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British Energy Generation Ltd

Stress Intensity Factor and Limit Load Handbook

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By: S Al Laham
Structural Integrity Branch

Authorised By: R A Ainsworth
Title: Group Head, Assessment Technology Group

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Stress Intensity Factor and Limit Load Handbook.

By Dr S Al Laham, Structural Integrity Branch

Issue 2

Date: 15 April 1999

I confirm this document has been subject to verification and validation by internal review within Nuclear Electric Ltd.

Dr R A Ainsworth, Group Head, Structural Integrity Branch

Dr M J H Fox, Team Leader, Structural Integrity Branch

Date:

Approved for Issue:

Date:

Dr R A Ainsworth, Group Head, Structural Integrity Branch

SUMMARY

This report provides a collation of stress intensity factor and limit load solutions for defective components. It includes the Stress Intensity Factor (SIFs) in the R6 Code software and in other computer programs, which have not previously been contained in a single source reference. This document has been produced as part of the BRITE-EURAM project SINTAP which aims to develop a defect assessment approach for the European Community. Most of the solutions presented in this document were collated from industry and establishments in the UK (Nuclear Electric Ltd, Magnox Electric Plc and HSE), Sweden (SAQ Kontroll AB) and Germany (Fraunhofer IWM, and GKSS). The solutions are compared to standard solutions published elsewhere and to those in the American Petroleum Institute document API 579. In this second issue, the quality of the figures has been improved, minor typographical errors found in the previous issue have been corrected, and comments from partners in SINTAP have been addressed.

REVISION/REVIEW REGISTER

Issue No.	Revision No.	Date	Page No.	Summary of Revision	Approved
Issue 2	Revision 1	15/4/1999	Summary (i)	Summary amended to reflect changes.	RAA
			AI.43.	Specimen width changed in figure to 2W. Equation for K edited by removing (2) from the denominator.	RAA
			AI.46.	Specimen width changed to 2W in figure. Remarks added.	RAA
			AI.56, 58.	Range of applicability modified to remove confusion.	RAA
			AI.43, 44, 46, 47, 49, 50, 52 & 54.	Range of applicability modified.	RAA
			AII.43 & 50.	The word Compression changed to Tension.	RAA
			AIII.22, 26 & 30.		RAA

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1. INTRODUCTION

The wide range of structural configurations, loading conditions and crack geometries, together with the material and geometric non-linearities which characterise response under loads, has made the analytical prediction of both the strength and Stress Intensity Factors (SIFs) difficult.

Generally fatigue cracks initiate at several locations, mostly around the weld region in joints and areas of discontinuities, due to the high bending, welding residual stresses and weld notch stresses. These cracks eventually coalesce to form a single crack which grows in both the length and depth directions and which may finally become a through thickness crack. In order to assess the integrity of structures containing defects, it is necessary to be able to estimate both plastic collapse and fracture strengths of the critical members containing defects.

Stress Intensity Factors (SIFs) can be calculated in the Nuclear Electric's R6 Code software⁽¹⁾ and other computer programs. Further, a number of methods are now available for evaluating stress intensity factors^(2 to 8) and limit loads^(9 to 15) of structures containing flaws.

In order to provide a single source reference for use in a procedure being developed under the Brite-Euram project SINTAP, this report collates solutions for stress intensity factors and limit loads for different cracked geometries and structures. In this document only one solution is presented for each cracked geometry/loading combination. This is the result of detailed evaluations and comparisons of available solutions. It should not be inferred that the solution selected is the only satisfactory one. Solutions other than those given here may be used in the analysis provided they are validated.

Most of the work presented in this document has been collated from industry and establishments in the UK (Nuclear Electric Ltd, Magnox Electric Plc and HSE), Sweden (SAQ Kontroll AB) and Germany (Fraunhofer IWM, and GKSS). In developing this source reference, care has been taken to ensure that, wherever possible, the solutions recommended have been validated. The recommended compendia of SIF and limit load solutions are given in four separate appendices. Appendix I gives the recommended solutions for SIFs, while guidance on calculating the limit loads is given in Appendix II. The assessment of tubular joints used in the offshore industry also requires specialist guidance due to the complexity of the joint geometry and the applied loading, and the current guidance for offshore structures is contained in Appendix III. Limit load solutions with the presence of material mismatch are given in Appendix IV of this report. Finally, the results of the comparison of the stress intensity factors from different sources are given in Appendix V. It should be noted that the scope of Appendix III is limited to the assessment of known or assumed weld toe flaws, including fatigue cracks found in service, in brace or chord members of T, Y, K or KT joints between circular section tubes under axial and/or bending loads.

These five appendices form the bulk of this report. In the main text, brief sections deal with the loading, behaviour, failure of structures and a description of the methodology used in this study. It should be noted that it is intended to update this document as and when knowledge and techniques improve.

2. Loading and Stresses Considered

Loading of a structure includes all forces and other effects which cause an increase of the strain on the part of the structure under assessment. The stresses to be considered in the assessment of the integrity of structures containing defects may be treated directly, or after resolution into the following four components⁽¹⁶⁾:

- a) **Membrane Stresses:** The component of uniformly distributed stress which is equal to the average value of stress across the section thickness and is necessary to satisfy the simple laws of equilibrium of internal and external forces.
- b) **Bending Stresses:** The component of stress due to imposed loading which varies across the section thickness.
- c) **Secondary Stresses:** The secondary stresses are self equilibrating stresses necessary to satisfy compatibility in the structure. Thermal and residual stresses are usually considered secondary.
- d) **Peak Stresses:** The peak stress is the increment of stress that is added to the primary membrane and bending stresses and secondary stresses due to concentration at local discontinuities.

3. Analysis and Assessment of the Integrity of Structures

The integrity of a structure containing defects may be evaluated by reference to two criteria^(1 and 17), fracture and plastic collapse. This may be carried out by obtaining the fracture and the collapse parameters K_r and L_r respectively. The L_r parameter is a measure of plasticity effects which gauges the closeness to plastic yielding of the structure, and is defined as the ratio of the loading condition being assessed to that required to cause plastic yielding of the structure. The fracture parameter K_r is a measure of the proximity to linear elastic fracture mechanics (LEFM) failure of the structure. K_r is simply the ratio of the linear elastic stress intensity factor to the fracture toughness of the material used. Structural integrity relative to the limiting condition may be evaluated by means of a Failure Assessment Diagram (FAD) using the procedures outlined in R6. These procedures require assessment points to be plotted on the FAD, the location of each assessment point depending upon the applied load, flaw size, material properties, etc. A necessary criterion of acceptance is that the assessment point of interest should lie within the area bounded by the axes of the failure assessment diagram and the assessment diagram line.

There are various stress intensity factor solutions, particularly for flat plates and pressure vessels with various cracked geometries. Some of these solutions are based on the use of thin-shell theory⁽¹⁸⁾, which does not take into account the three dimensional nature of the highly localised stresses in the vicinity of the crack front. Further, thin-shell theory does not take into account the effect of transverse shear acting along the crack front. In recent years three-dimensional finite element analyses have been performed by a number of analysts^(19 to 21). One advantage of the use of 3-D finite elements is that it is possible to take into account the effect of the 3-D nature of the stress state in the vicinity of the crack front. As part of the SINTAP project, three-dimensional finite element models have been used to obtain solutions of the stress intensity factors for through-thickness cracks in cylinders^(18 and 22).

As far as limit load solutions are concerned, a number of approaches have been used to estimate plastic limit loads. The upper and lower bound theorems of plasticity involve approximate modelling of the deformations or the stress distributions, respectively, and can provide approximate estimates of limit loads. Direct modelling of the plastic stress and strain distributions for given loading conditions through the use of constitutive equations can be accomplished analytically only for very simple undefective structures. Experimental determinations of limit loads involves correlating applied loads with measured plastic deformations. Three-dimensional finite element analyses have also been used. For example, finite element analysis has recently been employed to assess the integrity of tubular joints containing defects^(23 to 27).

Each method has its limitations and usually involves some form of idealisation and approximation. Typically, these relate to the representation of material properties, estimation of hardening effects, the allowance for change of shape of a deforming structure (geometrical non-linearities), and the definition of the state of deformation or stress distribution corresponding to the limit condition.

The plastic yield load (as referred to in R6⁽¹⁷⁾) depends on the yield or proof stress of the material, σ_y , and also on the nature of the defect to be assessed. For through thickness cracks or for defects which are characterised as through cracks, the yield load is the so-called “global” yield load, i.e. the rigid-plastic limit load of the structure, calculated for a rigid-plastic material with a yield stress equal to σ_y . For part through cracks, the yield load is the “local” limit load, i.e. the load needed to cause plasticity to spread across the remaining ligament, calculated for an elastic-perfectly plastic material with a yield stress equal to σ_y .

4. Methodology Used in Collating Solutions

It is convenient for both stress intensity factor and limit load solutions from various sources to be collected into a single document. Those sources normally contain estimates of both stress intensity factors and limit loads for a wide range of defective structures. It is common practice to express the stress intensity factors and limit load solutions in terms of simple mathematical expressions involving geometrical parameters describing the structure and the details of the defect contained. This makes them useful for studying the effect of changes in the structural geometry and defect sizes on the integrity of the structure. These stress intensity factor and limit load solutions form the basis of the present compendium.

The approach involved collating stress intensity factor and limit load solutions from different sources. Solutions for SIFs were compared where applicable, within the range of validity, and a set of solutions were later recommended.

The bulk of the compendium contains solutions for stress intensity factors and limit load solutions for both pressure vessels and offshore structures. The stress intensity factor solutions for pressure vessels are given in Appendix I. Solutions of limit loads for pressure vessels are given in Appendix II. For offshore structures general guidance and recommendations on the prediction of stress intensity factors and plastic collapse loads are given in the new British Standard BS 7910⁽²⁸⁾; this is summarised in Appendix III. Limit load solutions in the presence of material mismatch are listed separately in Appendix IV of this report. The results of the comparisons of stress intensity factors from different sources are given in Appendix V.

5. Computer Programs

A number of computer programs are available for performing fracture assessments. These programs are updated frequently. The following computer programs contain stress intensity factor and limit load solutions:

1. R6-Code⁽¹⁾, developed and marketed by Nuclear Electric Ltd (England).
2. CrackWise, developed and marketed by the Welding Institute TWI (England). This program is based on the British Standard Published Document PD 6493⁽¹⁶⁾.
3. The computer program SACC, which is developed by SAQ in Sweden.
4. The computer program PREFIS which carries out an assessment based on API 579 for the petrochemical industry.

It should be noted that MCS in Ireland are developing computer software which will be used as a vehicle to demonstrate SINTAP results.

Information in these computer programs has been used in producing the compendia in this document.

6. Conclusions

Various stress intensity factor and limit load solutions exist, and users need to find the appropriate solutions to apply fracture mechanics procedures. This document is the first step towards establishing a single source of reference to be used by European industry for carrying out structural integrity assessment in accordance with procedures being developed by SINTAP. In the current work the following tasks were carried out:

- Stress Intensity Factor (SIF) solutions from databases for cracks in pipes, flat plates and spheres were collated and presented in Appendix I.
- Limit Load (LL) solutions from databases for cracks in pipes, flat plates and spheres were collated and presented in Appendix II of this report.
- Stress Intensity Factor and Limit Load solutions for offshore tubular joints were collated and presented in Appendix III.
- The effects of material mismatch on the limit load solutions for different cracked geometries were presented in Appendix IV.
- The collated stress intensity factor solutions were compared to published data, and based on the results of the comparison, (Appendix V) preferred solutions were chosen and recommended for use, as presented in Appendix I.

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APPENDIX I

**STRESS INTENSITY FACTOR SOLUTIONS FOR PRESSURE VESSELS,
FLAT PLATES AND SPHERES**

CONTENTS

AI.1. INTRODUCTION

AI.2. STRESS INTENSITY FACTOR SOLUTIONS FROM SAQ

AI.2.1 CRACKS IN A PLATE

AI.2.2. AXIAL CRACKS IN A CYLINDER

AI.2.3. CIRCUMFERENTIAL CRACKS IN A CYLINDER

AI.2.4. CRACKS IN A SPHERE

AI.3. ADDITIONAL SIF SOLUTIONS FROM R6-CODE

AI.4. REFERENCES

AI.1. INTRODUCTION

A collation of solutions for stress intensity factors is presented in this appendix. Most solutions are for cracks in an infinite plate or an infinite long cylinder. Therefore boundary effects on the solutions are not included. Most of the results presented are from an earlier collation by Andersson et al [AI.1]. Solutions for through-wall cracks in cylinders can be obtained from finite element calculations by Zang [AI.2] as a part of the SINTAP project. However, for the purpose of this compendium these were extracted from the R6.CODE.

It should be noted that solutions are generally presented in terms of weight functions. Thus, stress intensity factors can be evaluated for arbitrary stress fields directly, without the need to resolve the stress fields into membrane and bending components. Polynomial fits to the stress field are, however, required for some solutions.

Solutions are given for both semi-elliptical surface and fully extended flaws. In the former case, values of stress intensity factor are provided for the surface point and for the deepest point of the flaw. In Section AI.2 of this appendix, SAQ solutions for some geometries are presented. Additional solutions for different cracked geometries, obtained from R6.CODE and presented in Section AI.3. Finally, source references are listed in Section AI.4.

AI.2. STRESS INTENSITY FACTOR SOLUTIONS FROM SAQ

AI.2.1 CRACKS IN A PLATE

Description: *Finite surface crack*

Schematic:

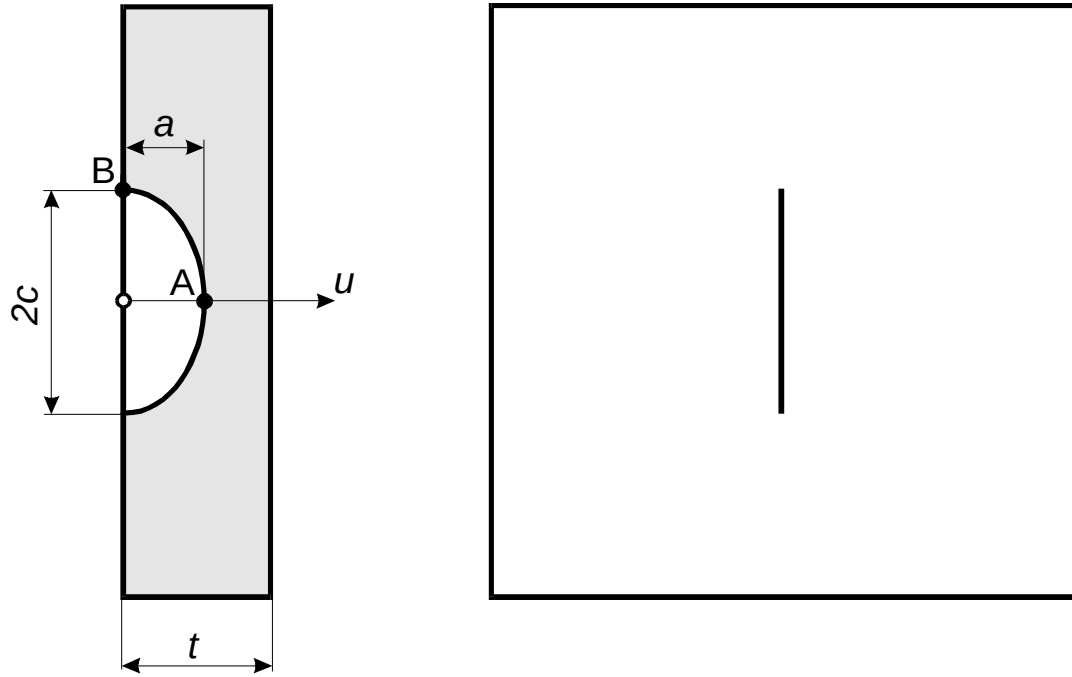


Figure AI.1. Finite surface crack in a plate.

Solution:

The stress intensity factor K_I is given by

$$K_I = \sqrt{pa} \sum_{i=0}^5 s_i f_i \left(\frac{a}{t}, \frac{2c}{a} \right) \quad (\text{AI.1})$$

s_i ($i = 0$ to 5) are stress components which define the stress state s according to

$$s = s(u) = \sum_{i=0}^5 s_i \left(\frac{u}{a} \right)^i \quad \text{for } 0 \leq u \leq a \quad (\text{AI.2})$$

s is to be taken normal to the prospective crack plane in an uncracked plate. s_i is determined by fitting s to Equation (AI.2). The co-ordinate u is defined in Figure AI.1.

f_i ($i = 0$ to 5) are geometry functions which are given in Tables AI.1 and AI.2 below for the deepest point of the crack (f^A), and at the intersection of the crack with the free

surface (f^B), respectively. The parameters used in the Tables are defined in Figure AI.1.

Table AI.1. Geometry functions for a finite surface crack in a plate - deepest point of the crack.

$2c/a = 2$						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.659	0.471	0.387	0.337	0.299	0.266
0.2	0.663	0.473	0.388	0.337	0.299	0.269
0.4	0.678	0.479	0.390	0.339	0.300	0.271
0.6	0.692	0.486	0.396	0.342	0.304	0.274
0.8	0.697	0.497	0.405	0.349	0.309	0.278
$2c/a = 5/2$						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.741	0.510	0.411	0.346	0.300	0.266
0.2	0.746	0.512	0.413	0.352	0.306	0.270
0.4	0.771	0.519	0.416	0.356	0.309	0.278
0.6	0.800	0.531	0.422	0.362	0.317	0.284
0.8	0.820	0.548	0.436	0.375	0.326	0.295
$2c/a = 10/3$						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.833	0.549	0.425	0.351	0.301	0.267
0.2	0.841	0.554	0.430	0.359	0.309	0.271
0.4	0.885	0.568	0.442	0.371	0.320	0.285
0.6	0.930	0.587	0.454	0.381	0.331	0.295
0.8	0.960	0.605	0.476	0.399	0.346	0.310
$2c/a = 5$						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.939	0.580	0.434	0.353	0.302	0.268
0.2	0.957	0.595	0.446	0.363	0.310	0.273
0.4	1.057	0.631	0.475	0.389	0.332	0.292
0.6	1.146	0.668	0.495	0.407	0.350	0.309
0.8	1.190	0.698	0.521	0.428	0.367	0.324
$2c/a = 10$						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	1.053	0.606	0.443	0.357	0.302	0.269
0.2	1.106	0.640	0.467	0.374	0.314	0.277
0.4	1.306	0.724	0.525	0.420	0.348	0.304
0.6	1.572	0.815	0.571	0.448	0.377	0.327
0.8	1.701	0.880	0.614	0.481	0.399	0.343

Table AI.1. Geometry functions for a finite surface crack in a plate - deepest point of the crack. (Continued)

$2c/a = 20$						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	1.103	0.680	0.484	0.398	0.344	0.306
0.2	1.199	0.693	0.525	0.426	0.364	0.323
0.4	1.492	0.806	0.630	0.499	0.417	0.364
0.6	1.999	1.004	0.838	0.631	0.514	0.437
0.8	2.746	1.276	1.549	1.073	0.817	0.660
$2c/a = 40$						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	1.120	0.686	0.504	0.419	0.365	0.325
0.2	1.245	0.708	0.553	0.452	0.389	0.346
0.4	1.681	0.881	0.682	0.538	0.451	0.394
0.6	2.609	1.251	0.971	0.722	0.583	0.493
0.8	4.330	1.885	2.016	1.369	1.026	0.819
$2c/a \rightarrow \infty$						
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	1.123	0.682	0.524	0.440	0.386	0.344
0.2	1.380	0.784	0.582	0.478	0.414	0.369
0.4	2.106	1.059	0.735	0.578	0.485	0.423
0.6	4.025	1.750	1.105	0.814	0.651	0.548
0.8	11.92	4.437	2.484	1.655	1.235	0.977

Table AI.2. Geometry functions for a finite surface crack in a plate - intersection of crack with free surface.

$2c/a = 2$						
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_4^B	f_5^B
0	0.716	0.118	0.041	0.022	0.014	0.010
0.2	0.729	0.123	0.045	0.023	0.014	0.010
0.4	0.777	0.133	0.050	0.026	0.015	0.011
0.6	0.839	0.148	0.058	0.029	0.018	0.012
0.8	0.917	0.167	0.066	0.035	0.022	0.015
$2c/a = 5/2$						
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_4^B	f_5^B
0	0.730	0.124	0.041	0.021	0.013	0.010
0.2	0.749	0.126	0.046	0.023	0.014	0.010
0.4	0.795	0.144	0.054	0.028	0.017	0.012
0.6	0.901	0.167	0.066	0.033	0.021	0.015
0.8	0.995	0.193	0.076	0.042	0.026	0.017
$2c/a = 10/3$						
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_4^B	f_5^B
0	0.723	0.118	0.039	0.019	0.011	0.008
0.2	0.747	0.125	0.044	0.022	0.014	0.010
0.4	0.803	0.145	0.056	0.029	0.018	0.012
0.6	0.934	0.180	0.072	0.037	0.023	0.016
0.8	1.070	0.218	0.087	0.047	0.029	0.020
$2c/a = 5$						
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_4^B	f_5^B
0	0.673	0.104	0.032	0.015	0.009	0.006
0.2	0.704	0.114	0.038	0.018	0.011	0.007
0.4	0.792	0.139	0.053	0.027	0.016	0.011
0.6	0.921	0.183	0.074	0.038	0.024	0.017
0.8	1.147	0.244	0.097	0.052	0.032	0.021
$2c/a = 10$						
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_4^B	f_5^B
0	0.516	0.069	0.017	0.009	0.005	0.004
0.2	0.554	0.076	0.022	0.011	0.007	0.005
0.4	0.655	0.099	0.039	0.019	0.012	0.008
0.6	0.840	0.157	0.063	0.032	0.020	0.013
0.8	1.143	0.243	0.099	0.055	0.034	0.023

Table AI.2. Geometry functions for a finite surface crack in a plate - intersection of crack with free surface (continued).

$2c/a = 20$						
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_4^B	f_5^B
0	0.384	0.067	0.009	0.004	0.003	0.002
0.2	0.422	0.074	0.011	0.006	0.004	0.003
0.4	0.546	0.096	0.020	0.010	0.006	0.004
0.6	0.775	0.136	0.031	0.016	0.010	0.007
0.8	1.150	0.202	0.050	0.028	0.017	0.011
$2c/a = 40$						
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_4^B	f_5^B
0	0.275	0.048	0.004	0.002	0.001	0.001
0.2	0.310	0.054	0.006	0.003	0.002	0.001
0.4	0.435	0.075	0.010	0.005	0.003	0.002
0.6	0.715	0.124	0.016	0.008	0.005	0.003
0.8	1.282	0.221	0.025	0.014	0.009	0.006
$2c/a \rightarrow \infty$						
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_4^B	f_5^B
0	0.000	0.000	0.000	0.000	0.000	0.000
0.2	0.000	0.000	0.000	0.000	0.000	0.000
0.4	0.000	0.000	0.000	0.000	0.000	0.000
0.6	0.000	0.000	0.000	0.000	0.000	0.000
0.8	0.000	0.000	0.000	0.000	0.000	0.000

Remarks: The plate should be large in comparison to the length of the crack so that edge effects do not influence the results.
Taken from References AI.2, AI.3 and AI.7.

Description: *Infinite surface crack*

Schematic:

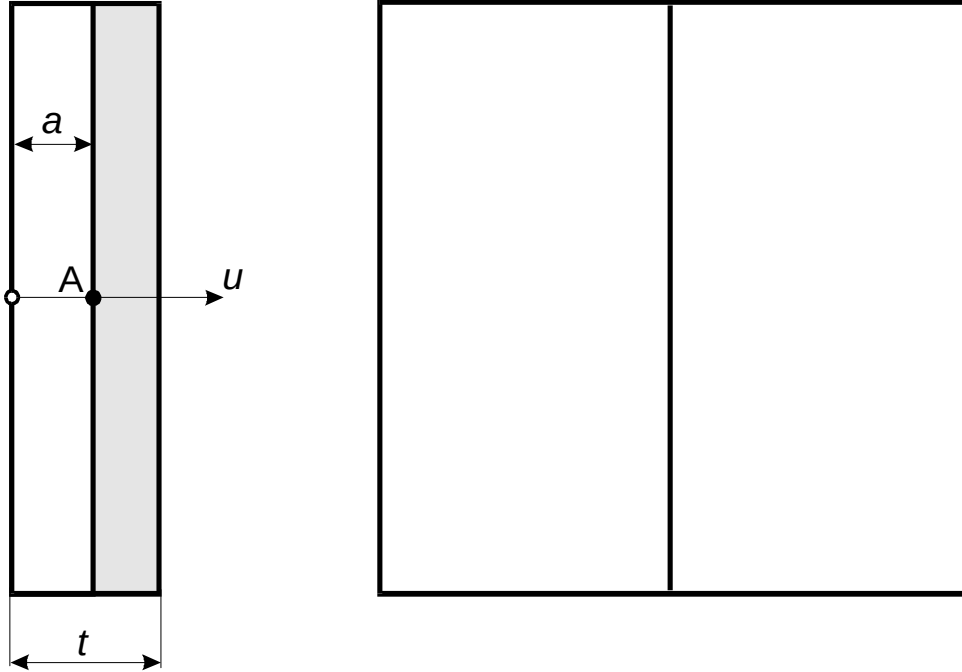


Figure AI.2. Infinite surface crack in a plate.

Solution:

The stress intensity factor K_I is given by

$$K_I = \frac{1}{\sqrt{2pa}} \int_0^a s(u) \sum_{i=1}^{i=5} f_i(a/t) \left(1 - \frac{u}{a}\right)^{i-\frac{3}{2}} du \quad (\text{AI.3})$$

The stress state $s = s(u)$ is to be taken normal to the prospective crack plane in an uncracked plate. The co-ordinate u is defined in Figure AI.2.

The geometry functions f_i ($i = 1$ to 5) are given in Table AI.3 for the deepest point of the crack (f^A). Parameters used in the Table are defined in Figure AI.2.

Table AI.3. Geometry functions for an infinite surface crack in a plate.

a/t	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	2.000	0.977	1.142	-0.350	-0.091
0.1	2.000	1.419	1.138	-0.355	-0.076
0.2	2.000	2.537	1.238	-0.347	-0.056
0.3	2.000	4.238	1.680	-0.410	-0.019
0.4	2.000	6.636	2.805	-0.611	0.039
0.5	2.000	10.02	5.500	-1.340	0.218
0.6	2.000	15.04	11.88	-3.607	0.786
0.7	2.000	23.18	28.03	-10.50	2.587
0.8	2.000	38.81	78.75	-36.60	9.871
0.9	2.000	82.70	351.0	-207.1	60.86

Remarks: The plate should be large in the transverse direction to the crack so that edge effects do not influence the results.
Taken from Reference AI.4.

Description: *Embedded crack*

Schematic:

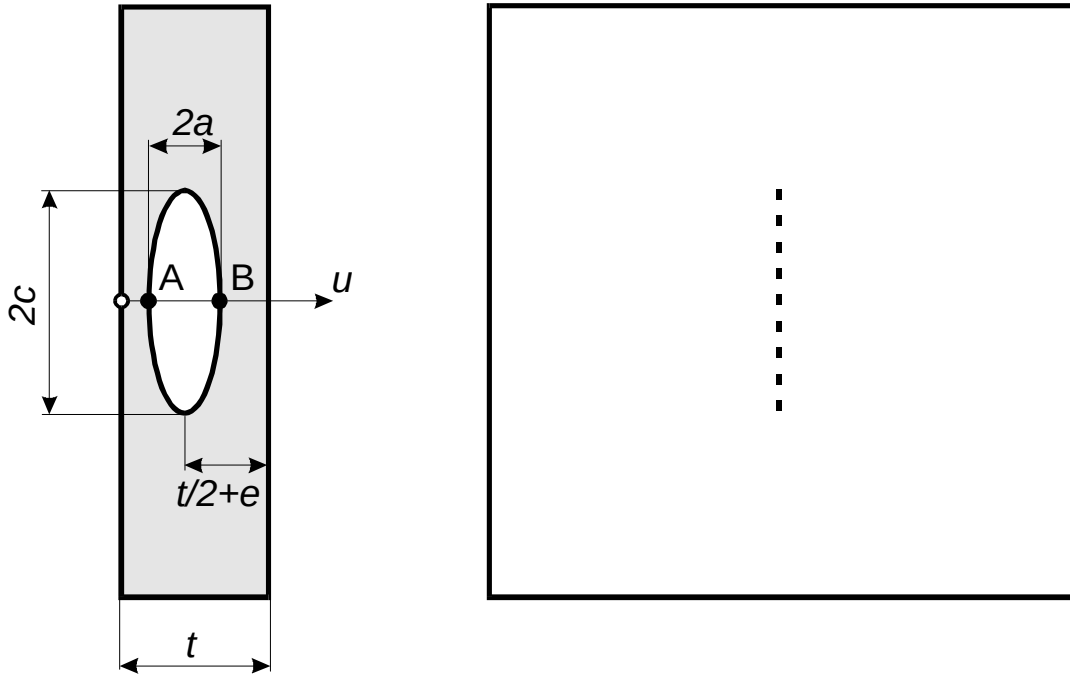


Figure AI.3. Embedded crack in a plate.

Solution:

The stress intensity factor K_I is given by

$$K_I = \sqrt{pa} \left(s_m f_m \left(\frac{2a}{t}, \frac{c}{a}, \frac{e}{t} \right) + s_b f_b \left(\frac{2a}{t}, \frac{c}{a}, \frac{e}{t} \right) \right) \quad (\text{AI.4})$$

In Equation (AI.4), s_m and s_b are the membrane and bending stress components respectively, which define the stress state s according to

$$s = s(u) = s_m + s_b \left(1 - \frac{2u}{t} \right) \quad \text{for } 0 \leq u \leq t \quad (\text{AI.5})$$

The stress s is to be taken normal to the prospective crack plane in an uncracked plate. s_m and s_b are determined by fitting s to Equation (AI.5). The co-ordinate u is defined in Figure AI.3.

The geometry functions f_m and f_b are given in Tables AI.4 and AI.5 for points A and B respectively, see Figure AI.3.

Table AI.4. Geometry functions for an embedded crack in a plate at point A which is closest to $u = 0$.

$c/a = 1$						
$2a/t$	$e/t = 0$		$e/t = 0.15$		$e/t = 0.3$	
	f_m^A	f_b^A	f_m^A	f_b^A	f_m^A	f_b^A
0	0.638	0.000	0.638	0.191	0.638	0.383
0.2	0.649	0.087	0.659	0.286	0.694	0.509
0.4	0.681	0.182	0.725	0.411	-	-
0.6	0.739	0.296	0.870	0.609	-	-
$c/a = 2$						
$2a/t$	$e/t = 0$		$e/t = 0.15$		$e/t = 0.3$	
	f_m^A	f_b^A	f_m^A	f_b^A	f_m^A	f_b^A
0	0.824	0.000	0.824	0.247	0.824	0.494
0.2	0.844	0.098	0.862	0.359	0.932	0.668
0.4	0.901	0.210	0.987	0.526	-	-
0.6	1.014	0.355	1.332	0.866	-	-
$c/a = 4$						
$2a/t$	$e/t = 0$		$e/t = 0.15$		$e/t = 0.3$	
	f_m^A	f_b^A	f_m^A	f_b^A	f_m^A	f_b^A
0	0.917	0.000	0.917	0.275	0.917	0.550
0.2	0.942	0.102	0.966	0.394	1.058	0.749
0.4	1.016	0.220	1.129	0.584	-	-
0.6	1.166	0.379	1.655	1.034	-	-
$c/a = \infty$						
$2a/t$	$e/t = 0$		$e/t = 0.15$		$e/t = 0.3$	
	f_m^A	f_b^A	f_m^A	f_b^A	f_m^A	f_b^A
0	1.010	0.000	1.010	0.303	1.010	0.606
0.2	1.041	0.104	1.071	0.428	1.189	0.833
0.4	1.133	0.227	1.282	0.641	-	-
0.6	1.329	0.399	2.093	1.256	-	-

Table AI.5. Geometry functions for an embedded crack in a plate at point B furthest from $u = 0$.

$c/a = 1$						
$2a/t$	$e/t = 0$		$e/t = 0.15$		$e/t = 0.3$	
	f_m^B	f_b^B	f_m^B	f_b^B	f_m^B	f_b^B
0	0.638	0.000	0.638	0.191	0.638	0.383
0.2	0.649	-0.087	0.646	0.108	0.648	0.303
0.4	0.681	-0.182	0.668	0.022	-	-
0.6	0.739	-0.296	0.705	-0.071	-	-
$c/a = 2$						
$2a/t$	$e/t = 0$		$e/t = 0.15$		$e/t = 0.3$	
	f_m^B	f_b^B	f_m^B	f_b^B	f_m^B	f_b^B
0	0.824	0.000	0.824	0.247	0.824	0.494
0.2	0.844	-0.098	0.844	0.155	0.866	0.418
0.4	0.901	-0.210	0.902	0.060	-	-
0.6	1.014	-0.355	1.016	-0.051	-	-
$c/a = 4$						
$2a/t$	$e/t = 0$		$e/t = 0.15$		$e/t = 0.3$	
	f_m^B	f_b^B	f_m^B	f_b^B	f_m^B	f_b^B
0	0.917	0.000	0.917	0.275	0.917	0.550
0.2	0.942	-0.102	0.945	0.181	0.980	0.482
0.4	1.016	-0.220	1.029	0.086	-	-
0.6	1.166	-0.379	1.206	-0.030	-	-
$c/a \rightarrow \infty$						
$2a/t$	$e/t = 0$		$e/t = 0.15$		$e/t = 0.3$	
	f_m^B	f_b^B	f_m^B	f_b^B	f_m^B	f_b^B
0	1.010	0.000	1.010	0.303	1.010	0.606
0.2	1.041	-0.104	1.048	0.210	1.099	0.550
0.4	1.133	-0.227	1.162	0.166	-	-
0.6	1.329	-0.399	1.429	0.000	-	-

Remarks: The plate should be large in comparison to the length of the crack so that edge effects do not influence the results.
Taken from Reference AI.5.

Description: *Through-thickness crack*

Schematic:

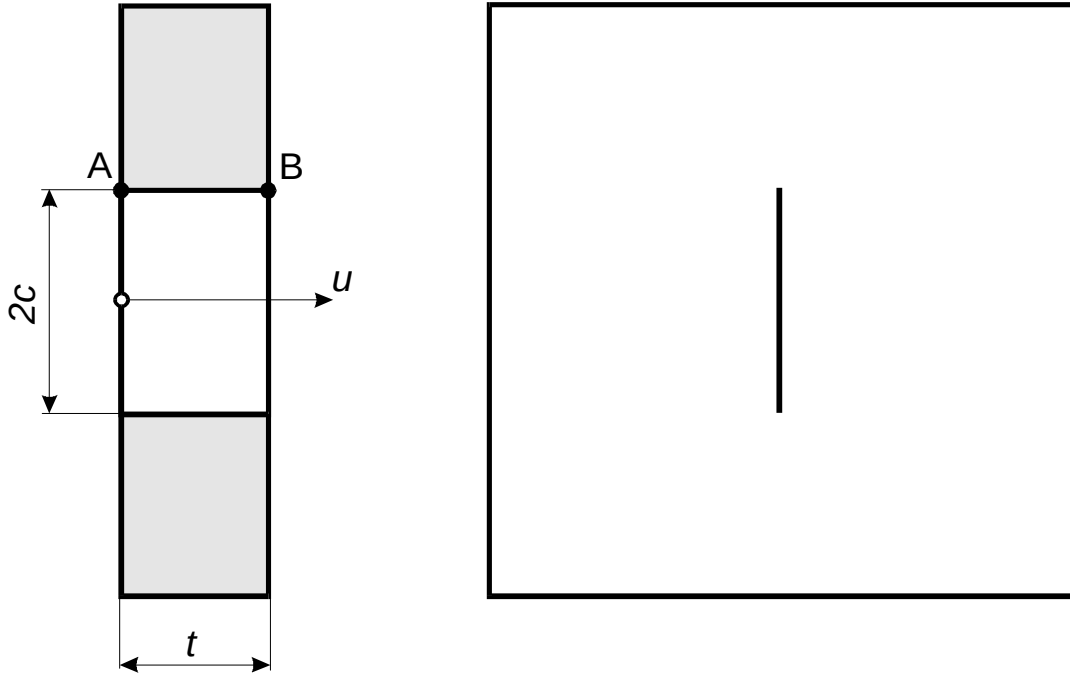


Figure AI.4. Through-thickness crack in a plate.

Solution:

The stress intensity factor K_I is given by

$$K_I = \sqrt{pc}(s_m f_m + s_b f_b)$$

In Equation (AI.6), s_m and s_b are the membrane and bending stress components respectively, which define the stress state s according to

$$s = s(u) = s_m + s_b \left(1 - \frac{2u}{t}\right) \quad \text{for } 0 \leq u \leq t \quad (\text{AI.7})$$

s is to be taken normal to the prospective crack plane in an uncracked plate. s_m and s_b are determined by fitting s to Equation (AI.7). The co-ordinate u is defined in Figure AI.4.

The geometry functions f_m and f_b are given in Table AI.6 for points at the intersections of the crack with the free surface at $u = 0$ (A) and at $u = t$ (B), see Figure AI.4.

Table AI.6. Geometry functions for a through-thickness crack in a plate.

f_m^A	f_b^A	f_m^B	f_b^B
1.000	1.000	1.000	-1.000

Remarks: The plate should be large in comparison to the length of the crack so that edge effects do not influence the results.
Taken from Reference AI.6.

AI.2.2. AXIAL CRACKS IN A CYLINDER

Description: *Finite internal surface crack*

Schematic:

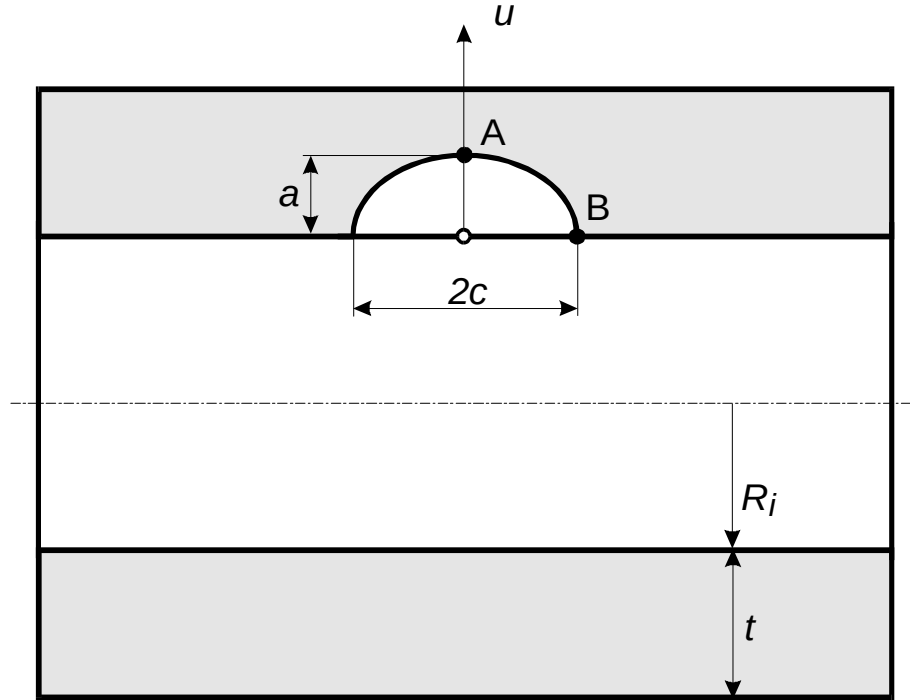


Figure AI.5. Finite axial internal surface crack in a cylinder.

Solution:

The stress intensity factor K_I is given by

$$K_I = \sqrt{pa} \sum_{i=0}^3 s_i f_i \left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t} \right) \quad (\text{AI.8})$$

s_i ($i = 0$ to 3) are stress components which define the stress state s according to

$$s = s(u) = \sum_{i=0}^3 s_i \left(\frac{u}{a} \right)^i \quad \text{for } 0 \leq u \leq a \quad (\text{AI.9})$$

s is to be taken normal to the prospective crack plane in an uncracked cylinder. s_i is determined by fitting s to Equation (AI.9). The co-ordinate u is defined in Figure AI.5.

The geometry functions f_i ($i = 0$ to 3) are given in Tables AI.7 and AI.8 for the deepest point of the crack (A) and at the intersection of the crack with the free surface (B) respectively, see Figure AI.5.

Table AI.7. Geometry functions for a finite axial internal surface crack in a cylinder at point A.

$2c/a = 2, R_i/t = 4$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	0.659	0.471	0.387	0.337
0.2	0.643	0.454	0.375	0.326
0.5	0.663	0.463	0.378	0.328
0.8	0.704	0.489	0.397	0.342
$2c/a = 2, R_i/t = 10$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	0.659	0.471	0.387	0.337
0.2	0.647	0.456	0.375	0.326
0.5	0.669	0.464	0.380	0.328
0.8	0.694	0.484	0.394	0.339
$2c/a = 5, R_i/t = 4$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	0.939	0.580	0.434	0.353
0.2	0.919	0.579	0.452	0.382
0.5	1.037	0.622	0.474	0.395
0.8	1.255	0.720	0.534	0.443
$2c/a = 5, R_i/t = 10$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	0.939	0.580	0.434	0.353
0.2	0.932	0.584	0.455	0.383
0.5	1.058	0.629	0.477	0.397
0.8	1.211	0.701	0.523	0.429
$2c/a = 10, R_i/t = 4$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	1.053	0.606	0.443	0.357
0.2	1.045	0.634	0.487	0.406
0.5	1.338	0.739	0.540	0.438
0.8	1.865	0.948	0.659	0.516
$2c/a = 10, R_i/t = 10$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	1.053	0.606	0.443	0.357
0.2	1.062	0.641	0.489	0.417
0.5	1.359	0.746	0.544	0.440
0.8	1.783	0.914	0.639	0.504

Table AI.8. Geometry functions for a finite axial internal surface crack in a cylinder at point B.

$2c/a = 2, R_i/t = 4$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.716	0.118	0.041	0.022
0.2	0.719	0.124	0.046	0.024
0.5	0.759	0.136	0.052	0.027
0.8	0.867	0.158	0.062	0.032
$2c/a = 2, R_i/t = 10$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.716	0.118	0.041	0.022
0.2	0.726	0.126	0.047	0.024
0.5	0.777	0.141	0.054	0.028
0.8	0.859	0.163	0.063	0.033
$2c/a = 5, R_i/t = 4$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.673	0.104	0.032	0.016
0.2	0.670	0.107	0.037	0.018
0.5	0.803	0.151	0.059	0.031
0.8	1.060	0.229	0.095	0.051
$2c/a = 5, R_i/t = 10$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.673	0.104	0.032	0.015
0.2	0.676	0.109	0.037	0.018
0.5	0.814	0.153	0.060	0.031
0.8	1.060	0.225	0.092	0.049
$2c/a = 10, R_i/t = 4$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.516	0.069	0.017	0.009
0.2	0.577	0.075	0.022	0.010
0.5	0.759	0.134	0.051	0.027
0.8	1.144	0.250	0.103	0.056
$2c/a = 10, R_i/t = 10$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.516	0.069	0.017	0.009
0.2	0.578	0.075	0.022	0.010
0.5	0.753	0.131	0.050	0.026
0.8	1.123	0.241	0.099	0.053

Remarks: The cylinder should be long in comparison to the length of the crack so that edge effects do not influence the results.
Taken from References AI.3 and AI.7.

Description: *Infinite internal surface crack*

Schematic:

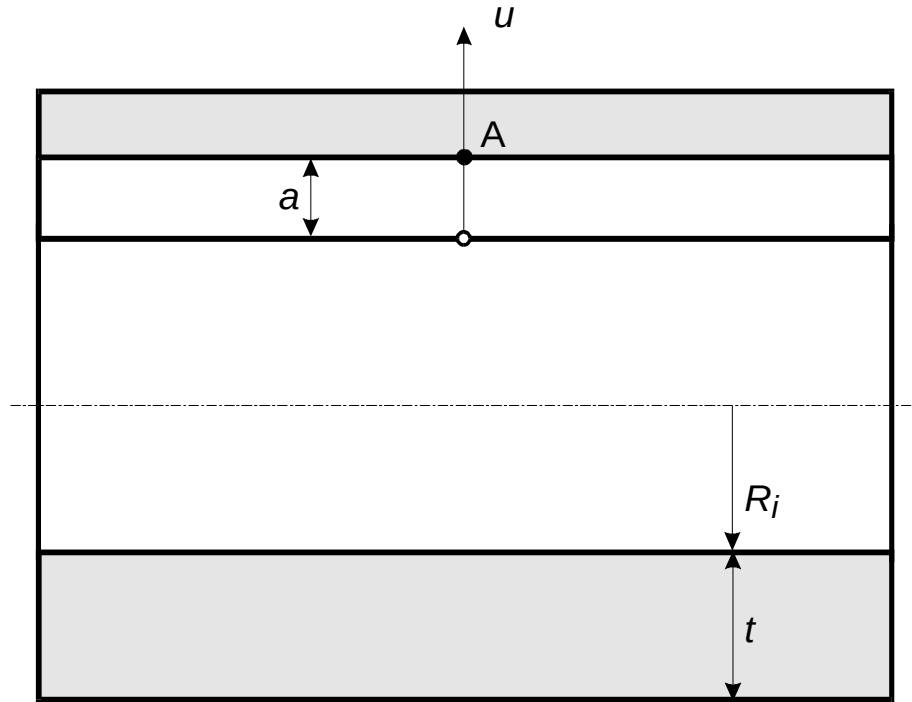


Figure AI.6. Infinite axial internal surface crack in a cylinder.

Solution:

The stress intensity factor K_I is given by

$$K_I = \frac{1}{\sqrt{2pa}} \int_0^a s(u) \sum_{i=1}^{i=3} f_i(a/t, R_i/t) \left(1 - \frac{u}{a}\right)^{i-\frac{3}{2}} du \quad (\text{AI.10})$$

The stress state $s = s(u)$ is to be taken normal to the prospective crack plane in an uncracked cylinder. The co-ordinate u is defined in Figure AI.6.

The geometry functions f_i ($i = 1$ to 3) are given in Table AI.9 for the deepest point of the crack (A), see Figure AI.6.

Table AI.9. Geometry functions for an infinite axial internal surface crack in a cylinder.

a/t	$R_i/t = 0.5$			$R_i/t = 1$		
	f_1^A	f_2^A	f_3^A	f_1^A	f_2^A	f_3^A
0	2.000	1.328	0.220	2.000	1.336	0.220
0.1	2.000	0.890	0.155	2.000	1.271	0.184
0.2	2.000	0.895	0.193	2.000	1.566	0.237
0.3	2.000	1.032	0.252	2.000	1.997	0.360
0.4	2.000	1.329	0.210	2.000	2.501	0.542
0.5	2.000	1.796	0.093	2.000	3.072	0.762
0.6	2.000	2.457	-0.074	2.000	3.807	0.892
0.7	2.000	3.597	-0.618	2.000	4.877	0.825
0.75	2.000	4.571	-1.272	2.000	5.552	0.786
a/t	$R_i/t = 2$			$R_i/t = 4$		
	f_1^A	f_2^A	f_3^A	f_1^A	f_2^A	f_3^A
0	2.000	1.340	0.219	2.000	1.340	0.219
0.1	2.000	1.519	0.212	2.000	1.659	0.217
0.2	2.000	2.119	0.322	2.000	2.475	0.358
0.3	2.000	2.934	0.551	2.000	3.615	0.709
0.4	2.000	3.820	1.066	2.000	4.982	1.499
0.5	2.000	4.692	1.853	2.000	6.455	2.936
0.6	2.000	5.697	2.600	2.000	7.977	5.018
0.7	2.000	6.995	3.224	2.000	9.513	7.637
0.75	2.000	7.656	3.733	2.000	10.24	9.134

Remarks: Taken from Reference AI.4.

Description: *Finite external surface crack*

Schematic:

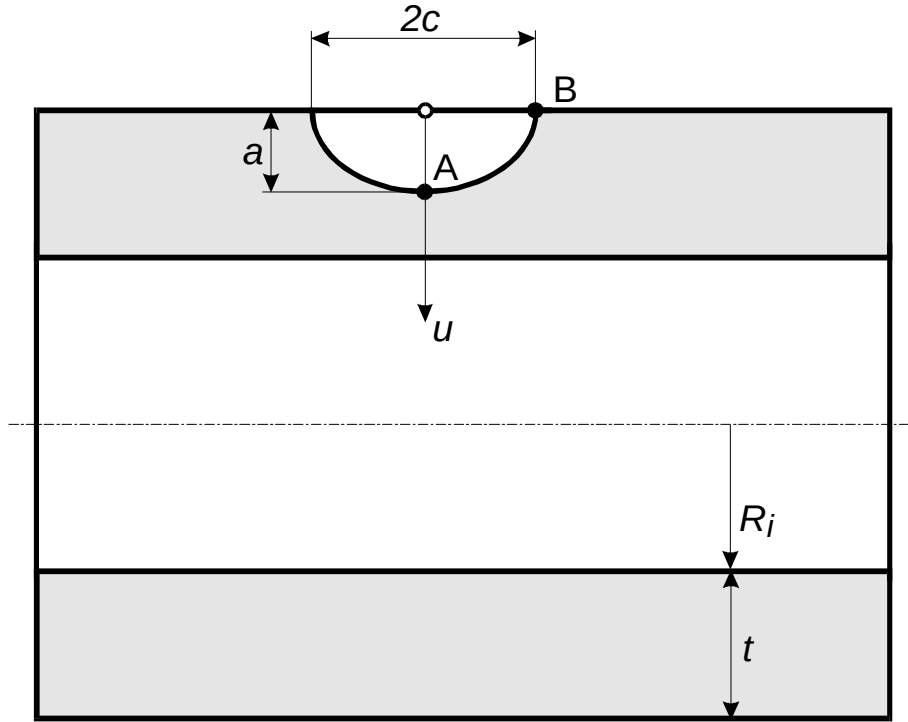


Figure AI.7. Finite axial external surface crack in a cylinder.

