

STATISTICAL ASPECTS OF MECHANICAL PROPERTY ASSURANCE

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This paper discusses the nature of the variability of the tensile properties of aluminum alloys, the part this variability plays in establishing guaranteed properties, and the problem of establishing sampling plans in specifications that would provide in themselves a high degree of assurance that all the material accepted under such plans meets the established guaranteed properties.

All metals, even when produced under good controls to established standard practices, will exhibit variability in their mechanical properties. This is due to random causes within the limits of good commercial control of melting and castings practices, composition, fabricating procedures, thermal treatment, testing procedures, etc. This random variability must be recognized and considered to be inherent in the product.

An individual piece of product will exhibit mechanical property variability if a large number of tests are performed on it. This is normally referred to as *within-piece* variability. Also, average properties of individual pieces in a lot might vary from *piece-to-piece* within a lot, and such variability shall be referred to as *among-pieces—within-lot* variability. In addition to these sources of variability, there may also be differences among the average properties from one lot to another. This variability will be referred to as *lot-to-lot* variability. The over-all variability that is experienced in routine data accumulated by testing random samples over a large number of lots of a given product is considered as being composed of these three basic components of variability.

Several examples of analyses of data on commercial aluminum alloy sheet are given to

show the relative magnitudes of the above three types of variability.

Figures 1 and 2 show the theoretical normal distribution of tensile ultimate and yield strengths, respectively, of these properties for a single sheet of 0.125 in. Alclad 7075-T6 material as compared with a similar distribution representative of a large number of lots of this same product in the thickness range of 0.040 to 0.249 in. The single-sheet distribution is based on the standard deviation of the results of 178 tests made throughout the one sheet and represents the within-piece variability of the property. The multi-lot distribution is based on the standard deviation of the results of 4290 tests (one test for each of several pieces randomly selected from a large number of lots) and, therefore, represents the combined within-piece, among-pieces—within-lot, and lot-to-lot variabilities.

A knowledge of the variability of mechanical properties within a given piece is of some importance in developing a philosophy of mechanical property assurance. It is more important, perhaps, to have a knowledge of the within-lot variability, which is composed of within-piece and among-pieces—within-lot variabilities, if lot-by-lot evaluation is to be considered.

Figures 3 and 4 show the single-lot and multi-lot theoretical distributions of the tensile ultimate and yield strengths of Alclad 2024-T3 sheet. The within-lot distribution is based on the average range (converted to standard deviation) of a control chart (1)²

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² The italic numbers in brackets refer to the list of references appended to this paper.

developed over a large number of production lots of 0.064-in. sheet which indicated a good state of control with respect to within-lot variability. This chart was based on the range of one test per sheet for three randomly selected sheets from each lot. The multi-lot distribution is based on the standard deviation of a large amount of data accumulated over a 6-month production period of the same product in the thickness range of 0.063 to 0.249 in.

Another set of data that permitted analysis to develop the several types of variability consisted of 12 lots of 0.064-in. Alclad 2024-T86 sheet from which 12 test specimens for each of 3 sheets per lot were tested. These data were analyzed by the method of components of variance (2) to obtain the within-piece, among-pieces—within-lot, and the lot-to-lot components of the over-all variance. Having these components, the within-lot variance was obtained by adding the within-piece and the among-pieces—within-lot components. From these variances the single-sheet, single-lot, and multi-lot standard deviations were obtained as a basis for the theoretical distributions shown in Figs. 5 and 6.

The above examples give some feel of the general magnitude of the three types of variability under discussion. The within-lot variability, which is perhaps the one of greatest significance in the problem being discussed, will vary among alloys and products, and more extensive analyses would be required to cover the complete area of aluminum alloy products. For heat-treated aluminum alloy sheet and plate products, however, perhaps a general statement could be made that the within-lot standard deviation for tensile ultimate and yield strengths would be of the order of 1 ± 0.5 ksi. For non-heat-treated sheet and plate, perhaps a somewhat lower value would apply.

Westcott (3) in discussing the subject of specifications, stated that, "The realistic viewpoint is to recognize that variation governs a specification, not that a specification governs variation." This concept, that specification requirements must be governed by the capability of the process with respect to the characteristics of interest, has always been the philosophy of the aluminum industry. For many years before World War II, it was Alcoa's practice to establish its mechanical

property guarantees by studying frequency distributions of its accumulated data on production material and selecting therefrom values for the respective properties that would be expected to be met by at least 99 per cent of good production material.

