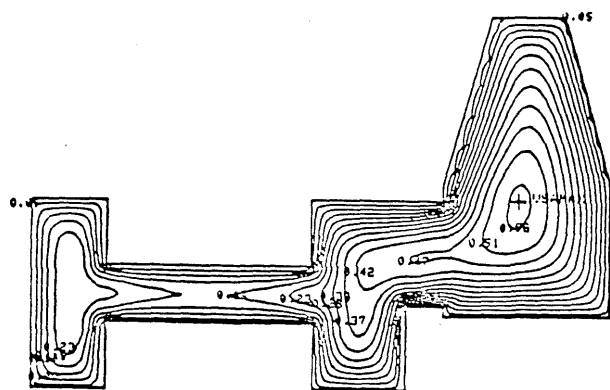


Fig. 15. Effect of tapered riser shape on the solidification wave fronts shown in Fig. 14.

results of solidification front analysis are shown in Fig. 13. The program indicates a maximum section modulus of 0.872 in. in the flange T-junction and a freezing order the same as in Fig. 4. The small difference between section modulus in the two different methods of analysis result from a slightly different scale input. Actually the difference in calculating $D_R = 4 \times V/A + D_p$ is only 4×0.872 vs. 4×0.9 or 3.48 vs. 3.60 in. with D_p remaining the same.

The software, called SOLIDIFICATION ANALYSIS [8] in Fig. 1, produces 2D solidification wave fronts in the gear blank cross section as shown in Fig. 13. Seven wave fronts and the last point to freeze are shown. Section modulus of each front and the maximum for the last point to freeze is tabulated on Fig. 13. Freezing order by the SOLIDIFICATION ANALYSIS program is the same as with the ORDER program and the section modulus values are nearly identical. The program permits analysis of regions of interest with more closely spaced wave fronts.

Adding risers and riser contacts to the casting results in interesting changes in solidification wave fronts depending on the location of the riser attachments. Only the flange riser location will be analyzed. Three points of attachment may be considered as shown in Fig. 14. The top riser would obviously be preferred from a yield viewpoint though it presents molding problems. The riser contact which produces an x-junction, Fig. 14 middle, leads to solidification delayed longer than if there were no riser attached and shrinkage would remain in the casting.

The riser contact which produces an L-junction, Fig. 14 lower, has favorable directional solidification although a slightly thicker contact would provide a margin of safety. Changing to the tapered riser shape alters the solidification fronts in the top and L-junction riser locations as shown in Fig. 15. Note that the final point of solidification (or maximum section modulus) moves closer to the riser contact in Fig. 15 vs.

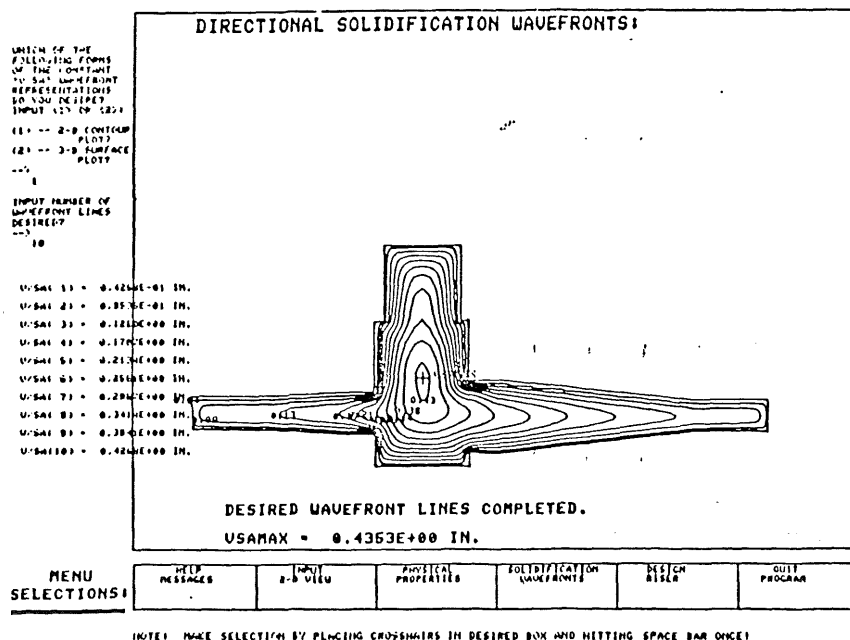


Fig. 16. Solidification wave fronts in cross section of tapered 1.0 in. plate and stepped riser as shown in Fig. 11.

Fig. 14. The tapered riser results in directional solidification in the riser from the top toward the riser contact. Liquid metal leaving the riser reveals the solidified thickness at its time of departure, Fig. 6, 10 and 12.

Geometric modeling of risers and riser contacts can lead to the most efficient use of feed metal and maximum yield. Obviously the casting exerts an influence on solidification of the contact and the riser because it alters the geometric configuration at the location where the casting must be fed. To use the principle of directional solidification at maximum yield efficiency apparently requires the kind of solidification front analysis just described. For example, tapered or stepped risers can cause directional solidification to be completed just outside the riser/casting contact. A solidification front analysis of the tapered plate—stepped riser combinations in Fig. 12 is shown in Fig. 16. Note the way the solidification fronts in Fig. 16 follow the step in the riser as revealed by the cross section of the riser in Fig. 12. Full utilization of these principles will require a 3D solid geometric analysis in some geometries while many more can be satisfied by the 2D method.

CMP SOFTWARE

As might be expected, the CMP software programs are being steadily modified to improve their usefulness. The WEIGHT ORDER program is presently being field tested by sponsors for foundry use. Modification of the program toward significantly increasing its capability for dealing with more complex shapes is under way. When the programs are satisfactory in practice, it is the intention of the sponsors and investigators to make them available through the AFS. The ORDER portion of the program will be followed by a RISER program that is currently being written.⁹

The SOLIDIFICATION ANALYSIS program is still in the early stages of development. Research with this program will continue to explore both the 2D and 3D capabilities of this type of solidification front analyses as applied to risering, effect of chills, insulation, feeding aids, etc. as well as the many geometric questions such as padding, cooling fins, core effects, riser contacts, riser configuration and riser design. Gating and the effects of gating on feeding from riser is another research area.

Presently, the SOLIDIFICATION ANALYSIS program uses a main frame computer. However, it may soon be possible to adapt the program to a minicomputer and this attempt is under study. Correlation of solidification front analysis with actual thermal analysis of solidifying castings is a desirable and necessary step to determine the extent to which the methods are basically sound.

The equations and language of the programs can be modified in many ways to ensure basic soundness in principle. The intent of this paper is not to examine questions of that nature but rather present useful engineering tools appearing on the computer horizon through the Cast Metals Program in the Computer Aided Engineering Center of the University of Wisconsin College of Engineering at Madison.

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Extending the Modulus Approach to Feeding to Account for Corner Effects

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ABSTRACT

The Chvorinov-based modulus method has been used successfully in riser design now for a number of years. The questions of correction for casting shape and end effects, which represent areas of uncertainty, have generally been handled by artificially adjusting the magnitude of the "mold constant." As a result of a recent study of the edge effect on casting solidification a new approach to such problems has emerged.

It has been shown in the present work that for a wedge-shaped mold intrusion or extrusion in contact with a molten metal, the difference between its absorption of heat per unit time and depth, and that heat absorption rate occurring over a corresponding planar mold, does not vary with time. That is, although both heat transfer rates vary with time, the extent of their difference is invariable with respect to time. Based on this theoretical finding, an analysis of the extent of the corner effect on the freezing time of bar castings with polygonal cross-section has been undertaken, and a Chvorinov-type formula derived. This shorthand formula uncovers a previously inconspicuous size effect on correlation of the modulus and the square-root-freezing-time. Further quantitative study of the edge effect on the freezing of a polygonal cross-section reveals an important geometric parameter, termed the "shape index," which is defined as $\text{volume}/(\text{surface area})^2$ and used in correlating the edge-induced freezing time to the edge-free freezing time.

It is also found that several thermophysical properties of the cast metal, in particular the thermal conductivity, affect the solidification process significantly. Four cast metals (Al, Sn, Pb and 0.25% carbon steel) have been selected to represent different groups of properties, in a series of numerical analyses designed to study the property influence. The properties of silica sand molds are used to simulate the chilling process. The results reveal how the "shape index" affects the proportionality of the modulus and the square-root-freezing-time and further how cast metal properties can bias the influence.

INTRODUCTION

One of the most significant applications of the knowledge of the heat transport in sand casting systems to the rationalization of the founding art has been the dimensioning of risers. A great number of researchers have focused on the subject of risering techniques. The original endeavor in rationalizing the art of risering castings was due to Farquhar and Chvorinov. Farquhar¹ empirically set forth criteria for risering castings. Chvorinov² procured the risering principles

via the mathematical deduction of Schwarz's analysis³ and set up a rule that relates the freezing time of a slab casting to its volume to surface area ratio, which is commonly known as the "modulus."

Modified forms of Chvorinov's one-dimensional rule that account for simple surface curvature effects have been established.^{4,5} It is notable that the Chvorinov's rule has since been the rationale of all risering principles, even with non-slab shaped castings. The first risering equation taking into consideration the amount of contraction on solidification was due to Caine.⁶ Caine's empirical equation was based on Chvorinov's concept of relative freezing time of riser and casting. Adams and Taylor⁷ subsequently derived an analytical equation for risering simply-shaped castings and compared the results with that given by Caine's equation. Good agreement between the two formulae for steel castings poured in sand molds was obtained. The concept of using a correction factor in the basic risering equation to account for the effect of thermal additives applied to the riser and casting such as insulators, chills and riser toppings was also discussed by Adams and Taylor.⁷

In their extensive studies of solidification rates and risering criteria, Pellini and his co-workers⁸⁻¹¹ have established several practical rules for producing sound castings. The role played by temperature gradient within the casting in determining the feeding capacity was examined.⁸ To generalize the often empirical estimation of the shape effect on feeding Bishop, Myskowski and Pellini¹¹ introduced a shape factor, $(\text{casting length} + \text{casting width})/\text{casting thickness}$, to represent the ranginess of shapes for estimating the required size of cylindrical risers. Risering of sections with appendages was also studied.¹¹ Merchant¹² reorganized the data obtained by Pellini and company⁸⁻¹¹ and proposed a general equation for risers which is similar to Caine's equation.⁶ Parallel with the advancement of knowledge of risering, the technique of applying exothermic additives to risers in order to improve both yield and soundness had been advanced.¹³ The part played by mold dilatation effects on risering requirements has also been noticed.^{14,15}

The use of moduli in calculating the size, shape and location of risers provides an economical approach to the design of optimum risers. A systematic procedure for the modulus method has been provided by Wlodawer.¹⁶ Heine and his co-workers^{17,18} have independently promoted the concept of feeding path for predicting the requirement of risers, as a modification of the modulus method. The modulus approach has thus long been adopted as a practical approach to risering design by modern foundrymen,^{19,20} who have also recognized the economical aspect of using insulating materials for feeder heads in commercial foundry production. As a whole, the empirical science of risering is well established, particularly so for steel sand castings. However, the only available literature on the quantification of the role played by the corner effect in determining the chilling power of sand molds seems to be the works by Ruddell and Skinner,²¹ and by Brandt, Bishop and Pellini.²² This notable lack of information is due to the unavailability of a general solution for the heat diffusion within non-rectangular corners.

A generalized analysis of the transient edge effect on

heat conduction in wedges has recently been undertaken.²³ The analysis reveals the fact that, for a wedge subjected to constant surface temperature, the *difference* between its loss or absorption of heat per unit time and depth, and that heat loss or absorption rate occurring over a corresponding *planar* surface does not vary with time. Based on the theoretical finding, this paper is intended to analyze the extent of the corner effect on the freezing time of shaped bar castings with polygonal cross-sections, for which the end effect on the freezing of the mid cross-section is insignificant. The analysis leads to a Chvorinov-type formula. This shorthand formula uncovers an otherwise inconspicuous size effect on correlation of the modulus and the square-root-freezing-time. Further quantitative study of the edge effect on the freezing of a polygonal cross-section reveals an important geometric parameter, which has been termed the "shape index." This is defined as volume/(surface area)² and is involved in correlating the edge induced freezing time to the edge-free freezing time. It is also found that several thermophysical properties of a cast metal, in particular the heat conductivity, interact with the shape effect and contribute significantly to its solidification time. Four casting materials (Al, Sn, Pb and a carbon steel) are selected to represent different groups of properties, in a series of numerical analyses designed to study the property influence. The properties of silica sand molds are used in these computations to simulate the chilling process. The results elucidate how the "shape index" affects the proportionality of the modulus and the square-root-freezing-time and how cast metal properties can bias the influence. This new approach should prove to be of considerable utility in basic riser design problems.

SOLIDIFICATION TIMES OF SIMPLY SHAPED CASTINGS

In an ideal solidification process where a semi-infinite mold and a semi-infinite molten metal are brought into perfect contact, Schwarz³ has shown, having assumed constant properties for mold, solidified metal and liquid metal, that the mold-metal interface temperature is constant at all times during solidification. Chvorinov² further demonstrated the practicality of Schwarz's analytical solution in steel sand castings of finite thickness. An averaged temperature of the liquidus and the solidus was used as the freezing temperature required in Schwarz's formula for the mold-metal interface temperature. The balancing of loss and gain of heat in the casting and the mold has led to the well-known Chvorinov's rule, which states that the modulus of a casting, defined as volume/surface area, is linearly proportional to the square root of its solidification time for a specified set of mold and casting conditions. A straightforward application of the rule to the design of risers is to determine the minimum riser size required so that it (the riser) does not solidify before the casting which is to be fed. As a first approximation, the Chvorinov's rule for a large slab sand casting can be put into the following simple form:^{5,24}

$$\frac{V}{A} = \frac{2}{\sqrt{\pi}} \left[\frac{T_i - T_\infty}{\rho_c L} \right] \sqrt{K_m \rho_m C_m} \sqrt{t_f} \quad (1)$$

The symbols used are listed in the table of nomenclature. The assumptions implied in the above formula are: (1) all

thermophysical properties are independent of temperature; (2) the thermal conductivity of the cast material is infinitely large so that the mold-metal interface temperature is constant, either at the melting temperature of pure metals or an averaged temperature of the solidus and the liquidus for alloys,² throughout the solidification period; (3) the convection flow effects due to pouring and density differences are neglected and the superheat may be incorporated in the latent heat term; and (4) the sand mold is sufficiently large so that no temperature variation occurs at the outer surface of the mold. Corresponding formulae for cylinder or sphere castings poured in large sand molds have the general form,^{5,24}

$$\frac{V}{A} = \frac{T_i - T_\infty}{\rho_c L} \left[\frac{2}{\sqrt{\pi}} \sqrt{K_m \rho_m C_m} \sqrt{t_f} + \frac{n K_m t_f}{2r} \right] \quad (2)$$

where r is the casting radius and $n = 1$ for cylinder, and 2 for sphere. It ought to be noted that, for cylinders, the above formula is valid only for small time intervals.^{5,25}

TWO-DIMENSIONAL CORNER EFFECT ON SOLIDIFICATION TIMES

A recent theoretical study of the transient edge effect on heat conduction in wedges²³ has uncovered a significant edge effect on the amount of heat transfer across the surface of a wedge which was instantaneously brought into contact with an isothermal heat source. The nature of sand casting systems in maintaining the mold-metal interface temperature near the solidification temperature whilst the casting solidifies allows one to separate the heat transfer phenomenon occurring within a two-dimensional dry sand mold and casting system into two isolated ideal heat transfer sub-systems. With the same assumptions made earlier for deriving the Chvorinov's rule, the casting is to dissipate its latent heat into an isothermal heat sink, at the casting boundary, at the solidification temperature. The heat absorbed by the sand mold can be studied, separately, by consideration of the mold at its initial temperature being suddenly brought into contact with an isothermal heat source at the solidification temperature. By equating the heat loss and the heat gain in these two isolated sub-systems, a two-dimensional Chvorinov-type rule can be formulated.