Solution:

The stress intensity factor K_I is given by

$$K_I = \sqrt{pa} \sum_{i=0}^3 s_i f_i \left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t} \right) \quad (\text{AI.11})$$

s_i ($i = 0$ to 3) are stress components which define the stress state s according to

$$s = s(u) = \sum_{i=0}^3 s_i \left(\frac{u}{a} \right)^i \quad \text{for } 0 \leq u \leq a \quad (\text{AI.12})$$

s is to be taken normal to the prospective crack plane in an uncracked cylinder. s_i is determined by fitting s to Equation (AI.12). The co-ordinate u is defined in Figure AI.7.

f_i ($i = 0$ to 3) are geometry functions which are given in Tables AI.10 and AI.11 for the deepest point of the crack (A), and at the intersection of the crack with the free surface (B), respectively, see Figure AI.7.

Table AI.10. Geometry functions at point A for a finite axial external surface crack in a cylinder.

$2c/a = 2, R_i/t = 4$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	0.659	0.471	0.387	0.337
0.2	0.656	0.459	0.377	0.327
0.5	0.697	0.473	0.384	0.331
0.8	0.736	0.495	0.398	0.342
$2c/a = 2, R_i/t = 10$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	0.659	0.471	0.387	0.337
0.2	0.653	0.457	0.376	0.327
0.5	0.687	0.470	0.382	0.330
0.8	0.712	0.487	0.394	0.340
$2c/a = 5, R_i/t = 4$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	0.939	0.580	0.434	0.353
0.2	0.964	0.596	0.461	0.387
0.5	1.183	0.672	0.500	0.410
0.8	1.502	0.795	0.568	0.455
$2c/a = 5, R_i/t = 10$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	0.939	0.580	0.434	0.353
0.2	0.953	0.591	0.459	0.386
0.5	1.139	0.656	0.491	0.405
0.8	1.361	0.746	0.543	0.439
$2c/a = 10, R_i/t = 4$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	1.053	0.606	0.443	0.357
0.2	1.107	0.658	0.499	0.413
0.5	1.562	0.820	0.584	0.465
0.8	2.390	1.122	0.745	0.568
$2c/a = 10, R_i/t = 10$				
a/t	f_0^A	f_1^A	f_2^A	f_3^A
0	1.053	0.606	0.443	0.357
0.2	1.092	0.652	0.496	0.411
0.5	1.508	0.799	0.571	0.457
0.8	2.188	1.047	0.704	0.541

Table AI.11. Geometry functions at point B for a finite axial external surface crack in a cylinder.

$2c/a = 2, R_i/t = 4$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.716	0.118	0.041	0.022
0.2	0.741	0.130	0.049	0.026
0.5	0.819	0.155	0.061	0.033
0.8	0.954	0.192	0.078	0.041
$2c/a = 2, R_i/t = 10$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.716	0.118	0.041	0.022
0.2	0.736	0.129	0.048	0.025
0.5	0.807	0.150	0.059	0.031
0.8	0.926	0.182	0.072	0.038
$2c/a = 5, R_i/t = 4$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.673	0.104	0.032	0.015
0.2	0.690	0.113	0.039	0.019
0.5	0.864	0.170	0.068	0.036
0.8	1.217	0.277	0.117	0.064
$2c/a = 5, R_i/t = 10$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.673	0.104	0.032	0.015
0.2	0.685	0.111	0.039	0.019
0.5	0.856	0.167	0.066	0.035
0.8	1.198	0.269	0.112	0.061
$2c/a = 10, R_i/t = 4$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.516	0.069	0.017	0.009
0.2	0.583	0.076	0.022	0.010
0.5	0.748	0.128	0.047	0.024
0.8	1.105	0.230	0.092	0.049
$2c/a = 10, R_i/t = 10$				
a/t	f_0^B	f_1^B	f_2^B	f_3^B
0	0.516	0.069	0.017	0.009
0.2	0.583	0.076	0.022	0.010
0.5	0.768	0.135	0.051	0.027
0.8	1.202	0.264	0.109	0.059

Remarks: The cylinder should be long in comparison to the length of the crack so that edge effects do not influence the results.
Taken from Reference AI.3 and AI.7.

Description: *Infinite external surface crack*

Schematic:

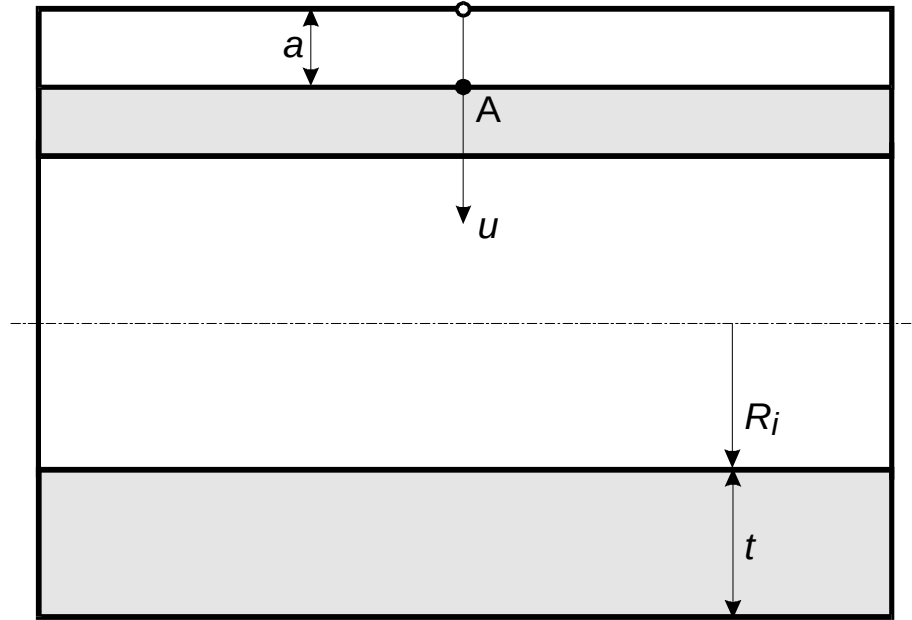


Figure AI.8. Infinite axial external surface crack in a cylinder.

Solution:

The stress intensity factor K_I is given by

$$K_I = \frac{1}{\sqrt{2pa}} \int_0^a s(u) \sum_{i=1}^{i=4} f_i(a/t, R_i/t) \left(1 - \frac{u}{a}\right)^{i-\frac{3}{2}} du \quad (\text{AI.13})$$

The stress state $s = s(u)$ is to be taken normal to the prospective crack plane in an uncracked cylinder. The co-ordinate u is defined in Figure AI.8.

f_i ($i = 1$ to 4) are geometry functions which are given in Table AI.12 for the deepest point of the crack (A). See Figure AI.8.

Table AI.12. Geometry functions for an infinite axial external surface crack in a cylinder.

a/t	$R_i/t = 0.5$				$R_i/t = 1$			
	f_1^A	f_2^A	f_3^A	f_4^A	f_1^A	f_2^A	f_3^A	f_4^A
0	2.000	0.901	1.401	-0.620	2.000	0.901	1.401	-0.620
0.1	2.000	1.359	1.376	-0.585	2.000	1.331	1.365	-0.584
0.2	2.000	1.933	1.387	-0.549	2.000	1.967	1.369	-0.543
0.3	2.000	2.614	1.422	-0.510	2.000	2.766	1.484	-0.512
0.4	2.000	3.408	1.541	-0.481	2.000	3.708	1.759	-0.505
0.5	2.000	4.321	1.799	-0.472	2.000	4.787	2.238	-0.528
0.6	2.000	5.459	2.101	-0.456	2.000	6.055	2.904	-0.577
0.7	2.000	7.145	2.187	-0.361	2.000	7.726	3.601	-0.605
0.75	2.000	8.355	2.112	-0.265	2.000	8.853	3.901	-0.590
a/t	$R_i/t = 2$				$R_i/t = 4$			
	f_1^A	f_2^A	f_3^A	f_4^A	f_1^A	f_2^A	f_3^A	f_4^A
0	2.000	0.901	1.401	-0.620	2.000	0.900	1.400	-0.620
0.1	2.000	1.330	1.370	-0.585	2.000	1.335	1.382	-0.587
0.2	2.000	2.086	1.403	-0.542	2.000	2.219	1.416	-0.535
0.3	2.000	3.095	1.580	-0.510	2.000	3.464	1.658	-0.501
0.4	2.000	4.307	2.054	-0.524	2.000	4.993	2.412	-0.549
0.5	2.000	5.643	3.004	-0.625	2.000	6.823	3.794	-0.704
0.6	2.000	7.103	4.376	-0.802	2.000	8.984	6.051	-1.011
0.7	2.000	8.976	5.735	-0.949	2.000	11.10	10.07	-1.674
0.75	2.000	10.28	6.243	-0.963	2.000	11.80	13.08	-2.229

Remarks: Taken from Reference AI.2.

AI.2.3. CIRCUMFERENTIAL CRACKS IN A CYLINDER

Description: *Part circumferential internal surface crack*

Schematic:

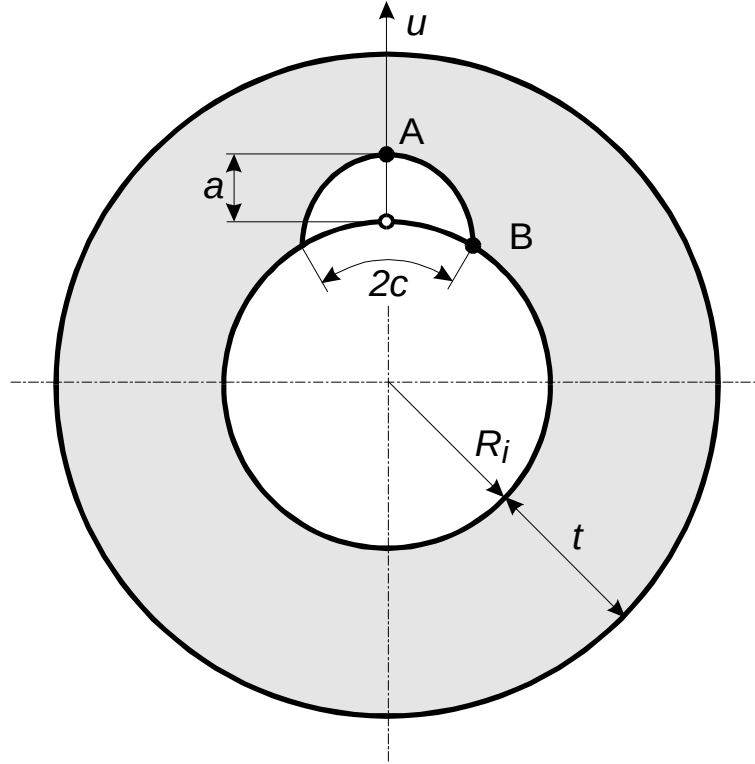


Figure AI.9. Part circumferential internal surface crack in a cylinder.

Solution:

The stress intensity factor K_I is given by

$$K_I = \sqrt{pa} \left(\sum_{i=0}^3 s_i f_i \left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t} \right) + s_{bg} f_{bg} \left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t} \right) \right) \quad (\text{AI.14})$$

s_i ($i = 0$ to 3) are stress components which define the axisymmetric stress state s according to

$$s = s(u) = \sum_{i=0}^3 s_i \left(\frac{u}{a} \right)^i \quad \text{for } 0 \leq u \leq a \quad (\text{AI.15})$$

and s_{bg} is the global bending stress, i.e. the maximum outer fibre bending stress. s and s_{bg} are to be taken normal to the prospective crack plane in an uncracked cylinder. s_i is determined by fitting s to Equation (AI.15). The co-ordinate u is defined in Figure AI.9. It should be noted that the solution for global bending stress assumes that the crack is symmetrically positioned about the global bending axis as shown in Figure AI.9. f_i ($i = 0$ to 3) and f_{bg} are geometry functions which are given in Tables AI.13 and AI.14 for the deepest point of the crack (A), and at the intersection of the crack with the free surface (B), respectively, see Figure AI.9.

Table AI.13. Geometry functions at point A for a part circumferential internal surface crack in a cylinder.

$2c/a = 2, R_i/t = 5$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	0.659	0.471	0.387	0.337	0.549
0.2	0.665	0.460	0.371	0.316	0.570
0.4	0.682	0.471	0.381	0.327	0.600
0.6	0.700	0.481	0.390	0.335	0.632
0.8	0.729	0.506	0.410	0.352	0.675
$2c/a = 2, R_i/t = 10$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	0.659	0.471	0.387	0.337	0.599
0.2	0.664	0.459	0.370	0.315	0.613
0.4	0.680	0.469	0.379	0.325	0.636
0.6	0.696	0.478	0.387	0.333	0.659
0.8	0.714	0.497	0.403	0.347	0.685
$2c/a = 4, R_i/t = 5$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	0.886	0.565	0.430	0.352	0.738
0.2	0.890	0.556	0.424	0.347	0.761
0.4	0.934	0.576	0.440	0.362	0.817
0.6	0.991	0.602	0.457	0.377	0.885
0.8	1.066	0.653	0.496	0.409	0.973
$2c/a = 4, R_i/t = 10$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	0.886	0.565	0.430	0.352	0.806
0.2	0.895	0.557	0.424	0.347	0.825
0.4	0.947	0.580	0.441	0.363	0.883
0.6	1.008	0.605	0.458	0.377	0.950
0.8	1.482	0.647	0.492	0.406	1.012
$2c/a = 8, R_i/t = 5$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.025	0.600	0.441	0.356	0.854
0.2	1.041	0.625	0.469	0.381	0.890
0.4	1.142	0.666	0.496	0.403	0.995
0.6	1.274	0.718	0.527	0.427	1.126
0.8	1.463	0.813	0.589	0.471	1.310

**Table AI.13. Geometry functions at point A for a part circumferential internal surface crack in a cylinder.
(Continued)**

$2c/a = 8, R_i/t = 10$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.025	0.600	0.441	0.356	0.931
0.2	1.053	0.629	0.471	0.382	0.970
0.4	1.180	0.678	0.502	0.407	1.097
0.6	1.335	0.737	0.536	0.431	1.253
0.8	1.482	0.814	0.587	0.469	1.402
$2c/a = 16, R_i/t = 5$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.079	0.635	0.473	0.388	0.899
0.2	1.130	0.665	0.493	0.398	0.964
0.4	1.294	0.732	0.537	0.433	1.120
0.6	1.521	0.820	0.587	0.468	1.321
0.8	1.899	0.987	0.690	0.541	1.633
$2c/a = 16, R_i/t = 10$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.079	0.635	0.473	0.388	0.981
0.2	1.150	0.672	0.498	0.401	1.059
0.4	1.366	0.756	0.549	0.441	1.267
0.6	1.643	0.859	0.606	0.479	1.531
0.8	1.972	1.002	0.694	0.541	1.842
$2c/a = 32, R_i/t = 5$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.101	0.658	0.499	0.413	0.918
0.2	1.180	0.690	0.512	0.414	1.004
0.4	1.521	0.775	0.564	0.453	1.188
0.6	1.707	0.902	0.638	0.505	1.430
0.8	2.226	1.137	0.783	0.609	1.794
$2c/a = 32, R_i/t = 10$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.101	0.658	0.499	0.413	1.001
0.2	1.209	0.701	0.518	0.418	1.112
0.4	1.490	0.810	0.582	0.464	1.377
0.6	1.887	0.958	0.665	0.520	1.737
0.8	2.444	1.187	0.799	0.613	2.219

Table AI.14. Geometry functions at point B for a part circumferential internal surface crack in a cylinder.

$2c/a = 2, R_i/t = 5$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.718	0.117	0.041	0.020	0.598
0.2	0.746	0.125	0.046	0.023	0.625
0.4	0.774	0.133	0.051	0.026	0.652
0.6	0.882	0.147	0.058	0.031	0.696
0.8	0.876	0.161	0.064	0.034	0.746
$2c/a = 2, R_i/t = 10$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.716	0.116	0.041	0.020	0.652
0.2	0.747	0.125	0.046	0.023	0.682
0.4	0.778	0.134	0.051	0.026	0.712
0.6	0.831	0.148	0.058	0.031	0.763
0.8	0.890	0.163	0.064	0.033	0.820
$2c/a = 4, R_i/t = 5$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.664	0.091	0.029	0.013	0.555
0.2	0.716	0.108	0.039	0.019	0.599
0.4	0.768	0.125	0.049	0.025	0.643
0.6	0.852	0.152	0.062	0.033	0.712
0.8	0.944	0.179	0.075	0.040	0.788
$2c/a = 4, R_i/t = 10$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.657	0.089	0.030	0.014	0.598
0.2	0.719	0.109	0.040	0.020	0.656
0.4	0.781	0.129	0.050	0.026	0.714
0.6	0.883	0.160	0.066	0.035	0.809
0.8	0.995	0.191	0.079	0.042	0.913
$2c/a = 8, R_i/t = 5$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.541	0.054	0.014	0.004	0.461
0.2	0.598	0.072	0.023	0.010	0.496
0.4	0.655	0.090	0.032	0.016	0.531
0.6	0.737	0.116	0.045	0.023	0.576
0.8	0.846	0.151	0.062	0.033	0.634

**Table AI.14. Geometry functions at point B for a part circumferential internal surface crack in a cylinder.
(Continued)**

$2c/a = 8, R_i/t = 10$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.527	0.047	0.010	0.002	0.481
0.2	0.602	0.072	0.023	0.010	0.547
0.4	0.677	0.097	0.036	0.018	0.613
0.6	0.788	0.131	0.052	0.027	0.710
0.8	0.927	0.172	0.070	0.037	0.829
$2c/a = 16, R_i/t = 5$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.417	0.027	0.004	0.000	0.381
0.2	0.447	0.037	0.009	0.003	0.357
0.4	0.477	0.047	0.014	0.006	0.333
0.6	0.528	0.062	0.021	0.010	0.292
0.8	0.600	0.085	0.032	0.017	0.236
$2c/a = 16, R_i/t = 10$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.413	0.025	0.003	0.000	0.387
0.2	0.455	0.039	0.010	0.004	0.411
0.4	0.497	0.053	0.017	0.008	0.435
0.6	0.568	0.073	0.026	0.013	0.475
0.8	0.670	0.104	0.041	0.021	0.531
$2c/a = 32, R_i/t = 5$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.276	0.010	0.000	0.000	0.313
0.2	0.294	0.014	0.002	0.000	0.200
0.4	0.312	0.018	0.004	0.001	0.087
0.6	0.331	0.023	0.006	0.003	0.056
0.8	0.348	0.026	0.009	0.003	0.276
$2c/a = 32, R_i/t = 10$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.275	0.009	0.001	0.000	0.276
0.2	0.298	0.015	0.003	0.000	0.258
0.4	0.321	0.021	0.005	0.002	0.240
0.6	0.352	0.028	0.009	0.004	0.200
0.8	0.389	0.038	0.012	0.006	0.139

Remarks: The cylinder should be long in the transverse direction to the crack so that edge effects do not influence the results.
Taken from Reference AI.3 and AI.9.

Description: *Complete circumferential internal surface crack*

Schematic:

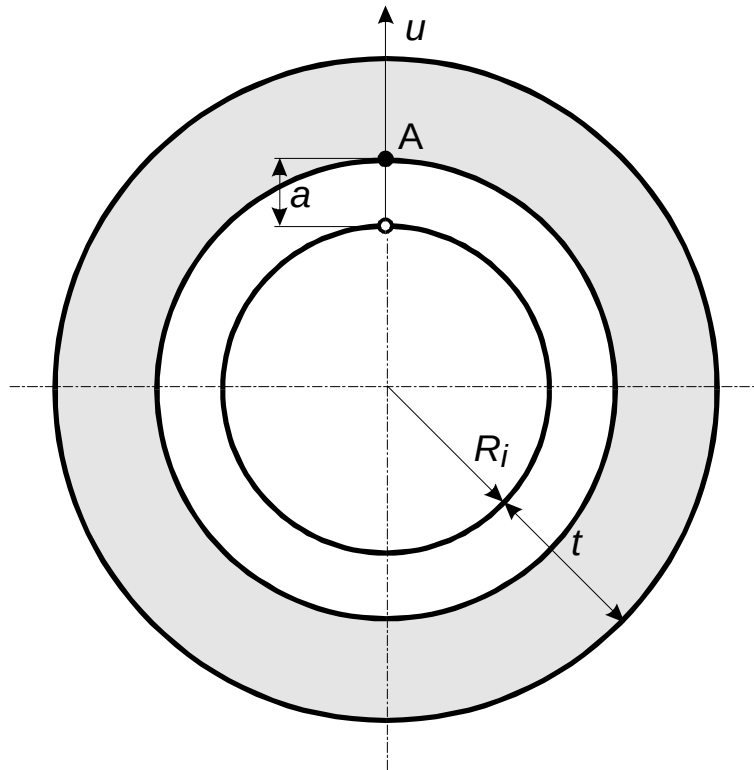


Figure AI.10. Complete circumferential internal surface crack in a cylinder.

Solution:

The stress intensity factor K_I is given by

$$K_I = \frac{1}{\sqrt{2pa}} \int_0^a s(u) \sum_{i=1}^{i=3} f_i(a/t, R_i/t) \left(1 - \frac{u}{a}\right)^{i-\frac{3}{2}} du \quad (\text{AI.16})$$

The stress state $s = s(u)$ is to be taken normal to the prospective crack plane in an uncracked cylinder. The co-ordinate u is defined in Figure AI.10.

f_i ($i = 1$ to 3) are geometry functions which are given in Table AI.15 for the deepest point of the crack (A). See Figure AI.10.

Table AI.15. Geometry functions for a complete circumferential internal surface crack in a cylinder.

a/t	$R_i/t = 7/3$		
	f_1^A	f_2^A	f_3^A
0	2.000	1.327	0.218
0.1	2.000	1.337	0.200
0.2	2.000	1.543	0.201
0.3	2.000	1.880	0.228
0.4	2.000	2.321	0.293
0.5	2.000	2.879	0.373
0.6	2.000	3.720	0.282
a/t	$R_i/t = 5$		
	f_1^A	f_2^A	f_3^A
0	2.000	1.336	0.218
0.1	2.000	1.460	0.206
0.2	2.000	1.839	0.241
0.3	2.000	2.359	0.353
0.4	2.000	2.976	0.556
0.5	2.000	3.688	0.837
0.6	2.000	4.598	1.086
a/t	$R_i/t = 10$		
	f_1^A	f_2^A	f_3^A
0	2.000	1.346	0.219
0.1	2.000	1.591	0.211
0.2	2.000	2.183	0.279
0.3	2.000	2.966	0.518
0.4	2.000	3.876	0.956
0.5	2.000	4.888	1.614
0.6	2.000	5.970	2.543

Remarks: The cylinder should be long in the transverse direction to the crack so that edge effects do not influence the results.
Taken from Reference AI.4.

Description: *Part circumferential external surface crack*

Schematic:

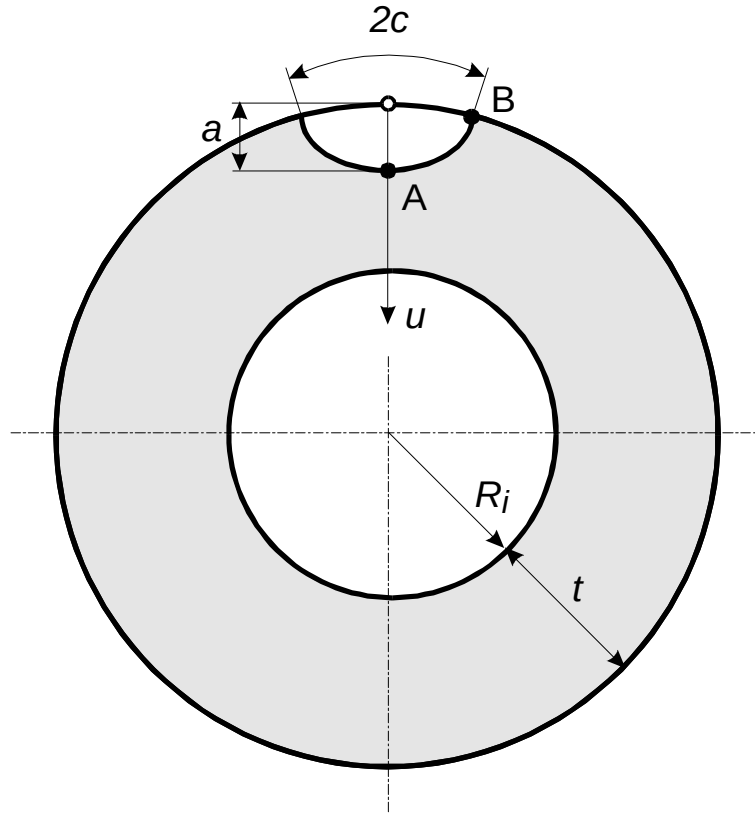


Figure AI.11. Part circumferential external surface crack in a cylinder.

Solution:

The stress intensity factor K_I is given by

$$K_I = \sqrt{pa} \left(\sum_{i=0}^3 s_i f_i \left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t} \right) + s_{bg} f_{bg} \left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t} \right) \right) \quad (\text{AI.17})$$

s_i ($i = 0$ to 3) are stress components which define the axisymmetric stress state s according to

$$s = s(u) = \sum_{i=0}^3 s_i \left(\frac{u}{a} \right)^i \quad \text{for } 0 \leq u \leq a \quad (\text{AI.18})$$

and s_{bg} is the global bending stress, i.e. the maximum outer fibre bending stress. s and s_{bg} are to be taken normal to the prospective crack plane in an uncracked cylinder. s_i is determined by fitting s to Equation (AI.18). The co-ordinate u is defined in Figure AI.11. It should be noted that the solution for global bending stress assumes that the crack is symmetrically positioned about the global bending axis as shown in Figure AI.11. f_i ($i = 0$ to 3) and f_{bg} are geometry functions which are given in Tables AI.16 and AI.17 for the deepest point of the crack (A), and at the intersection of the crack with the free surface (B), respectively. See Figure AI.11.

Table AI.16. Geometry functions at point A for a part circumferential external surface crack in a cylinder.

$2c/a= 2, R_i/t = 5$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	0.659	0.471	0.387	0.337	0.659
0.2	0.661	0.455	0.367	0.313	0.645
0.4	0.673	0.462	0.374	0.321	0.642
0.6	0.686	0.467	0.378	0.325	0.638
0.8	0.690	0.477	0.387	0.333	0.626
$2c/a= 2, R_i/t = 10$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	0.659	0.471	0.387	0.337	0.659
0.2	0.662	0.456	0.368	0.313	0.653
0.4	0.676	0.464	0.376	0.322	0.659
0.6	0.690	0.470	0.381	0.328	0.664
0.8	0.695	0.482	0.392	0.337	0.660
$2c/a= 4, R_i/t = 5$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	0.886	0.565	0.430	0.352	0.886
0.2	0.905	0.560	0.425	0.347	0.885
0.4	0.972	0.586	0.443	0.363	0.932
0.6	1.060	0.618	0.462	0.378	0.995
0.8	1.133	0.659	0.493	0.403	1.041
$2c/a= 4, R_i/t = 10$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	0.886	0.565	0.430	0.352	0.886
0.2	0.903	0.559	0.425	0.347	0.891
0.4	0.969	0.586	0.443	0.363	0.947
0.6	1.051	0.616	0.462	0.378	1.016
0.8	1.108	0.654	0.491	0.403	1.059
$2c/a= 8, R_i/t = 5$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.025	0.600	0.441	0.356	1.025
0.2	1.078	0.638	0.476	0.386	1.055
0.4	1.253	0.702	0.513	0.413	1.202
0.6	1.502	0.790	0.561	0.446	1.413
0.8	1.773	0.900	0.625	0.490	1.631

**Table AI.16. Geometry functions at point A for a part circumferential external surface crack in a cylinder.
(Continued)**

$2c/a = 8, R_i/t = 10$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.025	0.600	0.441	0.356	1.025
0.2	1.073	0.637	0.475	0.386	1.060
0.4	1.246	0.700	0.512	0.413	1.219
0.6	1.489	0.786	0.559	0.445	1.443
0.8	1.711	0.880	0.616	0.484	1.640
$2c/a = 16, R_i/t = 5$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.079	0.635	0.473	0.388	1.079
0.2	1.186	0.685	0.504	0.406	1.162
0.4	1.482	0.797	0.570	0.454	1.419
0.6	1.907	0.951	0.654	0.508	1.779
0.8	2.461	1.166	0.776	0.591	2.220
$2c/a = 16, R_i/t = 10$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.079	0.635	0.473	0.388	1.079
0.2	1.182	0.684	0.504	0.405	1.168
0.4	1.491	0.800	0.571	0.454	1.458
0.6	1.949	0.962	0.658	0.511	1.883
0.8	2.479	1.165	0.772	0.587	2.363
$2c/a = 32, R_i/t = 5$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.101	0.658	0.499	0.413	1.101
0.2	1.252	0.716	0.525	0.422	1.225
0.4	1.599	0.854	0.607	0.482	1.525
0.6	2.067	1.036	0.713	0.555	1.926
0.8	2.740	1.313	0.875	0.666	2.491
$2c/a = 32, R_i/t = 10$					
a/t	f_0^A	f_1^A	f_2^A	f_3^A	f_{bg}^A
0	1.101	0.658	0.499	0.413	1.101
0.2	1.252	0.716	0.525	0.421	1.237
0.4	1.651	0.869	0.614	0.485	1.611
0.6	2.243	1.089	0.736	0.566	2.157
0.8	3.011	1.387	0.904	0.678	2.845

Table AI.17. Geometry functions at point *B* for a part circumferential external surface crack in a cylinder.

$2c/a=2, R_i/t=5$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.715	0.117	0.040	0.020	0.717
0.2	0.748	0.125	0.045	0.023	0.744
0.4	0.781	0.133	0.050	0.026	0.771
0.6	0.837	0.147	0.057	0.030	0.821
0.8	0.905	0.163	0.063	0.033	0.880
$2c/a=2, R_i/t=10$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.713	0.117	0.041	0.020	0.713
0.2	0.748	0.125	0.046	0.023	0.745
0.4	0.783	0.133	0.051	0.026	0.777
0.6	0.841	0.149	0.058	0.030	0.832
0.8	0.912	0.166	0.064	0.033	0.898
$2c/a=4, R_i/t=5$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.654	0.088	0.028	0.013	0.657
0.2	0.724	0.110	0.040	0.020	0.719
0.4	0.794	0.132	0.052	0.027	0.781
0.6	0.915	0.168	0.069	0.037	0.888
0.8	1.059	0.208	0.087	0.046	1.012
$2c/a=4, R_i/t=10$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.649	0.087	0.028	0.013	0.649
0.2	0.723	0.110	0.040	0.020	0.720
0.4	0.797	0.133	0.052	0.027	0.791
0.6	0.925	0.172	0.071	0.038	0.912
0.8	1.081	0.215	0.089	0.048	1.058
$2c/a=8, R_i/t=5$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.527	0.047	0.010	0.003	0.537
0.2	0.610	0.074	0.024	0.011	0.603
0.4	0.693	0.101	0.038	0.019	0.669
0.6	0.818	0.139	0.055	0.029	0.762
0.8	0.972	0.185	0.077	0.041	0.868

Table AI.17. Geometry functions at point *B* for a part circumferential external surface crack in a cylinder.
(Continued)

$2c/a = 8, R_i/t = 10$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.518	0.043	0.009	0.002	0.521
0.2	0.610	0.074	0.024	0.011	0.607
0.4	0.702	0.105	0.039	0.020	0.693
0.6	0.856	0.152	0.062	0.033	0.834
0.8	1.060	0.211	0.088	0.047	1.019
$2c/a = 16, R_i/t = 5$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.425	0.029	0.004	0.001	0.454
0.2	0.459	0.040	0.010	0.004	0.443
0.4	0.493	0.050	0.016	0.007	0.432
0.6	0.529	0.058	0.018	0.008	0.390
0.8	0.542	0.057	0.016	0.006	0.294
$2c/a = 16, R_i/t = 10$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.409	0.023	0.003	0.000	0.417
0.2	0.461	0.040	0.011	0.004	0.455
0.4	0.513	0.057	0.019	0.009	0.493
0.6	0.589	0.078	0.028	0.014	0.542
0.8	0.671	0.099	0.037	0.018	0.582
$2c/a = 32, R_i/t = 5$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.307	0.017	0.005	0.000	0.379
0.2	0.306	0.016	0.003	0.000	0.265
0.4	0.305	0.014	0.001	0.000	0.151
0.6	0.299	0.008	0.000	0.000	0.024
0.8	0.292	0.003	0.000	0.000	0.255
$2c/a = 32, R_i/t = 10$					
a/t	f_0^B	f_1^B	f_2^B	f_3^B	f_{bg}^B
0	0.299	0.021	0.002	0.000	0.323
0.2	0.309	0.020	0.003	0.000	0.296
0.4	0.319	0.019	0.004	0.000	0.269
0.6	0.322	0.016	0.002	0.000	0.208
0.8	0.305	0.005	0.000	0.000	0.103

Remarks: The cylinder should be long in the transverse direction to the crack so that edge effects do not influence the results.
Taken from Reference AI.3 and AI.9.

Description: *Complete circumferential external surface crack*

Schematic:

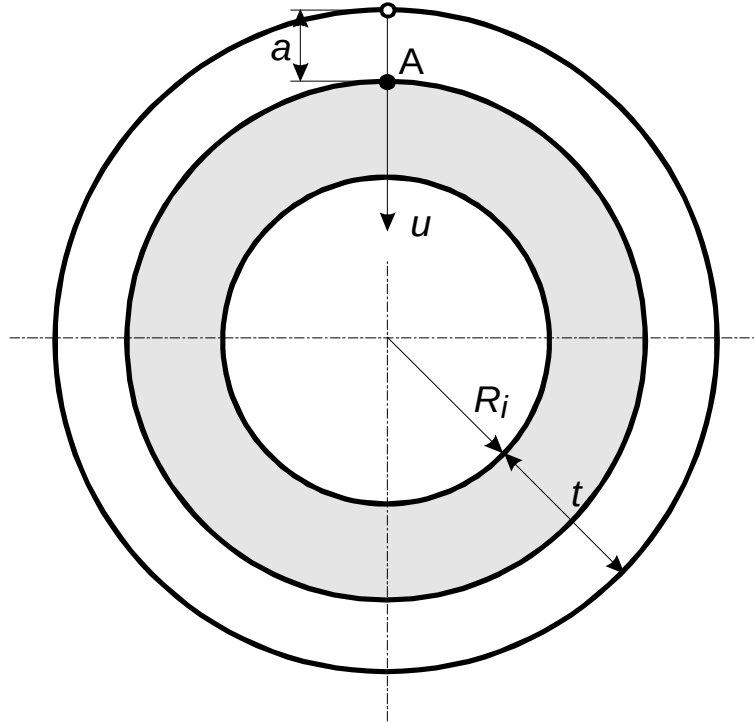


Figure AI.12. Complete circumferential external surface crack in a cylinder.

Solution:

The stress intensity factor K_I is given by

$$K_I = \frac{1}{\sqrt{2pa}} \int_0^a s(u) \sum_{i=1}^{i=3} f_i(a/t, R_i/t) \left(1 - \frac{u}{a}\right)^{\frac{3}{2}} du \quad (\text{AI.19})$$

The stress state $s = s(u)$ is to be taken normal to the prospective crack plane in an uncracked cylinder. The co-ordinate u is defined in Fig. AI.12.

f_i ($i = 1$ to 3) are geometry functions which are given in Table AI.18 for the deepest point of the crack (A). See Figure AI.12.

Table AI.18. Geometry functions for a complete circumferential external surface crack in a cylinder.

a/t	$R_i/t = 7/3$		
	f_1^A	f_2^A	f_3^A
0	2.000	1.359	0.220
0.1	2.000	1.642	0.236
0.2	2.000	2.127	0.307
0.3	2.000	2.727	0.447
0.4	2.000	3.431	0.668
0.5	2.000	4.271	0.951
0.6	2.000	5.406	1.183
a/t	$R_i/t = 5$		
	f_1^A	f_2^A	f_3^A
0	2.000	1.362	0.221
0.1	2.000	1.659	0.221
0.2	2.000	2.220	0.303
0.3	2.000	2.904	0.535
0.4	2.000	3.701	0.857
0.5	2.000	4.603	1.311
0.6	2.000	5.671	1.851
a/t	$R_i/t = 10$		
	f_1^A	f_2^A	f_3^A
0	2.000	1.364	0.220
0.1	2.000	1.694	0.211
0.2	2.000	2.375	0.310
0.3	2.000	3.236	0.630
0.4	2.000	4.252	1.136
0.5	2.000	5.334	1.972
0.6	2.000	6.606	2.902

Remarks: The cylinder should be long in the transverse direction to the crack so that edge effects do not influence the results.
Taken from Reference AI.4.

AI.2.4. CRACKS IN A SPHERE

Description: *Through-thickness crack*

Schematic:

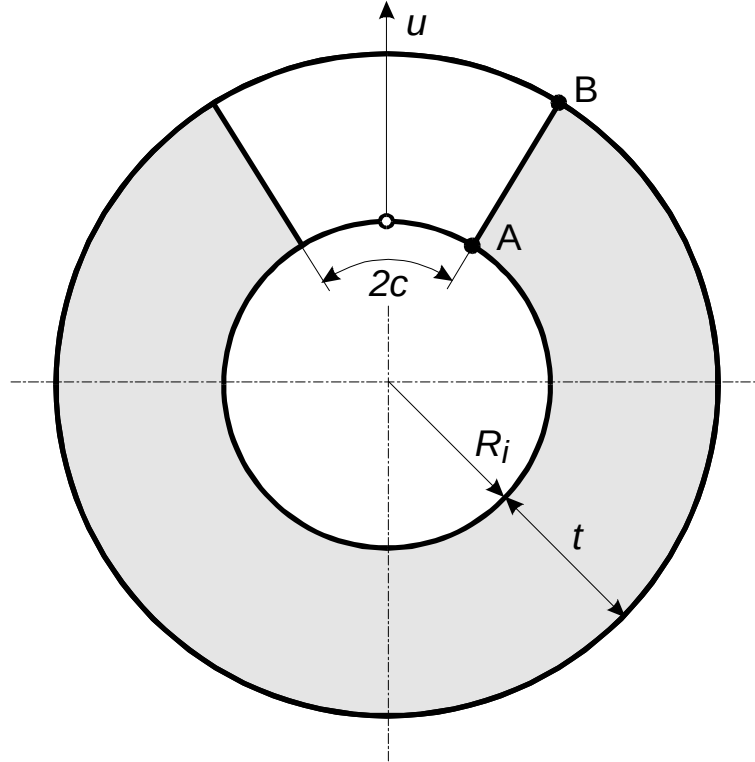


Figure AI.13. Circumferential through-thickness crack in a sphere.

Solution:

The stress intensity factor K_I is given by

$$K_I = \sqrt{pc} \left(s_m f_m \left(\frac{2c}{t}, \frac{R_i}{t} \right) + s_b f_b \left(\frac{2c}{t}, \frac{R_i}{t} \right) \right) \quad (\text{AI.20})$$

s_m and s_b are the membrane and through-thickness bending stress components, respectively, which define the axisymmetric stress state s according to

$$s = s(u) = s_m + s_b \left(1 - \frac{2u}{t} \right) \quad \text{for } 0 \leq u \leq t \quad (\text{AI.21})$$

s is to be taken normal to the prospective crack plane in an uncracked sphere. s_m and s_b are determined by fitting s to Equation (AI.21). The co-ordinate u is defined in Figure AI.13.

f_m and f_b are geometry functions which are given in Table AI.19 for the intersections of the crack with the free surface at $u = 0$ (A) and at $u = t$ (B). See Figure AI.13.

Table AI.19. Geometry functions for a through-thickness crack in a sphere.

l/t	$R/t = 10$				$R/t = 20$			
	f_m^A	f_b^A	f_m^B	f_b^B	f_m^A	f_b^A	f_m^B	f_b^B
0	1.000	1.000	1.000	-1.000	1.000	1.000	1.000	-1.000
2	0.919	0.993	1.240	-1.031	0.941	0.995	1.144	-1.020
4	0.894	0.993	1.637	-1.074	0.897	0.992	1.401	-1.050
6	0.944	0.997	2.083	-1.111	0.895	0.993	1.700	-1.080
8	1.059	1.003	2.549	-1.143	0.932	0.996	2.020	-1.106
10	1.231	1.011	3.016	-1.170	1.003	1.001	2.351	-1.130
15	1.915	1.031	4.124	-1.226	1.309	1.014	3.186	-1.180
20	2.968	1.050	5.084	-1.272	1.799	1.028	3.981	-1.219

Remarks: Taken from Reference AI.8.

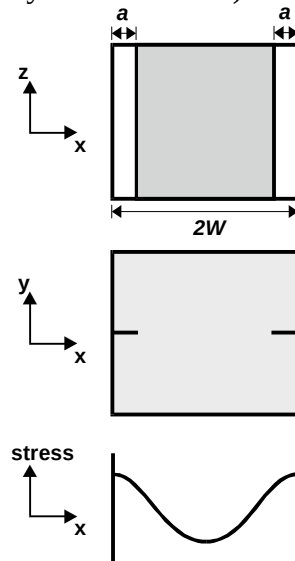
AI.3. ADDITIONAL SOLUTIONS FROM R6 CODE

Further solutions for stress intensity factors were extracted directly from the R6.CODE software and are presented in this section. Those solutions are presented graphically and algebraically. It should be noted that although R6.CODE allows for varying thicknesses to be considered, the solutions presented in this appendix are only for uniform thickness.

Stress Intensity Factor Handbook

Description: Extended Double Edge Cracked Finite Width Plate (For Symmetric Stress)

Schematic:



σ_0 = The Uncracked Body Stress at Mouth of Crack ($x=0$)

Equation:

$$K = \sqrt{\frac{\pi a}{\left(1 - \frac{a}{W}\right)}} \left(\sigma_0 + \frac{F}{W} \right) \times Z$$

Where

$$Z = 1.122 \left(1 - 0.5 \left(\frac{a}{W} \right) \right) - 0.015 \left(\frac{a}{W} \right)^2 + 0.091 \left(\frac{a}{W} \right)^3$$

and

$$F = \int_0^a \left(\frac{W-x}{\frac{\pi}{2}} \right) \arccos \left(\frac{x}{a} \cdot \frac{(W-a)}{(W-x)} \right) \left(\frac{d\sigma}{dx} \right) dx$$

Range of Applicability

The defect depth should be less than half the specimen width $2W$

References

Function is given in Reference AI.10. For uniform stressing the solution is the same as that given in Reference AI.11

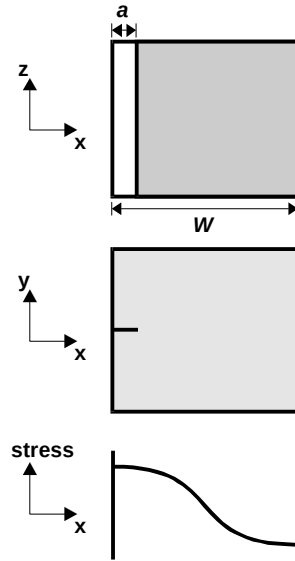
Validation

Reference AI.14 Pg. 111

Stress Intensity Factor Handbook

Description: Extended Surface Defect in Finite Width Plate

Schematic:



σ_0 = The Uncracked Body Stress at Mouth of Crack ($x=0$)

Equation:

$$K = YZA\sqrt{a}\left(\sigma_0 + \frac{F}{W}\right)$$

Where

$$F = \int_0^a \frac{(W-x)^2}{\frac{\pi W}{2}} \operatorname{acos}\left(\frac{x}{a} \frac{(W-a)}{(W-x)}\right) \left(\frac{d\sigma}{dx}\right) dx$$

and

$$YZA = \frac{\sqrt{\pi}\left(1 + 2\left(\frac{a}{W}\right)\right)}{\left(1 - \left(\frac{a}{W}\right)\right)^{\frac{3}{2}}} U$$

Where

$$U = 1.12078 - 3.68220\left(\frac{a}{W}\right) + 11.9543\left(\frac{a}{W}\right)^2 - 25.8521\left(\frac{a}{W}\right)^3 \\ + 33.09762\left(\frac{a}{W}\right)^4 - 22.4422\left(\frac{a}{W}\right)^5 + 6.17836\left(\frac{a}{W}\right)^6$$

Range of Applicability

The defect depth should be less than the specimen width W

References

Function is approximate and given in Reference AI.10 . The function is based on a bar of constant thickness so there are errors in using this in calculations with thickness variations.