With the advent of more aluminum producers in the field, this basic concept of 99 per cent conformance by good production material was generally adopted by the industry as the goal for the selection of mechanical property guarantees. This basic concept has continued to date. Some refinements have been added in our procedures of analysis, however, to audit the accumulated data in the interest of making them more truly representative of process capability and in the application of formal statistical methods in predicting the guaranteed value that would meet the 99 per cent conformance criterion.

The normal curve (4) has some very useful characteristics, and, fortunately, the mechanical properties distributions for a given alloy, temper, and size range of an aluminum alloy product often conform closely to such a frequency curve. In such cases, statistical methods based on known normal distribution characteristics are used in predicting from accumulated data the mechanical property values that will be met by at least 99 per cent of good production material. This 99 per cent of the distribution of a normal curve will fall above the value of the average, minus 2.33 times the standard deviation. This is based, however, on the true average and true standard deviation of the normal distribution. In making predictions from accumulated data we do not know the true average or the true standard deviation of the process population but only an estimate of these values based on the sample of data available for analysis.

Toward the end of World War II, statisticians developed the subject of statistical tolerance limits (5,6) to handle problems such as this. Procedures were developed for taking into account the potential errors in estimated population average and standard deviation depending on the number of data going into the calculation of these values. These procedures made available a method for predicting from sample data a limit above which one could expect, with a degree of confidence, that at least a certain per cent of the parent popu-

lation would fall. This is accomplished by subtracting K standard deviations from the sample average. The K factor used depends on the number of test data, the per cent of the distribution it is desired to have above the limit, and the confidence level of the prediction. About 1950, Alcoa adopted this statistical method to predict its guaranteed properties using a K factor that afforded a 95 per cent confidence that at least 99 per cent of the distribution would fall above the predicted minimum value (below the predicted maximum value if only maximum is involved, or within the minimum and maximum predicted value if both minimum and maximum are involved). This basis has continued in use and has been generally adopted by the aluminum industry.

If data are to be analyzed statistically to determine the capability of a process, it is highly important that care be taken to assure that the data used in the analysis are representative of that capability. To this end, Alcoa has a *Standard Practice for Physical Properties Summary Reports*, which establishes rules that govern the accumulation and reporting of data at all of its production plants. Other producers take similar precautions.

Once guaranteed mechanical property limits are established, some system of continuing review of process capability is desirable to confirm the correctness of the selected values and to detect any change in process capability with time. In 1956, Alcoa instituted a system of charting on a cyclical basis the average, standard deviation, number of tests, and number of failures against established limits. These statistics are taken from our routine *Physical Properties Summary Reports* which are submitted by all of our producing plants every 6 months. The data going into these reports are controlled by the standard practice referred to before and are in standard form, giving the frequency distributions and calculated average and standard deviation values for each alloy, temper, product, and size classification. About 15,000 such charts are routinely maintained and given surveillance for indications such as shifts in property level or variability, any indication of inappropriateness of the current guarantee, and differences among producing plants, etc. Approp-

riate action is taken where indicated by this surveillance.

It would be ideal if a testing frequency were followed that would prove, in itself, that all (or a very high per cent of all) increments of metal in each inspection lot possessed properties within the guaranteed limits. We must, however, ask whether it is justified to require such demonstrated performance when the method of evaluation is destructive by nature and costly. If this positive proof of each lot's performance were required, we would be disregarding all other evidence that has been developed about the mechanical properties of the product. Such a procedure would result in more testing than is current procedure and would require prohibitive expansion of testing facilities and personnel. Also, it does not impress one as being a good quality control procedure to disregard the wealth of supporting knowledge of process capability that has been used in establishing the guarantees originally, the process controls used to assure continued performance of the process at its demonstrated level, and the surveillance methods used to police such performance.

