

Modeling of metal forming: a review

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1.1 Introduction

Manufacturing of a product by plastic deformation of metals has been performed for ages. Plastic deformation can be accomplished with or without heating the metal. Accordingly, two broad categories of metal forming processes are hot metalworking and cold metalworking. A more refined classification includes warm metalworking in between hot and cold metalworking. In most of the metal forming processes, deformation is carried out by the application of a mechanical load; the role of heat is limited to reducing the flow stress (required to start the plastic deformation of the metal). However, there are processes, where the plastic deformation is achieved by the heat alone; an example is laser bending, where a sheet is bent by laser irradiation that creates a sufficient amount of thermal stresses to bend the sheet [1].

Based on raw material, desired final product and metal flow pattern, the metal forming processes are classified into bulk and sheet metal forming processes. Bulk metal forming processes deform high volume-to-surface area ratio raw materials resulting in a change of surface area. In the sheet metal forming processes, the raw material has a low volume-to-surface area ratio, and material deformation does not intend to change the surface area implying that the sheet thickness remains more or less unaltered. Recently, another category, namely, sheet-bulk metal forming has been introduced [2]. In sheet-bulk metal forming processes, the bulk deformation of sheet is carried out that invariably brings out the intended changes in the thickness as well. Some examples of bulk metal forming are forging, rolling, extrusion, and wire drawing. Sheet metal forming processes include deep drawing, bending, and spinning [3]. Coining, flow forming, and ironing are examples of sheet-bulk metal forming.

Modeling of metal forming started since the beginning of the 20th century [4]. Initial attempts were directed to estimate the load required for plastic deformation. Prominent methods were slip-line field method, slab method, and upper bound method. These methods involved several assumptions and were incapable of providing detailed information about stress–strain distribution in the material. The technique of viscoplasticity was introduced in the late 1950s to get detailed information about the deformation. In a viscoplasticity method a time-dependent velocity field is

calculated based on a series of photographs of a grid pattern on the metal being processed. Analytical and/or numerical techniques are used to estimate the stress–strain distribution and other derivative information. The decade 1950–60 saw the emergence of finite element method (FEM). A number of articles were published in the 1970s and 1980s on the modeling of metal forming processes. These methods were perfected in the 1990s, and nowadays, there are several commercial FEM packages dedicated to modeling of metal forming processes. Since the last two decades, there have been attempts to develop advanced FEM techniques and several meshless methods. However, the requirement of huge computational time is a hindrance in the popularization of these methods.

Some developments have taken place in carrying out microstructural modeling of metal forming processes. Classical plasticity theory could account for distinct material behavior based on microstructural features. Molecular dynamics simulations (MDSs) and crystal plasticity are two recently developed methods for realistic simulation of metal forming processes, particularly for modeling of microforming. However, these methods are still at a nascent stage of development for solving metal forming problems, and there are several computational difficulties to be overcome.

This chapter provides an overview of modeling of well-known metal forming processes. Capabilities and limitations of various modeling techniques are highlighted. Finally, the directions for further research are provided.

1.2 Modeling issues in various metal forming processes

Why do we model metal forming processes? What is expected out of a model? In general, modeling is expected to improve the efficiency of overall manufacturing system and reduce the dependence on costly hit and trial experiments. Modeling can provide the following valuable information: (1) required deformation load; (2) energy consumption in the process; (3) stresses on the dies and tools; (4) defects in the process; (5) quality of the product, particularly in terms of dimensional accuracy and surface integrity; (6) properties of the product; (7) stress, strain, strain rate, and temperature distribution in the product as well as tooling; and (8) life assessment of the tooling and machine. Adequate modeling can help in the design and optimization of metal forming process, machines, and tooling. In the sequel, salient modeling issues of popular metal forming processes are discussed.

1.2.1 Forging

Forging is a process of plastically deforming the metal by pressing and hammering. It may be performed in cold, warm, or hot state of the metal. There are mainly two types of forging processes: (1) open die or free forging and (2) closed die or impression die forging. Open die forging is the process of deforming the metal between multiple dies that do not completely enclose the material. It is often

employed to preform material for subsequent metal forming. There are several different types of open die forging processes. Fig. 1.1 shows schematic diagrams of three types of open die forging—upset forging or upsetting, cogging, and orbital or rotary forging. In upsetting the complete or partial portion of the workpiece is compressed between a fixed die and moving ram in order to increase the cross section of the desired portion. In Fig. 1.1A, a rod is being upset forged to make a head. In Fig. 1.1B, cogging process is being carried out in order to make a stepped bar. It is basically an incremental forming; a portion of the workpiece is compressed between the dies, dies retract, and the workpiece is advanced axially for next compression operation. Fig. 1.1C depicts upsetting of a workpiece by orbital forging. Here the lower die is fixed. The upper die rotates about an axis slightly inclined to workpiece-axis; hence, at a time it compresses only a small portion of the workpiece. This is also an incremental forming and load requirement gets reduced.

In the closed die forging or impression die forging, metal is compressed in the enclosed dies. In this process the metal is fully compacted to acquire the shape governed by the die cavities. Excess material comes out as flash and is trimmed.

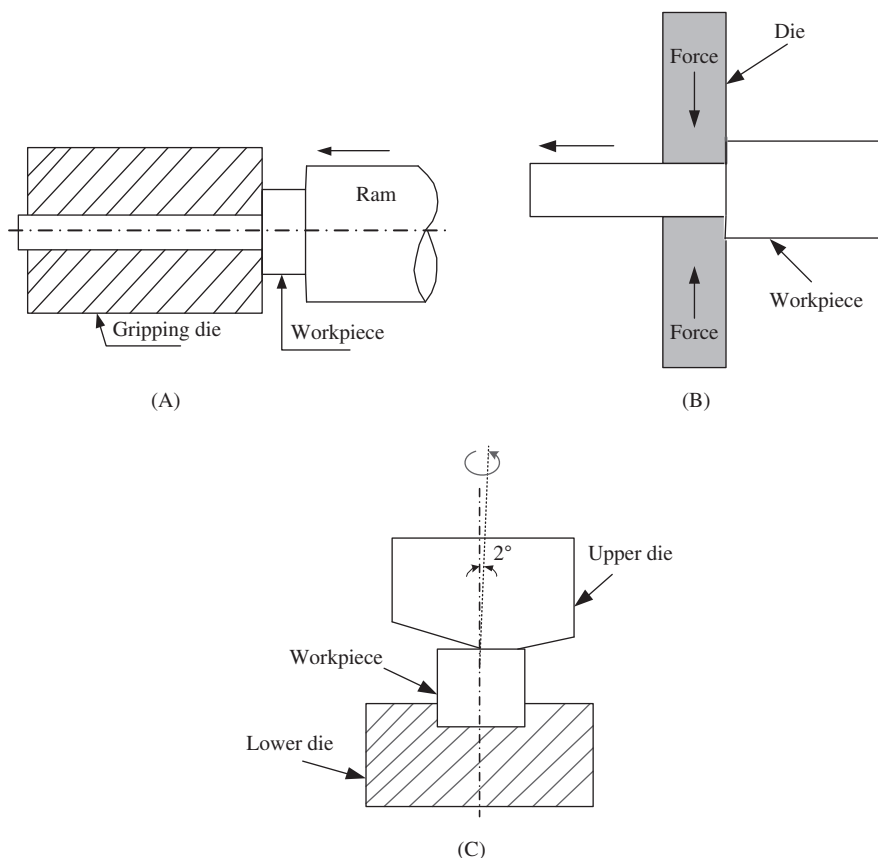


Figure 1.1 Open die forging: (A) upsetting, (B) cogging, (C) orbital forging.