Figure 1 shows two fictitious, unbounded wedge-shaped molds. The "unboundness" idealization is easily achieved in practice. Ruddle and Skinner,²¹ in their study of the heat extraction at rectangular corners in sand molds filled with

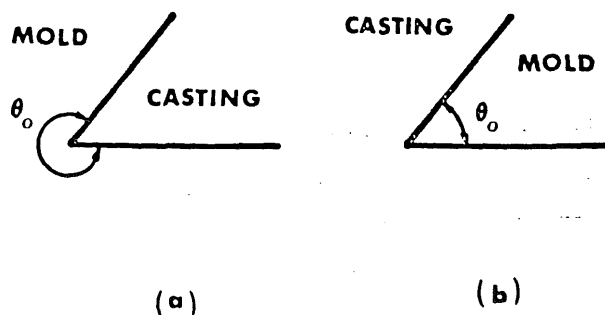


Fig. 1. (a) Semi-infinite mold intrusion; (b) semi-infinite mold extrusion.

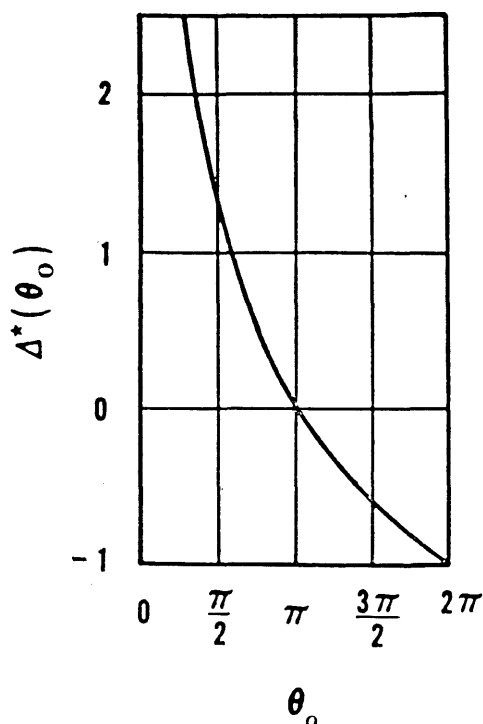


Fig. 2. Plot of $\Delta^*(\theta_0)$ vs. θ_0 ; after Hurwitz and Roe.²⁷

molten aluminum-30% copper alloy, observed that the edge influence on the isotherms measured diffused only 3-5 cm away from the edge in 15 minutes after pouring. A more general, theoretical analysis of the localization of the edge effect has also been realized.²⁶ For a wedge-shaped mold subjected to an isothermal heat source, it has been shown²³ that the *difference* between its absorption of heat per unit time and depth, and that heat absorption rate occurring over a corresponding *planar* mold, does not vary with time. This *difference* in heat absorption rate is a function of the wedge angle (θ_0) only and equals to $-K_m(T_i - T_\infty)\Delta^*(\theta_0)$, where²³

$$\Delta^*(\theta_0) = 2 \int_{-\infty}^{\infty} \coth \eta \left[\frac{1}{\pi \sinh \eta} - \frac{1}{\theta_0 \sinh \frac{\pi \eta}{\theta_0}} \right] d\eta \quad (3)$$

The above has been termed the wedge invariant and can be plotted²⁷ against θ_0 , as shown in Figure 2. The numerical values of $\Delta^*(\theta_0)$ are dimensionless.

The fact that the edge influence on heat transport is localized to the vicinity of the edge permits one to extend the result for a single wedge to the analysis of finite geometries, such as a long bar casting with a polygonal cross-section for which the end effect on the freezing time is negligible. Figure 3 shows such a cross-section, surrounded by a sufficiently large sand mold. Provided that no interactions occur among the neighboring edges during the solidification period, the total *difference* in heat absorption rate within the mold, as compared with that in a *planar* mold, is also invariant with respect to time and is equal to the algebraic sum of the contributing wedge invariants, multiplied by the thermal conductivity of the mold and the temperature difference. Symbolically,

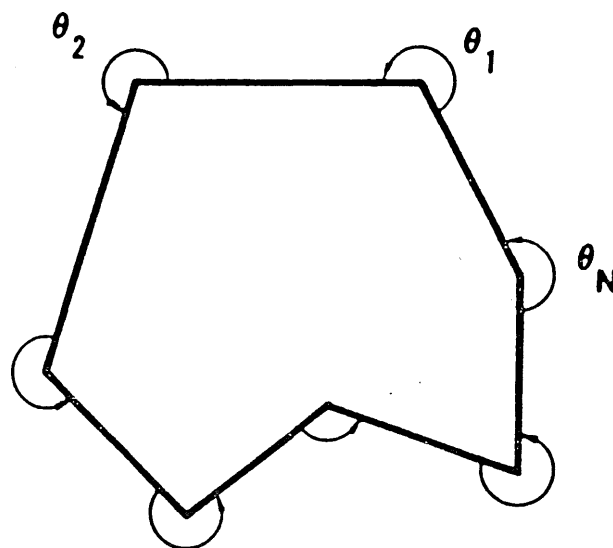


Fig. 3. Cross-section of a bar cast in a sufficiently large sand mold.

$$Q_{2d}(t) - Q_{1d}(t) = K_m (T_i - T_\infty) \left[-\sum_{n=1}^N \Delta^*(\theta_n) \right] \quad (4)$$

where Q_{2d} represents the total heat flow per unit time and depth, across the mold-metal interface, and Q_{1d} the reference heat flow rate across a corresponding planar mold of the same interface area.

Given the reference, one-dimensional heat flow rate,²⁵ one can integrate equation (4) with respect to time and obtain the total heat transported across the interface as a function of time. At the final freezing time t_f , this accumulated heat would be equivalent to the latent heat of the casting ($\rho_c V L$), and therefore leads to a modified Chvorinov's rule, namely:

$$\frac{V}{A} = \frac{T_i - T_\infty}{\rho_c L} \left\{ \frac{2}{\sqrt{\pi}} \sqrt{K_m \rho_m C_m} \sqrt{t_f} + \frac{K_m}{A} \left[-\sum_{n=1}^N \Delta^*(\theta_n) \right] t_f \right\} \quad (5)$$

Depending on the net geometric effect on diverging or converging the heat flow into the mold, the above equation states that the freezing time of the casting may be less or greater than that of a large slab having the same modulus.

CORNER AND SHAPE INDICES

The linear relationship existing between the modulus and the square-root-freezing-time, as exhibited in equation (1), does not hold for the two-dimensional formula (5) because the coefficient of the freezing time (t_f) is size-dependent. The implicit interplay of the corner and the size effects in formula (5) can be disentangled by introducing a heat transport parameter and a shape parameter. On the right hand side of (5), the first term corresponds to the heat that would have been transported had the corners been replaced by planar mold walls. It then follows that, on the right hand side of (5), the ratio of the second term to the first term would represent the extent of the corner induced diverging or converging heat flow. This ratio is called the corner index, denoted by ξ , and

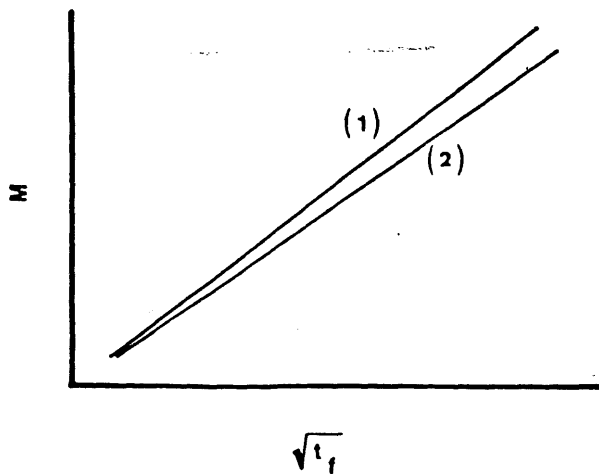


Fig. 4. A graphical representation of V/A vs. $\sqrt{t_f}$ for (1) a square bar, and (2) a large plate cast in sufficiently large molds.

$$\xi = \frac{\sqrt{\pi} K_m / \rho_m C_m}{2A} \left[\sum_{n=1}^N \Delta^*(\theta_n) \right] \sqrt{t_f} \quad (6)$$

The above expression is useful in determining the error incurred when one calculates the modulus without considering the corner effect. Another point of significance of the corner index is that it manifests the system variables comprising the corner effect.

Solving for the square-root-freezing-time in the quadratic equation (5) and substituting the result into equation (6) yields

$$\xi = -\frac{1}{2} + \frac{1}{2} \left\{ 1 + \frac{\pi \rho_c L}{\rho_m C_m (T_i - T_\infty)} \left[\sum_{n=1}^N \Delta^*(\theta_n) \right] \frac{V}{A^2} \right\}^{1/2} \quad (7)$$

The above expression indicates that the corner index ξ depends, geometrically, only on the product of the sum of the wedge invariants and a shape parameter, V/A^2 , which has been named the shape index.²⁶ It then follows that the corner index for linearly similar polygonal cross-sections is an invariant.

The concept of the corner index is helpful in quantifying the error incurred when one calculates the modulus without consideration of geometric effects. By denoting the solidification time resulting from the Chvorinov rule in equation (1) as t_f and that resulting from equation (5) as t_f' , one can derive a relation among t_f' , t_f and ξ by equating (1) and (5) which yields

$$\sqrt{\frac{t_f}{t_f'}} = 1 + \xi \equiv \frac{M}{M'} \quad (8)$$

where M/M' is used to represent the corner effect on the modulus of the casting. It is convenient to regard M as the virtual modulus obtained taking no account of the corner effect and M' as the effective modulus which is no longer just a volume to apparent surface area ratio. The effective modulus is a useful indicator for estimating the freezing time of a bar casting, on a V/A vs. $\sqrt{t_f}$ graph, from the straight line generated by equation (1).

Table 1. Latent Heat of Selected Cast Materials

Metal or Alloy	Latent Heat (J/g)	Density (g/cm ³)	Volume Latent Heat (J/cm ³)
Al	401.8	2.4	956
Sn	60.7	6.8	414
Pb	24.7	10.5	259
0.25%-C Steel	272.1	7.1	1940

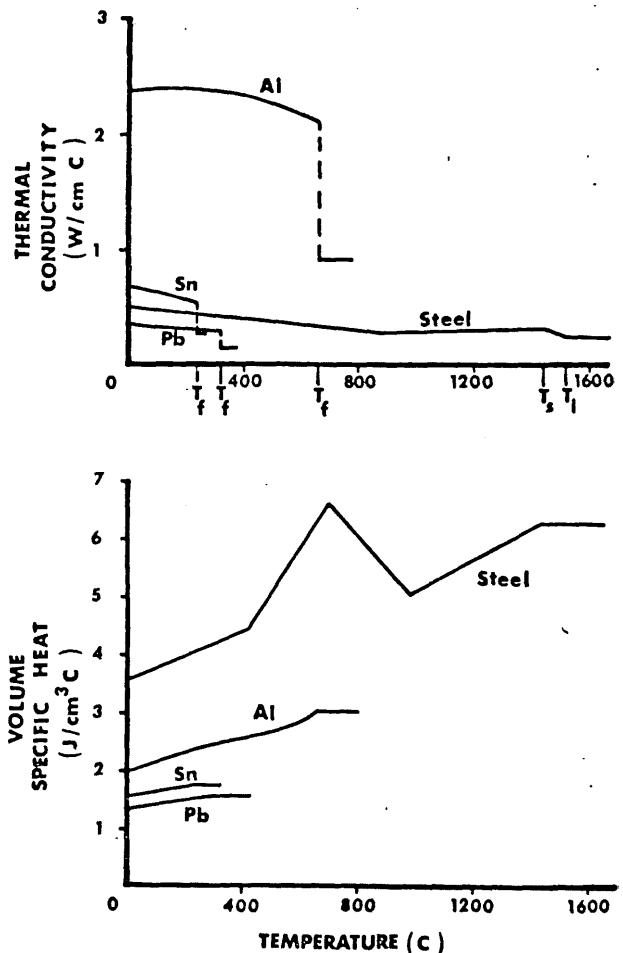


Fig. 5. Thermal conductivity and volume specific heat, as functions of temperature, for selected cast materials (T_f = freezing temperature, T_l = liquidus, T_s = solidus). After Refs. 19 and 30.

The above analysis states that the linear relationship between the modulus and the square-root-freezing-time still holds among a set of linearly similar polygonal cross-sections. Given a specified shape index, equation (5) traces a straight line on a V/A vs. $\sqrt{t_f}$ plane which is equivalent to a rotation of the line generated by equation (1) around the origin. Figure 4 illustrates a graphical representation of V/A vs. $\sqrt{t_f}$ for a square-sectioned bar and a large plate cast in sufficiently large molds.

ANALYTICAL STUDY OF THE PROPERTY INFLUENCE

The above theoretical analysis provides a solid basis for the

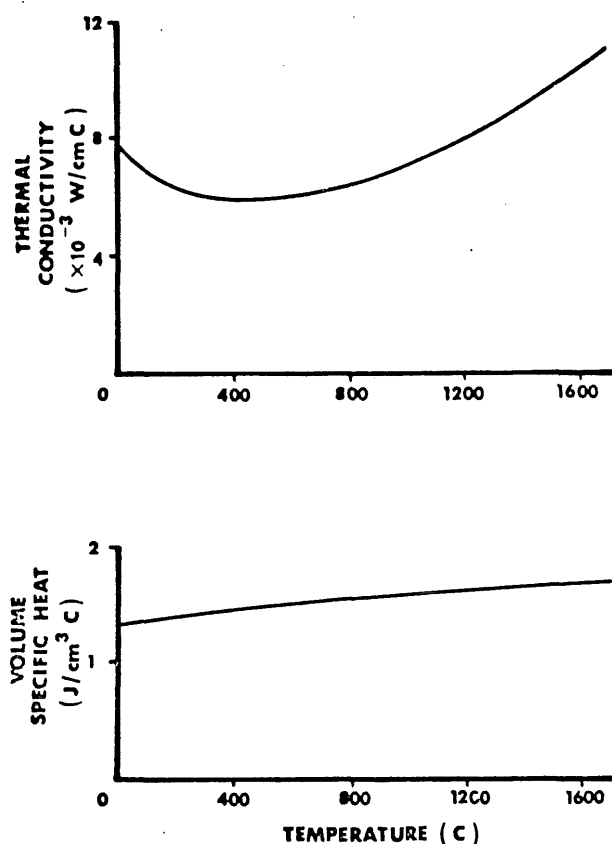


Fig. 6. Thermal conductivity and volume specific heat, as functions of temperature, for dried silica sand molds,³¹ density = 1.52 g/cm³.

quantification of the two-dimensional corner effect on freezing times. In practice, however, one particular assumption made earlier when deriving the shorthand rule needs to be refined. It was assumed that the thermal conductivity of the cast material was so great that little temperature gradient occurs across the casting during the solidification process. It follows intuitively that a poor heat conductivity would have a two fold effect on the freezing time: a larger temperature gradient which requires the mold to absorb a certain amount of sensible heat in the solidified layer and a lower mold-metal interface temperature which diminishes the chilling power of the mold. Other influential properties of the cast material on the solidification process may be the amount of volume latent heat and the solidification temperature.²⁸ Unfortunately, a closed form solution for the solidification process in the form of a shaped mold and casting system is far from one's reach. An analytical study of the influence exerted by material properties upon the corner effect has to rely on discretized, numerical approaches.

