

Mechanical Engineering Series

Zainul Huda

Metal Forming Processes

Fundamentals, Analysis, Calculations



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Mechanical Engineering Series

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Fundamentals, Analysis, Calculations

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Preface

Metal Forming is taught either as a full core course or as a major part of the manufacturing technology core course in BS (Mechanical/Manufacturing Engineering) programs in almost all reputed universities. Additionally, Applied Plasticity and Metal Forming is taught as a core course in MS (Mechanical Engineering) programs at reputed universities. This book, therefore, is equally useful to students of Mechanical/Manufacturing Engineering at both undergraduate and graduate levels. Besides students, practicing engineers in industry would also find this volume highly beneficial since there are three industrially oriented projects with their own key solutions.

This volume can be recommended as a textbook at universities and IHL since it covers both conventional metal-forming processes as well as recent advances in metal-forming technologies. The unique textbook features fundamentals, analyses, and calculations of metal-forming processes supported by 200 worked numerical examples and 100 MCQs. All three important groups of metal-forming processes (bulk-metal forming, sheet-metal forming, and sheet-bulk-metal forming) are covered. Theory of metal forming is presented by discussing deformation behavior, plasticity, and formability with detailed mathematical analyses and calculations. The mechanics of sheet metal forming is also covered by including principal strain increments in uniaxial loading as well as plane stress deformation. There are over 125 diagrammatic illustrations, which have been properly labeled to enhance the understanding of readers. One of the salient features of the book is the inclusion of industrially oriented projects, which cover both technological and business projects; the key solutions of these projects are also presented with the aid of mathematical analysis and diagrams. There are 100 multiple-choice questions (MCQs), whose answers are given at the end of the book along with the answers to selected problems.

There are 14 chapters in this book; each chapter starts with a list of nomenclature. The book is divided into an introductory chapter and three parts. In the first (introductory) chapter, the classification and the basics of the three principal metal-forming processes (bulk-deformation metal forming, sheet-metal forming, and bulk-sheet-metal forming) are covered. Part I re-enforces the theoretical concepts of metal forming by discussing deformation behavior of metals (Chap. 2), plasticity and

formability (Chap. 3), and cold working and hot working (Chap. 4). This part is supported by 34 worked numerical examples, 25 diagrams, and 32 MCQs. Mechanical properties data are also included in this part.

Part II focuses on bulk-metal-forming processes; this part explains the fundamentals and analyses of rolling (Chap. 5), forging (Chap. 6), extrusion (Chap. 7), and rod/wire drawing (Chap. 8). Chapter 5 contains a real-life picture from a rolling mill illustrating hot rolling of bars. Chapter 6 includes the author's industrial experience of the manufacture of forged axles for modern motor-cars. Part II contains 82 worked numerical examples, 36 diagrams, and 37 MCQs.

Part III deals with the qualitative and the quantitative aspects of sheet-metal fabrication by covering the mechanics of sheet metal forming (Chap. 9), shearing, blanking, and piercing (Chap. 10), bending (Chap. 11), and deep drawing (Chap. 12). Chapter 9 requires a good knowledge of integral calculus to be possessed by the reader. Photographs of equipment/machines from industry and laboratories are also included in this part. It contains 84 worked numerical examples, 58 diagrams, and 27 MCQs.

Part IV serves as the heart of this book since it presents industrially/business-oriented projects so as to develop, in the reader, the skills of designing the manufacturing/metal-forming processes for specific products as well as successfully completing technological/business projects. These projects involve combinations of bulk-metal-forming and sheet-metal-forming operations. In these projects, the reader is required to assume her/himself as a manufacturing engineer/manager in a metal-forming industry. S/he is required to draw the process flowcharts for the manufacture of (a) bolt (Chap. 13), (b) cup (Chap. 13), and (c) L-beam (business approach) (Chap. 14). The engineering manager is requested to conduct complete analyses involving the calculations of the pressure, force, power, etc. for each metal-forming operation, including the costs of each metal-forming operation for the business project for economic decision making.

Each chapter of this book ends with a Questions-and-Problems section as well as References so as to polish and widen the knowledge of the reader. Answers to all MCQs and selected problems are provided at the end of the book. It is expected that the reader has adequate knowledge of calculus since some analyses and the worked numerical examples and problems involve a great deal of both differential calculus and integral calculus. Another pre-requisite of this book is the readers' acquaintance with the basic principles of materials science/metallurgy. Based on my long-term associations with academia (40 years) and industry (2 years), I hope that this volume proves a useful literature for engineering students, academicians, researchers, and engineers practicing in industry.

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Chapter 1

Introduction



1.1 Metal Forming and Its Industrial Importance

Metal forming is widely used to manufacture machine components and structural parts; it finds applications in building construction, automotive industry, aerospace industry, defense sector, and the like. *Metal forming* refers to the plastic deformation of a metallic material to produce a product with a useful size and shape; it involves a great deal of plasticity behavior of metals (Służalec, 2004). In general, a metal forming process involves the use of a die and the application of force to deform the material. Examples of metal forming operations are rolling, forging, and sheet-metal forming processes (e.g. shearing, blanking, bending). There are many different types of metal forming processes, which are discussed with the aid of a classification chart in the subsequent section. Metal forming is a technologically important group of manufacturing processes, since it enjoys a great industrial significance among various production processes. Some distinct advantages of metal forming processes include enhanced mechanical properties, flexible operations, higher productivity, cost-effectiveness, and considerable material saving.

Many equipment and machinery, used in process industries, are made by metal forming. For example, pressure vessels, used in chemical plants, can be manufactured by open-die forging. Rolled metallic products find wide applications in building construction, structural engineering, ship building, automotive engineering, and the like. For instance, hot rolling process is used to produce steel bars for building construction (see Fig. 1.1). The metal sheets, for vehicle bodies and aircraft structures (Fig. 1.1), are also produced by flat rolling (Huda, 2016). Steel I-beams for industrial structures are manufactured by shape rolling. Metal formed products are also used in process industries and other engineering applications. For example, high-strength complex-shaped components (e.g. crane hooks, spanners) are produced by closed-die forging.



Fig. 1.1 Some metal formed products

1.2 Metal Forming Processes: Classification and Overview

1.2.1 Classification of Metal Forming Processes

Based on the working temperature, metal forming can be one of the following types: (a) cold working (CW), (b) warm working, and (c) hot working (HW). Based on the type of operation, there is a wide variety of different types of metal forming processes; these include rolling, forging, extrusion, bar/wire drawing, shearing, blanking, piercing, bending, and deep drawing (Nielsen & Martins, 2021; Huda, 2018, Ares, 2006). The classification of various metal forming processes is illustrated in Fig. 1.2.

1.2.2 Cold Working and Hot Working

Hot working (HW) refers to metal forming that is carried out at a temperature above the recrystallization temperature of the metal. Recrystallization temperature is the temperature at which atomic mobility is effective enough to reorient/realign the grains (crystals) so as to restore (increase) the ductility of the material. The recrystallization temperature (T_r) is given by:

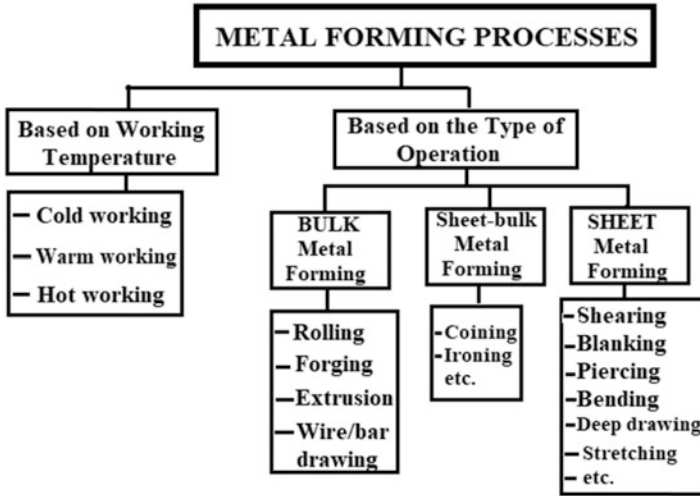


Fig. 1.2 Classification of metal forming processes

$$T_r = 0.5 T_m \quad (1.1)$$

where T_m is the melting temperature (in K). Cold working (CW) refers to a metal forming operation performed at a temperature below 0.3 times its melting temperature (in K). Warm working is the metal forming done at a temperature between 0.3 T_m and 0.5 T_m . For example, aluminum (a highly ductile metal) is usually formed by cold working at ambient temperature. On the other hand, high-strength alloy steels need to be formed by HW at a temperature in the range of 1200–1300 °C. The difference between CW, warm working, and HW is explained in detail in Chap. 4.

1.3 Bulk-Metal Forming Processes

Bulk-metal forming processes refer to those metal forming operations that involve a significant change in the thickness or cross-sectional area of the work-piece. There are primarily four types of bulk-metal forming processes: (a) rolling, (b) forging, (c) extrusion, and (d) rod/wire drawing; these processes are illustrated in Fig. 1.3. *Rolling* involves plastic deformation of a metallic work-piece by passing it between two or multiple rolls so as to change the thickness or cross-sectional area to obtain the desired shape and geometry (see Fig. 1.3a). Examples of rolled products include billets and bars of various geometries and cross-sectional areas; these products find applications in building construction, and other structural engineering. *Forging* involves the application of compressive forces to form a metallic work-piece by using dies for producing a product/semi-finished product (see Fig. 1.3b). Forged

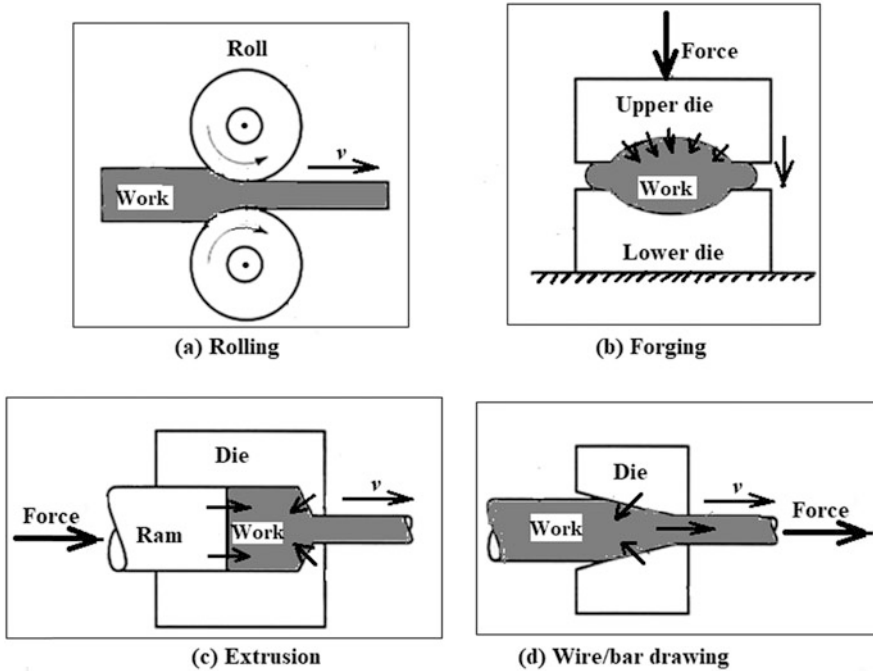


Fig. 1.3 Schematic illustration of bulk deformation processes. (a) Rolling. (b) Forging. (c) Extrusion. (d) Rod/bar drawing

products are noted for their high strength; examples are crane-hooks, spanners, and bolts. *Extrusion* involves forming a metallic work-piece to pass through a die opening of the desired cross-section by applying a pushing force (see Fig. 1.3c). Extruded products have excellent surface finish and dimensional accuracy; examples are welding electrodes and extruded tubes. Bar /wire drawing involves forming a metallic work-piece to pass through a die opening of the desired cross-section by applying a pulling force (see Fig. 1.3d).

1.4 Sheet-Metal Forming Processes

Sheet-metal forming refers to those metal forming processes that involve no or a little change in the thickness or cross-sectional area of the work-piece. Sheet-metal forming operations include shearing, blanking, piercing, bending, deep drawing, and stretching (see Fig. 1.4). *Shearing* involves cutting a sheet metal along a straight line between two cutting edges (Fig. 1.4a). *Blanking* involves cutting up a large metal sheet (along a closed path) into smaller pieces for further sheet-metal operation; the part that is cut out is the desired product—called the *blank* (see Fig. 1.4b). *Bending* is

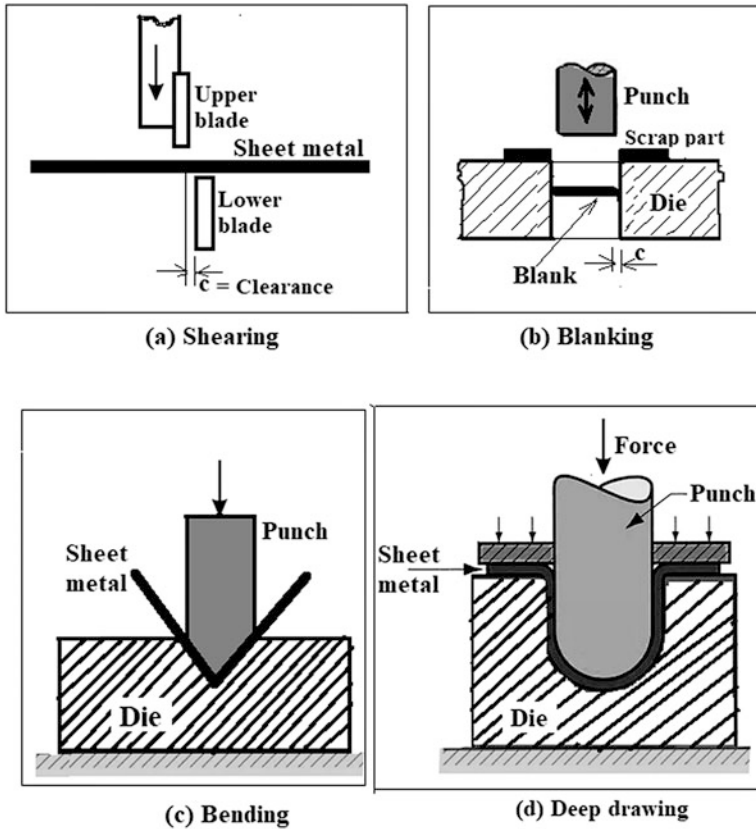


Fig. 1.4 Some sheet-metal forming operations. (a) Shearing. (b) Blanking. (c) Bending. (d) Deep drawing

a sheet-metal forming operation that involves the straining of the metal around a straight axis by using a punch and die (Fig. 1.4c). *Deep drawing* involves placing a sheet-metal piece onto a die and then forcing the metal into the die cavity with a punch (see Fig. 1.4d).

1.5 Sheet-Bulk Forming Processes

Sheet-bulk-metal forming (*SBMF*) processes involve forming of metal sheets with an intended three-dimensional material flow as in bulk forming processes (Merklein et al., 2012). Based on the tool motion, *SBMF* processes can be classified into two groups: (a) *SBMF* involving linear tool motion (e.g., ironing, coining), and (b) *SBMF* processes involving rotational tool motion (e.g., flow forming, orbital

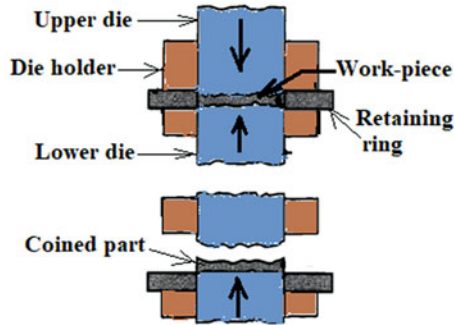


Fig. 1.5 Coining SBMF process

forming). The common characteristic of all SBMF processes is that there is a complex interaction between forming zones of high and low strains that possess three-dimensional as well as two-dimensional stress and strain states. The coining SBMF process is illustrated in Fig. 1.5.

Questions

- 1.1 (a) What are the advantages of metal forming processes as compared to other manufacturing processes? (b) List at least five applications of metal forming processes.
- 1.2 Draw sketches illustrating four (a) bulk metal forming operations and (b) sheet-metal forming operations.
- 1.3 Differentiate between the following groups of processes: (a) cold working and hot working and (b) bulk-metal forming and sheet-metal forming.
- 1.4 (MCQs). Select the most appropriate answers for the following statements.
 1. Which metal forming operation is the most suitable for manufacturing a car-radiator cap?
 - (a) Blanking, (b) deep drawing, (c) extrusion, (d) forging.
 2. Which operation is the most suitable for forming a head in a bolt?
 - (a) Blanking, (b) deep drawing, (c) extrusion, (d) upset forging.
 3. Which operation is the most suitable for manufacturing a welding electrode?
 - (a) Blanking, (b) deep drawing, (c) extrusion, (d) forging.
 4. Which metal forming operation falls under the category of SBMF?
 - (a) Rolling, (b) coining, (c) deep drawing, (d) blanking.

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Part I
Theory of Metal Forming

Chapter 2

Deformation Behaviors of Metals



Nomenclature

E	Young's modulus
H	plastic modulus
K	tangent modulus = strength coefficient
F	tensile force
A_o	original cross-sectional area
A	instantaneous cross-sectional area
A_f	final cross-sectional area
L_o	original gage length
L	gage length at an instant
L_f	final length
σ_{eng}	engineering stress
ϵ_{eng}	engineering strain
σ_{true}	true stress
ϵ_{true}	true strain
$d\sigma$	increment in stress
$d\epsilon_e$	increment in elastic strain
$d\epsilon_p$	increment in plastic strain
$d\epsilon$	overall increment in strain
n	strain hardening exponent
σ_{ys}	yield stress or yield strength
τ	shear stress
F_s	shear force
τ_r	resolved shear stress
ψ	angle between the tensile axis and normal to slip plane
λ	angle between the tensile axis and the slip direction
τ_{crss}	critical resolved shear stress

Abbreviations

ASTM	American Society of Testing of Materials
CW	cold working
HW	hot working

2.1 Stress and Strain

A tension test is the most fundamental test to determine the tensile mechanical properties of a metal. In order to conduct a tension test (or tensile test), it is necessary to prepare a tensile-test specimen, as shown in Fig. 2.1. The dimensions of the test specimen must be in accordance with the standards prescribed by the ASTM (American Society of Testing of Materials) (ASTM Committee, 2013). In tensile testing, the specimen is gripped in a tensile testing machine; one end of the specimen is held firm, whilst the other end is pulled by applying a tensile force. Once the specimen has fractured, it is reassembled and the gage length at failure (L_f), and the final cross-sectional area (A_f) are measured (see Fig. 2.1).

By recording the force and dimensional changes data, the engineering stress (σ_{eng}) and engineering strain (ϵ_{eng}) can be calculated by using the following relationships:

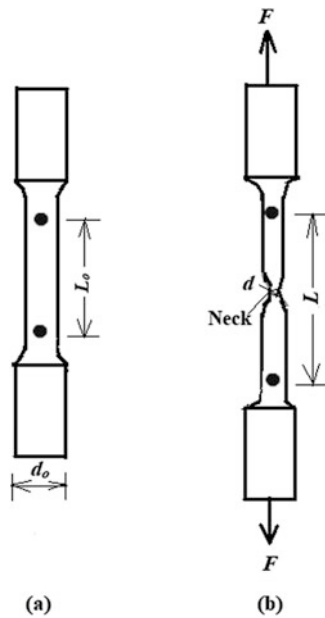


Fig. 2.1 Tensile test specimen before loading (a) and after loading (b)

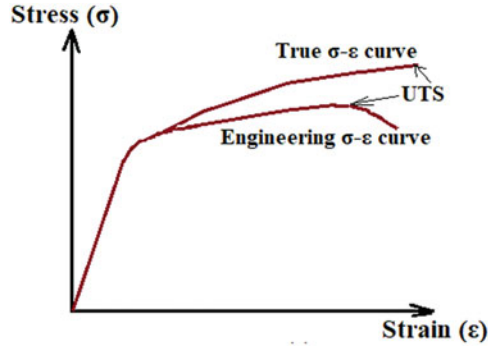


Fig. 2.2 Engineering and true stress–strain curves for a ductile material. *UTS* ultimate tensile strength

$$\text{Engineering stress} = \sigma_{\text{eng}} = \frac{F}{A_0} \quad (2.1)$$

$$\text{Engineering strain} = \epsilon_{\text{eng}} = \frac{\Delta L}{L_0} = \frac{L_f - L_0}{L_0} \quad (2.2)$$

where σ_{eng} is the engineering stress (MPa), F is the applied force (N), A_0 is the original cross-sectional area of the specimen (mm^2), ϵ_{eng} is the engineering strain, L_0 is the original length of the specimen (mm), and L_f is the final gage length just before fracture (mm) (see Example 2.1).

The relationship between load applied and deformation of the material is graphically expressed as a stress–strain curve, as shown in Fig. 2.2. The engineering stress–strain curve is based on the engineering stress (σ_{eng}) and the engineering strain (ϵ_{eng}), whilst the true stress–strain curve represents the variations in the true stress (σ_{true}) and the true strain (ϵ_{true}).

The true stress and the true strain represent the actual (instantaneous) values of stress and strain, respectively. The true stress and the true strain can be calculated by the following formulae:

$$\text{True stress} = \sigma_{\text{true}} = \frac{F}{A} \quad (2.3)$$

where F is the tensile force and A is the cross-sectional area at any instant during the tension test.

$$\text{True strain} = \epsilon_{\text{true}} = \int_{L_0}^L \frac{dL}{L} = \ln \left(\frac{L}{L_0} \right) = \ln \left(\frac{A_0}{A} \right) \quad (2.4)$$

where L_0 is the original gage length (mm) and L is the length at any instant during the test (mm). The true stress and strain can be related to the engineering stress and strain by (Huda, 2022):

$$\sigma_{\text{true}} = \sigma_{\text{eng}} (1 + \epsilon_{\text{eng}}) \quad (2.5)$$

$$\epsilon_{\text{true}} = \ln (1 + \epsilon_{\text{eng}}) \quad (2.6)$$

The significance of Eqs. 2.5 and 2.6 is illustrated in Example 2.2.

2.2 Elastic and Plastic Deformations

The tension test of a metal involves deformations in two stages: (a) elastic deformation and (b) plastic deformation (see Fig. 2.3). Elastic deformations are reversible, i.e., the energy expended in deformation is stored as elastic strain energy and is completely recovered upon the removal of load. On the other hand, plastic deformations are irreversible (permanent), i.e., the energy stored in the deformation remains even after the removal of load. In general, almost all engineering materials undergo some plastic deformation. The elasticity or stiffness of the material is generally expressed by the Young's modulus (E), which is the slope of the stress–strain curve in the elastic region. Mathematically,

$$E = \frac{d\sigma}{d\epsilon_e} = \frac{\sigma_2 - \sigma_1}{\epsilon_2 - \epsilon_1} \quad (2.7)$$

where $d\sigma$ is the increment in stress and $d\epsilon_e$ is the increment in elastic strain (see Example 2.3).

The Young's moduli and yield strengths of some metallic materials are presented in Table 2.1.

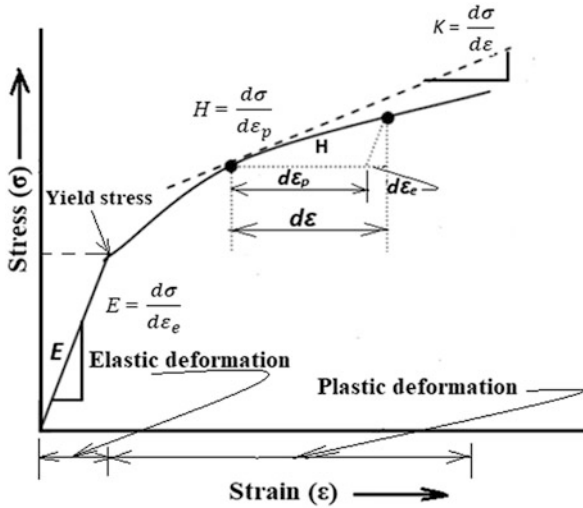


Fig. 2.3 The stress–strain curve showing the elastic- and the plastic deformation ranges. $d\epsilon_e$ increment in elastic strain, $d\epsilon_p$ increment in plastic strain, E Young's modulus, H plastic modulus, K tangent modulus

Table 2.1 The Young's moduli (E) and yield strengths of some metallic materials at 21 °C

Material	Gray cast iron	MS	Stainless steel	Bronze	Aluminum	Brass	Nickel	Magnesium
E , GPa ^a	130	200	180	116	70	100	170	45
Yield strength	65–420 MPa	250 MPa	175–300 MPa	125–800 MPa	35 MPa	95–124 MPa	70 MPa	20 MPa

^a1 GPa = 10^9 N/m². MS mild steel = very-low-carbon steel

The plasticity or formability can be numerically expressed by the plastic modulus (H), which is the slope of the stress–strain curve in the plastic region i.e.

$$H = \frac{d\sigma}{d\epsilon_p} \quad (2.8)$$

where $d\epsilon_p$ is the increment in plastic strain (see Fig. 2.3). The overall increment in strain ($d\epsilon$) is the sum of the increment in elastic strain ($d\epsilon_e$) and the increment in plastic strain ($d\epsilon_p$), i.e.,

$$d\epsilon = d\epsilon_e + d\epsilon_p \quad (2.9)$$

The overall increments in the stress ($d\sigma$) and the strain ($d\epsilon$) can be expressed by:

$$K = \frac{d\sigma}{d\epsilon} \quad (2.10)$$

where K is the tangent modulus or the strength coefficient of the material, $d\sigma$ is the increment in stress, and $d\epsilon$ is the overall increment in strain.

By combination of Eqs. 2.7–2.10, we obtain:

$$\frac{1}{K} = \frac{1}{E} + \frac{1}{H} \quad (2.11)$$

The significance of Eqs. 2.7–2.11 is illustrated in Examples 2.4

Many strain-hardened metals and alloys follow the deformation behavior according to the stress–strain curves, as shown in Fig. 2.3. The deformation behavior of these stress–strain curves can be represented by the following power law stress–strain relationship (Huda & Bulpett, 2012):

$$\sigma_{\text{true}} = K \epsilon_{\text{true}}^n \quad (2.12)$$

where σ_{true} is the true stress or the yield stress (MPa), ϵ_{true} is the true strain, K is the strength coefficient (or the tangent modulus) (MPa), and n is the work-hardening exponent (see Examples 2.5 and 2.6). It is useful to express Eq. 2.12 in logarithmic form, as follows:

$$\log \sigma_{\text{true}} = \log K + n \log \epsilon_{\text{true}} \quad (2.13)$$

The significance of Eq. 2.13 is illustrated in Examples 2.7 and 2.8. A metal (or alloy) can be strengthened by strain hardening by selecting an n value in the range of 0.10–0.50. For a perfectly plastic solid, $n = 0$, whilst $n = 1$ represents a 100% elastic solid. The data for n and K for various alloys are presented in Table 2.2.

Table 2.2 The data for n and K for some formable alloys (Callister, 2007)

Alloy	Copper (annealed)	Naval brass (annealed)	MS	304 stainless steel	4340 alloy steel	MgAZ31B (annealed)	2024-T3 Al alloy
n	0.44	0.21	0.21	0.44	0.12	0.16	0.17
K (MPa)	530	585	600	1400	2650	450	780

n strain-hardening exponent, MS mild steel

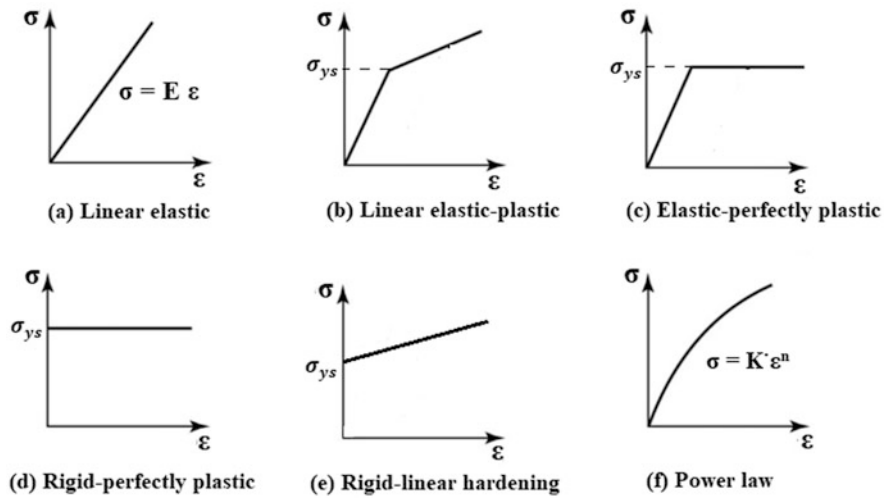


Fig. 2.4 The stress–strain behaviors for the six deformation models. (a) Linear elastic. (b) Linear elastic-plastic. (c) Elastic-perfectly plastic. (d) Rigid-perfectly plastic. (e) Rigid-linear hardening. (f) Power law

2.3 Deformation Models

Many engineering materials exhibit both elastic and plastic deformation behaviors. However, there are some exceptions to these deformation behaviors. For example, in some industrial metal-forming operations involving severe plastic deformation, the elastic deformation is ignored (Doltsinis, 2004). The various deformation behaviors, in metallic materials, may be represented by the following six deformation models: (a) linear elastic, (b) linear elastic/plastic, (c) elastic/perfectly plastic, (d) rigid/perfectly plastic, and (e) rigid/linear strain hardening, and (f) power law (see Fig. 2.4).

Figure 2.4a shows linear elastic deformation behavior of a material, i.e., Hook’s law is obeyed during the deformation without yielding. On the other hand, the deformation behaviors in Fig. 2.4b–e illustrate that yielding occurs when deformation crosses the elastic limit to begin plastic behavior. The six deformation behaviors (shown in Fig. 2.4) are based on the following assumptions: (a) the material is

isotropic, (b) the deformation is independent of rate of loading effects, and (c) the yield stress is independent of hydrostatic pressure.

Figure 2.4b representing linear elastic/plastic deformation indicates that Hook's law is obeyed up to the elastic limit, beyond which yielding occurs, followed by linear plastic behavior due to strain hardening; this deformation behavior, for example, is observed in tensile testing of ductile materials (e.g., mild steel) (see Chap. 3 for details). The elastic/perfectly plastic deformation model (Fig. 2.4c) indicates the absence of strain hardening, i.e., the yield stress (σ_{ys}) is constant after initial yield. Hot working (HW) of metals is an example of the elastic/perfectly plastic deformation behavior. The rigid/perfectly plastic deformation behavior (Fig. 2.4d) occurs when the plastic deformation is severe; here, the plastic strains are very large as compared to the elastic strains. The rigid/perfectly plastic deformation behavior is encountered in the engineering analysis of soil and rock stability. Fig. 2.4e showing rigid/linear strain hardening illustrates linear strain hardening without elastic deformation; this deformation behavior is observed in cold-working (CW) metal-forming processes (e.g., wire drawing, cold extrusion) involving significantly large strains up to 50% cold work (CW) (Doltsinis, 2004). Fig. 2.4f represents the power law stress–strain relationship (see Eq. 2.12).

The six deformation models, as discussed in the preceding paragraph, just represent simple models; however, the actual deformation behaviors in industrial practice may slightly deviate from the six models. For instance, sheet-metal forming processes generally involve large deformations together with complex loading sequences. For such conditions, an advanced anisotropic elastic–plastic model has been proposed by Haddag and co-researchers; this model has been formulated within the large strain framework and taking strain-path changes into account, coupled with an isotropic damage model (Haddag et al., 2009).

2.4 Deformation Mechanism: Dislocation Movement

All metal-forming processes involve plastic deformation, which in turn results from dislocation movements. A *dislocation* is a line defect (crystal imperfection) that occurs along a row of atoms. The application of shear stress, to a crystalline solid, results in atomic displacements due to dislocation movement. There are three types of dislocation in a crystalline solid: (a) edge dislocation, (b) screw dislocations, and (c) mixed dislocation. An *edge dislocation*, in a crystal lattice, results due to the introduction of an extra half plane of atoms in a crystal lattice (see Figs. 2.5). A *screw dislocation* is a dislocation in which the atoms are arranged in a helical pattern that is normal to the direction of the stress (see Fig. 2.6). A *mixed dislocation* refers to the combination of edge dislocation and screw dislocation.

When a shear stress (τ) is applied to a crystalline solid, atoms move (displace) in a direction parallel to the slip direction. On increasing the shear force, the atoms continue to move (slip) resulting in dislocation motion. As a row of the atoms find their way back into a proper spot in the lattice, another row of the atoms slip out of

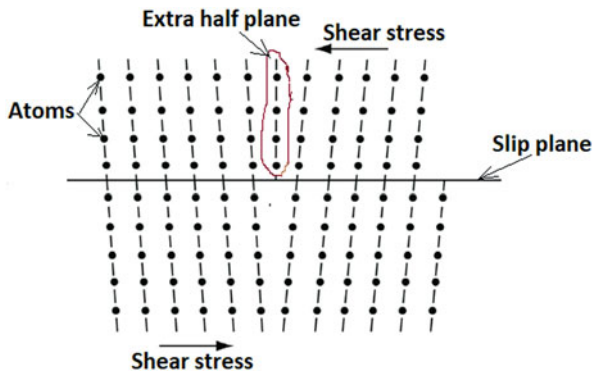


Fig. 2.5 An edge dislocation showing the formation of an extra half plane

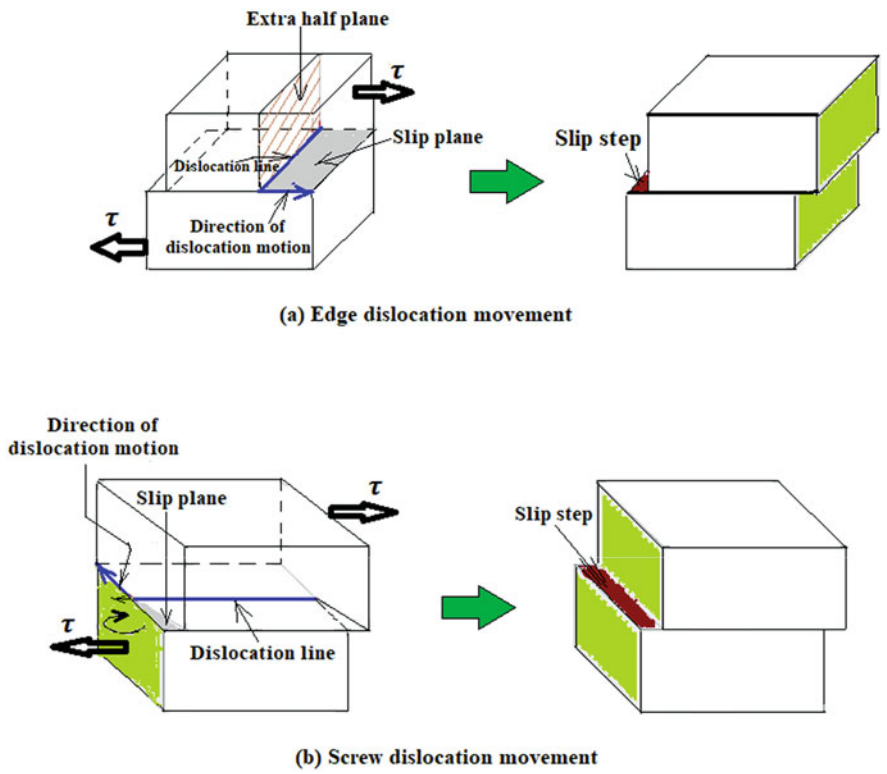


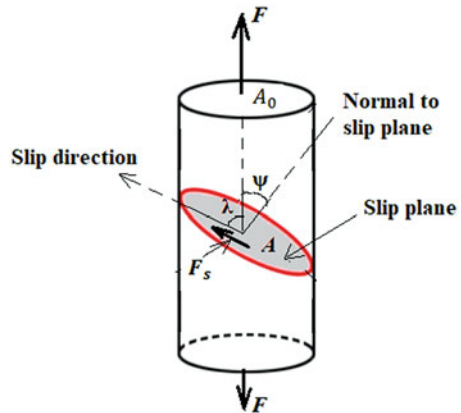
Fig. 2.6 Plastic deformation by slip with an edge dislocation movement (a) and screw dislocation movement (b)

position forming a *slip step* (see Fig. 2.6). The dislocation movement produces plastic deformation by *slip*, which occurs along specified *slip planes*. A *slip plane* is a crystallographic plane with the closest atomic packing. The slip across a slip plane occurs in specified directions, called *slip directions*; a slip direction is the direction of the maximum atomic density (Huda, 2020). In an *edge dislocation*, the dislocation line moves parallel to the applied stress (τ) (Fig. 2.6a). In a screw dislocation, the dislocation line moves in a direction perpendicular to the applied stress (Fig. 2.6b). The examples of plastic deformation by slip include metal rolling, extrusion, and deformation of a single crystal. In nano/micrometer-scale materials, dislocations accounting for crystal slip deformation are introduced from the surface because of the lack of preexisting dislocations inside the materials (Kawai et al., 2023).

2.5 Deformation of Single Crystal

It has been learnt in the preceding section that plastic deformation by slip occurs as a result of dislocation motion across slip planes under the action of shear stresses. The deformation by slip also occurs under the action of normal stress (either tensile or compressive stress); in this case, there exists a shear-stress component in the slip direction inclined at an angle to the direction of applied stress. This shear-stress component is called the *resolved shear stress*, τ_r (Gottstein, 2004). Consider a load F that is applied along the tensile axis of a single crystal, as shown in Fig. 2.7. The applied tensile force per unit original cross-sectional area is the tensile stress, σ ($\sigma = \frac{F}{A_0}$). The tensile force (F) can be resolved into the shear force (F_s) onto the slip plane (see Fig. 2.7). The resolved shear force per unit cross-sectional area in the slip plane is the resolved shear stress, τ_r ($\tau_r = \frac{F_s}{A}$).

Fig. 2.7 Resolution of tensile stress (σ) into shear stress (τ_r) under tensile loading



By reference to Fig. 2.7, the tensile force (F) is related to the tensile stress (σ) by:

$$F = \sigma A_0 \quad (2.14)$$

The shear force (F_s) can be related to the tensile force (F) by:

$$F_s = F \cos \lambda \quad (2.15)$$

where λ is the angle between the tensile axis and the slip direction (see Fig. 2.7).

By the combination of Eqs. 2.14 and 2.15, we obtain:

$$F_s = \sigma A_0 \cos \lambda \quad (2.16)$$

Since

$$A_0 = A \cos \psi \quad (2.17)$$

Equation 2.16 can be rewritten as:

$$F_s = \sigma A (\cos \psi) (\cos \lambda) \quad (2.18)$$

Or

$$\frac{F_s}{A} = \sigma (\cos \psi) (\cos \lambda) \quad (2.19)$$

Or

$$\tau_r = \sigma (\cos \psi) (\cos \lambda) \quad (2.20)$$

where τ_r is the the resolved shear stress, σ is the tensile stress, ψ is the angle between the tensile axis and normal to slip plane, and λ is the angle between the tensile axis and the slip direction (see Example 2.10).

There are a number of possible slip systems in a single crystal; however, there is one slip system that is the most favorably oriented to initiate slip. The minimum shear stress that is necessary to initiate slip in a single crystal under an applied normal stress, is called the *critical resolved shear stress*, τ_{crss} . In other words, the *critical resolved shear stress* (τ_{crss}) may be defined as the maximum value of the resolved shear stress that results in slip of the single crystal by yielding. Hence, Eq. 2.20 can be expressed by *Schmid's law*, as follows:

$$\tau_{\text{crss}} = \sigma_{\text{ys}} [(\cos \psi) (\cos \lambda)]_{\text{max}} \quad (2.21)$$

where σ_{ys} is the yield strength of the single crystal (see Example 2.11).

2.6 Worked Numerical Examples

Example 2.1: Computing the Engineering Stress and Engineering Strain

A tension-test metal specimen has a gage length of 62.5 mm and diameter of 12.5 mm. The specimen was subjected to a tensile load of 1700 N. The final length at fracture is 75 mm. Compute the engineering stress, engineering strain, and the final diameter at fracture.

Solution

$$L_o = 62.5 \text{ mm}, L_f = 75 \text{ mm}, d_o = 12.5 \text{ mm}, F = 1700 \text{ N}, \sigma_{\text{eng}} = ?, \epsilon_{\text{eng}} = ?, A = ?$$

$$\text{Original cross-sectional area} = A_o = \frac{\pi}{4} d_o^2 = \frac{\pi}{4} (12.5)^2 = 122.72 \text{ mm}^2$$

By using Eqs. 2.1 and 2.2,

$$\text{Engineering stress} = \sigma_{\text{eng}} = \frac{F}{A_o} = \frac{1700}{122.72} = 13.85 \text{ MPa}$$

$$\text{Engineering strain} = \epsilon_{\text{eng}} = \frac{L_f - L_o}{L_o} = \frac{75 - 62.5}{62.5} = 0.2$$

$$\text{Original volume} = \text{Final volume}$$

$$A_o L_o = A_f L_f$$

$$122.72 \times 62.5 = A_f (75)$$

$$A_f = 102.27 \text{ mm}^2$$

$$\frac{\pi}{4} d_f^2 = 102.27$$

$$d_f = \sqrt{\frac{A_f}{0.785}} = \sqrt{\frac{102.27}{0.785}} = 11.41 \text{ mm}$$

$$\text{The final diameter at fracture} = d_f = 11.41 \text{ mm}$$

Example 2.2: Calculating the True Stress and the True Strain

By using the data in Example 2.1, calculate the true stress and the true strain.

Solution

$$\sigma_{\text{eng}} = 13.85 \text{ MPa}, \epsilon_{\text{eng}} = 0.2, \sigma_{\text{true}} = ?, \epsilon_{\text{true}} = ?$$

By using Eqs. 2.5 and 2.6,

$$\sigma_{\text{true}} = \sigma_{\text{eng}} (1 + \epsilon_{\text{eng}}) = 13.85 (1 + 0.2) = 16.62 \text{ MPa}$$

$$\epsilon_{\text{true}} = \ln (1 + \epsilon_{\text{eng}}) = \ln (1 + 0.2) = 0.1823$$

Example 2.3: Determining the Young's Modulus in Compression for Gray Cast Iron

Figure 2.8 shows the stress–strain curve in compression for gray cast iron. Determine the Young's modulus of the cast iron.

Solution

In order to determine Young's modulus, we can use Eq. 2.7 by taking both σ_1 and ε_1 as zero. For determining σ_2 and ε_2 , Fig. 2.8 can be redrawn as shown in Fig. 2.9.

By using Eq. 2.7,

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{200 - 0}{0.0022 - 0} = 90909.1 \text{ MPa} = 90.909 \text{ GPa}$$

Example 2.4: Calculating the Plastic Modulus of Brass

By using the data in Tables 2.1 and 2.2, calculate the plastic modulus of naval brass.

Solution

$$E = 100 \text{ GPa} = 100 \times 10^9 \text{ Pa} = 100,000 \text{ MPa}, K = 585 \text{ MPa}, H = ?$$

Fig. 2.8 The stress–strain curve in compression for gray cast iron

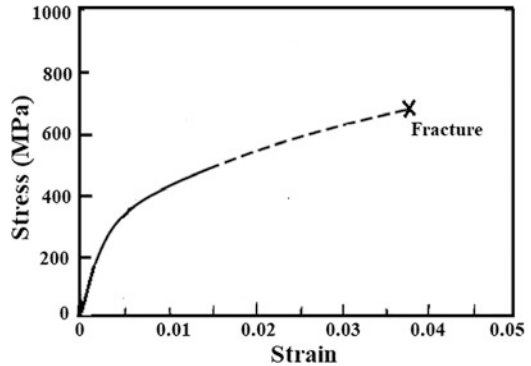
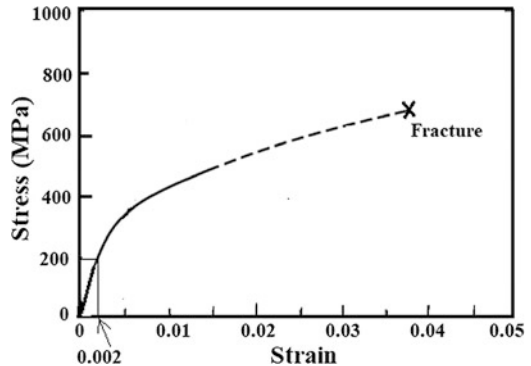


Fig. 2.9 Redrawn Fig. 2.8 to show $\sigma_2 = 200 \text{ MPa}$ and $\varepsilon_2 = 0.0022$



By using Eq. 2.11,

$$\begin{aligned}\frac{1}{585} &= \frac{1}{100000} + \frac{1}{H} \\ 0.00171 &= 0.00001 + \frac{1}{H} \\ \frac{1}{H} &= 0.0017\end{aligned}$$

The plastic modulus = $H = 588.23$ MPa

Example 2.5: Calculating the True Strain When the Yield Stress Is Known

The work-hardening stress–strain behavior of annealed 304 stainless steel can be represented by:

$$\sigma_{\text{true}} = 1275 (\epsilon_{\text{CW}} + 0.002)^{0.45}$$

where σ_{true} is the true stress and ϵ_{CW} is the cold-work strain. Calculate the true strain when the yield stress is 800 MPa.

Solution

By comparison of the given equation with Eq. 2.12, we obtain: $K = 1275$ MPa.

$\sigma_{\text{true}} = \text{yield stress} = 800$ MPa. By substituting $\sigma_{\text{true}} = 800$ MPa in the given equation,

$$\begin{aligned}800 &= 1275 (\epsilon_{\text{CW}} + 0.002)^{0.45} \\ (\epsilon_{\text{CW}} + 0.002)^{0.45} &= 0.627 \\ (\epsilon_{\text{CW}} + 0.002)^{\frac{0.45}{0.45}} &= 0.627^{\frac{1}{0.45}} \\ \epsilon_{\text{CW}} &= 0.3544 - 0.002 = 0.3524\end{aligned}$$

By comparing the given equation with Eq. 2.12,

$$\text{True strain} = \epsilon_{\text{true}} = \epsilon_{\text{CW}} + 0.002 = 0.3524 + 0.002 = 0.3544$$

Example 2.6: Calculating the Percent Cold Work When the True Strain Is Known

Refer to the data in Example 2.5. What percent cold work (% CW) must be inflicted to induce the required true strain in the steel?

Solution

$$\epsilon_{\text{true}} = 0.35476, \% \text{CW} = ?$$

By using Eq. 2.4,

$$\begin{aligned}\epsilon_{\text{true}} &= \ln \left(\frac{A_0}{A} \right) \\ 0.35476 &= \ln \left(\frac{A_0}{A_{\text{CW}}} \right) \\ \frac{A_0}{A_{\text{CW}}} &= e^{0.35476} = 1.42584 \\ \% \text{CW} &= \frac{A_0 - A_{\text{CW}}}{A_0} \times 100\end{aligned}$$

or

$$\% \text{CW} = \left[1 - \left(\frac{A_{\text{CW}}}{A_0} \right) \right] \times 100 = \left[1 - \left(\frac{1}{1.42584} \right) \right] \times 100 = 29.86$$

The degree of cold work to be inflicted = 29.86%

Example 2.7: Calculating the True Strain for an Alloy by Using the Values of n and K

A mild-steel sample was subjected to a true stress of 500 MPa. Calculate the true strain that was produced in the material. (Hint: Refer to Table 2.2).

Solution

$n = 0.21$ (see Table 2.2), $K = 600$ MPa (see Table 2.2), $\sigma_{\text{true}} = 500$ MPa, $\epsilon_{\text{true}} = ?$

By using Eq. 2.13,

$$\begin{aligned}\log \sigma_{\text{true}} &= \log K + (n \log \epsilon_{\text{true}}) \\ \log 500 &= \log 600 + (0.21 \log \epsilon_{\text{true}}) \\ 0.21 \log \epsilon_{\text{true}} &= 2.69 - 2.77 \\ \log \epsilon_{\text{true}} &= -0.381 \\ \text{True strain} = \epsilon_{\text{true}} &= 10^{-0.381} = 0.416\end{aligned}$$

Example 2.8: Calculating the True Stress When the True Strain Is Known

What true stress is necessary to produce a true plastic strain of 0.28 in a sample of annealed copper?

Solution

$$\epsilon_{\text{true}} = 0.28, n = 0.44, K = 530 \text{ MPa}, \sigma_{\text{true}} = ?$$

By using Eq. 2.12,

$$\sigma_{\text{true}} = K \epsilon_{\text{true}}^n = 530 \times 0.28^{0.44} = 302.71 \text{ MPa}$$

Example 2.9: Identifying the Material by Calculating Its Tangent Modulus

A stress increment of 230 MPa was applied to a metal-tensile-specimen. The resulting increments in the elastic strain and the plastic strain were recorded as 0.02 and 0.38, respectively. Identify the most-likely material that was tensile tested.

Solution

$$d\sigma = 230 \text{ MPa}, d\epsilon_e = 0.02, d\epsilon_p = 0.38, K = ?$$

By using Eqs. 2.9 and 2.10,

$$d\epsilon = d\epsilon_e + d\epsilon_p = 0.02 + 0.38 = 0.40$$

$$K = \frac{d\sigma}{d\epsilon} = \frac{230}{0.40} = 575 \text{ MPa}$$

By reference to Table 2.2, the tested material corresponds to annealed naval brass.

Example 2.10: Computing the Resolved Shear Stress in a Single Crystal

A tensile stress of 3 MPa is applied to a single crystal. The angle between the tensile axis and the slip direction is 55° . The angle between the tensile axis and normal to the slip plane is 65° . Compute the resolved shear stress along the slip plane.

Solution

By using Eq. 2.20,

$$\tau_r = \sigma (\cos \psi) (\cos \lambda) = 3 \times \cos 65^\circ \times \cos 55^\circ = 3 \times 0.4226 \times 0.5736 = 0.727 \text{ MPa}$$

The resolved shear stress = 0.727 MPa

Example 2.11: Calculating the Yield Strength of a Single Crystal When τ_{crss} Is Known

The *critical resolved shear stress* for the metal is 5 MPa. The metal single crystal is oriented such that the angle between the tensile axis and normal to slip plane is 60° . There are two possible slip directions that make angles of 40° and 50° with the tensile axis. Calculate the yield strength of the metal.

Solution

$$\tau_{\text{crss}} = 5 \text{ MPa}, \psi = 60^\circ, \lambda_1 = 40^\circ, \lambda_2 = 50^\circ, \sigma_y = ?$$

By using $\lambda_1 = 40^\circ$, $(\cos \psi) (\cos \lambda) = (\cos 60^\circ) \times (\cos 40^\circ) = 0.5 \times 0.766 = 0.383$

By using $\lambda_2 = 50^\circ$, $(\cos \psi) (\cos \lambda) = (\cos 60^\circ) \times (\cos 50^\circ) = 0.5 \times 0.6428 = 0.3214$

Hence,

$$[(\cos \psi) (\cos \lambda)]_{\max} = 0.383$$

By using the modified form of Eq. 2.21,

$$\text{Yield strength of the metal} = \sigma_y = \frac{\tau_{\text{crss}}}{[(\cos \psi) (\cos \lambda)]_{\max}} = \frac{5}{0.383} = 13.055 \text{ MPa}$$

Questions and Problems

2.1 (MCQs). Choose the best answers from each of the following questions.

1. Which one of the following terms refers to the slope of the stress–strain curve in the plastic region?
 - (a) Young's modulus, (b) plastic modulus, (c) tangent modulus, (d) rigidity modulus.
2. Which one of the following terms refers to the ratio of the overall increment in stress to the overall increment in strain?
 - (a) Young's modulus, (b) plastic modulus, (c) tangent modulus, (d) rigidity modulus.
3. Which one of the following terms refers to the slope of the stress–strain curve in the elastic region?
 - (a) Young's modulus, (b) plastic modulus, (c) tangent modulus, (d) rigidity modulus.
4. Which deformation behavior is observed in hot working of metals?
 - (a) Linear elastic, (b) linear elastic/plastic, (c) elastic/perfectly plastic, (d) rigid/perfectly plastic.
5. Which deformation behavior is observed in cold working of metals?
 - (a) Linear elastic, (b) linear elastic/plastic, (c) elastic/perfectly plastic, (iv) rigid/linear strain hardening.
6. Which behavior represents obedience to Hook's law during the deformation without yielding?
 - (a) Linear elastic, (b) linear elastic/plastic, (c) elastic/perfectly plastic, (d) rigid/perfectly plastic.

7. Which deformation behavior is observed in tensile testing of ductile materials?
 - (a) Linear elastic, (b) linear elastic/plastic, (c) elastic/perfectly plastic, (d) rigid/perfectly plastic.
8. Which deformation behavior is observed in the engineering analysis of soil and rock stability?
 - (a) Linear elastic, (b) linear elastic/plastic, (c) elastic/perfectly plastic, (d) rigid/perfectly plastic.
9. Which strain-hardening-exponent value represents a perfectly plastic solid?
 - (a) $n = 0$, (b) $n = 1$, (c) $n = 0.10\text{--}0.50$, (d) $n = 0.6\text{--}0.9$.
10. Which strain-hardening-exponent value should be selected to strengthen a metal (or an alloy) by strain hardening?
 - (a) $n = 0$, (b) $n = 1$, (c) $n = 0.10\text{--}0.50$, (d) $n = 0.6\text{--}0.9$.
11. Which strain-hardening-exponent value represents a perfectly elastic solid?
 - (a) $n = 0$, (b) $n = 1$, (c) $n = 0.10\text{--}0.50$, (d) $n = 0.6\text{--}0.9$.
- 2.2 (a) Why is the true stress greater than the engineering stress beyond the yield point?
 - (b) Prove the following relationship by using the definitions of stress and strain:

$$\sigma_{\text{true}} = \sigma_{\text{eng}} (1 + \epsilon_{\text{eng}}).$$

- 2.3 (a) List the six the deformation models for metallic materials.
 - (b) What assumptions are made in representing the deformation models in (a) above?
 - (c) Are the six deformation models strictly applicable to industrial sheet-metal forming? Explain.
- 2.4 Draw the stress–strain curve illustrating the Young’s modulus (E), the plastic modulus (H), and the tangent modulus (K).
- 2.5 Prove mathematically the following relationship:

$$\frac{1}{K} = \frac{1}{E} + \frac{1}{H}$$

- P2.6 A naval-brass sample was subjected to a true stress of 400 MPa. Calculate the true strain that was produced in the material. (Hint: Refer to Table 2.2).

- P2.7 A tensile-test specimen has a gage length of 45 mm and a diameter of 9 mm. The specimen was subjected to a tensile load of 2000 N. The final length at fracture is 54 mm.
Calculate the true stress and true strain.
- P2.8 A tensile-test specimen made of 304 stainless steel was subjected to a stress increment of 200 MPa. (a) What is the overall increment in strain? (b) Which one of the six deformation models (Fig. 2.4) corresponds to the behavior in (a) above?.
- P2.9 A stress increment of 180 MPa was applied to a metal-tensile-specimen. The resulting increments in the elastic strain and the plastic strain were recorded as 0.01 and 0.29 respectively. Identify the most likely material that was tensile tested.
- P2.10 A tensile stress of 2.4 MPa is applied to a single crystal. The angle between the tensile axis and the slip direction is 43° . The angle between the tensile axis and normal to the slip plane is 58° . Compute the resolved shear stress along the slip plane.

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Chapter 3

Plasticity, Formability, and Materials



Nomenclature

A	instantaneous area of cross-section.
ϵ_e	elastic strain,
ϵ_p	plastic strain
σ	true stress
σ_{ys}	yield strength
σ_{UTS}	ultimate tensile strength
σ_b	breaking strength
F	load (tensile load)
dF	change in the load
ϵ	true strain,
n	strain-hardening exponent
K	strength coefficient
$\dot{\epsilon}$	strain rate
m	strain rate sensitivity index
$\frac{d\sigma}{d\epsilon}$	rate of strain hardening

Abbreviations

% RA	percent reduction in area
AISI	American Iron & Steel Institute
ASS	austenitic stainless steel
CI	cast iron
FEA	Finite Element Analysis
FLD	Forming Limit Diagram
FSS	ferritic stainless steel
GOES	grain-oriented electrical steels

LAHS steel	Low-alloy high-strength steel
LCES	low-carbon electrical steels
LYP	Lower Yield Point
MS	mild steel
MSS	martensitic stainless steel
NGOES	non-grain-oriented electrical steels
SAE	Society of Automotive Engineers
SCC	stress corrosion cracking
SPF	Super-plastic forming
TRIP steel	TRansformation Induced Plasticity steel
UYP	Upper Yield Point

3.1 What Is Plasticity and Formability?

It has been learnt in the preceding chapter that many engineering materials exhibit both elastic and plastic deformation behaviors under the action of stress. The elastic deformation is reversible, whereas the plastic one is irreversible. The ability of a material to be plastically deformed, under the action of a stress, is called *plasticity*. Metal forming processes involve a great deal of plasticity, since many metallic products are manufactured by permanently deforming the metal. *Formability* is the ability of a material to be plastically deformed economically without defect. It has been experimentally proved that the higher the ductility and the lower the yield stress, the better is the formability of a material. *Formability* enables us to manufacture a product by metal forming with lower forces, power, and cost. Formability can be measured by forming limit diagrams (FLDs). An *FLD* is a [graphical representation](#) of the limits to forming, i.e. the major and minor stresses, where local necking occurs (Uijl & Carless, 2012). FLDs are extensively used in the analysis of sheet metal forming to define the limit of deformation of materials without necking or fracture (Paul, 2013).

3.2 Stress–Strain Curve: Plastic Flow and Strain Hardening

3.2.1 The Stress–Strain Curve and Plastic Flow

The plasticity of a material strongly depends on its ductility—% elongation and % reduction in area, which are generally determined by using a stress–strain curve. It is, therefore, important to get an insight into the plastic-deformation region of the stress–strain curve that results from a tension test. The engineering stress–strain curve for annealed mild steel (a ductile material) is shown in Fig. 3.1, which shows

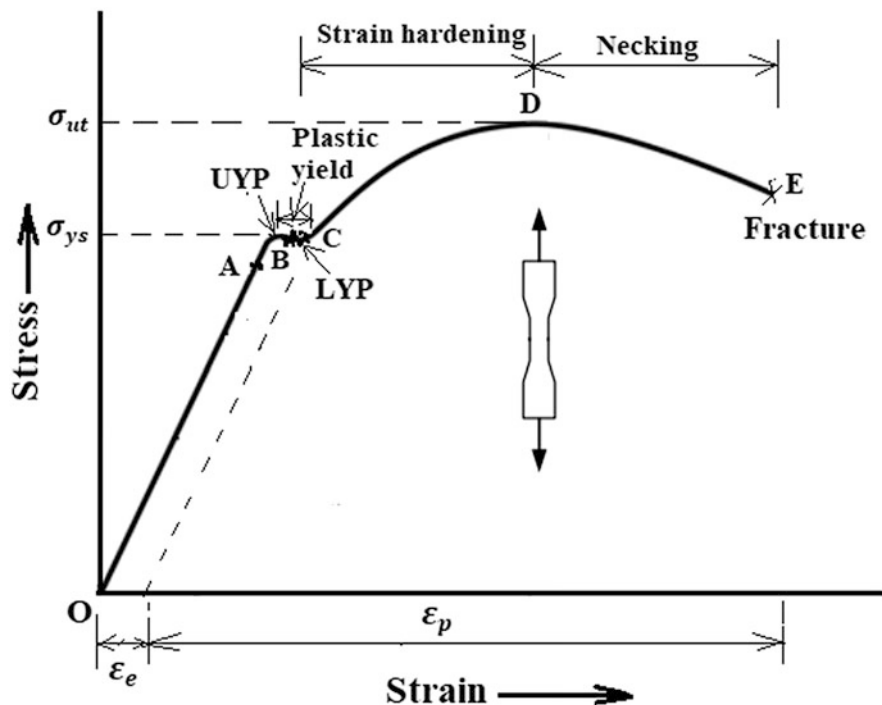


Fig. 3.1 Stress–strain curve for mild steel. UYP UPPER yield point, LYP lower yield point, ϵ_e = elastic strain, ϵ_p = plastic strain

that the total strain at any point is the sum of the elastic strain (ϵ_e) and the plastic strain (ϵ_p). The curve indicates that the stress–strain relation from O to A is linear elastic. The point A is the proportional limit, beyond which plastic deformation by yielding starts at the upper yield point (UYP) (see Fig. 3.1).

The stress at the UYP (point B) is called the yield stress (σ_{ys}), beyond which the yield stress drops to the lower yield point (LYP); the stress remains constant up to point C. The stretch BC is called the *plastic yield* or *plastic flow*. The yielding of steel is accompanied by the formation of so-called *Lüder's lines*, which have an angle of about 45° with the axis of the test sample. It must be noted that the curve in Fig. 3.1 represents the deformation behavior during loading; however the deformation behavior during unloading differs from that of the behavior for loading due to *Bauschinger effect*. The *Bauschinger effect* is the phenomenon by which plastic deformation of metal results in an increase in the yield strength in the direction of plastic flow and a decrease in the yield strength in the opposite direction (Huda, 2022).

3.2.2 Stress–Strain Curve and Strain Hardening

The curve CD, in Fig. 3.1, indicates that the stress increases with increasing strain with a nonlinear behavior; this phenomenon is called “*work hardening*” or “*strain hardening*.” The maximum engineering stress is reached at point *D*, after which the stress reduces because of necking of the test sample, until fracture occurs at *E*. The stress–strain curve (Fig. 3.1) enables us to determine both strength and ductility properties. For example, the strength properties that can be determined by stress–strain curve include the yield strength (σ_{ys}), the ultimate tensile strength (σ_{UTS}), and the fracture strength or breaking strength (σ_b); these tensile-mechanical properties can be determined by the following formulae:

$$\sigma_{ys} = \frac{F_{UYP}}{A_o} \quad (3.1)$$

$$\sigma_{UTS} = \frac{F_{max}}{A_o} \quad (3.2)$$

$$\sigma_b = \frac{F_b}{A_o} \quad (3.3)$$

where F_{UYP} is the force at the upper yield point, F_{max} is the maximum force during the tensile test, and F_b is the force at break. The yield strength and the ultimate tensile strength of mild steel (MS) are in the order of 200–400 MPa and 400–600 MPa, respectively. Table 3.1 lists the yield strength, the ultimate tensile strength, and ductility of some commonly used metals and alloys.

The ductility properties that can be determined by the stress–strain curve include the % elongation and the % reduction in area (% RA), which can be calculated as follows:

$$\% \text{Elongation} = \frac{L_f - L_o}{L_o} \times 100 \quad (3.4)$$

$$\% \text{RA} = \frac{A_o - A_f}{A_o} \times 100 \quad (3.5)$$

Table 3.1 Tensile mechanical properties of some metals and alloys

Material	Copper	Nickel	Aluminum	M.S	Brass (annealed)	Titanium	Cast iron
Yield strength (σ_{ys}), MPa	70	140	35	200–400	110	450	100–150
Tensile strength (σ_{UTS}), Mpa	200	480	90	400–600	330	520	300–900
% elongation (in $l_0 = 50$ mm)	45	40	40	30–50	62	25	0–7

where L_o is the original gage length, L_f is the final gage length at fracture, A_o is the original cross-sectional area, and A_f is the final cross-sectional area at fracture. The % elongation of mild steel is in the range of 30–50. It is important to note that a material that is weak and brittle in tension may behave as a stronger material with better ductility in compression; gray cast iron is an example of material with this plasticity behavior (see Fig. 3.10) (see Examples 3.1–3.4).

3.3 Plastic Instability in Uniaxial Tension

It has been learnt in the preceding section that as the “neck” appears during the tension test, the stress reduces and strain dramatically increases until fracture occurs; this *necking* phenomenon is called *plastic instability*. Thus, *plastic instability* may be defined as the rapid growth of a “neck” in a material upon the application of the maximum stress (see Fig. 3.2).

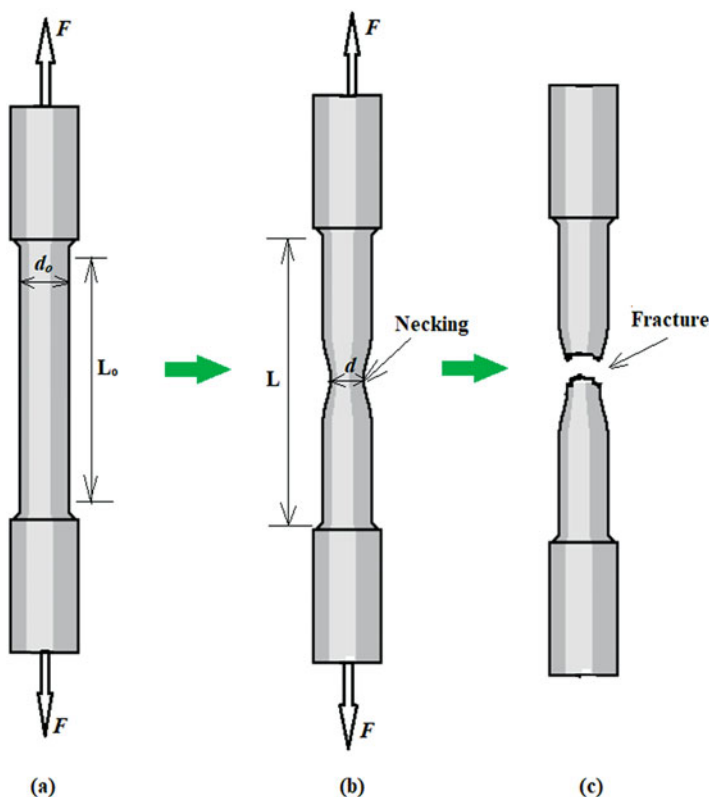


Fig. 3.2 Three main stages of the specimen during tension test; (a) original specimen, (b) plastic instability due to growth of neck, (c) fracture

In order to study the stress behavior during necking, we refer to Fig. 3.1. It is evident in Fig. 3.1 that the stress decreases with strain during necking; the fall in the stress continues until fracture (see Fig. 3.2). Now, a useful mathematical relationship for the rate of decrease in stress with respect to area during necking can be derived, as follows.

By using the definition of true stress, we can write:

$$F = \sigma A \quad (3.6)$$

Differentiating both sides of Eq. 3.6, we obtain:

$$dF = \sigma dA + A d\sigma \quad (3.7)$$

Because *necking* starts after reaching the maximum load, F , and since at the maximum load, F , there is no change in the load F , i.e., $dF = 0$, Eq. 3.7 can be rewritten as follows:

$$0 = \sigma dA + A d\sigma \quad (3.8)$$

or

$$A d\sigma = -\sigma dA$$

or

$$\frac{d\sigma}{\sigma} = -\frac{dA}{A} \quad (3.9)$$

$$\frac{d\sigma}{dA} = -\frac{\sigma}{A} \quad (3.10)$$

where σ is the true stress, and A is the instantaneous area of cross-section. According to Eq. 3.10, at necking the rate of decrease of the true stress with respect to the instantaneous area of cross-section is equal to the ratio of the true stress σ and the instantaneous area A (see Examples 3.5). By using the definition of true strain, we have:

$$\varepsilon = \ln \left(\frac{A_0}{A} \right)$$

Differentiating both sides, and by using the constant-volume relationship, we obtain:

$$-\frac{dA}{A} = \frac{dL}{L} = d\varepsilon \quad (3.11)$$

where L is the gage length at any instant of tensile testing, and ε is the true strain (see Example 3.6). By combination of Eqs. 3.9 and 3.11, we obtain:

$$\frac{d\sigma}{\sigma} = d\varepsilon \quad (3.12)$$

or

$$\frac{d\sigma}{d\varepsilon} = \sigma = \sigma_{\text{UTS}} \quad (3.13)$$

Equation 3.13 states that the rate of change of true stress with true strain (or the rate of strain hardening) is equal to the true stress, which corresponds to the ultimate tensile strength (due to necking); this relationship refers to the *condition of plastic instability* (see Example 3.7).

3.4 Formability and Its Improvement Techniques

Formability can be defined as the ability of a material to be deformed into a specific shape and size at a reasonably low force, power, and cost. It is the property of a material that enables us to manufacture complex parts with the utmost accuracy. This is why the trend in industry has been continuing towards the development of materials with improved formability. The formability of a material strongly depends on its yield strength and ductility; the higher the ductility and the lower the yield stress, the better is the formability of a material. It has been experimentally proved that the formability of a material improves with increasing temperature; this metal forming behavior can be verified by the plot in Fig. 3.3.

It is evident in Fig. 3.3 that the yield strength and the tensile strength of a metal decrease with temperature, whereas its ductility (% elongation) increases with temperature. Since a low yield stress and high ductility favor formability, metal forming at elevated temperatures results in improved formability. Sheet-metal forming finds wide applications in automotive, aerospace, and other engineering industries. Keum and co-researchers (2001) undertook a finite element analysis (FEA) of aluminum alloy sheet AA5052-H32 for non-isothermal warm forming; they conducted tensile tests at temperatures in the range of 18–300 °C. In the study, the following rate-sensitive power-law type constitutive equation was used (Keum et al., 2001):

$$\sigma = K \cdot \varepsilon^n \cdot (\dot{\varepsilon})^m \quad (3.14)$$

where σ is the true stress, ε is the true strain, n is the strain-hardening exponent of the material, K is the strength coefficient, $\dot{\varepsilon}$ is the strain rate, and m is the strain rate sensitivity index (see Sect. 3.5); the constants K , n , and m were expressed as functions of temperature.

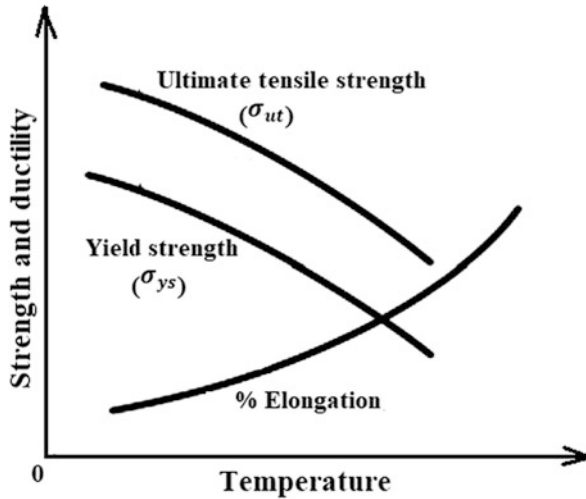


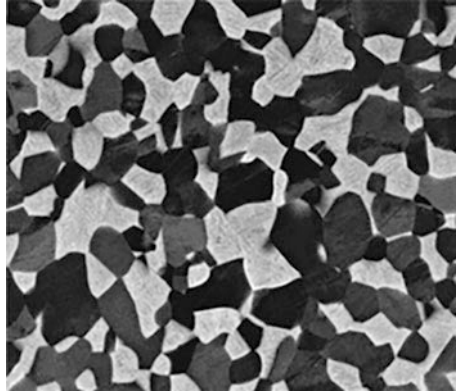
Fig. 3.3 Effects of temperature on the yield strength and ductility of alloys

Bagheriasl (2012) conducted forming experiments on a sheet-metal aluminum alloy: AA3003; he has reported that the key factors controlling warm formability are the dramatic reduction in flow stress and increase in positive rate sensitivity at elevated (warm) forming temperatures. The increased rate sensitivity promotes significant increases in post-uniform elongation (Bagheriasl, 2012). He has also reported that lubrication has an important (favorable) effect on formability since lower friction reduces thinning at the punch profile radius area but can cause a higher extent of wrinkling at the flange area; *Teflon* was found to be a much superior lubricant over the entire range of temperatures (100–300 °C) tested.

3.5 Superplasticity

Superplasticity, also called super-plastic forming (*SPF*), refers to the extraordinary plastic deformation (% elongation) of a material at high temperatures ($T > 0.5 T_m$) before failure, where T_m is the melting temperature in K. For example, the % elongation of mild steel is in the range of 30–50; but superplastic materials show extraordinarily high elongation approaching to 1000% under tension. *Superplasticity* enables us to manufacture parts with complex geometries at high temperatures by using fairly low stress. In order for a material to be superplastic, the following microstructural requirements must be fulfilled: (a) a fine grain size with an average grain diameter of up to 10 μm , (b) equi-axed grains, and (c) multi-phase microstructure (see Fig. 3.4).