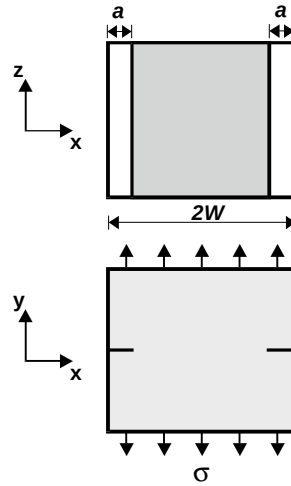
Validation

Reference AI.14 pg. 84

Stress Intensity Factor Handbook

Description: Double Edge Notched Tension Specimen (Extended Crack)

Schematic:



σ = The Uncracked Body Uniform Stress

Equation:

$$K = \sigma ZY \sqrt{a}$$

Where

$$ZY = \sqrt{\frac{p}{1 - \left(\frac{a}{W}\right)}} \left(1.122 \left(1 - 0.5 \left(\frac{a}{W} \right) \right) - 0.015 \left(\frac{a}{W} \right)^2 + 0.091 \left(\frac{a}{W} \right)^3 \right)$$

Range of Applicability

The defect depth should be less than half the specimen width $2W$

References

Validation

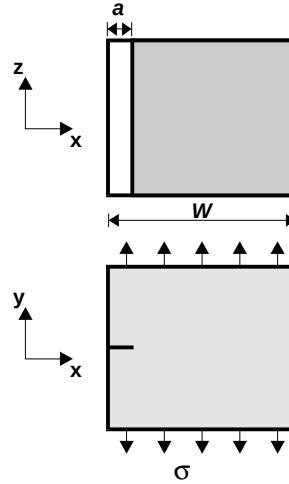
Reference AI.12 eqn. 1 pg. 6

Reference AI.12 eqn. 2 pg. 6

Stress Intensity Factor Handbook

Description: Single edge Notched Tension Specimen (Extended Crack)

Schematic:



Equation:

s = The Uncracked Body Uniform Stress
 $K = \sigma ZY \sqrt{a}$

Where

$$ZY = \frac{\sqrt{p} \left(1 + 2 \left(\frac{a}{W} \right) \right)}{\left(1 - \left(\frac{a}{W} \right) \right)^{3/2}} \times V$$

Where

$$V = 1.12078 - 3.68220 \left(\frac{a}{W} \right) + 11.95434 \left(\frac{a}{W} \right)^2 - 25.85210 \left(\frac{a}{W} \right)^3 \\ + 33.09762 \left(\frac{a}{W} \right)^4 - 22.4422 \left(\frac{a}{W} \right)^5 + 6.17836 \left(\frac{a}{W} \right)^6$$

Range of Applicability

The defect depth should be less than the specimen width W

References

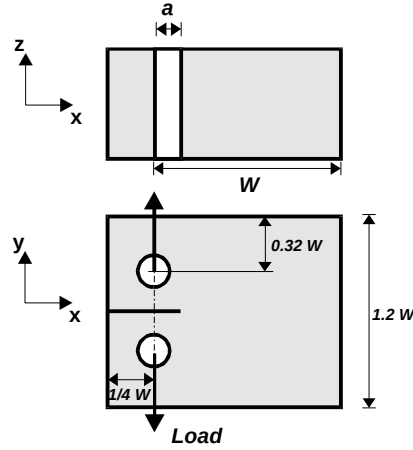
Validation

Reference AI.13, Section 2.11

Stress Intensity Factor Handbook

Description: Compact Tension Specimen (Extended Crack)

Schematic:



Equation:

s = The Uncracked Body Constant Stress (= Load / (Thickness x W))

$$K = \sigma ZY\sqrt{a}$$

Where

$$\text{If } \left(\frac{a}{W}\right) < 0.701 \quad \text{Then} \quad ZY = Y3\left(\frac{a}{W}\right)$$

$$\text{If } \left(\frac{a}{W}\right) > 0.701 \quad \text{Then} \quad ZY = Y4\left(\frac{a}{W}\right) \times Y\left(\frac{a}{W}\right)$$

Where

$$Y3\left(\frac{a}{W}\right) = 29.6 - 185.5\left(\frac{a}{W}\right) + 655.7\left(\frac{a}{W}\right)^2 - 1017\left(\frac{a}{W}\right)^3 + 638.9\left(\frac{a}{W}\right)^4$$

$$Y4\left(\frac{a}{W}\right) = 4 - 6\left(\frac{a}{W}\right) \left(0.6366 - 0.365\left(\frac{a}{W}\right) + 0.0581\left(\frac{a}{W}\right)^2 \right)$$

and

$$Y\left(\frac{a}{W}\right) = \frac{\sqrt{p}\left(1 + 2\left(\frac{a}{W}\right)\right)}{\left(1 - \left(\frac{a}{W}\right)\right)^{3/2}} \times V$$

Where

$$V = 1.12078 - 3.68220\left(\frac{a}{W}\right) + 11.95434\left(\frac{a}{W}\right)^2 - 25.85210\left(\frac{a}{W}\right)^3 \\ + 33.09762\left(\frac{a}{W}\right)^4 - 22.4422\left(\frac{a}{W}\right)^5 + 6.17836\left(\frac{a}{W}\right)^6$$

**Range of
Applicability**

The defect depth should be greater than 0.3 and less than 0.7 times the specimen width W

References

Reference AI.13

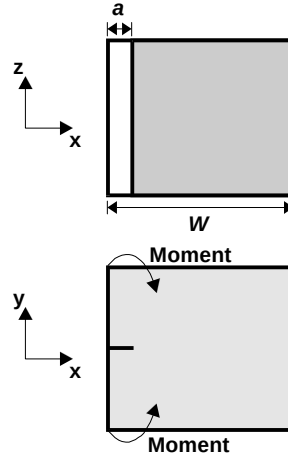
Validation

Reference AI.13, Section 2.20

Stress Intensity Factor Handbook

Description: Pure Bend Specimen (Extended Crack)

Schematic:



s = The Uncracked Body Extreme Fibre Tensile Stress

Equation: $K = \sigma ZY\sqrt{a}$

Where

$$ZY = Y2\left(\frac{a}{W}\right) \times Y\left(\frac{a}{W}\right)$$

Where

$$Y\left(\frac{a}{W}\right) = \frac{\sqrt{p}\left(1 + 2\left(\frac{a}{W}\right)\right)}{\left(1 - \left(\frac{a}{W}\right)\right)^{3/2}} \times V$$

Where

$$V = 1.12078 - 3.68220\left(\frac{a}{W}\right) + 11.95434\left(\frac{a}{W}\right)^2 - 25.85210\left(\frac{a}{W}\right)^3 \\ + 33.09762\left(\frac{a}{W}\right)^4 - 22.4422\left(\frac{a}{W}\right)^5 + 6.17836\left(\frac{a}{W}\right)^6$$

and

$$Y2 = 1 - 2\left(\frac{a}{W}\right)\left(0.6366 - 0.365\left(\frac{a}{W}\right) + 0.0581\left(\frac{a}{W}\right)^2\right)$$

**Range of
Applicability
References**

The defect size should be less than the specimen width W

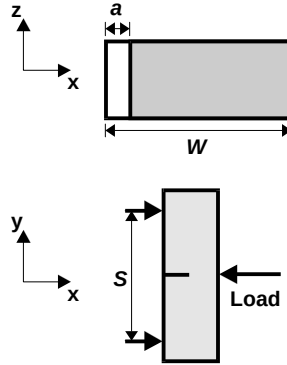
Validation

Reference AI.13 Section 2.14

Stress Intensity Factor Handbook

Description: Three Point Bend ($s/W = 8$) Specimen (Extended Crack)

Schematic:



s = The Uncracked Body Extreme Fibre Tensile Stress

Equation:

$$K = \sigma ZY\sqrt{a}$$

Where

$$\text{If } \left(\frac{a}{W}\right) < 0.651 \quad \text{Then} \quad ZY = Y5\left(\frac{a}{W}\right)$$

$$\text{If } \left(\frac{a}{W}\right) > 0.651 \quad \text{Then} \quad ZY = ZZ \times Y2\left(\frac{a}{W}\right) \times Y\left(\frac{a}{W}\right)$$

Where

$$Y\left(\frac{a}{W}\right) = \frac{\sqrt{p}\left(1 + 2\left(\frac{a}{W}\right)\right)}{\left(1 - \left(\frac{a}{W}\right)\right)^{3/2}} \times V$$

Where

$$V = 1.12078 - 3.68220\left(\frac{a}{W}\right) + 11.95434\left(\frac{a}{W}\right)^2 - 25.85210\left(\frac{a}{W}\right)^3 \\ + 33.09762\left(\frac{a}{W}\right)^4 - 22.4422\left(\frac{a}{W}\right)^5 + 6.17836\left(\frac{a}{W}\right)^6$$

and

$$Y2 = 1 - 2\left(\frac{a}{W}\right)\left(0.6366 - 0.365\left(\frac{a}{W}\right) + 0.0581\left(\frac{a}{W}\right)^2\right)$$

$$Y5 = 1.96 - 2.75\left(\frac{a}{W}\right) + 13.66\left(\frac{a}{W}\right)^2 - 23.98\left(\frac{a}{W}\right)^3 + 25.22\left(\frac{a}{W}\right)^4$$

$$ZZ = 0.9738993$$

**Range of
Applicability**

The defect depth should be less than 0.65 times the specimen width W

References

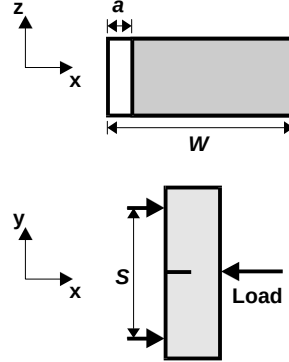
Validation

Reference AI.13, Section 2.16

Stress Intensity Factor Handbook

Description: Three Point Bend ($s/W = 4$) Specimen (Extended Crack)

Schematic:



s = The Uncracked Body Extreme Fibre Tensile Stress

Equation:

$$K = \sigma ZY \sqrt{a}$$

Where

$$\text{If } \left(\frac{a}{W}\right) < 0.651 \quad \text{Then} \quad ZY = Y6\left(\frac{a}{W}\right)$$

$$\text{If } \left(\frac{a}{W}\right) > 0.651 \quad \text{Then} \quad ZY = ZZ \times Y2\left(\frac{a}{W}\right) \times Y\left(\frac{a}{W}\right)$$

Where

$$Y\left(\frac{a}{W}\right) = \frac{\sqrt{p}\left(1 + 2\left(\frac{a}{W}\right)\right)}{\left(1 - \left(\frac{a}{W}\right)\right)^{3/2}} \times V$$

Where

$$V = 1.12078 - 3.68220\left(\frac{a}{W}\right) + 11.95434\left(\frac{a}{W}\right)^2 - 25.85210\left(\frac{a}{W}\right)^3 \\ + 33.09762\left(\frac{a}{W}\right)^4 - 22.4422\left(\frac{a}{W}\right)^5 + 6.17836\left(\frac{a}{W}\right)^6$$

and

$$Y_2 = 1 - 2\left(\frac{a}{W}\right)\left(0.6366 - 0.365\left(\frac{a}{W}\right) + 0.0581\left(\frac{a}{W}\right)^2\right)$$

$$Y_6 = 1.93 - 3.07\left(\frac{a}{W}\right) + 14.53\left(\frac{a}{W}\right)^2 - 25.11\left(\frac{a}{W}\right)^3 + 25.8\left(\frac{a}{W}\right)^4$$

$$ZZ = \frac{Y_6(0.65)}{Y_2(0.65) \times Y(0.65)}$$

Where $Y_6(0.65)$, $Y_2(0.65)$ and $Y(0.65)$ are the values of

Y_6 , Y_2 and Y for $\frac{a}{W} = 0.65$

**Range of
Applicability
References**

The defect depth should be less than 0.65 times the specimen width W

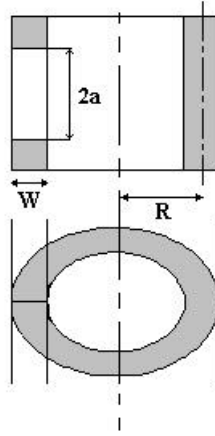
Validation

Reference AI.13, Section 2.16

Stress Intensity Factor Handbook

Description: Axial Through Thickness Defect in a Cylinder

Schematic:



R = The Mean Radius

σ_h, σ_b^s = The Average Uniform Hoop Stress, and the Extreme Fibre Bending Stress of the Uncracked Body, Respectively.

Equation:

For hoop stresses :

$$K_{in} = \sigma_h \cdot \sqrt{\pi a} (G1(\rho) - g1(\rho))$$

$$K_{out} = \sigma_h \cdot \sqrt{\pi a} (G1(\rho) + g1(\rho))$$

For through wall self - equilibrated bending stresses :

$$K_{in} = \sigma_b^s \cdot \sqrt{\pi a} (H1(\rho) - h1(\rho))$$

$$K_{out} = \sigma_b^s \cdot \sqrt{\pi a} (H1(\rho) + h1(\rho))$$

Where

$$r = \frac{a}{\sqrt{R \cdot W}}$$

$$G1(\rho) = \sqrt{1 + 0.7044\rho + 0.8378\rho^2}$$

$$g1(\rho) = -0.035211 + 0.39394\rho - 0.20036\rho^2 + 0.028085\rho^3$$

$$-0.0018763\rho^4 + \frac{(3.912 - \ln(R/W))}{1.6094} \cdot \left(\begin{aligned} &0.01556 - 0.05202\rho + \\ &.0381\rho^2 - .012782\rho^3 \\ &+ 0.001246\rho^4 \end{aligned} \right)$$

$$H1(p) = 0.76871 - 0.27718p + 0.14343p^2 - 0.037505p^3 + 0.0035194p^4$$

$$+ \frac{(3.912 - \ln(R/W))}{1.6094} \cdot \begin{pmatrix} 0.09852 - 0.16404p + \\ 0.10378p^2 - 0.027703p^3 \\ + 0.002597p^4 \end{pmatrix}$$

$$h1(p) = -0.0030702 + 0.074457p - 0.018716p^2 + 0.0025344p^3$$

$$- 0.00014028p^4 + \frac{(3.912 - \ln(R/W))}{1.6094} \cdot \begin{pmatrix} .0005847 + .010301p \\ - 0.007184p^2 \\ + 0.0019107p^3 \\ - 0.00017655p^4 \end{pmatrix}$$

**Range of
Applicability
References**

$0 \leq p \leq 4.4$

based on Reference AI.15

Remarks

A more complete and accurate solution covering a wider range of geometry and load configuration may be obtained following the results of the finite element study contained in Reference AI.2. These results are not included in this compendium due to the large amount of normalised stress intensity factors presented in the form of figures and tables in the reference.

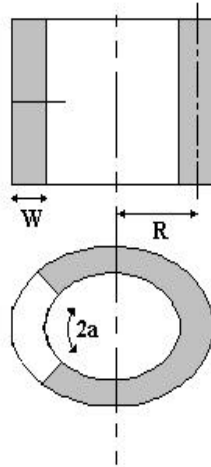
Validation

Reference AI.16

Stress Intensity Factor Handbook

Description: Circumferential Through Thickness Defect in a Cylinder

Schematic:



Equation:

σ_a, σ_b^s = The Average Uniform Hoop Stress, and the Extreme Fibre Bending Stress of the Uncracked Body, Respectively.

For hoop stresses :

$$K_{in} = \sigma_a \cdot \sqrt{\pi a} (G_2(\rho) - g_2(\rho))$$

$$K_{out} = \sigma_a \cdot \sqrt{\pi a} (G_2(\rho) + g_2(\rho))$$

For through wall self - equilibrated bending stresses :

$$K_{in} = \sigma_b^s \cdot \sqrt{\pi a} (H_2(\rho) - h_2(\rho))$$

$$K_{out} = \sigma_b^s \cdot \sqrt{\pi a} (H_2(\rho) + h_2(\rho))$$

For bending stresses on cracked section :

$$K_a = \sigma_b \sqrt{\pi a} \cdot G^* 2(\rho, \beta)$$

Where :

$$G^* 2(\rho, \beta) = G_2(\rho) \cdot \sin(\beta) \cdot C_2(\beta) / (\beta \cdot C_1(\beta))$$

Where :

$$\beta = a/R \text{ (Half angle subtended by defect)}$$

Where

$$\rho = \frac{a}{\sqrt{RW}}$$

$$G2(\rho) = 1 + 0.19\rho + 0.01\rho^2$$

$$g2(\rho) = -0.010195 + 0.2965\rho + 0.20036\rho^2 + 0.030839\rho^3 \\ - 0.0012261\rho^4$$

$$H2(\rho) = 0.81978 - 0.57979\rho + 0.28201\rho^2 - 0.068923\rho^3 + 0.0063193\rho^4 \\ + \frac{(4.60517 - \ln(R/W))}{2.30259} \cdot \begin{pmatrix} 0.1183 - 0.21012\rho + \\ 0.13265\rho^2 - 0.034987\rho^3 \\ + 0.0032506\rho^4 \end{pmatrix}$$

$$h2(\rho) = -0.0016231 + 0.058527\rho - 0.027002\rho^2 + 0.0044161\rho^3 \\ - 0.00021917\rho^4$$

$$C1(\beta) = 1 + \frac{0.7071(1 - \beta \cdot \text{Cot}(\beta))}{\left(\frac{\text{Cot}(\pi - \beta)}{\sqrt{2}} + \sqrt{2} \cdot \text{Cot}(\beta) \right) \cdot \beta}$$

$$C2(\beta) = 1 + \frac{0.35355(\beta + \beta \cdot \text{Cot}^2(\beta) - \text{Cot}(\beta))}{\left(\frac{\text{Cot}(\pi - \beta)}{\sqrt{2}} + \text{Cot}(\beta) \right)}$$

**Range of
Applicability**

$$0 \leq \rho \leq 4.4$$

References

based on Reference AI.15

Remarks

A more complete and accurate solution covering a wider range of geometry and load configuration may be obtained following the results of the finite element study contained in Reference AI.2. These results are not included in this compendium due to the large amount of normalised stress intensity factors presented in the form of figures and tables in the reference.

Validation

Reference AI.16

AI.4. REFERENCES

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APPENDIX II
**LIMIT LOAD SOLUTIONS FOR PRESSURE VESSELS,
FLAT PLATES AND SPHERES**

CONTENTS

NOMENCLATURE	AII.2
AII.1. INTRODUCTION	AII.2
AII.2. PLASTIC ANALYSIS OF STRUCTURES	AII.3
AII.3. LIMIT LOAD COMPENDIA	AII.3
AII.4. PROCEDURE FOR CONVERTING L_r TO LIMIT LOAD SOLUTIONS	AII.4
AII.5. LIMIT LOAD SOLUTIONS	AII.7
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NOMENCLATURE

The following are some of the symbols used in this appendix. Other symbols are defined where they appear.

b, c and d	these are geometrical variables, defined in the figures
M_{app}	applied bending moment
M_L	limit bending moment
m	applied axisymmetric through wall bending moment per unit angle of cross section
m_L	limit axisymmetric through wall bending moment per unit angle of cross section
N_L	limit force
P_L	limit pressure
Q	applied shear force
Q_L	limit shear force
R_1	inner radius
R_2	outer radius
R_m	mean radius
T	applied torque
T_L	limit torque
w	wall thickness
σ_m	membrane stress
σ_b	bending stress

INTRODUCTION

The plastic limit load of a structure is an important component in the analysis of structural integrity. Design and operating loads are generally related to the limit load by factors defined to prevent the attainment of the limit load under operating and most fault conditions. For defective structures, the limit load is potentially reduced, and this must be taken into account in safety cases. R6 [AII.1] provides a methodology for determining the limiting conditions for defective structures based on fracture mechanics. It assesses the load required to cause potential failure

by crack initiation and propagation. The methodology explicitly requires an estimate of the plastic limit load of the defective structure. The purpose of this appendix is to give a compendium of plastic limit loads for a variety of defective structures for use in structural integrity analysis.

PLASTIC ANALYSIS OF STRUCTURES

The need to estimate plastic limit loads has given rise to a considerable amount of work in plastic stress analysis. A number of approaches have been used. Direct modelling of the plastic stress and strain distributions for given loading conditions through the use of constitutive equations can be accomplished analytically only for very simple undefective structures, but finite element plastic stress analysis can be used for more complex cases. The upper and lower bound theorems of plasticity theory involve approximate modelling of the deformation or the stress distributions, respectively, and can provide approximate estimates of limit loads. Experimental determinations of limit loads involve correlating applied loads with measured plastic deformations. Each method has its limitations and usually involves some form of idealisation and approximation which users should be aware of. Typically, these relate to the representation of material properties, the estimation of hardening effects, the allowance for changes of shape of a deforming structure, and the definition of the state of deformation or stress distribution corresponding to the limit condition.

LIMIT LOAD COMPENDIA

It is convenient for plastic analysis results from various sources to be collected into a single document, such as Miller's review of limit loads [AII.2] which contains estimates of limit loads for a wide range of defective structures. The review also contains discussion and references on the methods used in analysis. More recently, Carter [AII.3] has derived a library of limit loads for use in the structural analysis program R6.CODE [AII.4]. The limit loads in [AII.3] can be written as simple mathematical expressions involving geometrical variables describing the structure and the details of the defect. This makes them useful when it is required to study the effect of changes in the structural geometry and defect size. These limit loads form the basis of the present compendium.

The derivation of plastic limit loads in [AII.3] was mainly achieved using a number of methods based on the lower bound theorem. Yielding stress distributions in equilibrium with applied loads were postulated, and simple cases combined together to obtain solutions for more complex geometries. Some solutions are taken directly from [AII.2]; for example, those for some test specimen geometries, and for fully penetrating defects in the walls of pressurised cylinders and spheres. For pressurised pipes with circumferential defects, the limit loads derived in [AII.3] neglected the hoop and radial components of stress. This has a significant effect and, for this reason, lower bound alternatives from [AII.5] are provided here.

In most cases, the solution for a given case is presented as the value of a limiting force, N_L , pressure, P_L , bending moment, M_L , or, in the case of axisymmetric through wall bend, bending moment per unit angle of wall subtended at the centre of the section, m_L . Solutions for these cases have been obtained from [AII.2] and [AII.3] which are mainly incorporated in R6.CODE. Tensile forces are assumed to act normally to the plane of the defect. Bending moments are

assumed to be positive when the stress in the undefective structure due to bending at the site of the defect is predominantly tensile.

Solutions for other cases have been obtained from an SAQ document and internal Nuclear Electric publications [AII.6] and [AII.7], respectively. The solutions which have been obtained from [AII.6] are presented in terms of the parameter L_r which can be directly input to R6.CODE as a user specified equation. The methodology to be used in converting the presented L_r equation into a suitable limit load solution, or vice versa, is described in Section AII.4.

In cases of bending loads, it is sometimes convenient to express the limit load in terms of an equivalent outer-fibre bending stress, σ_b^L , for a postulated linearly varying elastic stress distribution which has no net force on an element of the wall. Formulae for these are given in Table AII.1 for a number of structures.

It is intended that further issues of the compendium will have additional solutions.

Procedure for Converting L_r to Limit Load Solutions

The solutions which have been obtained from [AII.6] are presented in terms of the parameter L_r . This brief section clarifies the methodology to be used in converting the presented L_r equation into a suitable limit load solution, by means of an example.

Consider the following L_r solution:

$$L_r = \frac{g(\zeta)\frac{\sigma_b}{3} + \sqrt{g^2(\zeta)\frac{\sigma_b^2}{9} + (1-\zeta)^2\sigma_m^2}}{(1-\zeta)^2\sigma_y}$$

where $g(\zeta)$ is a geometrical function of some form, s_m and s_b are the applied membrane and bending stresses, respectively.

The measure of proximity to plastic collapse parameter L_r is given by:

$$L_r = \frac{P}{P_L} = \frac{s_m}{s_m^L} = \frac{s_b}{s_b^L}$$

Then the limiting bending stress for the given ratio of membrane to bending stress $\frac{\sigma_m}{\sigma_b}$ would be:

$$\sigma_b^L = \frac{(1-\zeta)^2 \cdot \sigma_y}{\frac{g(\zeta)}{3} + \sqrt{\frac{g^2(\zeta)}{9} + (1-\zeta)^2 \left(\frac{\sigma_m}{\sigma_b}\right)^2}}$$

This indicates, for example, that when the membrane stress is $s_m = 0$, in the absence of a defect ($\zeta = 0, g(\zeta) = 1$) the limiting elastic bending stress is $1.5\sigma_y$. Similarly the limiting membrane stress can be derived.

Table AII.1: Limit Bending Stresses as Functions of Limit Moments

Structure Type	Limit Bending Stress, σ_b^L	Location
Planar	$\left(\frac{6}{dw^2} \right) M_L$	tensile stress at wall surface (d is plate width)
Pipe with internal circumferential defect (axisymmetric bend)	$\frac{m_L}{A_b}$	tensile stress at inner wall surface (A_b is defined on the following page)
Pipe with external circumferential defect (axisymmetric bend)	$\frac{m_L}{B_b}$	tensile stress at outer wall surface (B_b is defined on the following page)
Pipe with internal or external circumferential defect (cantilever bend)	$\left[\frac{4R_2}{\pi(R_2^4 - R_1^4)} \right] M_L$	peak tensile stress at outer wall surface
Solid round bar with centrally embedded circular defect (axisymmetric bend)	$\left(\frac{192}{w^3} \right) m_L$	tensile stress at centre of bar
Solid round bar with external circumferential defect (axisymmetric bend)	$\left(\frac{96}{w^3} \right) m_L$	tensile stress at surface of bar
Solid round bar (cantilever bend)	$\left(\frac{32}{\pi w^3} \right) M_L$	peak tensile stress at surface of bar

In Table AII.1, A_b and B_b are functions of pipe geometry given by:

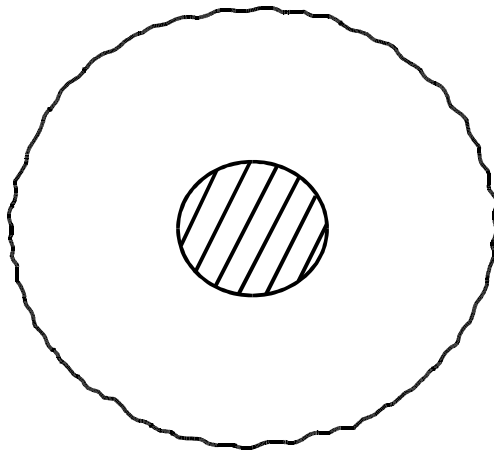
$$A_b = \frac{R_1 w^2}{6} \left[2 \left(\frac{R_m}{\frac{R_1}{2} + \frac{w}{3}} \right) - 3 \right] + \frac{w^3}{12} \left[3 \left(\frac{R_m}{\frac{R_1}{2} + \frac{w}{3}} \right) - 4 \right]$$

$$B_b = \frac{R_2 w^2}{6} \left[2 \left(\frac{R_m}{\frac{R_2}{2} - \frac{w}{3}} \right) - 3 \right] + \frac{w^3}{12} \left[4 - 3 \left(\frac{R_m}{\frac{R_2}{2} - \frac{w}{3}} \right) \right]$$

AII.5 LIMIT LOAD SOLUTIONS

Description: Infinite Axisymmetric Body; Embedded Defect; Through Wall Bending

Schematic:



Embedded Defect in an Infinite Axisymmetric Body

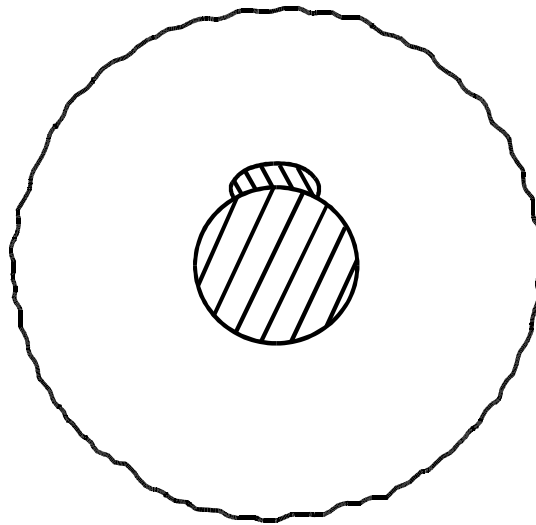
Solution:

$$\sigma_b^L = \frac{8\sigma_y(\sqrt{2}-1)}{\sqrt{2}}$$

Remarks: Taken from reference AII.3.

Description: Infinite Axisymmetric Body; Surface Defect; Through Wall Bending

Schematic:



Example of a Surface Defect in an Infinite Axisymmetric Body

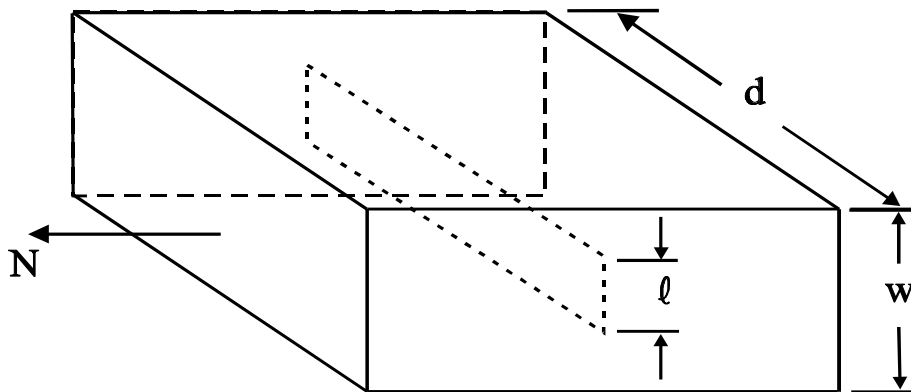
Solution:

$$\sigma_b^L = \frac{8\sigma_y(\sqrt{2}-1)}{\sqrt{2}}$$

Remarks: Taken from Reference AII.3.

Description: Plate; Centrally Embedded Extended Defect; Tension; Global Collapse;
Plane Stress (Tresca and Mises); Plane Strain (Tresca)

Schematic:



Centrally Embedded Extended Defect in a Plate

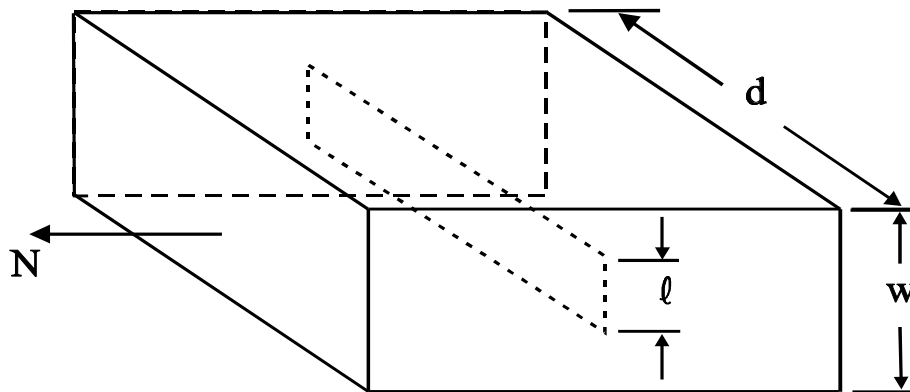
Solution:

$$N_L = \sigma_y (w - \ell) d$$

Remarks: Taken from Reference AII.3.

Description: Plate; Centrally Embedded Extended Defect; Tension; Global collapse;
Plane Strain (Mises)

Schematic:



Centrally Embedded Extended Defect in a Plate

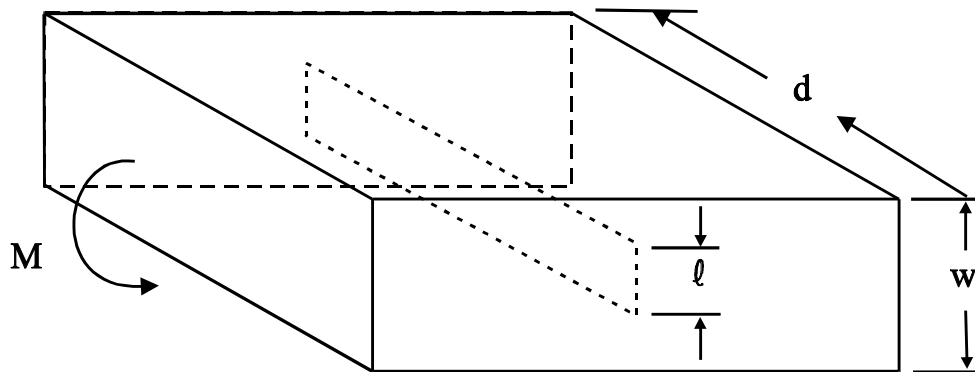
Solution:

$$N_L = 1.155\sigma_y(w - \ell)d$$

Remarks: Taken from Reference AII.3.

Description: Plate; Centrally Embedded Extended Defect; Through Wall Bend; Global Collapse

Schematic:



Centrally Embedded Extended Defect in a Plate

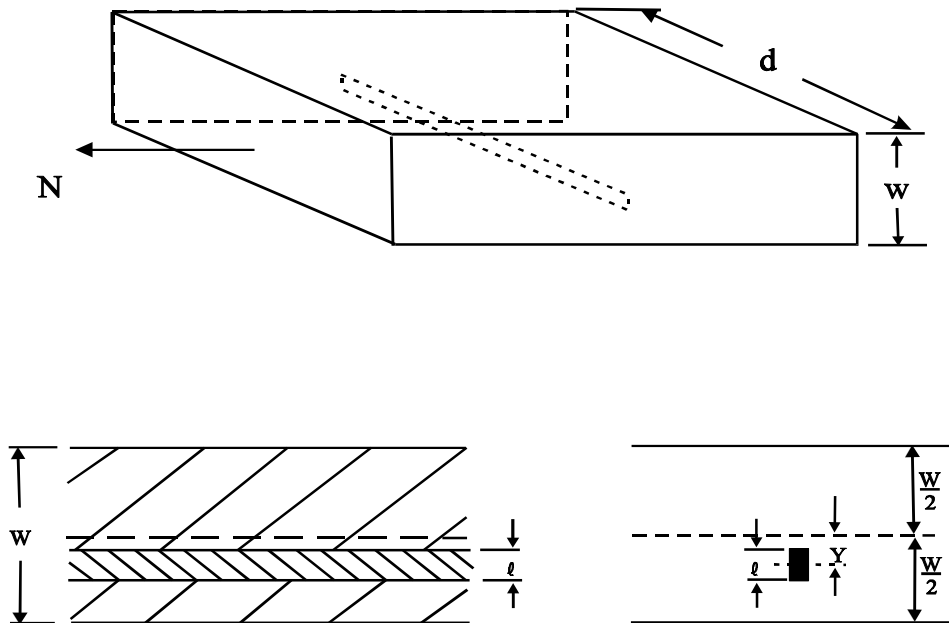
Solution:

$$M_L = \frac{\sigma_y w^2 d}{4} \left(1 - \frac{\ell^2}{w^2} \right)$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Defect; Pin Loaded Tension; Global Collapse

Schematic:



Off-Set Embedded Defect in a Plate

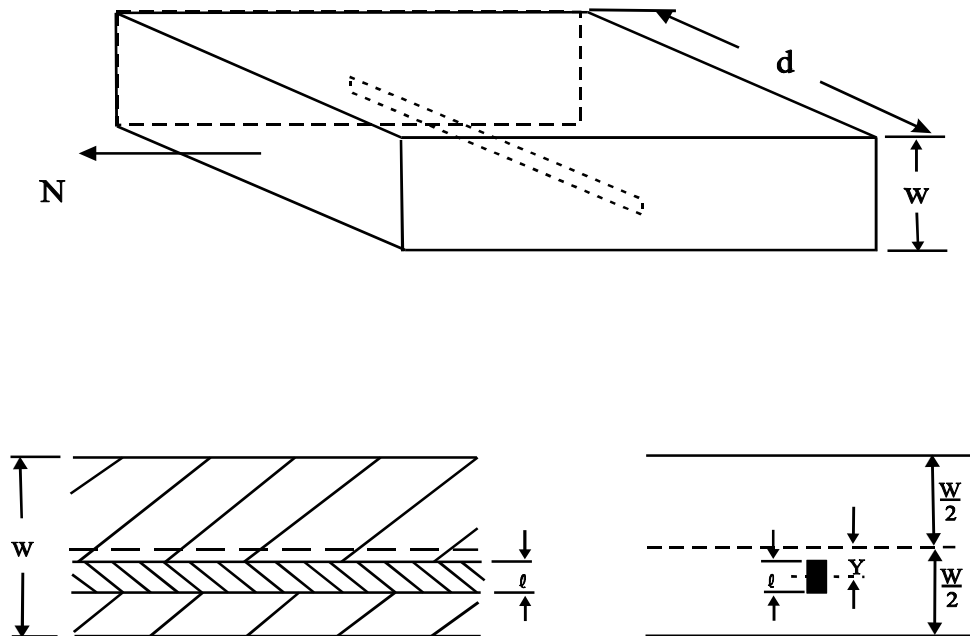
Solution:

$$N_L = \sigma_y w d \left[\left(1 - \frac{4\ell Y}{w^2} \right)^{1/2} - \frac{\ell}{w} \right]$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Defect; Fixed Grip Tension; Global Collapse

Schematic:



Off-Set Embedded Extended Defect in a Plate

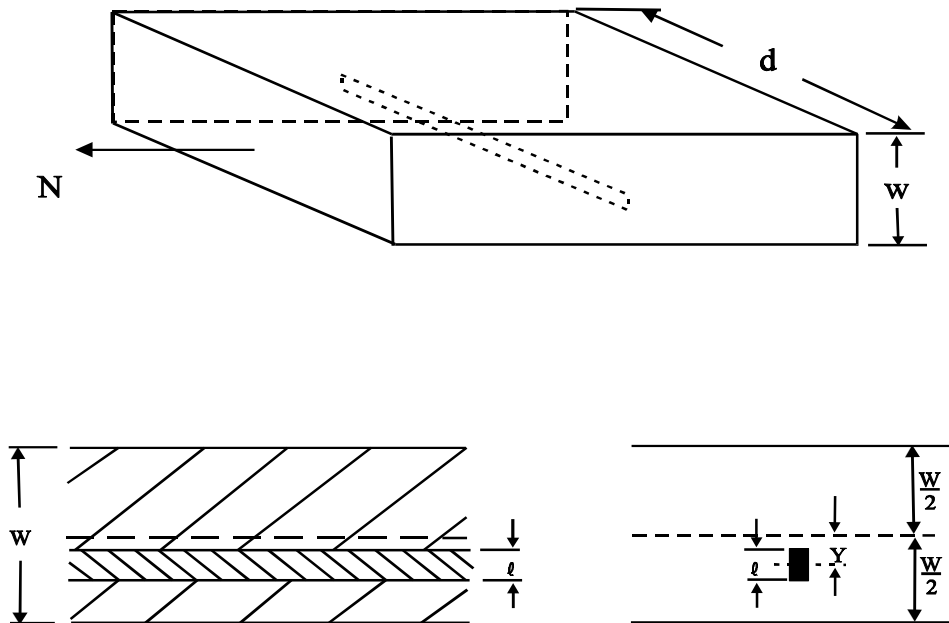
Solution:

$$N_L = \sigma_y w d \left(1 - \frac{\ell}{w}\right)$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Defect; Pin Loaded Tension; Local Collapse

Schematic:



Off-Set Embedded Extended Defect in a Plate

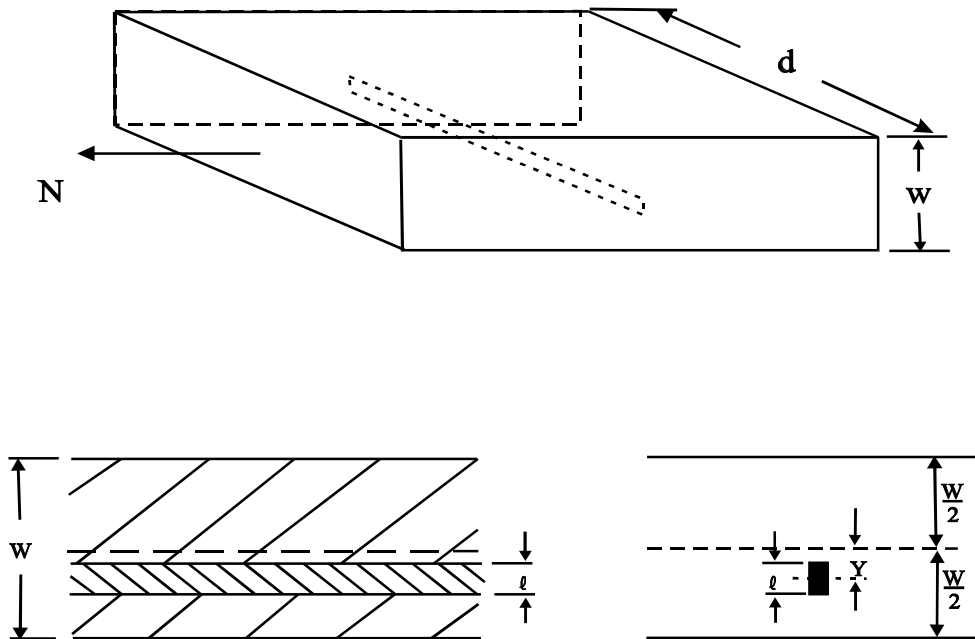
Solution:

$$N_L = \sigma_y w d \left(1 - \frac{\ell}{w - 2Y} \right)$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Defect; Fixed Grip Tension; Local Collapse

Schematic:



Off-Set Embedded Extended Defect in a Plate

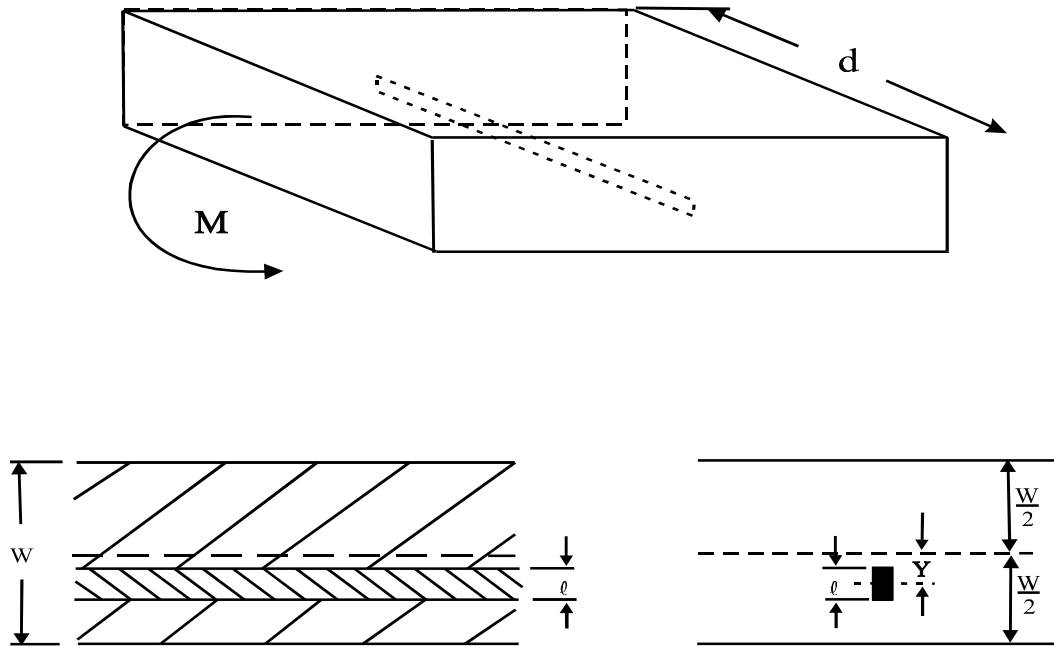
Solution:

$$N_L = \sigma_y w d \left(1 - \frac{\ell}{w - 2Y} \right)$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Extended Defect; Through Wall Bend; Global Collapse

Schematic:



Off-Set Embedded Extended Defect in a Plate

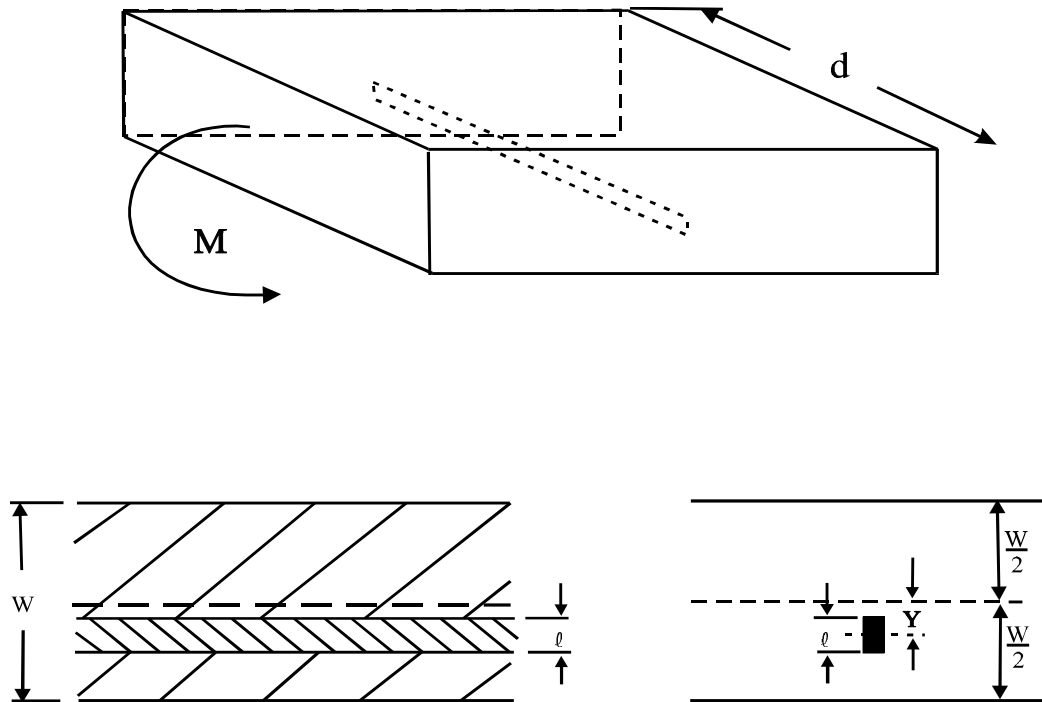
Solution:

$$M_L = \sigma_y d \left(\frac{W^2 - \ell^2}{4} - Y \ell \right)$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Extended Defect; Through Wall Bend;
Local Collapse

Schematic:



Off-Set Embedded Defect in a Plate

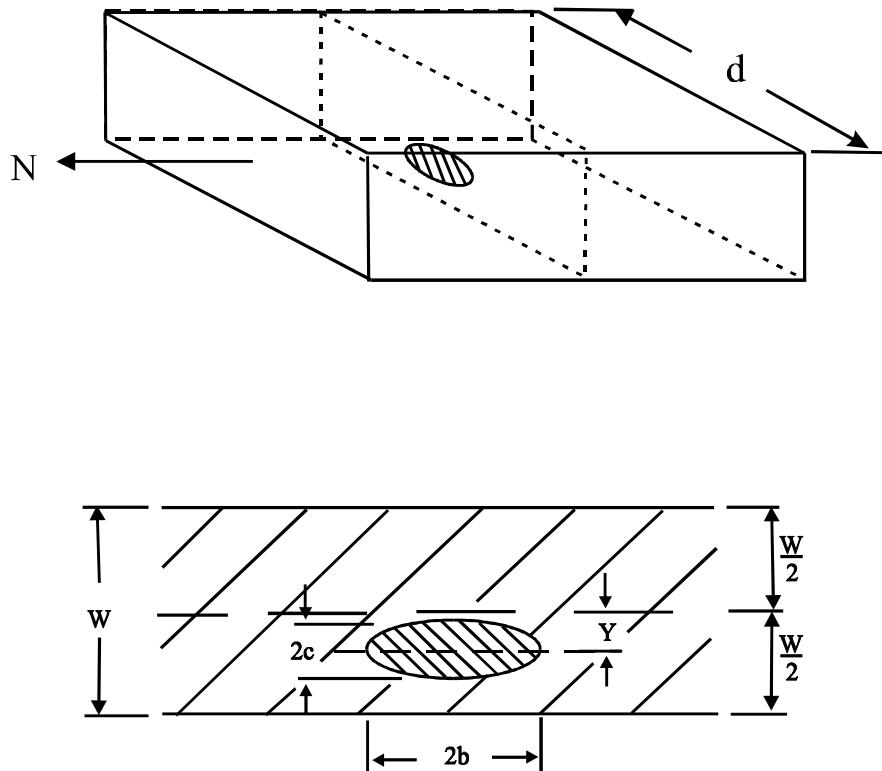
Solution:

$$M_L = \frac{\sigma_y d}{4} ((w - 2Y)^2 - \ell^2)$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Elliptical Defect; Tension; Global Collapse

Schematic:



Off-Set Embedded Elliptical Defect in a Plate

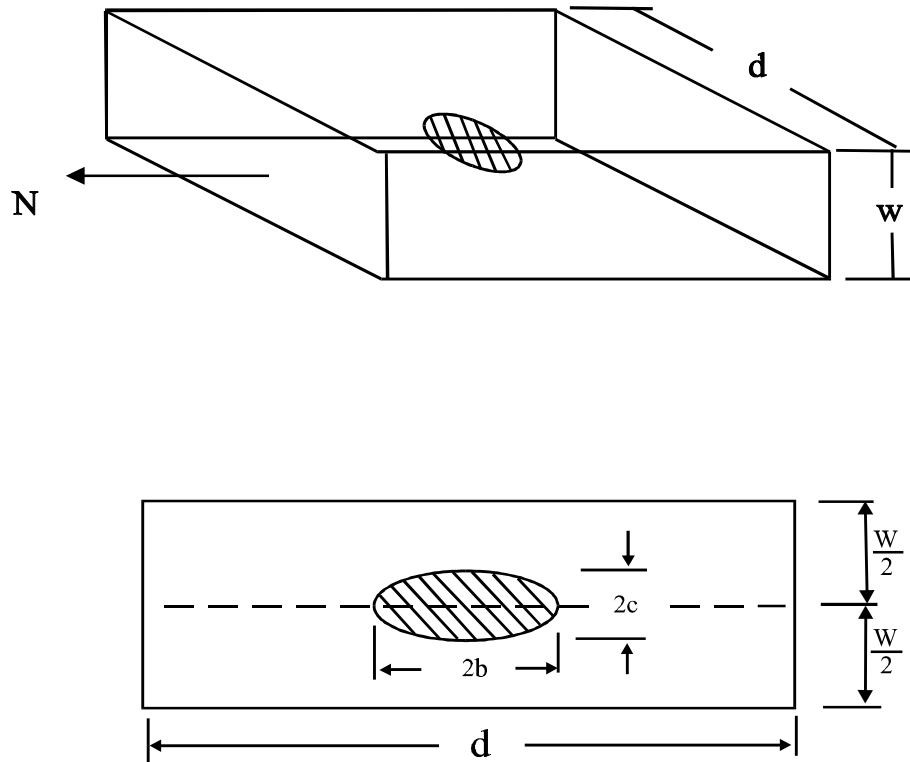
Solution:

$$N_L = \sigma_y d \frac{(w^2 + b(w - 2c))}{(w + b)}$$

Remarks: Taken from Reference AII.3.

Description: Plate; Centrally Embedded Elliptical Defect; Tension; Local Collapse

Schematic:



Centrally Embedded Elliptical Defect in a Plate

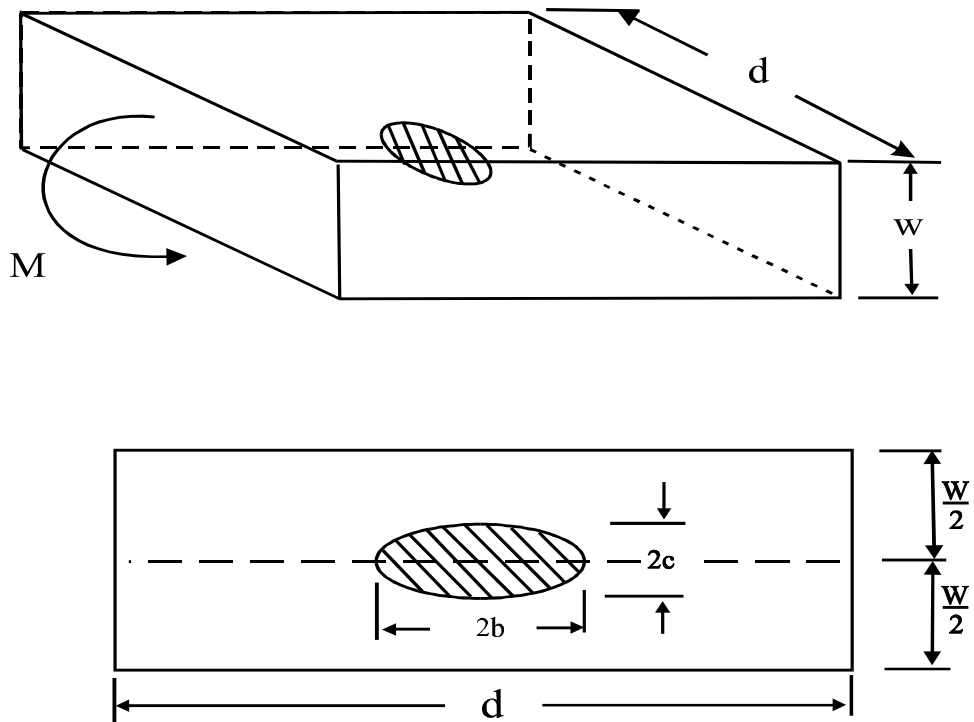
Solution:

$$N_L = \sigma_y w d \cdot \frac{(w - 2c)(1 + b/w)}{(w - 2c + b)}$$

Remarks: Taken from Reference AII.3.