Four cast materials have been selected to represent different groups of properties in a series of numerical experiments. Table 1 and Figure 5 list the properties^{29,30} that have been used in the analysis. A set of recently acquired property data of dried silica sand molds,³¹ as shown in Figure 6, has been used to simulate the chilling process. A reliable explicit finite difference enthalpy model,³² has been used to simulate the heat transport within the mold and casting systems. The advantages of the enthalpy ap-

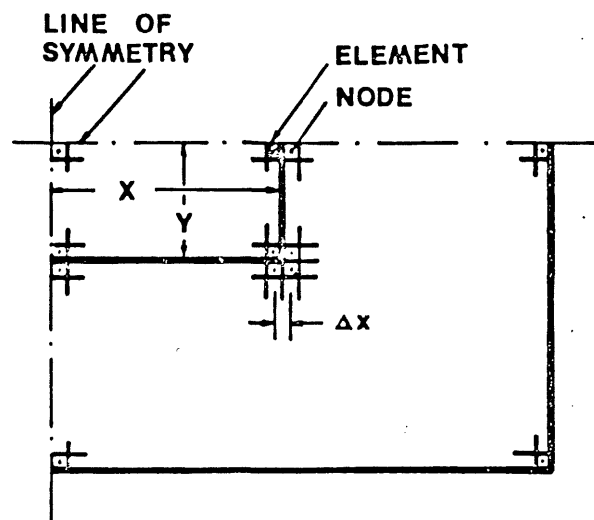


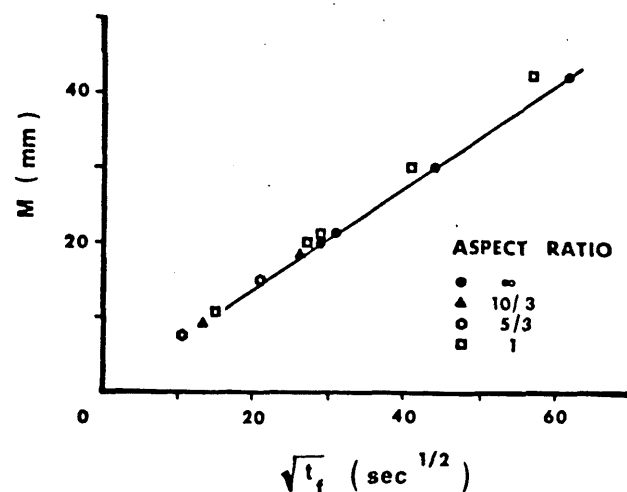
Fig. 7. Discretized mold and casting cross-section (aspect ratio = X/Y).

proach can be viewed in two major respects: the convenience in incorporating in the computation the temperature dependence of material properties, and the reliability in handling the loss of latent heat. The computations have been performed on rectangular cross-sections. Figure 7 illustrates the discretization of a rectangular cross-section surrounded by a sufficiently large mold. Equally spaced square elements and nodes are used to represent the continuous space. The temperature-time curves for all nodal points can be obtained by solving the discretized heat equations in the castings and the mold regions. The continuity of temperature and heat flux at the mold-metal interface are enforced upon the interface nodes at every discretized time step. The resulting freezing time is taken as the time step at which the temperature of the node nearest to the center first drops below the melting point or the solidus. Typical nodal distance Δx ranges from 2 to 10 mm. The solidus temperature of the low carbon steel was estimated, via Scheil's equation,³² as the temperature corresponding to a solid fraction of 90%. Zero superheat was used in these computations to avoid the complication caused by convection flows. The initial mold temperature was taken as the room temperature.

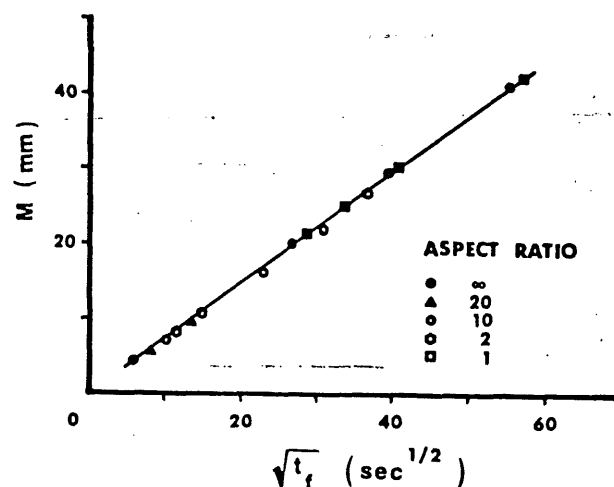
Figure 8 shows the analytical results. For rectangular cross-sections, the shape index varies only with the aspect ratio. In Figure 8, the aspect ratio is plotted as a parameter, and it will be noted that an infinite aspect ratio corresponds to infinitely large plates. Reference lines are also drawn through the points corresponding to these idealized systems.

DISCUSSION

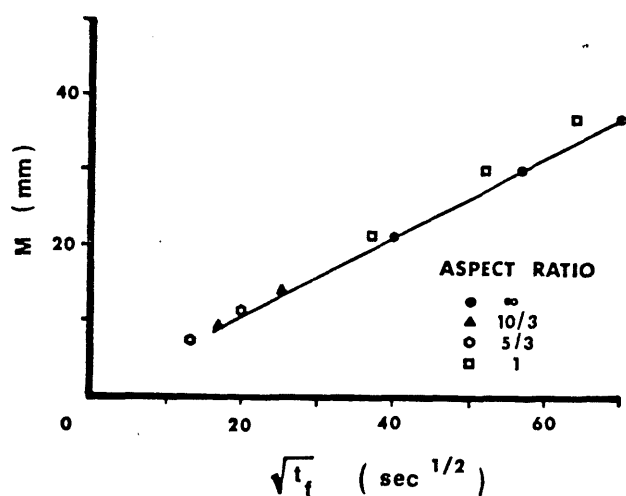
The validity of the theoretical assertions made herein on the geometric influence of corners upon casting solidification can be examined via the numerical experiments, which take into account the temperature dependence of properties of cast and mold materials. The roles played by thermal conductivity, solidification temperature and volume latent heat of a cast metal in pacing the solidification process can also be identified. Although differences in chilling power exist among various molding sands,^{2,5,31} the interplay of



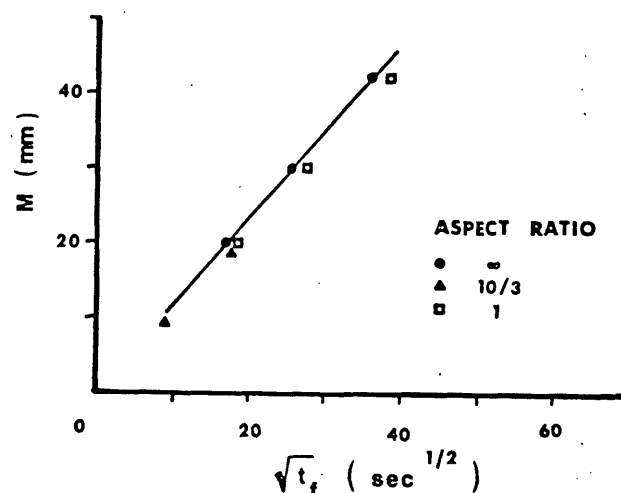
(a)



(c)



(b)



(d)

Fig. 8. Computed freezing times for various aspect ratios for (a) aluminum, (b) tin, (c) 0.25% carbon steel, and (d) lead. (Note and aspect ratio of ∞ represents an infinite plate, of one a square-sectioned bar.)

the property and geometric effects as revealed in Fig. 8 by adopting only dried silica sand molds will not be substantially affected.

The analytical results presented in Fig. 8 prompt the statement that the thermal conductivity of a cast metal is the most influential property in biasing the idealized edge effect with respect to the freezing time, as described by formula (5). On comparison of Fig. 8 with Fig. 4, several other factors emerge. For example, it appears that highly heat-conductive materials such as aluminum and tin exhibit the same corner effect as seen in an idealized cast material having an infinitely large thermal conductivity. The similarity in appearance of the numerically computed and analytical sets of results demonstrates the practicality of the theoretical approach.

Increasing the solidification temperature of a casting would normally enhance the chilling power of the sand

mold and, in effect, shorten the freezing time, while increasing the latent heat would do the reverse. However, as revealed in equation (6), a decrease in freezing time would result in a smaller corner index and therefore a less significant corner effect. Figures 8(a) and (b) reveal that the solidification temperature has been the dominant factor in determining these freezing times. It should be noted that all observations are based on the assumption of a perfect contact at the mold-metal interface. Generally speaking, the relatively low thermal conductivity of sand molds tends to veil the role played by the interface air gap due to density changes during the chilling process. In practice, however, the contribution of air gap in diminishing the heat flow may become significant in systems with low solidification temperature because of the fourth power relationship in the thermal radiation heat transfer across the air gap.

The geometric effect as illustrated in Figs. 8(a) and (b) is significantly biased in systems of poor heat conductive

metals, such as low carbon steel and lead. These metals exhibit large temperature gradients across the hot and cold regions in the casting, particularly at the corners, which would lower the mold-metal interface temperature and require the mold to absorb a certain quantity of sensible heat in the solidified layer during the solidification process. These effects jointly reduce the chilling power of the mold and prolong the freezing process. The counteraction of the enhanced heat transport in the mold due to its geometry with the impaired chilling power ascribed to the poor heat conductive metal determines the final corner effect on the freezing time. Both low carbon steel and lead systems exhibit a biased corner effect. The property effect overcomes the geometric contribution and results in an apparently reversed corner effect. In Fig. 8(c), the balance of these two contributing factors has produced a narrow distribution that appears to conceal the corner effect. This observation provides an analytical reasoning for the success in applying the modulus technique to feeding shaped steel castings without consideration of corner effects.¹⁶

Despite the influence exerted by properties of cast metals upon the corner effect on freezing times, the linearity with respect to the shape index, a remark that followed equation (8), holds for all cases. It is worth noting that this linearity also applies to rangy cross-sections for which the presumption of independent edge influences may be oversimplifying. In Fig. 8, all points representing a specific aspect ratio and therefore a specific shape index lie approximately on a straight line. The counteraction of the casting geometry with the contributing properties of the casting in determining the freezing time of a square-sectioned bar, in terms of the analytical equation (7) and the computed results, is summarized in Fig. 9.

CONCLUSIONS

This paper has presented a new, theoretical approach to understanding the corner effect on the freezing time of bar castings having polygonal cross-sections and poured in sufficiently large sand molds. It has also been demonstrated that several thermophysical properties of cast materials, in particular the thermal conductivity, play important parts in composing the corner effect. A rule of thumb that can be drawn from the analysis is that the linearity of the virtual modulus (V/A) with the square-root-freezing-time holds for all cross-sections having the same shape index, i. e., for linearly similar cross-sections. The interplay of the thermal conductivity of a cast metal with the casting geometry in producing reversed corner effects is examined. The quantification of how the shape index affects the solidification system is found to be material dependent. The new approach should prove to be fundamental for further studies of corner effects on casting solidification.

ACKNOWLEDGMENTS

The authors would like to acknowledge the financial support provided by the National Science Foundation under the Grant No. DAR 78-24301 (Program Manager, Dr. W. M. Spurgeon), and the valuable discussions shared with Dr. P. N. Hansen at the Technical University of Denmark.

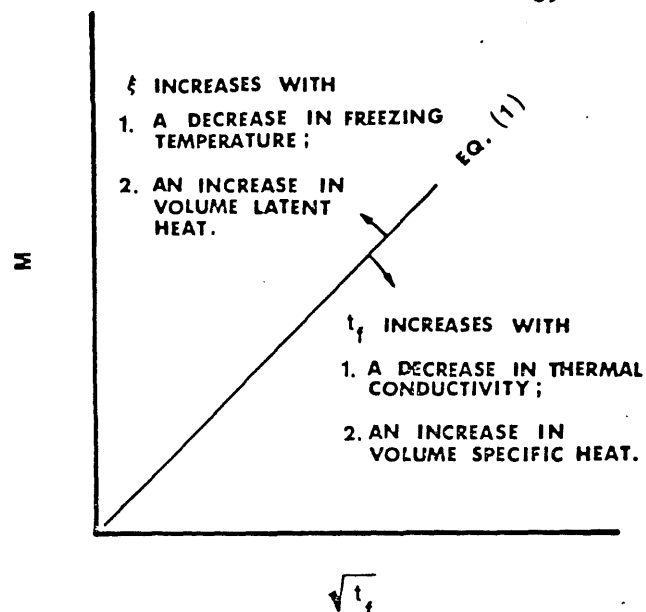


Fig. 9. Counteraction of the casting geometry with the contributing properties of the casting in determining the freezing time of a square-sectioned bar.