Fig. 3.4 The multi-phase microstructure of an SPF material



Examples of alloys exhibiting superplasticity include selected high-strength low alloy (HSLA) steels, titanium alloys, aluminum alloys, magnesium alloys, and the like. An aerospace application of *SPF* is the manufacture of *Ti-6Al-4 V* aircraft-engine's compressor blades using *SPF* combined with diffusion bonding. *Superplasticity* is measured as the strain-rate sensitivity index (m), which is related to the true stress and the strain rate ($\dot{\epsilon}$) by:

$$\sigma = C (\dot{\epsilon})^m \quad (3.15)$$

where σ is the true stress; $\dot{\epsilon} = \frac{d\epsilon}{dt}$, and C is a temperature-dependent constant expressed as:

$$C = K \cdot \epsilon^n \quad (3.16)$$

The combination of Eqs. 3.15 and 3.16 yields Eq. 3.14. The significance of Eq. 3.15 is illustrated in Example 3.8. By taking logarithm of both sides of Eq. 3.15, we obtain:

$$\log \sigma = m \log \dot{\epsilon} + \log C \quad (3.17)$$

By plotting a graph of $\log \sigma$ versus $\log \dot{\epsilon}$, the strain rate sensitivity index can be calculated by:

$$m = \text{slope} = \frac{\Delta \log \sigma}{\Delta \log \dot{\epsilon}} = \frac{\log \sigma_2 - \log \sigma_1}{\log \dot{\epsilon}_2 - \log \dot{\epsilon}_1} \quad (3.18)$$

The essential condition for superplastic forming (*SPF*) is that the strain-rate sensitivity index (m) must be in the range of 0.4–0.9; higher the m value (in the specified range), greater would be the % elongation. The processing conditions for the *SPF* alloy *Ti-6Al-4 V* is presented in Table 3.2.

Table 3.2 Processing conditions for the SPF alloy Ti-6Al-4 V (Kalpakjian & Schmid, 2017)

Parameters	True strain	Strain rate, s^{-1}	Temperature, $^{\circ}C$	C, MPa	m
Value ranges	0.2–3	2×10^{-4}	815–930	65–11	0.5–0.8

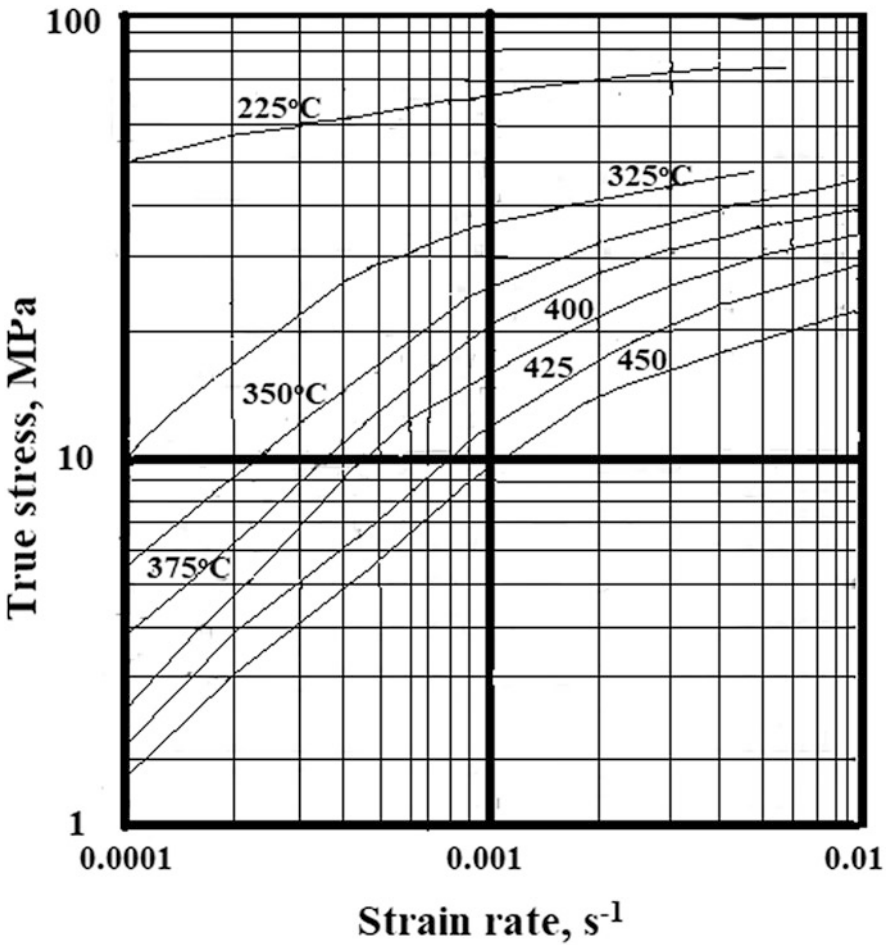


Fig. 3.5 Variation of true stress with strain rate and temperature for the alloy MgAZ31

The effects of elevated temperatures on the strain rate of a superplastic magnesium alloy MgAZ31 is illustrated in the logarithmic graphical plot, as shown in Fig. 3.5. The plot clearly indicates that the strain rate increases with temperature, provided the true stress is constant. Additionally, the true stress required for deformation decreases with increase in temperature (at a given strain rate), which means

that the formability is improved at elevated temperatures. The plot in Fig. 3.5 enables us to determine the ability for SPF at a particular temperature on the basis of the value of the strain-rate sensitivity index (m). It means that the MgAZ31 will behave as an SPF alloy if the m is in the recommended the range of 0.4–0.9 (see Examples 3.5–3.7).

3.6 Metallic Materials

Metallic materials (or metals and alloys) are inorganic materials having a crystalline structure as well as adequate strength and ductility; they have high electrical and thermal conductivities. Metallic materials find wide applications in structural engineering, automotive engineering, the energy sector, aerospace, biomedical, electrical, transport, beverage industries, and many more. Examples of metals are iron, aluminum, copper, zinc, nickel, and titanium. Alloys are solid solutions of one or more metals; they may also contain some nonmetallic elements. Examples of alloys are steels, brasses, bronzes, superalloys, and aluminum alloys. Steels are iron-based alloys; carbon steels contain carbon (up to 1.8 wt% C) and small amounts of impurities, whereas alloy steels contain substantial amounts of alloying elements (e.g., nickel, chromium, silicon, tungsten, vanadium). Brasses are alloys of copper and zinc.

Metals and alloys may be classified into two groups: *ferrous materials* and *nonferrous materials*. Ferrous materials contain a large percentage of iron (> 50 wt% Fe); examples of ferrous alloys are cast irons, carbon steels, and alloy steels. Non-ferrous metallic materials either contain no or a small amount of iron; examples of nonferrous materials are gold, silver, aluminum and their alloys, and copper and their alloys (see Fig. 3.6). Since cast irons are processed by the casting process, their discussion is out of scope of this book. An account of other metals and alloys is given in the following section.

3.7 Formable Ferrous Alloys

3.7.1 Steels: Definition, Classification, and Designation Systems

Steels are ferrous alloys containing less than 2 wt% carbon with small amounts of impurities; alloy steels contain large amounts (up to 50 wt%) of alloying elements in steels. They are broadly classified into two groups: (a) carbon steels, and (b) alloy steels (see Fig. 3.6). Steels are generally designated based on AISI (American Iron & Steel Institute) and SAE (Society of Automotive Engineers) numbering systems, which generally use four-digit numbers. The first two digits indicate the major

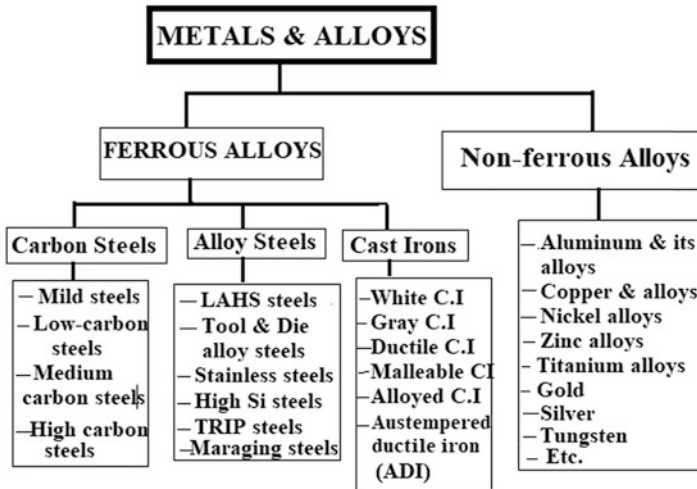


Fig. 3.6 Classification of metallic materials. *LAHS* low-allow high strength, *TRIP* transformation induced plasticity

alloying elements, and the last two digits refer to the wt% carbon in the steel. The digit series $1 \times \times \times$ indicates carbon steels; the $10 \times \times$ series refers to carbon steels containing 1.00% manganese (max.). For example, an AISI 1030 steel is a plain-carbon steel with 0.30 wt.% carbon. The AISI-SAE system classifies alloy steels using first digit for a specific alloying element (see Sect. 3.7.3).

3.7.2 Carbon Steels

Carbon steels are the steels that contain 0.05–1.5 wt% C; these steels generally contain less than 0.5% wt% silicon (Si) and 1.5% wt% manganese (Mn). In addition to C, Si, Mn, there may be small amounts of sulfur (S), phosphorous (P), oxygen (O), nitrogen (N), and hydrogen (H) in steels. In particular, the presence of significant hydrogen (> 0.002 wt% H) in steel is extremely detrimental to mechanical behavior of the material.

An increase in the carbon content from 0.01 to 1.5 wt% in carbon steel generally results in an increase in its hardness and strength with a corresponding decrease in ductility and toughness. The effects of carbon content on the strength and ductility of ferrite-pearlite steel is illustrated in Fig. 3.7. It is evident in Fig. 3.7 that the tensile strength (σ_{ut}) of steel rises linearly as its carbon content increases from 0.1 to 0.5 wt. %; beyond which, the rise in σ_{ut} follows a curve. However, ductility (% reduction in area) significantly decreases as carbon content is increased from 0.1 to 0.7 wt%.

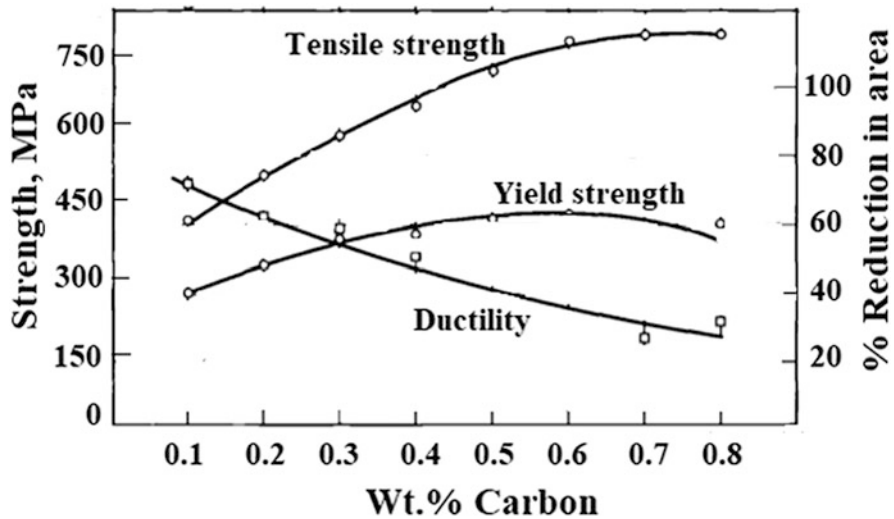


Fig. 3.7 The effects of carbon contents on the mechanical behavior of annealed steels

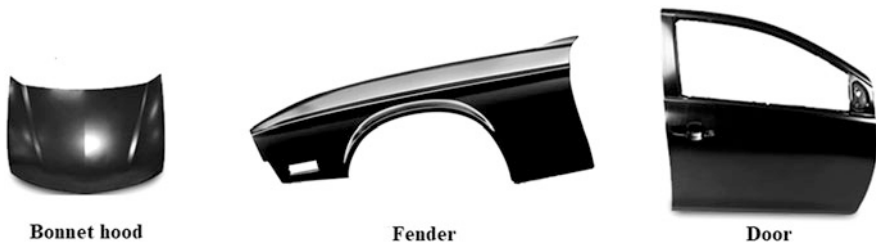


Fig. 3.8 Some of the low-carbon-steel parts that are used in car bodies

There are three main types of carbon steels: (a) low-carbon steels, (b) medium carbon steels, and (c) high carbon steels. Low carbon steels contain carbon in the range of 0.05–0.3 wt% (e.g., AISI 1004 (0.04% C steel), AISI 1008, AISI 1018 steel); they are ductile and highly formable. These steels cannot be effectively heat treated. *Low-carbon steels* find applications in screws, nails, wires, automobile body parts, and the like (see Fig. 3.8). *Medium carbon steels* contain carbon in the range of 0.2–0.6 wt% and manganese in the range of 0.6–1.6 wt% (e.g., AISI 1025, AISI 1050, AISI 1060). They have higher hardness and strength, and can be effectively heat treated; they are used for manufacturing gears, shafts, axles, and the like. *High carbon steels* contain 0.8–1.5 wt% carbon. They can be effectively hardened by heat treatment. Hardened (and tempered) high-carbon steels are used in making knives,

files, chisels, springs, etc. Low-carbon steels and medium-carbon steels can be easily processed by metal forming, whereas high-carbon steels are much harder to shape by metal forming.

3.7.3 Alloy Steels

An *alloy steel* is the steel to which one or more alloying elements (up to 50 wt.%) have been intentionally added for obtaining improved mechanical properties. The main alloying elements added to produce an alloy steel include manganese, silicon, nickel, chromium, molybdenum, tungsten, vanadium, cobalt, titanium, boron, copper, aluminum, and niobium. In particular, alloying with nickel (Ni), niobium (Nb), or vanadium (V) results in an increase in hardness and strength of steel without decreasing its ductility.

Alloy steels are classified based on the *AISI* designation system by using the first digit for alloying elements as follows: 2 for Ni steels, 3 for Ni-Cr steels, 4 for Mo steels, 5 for Cr steels, 6 for Cr-V steels, 7 for W-Cr steels, and 9 for Si-Mn steels. For example, a typical alloy steel with designation number AISI-SAE 4340 has the following composition ranges (wt.%): 0.37–0.43 C, 0.2–0.3 Mo, 0.6–0.8 Mn, 1.6–2.0 Ni, 0.7–0.9 Cr, 0.15–0.3 Si, 0.04 S, 0.035 P, balance: Fe (DeGarmo et al., 2003). The mechanical properties of alloy steels strongly depend on the nature and the amount of alloying elements and the heat treatment given to the steel. The alloy steel AISI-SAE 4140 is used in shafts, gears, and forgings; the steel 4340 is used in aircraft tubes, bushings, etc.

Alloy steels may be divided into the following types: (a) low-alloy high-strength (LAHS) steels, (b) stainless steels, (c) alloy tool and die steels, (d) Maraging steels, (e) Hadfields manganese steels, (f) high-silicon electrical steels, and (g) TRIP steels (see Fig. 3.9). Low alloy high strength (*LAHS*) steels are low-carbon steels

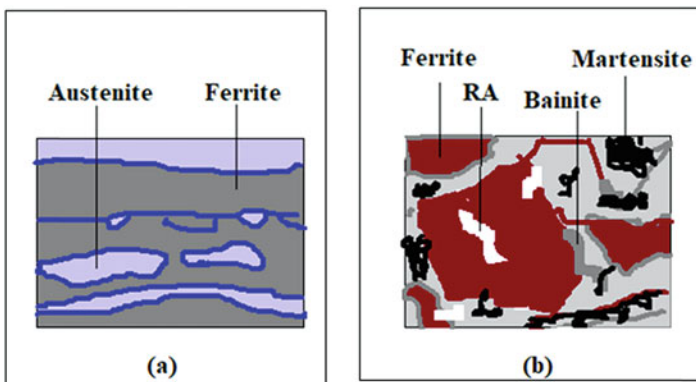


Fig. 3.9 Schematic microstructure of duplex stainless steel (a), and TRIP steel (b). RA retained austenite

strengthened by the addition of 1–4 wt% alloying elements (with up to 1.65 wt% Mn); these steels are sometimes strengthened by special rolling and heat-treatment techniques. Air hardening *LAHS* steel contains about 4.25 wt.% Ni and 1.25 wt.% Cr; an addition of 0.3 wt.% Mo renders nickel-chrome-molybdenum steels suitable for use in axles, shafts, gears, connecting rods, and other automotive applications.

Tool and die steels contain Cr, W, Mo, and V; which are carbide formers and stabilize ferrite and martensite. They possess high hardness, toughness, hardenability, and resistance to wear and corrosion. A typical composition of tool steel is as follows: Fe–0.8%Cr–18%W–4%Cr–1%V, which finds application as high-speed steel in making cutting tools. *Hadfield manganese steels* are the alloy steels that contain Mn (12–14 wt%) and C (1 wt%); these steels are austenitic at all temperatures and are nonmagnetic. Hadfield manganese steels are used in pneumatic drill bits, excavator bucket teeth, rock crusher jaws, ball mill linings and railway points and switches. When Hadfield manganese steel is water quenched from 1050 °C to retain carbon in solution, the soft core has a tensile strength of 849 MPa, a ductility of 40% and a Brinell hardness of 200; but after abrasion the surface hardness increases to 550 BHN. *Maraging steels* contain high Ni, Co, and Mo. A typical composition of Maraging steel is as follows: Fe–18%Ni–8%Co–5%Mo–0.4%Ti–0.1%Al and up to 0.05% C. Heat treatment involves solution treatment followed by quenching of the austenite to give a BCT martensitic structure. An additional aging heat treatment can produce finely dispersed precipitates of complex intermetallics (e.g., TiNi₃); this precipitation-strengthening results in high tensile strengths of around 2000 MPa. These steels are relatively tough with good corrosion resistance and good weldability, and find applications in aircraft undercarriage components, wing fittings, extrusion-press rams, and the like.

Stainless steels (SS) are corrosion-resistant alloy steels containing more than 12% Cr with nickel (Ni) (2–35 wt.%). Stainless steels can be classified into four types: (a) ferritic SS, (b) martensitic SS, (c) austenitic SS, and (d) duplex SS. A ferritic stainless steel (*FSS*) contains 12–25% Cr and up to 0.1% C. A typical composition of *FSS* is as follows: Fe–18%Cr–8%Ni–0.1%C; which is called 18–8 stainless steel. A martensitic stainless steel (*MSS*) contains 12–25% Cr and 0.1–1.5%C. It is produced by heating the steel into the austenite region and then quenched to give a martensitic structure. *MSS* has high strength and wear resistance with corrosion resistance; it is suitable for applications in hydroelectric turbines, knives, cutlery, and the like. Austenitic stainless steel (*ASS*) is a nonmagnetic SS that contains 16–26% Cr and up to 35% Ni. *ASS* has higher ductility than other types of SS, and can be cold-worked to produce deep-drawn shapes used in chemical plant, kitchenware, and architectural work. A duplex stainless steel (*DSS*) has a two-phase (duplex) microstructure consisting of ferrite and austenite (see Fig. 3.8a). *DSS* contains Cr, Mo, and Ni; *DSS* is about twice as strong as *ASS* or *FSS*. A *DSS* has significantly better impact toughness and ductility than *FSS*, and shows excellent resistance to stress corrosion cracking (SCC).

High silicon electrical steels are low-carbon steels containing up to 4 wt.% Si. These steels help to reduce eddy current losses. Electrical steels may be divided into three types: (a) non-grain oriented electrical steels (NGOES), (b) grain-oriented

electrical steels (GOES), and (c) low-carbon electrical steels (LCES). The *GOES* is produced by the following processing steps: (I) a hot-rolled coil of thick steel sheet is trimmed, (II) the sheet piece is annealed, de-scaled, and cleaned, and (III) the sheet metal is cold rolled via several passes to obtain a sheet thickness in the range of 0.20–0.35 mm (Mazurek, 2012). GOES possess magnetic properties that are strongly oriented along a [100] rolling direction, which is beneficial for electrical applications (e.g., transformer cores, electrical drive components).

Transformation Induced Plasticity (TRIP) steels are highly alloyed steels having high strength and excellent formability. *TRIP steels* possess a multi-phase microstructure: α -Fe (ferrite) + bainite + retained (metastable) γ -Fe (austenite) + martensite (see Fig. 3.8b). TRIP steels possess high energy absorption capacity and fatigue strength; they are well-suited for automotive structural and safety parts such as cross-members, longitudinal beams, bumper reinforcements, and frame rails.

3.8 Formable Non-ferrous Alloys

3.8.1 Aluminum and Its Alloys

Aluminum and its alloys are the most widely used non-ferrous materials. Aluminum (99.9% pure) has good ductility, corrosion resistance, and electrical conductivity; it finds applications in foils and conductor cables. Aluminum alloys (with proper heat treatment) are light-weight materials, and possess good strength and corrosion resistance; they find applications in aircraft structures, building construction, automotive engine components, etc. (Kaufman, 2000; Huda, 2016).

The classification systems for aluminum alloys have been developed for wrought alloys, cast alloys, and heat-treated alloys. Wrought aluminum alloys are strengthened by cold work; they have a 4-digit system. The cast alloys have a 3-digit and 1-decimal place system (see Table 3.3). In each designation system, the first digit indicates the principal alloying element. The heat treatable aluminum alloys have suffixes T3, T-36, T-351, etc. The series 1xxx *Al* alloys have good corrosion resistance; they are used in chemical-handling equipment. The series 6xxx alloys (e.g., 6061-T4) are used in automotive engines, pipelines, and aircraft fittings. The cast alloy 356.0 is used in automotive transmission cases and cylinder heads. The heat treatable alloys 2024-T3, 2024-T81, and 7075-T6 *Al* alloys are used in aircraft structures and other stressed applications. In particular, 7075-T6 is the most commonly used aerospace *Al* alloy owing to its high ultimate tensile strength of 510–535 MPa and yield strength of at least 434 MPa; it has elongation of 5–8%.

Table 3.3 The designations systems for cast, heat-treated, and wrought Al alloys

Cast alloys series	Principal alloying elements	Heat-treated alloys suffix codes	Heat treatment, temper, and post-process	Wrought alloys series	Principal alloying elements
1xx.x	99%Al	xxxx-T3	Solution heat treated (SHT), then cold worked (CW)	1xxx	99.xx%Al
2xx.x	Copper	xxxx-T36	SHT and CW (controlled)	2xxx	Copper
3xx.x	Si + Cu and/or Mg	xxxx-T351	SHT, stress-relieved stretched (SRS), then CW	3xxx	Manganese
4xx.x	Silicon	xxxx-T4	SHT then naturally aged	4xxx	Silicon
5xx.x	Magnesium	xxxx-T451	SHT then SRS	5xxx	Magnesium
6xx.x	Unused series	xxxx-T5	Artificially aged only	6xxx	Mg + Si
7xx.x	Zinc	xxxx-T6	SHT, artificial aging	7xxx	Zinc
8xx.x	Other elements	xxxx-T6	SHT, then artificially aged	8xxx	Other elements (e.g., Li)

3.8.2 Copper and Its Alloys

Copper has excellent electrical conductivity, and hence finds wide applications in electrical industry. Commonly used copper alloys include brasses (copper-zinc), bronzes (copper-tin), gun metal (Cu-Sn-Zn-Pb), Cu-Ni alloys, and nickel silver (Cu-Ni-Zn). Examples of copper alloys for high temperature applications (e.g., in boilers) include silicon bronzes, aluminum brasses, and copper nickels. Copper and its alloys find applications in the electrical industry, architecture, automotive, telecommunication, and mechanical engineering.

Brasses are the most important copper alloys; these include red brass, cartridge brass, clock brass, and *Muntz metal*. Alpha (α) brasses contain up to 38 wt.% zinc; they are relatively ductile with good formability, and find applications in deep drawn and cold-worked components (e.g., automotive radiator caps, battery terminals). Cartridge brass is an alpha brass with the composition: 70%Cu-30%Zn. Besides alpha phase, brasses also have other phases (β , γ , ϵ , η) that are stable at various temperatures corresponding to various compositions of Cu-Zn alloys. Beta (β) brasses are suitable for highly stressed applications, but they must be hot-worked due to their restricted ductility below 450 °C (if cold worked). Cold working significantly improves the strength of brass. The tensile strength of cold-worked cartridge brass wire may be as high as 900 MPa. Some alloying elements (lead, arsenic, nickel, iron, etc.) are added to the copper-zinc alloys for improving tensile strength, machinability, and the resistance to wear and corrosion.

Bronzes (or copper-tin alloys) have excellent corrosion resistance and good tensile strength and ductility. Bronzes, containing up to 15.8% Sn, retain the FCC

structure of alpha copper. Bronzes are used in bearings, gears, piston rings, valves, and fittings. Lead is added to bronzes to improve machinability for application in sleeve bearings. Since lead lowers the tensile strength, the composition of bronze is usually adjusted to balance machinability and strength.

3.8.3 *Nickel and Its Alloys*

Nickel has a low coefficient of thermal expansion, and finds application in bimetallic thermocouple sensors. Nickel alloys have an excellent tensile strength and resistance to creep and hot corrosion. *Monel metal* is a nickel-copper alloy with excellent corrosion resistance. *Monel 400* (63% Ni, 30% Cu, 2.3% Fe, and 1.8% Mn) is an exceptional alloy that is noted for maintaining its strength at sub-zero temperatures. *Superalloys* are the most important group of nickel alloys, which possess high tensile and creep strength with high resistance to hot corrosion. The main alloying elements in nickel-base superalloys include Cr, Fe, Mo, W, Ti, Al, C, Ta, Zr, Nb, and B. Ni-base superalloys find applications in hot-section components of gas-turbine (GT) engines and nuclear reactors. Some nickel-base superalloys developed for application in the GT blades include: *GTD-111*, Allvac 718Plus, and the single crystal superalloys (e.g., Rene-N6 and MC-534).

3.8.4 *Titanium and Its Alloys*

Commercial-purity 99 – 99.5% titanium has a low tensile strength ($\sigma_{ut} = 330 - 500$ MPa); however, when titanium is properly alloyed with Al, Cu, Mn, Mo, Sn, V, or Zr, its tensile strength significantly increases to 600 MPa (Ti–3.0Al–2.5 V) to as high as 1250 MPa (Ti–15Mo–5Zr–3Al). Titanium alloys have a good high-temperature strength and excellent stiffness ($E = 125$ GPa for the Ti–6Al–4 V alloy). The fatigue endurance strength at 10^7 cycles for titanium alloys is about 2/3rd of tensile strength. Titanium alloys have a low density of about 4500 kg/m^3 , which is slightly over half that of steel. Owing to their excellent specific strength combined and corrosion resistance, titanium alloys are extensively used in aerospace structures, gas turbine components (e.g., compressor blades), guided missiles, and the like. Additionally, a good biocompatibility of titanium alloys enables us to use them as biomaterials in artificial hips and other orthopedic bioengineering applications (Huda, 2020).

3.9 Worked Numerical Examples

Example 3.1: Determining the Tensile Strength and Compressive Strength of Gray C.I

Figure 3.10 shows the stress–strain curves for gray cast iron. Determine the (a) tensile strength, (b) breaking strength in tension, (c) yield strength in compression, (d) compressive strength, and (e) breaking strength in compression for the tested cast iron.

Solution

- (a) Tensile strength = 225 MPa
- (b) Breaking strength in tension = 225 MPa
- (c) Yield strength in compression = 310 MPa
- (d) Compressive strength = 700 MPa
- (e) Breaking strength in compression = 700 MPa

Example 3.2: Determining the Yield Strength, Tensile Strength, and Breaking Strength

A cylindrical specimen of stainless steel is pulled in tension. The test-specimen's original gage length is 50.8 mm and the original diameter is 12.8 mm. The force at the yield point is 93 kN. The maximum force during the tensile test is 160 kN, and the force at fracture is 125 kN. The final gage length at fracture is 57 mm. Determine the yield strength, the ultimate tensile strength, and the breaking strength of the steel.

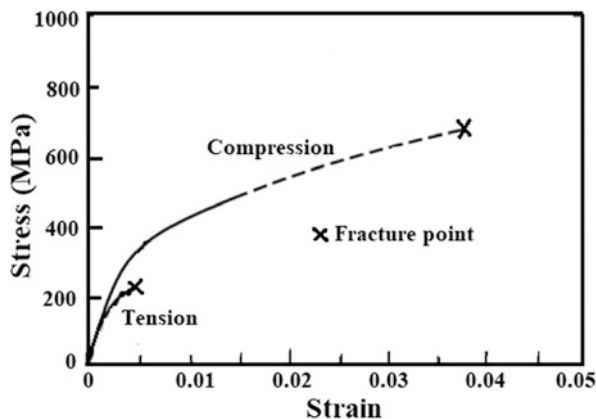


Fig. 3.10 Stress–strain curves for tension and compression for gray cast iron

Solution

$$L_o = 50.8 \text{ mm}, d_o = 12.8 \text{ mm}, L_f = 57 \text{ mm}, F_{\text{UYP}} = 93 \text{ kN}, F_{\text{max}} = 160 \text{ kN}, F_b = 125 \text{ kN}$$

$$A_o = \frac{\pi}{4} d_o^2 = 0.785 \times 12.8^2 = 128.614 \text{ mm}^2$$

By using Eqs. 3.1–3.3,

$$\sigma_{\text{ys}} = \frac{F_{\text{UYP}}}{A_o} = \frac{93000}{128.614} = 723.094 \text{ N/mm}^2 = 723.094 \text{ MPa}$$

$$\sigma_{\text{UTS}} = \frac{F_{\text{max}}}{A_o} = \frac{160000}{128.614} = 1244.032 \text{ MPa}$$

$$\sigma_b = \frac{F_b}{A_o} = \frac{125000}{128.614} = 971.9 \text{ MPa}$$

Example 3.3: Determining the Final Cross-sectional Area and the Final Diameter

By using the data in Example 3.2, determine the final cross-sectional area and the diameter at fracture for the specimen.

Solution

$$L_o = 50.8 \text{ mm}, A_o = 128.614 \text{ mm}^2, L_f = 57 \text{ mm}, A_f = ?$$

$$\text{Pre-test volume} = \text{Post-test volume}$$

$$L_o A_o = L_f A_f$$

$$A_f = \frac{L_o \cdot A_o}{L_f} = \frac{50.8 \times 128.614}{57} = 114.624 \text{ mm}^2$$

$$A_f = \frac{\pi}{4} d_f^2$$

$$\text{Final diameter} = d_f = \sqrt{\frac{A_f}{0.785}} = \sqrt{\frac{114.624}{0.785}} = \sqrt{146.018} = 12.084 \text{ mm}$$

Example 3.4: Determining the % Elongation and the % Reduction in Area

By using the data in Example 3.2, determine the % elongation and the % reduction in area for the stainless steel.

Solution

$$L_o = 50.8 \text{ mm}, A_o = 128.614 \text{ mm}^2, L_f = 57 \text{ mm}, A_f = 114.624 \text{ mm}^2$$

By using Eqs. 3.4 and 3.5,

$$\% \text{Elongation} = \frac{L_f - L_o}{L_o} \times 100 = \frac{57 - 50.8}{50.8} \times 100 = 12.2$$

$$\% \text{RA} = \frac{A_o - A_f}{A_o} \times 100 = \frac{128.614 - 114.624}{128.614} \times 100 = 10.877$$

Example 3.5: Calculating the Rate of Change of True Stress w.r.t the Instantaneous Area

A true stress of 800 MPa acts on a round specimen with a diameter of 5 mm at the start of necking during tensile testing (instantaneous gage length = 60 mm). Calculate the rate of change of stress with respect to instantaneous area of cross-section.

Solution

$$\text{The instantaneous area of cross-section} = A = \frac{\pi}{4} d^2 = 0.785 \times 5^2 = 19.625 \text{ mm}^2$$

By using Eq. 3.10,

$$\begin{aligned} \text{The rate of change of stress w.r.t. the instantaneous area} &= \frac{d\sigma}{dA} = -\frac{\sigma}{A} \\ &= -\frac{800}{19.625} = -40.76 \text{ N/mm}^4 \end{aligned}$$

Example 3.6: Calculating the Change in True Strain During Necking

By using the data in Example 3.5, calculate the small change in the true strain during necking.

Solution

$$A = 19.625 \text{ mm}^2, L = 60 \text{ mm}, d\varepsilon = ?$$

Equation 3.11 can be rewritten as follows:

$$-\frac{dA}{A} = \frac{dL}{L} = d\varepsilon$$

$$-\frac{dA}{dL} = \frac{A}{L} = d\epsilon$$

$$\text{The small change in true strain} = d\epsilon = \frac{A}{L} = \frac{19.625}{60} = 0.327$$

Example 3.7: Calculating the Rate of Strain Hardening at the Plastic Instability

An alloy specimen with original diameter of 12.5 mm is subjected to tension test. The maximum force during the test is 30 kN. Compute is the rate of strain hardening at the plastic instability.

Solution

$$\text{Original cross-sectional area} = A_o = \frac{\pi}{4} (d_o)^2 = 0.785 \times 12.5^2 = 122.656 \text{ mm}^2$$

$$\sigma_{ut} = \frac{F_{\max}}{A_o} = \frac{30000}{122.656} = 244.6 \text{ MPa}$$

By using Eq. 3.13,

$$\text{Therate of strain hardening} = \frac{d\sigma}{d\epsilon} = \sigma_{ut} = 244.6 \text{ MPa}$$

Example 3.8: Calculating the True Stress for SPF

A superplastic alloy is to be formed (processed) at a strain rate of $2 \times 10^{-4} \text{ s}^{-1}$ at a temperature of 900 °C. The constant $C = 60 \text{ MPa}$, and the strain-rate sensitivity index is 0.7. What true stress must be applied for the SPF?

Solution

By using Eq. 3.15,

$$\text{True stress} = \sigma = C (\dot{\epsilon})^m = 60 \times (2 \times 10^{-4})^{0.7} = 0.1545 \text{ MPa} = 154.5 \text{ kPa}$$

Example 3.9: Checking for SPF for the Alloy MgAZ31 by Calculating m at 225 °C

By reference to Fig. 3.5, calculate the strain-rate sensitivity index (m) of the magnesium alloy at 225 °C, and hence check for its behavior for SPF.

Solution

By using Fig. 3.5 for the temperature 325 °C, we obtain:

$$\sigma_1 = 10 \text{ MPa}, \dot{\epsilon}_1 = 0.0001 \text{ s}^{-1}, \sigma_2 = 78 \text{ MPa}, \dot{\epsilon}_2 = 0.001 \text{ s}^{-1}, m = ?$$

By using Eq. 3.18,

$$m = \frac{\log \sigma_2 - \log \sigma_1}{\log \dot{\epsilon}_2 - \log \dot{\epsilon}_1} = \frac{\log 78 - \log 50}{\log 0.001 - \log 0.0001} = \frac{1.892 - 1.699}{-3 + 4} = 0.193$$

Since m is out of the recommended range (0.4–0.9), the alloy would not behave SPF.

Example 3.10: Checking for SPF for the Alloy MgAZ31 by Calculating m at 325 °C

By reference to Fig. 3.5, calculate the strain-rate sensitivity index (m) of the magnesium alloy at 325 °C, and hence check for its behavior for SPF.

Solution

By using Fig. 3.5 for the temperature 325 °C, we obtain:

$$\sigma_1 = 10 \text{ MPa}, \dot{\epsilon}_1 = 0.0001 \text{ s}^{-1}, \sigma_2 = 38 \text{ MPa}, \dot{\epsilon}_2 = 0.001 \text{ s}^{-1}, m = ?$$

By using Eq. 3.18,

$$m = \frac{\log \sigma_2 - \log \sigma_1}{\log \dot{\epsilon}_2 - \log \dot{\epsilon}_1} = \frac{\log 38 - \log 10}{\log 0.001 - \log 0.0001} = \frac{1.58 - 1.00}{-3 + 4} = 0.58$$

Since m is within the recommended range (0.4–0.9), the alloy would behave SPF.

Example 3.11: Checking for SPF for the Alloy MgAZ31 by Calculating m at 400 °C

By reference to Fig. 3.5, calculate the strain-rate sensitivity index (m) of the magnesium alloy at 400 °C, and hence check for its behavior for SPF.

Solution

By using Fig. 3.5 for the temperature 400 °C, we obtain:

$$\sigma_1 = 2.7 \text{ MPa}, \dot{\epsilon}_1 = 0.0001 \text{ s}^{-1}, \sigma_2 = 17 \text{ MPa}, \dot{\epsilon}_2 = 0.001 \text{ s}^{-1}, m = ?$$

By using Eq. 3.18,

$$m = \frac{\log \sigma_2 - \log \sigma_1}{\log \dot{\epsilon}_2 - \log \dot{\epsilon}_1} = \frac{\log 17 - \log 2.7}{\log 0.001 - \log 0.0001} = \frac{1.23 - 0.431}{-3 + 4} = 0.799$$

Since m is within the recommended range (0.4–0.9), the alloy would behave SPF.

Example 3.12: Calculating the Constant C for the SPF Alloy MgAZ31

By using the data in Example 3.11, calculate the constant C for SPF alloy MgAZ31.

Solution

$$\sigma_1 = 2.7 \text{ MPa}, \dot{\epsilon}_1 = 0.0001 \text{ s}^{-1}, m = 0.799, C = ?$$

By using the modified form of Eq. 3.12,

$$C = \sigma (\dot{\epsilon})^{-m} = 2.7 \times 0.0001^{-0.799} = 4240$$

Questions and Problems

- 3.1 Define the following terms:
 (a) Plasticity, (b) formability, (c) forming limit diagram, and (d) superplasticity.
- 3.2 (a) Draw the engineering stress–strain curve for a ductile material (mild steel).
 (b) Explain the following regions of the curve: *plastic yield*, *strain hardening*, and *necking*.
- 3.3 Derive the mathematical relationship showing that at necking the rate of decrease of the true stress *w.r.t* the instantaneous area of cross-section is equal to the ratio of the true stress σ and the instantaneous area A .
- 3.4 What mechanical properties determine the formability of metals? Explain the effects of temperature and lubrication on formability?
- 3.5 (a) What are the microstructural requirements for an alloy to behave SPF?
 (b) List some alloys that behave SPF, and highlight at least an industrial application.
- P3.6 A superplastic alloy is to be formed (processed) at a strain rate of $2 \times 10^{-4} \text{ s}^{-1}$ at a temperature of 850 °C. The constant $C = 55 \text{ MPa}$, and the strain-rate sensitivity index is 0.6. What true stress must be applied for the SPF?
- P3.7 By reference to Fig. 3.5, calculate the strain-rate sensitivity index (m) of the magnesium alloy at 375 °C, and hence check for its behavior for SPF.
- P3.8 A true stress of 700 MPa acts on a round specimen with a diameter of 6 mm at the start of necking during tensile testing (instantaneous gage length = 70 mm). Calculate the rate of change of stress with respect to instantaneous area of cross-section.
- P3.9 By using the data in P3.8, determine the small change in the true strain during necking.
- P3.10 An alloy specimen with original diameter of 12,5 mm is being subjected to tension test.
 The maximum force during the test is 40 kN. Compute is the rate of strain hardening at the plastic instability.
- 3.11 (MCQs). Select the most appropriate answer for the following statements.
- The rapid growth of a “neck” upon the application of the maximum stress is called _____.
 (a) superplasticity, (b) plastic instability, (c) formability, (d) strain hardening, (e) FLD

2. The graphical representation of the limits to forming is called _____.
(a) SPF, (b) plastic instability, (c) formability, (d) strain hardening, (e) FLD
3. The region of engineering stress–strain curve where the stress increases with increasing strain with a nonlinear behavior is called _____.
(a) SPF, (b) plastic instability, (c) formability, (d) strain hardening, (e) FLD.
4. The extraordinary % elongation of a material at high temperatures before failure is called:
(a) SPF, (b) plastic instability, (c) formability, (d) strain hardening, (e) FLD
5. The ability of a material to be plastically deformed with lower forces and power is called:
(a) SPF, (b) plastic instability, (c) formability, (d) strain hardening, (e) FLD
6. Which of the following conditions favors the best formability?
(a) High σ_{ys} and high ductility, (b) Low σ_{ys} and high ductility, (c) low σ_{ys} and low ductility.
7. The constant-stress region between the UYP and the LYP in the stress–strain curve is called:
(a) Plastic flow, (b) plastic instability, (c) yield stress, (d) strain hardening.
8. Which type of steel has the best ductility and formability?
(a) AISI 1004, (b) AISI 1018, (c) AISI 1050, (d) AISI 1070
9. Which property of titanium renders it suitable for aerospace application?
(a) High strength, (b) low density, (c) high specific strength, (d) high stiffness
10. Which steel is used in making transformer cores?
(a) NGOES, (b) LCES, (c) GOES, (d) MSS
11. Which stainless steel is the most suitable for manufacturing deep-drawn products?
(a) FSS, (b) MSS, (c) DSS, (d) ASS

12. Which alloy steel possesses the best formability and strength combined with toughness?
 - (a) Stainless steels (b) TRIP steels, (c) LAHS steels, (d) Maraging steels
13. Which aluminum alloy is most widely used for aerospace structural applications?
 - (a) 2017, (b) 2024-T3, (c) 7075-T6, (d) 6061
14. Which copper alloy contains up to 38 wt% zinc?
 - (a) Commercial-purity copper, (b) beta brass, (c) bronze, (d) alpha brass.
- 3.12 Why is it much harder to process high-carbon steels by metal forming as compared to low- carbon and medium-carbon steels? Explain with reference to microstructures.
- 3.13 Why are high silicon steels used in making transformer cores? Explain the steps in manufacturing electrical steels.

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Chapter 4

Temperature-Dependence of Metal Forming



Nomenclature

A_o	original cross-sectional area
A_{CW}	cold-worked cross-sectional area
T_r	recrystallization temperature
T_m	melting temperature
ρ_D	dislocation density
σ_{flow}	flow stress
X	fraction of recrystallization
t	annealing time
ReX	recrystallization
Rate _{ReX}	rate of recrystallization
$t_{0.5}$	time required for 50% recrystallization
D_o	pre-annealing average grain diameter
D_t	average grain diameter after annealing for time-duration t
n	grain-growth exponent.

Abbreviations

CW	cold working
HW	hot working
RD	rolling direction
TD	transverse direction

4.1 Cold Working (CW) and Hot Working (HW)

4.1.1 Definitions of CW and HW Based on Working Temperature Ranges

The formability of a metal strongly depends on temperature. Based on the working temperature, metal forming can be divided into three groups: (a) cold working, (b) warm working, and (c) hot working. *Cold working* (CW) refers to metal forming at a temperature below $0.3 T_m$, where T_m is the melting temperature (K). Metal forming that is performed at a temperature between $0.3 T_m$ and $0.5 T_m$, may be termed as *warm working* (see Fig. 4.1). *Hot working* (HW) is the metal forming at a temperature between $0.5 T_m$ and $0.75 T_m$. In industrial practice, HW processes include hot rolling, hot forging, and hot extrusion; the examples of CW processes include cold rolling, cold extrusion, wire drawing, and sheet-metal forming processes (e.g. shearing, bending, deep drawing).

4.1.2 Definitions of CW and HW Based on Recrystallization Temperature

The distinction between cold working (CW) and hot working (HW) is actually based on the recrystallization temperature, which is explained as follows. *Cold working* is the metal forming that is performed at a temperature *below* the recrystallization temperature (T_r) of the metal. Warm working is done at temperatures around T_r . On the other hand, *hot working* is done at a temperature *above* the recrystallization temperature. The recrystallization temperature for pure metals can be related to their melting temperature, as follows:

$$T_r \approx 0.4 T_m \quad (4.1)$$

where T_r is the recrystallization temperature (measured in Kelvin, K) and T_m is the melting temperature (in K) (see Example 4.1). It may be noted that the

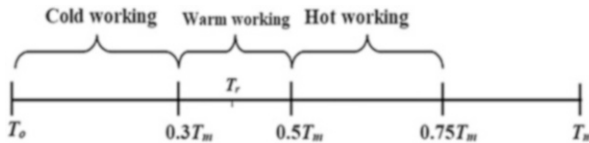


Fig. 4.1 Working temperature ranges for metal forming. T_m melting temperature, K, T_r recrystallization temperature for pure metals, K, T_o ambient temperature, K)

recrystallization temperature is not constant for a metal; it depends on the amount of cold work, the annealing time, and other factors. The terms “*recrystallization*” and the “*recrystallization temperature*” are explained in detail in Sect. 4.3.

The recrystallization temperatures (T_r) of ferrous metals are essentially above ambient temperature; however, T_r of non-ferrous metals may be either above or below the ambient temperature. For example, T_r of aluminum and copper are around 100 °C and 270 °C, respectively; but T_r of tin is around −71 °C (see Example 4.1). In general, most ductile materials are cold worked at ambient temperature. For example, aluminum ($T_r \approx 100^\circ\text{C}$) is cold worked at ambient temperature. However, there are exceptions to this rule in the case of metals with sub-zero recrystallization temperatures. For example, tin ($T_r \approx -71^\circ\text{C}$) is *hot worked* at ambient temperature.

4.1.3 Advantages and Disadvantages of Cold Working and Hot Working

Having distinguished between CW and HW, it is now important to highlight the advantages and disadvantages of the two groups of metal forming. Cold working is usually performed at ambient temperature for forming ductile materials (e.g. aluminum, mild steel, annealed brass); a greater force is required in CW as compared to HW. On the other hand, hot working is performed for deforming high-strength alloys (e.g. medium-carbon steel, alloy steels, superalloys). A lower force is required in HW since metals are easier to deform at high temperatures. The advantages and the disadvantages of the two groups of metal forming are presented in Table 4.1.

Table 4.1 Advantages and disadvantages of cold working and hot working

	Advantages	Disadvantages
Cold working	1. Results in enhanced strength 2. Results in better surface finish 3. No pre-heating is required 4. Suitable for forming ductile materials	1. Results in loss of ductility 2. Requires a greater force 3. Requires additional processing of recrystallization annealing; 4. Unsuitable for forming high-strength alloy
Hot Working	1. Requires a lower force; 2. Suitable for forming high- strength alloys; 3. No additional (anneal) processing is required after HW	1. Requires pre-heating in a furnace; 2. Surface finish is just fair. 3. Scale (oxide layer) may be formed after HW

4.2 Cold Working and Strain Hardening

4.2.1 Microstructural Changes During Cold Working

It is learnt in the preceding section that cold working (CW) is the metal forming at a temperature below $0.3 T_m$ (see Fig. 4.1). When a metallic material is cold worked (plastically deformed) beyond its yield point, dislocations moving through the material entangle with one another thereby hindering dislocation motion (see Fig. 4.2); thus, a greater stress is required to produce additional plastic deformation. This phenomenon of strengthening is called *strain hardening* (Huda, 2018; Guo, 2005).

A small percentage of the energy expended in plastically deforming a material, during cold working, remains stored in the metal as an increase in internal energy. The increment in the stored internal energy results in an increase in the dislocation density; the latter is defined as the number of dislocations in a unit area of the material's sample. Mathematically (Callister, 2007),

$$\rho_D = \frac{\text{Number of dislocations}}{\text{Area}} \quad (4.2)$$

where ρ_D is the dislocation density (lines/cm²), and the “area” is the area of the etched sample of the material (cm²); the number of dislocations is measured as the number of etch pits, (see Example 4.2). The dislocation density, for most annealed metals, increases from about 10^8 lines/cm² to around 10^{10} lines/cm² after a small degree of deformation; however, the dislocation density may increase to 10^{14} lines/cm² after heavy deformation (Huda & Bulpett, 2012).

When a metal is cold worked, there is a significant change in its microstructure. For example, when a metallic material is cold rolled, the grains are elongated in the rolling direction (RD), as shown in Fig. 4.3. The elongated-grained microstructure results in directional properties in the CW material. In particular, the hardening and

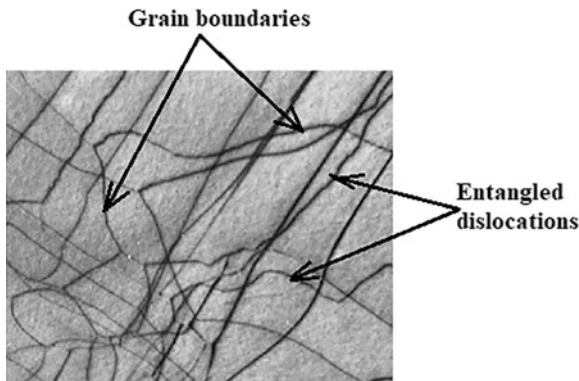


Fig. 4.2 TEM micrograph showing entangled dislocations due to strain hardening

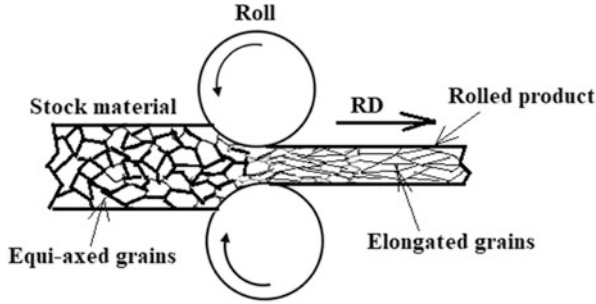


Fig. 4.3 The effect of cold rolling on the microstructure. *RD* rolling direction

strengthening are more effective in the RD as compared to the transverse direction (TD) (Ağca & Çankaya, 2017).

4.2.2 Mechanical Properties' Changes During Cold Working

Strain hardening of metal results in a sudden rise in the dislocation density, which in turn results in an increase in hardness and the flow stress of the material. The flow stress, for many metals, may be related to the dislocation density by the following formula (Schaffer et al., 1999):

$$\sigma_{\text{flow}} = \sigma_0 + k \sqrt{\rho_D} \quad (4.3)$$

where σ_{flow} is the flow stress (MPa), ρ_D is the dislocation density (lines/cm²), and σ_0 and k are the constants for the material with units of MPa-cm and MPa, respectively (see Example 4.3).

Cold working results in significant changes in both physical and mechanical properties of the material. In particular, there is a marked increase in hardness and electrical resistivity with the amount of cold working. *Strain hardening* of metal results in an increase in hardness and strength but a reduction in ductility. Thus, a cold-worked metal is harder (more difficult) to further deform by cold working (CW); this problem can be overcome by an additional process of recrystallization annealing (see Sect. 4.3). The degree of cold work or % cold work (% CW) can be calculated by the following formula:

$$\%CW = \left(\frac{A_o - A_{CW}}{A_o} \right) \times 100 \quad (4.4)$$

where A_o is the original cross-sectional area, and A_{CW} is the cold-worked (deformed) cross-sectional area (see Example 4.4).

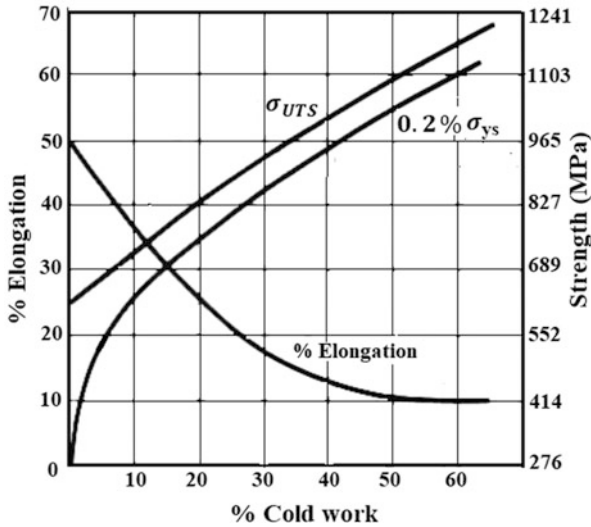


Fig. 4.4 The effects of degree of cold work on the tensile mechanical properties of rebar. σ_{UTS} ultimate tensile strength, σ_{ys} yield strength

The tensile mechanical properties of a cold-worked material strongly depend on the degree of cold work (%CW). The variations of the tensile strength, the yield strength, and % elongation with the % CW for reinforced iron bar (rebar) are illustrated in Fig. 4.4. It is evident in the graphical plot that the strength properties increase with the degree of deformation (% CW), whereas the ductility (% elongation) decreases with the % CW. The rise in strength properties is due to strain hardening, whereas the fall in ductility is due to the stored (internal) energy that makes the material difficult to further deformation by CW. The significance of the graphical plot (Fig. 4.4) is illustrated in Examples 4.5 and 4.6.

4.3 Annealing: Recovery, Recrystallization, and Grain Growth

4.3.1 What Is Annealing?

It is learnt in the preceding section that cold working of a material results in strain hardening, i.e. in an increase in hardness and strength, but a reduction in ductility. Thus, a cold-worked material is difficult to deform for further processing (CW); this difficulty can be overcome by treating the cold-worked material by recrystallization annealing. Annealing is a thermal treatment that involves three stages: (I) recovery, (II) recrystallization, and (III) grain growth; each of these stages of annealing is

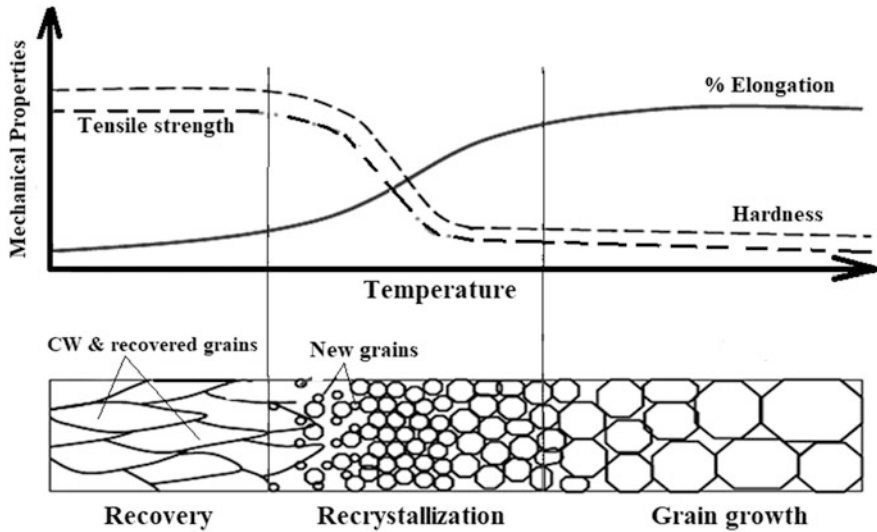


Fig. 4.5 The effects of annealing temperature on the microstructure and the mechanical properties

explained in the following sub-section (Sect. 4.3.2). Annealing treatment of cold-worked metals results in significant changes in the microstructure and mechanical properties. In particular, the annealing temperature and the duration control the grain size, grain geometry, and the mechanical properties (hardness, strength, and ductility) (see Fig. 4.5).

4.3.2 Recovery

Recovery is the first stage of annealing of cold-worked metals; it is carried out at temperatures around $0.25 T_m$, where T_m is the melting temperature (K). *Recovery* results in release of the stored internal energy; however, there is no change in the microstructure and mechanical properties of the metal. The recovery processes, which lower the internal energy, are associated with the motion of atoms and vacancies; there is a rearrangement of defects in the cold-worked (CW) grains. Thus, recovery eliminates the residual stresses in the material.

4.3.3 Recrystallization

Recrystallization (ReX), the second stage of annealing, is carried out at a temperature slightly above $0.4 \cdot T_m$, which is called the recrystallization temperature (T_{ry}) (see

Eq. 4.1). At the temperature T_{ry} , atomic mobility becomes high enough to allow recrystallization (re-arrangement of grains). The recrystallization temperature is not a constant for a material; it depends on the degree of cold work (% CW), the annealing time, and other factors. The recrystallization temperature is usually one-third to one-half the melting temperature (in K).

Recrystallization treatment involves nucleation and growth of new grains; thus, a completely new set of strain-free grains is developed. The cold-worked (elongated) grains are transformed to equi-axed grains with some grain refinement during recrystallization (see Fig. 4.5). It is the recrystallization stage of annealing that mainly restores the ductility of the CW metal. Recrystallization temperature, T_{ry} , is the minimum temperature at which complete recrystallization occurs in an annealed cold-worked metal within a specified time. Recrystallization (ReX) is a time-dependent process; the time dependence of the fraction of statically recrystallized material is commonly described through Avrami's general equation, as follows (Tajjaly & Huda, 2011; Callister, 2007; Huda, 2020):

$$X = 1 - \exp(-k t^n) \quad (4.5)$$

where X is the fraction of recrystallization, t is the annealing time, and k and n are time-independent constants for the particular process (see Examples 4.7 and 4.8). The kinetics of static recrystallization is often characterized in terms of the time required for recrystallization of 50% of the material, i.e. $t_{0.5}$. The rate of recrystallization ($Rate_{ReX}$) can be computed as the reciprocal of the time required for recrystallization of 50% of the material, i.e. $t_{0.5}$, as follows:

$$Rate_{ReX} = \frac{1}{t_{0.5}} \quad (4.6)$$

The significance of Eq. 4.6 is illustrated in Example 4.9.

4.3.4 Grain Growth

Grain growth, the third stage of annealing, refers to the increase in the average grain size of the metal when the metal is heated beyond the completion of recrystallization. There is a significant loss of tensile strength and hardness due to grain growth (see Fig. 4.5). The control of grain growth is of great technological importance because many mechanical properties strongly depend on the grain size and its distribution (Huda et al., 2014; Huda & Ralph, 1990).

Grain growth involves movement of grain boundaries, resulting in reduction of grain boundary area. A metal consisting of small grains has a large surface energy, i.e. large grain boundary area per volume; this large amount of surface energy in the

material provides a driving force for grain growth. Grain diameters, during grain growth, can be related to grain-growth exponent n by (Huda et al., 2023):

$$D_t - D_o = k \cdot t^n \quad (4.7)$$

where D_o is the pre-annealing average grain diameter, D_t is the average grain diameter after annealing for time-duration t , k is the constant, and n is the grain-growth exponent.

4.4 Hot Working of Metals

It has been learnt in Sect. 4.1 that hot working (HW) refers to metal forming at temperatures above the recrystallization temperature. In the case of a perfectly plastic material with strain hardening exponent $n \cong 0$, cold working is not feasible, i.e. metal must be formed by HW. In comparison to CW, lower forces and power are required in HW; here no strain hardening occurs. It means that the metal is still ductile on completion of HW. A hot-rolled part can be subsequently cold formed. Major limitations of HW include lower dimensional accuracy, higher total energy required, and poorer surface finish, including the formation of oxide scale.

Carbon steels, for HW, are generally heated in the temperatures range of 1000–1250 °C in reheating furnaces; these furnaces are designed with three burner zones: (I) preheating zone, (II) heating zone, and (III) soaking zone (Davies, 2016). Great care must be exercised in heating the metal for hot work (HW) with particular reference to temperature and time duration. The melting temperatures of most carbon steels are in the range of 1410–1500 °C. This is why these steels are generally hot worked at temperatures in the range of 1000–1250 °C (see Example 4.10). Any attempt to carry out hot working of steel below 1000 °C may be harmful to the equipment. On the other hand, grain growth will occur if the hot-working temperature is above 1250 °C; prolonged annealing at a reasonably high temperature (overheating) will also result in grain growth.

Hot working of metals is associated with significant changes in the microstructure (see Fig. 4.6). The stock material, prior to rolling, is at a temperature remarkably above the recrystallization temperature, which is why the grains are equi-axed (Fig. 4.6). On rolling, the crystals (grains) tend to become elongated in the rolling direction (RD); however, a few moments later, new grains appear due to the beginning of recrystallization. As hot rolling progresses, recrystallization is completed resulting in refined equi-axed (recrystallized) grains (see the microstructure of the hot-rolled product in Fig. 4.6).

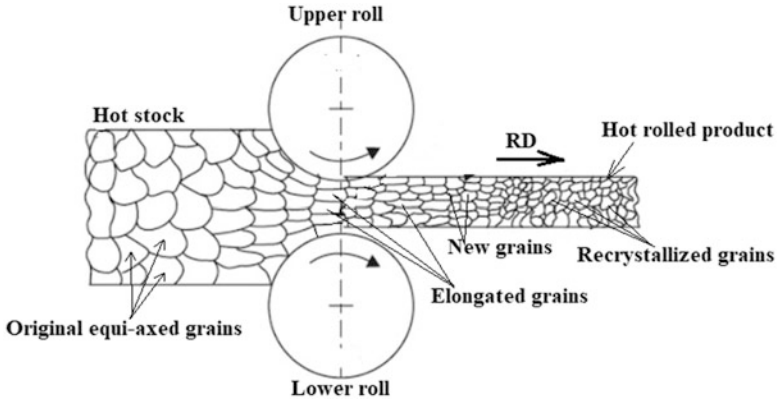


Fig. 4.6 The schematic of microstructural changes during hot rolling of steel

4.5 Worked Numerical Examples

Example 4.1: Calculating the Recrystallization Temperatures of Copper and Tin

The melting temperature of copper is 1085°C and that of tin is 232°C . Calculate the recrystallization temperatures of the two metals.

Solution

$$T_{m(\text{copper})} = 1085^{\circ}\text{C} = 1085 + 273 = 1358\text{ K}, T_{m(\text{tin})} = 232^{\circ}\text{C} = 232 + 273 = 505\text{ K}$$

By using Eq. 4.1,

$$\begin{aligned}\text{Recrystallization temperature of copper} = T_r &\approx 0.4 T_m = 0.4 \times 1358 = 543.2\text{ K} \\ &= 270.2^{\circ}\text{C}\end{aligned}$$

$$\text{Recrystallization temperature of tin} = T_r \approx 0.4 T_m = 0.4 \times 505 = 202\text{ K} = -71^{\circ}\text{C}$$

Example 4.2: Calculating the Dislocation Density

There are 7×10^6 dislocations (etch pits) in a 3 mm^2 area in an etched sample of a deformed metal. What is the dislocation density in the metal's sample?

Solution

$$\text{Area} = 3\text{ mm}^2 = 3 \times (10^{-1}\text{ cm})^2 = 3 \times 10^{-2}\text{ cm}^2, \rho_D = ?$$

By using Eq. 4.2,

$$\begin{aligned}\text{Dislocation density} = \rho_D &= \frac{\text{Number of dislocations}}{\text{Area}} = \frac{7 \times 10^6}{3 \times 10^{-2}} \\ &= 2.33 \times 10^8 \text{ lines} \cdot \text{cm}^{-2}\end{aligned}$$

Example 4.3: Calculating the Flow Stress When the Dislocation Density Is Known

Calculate the flow stress of the cold-worked metal with a dislocation density of 2×10^{11} lines/cm². The constants are $k = 3.82 \times 10^{-4}$ MPa · cm and $\sigma_0 = 1.8$ MPa.

Solution

$$\rho_D = 2 \times 10^{11} \text{ lines/cm}^2, k = 3.82 \times 10^{-4} \text{ MPa-cm}, \sigma_0 = 1.8 \text{ MPa}, \sigma_{\text{flow}} = ?$$

By using Eq. 4.3,

$$\sigma_{\text{flow}} = \sigma_0 + k \sqrt{\rho_D} = 1.8 + \left(3.82 \times 10^{-4} \times \sqrt{2 \times 10^{11}} \right) = 1.8 + 170.83 = 172.63 \text{ MPa}$$

Example 4.4: Calculating the Percent Cold Work

A 3.5-mm-thick sheet metal was cold rolled to 2-mm thickness with no change in width. Calculate the percent cold work.

Solution

$$A_0 = t_0 \cdot w = 3.5 \times w \text{ mm}^2 = 3.5 w$$

$$A_{\text{CW}} = t_f \cdot w = 2 \times w \text{ mm}^2 = 2 w$$

By using Eq. 4.4,

$$\% \text{CW} = \left(\frac{A_0 - A_{\text{CW}}}{A_0} \right) \times 100 = \left(\frac{3.5 w - 2 w}{3.5 w} \right) \times 100 = 42.86$$

Example 4.5: Determining the Tensile Mechanical Properties When % CW Is Known

Rebar was deformed by 40% CW. What are the approximate tensile mechanical properties after cold working?

Solution

By reference to Fig. 4.4, the 40% CW rebar has the following approximate tensile properties:

$$\begin{aligned}\text{Ultimate tensile strength} &= 1050 \text{ MPa}, 0.2\% \text{ yield strength} \\ &= 940 \text{ MPa}, \% \text{ elongation} = 13.\end{aligned}$$

Example 4.6: Selecting the Best Combination of Tensile Properties in CW Metal

By reference to Fig. 4.4, determine the % CW to obtain the best combination of tensile strength and ductility.

Solution

From Fig. 4.4, the % CW, to obtain the best combination of tensile strength and ductility, is around 13; the 13%CW corresponds to the following mechanical properties:

$$\sigma_{UTS} = 710 \text{ MPa and } \% \text{ elongation} = 34$$

Example 4.7: Determination of the Constant k in the Avrami Equation

The kinetics of recrystallization for an alloy follows the Avrami relationship (Eq. 4.5), and that the value of the numerical exponent n is 3.5. Calculate the constant k , if 50% recrystallization is complete after 150 s.

Solution

$$n = 3.5, t = 150 \text{ s}, X = 50\% = 0.50, k = ?$$

By using the Avrami equation (Eq. 4.5),

$$1 - \exp(-k t^n) = X$$

or

$$\exp(-k t^n) = 1 - X$$

or

$$-k t^n = \ln (1 - X)$$

or

$$k = -\frac{\ln(1 - X)}{t^n} = -\frac{\ln(1 - 0.5)}{150^{3.5}} = -\frac{-0.693}{41335139.4} = 1.67 \times 10^{-8}$$

Example 4.8: Determination of the Recrystallization Time by Using the Avrami Equation

By using the data in Example 4.7, determine the time required to complete 98% recrystallization.

Solution

$$n = 3.5, X = 98\% = 0.98, k = 1.67 \times 10^{-8}, t = ?$$

The Avrami equation can be modified as follows:

$$t^n = - \frac{\ln(1-X)}{k}$$

$$t^{3.5} = - \frac{\ln(1-0.98)}{1.67 \times 10^{-8}} = - \frac{-3.91 \times 10^8}{1.67} = 2.341 \times 10^8$$

$$3.5 \log t = \log 2.341 + 8 = 0.3694 + 8 = 8.3694$$

$$\log t = 2.391$$

$$t = 10^{2.391} = 246$$

The time to complete 98% recrystallization = 246 s = 4.1 min.

Example 4.9: Computing the Rate of Recrystallization

Compute the rate of recrystallization of the alloy with the parameters stated in Example 4.7.

Solution

The time to complete 50% recrystallization = $t_{0.5} = 150$ s.

By using Eq. 4.6,

$$\text{Rate}_{\text{ReX}} = \frac{1}{t_{0.5}} = \frac{1}{150} = 0.0066/\text{s}$$

The rate of recrystallization of the alloy = 0.0066 s^{-1}

Example 4.10: Calculating the Recrystallization Temperature of Steel for Hot Working

The melting temperature of a carbon-steel is 1450 °C. Calculate the recrystallization temperature of the steel. Will it be feasible to perform hot working of steel at 400 °C?

Solution

$$T_m = 1450^\circ \text{C} = 1450 + 273 = 1723 \text{ K}$$

By using Eq. 4.1,

$$\text{Recrystallization temperature} = T_r \approx 0.4 T_m = 0.4 \times 1723 = 689 \text{ K} = 416^\circ \text{C}$$

It is not feasible to perform hot working of steel at 400 °C.

Example 4.11: Calculating the Grain-Growth Exponent and the Constant k

The pre-growth grain diameter of the steel alloy 347H is measured to be 31.4 μm . The grain diameter after grain-growth annealing at 900 $^{\circ}\text{C}$ /30 min is 48.3 μm . In another annealing experiment at 900 $^{\circ}\text{C}$ /480 min, the post-growth grain diameter is 54.2 μm . Calculate the grain-growth exponent n and the constant k .

Solution

$$D_o = 31.4 \mu\text{m}, t_1 = 30 \text{ min}, D_{t1} = 48.3 \mu\text{m}, t_2 = 480 \text{ min}, D_{t2} = 54.2 \mu\text{m}, \\ n = ?, k = ?$$

By using Eq. 4.7,

$$\begin{aligned} D_{t1} - D_o &= k \cdot t_1^n \\ 48.3 - 31.4 &= k \cdot 30^n \\ 16.9 &= k \cdot 30^n \\ \log k + n (\log 30) &= \log 16.9 \\ \log k + 1.477 n &= 1.228 \end{aligned} \quad (4.8)$$

$$\begin{aligned} D_{t2} - D_o &= k \cdot t_2^n \\ 54.2 - 31.4 &= k \cdot 480^n \\ \log k + n (\log 480) &= \log 22.8 \\ \log k + 2.68 n &= 1.358 \end{aligned} \quad (4.9)$$

By simultaneously solving Eqs. 4.8 and 4.9, we obtain:
 $k = 11.708$, and $n = \text{grain-growth exponent} = 0.108$

Questions and Problems

- 4.1 Differentiate between cold working, warm working, and hot working.
- 4.2 Compare cold working and hot working highlighting their advantages and disadvantages.
- 4.3 Is it possible to do hot working at ambient temperature? Explain.
- 4.4 (a) What is the effect of CW on dislocation density?
 (b) Explain the effect of cold rolling on the microstructure with the aid of a sketch.
- 4.5 (a) What changes in mechanical properties are associated with strain hardening?
 (b) How is a strain hardened material treated?
- 4.6 Explain recrystallization annealing process with particular reference to energy changes.
- 4.7 Why is it important to exercise great care in heating the metal for hot work (HW) with particular reference to the temperature and time duration?

P4.8 There are 9×10^7 dislocations (etch pits) in 4 mm^2 area in an etched sample of a deformed metal. Calculate the flow stress of the cold-worked metal, if the constants are:

$$k = 3.82 \times 10^{-4} \text{ MPa-cm, and } \sigma_0 = 1.8 \text{ MPa.}$$

P4.9 A 5-mm-diameter rebar was cold rolled to 3-mm diameter. What are the approximate tensile mechanical properties after the CW? Hint: Refer to Fig. 4.4.

P4.10 The kinetics of recrystallization for an alloy follows the Eq. 4.5, with the numerical exponent $n = 3.5$. It is observed that 50% recrystallization is completed after 150 s.

Determine the time required to complete 95% recrystallization.

P4.11 The melting temperature of lead is 327°C . (a) Calculate the recrystallization temperature of lead. (b) Which method of metal forming will you prefer? HW or CW?

P4.12 The pre-growth grain diameter of an alloy is measured to be $31.4 \mu\text{m}$. The grain diameter after grain-growth annealing at 600°C for 120 min is $35.4 \mu\text{m}$. In another annealing experiment at $600^\circ\text{C}/1200 \text{ min}$, the post-growth grain diameter is $38.3 \mu\text{m}$. Calculate the grain-growth exponent n and the constant k .

4.13 (MCQs). Choose the most appropriate answers for the following statements.

1. Which type of metal forming is done at the temperature $0.65 T_m$, if T_m is measured in K ?

(a) Cold working, (b) warm working, (c) hot working, (d) none.

2. Which type of metal forming results in elongated grains?

(a) Cold rolling, (b) warm rolling, (c) hot rolling, (d) none.

3. Which mechanical property is increased by recrystallization annealing?

(a) Hardness, (b) ductility, (c) tensile strength, (d) yield strength

4. Which stage of annealing results in no change in the microstructure?

(a) Recovery, (b) recrystallization, (c) grain growth, (d) full annealing

5. Which type of metal forming results in refined equi-axed grains?

(a) Cold rolling, (b) warm rolling, (c) hot rolling, (d) none.

6. Which stage of annealing results in loss of strength and hardness?

(a) Recovery, (b) recrystallization, (c) grain growth, (d) full annealing.

7. Which temperature is referred by $0.4 T_m$, if T_m is measured in K ?

(a) Melting temperature, (b) recrystallization temperature, (c) grain growth temperature

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Part II

Bulk-Metal Forming Processes

Chapter 5

Metal Rolling



Nomenclature

t_o	original thickness
t_f	final thickness
d	draft
μ	friction coefficient
L_o	original length
w_o	original width
L_f	final length
w_f	final width
v_o	entry velocity of the work
v_f	exit work velocity
v_r	roll (linear) speed
ε	true strain
L_c	contact length
N	roll rotational speed
N	neutral point
R	roll radius
S	forward slip
$\bar{Y}_f$	average flow stress
K	strength coefficient
F	rolling force
T	roll torque
P	power
$d_{\max}$	maximum draft

Abbreviations

- CR cold rolling
- HR hot rolling
- RD rolling direction
- TD transverse direction

5.1 Metal Rolling and Friction

Rolling is the most commonly used and economical metal forming process. Metal rolling involves squeezing a metallic work-piece between two rotating rolls, which exert compressive stresses, thereby reducing the thickness or cross-sectional area of the work. The reduction in thickness is called the draft, which can be measured by the difference in the pre-rolling thickness (t_o) and the post-rolling thickness (t_f) (see Fig. 5.1).

It is evident in Fig. 5.1 that the rolls not only perform to reduce thickness but also act to feed the material in the rolling direction (RD) as they rotate in opposite directions to each other (Huda, 2017). Friction is, therefore, necessary for the rolling operation. It is essential that, in a metal rolling process, the level of friction between the rolls and work material is controlled (Ray, 2016). For cold rolling (CR) with lubricants, the coefficient of friction is in the range of $\mu \sim 0.05 - 0.10$; for hot rolling (HR) with sticky conditions, $\mu \sim 0.2$ (see Table 5.1).