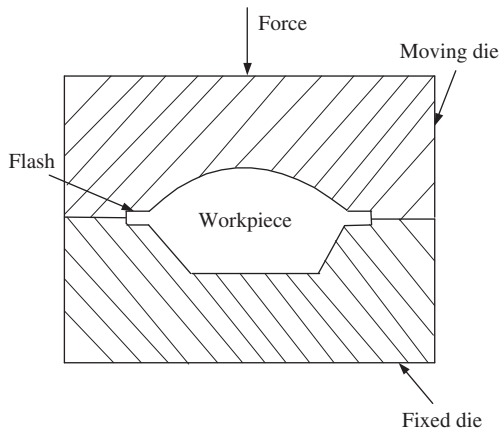


Figure 1.2 Closed die forging.

Usually, the mass of flash can be as much as 20%. Fig. 1.2 shows a schematic of closed die forging with flash generation. Flash-less forging is also possible but requires a careful tooling design.

In forging process, strain rate may lie in the range 10^{-3} – 10^{-2} s^{-1} , depending on whether it is press forging or hammer forging. In the cold forging the effective Coulomb's coefficient of friction ranges from 0.05 to 0.15 and 0.1 to 0.5 in hot forging. In most of the cases, Coulomb's friction model is inappropriate. Frictional behavior may change from sticking to sliding while moving outwardly from center in a direction normal to the load.

Estimation of forging load has been the focus of attention in forging. Another interest is to find outflow patterns in order to design dies. Forging process suffers from various defects that need to be controlled. Surface cracking may occur due to thermomechanical effects. Poor material flow may cause folding or overlapping of one region of metal onto another causing cold shut. It may also result in underfilling of die cavities. Surface may get roughened due to the deformation of coarse grains, which is called orange peel defect. Although forging load estimation techniques are sufficiently refined, prediction of defects is still a challenging task.

The simplest open die forging, namely upsetting is often used as a benchmark test for studying the material and friction behavior. In a typical compression test the specimen is compressed between two lubricated platens to find out the deformation behavior of metals. In a ring compression test a hollow cylinder specimen is compressed between two plates and friction is estimated based on the change in the hole diameter. In the cylindrical specimen of a ring compression test, typically the hole diameter is half of the outer diameter and height is one-third the outer diameter.

1.2.2 Rolling

Rolling shapes materials by passing it between counterrotating rolls. It has been in wide use since the 14th century. In this process, slabs, billets, blooms, or rods are

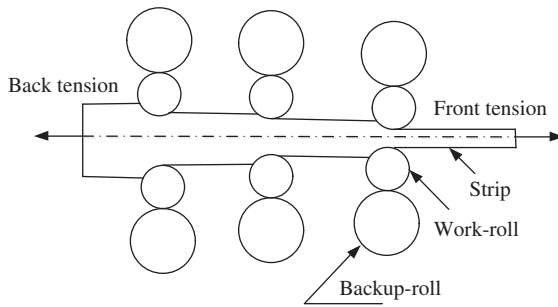


Figure 1.3 A tandem rolling mill.

rolled into plates, sheets, strips, rods, and tubes. The profiles can also be produced by rolling. Because of the limitation of the maximum possible reduction in one pass, usually multipass rolls with a number of stands in series are employed. This is called tandem rolling and is schematically depicted in Fig. 1.3. It has three stands; in each stand, there are two work-rolls and two backup rolls to prevent work-roll deflection. Rolling can be employed with or without front and back tensions.

Rolling process looks deceptively simple. However, a realistic simulation needs to focus on the following three complex tasks:

1. proper elastic–plastic modeling of the material to be processed,
2. modeling of friction behavior with a proper assessment of neutral zone in which the direction of frictional stresses changes from facilitating the movement of the material to opposing it, and
3. application of elasticity theory for estimating the roll flattening and roll deflection.

A proper model of rolling process estimates the roll torque, roll separating force, and roll pressure distribution accurately. The common defects in rolling process are edge cracking, wavy edge, central burst, and alligatoring. Edge cracking refers to cracking at the edges of the rolled products and occurs because of nonhomogeneous deformation due to wrong design of rolls or improper management of friction. Wavy edge occurs mainly because of roll deflection. Due to nonuniform roll gap, edges tend to elongate more than the center. To maintain continuity, edges get compressed and produce a wavy pattern. Central burst is a ductile fracture that initiates from a void at the center. In alligatoring a crack forms along the central plane and splits the ends. Excessive front or back tension may cause the tearing of the sheet.

Many times, rolling process is employed for improving the material properties. Asymmetric sheet rolling, in which the surface speed of rolls or friction differs on the two sides of the sheet, has been used to improve the microstructure [5,6]. In temper or skin-pass rolling, 0.5%–4% reduction in the sheet thickness is carried out to provide a degree of hardening to sheet, to prevent stretcher strains or Lüders band, and to improve the surface integrity of the sheet [7]. Accumulative roll bonding is a severe deformation process [8]. In this process a sheet is passed between two counterrotating cylinders to impart 50% reduction to it. Elongated sheet is cut into two pieces of equal length and stacked together to make it of same dimension

as the original sheet. It is further passed through the roll to impart 50% reduction. The procedure is repeated several times, which results in properly bonded thin stacked sheets with a large accumulated strain. With this procedure, grain size gets reformed, and strength gets improved.

1.2.3 Wire drawing

Wire drawing process pulls a wire through a die to reduce its cross section. It can be carried out in the presence of a back tension that helps in reducing the die pressure and providing dimensional stability. Theoretically maximum possible reduction in one pass is 63%, but a practical limit is 45%. Hence, for getting higher reduction, multistage wire drawing is employed. Fig. 1.4 depicts a schematic of two-stage wire drawing process to reduce the cross-sectional area of wire from A_i to A_f . Wire drawing is usually carried out at room temperature, but occasionally warm wire drawing is also performed.

Rod drawing is similar to wire drawing. Here instead of a wire (diameter less than about 6 mm), a rod is pulled through dies. In case of tube drawing, a tube is drawn over a mandrel. In tube, sinking no mandrel is used and outer diameter reduced with increase in length; tube thickness may reduce or increase depending on the process parameters.

Die design is very important in wire or rod drawing. Most of the researchers have optimized the die shape with an objective of minimizing energy. However, the shape of die has a large bearing on the quality of the product. Certain die shapes, although may increase the required power, reduce the defects and improve the mechanical properties such as tensile strength and hardness of the drawn wire/rod. There are several types of defects in a wire/rod drawing process. Some defects such as scabs (irregularly shaped flattened protrusions) occur due to defective raw material. Some defects occur due to faulty design of die and process. For example, a combination of large die angle and small reduction results in the narrowing of plastic zone in the vicinity of centerline and enhances the chance of central burst [9]. Friction is undesirable in wire drawing and usually the Coulomb's coefficient of friction ranges from 0.01 to 0.1 with proper lubrication. In the dry wire drawing the

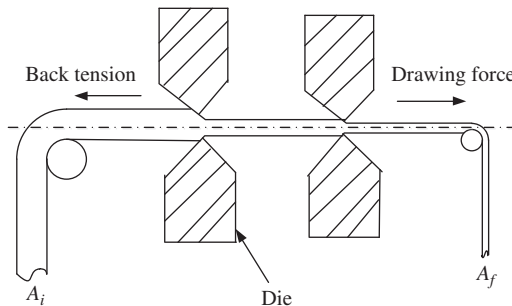


Figure 1.4 A two-stage wire drawing.

lubricant is coated on the wire before entering the die. In the wet drawing, die and wire are submerged in the lubricants. It is also important to control the residual stresses in the wire by proper design of the die and process. Recently dieless drawing of wires and tubes is also gaining importance [10].