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NOMENCLATURE

A	Surface area;
C_m	Specific heat of the mold;
K_m	Thermal conductivity of the mold;
L	Latent heat per unit mass;
M	Modulus (V/A);
M'	Effective modulus;

$Q_{1d}(t)$	Heat flow per unit time and depth, over a planar mold;
$Q_{2d}(t)$	Heat flow per unit time and depth, over a wedge-shaped mold;
t	Time;
t_f	Freezing time;
T_i	Interface temperature;
T_∞	Initial mold temperature;
V	Volume;
$\Delta^*(\theta_0)$	Wedge invariant;
θ_0	Wedge angle;
ξ	Corner index;
ρ_c	Density of the cast metal;
ρ_m	Density of the mold

EQUATIONS

$$\frac{V}{A} = \frac{2}{\sqrt{\pi}} \left(\frac{T_i - T_\infty}{\rho_c L} \right) \sqrt{K_m \rho_m C_m} \sqrt{t_f} \quad (1)$$

$$\frac{V}{A} = \frac{T_i - T_\infty}{\rho_c L} \left(\frac{2}{\sqrt{\pi}} \sqrt{K_m \rho_m C_m} \sqrt{t_f} + \frac{n K_m t_f}{2 r} \right) \quad (2)$$

$$\Delta^*(\theta_0) = 2 \int_{-\infty}^{\infty} \coth \eta \left[\frac{1}{\pi \sinh \eta} - \frac{1}{\theta_0 \sinh \frac{\pi \eta}{\theta_0}} \right] d\eta \quad (3)$$

$$Q_{2d}(t) - Q_{1d}(t) = K_m (T_i - T_\infty) \left[-\sum_{n=1}^N \Delta^*(\theta_n) \right] \quad (4)$$

$$\frac{V}{A} = \frac{T_i - T_\infty}{\rho_c L} \left\{ \frac{2}{\sqrt{\pi}} \sqrt{K_m \rho_m C_m} \sqrt{t_f} + \frac{K_m}{A} \left[-\sum_{n=1}^N \Delta^*(\theta_n) \right] t_f \right\} \quad (5)$$

$$\xi = \frac{\sqrt{\pi K_m / \rho_m C_m}}{2A} \left[-\sum_{n=1}^N \Delta^*(\theta_n) \right] \sqrt{t_f} \quad (6)$$

$$\xi = -\frac{1}{2} + \frac{1}{2} \left\{ 1 + \frac{\pi \rho_c L}{\rho_m C_m (T_i - T_\infty)} \left[-\sum_{n=1}^N \Delta^*(\theta_n) \right] \frac{V}{A^2} \right\}^{1/2} \quad (7)$$

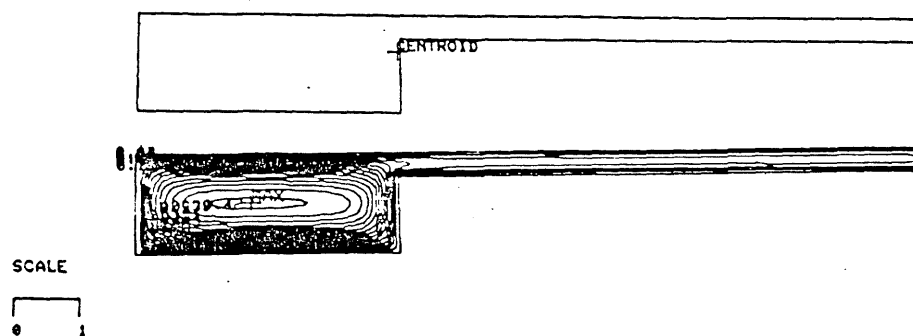
$$\sqrt{\frac{t_f}{t_f'}} = 1 + \xi \equiv \frac{M}{M'} \quad (8)$$

MAIN MENU LEVEL

PHYSICAL PROPERTIES:

X-SECT AREA = 8.9491 SQ. IN.
 VOLUME = 53.695 CU. IN.
 SURFACE AREA = 179.78 SQ. IN.
 WEIGHT = 4.8325 LB.
 GROSS U/SA = 0.29867 IN.
 X-CENTROID = -1.2073 IN.
 Y-CENTROID = -2.7678 IN.

SOLID OF EXTRUSION:
 LENGTH OF EXTRUSION = 6.0000 IN.



STATUS: CASTING NOT MOLD RAW NO SUPERHEAT

Fig. 3. Properties of volume, surface area, weight, gross section modulus, and centroids printed from graphics screen (upper). Wavefronts are shown in the lower figure reproduced from a separate graphics screen.

Table 1. Volume Percentage Feed Metal

Volume Percentage Feed Metal						
Metal-section	Mold	% Feed Metal vol. feed metal vol. casting plus risers	* Casting Soundness	** Min. Riser Top 0, in.	Reference	
1. Carbon steel- 1.0 inch 2.0 inches	green sand	3.7 5.4	S S	2.0 2.0	1	
2. 316 stainless 1.0 inch	green sand	3.8	U	2.0	1	
3. Ductile iron 10 to 100 lbs.	green sand	2.05	S	1.0		
4. A356 aluminum 1.0 inch	no-bake	2.12	U	1.5	1	
1. A356 Aluminum 0.375 to 1.5 in.	green sand	2.9	S	1.75	2	
2. 12% Si-Aluminum 0.375 to 1.5 in.	green sand	3.0% > 1350°F pour 4.05% > 1500°F pour	S	1.75	2	
3. Ductile iron 0.375 to 1.5 in.	green sand	4.75%	S	1.25	2	
4. Gray iron 0.375 to 1.5 in.	green sand	1.7-1.9%	S		2	

*S = sound, U = unsound, ** tapered risers.

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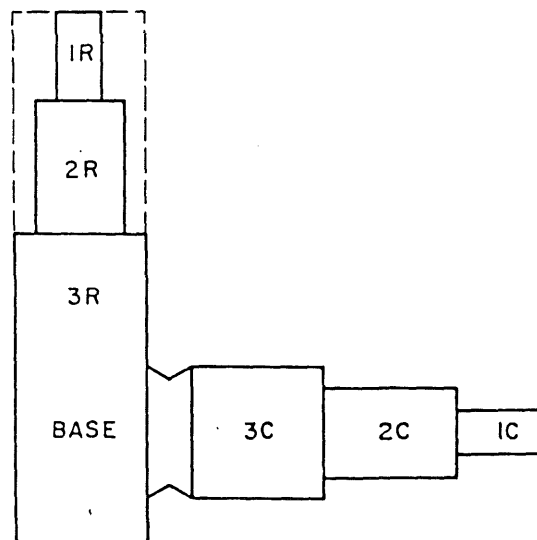
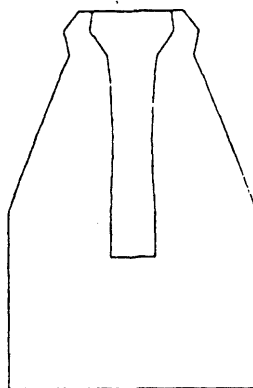


Fig. 4. Step-block casting fed by a step block side riser, or alternately a cylinder.

CURUTR
FEED
GRADNT
GRAPH
NEW
PROPTY
REVISE
RISER
USA
CLEAR
EXIT
HELP
INPUT
RECALL
ROTATE
S-HEAT
STORE
TRANS
ZOOM

SORTING USA VALUES
 TOP OR SIDE RISER?
 S, T --> S
 PRESSURE SECTION?
 IN. --> .5
 ASPECT RATIO?
 --> 1.5
 FEED METAL %?
 % --> 3.5
 # OF SECTIONS?
 # --> 8
 CURRENT USAMAX = 0.33169 IN.
 CHANGE CURRENT USAMAX?
 Y, N --> Y
 NEW VALUE?
 --> .75

RISERING



SCALE



ASPECT
DRAW
PREGSR
RESULT
SECTMS
SHRINK
TYPE
USAMAX
CLEAR
EXIT
HELP
INPUT
RECALL
ROTATE
S-HEAT
STORE
TRANS
ZOOM

Fig. 5. Risering menu level, information which must be provided and riser drawing for the castings of Figures 1-3.

Figure 4 with the example of a step-block casting. The smallest section modulus part of the casting, 1C, is fed by the smallest riser, 1R. The intermediate casting section modulus, 2C, is fed by the intermediate riser size, 2R; and so forth. Since this is a side riser, a base, B, must be added. With this geometry the yield will be less than 50% unless each casting step is lengthened to its maximum feeding distance. If the riser had no steps, it would be cylindrical riser of the maximum diameter shown. Each step is a disk that matches the corresponding VSA values in the casting.

The computer program provides a choice of input of the number of sections or disks that are stacked to form the riser. It also permits choice of the ratio of disk diameter to height, which is called the *aspect ratio*. The program also calls for a choice of top or side riser, the addition of a pressure section to the riser to provide pressure head, the percentage feed metal requirement, and a choice of the section modulus at the riser location or the average VSA computed by the program (Fig. 3). Typical feed metal requirements determined from the research are listed in Table 1.

The riser menu level is called up and the data entered as listed in Figure 5. The conditions chosen were as follows:

- 1) Top or Side Riser: S, T - S
- 2) Pressure Section Inches: 0.50
- 3) Aspect Ratio: 1.5
- 4) Feed Metal %: 3.5
- 5) Number of Sections: 8
- 6) Current VSA Maximum
- 7) Change Current VSA Max: Y, N - Y
- 8) New Value: 0.75

The conditions entered above and a drawing of the riser can then be displayed as in Figure 5, and the dimensional results as in Figure 6. Note the funnel shape at the top of the riser (Fig. 5) and its dimensions (Fig. 6). The top shape reflects the high early feed metal demand of the low VSA thin plate added to that of the heavy plate. Correlation of the riser design with casting behavior will be discussed later.

If a cylindrical riser is desired, an entry of one disk section instead of the eight disk sections entered for Figure 5 will then produce the output shown in Figure 7. However, a 6% reduction in yield results when the change from a tapered to a cylindrical riser is made in this case. An increase of about 4-11% yield is typical of the change from cylindrical to tapered risers.⁹

Using the dimensions of the riser from the program (Figs. 5 and 6) a drawing of the casting and tapered riser assembly as in Figure 1 may be entered into the program to determine that solidification is occurring directionally from the castings into the riser. When the riser is attached to the casting in the drawing, the riser diameter is halved. This is because in the 2-D representation of a riser $T/2 = D/4$ or $T = 0.5D$. The 2-D drawing sees all sections as plates and none as cylinders. Figure 8 shows the solidification wave fronts when no superheat is involved and Figure 9 shows them for 1.5% superheat. The latter is a program feature described in reference 4. In addition a surface coefficient of 0.02 was introduced at the top surface of the riser to simulate the air gap formed when the metal draws away because of piping action from feeding the casting. Figures 8 and 9 show good directional solidification with the final location in the riser outside the casting (mark +).

RISERING

RISER TYPE = SIDE FEED		PRESSURE SECT = 0.50		PAGE 1	
SUBRISER #	SUBRISER DIA	SUBRISER HEIGHT	TOTAL HEIGHT	PIPE DIA	
1	1.58	0.30	0.30	1.22	
2	2.05	0.38	0.68	1.22	
3	1.83	0.35	1.03	0.75	
4	2.19	0.41	1.44	0.67	
5	2.53	0.47	1.92	0.63	
6	2.88	0.54	2.46	0.61	
7	3.28	0.61	3.07	0.62	
8	3.72	0.70	3.77	0.69	
9	3.72	2.02	5.79	0.00	
CAST WEIGHT	RISER WEIGHT	YIELD	FEED %	ASPECT RATIO	DENS
4.83	5.87	45.14	3.50	1.50	0.05

Fig. 6. Dimensional results and yield for riser drawing of Figure 5.

RISERING

RISER TYPE = SIDE FEED		PRESSURE SECT = 0.50		PAGE 1	
SUBRISER #	SUBRISER DIA	SUBRISER HEIGHT	TOTAL HEIGHT	PIPE DIA	
1	3.06	8.96	2.96	0.91	
2	3.06	8.03	4.98	0.00	
CAST WEIGHT	RISER WEIGHT	YIELD	FEED %	ASPECT RATIO	DENS
4.97	7.55	39.63	3.50	0.75	0.00

SCALE

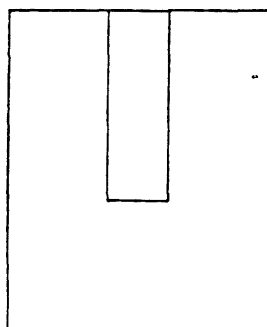


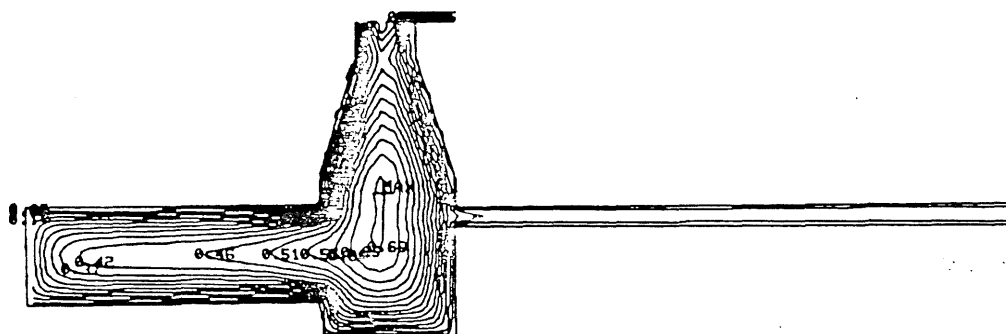
Fig. 7. Cylindrical riser designed to replace the tapered riser of Figure 5.

ASPECT
DRAW
PRESSR
RESULT
SECTNS
SHRINK
TYPE
USAMAX
CLEAR
EXIT
HELP
INPUT
RECALL
ROTATE
S-HEAT
STORE
TRANS
ZOOM

ASPECT
DRAW
PRESSR
RESULT
SECTNS
SHRINK
TYPE
USAMAX
CLEAR
EXIT
HELP
INPUT
RECALL
ROTATE
S-HEAT
STORE
TRANS
ZOOM

PAW DATA
COMPUTING VALUES.
FILL = 8679
OF CONTOURS? 15

SECTION MODULUS



SCALE



STATUS: CASTING NOT MOLD NORMALIZED NO SUPERHEAT

USA (IN.)
0.4618E+01
0.9236E+01
0.1385E+00
0.1847E+00
0.2309E+00
0.2771E+00
0.3233E+00
0.3694E+00
0.4156E+00
0.4618E+00
0.5080E+00
0.5542E+00
0.6003E+00
0.6465E+00
0.6927E+00
0.7019E+00

OPEN
CAST
NORM
RANGE
SAND
WAVE3D
WAVES
CLEAR
EXIT
HELP
INPUT
RECALL
ROTATE
S-HEAT
STORE
TRANS
ZOOM

Fig. 8. Riser with thick and thin plate castings attached as in Figure 1 and showing directional solidification toward riser. No superheat. Note cooling fin effect of thin plate shown by wave fronts.

The wave fronts in Figures 8 and 9 indicate that the thin plate is functioning as a cooling fin, thus shifting the final solidification location in the riser toward the heavy plate. This actually occurred in the casting experiments. If the thin plate is replaced with a plate 1.5-in. thick then the thermal condition on both sides of the riser is balanced. The wavefronts are then nearly symmetrical around the riser axis, as seen in Figure 10. Also, after designing the riser for thick/thick assembly it is evident that its shape at the top changes significantly (Fig. 11) from that of the thick/thin riser assembly (Fig. 5). Cavities in risers made with the thick/thin and thick/thick plate casting combinations bear out the wave front predictions (Fig. 12). An Al-Si eutectic alloy was used to make these castings. The thin plate is seen to cause the pipe cavity to develop by feeding toward the heavy casting (Fig. 12). In contrast, feeding in the thick/thick plate combination produces a vertical pipe cavity in the riser (Fig. 12). The thick castings were found to be metallographically sound.

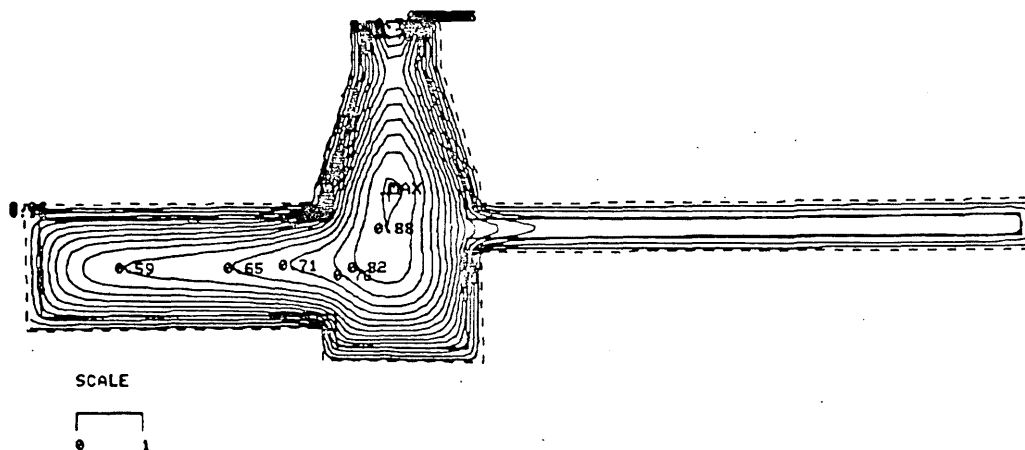
Risers designed with the wave front program do not display the riser contact graphically or dimensionally at this time. However, the riser base is included in side risers (Figs. 5-7 and 11). The contact used would follow the principles presented in references 1 and 2.