Recognizing that it might not be economically or otherwise practical to adopt a testing procedure that *in itself* would evaluate the acceptability of each lot with a high degree of assurance, we must pursue a different philosophy on the purpose of mechanical property testing. In the use of statistical quality control charts, it is customary to establish control limits such that, if the plot of a small-sample average value falls outside these limits, there is strong evidence of the material represented not being from the same population as the material used as the basis for establishing these limits. It is recognized, however, if the plot of the average of a small sample falls within the control limits, this in itself is not strong evidence that the material represented by the sample *is* from the same population. Here we recognize that once it is demonstrated that a process can operate in control, we establish a procedure that is searching for strong evidence of a change in the process capability, and in the absence of such evidence, we assume that production continues at the demonstrated level of capability.

Mechanical property testing as now prescribed by specifications can be viewed in this

same light; that is, it is a check on a process to see if there is any evidence that something has gone wrong in the production of a given lot. In the absence of such evidence, we are willing to assume that it possesses the characteristics demonstrated by the capability studies on which the specification guarantees are based. If we accept this view, we would conclude that the purpose of lot-by-lot testing is to detect any gross change in property level of the lot that would reflect some inadvertent failure to follow the standard practice prescribed for the product. We suggest that this is the purpose of the current test requirements in aluminum product specifications.

The number of samples tested under current specification requirements varies with lot size and is normally from 1 to 5 samples. Inasmuch as one failure to meet specified values is cause for rejection or, under some conditions, retesting, Fig. 7 is presented to show the operating characteristic curves (7) for such statistical plans.

When considering such plans as applicable to mechanical properties, it is important to appreciate what is meant by per cent defective. In this case it means that if the distribution of properties of each increment of metal represented by the size of a test specimen in the lot is so located with respect to the minimum specification that a certain per cent of the distribution is below the specification, the proportional area of that part of the distribution is the per cent defective. It is suggested, then, that the important point for consideration is not so much the per cent defective, as *how far* in terms of mechanical property units the lowest increment of that property in the lot might fall below the specification minimum. With this concept in mind, there have been entered on these operating characteristic curves in addition to the usual per cent defective, the comparable extent in terms of standard deviation that the lower tail (average-3 sigma) of the distribution of the lot might extend below the specification minimum for the respective per cent defective values.

The interpretation of these standard deviation values is perhaps best shown by example. In Fig. 7 the standard deviation value comparable to 20 per cent defective is 2.16. If we

consider the standard deviation to be 1 ksi, this means that the *lowest strength increment of metal* in a lot evaluated as being 20 per cent defective is 2.16 ksi below the minimum guarantee.

Although most current specifications for metal products have attribute-type-testing requirements for mechanical properties, there have been discussions of the potential of using variable-type sampling plans for this purpose. With attribute-type plans, the lot is judged on the basis of whether or not the individual tests meet the guaranteed value. With variable-type plans, the average and standard deviation or range of the test results are used to predict whether or not the distribution of properties of the lot is such as to be acceptable under the guaranteed requirements. Figure 8 shows a comparison of the operating characteristic curves for the tightest sampling plans from MIL-STD-414 (8) for "variability unknown" with sample sizes of 3, 4, and 5 with the similar curves from Fig. 7 for the same sample sizes.

It is obvious from Fig. 8 that for the sample sizes being considered here, there is not too much to be gained by going to a variables plan of the "variability unknown" type. This is particularly true if the respective plans are evaluated in terms of the lowest strength increment indicated by the standard deviations shown on the operating characteristic curve as discussed earlier. Some additional advantage in the use of variables plans could potentially be obtained by using a plan based on "known variability". MIL-STD-414 shows that the same operating characteristic curve as shown for a sample size of 5 would apply to a sample size of 2 if the "known variability" approach were applicable. The problem here would be in arriving at the known standard deviation that would be applicable to all suppliers for each and every alloy, temper, size group, and product of interest.

In view of the sound basis on which the mechanical property guarantees of aluminum alloy products are established, the relatively small variability in these properties, and the other considerations presented, it is felt that the current procedure for mechanical property assurance of these products is adequate. The

possibility of improving these procedures should not be overlooked, however, and consideration should be given to any practical

approach that may be forthcoming with new developments in this area of quality control.

