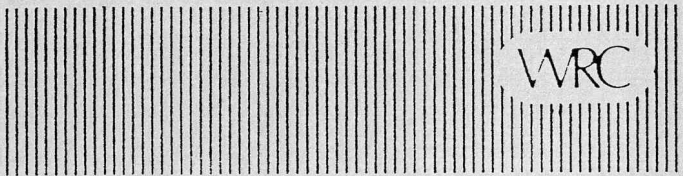


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October 2002 Update
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Welding Research Council
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LOCAL STRESSES IN SPHERICAL
AND CYLINDRICAL SHELLS
DUE TO EXTERNAL LOADINGS

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FOREWORD

October 2002 Update of March 1979 Revision

The October 2002 Update to the March 1979 Revision of WRC Bulletin 107 includes minor editorial changes for improvement and readability of several equations, curves and some text. There are NO technical changes.

- The calculation forms (Tables 2, 3 and 5) are improved, particularly to show the "+" and "-" quantities more definitively.
- The equation for stress in paragraph 3.6.3 is revised to be on one line.
- The parameter definitions on several of the curves (beginning with figure SR-1) are improved and clarified.
- Appendix B, exponents in Equations 1, 2, 3 and 4 are enlarged for readability.

PVRC thanks Mr. James R. Farr, Honorary Emeritus Member of the Pressure Vessel Research Council, for his assistance in preparing this update.

NOTE: WRC Bulletins 107 and 297 should be considered (and purchased) as an integral set. In addition, PVRC Technical Committees are working on a project that is envisioned to culminate in a new publication to add to the WRC Bulletin 107 and 297 set. The new publication will provide significant new technical information on local shell stresses from nozzles and attachments.

Greg L. Hollinger
The Pressure Vessel Research Council

FOREWORD

to March 1979 Revision

Welding Research Council Bulletin No. 107 has been one of the most widely used bulletins ever published by WRC. The original bulletin was published in August 1965. Since that time, a revised printing was issued in December 1968; a second revised printing was issued in July 1970; a third revised printing was released in April 1972; and a June 1977 reprint of the third revised printing was issued. As sometimes happens with publications of this type, some errors were detected and then corrected in subsequent revised printings.

In this March 1979 Revision of Bulletin 107, there are some additional revisions and clarifications. The formulations for calculation of the combined stress intensity, S , in Tables 2, 3, and 5 have been clarified. Changes in labels in Figures 1C-1, 2C-1, 3C, and 4C have been made and the calculated stresses for Model "R" in Table A-3 and Model "C-1" in Table A-4 have been revised accordingly. The background for the change in labels is given in a footnote on p. 66.

Present plans call for a review and possible extension of curves to parameters which will cover the majority of openings in nuclear containment vessels and large storage tanks. Plans are to extend R/T from 300 to 600 and to extend d/D range from 0.003 to 0.10 for the new R/T range, review available test data to establish limits of applicability, and develop some guidance for pad reinforcements.

Long range plans are to review shell theory in general, and Bijlaard's method in particular. The goal is to extend the R/t up to 1200 for a d/D up to 0.1. This will include large deflection theory and other nonlinear effects. In addition, available computer programs will be studied in hope of developing one which will be an appropriate supplement to Bijlaard's method. Finally, a review will be made of limit loads related to large R/T and small d/D .

J.R. Farr, *Chairman*
PVRC Design Division

FOREWORD

Several years ago, the Pressure Vessel Research Committee sponsored an analytical and experimental research program aimed at providing methods of determining the stresses in pressure vessel nozzle connections subjected to various forms of external loading. The analytical portion of this work was accomplished by Prof. P. P. Bijlaard of Cornell University, and was reported in References 1 to 8 inclusive. Development of the theoretical solutions involved a number of simplifying assumptions, including the use of shallow shell theory for spherical vessels and flexible loading surfaces for cylindrical vessels. These circumstances limited the potential usefulness of the results to d_i/D_i ratios of perhaps 0.33 in the case of spherical shells and 0.25 in the case of cylindrical shells. Since no data were available for the larger diameter ratios, Prof. Bijlaard later supplied data, at the urging of the design engineers, for the values of $\beta = 0.375$ and 0.50 (d_i/D_i ratios approaching 0.60) for cylindrical shells, as listed on page 12 of Reference 10. In so doing, Prof. Bijlaard included a specific warning concerning the possible limitations of these data, as follows: "The values for these large loading surfaces were computed on request of several companies. It should be remembered, however, that they actually apply to flexible loading surfaces and, for radial load, to the center of the loading surface. It should be understood that using these values for the edge of the attachment, as was recommended for small loading surfaces, may be unconservative."

Following completion of the theoretical work, experimental work was undertaken in an effort to verify the theory, the results of which were published in References 17 and 18. Whereas this work seemingly provided reasonable verification of the theory, it was limited to relatively small d_i/D_i ratios—0.10 in the case of spherical shells and 0.126 in the case of cylindrical shells. Since virtually no data, either analytical or experimental, were available covering the larger diameter ratios, the Bureau of Ships sponsored a limited investiga-

tion of this problem in spheres, aimed at a particular design problem, and the Pressure Vessel Research Committee undertook a somewhat similar investigation in cylinders. Results of this work have recently become available emphasizing the limitations in Bijlaard's data on cylindrical shells, particularly as it applies to thin shells over the "extended range" (page 12 of Reference 10).

Incident to the use of Bijlaard's data for design purposes, it has become apparent that design engineers sometimes have difficulty in interpreting or properly applying this work. As a result of such experience, PVRC has felt it desirable that all of Bijlaard's work be summarized in convenient, "cook-book" form to facilitate its use by design engineers. However, before this document could be issued, the above mentioned limitations became apparent, presenting an unfortunate dilemma, viz., the test data indicate that the calculated data are partially inadequate, but the exact nature and magnitude of the error is not known, nor is any better analytical treatment of the problem available (for cylinders).

Under these circumstances, it was decided that the best course was to proceed with issuing the "cook-book," extending Bijlaard's curves as best we can on the basis of available test data. This decision was based on the premise that all of the proposed changes would be toward the conservative (or "safe") side and that design engineers would continue to use Bijlaard's extended range data unless some alternative were offered. The following paper is therefore presented in the hope that it will facilitate the use of Bijlaard's work by design engineers. Every effort has been made to point out any known limitations in the work and to explain the exact nature of the changes which have been made to Bijlaard's original curves and data; however, users are warned that the resulting work is not necessarily adequate for all cases. It is the hope of the Subcommittee that additional theoretical work can be undertaken to provide more adequate data on various phases of this problem.

F. S. G. WILLIAMS, *Chairman*
PVRC Subcommittee on Reinforced
Openings and External Loadings

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Local Stresses in Spherical and Cylindrical Shells due to External Loadings

by K. R. Wichman, A. G. Hopper, and J. L. Mershon

1. Nomenclature

Symbols used in the formulas and equations in the text are listed below:

1.1 General Nomenclature

σ_i	= normal stress in the i th direction on the surface of the shell, psi
τ_{ij}	= shear stress on the i th face of the j th direction
S	= stress intensity = twice maximum shear stress, psi
N_i	= membrane force per unit length in the i th direction, lb/in.
M_i	= bending moment per unit length in the i th direction, in. lb/in.
K_n	= membrane stress concentration factor (pure tension or compression)
K_b	= bending stress concentration factor
i	= denotes direction. In the case of spherical shells, this will refer to the tangential and radial directions with respect to an axis normal to the shell through the center of the attachment as shown in Fig. 1. In the case of cylindrical shells, this will refer to longitudinal and circumferential directions with respect to the axis of the cylinder as shown in Fig. 2.
+	= denotes tensile stress (when associated with σ_i)
-	= denotes compressive stress (when associated with σ_i)
θ	= angle around attachment, degrees (see Figs. 1 and 2)
E	= modulus of elasticity, psi
P	= concentrated radial load or total distributed radial load, lb

V	= concentrated external shear load, lb
M	= concentrated external overturning moment, in lb
M_T	= concentrated external torsional moment, in. lb
d_i, d_m	= inside diameter and mean diameter, respectively, of the nozzle, in.
D_i, D_m	= inside diameter and mean diameter, respectively, of the shell, in.

1.2 Nomenclature Applicable to Spherical Shells

V_1	= concentrated external shear load in 2-2 direction, lb
V_2	= concentrated external shear load in 1-1 direction, lb
M_1	= external overturning moment in 1-1 direction, in. lb
M_2	= external overturning moment in 2-2 direction, in. lb
R_m	= mean radius of spherical shell, in.
T	= thickness of spherical shell, in.
r_0	= outside radius of cylindrical attachment, in.
r_m	= mean radius of hollow cylindrical attachment, in.
t	= thickness of hollow cylindrical attachment, in.
γ	= r_m/t
ρ	= T/t
U	= $r_0/\sqrt{R_m T}$
N_x, N_y	= membrane force in shell wall in radial and circumferential directions, respectively, lb/in. (see Fig. 1)
M_r, M_y	= bending moment in shell wall in radial and circumferential directions, respectively, in. lb/in. (see Fig. 1)
σ_r	= normal stress in radial direction, psi (see Fig. 1)
σ_y	= normal stress in circumferential direction, psi (see Fig. 1)

K.R. WICHMAN is with the Office of Standards Development of the U.S. Nuclear Regulatory Commission, Washington, DC. A.G. HOPPER, present location unknown. J.L. MERSHON, Consultant,

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τ_{xy}	= shear stress on the x face in the y direction, psi
τ_{yz}	= shear stress on the y face in the x direction, psi
τ_1	= shear stress on the 1-1 face
τ_2	= shear stress on the 2-2 face

1.3 Nomenclature Applicable to Cylindrical Shells

V_c	= concentrated shear load in the circumferential direction, lb
V_L	= concentrated shear load in the longitudinal direction, lb
M_c	= external overturning moment in the circumferential direction with respect to the shell, in. lb
M_L	= external overturning moment in the longitudinal direction with respect to the shell, in. lb
R_m	= mean radius of cylindrical shell, in.
l	= length of cylindrical shell, in.
r_0	= outside radius of cylindrical attachment, in.
c_1	= half length of rectangular loading in circumferential direction, in.
c_2	= half length of rectangular loading in longitudinal direction, in.
T	= wall thickness of cylindrical shell, in.
x	= coordinate in longitudinal direction of shell
y	= coordinate in circumferential direction of shell
ϕ	= cylindrical coordinate in circumferential direction of shell
α	= l/R_m
β	= attachment parameter
β_1	= c_1/R_m
β_2	= c_2/R_m
γ	= R_m/T ; shell parameter
C_c, C_l	= multiplication factors for N_ϕ and N_r for rectangular surfaces given in Tables 7 and 8
K_c, K_l	= coefficients given in Tables 7 and 8
M_ϕ, M_x	= bending moments in shell wall in the circumferential and longitudinal direction with respect to the shell
N_ϕ, N_r	= membrane forces in shell wall in the circumferential and longitudinal direction with respect to the shell
σ_ϕ	= normal stress in the circumferential direction with respect to the shell, psi
σ_x	= normal stress in the longitudinal direction with respect to the shell, psi
$\tau_{x\phi}$	= shear stress on the x face in the ϕ direction with respect to the shell, psi
$\tau_{\phi x}$	= shear stress on the ϕ face in the x

direction with respect to the shell, psi

2. General Equation

In the analysis of stresses in thin shells, one proceeds by considering the relation between internal membrane forces, internal bending moments and stress concentrations in accordance with the following:

$$\sigma_t = K_n \frac{N_t}{T} \pm K_b \frac{6M_t}{T^2}$$

Stress concentration factors should be considered in the following situations:

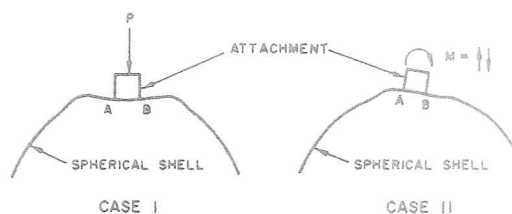
- the vessel is constructed of a brittle material.
- a fatigue evaluation is to be undertaken. The designer may find the data on stress concentrations contained in Appendix B to be helpful.

Much of the work contained in this Bulletin is devoted to a synopsis of methods for obtaining membrane forces (N_t) and bending moments (M_t) which have been developed by Professor P. P. Bijlaard in his numerous papers written on this subject. This data has been obtained for a wide range of cases by use of an electronic computer and is presented here in the form of nondimensional curves.

3. Spherical Shells

3.1 Sign Convention

For the most part, stresses will be considered in the vessel wall at the attachment-to-shell juncture. Here a biaxial state of stress exists on the inside and outside surfaces, so that one is concerned with radial and tangential (principal) stresses as indicated in Fig. 1. At this location, one can predict whether the stresses will be tensile (+) or compressive (-) by considering the deflection of the shell resulting from the various modes of loading.



To illustrate, consider Case I showing a direct radial inward load (P) transmitted to the shell by the attachment. Here the load acts similar to a local external pressure load on the shell causing compressive membrane stresses. Also, local bending occurs so that tensile bending stresses result on the inside of the vessel at A and B while compressive bending stresses result on the outside.

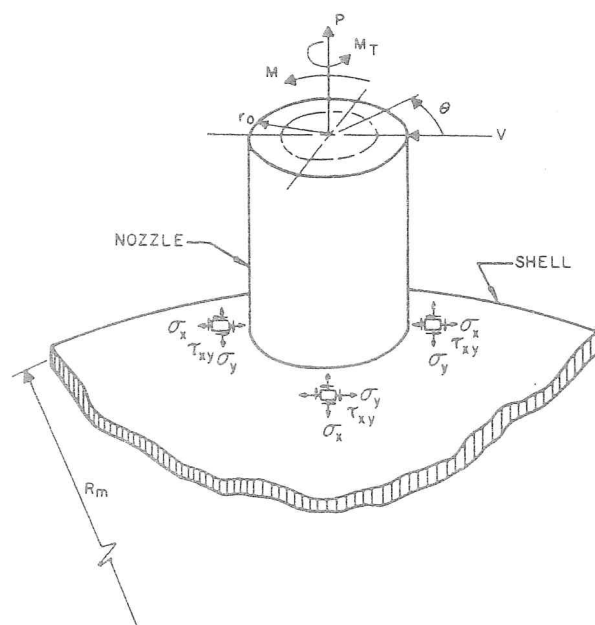


Fig. 1—Types of loading conditions at an attachment to a spherical shell

In Case II, the overturning moment may be considered to act as a couple composed of equal and opposite radial forces. Hence, tensile membrane stresses result at A while compressive membrane stresses result at B. As in Case I, local bending also occurs so that tensile bending stresses develop at A on the outside of the vessel and at B on the inside, while compressive bending stresses develop at A on the inside and B on the outside.

In this manner, the signs (tensile +, compressive -) of stresses resulting from various external loading conditions may be predicted. It is to be noted that these stresses are located in the vessel wall at its juncture with the attachment.

Table 1 shows the signs of stresses resulting from radial load and overturning moment. This table will facilitate the use of the nondimensional curves (presented in the following procedure) and minimize concern for the signs of the calculated stresses.

3.2 Parameters

The results of Bijlaard's work have been plotted in terms of nondimensional geometric parameters by use of an electronic computer. Hence, the first step in this procedure is to evaluate the applicable geometric parameters.

3.2.1 SHELL PARAMETER (U). The shell parameter is given by the ratio of the nozzle outside radius to the square root of the product of shell radius and thickness, thus:

$$U = \frac{r_0}{\sqrt{R_m T}}$$

If a square attachment is to be considered, U may be approximated as follows:

$$U = \frac{c_1}{0.875 \sqrt{R_m T}}$$

3.2.2 ATTACHMENT PARAMETERS. For spherical shells, either round or square attachments may be considered.

3.2.2.1 Rigid Attachments: In the case of a rigid attachment, no attachment parameter is required to use the nondimensional curves.

3.2.2.2 Nozzles: For a hollow cylindrical attachment such as a nozzle, the following parameters must be evaluated:

$$T = \frac{r_m}{t}$$

$$\rho = \frac{T}{t}$$

3.2.2.3 Hollow Square Attachment: If a hollow square attachment such as a box beam is to be considered, the required parameters may be approximated as follows:

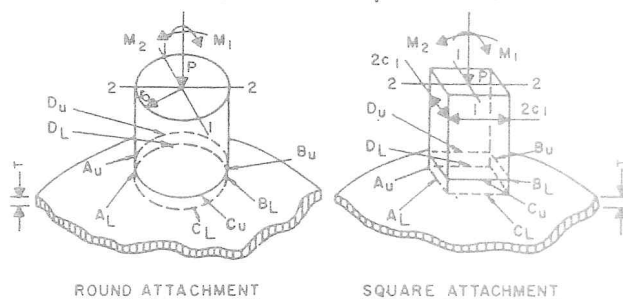
$$T = \frac{r_m}{0.875t}$$

$$\rho = \frac{T}{t}$$

3.3 Calculation of Stresses

3.3.1 STRESSES RESULTING FROM RADIAL LOAD, P

Table 1—Sign Convention for Stresses Resulting from Radial and Moment Load on a Spherical Shell



STRESS	LOCATION	LOADING		
		P	M ₁	M ₂
Membrane $\frac{N_x}{T} \text{ \& } \frac{N_y}{T}$	A _u AL	—	—	—
	B _u BL	—	—	—
	C _u CL	—	—	—
	D _u DL	—	—	—
Bending $\frac{6M_x}{T^2}$	A _u	+	+	—
	A _l	—	—	—
	B _u	+	+	—
	B _l	—	—	—
Bending $\frac{6M_y}{T^2}$	C _u	+	+	—
	C _l	—	—	—
	D _u	+	+	—
	D _l	—	—	—

NOTE: 1) Sign convention for stresses: + tension, — compression.

2) If load P reverses, all signs in column P reverse.

3) If overturning moment M₁ reverses, all signs in column M₁ reverse.

4) For round attachment, overturning moments M₁ and M₂ may be combined vectorially, thus:

$$M = \sqrt{M_1^2 + M_2^2}$$

3.3.1.1 Radial Stresses (σ_r):

Step 1. Using the applicable values of U , r and ρ^* , read off the dimensionless membrane force ($N_r T/P$) from the applicable curve which will be found in one of the following figures: Figure SR-2 or SP-1 to SP-10, inclusive.

Step 2. By the same procedure used in Step 1 above, read off the value of dimensionless bending moment (M_r/P) from the applicable curve. This value will be found in the same figure used in Step 1.

Step 3. Using the applicable values of P and T , calculate the radial membrane stress (N_r/T) by:

$$\frac{N_r}{T} = \left(\frac{N_r T}{P} \right) \cdot \left(\frac{P}{T^2} \right)$$

Step 4. By a procedure similar to that used in Step 3; calculate the radial bending stress ($6M_r/T^2$) thus:

$$\frac{6M_r}{T^2} = \left(\frac{M_r}{P} \right) \cdot \left(\frac{6P}{T^2} \right)$$

Step 5. Combine the radial membrane and bending stresses by use of the general stress equation (Section 2) together with the proper choice of sign (see Table 1); i.e.,

$$\sigma_r = K_n \frac{N_r}{T} \pm K_b \frac{6M_r}{T^2}$$

3.3.1.2 Tangential Stress (σ_θ): Follow the five (5) steps outlined in 3.3.1.1 using the same figure to obtain ($N_\theta T/P$) and (M_θ/P) as was used to obtain ($N_r T/P$) and (M_r/P). It follows that:

$$\frac{N_\theta}{T} = \left(\frac{N_\theta T}{P} \right) \cdot \left(\frac{P}{T^2} \right)$$

$$\frac{6M_\theta}{T^2} = \left(\frac{M_\theta}{P} \right) \cdot \left(\frac{6P}{T^2} \right)$$

$$\sigma_\theta = K_n \frac{N_\theta}{T} \pm K_b \frac{6M_\theta}{T^2}$$

3.3.2 STRESSES RESULTING FROM OVERTURNING MOMENT, M .

3.3.2.1 Radial Stresses (σ_r):

Step 1. Using the applicable values of U , r and ρ^* , read off the dimensionless membrane force ($N_r T \sqrt{R_m T}/M$) from the applicable curve which will be found in one of the following figures: Figure SR-3 or SM-1 to SM-10, inclusive.

Step 2. By the same procedure used in Step 1 above, read off the value of dimensionless bending moment ($M_r \sqrt{R_m T}/M$) from the applicable curve. This value will be found in the same figure used in Step 1.

Step 3. Using the applicable values of M , R_m and T , calculate the radial membrane stress

(N_r/T) by:

$$\frac{N_r}{T} = \left[\frac{N_r T \sqrt{R_m T}}{M} \right] \cdot \left[\frac{M}{T^2 \sqrt{R_m T}} \right]$$

Step 4. By a procedure similar to that used in Step 3, calculate the radial bending stress ($6M_r/T^2$), thus:

$$\frac{6M_r}{T^2} = \left[\frac{M_r \sqrt{R_m T}}{M} \right] \cdot \left[\frac{6M}{T^2 \sqrt{R_m T}} \right]$$

Step 5. Combine the radial membrane and bending stresses by use of the general stress equation (para. 2) together with the proper choice of sign (see Table 1); i.e.,

$$\sigma_r = K_n \frac{N_r}{T} \pm K_b \frac{6M_r}{T^2}$$

3.3.2.2 Tangential Stress (σ_θ): Follow the five steps outlined in 3.3.2.1, using the same figure to obtain ($N_\theta T \sqrt{R_m T}/M$) and ($M_\theta \sqrt{R_m T}/M$) used to obtain ($N_r T/P$) and (M_r/P). It follows that:

$$\frac{N_\theta}{T} = \left[\frac{N_\theta T \sqrt{R_m T}}{M} \right] \cdot \left[\frac{M}{T^2 \sqrt{R_m T}} \right]$$

$$\frac{6M_\theta}{T^2} = \left[\frac{M_\theta \sqrt{R_m T}}{M} \right] \cdot \left[\frac{6M}{T^2 \sqrt{R_m T}} \right]$$

$$\sigma_\theta = K_n \frac{N_\theta}{T} \pm K_b \frac{6M_\theta}{T^2}$$

3.3.3 STRESSES RESULTING FROM TORSIONAL MOMENT, M_T . In the case of a round attachment (such as a pipe), torsional moment is assumed to induce pure shear stresses, so that shear stress (τ) in the shell at the attachment-to-shell juncture is given by:

$$\tau_{\theta z} = \tau_{z\theta} = \frac{M_T}{2\pi r_0^2 T}$$

If only shear stresses are being considered, it is to be noted that the equivalent stress intensity is twice the above calculated shear stress.

In the case of rectangular attachments, torsional moment produces a complex stress field in the shell. Acceptable methods of analyzing this situation are not available at this time. If the designer has reason for concern, the problem should be resolved by testing in accordance with established code procedures.

3.3.4 STRESSES RESULTING FROM SHEAR LOAD, V . Bijlaard has proposed¹⁴ that shear force (V) can be assumed transmitted to the shell entirely by membrane shear force. Therefore, stresses in the shell at the attachment-to-shell juncture can be approximated as follows:

3.3.4.1 Round Attachment:

$$\tau_{xy} = \frac{V}{\pi r_0 T} \sin \theta \quad (\text{refer to Fig. 1})$$

* NOTE: T and ρ not required in the case of a rigid insert.

3.3.4.2 Square Attachment:

$$\tau_{xy} = \frac{V}{4c_1T} \quad (\text{at } \theta = 90^\circ \text{ and } 270^\circ)$$

3.3.5 STRESSES RESULTING FROM ARBITRARY LOADING. In the general case, all applied loads and moments must be resolved (at the attachment-shell interface) in the three principal directions; i.e., they must be resolved into components P , V_1 , V_2 , M_1 , M_2 and M_T . If one then proceeds in the manner previously outlined, membrane, bending and shear stresses can be evaluated at eight distinct points in the shell at its juncture with the attachment. These eight points are shown in the sign convention chart, Table 1.

The numerous stress components can be readily accounted for if a scheme similar to that shown in Table 2 and 3 is adopted. In using this scheme, it is to be noted that the Maximum Shear Theory has been used to determine equivalent stress intensities. Also, it is to be noted that evaluation of stresses resulting from internal pressure has been omitted.

Test work conducted by PVRC has shown that stresses attenuate rapidly at points removed from the attachment-to-shell juncture, the maximum stress frequently being located at the juncture.* However, in the general case of arbitrary loading, one has no assurance that the absolute maximum stress intensity in the shell will be located at one of the eight points considered in the above discussion.

3.4 List of Nondimensional Curves for Spherical Shells

Solid Attachment	
Fig. No.	
SR-1	Maximum Stress due to external loading (Radial load and overturning moment combined)
SR-2	Stress due to radial load P
SR-3	Stress due to overturning moment M

Hollow Attachment Stresses Due to Radial Load P on Nozzle Connection

T	ρ	Fig. No.
5	0.25	SP-1
5	1.00	SP-2
5	2.00	SP-3
5	4.00	SP-4
15	1.00	SP-5
15	2.00	SP-6
15	4.00	SP-7
15	10.00	SP-8
50	4.00	SP-9
50	10.00	SP-10

Stress Due to Overturning Moment M on Nozzle Connection

T	ρ	Fig. No.
5	0.25	SM-1
5	1.00	SM-2
5	2.00	SM-3
5	4.00	SM-4
15	1.00	SM-5

* Under certain conditions stresses may be higher in the nozzle wall than they are in the vessel wall. This possibility is most likely if the nozzle opening is not reinforced or if the reinforcement is placed on the vessel wall and not on the nozzle.

T	ρ	Fig. No.
15	2.00	SM-6
15	4.00	SM-7
15	10.00	SM-8
50	4.00	SM-9
50	10.00	SM-10

3.5 Limitations on Application

In general, the foregoing procedure is applicable to relatively small attachments on large spherical shells. Where relatively large attachments are considered, or when situations are encountered that deviate considerably from the idealized cases presented herein, the designer should refer to para. A.2 and Fig. A-1 in Appendix A or to the original references to ascertain the limitations of applicability for the procedure used.

Under certain conditions, it is possible that stresses will be higher at points removed from the attachment-to-shell juncture than they are at the juncture itself (as assumed in the foregoing procedure). Of notable concern are the following:

3.5.1 NOZZLE STRESS. The foregoing procedure provides one with a tool to find stresses in the shell, but not in the nozzle. In some instances, stresses will be higher in the nozzle wall than they are in the vessel wall. This possibility is most likely if the the nozzle opening is not reinforced, or if the reinforcement is placed on the vessel wall and not on the nozzle.

3.5.2 SHELL STRESSES. It has been found in some cases that certain of the stress components (e.g., N_t or M_t) may peak at points slightly removed from the attachment. Such situations are indicated in the accompanying curves by a dashed line for the stress component(s) in question. If this situation is encountered, the designer could use the maximum value(s) and obtain a conservative result. But in doing so, he should recognize that stress components from different points in the vessel would be combined, and although conservative, the procedure is not theoretically correct. When a rigorous solution of the problem is desired, Bibliographical References 4-9, 12, or 18 should be consulted.

3.5.3 ELLIPSOIDAL SHELLS. The method described in the text may be applied to ellipsoidal pressure vessel heads with reasonable accuracy if the mean shell radius R_m at the juncture with the attachment is used in the applicable formulas.

3.6 Abridged Calculation for Maximum Stresses Due to Radial and Moment Loading Only at a Rigid Attachment

In the case of a rigid attachment, it has been found that the radial stresses (σ_r) at the juncture are always larger than the tangential stresses (σ_θ). Hence, in situations where only radial and moment loading are involved, it is possible to find the maximum stresses by considering only the radial stresses (σ_r). Figure SR-1 has been plotted by combining the nondimensional radial membrane

Table 2—Computation Sheet for Local Stresses in Spherical Shells (Solid Attachment)

1. Applied Loads *

Radial Load, $P =$ _____ lb.
 Shear Load, $V_1 =$ _____ lb.
 Shear Load, $V_2 =$ _____ lb.
 Overturning Moment, $M_1 =$ _____ in.lb.
 Overturning Moment, $M_2 =$ _____ in.lb.
 Torsional Moment, $M_T =$ _____ in.lb.

3. Geometric Parameters

$$U = \frac{r_o}{\sqrt{R_m T}} = \text{_____}$$

$$\sqrt{R_m T} = \text{_____}$$

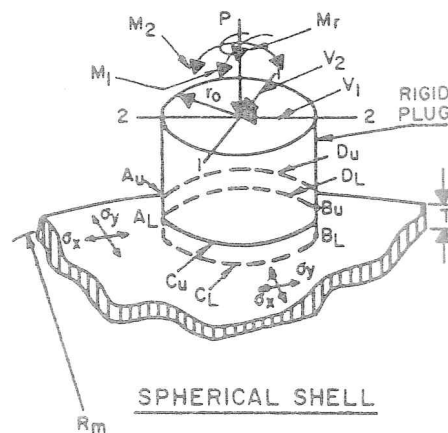
$$T \sqrt{R_m T} = \text{_____}$$

2. Geometry

Vessel Thickness, $T =$ _____ in.
 Vessel Mean Radius, $R_m =$ _____ in.
 Attachment Outside Radius, $r_o =$ _____ in.

4. Stress Concentration Factors

due to:
 membrane load, $K_n =$ _____
 bending load, $K_b =$ _____
 *NOTE: Enter all force values in accordance with sign convention



Reference Fig. Nos.	Read curves for	Calculate absolute values of stress and enter result *	STRESSES - if load is opposite that shown, reverse signs shown							
			Au	AL	Bu	BL	Cu	CL	Du	DL
SR - 2	$\frac{N_x T}{P} =$	$K_n \left(\frac{N_x T}{P} \right) \cdot \frac{P}{T^2} =$	-	-	-	-	-	-	-	-
	$\frac{M_x}{P} =$	$K_b \left(\frac{M_x}{P} \right) \cdot \frac{6P}{T^2} =$	-	+	-	+	-	+	-	+
SR - 3	$\frac{N_x T \sqrt{R_m T}}{M_1} =$	$K_n \left(\frac{N_x T \sqrt{R_m T}}{M_1} \right) \cdot \frac{M_1}{T^2 \sqrt{R_m T}} =$	-	-	-	-	-	-	+	+
	$\frac{M_x \sqrt{R_m T}}{M_1} =$	$K_b \left(\frac{M_x \sqrt{R_m T}}{M_1} \right) \cdot \frac{6M_1}{T^2 \sqrt{R_m T}} =$	-	-	-	-	-	+	+	-
	$\frac{N_x T \sqrt{R_m T}}{M_2} =$	$K_n \left(\frac{N_x T \sqrt{R_m T}}{M_2} \right) \cdot \frac{M_2}{T^2 \sqrt{R_m T}} =$	-	-	+	+	-	-	-	-
	$\frac{M_x \sqrt{R_m T}}{M_2} =$	$K_b \left(\frac{M_x \sqrt{R_m T}}{M_2} \right) \cdot \frac{6M_2}{T^2 \sqrt{R_m T}} =$	-	+	+	-	-	-	-	-
Add algebraically for summation of radial stresses, $\sigma_x =$										
SR - 2	$\frac{N_y T}{P} =$	$K_n \left(\frac{N_y T}{P} \right) \cdot \frac{P}{T^2} =$	-	-	-	-	-	-	-	-
	$\frac{M_y}{P} =$	$K_b \left(\frac{M_y}{P} \right) \cdot \frac{6P}{T^2} =$	-	+	-	+	-	+	-	+
SR - 3	$\frac{N_y T \sqrt{R_m T}}{M_1} =$	$K_n \left(\frac{N_y T \sqrt{R_m T}}{M_1} \right) \cdot \frac{M_1}{T^2 \sqrt{R_m T}} =$	-	-	-	-	-	-	+	+
	$\frac{M_y \sqrt{R_m T}}{M_1} =$	$K_b \left(\frac{M_y \sqrt{R_m T}}{M_1} \right) \cdot \frac{6M_1}{T^2 \sqrt{R_m T}} =$	-	-	-	-	-	+	+	-
	$\frac{N_y T \sqrt{R_m T}}{M_2} =$	$K_n \left(\frac{N_y T \sqrt{R_m T}}{M_2} \right) \cdot \frac{M_2}{T^2 \sqrt{R_m T}} =$	-	-	+	+	-	-	-	-
	$\frac{M_y \sqrt{R_m T}}{M_2} =$	$K_b \left(\frac{M_y \sqrt{R_m T}}{M_2} \right) \cdot \frac{6M_2}{T^2 \sqrt{R_m T}} =$	-	+	+	-	-	-	-	-
Add algebraically for summation of tangential stresses, $\sigma_y =$										
Shear stress due to load, V_1	$\tau_1 = \frac{V_1}{\pi r_o T} =$	$\tau_1 =$	-	-	-	-	-	+	+	
Shear stress due to load, V_2	$\tau_2 = \frac{V_2}{\pi r_o T} =$	$\tau_2 =$	+	+	-	-	-	-	-	-
Shear stress due to Torsion, M_T	$\tau_2 = \tau_1 = \frac{M_T}{2\pi r_o^2 T} =$	$\tau_2 =$	+	+	+	+	+	+	+	+
Add algebraically for summation of shear stresses, $\tau =$										
COMBINED STRESS INTENSITY - S										
1) When $\tau \neq 0$, $S =$ largest absolute magnitude of either $S = 1/2 [\sigma_x + \sigma_y \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}]$ or $\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$.										
2) When $\tau = 0$, $S =$ largest absolute magnitude of either $S = \sigma_x, \sigma_y$ or $(\sigma_x - \sigma_y)$.										

and radial bending stresses given in Figs. SR-2 and SR-3, so that the following simplified procedure for calculating maximum stresses has been developed.

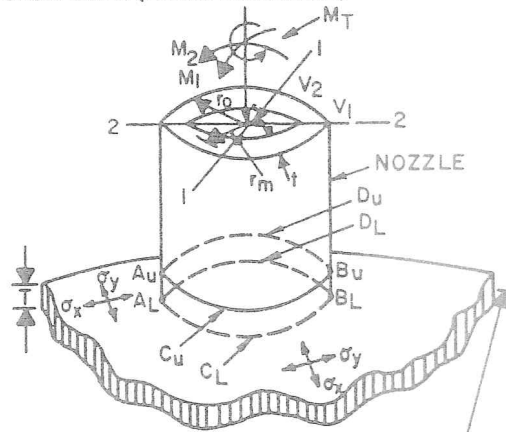
3.6.1 MAXIMUM STRESS RESULTING FROM RA-

DIAL LOAD (P).

Step 1. Calculate the value of the applicable shell parameter (U) as given in para. 3.2.1.

Step 2. Enter Fig. SR-1 at the value of U found

Table 3—Computation Sheet for Local Stresses in Spherical Shells (Hollow Attachment)



1. Applied Loads *

Radial Load, $P =$ _____ lb.
 Shear Load, $V_1 =$ _____ lb.
 Shear Load, $V_2 =$ _____ lb.
 Overturning Moment, $M_1 =$ _____ in.lb.
 Overturning Moment, $M_2 =$ _____ in.lb.
 Torsional Moment, $M_T =$ _____ in.lb.

Geometric Parameters

$$T = \frac{r_m}{t} = \text{_____}$$

$$\rho = \frac{T}{t} = \text{_____}$$

$$U = \frac{V_0}{\sqrt{R_m T}} = \text{_____}$$

2. Geometry

Vessel Thickness, $T =$ _____ in.
 Vessel Mean Radius, $R_m =$ _____ in.
 Nozzle Thickness, $t =$ _____ in.
 Nozzle Mean Radius, $r_m =$ _____ in.
 Nozzle Outside Radius, $r_o =$ _____ in.