RISERING BY THE SECTION MODULUS MICROCOMPUTER PROGRAM

The program principles involved are described in reference 3. The program is called RISER II and has been made available to AFS. Again, a drawing of the shape is needed to place on the microcomputer digitizing table so the dimensions and configuration can be input. The volume, surface area, and gross section modulus are determined by vector geometry (as in the wave front program). Then the section modulus of all sections of the casting and their freezing order must be determined. Figure 13 shows printouts of the total casting volume, section modulus, and also the section modulus of the two different sections of the casting. The overall section modulus is much lower than the section modulus of the thick section because of the very low section modulus thin section. The larger section modulus must be used to design the riser, not the average or overall modulus; $D_R = 4SM + D_p$. Much too small a riser would result if the average SM were used. Using the section modulus and freezing order SM data, the riser(s) can then be designed.

SUPERHEAT
PERCENT SUPERHEAT?
1.5
ORIGINAL AREA = 17.645 IN2
SUPERHEATED AREA = 26.571 IN2
COMPUTING VALUES.
FILL = 11689
OF CONTOURS?
15

SECTION MODULUS

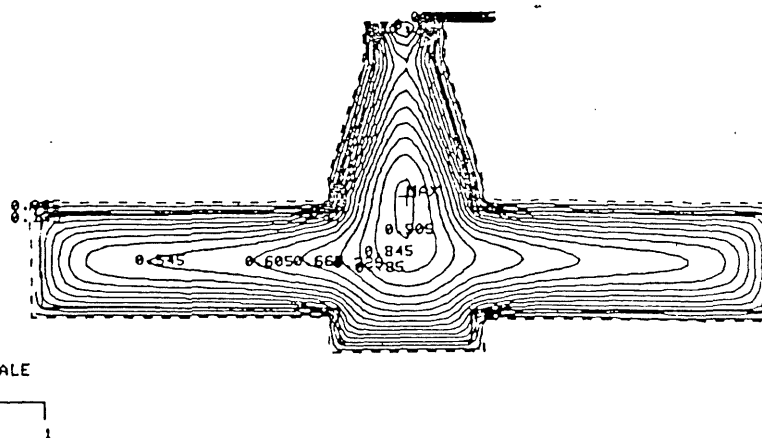


STATUS: CASTING NOT MOLD RAW NO SUPERHEAT

Fig. 9. Same as Figure 8 but with 1.5% superheat.

RAW DATA
SUPERHEAT
PERCENT SUPERHEAT?
1.5
ORIGINAL AREA = 21.435 IN2
SUPERHEATED AREA = 26.516 IN2
COMPUTING VALUES.
FILL = 16255
OF CONTOURS?
15

SECTION MODULUS



STATUS: CASTING NOT MOLD NORMALIZED NO SUPERHEAT

Fig. 10. Riser with two thick plate castings attached. Note balanced directional solidification compared with Figures 8 and 9. Superheat of 1.5%.

USA (IN.)

0.5879E+01	CAST
0.1176E+00	
0.1764E+00	NORM
0.2352E+00	
0.2939E+00	RANGE
0.3527E+00	
0.4115E+00	SAND
0.4703E+00	
0.5291E+00	WAVE3D
0.5879E+00	
0.6467E+00	WAVES
0.7055E+00	
0.7642E+00	CLEAR
0.8230E+00	
0.8818E+00	EXIT
0.8936E+00	

HELP

INPUT

RECALL

ROTATE

S-HEAT

STORE

TRANS

ZOOM

USA (IN.)

0.6018E+01	CAST
0.1204E+00	
0.1806E+00	NORM
0.2407E+00	
0.3009E+00	RANGE
0.3611E+00	
0.4213E+00	SAND
0.4815E+00	
0.5417E+00	WAVE3D
0.6018E+00	
0.6620E+00	WAVES
0.7222E+00	
0.7824E+00	CLEAR
0.8426E+00	
0.9028E+00	EXIT
0.9148E+00	

HELP

INPUT

RECALL

ROTATE

S-HEAT

STORE

TRANS

ZOOM

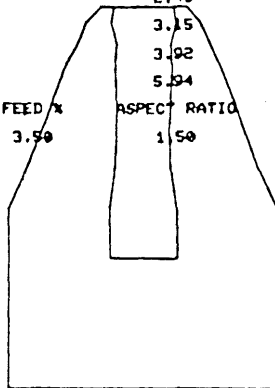
RISERING

RISER TYPE - SIDE FEED

PRESSURE SECT - 0.50

PAGE 1

SUBRISER #	SUBRISER DIA	SUBRISER HEIGHT	TOTAL HEIGHT	PIPE DIA	
1	1.32	0.65	0.25	0.94	
2	1.76	0.33	0.58	0.99	
3	2.00	0.37	0.95	0.86	
4	2.59	0.45	1.40	0.87	
5	2.73	0.51	1.91	0.83	
6	3.10	0.58	2.49	0.82	
7	3.52	0.66	3.15	0.86	
8	4.09	0.77	3.92	1.05	
9	4.09	2.02	5.94	0.00	
CAST WEIGHT	RISER WEIGHT	YIELD	FEED %	ASPECT RATIO	DENS
6.53	7.09	47.96	3.50	1.50	0.00



SCALE



ASPECT
DRAW
PRESSR
RESULT
SECTNS
SHRINK
TYPE
USAMAX
CLEAR
EXIT
HELP
INPUT
RECALL
ROTATE
S-HEAT
STORE
TRANS
ZOOM

Fig. 11. Riser design for the thick/thick plate casting assembly. Compare with Figures 5 and 6 and note change in funnel shape at the top.

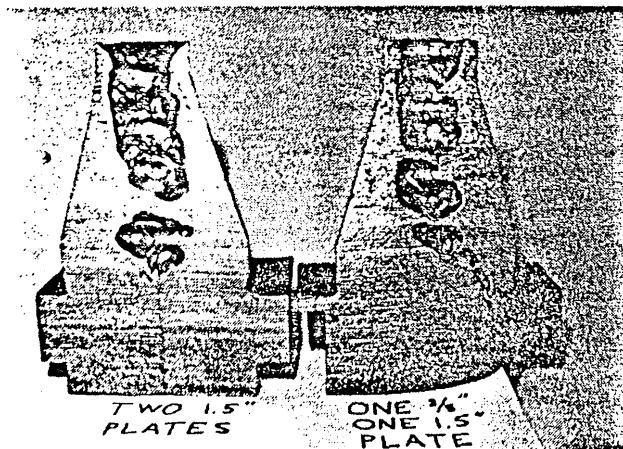
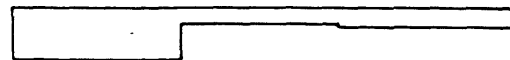


Fig. 12. Sectioned Al-Si eutectic alloy risers. Left, riser used to feed two 1.5-in. plates for comparison with Figure 10. Right, riser used to feed one 1.5-in. thick plate and one 0.375-in. thick plate for comparison with Figure 9.

SM = 0.301

VOL = 54.44



(3) New Subsection

(4) New Shape

Choose

SM = 0.298

VOL = 4.45



SM = 0.598 0.747 0.173

(3) New Subsection

(4) New Shape

Choose

Fig. 13. Printout of microcomputer screen showing the shape input, its volume and overall section modulus and the modulus of the two different sections of the casting and consequently their freezing order.

SM = 0.6						SM = 0.75					
RSM	HP	DP	HP	DR	YIELD	RSM	HP	DP	HP	DR	YIELD
0.26	0.33	0.81	4.84	51.39 *		0.28	0.33	0.81	5.44	42.08 *	
0.41	0.93	1.61	4.13	56.11 *		0.46	0.93	1.61	4.73	46.38 *	
0.52	1.71	2.42	3.81	57.39 *		0.59	1.71	2.42	4.41	47.56 *	
0.59	2.64	3.22	3.62	57.59 *		0.68	2.64	3.22	4.22	47.72 *	
0.63	3.69	4.03	3.49	57.33 *		0.74	3.69	4.03	4.09	47.44 *	
0.65	4.85	4.84	3.40	56.83 *		0.77	4.85	4.84	4.00	46.93 *	
0.67	6.11	5.64	3.32	56.21 *		0.79	6.11	5.64	3.92	46.31 *	
0.67	7.47	6.45	3.26	55.53 *		0.80	7.47	6.45	3.86	45.63 *	
0.67	8.91	7.26	3.21	54.81 *		0.80	8.91	7.26	3.81	44.92 *	
0.66	10.43	8.06	3.17	54.08 *		0.80	10.43	8.06	3.77	44.19 *	
0.66	12.04	8.87	3.14	53.34 *		0.80	12.04	8.87	3.74	43.47 *	
0.65	13.71	9.67	3.11	52.61 *		0.79	13.71	9.67	3.71	42.75 *	
0.64	15.46	10.48	3.08	51.88 *		0.79	15.46	10.48	3.68	42.05 *	
0.64	17.28	11.29	3.05	51.17 *		0.78	17.28	11.29	3.65	41.36 *	
0.63	19.17	12.09	3.03	50.47 *		0.77	19.17	12.09	3.63	40.69 *	
0.62	21.12	12.90	3.01	49.79 *		0.77	21.12	12.90	3.61	40.03 *	
0.62	23.13	13.70	2.99	49.12 *		0.76	23.13	13.70	3.59	39.39 *	
0.61	25.20	14.51	2.98	48.47 *		0.75	25.20	14.51	3.58	38.77 *	
(* - marks where max RSM: 1.3*SM being fed)						(* - marks where max RSM: 1.3*SM being fed)					
Press return to continue						Press return to continue					
Designing Riser/s for Path 1						Designing Riser/s for Path 1					
Number of riser's = 1, 2 (1)						Number of riser's = 1, 1 (0.666)					
Height of feeding section = 3.22 in.						Height of feeding section = 3.22 in.					
Height of pressure section = 0.5 in.						Height of pressure section = 0.5 in.					
Top diameter = 1.22, 0.86 in.						Top diameter = 1.22 in.					
Base diameter = 3.62, 3.26 in.						Base diameter = 4.22 in.					
Total riser height = 5.52 in.						Total riser height = 5.970 in.					
Height of contact = 1.2 in.						Height of contact = 1.5 in.					
Width of contact = 3 in.						Width of contact = 3.75 in.					
Length of contact = 0.5 in.						Length of contact = 0.5 in.					
Riser volume = 39.76, 31.24 cubic in.						Riser volume = 59.15 cubic in.					
Path volume = 54 cubic in.						Path volume = 54 cubic in.					
Riser yield = 57.59, 46.36%						Riser yield = 47.72%					
Enter-yes-for menu						Enter-yes-for menu					

Fig. 14. Alternate choices of tapered riser diameter, Hp/Dp, Hp and yield computed by riser program, (upper). Yield usually maximizes near Hp/Dp = 2.5. Riser dimensions, (lower) are shown after a choice of Hp is entered.

For the successful operation of this program, there must be a priori knowledge of:

- 1) The number of feeding paths in the casting: The SM values calculated are an indication of relative solidification times involved in all parts of a particular casting that determine the number of feeding paths.
- 2) The percentage feed metal required for the material chosen. The feed metal requirement is used in determining the size of the riser pipe.
- 3) The expected riser yield, as a percentage. A safe initial estimate is 50%.
- 4) The height of the pressure section. This height depends upon the location of the highest point to be fed in the casting.
- 5) The maximum feeding distance. The feeding distance over which a riser can feed a casting effectively must be known, or determined by experiment in order to ascertain the number of risers required for a given feeding path. This distance is a capability characteristic of the metal and the section thickness.

With the above information operation of the program can proceed. The program asks for the information needed to produce the output shown in Figure 14, top columns. A specific Hp, feeding section height, must be chosen and input from the list in order for the program to continue.

SM = 0.6						SM = 0.75					
RSM	HP	DP	HP	DR	YIELD	RSM	HP	DP	HP	DR	YIELD
0.87	0.33	0.81	4.84	48.55 *		0.96	0.33	0.81	5.44	39.19 *	
0.82	0.93	1.61	4.13	50.77 *		0.93	0.93	1.61	4.73	41.35 *	
0.79	1.71	2.42	3.81	50.09 *		0.91	1.71	2.42	4.41	40.61 *	
0.78	2.64	3.22	3.62	48.69 *		0.90	2.64	3.22	4.22	39.21 *	
0.77	3.69	4.03	3.49	47.10 *		0.89	3.69	4.03	4.09	37.71 *	
0.76	4.85	4.84	3.40	45.49 *		0.88	4.85	4.84	4.00	36.19 *	
0.75	6.11	5.64	3.32	43.94 *		0.88	6.11	5.64	3.92	34.73 *	
0.75	7.47	6.45	3.28	42.45 *		0.87	7.47	6.45	3.86	33.36 *	
0.74	8.91	7.26	3.21	41.05 *		0.87	8.91	7.26	3.81	32.08 *	
0.74	10.43	8.06	3.17	39.73 *		0.87	10.43	8.06	3.77	30.88 *	
0.73	12.04	8.87	3.14	38.49 *		0.86	12.04	8.87	3.74	29.77 *	
0.73	13.71	9.67	3.11	37.32 *		0.86	13.71	9.67	3.71	28.73 *	
0.73	15.46	10.48	3.08	36.22 *		0.86	15.46	10.48	3.68	27.76 *	
0.72	17.28	11.29	3.05	35.19 *		0.86	17.28	11.29	3.65	26.85 *	
0.72	19.17	12.09	3.03	34.21 *		0.86	19.17	12.09	3.63	26.00 *	
0.72	21.12	12.90	3.01	33.29 *		0.85	21.12	12.90	3.61	25.21 *	
0.71	23.13	13.70	2.99	32.42 *		0.85	23.13	13.70	3.59	24.46 *	
0.71	25.20	14.51	2.98	31.59 *		0.85	25.20	14.51	3.58	23.75 *	
(* - marks where max RSM: 1.3*SM being fed)						(* - marks where max RSM: 1.3*SM being fed)					
Press return to continue						Press return to continue					
Designing Riser/s for Path 1						Designing Riser/s for Path 1					
Number of riser's = 1, 1 (0.666)						Number of riser's = 1, 1 (0.666)					
Height of feeding section = 3.22 in.						Height of feeding section = 3.22 in.					
Height of pressure section = 0.5 in.						Height of pressure section = 0.5 in.					
Top and bottom dia. = 3.62, 3.623 in.						Top and bottom dia. = 4.22, 4.223 in.					
Total riser height = 5.52 in.						Total riser height = 5.970 in.					
Height of contact = 1.2 in.						Height of contact = 1.5 in.					
Width of contact = 3 in.						Width of contact = 3.75 in.					
Length of contact = 0.5 in.						Length of contact = 0.5 in.					
Riser volume = 56.89 cubic in.						Riser volume = 83.60 cubic in.					
Path volume = 54 cubic in.						Path volume = 54 cubic in.					
Riser yield = 48.70%						Riser yield = 39.21%					
Enter-yes-for menu						Enter-yes-for menu					

Fig. 15. Same as Figure 14 but for cylindrical risers.