1.2.4 Extrusion

In extrusion process the raw material is compressed through a die to reduce the cross-sectional area of the material or to generate special profile of the cross section. Theoretically, there is no limit to the maximum possible reduction in extrusion. The extrusion can be performed in cold, warm, or hot state of the metal.

There are a lot of variants of the extrusion process. In the forward or direct extrusion the metal flows in the direction of ram motion. Fig. 1.5 shows a schematic diagram of the process. To avoid sticking of ram with raw material, a dummy block is inserted in between. Movement of material through the container causes a lot of friction between the container wall and raw material. To alleviate this problem, backward or indirect extrusion is used, in which the ram and extruded material move in different directions as shown in Fig. 1.6; however, the ram needs to be hollow, which weakens it. In lateral extrusion, material flows sideways, usually perpendicular to ram motion; it helps to reduce the frictional losses. A schematic diagram is shown in Fig. 1.7, where lateral extrusion is taking place from both sides, but it can take place from one side as well. In impact extrusion, material is extruded with the impact force of the ram. Fig. 1.8 shows a schematic of impact extrusion used to make a thin-walled tube open at one end. In hydrostatic extrusion (Fig. 1.9) the raw material is placed in a sealed chamber containing liquid and a moving ram pressurizes the liquid; the raw material is forward extruded by the pressure of the liquid. Hydrostatic extrusion reduces the friction and also enhances the ductility of the raw material. In the multi-hole extrusion (Fig. 1.10) the die has more than one opening, causing more than one product to be extruded

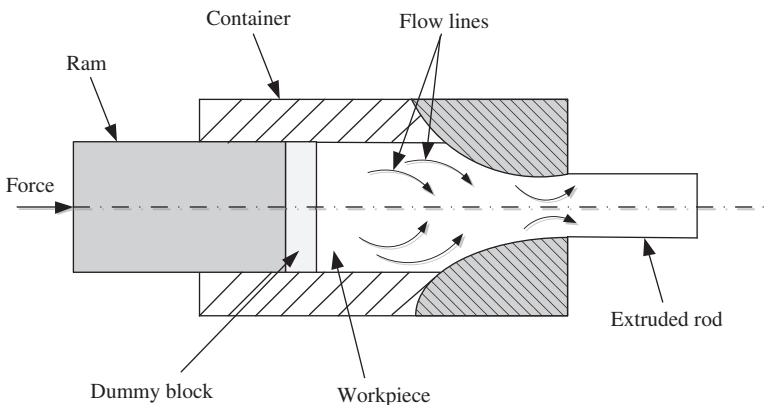


Figure 1.5 Direct or forward extrusion.

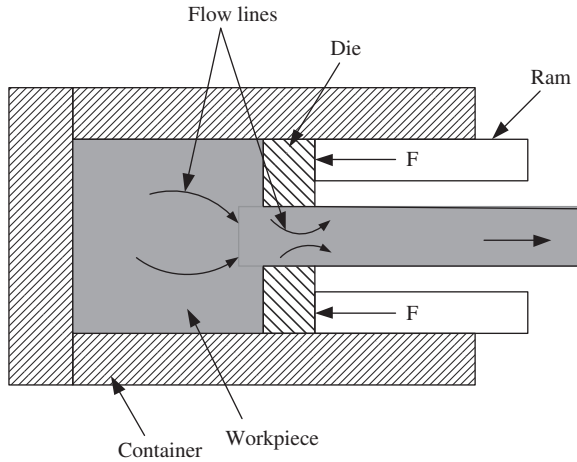


Figure 1.6 Indirect or backward extrusion.

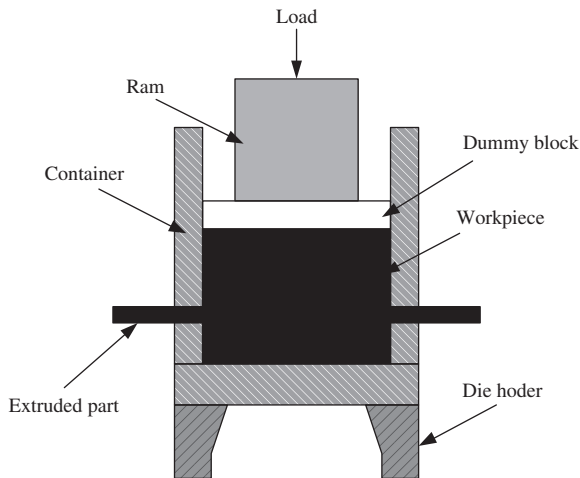


Figure 1.7 Lateral extrusion.

simultaneously. Multihole extrusion reduces the extrusion force, but improper die design can lead to poor mechanical properties [11].

Prominent defects in extrusion are piping, surface cracking, and internal cracking. In the piping defect, also known as fishtailing, tailpipe, or extrusion defect, metal flow tends to draw surface oxide and impurities toward the center of the product like a funnel. It occurs at the end of the extruded product and can be eliminated by controlling the flow pattern or using a proper dummy block. Surface cracking occurs due to thermal stresses and friction. Internal cracks are in the form

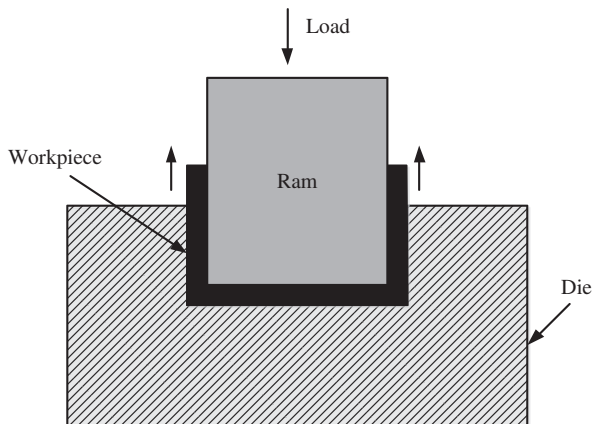


Figure 1.8 Impact extrusion.

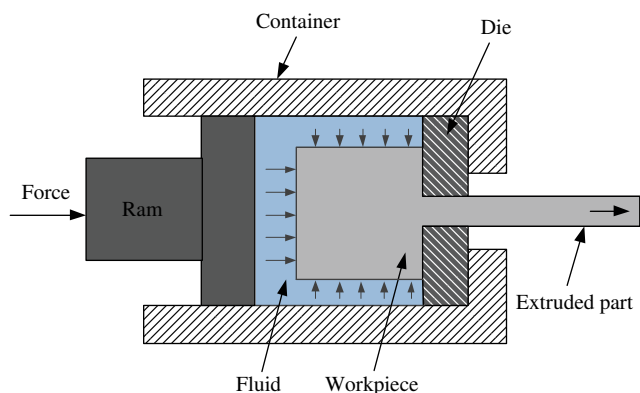


Figure 1.9 Hydrostatic extrusion.

of chevron cracking or central burst; high die angles and low reduction promote this defect [12].