Stress Concentration Factors

due to:
 membrane load, $K_n =$ _____
 bending load, $K_b =$ _____
 NOTE: Enter all force values in accordance with sign convention

From Fig.	Read curves for	Compute absolute values of stress and enter result	STRESSES — if load is opposite that shown, reverse signs shown							
			A _u	A _L	B _u	B _L	C _u	C _L	D _u	D _L
SP-1 to 10	$\frac{N_x T}{P} =$	$K_n \left(\frac{N_x T}{P} \right) \cdot \frac{P}{T^2} =$	—	—	—	—	—	—	—	—
↓	$\frac{M_x}{P} =$	$K_b \left(\frac{M_x}{P} \right) \cdot \frac{6P}{T^2} =$	—	+	—	+	—	+	—	+
SM-1 to 10	$\frac{N_x T \sqrt{R_m T}}{M_1} =$	$K_n \left(\frac{N_x T \sqrt{R_m T}}{M_1} \right) \cdot \frac{M_1}{T^2 \sqrt{R_m T}} =$	Hatched				—	—	+	+
↓	$\frac{M_x \sqrt{R_m T}}{M_1} =$	$K_b \left(\frac{M_x \sqrt{R_m T}}{M_1} \right) \cdot \frac{6M_1}{T^2 \sqrt{R_m T}} =$					—	+	+	—
↓	$\frac{N_x T \sqrt{R_m T}}{M_2} =$	$K_n \left(\frac{N_x T \sqrt{R_m T}}{M_2} \right) \cdot \frac{M_2}{T^2 \sqrt{R_m T}} =$	—	—	+	+	Hatched			
↓	$\frac{M_x \sqrt{R_m T}}{M_2} =$	$K_b \left(\frac{M_x \sqrt{R_m T}}{M_2} \right) \cdot \frac{6M_2}{T^2 \sqrt{R_m T}} =$	—	+	+	—				
Add algebraically for summation of $\sigma_x =$										
SP-1 to 10	$\frac{N_y T}{P} =$	$K_n \left(\frac{N_y T}{P} \right) \cdot \frac{P}{T^2} =$	—	—	—	—	—	—	—	—
↓	$\frac{M_y}{P} =$	$K_b \left(\frac{M_y}{P} \right) \cdot \frac{6P}{T^2} =$	—	+	—	+	—	+	—	+
SM-1 to 10	$\frac{N_y T \sqrt{R_m T}}{M_1} =$	$K_n \left(\frac{N_y T \sqrt{R_m T}}{M_1} \right) \cdot \frac{M_1}{T^2 \sqrt{R_m T}} =$	Hatched				—	—	+	+
↓	$\frac{M_y \sqrt{R_m T}}{M_1} =$	$K_b \left(\frac{M_y \sqrt{R_m T}}{M_1} \right) \cdot \frac{6M_1}{T^2 \sqrt{R_m T}} =$					—	+	+	—
↓	$\frac{N_y T \sqrt{R_m T}}{M_2} =$	$K_n \left(\frac{N_y T \sqrt{R_m T}}{M_2} \right) \cdot \frac{M_2}{T^2 \sqrt{R_m T}} =$	—	—	+	+	Hatched			
↓	$\frac{M_y \sqrt{R_m T}}{M_2} =$	$K_b \left(\frac{M_y \sqrt{R_m T}}{M_2} \right) \cdot \frac{6M_2}{T^2 \sqrt{R_m T}} =$	—	+	—	—				
Add algebraically for summation of $\sigma_y =$										
Shear stress due to load, V_1	$\tau_1 = \frac{V_1}{\pi r_o T} =$		Hatched				—	—	+	+
Shear stress due to load, V_2	$\tau_2 = \frac{V_2}{\pi r_o T} =$						+	+	—	—
Shear stress due to torsion, M_T	$\tau_2 = \tau_1 = \frac{M_T}{2\pi r_o^2 T} =$		+	+	+	+	+	+	+	+
Add algebraically for summation of $\tau =$										

COMBINED STRESS INTENSITY — S.

- When $\tau \neq 0$, $S =$ largest absolute magnitude of either $S = 1/2 [\sigma_x + \sigma_y \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}]$ or $\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$.
- When $\tau = 0$, $S =$ largest absolute magnitude of either $S = \sigma_x, \sigma_y$ or $(\sigma_x - \sigma_y)$.

in Step 1, and using the curve marked "radial load P ," read off the value of the nondimensional stress ($\sigma_r T^2/P$).

Step 3. Using the applicable value of load (P), shell thickness (T), and stress concentration factor (K_n), calculate the maximum combined stress (σ) thus:

$$\sigma = K_n \left(\frac{\sigma_i T^2}{P} \right) \left(\frac{P}{T^2} \right)$$

In the case of a cylindrical attachment, this stress will be located on the outside surface of the vessel all the way around at its intersection with the attachment. If P is acting radially inward, a compressive (—) stress results; if P acts radially outward, a tensile (+) stress results. Since the stress normal to the surface of the vessel is zero, σ is the maximum stress intensity; i.e., $S = \sigma$.

3.6.2 MAXIMUM STRESS RESULTING FROM OVERTURNING MOMENTS, M_1 AND M_2 .

Step 1. Resolve moments M_1 and M_2 vectorially into a single moment (M) thus:

$$M = \sqrt{M_1^2 + M_2^2}$$

NOTE: It is assumed that M_1 and M_2 are orthogonally oriented. Also, M must not include torsional moment (M_T) as shown in Fig. 1.

Step 2. Calculate the value of the applicable shell parameter (U) as given in para. 3.2.1.

Step 3. Enter Fig. SR-1 at the value of U found in Step 2, and using the curve marked "external moment M " read off the value of the nondimensional stress ($\sigma_x T^2 \sqrt{R_m T/M}$).

Step 4. Using the applicable value of moment (M), shell thickness (T), shell mean radius (R_m), and stress concentration factor (K_n), calculate the maximum combined stress (σ_x) thus:

$$\sigma = K_n \left(\frac{\sigma_r T^2 \sqrt{R_m T}}{M} \right) \times \left(\frac{M}{T^2 \sqrt{R_m T}} \right)$$

In the case of a cylindrical attachment, this stress will be located on the outside surface of the vessel, at its intersection with the attachment, on the "forward side" of the moment (M). The stress (σ) will be distributed sinusoidally around the attachment. Since the stress normal to the surface of the vessel is zero, σ is the maximum stress intensity, i.e., $S = \sigma$.

3.6.3 MAXIMUM STRESS RESULTING FROM COMBINED LOAD (P) AND OVERTURNING MOMENT (M). If load (P) and moment (M) are considered separately as outlined in 3.6.1 and 3.6.2 above, it is possible to consider the combined loading condition by superposing results of the two cases as follows:

$$\sigma = K_n \left[\left(\frac{\sigma_x T^2}{P} \right) \left(\frac{P}{T^2} \right) + \left(\frac{\sigma_x T^2 \sqrt{R_m T}}{M} \right) \left(\frac{M}{T^2 \sqrt{R_m T}} \right) \right]$$

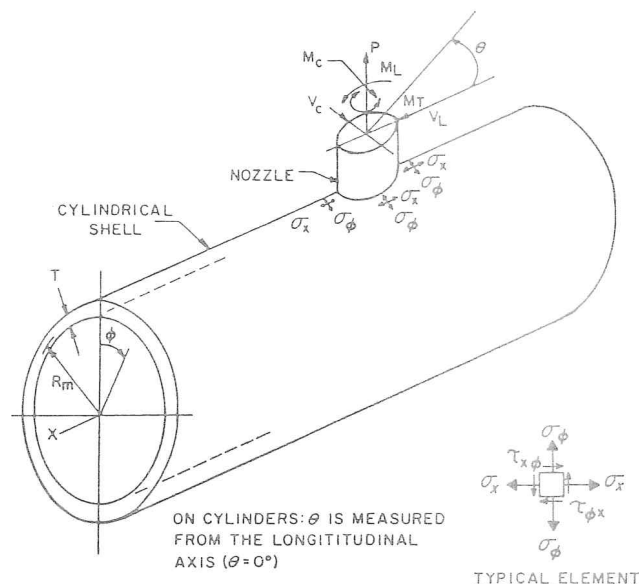


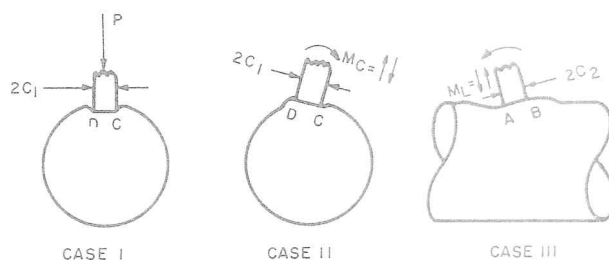
Fig. 2—Types of loading conditions at an attachment to a cylindrical shell

In using this formula, the sign conventions established in Table 1 should be used.

4. Cylindrical Shells

4.1 Sign Convention

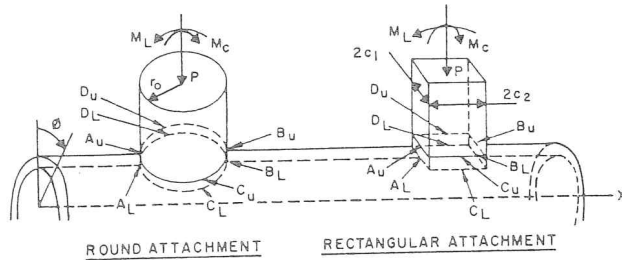
Stresses will be considered in the shell at the attachment-to-shell juncture in both the circumferential and longitudinal directions as shown in Fig. 2. A knowledge of the shell deflections resulting from various modes of loading permits one to predict whether resulting stresses will be tensile (+) or compressive (-).



Consider Case I showing a direct radial inward load, P . Here P acts similar to a local external pressure on the shell causing compressive membrane stresses. Furthermore, local bending occurs so that tensile bending stresses result on the inside of the vessel at C and D while compressive bending stresses result on the outside.

In Cases II and III, the applied moments are considered to act as couples composed of equal and opposite radial forces. Hence, tensile membrane stresses result at B and D while compressive membrane stresses result at A and C . As in Case I

Table 4—Sign Convention for Stresses Resulting from Radial and Moment Loading on a Cylindrical Shell



STRESS	LOCATION	LOADING		
		P	M _L	M _C
Membrane $\frac{N_x}{T}$ & $\frac{N_y}{T}$	A _U AL B _U BL	—	+	+
	C _U CL D _U DL	—	+	+
Bending $\frac{6M_x}{T^2}$	A _U AL B _U BL	++	+	+
	C _U CL D _U DL	++	+	+
Bending $\frac{6M_y}{T^2}$	A _U AL B _U BL	++	+	+
	C _U CL D _U DL	++	+	+

NOTE: 1) Sign convention for stresses: + tension, — compression.
2) If load or moment directions reverse, all signs in applicable column reverse.

tensile bending stresses result at A and C on the inside of the vessel, and B and D on the outside of the vessel. Similarly, compressive bending stresses result at A and C on the outside and B and D on the inside.

In this manner Table 4 has been developed to show the signs of stresses resulting from various external loading conditions. These stresses are located in the vessel wall at its juncture with the attachment. Use of Table 4 permits one to use the nondimensional curves presented in the following procedure with a minimum of encumbrance and concern for sign convention.

The numerous stress components can be readily accounted for if a scheme similar to that shown in Table 5 is adopted. In using this scheme it is to be noted that the Maximum Shear Theory has been used to determine equivalent stress intensities. Also it is to be noted that evaluation of stresses resulting from internal pressure has been omitted.

Test work conducted by PVRC has shown that stresses attenuate rapidly at points removed from the attachment-to-shell juncture, the maximum stress usually being located at the juncture. However, in the general case of arbitrary loading, one has no assurance that the absolute maximum stress intensity will be located at one of the eight points considered in the above discussion. The maximum stress intensity could be located at some intermediate point around the juncture under an arbitrary load, or under a longitudinal moment with the circumstances outlined in para. 4.4 and

Appendix A.

4.2 Parameters

The results of Bijlaard's work have been plotted in terms of nondimensional geometric parameters by use of an electronic computer. Hence, the first step in this procedure is to evaluate the applicable geometric parameters γ and β .

4.2.1 SHELL PARAMETER (γ). The shell parameter is given by the ratio of the shell mid-radius to shell thickness thus:

$$\gamma = R_m/T$$

4.2.2 ATTACHMENT PARAMETER (β). For cylindrical shells, either round or rectangular attachments may be considered in the following manner:

4.2.2.1 Round Attachment: For a round attachment the parameter β is evaluated using the expression:

$$\beta = \frac{0.875 r_0}{R_m}$$

4.2.2.2 Square Attachment: For a square attachment the parameter is evaluated by:

$$\beta = \beta_1 = \beta_2 = \frac{c_1}{R_m} = \frac{c_2}{R_m}$$

4.2.2.3 Rectangular Attachment Subject to Radial Load (P): For this case β is evaluated as follows:

$$\beta_1 = \frac{c_1}{R_m} \quad \beta_2 = \frac{c_2}{R_m}$$

If $\frac{\beta_1}{\beta_2} \geq 1$, $\beta =$

$$\left[1 - \frac{1}{3} \left(\frac{\beta_1}{\beta_2} - 1 \right) (1 - K_1) \right] \sqrt{\beta_1 \beta_2}$$

If $\frac{\beta_1}{\beta_2} < 1$, $\beta =$

$$\left[1 - \frac{4}{3} \left(1 - \frac{\beta_1}{\beta_2} \right) (1 - K_2) \right] \sqrt{\beta_1 \beta_2}$$

where K values are obtained from Table 6.

4.2.2.4 Rectangular Attachment Subject to Circumferential Moment (M_c).

4.2.2.4.1: When considering membrane forces (N_i): $\beta = \sqrt{\beta_1^2 + \beta_2^2}$. Then multiply values of $N_i/(M_c/R_m^2\beta)$ so determined by C_c from Table 7 (see para. 4.3).

4.2.2.4.2: When considering bending moment (M_i): $\beta = K_c \sqrt{\beta_1^2 + \beta_2^2}$ where K_c is given in Table 7.

4.2.2.5 Rectangular Attachment Subject to Longitudinal Moment (M_L).

4.2.2.5.1: When considering membrane forces (N_i): $\beta = \sqrt{\beta_1^2 + \beta_2^2}$. Then multiply values of

Table 5—Computation Sheet for Local Stresses in Cylindrical Shells

1. Applied Loads*

Radial load, $P = \text{--- lb.}$
 Circ. Moment, $M_c = \text{--- in. lb.}$
 Long. Moments, $M_L = \text{--- in. lb.}$
 Torsion Moment, $M_T = \text{--- in. lb.}$
 Shear Load, $V_c = \text{--- lb.}$
 Shear Load, $V_L = \text{--- lb.}$

2. Geometry

Vessel thickness, $T = \text{--- in.}$
 Attachment radius, $r_o = \text{--- in.}$
 Vessel radius, $R_m = \text{--- in.}$

3. Geometric Parameters

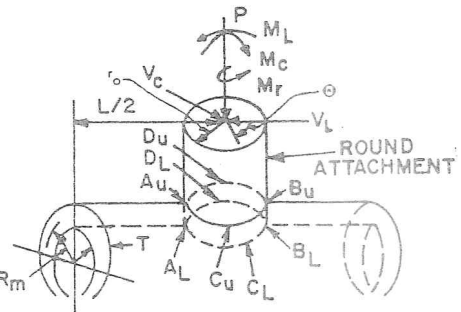
$$\gamma = \frac{R_m}{T} = \text{---}$$

$$\beta = (0.875) \frac{r_o}{R_m} = \text{---}$$

Stress Concentration due to:

- a) membrane load, $K_n = \text{---}$
 b) bending load, $K_b = \text{---}$

*NOTE: Enter all force values in accordance with sign convention



CYLINDRICAL SHELL

From Fig.	Read curves for	Compute absolute values of stress and enter result--	STRESSES - if load is opposite that shown, reverse signs shown											
			A _u	A _L	B _u	B _L	C _u	C _L	D _u	D _L				
3C or 4C	$\frac{N\phi}{P/R_m} =$	$K_n \left(\frac{N\phi}{P/R_m} \right) \cdot \frac{P}{R_m T} =$	-	-	-	-	-	-	-	-				
1C or 2C-1	$\frac{M\phi}{P} =$	$K_b \left(\frac{M\phi}{P} \right) \cdot \frac{6P}{T^2} =$	-	+	-	+	-	+	-	+				
3A	$\frac{N\phi}{M_c/R_m^2\beta} =$	$K_n \left(\frac{N\phi}{M_c/R_m^2\beta} \right) \cdot \frac{M_c}{R_m^2\beta T} =$					-	-	+	+				
1A	$\frac{M\phi}{M_c/R_m\beta} =$	$K_b \left(\frac{M\phi}{M_c/R_m\beta} \right) \cdot \frac{6M_c}{R_m\beta T^2} =$					-	+	+	-				
3B	$\frac{N\phi}{M_L/R_m^2\beta} =$	$K_n \left(\frac{N\phi}{M_L/R_m^2\beta} \right) \cdot \frac{M_L}{R_m^2\beta T} =$	-	-	+	+								
1B or 1B-1	$\frac{M\phi}{M_L/R_m\beta} =$	$K_b \left(\frac{M\phi}{M_L/R_m\beta} \right) \cdot \frac{6M_L}{R_m\beta T^2} =$	-	+	+	-								
Add algebraically for summation of ϕ stresses, $\sigma_\phi =$														
3C or 4C	$\frac{N_x}{P/R_m} =$	$K_n \left(\frac{N_x}{P/R_m} \right) \cdot \frac{P}{R_m T} =$	-	-	-	-	-	-	-	-				
1C-1 or 2C	$\frac{M_x}{P} =$	$K_b \left(\frac{M_x}{P} \right) \cdot \frac{6P}{T^2} =$	-	+	-	+	-	+	-	+				
4A	$\frac{N_x}{M_c/R_m^2\beta} =$	$K_n \left(\frac{N_x}{M_c/R_m^2\beta} \right) \cdot \frac{M_c}{R_m^2\beta T} =$					-	-	+	+				
2A	$\frac{M_x}{M_c/R_m\beta} =$	$K_b \left(\frac{M_x}{M_c/R_m\beta} \right) \cdot \frac{6M_c}{R_m\beta T^2} =$					-	+	+	-				
4B	$\frac{N_x}{M_L/R_m^2\beta} =$	$K_n \left(\frac{N_x}{M_L/R_m^2\beta} \right) \cdot \frac{M_L}{R_m^2\beta T} =$	-	-	+	+								
2B or 2B-1	$\frac{M_x}{M_L/R_m\beta} =$	$K_b \left(\frac{M_x}{M_L/R_m\beta} \right) \cdot \frac{6M_L}{R_m\beta T^2} =$	-	+	+	-								
Add algebraically for summation of X stresses, $\sigma_x =$														
Shear stress due to Torsion, M_T		$\tau \times \phi = \tau \times \phi = \frac{M_T}{2\pi r_o^2 T} =$	+	+	+	+	+	+	+	+				
Shear stress due to load, V_c		$\tau \times \phi = \frac{V_c}{\pi r_o T} =$	+	+	-	-								
Shear stress due to load, V_L		$\tau \times \phi = \frac{V_L}{\pi r_o T} =$									-	-	+	+
Add Algebraically for summation of shear stresses, $\tau =$														
COMBINED STRESS INTENSITY - S														
1) When $\tau \neq 0$, $S =$ largest absolute magnitude of either $S = 1/2 [\sigma_x + \sigma_\phi \pm \sqrt{(\sigma_x - \sigma_\phi)^2 + 4\tau^2}]$ or $\sqrt{(\sigma_x - \sigma_\phi)^2 + 4\tau^2}$.														
2) When $\tau = 0$, $S =$ largest absolute magnitude of either $S = \sigma_x, \sigma_\phi$ or $(\sigma_x - \sigma_\phi)$.														

$N_i/(M_L/R_m^2\beta)$ so determined by (C_L) from Table 8 (see para. 4.3).

4.2.2.5.2: When considering bending moment (M_i): $\beta = K_L/\beta_1\beta_2$, where K_L is given in Table 8.

4.3 Calculation of Stresses

4.3.1 STRESSES RESULTING FROM RADIAL LOAD, P.

4.3.1.1 Circumferential Stresses (σ_ϕ):

Step 1. Using the applicable values of β and γ

Table 6—Radial Load (P)

	N_ϕ	N_x	M_ϕ	M_x
K_1	0.91	1.68	1.76	1.2
K_2	1.48	1.2	0.88	1.25

NOTE: Above holds approximately within limits $4 \geq \beta_1/\beta_2 \geq 1/4$.

calculated in para. 4.2, enter Fig. 3C and read off the dimensionless membrane force $\left[\frac{N_\phi}{P/R_m} \right]$

Step 2. By the same procedure used in Step 1, enter Fig. 1C or 2C-1 and find the dimensionless bending moment $[M_\phi/P]$.

Step 3. Using applicable values of P , R_m , and T , find the circumferential membrane stress (N_ϕ/T) by:

$$\frac{N_\phi}{T} = \left[\frac{N_\phi}{P/R_m} \right] \cdot \left[\frac{P}{R_m T} \right]$$

Step 4. By a procedure similar to that used in Step 3, find the circumferential bending stress ($6M_\phi/T^2$) thus:

$$\frac{6M_\phi}{T^2} = \left[\frac{M_\phi}{P} \right] \cdot \left[\frac{6P}{T^2} \right]$$

Step 5. Combine the circumferential membrane and bending stresses by use of the general stress

equation (Section 2), together with the proper choice of sign (Table 4); i.e.:

$$\sigma_\phi = K_n \frac{N_\phi}{T} \pm K_b \frac{6M_\phi}{T^2}$$

4.3.1.2 Longitudinal Stresses (σ_x): Follow the 5 steps outlined in 4.3.1.1 except that $[N_x/P/R_m]$ is obtained using Fig. 4C; and $[M_x/P]$, using Fig. 2C or 2C-1. It follows that:

$$\frac{N_x}{T} = \left[\frac{N_x}{P/R_m} \right] \cdot \left[\frac{P}{R_m T} \right],$$

$$\frac{6M_x}{T^2} = \left[\frac{M_x}{P} \right] \cdot \left[\frac{6P}{T^2} \right] \text{ and}$$

$$\sigma_x = K_n \frac{N_x}{T} \pm K_b \frac{6M_x}{T^2}$$

4.3.2 STRESSES RESULTING FROM CIRCUMFERENTIAL MOMENT, M_c .

4.3.2.1 Circumferential Stresses (σ_ϕ):

Step 1. Using the applicable values of β and γ calculated in para. 4.2, enter Fig. 3A and read off the dimensionless membrane force $N_\phi/(M_c/R_m^2\beta)$.

Step 2. By the same procedure used in Step 1, enter Fig. 1A and find the dimensionless bending moment $M_\phi/(M_c/R_m\beta)$.

Step 3. Using applicable values of M_c , R_m , β

Table 7—Circumferential Moment (M_c)

β_1/β_2	γ	K_c for θ	K_c for M_ϕ	K_c for M_x	C_c for N_ϕ	C_c for N_x
$1/4$	15	1.09	1.31	1.84	0.31	0.49
	50	1.04	1.24	1.62	0.21	0.46
	100	0.97	1.16	1.45	0.15	0.44
	300	0.92	1.02	1.17	0.09	0.46
$1/2$	15	1.00	1.09	1.36	0.64	0.75
	50	0.98	1.08	1.31	0.57	0.75
	100	0.94	1.04	1.26	0.51	0.76
	300	0.95	0.99	1.13	0.39	0.77
2	15	(1.00)	(1.20)	(0.97)	(1.7)	(1.3)
	100	1.19	1.10	0.95	1.43	1.12
	300	...	(1.00)	(0.90)	(1.3)	(1.00)
	15	(1.00)	(1.47)	(1.08)	(1.75)	(1.31)
4	100	1.49	1.38	1.06	1.49	0.81
	300	...	(1.27)	(0.98)	(1.36)	(0.74)

NOTE: The values in parenthesis determined by an approximate solution.

Table 8—Longitudinal Moment (M_L)

β_2/β_1	γ	K_L for θ	K_L for M_ϕ	K_L for M_x	C_L for N_ϕ	C_L for N_x
$1/4$	15	1.14	1.80	1.24	0.75	0.43
	50	1.13	1.65	1.16	0.77	0.33
	100	1.18	1.59	1.11	0.80	0.24
	300	1.31	1.56	1.11	0.90	0.07
$1/2$	15	(1.00)	(1.08)	(1.04)	(0.90)	(0.76)
	100	1.00	1.07	1.02	0.97	0.68
	300	(1.00)	(1.05)	(1.02)	(1.10)	(0.60)
	15	...	(0.94)	(1.12)	(0.87)	(1.30)
2	100	1.09	0.89	1.07	0.81	1.15
	300	...	(0.79)	(0.90)	(0.80)	(1.50)
	15	1.39	0.90	1.24	0.68	1.20
	100	1.18	0.81	1.12	0.51	1.03
4	300	...	(0.64)	(0.83)	(0.50)	(1.33)

NOTE: The values in parenthesis determined by an approximate solution.

and T , find the circumferential membrane stress (N_ϕ/T) by:

$$\frac{N_\phi}{T} = \left[\frac{N_\phi}{M_c/R_m^2\beta} \right] \cdot \left[\frac{M_c}{R_m^2\beta T} \right]$$

Step 4. By a procedure similar to that used in Step 3, find the circumferential bending stress ($6M_\phi/T^2$). Thus:

$$\frac{6M_\phi}{T^2} = \left[\frac{M_\phi}{M_c/R_m\beta} \right] \cdot \left[\frac{6M_c}{R_m T^2 \beta} \right]$$

Step 5. Combine the circumferential membrane and bending stresses by use of the general stress equation (Section 2) together with proper choice of sign (Table 4); i.e.:

$$\sigma_\phi = K_n \frac{N_\phi}{T} \pm K_b \frac{6M_\phi}{T^2}$$

4.3.2.2 *Longitudinal Stresses (σ_x)*: Follow the 5 steps outlined in 4.3.2.1 except that $[N_x/(M_c/R_m^2\beta)]$ is obtained using Fig. 4A; and $[M_x/(M_c/R_m\beta)]$, using Fig. 2A. It follows that:

$$\frac{N_x}{T} = \left[\frac{N_x}{M_c/R_m^2\beta} \right] \cdot \left[\frac{M_c}{R_m^2\beta T} \right]$$

$$\frac{6M_x}{T^2} = \left[\frac{M_x}{M_c/R_m\beta} \right] \cdot \left[\frac{6M_c}{R_m T^2 \beta} \right]$$

$$\sigma_x = K_n \frac{N_x}{T} \pm K_b \frac{6M_x}{T^2}$$

4.3.3 STRESSES RESULTING FROM LONGITUDINAL MOMENT, M_L .

4.3.3.1 *Circumferential Stresses (σ_ϕ)*: Follow the 5 steps outlined in 4.3.2.1 except that $[N_\phi/(M_L/R_m^2\beta)]$ is obtained using Fig. 3B; and $[M_\phi/(M_L/R_m\beta)]$, using Fig. 1B or 1B-1. It follows that:

$$\frac{N_\phi}{T} = \left[\frac{N_\phi}{M_L/R_m^2\beta} \right] \cdot \left[\frac{M_L}{R_m^2\beta T} \right]$$

$$\frac{6M_\phi}{T^2} = \left[\frac{M_\phi}{M_L/R_m\beta} \right] \cdot \left[\frac{6M_L}{R_m\beta T^2} \right]$$

4.3.3.2 *Longitudinal Stresses (σ_x)*: Follow the 5 steps outlined in 4.3.2.1 except that $[N_x/(M_L/R_m^2\beta)]$ is obtained using Fig. 4B; and $[M_x/(M_L/R_m\beta)]$, using Fig. 2B or 2B-1. It follows that:

$$\frac{N_x}{T} = \left[\frac{N_x}{M_L/R_m^2\beta} \right] \cdot \left[\frac{M_L}{R_m^2\beta T} \right]$$

$$\frac{6M_x}{T^2} = \left[\frac{M_x}{M_L/R_m\beta} \right] \cdot \left[\frac{6M_L}{R_m\beta T^2} \right]$$

4.3.4 STRESSES RESULTING FROM TORSIONAL MOMENT, M_T . In the case of a round attachment (such as a pipe), torsional moment is assumed to induce only shear stresses, so that shear stress (τ) in the shell at the attachment-to-shell juncture is given by:

$$\tau_{\phi z} = \tau_{z\phi} = \frac{M_T}{2\pi r_0^2 T}$$

If only shear stresses are being considered, it is to be noted that the equivalent stress intensity is twice the above calculated shear stress.

In the case of rectangular attachments, torsional moment produces a complex stress field in the shell. Acceptable methods of analyzing this situation are not available at this time. If the designer has reason for concern, the problem should be resolved by testing in accordance with established code procedures.

4.3.5 STRESSES RESULTING FROM SHEAR LOADS, V_c AND V_L . Bijlaard has proposed* that shear force (V) can be assumed transmitted to the shell entirely by membrane shear force. Therefore stresses in the shell at the attachment-to-shell juncture can be approximated as follows:

4.3.5.1 Round Attachment

$$\tau_{x\phi} = \frac{V_c}{\pi r_0 T} \cos \theta \quad (\text{max. at A and B})$$

$$\tau_{\phi x} = \frac{V_L}{\pi r_0 T} \sin \theta \quad (\text{max. at C and D})$$

4.3.5.2 Rectangular Attachment

$$\tau_{x\phi} = \frac{V_c}{4c_1 T}$$

$$\tau_{\phi x} = \frac{V_L}{4c_2 T}$$

4.3.6 STRESSES RESULTING FROM ARBITRARY LOADING. In the general case, all applied loads and moments must be resolved (at the attachment-to-shell interface) in the three principal directions; i.e., they must be resolved into components P , V_c , V_L , M_c , M_L and M_T . If one then proceeds in the manner previously outlined (e.g., para. 4.3.1.1), membrane, bending and shear stresses can be evaluated at eight points in the shell at its juncture with the attachment. These eight points are shown in the sign convention chart, Table 4.

4.4 Nondimensional Curves for Cylindrical Shells

The nondimensional curves which follow constitute, in general, a replot of Bijlaard's data to a semilog scale in order that certain portions of the curves can be read with greater facility. Those portions of the curves which are taken directly from Bijlaard's work are shown as solid curves; those portions of the curves which have been modified on the basis of recent experimental data, as discussed in Appendix A, are shown as dotted curves.

In the case of longitudinal moment loading and axial loading (thrust), two sets of curves are shown for the bending components of stress—one set ap-

* See reference 14.

plying to the longitudinal axis, and the other applying to an area of maximum stress off the axes of symmetry (longitudinal moment), or to the transverse axis (thrust). In the latter case, a portion of the original curves has been deleted in order to emphasize that the curves should not be used beyond the limits indicated. This was done because the available data indicated that the "outer limits" of the curves were appreciably unconservative, with no feasible manner to "correct" them (as explained in Appendix A).

In the case of longitudinal moment, the exact location of the maximum stress cannot be defined with certainty, but Fig. A-14 will provide an estimate of its location (considering that the location of maximum stress under internal pressure and longitudinal moment was essentially the same on IIT model "C-1," as shown on Figs. A-2 and A-3). It should also be noted that, to the best of our knowledge, the curves for "maximum stresses off the axes of symmetry" (Figs. 1B-1 and 2B-1) would apply only to the case of a round, flexible nozzle connection; it is conceivable that a similar effect might apply to a rigid square or rectangular attachment, for which the shell at the outer edges of the attachment might take a greater part of the load than that portion of the shell adjacent to the longitudinal centerline. However, we know of no direct evidence to support such an assumption.

4.4.1 LIST OF NONDIMENSIONAL CURVES FOR CYLINDRICAL SHELLS

Fig. 1A	Moment $M_\phi / (M_c / R_m \beta)$ due to M_c
Fig. 2A	Moment $M_x / (M_c / R_m \beta)$ due to M_c
Fig. 3A	Membrane force $N_\phi / (M_c / R_m^2 \beta)$ due to M_c
Fig. 4A	Membrane force $N_x / (M_c / R_m^2 \beta)$ due to M_c
Fig. 1B or 1B-1	Moment $M_\phi / (M_L / R_m \beta)$ due to M_L
Fig. 2B or 2B-1	Moment $M_x / (M_L / R_m \beta)$ due to M_L
Fig. 3B	Membrane force $N_\phi / (M_L / R_m^2 \beta)$ due to M_L
Fig. 4B	Membrane force $N_x / (M_L / R_m^2 \beta)$ due to M_L
Fig. 1C	Moment M_ϕ / P due to P
Fig. 1C-1	Moment M_x / P due to P
Fig. 2C	Moment M_ϕ / P due to P
Fig. 2C-1	Moment M_x / P due to P
Fig. 3C	Membrane force $N_\phi / P / R_m$ and $N_x / P / R_m$ due to P
Fig. 4C	Membrane force $N_\phi / P / R_m$ and $N_x / P / R_m$ due to P

4.5 Limitations on Application

Where relatively large attachments are considered, or when situations are encountered that deviate considerably from the idealized cases presented herein, the designer should refer to para. A.3

in Appendix A and to the original references to ascertain the limitations of applicability for the procedure used.

However, there are a few generalizations that can safely be made regarding vessel and attachment geometry.

4.5.1 EXTERNAL RADIAL LOAD. Stresses are affected very little by the ratio of shell length to shell radius (l/R_m). Therefore, no restriction is made on the point of load application except in very extreme cases. The curves included in this report are for an l/R_m ratio of 8, which is sufficient for most practical applications. On the basis of data presented in Bibliographical Reference 2, results based on an l/R_m ratio of 8 will be slightly conservative for lesser values of l/R_m ratio and unconservative for greater values of l/R_m ratio. However, the error involved does not exceed approximately 10% of all l/R_m values greater than 3, which should be sufficiently accurate for most calculations. Since for lesser values of l/R_m , the results are conservative, no restriction will ordinarily be necessary on l/R_m ratio or the point of load application. For extreme cases or for "off center" loading, one may make corrections by use of the curves presented on page 8 of Bibliographical Reference 2, if desired.

Results are not considered applicable in cases where the length of the cylinder (l) is less than its radius (R_m). This applies either to the case of an open ended cylinder or closed ended cylinder where the stiffness is appreciably modified from the case considered.

4.5.2 EXTERNAL MOMENT. Results are applicable in the case of longitudinally off center attachments (a more usual case) provided that the attachment is located at least half the shell radius ($1/2 R_m$) from the end of the cylinder.

4.5.3 ATTACHMENT STRESSES. The foregoing procedure provides one with a tool to find stresses in the shell, but not in the attachment. Under certain conditions, stresses may be higher in the attachment than they are in the vessel. For example, in the case of a nozzle, it is likely that the stresses will be higher in the nozzle wall than they are in the vessel wall if the nozzle opening is unreinforced or if the reinforcement is placed on the vessel wall and not on the nozzle.