The output provides several alternative riser diameters and Hp heights. Based on mold requirements, a specific diameter/height combination is chosen. In general, best results are obtained when the ratio of Hp/Dp is 2 to 4:1, where Hp is the height of the feeding section, and Dp is the diameter of the pipe. A possible range is marked * in the output column. However, yield is maximized when Hp/Dp $\approx$ 2.5. After selecting and entering Hp, the final design is output and listed as in Figure 14 (lower). Figure 14 shows results for tapered risers at two different SM values, 0.6 and 0.75. A $\approx$ 10% loss in yield occurs when the 0.75 SM is used to design the riser. This points to the tremendous importance of choosing the correct SM value for riser design. All dimensions including the riser contact are given. The contact dimensions are for a full geometric contact. They may be reduced by necks or notches according to foundry experience.¹

The microcomputer risering program also provides a design choice for either tapered or cylindrical risers. If the risers are designed to be cylindrical, the output then becomes as shown in Figure 15. Again there is a $\approx$ 10% yield loss if a 0.75-in. SM is used instead of 0.60 in. The cylindrical risers also have about 6% lower yield than the tapered risers.

Comparison of the riser dimensions in Figure 14 with those in Figure 6 indicate that the dimensions are nearly identical from both the microcomputer and wave front programs. The assembly of plates and risers can be checked for section modulus and freezing order to determine that solidification proceeds directionally with both programs. This is done on the assembly in exactly the same way as the plates in Figure 13.

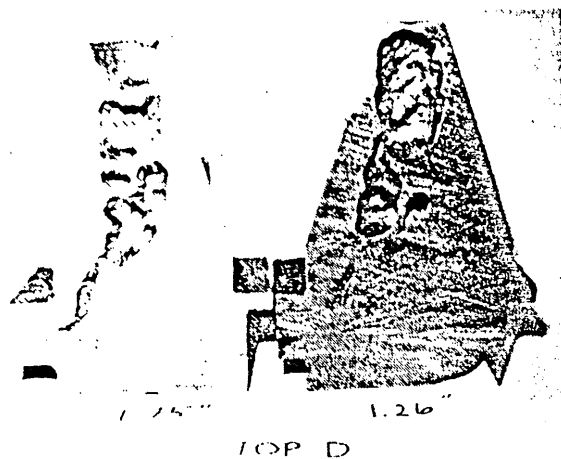


Fig. 16. Effect of top diameters of 1.26 and 1.75 in. on riser piping behavior of the thick/thin plate assembly. The 1.75-in. top dia. was obtained by capping the 1.26-in. dia. tapered riser.

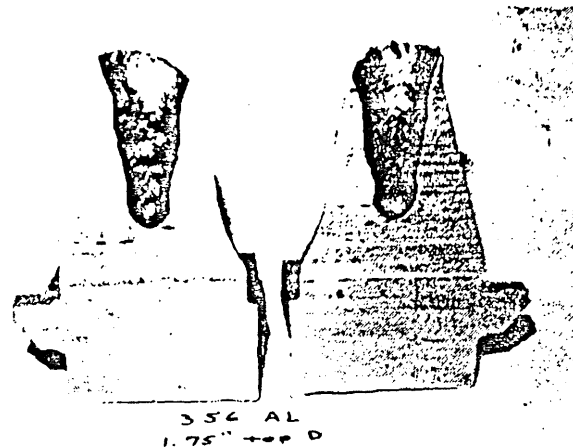


Fig. 17. Failure to pipe in A356 alloy riser with top diameter of 1.26 in.

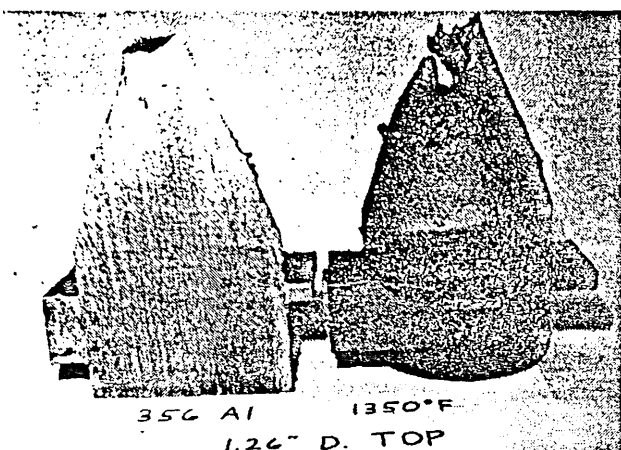


Fig. 18. Effective piping in A356 alloy riser with top diameter of 1.75 in. Compare with Figure 17. Poured at 1450F.

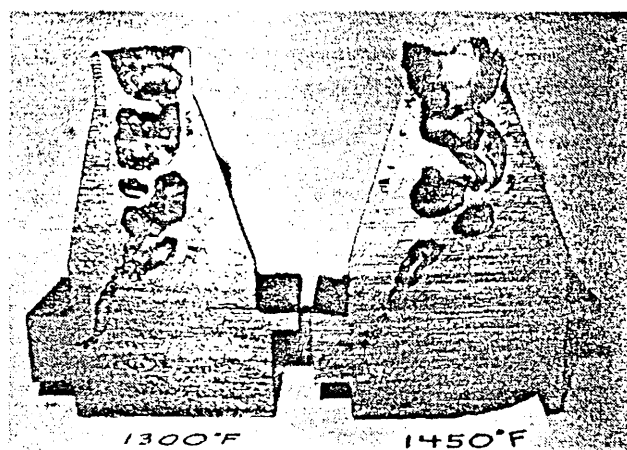


Fig. 19. Eutectic aluminum alloy risers showing increased piping and funnel top with increased pouring temperature.

CASTING EXPERIMENTS

Castings were produced in green sand to check the validity of the riser sizes and the piping behavior predicted by the computed results. In addition, certain other variables involved in risering were investigated. The alloys used were Al-Si eutectic alloy, A356 aluminum alloy, ductile iron and gray iron. The test pattern is shown in Figure 1.

Top Diameter of Tapered Risers

The top diameter of tapered risers must not be so small that it freezes off before piping begins. For cast steel, earlier work has shown that the top diameter should not be less than 1.75 in. for castings of 1.0–2.0 section thickness.⁸

Top diameters of 1.26 in. and 1.75 in. were compared with Al-Si eutectic castings (Fig. 16). A top diameter of 1.26 in. resulted in freezing over and failure to pipe adequately. Even increasing pouring temperature to 1550F did not produce consistent piping at the top. When the 1.26-in. dia. tapered riser was fitted with a 1.75-in. dia. cap, it piped successfully at all superheat temperatures of 1300–1500F (Fig. 16). Alloy 356 produced the same results

(Figs. 17 and 18). Note the smooth wall pipe of the non-eutectic alloy. The top diameter limit seems to be the same for Al and cast steel: between about 1.5 and 2.0 in.

Pouring Temperature

Providing the top diameter is sufficient; the major effect of pouring temperature seems to be on piping behavior. This is especially true of the tapered top where higher pouring temperature exaggerates the funnel effect at the top (Fig. 19). Also, more pipe volume occurs higher in the riser and a greater percentage of feed metal is required for the Al-Si alloy (Fig. 20). Note also the very smooth surface of the top funnel cavity. These effects are noted with all alloys cast thus far.

Gating Choke

Feed metal for the solidifying casting actually comes from the sprue and runners until they can no longer transport liquid because they have frozen. A choke in the gating system can be used to shut off flow from the sprue.² Early piping and maximum delivery

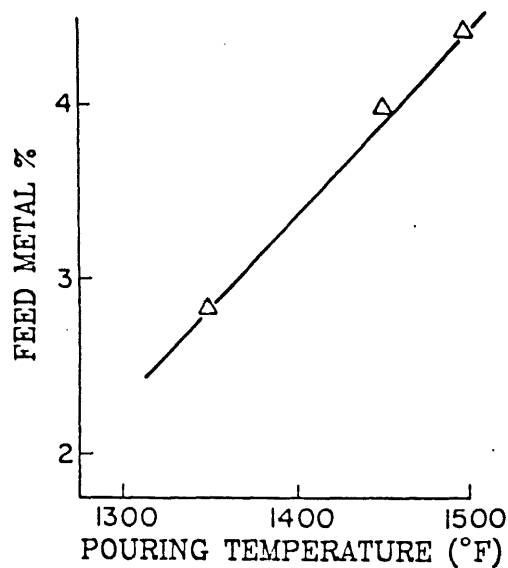


Fig. 20. Effect of pouring temperature on feed metal percentage required by eutectic Al-Si alloy.

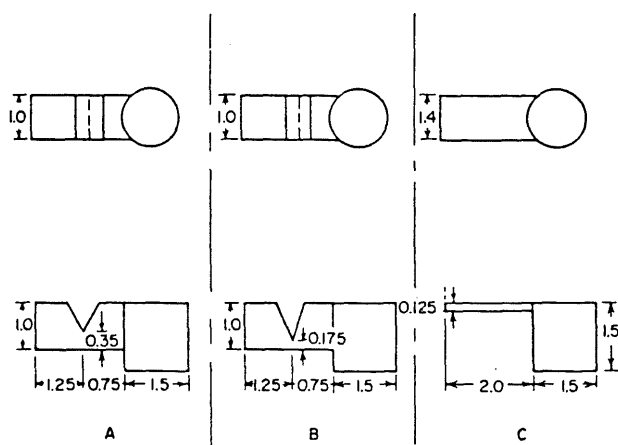


Fig. 21. Choke arrangements studied. Choke C was used as the most effective for piping.

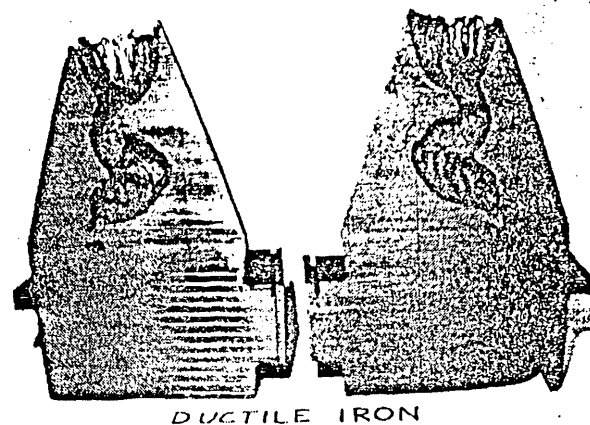


Fig. 22. Riser cross-section from thick/thin plate ductile iron casting assembly; 1.26-in. dia. top.

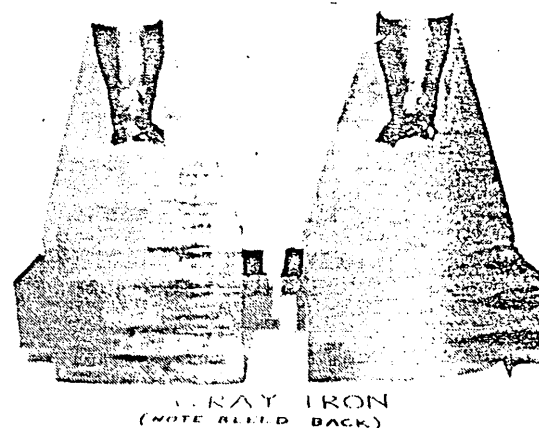


Fig. 23. Riser cross-section from thick/thin plate gray iron casting. Note bleed back, 1.26-in. dia. top.

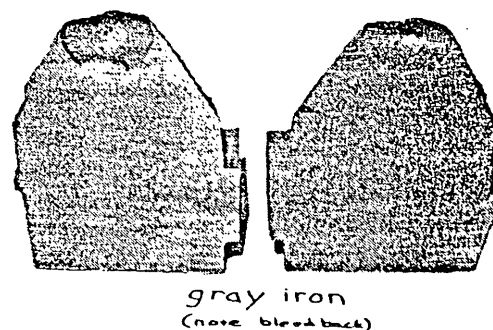


Fig. 24. Riser cross-section from thick/thin plate gray iron casting designed to produce a shorter riser, larger D_p , and shorter H_p .

of feed metal from the risers will then occur. The gating chokes were briefly considered in this research (Fig. 21). The thinner choke resulted in about 1% more feed metal being delivered by the riser.

Casting Alloy

Al-Si eutectic and A356 aluminum alloy casting results are depicted in Figures 16–20. A section risered from a ductile iron casting of the thick/thin plate type is shown in Figure 22. Piping occurred even with the 1.26-in. dia. top and a substantial part of the top of the tapered riser is missing. Other reports have shown that tapered risers with top diameters as little as 1.0 in. will function.^{2,8} Gray iron showed the same behavior at the top but the pipe cavity revealed bleeding back (Fig. 23). Feed metal percentage was 1.7. The gray iron riser appears unnecessarily large because of the bleeding back phenomenon. To check this, the riser was redesigned using the microcomputer program so that the taper was much greater; this leads to a shorter riser feeding section as shown in Figure 24. Bleed back again occurred, D_p was larger, H_p smaller, H_p/D_p smaller, the feed metal percentage was 1.9% and yield increased 5%.

Feed Metal Percentage and Piping

The volume percentage metal necessary to feed both the casting and the riser was determined by measuring the volume of the pipe in the riser. Feed metal percentage equals (pipe volume ÷ casting plus riser volume) × 100. The results of these experiments are listed in Table 1. The effect of pouring temperature on feed metal percentage for the Al-Si eutectic alloy was shown in Figure 20. The other alloys were poured at representative temperatures such as 1450F for A356 and 2500F for the cast irons.

The computer program calculation and output include the pipe dimensions D_p and H_p . In tapered risers the aluminum castings reproduce the calculated D_p and H_p values very well (Figs. 13 and 20). In ductile and gray cast iron graphitization during solidification can lead to bleed back, which displaces the pipe cavity. Cast steel has been previously shown to pipe as predicted.^{3,8} Apparently, the assumptions of a D_p and H_p which can be calculated and programmed based on section modulus are valid.

Yield

Depending on the choice of tapered or cylindrical shape and section modulus, the riser dimensions calculated and chosen, yield for this particular test pattern may vary from about 39.2–57.6%. The choice of a tapered riser instead of a cylinder increases yield about 4–8% for this pattern. The upper limit of yield with this pattern is limited by its dimensions. The thick plate provides only 4 x 6-in. feeding distance. The maximum feeding distance for a 1.5-in. plate is at least 7.5 in. in steel. Feeding two 7.5 x 7.5-in. plates from a central riser would permit yields approaching 70%. Published research has shown that yields of 75–80% are possible from a central riser if the maximum feeding distance capability of the metal is built into the casting feeding path distance.^{10,1} Hence, 39–55% yield shown in this work is lower because it is inherent in the plate design shown. Nevertheless, yield improvement available from the use of tapered risers, about 4–11%, is available in most cases.