REFERENCES

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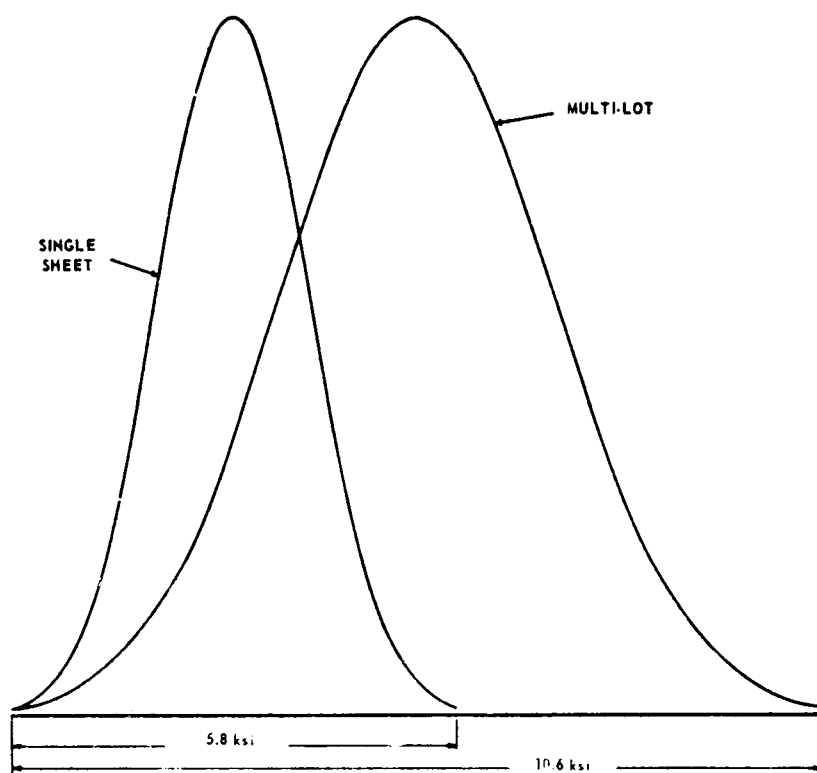


FIG. 1—ALC 7075-T6 ultimate tensile strength.

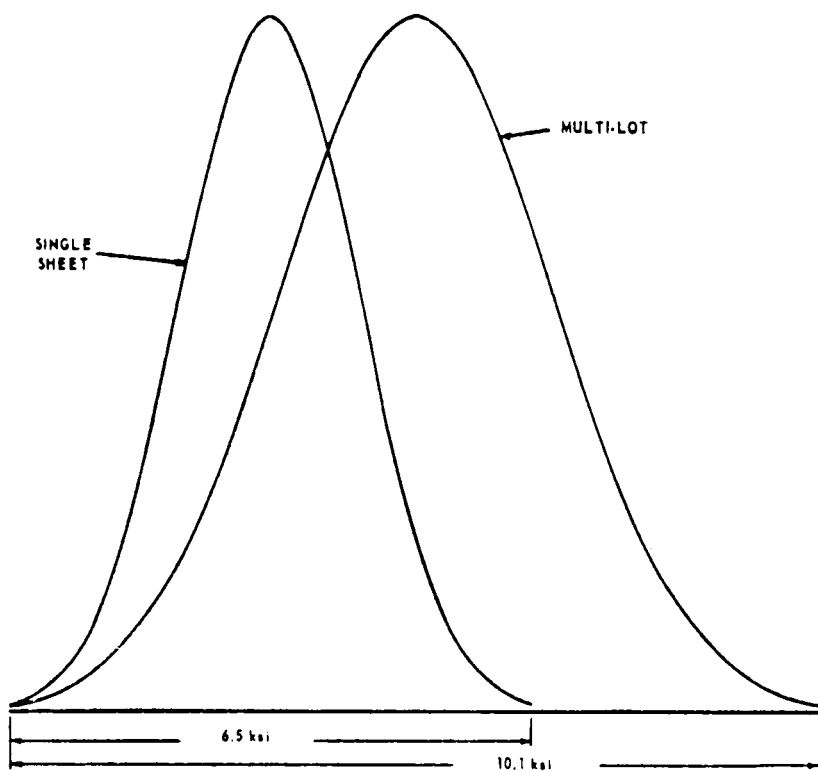


FIG. 2—ALC 7075-T6 yield tensile strength.