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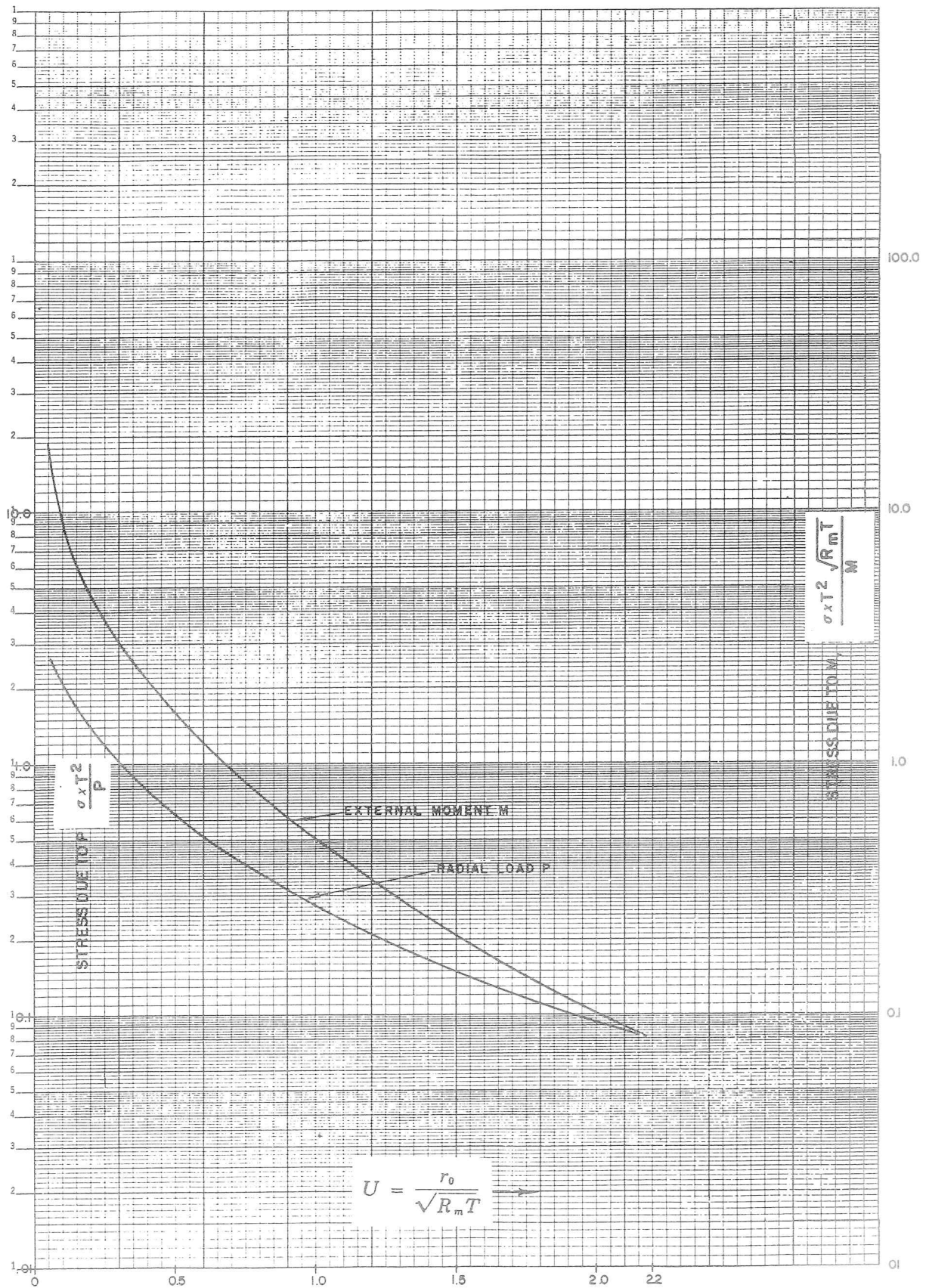


Fig. SR-1—Maximum stress due to external loading on a spherical shell (rigid plug)

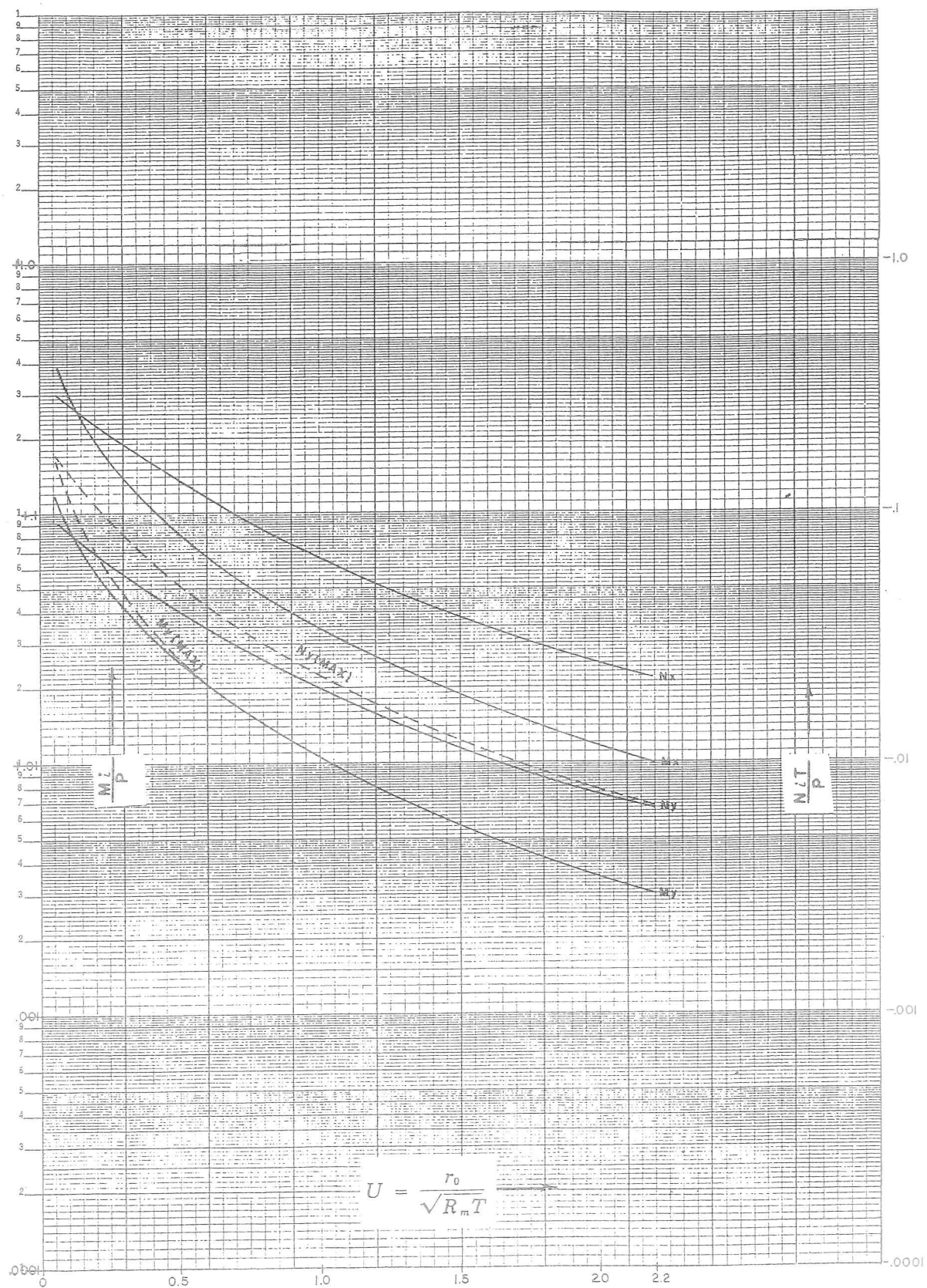


Fig. SR-2—Stresses in spherical shell due to a radial load P on a nozzle connection (rigid plug)

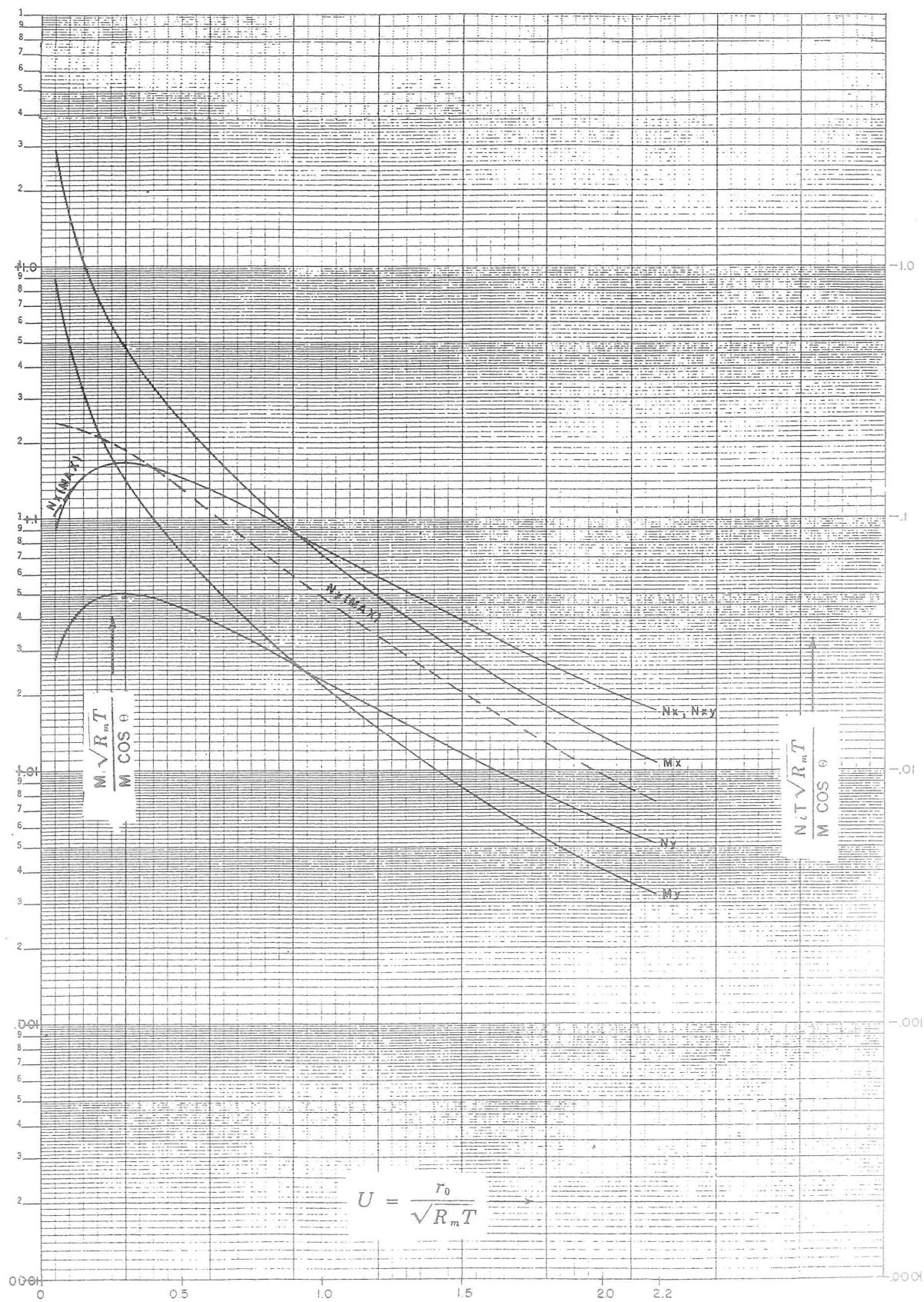


Fig. SR-3—Stresses in spherical shell due to overturning moment M on nozzle connection (rigid plug)

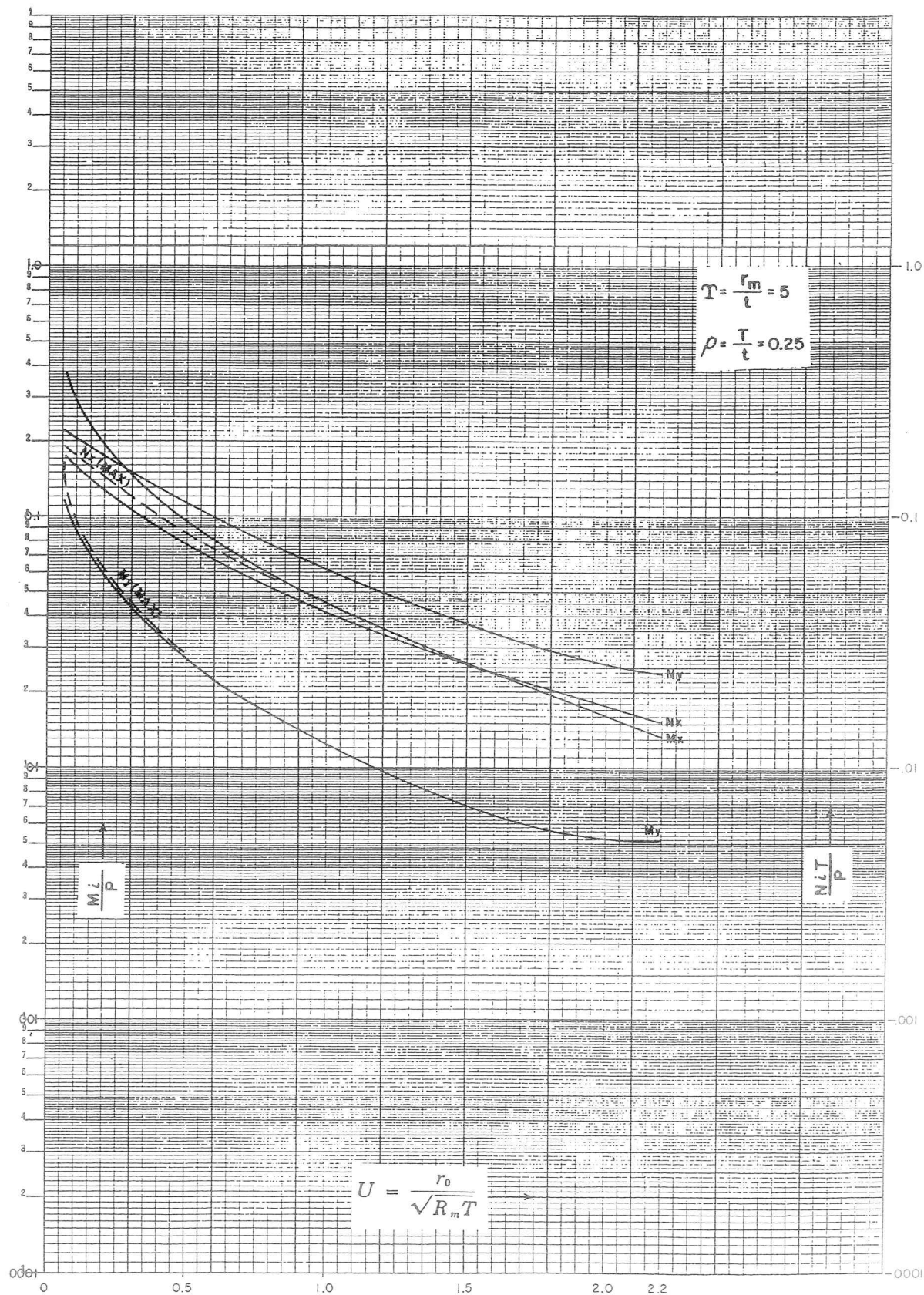


Fig. SP-1—Stresses in spherical shell due to radial load P on a nozzle connection

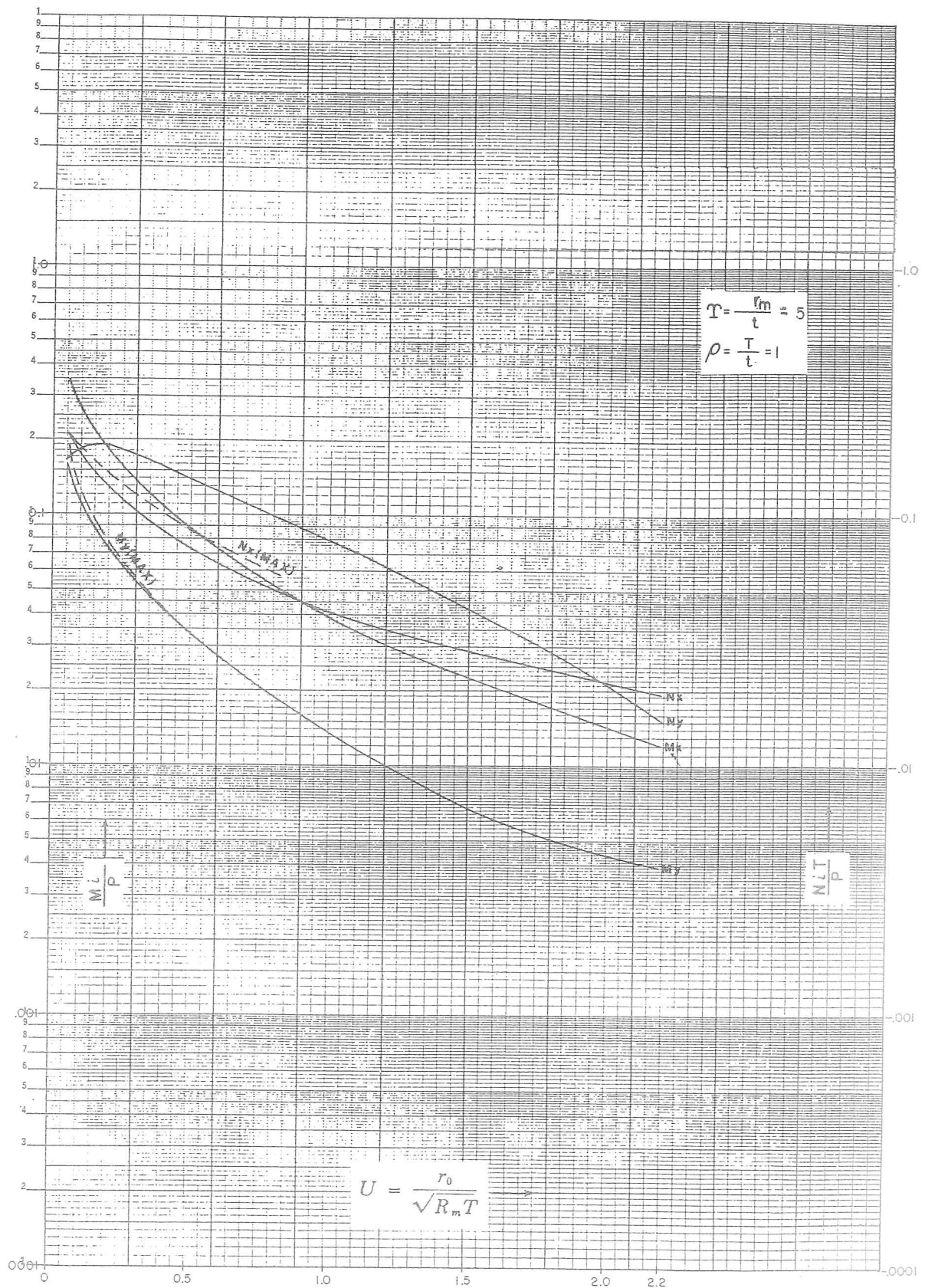


Fig. SP-2—Stresses in spherical shell due to radial load P on a nozzle connection

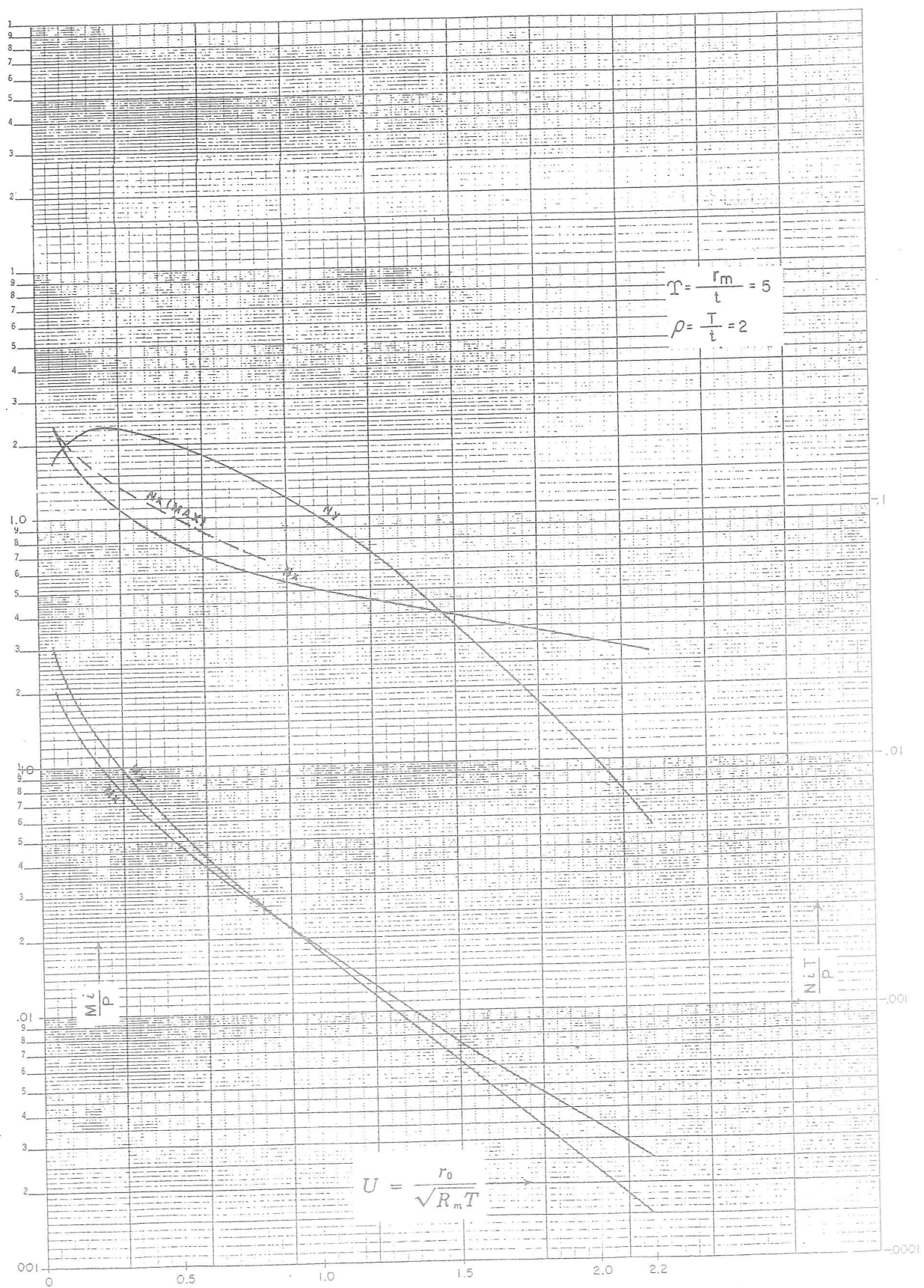


Fig. SP-3—Stresses in spherical shell due to radial load P on a nozzle connection

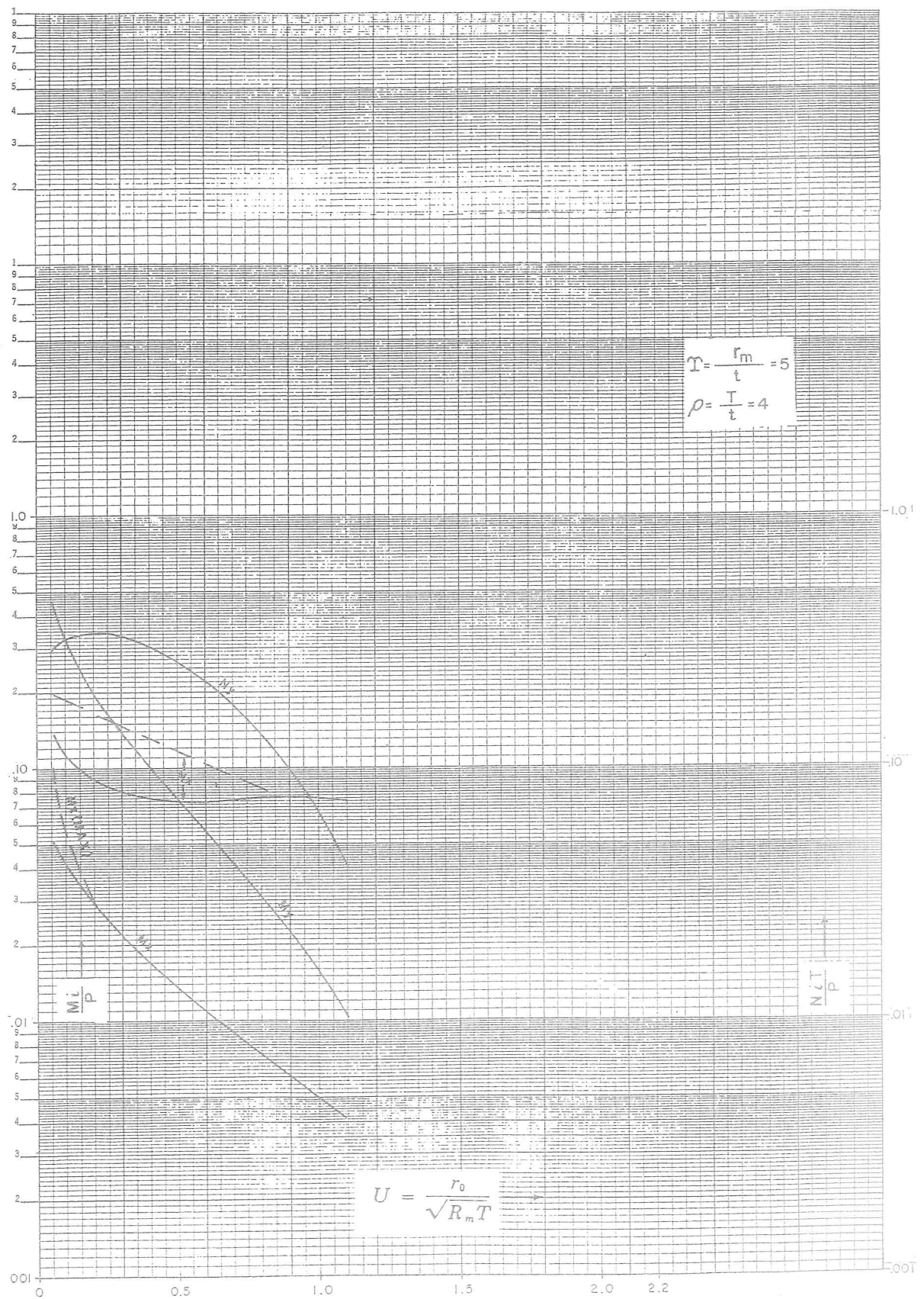


Fig. SP-4—Stresses in spherical shell due to radial load P on a nozzle connection

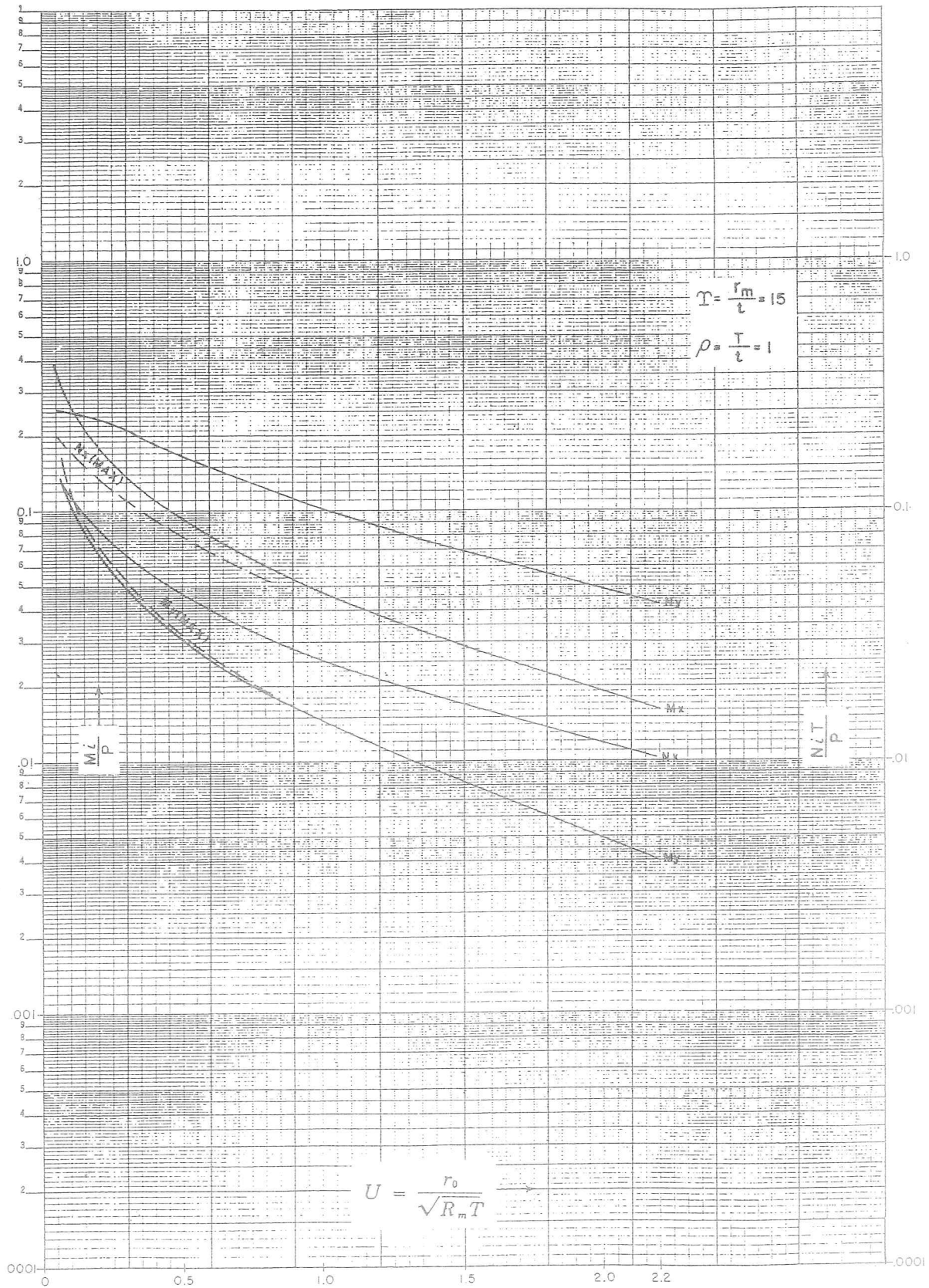


Fig. SP-5—Stresses in spherical shell due to radial load P on a nozzle connection

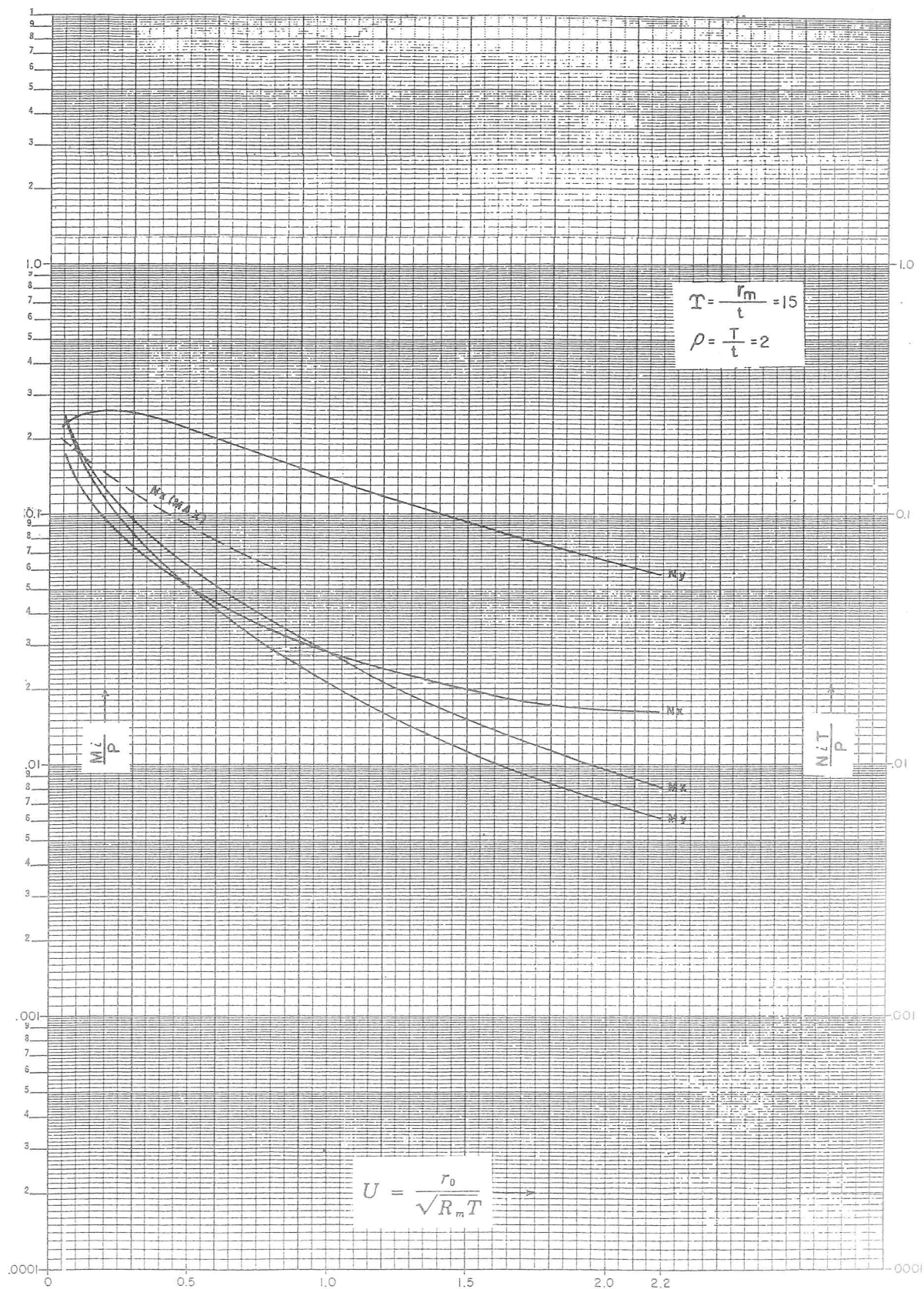


Fig. SP-6—Stresses in spherical shell due to radial load P on a nozzle connection

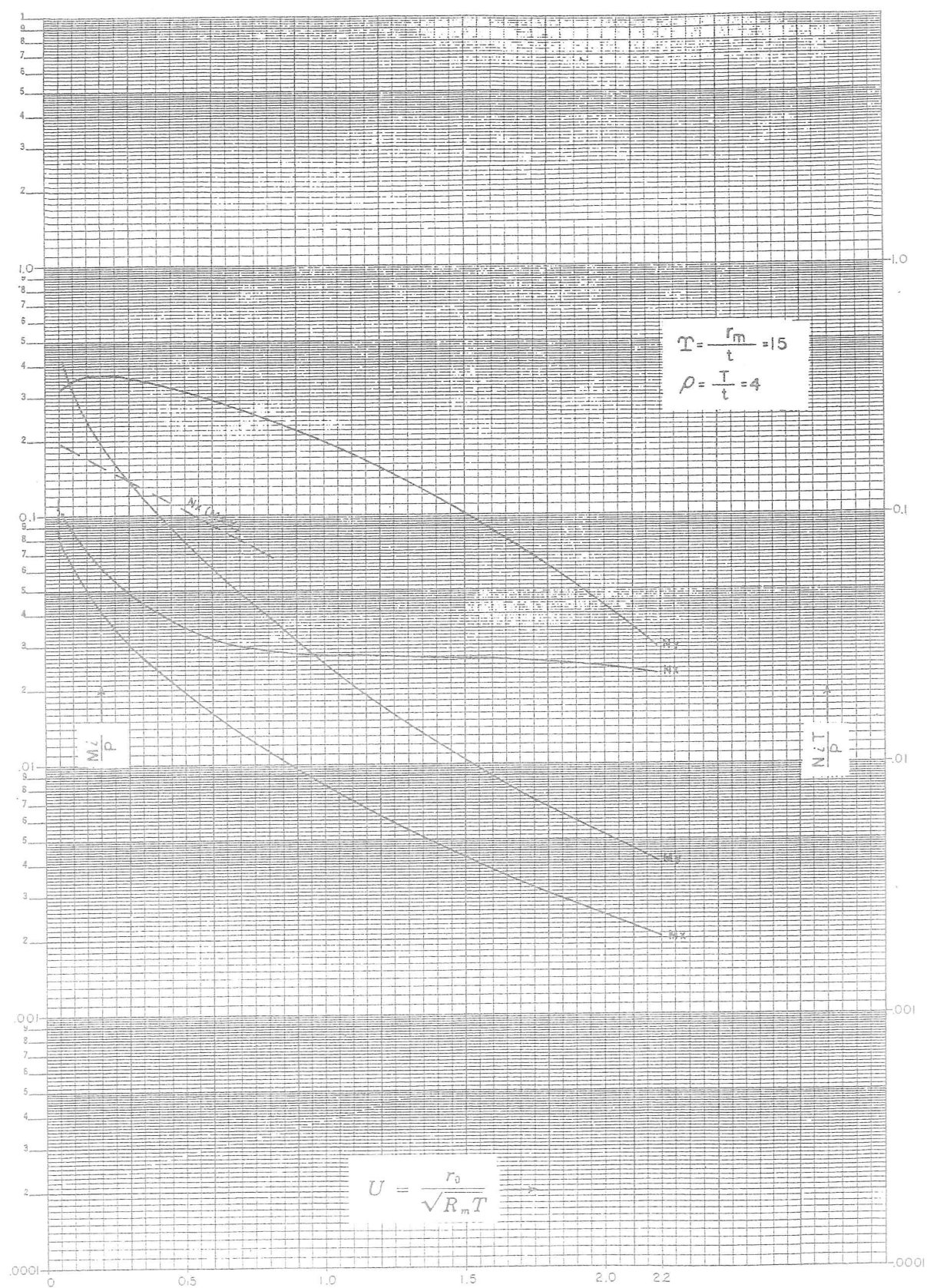


Fig. SP-7—Stresses in spherical shell due to radial load P on a nozzle connection

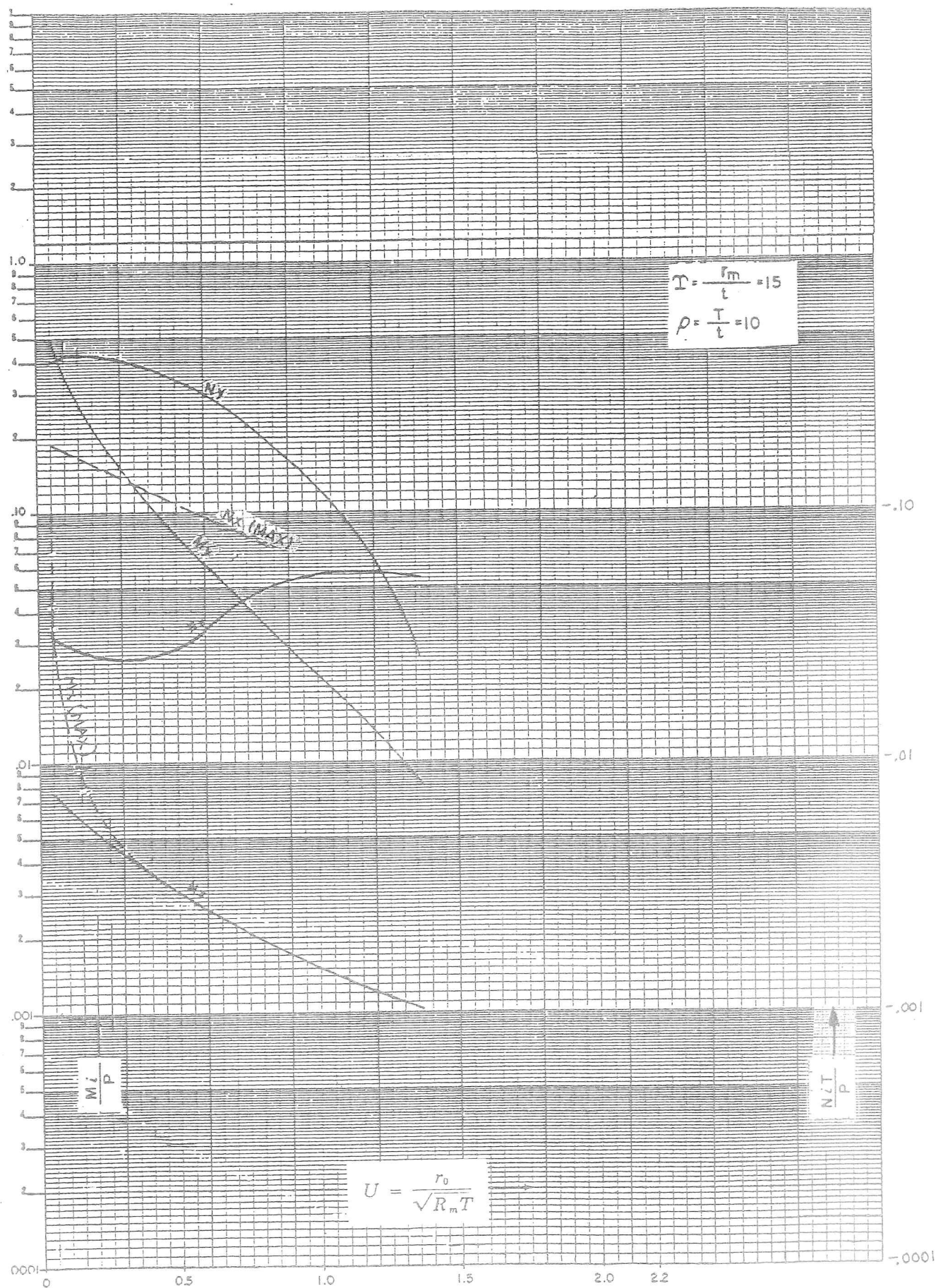


Fig. SP-8—Stresses in spherical shell due to radial load P on a nozzle connection