Energy Conservation

Yield improvement leads to lower energy consumption and increased productivity. In ferrous castings, energy savings amount

to about 15 KWH/Ton of casting mold weight poured per 1% yield improvement.⁹

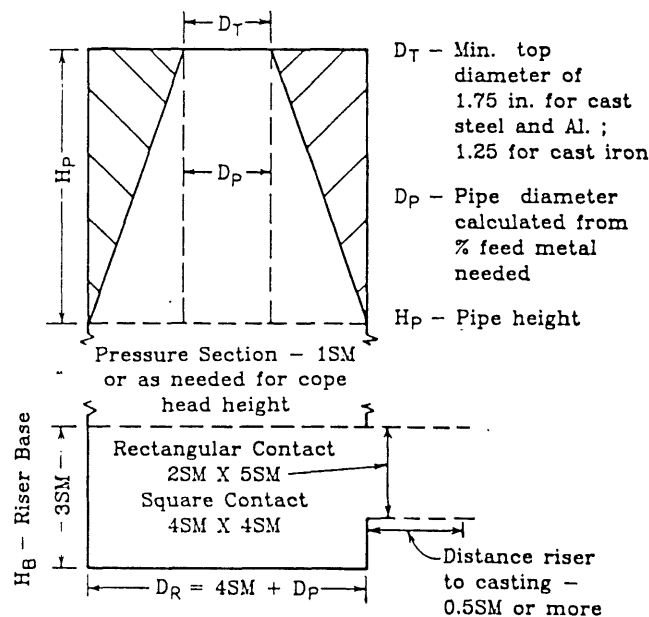
SUMMARY

The relationships between risers designed by two different types of computer programs and the piping and feed volume percentage requirements for Al-Si eutectic and A356 Al alloys has been established. Piping behavior in cross-sectioned risers has validated the predictions of the solidification wave front program as well as the riser dimensions calculated. The same riser dimensions are calculated by the microcomputer section modulus computer program although its output does not display whether the directional solidification will occur. The wave front program can prove directional solidification will occur. Both programs provide valid means of calculating riser dimensions for castings.

ACKNOWLEDGMENTS

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APPENDIX



Effective section thickness of casting at riser contact = 2SM

Dimensions of riser and contact for full size cylindrical or tapered size risers geometrically modeled

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The M-C Method for Riser Design for Non-Ferrous and Ferrous Metals

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ABSTRACT

A new method for riser design for non-ferrous as well as ferrous metals has been developed. This method was developed as an extension to the Q-Number Method, which was developed by McAdam for steel riser design. The Q-Number Method gives results similar to that of Caine, but is easier to use and covers a wider range of casting conditions.

One of the steps in the development of the Q-Method was the relationship between the freezing ratio and the Q-Number, which is:

$$V_f/V_z = 200/Q + 0.15$$

The relationship can then be modified into an expression for the volume of the riser, which is:

$$V_r = 200 M_c^3 + bV_z$$

This expression indicates that the riser volume can be expressed as the sum of two terms, a modulus term (M_c^3), which represents the solidification time factor for the critical section of the casting section being fed by the riser, and a second term which represents the feed metal requirements (V_z) of the casting section, which includes the critical section plus any appendages being fed. The coefficient "a," which is the solidification adjustment factor, is a function of the type of riser, such as side riser on top riser, the solidification conditions, such as the use of insulating sleeves, and various other factors. The coefficient "b," which is the feed metal adjustment factor, is a function of various factors, such as the solidification shrinkage characteristics of the alloy and the feed metal capacity of the riser.

The paper shows the development of these two coefficients for various riser types and for several non-ferrous alloys. The model is applied to a relatively simple problem to demonstrate how the method can be applied.

INTRODUCTION

The Q-Number system was developed by J. McAdam¹ for steel riser design. This method produces results similar to that of Caine's Method,^{2,3} but it is much easier to use and includes many additional factors such as feeding distance, chilling, cores, and junctions. The purpose of this study is to modify the Q-Method so it can be used for other metals, especially non-ferrous metals. The modified version of the Q-Method will be referred to as the M-C Method, which is a modulus (M) method that is corrected (C) for solidification shrinkage.

The Q-Number system^{1,4} was developed because the Modulus System (based upon feed metal requirements) and the Shape Factor System (based upon solidification requirements) were adequate for the low yield and high yield ranges respectively, but neither method was satisfactory for all steel castings. This problem was also observed in the design results obtained by geometric programming^{5,6} and a short iterative approach was presented for obtaining a solution, but iterative solutions are not accepted by most foundrymen.

The Q-Number uses the modulus and volume of the section being fed by the riser. The nomenclature for the variables in the equations developed is in Table 1. The Q-Number procedure is easy to apply and can be represented by the following three steps:

Step 1: Determine M_c and V_z

where M_c = modulus of section to which the feeder is attached

V_z = volume of section plus appendages fed by feeder

Step 2: Calculate M_f

$$M_f = 0.181 \sqrt[3]{200 M_c^3 + 0.15 V_z} \quad (1)$$

where M_f = modulus of the feeder

Step 3: Calculate D

$$D = 6.0 M_f \text{ when } H = D \quad (2)$$

$$D = 5.35 M_f \text{ when } H = 1.5D \quad (3)$$

$$D = 5.0 M_f \text{ when } H = 2D \quad (4)$$

where D = diameter of cylindrical side feeder

H = height of cylindrical side feeder

McAdam¹ then considers modifications to the formula for chilling, cores, and etc. Most of the modifications are adjustments to M_f and the last step remains the same.

MODEL DEVELOPMENT

The Q-Number is not directly used in the solution procedure, but it is necessary for the understanding of the M-C method and must be developed. The Q-Number is defined as:

$$Q = V_z/M_c^3 \quad (5)$$

$$V_f/V_z = 200/Q + 0.15 \quad (6)$$

Figure 1 shows a comparison of curves expressing the volume ratio, V_f/V_z , as a function of the Freezing Ratio (Caine's Method) and as a function of the Q-Number. The Freezing Ratio can also be expressed as the ratio of the feeder modulus to the casting modulus, M_f/M_c . If the updated coefficients for Caine's Formula are used, the curves for the Q-Number and Freezing Ratio with Volume Ratio are nearly identical, as indicated by Figure 2.

If one substitutes Equation 5 into Equation 6 and rearranges the terms, one obtains:

$$V_f = 200 M_c^3 + .15V_z \quad (7)$$

This expression indicates that the volume of the riser is the sum of two terms, a modulus term which represents the solidification time factor and a volume term which represents the feed metal requirements of the casting. This expression can be expressed in a generalized form of:

$$V_r = aM_c^3 + bV_z \quad (8)$$

Where a = solidification adjustment factor (*saf*)
 b = feed metal adjustment factor (*fmaf*)

The *saf* is approximately 200 for most cylindrical side riser shapes. The *saf* values for various cylindrical riser types and H/D ratios are presented in Table 2. The *saf* values will be affected by riser sleeves, hot topping, and other parameters, but these effects have not been developed at this time.

The *fmaf* will be primarily dependent upon the solidification shrinkage characteristics of the alloy and the feed metal capacity of the

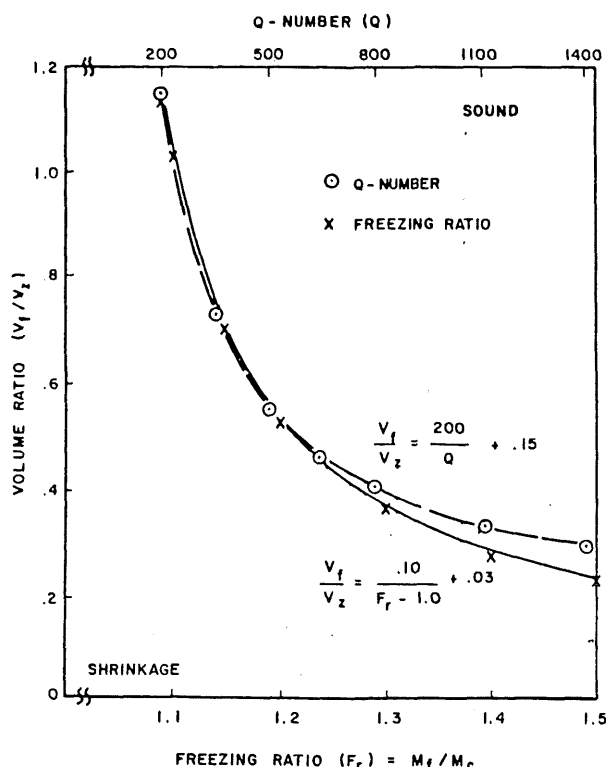


Fig. 1. Volume ratio as a function of the freezing ratio (F_r) and as a function of the Q-Number (Q).

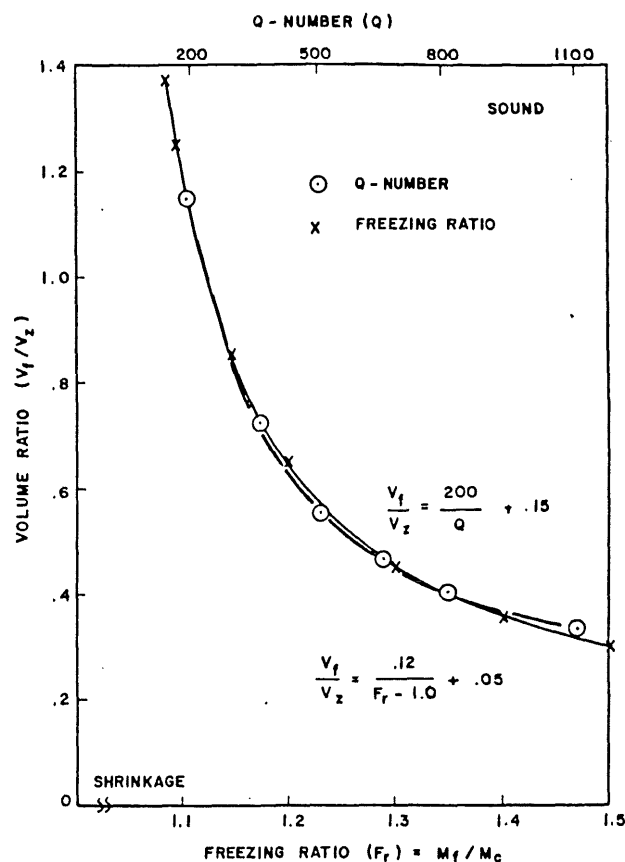


Fig. 2. Volume ratio as a function of the freezing ratio (F_r) using the updated version of Caine's formula and as a function of the Q-Number (Q).

riser. The $fmaf$ can be approximated by the ratio of the mean solidification shrinkage to the feed metal capacity of riser shape. For example, the feed metal capacity for a cylindrical riser has been shown by Umble⁷ to be approximately 1/6th of the riser volume for sand risers. The values of solidification shrinkage for various alloys was reported by Merchant⁸ and are included in Table 2. The values of the feed metal adjustment factors ($fmaf$) for various alloys are presented in Table 3. These $fmaf$ values would also change with insulating aids such as riser sleeves, because the feed metal capacity would change. The calculated $fmaf$ value for steel fortunately was the same as the value in the original expression represented by Equation 7.

MODEL APPLICATION

To illustrate the application of the model to a foundry situation, consider the casting of a 3-in. $\times$ 5-in. $\times$ 5-in. brass bar in a sand mold with a cylindrical side riser. The values of V_z , SA_c , and M_c would be 75 in³, 110 in², and 0.68 in. If a side riser is used, the value for "a" is 200 and the $fmaf$ value, or "b", would be 0.135 for brass. Inserting these values into Equation 8, a riser volume of 73.5 in³ would be required. The value of the cylindrical riser, V_r , can be expressed in terms of the riser height and diameter by:

$$V_r = \pi D^2 H/4 \quad (9)$$

The H/D ratio must be selected before the values of H and D can be determined. The non-ferrous risers have H/D ratios greater than

unity. If a value of $H/D = 2$ is selected, then the values for D and H would be 3.6-in. and 7.2-in. respectively. The modulus of the riser should be checked to verify that it is greater than the modulus of the casting. An expression for the modulus of a side riser is:

$$M_r = DH/(2D + 4H) \quad (10)$$

The value of M_r is 0.72 in., which is still greater than the 0.68 in. for M_c . The feeding distance should also be calculated to ascertain that the riser will properly feed the casting.

DISCUSSION

The proposed M-C Method involves the calculations of the calculation of the riser volume based upon the sum of two components, one representing the effect of solidification time and the other representing the effect of feed metal requirements. The solidification time adjustment factor is determined by the type of riser, such as a side on top riser, and is affected by insulating sleeves, chills, and other items which alter the solidification time. The feed metal adjustment factors is determined primarily by the alloy and the feed metal capacity, but other factors such as superheat and mold hardness could also be considered. Although the H/D ratio is permitted to vary, it must be restricted so the modulus of the riser exceeds that of the casting and also so pipe shrinkage would not extend into the casting.

Table 1. Table of Nomenclature

Symbol	Term
a	solidification adjustment factor (saf)
b	feed metal adjustment factor (fmaf)
M_c	modulus of section of casting under consideration (V_c/SA_c)
V_c	volume of casting section for modulus calculation
SA_c	surface area of casting section for modulus calculation
V_z	volume of casting section plus appendages fed by riser
M_f	modulus of riser
D	diameter of cylindrical riser
H	height of cylindrical riser
Q	Q-number
V_f	volume of riser
F_r	freezing ratio (M_f/M_c)

The riser design results presented are restricted to alloys which have a narrow freezing range and a low centerline feeding resistance. Alloys which freeze in a mushy manner, have a wide freezing range and high centerline feeding resistance will have dispersed porosity regardless of the riser design.

SUMMARY

A simple, direct expression for determining the riser volume has been obtained from the Q-Method and has become the basis of the M-C Method. This expression, in its generalized form, from Equation 8 is:

$$V_r = aM_c^3 + bV_z \quad (11)$$

Values for "a" and "b," the solidification adjustment factor and feed metal adjustment factor, respectively, are presented in Table 2 and 3. This system allows for a variation in the height-to-diameter ratio and can be applied to alloys that have a narrow freezing range and low centerline feeding resistance. Experimental testing needs to be performed to verify the validity of the M-C Method and to quantify the effect of solidification aids, chills, and superheat upon the two adjustment factors.