Description: Plate; Centrally Embedded Elliptical Defect; Through Wall Bend;
Global Collapse

Schematic:



Centrally Embedded Elliptical Defect in a Plate

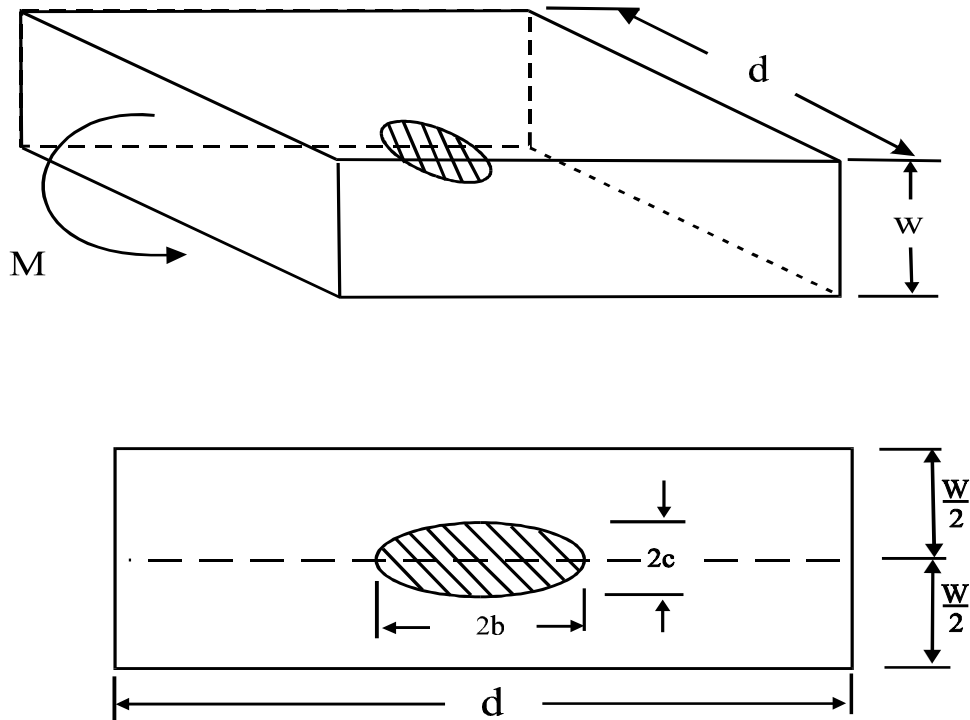
Solution:

$$M_L = \frac{\sigma_y w^2 d}{4} \cdot \frac{(w + b(1 - 4c^2/w^2))}{(w + b)}$$

Remarks: Taken from Reference AII.3.

Description: Plate; Centrally Embedded Elliptical Defect; Through Wall Bend; Local Collapse

Schematic:



Centrally Embedded Elliptical Defect in a Plate

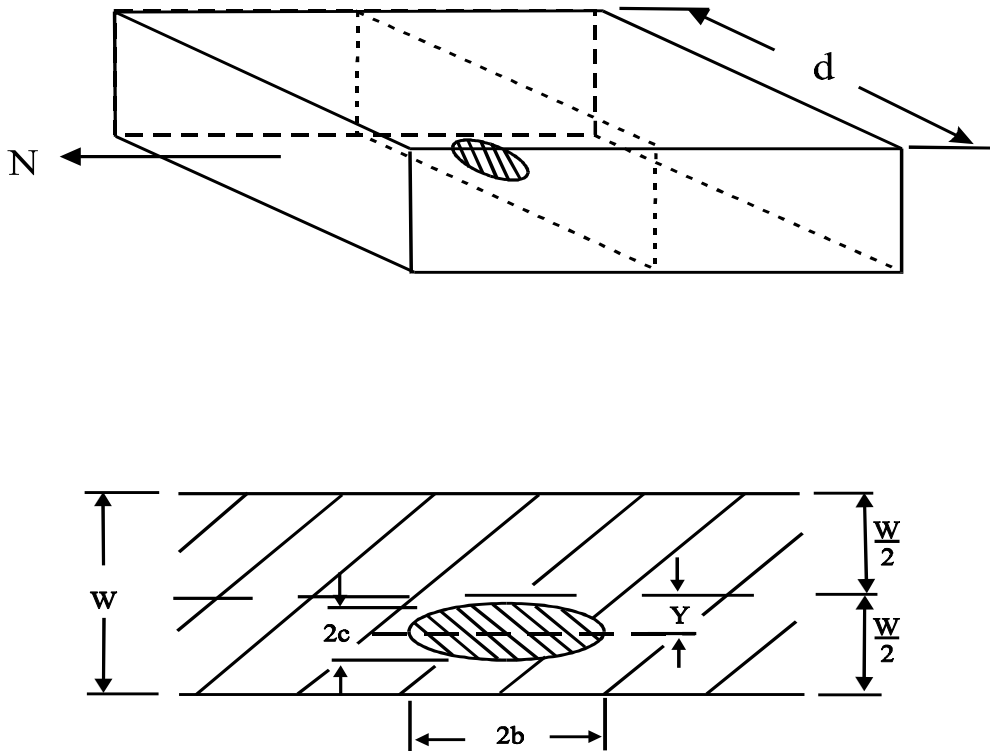
Solution:

$$M_L = \frac{\sigma_y w^2 d}{4} \cdot \frac{(w - 2c + b(1 - 4c^2/w^2))}{(w - 2c + b)}$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Elliptical Defect; Pin-Loaded Tension; Global Collapse

Schematic:



Off-Set Embedded Elliptical Defect in a Plate

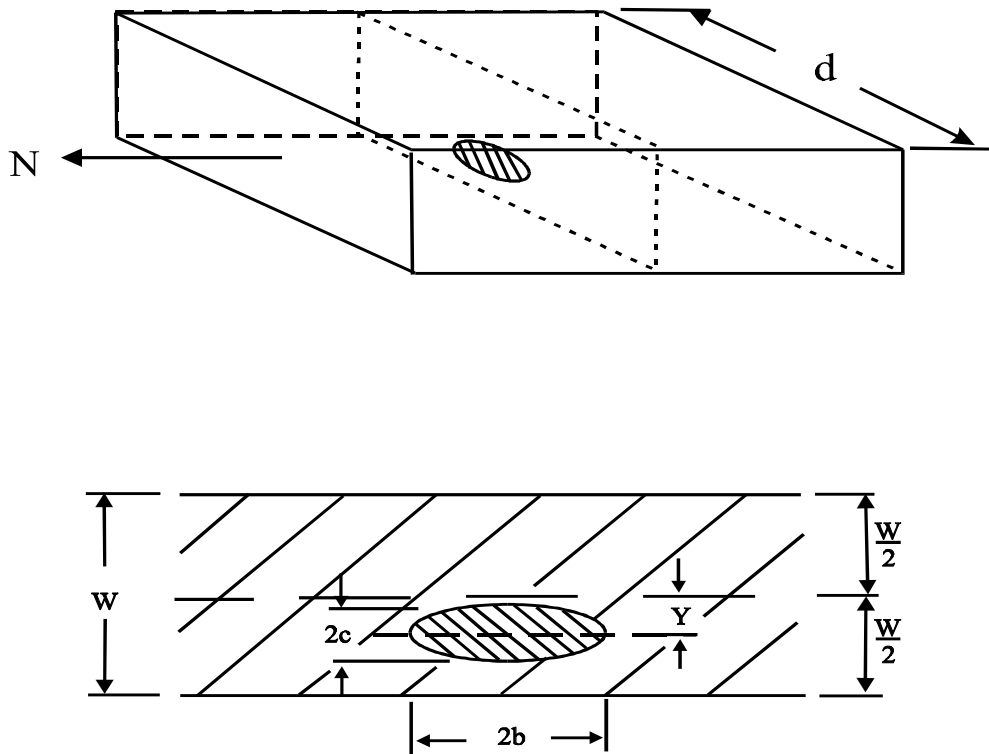
Solution:

$$N_L = \sigma_y w d \cdot \frac{(w + b((1 - 8cY/w^2)^{1/2} - 2c/w))}{(w + b)}$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Elliptical Defect; Pin-Loaded Tension; Local Collapse

Schematic:



Off-Set Embedded Elliptical Defect in a Plate

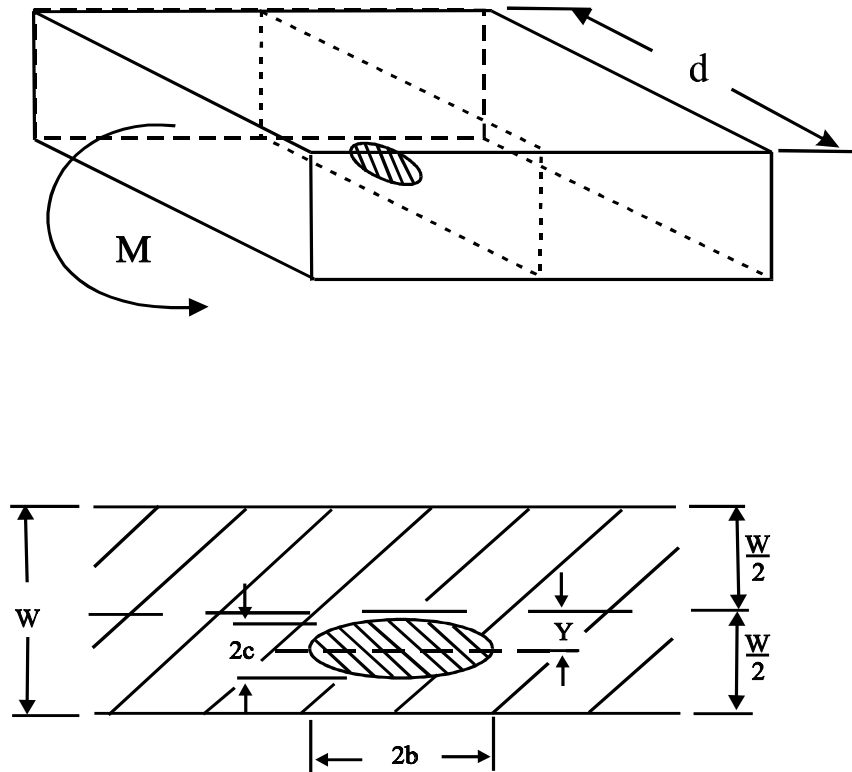
Solution:

$$N_L = \sigma_y w d \left(\frac{\left(1 - \frac{2c}{w - 2Y}\right)(w + b)}{w\left(1 - \frac{2c}{w - 2Y}\right) + b} \right)$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Elliptical Defect; Through Wall Bend; Global Collapse

Schematic:



Off-Set Embedded Elliptical Defect in a Plate

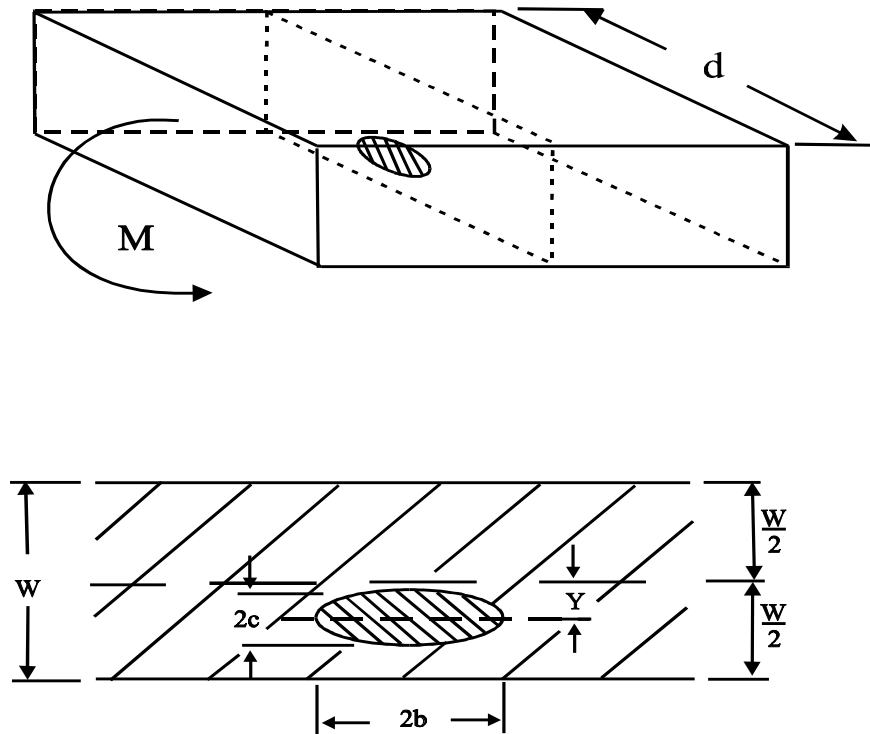
Solution:

$$M_L = \frac{\sigma_y d}{4(w + b)} (w^3 + b(w^2 - 4c^2 - 8Yc))$$

Remarks: Taken from Reference AII.3.

Description: Plate; Off-Set Embedded Elliptical Defect; Through Wall Bend; Local Collapse

Schematic:



Off-Set Embedded Elliptical Defect in a Plate

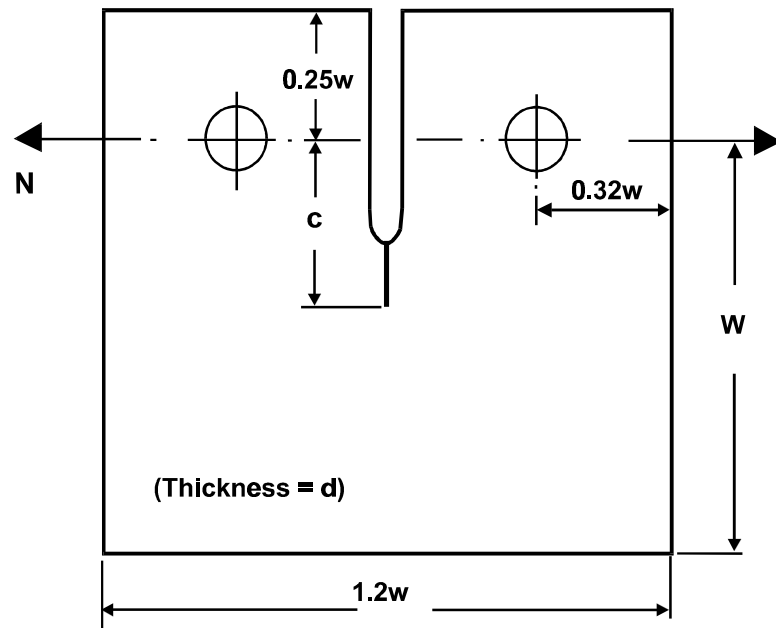
Solution:

$$M_L = \frac{\sigma_y d (w - 2Y)^2}{4} \left(\frac{w \left(1 - \frac{2c}{w - 2Y} \right) + b \left(1 - \frac{4c^2}{(w - 2Y)^2} \right)}{w \left(1 - \frac{2c}{w - 2Y} \right) + b} \right)$$

Remarks: Taken from Reference AII.3.

Description: Compact Tension Specimen; Tension; Plane Stress (Mises)

Schematic:



Compact Tension Specimen

Solution:

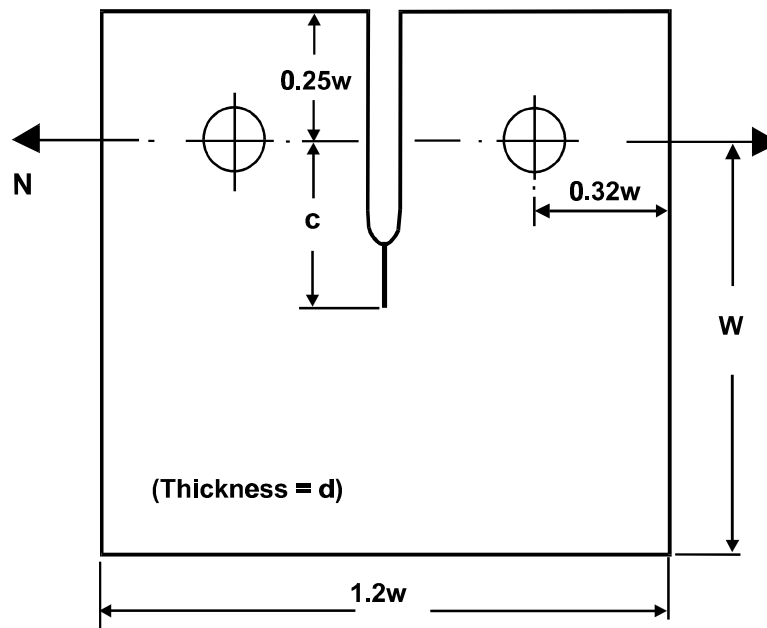
$$N_L = \sigma_y w d \left(\left((1 + \tilde{a}) \left(1 + \tilde{a} \left(\frac{c}{w} \right)^2 \right) \right)^{\frac{1}{2}} - 1 - \frac{\tilde{a}c}{w} \right)$$

$$\text{where } \gamma = \frac{2}{\sqrt{3}}$$

Remarks: Taken from Reference AII.3.

Description: Compact Tension Specimen; Tension; Plane Stress (Tresca)

Schematic:



Compact Tension Specimen

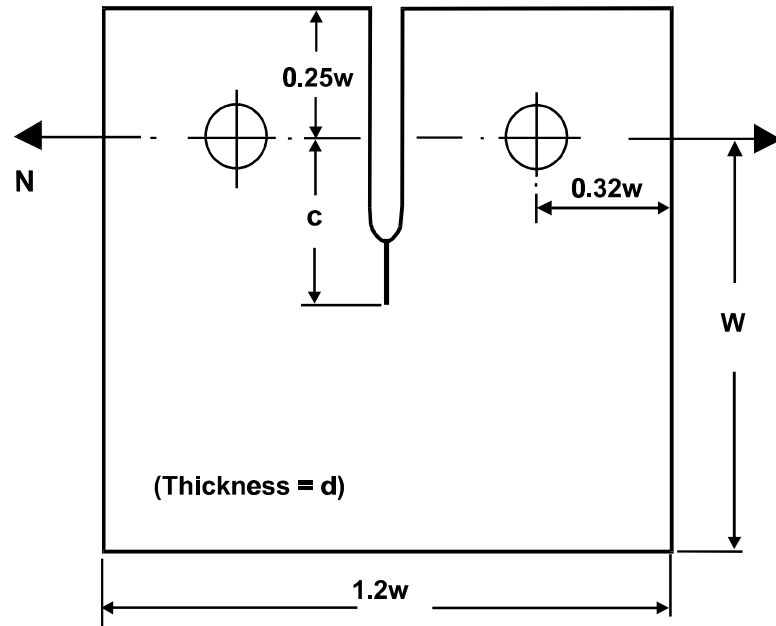
Solution:

$$N_L = \sigma_y w d \left(\left(2 + 2 \left(\frac{c}{w} \right)^2 \right)^{\frac{1}{2}} - 1 - \frac{c}{w} \right)$$

Remarks: Taken from Reference AII.3.

Description: Compact Tension Specimen; Tension; Plane Strain (Tresca)

Schematic:



Compact Tension Specimen

Solution:

$$N_L = \sigma_y w d \left(0.634 - 1.482 \left(\frac{c}{w} \right) + 0.134 \left(\frac{c}{w} \right)^2 + 0.25 \left(\frac{c}{w} \right)^3 \right)$$

for $0 \leq c/w \leq 0.09$,

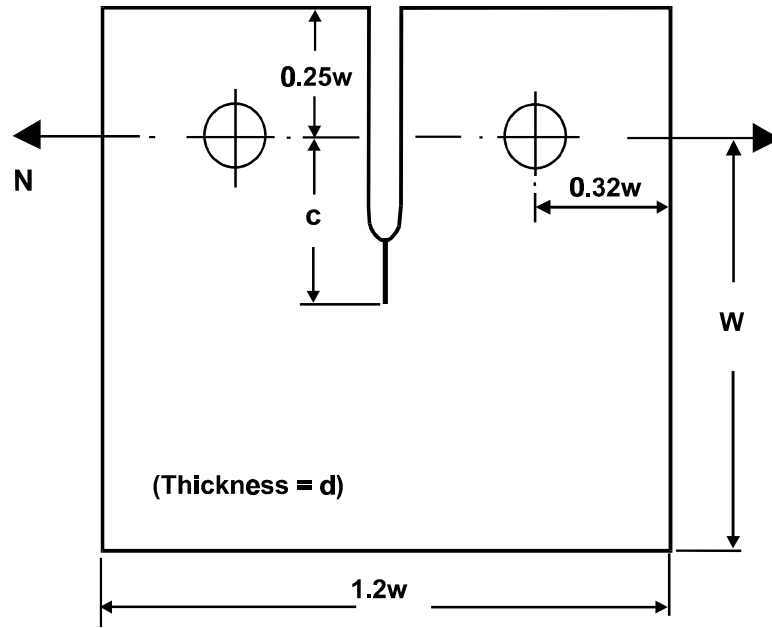
$$\text{and } N_L = \sigma_y w d \left(\left(2.702 + 4.599 \left(\frac{c}{w} \right)^2 \right)^{1/2} - 1 - 1.702 \left(\frac{c}{w} \right) \right)$$

for $0.09 \leq c/w \leq 1.0$

Remarks: Taken from Reference AII.3.

Description: Compact Tension Specimen; Tension; Plane Strain (Mises)

Schematic:



Compact Tension Specimen

Solution:

$$N_L = \sigma_y w d \gamma \left(0.634 - 1.482 \left(\frac{c}{w} \right) + 0.134 \left(\frac{c}{w} \right)^2 + 0.25 \left(\frac{c}{w} \right)^3 \right)$$

for $0 \leq c / w \leq 0.09$,

$$\text{and } N_L = \sigma_y w d \gamma \left(\left(2.702 + 4.599 \left(\frac{c}{w} \right)^2 \right)^{1/2} - 1 - 1.702 \left(\frac{c}{w} \right) \right)$$

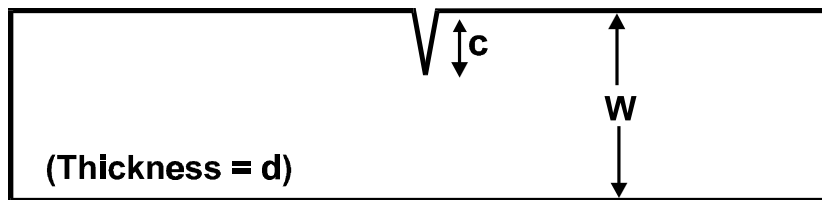
for $0.09 \leq c / w \leq 1.0$

where $\gamma = \frac{2}{\sqrt{3}}$

Remarks: Taken from Reference AII.3.

Description: Charpy Specimen; Three Point Bend; Plane Strain (Tresca)

Schematic:



Three Point Bend Specimen (Charpy)

Solution:

$$M_L = \frac{\sigma_y w^2 d}{4} \left(1 - \frac{c}{w}\right)^2 \left(1.12 + 1.13\left(\frac{c}{w}\right) - 3.194\left(\frac{c}{w}\right)^2\right)$$

$$\text{for } 0 \leq c/w \leq 0.18 ,$$

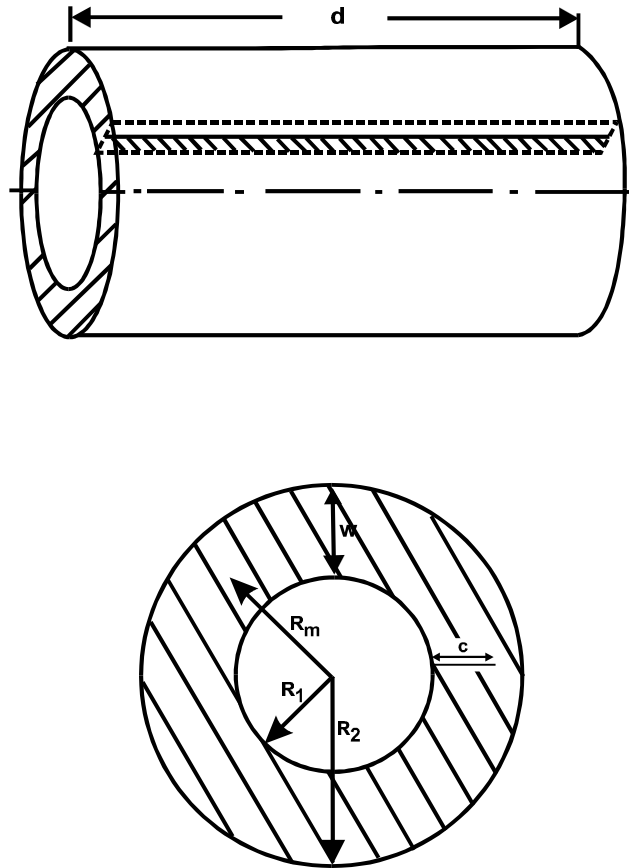
$$\text{and } M_L = \frac{\sigma_y w^2 d}{4} 1.22 \left(1 - \frac{c}{w}\right)^2$$

$$\text{for } 0.18 \leq c/w \leq 1.0$$

Remarks: Taken from Reference AII.3.

Description: Pipe; Internal Axial Extended Surface Defect; Pressure-Excluding Crack Faces

Schematic:



Internal Axial Surface Extended Defect in a Pipe

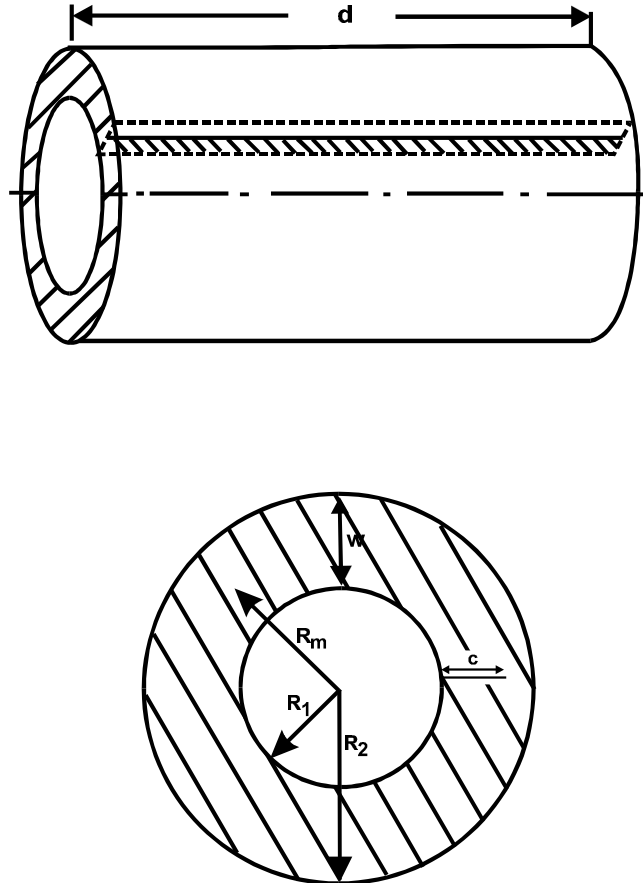
Solution:

$$P_L = \sigma_y \ell n \left(\frac{R_2}{R_1 + c} \right)$$

Remarks: Taken from Reference AII.3.

Description: Pipe; Internal Axial Extended Surface Defect; Pressure-Including Crack Faces

Schematic:



Internal Axial Surface Extended Defect in a Pipe

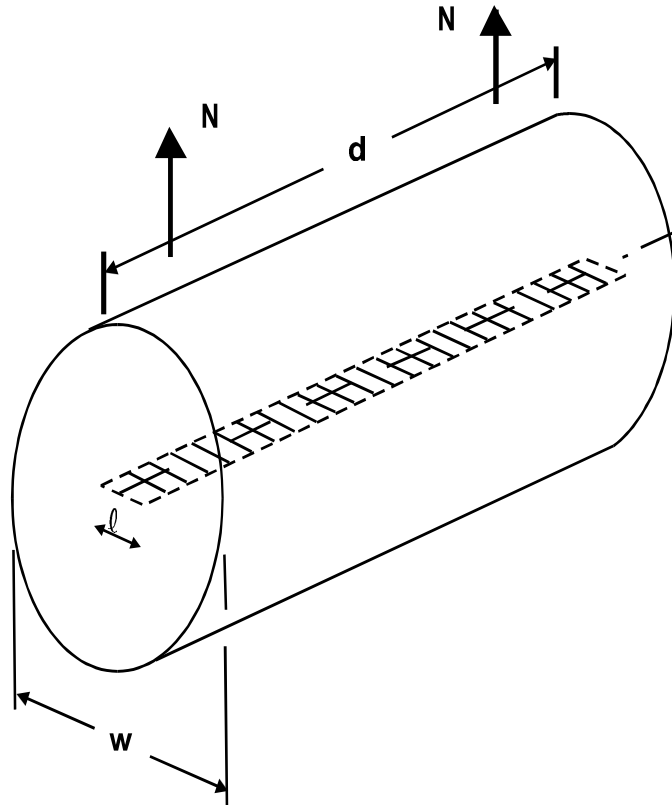
Solution:

$$P_L = \sigma_y \left(\frac{R_1}{R_1 + c} \right) \ln \left(\frac{R_2}{R_1 + c} \right)$$

Remarks: Taken from Reference AII.3.

Description: Solid Round Bar; Centrally Embedded Extended Defect; Tension

Schematic:



Centrally Embedded Extended Defect in a Round Bar

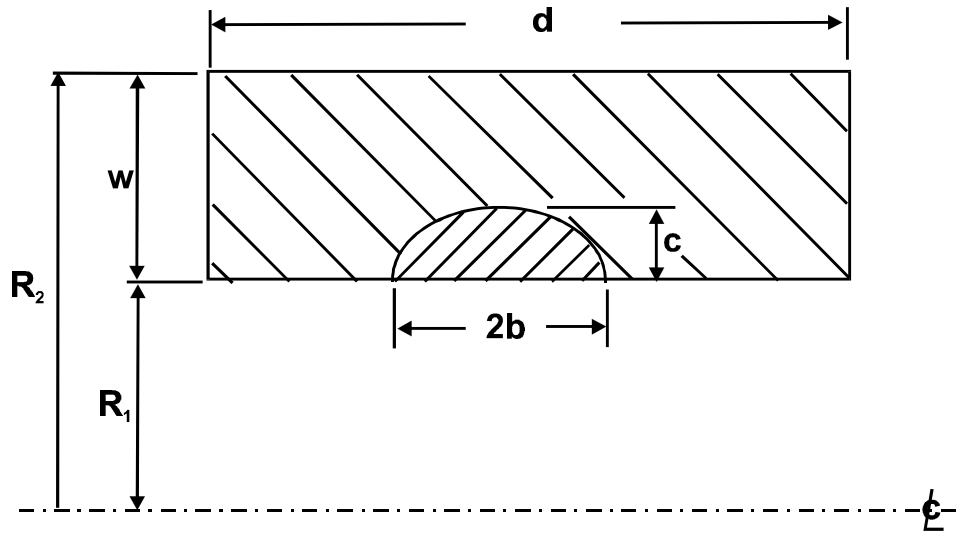
Solution:

$$N_L = \sigma_y w d \left(1 - \frac{\ell}{w} \right)$$

Remarks: Taken from Reference AII.3.

Description: Pipe; Internal Axial Semi-Elliptical Surface Defects; Pressure-Excluding Crack Faces; Global Collapse

Schematic:



Axial Semi-Elliptical Defect in the Inner Wall Surface of a Pipe

Solution:

$$P_L = \sigma_y \left(\frac{c}{R_1 M} + \ell n \left(\frac{R_2}{R_1 + c} \right) \right)$$

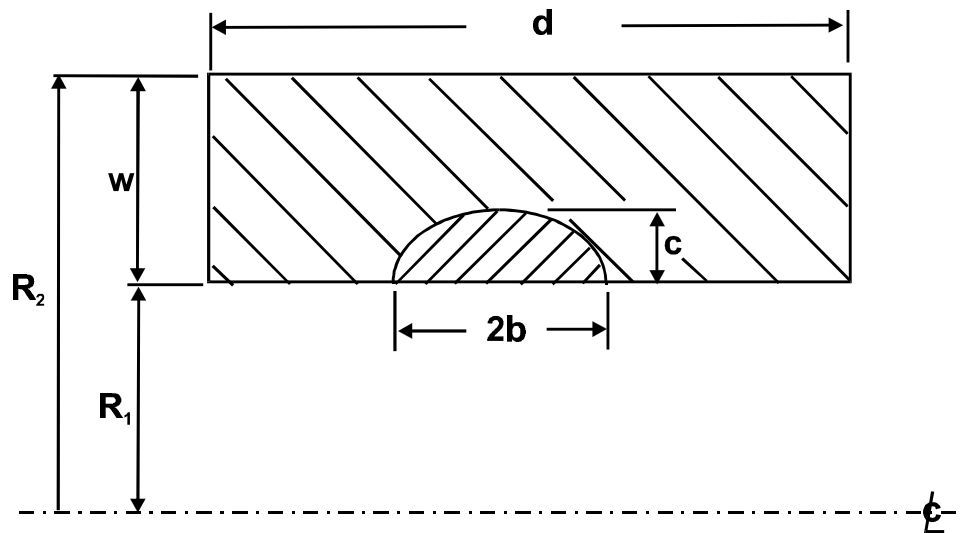
Where

$$M = \left(1 + \frac{1.61}{R_1 c} b^2 \right)^{\frac{1}{2}}$$

Remarks: Taken from Reference AII.3.

Description: Pipe; Internal Axial Semi-Elliptical Surface Defects; Pressure-Including Crack Faces; Global Collapse

Schematic:



Axial Semi-Elliptical Defect in the Inner Wall Surface of a Pipe

Solution:

$$P_L = \sigma_y \left(\frac{c}{R_1 M} + \left(\frac{R_1}{R_1 + c} \right) \ln \left(\frac{R_2}{R_1 + c} \right) \right)$$

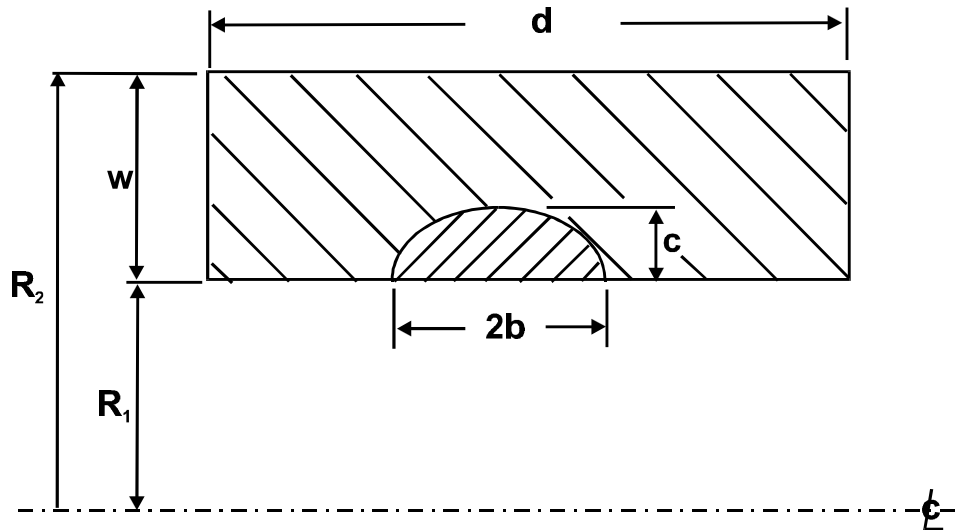
Where

$$M = \left(1 + \frac{1.61 b^2}{R_1 c} \right)^{1/2}$$

Remarks: Taken from Reference AII.3.

Description: Pipe; Internal Axial Semi-Elliptical Surface Defects; Pressure-Excluding Crack Faces; Local Collapse

Schematic:



Axial Semi-Elliptical Defect in the Inner Wall Surface of a Pipe

Solution:

$$P_L = \frac{\sigma_y}{2(s+b)} \cdot \left(s \ln \left(\frac{R_2}{R_1} \right) + 2b \ln \left(\frac{R_2}{R_1 + c} \right) \right)$$

where

$$s = \frac{bc(1 - c/w)}{MR_1 \left(\ln \left(\frac{R_2}{R_1} \right) - \ln \left(\frac{R_2}{R_1 + c} \right) \right) - c}$$

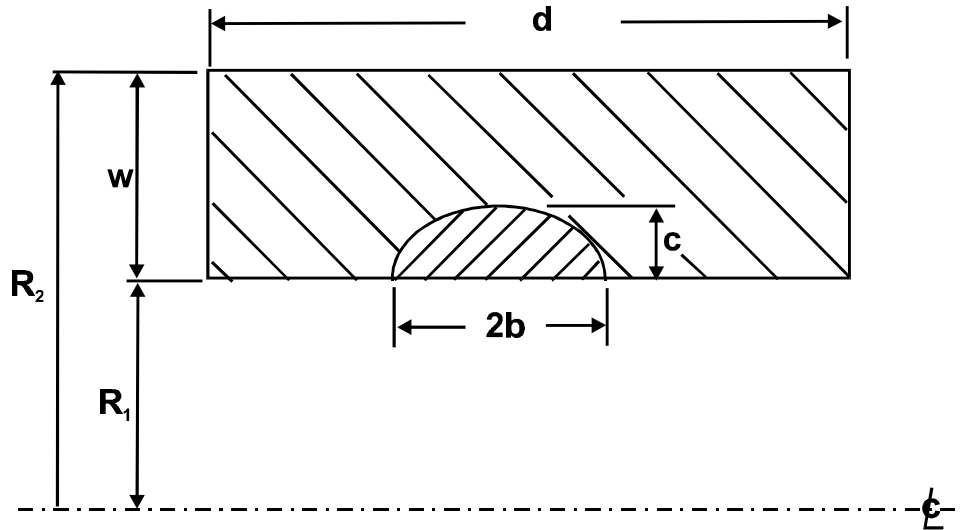
and

$$M = \left(1 + \frac{1.61 b^2}{R_1 c} \right)^{1/2}$$

Remarks: Taken from Reference AII.3.

Description: Pipe; Internal Axial Semi-Elliptical Surface Defects; Pressure-Including Crack Faces; Local Collapse

Schematic:



Axial Semi-Elliptical Defect in the Inner Wall Surface of a Pipe

Solution:

$$P_L = \frac{\sigma_y}{(s+b)} \cdot \left(s \ln \left(\frac{R_2}{R_1} \right) + b \left(\frac{R_1}{R_1+c} \right) \ln \left(\frac{R_2}{R_1+c} \right) \right)$$

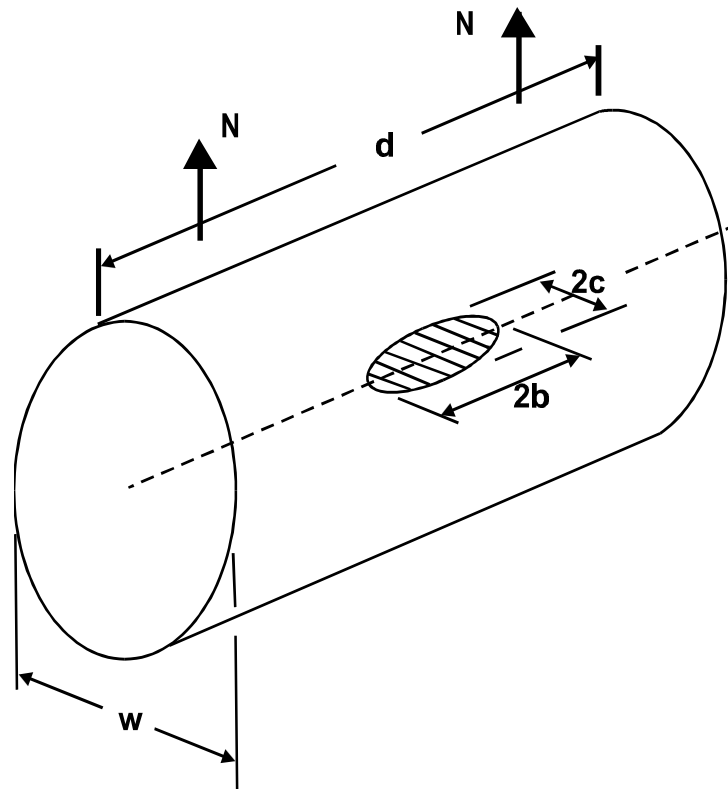
$$\text{Where } s = \frac{bc(1-c/w)}{MR_1 \left(\ln \left(\frac{R_2}{R_1} \right) - \left(\frac{R_1}{R_1+c} \right) \ln \left(\frac{R_2}{R_1+c} \right) \right) - c}$$

$$\text{and } M = \left(1 + \frac{1.61 b^2}{R_1 c} \right)^{1/2}$$

Remarks: Taken from Reference AII.3.

Description: Solid Round Bar; Centrally Embedded Axial Elliptical Defects; Tension;
Global Collapse

Schematic:



Centrally Embedded Elliptical Defect in the Round Bar

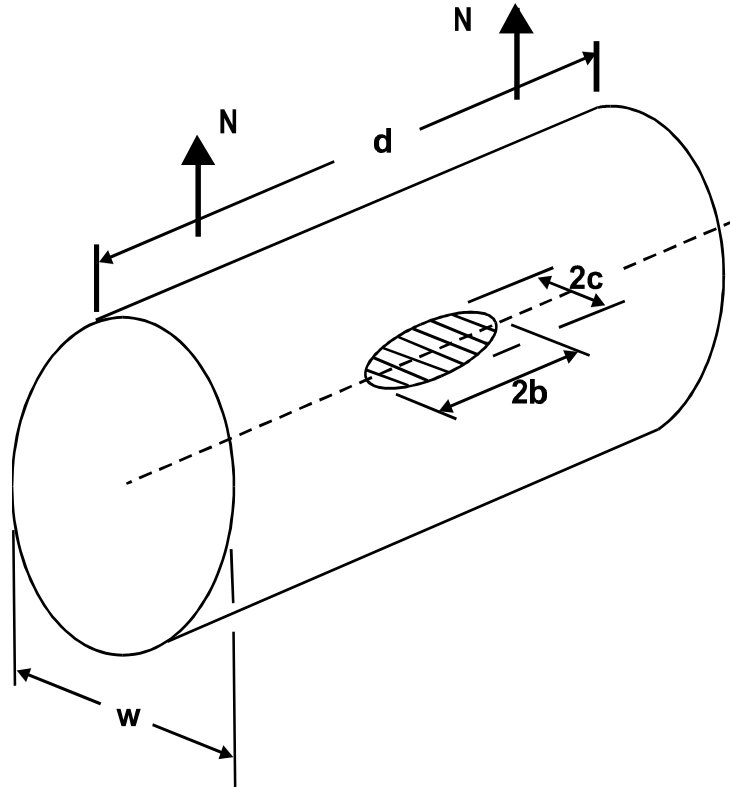
Solution:

$$N_L = \sigma_y w d \left(1 - \frac{2bc}{w(w+b)} \right)$$

Remarks: Taken from Reference AII.3.

Description: Solid Round Bar; Centrally Embedded Axial Elliptical Defects; Tension;
Local Collapse

Schematic:



Centrally Embedded Elliptical Defect in the Round Bar

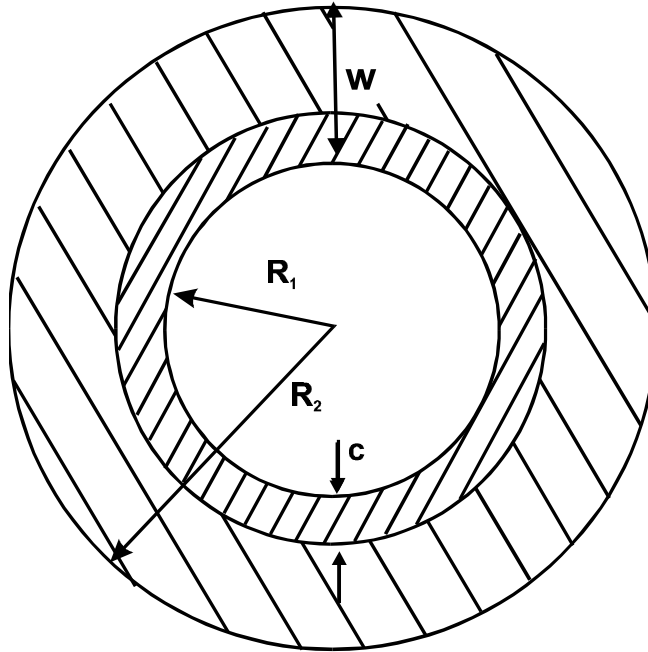
Solution:

$$N_L = \sigma_y w d \left(1 - \frac{2bc}{w(w - 2c + b)} \right)$$

Remarks: Taken from Reference AII.3.

Description: Pipe; Internal Fully Circumferential Surface Defect in a Thick Pipe;
Internal Pressure

Schematic:



Internal Fully Circumferential Surface Defect in a Thick Pipe

Solution:

$$P_L = \sigma_y \left(\ln \left(\frac{R_2}{R_1 + c} \right) + \frac{1}{2} \left(\left(\frac{R_2}{R_1 + c} \right)^2 - 1 \right) \right)$$

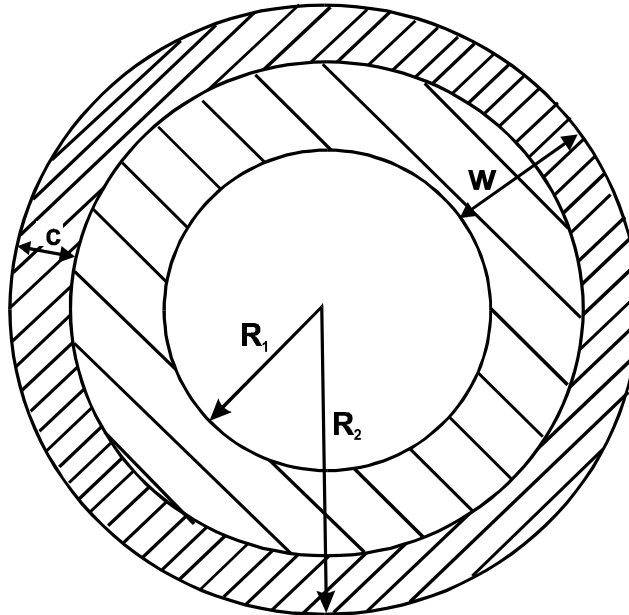
$$\text{if} \quad \left(1 - \frac{R_1}{R_1 + c} \right) > \frac{1}{2} \left(\left(\frac{R_2}{R_1 + c} \right)^2 - 1 \right),$$

$$\text{otherwise} \quad P_L = \sigma_y \left(\ln \left(\frac{R_2}{R_1 + c} \right) + 1 - \frac{R_1}{R_1 + c} \right)$$

Remarks: Taken from Reference AII.5. The above result is for the case where there is crack face pressure and the pipe has closed ends. The result for the crack sealed is contained in [AII.5]

Description: Pipe External Fully Circumferential Surface Defect in a Thick Pipe;
Internal Pressure

Schematic:



External Fully Circumferential Surface Defect in a Thick Pipe

Solution:

$$P_L = \sigma_y \left(\ln \left(\frac{R_2 - c}{R_1} \right) + \frac{1}{2} \left(1 - \left(\frac{R_1}{R_2 - c} \right)^2 \right) \right)$$

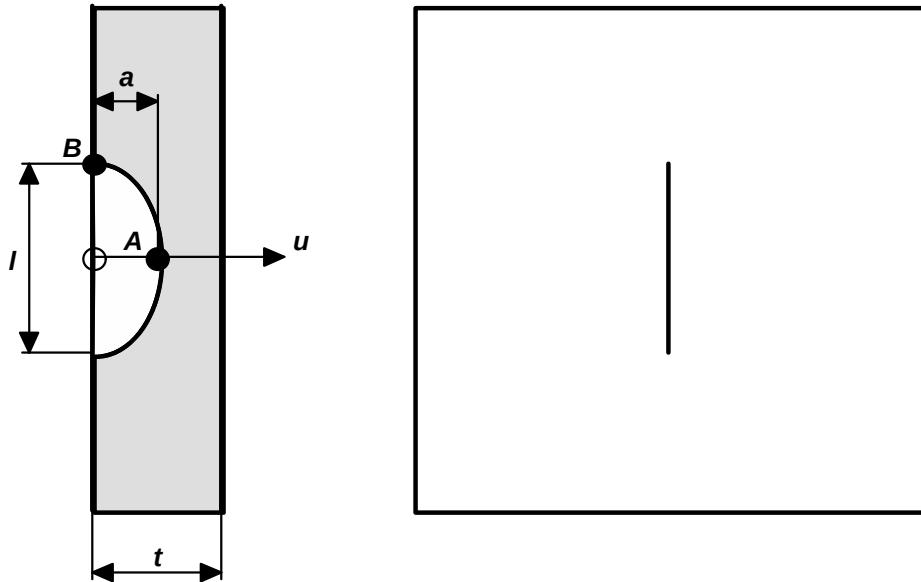
if $\ln \left(\frac{R_2}{R_2 - c} \right) > \frac{1}{2} \left(1 - \left(\frac{R_1}{R_2 - c} \right)^2 \right),$

otherwise $P_L = \sigma_y \ln \left(\frac{R_2}{R_1} \right)$

Remarks: Taken from Reference AII.5. The pipe has sealed ends.

