

RESTRICTION ORIFICE**JGS 210-120-1-73E**

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1. SCOPE

Of many kinds of flow restriction devices, restriction orifices (hereinafter referred to as “RO”) are frequently used, because they are simple and economical devices. This standard practice provides the guidelines for sizing of RO. It should be noted that this standard practice is applicable to single-phase fluids only.

2. WORK PROCEDURE

2.1 Input to the Design

The following is a summary of input data to be prepared for the design of RO:

- (1) Operating Conditions
 - Flow rate
 - Upstream & downstream pressure
 - Temperature (for vapor)
 - Line Size
- (2) Physical properties
 - For liquid service : density, vapor pressure
 - For vapor service : molecular weight, Cp/Cv, Z-factor, viscosity
- (3) Minimum allowable value of cavitation index for liquid service

2.2 Output from the Design

- (1) Single orifice
 - Orifice diameter
 - Pressure at vena contracta
 - Velocity at orifice
 - Calculated cavitation index (for liquid)
 - Critical or non-critical (for vapor)
- (2) Multi-stage orifice
 - Required stage number
 - Orifice diameter of each orifice
 - Distance between adjacent orifice plates
 - Inlet and outlet pressures of each orifice
 - Pressure at vena contracta of each orifice
 - Velocity at each orifice
 - Calculated cavitation index of each orifice

2.3 Computer Programs

The computer program (BEST-RO) is available for the design of RO.

3. DESIGN

3.1 Application of RO

RO is applied to the following services to regulate the flow rate or pressure. The typical example of RO application is illustrated in Fig. 1.

	Reference No. in Fig.-1
(1) To regulate the flow rate under a constant pressure drop - Pump minimum flow - Flushing oil injection - Cooling water supply to rotary machinery - Sampling connection - Steam turbine warming-up bypass - Purge gas injection	RO-1 RO-2 & 3 RO-4 RO-5 RO-6 RO-9
(2) To regulate the pressure under a constant flow rate - Vent to atmosphere - Steam letdown - Seal gases such as N₂ and fuel gas	RO-7 RO-8 RO-17
(3) Others - Uniform flow distribution from header - Support of control valves including noise control - For static mixer - Instrument tapping for volatile service	RO-10, 11, 12 & 13 RO-14 RO-15 & 16 RO-18

3.2 Principle of RO Calculation

3.2.1 Gas Service

3.2.1.1 Critical Pressure Ratio

The critical pressure ratio, r_c can be obtained from the following equation.

$$r_c = \frac{P_c}{P_1} = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}} \quad (1)$$

$$r_4 = \frac{P_4}{P_1} \quad (2)$$

If $r_4 \leq r_c$, then flow is the critical (sonic region).

And $r_4 > r_c$, then flow is the non-critical (subsonic region).

3.2.1.2 Orifice Diameter

Equation for orifice diameter should be selected using equation (2), depending on whether the flow is critical or non-critical.

(A) Critical flow (sonic region)

When ratio of downstream pressure to upstream pressure, r_4 , is smaller than or equal to critical pressure ratio, r_c , the following equation of orifice diameter for critical flow should be used.

$$W = C_v C_c A \sqrt{k P_1 C_1 \rho \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}} = 0.97 * C_c A \sqrt{k P_1 C_1 \rho \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}} \quad (3)$$

Therefore, orifice diameter d_o shall be as follows:

$$d_o = \sqrt{\frac{4W}{\pi * 0.97 C_c}} / \sqrt{k P_1 C_1 \rho \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}} \quad (4)$$

Where,

$$C_c = \frac{1}{2f} (r_c)^{-\frac{1}{k}} \left[1 + \frac{r_c - r_4}{k * r_c} - \sqrt{\left\{ 1 + \frac{r_c - r_4}{k * r_c} \right\}^2 - 2f \left(\frac{k+1}{k} \right) (1 - r_4)} \right] \quad (5)$$

Where,

$$f = \frac{1}{C_{Ci}} - \frac{1}{2C_{Ci}^2} \quad (6)$$

$$C_{Ci} = \frac{C_D}{\sqrt{\{C_v^2 - \beta^4 (C_v^2 - C_D^2)\}}} = \frac{0.61}{\sqrt{\{0.97^2 - \beta^4 (0.97^2 - 0.61^2)\}}} \quad (7)$$

$$\beta = \frac{d_o}{D} \quad (8)$$

Calculation procedure

- ① Calculate critical pressure ratio, r_c by equation (1).
- ② Calculate ratio of downstream pressure to upstream pressure, r_4 by equation (2).
- ③ Compare r_4 to r_c and select the equation of orifice diameter calculation by the flow regime.
 $r_4 \leq r_c$: (critical flow) → Calculate orifice diameter(d_o) by equation (4)
 $r_4 > r_c$: (non-critical flow) → Go to step (B)
- ④ Assume d_o and calculate β by equation (8).
- ⑤ Incompressible coefficient of contraction C_{Ci} shall be calculated by equation (7).
- ⑥ Force defect coefficient (f) shall be calculated by equation (6).
- ⑦ Coefficient of contraction (C_c) shall be calculated by equation (5).
- ⑧ Orifice diameter (d_o) shall be calculated by equation (4)
- ⑨ Compare the assumed d_o with the calculated d_o in step ⑦.
- ⑩ Calculated d_o shall be replaced with previous d_o and repeat step ③ to step ⑧ until the d_o agrees with calculated d_o .

(B) Non-critical flow (subsonic region)

The following equation of orifice diameter can be used for non-critical flow region.

$$\begin{aligned} W &= C_v C_c A \sqrt{\frac{2kP_1 C_1 \rho}{k-1} (r_4)^{\frac{2}{k}} \left\{ 1 - (r_4)^{\frac{k-1}{k}} \right\}} \\ &= 0.97 * C_c A \sqrt{\frac{2kP_1 C_1 \rho}{k-1} (r_4)^{\frac{2}{k}} \left\{ 1 - (r_4)^{\frac{k-1}{k}} \right\}} \end{aligned} \quad (9)$$

Therefore, orifice diameter do shall be as follows:

$$d_o = \sqrt{\frac{4W}{\pi * 0.97 * C_c}} / \sqrt{\frac{2kP_1 C_1 \rho}{k-1} (r_4)^{\frac{2}{k}} \left\{ 1 - (r_4)^{\frac{k-1}{k}} \right\}} \quad (10)$$

Where,

$$C_c = \frac{1}{2f} (r_4)^{-\frac{1}{k}} \left[1 - \sqrt{1 - \frac{2f(k-1)}{k} (1 - r_4)} / \left\{ 1 - (r_4)^{\frac{k-1}{k}} \right\} \right] \quad (11)$$

The force defect coefficient, f , can be calculated using the same equations as above (6), (7) and (8).