APPENDIX A

The Q-Number System

The Q-Number is a dimensionless, modulus-based shape indicator, which is calculated by

$$Q = V_r/M_c^3 \quad (A-1)$$

Table 2. Solidification Adjustment Factors (saf) for Cylindrical Risers

Riser Type	H/D Ratio	saf Value (calculated)	saf Value (recommended)
Side	1.0	169	200
Side	.5	201	200
Side	2.0	196	200
Top	.5	84.5	100
Top	1.0	98	100

Table 3. Feed Metal Adjustment Factors (fmaf) for Various Alloys

Alloy	Solidification Shrinkage	Mean Solidification Shrinkage	Feed Metal Capacity	fmaf value
[1]	[2]	[3] = [2]/2	[4]	[5] = [3]/[4]
Steel	.05	.025	1/6	.15
Brass (60-40)	.045 [8]	.0225	1/6	.135
Aluminum	.057 [8]	.0285	1/6	.171
Bronze (88-10-2)	.065 [8]	.0325	1/6	.195
Al - 4.5 Cu	.065 [8]	.0325	1/6	.195
Al - 8.5 Mg	.062 [8]	.031	1/6	.186
Copper	.04 [8]	.02	1/6	.12

where

V_z = volume of casting section plus appendages

M_c = modulus of casting section under consideration

The modulus of the casting is based upon the total volume of the casting section considered and the total cooling surface area. The cooling surface area may be adjusted for chills and cores. The volume ratio, V_f/V_z , can be expressed by

$$V_f/V_z = 200/Q + .15 \quad (A-2)$$

where

V_r = volume of riser (feeder)

Thus, by knowing V_z and M_c , one can determine Q and then V_r , the volume of the feeder. The optimal ratio for the side riser is $H = D$ and for the top riser is $H = D/2$, so one can determine the appropriate values of H and D by

$$D = (4 V_r / (\pi (H/D)))^{.333} \quad (A-3)$$

where

H/D = height: diameter ratio for the riser shape selected

In the original presentation, the author used for Q-Number to calculate the riser modulus, M_r , and then calculated the riser dimensions from the riser modulus. The expression for M_r is

$$M_r = .19 \sqrt[3]{200 M_c^3 + .15 V_r} \quad (A-4)$$

The values of .19 and 200 both include safety factors, so the diameter calculated is at least 5% greater than the optimal and may be even 10 to 12% greater. These factors may result in riser volumes of 15 to 40% above optimal. If one wants to apply a safety factor, it should be on the volume rather than the modulus. Also, one can calculate the riser volume easier and it will have one less adjustment.

Adjustments for chills and cores can be made to the cooling surface area in the casting modulus; that is

$$M_c = V_r/SA \quad (A-5)$$

where

SA = cooling surface area

For chills the surface area would be increased and for small cores the area would be decreased. The modulus can also be decreased by a modulus alteration factor for insulating materials.

Thus, the purpose of the modified method is to calculate the volume of the riser more directly and use one less correction factor. The modulus of the riser is not as critical as the volume of the riser, and the Q-Method uses the volume to calculate the modulus and then the modulus to calculate the diameter. On the other hand, the M-C Method calculates the riser volume and then calculates the riser diameter; it does not use the riser modulus directly except as a check to prevent a poor H/D selection.

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Cylindrical Top Riser Design Relationships for Evaluating Insulating Materials

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ABSTRACT

A cylindrical top riser model was analyzed with geometric programming to determine the effect of insulating materials and solidification shrinkage upon the riser dimensions. The riser and casting were both assumed to follow Chvorinov's Rule for solidification and the feed metal supplied by the riser to the casting was assumed to be one-sixth of the riser volume. The solidification constants (B) and solidification exponents (n) in Chvorinov's Rule, $t = B(V/SA)^n$, were assumed to be equal for the riser and the casting. The volume of the casting and riser

were adjusted for the volumetric contraction during solidification such that the final values would be:

$$\begin{aligned} V_{R(f)} &= V_{R(i)} - \beta V_{c(i)} = V_{R(i)} - 1/6 V_{R(i)} = 5/6 V_{R(i)} \\ V_{c(f)} &= V_{c(i)} + \beta V_{c(i)} = V_{c(i)} (1 + \beta) \end{aligned}$$

The volume of the cylindrical riser was

$$V_R = \pi D^2 H / 4$$

and the effective surface area of the riser was

$$SA_R = a\pi DH + b\pi D^2 / 4$$

where a and b were the apparent surface alteration factors for the side and top of the riser.

The geometric programming model was then formulated with the objective function to minimize the riser volume and require the riser to solidify after the casting and to supply the necessary feed metal. The geometric programming problem was first formulated in the primal form and converted into the dual form for the dual solution. The dual variables were then converted into the primal variables H and D . Fortunately, an exact solution for H and D could be found in terms of the apparent surface alteration factors, a and b , of the riser, the volume and surface area of the casting, and the volumetric contraction of the metal.

INTRODUCTION

The geometric programming method of riser design is based upon the mathematical techniques of geometric programming developed by Zener,¹ Beightler,² Phillips³ and others which are based upon the arithmetic-geometric mean inequality. This technique is not the geometric method of riser design used by Heine,⁴ which is based upon a specific geometric shape for the pipe in the riser cavity. The technique of geometric programming had previously been applied to riser design,⁵⁻⁷ but it has not included the effect of the riser connection area upon the casting surface area. Although the effect of the riser connection area is small upon plate shaped castings, it can be considerable on chunky castings.

Geometric Programming Model Development

The model developed was based upon Chvorinov's Rule, which has been the basis of models by Poirier and Gandhi,⁸ Ruddle,⁹ and Shenoy and Panchanathan.¹⁰ These models, however, result in the solution of cubic equations, which are presented in the form of nomograms for specific H/D ratios and are somewhat complex.

On the other hand, geometric programming can often lead to simpler expressions for the riser parameters H and D, although the process for obtaining these expressions is rather complex. The model does not need to be resolved once the design relationships are obtained.

The model developed was designed to determine the minimum riser volume and meet the solidification constraint

$$t_R \geq t_C \quad (1)$$

where

t = solidification time
R = riser C = casting.

From Chvorinov's Rule,

$$t = B(V/SA)^n \quad (2)$$

Equation 1 can be rewritten as

$$B_R(V_R/SA_R)^{n_R} \geq B_C(V_C/SA_C)^{n_C} \quad (3)$$

where

B_R = solidification constant for the riser
 V_R = volume of riser
 SA_R = surface area of the riser
 n_R = solidification exponent for the riser
 B_C = solidification constant for the casting
 V_C = volume of the casting
 SA_C = surface area of the casting, and
 n_C = solidification exponent for the casting.

There is a transfer of metal from the riser to the casting as the casting directionally solidifies to the riser. Under conditions of complete feeding, the amount of metal transferred to the casting would be βV_C , where β is the volumetric contraction during freezing. The volumetric contraction, as defined by Ruddle,¹¹ comprises the liquid shrinkage, the freezing shrinkage, a minor amount of the solid shrinkage, and a component due to mold wall movement. Also, the connecting area of the riser must be subtracted from the initial surface area of the casting. Thus, equation 3 becomes:

$$B_R((V_R - \beta V_C)/SA_R)^{n_R} \geq B_C((V_C + \beta V_C)/(SA_C - A_C))^{n_C} \quad (4)$$

where

β = volumetric contraction, and
 A_C = connecting area.

Some simplifying assumptions, which are usually made in most riser models, can greatly reduce the complexity of the model. The simplifying assumptions made were:

- 1) $n_R = n_C$; the solidification exponents of the riser and casting are equal.
- 2) $B_R = B_C$; the solidification constants of the riser and casting are equal.
- 3) The feed metal from the riser leaves a parabolic pipe equal to one sixth of the riser volume. This analysis has been applied by Johns¹² and Umble.¹³
- 4) The area of the casting surface where the riser is connected is at least equal to the connecting area.

If assumptions (1) and (2) are applied, equation 4 becomes

$$(V_R - \beta V_C)/SA_R \geq V_C(1 + \beta)/(SA_C - A_C) \quad (5)$$

For a cylindrical top riser, one has:

$$V_R = \pi D^2 H/4 \quad (6)$$

$$SA_R = a\pi DH + b\pi D^2/4 \quad (7)$$

$$A_C = \pi D^2/4 \quad (8)$$

where

H = height of the cylindrical top riser,
D = diameter of the cylindrical top riser,
a = apparent surface alteration factor for the riser side, and
b = apparent surface alteration factor for the riser top.

Using assumption (3) and equations 6 through 8, equation 5 can be rewritten as:

$$(\pi D^2 H/4 - 1/6 \pi D^2 H/4)/(a\pi DH + b\pi D^2/4) \geq V_C(1 + \beta)/(SA_C - \pi D^2/4) \quad (9)$$

Reducing the expression and rewriting the equation in the standard geometric programming form, equation 9 becomes:

$$(\pi/(4SA_C))D^2 + (24aV_C(1 + \beta)/(5SA_C))D^{-1} + (6bV_C(1 + \beta)/(5SA_C))H^{-1} \leq 1 \quad (10)$$

The geometric programming primal representation for this riser design model is:

$$\text{Minimize: } V_R = \pi D^2 H/4 \quad (11)$$

$$\text{Subject to: } (\pi/(4SA_C))D^2 + (24aV_C(1 + \beta)/(5SA_C))D^{-1} + (6bV_C(1 + \beta)/(5SA_C))H^{-1} \leq 1 \quad (12)$$

The standard dual form of this geometric programming model is:

$$W_{01} = 1 \quad (14)$$

$$2W_{01} + 2W_{11} - W_{12} = 0 \quad (15)$$

$$W_{01} - W_{13} = 0, \quad (16)$$

where W_{01} , W_{11} , W_{12} and W_{13} are the dual variables.

By combining the relationships between the primal and dual, an additional equation was developed and the solution for the dual was obtained via the procedure used previously.¹⁴ The results obtained from the solution gave exact solutions for D and H. The expression for H was complex, but could be simplified if expressed in terms of D. The expressions obtained were:

$$D = 6(6/5)(a)V_C(1 + \beta)/SA_C \quad (17)$$

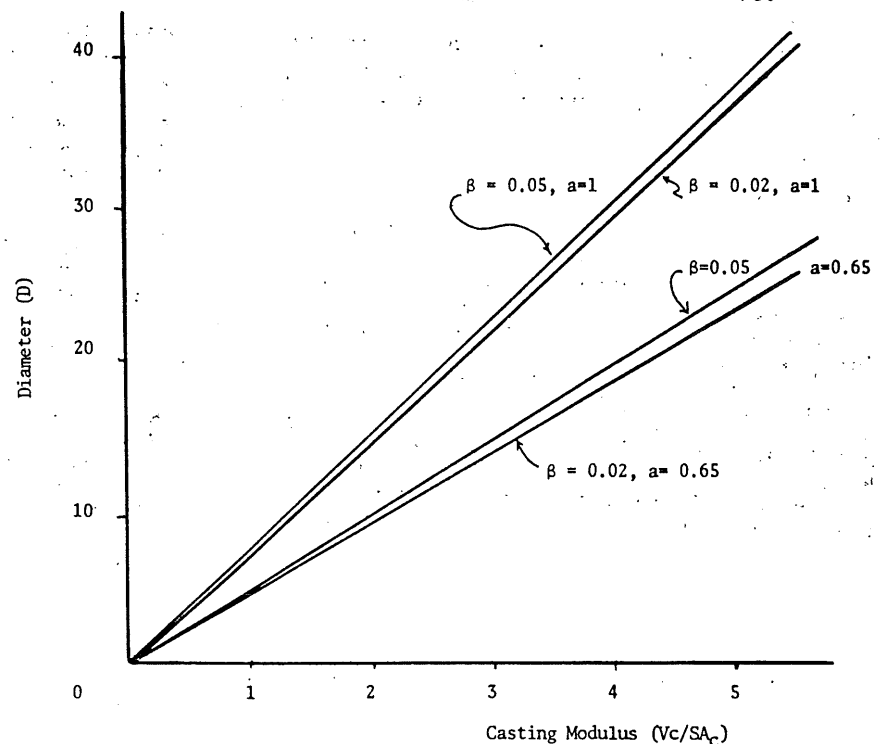
$$H = 3(6/5)(b)V_C(1 + \beta)/(SA_C - 3\pi D^2/4) \quad (18)$$

These expressions are much easier for the foundryman to use than the previous methods and nomograms are not required. The expressions clearly indicate the effect of insulating materials upon the riser diameter and height.

Interpretation of the Results

These results are quite consistent with those developed by Wlodawer¹⁵ and those previously developed by geometric

Fig. 1. The effect of solidification shrinkage and insulating walls upon the cylindrical top riser diameter.



programming.⁵⁻⁷ For example, the diameter is often expressed as six times the casting modulus. Thus, if

$$d = 6(V_c/SA_c) (6/5) a (1 + \beta) \quad (19)$$

the first part of the equation is the standard modulus expression. The $(1 + \beta)$ term represents the adjustment for the volumetric contraction, the a term represents the adjustment for the apparent surface alteration factor of the side wall material when sleeves are used, and the $6/5$ represents the 1.2 factor which is commonly used as a "factor of safety."¹¹⁻¹³ It is important to note that this "factor of safety" is not really a factor of safety, but represents the adjustment in the solidification time of the riser for the feed metal supplied to the casting by the riser. The 1.2 results from the assumption that the parabolic pipe shrinkage was one-sixth of the riser volume.

Figure 1 shows the effects of the volumetric contraction and the apparent surface alteration factor of the side wall material upon the riser diameter. Thus, Fig. 1 is a plot of equation 19 for two values of β and a can easily be made for any alloy.

The interpretation for the riser height is somewhat more complex, but is still consistent with the previous works.^{5-7,15} If equation 18 is rewritten as:

$$H = 3(V_c/SA_c - 3 D^2/4) (6/5) (b) (1 + \beta) \quad (20)$$

the first part of the equation is similar to the standard top riser relation that $H = D/2$ or three times the casting modulus. The surface area of the casting is adjusted by three times the area of the riser connection. The reason for the factor of 3 in adjusting the surface area is that all of the adjustment for the riser connection area is made on the riser height and none on the riser diameter. Since the riser volume is dimensionally related to the riser height by a power of 3, the surface area adjustment must also be made in the same ratio. The $(1 + \beta)$ term represents the adjustment for the volumetric contraction, the b term represents the adjustment for the apparent surface alteration factor of the top material (such as hot topping or insulation), and the $6/5$ or 1.2 represents the adjustment in the solidification time of the

riser for the feed metal supplied to the casting by the riser.

The H/D ratio was also evaluated to be:

$$H/D = (1/2) (b/a) (SA_c/(SA_c - 3\pi D^2/4)) \quad (21)$$

This expression shows the effect of the insulating materials upon the H/D ratio. This expression indicates why chunky castings where the $3\pi D^2/4$ correction term is a large fraction of SA_c , require larger H/D ratios and why insulating aids are not used on plate castings where the $3\pi D^2/4$ approaches a H/D lower limit of 0.5.

SUMMARY AND CONCLUSIONS

A geometric programming model of the cylindrical top riser was formulated and solved. The simplified general relationships developed for the diameter and height of the top riser were:

$$\begin{aligned} D &= 6(V_c/SA_c) (1.2) (a) (1 + \beta) \\ H &= 3(V_c/(SA_c - 3 D^2/4)) (1.2) (b) (1 + \beta) \end{aligned}$$

The effect of insulating materials (such as insulating sleeves, exothermic compounds, and topping materials) upon the dimensions of a top riser can be easily obtained from these design relationships. The various terms in the design relationships were analyzed in terms of adjustment factors to the casting modulus relationship.

Some of the conclusions are:

- the riser diameter is directly proportional to the apparent surface alteration factor for the riser side;
- the riser diameter is unaffected by the connecting area between the riser and the casting, whereas the height of the riser is affected by the connecting area;
- the riser height is in direct proportion to the apparent surface alteration factor for the riser top, whereas the riser diameter is unaffected by this factor;
- the H/D ratio for the riser is independent of the casting volume as it approaches $(b/a)/2$ as a lower limit.

With this information obtained from this study, foundrymen will be able to evaluate the effectiveness of various insulating materials and foundry suppliers can better serve their customers.

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