

Feed Tray

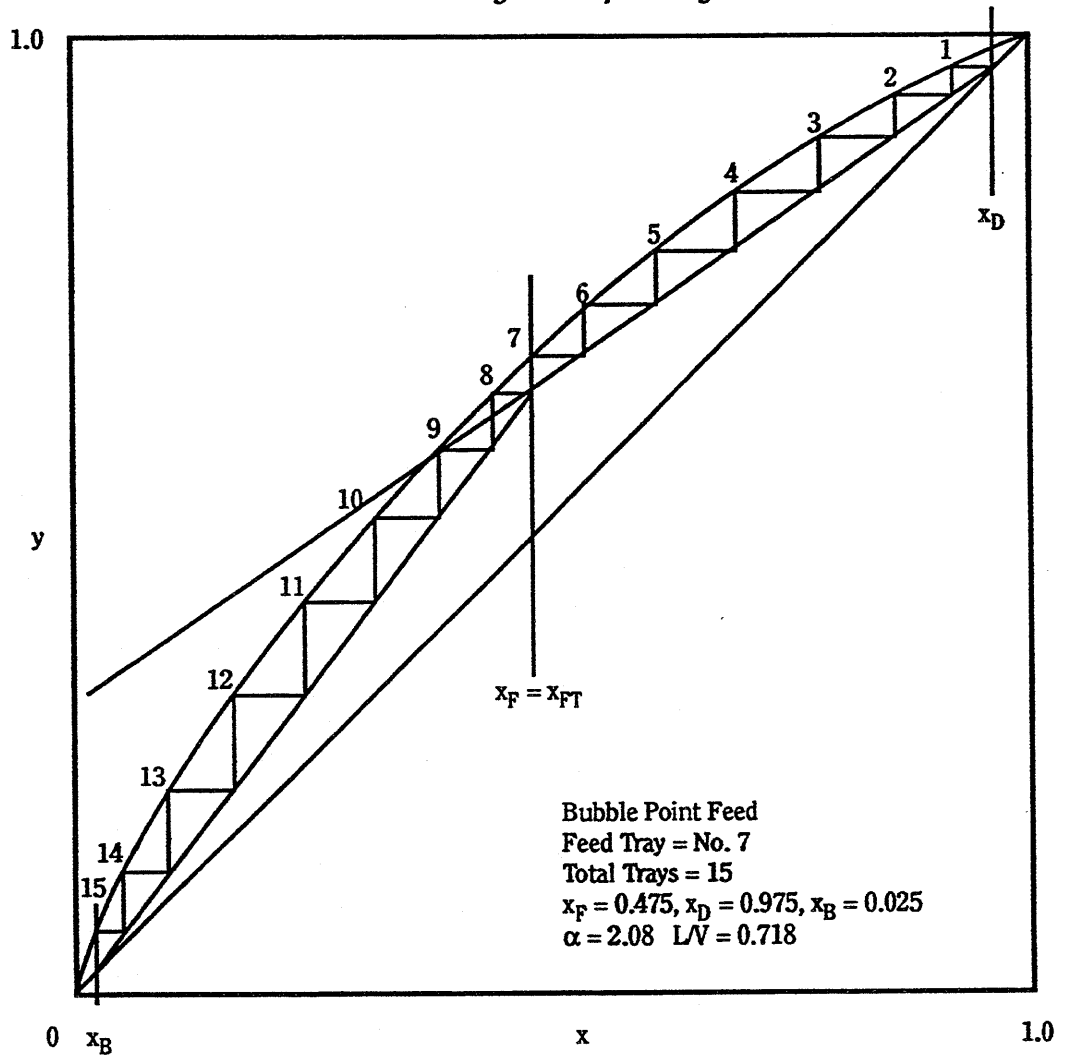
When stepping off trays on a McCabe-Thiele diagram, one automatically gets the correct feed tray.

The slope of the operating line always makes its shift from the upper to the lower lines at the tray where the feed is introduced. A "Feed Mismatch" occurs for bubble point feed when the feed composition, x_F , is not equal to x_{FT} . When the feed is not introduced on the proper tray, either more trays are required or poorer products will be made.

In stepping off the trays, go to the lower operating line when $x = x_{FT}$ is reached (bubble point feed only).

Figure 8

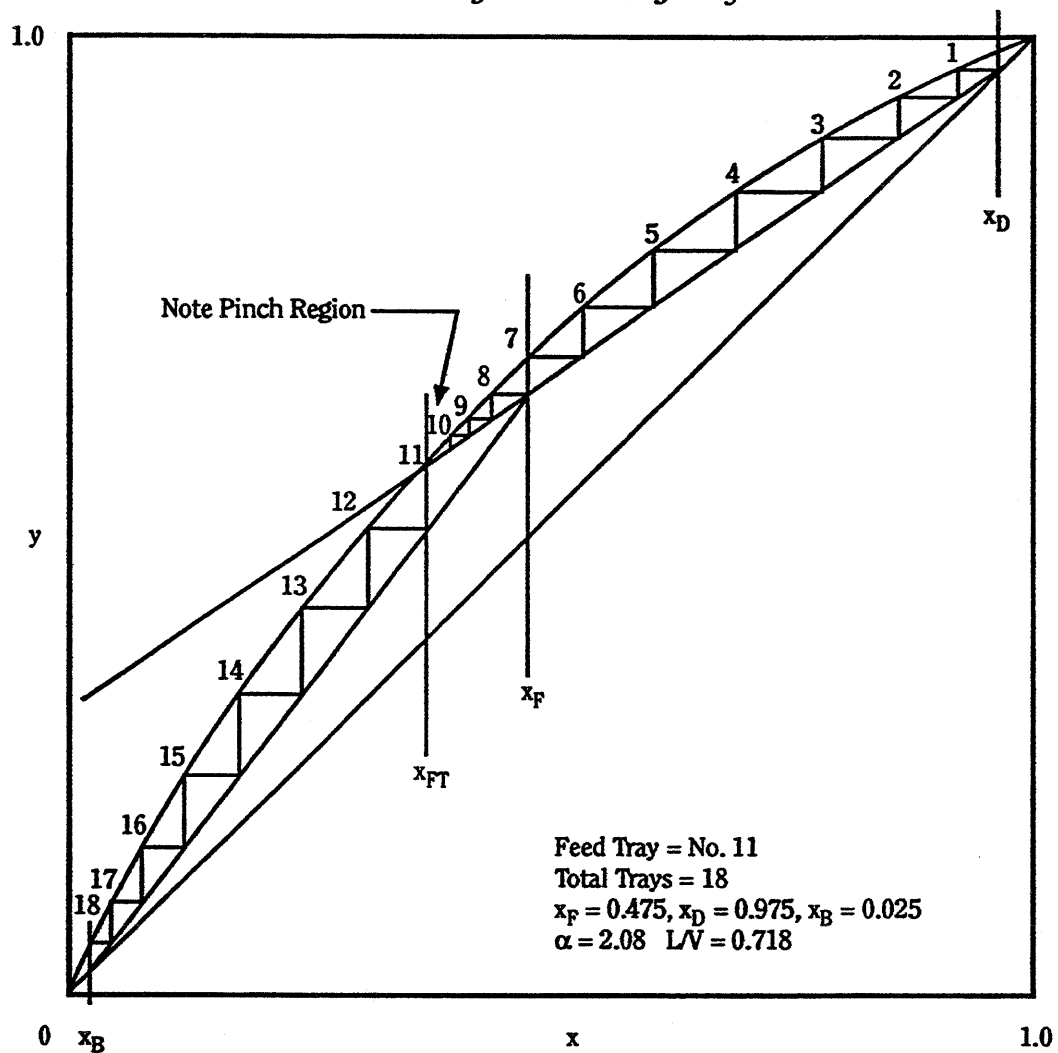
Feeding on Proper Tray



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Figure 9

Feeding on the Wrong Tray

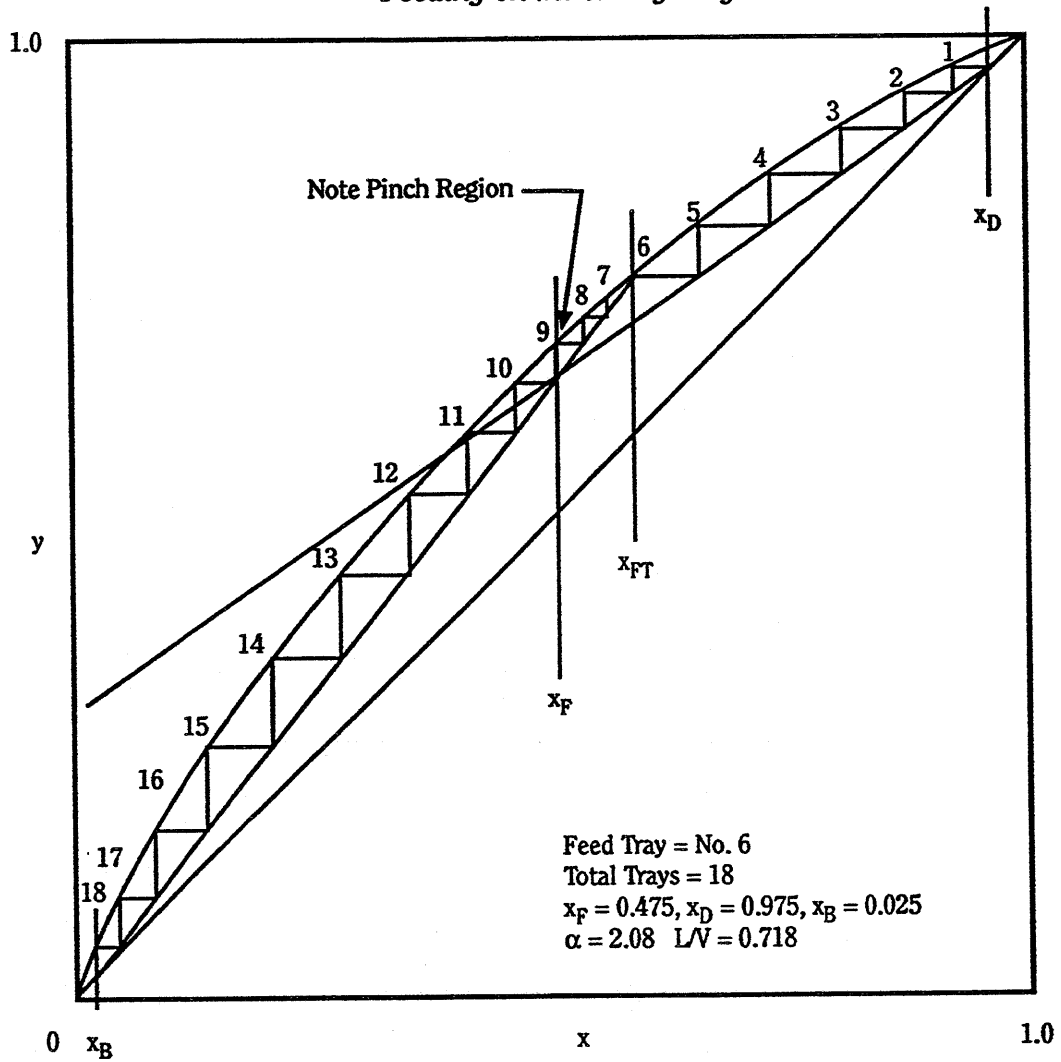


Feed is lighter than feed tray liquid.

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Figure 10

Feeding on the Wrong Tray



Feed is heavier than feed tray liquid.

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Log-Log Plots

Introduction

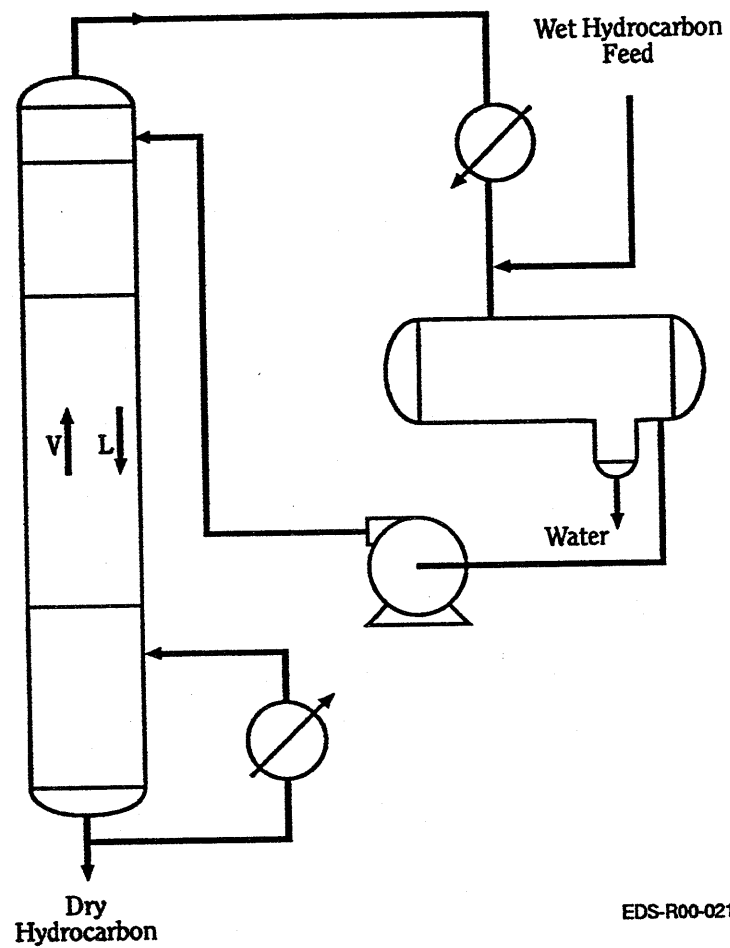
If a distillate of extreme purity is required, log-log paper should be used, plotting $1/(1-y)$ against $1/(1-x)$ in the top section. Thus, if the distillate is to be 99.99% purity, the value of $1/(1-x)$ is 10,000. The scale opens up to accommodate such high purity. On the other hand, if extreme purity of the bottoms is needed, for the same reason a plot of y against x is useful (example, distillation drying). The intercept on the ordinate will be a and the slope will be 1.0 (45° angle). On log-log paper, the operating lines are never really straight, but often very close to it. So, when using log-log paper, the operating lines are plotted by calculating a number of points.

Distillation Drying of Hydrocarbons

Distillation drying is an example of a class of commercial fractionators where binary distillation analysis and McCabe-Thiele diagrams are used in design. It also illustrates the use of a McCabe-Thiele diagram in non-rectangular coordinates.

Figure 11

Typical Distillation Drying Column

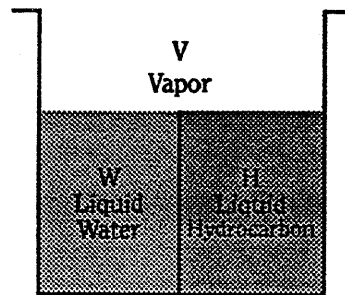


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Equilibrium Relationship

Consider a vapor phase, a liquid water phase, and a liquid hydrocarbon phase to all be in equilibrium at a constant temperature.

Figure 12



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K_w , equilibrium constant for water between phases V and H is:

$$K_w = \frac{(1 - x_{hw})p^o}{x_{wh}\pi}$$

The vapor and liquid hydrocarbon phase water concentrations are small, which allows some simplifications. With the hydrocarbon phase regarded as a pure component, the K of the hydrocarbon is essentially 1.0, thus giving $\alpha = K_w/K_h = K_w$. Furthermore, as $x \rightarrow 0$ and $y \rightarrow 0$, the equilibrium line equation becomes:

$$y = \frac{\alpha x}{1 + (\alpha - 1)x} \quad x \rightarrow 0, \quad y \rightarrow 0, \quad y = \alpha x$$

therefore; $y = K_w x$

McCabe-Thiele Diagram

Because of the wide range from ambient temperature water saturation (as high as 10,000 ppm) to the dryness required (as low as 1 ppm), the use of log-log coordinates allows for a more convenient determination of the number of plates. The equilibrium relationship will be a straight line on log-log paper.

$$\ln y = \ln x + \ln \alpha$$

The stripper section material balance relationship should be used.

$$y = (L/V)x - (B/V)x_B$$

Note:

$$L/V = \frac{V+B}{V} = 1 + \frac{B}{V}$$

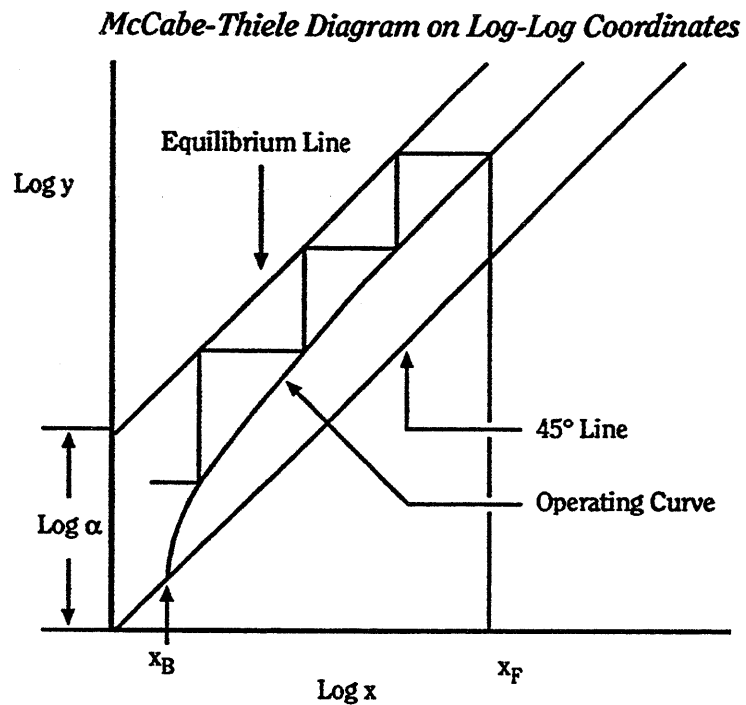
$$B/V = (L/V - 1)$$

Substitution gives $y = (L/V)x - (L/V - 1)x_B$

On log-log paper, this operating line will not be straight, although where $x \gg x_B$, the curve will approach a straight line. Plotting the operating line is best done by curve fitting a series of calculated points.

The relationships between internal L/V and external reflux and reboiler duty are determined by the appropriate heat and material balances.

Figure 13



Analytical Shortcut to Trays Required

(Conservative Estimate)

Assume Operating Line is straight $y = (L/V)x$

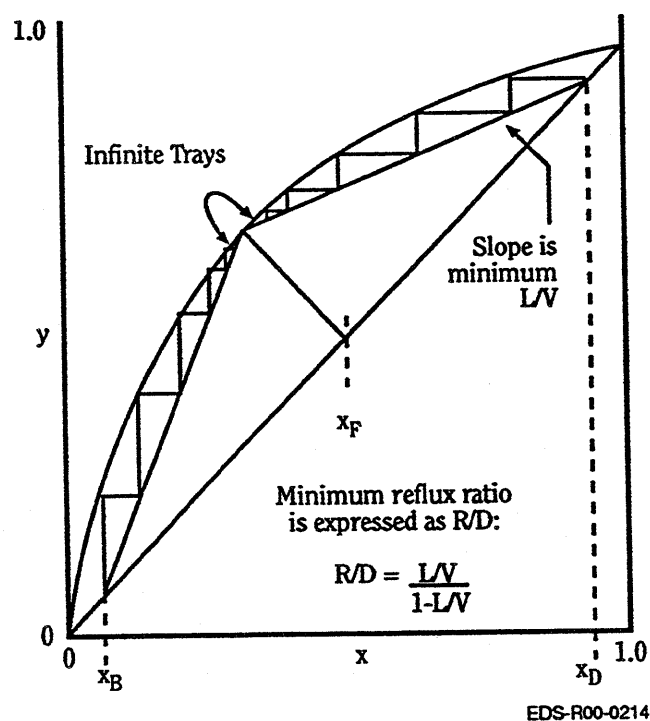
then:

$$n = \frac{\ln(x_F/x_B)}{\ln(\alpha V/L)}$$

Minimum Reflux/Infinite Trays

There is such a thing as a minimum reflux ratio below which it is impossible to achieve the required product purities for a given feed composition. This will be true regardless of how many plates are installed. Minimum reflux ratio is represented by that operating line which intersects the equilibrium curve with the feed line. The expression "operating line" has no logical etymology. It is merely a term used to show reflux relationships and, since these lines can be varied during operation, they get that name. The condition of minimum reflux is shown on McCabe-Thiele diagram below.

Figure 14



Minimum reflux ratio is expressed as R/D :

$$R/D = \frac{L/V}{1-L/V}$$

- Notes:
1. Minimum reflux is a function of feed thermal condition.
 2. Minimum reflux occurs whenever an operating line intersects, is coincident to, or is tangent to the equilibrium curve.

ANALYTICAL TECHNIQUES

Minimum Reflux/Infinite Trays

For the general case, the **Underwood equations** (constant molal overflow and relative volatility are assumed) can be used.

- (a) Find ϕ value that satisfies:

$$\frac{1-x_F}{1-\phi} + \frac{\alpha x_F}{\alpha-\phi} = 1-q \text{ (correct } \phi \text{ is between 1 and } \alpha\text{)}$$

- (b) With determined ϕ value, calculate minimum reflux:

$$(L/D)_m = \left[\frac{1-x_D}{1-\phi} \right] + \frac{\alpha x_D}{\alpha-\phi} - 1$$

Nomenclature:

ϕ = Underwood parameter

x_F, x_D = Light component mol fraction in total feed, total distillate

$(L/D)_m$ = Molal ratio of minimum internal reflux to total distillate

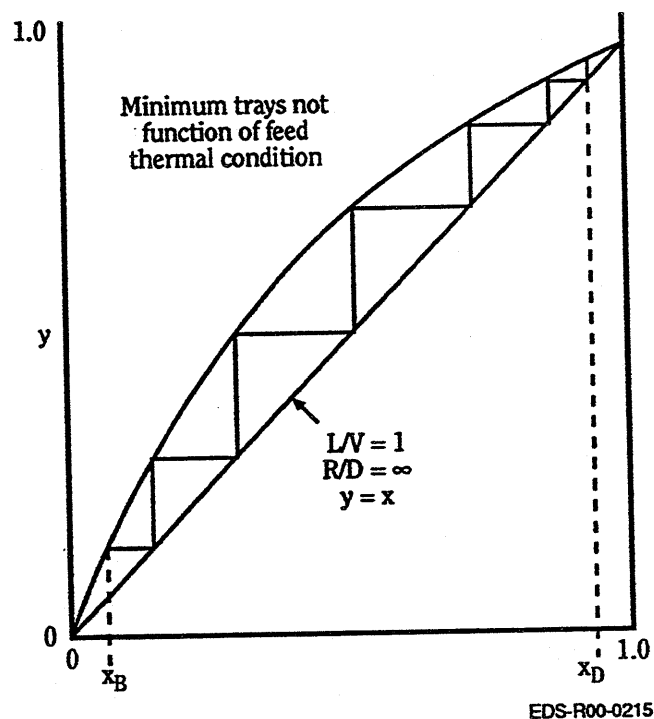
- (c) For a bubble point feed, the Underwood equations reduce to:

$$(L/D)_m = \frac{1}{(\alpha-1)} \left[\frac{x_D}{x_F} - \frac{\alpha(1-x_D)}{(1-x_F)} \right]$$

Minimum Trays/Total Reflux

If the feed to the column is shut off and no net overhead (D) or bottoms (B) is taken off, and if all of the material accumulating in the receiver is pumped back as reflux (just enough to heat balance the reboiler), we have a condition known as total reflux. This often occurs in a refinery when feed is temporarily unavailable. Under these conditions, the L/V in the top and bottom sections will be equal to 1.0. The operating line will then coincide with the 45° diagonal (where $x = y$). The trays will then produce some particular finite overhead and bottoms product purities. The condition of total reflux and minimum theoretical trays is shown on a McCabe-Thiele diagram below:

Figure 15



Note: Minimum trays not a function of feed thermal conditions.

The minimum number of theoretical trays at total reflux can be calculated from the Fenske relation.

$$n_m = \frac{\ln(r_D/r_B)}{\ln \alpha}$$

Where r_D is the ratio of the mols of the components in the distillate and r_B the ratio in the bottoms.

Feed Tray

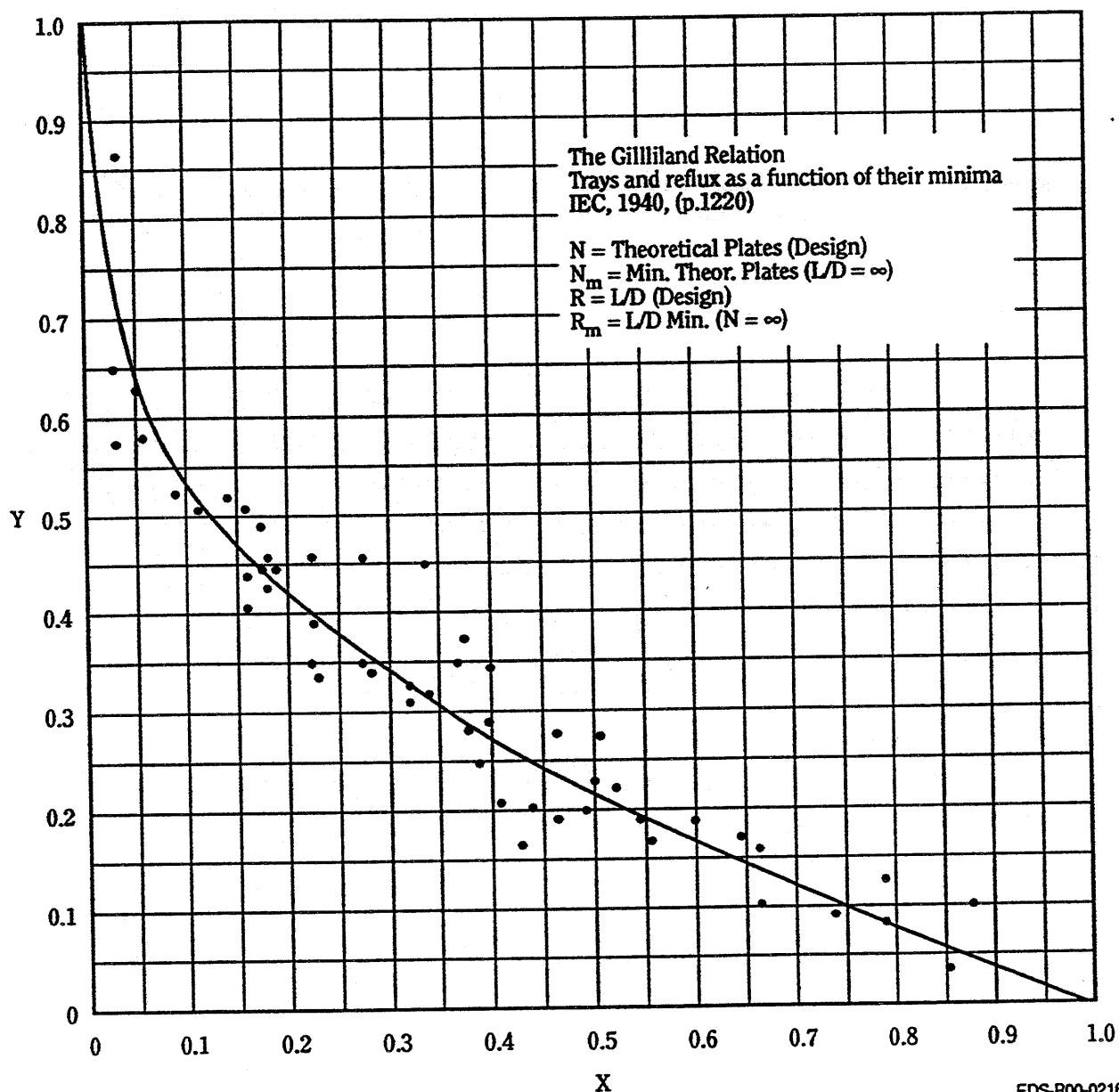
The relative feed tray location can be estimated by utilizing the Fenske equation with substitution of the feed composition for the bottoms composition. This will determine the minimum theoretical trays required for the rectifying section (assuming bubble point feed). The ratio of minimum rectifying trays to minimum total trays can be calculated and applied to the real trays actually installed for location of the feed point.

$$(a) \quad n_r = \frac{\ln(r_D/r_F)}{\ln \alpha}$$

$$(b) \quad n_r/n = \frac{\ln(r_D/r_F)}{\ln(r_D/r_B)} \quad \text{Note: Not a function of } \alpha$$

One interesting point is that as $X_D \rightarrow 1.0$, $X_B \rightarrow 1.0$, $n_r/n \rightarrow 0.5$. This leads to the maxim – "when in doubt, put the feed in the middle."

Figure 16



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$$X = \frac{R - R_m}{R + 1}$$

$$R = \frac{R_m + X}{1 - X}$$

Determine:

R_m from Underwood Equations

N_m from Fenske Equation

$$Y = \frac{N - N_m}{N + 1}$$

$$N = \frac{N_m + Y}{1 - Y}$$

Design Reflux and Trays

Between the limits of minimum trays/total reflux and minimum reflux/infinite trays, there are an infinite number of combinations of reflux and trays which can be used to effect the separation. The optimum ratio of design reflux to minimum reflux usually falls within the range of 1.05–1.50. The proper ratio depends on tangible factors such as utility costs vs capital costs and intangible factors such as the accuracy of equilibrium data and calculation methods.

For estimating, and as a starting point for more detailed rigorous design, a knowledge of the relationship between design reflux/trays as a function of minimum reflux/trays is useful. The first correlation that gained acceptance was one by Gilliland. This was based on many tray to tray calculations. Other correlations have since been published purported to be more accurate (Erber-Maddox considered the best) but, since any answers are still approximate, the Gilliland correlation is still preferred by most UOP engineers.

Reflux to Feed Ratio

Most textbooks and articles define the term "reflux ratio" as the ratio of reflux to distillate, L/D . The drawback to using this parameter as a reference is that for fixed overhead purity, x_D , the required L/D varies greatly as the feed composition, x_F , changes. For fractionation systems producing high purity product, that is as $x_D \rightarrow 1.0$, $x_B \rightarrow 0.0$, we find that $D/F \rightarrow x_F$. If we look at the Underwood equation for bubble point feed:

$$(a) \quad (L/D)_m = \frac{1}{x_F(\alpha-1)} \text{ when } x_D = 1.0 \text{ and } q = 1.0.$$

For fractionation systems producing high purity product, that is as $x_D \rightarrow 1.0$, $x_B \rightarrow 0.0$, we find:

$$(b) \quad D/F = x_F$$

Combining equation (a) and (b) leads to the relationship:

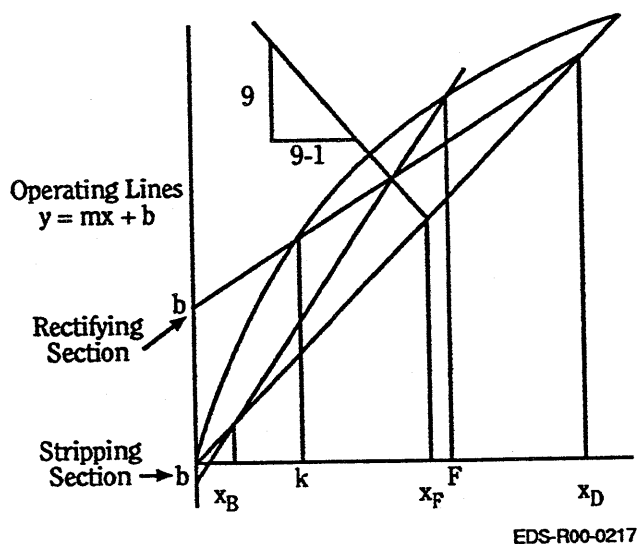
$$(L/F)_m = \frac{1}{\alpha - 1} \text{ pure products and bubble point feed}$$

Thus, L/F is constant for perfect fractionation and bubble point feed. Many commercial fractionators are designed and operated for conditions not that far from the above assumptions. This is the reason that UOP prefers to define "reflux ratio" as the ratio of reflux to feed; fractionators such as Gas Con Debutanizers, Platformer Stabilizers, Depropanizers, etc., can be classified as having so many trays and such a reflux ratio almost regardless of the feed source, feed composition, and operating severity.

Smoker's Method

There are a number of analytic computation methods suitable for a programmable calculator that can be used to solve distillation problems. Following, as an example, is a summary of the equations proposed by E. H. Smoker that can be programmed to solve binary distillation problems (a copy of Smoker's original article is appended). Smoker's method is essentially an analytical solution of the McCabe-Thiele method and is, therefore, subject to many of the same limitations.

Figure 17



(a) $m = L/V$ rectifying section

$m = L'/V'$ stripping section

(b) $b = x_D(1-m)$ rectifying section

$b = x_B(1-m)$ stripping section

(c)
$$A = \frac{m + b(\alpha - 1) - \alpha}{m(\alpha - 1)}$$

$$B = \frac{b}{m(\alpha - 1)}$$

(d) $K^2 + AK + B = 0$

$$K = \frac{-A}{2} \pm \sqrt{\frac{A^2}{4} - B} \quad 0 < K < 1$$

(e) $C = 1 + K(\alpha - 1)$

(f) $E = mc^2/\alpha$

(g) $x'_1 = x_1 - K$

$$x'_2 = x_2 - K$$

(h)
$$D = \frac{E(\alpha - 1)}{C(1 - E)}$$

(i) $d = Dx'_1, x'_2$

$$(j) \quad \Delta S = \frac{\ln \frac{x'_2 - d}{x'_1 - d}}{\ln E}$$

$$(k) \quad S = \sum \Delta S$$

Procedure

Calculate parameters for equations (a) – (f) for the rectifying section.

Set $x_1 = x_D$, choose $x_2 \geq x_F$.

Solve equations (g) – (j); ΔS is then the number of stages required to go from a liquid purity of x_1 to a liquid purity of x_2 .

If x_2 was not chosen equal to x_F , repeat the third step in increments whereby x_1 equals previous x_2 and a new x_2 is chosen greater than or equal to x_F .

NOTE: Generally, x_2 can be set equal to x_F in the second step and incremental calculations are not necessary; however, if a or L/V varies significantly with liquid composition, it is desirable to proceed in stages but recalculating equations (a) – (f) each time.

Sum the incremental ΔS to get the number of trays required for the rectifying section.

Repeat the above procedure for the stripping section starting with $x_1 = x_F$ and proceeding until $x_2 = x_B$.

TRAY EFFICIENCY

After running a McCabe-Thiele diagram on a particular binary and the number of theoretical trays are found, the problem of tray efficiency still remains. The question is still asked, "How many real trays shall I use?" The problem has never been solved. Tray efficiency is definitely dependent on system properties, size of unit, tray geometry and certain velocities in the column. With all the research work attempting to correlate something against tray efficiency for the last half century, there remains no answer. There seems to be one system property that might indicate a trend. We have never seen high efficiencies in systems with high values of α , i.e., from 4.0 to 20.0. Also, conversely, the only time we have seen high efficiencies (but not always) was for systems with low values of α – such as in isomer separations. Tray efficiencies vary between 10–140 pct over the very wide range of units which we have seen. Experience is the only reliable basis for setting tray efficiencies.