Apart from forming a desired shape, extrusion is also used for improving the microstructure and mechanical properties of the product. Equal-channel angular extrusion (ECAE), invented by Segal in 1972, is widely used for producing ultra-fine grained structures due to dynamic recrystallization (DRX) [13–15]. Other names for ECAE are equal-channel angular forging and equal-channel angular pressing. Fig. 1.11 depicts a schematic of ECAE. Here a billet is passed through two equal sized channels intersecting at an angle φ , which induces a lot of shear in the billet; however, the final size of the billet remains the same as before. The billet again passes through the channels to undergo further straining. After several passes through the channels, the billet gets heavily strained, and its grain sized is

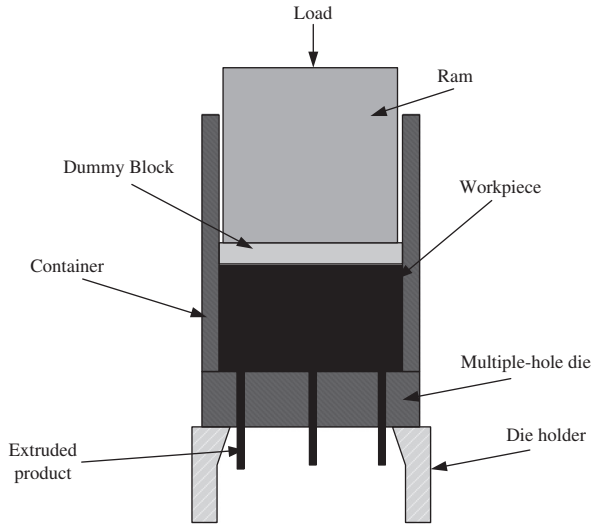


Figure 1.10 Multi-hole extrusion.

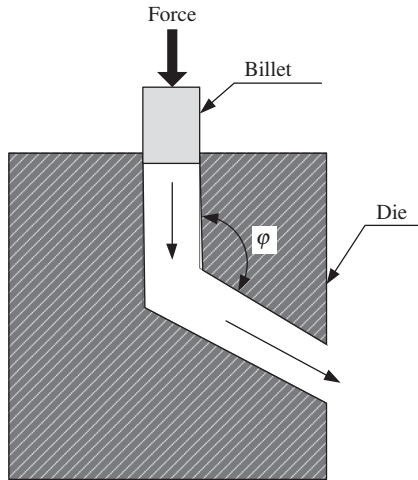


Figure 1.11 Equal-channel angular pressing.

significantly refined resulting in the enhancement of its strength and sometimes the ductility as well.

1.2.5 Deep drawing

Deep drawing process manufactures cup- or boxlike products by pushing a flat sheet through a die with the help of a punch while holding the sheet in a blank

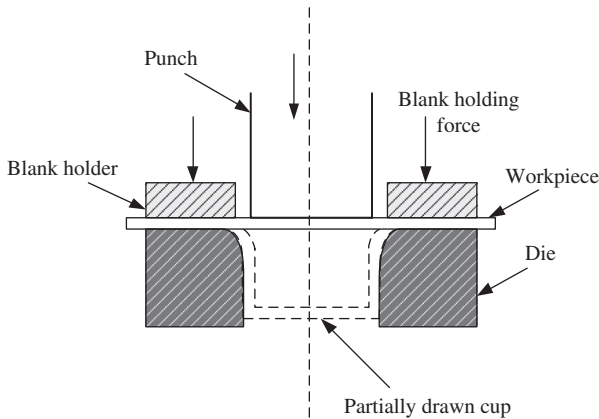


Figure 1.12 Deep drawing.

holder. Fig. 1.12 shows a schematic diagram of the process. The blank holder force (BHF) and clearance between punch and die should be carefully decided. Excessive BHF causes tearing, and too less BHF causes wrinkling. Clearance is usually 7%–15% of sheet thickness. Too less clearance causes ironing; sometimes, it may be desirable. Anisotropy of the sheet has a large influence on the deep drawing ability of the sheet. Planar anisotropy, in which properties differ with direction in the plane of the sheet, causes earlike formation on the drawn product; this defect is called earing. Normal anisotropy refers to the situation in which properties in the thickness direction differ from those in the plane of the sheet. A large flow stress in the thickness direction compared to that in the plane of the sheet provides better performance in deep drawing as it avoids tearing. A measure of normal anisotropy is the plastic strain ratio, which is the ratio of true width strain to true thickness strain for a material strained in the longitudinal direction. Automobile manufacturers prefer a plastic strain ratio of 1.4 or more for steel sheets. A measure of formability is the limiting draw ratio (LDR), which is the ratio of the diameter D of the largest blank that can be successfully drawn to the diameter of the punch d . Theoretical limit for LDR is 2.7.

Besides wrinkling, tearing, and earing, other common defect is the orange peeling. Orange peeling is the generation of high surface roughness in the region of the sheet that has undergone large deformation. This defect is more prominent with large grain size materials. Ironing can eliminate this defect.

1.2.6 Bending

Bending is the process by which a straight object is transformed to a curve object. Bending of a tube is called tube bending, and bending of a sheet is called sheet bending. In sheet bending the bend radius is expressed in terms of sheet thickness. For example, if the minimum bend radius is $4t$, then the sheet can be bent with a

minimum radius of four times the sheet thickness without cracking. In the tube bending, bend radius is expressed in terms of outer diameter of the tube. Bendability of a sheet can be improved by heating, applying hydrostatic pressure, or by applying compressive forces in the plane of the sheet. Sometimes, a tube is bent by filling the sand in the tube and heating it. During mechanical bending a provision has to be kept for elastic recovery, which is called springback. Sheet is bent up to the desired bend plus springback.

Nowadays, laser-based bending is getting prominence. There are two types of laser-based bending processes (Fig. 1.13). One is called laser bending, where a laser beam irradiates the surface of the workpiece to be bent, which produces thermal stresses responsible for bending [16]. Fig. 1.13A shows a schematic diagram of laser bending. In the other type of processes called laser-assisted bending, the task of laser is the local heating of material to soften it, and bending is carried out by applying external load. External load can be applied by hanging a load [17], by pushing the sheet mechanically, hydraulically, pneumatically or magnetically [18–21]. Fig. 1.13B shows a schematic diagram of a laser-assisted bending. Process modeling of laser-based bending is required to get accurate bend angle.