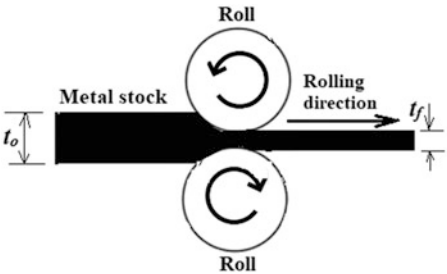


Fig. 5.1 Metal rolling to reduce thickness—flat rolling

Table 5.1 The coefficient-of-friction ranges for CR, warm rolling, and HR

Type of rolling	Cold rolling	Warm rolling	Hot rolling
Coefficient-of-friction (μ)	0.05–0.10	0.11–0.19	0.20

5.2 Industrial Applications of Rolled Products

Rolled metallic products include bars, plates, sheets, structural shapes (rails, I-beams, channels, etc.) as well as intermediate shapes for wire drawing or forging (Ginzburg, 1989). Rolled steel and aluminum-alloy products find wide industrial applications in building construction, the automotive industry, the aerospace industry, and the like. Based on the geometry of the rolled product, rolling may be classified into two types: (a) flat rolling, and (b) shape rolling. Flat rolling involves reduction in thickness of a metal strip with rectangular cross sections (with the width greater than the thickness) to obtain a plate or a sheet. In shape rolling, a square cross-section (e.g. bloom or billet) is formed into a shape with the required geometry (e.g., I-beam, channels, rail) by using grooved rolls. Figure 5.2 illustrates some steel rolled products with different geometrical shaped cross-sections (see also, Table 5.2).

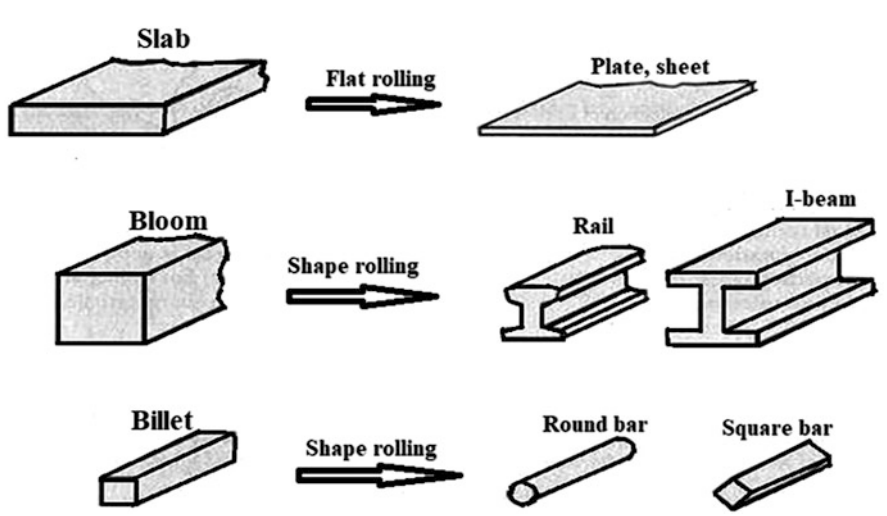


Fig. 5.2 Some steel products’ geometrical shapes manufactured by flat rolling and shape rolling

Table 5.2 Steel products manufactured by flat rolling and shape rolling

No.	Stock type	Stock cross-section	Rolling type	Rolled products
1.	Slab	$>25 \times 4 \text{ cm}^2$	Flat rolling	Plates, sheets, foils
2.	Bloom	$>15 \times 15 \text{ cm}^2$	Shape rolling	Rails, channels, I-beam, T-section, L-section, etc.
3.	Billet	$>4 \times 4 \text{ cm}^2$	Shape rolling	Round bar, square bar, hexagonal bar, etc.

5.3 Rolling Mills

5.3.1 Two-High, Three-High, and Four-High Rolling Mills

The equipment used in rolling is called a *rolling mill* (see Fig. 5.3). Depending upon the number of rolls used, rolling mills may be divided into the following types: (a) 2-high single-pass or pull-over rolling mill, (b) 2-high-reversing rolling mill, (c) 3-high rolling mill, (d) 4-high rolling mill, (e) cluster rolling mill, and (f) tandem rolling mill. Figure 5.4 illustrates the working principles of the 2-high pull-over mill and the 2-high reversing rolling mill. In the 2-high pull-over rolling mill, two rolls of equal size rotate in opposite directions with no reversing of direction; the work is



Fig. 5.3 Rolling process to produce hot steel bars. (Courtesy: Viktor Mácha/viktormacha.com)

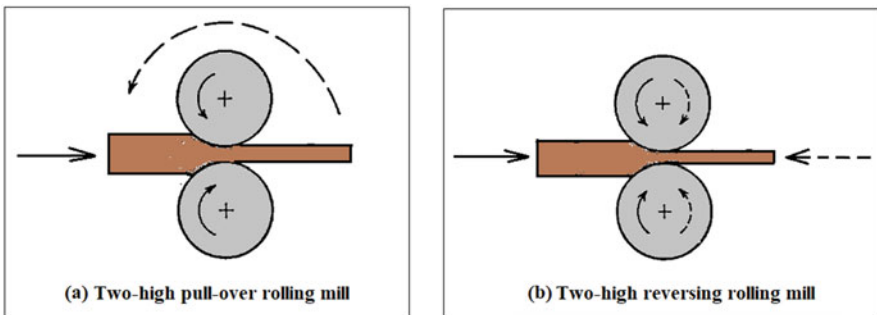


Fig. 5.4 Working principles of 2-high rolling mills; (a) 2-high pull over mill, and (b) 2-high reversing mill

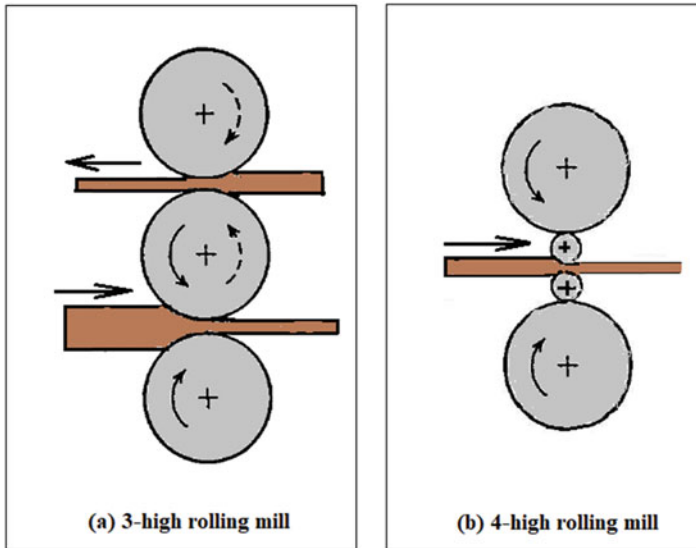


Fig. 5.5 3-high rolling mill (a) and 4-high rolling mill (b)

returned to the entrance side (pulled over) for further reduction (see Fig. 5.4a). The 2-high pull-over mill is the simplest and the most common type of rolling mill. In the 2-high reversing mill, the stock can be passed back and forth the rolls by reversing their direction of rotation (see Fig. 5.4b); this type of rolling mill has higher productivity.

The 3-high rolling mill operates on three working rolls; it consists of upper and lower driven rolls and the middle roll, which rotates by friction (see Fig. 5.5a). The 4-high mills use four rolls (two working rolls and two back-up rolls); the working rolls are smaller in diameter as compared to the rolls in 3-high mills (Huda, 2018; Ginzburg & Ballas, 2000). Since smaller-diameter rolls have lesser strength and rigidity, they are supported by larger-diameter backup rolls (see Fig. 5.5b). The 4-high rolling mill has reduced power requirements owing to the use of small-diameter working rolls (see Sect. 5.4.2).

5.3.2 Cluster Mill and Tandem Mill

A cluster rolling mill may have six rolls (two working rolls and four back-up rolls); the working rolls are very-small-diameter rolls for higher productivity. The back-up rolls may be further supported by larger-diameter supporting rolls (see Fig. 5.6). A cluster mill is capable of producing thin sheet metal with high dimensional accuracy (e.g. aluminum foil for food packaging). A tandem or continuous rolling mill has a series of rolling mills that are arranged one after another (Fig. 5.7). Since different

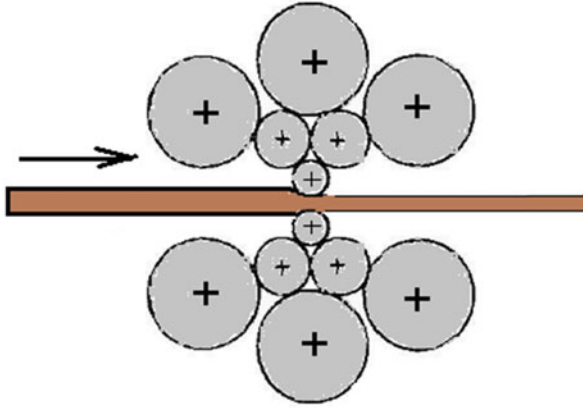


Fig. 5.6 The working principle of a cluster rolling mill

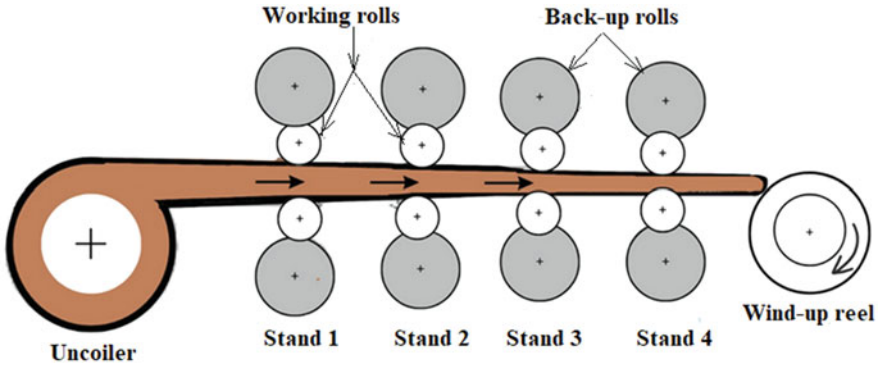


Fig. 5.7 Tandem rolling mill

reduction is taken at each stand, the strip will be moving with different velocities at each stage in the mill. The tandem cold mill is capable of progressively and effectively reducing the thickness of the strip in a single pass.

5.4 Engineering Analysis of Flat Rolling

5.4.1 Analysis for Percent Reduction and Slip

Consider a metal-strip of length L_o , width w_o , and thickness t_o . The strip is flat rolled to the length L_f , width w_f , and thickness t_f . The entry velocity of the work is v_o and the exit work velocity is v_f . The analysis of the flat rolling is illustrated in Fig. 5.8.

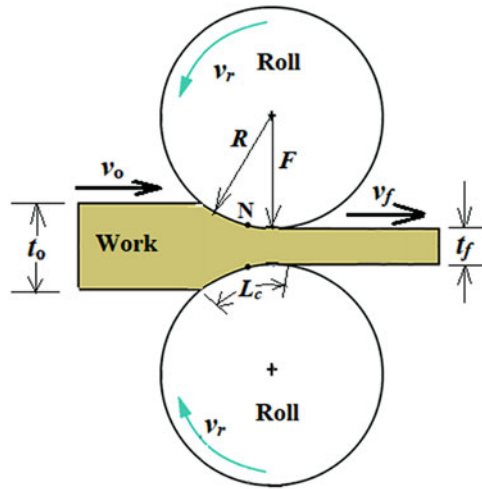


Fig. 5.8 Analysis of flat rolling. F rolling force, v_r roll speed, v_o pre-rolling work velocity, v_f post-rolling work velocity

Since the volume remains constant during rolling, the following relationship holds good:

$$t_o w_o L_o = t_f w_f L_f \quad (5.1)$$

where t_o , w_o , and L_o are the pre-rolling thickness, the pre-rolling width, and the pre-rolling length, respectively; and t_f , w_f , and L_f are the post-rolling thickness, the post-rolling width, and the post-rolling length, respectively. Accordingly, the post-rolling length can be computed by:

$$L_f = \frac{t_o \cdot w_o \cdot L_o}{w_f \cdot t_f} \quad (5.2)$$

The significance of Eq. 5.2 is illustrated in Example 5.1.

The amount of reduction in thickness is expressed as draft, d , as follows:

$$d = t_o - t_f \quad (5.3)$$

By assuming no change in width during rolling, the degree of deformation in rolling can be determined by percent reduction, as follows:

$$\% \text{reduction} = \frac{d}{t_o} \times 100 \quad (5.4)$$

where d is draft (mm), and t_o is the pre-rolling thickness of the work (mm) (see Example 5.2).

By knowing the roll radius and the roll rotational speed, the roll speed can be determined by:

$$v_r = 2\pi \cdot N \cdot R \quad (5.5)$$

where v_r is the roll (surface) speed (m/s), N is the roll rotational speed (rev/s), and R is the roll radius (m). The entry velocity of the work is v_o and its exit velocity is v_f . In order to derive a workable expression for the exit velocity, we may rewrite Eq. 5.1 as follows:

$$t_o w_o \left(\frac{L_o}{\text{Time duration}} \right) = t_f w_f \left(\frac{L_f}{\text{Time duration}} \right) \quad (5.6)$$

or

$$t_o w_o v_o = t_f w_f v_f \quad (5.7)$$

By assuming no change in the width during rolling i.e., $w_o = w_f = w$, Eq. 5.7 simplifies to:

$$v_f = v_o \left(\frac{t_o}{t_f} \right) \quad (5.8)$$

The significance of Eqs. 5.5–5.8 is illustrated in Example 5.3.

The forward slip, S , is defined as the difference between the velocity of the work at exit and the roll speed divided by the roll speed; the forward slip can be computed by (Lu, 2007):

$$S = \frac{v_f - v_r}{v_r} \quad (5.9)$$

where v_f is the work velocity at exit (m/s) and v_r is the roll speed (m/s) (see Example 5.4). At the roll exit, the forward slip is positive, which means that the work moves faster than roll. The forward slip in cold rolling can also be measured by using a high-speed digital camera (Lu, 2007). At the *neutral point* (N) along the arc, the work speed is equal to the roll speed (see Fig. 5.8), i.e. the slip is zero at the neutral point.

5.4.2 Analysis for Rolling Force

True strain ϵ in the work (strip) can be calculated by:

$$\epsilon = \ln \left(\frac{t_o}{t_f} \right) \quad (5.10)$$

The contact length, L_c , is related to the roll radius R and the draft d by (see Fig. 5.8):

$$L_c = \sqrt{Rd} \quad (5.11)$$

The significance of Eq. 5.11 is explained in Example 5.5. In the case of cold rolling, the average flow stress can be computed by:

$$\bar{Y}_f = \frac{K \cdot \epsilon^n}{1+n} \quad (5.12)$$

where $\bar{Y}_f$ is the average flow stress (MPa), K is the strength coefficient (MPa), and n is the strain-hardening exponent (see Example 5.6). The rolling force F is the load with which the rolls press against the work (see Fig. 5.8); it can be calculated by:

$$F = \bar{Y}_f w L_c \quad (5.13)$$

where F is the rolling force (N), $\bar{Y}_f$ is the average flow stress (MPa), and w is work width (mm) (see Example 5.7). By combination of Eqs. 5.11 and 5.13, we obtain:

$$F = \bar{Y}_f w \sqrt{Rd} \quad (5.14)$$

According to Eq. 5.14, the rolling force F varies directly as the square root of the roll radius R . It means that the smaller the roll radius, the lower will be the force required for rolling. This manufacturing principle has been applied to the development of 4-high rolling mills with smaller roll radii (see Figs. 5.5b–5.6).

5.4.3 Analysis for Rolling Torque and Power

Roll torque can be determined by multiplying the roll force F by the perpendicular distance from the axis of rotation (of the roll) to the point of application of the force. Thus, roll torque can be expressed (by reference to Fig. 5.9) as:

$$T = F \cdot x \approx F (0.5L_c) = 0.5 F \cdot L_c \quad (5.15)$$

where T is the roll torque (N-m), F is roll force (N), and L_c is contact length (m). Since two rolls are used in rolling, the rolling torque can be determined by:

$$\text{Rolling torque} = 2T = F L_c \quad (5.16)$$

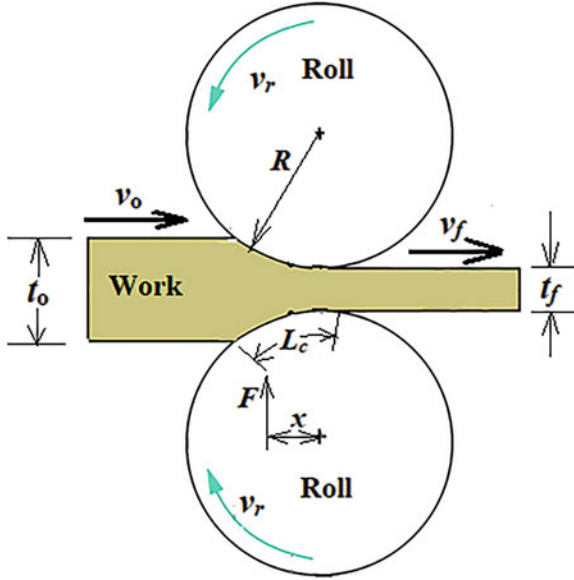


Fig. 5.9 The mechanics of flat rolling. F rolling force, x moment arm

The power required for rolling can be calculated by:

$$\text{Power} = P = (\text{Rolling torque}) \omega = (FL_c) (2\pi N) = 2\pi N F L_c \quad (5.17)$$

where P is the power (W), T is the rolling torque ($N \cdot m$), ω is the roll's angular speed (radians/s), N is the roll rotational speed (rev/s), F is rolling force (N), and L_c is the contact length (m) (see Example 5.8).

5.4.4 Analysis for Rolling Feasibility and Passes

The feasibility or success of rolling strongly depends on the coefficient of friction between the surfaces along the contact length. The maximum draft is related to the coefficient of friction by:

$$d_{\max} = \mu^2 R \quad (5.18)$$

where $d_{\max}$ is the maximum draft (mm), μ is the coefficient of friction between the surfaces along the contact length (see Fig. 5.9), and R is the roll radius (mm). In order for the rolling to be feasible in one pass, the following condition must be satisfied:

$$d_{\max} \geq d \quad (5.19)$$

The significance of Eqs. 5.18 and 5.19 is illustrated in Example 5.9. It can be concluded from Eqs. 5.18 and 5.19 that a larger roll radius (R) would result in a higher value of $d_{\max}$ thereby favoring success (feasibility) of rolling. However, a large roll radius would result in an increase in rolling force and power (see Eq. 5.14); this means that it is necessary to strike a compromise in roll radius by determining an optimum roll radius.

In the case rolling is not feasible in one pass, the minimum number of rolling passes can be determined by:

$$\text{Minimum number of rolling passes} = \frac{d}{d_{\max}} \quad (5.20)$$

In the case Eq. 5.20 yields a fraction, it is converted to a higher whole number (see Example 5.10). In the case Eq. 5.20 yields a whole number, $d_{\max}$ = draft for one (each) pass (see Example 5.13). It must be noted that Eq. 5.20 holds good for flat rolling.

5.5 Engineering Analysis for Shape Rolling: Roll Pass Design for Bar Mills

Shape rolling has been introduced to the reader in Sect. 5.2. In a rolling mill, the rolls rest on bearings at their ends (see Fig. 5.3). The size of a rolling mill is determined by the average distance from the center of one roll to the center of the other (see Fig. 5.9). The number of passes required in a bar tandem mill (Fig. 5.7) can be determined by the following formula:

$$\text{No. of passes required for a bar tandem mill} = \frac{\log (\text{Coeff. of elongation})}{\log (1.29)} \quad (5.21)$$

The coefficient of elongation (also called elongation factor) in hot rolling can be calculated by:

$$\text{Coeff. of elongation} = \frac{A_o}{A_{f(\text{actual})}} \quad (5.22)$$

where A_o is the original cross-sectional area and $A_{f(\text{actual})}$ is the actual finished cross-sectional area; the latter can be determined by the following formula:

$$A_{f(\text{actual})} = \frac{A_f}{\text{Shrinkage factor for hot rolling}} \quad (5.23)$$

where A_f is the ideal final cross-sectional area (see Examples 5.19–5.21).

5.6 Worked Numerical Examples in Metal Rolling

Example 5.1: Calculating the Post-rolling Length and the Draft

A 3-m long steel strip, with a thickness of 30 mm, is formed by flat rolling to 22-mm final thickness in one pass. The strip widens by 5% after rolling. Calculate the final length after rolling.

Solution

$$w_o = w, t_o = 30 \text{ mm}, L_o = 3 \text{ m} = 3000 \text{ mm}, t_f = 22 \text{ mm}, w_f = w + 0.05 w, \\ = 1.05 w, L_f = ?$$

By using Eq. 5.2,

$$L_f = \frac{t_o \cdot w_o L_o}{w_f \cdot t_f} = \frac{30 \times w \times 3000}{1.05 \times w \times 22} = \frac{30 \times 3000}{1.05 \times 22} = \frac{90,000}{23.1} = 3,896 \text{ mm} = 3.896 \text{ m}$$

Example 5.2: Calculating the Percent Reduction in Rolling

By using the data in Example 5.1, calculate the percent reduction in rolling.

Solution

$$t_o = 30 \text{ mm}, t_f = 22 \text{ mm}, \% \text{reduction} = ?$$

By using Eqs. 5.3 and 5.4,

$$\text{Draft} = d = t_o - t_f = 30 - 22 = 8 \text{ mm}$$

$$\% \text{Reduction} = \frac{d}{t_o} \times 100 = \frac{8}{30} \times 100 = 26.67$$

$$\text{Reduction} = 26.67\%$$

Example 5.3: Calculating the Roll Speed and the Work Exit Velocity

A steel strip 42-mm thick is to be rolled by flat rolling to 30-mm final thickness in one pass. The entrance velocity of the strip is 32 m/min. The roll radius is 250 mm and the rotational speed is 25 rev/min. Calculate the (a) roll speed and (b) exit velocity of the strip.

Solution

$$t_o = 42 \text{ mm}, t_f = 30 \text{ mm}, v_o = 32 \text{ m/min} = 32,000 \text{ mm/min}, R = 250 \text{ mm}, N = 25 \text{ rev/min}$$

(a) By using Eq. 5.5,

$$v_r = 2\pi \cdot N \cdot R = 2\pi \times 25 \times 250 = 39275 \text{ mm/min} = 39.275 \text{ m/min}$$

(b) By using Eq. 5.8,

$$v_f = v_o \left(\frac{t_o}{t_f} \right) = \frac{32 \times 42}{30} = 44.8 \text{ m/min}$$

Example 5.4: Determining the Forward Slip

By using the data in Example 5.3, determine the forward slip for the rolling operation.

Solution

$$v_r = 39.275 \text{ m/min}, v_f = 44.8 \text{ m/min}, S = ?$$

By using Eq. 5.9,

$$\text{Forward slip} = S = \frac{v_f - v_r}{v_r} = \frac{44.8 - 39.275}{39.275} = 0.14$$

Example 5.5: Calculating the Contact Length for a Rolling Operation

A 300-mm wide and 24-mm thick strip is fed through a 2-high rolling mill with rolls each of 260-mm radius. It is required to reduce the strip thickness to 19 mm in one pass. Calculate the contact length for the rolling operation.

Solution

$$t_o = 24 \text{ mm}, t_f = 19 \text{ mm}, R = 260 \text{ mm}, L_c = ?$$

By using Eqs. 5.3 and 5.11,

$$d = t_o - t_f = 24 - 19 = 5 \text{ mm}$$

$$\text{Contact length} = L_c = \sqrt{R \cdot d} = \sqrt{260 \times 5} = 36 \text{ mm}$$

Example 5.6: Determining the Average Flow Stress for a Rolling Operation

A metal plate that is 230-mm wide and 22-mm thick is required to be reduced in a single pass to a thickness of 18 mm by using a 2-high rolling mill. The strength coefficient of the work material is 530 MPa and the strain hardening exponent is 0.44. Determine the average flow stress for the rolling operation.

Solution

$$w = 230 \text{ mm}, t_o = 22 \text{ mm}, t_f = 18 \text{ mm}, K = 530 \text{ MPa}, n = 0.44, \bar{Y}_f = ?$$

By using Eqs. 5.10 and 5.12,

$$\varepsilon = \ln \left(\frac{t_o}{t_f} \right) = \ln \left(\frac{22}{18} \right) = \ln 1.22 = 0.1988$$

$$\bar{Y}_f = \frac{K \cdot \varepsilon^n}{1 + n} = \frac{530 \times 0.1988^{0.44}}{1 + 0.44} = 180.81 \text{ MPa}$$

Example 5.7: Calculating the Rolling Force

By using the data in Example 5.6, calculate the rolling force if the roll radius is 450 mm.

Solution

$$w = 230 \text{ mm}, t_o = 22 \text{ mm}, t_f = 18 \text{ mm}, R = 450 \text{ mm}, \bar{Y}_f = 180.81 \text{ MPa}, F = ?$$

By using Eqs. 5.3 and 5.14,

$$d = t_o - t_f = 22 - 18 = 4 \text{ mm}$$

$$F = \bar{Y}_f w \sqrt{R d} = 180.81 \times 230 \times \sqrt{450 \times 4} = 1764357.28 \text{ N} = 1.764357 \text{ MN}$$

Example 5.8: Calculating the Rolling Torque and Power

By using the data in Example 5.7, calculate the rolling torque and the power required for the rolling operation, if the roll's rotational speed is 50 rev/min.

Solution

$$F = 1764357.28 \text{ N}, N = 50 \text{ rev/min} = 0.833 \text{ rev/s}, R = 450 \text{ mm}, d = 4 \text{ mm}$$

By using Eqs. 5.11, 5.15, and 5.16, we obtain:

$$L_c = \sqrt{R d} = \sqrt{450 \text{ mm} \times 4 \text{ mm}} = 42.43 \text{ mm} = 0.04243 \text{ m}$$

$$\text{Rolling torque} = F L_c = 1764357.28 \times 0.04243 = 74861.68 \text{ N}\cdot\text{m}$$

$$\text{Rolling power} = 2\pi N F L_c = 2 \times \pi \times 0.833 \times 1764357.28 \times 0.04243 = 391868.85 \text{ W}$$

$$\text{Rolling power} = 391.868 \text{ kW}$$

Example 5.9: Determining the Feasibility of a Rolling Operation

A 300-mm wide and 24-mm thick strip is fed through a 2-high rolling mill with rolls each of 260-mm radius. It is required to reduce the strip thickness to 19 mm. The coefficient of friction between the surfaces of rolls and the strip is 0.12. Is the rolling operation feasible in one pass?

Solution

$$t_o = 24 \text{ mm}, t_f = 19 \text{ mm}, R = 260 \text{ mm}, \mu = 0.12$$

By using Eqs. 5.3 and 5.18,

$$d = t_o - t_f = 24 - 19 = 5 \text{ mm}$$

$$d_{\max} = \mu^2 R = 0.12^2 \times 260 = 3.744 \text{ mm}$$

Since $d_{\max} \nless d$ and $d_{\max} \neq d$, the rolling operation is not feasible in one pass (see Eq. 5.19).

Example 5.10: Determining the Number of Passes and the Draft for Each Pass

By using the data in Example 5.9, determine the number of minimum rolling passes. The draft is to be equal on each pass. Also, determine the draft for each pass.

Solution

$$t_o = 24 \text{ mm}, t_f = 19 \text{ mm}, d = 5 \text{ mm}, d_{\max} = 3.744 \text{ mm}$$

By using Eq. 5.20,

$$\text{Minimum number of rolling passes} = \frac{d}{d_{\max}} = \frac{5}{3.744} = 1.33 \rightarrow 2 \text{ passes}$$

$$\frac{5 \text{ mm}}{d_{\max}} = 2 \text{ passes}$$

$$d_{\max} = \frac{5}{2} = 2.5 \text{ mm}$$

$$\text{Draft for each pass} = 2.5 \text{ mm}$$

$$\text{Draft for 1st pass} = \text{Draft for 2nd pass} = 2.5 \text{ mm}$$

Example 5.11: Calculating the Final Length of Rolled Product After Multi-Passes Rolling

A 4-cm-thick slab is 25-cm wide and 90-cm long. The thickness is to be reduced at two passes in a rolling operation; each pass will reduce the slab to 80% of its previous thickness. It is estimated that the slab will widen by 4% in each pass of rolling. The entrance speed of the slab on the first step is 3 m/min, and roll speed is the same for each pass of rolling. Calculate the final length of the rolled product.

Solution

$$\begin{aligned}
 t_o &= 4 \text{ cm}, w_o = 25 \text{ cm}, L_o = 90 \text{ cm}, t_1 = 0.8t_o = 0.8 \times 4 = 3.2 \text{ mm}, \\
 w_1 &= w_o + 0.04 w_o = 26 \text{ cm}, L_1 = ?, \\
 t_2 &= 0.8t_1 = 0.8 \times 3.2 = 2.56 \text{ cm}, w_2 = w_1 + 0.04w_1 = 26 + (0.04 \times 26) \\
 &= 27.04 \text{ cm}, L_2 = L_f = ?
 \end{aligned}$$

By using Eq. 5.2 for 1st pass of rolling,

$$L_1 = \frac{t_o \cdot w_o L_o}{w_1 \cdot t_1} = \frac{4 \times 25 \times 90}{26 \times 3.2} = 108.173 \text{ cm}$$

By using Eq. 5.2 for 2nd pass of rolling,

$$L_f = \frac{t_1 \cdot w_1 L_1}{w_f \cdot t_f} = \frac{3.2 \times 26 \times 108.173}{27.04 \times 2.56} = 130 \text{ cm}$$

The final length of the rolled product = $L_f = 130 \text{ cm} = 1.3 \text{ m}$

Example 5.12: Calculating the Exit Velocity of Work After the Final Reduction

By using the data in Example 5.11, calculate the exit velocity of the work after the final reduction.

Solution

$$t_o = 4 \text{ cm}, t_f = t_2 = 2.56 \text{ cm}, v_o = 3 \text{ m/min}, v_f = ?$$

By using Eq. 5.8,

$$v_f = v_o \left(\frac{t_o}{t_f} \right) = \frac{3 \times 4 \text{ cm}}{2.56 \text{ cm}} = 4.687 \text{ m/min}$$

The exit velocity of the work after the final reduction = 4.687 m/min.

Example 5.13: Determining the Thickness After Each Pass in a Multi-pass Rolling Job A 200 mm wide, 15 mm thick, and 1 m long strip is fed through a 2-high rolling mill with rolls each of 300 mm radius. It is required to reduce the strip thickness to 3 mm. The roll rotational speed is 25 rev/min. The coefficient of friction between the surfaces of the rolls and the strip is 0.1. Determine the (a) minimum number of rolling passes, (b) draft for each pass, assuming that the draft is equal on each pass, and (c) thickness after each pass.

Solution

$t_o = 15$ mm, $t_f = 3$ mm, $R = 300$ mm, $\mu = 0.1$, No. of passes = ?, $d = ?$, $t_1 = ?$, $t_2 = ?$, $t_3 = ?$, $t_4 = ?$

By using Eqs. 5.3, 5.18, and 5.20,

(a) Draft = $d = t_o - t_f = 15 - 3 = 12$ mm

$$d_{\max} = \mu^2 R = 0.1^2 \times 300 = 3 \text{ mm}$$

$$\text{Minimum number of rolling passes} = \frac{d}{d_{\max}} = \frac{12}{3} = 4 \text{ passes}$$

(b) By reference to Sect. 5.3, Draft for each pass = $d_{\max} = 3$ mm

(c) Let the thickness after 1st pass = t_1

$$\text{Draft for 1st pass} = t_o - t_1$$

Or

$$3 = 15 - t_1$$

$$t_1 = 15 - 3 = 12 \text{ mm}$$

The thickness after 1st pass = $t_1 = 12$ mm

The thickness after 2nd pass = $t_2 = t_1 - 3 = 12 - 3 = 9$ mm

The thickness after 3rd pass = $t_3 = t_2 - 3 = 9 - 3 = 6$ mm

The thickness after 4th pass = $t_4 = t_3 - 3 = 6 - 3 = 3$ mm = t_f (confirmed)

Example 5.14: Determining the Length After Each Pass in a Multi-pass Rolling Job

By using the data in Example 5.13, determine the length after each pass. Assume no widening of the strip during rolling.

Solution

$w_o = w_1 = w$, $L_o = 1000$ mm, $t_o = 15$ mm, $t_1 = 12$ mm, $t_2 = 9$ mm, $t_3 = 6$ mm, $t_4 = t_f = 3$ mm

By using Eq. 5.2,

$$\text{The length after 1st pass} = L_1 = \frac{t_o \cdot w_o L_o}{w_1 \cdot t_1} = \frac{t_o \cdot L_o}{t_1} = \frac{15 \times 1000}{12} = 1250 \text{ mm}$$

$$\text{The length after 2nd pass} = L_2 = \frac{t_1 \cdot L_1}{t_2} = \frac{12 \times 1250}{9} = 1666 \text{ mm}$$

$$\text{The length after 3rd pass} = L_3 = \frac{t_2 \cdot L_2}{t_3} = \frac{9 \times 1666}{6} = 2499 \text{ mm}$$

$$\text{The length after 4th (final) pass} = L_f = \frac{t_3 \cdot L_3}{t_f} = \frac{6 \times 2499}{3} = 4998 \text{ mm}$$

Example 5.15: Determining the Duration for Each Pass in a Multi-pass Rolling Job

Refer to the data in Examples 5.13 and 5.14. In view of four passes, it is recommended to use a continuous hot-rolling mill with four stands. It is assumed that the forward slip is equal at each stand. Determine the (a) roll speed v_r at each stand, and (b) duration for each rolling pass.

Solution

$$R = 0.3 \text{ m}, N = 25 \text{ rev/min}, L_o = 1 \text{ m}, L_1 = 1.25 \text{ m}, L_2 = 1.66 \text{ m}, L_3 = 2.499 \text{ m}$$

$$\text{Roll speed} = v_r = 2\pi \cdot N \cdot R = 2\pi \times 25 \times 300 = 47130 \text{ mm/min} = 47.13 \text{ m/min}$$

$$\text{Duration} = \frac{\text{Length rolled}}{\text{Roll speed}} \quad (\text{By definition of speed})$$

$$\text{Duration for 1st rolling pass} = \text{time}_1 = \frac{L_o}{v_r} = \frac{1}{47.13} = 0.0212 \text{ min} = 353.33 \times 10^{-6} \text{ h}$$

$$\text{Duration for 2nd rolling pass} = \text{time}_2 = \frac{L_1}{v_r} = \frac{1.25}{47.13} = 0.0265 \text{ min} = 441.67 \times 10^{-6} \text{ h}$$

$$\text{Duration for 3rd rolling pass} = \text{time}_3 = \frac{L_2}{v_r} = \frac{1.66}{47.13} = 0.035 \text{ min} = 583.33 \times 10^{-6} \text{ h}$$

$$\text{Duration for 4th rolling pass} = \text{time}_4 = \frac{L_3}{v_r} = \frac{2.499}{47.13} = 0.053 \text{ min} = 889.33 \times 10^{-6} \text{ h}$$

Example 5.16: Determining the Power for Each Pass in a Multi-pass Rolling Job

By using the data in Example 5.13, calculate the (a) rolling force and (b) rolling power for each rolling pass. The strength coefficient of the material, in hot condition, is 140 MPa and its strain-hardening exponent is zero ($n = 0$).

$t_o = 15 \text{ mm}$, $R = 300 \text{ mm}$, $N = 25 \text{ rev/min}$, $K = 140 \text{ MPa}$, $w = 200 \text{ mm}$, draft for each pass = 3 mm

(a) For the 1st (each) pass,

$$L_c = \sqrt{R d} = \sqrt{300 \text{ mm} \times 3 \text{ mm}} = 30 \text{ mm}$$

$$\varepsilon = \ln \left(\frac{t_o}{t_1} \right) = \ln \left(\frac{15}{12} \right) = \ln 1.25 = 0.223$$

$$\overline{Y}_f = \frac{K \cdot \varepsilon^n}{1 + n} = \frac{140 \times 0.233^0}{1 + 0.233} = 113.544 \text{ MPa}$$

$$F = \overline{Y}_f w L_c = 113.544 \times 200 \times 30 = 681265.2 \text{ N}$$

(b) For each rolling pass,

$$\text{Power} = 2\pi N F L_c = 2 \times 3.142 \times \frac{25}{60} \text{ rev/s} \times 681265.2 \times 0.03 \text{ m} = 53513.38$$

$$W = 53.513 \text{ kW}$$

Example 5.17: Determining the Energy Consumed in Each Pass in a Multi-pass Rolling Job

By using the data in Examples 5.15 and 5.16, determine the energy consumed (KWh) in each rolling pass. Also calculate the total energy (kWh) consumed in the rolling job.

Solution

Power in each pass = 53.513 kW, time₁ = 353.33 × 10⁻⁶ h, time₂ = 441.67 × 10⁻⁶ h
 Energy consumed in 1st pass = Power × time₁ = 53.513 kW × 353.33 × 10⁻⁶ h
 = 0.01891 kWh

Energy consumed in 2nd pass = Power × time₂ = 53.513 kW × 441.67 × 10⁻⁶ h
 = 0.023635 kWh

Energy consumed in 3rd pass = Power × time₃ = 53.513 kW × 583.33 × 10⁻⁶ h
 = 0.031215 kWh

Energy consumed in 4th pass = Power × time₄ = 53.513 kW × 883.33 × 10⁻⁶ h
 = 0.04727 kWh

Total energy consumed in the rolling job = 0.121 kWh

Example 5.18: Calculating the Draft and Final Thickness When % Reduction Is Known

A steel sheet is hot rolled 35% from a 32-mm-thick slab. Calculate the (a) draft for the rolling operation, and (b) final thickness of the sheet metal.

Solution

(a) By using Eq. 5.4,

$$35 = \frac{d}{32} \times 100 = 3.125 \ d$$

$$d = \frac{35}{3.125} = 11.2 \text{ mm} = \text{draft}$$

$$d = t_o - t_f = 11.2$$

$$32 - t_f = 11.2$$

$$t_f = 20.8 \text{ mm} = \text{Final thickness}$$

Example 5.19: Determining Original C.S.A and Actual Finished C.S.A for Hot Bar Rolling

A steel billet of dimensions 150×150 mm is to be hot rolled, using a tandem mill, to a bar with dimensions 12×12 mm. Determine the original cross-sectional area and the actual finished cross-sectional area for the operation. The shrinkage factor for hot rolling = 1.2727.

Solution

$$A_o = 150 \text{ mm} \times 150 \text{ mm} = 22,500 \text{ mm}^2, A_f = 12 \text{ mm} \times 12 \text{ mm} = 144 \text{ mm}^2$$

By using Eq. 5.23,

$$A_{f(\text{actual})} = \frac{A_f}{\text{Shrinkage factor for hot rolling}} = \frac{A_f}{1.2727} = \frac{144}{1.2727} = 113.145 \text{ mm}^2$$

$$A_o = 22,500 \text{ mm}^2, A_{f(\text{actual})} = 113.145 \text{ mm}^2$$

Example 5.20: Determining the Coefficient of Elongation for Hot Bar Rolling

By using the data in Example 5.19, determine the coefficient of elongation for the rolling job.

Solution

By using Eq. 5.22,

$$\text{Coeff. of elongation} = \frac{A_o}{A_{f(\text{actual})}} = \frac{22500}{113.145} = 198.86$$

The coefficient of elongation = 198.86

Example 5.21: Determining the Number of Passes Required in Hot Tandem Bar Rolling Mill

By using the data in Example 5.20, determine the number of passes required for hot rolling in using a tandem mill.

Solution

By using Eq. 5.21,

$$\begin{aligned}\text{No. of passes required for a tandem mill} &= \frac{\log (\text{Coeff. of elongation})}{\log (1.29)} \\ &= \frac{\log (198.86)}{\log (1.29)} = 20.78\end{aligned}$$

The number of passes required = 21

Questions and Problems

5.1 (MCQs). Select the most appropriate answers for each of the following statements:

- Which type of rolling mill involves reversing of roll's rotational direction?
(a) 2-high mill, (b) 3-high mill, (c) 4-high mill, (d) cluster mill, (e) tandem mill.
- Which type of rolling mill has the smallest-diameter working rolls?
(a) 2-high mill, (b) 3-high mill, (c) 4-high mill, (d) cluster mill, (e) tandem mill.
- Which type of rolling mill has multiple stands?
(a) 2-high mill, (b) 3-high mill, (c) 4-high mill, (d) cluster mill, (e) tandem mill.
- In which type of rolling mill, the middle roll rotates by friction?
(a) 2-high mill, (b) 3-high mill, (c) 4-high mill, (d) cluster mill, (e) tandem mill.
- Which type of rolling mill has two supporting rolls?
(a) 2-high mill, (b) 3-high mill, (c) 4-high mill, (d) cluster mill.
- Which type of bloom cross-section should be used to manufacture an I-beam by rolling?
(a) Rectangular, (b) square, (c) triangular, (d) hexagonal, (e) pentagonal
- In flat rolling, the ratio of the draft to the original thickness is called the:
(a) Slip, (b) reduction, (c) deformation, (d) none.

8. What is the value of slip at the neutral point in rolling?

(a) $S = 1$, (b) $S = -1$, (c) $S = 0$, (d) $S = \infty$.

9. Which condition refers to unsuccessful rolling?

(a) $d_{\max} < d$, (b) $d_{\max} = d$, (c) $d_{\max} > d$, (d) none.

10. Which parameters determine the maximum draft (more than one)?

(a) Strain-hardening exponent, (b) friction, (c) strength coefficient, (d) roll diameter.

5.2 Draw a labeled sketch showing flat rolling of a metallic work.

5.3 Draw sketches showing the products produced by (a) flat rolling and (b) shape rolling.

5.4 List at least four industrial applications of rolled products.

5.5 Differentiate between the two principal types of 2-high rolling mill with the aid of sketches.

5.6 Why are small-diameter rolls used in 4-high mills and cluster mills?

5.7 What are the roles of working rolls and back-up rolls in 4-high rolling mills?

P5.8 A steel strip 2-m long and with a thickness of 25 mm is rolled by flat rolling to 19 mm final thickness in one pass. It is assumed that the strip will widen by 3% after rolling. Calculate the: (a) final length after rolling, and (b) % reduction.

P5.9 A steel strip 40-mm thick is to be rolled by flat rolling to 27-mm final thickness in one pass. The entrance velocity of the strip is 28 m/min. The roll radius is 230 mm and the rotational speed is 22 rev/min. Calculate the forward slip for the rolling operation.

P5.10 An aluminum plate, 30-mm thick, is fed through a 2-high rolling mill using rolls each of 270-mm radius. It is required to reduce the plate's thickness to 22 mm. The coefficient of friction between the surfaces of the rolls and the strip is 0.11. Is the rolling operation feasible in one pass?

P5.11 A metal plate that is 260-mm wide and 25-mm thick is required to be reduced in one pass to a thickness of 20 mm by using a 2-high rolling mill with a roll radius of 300 mm. The strength coefficient of the work material is 600 MPa and the strain-hardening exponent is 0.21. Determine the (a) rolling force, (b) rolling torque, and (c) power for the rolling operation.

P5.12 By using the data in P5.10, determine the number of rolling passes for the job. Also, determine the draft for each pass. Assume draft, for each pass, is equal.

P5.13 A 3.5-cm-thick slab is 23-cm wide and 80-cm long. The thickness is to be reduced in two passes in a rolling operation; each pass will reduce the slab to 75% of its previous thickness. It is estimated that the slab will widen by 5% in each pass of rolling. The entrance speed of the slab in the first step is 2.5 m/min, and roll speed is the same for each pass of rolling. Calculate the final length of the rolled product.

- P5.14 A steel sheet is hot rolled 42% from a 28-mm-thick slab. Calculate the (a) draft for the rolling operation and (b) final thickness of the sheet metal.
- P5.15 A steel billet of dimensions $(200 \times 200 \text{ mm})$ is to be hot rolled, using a tandem mill, to a bar with dimensions $(16 \times 16 \text{ mm})$. Determine the number of rolling passes required for the job. The shrinkage factor for hot rolling = 1.2727.

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Chapter 6

Metal Forging



Nomenclature

T_r	recrystallization temperature
ϵ	true strain
h_o	pre-forging height
h	height of the work at any instant during forging
h_f	final height
D_o	original diameter
D	work-part diameter at any instant during forging
D_f	final work diameter
Y_f	flow stress
K	strength coefficient
n	strain hardening exponent
K_f	forging shape factor
μ	coefficient of friction
F_{forge}	forging force
A	cross-sectional area of the work at any instant
σ_{ys}	yield strength
$F_{\text{all sticky}}$	forging force in the all sticky region
$\bar{Y}_f$	average flow stress

Abbreviation

ODF open die forging

6.1 Forging: Introduction and Industrial Importance

Forging is a bulk-metal forming operation that involves compression of the work between two dies, using either gradual pressure or impact to produce the part. It is the oldest of all metal forming operations. Forging was first used around 4000 B.C. to manufacture coins, jewelry, and utensils by pounding metal with stone tools (CFS Forge, 2018). Today, metal forging is an important industrial process used to manufacture a variety of high-strength components for a wide range of applications in the automotive industry, the aerospace industry, the defense sector, and the like. The examples of forged products include automotive components (e.g. axle hubs, crankshafts, connecting rods), gears, jet engine turbine-blades, aircraft structural components, thin-walled pressure vessels, and the like. This author has successfully developed the manufacturing process/heat-treatment cycle (by ensuring tempered martensite microstructure) in forged-hardened 1050-steel axle-hubs of modern motor-cars first-ever implemented in Pakistan (through Indus Motor Company (Pvt) Ltd./Transmission Engineering Industries (Pvt), Karachi) (Huda, 2012). In addition, the iron and steel industries commonly use forging to manufacture high-strength components (e.g. spanners, crane-hooks, bolts, and other highly stressed parts).

6.2 Advantages and Limitations of Forging

6.2.1 Advantages of Forging

Forging (especially closed-die drop forging) has several distinct advantages as compared to cast components and rolled products. The following are the main advantages of forging: (a) improved specific strength, (b) better structural integrity, (c) productivity, and (d) close dimensional accuracy. These advantages are briefly explained in the following paragraphs.

- (a) *Improved Specific Strength.* The closed-die hot forging process results in a refined- grained metallurgical structure due to favorable grain flow. During the hot plastic deformation process, the material's grain structure becomes compressed and aligned according to the component's shape (see Fig. 6.1); the grain-flow structure remarkably increases the strength-to-weight ratio (specific strength) and imparts high directional strength with reduced stress concentrations at corners and fillets. There is an absence of grain flow structure in cast and rolled products.
- (b) *Better Structural Integrity.* Metal castings often contain porosity and alloy segregation defects. Forging operation removes porosity, internal cavities, and segregation. Forging also draws out unwanted (nonmetallic) constituents from steel parts. All these benefits contribute to better structural integrity of forged products.

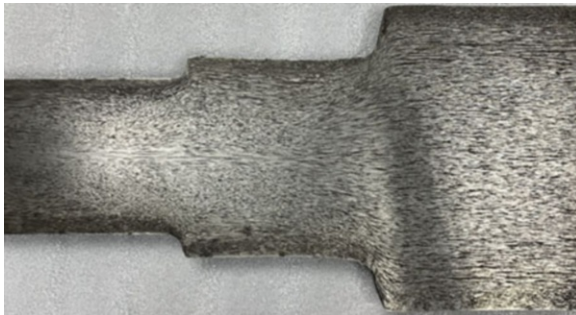


Fig. 6.1 Grain flow structure in a macro-etched forged metallic component

- (c) *Productivity*. There is a minimized amount of material waste in forging owing to high dimension-shape precision (Hawryluk et al., 2023). Forging cycle times are low, resulting in high productivity. A typical forging operation is completed in a few seconds. For example, in drop hammer forging, the hammer can strike in the range of 60–150 blows/min. Hence, forging is an efficient metal forming process.

6.2.2 Limitations of Forging

Forging is generally performed as a hot working operation; this is why it is a relatively dangerous process to work. However, health and safety can be maintained by operator training, controlled production procedures, and involvement of the expert workforce. One of the limitations of forging is the high cost of complex die making. There may also be issues of noise and vibration in forging operation, but they can be overcome with up-to-date damping under the hammer unit and the correct building structure.

6.3 Types of Forging Operations

There are several types of forging operations that are practiced in industry (Ares, 2006). The classification chart showing the various types of forging methods is illustrated in Fig. 6.2. Based on working temperature, forging may be done either as hot forging (above the recrystallization temperature of the metal, T_r) or as cold forging (below T_r). Based on the type of forging operation, there are three types of forging operations: (a) open-die forging, (b) impression-die forging or closed die forging, and (c) flash-less forging (Groover, 2019). Based on the type of equipment, forging may be either drop-hammer forging or press forging.

Open-die forging (ODF) involves placing a solid cylindrical piece between two flat (or almost flat) dies and reducing its height by compressing it (see Fig. 6.3a).

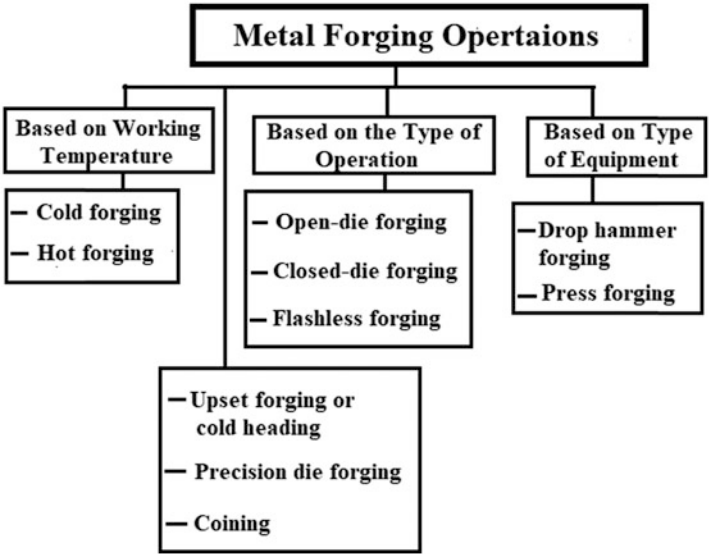


Fig. 6.2 Classification of different forging operations

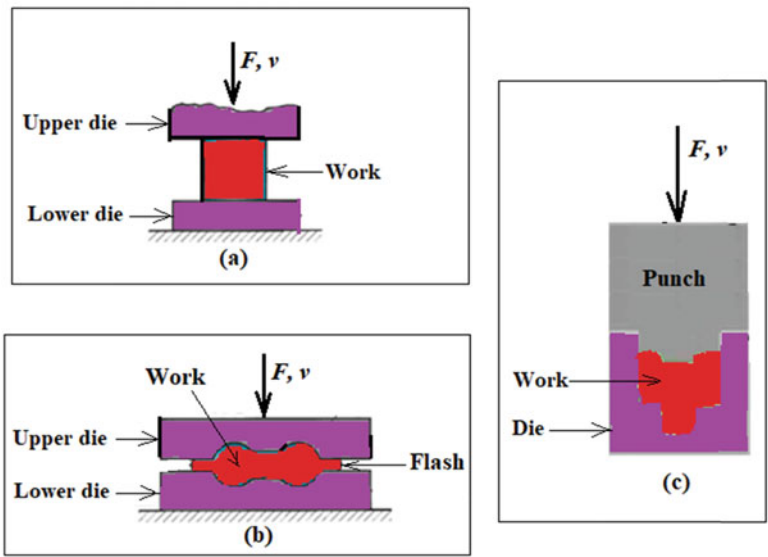


Fig. 6.3 The three types forging operations; (a) open-die forging, (b) closed-die forging, and (c) flash-less forging

Examples of open-die forged products include rings, pressure vessels, gas-turbine shafts, and the like. *Closed-die forging* or *impression-die forging* involves the use of a closed die that contains an impression that is imparted to the work-piece (see Fig. 6.3b). Examples of closed-die forgings include crane-hooks, spanners, connecting rods, and the like. *Flash-less* forging is a special closed-die forging with no allowance of flash; here, the volume of the stock must be exactly equal to the volume of the forged product (see Fig. 6.3c). In *drop hammer forging*, the metal billet is inserted into the die and hammered with multiple blows. In *press forging*, the steel work is formed between two dies that are hydraulically or mechanically controlled; it involves only a single stroke per die impression. *Upset forging* (or *cold heading*) is a special open-die forging operation, which is commonly used to manufacture bolts and nails.

6.4 Hammer Forging Versus Press Forging

Hammer forging, also called drop hammer forging, involves insertion of a metal billet between two dies and then hammering (impact loading). The *drop forging hammer* can be powered by gravity alone (Fig. 6.4), or by additional sources of power using either a hydraulic or pneumatic system (Fig. 6.5). The forging hammer can strike multiple blows in the range of 60–150 blows/min, depending on size and capacity. Experiments on hammer forging have shown that there is an increase in the impact efficiency and a reduction in the number of required impacts when using a hammer head filled with steel beads (Lavrinenko & Shagaleev, 2019). It is highly

Fig. 6.4 Schematic of a drop forging hammer working on gravity

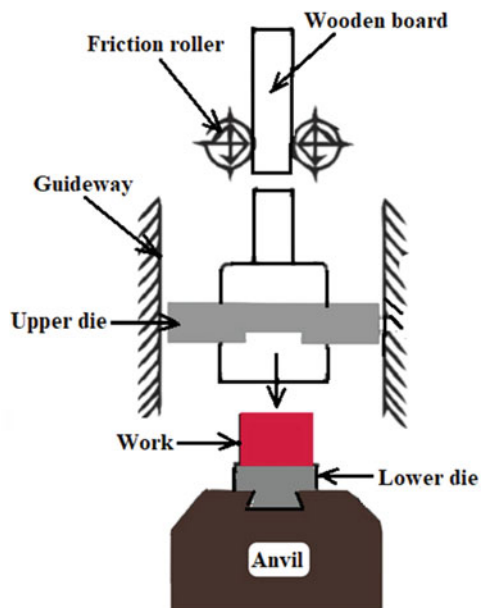
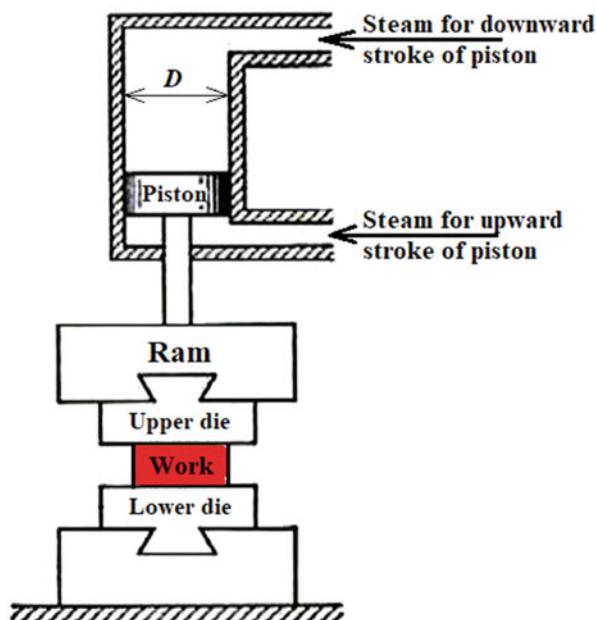


Fig. 6.5 Working principle of a steam-powered (drop) forging hammer



important that the forging dies are correctly designed, manufactured, and in-service used because forging dies may fail prematurely in-service if they are not correctly designed, manufactured, or in-service used (Irisarri & Pelayo, 2009).

It is evident in Fig. 6.5 that the hammer ram's total energy (at a height h above the upper die) is the sum of its potential energy and the work done for power, i.e.,

$$\text{Total energy} = m \cdot g \cdot h + p \cdot A \cdot h = (m \cdot g + p \cdot A) h \quad (6.1)$$

where m is the ram mass (kg), g is the acceleration of gravity (m/s^2), p is the steam (or air) pressure acting on the ram cylinder on down stroke (Pa), A is the area of the ram cylinder (m^2), and h is the height of the ram drop (m) (see Example 6.1).

Press forging applies continuous pressure on the metal so as to manipulate the interior and exterior of the work-piece. It involves only a single stroke per die impression. Table 6.1 presents a comparison between hammer forging and press forging.

6.5 Engineering Analysis of Open-Die Forging

6.5.1 Analysis of General Open-Die Forging

In open-die forging, a cylindrical work-piece is placed between two flat dies and compressive force is applied so as to reduce its height and increase the cross-

Table 6.1 Comparative advantages of hammer forging and press forging

Hammer forging	Press forging
1. It involves multiple blows per die impression.	1. It involves a single stroke per die impression.
2. It eliminates porosity that could otherwise cause failures under stress or impact.	2. In press forging, an expert worker can easily control strain rate.
3. The grain flow structure in hammer-forged metal conforms to the part’s shape. An expert metalworker can optimize the grain flow for maximum strength and reliability.	3. Press forged products are typically more near net, especially for thicker parts, since they require fewer draft angles.
4. Hammer forged parts carry greater value (price) owing to superior mechanical Properties, as compared to press forging.	4. At high-volume production, a press forging operation will eventually yield parts at a lower cost per piece than hammer forging.

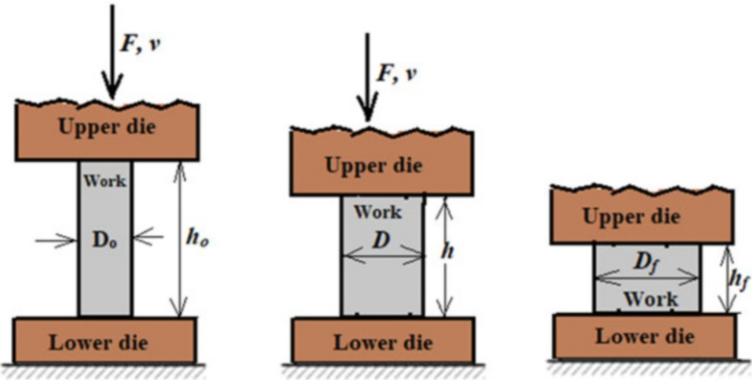


Fig. 6.6 Geometrical changes in the work in open die forging; (a) Original geometry, (b) geometry at an instant during forging, (c) final stage of forging

sectional area (see Fig. 6.6). The *true strain* ϵ in the work during the forging process can be determined by:

$$\epsilon = \ln \frac{h_o}{h} \tag{6.2}$$

where h_o is the pre-forging height (mm) and h is the height of the work either at any instant during forging or the final height (mm).

In the case of *cold forging* (cold working), the flow stress (Y_f) can be calculated by:

$$Y_f = K \epsilon^n \tag{6.3}$$

where K is the strength coefficient (MPa), n is the strain-hardening exponent (see Example 6.2).

The forging shape factor K_f can be determined by:

$$K_f = 1 + \frac{0.4\mu D}{h} \quad (6.4)$$

where μ is the coefficient of friction and D is the work-part diameter or other dimension representing the contact length with the die surface (mm). Once the values of Y_f and K_f have been determined by using Eqs. 6.3 and 6.4, the forging force can be computed by the following formula:

$$F_{\text{forge}} = K_f Y_f A \quad (6.5)$$

where F_{forge} is the forging force (N) and A is the cross-sectional area of the work under consideration (mm^2) (see Examples 6.3 and 6.4).

In the case of *hot forging* (hot working), the flow stress (Y_f) is given by:

$$Y_f = \text{Yield strength} = \sigma_{ys} \quad (6.6)$$

The significance of Eq. 6.6 is illustrated in Example 6.5.

6.5.2 Analysis of Upset Forging

Upset forging, or *cold heading*, is a special open-die forging operation, which is commonly used to manufacture bolts and nails. *Upset forging* involves the use of a grip die and a heading tool. The starting stock is a bar, which is gripped on the die; care must be taken to ensure that the projection length of the bar out of the die must not exceed three times the diameter of the bar. Force is applied by the heading tool to deform the material to form the head by increasing the diameter and reducing the length out of the die (see Fig. 6.7).

Figure 6.8 shows the changes in the shape and geometry of the work during *upset forging*. The starting material is a bar stock. Assuming that the product is a bolt, and that the main deformation occurs in forming the head, we can write:

Volume of the bar out of the die = Volume of the head of the bolt

$$\frac{\pi}{4} D_o^2 h_o = \frac{\pi}{4} D_f^2 h_f \quad (6.7)$$

where D_o is the original diameter of the bar stock, h_o is the bar's projection length out of the die, h_f is the thickness of the head (of the bolt), and D_f is the head's diameter (see Example 6.6).

The number of operations needed to complete an upset forging job can be determined by the following rules. Consider bolt head before and after heading. In case $\frac{h_o}{D} \leq 2$, a single operation in heading is feasible. On the other hand, two operations may be assumed if $\frac{h_o}{D} > 2$.

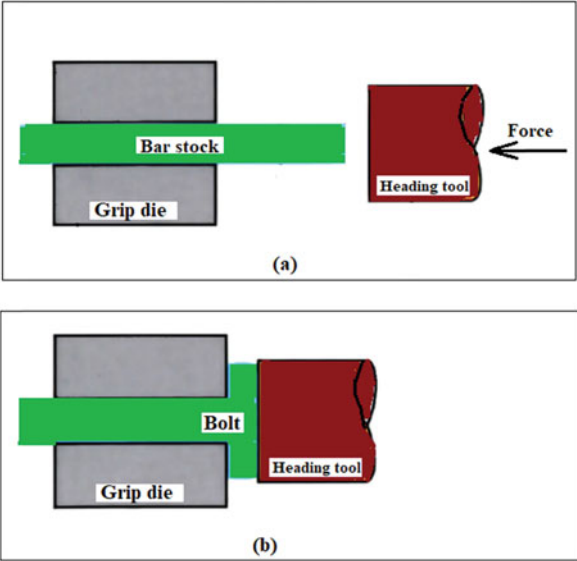


Fig. 6.7 Manufacture of bolt by upset forging; (a) original geometry of the stock, (b) the bolt

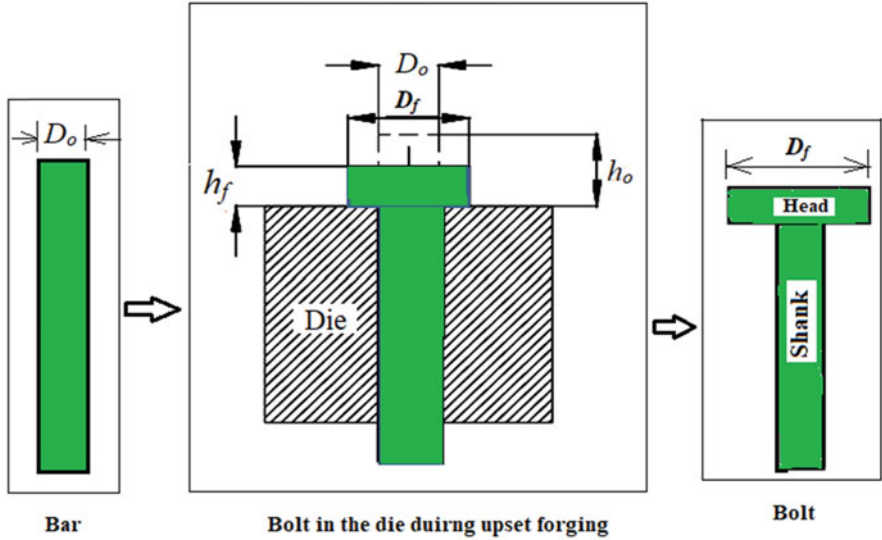


Fig. 6.8 The changes in the work's geometry during a bolt's manufacture by *upset forging*

In the Case, Two Operations Are Assumed

For the first operation,

$$h_o = h_{o1}, \quad h_f = h_{f1}, \quad D = D_o, D_f = D_{f1}$$

where

$$h_{f1} = \frac{h_o - h_f}{2}$$

D_{f1} can be calculated by constant volume relationship.

For the second operation,

$$h_o = h_{f1}, \quad h_f = h_{f2}, \quad D = D_{f1}, D_f = D_{f2}$$

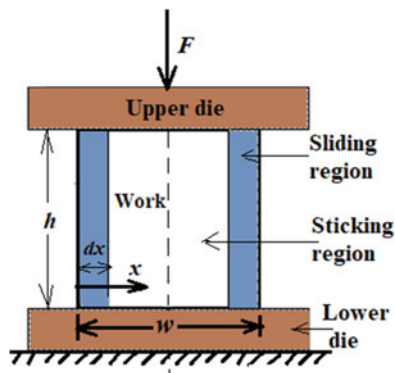
Check second operation for $\frac{h_o}{D} \leq 2$ or not.

The application of the above rules is illustrated in Example 6.10.

6.5.3 Role of Friction in Forging

Friction between work–die interfaces plays an important role in the forging operation. Consider a rectangular work-piece with dimensions ($h \times w \times \text{Unit depth}$) that is placed between two flat dies for open-die forging, as shown in Fig. 6.9. The forging pressure may act in the sliding region as well as in the sticking region of the work-piece. For low coefficient of friction (μ), the forging pressure will act in the sliding region, thereby resulting in lower forces. On the other hand, for higher friction (μ), the pressure will act in the sticking region, requiring higher forging forces. There

Fig. 6.9 Role of friction in open-die forging



exists a point x_i , where there occurs a transition from a sliding region to a sticking region in the work-piece. The distance x_i can be determined by:

$$x_i = \frac{h}{2\mu} \left(\ln \frac{1}{2\mu} \right) \quad (6.8)$$

where h is the height of the work at any instant during forging, μ is the coefficient of friction between the work–die interface (see Example 6.13).

It can be shown that for all sticky region, the forging force can be expressed as:

$$F_{\text{all sticky}} = 1.15 \bar{Y}_f \left(1 + \frac{w}{4h} \right) w \cdot \text{depth} \quad (6.9)$$

where $F_{\text{all sticky}}$ is the forging force in the all sticky region (*lbs*), $\bar{Y}_f$ is the average flow stress (*psi*), w is the width of the work (*in.*), and the depth is in inches (see Example 6.14). For all sliding region, the forging force can be expressed as:

$$F_{\text{all sliding}} = 1.15 \bar{Y}_f \left(1 + \frac{\mu \cdot w}{2h} \right) w \cdot \text{depth} \quad (6.10)$$

where μ is the coefficient of friction between the work–die interface (see Example 6.15).

6.6 Worked Numerical Examples in Metal Forging

Example 6.1: Calculating the Total Energy Required per Blow for Drop Forging

A power (drop) forging hammer has a 130-kgf ram. The ram's down-stroke is accelerated by steam pressure of 780 kPa. The ram has a height of 2 m. The ram cylinder has a diameter of 12 cm. Calculate the total energy required per blow for forging.

Solution

$$\begin{aligned} m &= 130 \text{ kg}, p = 780,000 \text{ Pa}, h = 2 \text{ m}, \text{Ram dia.} = D = 12 \text{ cm} \\ &= 0.12 \text{ m}, \text{Total Energy} = ? \end{aligned}$$

$$\text{Cross-sectional area of the ram cylinder} = A = \frac{\pi}{4} D^2 = 0.785 \times 0.12^2 = 0.0113 \text{ m}^2$$

By using Eq. 6.1,

$$\begin{aligned} \text{Total energy} &= (m g + p A) h = [(130 \times 9.81) + (780000 \times 0.0113)] \times 2 \\ &= 20178.6 \text{ J} = 20.178 \text{ kJ} \end{aligned}$$

Example 6.2: Determination of the Flow Stress in Open-Die Forging

An alloy cylindrical work is subjected to an open-die forging operation. The pre-forging height is 70 mm and the starting diameter is 45 mm. The post-forging (final) height of the work is 30 mm. The strength coefficient of the alloy is 380 MPa and the strain-hardening exponent is 0.17. Calculate the flow stress at a height of (a) 58 mm and (b) 30 mm.

Solution

$$h_o = 70 \text{ mm}, D_o = 45 \text{ mm}, h = 58 \text{ mm}, h_f = 30 \text{ mm}, K = 380 \text{ MPa}, n = 0.17, Y_f = ?$$

(a) By using Eqs. 6.2 and 6.3 for $h = 58 \text{ mm}$,

$$\varepsilon = \ln \frac{h_o}{h} = \ln \frac{70}{58} = \ln 1.21 = 0.191$$

$$Y_f = K \cdot \varepsilon^n = 380 \times 0.191^{0.17} = 286.78 \text{ MPa}$$

(b) By using Eqs. 6.2 and 6.3 for $h_f = 30 \text{ mm}$,

$$\varepsilon = \ln \frac{h_o}{h_f} = \ln \frac{70}{30} = \ln 2.33 = 0.846$$

$$Y_f = K \cdot \varepsilon^n = 380 \times 0.846^{0.17} = 369.35 \text{ MPa}$$

Example 6.3: Determination of Intermediate Diameter During Open-Die Forging

By using the data in Example 6.2, determine the (a) intermediate diameter when the intermediate height is 58 mm, and (b) final diameter when the final height is 30 mm.

Solution

$$h_o = 70 \text{ mm}, D_o = 45 \text{ mm}, h = 58 \text{ mm}, D = ?, h_f = 30 \text{ mm}, D_f = ?$$

(a) By reference to Fig. 6.6 and by using constant-volume relationship,

$$h_o \left(\frac{\pi}{4} D_o^2 \right) = h \left(\frac{\pi}{4} D^2 \right)$$

$$70 \times 45^2 = 58 \times D^2$$

$$D^2 = \frac{70 \times 45^2}{58} = 2444$$

At a height of $h = 58 \text{ mm}$, the intermediate diameter $= D = 49.436 \text{ mm}$

(b) By reference to Fig. 6.6 and by using constant-volume relationship,

$$h_o \left(\frac{\pi}{4} D_o^2 \right) = h_f \left(\frac{\pi}{4} D_f^2 \right)$$

$$70 \times 45^2 = 30 \times D_f^2$$

$$D_f^2 = 4725$$

$$D_f = 68.738 \text{ mm}$$

Example 6.4: Determining the Forging Force in an Open-Die Forging

By using the data in Examples 6.2 and 6.3, determine the forging force at: (a) intermediate height of 58 mm and (b) final height of 30 mm. The coefficient of friction is 0.11.

Solution

$$h_o = 70 \text{ mm}, D_o = 45 \text{ mm}, h = 58 \text{ mm}, D = 49.436 \text{ mm}, h_f = 30 \text{ mm}, D_f = 68.738 \text{ mm}, \mu = 0.11$$

(a) By using Eqs. 6.4 and 6.5 for $h = 58 \text{ mm}$,

$$K_f = 1 + \frac{0.4 \mu D}{h} = 1 + \frac{0.4 \times 0.11 \times 49.436}{58} = 1 + 0.0375 = 1.0375$$

$$A = \frac{\pi}{4} D^2 = 0.785 \times 49.436^2 = 1918.47 \text{ mm}^2$$

$$F_{\text{forge}} = K_f Y_f A = 1.0375 \times 286.78 \times 1918.47 = 570812.23 \text{ N} = 570.81 \text{ kN}$$

(b) By using Eqs. 6.4 and 6.5 for $h_f = 30 \text{ mm}$,

$$K_f = 1 + \frac{0.4 \mu D_f}{h_f} = 1 + \frac{0.4 \times 0.11 \times 68.738}{30} = 1 + 0.1008 = 1.1008$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 68.738^2 = 3709.05 \text{ mm}^2$$

$$F_{\text{forge}} = K_f Y_f A_f = 1.1008 \times 369.35 \times 3709.05 = 1508029.94 \text{ N} = 1.508 \text{ MN}$$

Example 6.5: Calculating the Forging Force in Open-Die Forging

An alloy cylindrical work is subjected to hot open-die forging operation. The pre-forging height is 70 mm and the starting diameter is 45 mm. The post-forging (final) height of the work is 30 mm. The yield strength of the alloy is 240 MPa. Calculate the forging force at a height of (a) 58 mm, and (b) 30 mm.

Solution

(a) By reference to Examples 6.3 and 6.4 and by using Eq. 6.4 for $h = 58$ mm,

$$K_f = 1 + \frac{0.4 \mu D}{h} = 1 + \frac{0.4 \times 0.11 \times 49.436}{58} = 1 + 0.0375 = 1.0375$$

$$A = \frac{\pi}{4} D^2 = 0.785 \times 49.436^2 = 1918.47 \text{ mm}^2$$

By using Eq. 6.6, $Y_f = \sigma_{ys} = 240$ MPa

By using Eq. 6.5 for $h = 58$ mm,

$$F_{\text{forge}} = K_f Y_f A = 1.0375 \times 240 \times 1918.47 = 477699.03 \text{ N} = 477.7 \text{ kN}$$

(b) By reference to Examples 6.3 and 6.4 and by using Eq. 6.4 for $h = 30$ mm,

$$K_f = 1 + \frac{0.4 \mu D_f}{h_f} = 1 + \frac{0.4 \times 0.11 \times 68.738}{30} = 1 + 0.1008 = 1.1008$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 68.738^2 = 3709.05 \text{ mm}^2$$

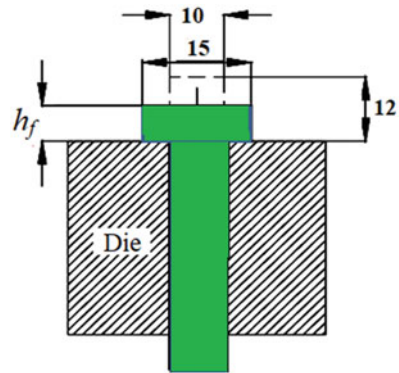
By using Eq. 6.5 for $h = 30$ mm,

$$F_{\text{forge}} = K_f Y_f A = 1.1008 \times 240 \times 3709.05 = 979901.33 \text{ N} = 979.9 \text{ kN}$$

Example 6.6: Determination of the Final Thickness of a Bolt's Head after Upset Forging

The dimensions during an upset forging operation, for manufacturing a bolt, are shown in Fig. 6.10. Determine the final thickness of the bolt's head. (Hint: Refer to Fig. 6.8).