Typical Observed Tray Efficiency (Simplistic Overall)

Xylene Isomer Fractionators	80-100
Deisopentanizers/Deisobutanizers	80-100
Benzene/Toluene/Xylene	75-80
Depropanizers/Debutanizers	75-80
Naphtha Fractionators	65-85
High Pressure Deethanizers	50-60
Low Pressure Drop Columns	40-60
Distillation Dryers	15
Gas Strippers	7-10
Gas Con Absorbers	
Primary	30-35
Sponge	20-25
Benzene/Cumene Splitters	40-50

General Guide for Tray Efficiency as Function of α Alone

Alpha	Tray Efficiency
1.2	90
2.0	70
3.0	50
5.0	20
15.0	10

SUMMARY OF EQUATIONS

(a) $F = D + B$

(b) $x_1 + x_2 = 1$

$y_1 + y_2 = 1$

(c) $x_F F = x_D D + x_B B$

(d) $\frac{D}{F} = \frac{x_F - x_B}{x_D - x_B}$

(e) $y^{n+1} = (L/V)x^n + (D/V)x_D$ (Rectifying Section)

$y^{n+1} = (L'/V')x^n - (B/V')x_B$ (Stripping Section)

(f) $q = \frac{H_F^V - H_F}{\lambda_F}$

(g) $L' = L + qF$

(h) $y^{n+1} = [q/(q-1)]x^n - [1/(q-1)]x_F$ (Feed Tray)

(i) $K = y/x$

$$(j) \quad \alpha = K_1/K_2$$

$$(k) \quad x = \frac{1-K_2}{K_1-K_2}$$

$$y = K_1 x$$

$$(l) \quad y = \frac{\alpha x}{1+(\alpha-1)x}$$

$$(m) \quad L/D = \frac{L/V}{1-L/V}$$

$$L/V = \frac{L/D}{1+L/D}$$

$$(n) \quad r_D = x_D/(1-x_D)$$

$$r_B = x_B/(1-x_B)$$

$$(o) \quad \frac{1-x_F}{1-\phi} + \frac{\alpha x_F}{\alpha-\phi} = 1-q \quad \text{Correct } \phi \text{ is between } 1 \text{ and } \alpha$$

$$(L/D)_m = \frac{1-x_D}{1-\phi} + \frac{\alpha x_D}{\alpha-\phi} - 1$$

$$(p) \quad n_m = \frac{\ln(r_D/r_B)}{\ln \alpha}$$

$$n_r/n = \frac{\ln(r_D/r_F)}{\ln(r_D/r_B)}$$

BINARY DISTILLATION

Problems

1. Feed is 54 mol pct propane, 46 mol pct isobutane. Desired overhead purity is 97 mol pct propane, desired bottoms purity is 95.5 mol pct isobutane. Assume:

Column pressure = 250 psia = constant

Reflux is not subcooled

Overhead product is withdrawn as a liquid

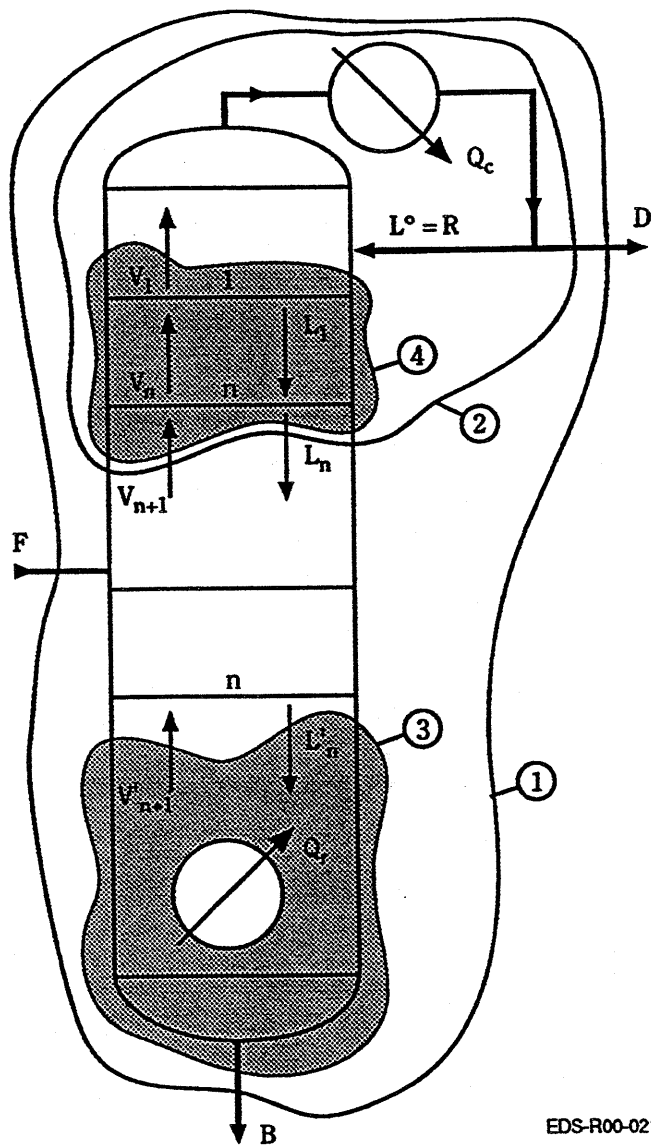
Determine (both graphically and analytically):

- a. the minimum trays required (total reflux).
 - b. the minimum reflux (L/D) required (infinite trays) for a bubble point liquid feed, for a feed that is 50 mol pct vapor, and for a saturated vapor feed.
 - c. the number of theoretical trays required when the reflux is 20 pct higher than the minimum for a saturated liquid feed and for a dew point vapor feed.
 - d. the optimum feed tray for the two cases of part c.
2. Wet propane is to be distillation dried to 1 mol ppm. The receiver pressure and temperature are to be controlled at 350 psia and 100°F respectively. Determine the required number of theoretical trays if the column is to be operated at an internal L/V of 2.0.

BINARY FRACTIONATION

- I. Heat and Material Balance
- II. Equilibrium
- III. Concept of Equimolar Overflow
- IV. McCabe-Thiele Graphical Method
- V. Analytical Methods
- VI. Tray Efficiency

Figure 18



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Envelope 1

Mass Balance:

$$F = D + B$$

$$x_i^F F = x_i^D D + x_i^B B \quad (i = 1 \text{ to } N \text{ components})$$

Envelope 2

Mass Balance:

$$V^{n+1} = L^n + D$$

$$y_i^{n+1} V^{n+1} = x_i^n L^n + x_i^D D$$

Heat Balance:

$$h_V^{n+1} V^{n+1} = h_L^n L^n + h_D D + Q_c$$

Envelope 3

Mass Balance:

$$L^{n'} = V^{n+1} + B$$

$$x_i^n L^{n'} = y_i^{n+1} V^{n+1} + x_i^B B$$

Heat Balance:

$$Q_R + h_L L^{n'} = h_V V^{n+1} + h_B B$$

Envelope 4

Mass Balance:

$$V^{n+1} + L^o = V' + L^n$$

Heat Balance:

$$h_V^{n+1}V^{n+1} + h_L^oL^o = h_V'V' + h_L^nL^n$$

Envelope 4

Mass Balance:

$$V^{n+1} = V' + L^n - L^o$$

Heat Balance:

$$h_V^{n+1}V' + h_V^{n+1} - h_V^{n+1}L^o = h_V'V' + h_L^nL^n$$

Assume

Sensible heat changes are negligible:

$$h_L^o = h_L' = \dots = h_L^n = h_L$$

Molal heats of vaporization are constant:

$$\lambda^o = \lambda' = \dots = \lambda^n$$

Since:

$$\lambda^n = h_V^n - h_L^n$$

$$h_V^o = h_V' = \dots = h_V^n = h_V$$

Heat Balance:

$$h_V V' + h_V L^n - h_V L^o + h_L L^o = h_V V' + h_L L^n$$

$$(h_V - h_L) L^n = (h_V - h_L) L^o$$

Therefore:

$$L^n = L^o = \text{Constant}$$

$$L^n = L^o = \text{Constant}$$

Sensible heat changes are negligible.

Molal heats of vaporization are constant.

Validity?

Boiling point range of components is narrow.

Molecular characteristics of components are similar.

For example, all paraffinic hydrocarbons or all aromatic hydrocarbons not mixture of paraffins and aromatics.

Envelope 2

Mass Balance:

$$y_i^{n+1} V^{n+1} = x_i^n L^n + x_i^D D$$

Equimolal overflow:

$$L = \text{Constant}$$

$$V = \text{Constant}$$

$$y_i^{n+1} = (L/V)x_i^n + (D/V)x_i^D$$

Binary

$$x_i \rightarrow x_1, x_2$$

$$x_2 = 1 - x_1$$

Use convention of dropping subscript i

$$x = x_1$$

> mol fraction of more volatile (lower boiling point) component

$$y = y_1$$

$$y_{n+1} = (L/V)x_n + (D/V)x_D \quad \text{Trays above feed}$$

Envelope 3

Mass Balance:

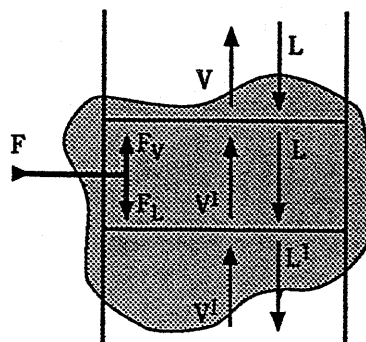
$$x_i^n L^{n'} = y_i^{n+1} V^{n+1} + x_i^B B$$

$$L' = \text{Constant} \neq L$$

$$V' = \text{Constant} \neq V$$

$$y_{n+1} = (L'/V')x_n - (B/V')x_B \quad \text{Trays below feed}$$

Figure 19



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Mass Balance: $F + V' + L = V + L'$

Heat Balance: $h_F F + h'_V V' + h_L L = h_V V + h'_L L'$

$$V = L + D$$

$$V' = L' - B$$

Assume

$$h'_V = h_V$$

$$> h_F F + h_V (L' - B) + h_L L = h_V (L + D) + h_L L'$$

$$h'_L = h_L$$

rearranging: $h_V L' - h_L L' = h_V L - h_L L + h_V D + h_V B - h_F F$

$$(h_V - h_L) L' = (h_V - h_L) L + h_V (D + B) - h_F F$$

$$D + B = F$$

$$L' = L + \left(\frac{h_V - h_F}{h_V - h_L} \right) F$$

Define:

$$q = \frac{h_F^V - h_F}{h_F^V - h_F^L}$$

Then:

$$L' = L + qF$$

Also:

$$y^{n+1} = \left[\frac{q}{(q-1)} \right] x^n - \left[\frac{1}{(q-1)} \right] x_F \quad \text{At feed tray}$$

EQUILIBRIUM

$$y = K_1 x$$

$$(1-y) = K_2(1-x)$$

$$x = \frac{1-K_2}{(K_1-K_2)}$$

$$y = K_1 x$$

$$(1-y) = K_2(1-x)$$

$$\frac{y}{1-y} = \frac{K_1}{K_2} \frac{x}{1-x}$$

$$\text{Define } \alpha = \frac{K_1}{K_2}$$

$$y = \frac{\alpha x}{1+(\alpha-1)x}$$

$$x = \frac{y}{K_1}$$

BINARY SYSTEM: PROPANE/ISOBUTANE

Specified

Feed = 200 mol/h C₃ + 200 mol/h iC₄ at bubble point

Distillate = 196 mol/h

Reflux = 400 mol/h at 100 °F

Column pressure = 250 psia

Number of trays = 24, feed on 13

Problem

Find x_D and x_B

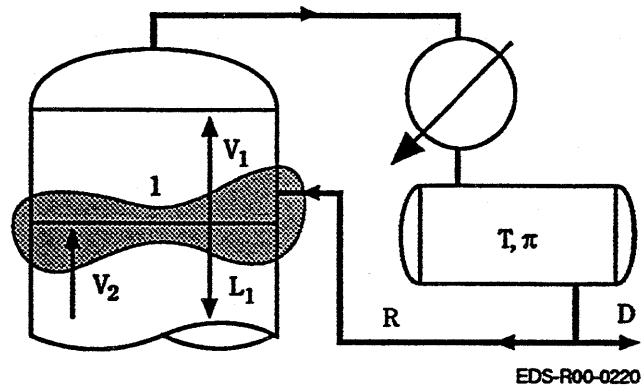
$$L = 1.16R = 464 \text{ mol/h}$$

$$L/F = \frac{464}{400} = 1.16$$

	Rigorous Heat & Weight Balanced Tray-to-Tray Method	Equimolal Overflow Simplified Tray-to-Tray Method
$x_D =$	0.927	0.917
$x_B =$	0.090	0.098

INTERNAL VS EXTERNAL REFLUX

Figure 20



$$h_{V_2}V_2 + h_R R = h_{V_1}V_1 + h_{L_1}L_1$$

$$V_1 = R + D; \quad V_2 = L_1 + D$$

$$\text{Assume: } h_{V_1} = h_{V_2}$$

$$L = \frac{(h_V - h_R)}{(h_V - h_L)} R$$

h_V = Saturated vapor enthalpy @ π

h_L = Saturated liquid enthalpy @ π

h_R = Liquid enthalpy @ T

Receiver temperature = 100 °F

Receiver pressure = 250 psia

Assume: receiver liquid is 100 pct propane

$$h_V = 168 \text{ btu/lb}; \quad h_L = 47 \text{ btu/lb}; \quad h_R = 27 \text{ btu/lb}$$

$$L = 1.16 R$$

BINARY SYSTEM: PROPANE/BENZENE

Specified

Feed = 50 mol/h C₃ + 50 mol/h Bz at bubble point

Distillate = 50 mol/h

$$x_D = 0.99$$

Column pressure = 215 psia

Number of trays = 10, feed on 6

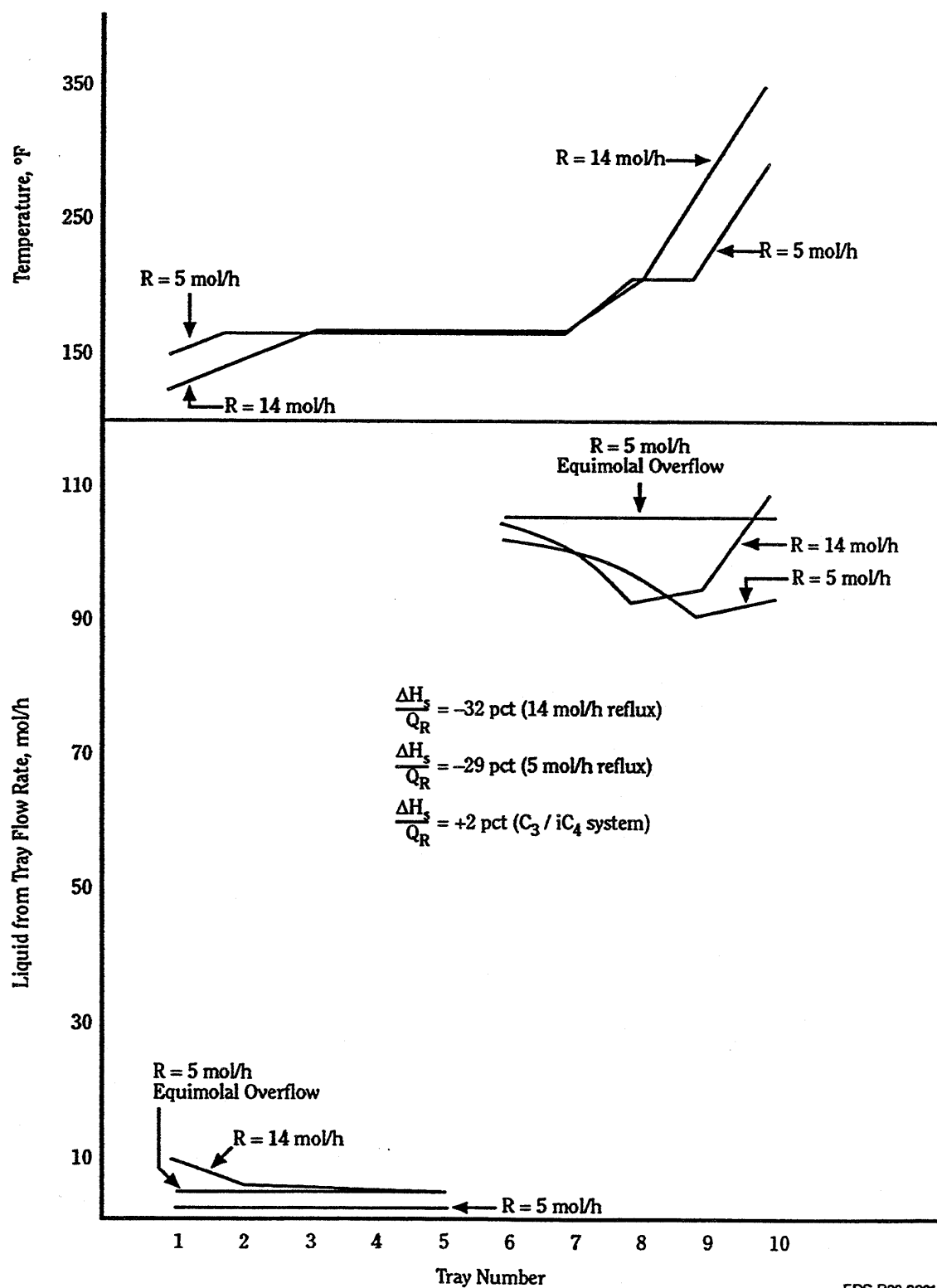
Problem

Find reflux rate (temperature = 100°F)

	Equimolal Overflow Simplified Tray-to-Tray <u>Method</u>	Rigorous Heat & Weight Balanced Tray-to-Tray <u>Method</u>
R =	5 mol/h	14 mol/h

if 5 mol/h, $x_D = 0.967$

Figure 21



SUMMARY OF EQUATIONS

Rectifying Section

$$y^{n+1} = (L/V)x^n + (D/V)x_D, \quad V = L + D$$

Stripping Section

$$y^{n+1} = (L'/V')x^n - (B/V')x_B, \quad V' = L' - B$$

Feed Tray

$$y^{n+1} = [q/(q-1)]x^n - [1/(q-1)]x_F, \quad q = \frac{h_R^V - h_F}{x_F}$$

Equilibrium

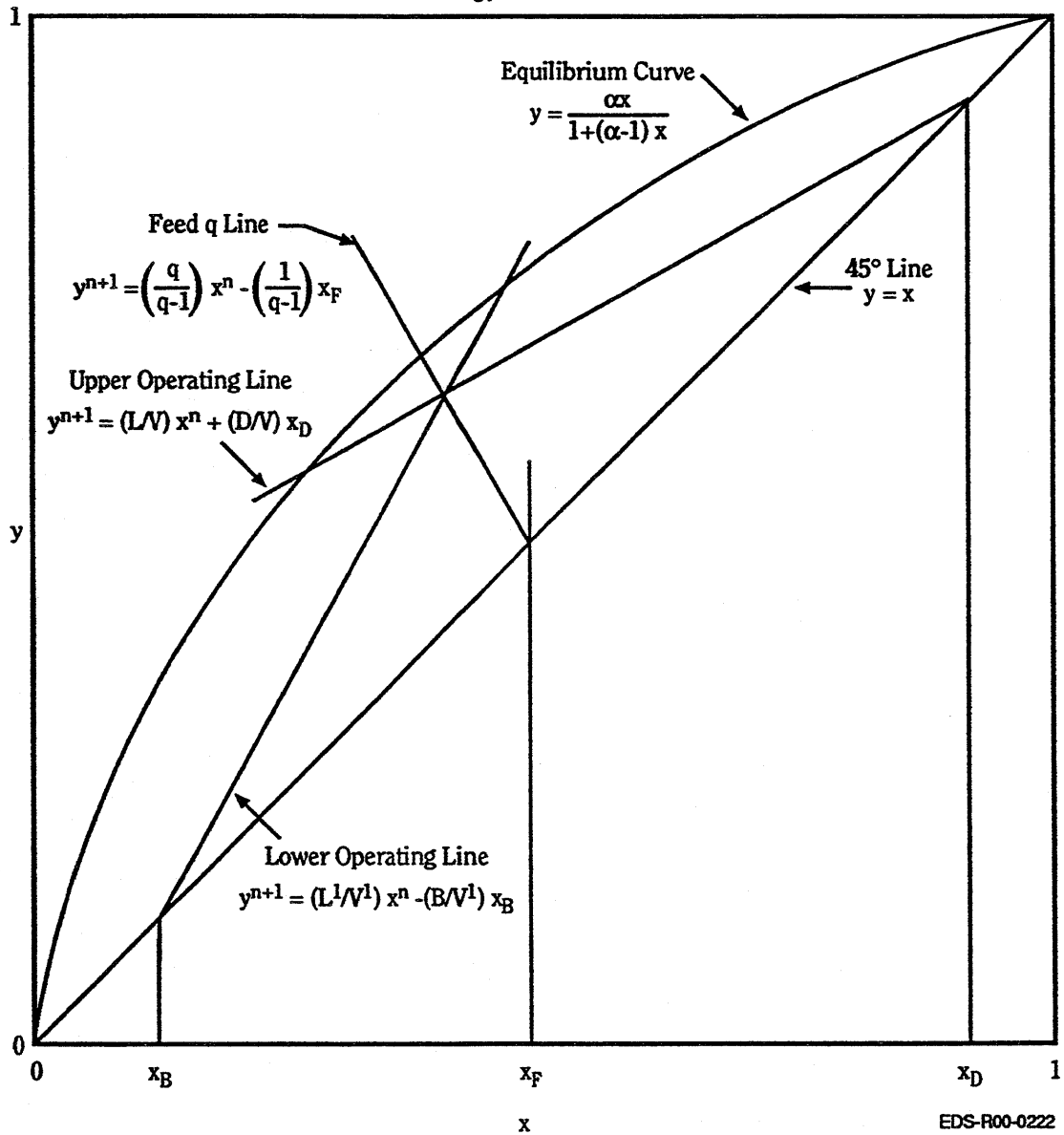
$$x^n = \frac{(1-K_2)}{(K_1-K_2)}, \quad y^n = K_1 x^n$$

or

$$y^n = \frac{\alpha x^n}{1 + (\alpha - 1)x^n}, \quad \alpha = K_1/K_2$$

Figure 22

*Mccabe - Thiele Method
Typical Plot*



Feed q Line

$$y^{n+1} = \frac{q}{(q-1)} x^n - \frac{1}{(q-1)} x_F, \quad q = \frac{h_F^V - h_F}{h_F^V - h_F^L}$$

Note: When

$$x^n = x_F, \quad y^{n+1} = \left[\frac{q}{q-1} - \frac{1}{q-1} \right] x_F = x_F$$

Equation will plot as a straight line with slope equal to $q/(q-1)$ passing through the point x_F, y_F .

For this problem:

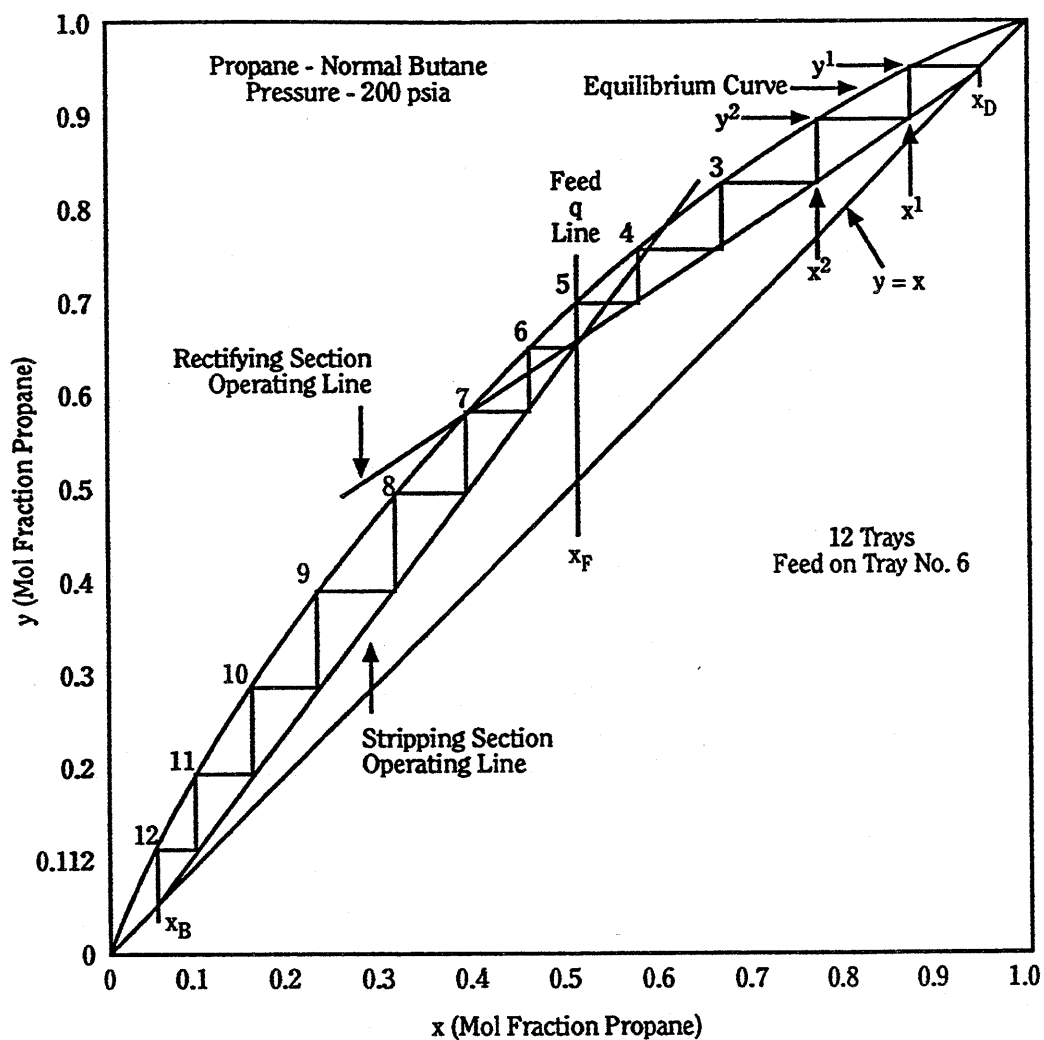
$$h_F = h_F^L \text{ (bubble point liquid feed)}$$

Therefore,

$$q = 1$$

$$\text{Slope} = \frac{1}{1-1} = \infty$$

Figure 23



EDS-R00-0223

PROBLEM

Binary System: Propane-Normal Butane

System pressure = 200 psia

Feed = 50 mol/h C₃ + 50 mol/h nC₄ at bubble point

Desired purities top and bottom

$$x_D = 0.95 \quad x_B = 0.05$$

Reflux rate (internal)

$$R = L = 100 \text{ mol/h}$$

Question: How many trays?

$$x_F = 50/100 = 0.50$$

$$x_F F = x_D D + x_B B;$$

$$B = F - D$$

$$(0.5)(100) = (0.95)D + 0.05(100 - D) \Rightarrow D = 50 \text{ mol/h}$$

PROPANE-NORMAL BUTANE

$$\pi = 200 \text{ psia}$$

$^{\circ}\text{F}$	K_1	K_2	α	x_1	y_1
110	1.058	0.4098	2.58	0.911	0.963
120	1.151	0.4622	2.49	0.781	0.899
130	1.249	0.5180	2.41	0.659	0.824
140	1.350	0.5769	2.34	0.547	0.739
150	1.456	0.6389	2.28	0.442	0.643
160	1.566	0.7036	2.23	0.344	0.538
170	1.681	0.7710	2.18	0.252	0.423
180	1.801	0.8408	2.14	0.166	0.299
190	1.925	0.9130	2.11	0.086	0.165
200	2.051	0.9875	2.08	0.012	0.024

Rectifying Section

$$y^{n+1} = (L/V)x^n + (D/V)x_D, \quad V = L + D$$

Note: When

$$x^n = x_D, \quad y^{n+1} = \left(\frac{L}{L+D} + \frac{D}{L+D} \right) x_D = x_D$$

Can plot equation as a straight line with slope equal to $L/(L+D)$ that passes through the point x_D, y_D

For this problem:

$$L = 100 \text{ mol/h}$$

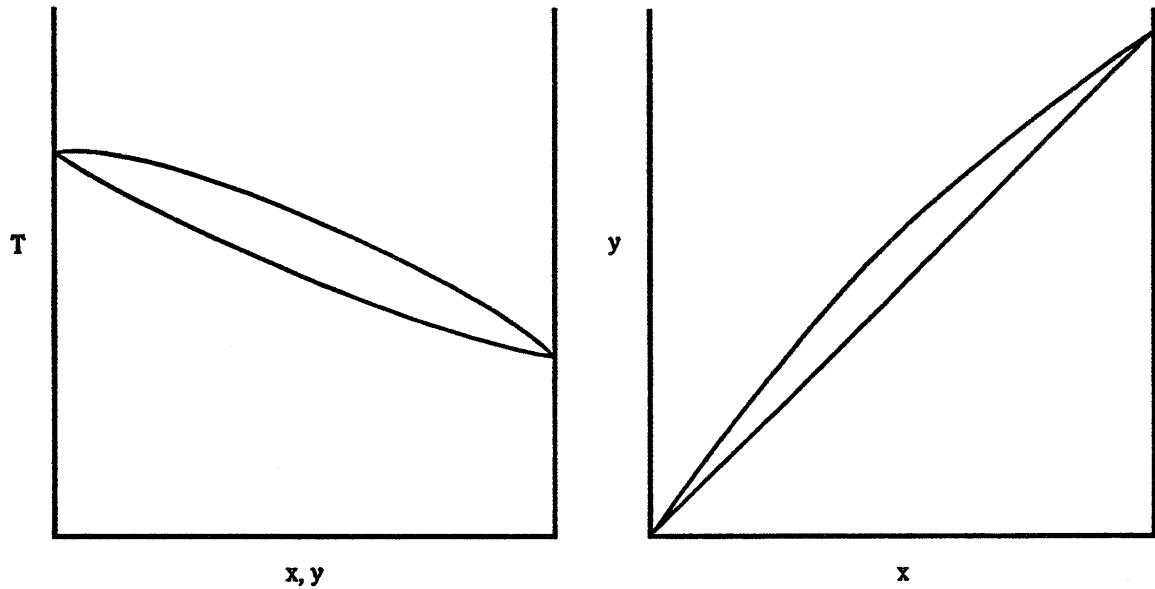
$$D = 50 \text{ mol/h}$$

$$\text{Slope} = \frac{100}{100 + 50} = 0.67$$

Figure 24

Equilibrium

Pressure Constant



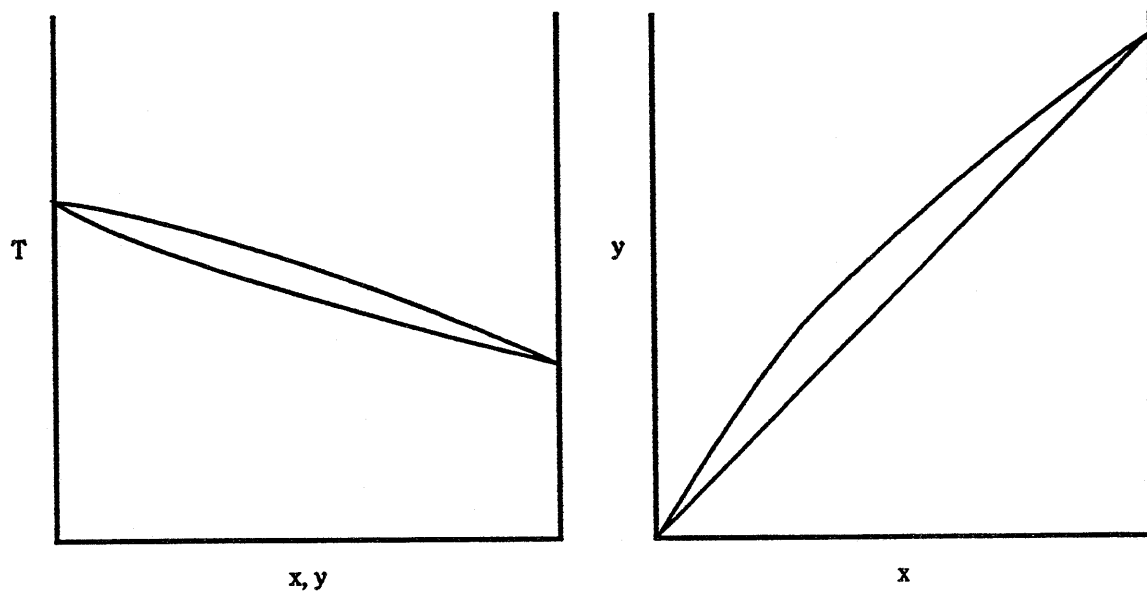
Ideal Vapor/Liquid Equilibria:
Systems that conform to Raoult's Law
(i.e. $p^* = P_v x$, $\therefore \alpha = \frac{P_v}{P} = \text{constant}$)

EDS-R00-0224

Figure 25

Equilibrium

Pressure Constant



Slight Deviation from Ideality:

$$p^* = \gamma P_v x$$

Activity Coefficient:

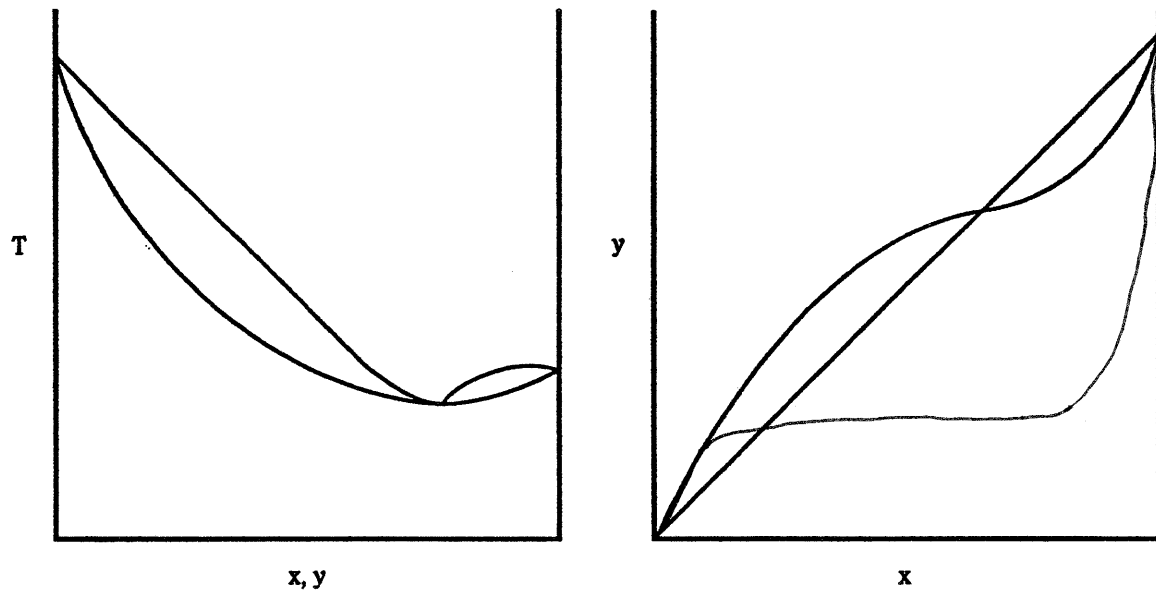
$$\gamma = f(x, T), \alpha \neq \text{constant}$$

