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STRUCTURAL INTEGRITY ASSESSMENT PROCEDURES FOR EUROPEAN INDUSTRY

SINTAP

MILESTONE REPORT : TASK 1.5

DRAFT PROCEDURE FOR MIS-MATCH

(1ST REVISION)

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SUMMARY

This report describes the SINTAP procedure for strength mismatched structures. Following the structure of the SINTAP homogeneous procedure, various levels of the mismatch procedure are set out, depending on how much information on (material stress strain) data is available:

- only yield stress is available
- both yield and ultimate stresses are available
- full stress strain data are available
- the J integral is available

In the lower levels, the assessment equations are specific to the type of materials:

- both materials exhibit a Lüders strain
- both materials do not exhibit a Lüders strain
- only one material exhibits a Lüders strain.

Detailed assessment equations are given in Section 2. The resulting assessment curves for various levels are compared in Section 3 through worked examples.

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1. INTRODUCTION

1.1 Brief Review of Existing Procedures : modified R6 and ETM-MM methods

The SINTAP procedure for mismatch, described in this report, is based on two existing assessment procedures specific to the strength mismatched structures, the Engineering Treatment Model for Mis-Match (ETM-MM) [1,2] and the R6 method modified for strength mismatched structures (modified R6 method) [3], which are two most complete procedures for defective welded structures.

One of the main features of the ETM-MM method is to introduce the mismatch strain hardening exponent from which the crack driving force of the defective structures with strength mismatch is extrapolated. On the other hand, in the modified R6 method, the concept of an equivalent material is introduced, which makes the homogeneous assessment procedure applicable. Most importantly, both methods share one common feature, mismatch effect on yield (limit) loads.

1.2 Mismatch Effect on Yield Load

It is now widely accepted that the mismatch effect on the yield (limit) load is the most crucial parameter for defect assessment methods specific to strength mismatched structures. Within SINTAP, such mismatch yield load solutions have been generated for a number of geometries. Those solutions have been reported to Task 2.6: Collation of limit load solutions (see [4]).

1.3 Compatibility between FAD and CDF

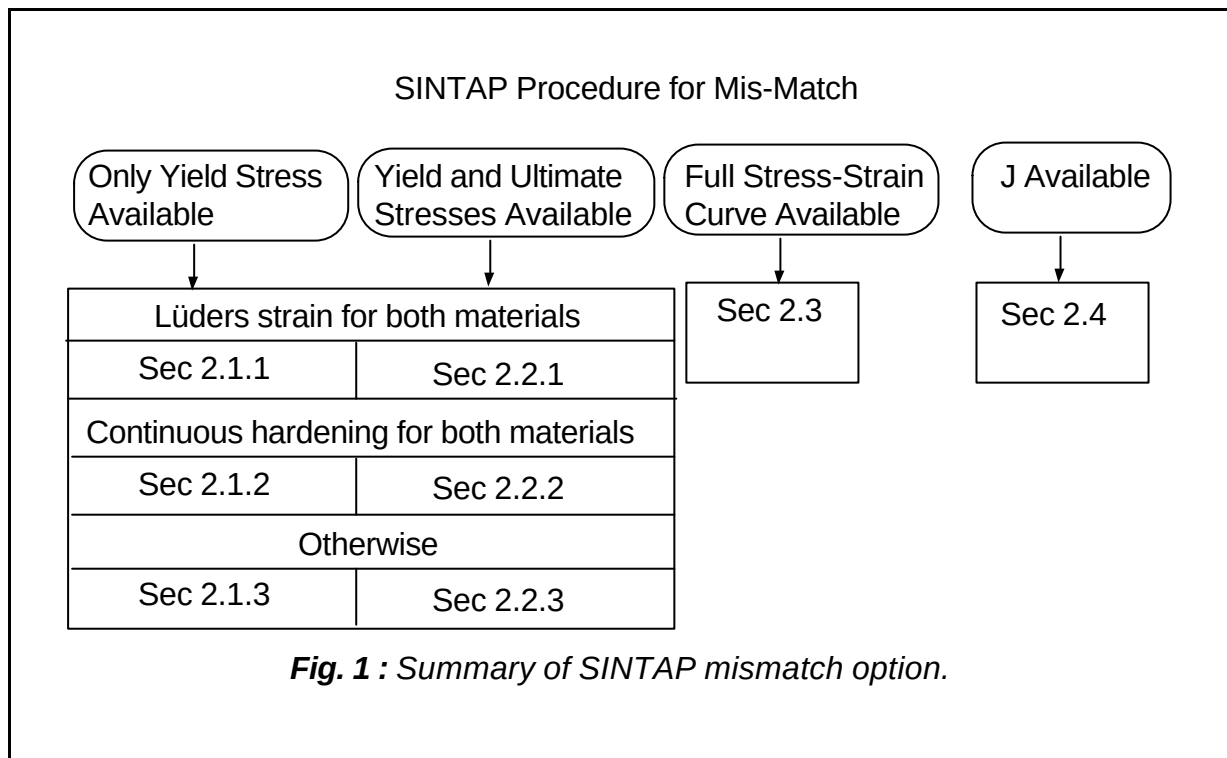
The assessment results can be interpreted either in terms of Failure Assessment Diagram (FAD) or in terms of Crack Driving Force (CDF) diagram. The compatibility between FAD and CDF representations is discussed in [5].

1.4 Compatibility between ETM-MM and modified R6 methods

Although the way to treat the strength mismatch effect in the ETM-MM method is different from that in the modified R6 method, it has been shown that both ETM-MM and modified R6 curves are very close and show excellent agreement with the FE results [6,7]. Such compatibility provides confidence of existing procedures. The SINTAP procedure for mismatch which will be provided in this report is developed by blending the essence of two existing assessment procedures with minor adjustments.

2. RECOMMENDED ASSESSMENT CURVES

Detailed assessment equations specific to the strength mismatched structures are given below. A general structure of mismatch procedure follows that of homogeneous procedure: the assessment curves depends on how much information on material stress strain data is available and on whether the material is expected to exhibit a Lüders strain or not. The levels of the mismatch option are illustrated in Fig. 1, together with the appropriate section number in this report.