(C) Calculation procedure

- ① Calculate critical pressure ratio, r_c by equation (1).
- ② Compare r_4 to r_c and select the equation of orifice diameter by the flow regime.
 $r_4 \leq r_c$: (critical flow) $\rightarrow$ Calculate orifice diameter(d_o) by equation (3)
 $r_4 > r_c$: (non-critical flow) $\rightarrow$ Go to step (A)
- ③ Assume d_o and calculate β by equation (8).
- ④ Incompressible coefficient of contraction C_{Ci} shall be calculated by equation (7).
- ⑤ Force defect coefficient (f) shall be calculated by equation (6).
- ⑥ Coefficient of contraction (C_C) shall be calculated by equation (11).
- ⑦ Orifice diameter (d_o) shall be calculated by equation (10).
- ⑧ Compare the assumed d_o with the calculated d_o in step ⑦.
- ⑨ Calculated d_o shall be replaced with previous d_o and repeat step ③ to step ⑧ until the d_o agrees with calculated d_o .

3.2.2 Liquid Service**3.2.2.1 Orifice Diameter**

The following equation of orifice diameter can be used for liquid services.

$$W = C_v C_{Ci} A \sqrt{2C_1 \rho (P_1 - P_4)} = 0.97 * C_{Ci} A \sqrt{2C_1 \rho (P_1 - P_4)} \quad (12)$$

Therefore, orifice diameter d_o shall be as follows:

$$d_o = \sqrt{\frac{4W}{\pi * 0.97 C_{Ci}}} / \sqrt{2C_1 \rho (P_1 - P_4)} \quad (13)$$

C_{Ci} shall be calculated by the equation (7).

3.2.2.2 Cavitation Index

In order to avoid cavitation problem, minimum allowable value of cavitation index, K_d , should be selected based on the following:

- (1) Cavitation index $K_d=0.37$ shall be used for usual case. At this critical cavitation condition, the noise is steady but still light. No erosion will occur. (See Note)
- (2) In some occasion such as the following cases, use $K_d=0.93$ as an incipient cavitation condition in order to avoid severe economical risk.
 - Material is high grade such as stainless steel or higher and pipe size is larger than 12".

For the equation of cavitation index, refer to Section 4.2.

Note: Once the orifice chokes and supercavitation occurs, no damage by erosion will exist near the orifice. This is because the damage is caused by the collapse of the cavities and the collapse occurs far downstream during supercavitation.¹

3.2.2.3 Interval of Orifices

In case of multi-stage orifice, the minimum distance, as shown below, should be provided between orifices, to avoid the reduction in RO performance.

- For concentric orifice : L 5.4*D*(1- β)
- For eccentric orifice : L D

¹ J. P. Tullis and R. Govindarajan, "Cavitation and Size Scale Effects for Orifices", Journal of the Hydraulic Division, Proceedings of the American Society of Civil Engineers (1973)

3.2.2.4 Deflection Ratio of Eccentric Orifice

The deflection ratio of the eccentric orifice “e” is 0.75.

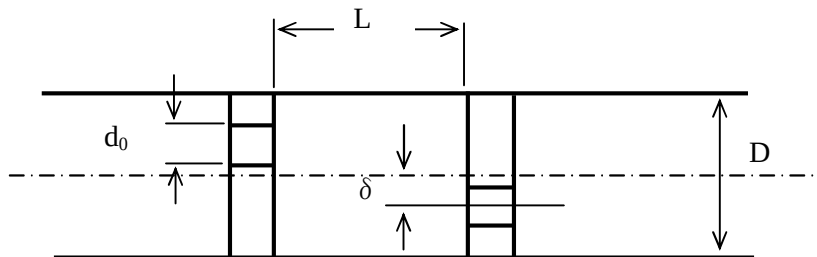
$$e = 2\delta / (D - d_o) = 0.75 \quad (14)$$

Where,

δ : Pipe center to orifice center length (m)

d_o : Orifice hole diameter (m)

D: Pipe inside diameter (m)

**3.2.2.5 Calculation Procedure****A. Single RO system**

- ① Decide minimum allowable value of cavitation index to meet a given situation.
Kd = 0.37: for usual case
Kd = 0.93: for conservative case
- ② Assume d_o and calculate β by equation (8).
- ③ Incompressible coefficient of contraction C_{Ci} shall be calculated by equation (7).
- ④ Orifice diameter (d_o) shall be calculated by equation (13).
- ⑤ Compare the assumed d_o with the calculated d_o in step ④.
- ⑥ Calculated d_o shall be replaced with previous d_o and repeat step ② to step ⑤ until the d_o agrees with calculated d_o .

Calculate cavitation index, Kd by equation (15), and compare with minimum allowable value.

If cavitation index 0.37 (or 0.93), then the orifice diameter is acceptable.

If cavitation index < 0.37 (or 0.93), then single orifice is unable to accommodate the required pressure drop. In that case, multi-stage orifice system should be applied.

B. Multi-stage RO system

- ① Decide minimum allowable value of cavitation index to meet a given situation.
Kd = 0.37: for usual case
Kd = 0.93: for conservative case
- First stage orifice: located at outlet
- ② Assume upstream pressure of the first stage orifice.
- ③ Assume d_o and calculate β by equation (8) for the first stage orifice.
- ④ Incompressible coefficient of contraction C_{Ci} shall be calculated by equation (7).
- ⑤ Orifice diameter (d_o) shall be calculated by equation (13) using assumed upstream pressure.
- ⑥ Compare the assumed d_o with the calculated d_o in step ⑤.
- ⑦ Calculated d_o shall be replaced with previous d_o and repeat step ③ to step ⑥ until the d_o agrees with calculated d_o .

- ⑧ Calculate cavitation index, K_d , using Equation (15), and compare with minimum allowable value.
 If cavitation index > 0.37 (or 0.93), increase the upstream pressure and repeat the steps from ③ to ⑧.
 If cavitation index < 0.37 (or 0.93), decrease the upstream pressure and repeat the steps from ③ to ⑧.
 If cavitation index equals to or slightly bigger than 0.37 (or 0.93), the design of first stage RO is completed and go to step ⑨.

n-th stage orifice

- ⑨ Set the upstream pressure of (n-1)-th stage orifice for the downstream pressure of n-th stage orifice.
 ⑩ Assume the upstream pressure of n-th stage orifice.
 ⑪ Repeat the steps from ③ to ⑧, until the cavitation index is equivalent to the minimum allowable value.

3.3 Special Consideration of RO Design

- (1) Minimum hole diameter of RO
 To prevent plugging problem with RO caused by debris, the hole diameter should be greater than the following values:

For the clean liquid service : 2mm
 For the clean Gas service : 1mm

When the diameter smaller than the above values is required, the strainer or filter to remove debris should be provided upstream of RO.

- (2) Necessity of minimum straight run length
 Basically, the objective of RO is rough control of flow rate and should not be used for strict control of flow rate. Therefore, it should not be necessary to take straight run of piping both upstream and downstream of RO to keep performance. However, for the erosional services such as slurry or flush services, countermeasures for erosion shall be considered.
- (3) Calculated hole diameter of RO should be rounded to the conservative size for easy manufacturing.