EDS-R00-0225

Figure 26

Equilibrium

Pressure Constant



Large Deviation from Ideality:
e.g. Minimum boiling azeotrope

EDS-R00-0226

Figure 27

Feeding on Proper Tray

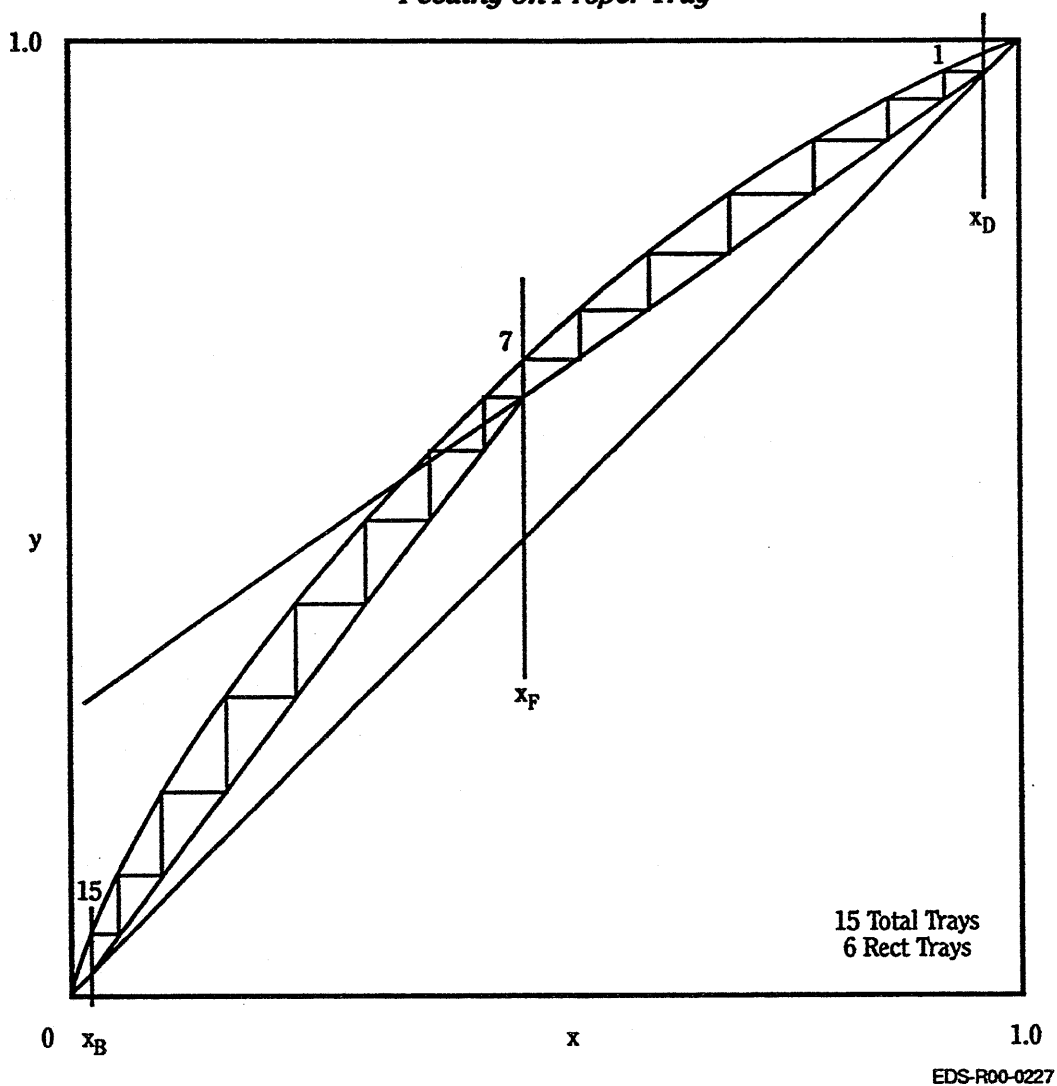
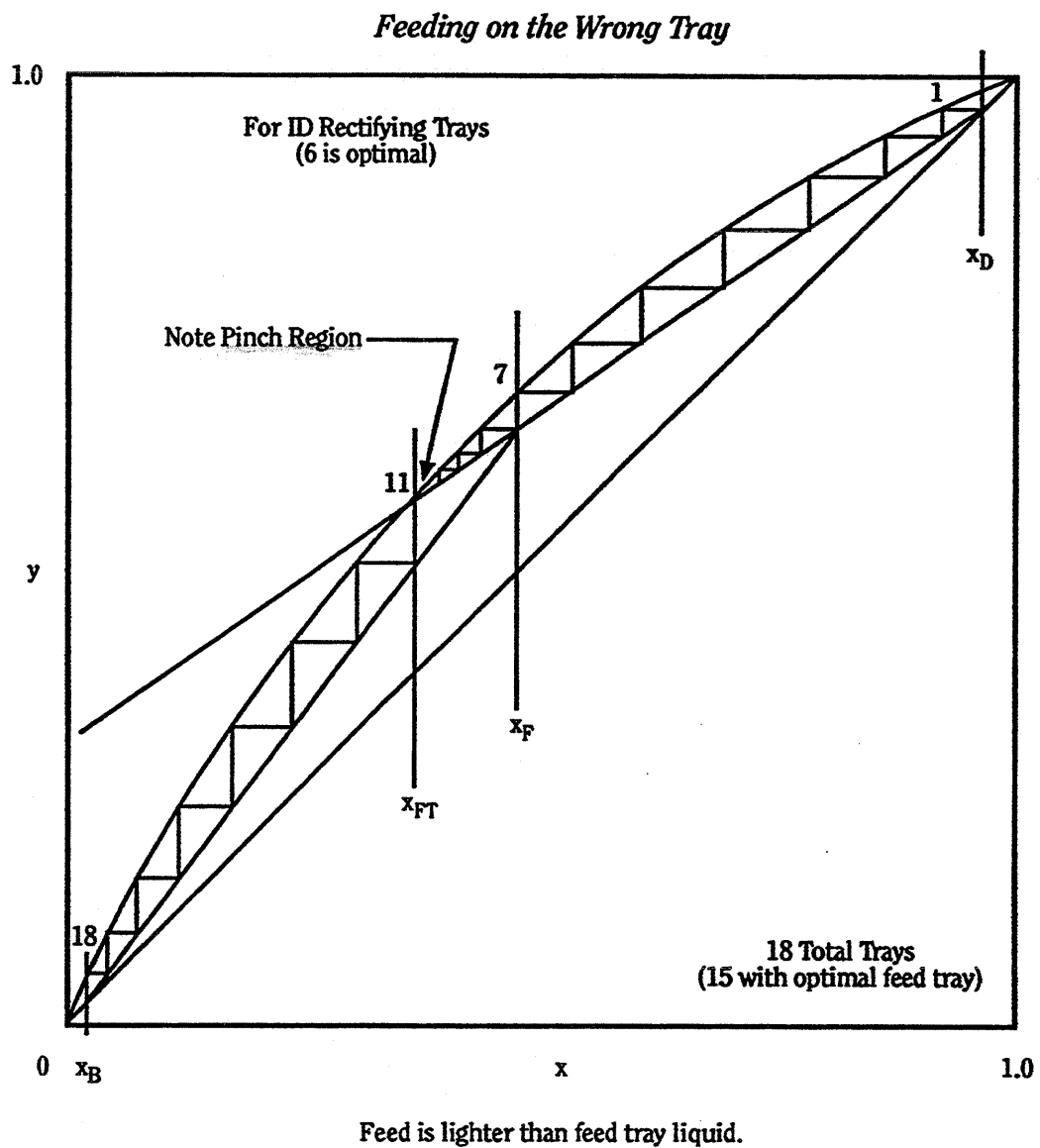


Figure 28



EDS-R00-0228

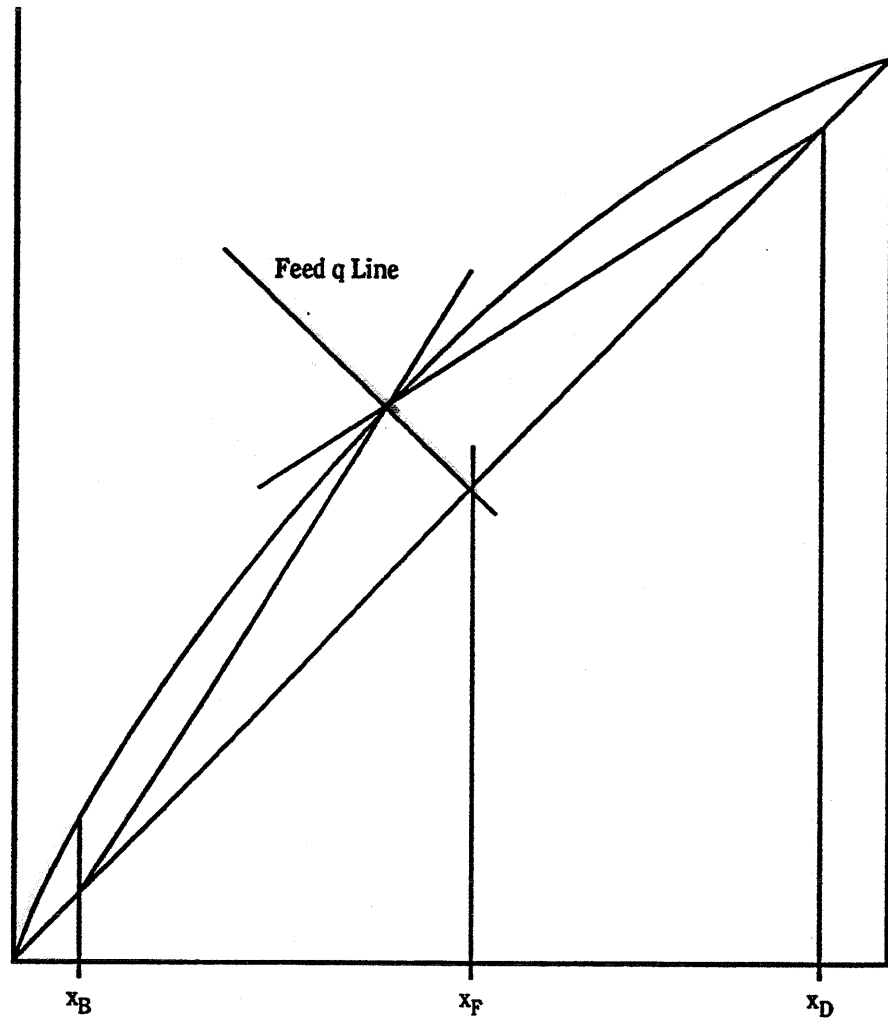
2



Actual frang
= 3 ~ 4 x ~~Teeth~~ frang.

Figure 30

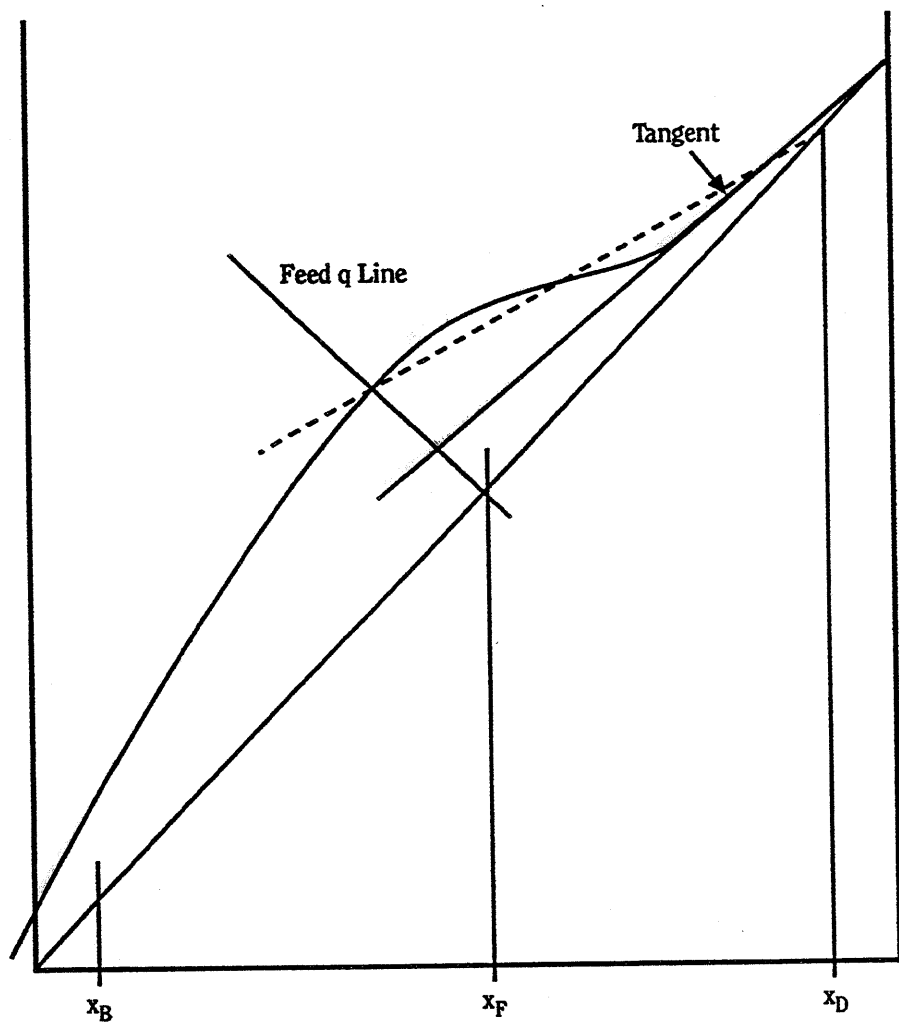
Minimum Reflux at Infinite Trays



EDS-R00-0230

Figure 31

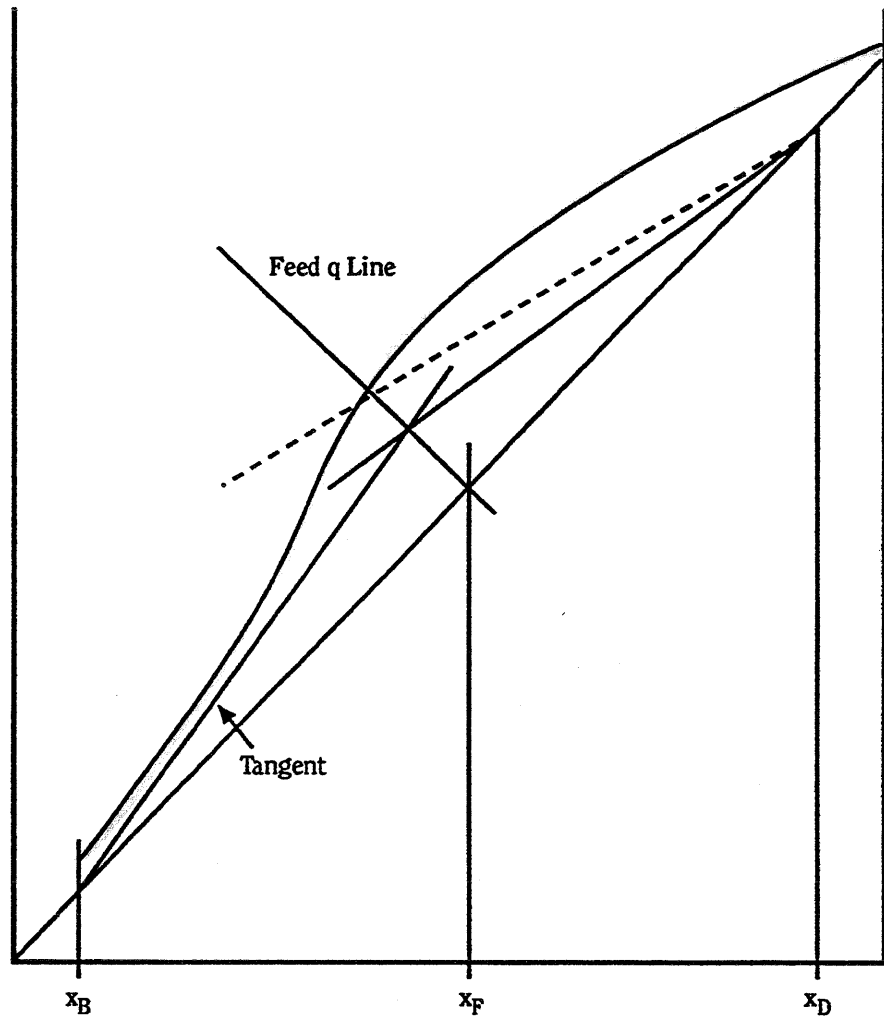
Minimum Reflux at Infinite Trays



EDS-R00-0231

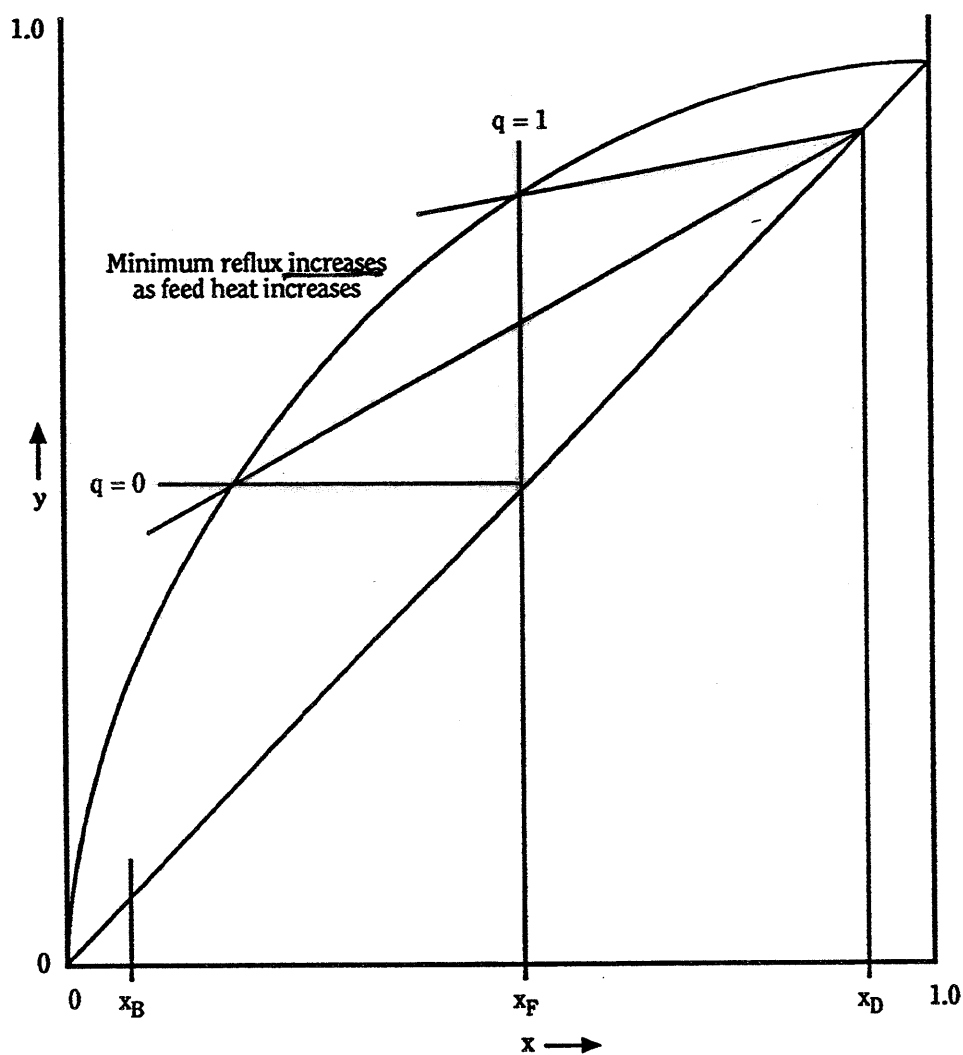
Figure 32

Minimum Reflux at Infinite Trays



EDS-R00-0232

Figure 33

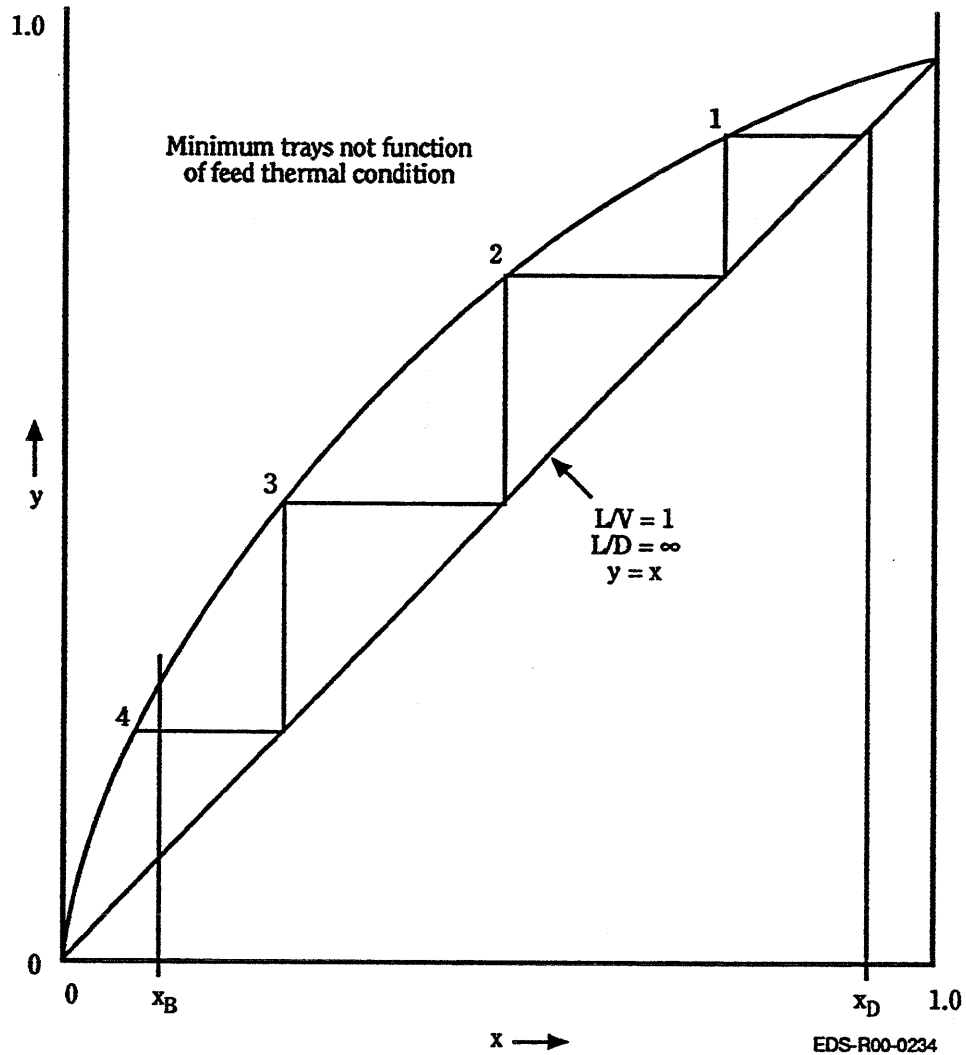


EDS-R00-0233

Feed preheat. vs. Reboiler duty
 efficiency? Condenser duty

Figure 34

Minimum Trays



Fenske Equation

$$n_m = \frac{\ln(r_D/r_B)}{\ln \alpha}$$

$$r_D = \frac{x_D}{1-x_D}$$

$$r_B = \frac{x_B}{1-x_B}$$

FEED TRAY LOCATION

For bubble point feed:

$$n_m^R = \frac{\ln(r_D/r_F)}{\ln \alpha}$$

(Fenske equation applied to rectifying section)

Then:

$$\frac{n_m^R}{n_m} = \frac{\ln(r_D/r_F)}{\ln(r_D/r_B)}$$

(Assume ratio holds for any reflux ratio)

DISA
K5

MINIMUM REFLUX

Underwood Method

$$(a) \quad \frac{1-x_F}{1-\phi} + \frac{\alpha x_F}{\alpha-\phi} = 1-q, \quad 1 < \phi < \alpha$$

$$(b) \quad (L/D)_{min} = \frac{(1-x_D)}{(1-\phi)} + \frac{\alpha x_D}{(\alpha-\phi)} - 1$$

Solve equation 1 for the proper root of the quadratic for ϕ .

Solve equation 2 for $(L/D)_{min}$ using the ϕ from equation 1.

Note: If terms are multiplied out, equation 1 becomes:

$$\underbrace{(1-q)}_A \phi^2 + \underbrace{(q + \alpha q + \alpha x_F - \alpha - x_F)}_B \phi - \underbrace{q\alpha}_C = 0$$

$$\phi = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

If feed is bubble point liquid:

$$q = 1$$

The Underwood equations reduce to:

$$(L/D)_{min} = \frac{1}{\alpha-1} \left[\frac{x_D}{x_F} - \frac{\alpha(1-x_D)}{(1-x_F)} \right]$$

Note: For perfect separation ($x_D=1$)

$$(L/D)_m = \left(\frac{1}{\alpha-1} \right) \left(\frac{1}{x_F} \right)$$

Since:

$$x_F F = x_D D + x_B B$$

then:

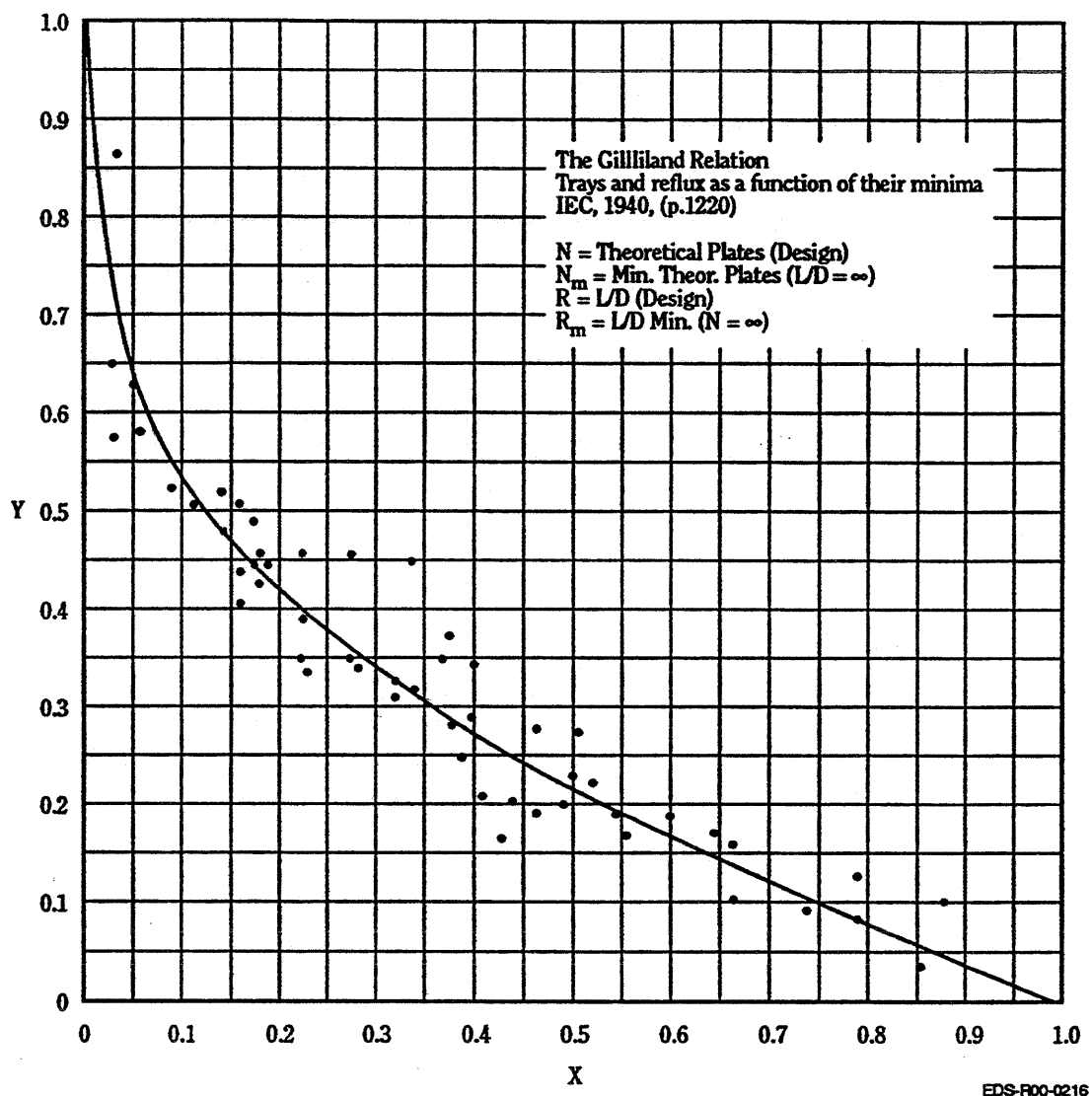
$$x_F = D/F \text{ when } x_D = 1$$

Therefore:

$$(L/F)_m = \frac{1}{\alpha - 1}$$

for perfect separation with bubble point feed.

ANALYTICAL TECHNIQUES



$$X = \frac{R - R_m}{R + 1}$$

$$R = \frac{R_m + X}{1 - X}$$

$$Y = \frac{N - N_m}{N + 1}$$

$$N = \frac{N_m + Y}{1 - Y}$$

Determine:

R_m from Underwood Equations

N_m from Fenske Equation

Example

$$\begin{array}{lll}x_F = 0.5 & r_F = 1.0 & \\x_D = 0.927 & r_D = 12.7 & D/F = 0.49 \\x_B = 0.090 & r_B = 0.0989 & \\\alpha = 1.76 & q = 1.0 & n = 24\end{array}$$

Underwood:

$$(L/D)_m = \frac{1}{(\alpha - 1)} \left[\frac{x_D}{x_F} - \frac{\alpha(1 - x_D)}{(1 - x_F)} \right] = 2.10$$

Fenske:

$$n_m = \frac{\ln(r_D/r_B)}{\ln \alpha} = 8.59$$

$$\frac{n_m^r}{n_m} = \frac{\ln(r_D/r_F)}{\ln(r_D/r_B)} = 0.523$$

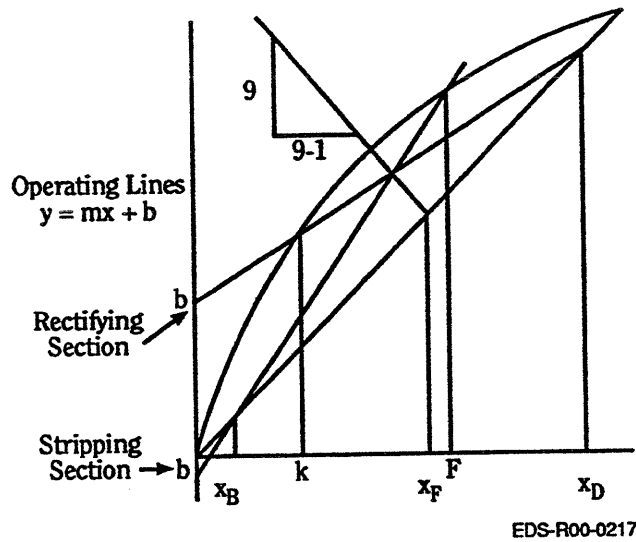
$$n \text{ (real in rectifying section)} = 0.523 \times 24 = 12.6$$

Gilliland:

$$Y = \frac{N - N_m}{N + 1} = 0.616 \rightarrow x = 0.055$$

$$L/D = \frac{(L/D)_m + x}{1 - x} = 2.28 \rightarrow R/F = 1.12$$

Smoker's Method



(a) $m = L/V$ rectifying section
 $m = L'/V'$ stripping section

(b) $b = x_D(1-m)$ rectifying section
 $b = x_B(1-m)$ stripping section

(c)
$$A = \frac{m + b(\alpha - 1) - \alpha}{m(\alpha - 1)}$$

$$B = \frac{b}{m(\alpha - 1)}$$

(d)
$$K^2 + AK + B = 0$$

$$K = \frac{-A}{2} \pm \sqrt{\frac{A^2}{4} - B} \quad 0 < K < 1$$

(e)
$$C = 1 + K(\alpha - 1)$$

(f)
$$E = mc^2/\alpha$$

$$(g) \quad x'_1 = x_1 - K$$

$$x'_2 = x_2 - K$$

$$(h) \quad D = \frac{E(\alpha - 1)}{C(1 - E)}$$

$$(i) \quad d = D x'_1 x'_2$$

$$(j) \quad \Delta S = \frac{\ln \frac{(x'_2 - d)}{(x'_1 - d)}}{\ln E}$$

$$(k) \quad S = \sum \Delta S$$

Procedure

Calculate parameters for equations 1–6 for the rectifying section.

Set $x_1 = x_D$, choose $x_2 \geq x_F$.

Solve equations 7–10; ΔS is then the number of stages required to go from a liquid purity of x_1 to a liquid purity of x_2 .

If x_2 was not chosen equal to x_F , repeat step 3 in increments whereby x_1 equals previous x_2 and a new x_2 is chosen greater than or equal to x_F .

Note: Generally, x_2 can be set equal to x_F in step 2 and incremental calculations are not necessary; however, if α or L/V varies significantly with liquid composition, it is desirable to proceed in stages but recalculating equations 1-6 each time.

Sum the incremental ΔS to get the number of trays required for the rectifying section.

Repeat steps 1 through 5 for the stripping section starting with $x_1 = x_F$ and proceeding until $x_2 = x_B$.