Description: Finite surface crack in a plate

Schematic:



Finite surface crack in a plate.

Solution:

L_r is given by:

$$L_r = \frac{g(z) \frac{s_b}{3} + \sqrt{g^2(z) \frac{s_b^2}{9} + (1-z)^2 s_m^2}}{(1-z)^2 s_Y},$$

where

$$g(z) = 1 - 20z^3 \left(\frac{a}{l} \right)^{0.75},$$

$$z = \frac{al}{t(l+2t)}.$$

σ_m and σ_b are the membrane and bending stress components, respectively. These stresses define the stress state σ according to:

$$s = s(u) = s_m + s_b \left(1 - \frac{2u}{t}\right) \quad \text{for } 0 \leq u \leq t.$$

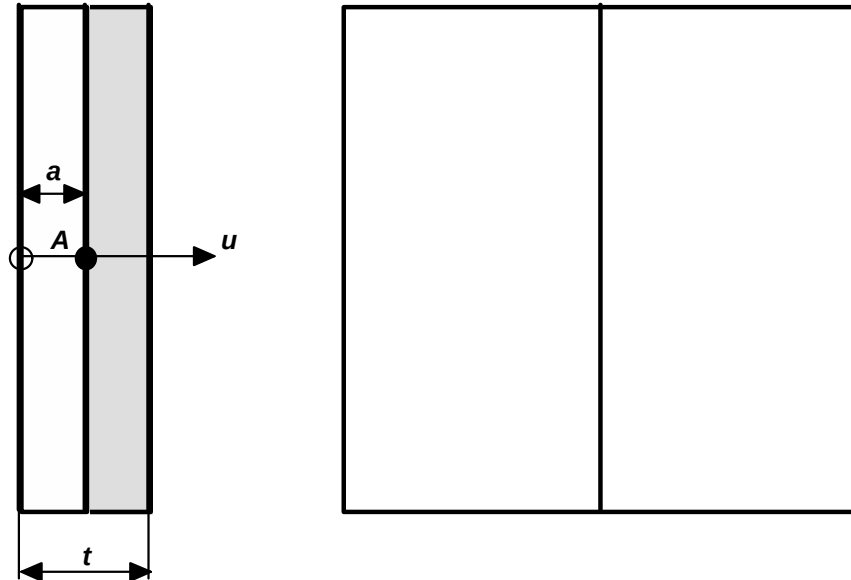
σ is to be taken normal to the prospective crack plane in an uncracked plate. σ_m and σ_b are determined by fitting s to the above equation. The co-ordinate u is defined in the figure.

Remarks: The solution is limited to $a/t \leq 0.8$, for pure tension. If bending is present, the solution is limited to $a/t \leq 0.6$. Also, the plate should be large in comparison to the length of the crack so that edge effects do not influence the results.

Taken from Reference AII.8.

Description: Infinite surface crack in a plate

Schematic:



Infinite surface crack in a plate.

Solution:

L_r is given by:

$$L_r = \frac{z s_m + \frac{s_b}{3} + \sqrt{\left(z s_m + \frac{s_b}{3}\right)^2 + (1-z)^2 s_m^2}}{(1-z)^2 s_Y},$$

where

$$z = \frac{a}{t}.$$

σ_m and σ_b are the membrane and bending stress components, respectively, which define the stress state σ according to:

$$s = s(u) = s_m + s_b \left(1 - \frac{2u}{t}\right) \quad \text{for } 0 \leq u \leq t.$$

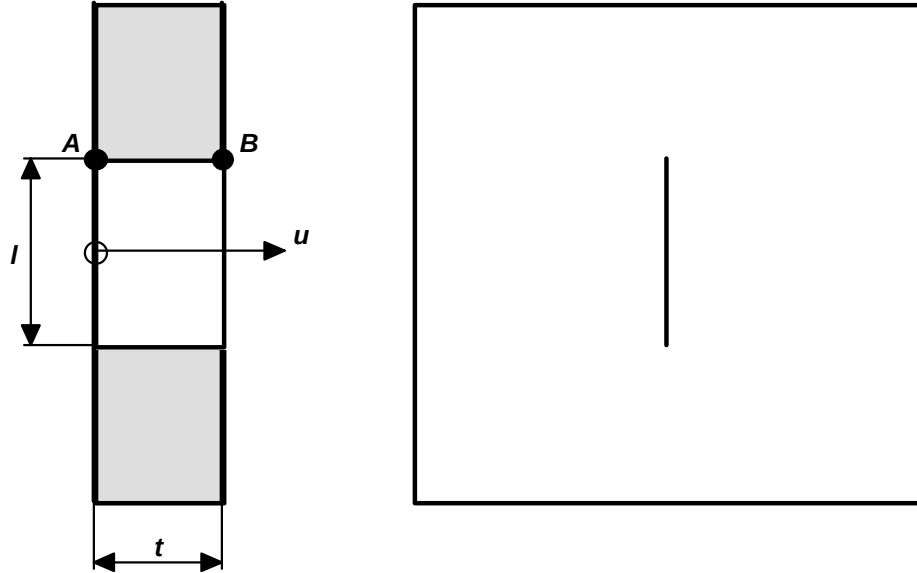
σ is to be taken normal to the prospective crack plane in an uncracked plate. σ_m and σ_b are determined by fitting σ to the above equation. The co-ordinate u is defined in the figure.

Remarks: The solution is limited to $a/t \leq 0.8$. Also, the plate should be large in the transverse direction to the crack so that edge effects do not influence the results.

Taken from Reference AII.9.

Description: Through-thickness crack in a plate

Schematic:



Through-thickness crack in a plate.

Solution:

L_r is given by:

$$L_r = \frac{\frac{s_b}{3} + \sqrt{\frac{s_b^2}{9} + s_m^2}}{s_Y}.$$

σ_m and σ_b are the membrane and bending stress components respectively, which define the stress state σ according to:

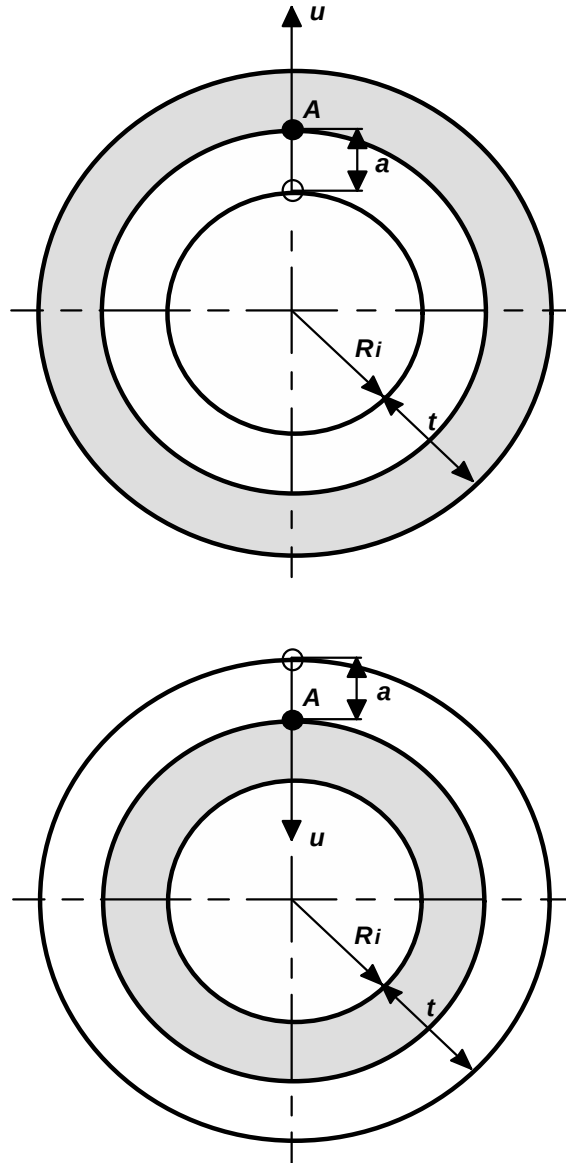
$$s = s(u) = s_m + s_b \left(1 - \frac{2u}{t}\right) \quad \text{for } 0 \leq u \leq t.$$

s is to be taken normal to the prospective crack plane in an uncracked plate. σ_m and σ_b are determined by fitting σ to the above equation. The co-ordinate u is defined in the figure provided.

Remarks: The plate should be large in comparison to the length of the crack so that edge effects do not influence the results.

Description: Complete circumferential internal or external surface crack in a thin-walled cylinder

Schematic:



Complete circumferential internal or external surface crack in a cylinder.

For a cylinder of mean radius R under axial load F with a fully circumferential internal or external crack, a lower bound limit load has been derived [AII.7] for a thin-walled cylinder using the von Mises yield criterion and it has been shown that this can exceed the net section collapse formula by a factor of up to $(2/\sqrt{3})$.

Solution:

$$F_L = 2\pi R(t-a)\sigma_y \left\{ \left[\frac{a}{2(t-a)} \right] + \left[1 - \frac{3}{4} \left(\frac{a}{t-a} \right)^2 \right]^{1/2} \right\} \quad \text{for } a \leq \frac{t}{1+\sqrt{3}}$$

$$F_L = \frac{2}{\sqrt{3}} \sigma_y [2\pi R(t-a)] \quad \text{for } a \geq \frac{t}{1+\sqrt{3}}$$

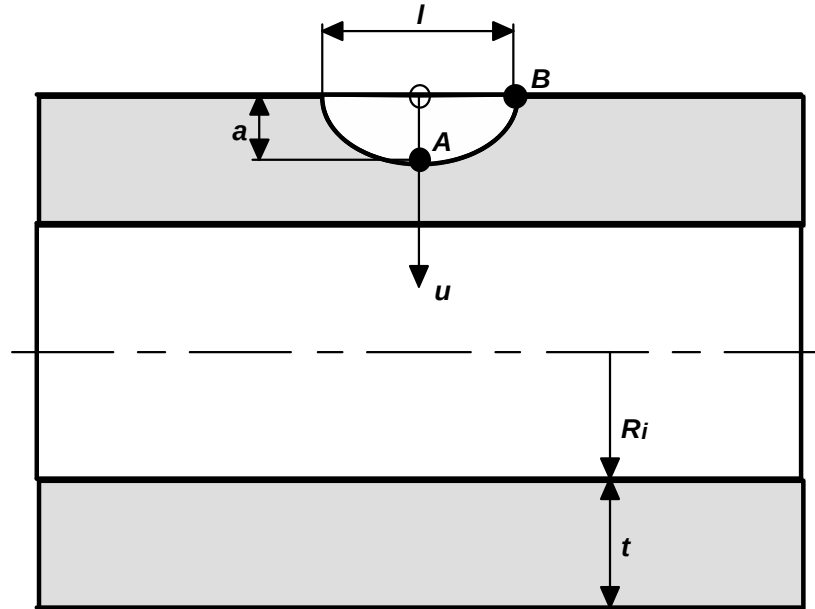
where R is the mean radius.

Remarks: The solution is believed to be conservative for thick-walled pipes due to the radial stresses.

Taken from Reference AII.7.

Description: Finite external surface crack in a cylinder

Schematic:



Finite axial external surface crack in a cylinder.

Solution:

L_r is given by:

$$L_r = \frac{g(z) \frac{s_b}{3} + \sqrt{g^2(z) \frac{s_b^2}{9} + (1-z)^2 s_m^2}}{(1-z)^2 s_Y},$$

where

$$g(z) = 1 - 20z^3 \left(\frac{a}{l} \right)^{0.75},$$

$$z = \frac{al}{t(l+2t)}.$$

σ_m and σ_b are the membrane and bending stress components, respectively, which define the stress state σ according to:

$$s = s(u) = s_m + s_b \left(1 - \frac{2u}{t}\right) \quad \text{for } 0 \leq u \leq t.$$

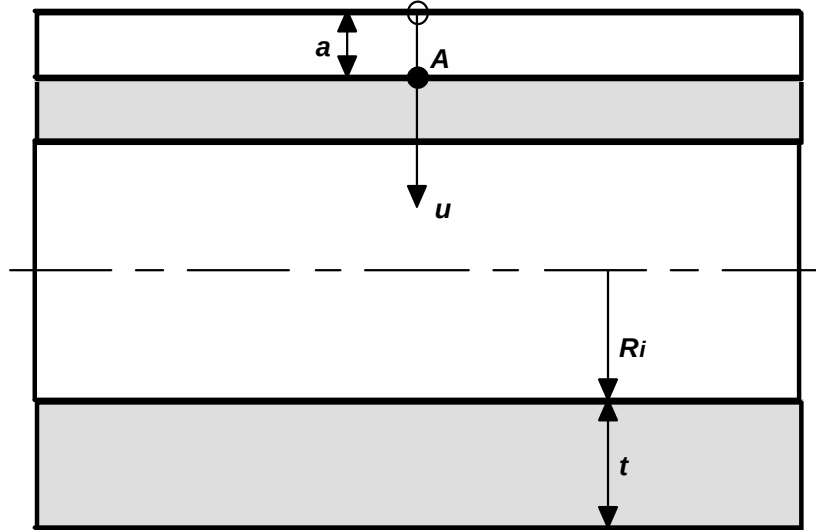
σ is to be taken normal to the prospective crack plane in an uncracked cylinder. σ_m and σ_b are determined by fitting σ to the above equation. The co-ordinate u is defined in the figure.

Remarks: The solution is limited to $a/t \leq 0.8$, for pure tension. If bending is present, the solution is limited to $a/t \leq 0.6$. Also, the cylinder should be long in comparison to the length of the crack so that edge effects do not influence the results.

Taken from Reference AII.8.

Description: Infinite external surface crack in a cylinder

Schematic:



Infinite axial external surface crack in a cylinder.

Solution:

L_r is given by:

$$L_r = \frac{z s_m + \frac{s_b}{3} + \sqrt{\left(z s_m + \frac{s_b}{3}\right)^2 + (1-z)^2 s_m^2}}{(1-z)^2 s_y},$$

where

$$z = \frac{a}{t}.$$

σ_m and σ_b are the membrane and bending stress components respectively. The stresses define the stress state σ according to:

$$s = s(u) = s_m + s_b \left(1 - \frac{2u}{t}\right) \quad \text{for } 0 \leq u \leq t.$$

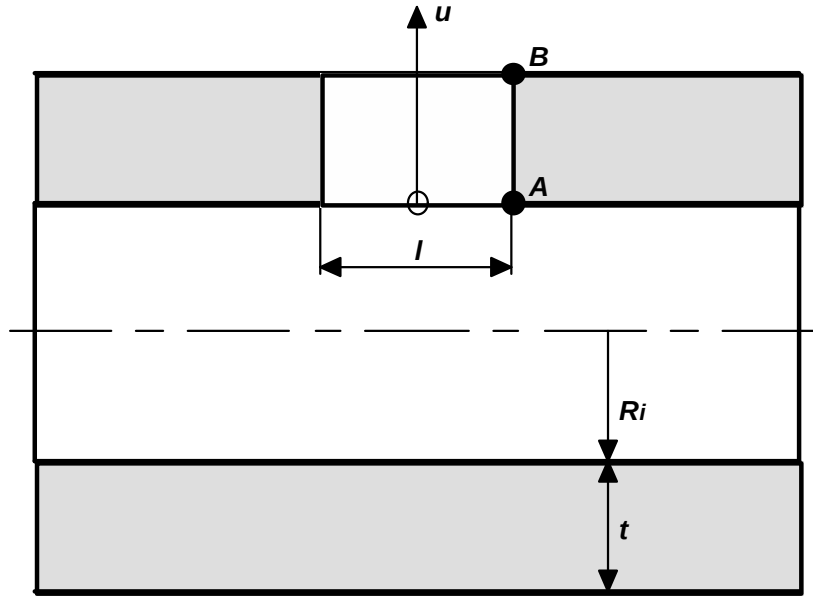
σ is to be taken normal to the prospective crack plane in an uncracked cylinder. σ_m and σ_b are determined by fitting σ to the above equation. The co-ordinate u is defined in the figure.

Remarks: The solution is limited to $a/t \leq 0.8$.

Taken from Reference AII.10.

Description: Through-thickness crack in a cylinder

Schematic:



Axial through-thickness crack in a cylinder.

Solution:

L_r is given by:

$$L_r = \frac{s_m}{s_y} \sqrt{1 + 1.05 l^2},$$

where

$$l = \frac{l}{2\sqrt{R_i t}}.$$

σ_m is the membrane stress component which defines the stress state σ according to:

$$s = s(u) = s_m \quad \text{for } 0 \leq u \leq t.$$

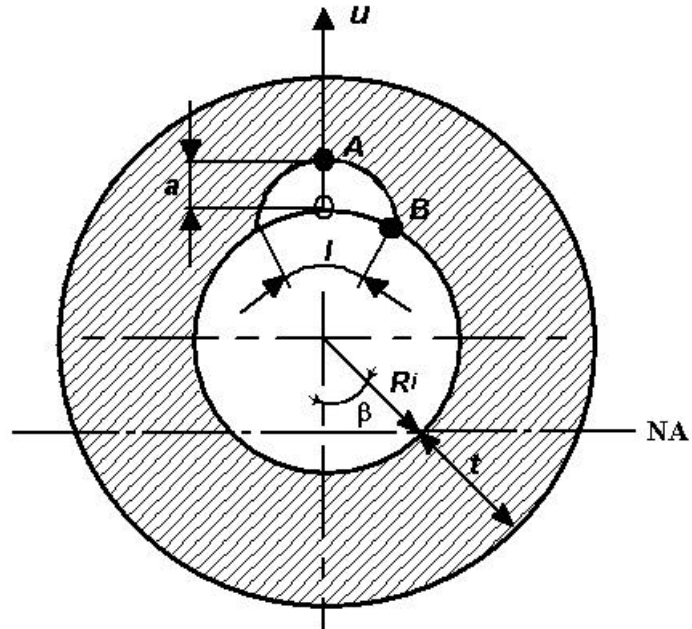
σ is to be taken normal to the prospective crack plane in an uncracked cylinder. σ_m is determined by fitting σ to the above equation. The co-ordinate u is defined in the figure.

Remarks: The cylinder should be long in comparison to the length of the crack so that edge effects do not influence the results.

Taken from Reference AII.10.

Description: Part circumferential internal surface crack in a cylinder

Schematic:



Part circumferential internal surface crack in a cylinder.

Solution:

L_r is given by:

$$L_r = \frac{\sigma_m}{s_m} = \frac{\sigma_{bg}}{s_{bg}}$$

where the parameters s_m and s_{bg} are obtained by solving the equation system:

$$\frac{s_m}{\sigma_y} = 1 - 2 \frac{\beta}{\pi} - \frac{a}{t} \frac{\alpha}{\pi}$$

$$\frac{s_{bg}}{\sigma_y} = \frac{4}{\pi} \sin \beta - \frac{2a}{\pi t} \sin \alpha$$

$$\theta = \frac{l}{2R_i}$$

$$\alpha = \begin{cases} \theta & \text{if } \theta \leq \pi - \beta \\ \pi - \beta & \text{if } \theta > \pi - \beta \end{cases}$$

$$\sigma_m s_{bg} - \sigma_{bg} s_m = 0$$

where β is half the angle subtended by the neutral axis of the cylinder, θ is half the angle subtended by the crack.

σ_m and σ_{bg} are the membrane and global bending stress components respectively. The stress σ_m defines the axisymmetric stress state σ according to:

$$\sigma = \sigma(u) = \sigma_m \quad \text{for } 0 \leq u \leq t.$$

σ is to be taken normal to the prospective crack plane in an uncracked cylinder. σ_m is determined by fitting σ to the above equation. The co-ordinate u is defined in the figure.

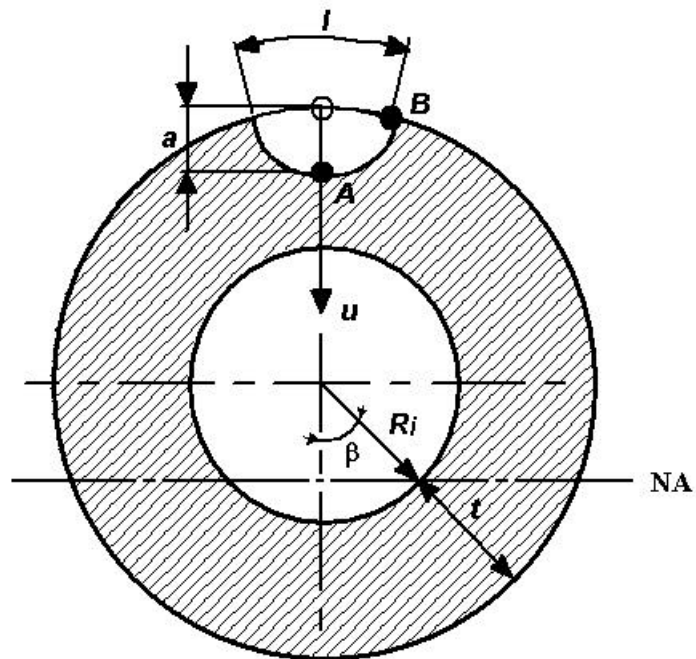
Remarks: The cylinder should be thin-walled. Also, the cylinder should be long in the transverse direction to the crack so that edge effects do not influence the results.

Taken from Reference AII.11.

When $\sigma_{bg} = 0$ then L_r is simply σ_m / s_m ; similarly when $\sigma_m = 0$ then $L_r = \sigma_{bg} / s_{bg}$; when $\sigma_{bg} \neq 0$ and $\sigma_m \neq 0$ then $\sigma_m / s_m = \sigma_{bg} / s_{bg}$ and either equation can be used to evaluate L_r .

Description: Part circumferential external surface crack in a cylinder

Schematic:



Part circumferential external surface crack in a cylinder.

Solution:

L_r is given by:

$$L_r = \frac{\sigma_m}{s_m} = \frac{\sigma_{bg}}{s_{bg}}$$

where the parameters s_m and s_{bg} are obtained by solving the equation system:

$$\frac{s_m}{\sigma_y} = 1 - 2 \frac{\beta}{\pi} - \frac{a}{t} \frac{\alpha}{\pi}$$

$$\frac{s_{bg}}{\sigma_y} = \frac{4}{\pi} \sin \beta - \frac{2}{\pi} \frac{a}{t} \sin \alpha$$

$$\theta = \frac{l}{2(R_i + t)}$$

$$\alpha = \begin{cases} \theta & \text{if } \theta \leq \pi - \beta \\ \pi - \beta & \text{if } \theta > \pi - \beta \end{cases}$$

$$\sigma_m s_{bg} - \sigma_{bg} s_m = 0$$

where β is half the angle subtended by the neutral axis of the cylinder, θ is half the angle subtended by the crack.

σ_m and σ_{bg} are the membrane and global bending stress components respectively. The stress σ_m defines the axisymmetric stress state σ according to:

$$\sigma = \sigma(u) = \sigma_m \quad \text{for } 0 \leq u \leq t.$$

σ is to be taken normal to the prospective crack plane in an uncracked cylinder. σ_m is determined by fitting σ to the above equation. The co-ordinate u is defined in the figure.

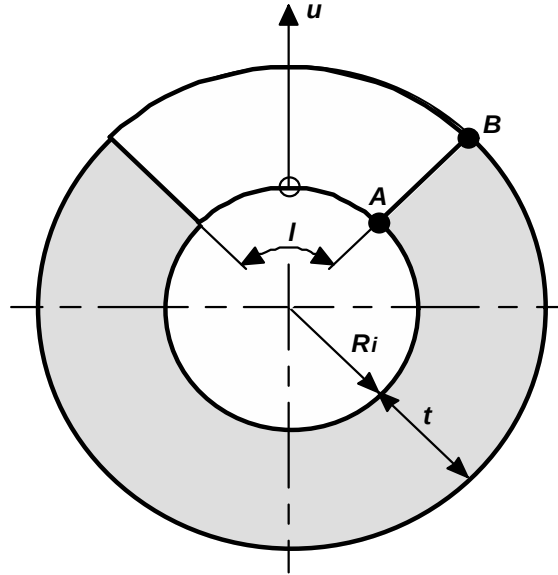
Remarks: The cylinder should be thin-walled. Also, the cylinder should be long in the transverse direction to the crack so that edge effects do not influence the results.

Taken from Reference AII.11.

When $\sigma_{bg} = 0$ then L_r is simply σ_m / s_m ; similarly when $\sigma_m = 0$ then $L_r = \sigma_{bg} / s_{bg}$; when $\sigma_{bg} \neq 0$ and $\sigma_m \neq 0$ then $\sigma_m / s_m = \sigma_{bg} / s_{bg}$ and either equation can be used to evaluate L_r .

Description: Through-thickness crack in a cylinder

Schematic:



Circumferential through-thickness crack in a cylinder.

Solution:

L_r is given by:

$$L_r = \frac{\sigma_m}{s_m} = \frac{\sigma_{bg}}{s_{bg}}$$

where the parameters s_m and s_{bg} are obtained by solving the equation system:

$$\frac{s_m}{\sigma_y} = 1 - 2 \frac{\beta}{\pi} - \frac{\theta}{\pi}$$

$$\frac{s_{bg}}{\sigma_y} = \frac{4}{\pi} \sin \beta - \frac{2}{\pi} \sin \theta$$

$$\theta = \frac{l}{2R_i}$$

$$\sigma_m s_{bg} - \sigma_{bg} s_m = 0$$

where β is half the angle subtended by the neutral axis of the cylinder, θ is half the angle subtended by the crack.

σ_m and σ_{bg} are the membrane and global bending stress components respectively. The stress σ_m defines the axisymmetric stress state σ according to:

$$\sigma = \sigma(u) = \sigma_m \quad \text{for } 0 \leq u \leq t.$$

σ is to be taken normal to the prospective crack plane in an uncracked cylinder. σ_m is determined by fitting σ to the above equation. The co-ordinate u is defined in the figure.

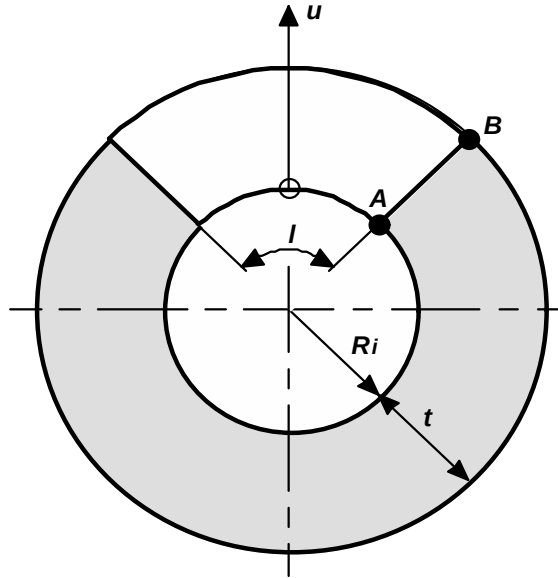
Remarks: The cylinder should be thin-walled. Also, the cylinder should be long in the transverse direction to the crack so that edge effects do not influence the results.

Taken from Reference AII.11.

When $\sigma_{bg} = 0$ then L_r is simply σ_m / s_m ; similarly when $\sigma_m = 0$ then $L_r = \sigma_{bg} / s_{bg}$; when $\sigma_{bg} \neq 0$ and $\sigma_m \neq 0$ then $\sigma_m / s_m = \sigma_{bg} / s_{bg}$ and either equation can be used to evaluate L_r .

Description: Through-thickness crack in a sphere

Schematic:



Circumferential through-thickness crack in a sphere.

Solution

L_r is given by:

$$L_r = \frac{s_m}{s_Y} \frac{1 + \sqrt{1 + 8(l / \cos q)^2}}{2},$$

where

$$l = \frac{l}{2\sqrt{R_i t}},$$

$$q = \frac{l}{2R_i}.$$

σ_m is the membrane stress components. σ_m defines the axisymmetric stress state σ according to:

$$s = s(u) = s_m \quad \text{for } 0 \leq u \leq t.$$

σ is to be taken normal to the prospective crack plane in an uncracked sphere. σ_m is determined by fitting σ to the above equation. The co-ordinate u is defined in the figure.

Remarks: The sphere should be thin-walled.

Taken from Reference AII.12.

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APPENDIX III

**STRESS INTENSITY FACTOR AND LIMIT LOAD SOLUTIONS
FOR OFFSHORE TUBULAR JOINTS**

CONTENTS

AIII.1 INTRODUCTION.....	AIII.2
AIII.2 STRESS ANALYSIS.....	AIII.3
AIII.3 STRESS INTENSITY FACTOR SOLUTIONS.....	AIII.4
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AIII.1 INTRODUCTION

This appendix presents guidance on Stress Intensity Factor (SIF) and Limit Load (LL) solutions for flaws in offshore structures. The assessment of fatigue crack growth and fracture in tubular joints requires specialist guidance due to the complexity of the joint geometry and the applied loading and this appendix provides supplementary guidance on the SIF and LL used for the application of the PD6493^(AIII.1) procedure to tubular joints. Its scope is limited to the assessment of known or assumed weld toe flaws, including fatigue cracks found in service, in brace or chord members of T, Y, K or KT joints between circular section tubes under axial and / or bending loads. Further information concerned with the design, assessment and certification of offshore installation is given in [AIII.2].

The determination of plastic collapse parameters should be based on conditions for local collapse in the neighbourhood of the crack. This recommendation is satisfactory for structures where yielding of a ligament causes complete plastic collapse to occur. Where the first yielding of a ligament is contained by surrounding elastic material such that the plastic strains are limited to levels not much beyond the elastic range, the adoption of first yielding may be very conservative.

The assessment of the significance of flaws requires information on the plastic collapse strength of the cracked geometry. The major effort in this area has been through the work of Burdekin and Frodin^(AIII.3), Cheaitani^(AIII.4), Al Laham and Burdekin^(AIII.5). Frodin's work was concerned with T and double T joints under axial tension, whilst Cheaitani examined balanced 45° K joints under axial loading. In both cases they examined three different brace to chord diameter ratios ($\beta = 0.35, 0.53, 0.8$ approximately). The plastic collapse ultimate strength was determined for each of the uncracked geometries and for three different through thickness cracks lengths at the chord weld toe in the range of 15% to 35% of the weld perimeter length. In both cases the work was carried out by using 3-D elastic plastic finite element analysis and by experimental tests at model scale on each geometry and crack case considered. Al Laham's work was concerned with 45° K joints under axial, in-plane and out of plane bending loading, and examined higher brace to chord diameter ratios ($\beta = 0.53 - 0.95$). The results illustrated the effects of cracks of different sizes on the ultimate strength of the uncracked geometry.

Since several parametric equations are available for the design strength of the uncracked geometry [HSE^(AIII.6), UEG^(AIII.7), API^(AIII.8) and others], the main objectives of the above research programmes were to determine correction factors to give the plastic collapse strength of the cracked geometry as a proportion of the uncracked strength.

AIII.2 STRESS ANALYSIS

Results of structural analysis of the overall frame under the chosen critical loading conditions must be available to give the forces and moments in the members in the region being assessed. These should be provided as axial force, in-plane and out-of-plane bending moments.

AIII.3 STRESS INTENSITY FACTOR SOLUTIONS

AIII.3.1 EVALUATION METHODS

The principal methods used to determine stress intensity factors for weld toe surface cracks in tubular joints are:

Numerical (i.e. finite element or boundary element) analysis of tubular joints.

Standard and analytical (e.g. weight function) solutions for semi-elliptical cracks in plates.

AIII.3.2 NUMERICAL SOLUTIONS FOR TUBULAR JOINTS

The determination of stress intensity factor solutions for surface cracks in tubular joints by numerical methods requires complex modelling and stress analysis and consequently only a limited number of solutions are available^(AIII.9, AIII.10 and AIII.11). The most extensive solutions are those obtained from finite element analysis performed on T-joints^(AIII.10) and Y-joints^(AIII.11). The collected solutions are given in Section AIII.5.

AIII.4 LIMIT LOAD SOLUTIONS

The collapse parameter L_r for tubular joints may be calculated using either local collapse analysis or global collapse analysis^[AIII.2]. The local collapse approach will usually be very conservative, whilst the use of the global approach tends to give more realistic predictions of plastic collapse in tubular joints.

As far as the global collapse analysis is concerned, the lower bound characteristic ultimate strength, for the uncracked geometry and the specified minimum yield strength concerned, should be calculated using the Health and Safety Executive characteristic strength or API RP 2A equations^(AIII.6 and AIII.8). The plastic collapse strength of cracked tubular joints can be obtained by multiplying the strength of the uncracked joints, with the same geometry, by an appropriate strength reduction factor. These strength reduction factors depend upon the loading condition as well as the type of joint considered. For axially loaded joints Area Reduction Factor (ARF) should be used, while for bending loaded joints Inertia Reduction Factor (IRF) should be applied. Hence, the limit strength of a cracked joint is obtained simply by calculating the characteristic strength of the uncracked joint, using the Health and Safety Executive characteristic strength or API RP 2A equations^(AIII.6, AIII.7), which is then reduced by an appropriate factor depending on the loading and type of joint considered.

Lower bound collapse loads should be calculated separately for axial loading, in-plane and out-of-plane bending for the overall cross-section of the member containing the flaw, based on net area (for axial loading)/inertia (for bending loading) and yield strength. The contribution of the net area for axial loading should be taken as the full area of the cross-section of the joint minus the area of rectangle containing the flaw^(AIII.4). For joints subjected to bending moment, the fully plastic moment of the cross-section of the joint should be calculated for in-plane or out-of-plane loads, based on the net cross-sectional inertia of the section: a rectangle containing the flaw should be considered which will reduce the moment of inertia of the section^(AIII.5).

For simple T- DT- and gapped K-joints under axial loading, Cheaitani^(AIII.4) suggested the use of the following area reduction factors to be applied to parametric formulae for the uncracked strength:

$$ARF = \left(1 - \frac{\text{Crack Area}}{\text{Weld Length} \times T} \right) \left(\frac{1}{Q_\beta} \right)^m$$

where:

- ARF is an Area Reduction Factor to allow for the effect of the crack on net cross-sectional area.
- Q_β is the factor used in the various parametric formulae to allow for the increased strength observed at β (the ratio of brace to chord diameter) values above 0.6. The

factor Q_β is given together with the recommended solutions for the uncracked joints in Section AIII.6.

- T is the chord thickness.

The exponent, m , depends on the use of either Health and Safety Executive characteristic strength or API RP 2A equations^(AIII.6 and AIII.8). $m=1.0$ when Health and Safety Executive characteristic strength is adopted, while $m=0$ when API RP 2A is used.

For K-joints under in-plane and out-of-plane bending loading, a different correction factor is proposed by Al Laham and Burdekin^(AIII.5) based on the effect of the crack in reducing the fully plastic moment of resistance of the tubular joint. Although the position of the cracks considered in this work is around the toe of the brace to chord weld in the chord, the major effect is assumed to be equivalent to a reduction in bending strength of the brace because the part of the brace circumference corresponding to the crack cannot transmit forces to the chord. The strength reduction factor for these bending cases becomes:

$$\text{Inertia Reduction Factor} = \cos\left(\frac{\Theta}{2}\right) \left(1 - \sin\left(\frac{\Theta}{2}\right)\right)$$

where Θ is the cracked angle subtended by defect.

For cracked joints the use of HSE characteristic strength predictions of joints, modified by an area reduction for tension/compression^(AIII.4) or a moment reduction factor for bending^(AIII.5) gave calculated curves close to or outside the standard PD6493 level 2 curve indicating that this basis for calculating L_r with the standard curve would be expected to give safe results.

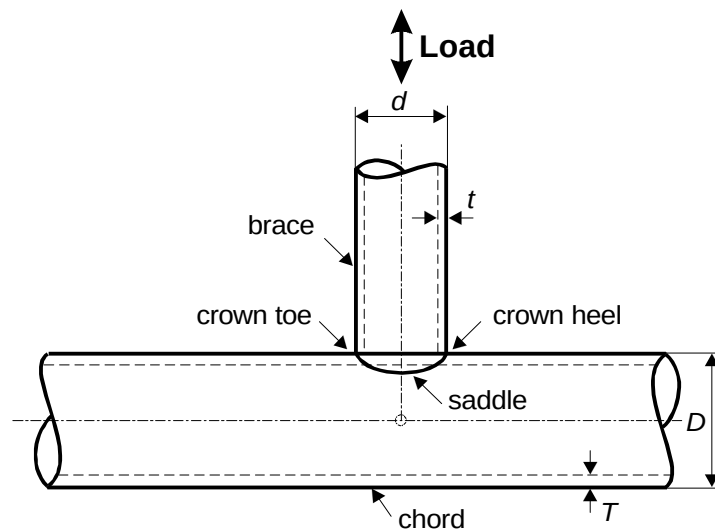
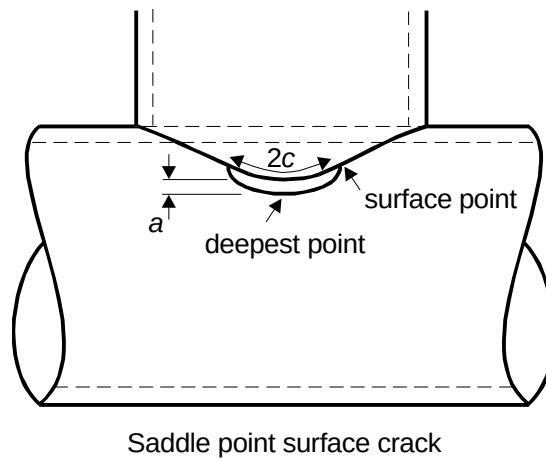
The limit loads solutions collected for the purpose of this compendium are given in Section AIII.6 of this appendix.

AIII.5 STRESS INTENSITY FACTOR SOLUTIONS

Description: Surface Crack at the Saddle Point of T-Joints
(Deepest Point)

Loading: Axial

Schematic:



Notation:

a	crack depth
$2c$	surface crack length
d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
α	$2L/D$
β	d/D
γ	$D/2T$
τ	t/T
σ_n	brace nominal stress

Stress Intensity Factor Solution:

at the deepest point under axial loading:

$$K_e = \sigma_n F_g F_i F_s \sqrt{\delta a}$$

$$F_g = 0.2749\beta (-0.6225 - 1.2685 \ln \beta) \gamma (1.3191 - 0.1661 \ln \tau) \tau (1.6621 + 0.3704 \ln \beta)$$

$$F_i = \beta (0.3561 A - 0.0956 C) \gamma (0.0983 A + 0.2298 C + 0.0817 C^2) \tau - 0.0762 A$$

$$F_s = (a/T)^p (3c/d)^r$$

$$p = -0.8669 - 0.2198A - 0.0162A^2 - 0.4750C^2 - 0.1667C^3 - 0.0193C^4$$

$$r = 0.0777 + 1.0531A + 0.5820A^2 + 0.0810A^3 - 0.07001C - 0.0604C^2 + 0.0060C^3$$

$$A = \ln (a/T)$$

$$C = \ln (3c/d)$$

For Axial Tension (AT)
$$s_n = \frac{4P}{p[d^2 - (d - 2t)^2]}$$

where σ_n is the nominal stress and P is the applied load in the brace.

K_e combines the contributions of the stress intensity factor components for modes I, II and III, i.e.

$$K_e = \left[\frac{K_I^2 + K_{II}^2 + K_{III}^2}{(1 - \nu)} \right]^{1/2}$$

Limits to Stress Intensity Factor Solution:

$$\alpha = 12$$

$$0.4 \leq \beta \leq 0.8$$

$$10 \leq \gamma \leq 20$$

$$0.3 \leq \tau \leq 1.0$$

$$0.05 \leq a/T \leq 0.80$$

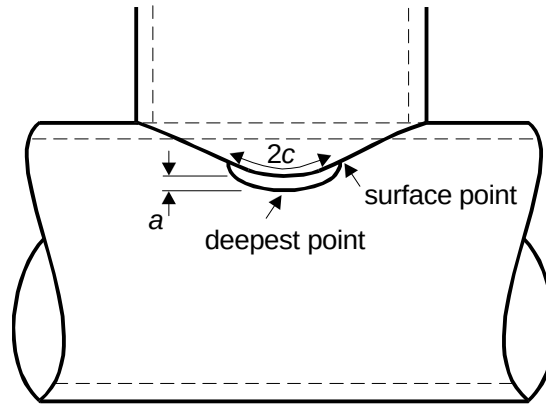
$$0.05 \leq 3c/d \leq 1.20$$

Remarks: Taken from Reference AIII.10.

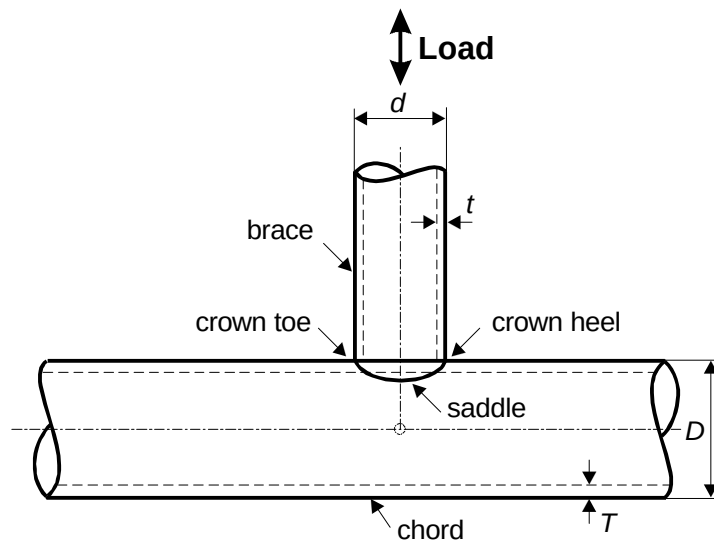
Description: Surface Crack at the Saddle Point of T-Joints
(Surface Point)

Loading: Axial

Schematic:



Saddle point surface crack



Notation:

a	crack depth
$2c$	surface crack length
d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
α	$2L/D$
β	d/D
γ	$D/2T$
τ	t/T
σ_n	brace nominal stress

Stress Intensity Factor Solution:

$$K_e = s_n F_g F_i F_s \sqrt{\delta a}$$

$$F_g = 204.08 \beta^{(-0.5858 - 0.7492 \ln \beta)} \gamma^{(-2.6713 - 0.2884 \ln \beta + 0.5646 \ln \gamma)} \tau^{(1.1491 - 0.2936 \ln \gamma - 0.5043 \ln \tau)}$$

$$F_i = \beta^{0.0680} A \gamma^{(0.0473 A - 0.5344 C - 0.1218 C^2)} \tau^{(-0.1299 A - 0.0370 C)}$$

$$F_s = (a/T)^p (3c/d)^r$$

$$p = 1.0787 + 0.6397A + 0.1569A^2 + 0.0186A^3 - (0.0770 + 0.0478A + 0.0099A^2) C^2$$

$$r = 0.8617 + 0.4888A + 0.1816A^2 + 0.0123A^3 - 0.3252C - 0.2210C^2 - 0.0275C^3$$

$$A = \ln (a/T)$$

$$C = \ln (3c/d)$$

For Axial Tension (AT)
$$s_n = \frac{4P}{p[d^2 - (d - 2t)^2]}$$

where σ_n is the nominal stress and P is the applied load in the brace.

K_e combines the contributions of the stress intensity factor components for modes I, II and III, i.e.

$$K_e = \left[\frac{K_I^2 + K_{II}^2 + K_{III}^2}{(1 - \nu)} \right]^{1/2}$$

Limits to Stress Intensity Factor Solution:

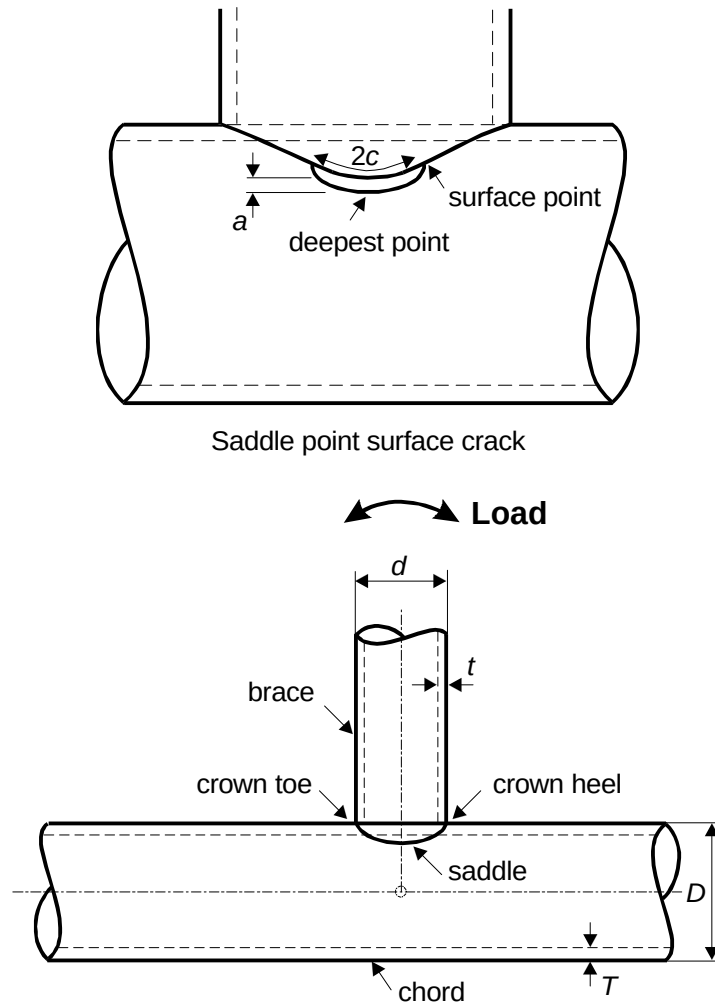
$$\begin{aligned} \alpha &= 12 \\ 0.4 &\leq \beta \leq 0.8 \\ 10 &\leq \gamma \leq 20 \\ 0.3 &\leq \tau \leq 1.0 \\ 0.05 &\leq a/T \leq 0.80 \\ 0.05 &\leq 3c/d \leq 1.20 \end{aligned}$$

Remarks: Taken from Reference AIII.10.

Description: Surface Crack at the Saddle Point of T-Joints
(Deepest point)

Loading: In-plane bending

Schematic:



Notation:

a	crack depth
$2c$	surface crack length
d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
α	$2L/D$
β	d/D
γ	$D/2T$
τ	t/T
σ_n	brace nominal stress

Stress Intensity Factor Solution:

$$K_e = s_n F_g F_i F_s \sqrt{\delta a}$$

$$F_g = 0.0887\beta (1.3433 - 0.4798 \ln \beta) \gamma (5.2247 - 0.5555 \ln \beta - 0.8310 \ln \gamma) \tau (0.6928 - 0.4302 \ln \beta)$$

$$F_i = 0.0887\beta (-0.0758 A - 0.2391 C) \gamma (0.14106 A + 0.4341 C + 0.1543 C^2) \tau -0.1771 A$$

$$F_s = 0.0887(a/T)^p (3c/d)^r$$

$$p = 1.8586 + 2.2859A + 0.9035A^2 + 0.1215A^3 - 1.0918C - 0.4785C^2$$

$$r = -1.0298 - 0.3040A^2 + 0.4834C + 0.7030C^2 + 0.1130C^3 - 0.1207A^2C$$

$$A = \ln (a/T)$$

$$C = \ln (3c/d)$$

For In-plane bending (IPB)
$$s_n = \frac{32 d M_i}{p [d^4 - (d - 2t)^4]}$$

where σ_n is the nominal stress and M_i is the brace in-plane bending moment.

K_e combines the contributions of the stress intensity factor components for modes I, II and III, i.e.

$$K_e = \left[\frac{K_I^2 + K_{II}^2 + K_{III}^2}{(1-\nu)} \right]^{1/2}$$

Limits to Stress Intensity Factor Solution:

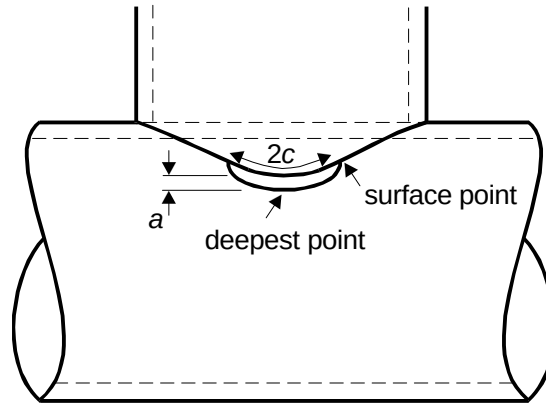
$$\begin{aligned} \alpha &= 12 \\ 0.4 &\leq \beta \leq 0.8 \\ 10 &\leq \gamma \leq 20 \\ 0.3 &\leq \tau \leq 1.0 \\ 0.05 &\leq a/T \leq 0.80 \\ 0.05 &\leq 3c/d \leq 1.20 \end{aligned}$$

Remarks: Taken from Reference AIII.10.