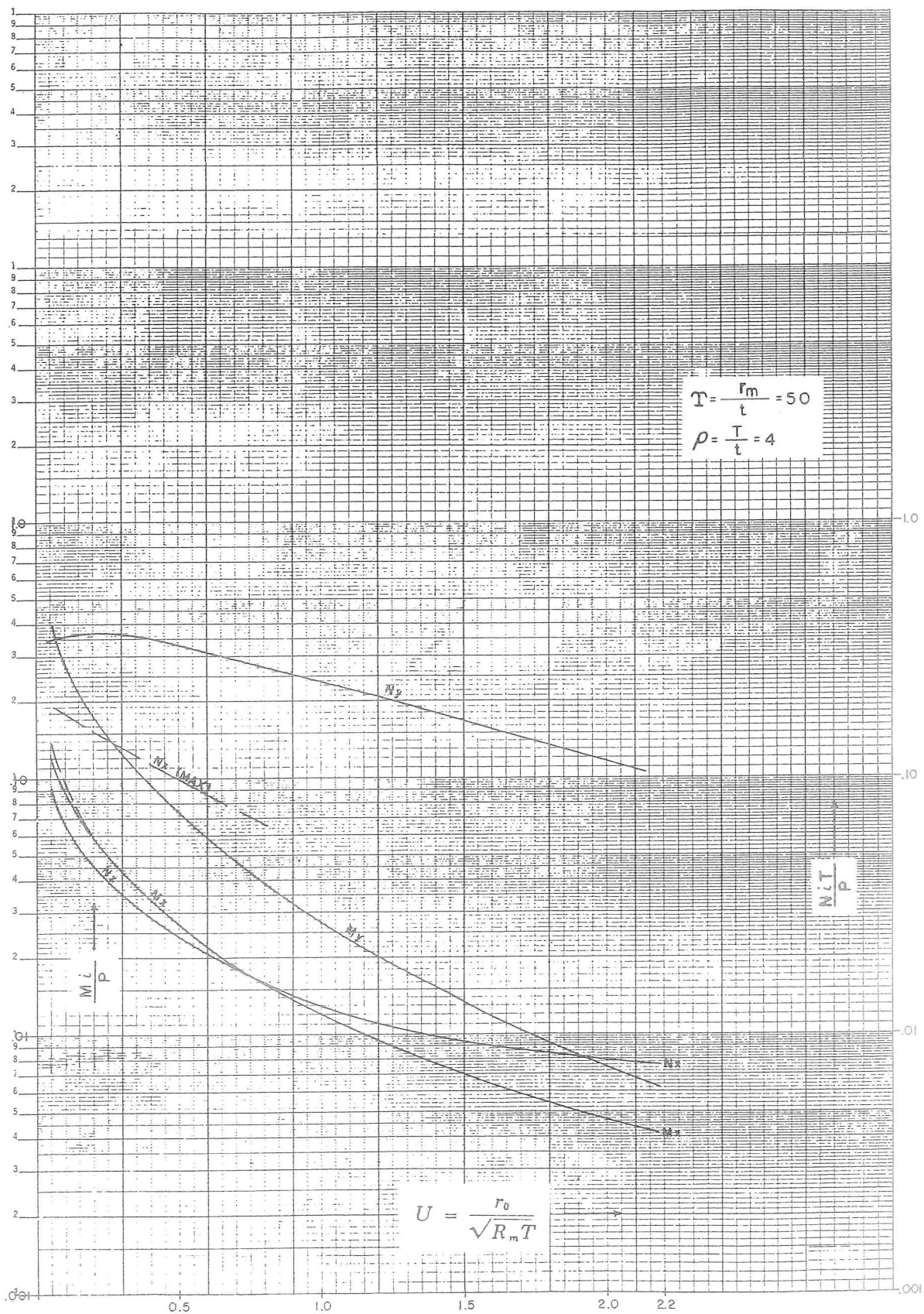


Fig. SP-9—Stresses in spherical shell due to radial load P on a nozzle connection

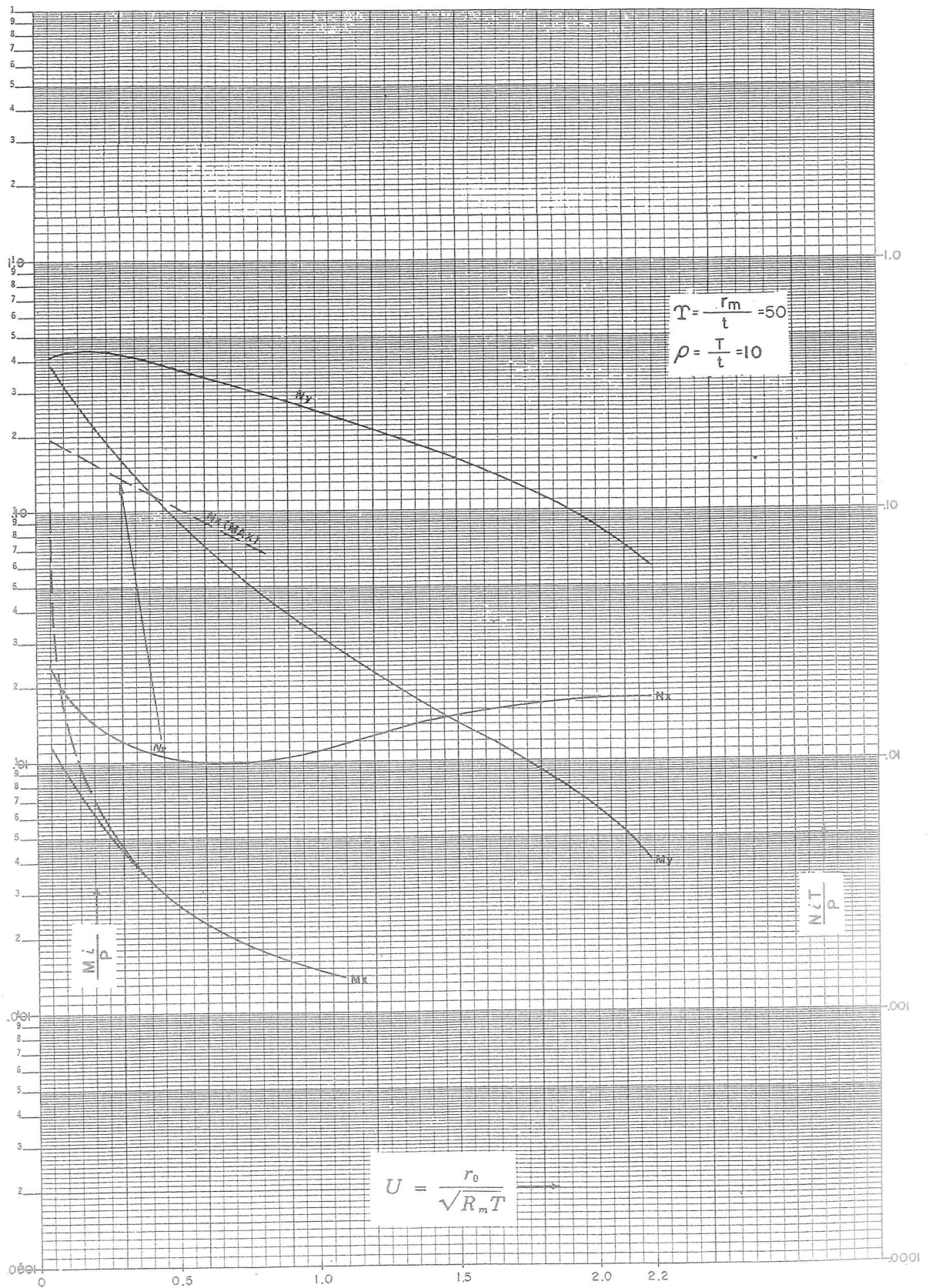


Fig. SP-10—Stresses in spherical shell due to radial load P on a nozzle connection

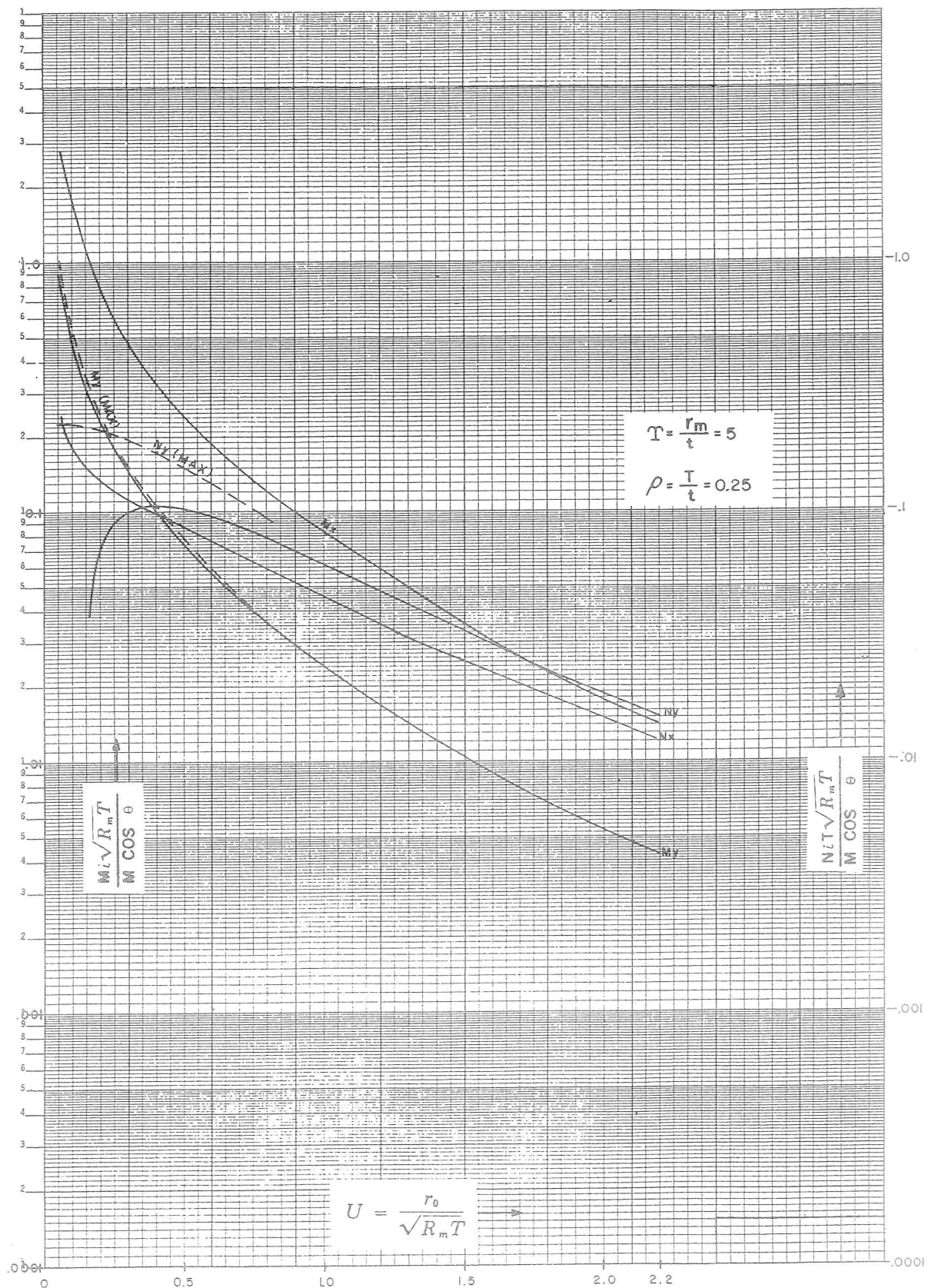


Fig. SM-1—Stresses in spherical shell due to overturning moment M on nozzle connection

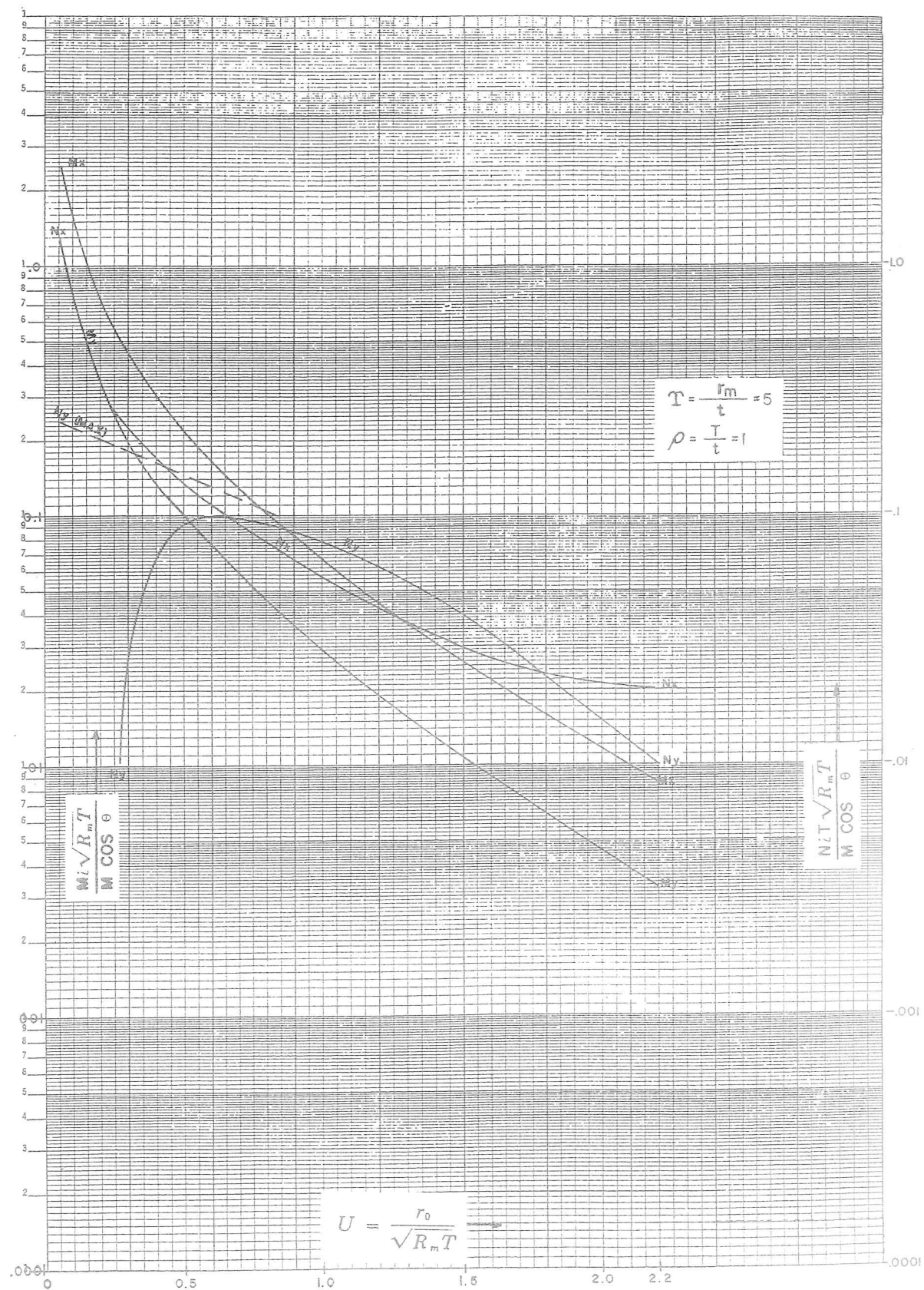


Fig. SM-2—Stresses in spherical shell due to overturning moment M on nozzle connection

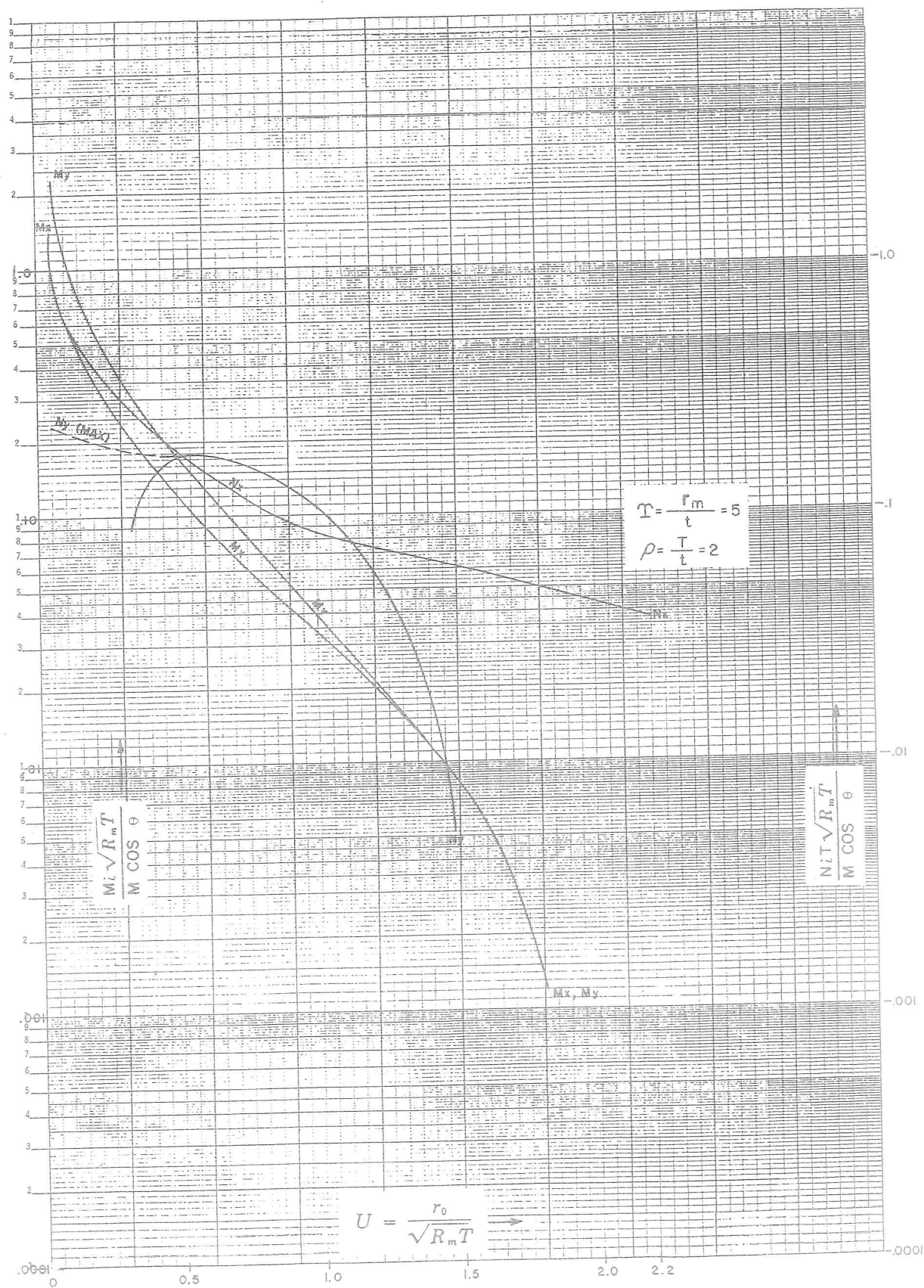


Fig. SM-3—Stresses in spherical shell due to overturning moment M on nozzle connection

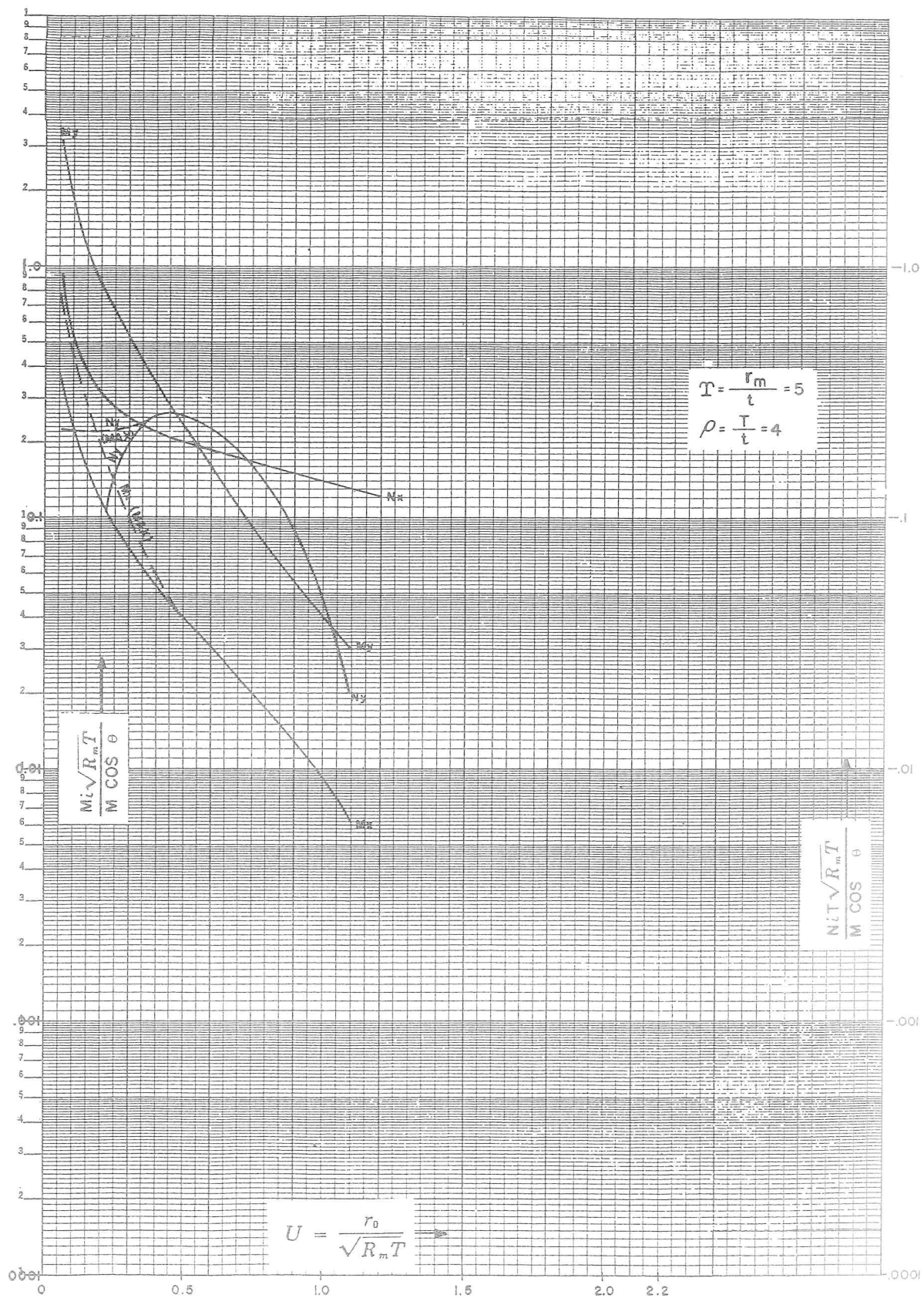


Fig. SM-4—Stresses in spherical shell due to overturning moment M on nozzle connection

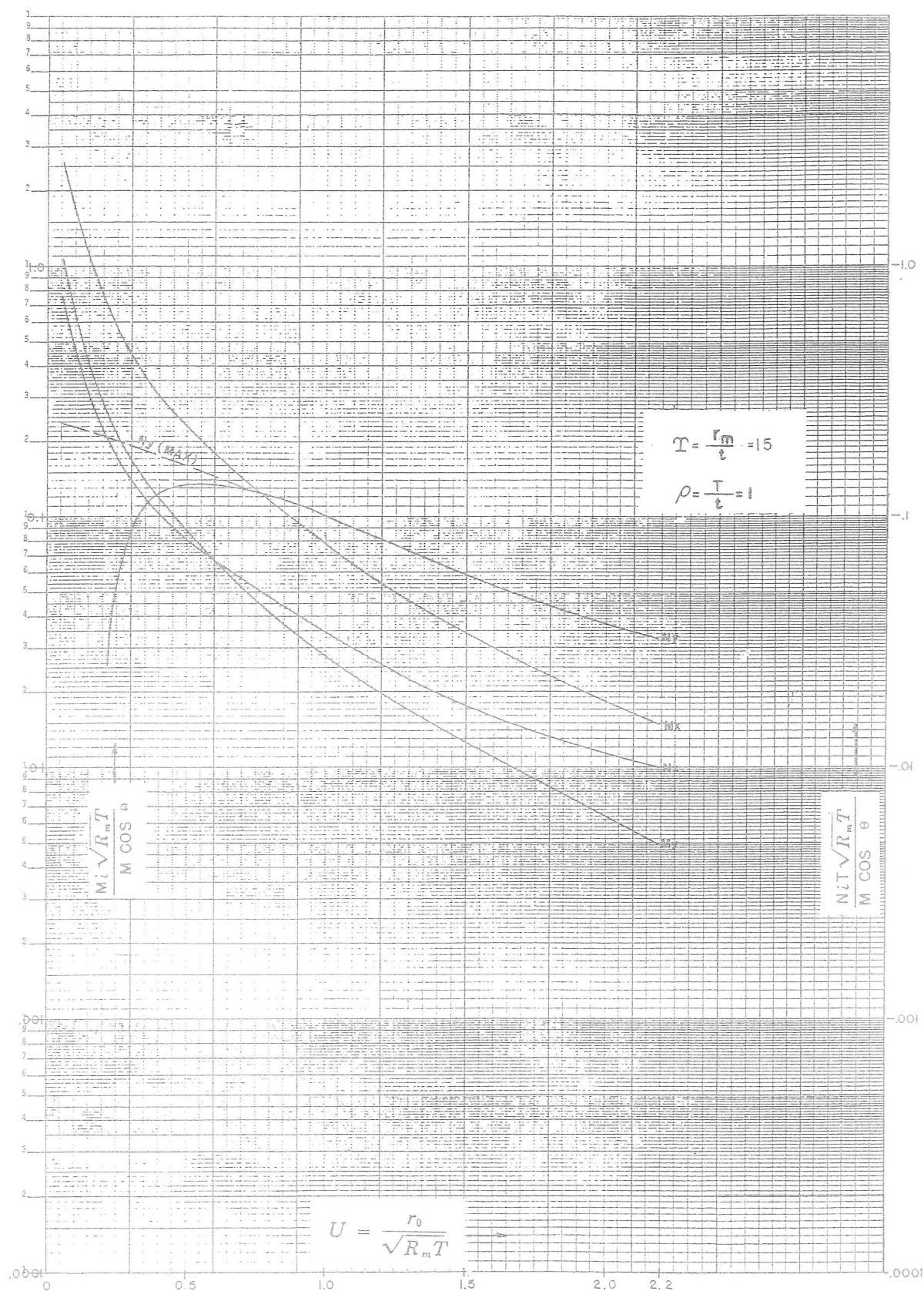


Fig. SM-5—Stresses in spherical shell due to overturning moment M on nozzle connection



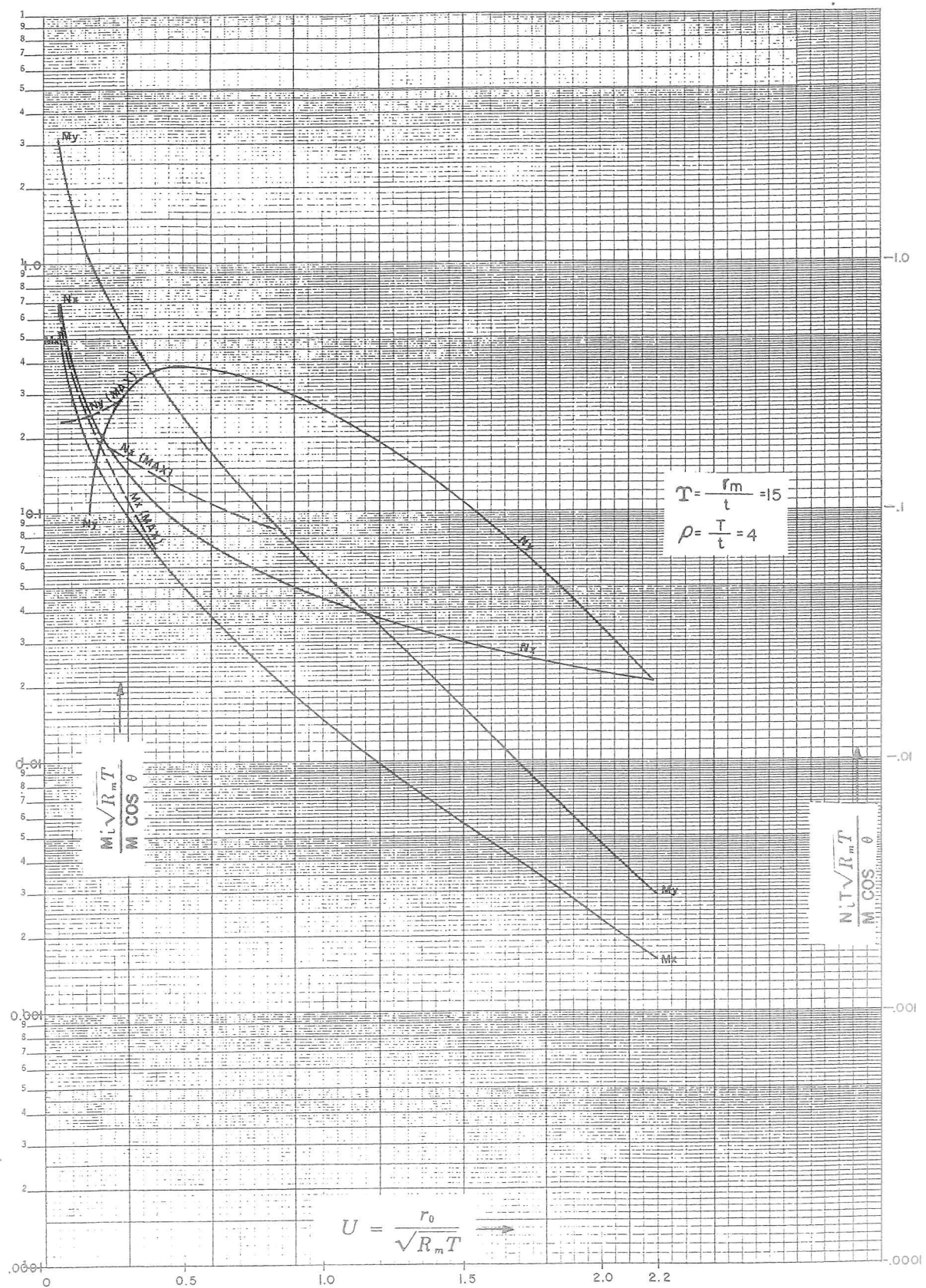


Fig. SM-7—Stresses in spherical shell due to overturning moment M on nozzle connection

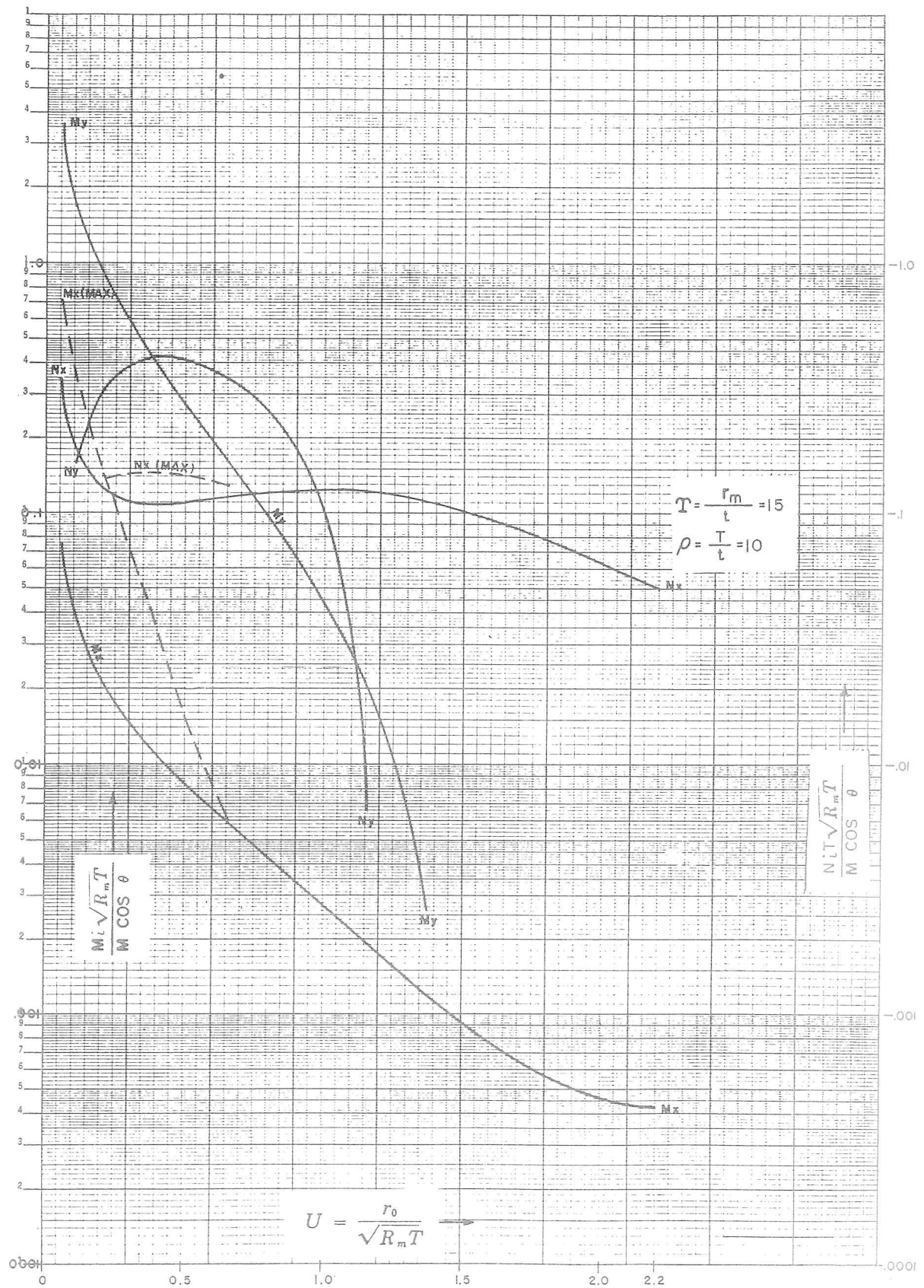


Fig. SM-8—Stresses in spherical shell due to overturning moment M on nozzle connection