TRAY EFFICIENCY

After running a McCabe-Thiele diagram on a particular binary and the number of theoretical trays is found, the problem of tray efficiency still remains. The question is still asked, "How many real trays shall I use?" The problem has never been solved. Tray efficiency is definitely dependent on system properties, size of unit, tray geometry, and certain velocities in the column. With all the research work, attempting to correlate something against tray efficiency, for the last half century, there remains no answer. There seems to be one system property that might indicate a trend – we have never seen high efficiencies in systems with high values of α , i.e., from 4.0 to 20.0. Also, conversely, the only time we have seen high efficiencies (but not always), was for systems with low values of α – such as in Isomer separations. Tray efficiencies vary between 10-140 pct over the very wide range of units which we have seen. Experience is the only reliable basis for setting tray efficiencies.

TYPICAL OBSERVED TRAY EFFICIENCY (Simplistic Overall)

Xylene Isomer Fractionators	80-100
Deisopentanizers/Deisobutanizers	80-100
Benzene/Toluene/Xylene	75-80
Depropanizers/Debutanizers	75-80
Naphtha Fractionators	65-85
High Pressure Deethanizers	50-60
Low Pressure Drop Columns	40-60
Distillation Dryers	15
Gas Strippers	7-10
Gas Con Absorbers	
Primary	30-35
Sponge	20-25

**GENERAL GUIDE FOR TRAY EFFICIENCY
AS FUNCTION OF α ALONE**

Alpha	Tray Efficiency
1.2	90
2.0	70
3.0	50
5.0	20
15.0	10

Tray Efficiency

Figure 12 shows a plot of overall tray efficiency of the V-1 Ballast tray obtained in a 4'-0" diameter fractionating column on the isobutane/n-butane and cyclohexane/n-heptane systems at several pressures. The results of these tests are described fully in *Glitsch Bulletin No. 160*.

This figure may be used as a guide in selecting tray efficiency for design of commercial columns for similar systems. However, the usual practice has been to establish the number of trays required to obtain the desired separation with bubble cap or sieve trays and to use the same number of Ballast trays. The V-4 Ballast tray has been used to separate the ethylbenzene-styrene system in a single column without having an excessive bottom tray pressure.

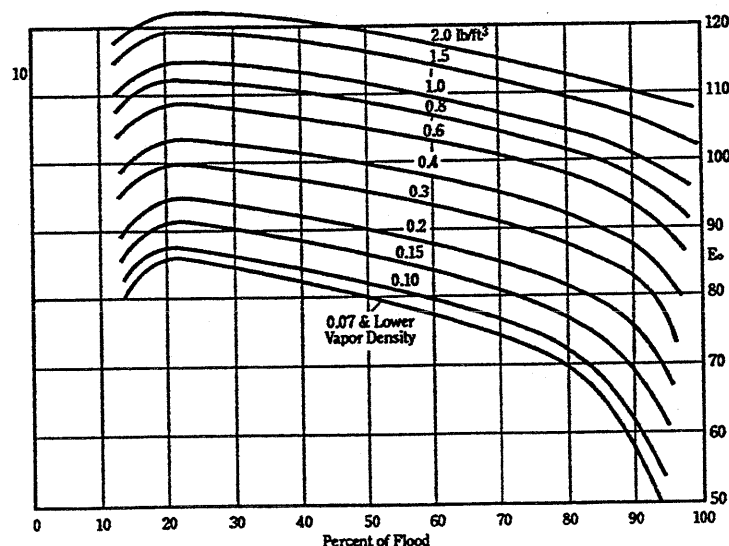
Large size V-2 Ballast units have been used to replace bubble cap and riser assemblies in existing columns. The efficiency and capacity of the installation have exceeded that of the original bubble cap tray.

Extra trays to compensate for mislocation of the feed tray and instrumentation swings should be considered when the number of trays is established.

Figure 12

*Glitsch Bulletin 4900
Fifth Edition*

**V-1 Ballast Tray
Overall Tray Efficiency
vs
Vapor Density & Load
Experimental Data at Total
Reflex 2" Weir Height, 30" FPL**



EDS-FR00-0245

DESIGNING A COLUMN

- Define feed
- Define product specification
- Set column pressure
- Optimize column design
- Set composition control

Define Feed

- **Composition**
- **Flow rate**
- **Temperature**
- **Pressure**
- **Enthalpy**

Define Product Specifications

- Receiver temperature
- Product purities and recoveries
- Zero purity specification is not acceptable

Set Column Pressure

- Maximize alpha value
- Minimize column cost
- Keep flare material out of overhead
- Totally condense overhead products
- Prevent need for net gas compression
- Minimize net overhead vapor
- Use condenser as heat source
- Use bottoms as hot oil
- Limit bottom temperature

Cracking

Polymerization

Approach to critical

Optimize Column Design

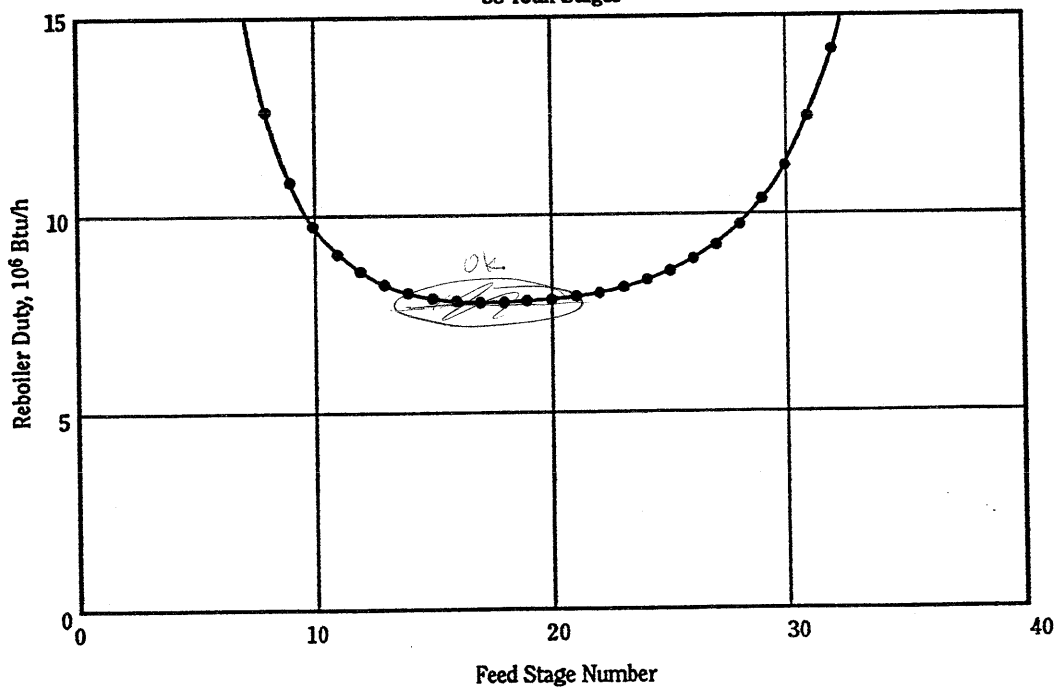
- Best trays versus reboiler
- Best feed tray location
- Feed preheat

Best Feed Tray Location

C₃/iC₄ Splitter
38 Total Stages

Feed Stage	Reboiler Duty 1 x 10 ⁶ Btu/h	Feed Stage	Reboiler Duty 1 x 10 ⁶ Btu/h	Feed Stage	Reboiler Duty 1 x 10 ⁶ Btu/h
1	288.701	14	8.141	27	9.309
2	148.546	15	8.008	28	9.785
3	83.707	16	7.929	29	10.417
4	49.136	17	7.891	30	11.277
5	30.807	18	7.885	31	12.48
6	21.036	19	7.905	32	14.223
7	15.723	20	7.957	33	16.847
8	12.72	21	8.032	34	20.964
9	10.944	22	8.134	35	27.665
10	9.84	23	8.269	36	38.831
11	9.13	24	8.442	37	57.548
12	8.66	25	8.663	38	88.681
13	8.348	26	8.945		

Best Feed Tray
38 Total Stages



EDS-R00-2513

Trays Versus Reboiler Duty Selection

C₃/C₄ Splitter

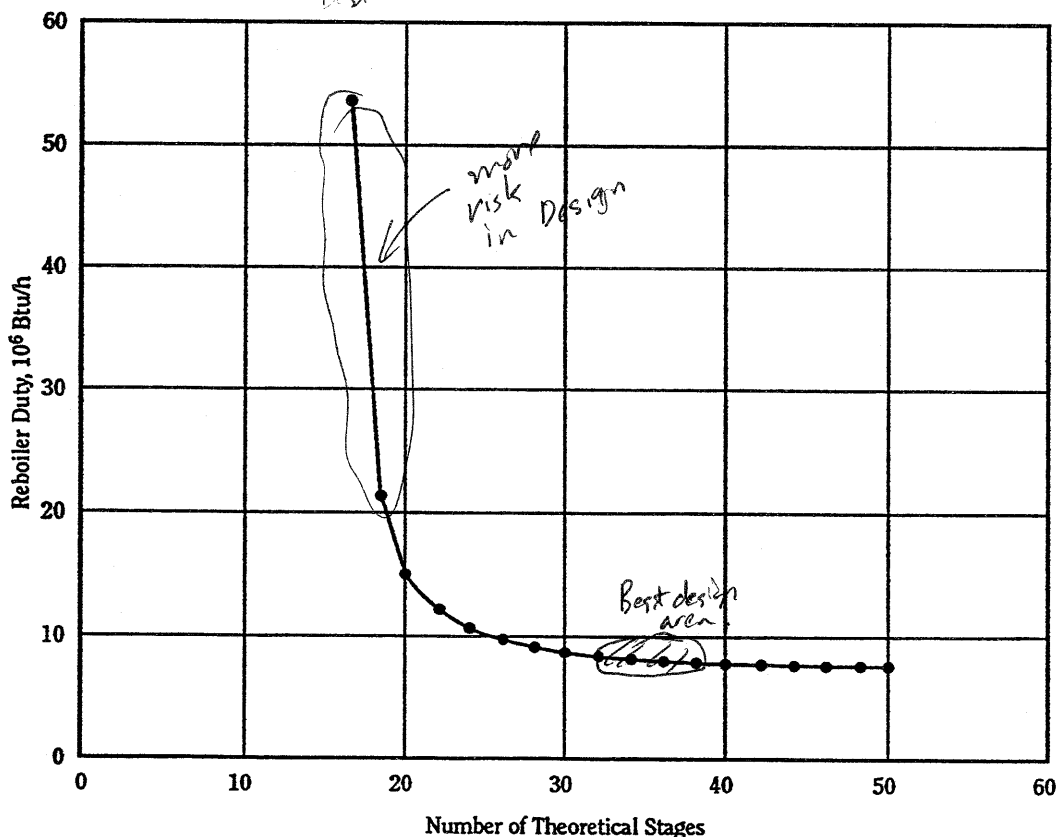
File Name: REFLUXSL.WK3

Feed Stage	Total Stages	Reboiler 1 x 10 ⁶ Btu/h	pct Delta Reboiler/ Theo Tray	Feed Stage	Total Stages	Reboiler 1 x 10 ⁶ Btu/h	pct Delta Reboiler/ Theo Tray
25	50	7.582	0.18	15	30	8.689	2.38
24	48	7.609	0.21	14	28	9.102	3.28
23	46	7.641	0.31	13	26	9.699	4.74
22	44	7.689	0.36	12	24	10.619	7.17
21	42	7.744	0.43	11	22	12.141	11.70
20	40	7.811	0.61	10	20	14.982	22.66
19	38	7.907	0.77	9	18	21.771	72.92
18	36	8.028	0.99	8	16	53.521	770.88
17	34	8.187	1.28	7	15	466.104	
16	32	8.397	1.74	7	14	Will not solve	

possible

more risk

Trays vs Reboiler Duty

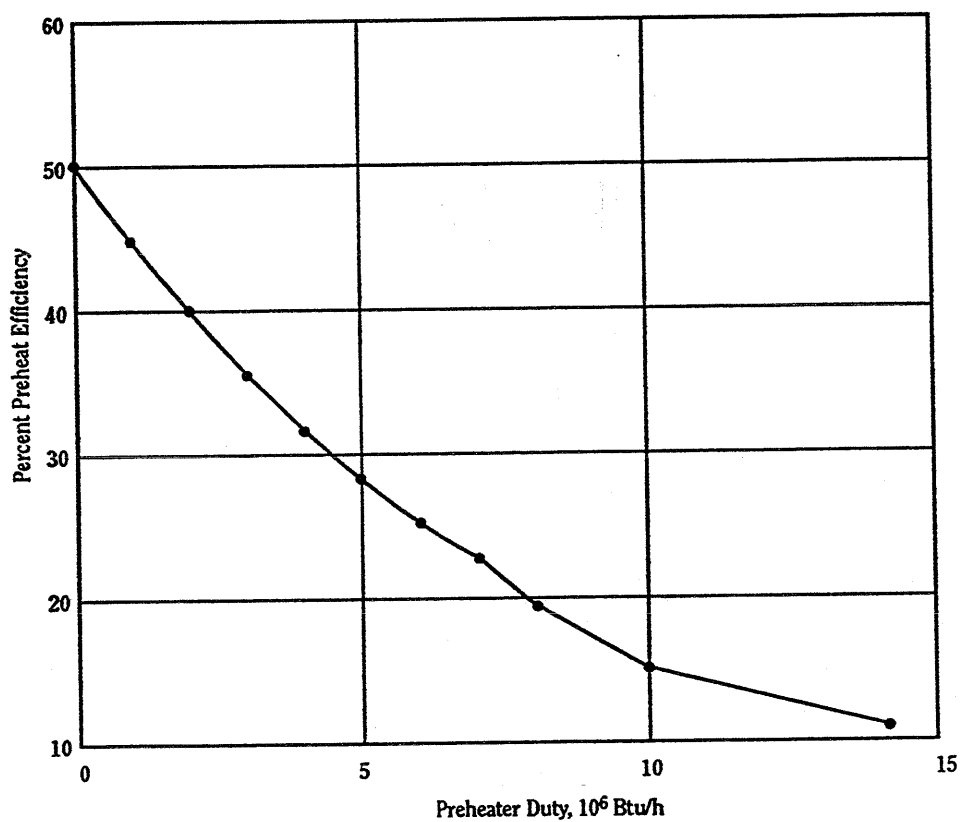


EDS-R00-0235

FEED PREHEAT

Preheater Duty 10 ⁶ Btu/h	Feed Vapor Fraction	Reboiler Duty 10 ⁶ Btu/h	Preheat Eff, pct
0	0	8.741	50.5
1	0	8.236	45.1
2	0.0084	7.785	40.2
3	0.2363	7.383	35.7
4	0.4645	7.026	31.7
5	0.6932	6.709	28.2
6	0.9227	6.427	25.3
7	1	6.174	22.8
8	1	5.946	19.6
10	1	5.555	15.2
14	1	4.947	11.0
20	1	4.289	

Feed Preheat Efficiency



EDS-R00-2515

FRACTIONATION BOOKS OF INTEREST

Distillation Operation

By Henry Z. Kister

McGraw Hill

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Includes chapters on:

Distillation Troubleshooting

Reflux and Intermediate Feed Inlets for Tray Columns

Reflux and Intermediate Feed Distribution and Liquid Redistribution in Packed Towers

Bottom Section and Column Outlets

Gravity Lines and Instrument and Access Connections

Tray and Downcomer Layout

Mechanical Requirements for Trays

Internals Unique to Packed Towers

Distillation Overpressure Relief

Column Assembly and Preparation for Commissioning

Column Commissioning

Column Start-up and Shutdown

Operation Difficulties

Column Field Testing: Flooding, Foaming, Efficiency, and Troubleshooting

Reboilers and Condensers Operation and Troubleshooting

Basic Distillation Control Philosophy

Reboiler, Condenser, and Pressure Controls

Temperature and Composition Control

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Columns That Did Not Work: Case Histories

Distillation Design

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Introduction to Distillation

Key Fractionation Concepts

Column Process Design, Optimization, and Shortcut Calculations

Rigorous Distillation Calculations

Batch Distillation

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Packing Capacity and Pressure Drop GPDC Interpolation Charts Atlas

Packing Efficiency Data

Petroleum Refinery Distillation

Process Design for Reliable Operation

2nd Edition

By Norman P. Lieberman

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What is Process Design?

Packed Column Pitfalls

Vacuum Tower Design

Trayed Tower Internals

Washing Flash Zone Vapors

Revamping Distillation Columns

Distillation Tower Reboiler Details

Sizing Vessels

Steam Turbines and Surface Condensers

How to Pull Deep Vacuum

Centrifugal Compressors: Designing to Avoid Surge

Sizing Centrifugal Pumps

Shell-and-tube Heat Exchangers – The Pitfalls of Oversizing

Revamp Errors Made in Expanding Condensers

Fired Heater Design for Maximum Run Lengths

Reboiler Design Errors

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Safety

Liquid Level Control

Saving Energy Through Flexibility

Cost Estimation – How to Allow for Installation Factors

Petroleum Refinery Distillation

By R. N. Watkins

Gulf Publishing Company

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Covers column designs where the feed and products are related in terms of ASTM, TBP and EFV curves. Covers the use of Packie-type analysis. Includes chapters on Atmospheric Towers, Vacuum Towers, Main Fractionator – Catalytic Cracking Unit, Refinery Light Ends Fractionation, Main Fractionator – Delayed Coking Unit and Pyrolysis Gas Quench Tower. The appendix includes a section on Crude Assay and a section on Enthalpy and Total Heat of Vaporization of Petroleum Fractions.

Random Packing and Packed Towers

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by Ralph F. Strigle, Jr.

Gulf Publishing Company

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Hydraulics of Operation

Air Pollution Control

Gas Absorption

Liquid Stripping

Heat Transfer

Atmospheric Distillation

Vacuum Distillation

Pressure Distillation

Column Internals

Olefins Production

Liquid-Liquid Extraction

Mass Transfer Operation

by Robert E. Treybal

McGraw Hill

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Includes a section on Diffusion and Mass Transfer, Gas-Liquid Operations, Liquid-Liquid Operations, Solid-Fluid Operations and The Less Conventional Operations.

Applied Hydrocarbon Thermodynamics

Volume 1, 2nd Edition

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Homogeneous Closed Systems

Heterogeneous Closed Systems

PVT Data and Correlations

Equations of State

Corresponding States Methods

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EDS-R00-02(Cover)

F.R.I. MEETS INDUSTRY NEEDS

Fractionation is the most versatile, economical and often only practical method used in 95 pct of all separations in the chemical and petroelum industries. F.R.I. provides commercial scale research quality data to support this basic unit operation.

Member companies benefit from reduced costs of engineering time when the operating, engineering and equipment manufacturing companies all work from the same internationally recognized F.R.I. database.

Research objectives are determined by the expressed needs of member companies. Each member can contribute ideas and influence program content. Membership direction and control of the F.R.I. research program assures continuous review.

F.R.I. data and models facilitate accurate predictions for column modifications needed to respond to feedstock changes, increased throughput requirements, or increased product purity specifications.

Uncertainty in design increases both initial capital costs and operating expense throughout project life. A primary function of F.R.I. research is to obtain the data needed to improve and extend present modeling to decrease design uncertainty.

While the basic elements of fractionation are mature, there are significant opportunities to save energy and capital costs with new, efficient trays and packings. F.R.I. commercial scale data characterizes new hardware performance and provides confidence that installations will perform as expected.

The F.R.I. staff serves as a direct source of information about applications of the F.R.I. research results. Their familiarity with the F.R.I. database and current models is of value especially to new members.

F.R.I. supplements its small, versatile staff with a worldwide network of consultants, each of whom is retained because of outstanding expertise in a particular field. This approach involves the best available talent in each facet of a diverse program while achieving maximum utilization of available funds.

Participation in high value distillation research is achieved through F.R.I. at a comparatively low price. Not so obvious, perhaps, is that the value of this research is greatly enhanced by the guidance given by the elected Technical Committee. These experienced industry experts examine the data and models from many viewpoints. The results to the members are often of much higher quality and broader application than research from universities, the government and, sometimes, internal corporate research.

Costly unit downtime to modify internals of a single fractionation column designed without the benefit of a member's direct access to F.R.I. data and models could more than pay for membership.

EXECUTIVE SUMMARY

Fractionation Research, Inc. invites your participation in a Tenth Extension of its program, building upon forty years of member-oriented operation. Participation will bring you the following benefits:

Results of the F.R.I. experimental research program, developed and monitored by the Technical Director and the distillation specialists of the Technical Committee, costing approximately \$1.1 million per year, and conducted in plant-size columns of the recently upgraded experimental facility.

Interpretation of experimental results, motion pictures, models of performance of tested devices, computer programs and handbook methods for rating these devices.

Complete results of impartial tests of proprietary devices, information which will facilitate valid comparisons between these devices and generic equipment.

Participation, through the Board of Directors, in determining F.R.I. business and administrative policies; participation, through the Technical Advisory Committee, in determining the content and priorities of the experimental program.

Industry contacts and the experience of other companies, gained through your representatives' attendance at F.R.I. meetings and participation in informal sessions.

Following renovation, upgrading, and relocation to Stillwater, Oklahoma, a step requiring approximately 1-3/4 years, the F.R.I. experimental facility is in regular operation and producing results consistent with previous performance. Operating cost at Stillwater is lower than at the previous location, and the improved facility produces higher quality and more complete data. The experimental program planned for the current extension has been monitored carefully, and we have forecasted how much will be accomplished in the time remaining. The program proposed for the next extension is coordinated with the present program. Each research item is described in detail and reasons for its importance are given.

F.R.I. remains committed to the principles of accountability and member direction.

You know the cost and length of your commitment to membership.

You know the research program to be undertaken and how its cost and length were estimated.

Your representative will participate in deciding the priority of research items.

You will be able to monitor progress.

RECENT ACCOMPLISHMENTS

Relocation Benefits

Increased reliability and operating efficiency were achieved through instrumentation modernization, addition of a microprocessor based data acquisition/control system and complete redesign of the piping and structure. The reconstructed unit conforms to the latest codes and standards.

Structured Packing

High pressure operation completed the basic program. The commercially observed efficiency loss at high pressure/high liquid load was conformed.

Maldistribution tests produced distributor guidelines and highlighted random/structured packing differences.

Good performance at lower liquid loads than previous research was demonstrated providing a basis for further work.

First gamma scan results under good and maldistributed flow conditions were obtained.

Random Packing

Tests at varying bed lengths with and without a redistributor quantified conditions under which redistribution is required as well as provided data to test various scale-up models.

Modeling and Computer

PC versions of rating programs were issued.

A project is underway to provide the entire F.R.I. data bank to the membership in a PC database format. This will permit easy search for the closest experimental result to a design problem or model prediction.

HIGHLIGHTS OF PROPOSED NEW EXTENSION PROGRAM

The primary thrust of the experimental program is to expand the existing database to provide information required for more confident decisions.

Structured and Random Packing

Vapor maldistribution, liquid spreading and wall flow studies will be employed to test existing design rules and models for both random and structured packings.

The structured packing database will be expanded to add other sizes of sheet metal packing, performance of grids and heat transfer studies.

Plastic packing will be added to the random packing database and gamma ray studies will be performed under controlled conditions to increase confidence in the interpretation of commercial troubleshooting results.

Trays

A generic valve tray database will be generated to provide a reference for the evaluation of proprietary offerings.

The sieve tray database will be expanded to include additional hardware parameters often encountered commercially (deck thickness, hole sizes, modified weirs).

Modeling

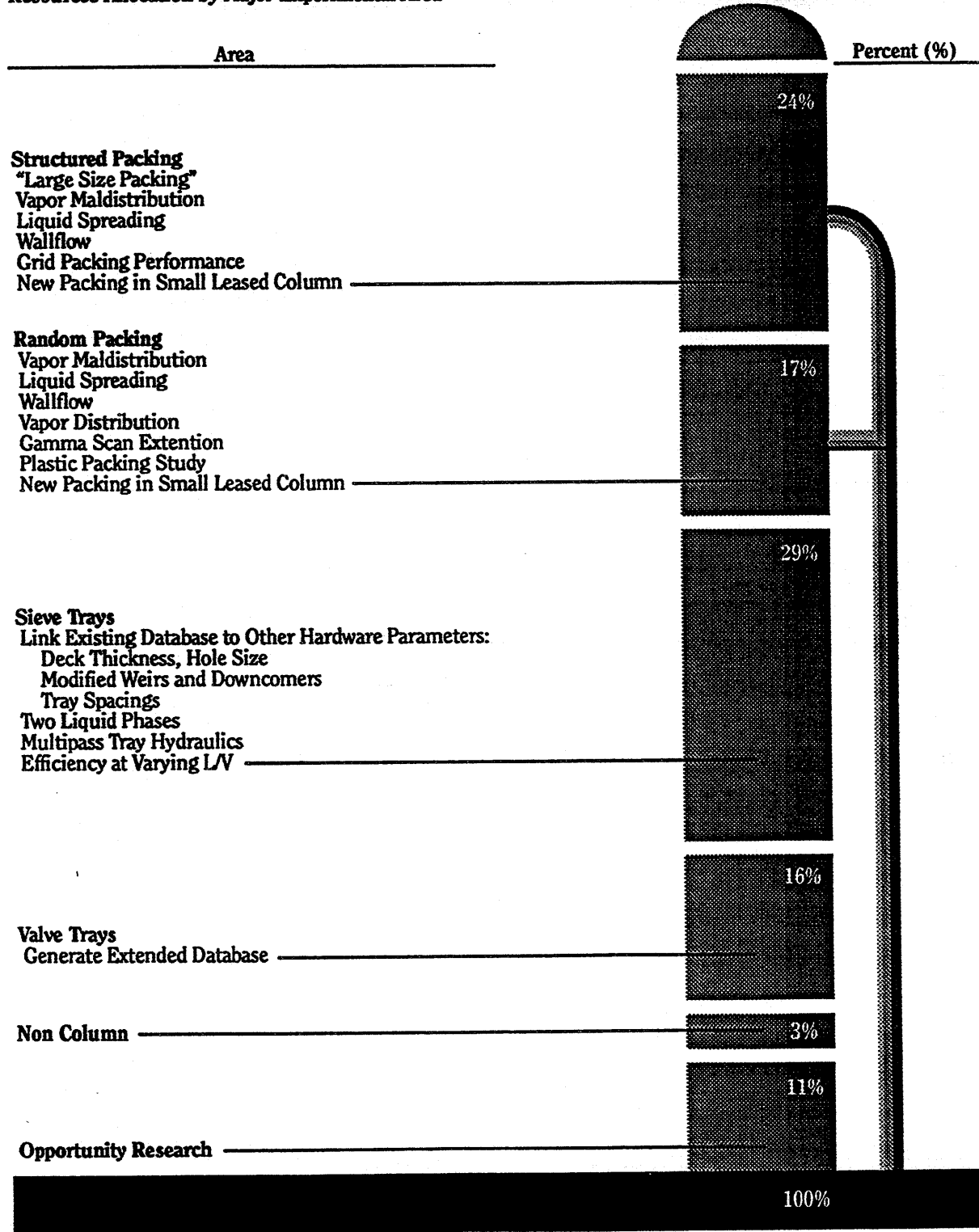
A consistency and reliability analysis of existing models will be performed and statistically based safety factors will be derived.

New Initiatives

Leasing of a small (1-2 ft) diameter column will be pursued. Mass transfer efficiency tests in equipment at this scale are much less expensive than in the F.R.I. full commercial scale equipment. It is believed that with appropriate calibration runs and with F.R.I. control of the tests, the results can be related to commercial column performance. This would permit more devices to be tested for the same amount of expenditures.

Proposed Extension

Resources Allocation by Major Experimental Area



EDS-R00-0237

HISTORICAL PERSPECTIVE

Forty Years of Progress

In the early 1950's, Dr. Karl Hachmuth, of Phillips Petroleum Company, was studying the effectiveness of distillation devices and decided that too little was known for design engineers to properly predict distillation results. He concluded that research was needed on plant-scale kequipment and also concluded that such research was too expensive for any one company. So, he promoted the concept of cooperative research to technical representatives of other companies, who found the idea good. Mr. T. B. Hudson, of Phillips, did much of the organizational work that developed the structure of F.R.I. and became its first President. By February 1, 1952, 15 companies had executed the F.R.I. agreement and incorporation was completed on April 4, 1952. By the first stockholder meeting, November 10, 1952, there were 37 member companies.

From the start, F.R.I. was organized as a corporation with a limited life unless extended by the membership. The first operation was envisioned as a five-year research program. Since then, F.R.I.'s life has been extended, in periods of three or five years, because the members continued to see important research projects of interest to a large group of member companies. The research proposed for each extension is outlined in prospectus form for membership approval. Since the research is membership directed, the research program can be modified during an extension by vote of the Technical Advisory Committee.

Dr. E. H. Amick of Columbia University was the first Technical Director. He was followed by Mr. F. W. Winn. Mr. George Keller was employed in August, 1954, and became Technical Director in January, 1959. Dr. John Kunesh became Technical Director in October, 1984.

In January, 1954, F.R.I. entered into a contract with C. F. Braun & Co., Alhambra, California, to use their simulator. The first four foot (1.2m) diameter F.R.I. column was contracted in March, 1954, and operations began September 30, 1954. An eight foot (2.4m) diameter extension to the research column was made in 1960. A four foot diameter high pressure column was added in 1963. Operations continued at C. F.

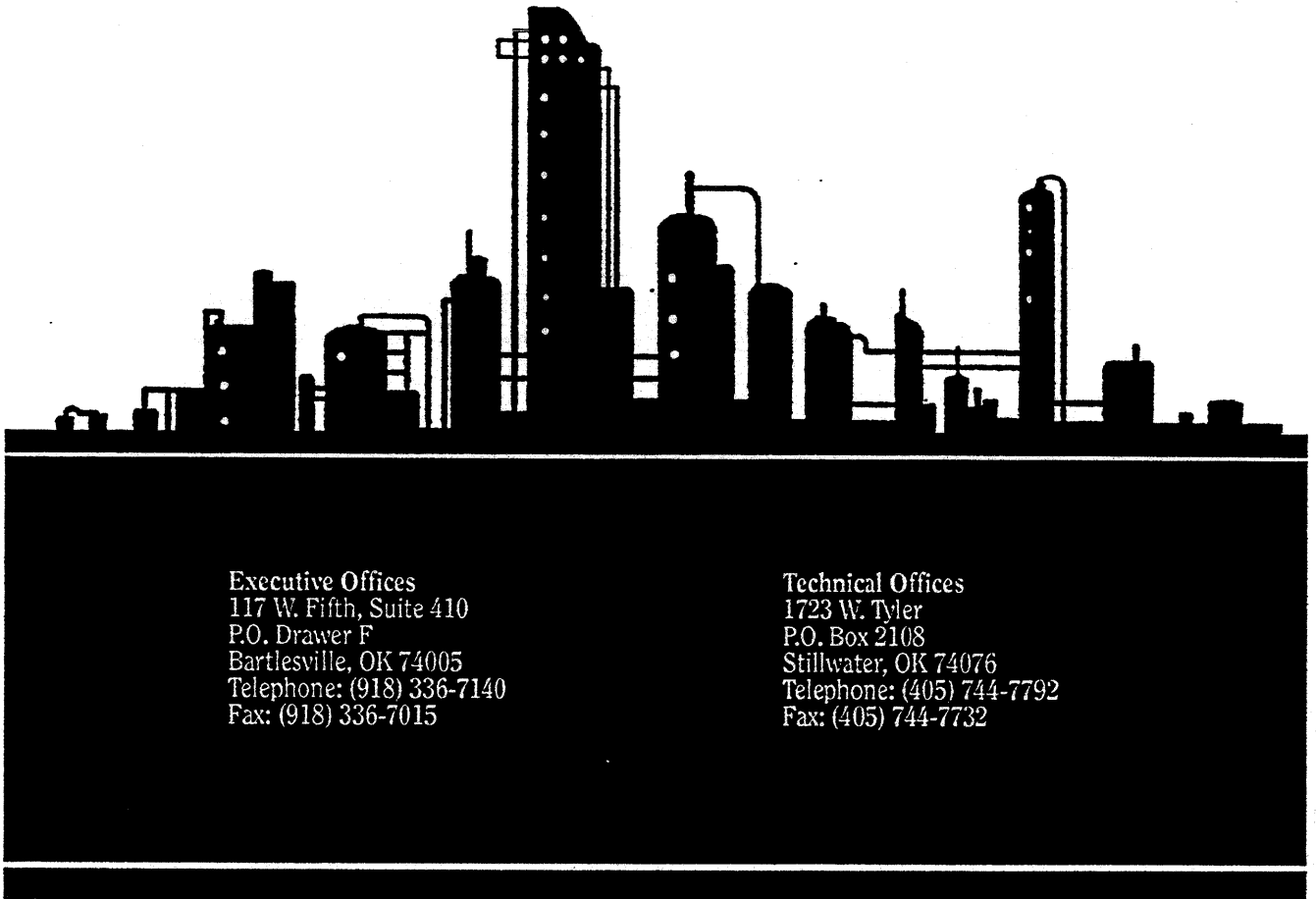
Braun until November, 1989. The equipment was then moved to a site at Oklahoma State University, Stillwater, Oklahoma, and began operating in August, 1991.

How F.R.I. Works Today

Fractionation Research, Inc. is directed by its membership in both business and technical activities. The corporation is organized to make this direction effective. F.R.I. is unique in that it legally exists only for a specific contract period. This has typically been 3-5 years. It continues to exist only if the membership finds it desirable for this to occur. A prospectus is issued for each extension which outlines the research planned for that extension. The membership then agrees to participate during that extension.

To oversee its business, F.R.I. has a Board of Directors. Each member company may have a member on this Board if it desires. The Board then elects an Executive Committee which directs the detailed business of the Corporation through the President and the Secretary of the Corporation, subject to Board approval.

To oversee the research, there is a Technical Advisory Committee which plans, approves, and receives the reports on the research program. It meets two times per year. Each member company has a representative on this committee. The TAC elects a Chairman and a Technical Committee from its membership. The Chairman chairs both Committees. The Technical Committee has three representatives from each industry classification – petroleum, chemical and engineering. One from each category is elected each year. The Chairman serves throughout a contract period. The Chairman and the Technical Committee meet four times per year and work closely with the F.R.I. Technical Director to execute the research program approved by the TAC. The Technical Director directs the staff in executing the research program.