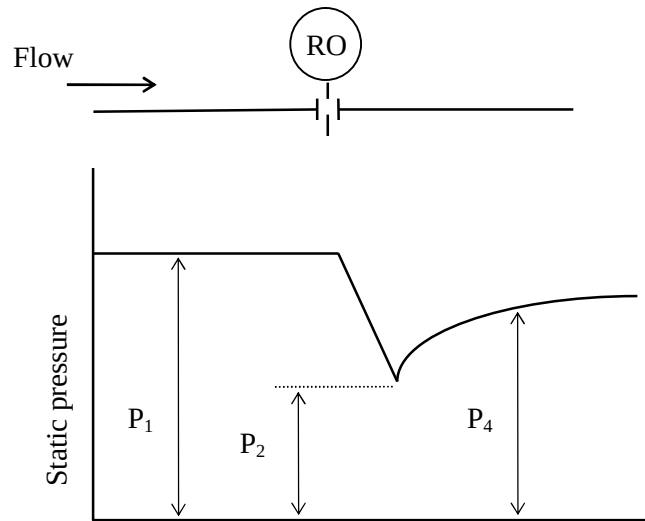
4. DEFINITIONS

4.1 Multi-Stage RO

In liquid services, a single RO in a hydraulic system is unable to accommodate the required pressure drop because of restrictions on noise, vibration, and erosion. Such phenomena are called orifice cavitation. To avoid orifice cavitation, it is necessary to use multi-stage RO. Consequently, pressure drop in each orifice is decreased, and orifice cavitation is prevented.

4.2 Cavitation Index

The figure below illustrates liquid flow through a single orifice in a pipe.



Where,

P_1 is an upstream pressure.

P_2 is a vena contracta pressure.

P_4 is downstream pressure of the orifice.

If P_2 falls below P_v , which is a vapor pressure of fluid, bubbles form the first stage of cavitation. As the fluid moves downstream, the bubbles collapse under the higher pressure, P_4 , the final stage of cavitation.

To avoid cavitation, the orifice size selected for the pressure drop requirement must be checked against cavitation index. The cavitation index K_d is defined as

$$K_d = \frac{H_0 - H_v}{\frac{V_0^2}{2g_c(0.61)^2}} \quad (15)$$

Where,

$$V_0 = \frac{W}{\rho} \left/ \left(\frac{\pi d_o^2}{4} \right) \right. \quad (16)$$

$$H_0 = \frac{C^2 P_2}{\rho} \quad (17)$$

$$H_v = \frac{C^2 P_v}{\rho} \quad (18)$$

$$P_2 = P_1 - \frac{P_1 - P_4}{1 - \beta^2} \quad (19)$$

4.3 Vena Contracta

The downstream pressure of the orifice reaches the minimum (P_2) at vena contracta. Pressure profiles are shown in Fig. 2.

4.4 Critical Pressure Ratio

If gas is passing through a RO at a velocity equals to the velocity of sound, the flow through this RO is critical. The ratio of the absolute downstream pressure to the absolute upstream pressure (P_4/P_1) for which the velocity of the fluid through the RO is equal to the sonic velocity is called the “critical pressure ratio (r_c)”.

5. NOMENCLATURE

		Metric	SI
d_o	: Orifice diameter	m	m
e	: Deflection ratio of the eccentric orifice		
f	: Force defect coefficient	-	-
g_c	: Gravity constant	9.807	1.000
k	: Ratio of specific heats (C_p/C_v)	-	-
r_4	: Ratio of downstream pressure to upstream pressure	-	-
r_c	: Critical pressure ratio	-	-
z	: Z-factor	-	-
A	: Orifice Area	m ²	m ²
C	: Conversion constant for calculation of H_0 and H_v	100	10.098
C_1	: Conversion constant for calculation of gas services	98,066.5	1,000
C_C	: Coefficient of contraction	-	-
C_{Ci}	: Incompressible coefficient of contraction	-	-
C_D	: Coefficient of discharge	0.61	0.61
C_v	: Coefficient of velocity	0.97	0.97
D	: Inside diameter of pipe	m	m
H_0	: Pressure head at orifice	m	m
H_v	: Vapor pressure head of liquid	m	m
K	: Flow coefficient	-	-
K_d	: Cavitation index	-	-
L	: Interval of orifices	m	m
M	: Molecular weight	-	-
P_1	: Upstream pressure	kg/cm ² A	kPa
P_2	: Pressure at Vena Contracta	kg/cm ² A	kPa
P_4	: Downstream pressure	kg/cm ² A	kPa
P_C	: Critical pressure	g/cm ² A	kPa
P_v	: Vapor pressure	kg/cm ² A	kPa
R	: Gas Constant	847.84	8,314.47
T	: Gas temperature	°K	°K
V_o	: Volumetric flow rate at orifice	m ³ /s	m ³ /s
W	: Weight flow rate at orifice	kg/s	kg/s
Y	: Expansion factor	-	-
β	: Ratio of diameter (d_o/D)	-	-
δ	: Pipe center to orifice center length	m	m
ρ	: Vapor density at upstream	kg/m ³	kg/m ³