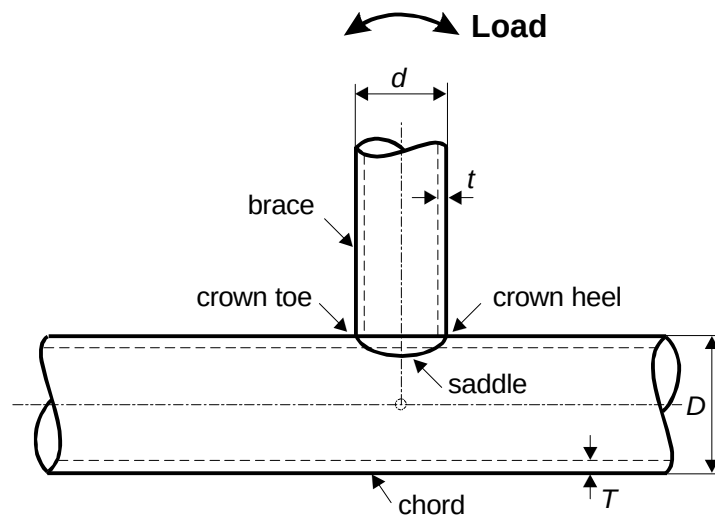
Description: Surface Crack at the Saddle Point of T-Joints
(Surface point)

Loading: In-plane bending

Schematic:



Saddle point surface crack



Notation:

a	crack depth
$2c$	surface crack length
d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
α	$2L/D$
β	d/D
γ	$D/2T$
τ	t/T
σ_n	brace nominal stress

Stress Intensity Factor Solution:

$$K_e = s_n F_g F_i F_s \sqrt{\delta a}$$

$$F_g = 0.1395\beta^{(-0.6498 - 1.1883 \ln \beta)} \gamma^{(1.0779 - 0.3414 \ln \beta)} \tau^{(0.8168 - 0.2149 \ln \beta)}$$

$$F_i = \beta^{(0.0422A - 0.2452 C)} \gamma^{(1.4558A + 0.4173 A - 0.9276C - 0.3297C^2)} \tau^{(-0.0905A - 0.0338 C)}$$

$$F_s = (a/T)^p (3c/d)^r$$

$$p = -2.4921 - 0.0063A + 0.2056A^2 + 0.9804C + 0.3916C^2 + 0.0620C^3 - 0.0110C^4$$

$$r = 2.8298 + 0.5682A^2 + 0.0704A^3 + 0.6562C - 0.0453C^2 + 0.0022C^3 \\ + (0.1621 + 0.0384C) A^2 C$$

$$A = \ln (a/T)$$

$$C = \ln (3c/d)$$

$$\text{For In-plane bending (IPB)} \quad s_n = \frac{32 d M_i}{p [d^4 - (d - 2t)^4]}$$

where σ_n is the nominal stress and M_i is the brace in-plane bending moment.

K_e combines the contributions of the stress intensity factor components for modes I, II and III, i.e.

$$K_e = \left[\frac{K_I^2 + K_{II}^2 + K_{III}^2}{(1 - \nu)} \right]^{1/2}$$

Limits to Stress Intensity Factor Solution:

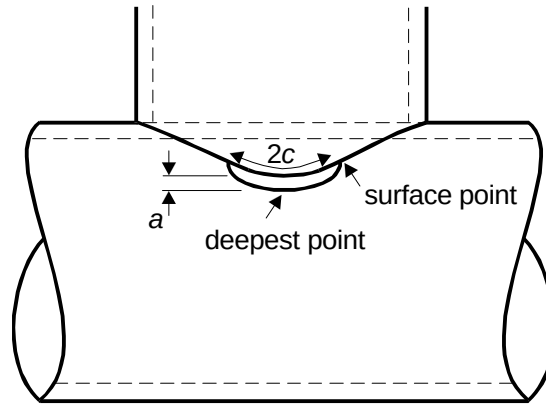
$$\begin{aligned} \alpha &= 12 \\ 0.4 &\leq \beta \leq 0.8 \\ 10 &\leq \gamma \leq 20 \\ 0.3 &\leq \tau \leq 1.0 \\ 0.05 &\leq a/T \leq 0.80 \\ 0.05 &\leq 3c/d \leq 1.20 \end{aligned}$$

Remarks: Taken from Reference AIII.10.

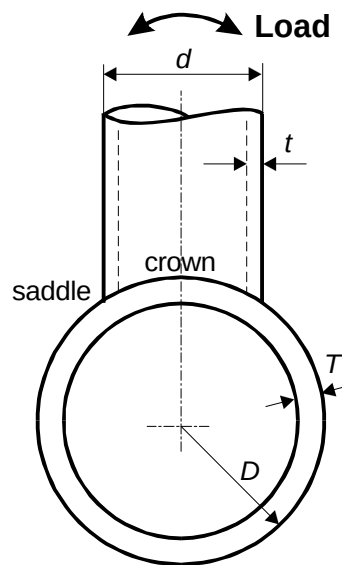
Description: Surface Crack at the Saddle Point of T-Joints
(Deepest point)

Loading: Out-of-plane bending

Schematic:



Saddle point surface crack



Notation:

a	crack depth
$2c$	surface crack length
d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
α	$2L/D$
β	d/D
γ	$D/2T$
τ	t/T
σ_n	brace nominal stress

Stress Intensity Factor Solution:

$$K_e = s_n F_g F_i F_s \sqrt{\delta a}$$

$$F_g = 0.1718 \beta (0.9626 - 0.5003 \ln \beta) \gamma^{1.5274} \tau (0.6488 + 0.3353 \ln \beta - 0.2962 \ln \tau)$$

$$F_i = b^{(0.3066 A - 0.0598 C)} \left(\frac{a}{T} \right)^{(0.1315 \ln g - 0.0775 \ln t)}$$

$$F_s = (a/T)^p (3c/d)^r$$

$$p = -1.3130 - 0.4253A - 0.0584A^2 + 0.9843C - 0.3278C^2 - 0.0308C^3$$

$$r = 0.7184 + 0.5401A^2 + 0.0889A^3 - 0.4186C - 0.0496C^2 - 0.04210A^2C$$

$$A = \ln(a/T)$$

$$C = \ln(3c/d)$$

For Out-of-plane bending (OPB)
$$s_n = \frac{32 d M_o}{p [d^4 - (d - 2t)^4]}$$

where σ_n is the nominal stress and M_o is the brace out-of-plane bending moment.

K_e combines the contributions of the stress intensity factor components for modes I, II and III, i.e.

$$K_e = \left[\frac{K_I^2 + K_{II}^2 + K_{III}^2}{(1 - \nu)} \right]^{1/2}$$

Limits to Stress Intensity Factor Solution:

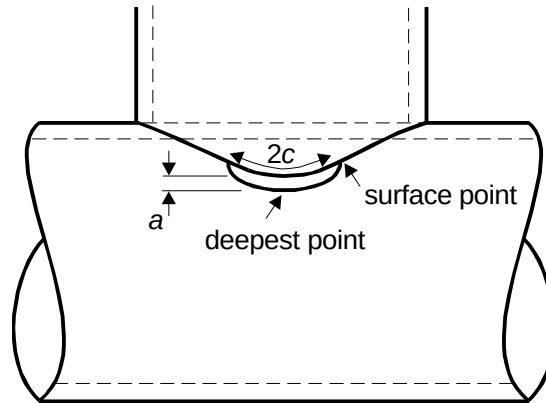
$$\begin{aligned} \alpha &= 12 \\ 0.4 &\leq \beta \leq 0.8 \\ 10 &\leq \gamma \leq 20 \\ 0.3 &\leq \tau \leq 1.0 \\ 0.05 &\leq a/T \leq 0.80 \\ 0.05 &\leq 3c/d \leq 1.20 \end{aligned}$$

Remarks: Taken from Reference AIII.10.

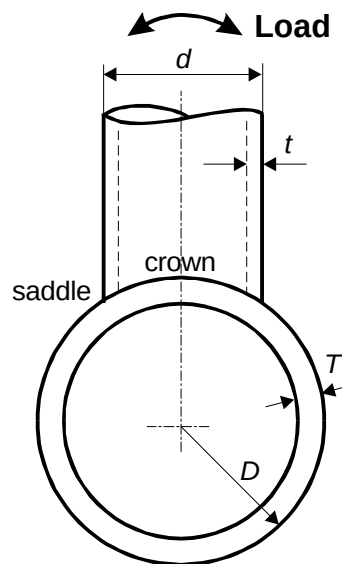
Description: Surface Crack at the Saddle Point in T-Joints
(Surface point)

Loading: Out-of-plane bending

Schematic:



Saddle point surface crack



Notation:

a	crack depth
$2c$	surface crack length
d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
α	$2L/D$
β	d/D
γ	$D/2T$
τ	t/T
σ_n	brace nominal stress

Stress Intensity Factor Solution:

$$K_e = s_n F_g F_i F_s \sqrt{\delta a}$$

$$F_g = 4.7016 b^{(0.7362 - 0.9523 \ln b)} g^{(0.2227 - 0.7169 \ln b)} t^{(0.6663 - 0.1040 \ln g - 0.3802 \ln t)}$$

$$F_i = b^{(0.1388 A - 0.2143 C)} g^{(0.0573 A - 0.5026 C - 0.1175 C^2)} t^{-0.1548 A}$$

$$F_s = (a/T)^p (3c/d)^r$$

$$p = 1.5044 + 0.8350A + 0.1258A^2 + 0.6624C - 0.0202C^2$$

$$r = 0.2954 + 0.3328A^2 + 0.0453A^3 - 0.6990C - 0.3648C^2 - 0.0473C^3$$

$$A = \ln(a/T)$$

$$C = \ln(3c/d)$$

For Out-of-plane bending (OPB)
$$s_n = \frac{32 d M_o}{p [d^4 - (d - 2t)^4]}$$

where σ_n is the nominal stress and M_o is the brace out-of-plane bending moment.

K_e combines the contributions of the stress intensity factor components for modes I, II and III, i.e.

$$K_e = \left[\frac{K_I^2 + K_{II}^2 + K_{III}^2}{(1 - \nu)} \right]^{1/2}$$

Limits to Stress Intensity Factor Solution:

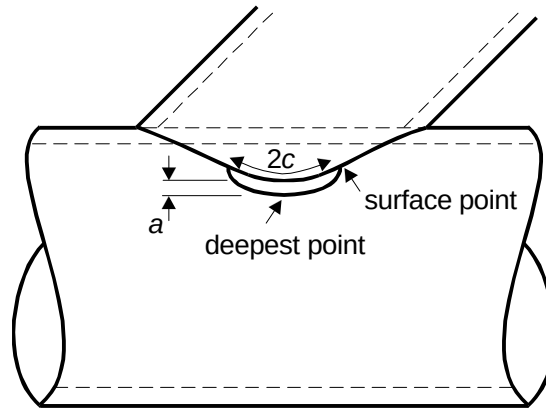
$$\begin{aligned} \alpha &= 12 \\ 0.4 &\leq \beta \leq 0.8 \\ 10 &\leq \gamma \leq 20 \\ 0.3 &\leq \tau \leq 1.0 \\ 0.05 &\leq a/T \leq 0.80 \\ 0.05 &\leq 3c/d \leq 1.20 \end{aligned}$$

Remarks: Taken from Reference AIII.10.

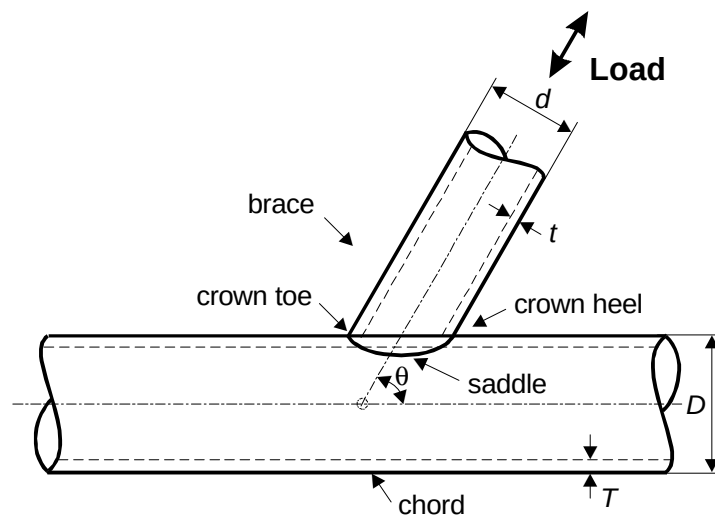
Description: Surface Crack at the Saddle Point of Y-Joints
(Deepest point)

Loading: Axial

Schematic:



Saddle point surface crack



Notation:

a	crack depth
$2c$	surface crack length
d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
α	$2L/D$
β	d/D
γ	$D/2T$
τ	t/T
θ	Angle between chord and brace
σ_n	brace nominal stress

Stress Intensity Factor Solution:

The mode I stress intensity factor is:

$$K_I = Y s_n \sqrt{pa}$$

where

$$\frac{Y}{k_{t,HS}} = A - B \left(\frac{a}{T} \right)$$

a/c	A	B
0.10	1.22	0.69
0.20	1.07	0.84
0.30	0.96	0.83
0.40	0.87	0.81

and $k_{t,HS}$ is the stress concentration factor at the hot spot, which can be obtained from [AIII.12].

For Axial Tension (AT)
$$s_n = \frac{4P}{p[d^2 - (d - 2t)^2]}$$

where σ_n is the nominal stress in the brace, and P is the applied load in the brace.

Limits to Stress Intensity Factor Solution:

$$\begin{aligned} \theta &= 60^\circ \\ \alpha &= 12 \\ 0.6 &\leq \beta \leq 0.8 \\ 10 &\leq \gamma \leq 35 \\ 0.2 &\leq \tau \leq 1.0 \\ 0.1 &\leq a/T \leq 0.8 \\ 0.1 &\leq a/c \leq 0.4 \end{aligned}$$

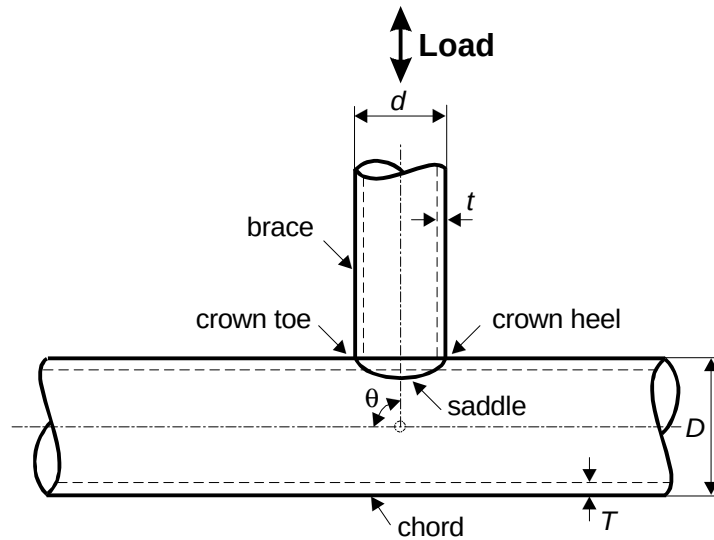
Remarks: Taken from Reference AIII.11.

AIII.6 LIMIT LOAD SOLUTIONS

Description: T- and Y-Joints

Loading: Axial

Schematic:



Notation:

d	Brace diameter
D	Chord diameter
L	Chord length
t	Brace thickness
T	Chord thickness
β	d/D
γ	$D/2T$
τ	t/T
θ	Angle between brace and chord

Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$P_k = Q_u Q_f \frac{s_y T^2 K_a}{\sin q}$$

where

- P_k = characteristic strength for brace axial load
- σ_y = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

$$K_a = \frac{\left(1 + \frac{1}{\sin q}\right)}{2}$$

Q_f = is a factor to allow for the presence of axial and moment loads in the chord. Q_f is defined as:

$$\begin{aligned} Q_f &= 1.0 - 1.638 \lambda \gamma U^2 && \text{for extreme conditions} \\ &= 1.0 - 2.890 \lambda \gamma U^2 && \text{for operating conditions} \end{aligned}$$

where

$$\begin{aligned} \lambda &= 0.030 \text{ for brace axial load} \\ &= 0.045 \text{ for brace in-plane moment load} \\ &= 0.021 \text{ for brace out-of-plane moment load} \end{aligned}$$

$$\text{and } U = \frac{\sqrt{(0.23PD)^2 + M_i^2 + M_o^2}}{0.72D^2T \phi_y}$$

with all forces (P , M_i , M_o) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$$\text{chord axial tension force} \geq \frac{1}{0.23D} (M_i^2 + M_o^2)^{0.5}$$

with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$$\begin{aligned} Q_u &= (2 + 20\beta) \sqrt{Q_\beta} && \text{(for Axial Compression)} \\ Q_u &= (8 + 22\beta) && \text{(for Axial Tension)} \end{aligned}$$

Q_β = is the geometric modifier defined as follows

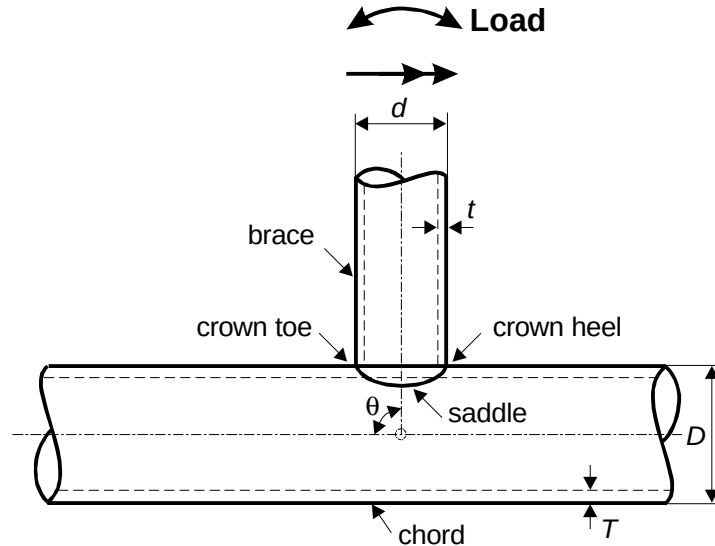
$$\begin{aligned} Q_b &= 1.0 && \text{for } b \leq 0.6 \\ &= \frac{0.3}{b(1 - 0.833b)} && \text{for } b > 0.6 \end{aligned}$$

Remarks: Taken from Reference AIII.6.

Description: T- and Y-Joints

Loading: In-plane and out-of-plane bending

Schematic:



Notation:

d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
β	d/D
γ	$D/2T$
τ	t/T
θ	Angle between brace and chord

Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$M_{ki} = M_{ko} = Q_u Q_f \frac{s_y T^2 d}{\sin \epsilon}$$

where

M_{ki} = characteristic strength for brace in-plane moment load

M_{ko} = characteristic strength for brace out-of-plane moment load

σ_y = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

Q_f = is a factor to allow for the presence of axial and moment loads in the chord. Q_f is defined as:

$$\begin{aligned} Q_f &= 1.0 - 1.638 \lambda \gamma U^2 && \text{for extreme conditions} \\ &= 1.0 - 2.890 \lambda \gamma U^2 && \text{for operating conditions} \end{aligned}$$

where

$$\begin{aligned} \lambda &= 0.030 \text{ for brace axial load} \\ &= 0.045 \text{ for brace in-plane moment load} \\ &= 0.021 \text{ for brace out-of-plane moment load} \end{aligned}$$

and

$$U = \frac{\sqrt{(0.23PD)^2 + M_i^2 + M_o^2}}{0.72D^2 T s_y}$$

with all forces (P, M_i, M_o) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$$\text{chord axial tension force} \geq \frac{1}{0.23D} (M_i^2 + M_o^2)^{0.5}$$

with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$$Q_u = 5 \beta \gamma^{0.5} \sin \theta \quad \textbf{(for In-Plane Bending)}$$

$$Q_u = (1.6 + 7 \beta) Q_\beta \quad \textbf{(for Out-of Plane Bending)}$$

Q_β = is the geometric modifier defined as follows

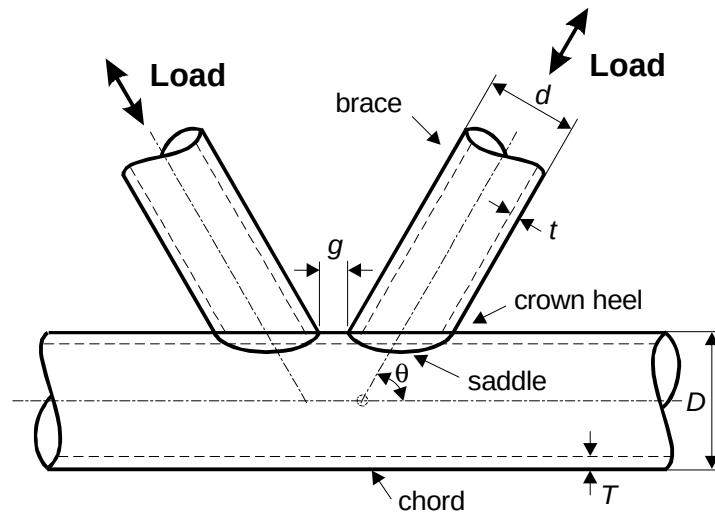
$$\begin{aligned} Q_b &= 1.0 && \text{for } b \leq 0.6 \\ &= \frac{0.3}{b(1 - 0.833b)} && \text{for } b > 0.6 \end{aligned}$$

Remarks: Taken from Reference AIII.6.

Description: K-Joints

Loading: Axial

Schematic:



Notation:

d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
β	d/D
γ	$D/2T$
τ	t/T
ζ	g/d
θ	Angle between braces and chord

Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$P_k = Q_u Q_f \frac{s_y T^2 K_a}{\sin q}$$

where

- P_k = characteristic strength for brace axial load
 σ_y = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

$$K_a = \frac{\left(1 + \frac{1}{\sin q}\right)}{2}$$

Q_f = is a factor to allow for the presence of axial and moment loads in the chord.
 Q_f is defined as:

$$\begin{aligned} Q_f &= 1.0 - 1.638 \lambda \gamma U^2 && \text{for extreme conditions} \\ &= 1.0 - 2.890 \lambda \gamma U^2 && \text{for operating conditions} \end{aligned}$$

where

$$\begin{aligned} \lambda &= 0.030 \text{ for brace axial load} \\ &= 0.045 \text{ for brace in-plane moment load} \\ &= 0.021 \text{ for brace out-of-plane moment load} \end{aligned}$$

and

$$U = \frac{\sqrt{(0.23PD)^2 + M_i^2 + M_o^2}}{0.72D^2 T \sigma_y}$$

with all forces (P , M_i , M_o) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$$\text{chord axial tension force} \geq \frac{1}{0.23D} (M_i^2 + M_o^2)^{0.5}$$

with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$$\begin{aligned} Q_u &= (2 + 20\beta) Q_g \sqrt{Q_\beta} && \text{(for Axial Compression)} \\ Q_u &= (8 + 22\beta) Q_g && \text{(for Axial Tension)} \end{aligned}$$

Q_β = is the geometric modifier defined as follows

$$\begin{aligned} Q_\beta &= 1.0 && \text{for } b \leq 0.6 \\ &= \frac{0.3}{b(1 - 0.833b)} && \text{for } b > 0.6 \end{aligned}$$

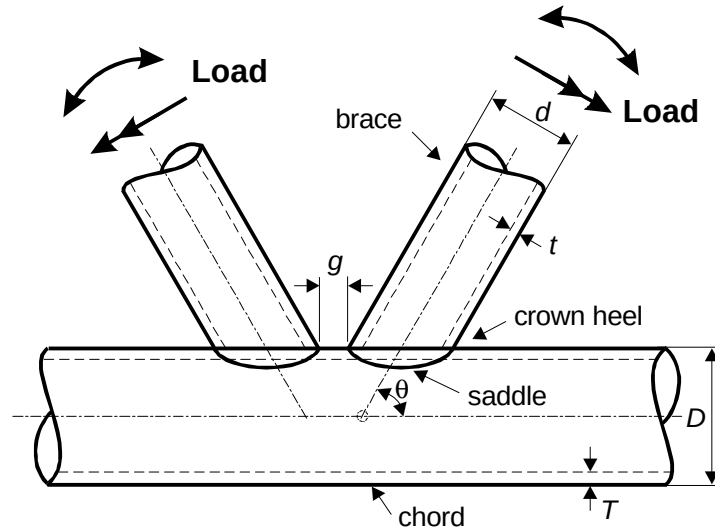
$$Q_g = 1.7 - 0.9\zeta^{0.5} \quad \text{but should not be taken as less than 1.0}$$

Remarks: Taken from Reference AIII.6.

Description: K-Joints

Loading: In-plane and out-of-plane bending

Schematic:



Notation:

d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
β	d/D
γ	$D/2T$
τ	t/T
ζ	g/d
θ	Angle between braces and chord

Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$M_{ki} = M_{ko} = Q_u Q_f \frac{s_y T^2 d}{\sin q}$$

where

M_{ki} = characteristic strength for brace in-plane moment load

M_{ko} = characteristic strength for brace out-of-plane moment load

σ_y = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

Q_f = is a factor to allow for the presence of axial and moment loads in the chord.
 Q_f is defined as:

$$Q_f = 1.0 - 1.638 \lambda \gamma U^2 \text{ for extreme conditions}$$

$$= 1.0 - 2.890 \lambda \gamma U^2 \text{ for operating conditions}$$

where

$$\lambda = 0.030 \text{ for brace axial load}$$

$$= 0.045 \text{ for brace in-plane moment load}$$

$$= 0.021 \text{ for brace out-of-plane moment load}$$

and

$$U = \frac{\sqrt{(0.23PD)^2 + M_i^2 + M_o^2}}{0.72D^2 T s_y}$$

with all forces (P , M_i , M_o) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$$\text{chord axial tension force} \geq \frac{1}{0.23D} (M_i^2 + M_o^2)^{0.5}$$

with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$$Q_u = 5\beta \gamma^{0.5} \sin \theta \quad \textbf{(for In-Plane Bending)}$$

$$Q_u = (1.6 + 7\beta) Q_\beta \quad \textbf{(for Out-of Plane Bending)}$$

Q_β = is the geometric modifier defined as follows

$$Q_\beta = 1.0 \quad \text{for } b \leq 0.6$$

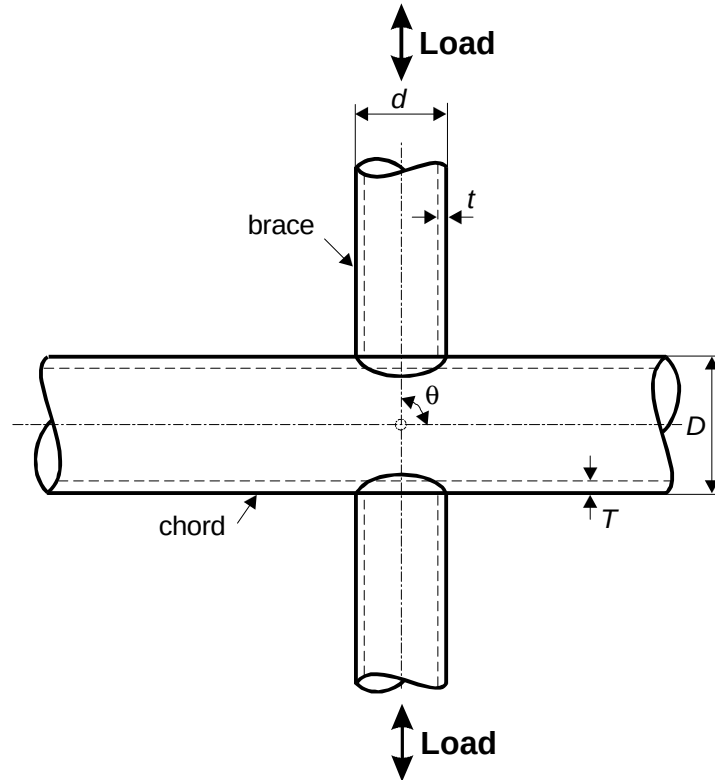
$$= \frac{0.3}{b(1 - 0.833b)} \quad \text{for } b > 0.6$$

Remarks: Taken from Reference AIII.6.

Description: X- and DT-Joints

Loading: Axial

Schematic:



Notation:

d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
β	d/D
γ	$D/2T$
τ	t/T
θ	Angle between braces and chord

Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$P_k = Q_u Q_f \frac{s_y T^2 K_a}{\text{Sin} q}$$

where

P_k = characteristic strength for brace axial load
 σ_y = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

$$K_a = \frac{\left(1 + \frac{1}{\sin q}\right)}{2}$$

Q_f = is a factor to allow for the presence of axial and moment loads in the chord.
 Q_f is defined as:

$$Q_f = 1.0 - 1.638 \lambda \gamma U^2 \text{ for extreme conditions}$$

$$= 1.0 - 2.890 \lambda \gamma U^2 \text{ for operating conditions}$$

where

$$\lambda = 0.030 \text{ for brace axial load}$$

$$= 0.045 \text{ for brace in-plane moment load}$$

$$= 0.021 \text{ for brace out-of-plane moment load}$$

and

$$U = \frac{\sqrt{(0.23PD)^2 + M_i^2 + M_o^2}}{0.72D^2 T \sigma_y}$$

with all forces (P, M_i , M_o) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$$\text{chord axial tension force} \geq \frac{1}{0.23D} (M_i^2 + M_o^2)^{0.5}$$

with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$$Q_u = (2.5 + 14\beta) Q_\beta \quad \textbf{(for Axial Compression)}$$

$$Q_u = (7 + 17\beta) Q_\beta \quad \textbf{(for Axial Tension)}$$

Q_β = is the geometric modifier defined as follows

$$Q_\beta = 1.0 \quad \text{for } b \leq 0.6$$

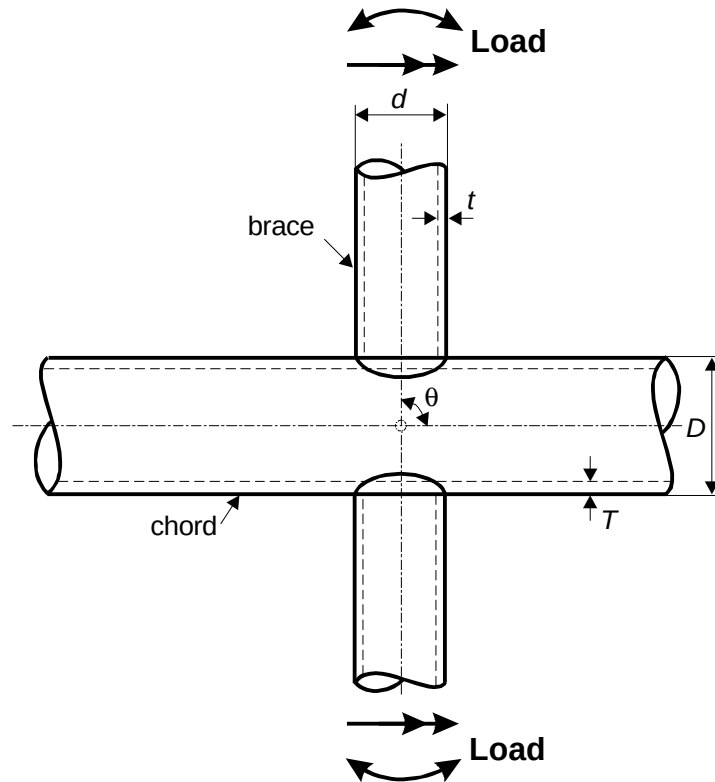
$$= \frac{0.3}{b(1 - 0.833b)} \quad \text{for } b > 0.6$$

Remarks: Taken from Reference AIII.6.

Description: X- and DT-Joints

Loading: In-plane and out-of-plane bending

Schematic:



Notation:

d	brace diameter
D	Chord diameter
L	Chord length
t	brace thickness
T	Chord thickness
β	d/D
γ	$D/2T$
τ	t/T
θ	Angle between braces and chord

Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$M_{ki} = M_{ko} = Q_u Q_f \frac{s_y T^2 d}{\sin q}$$

where

M_{ki} = characteristic strength for brace in-plane moment load

M_{ko} = characteristic strength for brace out-of-plane moment load
 σ_y = characteristic yield stress of the chord member at the joint (or .7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

Q_f = is a factor to allow for the presence of axial and moment loads in the chord.
 Q_f is defined as:

$$Q_f = 1.0 - 1.638 \lambda \gamma U^2 \text{ for extreme conditions}$$

$$= 1.0 - 2.890 \lambda \gamma U^2 \text{ for operating conditions}$$

where

$$\lambda = 0.030 \text{ for brace axial load}$$

$$= 0.045 \text{ for brace in-plane moment load}$$

$$= 0.021 \text{ for brace out-of-plane moment load}$$

and

$$U = \frac{\sqrt{(0.23PD)^2 + M_i^2 + M_o^2}}{0.72D^2T\sigma_y}$$

with all forces (P , M_i , M_o) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$$\text{chord axial tension force} \geq \frac{1}{0.23D} (M_i^2 + M_o^2)^{0.5}$$

with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$$Q_u = 5\beta \gamma^{0.5} \sin \theta \quad \text{(for In-Plane Bending)}$$

$$Q_u = (1.6 + 7\beta) \sqrt{Q_\beta} \quad \text{(for Out-of Plane Bending)}$$

Q_β = is the geometric modifier defined as follows

$$Q_b = 1.0 \quad \text{for } b \leq 0.6$$

$$= \frac{0.3}{b(1 - 0.833b)} \quad \text{for } b > 0.6$$

Remarks: Taken from Reference AIII.6.

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**APPENDIX IV
LIMIT LOAD SOLUTIONS FOR MATERIAL MISMATCH**

CONTENTS

AIV.1 INTRODUCTION

AIV.2 METHODOLOGY USED IN COLLATING THE SOLUTIONS

AIV.3 FURTHER RECOMMENDATION

AIV.4. LIMIT LOAD SOLUTIONS

AIV.5 REFERENCES

AIV.1 INTRODUCTION

Unlike homogeneous plates, welded plates exhibit various patterns of plasticity development, which are due to the presence of material mismatch. The occurrence of the various plasticity patterns depends on the following:

1. the strength mismatch factor or the mismatch ratio M , which is the ratio of the yield strength of the weld metal to that of the base material
2. the geometrical parameters such as (W) half the plate width, (a) half the crack size and (h) half the weld width.

Such plasticity development patterns play an important role in determining the mismatch limit load. Fig. IV-1 depicts possible patterns of plasticity development for the mismatched plate with a crack in the centre line of the weld metal. For other cases such as bimaterial joints with an interface crack between weld metal and base plate, there are similar patterns of plasticity development.

For undermatching, plastic deformations may either be confined to the weld metal (Fig. IV-1.a) or penetrate to the base plate (Fig. IV-1.b). Solutions have to be derived for both cases and the lower of the two should be adopted as the limit load. For overmatching, plastic deformations may either spread to the base plate (Fig. IV-1.c) or be confined to the base plate (Fig. IV-1.d). Again solutions have to be derived for both cases and the lower of the two should be adopted as the limit load.

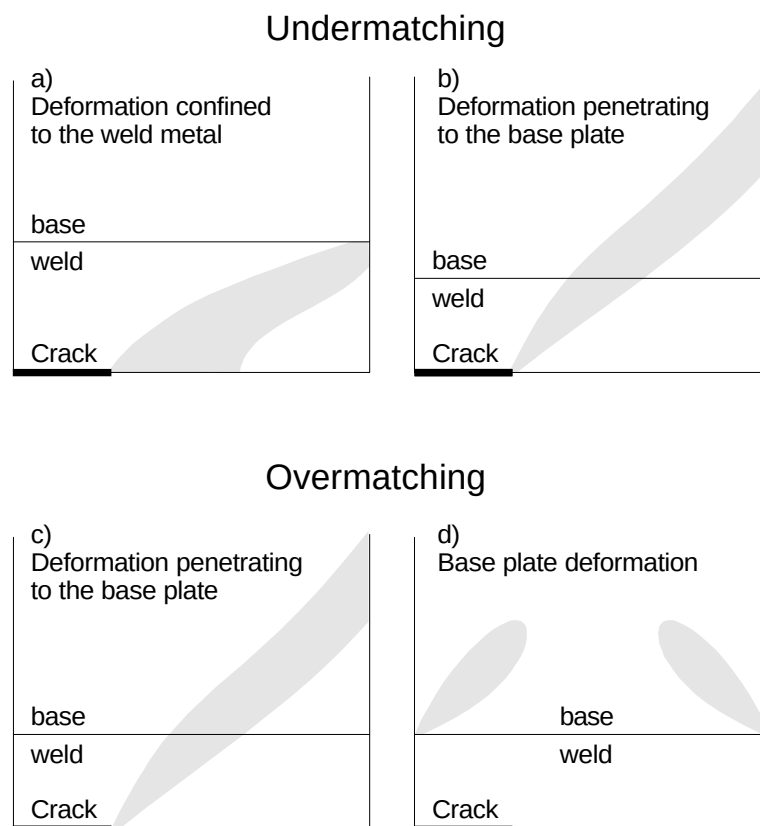


Fig. IV-1: Classification of plasticity deformation patterns for mismatched plates.

AIV.2 METHODOLOGY USED IN COLLATING THE SOLUTIONS

As with homogeneous components, the limit load may be evaluated using a number of approaches: plastic limit analysis, non-linear finite element analysis or scaled model tests. The methods that have been used for mismatched components are mainly plastic limit analysis and finite element analysis. These solutions have been fitted by equations for ease of application. It should be noted that all solutions presented in this appendix were taken from Reference [IV.1].

AIV.3 FURTHER RECOMMENDATION

At present, limit load solutions for mismatched components are limited to simple geometries. Thus the mismatch limit load solutions for more complex geometries are subject to further development. Pending such solutions, when solutions are not available for the particular geometry of interest, the mismatch effect on the limit load could be roughly estimated from the existing solutions listed in this Appendix. For instance, for the HAZ crack in overmatched plates, the existing solutions indicate that the limit load solution based on all base plate would be sufficient for all cases.

AIV.4. LIMIT LOAD SOLUTIONS

DESCRIPTION: CENTRE CRACKED PLATES IN TENSION

Schematic:

Fig. IV-2.a
Crack in the centre line
of the weld material

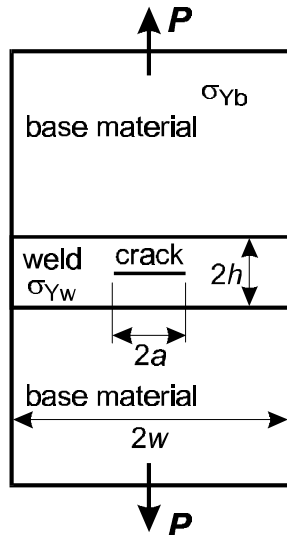


Fig. IV-2.b
Crack in the interface between
weld metal and base plate

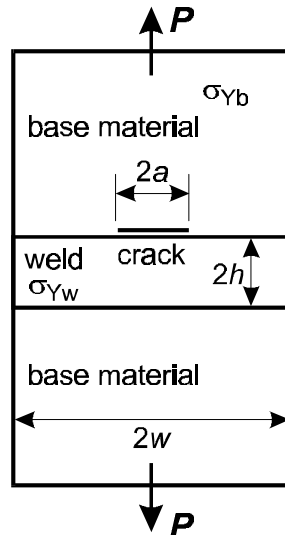
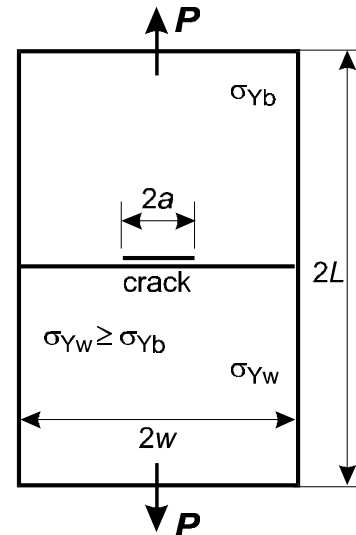


Fig. IV-2.c
Crack in the interface
of a bimaterial joint



Notation:

$2a$	total defect length
B	thickness of plate
$2h$	total width of weld
$2L$	total length of plate
M	$=\sigma_{Yw}/\sigma_{Yb}$, strength mismatch factor
P	total applied end load
$2W$	total width of plate
σ_{Yb}	yield strength of the base plate
σ_{Yw}	yield strength of the weld metal
y	$=(W-a)/h$

Solution: (crack in the centre line of the weld metal, Fig. IV-2.a)

(i) Plane Stress

The limit load for the plate made wholly of material *b* is

$$P_{Lb} = 2s_{yb} \cdot B \cdot (W - a)$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq 1.43 \\ \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\} & \text{for } 1.43 \leq y \end{cases}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = M \cdot \left[\frac{2}{\sqrt{3}} - \left(\frac{2 - \sqrt{3}}{\sqrt{3}} \right) \frac{1.43}{y} \right]$$

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = 1 - (1 - M) \cdot \frac{1.43}{y}$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(3)}}{P_{Lb}}, \frac{1}{1 - a/w} \right\}$$

$$\frac{P_{Lmis}^{(3)}}{P_{Lb}} = \begin{cases} M & \text{for } y \leq y_1 = (1 + 0.43e^{-5(M-1)}) \cdot e^{-(M-1)/5} \\ \frac{24(M-1)}{25} \cdot \frac{y_1}{y} + \frac{M+24}{25} & \text{for } y \geq y_1 = (1 + 0.43e^{-5(M-1)}) \cdot e^{-(M-1)/5} \end{cases}$$

(ii) Plane Strain

The limit load for the plate made wholly of material *b* is

$$P_{Lb} = \frac{4}{\sqrt{3}} s_{yb} \cdot B \cdot (W - a)$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq 1 \\ \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\} & \text{for } 1 \leq y \end{cases}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = 1 - (1 - M) \cdot \frac{1}{y}$$

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = \begin{cases} M \cdot \left[1.0 + 0.462 \frac{(y-1)^2}{y} - 0.044 \frac{(y-1)^3}{y} \right] & \text{for } 1 \leq y \leq 3.6 \\ M \cdot \left[2.571 - \frac{3.254}{y} \right] & \text{for } 3.6 \leq y \leq 5.0 \\ M \cdot \left[0.125y + 1.291 + \frac{0.019}{y} \right] & \text{for } 5.0 \leq y \end{cases}$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(3)}}{P_{Lb}}, \frac{1}{1 - a/w} \right\}$$

$$\frac{P_{Lmis}^{(3)}}{P_{Lb}} = \begin{cases} \frac{M}{25} & \text{for } y \leq y_1 = e^{-(M-1)/5} \\ \frac{24(M-1)}{25} \cdot \frac{y_1}{y} + \frac{M+24}{25} & \text{for } y \geq y_1 = e^{-(M-1)/5} \end{cases}$$

Solution (crack in the interface between weld metal and base plate, Fig. IV-2.b)**(i) Plane Stress**

The limit load for the plate made wholly of material b is

$$P_{Lb} = 2s_{yb} \cdot B \cdot (W - a)$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = M \cdot [1.095 - 0.095 \cdot \exp[-(1 - M)/0.108M]]$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{1}{(1 - a/w)} \right\}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = 1.095 - 0.095 \cdot \exp[-(M - 1)/0.108]$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$P_{Lb} = \frac{4}{\sqrt{3}} s_{yb} \cdot B \cdot (W - a)$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = \begin{cases} f & \text{for } 0 \leq y \leq y_1 = 2[1 - (2 - \sqrt{2}) \cdot (1 - M)] \\ 1 - (1 - f) \cdot \frac{y_1}{y} & \text{for } y \geq y_1 = 2[1 - (2 - \sqrt{2}) \cdot (1 - M)] \end{cases}$$

$$f = \begin{cases} M \cdot \left[1 + 0.52 \left(\frac{1 - M}{M} \right) - 0.22 \left(\frac{1 - M}{M} \right)^2 \right] & \text{for } 0.5 \leq M \leq 1 \\ 1.30 \cdot M & \text{for } M \leq 0.5 \end{cases}$$

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = \begin{cases} \left[1.30 + 0.394 \frac{1.30 \cdot M}{y} - 0.027 \frac{(y - \sqrt{2})^3}{y} \right] & \text{for } 0 \leq y \leq \sqrt{2} \\ M \cdot \left[2.881 - \frac{4.123}{y} \right] & \text{for } \sqrt{2} \leq y \leq 4.2 \\ M \cdot \left[0.125y + 1.294 + \frac{0.909}{y} \right] & \text{for } 4.2 \leq y \leq 6.2 \\ & \text{for } 6.2 \leq y \end{cases}$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(3)}}{P_{Lb}}, \frac{1}{(1 - a/w)} \right\}$$

$$\frac{P_{Lmis}^{(3)}}{P_{Lb}} = \begin{cases} \left(f - \frac{M + 24}{25} \right) \cdot \exp \left[-\frac{y - \sqrt{2}}{4M - 1} \right] + \frac{M + 24}{25} & \text{for } 0 \leq y \leq \sqrt{2} \\ & \text{for } \sqrt{2} \leq y \end{cases}$$

$$f = \begin{cases} 1 + 0.52(M - 1) - 0.22(M - 1)^2 & \text{for } 1 \leq M \leq 2 \\ 1.30 & \text{for } M \geq 2 \end{cases}$$

Solution (crack in the interface of a bimaterial joint, Fig. IV-2.c)

(i) Plane Stress

The limit load for the plate made wholly of material b is

$$P_{Lb} = 2s_{yb} \cdot B \cdot (W - a)$$

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{1}{(1 - a/w)} \right\}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = 1.095 - 0.095 \cdot \exp \left[-\frac{(M - 1)}{0.108} \right]$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$P_{Lb} = \frac{4}{\sqrt{3}} s_{yb} \cdot B \cdot (W - a)$$

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{1}{(1 - a/w)} \right\}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = \begin{cases} 1 + 0.52(M - 1) - 0.22(M - 1)^2 & \text{for } 1 \leq M \leq 2 \\ 1.30 & \text{for } M > 2 \end{cases}$$

DESCRIPTION: DOUBLE EDGE NOTCHED PLATE IN TENSION

Schematic:

Fig. IV-3.a
Crack in the centre line
of the weld material

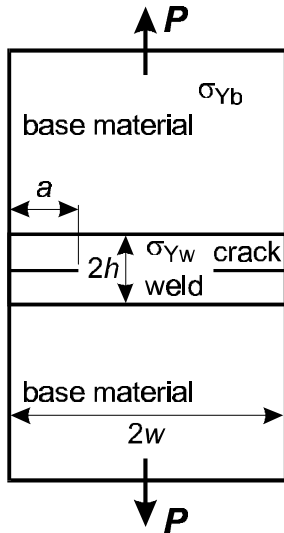


Fig. IV-3.b
Crack in the interface between
weld metal and base plate

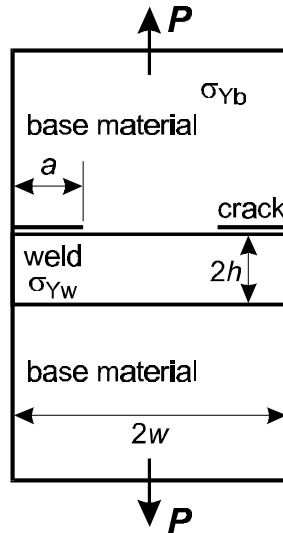
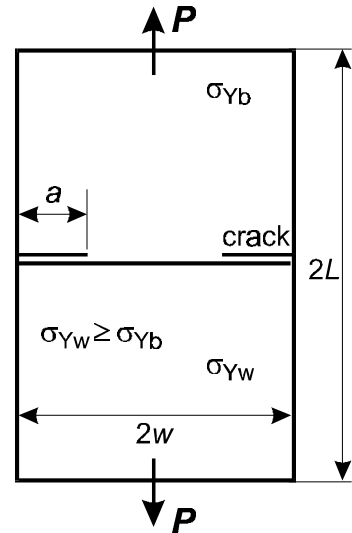


Fig. IV-3.c
Crack in the interface
of a bimaterial joint



Notation:

a	defect length
B	thickness of plate
$2h$	total width of weld
$2L$	total length of plate
M	$=\sigma_{Yw}/\sigma_{Yb}$, strength mismatch factor
P	total applied end load
$2W$	total width of plate
σ_{Yb}	yield strength of the base plate
σ_{Yw}	yield strength of the weld metal
y	$=(W-a)/h$

Solution: (crack in the centre line of the weld metal, Fig. IV-3.a)

(i) Plane Stress

The limit load for the plate made wholly of material b is

$$P_{Lb} = b \cdot 2s_{yb} \cdot B \cdot (W - a) ; \quad b = \begin{cases} 1 + 0.54 \left(\frac{a}{w} \right) & \text{for } 0 < \frac{a}{w} \leq 0.286 \\ \frac{2}{\sqrt{3}} & \text{for } 0.286 < \frac{a}{w} < 1 \end{cases}$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = M \quad \text{for all } y$$

Overmatching ($M > 1$),

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{1}{b(1 - a/w)} \right\}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq y_1 = e^{-2(M-1)/5} \\ \frac{M + 24}{25} + \left[\frac{24(M-1)}{25} + 0.1(M-1) \right] \cdot \frac{y_1}{y} - 0.1(M-1) \cdot \left(\frac{y_1}{y} \right)^M & \text{for } y \geq y_1 = e^{-2(M-1)/5} \end{cases}$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$P_{Lb} = b \cdot \frac{4}{\sqrt{3}} s_{yb} \cdot B \cdot (W - a) ; \quad b = \begin{cases} 1 + \ln \left(\frac{2w - a}{2(w - a)} \right) & \text{for } 0 < \frac{a}{w} \leq 0.884 \\ 1 + \frac{p}{2} & \text{for } 0.884 < \frac{a}{w} < 1 \end{cases}$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq 0.5 \\ \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\} & \text{for } 0.5 \leq y \end{cases}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = 1 - (1 - M) \cdot \frac{0.5}{y}$$