Executive Offices
117 W. Fifth, Suite 410
P.O. Drawer F
Bartlesville, OK 74005
Telephone: (918) 336-7140
Fax: (918) 336-7015

Technical Offices
1723 W. Tyler
P.O. Box 2108
Stillwater, OK 74076
Telephone: (405) 744-7792
Fax: (405) 744-7732

EDS-R00-02(Bk Cover)

Problems

Pressure = 250 psia
Temperature = 100° F

Depropanizer

1

N_F

N_T

Butane Product to Storage

Mixed LPG from Mercox Unit
(Temperature = 100° F)

Steam

Propane Drying Column

1

N_T

Steam

Propane Product to Storage

Water

Water

EDS-R00-0238

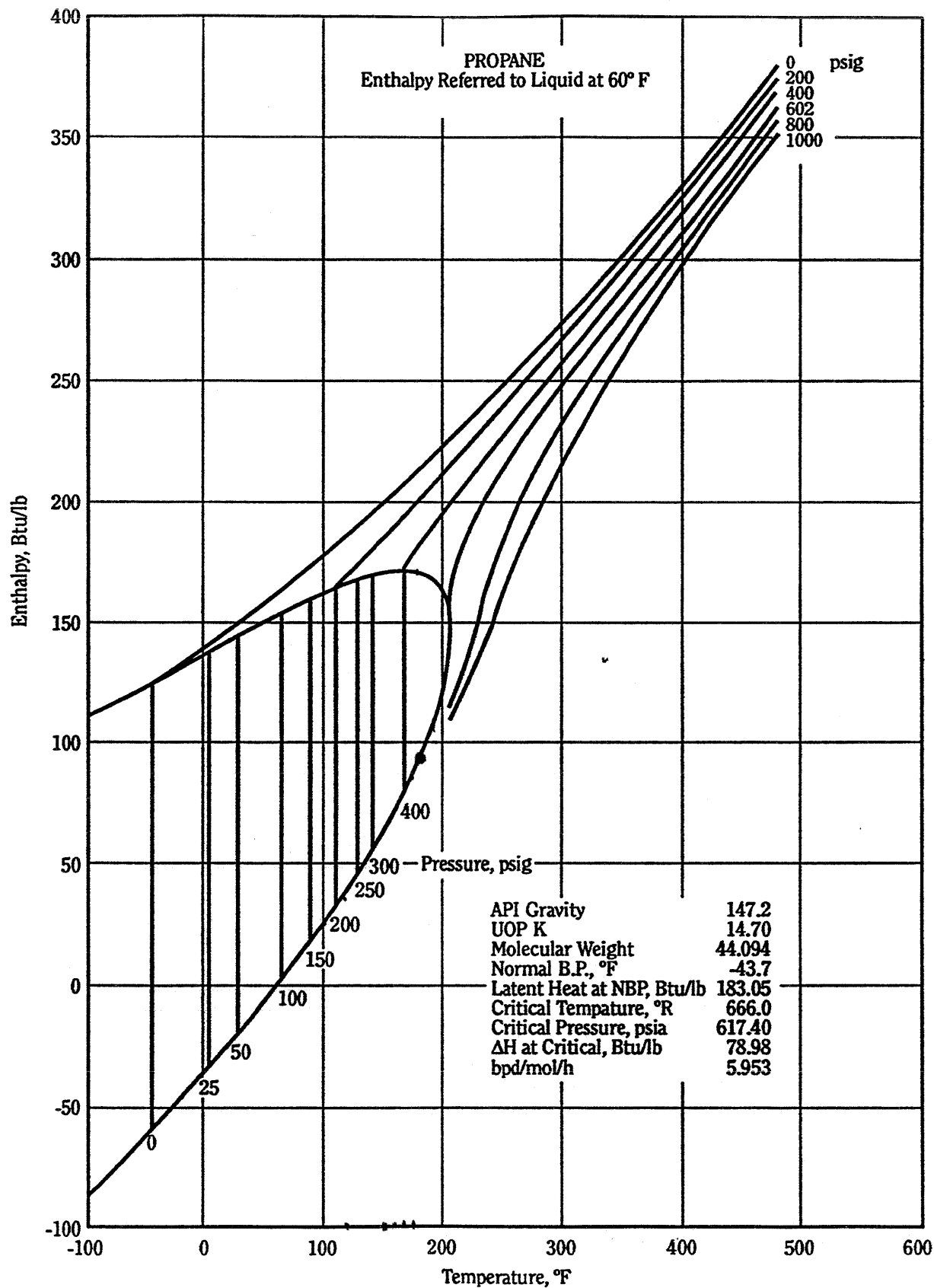
You are to determine the following (use both graphical and analytical methods):

1. The minimum number of theoretical equilibrium stages required for the Depropanizer (total reflux operation).
2. The minimum internal reflux ratio (L/D) required for the Depropanizer (infinite stages) when the feed exchanger is designed such that the bottoms outlet temperature is 120°F. *Calculate the Depropanizer feed-bottoms design duty.*
3. The actual number of stages and optimum feed stage for the Depropanizer when the design reflux ratio is 1.2 times the minimum ratio of part 2.
4. The actual number of stages required for the Propane Drying Column if the column is to be designed for an average V/L of 1/3.

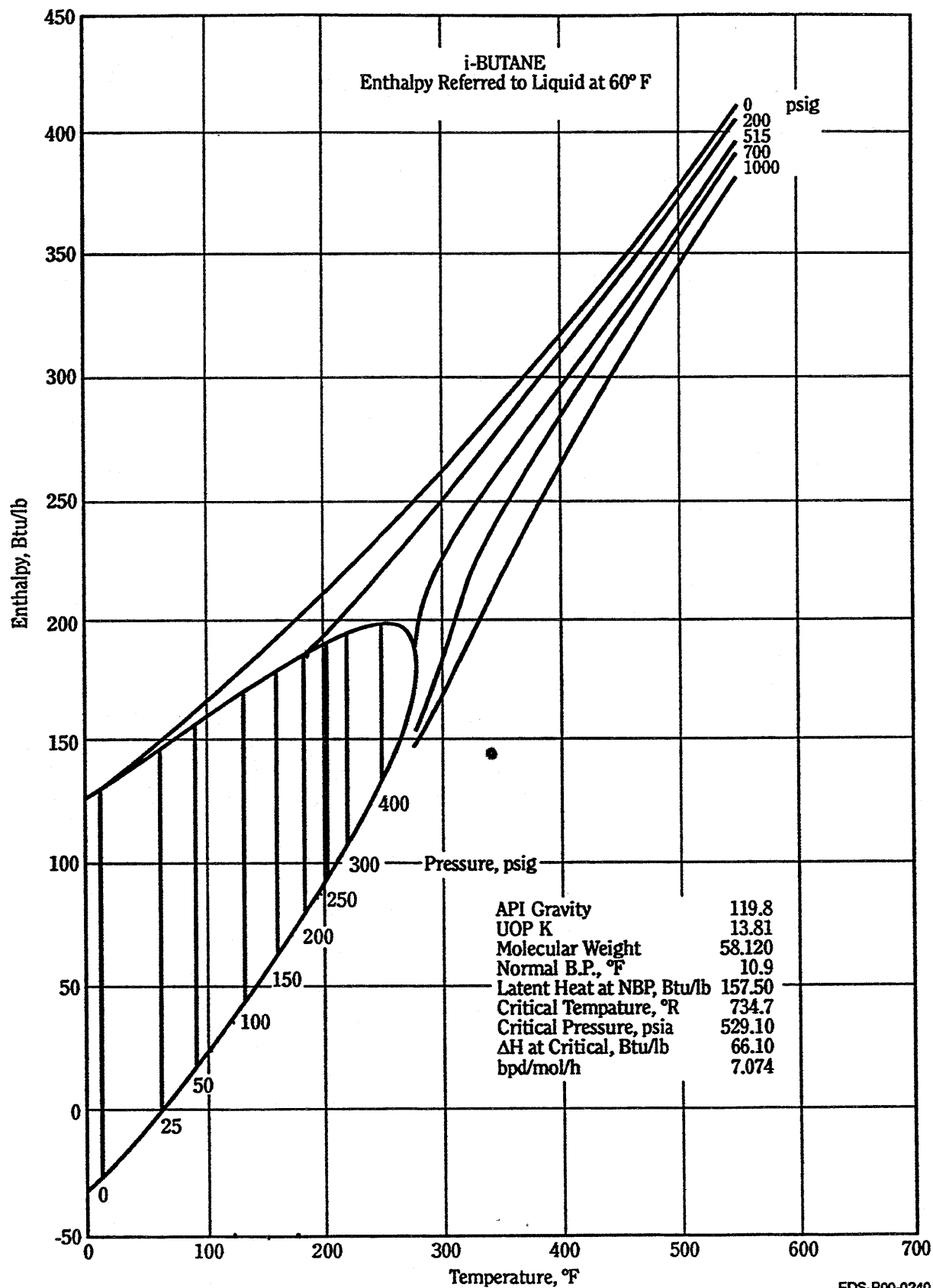
Notes

- a. Assume that the operating pressure throughout both columns is constant and equal to the receiver pressure.
- b. Assume that the NGPA cobalt bromide test for water content of the propane corresponds to a specification of 5 wt-ppm water.
- c. Assume that the feed to the drying column does not contain any entrained water.
- d. Assume that the water contained in the mixed LPG feed has an insignificant effect on the design of the Depropanizer and, therefore, can be ignored.
- e. The NGPA specification for maximum C_4^+ in propane is 2.5 liquid volume pct. This corresponds to a specification of 2.1 mol pct isobutane in the propane distillate of our problem.

- f. Use the API data for determining water solubilities in hydrocarbon.
- g. Assume the hydrocarbon composition at the bottom of the drying column is the same as in the receiver.



EDS-R00-0239



ENGINEERING DESIGN SEMINAR

SEPTEMBER 6, 1994 – NOVEMBER 11, 1994

BINARY FRACTIONATION

ANSWERS TO PROBLEMS

Material Balance

Feed

$$39870 \text{ lb/h}, \quad 27.9 \text{ wt pct } C_3, \quad 72.1 \text{ wt pct } iC_4$$

$$11124 \text{ lb/h } C_3 \times \frac{1 \text{ lb mol } C_3}{44.094 \text{ lb } C_3} = 252.3 \text{ mol/h } C_3$$

$$28746 \text{ lb/h } iC_4 \times \frac{1 \text{ lb mol } iC_4}{58.12 \text{ lb } iC_4} = 494.6 \text{ mol/h } iC_4$$

$$\text{Total feed mol/h} = 746.9$$

$$33.8 \text{ mol pct } C_3$$

$$\left. \begin{array}{l} 33.8 \text{ mol pct } C_3 \\ 66.2 \text{ mol pct } iC_4 \end{array} \right\} x_F = 0.338$$

Distillate (Overhead)

$$2.1 \text{ mol pct } iC_4 > x_D = 0.979$$

Distillate/Feed Ratio (molar)

95 pct of C_3 in feed is recovered in overhead:

$$C_3 \text{ in overhead} = 0.95 \times 252.3 \text{ mol/h} = 239.7 \text{ mol/h}$$

$$\text{Total distillate} = \frac{239.7 \text{ mol/h } C_3}{0.979 \text{ mol } C_3/\text{mol distillate}} = 244.8 \text{ mol/h}$$

$$D/F = \frac{244.8}{746.9} = 0.328$$

Bottoms

$$\text{Bottoms} = \text{Feed} - \text{Distillate}$$

$$\text{Bottoms (Total mol/h)} = 746.9 - 244.8 = 502.1$$

$$\text{Bottoms } C_3 = \text{Feed } C_3 - \text{Distillate } C_3$$

$$\text{Bottoms } C_3 (\text{mol/h}) = 252.3 - 239.7 = 12.6$$

$$\text{Bottoms } C_3 \text{ fraction} = \frac{12.6 \text{ mol/h } C_3}{502.1 \text{ mol/h}} = 0.025$$

$$x_B = 0.025$$

Summary

$$x_F = 0.337$$

$$x_D = 0.979 \quad \left. \vphantom{x_D} \right\} \text{Molar fraction of propane}$$

$$x_B = 0.025$$

Heat Balance

$$\text{Feed} \Rightarrow \text{Temperature} = 100^\circ \text{F}$$

Find bubble point temperature @ 250 psia

	Mol Fraction	$K\pi$ @ 160°F		$K\pi$ @ 170°F	
C_3	0.337	331.69	111.78	354.79	119.56
iC_4	0.663	190.25	<u>126.14</u>	205.69	<u>136.37</u>
			237.92		255.94

$$\text{Interpolate for sum} = 250 \Rightarrow T = 167^\circ \text{F}$$

Enthalpy @ 167°F

	Wt Fraction	Liquid (Btu/lb)		Vapor (Btu/lb)	
C ₃	0.279	80	22.32	190	53.01
iC ₄	0.721	67	<u>48.31</u>	180	<u>129.78</u>
			70.63		182.79

$$h_F^L = 70.6 \text{ Btu/lb}$$

$$h_F^V = 182.8 \text{ Btu/lb}$$

Enthalpy @ 100°F

	Wt Fraction	x	Liquid Btu/lb =	
C ₃	0.279		27	7.53
iC ₄	0.721		24	<u>17.30</u>
				24.83

$$\text{Feed } H = 24.8 \text{ Btu/lb} \times 39870 \text{ Lb/h} = 0.99 \times 10^6 \text{ Btu/h}$$

Bottoms

$$C_3 \text{ in bottoms} = 12.6 \text{ mol/h} \times 44.094 = 556 \text{ Lb/h}$$

$$\begin{aligned} iC_4 \text{ in bottoms} &= 502.1 \text{ mol/h total} - 12.6 \text{ mol/h} = 489.5 \text{ mol/h} \\ &= 489.5 \text{ mol/h} \times 58.12 = 28450 \text{ Lb/h} \end{aligned}$$

$$\text{Total mass flow} = 556 + 28450 = 29006 \text{ Lb/h}$$

$$\text{Wt fraction } C_3 = \frac{556}{29006} = 0.019$$

$$\text{Wt fraction } iC_4 = 0.981$$

Bubble point temperature @ 250 psia

	Mol Fraction	K π @ 190°F		K π @ 200°F	
C ₃	0.025	403.23	10.1	427.81	10.7
iC ₄	0.975	237.75	<u>231.8</u>	254.42	<u>248.1</u>
			241.9		258.8

Interpolate for sum = 250 $\Rightarrow T = 195^\circ F$

	Wt Fraction	Liquid Enthalpy @ 195°F Btu/lb		Vapor Enthalpy @ 120°F Btu/lb	
C ₃	0.019	110	2.09	41	0.8
iC ₄	0.981	88	<u>86.33</u>	36	<u>35.3</u>
			88.42		36.1

$$H @ 195^\circ F = 88.4 \text{ Btu/lb} \times 29006 \text{ lb/h} = 2.56 \times 10^6 \text{ Btu/h}$$

$$H @ 120^\circ F = 36.1 \text{ Btu/lb} \times 29006 \text{ lb/h} = 1.05 \times 10^6 \text{ Btu/h}$$

$$\text{Feed/bottoms exchanger duty} = 2.56 - 1.05 = 1.51 \times 10^6 \text{ Btu/h}$$

$$\begin{aligned} \text{Feed to column } H &= \text{Feed } H @ 100^\circ F + \text{Exchanger Duty} \\ &= 0.99 + 1.51 = 2.50 \times 10^6 \text{ Btu/h} \end{aligned}$$

$$\text{Enthalpy of feed to column} = \frac{2.50 \times 10^6 \text{ Btu/h}}{39870 \text{ lb/h}} = 62.7 \text{ Btu/lb}$$

$$\text{Feed } q = \frac{h_F^V - h_F}{h_F^V - h_F^L} = \frac{182.8 - 62.7}{182.8 - 70.6} = 1.07$$

$$q = 1.07$$

Equilibrium

Vapor pressure graph:

Boiling point of propane @ 250 psia = 122 °F

Boiling point of isobutane @ 250 psia = 199 °F

Equilibrium coefficients (K) for propane – isobutane system @ 250 psia

T, °F	K(C ₃)	K(iC ₄)	α	x	y
130	1.069	0.586	1.824	0.857	0.916
140	1.152	0.642	1.794	0.702	0.809
150	1.238	0.701	1.766	0.557	0.689
160	1.327	0.761	1.744	0.422	0.560
170	1.419	0.823	1.724	0.297	0.421
180	1.514	0.886	1.709	0.182	0.275
190	1.612	0.951	1.695	0.074	0.119

$$\alpha_{\text{MEAN}} = [(1.824)(1.794)(1.766)(1.744)(1.724)(1.709)(1.695)]^{1/7}$$

$$\alpha = 1.75$$

Minimum Stages (Total Reflux)

Fenske equations

$$n_m = \frac{\ln(r_D/r_B)}{\ln \alpha}, \quad r_D = \frac{x_D}{1-x_D}, \quad r_B = \frac{x_B}{1-x_B}$$

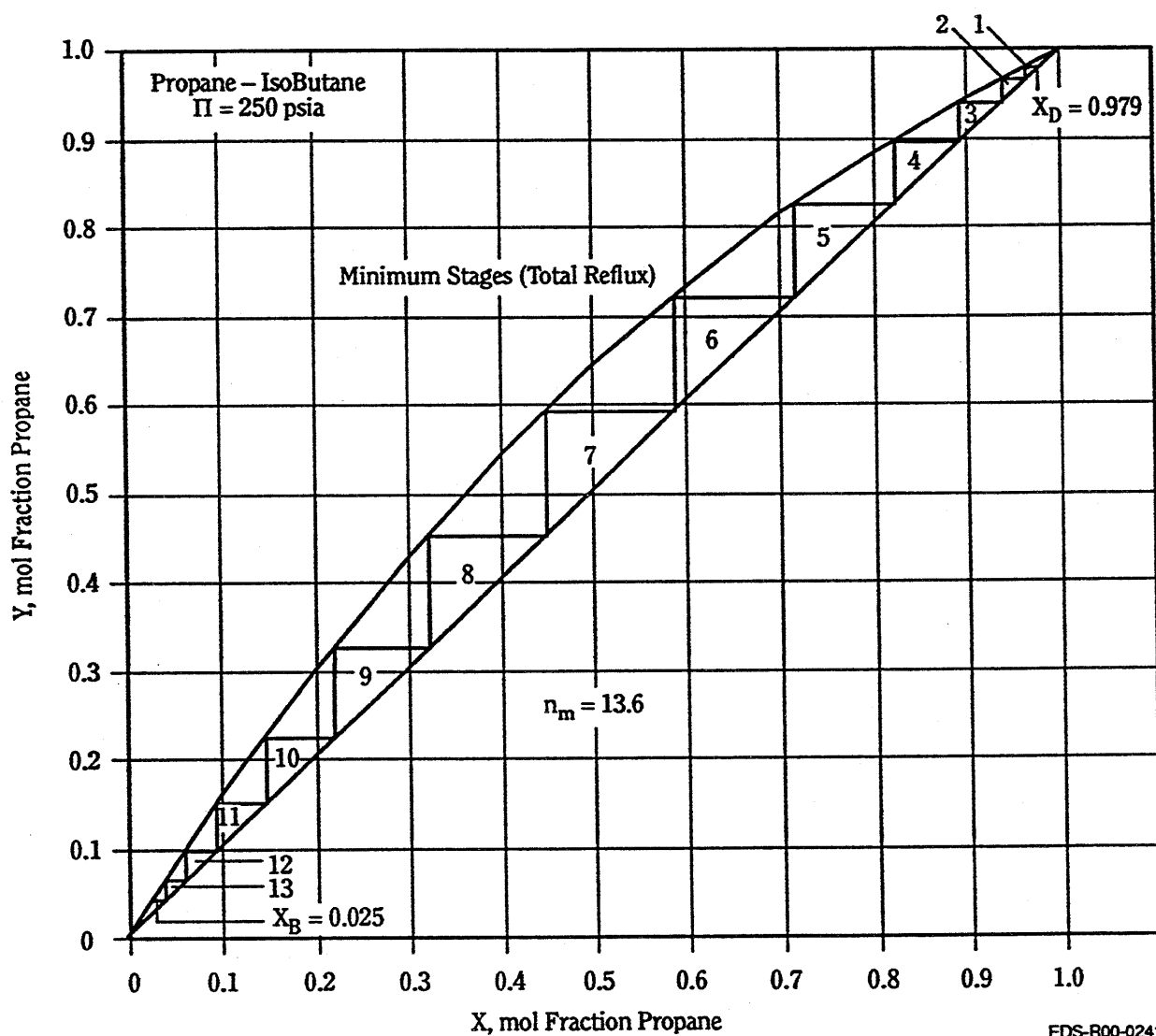
$$r_D = \frac{0.979}{1-0.979} = 46.62 \quad r_B = \frac{0.025}{1-0.025} = 0.02564$$

$$n_m = \frac{\ln(46.62/0.02564)}{\ln 1.75} = 13.4$$

Graphical

$$n_m = 13.4 \text{ (Analytic)}$$

$$n_m = 13.6 \text{ (Graphical)}$$



Minimum Reflux (Infinite Stages)

Underwood equations:

$$\frac{1-x_F}{1-\phi} + \frac{\alpha x_F}{\alpha-\phi} = 1-q; \quad (L/D)_m = \frac{1-x_D}{1-\phi} + \frac{\alpha x_D}{\alpha-\phi} - 1$$

$$\frac{1-0.337}{1-\phi} + \frac{(1.75)(0.337)}{1.75-\phi} = 1-1.07$$

$$\frac{1.1603 - 0.663\phi + 0.5898 - 0.5898\phi}{1.75 - 2.75\phi + \phi^2} = -0.07$$

$$1.7501 - 1.2528\phi = -0.1225 + 0.1925\phi - 0.07\phi^2$$

$$0.07\phi^2 - 1.4453\phi + 1.8726 = 0$$

$$\phi = \frac{1.4453 \pm \sqrt{(1.4453)^2 - (4)(0.07)(1.8726)}}{(2)(0.07)}$$

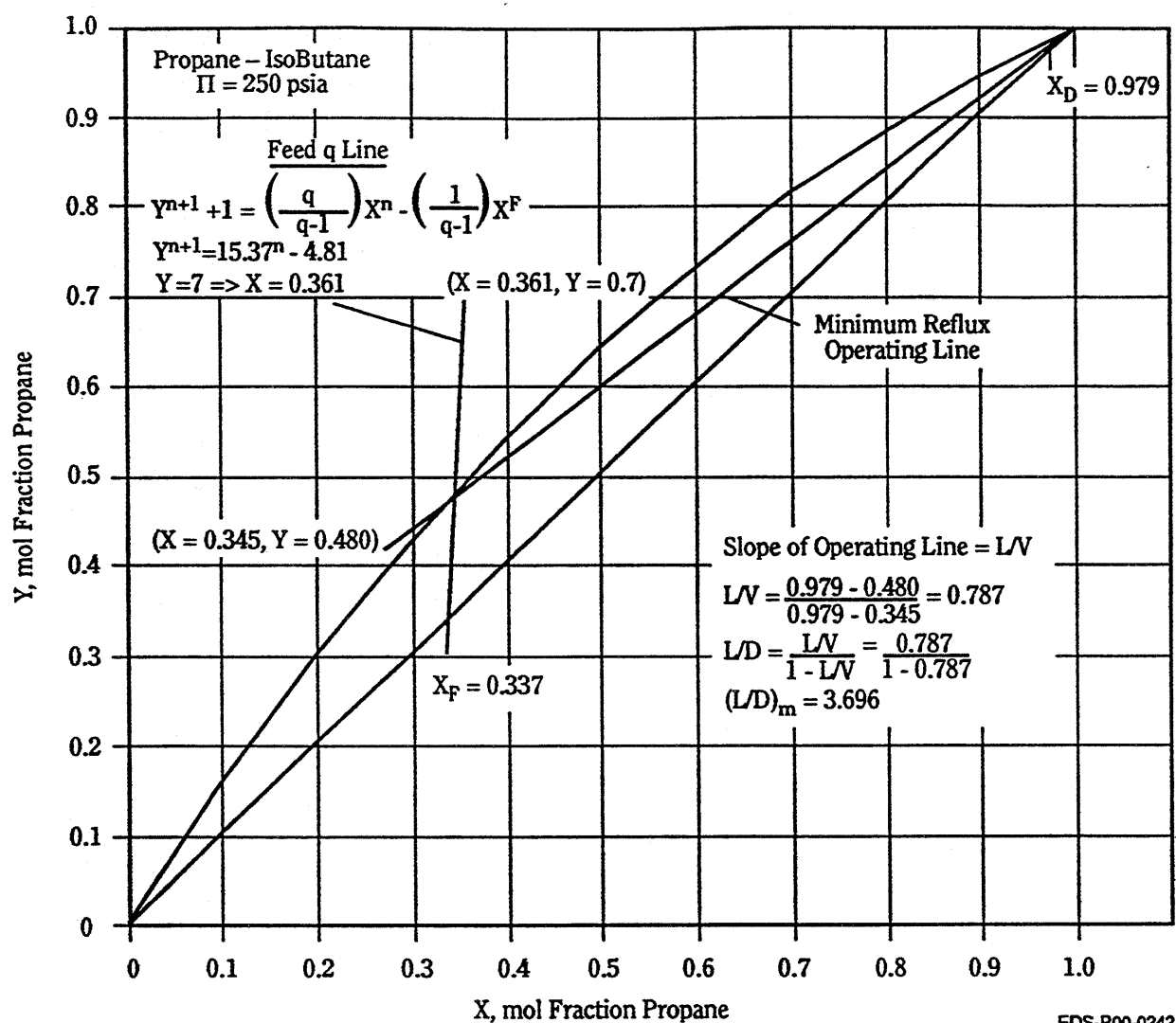
$$\phi = 10.3236 \pm 8.9345 \quad 1 < \phi < \alpha$$

$$\phi = 1.3891$$

$$(L/D)_m = \frac{1-0.979}{1-1.3891} + \frac{(1.75)(0.979)}{1.75-1.3891} - 1$$

$$(L/D)_m = 3.693 \text{ (Analytic)}$$

$$(L/D)_m = 3.696 \text{ (Graphical)}$$



Actual Stages (1.2 x Minimum Reflux)

Gilliland correlation

$$X = \frac{R - R_m}{R + 1} = \frac{L/D - (L/D)_m}{L/D + 1}$$

$$L/D = 1.2(L/D)_m = (1.2)(3.693) = 4.432$$

$$X = \frac{4.432 - 3.693}{4.432 + 1} = 0.136$$

Minimum Reflux (Infinite Stages)

Underwood equations:

$$\frac{1-x_F}{1-\phi} + \frac{\alpha x_F}{\alpha-\phi} = 1-q; \quad (L/D)_m = \frac{1-x_D}{1-\phi} + \frac{\alpha x_D}{\alpha-\phi} - 1$$

$$\frac{1-0.337}{1-\phi} + \frac{(1.75)(0.337)}{1.75-\phi} = 1-1.07$$

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$$\phi = \frac{1.4453 \pm \sqrt{(1.4453)^2 - (4)(0.07)(1.8726)}}{(2)(0.07)}$$

$$\phi = 10.3236 \pm 8.9345 \quad 1 < \phi < \alpha$$

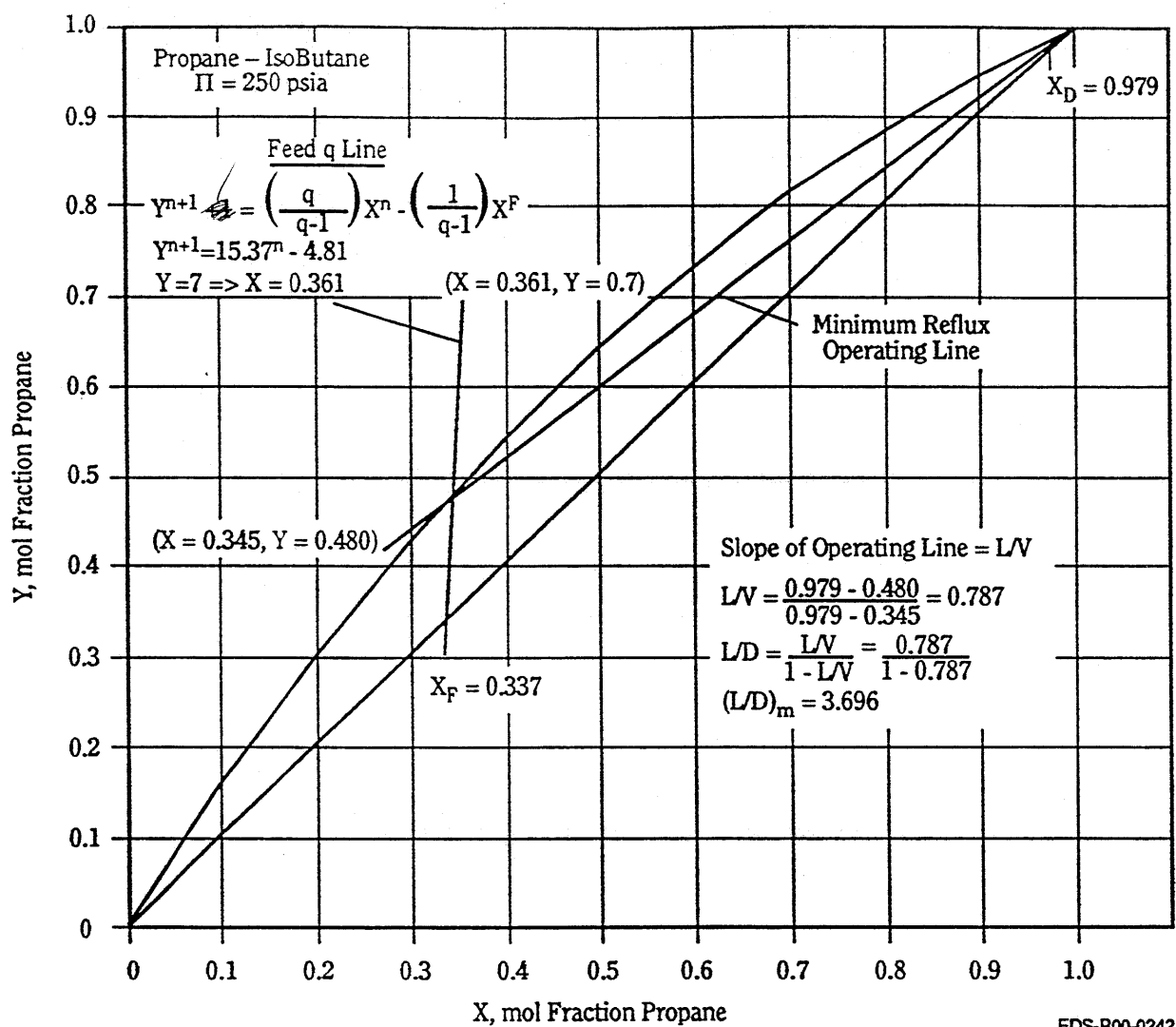
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$$(L/D)_m = \frac{1-0.979}{1-1.3891} + \frac{(1.75)(0.979)}{1.75-1.3891} - 1$$

$$(L/D)_m = 3.693 \text{ (Analytic)}$$

$$(L/D)_m = 3.696 \text{ (Graphical)}$$

Sept 80
Revision
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Actual Stages (1.2 x Minimum Reflux)

Gilliland correlation

$$X = \frac{R - R_m}{R + 1} = \frac{L/D - (L/D)_m}{L/D + 1}$$

$$L/D = 1.2(L/D)_m = (1.2)(3.693) = 4.432$$

$$X = \frac{4.432 - 3.693}{4.432 + 1} = 0.136$$

From graph:

$$X = 0.136 \rightarrow Y = 0.49$$

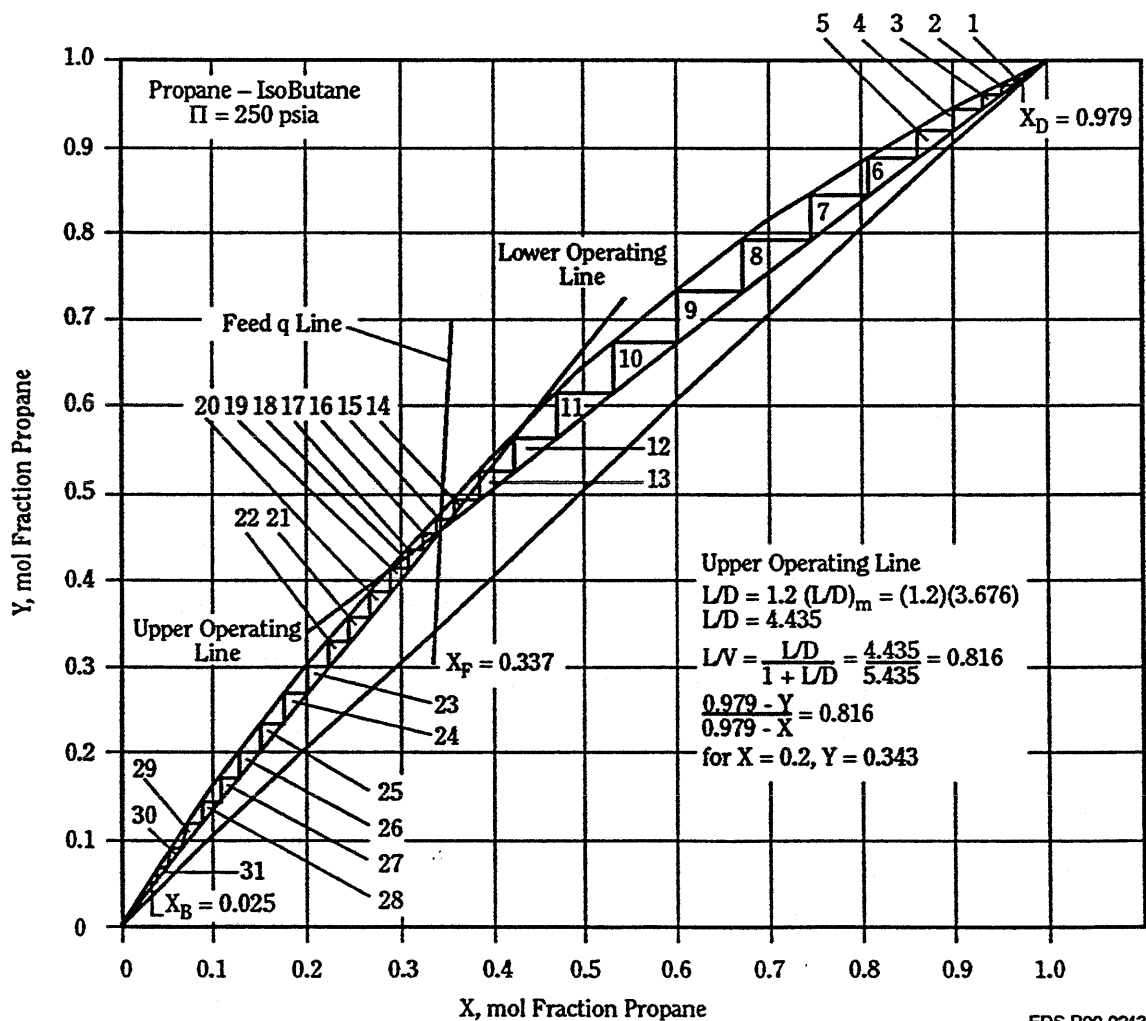
$$N = \frac{N_m + Y}{1 - Y} = \frac{13.4 + 0.49}{1 - 0.49}$$

$$n = 27.2 \text{ (Theoretical stages)}$$

Feed tray:

$$\frac{n_R}{n} = \frac{\ln(r_D/r_F)}{\ln(r_D/r_B)} = \frac{\ln(46.62/0.5083)}{\ln(46.62/0.02564)} = 0.602$$

$$n_R = 16.4$$



Water Stripping

Feed:

Receiver liquid is taken to be essentially 100 pct propane plus water of saturation.