6. REFERENCES

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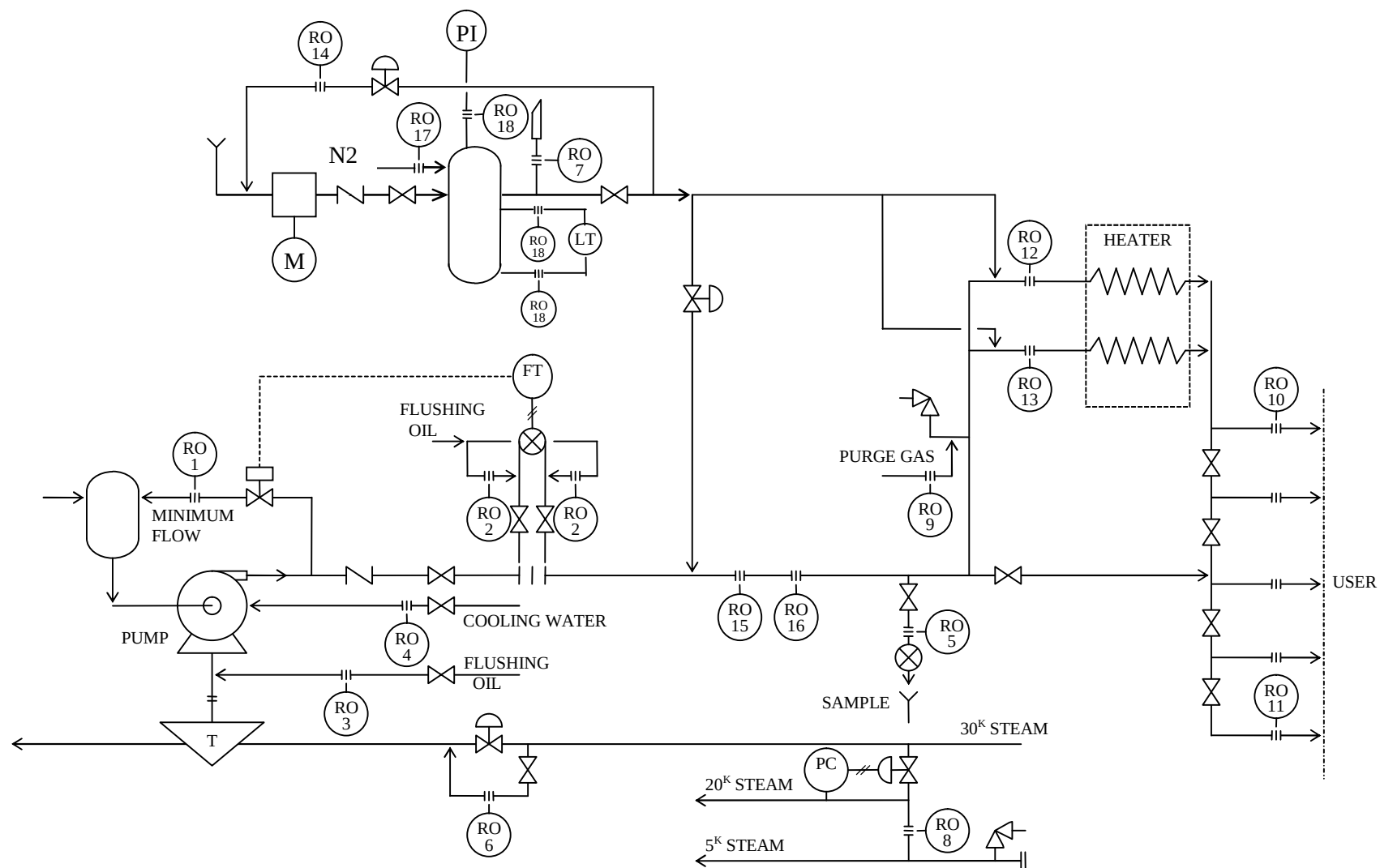
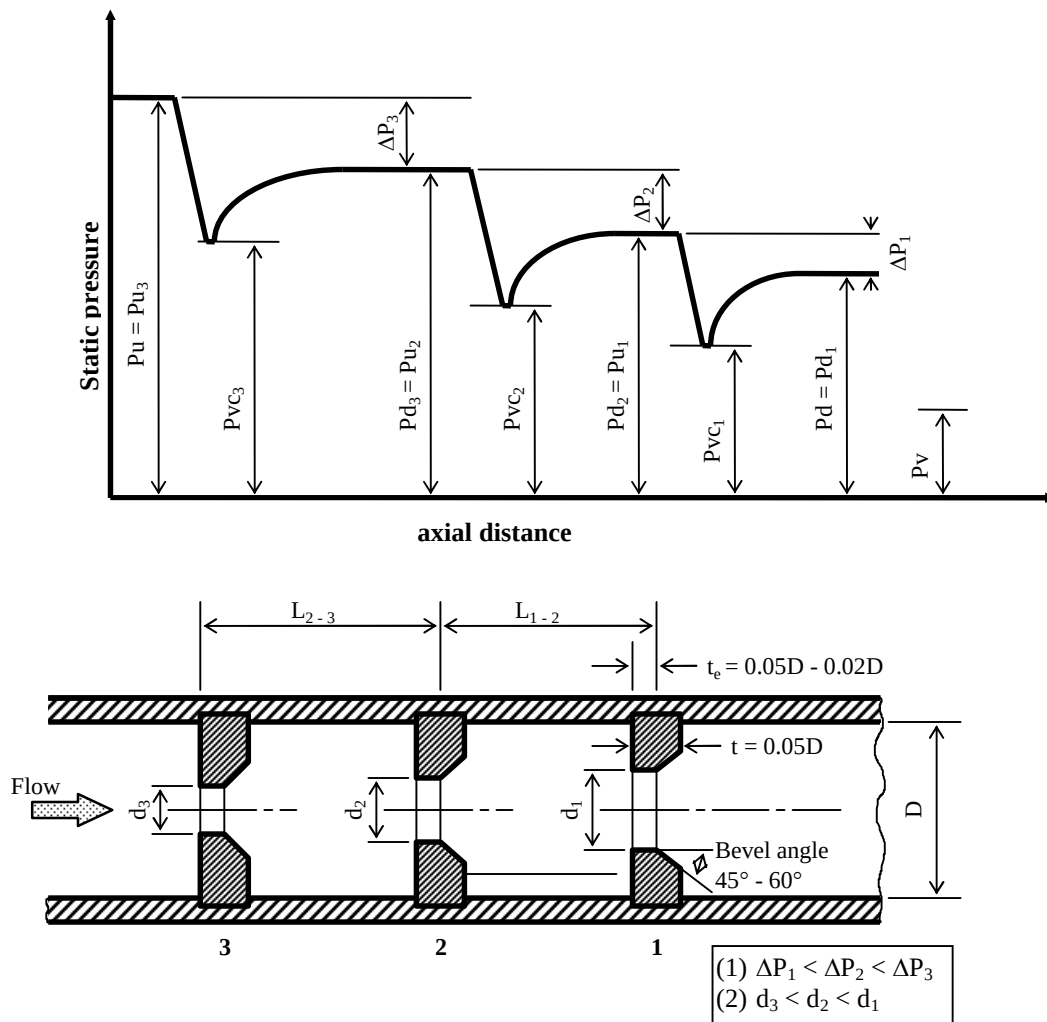
Fig. 1 Example of RO Application

Fig. 2 Multi-stage Orifice Hydraulic System

APPENDIX 1: EXAMPLE OF RO CALCULATION**A1.1 Example-1 (Gas Service --- Sonic Region)**Design basis

- Pipe I.D.	(mm)	102.2
- Flow rate	(kg/Hr)	500.0
- Upstream Pressure (P1)	(kg/cm2-A)	25
- Down Stream Pressure (P4)	(kg/cm2-A)	10
- Temperature	(°C)	61.6
- M.W.	-	30.0
- Cp/Cv	-	1.28
- Z-factor	-	0.91

(1) Critical pressure ratio

$$r_c = \left(\frac{2.0}{1.28 + 1.0} \right)^{\frac{1.28}{0.28}} = 0.549$$

$$P_4 / P_1 = r_4 = (10) / (25) = 0.4$$

$$r_4 < r_c$$

Therefore, the equation for critical flow should be applied.