2.1 Level 1 (Only Yield Stress Data Available)

2.1.1 Lüders strain expected for both base and weld materials

Where only yield stress is available and **both materials are expected to exhibit a Lüders plateau**, then the function

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2\right)^{-1/2} \quad (2.1)$$

is used up to $L_r = L_r^{\max} = 1$ (or equivalently $F = F_{YM}$).

2.1.2 Lüders strain not expected for both materials

Where only yield stress is available and **Lüders strain is not expected for both materials**, then the function

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2\right)^{-1/2} \cdot \left[0.3 + 0.7 \exp\left(-0.6 \cdot L_r^6\right)\right] \quad (2.2)$$

is used up to $L_r = L_r^{\max} = 1 + (150/YS)^{2.5}$, with YS being the lower of the yield stresses of the two materials.

2.1.3 Lüders strain not expected only for one material

Where only yield stress is available and **Lüders strain is not expected only for one material**, then eqn. (2.2) is used up to $L_r = L_r^{\max} = 1$:

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2\right)^{-1/2} \cdot \left[0.3 + 0.7 \exp\left(-0.6 \cdot L_r^6\right)\right] \quad (2.2)$$

2.2 Level 2 (Yield and Ultimate Stress Available)

2.2.1 Lüders strain expected for both base and weld materials

If both materials exhibit a Lüders plateau and when yield and ultimate stresses are available for both materials, then eqn. (2.1) is used for $L_r < 1$ (or $F < F_{YM}$):

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2 \right)^{-1/2} \quad (2.1)$$

At $L_r = 1$ (or $F = F_{YM}$), the function f is taken as discontinuous and reduces to the value $f(1)$, which is dependent on the extent of the Lüders strain. Denoting $\Delta\epsilon_W$ and $\Delta\epsilon_B$ as the extent of the Lüders strain for the weld metal and base plate, respectively, the function $f(1)$ is expressed as

$$\begin{aligned} f(1) &= \left(\lambda_M + \frac{1}{2\lambda_M} \right)^{-1/2} \\ \lambda_M &= \frac{(F_{YM}/F_{YB} - 1)\lambda_W + (M - F_{YM}/F_{YB})\lambda_B}{(M - 1)} \\ \lambda_W &= 1 + \frac{E_W \Delta\epsilon_W}{\sigma_{YW}} \quad \lambda_B = 1 + \frac{E_B \Delta\epsilon_B}{\sigma_{YB}} \\ \Delta\epsilon_W &= 0.0375 \left(1 - \frac{\sigma_{YW}}{1000} \right) \quad \Delta\epsilon_B = 0.0375 \left(1 - \frac{\sigma_{YB}}{1000} \right) \end{aligned} \quad (2.3)$$

where $M = \sigma_{YW}/\sigma_{YB}$ and the corresponding F_{YM}/F_{YB} is evaluated at M .

For $L_r > 1$ (or $F > F_{YM}$), the following function is used up to $L_r = L_r^{\max}$;

$$f(L_r) = f(1) (L_r)^{(N_M - 1)/2N_M} \quad (2.4)$$

$$L_r^{\max} = \frac{1}{2} \left(1 + \frac{0.3}{0.3 - N_M} \right) \quad (2.5)$$

The strain hardening exponent for mismatched component, N_M , in eqn.(2.4) can be estimated as follows. The hardening exponent for the weld and base material, N_W and N_B , can be estimated from

$$N_W = 0.3 \left(1 - \frac{\sigma_{YW}}{\sigma_{UW}} \right) ; \quad N_B = 0.3 \left(1 - \frac{\sigma_{YB}}{\sigma_{UB}} \right) \quad (2.6)$$

The hardening behaviour of the mismatched component depends not only on the hardening capacity of individual material but also M and the resulting F_{YM}/F_{YB} :

$$N_M = \frac{(M-1)}{(F_{YM}/F_{YB} - 1)N_W + (M - F_{YM}/F_{YB})N_B} \quad (2.7)$$

Note that $M = \sigma_{YW}/\sigma_{YB}$ and the corresponding F_{YM}/F_{YB} is evaluated at M.

2.2.2 Lüders strain **NOT** expected for both base and weld materials

(Level 2-1) If both materials do not exhibit a Lüders plateau, then the function given by eqn. (2.2) can be extended up to $L_r = L_r^{\max}$, defined by eqn.(2.5):

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2 \right)^{-1/2} \cdot \left[0.3 + 0.7 \exp \left(-0.6 \cdot L_r^6 \right) \right] \quad (2.2)$$

$$L_r^{\max} = \frac{1}{2} \left(1 + \frac{0.3}{0.3 - N_M} \right) \quad (2.5)$$

Note that N_M is determined according to eqns. (2.6) and (2.7).

(Level 2-2) Alternatively, for $L_r \geq 1$,

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2 \right)^{-1/2} \cdot \left[0.3 + 0.7 \exp \left(-\mu_M \cdot L_r^6 \right) \right] \quad (2.8)$$

with μ_M calculated from

$$\mu_M = \frac{(M-1)}{(F_{YM}/F_{YB} - 1)\mu_W + (M - F_{YM}/F_{YB})\mu_B} \quad (2.9)$$

$$\mu_W = 0.001 \cdot \frac{E_W}{\sigma_{YW}} ; \quad \mu_B = 0.001 \cdot \frac{E_B}{\sigma_{YB}}$$

Note that when the calculated value of μ_M , μ_W , or μ_B can not exceed 0.6; 0 μ_M , μ_W , μ_B 0.6.

For $L_r > 1$, the equation is then continued by eqn. (2.4) up to $L_r = L_r^{\max}$, defined by eqn.(2.5):

$$f(L_r) = f(1) (L_r)^{(N_M - 1)/2 N_M} \quad (2.4)$$

$$L_r^{\max} = \frac{1}{2} \left(1 + \frac{0.3}{0.3 - N_M} \right) \quad (2.5)$$

In eqn. (2.4), the value $f(1)$ is evaluated from eqn.(2.8) with $L_r = 1$. The value of N_M is estimated from eqns. (2.6) and (2.7).