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = \begin{cases} M \cdot \left[b + A \cdot (y - 0.5) + B \cdot (y - 0.5)^2 \right] / b & \text{for } 0.5 \leq y \leq y_o \\ M \cdot (0.25y + 2.2172) / b & \text{for } y \geq y_o \end{cases}$$

$$A = \begin{cases} 0.25 - \frac{(b - 2.3422)}{(y_o - 0.5)} & \text{for } 0 < \frac{a}{w} < 0.35 \\ 0.25 - \frac{2(b - 2.3422)}{(y_o - 0.5)} & \text{for } \frac{a}{w} > 0.35 \end{cases}$$

$$B = \begin{cases} 0 & \text{for } 0 < \frac{a}{w} < 0.35 \\ \frac{b - 2.3422}{(y_o - 0.5)^2} & \text{for } \frac{a}{w} > 0.35 \end{cases}$$

$$y_o = 16.3 - 35.2(a/w) + 19.9(a/w)^2$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(3)}}{P_{Lb}}, \frac{1}{b(1 - a/w)} \right\}$$

$$\frac{P_{Lmis}^{(3)}}{P_{Lb}} = \begin{cases} \frac{M}{50} & \text{for } y \leq y_1 = 0.3e^{-(M-1)/0.5} + 0.2 \\ \frac{49(M-1)}{50} \cdot \frac{y_1}{y} + \frac{M+49}{50} & \text{for } y \geq y_1 = 0.3e^{-(M-1)/0.5} + 0.2 \end{cases}$$

Solution: (crack in the interface between weld metal and base plate, Fig. IV-3.b)

(i) Plane Stress

The limit load for the plate made wholly of material b is

$$P_{Lb} = b \cdot 2s_{yb} \cdot B \cdot (W - a) ; \quad b = \begin{cases} 1 + 0.54 \left(\frac{a}{w} \right) & \text{for } 0 < \frac{a}{w} \leq 0.286 \\ \frac{2}{\sqrt{3}} & \text{for } 0.286 < \frac{a}{w} < 1 \end{cases}$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = M \quad \text{for all } y$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = 1 \quad \text{for all } y$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$P_{Lb} = b \cdot \frac{4}{\sqrt{3}} s_{yb} \cdot B \cdot (W - a) ; \quad b = \begin{cases} 1 + \ln \left(\frac{2w - a}{2(w - a)} \right) & \text{for } 0 < \frac{a}{w} \leq 0.884 \\ 1 + \frac{p}{2} & \text{for } 0.884 < \frac{a}{w} < 1 \end{cases}$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq 1 \\ \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\} & \text{for } 1 \leq y \end{cases}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = 1 - (1 - M) \cdot \frac{1}{y}$$

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = \begin{cases} M \cdot \left[b + A \cdot (y - 1) + B \cdot (y - 1)^2 \right] / b & \text{for } 1 \leq y \leq y_o \\ M \cdot (0.125y + 2.2172) / b & \text{for } y \geq y_o \end{cases}$$

$$A = \begin{cases} 0.125 - \frac{(b - 2.3422)}{(y_o - 1)} & \text{for } 0 < \frac{a}{w} < 0.35 \\ 0.125 - \frac{2(b - 2.3422)}{(y_o - 1)} & \text{for } \frac{a}{w} > 0.35 \end{cases}$$

$$B = \begin{cases} 0 & \text{for } 0 < \frac{a}{w} < 0.35 \\ \frac{b - 2.3422}{(y_o - 1)^2} & \text{for } \frac{a}{w} > 0.35 \end{cases}$$

$$y_o = 32.6 - 70.4(a/w) + 39.8(a/w)^2$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = 1 \quad \text{for ally}$$

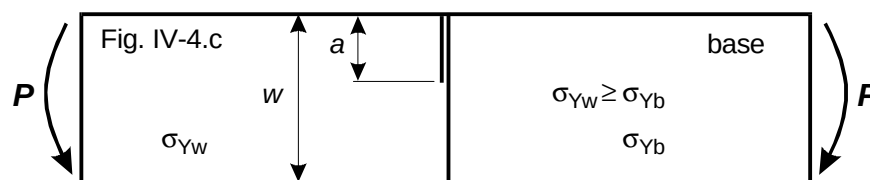
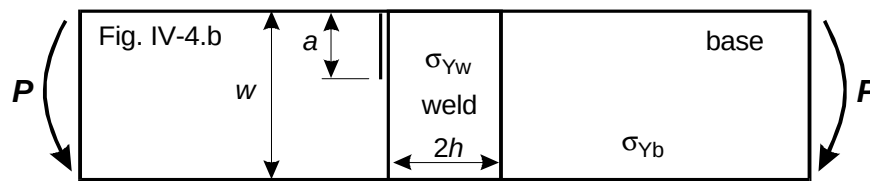
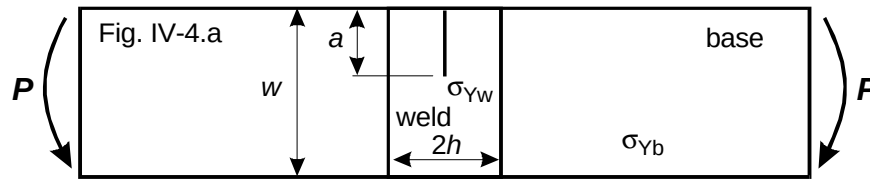
Solution: (crack in the interface of a bimaterial joint, Fig. IV-3.c)

(i) Plane Stress

$$P_{Lmis} = b \cdot 2s_{yb} \cdot B \cdot (W - a) ; \quad b = \begin{cases} 1 + 0.54 \frac{a}{w} & \text{for } 0 < \frac{a}{w} \leq 0.286 \\ \frac{2}{\sqrt{3}} & \text{for } 0.286 < \frac{a}{w} < 1 \end{cases}$$

(ii) Plane Strain

$$P_{Lmis} = b \cdot \frac{4}{\sqrt{3}} s_{yb} \cdot B \cdot (W - a) ; \quad b = \begin{cases} 1 + \ln \left(\frac{2w - a}{2(w - a)} \right) & \text{for } 0 < \frac{a}{w} \leq 0.884 \\ 1 + \frac{p}{2} & \text{for } 0.884 < \frac{a}{w} < 1 \end{cases}$$

DESCRIPTION: SINGLE EDGE NOTCHED PLATES IN PURE BENDING**Schematic:****Notation:**

a	total defect length
B	thickness of plate
$2h$	total width of weld
M	$=\sigma_{Yw}/\sigma_{Yb}$, strength mismatch factor
P	total applied end moment
W	total width of plate
σ_{Yb}	yield strength of the base plate
σ_{Yw}	yield strength of the weld metal
y	$=(W-a)/h$

Solution: (crack in the centre line of the weld metal, Fig. IV-4.a)

(i) Plane Stress

The limit load for the plate made wholly of material b is

$$P_{Lb} = 0.4641 \cdot \frac{S_{yb}}{\sqrt{3}} \cdot B \cdot (W - a)^2$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = M \quad \text{for all } y$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{1}{(1 - a/w)^2} \right\}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq y_1 \\ \left(\frac{M + 49}{50} \right) + \left[\frac{49(M - 1)}{50} + 1 - \sqrt{M - 1} \right] \cdot \frac{y_1}{y} + (1 + \sqrt{M - 1}) \cdot \left(\frac{y_1}{y} \right)^M & \text{for } y \geq y_1 \end{cases}$$

$$y_1 = (2.0 + 0.7e^{-(M-1)}) \cdot e^{-(M-1)/8}$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$P_{Lb} = b \cdot \frac{S_{yb}}{\sqrt{3}} \cdot B \cdot (W - a)^2 ; b = \begin{cases} 0.50 + 0.808 \left(\frac{a}{w} \right) - 1.245 \left(\frac{a}{w} \right)^2 & \text{for } 0 < \frac{a}{w} \leq 0.3 \\ 0.631 & \text{for } 0.3 \leq \frac{a}{w} < 1 \end{cases}$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq 2.0 \\ \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\} & \text{for } 2.0 \leq y \end{cases}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = \left[\frac{9(M - 1)}{10} \right] \cdot \exp \left[-\frac{1}{20(1 - M)} \cdot (y - 2) \right] + \frac{M + 9}{10}$$

For $0 < \frac{a}{w} \leq 0.3$,

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = \begin{cases} M \cdot \left[1 + \frac{(-3b + 5.4)}{1.69b} \cdot \left(\frac{y}{10} - 0.2 \right)^2 - \frac{(2b - 3.33)}{2.2b} \cdot \left(\frac{y}{10} - 0.2 \right)^3 \right] & \text{for } 2.0 \leq y \leq 15.0 \\ M \cdot \left[1.1345 + 0.623 \frac{y}{10} \right] / b & \text{for } 15.0 \leq y \end{cases}$$

For $0.3 < \frac{a}{w}$,

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = \begin{cases} M \cdot \left[1.094 - 1.017 \left(\frac{y}{10} \right) + 3.129 \left(\frac{y}{10} \right)^2 - 1.952 \left(\frac{y}{10} \right)^3 \right] & \text{for } 2.0 \leq y \leq 7.0 \\ M \cdot \left[0.900 + 0.494 \left(\frac{y}{10} \right) \right] & \text{for } 7.0 \leq y \end{cases}$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(3)}}{P_{Lb}}, \frac{1}{2b(1-a/w)^2} \right\}$$

$$\frac{P_{Lmis}^{(3)}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq y_1 \\ A + B \cdot \frac{y_1}{y} + C \cdot \left(\frac{y_1}{y} \right)^M & \text{for } y_1 \leq y \end{cases}$$

$$y_1 = \begin{cases} 2e^{-(M-1)/(10 \cdot a/w)} & \text{for } 0 < a/w \leq 0.4 \\ 2e^{-(M-1)/8} & \text{for } 0.4 < a/w \end{cases}$$

$$A = \frac{M + 49}{50}; B = \frac{49(M - 1)}{50} - C; C = 0.3(M - 1)\sqrt{M - 1}$$

Solution: (crack in the interface between weld metal and base plate, Fig. IV-4.b)

(i) Plane Stress

The limit load for the plate made wholly of material b is

$$P_{Lb} = 0.4641 \cdot \frac{S_{yb}}{\sqrt{3}} \cdot B \cdot (W - a)^2$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = M \cdot \left[1.04 - 0.04e^{-(1-M)/0.13M} \right] \quad \text{for all } y$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = -0.04e^{-(M-1)/0.13} + 1.04 \quad \text{for all } y$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$P_{Lb} = b \cdot \frac{S_{yb}}{\sqrt{3}} \cdot B \cdot (W - a)^2 ; \quad b = \begin{cases} 0.50 + 0.808\left(\frac{a}{w}\right) - 1.245\left(\frac{a}{w}\right)^2 & \text{for } 0 < \frac{a}{w} \leq 0.3 \\ 0.631 & \text{for } 0.3 \leq \frac{a}{w} < 1 \end{cases}$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq 4 \\ \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\} & \text{for } 4 \leq y \end{cases}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = A \cdot e^{-B(y-4)} + C$$

$$f = \begin{cases} M & \text{for } 0 < a/w \leq 0.3 \\ M \cdot \left[1.06 - 0.06e^{-(1-M)/0.3M} \right] & \text{for } 0.3 \leq a/w \end{cases}$$

$$A = (f - C) \cdot [1 + B \cdot (y - 4)] ; \quad B = \frac{1}{8.5\sqrt{1-M}} ; \quad C = \frac{M+9}{10}$$

For $0 < a/w \leq 0.3$,

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = \begin{cases} M \cdot \left[1 + \frac{(2b - 3.377)}{b} \cdot \left(\frac{y}{10} \right)^2 + \frac{(5.377 - 3b)}{b} \cdot \left(\frac{y}{10} \right)^3 \right] & \text{for } 4.0 \leq y \leq 14.0 \\ M \cdot \left[0.623 \left(\frac{y}{10} \right) + 1.377 \right] / b & \text{for } 14.0 \leq y \end{cases}$$

For $0.3 \leq a/w$,

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = \begin{cases} M \cdot \left[1.06 + 0.522 \left(\frac{y}{10} \right)^2 - 0.133 \left(\frac{y}{10} \right)^3 \right] & \text{for } 4.0 \leq y \leq 14.0 \\ M \cdot \left[0.494 \left(\frac{y}{10} \right) + 1.00 \right] & \text{for } 14.0 \leq y \end{cases}$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} \approx \begin{cases} 1 & \text{for } 0 < \frac{a}{w} < 0.3 \\ -0.06e^{-(M-1)/0.3} + 1.06 & \text{for } 0.3 \leq \frac{a}{w} \end{cases}$$

Solution: (crack in the interface of a bimaterial joint, Fig. IV-4.c)

(i) Plane Stress

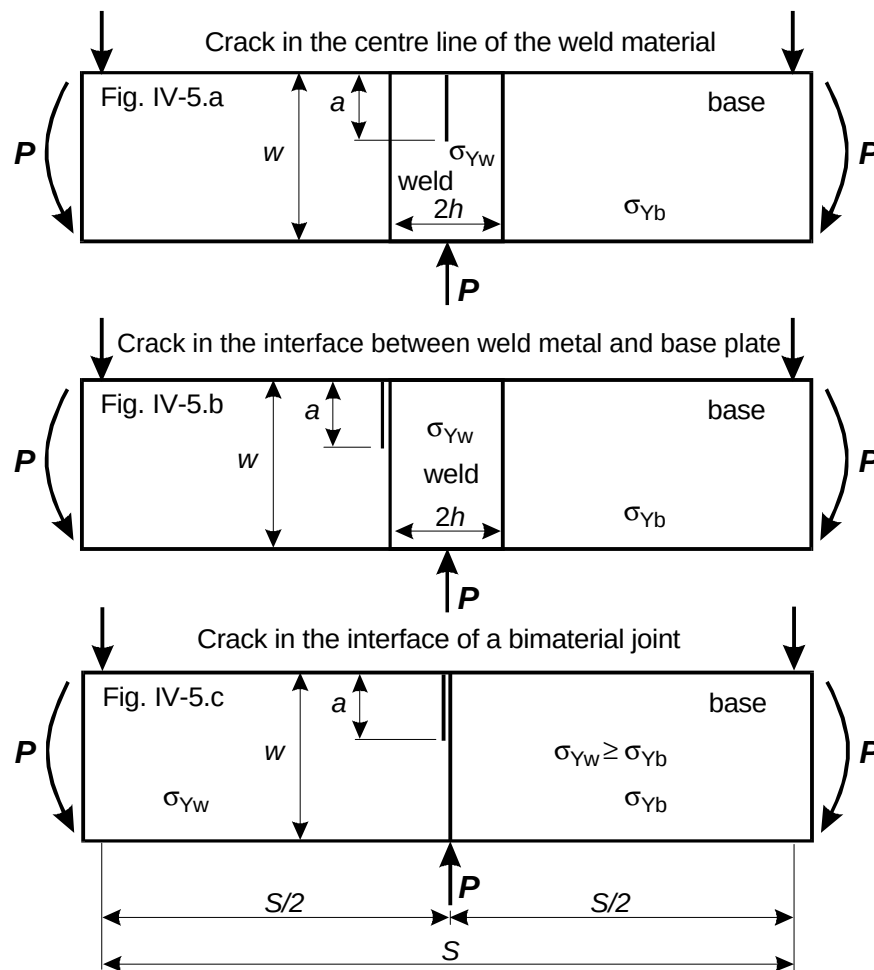
$$P_{Lmis} = 0.4641 \cdot b \cdot \frac{S_{yb}}{\sqrt{3}} \cdot B \cdot (W - a)^2 \quad ; \quad b = -0.04e^{-(M-1)/0.13} + 1.04$$

(ii) Plane Strain

$$P_{Lmis} = b \cdot \frac{S_{yb}}{\sqrt{3}} \cdot B \cdot (W - a)^2 ; b = \begin{cases} (b_1 - b_\infty) \cdot e^{-(M-1)/(a/w)} + b_\infty & \text{for } 0 < \frac{a}{w} \leq 0.3 \\ (b_1 - b_\infty) \cdot e^{-(M-1)/0.3} + b_\infty & \text{for } 0.3 < \frac{a}{w} \leq 1 \end{cases}$$

$$b_1 = \begin{cases} 0.500 + 0.808 \left(\frac{a}{w} \right) - 1.245 \left(\frac{a}{w} \right)^2 & \text{for } 0 < \frac{a}{w} \leq 0.3 \\ 0.631 & \text{for } 0.3 < \frac{a}{w} \leq 1 \end{cases}$$

$$b_\infty = \begin{cases} 0.500 + 0.890 \left(\frac{a}{w} \right) - 1.165 \left(\frac{a}{w} \right)^2 & \text{for } 0 < \frac{a}{w} \leq 0.4 \\ 0.670 & \text{for } 0.4 < \frac{a}{w} \leq 1 \end{cases}$$

DESCRIPTION: SINGLE EDGE CRACKED IN THREE POINT BENDING**Schematic:****Notation:**

a	total defect length
B	thickness of plate
$2h$	total width of weld
M	$=\sigma_{Yw}/\sigma_{Yb}$, strength mismatch factor
P	total applied load
S	total span
W	total width of plate
σ_{Yb}	yield strength of the base plate
σ_{Yw}	yield strength of the weld metal
y	$=(W-a)/h$

Solution: (crack in the centre line of the weld metal, Fig. IV-5.a)

(i) Plane Stress

The limit load for the plate made wholly of material b is

$$P_{Lb} = 0.960 \cdot \frac{s_{yb}}{\sqrt{3}} \cdot \frac{B \cdot (W - a)^2}{(S/2)}$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = M \quad \text{for all } y$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq y_1 \\ \frac{M + 49}{50} + \left[\frac{49(M - 1)}{50} - 0.2(M - 1) \right] \cdot \frac{y_1}{y} + 0.2(M - 1) \cdot \left(\frac{y_1}{y} \right)^M & \text{for } y_1 \leq y \end{cases}$$

$$y_1 = (2.5 + 0.5e^{-(M-1)}) \cdot e^{-(M-1)/4}$$

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = \frac{b_b}{0.960} \cdot \frac{1}{(1 - a/w)^2}$$

$$b_b = 4.00 - 2.60 \cdot \left(2 - \frac{H}{W} \right) + 0.54 \cdot \left(2 - \frac{H}{W} \right)^2$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$P_{Lb} = b \cdot \frac{s_{yb}}{\sqrt{3}} \cdot \frac{B(W - a)^2}{(S/2)} \quad ; \quad b = \begin{cases} 1.125 + 0.892 \left(\frac{a}{w} \right) - 2.238 \left(\frac{a}{w} \right)^2 & \text{for } 0 < \frac{a}{w} < 0.172 \\ 1.199 + 0.096 \left(\frac{a}{w} \right) & \text{for } 0.172 \leq \frac{a}{w} < 1 \end{cases}$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \begin{cases} M & \text{for } 0 < y < 2.0 \\ \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\} & \text{for } 2.0 \leq y \end{cases}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = \begin{cases} M \cdot \left[1 + \frac{(5.384 - 3b)}{b} \cdot \left(\frac{y-2}{10} \right)^2 + \frac{(2b - 3.384)}{b} \cdot \left(\frac{y-2}{10} \right)^3 \right] & \text{for } 2.0 \leq y \leq 12.0 \\ M \cdot \left(1.384 + 0.616 \cdot \left(\frac{y-2}{10} \right) \right) / b & \text{for } 12.0 \leq y \end{cases}$$

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = \frac{9(M-1)}{10} \cdot \exp \left[-\frac{(y-2)}{20(1-M)} \right] + \frac{M+9}{10}$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(3)}}{P_{Lb}}, \frac{P_{Lmis}^{(4)}}{P_{Lb}} \right\}$$

$$\frac{P_{Lmis}^{(3)}}{P_{Lb}} = \frac{M+49}{50} + \left[\frac{49(M-1)}{50} - 0.3(M-1)\sqrt{M-1} \right] \cdot \frac{y_1}{y} + 0.3(M-1)\sqrt{M-1} \cdot \left(\frac{y_1}{y} \right)^M$$

$$y_1 = \begin{cases} 2e^{-(M-1)/(4a/w)} & \text{for } 0 < \frac{a}{w} < 0.172 \\ 2e^{-(M-1)/8} & \text{for } 0.172 \leq \frac{a}{w} < 1 \end{cases}$$

$$\frac{P_{Lmis}^{(4)}}{P_{Lb}} = \frac{b_b}{b} \cdot \frac{1}{(1-a/w)^2}$$

$$b_b = 4.5557 - 3.6072 \cdot \left(2 - \frac{H}{W} \right) + 1.3095 \cdot \left(2 - \frac{H}{W} \right)^2 - 0.1818 \cdot \left(2 - \frac{H}{W} \right)^3$$

Solution: (crack in the interface between weld metal and base plate, Fig. IV-5.b)

(i) Plane Stress

The limit load for the plate made wholly of material b is

$$P_{Lb} = 0.960 \cdot \frac{s_{yb}}{\sqrt{3}} \cdot \frac{B \cdot (W - a)^2}{(S/2)}$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = M \quad \text{for all } y$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = 1 \quad \text{for all } y$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$P_{Lb} = b \cdot \frac{s_{yb}}{\sqrt{3}} \cdot \frac{B(W - a)^2}{(S/2)} \quad ; \quad b = \begin{cases} 1.125 + 0.892 \left(\frac{a}{w} \right) - 2.238 \left(\frac{a}{w} \right)^2 & \text{for } 0 < \frac{a}{w} < 0.172 \\ 1.199 + 0.096 \left(\frac{a}{w} \right) & \text{for } 0.172 \leq \frac{a}{w} < 1 \end{cases}$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \begin{cases} M & \text{for } 0 < y < 4.0 \\ \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\} & \text{for } 4.0 \leq y \end{cases}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = \begin{cases} M \cdot \left[1 + \left(\frac{9.08 - 3b}{8b} \right) \cdot \left(\frac{y - 4}{10} \right)^2 + \left(\frac{b - 2.616}{16b} \right) \cdot \left(\frac{y - 4}{10} \right)^3 \right] & \text{for } 4.0 \leq y \leq 12.0 \\ M \cdot \left(2.0 + 0.616 \cdot \left(\frac{y - 4}{10} \right) \right) / b & \text{for } 12.0 \leq y \end{cases}$$

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = \frac{9(M - 1)}{10} \cdot \exp \left[-\frac{(y - 4)}{20(1 - M)} \right] + \frac{M + 9}{10}$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = 1 \quad \text{for all } y$$

Solution: (crack in the interface of a bimaterial joint, Fig. IV-5.c)

(i) Plane Stress

$$P_{Lmis} = 0.960 \cdot \frac{s_{yb}}{\sqrt{3}} \cdot \frac{B \cdot (W - a)^2}{(S/2)}$$

(ii) Plane Strain

$$P_{Lmis} = b \cdot \frac{s_{yb}}{\sqrt{3}} \cdot \frac{B \cdot (W - a)^2}{(S/2)} \quad ; \quad b = (b_1 - b_\infty) \cdot e^{-(M-1)/0.23} + b_\infty$$

$$b_1 = \begin{cases} 1.125 + 0.892 \left(\frac{a}{w} \right) - 2.238 \left(\frac{a}{w} \right)^2 & \text{for } 0 < \frac{a}{w} \leq 0.172 \\ 1.199 + 0.096 \left(\frac{a}{w} \right) & \text{for } 0.172 < \frac{a}{w} \leq 1 \end{cases}$$

$$b_\infty = \begin{cases} 1.125 + 1.108 \left(\frac{a}{w} \right) - 2.072 \left(\frac{a}{w} \right)^2 & \text{for } 0 < \frac{a}{w} \leq 0.172 \\ 1.238 + 0.107 \left(\frac{a}{w} \right) & \text{for } 0.172 < \frac{a}{w} \leq 1 \end{cases}$$

DESCRIPTION: FULL CIRCUMFERENTIAL SURFACE CRACK IN PIPES UNDER TENSION

Schematic:

Fig. IV-6.a
Crack in the centre line
of the weld material

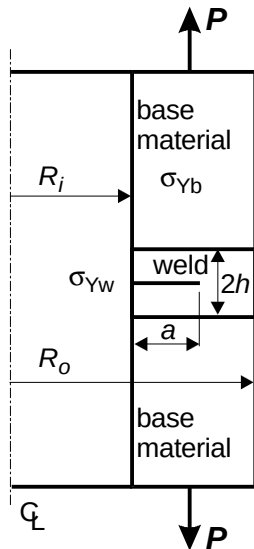


Fig. IV-6.b
Crack in the interface between
weld metal and base plate

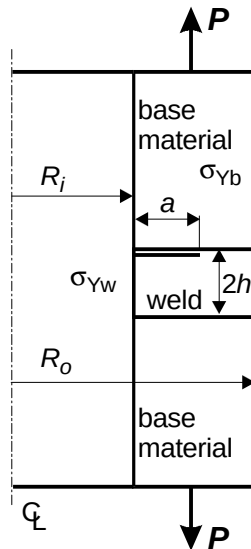
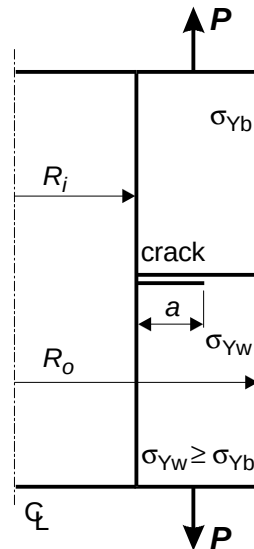


Fig. IV-6.c
Crack in the interface
of a bimaterial joint



Notation:

a	total defect length
$2h$	total width of weld
M	$=\sigma_{Yw}/\sigma_{Yb}$, strength mismatch factor
P	total applied end load
t	$=(R_o-R_i)$ thickness of the pipe
σ_{Yb}	yield strength of the base plate
σ_{Yw}	yield strength of the weld metal
ψ	$=(t-a)/h$
R_i	internal radius
R_o	external radius

Solution: (crack in the centre line of the weld metal, Fig. IV-6.a)

The limit load for the pipe made wholly of material *b* is

$$P_{Lb} = \frac{2}{\sqrt{3}} s_{yb} \cdot p \cdot [R_o^2 - (R_i + a)^2]$$

Undermatching ($M < 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq 1 \\ \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\} & \text{for } y \geq 1 \end{cases}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = M \cdot \left[1 + \frac{(y-1)}{3\sqrt{3}} \right]$$

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = 1 - (1-M) \cdot \frac{1}{y}$$

Overmatching ($M > 1$)

$$\frac{P_{Lmis}}{P_{Lb}} = \min \left\{ \frac{P_{Lmis}^{(3)}}{P_{Lb}}, \frac{1}{(1-a/w)} \right\}$$

$$\frac{P_{Lmis}^{(3)}}{P_{Lb}} = \begin{cases} M & \text{for } y \leq y_1 = e^{-2(M-1)/5} \\ \frac{24(M-1)}{25} \cdot \left(\frac{y_1}{y} \right) + \frac{M+24}{25} & \text{for } y \geq y_1 = e^{-2(M-1)/5} \end{cases}$$

Solution (crack in the interface between weld metal and base pipe, Fig. IV-6.b)

The limit load for the pipe made wholly of material *b* is

$$P_{Lb} = \frac{2}{\sqrt{3}} s_{yb} \cdot p \cdot [R_o^2 - (R_i + a)^2]$$

Undermatching (M<1)

$$\frac{P_{Lmis}}{P_{Lb}} = \begin{cases} M & \text{for } 0 \leq y \leq 2 \\ \min \left\{ \frac{P_{Lmis}^{(1)}}{P_{Lb}}, \frac{P_{Lmis}^{(2)}}{P_{Lb}} \right\} & \text{for } y \geq 2 \end{cases}$$

$$\frac{P_{Lmis}^{(1)}}{P_{Lb}} = M \cdot \left[1 + \frac{(y - 2)}{6\sqrt{3}} \right]$$

$$\frac{P_{Lmis}^{(2)}}{P_{Lb}} = 1 - (1 - M) \cdot \frac{2}{y}$$

Overmatching (M>1)

$$\frac{P_{Lmis}}{P_{Lb}} = 1 \quad \text{for all } y$$

Solution: (crack in the interface of a bimaterial joint, Fig. IV-6.c)

$$P_{Lmis} = \frac{2}{\sqrt{3}} s_{yb} \cdot p \cdot [R_o^2 - (R_i + a)^2]$$

Remarks: Solutions are valid for thin-walled pipes with deep cracks, $\frac{a}{t} \geq 0.3$.

AIV.5 REFERENCES

- AIV.1. H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM - The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

APPENDIX V

**COMPARISON BETWEEN DIFFERENT STRESS INTENSITY FACTOR
SOLUTIONS**

AV.1. INTRODUCTION

The purpose of this appendix is to provide confidence in the solutions to be adopted for the SINTAP project. A large number of different test cases has been run, comparing the SAQ, R6.CODE, IWM and API results with those found in handbooks and other references. The cases presented in this appendix are most likely to be of practical use, that is, flat plate and cylinder geometries. A list of cases covered is provided in Section AV.2. The results of the comparison are provided in Section AV.3. The conclusions of the comparison are presented in Section AV.4.

AV.2 CASES CONSIDERED

Details of the cases which were considered in the present work are given in Table AV.1 on the following pages. The cases were divided into four categories: through thickness defects, extended defects, embedded and surface defects. The table shows the structural component type, the crack location and orientation, and the loading condition. All geometries in this appendix were subjected to tensile polynomial stresses. These polynomial stresses were taken to be constant. One geometry, however, was subjected to a linearly varying stress polynomial, which is the case of a semi-elliptical circumferential internal surface crack in cylinder with $R_i/t=10$ and $a/c=1.0$. Most of the extended and through thickness defect cases were run. Some semi-elliptical geometrical cases were not run due to the lack of handbook solutions. Some of the comparisons were carried out partially due to the different applicability ranges.

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Table AV.1. Wide plate, and cylinder cracked cases considered

Crack Category	Structure	Location	Orientation	Geometrical Parameters	Loading	Comments
Through Thickness Crack	Wide Plate	Central	-	-	Tension	For Different Ratio of $2a/W$
	Cylinder	-	Circumferential	$Ri/t=10$	Tension	For Different Ratio of $2\theta/t$
	Cylinder	-	Axial	$Ri/t=10$	Tension	For Different Ratio of $2a/t$
Extended Defects	Wide Plate	Central	-	-	Tension	For Different Ratio of a/t
	Cylinder	External	Axial	$Ri/t=4$	Tension	For Different Ratio of a/t
	Cylinder	External	Axial	$Ri/t=10$	Tension	For Different Ratio of a/t
	Cylinder	Internal	Axial	$Ri/t=4$	Tension	For Different Ratio of a/t
	Cylinder	Internal	Axial	$Ri/t=10$	Tension	For Different Ratio of a/t
	Cylinder	External	Complete Circumferential	$Ri/t=2-2.33$	Tension	For Different Ratio of a/t
	Cylinder	Internal	Complete Circumferential	$Ri/t=10$	Tension	For Different Ratio of a/t
Embedded Defects	Wide Plate	Central	-	$a/c=0.05$	Tension	For Different Ratio of a/t
	Wide Plate	Central	-	$a/c=0.5$	Tension	For Different Ratio of a/t
	Wide Plate	Central	-	$a/c=1.0$	Tension	For Different Ratio of a/t

Table AV.1. Wide plate, and cylinder cracked cases considered (Continued)

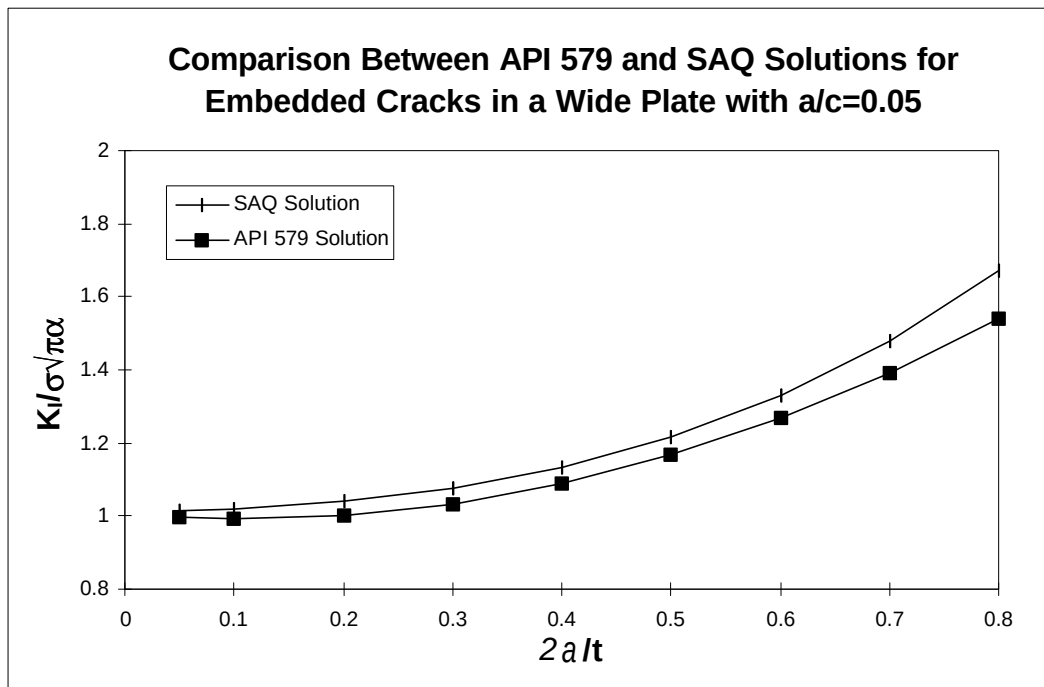
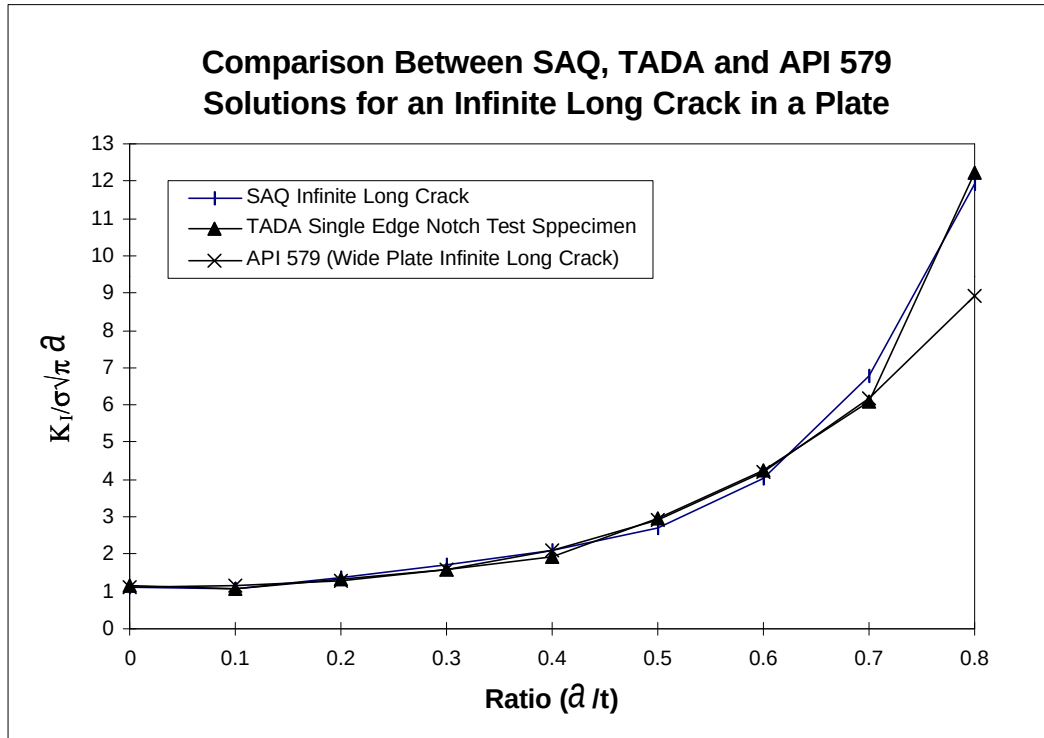
Crack Category	Structure	Location	Orientation	Geometrical Parameters	Loading	Comments
Semi-Elliptical Surface Defects	Wide Plate	Central	-	a/c=0.1	Tension	For Different Ratio of a/t
	Wide Plate	Central	-	a/c=0.2	Tension	For Different Ratio of a/t
	Wide Plate	Central	-	a/c=0.6	Tension	For Different Ratio of a/t
	Wide Plate	Central	-	a/c=0.8	Tension	For Different Ratio of a/t
	Wide Plate	Central	-	a/c=1.0	Tension	For Different Ratio of a/t
	Cylinder	External	Axial	Ri/t=10 a/c=0.2	Tension	For Different Ratio of a/t
	Cylinder	Internal	Axial	Ri/t=10 a/c=0.2	Tension	For Different Ratio of a/t
	Cylinder	Internal	Axial	Ri/t=10 a/c=0.4	Tension	For Different Ratio of a/t
	Cylinder	Internal	Circumferential	Ri/t=10 a/c=1.0	Linearly Varying tensile Stress	For Different Ratio of a/t
Only Between SAQ and IWM						
Semi-Elliptical Surface Defects	Cylinder	Internal	Circumferential	Ri/t=10 a/c=0.125	Tension	For Different Ratio of a/t
	Cylinder	Internal	Circumferential	Ri/t=10 a/c=1.0	Tension	For Different Ratio of a/t

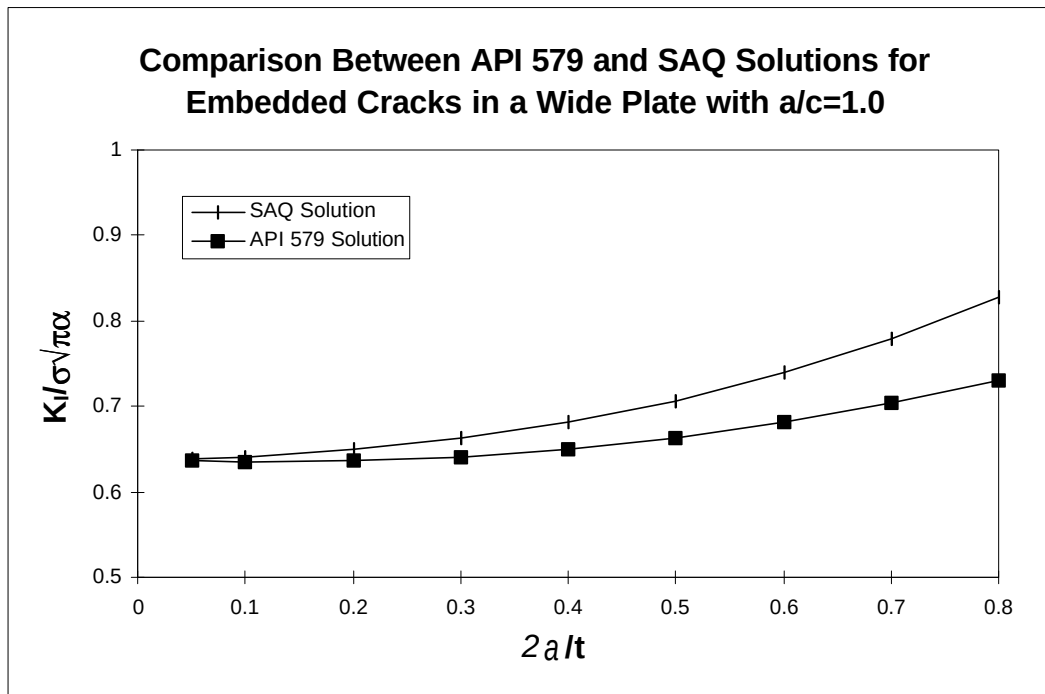
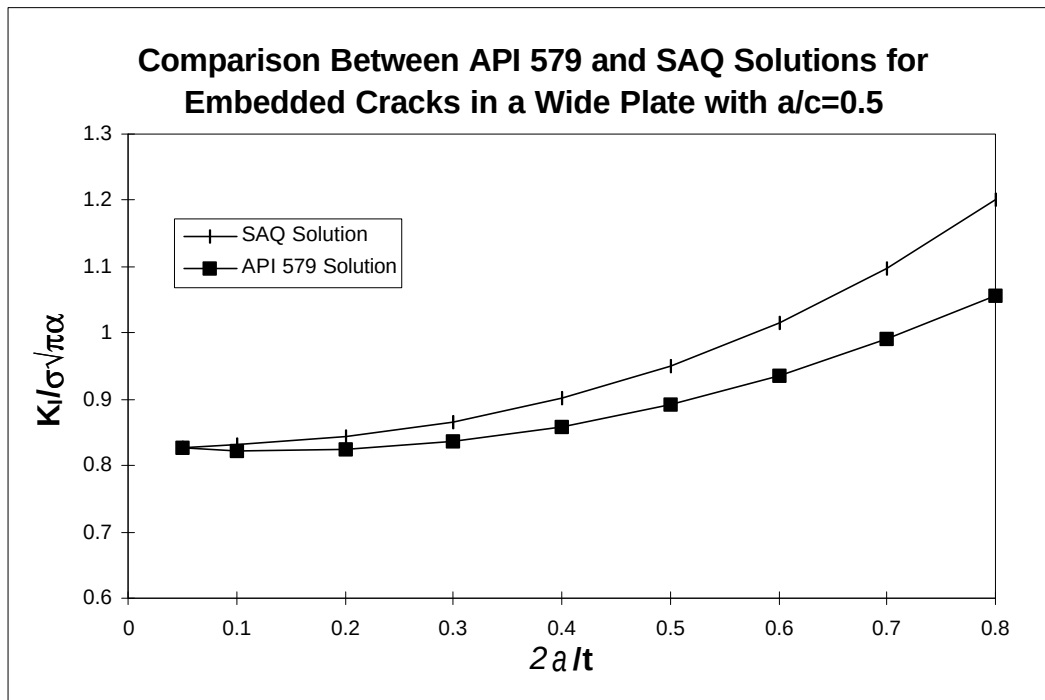
AV.3. RESULTS OF THE COMPARISON

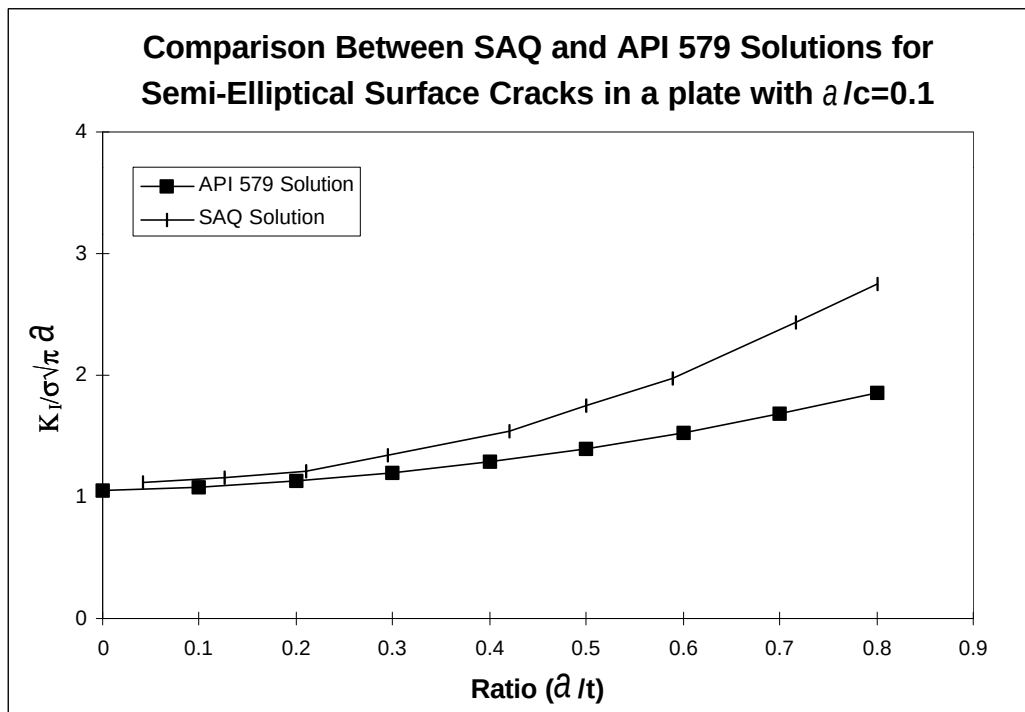
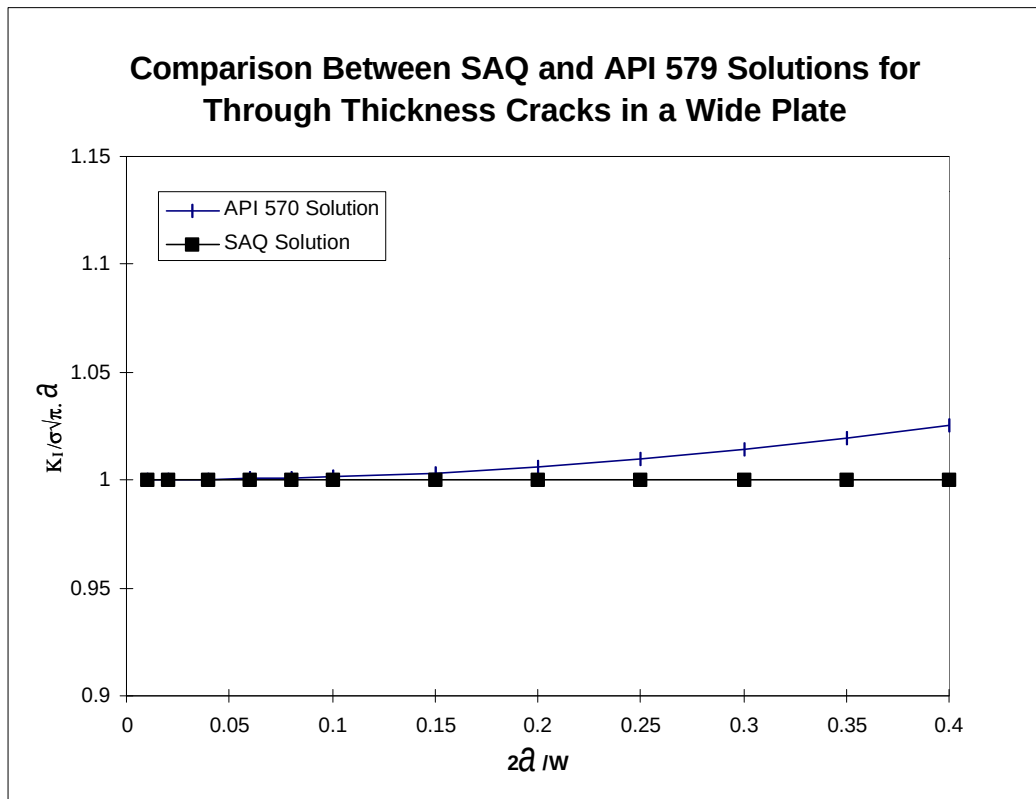
AV.3.1 Flat Plates