(2) Orifice diameter

(a) 1ST trial

Assume $d_0 = 10\text{mm}$

Therefore, assumed β is $10/102.2 = 0.0978$

$$\text{From equation (7): } C_{Ci} = \frac{0.61}{\sqrt{[0.97^2 - 0.0978^4(0.97^2 - 0.61^2)]}} = 0.629$$

$$\text{From equation (6): } f = \frac{1}{0.629} - \frac{1}{2 * 0.629^2} = 0.326$$

From equation (5):

$$C_c = \frac{1}{2 * 0.326} (0.549)^{-\frac{1}{1.28}} \left[1 + \frac{0.549 - 0.4}{1.28 * 0.549} - \sqrt{\left\{ 1 + \frac{0.549 - 0.4}{1.28 * 0.549} \right\}^2 - 2 * 0.326 \left(\frac{1.28 + 1}{1.28} \right) (1 - 0.4)} \right]$$

$$= 0.816$$

Therefore, 1ST result by equation (4)

$$d_o = \sqrt{\frac{4 * 500 / 3600}{\pi * 0.97 * 0.816}} / \sqrt{1.28 * 25 * 98066.5 * 29.04 \left(\frac{2}{1.28 + 1} \right)^{\frac{1.28 + 1}{1.28 - 1}}} = 0.00632 \text{ m} = 6.32\text{mm}$$

$$d_0 / \text{Assumed } d_0 = 6.32 / 10 = 0.632$$

(b) 2ND trialAssumed $d_0 = 6.32\text{mm}$ Therefore, assumed β is $6.32/102.2=0.0618$

$$\text{From equation (7): } C_{Ci} = \frac{0.61}{\sqrt{[0.97^2 - 0.0618^4(0.97^2 - 0.61^2)]}} = 0.629$$

$$\text{From equation (6): } f = \frac{1}{0.629} - \frac{1}{2 * 0.629^2} = 0.326$$

From equation (5)

$$C_C = \frac{1}{2 * 0.326} (0.549)^{-\frac{1}{1.28}} \left[1 + \frac{0.549 - 0.4}{1.28 * 0.549} - \sqrt{\left\{ 1 + \frac{0.549 - 0.4}{1.28 * 0.549} \right\}^2 - 2 * 0.326 \left(\frac{1.28 + 1}{1.28} \right) (1 - 0.4)} \right]$$

$$= 0.816$$

Therefore, 2ND result by equation (4)

$$d_o = \sqrt{\frac{4 * 500 / 3600}{\pi * 0.97 * 0.816}} / \sqrt{1.28 * 25 * 98066.5 * 29.04 \left(\frac{2}{1.28 + 1} \right)^{\frac{1.28 + 1}{1.28 - 1}}} = 0.00632 \text{ m} = 6.32\text{mm}$$

$$d_0 / \text{Assumed } d_0 = 6.32 / 6.32 = 1.00$$

(c) Final results

The orifice diameter is 6.3 mm.

A1.2 Example-2 (Gas Service --- Sub-Sonic Region)Design basis

- Pipe I.D.	(mm)	102.2
- Flow rate	(kg/Hr)	500.0
- Upstream Pressure (P1)	(kg/cm2-A)	21.70
- Down Stream Pressure (P4)	(kg/cm2-A)	14.50
- Temp.	(°C)	61.6
- M.W	-	30.0
- Cp/Cv	-	1.28
- Z-factor	-	0.91

(1) Critical pressure ratio

$$r_c = \left(\frac{2.0}{1.28 + 1.0} \right)^{\frac{1.28}{0.28}} = 0.549$$

$$P_4 / P_1 = r_4 = 14.50 / 21.70 = 0.668$$

$$r_4 > r_c$$

Therefore, the equation for non-critical flow should be applied.

(2) Orifice diameter

(a) 1ST trial

Assume $d_0 = 10\text{mm}$

Therefore, assumed β is $10/102.2 = 0.0978$

$$\text{From equation (7): } C_{Ci} = \frac{0.61}{\sqrt{[0.97^2 - 0.0978^4(0.97^2 - 0.61^2)]}} = 0.629$$

$$\text{From equation (6): } f = \frac{1}{0.629} - \frac{1}{2 * 0.629^2} = 0.326$$

From equation (11):

$$C_c = \frac{1}{2 * 0.326} (0.668)^{-\frac{1}{1.28}} \left[1 - \sqrt{1 - \frac{2 * 0.326 * (1.28 - 1)}{1.28} (1 - 0.668)} / \left\{ 1 - (0.668)^{\frac{1.28-1}{1.28}} \right\} \right]$$

$$= 0.708$$

Therefore, 1ST result by equation (9)

$$d_o = \sqrt{\frac{4 * 500 / 3600}{\pi * 0.97 * 0.708}} / \sqrt{\frac{2 * 1.28 * 21.7 * 98066.5 * 25.21}{1.28 - 1} (0.668)^{\frac{2}{1.28}} \left\{ 1 - (0.668)^{\frac{1.28-1}{1.28}} \right\}}$$

$$= 0.00740\text{m} = 7.40\text{mm}$$

$$d_0 / \text{Assumed } d_0 = 7.40 / 10 = 0.740$$

(b) 2ND trial

Assume $d_0=7.40\text{mm}$

Therefore, assumed β is $7.40/102.2=0.0724$

$$\text{From equation (6): } C_{Ci} = \frac{0.61}{\sqrt{[0.97^2 - 0.0724^4(0.97^2 - 0.61^2)]}} = 0.629$$

$$\text{From equation (5): } f = \frac{1}{0.629} - \frac{1}{2 * 0.629^2} = 0.326$$

From equation (10):

$$C_c = \frac{1}{2 * 0.326} (0.668)^{-\frac{1}{1.28}} \left[1 - \sqrt{1 - \frac{2 * 0.326 * (1.28 - 1)}{1.28} (1 - 0.668)} / \left\{ 1 - (0.668)^{\frac{1.28-1}{1.28}} \right\} \right]$$

$$= 0.708$$

Therefore, 1ST result by equation (9)

$$d_o = \sqrt{\frac{4 * 500 / 3600}{\pi * 0.97 * 0.708}} / \sqrt{\frac{2 * 1.28 * 21.7 * 98066.5 * 25.21}{1.28 - 1} (0.668)^{\frac{2}{1.28}} \left\{ 1 - (0.668)^{\frac{1.28-1}{1.28}} \right\}}$$

$$= 0.00740\text{m} = 7.40\text{mm}$$

$$d_0/\text{Assumed } d_0 = 7.40/7.40 = 1.00$$

(c) Final results

The orifice diameter is 7.4 mm.

A1.3 Example-3 (Liquid Service----Single Stage)Design basis

- Pipe I.D.	(mm)	290.0
- Flow rate	(kg/Hr)	688,500
- Upstream Pressure	(kg/cm ² A)	110.0
- Down stream Pressure	(kg/cm ² A)	75.0
- Density	(kg/m ³)	928.0
- Vapor pressure	(kg/cm ² A)	3.47
- Cavitation index	-	0.37

(2) Orifice diameter

$$C = 100 \text{ (Metric)}$$

$$W = 688500/3600 = 191.25 \text{ kg/sec}$$

Trial No.	1 ST	2 ND	3 RD
Assumed d _o	0.03	0.07072	0.07068
β	0.1034	0.2439	0.2437
C _{ci}	0.6289	0.6295	0.6295
d _o	0.07072	0.07068	0.07068
d _o /Assumed d _o	2.357	0.999	1.000

Therefore, the orifice diameter is 70.68mm.