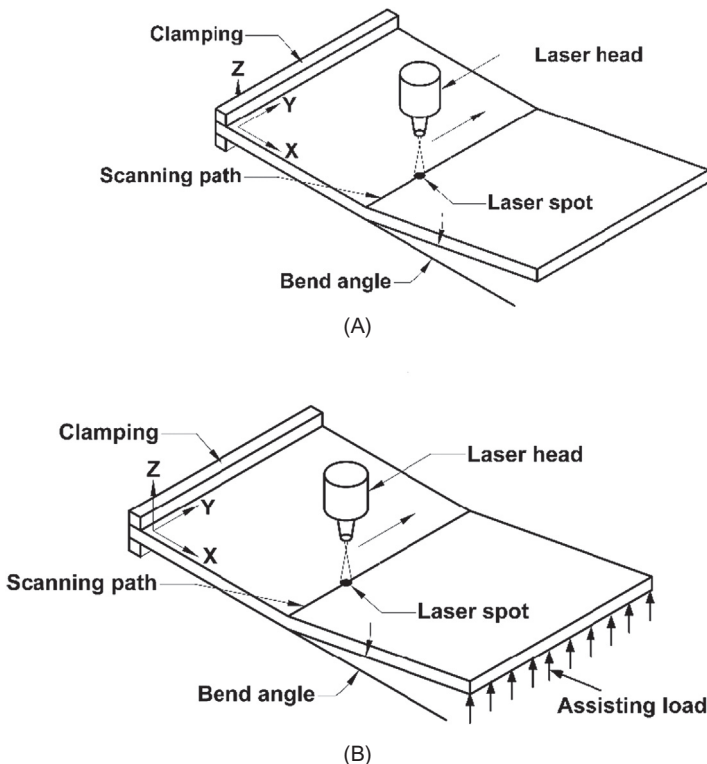


Figure 1.13 Bending of a sheet with the help of a laser beam: (A) without any mechanical load; (B) with mechanical loading at one end.

Due to high temperatures involved in the process, temperature-dependent properties of the sheet are needed.

1.3 Various modeling techniques

In general, the modeling of metal forming is a thermomechanical problem. A lot of heat is generated due to deformation of material, and about 95% of it is dissipated as heat raising the temperature of workpiece and tooling. Friction between workpiece and tooling also produces heat. However, in cold metalworking, temperature rise may not be enough to alter the mechanical properties of the material. In view of it, most of the times metal forming processes are analyzed without considering the heat transfer phenomenon. There are various modeling techniques differing in their abilities and capabilities. They can be classified into analytical, semianalytical, numerical, empirical, data based, and atomistic. However, often the distinction is blurred; in fact, of-late hybrid techniques are gaining more popularity. In this section, some methods are briefly described.

1.3.1 *Slab method*

In this method the free body diagram of an infinitesimal slab of the deforming material is made, and the force balance is carried out. This produces one or more differential equations that are solved analytically or numerically. Slab method has been used for the modeling of various processes such as wire drawing, extrusion, forging, and rolling (both symmetrical and asymmetrical). Although it is a very old technique and employs several assumptions (such as uniform or linearly varying stress, and less-complicated friction law), it is still being used for analyzing the metal forming process for getting quick estimates with reasonable accuracy. For example, roll force, roll torque, and curvature of the rolled sheet in asymmetric rolling have been predicted with reasonable accuracy using the slab method [22–24]. Based on a radial gradient slab method, Hongyu et al. [25] have analyzed a dieless wire drawing process and obtained results very close to those calculated using an FEM model.

1.3.2 *Slip-line field method*

Slip-line field method is mainly used for modeling of plane strain rigid–plastic metal forming processes. It is based on the assumption that at each point, a pair of orthogonal curves can be drawn along which the shear stress has the maximum value. These curves are called slip lines, and the material is assumed to flow along these lines. In essence, the slip line method solves the following equations:

- yield criterion for plane strain
- equilibrium equations along two orthogonal directions in the plane

- continuity equation (volume conservation as the density remains practically unchanged during metal forming for most of the metals)
- an equation indicating that directions of principal stress and corresponding principal strains coincide

Usually, slip-line field satisfying the stress boundary conditions is assumed and tested to ensure that it also satisfies the velocity boundary conditions. Construction of proper slip-line field is nontrivial. Slip-line field method is also an old technique, which is still in vogue. However, most of the engineers are disinclined to use this technique due to following reasons:

- It is limited in scope as it is conveniently applied only for the modeling of plane strain metal forming of rigid–plastic metals, notwithstanding that it has been applied for analyzing axisymmetric metal forming and hardening metals also by a few researchers [26–28].
- Construction of slip-line field requires a lot of skill and efforts.

1.3.3 *Visioplasticity*

The visioplasticity method, introduced by Thomsen et al. [29], combines experiments and analysis. In this method a velocity field is obtained from a series of photographs of the instantaneous grid pattern during an actual forming process. Thus the deformation field is obtained, from which strain rates and strains can be calculated. Using the equations from theory of plasticity, stress field can be obtained. This method has often been used only to study the flow pattern [30], because the calculations involved are too cumbersome. Moreover, with the advent of finite element, prominence of such type of experimental technique has diminished.

1.3.4 *Upper bound method*

Upper bound theorem is used to find out an upper estimate of deformation field. In this a deformation field satisfying continuity equation and velocity boundary condition is assumed. Such a field is called kinematically admissible velocity field. Based on the assumed velocity field, the total energy for deformation is included. Total energy consists of energy for plastic deformation as well as energy to overcome the friction. Sometimes, discontinuous velocity field may also be assumed. However, only the tangential velocity discontinuity is permitted, not the normal velocity. This means that a portion of metal can slide against another portion, but one portion can neither penetrate into the adjoining portion nor can separate leaving a void. When the discontinuous velocity field is assumed, then the energy dissipation at the line of velocity discontinuities should also be included in the total energy. Per unit time energy dissipation at an infinitesimal surface area of velocity discontinuity is equal to the product of yield shear strength, magnitude of velocity discontinuity, and the surface area. For obtaining the friction energy, a friction factor model is assumed, in which the frictional stress is equal to some fraction of yield shear strength of the material. The advantage of using this friction model is

that the normal stress at the interface is not required. The upper bound theorem states that the total energy calculated by an assumed velocity field will be greater than or equal to actual energy needed for deforming the metal. The upper bound theorem has been used for estimating the deformation load in processes such as forging [31–35], wire drawing [36,37], extrusion [38–40], and rolling [41–46]. Attempts have also been made to include the effect of strain hardening in this method [47,48].

A generalized upper bound theorem has also been proposed [49]. In the generalized upper bound theorem, the assumed velocity field satisfies the continuity equation but need not satisfy the velocity boundary conditions. The power imparted by the actual traction on surface due to assumed velocity field will be less or equal to actual power of plastic deformation. Using this theorem, one can use Coulomb's friction model as well.

1.3.5 Finite difference method

Finite difference method is a method for solving the differential equations. Hence, the first task while solving the metal forming problem by this method is to write down the governing differential equations. These differential equations are converted to difference equations from the basic definitions of derivatives, albeit considering approximation. This can be carried out by considering first few terms of a Taylor series. As a simple example, consider a one-dimensional case of the displacement u as a function of $x \in R$, where R is the set of real numbers. From the basic principle of calculus, the exact first derivative at a point x is given by the following equation:

$$\frac{du}{dx} = \lim_{h \rightarrow 0} \frac{u(x+h) - u(x)}{h}. \quad (1.1)$$

In the finite difference method the same equation is used but instead of h tending to zero, a small value of h is chosen. Entire domain is discretized into a grid having lot of points (called node) separated by small distance. Difference equations can be written at each node and then assembled. Boundary conditions can be suitably employed. The assembled system of algebraic equations can be solved by a suitable numerical method giving the results at the nodes. If a result is needed at an intermediate location, interpolation can be used.

Finite difference has been used for solving the metal forming problems, particularly heat transfer problem [50–52]. However, it is not convenient to make a proper grid if the domain boundaries are curved, and often the method needs a lot of computational time. It has been surpassed by FEM in many situations.