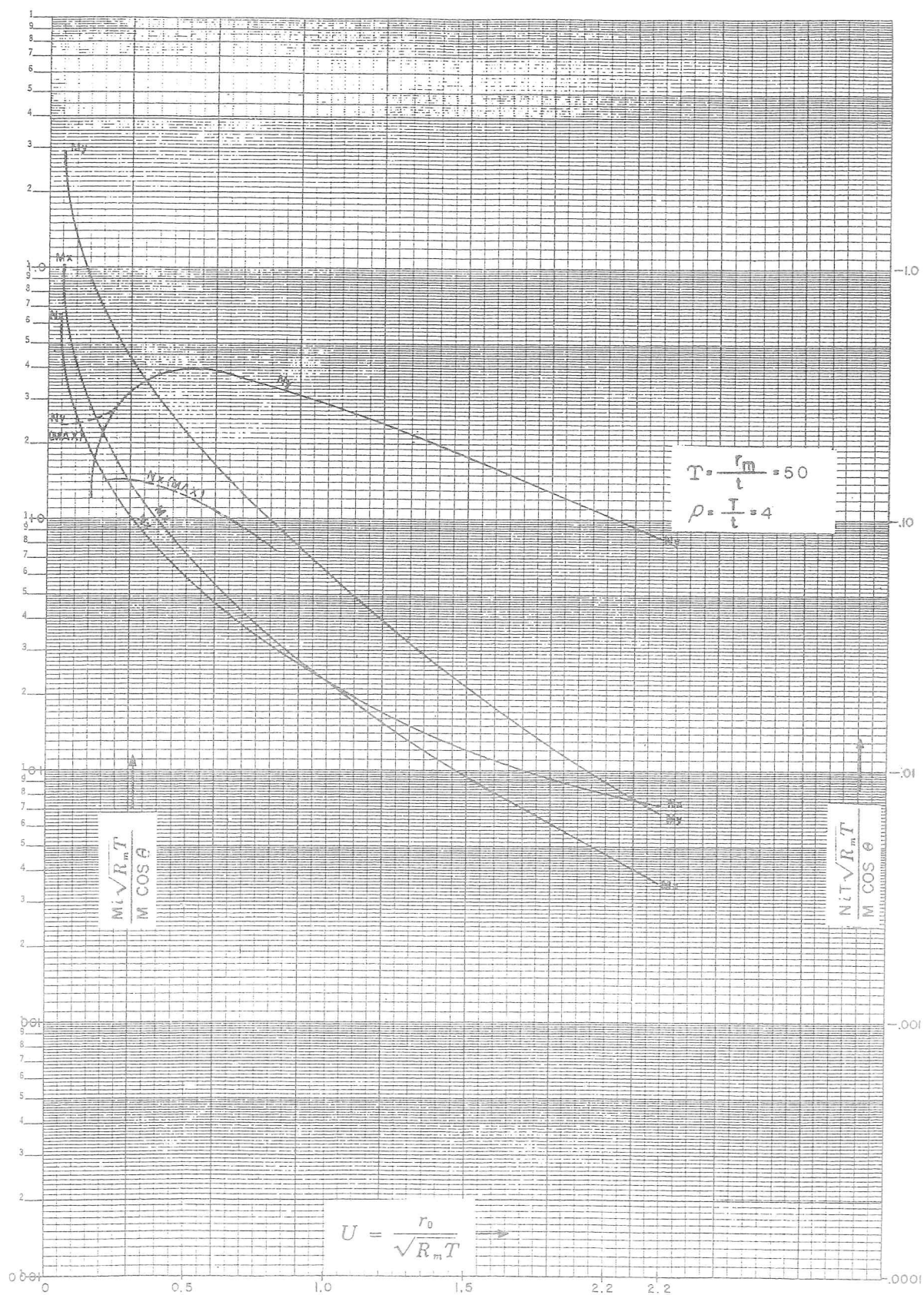


Fig. SM-9—Stresses in spherical shell due to overturning moment M on nozzle connection

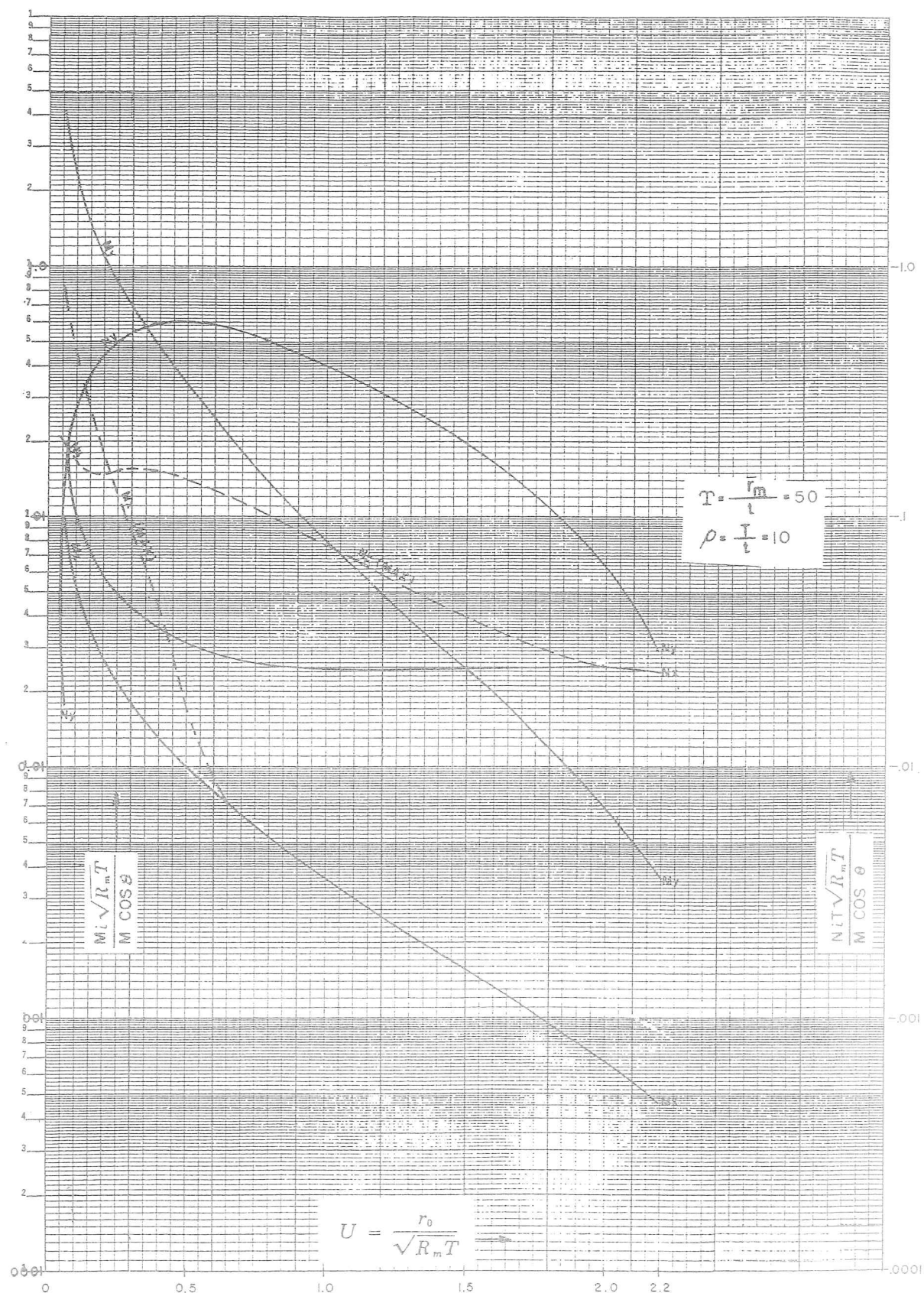


Fig. SM-10—Stresses in spherical shell due to overturning moment M on nozzle connection

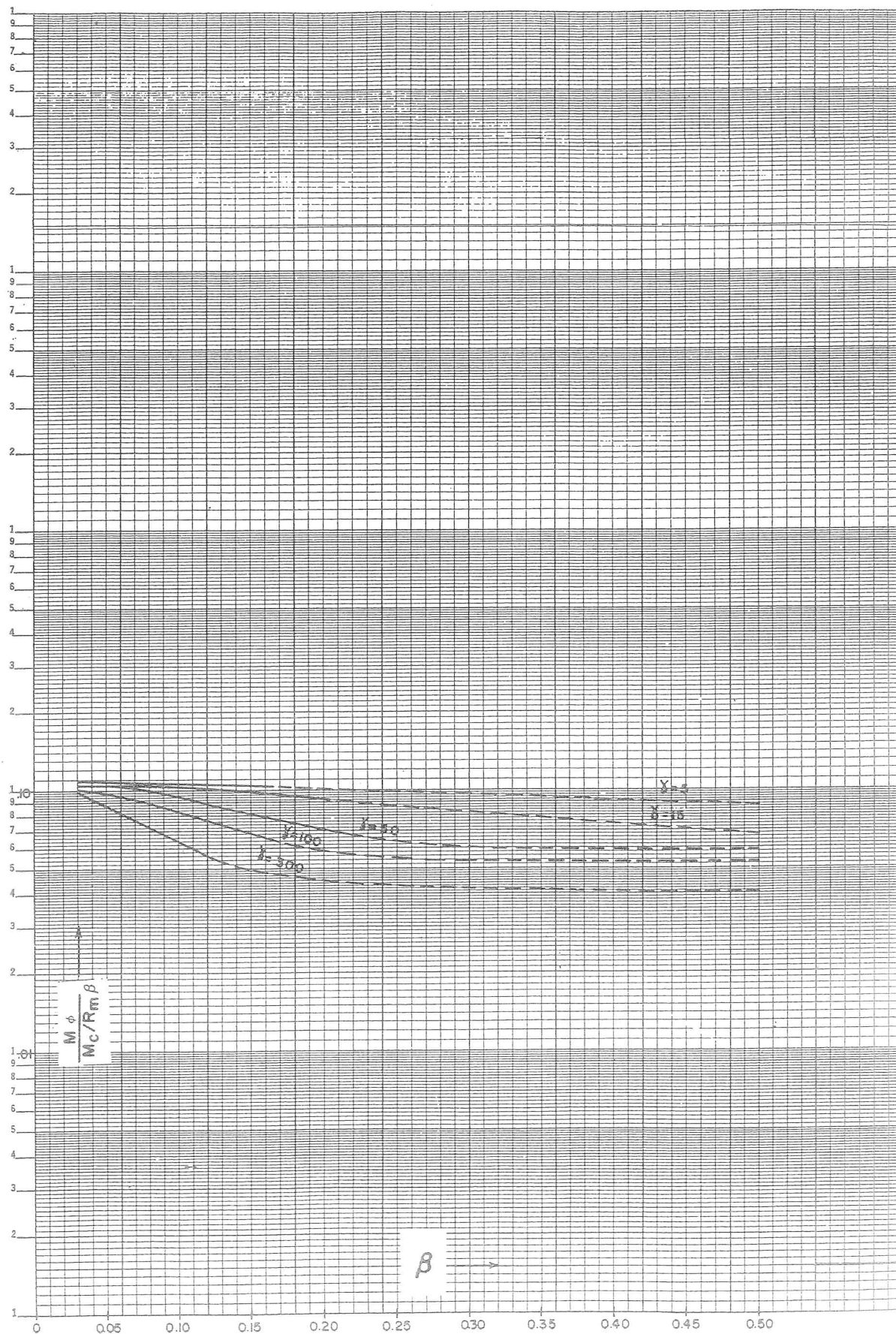


Fig. 1A—Moment $M_\phi / (M_c / R_m \beta)$ due to an external circumferential moment M_c on a circular cylinder

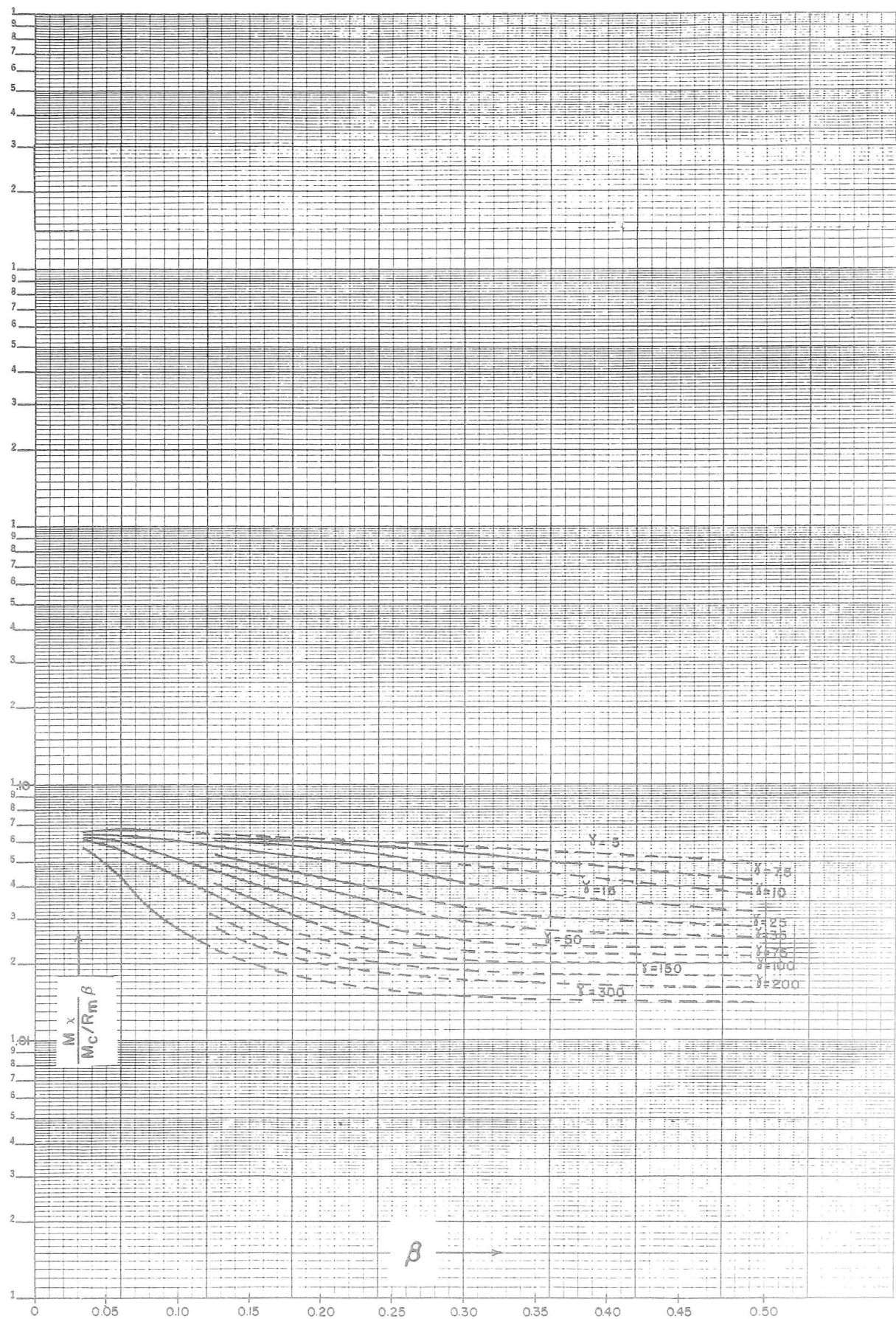


Fig. 2A—Moment $M_x/(M_c/R_m)\beta$ due to an external circumferential moment M_c on a circular cylinder

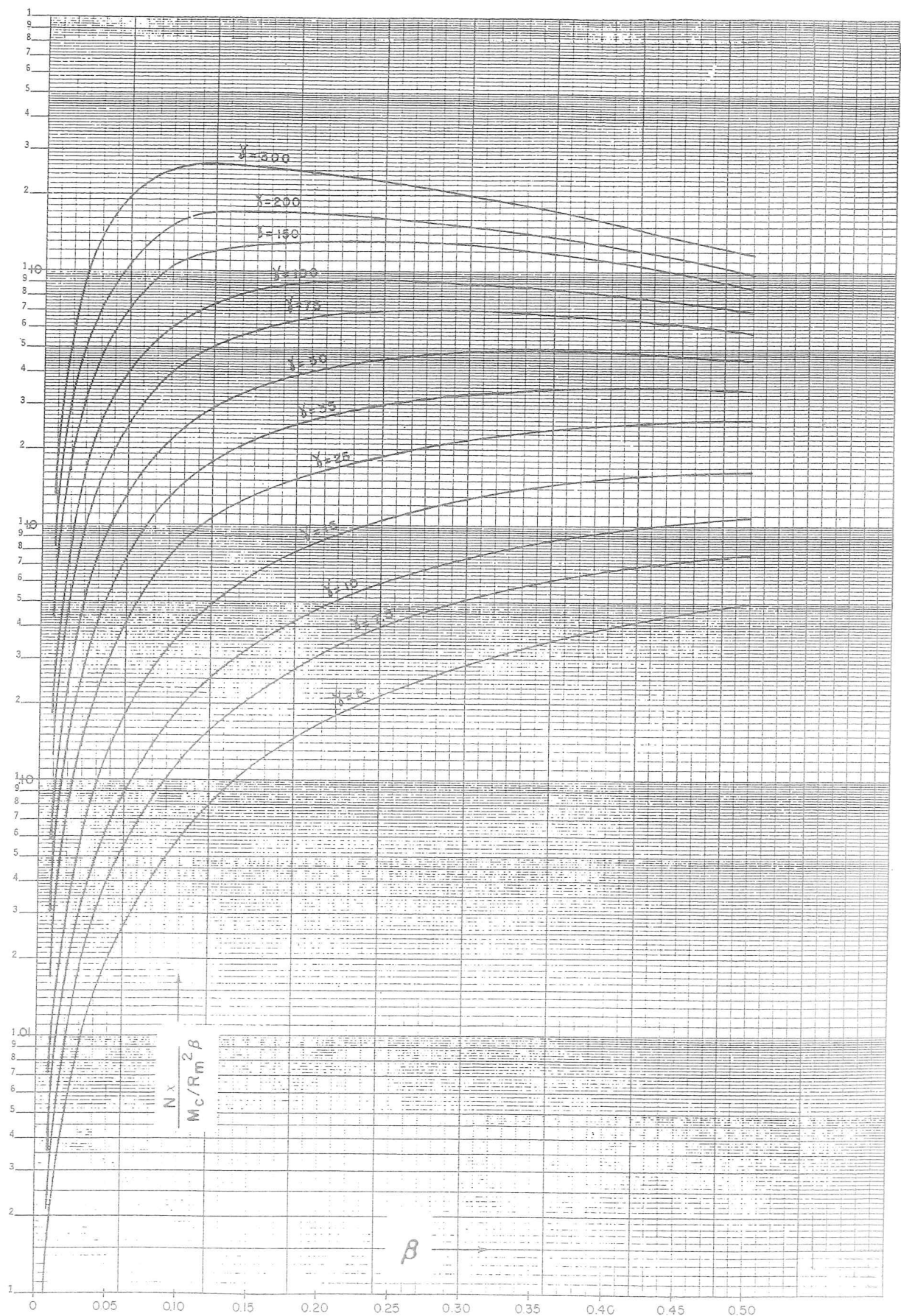


Fig. 4A—Membrane force $N_x / (M_c / R_m^2 \beta)$ due to an external circumferential moment M_c on a circular cylinder

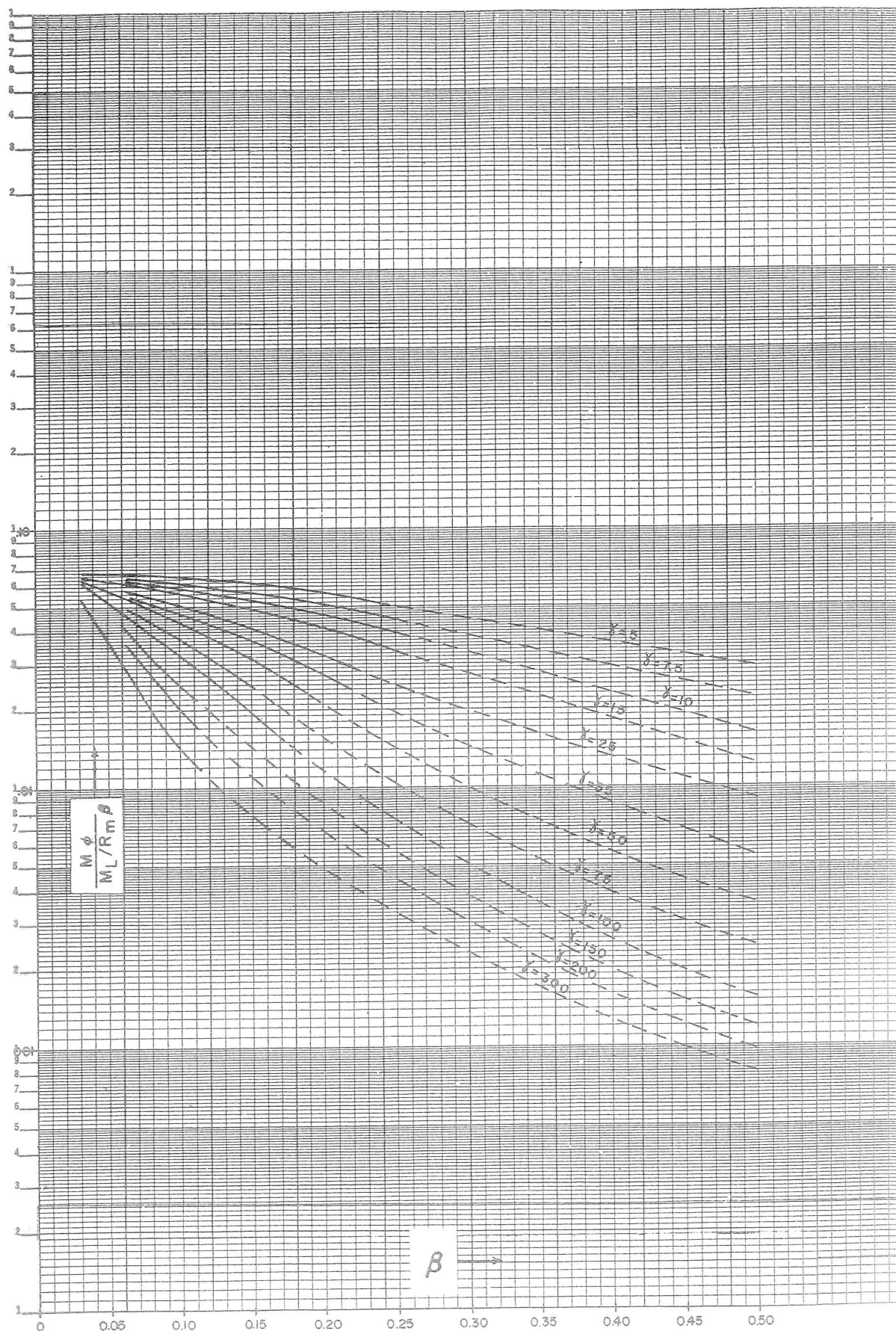


Fig. 1B—Moment $M_\phi / (M_L / R_m \beta)$ due to an external longitudinal moment M_L on a circular cylinder (Stress on the longitudinal plane of symmetry)

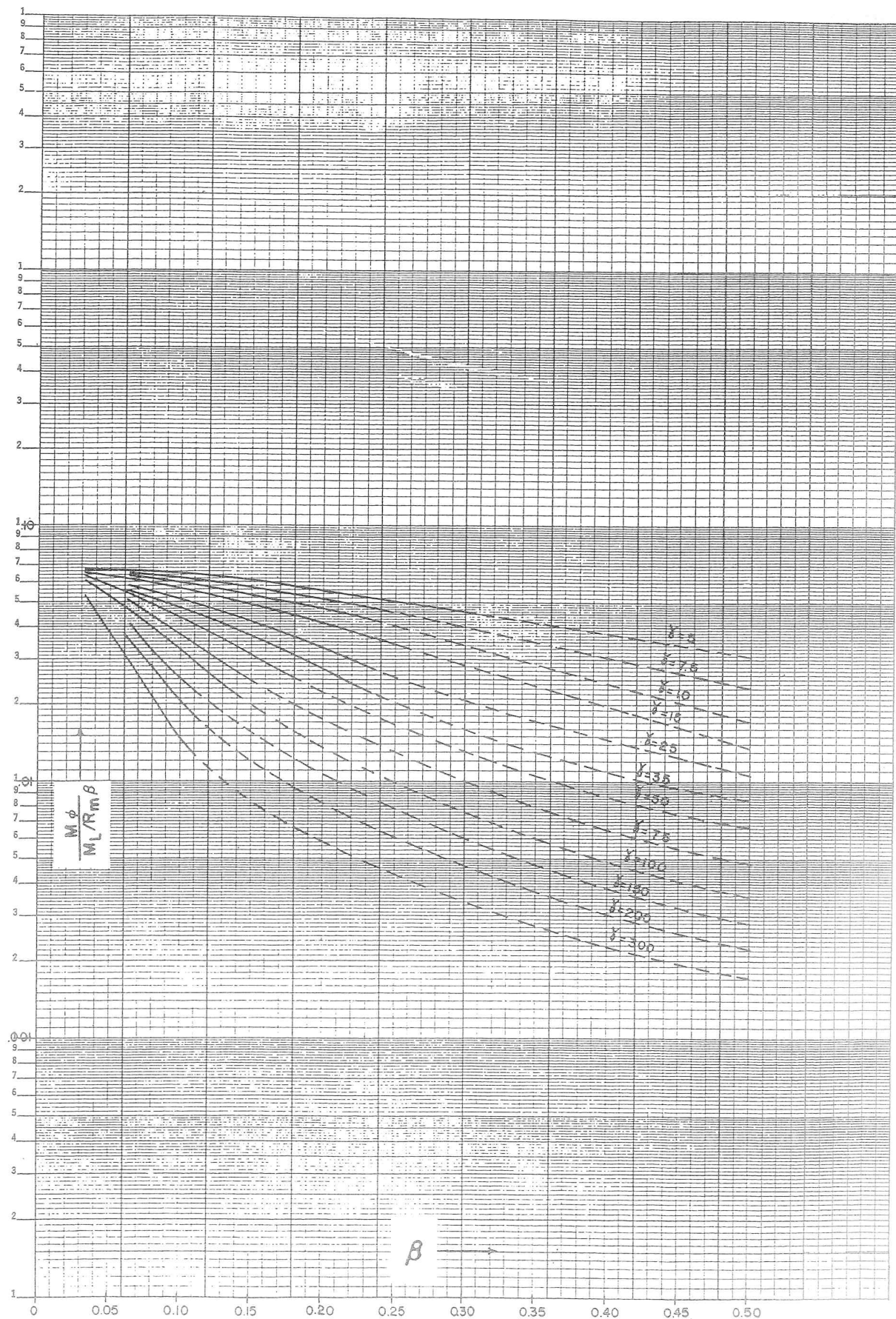


Fig. 1B-1—Moment $M_\phi / (M_L / R_m \beta)$ due to an external longitudinal moment M_L on a circular cylinder

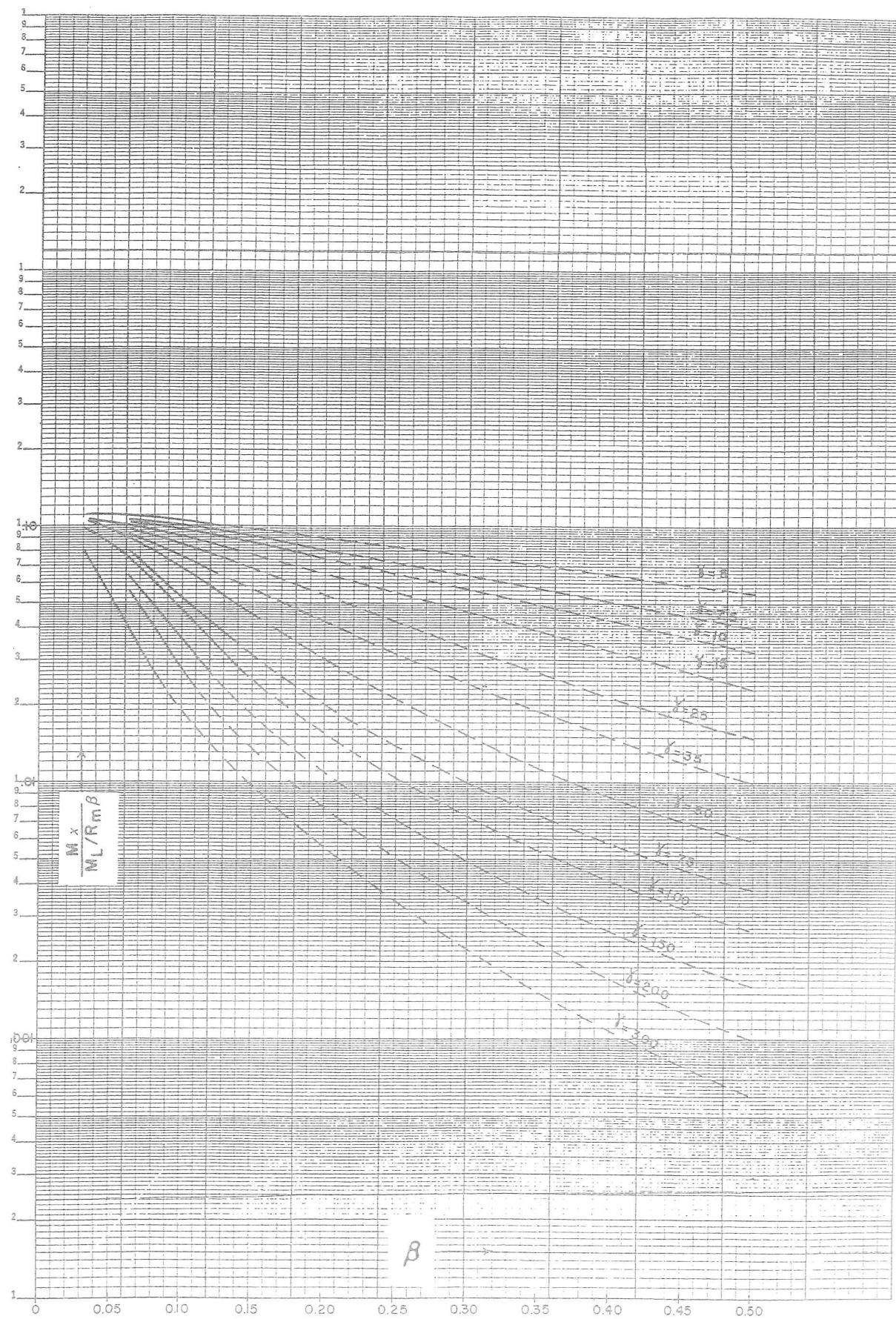


Fig. 2B-Moment $M_x/(M_t/R_m\delta)$ due to an external longitudinal moment M_t on a circular cylinder (Stress on longitudinal plane of symmetry)

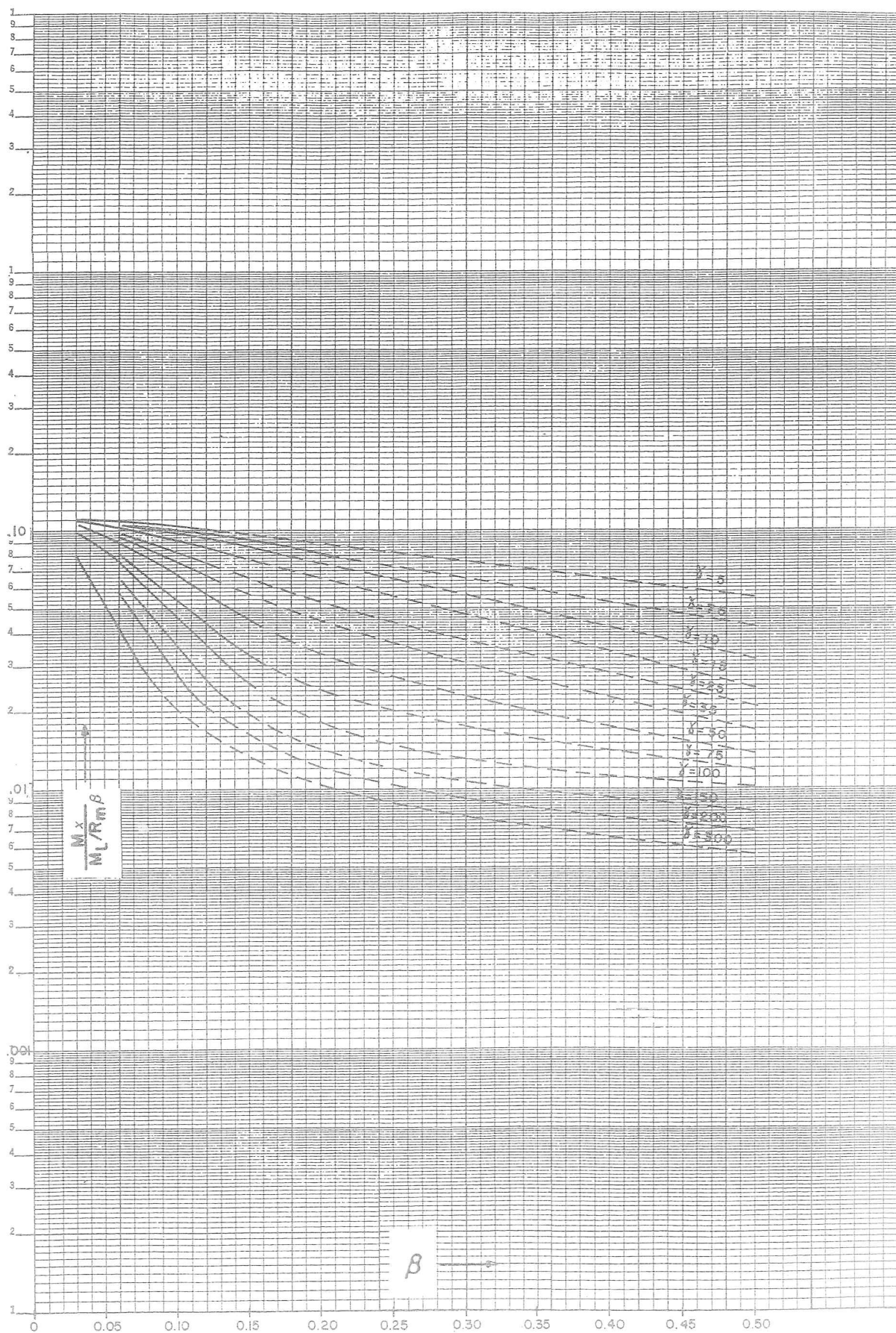


Fig. 2B-1—Moment $M_x/(M_1/R_m\beta)$ due to an external longitudinal moment M_1 on a circular cylinder

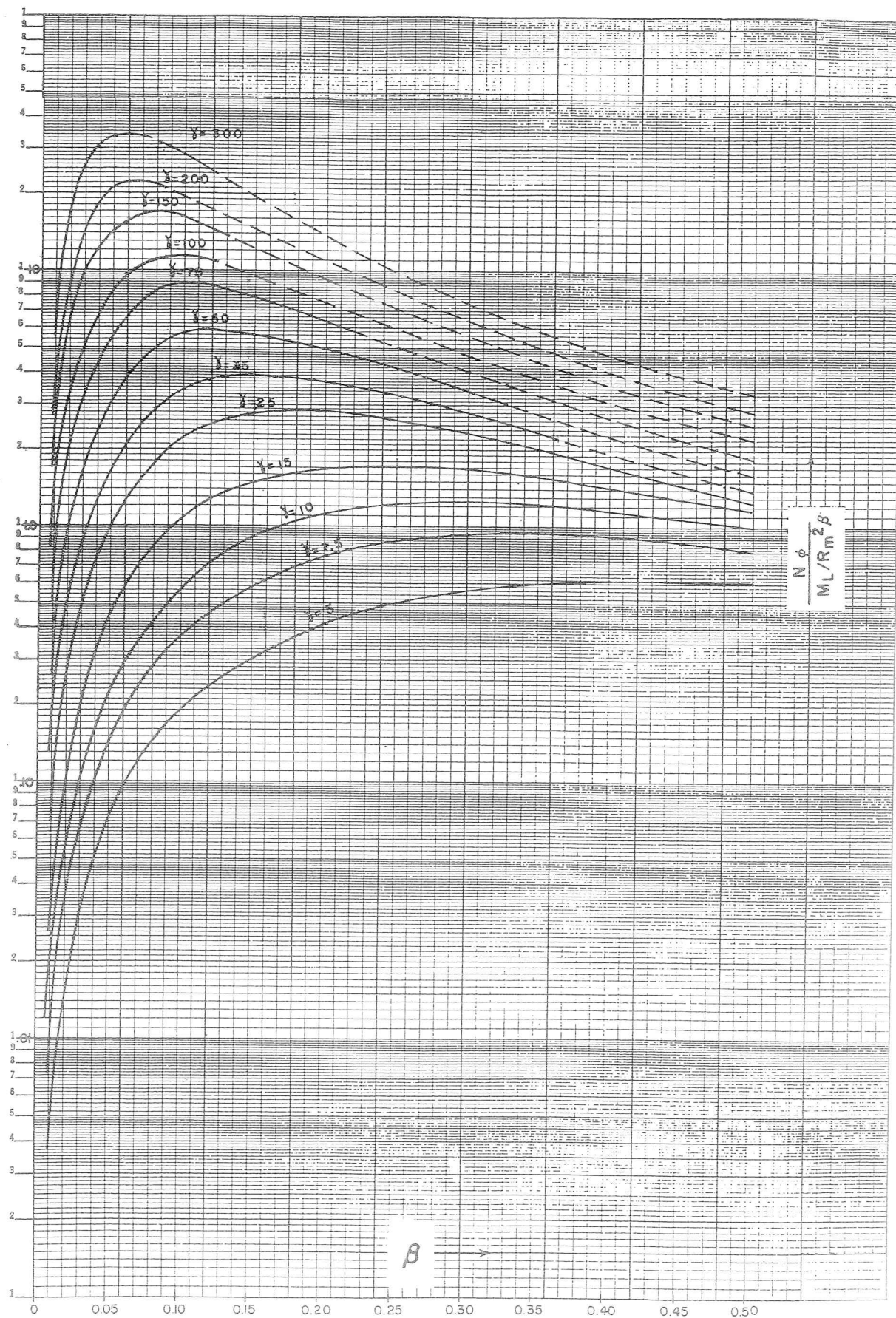


Fig. 3B—Membrane force $N_\phi / (M_L / R_m^2 \beta)$ due to an external longitudinal moment M_L on a circular cylinder