(3) Check cavitation index

$$\beta = 0.2337$$

$$H_o = \frac{100^2}{928} \times \left(110.0 - \frac{110.0 - 75.00}{1 - 0.239^2} \right) = 784.36$$

$$H_v = \frac{100^2 \times 3.47}{928} = 37.39$$

$$V_o = \frac{191.25}{928} \div \left(\frac{\pi \times 0.07068^2}{4} \right) = 52.52$$

$$K_d = \frac{784.36 - 37.39}{\frac{52.52^2}{2 \times 9.806 \times 0.61^2}} = 1.976$$

Since $K_d > 0.37$, single stage RO is sufficient.

A1.4 Example-4 (Liquid Service----Multi-Stage)Design basis

- Pipe I.D.	(mm)	77.9
- Flow rate	(kg/Hr)	22440.0
- Upstream Pressure	(kg/cm ² A)	10.0
- Down stream Pressure	(kg/cm ² A)	2.0
- Density	(kg/m ³)	988.0
- Vapor pressure	(kg/cm ² A)	0.12
- Cavitation index	-	0.37

A1.4.1 Check Single Stage

(1) Orifice diameter

C = 100 (Metric)

W = 22440 / 3600 = 6.23 kg / sec

Trial No.	1 ST	2 ND
Assumed d _o	0.01	0.01818
β	0.1284	0.2333
C _{ci}	0.6289	0.6294
d _o	0.01818	0.01817
d _o /Assumed d _o	1.818	1.000

Therefore, the orifice diameter is 18.17mm.

(2) Check cavitation index

β = 0.2334

$$H_o = \frac{100^2}{988} \times \left(10.0 - \frac{10.0 - 2.0}{1 - 0.2333^2} \right) = 15.58$$

$$H_v = \frac{100^2 \times 0.12}{988} = 1.215$$

$$V_o = \frac{6.23}{988} \div \left(\frac{\pi \times 0.01817^2}{4} \right) = 22.33$$

$$K_d = \frac{15.58 - 1.215}{\frac{22.33^2}{2 \times 9.806 \times 0.61^2}} = 0.1772$$

Since K_d < 0.37, single stage RO is not sufficient.

A1.4.2 Multi - Stage RO Calculation**A1.4.2.1 Second Stage**

Down stream pressure (P4) = 2.0 kg / cm²A

(1) Assume upstream pressure (P1) as 5.0 kg / cm²A

Orifice diameter

Trial No.	1 ST	2 ND	3 RD
Assumed d _o	0.01	0.02323	0.02320
β	0.1284	0.2982	0.2978
C _{Ci}	0.6289	0.6304	0.6304
d _o	0.02323	0.02320	0.02320
d _o /Assumed d _o	2.323	0.999	1.000

Therefore, the orifice diameter is 23.2mm.

Check cavitation index

V_o = 14.92
P₂ = 1.708
H_o = 17.29
H_v = 1.215
K_d = 0.527

Since K_d > 0.37, assumed upstream pressure should be increased.

(2) Assume upstream pressure (P1) as 6.15 kg / cm²A

Orifice diameter

Trial No.	1 ST	2 ND	3 RD
Assumed d _o	0.01	0.02142	0.02140
β	0.1284	0.2750	0.2747
C _{Ci}	0.6289	0.6300	0.6300
d _o	0.02142	0.02140	0.02140
d _o /Assumed d _o	2.142	0.999	1.000

Therefore, the orifice diameter is 21.40mm.

Check the cavitation index

V_o = 17.54
P₂ = 1.66
H_o = 16.81
H_v = 1.215
K_d = 0.370

Since K_d is slightly bigger than 0.37, the solution of the second stage RO is obtained.

A1.4.2.2 First Stage

Upstream pressure (P1) = 10.0 kg / cm²A

Down stream pressure (P4) = 6.15 kg / cm²A

Orifice diameter

Trial No.	1 ST	2 ND	3 RD
Assumed d _o	0.01	0.02182	0.02180
β	0.1284	0.2802	0.2799
C _{Ci}	0.6289	0.6300	0.6300
d _o	0.02182	0.02180	0.02180
d _o /Assumed d _o	2.182	0.999	1.000

Therefore, the orifice diameter is 21.80mm.

Check the cavitation index

V_o = 16.90

P₂ = 5.82

H_o = 58.93

H_v = 1.215

K_d = 1.476

Since K_d > 0.37, two stages are sufficient.

A1.4.2.3 Summary of Calculation Results

No of stage	1 ST	2 ND
Pressure (kg / cm ² A)		
Upstream	10.0	6.15
Downstream	6.15	2.0
Orifice diameter (mm)	21.8	21.4
Calculated cavitation index	1.48	0.93

APPENDIX 2: STUDY ON CAVITATION INDEX

2005/10/05

Refinery Process K.Nagano

A2.1 Purpose

Regarding the cavitation index for restriction orifices, a new design criterion is established based on available experimental data included in the following literatures.

- [1] J. P. Tullis and R. Govindarajan, "Cavitation and Size Scale Effects for Orifices", Journal of the Hydraulic Division, Proceedings of the American Society of Civil Engineers (1973)
- [2] S. Numachi et. al., "Effect of Cavitation on Orifices in Pipes", The Memoirs of the Institutes of High Speed Mechanics, Vol.14 (1959)
- [3] I. Yamada, "Design of Pressure Reduction Facility", Haikan-Gijutu, '81, No.2

In addition to the above, although actual experimental data is not included, the following literatures are referred:

- [4] S. H. Lin, "Preventing Cavitation with Multistage Orifices", Hydrocarbon Processing, July (1985)
- [5] P. G. Goksem and I. D. Patel, "Program gives design data for multiple-orifice system to avoid cavitation", O.G.J., April 15 (1985)

As we know, although there were many studies for the cavitation index, we have still no concrete design criteria on this matter. This was because we could not start from actual experimental data those were indicated in different expressions. In this report, we will start how to convert one experimental data expression to the others so that we can compare all of experimental data in one expression.

A2.2 Conclusion

- (1) Cavitation index $K_d=0.37$ shall be used for usual case. At this critical cavitation condition, the noise is steady but still light. No erosion will occur.
- (2) In some occasion such as the following cases, use $K_d=0.93$ as an incipient cavitation condition in order to avoid big economical risk.
 - Material is high grade such as stainless steel or higher and pipe size is larger than 12"

A2.3 Study**A2.3.1 Conversion of Various Cavitation Indexes****A2.3.1.1 Basic Expressions**

Several expressions of cavitation index have been proposed from many authors as follows:

Symbol	Expression	Meaning	Reference
K_d	$K_d = \frac{H_c - H_v}{\frac{v_0^2}{2gC_c^2}}$	Pressure difference between vena contracta (lowest pressure) and vapor pressure is compared with velocity head of vena contracta velocity.	[4] JGC applying.
K_d'	$K_d' = \frac{H_c - H_v}{\frac{v_0^2}{2g}}$	Pressure difference between vena contracta (lowest pressure) and vapor pressure is compared with velocity head of average orifice velocity.	[2]
σ	$\sigma = \frac{H_d - H_v}{H_u - H_d}$	Pressure difference between orifice downstream pressure and vapor	[1], [5]