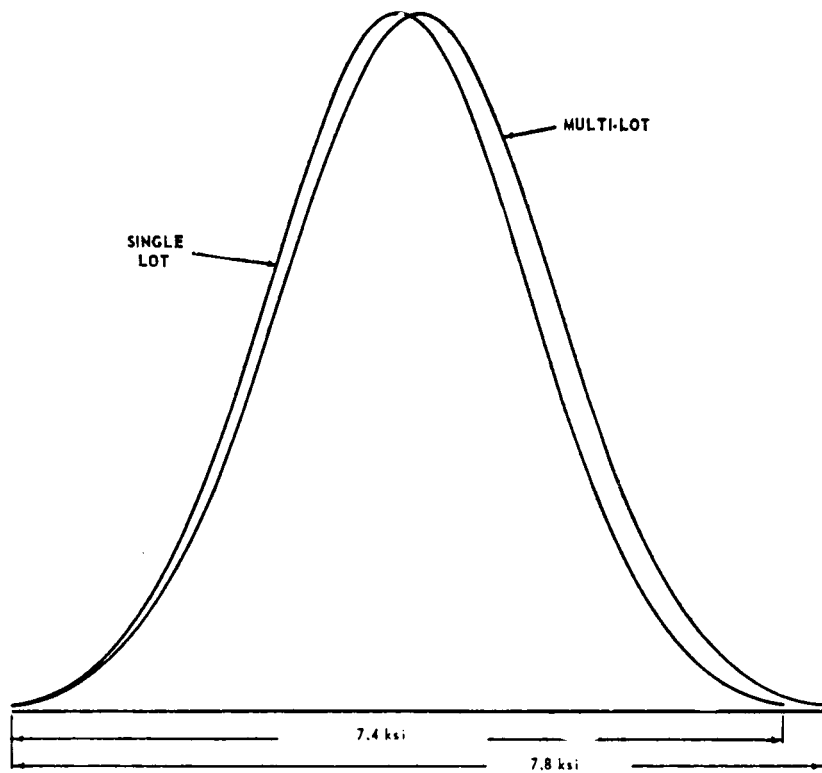


FIG. 3—2024-T3 ultimate tensile strength.

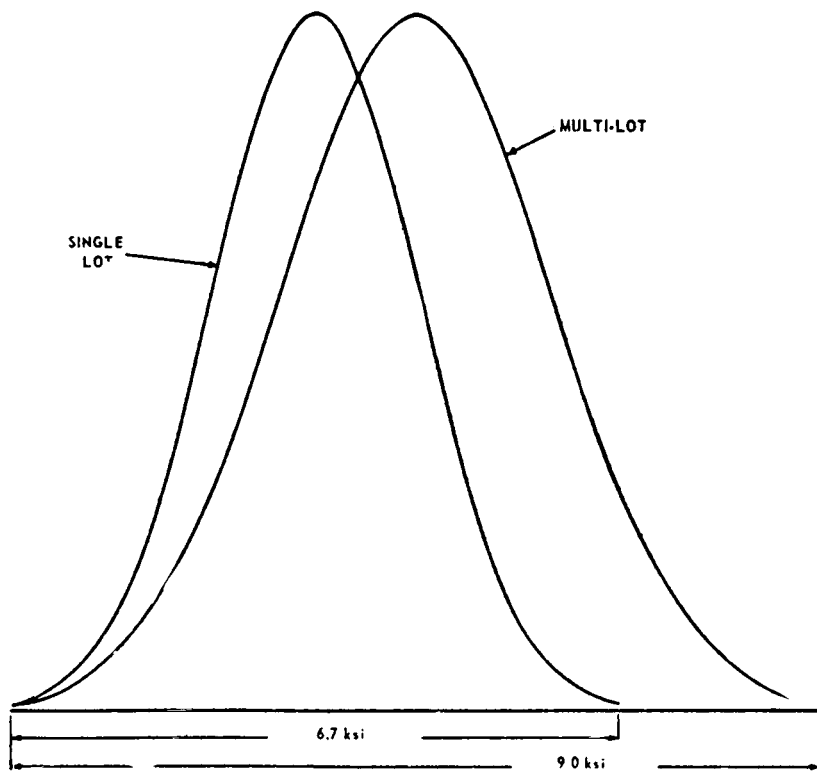


FIG. 4—2024-T3 yield strength.