2.2.3 Lüders strain expected for only one material

This is the case when yield and ultimate stresses are available for both materials, and only one material is expected to exhibit a Lüders plateau.

For $L_r < 1$ (or $F > F_{YM}$), eqn.(2.8) is used with the term involving μ for the material with a Lüders plateau set to zero. For instance, if the weld metal does not have any Lüders strain, then

$$\mu_M = \frac{(M - 1)}{(F_{YM} / F_{YB} - 1)} \mu_W \quad ; \quad \mu_W = 0.001 \cdot \frac{E_W}{\sigma_{YW}} \quad (2.10)$$

At $L_r = 1$, the function f can be discontinuous and reduced to the value $f(1)$ given by eqn.(2.3) with $\lambda = 0$ for the continuous hardening material; for instance, if the weld metal does not have any Lüders strain, then

$$f(1) = \left(\lambda_M + \frac{1}{2\lambda_M} \right)^{-1/2} \quad (2.11)$$

$$\lambda_M = \frac{(M - F_{YM} / F_{YB}) \lambda_B}{(M - 1)} ; \lambda_B = 1 + \frac{0.0375 \cdot E_B}{\sigma_{YB}} \left(1 - \frac{\sigma_{YB}}{1000} \right)$$

For $L_r > 1$, the function given by eqn. (2.4) is used up to $L_r = L_r^{\max}$, defined by eqn.(2.5).

2.3 Level 3 (Full Stress-Strain Data Available)

Where full stress-strain data for both materials are available, the function

$$f(L_r) = \left(\frac{E \varepsilon^{(e)}}{\sigma^{(e)}} + \frac{0.5 L_r^2}{\left(E \varepsilon^{(e)} / \sigma^{(e)} \right)} \right)^{-1/2} \quad (2.12)$$

can be used up to $L_r = L_r^{\max}$, given in eqn. (2.14) below. In eqn. (2.12), $\varepsilon^{(e)}$ is the true strain obtained from the uniaxial stress strain curve for the *equivalent* material at a true stress $\sigma^{(e)} = L_r \sigma_{Ye}$. The stress strain curve for the equivalent material is given by

$$\sigma^{(e)}(\varepsilon^p) = \frac{(F_{YM}/F_{YB} - 1) \cdot \sigma_W(\varepsilon^p) + (M - F_{YM}/F_{YB}) \cdot \sigma_B(\varepsilon^p)}{(M - 1)} \quad (2.13)$$

Note that F_{YM}/F_{YB} in eqn. (2.13) is defined for $M = M(\varepsilon^p) = \sigma_W(\varepsilon^p)/\sigma_B(\varepsilon^p)$ at a number of plastic strain values ε^p . Accordingly, the cut-off $L_r^{\max}$ is defined by the equivalent yield stress σ_{Ye} and the equivalent flow stress $\bar{\sigma}_e$ as

$$L_r^{\max} = \bar{\sigma}_e / \sigma_{Ye} \quad (2.14)$$

$$\bar{\sigma}_e = \left[F_{YM}(\varepsilon^p) / F_{YB}(\varepsilon^p) \right] \cdot \sigma_B(\varepsilon^p); \quad \sigma_{Ye} = (F_{YM}/F_{YB}) \cdot \sigma_{YB}$$

2.4 Level 4 (J available)

Where a solution for J from mismatched geometry is available, the function f is defined by

$$f(L_r) = \left(\frac{J}{J_e} \right)^{-1/2} \quad (2.15)$$

up to $L_r = L_r^{\max}$ given by eqn. (2.14).

3. WORKED EXAMPLE

In this section, the various levels of assessment curves given in the previous section are compared. Using various “real” stress strain data, simulated welds were produced with an idealised weld configuration (rectangular, Fig. 2), having a wide range of mismatch ratios. The level 2 and 3 curves were generated according to the equations given in the previous section, whereas the level 4 curve was obtained from the J integral determined from the FE analysis.

Note that, unlike the homogeneous assessment curves, the assessment curves (levels 1,2 and 3) for the mismatch option are geometry dependent through the mismatch specific limit load solution. In the examples shown in this section, the centre cracked tensile (CCT) plate with $a/W=0.5$ and $(W-a)/H=3$ was considered, of which the geometry is depicted in Fig. 2. Both plane strain and plane stress conditions were considered.

Figures 3a-3c show the results for the case where both materials exhibit Lüders strains. Figures 4a-4c show the results for the case where both materials do not exhibit Lüders strains. It can be seen that the higher level curves lead to less conservatism. It should be noted that the tendency is very similar to that of the homogeneous SINTAP assessment curves.