1.3.6 Finite element method

Like finite difference method, FEM is also a numerical method of solving differential equations. Due to its ability to provide detailed information, this method has

attained a large amount of popularity, and there are several commercial as well open-source software FEM packages suitable for modeling the metal forming. Hence, this method is being described relatively elaborately.

There are two prominent FEM approaches for solving metal forming processes—updated Lagrangian approach and Eulerian approach [3]. In updated Lagrangian approach the domain consists of a set of particles that may keep changing their position in the space. At each time increment, the current configuration of the domain is taken as the reference configuration, that is, the reference configuration keeps updating with the time. An incremental strain tensor is taken as the measure of deformation. Accordingly, the constitutive equation is expressed in terms of incremental stress and incremental strain tensor.

Following equations need to be solved using FEM:

1. Incremental strain-displacement relation: six scalar equations.
2. Incremental elastic–plastic stress–strain relations: six scalar equations. Three different sets of equations are needed as follows: (1) equations applicable during loading with plastic deformation, (2) equations applicable for loading without plastic deformation, that is, with elastic deformation alone, and (3) equations applicable during unloading. In most of the cases unloading is considered to be elastic, and there is no difference in the equations for elastic loading and unloading.
3. Incremental equations of motion: three scalar equations.

Another type of formulation is Eulerian formulation, in which the domain of the analysis is a region fixed in space. All these equations are written in rate form instead of incremental form. Thus there are following set of equations:

1. Strain rate-velocity relations: six scalar equations.
2. Elastic–plastic stress–strain relations: six scalar equations. There are different relations for elastic and elastic–plastic case. Plastic portion of deviatoric strain-rate is proportional to deviatoric stress, whereas the elastic portion of deviatoric stress is proportional to rate of deviatoric part of stress. Cauchy stress used in the analysis is objective (frame-invariant) but not its rate. Hence, a different stress rate measure has to be used such as Jaumann stress rate.
3. Incompressibility constraint: one equation stating that sum of direct strain rates is zero, that is,

$$\dot{\epsilon}_{kk} = \dot{\epsilon}_{11} + \dot{\epsilon}_{22} + \dot{\epsilon}_{33} = 0. \quad (1.2)$$

The use of Eq. (1.2) yields to ill conditioning. An alternate formulation (called penalty method) transforms Eq. (1.2) in the following form:

$$\dot{\epsilon}_{kk} + \frac{p}{\lambda} = 0, \quad (1.3)$$

where p is called pressure (negative of hydrostatic stress), and λ is a large number called penalty number. Choosing a proper value of penalty parameter requires skill and is dependent on the computer system used. Whatever method is used for

solving the equations (mixed or penalty formulation), pressure values usually contain errors. One way is to correct the pressure values by solving the equations again by finite difference method.

Solving the metal forming problems by FEM requires a lot of skill. There are different ways to achieve the goal, and decision has to be taken carefully. For example, the deformation load can be calculated by integrating stresses over an area. However, the stresses, being a derivative quantity in FEM formation, are not as accurately determined as the primary variables. Moreover, they have generally more inaccuracy at the boundary, and the effect of local errors may introduce a significant error in the load estimation. An alternate approach is to find out total energy (or power) of deformation and from that find out the deformation load. This approach has provided significantly better accuracy.

Decision to use updated Lagrangian or Eulerian FEM requires careful considerations. Updated Lagrangian method can provide the detailed information about the deformation, including the proper identification of elastic–plastic boundaries and residual stresses. However, the method is computationally intensive. Moreover, most of the reported results in the literature could not provide confidence in the accurate estimation of residual stresses. Eulerian method is computationally faster, but not very suitable for analyzing the stresses in elastic zone including residual stresses. A proper methodology for faster computation of residual stresses is the need of the hour.

Arbitrary Lagrangian–Eulerian (ALE) FEM takes the advantages of both updated Lagrangian and Eulerian methods. In the ALE method the nodes of the computational mesh may be moved in an arbitrary specified fashion, such that new mesh can be generated conveniently. Updated Lagrangian and Eulerian methods become the special case of this method. The method was introduced by Trulio [53] and developed in the 1997s by Hirt et al. [54]. A lot of researchers have applied this method to metal forming problems [55–57].

1.3.7 Meshless method

In the updated Lagrangian method, mesh has to be generated again and again to compensate for mesh distortion due to large deformation. Moreover, in certain situations, mesh generation requires a lot of effort. This motivated the researchers to develop meshless methods. Some meshless methods are smoothed particle hydrodynamics [58], diffuse element method [59], material point method [60], element free Galerkin (EFG) [61], HP Clouds [62], reproducing kernel particle method [63], partition of unity method [64], finite point method [65], and local boundary integration equation method [66]. Among these methods, EFG method has gained a lot of popularity. These methods require a lot of time. Mahadevan et al. [67] used a meshless method, radial basis function (RBF) collocation, for solving a problem of axisymmetric cold forging. For 20% reduction the required CPU time was about 7 hours on a Pentium IV machine.

1.3.8 Molecular dynamics simulation

MDS is a useful simulation technique for atomic and molecular simulation at small-length scale [68]. In this the evolution of an ensemble of atoms subjected to boundary conditions is simulated. System evolution is governed by Newton's second law of motion. Force on an atom due to interaction with other atom is obtained by taking the gradient of a suitable potential function. The selection of a proper potential function is very crucial. With the net forces known on each molecule, the acceleration can be calculated using Newton's second law. Time integration provides updated velocity and position of each particle.

MDS is useful when underlying deformation mechanism is not well understood. It gives a fundamental insight into the process even in the absence of prior knowledge of system evolution. However, it requires a lot of computational time and very small domain size (of the order of 100 nm). Hence, the method is suitable for nano-forming or when specific information is needed in a small local area. MDS has been applied in the modeling of nano-forming [69].

1.3.9 Soft computing

Soft computing techniques rely more on data than the physics of the process, notwithstanding that knowledge of the physics of the process augments the effectiveness of the soft computing. Three popular soft computing methods are artificial neural network (ANN), fuzzy set theory, and genetic algorithms [3]. Neural network and fuzzy set theory have been used for performance prediction in metal forming. Among these two, neural networks are good at learning from the data, and fuzzy set theory is good in tackling uncertainty. Genetic algorithms have been used for optimization of metal forming processes [70,71].

ANN is motivated by the working of the human brain, mimicking its learning behavior. Two popular neural networks are multilayer perceptron and RBF neural networks. They have been used in load prediction in metal forming [72,73]. Fuzzy set theory has been applied in three ways. One way is the use of fuzzy set operations in proper decision-making. For example, the specifications of a rolling mill were decided by treating the requirement of customers as fuzzy [74]. The second way is the use of fuzzy arithmetic for modeling uncertainty. For example, roll force and roll torques were computed by treating material and friction parameters as fuzzy numbers [75]. The third way is the use of fuzzy logic, which is useful in making inferences through fuzzy rules. Gudur and Dixit [73] have used fuzzy logic for the estimation of roll force and roll torque in rolling.