Solubility of water in propane at the receiver temperature (100°F) is 0.00061 mol fraction.

$$x_F = 0.00061 = 610 \text{ mol ppm}$$

Equilibrium:

Since water concentration is very low, the column will operate at essentially the boiling point of propane at the column pressure.

$$T_{BP}(C_3) @ 250 \text{ psia} = 122^\circ F$$

$$K(C_3) @ 122^\circ F \text{ and } 250 \text{ psia} = 1$$

$$K_{H_2O} @ 122^\circ F \text{ and } 250 \text{ psia} = \frac{P_{H_2O}^o(T = 122^\circ F) \left[1 - x_s^{C_3/H_2O @ 122^\circ F} \right]}{\pi(250 \text{ psia}) x_s^{H_2O/C_3 @ 122^\circ F}}$$

$$K_{H_2O} = \frac{(1.7891)(1 - 0.0002)}{(250)(0.0011)} = 6.5$$

$$\alpha = K_{(H_2O)} / K_{(C_3)} = 6.5$$

$$\text{Equilibrium line: } y = \alpha x = 6.5x$$

Bottoms:

Specification is 5 wt-ppm water

$$x_B = \frac{5 \text{ lb } H_2O}{10^6 \text{ lb } C_3} \times \frac{44.094 \text{ lb } C_3}{\text{lb mol } C_3} \times \frac{\text{lb mol } H_2O}{18.016 \text{ lb } H_2O}$$

$$x_B = 0.0000122 = 12.2 \text{ mol ppm}$$

Operating Line:

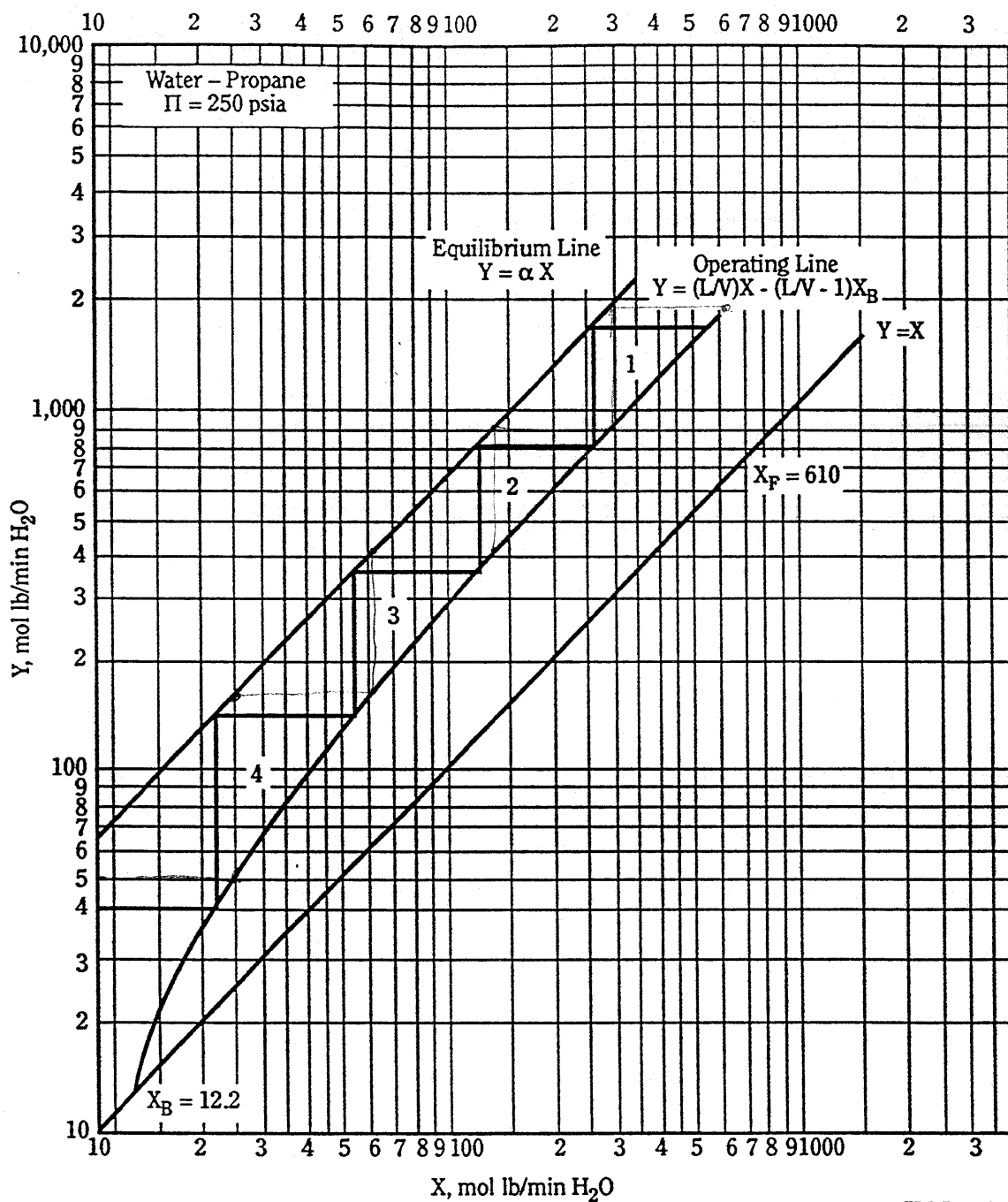
$$y = (L/V)x - (L/V - 1)x_B$$

$$L/V = 3$$

$$y = 3x - 2x_B = 3x - 24.4$$

x	13.5	15	17.5	20	25	30	50	100	300	610
y	16.1	20.6	28.1	35.6	50.6	65.6	126	276	876	1810

sent to
revision
9/20/94



EDS-R00-0244

$n = 4.5$ stages

Analytical Approximation

$$n = \frac{\ln(X_F/X_B)}{\ln(\alpha V/L)} = \frac{\ln(610/12.2)}{\ln(6.5/3)}$$

$n = 5.1$ stages

Bottoms:

Specification is 5 wt-ppm water

$$x_B = \frac{5 \text{ lb } H_2O}{10^6 \text{ lb } C_3} \times \frac{44.094 \text{ lb } C_3}{\text{lb mol } C_3} \times \frac{\text{lb mol } H_2O}{18.016 \text{ lb } H_2O}$$

$$x_B = 0.0000122 = 12.2 \text{ mol ppm}$$

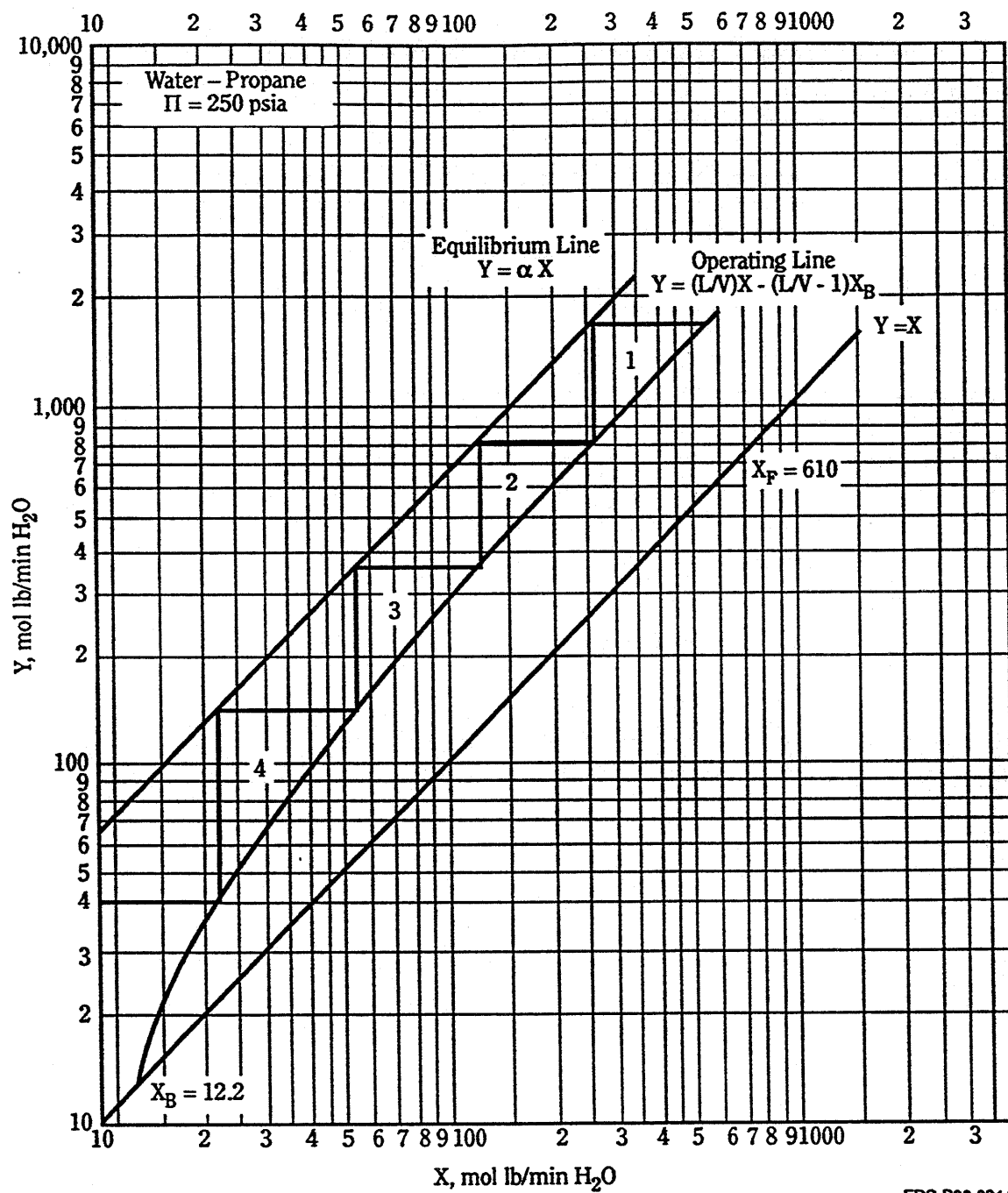
Operating Line:

$$y = (L/V)x - (L/V - 1)x_B$$

$$L/V = 3$$

$$y = 3x - 2x_B = 3x - 24.4$$

x	13.5	15	17.5	20	25	30	50	100	300	610
y	16.1	20.6	28.1	35.6	50.6	65.6	126	276	876	1810



EDS-R00-0244

Smoker's Solutions

Date: 20-Feb-89
Time: 04:42:38 PM
Name: PC Steacy

FOR COL TOP M=LV					
FOR COL BOT M=LPVP					
INPUT					
FEED	FEED	746.9	746.9	746.9	746.9
FEED COMP	XFEED	0.338	0.338	0.338	0.338
DIST COMP	XDIST	0.979	0.979	0.979	0.979
BOT COMP	XBOT	0.025	0.025	0.025	0.025
DISTILLATE	DIST	244.8	244.8	244.8	244.8
REFLUX	REFLUX	1085.688	1085.688	1085.688	1085.688
FEED Q	QFEED	1.07	1.07	1.07	1.07
IF 1, THEN RECT SECT	COLSECT	1	1	1	1
RELATIVE VOLITILITY	ALPHA	1.824	1.794	1.766	1.744
X1	COMP1	0.979	0.857	0.702	0.557
X2	COMP2	0.857	0.702	0.557	0.346150
OUTPUT					
TRAYS		4.10	2.52	2.13	5.37
COLUMN SECTION	RECT	RECT	RECT	RECT	RECT
Q-OPER LINE INTERCEPT		0.3462	0.3462	0.3462	0.3462
CALCULATIONS					
BOT MOL/HR	BOT	502.1	502.1	502.1	502.1
RECT VAPOR RATE	V	1330.488	1330.488	1330.488	1330.488
L/V	LV	0.816007	0.816007	0.816007	0.816007
L'	LP	1884.871	1884.871	1884.871	1884.871
V'	VP	1382.771	1382.771	1382.771	1382.771
L'/V'	LPVP	1.363111	1.363111	1.363111	1.363111
CONST FOR XINT CALC	XINTA	5.008700	5.008700	5.008700	5.008700
CONST FOR XINT CALC	XINTB	14.46970	14.46970	14.46970	14.46970
Q OPER LINE INTERCEPT	XINT	0.346150	0.346150	0.346150	0.346150
OPER LINE SLOPE	M	0.816007	0.816007	0.816007	0.816007
OPER LINE X=0 INTR	B	0.180128	0.180128	0.180128	0.180128
ALPHA-1	ALPHA1	0.824	0.794	0.766	0.744
	CAPA	-1.27837	-1.28871	-1.29909	-1.30779
	CAPB	0.267893	0.278015	0.288177	0.296699
	SQRT	0.375056	0.370379	0.365696	0.361780
	KROOT1	1.014243	1.014735	1.015243	1.015679
	KROOT2	0.264131	0.273977	0.283850	0.292118
	K	0.264131	0.273977	0.283850	0.292118
	CAPC	1.217644	1.217538	1.217429	1.217336
	CAPE	0.663299	0.674274	0.684843	0.693375
	X1P	0.714868	0.583022	0.418149	0.264881
	X2P	0.592868	0.428022	0.273149	0.054032
	CAPD	1.333133	1.349968	1.367254	1.382053
	D	0.565013	0.336879	0.156163	0.019780
	TRAYS	4.098710	2.520789	2.129724	5.374160

Date: 20-Feb-89

Time: 04:42:38 PM

Name: PC Steacy

FOR COL TOP M=LV					
FOR COL BOT M=LPVP					
INPUT					
FEED	FEED	746.9	746.9	746.9	
FEED COMP	XFEED	0.338	0.338	0.338	
DIST COMP	XDIST	0.979	0.979	0.979	
BOT COMP	XBOT	0.025	0.025	0.025	
DIST.	DIST	244.8	244.8	244.8	
REFLUX	REFLUX	1085.688	1085.688	1085.688	
FEED Q	QFEED	1.07	1.07	1.07	
IF 1, THEN RECT SECT	COLSECT				
RELATIVE VOLITILITY	ALPHA	1.724	1.709	1.695	
X1	COMP1	0.3461250	0.297	0.182	
X2	COMP2	0.297	0.182	0.025	
					14.12338
OUTPUT					15.81358
TRAYS		3.10	4.82	7.90	29.94
COLUMN SECTION	Strp		Strp	Strp	
Q-OPER LINE INTERCEPT		0.3462	0.3462	0.3462	
CALCULATIONS					
BOT MOL/HR	BOT	502.1	502.1	502.1	
RECT VAPOR RATE	V	1330.488	1330.488	1330.488	
L/V	LV	0.816007	0.816007	0.816007	
L'	LP	1884.871	1884.871	1884.871	
V'	VP	1382.771	1382.771	1382.771	
L'/V'	LPVP	1.363111	1.363111	1.363111	
CONST FOR XINT CALC	XINTA	5.008700	5.008700	5.008700	
CONST FOR XINT CALC	XINTB	14.46970	14.46970	14.46970	
Q OPER LINE INTERCEPT	XINT	0.346150	0.346150	0.346150	
OPER LINE SLOPE	M	1.363111	1.363111	1.363111	
OPER LINE X=0 INTR	B	0.00907	0.00907	0.00907	
ALPHA-1	ALPHA1	0.724	0.709	0.695	
	CAPA	-0.37234	-0.36455	-0.35698	
	CAPB	-0.00919	-0.00939	-0.00958	
	SQRT	0.209422	0.206442	0.203574	
	KROOT1	0.395593	0.388720	0.382068	
	KROOT2	-0.02325	-0.02416	-0.02507	
	K	0.395593	0.388720	0.382068	
	CAPC	1.286409	1.275603	1.265537	
	CAPE	1.308436	1.297837	1.287987	
	X1P	-0.04944	-0.09172	-0.20006	
	X2P	-0.09859	-0.20672	-0.35706	
	CAPD	-2.38751	-2.42198	-2.45610	
	D	-0.01163	-0.04592	-0.17545	
	TRAYS	3.098475	4.817423	7.897691	

Smoker's Equations for Symphony PC Program

BOT	+FEED-DIST
V	+REFLUX+DIST
LV	+REFLUX/(REFLUX+DIST)
LP	+REFLUX+QFEED*FEED
VP	+LP-BOT
LPVP	+LP/VP
XINTA	@IF(QFEED=1,+XFEED,+DIST*XDIST/V+XFEED/(QFEED-1))
XINTB	@IF(QFEED=1,1,+QFEED/(QFEED-1)-LV)
XINT	+XINTA/XINTB
M	@IF(COLSECT=1,+LV,+LPVP)
B	@IF(COLSECT=1,XDIST*(1-M),XBOT*(1-M))
ALPHA1	+ALPHA-1
CAPA	(M+B*ALPHA1-ALPHA)/(M*ALPHA1)
CAPB	+B/(M*ALPHA1)
SQRT	@SQRT(CAPA^2/4-CAPB)
KROOT1	-CAPA/2+SQRT
KROOT2	-CAPA/2-SQRT
K	@IF(KROOT1>1,+KROOT2,+KROOT1)
CAPC	1+K*ALPHA1
CAPE	+M*CAPC^2/ALPHA
X1P	+COMP1-K
X2P	+COMP2-K
CAPD	+CAPE*ALPHA1/(CAPC*(1-CAPE))
D	+CAPD*X1P*X2P
TRAYS	@LN(X2P-D)/(X1P-D))/@LN(CAPE)

Graphical Design of Fractionating Columns¹

By W. L. McCabe and E. W. Thiele

MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE, MASS.

THE rational design of column apparatus for separating the components of binary mixtures of volatile, miscible components dates back to Sorel.² He assumed that equilibrium is established between the liquid on each plate and the vapor rising from it, and calculated the enrichment from plate to plate by heat and material balances. Sorel's method, though sound, is tedious when applied analytically. Many writers have simplified or modified Sorel's method, among them being Lewis,³ Leslie,⁴ Peters,⁵ Van Nys,⁶ and Gay,⁷ who have all used analytical methods. Lewis's method is typical of many of them. Lewis assumed that the enrichment per plate was small, wrote a general enrichment equation differentially, integrated it, and so determined the number of plates. The larger the number of plates the more accurate is this method, but for columns of but a few plates the error introduced by assuming continuous for stepwise conditions is appreciable.

In all the procedures mentioned above, perfect functioning of the column, and hence complete equilibrium between the vapor and liquid of each plate, is assumed, and the number of plates so determined is the number of theoretical or perfect plates necessary to effect the desired separation. If an actual column is one of the plate type, the number of theoretical plates is divided by an average plate efficiency to translate it into the number of actual plates. If the column is of the packed type the number of theoretical plates is multiplied by the height of column equivalent to one theoretical plate (H. E. T. P.)⁸ if the height of the column is desired. Both factors are experimentally determined and both vary with the design of the column and with the mixture distilled. Since they have not been accurately determined for all types of columns or for all mixtures, there is little practical value in determining theoretical plates with great precision and a method such as Lewis's offers sufficient accuracy in spite of the approximation inherent in it. However, since the equations used in all methods are linear, they adapt themselves to stepwise graphical manipulations, with consequent simplicity without the assumption of continuity from plate to plate.

Graphical methods have been presented by Ponchon,⁹

Rodebush,¹⁰ and Savarit.¹¹ However, it is felt that the following method is simpler, exhibits its results in a plainer manner than any method, analytical or graphical, so far proposed, and is accurate enough for all practical use.

This paper is to consider the determination of the number of theoretical plates only, and it is assumed throughout that the plate efficiency, or the H. E. T. P., for the type of column and the mixture under consideration is approximately known.

A rapid and accurate graphical method is described for computing the theoretical number of plates required in a column for performing a given separation of a binary mixture. The method consists essentially in drawing, on the same rectangular plot, the equilibrium curve for vapor and liquid compositions and straight lines representing the equations for enrichment from plate to plate, and passing from one to the other in a series of steps. It is shown that the effect of varying feed, product, waste, and overflow, as well as the effect of varying temperature of feed, may be represented by very simple geometrical constructions. The application of the method to some more complicated problems is discussed.

Preliminary Assumptions

For the purpose of initial simplification certain assumptions will be made, which are commonly realized or approached in ordinary practice within the precision of these data. It will be shown later that, if necessary for any particular case, the restrictions so imposed can be removed without greatly increasing the complexity of the method. The assumptions are:

1—The number of mols of vapor ascending the column, and hence, the molal overflow except for the change at the feed plate, is constant from plate to plate. Since heat losses from the column can ordinarily be neglected either because of lagging or large ratio of volume to surface, and since the absolute boiling points of the components are not far apart (otherwise separation is easy in any case), Trouton's rule requires constant molal vaporization from plate to plate.³

Note—Trouton's rule states that for normal liquids the molal entropy of vaporization is nearly constant, or $(M/T) = \text{constant}$, where M = molecular weight, T = boiling temperature (absolute), and r = latent heat per unit weight.

2—The feed enters the column at a temperature equal to the boiling point of the liquid on the feed plate. The feed is usually preheated very nearly to this temperature by the hot waste.

3—The only condenser is a simple or total condenser, so the composition of the product is the same as that of the vapor from the top plate.

Note—A simple condenser condenses all the vapor fed to it, but removes none of the sensible heat of the condensed liquid.

4—The heat is supplied at the base of the column by means of "closed steam," so the condensing steam does not dilute the waste.

Note—Obviously, open steam can be used only when the less volatile component is water, if the mixture is to remain one of two components only.

Nomenclature

The nomenclature is a modification of that used by Lewis. That part of the apparatus above the feed plate will be referred to as the rectifying column, and that part below the feed plate as the exhausting column. All concentrations are expressed as mol fractions of the more volatile component. Liquid compositions are denoted by x , and vapor compositions by y . All quantities pertaining to the exhausting column are distinguished from corresponding quantities pertaining to the

¹ Received February 19, 1925. Presented before the Division of Industrial and Engineering Chemistry at the 69th Meeting of the American Chemical Society, Baltimore, Md., April 6 to 10, 1925.

² *Compt. rend.*, 58, 1128, 1204, 1317 (1889); 68, 1213 (1894).

³ *THIS JOURNAL*, 14, 492 (1922).

⁴ "Motor Fuels," 1923, p. 70. Chemical Catalog Co., New York.

⁵ *THIS JOURNAL*, 15, 402 (1923).

⁶ *Chem. Met. Eng.*, 28, 207, 255, 311, 359, 408 (1923).

⁷ *Chimie & industrie*, 3, 157; 4, 178, 735 (1920); 6, 567 (1921); 10, 811, 1026 (1923).

⁸ Peters, *THIS JOURNAL*, 14, 476 (1922).

⁹ *Tech. moderne*, 13, 20, 55 (1921).

¹⁰ *THIS JOURNAL*, 14, 1036 (1922).

¹¹ *Chimie & industrie*, Special No. 737-56, May, 1923.

rectifying column by a line written over them. The plates are numbered from the top plate of the rectifying column down to the bottom plate of the exhausting column. The feed plate, which is the top plate of the exhausting column, will simply be a plate with a definite number. A "general" plate will be denoted by n , if in the rectifying column, or by $\bar{n}$ if in the exhausting column. Where any quantity is written with a subscript, the subscript refers to the point of origin of that quantity; thus y_n means the composition of the vapor rising from plate n and $\bar{x}_{n+1}$ represents the composition of the liquid descending from plate $\bar{n} + 1$. Other symbols are:

- P = mols of distillate withdrawn as product in a given time
 x_p = composition of product
 F = mols of mixture fed to the apparatus in a given time
 x_f = composition of feed mixture
 W = mols of waste withdrawn from column in a given time
 x_w = composition of waste
 $\bar{O}$ = mols overflow from any plate in the rectifying column
 $\bar{O}$ = mols overflow from any plate in the exhausting column
 V = mols of vapor rising from any plate in the rectifying column
 $\bar{V}$ = mols of vapor rising from any plate in the exhausting column
 A, A', B, B' = coefficients in the enrichment equations

In the whole discussion, the basis will be 1 mol of product—i. e., $P = 1$.

Derivation of Equations

The following equations have been developed and published many times.

Figure 1 represents a "perfect column" performing in accordance with the assumption listed above. By means of balances of total material and more volatile component it has been repeatedly shown that, for the rectifying column

$$y_{n+1} = \frac{O}{O+1} x_n + \frac{x_p}{O+1} \quad (1)$$

$$\text{Or, if } A = \frac{O}{O+1} \text{ and } B = \frac{x_p}{O+1} \quad (2)$$

$$\text{then } y_{n+1} = Ax_n + B \quad (3)$$

which, when plotted with y as ordinates versus x as abscissas, is the equation of a straight line having a slope of A and a y -axis intercept of B . This equation will be called the enrichment equation of the rectifying column, or simply the rectifying equation, and the straight line representing it the enrichment line of the rectifying column or the rectifying line.

Consider the entire apparatus below a section drawn between plates $\bar{n} + 1$ and n . By material balances, it can be shown that

$$\bar{x}_n = \frac{\bar{O} - W}{O} y_{n+1} + \frac{W}{O} x_w \quad (4)$$

When Assumptions 1 and 2 are made, a heat balance shows

$$\bar{V} = V \quad (5)$$

and a material balance around the feed plate (Figure 1) shows that

$$\bar{O} = F + O \quad (6)$$

Also, by a material balance on the entire apparatus

$$F = W + 1 \quad (7)$$

Substituting values for $\bar{O}$ and W from Equations 6 and 7 into Equation 4, there results

$$\bar{x}_n = \frac{O+1}{O+F} y_{n+1} + \frac{F-1}{O+F} x_w \quad (8)$$

$$\text{Let } A' = \frac{O+1}{O+F} \text{ and } B' = \frac{F-1}{O+F} x_w \quad (9)$$

$$\text{Then } \bar{x}_n = A' y_{n+1} + B' \quad (10)$$

which is the equation of another straight line having a slope

of $1/A'$ and an x -axis intercept of B' . This equation will be referred to as the enrichment equation of the exhausting column or the exhausting equation, and its straight line as the enrichment line of the exhausting column or the exhausting line.

Use of Enrichment Lines

For every binary mixture there is a definite relationship between x , the composition of the liquid phase, and y , the composition of the vapor phase in equilibrium with it. This relationship is best exhibited in the form of a rectangular plot of y as ordinates versus x as abscissas. The curve so formed is known as the equilibrium curve, and gives the relation between the composition of the liquid on any plate and that of the vapor rising from the same plate.

The equilibrium curve and Equations 3 and 10 are all that is necessary to calculate analytically the compositions of liquid and vapor pertaining to each plate, and hence the number of theoretical plates, once the composition of feed, waste, and product, and magnitudes of feed and overflow have been chosen.

ANALYTICAL METHOD—Plot the equilibrium curve on an x versus y plot, as in Figure 2. Since the condenser used is a simple one, y_1 , the concentration of the vapor from the top plate, is equal to x_p , the composition of the product, and therefore is known. Accordingly, the value of x_1 , the composition of the liquid on the top plate, can be read directly from the equilibrium curve, since it is represented by the abscissa of the point on the curve which has the ordinate y_1 . The coefficients A and B of Equation 3 are calculated from x_p and O . If the equation is written as

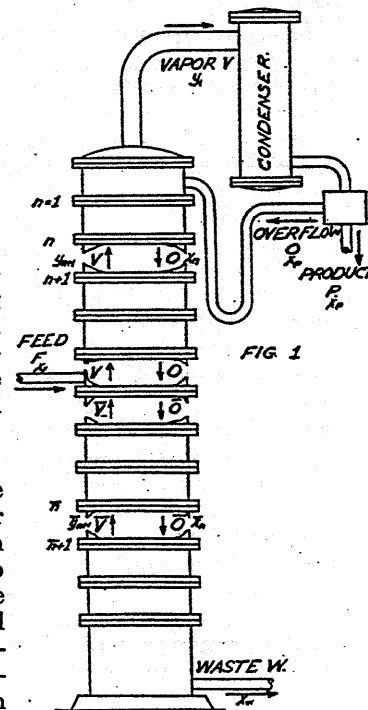
$$y_2 = Ax_1 + B$$

and if the known values of A , B , and x_1 are substituted therein, y_2 , the composition of the vapor rising from plate 2, can be calculated. Since y_2 is determined, x_2 can be read from the equilibrium curve by finding the point thereon with an ordinate y_2 , and reading its abscissa. The value of x_2 is substituted in the equation

$$y_3 = Ax_2 + B$$

and y_3 calculated. The process is repeated for each plate, and for each plate it is necessary to read one value from the equilibrium curve, and carry out the one calculation by Equation 3. Suppose this process has been carried out for a number of plates, and $x_1, y_1, x_2, y_2, x_3, y_3$, etc., have been so determined. Suppose the following points are plotted with the coordinates indicated: a , coordinates x_1, y_1 ; b , coordinates x_1, y_2 ; c , (x_2, y_2); d , (x_2, y_3); and e , (x_3, y_3). The points so plotted can be connected by short horizontal and vertical lines, such as ab, bc, cd , and de , since the two points connected by any line have either abscissas or ordinates in common. Furthermore, the points b and d are on a straight line, the equation of which is

$$y = Ax + B$$



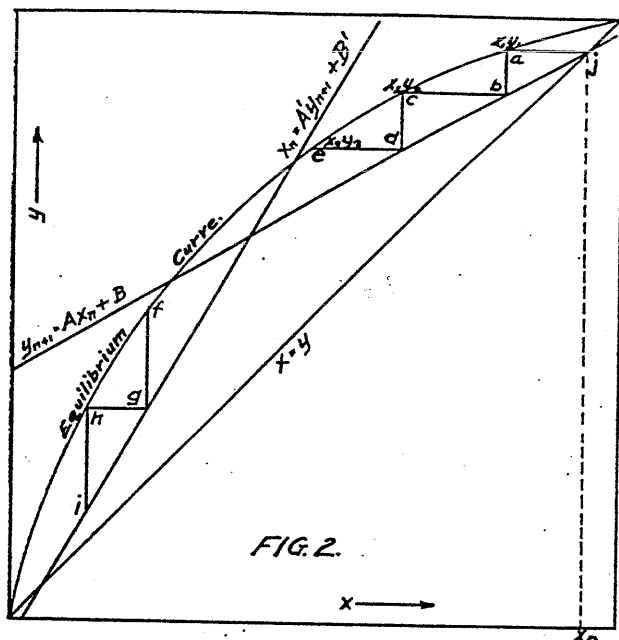


FIG. 2.

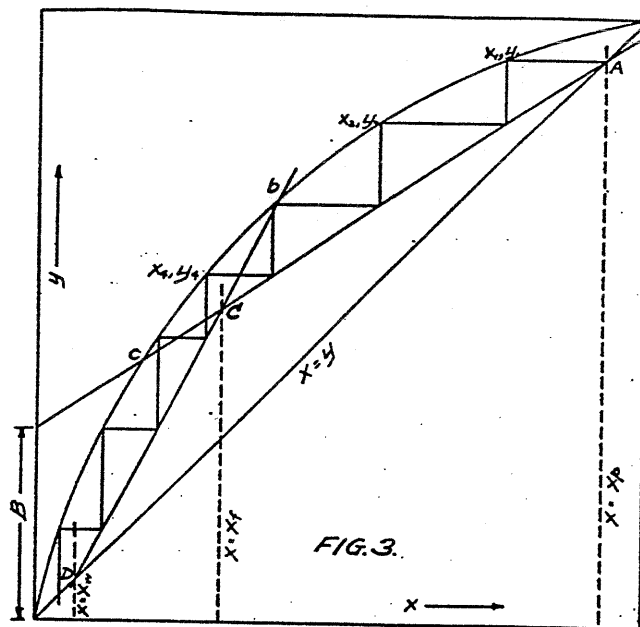


FIG. 3.

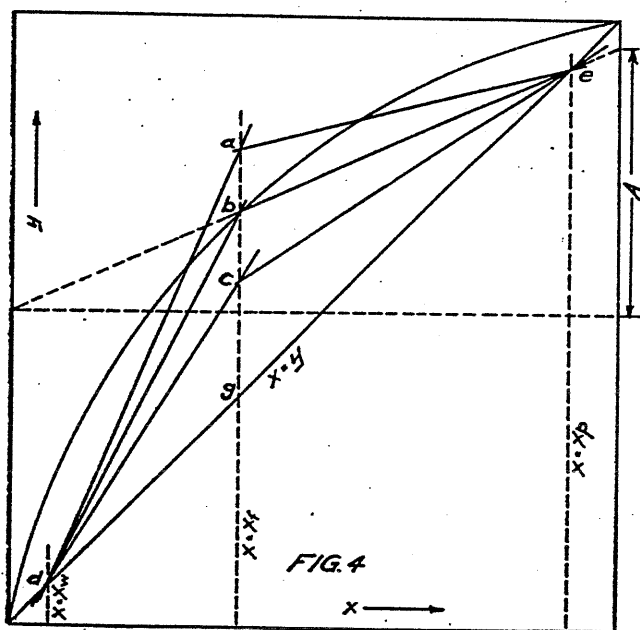


FIG. 4.

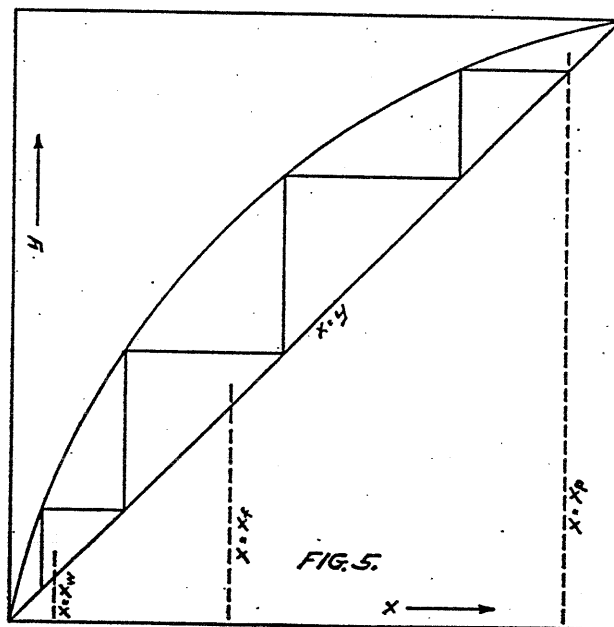


FIG. 5.

since their ordinates were calculated from Equation 3, which is represented by this same straight line. The foregoing construction can, therefore, be made much more simply without analytical calculations at all by drawing the line representing Equation 3 in the first place, starting with the point a , and drawing a series of steps composed of alternate vertical and horizontal segments, each step representing a perfect plate, and defining the compositions of the liquid on each plate and the vapor rising to that plate from the plate below.