Fig. 6.10 The dimensions (in mm) during an upset forging operation for manufacturing a bolt



Solution

By reference to Figs. 6.8 and 6.10, $h_o = 12$ mm, $D_o = 10$ mm, $D_f = 15$ mm, $h_f = ?$

$$\begin{aligned}\frac{\pi}{4} D_o^2 h_o &= \frac{\pi}{4} D_f^2 h_f \\ D_o^2 h_o &= D_f^2 h_f \\ h_f &= \frac{D_o^2 h_o}{D_f^2} = \frac{10^2 \times 12}{15^2} = \frac{1200}{225} = 5.33 \text{ mm}\end{aligned}\quad (6.7)$$

The bolt's final thickness = 5.33 mm

Example 6.7: Calculating the True Strain and the Forging Shape Factor for Upset Forging

By using the data in Example 6.6, calculate the true strain and the forging shape factor for the upset forging operation. The coefficient of friction is 0.12.

Solution

$$\begin{aligned}h_o &= 12 \text{ mm}, D_o = 10 \text{ mm}, D_f = 15 \text{ mm}, h_f = 5.33 \text{ mm}, \mu = 0.12 \\ \varepsilon &= \ln \frac{h_o}{h_f} = \ln \frac{12}{5.33} = \ln 2.2514 = 0.8115 \\ K_f &= 1 + \frac{0.4 \mu D_f}{h_f} = 1 + \frac{0.4 \times 0.12 \times 15}{5.33} = 1 + 0.135 = 1.135\end{aligned}$$

Example 6.8: Determining the Maximum Punch Force in Upset Forging

By using the data in Example 6.7, calculate the maximum punch force to form the head. The strength coefficient of the work material is 480 MPa and the strain-hardening exponent is 0.18.

Solution

$$\begin{aligned}\varepsilon &= 0.8115, K_f = 1.135, K = 480 \text{ MPa}, n = 0.18, D_f = 15 \text{ mm} \\ Y_f &= K \cdot \varepsilon^n = 480 \times 0.8115^{0.18} = 462.288 \text{ MPa} \\ A_f &= \frac{\pi}{4} D_f^2 = 0.785 \times 15^2 = 176.625 \text{ mm}^2\end{aligned}$$

The maximum force is applied at the final cross-sectional area.

$$F_{\text{forge}} = K_f Y_f A_f = 1.135 \times 462.288 \times 176.625 = 92674.58 \text{ N}$$

Example 6.9: Determination of the Number of Bolts That Can Be Produced by Upset Forging

A metal bar of length 44,531 mm with 3.2-mm diameter is available to manufacture bolts by upset forging. Each bolt's design requirements demand a head thickness of 3 mm, a head diameter of 7.25 mm, and a shank length of 72 mm. The initial projection length of the bar out of the forging-die is 12 mm. How many bolts can be manufactured to meet the design requirements?

Solution

$$D_{\text{bar}} = 3.2 \text{ mm}, h_o = 12 \text{ mm}, D_f = 7.25 \text{ mm}, h_f = 3 \text{ mm}$$

$$\text{Volume of the bar stock} = L_{\text{bar}} \left(\frac{\pi}{4} D_{\text{bar}}^2 \right) = 44,531 \times 0.785 \times 3.2^2 = 357958 \text{ mm}^3$$

$$\text{Volume of a bolt} = \text{Volume of shank} + \text{Volume of head}$$

$$\begin{aligned} \text{Volume of a bolt} &= \left(L_{\text{shank}} \times \frac{\pi}{4} D_{\text{bar}}^2 \right) + \left(h_f \times \frac{\pi}{4} D_f^2 \right) \\ &= \left(72 \times \frac{\pi}{4} 3.2^2 \right) + \left(3 \times \frac{\pi}{4} 7.25^2 \right) = 578.76 + 123.78 = 702.54 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} \text{Number of bolts that can be manufactured} &= \frac{\text{Volume of bar stock}}{\text{Volume of a bolt}} = \frac{357958}{702.54} \\ &= 509.52 \cong 500 \end{aligned}$$

Example 6.10: Determination of the Number of Operations Needed for Upset-Forging Job

A metal bar of diameter 3.2-mm is to be upset forged to produce a bolt with a head thickness of 3 mm and head diameter of 7.25 mm. The initial projection length of the bar out of the forging-die is 12 mm. How many operations are needed for this forging job?

Solution

$$h_o = 12 \text{ mm}, D = D_o = 3.2 \text{ mm}, h_f = 3 \text{ mm}, D_f = 7.25 \text{ mm}$$

$$\frac{h_o}{D} = \frac{12}{3.2} = 3.75 > 2$$

Since $\frac{h_o}{D} > 2$, we assume two operations to perform this job of upset forging.

For first operation,

$$h_o = h_{o1} = 12 \text{ mm}, h_f = h_{f1}, D = D_o = 3.2 \text{ mm}, D_f = D_{f1}$$

$$h_{f1} = \frac{h_o - h_f}{2} = \frac{12 - 3}{2} = 4.5 \text{ mm}$$

D_{f1} can be calculated by constant volume relationship, as follows:

$$h_{o1} \left(\frac{\pi}{4} D_o^2 \right) = h_{f1} \left(\frac{\pi}{4} D_{f1}^2 \right)$$

$$D_{f1}^2 = \frac{h_{o1} D_o^2}{h_{f1}} = \frac{12 \times 3.2^2}{4.5} = 27.3 \text{ mm}$$

$$D_{f1} = 5.22 \text{ mm}$$

For second operation,

$$h_o = h_{f1} = 4.5 \text{ mm}, \quad h_f = h_{f2} = 3 \text{ mm}, \quad D_o = D_{f1} = 5.22 \text{ mm}, \quad D_f = D_{f2}$$

$$\text{Checking for } \frac{h_o}{D} = \frac{h_{f1}}{D_{f1}} = \frac{4.5}{5.22} = 0.86 < 2 \rightarrow \text{single operation}$$

Hence, 2 operations are needed to perform this forging job.

Example 6.11: Determining the Projection Length of Bar Stock Out of Die in Upset Forging

An upset forging operation is required to be performed to produce the head on a steel bolt. The strength coefficient for the steel is 560 MPa, and the strain-hardening exponent is 0.2. The coefficient of friction at the die–work interface is 0.13. The starting bar stock has a diameter of 4.50 mm. The final diameter of the head is 10 mm and the final thickness of the head is 1.8 mm. The final length of the bolt is 130 mm. What length of the bar stock must project out of the die for this forging operation?

Solution

$$D_o = 4.50 \text{ mm}, D_f = 10 \text{ mm}, h_f = 1.8 \text{ mm}, h_o = \text{The projection length out of the die} = ?$$

By using Eq. 6.7,

Volume of the bar out of the die = Volume of the head of the bolt

$$\frac{\pi}{4} D_o^2 h_o = \frac{\pi}{4} D_f^2 h_f$$

$$D_o^2 h_o = D_f^2 h_f$$

$$h_o = \left(\frac{D_f}{D_o} \right)^2 h_f = \left(\frac{10}{4.5} \right)^2 \times 1.8 = 2.22^2 \times 1.8 = 8.87 \text{ mm}$$

The projection length out of the die = 8.87 mm.

Example 6.12: Determining the Maximum Punch Force in Upset Forging

By using the data in Example 6.11, determine the maximum punch force for this forging operation.

Solution

$$D_o = 4.50 \text{ mm}, D_f = 10 \text{ mm}, h_f = 1.8 \text{ mm}, h_o = 8.87 \text{ mm}, K = 560 \text{ MPa}, n = 0.2, \\ \mu = 0.13, F = ?$$

$$\varepsilon = \ln \frac{h_o}{h_f} = \ln \frac{8.87}{1.8} = \ln 4.93 = 1.595$$

$$Y_f = K \cdot \varepsilon^n = 560 \times 1.595^{0.2} = 614.81 \text{ MPa}$$

$$K_f = 1 + \frac{0.4 \mu D_f}{h_f} = 1 + \frac{0.4 \times 0.13 \times 10}{1.8} = 1 + 0.2889 = 1.2889$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 10^2 = 78.5 \text{ mm}^2$$

$$F = K_f Y_f A_f = 1.2889 \times 614.81 \times 78.5 = 62205.64 \text{ N}$$

Example 6.13: Determining the Maximum Forging Force for a Lead Strip

An open-die forging operation is performed on a lead strip with initial dimensions 26 mm × 25 mm × 180 mm at room temperature (25 °C). The final strip dimensions are: 8 mm × 100 mm × 152 mm. The yield strength of lead is 7 MPa and the coefficient of friction at the die–work interface is 0.2. Determine the maximum forging force for the operation.

Solution

$$h_0 = 26 \text{ mm}, w_0 = 25 \text{ mm}, L_0 = 180 \text{ mm}; h_f = 8 \text{ mm}, w_f = 100 \text{ mm}, L_f \\ = 152 \text{ mm}, \sigma_y = 7 \text{ MPa}, \mu = 0.2$$

Since forming of lead at room temperature is actually hot working (HW),

$$Y_f = \text{yield strength} = 7 \text{ MPa}$$

The maximum forging force corresponds to the final area.

$$\text{Final cross-sectional area} = A_f = 100 \times 152 = 15200 \text{ mm}^2$$

$$D_f^2 = 100^2 + 152^2, \quad D_f = 181.94 \text{ mm}$$

$$K_f = 1 + \frac{0.4 \mu D_f}{h_f} = 1 + \frac{0.4 \times 0.2 \times 181.94}{8} = 2.8194$$

$$F_f = K_f Y_f A_f = 2.8194 \times 7 \times 15200 = 299984.16 \text{ N}$$

Example 6.14: Determining the Transition Point from Sliding to Sticking Region in O.D.F

The work-piece height, at the end of an open-die forging (ODF) operation, is 8 mm. The coefficient of friction at the work–die interface is 0.2. Determine the point of transition from sliding region to the sticking region in the work-piece.

Solution

By using Eq. 6.8,

$$x_i = \frac{h}{2\mu} \left(\ln \frac{1}{2\mu} \right) = \frac{8}{2 \times 0.2} \left(\ln \frac{1}{2 \times 0.2} \right) = 20 \ln 2.5 = 18.32 \text{ mm}$$

Example 6.15: Determine the Forging Force in the All Sticky Region in Open-Die Forging

A metallic work-piece is to be subjected to open-die forging to the final dimensions: width = 3 in., height = 0.333 in, and depth = 2 in. The average flow stress of the work material is 1200 lbs./in². Determine the forging force in the all sticky region required for this operation.

Solution

By using Eq. 6.9,

$$F_{\text{all sticky}} = 1.15 \bar{Y}_f \left(1 + \frac{w}{4h} \right) w \cdot \text{depth} = 1.15 \times 1200 \times \left(1 + \frac{3}{4 \times 0.333} \right) \times 3 \times 2$$

$$F_{\text{all sticky}} = 8280 \times (1 + 2.2522) = 26928.64 \text{ lbs.}$$

Example 6.16: Determining the Forging Force in the All Sliding Region in Open-Die Forging

By using the data in Example 6.15, determine the forging force in the all sliding region, if the coefficient of friction between the work–die interface is 0.10.

Solution

$$w = 3 \text{ in.}, h = 0.333, \text{depth} = 2 \text{ in.}, \bar{Y}_f = 1200 \text{ psi}, \mu = 0.1, F = ?$$

By using Eq. 6.10,

$$F_{\text{all sliding}} = 1.15 \bar{Y}_f \left(1 + \frac{\mu \cdot w}{2h} \right) w \cdot \text{depth} = 1.15 \times 1200 \times \left(1 + \frac{0.1 \times 3}{2 \times 0.333} \right) \times 3 \times 2$$

$$F_{\text{all sliding}} = 8280 \times (1 + 0.45) = 12006 \text{ lbsf}$$

Questions and Problems

6.1 (MCQs). Select the most appropriate answers for the following statements.

1. When was forging first used?
(a) 1000 A.D, (b) 2000 A.D, (c) 4000 B.C, (d) 1000 B.C.
 2. Which type of structure does mainly contribute to high strength of forged products?
(a) Elongated grains, (b) grain flow, (c) coarse grains, (d) fine grains.
 3. Which type of forging operation best corresponds to cold heading?
(a) Open-die forging, (b) closed-die forging, (c) impression-die forging, (d) flash-less.
 4. Which measure will best overcome the issues of noise and vibration in forging operation?
(a) Correct equipment, (b) correct forging force, (c) correct building structure.
 5. Which forging operation is the best for manufacturing bolts and nails?
(a) Upset forging, (b) flash-less forging, (c) closed-die forging, (d) coining
 6. Which deformation behavior is observed in hot forging?
(a) Strain hardening, (b) yielding, (c) work hardening, (d) plastic flow
 7. Which manufacturing process is the most suitable for producing crane hooks?
(a) Closed-die forging, (b) upset forging, (c) casting, (d) open-die forging
 8. Which forging operation is the best for manufacturing pressure vessels?
(a) Closed-die forging, (b) upset forging, (c) coining, (d) open-die forging
 9. In upset forging, the projection length of the bar-stock out of the die must be less than x times the diameter of the bar stock. What is the value of x ?
(a) 1, (b) 2, (c) 3, (d) 4.
- 6.2 Highlight the advantages and limitations of metal forging.
- 6.3 Differentiate between hammer forging and press forging.
- 6.4 Give at least three examples of components that are manufactured by
(a) open-die forging and (b) closed-die forging.

- 6.5 Draw labeled sketches showing the three forging operations: (a) open-die forging, (b) closed-die forging, and (c) flash-less forging.
- 6.6 Describe upset forging operation with the aid of sketches.
- 6.7 Explain the role of friction in the open-die forging operation.
- P6.8 A power (drop) forging hammer has a 120-kgf ram. The ram's down-stroke is accelerated by steam pressure of 760 kPa. The ram has a rise of height of 1.8 m. The ram cylinder has a diameter of 10 cm. Calculate the total energy required per blow for forging.
- P6.9 An alloy cylindrical work is subjected to an open-die forging operation. The pre-forging height is 65 mm and the starting diameter is 40 mm. The post-forging (final) height of the work is 27 mm. The strength coefficient of the alloy is 400 MPa and the strain-hardening exponent is 0.18. The coefficient of friction between the die-work surface is 0.09. Calculate the forging force at a height of (a) 55 mm, and (b) 28 mm.
- P6.10 A metal bar of diameter 3.4-mm is to be upset forged to produce a bolt with a head diameter of 8 mm. The initial projection length of the bar out of the forging-die is 16 mm.
Determine the final head thickness of the bolt.
- P6.11 A metal bar of diameter 3 mm is to be upset forged to produce a bolt with a head thickness of 2.8 mm and head diameter of 7 mm. The initial projection length of the bar out of the forging-die is 11 mm. How many operations are needed for this forging job?
- P6.12 An upset forging operation is required to be performed to produce the head on a steel bolt.
The starting bar stock has a diameter of 4.2 mm. The final diameter of the head is 9 mm with the final thickness of the head is 1.5 mm. What length of the bar stock must project out of the die for this forging operation?
- P6.13 A metallic work-piece is to be subjected to open-die forging to the final dimensions: width = 4 in., height = 0.25 in, and depth = 2 in. The average flow stress of the work material is 1100 lbs./in². Determine the required forging force in the all sticky region.
- P6.14 The work-piece height, at the end of an open-die forging (ODF) operation, is 7 mm. The coefficient of friction at the work–die interface is 0.22. Determine the point of transition from sliding region to the sticking region in the work-piece.
- P6.15 By using the data in P6.13, determine the forging force in the all sliding region, if the coefficient of friction between the work–die interface is 0.14.

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Chapter 7

Metal Extrusion



Nomenclature

r_x	extrusion ratio
A_o	original cross-sectional area of billet
A_f	cross-sectional area of the extruded part
α	die angle
ϵ	true strain
ϵ_x	extrusion strain
a and b	the constants in the Johnson's formula
$\bar{Y}_f$	average flow stress
K	strength coefficient
n	strain-hardening exponent
$p_{d. e}$	ram pressure for direct extrusion
L	length of work-piece
D_o	original diameter of the work
K_x	shape factor for the complex die orifice in extruder
C_x	perimeter of the extruded cross-section
C_c	perimeter of a circle of the same area as the extruded shape
$p_{i.e}$	ram pressure for <i>indirect extrusion</i>
$p'_{i.e}$	ram pressure for indirect extrusion using an extruder with complex die orifice
F	ram force in extrusion
P	power for an extrusion operation
v	ram velocity
$\dot{\epsilon}$	true strain rate

Abbreviations

CW	cold working
HW	hot working

7.1 Introduction

7.1.1 *Extrusion and Its Products*

Extrusion is a metal forming process in which the cylindrical work metal (billet) is forced to flow by pushing through a die opening to produce a desired cross-section. The fixed cross-sectional profile extruded parts are called “extrudates”; they are pushed out using either a mechanical or hydraulic press. Some examples of extrudates include welding electrodes, solid square rods, hexagonal rods, tubes, aluminum pipe fittings, and a variety of complex cross-sections (see Fig. 7.1).



Fig. 7.1 Some metal-extruded products with complex cross-sections

7.1.2 Advantages of Extrusion

Metal extrusion has several advantages, which are given below.

- (a) Extrusion forms finished parts with an excellent surface finish (e.g., welding rods)
- (b) Extrusion can produce parts with very complex cross-sections (e.g., aircraft parts).
- (c) In extrusion, a little or no wasted material is created.
- (d) Although aluminum is the most extensively used metal in the extrusion industry (Saha, 2000), extrusion is also increasingly being used in the working of difficult-to-form metals (e.g. stainless steels, Ni-based alloys, and other high-strength alloys). In particular, cold extrusion enables us to extrude copper, tin, titanium, steel, and alloys of aluminum (Navaneetha & Lakshmi, 2023).

7.2 Classification of Extrusion Processes

Extrusion processes may be classified based on temperature, deformation type, and the process type. Based on temperature, an extrusion process may be cold working, warm working, or hot working (see Chap. 4). Based on deformation type, an extrusion process may be direct extrusion or indirect extrusion. Based on the type of process, extrusion may be a continuous extrusion process or a discrete extrusion process. In addition to these processes, there are some related processes, such as hydrostatic extrusion, and lateral extrusion. The classification chart for various extrusion processes is presented in Fig. 7.2.

Cold extrusion is carried out at a temperature below the *recrystallization temperature* of the metal (see Fig. 4.1). The advantages of *cold extrusion* include (a) high dimensional accuracy/close tolerance, (b) increased work strength due to strain hardening, and (c) improved surface finish. *Hot extrusion* is performed at a temperature above the recrystallization temperature of the metal. The following are the advantages of *hot extrusion*: (a) greater operational economy due to lower ram force and power requirement, (b) higher ram velocity resulting in higher production rate, and (c) the ability to produce long parts with more complex shapes and geometry. *Warm extrusion* is carried out at a temperature around $0.3T_m$, where T_m is the melting temperature of the metal in K; generally, the temperature ranges from 424 to 975 °C (see Fig. 4.1). *Warm extrusion* is mainly used to manufacture aluminum extruded tubing, which is cylindrical in most cases, but aluminum rectangular tubing and aluminum square tubing are also extruded for specialty applications, such as in building construction.

Based on deformation, there are two types of extrusion: (a) direct extrusion, and (b) indirect extrusion. In the case of *direct extrusion*, a ram pushes the material, thereby forcing it to flow through one or more die-openings in the same direction as the ram's motion. In the case of *indirect extrusion*, the ram forces the metal to flow through the die opening in a direction opposite to the ram motion.

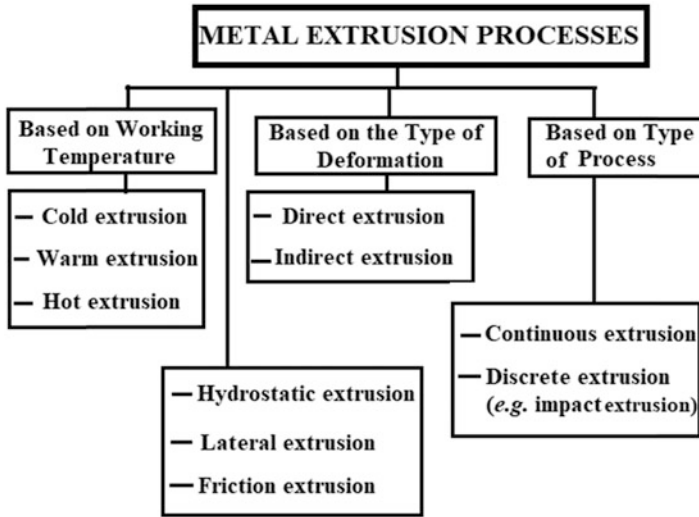


Fig. 7.2 The classification chart for metal extrusion processes

Fig. 7.3 Direct extrusion – working principle



Based on the type of process, an extrusion process may be either a *continuous extrusion* process or a *discrete extrusion* process. A *continuous extrusion* process is theoretically capable of producing infinitely long bars in one cycle; this process is ultimately limited by the size of the billet that can be loaded into the extrusion container. A *discrete extrusion* operation involves production of a single part in each extrusion cycle; *impact extrusion* is an example of a discrete extrusion process.

7.3 Direct Extrusion of Metals

Direct extrusion, also called *forward extrusion*, involves loading a metal billet into a container, and then compressing the work using a ram. The material is forced to escape through one or more die openings in the same direction as the ram motion (see Fig. 7.3). The diameter of the extruded part (extrudate) is the same as that of the die opening. Since there is a significant friction between the billet surface and wall of the container, a greater ram force is required in direct extrusion. The equipment for direct extrusion is simpler, and hence less costly, as compared to the equipment for indirect extrusion.

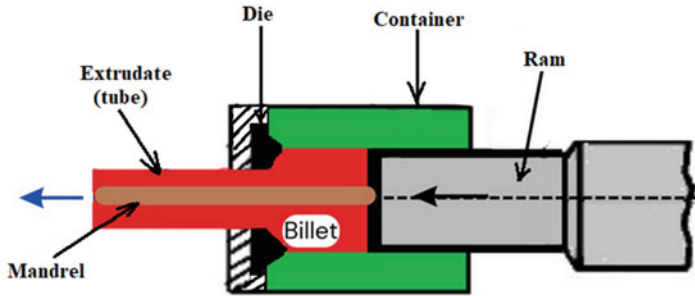
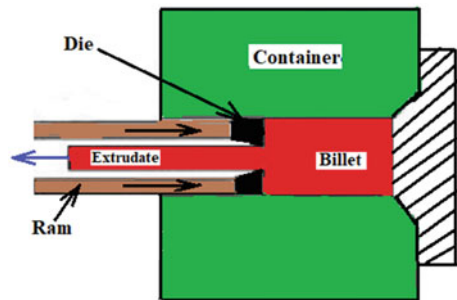


Fig. 7.4 Tube manufacturing by direct extrusion

Fig. 7.5 Indirect extrusion – working principle (Note: Die angle $< 90^\circ$)



Tube Manufacturing Direct extrusion process is also employed for manufacturing tubes. The starting material (stock) is in the form of a round steel billet. In case of hot extrusion, the stock is heated to the forming temperature, and then inserted into the container, which features a round-bored die (see Fig. 7.4). The billet is initially pierced through the center by a mandrel driven by a hydraulic ram. As the mandrel passes through the die, it forms an annular gap through which the material is deformed under the pressure exerted by the ram to form the tube.

7.4 Indirect Extrusion of Metals

Indirect extrusion, also called *backward extrusion*, involves mounting of the die to the ram rather than at the opposite end of the container (see Fig. 7.5). The die is stationary whereas the billet and the container move together. As the ram penetrates into the work (billet), the metal is forced to flow through the die opening in a direction opposite to the ram's motion. In order to keep the die stationary, a "stem" is used which must be longer than the length of the container. Since the billet movement is along with the container, the frictional forces are significantly reduced.

The low friction in indirect extrusion results in the following advantages: (a) 25–30% reduction of friction, thereby allowing extrusion of larger billets, enhanced speed, and an increased ability to extrude smaller cross-sections, (b) smaller ram force and power is required, which results in saving of operational cost, (c) less tendency for extrusions to crack, since no frictional heat generation occurs, and (d) the container liner's life is longer, due to less wear.

7.5 Miscellaneous Extrusion Processes

Miscellaneous extrusion processes include impact extrusion, hydrostatic extrusion, and lateral extrusion. *Impact extrusion* involves impacting the metallic work-piece at a high strain rate by use of a die and a punch (Fig. 7.6). In order to perform impact extrusions, one needs either a mechanical or hydraulic press. Firstly, the work-piece is positioned between a punch and a die. On activation, the press will move the punch down onto the work, thereby forcing the metal through the die. The work will then take the shape of the die. Once the work has been forced through the die, it is removed via an ejector. *Impact extrusion* has several advantages. For example, it is a cold-working (CW) process, thus requiring no preheating of the metal stock.

Impact extrusion only requires a single impact, during which, the metal will contour to the shape and dimensions of the die with which it is used. Impact extrusion is often used to create fasteners (e.g., screws, bolts). In order to create a fastener, a metal stock with a smooth finish is used; here, the die contains a grainy texture that results in creating the fastener's external threading.

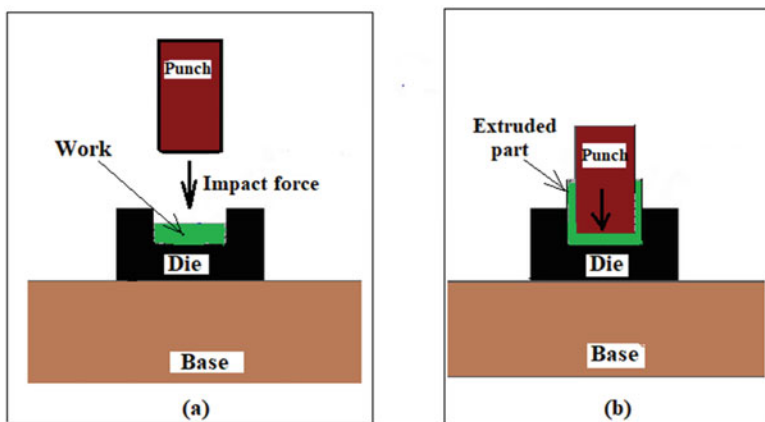


Fig. 7.6 Impact extrusion; (a) punch approaches work at a high velocity, (b) part is extruded by the impact

Hydrostatic extrusion involves the introduction of a fluid inside the container; the fluid surrounds the billet. The ram pressurizes the fluid, which in turn forces the billet to squeeze through the die opening. *Hydrostatic extrusion* is a direct extrusion process, but the friction is almost eliminated thereby significantly lowering the ram force. *Lateral extrusion* involves the use of a vertical container with the die located on the side. As the punch pushes down on the billet, the flow of metal is perpendicular to the punch direction; the extrudate comes out the side of the container forming an L-shaped part. *Friction extrusion* is an extrusion process that can be used to form fully consolidated rods, wires, tubes, or other noncircular metal shapes directly from a variety of precursor charges, including metal powder, flake, metal scrap, or solid billet. Like conventional extrusion processes, in *friction extrusion*, a shape change is accomplished on the charge by forcing its passage through a die. However, friction extrusion differs from conventional extrusion in that the extrusion charge (billet or other precursor) rotates relative to the extrusion die.

7.6 Engineering Analysis of Extrusion

The extrusion process can be mathematically analyzed by considering the geometric design features of an extrusion (direct extrusion) machine and the metal stock, as shown in Fig. 7.7. An important extrusion-process parameter is the *extrusion ratio* that is given by (Huda, 2018):

$$r_x = \frac{A_o}{A_f} \quad (7.1)$$

where r_x is the extrusion ratio, A_o is the original cross-sectional area of work (mm^2), and A_f is the cross-sectional area of the extruded part (mm^2) (see Example 7.1).

Once the extrusion ratio has been calculated using Eq. 7.1, the true strain and the extrusion strain can be determined by the following mathematical relationships (Groover, 2019):

$$\epsilon = \ln r_x \quad (7.2)$$

$$\epsilon_x = a + b \epsilon \quad (7.3)$$

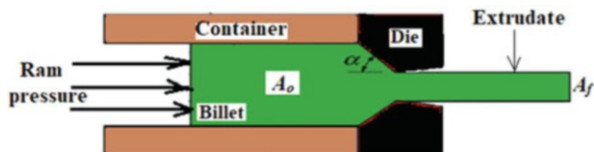


Fig. 7.7 Geometric design features of extrusion machine and the billet (α = die angle)

where ε is the true strain, ε_x is the extrusion strain, and a and b are the constants in the Johnson's formula (see Example 7.2). In case of cold extrusion, the average flow stress can be found by:

$$\bar{Y}_f = \frac{K \cdot \varepsilon^n}{1+n} \quad (7.4)$$

where $\bar{Y}_f$ is the average flow stress (MPa), K is the strength coefficient (MPa), and n is the strain-hardening exponent (see Example 7.3).

The expression for the *extrusion pressure* depends on whether extrusion is direct or indirect. The pressure also depends whether the die orifice is circular or has complex geometry. In case of *direct extrusion* with circular die orifice, *ram pressure* is given by:

$$p_{d,e} = \bar{Y}_f \left(\varepsilon_x + \frac{2L}{D_o} \right) \quad (7.5)$$

where $p_{d,e}$ is the ram pressure for direct extrusion machine with circular orifice (MPa), $\bar{Y}_f$ is the average flow stress (MPa), ε_x is the extrusion strain; L is work length (mm), and D_o is original diameter of the work (mm) (see Examples 7.4 and 7.5). In case of *direct extrusion* by using an extruder with complex die orifice, *ram pressure* can be determined by (Altan et al., 1983):

$$p'_{d,e} = K_x \bar{Y}_f \left(\varepsilon_x + \frac{2L}{D_o} \right) \quad (7.6)$$

where $p'_{d,e}$ is the ram pressure for direct extrusion machine with complex die orifice, K_x is the shape factor for the complex die orifice in extruder, which is given by:

$$K_x = 0.98 + 0.02 \left(\frac{C_x}{C_c} \right)^{2.25} \quad (7.7)$$

where C_x is the perimeter of the extruded cross-section (mm) and C_c is the perimeter of a circle of the same area as the extruded shape (mm) (see Examples 7.6–7.8). The shape factor for circular die orifice is unity ($K_x = 1$).

In case of *indirect extrusion* by using an extruder with circular die orifice, *pressure* is given by:

$$p_{i,e} = \bar{Y}_f \varepsilon_x \quad (7.8)$$

where $p_{i,e}$ is the ram pressure for *indirect extrusion* (MPa) (see Example 7.9).

For *indirect extrusion* by using an extruder with complex die orifice, *ram pressure* is given by:

$$p'_{i,e} = K_x \bar{Y}_f \varepsilon_x \quad (7.9)$$

where $p'_{i,e}$ is the ram pressure for indirect extrusion using an extruder with complex die orifice, and K_x is the shape factor for the complex die orifice cross-section (see Example 7.10).

The *Ram force* in extrusion can be computed by the relation:

$$F = p A_o \quad (7.10)$$

where F is the ram force (N), p is the ram pressure (MPa), and A_o is the billet's original cross-sectional area (mm^2) (see Example 7.13). The *power* for an extrusion operation is given by:

$$P = F v \quad (7.11)$$

where P is the power (W), F is the ram force (N), and v is ram velocity (m/s) (see Example 7.14).

In case of hot extrusion for high extrusion ratios and a die angle of 0.7854 rad, as the case may be, with poor lubrication, the true strain rate can be determined by (Kalpakjian & Schmid, 2008):

$$\dot{\epsilon} = \frac{6v}{D_o} \ln r_x \quad (7.12)$$

where $\dot{\epsilon}$ is the true strain rate (/s) (see Example 7.21).

7.7 Worked Numerical Examples in Extrusion

Example 7.1: Determining the Extrusion Ratio

A cylindrical billet that is 110-mm long and 60 mm in diameter is to be reduced to 28 mm diameter by direct extrusion. Determine the extrusion ratio.

Solution

$$D_o = 60 \text{ mm}, D_f = 28 \text{ mm}, r_x = ?$$

$$\text{The pre-extrusion cross-sectional area} = A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 60^2 = 2826 \text{ mm}^2$$

$$\text{The post-extrusion cross-sectional area} = A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 28^2 = 615.44 \text{ mm}^2$$

By using Eq. 7.1,

$$\text{Extrusion ratio} = r_x = \frac{A_o}{A_f} = \frac{2826}{615.44} = 4.59$$

Example 7.2: Determining the True Strain and the Extrusion Strain

By using the data in Example 7.1, determine the (a) true strain (homogeneous deformation) and (b) extrusion strain. The constants in the Johnson's formula are $a = 0.8$ and $b = 1.5$.

Solution

By using Eqs. 7.2–7.3,

$$\text{True strain} = \epsilon = \ln r_x = \ln 4.59 = 1.524$$

$$\text{Extrusion strain} = \epsilon_x = a + b \epsilon = 0.8 + (1.5 \times 1.524) = 3.086$$

Example 7.3: Determining the Average Flow Strain in Direct Extrusion

By using the data in Example 7.2, determine the average flow stress in the extrusion operation.

The work material is naval brass with the strain hardening exponent of 0.21 and the strength coefficient of 585 MPa.

Solution

$$K = 585 \text{ MPa}, n = 0.21, \epsilon = 1.524, \bar{Y}_f = ?$$

By using Eq. 7.4,

$$\bar{Y}_f = \frac{K \cdot \epsilon^n}{1 + n} = \frac{585 \times 1.524^{0.21}}{1 + 0.21} = 528.2 \text{ MPa}$$

Example 7.4: Determining the Ram Pressure in Direct Extrusion with Round Die Orifice

A naval-brass billet that is 110-mm long and 60 mm in diameter is reduced by direct extrusion to a 28 mm diameter. The constants in the Johnson's formula are $a = 0.8$, and $b = 1.5$. Determine the pressure applied to the end of the billet as the ram moves forward.

Solution

By reference to Examples 7.2 and 7.3,

$$L = 110 \text{ mm}, D_o = 60 \text{ mm}, \epsilon_x = 3.086, \bar{Y}_f = 528.2 \text{ MPa}, p = ?$$

By using Eq. 7.5,

$$p_{d.e} = \bar{Y}_f \left(\epsilon_x + \frac{2L}{D_o} \right) = 528.2 \times \left(3.086 + \frac{2 \times 110}{60} \right) = 528.2 \times 6.753 = 3566.76 \text{ MPa}$$

Example 7.5: Determining the Ram Pressures at Various Billet Lengths, Including $L = 0$

By using the data in Example 7.4, determine the ram pressures for billet length of: (a) $L = 90 \text{ mm}$, (b) $L = 60 \text{ mm}$, (c) $L = 0$.

Solution

(a) $L = 90 \text{ mm}$, $D_o = 60 \text{ mm}$, $\epsilon_x = 3.086$, $\bar{Y}_f = 528.2 \text{ MPa}$, $p = ?$

$$p_{d,e} = \bar{Y}_f \left(\epsilon_x + \frac{2L}{D_o} \right) = 528.2 \times \left(3.086 + \frac{2 \times 90}{60} \right) = 3214.62 \text{ MPa}$$

(b) $L = 60 \text{ mm}$, $D_o = 60 \text{ mm}$, $\epsilon_x = 3.086$, $\bar{Y}_f = 528.2 \text{ MPa}$, $p = ?$

$$p_{d,e} = \bar{Y}_f \left(\epsilon_x + \frac{2L}{D_o} \right) = 528.2 \times \left(3.086 + \frac{2 \times 60}{60} \right) = 2686.42 \text{ MPa}$$

(c) $L = 0$, $D_o = 60 \text{ mm}$, $\epsilon_x = 3.086$, $\bar{Y}_f = 528.2 \text{ MPa}$, $p = ?$

$$p_{d,e} = \bar{Y}_f \left(\epsilon_x + \frac{2L}{D_o} \right) = 528.2 \times \left(3.086 + \frac{2 \times 0}{60} \right) = 1630.02 \text{ MPa}$$

This is also the value of ram pressure ($p = p_{i,e} = \bar{Y}_f \epsilon_x$) that would be associated with indirect extrusion throughout the length of the billet.

Example 7.6: Determining the Shape Factor for an Extruder with a Complex Die Orifice

A naval-brass billet that is 110-mm long and 60 mm in diameter is to be extruded by direct extrusion using an extrusion machine with a die orifice of cross-section 20 mm $\times$ 20 mm. Determine the shape factor for the extrusion operation.

Solution

$C_x = \text{perimeter of the extruded cross-section} = 4 \times 20 \text{ mm} = 80 \text{ mm}$

Area of the square cross-section = 20 mm $\times$ 20 mm = 400 mm²

Area of the circle = $\pi r^2 = \text{area of the complex cross-section} = 400 \text{ mm}^2$

Radius of the circle of the same area = $r = \sqrt{\frac{400}{\pi}} = 11.284 \text{ mm}$

$C_c = \text{perimeter of a circle of the same area as the extruded shape}$
 $= 2\pi r = 2 \times \pi \times 11.28 = 70.9 \text{ mm}$

By using Eq. 7.7,

$$\begin{aligned} K_x &= 0.98 + 0.02 \left(\frac{C_x}{C_c} \right)^{2.25} = 0.98 + 0.02 \times \left(\frac{80}{70.9} \right)^{2.25} \\ &= 0.98 + 0.02 \times (1.128)^{2.25} = 1.0062 \end{aligned}$$

Example 7.7: Determining r_x for an Extruder with a Die Orifice of Square Cross-Section

A naval-brass billet that is 110-mm long and 60 mm in diameter is to be extruded by direct extrusion by passing through an extruder's die orifice of square cross-section: 20 mm $\times$ 20 mm. The constants in the Johnson's formula are $a = 0.8$ and $b = 1.5$. The strain hardening exponent is 0.21 and the strength coefficient is 585 MPa. Determine the extrusion ratio.

Solution

$$A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 60^2 = 2826 \text{ mm}^2$$

$$A_f = 20 \text{ mm} \times 20 \text{ mm} = 400 \text{ mm}^2$$

$$r_x = \frac{A_o}{A_f} = \frac{2826}{400} = 7.1$$

Example 7.8: Determining the Ram Pressure for Direct Extrusion Using an Extruder with a Die Orifice of Square Cross-Section

By using the data in Examples 7.6 and 7.7, determine the ram pressure for the direct extrusion operation by using the extruder with the die orifice of square cross-section.

Solution

$$K_x = 1.0062, \quad r_x = 7.1, \quad a = 0.8, \quad b = 1.5, \quad n = 0.21, \quad K = 585 \text{ MPa}, \\ L = 110 \text{ mm}, \quad D_o = 60 \text{ mm}.$$

$$\epsilon = \ln r_x = \ln 7.1 = 1.96$$

$$\epsilon_x = a + (b \epsilon) = 0.8 + (1.5 \times 1.96) = 3.74$$

$$\bar{Y}_f = \frac{K \cdot \epsilon^n}{1 + n} = \frac{585 \times 1.96^{0.21}}{1 + 0.21} = 556.86 \text{ MPa}$$

By using Eq. 7.6,

$$p_{d.e} = K_x \bar{Y}_f \left(\epsilon_x + \frac{2L}{D_o} \right) = 1.0062 \times 556.86 \times \left(3.74 + \frac{2 \times 110}{60} \right) = 4150.048 \text{ MPa}$$

Example 7.9: Determining the Ram Pressure for Indirect Extrusion with Round Die Orifice

An annealed-copper billet that is 100 mm long and 55 mm in diameter is to be extruded to a diameter of 25 mm by indirect extrusion. The strain hardening exponent of the work material is 0.44 and its strength coefficient is 530 MPa. The constants in the Johnson's formula are $a = 0.8$ and $b = 1.5$. Determine the ram pressure for the indirect extrusion operation.

Solution

$$L = 100 \text{ mm}, D_o = 55 \text{ mm}, D_f = 25 \text{ mm}, n = 0.44, K = 530 \text{ MPa},$$

$$\text{The pre-extrusion cross-sectional area} = A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 55^2 = 2374.63 \text{ mm}^2$$

$$\text{The post-extrusion cross-sectional area} = A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 25^2 = 490.63 \text{ mm}^2$$

$$r_x = \frac{A_o}{A_f} = \frac{2374.62}{490.62} = 4.84$$

$$\varepsilon = \ln r_x = \ln 4.84 = 1.577$$

$$\varepsilon_x = a + (b \varepsilon) = 0.8 + (1.5 \times 1.577) = 3.166$$

$$\bar{Y}_f = \frac{K \cdot \varepsilon^n}{1 + n} = \frac{530 \times 1.577^{0.44}}{1 + 0.44} = 449.74 \text{ MPa}$$

By using Eq. 7.8,

$$p_{i.e} = \bar{Y}_f \varepsilon_x = 449.74 \times 3.165 = 1423.43 \text{ MPa}$$

The ram pressure for the indirect extrusion operation = 1423.43 MPa.

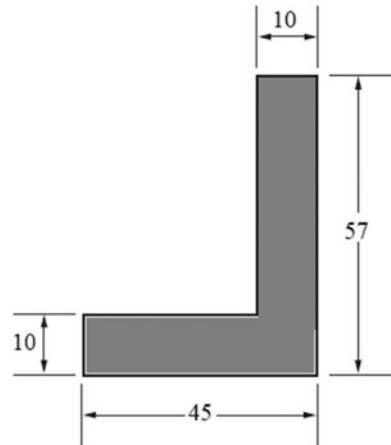
Example 7.10: Determining Shape Factor for Extruder's Complex Die Orifice Cross-Section

Determine the shape factor for an extruder's complex die orifice cross-section, as shown in Fig. 7.8.

Solution

$$C_x = \text{perimeter of the extruded cross-section} = 57 + 10 + 47 + 35 + 10 + 45 = 204 \text{ mm}$$

Fig. 7.8 The extruder's die orifice cross-section for Example 7.10



Area of the complex cross-section = $(57 \times 10) + (35 \times 10) = 570 + 350 = 920 \text{ mm}^2$

Area of the circle = πr^2 = area of the complex cross-section = 920 mm^2

$$r = \sqrt{\frac{920}{\pi}} = 17.11 \text{ mm}$$

$$\begin{aligned} C_c &= \text{perimeter of a circle of the same area as the extruded shape} = 2\pi r \\ &= 2 \times \pi \times 17.11 = 107.52 \text{ mm} \end{aligned}$$

By using Eq. 7.7,

$$\begin{aligned} K_x &= 0.98 + 0.02 \left(\frac{C_x}{C_c} \right)^{2.25} = 0.98 + 0.02 \times \left(\frac{204}{107.52} \right)^{2.25} \\ &= 0.98 + 0.02 \times (1.897)^{2.25} = 1.0645 \end{aligned}$$

Example 7.11: Determining the Extrusion Ratio for Indirect Extrusion with Complex Die

An annealed-copper billet that is 100-mm long and 55 mm in diameter is to be extruded by indirect extrusion using an extruder with a rectangular die orifice cross-section: 55 mm $\times$ 15 mm. The strain hardening exponent of the work material is 0.44 and its strength coefficient is 530 MPa. The constants in the Johnson's formula are $a = 0.8$ and $b = 1.5$. Determine the (a) shape factor for the complex die orifice and (b) extrusion ratio.

Solution

$$C_x = \text{Perimeter of the extruded cross-section} = (2 \times 55) + (2 \times 15) = 140 \text{ mm}$$

$$\text{Area of the extruded cross-section} = 55 \times 15 = 825 \text{ mm}^2$$

$$r = \sqrt{\frac{825}{\pi}} = 16.205 \text{ mm}$$

$$\begin{aligned} C_c &= \text{perimeter of a circle of the same area as the extruded shape} = 2\pi r \\ &= 2\pi \times 16.204 = 101.826 \text{ mm} \end{aligned}$$

$$\begin{aligned} K_x &= 0.98 + 0.02 \left(\frac{C_x}{C_c} \right)^{2.25} = 0.98 + 0.02 \times \left(\frac{140}{101.826} \right)^{2.25} \\ &= 0.98 + 0.02 \times (1.375)^{2.25} = 1.021 \end{aligned}$$

The shape factor = $K_x = 1.021$

$$\text{Original cross-sectional area} = A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 55^2 = 2374.63 \text{ mm}^2$$

Cross-sectional area of the extruded part = $A_f = 55 \times 15 = 825 \text{ mm}^2$

$$\text{The extrusion ratio} = r_x = \frac{A_o}{A_f} = \frac{2374.62}{825} = 2.88$$

Example 7.12: Determining the Ram Pressure for Indirect Extrusion with Complex Die

By using the data in Example 7.11, determine the ram pressure for the indirect extrusion operation with the complex die orifice.

Solution

$$K_x = 1.021, r_x = 2.88, n = 0.44, K = 530 \text{ MPa}, a = 0.8, b = 1.5.$$

$$\varepsilon = \ln r_x = \ln 2.88 = 1.05779$$

$$\varepsilon_x = a + (b \varepsilon) = 0.8 + (1.5 \times 1.05779) = 2.3867$$

$$\bar{Y}_f = \frac{K \cdot \varepsilon^n}{1 + n} = \frac{530 \times 1.05779^{0.44}}{1 + 0.44} = 377.267 \text{ MPa}$$

$$p'_{i.e} = K_x \bar{Y}_f \varepsilon_x = 1.021 \times 377.267 \times 2.3867 = 919.33 \text{ MPa}$$

Example 7.13: Determining the Ram Force for an Extrusion Operation

By using the data in Examples 7.11 and 7.12, determine the ram force for the extrusion operation.

Solution

$$p'_{i.e} = 919.33 \text{ MPa}, A_o = 2374.62 \text{ mm}^2, F = ?$$

By using Eq. 7.10,

$$F = p A_o = 919.33 \times 2374.62 = 2183059.4 \text{ N} = 2.183 \text{ MN}$$

The ram force = $F = 2,183,059.4 \text{ N}$.

Example 7.14: Determining the Ram Power for an Extrusion Operation

By using the data in Example 7.13, determine the ram power, if it takes 50 s in the extrusion operation.

Solution

$$F = 2183059.4 \text{ N, time in extrusion} = 50 \text{ s, } L = 100 \text{ mm} = 0.1 \text{ m, } P = ?$$

$$\text{Ram velocity} = v = \frac{\text{Billet length}}{\text{Time in extrusion}} = \frac{0.1 \text{ m}}{50 \text{ s}} = 0.002 \text{ m/s}$$

By using Eq. 7.11,

$$\text{Power} = P = F v = 2183059.4 \times 0.002 = 4366.12 \text{ W} = 4.366 \text{ kW}$$

Example 7.15: Determining the Final Length of the Extrudate

An aluminum alloy (2024-T3) billet that is 285 mm long and 40 mm in diameter is to be extruded by direct extrusion using an extruder with a die orifice diameter of 8 mm and die angle of 90° . The ram velocity is 5 mm/s. The Johnson parameters are $a = 0.66$ and $b = 1.3$. The work material has $n = 0.17$ and $K = 780$ MPa. Assuming that no butt is left in the container, determine the final length of the extrudate.

Solution

$$L_o = 285 \text{ mm}, D_o = 40 \text{ mm}, D_f = 8 \text{ mm}, L_f = ?$$

$$\text{Original volume of the billet} = V_o = \frac{\pi}{4} D_o^2 L_o = \frac{\pi}{4} \times 40^2 \times 285 = 357,960 \text{ mm}^3$$

$$\text{Volume of the extrudate} = V_f = \frac{\pi}{4} D_f^2 L_f = \frac{\pi}{4} \times 8^2 \times L_f = 50.24 L_f$$

$$\text{Original volume of the billet} = \text{Volume of the extrudate}$$

$$357,960 = 50.24 L_f$$

$$L_f = \text{Final length of the extrudate} = 7125 \text{ mm}$$

Example 7.16: Determining the Ram Power for Direct Extrusion

By using the data in Example 7.15, determine the power required for the extrusion operation.

Solution

$L = 285 \text{ mm}$, $D_o = 40 \text{ mm}$, $D_f = 8 \text{ mm}$, $a = 0.66$, $b = 1.3$, $n = 0.17$ and $K = 780 \text{ MPa}$, $v = 5 \text{ mm/s}$.

$$\epsilon = \ln \frac{A_o}{A_f} = \ln \frac{\frac{\pi}{4} D_o^2}{\frac{\pi}{4} D_f^2} = \ln \frac{\frac{\pi}{4} 40^2}{\frac{\pi}{4} 8^2} = \ln \frac{1600}{64} = \ln 25 = 3.2189$$

$$\epsilon_x = a + b \cdot \epsilon = 0.66 + (1.3 \times 3.2189) = 4.8445$$

$$\bar{Y}_f = \frac{K \epsilon^n}{1 + n} = \frac{780 \times 3.2189^{0.17}}{1 + 0.17} = \frac{951.49}{1.17} = 813.24 \text{ MPa}$$

$$\text{Ram pressure} = p = \bar{Y}_f \left(\epsilon_x + \frac{2L}{D_o} \right) = 813.24 \times \left(4.8445 + \frac{2 \times 285}{40} \right) = 15528.41 \text{ MPa}$$

$$\text{Ram Force} = F = p A_o = p \left(\frac{\pi}{4} D_o^2 \right) = 15528.41 \times 0.785 \times 40^2 = 19503682.96 \text{ N}$$

$$\text{Power} = P = F v = 19503682.96 \text{ N} \times 0.005 \text{ m/s} = 97518.415 \text{ W} = 97.518 \text{ kW}$$

Example 7.17: Determining the % Saving in Power by Switching from Direct to Indirect Extrusion

Refer to the data in Example 7.16. If we switched from the direct extrusion machine to an indirect machine with the same specifications, how much is the % saving in power?

Solution

For indirect extrusion,

$$\text{Ram pressure} = p = \bar{Y}_f \epsilon_x = 813.24 \text{ MPa} \times 4.8445 = 3939.74 \text{ MPa}$$

$$\text{Ram Force} = F = p A_o = p \left(\frac{\pi}{4} D_o^2 \right) = 3939.74 \times 0.785 \times 40^2 = 4948314.922 \text{ N}$$

$$\text{Power} = P = F v = 4948314.922 \text{ N} \times 0.005 \text{ m/s} = 24741.57 \text{ W} = 24.741 \text{ kW}$$

For direct extrusion, the power was calculated to be 97.518 kW.

$$\% \text{Saving in power} = \frac{(\text{Power in direct extrusion}) - (\text{Power in indirect extrusion})}{\text{Power in direct extrusion}} \times 100$$

$$\% \text{ Saving in power} = \frac{97.518 - 24.741}{97.518} \times 100 = 74.63$$

Hence, the saving would be 74.63%.

Example 7.18: Determining the Height of Slug to Manufacture a Cup by Impact Extrusion

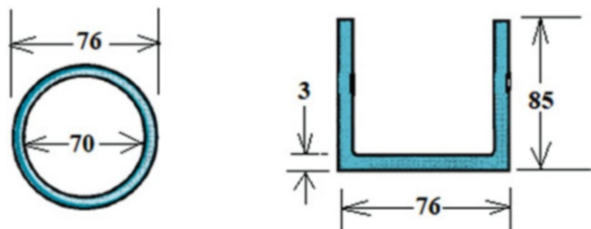
A cup-shaped part is extruded by indirect (impact) extrusion from an aluminum slug that is 76 mm in diameter. The final dimensions of the cup are shown in Fig. 7.9. Determine the height of starting slug required to achieve the final dimensions.

Solution

$$\text{OD} = 76 \text{ mm, ID} = 70 \text{ mm, } 2t = 76 - 70, t = 3 \text{ mm, } h = 85 \text{ mm}$$

$$\begin{aligned} \text{Volume of the extruded cup} &= \frac{\pi}{4} \cdot \text{OD}^2 \cdot t + \left\{ \frac{\pi}{4} \cdot (\text{OD}^2 - \text{ID}^2) \cdot (h - t) \right\} \\ &= (0.785 \times 76^2 \times 3) + [0.785 \times (76^2 - 70^2) (85 - 3)] \end{aligned}$$

Fig. 7.9 The extruded cup dimensions (in mm) for Example 7.18



$$\text{Volume of the extruded cup} = 13602.48 + (0.785 \times 876 \times 82) = 69,990.6 \text{ mm}^3$$

$$\text{Volume of the slug} = \frac{\pi}{4} D^2 \cdot h = 0.785 \times 76^2 \cdot h = 4534.16 \cdot h$$

$$\text{Volume of the slug} = \text{Volume of } 0.785 \times 76^2 \text{ the extrudate}$$

$$4,534.16 \cdot h = 69,990.6$$

$$h = 15.44 \text{ mm}$$

The height of starting slug = 15.44 mm.

Example 7.19: Determining the Extrusion Ratio to Make a Cup by Impact Extrusion

By using the data in Example 7.18, determine the extrusion ratio for the extrusion operation.

Solution

$$\text{Cross-sectional area of the slug} = A_o = \frac{\pi}{4} D^2 = 0.785 \times 76^2 = 4534.16 \text{ mm}^2$$

$$\begin{aligned} \text{Cross-sectional area of the cup} = A_f &= \frac{\pi}{4} \cdot (\text{OD}^2 - \text{ID}^2) = 0.785 \times (76^2 - 70^2) \\ &= 687.66 \text{ mm}^2 \end{aligned}$$

$$\text{Extrusion ratio} = r_x = \frac{A_o}{A_f} = \frac{4534.16}{687.66} = 6.59$$

Example 7.20: Determining Final Length of Extrudate When Butt Is Left in the Extruder

By using the data in Example 7.15, determine the final length of the extrudate, if 10,000 mm³ butt is left in the container, at the end of extrusion operation.

Solution

$$\text{Original volume of the billet} = 357,960 \text{ mm}^3; \text{ Volume of the extrudate} = 50.24 L_f$$

$$\text{Volume of the extrudate}$$

$$= \text{Original volume of the billet} - \text{Volume of the butt left in extruder}$$

$$50.24 L_f = 357960 - 10,000 = 347,960$$

$$L_f = \frac{347960}{50.24} = 6925.96 \text{ mm} \cong 6926 \text{ mm}$$

The final length of the extrudate = 6926 mm.

Example 7.21: Determining the True Strain Rate for Hot Extrusion

A metal billet 200-mm long and 100 mm in diameter is hot extruded at a ram speed of 250 mm/s. The extruder has square dies and poor lubrication with a die orifice diameter of 50 mm. Determine the true strain rate for the extrusion operation.

Solution

$$D_o = 100 \text{ mm}, D_f = 50 \text{ mm}, v = 250 \text{ mm/s.}$$

$$A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 100^2 = 7850 \text{ mm}^2$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 50^2 = 1962.5 \text{ mm}^2$$

$$r_x = \frac{A_o}{A_f} = \frac{7850}{1962.5} = 4$$

By using Eq. 7.12,

$$\dot{\epsilon} = \frac{6}{D_o} v \ln r_x = \frac{6 \times 250}{100} \ln 4 = 15 \times 1.386 = 20.794/\text{s}$$

Questions and Problems

7.1. (MCQs). Select the most appropriate answers for each of the following statements.

- Which extrusion operation is the most suitable for producing L-shaped products?
 - Hydrostatic extrusion, (b) lateral extrusion, (c) impact extrusion, (d) none.
- Which one is an example of discrete extrusion process?
 - Hydrostatic extrusion, (b) lateral extrusion, (c) impact extrusion, (d) none.
- Which metal is the most extensively extruded in industry?
 - Aluminum, (b) steel, (c) copper, (d) titanium.
- Which type of extrusion is the best for high ram velocity and low operational cost?
 - Hot extrusion, (b) warm extrusion, (c) cold extrusion.
- Which extrusion operation involves the minimum friction?
 - Direct extrusion, (b) lateral extrusion, (c) hydrostatic extrusion, (d) none.

6. Impact extrusion is an example of.
 - (a) Continuous extrusion, (b) direct extrusion, (c) discrete extrusion, (d) indirect extrusion.
 7. What is the shape factor for circular die orifice of an extruder?
 - (a) 1, (b) 2, (c) 3, (d) 4.
 8. Which process requires the use of a mandrel in the extruder?
 - (a) Cup manufacture, (b) tube manufacture, (c) bar manufacture, (d) L-shape.
 9. Which process is capable of producing infinitely long bars in one cycle?
 - (a) impact extrusion process, (b) discrete extrusion, (c) continuous extrusion process.
 10. Which type of extrusion involves the introduction of a fluid inside the container?
 - (a) Lateral extrusion, (b) hydrostatic extrusion, (c) impact extrusion, (d) none.
- 7.2. Highlight the advantages of extrusion over other metal forming processes.
- 7.3. Compare cold extrusion and hot extrusion with respect to their advantages.
- 7.4. What are the merits and demerits of direct extrusion and indirect extrusion?
- 7.5. Draw a labeled sketch showing tube manufacturing by direct extrusion.
- 7.6. Draw the classification chart showing various types of extrusion processes.
- 7.7. Explain impact extrusion operation with the aid of sketches.
- P7.8. A cylindrical billet that is 120-mm long and 65 mm in diameter is to be reduced to 30 mm diameter by direct extrusion. Determine the extrusion ratio.
- P7.9. By using the data in P7.8, determine the pressure applied to the end of the billet as the ram moves forward. The Johnson's formula constants are $a = 0.8$ and $b = 1.5$. The strain hardening exponent of the work material is 0.18 and its strength coefficient is 500 MPa.
- P7.10. A metallic billet that is 150-mm long and 70 mm in diameter is to be extruded by direct extrusion using an extrusion machine with a die orifice of cross-section 25 mm $\times$ 40 mm. Determine the shape factor for the extrusion operation.
- P7.11. By using the data in P7.10, determine the ram power for the extrusion operation, if the ram speed is 4 mm/s. The Johnson's formula constants are $a = 0.8$ and $b = 1.5$. The strain hardening exponent of the work material is 0.18 and its strength coefficient is 500 MPa.
- P7.12. An annealed-copper billet that is 130-mm long and 68 mm in diameter is to be extruded to a diameter of 27 mm by indirect extrusion. The strain hardening exponent of the work material is 0.44 and its strength coefficient is 530 MPa. The constants in the Johnson's formula are $a = 0.8$ and $b = 1.5$. Determine the true strain for the extrusion operation.

- P7.13. By using the data in Example 7.12, determine the ram pressure for the indirect extrusion operation.
- P7.14. A metallic billet that is 230 mm long and 45 mm in diameter is to be extruded by direct extrusion using an extruder with a die orifice diameter of 10 mm. Assuming that no butt is left in the container, determine the final length of the extrudate.
- P7.15. By using the data in P7.11, determine the % saving in power if the operation is switched from direct to indirect extrusion.
- P7.16. A metal billet that is 180-mm long and 90 mm in diameter is hot extruded at a ram speed of 200 mm/s. The extruder has square dies and poor lubrication with a die orifice diameter of 45 mm. Determine the true strain rate for the extrusion operation.

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Chapter 8

Rod/Wire Drawing



Nomenclature

r	area reduction = reduction ratio
A_o	original cross-sectional area of the metal stock
A_f	final cross-sectional area
$r_{\max}$	maximum limit for reduction ratio
d	draft
ε	true strain
σ	ideal draw stress
$\bar{Y}_f$	average flow stress
K	strength coefficient
n	strain hardening exponent
F	ideal draw force
F_d	actual draw force
σ_d	actual draw stress
μ	die-work coefficient of friction
α	entrance angle
ϕ	factor for nonhomogeneous deformation
D_o	original work diameter
D_f	final work diameter
D	average diameter of work
L_c	contact length
D_{in}	pre-draw wire diameter in continuous wire drawing
D_{out}	wire diameter after a die's draw in continuous wire drawing

8.1 Rod/Wire Drawing and Its Industrial Importance

Rod/wire drawing is a bulk-metal forming operation in which the cross section of a bar/rod is reduced by pulling it through a die to obtain a rod or a wire. The diameter of a bar is in the range of 15–50 mm, whereas a rod’s diameter is in the range of 5.5–15 mm (see Table 8.1). *Continuous wire drawing* involves the use of a series of dies (in the range of 4–12). The term *continuous drawing* is used because of the long production runs that are achieved with the wire coils. Rod/wire drawing is usually performed as a cold working (CW) operation (Lange, 2006).

Figure 8.1 illustrates a rod/wire drawing operation with a single drawing die. Although there are tensile stresses in wire drawing, compression also plays a significant role because the metal is squeezed while passing through the die opening. Rod/wire drawing is an important industrial process for manufacturing electrical wire and cable, wire stock for fences, and rod stock to produce nails, screws, rivets, and the like.

8.2 Advantages and Developments in Wire Drawing

Wire drawing is characterized by its high deformation rate and excellent surface quality (Alexandrov et al., 2022). There are a number of advantages of the rod/wire drawing process, especially due to cold working; some of the advantages include: (a) close dimensional control; (b) good surface finish; (c) improved mechanical properties, such as strength and hardness; (d) improved machinability; (e) adaptability to economical batch or mass production; and (f) the achievement of drawing speeds as high as 50 m/s for very fine wires.

Table 8.1 Diameter ranges for a bar, a rod, and a wire

Structural element	Bar	Rod	Wire
Diameter (mm)	15–50	5.5–15	<5.5

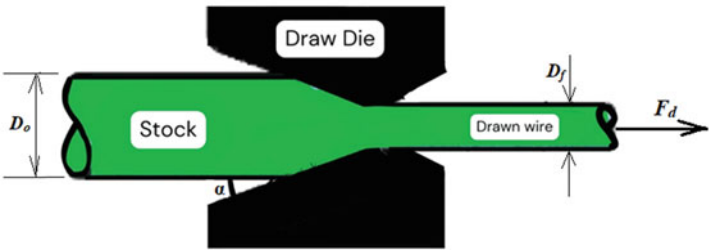


Fig. 8.1 Wire drawing operation using a single draw die (α = entrance angle)

The history of wire dates back to 3000 BC; gold wire was present in the jewelry of the Pharaohs and the Assyrians (Cosslett, 1971); the wire was produced by beating out small ingots into plates and strips and finally drawing them into wire. Wire drawers were firmly established in Europe by the Middle Ages, but all the functions were still carried out by hand—with a hammer and anvil or with a drawplate. The presently used continuous wire drawing machines were first developed in the USA in 1925.

Recently (2022), Li and co-researchers have reported a novel die-less drawing technique using lasers for processing contamination-free metal wires with controlled surface chemistry (Li et al., 2022). These die-less laser-drawn wires are particularly useful to produce filters for high-pressure and high-temperature corrosive filtration applications, catalysts, micron-sized electrical conductors, and ornaments with gold fibers requiring good optical surface properties.

8.3 Engineering Analysis of Rod/Wire Drawing

The *area reduction*, r (also called reduction ratio, r), in rod/wire drawing is expressed as:

$$r = \frac{A_o - A_f}{A_o} \quad (8.1)$$

where A_o is the original cross-sectional area of the metal stock, and A_f is the final cross-sectional area of the rod or wire (see Example 8.1). The maximum limits of 0.5 for reduction ratio ($r_{\max} = 0.5$) for a single die and $r_{\max} = 0.3$ for multiple dies are usually practiced in wire drawing industry. The *draft* d , in rod/wire drawing, is given by (see Fig. 8.2):

$$d = D_o - D_f \quad (8.2)$$

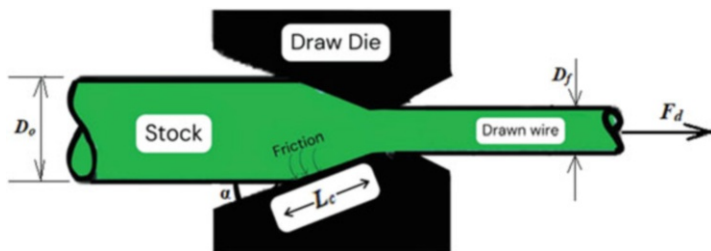


Fig. 8.2 Schematic of rod/wire drawing (L_c = contact length; α = entrance angle)

If no friction or redundant work is done, the deformation is homogeneous; the *true strain*, ϵ , is given by (Groover, 2019):

$$\epsilon = \ln \left(\frac{A_o}{A_f} \right) \quad (8.3)$$

The significance of Eq. 8.3 is illustrated in Example 8.2.

The *ideal draw stress* (in case of homogeneous deformation) is given by (Huda, 2018):

$$\sigma = \bar{Y}_f \cdot \epsilon \quad (8.4)$$

where σ is the ideal draw stress (MPa), and $\bar{Y}_f$ is the average flow stress (MPa) given by:

$$\bar{Y}_f = \frac{K \cdot \epsilon^n}{1+n} \quad (8.5)$$

where K is the strength coefficient (MPa) and n is the strain hardening exponent (see Example 8.3). The *draw force* is determined by:

$$F = \sigma A_f \quad (8.6)$$

where F is the ideal draw force (N), and A_f is the final cross-sectional area of the drawn part (mm^2) (see Example 8.4).

In industrial practice, there exists friction in drawing, which results in nonhomogeneous deformation experienced by the work metal. Thus, the actual draw stress depends not only on the ratio A_o/A_f but also on die angle and coefficient of friction at the work–die interface. *Actual draw stress* can be given by the Schey Equation as follows:

$$\sigma_d = \bar{Y}_f \left(1 + \frac{\mu}{\tan \alpha} \right) \phi \epsilon \quad (8.7)$$

where σ_d is the actual draw stress (MPa), μ is the die-work coefficient of friction, α is the entrance angle, ϕ is the factor for nonhomogeneous deformation, and ϵ is *true strain*. The factor for nonhomogeneous deformation, ϕ , can be determined by:

$$\phi = 0.88 + \left(0.12 \frac{D}{L_c} \right) \quad (8.8)$$

where D is the average diameter, and L_c is the contact length (see Fig. 8.2).

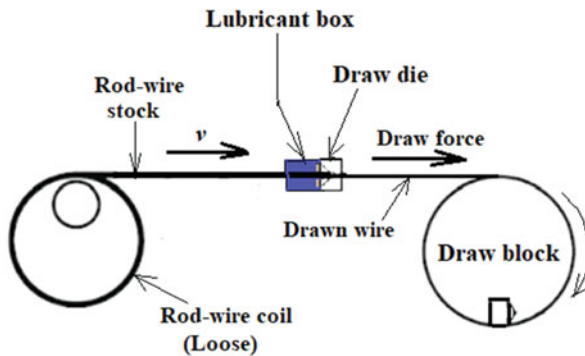


Fig. 8.3 Schematic of wire drawing equipment with a single draw die

The contact length can be determined by:

$$L_c = \frac{D_o - D_f}{2 \sin \alpha} \quad (8.9)$$

The significance of Eqs. 8.7–8.9 is illustrated in Examples 8.5–8.9.

Once the actual draw stress and the final cross-sectional area have been determined, the actual draw force can be determined by the following formula (see Fig. 8.3):

$$F_d = \sigma_d \cdot A_f \quad (8.10)$$

where F_d is the actual draw force (N), σ_d is the actual draw stress (MPa), and A_f is the final cross-sectional area (mm^2) (see Example 8.10).

8.4 Continuous Wire Drawing

Continuous wire drawing involves the use of a series of dies in the range of 4–12. A continuous wire drawing machine is a multi-die multi-drum wire drawing machine equipped with a water-cooling device (see Fig. 8.4). Proper lubrication of the die box is essential for improving die life, and for reducing drawing forces and temperature. This is why the die box is generally wet lubricated i.e., dies and rods are completely immersed in lubricant.

It is evident in Fig. 8.4 that each capstan drum is motor driven to provide the proper pull force to draw the wire stock. Since each die allows a certain amount of reduction in the wire, the desired total reduction is achieved by the series of dies. Since wire drawing is a cold working (CW) operation, recrystallization annealing of the wire is required between groups of dies in the series. This author, during his visit to a continuous wire drawing factory, witnessed annealing of fine wire between

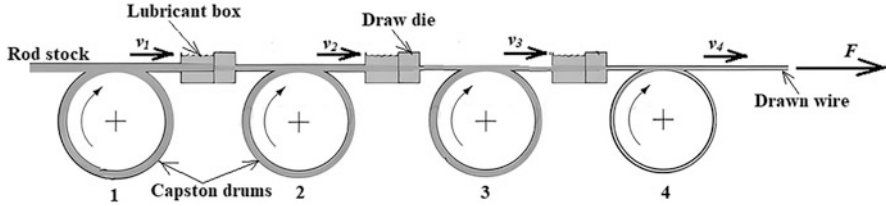


Fig. 8.4 Schematic of continuous wire drawing equipment

groups of dies by heating the wire (during drawing) by passing the current of correct amperage through it.

The number of dies required for continuous wire drawing equipment can be determined by using the following formula, which has been obtained by rearranging the terms in Eq. 8.1:

$$D_{\text{out}} = D_{\text{in}} \sqrt{1 - r} \quad (8.11)$$

where D_{in} is the pre-draw wire diameter, D_{out} is the wire diameter after a die's draw, and r is the area reduction (see Example 8.11). The number of capstan drums is given by (see Fig. 8.4):

$$\text{Number of capstan drums} = (\text{number of dies}) + 1 \quad (8.12)$$

8.5 Worked Numerical Examples

Example 8.1: Determining the Area Reduction and the Draft in Wire Drawing

A metal rod is drawn through a draw die to produce a wire. The rod's starting diameter is 6 mm and the wire's final diameter is 3.5 mm. The metal has a strength coefficient of 200 MPa and a strain hardening exponent of 0.21. Determine the area reduction and the draft for the operation.

Solution

$$D_o = 6 \text{ mm}, D_f = 3.5 \text{ mm}, r = ?, d = ?$$

By using Eqs. 8.1 and 8.2,

$$\text{Draft} = d = D_o - D_f = 6 - 3.5 = 2.5 \text{ mm}$$

$$A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 6^2 = 28.26 \text{ mm}^2$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 3.5^2 = 9.616 \text{ mm}^2$$

$$\text{Area reduction} = r = \frac{A_o - A_f}{A_o} = \frac{28.26 - 9.616}{28.26} = 0.6597$$

Example 8.2: Determining the True Strain in Wire Drawing

By using the data in Example 8.1, determine the true strain for the drawing operation.

Solution

$$A_o = 28.26 \text{ mm}^2, A_f = 9.616 \text{ mm}^2, \varepsilon = ?$$

By using Eq. 8.3,

$$\text{True strain} = \varepsilon = \ln\left(\frac{A_o}{A_f}\right) = \ln\left(\frac{28.26}{9.616}\right) = \ln 2.9388 = 1.078$$

Example 8.3: Determining the Ideal Draw Stress for a Wire Drawing Operation

By using the data in Examples 8.1 and 8.2, determine the average flow stress and the ideal draw stress for the drawing operation.

Solution

$$K = 200 \text{ MPa}, n = 0.21, \varepsilon = 1.078, \bar{Y}_f = ?, \sigma = ?$$

By using Eqs. 8.4 and 8.5,

$$\text{Average flow stress} = \bar{Y}_f = \frac{K \cdot \varepsilon^n}{1 + n} = \frac{200 \times 1.078^{0.21}}{1 + 0.21} = 167.917 \text{ MPa}$$

$$\text{Ideal draw stress} = \sigma = \bar{Y}_f \cdot \varepsilon = 167.917 \times 1.078 = 181.014 \text{ MPa}$$

Example 8.4: Determining the Ideal Draw Force and Power for a Wire Drawing Operation

By using the data in Examples 8.1 and 8.3, determine the ideal draw force. Also determine the ideal drawing power, if the drawing speed is 50 m/s.

Solution

$$\sigma = 181.014 \text{ MPa}, A_f = 9.616 \text{ mm}^2, v = 50 \text{ m/s}, F = ?, \text{Power} = ?$$

By using Eq. 8.6,

$$F = \sigma \cdot A_f = 181.014 \times 9.616 = 1740.63 \text{ N}$$

$$\text{Power} = F \cdot v = 1740.63 \times 50 = 87,031.5 \text{ W} = 87.031 \text{ kW}$$

Example 8.5: Determining the Average Diameter and the Av. Flow Stress for Wire Drawing

A metal wire is drawn through a draw die with a die angle of 13° . The starting diameter of the wire is 3.0 mm and the final diameter is 1.8 mm. The coefficient of friction at the work–die interface is 0.09. The metal has a strength coefficient of

230 MPa and a strain hardening exponent of 0.19. Determine the original cross-sectional area, the final cross-sectional area, and the true strain for the drawing operation.

Solution

$$D_o = 3.0 \text{ mm}, D_f = 1.8 \text{ mm}, A_o = ?, A_f = ?, \varepsilon = ?,$$

$$A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 3^2 = 7.065 \text{ mm}^2$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 1.8^2 = 2.543 \text{ mm}^2$$

$$\text{True strain} = \varepsilon = \ln\left(\frac{A_o}{A_f}\right) = \ln\left(\frac{7.065}{2.543}\right) = \ln 2.778 = 1.0217$$

Example 8.6: Determining the Average Diameter and the Av. Flow Stress for Wire Drawing

By using the data in Example 8.5, determine the average diameter and the average flow stress for the operation.