Use of more than one techniques in solving a problem is called solving using hybrid methods. There are three interpretations of hybrid methods. In one interpretation, hybrid method is the method in which a method assists the main method. For example, Gudur and Dixit [76] have used neural networks for enhancing the speed of an FEM code. An RBF neural network was used by the velocity field generated by an FEM model. Trained neural network was used to provide a guess value of velocity field and location of neutral point. From the guess value the FEM model

was used to generate an accurate solution. Computational time got reduced by a factor of 10. The second way of hybrid modeling is the use of two methods together. The third way of using hybrid methods is to develop a method that is inspired by more than one method. One example is adaptive neuro-fuzzy inference system (ANFIS). Baseri et al. [77] used ANFIS for the modeling of springback in bending.

1.4 Inverse modeling

Metal forming models are often quite complex. They require a lot of input variables for predicting the responses. It may be difficult to get all the necessary data. Even if the data are available, it may be inaccurate. Inverse modeling helps in getting the input data based on experimental observations of the process. It also helps in fine-tuning the model with available experimental data. Sometimes, the inversely obtained data of input variables may not be accurate, but it may provide better results by the model than if the accurate values of the data were used. In other words, inversely obtained data compensates for modeling errors.

Essentially, inverse modeling is an optimization problem. Variables to be inversely determined are treated as design variables. The objective function is the error between predicted and experimentally observed response, based on several experiments. The objective function is minimized subject to constraints and variable bounds. Sometimes, there may not be a unique solution to the problem. In that case, either the objective function should be modified by incorporating additional types of responses and/or more number of experimental data or suitable guess values should be chosen such that obtained solution is realistic.

In metal forming problems, particularly in rolling, a number of researchers have carried out inverse modeling. Several researchers estimated the coefficient of friction, heat flux input into the roll, and the material properties of the strip by using an inverse analysis. Kusiak et al. [78] proposed a methodology for the evaluation of material parameters in hot forming of metals by an inverse analysis. The methodology was tested for hot compression of medium carbon tool steel. The combined effect of hardening and recovery was modeled by a hyperbolic sine equation. Cho et al. [79] proposed an inverse method to determine the flow stresses and the coefficient of friction. The inverse method was based on the minimization of the difference between the experimental loads and the corresponding FEM predictions. For bulk forming the basis for the inverse estimation was a ring compression test, and for sheet metal forming, it was a modified limiting dome height test. Inversely estimated data were tested on cylinder compression and hydraulic bulge test, as representative of bulk and sheet metal forming, respectively. Han [80] applied a modified two-specimen method for the online determination of flow stresses and the coefficient of friction in rolling. In their method the strip is rolled twice with two different sets of roll radii. Anisotropy of the sheet has been considered in this work using Hill's 1948 criterion. Cho and Altan [81] proposed an inverse method

to determine flow stress and friction factor of the bulk and sheet materials under isothermal condition. First, the material parameters were determined by minimizing the difference between the experimental and calculated loads. Afterward, the shape of the deformed specimen was compared with the computed shape. If they did not match, the friction factor was adjusted in order to reduce the difference in the next iteration. The methodology was applied to the ring compression and the modified limiting dome height test. Flow stress was modeled as power law as a function of strain and strain rate. Byon et al. [82] proposed an inverse method for the prediction of flow stress–strain curve and coefficient of friction using actual mill data. The roll force and forward slip were taken as the basis for inverse estimation.

Most of these models find out the parameters of a material behavior by assuming a constitutive model. Even the parameters involved in famous Johnson–Cook (JC) model can be obtained inversely, as conducting the test at high temperature, and high strain rate is time-consuming and costly. However, inverse determination of JC model parameters has been mostly carried out in machining. Recently, Yadav et al. [83] determined the material parameters as well as coefficient of friction in a warm rolling process by measuring the exit temperature and the slip. Friction has been earlier also estimated based on the measurement of forming load in rolling and/or slip. Lenard and Zhang [84] estimated the friction through an inverse technique by minimizing the error between the measured and computed roll force, roll torque, and forward slip. However, this was based only on a one-dimensional model of rolling. Lenard and Nad [85] estimated the coefficient of friction in rolling as a function of process and material parameters by an inverse analysis by minimizing the error between the measured and calculated roll force and roll torque. Friction has been inversely determined based on the measurement of curvature of rolled sheet in an asymmetric cold flat rolling process [86]. In the open die forging process the friction has been determined by the measurement of bulge [87]. In laser forming, absorptivity is determined often in an inverse manner [88]. Recently, thermal and mechanical properties of the sheet have been determined based on the measurement of bend angle and temperature profile in the laser bending process [89–92].

1.5 Modeling of microstructure and surface integrity

Microstructure describes the appearance of a material at a nm–cm length scale. It is basically the arrangement of phases and defects within a material. Consideration of microstructure is of paramount importance in hot working. Microstructure during a metalworking depends on the temperature, cooling rate, and also on the deformation. However, generally, temperature and cooling rate have more effect on microstructure. Microstructure is predicted considering phase transformation kinetics, which describes the rate of transformation of a parent phase into its microstructural constituents at different temperatures. Phase transformation is of two types—diffusional and displacive. In diffusional transformation the atomic bonds break and

individual atoms move to rearrange themselves for forming new microstructures; it occurs by nucleation and grain growth. For example, in steel, diffusional transformation transforms austenite to pearlite, bainite, ferrite, or cementite. Diffusional transformation may be isothermal or anisothermal. In displacive transformation, ordered groups of atom displace without actually breaking the bonds. For example, at rapid cooling rate, austenite in steel gets converted to martensite.

For predicting phase transformation, time-temperature transformation (TTT) and/or continuous cooling transformation diagrams are required. These are material specific and are obtained experimentally. Diffusional transformation under isothermal conditions is governed by the classical Johnson–Mehl–Avrami–Kolmogorov (JMAK) model, which is stated as [93]

$$y = 1 - \exp(-kt^n), \quad (1.4)$$

where y is the fraction of transformed constituent, k is a diffusion coefficient, and n is the Avrami exponent of transformation. Both k and n are constants dependent on temperature and can be obtained through TTT diagram by noting the times for 1% and 99% transformation. Displacive transformation is not dependent on the time, but it depends on temperature. An empirical equation can be used for each material.

Other important phenomena in metal forming are strain hardening, dynamic recovery, static recrystallization, and DRX. Strain hardening is incorporated in classical plasticity in a phenomenological way. By dynamic recovery energy stored in the form of dislocation is released without involving the movement of high-angle boundaries. During recovery, reduction in point defect density and annihilation of dislocations takes place. Recrystallization is the formation and migration of high-angle boundary grains due to stored energy of deformation. Nucleation and growth of new grains may occur during deformation, which is called DRX. DRX may be described by the JMAK equation. The temperature at which a given material almost completely recrystallizes in one hour is called the recrystallization temperature. Increasing the degree of deformation and reducing the temperature of deformation lowers the recrystallization temperature. Recrystallization temperature is less if the initial grain size is small. Recovery and recrystallization are not a phase transformation, but microstructural transformation.

Cold work does not involve phase transformation. During cold work, point defects and dislocation density increase, which causes strain hardening. When a material is subjected to severe plastic deformation, new grains are formed. There is a lot of severe plastic deformation processes [94] by which ultrafine grains can be produced. By this method the strength of the metal can be enhanced by two-to-three times mainly due to Hall–Petch effect. As per Hall–Petch effect, the strength (yield as well as tensile) of the material increases with the reduction in its grain size. For a number of materials, the yield strength σ_Y is related to its grain size in the following manner [93]:

$$\sigma_Y = \sigma_0 + \frac{k_y}{\sqrt{d}}, \quad (1.5)$$

where d is the average grain diameter, k_y is the strengthening coefficient, and σ_0 is called the friction stress. As the grain size decreases, the total grain boundary area increases that causes hindrance to the movement of dislocations. This is the main reason for the appearance of Hall–Petch effect.