Symbol	Expression	Meaning	Reference
		pressure is compared with pressure drop of orifice.	
K	$K = \frac{H_d - H_v}{H_T - H_d}$	Pressure difference between orifice downstream pressure and vapor pressure is compared with pressure drop of orifice.	[3]

Where,

- H_c = Pressure head at vena contracta [m]
 H_v = Vapor pressure head [m]
 H_u = Pressure head at upstream of orifice [m]
 H_d = Pressure head at downstream of orifice [m]
 H_T = Total pressure at upstream of orifice [m]
 v_0 = Velocity through orifice hole [m/s]
 C_c = Contraction coefficient [-]
 g = Gravitational constant [m/s^2]

A2.3.1.2 Orifice Formulas

Pressure drop of orifice is calculated as follows:

$$\Delta H = \zeta \cdot \frac{v^2}{2g} \quad (A1)$$

Where,

- ΔH = Pressure drop across orifice $= H_u - H_d$ [m]
 ζ = Loss factor [-]
 v = Fluid velocity in pipe [m/s]

If we introduce orifice diameter ratio, β , from the continuity,

$$v \cdot A = v_0 \cdot A_0 = v_c \cdot A_c \quad (A2)$$

Where,

- A = Pipe area [m^2]
 A_0 = Orifice area [m^2]
 A_c = Area of vena contracta $= C_c A_0$ [m^2]
 D = Pipe diameter [m]
 d_0 = Orifice diameter [m]
 $\beta = d_0/D$ = Orifice diameter ratio [-]

Finally, loss factor can be calculated as follows [3]:

$$\zeta = \left(\frac{1}{\beta^2} - 1 \right) \left(\frac{2.75}{\beta^2} - 1.56 \right) \quad (A3)$$

A2.3.1.3 Conversion of Cavitation Index

Based on these definitions, we can convert cavitation indexes to K_d as follows:

$$K_d = C_c^2 \cdot K_d' \quad (A4)$$

$$K_d = \zeta \beta^4 C_c^2 \cdot \left[\sigma - \frac{\frac{1}{\beta^4 C_c^2} - 1 - \zeta}{\zeta} \right] \quad (A5)$$

$$K_d = \beta^4 C_c^2 \cdot \left[(1 + \zeta)(1 + K) - \frac{1}{\beta^4 C_c^2} \right] \quad (A6)$$

Conversion to σ will be as follows:

$$\sigma = \frac{\frac{1}{\beta^4 C_c^2} (1 + K_d) - 1 - \zeta}{\zeta} \quad (A7)$$

A2.3.1.4 Contraction Coefficient

Contraction coefficient can be approximated with the orifice diameter ratio as follows:

$$C_c = \frac{0.61}{\sqrt{1 - \beta^4 (1 - 0.61^2)}} \quad (A8)$$

A2.3.2 Comparison of Cavitation Index

A2.3.2.1 Origin of JGC Design Criteria

Numachi [2] observed values for cavitation from their 4" apparatus as follows:

Kd'=2.5 for incipient cavitation

Kd'=0.8 - 1.0 for critical cavitation (smaller β is smaller Kd')

Lin [4] converted these values, applying $C_c=0.61$, to Kd as follows:

Kd=0.93 (= 2.5×0.61^2)

Kd=0.37 (= 1.0×0.61^2)

This means that the original source is Numachi's experimental data and Lin's values are consistent to the Numachi's work. These are applied as JGC's design criteria.

A2.3.2.2 Base Experimental Data

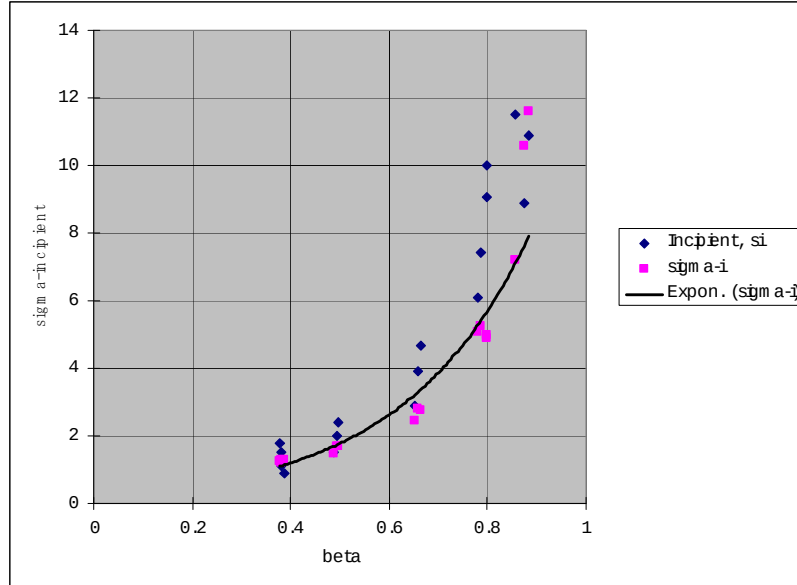
Tullis [1] gave comprehensive and accurate experimental data as shown below.

D (mm)	β	Incipient, σ_i	Critical, σ_c	Choking, σ_{ch}	Remark
27.4	0.386	0.90	0.80	0.200	
78.0	0.380	1.09	0.93	0.264	
78.0	0.489	1.53	1.15	0.381	
78.0	0.651	2.87	1.96	0.665	
78.0	0.782	6.1	3.68	1.65	
78.0	0.875	8.89	7.27	-	
154	0.381	1.21	1.08	0.197	
154	0.493	2.01	1.50	0.356	
154	0.658	3.89	2.64	0.800	
154	0.787	7.44	5.17	1.69	
154	0.884	10.9	8.2	4.44	
305	0.381	1.51	1.35	0.19	
305	0.497	2.41	1.91	-	
305	0.665	4.68	3.83	-	
305	0.800	9.05	6.39	-	
305	0.858	11.5	9.00	-	
597	0.379	1.8	1.60	-	
597	0.800	10	7.50	-	

A2.3.2.3 Incipient Cavitation

Incipient cavitation is defined as a start point of cavitation where small bubbles will appear without noise. Incipient cavitation is a function of the orifice diameter ratio and the pipe size.