Solution

$$D_o = 3.0 \text{ mm}, D_f = 1.8 \text{ mm}, \varepsilon = 1.0217, K = 230 \text{ MPa}, n = 0.19, D = ?, \bar{Y}_f = ?$$

$$\text{Average diameter} = D = \frac{D_o + D_f}{2} = \frac{3.0 + 1.8}{2} = 2.4 \text{ mm}$$

$$\text{Av. flow stress} = \bar{Y}_f = \frac{K \cdot \varepsilon^n}{1 + n} = \frac{230 \times 1.0217^{0.19}}{1 + 0.19} = 194.067 \text{ MPa}$$

Example 8.7: Determining the Contact Length for a Wire Drawing Operation

By using the data in Examples 8.5 and 8.6, determine the contact length for the drawing operation.

Solution

$$D_o = 3.0 \text{ mm}, D_f = 1.8 \text{ mm}, \alpha = 13^\circ, L_c = ?$$

By using Eq. 8.9,

$$\text{Contact length} = L_c = \frac{D_o - D_f}{2 \sin \alpha} = \frac{3 - 1.8}{2 \sin 13} = \frac{1.2}{2 \times 0.22} = 2.73 \text{ mm}$$

Example 8.8: Determining the Factor for Nonhomogeneous Deformation in Wire Drawing

By using the data in Examples 8.5–8.7, determine the factor for nonhomogeneous deformation in the wire drawing operation.

Solution

$D = 2.4 \text{ mm}$, $L_c = 2.73 \text{ mm}$, $\phi = ?$

By using Eq. 8.8,

$$\phi = 0.88 + \left(0.12 \frac{D}{L_c} \right) = 0.88 + \left(\frac{0.12 \times 2.4}{2.73} \right) = 0.986$$

Example 8.9: Determining the Actual Draw Stress in Wire Drawing

By using the data in Examples 8.5–8.8, determine the actual draw stress for the operation.

Solution

$\bar{Y}_f = 194.067 \text{ MPa}$, $\alpha = 13^\circ$, $\mu = 0.09$, $\phi = 0.986$, $\varepsilon = 1.0217$.

By using Eq. 8.7,

$$\sigma_d = \bar{Y}_f \left(1 + \frac{\mu}{\tan \alpha} \right) \phi \varepsilon = 194.067 \times \left(1 + \frac{0.09}{\tan 13} \right) \times 0.986 \times 1.0217$$

$$\sigma_d = 195.5 \times \left(1 + \frac{0.09}{0.231} \right) = 271.669 \text{ MPa}$$

The actual draw stress $= \sigma_d = 271.669 \text{ MPa}$.

Example 8.10: Determining the Actual Draw Force and Actual Drawing Power

By using the data in Examples 8.5–8.9, determine the actual draw force for the operation. Also determine the actual drawing power, if the drawing speed is 45 m/s.

Solution

$\sigma_d = 271.669 \text{ MPa}$, $A_f = 2.543 \text{ mm}^2$, $v = 45 \text{ m/s}$, $F_d = ?$, Actual draw power = ?

By using Eq. 8.10,

$$F_d = \sigma_d \cdot A_f = 271.669 \times 2.543 = 690.854 \text{ N}$$

The actual draw force $= 690.854 \text{ N}$.

$$\text{Actual draw power} = F_d \cdot v = 690.854 \times 45 = 31,088.43 \text{ W} = 31.088 \text{ kW}$$

Example 8.11: Determining the Number of Dies and the Number of Capstan Drums

A metal rod is drawn by using a continuous wire drawing machine to obtain wire. The starting diameter of the rod is 8.0 mm and the final diameter of the drawn wire is 3.2 mm. The area reduction is 0.2. How many dies and capstan drums are required in the continuous drawing machine for this job?

Solution

Area reduction $= r = 0.2$, $D_{\text{in}} = D_o = 8 \text{ mm}$, $D_{\text{out}} = D_f = 3.2 \text{ mm}$.

We can determine the diameter-out for each draw die by using Eq. 8.11, as follows.

First draw die

$$D_{\text{out}} = D_{\text{in}} \sqrt{1 - r} = 8 \times \sqrt{1 - 0.2} = 8 \times 0.894 = 7.155 \text{ mm}$$

Second draw die

$$D_{\text{out}} = D_{\text{in}} \sqrt{1 - r} = 7.155 \times 0.894 = 6.3969 \text{ mm}$$

Third draw die

$$D_{\text{out}} = D_{\text{in}} \sqrt{1 - r} = 6.3969 \times 0.894 = 5.7188 \text{ mm}$$

Fourth draw die

$$D_{\text{out}} = D_{\text{in}} \sqrt{1 - r} = 5.7188 \times 0.894 = 5.11 \text{ mm}$$

Fifth draw die

$$D_{\text{out}} = D_{\text{in}} \sqrt{1 - r} = 5.11 \times 0.894 = 4.571 \text{ mm}$$

Sixth draw die

$$D_{\text{out}} = D_{\text{in}} \sqrt{1 - r} = 4.571 \times 0.894 = 4.086 \text{ mm}$$

Seventh draw die

$$D_{\text{out}} = D_{\text{in}} \sqrt{1 - r} = 4.086 \times 0.894 = 3.65 \text{ mm}$$

Eighth draw die

$$D_{\text{out}} = D_{\text{in}} \sqrt{1 - r} = 3.65 \times 0.894 = 3.26 \text{ mm}$$

Since the D_{out} for the eighth draw die is very close to the final diameter ($D_f = 3.2 \text{ mm}$), the number of dies required = 8. The number of capstan drums required = $8 + 1 = 9$ (see Eq. 8.12).

Example 8.12: Determining the Total Strain for a Multiple-Die Wire Drawing Operation a

A rod stock of initial diameter = 7.0 mm is drawn through two dies each providing a 0.25 reduction ratio. Determine the total true strain experienced by the work metal after the second draw.

Solution

$D_o = 7$ mm, $r = 0.25$, strain for first draw = ?, strain for second draw = ?, Total strain after second draw = ?

First draw

$$D_{out} = D_{in} \sqrt{1 - r} = 7 \times \sqrt{1 - 0.25} = 6.06 \text{ mm} = D_f$$

$$A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 7^2 = 38.465 \text{ mm}^2$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 6.06^2 = 28.828 \text{ mm}^2$$

$$\text{True strain in the first draw} = \epsilon_1 = \ln \left(\frac{A_o}{A_f} \right) = \ln \left(\frac{38.465}{28.828} \right) = \ln 1.334 = 0.288$$

Second draw

$$D_{out} = D_{in} \sqrt{1 - r} = 6.06 \times \sqrt{1 - 0.25} = 5.25 \text{ mm}$$

$$A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 6.06^2 = 28.828 \text{ mm}^2$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 5.25^2 = 21.636 \text{ mm}^2$$

$$\text{True strain in the second draw} = \epsilon_2 = \ln \left(\frac{28.828}{21.636} \right) = \ln 1.332 = 0.2867$$

Total true strain experienced by the work metal after the second draw = $\epsilon_1 + \epsilon_2$

$$\epsilon = 0.2880 + 0.2867 = 0.5687$$

Example 8.13: Determining the Av. Flow Stress for a Multiple-Die Wire-Drawing Operation

By using the data in Example 8.12, determine the average flow stress for the second draw. The metal has a strength coefficient of 350 MPa and a strain hardening exponent of 0.20.

Solution

$$K = 350 \text{ MPa}, n = 0.2, \epsilon = 0.5687, \bar{Y}_f = ?$$

$$\bar{Y}_f = \frac{K \cdot \epsilon^n}{1 + n} = \frac{350 \times 0.5687^{0.2}}{1 + 0.2} = 260.533 \text{ MPa}$$

Example 8.14: Determining the Final Length of the Drawn Wire

By using the data in Example 8.11, determine the final length of the wire if the length of the rod stock is 7125 mm.

Solution

$$L_o = 7125 \text{ mm}, D_o = 8 \text{ mm}, D_f = 3.2 \text{ mm}, L_f = ?$$

Volume of the rod stock = Volume of the drawn wire

$$L_o \left(\frac{\pi}{4} D_o^2 \right) = L_f \left(\frac{\pi}{4} D_f^2 \right)$$

$$L_o (D_o^2) = L_f (D_f^2)$$

$$L_f = \frac{L_o D_o^2}{D_f^2} = \frac{7125 \times 8^2}{3.2^2} = \frac{456,000}{10.24} = 44,531.25 \text{ mm} = 44.531 \text{ m}$$

The length of the drawn wire = 44,531.25 mm = 44.531 m.

Example 8.15: Determining the Exit Speed of Drawn Wire

The draw force in a single-die wire drawing operation is 2000 N. The motors driving the draw block can deliver 70 kW power at 85% efficiency. Determine the maximum possible speed of the wire as it exits the draw die.

Solution

Motor gross power = $P_g = 70,000 \text{ W}$, Motor efficiency = $\eta = 85 \% = 0.85$, $F = 2000 \text{ N}$, $v = ?$

$$P_g = \frac{\text{Power}}{\eta}$$

$$\text{Power} = P = P_g \times \eta = 70,000 \times 0.85 = 59,500 \text{ W}$$

$$\text{Power} = F v$$

$$\begin{aligned} \text{The maximum possible speed as the wire exits the die} &= v = \frac{\text{Power}}{F} = \frac{59,500}{2000} \\ &= 29.75 \text{ m/s} \end{aligned}$$

Example 8.16: Determining the Motor Gross Power with the Aid of Wire Drawing Sketch

Determine the motor gross power (kW) with the aid of data shown in Fig. 8.5. The motor mechanical efficiency = 90%.

Solution

$$F = 1800 \text{ N}, v = 30 \text{ m/s}, \eta = 90\%, \text{ Gross power} = P_g = ?$$

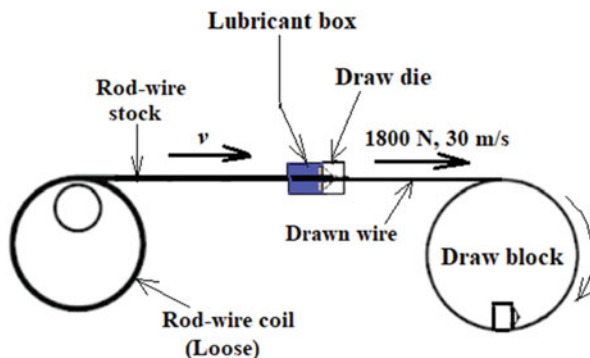


Fig. 8.5 Wire drawing sketch showing the data for Example 8.16

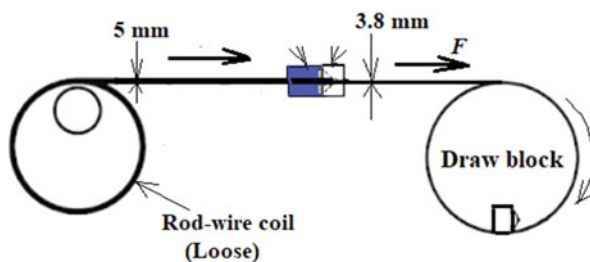


Fig. 8.6 The wire drawing sketch showing wire diameters for Example 8.17

$$\text{Power} = F v = 1800 \times 30 = 54,000 \text{ W}$$

$$P_g = \frac{\text{Power}}{\eta} = \frac{54,000}{0.90} = 60,000 = 60 \text{ kW}$$

Example 8.17: Determining Reduction Ratio, Draft, and ϵ with Reference to a Sketch

Determine the reduction ratio, the draft, and the true strain with the aid of wire drawing data shown in Fig. 8.6.

Solution

$$D_o = 5 \text{ mm}, D_f = 3.8 \text{ mm}, d = ?, r = ?, \epsilon = ?$$

$$\text{Draft} = d = D_o - D_f = 5 - 3.8 = 1.2 \text{ mm}$$

$$A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 5^2 = 19.625 \text{ mm}^2$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 3.8^2 = 11.335 \text{ mm}^2$$

$$r = \frac{A_o - A_f}{A_o} = \frac{19.625 - 11.335}{19.625} = 0.4224$$

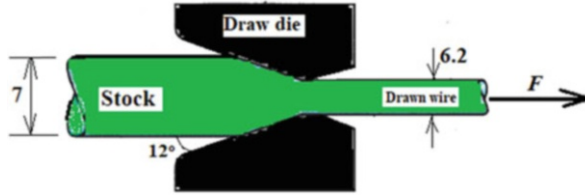


Fig. 8.7 Wire drawing data (all dimensions in mm) for Example 8.18

$$\epsilon = \ln\left(\frac{A_o}{A_f}\right) = \ln\left(\frac{19.625}{11.335}\right) = \ln 1.7314 = 0.5489$$

Example 8.18: Determining the Contact Length with Reference to a Sketch

Determine the contact length with the aid of wire drawing data shown in Fig. 8.7.

Solution

$$D_o = 7.0 \text{ mm}, D_f = 6.2 \text{ mm}, \alpha = 12^\circ, L_c = ?$$

By using Eq. 8.9,

$$\text{Contact length} = L_c = \frac{D_o - D_f}{2 \sin \alpha} = \frac{7 - 6.2}{2 \sin 12} = \frac{0.8}{2 \times 0.2079} = 1.924 \text{ mm}$$

Example 8.19: Determining the Draw Force Using the Imperial System of Units

A metallic rod stock is drawn through a draw die with an entrance angle of 13° . The initial diameter = 0.48 in. and the final diameter = 0.32 in. The coefficient of friction at the work–die interface is 0.12. The strength coefficient of the metal is 45,000 lb/in.² and a strain hardening exponent = 0.22. Determine the draw force (in *lbs*) for the operation.

Solution:

$$\alpha = 13^\circ, D_o = 0.48 \text{ in.}, D_f = 0.32 \text{ in.}, \mu = 0.12, n = 0.22, K = 45,000 \text{ lb/in.}^2, F = ?$$

$$A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 0.48^2 = 0.181 \text{ in.}^2$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 0.32^2 = 0.080 \text{ in}^2$$

$$\epsilon = \ln\left(\frac{A_o}{A_f}\right) = \ln\left(\frac{0.181}{0.080}\right) = \ln 2.262 = 0.816$$

$$\bar{Y}_f = \frac{K \cdot \epsilon^n}{1 + n} = \frac{45,000 \times 0.816^{0.22}}{1 + 0.22} = 35,271.5 \text{ lb/in}^2$$

$$D = \frac{D_o + D_f}{2} = \frac{0.48 + 0.32}{2} = 0.4 \text{ in.}$$

$$L_c = \frac{D_o - D_f}{2 \sin \alpha} = \frac{0.48 - 0.32}{2 \sin 13} = \frac{0.16}{2 \times 0.225} = 0.355$$

$$\phi = 0.88 + \left(0.12 \frac{D}{L_c}\right) = 0.88 + \left(\frac{0.12 \times 0.4}{0.355}\right) = 1.015$$

$$\sigma_d = \bar{Y}_f \left(1 + \frac{\mu}{\tan \alpha}\right) \phi \quad \varepsilon = 35,271.5 \times \left(1 + \frac{0.12}{\tan 13}\right) \times 1.015 \times 0.816$$

$$\sigma_d = 29,219.3477 \times (1 + 0.51977) = 44,406.895 \text{ lb/in.}^2$$

$$F_d = \sigma_d \cdot A_f = 44,406.895 \times 0.08 = 3552.55 \text{ lb}$$

Example 8.20: Determining the Horsepower in Wire Drawing Operation

By using the data in Example 8.19, determine the power (in horsepower) to perform the operation if the exit velocity of the drawn work = 2.2 ft/s.

Solution

$$F = 3552.55 \text{ lb}, v = 2.2 \text{ ft/s}, \text{ horsepower} = ?$$

$$\begin{aligned} \text{Power} &= F v = 3552.55 \text{ lb} \times 2.2 \frac{\text{ft}}{\text{s}} = 3552.55 \text{ lb} \times 2.2 \times 60 \frac{\text{ft}}{\text{min}} \\ &= 468,936 \text{ ft} \times \text{lb} / \text{min} \end{aligned}$$

$$33,000 \text{ ft} \times \text{lb} / \text{min} = 1 \text{ horse-power} = 1 \text{ HP}$$

$$468,936 \text{ ft} \cdot \text{lb} / \text{min} = \frac{468,936}{33,000} = 14.21 \text{ HP}$$

$$\text{Power} = 14.21 \text{ HP} = 14.21 \text{ horsepower.}$$

Example 8.21: Determining the Contact Length When the Die Angle Is Unknown

A machine-shop has received a work order to draw a wire for a client. The drawing process is dry and the wire starting diameter is 3.0 mm and the final diameter = 2.5 mm. The working metal has a strength coefficient of 205 MPa and a strain hardening exponent of 0.20. The Schey's factor of non-homogeneous deformation is 1.15. Determine the contact length of the work with the draw die.

Solution

$$D_o = 3 \text{ mm}, D_f = 2.5 \text{ mm}, \phi = 1.15, L_c = ?$$

$$D = \frac{D_o + D_f}{2} = \frac{3 + 2.5}{2} = 2.75 \text{ mm}$$

$$\phi = 0.88 + \left(0.12 \frac{D}{L_c}\right)$$

$$\left(0.12 \frac{D}{L_c}\right) = \phi - 0.88$$

$$\frac{D}{L_c} = \frac{\phi - 0.88}{0.12}$$

$$\frac{L_c}{D} = \frac{0.12}{\phi - 0.88}$$

$$L_c = \left(\frac{0.12}{\phi - 0.88}\right) \cdot D = \left(\frac{0.12}{1.15 - 0.88}\right) \times 2.75 = \frac{0.33}{0.27} = 1.22 \text{ mm}$$

The contact length = 1.22 mm.

Example 8.22: Determining the Die Angle for a Wire Drawing Operation

By using the data in Example 8.21, determine the drawing die angle.

Solution

$$D_o = 3 \text{ mm}, D_f = 2.5 \text{ mm}, L_c = 1.22 \text{ mm}, \alpha = ?$$

$$L_c = \frac{D_o - D_f}{2 \sin \alpha}$$

$$2 \sin \alpha = \frac{D_o - D_f}{L_c}$$

$$\alpha = \sin^{-1} \frac{D_o - D_f}{2 L_c} = \sin^{-1} \frac{3.0 - 2.5}{2 \times 1.22} = \sin^{-1} 0.2049 = 11.8^\circ$$

The draw die angle = 11.8°.

Example 8.23: Determining the Actual Draw Stress When the True Strain Is Unknown

Refer to the data in Examples 8.21 and 8.22. If the maximum force that can be applied by the drawing machine is 500 N, what is the maximum stress that can be induced in the given working wire?

Solution

$$F_d = 500 \text{ N}, D_f = 2.5 \text{ mm}, \sigma_d = ?$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 2.5^2 = 4.906 \text{ mm}^2$$

$$F_d = \sigma_d \cdot A_f$$

$$\sigma_d = \frac{F_d}{A_f} = \frac{500}{4.906} = 101.91 \text{ N/mm}^2 = 101.91 \text{ MPa}$$

The maximum draw stress = 101.91 MPa.

Example 8.24: Determining the Drawing Force for Wire Drawing with Lubricant

Refer to the data in Examples 8.21 and 8.23. If lubricant is used instead of dry operation, what would be the effect on the drawing force?

Solution

$$D_o = 3 \text{ mm}, D_f = 2.5 \text{ mm}, K = 205 \text{ MPa}, n = 0.2, F = ?$$

If lubricant is used, the deformation would be homogeneous; it means that the stress and the force would be ideal. The drawing force can be determined as follows.

$$A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 3^2 = 7.065 \text{ mm}^2$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 2.5^2 = 4.906 \text{ mm}^2$$

$$\text{True strain} = \epsilon = \ln\left(\frac{A_o}{A_f}\right) = \ln\left(\frac{7.065}{4.906}\right) = 0.3647$$

$$\text{Average flow stress} = \bar{Y}_f = \frac{K \cdot \epsilon^n}{1 + n} = \frac{205 \times 0.3647^{0.20}}{1 + 0.20} = 139.62 \text{ MPa}$$

$$\text{Ideal draw stress} = \sigma = \bar{Y}_f \cdot \epsilon = 139.62 \times 0.3647 = 50.92 \text{ MPa}$$

$$\text{Ideal force} = F = \sigma \cdot A_f = 50.92 \times 4.906 = 249.81 \text{ N}$$

Questions and Problems

- 8.1. (a) What is the difference between a bar, a rod, and a wire?
(b) Explain rod/wire drawing operation with the aid of a labeled sketch.
- 8.2. (a) Draw a sketch illustrating continuous wire drawing (CWD).
(b) What additional processing is required between two consecutive dies in CWD? And why?
- 8.3. List six advantages of rod/wire drawing operation.
- 8.4. Is it possible to perform a wire drawing operation without a die? Explain!
- 8.5. What is the difference between ideal draw stress and actual draw stress?
- P8.6. A metal rod is drawn through a draw die to produce a wire. The rod's starting diameter is 5.8 mm and the wire's final diameter is 4.5 mm. The metal has a strength coefficient of 280 MPa and a strain hardening exponent of 0.18. Determine the draft, the original cross-sectional area, and the final c.s. area for the operation.
- P8.7. By using the data in P8.6, determine the (a) reduction ratio, (b) true strain, and (c) average flow stress for the operation.
- P8.8. By using the data in P8.6, determine the final length of the wire if the length of the rod stock is 6 m.
- P8.9. A metal wire is drawn through a draw die with a die angle of 11° . The starting diameter of the wire is 3.8 mm and the final diameter is 1.4 mm. The coefficient of friction at the work–die interface is 0.10. The metal has a

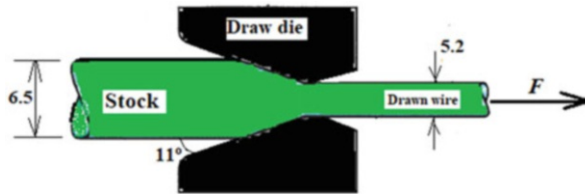


Fig. 8.8 The sketch showing the wire drawing data (in mm) for P8.12

- strength coefficient of 280 MPa and a strain hardening exponent of 0.18. Determine the (a) actual draw stress and (b) the draw force for the operation.
- P8.10. The draw force in a single-die wire drawing operation is 1800 N. The motors driving the draw block can deliver 65 kW power at 88% efficiency. Determine the maximum possible speed of the wire as it exits the draw die.
- P8.11. A metallic rod stock is drawn through a draw die with an entrance angle of 10° . The initial diameter = 0.50 in and the final diameter = 0.28 in. The coefficient of friction at the work–die interface is 0.11. The strength coefficient of the metal is 44,500 lb/in.² and a strain hardening exponent = 0.21. Determine the draw force (in lbs) for the operation.
- P8.12. Refer to Fig. 8.8 and determine the draft and the contact length for the operation.
- P8.13. By using the data in P8.11, determine the power (in horsepower) to perform the operation, if the exit velocity of the drawn work = 1.9 ft/s.
- 8.14. (MCQs). Select the most appropriate answer for each of the following statement.
1. A diameter of 40 mm corresponds to:
 - (a) Rod, (b) bar, (c) wire, (d) none.
 2. A diameter of 10 mm corresponds to:
 - (a) Rod, (b) bar, (c) wire, (d) none.
 3. The difference of the original and the final diameters is called:
 - (a) Reduction ratio, (b) average diameter, (c) draft, (d) reduction ratio.
 4. What maximum reduction ratio is allowed in a single die wire drawing practice?
 - (a) 0.2, (b) 0.3, (c) 0.4, (d) 0.5
 5. What maximum reduction ratio is allowed in continuous wire drawing practice?
 - (a) 0.2, (b) 0.3, (c) 0.4, (d) 0.5.

6. The history of wire dates back:
(a) 3000 BC, (b) 4000 BC, (c) 5000 BC, (d) 6000 BC
7. Which annealing process is required between groups of dies in continuous wire drawing?
(a) Recovery, (b) recrystallization, (c) grain growth, (d) all.
8. Wire drawing is usually performed as:
(a) Cold working, (b) warm working, (c) hot working, (d) none.

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Part III

Sheet Metal Forming

Chapter 9

Mechanics of Sheet Metal Forming



Nomenclature

l_o	pre-tensile-test length
w_o	pre-test width
t_o	pre-test thickness
l	post-tensile-test length
w	post-test width
t	post-test thickness
dl	increase in length
dw	increase in width
dt	increase in thickness
σ_1	tensile stress in direction 1
σ_2	tensile stress in direction 2
σ_3	tensile stress in direction 3
ϵ_1	principal strain in direction 1
ϵ_2	principal strain in direction 2
ϵ_3	principal strain in direction 3
$d\epsilon_1$	principal strain increment in direction 1
$d\epsilon_2$	principal strain increment in direction 2
$d\epsilon_3$	principal strain increment in direction 3
ϵ_w	width strain
ϵ_t	thickness strain
α	stress ratio
β	strain ratio
σ_h	hydrostatic stress
σ'	deviatoric stress
σ'_1	deviatoric stress in direction 1
σ'_2	deviatoric stress in direction 2
σ'_3	deviatoric stress in direction 3

$\sigma_{\max}$	maximum tensile stress
$\sigma_{\min}$	minimum tensile stress
σ_{ys}	yield stress
$\bar{\sigma}$	effective stress
$\bar{\epsilon}$	effective strain
F_{sf}	stretch forming force
σ_{UTS}	ultimate tensile strength

9.1 Introduction

Sheet metal forming is widely used in a multitude of industries across the world; these industries include automotive, aerospace, electronics, telecommunications, lighting, construction, energy sector, and food-packaging industries (see Fig. 9.1). It has been learned in Chap. 1 that sheet metal forming refers to those metal forming processes that involve no or a little change in the thickness or cross-sectional area; these processes include: shearing, blanking, piercing, bending, deep/cup drawing, and stretching; however, the mechanics of all sheet-metal forming basically consists of bending and stretching (Kalpakjian & Schmid, 2008).



Fig. 9.1 Some examples of products/machines manufactured by sheet-metal forming

There are two regimes of interest in sheet-metal forming: (a) elastic deformation and (b) plastic deformation (see Chap. 2). Sheet-metal forming operations generally involve plastic deformation owing to permanent ‘plastic’ flow. On the other hand, elastic strains are generally small and hence they can be ignored; however, there is an exception in the case when forming forces are unloaded, thereby resulting in large elastic shape change—‘springback’ (ASM, 2006).

Three types of loading conditions are generally encountered in sheet-metal forming processes: (a) uniaxial tension, (b) plane-strain deformation, and (c) biaxial stretching (Chow et al., 2003); however, the following two conditions are more common in sheet-metal forming: (a) principal strain increments in uniaxial loading, and (b) plane stress deformation. These loading aspects are discussed and analyzed in the following sections.

9.2 Principal Strain Increments in Uniaxial Loading

9.2.1 Tensile Test Using a Flat Specimen

Many mechanical properties (especially strength and ductility properties) of a material are based on measurements made in the tensile test. It is, therefore, useful to consider a tensile test sample, as an example of uniaxial loading. Figure 9.2 shows a flat tensile-test sample with original length l_o that is at least four times the original width w_o . The application of tensile force results in increase in the dimensions to l , w , and t ; the corresponding increments in dimensions are dl , dw , and dt in the three directions 1, 2, and 3, respectively (see Fig. 9.2).

Since the volume remains constant, we can write:

$$\begin{aligned} \text{Pre-test volume} &= \text{Post-test volume} \\ l_o w_o t_o &= l \cdot w \cdot t \end{aligned} \quad (9.1)$$

where $l = l_o + dl$, $w = w_o + dw$, and $t = t_o + dt$ (see Example 9.1). The percent elongation of the test material can be determined by:

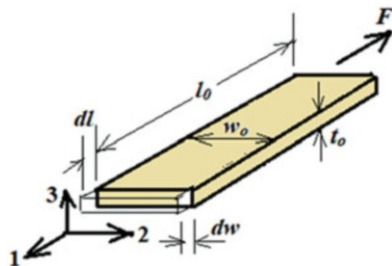


Fig. 9.2 A tensile-test flat-sample showing the gage elements and the principal directions

$$\% \text{Elongation} = \frac{l - l_o}{l_o} \times 100 \quad (9.2)$$

The significance of Eq. 9.2 is given in Example 9.2. By reference to Fig. 9.2, the principal strain increments in directions 1, 2, and 3 can be expressed as follows:

$$\text{The principal strain increment in direction 1} = d\varepsilon_1 = \frac{dl}{l} \quad (9.3)$$

$$\text{The principal strain increment in direction 2} = d\varepsilon_2 = \frac{dw}{w} \quad (9.4)$$

$$\text{The principal strain increment in direction 3} = d\varepsilon_3 = \frac{dt}{t} \quad (9.5)$$

The three principal strains can be determined by integrating Eqs. 9.3–9.5, as follows:

$$\int d\varepsilon_1 = \int_{l_o}^l \frac{dl}{l} \quad (9.6)$$

or

$$\varepsilon_1 = \ln \frac{l}{l_o} \quad (9.7)$$

$$\int d\varepsilon_2 = \int_{w_o}^w \frac{dw}{w} \quad (9.8)$$

or

$$\varepsilon_2 = \ln \frac{w}{w_o} \quad (9.9)$$

$$\int d\varepsilon_3 = \int_{t_o}^t \frac{dt}{t} \quad (9.10)$$

or

$$\varepsilon_3 = \ln \frac{t}{t_o} \quad (9.11)$$

The significance of Eqs. 9.3–9.11 is illustrated in Example 9.4.

Equation 9.1 can be reworked by using differential calculus, as follows:

$$d(l_o w_o t_o) = d(l \cdot w \cdot t) \quad (9.12)$$

Because the product $l_0 w_0 t_0 = l \cdot w \cdot t = \text{constant volume}$, and since the derivative of a constant $= 0$,

$$d(l \cdot w \cdot t) = 0 \quad (9.13)$$

or

$$l \cdot w \, dt + l \cdot t \, dw + t \cdot w \, dl = 0 \quad (9.14)$$

By dividing both sides by $l \cdot w \cdot t$, we obtain:

$$\frac{dl}{l} + \frac{dw}{w} + \frac{dt}{t} = 0 \quad (9.15)$$

By using Eqs. 9.3–9.5, we get:

$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0 \quad (9.16)$$

Equation 9.16 implies that the sum of the principal strain increments in directions 1, 2, 3 is zero.

The integration of both sides of Eq. 9.16 yields:

$$\int d\varepsilon_1 + \int d\varepsilon_2 + \int d\varepsilon_3 = \int 0$$

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0 \quad (9.17)$$

Equation 9.17 implies that the sum of the principal strains, in directions 1, 2, 3, is zero.

In the case of uniaxial loading, no stress acts in direction 2 or direction 3 i.e., $\sigma_2 = 0$, $\sigma_3 = 0$ (see Figs. 9.1 and 9.2); which means that the strains in directions 2 and 3 are equal, i.e. $\varepsilon_2 = \varepsilon_3$. Thus, for uniaxial loading (during the tensile test), Eq. 9.17 can be rewritten as follows:

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_2 = 0 \quad (9.18)$$

or

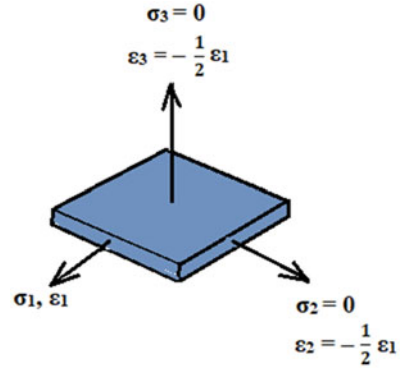
$$\varepsilon_2 = -\frac{1}{2}\varepsilon_1 \quad (9.19)$$

Similarly,

$$\varepsilon_3 = \varepsilon_2 = -\frac{1}{2}\varepsilon_1 \quad (9.20)$$

The stresses and strains acting on a tensile-test flat-specimen are shown in Fig. 9.3. The significance of Eqs. 9.19 and 9.20 is illustrated in Example 9.5.

Fig. 9.3 Stresses and strains acting in uniaxial loading in tensile testing



9.2.2 Isotropy and Anisotropy

A material is said to be *isotropic* when the properties measured in any direction are the same, i.e., the width and thickness strains are the same. On the other hand, in the case of *anisotropic* material, the width strain is different from the thickness strain. This means that the properties of *anisotropic* materials depend on the alignment or orientation of the test specimen. The state of *anisotropy* is represented by the *R-value*, which is defined as the ratio of the width strain to thickness strain.

The *R-value* can be mathematically expressed by (Hu et al., 2002):

$$R\text{-value} = \frac{\epsilon_w}{\epsilon_t} \quad (9.21)$$

where ϵ_w is the width strain, and ϵ_t is the thickness strain. By combination of Eqs. 9.9, 9.11, and 9.21, we obtain:

$$R\text{-value} = \frac{\ln \frac{w}{w_o}}{\ln \frac{t}{t_o}} \quad (9.22)$$

The significance of Eq. 9.22 is illustrated in Example 9.7.

Since the strains in the width and thickness directions are equal in case of isotropic material, Eq. 9.20 can be rewritten as follows:

$$\epsilon_2 = \epsilon_3 = -\frac{1}{2} \epsilon_1 \quad (9.23)$$

The significance of Eq. 9.23 is illustrated in Example 9.8.

9.3 Plane Stress Deformation in Sheet Metal Forming

9.3.1 Plane Stress and Membrane Stresses

Sheet metal forming processes involve a great deal of plastic deformation; these processes include: bending, deep drawing (with a flat-headed punch), stretch forming, and general sheet forming (Mellor & Parmar, 1978). Most of these sheet forming process, in general, are in the state of *plane stress*, i.e., the stress, normal to the surface of the sheet, is negligibly small ($\sigma_3 = 0$). The stresses in the plane of the sheet (σ_1, σ_2) are called the *membrane stresses* (see Fig. 9.4). This is why in a typical plane-stress sheet-metal forming process, most work-materials deform under the membrane stresses.

9.3.2 Stress Ratio and Strain Ratio Relationships

It is useful to define two important terms associated with the state of plane stress; these terms are the *stress ratio* and the *strain ratio* (see Fig. 9.4). The ratio of the stress in direction 2 to the stress in direction 1 is called the *stress ratio* (α). Similarly, the ratio of strain in direction 2 to the strain in direction 1 is called the *strain ratio* (β). The stress ratio and the strain ratio can be numerically expressed as:

$$\alpha = \frac{\sigma_2}{\sigma_1} \quad (9.24)$$

$$\beta = \frac{\varepsilon_2}{\varepsilon_1} = \frac{d\varepsilon_2}{d\varepsilon_1} \quad (9.25)$$

For uniaxial tension, $\sigma_2 = 0$ giving $\alpha = 0$. By reference to Eq. 9.23, for isotropic materials $\beta = -1/2$.

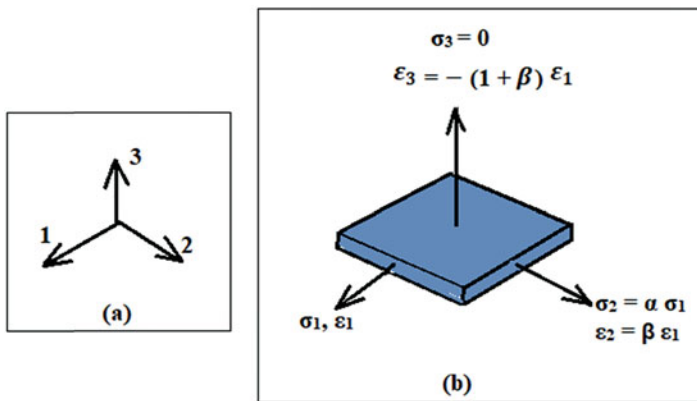


Fig. 9.4 (a) The three directions, and (b) the state of plane stress in sheet metal forming

By a combination of Eqs. 9.21, 9.24, and 9.25 in the state of plane stress, we obtain:

$$\sigma_2 = \alpha \cdot \sigma_1 \quad (9.26a)$$

$$\sigma_3 = 0 \quad (9.26b)$$

$$\varepsilon_2 = \beta \cdot \varepsilon_1 \quad (9.27a)$$

$$\varepsilon_3 = -(\varepsilon_1 + \varepsilon_2) = -(\varepsilon_1 + \beta\varepsilon_1) = -(1 + \beta) \varepsilon_1 \quad (9.27b)$$

The significance of Eqs. 9.26 and 9.27 is illustrated in Examples 9.9–9.11.

9.4 Principal Normal Stresses, Hydrostatic Stress and Deviatoric Stresses

The stresses σ_1 , σ_2 , and σ_3 , as shown in Fig. 9.5, are called the *principal normal stresses*. The average of the principal normal stresses is called the *hydrostatic stress*. Numerically, the hydrostatic stress (σ_h) is expressed as:

$$\sigma_h = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad (9.28)$$

The difference between the principal normal stress and the hydrostatic stress is called the *deviatoric stress* (σ'_i) (see Fig. 9.6).

By reference to Fig. 9.6, the deviatoric stresses (σ'_1 , σ'_2 , σ'_3) can be expressed as:

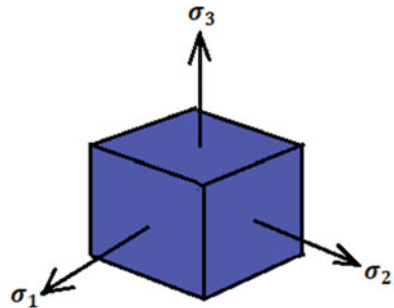
$$\sigma'_1 = \sigma_1 - \sigma_h \quad (9.29a)$$

$$\sigma'_2 = \sigma_2 - \sigma_h \quad (9.29b)$$

$$\sigma'_3 = \sigma_3 - \sigma_h \quad (9.29c)$$

The significance of Eqs. 9.28 and 9.29 is illustrated in Examples 9.12 and 9.13.

Fig. 9.5 Principal normal stresses in 3-dimensions (3D)



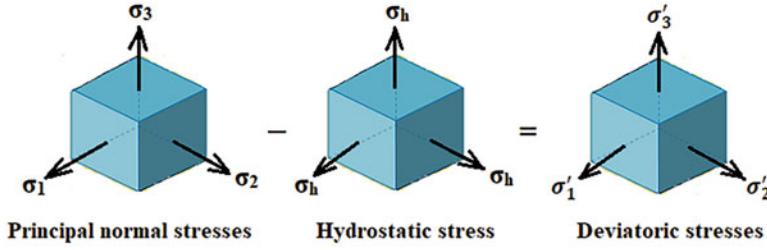


Fig. 9.6 The relationship between the principal normal stresses, hydrostatic stress (σ_h) and the deviatoric stresses (σ'_1 , σ'_2 , and σ'_3)

Since most sheet metal forming processes are in the state of *plane stress*, the deviatoric stresses may be expressed in terms of the stress ratio α by rewriting Eqs. 9.29(a-c), as follows:

$$\sigma'_1 = \sigma_1 - \sigma_h = \sigma_1 - \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \right) = \frac{3\sigma_1 - \sigma_1 - \sigma_2 - 0}{3} = \frac{2\sigma_1 - \alpha\sigma_1}{3} = \left(\frac{2-\alpha}{3} \right) \sigma_1$$

$$\begin{aligned} \sigma'_2 &= \sigma_2 - \sigma_h = \sigma_2 - \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \right) = \frac{3\sigma_2 - \sigma_1 - \sigma_2 - 0}{3} = \frac{2\sigma_2 - \sigma_1}{3} = \frac{2\alpha\sigma_1 - \sigma_1}{3} \\ &= \left(\frac{2\alpha-1}{3} \right) \sigma_1 \end{aligned}$$

$$\sigma'_3 = \sigma_3 - \sigma_h = 0 - \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \right) = \frac{-\sigma_1 - \sigma_2 - 0}{3} = \frac{-\sigma_1 - \alpha\sigma_1}{3} = -\left(\frac{1+\alpha}{3} \right) \sigma_1$$

The above equations may be expressed in simplified form, as follows:

$$\sigma'_1 = \left(\frac{2-\alpha}{3} \right) \sigma_1 \quad (9.30a)$$

$$\sigma'_2 = \left(\frac{2\alpha-1}{3} \right) \sigma_1 \quad (9.30b)$$

$$\sigma'_3 = -\left(\frac{1+\alpha}{3} \right) \sigma_1 \quad (9.30c)$$

The significance of Eqs. 9.30(a-c) is illustrated in Example 9.14.

9.5 Levy-Mises Flow Rule and Relation Between α and β

Sheet metal forming involves a great deal of plastic flow. The Levy-Mises flow rule relates the strain rate ($\dot{\epsilon}$) to the deviatoric stress, as follows:

$$\frac{\dot{\epsilon}_1}{\sigma'_1} = \frac{\dot{\epsilon}_2}{\sigma'_2} = \frac{\dot{\epsilon}_3}{\sigma'_3} \quad (9.31)$$

where $\dot{\epsilon}_1$, $\dot{\epsilon}_2$, and $\dot{\epsilon}_3$ are the strain rates in directions 1, 2, and 3, respectively. Equation 9.31 can be rewritten as:

$$\frac{\frac{d\epsilon_1}{dt}}{\sigma'_1} = \frac{\frac{d\epsilon_2}{dt}}{\sigma'_2} = \frac{\frac{d\epsilon_3}{dt}}{\sigma'_3} \quad (9.32)$$

or

$$\frac{d\epsilon_1}{\sigma'_1} = \frac{d\epsilon_2}{\sigma'_2} = \frac{d\epsilon_3}{\sigma'_3} \quad (9.33)$$

By combination of Eqs. 9.30(a–c) and 9.33, we obtain:

$$\frac{d\epsilon_1}{2-\alpha} = \frac{d\epsilon_2}{2\alpha-1} = \frac{d\epsilon_3}{-(1+\alpha)} \quad (9.34)$$

By using integral calculus,

$$\int \frac{d\epsilon_1}{2-\alpha} = \int \frac{d\epsilon_2}{2\alpha-1} = \int \frac{d\epsilon_3}{-(1+\alpha)} \quad (9.35)$$

$$\frac{\epsilon_1}{2-\alpha} = \frac{\epsilon_2}{2\alpha-1} = \frac{\epsilon_3}{-(1+\alpha)} \quad (9.36)$$

By combining Eqs. 9.27(a–b) and 9.36, we get:

$$\frac{\epsilon_1}{2-\alpha} = \frac{\epsilon_2}{2\alpha-1} = \frac{\beta\epsilon_1}{2\alpha-1} = \frac{\epsilon_3}{-(1+\alpha)} = \frac{-(1+\beta)\epsilon_1}{-(1+\alpha)} \quad (9.37)$$

or

$$\alpha = \frac{2\beta+1}{2+\beta} \quad (9.38)$$

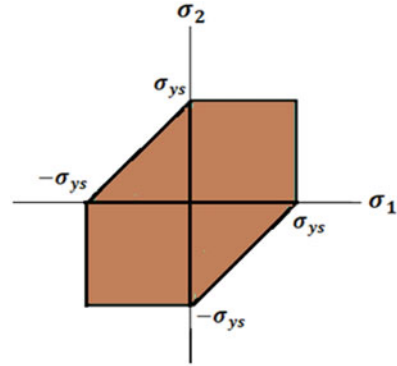
where α is the stress ratio and β is the strain ratio (see Example 9.15).

9.6 Theories of Yielding for Sheet Metal Forming

9.6.1 Tresca Yield Condition for Plane Stress

In sheet metal forming, force is applied to a piece of sheet metal so as to stress the metal beyond its yield strength, thereby causing the material to plastically deform, but not to fail. The term *plastic yield* or *plastic flow* has been explained in Chap. 3 (Sect. 3.2). The term *flow stress* or *yield stress* (σ_{ys}) may be defined as the stress at

Fig. 9.7 Graphical representation of the Tresca yield criterion



which the material will yield in simple tension (uniaxial loading), i.e., when $\alpha = 0$ (see Sect. 9.3.2). According to the *Tresca yield theory*, yielding would occur in any deformation process when:

$$|\sigma_{\max} - \sigma_{\min}| = \sigma_{ys} \quad (9.39)$$

In the *plane stress* state, $\sigma_{\max} = \sigma_1$ and $\sigma_3 = 0$. $\sigma_{\min} = \sigma_3$ when $\sigma_2 > 0$. On the other hand, $\sigma_{\min} = \sigma_2$ when $\sigma_2 < 0$ (see Example 9.16 and 9.17). The Tresca theory is graphically shown in Fig. 9.7.

9.6.2 von Mises Yield Theory

According to the *von Mises yield criterion*, yielding will occur when the root-mean-square (*r.m.s*) value of the maximum shear stresses reaches a critical value. It can be mathematically proved that for the *plane stress* state (Fig. 9.3), the following relationship holds good:

$$\left(\sqrt{1 - \alpha + \alpha^2} \right) \sigma_1 = \sigma_{ys} \quad (9.40)$$

The significance of Eq. 9.40 is illustrated in Examples 9.18 and 9.19.

9.7 Effective Stress, Effective Strain, and Work of Plastic Deformation

In sheet metal forming, it is often useful to determine the *effective strain* and the *effective stress*; the latter are expressed as follows:

$$\text{Effective strain} = \bar{\varepsilon} = \sqrt{\frac{4}{3}(1 + \beta + \beta^2)} \cdot \varepsilon_1 \quad (9.41)$$

$$\text{Effective stress} = \bar{\sigma} = K(a + \bar{\varepsilon})^n = \left(\sqrt{1 - \alpha + \alpha^2}\right) \sigma_1 \quad (9.42)$$

where K is the strength coefficient (MPa), a and n are constants, ε_1 is the principal strain in direction 1, and σ_1 is the principal normal stress in direction 1 (MPa). By rearrangement of the terms for the principal normal stress σ_1 , Eq. 9.42 can be rewritten as:

$$\sigma_1 = \frac{\bar{\sigma}}{\sqrt{1 - \alpha + \alpha^2}} \quad (9.43)$$

The significance of Eqs. 9.41–9.43 is illustrated in Examples 9.20–9.23.

The work done in plastic deformation in a sheet-metal forming process can be determined by:

$$\text{Work done per unit volume} = \int_0^{\bar{\varepsilon}} \bar{\sigma} \cdot d\bar{\varepsilon} \quad (9.44)$$

The significance of Eq. 9.44 is illustrated in Example 9.24.

9.8 Mechanics of Stretch Forming

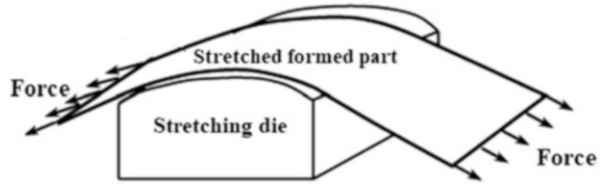
Stretch forming of a sheet metal is a forming process that involves stretching a metal sheet over a die to produce complex and contoured shapes. It is commonly used in the automotive, aerospace, and architectural industries to produce components with precise shapes and tight tolerances. The process begins with a flat metal-sheet that is clamped around its edges and secured in place. It is then subjected to a stretching force that pulls the material over the die. The stretching operation causes the material to deform and take on the shape of the die (see Fig. 9.8).

The stretch forming force can be determined by the following formula:

$$F_{sf} = \left(\frac{\sigma_{ys} + \sigma_{UTS}}{2}\right) \cdot A \quad (9.45)$$

where F_{sf} is the stretch forming force (N), σ_{ys} is the material's yield strength (MPa), σ_{UTS} is the material's ultimate tensile strength (MPa), and A is the cross-sectional area of the work (mm^2) (see Example 9.25).

Fig. 9.8 Stretch forming of a sheet metal



9.9 Worked Numerical Examples

Example 9.1: Determining the Final Thickness in a Sheet-Metal Formation Operation

A square element $9 \text{ mm} \times 9 \text{ mm}$ in an un-deformed metal sheet of 1.0-mm -thickness becomes a rectangle, $7.8 \text{ mm} \times 11.6 \text{ mm}$ after forming. Assume that the stress normal to the sheet is zero. Determine the final thickness of the sheet after the sheet forming process.

Solution

$l_0 = 9 \text{ mm}$, $w_0 = 9 \text{ mm}$, $t_0 = 1.0 \text{ mm}$, $l = 11.6 \text{ mm}$, $w = 7.8 \text{ mm}$, $t = ?$

By using the modified form of Eq. 9.1,

$$\text{Final thickness of the sheet} = t = \frac{l_0 w_0 t_0}{lw} = \frac{9 \times 9 \times 1.0}{11.6 \times 7.8} = 0.895 \text{ mm}$$

Example 9.2: Determining the Ductility of a Metal by Finding Its % Elongation

A tensile-test flat-metal-sample has the gage elements $l_o = 50 \text{ mm}$, $w_o = 12.5 \text{ mm}$, and $t_o = 0.8 \text{ mm}$. The final gage length after the tensile test is 62 mm . Determine the percent elongation of the metal.

Solution

By using Eq. 9.2,

$$\% \text{Elongation} = \frac{l - l_o}{l_o} \times 100 = \frac{62 - 50}{50} \times 100 = 24$$

or

$$\text{Elongation} = 24\%$$

Example 9.3: Determining the % Reduction in Area Based on Tensile Flat Sample

By using the data in Example 9.2, determine the percent reduction in area for the tensile test.

Solution

$l_o = 50 \text{ mm}$, $w_o = 12.5 \text{ mm}$, $t_o = 0.8 \text{ mm}$, $l = 62 \text{ mm}$, $\% \text{RA} = ?$

$$A_o = w_o \cdot t_o = 12.5 \times 0.8 = 10 \text{ mm}^2$$

Pre-test volume = post-test volume

$$l_o \cdot A_o = l_f \cdot A_f$$

$$A_f = \frac{l_o \cdot A_o}{l_f} = \frac{50 \times 10}{62} = 8.064 \text{ mm}^2$$

$$\% \text{Reduction in area} = \frac{A_o - A_f}{A_o} \times 100 = \frac{10 - 8.064}{10} \times 100 = 19.36$$

Example 9.4: Computing Principal Strains in Directions 1, 2, and 3 in Sheet Metal Forming

By using the data in Example 9.1, calculate the principal strains in the directions 1, 2, and 3 in the sheet forming process.

Solution

$l_o = 9 \text{ mm}$, $w_o = 9 \text{ mm}$, $t_o = 1.0 \text{ mm}$, $l = 11.6 \text{ mm}$, $w = 7.8 \text{ mm}$, $t = 0.895 \text{ mm}$
By using Eqs. 9.7, 9.9, and 9.11,

$$\text{The principal strain in direction 1} = \varepsilon_1 = \ln \frac{l}{l_o} = \ln \frac{11.6}{9} = 0.253$$

$$\text{The principal strain in direction 2} = \varepsilon_2 = \ln \frac{w}{w_o} = \ln \frac{7.8}{9} = -0.1431$$

$$\text{The principal strain in direction 3} = \varepsilon_3 = \ln \frac{t}{t_o} = \ln \frac{0.895}{1.0} = -0.111$$

Example 9.5: Determining the Stresses and Strains in Directions 1,2,3 in Uniaxial Loading

A tensile stress of 300 MPa acts in uniaxial loading during a tensile test using a flat specimen. The resulting strain, in the loading direction, is 0.03. Determine the stresses and strains in all the three directions and show them in a sketch.

Solution

$$\sigma_1 = 300 \text{ MPa}, \sigma_2 = 0, \sigma_3 = 0, \varepsilon_1 = 0.03.$$

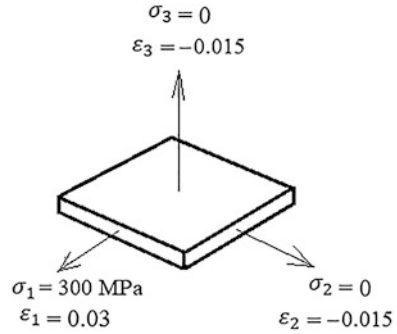
By using Eqs. 9.19 and 9.20,

$$\varepsilon_2 = -\frac{1}{2}\varepsilon_1 = -\frac{1}{2} \times 0.03 = -0.015 \text{ MPa}$$

$$\varepsilon_3 = \varepsilon_2 = -0.015 \text{ MPa}$$

The stresses and strains in all the three directions are shown in Fig. 9.9.

Fig. 9.9 The magnitude of stresses and strains in uniaxial loading for Example 9.5



Example 9.6: Determining the Yield Strength, σ_{UTS} , and σ_{break} for Flat Sample
 A tensile-test flat-specimen is cut from a steel sheet of 0.9-mm thickness. The gage length is 50 mm and the initial width is 12.5 mm. The load at the upper yield point (UYP), during the test, is 2.80 kN. The maximum load is 2.92 kN, and the specimen fractures at a load of 2.40 kN. Determine the yield strength (σ_{ys}), the ultimate tensile strength (σ_{UTS}), and the breaking strength.

Solution

$l_o = 50 \text{ mm}$, $w_o = 12.5 \text{ mm}$, $t_o = 0.9 \text{ mm}$, $F_{\text{UYP}} = 2.80 \text{ kN} = 2800 \text{ N}$,
 $F_{\text{max}} = 2.92 \text{ kN}$, $F_b = 2.4 \text{ kN}$

The original cross-sectional area $A_o = w_o \cdot t_o = 12.5 \times 0.9 = 11.25 \text{ mm}^2$

By using Eqs. 3.1–3.3,

$$\sigma_{\text{ys}} = \frac{F_{\text{UYP}}}{A_o} = \frac{2800}{11.25} = 248.889 \text{ MPa}$$

$$\sigma_{\text{UTS}} = \frac{F_{\text{max}}}{A_o} = \frac{2920}{11.25} = 259.555 \text{ MPa}$$

$$\text{Breaking strength} = \frac{F_b}{A_o} = \frac{2400}{11.25} = 213.33 \text{ MPa}$$

Example 9.7: Determining the R -value for a Sheet Forming Process

By using the data in Example 9.1, determine the R -value for the sheet forming process.

Solution

$l_o = 9 \text{ mm}$, $w_o = 9 \text{ mm}$, $t_o = 1.0 \text{ mm}$, $l = 8.8 \text{ mm}$, $w = 6.8 \text{ mm}$, $t = 1.35 \text{ mm}$

By using Eq. 9.22,

$$R\text{-value} = \frac{\ln \frac{w}{w_o}}{\ln \frac{l}{l_o}} = \frac{\ln \frac{6.8}{9}}{\ln \frac{1.35}{1.0}} = \frac{-0.2803}{0.3001} = -0.934$$

Example 9.8: Determining Strains in the Three Directions for an Isotropic Material

Use the data in Example 9.2. Assuming the material to be isotropic, determine the strains in the three directions for uniaxial loading.

Solution

$$l_o = 50 \text{ mm}, l = 62 \text{ mm}, \varepsilon_1 = ?, \varepsilon_2 = ?, \varepsilon_3 = ?$$

By using Eq. 9.7,

$$\varepsilon_1 = \ln \frac{l}{l_o} = \ln \frac{62}{50} = 0.2151$$

By using Eq. 9.23,

$$\varepsilon_2 = -\frac{1}{2} \varepsilon_1 = -\frac{1}{2} \times 0.2151 = -0.10755$$

$$\varepsilon_3 = \varepsilon_2 = -0.10755$$

Example 9.9: Determining the Stress Ratio and the Stress in Direction 3

The membrane stresses in directions 1 and 2, in a plane-stress sheet-metal forming process, are 425 MPa and 370 MPa, respectively. Determine the stress ratio and the stress in direction 3.

Solution

$$\sigma_1 = 425 \text{ MPa}, \sigma_2 = 370 \text{ MPa}.$$

By using Eq. 9.24,

$$\text{Stress ratio} = \alpha = \frac{\sigma_2}{\sigma_1} = \frac{370 \text{ MPa}}{425 \text{ MPa}} = 0.87$$

$$\sigma_3 = 0 \quad (\text{since the state is plane stress})$$

Example 9.10: Calculating the Strain Ratio When the Principal Strain Increments Are Given

The principal strain increments in directions 1 and 2, in a plane-stress sheet-metal forming process, are 0.013 and 0.007, respectively. Calculate the strain ratio for the forming process.

Solution

By using Eq. 9.25,

$$\text{Strain ratio} = \beta = \frac{d\varepsilon_2}{d\varepsilon_1} = \frac{0.007}{0.013} = 0.538$$

Example 9.11: Determining the Principal Strain in Direction 3

The principal strains in directions 1 and 2, in a plane-stress sheet-metal forming process, are 0.171 and -0.206 , respectively. Determine the principal strain in direction 3.

Solution

$$\varepsilon_1 = 0.171, \varepsilon_2 = -0.206, \varepsilon_3 = ?$$

By using Eqs. 9.25 and 9.27(b),

$$\beta = \frac{\varepsilon_2}{\varepsilon_1} = \frac{-0.206}{0.171} = -1.2047$$

$$\varepsilon_3 = -(1 + \beta) \varepsilon_1 = -[1 + (-1.2047)] \times 0.171 = (-0.205) \times 0.171 = 0.035$$

The principal strain in direction 3 = 0.035.

Example 9.12: Determining the Hydrostatic Stress during a Metal Forming Process

The principal normal stresses acting on an element of material during a metal forming operation in directions 1, 2, and 3 are shown in Fig. 9.10. Determine the hydrostatic stress.

Solution

By reference to Fig. 9.10, $\sigma_1 = 280$ MPa, $\sigma_2 = -210$ MPa, $\sigma_3 = 150$ MPa, $\sigma_h = ?$

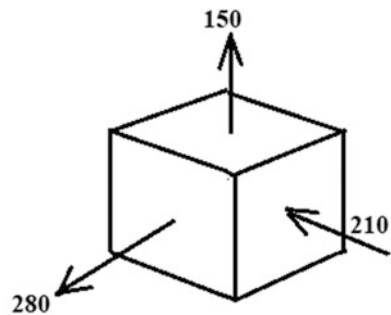
By using Eq. 9.28,

$$\sigma_h = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{280 - 210 + 150}{3} = 73.33 \text{ MPa}$$

Example 9.13: Determining the Deviatoric Stresses in Metal Forming Process

By using the data in Example 9.12, determine the deviatoric stresses in the forming process.

Fig. 9.10 The sketch showing the principal normal stresses data for Example 9.12



Solution

$$\sigma_1 = 280 \text{ MPa}, \sigma_2 = -210 \text{ MPa}, \sigma_3 = 150 \text{ MPa}, \sigma_h = 73.33 \text{ MPa}, \sigma'_1 = ?, \sigma'_2 = ?, \sigma'_3 = ?$$

By using Eqs. 9.29(a–c),

$$\sigma'_1 = \sigma_1 - \sigma_h = 280 - 73.33 = 206.67 \text{ MPa}$$

$$\sigma'_2 = \sigma_2 - \sigma_h = -210 - 73.33 = -283.33 \text{ MPa}$$

$$\sigma'_3 = \sigma_3 - \sigma_h = 150 - 73.33 = 76.67 \text{ MPa}$$

Example 9.14: Determining the Deviatoric Stresses When σ_3 Is Unknown

By using the data in Example 9.9, determine the deviatoric stresses for the sheet metal forming process.

Solution

$$\sigma_1 = 425 \text{ MPa}, \sigma_2 = 370 \text{ MPa}, \alpha = 0.87$$

By using Eqs. 9.30(a–c),

$$\sigma'_1 = \left(\frac{2-\alpha}{3}\right)\sigma_1 = \left(\frac{2-0.87}{3}\right) \times 425 = 160.083 \text{ MPa}$$

$$\sigma'_2 = \left(\frac{2\alpha-1}{3}\right)\sigma_1 = \left(\frac{2 \times 0.87-1}{3}\right) \times 425 = 104.833 \text{ MPa}$$

$$\sigma'_3 = -\left(\frac{1+\alpha}{3}\right)\sigma_1 = -\left(\frac{1+0.87}{3}\right) \times 425 = -264.92 \text{ MPa}$$

Example 9.15: Determining the Stress Ratio When the Strain Ratio Is Known

By using the data in Example 9.11, determine the stress ratio for the sheet metal forming process.

Solution

$$\beta = -1.2047, \alpha = ?$$

By using Eq. 9.38,

$$\text{Stress ratio} = \alpha = \frac{2\beta + 1}{2 + \beta} = \frac{(2 \times -1.2047) + 1}{2 - 1.2047} = \frac{-2.4094 + 1}{0.795} = -1.7728$$

Example 9.16: Determination of Flow Stress When the Stress σ_2 Is Positive

The membrane stresses in directions 1 and 2, in a plane-stress sheet-metal forming process, are 350 MPa and 230 MPa, respectively. Determine the flow stress for the forming process.

Solution

$$\sigma_1 = 350 \text{ MPa}, \sigma_2 = 230 \text{ MPa}, \sigma_{ys} = ?$$

$$\sigma_{\max} = \sigma_1 \text{ and } \sigma_3 = 0.$$

$$\sigma_{\min} = \sigma_3 = 0$$

By using Eq. 9.39,

$$\sigma_{ys} = |\sigma_{\max} - \sigma_{\min}| = |\sigma_1 - \sigma_3| = |350 - 0| = |350| = 350 \text{ MPa}$$

The flow stress = 350 MPa.

Example 9.17: Determination of Flow Stress When the Stress σ_2 Is Negative

The membrane stresses in directions 1 and 2, in a plane-stress sheet-metal forming process, are 250 MPa (tension) and 180 MPa (compression), respectively. Determine the flow stress for the forming process.

Solution

$$\sigma_1 = 250 \text{ MPa}, \sigma_2 = -180 \text{ MPa}, \sigma_{ys} = ?$$

$$\sigma_{\max} = \sigma_1 \text{ and } \sigma_3 = 0.$$

$$\sigma_{\min} = \sigma_2$$

By using Eq. 9.39,

$$\sigma_{ys} = |\sigma_{\max} - \sigma_{\min}| = |\sigma_1 - \sigma_2| = |250 - (-180)| = 430 \text{ MPa}$$

The flow stress = 430 MPa.

Example 9.18: Determining the Stress Ratio When Principal Strain Increments Are Known

The current flow stress of an element of material is 280 MPa. The principal strain increments, in a plane-stress sheet-metal forming process, are 0.014 and 0.006 in the directions 1 and 2, respectively. Determine the stress ratio.

Solution

$$d\varepsilon_1 = 0.014, d\varepsilon_2 = 0.006, \alpha = ?$$

$$\beta = \frac{d\varepsilon_2}{d\varepsilon_1} = \frac{0.006}{0.014} = 0.4285$$

$$\alpha = \frac{2\beta + 1}{2 + \beta} = \frac{(2 \times 0.4285) + 1}{2 + 0.4285} = \frac{1.857}{2.4285} = 0.7646$$

Example 9.19: Determining the Membrane Stresses When the Stress Ratio Is Known

By using the data in Example 9.18, determine the membrane stresses associated with the strains in the plane-stress process.

Solution

$$\sigma_{ys} = 280 \text{ MPa}, \alpha = 0.7646, \sigma_1 = ?, \sigma_2 = ?$$

By using Eq. 9.40,

$$\left(\sqrt{1 - \alpha + \alpha^2} \right) \sigma_1 = \sigma_{ys}$$

$$\sigma_1 = \frac{\sigma_{ys}}{\sqrt{1 - \alpha + \alpha^2}} = \frac{280}{\sqrt{1 - 0.7646 + 0.7646^2}} = \frac{280}{\sqrt{0.820013}} = \frac{280}{0.9055} = 309.22 \text{ MPa}$$

By using the modified form of Eq. 9.24,

$$\sigma_2 = \alpha \cdot \sigma_1 = 0.7646 \times 309.22 = 236.43 \text{ MPa}$$

The membrane stresses are $\sigma_1 = 309.22 \text{ MPa}$, $\sigma_2 = 236.43 \text{ MPa}$.

Example 9.20: Determining the Strain Ratio in Sheet Metal Forming Process

A square element 12 mm $\times$ 12 mm in an un-deformed metal sheet of 1.3-mm-thickness becomes a rectangle with dimensions 10 mm $\times$ 14 mm after forming. Assume that the state of *plane stress* in the sheet metal forming process. Determine the strain ratio for the forming process. The material obeys the following stress-strain law: $\bar{\sigma} = 450 (0.007 + \bar{\epsilon})^{0.19} \text{ MPa}$.

Solution

$$l_0 = 12 \text{ mm}, w_0 = 12 \text{ mm}, t_0 = 1.3 \text{ mm}, l = 14 \text{ mm}, w = 10 \text{ mm}, \sigma_3 = 0, \bar{\epsilon} = ?$$

$$\epsilon_1 = \ln \frac{l}{l_0} = \ln \frac{14}{12} = 0.1541$$

$$\epsilon_2 = \ln \frac{w}{w_0} = \ln \frac{10}{12} = -0.1823$$

$$\beta = \frac{\epsilon_2}{\epsilon_1} = \frac{-0.1823}{0.1541} = -1.183$$

Example 9.21: Determining the Effective Strain for a Sheet Metal Forming Process

By using the data in Example 9.20, determine the effective strain for the sheet forming process.

Solution

$$\epsilon_1 = 0.1541, \beta = -1.183, \bar{\epsilon} = ?$$

By using Eq. 9.41,

$$\bar{\epsilon} = \sqrt{\frac{4}{3} (1 + \beta + \beta^2)} \cdot \epsilon_1 = \sqrt{\frac{4}{3} (1 - 1.183 + 1.183^2)} \times 0.1541$$

$$\bar{\epsilon} = \sqrt{\frac{4}{3} \times 1.21649} \times 0.1541 = \sqrt{1.622} \times 0.1541 = 0.1962$$

$$\text{Effective strain} = \bar{\epsilon} = 0.1962$$

Example 9.22: Determining the Effective Stress for a Sheet Forming Process

By using the data in Example 9.21, determine the effective stress for the sheet forming process.

Solution

$$\bar{\sigma} = 450 (0.007 + \bar{\epsilon})^{0.19} \text{ MPa}, \bar{\epsilon} = 0.1962$$

$$\begin{aligned} \bar{\sigma} &= 450 (0.007 + \bar{\epsilon})^{0.19} \text{ MPa} = 450 (0.007 + 0.1962)^{0.19} = 450 \times 0.7388 \\ &= 332.44 \text{ MPa} \end{aligned}$$

$$\text{Effective stress} = \bar{\sigma} = 332.44 \text{ MPa}$$

Example 9.23: Determining the Membrane Stresses When the Effective Stress Is Known

By using the data in Examples 9.20–9.22, determine the (a) stress ratio, and (b) membrane stresses for the sheet forming process.

Solution

$$\beta = -1.183, \bar{\sigma} = 333.38 \text{ MPa}, \alpha = ?, \sigma_1 = ?, \sigma_2 = ?$$

$$(a) \alpha = \frac{2\beta+1}{2+\beta} = \frac{(2 \times -1.183)+1}{2+(-1.183)} = \frac{-2.366+1}{0.817} = -1.672$$

(b) By using Eq. 9.43,

$$\begin{aligned} \sigma_1 &= \frac{\bar{\sigma}}{\sqrt{1-\alpha+\alpha^2}} = \frac{333.38}{\sqrt{1-(-1.672)+1.672^2}} = \frac{333.38}{\sqrt{5.4676}} = \frac{333.38}{2.3382} \\ &= 142.58 \text{ MPa} \end{aligned}$$

$$\sigma_2 = \alpha \cdot \sigma_1 = -1.672 \times 142.58 \text{ MPa} = -238.4 \text{ MPa}$$

$$\alpha = -1.672, \sigma_1 = 142.58 \text{ MPa (tension)}, \sigma_2 = 238.4 \text{ MPa (compression)}.$$

Example 9.24: Determining the Work Done in Plastic Deformation in Sheet Metal Forming

By using the data in Example 9.22, determine the work done in plastic deformation for the plane-stress sheet-metal forming process.

Solution

$$l_0 = 12 \text{ mm}, w_0 = 12 \text{ mm}, t_0 = 1.3 \text{ mm}, l = 14 \text{ mm}, w = 10 \text{ mm}, \bar{\sigma} = 450 (0.007 + \bar{\epsilon})^{0.19} \text{ MPa}, \bar{\epsilon} = 0.1962$$

By using Eq. 9.44,

$$\begin{aligned}
\frac{\text{Work}}{\text{Volume}} &= \int_0^{\bar{\epsilon}} \bar{\sigma} \cdot d\bar{\epsilon} = \int_0^{0.1962} 450 (0.007 + \bar{\epsilon})^{0.19} \cdot d\bar{\epsilon} \\
&= 450 \int_0^{0.1962} (0.007 + \bar{\epsilon})^{0.19} \cdot d\bar{\epsilon} = 450 \cdot \left| \frac{(0.007 + \bar{\epsilon})^{0.19+1}}{1 + 0.19} \right| \\
&= 450 \left\{ \left[\frac{(0.007 + 0.1962)^{1.19}}{1.19} \right] - \left[\frac{(0.007 + 0)^{1.19}}{1.19} \right] \right\} = 450 \left[\frac{0.2032^{1.19}}{1.19} - \frac{0.007^{1.19}}{1.19} \right] \\
\frac{\text{Work}}{\text{Volume}} &= 450 (0.1261 - 0.0023) = 55.71 \text{ MPa} = 55.71 \text{ N/mm}^2
\end{aligned}$$

$$\text{Volume} = 12 \text{ mm} \times 12 \text{ mm} \times 1.3 \text{ mm} = 187.2 \text{ mm}^3$$

$$\begin{aligned}
\text{Work done} &= 55.71 \times 187.2 \text{ N} \cdot \text{mm} = 10,428.91 \text{ N} \cdot \text{mm} = 10,428.91 \times 10^{-3} \\
&= 10.43 \text{ N} \cdot \text{m}
\end{aligned}$$

Work done in plastic deformation = 10.43 J.

Example 9.25: Determining the Stretch Forming Force for an Aluminum Sheet

A commercially pure aluminum sheet with a width of 80 mm and thickness of 1.2 mm is to be stretched formed. Determine the force required for the operation.

Solution

By reference to Table 3.1, $\sigma_{ys} = 35 \text{ MPa}$, $\sigma_{UTS} = 90 \text{ MPa}$.

$$\text{Cross-sectional area} = A = 1.2 \times 80 = 96 \text{ mm}^2$$

By using Eq. 9.45,

$$F_{sf} = \left(\frac{\sigma_{ys} + \sigma_{UTS}}{2} \right) \cdot A = \left(\frac{35 + 90}{2} \right) \times 96 = 6000 \text{ N} = 6 \text{ kN}$$

The stretch forming force = 6 kN.

Example 9.26: Determining the Ratio $\frac{d\epsilon_1}{d\epsilon_3}$ When the Membrane Stresses Are Known

Membrane stresses ($\sigma_1 = 450 \text{ MPa}$, $\sigma_2 = 150 \text{ MPa}$) are applied to a sheet metal in plane stress state. Determine the ratio $\frac{d\epsilon_1}{d\epsilon_3}$.

Solution

$$\sigma_1 = 450 \text{ MPa}, \sigma_2 = 150 \text{ MPa}, \sigma_3 = 0, \frac{d\epsilon_1}{d\epsilon_3} = ?$$

$$\text{Stress ratio} = \alpha = \frac{\sigma_2}{\sigma_1} = \frac{150 \text{ MPa}}{450 \text{ MPa}} = 0.333$$

By using Eq. 9.37,

$$\frac{\varepsilon_1}{2 - \alpha} = \frac{\varepsilon_3}{-(1 + \alpha)}$$

or

$$\frac{\varepsilon_1}{\varepsilon_3} = \frac{2 - \alpha}{-(1 + \alpha)}$$

or

$$\frac{\varepsilon_1}{\varepsilon_3} = \frac{d\varepsilon_1}{d\varepsilon_3} = \frac{2 - \alpha}{-(1 + \alpha)} = \frac{2 - 0.333}{-(1 + 0.333)} = \frac{1.667}{-1.333} = -1.25$$

$$\frac{d\varepsilon_1}{d\varepsilon_3} = -1.25$$

Questions and Problems

- 9.1. (a) List six sheet-metal forming operations.
 (b) List the three types of loading conditions generally encountered in sheet-metal forming
- 9.2. (a) What is meant by *plane stress* state?
 (b) Draw a sketch showing the stresses and strains in the three directions in a plane-stress sheet forming process.
- 9.3. Prove that the sum of the principal strain increments in directions 1, 2, 3 is zero.
- 9.4. Diagrammatically illustrate the relationship between the principal normal stresses, hydrostatic stress, and the deviatoric stresses.
- 9.5. Describe Tresca yield theory with the aid of a graphical sketch.
- 9.6. (MCQs). Select the most appropriate answers for each of the following statement:
 1. Which type of loading is most commonly encountered in sheet-metal forming?
 (a) Uniaxial tension, (b) plane stress, (c) biaxial stretching, (d) none.
 2. The ratio of the width strain to thickness strain represents:.....
 (a) R-value, (b) strain ratio, (c) stress ratio, (d) strain rate.
 3. Zero stresses in the directions 2 and 3 i.e., $\sigma_2 = 0$, $\sigma_3 = 0$ represent:.....
 (a) Tri-axial stress, (b) plane stress, (c) biaxial stretching, (d) uniaxial loading.
 4. The average of the principal normal stresses is called:.....
 (a) Membrane stress, (b) deviatoric stress, (c) hydrostatic stress, (d) shear stress.