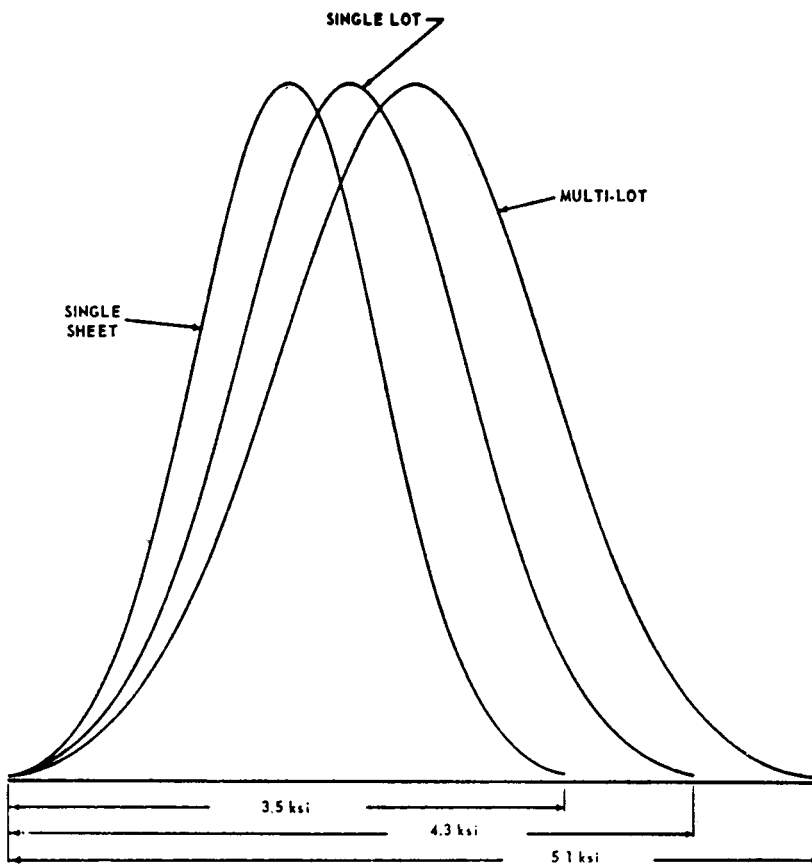


FIG. 5—ALC 2024-T86 ultimate tensile strength.

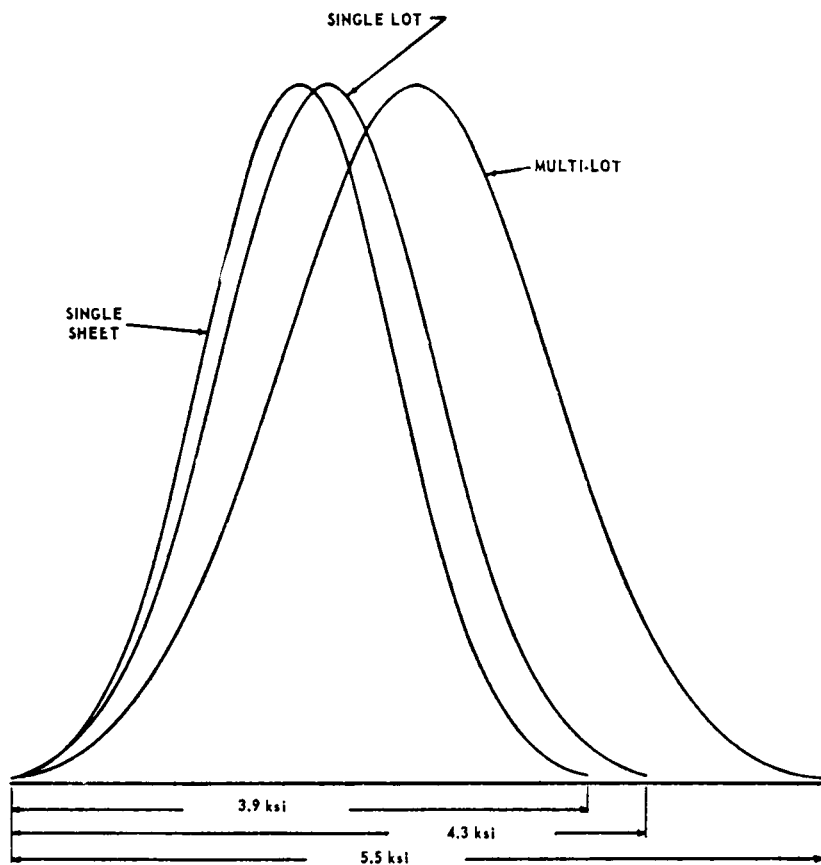


FIG. 6—ALC 2024-T86 yield tensile strength.