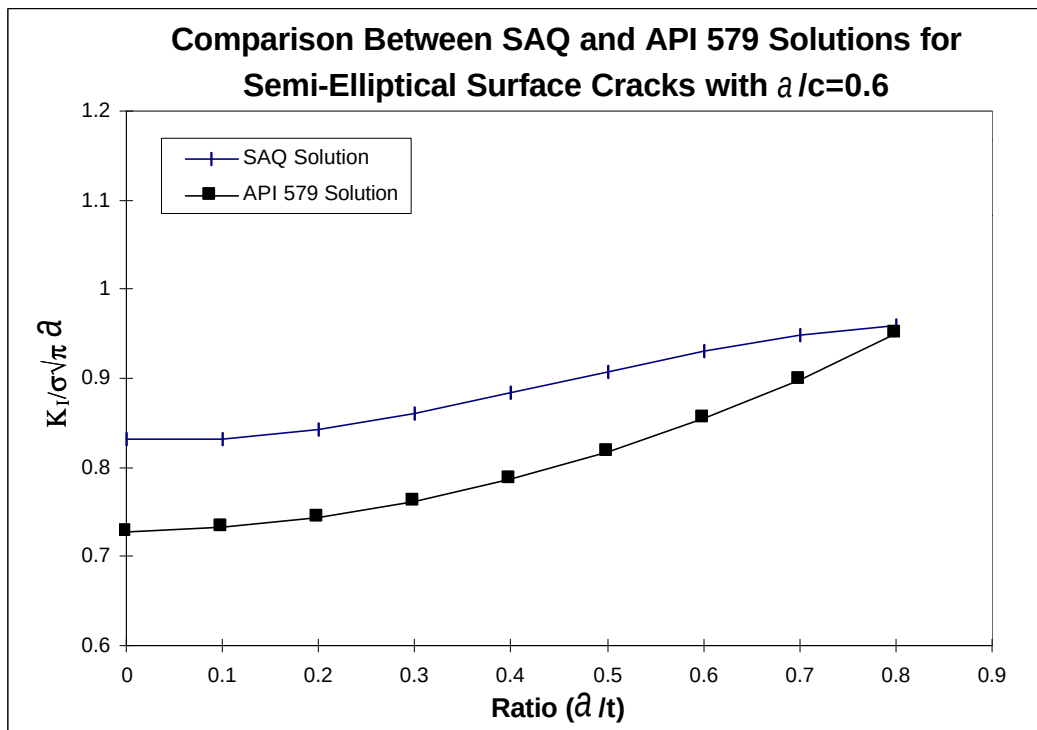
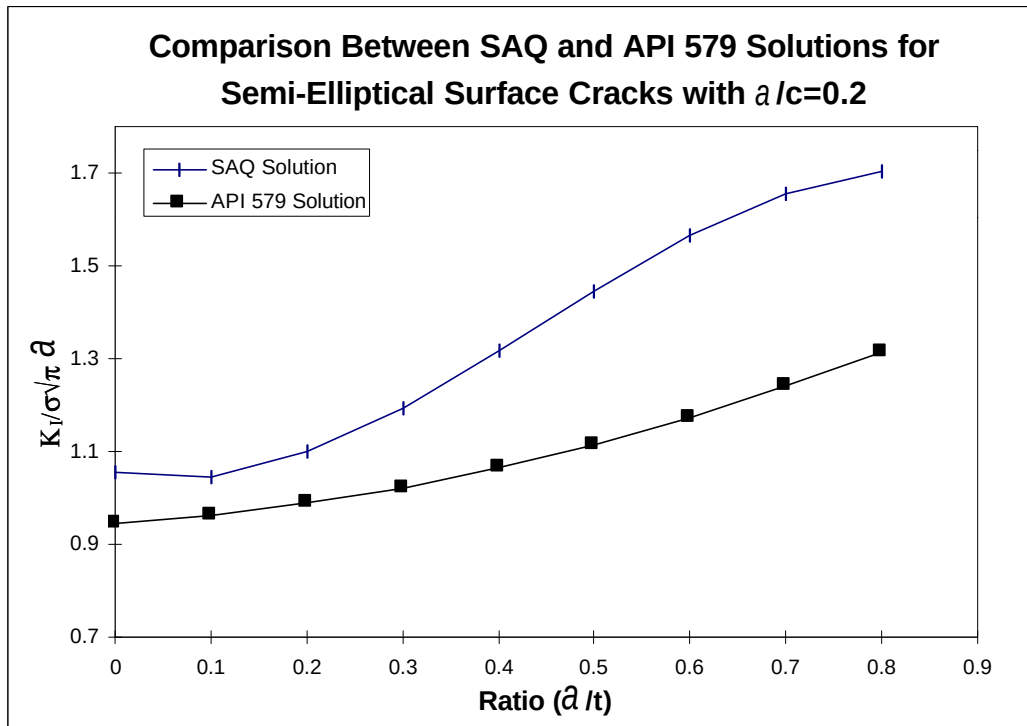
In this section the results of the comparison for flat plates with extended, surface, embedded and through thickness cracks are presented. These are given on the following pages. The equation used to obtain the normalised stress intensity factor is given as follows:

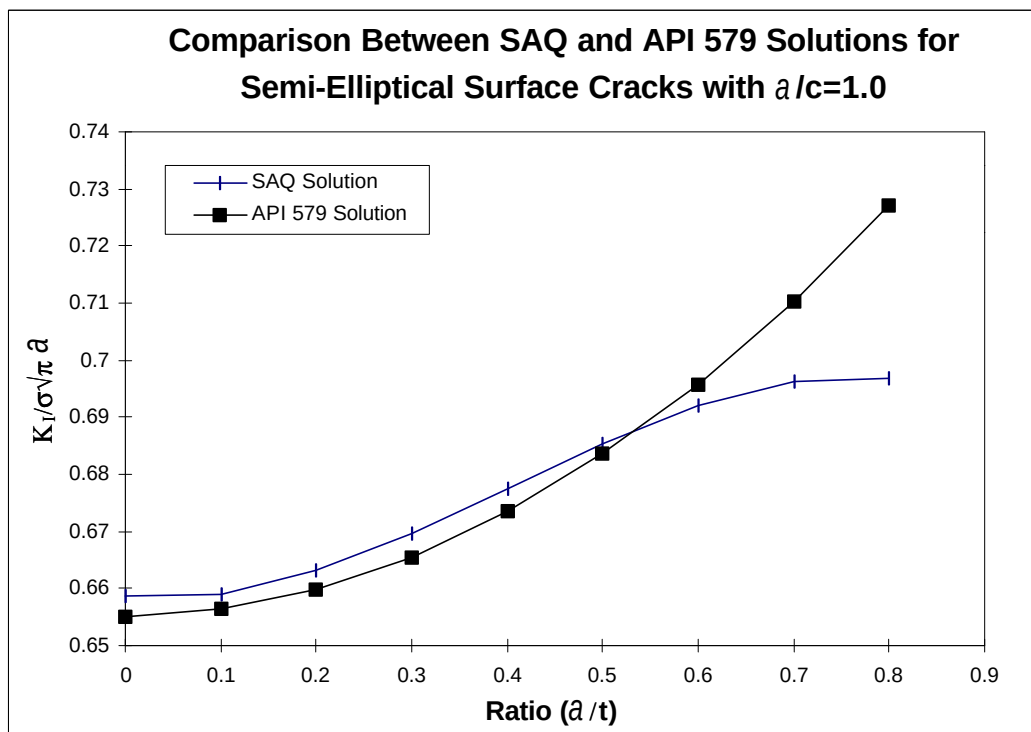
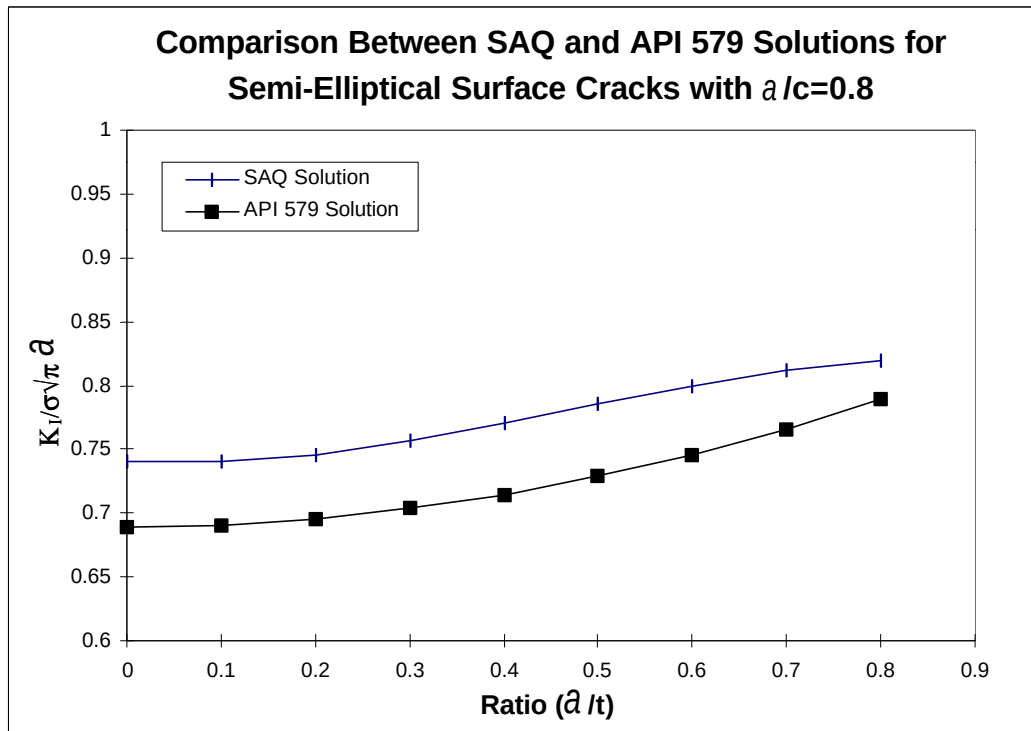
$$K_{Norm} = \frac{K}{s \sqrt{p \cdot a}}$$







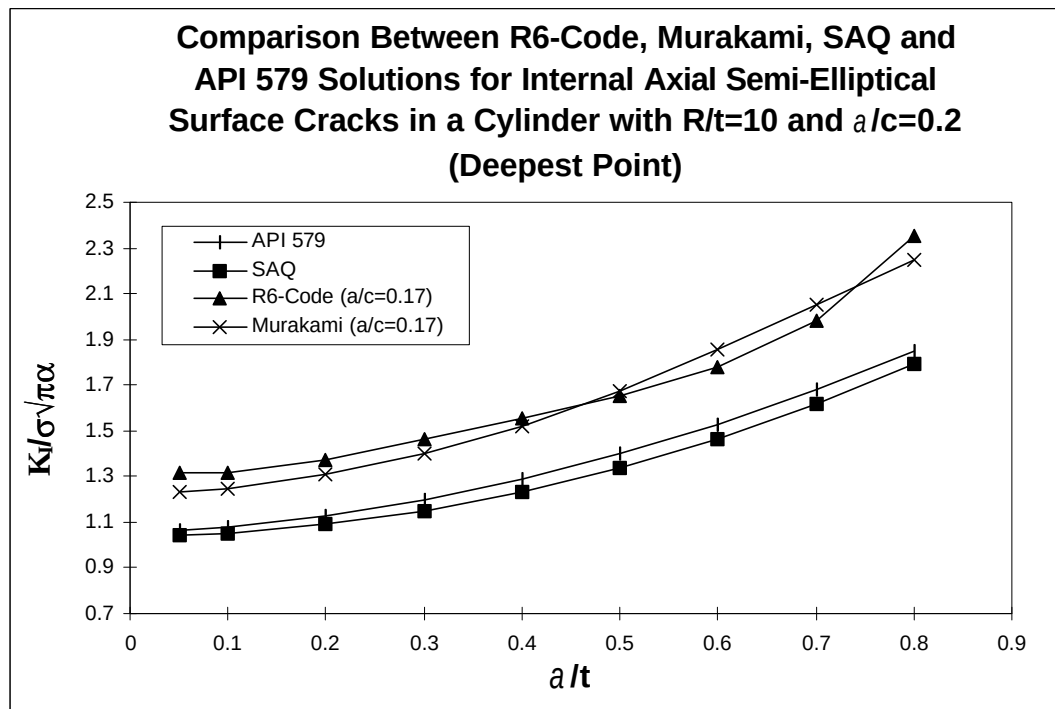


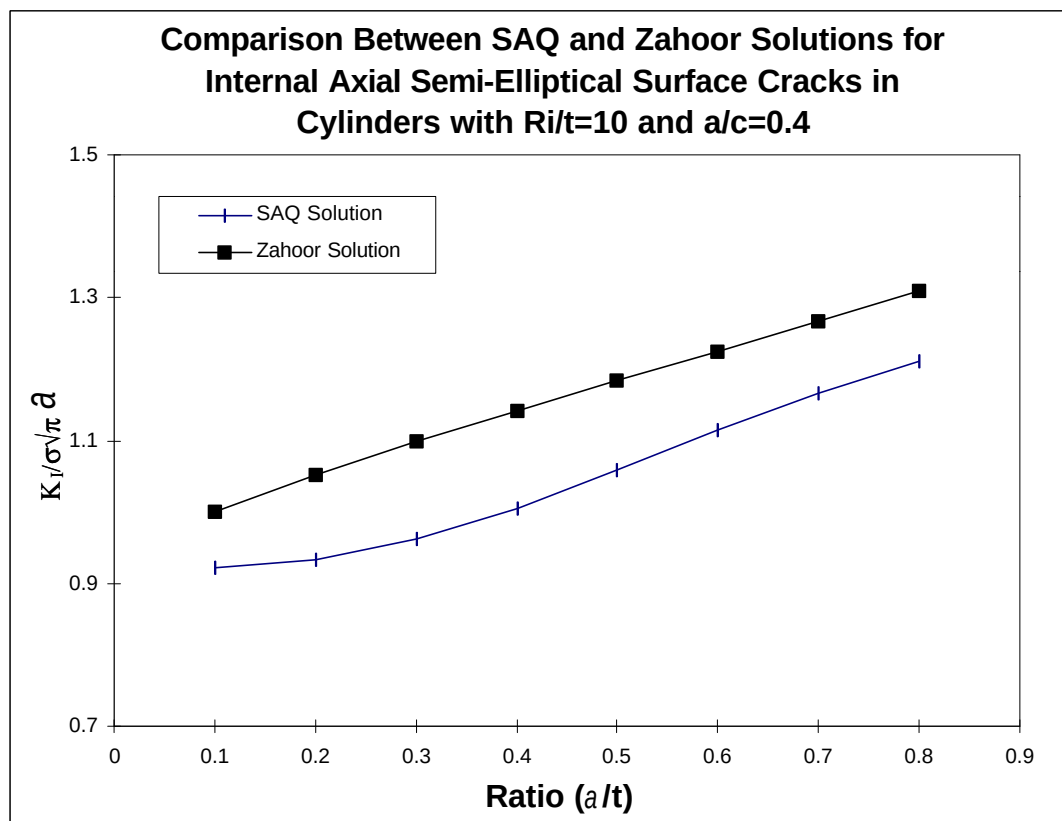
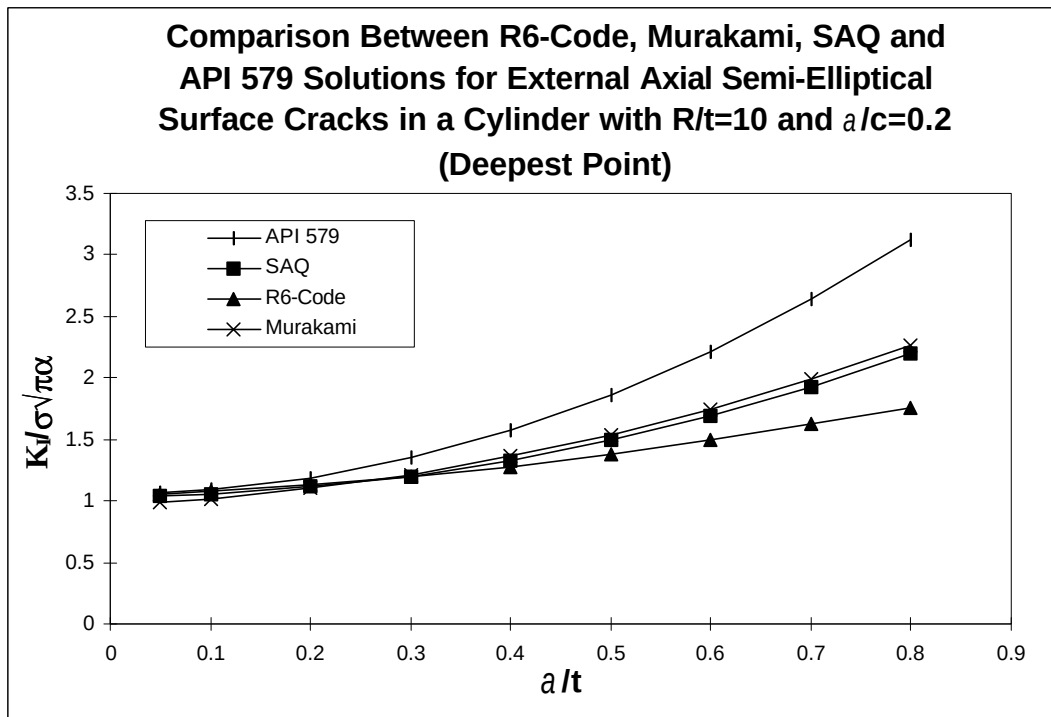


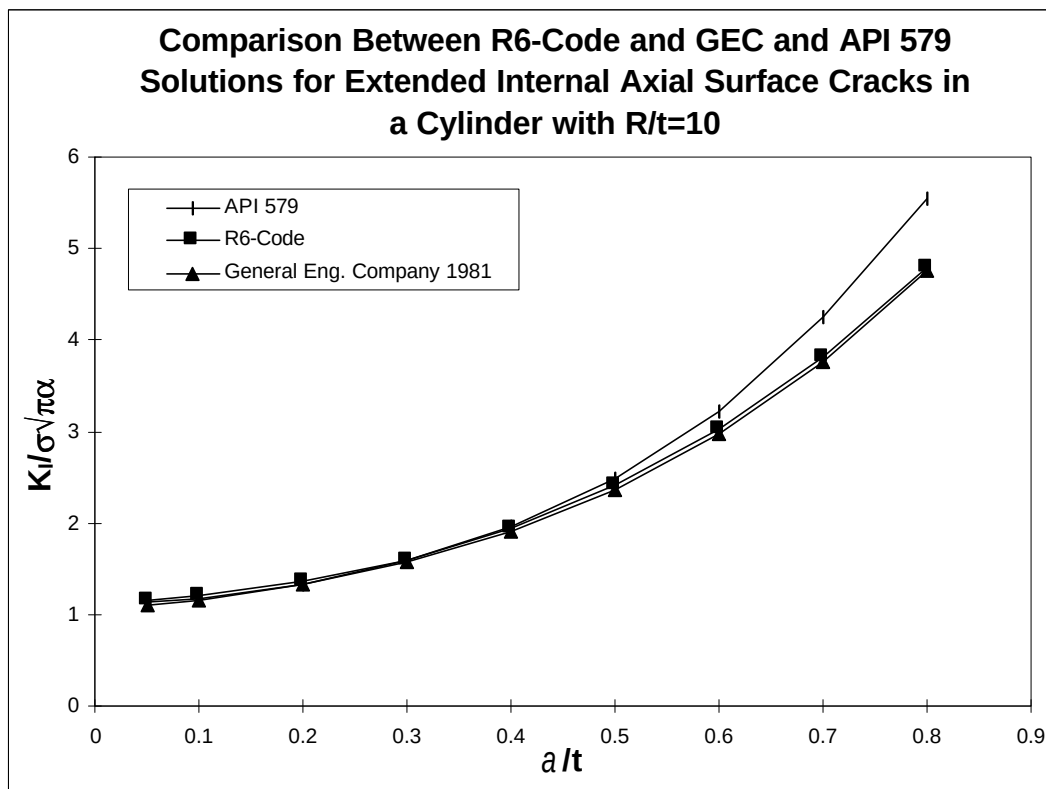
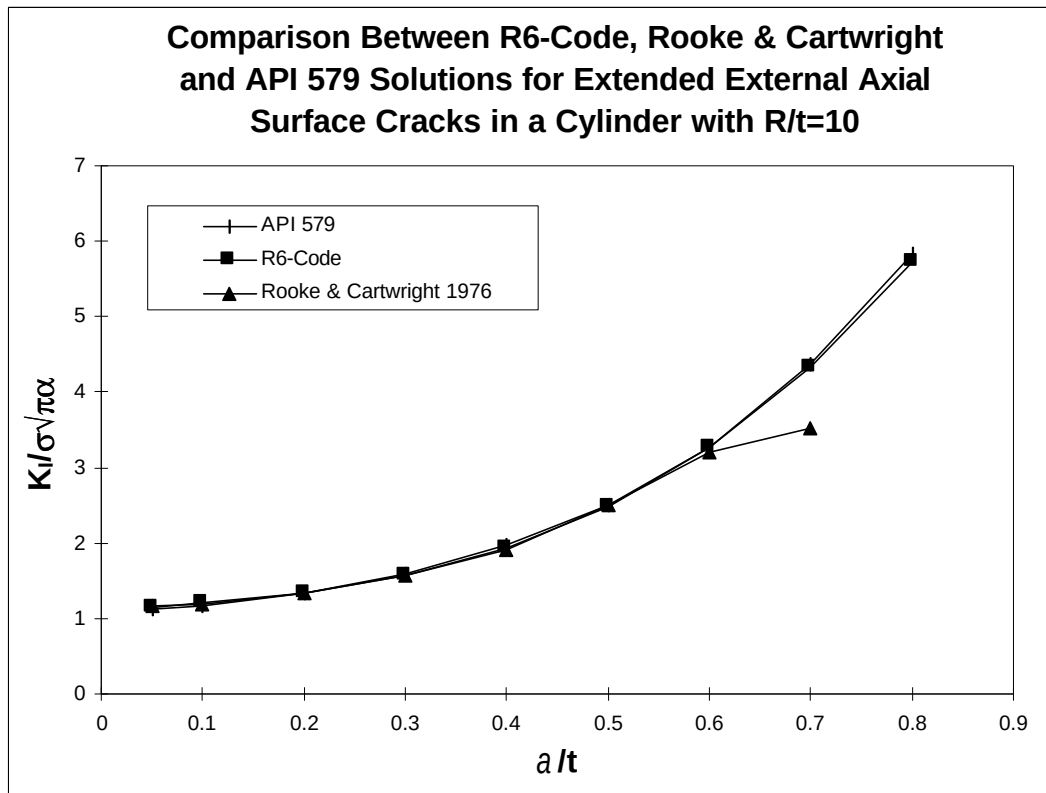
AV.3.2 Cylinders

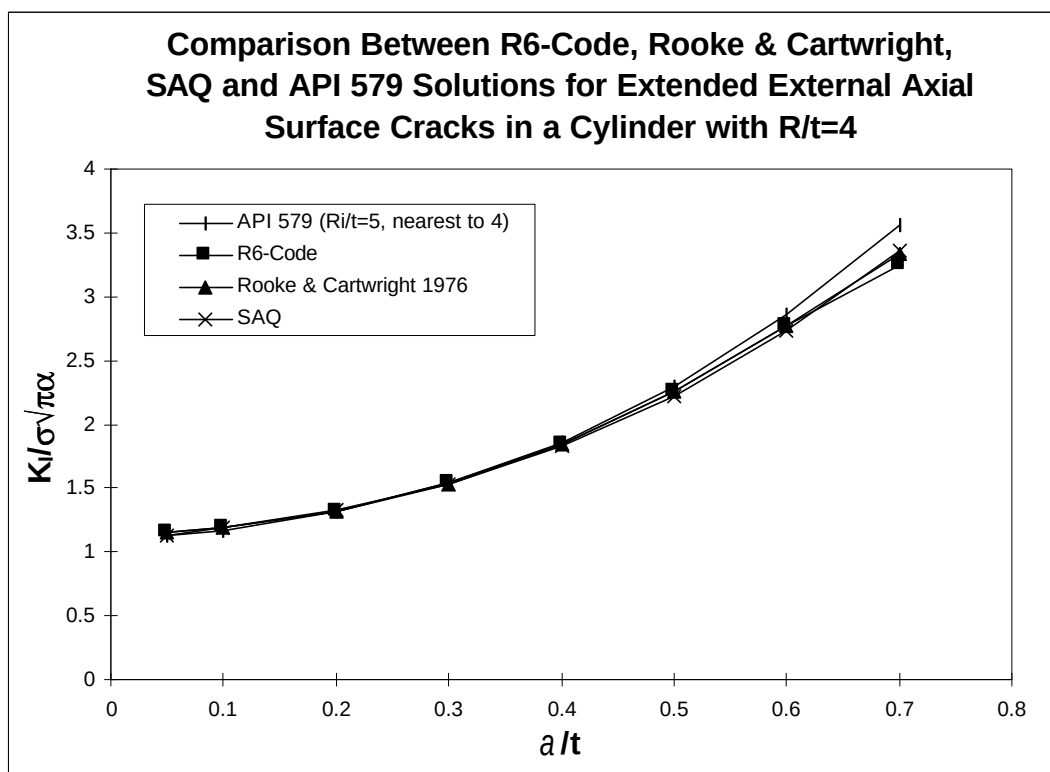
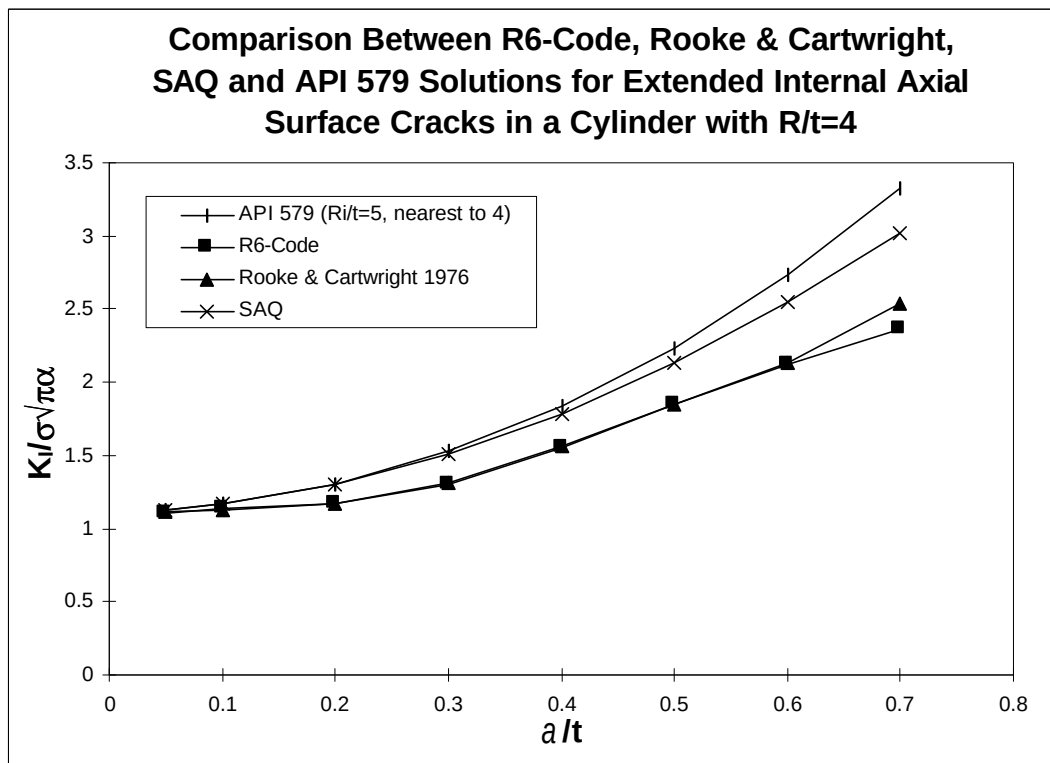
In this section the results of the comparisons for cylinders with extended, surface and through thickness cracks are presented for axial and circumferential cracks. The equation used to obtain the normalised stress intensity factor is given as follows:

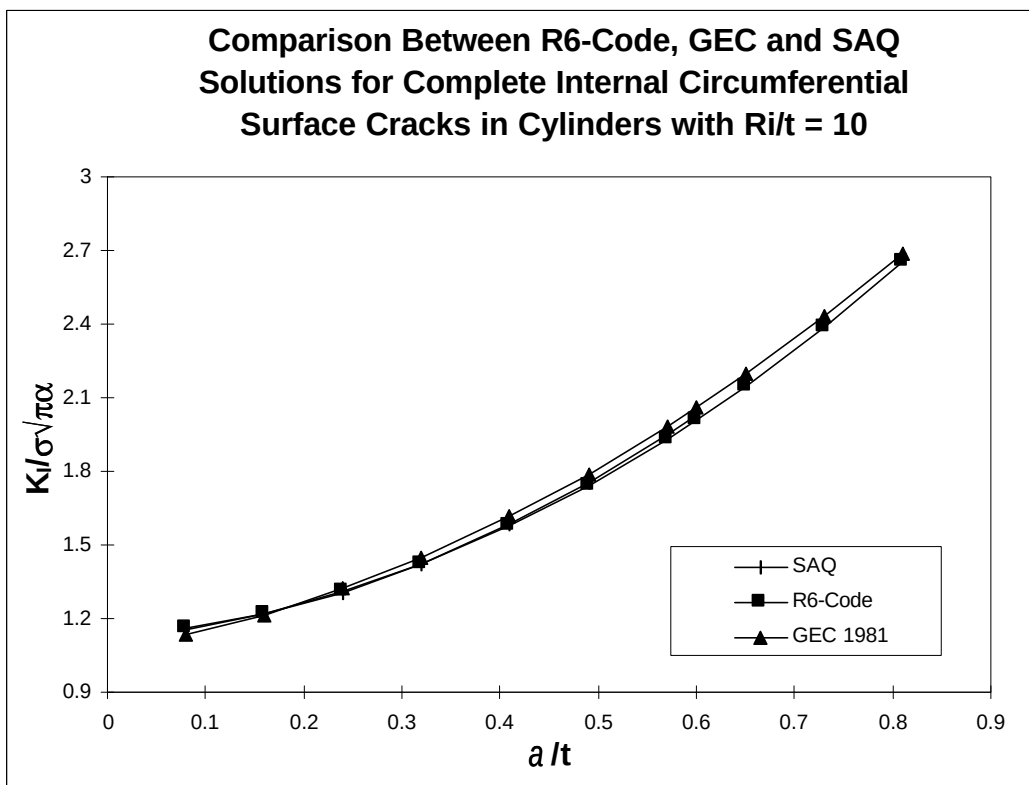
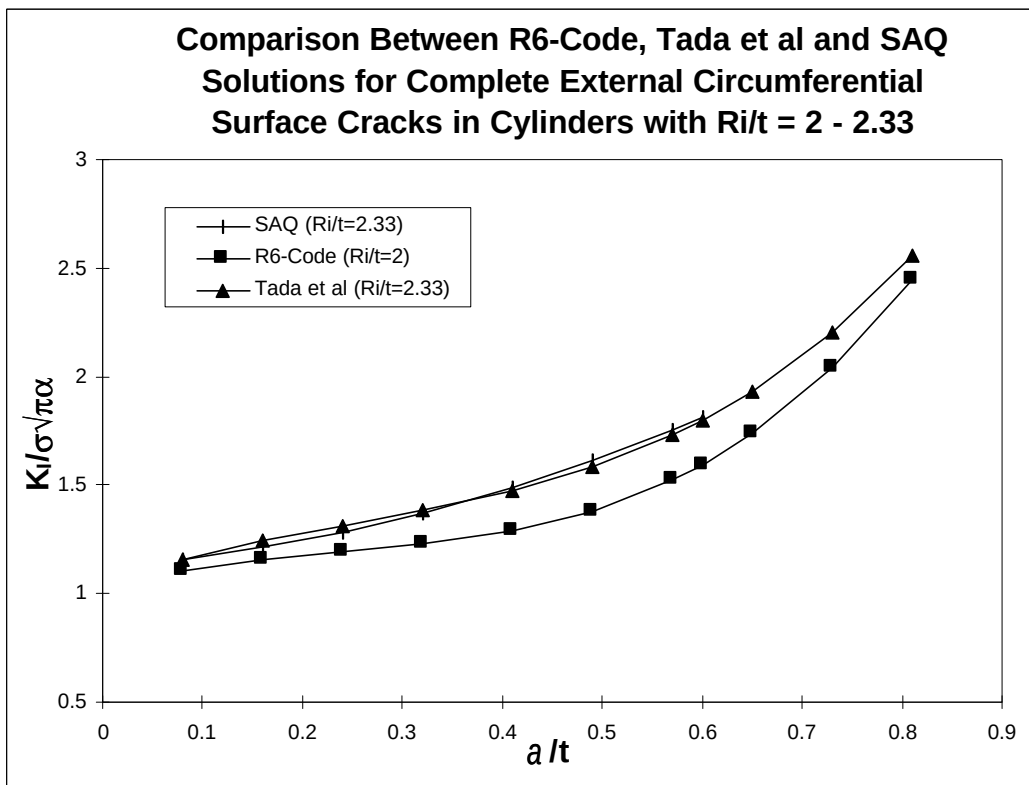
$$K_{Norm} = \frac{K}{s \sqrt{p \cdot a}}$$

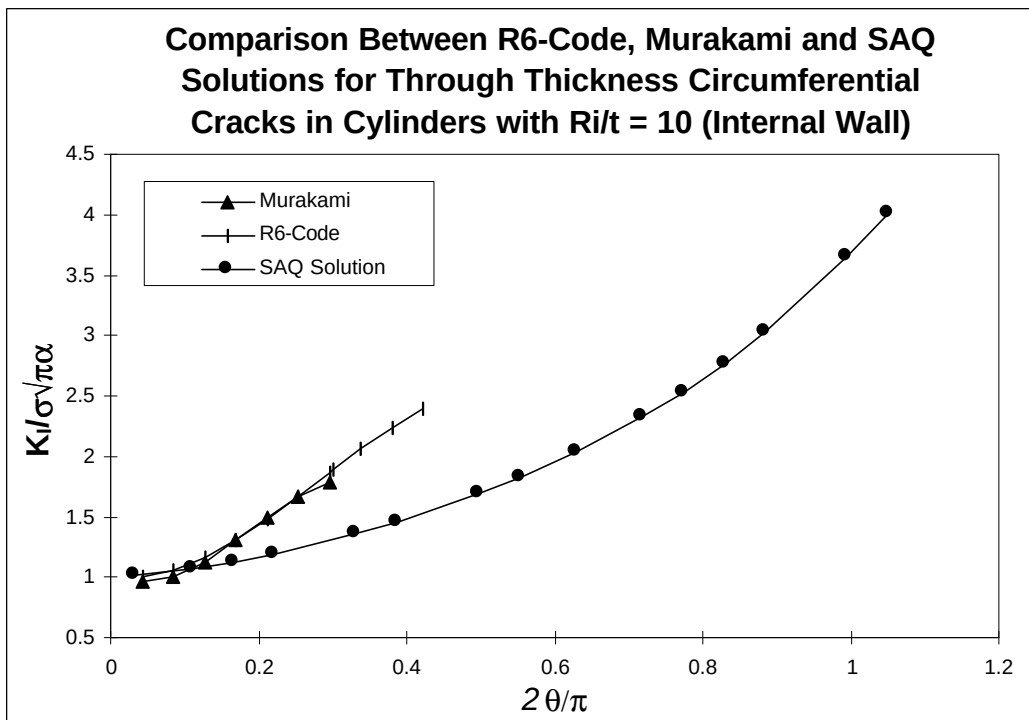
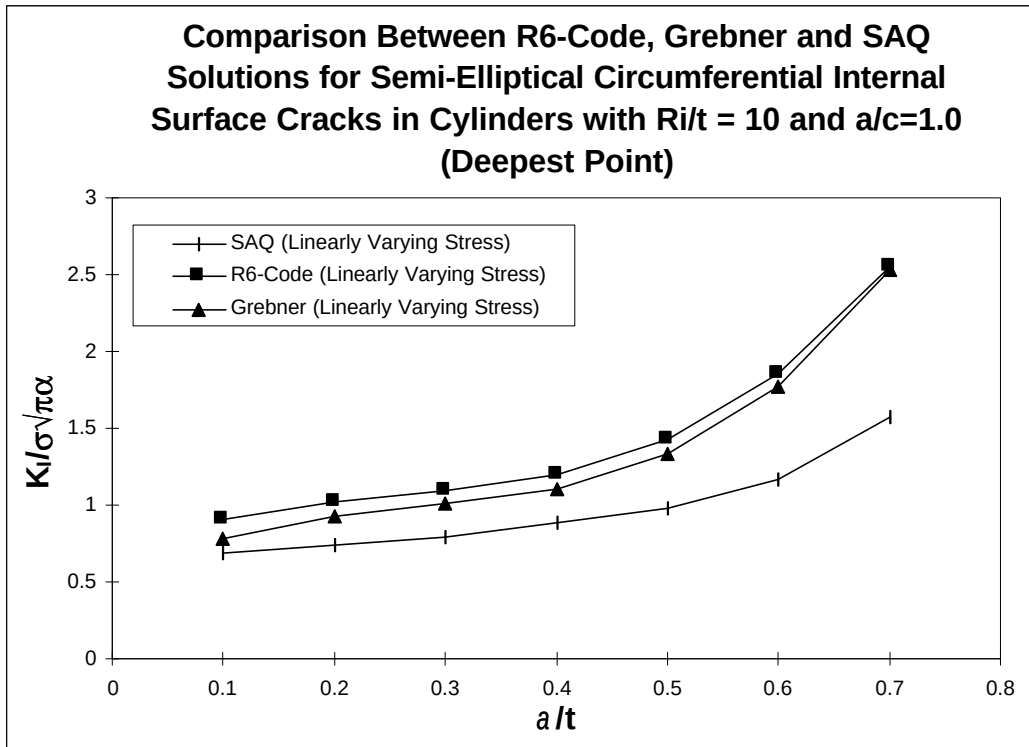


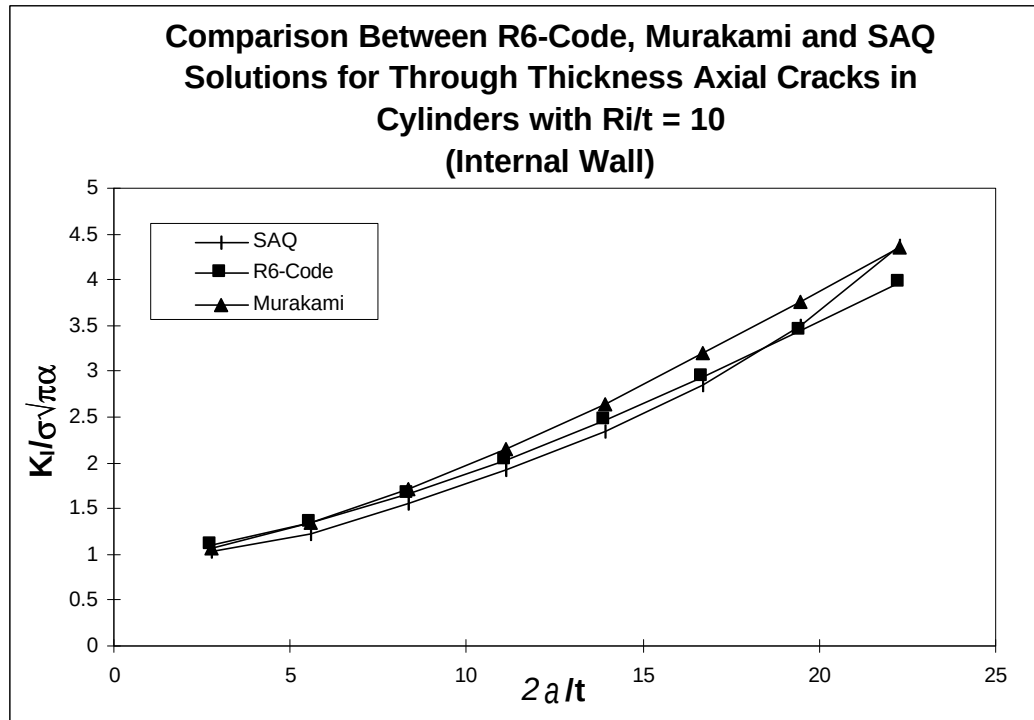








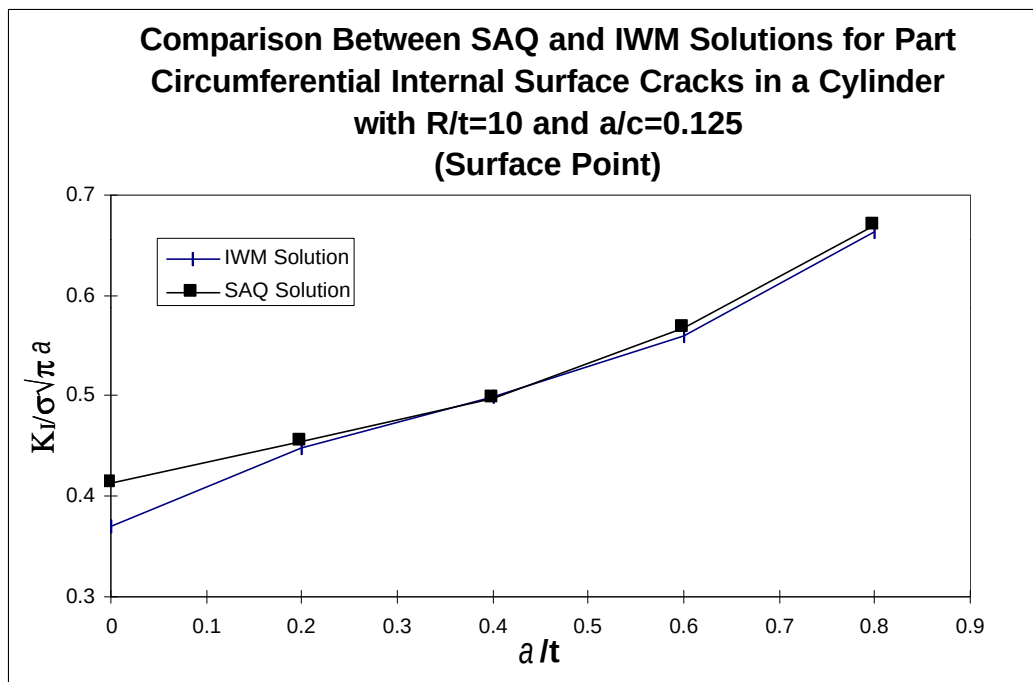
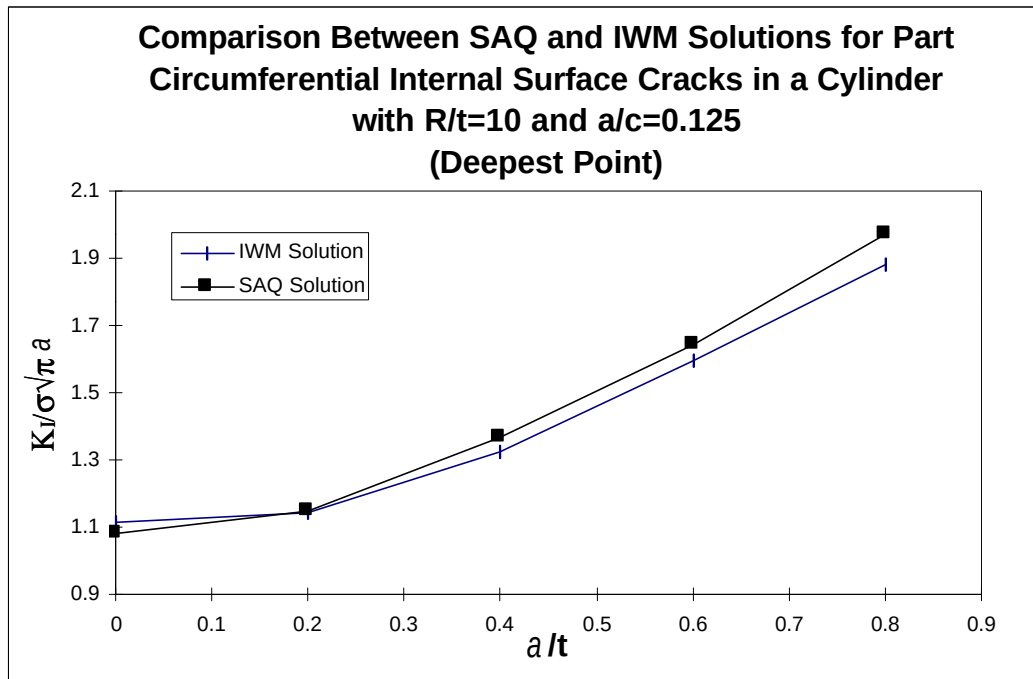


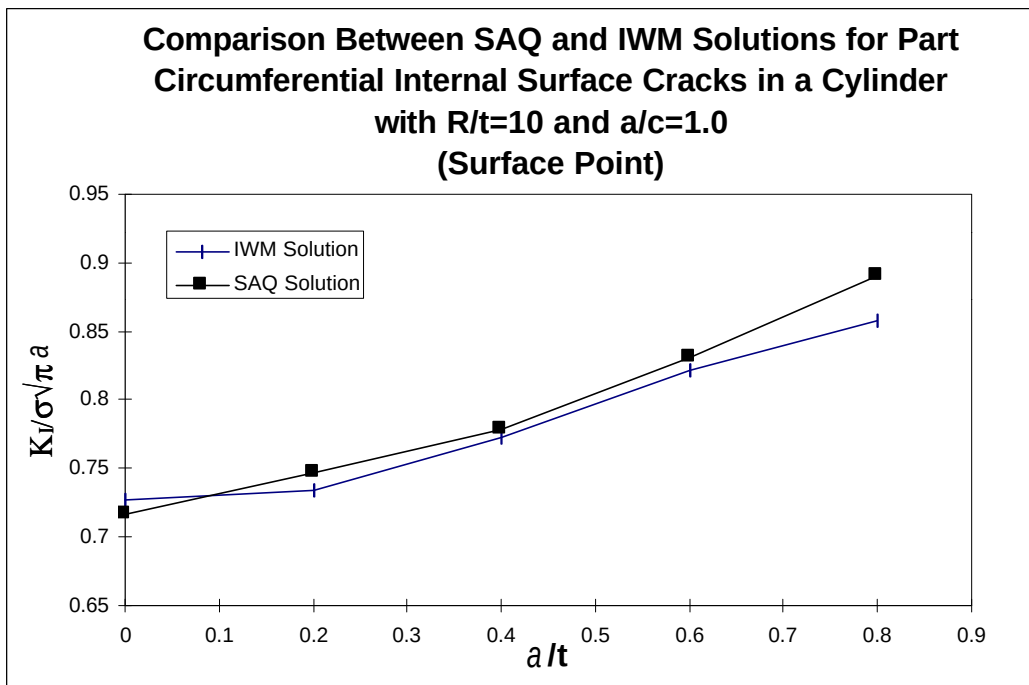
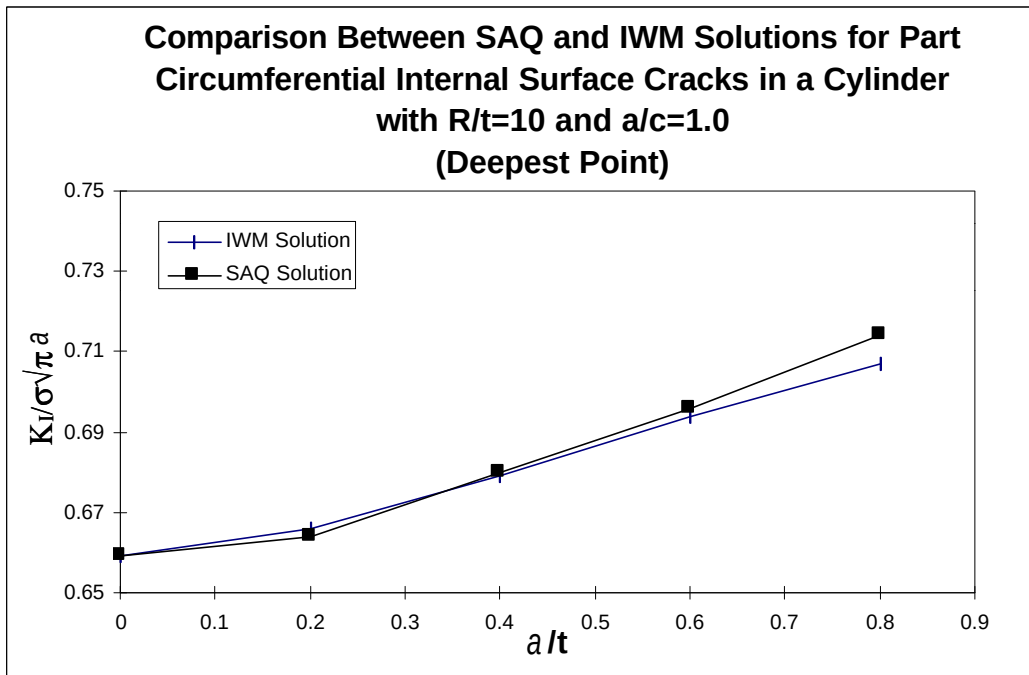


AV.3.3 Comparison between SAQ and IWM solutions only

In this section the results of the comparison for cylinders with semi-elliptical circumferential surface cracks between SAQ and IWM solutions are presented. The equation used to obtain the normalised stress intensity factor is given as follows:

$$K_{Norm} = \frac{K}{s \sqrt{p \cdot a}}$$





AV.4. CONCLUDING REMARKS

R6.CODE, the API code (PREFIS), SAQ and IWM solutions have been used to generate the results for the different geometrical arrangements given in Table AV.1. These included cases which are through thickness cracked, extended cracked, embedded cracked and semi-elliptically cracked geometries. The results obtained from the different sources were compared with handbook solutions and other references. The following conclusions can be drawn:

There is excellent agreement between SAQ results and those obtained using the IWM solutions, for cylinders with semi-elliptical circumferential surface cracks.

The comparison between SAQ and API 579 solutions, for flat plates with semi-elliptical surface cracks, showed very good agreement in most cases. The results, however, did not agree in one case, where the crack depth to length ratio a/c is as low as 0.1. In this case better agreement between SAQ and other solutions was found.

Generally, good agreement was found between the results of R6.CODE, API 579, SAQ and other published handbook solutions.

API solutions are more conservative than other solutions for the case of externally axially cracked cylinders, particularly at low a/c ratio where the crack tends to be extended. The large difference may be due to the fact that SAQ and others used more accurate solid modelling to obtain their K solutions, rather than relying on solutions which are often based on less accurate thin shell theory.

Based on the results of this comparison, some SAQ solutions supplemented by solutions from R6-Code were recommended in Appendix I.

AV.5. REFERENCES

- AV.1. User Guide of R6-Code. Software for Assessing the Integrity of Structures Containing Defects. Version 1.4x, Nuclear Electric Ltd (1996).
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- AV.5. General Electric Company, An Engineering Approach for Elastic-Plastic Fracture Analysis, EPRI Report NP-1931 (1981).
- AV.6. P. Andersson, M. Bergman, B. Brickstad, L. Dahlberg, P. Delfin, I. Sattari-Far and W. Zang, Collation of Solutions for Stress Intensity Factors and Limit Loads, Report No SINTAP/SAQ/05, SAQ Kontroll AB, Sweden (1997).
- AV.7. L. Hodulak and I Varfolomeyev, A Contribution to Collation of Stress Intensity Factors, SINTAP/IWM/01, Fraunhofer IWM Report V00/97 (1997).