5. According to the *Tresca failure theory*, the absolute value of the difference between the maximum stress and the minimum stress is equal to:
 - (a) Membrane stress, (b) deviatoric stress, (c) hydrostatic stress, (d) yield stress.
6. In simple tension (uniaxial loading), the stress ratio is equal to:
 - (a) Zero, (b) unity, (c) ∞ , (d) 2.
- P9.7. A tensile-test flat-metal-sample has the gage elements $l_o = 50$ mm, $w_o = 12.5$ mm, and $t_o = 0.9$ mm. The final gage length after the tensile test is 68 mm. Determine the percent reduction in area of the metal.
- P9.8. A square-element with dimensions 8 mm $\times$ 8 mm in an un-deformed metal sheet of 1.0-mm-thickness becomes a rectangle, 6.8 mm $\times$ 10.6 mm after forming. Assume that the element is in the plane-stress state. Determine the final thickness after the forming process.
- P9.9. A tensile-test flat-specimen is cut from a steel sheet of 1.0-mm thickness. The gage length is 50 mm and the initial width is 12.5 mm. The load at the upper yield point (UYP), during the test, is 2.4 kN. The maximum load is 3 kN, and the specimens fractures at a load of 2.7 kN. Determine the yield strength (σ_{ys}), the ultimate tensile strength (σ_{UTS}), and the breaking strength of the steel.
- P9.10. By using the data in P9.8, determine the *R-value* for the sheet forming process.
- P9.11. The principal strains in directions 1 and 2, in a plane-stress sheet-metal forming process, are 0.163 and -0.197 , respectively. Determine the principal strain in direction 3.
- P9.12. The principal strain increments in directions 1 and 2, in a plane-stress sheet forming process, are 0.018 and 0.010, respectively. Calculate the strain ratio for the process.
- P9.13. The membrane stresses in directions 1 and 2, in a plane-stress sheet-metal forming process, are 440 MPa and 320 MPa, respectively. Determine the deviatoric stresses.
- P9.14. The membrane stresses in directions 1 and 2, in a plane-stress sheet-metal forming process, are 380 MPa and 250 MPa, respectively. Determine the stress ratio and the stress in direction 3.
- P9.15. The current flow stress of an element of material is 250 MPa. The principal strain increments, in a plane-stress sheet-metal forming process, are 0.020 and 0.009 in the directions 1 and 2, respectively. Determine the stress ratio.
- P9.16. The membrane stresses in directions 1 and 2, in a plane-stress sheet-metal forming process, are 375 MPa and 260 MPa, respectively. Determine the flow stress for the forming process.
- P9.17. The membrane stresses in directions 1 and 2, in a plane-stress sheet-metal forming process, are 270 MPa (tension) and 140 MPa (compression), respectively. Determine the flow stress for the forming process.
- P9.18. The principal normal stresses acting on a material's element are 230 MPa, -170 MPa, and 140 MPa, respectively. Determine the hydrostatic stress.

- P9.19. By using the data in P9.15, determine the membrane stresses associated with the strains in the plane-stress process.
- P9.20. A square element 11 mm × 11 mm in an un-deformed metal sheet of 1.1-mm-thickness becomes a rectangle with dimensions 9 mm × 13 mm after forming. Assume that the state of *plane stress* in the sheet metal forming process. Determine the strain ratio for the forming process. The material obeys the following stress-strain law:

$$\bar{\sigma} = 450 (0.007 + \bar{\epsilon})^{0.19} \text{ MPa.}$$

- P9.21. By using the data in P9.20, determine the (a) effective strain, (b) effective stress, and (c) work done in the plastic deformation.
- P9.22. A mild-steel sheet having a yield strength of 300 MPa and the tensile strength of 500 MPa with a width of 80 mm and thickness of 1.2 mm is to be stretched formed. Determine the force required for the stretch-forming operation.

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Chapter 10

Shearing, Blanking, and Piercing



Nomenclature

c	clearance
A_c	clearance allowance
t	thickness of the sheet metal
F_s	shearing force or blanking force or piercing force
L	length of cut
τ	shear strength of the metal
σ_{UTS}	ultimate tensile strength
D_b	blank diameter
D_p	punch diameter
D_h	hole diameter

10.1 Introduction to Shearing and Related Operations

An overview of sheet metal forming processes and their mechanics have been given in the preceding chapter. There are five types of processes that are generally used in press-work sheet-metal forming: (a) open shearing, (b) blanking, (c) piercing, (d) bending, and (e) deep drawing. Prior to discussing these processes, it is useful to make a distinction between a metal-plate and a metal-sheet. Sheet-metal thicknesses are typically in the range of 0.4–6.0 mm. On the other hand, when thickness exceeds 6 mm, the stock is usually referred to as a plate. Metal plates and sheets are produced by flat rolling; they are the backbones of automotive and aerospace structures. They are extensively used in the manufacture of bodies of automobiles, railway cars, aircraft structures, office furniture, and the like (see Chap. 9, Fig. 9.1).

10.2 Shearing, Blanking, and Piercing

Strictly speaking, there are three types of shearing operations in sheet-metal forming practice: (a) open shearing, (b) blanking, and (c) piercing. *Open shearing* (also called *shearing*) involves cutting off smaller pieces from a large sheet metal with an open cutting path. *Blanking* is a sheet-metal forming operation, during which a metal work-piece is removed from the primary metal sheet or strip when it is punched through a closed cutting path. The material that is removed is called *blank*. In the *blanking* operation, the cut-out part (blank) is used for further press-work operations. On the other hand, if the cut-out part is a scrap, the operation is called *piercing*.

10.3 Shearing (Open Shearing)

10.3.1 Shearing: Principle of Operation

Shearing, or *open shearing*, is a sheet-metal cut-off operation along a straight line between two cutting edges. In *shearing*, a piece of sheet metal is separated by applying a *shearing force* that is high enough to exceed the ultimate shear strength of the material; the excessive applied shear stress causes fracture and separation at the cut location (see Fig. 10.1). Basically, shearing involves a moving blade (upper blade) applying pressure and pushing a work-piece up against a fixed blade (lower blade). The moving blade is called a *punch* and the fixed blade is called a *die* (Altan & Tekkaya, 2012).

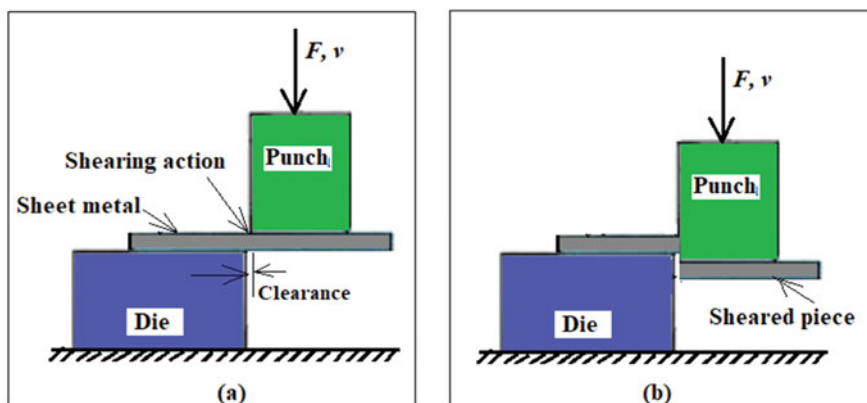


Fig. 10.1 Shearing operation; (a) Pre-shearing condition, (b) post-shearing condition

A small clearance must be allowed between the edges of the upper and lower blades, which facilitates the fracture of the material. The clearance size is in the range of 2–10% of the material thickness, and it depends on the sheet thickness, the specific shearing process, and the material. When the punch (or upper blade) impacts the sheet, the clearance between the blades enables the sheet to plastically deform and “rollover” the edge.

10.3.2 Advantages and Limitations of Shearing and Related Operation

Shearing is good for creating simple shapes, but it is not always a good choice. The biggest advantage of shearing is that it produces minimal or no *kerf*, with virtually no loss of material. For example, in shearing a rod, the typical tolerance is ± 0.127 mm. A notable advantage of shearing is that it performs straight-line cuts without chip formation or burning or melting the material. This feature allows the process to work well with many soft and ductile metals and alloys (e.g., aluminum, brass, bronze, and mild steel). The metal shearing process is especially cost-effective for mass production; it can produce thousands of pieces per hour. The shearing force (and the operating cost) can be reduced by mounting the blades at a shear angle of about 15° with the axis parallel to the sheet length (see Fig. 10.2).

The shearing process also has some disadvantages. Shearing is not preferred for cutting hard and brittle metals, such as cutting tungsten. Shearing a hard and brittle metal may result in fracturing of the part, as well as significant wear on the tool (blade) itself. Although the metal shearing process is often cost-effective for high-volume operations, shearing is not ideal for lengths under 3.175 mm. Sometimes burrs are formed in shearing, i.e. shearing cannot be considered burr-free cutting. This is why shearing may not be the best choice for applications where a clean end finish is required.

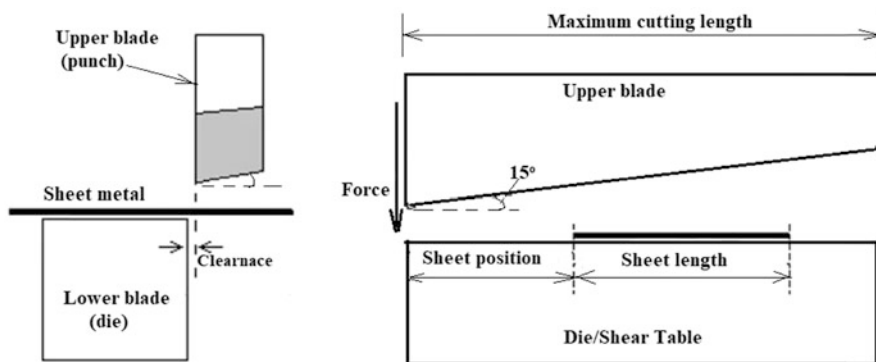


Fig. 10.2 Views of shearing operation showing the shear angle to reduce shearing force

10.3.3 Shearing Machines

The sheet-metal shearing machines can be classified into two types: (a) hydraulic shearing machines and (b) mechanical shearing machines. Hydraulic shearing machines are more commonly used due to their reduced noise and better energy economy, as compared to mechanical shearing machines. A hydraulic shearing machine uses a moving upper blade (also called *punch*) and a fixed lower blade (also called *die*) to apply a shearing force to metal sheets, resulting in the separation of the sheets into the desired size by utilizing an appropriate blade clearance. The frame of the shearing machine is constructed using steel plates; it includes the left and right boards, a work-table, a clamp holder, and a fuel tank (see Fig. 10.3). The hold-down cylinder of a hydraulic shearing machine is powered by the hydraulic system to clamp down on the steel plate; the left and right cylinders drive the movement of the blade up and down.

Hydraulic shearing machines are capable of cutting sheets with widths of up to 6000 mm. The shearing force is reduced by mounting the upper blade at a shear angle of about 15° with the axis parallel to the sheet length (Fig. 10.2). However, in order to reduce bending and distortion in the sheared sheet metal, a smaller shear angle is generally used, which may increase the shear force but also improves the quality of the sheared sheet.



Fig. 10.3 A hydraulic shearing machine. (Courtesy: King Abdulaziz University)

Table 10.1 Clearance allowance (A_c) data for shearing of various materials

Material	Aluminum alloys 1100S and 5052S	Cold-rolled (CR) mild steels and stainless steels (half hard, full hard)	Aluminum alloys 2024ST and 6061ST; brass, soft CR steel, soft stainless steel
Clearance allowance	0.045	0.075	0.06

10.3.4 Engineering Analysis of Shearing

It is important to allow appropriate clearance during shearing or blanking operation since there are effects of clearance on tool life, shearing force, and power (Semiatin, 2006). The *clearance*, in shearing and related operations, depends on the sheet thickness and the type of metal being formed; it is determined by Huda (2017, 2018):

$$c = A_c \cdot t \quad (10.1)$$

where c is the clearance (mm), A_c is the clearance allowance, and t is the thickness of the sheet metal (mm). The *clearance allowance* depends on the type of metal to be sheared; the values of A_c for various alloys are presented in Table 10.1.

The significance of Eq. 10.1 and Table 10.1 is illustrated in Examples 10.1 and 10.2. The *shearing force* or *punching force* depends on the sheet dimensions and strength of the sheet metal; it can be determined by using the following formula:

$$F_s = \tau \cdot t \cdot L = 0.7\sigma_{UTS} \cdot t \cdot L \quad (10.2)$$

where F_s is the shearing force (N), t is the thickness of the sheet metal (mm), L is the length of cut (mm), τ is the shear strength of the metal (MPa), and σ_{UTS} is the ultimate tensile strength (MPa) (see Examples 10.3–10.5).

10.4 Blanking/Piercing Operation

10.4.1 Blanking Operation

Blanking involves the removal of a work-piece from a primary metal sheet or strip when it is punched with a closed cutting path. The cut-out material that is removed is called *blank* (see Fig. 10.4). In the *blanking* operation, the cut-out part (blank) is used for further press-work operations. The blanking operation can be performed on any type of sheet metal that is required to be sheared using punches and dies. During the metal blanking operation, the punches and dies work differently depending on the material and the tool steel's ability to withstand the stresses that arise on the cutting edges.

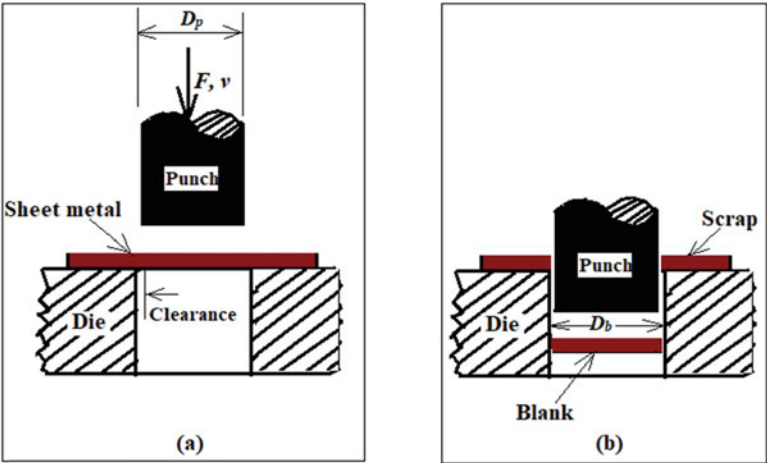


Fig. 10.4 Blanking operation; (a) Pre-blanking position, (b) post-blanking position (D_p = punch diameter, D_b = blank diameter)

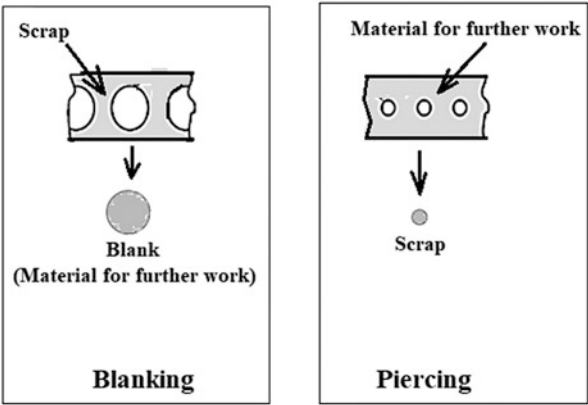


Fig. 10.5 Blanking and piercing operations

10.4.2 Blanking Versus Piercing

Blanking and *piercing* are shearing operations in which a punch and die are used to produce parts from metal sheet stock. *Blanking* produces the outside features of the component, while *piercing* produces holes or internal shapes. The *piercing* operation is basically the same as the blanking operation, but the cut-out part is a scrap (see Fig. 10.5).

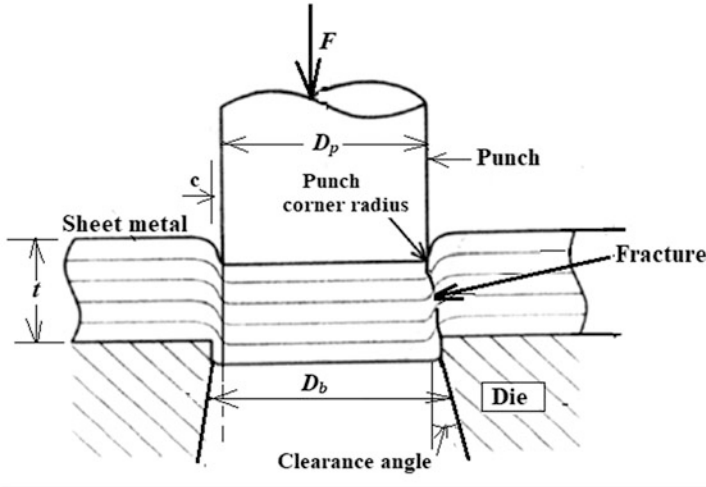


Fig. 10.6 The geometric design parameters in blanking

Figure 10.6 illustrates the geometric design parameters (clearance c , and punch radius, and sheet thickness t) in a blanking/piercing operation. The clearance (c) and the punching force (F), in blanking/piercing, can be determined by using Eqs. 10.1 and 10.2 (see Examples 10.6–10.9).

In blanking operation, the die diameter is the same as the blank diameter (D_b) (see Fig. 10.6).

The punch diameter (D_p) can be determined as follows (see Fig. 10.6):

$$D_p + 2c = D_b \quad (10.3)$$

or

$$D_p = D_b - 2c \quad (10.4)$$

Thus, it is established that for blanking:

$$\text{Blanking die diameter} = D_b \quad (10.5)$$

$$\text{Blanking punch diameter} = D_b - 2c \quad (10.6)$$

In a punching operation, the punch and die sizes are determined on the basis of hole diameter D_h , as follows (Groover, 2014):

$$\text{Piercing die diameter} = D_h + 2c \quad (10.7)$$

$$\text{Piercing punch diameter} = D_h \quad (10.8)$$

where D_h is the hole diameter and c is the clearance. The significance of Eqs. 10.4–10.7 is illustrated in Examples 10.10 and 10.11.

10.5 Dies for Blanking and Piercing

10.5.1 *Progressive Die for Blanking and Piercing*

A blanking/piercing operation is generally performed by the use of a *progressive die*. Modern day press tool shops are using progressive dies to improve productivity (Arora et al., 2022). In a progressive die, each tool/station can perform only a single operation; it means that for multiple operations, the progressive dies are designed with two or more stations (e.g. one station for blanking and the other station for piercing (see Fig. 10.7). The force required in a progressive die is taken as the greater of the blanking or piercing force.

10.5.2 *Compound Dies for Blanking and Piercing*

A *compound die* enables us to perform several operations (e.g. blanking, piercing) on the same strip in one stroke at one station; it means that a compound die is a type of die set that has multiple shearing operations built into a single tool/station (see Fig. 10.8). For example, both blanking and piercing operations can be performed at one station by using a compound die. It has been reported that a blanking/forming

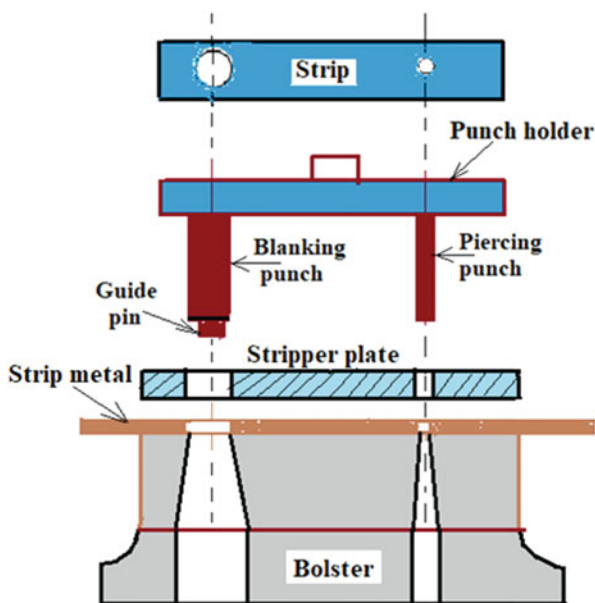


Fig. 10.7 Schematic of a progressive die for blanking and piercing operations

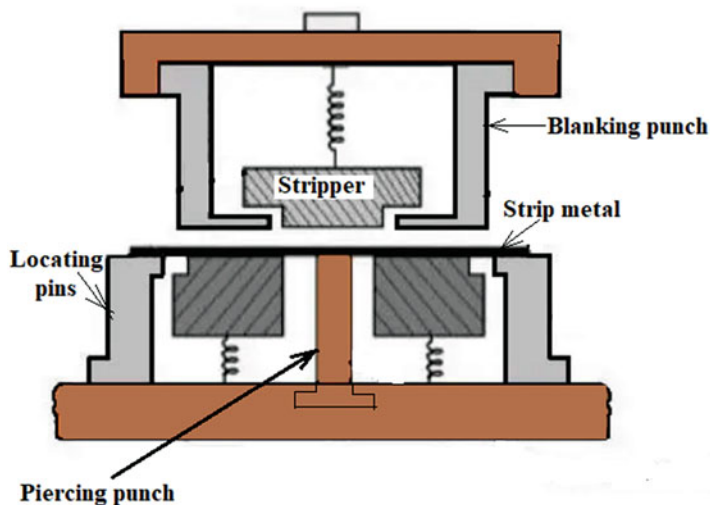


Fig. 10.8 Compound die—working principle

compound die can be used to fabricate micro parts of thin metal foils by laser shock waves (Wang et al., 2015). The force required in a compound die is the sum of the forces required for blanking and piercing i.e.,

$$\text{Force required in a compound die} = \text{Blanking force} + \text{Piercing force} \quad (10.9)$$

The significance of Eq. 10.9 is illustrated in Examples 10.13–10.15.

10.6 Worked Numerical Examples

Example 10.1: Determining the Clearance for a Shearing Operation

A cold-rolled mild steel sheet that is 4-mm thick is required to be sheared. Determine the clearance for the shearing operation.

Solution

By reference to Table 10.1, $A_c = 0.075$ mm. $t = 4$ mm, $c = ?$

By using Eq. 10.1,

$$c = A_c \cdot t = 0.075 \times 4 = 0.3 \text{ mm}$$

The clearance = 0.3 mm.

Example 10.2: Determining the Clearance When the Clearance Allowance (A_c) Is Unknown

A length of 8 cm is required to be sheared from a 5-mm-thick titanium sheet. Determine the clearance for the shearing operation.

Solution

$$t = 5 \text{ mm}, A_c = ?, c = ?$$

Since the clearance is unknown, we refer to the rule $c = 2\text{--}10\%$ of thickness (see Sect. 10.3.1).

Thus,

$$c = 5\% \text{ of } t = 0.05 \times 5 = 0.25 \text{ mm}$$

Example 10.3: Determining the Shearing Force for a Metal Shearing Operation

By using the data in Example 10.2, determine the shearing force for the operation.

Solution

$$L = 8 \text{ cm} = 80 \text{ mm}, t = 5 \text{ mm}, \sigma_{\text{UTS}} = 520 \text{ MPa (from Table 3.1, Chap. 3)}, F_s = ?$$

By using Eq. 10.2,

$$F_s = 0.7\sigma_t \cdot t \cdot L = 0.7 \times 520 \times 5 \times 80 = 145,600 \text{ N} = 145.6 \text{ kN}$$

The shearing force = 145.6 kN.

Example 10.4: Determining the Shearing Force When the Length of Cut Is Not Stated

A 4-m-long brass strip is to be sheared into two pieces each of length 2 m, as shown in Fig. 10.9. The sheet thickness is 3 mm. The tensile strength of the material is 400 MPa. Determine the shearing force for the operation.

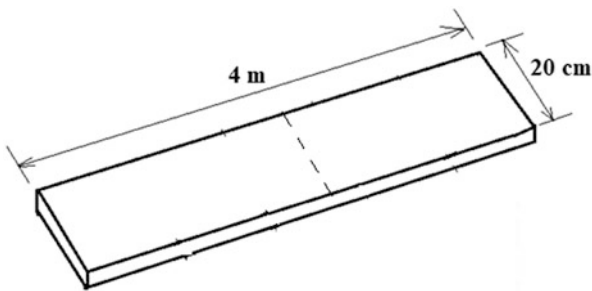


Fig. 10.9 Brass sheet that is to be sheared into two pieces of equal length for Example 10.4

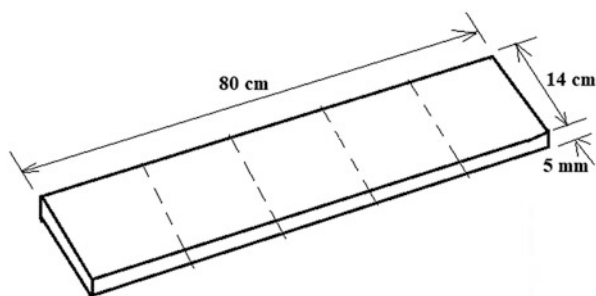


Fig. 10.10 Brass sheet that is to be sheared into five pieces of equal length for Example 10.5

Solution

Length to be sheared = width of the sheet = $L = 20 \text{ cm} = 200 \text{ mm}$, $t = 3 \text{ mm}$,
 $\sigma_{\text{UTS}} = 400 \text{ MPa}$

$$F_s = 0.7\sigma_t \cdot t \cdot L = 0.7 \times 400 \times 3 \times 200 = 168,000 \text{ N} = 168 \text{ kN}$$

The shearing force = 168 kN.

Example 10.5: Determining the Shearing Force for a Multi-stroke Job Shown by a Sketch

An 80-cm long brass sheet is required to be sheared into five pieces each of equal length, as shown in Fig. 10.10. Determine the total shearing force for the job.
 $\sigma_{\text{UTS}} = 400 \text{ MPa}$.

Solution

Total length sheared = $L = 4 \times 14 \text{ cm} = 56 \text{ cm} = 560 \text{ mm}$, $t = 5 \text{ mm}$,
 $\sigma_{\text{UTS}} = 400 \text{ MPa}$

$$F_s = 0.7\sigma_t \cdot t \cdot L = 0.7 \times 400 \times 5 \times 560 = 784,000 \text{ N} = 784 \text{ kN}$$

Example 10.6: Determining the Clearance, and the Length of Cut for a Blanking Operation

A round disk of 130-mm diameter is to be blanked from a cold-rolled mild-steel strip that is 4-mm thick with shear strength of 315 MPa. Determine the clearance, and the length of cut.

Solution

$D_b = 130 \text{ mm}$, $t = 4 \text{ mm}$, $A_c = 0.075$ (see Table 10.1), $\tau = 315 \text{ MPa}$, $c = ?$, $L = ?$

By using Eq. 10.1,

$$c = A_c \cdot t = 0.075 \times 4 = 0.3 \text{ mm}$$

Since the disk is round, the length of cut is equal to the circumference of the circle i.e.,

$$\text{Length of cut} = L = \text{Circumference} = 2\pi r = \pi \cdot D_b = 3.143 \times 130 = 408.6 \text{ mm}$$

Example 10.7: Determination of the Blanking Force for a Round Blank

By using the data in Example 10.6, determine the force required for the blanking operation.

Solution

$$t = 4 \text{ mm}, \tau = 315 \text{ MPa} = 315 \text{ N/mm}^2, L = 102.05 \text{ mm}, F = ?$$

By using Eq. 10.2,

$$F_s = \tau \cdot t \cdot L = 315 \times 4 \times 102.05 = 128,583 \text{ N}$$

The blanking force = 128,583 N

Example 10.8: Determination of the Piercing Force

A hole of 2-mm diameter is to be pierced from an aluminum alloy-2024 strip that is 1.5-mm thick with shear strength = 250 MPa. Determine the piercing force.

Solution

$$D_h = 2 \text{ mm}, t = 1.5 \text{ mm}, \tau = 250 \text{ MPa}, F = ?$$

$$\text{Length of cut} = L = \text{Circumference} = 2\pi r = \pi \cdot D_h = 3.143 \times 2 = 6.28 \text{ mm}$$

$$F_s = \tau \cdot t \cdot L = 250 \times 1.5 \times 6.28 = 2357.14 \text{ N}$$

Piercing force = 2357.14 N

Example 10.9: Determination of the Blanking Force for a Square Blank

A square disk (40 mm × 40 mm) is to be blanked from a cold-rolled mild-steel strip that is 4-mm thick with shear strength of 315 MPa. Determine the blanking force.

Solution

$$\text{Blank dimensions: } 40 \text{ mm} \times 40 \text{ mm}, t = 4 \text{ mm}, \tau = 315 \text{ MPa}, F = ?$$

Since the disk is square, the length of cut is equal to the perimeter of the square shape i.e.,

$$\text{Length of cut} = L = \text{perimeter} = 4 \times 40 = 160 \text{ mm}$$

$$F_s = \tau \cdot t \cdot L = 315 \times 4 \times 160 = 201,600 \text{ N}$$

Example 10.10: Designing a Blanking Die for a Round Blank

By using the data in Example 10.6, design the blanking die for the blanking operation.

Solution

$$D_b = 130 \text{ mm}, t = 4 \text{ mm}, c = 0.3 \text{ mm}.$$

By using Eqs. 10.5 and 10.6,

$$\text{Blanking die diameter} = D_b = 130 \text{ mm}$$

$$\text{Blanking punch diameter} = D_b - 2c = 130 - (2 \times 0.3) = 129.4 \text{ mm}$$

Example 10.11: Determining the Piercing Die Diameter and the Piercing Punch Diameter

A hole of 4-mm diameter is to be pierced from an aluminum alloy-2024 strip that is 3-mm thick with shear strength = 250 MPa. Determine the piercing die diameter and the punch diameter.

Solution

$$D_h = 4 \text{ mm}, t = 3 \text{ mm}, A_c = 0.06 \text{ (from Table 10.1)}, \tau = 250 \text{ MPa}.$$

$$c = A_c \cdot t = 0.06 \times 3 = 0.18 \text{ mm}$$

By using Eqs. 10.7 and 10.8,

$$\text{Piercing die diameter} = D_h + 2c = 4 + (2 \times 0.18) = 4.36 \text{ mm}$$

$$\text{Piercing punch diameter} = D_h = 4 \text{ mm}$$

Example 10.12: Determining the Punching Force in US Customary Units

A hole of 0.375-in. diameter is to be pierced from an AISI-1020 cold-rolled steel strip that is 0.03-in. thick with shear strength = 52,000 lbs./in.². Determine the punching force in tons (US customary units).

Solution

$$D_h = 0.375 \text{ in.}, t = 0.03 \text{ in.}, \tau = 52,000 \text{ lbs./in.}^2, F = ?$$

$$\text{Length of cut} = L = \pi \cdot D_h = 3.143 \times 0.375 = 1.178 \text{ in}$$

$$F_s = \tau \cdot t \cdot L = 52,000 \times 0.03 \times 1.178 = 1837.68 \text{ lbs.}$$

$$2000 \text{ lbs} = 1 \text{ ton (in US Customary Units)}$$

$$\text{Force} = \frac{1837.68}{2000} = 0.9188 \text{ ton} \cong 1 \text{ ton}$$

Example 10.13: Designing a Compound Die for Manufacturing a Washer

A compound die will be used to blank and pierce a washer out of a soft stainless steel sheet stock that is 4-mm thick with shear strength of 620 MPa. The dimensions of the washer are as follows: OD = 55 mm, ID = 20 mm. Determine the (a) die and the punch diameters for the blanking operation and (b) the die and the punch diameters for the piercing operation.

Solution

$$A_c = 0.06 \text{ (Table 10.1)}, t = 4 \text{ mm}, D_b = 55 \text{ mm}, D_h = 20 \text{ mm}.$$

$$c = A_c \cdot t = 0.06 \times 4 = 0.24 \text{ mm}$$

(a) Blanking die diameter = $D_b = 55 \text{ mm}$

$$\text{Blanking punch diameter} = D_b - 2c = 55 - (2 \times 0.24) = 54.52 \text{ mm}$$

(b) Piercing die diameter = $D_h + 2c = 20 + (2 \times 0.24) = 20.48 \text{ mm}$

$$\text{Piercing punch diameter} = D_h = 20 \text{ mm}$$

Example 10.14: Determining the Blanking and Piercing Forces for Manufacturing Washer

By using the data in Example 10.13, determine the: (a) blanking force and (b) piercing force. The shear strength of the material is 620 MPa.

Solution

$$t = 4 \text{ mm}, D_b = 55 \text{ mm}, D_h = 20 \text{ mm}, \tau = 620 \text{ MPa}.$$

(a) Length of cut for blanking = $\pi D_b = 3.143 \times 55 = 172.86 \text{ mm}$

$$\text{Blanking force} = F_s = \tau \cdot t \cdot L = 620 \times 4 \times 172.86 = 428,705.2 \text{ N}$$

(b) Length of cut for piercing = $\pi D_h = 3.143 \times 20 = 62.86 \text{ mm}$

$$\text{Piercing force} = F_s = \tau \cdot t \cdot L = 620 \times 4 \times 62.86 = 155,892.8 \text{ N}$$

Example 10.15: Determining the Required Forces for Compound and Progressive Dies

By using the data in Examples 10.13 and 10.14, determine the minimum press force required for manufacturing the washer. If a progressive punch press were used instead of the compound die, what should the minimum press force required to produce the same washer?

Table 10.2 The strength data for C20 steel

Materials	Shear strength, MPa (annealed)	Shear strength, MPa (un-annealed)	Tensile strength, (MPa)
C10 steel	235	314	400
C20 steel	294	390	550

Solution

Blanking force = 428,705.2 N, Piercing force = 155,892.8 N.

By using Eq. 10.9,

Force required for compound die = Blanking force + Piercing force = 428,705.2 + 155,892.8

Force required for the compound die = 584,598 N = 584.6 kN

Force required for the progressive die = 428,705.2 N = 428.71 kN

Example 10.16: Designing a Blanking Die and Computing Blanking Force for Circular Disc

Consider a C20 steel (annealed) sheet stock with dimensions $30 \times 30 \times 1.5$ mm. The clearance allowance is 0.0823. It is required to blank a circular disc of 20 mm diameter from the sheet stock. Determine the punch size, the die size, and the punching force. The strength data of the steel is given in Table 10.2.

Solution

$$t = 1.5 \text{ mm}, A_c = 0.0823, D_b = 20 \text{ mm}, \tau = 294 \text{ MPa}.$$

$$c = A_c \cdot t = 0.0823 \times 1.5 = 0.123 \text{ mm}$$

$$\text{Blanking die diameter} = D_b = 20 \text{ mm}$$

$$\text{Blanking punch diameter} = D_b - 2c = 20 - (2 \times 0.123) = 19.75 \text{ mm}$$

$$\text{Length of cut for blanking} = L = \pi D_b = \pi \times 20 = 62.8 \text{ mm}$$

$$\text{Blanking force} = F_s = \tau \cdot t \cdot L = 294 \times 1.5 \times 62.8 = 27,694.8 \text{ N} = 27.695 \text{ kN}$$

Example 10.17: Designing a Piercing Die and Computing the Piercing Force

Refer to the data in Example 10.16. If a rectangular sheet with a circular hole of a 20 mm diameter is to be pierced from the sheet stock, determine the punch size, the die size, and the punching force.

Solution

$$c = 0.123 \text{ mm}, D_h = 20 \text{ mm}, \tau = 294 \text{ MPa}$$

$$\text{Piercing die diameter} = D_h + 2c = 20 + (2 \times 0.123) = 20.246 \text{ mm}$$

$$\text{Piercing punch diameter} = D_h = 20 \text{ mm}$$

$$\text{Length of cut for piercing} = L = \pi D_h = \pi \times 20 = 62.8 \text{ mm}$$

$$\text{Piercing force} = F_s = \tau \cdot t \cdot L = 294 \times 1.5 \times 62.8 = 27,694.8 \text{ N} = 27.695 \text{ kN}$$

Example 10.18: Determining the Blanking and Piercing Forces for Un-annealed Steel

Refer to the data in Example 10.16. If the starting material were un-annealed, what would have been the punching forces for: (a) blanking, and (b) piercing?

Solution

$$D_b = 20 \text{ mm}, D_h = 20 \text{ mm}, \tau = 390 \text{ MPa.}$$

$$(a) \text{ Blanking force} = F_s = \tau \cdot t \cdot L = 390 \times 1.5 \times 62.8 = 36,738 \text{ N} = 36.738 \text{ kN}$$

$$(b) \text{ Piercing force} = F_s = \tau \cdot t \cdot L = 390 \times 1.5 \times 62.8 = 36,738 \text{ N} = 36.738 \text{ kN}$$

Example 10.19: Designing a Blanking Die and Computing the Force to Blank a Square Part

You are requested to design a blanking die to blank the square part with each side = 70 mm. The material is 4-mm-thick cold-rolled mild steel. Determine the dimensions of the blanking die and the punch for the job. Also determine the punching force, if the shear strength of the material is 440 MPa.

Solution

$A_c = 0.075$, $t = 4 \text{ mm}$, $\tau = 440 \text{ MPa}$, Blanking die size = ? Punch size = ?
Punching force = ?

$$c = A_c \cdot t = 0.075 \times 4.0 = 0.3 \text{ mm}$$

Blanking die: The dimensions are the same as for the square part with each side = 70 mm.

Blanking punch: The 70-mm length dimension = $70 - 2c = 70 - (2 \times 0.3) = 69.4 \text{ mm}$.

Each side of the blanking punch must have dimension = 69.4 mm.

$$\text{Length of cut} = L = \text{perimeter} = 4 \times 70 = 280 \text{ mm}$$

$$\text{Punching force} = F_s = \tau \cdot t \cdot L = 440 \times 4 \times 280 = 492,800 \text{ N} = 492.8 \text{ kN}$$

Example 10.20: Designing a Blanking Die for a Part with Complex Geometry

You are requested to design a blanking die to blank the part, as shown in Fig. 10.11. The material is 3-mm-thick aluminum alloy 6061 ST. Determine the dimensions of the blanking die and the blanking punch for the job.

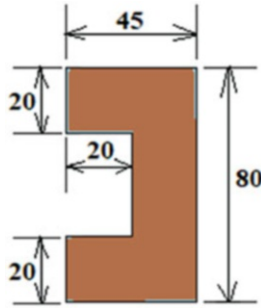


Fig. 10.11 Blanked part for Example 10.20

Solution

$t = 3.0$ mm, $A_c = 0.06$ (from Table 10.1), Blanking die size = ?, Punch size = ?

$$c = A_c \cdot t = 0.06 \times 3.0 = 0.18 \text{ mm}$$

Blanking die: The dimensions are the same as for the part shown in Fig. 10.11.
Blanking punch:

The 80 - mm length dimension = $80 - 2c = 80 - (2 \times 0.18) = 79.64$ mm

The 45 - mm width dimension = $45 - (2 \times 0.18) = 44.64$ mm

The top and bottom 20 - mm extension widths = $20 - (2 \times 0.18) = 19.64$ mm

The 20-mm inset dimension remains the same.

Example 10.21: Determining the Blanking Force to Blank a Part with Complex Geometry

Determine the blanking force to blank the part shown in Fig. 10.11. The tensile strength of the material is 310 MPa.

Solution

$t = 3.0$ mm, $\sigma_{UTS} = 310$ MPa, $F_s = ?$

Length of cut = $L = \text{perimeter} = 80 + 45 + 20 + 20 + 40 + 20 + 20 + 45 = 290$ mm

Punching force = $F_s = 0.7 \sigma_{UTS} \cdot t \cdot L = 0.7 \times 310 \times 3 \times 290 = 188,790$ N = 188.80 kN

Example 10.22: Determining the Force When Blanking and Piercing Occur Simultaneously

By using the data in Example 10.13, determine the minimum press force to produce the washer, if the blanking and piercing operations occur simultaneously in the compound die.

Solution

$$t = 4 \text{ mm}, \tau = 620 \text{ MPa}, D_b = 55 \text{ mm}, D_h = 20 \text{ mm}.$$

$$\text{Length of cut} = L = \pi \cdot D_b + \pi \cdot D_h = 3.143 \times (55 + 20) = 235.62 \text{ mm}$$

$$\text{Press force} = F_s = \tau \cdot t \cdot L = 620 \times 4 \times 235.62 = 584,336 \text{ N} = 584.34 \text{ kN}$$

Example 10.23: Determining the Clearance for Shearing with Reference to Sketch

It is required to perform shearing and blanking operations by first cutting many square pieces (145 mm $\times$ 145 mm) out of the sheet stock shown in Fig. 10.12; and then producing blanks (each blank diameter = 137 mm). Each squared sheared piece will be used to produce a blank. The sheet material is naval brass (annealed). What clearance must be allowed in the shearing or blanking operation?

Solution

$$A_c = 0.06 \text{ (from Table 10.1)}, t = 2 \text{ mm (from Fig. 10.12)}, c = ?$$

$$c = A_c \cdot t = 0.06 \times 2.0 = 0.12 \text{ mm}$$

Example 10.24: Determining the Shearing Force with Reference to a Sketch

By using the data in Example 10.23, determine the shearing force to cut a square piece (145 mm $\times$ 145 mm) out of the sheet stock shown in Fig. 10.12.

Solution

$$\text{Brass } \sigma_{\text{UTS}} = 330 \text{ MPa (from Table 3.1)}, L = 145 \text{ mm (from Fig. 10.12)}, \\ t = 2 \text{ mm}, F_s = ?$$

$$F_s = 0.7\sigma_{\text{UTS}} \cdot t \cdot L = 0.7 \times 330 \times 2 \times 145 = 66,990 \text{ N} \cong 67 \text{ kN}$$

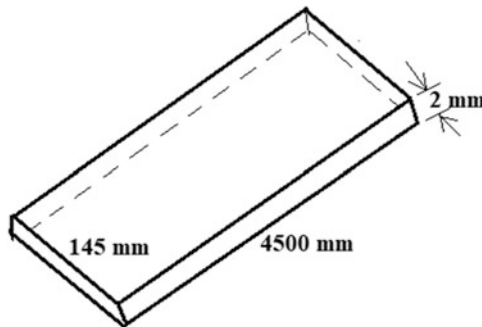


Fig. 10.12 Brass sheet for Example 10.23

Example 10.25: Determining the Dimensions of the Sheared Piece with Reference to Sketch

Refer to the data in Examples 10.23 and 10.24. What are the dimensions of the sheared piece?

Solution

By reference to Fig. 10.12 and by knowing that there is no significant change in thickness in sheet metal forming (here, shearing),

The dimensions of the sheared piece are $145 \text{ mm} \times 145 \text{ mm} \times 2 \text{ mm}$.

Example 10.26: Calculating the Volumes of the Sheared Piece and the Sheet Stock

By using the data in Examples 10.23–10.25, calculate the volume of the given starting (stock) sheet and the volume of the sheared piece.

Solution

$$\text{Volume of the sheet stock} = 4500 \text{ mm} \times 145 \text{ mm} \times 2 \text{ mm} = 1,305,000 \text{ mm}^3$$

$$\text{Volume of the sheared piece} = 145 \text{ mm} \times 145 \text{ mm} \times 2 \text{ mm} = 42,050 \text{ mm}^3$$

Example 10.27: Determining the Number of Pieces That Can Be Sheared Out of Sheet Stock

By using the data in Example 10.26, determine the number of square pieces can be sheared out of the sheet stock.

Solution

$$\begin{aligned} \text{Number of pieces that can be sheared} &= \frac{\text{Volume of the sheet stock}}{\text{Volume of the sheared piece}} \\ &= \frac{1,305,000}{42,050} = 31 \end{aligned}$$

The number of sheared pieces = 31

Example 10.28: Determining the Dimensions of the Blank and Drawing the Sketches

Refer to the data in Examples 10.23–10.27. It is required to perform blanking operation on each and every sheared piece to produce blanks with each blank diameter = 137 mm. What are the dimensions of each blank? Draw the sketch showing the dimensions of the blank. Also draw the sketch of the sheared piece from which the blank is produced.

Solution

Blank diameter = 137 mm, thickness = 2 mm.

The sketches of the sheared piece and the blank with dimensions are shown in Fig. 10.13.

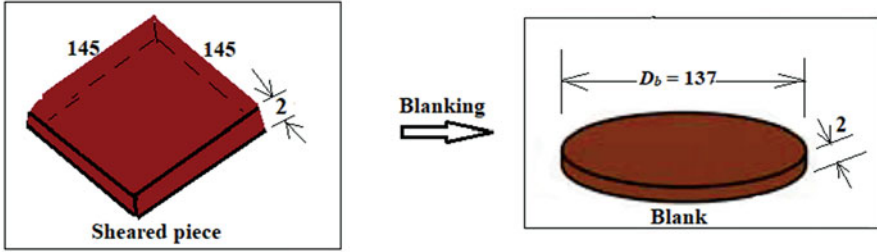


Fig. 10.13 The sketches of the sheared piece and the blank with dimensions for Example 10.28 (dimensions in mm)

Example 10.29: Designing a Blanking Die and Computing the Force with the Aid of Sketch

With reference to the sketch in Fig. 10.13 and by using the data in Examples 10.23 and 10.24, determine the blanking force. Also design the die for the blanking operation.

Solution

By reference to Fig. 10.13, $D_b = 137$ mm, $t = 2$ mm, $\sigma_{UTS} = 330$ MPa, $c = 0.12$ mm.

$$\text{Length of cut} = L = \pi \cdot D_b = 3.143 \times 137 = 430.4 \text{ mm}$$

$$\text{Blanking force} = F_s = 0.7 \cdot \sigma_{UTS} \cdot t \cdot L = 0.7 \times 330 \times 2 \times 430.4 = 198,844.8 \text{ N}$$

$$\text{Blanking die diameter} = D_b = 137 \text{ mm}$$

$$\text{Blanking punch diameter} = D_b - 2c = 137 - (2 \times 0.12) = 136.76 \text{ mm}$$

Example 10.30: Determining the Volume of Scrap after a Blanking Operation

By using the data in Examples 10.26–10.28, determine the volume of the scrap after each blanking operation.

Solution

$$\text{Volume of each sheared piece} = 42,050 \text{ mm}^3$$

The volume of each blank can be determined by reference to Fig. 10.13, as follows.

$$\text{Volume of a blank} = \frac{\pi}{4} \cdot D_b^2 \cdot t = 0.785 \times (137)^2 \times 2 = 29,467.33 \text{ mm}^3$$

$$\begin{aligned} \text{Volume of scrap after a blanking operation} \\ = \text{Volume of a sheared piece} - \text{volume of a blank} \end{aligned}$$

$$\begin{aligned} \text{Volume of scrap after a blanking operation} &= 42,050 \text{ mm}^3 - 29,467.33 \text{ mm}^3 \\ &= 12,582.67 \text{ mm}^3 \end{aligned}$$

Example 10.31: Determining the Mass of Each Scrap Piece after a Blanking Operation

By using the data in Example 10.30, determine the mass of each scrap piece after a blanking operation.

Solution

Volume of the brass scrap after each blanking operation = $12,582.67 \text{ mm}^3$.

$$\begin{aligned}\text{Density of brass} &= 8.4 \text{ g/cm}^3 = 8.4 \times \frac{1 \text{ g}}{(1 \text{ cm})^3} = 8.4 \times \frac{1 \text{ g}}{(10 \text{ mm})^3} \\ &= 8.4 \times \frac{1 \text{ g}}{1000 \text{ mm}^3} = 0.0084 \text{ g/mm}^3\end{aligned}$$

$$\text{Mass} = \text{density} \times \text{volume}$$

$$\begin{aligned}\text{Mass of each brass scrap after a blanking operation} &= 0.0084 \text{ g/mm}^3 \\ &\times 12,582.67 \text{ mm}^3 = 105.7 \text{ g}.\end{aligned}$$

Example 10.32: Determining the Total Scrap Mass after Completion of the Blanking Job

By using the data in Examples 10.23–10.31, determine the total scrap mass on completion of the job.

Solution

Mass of each brass scrap after a blanking operation = 105.7 g.

The number of sheared pieces = 31

Since each sheared piece is used to produce a blank, the number of blanks = 31

$$\text{Total scrap mass on completion of the job} = 105.7 \text{ g} \times 31 = 3276.5 \text{ g} = 3.276 \text{ kg}$$

Example 10.33: Determining the Total Price of the Scrap after Completion of Blanking Job

By using the data in Example 10.32, determine the total price of the scrap metal after completion of the blanking job. Assume the price of brass scrap in USA = \$ 2.0/lb.

Solution

Price of brass scrap = \$ 2.0/lb. The total scrap mass on completion of the job = 3.276 kg.

$$0.4536 \text{ kg} = 1 \text{ lb}$$

$$3.276 \text{ kg} = \frac{3.276}{0.4536} \text{ lb} = 7.22 \text{ lbs.}$$

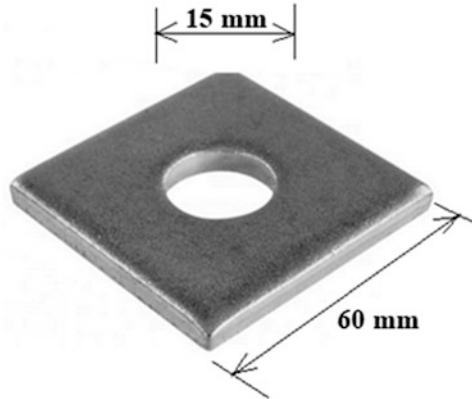


Fig. 10.14 Square washer with dimensions for Example 10.34

Total scrap mass on completion of the job = 3.276 kg = 7.22 lbs.

Total price of the brass scrap = \$ 2.0 $\times$ 7.22 = \$ 14.44

Example 10.34: Designing a Compound Die for a Square Washer

A compound die will be used to blank and pierce a washer out of a soft stainless steel sheet stock that is 4.5-mm thick. The dimensions of the washer are shown in Fig. 10.14. Determine the (a) blanking die and the punch sizes and (b) piercing die and the punch diameters.

Solution

$$t = 4.5 \text{ mm}, A_c = 0.06,$$

$$c = A_c \cdot t = 0.06 \times 4.5 = 0.27 \text{ mm}$$

(a) Blanking

Blanking die: The dimensions are the same as for the square part with each side = 60 mm.

Blanking punch: The 60 - mm length dimension = $60 - 2c = 60 - (2 \times 0.27) = 59.46 \text{ mm}$.

Each side of the blanking die must have dimension = 60 mm

Each side of the blanking punch must have dimension = 59.46 mm

(b) Piercing

$$\text{Piercing die diameter} = D_h + 2c = 15 + (2 \times 0.27) = 15.54 \text{ mm}$$

$$\text{Piercing punch diameter} = D_h = 15 \text{ mm}$$

Example 10.35: Determining the Blanking Force for a Square Washer

By using the data in Example 10.34, determine the blanking force, if the ultimate tensile strength of the material is 760 MPa.

Solution

$$t = 4.5 \text{ mm}, \sigma_{\text{UTS}} = 760 \text{ MPa}.$$

$$L = \text{length of cut for blanking} = \text{perimeter} = 4 \times 60 = 240 \text{ mm}$$

$$\begin{aligned} \text{Blanking force} = F_s &= 0.7 \sigma_{\text{UTS}} \cdot t \cdot L = 0.7 \times 760 \times 4.5 \times 240 = 574,560 \text{ N} \\ &= 574.56 \text{ kN} \end{aligned}$$

Example 10.36: Determining the Press Force to Produce a Square Washer

By using the data in Example 10.34, determine the minimum press force to produce the square washer.

Solution

$$t = 4.5 \text{ mm}, \sigma_{\text{UTS}} = 760 \text{ MPa}, D_h = 15 \text{ mm}.$$

$$\text{Length of cut for piercing} = L = \pi \cdot D_h = \pi \times 15 = 47.124 \text{ mm}$$

$$\begin{aligned} \text{Piercing force} = F_s &= 0.7 \sigma_{\text{UTS}} \cdot t \cdot L = 0.7 \times 760 \times 4.5 \times 47.124 = 112,814.86 \text{ N} \\ &= 112.815 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{The minimum press force to produce the washer} &= 574.56 + 112.815 \\ &= 687.375 \text{ kN} \end{aligned}$$

Questions and Problems

- 10.1. (a) Differentiate between metal plates and metal sheets.
(b) Distinguish between open shearing, blanking, and piercing.
- 10.2. (a) Explain the open shearing operation with the aid of sketches.
(b) Highlight the advantages and limitations of shearing operation.
- 10.3. (a) Describe the blanking operation with the aid of sketches.
(b) Differentiate between blanking and piercing with the aid of a sketch.
(c) Draw a diagram showing the geometric design parameters in blanking.
- 10.4. Distinguish between a progressive die and a compound die used in sheet metal forming.
- P10.5. Name the type of dies (A and B), and write the missing labels in Fig. 10.15.
- P10.6. A 5-mm-thick aluminum alloy 5052S sheet is to be sheared. Determine the clearance for the shearing operation.
- P10.7. A 4-mm-thick nickel sheet is to be sheared. Determine the clearance for the shearing operation.

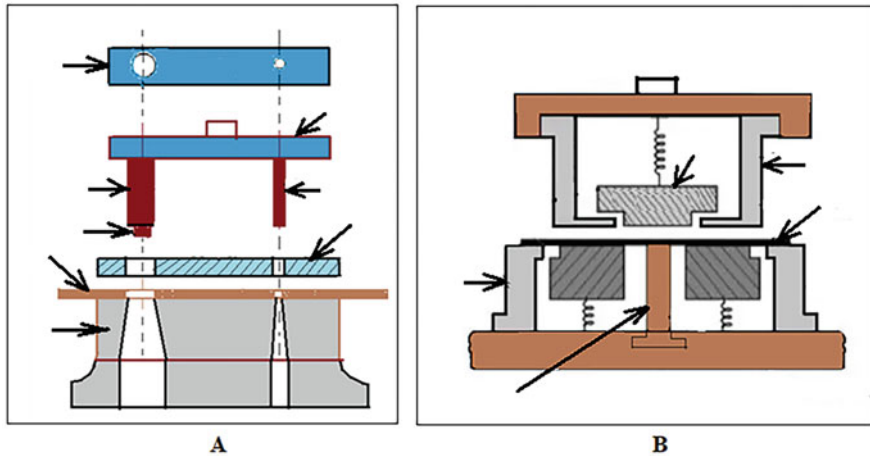
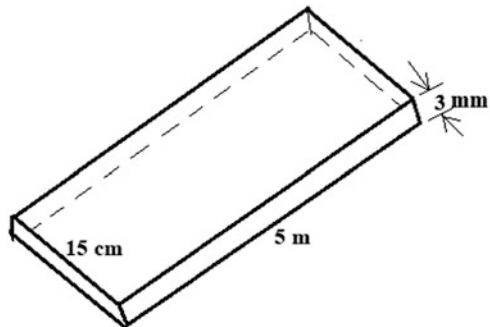


Fig. 10.15 Two types of die used in sheet metal forming for Problem P-10.5

- P10.8. A length of 10 cm is required to be sheared from a 3-mm-thick brass sheet. Determine the shearing force. The shear strength of the free-machining brass is 270 MPa.
- P10.9. A 3.5-mm-thick disc of 140-mm diameter is to be blanked from an aluminum alloy 6061 strip. Determine the die diameter, the punch diameter, and the blanking force.
- P10.10. A hole of 3-mm diameter is to be pierced from a brass strip that is 2-mm thick. Determine the piercing die diameter and the punch diameter.
- P10.11. A compound die will be used to blank and pierce a washer out of an aluminum alloy 1100S sheet stock that is 5-mm thick. The dimensions of the washer are as follows: OD = 45 mm, ID = 10 mm. Determine the (a) blanking die and the punch diameters and (b) piercing die and the punch diameters.
- P10.12. A hole of 0.4-in. diameter is to be pierced from an AISI-1020 cold-rolled steel strip that is 0.02-in. thick with shear strength = 52,000 lbs./in.². Determine the punching force in tons (US customary units).
- P10.13. By using the data in P10.11, determine the minimum press force required to produce the washer, if the shear strength of the sheet material is 62 MPa.
- P10.14. Multiple blanking operations on a large aluminum strip resulted in 4 kg scrap. Determine the price of the scrap, if the price of aluminum scrap in USA is assumed to be \$ 0.20/lb.
- P10.15. By using the data in P10.13, determine the minimum press force to produce the washer, if the blanking and piercing operations occur simultaneously in the compound die.

Fig. 10.16 Brass sheet for P10.16



- P10.16. It is required to perform shearing and blanking operations by first cutting many square pieces ($15\text{ cm} \times 15\text{ cm}$) out of the sheet stock shown in Fig. 10.16; and then producing blanks (each blank diameter = 140 mm). Each squared sheared piece will be used to produce a blank. The sheet material is naval brass (annealed). What clearance must be allowed in the shearing or blanking operation?
- P10.17. By using the data in P10.16, determine: (a) the shearing force to cut a square piece ($15\text{ cm} \times 15\text{ cm}$) out of the sheet stock shown in Fig. 10.16, (b) the dimensions of the sheared piece, (c) the volume of the given starting (stock) sheet and the volume of the sheared piece, (d) the number of square pieces can be sheared out of the sheet stock, (e) dimensions of each blank, (f) blanking force, (g) blanking die and punch diameters, (h) the volume of the scrap after each blanking operation, (i) mass of each scrap piece after a blanking operation., (j) the total scrap mass on completion of the job, and (k) the total price of the scrap metal after completion of the blanking job, assuming that the price of brass scrap in USA = \$ 2.0/lb.
- 10.18. (MCQs). Select the most appropriate answers for the following statements.
- Which operation is used to produce holes in a blank?
(a) Shearing, (b) blanking, (c) piercing, (d) bending.
 - Which operation involves cutting off smaller pieces with an open cutting path?
(a) Shearing, (b) blanking, (c) piercing, (d) bending.
 - In which operation the closed cut-out part is used for further press-work?
(a) Shearing, (b) blanking, (c) piercing, (d) bending.
 - Which type of die is the most suitable for producing a washer?
(a) Blanking die, (b) progressive die, (c) piercing die, (d) forging die.

5. Which type of die facilitates simultaneous blanking and piercing operations?

- (a) Blanking die, (b) progressive die, (c) piercing die, (d) compound die.

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Chapter 11

Bending of Sheet Metal



Nomenclature

α'	included angle
α	bend angle
R	bend radius
F	bending force
F_h	holding force
K_{bf}	bending factor
w	width of the sheet metal in the direction of bend axis
t	sheet thickness
D	die-opening dimension
A_b	bend allowance
K_{ba}	stretching factor
$A_{b(\text{neutral})}$	bend allowance for neutral axis
S	degree of springback
α'_p	included angle of the punch
E	modulus of elasticity
σ_{ys}	yield strength
k	material factor
S_f	springback factor

Abbreviations

CR	cold rolled
MEMS	micro-electromechanical systems
MS	mild steel
SS	stainless steel

11.1 Introduction to Bending

The bending is a sheet-metal forming operation that involves deformation by applying force thereby causing the sheet-material to bend at an angle and form the anticipated shape. Press-bending is a key step in metal sheet manufacture (Corrieri, 2018). Bending, in sheet-metal work, involves the straining of the metal around a straight (neutral) axis. The neutral axis is the axis within the material where the stress and strain are zero. Sheet-metal bending often results in a ‘V’ or a ‘U’ shape. During bending, the metal on the outside of the neutral axis is stretched, whereas the metal on the inside of the neutral axis is compressed (see Fig. 11.1). A press brake is generally used as a tool (equipment) for bending metal sheets; the equipment uses a punch and a die to perform the bending operation (Benson, 1997).

Sheet-metal bending can be used for manufacturing automobile body panels, steel furniture, fridges, electronic casings, and brackets. For example, three-roll bending operations can be used for coil tubing or bar for refrigeration and cooling towers or to make 360° rings; they are also used for manufacturing the components of storage tanks, pressure vessels, race car bumpers, ornamental iron work, and hand-rail applications. Sheet metal bending is best suited for applications with lower production volumes or products using thin sheet metal for which casting is difficult to perform.

Bending is a key sheet-metal forming process commonly used in the fabrication of parts for diversified applications. Bending operations are generally performed using punches, dies, rolls, and other supporting systems to control the downward and upward movement of the bending tools, depending on the bending processes, i.e., V-bending, air bending, U-bending, edge bending, and the like. Bending operations are also used in the manufacture of micro-sheet metal products, such as electronics products and micro-electromechanical systems (MEMS). Typical examples of these products and systems include micro-electric-contacts, micro-electric switches, 3D sheet metal frames/housing for optical devices and micro-sensors, and 3D profiles for mechanical and thermo-mechanical sensors.

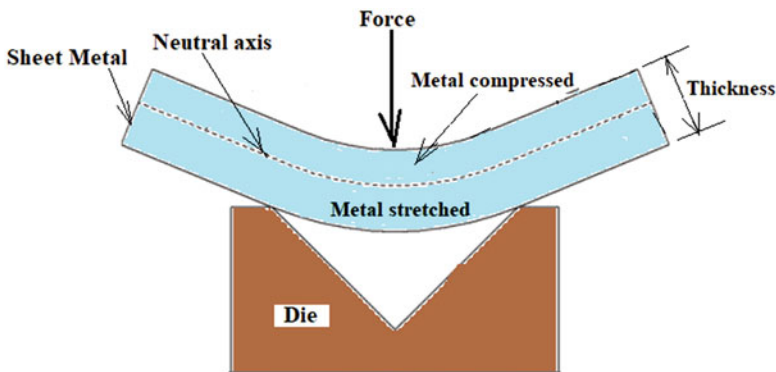


Fig. 11.1 Bending operation resulting in straining of the metal around the neutral axis

11.2 Methods of Sheet Metal Bending

11.2.1 The Five Methods of Bending

There are five commonly used methods of bending metal sheets: (1) V-bending, (2) air bending, (3) bottom bending (or bottoming), (4) edge bending (or wipe bending), and (5) roll bending. The five methods are briefly explained in the following sub-sections. Prior to explaining these bending methods, it is necessary to define the term *springback*. In simple terms, *springback* is a phenomenon that occurs when the material angularly tries to return to its original shape after being bent (Benson, 2014) (see Sect. 11.4 for details).

11.2.2 The V-Bending Operation

V-bending of metal sheets is the most commonly employed bending method. The v-bending method employs a V-shaped die and a V-shaped punch to bend metal sheets at desired angles (see Fig. 11.2a, b). The punch forces the metal sheet into the “V-shaped” groove in the die, thereby forming sheet metals with various bend angles; these angles may be acute, obtuse, or 90° bend angles, depending on the punch and die angle. The V-bending operation is performed by use of a press brake (Fig. 11.3a, b); it is ideal for low volume of production.

Recently (2021), Kuboki and coworkers reported a new V-bending method of sheet metal for forming diversified shapes and enhancing operability and safety (Kuboki et al., 2021). The new method, “asymmetric bending,” uses a punch and a

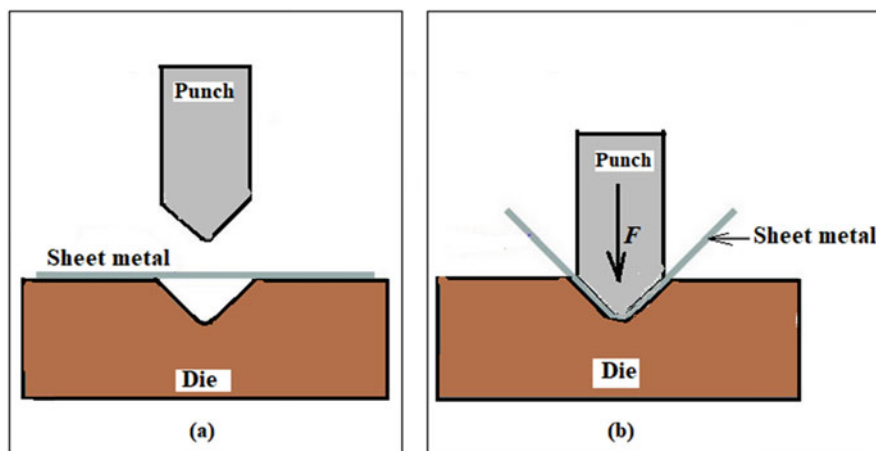


Fig. 11.2 V-bending of sheet metal; (a) pre-bending position, (b) post-bending position

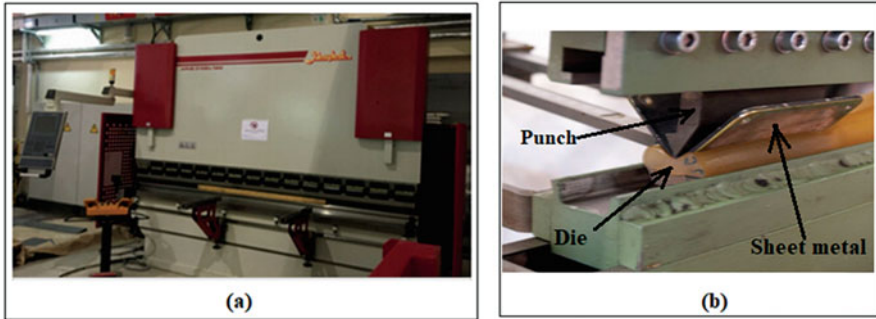


Fig. 11.3 A press brake (a), V-bending in action (b). (Courtesy: King Abdulaziz University, Dept. of Mech. Eng.)

pair of front and back dies. While the punch moves down for bending, the two dies move at different velocities so that the movement of the front side of the sheet metal can severely be restricted. The front side of the metal can be deformed with diversity in advance as it can escape from geometric interference with the bending tools (Kuboki et al., 2021).

11.2.3 The Air Bending Operation

Air bending is similar to the V-bending method. However, unlike the conventional v-bending process, the punch in the air bending method does not force the sheet into the bottom of the die-cavity. Although air bending relies on a V-shaped punch and a die to bend sheet metals, it leaves air (space) underneath the sheet, thereby allowing more bend angle control as compared to conventional v-bending (see Fig. 11.4).

11.2.4 Bottom Bending

In bottom bending (or *bottoming*), there is an angular clearance between the punch and die. For bottoming, the die angle should match the intended angle of the work piece to compensate a few degrees for *springback*. This is why there exists an 88° punch tooling to achieve a 90° angle in the bent sheet. For performing the bottom-bending operation, the sheet-metal is first bottomed against the die (Fig. 11.5a), then the radius of the punch is forced into the sheet-metal which achieves the angle of the punch (Fig. 11.5b). As the punch continues to apply pressure, the material is forced down (due to *springback*) to conform to the die angle (Fig. 11.5c).

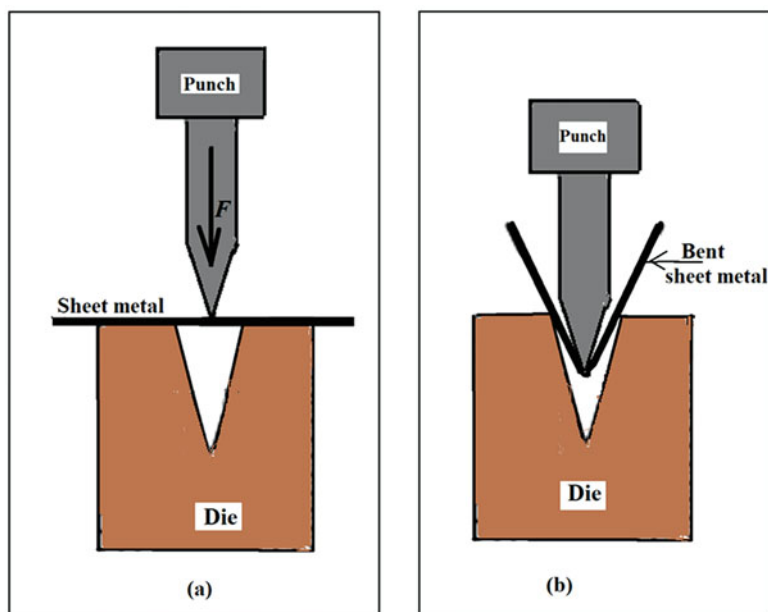


Fig. 11.4 Air bending of sheet metal—*principle of operation*. (a) start of the bending operation, (b) end of the operation

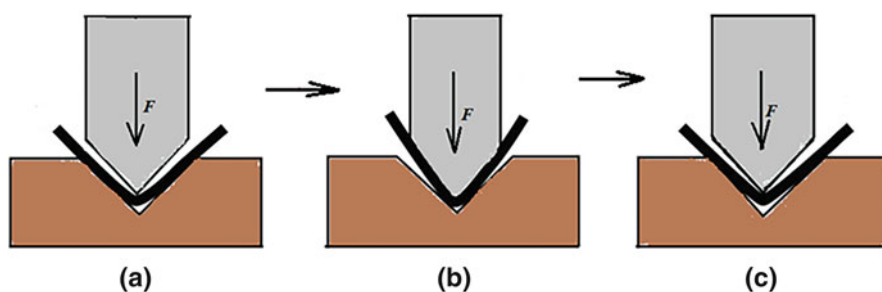


Fig. 11.5 Bottom bending—*principle of operation*. (a) start of the bending operation, (b) intermediate step at an instant, and (c) end of the operation

11.2.5 Edge Bending

In edge bending (or wipe bending), the sheet metal is held against an edge die by use of a pressure pad, which exerts holding force (F_h). The punch then applies a bending force (F) against the edge of the sheet that extends beyond the die and pressure pad (Fig. 11.6a); the force F causes the sheet-metal to bend over the end of the die (Fig. 11.6b). The edge bending method is well suited for mass production; however, it is limited to bends of 90° or less.

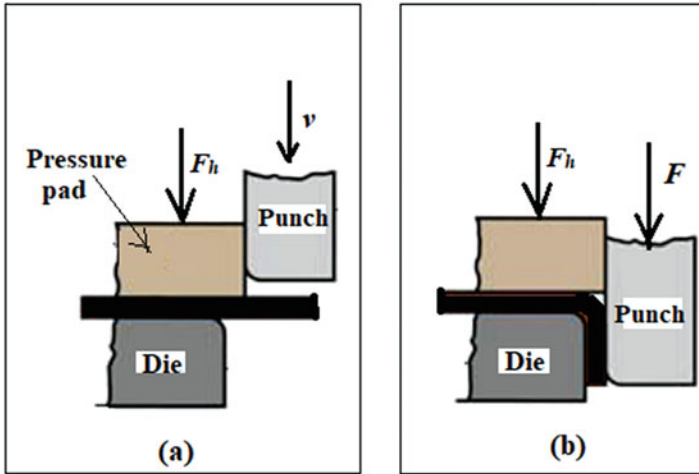


Fig. 11.6 Edge bending; (a) pre-bending position, (b) post-bending position

11.2.6 Roll Bending

Roll bending involves forming large sheet-metal parts into curved sections by passing between rolls; the distance between the rolls determines the desired radius of curvature. It is possible to achieve different bend radii by positioning the center roll up or down. Products of roll bending include components of storage tanks, pressure vessels, race car bumpers, ornamental iron work, and hand-rail applications.

Roll bending is performed by use of three-roll bending machines, as shown in Fig. 11.7. These machines are capable of producing large radius bends in tubing as well as in flat stock. Three-roll bending machines can also be used to coil tubing or bar for refrigeration and cooling towers or to make 360° rings.

11.3 Engineering Analysis of Bending

11.3.1 Terminologies in Sheet-Metal Bending

The terminologies in bending of sheet metal are illustrated in Fig. 11.8. The bend angle can be computed, with reference to Fig. 11.8, by:

$$\text{Bend angle} = \alpha = 180^\circ - \alpha' \quad (11.1)$$



Fig. 11.7 A roll bending machine. (Courtesy: King Abdulaziz University)

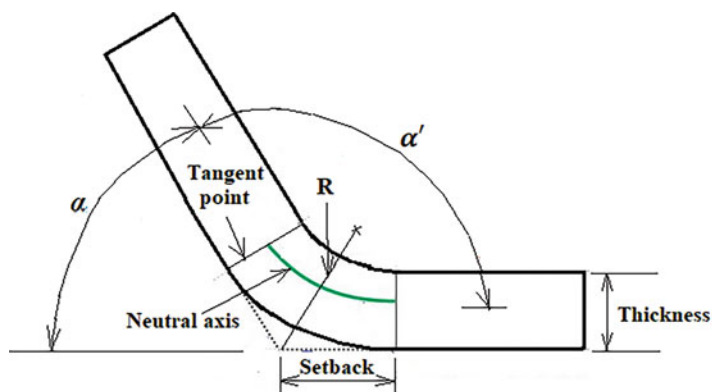


Fig. 11.8 The terminologies in bending of sheet metal (α' = included angle, α = bend angle, R = bend radius)

11.3.2 Bending Force

The bending force depends on a number of factors, including the die opening dimension and the bending factor (a constant for a specific method of bending). It is, therefore, useful to define the two terms. The die opening dimension can be measured in edge bending and V-bending, as shown in Fig. 11.9.