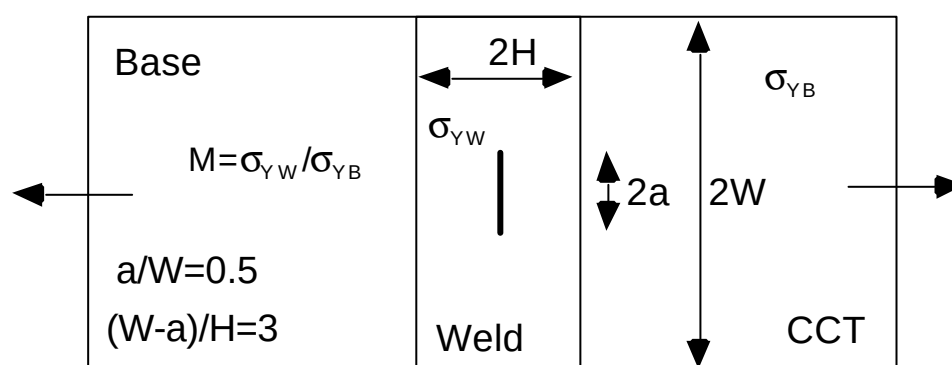


Fig. 2

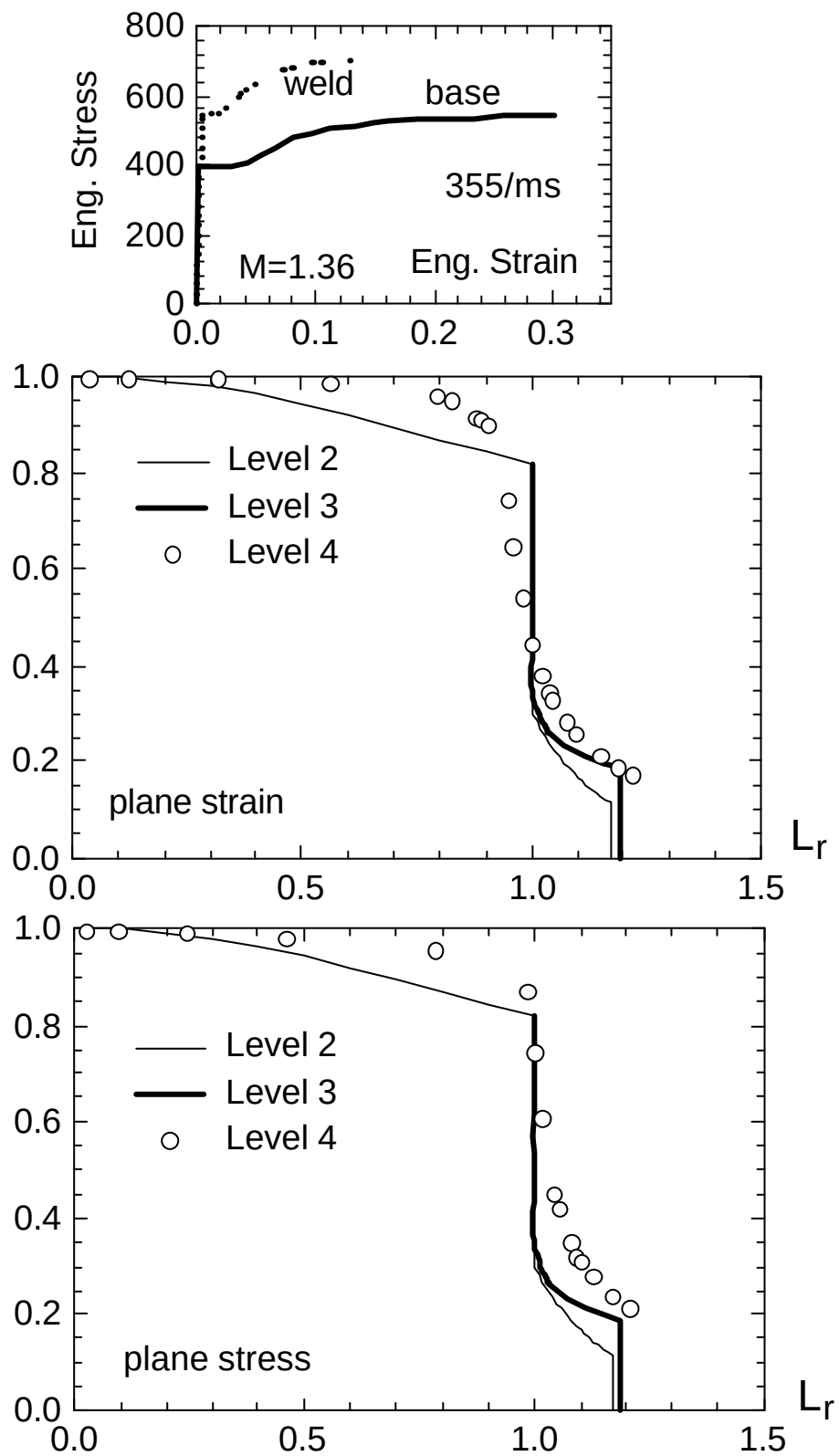


Fig. 3a

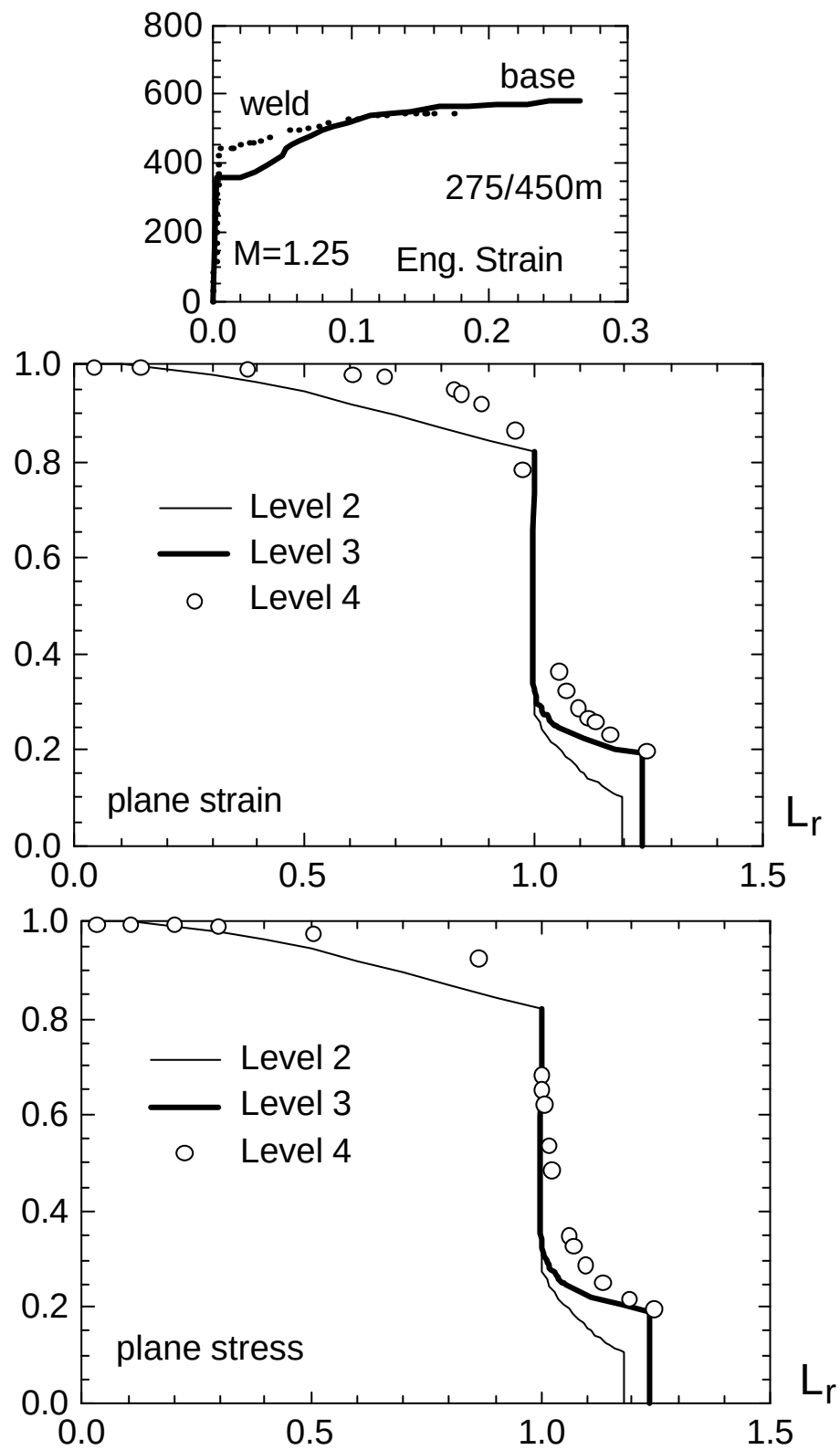


Fig. 3b

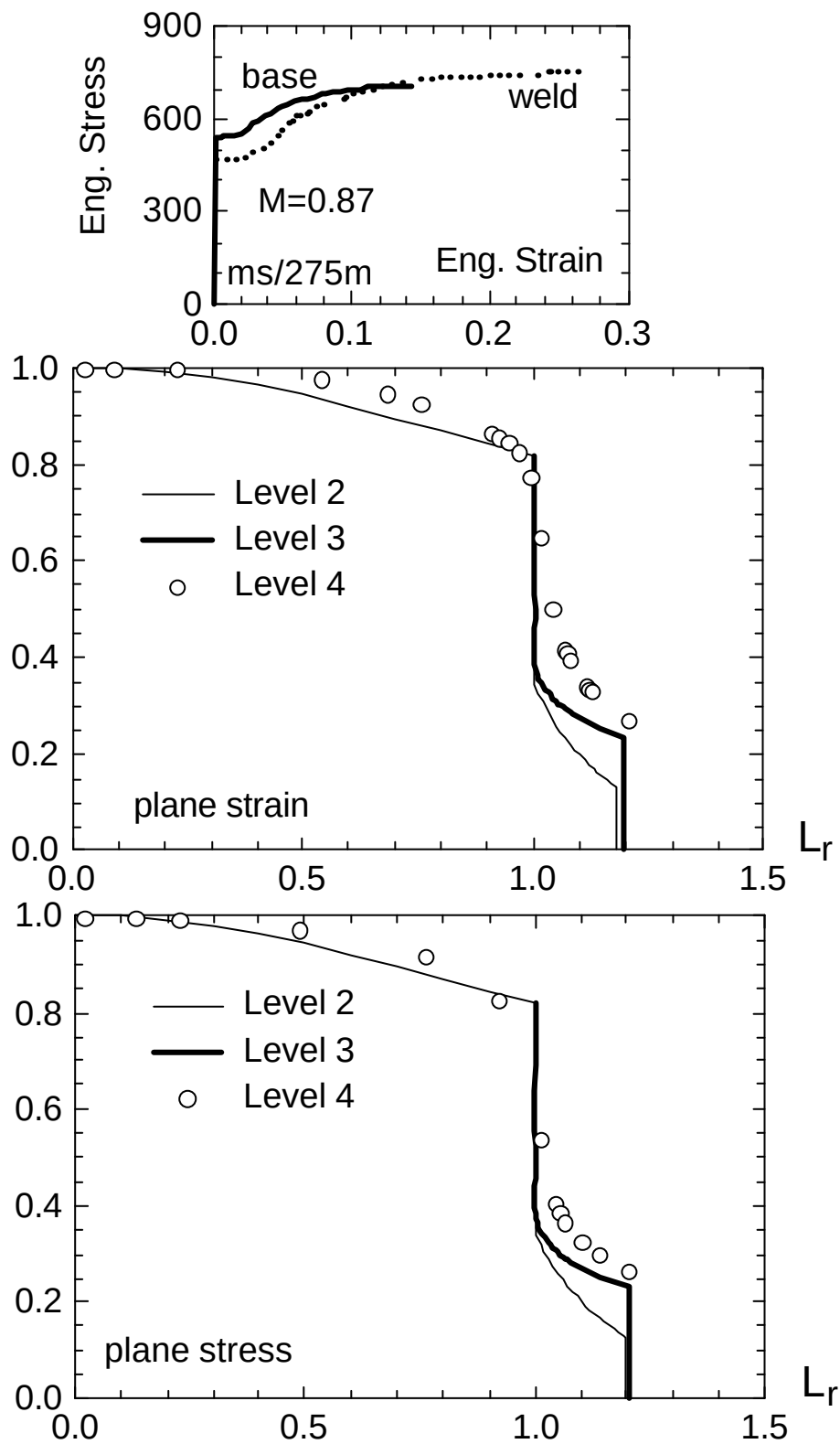


Fig. 3c

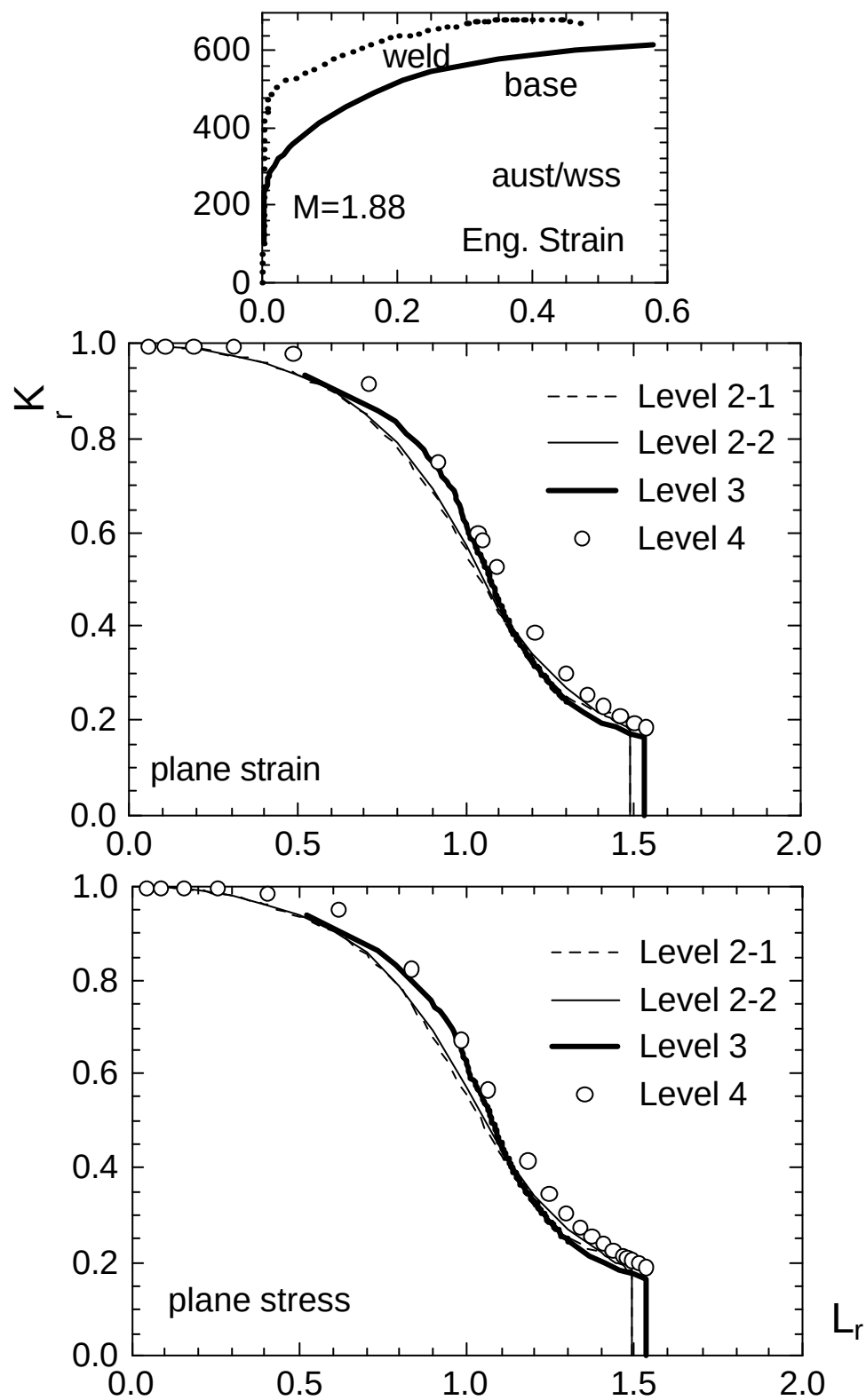


Fig. 4a

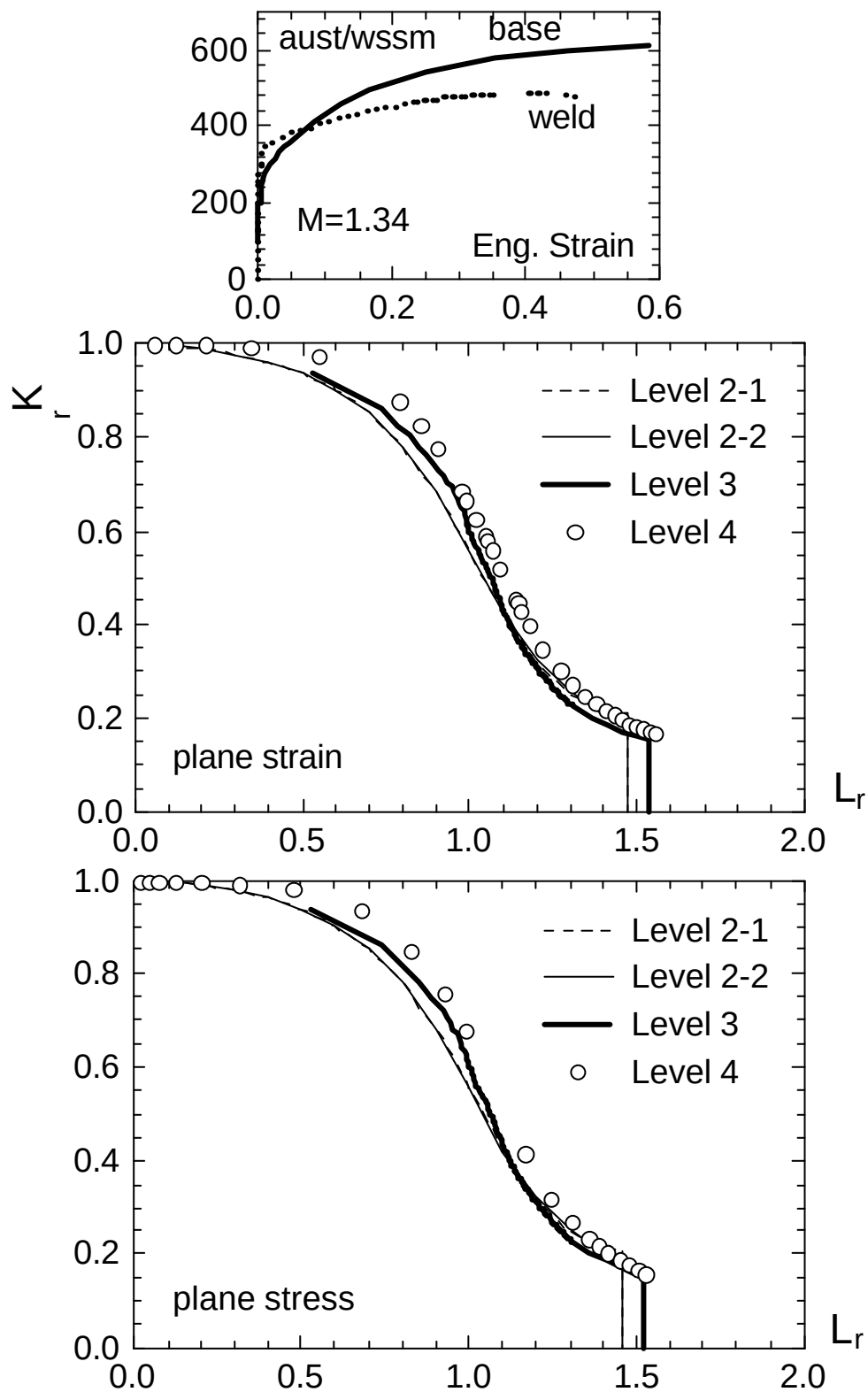