Giving $K_d=0.93$, we can compare experimental data (Incipient, σ_i) with calculated data (σ_{a-i}) as follows:



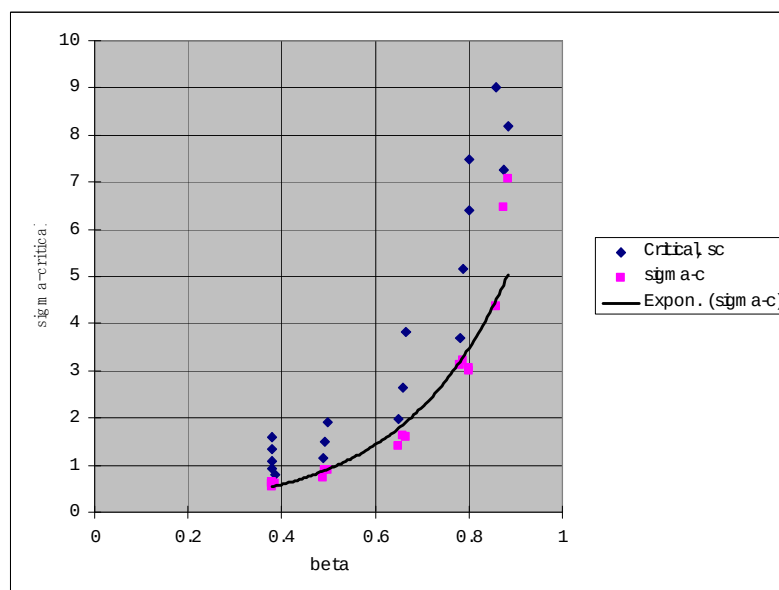
The calculated values well agree with the cavitation indexes for small pipe size because $K_d=0.97$ is an observed value from small size pipe [2].

A2.3.2.4 Critical Cavitation

At the critical cavitation, the noise is steady but still light. Therefore, operation at this condition is no problem because there is some margin until moderate or heavy cavitation.

Critical cavitation is a function of the orifice diameter ratio and the pipe size.

Similarly, we can prepare a comparison of critical cavitation with calculated values at $K_d=0.37$ as follows:

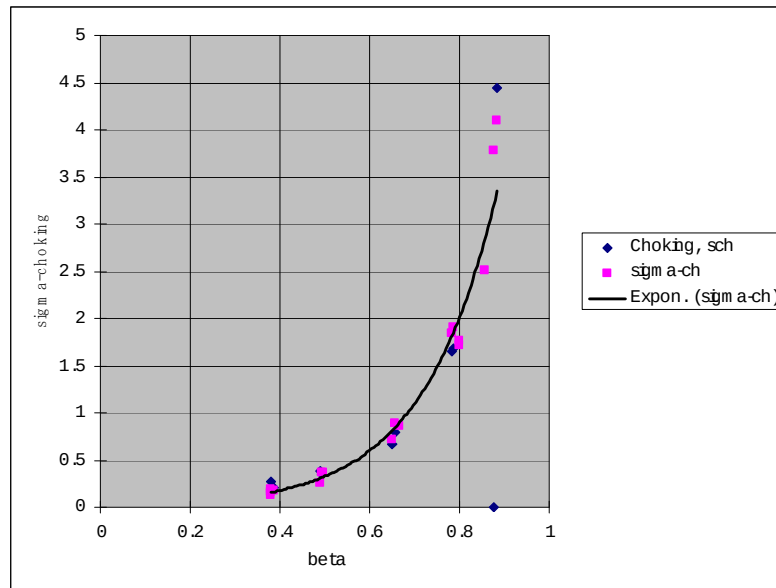


Calculated values (σ_{a-c}) well agree with the measured cavitation indexes for small size pipe too.

A2.3.2.5 Choking

The most severe cavitation will occur just before the orifice chokes. This condition is considered to be the worst possible operating range for the orifice. Once the orifice chokes and supercavitation occurs, no damage by erosion will exist near the orifice. This is because the damage is caused by the collapse of the cavities and the collapse occurs far downstream during supercavitation.

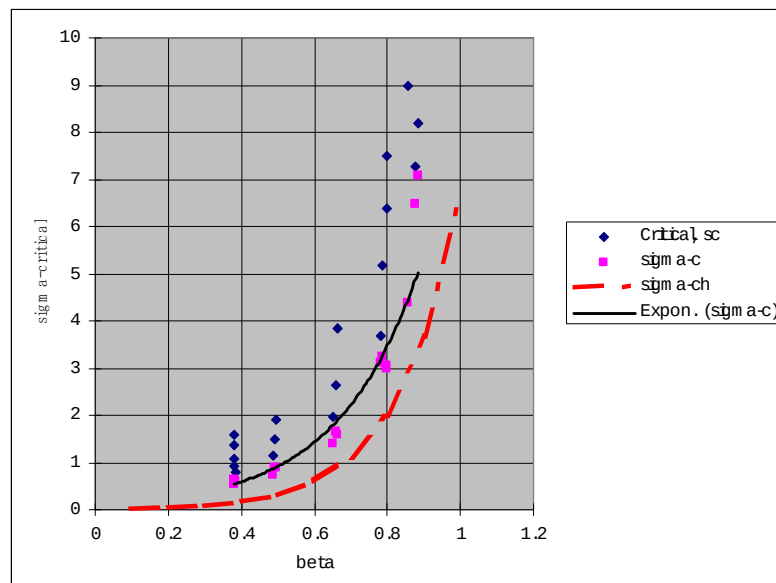
Similarly, we can prepare a comparison of supercavitation with calculated values at $K_d=0$ as follows:



Since choking is a function of the orifice diameter ratio only, the calculated values (σa -ch) well agree with the experimental values. This is a quite good agreement.

A2.3.3 Design Criteria

We shall avoid severe cavitation or choking. Cavitation index at the critical cavitation is almost two times of cavitation index at choking as shown below. Therefore, although we select the critical cavitation index $K_d=0.37$ as our design basis, we still have some design margin.



A2.4. Derivation of Conversion Equations

From Bernoulli's relation, we can get:

$$H_u + \frac{v^2}{2g} = H_c + \frac{v_c^2}{2g} = H_d + \frac{v^2}{2g} + \Delta H \quad (A9)$$

Combining (A1) and (A2) with (A9),

$$\begin{aligned} H_d - H_c &= \frac{v_c^2}{2g} - \frac{v^2}{2g} - \Delta H \\ &= \frac{v^2}{2g} \left(\frac{1}{\beta^2 C_c} \right)^2 - \frac{v^2}{2g} - \zeta \frac{v^2}{2g} \\ &= \frac{v^2}{2g} \cdot \left[\left(\frac{1}{\beta^2 C_c} \right)^2 - 1 - \zeta \right] \end{aligned} \quad (A10)$$

Then, finally we can get:

$$\begin{aligned} \sigma &= \frac{H_d - H_v}{H_u - H_d} = \frac{H_d - H_c + H_c - H_v}{H_u - H_d} \\ &= \frac{H_d - H_c}{H_u - H_d} + \frac{H_c - H_v}{H_u - H_d} \\ &= \frac{\left(\frac{1}{\beta^2 C_c} \right)^2 - 1 - \zeta}{\zeta} + \frac{H_c - H_v}{\zeta v^2 / 2g} \\ &= \frac{\left(\frac{1}{\beta^2 C_c} \right)^2 - 1 - \zeta}{\zeta} + \frac{\left(\frac{1}{\beta^2 C_c} \right)^2 K_d}{\zeta} \\ &= \frac{\left(\frac{1}{\beta^2 C_c} \right)^2 (1 + K_d) - 1 - \zeta}{\zeta} \end{aligned} \quad (A11)$$

The other equation can be derived as same manner as above.