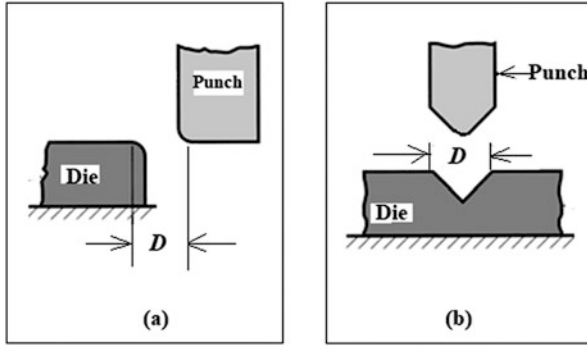


Fig. 11.9 The die opening dimensions for edge bending (a) and V-bending (b)

Depending upon whether the operation is V-bending or edge bending, there exists a constant called *bending factor* K_{bf} . For V-bending, $K_{bf} = 1.33$, whereas for edge bending, $K_{bf} = 0.33$ (Groover, 2014). The bending force can be determined by the following formula (Huda, 2018):

$$F = \frac{K_{bf} \cdot \sigma_{UTS} \cdot w \cdot t}{D} \quad (11.2)$$

where F is the bending force (N), σ_{UTS} is the ultimate tensile strength of the material (MPa), w is the width of the sheet metal (mm), t is the thickness (mm), and D is the die-opening dimension (mm) (see Examples 11.2 and 11.3).

11.3.3 Bend Allowance

It is learnt in the introductory section (Sect. 11.1) that, during bending, the metal on the outside of the neutral axis is stretched (see Fig. 11.10a). Hence, it is important to allow extra length on the starting blank sheet for the stretching; this allowance is called *bend allowance*, which is defined as *the distance on the flat sheet before bending which will take up the radius after bending* (see Fig. 11.10b).

The bend allowance can be determined by the following formula (Groover, 2014):

$$A_b = \frac{2\pi \cdot \alpha(R + K_{ba} \cdot t)}{360^\circ} \quad (11.3)$$

where A_b is the bend allowance (mm), α is the bend angle (deg.), R is the bend radius (mm), t is the sheet thickness (mm), and K_{ba} is the stretching factor ($K_{ba} = 0.33$ when $R < 2t$, whereas $K_{ba} = 0.5$ when $R \geq 2t$) (see Examples 11.4 and 11.5). Once the bend allowance has been determined by using Eq. 11.3, the starting blank size for the bending operation can be computed by:

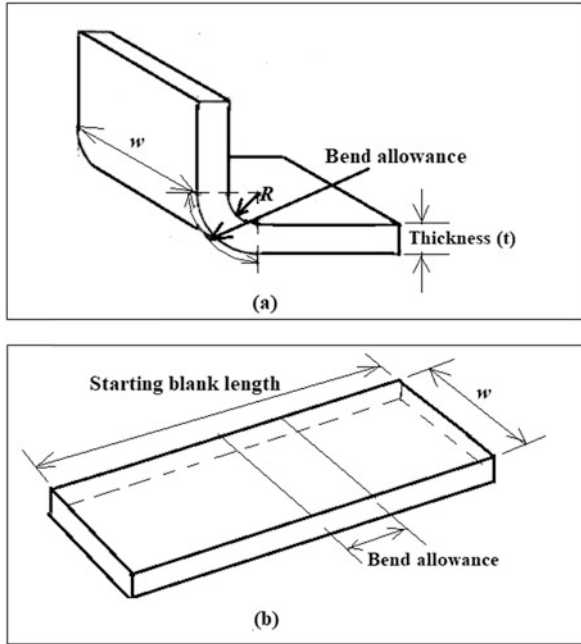


Fig. 11.10 Bent metal sheet (a), blank sheet metal (b). (R = bend radius)

$$\text{Starting (sheet) blank length} = \text{Total measured length} + \text{bend allowance} \quad (11.4)$$

The significance of Eq. 11.4 is illustrated in Examples 11.6 and 11.7.

The stretched length of the bend along the neutral axis or the bend allowance for neutral axis ($A_{b(\text{neutral})}$) can be determined by:

$$A_{b(\text{neutral})} = \frac{2\pi \cdot \alpha \cdot \left(R + \frac{t}{2}\right)}{360^\circ} \quad (11.5)$$

The significance of Eq. 11.5 is illustrated in Examples 11.8–11.10.

11.4 Springback

11.4.1 Bending Angle, Bent Angle, and Springback

Plastic deformation of a material is always followed by “elastic recovery” upon removal of load; this recovery, in bending, is called *springback* (Kalpakjian & Schmid, 2008). In other words, *springback* is a phenomenon that occurs when the

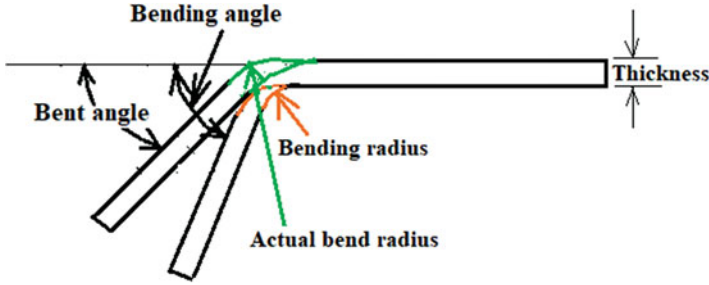


Fig. 11.11 Bent metal sheet illustrating springback

material angularly tries to return to its original shape after being bent. This is why a *bending angle* is different from a *bent angle*, as illustrated in Fig. 11.11. It is evident in Fig. 11.11 that the bending angle is the initial angle to which the operator overbends so that the metal springs back to the desired bent angle.

By reference to Fig. 11.11, the spring back S can be determined by:

$$S = \text{bending angle} - \text{bent angle} \quad (11.6)$$

11.4.2 Springback for V-bending

For a V-bending operation, the *springback* can be determined by (Groover, 2014):

$$S = \frac{\alpha' - \alpha'_p}{\alpha'_p} \quad (11.7)$$

where S is the degree of springback, α' is the included angle, α'_p is the included angle of the punch, and α is the bend angle (see Example 11.1). The degree of springback increases with the materials' modulus of elasticity (E), the yield strength (σ_{ys}), and the bend radius (R). The ratio of the bend radius to sheet thickness (R/t) plays an important role in determining the springback. For $\frac{R}{t} = 1$, the values of the springback for V-bending of various metals and alloys are presented in Table 11.1. However, as the bend-radius value increases to the point of becoming equal to or greater than 8 times the thickness ($\frac{R}{t} \geq 8$), the springback dramatically increases.

Table 11.1 The springback data for V-bending of various alloys for $\frac{R}{t} = 1$ (Benson, 2014)

Material	Aluminum	CR MS	Hot rolled MS	304 SS	Copper and brass
Springback	1.5–2°	0.75–1.0°	0.5–1.0°	2–3°	0–0.5°

Note: *MS* mild steel, *CR* cold rolled, *SS* stainless steel

Table 11.2 Material factor for springback (k) values for air bending of some alloys

Alloy	Cold rolled MS	304 stainless steel	H32 aluminum alloy
Material factor k	1.0	3.5	3.0

11.4.3 Determination of Springback for Air Bending

For air bending, springback can be determined by using the following formula (Benson, 2014):

$$S = k \left(\frac{R}{2.1t} \right) \quad (11.8)$$

where S is the *springback* (deg), R is the bend radius (mm), t is the sheet thickness (mm), and k is the material factor. It must be noted that Eq. 11.8 yields an approximate value of the springback because the degree of springback is also greatly influenced by the material's tensile strength and tooling type (die opening dimension). For cold rolled mild steel (CR MS), the value of the material factor $k = 1$; the k values for other alloys are listed in Table 11.2.

The significance of Eq. 11.8 and Table 2.1 is illustrated in Examples 11.11 and 11.12.

11.4.4 Springback Factor

It has been explained in Sect. 11.4.1 that a *bending angle* is different from a *bent angle*. The ratio of the bending angle to the bent angle is called the springback factor S_f . The springback factor can also be expressed as the ratio of the actual bend radius to the bending radius (see Fig. 11.11). Hence, the springback factor can be mathematically expressed as:

$$S_f = \frac{\text{Bending angle}}{\text{Bent angle}} = \frac{\text{Actual bend radius}}{\text{Bending radius}} \quad (11.9)$$

The significance of Eq. 11.9 is illustrated in Examples 11.13–11.15.

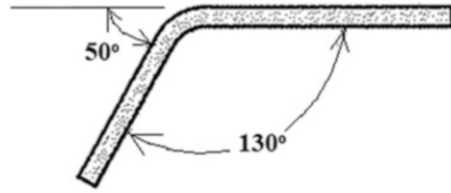


Fig. 11.12 The sketch showing the bending data for Example 11.1

11.5 Worked Numerical Examples

Example 11.1: Determining the Spring-Back and the Bend Angle in a Bending Operation

The included angle in a bent sheet metal was measured to be 130° . Determine the bend angle and draw a sketch showing the bending data.

Solution

By using Eqs. 11.1,

$$\text{Bend angle} = \alpha = 180^\circ - \alpha' = 180^\circ - 130^\circ = 50^\circ.$$

The sketch with the bending data is shown in Fig. 11.12.

Example 11.2: Determining the Bending Force for an Edge Bending Operation

A 3-mm-thick sheet-metal is to be bent by edge-bending method. The width of the sheet is 45 mm. The die-opening dimension is 26 mm. Determine the bending force, if metal has an ultimate tensile strength of 500 MPa.

Solution

$t = 3$ mm, $w = 45$ mm, $\sigma_{\text{UTS}} = 500$ MPa, $D = 26$ mm, $K_{\text{bf}} = 0.33$ (for edge bending).

By using Eq. 11.2,

$$F = \frac{K_{\text{bf}} \cdot \sigma_{\text{UTS}} \cdot w \cdot t}{D} = \frac{0.33 \times 500 \times 45 \times 3}{26} = 856.73 \text{ N}$$

Example 11.3: Determining the Bending Force with Reference to a Labeled Sketch

Refer to the labeled sketch in Fig. 11.13. Determine the bending force for the operation.

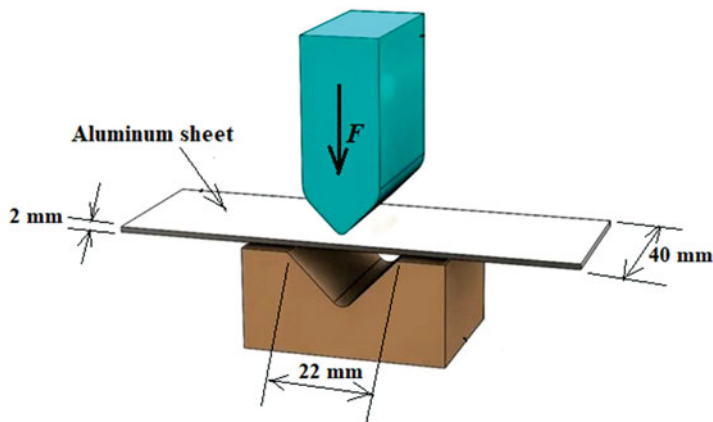


Fig. 11.13 V-edge bending data for Example 11.3

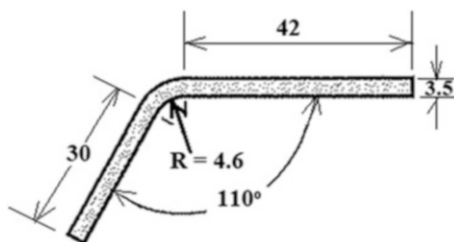


Fig. 11.14 Bent sheet data for Example 11.4 (dimensions in mm)

Solution

$t = 2 \text{ mm}$, $w = 40 \text{ mm}$, $D = 22 \text{ mm}$, $K_{bf} = 1.33$ (for V-bending), $\sigma_{UTS} = 90 \text{ MPa}$ (from Table 3.1)

$$F = \frac{K_{bf} \cdot \sigma_{UTS} \cdot w \cdot t}{D} = \frac{1.33 \times 90 \times 40 \times 2}{22} = 435.27 \text{ N}$$

Example 11.4: Determining the Stretching Factor and the Bend Angle with Reference to Sketch

Refer to Fig. 11.14. Determine the stretching factor and the bend angle.

Solution

By reference to Fig. 11.14, $t = 3.5 \text{ mm}$, $R = 4.6 \text{ mm}$, $\alpha' = 110^\circ$, $K_{ba} = ?$, $\alpha = ?$

$$2t = 2 \times 3.5 = 7 \text{ mm}, R = 4.6 \text{ mm}$$

$$\text{Since } R < 2t, K_{ba} = 0.33$$

$$\alpha = 180^\circ - \alpha' = 180^\circ - 110^\circ = 70^\circ$$

The stretching factor = $K_{ba} = 0.33$, bend angle = $\alpha = 70^\circ$.

Example 11.5: Determining the Bend Allowance in Sheet Metal Bending

By using the data in Example 11.4, determine the bend allowance for the bent sheet.

Solution

$$t = 3.5 \text{ mm}, R = 4.6 \text{ mm}, \alpha' = 110^\circ, K_{ba} = 0.33, \alpha = 70^\circ.$$

By using Eq. 11.3,

$$A_b = \frac{2\pi \cdot \alpha(R + K_{ba} \cdot t)}{360} = \frac{2\pi \times 70 \times (4.6 + 0.33 \times 3.5)}{360} = \frac{2\pi \times 70 \times 5.755}{360} = 7 \text{ mm}$$

The bend allowance = 7 mm.

Example 11.6: Determining the Starting Blank Length for a Bending Operation

By using the data in Examples 11.4 and 11.5, determine the starting blank length for the bending operation.

Solution

By reference to Fig. 11.14,

$$\text{The total measured length} = 42 + 30 = 72 \text{ mm}$$

By using Eq. 11.4,

$$\text{Starting blank length} = \text{Total measured length} + \text{bend allowance} = 72 + 7 = 79 \text{ mm}$$

Example 11.7: Determining the Blank Length with Reference to a Labeled Sketch

With reference to the bent sheet metal shown in Fig. 11.15, determine the blank size required for the bending operation.

Solution

$$t = 4 \text{ mm}, R = 7 \text{ mm}, \alpha' = 38^\circ, K_{ba} = 0.33 \text{ (since } R < 2t), \alpha = 180^\circ - \alpha' = 180^\circ - 38^\circ = 142^\circ.$$

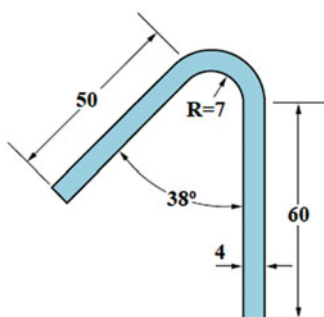


Fig. 11.15 The bent sheet for Example 11.7 (dimensions in mm)

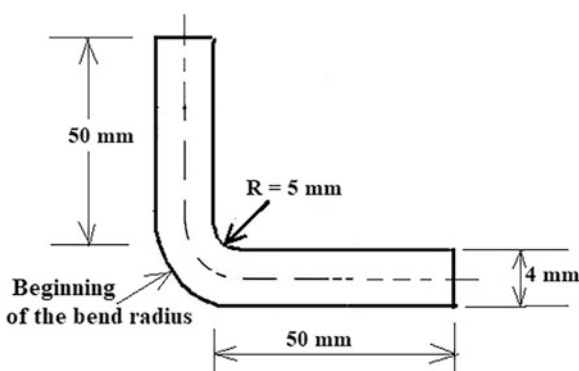


Fig. 11.16 The L-shaped bent part for Example 11.8

$$\begin{aligned} \text{Bend allowance} = A_b &= \frac{2\pi \cdot \alpha(R + K_{ba} \cdot t)}{360} = \frac{2\pi \times 142 \times (7 + 0.33 \times 4)}{360} \\ &= \frac{2\pi \times 142 \times 8.32}{360} = 20.6 \text{ mm} \end{aligned}$$

$$\text{Blank length} = \text{Total measured length} + \text{bend allowance}$$

$$\text{Blank length} = 50 + 60 + 20.6 = 130.6 \text{ mm}$$

Example 11.8: Drawing the Sketch Showing the Two Sides That Result after Bending

A flat blank is to be bent to an L-shaped part by the V-bending method on a press brake. The dimensions of the flat blank are 100 mm × 40 mm × 4 mm. The 90° bend is to be made in the middle of the 100-mm length. The bend radius is 5 mm. Draw a labeled sketch showing the two equal sides that will result after the bending operation, and the beginning point for the measurement of the bend radius.

Solution

The labeled sketch is shown in Fig. 11.16.

Example 11.9: Determining the Dimensions of the Two Sides That Result after the Bend

By using the data in Example 11.8, determine the dimensions of the two equal sides that will result after the bend.

Solution

$$t = 4 \text{ mm}, R = 5 \text{ mm}, \alpha = 180^\circ - \alpha' = 180^\circ - 90^\circ = 90^\circ, K_{ba} = 0.33 (R < 2t).$$

$$\begin{aligned} \text{Bend allowance} = A_b &= \frac{2\pi \cdot \alpha (R + K_{ba} \cdot t)}{360} = \frac{2\pi \times 90^\circ \times (5 + 0.33 \times 4)}{360} \\ &= \frac{2\pi \times 90 \times 6.32}{360} = 10 \text{ mm} \end{aligned}$$

By reference to Fig. 11.16,

$$\text{The dimension of each of the two equal sides} = 50 + \frac{10}{2} = 50 + 5 = 55 \text{ mm}.$$

Example 11.10: Determining the Stretched Length of the Bend Along the Neutral Axis

By using the data in Example 11.9, determine the length of the part's neutral axis after the bend.

Solution

$$t = 4 \text{ mm}, R = 5 \text{ mm}, \alpha = 90^\circ.$$

By using Eq. 11.5,

$$A_{b(\text{neutral})} = \frac{2\pi \cdot \alpha \cdot (R + \frac{t}{2})}{360^\circ} = \frac{2\pi \times 90^\circ (5 + \frac{4}{2})}{360^\circ} = 11 \text{ mm}$$

$$\text{The stretched length of the bend along the neutral axis} = A_{b(\text{neutral})} = 11 \text{ mm}$$

$$\text{Length of the part's neutral axis after the bend} = (2 \times 55) + 11 = 121 \text{ mm}$$

Example 11.11: Determining the Springback for a V-Bending Operation

In a V-bending operation, the included angle of the sheet-metal part is 140° and the punch's included angle is 88° . Determine the springback.

Solution

By using Eq. 11.7,

$$\text{Springback} = S = \frac{\alpha' - \alpha'_p}{\alpha'_p} = \frac{140 - 88}{88} = 0.59^\circ$$

Example 11.12: Determining the Springback for Air Bending of 304 Stainless Steel

It is required to perform air bending on a 1.0-mm-thick sheet metal. The bend radius is 1.5 mm and the sheet material is 304 stainless steel. Determine the springback for the bending operation.

Solution

$t = 1.0 \text{ mm}$, $R = 1.5 \text{ mm}$, $k = 3.5$ (from Table 11.2), $S = ?$

By using Eq. 11.8,

$$S = k \left(\frac{R}{2.1t} \right) = 3.5 \times \left(\frac{1.5}{2.1 \times 1.0} \right) = 2.5^\circ$$

The springback = 2.5° .

Example 11.13: Determining the Springback for Air Bending of Aluminum

Repeat Example 11.12 if the sheet material is H32 aluminum alloy.

Solution

$t = 1.0 \text{ mm}$, $R = 1.5 \text{ mm}$, $k = 3.0$ (from Table 11.2), $S = ?$

$$S = k \left(\frac{R}{2.1t} \right) = 3.0 \times \left(\frac{1.5}{2.1 \times 1.0} \right) = 2.1^\circ$$

The springback = 2.1° .

Example 11.14: Determining the Springback Factor in a Bending Operation

In a V-bending operation, there is a 90° bending angle, and the material springs back 2° . Determine the Springback factor.

Solution

Bending angle = 90° , Springback = 2° , $S_f = ?$

By using the modified form of Eq. 11.6,

$$\text{Bent angle} = \text{Bending angle} - \text{springback} = 90 - 2 = 88^\circ$$

By using Eq. 11.9,

$$S_f = \frac{\text{Bending angle}}{\text{Bent angle}} = \frac{90}{88} = 1.022^\circ$$

The springback factor = $S_f = 1.022^\circ$.

Example 11.15: Determining the Actual Bend Radius

By using the data in Example 11.14, determine the actual bend radius if the bending radius is 1.6 mm.

Solution

$S_f = 1.022^\circ$, bending radius = 1.6 mm, actual bend radius = ?

By using the modified form of Eq. 11.9,

$$\text{Actual bend radius} = S_f \times \text{bending radius} = 1.022 \times 1.6 = 1.63 \text{ mm}$$

Actual bend radius = 1.63 mm.

Questions and Problems

11.1. (MCQs). Select the most appropriate answers for the following statements:

1. The elastic recovery in sheet-metal bending operations is called:
(a) Modulus of elasticity, (b) springback, (c) elastic constant, (d) elasticity.
2. The equipment that is generally used in sheet metal bending is called:
(a) Forging press, (b) bending press, (c) hydraulic press, (d) press brake.
3. Which bending method does not force the sheet into the bottom of the die-cavity?
(a) Air bending, (b) V-bending, (c) roll bending, (d) none.
4. Which bending method is limited to bends of 90° or less?
(a) Air bending, (b) edge bending, (c) bottom bending, (d) V-bending.
5. The ratio of the bending angle to the bent angle is called:
(a) Pressing ratio, (b) springback, (c) springback factor, (d) material ratio.
6. Which bending method is the most suitable for manufacturing 360° rings?
(a) Three-roll bending, (b) edge bending, (c) bottom bending, (d) V-bending.
7. The axis within the material, where the stress and strain are zero, is called:
(a) Coordinate axis, (b) orthogonal axis, (c) bending axis, (d) neutral axis.

- 11.2. (a) Define the following terms: bending, springback, and press brake.
(b) List five (industrial) applications of sheet-metal bending.
- 11.3. Draw a sketch showing the various terminologies in sheet-metal bending.
- 11.4. Why should the starting blank for sheet-metal bending be oversized as compared to the total straight line length in the bent part? Explain with the aid of a sketch.
- 11.5. Explain one of the following sheet-metal bending methods with the aid of a sketch:
(a) V-bending, (b) air bending, (c) bottom bending, (d) Edge bending, (e) roll bending.
- 11.6. Draw the labeled sketch showing the following terms: bending angle, bent angle, bending radius, and actual bend radius.
- P11.7. With reference to the data in Fig. 11.17, determine the force required to perform the bending operation.
- P11.8. With reference to Fig. 11.18, determine the stretching factor and the bend angle.
- P11.9. By reference to the data in P11.8, determine the starting blank length for the bending operation.
- P11.10. In a V-bending operation, the included angle of the sheet-metal part is 150° and the punch's included angle is 86° . Determine the springback.
- P11.11. A flat sheet-metal blank is to be bent to an L-shaped part by the V-bending method on a press brake. The dimensions of the flat blank are $140 \text{ mm} \times 40 \text{ mm} \times 5 \text{ mm}$. The 90° bend is to be made in the middle of the 140-mm length, as shown in Fig. 11.19. Determine the (a) dimensions

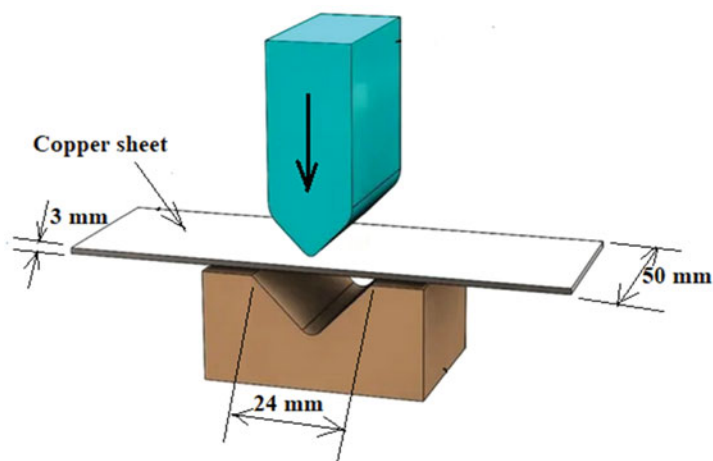


Fig. 11.17 The bending data for P11.7

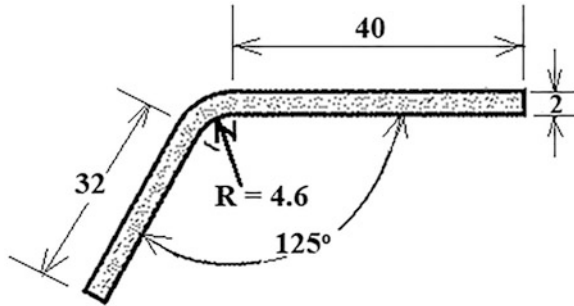


Fig. 11.18 Bent sheet data for P11.8

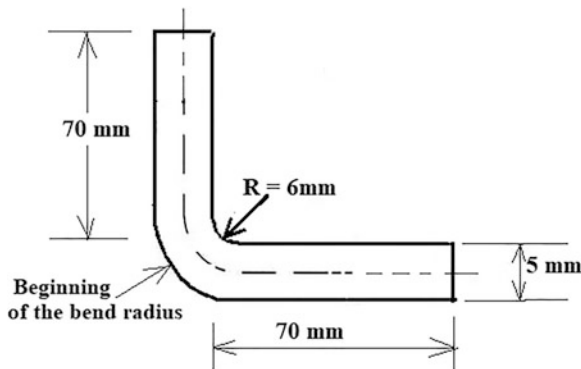


Fig. 11.19 The bent sheet data for Example P11.11

of the two equal sides that will result after the bend and (b) length of the part's neutral axis after the bend.

- P11.12. It is required to perform air bending operation on a 1.5-mm-thick sheet metal. The bend radius is 2 mm and the sheet material is cold rolled mild steel. Determine the springback for the bending operation.
- P11.13. In a V-bending operation, the bending angle is 100° , and the material springs back is 3° .

Determine the (a) springback factor and (b) actual bend radius if the bending radius is 1.8 mm.

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Chapter 12

Deep Drawing of Sheet Metal



Nomenclature

c	clearance
D_b	blank diameter
D_p	punch diameter
D_o	cup outside diameter
D_i	cup inside diameter
R_d	die corner radius
R_p	punch corner radius
h	cup height
t	sheet thickness
F	drawing force
F_h	blank holder force = BHF

Abbreviations

DD	deep drawing
MMDD	multi-step micro deep drawing

12.1 Introduction to Deep Drawing

Deep drawing is a sheet-metal forming process for manufacturing a seamless hollow part that is closed at one end and has a height (or depth) greater than its radius. Deep drawing involves drawing a sheet metal (blank) into a die cavity; the sheet thickness

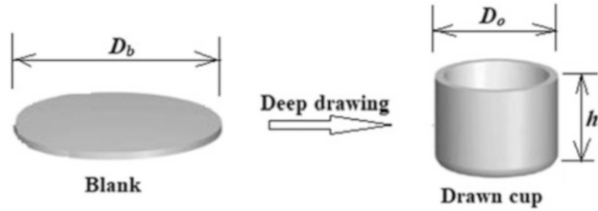


Fig. 12.1 The starting material (blank) and the product (cup) in deep drawing. D_b blank diameter, D_o outer diameter of cup, h height of cup

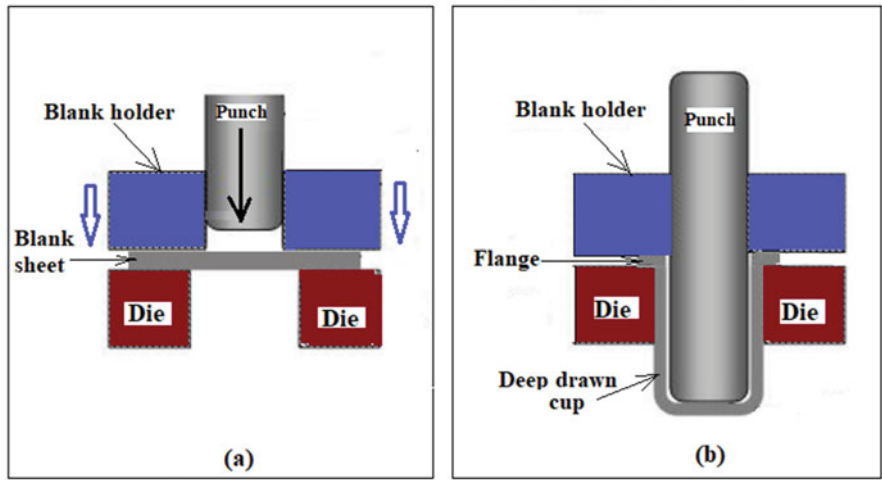


Fig. 12.2 Deep drawing operation; (a) start of deep drawing, (b) end of the operation

remains almost the same from start to finish during deep drawing. A deep drawn part can be recognized by its depth and its smooth corners with radius of curvature (see Fig. 12.1).

In *deep drawing*, a piece of sheet metal is placed onto a die cavity and then forcing the metal into the cavity using a punch; the sheet metal is radially drawn into the die cavity by the mechanical action of the punch (see Fig. 12.2). In order to perform a *deep drawing* operation, first the sheet metal is put onto the die, and then a blank-holder (cushion pad) is introduced on the top of the work-piece, which is not deformed by the punch. The blank-holder controls the sliding of the sheet metal during the deep-drawing operation. Once the blank-holder has been closed, the punch moves down and deforms the work-piece to its final shape. The punch geometry matches the shape of the product to be drawn. In particular, the smooth corner with radius of curvature in the deep-drawn cup (Fig. 12.1) is owing to the adequate punch corner radius (see Fig. 12.2). The sequence of operational steps in deep drawing is described in Sect. 12.4.



Fig. 12.3 Stainless steel deep-drawn sink (inverted view)

12.2 Applications and Advantages of Deep Drawing

The deep drawing operation is used for mass production of sheet-metal components for various industries. Some of the applications of deep-drawn products include automotive components (e.g. radiator caps, fluid and air control valves), household items (e.g. stainless-steel kitchen-sink, cooking utensils, metallic cups), defense components (e.g. ammunition shells), aeronautical, and food industries (Tricarico & Palmieri, 2023).

There are several distinct advantages of deep drawing operation as compared to other sheet metal forming processes. Some of the advantages include: (a) manufacture of hollow cylindrical, square, rectangular, and other geometries (see Fig. 12.3), (b) high specific strength of the deep drawn parts, (c) low manufacturing cost, (d) high productivity, and (e) low cost of tool construction as compared to the cost of progressive stamping tool.

12.3 Deep Drawing Equipment/Press

Sheet metal is drawn by deep drawing in either hydraulic or mechanical presses (Demeri, 2006). Hydraulic presses are also known as alt “double-acting presses” since they work with rollers that apply pressure from up, and support from the bottom. Figure 12.4 presents the picture of a hydraulic press Benedetti HPS 100 for deep drawing with the control of the blank-holding force; this press has the following specifications: nominal drawing force = 1000 kN, nominal blank holder force = 400 kN, bolster dimension = 1000 mm × 900 mm, maximum punch stroke = 500 mm.



Fig. 12.4 The hydraulic press “Benedetti HPS 100” for deep drawing of metal sheets. (Courtesy: Department of Manufacturing Engineering, Technical University of Cluj-apoca)

12.4 The Steps in Deep Drawing Operation

The following principal steps are involved in deep drawing of sheet metal parts.

Step I. The sheet metal (blank) is placed on the die cavity, as shown in Fig. 12.5a.

Step II. The blank holder grips the blank in between the die and the blank holder with the blank holder force (BHF) F_h , and the punch applies the downward punching force F for deep drawing [see Fig. 12.5b]. The blank-holding force is usually

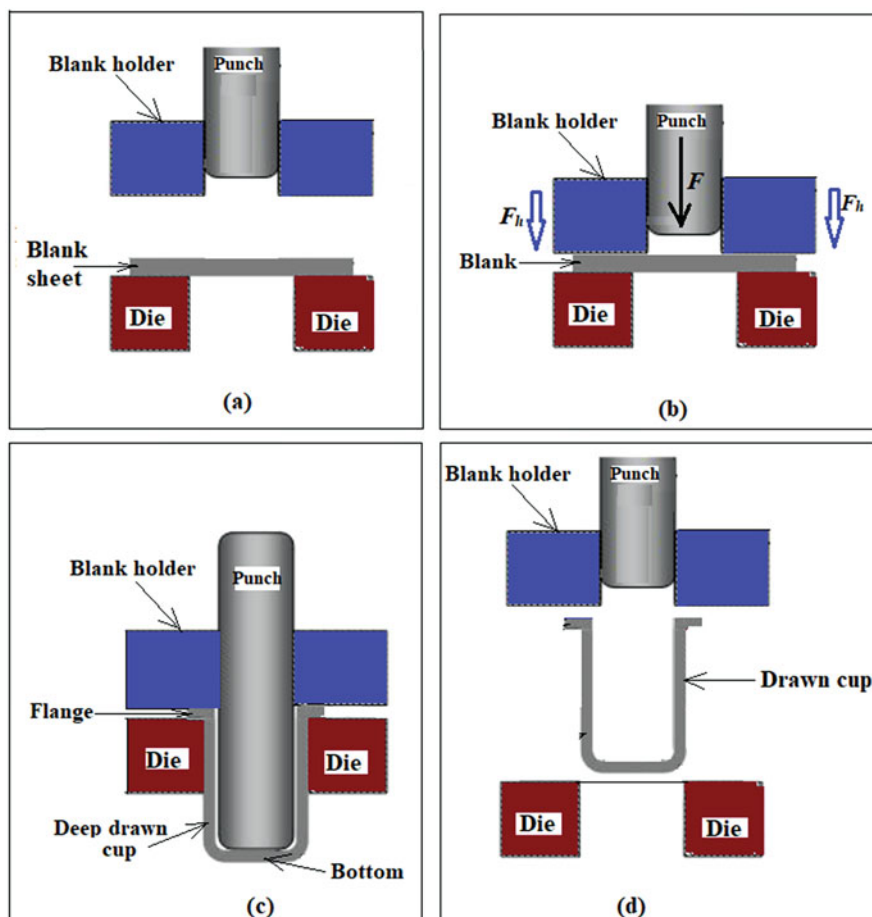


Fig. 12.5 The steps in deep drawing; (a) the sheet blank is placed over the die cavity, (b) the blank holder grips the blank in between the die and the blank holder with holding force F_h , and the punch applies the downward punching force F , (c) the punch deforms the material by deep drawing to produce the cup, and (d) the product is removed

applied by the outer ram of a double-action hydraulic press or by a cushion in a single-action press (Ramezani & Ripin, 2012).

Step III. As the punch moves downward toward its final bottom position, the material experiences a combination of tensile and compressive stresses resulting in straining of the work as it is gradually formed into the shape defined by the punch and die cavity [see Fig. 12.5c]. The work is simply bent over the corner of the punch and the corner of the die. With further straining, a straightening action occurs in the metal that was previously bent over the die radius. During this stage, the BHF is gradually reduced to allow free deformation of work material.

Steps IV. On completion of the deep drawing process, the product is removed (Fig. 12.5d).

The blank holder force (BHF) F_h plays an important role during deep drawing. For example, too small *BHF* will result in wrinkling of the drawn product. On the other hand, too large *BHF* will restrict the free flow of the metal, resulting in stretching and tearing of the sheet metal. There occurs stretching and thinning of the metal as it is pulled over the edge of the die opening. In practice, up to 25% thinning of the side wall may occur in a successful deep-drawing operation; thinning mostly occurs near the cup base.

Friction and compression play important roles in the flange of the blank during step III of deep drawing operation (see Fig. 12.5c). It is necessary to overcome friction between the work metal and the surfaces of the blank-holder and the die opening to allow free movement of the material in the flange towards the die opening. In order to reduce friction, lubricants or drawing compounds are generally used in deep drawing equipment. Besides friction, there is also compression on the outer edge of the blank. As the metal in this region of the blank is drawn toward the center, the outer perimeter becomes smaller. Since the volume of metal remains constant, the metal becomes thicker as the perimeter is reduced.

12.5 Engineering Analysis of Deep Drawing

12.5.1 Press Design and Process Parameters for Deep Drawing

There are a number of press design and process parameters for deep drawing of metal sheets; these parameters include the clearance (c), blank diameter (D_b), punch diameter (D_p), die corner radius (R_d), punch corner radius (R_p), drawing force (F) and the blank holder force (F_h) (see Fig. 12.6). Like blanking, it is important to allow adequate clearance between the punch and die surfaces for deep drawing. The clearance can be determined by:

$$c = 1.1 \ t \quad (12.1)$$

where c is the clearance and t is the sheet thickness.

It must be noted that the smooth corner with radius of curvature in the deep-drawn cup (Fig. 12.1) is owing to the adequate punch corner radius R_p (see Fig. 12.6). In case the punch corner radius is zero, the punch will act as a blanking punch instead of a deep-drawing punch. Hence, it is important that the press design for deep drawn must take into consideration an adequate punch corner radius.

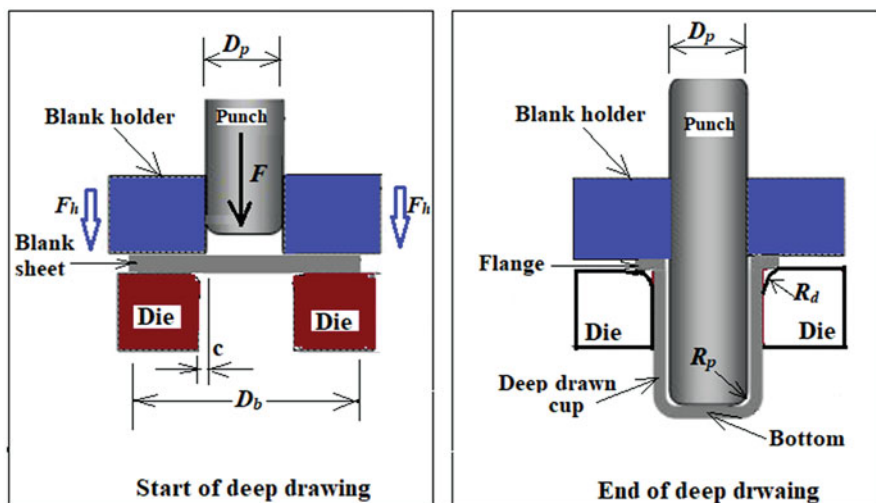


Fig. 12.6 Press design and process parameters for deep drawing. D_b blank diameter, D_p punch diameter, c clearance, R_d die corner radius, R_p punch corner radius, F drawing force, F_h blank holder force

12.5.2 Checking the Feasibility of Deep Drawing Operation for Quality

Prior to starting the deep drawing operation, the limitations on the amount of drawing must be assessed. It is also important to determine the starting blank size based on the dimensions of the cup/product to be produced. The severity of a deep drawing operation can be assessed by the drawing ratio (DR), which is defined for a cylindrical shape as the ratio of the blank diameter to the punch diameter, i.e.

$$DR = \frac{D_b}{D_p} \quad (12.2)$$

where D_b is the blank diameter and D_p is the punch diameter (see Fig. 12.6). In order for deep drawing to be feasible, the drawing ratio must be equal to or less than 2 ($DR \leq 2$). An increase in the DR beyond this limit will require a higher drawing force (F), which will result in excessive thinning or even cracks in the cup wall. The significance of Eqs. 12.1 and 12.2 is illustrated in Example 12.1.

The actual limits for a given deep-drawing operation depend on many factors, including the punch corner radius (R_p), the die corner radius (R_d), friction conditions, draw depth (cup height, h), sheet thickness (t), blank diameter (D_b), and mechanical characteristics of the sheet metal. Another measure of the severity of drawing is the reduction r , which is given by (Huda, 2018):

$$r = \frac{D_b - D_p}{D_b} \quad (12.3)$$

For a successful drawing operation, the value of r should be less than 0.5.

The thickness-to-diameter ratio (t/D_b) is also an important measure for checking the severity of deep drawing; the limit for the feasibility requires the ratio to be greater than 1%. A decrease in the thickness-to-diameter ratio [$(t/D_b) < 1\%$] may result in significant wrinkling in the drawn product. In industrial practice, forming of thin-walled dome parts often results in the occurrence of wrinkling defects, which is primarily attributed to circumferential compressive stress. Very recently (2024), Sun and coresearchers have proposed a novel deep-drawing technique that effectively suppresses wrinkles in thin-walled dome parts through the utilization of a local reverse bulging tool for active stress control (Sun et al., 2024).

Having discussed the conditions for severity of deep drawing in the preceding paragraph, the following conditions are summarized for a successful deep drawing operation:

$$\text{DR} = \frac{D_b}{D_p} < 2 \quad (12.4a)$$

$$r = \frac{D_b - D_p}{D_b} < 0.5 \quad (12.4b)$$

$$\frac{t}{D_b} > 1\% \quad (12.4c)$$

In cases where the limits outlined in Eqs. 12.4a, 12.4b, 12.4c are exceeded by the design of the drawn product, the blank must be deep-drawn in two or more steps. Since sheet-metal forming is a cold working process involving strain hardening, recrystallization annealing between these steps is required for restoring the ductility. The significance of Eqs. 12.4a, 12.4b, 12.4c is illustrated in Example 12.2. It must be, however, noted that Eqs. 12.4a, 12.4b, 12.4c hold good for feasibility of a deep drawing operation when the constant-volume relationship is satisfied.

12.5.3 *Determination of Drawing Force and the Blank Holding Force in Deep Drawing*

The drawing force required to perform a given deep-drawing operation can be estimated by the following formula:

$$F = \pi \cdot D_p \cdot t \cdot \sigma_{\text{UTS}} \cdot \left(\frac{D_b}{D_p} - 0.7 \right) \quad (12.5)$$

where F is the drawing force (N), D_p is the punch diameter (mm), t is the sheet thickness (mm), σ_{UTS} is the ultimate tensile strength (MPa), and D_b is blank diameter

(mm) (see Example 12.3). Equation 12.5 estimates the maximum force in a deep-drawing operation. In practice, the drawing force varies (gradually increases) as the punch moves downward during the operation. The drawing force usually reaches its maximum (F) at about one-third the length of the punch stroke (Groover, 2014).

Besides the drawing force, the blank holder force (BHF) is also an important parameter in deep drawing. The BHF can be roughly determined by the following mathematical relationship (Schey, 2000):

$$F_h = (\pi \cdot \sigma_{ys}) \cdot (\text{The original area of the blank that is to be held by the blank-holder}) \quad (12.6)$$

or

$$F_h = (0.015\pi \cdot \sigma_{ys}) \cdot [D_b^2 - (D_p + 2.2t + 2R_d)^2] \quad (12.7)$$

where F_h is the blank holder force (BHF) (in N), σ_{ys} is the yield strength of the sheet metal (MPa), D_p is the punch diameter (mm), t is the sheet thickness (mm), D_b is blank diameter (mm), and R_d is the die corner radius (mm) (see Example 12.4). The BHF is generally applied by a single blank holder (cushion pad). A proper BHF profile can prevent wrinkling of the drawn cup and delay necking. The proper BHF profile starts from zero at the beginning of drawing and reaches a maximum, and then reduces to zero at the end of the process. It is desirable to minimize friction at the flange area for eliminating the wrinkling defect (Ramezani & Ripin, 2012). The double-ring blank holder technique has been developed for preventing the wrinkling defect in deep-drawn cups (see Sect. 12.6).

The reliability in deep drawing is closely linked to the possibility of adequate control of the BHF. In order to avoid typical failure mechanisms, such as wrinkling and tearing, it is necessary to properly adjust the BHF during the process. Recently (2023), Brun and coresearchers presented the design of a new blank-holding set-up based on magneto-rheological actuators, which allow a flexible adjustment of the force during the process (see Sect. 12.6).

12.5.4 Determination of Blank Size for Deep Drawing

It is important to determine the correct starting blank size for deep drawing to achieve the accurate dimensions of a cylindrical drawn shape. The blank size must be sufficiently large to supply sufficient metal to complete the cup. Obviously, an oversize blank will result in waste of material. The correct blank size can be determined by using the constant-volume relationship between the starting blank and the drawn cup, i.e.

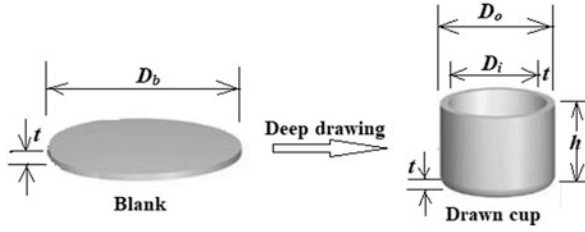


Fig. 12.7 Geometric parameters of starting blank and the drawn cup

$$\text{Volume of blank} = \text{Volume of drawn cup} \quad (12.8)$$

Equation 12.8 can then be solved for the blank diameter D_b with reference to Fig. 12.7, as follows.

By reference to Fig. 12.7, Eq. 12.8 can be rewritten as:

$$\begin{aligned} \frac{\pi}{4} \cdot D_b^2 \cdot t &= (\text{Volume of solid base of the cup}) \\ &+ (\text{Volume of the hollow portion}) \end{aligned} \quad (12.9)$$

or

$$\frac{\pi}{4} \cdot D_b^2 \cdot t = \left(\frac{\pi}{4} \cdot D_o^2 \cdot t \right) + \left\{ \frac{\pi}{4} (D_o^2 - D_i^2) (h - t) \right\} \quad (12.10)$$

or

$$D_b^2 = D_o^2 + \frac{(D_o^2 - D_i^2) (h - t)}{t} \quad (12.11)$$

or

$$D_b = \sqrt{D_o^2 + \frac{(D_o^2 - D_i^2) (h - t)}{t}} \quad (12.12)$$

Eq. 12.12 holds good when thinning of cup wall is ignored. The thickness t of the cup wall can be calculated with reference to Fig. 12.7, as follows:

$$D_i + 2t = D_o \quad (12.13)$$

or

$$t = \frac{D_o - D_i}{2} \quad (12.14)$$

Equations 12.12–12.14 enable us to compute the starting blank diameter when the height and outside and inside diameters of the cup, to be drawn, are given (see Example 12.5).

12.6 Recent Advances in Deep Drawing

12.6.1 *Use of Reverse Bulging Tool for Stress Control*

It has been learnt in earlier sections that wrinkling and tearing defects are major problems in thin-walled deep-drawn parts. In order to prevent the defects, a lot of research has been reported. For example, Sun and coresearchers have recently proposed a novel deep-drawing technique that effectively suppresses wrinkles in thin-walled dome parts through the utilization of a local reverse bulging tool for active stress control (Sun et al. 2024). They have developed an analytical model to describe this novel deep drawing method, enabling analysis of the radius variations in the reverse bulging area and the corresponding reverse bulging force.

12.6.2 *Multi-Step Micro Deep Drawing (MMDD)*

In another study, multi-step micro deep drawing (MMDD) has been proposed for optimum formability of copper foils (Zhao et al., 2024). In this work, the copper foils were annealed at temperatures ranging from 300 to 700 °C for 5 min, followed by tensile tests and electron back-scattered diffraction (EBSD) tests for the exploration of mechanical behavior and microstructural characterization of annealed specimens. The results show that the as-received copper foils exhibit poor formability and cannot form micro cups during MMDD. When the annealing temperature attains 500 °C, a homogeneous microstructure is formed inside copper foils, thereby enhancing the formability of material and improving the surface quality of formed products. In addition, the results show that the E texture component within the copper foils is favorable for MMDD.

12.6.3 *New Design of Blank Holder with Magneto-Rheological Actuators*

In order to avoid wrinkling and tearing, it is also necessary to properly adjust the blank holder force (BHF) during the deep-drawing operation. Recently (2023), Brun and coresearchers presented the design of a new blank-holding set-up based on magneto-rheological actuators, which allow a flexible adjustment of the force during the process. In this study, a closed-loop control, based on the direct detection of the

metal sheet displacement under the blank holder, was implemented to regulate in real time the actuator's load (Brun et al., 2023). The obtained experimental test results show that (i) the presented set-up allows us to apply blank holder pressures with an average error less than 5% with respect of the target forces; (ii) the low latency times to allow the force variations, and (iii) the possibility to vary in real time the blank holder load response during the deep drawing.

12.6.4 Blank Holder with Double Rings

Zhang and coresearchers (2021) have also reported a new blank holder to prevent the wrinkling defect in deep-drawn cups. In this work, the blank holder has been divided into double rings so that the blank holder force (BHF) can be independently and effectively applied on different flange regions (Zhang et al. 2021). In the study, the principle and characteristics of the new blank holder technique have been expatiated by theoretical derivation and stress analysis; all results indicated that the distribution of BHF applied by the double-rings blank holder is more reasonable than a single-ring blank holder under the condition of proper parameter selection, and the wrinkling can be better suppressed. The effectiveness of the double-rings blank holder has been proven by deep drawing experiments. Using the new blank holder technique, the flange wrinkling was demonstrated to be more eliminated as compared to the single-ring blank holder.

12.7 Worked Numerical Examples in Deep Drawing

Example 12.1: Determining the Clearance and the Drawing Ratio for Deep Drawing

A deep drawing operation is used to form a cylindrical cup with inside diameter of 70 mm. The starting blank size is 130 mm and the sheet thickness is 2.5 mm. Determine the clearance and the drawing ratio.

Solution

Cup inside diameter = punch diameter = $D_p = 70$ mm, $D_b = 130$ mm, $t = 2.5$ mm

By using Eqs. 12.1 and 12.2,

$$\text{Clearance} = c = 1.1 \ t = 1.1 \times 2.5 = 2.75 \text{ mm}$$

$$\text{Drawing ratio} = \text{DR} = \frac{D_b}{D_p} = \frac{130}{70} = 1.85$$

Example 12.2: Determining the Feasibility of a Deep Drawing Operation

Refer to the data in Example 12.1. Based on the data, is the operation feasible assuming that the constant-volume relationship holds good?

Solution

$$D_b = 130 \text{ mm}, D_p = \text{inside diameter of cup} = 70 \text{ mm}, t = 2.5 \text{ mm}$$

The feasibility of deep-drawing operation can be checked with reference to Eqs. 12.4a, 12.4b, 12.4c as follows:

$$\begin{aligned} \text{DR} &= \frac{D_b}{D_p} = \frac{130}{70} = 1.857 < 2 \\ r &= \frac{D_b - D_p}{D_b} = \frac{130 - 70}{130} = 0.46 < 0.5 \\ \frac{t}{D_b} &= \frac{2.5}{130} = 0.0192 = 1.92\% > 1\% \end{aligned}$$

Since all conditions of Eqs. 12.4a, 12.4b, 12.4c are satisfied, the deep-drawing operation is feasible.

Example 12.3: Determining the Drawing Force for a Deep Drawing Operation

A cylindrical cup, with inside diameter of 70 mm, is required to be formed by deep drawing. The starting blank diameter is 137 mm and the stock thickness is 2 mm. The ultimate tensile strength of the sheet metal (mild steel) is 450 MPa and its yield strength is 300 MPa. The die corner radius is 6 mm. Determine the required drawing force for the operation.

Solution

$$D_p = 70 \text{ mm}, D_b = 137 \text{ mm}, t = 2 \text{ mm}, \sigma_{\text{UTS}} = 450 \text{ MPa}, F = ?$$

By using Eq. 12.5,

$$\begin{aligned} F &= \pi \cdot D_p \cdot t \cdot \sigma_{\text{UTS}} \cdot \left(\frac{D_b}{D_p} - 0.7 \right) = 3.143 \times 70 \times 2 \times 450 \times \left(\frac{137}{70} - 0.7 \right) \\ &= 248,814 \text{ N} \end{aligned}$$

The drawing force = 248.814 kN $\cong$ 250 kN.

Example 12.4: Determining the Blank Holder Force in a Deep Drawing Operation

Refer to the data in Fig. 12.8. Determine the blank holder force (F_h) for the operation, if the sheet material is mild steel.

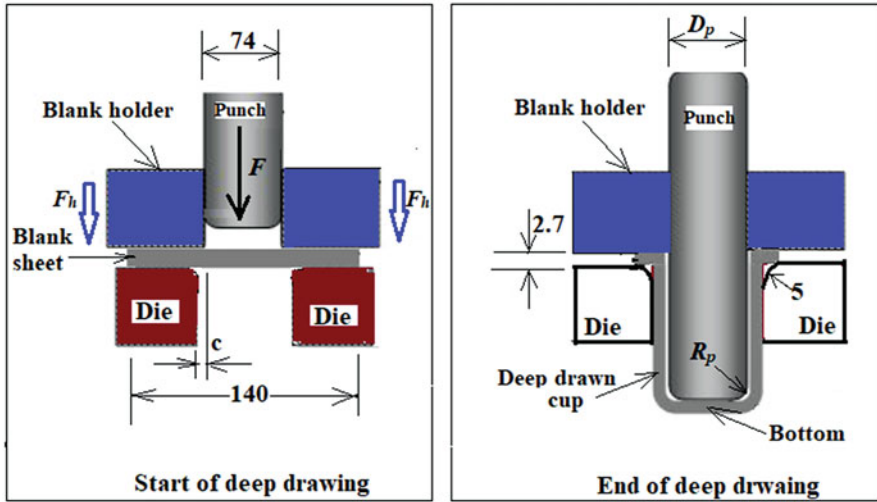


Fig. 12.8 Deep drawing process parameters' data for Example 12.4

Solution

$$D_p = 70 \text{ mm}, D_b = 137 \text{ mm}, t = 2 \text{ mm}, \sigma_{ys} = 300 \text{ MPa}, R_d = 6 \text{ mm}, F_h = ?$$

By using Eq. 12.7,

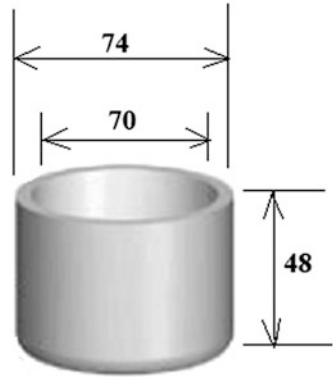
$$\begin{aligned} F_h &= (0.015\pi \cdot \sigma_{ys}) \cdot \left[D_b^2 - (D_p + 2.2t + 2R_d)^2 \right] = 0.015 \pi \times 300 \\ &\quad \times \left[137^2 - (70 + 2.2 \times 2 + 2 \times 6)^2 \right] \\ F_h &= 14.14 \times \left[18769 - (70 + 4.4 + 12)^2 \right] = 14.14 \times \left[18769 - (86.4)^2 \right] \\ &= 159,839 \text{ N} \end{aligned}$$

The blank holder force = 159.839 N $\cong$ 160 kN.

Example 12.5: Determining the Blank Diameter When the Cup Dimensions Are Known

A metallic cup is to be deep-drawn as shown in Fig. 12.9. Determine the starting blank diameter for the deep drawing operation. Ignore thinning effect during the cup drawing.

Fig. 12.9 The cup dimensions for Example 12.5 (dimensions in mm)



Solution

$$D_i = 70 \text{ mm}, D_o = 74 \text{ mm}, h = 48 \text{ mm}, D_b = ?$$

By using Eq. 12.14,

$$t = \frac{D_o - D_i}{2} = \frac{74 - 70}{2} = 2 \text{ mm}$$

By using Eq. 12.12,

$$\begin{aligned} D_b &= \sqrt{D_o^2 + \frac{(D_o^2 - D_i^2)(h - t)}{t}} = \sqrt{74^2 + \frac{(74^2 - 70^2)(48 - 2)}{2}} \\ &= \sqrt{5476 + \frac{(5476 - 4900)(48 - 2)}{2}} \\ D_b &= \sqrt{5476 + \frac{(5476 - 4900)(48 - 2)}{2}} = \sqrt{5476 + 13248} = 136.8 \text{ mm} \end{aligned}$$

The blank diameter = 136.8 mm $\cong$ 137 mm.

Example 12.6: Checking the Feasibility of Deep Drawing Operation by Using Constant–Volume Relationship

A metallic cup is to be deep drawn. The inside diameter of the cup is 95 mm and its height is 70 mm. The sheet-metal thickness is 2 mm. The blank diameter is 170 mm. Is the deep drawing operation feasible?

Solution

$$D_i = D_p = 95 \text{ mm}, D_b = 170 \text{ mm}, t = 2 \text{ mm}, h = 70 \text{ mm}$$

In order to check the feasibility, we first refer to Eqs. 12.4a, 12.4b, 12.4c.

$$\begin{aligned} \text{DR} &= \frac{D_b}{D_p} = \frac{170}{95} = 1.79 < 2 \\ r &= \frac{D_b - D_p}{D_b} = \frac{170 - 95}{170} = 0.44 < 0.5 \\ \frac{t}{D_b} &= \frac{2}{170} = 0.011 = 1.1\% > 1\% \end{aligned}$$

Although all conditions for feasibility of the operation seem to have satisfied, it is necessary to check the constant-volume relationship (volume of blank = volume of cup) by using Eqs. 12.12 and 12.13.

$$D_o = D_i + 2t = 95 + (2 \times 2) = 99 \text{ mm}$$

$$\begin{aligned} D_b &= \sqrt{D_o^2 + \frac{(D_o^2 - D_i^2)(h-t)}{t}} = \sqrt{99^2 + \frac{(99^2 - 95^2)(70-2)}{2}} = \\ &= \sqrt{9801 + \frac{(9801 - 9025)(70-2)}{2}} \quad D_b = \sqrt{9801 + 26384} = 190.22 \text{ mm} \end{aligned}$$

The cup design requires that the blank size should be at least 190.22 mm; but the given blank diameter is 170 mm. Hence, the deep drawing operation is *not* feasible.

Example 12.7: Determining the Ratio of the Drawing Force to the Blank Holder Force

A metallic cup is to be formed by deep drawing operation. The cup's inside diameter is 75 mm and its height is 45 mm. The sheet thickness is 2.8 mm, and the starting blank diameter is 145 mm. The punch corner radius is 3.8 mm and die corner radius is also 3.8 mm. The sheet material's tensile strength is 425 MPa and its yield strength is 190 MPa. Determine: (a) drawing ratio, (b) reduction, (c) drawing force, and (d) blank-holder force. What is the relationship between the drawing force and the blank-holder force?

Solution

$$D_i = D_p = 75 \text{ mm}, D_b = 145 \text{ mm}, t = 2.8 \text{ mm}, h = 45 \text{ mm}, \sigma_{\text{UTS}} = 425 \text{ MPa}, \sigma_{\text{ys}} = 190 \text{ MPa}$$

$$\begin{aligned} \text{DR} &= \frac{D_b}{D_p} = \frac{145}{75} = 1.93 \\ r &= \frac{D_b - D_p}{D_b} = \frac{145 - 75}{145} = 0.482 \end{aligned}$$

$$\begin{aligned} F &= \pi \cdot D_p \cdot t \cdot \sigma_{\text{UTS}} \cdot \left(\frac{D_b}{D_p} - 0.7 \right) = \pi \times 75 \times 2.8 \times 425 \times \left(\frac{145}{75} - 0.7 \right) \\ &= 345,810.8 \text{ N} \end{aligned}$$

$$F_h = (0.015\pi \cdot \sigma_{\text{ys}}) \cdot [D_b^2 - (D_p + 2.2t + 2R_d)^2]$$

$$F_h = 0.015 \pi \times 190 \times \left[145^2 - (75 + 2.2 \times 2.8 + 2 \times 3.8)^2 \right]$$

$$F_h = 8.9535 \times \left[21025 - (75 + 6.16 + 7.6)^2 \right] = 117,708.6 \text{ N}$$

The relationship between the drawing force and the blank-holder force is determined as follows.

$$\frac{F}{F_h} = \frac{345810.8}{117708.6} = 2.9 \cong 3$$

It means that the blank-holder force is about one-third of the drawing force.

Questions and Problems

- 12.1 Describe deep drawing of a metal sheet with the aid of sketches.
- 12.2 Why is it important to provide adequate punch corner radius in the press design for deep drawing? Explain.
- 12.3 Highlight the applications and advantages of deep drawing of metal sheets.
- 12.4 Illustrate diagrammatically the steps in the deep drawing operation.
- 12.5 Explain the proper BHF profile that can prevent wrinkling of the drawn cup and delay necking.
- 12.6 List four recently developed techniques to prevent the wrinkling defect in deep-drawn cups.
- P12.7 A deep drawing operation is to be used to form a cylindrical cup with inside diameter of 68 mm. The starting blank size is 127 mm and the sheet thickness is 2.2 mm. Based on the data, is the operation feasible? Ignore thinning.
- P12.8 By using the data in P12.7, determine the clearance and the drawing ratio for the deep-drawing operation.
- P12.9 A cylindrical cup, with inside diameter of 74 mm, is required to be formed by deep drawing. The starting blank diameter is 140 mm and the sheet thickness is 2.7 mm. The ultimate tensile strength of the sheet metal is 380 MPa and its yield strength is 210 MPa. The die corner radius is 5 mm. Determine the required drawing force for the operation.
- P12.10 Refer to the data in Fig. 12.10. Determine the blank holder force (F_h) for the deep-drawing operation.
- P12.11 A metallic cup is to be deep drawn. The inside diameter of the cup is 90 mm and its height is 72 mm. The sheet-metal thickness is 2.5 mm. The blank diameter is 160 mm. Is the deep- drawing operation feasible?
- P12.12 A metallic cup is to be deep-drawn as shown in Fig. 12.11. Determine the starting blank diameter for the deep drawing operation.
- P12.13 A metallic cup is to be formed by deep drawing operation. The cup's inside diameter is 73 mm and its height is 46 mm. The sheet thickness is 3 mm, and the starting blank diameter is 144 mm. The punch corner radius is 4 mm and

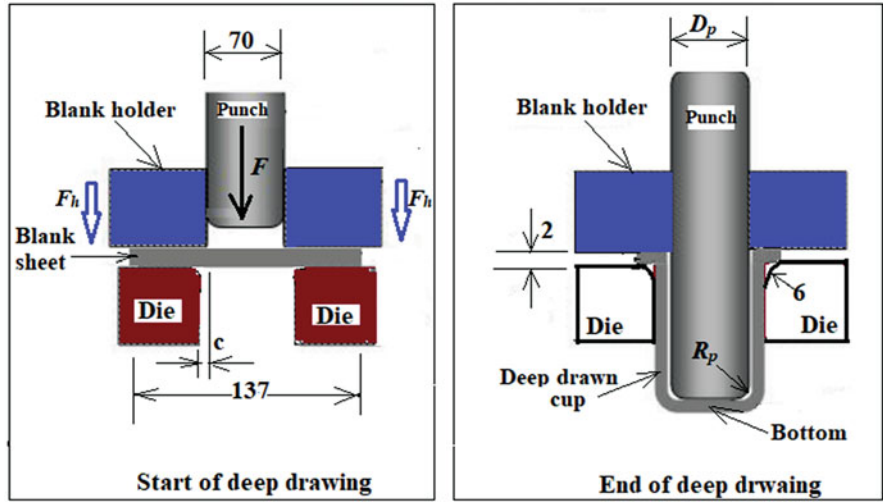


Fig. 12.10 Deep drawing data for P12.10

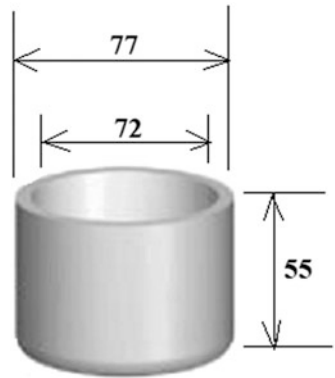


Fig. 12.11 The cup dimensions for P12.12 (dimensions in mm)

die corner radius is also 4 mm. The sheet material's tensile strength is 400 MPa and its yield strength is 180 MPa.

Determine: (a) drawing ratio, (b) reduction, (c) drawing force, and (d) blank-holder force. What is the relationship between the drawing force and the blank-holder force?

P12.14 (MCQs). Select the most appropriate answers for the following statements.

1. What is the relation between the height (h) and the radius (R) of a deep-drawn cup?
(a) $h < R$, (b) $h > R$, (c) $h = R$, (d) $h \leq R$
2. What is the relation between the cup's inside diameter (D_i) and the punch diameter (D_p)?
(a) $D_i = D_p$, (b) $D_i > D_p$, (c) $D_i < D_p$, (d) $D_i \leq D_p$
3. Which type of stress acts during deformation in deep drawing?
(a) Tensile stress, (b) compressive stress, (c) combination of tensile and compressive stresses.
4. What is the effect of too small blank holder force (BHF) during deep drawing?
(a) Tearing, (b) brittle fracture, (c) earing, (d) wrinkles.
5. What is the effect of too large blank holder force (BHF) during deep drawing?
(a) Tearing, (b) brittle fracture, (c) earing, (d) wrinkles.
6. Which technique should be used for success of a deep-drawing (DD) operation if the limits established in Eqs. 12.4a, 12.4b, 12.4c are exceeded?
(a) Double-action DD, (b) multi-step DD, (c) single action DD, (d) hydraulic press DD
7. Which technique is suitable for eliminating wrinkling defect in deep-drawn cups?
(a) Double-action press, (b) blank holder with double ring (c) single action DD.
8. Which deep-drawing technique is best suited for optimum formability of copper foils?
(a) Double-action DD, (b) multi-step DD, (c) single action DD, (d) multi-step micro DD.
9. Which equipment modification technique allows a flexible adjustment of BHF during DD?
(a) Magneto-rheological actuators, (b) ultrasonic actuators, (c) single-action cushion.

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Part IV
Worked Projects on Metal Forming
and Business

Chapter 13

Worked Metal Forming Projects



13.1 Metal Forming Project 1: Bolts Manufacturing

13.1.1 Project Description of Project 1

Assume you are a manufacturing manager in a metal forming industry. You are requested to manufacture the bolt as shown in Fig. 13.1a. The available material in the market is the aluminum alloy 2024-T3 solid cylindrical billet, as shown in Fig. 13.1b.

The following machines are at your disposal: (1) an extruder, (2) a wire-drawing machine, and (3) a heading (upset forging) machine. The machines' specifications are as follows:

Extrusion Machine:

Process type	: continuous extrusion
Extrusion type	: direct extrusion (die angle = 90°)
Die opening diameter	: 8 mm
Dies' Johnson constants	: $a = 0.66$ and $b = 1.3$
Ram velocity	: 5 mm/s

Wire Drawing Machine:

Wire out diameter	: 3.2 mm
Wire in diameter	: 8 mm
Reduction ratio	: 0.2 (for each die)

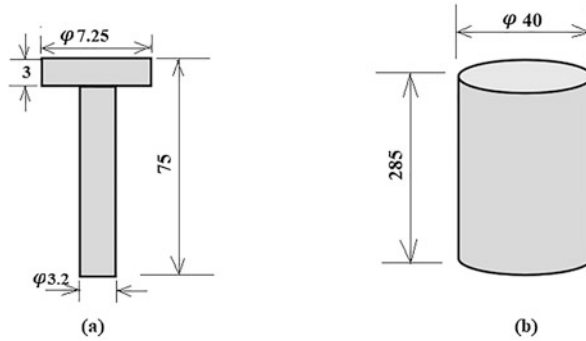


Fig. 13.1 (a) bolt dimension, (b) billet dimensions (all dimensions in mm)

Number of dies : You can use several numbers of dies up to 10 dies
 Dies' coefficient of friction : 0.09 (with lubricant)
 Dies' entrance angle : 11°

Heading (Upsetting):

Wire stock diameter : 3.2 mm
 Die's coefficient of friction : 0.12
 Initial wire stock projection length out of the die : 12 mm
 Bolt's final head diameter : 7.25 mm
 Bolt's final head height : 3 mm

13.1.2 Project Objectives/Requirements for Project 1

You are required to analyze and design a complete manufacturing process that can be applied to manufacture the **bolts** by using the available machines. The requirements of each manufacturing steps are as per the following:

Extrusion

- Determine the ram pressure, force, and power at billet lengths of $L = 285\text{ mm}$, $L = 150\text{ mm}$, $L = 20\text{ mm}$.
- Is it possible to have $L = 0\text{ mm}$?
- What is the final length of the extruded billet?
- At what temperature you prefer to do this process?
- If we switched from the direct extrusion machine to an indirect machine with the same specifications, how much is the saving in power?
- Which one do you prefer, direct or indirect?

Wire Drawing

- How many draw dies and capstan drums do you need to reduce the wire stock from 8 mm to 3.2 mm?
- What type of drawing lubricant should you use?
- How will you prepare the wire sock before drawing?
- Calculate the draw stress and the draw force for first and second steps of reduction.
- What additional processing do you recommend after the second draw? And why?
- What is the final length of the stock wire after exiting the final drawing step?

Heading (Upsetting) Machine

- Depending on the initial and final dimensions of the wire stock, what is the number of operations do you need to perform for this job?
- How many bolts can be manufactured from the wire stock by the wire drawing process?
- Calculate the maximum forging force.

The mechanical behavior data for various metals and alloys are presented in Table 13.1

For cold working, the strain hardening exponent n of an alloy should be in the range of 0.1–0.5.

Table 13.1 The data for n and K for some formable alloys

Alloy	Copper (annealed)	Naval brass (annealed)	MS	304 stainless steel	4340 alloy steel	MgAZ31B (annealed)	2024-T3 Al alloy
n	0.44	0.21	0.21	0.44	0.12	0.16	0.17
K (MPa)	530	585	600	1400	2650	450	780

n strain hardening exponent, MS mild steel

13.1.3 Key Solutions to Project 1

13.1.3.1 Calculations for Extrusion (Direct) Operation

- (a) To determine the ram pressure, the ram force, and power at billet lengths of $L = 285 \text{ mm}$, $L = 150 \text{ mm}$, 20 mm

For billet length, $L = 285 \text{ mm}$

$$\varepsilon = \ln \frac{A_o}{A_f} = \ln \frac{\frac{\pi}{4} D_o^2}{\frac{\pi}{4} D_f^2} = \ln \frac{\frac{\pi}{4} 40^2}{\frac{\pi}{4} 8^2} = \ln \frac{1600}{64} = \ln 25 = 3.2189$$

$$\varepsilon_x = a + b\varepsilon = 0.66 + (1.3 \times 3.2189) = 4.8445$$

$$\bar{Y}_f = \frac{K\varepsilon^n}{1+n} = \frac{780 \times 3.2189^{0.17}}{1+0.17} = \frac{951.49}{1.17} = 813.24 \text{ MPa}$$

$$\begin{aligned} \text{Ram pressure} = p &= \bar{Y}_f \left(\varepsilon_x + \frac{2L}{D_o} \right) = 813.24 \times \left(4.8445 + \frac{2 \times 285}{40} \right) \\ &= 15,528.41 \text{ MPa} \end{aligned}$$

$$\text{Ram force} = F = pA_o = p \left(\frac{\pi}{4} D_o^2 \right) = 15,528.41 \times 0.785 \times 40^2 = 19,503,682.96 \text{ N}$$

$$\text{Power} = P = Fv = 19,503,682.96 \text{ N} \times 0.005 \text{ m/s} = 97,518.415 \text{ W} = 97.518 \text{ kW}$$

For billet length, $L = 150 \text{ mm}$

$$\begin{aligned} \text{Ram pressure} = p &= \bar{Y}_f \left(\varepsilon_x + \frac{2L}{D_o} \right) = 813.24 \times \left(4.8445 + \frac{2 \times 150}{40} \right) \\ &= 10,039.04 \text{ MPa} \end{aligned}$$

$$\text{Ram force} = F = pA_o = p \left(\frac{\pi}{4} D_o^2 \right) = 10,039.04 \times 0.785 \times 40^2 = 12,609,035.72 \text{ N}$$

$$\text{Power} = P = Fv = 12,609,035.72 \text{ N} \times 0.005 \text{ m/s} = 63,045.17 \text{ W} = 63.045 \text{ kW}$$

For billet length, $L = 20 \text{ mm}$

$$\text{Ram pressure} = p = \bar{Y}_f \left(\varepsilon_x + \frac{2L}{D_o} \right) = 813.24 \times \left(4.8445 + \frac{2 \times 20}{40} \right) = 4752.98 \text{ MPa}$$

$$\text{Ram force} = F = pA_o = p \left(\frac{\pi}{4} D_o^2 \right) = 4752.98 \times 0.785 \times 40^2 = 5,969,742.88 \text{ N}$$

$$\text{Power} = P = Fv = 5,969,742.88 \text{ N} \times 0.005 \text{ m/s} = 29,848.71 \text{ W} = 29.848 \text{ kW}$$

(b) Yes! It is possible to have $L = 0$.

(c) *To determine the final length of the extruded billet*

$$\text{Volume of the original billet} = V_o = \frac{\pi}{4} D_o^2 h = \frac{\pi}{4} \times 40^2 \times 285 = 357,960 \text{ mm}^3$$

$$\text{Volume of the extruded billet} = \frac{\pi}{4} D_o^2 h = \frac{\pi}{4} \times 8^2 \times L = 50.24 L$$

$$\text{Volume of the original billet} = \text{Volume of the extruded billet}$$

$$50.24L = 357,960$$

$$L = \text{Final length of the extruded billet} = 7125 \text{ mm} = 7.125 \text{ m}$$

(d) *At what temperature the alloy should be processed (extruded)?*

Since the strain hardening exponent ($n = 0.17$) of aluminum alloy 2024-T3 is in the range of 0.10–0.50, it can be processed by CW (cold extrusion) at ambient temperature (see Chap. 2).