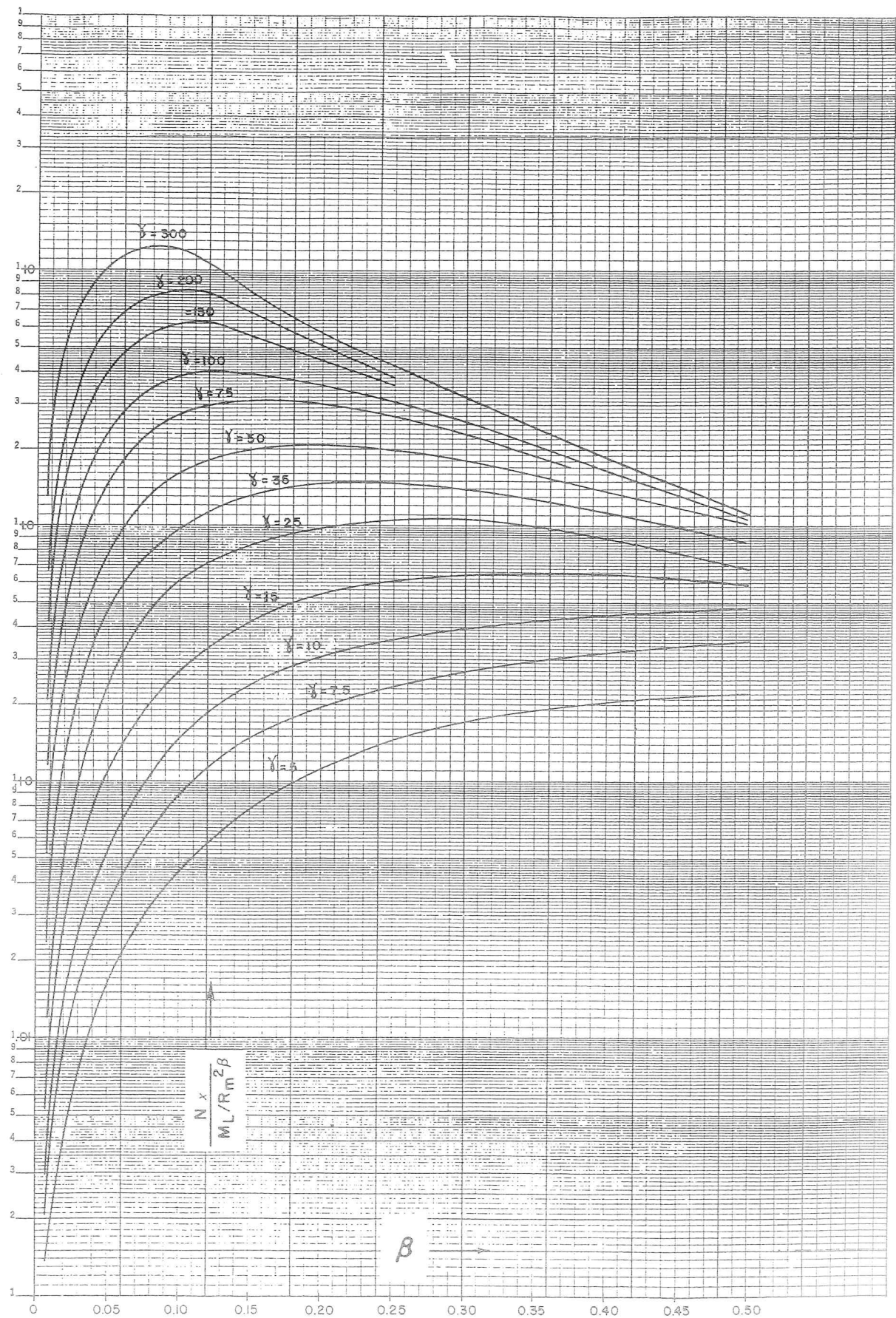


Fig. 4B—Membrane force $N_x / (M_L / R_m^2 \beta)$ due to an external longitudinal moment M_L on a circular cylinder

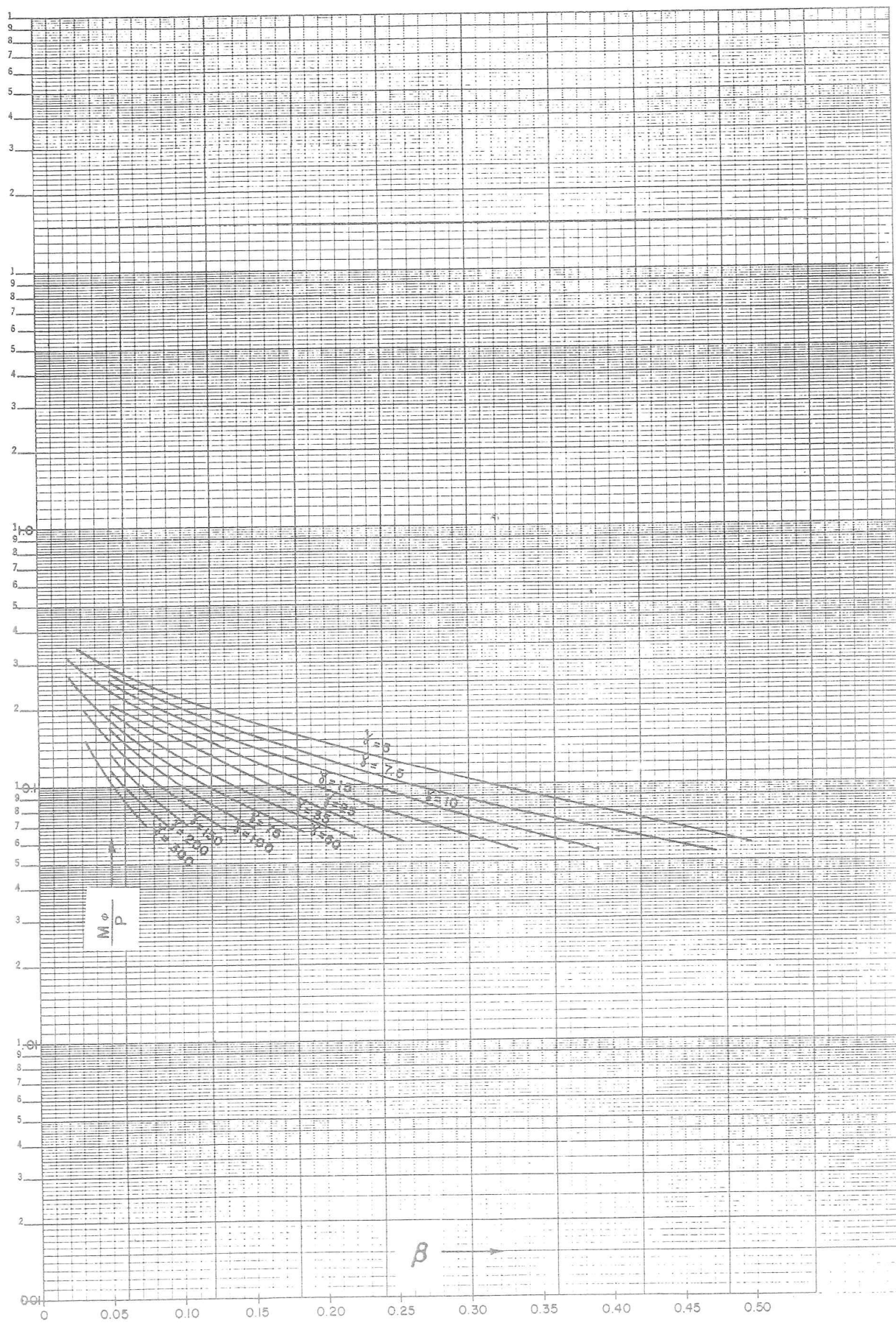


Fig. 1C—Bending moment M_ϕ/P due to an external radial load P on a circular cylinder (transverse axis)

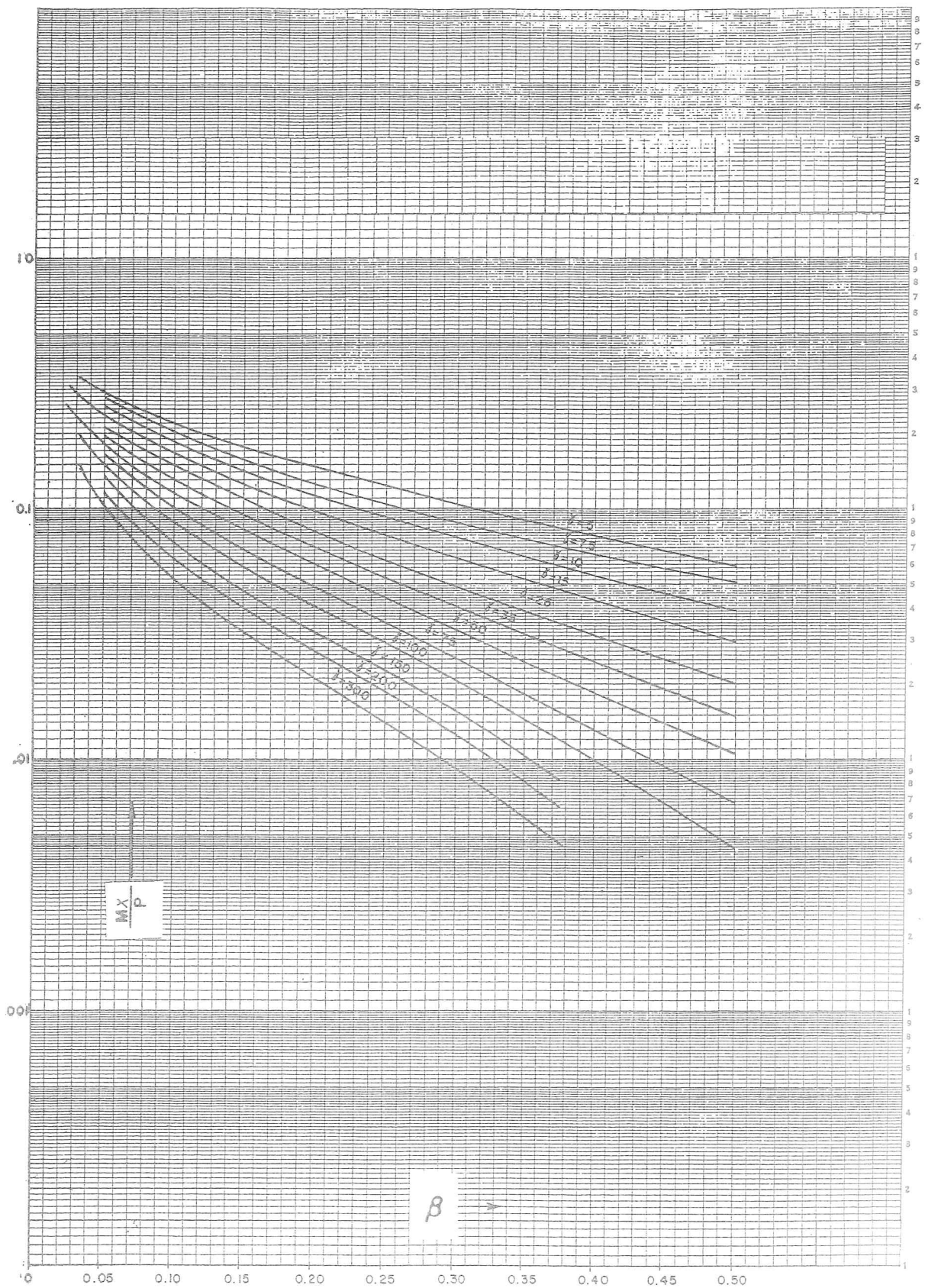


Fig. 1C-1—Bending moment M_x/P due to an external radial load P on a circular cylinder (longitudinal axis)

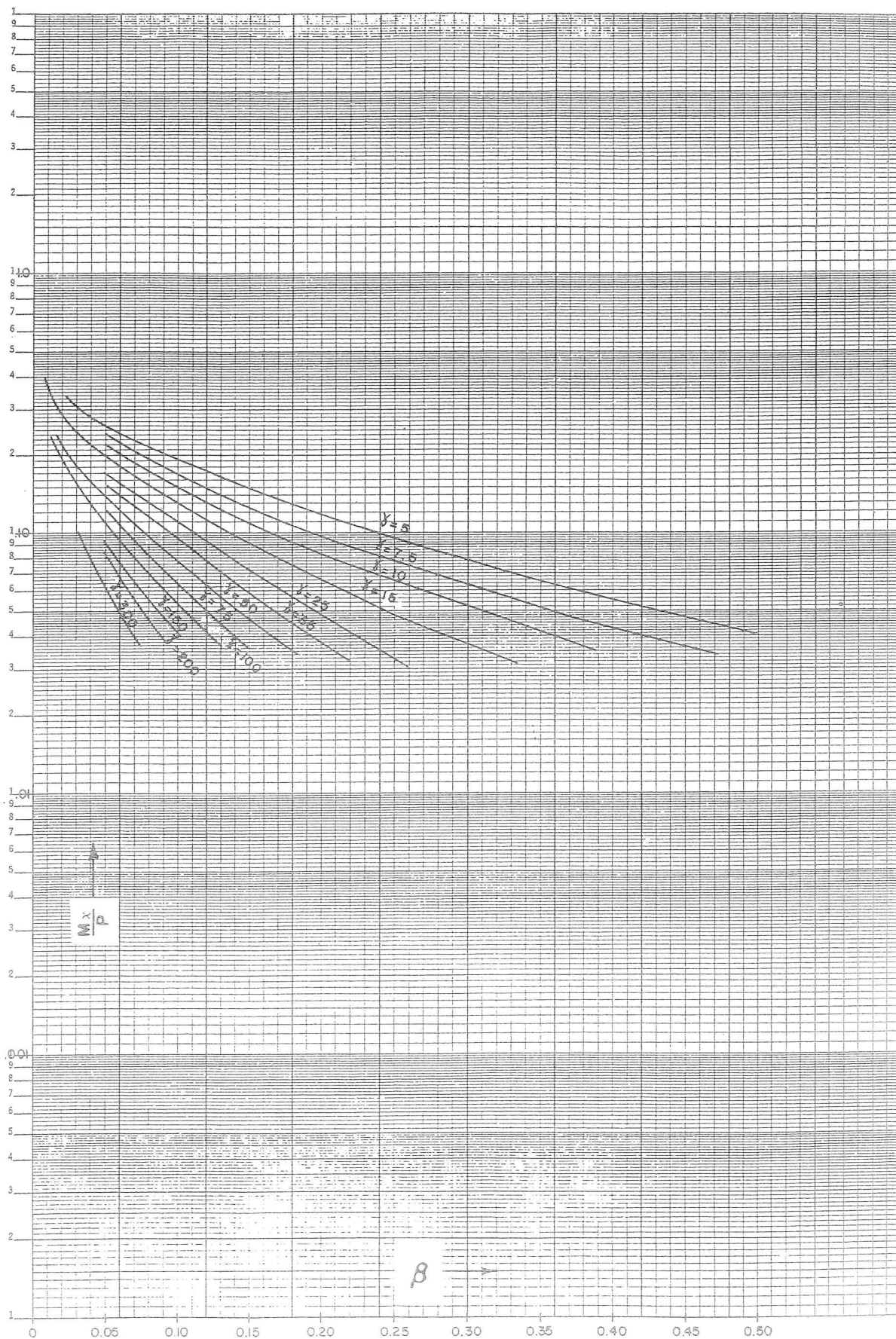


Fig. 2C—Bending moment M_x/P due to an external radial load P on a circular cylinder (transverse axis)

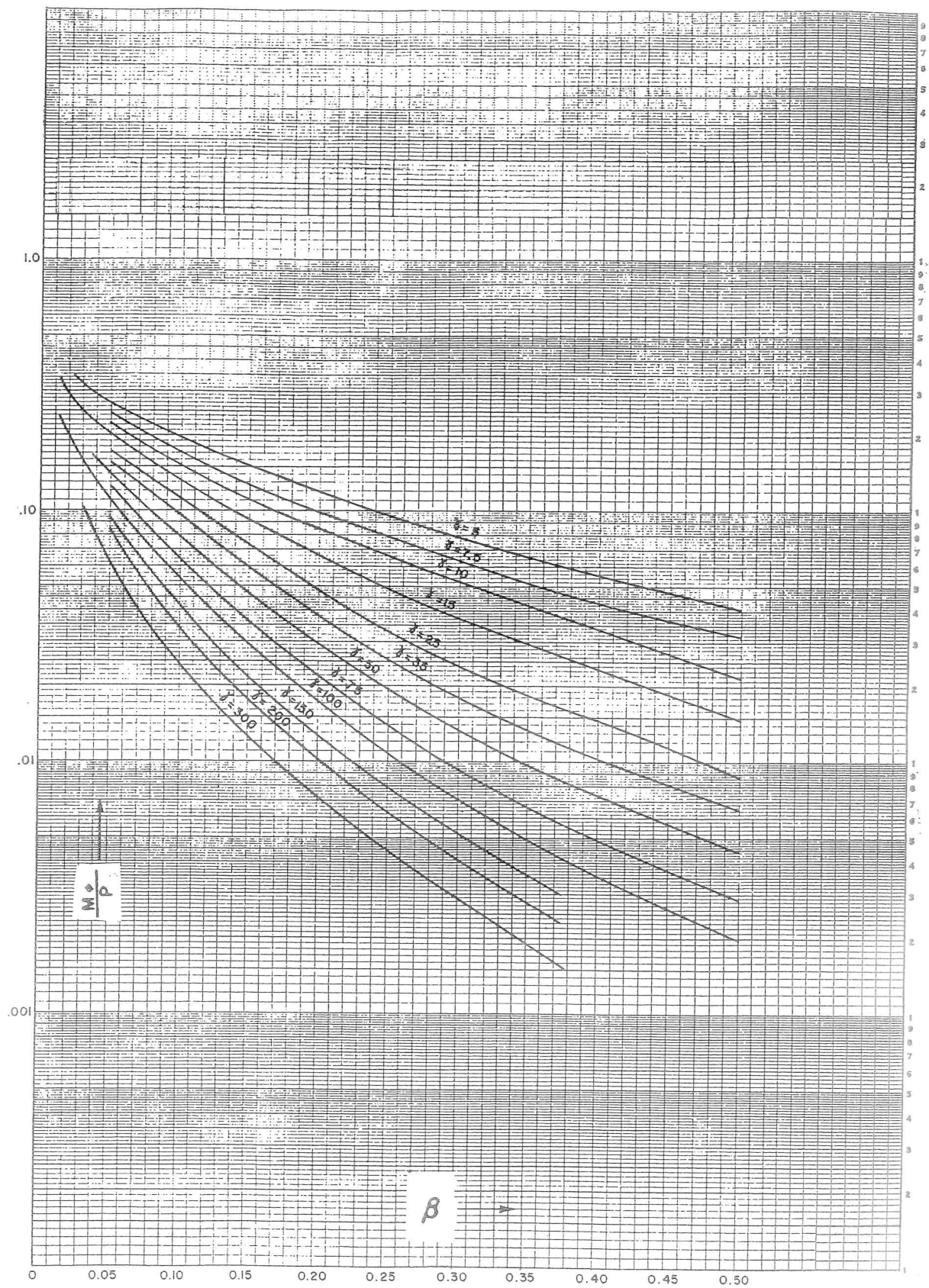


Fig. 2C-1—Bending moment M_ϕ/P due to an external radial load P on a circular cylinder (longitudinal axis)

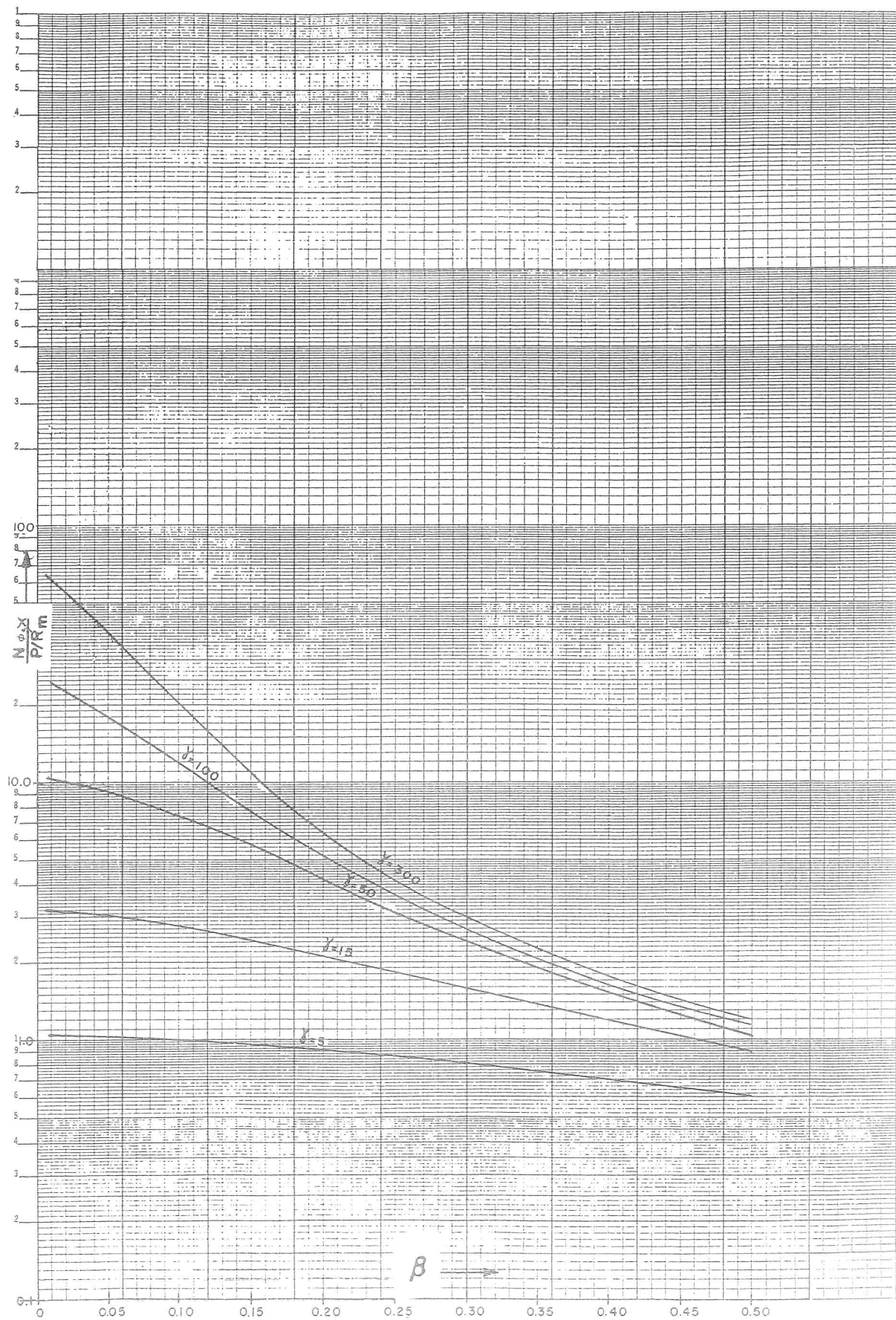


Fig. 3C—Membrane force $N_{\phi}/(P/R_m)$ due to an external radial load P on a circular cylinder (transverse axis)
 —Membrane force $N_x/(P/R_m)$ due to an external radial load P on a circular cylinder (longitudinal axis)

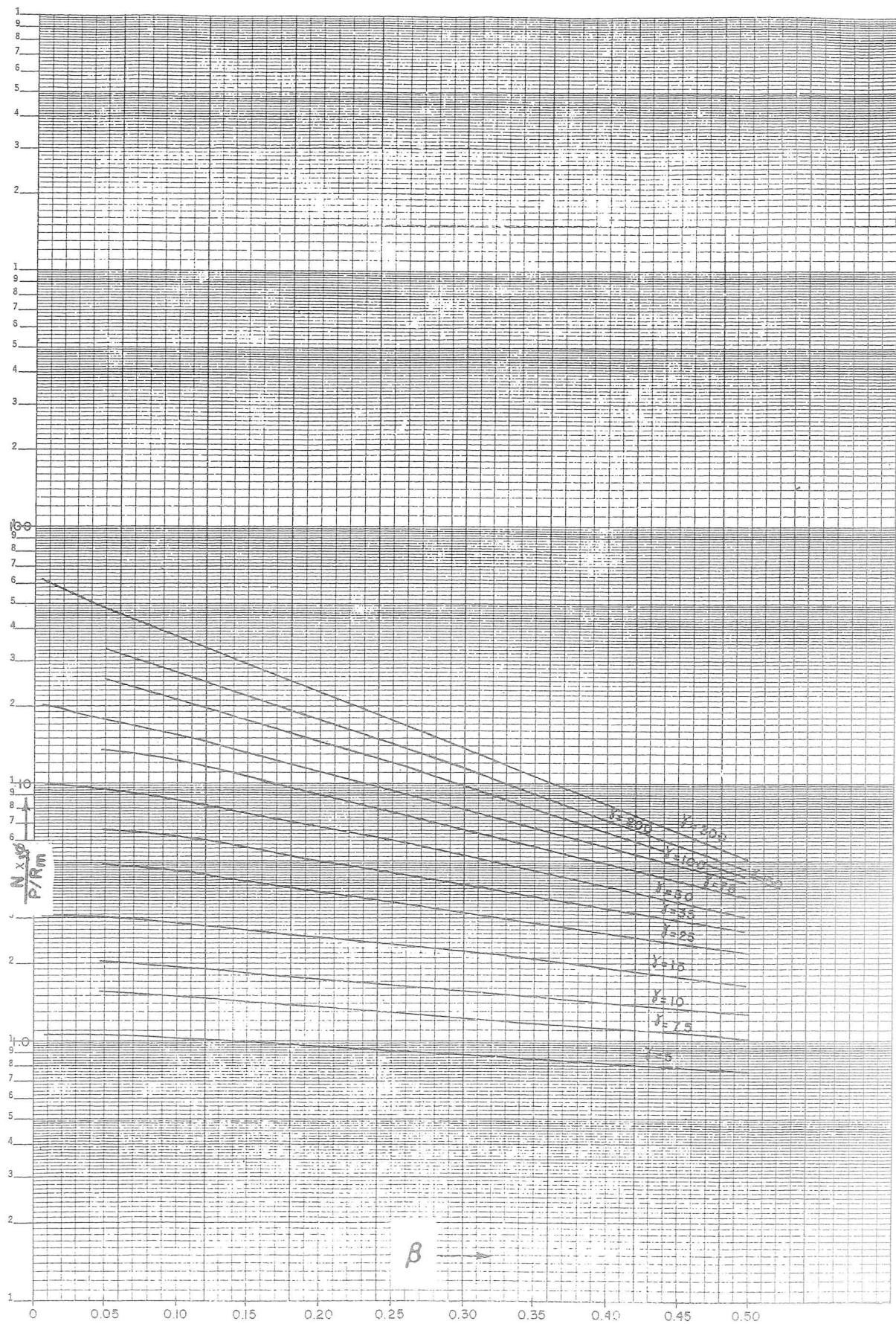


Fig. 4C—Membrane force $N_x/(P/R_m)$ due to an external radial load P on a circular cylinder (transverse axis)
 —Membrane force $N_l/(P/R_m)$ due to an external radial load R on a circular cylinder (longitudinal axis)

APPENDIX A—BASIS FOR “CORRECTIONS” TO BIJLAARD’S CURVES

A.1 Introduction

During the past year or more, data have become available indicating, or emphasizing, certain limitations in Bijlaard's work for external loadings on nozzle connections. In some cases, the discrepancies involved were quite large and seemingly indicated a necessity for providing interim modifications to a portion of Bijlaard's data pending development of a more adequate analytical treatment of the problem. The following summary is presented to document the nature of the discrepancies and to explain the manner in which the curves based on Bijlaard's data have been modified herein.

A.2 Spherical Shells

Bijlaard's work on spherical shells was based on shallow shell theory, and the limitations which he placed on the theory were essentially as indicated in Fig. A-1, from which it will be observed that the limiting d_i/D_i ratio is about $1/3$ for “thin” shells, and somewhat less in thicker shells (D_m/T ratios of 20–55). The experimental work at Cornell University which was performed to verify the theory, as reported in Reference 18, was for a d_i/D_i ratio of approximately 0.10 and D_m/T ratios of approxi-

mately 37, 80 and 92, which parameters are all well within Bijlaard's limits, as is indicated on Fig. A-1. Subsequently, Westinghouse Research Laboratories tested four photoelastic models for the Bureau of Ships, under moment loading only, as reported in Reference 24; these models had a D_m/T ratio of 51.0, one with a d_i/D_i ratio of 0.13, two with a d_i/D_i ratio of 0.27, and one with a d_i/D_i ratio of 0.50. Similarly, IIT Research Institute has tested one steel model for PVRC, having a D_m/T ratio of 236 and a d_i/D_i ratio of 0.50, the preliminary results of which are reported in Reference 23. As indicated on Fig. A-1, these models provide one point well within Bijlaard's limits, two approaching those limits and two well outside those limits.

Also recently, the work of Penny-Leckie²⁰ became available, which is based on “not-shallow shell” theory and which might offer hope of a more adequate treatment at the larger diameter ratios.

A summary of the parameters for the above mentioned models is contained in Table A-1, and a summary of the calculated and measured stresses in Table A-2. Reference 20 provides curves only for the maximum of the two stresses, and states that “...for small values of t/T , the hoop stress

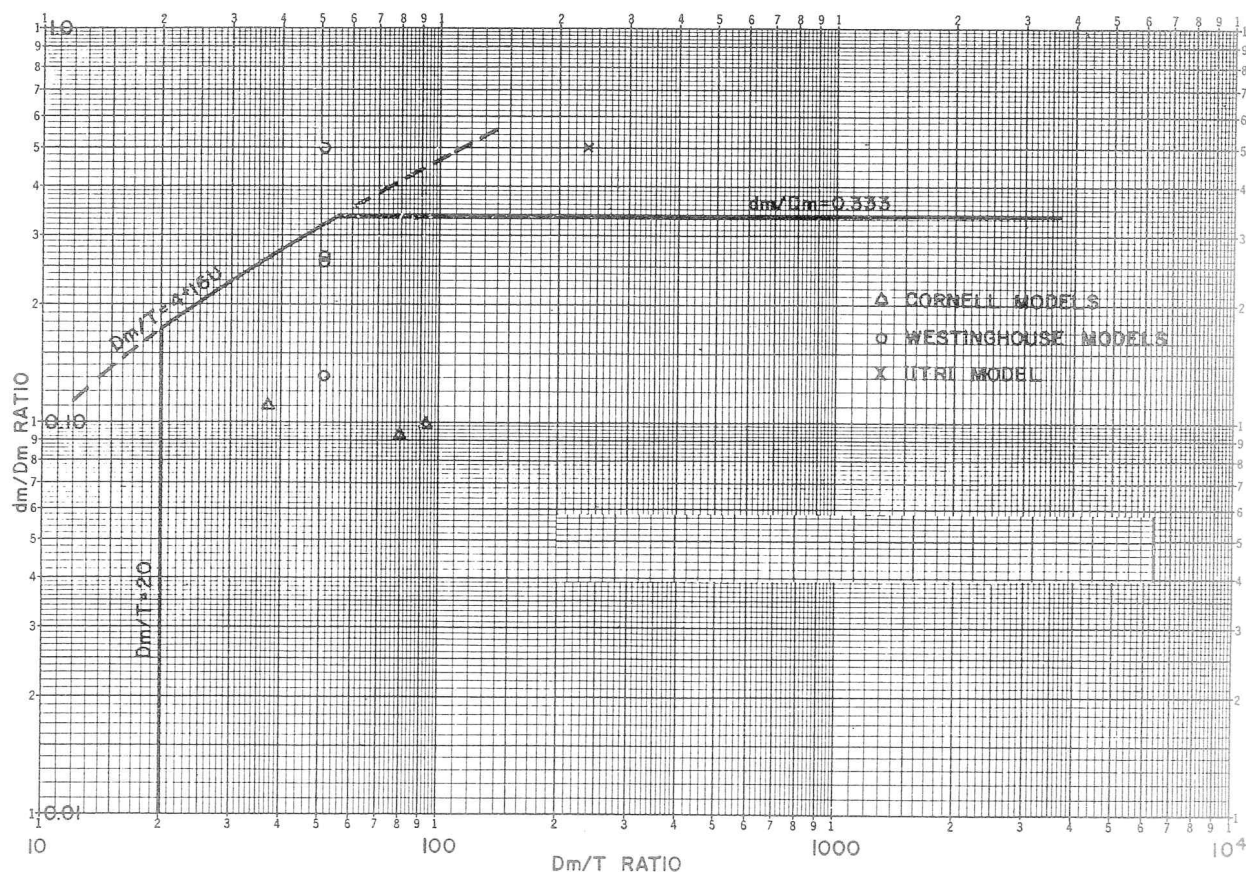


Fig. A-1—Relation of spherical test models to Bijlaard's "applicability limits"

Table A-1—Parameters for Model Vessels Tested with External Loads on Nozzles

Model	D_m/T	d_i/D_i	r_m/R_m	ρ^a ($= T/t$)	γ^a ($= r_m/t$)	u^a ($= 1.82$ $\frac{r_m}{R_m} \sqrt{R_m/T}$)	$\frac{r_m}{R_m} \sqrt{R_m/T}$	$Fillet$ radius, $\times T$
Spherical shell models								
West WN-50A	51.0	0.50	0.50	2.0	25.5	4.59	2.52	0.80
West WN-50B	51.0	0.27	0.2745	2.0	14.0	2.52	1.38	0.80
West WN-50C	51.0	0.27	0.269	4.0	27.5	2.475	1.36	0.80
West WN-50D	51.0	0.129	0.131	4.0	13.4	1.205	0.662	0.80
IITRI S-1	236.	0.496	0.498	1.01	59.5	9.87	5.41	~ 1.39
Model	D_m/T	d_i/D_i	r_m/R_m	t/T	s/S	β^a ($= 0.875$ $\frac{r_0}{R_m}$)	$Fillet$ radius, $\times T$	$\frac{r_m}{R_m} \sqrt{R_m/T}$
Cylindrical shell models								
Penn St. "L"	19.0	0.32	0.325	0.43	0.754	0.305	1.0	1.005
Penn St. "R"	19.0	0.63	0.634	0.687	0.926	0.585	0.75	1.95
Penn St. "S"	19.0	1.00	1.00	1.00	1.00	...	0.75	...
IITRI "C-1"	230	0.496	0.4975	0.98	0.508	0.439	~ 1.35	5.3

^a Bijlaard's parameters.

Table A-2—Comparison of Calculated and Measured Stresses in Spherical Models under External Nozzle Loadings

	Calculated stresses, ^a ksi			Measured stresses, ^a ksi		
	Bijlaard			Adjusted		
	σ_y	σ_x	Penny	σ_y	σ_z	σ_x
Moment loading						
WN-50A	2.72	2.13	3.64	4.81	6.05	5.03 ^b
WN-50B	3.18	2.51	3.37	4.59	5.73	4.78 ^b
WN-50C	2.40	0.554	2.52	3.83	4.45	3.97 ^c
WN-50D	2.11	0.447	2.09	2.73	3.15	2.81 ^c
S-1	...	...	14.5	9.95	14.4	11.8 ^d
Direct axial load (6000# pull)						
S-1	...	...	23.4	16.56 ^e	20.66 ^e	17.0 ^d
				18.3 ^f	23.5 ^f	19.3 ^d

NOTES:

^a Stresses due to moment loading are reported as a ratio of the stress in question to the calculated bending stress in the nozzle, as was reported by Westinghouse for the photoelastic models. These "base" nozzle stresses are as follows: WN-50A—0.00398Mb; WN-50B—0.0135Mb; WN-50C—0.0274Mb; WN-50D—0.118Mb; S-1—0.0903Mb, where Mb is the applied moment.

^b Based on local SCF of 1.20.

^c Based on local SCF of 1.12.

^d Based on local SCF of 1.22.

^e Average of eight separate measurements around nozzle.

^f Maximum of eight separate measurements around nozzle.

(σ_y) is higher than the meridional stress (σ_x). With increasing t/T , σ_y becomes smaller and σ_x larger, until σ_x begins to dominate. This change-over takes over when t/T is approximately 0.75 On this basis, the stresses from Penny-Leckie are presumably for σ_y for the four photoelastic models (WN-50 series), and σ_x for the steel model ("S-1").

In the case of the photoelastic models (moment loading), it will be noted that the calculated stresses from Bijlaard and Penny-Leckie agree almost exactly for the smallest d_i/D_i ratio (WN-50D), Bijlaard is about 5% lower than Penny-Leckie for models WN-50B and WN-50C, and 25% lower than Penny-Leckie for model WN-50A, indicating a progressive deterioration of shallow shell theory. However, it will be noted that for these four models, all of the calculated values for σ_y are somewhat below the measured values, and all of the

calculated values for σ_x are greatly below the measured values. As was explained in Reference 28, the primary reason for this difference is that both Bijlaard and Penny-Leckie provide *only for the calculation of the stresses in the shell*; however, for all four of these models, the maximum stress proved to be across the base of the nozzle, characterized by a high bending stress in the axial (σ_x) direction.

In the case of the steel model (S-1), the calculated stresses from Penny-Leckie agree almost exactly with the maximum measured stresses under both moment and axial loading, and would be about 25–30% conservative in relation to *average* measurements adjusted for local stress intensification. It should be emphasized, however, that present figures are preliminary and "average" figures may not be the most valid representation of the data.