Fig. 4b

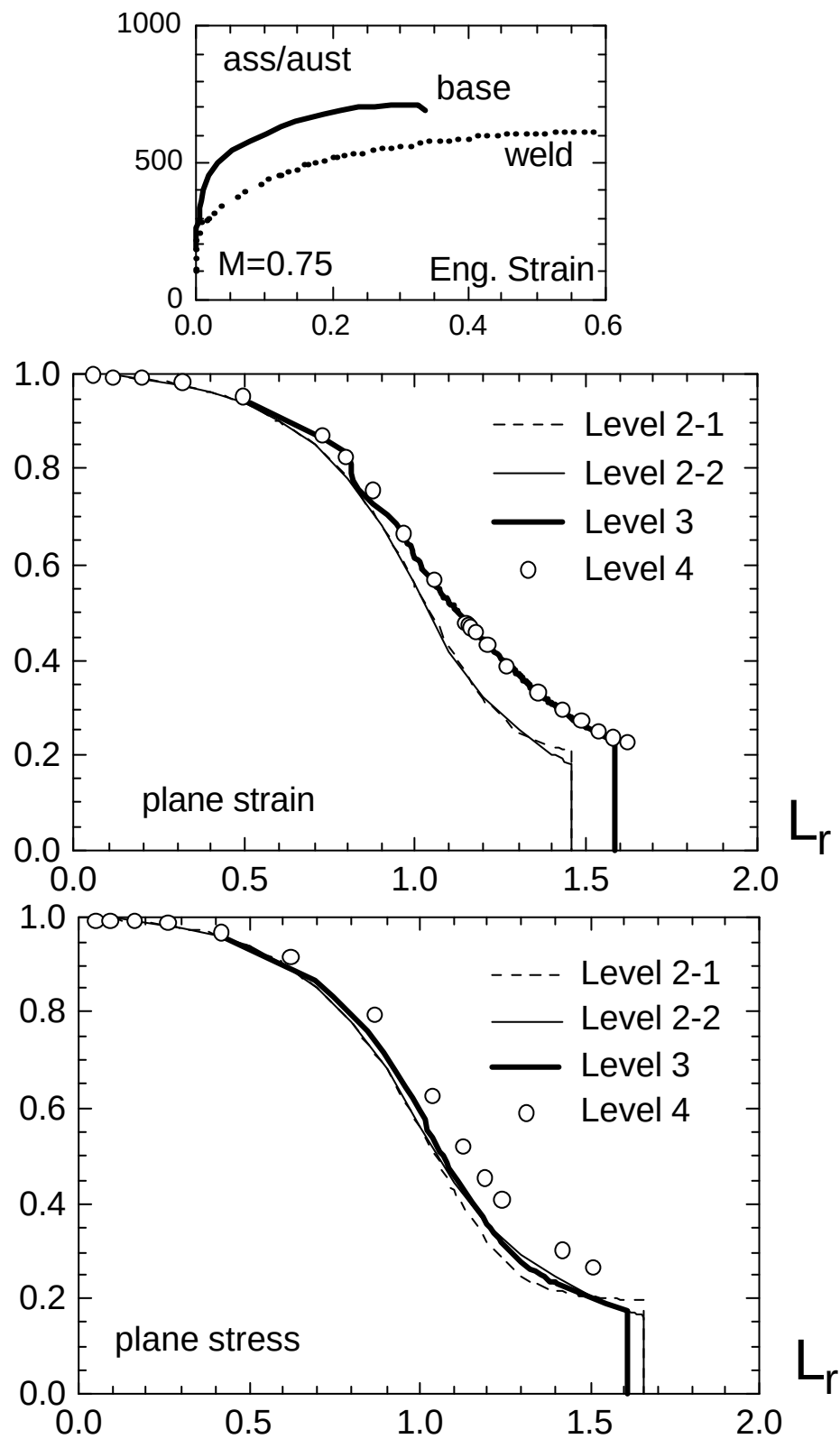


Fig. 4c

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APPENDIX : SUMMARY OF ASSESSMENT EQUATIONS

- SINTAP MISMATCH OPTION

Level 1

- Lüders strain expected for both base and weld metal

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2\right)^{-1/2} \quad \text{for} \quad L_r \leq L_r^{\max} = 1$$

- Lüders strain not expected for both base and weld metal

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2\right)^{-1/2} \cdot \left[0.3 + 0.7 \exp\left(-0.6 \cdot L_r^6\right)\right] \quad \text{for} \quad L_r \leq L_r^{\max} = 1 + \left(\frac{150}{Y_S}\right)^{2.5}$$

- Lüders strain expected only for one material

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2\right)^{-1/2} \cdot \left[0.3 + 0.7 \exp\left(-0.6 \cdot L_r^6\right)\right] \quad \text{for} \quad L_r \leq L_r^{\max} = 1$$