Similarly, the enrichment from plate to plate in the exhausting column can be determined by plotting the exhausting line to conform with Equation 10 and moving up or down in the same stepwise manner—namely, by drawing alternate horizontal and vertical segments between the equilibrium curve and the exhausting line. Thus, if the coordinates of point f are $\bar{x}_{n-1}$, $\bar{y}_{n-1}$, those of point g will be x_n , y_n ; those of h are $\bar{x}_n$, $\bar{y}_n$; and those of i are $\bar{x}_{n+1}$, $\bar{y}_{n+1}$.¹²

¹² Van Nys, in the course of his mathematical presentation of the theory of rectification, uses a diagram such as Figure 2. *Chem. Met. Eng.*, 28, 256 (1923).

The diagonal line, $x = y$, is drawn for convenience, since the point a is easily located by drawing the vertical line $x = x_p$, finding its intersection with the diagonal (point j), and drawing a horizontal line ja . The coordinates of point j may be considered to be x_p , y_1 , as $x_p = y_1$.

Plotting the Enrichment Lines

Inspection of Equations 1 and 8 shows that the orientation of the enrichment lines, if plotted on rectangular coordinates, depends only on the reflux, O , for any given values of F , x_p , and x_w .

The following algebraic properties of the enrichment lines facilitate plotting them:

If the subscripts are dropped from Equation 1, and if x is set equal to y , O vanishes, and it follows that

$$y = x = x_p \quad (11)$$

showing that the rectifying line crosses the diagonal ($x = y$ line) at the composition of the product, whatever the reflux may be.

Also, if $x = \text{zero}, y = \frac{x_p}{O+1} = B$ (12)

and so for any definite overflow the rectifying line is easily plotted by computing B , plotting it as in Figure 3, and drawing the straight line AB .

It can similarly be shown that the exhausting line crosses the diagonal at the point where $x = x_w$.

The intersection of the two enrichment lines is also very simply obtained. The locus of this intersection is found by eliminating O from Equations 1 and 8, and solving for x , obtaining

$$x = \frac{(F-1)x_w + x_p}{F} \quad (13)$$

By material balances over the entire apparatus

$$F = W + 1 \quad (14)$$

$$Fx_f = Wx_w + x_p \quad (15)$$

Eliminating W and solving for F

$$F = \frac{x_p - x_w}{x_f - x_w} \quad (16)$$

Substituting for F in Equation 13 and simplifying

$$x = x_f \quad (17)$$

or the enrichment lines intersect on the vertical line $x = x_f$, whatever the reflux may be.

Note—No points on the enrichment lines have physical significance except those determined by the steps themselves. The lines are simply a geometric convenience, useful because all points of coordinates, such as x_n, y_{n+1} or x_n, y_{n-1} , lie on them.

The enrichment line for the exhausting column is plotted by drawing a straight line connecting the intersection of the rectifying line and the vertical line $x = x_f$, with the intersection of the $x = y$ line and the vertical line $x = x_w$. (Line CD , Figure 3)

Location of the Feed Plate

Suppose it is desired to separate a mixture of composition x_f into a product of composition x_p and a waste of composition x_w , using a reflux O (Figure 3). The lines ACB and CD are plotted as above. The point A will have the coordinates x_p, y_1 , and the stepwise procedure is started from that point. Inspection shows that the rectifying line must be used as far as the point b , and can be used nearly as far as the point c , and that the change to the exhausting line must be made somewhere between these points; i. e., the feed must be introduced

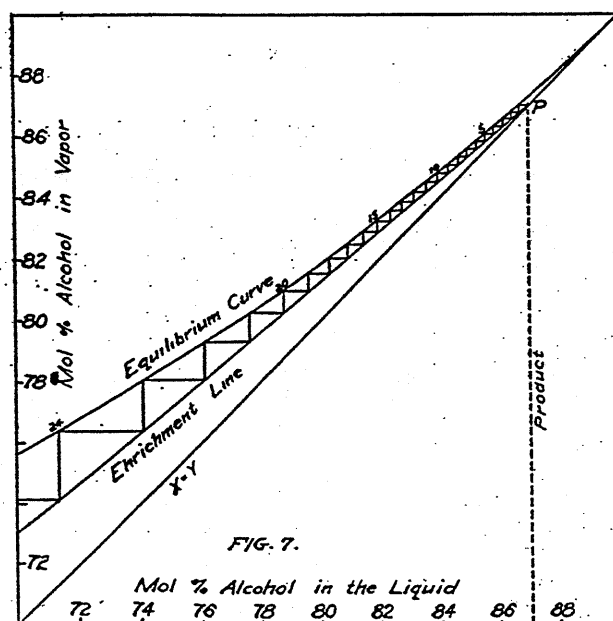
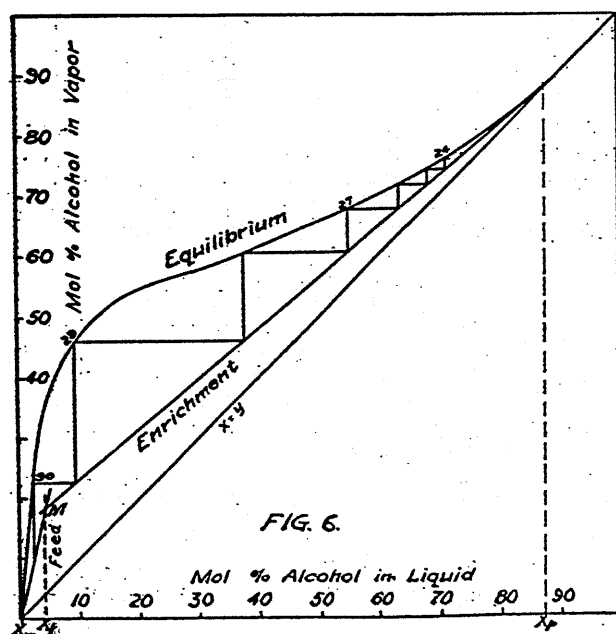
on a plate where the composition of the liquid is between c and b , if the column is to function. Economy demands that the feed plate be chosen such that the total number of plates is a minimum for the reflux chosen, which means that the enrichment per plate, and therefore the average length of the segments, must be as great as possible. It can be seen that the rectifying line should be used as far as point C and the exhausting line utilized from there on. Therefore, the feed should be introduced on that plate on which the composition of the liquid is just less than that of the feed. In Figure 3 the feed plate is the fourth one from the top. The steps are then carried below C , using the exhausting line, until one of the horizontal segments crosses the line $x = x_w$, and each step represents a perfect plate. In Figure 3 the total number of plates is $6\frac{1}{2}$.

Infinite or Minimum Overflow

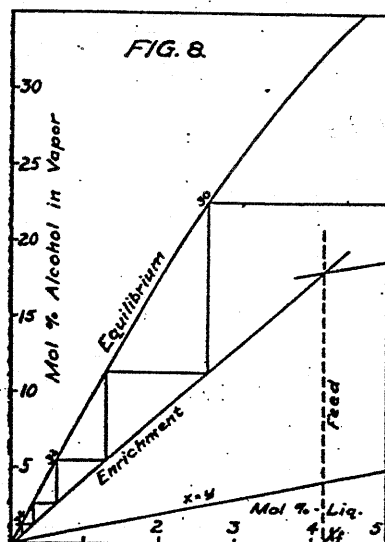
As pointed out by Lewis,² there are two limiting conditions in the design of a fractionating column, depending on the overflow. On one hand there is a theoretical minimum overflow which will effect the required separation in a perfect column containing an infinite number of plates. This minimum overflow for any given mixture depends upon the separation desired. On the other hand, if an infinitely large reflux is assumed, the separation will be effected in a column containing a minimum number of perfect plates. In the first case the cost of column, and the fixed charges on it, will be infinite, but the cost of heat is a minimum since a minimum quantity of vapor is generated at the bottom of the column. In the second case the number of plates in the column is a minimum, but the cost of heat is infinite, as an infinite amount of vapor must be generated. The designer should choose an overflow somewhere between these two limits such that the sum of the fixed costs and the heat cost is a minimum. The overflow so determined is known as the "optimum overflow," and the number of perfect plates corresponding to the optimum overflow is called the "optimum number of plates," and must be determined by trial.

Minimum Overflow

Suppose the concentrations of waste, feed, and product are plotted on the x versus y diagram (Figure 4). It has been shown that the enrichment lines pass through the points d



and e , respectively, and intersect on the line $x = x_f$. Their slope depends on the overflow only, since F is determined. From Figure 4 it is apparent that, if the lines intersect above the equilibrium curve as at a , the desired separation cannot be effected; if the lines intersect at the intersection of the equilibrium curve and the line $x = x_f$, as at point b , the separation can be attained, but only with an infinite number of perfect plates; and finally, if the lines intersect at any point below the equilibrium curve, as point c , the separation can be effected with a finite number of plates. As the only means of moving the intersection from a to c is by increasing the overflow, by definition the minimum overflow is that overflow for which the rectifying line will intersect the $x = x_f$ line at point b .



The magnitude of the minimum overflow is found by measuring the distance A and computing O from the equation

$$O = \frac{A}{1 - A} \quad (18)$$

which is obtained from Equation 2.

Note—Inspection of Figure 4 shows that for normal mixtures (equilibrium curves concave downward at all points) the largest, or controlling value of the minimum reflux is that corresponding to the composition of the feed.

Infinite Overflow

Inspection of Equations 1 and 8 shows that as O increases indefinitely, the slopes of the enrichment lines approach unity, and for an infinite overflow, the lines coincide with the $x = y$ line, and the minimum number of plates is determined by steps, as in Figure 5.

Constant Boiling Mixtures

Since a constant boiling mixture behaves like a pure component, an infinite number of plates would be necessary to reach the concentration corresponding to such a mixture by distillation alone. The diagram of such a system illustrates this, since if an attempt is made to attain the concentration of the constant boiling mixture, either with the product or the waste, the equilibrium curve, the enrichment line and the $x = y$ line all intersect at the point where x equals the above concentration, and the steps must become infinitely small to reach the apex of the angle in which they are confined.

Technic of Using the Plots

Usually the product is to be nearly pure and the water practically exhausted, speaking in terms of the more volatile component. This means that a single diagram will be too condensed at the ends of the equilibrium curve to use with precision, and separate, enlarged diagrams of these two sectors must be made. Ordinarily, the equilibrium curve will follow Raoult's law at the top and Henry's law at the bottom,¹³

¹³ Walker, Lewis, and McAdams, "Principles of Chemical Engineering," 1923, p. 576. McGraw-Hill Book Company.

and so can be calculated and plotted with any desired precision in these ranges.

As an illustration of the application of the method to an actual case, the example cited by Lewis² is worked out. An alcohol-water mixture containing 10 per cent alcohol by weight (4.17 mol per cent) is to be separated into a product containing 94.5 per cent alcohol by weight (87.05 mol per cent) and a waste containing not more than 0.1 per cent alcohol by weight (0.039 mol per cent). The overflow in the rectifying column is assumed to be 5 mols per mol of product.

When the equilibrium curve for alcohol-water mixtures is plotted, as in Figure 6, and the values of x_p , x_f , and x_w are plotted thereon, it is evident that the entire operation cannot be indicated on a single diagram of moderate size. Accordingly, three enlarged diagrams are drawn, as in Figures 7, 8, and 9, and used with Figure 6, Figure 7 being an enlargement between the limits $x = 70$ and $x = 90$ mol per cent, and Figures 8 and 9 from $x = 5$ to $x = 0$ mol per cent. The enrichment lines are then drawn, the rectifying line passing through the point P (coordinates x_p , y_1) (Figure 7), and with a slope of $5/(5 + 1) = 0.833$, and the exhausting line passing through the point M (the intersection of the line $x = x_f$ and the rectifying line) (Figure 6), and the point N (coordinates x_w , x_w) (Figure 9). The stepwise procedure is started at point P (Figure 7) and carried through Figures 7, 6, 8, and 9 in turn, until the composition of the waste is reached with, in this case, the thirty-fifth step, and so the entire column contains $34\frac{1}{2}$ perfect plates. The cross-over from the rectifying line to the exhausting line occurs at the thirtieth step, and so the feed plate is the thirtieth plate from the top.

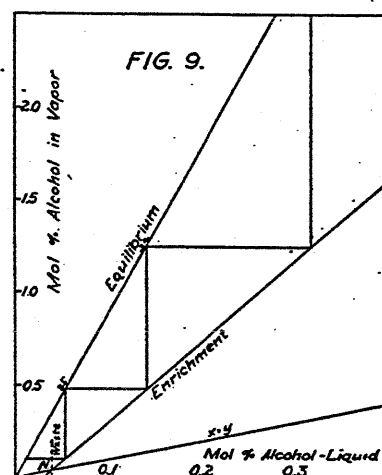
Most of the plates are in the range represented by Figure 7, and a small error in the equilibrium curve in this range introduces a relatively large error in the number of plates; but any method of calculation is subject to the same limitation.

For the sake of more open diagrams, the scale of the x -axis has been expanded to five times that of the y -axis, in Figures 8 and 9. This does not affect the construction or the result, except that it changes the angle of slope of the lines; thus, the $x = y$ line is no longer at 45 degrees.

For any binary combination, the equilibrium curve need be drawn only once, in ink perhaps; the enrichment lines and the steps may be drawn in lightly, with pencil, for any possible combination of overflow, and feed, product, and waste compositions.

If Lewis's results for the foregoing problem are multiplied by 0.8 to eliminate his assumption of 80 per cent plate efficiency, the results

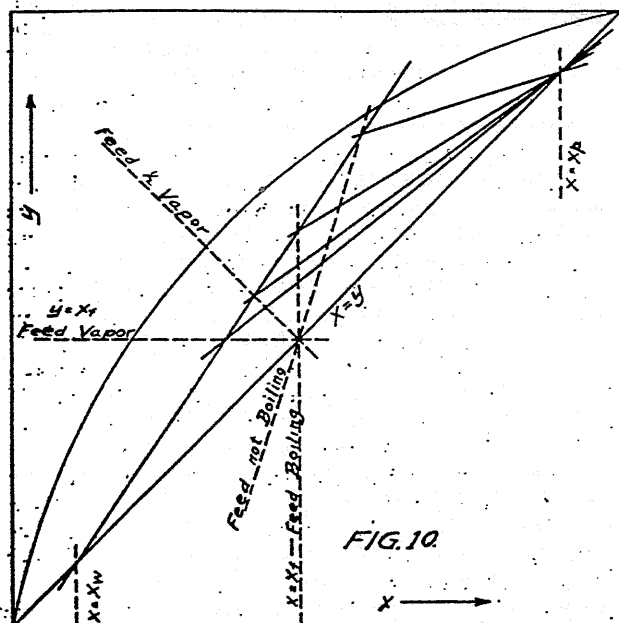
are comparable, as in the table; Lewis's feed plate comes at a composition $x = 2.05$ mol per cent.



Number of Theoretical Plates

	Lewis	Authors
Above feed	31.0	29
Below feed	3.3	6.3
TOTAL	34.3	35.3

The accuracy of the results obtained by the graphical method is theoretically limited only by the error introduced by assuming constant molal overflow, whereas the accuracy of the Lewis method as applied to the foregoing example is limited, not only by the assumption given above, but also



by an additional assumption of differential changes from plate to plate. Therefore, any discrepancy between the results obtained by the two methods is a measure of the error introduced by the assumption of a continuous for a step-wise régime.

The agreement above the feed is very good, as was to be expected, since when the enrichment per plate is small Lewis's method, which is based on an assumption of differential enrichment, is most accurate. Below the feed the steps are very large, and the percentage error of the Lewis method is larger. In this particular case the differences nearly compensate, and the total number of plates is nearly the same in each case.

Partial Condenser

The effect of removing the restrictions imposed by the four assumptions made above will now be considered. The result of using a partial condenser in place of a total condenser is that the vapor rising from the top plate is weaker than the product. The enrichment equation is not affected so long as the composition of the product is known. Hence, with a partial condenser, instead of beginning the step corresponding to the first plate along the line $y = x_p$, it is begun at a line $y = y_1$, where y_1 is less than x_p , and must be computed from a knowledge of the design of the partial condenser, which is not in the province of this article.

Use of "Open Steam"

When the less volatile component of the mixture under consideration is water, the heat is often supplied to the base of the column by blowing in steam through a perforated pipe, and the waste is diluted by the steam so condensed. The latent heat of condensation of the entering steam will vaporize an equal weight of mixture (which is nearly all water), and the mols of steam used will equal the mols of vapor sent up the column, $\bar{V}$, which is in turn equal to $O + 1$. If x_w' is the composition of the diluted waste, and W' is the number of mols of the same, then

$$Wx_w = W'x_w' \quad (19)$$

since the total amount of more volatile component lost in the waste is, in the case of closed steam, fixed as Wx_w .

It can be shown algebraically that in the case of open steam the exhausting line still passes through the point of intersection of the $x = y$ line and the line $x = x_w$, and also crosses the x -axis at the point where $x = x_w$. Therefore, it is only necessary to carry the steps, not only to where $x = x_w$, but to the x -axis to allow for the effect of open steam.

Note—Open steam must be used where a closed coil would become fouled by the waste, but when used, the number of plates is increased (though very slightly in most cases) and the waste is diluted.

Feed Not at Boiling Point

If the feed is not wholly a liquid at its boiling point, the number of mols of vapor rising from the feed plate is not the same as that rising to it. If the feed is below the boiling point, the vapor rising to the feed plate will heat the feed to the boiling point by condensation, thereby increasing the overflow below the feed and decreasing the vapor above the feed by the number of mols condensed. If the feed is partly or wholly vapor, the vapor above the feed will be increased by the amount of vapor admitted, while the overflow below the feed will be less than $O + F$. In either case the effect on the diagram is to cause the locus of the intersection of the two enrichment lines to incline from the vertical.

Let q = the total heat needed to convert 1 mol of feed into saturated vapor, divided by the molal latent heat.

Then in the case of a liquid at the boiling point, $q = 1$.

In the case of a feed of wholly saturated vapor, $q = 0$.

If the feed is partly liquid and partly vapor, q is the fractional part which is liquid.

If the feed is below the boiling point, then q is greater than 1, and is equal to 1 plus the fractional part of a mol of vapor needed to raise one mol of feed to the boiling point.

In place of Equation 6 we have

$$\bar{O} = O + qF \quad (20)$$

Hence Equation 8 becomes

$$x_n = \frac{(O - W + qF)y_{n+1}}{O + qF} + \frac{Wx_w}{O + qF} \quad (21)$$

To find the locus of the intersection (for different values of O) of this enrichment line with that represented by Equation 1, eliminate O from Equations 1 and 21 and substitute for F its value as given by Equation 16. When this is done there results

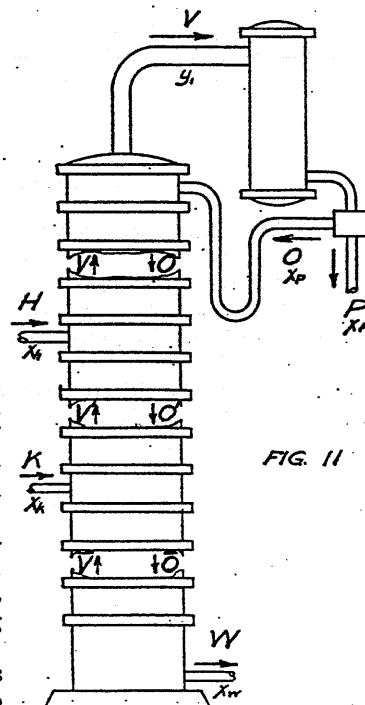
$$qx - (q-1)y = x_f \quad (22)$$

This is the equation of a straight line. If $x = y$, then $x = y = x_f$. Hence this line always crosses the $x = y$ line at x_f . If $q = 1$, $x = x_f$. This is the result already given in Equation 17.

If $q = 0$ (feed a saturated vapor), $y = x_f$. In this case the locus is a horizontal line passing through $x = y = x_f$.

If $q > 1$, the slope is positive—i. e., up and to the right.

If $q < 1$, the slope is negative—up to the left.



These four cases are represented in Figure 10, where the number of mols of vapor rising to feed plate per mol of product is the same for all cases.

Variation on Molal Overflow

The molal overflow may not be constant in a column, for three reasons. The first cause may be a difference in the molal heats of vaporization of the two components. Allowance may be made for this by computing all quantities, not in mols, but in "latent heat units,"¹⁸ that is, in the quantities of liquid evaporated by any definite quantity of heat. Thus, if the liquids have heats of vaporization (per gram) of 969 and 305 calories, we may adopt as our base 969 calories and the latent heat units will be 969/969 and 969/305 grams of the two liquids, respectively. They will then be treated exactly as if they had molecular weights of 1 and 3.177, respectively. Since no chemical reaction takes place, the only reason for using mols at all is the fact that they correspond nearly to the more accurate "latent heat units;" if this correspondence does not hold there is no object in using them. If latent heat units are chosen, they must be used throughout—in the construction of the equilibrium curve as well as in expressing the compositions on each plate and of the feed, product, and waste.

The second reason why the overflow may vary is the change in boiling point of the liquids on the successive plates, and the third cause may be heat losses from the column. To allow for these effects rigorously would require the construction of a different enrichment line for every plate, and so would be tedious. However, the effects are usually small and tend to balance each other, so they rarely require consideration.

Note—The graphical methods of Ponchon⁹ and Savarit¹¹ are rigorously applicable to cases where the overflow varies, but these methods are rather complicated.

Two Feeds

Suppose it is desired to rectify in the same column two mixtures of the same two liquids but of different compositions. Let the mols of the two feeds (per mol of product) be H and K . Let x_h and x_k be the compositions of the mixtures. Similarly, let G and x_g be the amount and composition of the mixture that would be produced if the two feeds were mixed. Then

$$G = H + K \quad (23)$$

$$Gx_g = Hx_h + Kx_k \quad (24)$$

Let $x_h > x_k$ so that the "H" feed will naturally be put in higher up the column. The column is diagrammatically represented by Figure 11.

The original assumptions will be used. Through material balances it can be shown, by deriving the enrichment equations for the three parts of the column (above both feeds, between the feeds, and below both feeds), and determining the loci of their intersections by eliminating the overflow, that the enrichment lines for the top and bottom sections of the column cross the $x = y$ line at x_g and x_w , respectively, and intersect on the vertical line $x = x_g$; and that the enrichment line for the middle part of the column (between the two feed plates) must pass through two points—namely, the intersection of the upper enrichment line with the vertical line $x = x_h$ and the intersection of the lower enrichment line with the vertical line $x = x_k$. For a two-feed column, then, these rules follow:

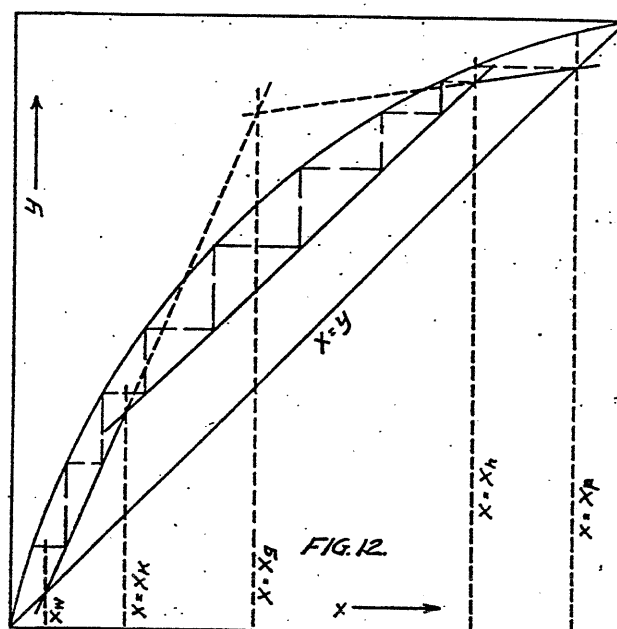
1—To construct the enrichment lines for a two-feed column, compute the composition that would result if the feeds were mixed. The enrichment lines for the ends of the column will be exactly the same as for a single-feed column that is fed with this mixture. The enrichment line for the middle of the column is found by joining the points where the enrichment lines for the ends intersect the lines $x = x_h$ and $x = x_k$, where x_h and x_k are the compositions of the two feeds.

2—If there are two feeds, each heated to boiling, each feed is to be admitted on the highest plate that has a composition lower than that feed.

These rules are illustrated in Figure 12. When the two feeds are very different in composition the column may function well when the feeds are put in at the proper points, though at the same overflow it would not function if the feeds were mixed.

If for any reason part of the liquid (not a third constituent) is withdrawn regularly from the column, the calculations are similar to those for two feeds, except that either H or K will be negative.

Mains¹⁴ has extended Lewis's method to the design of columns for the separation of mixtures such as furfural and water where the product, on cooling, forms two layers which can be separated in a continuous decanter and one of them returned to the column. This is a special case of a



two-feed column where the second feed is one of the layers formed in the decanter. If this second feed is not at the boiling point the enrichment line for that part of the column below the entrance of the decanted feed must be modified as described under the section "Feed Not at Boiling Point."

Acknowledgment

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¹⁴ *Chem. Met. Eng.*, 31, 307 (1924).

New Green Dye

The Dyestuffs Department of E. I. du Pont de Nemours & Company has just placed on the market a new chrome green, known as pontachrome green G, to take the place of pontachrome green GLO, which will be discontinued. It is bluer than pontachrome green GLO, but very similar in properties as well as in strength. Pontachrome green G can be applied as a straight color or can be used in combinations to produce browns, greens, or olives, and especially in mixtures to produce blacks that leave silk effects unstained.

ANALYTIC DETERMINATION OF PLATES IN FRACTIONATING COLUMNS*

By E. H. SMOKER† (Non-Member)

In distillation theory the basic equations for binary mixtures derived by Lewis³ have become the starting point for many further investigations. By assuming constant molal reflux McCabe and Thiele⁴ showed that the operating lines were straight lines. On this basis they were able to develop a graphic method for computing the number of theoretical plates required to produce any given separation of two components.

The method of McCabe and Thiele is excellent for most cases met in practice. However, it has the usual disadvantage of graphic methods in that the accuracy depends primarily on the care given to the construction of the plots. Also it is quite tedious and almost impractical to apply to systems involving a large number of theoretical plates. An analytic method avoids these disadvantages without introducing others if too many simplifying assumptions are not made.

In many of the binary mixtures requiring intensive fractionation the two components are sufficiently alike chemically that the vapor-liquid relationships of the solutions can be represented accurately over wide concentration ranges by Raoult's law,

$$y = \frac{\alpha x}{1 + (\alpha - 1)x} \quad (1)$$

Theoretically the value of α should vary somewhat with temperature, but if the mixture is difficult to separate, the variation of boiling point with composition is sufficiently small that α is essentially constant.

Assuming Raoult's law with constant α to represent the vapor-liquid equilibrium relationship, the purpose of this paper is to derive equations for determining analytically (as McCabe and Thiele do graphically) the number of plates necessary to produce any desired separation.

* This paper is printed prior to presentation at the Thirtieth Semi-Annual Meeting, White Sulphur Springs, May 9, 10, 11, 1938, in order to encourage discussion both verbal at time of presenting and written.

Written discussion should be sent to the Secretary's Office promptly as it will be submitted to the author before publication. Written discussion received prior to May 6th will be read at the time of presentation.

Discussion on this paper will be printed in Volume 34, No. 5 only.

† United Gas Improvement Co., Philadelphia, Pa.

GENERAL DERIVATION

Referring to Figure 1, equation (1) represents the vapor-liquid equilibrium diagram shown. Any operating line (either above or below the feed plate) is represented by the general equation of the straight line,

$$y = mx + b \quad (2)$$

Eliminating y between equations (1) and (2) results in the quadratic equation,

$$m(\alpha - 1)x^2 + [m + b(\alpha - 1) - a]x + b = 0 \quad (3)$$

In any problem having significance in distillation theory, one and only one of the two roots of equation (3) will lie between zero and one. Representing this root by k , and substituting in equation (2) there are obtained the equations,

$$x = k \quad (4)$$

and

$$y = mk + b \quad (5)$$

These values of x and y are the coordinates of the point of intersection of the vapor-liquid equilibrium curve and the operating line.

If the origin of coordinates is shifted from $x = y = 0$ to this intersection point,

$$x' = x - k \quad (6)$$

$$y' = y - (mk + b) \quad (7)$$

equations (1) and (2) become transformed as follows.

The operating line is

$$y' = mx' \quad (8)$$

The vapor-liquid equilibrium curve is

$$y' + mk + b = \frac{\alpha(x' + k)}{1 + (\alpha - 1)(x' + k)} \quad (9)$$

Since when $x' = 0$, $y' = 0$, and equation (9) becomes

$$mk + b = \frac{\alpha k}{1 + (\alpha - 1)k} \quad (10)$$

By eliminating b between equations (9) and (10) and rearranging

$$y' = \frac{ax'}{[1 + (a-1)k]^2 + (a-1)[1 + (a-1)k]x'} \quad (11)$$

Letting

$$c = 1 + (a-1)k \quad (12)$$

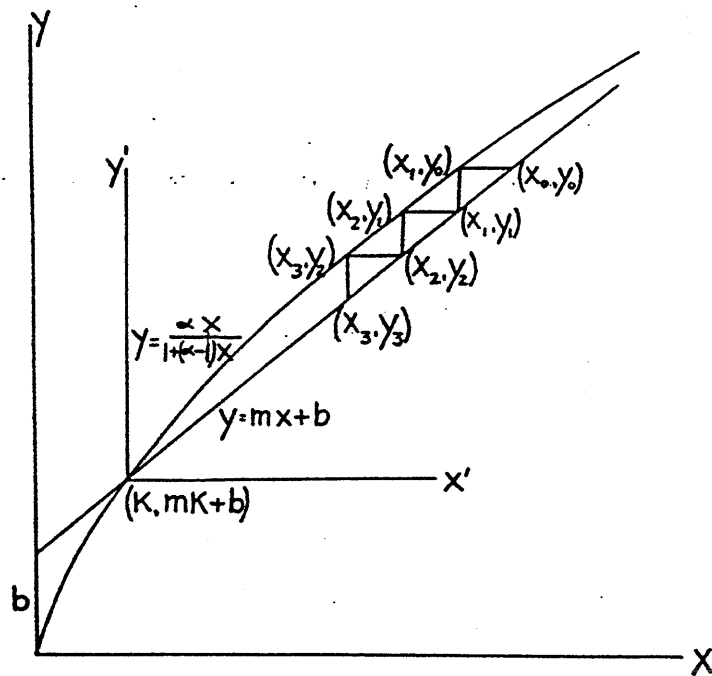


FIG. 1.

equation (11) simplifies to

$$y' = \frac{ax'}{c^2 + c(a-1)x'} \quad (13)$$

Equation (13) is Raoult's law expressed in coordinates determined by the intersection of the original Raoult equation and any operating line. Solving it for x' ,

$$x' = \frac{c^2 y'}{a - c(a-1)y'} \quad (14)$$

In Figure 1, let x'_0, y'_0 represent any point on the operating line $y' = mx'$. If the stepwise procedure of McCabe and Thiele is fol-

lowed, successive points on the operating line are $x'_0, y'_0; x'_1, y'_1; x'_2, y'_2; \dots x'_n, y'_n$. Corresponding successive points on the vapor-liquid equilibrium curve are $x'_1, y'_0; x'_2, y'_1; x'_3, y'_2; \dots x'_n, y'_{n-1}$. Since the coordinates of points on the operating line are related by equation (8) and those on the vapor-liquid equilibrium curve by equation (14)

$$y'_0 = mx'_0 \quad (15)$$

and

$$x'_1 = \frac{c^2 y'_0}{a - c(a-1)y'_0} \quad (16)$$

Eliminating y'_0 between (15) and (16) expresses x'_1 in terms of x'_0 ,

$$x'_1 = \frac{mc^2 x'_0}{a - mc(a-1)x'_0} \quad (17)$$

Similarly

$$y'_1 = mx'_1 \quad (18)$$

and

$$x'_2 = \frac{c^2 y'_1}{a - c(a-1)y'_1} = \frac{mc^2 x'_1}{a - mc(a-1)x'_1} \quad (19)$$

Eliminating x'_1 from equations (17) and (19) expresses x'_2 in terms of x'_0 ,

$$x'_2 = \frac{m^2 c^4 x'_0}{a^2 - mc(a-1)(a + mc^2)x'_0} \quad (20)$$

Proceeding in the same stepwise fashion it can be shown that

$$x'_3 = \frac{m^3 c^6 x'_0}{a^3 - mc(a-1)(a^2 + amc^2 + m^2 c^4)x'_0} \quad (21)$$

Inspection of equations (17), (20), and (21) results in the generalization for n steps,

$$x'_n = \frac{m^n c^{2n} x'_0}{a^n - mc(a-1) \left(\frac{a^n - m^n c^{2n}}{a - mc^2} \right) x'_0} \quad (22)$$

Solving equation (22) for n ,

$$n = \frac{\log \frac{x'_0 \left(1 - \frac{mc(a-1)}{a-mc^2} x'_n\right)}{x'_n \left(1 - \frac{mc(a-1)}{a-mc^2} x'_0\right)}}{\log \frac{a}{mc^2}} \quad (23)$$

Equation (23) is an accurate analytic expression of the number of theoretical plates necessary to effect the separation represented by the concentrations x'_0 and x'_n , the operating line $y = mx + b$, and the vapor-liquid equilibrium curve characterized by α . Except for Raoult's law and constant molal overflow it involves no assumptions, is valid for any reflux ratio, operates between any pair of concentrations, and is equally applicable to a discontinuous or to either section of a continuous still.

APPLICATION TO DISTILLATION

Applying to actual distillation conditions, for a rectifying column where a determination of the number of plates necessary to enrich from feed composition, x_F , to product composition, x_P , at fixed reflux ratio, R , is required,

$$m = \frac{R}{R+1} \quad (24)$$

and

$$b = \frac{x_P}{R+1} \quad (25)$$

$$\left. \begin{aligned} \text{Also } m(a-1)k^2 + [m + b(a-1) - a]k + b &= 0 \\ k \text{ is the root between 0 and 1} \end{aligned} \right\} \quad (26)$$

$$x_0 = x_P \quad (27)$$

$$x'_0 = x_P - k \quad (28)$$

$$x_n = x_F \quad (29)$$

$$x'_n = x_F - k \quad (30)$$

$$c = 1 + (a-1)k \quad (31)$$

For the stripping section the residue composition, x_W , as well as the feed and overhead compositions and the reflux ratio, fixes the operating line. The values for m and b are

$$m = \frac{Rx_F + x_P - (R + 1)x_W}{(R + 1)(x_F - x_W)} \quad (32)$$

and

$$b = \frac{(x_F - x_P)x_W}{(R + 1)(x_F - x_W)} \quad (33)$$

If the feed is not introduced at its boiling point, in place of x_F the x coordinate of the intersection of the operating line in the rectifying section and the q line¹ is used.

If an attempt is made to apply the calculations to a separation that is physically impossible the numerator of equation (23) involves the logarithm of a negative number which is a mathematical absurdity.