In normal design procedure, it must be assumed that the maximum pressure and maximum external loading stresses occur at the same point. In the case of Model "S-1," this was true insofar as can be determined from the preliminary data. In the case of the photoelastic models, a duplicate of Model WN-50B was tested under internal pressure; for this model, the points of maximum stress did not quite coincide. Assuming that the point of maximum pressure stress will be controlling, the external (moment) loading stress at that point was perhaps 10–15% less than its maximum. On the basis of present evidence, it should *not* be assumed that there is any large conservatism in considering the points of maximum stress as coinciding (in the case of the larger diameter ratios in spherical shells at least).

Quoting Reference 28, the status of the theoretical work on spherical shells can be summarized as follows:

"The theoretical solutions for the stresses and deflections in (spherical) pressure vessels produced by externally applied forces and moments have been developed to the point where they can be of considerable value to the designer if used with discretion. The discretion which must be used consists of cognizance of the following limitations:

(1) When the loads are applied through relatively thin walled nozzles, the *rigid-insert approximation*² suppresses the stresses circumferential to the nozzle. Whereas this approximation also exaggerates the meridional stresses, there is no reason to believe that the calculated meridional stress is a good approximation of the actual circumferential stress.

(2) When the loads are applied through relatively thin-walled nozzles, the highest stress may occur in the nozzle and a solution which gives only shell stresses (as do Bijlaard's and Penny-Leckie's solutions in their present form) may seriously underestimate the peak stress.

(3) None of the theories are capable of considering the geometry of the junction in detail. Therefore, the concentrating effect of a sharp corner must be estimated separately. Also, the addition of even a small fillet or weld bead can significantly affect the stiffness of the junction and result in discrepancies between the actual and calculated stress. . . ."

On the basis of the foregoing, no changes in Bijlaard's curves for spherical shells are considered necessary, but particular attention should be paid to these limitations, and to the limitations which Bijlaard placed on his own work (as summarized on Fig. A-1). Since Penny-Leckie's theory appears to give the same results as Bijlaard's at small diameter ratios but does not have the limitations of shallow shell theory, and in addition covers both flush and protruding (balanced) nozzles, the PVRC subcommittee hopes to provide, in the future, more

complete stress data based on this theory, including the stress distribution in both the shell and nozzle.

A.3 Cylindrical Shells

About two years ago, PVRC undertook testing of a series of simple, fabricated tee type models consisting of two models at a D_i/T ratio of 18.0, with d_i/D_i ratios of 0.63 and 1.00, and two models at a D_i/T ratio of 230, with d_i/D_i ratios of 0.50 and 1.00. The primary purpose of these models was to provide external loading data at the larger diameter ratios, in the hope of being able to extrapolate Bijlaard's curves for cylindrical shells on up to a d_i/D_i ratio of 1.0.

The results of this work have recently become available, the data on the two "thick" shell models being reported in Reference 22 and the *preliminary* data on the smaller of the two thin shell models in Reference 23. The results from the thick shell models indicate discrepancies in Bijlaard's "extended range" data (page 12 of Reference 10) of a magnitude consistent with that which would be expected from shallow shell theory (as indicated by the work on spherical shells). However in the case of thin shell model, the results indicate that some of this extended range data is greatly in error; in addition, it appears probable that some of the original curves^{2, 3} are significantly in error in the very thin shell region (say, for values of β greater than 0.15 and 0.10 at values of $\gamma = 100$ and 300, respectively). It appears that the basic reason for this discrepancy is that, in thin shells, the longitudinal axis is relatively flexible and free to deform in relation to the transverse axis, causing the transverse axis to carry a disproportionate share of the load. This effect was not fully provided for in Bijlaard's treatment of the problem, which treated the nozzle as an "equivalent" square attachment. Actually, from superficial examination, some of the test results appear so improbable as to create suspicion of major deficiencies in the test model. However, upon detailed comparison with available internal pressure data, there is very good reason to believe that the results are essentially correct.

A.3.1 "Thick-Walled" Model Data

A summary of the parameters for the models in question (Penn State Models "R" and "S" and IITRI Model "C-1") is contained in Table A-1, together with similar data (subsequently used for comparison purposes) for Penn State Model "L," reported in Reference 21.

A summary and comparison of calculated and measured data for the three "thick-walled" models is contained in Table A-3. These data indicate that for Model "L," which presumably is at about the upper limit of shallow shell theory, the calculated stresses under moment loading range from 10 to 50% conservative; for Model "R," which involves an extrapolation of Bijlaard's curves, the

Table A-3—Comparison of Calculated and Measured Stresses in Thick Walled Cylindrical Vessels with External Loads on Nozzles

		Calculated stresses, ksi			Maximum measured stresses, ksi	
		Membrane	Bending	Total	Gross	Adjusted
Model "L"						
Longitudinal moment	$-\sigma_\varphi$	10.45	17.1	27.55	20.5	20.5
(250,000 in.-lb)	$-\sigma_z$	3.36	26.9	30.26	35.4	27.2 ^d
Transverse moment	$-\sigma_\varphi$	3.41	45.4	48.8	44.7	34.4 ^d
(250,000 in.-lb)	$-\sigma_z$	6.37	24.7	31.1	21.2	21.2
Model "R"						
Longitudinal moment	$-\sigma_\varphi$	7.58	5.37	12.95	14.6	14.6
(500,000 in.-lb)	$-\sigma_z$	3.60	8.03	11.63	23.7	16.9 ^b
Transverse moment	$-\sigma_\varphi$	4.16	31.5	35.66	57.1	40.7 ^b
(500,000 in.-lb)	$-\sigma_z$	10.0	15.0	25.0	26.9	26.9
Radial load (30,000# pull)						
Longitudinal axis	$-\sigma_\varphi$	3.55	3.24	6.79	1.4	1.4
	$-\sigma_z$	2.08	5.78	7.86	2.2	1.57 ^a
Transverse axis	$-\sigma_\varphi$	2.08	5.78	7.86	14.4	10.3 ^a
	$-\sigma_z$	3.55	3.24	6.79	8.2	8.2
Model "S"						
Longitudinal moment	$-\sigma_\varphi$	...	...	...	4.8	4.8
(500,000 in.-lb)	$-\sigma_z$	...	...	...	9.7	6.9 ^b
Transverse moment	$-\sigma_\varphi$	...	...	...	9.8	8.15 ^c
(500,000 in.-lb)	$-\sigma_z$	...	...	...	3.3 (60° axis)	3.3

NOTES:

^a Based on S.C.F. of 1.30.^b Based on S.C.F. of 1.40.^c Based on S.C.F. of 1.20.^d Based on S.C.F. of 1.15.

calculated stresses under moment loading range from about 7 to 45% unconservative, with all four stress quantities being 38 to 55% less conservative than was the case for Model "L." This effect is believed attributable to limitations analogous to those of shallow shell theory, and is of a magnitude not inconsistent with the effect noted in spherical shells.

For the case of radial load, data were not obtained on Model "L." For Model "R," the maximum measured stresses on the longitudinal axis (both σ_φ and σ_z) are less than one-fourth the calculated values. On the transverse axis, the measured longitudinal stress, σ_z (longitudinal with respect to the shell but circumferential with respect to the nozzle) was of the same order of magnitude as the calculated stress, although there is perhaps an indication that the membrane portion of the calculated stress is "low" and the bending portion "high." In the case of the circumferential stress, σ_φ (circumferential with respect to the shell), the calculated stress is significantly lower than the measured stress, but in this case the maximum measured stress was across the base of the nozzle. The next highest reading was in the shell and would seem to be quite consistent with the calculated value.

A.3.2 "Thin-Walled" Model Data

A summary and comparison of the calculated and measured data for IITRI Model "C-1" is contained in Table A-4. From this comparison, it will be noted that in some instances there are very large discrepancies between the calculated and measured stress values. Further, it will be observed that under a longitudinal moment loading, the maximum stress occurs well off the longitudinal

axis of the vessel, with the maximum value being better than twice that directly on the longitudinal axis; a very similar effect was noted under internal pressure. Whereas we had forewarning of this possibility under internal pressure, the effect was quite unexpected under longitudinal moment. Because this result seems somewhat irrational, a special effort was made to evaluate the probable validity of the results, summarized as follows:

(a) The vessel was not a machined model and was slightly out-of-round. Such out-of-roundness may have some effect on the measured pressure stresses, but we do not believe that it would significantly affect the stresses due to external loading. The fillet and the area adjacent to the nozzle-shell juncture were checked with templates, and it is believed that any deviation in thickness or local contour is minor and does not constitute an explanation for the effects noted.

(b) In testing the model, only one quadrant was completely instrumented.* However, for the axes of symmetry, supplementary instrumentation was installed 180° opposite the primary instrumentation. Also, for the external loading tests, the loads were applied in both the "positive" and "negative" directions, giving a total of four readings for each nominal location on the axes of symmetry and two readings for locations off the axes of symmetry. In all cases, the critically stressed region was found to be directly in the fillet at the nozzle-shell juncture. A plot of the stresses along this fillet, under the four loading conditions used, is shown in Figs. A-2 to A-5 inclusive, based on arithmetic averages of the available data. The total scatter in the data for a given location slightly

* See note at the end of Section A.3.2.

Table A-4—Comparison of Calculated and Measured Stresses in Thin Walled Cylindrical Vessel (IITRI Model C-1) with External Loads on Nozzles

	Calculated stresses, ksi			Measured stresses, ksi	
	Membrane	Bending	Total	Gross	Adjusted
Longitudinal moment (18,000 in.-lb)					
Longitudinal axis— σ_φ	5.74	0.72	6.46	11.8	11.8
Longitudinal axis— σ_z	4.02	0.93	4.95	12.6	10.3 ^a
Maximum ^b — σ_n	...	...	...	15.9	15.9
Maximum ^b — σ_t	...	...	...	29.5	24.15 ^a
Transverse moment (3000 in.-lb)					
Transverse axis— σ_φ	0.62	6.54	7.16	23.85	19.5 ^a
Transverse axis— σ_z	3.97	2.30	6.27	10.5	10.5
Radial load (1000# pull)					
Longitudinal axis— σ_φ	4.16	1.44	5.60	2.7	2.7
Longitudinal axis— σ_z	1.12	3.66	4.78	3.0	2.46 ^a
Transverse axis— σ_φ	1.12	3.66	4.78	26.1	21.4 ^a
Transverse axis— σ_z	4.16	1.44	5.60	15.3	15.3

NOTES:

^a Based on local S.C.F. of 1.22.

^b Maximum stresses were located 60–70° off the longitudinal axis. These stresses are derived from strain gage measurements oriented radially and circumferentially with respect to the nozzle (or the hole in the shell). The maximum principal stress at this location may be somewhat higher (but by definition cannot be lower) than indicated by these measurements. σ_n designates the stress normal to a plane at this section (= circumferential with respect to the nozzle) and σ_t , the stress in the plane of such a section (axial with respect to the nozzle). Therefore, on the longitudinal axis, $\sigma_n = \sigma_\varphi$ and $\sigma_t = \sigma_z$; on the transverse axis, $\sigma_n = \sigma_z$ and $\sigma_t = \sigma_\varphi$.

exceeded 10% for the worst case (except in low stressed areas where considerable scatter—in per cent—is normal); in most cases, the scatter was less than 10%. From the plot of the data on Figs. A-2 to A-5 inclusive, it will be noted that the consistency of the data is good except for the case of the stress, σ_n [circumferential with respect to

the nozzle; see NOTE (2) in Table A-4] in the region 45–70° off the longitudinal axis under internal pressure and longitudinal moment (Figs. A-2 and A-3, respectively). The readings off the axes of symmetry were obtained with two-element rosettes, readings from the third element having been discarded because of excessive scatter; the maxi-

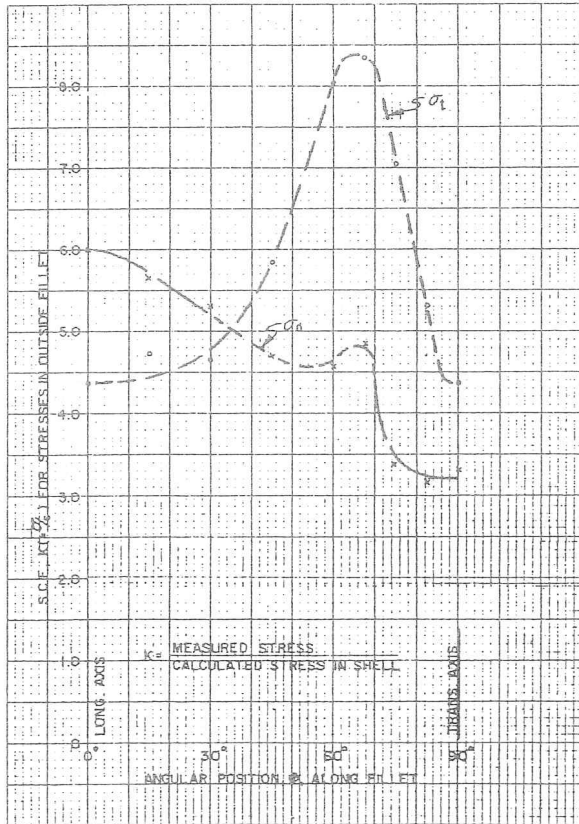


Fig. A-2—Measured stresses in fillet of IITRI model C-1 tested under internal pressure (30 psi pressure)

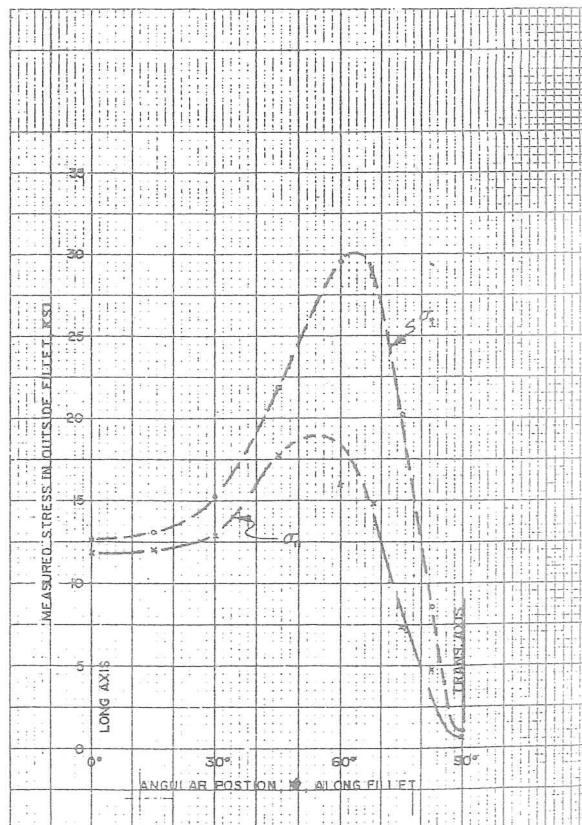


Fig. A-3—Measured stresses in outside fillet of IITRI cylindrical shell model C-1 under longitudinal moment loading of 18,000 in.-lb

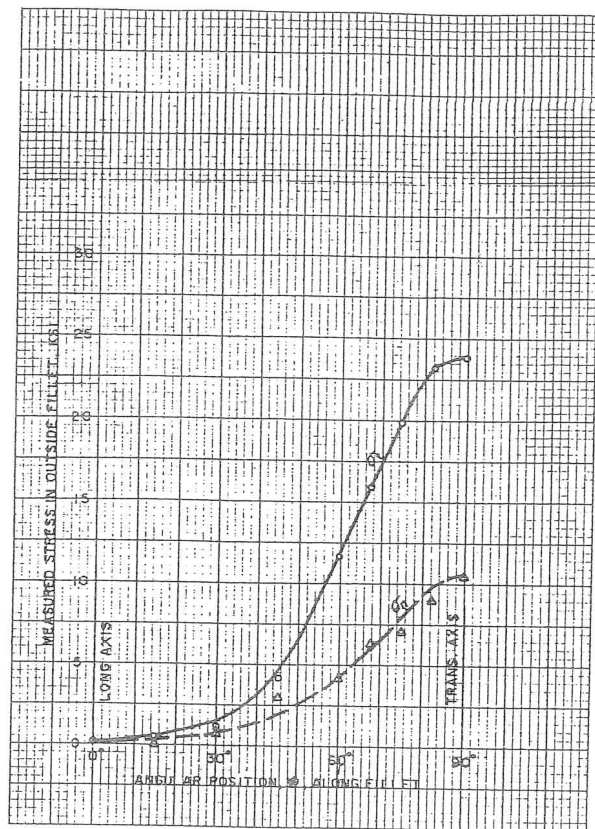


Fig. A-4—Measured stresses in outside fillet of IITRI cylindrical shell model C-1 under transverse moment loading of 3000 in-lb

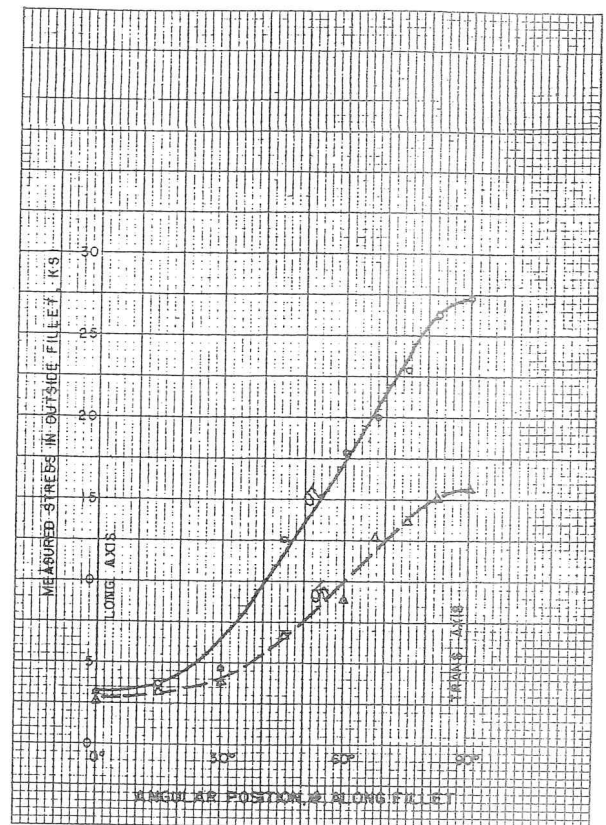


Fig. A-5—Measured stresses in outside fillet of IITRI cylindrical shell model C-1 under a radial load (pull) of 1000 lb

imum principal stress at each location may be higher than indicated by present readings, but cannot be lower (by definition, assuming the basic validity of the data). (NOTE: The basic reason for the scatter in the third element is believed attributable to difficulty in accurate orientation of the $1/32$ in. gages; these readings will be checked using $1/16$ in. pre-assembled, three-element rosettes).

(c) As a part of the PVRC reinforced openings program, an effort is being made under the direction of Dr. A. C. Eringen to provide an analytical solution for the cylinder-to-cylinder intersection problem *under internal pressure*. The basic theory and the first numerical results from this work are contained in References 25, 26 and 27. Dr. Eringen has shown that the stresses in such an opening can be related to a single parameter, $\Omega = (d_m/D_m)\sqrt{D_m/T}$. Although present numerical results cover only the case of an opening with a membrane closure ($t/T = 0$), these results show that as the parameter Ω increases, the maximum membrane stress and the maximum surface stresses both shift off the longitudinal axis. Although the numerical results presented in these reports cover values of Ω only up to a value of $\Omega = 2.8$, Van Dyke³¹ subsequently extended the range of calculated data up to a value of $\Omega = \sim 8.0$. The results from this work show that, as the parameter

Ω increases, the maximum membrane stress circumferential to the hole and the maximum stress on both surfaces all shift off the longitudinal axis. This "shift" develops first on the outside surface, followed by a shift in the membrane stress and finally by a shift in the inside surface stress. In each case, as the value of Ω increases, the maximum stress increases progressively in magnitude and also moves progressively farther away from the longitudinal axis. This is illustrated in typical fashion for the membrane stress, as shown on Fig. A-6. Stress profiles for all three stresses are shown on Fig. A-7 for a value of Ω closely approximating that of the IITRI Model C-1. It should be emphasized that these present results are for a hole with a membrane closure ($t/T = 0$), and that the work is based on shallow shell theory, which would presumably limit its validity to diameter ratios in the order of $1/3$. Nevertheless, the results may give qualitative trends for larger diameter ratios and they definitely indicate that, for the cases studied ($t/T = 0$), an instability or bulge of increasing severity develops as the value of Ω increases. Although it is unfortunate that similar data are not yet available for finite t/T ratios, it seems obvious that attachment of a nozzle will tend to restrain this localized deflection and rotation at the edge of the opening. In such case, there is every reason to believe that the high circumferential

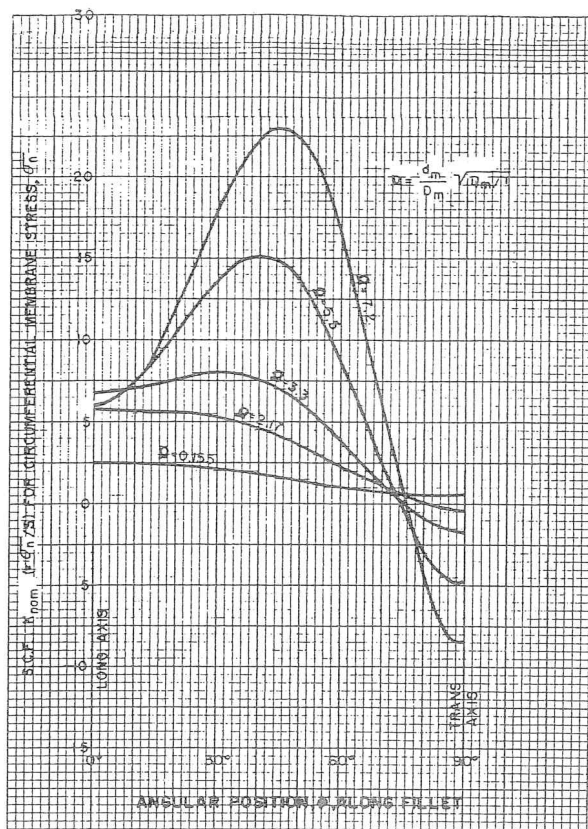


Fig. A-6—Development of critically stressed membrane area at edge of hole in cylindrical shell under internal pressure (Eringen's and Van Dyke's data, $t/T = 0$)

stress (in relation to the nozzle), would be partially replaced by an axial stress which should "peak" at essentially the same point. In this respect, then, it should be noted that the form of the curve for the axial stress, σ_z , shown on Fig. A-2, is quite consistent with the form of the curves on Figs. A-6 and A-7 [although the peak of the curve for the model (Fig. A-2) is farther from the longitudinal axis than indicated by the calculated data]. In spite of the latter difference, we feel that the calculated data provides good *qualitative* evidence of the validity of the experimental data under internal pressure loading.

(d) For the external loading conditions, judgment of the validity of the data must rest in considerable measure on the internal consistency of the data itself. In this regard, a detailed comparison of the stresses in the fillet on the axes of symmetry for the three external loading conditions is contained in Table A-5 for those cases where the stress was large enough to be significant (greater than 2.0 ksi). In each case, the value listed is an average of readings obtained in the positive and negative loading directions. For all such cases, agreement between one axis of symmetry and its counterpart 180° opposite is within the range of 3 to 12%. Considering variations in fillet radius and difficulty in exact placement of the gages, this is

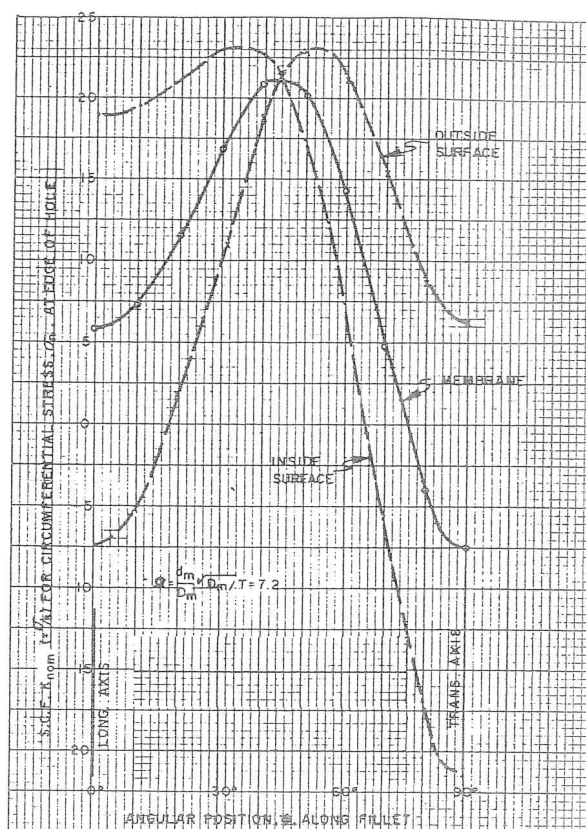


Fig. A-7—Circumferential stress (σ_n) at edge of hole with membrane closure ($t/T = 0$) in cylindrical shell under internal pressure (Van Dyke's data)

excellent agreement and there is nothing in the data which would indicate any serious deficiency in the model.

In the case of radial load on the nozzle, prior tests on Penn State Models "D," "E" and "R"^{21, 22} indicated stresses on the transverse axis 3–5 times those on the longitudinal axis. In the case of Model "C-1," the ratio is 5.7 for σ_ϕ and 8.7 for σ_z . The qualitative effect is therefore the same, but the difference is seemingly accentuated in the thin shell model.

In the case of moment loading, results from Penn State Model "R" gave maximum stresses under a transverse moment approximately 2–2.5 times those due to an equal longitudinal moment. Calculations based on Bijlaard's curves predicted a similar difference, although the absolute values of the calculated stresses were somewhat lower than the measured ones in both cases. For equivalent moments on Model "C-1," the maximum stresses due to a transverse moment are 4–5 times as great as those due to a longitudinal moment, *with the maximum stress being located $60\text{--}70^\circ$ off the longitudinal axis in the latter case*; for the stress directly on the longitudinal axis, the ratios are 12.1 and 5.0 for σ_ϕ and σ_z , respectively. The comparative effects noted in the Penn State and IIT models are therefore qualitatively similar, with the added fac-

Table A-5—Comparison of Measured Stresses on Opposite Sides of IITRI Cylindrical Vessel Model C-1

	Maximum measured stress, ksi			
	0°	180°	90°	270°
Longitudinal moment (18,000 in.-lb)				
σ_φ	11.65	12.0	...	...
σ_z	12.3	12.9	...	...
Transverse moment (3000 in.-lb)				
σ_φ	...	...	24.35	23.35
σ_z	...	...	11.1	9.95
Radial load (1000# pull)				
σ_φ	2.75	2.65	27.35	24.9
σ_z	3.15	2.9	15.65	14.85

tor of an apparent "instability" or local bulging in the thin shell model. In the latter connection, it should be noted that the stress pattern for the axial stress, σ_z , under longitudinal moment, as shown on Fig. A-3, is remarkably similar to that under internal pressure, Fig. A-2 (which, as has already been shown, would seem to be qualitatively consistent with calculated data).

On the basis of the foregoing, there seems to be no reason to question the qualitative validity of the data.* The reason for the high stresses on or adjacent to the transverse axis appears to be that, in a thin shell, the longitudinal axis is relatively flexible and free to deform, and that the loads are thereby transferred to (or toward) the transverse axis which is less free to deform. On the basis of the data available, it would seem that a large part of Bijlaard's "extended range" data may be unconservative; further it would appear that the very thin shell region of his original data^{2, 3} may be significantly low. That is, values of $\gamma = 300$ and $\beta = 0.25$ would correspond to a value of $\Omega = [(d_m/D_m) \sqrt{D_m/T}]$ of approximately 6.95. However, the IIT model, with a value of Ω only slightly higher than this, shows a very well developed condition of instability under longitudinal moment and a marked shift of the load to the transverse axis under the other two loading conditions. On the basis of the data reviewed above, it is considered that the original data is open to question at d_i/D_i ratios greater than 0.15 for $\gamma = 100$ and greater than 0.10

for $\gamma = 300$ (i.e., for values of Ω greater than say, 2.0 - 2.5).

A.3.3 Modification of Curves

Since the experimental data indicates that the extended range of Bijlaard's data may be in error by factors of as much as 5.0, it seemed necessary to provide interim "corrections" to Bijlaard's curves until such time as better analytical methods are developed to compute these stresses. Unfortunately, we have little basis beyond the experimental data on which to make these "corrections," and time may prove that their only virtue is that they are in the "safe direction." Because of this uncertainty, it seems necessary to document the exact manner in which the curves have been modified, as outlined in the following paragraphs.

A.3.3.1 REDUCTION OF DATA. In proposing any modification to Bijlaard's curves, the first problem to be faced is that of placing the experimental and calculated data on an equivalent basis, which we have endeavored to do by "correcting" the experimental data for local stress intensification. The next step is to try to break the experimental data down into membrane and bending components, in order to determine the nature of the specific modifications required. For both the Penn State and IITRI models, stress distributions were obtained in both the nozzle and shell on the axes of symmetry. In the case of the IIT model, all such data can be broken down into its membrane and bending components except for the reading directly in the fillet (the closest "valid" points being $1/4$ in. from the fillet, on both the nozzle and shell). Such data seems to indicate that the membrane components of stress as calculated from Bijlaard are relatively accurate, but that the bending components are sometimes greatly in error. Also, in a number of cases, the bending stress is large in relation to the membrane stress; for such cases, large percentage increases in membrane stress would be quite ineffectual in correcting the over-all total. For these two reasons, it was decided that the major corrections should be made to the bending stress curves, although relatively minor changes have been made to the membrane curves in a couple of cases.

* NOTE: Subsequently, this high stressed zone was instrumented in the remaining three quadrants on the model, two with $1/32$ in. two-element rosettes, and the other with $1/16$ in. three-element rosettes. The results are summarized as follows (each value being an average of readings in the positive and negative loading directions):

Position	Gage length, in.	Tangential stress, σ_t , ksi	Normal stress, σ_n , ksi
60° (original)	$1/32$	29.5	15.9
60° (retest)	$1/32$	30.6	17.1
120°	$1/32$	31.8	18.6
300°	$1/32$	27.8	15.3
240°	$1/16$	24.5	14.1

The $1/16$ in. three-element rosette confirmed that the stresses measured by the two-element rosettes were essentially the principal stresses. Although there is some scatter in the data, and the readings obtained with the $1/16$ in. rosette were 15-20% lower than the average of those obtained with $1/32$ in. rosettes, it is apparent that a high stressed zone exists at this location in all four quadrants (materially higher than on the longitudinal axis, itself). These results would seem to remove any question concerning an isolated local deficiency in the model and concerning the qualitative validity of the data.

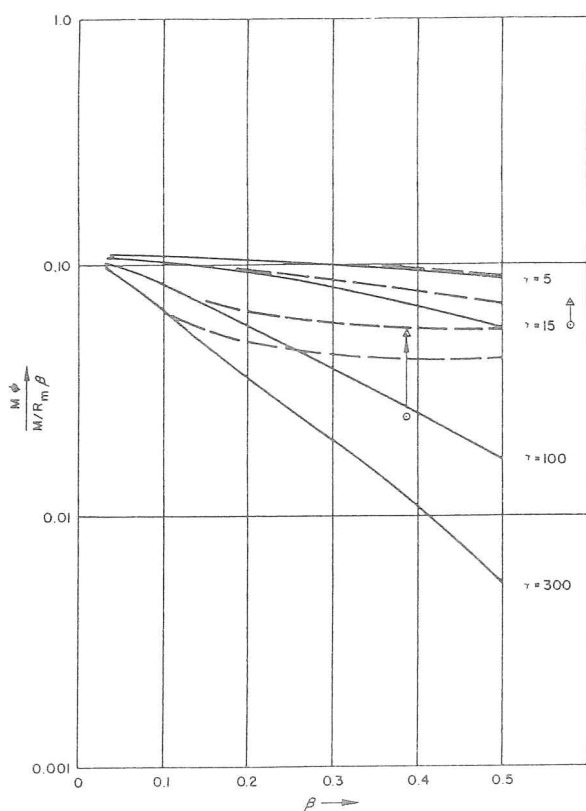


Fig. A-8—Moment $M_\psi/(M/R_m\beta)$ due to a circumferential moment M

A.3.3.2 CIRCUMFERENTIAL (TRANSVERSE) MOMENT

(a) *Circumferential Shell Stress, σ_ψ* : In the case of the thick shell model (Model "R"), the calculated stress, σ_ψ , was about 14% lower than the maximum measured value (after adjustment for local stress intensification). In the actual model, the location of maximum stress would perhaps be construed as being across the base of the nozzle rather than in the shell; however, Bijlaard's theory for cylindrical shells does not take into account the effect of relative stiffness of nozzle and shell (t/T ratio), and for a stiffer nozzle, the location of this maximum stress could very well be shifted down into the shell. Therefore, in the interest of conservatism, and until better methods of analysis become available, the maximum measured stress was treated as though it were in the shell. The bending component of the stress was about 90% of the total, and all of the correction was made to this stress component, amounting to an increase of approximately 16%. This relatively minor correction is indicated on Fig. A-8.

In the case of the thin shell model (model "C-1"), the calculated circumferential stress in the shell was low by a factor of 2.7 in relation to the measured value. The calculated value of the membrane stress was about 20–25% low in relation to a measured value $1/4$ inch away in the shell, indicat-

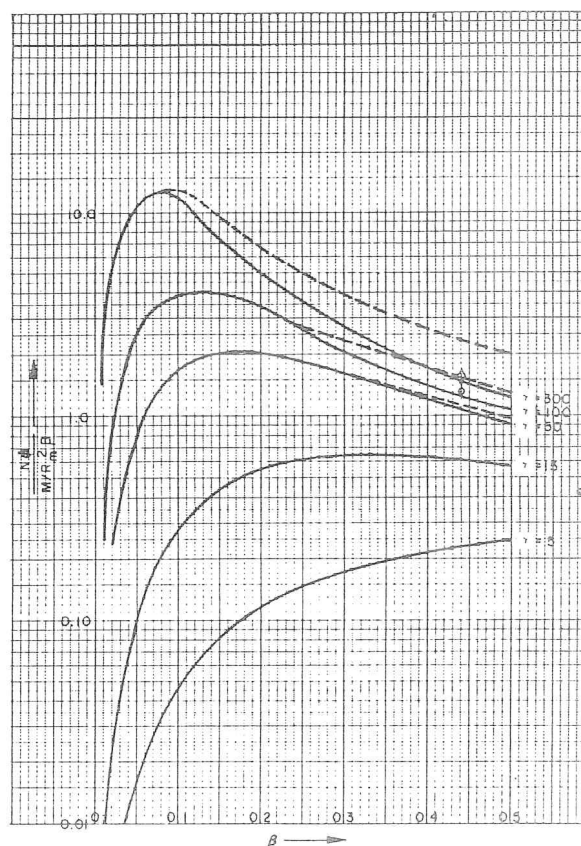


Fig. A-9—Membrane force $N_\phi/(M/R_m^2\beta)$ due to circumferential moment M

ing that the curves for the membrane stress may converge too rapidly in the thin shell region (which appears entirely possible, on the basis of simple inspection). This portion of the curve has been revised accordingly, as shown on Fig. A-9. Actually, the effect of this correction is almost negligible in terms of the total difference between the calculated and measured stresses, since the membrane stress is but a very small part of the total stress. The resulting correction required for the bending stress amounts to a factor of 2.85, as shown on Fig. A-8.

In the case of the bending stresses, the experimentally determined points from Models "R" and "C-1" pose a problem in that the curves are seemingly compressed into a very narrow band at high values of β ; also, if the thin shell curves are correct at small values of β , a drastic revision is required in the form of those curves. The revisions shown on Fig. A-8 represent the best judgment of the authors on the basis of the limited information available. However, it is warned that these curves are not necessarily correct and their only virtue may be that they are more conservative than the original curves.

(b) *Longitudinal Shell Stress, σ_x* : The required corrections for the longitudinal shell stress, σ_x , were very similar to those described above for σ_ψ ,

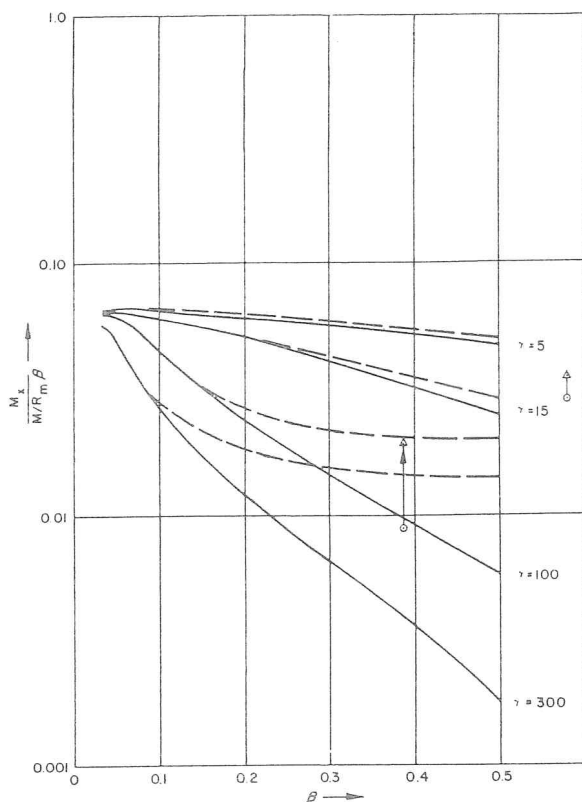


Fig. A-10—Moment $M_x/(M/R_m\beta)$ due to a circumferential moment M

except that no correction of the membrane stress was considered warranted. The corrections to the bending components of the stress are shown on Fig. A-10, amounting to roughly 10% for Model "R" and a factor of 2.72 for Model "C-1."