Level 2

- Lüders strain expected for both base and weld metal

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2\right)^{-1/2} \quad \text{for} \quad L_r < 1$$

$$f(1) = \left(\lambda_M + \frac{1}{2\lambda_M}\right)^{-1/2} \quad \text{for} \quad L_r = 1$$

$$\lambda_M = \frac{(F_{YM}/F_{YB} - 1)\lambda_W + (M - F_{YM}/F_{YB})\lambda_B}{(M - 1)}$$

$$\lambda_W = 1 + \frac{0.0375 \cdot E_W}{\sigma_{YW}} \left(1 - \frac{\sigma_{YW}}{1000}\right) \quad \lambda_B = 1 + \frac{0.0375 \cdot E_B}{\sigma_{YB}} \left(1 - \frac{\sigma_{YB}}{1000}\right)$$

$$f(L_r) = f(1) \cdot (L_r)^{(N_M - 1)/2N_M} \quad \text{for} \quad 1 < L_r \leq L_r^{\max} = \frac{1}{2} \left(1 + \frac{0.3}{0.3 - N_M}\right)$$

$$N_M = \frac{(M - 1)}{(F_{YM}/F_{YB} - 1)/N_W + (M - F_{YM}/F_{YB})/N_B}$$

$$N_W = 0.3 \left(1 - \frac{\sigma_{YW}}{\sigma_{UW}}\right); \quad N_B = 0.3 \left(1 - \frac{\sigma_{YB}}{\sigma_{UB}}\right)$$

- Lüders strain not expected for both base and weld metal

Level 2-1

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2\right)^{-1/2} \cdot \left[0.3 + 0.7 \exp\left(-0.6 \cdot L_r^6\right)\right] \quad \text{for} \quad L_r \leq L_r^{\max} = \frac{1}{2} \left(1 + \frac{0.3}{0.3 - N_M}\right)$$

Level 2-2

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2\right)^{-1/2} \cdot \left[0.3 + 0.7 \exp\left(-2\mu_M \cdot L_r^6\right)\right] \quad \text{for} \quad L_r \leq 1$$

$$\mu_M = \frac{(M-1)}{(F_{YM}/F_{YB} - 1)\mu_W + (M - F_{YM}/F_{YB})\mu_B}$$

$$\mu_W = 0.001 \cdot \frac{E_W}{\sigma_{YW}} \quad ; \quad \mu_B = 0.001 \cdot \frac{E_B}{\sigma_{YB}}$$

$$f(L_r) = f(1) \cdot (L_r)^{(N_M - 1)/2N_M} \quad \text{for} \quad 1 < L_r \leq L_r^{\max} = \frac{1}{2} \left(1 + \frac{0.3}{0.3 - N_M}\right)$$

$$N_M = \frac{(M-1)}{(F_{YM}/F_{YB} - 1)N_W + (M - F_{YM}/F_{YB})N_B}$$

$$N_W = 0.3 \left(1 - \frac{\sigma_{YW}}{\sigma_{UW}}\right) \quad ; \quad N_B = 0.3 \left(1 - \frac{\sigma_{YB}}{\sigma_{UB}}\right)$$

- Lüders strain expected only for one material, for instance, if the weld metal does not have a Lüders plateau,

$$f(L_r) = \left(1 + \frac{1}{2} L_r^2\right)^{-1/2} \cdot \left[0.3 + 0.7 \exp\left(-2\mu_M \cdot L_r^6\right)\right] \quad \text{for} \quad L_r < 1$$

$$\mu_M = \frac{(M-1)}{(F_{YM}/F_{YB} - 1) \mu_W} \quad ; \quad \mu_W = 0.001 \cdot \frac{E_W}{\sigma_{YW}}$$

$$f(1) = \left(\lambda_M + \frac{1}{2\lambda_M}\right)^{-1/2} \quad \text{for} \quad L_r = 1$$

$$\lambda_M = \frac{(M - F_{YM}/F_{YB}) \lambda_B}{(M-1)} \quad ; \quad \lambda_B = 1 + \frac{0.0375 \cdot E_B}{\sigma_{YB}} \left(1 - \frac{\sigma_{YB}}{1000}\right)$$

$$f(L_r) = f(1) \cdot (L_r)^{(N_M - 1)/2N_M} \quad \text{for} \quad 1 < L_r \leq L_r^{\max} = \frac{1}{2} \left(1 + \frac{0.3}{0.3 - N_M}\right)$$

$$N_M = \frac{(M-1)}{(F_{YM}/F_{YB} - 1) N_W + (M - F_{YM}/F_{YB}) N_B}$$

$$N_W = 0.3 \left(1 - \frac{\sigma_{YW}}{\sigma_{UW}}\right) \quad ; \quad N_B = 0.3 \left(1 - \frac{\sigma_{YB}}{\sigma_{UB}}\right)$$

Level 3

$$f(L_r) = \left(\frac{E \varepsilon^{(e)}}{\sigma^{(e)}} + \frac{0.5 L_r^2}{(E \varepsilon^{(e)} / \sigma^{(e)})}\right)^{-1/2} \quad \text{for} \quad L_r \leq L_r^{\max}$$

$$\sigma^{(e)}(\varepsilon^p) = \frac{(F_{YM}/F_{YB} - 1) \cdot \sigma_W(\varepsilon^p) + (M - F_{YM}/F_{YB}) \cdot \sigma_B(\varepsilon^p)}{(M-1)}$$

$$L_r^{\max} = \bar{\sigma}_e / \sigma_{ye}$$

$$\bar{\sigma}_e = \left[F_{YM}(\varepsilon^p) / F_{YB}(\varepsilon^p)\right] \cdot \sigma_B(\varepsilon^p) \quad ; \quad \sigma_{ye} = (F_{YM}/F_{YB}) \cdot \sigma_{YB}$$

Level 4

$$f(L_r) = \left(\frac{J}{J_e}\right)^{-1/2} \quad \text{for} \quad L_r \leq L_r^{\max}$$

$$L_r^{\max} = \bar{\sigma}_e / \sigma_{ye}$$

$$\bar{\sigma}_e = \left[F_{YM}(\varepsilon^p) / F_{YB}(\varepsilon^p)\right] \cdot \sigma_B(\varepsilon^p) \quad ; \quad \sigma_{ye} = (F_{YM}/F_{YB}) \cdot \sigma_{YB}$$