For distillation at total reflux $R = \infty$, $m = 1$, $b = 0$, $k = 0$, $c = 1$ and equation (23) simplifies to one derived by Underwood,⁶

$$n = \frac{\log \frac{x_0(1 - x_n)}{x_n(1 - x_0)}}{\log \alpha} \quad (34)$$

That equation (23) should reduce to equation (34) is to be expected, since both are derived by the identical stepwise procedure. The Underwood equation at total reflux therefore is only a special case of the more general equation (23).

OTHER EQUATIONS

Several previous investigators have derived equations for calculating the number of theoretical plates necessary to produce a given separation at finite reflux ratios. All have used Lewis' basic equations, Raoult's law, and constant molal reflux. However, in each case further simplifying assumptions were made which tend to limit either the applicability or accuracy of the result.

Underwood⁷ assumes good purity is obtained and thus limits the use of his equation to cases in which overhead composition approaches unity. For difficult separations such an assumption is an impractical one.

Peters⁵ assumes that the rate of enrichment is constant with height for a packed column. Although he does not so state explicitly in the text of his paper, the basic differential equation he uses necessitates the further assumption that slight differences in composition occur from plate to plate and that the column contains a large number of plates. Several errors of definition of terms and sign of algebraic quantities are also present, rendering useless any attempt to use his formulas quantitatively.

Dodge and Huffman² make the same basic assumptions and use the same primary equations as Peters, but perform the integration in faultless fashion. Because of the assumptions their calculations are inaccurate when the number of plates is small. Furthermore they find it necessary to derive three different equations, each to be applied under different conditions, so that none of their equations has the generality of application of equation (23).

SPECIFIC EXAMPLES

As an example of the use of equation (23), consider a continuous still fractionating a mixture of 40 per cent benzene and 60 per cent toluene, so that each product is 99 per cent pure, at a reflux ratio of 3 to 1. For this mixture the average value of α is 2.50. In the rectifying section, $m = .75$, $b = .2475$, $k = .2184$, $c = 1.3276$. Substituting in equation (23) $n = 7.9$ plates. In the stripping section $m = 1.3782$, $b = -.0038$, $k = .5486$, $c = 1.8229$ and $n = 7.8$ plates. These values check exactly with those obtained by a McCabe and Thiele construction.

For a case in which the McCabe-Thiele method is inapplicable, the problem of separating oxygen isotopes by distilling water cited by Dodge and Huffman is a good illustration. For conditions in which $\alpha = 1.006$, $R = 3920$, $x_0 = .0254$, $x_n = .0085$, by equation (23) $n = 521$ theoretical plates. Since the assumptions of slight enrichment from plate to plate and the presence of a large number of plates made by Dodge and Huffman are fulfilled in this case, they obtain the same result.

NOMENCLATURE

- x = mol fraction of lower boiling component in liquid phase
- y = mol fraction of lower boiling component in vapor phase
- x' = coordinate defined by equation (6)
- y' = coordinate defined by equation (7)

- α = relative volatility of the two components
 m = slope of a straight line
 b = intercept of a straight line with the y axis
 k = root of equation (3) between 0 and 1.
 c = constant defined by equation (12)
 n = number of plates between any two points in column under consideration
 R = reflux ratio, or mols reflux per unit time divided by mols of distillate per unit time.

Subscripts

- F refers to feed
 P " " overhead product or distillate
 W " " waste product or bottoms

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6. Underwood, A. J. V., *Trans. Inst. Chem. Eng.*, 10, 112-52 (1932), equation 29.
7. Underwood, A. J. V., *Ibid.*, 10, 112-52 (1932), equation 37.

On the following pages is discussion both oral and written on papers printed in Volume 34, No. 2, prior to presentation at the Thirtieth Semi-Annual Meeting, White Sulphur Springs, W. Va., May 9, 10, 11, 1938. The written discussion in all cases is submitted to the authors and their comments, if any, included. No further discussion of these papers will be printed.

DISCUSSION ON ANALYTIC DETERMINATION OF PLATES IN FRACTIONATING COLUMNS

By E. H. SMOKER

(Vol. 34, No. 2, Page 165)

Dr. Brown (University of Michigan, Ann Arbor, Michigan): This is an interesting development of an equation to give results similar to the well-known graphical method. The use of a constant α , taken as the arithmetic average between that on the top plate and that on the bottom plate of the section over which the computation is being made will, in many cases, give reliable results equivalent to those obtained from the graphical method in which the assumption of constant α is not made. But there are cases in which the use of a constant α introduces an appreciable error; for example, when separating a feed of 40 per cent benzene and 60 per cent toluene into distillate of 55 per cent benzene and bottoms of 5.9 per cent benzene.

However, in the case of a large number of plates where the feed is not introduced close to one product, I believe that this method will give results that are just as reliable as can be obtained by any other method, and it is applicable in cases where the graphical method would be almost impossible.

However, I wish to point out that in all of these fractionating calculations, any assumption of any kind which is introduced should be carefully pointed out because we find that certain assumptions give methods which are reliable and satisfactory under certain conditions, and under other conditions we have to make other assumptions and make other calculations.

There is an equation somewhat similar to this, derived by Underwood in 1933 in the Journal of the Society of Chemical Industry, by assuming constant α and also that the mole fraction of the more volatile component may be assumed to have an average value over that section of the column which is being computed. If this last assumption may be made, it is no longer necessary to solve the quadratic equation and the results are in many cases equally satisfactory.

In all of these design calculations, in so far as possible, it is desirable to include data on actual columns and to compare a proposed procedure against actual plant data, because there is a good deal of confusion in the interpretation of these methods of design and in many cases assumptions have crept in which are not clearly indicated; for example, Underwood in 1932 in the Transactions of The Institution of Chemical Engineers, presented an equation for minimum reflux, which included the assumption which was not pointed out and which probably crept in unawares, that the feed plate composition was the same as that of the feed. When calculations based on this equation are compared with commercial columns, we find actual columns operating below the minimum reflux so computed. If the feed plate composition is correctly computed by methods given in the same paper and used in the equation for minimum reflux the results are in good agreement with actual operation. The final proof of any method of design calculation is whether it works in practice, and not whether it gives the same result as some previous or other method.

Mr. Chilton (E. I. du Pont de Nemours & Co., Wilmington, Delaware): I think the author of this paper is to be felicitated on having arrived at a solution, by a rather elegant mathematical procedure, of a problem that has been apparent since the original procedures for the solution of column design were encountered.

He has made reference to the papers of Dodge and Huffman and of Peters, which give some solutions for finding a large number of plates, with practically identical numerical results.

For the sake of the record, it should be pointed out that Dodge and Huffman's solution does not give plates, but what we have been calling "transfer units," whereas Dr. Smoker's solution does give actual theoretical plates. It is true that the numerical answers obtained, where the number is large, turn out to be identical, but for small numbers of plates or transfer units, the answer should differ.

If my recollection serves me correctly, Dodge and Huffman did present alternative equations which can be used to give solutions involving fewer significant figures in the calculations than are required in the final equation which Dr. Smoker has presented, and I would express the hope that someone would be able to expand, by some conveniently converging series, logarithmic expressions so that one would not have to solve, in Equation 23, in the second example, the denominator for the logarithm of a quotient of α equals 1.006, m equals 0.9997 and c equals approximately 1.0001, which is obviously a difficult logarithm to evaluate.

Dr. Smoker: I might point out that although that logarithm might be difficult to evaluate, it would be practically impossible to get the answer by any other method.

WRITTEN DISCUSSION SUBMITTED BY
DR. BARNETT F. DODGE

Yale University, New Haven, Conn.

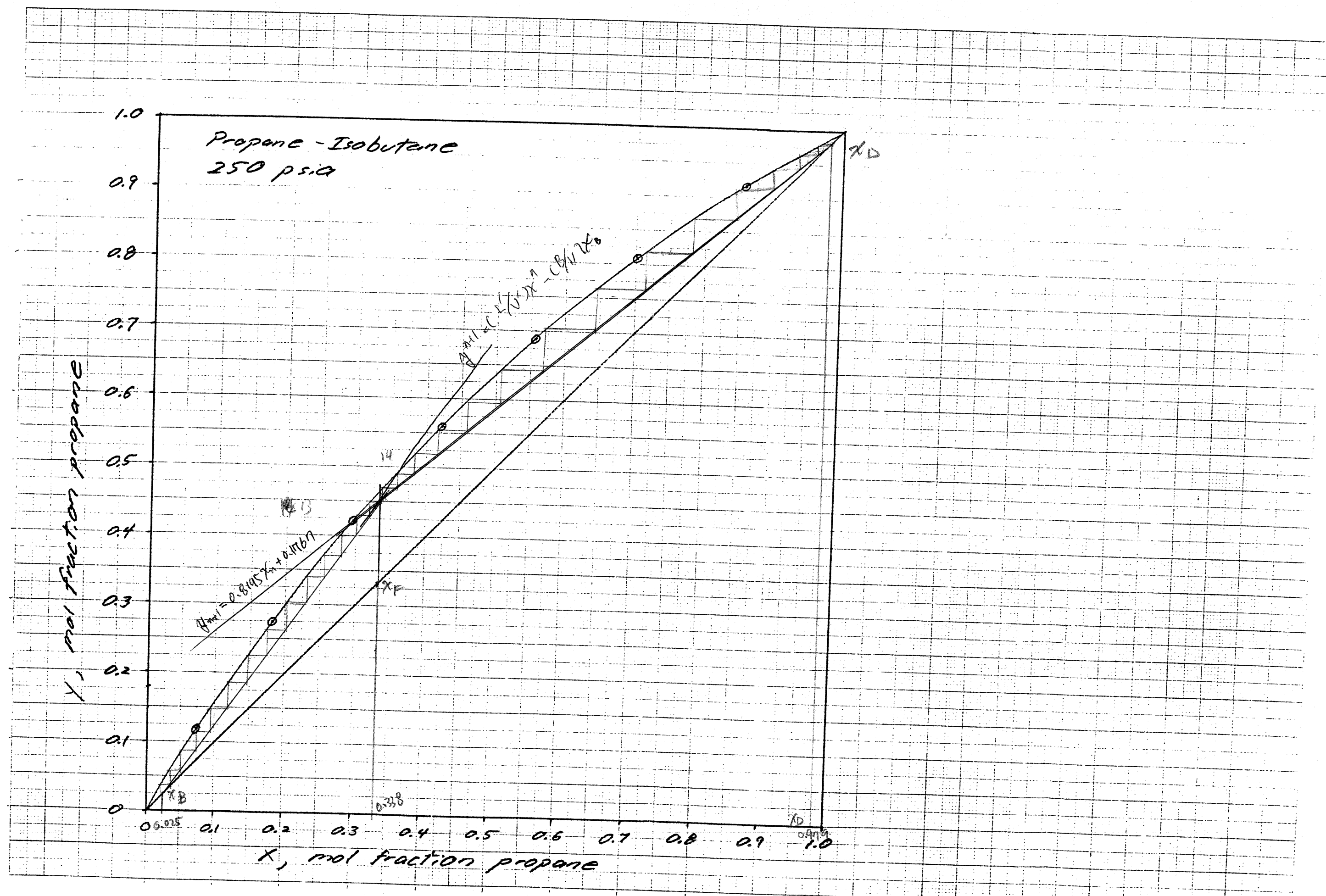
This paper presents a very ingenious analytical treatment of the usual stepwise graphical method for determining the number of theoretical plates in a rectifying column, for the special case where the solution being distilled may be assumed to be an ideal one. I wish merely to comment briefly on a few points mentioned by the author in referring to the paper by Huffman and myself, (*Ind. Eng. Chem.* 29, 1434-36 (1937)) which presents a different method for making the same calculation.

As the author points out, his equation gives a more accurate result than ours when the number of plates is small. Actually, however, the difference is not of much practical significance. For instance, in his first numerical example, Dr. Smoker obtains 7.9 plates by his equation whereas our equation gives 7.5 plates. Furthermore, it is probably true that both our methods are only useful when the number of plates is large and for such a case the two give substantially identical results.

The author also points out that we used three equations (nos. 9, 11 and 13, for example) to accomplish what he does with only one (his no. 23). Our equation (11) [or (12)] treats a special case which could also be handled by our general equation (9) but because of the small differences of large numbers which are involved, one has to retain about 7 significant figures in evaluating certain of the terms in order to obtain a reasonably accurate result. Our equation (11) was justified because it enables one to make the calculation for this special case in a much simpler manner than by the general equation.

It is true that Dr. Smoker's general equation (23) reduces in a simple manner when the special case of infinite reflux is treated whereas ours does not. Our general equation should presumably reduce to equation (13) [or (14)] for the special case of infinite reflux, but the reduction is not simple and we have not attempted it because the reduction seemed so much simpler if performed before integrating equation (15).

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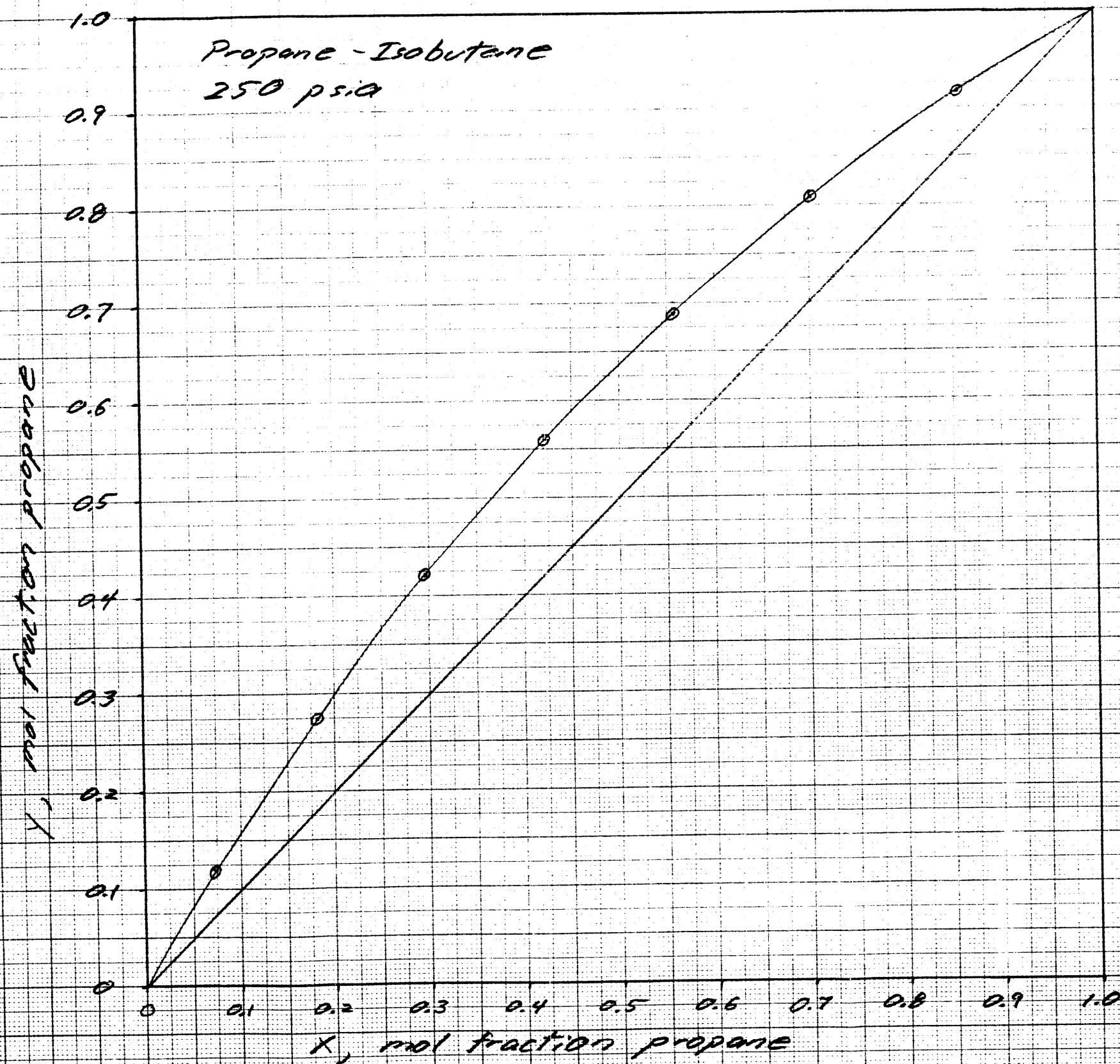


Figure 1

BINARY FRACTIONATION

$$L^{\eta} = L^0 = \text{constant}$$

VALIDITY ?

- 1) Boiling point range of components is narrow
- 2) Molecular characteristics of components are similar

for example : All paraffinic hydrocarbons
or all aromatic hydrocarbons
not mixture of paraffins / aromatics

Envelope ②

$$\text{Mass balance} \rightarrow y_i^{n+1} V^{n+1} = x_i^n L^n + x_i^D D$$

equimolar overflow

$$L = \text{constant}$$

$$V = \text{constant}$$

$$y_i^{n+1} = (L/V) x_i^n + (D/V) x_i^D$$

Binary

$$x_i \rightarrow x_1, x_2$$

$$x_2 = 1 - x_1$$

$x = x_1$ > mole fraction of
 $y = y_1$ more volatile (lower boiling point) component.

$$y_{n+1} = (L/V) x_n + (D/V) x_D$$

Tray above Feed.

Envelope ③

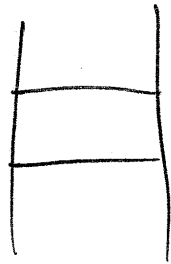
Mass balance $\rightarrow x_i^n L^{n'} = y_i^{n+1} V^{n+1} + x_i^B B$

$L' = \text{constant} \neq L$

$V' = \text{constant} \neq V$

$y_{n+1} = (V'/V) x_n - (B/V') x_B$

try below Feed.



Mass balance $\rightarrow F + V' + L = V + L'$

Heat balance $\rightarrow h_F F + h_V V' + h_L L = h_V V + h_L' L'$

$V = L + D$

$V' = L' - B$

Assume

$h_V = h_{v1}$

$h_L' = h_L$

$\left. \begin{array}{l} h_V = h_{v1} \\ h_L' = h_L \end{array} \right\} \rightarrow h_F F + h_V (L' - B) + h_L L = h_V (L + D) + h_L L'$

rearrange $\rightarrow h_V L' - h_L L' = h_V L - h_L L + h_V D + h_V B - h_F F$

$(h_V - h_L) L' = (h_V - h_L) L + h_V (D + B) - h_F F$

Define $q = \frac{h_F^V - h_F}{h_F^V - h_F^L}$

Then

$$L' = L + gF$$

Also

$$y^{n+1} = \left[\frac{g}{g-1} \right] x^n + \left[\frac{1}{g-1} \right] x_F \quad \text{at feed tray}$$

EQUILIBRIUM.

$$y = K_1 x$$

$$(1-y) = K_2 (1-x)$$

$$\boxed{\begin{aligned} x &= \frac{1-x_2}{(K_2 - K_1)} \\ y &= K_1 x \end{aligned}}$$

$$y = K_1 x$$

$$(1-y) = K_2 (1-x)$$

$$\frac{y}{(1-y)} = \frac{K_1}{K_2} \frac{x}{(1-x)}$$

Define $\alpha = \frac{K_1}{K_2}$

$$\boxed{\begin{aligned} y &= \frac{\alpha x}{1 + (\alpha - 1)x} \\ x &= \frac{y}{\alpha} \end{aligned}}$$

① Binary System = Propane / Iso-Butane
Specified.

Feed = 200 MPH C₃ + 200 MPH I₄
 at Bubble point

Distillate = 196 MPH.

Reflux = 100 MPH at 100°F

Column pressure = 250 psia,

No. of tray = 24, Feed on 13

Problem.

Find X_D & X_B

$$L = 1.16R = 464 \text{ mph}$$

$$L/F = 464/400 = 1.16$$

Rigorous

Heat & weight Balance

Tray-to-tray method

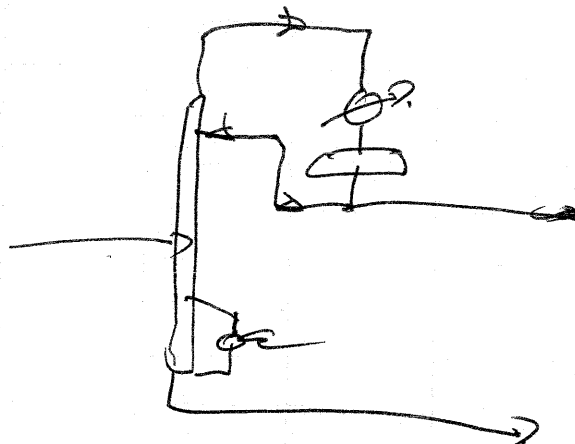
$$X_D = 0.927$$

$$X_B = 0.090$$

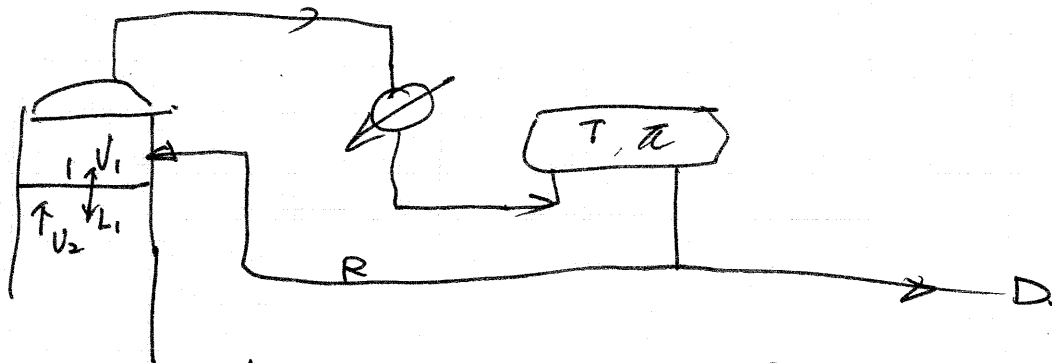
Equilibrium
 overflow
 Simplified
 Tray-to-tray
 Method

$$0.919$$

$$0.098$$



INTERNAL VS EXTERNAL REFLUX



$$h_{V2} V_2 + h_R R = h_{V1} V_1 + h_{L1} L_1$$

$$V_1 = R + D \quad ; \quad V_2 = L_1 + D$$

$$\text{Assume } h_{V1} = h_{V2}$$

$$h = \frac{Ch_V - h_R}{Ch_V - h_L} R$$

h_R = log. enthalpy @ T
 h_V = sat. vapor enthalpy @ P
 h_L = sat. liquid enthalpy @ P

Receiver Temp = $100^\circ F$
 Press = 250 psia.

Assume: Receiver liquid is 100% propane.

$$h_V = 168 \text{ Btu/lb}, \quad h_L = 47 \text{ Btu/lb}, \quad h_R = 27 \text{ Btu/lb}$$

$$L = 1.16 R$$

PROPANE / BENZENE SEPARATION

Feed 250 mph G + 50 mph RB
at Bubble point.

Distillate = 50 mph $x_D = 0.99$

Column Pressure = 225 psia.

No. of tray = 10, Feed on 6

Problem

Find Reflux Rate (Temp. = 100°F)

Empirical
overflow

$R = 5$ mph

Rigorous method

14 mph !!
if 5 mph $x_B = \underline{\underline{0.967}}$

Summary of Equations

Rectifying Section

$$y^{n+1} = (L/V) x^n + (L/V) x_D, \quad V = L + D$$

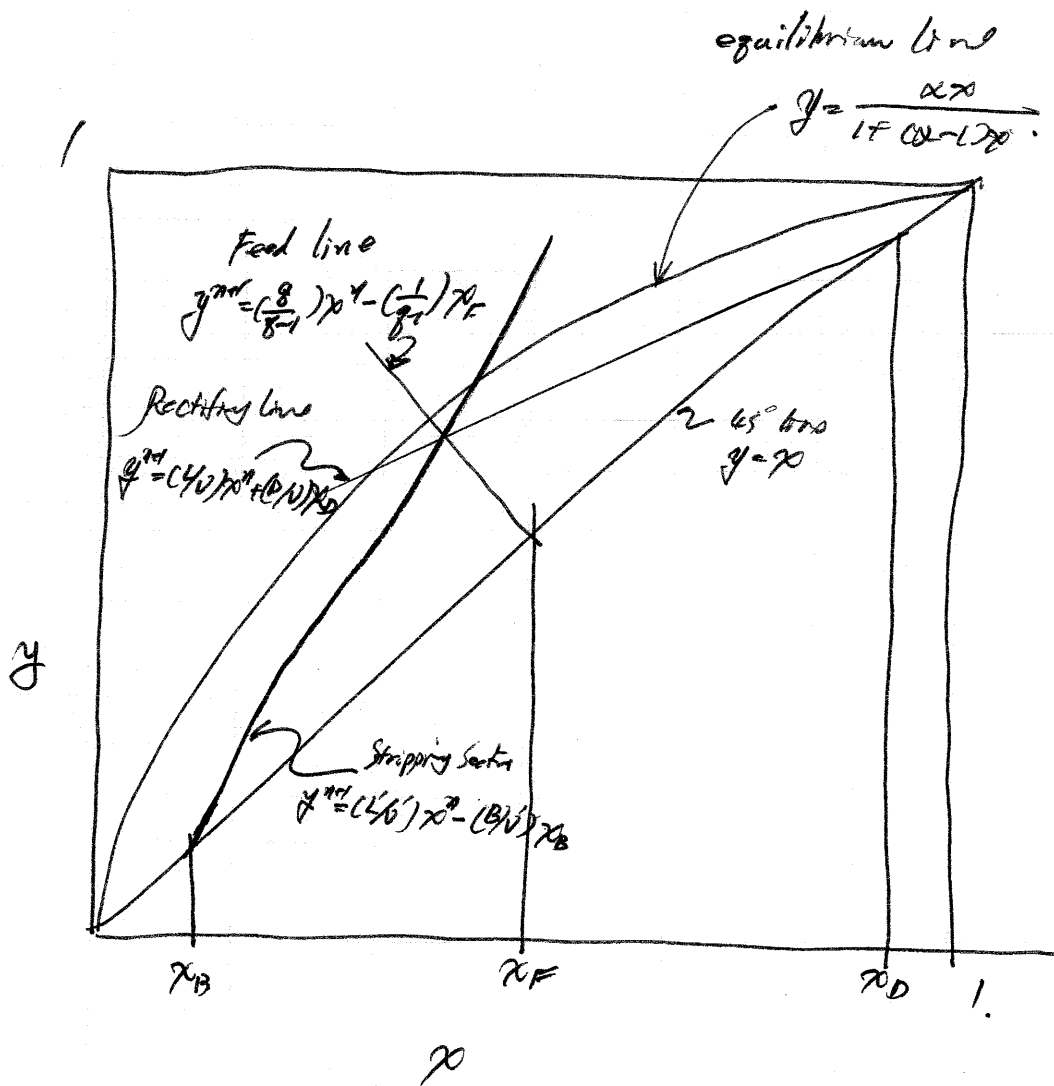
Stripping section

$$y^{n+1} = (L'/V') x^n - (B/V') x_B, \quad V' = L' - B$$

Feed tray

$$y^{n+1} = [g/(g-1)] x^n - (1/(g-1)) q_F,$$

$$g = \frac{h_i^V - h_F}{\lambda_F}$$



PROBLEM.

Propane - n -Butane

System press = 200 psia

Feed = 80 mph

Rectifying section

$$y^{n+1} = (L/D) x^n + (D/V) x_D, \quad V = L + D$$

Note: When $x^n = x_D$, $y^{n+1} = \left(\frac{L}{L+D} + \frac{D}{L+D}\right) x_D = x_D$

Can plot equation as a straight line with slope equal to $L/(L+D)$ that passes through the point x_D, x_D

For this problem

$$L = 100 \text{ mph}$$

$$D = 50 \text{ mph}$$

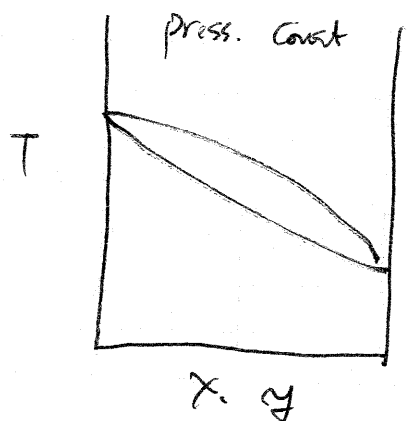
$$\frac{100}{100+50} = \frac{100}{150} = \frac{2}{3}$$

Feed q line

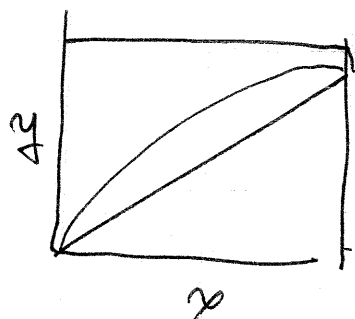
$$y^{n+1} = \frac{q}{q-1} x^n - \frac{1}{(q-1)} x_F, \quad q = \frac{h_v^F - h_F}{h_v^F - h_F^F}$$

Note: when $x^n = x_F$, $y^{n+1} = \left[\frac{q}{q-1} - \frac{1}{q-1}\right] x_F = x_F$

Equilibrium



IDEAL
Vapor/Liquid
equilibrium



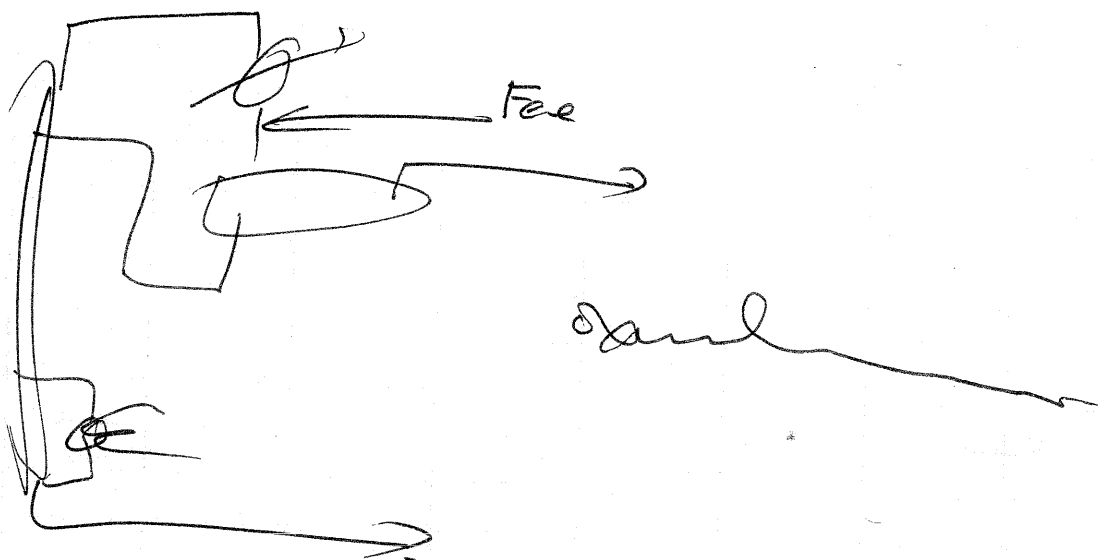
Slight
deviation
from ideality

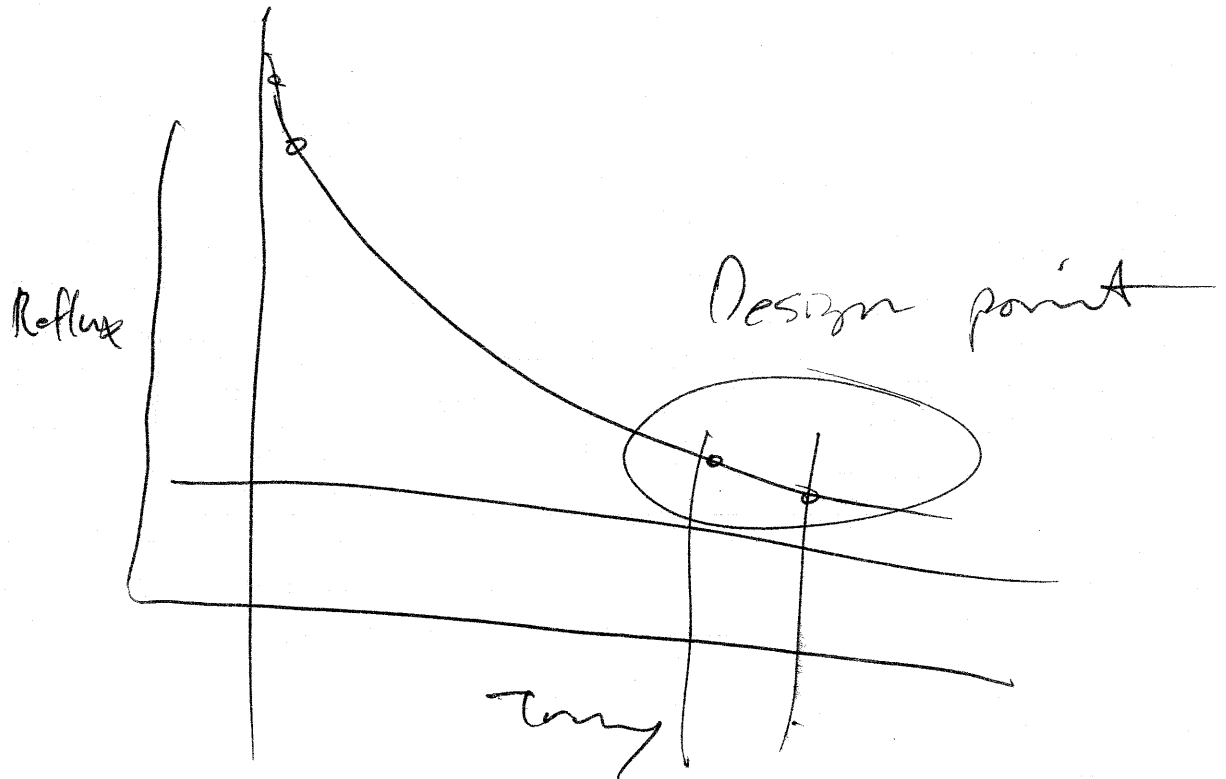
$$p^* = \gamma_R x$$

Activity coefficient

$$\gamma = f(x, T)$$

$\gamma \neq \text{constant}$





$$y[1 + (\alpha - 1)x] = ax$$

$$y + y(\alpha - 1)x = ax$$

$$[a - y(\alpha - 1)]x = \frac{y}{\alpha}$$

$$x = \frac{y}{[a - y(\alpha - 1)]}$$

DSQ

$$y = \frac{ax}{1 + (\alpha - 1)x}$$

$$D = 0.49$$

$$B = 0.51$$

$$\frac{2.89}{0.49}$$

$$R_m = \frac{(1-x_0)}{(1+\phi)} + \frac{x_0}{(x-\phi)} - 1$$

$$3.75817 = \frac{(1-)}{}$$

$$\alpha = 0.143$$

$$g = 4.143$$

$$x_F = 0.139$$

$$x_D = 0.140$$

$$x_B = 0.141$$