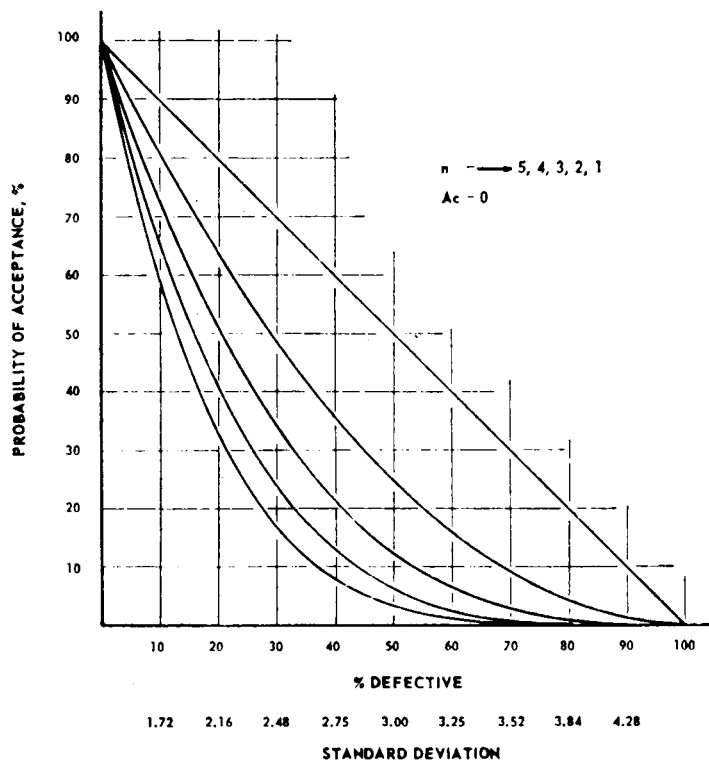


FIG. 7—Operating characteristic curves—attribute sampling plan.

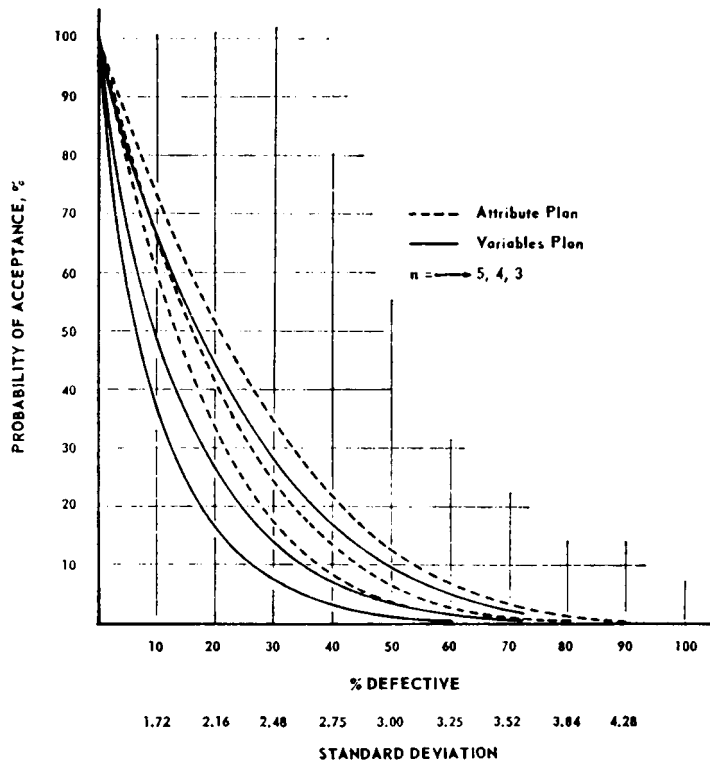


FIG. 8—Operating characteristic curves—variable sampling plan.