A.3.3.3 LONGITUDINAL (IN-PLANE) MOMENT. Consideration of corrections required to the curves for longitudinal moment is complicated by the fact that for the thin shell model, the maximum stresses were off the longitudinal axis. Under the circumstances, it was decided to provide two sets of curves, one applying to the longitudinal axis and the other covering the maximum stresses. Actually, it would appear that only the maximum stresses are of interest, since the available data (Figs. A-2 and A-3) indicate that the stresses due to internal pressure and longitudinal moment peak at the same location and must be considered additive (at least in the case of the axial stress, σ_z , which is the critical stress). A detailed description of the corrections follows:

(a) *Circumferential Shell Stress, σ_ϕ , on the Longitudinal Axis:* In the case of Model "R," the required correction was relatively modest, and was applied only to the bending component. This correction amounted to about 18%, as shown on Fig. A-11.

In the case of Model "C-1," measured data $1/4$

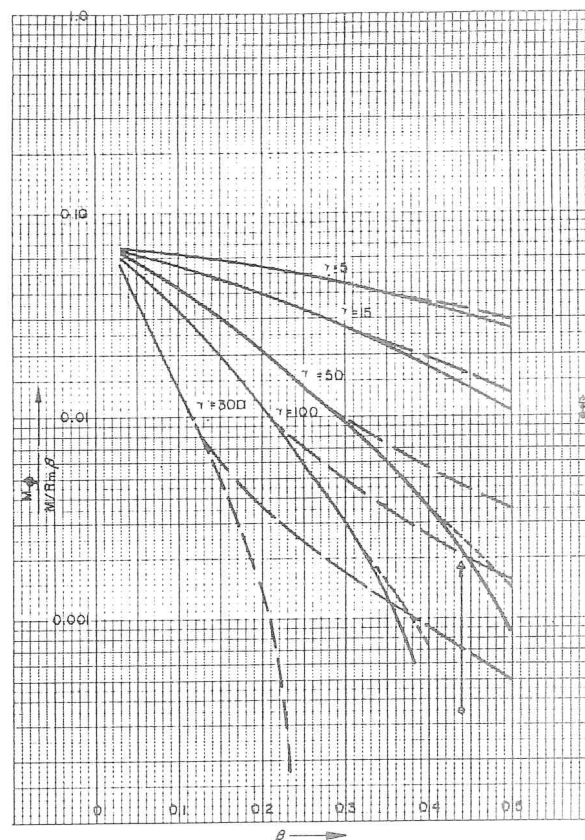


Fig. A-11—Moment $M_\phi/(M/R_m\beta)$ due to a longitudinal moment M (on longitudinal axis)

inch away in both the nozzle and shell gave membrane stresses approximately 30% higher than the calculated value; also, simple inspection of the original curves would indicate a possible too-rapid convergence in the thin-shell region. Under this circumstance, the membrane curves were adjusted upward a commensurate amount, as indicated on Fig. A-12. The remainder of the required correction was applied to the bending component, which was adjusted upward by a factor of 5.2, as indicated on Fig. A-11.

(b) *Longitudinal Shell Stress, σ_z , on the Longitudinal Axis:* Similar corrections were required for the longitudinal shell stress, except that no correction of the membrane curves were considered warranted. The increase in the bending component of the stress was approximately 66% in the case of Model "R," and a factor of 6.75 in the case of Model "C-1," as indicated in Fig. A-13.

(c) *Maximum Stresses:* The only basis for estimating the progressive divergence of the maximum stresses from the longitudinal axis is to assume that the effect is similar to that indicated by the calculated data under internal pressure (perhaps adjusted to be consistent with Model C-1), which is indicated graphically on Fig. A-14. As a corollary, it was also assumed that the maximum stresses due to internal pressure and longitudinal moment have

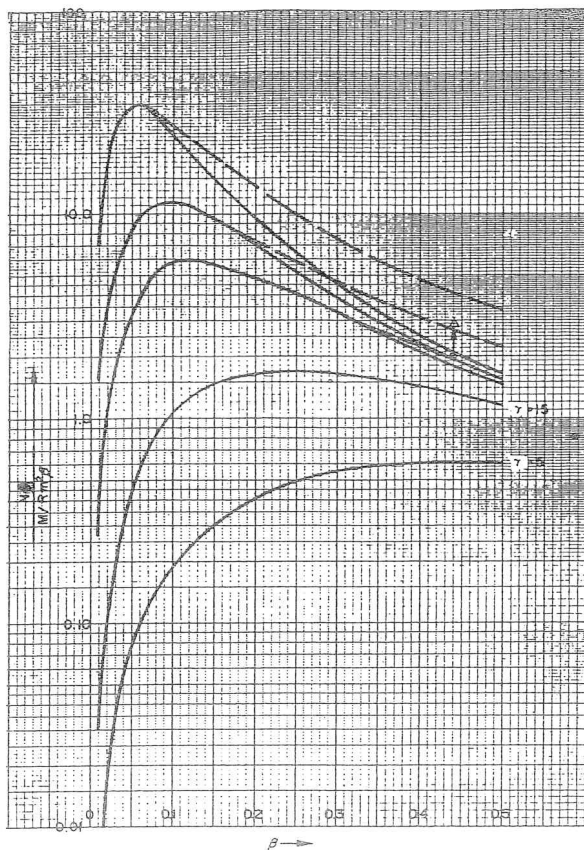


Fig. A-12—Membrane force $N_\phi/(M/R_m^2\beta)$ due to a longitudinal moment M

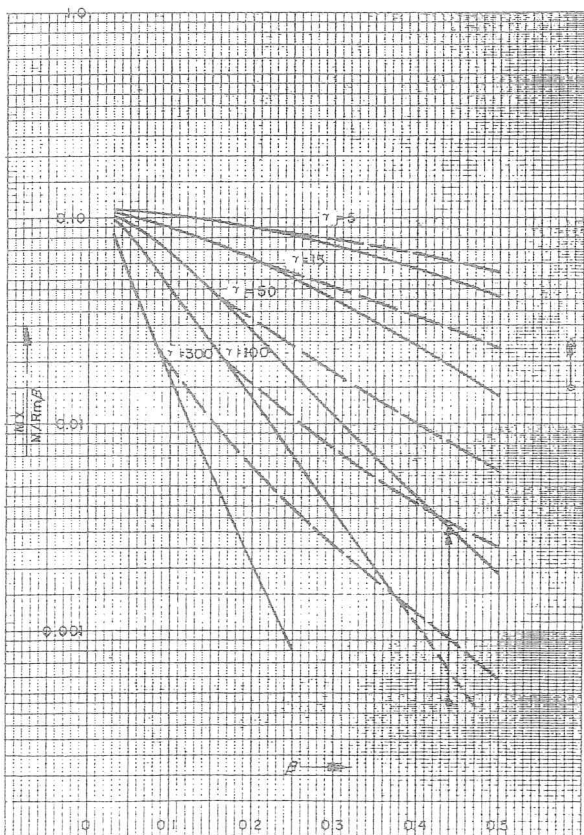


Fig. A-13—Moment $M_x/(M/R_m\beta)$ due to a longitudinal moment M (on longitudinal axis)

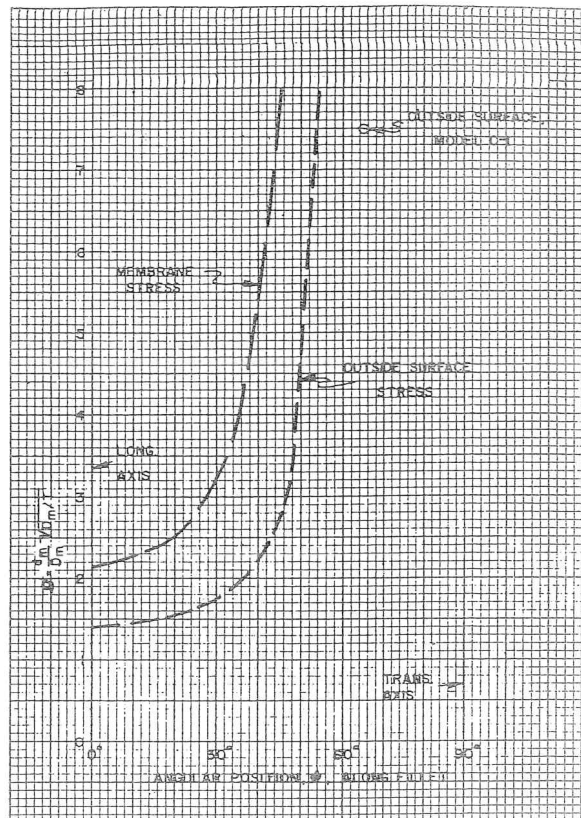


Fig. A-14—Location of maximum stress in cylinder under internal pressure (Eringen's and Van Dyke's data, $t/T = 0$)

the same orientation and are directly additive. Until further data become available, the orientation of these stresses will be taken as circumferential and axial *with respect to the nozzle*, σ_n and σ_t , respectively (which corresponds to the orientation of the strain gages on Model C-1 and to the terminology which has been generally used in the reinforced openings program). For relatively small values of β , where the maximum stresses are on the longitudinal axis, $\sigma_n = \sigma_\phi$ and $\sigma_t = \sigma_x$; curves for σ_n and σ_t were therefore obtained through modification of the curves for σ_ϕ and σ_x , respectively. Also, since no basis is available for modifying the membrane stress, and that component of the stress appears to be relatively small in relation to the bending component, the curves for membrane stress on the longitudinal axis were arbitrarily assumed to apply, and the necessary correction made to the bending curves. The resulting modifications to the curves are as shown on Figs. A-15 and A-16 for σ_n and σ_t , respectively.

A.3.3.4 DIRECT AXIAL LOAD. Bijlaard's treatment of axial load calculated the stress at the center of an attachment on an unpierced shell, having a uniformly distributed load. For the sake of conservatism, and in an effort to take into account the rigidity of the attachment, he then assumed that these values would apply at the edge of the attachment.^{2, 10} However, as noted in Reference 17,

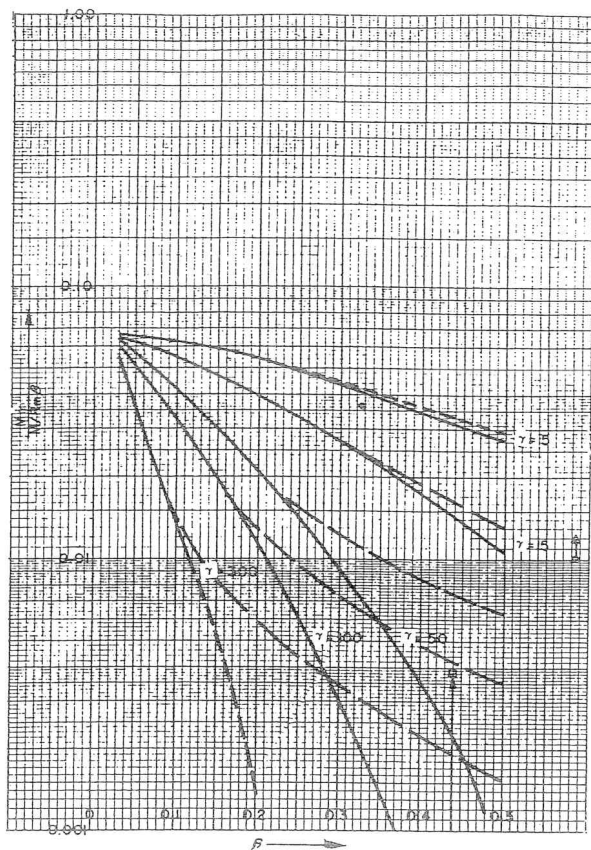


Fig. A-15—Moment $M_n/(M/R_m\beta)$ due to a longitudinal moment M

this procedure does not distinguish between the values at the edge of the attachment on the longitudinal axis of the shell vs. the transverse axis of the shell. A summary of the experimental results in comparison with the calculated (as taken from Table 3 of Reference 17, for Attachment 2) is shown in Table A-6. From this comparison, it will be noted that the agreement between theory and experiment was quite good on the transverse axis, but that the theoretical results were conservative by a factor of, say 1.5–2.0, as applied to the stresses on the longitudinal axis. Prof. Cranch therefore suggested that, in the case of the circumferential stress, σ_ϕ , on the longitudinal axis, no “shift” in the stress from the center of the attachment to its edge is necessary. However, the only calculated data

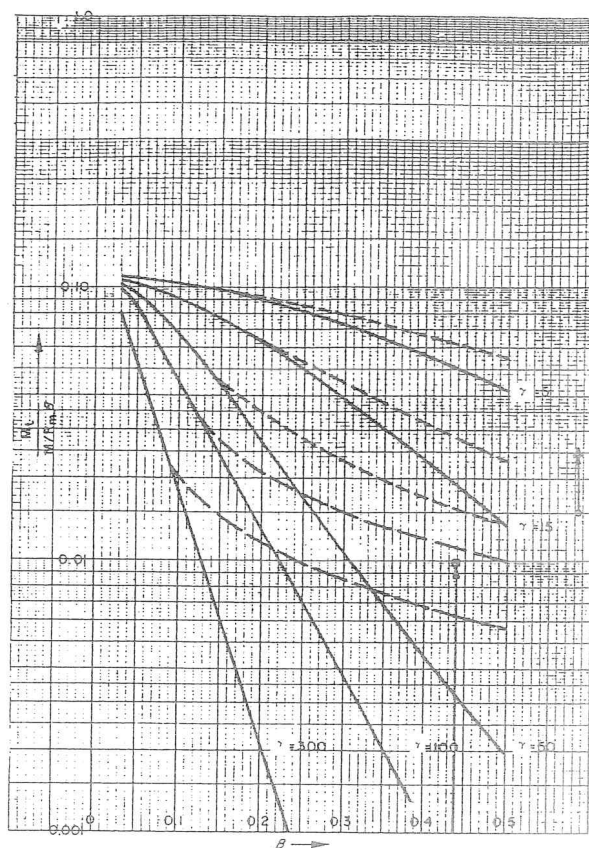


Fig. A-16—Moment $M_t/(M/R_m\beta)$ due to a longitudinal moment M

available for the edges of the attachment are those obtainable from a cross plot of the curves presented in Reference 10; further, the latter data were for a value of $\alpha = 4$ rather than 8, and were limited to values of β no greater than 0.25. Under these circumstances, the comparisons of Tables A-3 and -4 were made on the basis that Bijlaard's calculated stresses, for the center of the attachment, apply at the edge of the attachment on both the longitudinal and transverse axes (even though the available evidence for a model well within the presumed limits of Bijlaard's theory indicated that the calculated stresses might be appreciably conservative as applied to the longitudinal axis).

The test results on the longitudinal axis of both Models “R” and “C-1” indicate that Bijlaard's

Table A-6—Summary of Calculated and Measured Stresses for Cornell Attachment No. 2^a under Radial Load (Pull) of 17,700 Lb (WRC Bull. No. 60, Table 3)

	Calculated stresses, ksi			Measured ^b stresses, ksi		
	Membrane	Bending	Total	Membrane	Bending	Total
Transverse axis						
σ_ϕ	5.74	26.4	32.14	3.71	27.6	31.3
σ_z	6.7	17.9	24.6	5.3	16.8	22.1
Longitudinal axis						
σ_ϕ	5.74	26.4	32.14	4.95	10.4	15.35
σ_z	6.7	17.9 ^c	24.6	3.18	13.6	16.78

NOTES:

^a Model parameters: $D_m/T = 78$; $d_i/D_i = 0.126$; $t/T = 0.448$; $(d_m/D_m)\sqrt{D_m/T} = 1.14$; $\gamma = 39.0$; $\beta = 0.119$.

^b Tabulated stresses at edge of attachment were obtained by extrapolation of measured values from strain gages located $11/16$ in. away (outside the edge of weld fillet).

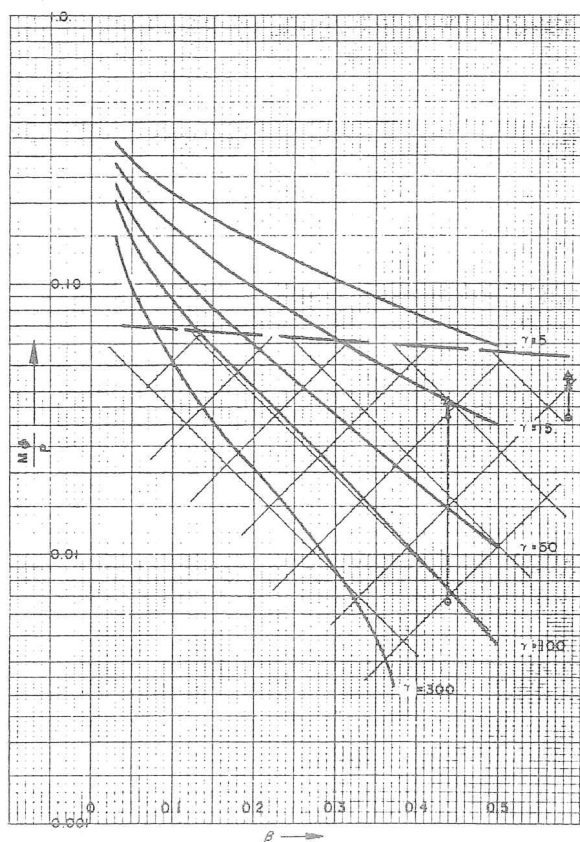


Fig. A-17—Bending moment M_ϕ/P due to a radial load P

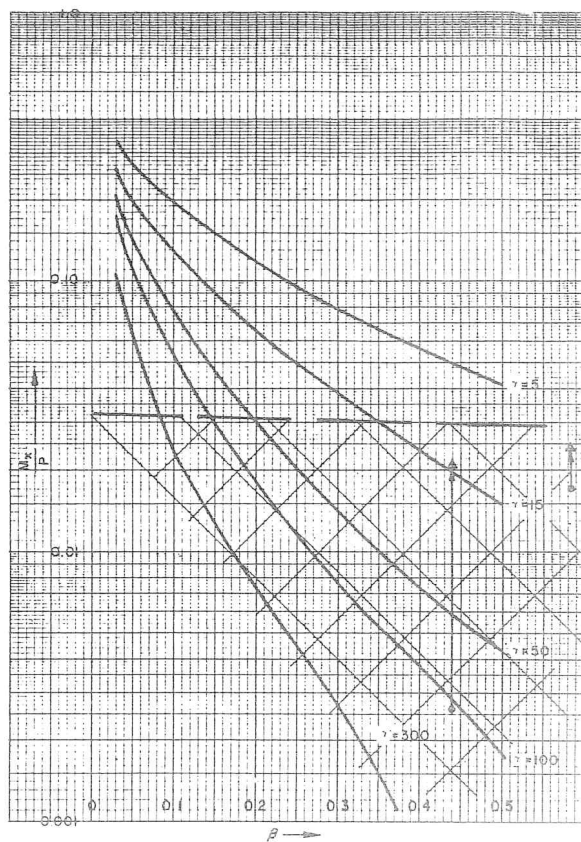


Fig. A-18—Bending moment M_x/P due to a radial load P

curves for axial load are appreciably conservative as applied to the stresses on the longitudinal axis, as was the case for Cornell Attachment 2. However, as applied to the stresses on the transverse axis, they are slightly inadequate for thick shells (Model "R"), and greatly inadequate for thin shells (Model "C-1"); in the latter case, the calculated values were low by a factor of 2.5–3.0 for $\sigma_x (= \sigma_n)$ and about 4.5 for $\sigma_\phi (= \sigma_t)$. Furthermore, a plot of the test results would seem to indicate a compression of the curves into a very narrow band, or (more likely) a "cross-over" of the curves. Under this circumstance, no "correction" to the

curves is considered feasible, and it can only be warned that for large values of $(d_m/D_m)\sqrt{D_m/T}$ say, above 2.0—the curves are probably significantly in error (unconservative).

Under these circumstances, two sets of curves are shown: (1) Bijlaard's original curves, which are considered adequate (or more than adequate) for the stresses on the longitudinal axis, and (2) a second set of curves for application to the transverse axis, which have been limited to "small" values of $(d_m/D_m)\sqrt{D_m/T}$, as indicated in principle by Figs. A-17 and A-18.

NOTE:

* Bijlaard's treatment of radial loading provided stress resultants at the edges of a rectangular loading surface. However, experimental data indicated that some of these values might not be adequately conservative. Therefore, in the interests of conservatism, he then recommended that the calculated stresses for the center of the loading surface be applied at its edges, both on the longitudinal and transverse axes. However, as noted above, this procedure does not allow for possible differences in the magnitude of the stresses on the two axes; also, it does not make any distinction in terms of possible differences in orientation of the maximum stresses on the two axes. In the latter respect, if one considers the case of a nozzle attached to a flat plate or a "small" nozzle on a cylinder, it should be apparent that the σ_x stress on the longitudinal axis is the equivalent of the $\sigma_\phi (= \sigma_n)$ stress on the transverse axis, both being radial with respect to the nozzle. As such, it can be anticipated that these two stresses will be most affected by the discontinuity between the nozzle and shell (or plate) and will have relatively high bending stresses as compared to the stresses oriented 90° thereto (circumferential with respect to the nozzle). Of six experimental models currently available, this is true in every case. From Table A-6, it will be noted that the calculated stresses are qualitatively consistent with the measured stresses on the transverse axis, but not consistent with those on the longitudinal axis (neither with respect to the bending stress nor even the membrane stress). This was also true for two other models having comparable diameter ratios, viz., ORNL-3 (Ref. 32) and Franklin Inst. model "EF" (Ref. 33). In these cases, the matter seems relatively unimportant, since the calculated stresses are appreciably

conservative as applied to the longitudinal axis. Other recent data at larger diameter ratios show this same general inconsistency, but in addition have disclosed cases where the calculated stresses are inadequate, such as for model ORNL-1 (Ref. 34), which had stresses as follows under a radial load (pull) of 300 lbs.:

	Calc. Stress, KSI		Exper. Stress, KSI	
	σ_x	σ_y	σ_x	σ_y
Outside surf.	3.25	3.54	2.30	2.50
Inside surf.	1.01	-1.94	-2.00	0.60
Membrane stress	2.13	0.80	0.15	1.55
Bending stress	1.12	2.74	2.15	0.95

It will be observed that if the "labels" of the calculated stresses were reversed, the qualitative consistency would be much improved, and the two cases of "low" calculated stress would then be adequate. This was also found true for the two other models of relatively large diameter ratio. In light of this, for this March 1979 Revision, the titles on the curves for radial load (Figs. 1C-1, 2C-1, etc.) have been revised to reverse the orientation of the stresses for the longitudinal axis. Whereas this will make the curves (calculated stresses) adequate or more than adequate for all presently known cases of shell stress on the longitudinal axis, it does not alter the possibility of inadequacy for very thin shells and flexible nozzles, or the fact that stresses in the nozzle can sometimes be considerably higher than in the shell, particularly when there is little or no reinforcement in the nozzle wall.

APPENDIX B—STRESS CONCENTRATION FACTORS FOR STRESSES DUE TO EXTERNAL LOADS

B.1 Introduction and Terminology

Presently available analytical methods for stresses in nozzles, shells, etc., do not take into account the localized stresses in fillets and transitions. The following data may prove useful in performing a fatigue analysis where such effects must be considered. In presenting these data, the following terminology will be used:

- H = thickness of thicker section of stepped bar, in.
- h = thickness of thinner section of stepped bar, in.
- r = fillet radius between two sections of bar, or between nozzle and shell, in.
- d_i = inside diameter of nozzle, in.
- d_n = outside diameter of nozzle, in. (see Fig. B-4)
- d_p = outside diameter of attached pipe, in. (see Fig. B-4)
- t = thickness of pipe, in.
- t^1 = thickness of reinforced section of nozzle, in. (see Fig. B-4)
- T = thickness of vessel wall, in.
- θ = angle of taper between two sections of bar or nozzle, degrees (see Figs. B-3 and B-4)
- K_T = stress concentration factor at fillet of a stepped bar in tension (as related to the stress in the *thinner* member)
- K_B = ditto, for a stepped bar in bending (as related to the surface stress of the *thinner* member)
- K_n = stress concentration factor applied to the membrane portion of the stress due to external nozzle loadings ("tension" curve on Fig. B-2; see para. B.3.1)
- K_b = stress concentration factor applied to the bending portion of the stress due to external nozzle loadings ("bending" curve on Fig. B-2; see para. B.3.1)
- K_0 = stress concentration factor at fillet of stepped bar for case of $\theta = 0^\circ$
- K^1 = stress concentration factor for inclined shoulder

radius between the two sections of the bar, as illustrated in Fig. B-1. Heywood's curves conform to the following formulas:

Tension case

$$K_T = 1 + \left[\frac{\frac{H}{h} - 1}{4 \left(1.4 \frac{H}{h} - 1 \right)} \times \frac{h}{r} \right]^{0.65} \quad (1)$$

Bending case

$$K_B = 1 + \left[\frac{\frac{H}{h} - 1}{2 \left(5.37 \frac{H}{h} - 4.8 \right)} \times \frac{h}{r} \right]^{0.85} \quad (2)$$

For the case of a plate of "infinite" width (H very large in relation to h), these formulas would reduce to:

$$\text{Tension} \quad K_T = 1 + \left[\frac{1}{5.6 \frac{r}{h}} \right]^{0.65} \quad (3)$$

$$\text{Bending} \quad K_B = 1 + \left[\frac{1}{10.74 \frac{r}{h}} \right]^{0.85} \quad (4)$$

Upon comparing Heywood's computed curves with the corresponding curves from Peterson,²⁹ it will be found that the tension curves are quite consistent with Peterson's data, but that the bending curves are seemingly somewhat "low" for small values of r/h . The following alternate formula provides a curve which is more consistent with Peterson's data (and somewhat conservative in relation to Heywood's data):

$$K_n = 1 + \left[\frac{1}{9.4 \frac{r}{h}} \right]^{0.80} \quad (5)$$

Curves based on formulas (3) and (5) are shown on Fig. B-2, taking H as infinite and taking h as equal to $2T$ when applied to the vessel shell and equal to

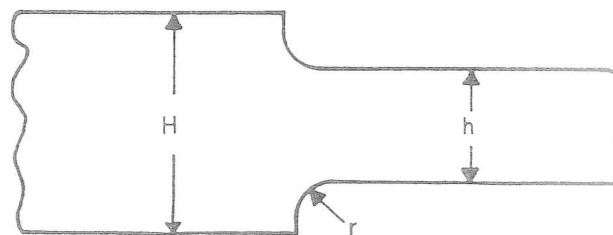


Fig. B-1—Stepped bar

B.2 Stress Concentration Factors

Peterson²⁹ and Heywood³⁰ provide a considerable amount of data covering the stress concentration factors for various design problems including the cases of two-dimensional, stepped bars in both tension and bending. In both cases, curves are provided giving stress concentration factors in terms of the H/h^* ratio of the bar and the fillet

* Terminology changed to avoid conflict with terminology in this text.

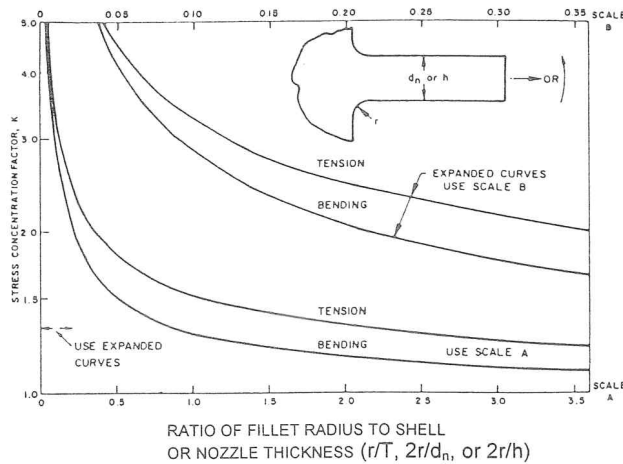


Fig. B-2 – Stress Concentration Factors for $D \gg d$

d_n when applied to the nozzle* (see Fig. B-4).

The case of an inclined shoulder is also of interest, as discussed on page 179 of Reference 30. If the stress concentration factor obtained from Fig. 57 or 60 of Reference 29 is designated K_0 , and the stress concentration factor for the inclined shoulder as K' , the following relationships apply:

- (1) If $r < (t' - t)$

$$\frac{K' - 1}{K_0 - 1} = 1 - \left[\frac{\theta}{90} \right]^n \quad (6)$$

where θ is the angle between the tapered shoulder and the square shoulder (see Fig. B-3), and $n = 1 + 2.4 \sqrt{r/(t' - t)}$

- (2) If $r > h$

$$\frac{K' - 1}{K_0 - 1} = 1 - \left[\frac{\theta - \alpha}{90} \right]^n \quad (7)$$

where

$$\alpha = \sin^{-1} \left[1 - \frac{(t' - t)}{r} \right]$$

- (3) If $H \gg h$ for use with Fig. B-2

$$\frac{K' - 1}{K_0 - 1} = 1 - \frac{\theta}{90} \quad (8)$$

The quantities on the left-hand side of the above eqs. 6, 7, and 8 are plotted in Fig. B-3 vs. the quantity of the right-hand side.

B.3 Application to External Loads on a Nozzle

B.3.1 Stresses at Fillet-Shell Junction

The calculated stresses in the shell due to an axial load or moment loading on a nozzle are assumed to apply to the juncture of the fillet and shell, point A in Fig. B-4. These stresses have been derived in terms of a membrane component (N/T) and a bending component ($6M/T^2$), in the

* Since less data are available for stress concentration factors in shafts than in bars or plates, Heywood (Reference 30, page 195) recommended that two-dimensional plate data be used. In general, it is believed that such data are slightly conservative in relation to three-dimensional data.

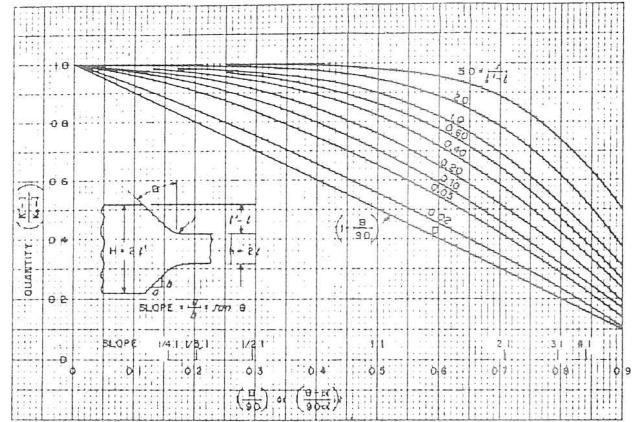


Fig. B-3—Effect of tapered shoulder

form:

$$\sigma_i = K_n \frac{Ni}{T} \pm K_b \frac{6Mi}{T^2}$$

The stress concentration factor, K_n , for the membrane component is that obtained from Fig. B-2 for the tensile case, using the appropriate ratio of fillet radius-to-plate thickness, r_A/T (see Fig. B-4). The factor, K_b , for the bending component is determined from the same figure for the bending case.

In both cases, the stress concentration factor is applied to the stresses which are *perpendicular to the change in section*. In the normal case, these are the stresses which are oriented *axially with respect to the nozzle*, i.e., σ_x in the case of a spherical vessel or the longitudinal axis of a cylindrical vessel, and σ_ϕ in the case of the transverse axis of a cylindrical vessel.

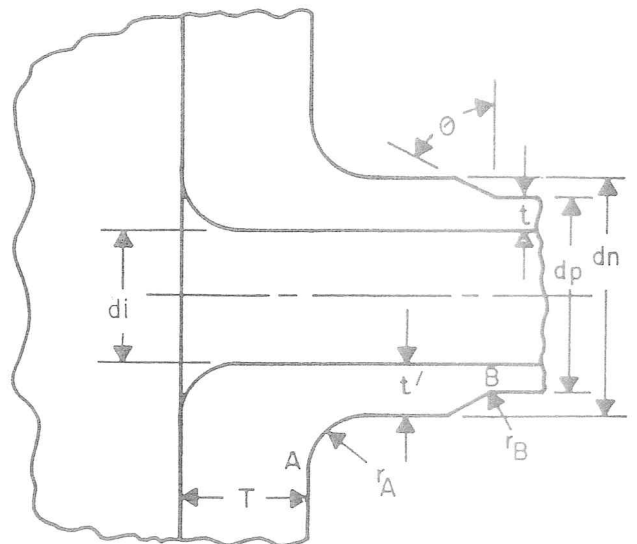


Fig. B-4—Nozzle configuration

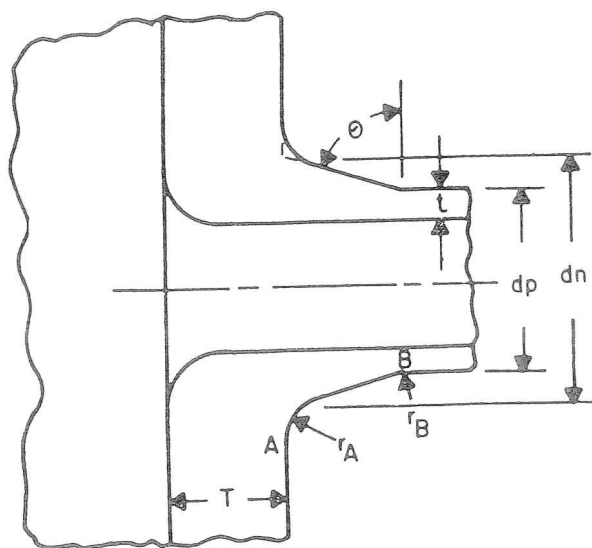


Fig. B-5—Nozzle configuration

B.3.2 Stresses at Nozzle-Pipe Junction

The stress concentration factor for an axial load, or for a bending moment, at the juncture of a nozzle and its attached pipe (Point B in Fig. B-4) may be obtained from Figs. 57 and 60 of Reference 29* and Fig. B-3 herein in terms of d_n , d_p , r_B and θ . These factors should be applied to the stress in the pipe (thinner member) calculated from the conventional P/A and Mc/I relationships.

B.3.3 Special Case of a Tapered Nozzle

The following procedure should be used for a special case of a tapered nozzle as shown in Fig. B-5:

- Calculate the stress concentration factor that would exist at Point A if the nozzle were of uniform diameter, d_n .
- Account for the taper by using the quantity $(90 - \theta)$ in place of θ in the abscissa of Fig. B-3.
- The stress at the nozzle-pipe juncture, point B, should be calculated in the same manner as in para. B.3.2.

B.4 Application to Bars and Structural Attachments

The stress concentration at the juncture of a

* When using data from Reference 29, changes in symbols as contained in this text should be noted.

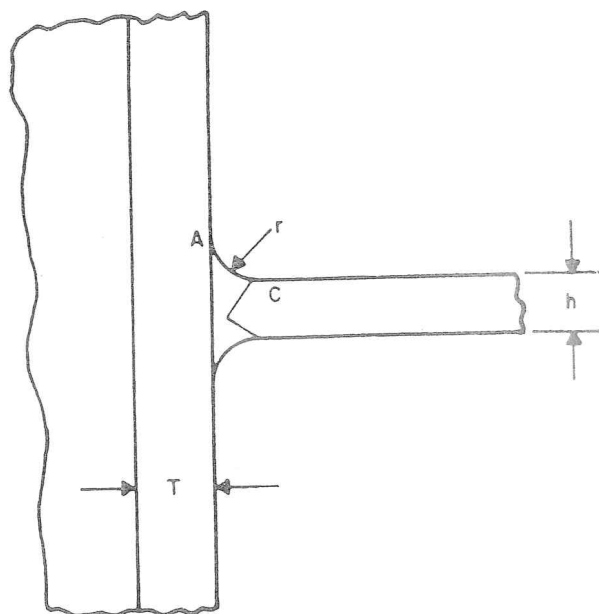


Fig. B-6—Structural attachment configuration

structural attachment to a shell, as shown in Fig. B-6, may be treated in the same way as for a nozzle, as covered in para. B.3.1. Another location on the attachment which may be critically stressed is point C, at the juncture of the fillet and the attachment. The stresses at this point may be calculated by applying the appropriate factor from Fig. B-2 to the conventionally calculated P/A and Mc/I stresses in the bar or attachment. [NOTE: This procedure can also be applied to a nozzle provided that the nozzle is relatively "rigid" (thick in relation to its diameter). However, experimental data indicate that such treatment would not be proper for relatively thin, flexible nozzles, which have high bending stresses *through the thickness* of the nozzle as differentiated from the beam action of a thick-walled nozzle.]

B.5 Acknowledgment

The foregoing material constitutes an adaptation of material originally presented in Reference 16, with added data documenting the genesis of the curves shown in Fig. B-2.

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