There have been some attempts to combine phase transformation models in a classical plasticity-based FEM model. Recent work is by Shufen and Dixit [95] and Kumar and Dixit [96]. Shufen and Dixit [95] modeled a thermal autofrettage process for cylinder along with a special heat treatment to introduce compressive residual stresses at the outer surface of a thick-walled cylinder. Phase transformation kinetics was implemented through a user-defined subroutine in a commercial finite element analysis (FEA) package. By following the similar procedure, Kumar and Dixit [96] developed a model to predict the hardness of a laser bent strip.

1.6 A note on multiscale modeling of metal forming

Materials are composed of atoms. In principle, it is possible to model the behavior of each and every atom. However, this will require huge computational time. For example, MDS studies the movement of atoms and molecules under the action of various forces of interaction and relies on Newtonian mechanics. Typical time steps in MDS are of the order of femtosecond and interatomic spacing are of the order of angstrom. At this scale, classical concept of stress does not have any relevance. However, carrying out MDS for the entire domain of metal forming is next to impossible. Multiscale modeling may be a right choice for modeling the overall process. Multiscale modeling involves modeling of the processes simultaneously at different scales for predicting various parameters of the interest. For example, texture of the material can be obtained by microstructural modeling, and total load requirement can be obtained by classical FEM following continuum mechanics. There are two broad approaches to carry out multiscale modeling. In sequential multiscale modeling, some parameters of a macroscale model can be determined by carrying out a microscale modeling of the process. Parameters obtained from microscale model are passed on to macroscale model. In concurrent multiscale modeling, micro- and macroscale models run concurrently.

The need for a multiscale modeling approach arises when one model is not adequate to accurately predict the behavior of the system [97]. For example, the classical theory of plasticity for macroscale analysis of elastic–plastic deformation in metals is capable of predicting the strain and stress with considerable accuracy. However, the mathematical formulations based on this approach may not be sufficient in case of thermal elastic–plastic deformations at elevated temperatures involving recrystallization. Above the recrystallization temperature, new grains appear in the material and based on the cooling rate, the different phases may be produced. Since each material phase has different properties, the overall macroscale behavior of the material is affected. In this case, one may be required to incorporate the phase transformation kinetics such as the JMAK model alongside classical

plasticity. The basic features of a multiscale modeling may be described by considering the thermomechanical analysis in heat treatment of steel as that used in Shufen and Dixit [95]. A theoretical model that is capable of describing the heat treatment process in steel must involve an interaction of thermal and mechanical science at the macroscale and metallurgical science at the microscale. The thermal equation predicts the change in temperature, which in turn affects the mechanical expansion and phase change. When there is plastic deformation, the mechanical equation predicts internal heat generation due to plastic deformation and the subsequent role in phase change. When there is phase change, the microstructural model predicts the internal heat generation due to release of latent heat during phase change that affects the thermal behavior and the dilatational strain due to phase change that affects the mechanical response.

There is not much literature on multiscale modeling of metal forming processes. However, some published articles indicate huge research potential in the area of multiscale modeling. McGinty and McDowell [98] have used multiscale crystal plasticity models to generate forming limit diagrams for sheet metal forming. Van Houtte et al. [99] modeled plastic anisotropy and deformation texture of polycrystalline materials following multiscale modeling. Results obtained from meso/microscale models were intended to be used in a macroscale model. Hol et al. [100] have presented a multiscale friction model for use in sheet metal forming. First, a microscale model predicts friction based on the texture of the sheet. Afterward the estimated friction is used in the macroscale model of sheet metal forming for computing other quantities of the interest. Recently, Kumar and Dixit [96] also has used a multiscale model for the estimation of hardness of strip after laser bending, although their micromodel is largely empirical.

1.7 Challenging issues

Compared to machining, metal forming processes have been accurately modeled. However, often large deformation metal forming processes are modeled by neglecting the elastic strains, that is, by rigid–plastic approach. Rigid plastic modeling is unable to predict residual stresses, for which elastic strains needs to be considered. Elastic strains are of the order of 10^{-3} , whilst the plastic strains are of the order 1. Huge difference in the values of strains creates numerical difficulties. There are very few attempts to model residual stresses in the metal forming processes [101]. Apart from numerical difficulties, there is difficulty in experimentally measuring the residual stresses.

Modeling often requires a huge amount of computational time. The attempts should be directed toward minimizing the computational time. There are many ways to achieve the objective of reducing the computational time. The development of faster modeling techniques can revolutionize this research area. Evolutionary progress can be made by parallel computing, developing efficient solvers, and using

hybrid techniques. One example of hybrid techniques is to use neural networks for enhancing the efficiency of a finite element model [76,102].

Friction is an important but the least understood aspect of manufacturing processes. It is still chosen in an ad hoc manner. Although a number of models have been developed, they have not been properly applied in the modeling of manufacturing processes. The difficulty in using various friction models is that they require certain parameters that are difficult to obtain. Obtain those parameters in an inverse manner from experiments seem to be the best option available. In future, it may be possible to develop more sophisticated models using the knowledge of mechanics as well as chemistry. Study of friction becomes more pressing in the modeling of microforming processes.

Modeling of microforming and nano-forming is quite different from the modeling of conventional metal forming processes. Scale effect comes into play and physical phenomena that are insignificant at macroscale gain significance at micro and nanoscale. At present, enough data are not available on the small-scale behavior of the material. For most of the materials, the conventional yield criteria may not be applicable and anisotropy may play a major role. Yield criterion has to incorporate the effect of strain gradient, which recently developed crystal plasticity-based models are able to capture.

Surface roughness is an important property of a precision product. However, it is very difficult to model the surface roughness based on the physics as the number of parameters comes into play. In the area of metal forming, very few articles have been published on the modeling of surface roughness [103]. There is ample scope to investigate not only the surface roughness but also entire surface integrity in metal forming.

In the past, microstructural aspects were not given enough attention while modeling of metal forming processes. Now, there is a growing realization that mechanical as well as microstructural modeling should be carried out concurrently for getting the proper benefit out of modeling. As an example, a number of papers have been published on the die design during extrusion, but all of them invariably attempt to reduce the power by minimizing the redundant work. However, the redundant work is not always wastage; it often helps to improve the mechanical properties. An integrated optimization scheme needs to trade-off between the power minimization and quality of the end product.

1.8 Conclusion

Metal forming is one of the oldest manufacturing processes. However, the modeling of metal forming processes, as a research area, is only about 100 years old. This article has summarized various analytical, semianalytical and numerical techniques employed for the modeling of metal forming. As of now, fairly accurate models are available, which can predict the forming load quite accurately. However, limited progress has been made in the prediction of residual stresses and surface integrity.

There is a need to carry out research for better understanding of the physics of the metal forming processes as well as enhancing the computational efficiency of the models. Recent technological developments in microforming and development of new materials, including functionally graded materials, further demand proper characterization of the material behavior. Crystal plasticity and MDS are the promising techniques that can unfold the intricacies of metal forming processes. However, extensive computational time required in these techniques limits their domain of application. Multiscale modeling of the metal forming process may be a viable panacea in future.

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