(e) *If you switch from the direct extrusion machine to an indirect machine with the same specifications, how much will be the saving in power?*

For indirect extrusion,

$$\text{Ram pressure} = p = \bar{Y}_f \epsilon_x = 813.24 \text{ MPa} \times 4.8445 = 3939.74 \text{ MPa}$$

$$\text{Ram force} = F = pA_o = p\left(\frac{\pi}{4} D_o^2\right) = 3939.74 \times 0.785 \times 40^2 = 4948314.922 \text{ N}$$

$$\text{Power} = P = Fv = 4,948,314.922 \text{ N} \times 0.005 \text{ m/s} = 24,741.57 \text{ W} = 24.741 \text{ kW}$$

For direct extrusion, power (based on $L = 285 \text{ mm}$) was calculated to be 97.518 kW

%Saving in power

$$= \frac{(\text{Power in direct extrusion}) - (\text{Power in indirect extrusion})}{\text{Power in direct extrusion}} \times 100$$

$$\% \text{Saving in power} = \frac{97.518 - 24.741}{97.518} \times 100 = 74.63$$

Hence, the saving in power would be 74.63%.

(f) *Which one do you prefer, direct or indirect?*

In view of the great saving in the power, I would prefer indirect extrusion.

13.1.3.2 Calculations for Wire Drawing Operation

- (a) *How many dies and capstan drums do you need to reduce the wire from 8 mm to 3.2 mm?*

Area reduction = $r = 0.2$, $D_{in} = D_o = 8$ mm, $D_{out} = D_f = 3.2$ mm.

We can determine the diameter-out for each draw die by using Eq. 8.11, as follows.

First draw die

$$D_{out} = D_{in} \sqrt{1 - r} = 8 \times \sqrt{1 - 0.2} = 8 \times 0.894 = 7.155 \text{ mm}$$

Second draw die

$$D_{out} = D_{in} \sqrt{1 - r} = 7.155 \times \sqrt{1 - 0.2} = 6.3969 \text{ mm}$$

Third draw die

$$D_{out} = D_{in} \sqrt{1 - r} = 6.3969 \times \sqrt{1 - 0.2} = 5.7188 \text{ mm}$$

Fourth draw die

$$D_{out} = D_{in} \sqrt{1 - r} = 5.7188 \times \sqrt{1 - 0.2} = 5.11 \text{ mm}$$

Fifth draw die

$$D_{out} = D_{in} \sqrt{1 - r} = 5.11 \times \sqrt{1 - 0.2} = 5.11 \times 0.894 = 4.571 \text{ mm}$$

Sixth draw die

$$D_{out} = D_{in} \sqrt{1 - r} = 4.571 \times \sqrt{1 - 0.2} = 4.571 \times 0.894 = 4.086 \text{ mm}$$

Seventh draw die

$$D_{out} = D_{in} \sqrt{1 - r} = 4.086 \times \sqrt{1 - 0.2} = 4.086 \times 0.894 = 3.65 \text{ mm}$$

Eighth draw die

$$D_{out} = D_{in} \sqrt{1 - r} = 3.65 \times 0.894 = 3.26 \text{ mm}$$

Since the D_{out} for the eighth draw die is very close to the final diameter ($D_f = 3.2$ mm), the number of dies required = 8. The number of capstan drums required = $8 + 1 = 9$ (see Equation 8.12).

(b) *What type of lubricant would you use?*

Wet lubricant drawing should be used, since it significantly reduces drawing force and power.

(The dies should be completely immersed in lubricant).

(c) *How will you prepare the wire sock before drawing?*

The wire can be prepared by shrinking the beginning end of it, by hammering, filing, rolling, or swaging so that it will fit through the die; the wire is then pulled through the die.

(d) *Calculate the draw stress and draw force for the first and second steps of reduction.*

Dies' coefficient of friction = $\mu = 0.09$ (with lubricant), die angle = $\alpha = 11^\circ$.

First step of reduction

$$D_{\text{in}} = D_o = 8 \text{ mm}, \quad D_{\text{out}} = 7.155 \text{ mm}$$

$$D = \frac{D_{\text{in}} + D_{\text{out}}}{2} = \frac{8 + 7.155}{2} = 7.577 \text{ mm}, \quad L_c = \frac{D_{\text{in}} - D_{\text{out}}}{2 \sin \alpha} = \frac{8 - 7.155}{2 \sin 11^\circ} = \frac{0.845}{0.3816} = 2.214 \text{ mm}$$

$$\phi = 0.88 + \frac{0.12D}{L_c} = 0.88 + \frac{0.12 \times 7.577}{2.214} = 0.88 + 0.41 = 1.29$$

$$\epsilon = \ln \frac{A_{\text{in}}}{A_{\text{out}}} = \ln \frac{\frac{\pi}{4} D_{\text{in}}^2}{\frac{\pi}{4} D_{\text{out}}^2} = \ln \frac{\frac{\pi}{4} 8^2}{\frac{\pi}{4} 7.155^2} = \ln \frac{64}{51.194} = \ln 1.25 = 0.223$$

$$\bar{Y}_f = \frac{K \epsilon^n}{1 + n} = \frac{780 \times 0.223^{0.17}}{1 + 0.17} = 516.56 \text{ MPa}$$

$$\sigma_d = \bar{Y}_f \left(1 + \frac{\mu}{\tan \alpha} \right) \phi \epsilon = 516.56 \times \left(1 + \frac{0.09}{\tan 11^\circ} \right) \times 1.29 \times 0.223$$

$$\sigma_d = 516.56 \times 1.464 \times 0.2877 \text{ MPa}$$

$$\text{Draw stress} = \sigma_d = 217.55 \text{ MPa}$$

$$\text{Draw force} = \sigma_d A_{\text{out}} = \sigma_d \left(\frac{\pi}{4} D_{\text{out}}^2 \right) = 217.55 \times (0.785 \times 7.155^2) = 8742.75 \text{ N}$$

Second step of reduction

$$D_{\text{in}} = 7.155 \text{ mm}, \quad D_{\text{out}} = 6.397 \text{ mm}$$

$$\begin{aligned}
 D &= \frac{D_{in} + D_{out}}{2} = \frac{7.155 + 6.397}{2} = 6.776 \text{ mm}, & L_c &= \frac{D_o - D_f}{2 \sin \alpha} \\
 &= \frac{7.155 - 6.397}{2 \sin 11^\circ} = \frac{0.758}{0.3816} = 1.986 \text{ mm} \\
 \emptyset &= 0.88 + \frac{0.12D}{L_c} = 0.88 + \frac{0.12 \times 6.776}{1.986} = 0.88 + 0.409 = 1.2894 \\
 \varepsilon &= \ln \frac{A_o}{A_f} = \ln \frac{\frac{\pi}{4} D_{in}^2}{\frac{\pi}{4} D_{out}^2} = \ln \frac{\frac{\pi}{4} 7.155^2}{\frac{\pi}{4} 6.397^2} = \ln \frac{51.194}{40.956} = \ln 1.25 = 0.2231
 \end{aligned}$$

Total strain after second draw = strain in first draw + strain in second draw
 $= 0.223 + 0.2231 = 0.4461$

$$\bar{Y}_f = \frac{K \varepsilon^n}{1 + n} = \frac{780 \times 0.4461^{0.17}}{1 + 0.17} = 581.182 \text{ MPa}$$

$$\sigma_d = \bar{Y}_f \left(1 + \frac{\mu}{\tan \alpha} \right) \emptyset \varepsilon = 581.182 \times \left(1 + \frac{0.09}{\tan 11^\circ} \right) \times 1.2894 \times 0.4461$$

$$\sigma_d = 581.182 \times 1.464 \times 0.4461 = 379.56 \text{ MPa}$$

$$\text{Draw stress} = \sigma_d = 379.56 \text{ MPa}$$

$$\text{Draw force} = \sigma_d A_f = \sigma_d \left(\frac{\pi}{4} D_{out}^2 \right) = 379.56 \times (0.785 \times 6.397^2) = 12,192.78 \text{ N}$$

- (e) *What additional processing should be recommended after second draw? And why?*

I would recommend recrystallization annealing of the metal after second draw because the draw force has significantly increased. One way of annealing is to pass current after each draw.

- (f) *What is the final length of the stock wire after exiting the final drawing step?*

L_o = Length of the stock (extruded billet) = 7125 mm

D_o = Diameter of the stock (extruded billet) = 8 mm

D_f = Diameter of the final wire after drawing = 3.2 mm

L_f = Length of the final wire after drawing = ?

Volume of the stock (extruded billet) = Volume of the drawn wire

$$L_o \left(\frac{\pi}{4} D_o^2 \right) = L_f \left(\frac{\pi}{4} D_f^2 \right)$$

$$L_o (D_o^2) = L_f (D_f^2)$$

$$L_f = \frac{L_o D_o^2}{D_f^2} = \frac{7125 \times 8^2}{3.2^2} = \frac{456,000}{10.24} = 44,531.25 \text{ mm} = 44.531 \text{ m}$$

The final length of the wire after drawing = 44,531.25 mm = 44.531 m.

13.1.3.3 Calculations for Heading (Upset Forging) Operation

(a) *How many operations are needed to perform heading operation?*

Initial wire stock projection length out of the die = $h_o = 12$ mm, $D_o = D = 3.2$ mm, $h_f = 3$ mm

$$\frac{h_o}{D} = \frac{12}{3.2} = 3.75 > 2$$

Since $\frac{h_o}{D} > 2$, we assume two operations to perform this job of upset forging.

For first operation,

$$h_o = h_{o1} = 12 \text{ mm}, \quad h_f = h_{f1}$$

$$\text{where } h_{f1} = \frac{h_o - h_f}{2} = \frac{12 - 3}{2} = 4.5 \text{ mm}$$

$$D = D_o = 3.2 \text{ mm}, \quad D_f = D_{f1}$$

D_{f1} can be calculated by constant-volume relationship

$$h_{o1} \left(\frac{\pi}{4} D_o^2 \right) = h_{f1} \left(\frac{\pi}{4} D_{f1}^2 \right)$$

$$h_{o1} (D_o^2) = h_{f1} (D_{f1}^2)$$

$$D_{f1}^2 = \frac{h_{o1} D_o^2}{h_{f1}} = \frac{12 \times 3.2^2}{4.5} = 27.3 \text{ mm}$$

$$D_{f1} = 5.22 \text{ mm}$$

For second operation,

$$h_o = h_{f1} = 4.5 \text{ mm}, \quad h_f = h_{f2} = 3 \text{ mm}$$

$$D_o = D_{f1} = 5.22 \text{ mm}, \quad D_f = D_{f2}$$

$$\text{Checking for } \frac{h_o}{D} = \frac{h_{f1}}{D_{f1}} = \frac{4.5}{5.22} = 0.86 < 2 \rightarrow \text{single operation}$$

Hence, two operations are needed to perform this forging job.

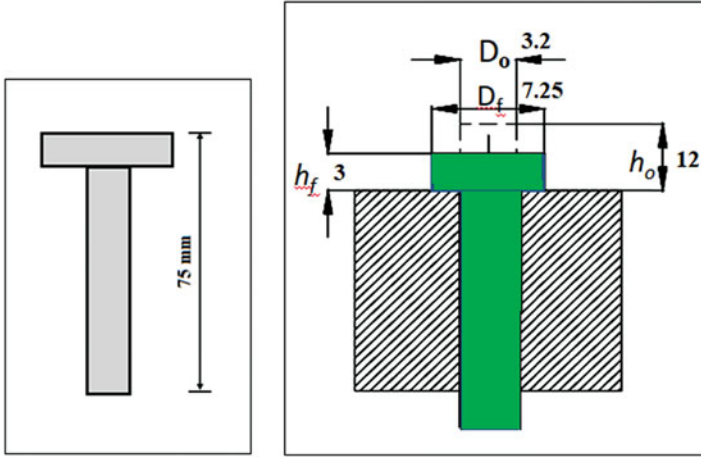


Fig. 13.2 Dimensions of bolt while in the upsetting die

- (b) *How many bolts can you make from the wire stock produced by the wire drawing process?*

$$\text{Volume of wire stock} = L_{\text{wire}} \left(\frac{\pi}{4} D_{\text{wire}}^2 \right) = 44,531.25 \times 0.785 \times 3.2^2 = 357,960 \text{ mm}^3$$

By reference to Fig. 13.2,

$$\text{Volume of a bolt} = \text{Vol. of shank} + \text{Vol. of head}$$

$$\begin{aligned} \text{Volume of a bolt} &= \left(72 \times \frac{\pi}{4} 3.2^2 \right) + \left(3 \times \frac{\pi}{4} 7.25^2 \right) = 578.76 + 123.78 \\ &= 702.54 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} \text{Number of bolts that can be made} &= \frac{\text{Volume of wire stock}}{\text{Volume of a bolt}} = \frac{357,960}{702.54} = 509.52 \\ &\cong 509 \end{aligned}$$

- (c) *Calculating the maximum forging force*

The maximum forging force will be at the final head height and at the final head diameter.

$$K_f = 1 + \frac{0.4 \mu D}{h} = 1 + \frac{0.4 \times 0.12 \times 7.25}{3} = 1.116$$

$$\epsilon = \ln \frac{h_o}{h_f} = \ln \left(\frac{12}{3} \right) = \ln 4 = 1.386$$

$$Y_f = K \varepsilon^n = 780 \times 1.386^{0.17} = 824.51 \text{ MPa}$$

$$A_f = \frac{\pi}{4} D_f^2 = 0.785 \times 7.25^2 = 41.261 \text{ mm}^2$$

$$F = K_f Y_f A_f = 1.116 \times 824.51 \times 41.261 = 37,966.95 \text{ N}$$

The maximum forging force = 37966.95 N $\cong$ 38 kN.

13.2 Metal Forming Project 2: Cup Manufacturing

13.2.1 Project Description of Project 2

As a manufacturing engineer in a metal forming industry, you are requested to manufacture a cup as shown in Fig. 13.3. The available material in the market is naval brass (annealed), in the form of the plate, as shown in Fig. 13.4.

Fig. 13.3 The brass cup dimensions for Metal Forming Project 2 (dimensions in mm)

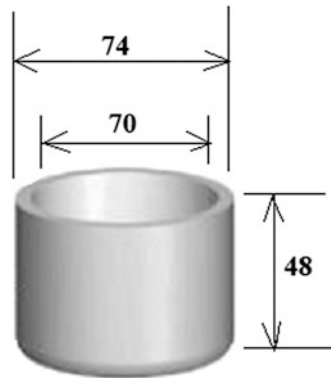
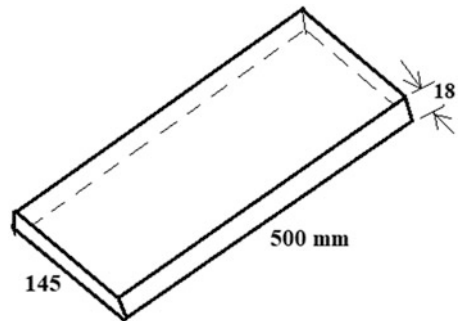


Fig. 13.4 The brass plate dimensions for Metal Forming Project 2 (dimensions in mm)



The following machines are at your disposal: (1) a rolling mill, (2) a shearing machine, (3) a blanking machine, and (4) a deep drawing machine with a die-corner radius of 5 mm.

The **rolling mill**'s specifications are as follows:

- Type of rolling mill: 2-high reversing mill for flat rolling
- Roll radius = 800 mm
- Roll rotational speed = 70 rev/min
- Coefficient of friction between rolls and work = 0.13
- In case of multiple passes, the draft is equal for each pass.

13.2.2 Project Objectives/Requirements for Project 2

You are required to analyze and design the complete manufacturing process (set of manufacturing operations) that can be applied to manufacture the cup by using the available machines. Ignore any change in thickness of the blank during deep drawing. The requirements of each manufacturing steps are as follows:

Rolling

Assume that you have a 2-high mill for flat rolling. Find the following:

- Determine the number of rolling passes required, and the draft for each pass. Assume draft is equal for each pass.
- Determine the rolling force, the rolling torque, and the rolling power for each pass.
- Determine the time taken in rolling for each pass.
- Estimate the total cost of rolling, if the electrical energy cost is SR 0.20/kwh.
- At what temperature you prefer to do this process and why?
- Assuming no change in width during rolling, what is the final length of the rolled sheet?

Shearing

- What clearance must be allowed in the shearing operation?
- Determine the shearing force to cut a square piece out of the rolled sheet.
- What are the dimensions of the sheared sheet?
- How many square pieces can be sheared out of the rolled sheet?

Blanking

- Determine the blanking force
- Determine the blanking punch diameter, and the blanking die diameter

Deep drawing

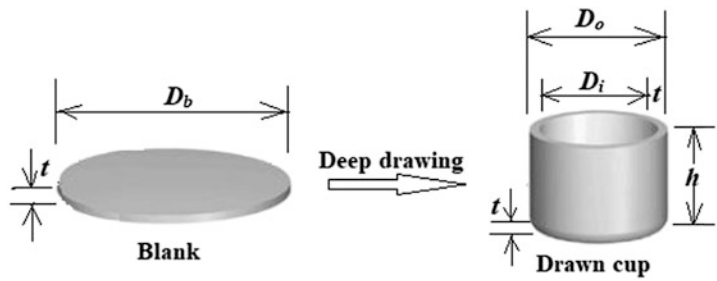
- Determine the punch diameter, and the die cavity diameter
- Is the deep drawing operation feasible?
- Determine the drawing force and the blank holder force (BHF).

Overall

- How many cups can you manufacture by using the given starting plate?
- Do you expect any defect in the manufactured cup? Why?
- Draw a flow chart showing the sequence of metal forming operations in manufacturing the cup, and the dimensions of the work after each operation.

Hint

A cup is manufactured by deep drawing a blank, as shown in Fig. 12.7, which is reproduced below.



$$\text{Volume of drawn cup} = \left(\frac{\pi}{4} \cdot D_o^2 \cdot t \right) + \left\{ \frac{\pi}{4} (D_o^2 - D_i^2) (h - t) \right\} \quad (13.1)$$

The diameter of blank can be determined by (Tables 13.2 and 13.3):

Table 13.2 Clearance allowance (A_c) data for shearing of various materials

Material	Aluminum alloys 1100S and 5052S	Cold-rolled (CR) mild steels and stainless steels (half hard, full hard)	Aluminum alloys 2024ST and 6061ST; brass, soft CR steel, soft stainless steel
Clearance allowance	0.045	0.075	0.06

Table 13.3 Tensile strength of some metals and alloys

Material	Copper	Nickel	Aluminum	Mild steel	Brass (annealed)	Titanium	Cast iron
Yield strength (σ_{ys}), MPa	70	140	35	200–400	110	450	100–150
Ultimate tensile strength (σ_{UTS}), MPa	200	480	90	400–600	330	520	300–900

$$D_b = \sqrt{D_o^2 + \frac{(D_o^2 - D_i^2)(h - t)}{t}}$$

13.2.3 Key Solutions to Project 2

13.2.3.1 Determination of Blank Diameter

The cup dimensions are:

$h = 48$ mm, $D_i = 70$ mm, $D_o = 74$ mm.

It would be useful to first determine the blank diameter. The wall thickness of the cup can be found as follows.

$$t = \frac{D_o - D_i}{2} = \frac{74 - 70}{2} = 2 \text{ mm}$$

By using Eq. 12.12,

$$\begin{aligned} D_b &= \sqrt{D_o^2 + \frac{(D_o^2 - D_i^2)(h - t)}{t}} = \sqrt{74^2 + \frac{(74^2 - 70^2)(48 - 2)}{2}} \\ &= \sqrt{5476 + \frac{(5476 - 4900)(48 - 2)}{2}} \\ D_b &= \sqrt{5476 + \frac{(5476 - 4900)(48 - 2)}{2}} = \sqrt{5476 + 13248} = 136.8 \text{ mm} \end{aligned}$$

The blank diameter = 136.8 mm $\cong$ 137 mm.

13.2.3.2 Calculations for the Rolling Operation

(a) *Determining the number of rolling passes required, and the draft for each pass.*

$R = 800$ mm, $\mu = 0.13$, $N = 70$ rev/min = 1.16 rev/s, $t_o = 18$ mm, $t_f = 2$ mm, $w = 145$ mm, $K = 585$ MPa, $n = 0.21$

By using Eqs. 5.3, 5.18, and 5.20,

$$\text{Draft} = d = t_o - t_f = 18 - 2 = 16 \text{ mm}$$

$$d_{\max} = \mu^2 R = 0.13^2 \times 800 = 13.52 \text{ mm}$$

$$\text{Minimum number of rolling passes} = \frac{d}{d_{\max}} = \frac{16}{13.52} = 1.18 \rightarrow 2$$

There must be two rolling passes because the number of passes must be a whole number.

$$\frac{d}{d_{\max}} = 2$$

$$d_{\max} = \frac{d}{2} = \frac{16}{2} = 8 \text{ mm}$$

Draft for each pass = $d = 8 \text{ mm}$

Draft for first pass = Draft for second pass = 8 mm

(b) *Determining the rolling force, the rolling torque, and the rolling power for each pass*

Let the thickness after first pass = t_1 .

Draft for first pass = $t_o - t_1$

$$8 = 18 - t_1$$

$$t_1 = 18 - 8 = 10 \text{ mm}$$

The thickness after first pass = $t_1 = 10 \text{ mm}$

The thickness after second pass = $t_2 = t_1 - 8 = 10 - 8 = 2 \text{ mm} = t_f$

$$L_c = \sqrt{Rd} = \sqrt{800 \times 8} = 80 \text{ mm} = 0.08 \text{ m}$$

For first pass

$$\epsilon_1 = \ln \frac{t_o}{t_1} = \ln \frac{18}{10} = \ln 1.8 = 0.587$$

$$\overline{Y}_{f1} = \frac{K \cdot \epsilon_1^n}{1 + n} = \frac{585 \times 0.587^{0.21}}{1 + 0.21} = 432.3 \text{ MPa}$$

Rolling force in first pass = $F_1 = \overline{Y}_{f1} \cdot w \cdot L_c = 432.3 \times 145 \times 80 = 5014680 \text{ N}$

Rolling torque in first pass = $T_1 = F_1 \cdot L_c = 5,014,680 \times 0.08 = 401,174.4 \text{ N} \cdot \text{m}$

Power in 1st rolling pass = $P_1 = \omega \cdot T_1 = 2\pi \cdot N \cdot T_1 = 2\pi \times 1.16 \times 401,174.4 = 2,923,957.6 \text{ W}$

$$P_1 = 2,923,957.6 \text{ W} = 2923.958 \text{ kW}$$

For second pass

$$\epsilon_2 = \ln \frac{t_1}{t_f} = \ln \frac{10}{2} = \ln 5 = 1.61$$

$$\text{Total strain after second pass} = \epsilon_1 + \epsilon_2 = 0.587 + 1.61 = 2.197$$

$$\overline{Y}_{f2} = \frac{K \cdot \epsilon^n}{1+n} = \frac{585 \times 2.197^{0.21}}{1+0.21} = 570.37 \text{ MPa}$$

$$\begin{aligned} \text{Rolling force in second pass} &= F_2 = \overline{Y}_{f2} \cdot w \cdot L_c = 570.37 \times 145 \times 80 \\ &= 6,616,263.56 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Rolling torque in second pass} &= T_2 = F_2 \cdot L_c = 6,616,263.56 \times 0.08 \\ &= 529,301.1 \text{ N} \cdot \text{m} \end{aligned}$$

$$\begin{aligned} \text{Power in second rolling pass} &= P_2 = 2\pi \cdot N \cdot T_2 = 2\pi \times 1.16 \times 529,301.1 \\ &= 3,857,808.29 \text{ W} \end{aligned}$$

$$P_2 = 3,857,808.29 \text{ W} = 3857.8083 \text{ kW}$$

(c) *Determining the time taken in rolling for each pass.*

$$\text{Roll surface speed} = v = \omega \cdot R = 2\pi \cdot N \cdot R = 2\pi \times 1.16 \times 0.08 = 0.583 \text{ m/s}$$

$$\text{Time} = \frac{\text{Length rolled}}{\text{Speed}} = \frac{\text{Length rolled}}{0.583 \text{ m/s}}$$

First rolling pass

$$\begin{aligned} \text{Time in first rolling pass} &= \frac{\text{Length rolled}}{\text{Speed}} = \frac{L_o}{0.583 \text{ m/s}} = \frac{0.5 \text{ m}}{0.583 \text{ m/s}} \\ &= 0.858 \text{ s} = 238.33 \times 10^{-6} \text{ h} \end{aligned}$$

Second rolling pass

$$L_o w t_o = L_1 w t_1$$

$$L_1 = \frac{L_o \cdot t_o}{t_1} = \frac{500 \times 18}{10} = 900 \text{ mm} = 0.9 \text{ m}$$

$$\begin{aligned} \text{Time in second rolling pass} &= \frac{\text{Length rolled}}{\text{Speed}} = \frac{L_1}{v} = \frac{0.9 \text{ m}}{0.583 \text{ m/s}} = 1.544 \text{ s} \\ &= 428.88 \times 10^{-6} \text{ h} \end{aligned}$$

(d) *Estimating the total cost of rolling, if the electrical energy cost is US\$ 0.12/kwh*

$$\begin{aligned} \text{Energy consumed in first rolling pass} &= P_1 \times \text{time}_1 = 2923.958 \text{ kW} \times \\ &238.33 \times 10^{-6} \text{ h} = 0.697 \text{ kWh} \end{aligned}$$

$$\text{Cost of rolling in first pass} = \$ 0.12/\text{kWh} \times 0.697 \text{ kWh} = \$ 0.084$$

$$\begin{aligned} \text{Energy consumed in second pass} &= P_2 \times \text{time}_2 = 3857.8083 \text{ kW} \times 428.88 \times 10^{-6} \text{ h} \\ &= 1.654 \text{ kWh} \end{aligned}$$

$$\text{Cost of rolling in second pass} = \$ 0.12/\text{kWh} \times 1.654 \text{ kWh} = \$ 0.198$$

$$\text{Total cost of rolling} = \$ 0.084 + \$ 0.198 = \$ 0.2825$$

(e) *At what temperature you prefer to do rolling and why?*

The strain hardening exponent of brass is 0.21, which is well in the range of 0.1–0.5. Hence, rolling should be done cold rolling.

(f) *Assuming no change in width during rolling, what is the final length of the rolled sheet?*

$$\text{Pre-rolling volume} = \text{post-rolling volume}$$

$$L_o w t_o = L_f w t_f$$

$$L_f = \frac{L_o \cdot t_o}{t_f} = \frac{500 \times 18}{2} = 4500 \text{ mm} = 4.5 \text{ m}$$

$$\text{Final length of the rolled sheet} = 4500 \text{ mm} = 4.5 \text{ m.}$$

13.2.3.3 Calculations for Shearing Operation

(a) *What clearance must be allowed in the shearing operation?*

$$\text{Clearance allowance for naval brass} = A_c = 0.06 \text{ (see Table 13.2).}$$

$$\text{Clearance} = c = A_c \cdot t = 0.06 \times 2 = 0.12 \text{ mm}$$

(b) *Determining the shearing force to cut a square piece out of the rolled sheet.*

Length of cut = $L = 145 \text{ mm}$ (see Fig. 13.3), $\sigma_{\text{UTS}} = 330 \text{ MPa}$ (see Table 13.3)

$$\text{Shearing force} = F_s = 0.7 \sigma_{\text{UTS}} \cdot t \cdot L = 0.7 \times 330 \times 2 \times 145 = 66,990 \text{ N}$$

(c) *What are the dimensions of the sheared sheet?*

$$\text{Dimensions of the sheared sheet} = 145 \text{ mm} \times 145 \text{ mm} \times 2 \text{ mm}$$

(d) *How many square pieces can be sheared out of the rolled sheet?*

$$\text{Volume of the rolled sheet} = 4500 \text{ mm} \times 145 \text{ mm} \times 2 \text{ mm} = 1,305,000 \text{ mm}^3$$

$$\text{Volume of a square piece} = 145 \text{ mm} \times 145 \text{ mm} \times 2 \text{ mm} = 42,050 \text{ mm}^3$$

$$\text{Number of square pieces that can be sheared} = \frac{1,305,000}{42,050} = 31$$

13.2.3.4 Calculations for Blanking Operation

(a) *Determining the blanking force*

Diameter of blank = $D_b = 137 \text{ mm}$ (see Sect. 13.2.3.1)

$$\text{Length of cut for blanking} = L = \pi \cdot D_b = \pi \times 137 = 430.4 \text{ mm}$$

$$\text{Blanking force} = 0.7\sigma_{\text{UTS}} \cdot t \cdot L = 0.7 \times 330 \times 2 \times 430.4 = 198,843.96 \text{ N}$$

(b) *Determining the blanking punch diameter and the blanking die diameter*

$$\text{Blanking punch diameter} = D_b - 2c = 137 - (2 \times 0.12) = 136.76 \text{ mm}$$

$$\text{Blanking die diameter} = D_b = 137 \text{ mm}$$

13.2.3.5 Calculations for Deep Drawing

(a) *Determining punch diameter, drawing ratio, and reduction for the deep drawing operation*

$$\text{Punch diameter} = D_p = \text{inside diameter of the cup} = 70 \text{ mm}$$

$$\text{Drawing ratio} = \text{DR} = \frac{D_b}{D_p} = \frac{137}{70} = 1.96$$

$$\text{Reduction} = r = \frac{D_b - D_p}{D_b} = \frac{137 - 70}{137} = 0.489$$

(b) *Is the deep drawing operation feasible?*

$$\text{Thickness-to-diameter ratio} = \frac{t}{D_b} = \frac{2}{137} = 0.014 = 1.4\%$$

$$\text{DR} = 1.96 < 2$$

$$r = 0.489 < 0.5$$

$$\frac{t}{D_b} = 1.4\% > 1\%$$

Hence, the deep drawing operation is feasible.

(c) *Determining the drawing force and the blank holder force (BHF)*

$D_p = 70$ mm, $D_b = 137$ mm, $\sigma_{UTS} = 330$ MPa, $t = 0.2$ mm, $\sigma_{ys} = 110$ MPa, $R_d = 5$ mm.

$$F = \pi \cdot D_p \cdot t \cdot \sigma_{UTS} \cdot \left(\frac{D_b}{D_p} - 0.7 \right) = \pi \times 70 \times 2 \times 330 \times \left(\frac{137}{70} - 0.7 \right)$$

$$= 182,463.7 \text{ N}$$

The drawing force $= F = 182,463.7 \text{ N} \cong 182.46 \text{ kN}$

The blank holder force F_h is calculated as follows:

$$F_h = (0.015\pi \cdot \sigma_{ys}) \cdot \left[D_b^2 - (D_p + 2.2t + 2R_d)^2 \right] = 0.015\pi \times 110$$

$$\times \left[137^2 - (70 + 2.2 \times 2 + 2 \times 5)^2 \right]$$

$$F_h = 5.1836 \times \left[137^2 - (84.4)^2 \right] = 5.1836 \times (18,769 - 7123.36) = 60,366.34 \text{ N}$$

(d) The blank holder force (BHF) $= 60,366.34 \text{ N} \cong 60.366 \text{ kN}$

13.2.3.6 Other Key Solutions to Project 2

Calculations for Number of Cups to Be Manufactured

$D_o = 74$ mm, $D_i = 70$ mm, $h = 48$ mm, $t = 2$ mm

By using Eq. 13.1,

$$\text{Volume of drawn cup} = \left(\frac{\pi}{4} \cdot D_o^2 \cdot t \right) + \left\{ \frac{\pi}{4} (D_o^2 - D_i^2) (h - t) \right\}$$

$$\text{Volume of cup} = (0.785 \times 74^2 \times 2) + \{0.785 \times (74^2 - 70^2) (48 - 2)\}$$

$$\text{Volume of a cup} = 8597.32 + \{0.785 \times (5476 - 4900) \times 46\} = 8597.32$$

$$+ 20,799.36 = 29396.68 \text{ mm}^3$$

$$\text{Volume of starting plate} = 500 \text{ mm} \times 145 \text{ mm} \times 18 \text{ mm} = 1,305,000 \text{ mm}^3$$

$$\text{No. of cups drawn} = \frac{1,305,000}{29,396.68} = 44.39$$

The number of cups that can be manufactured $= 44$.

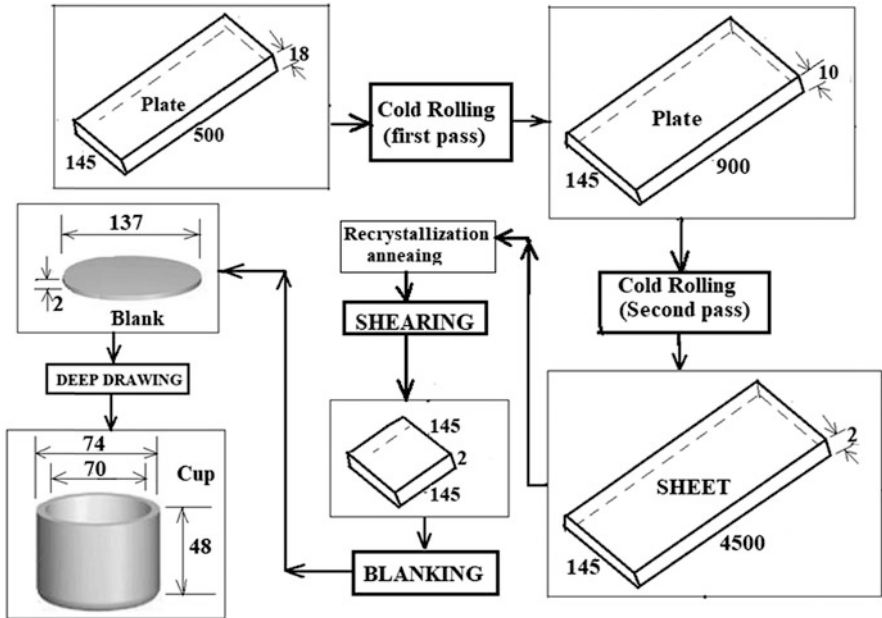


Fig. 13.5 The flow chart showing the sequence of operations in manufacturing the cup (dimensions in mm)

Checking for Defects

Since the feasibility of the deep drawing operation has been confirmed, I do not expect any defect in the deep drawn cup.

The Process Flow Chart for Manufacturing the Cup

The flow chart showing the sequence of metal forming operations in manufacturing the cup, and the dimensions of the work after each operation is presented in Fig. 13.5.

Practice Projects

- 13.1. Solve the metal forming project 1 using naval brass instead of aluminum alloy 2024-T3.
- 13.2. Solve the metal forming project 2 using copper instead of naval brass.

Chapter 14

Worked Metal-Forming-Cum-Business Project



14.1 The Project Description

Assume you are a manufacturing manager in a metal-forming industry. You are requested to manufacture the L-shaped beam, as shown in Fig. 14.1. You have to analyze manufacturing economics of the beam in three different methods of manufacturing processes by using the equipment available in your workshop. The available material in the market is an annealed aluminum alloy, and it comes in two shapes: (a) as a plate with the following dimensions: length = 550 mm, width = 200 mm, and thickness = 15 mm, and (b) as a cylindrical billet with a diameter of 120 mm and length of 100 mm. The material's properties are presented in Table 14.1.

In a meeting with your production team, you and your team decided to use the available aluminum plate and billet to manufacture the required L-shaped beam by three methods: (a) Method 1 (rolling, shearing, and bending), (b) Method 2 (extrusion and shearing) using a direct extrusion machine, and (c) Method 2 (extrusion and shearing) using an indirect extrusion machine. You decided to compare the three methods from the induced manufacturing costs perspective to explore the most economical method. The proposed methods are given as follows:

(a) *Method 1*

- All steps are cold working.
- Use the aluminum plate and roll it until the final thickness of 3 mm.
- Cut the aluminum sheet into strips with the length of cut = 200 mm.
- Use a press brake to bend the strips to the final shape, as shown in Fig. 14.1.

(b) *Method 2 with Direct Extrusion*

- All steps are cold working.
- Use the aluminum billet and extrude it (by direct extrusion) to obtain the shape as shown in Fig. 14.2.
- Cut the extruded metal into segments to produce the beam, as shown in Fig. 14.1.

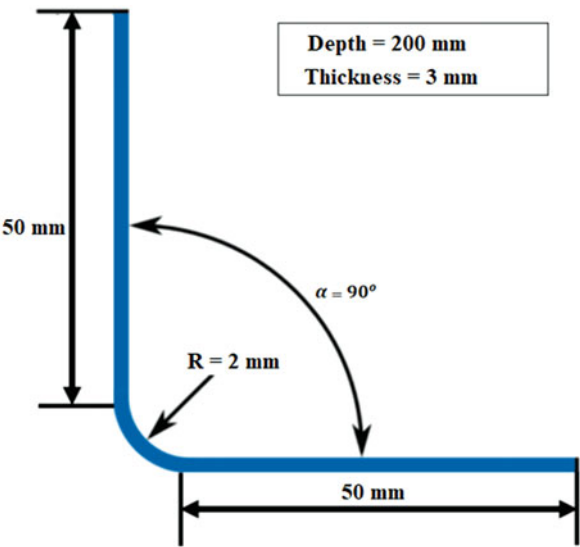


Fig. 14.1 The L-shaped beam geometric details for Chap. 14

Table 14.1 The relevant mechanical properties of the aluminum alloy

Mechanical properties	Values
Strength coefficient, K	240 MPa
Strain hardening exponent, n	0.15
Ultimate tensile strength, σ_{UTS}	455 MPa
Shear strength, τ	274 MPa

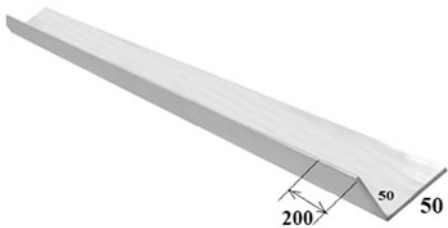


Fig. 14.2 The beam extruded directly from billet

(c) Method 2 with Indirect Extrusion

The following machines are at your disposal in the workshop: (1) rolling mill, (2) shearing machine, (3) extruder, and (4) press brake. The machine and process data is given below.

Rolling

- (a) Roll radius = 300 mm.
- (b) Roll speed = 20 rev/min.
- (c) $\mu = 0.1$.

Extrusion

- (a) Extrusion speed = 5 m/min.

Bending

- (a) V-bending.
- (b) Die opening dimension = 70.7 mm.

14.2 Project Objectives/Requirements

You are required to analyze and design a complete manufacturing process (set of manufacturing processes) that can be applied to manufacture the L-shaped beam by both proposed methods by using the available machines. The requirements of each manufacturing step are as follows:

Method 1*Rolling*

Assume that you have a 2-high mill for flat rolling; and find the following:

- Determine the number of rolling passes required, and the draft for each pass. Assume there is no change in width.
- Determine the rolling force, the rolling torque, and the rolling power for each pass.
- Determine the length after each pass. Assume no widening of the strip during rolling.
- Determine the energy consumed in each rolling pass.
- Determine the total cost of rolling if you know that the electricity tariff for industrial sector in the USA is \$ 0.12/kwh.

Shearing

- Determine the shearing force to cut the required strip out of the rolled sheet.
- What are the dimensions of the sheared strip? (Hint: Consider the bending allowance)
- How many strips can be sheared out from one rolled sheet?
- How many plates do we need to manufacture 25 beams?
- What is the cost of cutting the 25 strips out from the rolled sheet if the cost of shearing is US\$ 4.85/m?

Bending

- Determine the strip's length before bending. ($A_b = 5.5$ mm).
- Determine the bending force.
- Calculate the bending cost for all 25 pieces, if the bending will cost US\$ 5.30/m.

Overall

Draw the process flow chart showing the dimensions of the work after each step of operation.

Method 2*Extrusion*

- If the machine available is a direct continuous machine, calculate the extrusion force and power (take $a = 0.8$, $b = 1.5$).
- For the direct extrusion machine, calculate the cost of manufacturing the extruded part. (Use the rate \$ 0.12/kwh).
- If we replace the direct extrusion machine by indirect extrusion machine, how much is the saving in power?

Cutting

- How many beams can we have from the extruded part if we shear segments, as shown in Fig. 14.2?
- What is the cost of shearing the segments if you know the shearing cost is \$2.00/piece?

Overall

Calculate the cumulative manufacturing costs for each method and identify which method is the most economical, and why.

14.3 Key Solution for Method 1**14.3.1 Process Flow Chart for Method 1**

The process flow chart for manufacturing the L-beam, by using Method 1, is illustrated in Fig. 14.3.

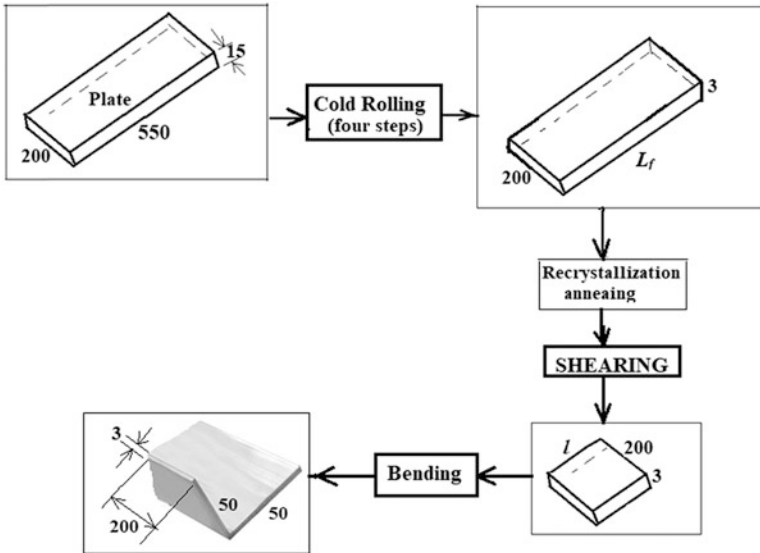


Fig. 14.3 The process flow chart for manufacturing the L-beam by using Method 1

14.3.2 Calculations for Rolling

(a) *Determining the number of rolling passes required, and the draft for each pass*

Aluminum alloy plate dimensions: $L_o = 550$ mm, $w = 200$ mm, $t_o = 15$ mm

$$R = 300 \text{ mm}, N = 20 \text{ rev/min} = \frac{20 \text{ rev}}{60 \text{ s}} = 0.33 \text{ rev/s}, \mu = 0.1, t_o = 15 \text{ mm}, t_f = 3 \text{ mm}$$

$$d_{\max} = \mu^2 R = (0.1)^2 \times 300 = 3 \text{ mm}$$

$$d = t_o - t_f = 15 - 3 = 12 \text{ mm}$$

Since $d_{\max} < d$, rolling is not feasible in single pass.

$$\text{Minimum number of rolling passes} = \frac{d}{d_{\max}} = \frac{12}{3} = 4$$

The number of rolling passes required = 4

The draft for each pass = 3 mm.

(b) *Determining the rolling force, the rolling torque, and the rolling power for each pass*

Original thickness = $t_o = 15$ mm; Let the thickness after 1st pass = t_1 .

$$\text{Draft for 1st pass} = 3 = t_o - t_1$$

$$t_1 = t_o - 3 = 15 - 3 = 12 \text{ mm}$$

The thickness after 1st pass = $t_1 = 12 \text{ mm}$

The thickness after 2nd pass = $t_2 = t_1 - 3 = 12 - 3 = 9 \text{ mm}$

The thickness after 3rd pass = $t_3 = t_2 - 3 = 9 - 3 = 6 \text{ mm}$

The thickness after 4th pass = $t_4 = t_3 - 3 = 6 - 3 = 3 \text{ mm} = t_f$

$t_o = 15 \text{ mm}$, $t_1 = 12 \text{ mm}$, $t_2 = 9 \text{ mm}$, $t_3 = 6 \text{ mm}$, $t_4 = 3 \text{ mm} = t_f$

$$\text{Contact length in each rolling pass} = L_c = \sqrt{Rd} = \sqrt{300 \times 3} = 30 \text{ mm} = 0.03 \text{ m}$$

First Rolling Pass

$$\varepsilon_1 = \ln \frac{t_o}{t_1} = \ln \frac{15}{12} = \ln 1.25 = 0.2231$$

$$\overline{Y}_{f1} = \frac{K \cdot \varepsilon_1^n}{1 + n} = \frac{240 \times 0.2231^{0.15}}{1 + 0.15} = 166.63 \text{ MPa}$$

$$\text{Rolling force in 1st pass} = F_1 = \overline{Y}_{f1} \cdot w \cdot L_c = 166.63 \times 200 \times 30 = 999793.66 \text{ N}$$

$$\text{Rolling torque in 1st pass} = T_1 = F_1 \cdot L_c = 999793.66 \times 0.03 = 29993.81 \text{ N} \cdot \text{m}$$

$$\text{Power in 1st rolling pass} = P_1 = \omega \cdot T_1 = 2\pi \cdot N \cdot T_1 = 2\pi \times 0.33 \times 29993.81 = 62190.7 \text{ W}$$

$$P_1 = 62190.7 \text{ W} = 62.1907 \text{ kW}$$

Second Rolling Pass

$$\varepsilon_2 = \ln \frac{t_1}{t_2} = \ln \frac{12}{9} = \ln 1.33 = 0.2852$$

$$\text{Total strain after 2nd pass} = \varepsilon_1 + \varepsilon_2 = 0.2231 + 0.2852 = 0.5083$$

$$\overline{Y}_{f2} = \frac{K \cdot \varepsilon^n}{1 + n} = \frac{240 \times 0.5083^{0.15}}{1 + 0.15} = 188.55 \text{ MPa}$$

$$\text{Rolling force in 2nd pass} = F_2 = \overline{Y}_{f2} \cdot w \cdot L_c = 188.55 \times 200 \times 30 = 1131312.715 \text{ N}$$

$$\text{Rolling torque in 2nd pass} = T_2 = F_2 \cdot L_c = 1131312.715 \times 0.03 = 33939.38 \text{ N} \cdot \text{m}$$

$$\text{Power in 2nd rolling pass} = P_2 = 2\pi \cdot N \cdot T_2 = 2\pi \times 0.33 \times 33939.38 = 70371.65 \text{ W}$$

$$P_2 = 70371.65 \text{ W} = 70.3716 \text{ kW}$$

Third Rolling Pass

$$\varepsilon_3 = \ln \frac{t_2}{t_3} = \ln \frac{9}{6} = \ln 1.5 = 0.4055$$

Total strain after 3rd pass = $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0.2231 + 0.2852 + 0.4055 = 0.9138$

$$\overline{Y}_{f3} = \frac{K \cdot \varepsilon^n}{1+n} = \frac{240 \times 0.9138^{0.15}}{1+0.15} = 205.89 \text{ MPa}$$

Rolling force in 3rd pass = $F_3 = \overline{Y}_{f3} \cdot w \cdot L_c = 205.89 \times 200 \times 30 = 1235356.56 \text{ N}$

Rolling torque in 3rd pass = $T_3 = F_3 \cdot L_c = 1235356.56 \times 0.03 = 37060.696 \text{ N} \cdot \text{m}$

Power in 3rd rolling pass = $P_3 = 2\pi \cdot N \cdot T_3 = 2\pi \times 0.33 \times 37060.696$
 $= 76843.54 \text{ W}$

$$P_3 = 76843.54 \text{ W} = 76.8435 \text{ kW}$$

Fourth Rolling Pass

$$\varepsilon_4 = \ln \frac{t_3}{t_f} = \ln \frac{6}{3} = \ln 2 = 0.6931$$

Total strain after 4th pass = $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 = 0.2231 + 0.2852 + 0.4055$
 $+ 0.6931 = 1.6069$

$$\overline{Y}_{f4} = \frac{K \cdot \varepsilon^n}{1+n} = \frac{240 \times 1.6069^{0.15}}{1+0.15} = 224.08 \text{ MPa}$$

Rolling force in 4th pass = $F_4 = \overline{Y}_{f4} \cdot w \cdot L_c = 224.08 \times 200 \times 30 = 1344506.727 \text{ N}$

Rolling torque in 4th pass = $T_4 = F_4 \cdot L_c = 1344506.727 \times 0.03 = 40335.20 \text{ N} \cdot \text{m}$

Power in 4th rolling pass = $P_4 = 2\pi \cdot N \cdot T_4 = 2\pi \times 0.33 \times 40335.20$
 $= 83633.071 \text{ W}$

$$P_4 = 83633.071 \text{ W} = 83.6331 \text{ kW}$$

(c) *Determining the length after each pass. Assume no widening of the strip during rolling* $t_o = 15 \text{ mm}$, $t_1 = 12 \text{ mm}$, $t_2 = 9 \text{ mm}$, $t_3 = 6 \text{ mm}$, $t_4 = 3 \text{ mm} = t_f$

$$\text{The length after 1st pass} = L_1 = \frac{t_o \cdot w_o L_o}{w_1 \cdot t_1} = \frac{t_o \cdot L_o}{t_1} = \frac{15 \times 550}{12} = 687.5 \text{ mm}$$

$$\text{The length after 2nd pass} = L_2 = \frac{t_1 \cdot L_1}{t_2} = \frac{12 \times 687.5}{9} = 916.66 \text{ mm}$$

$$\text{The length after 3rd pass} = L_3 = \frac{t_2 \cdot L_2}{t_3} = \frac{9 \times 916.66}{6} = 1375 \text{ mm}$$

$$\text{The length after 4th (final) pass} = L_f = \frac{t_3 \cdot L_3}{t_f} = \frac{6 \times 1375}{3} = 2750 \text{ mm}$$

(d) *Determining the energy consumed in each rolling pass*

$$\text{Surface speed of roll} = v_r = 2\pi \cdot N \cdot R = 2\pi \times 20 \text{ rev/min} \times 300$$

$$= 37,699.11 \text{ mm/min}$$

$$\text{Time taken in 1st rolling pass} = \text{time}_1 = \frac{L_o}{v_r} = \frac{550}{37699.11} = 0.01459 \text{ min} = 2.4315 \times 10^{-4} \text{ h}$$

$$\text{Time taken in 2nd rolling pass} = \text{time}_2 = \frac{L_1}{v_r} = \frac{687.5}{37699.11} = 0.01823 \text{ min} = 3.0394 \times 10^{-4} \text{ h}$$

$$\text{Time taken in 3rd rolling pass} = \text{time}_3 = \frac{L_2}{v_r} = \frac{916.66}{37699.11} = 0.0243 \text{ min} = 4.0525 \times 10^{-4} \text{ h}$$

$$\text{Time taken in 4th rolling pass} = \text{time}_4 = \frac{L_3}{v_r} = \frac{1375}{37699.11} = 0.0364 \text{ min} = 6.0788 \times 10^{-4} \text{ h}$$

$$\begin{aligned} \text{Energy consumed in 1st pass} &= P_1 \cdot \text{time}_1 = 62.1907 \text{ kW} \times 2.4315 \times 10^{-4} \text{ h} \\ &= 0.01512 \text{ kWh} \end{aligned}$$

$$\begin{aligned} \text{Energy consumed in 2nd pass} &= P_2 \cdot \text{time}_2 = 70.3716 \text{ kW} \times 3.0394 \times 10^{-4} \text{ h} \\ &= 0.0214 \text{ kWh} \end{aligned}$$

$$\begin{aligned} \text{Energy consumed in 3rd pass} &= P_3 \cdot \text{time}_3 = 76.8435 \text{ kW} \times 4.0525 \times 10^{-4} \text{ h} \\ &= 0.0311 \text{ kWh} \end{aligned}$$

$$\begin{aligned} \text{Energy consumed in 4th pass} &= P_4 \cdot \text{time}_4 = 83.6331 \text{ kW} \times 6.0788 \times 10^{-4} \text{ h} \\ &= 0.0508 \text{ kWh} \end{aligned}$$

(e) *Determining the total cost of rolling*

$$\begin{aligned} \text{Total energy consumed in rolling} &= 0.01512 + 0.0214 + 0.0311 + \\ &0.0508 = 0.11845 \text{ kWh} \end{aligned}$$

$$\text{Electricity tariff for industrial sector in the USA} = \$ 0.12/\text{kwh}$$

$$\text{Total cost in rolling} = \$ 0.12/\text{kwh} \times 0.11845 \text{ kWh} = \$ 0.014$$

14.3.3 Calculations for Shearing

(a) *Determining the shearing force to cut the required strip out of the rolled sheet*

$$\text{Length of cut} = L_s = 200 \text{ mm}$$

$$\text{Shearing force for a strip} = F_s = \tau \cdot t \cdot L_s = 274 \times 3 \times 200 = 164400 \text{ N}$$

(b) *Determining the dimensions of the sheared strip*

$$\text{Bend allowance} = A_b = 5.5 \text{ mm (given)}$$

$$\text{Length of the sheared strip (blank for the beam)} = l = 50 + 50 + A_b$$

$$l = 50 + 50 + 5.5 = 105.5 \text{ mm}$$

$$\text{Dimensions of the sheared strip are } 105.5 \text{ mm} \times 200 \text{ mm} \times 3 \text{ mm}$$

(c) *How many strips can be sheared out from one rolled sheet?*

$$\text{Volume of a sheared strip} = 105.5 \text{ mm} \times 200 \text{ mm} \times 3 \text{ mm} = 63,300 \text{ mm}^3$$

$$\text{Volume of the rolled sheet} = 2750 \text{ mm} \times 200 \text{ mm} \times 3 \text{ mm} = 1650000 \text{ mm}^3$$

$$\text{Number of strips that can be sheared} = \frac{1650000}{63,300} = 26 \cong 25$$

(d) *How many plates do we need to manufacture 25 beams?*

Each strip produces a beam. It means that volume of a beam = volume of a strip.

$$\text{Volume of a sheared strip} = 63,300 \text{ mm}^3 = \text{Volume of an L-beam}$$

$$\text{Volume of 25 beams} = \text{Volume of 25 strips} = 25 \times 63300 \text{ mm}^3 = 1582500 \text{ mm}^3$$

$$\text{Volume of a plate} = L w t = 550 \times 200 \times 15 = 1650000 \text{ mm}^3$$

$$\text{No. of plates needed to manufacture 25 beams} = \frac{1650000}{1582500} = 1.04$$

Hence, we need just ONE plate to manufacture 25 beams.

(e) *What is the cost of cutting the 25 strips out from the rolled sheet if the per meter cost of shearing is US\$ 4.75?.*

$$\text{Cost of cutting 1 m} = \$ 4.85$$

$$\text{Length of cut for ONE strip} = 200 \text{ mm}$$

$$\text{Length of cut for 25 strips} = 200 \text{ mm} \times 25 = 5000 \text{ mm} = 5 \text{ m}$$

$$\text{Shearing cost for 25 strips} = \$ 4.85 \times 5 = \$ 24.25$$

14.3.4 Calculations for Bending

(a) *Determining the strip length before bending. ($A_b = 5.5 \text{ mm}$)*

$$\text{Length of the sheared strip (blank for bending the beam)} = l = 50 + 50 + A_b$$

$$l = 50 + 50 + 5.5 = 105.5 \text{ mm}$$

(b) *Determining the bending force*

$$\begin{aligned} \text{V-bending, } K_{bf} &= 1.33, D = 70.7 \text{ mm, } w = 200 \text{ mm, } t = 3 \text{ mm,} \\ \sigma_{\text{UTS}} &= 455 \text{ MPa} \end{aligned}$$

$$\text{Bending force} = F = \frac{K_{bf} \cdot \sigma_{ut} \cdot w \cdot t}{D} = \frac{1.33 \times 455 \times 200 \times 3}{70.7} = 5135.64 \text{ N}$$

(c) *Determining the bending cost for all 25 pieces, if the bending will cost US\$ 5.30/m*

The length bent in 1 strip = 200 mm

The length bent in 25 strips = 5000 mm = 5 m

The cost of bending 1 m = \$ 5.30

The cost of bending 5 m for 25 strips = \$ (5.30 × 5) = \$ 26.50

14.3.5 The Process Flow Chart with Dimensions for Method 1

The process flow chart, showing the dimensions of the work after each step of operation using Method 1, is illustrated in Fig. 14.4.

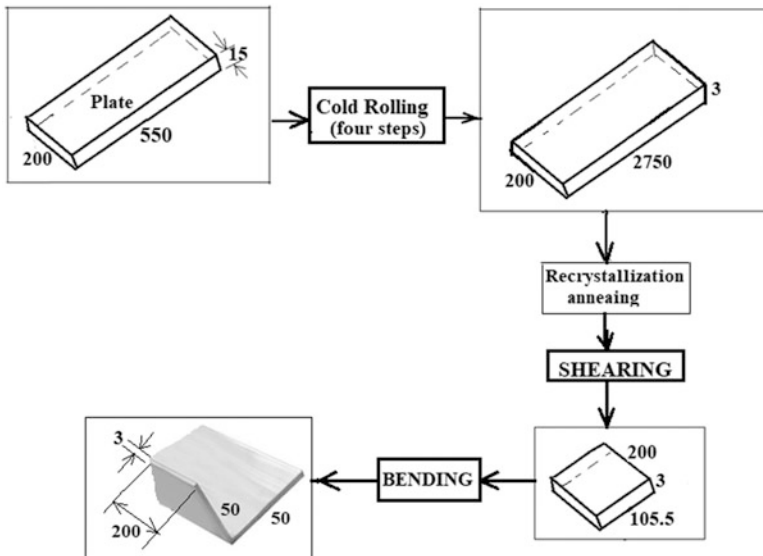


Fig. 14.4 Process flow chart, showing the dimensions of work after each step of operation using Method 1

14.4 Key Solution for Method 2

14.4.1 Process Flow Chart for Method 2

The process flow chart for manufacturing the L-beam by using Method 2 is illustrated in Fig. 14.5.

14.4.2 Calculations for Extrusion

- (a) If the machine available is a direct continuous extrusion machine, calculate the extrusion force and power (take $a = 0.8$, $b = 1.5$)
 $v = 5 \text{ m/min} = 0.083 \text{ m/s}$, $D_o = 120 \text{ mm}$, $L = 100 \text{ mm} = 0.1 \text{ m}$

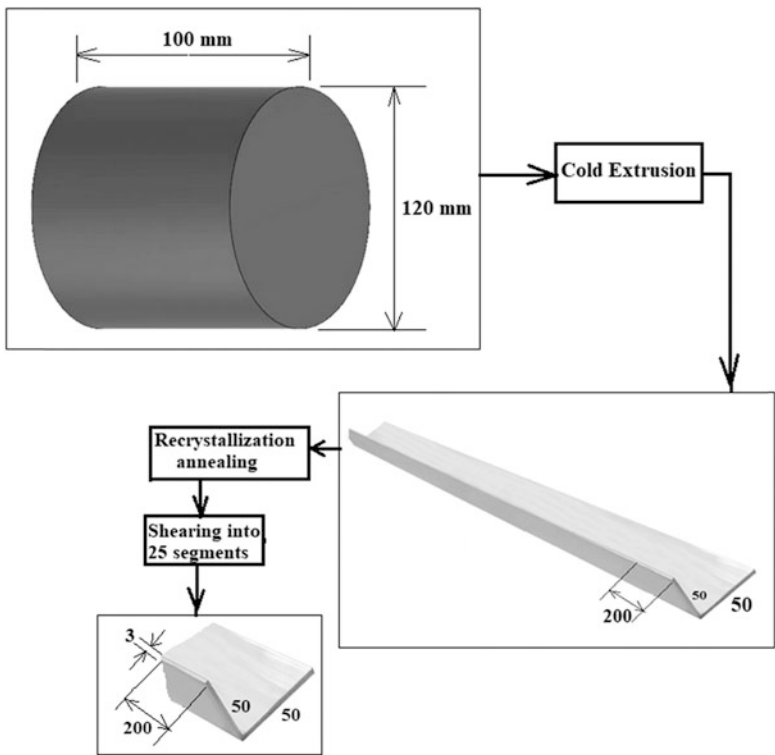


Fig. 14.5 The process flow chart for manufacturing L-beam by Method 2

$$\text{Pressure in direct extrusion} = p_{d.e} = K_x \bar{Y}_f \left(\epsilon_x + \frac{2L}{D_o} \right)$$

where K_x = shape factor for the complex die-orifice in extruder =
 $0.98 + 0.02 \left(\frac{C_x}{C_c} \right)^{2.25}$

$$\text{Original cross-sectional area} = A_o = \frac{\pi}{4} D_o^2 = 0.785 \times 120^2 = 11304 \text{ mm}^2$$

$$\text{Cross-sectional area of the extruded part} = A_f = (50 \times 3) + (50 \times 3) = 300 \text{ mm}^2$$

$$\text{Extrusion ratio} = r_x = \frac{A_o}{A_f} = \frac{11304}{300} = 37.68$$

$$\epsilon = \ln r_x = \ln 37.68 = 3.629$$

$$\epsilon_x = a + (b \epsilon) = 0.8 + (1.5 \times 3.629) = 6.2437$$

$$\bar{Y}_f = \frac{K \cdot \epsilon^n}{1 + n} = \frac{240 \times 3.629^{0.15}}{1 + 0.15} = 253.21 \text{ MPa}$$

$$C_x = \text{Perimeter of the extruded cross-section} = 50 + 50 + 3 + 50 + 50 + 3 = 206 \text{ mm}$$

$$C_c = \text{Perimeter of a circle of the same area as the extruded part} = 2\pi \cdot r$$

$$\text{Cross-sectional area of the extruded part} = 300 \text{ mm}^2 = \pi \cdot r^2$$

$$r = 9.77 \text{ mm}$$

$$C_c = 2\pi \cdot r = 2\pi \times 9.77 = 61.399 \text{ mm}$$

$$K_x = 0.98 + 0.02 \left(\frac{C_x}{C_c} \right)^{2.25} = 0.98 + 0.02 \left(\frac{206}{61.399} \right)^{2.25} = 0.98 + 0.02 (3.355)^{2.25} = 1.285$$

$$p_{d.e} = K_x \bar{Y}_f \left(\epsilon_x + \frac{2L}{D_o} \right) = 1.285 \times 253.21 \times \left(6.2437 + \frac{2 \times 100}{120} \right) = 2573.83 \text{ MPa}$$

$$\text{Extrusion force} = F = p_{d.e} A_o = 2573.83 \times 11304 = 29,094,574.32 \text{ N} = 29.0945 \text{ MN}$$

$$\text{Power} = F \cdot v = 29,094,574.32 \times 0.083 = 2414849.66 \text{ W} = 2414.8496 \text{ kW}$$

(b) *Determining the cost of extrusion using the direct extrusion machine (Rate \$ 0.12/kwh)*

$$\text{Time in extrusion} = \frac{\text{Length extruded}}{\text{Extrusion speed}} = \frac{0.1}{0.083} = 1.2 \text{ s} = 3.346 \times 10^{-4} \text{ h}$$

$$\text{Energy consumed in direct extrusion} = \text{power} \times \text{time} = 2414.8496 \text{ kW} \times 3.346 \times 10^{-4} \text{ h}$$

$$\text{Energy consumed in case of direct extrusion} = 0.8082 \text{ kWh}$$

$$\begin{aligned} \text{Cost of extrusion by using direct extrusion machine} &= \$ 0.12/\text{kwh} \\ &\times 0.8082 \text{ kWh} = \$ 0.097 \end{aligned}$$

(c) *If we replace the direct extrusion machine by indirect extrusion machine how much is the saving in power and the saving in cost?*

$$\begin{aligned} \text{Pressure in indirect extrusion} &= p'_{i,e} = K_x \bar{Y}_f \epsilon_x = 1.285 \times 253.21 \times 6.2437 \\ &= 2031.54 \text{ MPa} \end{aligned}$$

$$\text{Force in indirect extrusion} = F = p'_{i,e} \cdot A_o = 2031.54 \times 11304 = 22964561.5 \text{ N}$$

$$\begin{aligned} \text{Power in indirect extrusion} &= F \cdot v = 22964561.5 \times 0.0833 = 1912947.97 \text{ W} \\ &= 1912.948 \text{ kW} \end{aligned}$$

$$\% \text{Saving in power} = \frac{\text{Power in direct extrusion} - \text{Power in indirect extrusion}}{\text{Power in direct extrusion}} \times 100$$

$$\% \text{Saving in power} = \frac{2414.8496 \text{ kW} - 1912.948 \text{ kW}}{2414.8496 \text{ kW}} \times 100 = 0.2078 \times 100 = 20.78$$

$$\text{Saving in power} = 20.78\%$$

$$\begin{aligned} \text{Energy consumed in indirect extrusion} &= \text{power} \times \text{time} = 1912.948 \text{ kW} \times 3.346 \\ &\times 10^{-4} \text{ h} \end{aligned}$$

$$\text{Energy consumed in indirect extrusion} = 0.64 \text{ kWh}$$

$$\begin{aligned} \text{Cost of extrusion by using indirect extrusion machine} &= \$ 0.12/\text{kwh} \\ &\times 0.64 \text{ kWh} = \$ 0.0768 \end{aligned}$$

$$\% \text{Saving in cost} = \frac{\text{Cost in direct extrusion} - \text{Cost in indirect extrusion}}{\text{Cost in direct extrusion}} \times 100$$

$$\% \text{Saving in cost} = \frac{0.097 - 0.0768}{0.097} \times 100 = 0.208 \times 100 = 20.80$$

$$\text{Saving in cost by using indirect extrusion machine} = 20.8\%$$

14.4.3 Calculations for Cutting

- (a) *How many beams can we produce from the extruded part if we shear segments, as shown in Fig. 14.2?*

$$\text{Volume of billet} = \frac{\pi}{4} D^2 L = 0.785 \times 120^2 \times 100 = 1130400 \text{ mm}^3$$

$$\text{Volume of an L-beam} = \text{Volume of a sheared strip by Method 1} = 63,300 \text{ mm}^3$$

$$\text{Number of L-beams we can produce by Method 2} = \frac{1130400}{63,300} = 17.8$$

$$\text{Number of L-beams we can produce by Method 2} = 17$$

- (b) *What is the cost of shearing the segments if the shearing cost is US\$ 2.00/piece*

$$\text{Cost of shearing ONE segment to produce an L-beam} = \$ 2.00$$

$$\text{Cost of shearing 17 segments in Method 2} = \$ 2.00 \times 17 = \$ 34.00$$

14.5 Overall Economics of Manufacturing (Metal Forming)

$$\text{Cost of rolling in Method 1} = \$ 0.014$$

$$\text{Cost of shearing 25 strips in Method 1} = \$ 24.25$$

$$\text{Cost of bending 25 strips in Method 1} = \$ 26.50$$

$$\begin{aligned} \text{Total cost of metal forming 25 beams in Method 1} &= \$ 0.014 + \$ 24.25 \\ &+ \$ 26.50 = \$ 50.764 \end{aligned}$$

$$\text{Metal-forming cost per piece in Method 1} = \frac{50.764}{25} = \$ 2.0305$$

$$\text{Metal-forming cost per piece in Method 1} = \$ 2.0305$$

$$\text{Cost of extrusion by using direct extrusion machine in Method 2} = \$ 0.097$$

$$\text{Cost of shearing 17 segments (from direct extrusion) in Method 2} = \$ 34.00$$

$$\begin{aligned} \text{Total cost of metal forming for 17 beams by direct extrusion in Method 2} &= \$ \\ 0.097 + \$ 34.00 &= \$ 34.097 \end{aligned}$$

Table 14.2 The metal forming costs (US\$) for manufacturing the beam using the three methods

Metal forming method	Method 1	Method 2 using direct extrusion	Method 2 using indirect extrusion
Cost per piece	\$ 2.0305	\$ 2.0057	\$ 2.0045

Metal-forming cost per piece by direct extrusion in Method 2 = $\frac{34.097}{17}$
= \$ 2.0057

Metal-forming cost per piece **by direct extrusion in Method 2** = \$ 2.0057

Cost of extrusion by using indirect extrusion machine in Method 2 = \$ 0.0768

Cost of shearing 17 segments (from indirect extrusion) in Method 2 = \$ 34.00

Total cost of metal forming for 17 beams by indirect extrusion in Method 2
= \$ 34.0768

Metal-forming cost per piece by indirect extrusion in Method 2 = $\frac{34.0768}{17}$
= \$ 2.0045

Metal-forming cost per piece **by indirect extrusion in Method 2** = \$ 2.0045

The overall metal-forming costs for manufacturing the L-beam, using the three methods, are presented in Table 14.2.

It is apparent in the data in Table 14.2 that Method 2 using indirect extrusion machine is the most economical method. However, the manufacturing manager must take into consideration the higher price of indirect extrusion machine. It means that the Method 2 using indirect extrusion machine is the most economical for mass production, i.e. large quantities are produced with a significant market demand for mass production. The industrial engineer should conduct break even analysis for establishing the economics of manufacturing.

Projects for Practice

- 14.1 Repeat the project in this chapter for mild steel as the stock material.
- 14.2 Analyze the operations used in the metal-forming process for manufacturing the product in your industry. Try to find more economical manufacturing processes for the same product.

Answers to MCQs and the Selected Problems

Chapter 1

1.4.(MCQs). (1) b, (2) d, (3) c, (4) b.

Chapter 2

2.1.(MCQs). (1) b, (2) c, (3) a, (4) c, (5) d, (6) a, (7) b, (8) d, (9) a, (10) c, (11) d

P2.6. $\epsilon_{true} = 0.164$

P2.8. (a) 0.142, (b) Power-law deformation behavior

P2.10. $\tau_r = 0.93$ MPa

Chapter 3

P3.6. $\sigma = 331.88$ kPa

P3.8. $\frac{d\sigma}{dA} = -24.75$ N/mm⁴

P3.10. $\frac{d\sigma}{d\epsilon} = 326.11$ MPa

3.11(MCQs).

(1) b, (2) e, (3) d, (4) a, (5) c, (6) b, (7) a, (8) a, (9) c, (10) c, (11) d, (12) b, (13) c, (14) d.

Chapter 4

P4.8. $\sigma_{flow} = 19.92 \text{ MPa}$

P4.10. $\sigma_{UTS} = 1200 \text{ MPa}$, $\sigma_{ys} = 1120 \text{ MPa}$, % Elongation = 10

P4.12. $k = 1.285$, $n = \text{grain-growth exponent} = 0.237$

4.13(MCQs). (1) c, (2) a, (3) b, (4) a, (5) c, (6) c, (7) b.

Chapter 5

5.1(MCQs). (1) a, (2) d, (3) e, (4) b, (5) c, (6) b, (7) b, (8) c, (9) a, (10) b and d.

P5.8. (a) 2.555 m, (b) 24

P5.10. No! Rolling is not feasible in one pass.

P5.12. No. of passes = 3, Draft for each pass = 2.66 mm

P5.14. (a) 11.76 mm, (b) 16.24 mm

Chapter 6

6.1(MCQs). (1) c, (2) b, (3) a, (4) c, (5) a, (6) b, (7) a, (8) d, (9) c, (10) b.

P6.8. Total energy per blow = 12.857 kJ

P6.10. Final head thickness = 2.89 mm

P6.12. Projection length out of die = 6.87 mm

P6.14. $x_f = 13.064 \text{ mm}$

Chapter 7

7.1(MCQs). (1) b, (2) c, (3) a, (4) a, (5) c, (6) c, (7) a, (8) b, (9) c, (10) b.

P7.8. $r_x = 4.69$

P7.10. $K_x = 1.00787$

P7.12. $\epsilon = 1.847$

P7.14. $L_f = 4657.5 \text{ mm}$

P7.16. $\dot{\epsilon} = 15.674/\text{s}$

Chapter 8

P8.6. $d = 1.3 \text{ mm}$, $A_o = 26.4 \text{ mm}$, $A_f = 15.896 \text{ mm}$

P8.8. $L_f = 9.967 \text{ m}$

P8.10. $v = 31.77 \text{ m/s}$

P8.12. $d = 1.3 \text{ mm}$, $L_c = 3.4 \text{ mm}$

8.14(MCQs). (1) b, (2) a, (3) c, (4) d, (5) b, (6) a, (7) b, (8) a.

Chapter 9

9.6(MCQs). (1) b, (2) a, (3) d, (4) c, (5) d, (6) a.

P9.8. $t_f = 0.888 \text{ mm}$.

P9.10. $R\text{-value} = 1.395$

P9.12. $\beta = 0.555$

P9.14. $\alpha = 0.658$, $\sigma_3 = 0$

P9.16. $\sigma_{ys} = 375 \text{ MPa}$

P9.18. $\sigma_h = 66.67 \text{ MPa}$

P9.20. $\beta = -1.2036$

P9.22. $F_{sf} = 38.4 \text{ kN}$

Chapter 10

P10.6. $c = 0.225 \text{ mm}$.

P10.8. $F_s = 81,000 \text{ N}$

P10.10. Die size = 3.24 mm , punch size = 3 mm .

P10.12. $F_s = 0.653 \text{ ton}$

P10.14. Scrap price = \$ 1.76

P10.16. $c = 0.18 \text{ mm}$

Chapter 11

11.1(MCQs). (1) b, (2) d, (3) a, (4) b, (5) c, (6) a, (7) d.

P11.8. $K_{ba} = 0.5$, $\alpha = 55^\circ$

P11.10. $S = 0.74^\circ$

P11.12. $S = 0.63^\circ$

Chapter 12

P12.8. $c = 2.42 \text{ mm}$, $DR = 1.87$

P12.10. $F_h \cong 114 \text{ kN}$.

P12.12. $D_b = 147 \text{ mm}$.

12.14(MCQs): (1) b, (2) a, (3) c, (4) d, (5) a, (6) b, (7) b, (8) d, (9) a.

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